Capital Accumulation, Markups, and Trade Imbalances*

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Abstract

We study how changes in productivity and trade costs affect welfare when countries accumulate capital and borrow and lend internationally, and firms charge markups. We quantify a multi-country, multi-sector model with sector-specific capital accumulation and input-output linkages, and compare it with a static benchmark with fixed aggregate capital and balanced trade. We find that the productivity and trade barrier changes inferred over 2000–2014 generate consumption-equivalent welfare gains of more than 150 percent for China in the dynamic model, compared with around 100 percent in the static model. We find that aggregate capital accumulation generates most of the additional 50 percentage points by expanding future production and sales. Profit shifting accounts for two-thirds of the difference.

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1 Introduction

Changes in productivity and trade barriers shift production and sales across countries. When firms charge prices above marginal cost, profit income from these changes in sales, or profit shifting, affects how welfare is distributed across countries. Investment extends this effect over time because capital installed today changes future production and sales. Understanding profit shifting, therefore, requires accounting for how capital accumulates following changes in productivity and trade barriers.

This paper studies how capital accumulation and international borrowing and lending shape profit shifting and welfare following changes in productivity and trade barriers. We develop and quantify a dynamic multi-country, multi-sector model in which households accumulate sector-specific capital subject to adjustment costs and borrow and lend internationally. We compare the welfare effects with those from a static model that holds aggregate capital fixed and imposes balanced trade.

We begin with a two-country, two-sector model that isolates profit shifting. An increase in Home's productivity in one sector lowers its export price in that sector and raises its wage and export price in the other sector. Buyers shift expenditure toward Home producers, increasing the profits received by Home households. At the same time, the changes in export prices worsen Home's terms of trade. The model, therefore, separates the gain from profit shifting from the loss through the terms of trade.

The stylized model also shows how capital accumulation carries profit shifting into future periods. When the increase in productivity is anticipated, the higher expected return to capital induces investment in the affected sector. After productivity returns to its initial path, the additional capital continues to lower marginal cost and sustain production and sales. Its sector specificity keeps the capital in the affected sector and preserves the increase in production.

The quantitative model extends the stylized setting to multiple countries and sectors connected through domestic and international input-output linkages. Households accumulate sector-specific capital in every period subject to adjustment costs and trade a one-period international bond. International borrowing and lending allow domestic investment to differ from domestic saving and, therefore, affect the path of capital accumulation. Sectoral demand elasticities generate heterogeneous markups.

We map the model to data and calibrate it to a global economy comprising the United States, China, India, the European Union, and the rest of the world, each with eight sectors. We use the World Input-Output Database for production, expenditure, bilateral sourcing, input-output linkages, and sectoral capital stocks over 2000–2014 (Timmer et al., 2015). Observed net external positions from the External Wealth of Nations database initialize the model's international bond positions (Lane and Milesi-Ferretti, 2018). External estimates determine depreciation rates and trade elasticities, while simulated method of moments estimates capital adjustment costs and intertemporal preference wedges.

Using annual producer prices, input costs, and bilateral expenditure shares, we infer changes in productivity and bilateral trade barriers over 2000–2014. We compare this path with a reference path that assigns the average productivity growth rate in the world to every country and sector and holds bilateral trade barriers at their levels in 2000. Counterfactual permanent and temporary increases in Chinese manufacturing productivity and a permanent reduction in manufacturing trade barriers isolate the mechanisms.

For China, the dynamic model raises the consumption-equivalent welfare effect of the changes in productivity and trade barriers inferred over 2000–2014 by 49.5 percentage points, from 106.6 percent in the static model to 156.1 percent. Profit shifting contributes 32.4 percentage points, or 65.6 percent, of the difference between the dynamic and static models. Capital accumulation expands future production and sales, increasing the profit income received by Chinese households.

Counterfactual changes in productivity show that aggregate capital accumulation generates most of this difference. Under a permanent 10 percent increase in Chinese manufacturing productivity, the difference between dynamic and static welfare gains is 1.12 percentage points when international borrowing and lending are allowed, falling modestly to 1.05 percentage points under financial autarky. Sector specificity contributes less to lifetime welfare but determines where capital accumulates, while adjustment costs govern the pace of adjustment. When a ten-year increase in productivity ends, capital in the affected sector remains 11.0 percent above its reference level, output remains 2.6 percent higher, and export quantity remains 5.8 percent higher.

International borrowing and lending dampen domestic investment after temporary increases in productivity but support it after a permanent increase. After the ten-year increase, China lends part of its additional saving internationally, reducing capital accumulation relative to financial autarky and lowering the welfare effect by 0.13 percentage point. After the permanent increase, China borrows to finance investment, raising the

welfare effect by 0.04 percentage point.

Changes in trade barriers also generate profit shifting. A permanent 10 percent reduction in manufacturing trade barriers raises welfare in every economy. Direct resource savings account for most of the gains, while profit shifting contributes between 20.7 and 35.5 percent. Profit shifting, therefore, matters for the welfare effects of changes in both productivity and trade barriers.

The results also show that profit shifting depends more on a common markup than on differences in markups across sectors. A positive common markup is sufficient for changes in sales to redistribute profit income across countries. In the counterfactual increase in Chinese manufacturing productivity, sectoral markup differences reduce profit shifting because production expands relatively more in sectors with smaller markup wedges. These differences have little effect on world welfare; their larger effects for individual countries reflect the distribution of profit income.

Related Literature The paper builds on research that decomposes the gains from trade and trade policy under imperfect competition. Import tariffs can affect welfare through the terms of trade and profit shifting (Ossa, 2014), while subsequent work studies scale economies, markups, and production relocation (Lashkaripour and Lugovskyy, 2023; Firooz and Heins, 2026). Research on allocative efficiency evaluates production changes at price-cost wedges (Holmes et al., 2014; Baqaee and Farhi, 2020). We show how capital accumulation changes future production and how the resulting profit income is distributed across countries.

The paper also contributes to quantitative dynamic models of open economies with capital accumulation, trade imbalances, and transition dynamics (Dekle et al., 2007; Reyes-Heroles, 2016; Kehoe et al., 2018; Anderson et al., 2019; Ravikumar et al., 2019; Dix-Carneiro et al., 2022; Kleinman et al., 2026). Boehm et al. (2026) study adjustment through destination-specific entry in a dynamic Krugman model and find that international borrowing and lending mainly change the timing of the gains from trade. Our emphasis on sector-specific capital and heterogeneous sectoral markups instead connects trade imbalances to capital accumulation across countries and the profit income received by households.

Section 2 develops the stylized model. Section 3 presents the quantitative model and welfare decomposition. Section 4 maps the model to the data. Section 5 presents the quantitative analysis, and Section 6 concludes.

2 Stylized Model

2.1 Environment

There are two symmetric countries, Home and Foreign, indexed by $n \in \{h, f\}$, two sectors, $j \in \{1, 2\}$, and three dates, $t \in \{0, 1, 2\}$. Sector 1 uses capital and labor, whereas sector 2 uses labor only. Country n has a fixed labor endowment L_n and a sector-1 capital stock $k^1_{n,0}$ at date 0. Labor moves freely across sectors within each country but not across countries. We assume there are no international trade costs.

At date 0, households learn that Home's sector-1 productivity will rise at date 1 and return to its initial level at date 2. Households invest only at date 0, and capital depreciates at rate $\delta \in (0, 1)$.[1]

Final and Sectoral Goods The final consumption good is

$$C_{n,t} = \prod_{j \in \{1,2\}} \left(\frac{C^j_{n,t}}{a^j} \right)^{a^j},$$

where $C^j_{n,t}$ is consumption of the sector-j composite and $a^j > 0$ is its expenditure share, with $a^1 + a^2 = 1$. Sectoral consumption is

$$C^j_{n,t} = \left[\sum_{m \in \{h,f\}} \omega^{1/\sigma}_{mn} \left(c^j_{mn,t} \right)^{(\sigma-1)/\sigma} \right]^{\sigma/(\sigma-1)},$$

where $c^j_{mn,t}$ is country n's consumption of country m's sector-j variety, $\omega_{mn} > 0$ is its demand weight, and $\sigma > 1$ is the elasticity of substitution.[2] Let $\theta \equiv \sigma - 1$ denote the trade elasticity.

Investment uses the same final good. Let $P_{n,t}$ denote its price and $q^j_{mn,t}$ country n's demand for country m's sector-j variety for consumption and investment. Expenditure minimization subject to these aggregation technologies determines the price indices and variety demands.

[1] This timing isolates the effects of capital accumulated before the productivity increase. Section 3 allows investment in every period, convex capital adjustment costs, and international input–output linkages. Appendix A.1 provides the derivations and proofs for the stylized model.

[2] Bilateral variables are indexed by origin and then destination.

Production A sector-1 firm takes the wage $w_{n,t}$ and capital rental rate $r^1_{n,t}$ as given and minimizes the cost of producing $y^1_{n,t}$:

$$\min_{k^1_{n,t},\ell^1_{n,t}} \quad r^1_{n,t}k^1_{n,t} + w_{n,t}\ell^1_{n,t} \quad \text{s.t.} \quad A^1_{n,t}\left(k^1_{n,t}\right)^{\alpha}\left(\ell^1_{n,t}\right)^{1-\alpha} \geq y^1_{n,t}, \tag{1}$$

where $\ell^1_{n,t}$ is labor input, $A^1_{n,t}$ is productivity, and $\alpha \in (0,1)$ is capital's output elasticity. The resulting marginal cost is $\text{mc}^1_{n,t} = \frac{1}{A^1_{n,t}}\left(\frac{r^1_{n,t}}{\alpha}\right)^{\alpha}\left(\frac{w_{n,t}}{1-\alpha}\right)^{1-\alpha}$. Sector 2 produces $y^2_{,t} = \ell^2_{n,t}$, where $\ell^2_{n,t}$ is its labor input, and has marginal cost $\text{mc}^2_{n,t} = w_{n,t}$.

Profit-maximizing firms charge markups setting the price $p^j_{nm,t} = \mu^j \text{mc}^j_{n,t}$ in destination m, where μ^j is the gross markup. Sector 1 operates under monopolistic competition with $\mu^1 = \sigma/(\sigma-1) = 1 + 1/\theta$, while sector 2 is competitive with $\mu^2 = 1$.

Revenue and Profits Country n's sector-j revenue is $R^j_{n,t} = \sum_{m\in\{h,f\}} X^j_{nm,t}$, where $X^j_{nm,t} \equiv p^j_{nm,t}q^j_{nm,t}$ is expenditure in country m on its variety. Sector-1 profits are $\Pi^1_{n,t} = \varrho^1 R^1_{n,t}$, where $\varrho^1 \equiv 1 - 1/\mu^1$ is the profit share of revenue, while sector 2 earns zero profits.

Households The representative household owns domestic firms and capital and trades a one-period international bond. Each bond pays one unit of a reference country's final good, which serves as the numeraire: $P_{n^*,t} = 1$, where $n^* \in \{h,f\}$ denotes the reference country. Given its initial capital stock $k^1_{n,0}$, the household solves the following problem to maximize the life-time utility U_n:

$$\begin{aligned} U_n = \max_{\substack{\{C_{n,t}\}_{t=0}^2, i^1_{n,0},\\ b_{n,1}, b_{n,2}}} \quad & \sum_{t=0}^{2}\beta^t \log C_{n,t} \\ \text{s.t.} \quad & P_{n,t}\left(C_{n,t} + i^1_{n,t}\right) + Q_t b_{n,t+1} \\ & \qquad = w_{n,t}L_n + r^1_{n,t}k^1_{n,t} + \Pi^1_{n,t} + b_{n,t}, \quad t \in \{0,1,2\}, \\ & k^1_{n,1} = (1-\delta)k^1_{n,0} + i^1_{n,0}, \quad k^1_{n,2} = (1-\delta)k^1_{n,1}, \\ & i^1_{n,t} = 0, \quad t \in \{1,2\}, \quad b_{n,0} = 0, \quad b_{n,3} = 0, \end{aligned} \tag{2}$$

where $\beta \in (0,1)$ is the discount factor, $i^1_{n,t}$ is investment, $b_{n,t+1}$ is the number of bonds purchased at date t, and Q_t is the bond price.[3]

Equilibrium An equilibrium consists of prices and allocations that satisfy household optimization, firm cost minimization and pricing, and market clearing. Goods markets clear when $y^j_{n,t} = \sum_{m\in\{h,f\}} q^j_{nm,t}$, labor markets when $\ell^1_{n,t} + \ell^2_{n,t} = L_n$, and the international bond market when $\sum_{n\in\{h,f\}} b_{n,t} = 0$. Capital employed in sector 1 equals the stock supplied by domestic households.

2.2 Welfare

We suppress date subscripts, write $\bar{n}$ for country n's trading partner, and use $+1$ to denote the next date. Appendix A.1.4 derives the following welfare decomposition from the household budget, expenditure minimization, and firm cost minimization and pricing:

$$\begin{aligned} P_n C_n \,\mathrm{d}u_n = {} & \underbrace{V^1_n \,\mathrm{d}\log A^1_n}_{\text{Productivity}} + \underbrace{r^1_n \,\mathrm{d}k^1_n - P_n \,\mathrm{d}i^1_n}_{\text{Capital accumulation}} + \underbrace{\mathrm{d}\left(b_n - Q b_{n,+1}\right)}_{\text{Net bond receipts}} \\ & + \underbrace{\sum_{j\in\{1,2\}} \left(X^j_{n\bar{n}} \,\mathrm{d}\log \mathrm{mc}^j_n - X^j_{\bar{n}n} \,\mathrm{d}\log \mathrm{mc}^j_{\bar{n}} \right)}_{\equiv \mathrm{d}\mathcal{W}^{\mathrm{ToT}}_n \text{ (terms of trade)}} \\ & + \underbrace{\varrho^1 \left(p^1_{nn} \,\mathrm{d}q^1_{nn} + p^1_{n\bar{n}} \,\mathrm{d}q^1_{n\bar{n}} \right)}_{\equiv \mathrm{d}\mathcal{W}^{\Pi,1}_n \text{ (profit shifting)}}, \end{aligned} \tag{3}$$

where $\mathrm{d}u_n \equiv \mathrm{d}\log C_n$ denotes the change in period utility, $V^1_n \equiv R^1_n/\mu^1$ is sector-1 variable cost.[4] Lifetime welfare satisfies $\mathrm{d}U_n = \sum_{t=0}^{2} \beta^t \mathrm{d}u_{n,t}$.[5]

2.3 Terms of Trade and Profit Shifting

We evaluate changes around a symmetric balanced-trade allocation, holding relative expenditure fixed: $\mathrm{d}\log E_h - \mathrm{d}\log E_f = 0$, where $E_n \equiv P_n(C_n + i^1_n)$ denotes total final

[3] Investment reduces current consumption possibilities and generates future capital income. International borrowing and lending allow households to finance investment and smooth consumption.

[4] Fixed markups imply $\mathrm{d}\log p^j_{nm} = \mathrm{d}\log \mathrm{mc}^j_n$, so the terms-of-trade contribution values export and import price changes at initial quantities. The profit-shifting contribution satisfies $\mathrm{d}\mathcal{W}^{\Pi,1}_n = \mathrm{d}\Pi^1_n - \Pi^1_n \,\mathrm{d}\log \mathrm{mc}^1_n$ and measures the price-cost gap on changes in domestic and export sales.

[5] Each term on the right-hand side of equation (3) is measured in units of the reference country's final good. Dividing by $P_n C_n$ gives its contribution to period utility.

expenditure. At this allocation, the demand weights imply the same domestic expenditure share $s \in (1/2, 1)$ in both sectors. Each country has sector-j revenue R^j and exports $X^{j,x} \equiv (1-s)R^j$.

Define Home's relative sector-1 cost advantage as $\Delta \equiv \mathrm{d}\log \mathrm{mc}_f^1 - \mathrm{d}\log \mathrm{mc}_h^1 > 0$. Demand implies $\mathrm{d}\log R_h^1 - \mathrm{d}\log R_f^1 = \eta\Delta$, where $\eta \equiv 4s(1-s)\theta$. Labor-market clearing gives $\widetilde{w} = \lambda_L^1 \eta\Delta/[1 + (1-\lambda_L^1)\eta] > 0$, where $\widetilde{w} \equiv \mathrm{d}\log w_h - \mathrm{d}\log w_f$ is the relative wage change and λ_L^1 is sector 1's initial share of labor income.[6] Define relative profit shifting as the deviation from the two-country average:

$$\mathrm{d}\mathcal{W}_n^{\Pi,1,\mathrm{rel}} \equiv \frac{\mathrm{d}\mathcal{W}_n^{\Pi,1} - \mathrm{d}\mathcal{W}_{\bar{n}}^{\Pi,1}}{2}. \tag{4}$$

Proposition 1 (Relative profit shifting outweighs the terms-of-trade loss)**.** *Suppose $s > 1/2$ and $\mu^1 = 1 + 1/\theta$, with $\theta > 0$. At the symmetric balanced-trade allocation, Home's sector-1 cost advantage $\Delta > 0$ worsens its terms of trade but generates a relative profit-shifting gain that more than offsets the loss, provided relative expenditure remains fixed.:*

$$\begin{aligned} \mathrm{d}\mathcal{W}_h^{\mathrm{ToT}} &= -\overbrace{X^{1,x}\Delta}^{\text{Sector 1}} + \overbrace{X^{2,x}\widetilde{w}}^{\text{Sector 2}} < 0, \\ \mathrm{d}\mathcal{W}_h^{\Pi,1,\mathrm{rel}} &= \frac{\varrho^1 R^1}{2}(1+\eta)\Delta > -\mathrm{d}\mathcal{W}_h^{\mathrm{ToT}} > 0. \end{aligned} \tag{5}$$

Foreign's terms-of-trade and relative profit-shifting contributions are equal in magnitude and opposite in sign.

Appendix A.1.7 provides the proof of Proposition 1. Proposition 1 establishes that the sum of the terms-of-trade and relative profit-shifting contributions is positive for Home and negative for Foreign. Profit shifting includes changes in domestic and export sales, whereas the terms-of-trade contribution applies only to international trade. Under the stated home-bias and markup assumptions, Home's profit-shifting contribution exceeds the two-country average by more than its terms-of-trade loss.[7]

[6] The labor-income share is $\lambda_L^1 = [(1-\alpha)R^1/\mu^1]/[(1-\alpha)R^1/\mu^1 + R^2]$. Home's lower relative sector-1 marginal cost raises its relative revenue and wage. The wage increase raises the relative price of its sector-2 exports.

[7] This result does not determine either country's total welfare change, which also includes the common profit-shifting contribution and the remaining terms in equation (3).

2.4 Persistence after a Temporary Increase in Productivity

Proposition 1 shows that a relative sector-1 cost advantage generates a relative profit-shifting gain that exceeds the terms-of-trade loss. Investment in anticipation of a temporary productivity increase can preserve this cost advantage after productivity returns to baseline.

At date 0, households learn that Home's sector-1 productivity will rise at date 1 and return to baseline at date 2. Denote the proportional productivity increase by $\widehat{A}^1_{h,1} = \varepsilon > 0$, where $\widehat{x} \equiv \mathrm{d}\log x$ denotes the proportional change in a positive variable x. This anticipated increase raises Home's capital stock relative to Foreign's, so $\widetilde{k}^1 \equiv \widehat{k}^1_{h,1} - \widehat{k}^1_{f,1} > 0$.[8]

Common proportional depreciation preserves this relative capital difference at date 2: $\widehat{k}^1_{h,2} - \widehat{k}^1_{f,2} = \widetilde{k}^1$. With relative date-2 expenditure fixed, the remaining capital difference generates a marginal-cost advantage $\Delta_2 = \lambda_K \widetilde{k}^1 > 0$, where $\lambda_K > 0$ measures the effect of relative capital on Home's marginal cost.[9]

Corollary 1 (Persistence through capital accumulation)**.** *The anticipated productivity increase $\varepsilon > 0$ generates a positive relative capital response $\widetilde{k}^1 > 0$. With relative date-2 expenditure fixed, Home's terms-of-trade and relative profit-shifting contributions, after productivity levels return to baseline levels, satisfy*

$$\mathrm{d}\mathcal{W}^{\mathrm{ToT}}_{h,2} = \left[-X^{1,x} + X^{2,x}\frac{\lambda^1_L \eta}{1+(1-\lambda^1_L)\eta}\right]\underbrace{\lambda_K \widetilde{k}^1}_{\Delta_2} < 0,$$

$$\mathrm{d}\mathcal{W}^{\Pi,1,\mathrm{rel}}_{h,2} = \frac{\varrho^1 R^1}{2}(1+\eta)\lambda_K \widetilde{k}^1 > -\mathrm{d}\mathcal{W}^{\mathrm{ToT}}_{h,2} > 0, \tag{6}$$

where all coefficients are evaluated at the symmetric date-2 allocation.

Appendix A.1.8 provides the proof of Corollary 1. The corollary identifies capital accumulation as a source of persistence in the welfare effects characterized by Proposition 1: investment before the productivity increase sustains those effects after productivity returns to baseline, which would be omitted in a static model with fixed capital.

[8] The household's investment condition is $P_{n,0} = Q_0 r^1_{n,1} + (1-\delta)Q_0 Q_1 r^1_{n,2}$. Appendix A.1.6 derives the positive relative capital response from this condition and the household budget constraint.

[9] At the symmetric date-2 allocation, $\lambda_K = \alpha/\{1+\alpha\eta+(1-\alpha)\lambda^1_L\eta/[1+(1-\lambda^1_L)\eta]\}$. Home's larger relative capital stock lowers its relative rental rate and marginal cost after productivity returns to baseline.

3 Model

We extend the stylized setting to multiple countries and sectors with input-output linkages, sector-specific capital, and international borrowing and lending. Appendix A.2 derives the demand, cost, and pricing conditions. Appendix A.7 provides the solution method.

