Do Target-Date Funds Hit the Target?

By

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Abstract

Target-Date Funds (TDFs) are diversified investment funds that offer investors the advantage of professional portfolio management for their retirement savings. With TDFs, asset allocations follow a pre-determined "glide path" whereby portfolio risk is reduced as the investor approaches a target retirement date. This paper investigates how TDF asset allocations and investor welfare compare to those from an optimal dynamic investment policy in the presence of taxes. We find that optimal asset allocations and investor welfare can deviate substantially from those that emerge from a "one-size-fits-all" TDF, depending importantly on the individual's risk aversion, value of human capital, and split of wealth between taxable and tax-advantaged retirement accounts. Interestingly, the target retirement date (or investment horizon) is of secondary importance once other relevant state variables are considered.

1. Introduction

Target-Date Funds (TDFs) are diversified investment funds that offer investors a pre-determined path for asset allocations over their lifetimes. With TDFs, the asset allocations change over time (called the "glide path") to reduce portfolio risk as the investor approaches a "target retirement date." The rationale for a downward-sloping glide path (i.e., reduced equity exposure as the investor ages) is based upon the observation that the value of human capital (i.e., the present value of future labor income) is less risky than equity and declines over time as the investor ages. The implication is that young investors, who have more valuable human capital, should hold riskier financial portfolios than investors who are closer to their target retirement date and have less valuable human capital.

TDFs became a central feature in many 401(k) and 403(b) tax-deferred retirement programs after the enactment of the Pension Protection Act (PPA) of 2006.[1] Prior to the passage of the PPA, plan sponsors typically assigned participants who did not make an investment election for their retirement plans into money market funds to avoid fiduciary liability risk. After passage of the PPA, TDFs emerged as an important Qualified Default Investment Alternative (QDIA) for automatic enrollment without subjecting the plan sponsor to fiduciary liability. Since TDFs offer a better risk-return tradeoff for individuals as they age, TDFs are viewed as a more appropriate default option than the traditional money market fund.

The popularity of TDFs has grown steadily since the passage of the PPA (2006). Over the past 15 years, TDF assets have grown from $315 billion in 2010 to over $5.2 trillion in 2025, a compound annual growth rate over 20%. According to Vanguard, 96% of all retirement plans in 2024 offered a TDF option, and for those plans that specify a Qualified Default Investment Alternative (QDIA), 98% choose a TDF. Among plan participants, 84% utilize TDFs for their retirement investments when they are available, with 73% of TDF investors utilizing a single TDF for their entire retirement account balance. TDF utilization is particularly high among young investors – the group that is most affected by the post-PPA QDIA design.

TDFs are designed for tax-advantaged retirement accounts and for individuals with a common target retirement date. Participants choose (or are assigned to) the TDF that most

[1] Parker, Schoar, and Sun (2023) and Parker, Schoar, Cole and Simester (2025) emphasize the increased use of target-date funds since the enactment of the PPA and the latter also highlights the increase in equity ownership.

closely matches the participant's planned retirement date. Proprietary models of glide path construction have been developed by investment management firms using insights from models of lifetime consumption and portfolio choice developed by academic scholars.[2] The proprietary models often incorporate many key factors that influence portfolio choice, including risk aversion, labor income, savings rates, retirement dates, pension and social security benefits, mortality risk, and bequest motives.[3] However, to establish a single glide path for a target retirement date, TDFs typically use population averages for most of the important model inputs. This ignores any differences among individuals with the same target retirement date.

The biggest advantage of TDFs is that they allow plan participants, who may be less knowledgeable about financial investing, to delegate their portfolio decisions to professional fund managers.[4] At the same time, the construction of TDF glide paths raises several issues with respect to "hitting the target" for investors. While participants in a specific TDF are likely to have similar target retirement dates (and ages), it is highly unlikely that age alone is sufficient to determine optimal asset allocation. Differences in several other important investment-relevant factors may lead to optimal asset allocations that deviates from those that emerge from the "one-size-fits-all" TDF glide path.[5]

Another limitation of TDFs is that they are designed primarily for retirement accounts and, therefore, implicitly assume that individuals have no wealth, or future savings, in taxable accounts. Although many investors do hold a significant portion of their savings in tax-advantaged retirement accounts, it is obviously counterfactual to assume that taxable wealth is nonexistent. Dammon, Spatt and Zhang (2004) have shown that investors should optimally hold equity in their taxable accounts to the extent possible, with equity spilling over into their tax-deferred accounts only if the optimal holding of equity exceeds the

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[2] Samuelson (1969) and Merton (1969, 1971) developed foundational models of lifetime consumption and investment in continuous time. Bodie, Merton, and Samuelson (1992) and Gomes, Kotlikoff, and Viceira (2008) incorporate an endogenous labor-leisure choice to show how human capital risk can affect life-time consumption and portfolio decisions. Heaton and Lucas (1996, 2000) and Constantinides and Duffie (1996) are part of a large literature that incorporates uninsurable labor income risk to study life-time savings and portfolio decisions. Dammon, Spatt, and Zhang (2004) examine the life-time consumption and portfolio allocation decisions in an environment with both taxable and tax-deferred (retirement) savings accounts.

[3] The proprietary models of glide path construction typically take consumption and savings decisions to be exogenous. Aliaga-Diaz, Rayfield, Daga, and Dinerstein (2025) discuss Vanguard's proprietary life cycle investing model.

[4] The circumstances in which the participant selects the default portfolio rather than a customized portfolio are discussed in Spatt (2018).

[5] An important critique of the PPA framework is that it assumes that a "one-size-fits-all" TDF approach meets the needs of all (or most) individuals. This can be a misleading and potentially harmful message for plan participants who have chosen (or been assigned) to invest through standard TDFs.

amount of wealth in the taxable account.[6] This implies that with both taxable and tax-deferred accounts there needs to be an integrated approach to the design of optimal portfolios. Optimal asset allocation for the retirement account cannot be determined without consideration of the taxable account and vice versa. Optimal glide paths for the taxable and tax-deferred accounts will differ from each other and both will depend upon the split of wealth between the taxable and tax-deferred accounts.

The purpose of this paper is to investigate how TDF asset allocations and investor welfare compare to those from an optimal dynamic investment policy. To conduct the analysis, we employ an intertemporal model of investment choice, where the objective is to maximize the expected utility of terminal (retirement date) wealth. To simplify our analysis, we assume that there are only two assets: a risky stock index (equity) and a riskless taxable bond. Individuals earn riskless labor income each period until their retirement date and are allowed to save and invest in both a tax-advantaged retirement account and a taxable account. Assets held in the tax-advantaged retirement account earn pre-tax returns, while the return on assets held in the taxable account are taxed on an annual basis (i.e., no deferral option). We numerically solve for the optimal asset allocations over the investor's investment horizon and conduct simulation analysis to identify how asset allocations and investor welfare compare to those under a standard TDF investment policy.

We find that optimal asset allocations and investor welfare under the optimal dynamic investment policy can deviate substantially from those that emerge from a standard TDF investment policy. The deviations in asset allocation and investor welfare depend importantly on individual-specific characteristics and state variables, including the degree of risk aversion, the value of human capital, and the split of wealth between taxable and tax-advantaged retirement accounts. In contrast to TDF glide paths, the optimal glide paths for the taxable and retirement accounts are dynamic and depend upon the evolution of the underlying state variables. Interestingly, the target retirement date (or investment horizon) is of secondary importance once the other state variables are considered, suggesting that an individual's optimal asset allocation over time are (nearly) independent of the planned retirement date. We go on to show that a one-period myopic investment

[6] Equity is a desirable security for at least some portion of the taxable account because of its relatively low effective tax rate. Interest income on bonds and money market funds is taxed as ordinary income, while qualified dividends and capital gains are taxed at lower preferential rates (the effective tax rate on capital gains may be even lower due to the deferral of capital gains in practice). In practice, the investor possesses a valuable tax-timing option whereby capital losses can be realized when they occur (subject to loss limitations), while capital gains can be deferred indefinitely. See Constantinides (1983, 1984) and Dammon and Spatt (1996) for foundational work on tax-timing options.

policy performs nearly as well as the optimal dynamic investment policy that emerges from our model.

Our paper is closely related to several other research papers dealing with life-time portfolio allocation. Viceira (2001), Cocco, Gomes and Maenhout (2005), Gomes and Michaelides (2005), Gomes, Kotlikoff and Viceira (2008), Bodie, Detemple and Rindisbacher (2009), and Dahlquist, Setty and Vestman (2018) study optimal lifetime consumption and portfolio choice in models with non-tradeable labor income. These papers demonstrate that because of the "bond-like" nature of labor income the optimal share of wealth allocated to risky equities generally declines as the investor ages. At the same time, Gomes, Kotlikoff and Viceira (2008), Bodie, Detemple, and Rindisbacher (2009), and Dahlquist, Setty and Vestman (2018) show that strictly age-based asset allocations, such as those that emerge from TDFs, are not welfare maximizing because they do not consider other important individual-specific state variables. However, these studies either ignore taxes altogether or assume that tax-advantaged retirement accounts are non-existent. Our contribution is to extend the analysis of optimal lifetime portfolio choice by explicitly introducing taxes and allowing individuals to save and invest through both taxable and tax-advantaged retirement accounts. By doing so, we provide new insights regarding optimal lifetime asset allocations and the suitability of TDFs for tax-advantaged retirement accounts.

Dammon, Spatt and Zhang (2004) investigate the optimal intertemporal consumption and investment decisions in a model with both taxable and tax-deferred savings accounts and demonstrate that the preferred location for holding risky equity is the taxable account. Our model is similar to theirs, but our focus is different. We are interested in understanding the suitability of TDFs as an investment choice for individuals saving for retirement. We do this by investigating how the asset allocations and investor welfare from a TDF investment strategy compare to those that emerge from the optimal intertemporal investment strategy and identify the important factors that contribute to any deviations. Importantly, we also demonstrate that once the relevant state-variables are considered, a myopic one-period model performs nearly as well as the more complex multi-period model whose solution relies on dynamic programming.

The paper is organized as follows. In **Section 2**, we present our intertemporal model of portfolio choice and discuss the underlying assumptions. We also discuss the optimal asset location policy in which the taxable account is shown to be the preferred location to hold equity. To streamline the presentation, the details of the dynamic programming approach used to solve the model are presented in **Appendix A**.

In **Section 3**, we present numerical solutions to our model and show how the optimal dynamic asset allocations depend upon the degree of risk aversion and the underlying state variables. We also examine how the optimal dynamic asset allocations vary across different target retirement dates and show that the target retirement date is of secondary importance once the other state variables are considered. The details of the numerical solution methodology are presented in **Appendix B**.

In **Section 4**, we conduct a simulation analysis to investigate how asset allocations and investor welfare compare across the optimal dynamic investment policy and the TDF investment policy. We show that the deviations in asset allocation and investor welfare can be substantial across the two investment policies. We also investigate how a simple one-period myopic investment policy performs relative to the optimal dynamic investment policy. The welfare gains are shown to be robust to situations in which all wealth and future savings are restricted to the retirement account (i.e., no wealth or future savings in the taxable account). In **Section 5**, we offer some recommendations regarding how investor welfare might be improved through some additional customization of TDFs or through a "Do-It-Yourself" investment approach. **Section 6** concludes.

2. The Intertemporal Investment Model

Our main purpose is to investigate the suitability of Target-Date Funds (TDFs) as an investment choice for individuals saving for retirement. To do this, we first examine the optimal asset allocation decisions over an individual's working years using a simplified model of portfolio choice in which the individual maximizes the expected utility of terminal (retirement date) wealth. We incorporate labor income and savings exogenously into the model and, for simplicity, assume that future labor income is known with certainty.[7] Individuals can save for retirement through both a taxable account and a tax-advantaged retirement account. Although we could allow savings rates to differ at various ages, we assume that the savings rates, and the proportion of savings that goes into the taxable and retirement accounts, are held constant over the individual's working years.

[7] Exogenous labor income and savings rates imply that intertemporal consumption is also exogenous in our model. We could endogenize the consumption and savings decisions, as in Dammon, Spatt, and Zhang (2004), but this modification would not materially change our main conclusions regarding optimal asset allocation. Likewise, as other authors have shown, labor income risk would produce a precautionary savings motive and reduce the optimal holding of equity to lower the overall risk of the investor's total wealth (including human capital). Since labor income risk would needlessly complicate our model without adding new insights, we have chosen to abstract from labor income risk in our model.

TDFs typically invest in a host of asset classes, including domestic equity, international equity, domestic bonds, international bonds, inflation-indexed bonds, commodities, and, sometimes, alternative investments like private equity. We will simplify our analysis by assuming that there are only two asset classes: (1) a risky asset (which can broadly be thought of as a diversified portfolio of equity securities) and (2) a riskless asset (which can be broadly thought of as short-term government securities and money market funds). Interest rates, dividend yields, expected capital gain returns, and return volatilities are assumed to be constant over time.

Financial assets held in the taxable account are subject to taxation. Interest income from the riskless asset is taxed as ordinary income. In contrast, dividends and capital gains are taxed at preferential rates. Although in practice capital gains are taxed only when assets are sold, we assume that all capital gains and losses are taxed each year.[8] We assume that individuals contribute ***after-tax*** dollars to their retirement accounts (much like a Roth IRA) for which they earn pre-tax returns.[9]

At the end of each year t, the individual's financial wealth, $W_{F,t}$, is composed of the ***after-tax*** value of the taxable and retirement accounts. The after-tax wealth in these two accounts evolves through time as follows:[10]

$$W_{NR,t} = W_{NR,t-1}\{1 + x_{t-1}[d(1-t_d) + g_t(1-t_g)] + (1-x_{t-1})r(1-t_o)\} + y_{NR}L_t \qquad (1)$$

[8] This assumption greatly simplifies the model since it eliminates the tax basis as a state variable and allows us to make portfolio adjustments in the taxable account without the need to consider the tax consequences of doing so. However, it also results in an undervaluation of equity and a reduction in the optimal holding of equity in the taxable account.

[9] For simplicity, we treat all retirement contributions, including the employer match, to be on an ***after-tax*** basis. As long as the individual's tax rates are constant over time, the individual is indifferent to contributing pre-tax or after-tax dollars to his retirement account. Contributing a ***pre-tax*** dollar to the retirement account, deferring the payment of the tax until the money is withdrawn, results in the following distribution after T years: $[\$1(1+m)]R_{t,T}(1-t_o)$, where $R_{t,T}$ is the total gross return from date t to date T and m is the percentage employer match. On the other hand, paying the tax at date t and investing $\$1(1-t_o)$ of ***after-tax*** dollars into the retirement account provides the following distribution after T years: $[\$1(1-t_0)(1+m)]R_{t,T}$, which is identical to the final distribution of pre-tax investment dollars. There are two caveats to this, however. The first is that there will be a preference for investing ***pre-tax*** dollars in the retirement account if the individual's tax rate at the time of distribution is less than the tax rate at the time of contribution. Second, if there is an annual dollar contribution limit to the retirement account, then contributions of ***after-tax*** dollars will effectively increase the contribution limit by a factor of $1/(1-t_o)$ relative to ***pre-tax*** contributions.

[10] For retirement accounts in which the individual invests ***after-tax*** dollars (e.g., Roth IRAs) the after-tax value is simply the year-end balance of the account. For traditional ***pre-tax*** retirement accounts, the after-tax value is equal to the pre-tax balance multiplied by $(1-t_o)$, where t_o is the individual's ordinary tax rate. In essence, for pre-tax retirement accounts the government is effectively a co-investor owning the fraction t_o of the assets held in the account with the individual owning the remainder.

and

$$W_{R,t} = W_{R,t-1}\{1 + z_{t-1}(d + g_t) + (1 - z_{t-1})r\} + y_R L_t(1 + m) \quad (2)$$

where $W_{NR,t}$ is after-tax wealth in the taxable (non-retirement) account, $W_{R,t}$ is after-tax wealth in the retirement account, x_t and z_t are the fractions of taxable and retirement wealth invested in the risky asset, d is the constant dividend yield on the risky asset, g_t is the capital gain (loss) on the risky asset, r is the risk-free interest rate, t_d is the tax rate on dividend income, t_g is the tax rate on capital gains (losses), t_o is the tax rate on interest income, L_t is after-tax labor income, y_{NR} and $y_{R,}$ are the individual's savings rates (as a percentage of after-tax labor income) for the taxable and retirement accounts, respectively, and m is the employer match (as a percentage of the individual's contribution) in the retirement account.