3.1 Setting

Time is discrete, $t \in \{0, 1, \ldots\}$. Countries are indexed by $n, m \in \mathcal{N}$, and sectors by $j, k \in \mathcal{J}$. Each sector in each country has a unit mass of symmetric firms that compete monopolistically and produce differentiated varieties. Under symmetry, we suppress the firm index.[10]

Country n has a fixed labor endowment L_n, which households supply inelastically. Households own domestic firms and capital and trade a one-period risk-free bond internationally. Labor moves across sectors within a country, while capital is specific to each sector within a country. Households and firms have perfect foresight over the paths of productivity and trade barriers.

3.2 Demand

Final Good Let $Z_{n,t}$ denote the final good produced in country n, and let $Z^j_{n,t}$ denote the amount of the sector-j composite used in its production. The parameter $\alpha^{F,j}_n > 0$ is the expenditure share of sector j, with $\sum_j \alpha^{F,j}_n = 1$. Competitive final-good producers use the Cobb–Douglas technology

$$Z_{n,t} = \prod_{j \in \mathcal{J}} \left(Z^j_{n,t} \right)^{\alpha^{F,j}_n}. \tag{7}$$

Let $P_{n,t}$ denote the price of the final good and $P^j_{n,t}$ the price of the sector-j composite. Cost minimization gives

$$P_{n,t} = \prod_{j \in \mathcal{J}} \left(\frac{P^j_{n,t}}{\alpha^{F,j}_n} \right)^{\alpha^{F,j}_n} \quad \text{and} \quad P^j_{n,t} Z^j_{n,t} = \alpha^{F,j}_n P_{n,t} Z_{n,t}. \tag{8}$$

[10] The fixed firm mass allows operating profits to accrue to households; free entry would require entry and fixed costs.

Consumption, investment, and capital adjustment use the final good. Nominal expenditure on the final good is $F_{n,t} \equiv P_{n,t} Z_{n,t}$.

Sectoral Goods Sectoral goods are used in final demand and as intermediate inputs. Let $Q^j_{n,t}$ denote the sector-j composite available in country n, and let $q^j_{m,n,t}$ denote its quantity of the variety produced in country m. The demand weight on that variety is $\gamma^j_{m,n} > 0$, and the elasticity of substitution across origins is $\sigma^j > 1$. The sectoral composite is

$$Q^j_{n,t} = \left[\sum_{m \in \mathcal{N}} \left(\gamma^j_{m,n} \right)^{1/\sigma^j} \left(q^j_{m,n,t} \right)^{(\sigma^j - 1)/\sigma^j} \right]^{\sigma^j/(\sigma^j - 1)}, \tag{9}$$

Final-good producers and firms purchase the same sector-j composite for final demand and intermediate use.

Variety Demand Let $p^j_{m,n,t}$ denote the price in country n of the variety produced in country m. Cost minimization gives the sectoral price index and demand for each variety:

$$\begin{aligned} P^j_{n,t} &= \left[\sum_{m \in \mathcal{N}} \gamma^j_{m,n} \left(p^j_{m,n,t} \right)^{1-\sigma^j} \right]^{1/(1-\sigma^j)} \\ q^j_{m,n,t} &= \gamma^j_{m,n} \left(\frac{p^j_{m,n,t}}{P^j_{n,t}} \right)^{-\sigma^j} Q^j_{n,t}. \end{aligned} \tag{10}$$

If $X^j_{n,t} = P^j_{n,t} Q^j_{n,t}$ is country n's total expenditure on sector j, bilateral expenditure is $X^j_{m,n,t} = p^j_{m,n,t} q^j_{m,n,t}$.

3.3 Production

Let $Y^j_{m,t}$ denote output of country m, sector j, and let $A^j_{m,t}$ denote its productivity. Production uses labor $L^j_{m,t}$, sector-specific capital $K^j_{m,t}$, and the sector-k composite $M^{j,k}_{m,t}$ as an intermediate input. The corresponding output elasticities are ℓ^j_m, κ^j_m, and $\omega^{j,k}_m$, and they sum to one. Each firm has the constant-returns Cobb–Douglas production function

$$Y^j_{m,t} = A^j_{m,t} \left(L^j_{m,t} \right)^{\ell^j_m} \left(K^j_{m,t} \right)^{\kappa^j_m} \prod_{k \in \mathcal{J}} \left(M^{j,k}_{m,t} \right)^{\omega^{j,k}_m}. \tag{11}$$

The constant-returns restriction is $\ell^j_m + \kappa^j_m + \sum_k \omega^{j,k}_m = 1$.

Let $w_{m,t}$ denote the wage, $r^{K,j}_{m,t}$ the rental rate of sector-j capital, and $\text{mc}^j_{m,t}$ marginal cost in country m, sector j. Firms choose inputs to minimize variable cost at these factor prices and the intermediate-input prices $\{P^k_{m,t}\}_{k\in\mathcal{J}}$.[11] Constant returns imply that variable cost equals $\text{mc}^j_{m,t}Y^j_{m,t}$, and cost minimization gives

$$\text{mc}^j_{m,t} = \frac{1}{A^j_{m,t}}\left(\frac{w_{m,t}}{\ell^j_m}\right)^{\ell^j_m}\left(\frac{r^{K,j}_{m,t}}{\kappa^j_m}\right)^{\kappa^j_m}\prod_{k\in\mathcal{J}}\left(\frac{P^k_{m,t}}{\omega^{j,k}_m}\right)^{\omega^{j,k}_m}. \tag{12}$$

The production elasticities are also the shares of variable cost paid to labor, capital, and each intermediate input. Appendix A.2 reports the conditional input demands.

3.4 International Trade, Price Setting, and Profits

Shipping a sector-j good from country m to country n requires $\tau^j_{m,n,t} \geq 1$ units at the origin for one unit to arrive, with $\tau^j_{n,n,t} = 1$. The marginal cost of supplying country n is therefore $\tau^j_{m,n,t}\text{mc}^j_{m,t}$.

A firm takes the sectoral price index and expenditure in each destination as given and chooses a destination-specific price to maximize operating profits. Demand in equation (10) has elasticity $-\sigma^j$. Profit maximization gives

$$p^j_{m,n,t} = \underbrace{\mu^j}_{\text{gross markup}} \times \underbrace{\tau^j_{m,n,t}\text{mc}^j_{m,t}}_{\text{delivered marginal cost}}. \tag{13}$$

The gross markup is $\mu^j \equiv \sigma^j/(\sigma^j - 1)$. It is constant within a sector because demand has a constant elasticity and differs across sectors through σ^j. Defining the trade elasticity as $\theta^j \equiv \sigma^j - 1$ gives $\mu^j = 1 + 1/\theta^j$ (Armington, 1969; Dixit and Stiglitz, 1977).[12]

Substituting equation (13) into demand gives the bilateral expenditure share

$$\pi^j_{m,n,t} \equiv \frac{X^j_{m,n,t}}{X^j_{n,t}} = \frac{\gamma^j_{m,n}\left[\tau^j_{m,n,t}\text{mc}^j_{m,t}\right]^{-\theta^j}}{\sum_{o\in\mathcal{N}}\gamma^j_{o,n}\left[\tau^j_{o,n,t}\text{mc}^j_{o,t}\right]^{-\theta^j}}. \tag{14}$$

Higher productivity and lower trade barriers reduce the cost of supplying a destination

[11] A factor with a zero cost share is omitted from equation (12).

[12] The first-order condition is $0 = q^j_{m,n,t} + (p^j_{m,n,t} - \tau^j_{m,n,t}\text{mc}^j_{m,t})\partial q^j_{m,n,t}/\partial p^j_{m,n,t}$. Substituting the demand elasticity $(\partial q^j_{m,n,t}/\partial p^j_{m,n,t})(p^j_{m,n,t}/q^j_{m,n,t}) = -\sigma^j$ gives the markup.

and raise that destination's expenditure share on the origin. The sectoral markup does not appear in equation (14) because it is common across origins within a sector.

Let $R^j_{m,t} = \sum_n X^j_{m,n,t}$ denote revenue earned by country m's sector-j firms, $V^j_{m,t}$ their variable cost, and $\Pi^j_{m,t}$ their operating profits. Output at the origin satisfies $Y^j_{m,t} = \sum_n \tau^j_{m,n,t} q^j_{m,n,t}$. Equation (13) then implies

$$V^j_{m,t} = \frac{R^j_{m,t}}{\mu^j} \quad \text{and} \quad \Pi^j_{m,t} = R^j_{m,t} - V^j_{m,t} = \left(1 - \frac{1}{\mu^j}\right) R^j_{m,t}. \tag{15}$$

The domestic household receives $\Pi^j_{m,t}$. The production cost shares allocate revenue net of profits across labor, capital, and intermediate inputs.

3.5 Households

The representative household in country n chooses consumption, sectoral investment, and a position in a one-period risk-free international bond. The bond is denominated in units of the final good of a reference country $n^* \in \mathcal{N}$. Let $C_{n,t}$ be nominal consumption expenditure and $C^R_{n,t} = C_{n,t}/P_{n,t}$ real consumption. Let $I^j_{n,t}$ denote investment in sector j, $K^j_{n,t}$ its installed capital stock, and $B_{n,t}$ the beginning-of-period international bond position. A positive $B_{n,t}$ is a claim on the rest of the world, while a negative position is a liability. Let $\beta \in (0,1)$ be the discount factor and $\xi_{n,t}$ an intertemporal preference wedge, with $\xi_{n,0} = 0$.

Given initial capital stocks and bond positions, the household chooses $C_{n,t}$, $I^j_{n,t}$, $K^j_{n,t+1}$, and $B_{n,t+1}$ at every date to maximize

$$\max \sum_{t=0}^{\infty} \beta^t \exp\left(\sum_{s=0}^{t} \xi_{n,s}\right) \log C^R_{n,t}. \tag{16}$$

The household maximizes equation (16) subject to the budget and capital accumulation constraints below, together with the no-Ponzi and capital transversality conditions. A positive intertemporal preference wedge raises the household's relative weight on future consumption.

The beginning-of-period bond position pays the gross return $\mathcal{R}_t$, and $B_{n,t+1}$ is expenditure on the position carried into the next period. Let $\delta_n \in (0,1)$ denote the depreciation rate and $\chi^j_{n,t}$ the capital adjustment cost measured in units of the final good. The

household budget constraint and the law of motion for sector-specific capital are

$$\begin{aligned} C_{n,t} + P_{n,t} \sum_{j \in \mathcal{J}} \left(I^j_{n,t} + \chi^j_{n,t} \right) + B_{n,t+1} &= w_{n,t} L_n + \sum_{j \in \mathcal{J}} r^{K,j}_{n,t} K^j_{n,t} + \sum_{j \in \mathcal{J}} \Pi^j_{n,t} + \mathcal{R}_t B_{n,t}, \\ K^j_{n,t+1} &= (1 - \delta_n) K^j_{n,t} + I^j_{n,t}. \end{aligned} \tag{17}$$

The capital stock $K^j_{n,t}$ is predetermined at date t. Investment chosen at that date determines the next-period stock in the same country-sector pair, and installed capital cannot be transferred directly to another sector. Capital is sector-specific, so reallocation requires new investment, while the adjustment cost below makes that reallocation gradual.

Installing capital uses the final good. Let $\phi_n \geq 0$ govern the cost of changing investment relative to the capital stock. The adjustment cost is

$$\chi^j_{n,t} = \frac{\phi_n}{2} \left(\frac{I^j_{n,t}}{K^j_{n,t}} - \delta_n \right)^2 K^j_{n,t}, \tag{18}$$

Household optimization equates the intertemporal return on the bond with the return on sectoral investment. A higher expected rental rate raises current investment, while adjustment costs spread the response over time. Access to the international bond allows the timing of consumption to differ from the timing of domestic income and investment. Appendix A.2 reports the corresponding Euler equations.

Let $\lambda^{K,j}_{n,t} \equiv 1 + \phi_n (I^j_{n,t} / K^j_{n,t} - \delta_n)$ denote the marginal installation cost of sector-j capital. For a terminal horizon H, the bond and capital paths satisfy the following conditions as H tends to infinity:

$$\begin{aligned} \lim_{H \to \infty} \frac{B_{n,H+1}}{\prod_{s=1}^{H} \mathcal{R}_s} &\geq 0, \\ \lim_{H \to \infty} \beta^H \exp\left(\sum_{s=0}^{H} \xi_{n,s} \right) \frac{P_{n,H} \lambda^{K,j}_{n,H} K^j_{n,H+1}}{C_{n,H}} &= 0 \qquad \text{for every } j \in \mathcal{J}. \end{aligned} \tag{19}$$

The first condition is the no-Ponzi condition, and the second is the transversality condition for sector-specific capital.

3.6 Market Clearing and Equilibrium

Household choices imply $F_{n,t} = C_{n,t} + P_{n,t}\sum_{j\in\mathcal{J}}(I^j_{n,t} + \chi^j_{n,t})$. Sectoral expenditure combines final demand with intermediate-input demand:

$$X^j_{n,t} = \alpha^{F,j}_n F_{n,t} + \sum_{k\in\mathcal{J}} \omega^{k,j}_n \frac{R^k_{n,t}}{\mu^k}. \tag{20}$$

Revenue equals sales to all destinations. Labor clears within each country, and demand for capital services equals the installed sectoral stock:

$$R^j_{m,t} = \sum_{n\in\mathcal{N}} \pi^j_{m,n,t} X^j_{n,t}, \qquad \sum_{j\in\mathcal{J}} L^j_{n,t} = L_n, \qquad \kappa^j_n \frac{R^j_{n,t}}{\mu^j} = r^{K,j}_{n,t} K^j_{n,t}. \tag{21}$$

The global bond market clears, $\sum_n B_{n,t+1} = 0$. Let $TB_{n,t}$ denote country n's trade balance, defined as the value of its exports less the value of its imports:

$$TB_{n,t} = \sum_j \sum_{d\neq n} X^j_{n,d,t} - \sum_j \sum_{o\neq n} X^j_{o,n,t}. \tag{22}$$

Combining market clearing with the household budget gives $TB_{n,t} = B_{n,t+1} - \mathcal{R}_t B_{n,t}$. A trade surplus increases a country's international bond position, while a deficit reduces that position or increases its bond liabilities.

Definition 1 (Sequential equilibrium)**.** The fixed parameters are the discount factor, final-expenditure shares, demand weights and elasticities, production elasticities, depreciation rates, and capital adjustment-cost parameters. Given these parameters, initial capital stocks $\{K^j_{n,0}\}_{n\in\mathcal{N},j\in\mathcal{J}}$, initial international bond positions $\{B_{n,0}\}_{n\in\mathcal{N}}$, labor endowments $\{L_n\}_{n\in\mathcal{N}}$, exogenous sequences $\{A^j_{n,t}, \tau^j_{m,n,t}, \xi_{n,t}\}_{t\geq 0}$, a sequential equilibrium is a sequence of the prices and allocations defined above that satisfies the following conditions at every date and for all countries and sectors:

1. Final-good and sectoral-good producers minimize expenditure subject to equations (7) and (9). Their optimal demands and price indices satisfy equations (8) and (10).

2. Firms minimize variable cost subject to equation (11) and maximize profits in each destination subject to variety demand. Marginal costs, conditional input demands, destination prices, and profits satisfy equations (12), (A.68), (13), and (15), with

$\Upsilon^j_{m,t} = \sum_{n \in \mathcal{N}} \tau^j_{m,n,t} q^j_{m,n,t}$.

3. Households maximize equation (16) subject to equation (17), the adjustment cost in equation (18), and the no-Ponzi and transversality conditions in equation (19). Their choices satisfy the Euler equations (A.69).

4. Real and nominal quantities are mutually consistent, so $C^R_{n,t} = C_{n,t}/P_{n,t}$, $F_{n,t} = P_{n,t} Z_{n,t} = C_{n,t} + P_{n,t} \sum_{j \in \mathcal{J}} (I^j_{n,t} + \chi^j_{n,t})$, $X^j_{m,n,t} = p^j_{m,n,t} q^j_{m,n,t}$, and $X^j_{n,t} = P^j_{n,t} Q^j_{n,t}$. Bilateral expenditure shares satisfy equation (14).

5. Goods, labor, sector-specific capital, and the international bond clear according to equations (20) and (21) and $\sum_{n \in \mathcal{N}} B_{n,t+1} = 0$.

3.7 Welfare

For a sequential equilibrium $\mathcal{E}$, define the household's date-t welfare weight and the sum of these weights by

$$\varpi_{n,t} \equiv \beta^t \exp\left(\sum_{s=0}^{t} \xi_{n,s}\right) \quad \text{and} \quad \Omega_n \equiv \sum_{t=0}^{\infty} \varpi_{n,t} < \infty. \tag{23}$$

Country n's welfare is lifetime utility evaluated along the equilibrium path:

$$W_n(\mathcal{E}) = \sum_{t=0}^{\infty} \varpi_{n,t} \log C^R_{n,t}(\mathcal{E}) \quad \text{and} \quad C^R_{n,t}(\mathcal{E}) = \frac{C_{n,t}(\mathcal{E})}{P_{n,t}(\mathcal{E})}. \tag{24}$$

Welfare therefore depends on the entire path of real consumption, not only its value in a particular period. The timing of consumption matters through the weights in equation (23).

Consider two equilibria, $\mathcal{E}^0$ and $\mathcal{E}^1$, with the same welfare weights. Define their welfare difference and its consumption-equivalent representation by

$$\begin{aligned} \Delta W_n(\mathcal{E}^1, \mathcal{E}^0) &\equiv W_n(\mathcal{E}^1) - W_n(\mathcal{E}^0), \\ \mathcal{CE}_n(\mathcal{E}^1, \mathcal{E}^0) &\equiv 100 \left[\exp\left(\frac{\Delta W_n(\mathcal{E}^1, \mathcal{E}^0)}{\Omega_n}\right) - 1\right]. \end{aligned} \tag{25}$$

The consumption equivalent is the uniform percentage change in real consumption at every date along $\mathcal{E}^0$ that gives the household the same welfare change as moving to $\mathcal{E}^1$. It

expresses a lifetime welfare difference in units that are comparable across countries and equilibrium paths.

To attribute the welfare difference to its economic sources, let $\{\mathcal{E}(\zeta) : \zeta \in [0,1]\}$ be a smooth path of equilibria connecting $\mathcal{E}^0$ and $\mathcal{E}^1$, with the welfare weights held fixed. The fundamental theorem of calculus gives

$$\Delta W_n(\mathcal{E}^1, \mathcal{E}^0) = \int_0^1 \sum_{t=0}^{\infty} \varpi_{n,t} \frac{\partial \log C^R_{n,t}(\mathcal{E}(\zeta))}{\partial \zeta} \, \mathrm{d}\zeta. \tag{26}$$

The total is determined by the endpoint equilibria, although the contribution assigned to an individual component can depend on the connecting path. A local decomposition separates the change in real consumption into productivity, terms of trade, profit shifting, trade barriers, international bond payments, installed capital, and investment.

3.8 Welfare Decomposition

The household budget, expenditure function, firm profits, and cost-minimization conditions jointly determine the local change in real consumption. We decompose this change into productivity, terms of trade, profit shifting, trade barriers, and intertemporal components. We use o and d to index an origin and a destination, and Appendix A.7.5 provides the derivation.

3.8.1 Terms of Trade

Let $d\mathcal{W}^{\text{ToT}}_{n,t}$ denote country n's nominal terms-of-trade contribution at date t. Let $\bar{p}^j_{o,t} \equiv \mu^j \text{mc}^j_{o,t}$ be the producer price before trade costs. The price paid in destination d is $p^j_{o,d,t} = \tau^j_{o,d,t} \bar{p}^j_{o,t}$. The fixed sectoral markup implies that the percentage change in $\bar{p}^j_{o,t}$ equals the percentage change in marginal cost. The terms-of-trade contribution is therefore

$$\begin{aligned} d\mathcal{W}^{\text{ToT}}_{n,t} \equiv & \sum_{j \in \mathcal{J}} \sum_{d \neq n} X^j_{n,d,t} \, d \log \text{mc}^j_{n,t} \\ & - \sum_{j \in \mathcal{J}} \sum_{o \neq n} X^j_{o,n,t} \, d \log \text{mc}^j_{o,t}. \end{aligned} \tag{27}$$

The first line values changes in the prices received on country n's exports at the quantities exported. The second values changes in the prices paid for its imports at the quantities imported. These contributions sum to zero across countries when expressed in common nominal units.

3.8.2 Profit Shifting

Let $\varrho^j \equiv 1 - 1/\mu^j$ denote the share of sector-j revenue received as operating profits. Profits of sector-j producers in country o are

$$\Pi^j_{o,t} = \varrho^j \sum_{d \in \mathcal{N}} X^j_{o,d,t}. \tag{28}$$

Let $d\mathcal{W}^{\Pi,j}_{o,t}$ denote the profit-shifting contribution. Following Ossa (2014), it isolates the change in profits associated with changes in sales by subtracting the profit change caused by the change in the producer price. Since $d \log \bar{p}^j_{o,t} = d \log \mathrm{mc}^j_{o,t}$, this subtraction can be written using marginal cost:

$$\begin{aligned} d\mathcal{W}^{\Pi,j}_{o,t} &\equiv d\Pi^j_{o,t} - \Pi^j_{o,t}\, d \log \mathrm{mc}^j_{o,t} \\ &= \varrho^j \sum_{d \in \mathcal{N}} \left[dX^j_{o,d,t} - X^j_{o,d,t}\, d \log \mathrm{mc}^j_{o,t} \right]. \end{aligned} \tag{29}$$

Operating profits received by country o's household arise from sales to every destination, all of which enter equation (29). A rise in sales at home or abroad can therefore raise profit income even when the producer's marginal cost falls.

Using $X^j_{o,d,t} = p^j_{o,d,t} q^j_{o,d,t}$ and $p^j_{o,d,t} = \tau^j_{o,d,t} \bar{p}^j_{o,t}$, equation (29) is equivalently

$$d\mathcal{W}^{\Pi,j}_{o,t} = \varrho^j \sum_{d \in \mathcal{N}} \left[p^j_{o,d,t}\, dq^j_{o,d,t} + X^j_{o,d,t}\, d \log \tau^j_{o,d,t} \right]. \tag{30}$$

The change in $\bar{p}^j_{o,t}$ is common across destinations. Price changes caused by destination-specific trade barriers remain in the second term and combine with the resources required for international delivery in equation (32). When trade barriers remain fixed, each additional unit sold contributes its price-cost margin to the producer's profits. Equation (30) sums this contribution across domestic and foreign sales.

Summing equation (30) across origins and sectors when trade barriers are fixed gives

$$\sum_{o \in \mathcal{N}} \sum_{j \in \mathcal{J}} d\mathcal{W}^{\Pi,j}_{o,t} = \sum_{o \in \mathcal{N}} \sum_{j \in \mathcal{J}} \sum_{d \in \mathcal{N}} \left(p^j_{o,d,t} - \tau^j_{o,d,t} \mathrm{mc}^j_{o,t} \right) dq^j_{o,d,t}. \tag{31}$$

The price is the buyer's marginal value of an additional unit, and $\tau^j_{o,d,t} \mathrm{mc}^j_{o,t}$ is its marginal resource cost. The right-hand side therefore values each quantity change at its price-cost gap, the first-order reallocation term in measures of allocative efficiency (Holmes et al.,

2014; Baqaee and Farhi, 2020). When the markup is common across activities, a positive value reflects an expansion in quantities that the markup restricts. Differences in markups add a second force because production can move between activities with different price-cost gaps. The country terms assign profit income to the origin country's household and therefore show how the gains are distributed across countries.

3.8.3 Trade Barriers

A change in a trade barrier changes the resources required to receive an imported good. Let $d\mathcal{W}^{\tau}_{n,t}$ denote this direct contribution to country n:

$$d\mathcal{W}^{\tau}_{n,t} \equiv -\sum_{j\in\mathcal{J}}\sum_{o\neq n} X^{j}_{o,n,t}\, d\log \tau^{j}_{o,n,t}. \tag{32}$$

Equation (32) is positive when a reduction in the trade barrier lowers the resources required for delivery.

3.8.4 Common Markup and Markup Dispersion

Define the initial revenue-weighted profit share as $\bar{\varrho} \equiv \sum_{n,j} \varrho^{j} R^{j}_{n,0} / \sum_{n,j} R^{j}_{n,0}$. Profit shifting separates into a common-markup component and a component due to differences in markups across sectors:

$$\begin{aligned} d\mathcal{W}^{\Pi,\text{common}}_{n,t} &\equiv \bar{\varrho}\sum_{j\in\mathcal{J}}\sum_{d\in\mathcal{N}}\left[dX^{j}_{n,d,t} - X^{j}_{n,d,t}\, d\log \text{mc}^{j}_{n,t}\right], \\ d\mathcal{W}^{\Pi,\text{dispersion}}_{n,t} &\equiv \sum_{j\in\mathcal{J}}\left(\varrho^{j}-\bar{\varrho}\right)\sum_{d\in\mathcal{N}}\left[dX^{j}_{n,d,t} - X^{j}_{n,d,t}\, d\log \text{mc}^{j}_{n,t}\right]. \end{aligned} \tag{33}$$

The two expressions sum to country n's profit-shifting contribution. The first evaluates changes in production at a common profit share. Under fixed trade barriers, the second isolates the part of equation (31) generated by reallocation across sectors with different markups.

3.8.5 Productivity

Let $d\mathcal{W}^A_{n,t}$ denote the reduction in variable cost generated by higher productivity. Since $V^j_{n,t} = R^j_{n,t}/\mu^j$ is variable cost, this contribution is

$$d\mathcal{W}^A_{n,t} \equiv \sum_{j\in\mathcal{J}} V^j_{n,t}\, d\log A^j_{n,t}. \tag{34}$$

3.8.6 Intertemporal Components

Let $d\mathcal{W}^B_{n,t}$, $d\mathcal{W}^K_{n,t}$, and $d\mathcal{W}^I_{n,t}$ denote, respectively, the contributions from net international bond payments, installed capital, and the resources used for investment and capital adjustment.

$$\begin{aligned} d\mathcal{W}^B_{n,t} &\equiv d\left(\mathcal{R}_t B_{n,t} - B_{n,t+1}\right), \\ d\mathcal{W}^K_{n,t} &\equiv \sum_{j\in\mathcal{J}} r^{K,j}_{n,t}\, dK^j_{n,t}, \\ d\mathcal{W}^I_{n,t} &\equiv d\left[\sum_{j\in\mathcal{J}} \left(I^j_{n,t} + \chi^j_{n,t}\right)\right]. \end{aligned} \tag{35}$$

The bond term is the change in net receipts from international bond positions. The capital term values a change in installed capital at its rental rate, while the investment term records the final goods used for investment and capital adjustment.

Substituting these components into the differentiated household budget gives the local welfare identity

$$\begin{aligned} d\log C^R_{n,t} = &\frac{1}{P_{n,t}C^R_{n,t}}\left[d\mathcal{W}^A_{n,t} + d\mathcal{W}^{\text{ToT}}_{n,t} + d\mathcal{W}^{\Pi,\text{common}}_{n,t} + d\mathcal{W}^{\Pi,\text{dispersion}}_{n,t}\right] \\ &+ \frac{1}{P_{n,t}C^R_{n,t}}\left[d\mathcal{W}^{\tau}_{n,t} + d\mathcal{W}^B_{n,t} + d\mathcal{W}^K_{n,t}\right] - \frac{d\mathcal{W}^I_{n,t}}{C^R_{n,t}}. \end{aligned} \tag{36}$$

Multiplying each contribution by the welfare weights and integrating along the path in equation (26) gives the finite-change decomposition. Each contribution is expressed in consumption-equivalent percentage points, the same units as total welfare.

4 Quantification

4.1 Data and Aggregation

The World Input-Output Database provides the bilateral and sectoral accounts needed to construct the model's initial allocation in 2000 and its observed paths through 2014 (Timmer et al., 2015). We aggregate the data into five economies: the United States, China, India, the European Union, and the rest of the world. Appendix Table A.11 reports the country mapping.

We aggregate industries into eight sectors: agriculture, mining and utilities, three manufacturing groups, and three service and construction groups. This aggregation preserves differences in trade exposure, input use, capital intensity, and markups that are central to the quantitative analysis. Appendix Table A.12 provides the complete industry mapping.

The input-output tables provide bilateral intermediate and final expenditure, gross output, value added, consumption, and investment. The Socio-Economic Accounts provide employment, factor compensation, capital stocks, and sectoral output price indices. We obtain depreciation rates from the Penn World Table and observed net external positions from the External Wealth of Nations database (Feenstra et al., 2015; Lane and Milesi-Ferretti, 2018). Trade balances and investment shares are constructed from the same aggregated accounts as the model.

4.2 Parameters and Initial Conditions

The input-output accounts for 2000 determine initial final-expenditure shares, intermediate-input shares, production cost shares, and bilateral trade shares. Initial bilateral expenditures also determine the Armington weights that reproduce observed domestic and foreign sourcing. Labor and capital shares are constructed from factor payments after removing the operating profits implied by the sectoral markups. The annual discount factor is 0.98, while trade elasticities and depreciation rates come from external estimates.

The Socio-Economic Accounts for 2000 provide capital and investment, which we allocate across sectors using capital payments net of operating profits. We use observed net external positions from the External Wealth of Nations database to initialize the model's international bond positions (Lane and Milesi-Ferretti, 2018), and choose the rest of the world's bond position to clear the global bond market. The initial allocation is not

restricted to be a steady state. Appendix A.3 reports the complete parameter mapping and construction of the baseline accounts.

4.3 Trade Elasticities and Implied Markups

We set the long-run trade elasticities $\theta^j \equiv \sigma^j - 1$ using external estimates (Caliendo and Parro, 2015). Agriculture uses the estimate 9.11. The value 7.10 is the observation-weighted average of the estimates for the manufacturing industries in our three manufacturing aggregates. We assign the aggregate estimate 4.49 to services and construction. Mining and utilities use 9.11 because the data do not provide separate estimates for those industries.[13] The pricing condition $\mu^j = 1 + 1/\theta^j$ then gives the markups in Table 1.

Table 1: Trade Elasticities and Sectoral Markups

Sector group	Trade elasticity θ^j	Markup μ^j
Agriculture; mining and utilities	9.11	1.110
Manufacturing	7.10	1.141
Services and construction	4.49	1.223
Common markup (same aggregate profit share)		1.188

The common-markup comparison removes sectoral markup dispersion while preserving the baseline aggregate profit share. Perfect competition sets every markup to one and removes both the aggregate markup wedge and its dispersion. Both comparisons hold demand elasticities fixed, which isolates the welfare effects of the markup level and its sectoral heterogeneity. A separate sensitivity scales price-cost wedges around their baseline-revenue-weighted mean while preserving that mean, allowing markup dispersion to vary without changing demand responses. Appendix A.3 reports the construction of the common markup, and Appendix Table A.8 reports the sensitivity.

4.4 Productivity and Trade Barriers

Observed prices, input costs, and trade shares allow us to infer annual changes in productivity and bilateral trade barriers. We use exact-hat identities, which express these changes relative to the allocation in 2000 (Eaton and Kortum, 2002; Arkolakis et al., 2012; Costinot and Rodríguez-Clare, 2014; Caliendo and Parro, 2015). For any variable x, let

[13] Assigning the aggregate elasticity to services and the agriculture elasticity to mining and utilities are maintained aggregation assumptions. Section 5 reports comparisons that separate the common markup from sectoral markup dispersion.

$\widehat{x}_t = x_t / x_{2000}$. With domestic trade barriers normalized to one, changes in domestic expenditure shares identify marginal-cost changes relative to the sectoral price index.

The input-cost index combines changes in labor compensation per worker, capital compensation per unit of capital, and intermediate-input prices using the production cost shares. Denote the first two changes by $\widehat{w}_{n,t}^{\text{data},j}$ and $\widehat{r}_{n,t}^{\text{data},j}$. Substituting these input prices and the domestic-share marginal cost into equation (12) yields

$$\widehat{A}_{n,t}^{j} = \left(\widehat{\pi}_{n,n,t}^{j}\right)^{1/(\sigma^j-1)} \frac{\left(\widehat{w}_{n,t}^{\text{data},j}\right)^{\ell_n^j} \left(\widehat{r}_{n,t}^{\text{data},j}\right)^{\kappa_n^j} \prod_{k\in\mathcal{J}} \left(\widehat{P}_{n,t}^{k}\right)^{\omega_n^{j,k}}}{\widehat{P}_{n,t}^{j}}. \tag{37}$$

Productivity is the residual change in efficiency needed to reconcile observed marginal costs with observed input prices under the calibrated production structure. Appendix A.3 describes the underlying price series and cells with a zero capital share.

Relative foreign and domestic expenditure shares identify bilateral trade barriers. For origin m, destination n, and sector j,

$$\widehat{\tau}_{m,n,t}^{j} = \left(\frac{\widehat{\pi}_{m,n,t}^{j}}{\widehat{\pi}_{n,n,t}^{j}}\right)^{1/(1-\sigma^j)} \frac{\widehat{\text{mc}}_{n,t}^{j}}{\widehat{\text{mc}}_{m,t}^{j}}. \tag{38}$$

The normalization sets $\widehat{\tau}_{n,n,t}^{j} = 1$. The inferred bilateral path measures changes in policy and nonpolicy trade barriers relative to each destination's domestic sourcing option.

We express productivity relative to a world productivity index, defined as the geometric mean of productivity across countries and sectors weighted by gross output in 2000. A value above one means that productivity in a country and sector grows faster than the world index, while a value below one means that it grows more slowly. The common normalization preserves relative productivity paths, allocations, and welfare comparisons.

Along the reference path, productivity in every country and sector grows at the world rate, and bilateral trade barriers remain at their levels in 2000. The reported welfare effects compare the paths of productivity and trade barriers inferred from annual data over 2000–2014 with this reference path. A negative value indicates lower welfare than the reference path and can occur even when welfare rises in absolute terms from its level in 2000. Appendix Figures A.2–A.4 report the inferred paths and their continuation after 2014.

4.5 Capital Adjustment Costs and Intertemporal Preference Wedges

We estimate one adjustment-cost curvature ϕ_n for each economy and recover a sequence of intertemporal preference wedges $\xi_{n,t}$ jointly using the simulated method of moments (SMM). The estimation uses the model with one sectorally mobile domestic capital stock and minimizes weighted squared deviations between model-generated and observed moments.

The moment vector contains the annual trade-balance-to-GDP ratios for the four separately modeled economies and the standard deviation of the investment-to-GDP ratio for each economy over 2000–2014. The trade-balance moments identify the wedge sequences, while the investment-volatility moments identify the adjustment-cost curvatures. Global market clearing determines the rest of the world's trade balance.[14]

Table 2: Depreciation and Capital Adjustment Costs

Economy	Depreciation δ_n	Curvature ϕ_n
United States	0.040	2.459
China	0.055	0.877
India	0.050	1.684
European Union	0.039	7.472
Rest of World	0.044	7.455

The sector-specific capital model applies each country's estimated curvature ϕ_n to all active sectors. A lower curvature permits investment to respond more quickly, while a higher curvature spreads adjustment over more dates. Appendix Figure A.1 reports the fit of the trade-balance paths, and Appendix A.3 reports the SMM objective, weighting matrix, and remaining fit statistics.

4.6 Continuation and Reference Paths

The welfare calculation requires productivity and trade barriers after 2014. We project their growth rates with autoregressive processes until productivity and trade barriers converge to constant values. The equilibrium then converges to the stationary allocation associated with those values. Appendix A.3 describes the continuation rules, and Appendix A.7 reports the terminal and equilibrium checks.

[14] Only relative intertemporal wedges are identified, so we impose $\sum_n \xi_{n,t} = 0$ at each date. Appendix A.3 reports the weighting matrix.

The allocation in 2000 is not a steady state, so the inferred and reference paths begin from the same initial capital stocks and international bond positions. Their welfare difference isolates the effects of changes in productivity and trade barriers from the adjustment generated by the initial conditions.

5 Quantitative Analysis

5.1 Welfare Effects of Changes over 2000–2014

We first compare welfare under the paths of productivity and trade barriers inferred over 2000–2014 with welfare under the reference path. We then examine productivity and trade barriers separately and trace how capital accumulation and international bond positions transmit these changes over time.

Table 3: Welfare from Changes in Productivity and Trade Barriers over 2000–2014

Economy	Total	Productivity	Terms of trade	Profit shifting	Trade barriers	Bond, capital, and investment
Panel A. Consumption-equivalent percentage points						
United States	-4.164	-7.853	2.567	-1.451	2.682	-0.110
China	156.087	81.245	-6.562	69.996	9.863	1.546
India	-26.790	-27.388	12.356	-8.011	-3.693	-0.054
European Union	0.557	-1.456	-2.036	-0.379	4.584	-0.157
Rest of the world	21.196	22.108	-0.148	5.844	-6.631	0.023
Panel B. Percent of total welfare						
United States	100.0	188.6	-61.6	34.8	-64.4	2.6
China	100.0	52.1	-4.2	44.8	6.3	1.0
India	100.0	102.2	-46.1	29.9	13.8	0.2
European Union	—	—	—	—	—	—
Rest of the world	100.0	104.3	-0.7	27.6	-31.3	0.1

Notes: Welfare is measured relative to a reference path that assigns world-average productivity growth to every country and sector and holds bilateral trade barriers at their levels in 2000. Bond, capital, and investment sum net international bond payments, changes in installed capital valued at rental rates, and the resources used for investment and capital adjustment. Ratios are not reported for the European Union because its total welfare effect is close to zero relative to the offsetting components.

Table 3 shows that China has the largest welfare gain relative to the reference path. Productivity is the largest component, but profit shifting is nearly as important. Lower trade barriers contribute positively, while the deterioration in China's terms of trade offsets part of these gains.

The large profit-shifting contribution reflects an expansion of Chinese production at positive markup wedges. Higher sales at home and abroad raise the profits received

by Chinese households, while lower export prices transfer part of the benefit to foreign consumers through the terms of trade. The decomposition therefore separates the increase in production from the distribution of its benefits across countries.

The cross-country differences reinforce this interpretation. Economies that contract relative to the reference path lose profit income, while terms-of-trade movements redistribute some of those gains and losses across countries. Dynamic adjustment therefore changes both total profit income and its distribution across countries.

5.2 Dynamic versus Static Welfare

The dynamic model allows sector-specific investment, gradual capital adjustment, and international borrowing and lending. The static model has no intertemporal investment or international bond and allocates a fixed domestic capital stock across sectors within each period. The welfare difference between the two models captures the combined effect of these features.

Table 4: Welfare Effects in the Dynamic and Static Models, 2000–2014

	Consumption-equivalent percent		Percentage points
Economy	Dynamic	Static	Difference
United States	-4.164	-4.883	0.719
China	156.087	106.641	49.446
India	-26.790	-23.511	-3.280
European Union	0.557	0.141	0.416
Rest of the world	21.196	18.008	3.189

Components of the difference between the dynamic and static models, consumption-equivalent percentage points

Economy	Total	Productivity	Terms of trade	Profit shifting	Trade barriers	Bond, capital, and investment
United States	0.719	-0.073	0.253	0.421	0.225	-0.107
China	49.446	13.163	0.605	32.445	1.086	2.147
India	-3.280	-4.887	5.794	-3.040	-1.100	-0.047
European Union	0.416	0.597	-0.625	0.152	0.450	-0.157
Rest of the world	3.189	4.103	-0.016	0.407	-1.331	0.025

Components as a percent of the difference between the dynamic and static models

Economy	Total	Productivity	Terms of trade	Profit shifting	Trade barriers	Bond, capital, and investment
United States	100.0	-10.1	35.2	58.6	31.2	-14.9
China	100.0	26.6	1.2	65.6	2.2	4.3
India	100.0	149.0	-176.7	92.7	33.5	1.4
European Union	—	—	—	—	—	—
Rest of the world	100.0	128.7	-0.5	12.8	-41.7	0.8

Notes: The difference is the welfare effect in the dynamic model minus the welfare effect in the static model. Bond, capital, and investment sum net international bond payments, changes in installed capital valued at rental rates, and the resources used for investment and capital adjustment. Ratios are not reported for the European Union because its difference is small relative to the offsetting components.

Table 4 shows that the dynamic model substantially raises China's welfare gain relative to the static model. The difference is also positive for most other economies, although its size and decomposition vary across economies.

For China, profit shifting accounts for 65.6 percent of the difference between the dynamic and static models, while the bond, capital, and investment components together account for 4.3 percent. Capital accumulation raises future consumption, but the resources used for investment offset most of that direct gain. Investment instead changes the paths of capital and production, lowers costs, and raises sales at positive markup wedges.

This finding identifies the question for the remaining analysis. The key issue is not simply that households can move consumption across dates, but how capital accumulation changes production and how access to the international bond determines whether additional saving finances domestic investment or net international lending.

5.3 Counterfactual Changes in Trade Barriers and Productivity

The changes inferred over 2000–2014 vary across countries and sectors. Counterfactual changes in trade barriers and productivity clarify how each source of change affects production and welfare.

Table 5: Counterfactual Reduction in Manufacturing Trade Barriers

	Consumption-equivalent percentage points				Percent of total welfare	
Economy	Total	Terms of trade	Profit shifting	Trade barriers	Profit shifting	Trade barriers
Panel A. Permanent reduction						
United States	1.450	-0.042	0.301	1.193	20.7	82.3
China	3.921	-0.036	1.394	2.565	35.5	65.4
India	1.640	-0.003	0.374	1.271	22.8	77.5
European Union	1.338	0.045	0.280	1.015	20.9	75.8
Rest of the world	1.948	0.007	0.485	1.457	24.9	74.8
Panel B. Ten-year reduction						
United States	0.274	-0.006	0.064	0.215	23.4	78.7
China	0.723	-0.004	0.279	0.452	38.5	62.6
India	0.357	-0.003	0.093	0.271	26.0	76.0
European Union	0.246	0.004	0.052	0.190	21.2	77.3
Rest of the world	0.349	0.003	0.089	0.256	25.6	73.5

Notes: Manufacturing trade barriers fall by 10 percent for every international origin-destination pair. The permanent reduction remains in place, while the temporary reduction lasts ten years. Small differences between the total and the displayed components reflect international bond payments, changes in installed capital valued at rental rates, and the resources used for investment and capital adjustment.

Table 5 reports the welfare effects of a multilateral reduction in manufacturing trade barriers. The direct contribution of lower trade barriers is the largest source of welfare gains because lower barriers reduce the resources required to deliver traded goods. Profit shifting is the second-largest contribution: lower barriers expand trade and production, raising sales at positive markup wedges. The multilateral change produces comparatively small terms-of-trade effects.

Unlike a reduction in trade barriers, the increase in productivity does not reduce the resources required to deliver traded goods. Higher Chinese productivity lowers marginal cost and shifts production and profits toward China, but it also lowers the price received by Chinese exporters relative to the price paid for imports.

Table 6: Counterfactual Increase in Chinese Manufacturing Productivity

	Dynamic model, consumption-equivalent percentage points				Percentage points
Economy	Total	Productivity	Terms of trade	Profit shifting	Difference
Panel A. Permanent increase					
United States	0.008	0.000	0.009	-0.001	0.000
China	6.142	3.922	-0.401	2.622	1.115
India	-0.008	0.000	0.000	-0.006	0.005
European Union	0.008	0.000	0.007	0.001	0.000
Rest of the world	0.034	0.000	0.028	0.006	0.004
Panel B. Ten-year increase					
United States	0.003	0.000	0.002	0.001	0.002
China	1.049	0.719	-0.078	0.424	0.132
India	0.005	0.000	0.004	0.002	0.007
European Union	0.003	0.000	0.002	0.001	0.001
Rest of the world	0.009	0.000	0.005	0.002	0.003

Notes: Productivity in China's light and consumer manufacturing sector rises by 10 percent. The permanent increase remains in place, while the temporary increase lasts ten years. Difference is the welfare effect in the dynamic model minus the welfare effect in the static model.