The individual's total after-tax wealth, W_t, is composed of financial wealth, $W_{F,t}$, plus the present value of future ***after-tax*** labor income, $PV_t(L)$:

$$W_t = W_{F,t} + PV_t(L) = W_{NR,t} + W_{R,t} + PV_t(L) \quad (3)$$

The present value of future labor income, $PV_t(L)$, can be thought of as the value of the individual's human capital at date t and can be written as follows:

$$PV_t(L) = L_t A_t \quad (4)$$

where A_t is the date-t present value factor given by

$$A_t = \sum_{j=t+1}^{T} \left[\frac{\prod_{k=t+1}^{j}(1 + i_k)}{[1 + r(1 - t_0)]^{j-t}} \right]$$

and i_k is the growth rate in labor income at date k, k > t, which is assumed to be known with certainty (i.e., no risk in future labor income).

The individual's intertemporal optimization problem can be expressed as follows:

$$\underset{x_t,\, z_t}{MAX} \quad E_t[U(W_T)], \qquad t = 0, 1, \ldots, T - 1 \quad (5)$$

subject to the constraints in Equations (1) – (4) and

$$0 \leq x_t \leq 1 \text{ and } 0 \leq z_t \leq 1, \qquad t = 0, 1, \ldots, T-1 \tag{6}$$

The choice variables for the optimization problem are the fraction of wealth invested in equity in the taxable account, x_t, and the fraction of wealth invested in equity in the retirement account, z_t.[11] The relevant state variables are: $H_t = \{W_{F,t}, W_{NR,t}, A_tL_t, T\text{-}t\}$, where T-t is the time remaining until the target retirement date.

The solution to the individual's intertemporal investment problem requires the use of dynamic programming. **Appendix A** provides details of our dynamic programming approach. As a quick reference, **Table 1** provides a list of the variables and notation used in our model.

2.1. Optimal Asset Location

The first-order optimality conditions for x_t and z_t are given by Equations (A10) and (A11) in **Appendix A** and stated below for convenience:

$$E_t\{V'_W[d(1-t_d) + g_{t+1}(1-t_g) - r(1-t_o)]\} + \lambda_{1,t} - \lambda_{2,t} = 0 \tag{7a}$$

and

$$E_t\{V'_W[d + g_{t+1} - r]\} + \lambda_{3,t} - \lambda_{4,t} = 0 \tag{7b}$$

where V'_W is the derivative of the value function, $V(W_{t+1}, H_{t+1})$, with respect to total wealth, W_{t+1}, and l_j, j = 1,...,4, are Lagrange multipliers for the constraints: $0 \leq x_t^* \leq 1$ and $0 \leq z_t^* \leq 1$. In particular, $\lambda_1 \geq 0$ if $x_t^* = 0$, $\lambda_2 \geq 0$ if $x_t^* = 1$, $\lambda_3 \geq 0$ if $z_t^* = 0$, and $\lambda_4 \geq 0$ if $z_t^* = 1$.

Equations (7a) and (7b) imply that the optimal holding of equity in the taxable and retirement accounts cannot both be positive unless $x_t^* = 1$ (i.e., $\lambda_{2,t} > 0$) or all tax rates are equal (i.e., $t_d = t_g = t_o$). To see this, let $r > d$ and $t_g \leq t_d < t_o$. Then, if the optimal holding of

[11] Note that the individual's consumption at date t is exogenous and equal to $C_t = L_t[1 - y_{NR} - y_R]$. The exogeneity of consumption is due to the assumption that future labor income and savings rates are exogenous and known with certainty, which is the standard approach taken by financial firms in designing portfolio allocations for their clients. Dammon, Spatt, and Zhang (2004) endogenize both the consumption and portfolio allocation decisions in an environment with both a taxable and retirement account. Interestingly, the intertemporal asset allocations that arise in the current model are consistent with those derived in Dammon, Spatt, and Zhang (2004), even though we do not solve endogenously for optimal intertemporal consumption.

equity in the retirement account is strictly positive, $0 < z_t^* \leq 1$, $\lambda_{3,t} = 0$ and the optimality condition (7b) becomes:

$$E_t\{V'_W[d + g_{t+1} - r]\} - \lambda_{4,t} = 0 \tag{8}$$

where $\lambda_{4,t} \geq 0$. Substituting Equation (8) into Equation (7a) and simplifying terms yields:

$$E_t[V'_W][r(t_0 - t_g) - d(t_d - t_g)] + \lambda_{1,t} - \lambda_{2,t} + \lambda_{4,t}(1 - t_g) = 0 \tag{9}$$

Since $r(t_0 - t_g) > d(t_d - t_g)$, Equation (9) requires $l_{2,t} > 0$ (i.e., strictly positive). This implies that the optimal equity holding in the taxable account is at a corner solution in which $x_t^* = 1$. This leads to the following ***optimal asset location policy***:

> The optimal asset location policy involves holding equity in the taxable account to the extent possible, with equity spilling over into the retirement account only if the optimal overall holding of equity exceeds the individual's wealth in the taxable account. In other words, $z_t^* > 0\%$ only if $x_t^* = 100\%$.

This optimal asset location policy was first presented in Dammon, Spatt, and Zhang (2004) and is derived under the assumption that capital gains and losses are realized and taxed annually. However, if investors can defer capital gains and realize capital losses (tax-loss harvesting) then the effective capital gains tax rate will be close to zero ($t_g = 0$). [12] This makes equity even more attractive to hold in the taxable account as evidenced by the increasing value of the term within square brackets in Equation (9). This implies that the optimal asset location policy is robust to tax environments in which investors are allowed to defer capital gains and realize capital losses.

The optimal asset location policy was derived using a utility-based argument and stands in direct contrast to the (incorrect) argument that is made by some practitioners and academics that investors should hold the asset with the highest expected return (i.e., risky equity) in the retirement account. The (incorrect) argument compares the after-tax return on equity in the retirement account to the after-tax return in the taxable account as shown in Equation (10) below:

[12] In the U.S., net capital losses are limited to $3,000 per year, which severely limits the annual tax benefit from realizing net capital losses. However, any net capital loss greater than $3,000 each year can be carried forward indefinitely to offset future capital gains. In this tax environment it is not unrealistic to assume an effective capital gains tax rate of zero ($t_g = 0$).

$$\Delta E(r_P) = [d + E(g)] - \left[d(1 - t_d) + E(g)\left(1 - t_g\right)\right] = \left[dt_d + E(g)t_g\right] > 0 \qquad (10)$$

From Equation (10) it appears as though the strategy that maximizes the expected after-tax portfolio return, $E(r_P)$, is to invest in the asset with the highest expected return in the retirement account.

But this comparison is problematic for at least two reasons. First, by a simple adding up constraint, any increase (reduction) in the holding of equity must be met with an offsetting reduction (increase) in the holding of taxable bonds, which is not accounted for in Equation (10). Second, Equation (10) does not control for the overall risk of the investor's portfolio. To maintain the same risk exposure, a \$1 shift from equity to taxable bonds in the taxable account must be offset with a $(1 - t_g)$ shift from taxable bonds to equity in the retirement account.[13] Making these corrections to Equation (10) produces the following expression for $\Delta E(r_P)$:

$$\Delta E(r_P) = [d + E(g) - r](1 - t_g) - \left[d(1 - t_d) + E(g)\left(1 - t_g\right) - r(1 - t_o)\right] \qquad (11)$$

$$= d\left(t_d - t_g\right) - r(t_o - t_g) < 0$$

since $t_o > t_d \geq t_g$ and $r > d$. This, once again, demonstrates that shifting equity holdings from the taxable account to the retirement account, with offsetting shifts in the holdings of taxable bonds, is suboptimal.[14]

2.2. Savings in the Taxable and Retirement Accounts

Another issue that frequently arises in discussions of optimal lifetime savings is whether individuals should maximize the contributions to their retirement accounts before investing any savings in their taxable accounts. Of course, individuals may have a motive for maintaining some level of savings in taxable accounts for transaction purposes, to deal with unexpected consumption or income shocks, or to finance a major purchase (e.g., a home or higher education). But absent these motives, should individuals save anything in

[13] The risk of after-tax equity returns in the taxable account is $\sigma_s\left(1 - t_g\right)$, where σ_s is the standard deviation of pre-tax equity returns. Since pre-tax equity returns are earned in the retirement account, it is necessary to hold the fraction $(1\text{-}t_g)$ in equity in the retirement account to keep the risk of the overall portfolio constant.

[14] Even though Equation (11) is consistent with the optimal asset location policy derived from Equation (9), Equation (11) suffers from the limitation that it only captures the change in the expected after-tax portfolio return over a single period. In contrast, Equation (9), which is derived from the Bellman equation of optimality, ensures that the optimal asset location policy maximizes the investor's utility over the entire planning horizon.

their taxable accounts if they have not yet contributed the maximum to their retirement account?

We can gain some insight into this question by differentiating the value function, V_t, with respect to the savings rates, y_{NR} and y_R, to determine which adds the most value (in terms of welfare). Doing so reveals that the welfare benefits are higher for savings in the retirement account relative to savings in the non-retirement (taxable) account. [15] Savings in the retirement account has two distinct advantages over the taxable account. First, individuals may receive an employer match in the retirement account that is not available in the taxable account. Individuals should certainly take advantage of the employer match before contributing any savings to the taxable account. Second, individuals earn ***pre-tax*** returns on investments made in the retirement account, but not in the taxable account. Both advantages provide strong incentives for individuals to first save in their retirement accounts before saving in their taxable accounts. Interestingly, while tax-loss harvesting and capital gain deferral make equity more valuable by reducing the effective capital gains tax rate, it still does not alter the preference for saving in the retirement account before saving in the taxable account.

3. Numerical Solutions

As mentioned earlier, the solution to the intertemporal investment problem presented in **Section 2** requires the use of dynamic programming. To simplify the dynamic programming approach, we assume that the individual has Constant Relative Risk Averse (CRRA) preferences, which guarantees that the optimal equity allocations in the taxable and retirement accounts, x_t and z_t, are independent of total wealth.

In **Appendix A**, we normalize the intertemporal investment problem by total wealth, W_t. Doing so leads to the following normalized vector of state variables: $h_t = \{w_{NR,t}, A_t l_t, T - t\}$, where $w_{NR,t} = W_{NR,t}/W_{F,t}$ is the fraction of financial wealth held in the taxable (non-retirement) account and $A_t l_t = A_t L_t / W_t$ is the present value of future labor income (human capital) as a percent of total wealth. [16] Because analytical solutions to the normalized dynamic programming problem are not readily available, it is necessary to use numerical methods

[15] Details of the analysis are available from the authors upon request.

[16] Recall that financial wealth is the wealth held in the taxable and retirement accounts combined. Hence, $w_{R,t} = 1 - w_{NR,t}$, where $w_{R,t}$ is the fraction of financial wealth held in the retirement account. Also, total wealth is equal to financial wealth plus the present value of future labor income. Hence, $w_{F,t} = 1 - A_t l_t$, where $w_{F,t}$ is the financial wealth as a percent of total wealth. This implies that $h_t = \{w_{NR,t}, A_t l_t, T\text{-}t\}$ provides all the information we need at date t in making optimal decisions.

to derive solutions. **Appendix B** provides details of the numerical solution methodology we employ.

In this section, we present the optimal asset allocations in the taxable and retirement accounts that emerge from our numerical solutions and make comparisons with the asset allocations for a typical TDF. We begin by presenting the solution to the intertemporal investment problem for the base-case parameters in **Table 1**. In particular, the risk-free interest rate is assumed to be r = 3.5%. The risky equity portfolio is assumed to have a pre-tax dividend yield of d = 1.5%, an expected capital gain return of E(g) = 7.0%, and a standard deviation of σ = 20.0%. The ordinary tax rate is assumed to be t_o = 35%, while the dividend and capital gains tax rates are assumed to be $t_d = t_g$ = 20%. The individual in the base-case is assumed to have a target retirement date of T = 35 years (age 30) and CRRA preferences with a risk aversion parameter of γ = 3.

The **Bureau of Labor Statistics** and academic researchers have shown that labor income increases significantly with age but levels off (or even declines) after age 55. **Figure 1A** shows the assumed growth rates for labor income from age 20 to age 65. The resulting pattern of labor income is shown in **Figure 1B** and the present value of future labor income (i.e., value of human capital) is shown in **Figure 1C.** Both **Figure 1B** and **Figure 1C** are drawn for an initial (age 20) after-tax labor income that has been normalized to $50,000.

Vanguard's "How America Saves 2025" reports that the average aggregate contribution rate (including both employee and employer contributions) was 12.0% for active participants in 401(k) retirement plans in 2024. This contribution rate includes all active participants in 401(k) plans, including both voluntary and automatic enrollments. **Table 2** shows the average aggregate contribution rates to 401(k) retirement plans for 2024 as reported by ***Vanguard***. Aggregate contribution rates tend to increase with the participant's age and income level, which is not surprising since age and income are highly correlated. We simplify our analysis by assuming that the aggregate contribution rate to the retirement account is a constant $y_R(1+m)$ = 10.0% of ***after-tax*** labor income across all ages and income levels. This is equivalent to an aggregate contribution rate of 15.4% of ***pre-tax*** labor income.[17] In addition to retirement savings, the individual can also contribute savings to a taxable account. We assume that the individual contributes a constant y_{NR} = 2.0% of ***after-tax*** labor income to the taxable account across all ages and income levels.

[17] An aggregate contribution rate of 10% of ***after-tax*** income to the retirement account is equivalent to an aggregate contribution rate of .10/(1-t_o) = .10/(1-.35) = 15.4% of ***pre-tax*** income to the retirement account. This is because a 10% contribution of ***after-tax*** dollars generates the same future after-tax wealth as a 15.4% contribution of ***pre-tax*** dollars.

These assumptions produce an overall savings rate of 12.0% of ***after-tax*** labor income at all ages and income levels.

Figure 2 is a 3D representation of the optimal equity allocations in the taxable and retirement accounts for an individual at age 30 (T = 35 years until retirement). The optimal equity allocations are shown as a function of the remaining two state variables: (1) the fraction of financial wealth held in the taxable account and (2) the present value of future labor income (human capital) as a percent of total wealth. While the 3D surface in **Figure 2** illustrates how the optimal holdings of equity vary across the entire state space, a cross-section of **Figure 2** shown in 2D space can be easier to interpret.

Figure 3 shows the optimal holdings of equity (at age 30) in the taxable and tax-deferred accounts as a function of the value of human capital (as a percent of total wealth) for four different levels of taxable wealth. There are several interesting observations that can be made about the equity allocations shown in **Figure 3**. First, note that the optimal equity holdings are very different in the taxable (**Figure 3A**) and retirement (**Figure 3B**) accounts. As shown earlier, the optimal asset location policy is to hold equity in the taxable account to the extent possible, with equity spilling over into the retirement account only if the taxable account is invested entirely in equity. Second, the optimal overall holding of equity (**Figure 3C**) is higher for individuals with higher levels of human capital. This is because human capital is comparable to owning a riskless bond that generates a future stream of riskless income. This gives the individual an incentive to invest a higher percentage of financial wealth in risky equity to achieve the desired overall risk exposure. Finally, the optimal overall holding of equity is also increasing with the level of taxable wealth as shown in **Figure 3C**. Since the taxable account is the preferred location for equity, the higher the percentage of financial wealth held in the taxable account the higher the optimal overall holding of equity.