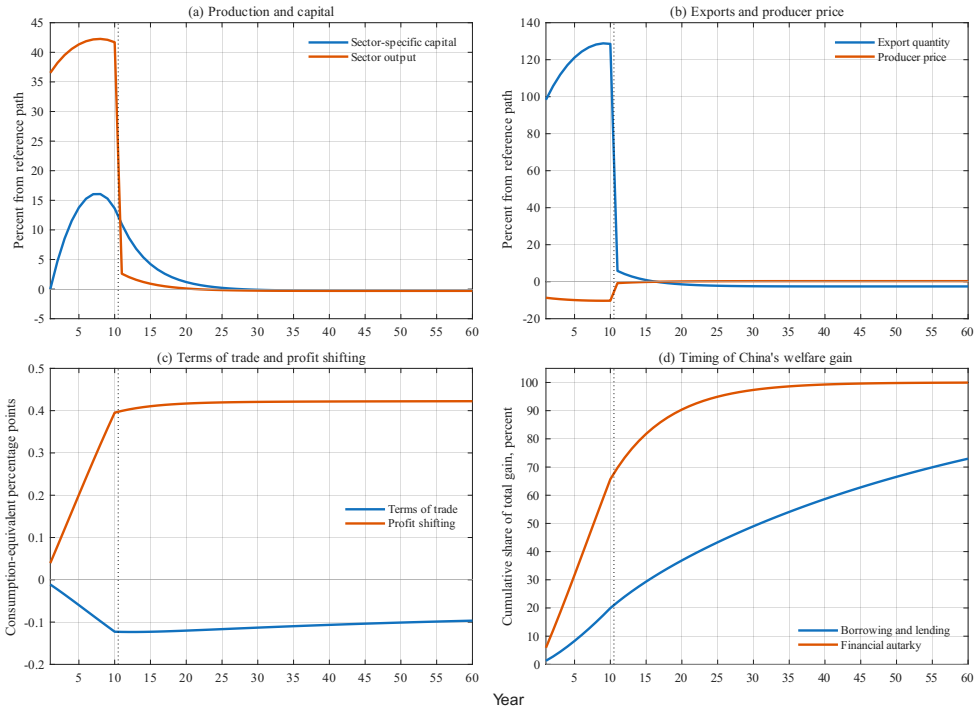
Table 6 shows this tradeoff for an increase in productivity in China's light and consumer manufacturing sector. Productivity and profit shifting raise China's welfare, while its terms of trade deteriorate. Profit shifting remains important under both permanent and temporary increases because higher productivity expands Chinese production at a positive average markup wedge.

The split between the common markup and markup dispersion components is informative. The common positive wedge accounts for more than the total contribution from profit shifting, while sectoral markup differences partially offset it. Markup dispersion is therefore not sufficient to predict the welfare contribution; what matters is whether production expands in the sectors with larger wedges.

5.4 Capital Accumulation, Markups, and Dynamic Adjustment

The ten-year increase in productivity shows how capital and international bond positions transmit a temporary change in productivity into later production and consumption. It also distinguishes the effect of a common markup on aggregate production from the effect of markup differences on the allocation of production across sectors. Figure 1 traces the state variables and their welfare implications.

Figure 1: Adjustment after a Temporary Increase in Productivity in China



Notes: Productivity in China's light and consumer manufacturing sector is 10 percent above its reference path in years 1–10 and returns to that path at the dotted line. Panels (a) and (b) report percentage differences from the reference path. Panel (c) reports cumulative contributions in consumption-equivalent percentage points. Panel (d) reports cumulative shares of the welfare effect over the full transition.

Panels (a) and (b) show the persistence of capital and production. When productivity returns to its initial path, capital in the affected sector remains 11.0 percent above its reference level, output remains 2.6 percent higher, and the export quantity remains 5.8 percent higher. The capital deviation has a four-year half-life and remains 0.9 percent ten years later. This comparative advantage is transitional: as productivity normalizes, excess capital lowers its own return, investment recedes, and the sector moves back toward its stationary allocation.

Panel (c) shows how this persistence carries the terms-of-trade and profit-shifting contributions forward. During the increase in productivity, capital accumulation expands Chinese production and shifts profits toward China. After productivity returns, inherited capital continues to lower Chinese marginal cost. China's terms of trade remain below their reference path, while profit shifting remains positive until capital and expenditure have adjusted.

Table 7: Sources of China's Welfare Difference between the Dynamic and Static Models

	Permanent increase		Ten-year increase	
Comparison	Total welfare	Profit shifting	Total welfare	Profit shifting
Dynamic model minus static model	1.115	0.862	0.132	0.101
Aggregate capital accumulation	1.054	0.801	0.242	0.207
Sector-specific capital	0.018	0.017	0.015	0.014
International bond access	0.044	0.045	−0.125	−0.120
Without capital adjustment costs				
Aggregate capital accumulation	1.045	0.810	0.251	0.222
Sector-specific capital	0.004	0.004	0.004	0.003
International bond access	0.077	0.076	−0.135	−0.115

Notes: Entries are consumption-equivalent percentage-point differences for China. The static model fixes aggregate capital, allows capital to move across sectors within each period, and imposes financial autarky. Aggregate capital accumulation compares the dynamic model with sectorally mobile capital under financial autarky with the static model. Sector-specific capital compares sector-specific with sectorally mobile capital under financial autarky. International bond access compares sector-specific capital with and without the bond. The three baseline rows sum to the full difference up to rounding. The final three rows repeat the sequential comparisons with zero capital adjustment costs.

Table 7 identifies the sources of the difference between the dynamic and static models. Aggregate capital accumulation is the dominant force: allowing the total capital stock to respond accounts for nearly all of the difference under the permanent increase and more than the full difference under the temporary increase before the offsetting effect of the international bond. Profit shifting follows the same pattern because a larger capital stock expands sales at positive markup wedges.

Sector-specific capital contributes much less to lifetime welfare than aggregate accumulation, but it determines which sectors accumulate capital and therefore how long the affected sector continues to expand after productivity returns to its reference path. Removing adjustment costs changes the speed of this transition without changing the relative importance of the mechanisms.

International bond access has opposite effects across the two productivity paths. It offsets part of the difference under the temporary increase because China undertakes net international lending, but it reinforces the difference under the permanent increase by allowing investment to exceed domestic saving. This contrast connects the model comparison in Table 7 to the saving and investment paths behind it.

International borrowing and lending change both the allocation of saving and the timing of consumption. During the temporary increase, China's additional saving exceeds its additional domestic investment. China uses the difference for net international lending, which produces a trade surplus and increases its international bond holdings. Financial

autarky keeps this saving at home, producing a larger domestic capital response but less scope to smooth consumption internationally.

The permanent increase reverses this response. Newly installed capital continues to operate with higher productivity, so desired investment rises above domestic saving. China borrows through a trade deficit, and financial openness finances capital accumulation beyond domestic saving. Trade imbalances therefore show whether the increase in productivity leads to net international lending or international borrowing to finance domestic investment.

This allocation also explains the timing in Figure 1. Interest receipts on China's international bond holdings and later reductions in its bond position support consumption after the temporary increase in productivity ends, even as capital in the affected sector gradually returns to its reference path. Financial autarky instead concentrates more of the welfare effect during the period of higher productivity by tying saving to domestic investment.

The two-period problem in Appendix A.1.9 shows why the response depends on persistence. The bond position equals desired saving minus domestic investment: higher current income raises saving, while higher future productivity raises the value of newly installed capital and desired investment. Investment becomes productive with a lag, so a more persistent increase in productivity raises desired investment by more.

The additional duration and magnitude exercises in Appendix A.4 confirm this distinction. Scaling a temporary increase changes the strength but not the direction of the bond response over the range considered, whereas making the increase permanent changes lending into borrowing. The model does not imply a general cutoff based on the size of a temporary increase alone because investment adjusts until its return is aligned with the bond return.

5.5 Markups, Financial Openness, and Welfare

Whether China lends or borrows is not determined by markups: temporary lending and permanent borrowing also occur under perfect competition. Markups determine the welfare consequences. When financial autarky redirects saving toward domestic capital after a temporary increase in productivity, the additional production raises welfare because price exceeds marginal resource cost.

Comparing the welfare effect of the increase in productivity across financial regimes does not reveal which regime leaves China better off because each effect is measured

relative to its own reference path. Table 8 instead gives every economy the same zero initial international bond position in both regimes and directly compares the resulting welfare levels.

Table 8: Welfare Levels under Financial Autarky and Openness

Markup regime	Reference path	Ten-year increase	Permanent increase
Panel A: China			
Heterogeneous markups	-0.012	0.112	-0.058
Common markup	0.060	0.182	0.015
Perfect competition	-0.226	-0.142	-0.259
Panel B: Consumption-weighted world			
Heterogeneous markups	-0.008	-0.006	-0.009
Common markup	-0.006	-0.004	-0.007
Perfect competition	-0.018	-0.016	-0.019

Notes: Entries are financial autarky minus financial openness in consumption-equivalent percent. Every economy starts with a zero international bond position in both financial regimes. The world statistic uses baseline consumption weights.

With positive markups, China has higher welfare under autarky after the ten-year increase in productivity. Autarky keeps more saving in domestic capital, and the additional production is valuable because markups otherwise restrict output. Under perfect competition, this benefit disappears and openness leaves China better off through consumption smoothing and investment financing. The result with a common markup shows that a positive average wedge is sufficient; sectoral markup dispersion is not required.

Sectoral dispersion matters through the alignment between markup wedges and changes in production. Wider markup differences amplify profit shifting when production expands relatively more in high-wedge sectors. In the counterfactual considered here, production instead expands in sectors whose wedges do not rise, so greater dispersion weakens the contribution.

Although China has higher welfare under financial autarky with positive markups, this result does not imply that financial autarky raises world welfare. Part of China's gain reflects profit income received by Chinese households and changes in the terms of trade, while consumption-weighted world welfare remains higher under openness. Appendix A.4 reports the supporting comparisons across countries and markup specifications.

6 Conclusion

This paper studies how markups affect welfare when capital and international bond positions adjust to changes in productivity and trade barriers. The quantitative analysis shows that dynamic adjustment matters because capital accumulation changes the capital stock, future production, and profit income, not simply the timing of consumption. Aggregate capital accumulation accounts for most of the difference between the dynamic and static models, while sector-specific capital determines which sectors accumulate capital and prolongs the effects of temporary changes. Markups make these production responses important for welfare because expanding sales also changes the profits received by households.

Access to the international bond dampens domestic capital accumulation when temporary income gains lead to net international lending, but supports it when persistent productivity improvements lead countries to borrow for domestic investment. The resulting trade imbalances show whether a country uses additional saving for net international lending or borrows internationally to finance domestic investment. Markups determine the welfare consequences for each country, but a gain for one country does not imply a gain in consumption-weighted world welfare. Future research could study optimal financial openness with additional financial instruments and borrowing frictions and allow markups, entry, innovation, or learning to generate lasting changes in comparative advantage.

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A Appendix

A.1 Stylized Model: Derivations

We first derive the household and firm optimality conditions, the welfare decomposition, and the local equilibrium responses used throughout the appendix. We then establish the capital response to an anticipated productivity increase and use these results to prove Proposition 1 and Corollary 1. The final subsection derives saving and investment responses in a two-period model. Comparative statics are local to the symmetric balanced-trade allocation, with relative expenditure endogenous unless explicitly held fixed.

A.1.1 Demand and Prices

We suppress date indices for within-period problems. Denote country m's final-good requirement by $F_m \equiv C_m + i_m^1$ and its sector-j input by F_m^j. For $\sigma > 1$, the sectoral aggregation function is $G_m^j(q) \equiv [\sum_n \omega_{nm}^{1/\sigma}(q_{nm}^j)^{(\sigma-1)/\sigma}]^{\sigma/(\sigma-1)}$. The household solves

$$\min_{\{q_{nm}^j\}_n} \quad \sum_n p_{nm}^j q_{nm}^j \quad \text{s.t.} \quad G_m^j(q) \geq F_m^j. \tag{A.1}$$

Denote the multiplier by γ_m^j. The Lagrangian is $\mathcal{L}_m^j = \sum_n p_{nm}^j q_{nm}^j + \gamma_m^j[F_m^j - G_m^j(q)]$. Differentiating the aggregator, $\partial G_m^j/\partial q_{nm}^j = [\sigma/(\sigma-1)](G_m^j)^{1/\sigma}\omega_{nm}^{1/\sigma}[(\sigma-1)/\sigma](q_{nm}^j)^{-1/\sigma} = \omega_{nm}^{1/\sigma}(G_m^j/q_{nm}^j)^{1/\sigma}$. Positive prices imply $G_m^j = F_m^j$ at the optimum, so $\partial\mathcal{L}_m^j/\partial q_{nm}^j = 0$ requires $p_{nm}^j = \gamma_m^j\omega_{nm}^{1/\sigma}(F_m^j/q_{nm}^j)^{1/\sigma}$. Raising both sides to σ and multiplying by q_{nm}^j, $(p_{nm}^j)^\sigma q_{nm}^j = (\gamma_m^j)^\sigma \omega_{nm} F_m^j$. Thus

$$q_{nm}^j = \omega_{nm}\left(\frac{\gamma_m^j}{p_{nm}^j}\right)^\sigma F_m^j. \tag{A.2}$$

To determine γ_m^j, insert this demand into the binding constraint. Raising the constraint to $(\sigma-1)/\sigma$ first makes the substitution explicit:

$$\begin{aligned}(F_m^j)^{(\sigma-1)/\sigma} &= \sum_n \omega_{nm}^{1/\sigma}\left[\omega_{nm}\left(\frac{\gamma_m^j}{p_{nm}^j}\right)^\sigma F_m^j\right]^{(\sigma-1)/\sigma} \\ &= (\gamma_m^j)^{\sigma-1}(F_m^j)^{(\sigma-1)/\sigma}\sum_n \omega_{nm}(p_{nm}^j)^{1-\sigma}.\end{aligned} \tag{A.3}$$

Divide by $(F_m^j)^{(\sigma-1)/\sigma} > 0$ to obtain $1 = (\gamma_m^j)^{\sigma-1}\sum_n \omega_{nm}(p_{nm}^j)^{1-\sigma}$. Solving for the multiplier,

$$\gamma_m^j = \left[\sum_n \omega_{nm}(p_{nm}^j)^{1-\sigma}\right]^{1/(1-\sigma)} \equiv P_m^j. \tag{A.4}$$

At these quantities, expenditure is $\sum_n p_{nm}^j q_{nm}^j = (\gamma_m^j)^\sigma F_m^j \sum_n \omega_{nm}(p_{nm}^j)^{1-\sigma} = (\gamma_m^j)^\sigma F_m^j (\gamma_m^j)^{1-\sigma} = P_m^j F_m^j$. Hence P_m^j is the sectoral unit expenditure.

Next, denote the final-good aggregator by $G_m(F) \equiv \prod_j (F_m^j/a^j)^{a^j}$, with $a^j > 0$ and $a^1 + a^2 = 1$. The household allocates expenditure across sectors by solving

$$\min_{\{F_m^j\}_j} \quad \sum_j P_m^j F_m^j \quad \text{s.t.} \quad G_m(F) \geq F_m. \tag{A.5}$$

With multiplier γ_m, the Lagrangian is $\mathcal{L}_m = \sum_j P_m^j F_m^j + \gamma_m[F_m - G_m(F)]$. Since $\partial G_m/\partial F_m^j = a^j G_m/F_m^j$, the binding constraint and first-order condition imply $0 = P_m^j - \gamma_m a^j F_m/F_m^j$, or $F_m^j/a^j = \gamma_m F_m/P_m^j$. Substituting into the constraint, $F_m = \prod_j (\gamma_m F_m/P_m^j)^{a^j}$. Factor out the common term and use $\sum_j a^j = 1$: $F_m = (\gamma_m F_m)^{\sum_j a^j}\prod_j (P_m^j)^{-a^j} = \gamma_m F_m \prod_j (P_m^j)^{-a^j}$. Dividing by $F_m > 0$ and solving,

$$\gamma_m = \prod_j (P_m^j)^{a^j} \equiv P_m. \tag{A.6}$$

Multiplying each first-order condition by F_m^j, $P_m^j F_m^j = \gamma_m a^j F_m$. Summing across sectors, total expenditure is $E_m \equiv \sum_j P_m^j F_m^j = \gamma_m F_m \sum_j a^j = P_m F_m$, so $P_m^j F_m^j = a^j E_m$. Substituting $F_m^j = a^j E_m/P_m^j$ into variety demand,

$$X_{nm}^j \equiv p_{nm}^j q_{nm}^j = p_{nm}^j \omega_{nm}\left(\frac{P_m^j}{p_{nm}^j}\right)^\sigma \frac{a^j E_m}{P_m^j} = \underbrace{\omega_{nm}\left(\frac{p_{nm}^j}{P_m^j}\right)^{1-\sigma}}_{\lambda_{nm}^j} a^j E_m. \tag{A.7}$$

The shares sum to one because $\sum_n \lambda_{nm}^j = (P_m^j)^{\sigma-1}\sum_n \omega_{nm}(p_{nm}^j)^{1-\sigma} = (P_m^j)^{\sigma-1}(P_m^j)^{1-\sigma} = 1$.

We will also use the price derivatives in the welfare decomposition and the equilibrium responses. Differentiating $(P_n^j)^{1-\sigma} = \sum_m \omega_{mn}(p_{mn}^j)^{1-\sigma}$ gives $(1-\sigma)(P_n^j)^{1-\sigma}\mathrm{d}\log P_n^j = (1-\sigma)\sum_m \omega_{mn}(p_{mn}^j)^{1-\sigma}\mathrm{d}\log p_{mn}^j$. Dividing by $(1-\sigma)(P_n^j)^{1-\sigma}$ and using the expenditure

shares, $\mathrm{d}\log P_n^j = \sum_m \lambda_{mn}^j \mathrm{d}\log p_{mn}^j$. Since $\mathrm{d}\log P_n = \sum_j a^j \mathrm{d}\log P_n^j$,

$$E_n \,\mathrm{d}\log P_n = \sum_{j,m} a^j E_n \lambda_{mn}^j \mathrm{d}\log p_{mn}^j = \sum_{j,m} X_{mn}^j \frac{\mathrm{d}p_{mn}^j}{p_{mn}^j} = \sum_{j,m} q_{mn}^j \,\mathrm{d}p_{mn}^j. \tag{A.8}$$

A.1.2 Production and Firm Pricing

A sector-1 firm with $\alpha \in (0,1)$ takes factor prices as given and chooses capital and labor to produce y_n^1. Its cost-minimization problem is

$$\min_{k_n^1,\, \ell_n^1} \quad r_n^1 k_n^1 + w_n \ell_n^1 \quad \text{s.t.} \quad A_n^1 (k_n^1)^\alpha (\ell_n^1)^{1-\alpha} \geq y_n^1. \tag{A.9}$$

Denote the multiplier by v_n. The Lagrangian is $\mathcal{L}_n^1 = r_n^1 k_n^1 + w_n \ell_n^1 + v_n[y_n^1 - A_n^1 (k_n^1)^\alpha (\ell_n^1)^{1-\alpha}]$. At an interior optimum, the constraint binds, and the first-order conditions are $0 = r_n^1 - v_n \alpha y_n^1 / k_n^1$ and $0 = w_n - v_n(1-\alpha) y_n^1 / \ell_n^1$. Solving for inputs, $k_n^1 = \alpha v_n y_n^1 / r_n^1$ and $\ell_n^1 = (1-\alpha) v_n y_n^1 / w_n$. Insert both expressions into production:

$$\begin{aligned} y_n^1 &= A_n^1 \left(\frac{\alpha v_n y_n^1}{r_n^1}\right)^\alpha \left(\frac{(1-\alpha) v_n y_n^1}{w_n}\right)^{1-\alpha} \\ &= A_n^1 v_n y_n^1 \left(\frac{\alpha}{r_n^1}\right)^\alpha \left(\frac{1-\alpha}{w_n}\right)^{1-\alpha}. \end{aligned} \tag{A.10}$$

Dividing by $y_n^1 > 0$, $1 = A_n^1 v_n (\alpha / r_n^1)^\alpha [(1-\alpha)/w_n]^{1-\alpha}$. Solving for the multiplier,

$$v_n = \frac{1}{A_n^1} \left(\frac{r_n^1}{\alpha}\right)^\alpha \left(\frac{w_n}{1-\alpha}\right)^{1-\alpha} \equiv \mathrm{mc}_n^1. \tag{A.11}$$

Minimized cost is $V_n^1 \equiv r_n^1 k_n^1 + w_n \ell_n^1 = [\alpha + (1-\alpha)] v_n y_n^1 = \mathrm{mc}_n^1 y_n^1$. The two first-order conditions, therefore, become

$$r_n^1 k_n^1 = \alpha \mathrm{mc}_n^1 y_n^1 = \alpha V_n^1, \qquad w_n \ell_n^1 = (1-\alpha) \mathrm{mc}_n^1 y_n^1 = (1-\alpha) V_n^1. \tag{A.12}$$

In sector 2, the firm solves $\min_{\ell_n^2} w_n \ell_n^2$ subject to $\ell_n^2 \geq y_n^2$. Its Lagrangian is $w_n \ell_n^2 + v_n^2 (y_n^2 - \ell_n^2)$, with conditions $w_n - v_n^2 = 0$ and $v_n^2 (y_n^2 - \ell_n^2) = 0$. Since $w_n > 0$, $v_n^2 = w_n > 0$ and $\ell_n^2 = y_n^2$. Thus $\mathrm{mc}_n^2 = \partial(w_n y_n^2)/\partial y_n^2 = w_n$.

Under monopolistic competition, sector-1 firms take aggregate expenditure and price indices as given. Denote $B_{nm}^1 \equiv \omega_{nm} (P_m^1)^\sigma F_m^1$. The firm chooses destination prices and

quantities to solve

$$\max_{\{p^1_{nm},q^1_{nm}\}_m} \quad \sum_m (p^1_{nm} - \text{mc}^1_n) q^1_{nm} \quad \text{s.t.} \quad q^1_{nm} = B^1_{nm}(p^1_{nm})^{-\sigma}, \quad m \in \{h, f\}. \tag{A.13}$$

Substituting demand into destination-m profit, $\Pi^1_{nm}(p^1_{nm}) = B^1_{nm}[(p^1_{nm})^{1-\sigma} - \text{mc}^1_n(p^1_{nm})^{-\sigma}]$. Differentiate with respect to its price and factor out the positive term $B^1_{nm}(p^1_{nm})^{-\sigma-1}$:

$$\begin{aligned} \frac{\partial \Pi^1_{nm}}{\partial p^1_{nm}} &= B^1_{nm}[(1-\sigma)(p^1_{nm})^{-\sigma} + \sigma \text{mc}^1_n (p^1_{nm})^{-\sigma-1}] \\ &= B^1_{nm}(p^1_{nm})^{-\sigma-1}[\sigma \text{mc}^1_n - (\sigma-1)p^1_{nm}]. \end{aligned} \tag{A.14}$$

The first-order condition requires $\sigma \text{mc}^1_n - (\sigma-1)p^1_{nm} = 0$. Hence the optimal price is

$$p^1_{nm} = \underbrace{\frac{\sigma}{\sigma-1}}_{\mu^1} \text{mc}^1_n. \tag{A.15}$$

The derivative is positive below this price and negative above it. Perfect competition in sector 2 requires $p^2_{nm} = \text{mc}^2_n = w_n$.

Denote revenue by $R^j_n \equiv \sum_m p^j_{nm} q^j_{nm}$. Goods-market clearing implies $y^j_n = \sum_m q^j_{nm}$, so $R^1_n = \sum_m \mu^1 \text{mc}^1_n q^1_{nm} = \mu^1 \text{mc}^1_n y^1_n = \mu^1 V^1_n$. Subtracting variable cost from revenue,

$$\Pi^1_n = R^1_n - \frac{R^1_n}{\mu^1} = \underbrace{\left(1 - \frac{1}{\mu^1}\right)}_{\varrho^1} R^1_n. \tag{A.16}$$

In sector 2, $R^2_n = w_n \sum_m q^2_{nm} = w_n \ell^2_n = V^2_n$. Consequently, $\Pi^2_n = R^2_n - V^2_n = 0$.

A.1.3 Household Optimality Conditions

The household owns domestic capital and firms and takes prices and distributed profits as given. Denote income by $Y_{n,t} \equiv w_{n,t} L_n + r^1_{n,t} k^1_{n,t} + \Pi^1_{n,t}$. Given $k^1_{n,0}$ and zero initial bonds, it solves the following problem with $\beta > 0$, $0 \leq \delta < 1$, and positive consumption

and capital:

$$\begin{aligned}\max_{\{C_{n,t}\}_{t=0}^{2},\, i_{n,0}^{1},\, b_{n,1},\, b_{n,2}} \quad & \sum_{t=0}^{2} \beta^t \log C_{n,t} \\ \text{s.t.} \quad & P_{n,t}(C_{n,t} + i_{n,t}^{1}) + Q_t b_{n,t+1} = Y_{n,t} + b_{n,t}, \quad t \in \{0,1\}, \\ & P_{n,2} C_{n,2} = Y_{n,2} + b_{n,2}, \\ & k_{n,1}^{1} = (1-\delta)k_{n,0}^{1} + i_{n,0}^{1}, \quad k_{n,2}^{1} = (1-\delta)k_{n,1}^{1}, \\ & i_{n,1}^{1} = i_{n,2}^{1} = 0, \qquad b_{n,0} = b_{n,3} = 0. \end{aligned} \tag{A.17}$$

First impose the capital laws in future income: $Y_{n,1} = w_{n,1}L_n + r_{n,1}^{1}[(1-\delta)k_{n,0}^{1} + i_{n,0}^{1}] + \Pi_{n,1}^{1}$ and $Y_{n,2} = w_{n,2}L_n + (1-\delta)r_{n,2}^{1}[(1-\delta)k_{n,0}^{1} + i_{n,0}^{1}] + \Pi_{n,2}^{1}$. Their investment derivatives are $\partial Y_{n,1}/\partial i_{n,0}^{1} = r_{n,1}^{1}$ and $\partial Y_{n,2}/\partial i_{n,0}^{1} = (1-\delta)r_{n,2}^{1}$. Denoting the budget multipliers by $\Lambda_{n,t}$, the Lagrangian is

$$\begin{aligned}\mathcal{L}_n = & \sum_{t=0}^{2} \beta^t \log C_{n,t} + \sum_{t=0}^{1} \Lambda_{n,t}[Y_{n,t} + b_{n,t} - P_{n,t}(C_{n,t} + i_{n,t}^{1}) - Q_t b_{n,t+1}] \\ & + \Lambda_{n,2}[Y_{n,2} + b_{n,2} - P_{n,2}C_{n,2}]. \end{aligned} \tag{A.18}$$

The consumption derivative is $\partial \mathcal{L}_n/\partial C_{n,t} = \beta^t/C_{n,t} - \Lambda_{n,t}P_{n,t} = 0$, so $\Lambda_{n,t} = \beta^t/(P_{n,t}C_{n,t})$ for $t \in \{0,1,2\}$. The bond derivative is $\partial \mathcal{L}_n/\partial b_{n,t+1} = -\Lambda_{n,t}Q_t + \Lambda_{n,t+1} = 0$, so $\Lambda_{n,t+1} = Q_t\Lambda_{n,t}$ for $t \in \{0,1\}$. Differentiating with respect to investment,

$$\begin{aligned} 0 = \frac{\partial \mathcal{L}_n}{\partial i_{n,0}^{1}} &= -\Lambda_{n,0}P_{n,0} + \Lambda_{n,1}\frac{\partial Y_{n,1}}{\partial i_{n,0}^{1}} + \Lambda_{n,2}\frac{\partial Y_{n,2}}{\partial i_{n,0}^{1}} \\ &= -\Lambda_{n,0}P_{n,0} + \Lambda_{n,1}r_{n,1}^{1} + (1-\delta)\Lambda_{n,2}r_{n,2}^{1}. \end{aligned} \tag{A.19}$$

Divide by $\Lambda_{n,0}$ and use $\Lambda_{n,1}/\Lambda_{n,0} = Q_0$ and $\Lambda_{n,2}/\Lambda_{n,0} = (\Lambda_{n,2}/\Lambda_{n,1})(\Lambda_{n,1}/\Lambda_{n,0}) = Q_1Q_0$. The investment condition becomes

$$P_{n,0} = Q_0 r_{n,1}^{1} + (1-\delta)Q_0Q_1 r_{n,2}^{1}. \tag{A.20}$$

For consumption, substitute the consumption multipliers into the bond condition: $\frac{\beta^t Q_t}{P_{n,t}C_{n,t}} = \frac{\beta^{t+1}}{P_{n,t+1}C_{n,t+1}}$. Cross-multiplication and cancellation of β^t imply

$$Q_t P_{n,t+1} C_{n,t+1} = \beta P_{n,t} C_{n,t}. \tag{A.21}$$

Taking the ratio across countries, $P_{h,t+1}C_{h,t+1}/(P_{f,t+1}C_{f,t+1}) = [(\beta/Q_t)P_{h,t}C_{h,t}]/[(\beta/Q_t)P_{f,t}C_{f,t}] = P_{h,t}C_{h,t}/(P_{f,t}C_{f,t})$. Relative nominal consumption expenditure is, therefore, constant across dates.

Finally, multiply the date-1 budget by Q_0 and the date-2 budget by Q_0Q_1 and add them to the date-0 budget. Net bond receipts cancel because $-Q_0 b_{n,1} + Q_0(b_{n,1} - Q_1 b_{n,2}) + Q_0Q_1 b_{n,2} = 0$. The resulting intertemporal budget is

$$\begin{aligned} P_{n,0}(C_{n,0} + i^1_{n,0}) + Q_0 P_{n,1}C_{n,1} + Q_0Q_1 P_{n,2}C_{n,2} \\ = Y_{n,0} + Q_0 Y_{n,1} + Q_0Q_1 Y_{n,2}. \end{aligned} \tag{A.22}$$

A.1.4 Welfare Decomposition

We suppress date indices and denote the change in period utility by $\mathrm{d}u_n \equiv \mathrm{d}\log C_n$. Differentiating $E_n = P_n(C_n + i^1_n)$ and using equation (A.8), $\mathrm{d}E_n = P_nC_n\mathrm{d}u_n + P_n\mathrm{d}i^1_n + \sum_{j,m} q^j_{mn}\mathrm{d}p^j_{mn}$. The household budget also implies $\mathrm{d}E_n = \mathrm{d}(w_nL_n + r^1_nk^1_n) + \mathrm{d}\Pi^1_n + \mathrm{d}(b_n - Qb_{n,+1})$, where $+1$ denotes the next date. Equating these expressions,

$$\begin{aligned} P_nC_n\,\mathrm{d}u_n = \mathrm{d}(w_nL_n + r^1_nk^1_n) + \mathrm{d}\Pi^1_n + \mathrm{d}(b_n - Qb_{n,+1}) \\ - P_n\,\mathrm{d}i^1_n - \sum_{j,m} q^j_{mn}\,\mathrm{d}p^j_{mn}. \end{aligned} \tag{A.23}$$

To express factor income in terms of marginal costs, log-differentiate equation (A.11): $\mathrm{d}\log \mathrm{mc}^1_n = -\mathrm{d}\log A^1_n + \alpha\mathrm{d}\log r^1_n + (1-\alpha)\mathrm{d}\log w_n$. Multiplication by V^1_n and substitution of the factor payments yield $V^1_n\mathrm{d}\log \mathrm{mc}^1_n = -V^1_n\mathrm{d}\log A^1_n + k^1_n\mathrm{d}r^1_n + \ell^1_n\mathrm{d}w_n$. In sector 2, $V^2_n\mathrm{d}\log \mathrm{mc}^2_n = \ell^2_n\mathrm{d}w_n$. Adding these identities, imposing $\ell^1_n + \ell^2_n = L_n$, and using fixed L_n,

$$\begin{aligned} \mathrm{d}(w_nL_n + r^1_nk^1_n) &= L_n\,\mathrm{d}w_n + k^1_n\,\mathrm{d}r^1_n + r^1_n\,\mathrm{d}k^1_n \\ &= V^1_n\,\mathrm{d}\log A^1_n + r^1_n\,\mathrm{d}k^1_n + \sum_j V^j_n\,\mathrm{d}\log \mathrm{mc}^j_n. \end{aligned} \tag{A.24}$$

Denote country n's trading partner by $\bar{n}$. Fixed markups imply $q^j_{mn}\mathrm{d}p^j_{mn} = X^j_{mn}\mathrm{d}\log \mathrm{mc}^j_m$. Substituting $V^1_n = R^1_n - \Pi^1_n$, $V^2_n = R^2_n$, and $R^j_n = X^j_{nn} + X^j_{n\bar{n}}$ into the price

terms,

$$\begin{aligned}&\sum_j V_n^j \,\mathrm{d}\log \mathrm{mc}_n^j - \sum_{j,m} q_{mn}^j \,\mathrm{d}p_{mn}^j \\ &\quad = \underbrace{\sum_j \left[X_{n\bar{n}}^j \,\mathrm{d}\log \mathrm{mc}_n^j - X_{\bar{n}n}^j \,\mathrm{d}\log \mathrm{mc}_{\bar{n}}^j\right]}_{\mathrm{d}\mathcal{W}_n^{\mathrm{ToT}}} - \Pi_n^1 \,\mathrm{d}\log \mathrm{mc}_n^1. \end{aligned} \tag{A.25}$$

The domestic-sales terms cancel because their revenue and expenditure changes use the same price. Denote the remaining profit contribution by $\mathrm{d}\mathcal{W}_n^{\Pi,1} \equiv \mathrm{d}\Pi_n^1 - \Pi_n^1 \mathrm{d}\log \mathrm{mc}_n^1$. Differentiating $\Pi_n^1 = \varrho^1 \sum_m p_{nm}^1 q_{nm}^1$,

$$\mathrm{d}\mathcal{W}_n^{\Pi,1} = \varrho^1 \sum_m \left[p_{nm}^1 \,\mathrm{d}q_{nm}^1 + \underbrace{q_{nm}^1 \,\mathrm{d}p_{nm}^1 - p_{nm}^1 q_{nm}^1 \frac{\mathrm{d}p_{nm}^1}{p_{nm}^1}}_{0}\right] = \varrho^1 \sum_m p_{nm}^1 \,\mathrm{d}q_{nm}^1. \tag{A.26}$$

Substituting equations (A.24)–(A.26) into the differentiated budget establishes

$$\begin{aligned}P_n C_n \,\mathrm{d}u_n = {} & V_n^1 \,\mathrm{d}\log A_n^1 + r_n^1 \,\mathrm{d}k_n^1 - P_n \,\mathrm{d}i_n^1 + \mathrm{d}(b_n - Qb_{n,+1}) \\ & + \mathrm{d}\mathcal{W}_n^{\mathrm{ToT}} + \mathrm{d}\mathcal{W}_n^{\Pi,1}. \end{aligned} \tag{A.27}$$

This is equation (3) in the main text. Since $\mathrm{d}U_n = \sum_{t=0}^2 \beta^t \mathrm{d}u_{n,t}$, each period's monetary contributions enter lifetime welfare with weight $\beta^t/(P_{n,t}C_{n,t})$.

A.1.5 Equilibrium Responses around Symmetry

We now derive the local equilibrium relations used in both proofs and in the capital-response calculation. Denote $\widehat{x}_n \equiv \mathrm{d}\log x_n$ and $\widetilde{x} \equiv \widehat{x}_h - \widehat{x}_f$. At the symmetric balanced-trade allocation, domestic expenditure shares equal $s \in (1/2, 1)$ in both sectors, so $R^j = a^j E$, $X^{j,x} = (1-s)R^j$, and $Y = R^1 + R^2 = E$. Throughout this subsection, relative expenditure can change.

Revenue and Factor Prices Denote $\theta \equiv \sigma - 1$ and suppress destination indices on producer prices, since trade is costless. The price derivatives established above specialize

to $\widehat{P}_h^j = s\widehat{p}_h^j + (1-s)\widehat{p}_f^j$ and $\widehat{P}_f^j = (1-s)\widehat{p}_h^j + s\widehat{p}_f^j$. Log-differentiating equation (A.7),

$$\widehat{X}_{nm}^j = \widehat{E}_m - \theta(\widehat{p}_n^j - \widehat{P}_m^j). \tag{A.28}$$

The domestic and export shares in Home's revenue are s and $1-s$. Substituting the price derivatives into $\widehat{R}_h^j = s\widehat{X}_{hh}^j + (1-s)\widehat{X}_{hf}^j$,

$$\begin{aligned}\widehat{R}_h^j &= s[\widehat{E}_h - \theta(1-s)(\widehat{p}_h^j - \widehat{p}_f^j)] + (1-s)[\widehat{E}_f - \theta s(\widehat{p}_h^j - \widehat{p}_f^j)] \\ &= s\widehat{E}_h + (1-s)\widehat{E}_f - 2s(1-s)\theta(\widehat{p}_h^j - \widehat{p}_f^j).\end{aligned} \tag{A.29}$$

Exchanging country indices gives Foreign's revenue response. Subtracting it from Home's and using fixed markups,

$$\widetilde{R}^j = (2s-1)\widetilde{E} - 4s(1-s)\theta\widetilde{\mathrm{mc}}^j = \kappa\widetilde{E} - \eta\widetilde{\mathrm{mc}}^j, \tag{A.30}$$

where $\kappa \equiv 2s-1$ and $\eta \equiv 4s(1-s)\theta$. Denote Home's relative sector-1 cost advantage by $\Delta \equiv -\widetilde{\mathrm{mc}}^1$, without restricting its sign. Then $\widetilde{R}^1 = \kappa\widetilde{E} + \eta\Delta$ and $\widetilde{R}^2 = \kappa\widetilde{E} - \eta\widetilde{w}$.

Labor-market clearing implies $w_n L_n = (1-\alpha)R_n^1/\mu^1 + R_n^2$. Denote sector 1's labor-income share by $\lambda_L^1 \equiv [(1-\alpha)a^1/\mu^1]/[(1-\alpha)a^1/\mu^1 + a^2]$. Differentiating with fixed labor supply and substituting the revenue responses, $\widetilde{w} = \lambda_L^1(\kappa\widetilde{E} + \eta\Delta) + (1-\lambda_L^1)(\kappa\widetilde{E} - \eta\widetilde{w})$. Collecting wage terms and denoting $D \equiv 1 + (1-\lambda_L^1)\eta$,

$$\widetilde{w} = \frac{\kappa\widetilde{E} + \lambda_L^1\eta\Delta}{D}. \tag{A.31}$$

Capital payments satisfy $r_n^1 k_n^1 = \alpha R_n^1/\mu^1$. Denote relative sector-1 capital and productivity changes by $\widetilde{K}$ and $\widetilde{A}$, and the relative rental-rate change by $\widetilde{r}$. Log differentiation yields $\widetilde{r} + \widetilde{K} = \widetilde{R}^1$, so $\widetilde{r} = \kappa\widetilde{E} + \eta\Delta - \widetilde{K}$.

Marginal Costs, Aggregate Prices, and Income The relative marginal-cost equation is $-\Delta = -\widetilde{A} + \alpha\widetilde{r} + (1-\alpha)\widetilde{w}$. Substituting both factor prices and collecting terms in Δ,

$$\begin{aligned}-\Delta &= -\widetilde{A} + \alpha(\kappa\widetilde{E} + \eta\Delta - \widetilde{K}) + \frac{1-\alpha}{D}(\kappa\widetilde{E} + \lambda_L^1\eta\Delta), \\ \underbrace{\left[1 + \alpha\eta + \frac{(1-\alpha)\lambda_L^1\eta}{D}\right]}_{H}\Delta &= \widetilde{A} + \alpha\widetilde{K} - \kappa\left[\alpha + \frac{1-\alpha}{D}\right]\widetilde{E}.\end{aligned} \tag{A.32}$$

Since $1+\eta = D+\lambda_L^1\eta$, the expenditure coefficient satisfies $(1+\eta)[\alpha+(1-\alpha)/D] = 1+\alpha\eta+(1-\alpha)\lambda_L^1\eta/D = H$. Denoting $\zeta \equiv \kappa/(1+\eta)$, division by H yields

$$\Delta = \frac{\widetilde{A}+\alpha\widetilde{K}}{H} - \zeta\widetilde{E}. \tag{A.33}$$

Substitute this expression into the factor-price equations. The expenditure coefficients simplify using $\kappa - \lambda_L^1\eta\zeta = D\zeta$ and $\kappa - \eta\zeta = \zeta$, so

$$\begin{aligned} \widetilde{w} &= \zeta\widetilde{E} + \frac{\lambda_L^1\eta}{DH}(\widetilde{A}+\alpha\widetilde{K}), \\ \widetilde{r} &= \zeta\widetilde{E} + \frac{\eta}{H}\widetilde{A} - \xi\widetilde{K}, \end{aligned} \tag{A.34}$$

where $\xi \equiv 1-\alpha\eta/H$. The same coefficients apply at each baseline date because the sectoral revenue shares are constant.

Subtracting the sectoral price derivatives across countries gives $\widetilde{P}^j = \kappa\widetilde{\mathrm{mc}}^j$, hence $\widetilde{P} = \kappa(-a^1\Delta + a^2\widetilde{w})$. Substitution of the cost and wage responses reduces the term in parentheses to $\zeta\widetilde{E} - [a^1 - a^2\lambda_L^1\eta/D](\widetilde{A}+\alpha\widetilde{K})/H$. Denote $J \equiv a^1 - a^2\lambda_L^1\eta/D$ and $\pi \equiv \kappa\zeta$. Then

$$\widetilde{P} = \pi\widetilde{E} - \frac{\kappa J}{H}(\widetilde{A}+\alpha\widetilde{K}). \tag{A.35}$$

Income satisfies $Y_n = R_n^1 + R_n^2$, with baseline shares a^1 and a^2. Therefore, $\widetilde{Y} = a^1\widetilde{R}^1 + a^2\widetilde{R}^2 = \kappa\widetilde{E} + \eta(a^1\Delta - a^2\widetilde{w})$. Using the same substitution and $\kappa - \eta\zeta = \zeta$,

$$\widetilde{Y} = \zeta\widetilde{E} + \nu(\widetilde{A}+\alpha\widetilde{K}), \tag{A.36}$$

where $\nu \equiv \eta J/H$. The cost, factor-price, and income responses above will be used without rederivation in the capital calculation and the proofs.