The optimal equity allocations illustrated in **Figure 3** have important implications for TDFs. TDFs are typically designed for retirement accounts and for individuals with similar retirement dates. But as **Figure 3** illustrates, the optimal asset allocations differ considerably between the taxable and retirement accounts and depend upon much more than just the planned retirement date.

The optimal overall holding of equity (**Figure 3C**) varies considerably across individuals with different levels of taxable wealth and human capital. Moreover, since it is optimal to hold equity in the taxable account to the extent possible, the optimal holding of equity in the retirement account also varies considerably across individuals with different levels of

taxable wealth and human capital. This variation in the optimal holding of equity occurs despite the common target retirement date across individuals.

TDFs attempt to appeal to a broad cross section of individuals and, therefore, cannot accommodate important differences across individuals when making asset allocation decisions. In particular, TDF managers allocate retirement savings to risky equity under the implicit assumptions that (1) all wealth and future savings are limited to the retirement account (i.e., no wealth or future savings in the taxable account) and (2) the value of human capital (as a percent of total wealth) is identical for all individuals with a common target retirement date. This practice likely leads to an over-investment in risky equity in the retirement account for individuals with some non-negligible taxable wealth and for individuals with less valuable human capital.

TDFs design portfolios for individuals based solely upon a target retirement date. To understand the importance of the target retirement date in the design of optimal portfolios, **Figure 4** illustrates the optimal equity holdings for the taxable and tax-deferred accounts, as well as the overall equity allocation, for four different target retirement dates (T = 1, 5, 20, and 35). **Figure 4** is drawn assuming that existing financial wealth is split evenly (50%) between the taxable and retirement accounts. All other variables are set at their base-case parameter values.

It is perhaps surprising how similar the optimal equity allocations are across all four target retirement dates. As we saw earlier in **Figure 3**, the two important state variables for determining the optimal equity allocations are (1) the split of wealth between the taxable and retirement accounts and (2) the value of human capital (as a percent of total wealth). Given these two state variables, the target retirement date is of secondary importance (as shown in **Figure 4**). Individuals with the same value of human capital and the same split of wealth between the taxable and retirement accounts will hold nearly identical portfolios regardless of their target retirement dates. [18]

Although the target retirement date is of secondary importance in the asset allocation decision, it is likely to be highly correlated with the value of human capital. Young investors, with more distant target retirement dates, are likely to have more valuable

[18] This suggests that the solution to the one-period (T = 1) portfolio problem may be a reasonably good approximation for the more complex intertemporal portfolio problem presented in **Section 2**. If this is the case, then the dynamic nature of the portfolio problem can safely be ignored and the solution to the one-period portfolio problem can be implemented myopically period by period. We will investigate the reasonableness of the myopic one-period investment policy when we conduct our simulation analysis in **Section 4**.

human capital and, therefore, higher equity holdings. In fact, this is the primary thesis of TDFs. TDFs design portfolios so that the exposure to risky assets declines as individuals age and the value of human capital declines. Of course, the appropriateness of relying on the target retirement date as a proxy for the value of human capital will obviously depend upon how diverse the value of human capital is across individuals with the same target retirement date. We will return to this issue in the next section where we conduct our simulation analysis.

Having discussed the impact of the three state variables (value of human capital, split of wealth between the taxable and tax-deferred accounts, and the target retirement date), we now turn our attention to another important factor that influences the optimal asset allocation decision: risk aversion. **Figure 5** shows the optimal equity holdings for three different risk aversion parameters: $\gamma = 1$ (log utility), $\gamma = 3$, and $\gamma = 5$. In constructing **Figure 5**, we assume that the target retirement date is T = 35 years (age 30) and that wealth in the taxable account is 50% of financial wealth. All other variables are set at their base-case parameter values.

Figure 5 illustrates the optimal equity holdings for both the taxable and retirement accounts, as well as the overall equity allocation. Not surprisingly, higher risk aversion reduces the optimal holding of equity in both the taxable and retirement accounts, except possibly when the optimal holding of equity hits a boundary constraint of either 0% or 100%.[19] Clearly, differences in risk aversion across investors will make it impossible for a TDF to invest optimally for all investors, even if all investors have identical target retirement dates, wealth levels, and value of human capital.

In this section, we have seen that the presence of both taxable and retirement savings necessitates an integrated approach to the design of optimal portfolios. Optimal asset allocations should utilize the retirement account to hold highly-taxed assets (e.g., taxable fixed-income securities), while the taxable account is more appropriate for tax-advantaged securities (e.g., equities and tax-exempt bonds). Moreover, the optimal holdings in these two accounts will depend primarily upon four individual-specific factors: risk aversion, the split of wealth between the taxable and retirement accounts, the value of human capital, and the future savings rates for the taxable and retirement accounts.[20] The target

[19] When $\gamma = 1$ (log utility) the optimal holding of equity is 100% in both the taxable and retirement accounts, as well as for the overall portfolio.

[20] Since labor income is risk-free in our model, an increase in the savings rates is comparable to having higher risk-free bond income. Consequently, higher savings rates will increase the optimal overall holding of equity.

retirement date (or the investor's age) is of secondary importance, despite it being the primary focus of most TDFs.

4. Simulation and Welfare Analysis

In this section, we conduct simulations to examine how the investor's optimal portfolio allocations in the taxable and retirement accounts evolve over time and compare these allocations to those for a typical TDF. We also conduct simulations to quantify the welfare gains of the optimal dynamic investment policy against a typical TDF investment policy. In our simulations, we consider an individual in their early 20s who has a target retirement date of T = 45 years. Our simulations are conducted using the base-case parameter values in **Table 1** and assume that equity returns follow a lognormal distribution.

We simulate returns, wealth accumulation, and utility of terminal wealth under three different investment policies for the taxable and retirement accounts: (1) a TDF investment policy, (2) an optimal dynamic asset allocation policy and (3) a myopic investment policy. A more detailed description of each of these policies is provided below:

4.1. Target-Date Fund Policy

For a typical TDF, the glide path for risky equity is a pre-determined function of the time remaining until the target retirement date. As the target retirement date gets closer, the TDF's allocation to risky equity declines in a predictable manner. We assume a typical TDF equity allocation that declines as a function of the investor's age as shown in **Figure 6**. The equity allocation remains constant at 95% from age 20 to age 30, gradually declines to 60% by age 60, and then declines further to 50% by age 65 (i.e., the "target retirement date"). Although different investment firms will have slightly different glide paths for their TDF offerings, the glide path shown in **Figure 6** is typical of a TDF with a target retirement date of 45 years.

Because TDFs are designed primarily for retirement accounts, they implicitly assume that either (1) investors do not have, and will never have, wealth invested in taxable accounts or (2) the equity allocations for both taxable and retirement accounts are the same. The assumption that individuals have no investable wealth outside their retirement accounts seems restrictive and counterfactual. It seems more natural to impose the restriction that the equity allocations for both the taxable and retirement accounts are the same when evaluating the TDF investment strategy.[21] However, we will also evaluate the asset

[21] It has been argued that TDFs, because of the capital gain realizations resulting from frequent rebalancing, are not an appropriate investment vehicle for taxable accounts. This argument is correct when the realization

allocations and welfare implications for TDFs when individuals have no wealth, or future savings, in the taxable account.

4.2. Optimal Dynamic Asset Allocation and Location

The optimal dynamic investment policy maximizes the expected utility of the investor's terminal (target date) after-tax wealth. It is the solution to the dynamic intertemporal investment problem presented in **Section 2**. Under this policy, the investor's optimal asset allocations in the taxable and retirement accounts will evolve over time as a function of (1) the split of wealth between the taxable and retirement accounts, (2) the value of human capital (as a percent of total wealth), and (3) the time remaining until retirement. Assets are held in the taxable and retirement accounts to maximize tax efficiency. Under the optimal dynamic investment policy, asset allocation and asset location are optimized throughout the investor's investment horizon.

4.3. Myopic Investment Policy

The optimal dynamic investment policy described above requires the implementation of dynamic programming to find the optimal solution at each age prior to retirement. However, as shown in **Section 3**, while the optimal asset allocations depend upon the split of wealth between the taxable and retirement accounts and the value of human capital, the individual's age (or target retirement date) itself is of secondary importance. This raises the question as to how a simple myopic investment policy that assumes a one-period horizon performs relative to the TDF investment policy and the optimal dynamic investment policy. Under the myopic investment policy, we consider the split of wealth between the taxable and retirement accounts, and the value of human capital, but rely on the solution to the one-period (T = 1) problem to determine asset allocations at each date prior to retirement.

4.4. Simulation Results and Welfare Comparisons

We run 5,000 simulations over a 45-year investment horizon for an individual in their early 20s. For individuals in their early 20s, who are just beginning their careers, total wealth is largely in the form of human capital with possibly a small amount of financial wealth. The initial conditions for our simulations assume that the value of human capital is 95% of total wealth and the fraction of financial wealth held in the taxable account is 50%. We also assume constant relative risk averse (CRRA) preferences and set all other parameters of our model to their base-case values shown in **Table 1**.

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of capital gains can be deferred. However, in our model, we assume that all capital gains (and losses) are realized annually, so that the deferral of capital gains in the taxable account is prohibited. This implies that the frequent trading and rebalancing of TDFs is not a disadvantage for taxable accounts in our model.

Figure 7 shows the ***average*** equity holdings in the taxable and retirement accounts, along with the ***average*** overall equity holdings, at each age under the optimal dynamic investment policy. The averages are taken over the 5,000 simulation trials. As a comparison, **Figure 7** also shows the equity allocations by age for the TDF. Note that the TDF equity allocations deviate substantially from the average equity allocations in the taxable and retirement accounts. At young and old ages, the TDF equity allocations are similar to the average overall equity allocations but deviate substantially from age 35 through age 55. For example, at age 45 the TDF equity allocation is 80%, while the average overall equity allocation is 64%. The distribution of overall equity allocations at age 45 over the 5,000 simulation trials is shown in **Figure 8**. Notice that nearly the entire distribution of optimal overall equity allocations is below the TDF equity allocation of 80%.[22]

As is the nature of TDFs, the asset allocations over time follow a fixed, pre-determined glide path as shown in **Figures 6 and 7**. When asset returns push the portfolio allocations away from the glide path, TDFs rebalance the portfolio back to the glide path. Once set, the glide path is rarely, if ever, changed. In contrast to the TDF equity allocations, the glide paths change dynamically over time under the optimal asset allocation policy (as well as the myopic investment policy).

Figure 9 shows the glide paths for the optimal dynamic investment policy for two unique patterns of equity returns over the 45-year period. **Figure 9A** shows the optimal equity allocations over time for a simulation in which equity returns are above average (average annual realized capital gain return of 16.7%) and **Figure 9B** shows the optimal equity allocations over time for a simulation in which equity returns are below average (average annual realized capital gain return of 4.8%). Notice that the glide paths are dynamic and depend upon the specific pattern of realized returns. This is because the optimal equity allocations depend upon both (1) the dynamics of the investor's wealth in the taxable and retirement accounts and (2) the value of human capital (as a percentage of total wealth). The dynamic glide paths shown in **Figure 9** show a very different pattern of equity holdings from the static glide path for the typical TDF.

Next, consider the difference in performance across the three investment strategies: (1) the TDF investment strategy, (2) the optimal dynamic investment strategy, and (3) the myopic

[22] The equity allocations shown in Figures 7 and 8 are derived under the assumption that labor income is riskless. If, alternatively, labor income is positively correlated with equity returns, the optimal overall holding of equity would be lower than those shown in Figures 7 and 8, leading to an even larger deviation from the TDF equity allocations.

investment strategy. We measure performance by first calculating the average utility of terminal wealth over the 5,000 simulation paths. Since utility values are difficult to interpret, the average utility of terminal wealth for each of the three alternative investment strategies is converted to ***certainty-equivalent wealth***. The certainty equivalent wealth, W_{CE}, is the after-tax wealth the investor would be willing to accept with certainty at the terminal date instead of the risky after-tax wealth generated by his portfolio. In other words, the certainty equivalent wealth generates the same expected utility as the risky terminal wealth for the investor.

$$U(W_{CE}) = E[U(\widetilde{W_T})]$$

The results over the 5,000 simulation trials are summarized in the table below. We report the percentage improvement (in the certainty equivalent wealth) over the TDF strategy for both the optimal dynamic investment strategy and the myopic investment strategy. The results are shown for three different risk aversion parameters: $\gamma = 1$, $\gamma = 3$, and $\gamma = 5$. Across all 5,000 simulation trials we assume that the initial (age 20) starting value of human capital (as a percent of total wealth) is 95% and the initial fraction of financial wealth held in the taxable account is 50%.

Percent Improvement Over the TDF Policy		
Risk Aversion (γ)	**Optimal Dynamic**	**Myopic**
1	20.1%	20.1%
3	2.8%	2.7%
5	16.3%	16.1%

The optimal dynamic investment policy generates the highest average utility and certainty equivalent wealth for investors. For investors with moderate risk aversion ($\gamma = 3$), the gains relative to the TDF are relatively modest. For investors who have higher or lower risk aversion, the utility gains are significantly higher. The glide path for the TDF is suboptimal for these investors, investing too much wealth in equity for investors with high risk aversion ($\gamma = 5$) and too little wealth in equity for investors with low risk aversion ($\gamma = 1$). [23] The utility gains for the optimal dynamic investment policy are 20% for investors with $\gamma = 1$ and over 16% for investors with $\gamma = 5$.

[23] The welfare gains are identical for the optimal dynamic investment policy and the myopic investment policy when $\gamma = 1$. This is because the optimal equity allocation in both the taxable and retirement accounts is 100% at all ages when $\gamma = 1$.

It is also interesting to note that the utility gains for the myopic investment policy (relative to the TDF investment policy) are nearly identical to those for the optimal dynamic investment policy. This reflects the fact that the investor's age (or target retirement date) is of secondary importance for determining optimal asset holdings once we have controlled for the other two state variables: (1) the split of wealth between the taxable and retirement accounts and (2) the value of human capital (as a percent of total wealth). This suggests that the implementation of an optimal investment policy can be simplified by imposing the one-period myopic solution each year prior to retirement rather than solving the more complicated dynamic programming problem.

As a final comparison, consider a situation in which all wealth and future savings are invested in the retirement account, so that current and future wealth in the taxable account are zero. This is basically the assumption made by TDFs when designing portfolio allocations at different ages. We run 5,000 simulation trials over a 45-year investment horizon (T = 45) for an individual in their early 20s. We assume that the initial value of human capital is 95% of total wealth at the start of each simulation trial. We also assume that the savings rate for the retirement account is y_R = 7.5% with an employer match of m = 60%, which leads to an identical overall savings rate (12.0%) as in our earlier simulations.

The average certainty equivalent wealth levels at the end of the 45-year investment horizon are compared across the three investment policies. The welfare gains (over the TDF investment policy) for the optimal dynamic investment policy and the myopic investment policy are reported below.

Percent Improvement Over the TDF Policy

Risk Aversion (γ)	Optimal	Myopic
1	19.2%	19.2%
3	2.6%	2.5%
5	22.6%	22.6%

The welfare gains for γ = 1 and γ = 3 are similar to those reported earlier when wealth and future savings were positive for both the taxable and retirement accounts. In contrast, the welfare gains for γ = 5 are substantially higher when the investor's wealth and future savings are restricted to the retirement account. Recall that when γ = 5, the TDF equity allocations are higher than those for the optimal dynamic investment policy. When wealth and future savings are restricted to the retirement account, the optimal holding of equity declines further, driving an even larger difference between the optimal dynamic investment policy and the TDF investment policy. This, then, produces a larger welfare gain for the optimal

dynamic investment policy over the TDF investment policy. Clearly, the welfare gains for the optimal dynamic investment policy and the myopic investment policy are not driven by the inclusion of a taxable account in our model.