The signs needed below follow from the primitive restrictions. In particular, $0 < \zeta < 1/(1+\eta)$ and $0 < \pi < 1$, while the labor-share identity implies

$$J = a^1\left[1 - \frac{1-\alpha}{\mu^1}\frac{(1-\lambda_L^1)\eta}{1+(1-\lambda_L^1)\eta}\right] \in (0, a^1). \tag{A.37}$$

Also, $H - \alpha\eta = 1+(1-\alpha)\lambda_L^1\eta/D > 1$, so $\xi > 0$. Combining this bound with $1-\zeta > \eta\zeta$

and $\alpha J < 1$,

$$\xi(1-\zeta) - \alpha\nu\zeta = \frac{(H-\alpha\eta)(1-\zeta) - \alpha\eta J\zeta}{H} > \frac{\eta\zeta(1-\alpha J)}{H} > 0. \tag{A.38}$$

Terms of Trade and Relative Profit Shifting At symmetry, exports and imports equal $X^{j,x}$ in each sector. Substitution into the terms-of-trade contribution defined above gives $\mathrm{d}\mathcal{W}_h^{\mathrm{ToT}} = X^{1,x}\widetilde{\mathrm{mc}}^1 + X^{2,x}\widetilde{w}$. Using the general wage response,

$$\mathrm{d}\mathcal{W}_h^{\mathrm{ToT}} = \frac{\kappa X^{2,x}}{D}\widetilde{E} - \left(X^{1,x} - \frac{\lambda_L^1\eta}{D}X^{2,x}\right)\Delta. \tag{A.39}$$

For profits, $\Pi_n^1 = \varrho^1 R_n^1$ implies $\mathrm{d}\mathcal{W}_n^{\Pi,1} = \varrho^1 R^1(\widehat{R}_n^1 - \widehat{\mathrm{mc}}_n^1)$. Denote the deviation from the two-country average by $\mathrm{d}\mathcal{W}_h^{\Pi,1,\mathrm{rel}} \equiv (\mathrm{d}\mathcal{W}_h^{\Pi,1} - \mathrm{d}\mathcal{W}_f^{\Pi,1})/2$. Subtracting the country responses and using equation (A.30),

$$\mathrm{d}\mathcal{W}_h^{\Pi,1,\mathrm{rel}} = \frac{\varrho^1 R^1}{2}(\widetilde{R}^1 - \widetilde{\mathrm{mc}}^1) = \frac{\varrho^1 R^1}{2}[\kappa\widetilde{E} + (1+\eta)\Delta]. \tag{A.40}$$

Exchanging country indices reverses the signs of $\widetilde{E}$ and Δ. Foreign's terms-of-trade and relative profit-shifting contributions are, therefore, the negatives of Home's.

Baseline Expenditure and Rental Weights At the symmetric baseline, equal bond holdings and bond-market clearing imply $b_{h,t} = b_{f,t} = 0$. Since investment occurs only at date 0, $E_t = P_t C_t$ at dates 1 and 2. The consumption Euler equation implies $Q_0 E_1 = \beta P_0 C_0$ and $Q_0 Q_1 E_2 = \beta^2 P_0 C_0$. Denote $c \equiv P_0 C_0/E_0 > 0$, $T \equiv \beta(1+\beta)$, and $g \equiv \alpha a^1/\mu^1$. Symmetric revenue shares imply $r_t^1 k_t^1 = \alpha R_t^1/\mu^1 = gE_t$. Multiplying the investment Euler equation by k_1^1,

$$\begin{aligned} P_0 k_1^1 &= Q_0 r_1^1 k_1^1 + Q_0 Q_1 r_2^1 \underbrace{(1-\delta)k_1^1}_{k_2^1} \\ &= g(Q_0 E_1 + Q_0 Q_1 E_2) = gTP_0 C_0. \end{aligned} \tag{A.41}$$

Thus $P_0 k_1^1/E_0 = gTc$. Dividing the date-1 rental payment by this capital value gives $Q_0 r_1^1/P_0 = gQ_0 E_1/(P_0 k_1^1) = \beta/T = 1/(1+\beta)$. The corresponding date-2 weight is $(1-\delta)Q_0 Q_1 r_2^1/P_0 = gQ_0 Q_1 E_2/(P_0 k_1^1) = \beta^2/T = \beta/(1+\beta)$.

A.1.6 Capital Response to an Anticipated Productivity Increase

At date 0, households learn that Home's sector-1 productivity will rise by $\widehat{A}^1_{h,1} = \varepsilon > 0$ at date 1 and return to baseline at date 2. Denote $K \equiv \widetilde{k}^1 \equiv \widehat{k}^1_{h,1} - \widehat{k}^1_{f,1}$. Since $k^1_{n,2} = (1-\delta)k^1_{n,1}$, differentiation gives $\widehat{k}^1_{n,2} = \widehat{k}^1_{n,1}$, so relative capital changes by K at both dates. We establish $K > 0$ while allowing relative expenditure to adjust.

Investment and Expenditure Equations Denote the common relative change in nominal consumption expenditure by e and relative total expenditure at date 0 by e_0. The consumption Euler equation established above implies $\widetilde{E}_1 = \widetilde{E}_2 = e$, since investment occurs only at date 0. Initial capital and date-0 productivity do not change, so the common equilibrium relations imply $\widetilde{P}_0 = \pi e_0$ and $\widetilde{Y}_0 = \zeta e_0$.

Differentiate the investment Euler equation using the baseline rental weights derived below equation (A.41): $\widehat{P}_{n,0} = \widehat{Q}_0 + [\widehat{r}^1_{n,1} + \beta \widehat{r}^1_{n,2} + \beta \widehat{Q}_1]/(1+\beta)$. Subtracting across countries eliminates the common bond-price changes. Substitute the rental responses from equation (A.34):

$$\pi e_0 = \frac{\zeta e + (\eta/H)\varepsilon - \xi K + \beta(\zeta e - \xi K)}{1+\beta} = \zeta e + \frac{\eta \varepsilon}{H(1+\beta)} - \xi K. \tag{A.42}$$

For date-0 expenditure, differentiate $E_{n,0} = P_{n,0}C_{n,0} + P_{n,0}i^1_{n,0}$. Fixed initial capital implies $\mathrm{d}i^1_{h,0} - \mathrm{d}i^1_{f,0} = k^1_1 K$. With the baseline coefficients c, g, and T defined above, division by E_0 yields $e_0 = ce + (1-c)\widetilde{P}_0 + gTcK$. Substituting $\widetilde{P}_0 = \pi e_0$ and collecting terms,

$$Ze_0 = ce + gTcK, \tag{A.43}$$

where $Z \equiv 1 - (1-c)\pi = 1 - \pi + \pi c > 0$. Thus $e_0 = c(e + gTK)/Z$.

Next differentiate the intertemporal budget. The bond-price terms cancel at the baseline because $Y_t = E_t$: $\mathrm{d}Q_0(Y_1 - E_1) + \mathrm{d}(Q_0Q_1)(Y_2 - E_2) = 0$. Subtracting across countries, dividing by E_0, and using $Q_0E_1/E_0 = c\beta$ and $Q_0Q_1E_2/E_0 = c\beta^2$ gives $e_0 + cTe = \widetilde{Y}_0 + c\beta\widetilde{Y}_1 + c\beta^2\widetilde{Y}_2$. Substituting the income responses,

$$\begin{aligned} e_0 + cTe &= \zeta e_0 + c\beta[\zeta e + \nu(\varepsilon + \alpha K)] + c\beta^2(\zeta e + \nu\alpha K) \\ &= \zeta e_0 + cT\zeta e + c\beta\nu\varepsilon + cT\nu\alpha K. \end{aligned} \tag{A.44}$$

Move the expenditure terms on the right to the left. Equations (A.42)–(A.44) are the three equations determining e_0, e, and K.

Solution and Sign of the Capital Response Eliminate e_0 using $e_0 = c(e + gTK)/Z$. In the investment equation, multiply by Z; in the lifetime budget, multiply by Z/c. Collecting the coefficients of e and K,

$$\begin{aligned}(\pi c - \zeta Z)e + (\xi Z + \pi g T c)K &= \frac{\eta Z}{H(1+\beta)}\varepsilon, \\ (1-\zeta)(1+TZ)e + T[g(1-\zeta) - \alpha\nu Z]K &= \beta\nu Z\varepsilon. \end{aligned} \tag{A.45}$$

Multiply the first equation by $(1-\zeta)(1+TZ)$ and the second by $\pi c - \zeta Z$, then subtract and divide by Z. Using $\nu = \eta J/H$, the resulting equation is $\mathcal{D}K = (\eta/H)\mathcal{N}\varepsilon$, where

$$\begin{aligned}\mathcal{N} &\equiv \frac{(1-\zeta)(1+TZ)}{1+\beta} - \beta J(\pi c - \zeta Z), \\ \mathcal{D} &\equiv \frac{(1-\zeta)(1+TZ)(\xi Z + \pi g T c) - (\pi c - \zeta Z)T[g(1-\zeta) - \alpha\nu Z]}{Z}. \end{aligned} \tag{A.46}$$

The signs are clearer after expanding these coefficients. Substituting $T = \beta(1+\beta)$ and $\pi c = Z - 1 + \pi$ into the numerator,

$$\begin{aligned}\mathcal{N} &= \frac{1-\zeta}{1+\beta} + \beta[(1-\zeta) + J\zeta]Z - \beta J(Z - 1 + \pi) \\ &= \frac{1-\zeta}{1+\beta} + \beta J(1-\pi) + \beta(1-\zeta)(1-J)Z > 0. \end{aligned} \tag{A.47}$$

Each term is positive by the bounds established in the common equilibrium derivation. For the denominator, expand its defining expression to obtain $\mathcal{D} = (1-\zeta)\xi(1+TZ) + gT(1-\zeta)(\pi c T + \zeta) + \alpha\nu T(\pi c - \zeta Z)$. Substituting $Z = 1 - \pi + \pi c$ and grouping terms,

$$\begin{aligned}\mathcal{D} = (1-\zeta)\xi + gT\zeta(1-\zeta) + T(1-\pi)[\xi(1-\zeta) - \alpha\nu\zeta] \\ + T\pi c(1-\zeta)(\xi + gT + \alpha\nu) > 0. \end{aligned} \tag{A.48}$$

All coefficients are positive; equation (A.38) establishes the sign of the bracketed term. Dividing the capital equation by $\mathcal{D}$, we obtain

$$K = \underbrace{\frac{\eta}{H}\frac{\mathcal{N}}{\mathcal{D}}}_{\chi_K > 0}\varepsilon > 0. \tag{A.49}$$

Thus the positive relative capital response follows from household optimization and market clearing at an interior investment optimum. To recover expenditure, substitute

$K = \chi_K \varepsilon$ into the second equation of (A.45) and then into the date-0 expenditure equation:

$$e = \frac{\beta \nu Z - T[g(1-\zeta) - \alpha \nu Z]\chi_K}{(1-\zeta)(1+TZ)}\varepsilon, \qquad e_0 = \frac{c(e + gT\chi_K \varepsilon)}{Z}. \tag{A.50}$$

A.1.7 Proof of Proposition 1

Set $\widetilde{E} = 0$ and $\Delta > 0$ in the equilibrium relations derived above. The wage response is

$$\widetilde{w} = \frac{\lambda_L^1 \eta}{D}\Delta > 0. \tag{A.51}$$

Equation (A.39) then reduces to $\mathrm{d}\mathcal{W}_h^{\mathrm{ToT}} = -X^{1,x}\Delta + X^{2,x}\widetilde{w}$. Using $X^{2,x}/X^{1,x} = R^2/R^1$ and $\lambda_L^1/(1-\lambda_L^1) = [(1-\alpha)R^1/\mu^1]/R^2$, the ratio of the sector-2 gain to the sector-1 loss is

$$\frac{X^{2,x}\widetilde{w}}{X^{1,x}\Delta} = \frac{R^2}{R^1}\frac{\lambda_L^1 \eta}{D} = \frac{1-\alpha}{\mu^1}\frac{(1-\lambda_L^1)\eta}{1+(1-\lambda_L^1)\eta} < 1. \tag{A.52}$$

Both factors are positive and less than one. Therefore, the total terms-of-trade contribution is negative.

Under the same expenditure restriction, equation (A.40) becomes

$$\mathrm{d}\mathcal{W}_h^{\Pi,1,\mathrm{rel}} = \frac{\varrho^1 R^1}{2}(1+\eta)\Delta > 0. \tag{A.53}$$

To compare this gain with the sector-1 terms-of-trade loss, substitute $\varrho^1 = 1/(1+\theta)$, $\eta = 4s(1-s)\theta$, and $X^{1,x} = (1-s)R^1$:

$$\begin{aligned}\mathrm{d}\mathcal{W}_h^{\Pi,1,\mathrm{rel}} - X^{1,x}\Delta &= \frac{R^1\Delta}{2(1+\theta)}[1 + 4s(1-s)\theta - 2(1-s)(1+\theta)] \\ &= \frac{R^1(2s-1)[1+2(1-s)\theta]}{2(1+\theta)}\Delta > 0.\end{aligned} \tag{A.54}$$

The relative profit gain exceeds the sector-1 loss alone. Adding the positive sector-2 contribution establishes $\mathrm{d}\mathcal{W}_h^{\Pi,1,\mathrm{rel}} > -\mathrm{d}\mathcal{W}_h^{\mathrm{ToT}} > 0$. The opposite signs for Foreign follow from the country symmetry established above.

A.1.8 Proof of Corollary 1

Equation (A.49) establishes $K = \widetilde{k}^1 > 0$, and common depreciation preserves this relative capital response at date 2. Productivity has returned to baseline, so $\widetilde{A}_2 = 0$. Holding

relative date-2 expenditure fixed, equation (A.33) reduces to

$$\Delta_2 = \underbrace{\frac{\alpha}{H}}_{\lambda_K>0} K > 0. \tag{A.55}$$

Apply Proposition 1 at the date-2 allocation and substitute this cost advantage:

$$\begin{aligned} \mathrm{d}\mathcal{W}_{h,2}^{\mathrm{ToT}} &= -\left(X_2^{1,x} - \frac{\lambda_L^1 \eta}{D} X_2^{2,x} \right) \lambda_K K < 0, \\ \mathrm{d}\mathcal{W}_{h,2}^{\Pi,1,\mathrm{rel}} &= \frac{\varrho^1 R_2^1}{2} (1+\eta) \lambda_K K > -\mathrm{d}\mathcal{W}_{h,2}^{\mathrm{ToT}} > 0. \end{aligned} \tag{A.56}$$

These are the claimed date-2 contributions. Their signs and ranking follow from the proposition, completing the proof.[15]

A.1.9 Saving and Investment with an International Bond

We derive a two-period household's choices and their differentials before considering changes in current income and future productivity. All quantities are measured in units of the reference country's final good, and the gross bond return $R > 0$ is fixed. The household solves

$$\begin{aligned} \max_{c_1, c_2, i, b} \quad \log c_1 + \beta \log c_2 \quad \text{s.t.} \quad & c_1 + j(i) + b = y_1(z_1), \\ & c_2 = x(\bar{k} + i, z_2) + Rb, \end{aligned} \tag{A.57}$$

where y_1 is current income, $j(i)$ is installation cost, $\bar{k}$ is predetermined capital, and $x(\bar{k} + i, z_2)$ is the future rental and continuation payoff. The variables z_1 and z_2 denote current and future productivity, and $b > 0$ denotes international lending. We consider an interior solution with positive consumption, $j' > 0$, $j'' \geq 0$, $x_k > 0$, $x_{kk} \leq 0$, and $Rj'' - x_{kk} > 0$.

Optimal Choices and Their Differentials With budget multipliers Λ_1 and Λ_2, the Lagrangian is $\mathcal{L} = \log c_1 + \beta \log c_2 + \Lambda_1[y_1 - c_1 - j(i) - b] + \Lambda_2[x(\bar{k} + i, z_2) + Rb - c_2]$. The consumption conditions are $1/c_1 - \Lambda_1 = 0$ and $\beta/c_2 - \Lambda_2 = 0$. Substituting them

[15] The capital response is determined with endogenous expenditure. The corollary evaluates its contribution holding relative date-2 expenditure fixed. With endogenous date-2 expenditure, $\Delta_2 = \alpha K/H - \zeta e$; $K > 0$ alone does not establish the sign of this total cost response.

into the bond condition $-\Lambda_1 + R\Lambda_2 = 0$ yields $1/c_1 = \beta R/c_2$, hence $c_2 = \beta R c_1$. The investment condition is $-\Lambda_1 j'(i) + \Lambda_2 x_k(\bar{k} + i, z_2) = 0$. Dividing by Λ_2,

$$x_k(\bar{k} + i, z_2) = \frac{\Lambda_1}{\Lambda_2} j'(i) = R j'(i). \tag{A.58}$$

Denote its solution by $i^*(z_2; R)$. Since $\partial[x_k - Rj']/\partial i = x_{kk} - Rj'' < 0$, there is at most one interior investment solution.

Eliminate bonds by substituting $b = y_1 - c_1 - j(i^*)$ into the second budget. This yields $c_1 + c_2/R = y_1 + x(\bar{k} + i^*, z_2)/R - j(i^*)$. Denote the investment's net present value by $\mathcal{V}(z_2; R) \equiv x(\bar{k} + i^*, z_2)/R - j(i^*)$. Substituting $c_2 = \beta R c_1$, we obtain $(1 + \beta)c_1 = y_1 + \mathcal{V}$, so $c_1 = (y_1 + \mathcal{V})/(1 + \beta)$ and $c_2 = \beta R(y_1 + \mathcal{V})/(1 + \beta)$. Consequently,

$$s_1 \equiv y_1 - c_1 = \frac{\beta y_1 - \mathcal{V}}{1 + \beta}, \qquad b = s_1 - j(i^*). \tag{A.59}$$

At fixed R and $\bar{k}$, differentiate the investment condition: $x_{kk}\mathrm{d}i^* + x_{kz_2}\mathrm{d}z_2 = Rj''\mathrm{d}i^*$. Collecting the investment terms gives

$$\mathrm{d}i^* = \frac{x_{kz_2}}{Rj'' - x_{kk}} \mathrm{d}z_2. \tag{A.60}$$

Next differentiate the value function, including the induced investment change. The investment condition cancels its coefficient:

$$\mathrm{d}\mathcal{V} = \underbrace{\left(\frac{x_k}{R} - j'\right)}_{0} \mathrm{d}i^* + \frac{x_{z_2}}{R}\mathrm{d}z_2 = \frac{x_{z_2}}{R}\mathrm{d}z_2. \tag{A.61}$$

Finally, differentiate the saving and bond identities: $\mathrm{d}s_1 = [\beta \mathrm{d}y_1 - \mathrm{d}\mathcal{V}]/(1 + \beta)$ and $\mathrm{d}b = \mathrm{d}s_1 - j'\mathrm{d}i^*$. Substituting the preceding two differentials,

$$\mathrm{d}b = \frac{\beta}{1 + \beta}\mathrm{d}y_1 - \left[\frac{x_{z_2}}{R(1 + \beta)} + \frac{j' x_{kz_2}}{Rj'' - x_{kk}}\right] \mathrm{d}z_2. \tag{A.62}$$

Current Income and Future Productivity For a current-income change, set $\mathrm{d}z_2 = 0$ in the differentials above. Then $\mathrm{d}i^* = \mathrm{d}\mathcal{V} = 0$, so $\partial i^*/\partial y_1 = 0$ and

$$\frac{\partial b}{\partial y_1} = \frac{\beta}{1 + \beta} > 0. \tag{A.63}$$

For future productivity, hold y_1 fixed and assume $x_{z_2} > 0$ and $x_{kz_2} > 0$. Equations (A.60) and (A.61) imply $\partial i^*/\partial z_2 = x_{kz_2}/(Rj'' - x_{kk}) > 0$ and $\partial \mathcal{V}/\partial z_2 = x_{z_2}/R > 0$. Setting $\mathrm{d}y_1 = 0$ in equation (A.62),

$$\frac{\partial b}{\partial z_2} = -\frac{x_{z_2}}{R(1+\beta)} - \frac{j' x_{kz_2}}{Rj'' - x_{kk}} < 0. \tag{A.64}$$

For a change in z affecting both income and future productivity, denote $y_{1z} \equiv \mathrm{d}y_1/\mathrm{d}z$ and $z_{2z} \equiv \mathrm{d}z_2/\mathrm{d}z$. Substitute $\mathrm{d}y_1 = y_{1z}\mathrm{d}z$ and $\mathrm{d}z_2 = z_{2z}\mathrm{d}z$ into the same bond differential and divide by $\mathrm{d}z$:

$$\frac{\mathrm{d}b}{\mathrm{d}z} = \underbrace{\frac{\beta y_{1z}}{1+\beta}}_{\text{Current income}} - \underbrace{\frac{x_{z_2} z_{2z}}{R(1+\beta)}}_{\text{Future income}} - \underbrace{\frac{j' x_{kz_2} z_{2z}}{Rj'' - x_{kk}}}_{\text{Domestic investment}}. \tag{A.65}$$

For $y_{1z} > 0$ and $z_{2z} > 0$, the first term is positive and the other two are negative. Their sum determines the sign of the bond response.[16]

Financial Autarky Imposing $b = 0$ in the budgets yields $c_1 = y_1 - j(i)$ and $c_2 = x(\bar{k} + i, z_2)$. Substituting into utility, the household solves

$$\max_i \quad \log\left[y_1 - j(i)\right] + \beta \log x(\bar{k} + i, z_2) \quad \text{s.t.} \quad y_1 - j(i) > 0, \quad x(\bar{k} + i, z_2) > 0, \qquad \bar{k} + i > 0. \tag{A.66}$$

Denote the objective by $u(i)$. Differentiation gives $u'(i) = -j'(i)/[y_1 - j(i)] + \beta x_k(\bar{k} + i, z_2)/x(\bar{k} + i, z_2)$. Setting this derivative to zero,

$$\frac{j'(i)}{c_1} = \beta \frac{x_k}{c_2}. \tag{A.67}$$

Differentiating the two terms of $u'(i)$ gives $u''(i) = -j''/c_1 - (j')^2/c_1^2 + \beta x_{kk}/c_2 - \beta x_k^2/c_2^2 < 0$. The interior optimum is, therefore, unique when it exists. The first budget also implies $s_1 = y_1 - c_1 = j(i)$.

[16] Higher current income increases saving, while higher future income reduces it. The induced increase in investment further reduces international lending. These comparative statics hold the bond return fixed.

A.2 Model Derivations

A.2.1 Demand

Given $Z_{n,t}$, the final-good producer minimizes $\sum_j P^j_{n,t} Z^j_{n,t}$ subject to equation (7). The first-order conditions imply $P^j_{n,t} Z^j_{n,t} = \alpha^{F,j}_n P_{n,t} Z_{n,t}$, and substituting these demands into expenditure gives the price index in equation (8).

The sectoral-good producer minimizes $\sum_m p^j_{m,n,t} q^j_{m,n,t}$ subject to equation (9). The first-order condition for an origin variety is

$$p^j_{m,n,t} = \lambda^j_{n,t} \left(\gamma^j_{m,n}\right)^{1/\sigma^j} \left(q^j_{m,n,t}\right)^{-1/\sigma^j} \left(Q^j_{n,t}\right)^{1/\sigma^j},$$

where $\lambda^j_{n,t}$ is the multiplier on the aggregation constraint. Solving the first-order conditions and imposing the constraint gives the price index and variety demand in equation (10).