5. Design Customization

The welfare gains we report for the optimal dynamic investment policy (and the myopic investment policy) can be interpreted as the value of customization in portfolio construction and management over an individual's working years. Customization involves the dynamic optimization of portfolio allocations considering the individual's risk aversion and the evolution of the relevant state variables. As we have seen, the welfare gains from this customization can be significant and more than outweigh the costs of providing a customized approach for investors.

To provide customized advice to clients, investment management firms would need to have information about the client's risk preferences, the split of wealth between taxable and retirement accounts, and estimates of future income.[24] For high-net-worth individuals, wealth management services are available that offer a fully customized approach. For less wealthy individuals, an investment management firm's robo-advising services can be designed to elicit the relevant information from clients and provide a semi-customized approach to asset allocation. Of course, effective portfolio management will require individuals to take the initiative to update their personal financial information over time.

Short of full customization, TDFs can add considerable value by designing separate portfolios for individuals with high, medium, and low risk aversion. In this way, individuals with the same target retirement date would have the ability to choose a portfolio that best matches their risk preferences. The welfare implications can be significant. To illustrate, we ran 5,000 simulation trials over a 45-year period for all three levels of risk aversion ($\gamma = 1$, $\gamma = 3$, and $\gamma = 5$) and recorded for each g the average overall equity allocation at each age under the optimal dynamic investment policy (similar to what is shown in **Figure 7** for $\gamma = 3$). Suppose we design three different TDFs, one for each level of risk aversion, that utilize the average overall equity allocations that emerge from the simulations. We then ran a second simulation with 5,000 trials for each g and recorded the certainty equivalent terminal

[24] Future income would include labor income prior to retirement, as well as any pension and social security income during retirement. In addition, information about the client's desired consumption and bequests would influence the required savings rates prior to retirement. While we have been focused on an individual's savings and investment decisions, investment management firms would likely want to focus their advice and management services at the household level.

wealth for the TDF, the optimal dynamic investment policy, and the myopic investment policy. The welfare gains (over the TDF) are shown below.

Percent Improvement Over the TDF Policy

Risk Aversion (γ)	Optimal	Myopic
1	0.0%	0.0%
3	1.5%	1.4%
5	2.8%	2.7%

Notice that the welfare gains for the optimal dynamic investment policy and the myopic investment policy are substantially reduced when the TDF is designed for a specific level of risk aversion.[25] Even though the TDF does not take into account the dynamics of the relevant state variables, using the average overall equity allocations at each age that emerge from the optimal dynamic investment policy, and controlling for differences in risk preferences, produces reasonably good welfare results. For individuals who do not declare an investment option within their employer-sponsored pension plan, the TDF with average risk aversion could be used as the default option.

Short of any customization in the design of TDFs, it is worth asking what an individual would need to implement their own dynamic investment strategy that outperforms the static, "one-size-fits-all" TDF. The relevant state variables for the "Do-It-Yourself" alternative are: (1) the split of wealth between the taxable and retirement accounts and (2) the value of human capital (as a percent of total wealth). The individual will also need to specify his future savings rates for the taxable and retirement accounts, a target retirement date, and estimates of tax rates on investment income. Future expected returns and volatilities can be estimated from historical returns or taken from estimates provided by several professional investment advisors. The myopic one-period asset allocation strategy performs nearly as well as the optimal dynamic investment policy but has the advantage of being easier to solve numerically and can accommodate more than two asset classes. The solution methodology can easily be automated using a computerized optimization program in Excel.

All the information needed to implement the one-period myopic investment strategy is observable to (or determined by) the individual, except for the value of human capital. There are available research sources that report total lifetime income for individuals with

[25] The welfare gains are zero when $\gamma = 1$ because the optimal overall equity allocation is 100% at all ages. Hence, the TDF asset allocation in this case is identical to the asset allocation in the optimal dynamic investment policy and the myopic investment policy.

different education levels and college majors. Some of these sources also report the year-by-year average income levels and present value of lifetime earnings for different education levels and college majors.[26] Utilizing the pattern of lifetime earnings reported by these sources, along with any private information that the individual has about her future prospects, should allow for a reasonable estimate of the present value of future lifetime earnings (human capital). Of course, as the individual's career develops, and new information emerges about the individual's future earnings potential, the value of human capital can be updated in determining the optimal asset allocations.

6. Conclusion

We developed a simple model of intertemporal asset allocation where individuals can save and invest in both a taxable account and a tax-advantaged retirement account. We use dynamic programming and numerical methods to derive an optimal dynamic investment policy. We show that because the preferred location for holding equity is in the taxable account (optimal asset location), the optimal equity allocations (glide paths) will differ between the taxable and retirement accounts. The optimal glide paths for the taxable and retirement accounts are dynamic and depend upon the investor's level of risk aversion and two underlying state variables: (1) the split of wealth between the taxable and retirement accounts and (2) the present value of future labor income (human capital). This contrasts with the glide path for a typical "one-size-fits-all" TDF that is static in nature, designed for an "average" investor, and advocates for a declining equity proportion as a function of age alone.

We use simulation analysis to investigate how TDF asset allocations and investor welfare compare to those from the optimal dynamic investment policy. We find that the optimal asset allocations and investor welfare can deviate substantially from a "one-size-fit-all" TDF investment policy. The deviations in asset allocations and investor welfare depend importantly on the individual's risk aversion, value of human capital, and split of wealth between the taxable and retirement accounts. Interestingly, the target retirement date is of secondary importance once the other relevant state variables are accounted for. A simple one-period myopic investment policy was shown to perform relatively well in comparison to the optimal dynamic investment policy.

The welfare gains for the optimal dynamic investment policy (and the myopic policy) are interpreted as the value of customization of asset allocation and portfolio management.

[26] See, for example, **The Hamilton Project**, https://www.hamiltonproject.org/data/career-earnings-by-college-major/

We argue that the value of full customization can easily exceed the cost for some investors. We also show that designing TDFs to account for differences in risk aversion across investors can substantially improve welfare and provide a better default option for employer-sponsored pension plans. Finally, we discuss how an individual could design their own "Do-It-Yourself" investment strategy that would out-perform the "one-size-fits-all" TDF. The "Do-It-Yourself" alternative is made easier by the fact that the myopic (one-period) investment policy performs nearly as well as the more complicated optimal dynamic investment policy. The information needed to implement the "Do-It-Yourself" alternative is readily available, or can easily be estimated, by the investor.

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Viceira, L. M., 2001, Optimal portfolio choice for long-horizon investors with nontradable labor income, *Journal of Finance* 56, 433–470. https://doi.org/10.1111/0022-1082.00333.

Table 1: Model Variables and Notation

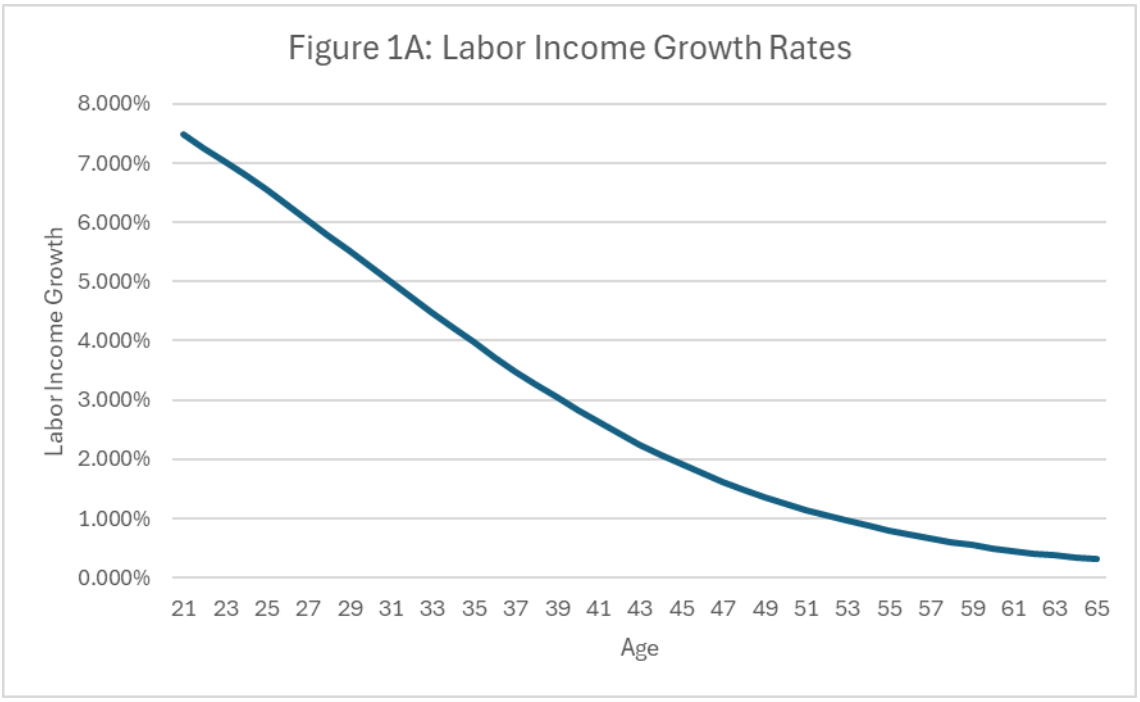
Variable	Notation	Base-Case Values
Return Variables:		
Risk-free interest rate	r	3.5%
Dividend yield on risky asset	d	1.5%
Capital gain return on risky asset	g	7.0%
Standard deviation of returns on risky asset	σ	20.0%
Tax Rates:		
Ordinary income tax rate	t_o	35.0%
Dividend tax rate	t_d	20.0%
Capital gains tax rate	t_g	20.0%
Utility and Value Function:		
Utility of terminal wealth	$U(W_T)$	
Value function at date t	$V(H_t)$	
Vector of state variables at date t	H_t	
Constant of relative risk aversion	γ	3
Target (retirement) date	T	35 years
Labor Income and Savings Rate Variables:		
After-tax labor income at date t	L_t	
Annual growth rate in labor income at date t	i_t	
Savings rate (as percent of labor income) in the retirement account	y_R	6.25%
Savings rate (as percent of labor income) in the taxable account	y_{NR}	2.00%
Employer match percentage for retirement savings	m	60.0%
Wealth Variables:		
Total wealth at date t	W_t	
Financial wealth at date t	$W_{F,t}$	
Wealth held in the retirement account at date t	$W_{R,t}$	
Wealth held in the taxable (non-retirement) account at date t	$W_{NR,t}$	
Present value of future labor income at date t	$PV_t(L)$	
After-tax growth rate (gross) in total wealth at date t	ρ_t	
After-tax growth rate (gross) in retirement wealth at date t	$\rho_{R,t}$	
After-tax growth rate (gross) in taxable wealth at date t	$\rho_{NR,t}$	
Choice Variables:		
Proportion of taxable account wealth invested in equity at date t	x_t	
Proportion of retirement account wealth invested in equity at date t	z_t	
Normalized Variables:		
Labor income-to-total wealth at date t	l_t	
Financial wealth as fraction of total wealth at date t	$w_{F,t}$	
Retirement wealth as fraction of financial wealth at date t	$w_{R,t}$	
Taxable wealth as fraction of financial wealth at date t	$w_{NR,t}$	
Value function normalized by total wealth at date t	$v(h_t)$	
Vector of normalized state variables at date t	h_t	

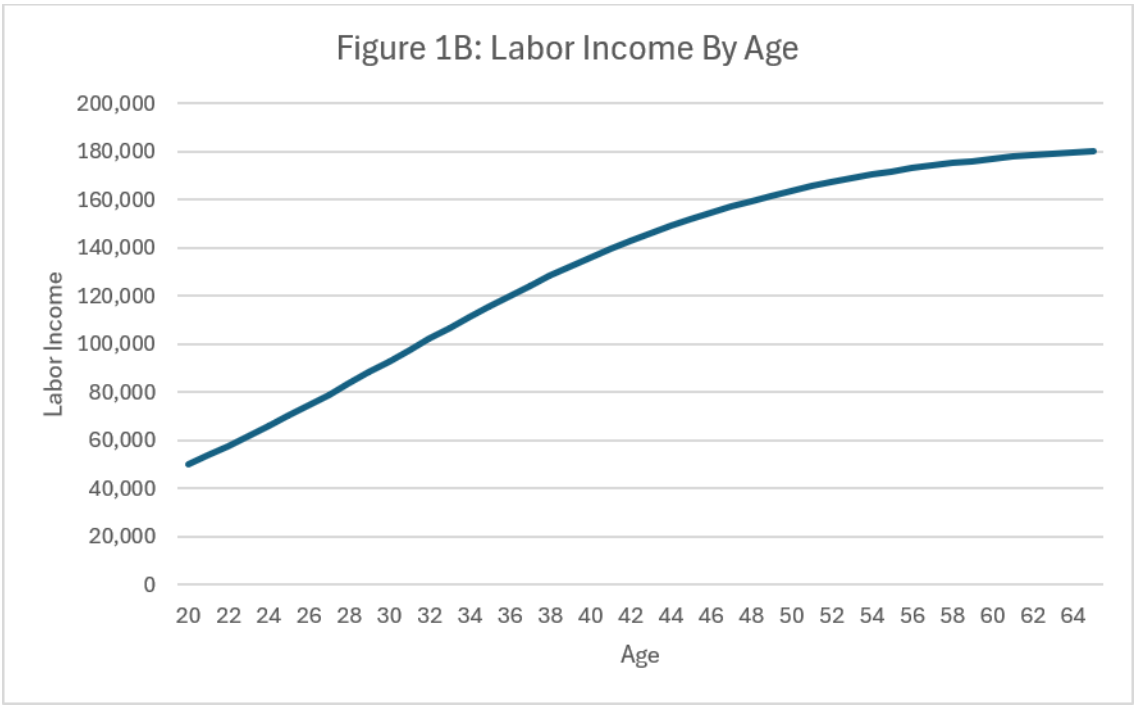
Table 2: Aggregate Contribution Rates to 401(k) Plans for 2024

	Range	Aggregate Contribution Rates (incl. employee and employer contributions)
All Participants		12.0%
Income Levels	<$15,000	10.3%
	$15,000-$29,999	9.7%
	$30,000-$49,999	9.4%
	$50,000-$74,999	10.8%
	$75,000-$99,999	12.1%
	$100,000-$149,999	13.2%
	$150,000+	13.9%
Age		
	<25	9.3%
	25-34	11.1%
	35-44	11.7%
	45-54	12.3%
	55-64	13.8%
	65+	14.6%