A.2.2 Production and Price Setting

For a given output level, a firm minimizes $w_{m,t} L^j_{m,t} + r^{K,j}_{m,t} K^j_{m,t} + \sum_k P^k_{m,t} M^{j,k}_{m,t}$ subject to equation (11). The Cobb–Douglas first-order conditions give equation (12) and the conditional input demands

$$w_{m,t} L^j_{m,t} = \ell^j_m V^j_{m,t}, \qquad r^{K,j}_{m,t} K^j_{m,t} = \kappa^j_m V^j_{m,t}, \qquad P^k_{m,t} M^{j,k}_{m,t} = \omega^{j,k}_m V^j_{m,t}. \tag{A.68}$$

Each production elasticity is therefore the corresponding input's share of variable cost.

In destination n, a firm chooses $p^j_{m,n,t}$ to maximize

$$\left[p^j_{m,n,t} - \tau^j_{m,n,t} \mathrm{mc}^j_{m,t}\right] q^j_{m,n,t}(p^j_{m,n,t}).$$

Variety demand has elasticity $-\sigma^j$, so the first-order condition is

$$q^j_{m,n,t} + \left[p^j_{m,n,t} - \tau^j_{m,n,t} \mathrm{mc}^j_{m,t}\right] \frac{\partial q^j_{m,n,t}}{\partial p^j_{m,n,t}} = 0.$$

Substituting the demand elasticity gives $p^j_{m,n,t} = \sigma^j (\sigma^j - 1)^{-1} \tau^j_{m,n,t} \mathrm{mc}^j_{m,t}$. Aggregating destination profits then gives equation (15).

A.2.3 Household Optimality Conditions

Let $m^j_{n,t} \equiv I^j_{n,t}/K^j_{n,t}$, $\lambda^{K,j}_{n,t} \equiv 1 + \phi_n(m^j_{n,t} - \delta_n)$, and $\chi^{K,j}_{n,t} \equiv \phi_n[\delta_n^2 - (m^j_{n,t})^2]/2$. Differentiating the household problem with respect to the international bond and sectoral investment gives

$$\begin{aligned} \frac{1}{C_{n,t}} &= \beta \mathcal{R}_{t+1} \exp(\xi_{n,t+1}) \frac{1}{C_{n,t+1}}, \\ \frac{P_{n,t}\lambda^{K,j}_{n,t}}{C_{n,t}} &= \beta \exp(\xi_{n,t+1}) \frac{r^{K,j}_{n,t+1} + P_{n,t+1}\left[(1-\delta_n)\lambda^{K,j}_{n,t+1} - \chi^{K,j}_{n,t+1}\right]}{C_{n,t+1}}. \end{aligned} \tag{A.69}$$

The first equation determines international consumption smoothing. The second equates the marginal installation cost of capital with its discounted rental and continuation value.

A.3 Quantification Details

The initial equilibrium combines the data sources and externally assigned parameters described in Section 4. Observed prices, input costs, and trade shares determine the paths of productivity and trade barriers through 2014. The simulated method of moments determines capital adjustment costs and intertemporal preference wedges, and the continuation rules extend productivity and trade barriers beyond 2014.

A.3.1 Parameter Mapping and Alternative Markups

Table A.1 summarizes the parameters set outside the model and the moments that identify the remaining objects. Baseline expenditure and production shares are constructed directly from the accounts described in Section 4.

Table A.1: Core Parameters and Identification

Object	Value or construction	Source or target
β	Annual discount factor, 0.98	Assigned externally
$\theta^j = \sigma^j - 1$	9.11 in agriculture, mining, and utilities; 7.10 in manufacturing; 4.49 in services and construction	External estimates and maintained assignments
μ^j	Markups implied by σ^j	CES pricing condition
δ_n	Country-specific depreciation rates	Penn World Table 10.01
$A^j_{n,t}$	Changes in productivity	Domestic-share exact-hat identity
$\tau^j_{m,n,t}$	Bilateral changes in trade barriers	Relative-share exact-hat identity
$\xi_{n,t}$	Intertemporal preference wedges	Trade-balance-to-GDP ratio
ϕ_n	Adjustment-cost curvatures	Standard deviation of the investment-to-GDP ratio

Notes: The agriculture and aggregate estimates are from Caliendo and Parro (2015). The manufacturing value is an observation-weighted average of their manufacturing estimates. The aggregate estimate is assigned to services and construction, while the agriculture estimate is assigned to mining and utilities.

The common-markup comparison preserves aggregate variable cost and the aggregate profit share at the allocation in 2000. With $R^j_{n,0}$ denoting baseline revenue, its markup is

$$\bar{\mu} = \frac{\sum_{n\in\mathcal{N}, j\in\mathcal{J}} R^j_{n,0}}{\sum_{n\in\mathcal{N}, j\in\mathcal{J}} R^j_{n,0}/\mu^j}. \tag{A.70}$$

The resulting value is 1.188. This comparison changes the distribution of markup wedges across sectors while holding the baseline aggregate profit share fixed.

A.3.2 Baseline Accounts and Initial Conditions

At the baseline markup, sector profit is $(1 - 1/\mu^j)R^j_{n,0}$, and variable cost is $R^j_{n,0}/\mu^j$. We subtract implied profit from WIOD capital compensation before constructing κ^j_n, then allocate the remaining variable cost to labor, capital, and intermediate inputs. Implied profit exceeds measured capital compensation in ROW agriculture; India light and consumer manufacturing; EU light and consumer manufacturing; and China local, public, real estate, and construction services. We set the capital share to zero in these cells and include the residual in labor and mixed income.

We convert industry capital stocks to the units of the input-output tables using the ratio of value added in those tables to total labor and capital compensation in the Socio-Economic Accounts. Country totals are allocated across sectors in proportion to capital payments net of profits, and initial investment is allocated using the same shares. The country depreciation rates are 0.03995 for the United States, 0.05494 for China, 0.05032

for India, 0.03864 for the European Union, and 0.04430 for the rest of the world.

Observed net external positions are expressed relative to GDP. The ratios in 2000 are -15.64 percent for the United States, 3.47 percent for China, -17.04 percent for India, and -9.94 percent for the European Union. We multiply these ratios by model value added and use the resulting values as the model's initial international bond positions. We choose the rest of the world's bond position to clear the global bond market.

A.3.3 Productivity, Trade Barriers, and Estimated Parameters

The input-cost index used in equation (37) combines labor compensation per worker, capital compensation net of profits per unit of capital, and WIOD sectoral price indices using initial production shares. The capital-price term is omitted in the four cells with a zero capital share. At each date, we remove the mean of log productivity across countries and sectors weighted by initial gross output. The resulting paths of productivity and bilateral trade barriers are therefore consistent with the construction of initial factor payments and the normalization by world productivity used in the welfare comparisons.

Let $\boldsymbol{\vartheta}$ collect the free intertemporal-wedge innovations and $\log \phi_n$, and let $\boldsymbol{m}(\boldsymbol{\vartheta})$ stack the differences between model-generated and observed moments. The SMM estimator solves

$$\widehat{\boldsymbol{\vartheta}} = \arg\min_{\boldsymbol{\vartheta}} \frac{1}{2} \boldsymbol{m}(\boldsymbol{\vartheta})' W' W \boldsymbol{m}(\boldsymbol{\vartheta}), \qquad W = \operatorname{diag}\Big(I_{60}, \sqrt{12}\, I_5 \Big) .$$

The first 60 moments are the annual trade-balance-to-GDP gaps for four economies over 2000–2014, and the remaining five are the country investment-volatility gaps. The weighting matrix gives the trade-balance and investment-volatility groups equal aggregate weight in the quadratic objective.

Only relative intertemporal preference wedges are identified, so we impose $\sum_n \xi_{n,t} = 0$ at each date. The largest gap between the model and observed trade balances is 0.059 percentage points of GDP, and the largest gap in investment volatility is 0.029 percentage points. Observed capital and investment determine the initial conditions, while the standard deviation of the investment-to-GDP ratio enters the SMM moment vector.

A.3.4 Continuation after 2014

For each country and sector, we estimate an autoregression for annual productivity growth in logs with a separate constant. The constant remains active for 20 years after 2014, after which growth follows the autoregressive decay without a constant. Growth

Table A.2: Fit of the Investment-Volatility Targets

Economy	Data volatility	Model volatility
United States	0.01836	0.01830
China	0.04153	0.04124
India	0.04297	0.04295
European Union	0.01013	0.01012
Rest of World	0.00714	0.00714

Notes: Volatility is the standard deviation of the annual investment-to-GDP ratio over 2000–2014.

in bilateral trade barriers follows an autoregressive decay without a constant, domestic trade barriers remain fixed at one, and all persistence estimates are bounded between -0.99 and 0.99.

The transition horizon extends until every endogenous variable reaches the stationary allocation associated with the terminal productivity and trade barriers. Convergence occurs before the terminal date, and Appendix A.7 reports the equilibrium and terminal-condition checks.

A.4 Additional Quantitative Results

This appendix reports additional quantitative results on financial openness, the duration of increases in productivity, capital adjustment, and markups. These results support the explanations developed in Section 5.

A.4.1 Financial Openness and the Duration of Increases in Productivity

Table A.3 varies the duration and magnitude of the increase in productivity. For every temporary duration considered, openness lowers China's welfare gain relative to autarky because part of the additional saving finances net international lending rather than domestic investment. A permanent increase reverses the comparison: the higher productivity of future capital raises desired investment above domestic saving, and international borrowing finances the difference.

Changing the magnitude of the ten-year increase does not change the sign of the welfare difference between openness and autarky. Larger changes scale both saving and investment, while additional saving remains greater than additional domestic investment throughout the reported range. Duration changes the balance because it determines how long capital installed today benefits from higher productivity.

Table A.3: Financial Openness across Increases in Productivity

Increase in productivity	Openness	Autarky	Difference		
	Welfare	Welfare	Welfare	Profit shifting	Share
	consumption-equivalent percent			or percentage points	percent
Panel A: Duration of a 10 percent increase in productivity					
One year	0.109	0.135	−0.026	−0.027	104.2
Five years	0.539	0.631	−0.092	−0.093	100.9
Ten years	1.049	1.175	−0.125	−0.120	96.0
Twenty years	1.958	2.090	−0.133	−0.112	84.2
Forty years	3.340	3.448	−0.108	−0.061	56.7
Permanent	6.142	6.098	0.044	0.045	103.9
Panel B: Magnitude of a ten-year increase in productivity					
1 percent	0.103	0.115	−0.013	−0.011	89.2
2.5 percent	0.257	0.289	−0.032	−0.029	90.2
5 percent	0.518	0.581	−0.063	−0.058	91.9
10 percent	1.049	1.175	−0.125	−0.120	96.0
20 percent	2.166	2.408	−0.242	−0.260	107.2

Notes: Productivity rises in China's light and consumer manufacturing sector. Financial openness allows every model economy to trade the one-period international bond, while financial autarky removes bond trade. The difference is welfare under openness minus welfare under autarky. Profit shifting reports its contribution to this difference, and Share is this contribution divided by the total difference. Each specification is compared with its own reference path.

Table A.4 reports the corresponding changes in saving, domestic investment, and the international bond position. Under the ten-year increase, China undertakes net international lending and increases its bond holdings under openness, ending year 10 with less aggregate and sectoral capital than under autarky. Under the permanent increase, China instead borrows, and openness supports more investment and capital. In each case, saving is allocated between domestic investment and net international lending.

Table A.4: Saving and Investment under Financial Openness and Autarky

Increase in productivity	Regime	Year	Consumption	Investment/GDP	Saving/GDP	Bond position /GDP	Capital	Sector capital
Ten-year increase	Open	1	0.660	1.986	3.913	1.744	0.000	0.000
	Autarky	1	3.437	1.576	1.576	0.000	0.000	0.000
	Open	10	1.580	−0.875	3.423	35.238	3.359	13.629
	Autarky	10	4.720	1.760	1.760	0.000	7.155	16.624
Permanent increase	Open	1	5.315	2.070	0.034	−2.275	0.000	0.000
	Autarky	1	3.917	1.139	1.139	0.000	0.000	0.000
	Open	10	5.980	0.428	0.319	−9.946	5.552	23.185
	Autarky	10	5.510	0.420	0.420	0.000	4.155	21.873

Notes: Entries are changes from the corresponding reference path for China. Consumption, aggregate capital, and capital in the affected sector are percentage changes. Investment/GDP, Saving/GDP, and the international bond position relative to GDP are percentage-point changes. A positive bond position is a claim on other economies. In the model, domestic saving is allocated between domestic investment and net international lending. Capital is predetermined in year 1, and the temporary increase remains active through year 10.

A.4.2 Capital Adjustment and International Borrowing and Lending

Table A.5 compares sector-specific and sectorally mobile capital, adjustment costs, and bond access across markup structures and changes in productivity and trade barriers. The differences between heterogeneous and common markups are small relative to the comparisons of these model features. Markup dispersion therefore has little effect on these comparisons, even though markups affect the welfare value of the resulting production changes.

Table A.5: Markups and Dynamic Adjustment in China

Change	Model feature	Heterogeneous markups	Common markup	Perfect competition	Heterogeneous minus common
		consumption-equivalent percentage points			
Permanent productivity	Sector-specific capital	0.018	0.023	0.001	-0.005
	Adjustment costs	-0.011	0.009	-0.018	-0.020
	International bond access	0.044	0.043	0.028	0.001
Ten-year productivity	Sector-specific capital	0.019	0.018	0.005	0.000
	Adjustment costs	0.012	0.020	-0.003	-0.009
	International bond access	-0.125	-0.125	-0.087	-0.001
Permanent trade barriers	Sector-specific capital	0.005	0.009	-0.003	-0.004
	Adjustment costs	-0.018	-0.007	-0.016	-0.011
	International bond access	0.018	0.015	0.014	0.003
Ten-year trade barriers	Sector-specific capital	0.006	0.007	-0.002	-0.002
	Adjustment costs	-0.027	-0.019	-0.021	-0.009
	International bond access	-0.018	-0.022	0.008	0.004

Notes: Each entry is welfare in the baseline model minus welfare when the indicated feature is removed. Sector-specific capital compares sector-specific with sectorally mobile capital. Adjustment costs compare the baseline model with costless adjustment. International bond access compares the baseline model with financial autarky. The changes reduce international manufacturing trade barriers by 10 percent.

Table A.6 separates welfare during the ten years of higher productivity from welfare during the subsequent adjustment. Most of the baseline welfare effect occurs after productivity returns to its reference path because the international bond position and inherited capital continue to affect consumption and production. Removing the international bond shifts substantially more of the effect into the period of higher productivity, whereas sectorally mobile capital, costless adjustment, and alternative markup specifications leave the broad timing pattern intact.

Table A.6: Welfare during and after the Temporary Increase in Productivity

Specification	Total gain consumption-equivalent percent	During increase percent of total gain	After increase
Baseline model	1.049	20.0	80.0
Sectorally mobile capital	1.031	18.2	81.8
No capital adjustment costs	1.038	19.6	80.4
Financial autarky	1.175	65.7	34.3
Common markup	1.080	19.5	80.5
Perfect competition	1.065	24.2	75.8

Notes: Productivity in China's light and consumer manufacturing sector is 10 percent above its reference path for ten years. Each specification is compared with its own reference path. The shares allocate the welfare effect over the full transition between the period of higher productivity and the subsequent adjustment.

A.4.3 Markup Wedges and Welfare Across Countries

Table A.7 isolates the welfare value of the average markup wedge by comparing a common markup with perfect competition. For China, the effect operates mainly through profit income from domestic sales, while the sum of the profit-shifting contributions across countries is economically negligible. The common markup therefore changes the distribution of profit income across countries without producing a comparable change in the world profit-shifting contribution.

Table A.7: Common Markups and Dynamic Adjustment

Panel A: Welfare effects for China

Increase in productivity	Model feature	Total	Productivity	Terms of trade	Profit shifting	Domestic sales	Foreign sales	Bond, capital, and investment
		consumption-equivalent percentage points						
Permanent	Sector-specific capital	0.0215	-0.0006	0.0019	0.0200	0.0273	-0.0074	0.0002
	International bond access	0.0145	-0.0436	0.0156	0.0451	0.0461	-0.0010	-0.0026
Ten-year	Sector-specific capital	0.0135	-0.0096	0.0057	0.0172	0.0292	-0.0120	0.0003
	International bond access	-0.0382	0.1170	-0.0307	-0.1266	-0.1297	0.0032	0.0021

Panel B: Profit shifting by country

Increase in productivity	Model feature	United States	China	India	European Union	Rest of world	World
		percent of world consumption					
Permanent	Sector-specific capital	-0.00011	0.00059	-0.00002	-0.00012	-0.00038	-0.00003
	International bond access	-0.00012	0.00173	0.00000	-0.00004	-0.00023	0.00133
Ten-year	Sector-specific capital	-0.00001	0.00059	-0.00001	-0.00003	-0.00007	0.00046
	International bond access	0.00052	-0.00511	0.00008	0.00020	0.00061	-0.00369

Notes: Panel A reports the effect under a common markup minus the effect under perfect competition for China. Sector-specific capital compares sector-specific and sectorally mobile capital, while international bond access compares the model with bond access to financial autarky. Domestic and foreign sales sum to profit shifting. The final column sums international bond payments, changes in installed capital valued at rental rates, and the resources used for investment and capital adjustment. Panel B reports profit shifting in percent of world consumption. The World column sums across countries and values quantity changes at the price-cost gaps implied by the common markup. All four world contributions are economically negligible; the permanent entry for sector-specific capital is also of the same order as the maximum numerical discrepancy in the decomposition.

Table A.8 varies sectoral markup dispersion while preserving the average price-cost wedge. Greater dispersion lowers profit shifting in both productivity exercises. The reason is not dispersion itself but the covariance between markup wedges and production changes: the expanding manufacturing sector has a wedge below the preserved mean, so widening dispersion lowers its wedge and makes the covariance less favorable.

Table A.8: Mean-Preserving Markup-Dispersion Sensitivity for China

		Permanent increase			Ten-year increase		
Scale λ	Markup range	Welfare	Profit shifting	Share	Welfare	Profit shifting	Share
0.0	1.188–1.188	6.299	3.039	48.2	1.080	0.499	46.2
0.5	1.148–1.205	6.215	2.828	45.5	1.063	0.460	43.3
1.0	1.110–1.223	6.142	2.622	42.7	1.049	0.424	40.4
1.5	1.074–1.241	6.080	2.419	39.8	1.038	0.388	37.4
2.0	1.041–1.259	6.028	2.219	36.8	1.028	0.353	34.3

Notes: The price-cost wedge is $\rho^j = 1 - 1/\mu^j$. The sensitivity sets $\rho^j(\lambda) = \bar{\rho} + \lambda(\rho^j - \bar{\rho})$, where $\bar{\rho}$ is the baseline-revenue-weighted mean. Thus $\lambda = 0$ gives the common markup, $\lambda = 1$ gives the baseline markups, and $\lambda = 2$ doubles wedge dispersion while preserving the mean. Demand and trade elasticities remain fixed. Welfare is a consumption-equivalent percent, profit shifting is in percentage points, and Share is profit shifting as a percent of welfare.

A.4.4 Additional Welfare Comparisons

Table A.9 evaluates the welfare effects of the changes inferred over 2000–2014 at shorter horizons. The values over the full transition differ materially from the values over five and ten years because capital and international bond positions continue to affect consumption after the observed changes end. This comparison motivates evaluating welfare over the complete transition rather than stopping in 2014.

Table A.9: Welfare Effects by Evaluation Horizon

Economy	5 years	10 years	20 years	Full transition
	consumption-equivalent percent			
United States	-13.97	-14.33	-12.52	-4.16
China	102.00	104.90	113.40	156.09
India	-10.57	-13.32	-19.59	-26.79
European Union	-10.74	-11.34	-9.45	0.56
Rest of World	12.24	13.20	11.01	21.20

Notes: Consumption-equivalent welfare effects from the paths of productivity and trade barriers inferred from annual data over 2000–2014, relative to a reference path in which productivity in every country and sector follows world productivity growth and bilateral trade barriers remain at their levels in 2000. Negative entries indicate welfare below this reference path. The first three columns evaluate welfare over the indicated horizon.

Table A.10 reports additional changes in productivity and trade barriers. Larger permanent changes strengthen the same qualitative responses as the baseline exercises, while temporary changes generate smaller welfare effects over the full transition. Bilateral U.S.–China barrier increases produce losses for the two directly affected economies, whereas multilateral reductions generate broader gains.

Table A.10: Welfare Effects of Changes in Productivity and Trade Barriers

Change	U.S.	China	India	EU	ROW
		consumption-equivalent percent			
Panel A. Increases in Productivity in China					
Manufacturing: +10%, permanent	0.008	6.142	-0.008	0.008	0.034
Manufacturing: +25%, permanent	0.033	16.48	-0.016	0.034	0.150
Manufacturing: +10%, ten-year	0.003	1.049	0.005	0.003	0.009
Agriculture: +10%, permanent	0.001	3.698	0.002	0.004	0.019
Panel B. Manufacturing Trade Barriers					
Global reduction: 10%, permanent	1.450	3.921	1.640	1.338	1.948
Global reduction: 25%, permanent	6.626	18.67	8.076	6.233	8.796
Global reduction: 10%, ten-year	0.274	0.723	0.357	0.246	0.349
U.S.–China increase: 10%, permanent	-0.023	-0.276	0.001	0.001	0.003
U.S.–China increase: 10%, ten-year	-0.005	-0.041	0.000	0.000	0.000

A.5 Country and Sector Aggregation

Table A.11 maps the WIOD countries to the five model economies, and Table A.12 maps the WIOD industries to the eight model sectors. The mappings remain fixed across years and welfare comparisons. The industry codes follow Revision 4 of the International Standard Industrial Classification (ISIC).