Source: "How America Saves 2025", Vanguard

Figure 1: Annual Labor Income Growth Rates





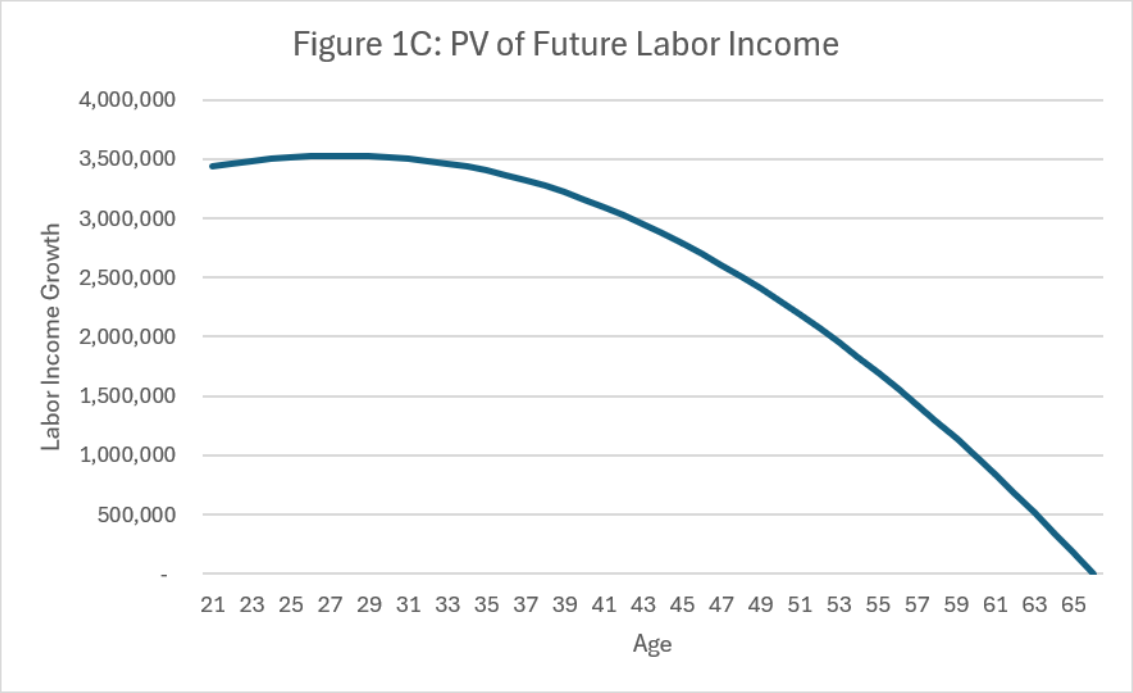
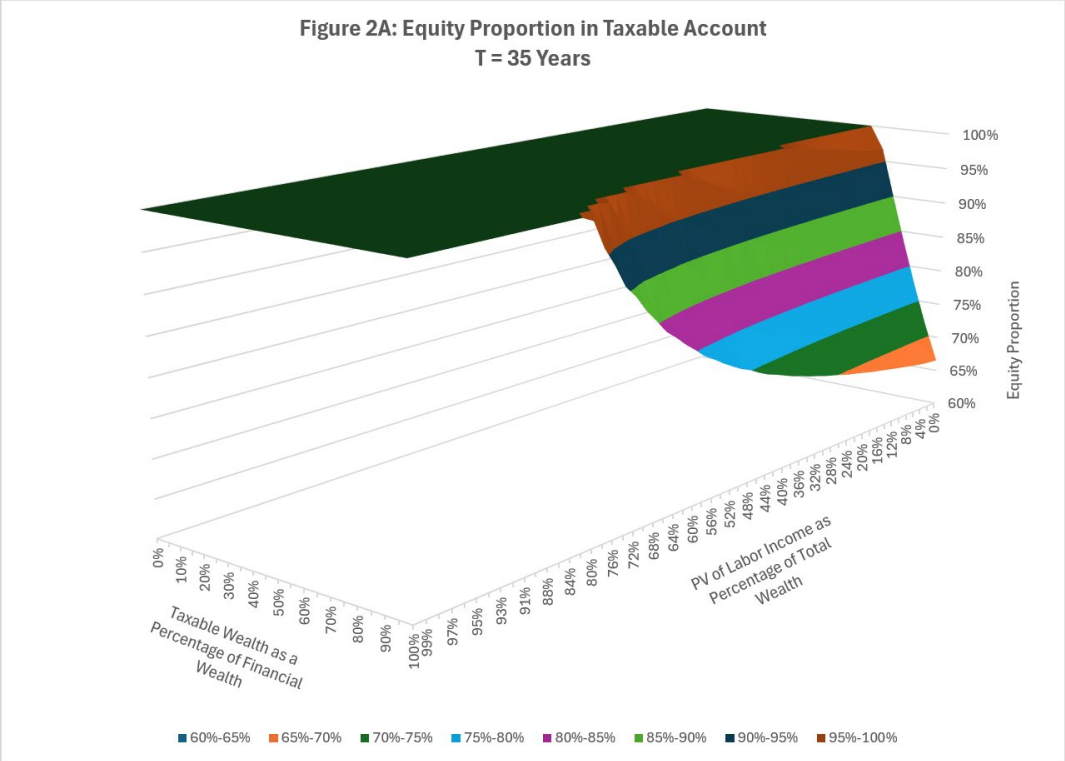
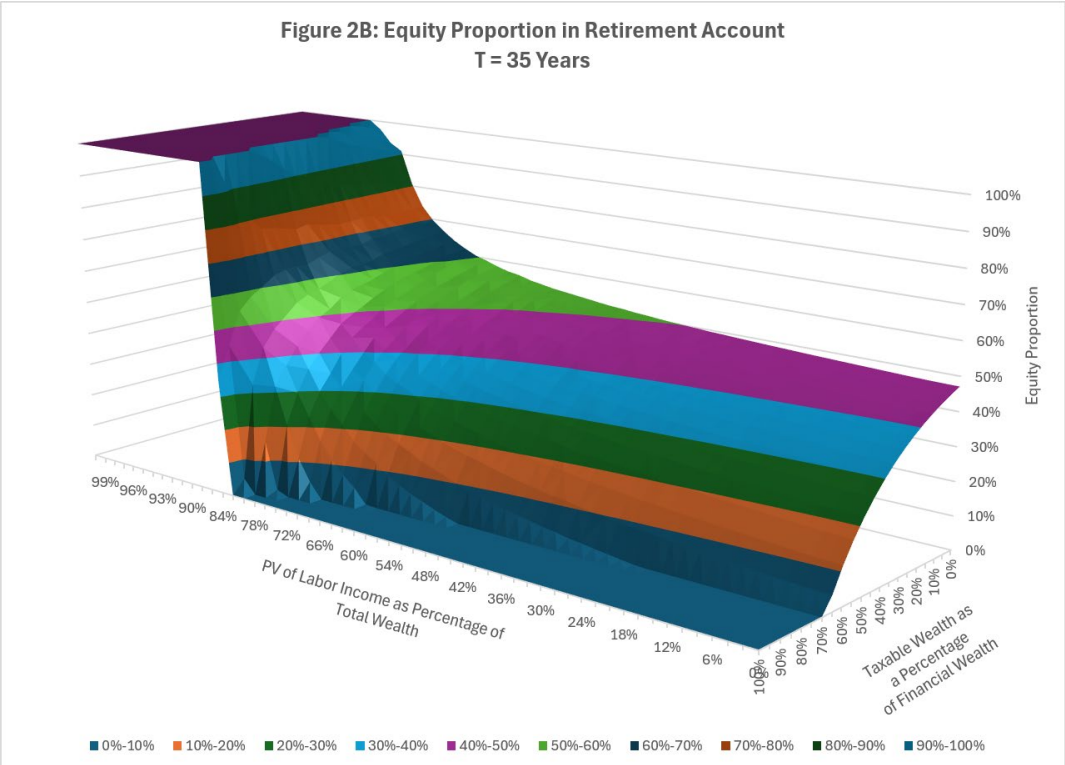

Figure 1C: PV of Future Labor Income
4,000,000
3,500,000
3,000,000
2,500,000
2,000,000
1,500,000
1,000,000
500,000
-
Labor Income Growth
21 23 25 27 29 31 33 35 37 39 41 43 45 47 49 51 53 55 57 59 61 63 65
Age

Figure 2: Optimal Equity Allocations





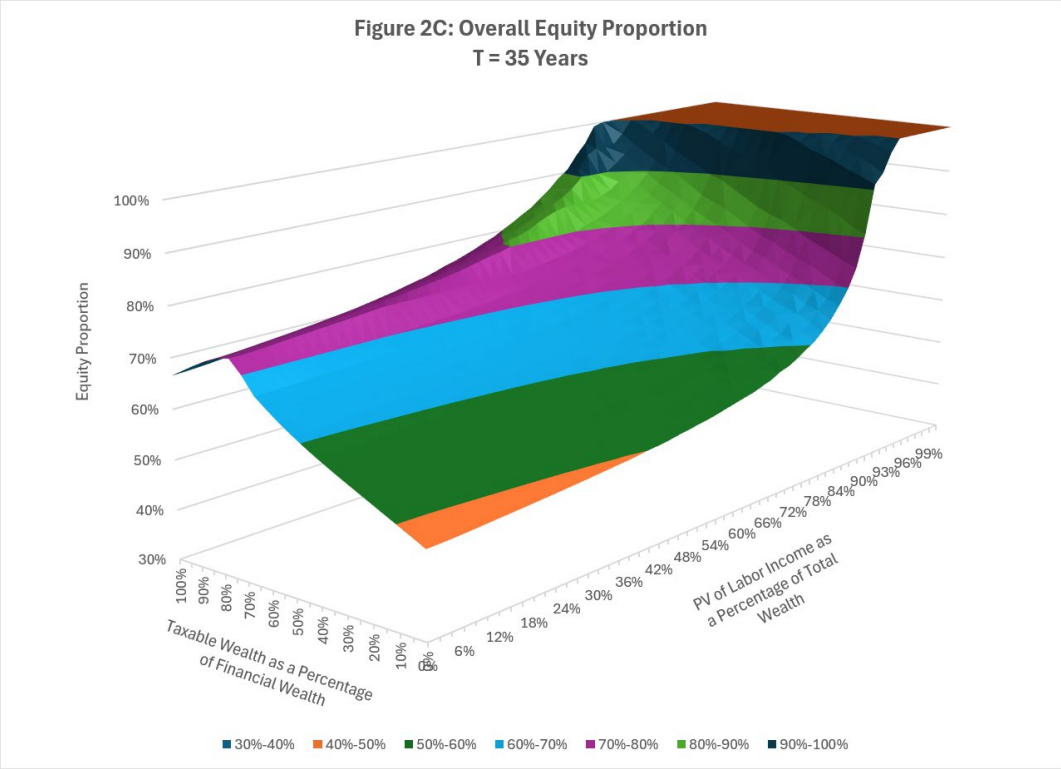
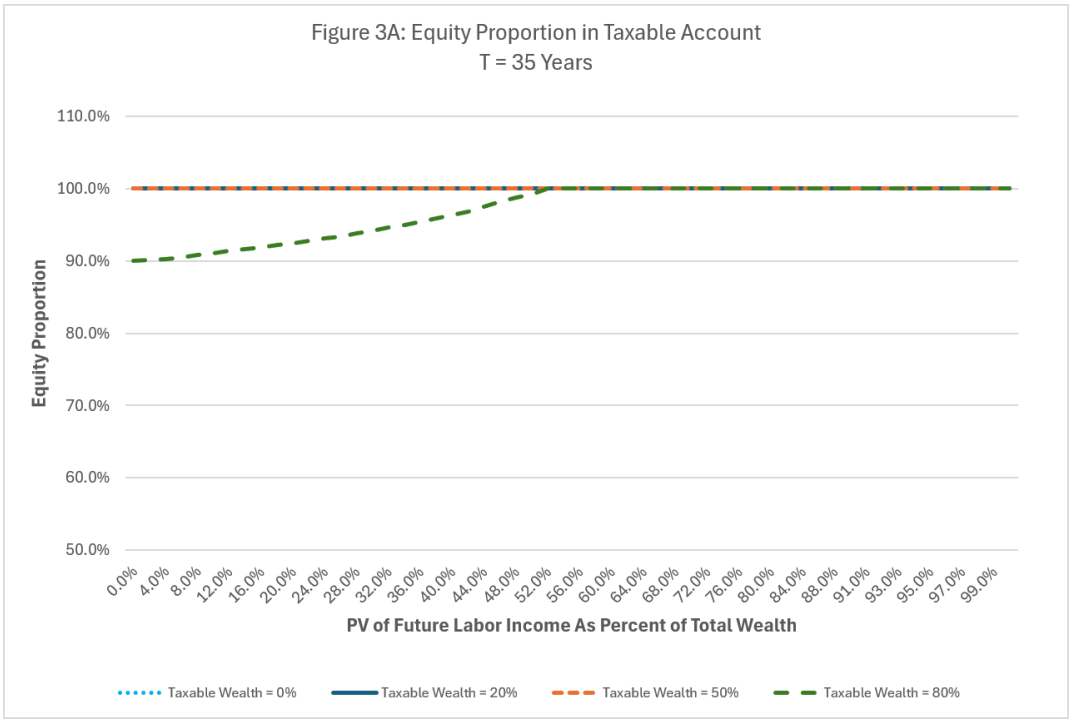
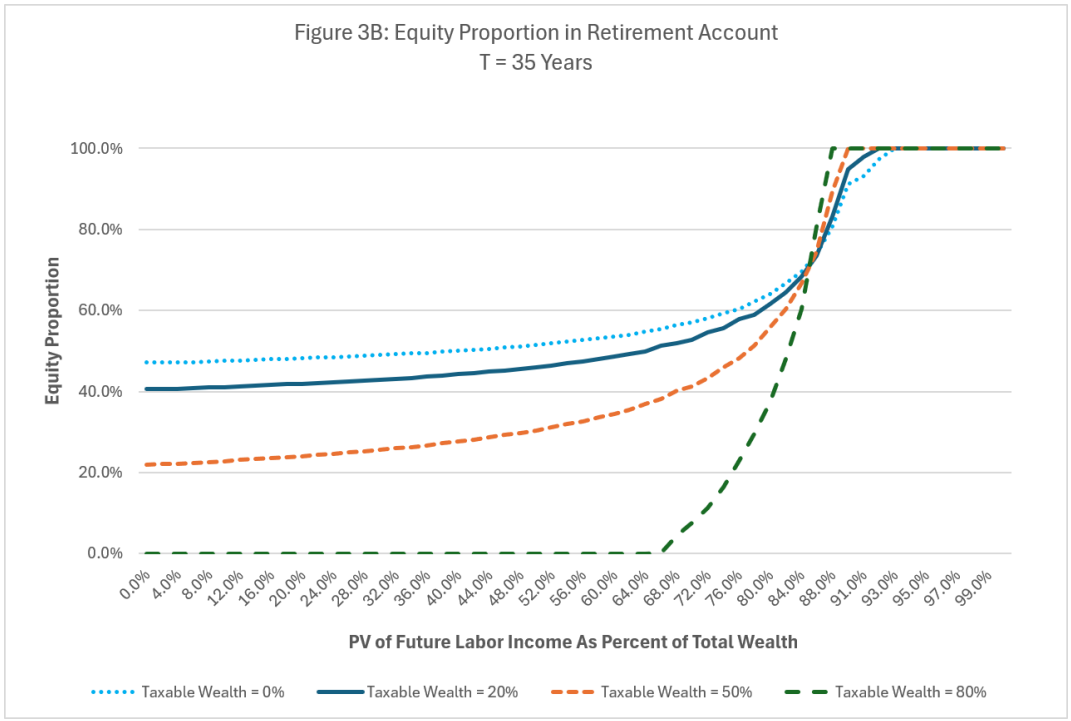

Figure 2C: Overall Equity Proportion
T = 35 Years
Equity Proportion
100%
90%
80%
70%
60%
50%
40%
30%
Taxable Wealth as a Percentage of Financial Wealth
100%
90%
80%
70%
60%
50%
40%
30%
20%
10%
0%
PV of Labor Income as a Percentage of Total Wealth
0%
6%
12%
18%
24%
30%
36%
42%
48%
54%
60%
66%
72%
78%
84%
90%
93%
96%
99%
30%-40%
40%-50%
50%-60%
60%-70%
70%-80%
80%-90%
90%-100%

Figure 3: Cross-Section of Optimal Equity Allocations





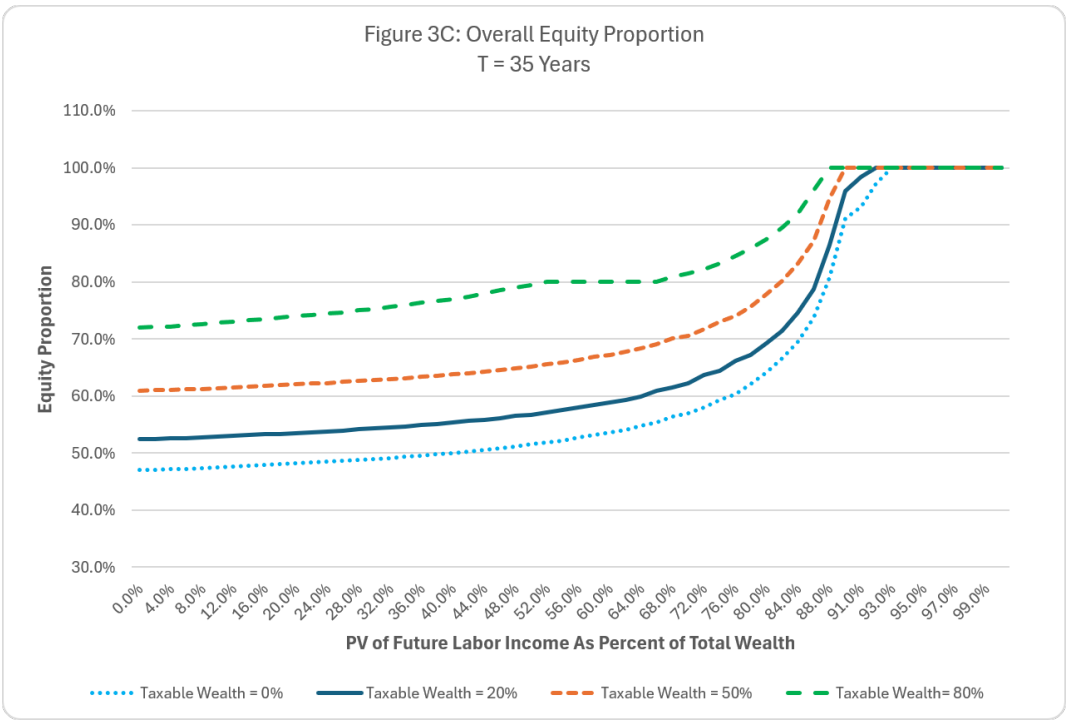
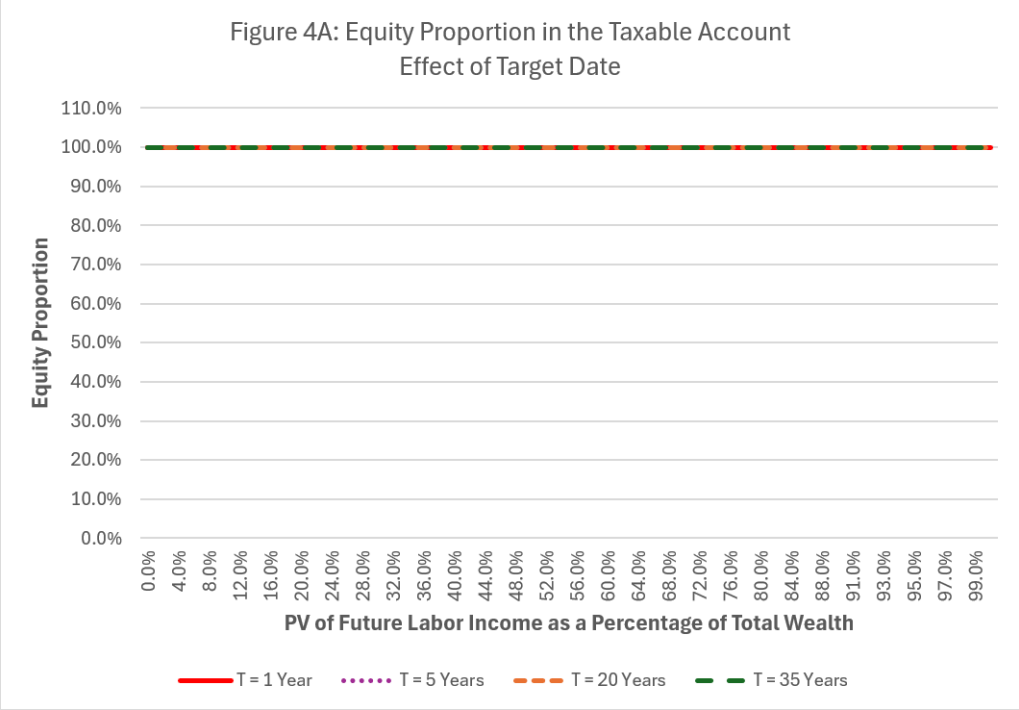
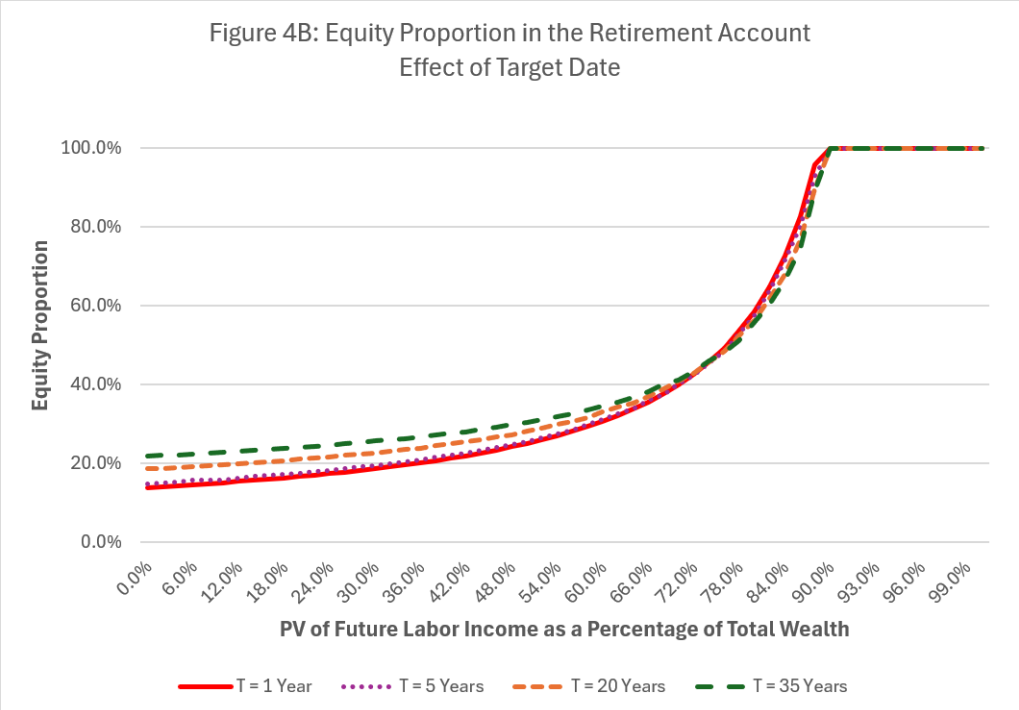

Figure 3C: Overall Equity Proportion
T = 35 Years
110.0%
100.0%
90.0%
80.0%
70.0%
60.0%
50.0%
40.0%
30.0%
Equity Proportion
0.0% 4.0% 8.0% 12.0% 16.0% 20.0% 24.0% 28.0% 32.0% 36.0% 40.0% 44.0% 48.0% 52.0% 56.0% 60.0% 64.0% 68.0% 72.0% 76.0% 80.0% 84.0% 88.0% 91.0% 93.0% 95.0% 97.0% 99.0%
PV of Future Labor Income As Percent of Total Wealth
Taxable Wealth = 0%
Taxable Wealth = 20%
Taxable Wealth = 50%
Taxable Wealth= 80%

Figure 4: Cross-Section of Optimal Equity Proportions
Effect of Target Date





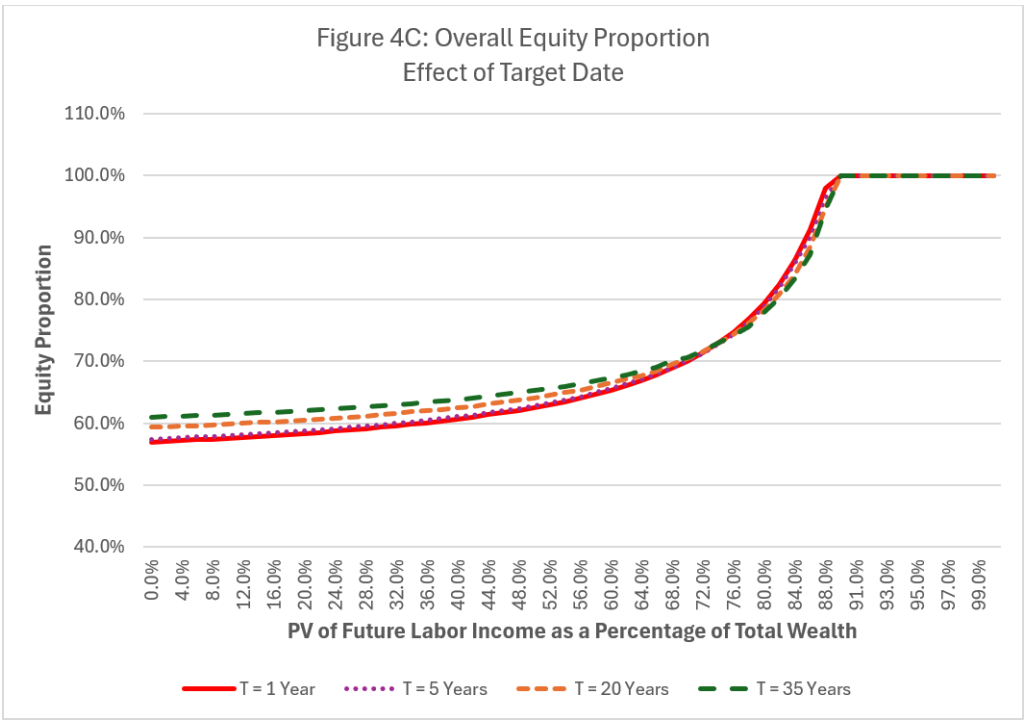
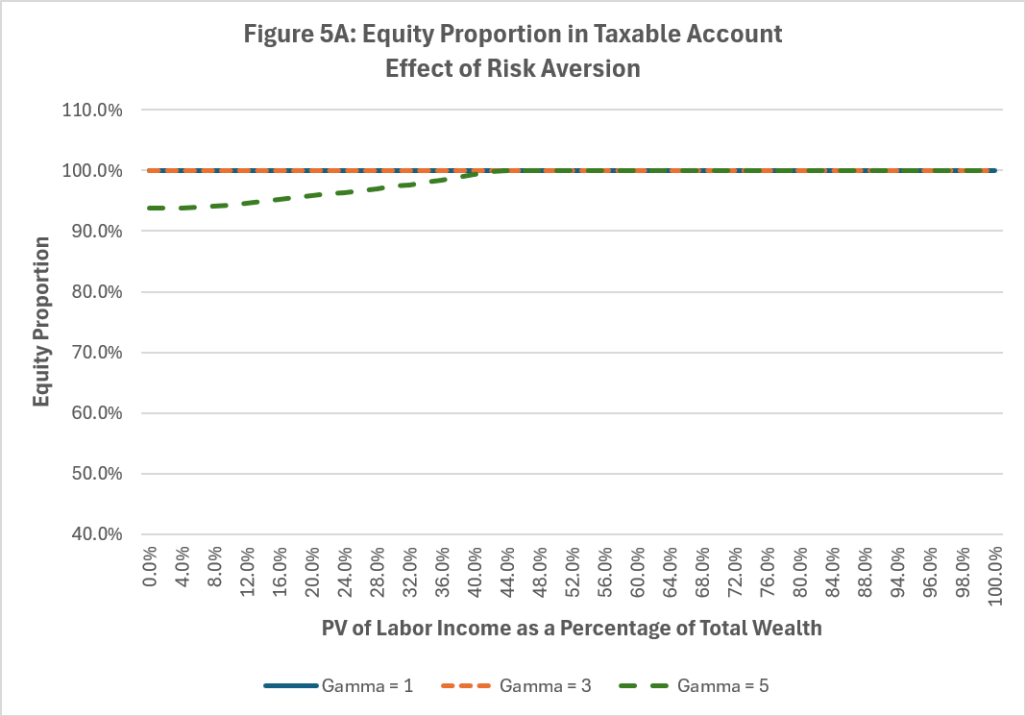
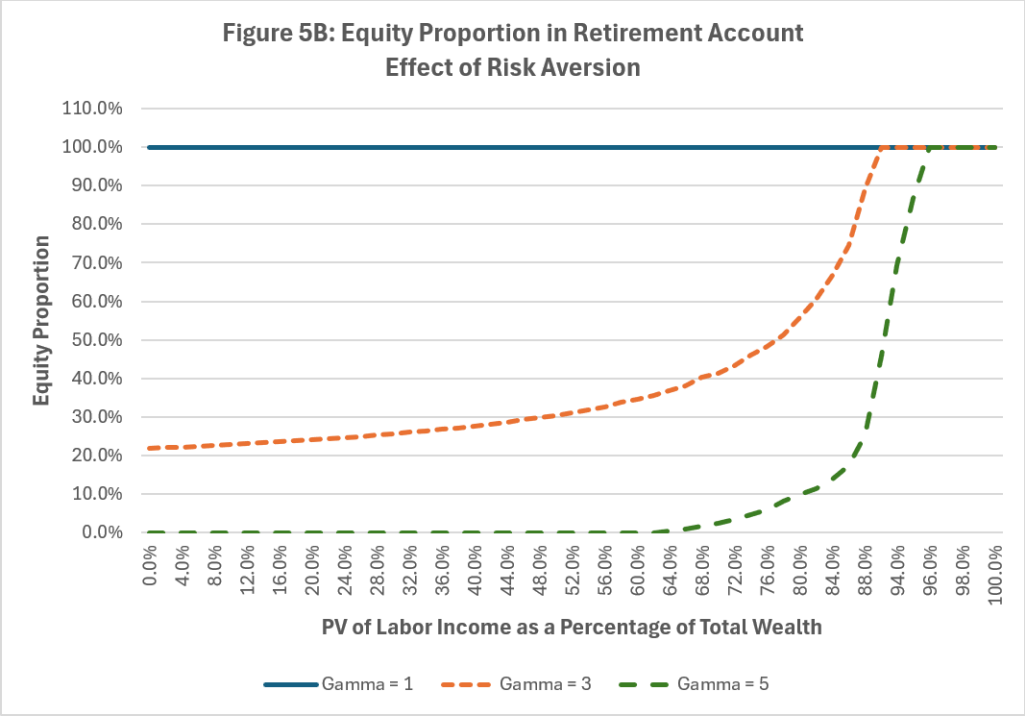

Figure 4C: Overall Equity Proportion
Effect of Target Date
110.0%
100.0%
90.0%
80.0%
70.0%
60.0%
50.0%
40.0%
Equity Proportion
0.0%
4.0%
8.0%
12.0%
16.0%
20.0%
24.0%
28.0%
32.0%
36.0%
40.0%
44.0%
48.0%
52.0%
56.0%
60.0%
64.0%
68.0%
72.0%
76.0%
80.0%
84.0%
88.0%
91.0%
93.0%
95.0%
97.0%
99.0%
PV of Future Labor Income as a Percentage of Total Wealth
T = 1 Year
T = 5 Years
T = 20 Years
T = 35 Years

Figure 5: Cross-Section of Optimal Equity Allocations
Effect of Risk Aversion





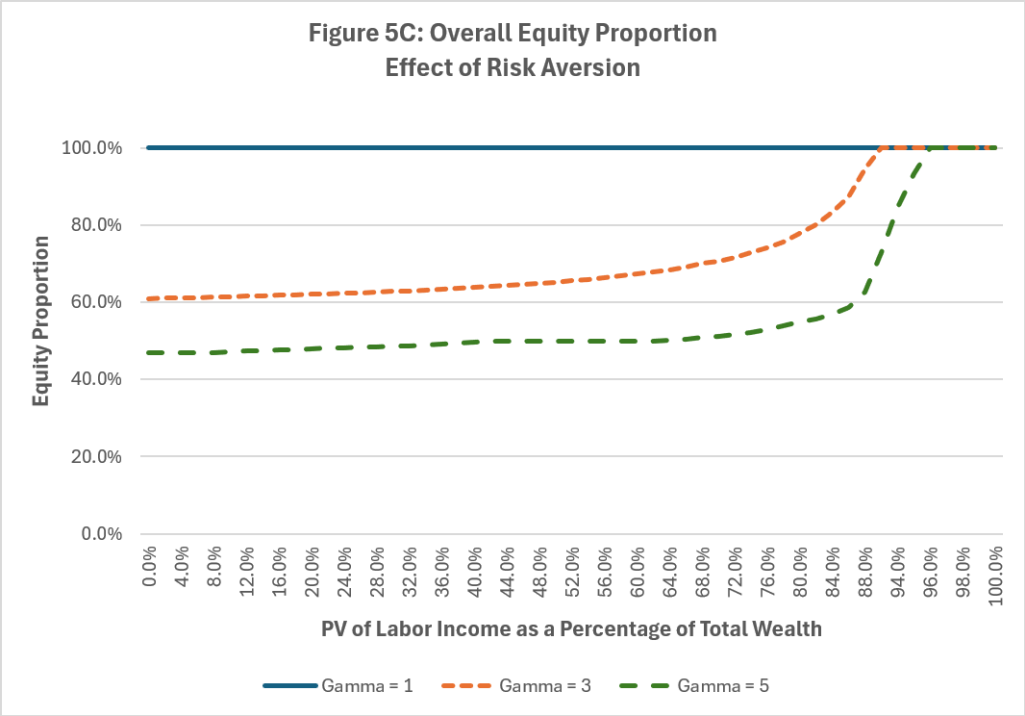

Figure 5C: Overall Equity Proportion
Effect of Risk Aversion
Equity Proportion
100.0%
80.0%
60.0%
40.0%
20.0%
0.0%
0.0% 4.0% 8.0% 12.0% 16.0% 20.0% 24.0% 28.0% 32.0% 36.0% 40.0% 44.0% 48.0% 52.0% 56.0% 60.0% 64.0% 68.0% 72.0% 76.0% 80.0% 84.0% 88.0% 94.0% 96.0% 98.0% 100.0%
PV of Labor Income as a Percentage of Total Wealth
Gamma = 1
Gamma = 3
Gamma = 5

Figure 6: Typical Target-Date Fund Glide Path

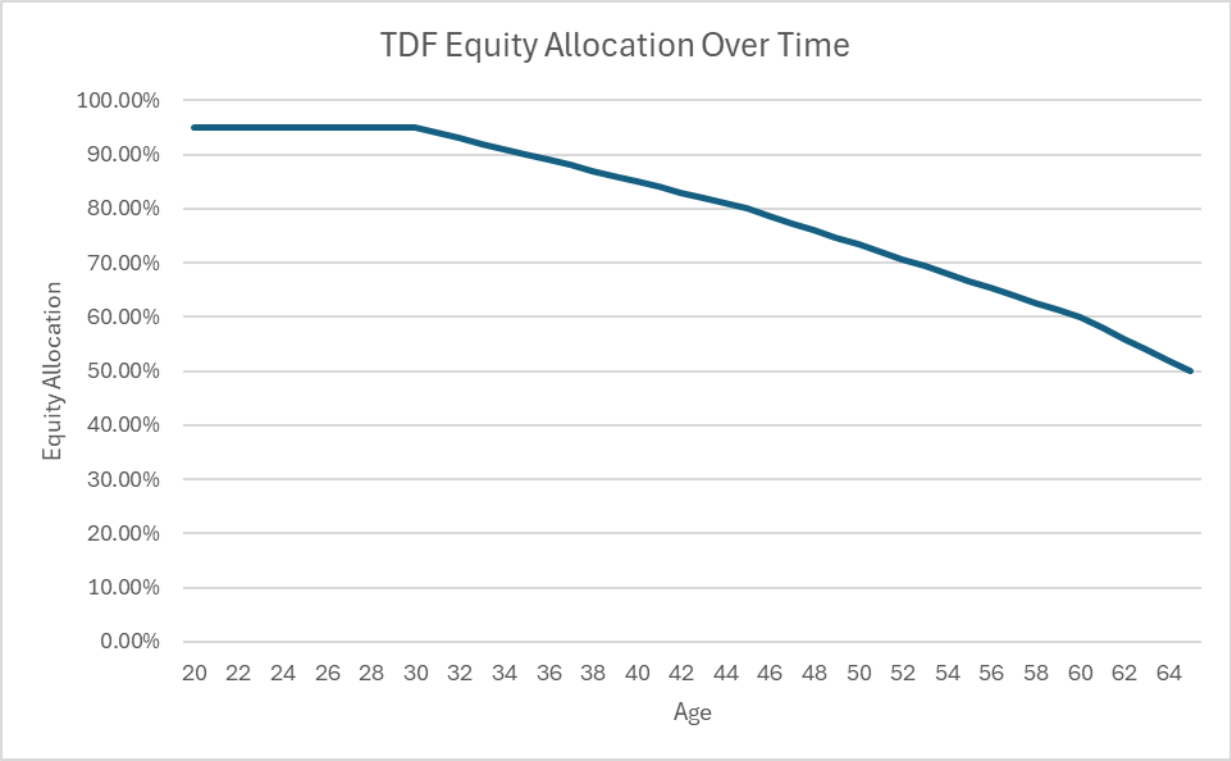


Figure 7: Average Equity Allocations at Each Age

The graph below shows the ***average*** equity allocations for the taxable and retirement accounts, as well as the overall equity allocation, at each age. The averages at each age are taken over the 5,000 simulation trials beginning at age 20 with an initial value of human capital equal to 95% of total wealth and 5% of financial wealth held in the taxable account. As a comparison, the equity allocations at each age for the Target-Date Fund are also shown.

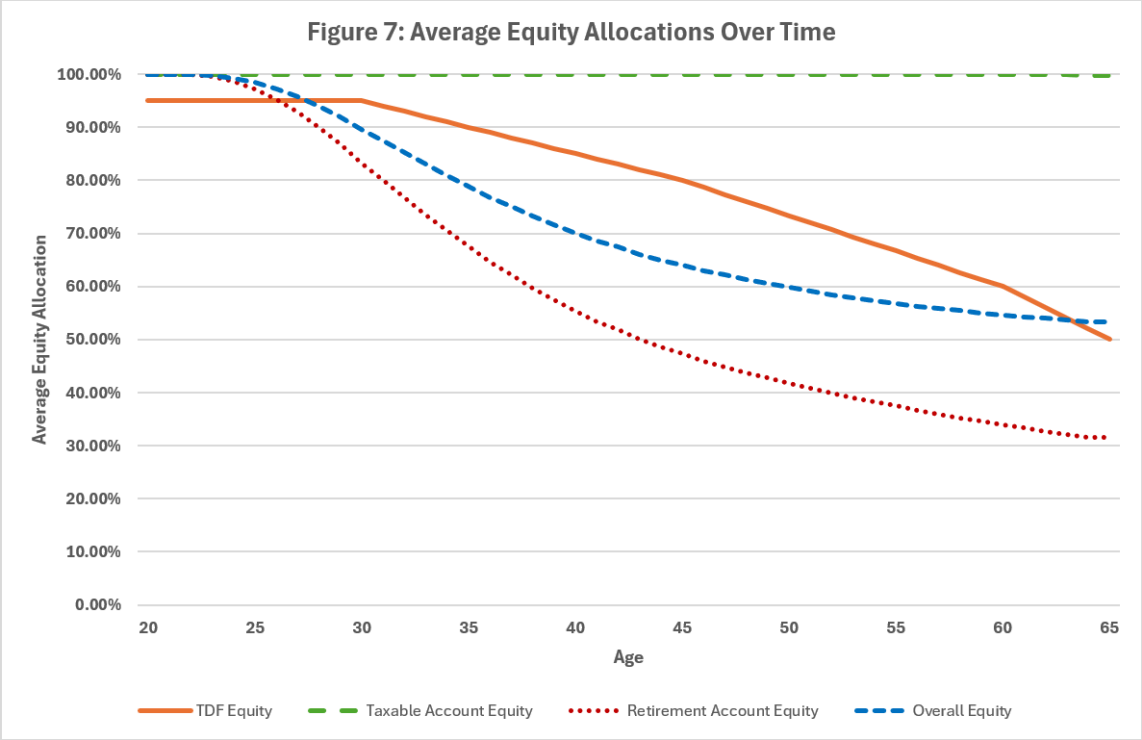


Figure 8: Distribution of Optimal Overall Equity Holdings at Age 45

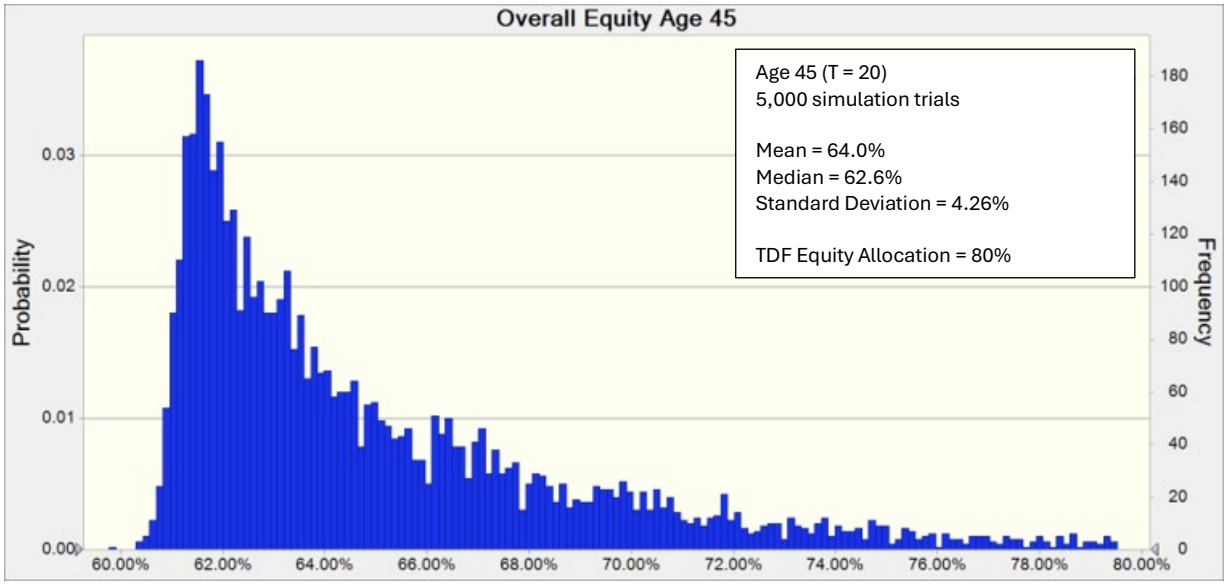
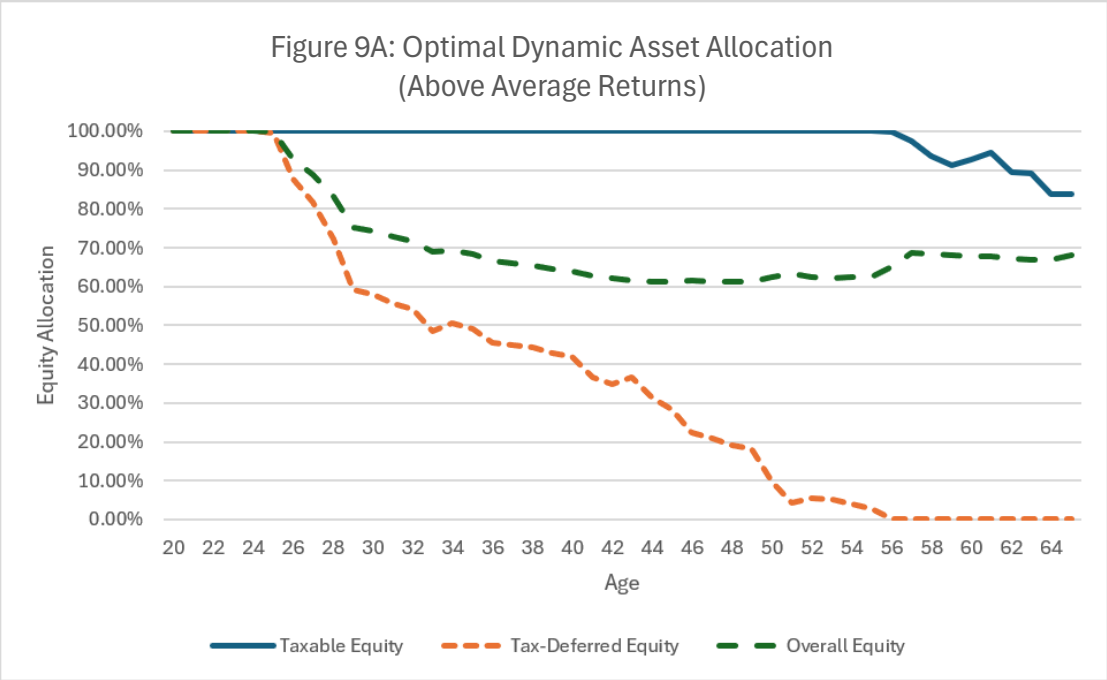
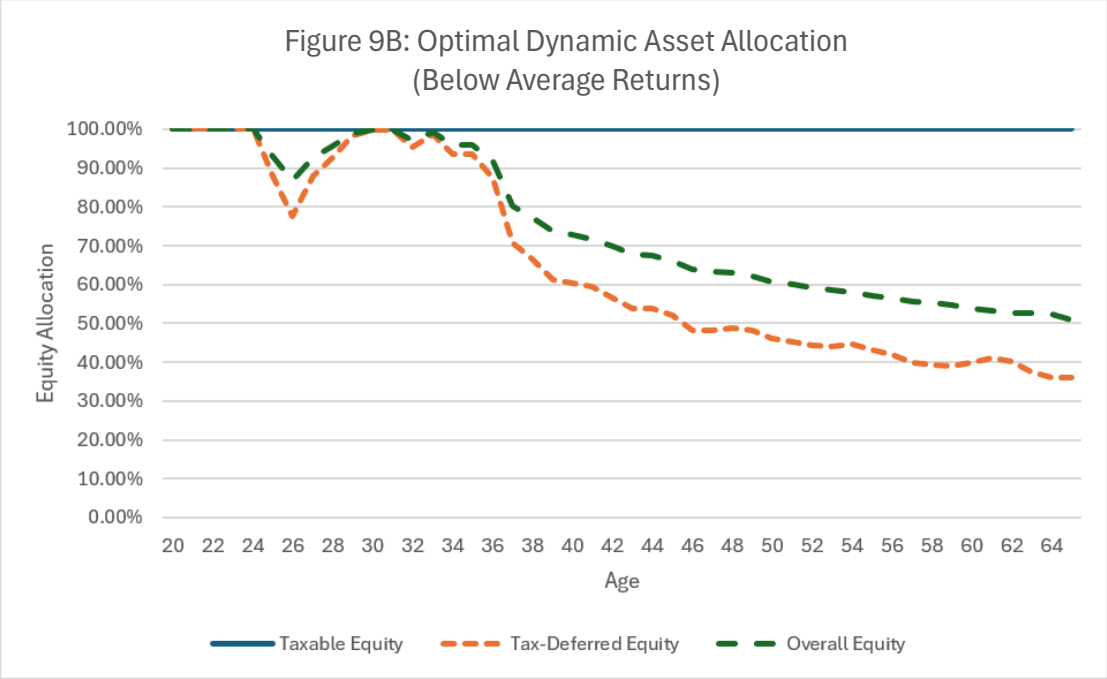


Figure 9: Glide Paths for the Dynamic Investment Policy





Appendix A: The Dynamic Programming Approach

The solution to the intertemporal investment problem requires the use of dynamic programming. Bellman's (1952) optimality condition allows us to define an intertemporal value function for the optimization problem presented in Equation (5):

$$V(W_t, H_t) = \underset{x_t,\, z_t}{MAX} \quad E_t[V(W_{t+1}, H_{t+1})], \qquad t = 0, 1, \ldots, T-1 \tag{A1}$$

subject to the constraints in Equations (1) - (4) and (6). At date T, the value function is $V(W_T, H_T) = U(W_T)$.

Define $w_{F,t} = W_{F,t}/W_t$ as the ratio of financial wealth to total wealth, $w_{NR,t} = W_{NR,t}/W_{F,t}$ as the fraction of financial wealth invested in the taxable (non-retirement) account, $w_{R,t} = W_{R,t}/W_{F,t} = (1-w_{NR,t})$ as the fraction of financial wealth invested the retirement account, and $l_t = L_t/W_t$ as the labor income-to-total wealth ratio. Then, the dynamic wealth equation can be written as follows:

$$W_{t+1} = W_{NR,t+1} + W_{R,t+1} + PV_{t+1}(L) \tag{A2}$$

$$= W_t\{[w_{F,t}w_{NR,t}\rho_{NR,t+1} + y_{NR}l_t(1 + i_{t+1})]$$

$$+[w_{F,t}(1 - w_{NR,t})\rho_{R,t+1} + y_R l_t(1 + i_{t+1})(1 + m)] + l_t(1 + i_{t+1})A_{t+1}\}$$

where $\rho_{NR,t+1}$ is one plus the after-tax return on taxable wealth at date t+1 and $\rho_{R,t+1}$ is one plus the return on retirement wealth at date t+1 given by

$$\rho_{NR,t+1} = \{1 + x_t[d(1 - t_d) + g_{t+1}(1 - t_g)] + (1 - x_t)r(1 - t_o)\} \tag{A3}$$

and

$$\rho_{R,t+1} = \{1 + z_t(d + g_{t+1}) + (1 - z_t)r\} \tag{A4}$$

Dividing the left- and right-hand sides of Equation (A2) by total wealth, W_t, produces the following expression:

$$\rho_{t+1} = w_{F,t}\{w_{NR,t}\rho_{NR,t+1} + (1 - w_{NR,t})\rho_{R,t+1}\} \tag{A5}$$

$$+l_t(1+i_{t+1})\{y_{NR}+y_R(1+m)+A_{t+1}\}$$

where $\rho_{t+1} = W_{t+1}/W_t$ is one plus the growth rate in total after-tax wealth.

Substituting Equation (A5) into Equation (A1) allows us to write the value function as follows:

$$V(W_t, H_t) = \underset{x_t,\, z_t}{MAX}\ E_t[V(W_t\rho_{t+1}, H_{t+1})]\,, \qquad t = 0, 1, \dots, T-1 \tag{A6}$$

Next, we augment the above maximization problem by introducing Lagrange multipliers into Equation (A6) to account for the upper and lower bounds for x_t and z_t.

$$V(W_t, H_t) = \underset{x_t,\, z_t}{MAX}\ E_t[V(W_t\rho_{t+1}, H_{t+1})] + W_{NR,t}[\lambda_{1,t}x_t + \lambda_{2,t}(1-x_t)] \tag{A7}$$

$$+W_{R,t}[\lambda_{3,t}z_t + \lambda_{4,t}(1-z_t)], \qquad t = 0, 1, \dots, T-1$$

where the second term in Equation (A7) deals with the restriction that the fraction of equity in the taxable account satisfies $0 \le x_t \le 1$ and the third term deals with the restriction that the fraction of equity in the retirement account satisfies $0 \le z_t \le 1$. In particular, $\lambda_{1,t} \ge 0$ if $x_t^* = 0$, $\lambda_{2,t} \ge 0$ if $x_t^* = 1$, $\lambda_{3,t} \ge 0$ if $z_t^* = 0$, and $\lambda_{4,t} \ge 0$ if $z_t^* = 1$.

The first-order condition for x_t is:

$$\frac{\partial V(W_t, H_t)}{\partial x_t} = W_t E_t\left\{V'_W\left(\frac{\partial \rho_{t+1}}{\partial x_t}\right)\right\} + W_{NR,t}(\lambda_{1,t} - \lambda_{2,t}) = 0 \tag{A8}$$

where $V'_W = \partial V(W_{t+1}, H_{t+1})/\partial W_{t+1}$. But from the definition of r_{t+1} in Equation (A5) we have:

$$\frac{\partial \rho_{t+1}}{\partial x_t} = w_{F,t}w_{NR,t}\left[d(1-t_d) + g_{t+1}(1-t_g) - r(1-t_o)\right] \tag{A9}$$

Substituting Equation (A9) into Equation (A8), recognizing that $w_{F,t}w_{NR,t} = W_{NR,t}/W_t$, we have the following first-order condition for x_t:

$$E_t\left\{V'_W\left[d(1-t_d) + g_{t+1}(1-t_g) - r(1-t_o)\right]\right\} + \lambda_{1,t} - \lambda_{2,t} = 0 \tag{A10}$$

Similarly, the first-order condition for z_t is:

$$E_t\{V'_W[d + g_{t+1} - r]\} + \lambda_{3,t} - \lambda_{4,t} = 0 \quad (A11)$$

In what follows, we assume that the individual has constant relative risk averse (CRRA) preferences, which ensures that our model has the property that the asset allocation decision rules $\{x_t, z_t\}$ are independent of total wealth, W_t.

With CRRA preferences, the utility of terminal wealth is:

$$U(W_T) = \left(\frac{W_T^{1-\gamma}}{1-\gamma}\right) = \left(\frac{W_t^{1-\gamma}}{1-\gamma}\right)\left[\prod_{j=t+1}^{T} \rho_j^{1-\gamma}\right] \quad (A12)$$

where $\rho_j = W_j/W_{j-1}$ is one plus the growth rate in total wealth at date j.

Dividing both sides of Equation (A12) by $W_t^{1-\gamma}$, utilizing the Bellman equation of optimality, and imposing the constraints that $0 \le x_t \le 1$ and $0 \le z_t \le 1$, for all $t = 0, \ldots, T-1$, allows us to write the normalized objective function as follows:

$$v(h_t) = \underset{x_t,\, z_t}{MAX}\; E_t[v(h_{t+1})] + [\lambda_{1,t}x_t + \lambda_{2,t}(1 - x_t)] + [\lambda_{3,t}z_t + \lambda_{4,t}(1 - z_t)] \quad (A13)$$

where

$$v(h_{t+1}) = \left(\frac{\rho_{t+1}^{1-\gamma}}{1-\gamma}\right)\phi_{t+1}(h_{t+1}) = \left(\frac{\rho_{t+1}^{1-\gamma}}{1-\gamma}\right)\prod_{j=t+2}^{T}\left(\rho_j^*\right)^{1-\gamma} \quad (A14)$$

At date T, $v(h_T) = \frac{\left(\rho_T^{1-\gamma}\right)}{1-\gamma}$. The term $\phi_{t+1}(h_{t+1})$ is a time-dependent function of future one-period growth rates in total wealth under the optimal investment policies: $\{x_j^*, z_j^*\}$, j = t+1,...,T-1. The vector of normalized state variables that we use is $h_t = \{A_t l_t, w_{NR,t}, T\text{-}t\}$, where $A_t l_t = A_t\left(\frac{L_t}{W_t}\right) = 1 - w_{F,t}$ is the present value of future labor income as a percentage of

total wealth and $w_{NR,t} = W_{NR,t}/W_{F,t}$ is wealth held in the taxable (non-retirement) account as a percentage of financial wealth.[27]

It is worth noting that ***if*** ρ_{t+1} and $\phi_{t+1}(h_{t+1})$ are independent at time t, then

$$\mathrm{E_t}\left[\left(\frac{\rho_{t+1}^{1-\gamma}}{1-\gamma}\right)\phi_{t+1}(h_{t+1})\right] = \mathrm{E_t}\left[\left(\frac{\rho_{t+1}^{1-\gamma}}{1-\gamma}\right)\right]\mathrm{E_t}[\phi_{t+1}(h_{t+1})] \qquad \text{(A14)}$$

This implies that the optimal asset allocations, x_t and z_t, can be found by solving a one-period optimization problem of the form:

$$\underset{x_t,\, z_t}{\mathrm{MAX}} \quad \mathrm{E_t}\left[\frac{\rho_{t+1}^{1-\gamma}}{1-\gamma}\right] + [\lambda_{1,t}x_t + \lambda_{2,t}(1-x_t)] + [\lambda_{3,t}z_t + \lambda_{4,t}(1-z_t)] \qquad \text{(A15)}$$

In this case, the optimal investment policy is myopic. In our model, however, ρ_{t+1} and $\phi_{t+1}(h_{t+1})$ are not independent since the realization of ρ_{t+1} will influence the vector of state variables at future dates.

Solving for the optimal holding of equity in the taxable and retirement accounts for each date t involves the use of dynamic programming. Because analytical solutions are not available, it is necessary to use numerical methods to derive solutions to the intertemporal optimization problem. **Appendix B** provides the details of the numerical solution methodology.

[27] Recall that the original vector of state variables is $H_t = \{W_{F,t}, W_{NR,t}, A_tL_t, T\text{-}t\}$. Notice that the normalized state variables, $h_t = \{A_t l_t, w_{NR,t}, T\text{-}t\}$, along with total wealth, W_t, are sufficient to recover the original vector of state variables $H_t = \{W_{F,t}, W_{NR,t}, A_tL_t, T\text{-}t\}$ since $W_{F,t} = (1 - A_t l_t)W_t$, $W_{NR,t} = w_{NR,t}W_{F,t}$, and $A_tL_t = (A_t l_t)W_t$.

Appendix B: Numerical Solution Methodology

Our objective is to solve the intertemporal investment problem stated below:

$$v(\mathrm{h_t}) = \underset{\mathrm{x_t},\, \mathrm{z_t}}{\mathrm{MAX}}\ \mathrm{E_t}[v(h_{t+1})] + [\lambda_{1,\mathrm{t}}\mathrm{x_t} + \lambda_{2,\mathrm{t}}(1-\mathrm{x_t})] + [\lambda_{3,\mathrm{t}}\mathrm{z_t} + \lambda_{4,\mathrm{t}}(1-z_t)] \tag{B1}$$

for all t = 0, …, T-1, where

$$v(h_{t+1}) = \left(\frac{\rho_{t+1}^{1-\gamma}}{1-\gamma}\right)\phi_{t+1}(h_{t+1}) = \left(\frac{\rho_{t+1}^{1-\gamma}}{1-\gamma}\right)\prod_{j=t+2}^{T}\left(\rho_j^*\right)^{1-\gamma} \tag{B2}$$

is the date-t+1 value function,

$$\rho_{t+1} = w_{F,t}\{w_{NR,t}\rho_{NR,t+1} + (1-w_{NR,t})\rho_R\} \tag{B3}$$

$$+l_t(1+i_{t+1})\{y_{NR} + y_{R.t+1}(1+m) + A_{t+1}\}$$

is the gross after-tax growth rate in total wealth between dates t and t+1, and

$$\rho_{NR,t+1} = \left\{1 + \mathrm{x_t}\left[\mathrm{d}(1-\mathrm{t_d}) + \mathrm{g_{t+1}}\left(1-\mathrm{t_g}\right)\right] + (1-\mathrm{x_t})\mathrm{r}(1-\mathrm{t_o})\right\} \tag{B4}$$

$$\rho_{R,t+1} = \{1 + \mathrm{z_t}(\mathrm{d} + \mathrm{g_{t+1}}) + (1-\mathrm{z_t})\mathrm{r}\} \tag{B5}$$

are the gross after-tax growth rates in taxable (non-retirement) wealth and retirement wealth, respectively, between dates t and t+1.

The control (choice) variables are the fraction of taxable wealth and retirement wealth invested in equity at each date: $\{\mathrm{x_t}, \mathrm{z_t}\}$, t = 1, …, T-1. The state variables that are relevant for solving this intertemporal investment problem are not unique. There are a variety of state variable combinations that provide the same relevant information. We have chosen to use the fraction of financial wealth invested in the taxable account, $\mathrm{w_{NR,t}}$, and the present value of future labor income as a fraction of total wealth, l_tA_t, as our state variables.[28] Of course, the number of years prior to the retirement date, T-t, is also a relevant state variable for the intertemporal investment problem.

[28] The advantage of using $\mathrm{w_{NR,t}}$ and l_tA_t as the normalized state variables is that they both have natural lower and upper bounds of 0% and 100%, which facilitates the implementation of our numerical methodology.

The objective is to find the optimal holdings of equity in both the taxable and retirement accounts at each date t = 0,1, …, T-1. Since the investment choices at date t affects the future state variables, it is necessary to use dynamic programming to solve the intertemporal investment problem. However, because we do not have an analytical solution, we must use numerical methods. To facilitate the implementation, we assume that equity returns follow a lognormal distribution and that the individual has constant relative risk averse preferences.

To conduct our numerical solutions, we must discretize the state variables $w_{NR,t}$ and $l_t A_t$. The state variable $w_{NR,t}$ is discretized by assuming 5% increments between 0% and 100%. The state variable $l_t A_t$ is discretized by assuming 2% increments between 0% and 90%, and 1% increments between 90% and 100%. The denser grid for $l_t A_t$ is needed because there is more variation in the value function in that dimension, especially near the boundary where $l_t A_t = 100\%$. This, then, gives us a 21 x 56 state variable grid.

We begin by solving the optimization problem at date T-1 and working backward period-by-period until we have solved for the optimal equity holdings and value functions for each date t = 0, 1, …, T-1. The dynamic programming approach accounts for the fact that current portfolio decisions affect future state variables. However, because there is no guarantee that the realization of returns in the current period will produce future state variables that lie on the discretized grid, we also need to utilize interpolation methods to find solutions that lie between grid points.

We formulate the intertemporal investment problem in Excel and utilize the VBA programming feature to design a Macro that finds the optimal equity allocations in the taxable and retirement accounts, as well as the resulting optimized value function, for each point on the 21x56 state-variable grid and for each date t = 0, 1, …, T. We use a simple bi-linear interpolation method to find optimal equity allocations and value functions for state variables that lie between grid points. Base-case parameter values are provided in **Table 1**.

There are points on the boundary of the 21x56 state variable grid where the optimal equity holdings for the taxable and retirement accounts are indeterminate. In particular, the optimal equity holdings in the taxable and retirement accounts are undefined when the present value of future labor income, $l_t A_t$, represents 100% of total wealth. Likewise, the optimal equity allocations in the retirement (taxable) account are undefined when the fraction of financial wealth held in the taxable account, w_{NR}, is 100% (0%). However, the optimal equity allocations at the boundary of the state variable grid are still important

because they are used for interpolation purposes. In assigning values for the optimal equity allocations at the boundary of the grid, we rely on economic arguments and the solutions at nearby grid points inside the boundary to ensure that there is a smooth transition to the boundary.[29]

[29] For example, although the optimal equity holdings in the taxable and retirement accounts are undefined when the present value of future labor income, $l_t A_t$, is 100% of total wealth, an economic argument can be made that supports using x_t^* = 100% and z_t^* = 100% when $l_t A_t$ = 100%. As $l_t A_t$ approaches 100%, so that the vast majority of the individual's total wealth is composed of risk-free future labor income, the optimal holding of equity in both the taxable and retirement accounts should also approach 100% to provide the individual with some risk exposure.