Table A.11: Country Aggregation

Model economy	WIOD countries
United States	United States
China	China
India	India
European Union	Austria, Belgium, Bulgaria, Croatia, Cyprus, Czech Republic, Denmark, Estonia, Finland, France, Germany, Greece, Hungary, Ireland, Italy, Latvia, Lithuania, Luxembourg, Malta, Netherlands, Poland, Portugal, Romania, Slovakia, Slovenia, Spain, Sweden, United Kingdom
Rest of the World	Australia, Brazil, Canada, Indonesia, Japan, Republic of Korea, Mexico, Norway, Russian Federation, Switzerland, Taiwan, Turkey, and the WIOD rest-of-world residual region

Notes: The United Kingdom is included in the European Union throughout the sample.

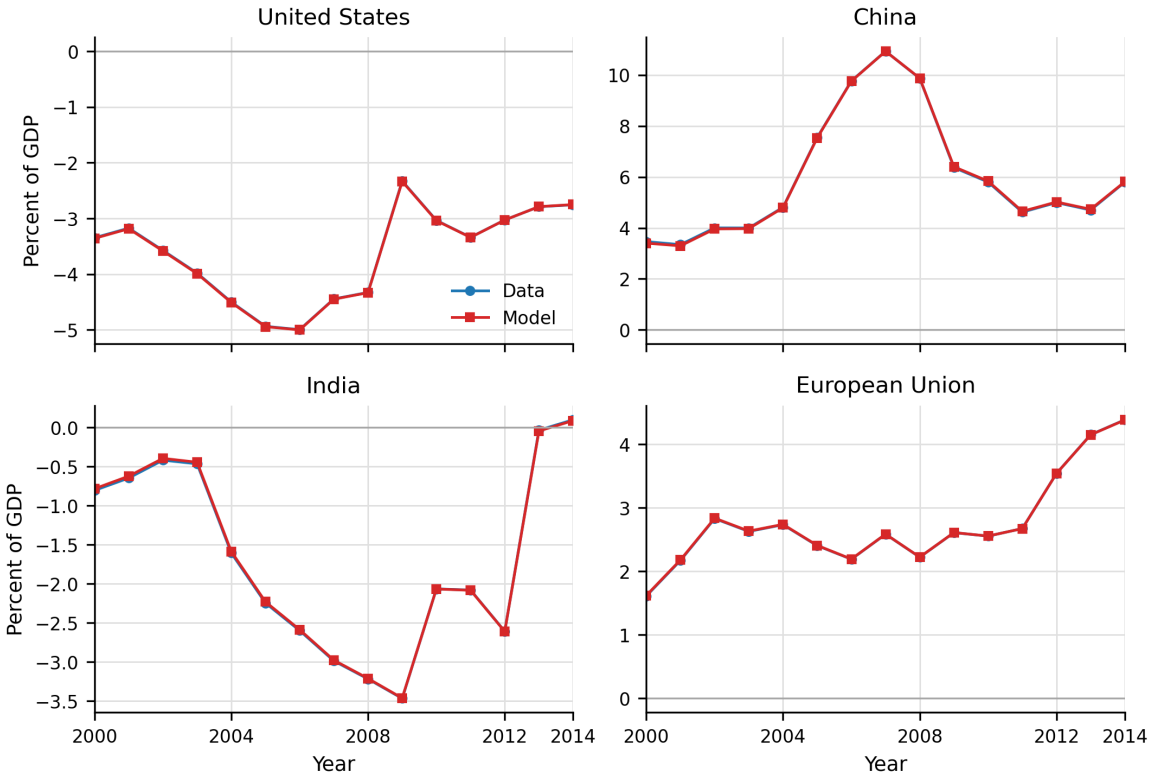
Table A.12: Sector Aggregation

Model sector	WIOD industries (ISIC Rev. 4)
Agriculture	A01, A02, A03
Mining and utilities	B, D35, E36, E37–E39
Light and consumer manufacturing	C10–C12, C13–C15, C16, C17, C18, C31–C32, C33
Materials and process manufacturing	C19, C20, C21, C22, C23, C24, C25
Machinery and durables manufacturing	C26, C27, C28, C29, C30
Distribution, transportation, and hospitality services	G45, G46, G47, H49, H50, H51, H52, H53, I
Information, finance, and business services	J58, J59–J60, J61, J62–J63, K64, K65, K66, M69–M70, M71, M72, M73, M74–M75, N
Local, public, real estate, and construction services	F, L68, O84, P85, Q, R–S, T, U

Notes: Industries follow ISIC Revision 4.

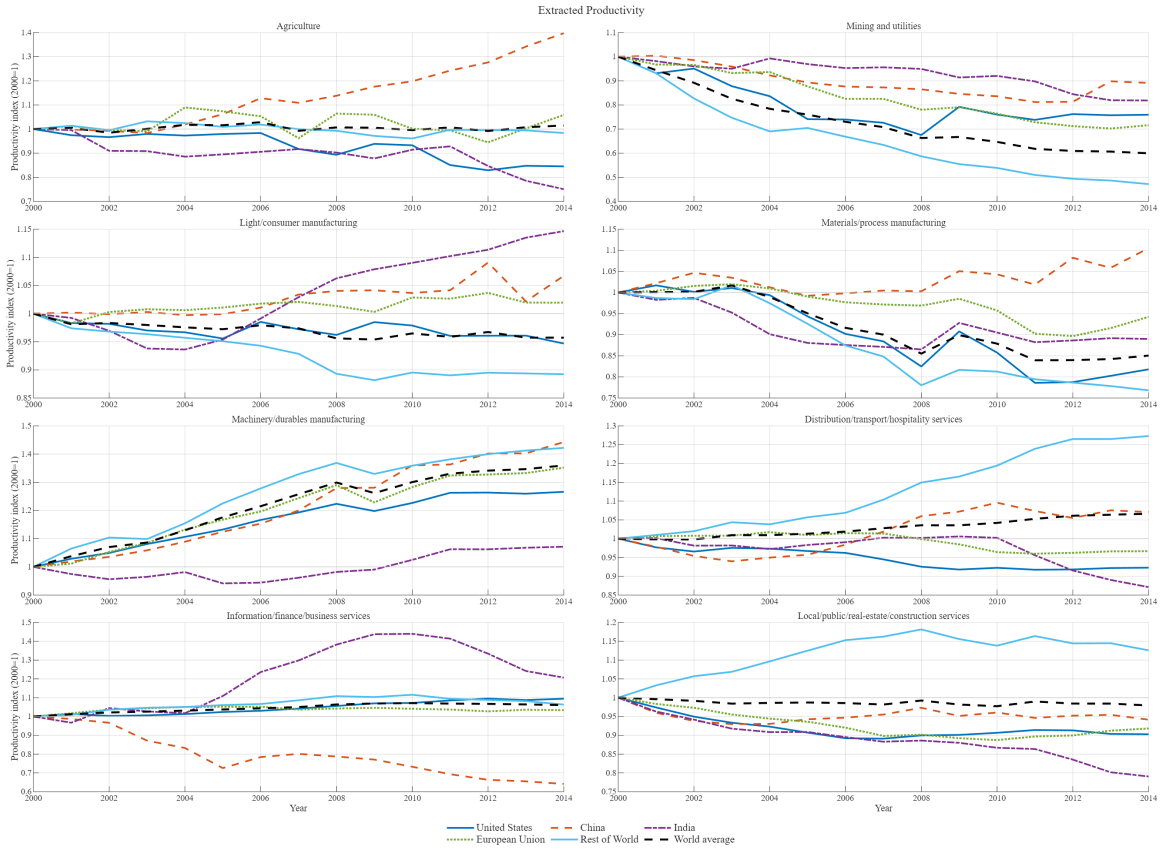
A.6 Additional Figures

Figure A.1: Trade Balances as Shares of GDP: Data and Model



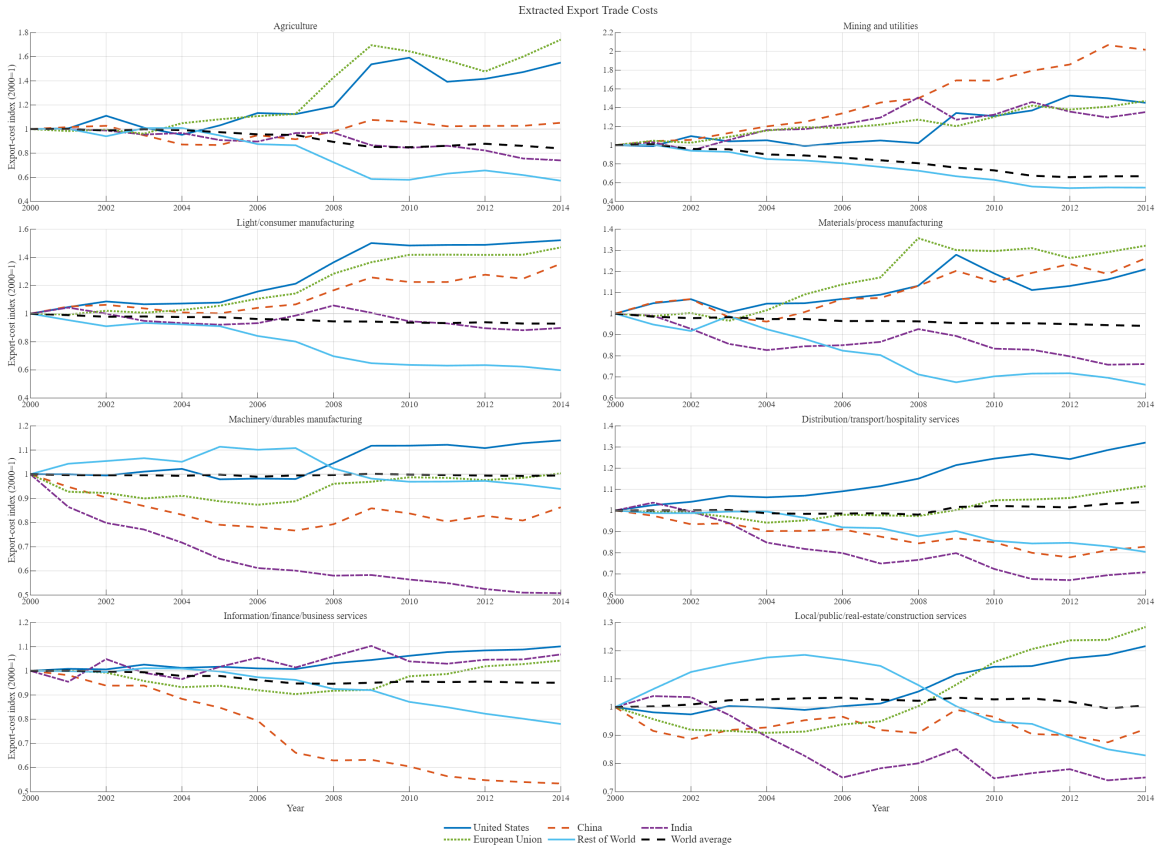
Notes: Each panel compares observed and model-implied trade balances, including goods and services, relative to GDP from 2000 to 2014 in the model with one sectorally mobile domestic capital stock used for SMM estimation. The annual trade-balance paths are included in the SMM moment vector.

Figure A.2: Productivity Paths



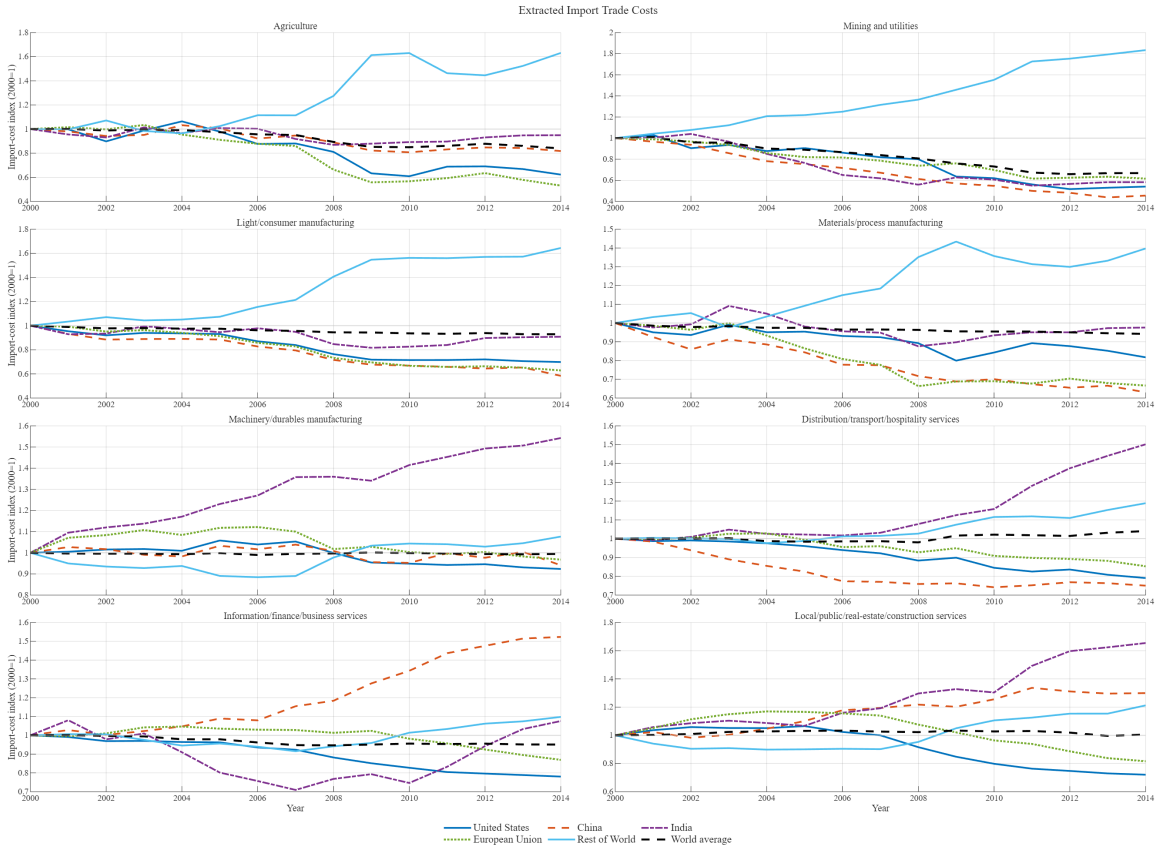
Notes: Each panel reports the productivity index $\widehat{A}^{j}_{n,t}$ for a model sector, normalized to one in 2000. The productivity paths exclude the world productivity trend used for the model numeraire; the trend is computed with baseline gross-output weights.

Figure A.3: Average Bilateral Trade Barriers by Origin Economy



Notes: Each panel reports the geometric mean of bilateral trade barriers $\widehat{\tau}^{j}_{n,d,t}$ from origin n to destinations $d \neq n$, weighted by 2000 bilateral trade shares and normalized to one in 2000.

Figure A.4: Average Bilateral Trade Barriers by Destination Economy



Notes: Each panel reports the geometric mean of bilateral trade barriers $\widehat{\tau}^{j}_{o,n,t}$ into destination n from origins $o \neq n$, weighted by 2000 bilateral trade shares and normalized to one in 2000.

A.7 Solution Method

We choose a terminal date by which all endogenous variables have reached their stationary values and impose the stationary allocation at that date. The resulting solution contains the complete adjustment path. The transition with sector-specific capital is solved as one nonlinear system. At each Newton iteration, the residual evaluator computes the equilibrium residuals, constructs the analytic sparse Jacobian, solves the Newton linear system with sparse matrix algebra, and uses a damped line search to improve convergence.

A.7.1 Transition Unknowns and Residuals

The computations use a terminal date T by which the equilibrium has reached its stationary allocation. In this appendix, transition dates are indexed by $t = 1, \ldots, T$, so $t = 1$ corresponds to $t = 0$ and the year 2000 in Section 3. Initial sector capital stocks $K^j_{n,1}$ and international bond positions $B_{n,1}$ are given. Let Q_{t-1} be the date-$t-1$ price, in units of the reference country's final good, of a bond that pays one unit of that good at date t. Its gross return is $\mathcal{R}_t = Q_{t-1}^{-1}$. Throughout the solution method, hatted variables denote ratios to their values in 2000; Appendix A.1 uses hats for log deviations. Let $W_{n,t}$ denote the nominal wage, while W_n continues to denote welfare as defined in equation (24). With N countries and J sectors, the transition unknown vector stacks

$$x = \left\{ \log \widehat{C}_{n,t},\ i^j_{n,t},\ B_{n,t+1}\ (n < N),\ \log \mathcal{R}_t\ (t \geq 2),\ \log \widehat{W}_{n,t},\ \log \widehat{r}^{K,j}_{n,t},\ \log \widehat{K}^j_{n,t+1} \right\}_{t=1}^{T}, \tag{A.71}$$

where $i^j_{n,t} \equiv I^j_{n,t} / I^j_{n,1}$ is sector investment relative to its value in 2000, kept in levels because disinvestment can occur, and global bond-market clearing determines the Nth economy's international bond position. The vector has $3NT - 1 + 3NJT$ unknowns. The residual vector contains labor-market-clearing conditions, sector capital-service-clearing conditions, household budgets for $n < N$, bond Euler equations, Euler equations for sector-specific capital, and the law of motion for sector-specific capital. It also contains terminal stationarity conditions for international bond positions and sector-specific capital, together with the nominal normalization. The residual and unknown vectors have the same dimension, so the system is square.[17]

[17] Four country-sector cells have a zero capital cost share and hold no capital. Their capital stocks are fixed at zero by identity rows.

A.7.2 Static Block Within Each Period

Given a candidate transition path, the static Armington input-output block determines prices, trade shares, revenues, factor payments, profits, income, and trade balances in each period from current state variables, productivity, and trade barriers. Unit costs satisfy

$$\log \widehat{\mathrm{mc}}^{j}_{n,t} = -\log \widehat{A}^{j}_{n,t} + \ell^{j}_{n} \log \widehat{W}_{n,t} + \kappa^{j}_{n} \log \widehat{r}^{K,j}_{n,t} + \sum_{k \in \mathcal{J}} \omega^{j,k}_{n} \log \widehat{P}^{k}_{n,t}, \tag{A.72}$$

with the sectoral rental rate entering in proportion to its cost share. Prices paid in each destination equal the marginal cost of supplying that destination multiplied by the sectoral markup. The Armington price indices and current trade shares follow from the CES structure evaluated at baseline shares. Prices are solved by fixed-point iteration over trade shares. Holding shares fixed makes the log price system linear, so it can be solved exactly; shares are then updated until convergence. The outer transition problem is solved by Newton's method. Revenues then solve the input-output system

$$R_t = (I - A_{R,t})^{-1} b_t, \tag{A.73}$$

where b_t collects final demand routed through trade shares and $A_{R,t}$ propagates intermediate demand, scaled by inverse markups. Both are $NJ \times NJ$ systems solved period by period.

A.7.3 Dynamic Residuals

Sector investment is $I^{j}_{n,t} = I^{j}_{n,1} i^{j}_{n,t}$, with investment-capital ratios $m^{j}_{n,t} = I^{j}_{n,t}/K^{j}_{n,t}$, marginal installation costs $\lambda^{K,j}_{n,t} = 1 + \phi_n(m^{j}_{n,t} - \delta_n)$, and $\chi^{K,j}_{n,t} = \frac{\phi_n}{2}(\delta_n^2 - (m^{j}_{n,t})^2)$. For $n < N$, the budget residual compares spending plus bond purchases to income plus maturing bonds, scaled by baseline magnitudes; the last country's budget is implied by Walras' law. The bond Euler residual is

$$f^{B}_{n,t} = \log C_{n,t+1} - \log C_{n,t} - \log(\beta \mathcal{R}_{t+1}) - \xi_{n,t+1}, \tag{A.74}$$

and the sector capital Euler residual is

$$\begin{aligned} f^{K,j}_{n,t} = &-\log C_{n,t} + \log C_{n,t+1} + \log(P_{n,t}\lambda^{K,j}_{n,t}) - \log\beta - \xi_{n,t+1} \\ &- \log\left[r^{K,j}_{n,t+1} + P_{n,t+1}\left((1-\delta_n)\lambda^{K,j}_{n,t+1} - \chi^{K,j}_{n,t+1}\right)\right]. \end{aligned} \tag{A.75}$$

The law of motion for sector-specific capital, terminal conditions $B_{n,T+1} = B_{n,T}$ and $K^j_{n,T+1} = K^j_{n,T}$, and the world-gross-output normalization $\sum_{n,j} R^j_{n,t} = \sum_{n,j} R^j_{n,1}$ complete the system.

A.7.4 Analytic Jacobian and Newton Method

The Newton method uses the analytic Jacobian F_x, with the static price block differentiated by the implicit function theorem. Let z_t collect current wages, sectoral rental rates, productivity, markups, and trade barriers. If $G_{P,t}$ is the derivative of the price mapping with respect to prices and $H_{P,t}$ is its direct derivative with respect to z_t, then $dp_t = (I - G_{P,t})^{-1} H_{P,t}\, dz_t$. The matrix $G_{P,t}$ transmits costs through intermediate-input prices, while $H_{P,t}$ contains the direct effects of z_t. A rental-rate change in origin o, sector j, affects destination d's costs in proportion to $\pi^j_{o,d,t}\kappa^j_o$.

Trade-share derivatives follow from CES demand, and revenue derivatives use one factorization of the revenue system per period with multiple right-hand sides. Dynamic rows link adjacent dates, yielding a block-banded Jacobian solved with sparse matrix factorizations. An Armijo line search improves convergence, and variable-specific scaling accounts for the larger magnitude of nominal bond positions.

With productivity and trade barriers fixed, the static block reproduces baseline revenues and bilateral trade shares with maximum discrepancies of 3.3×10^{-15} and 2.2×10^{-16}. The reference path has a maximum equilibrium residual of 2.7×10^{-9}, and its terminal stationarity residuals are near 10^{-16}. Graph-colored finite-difference checks produce relative Jacobian errors near 10^{-9}. The model with sectorally mobile capital also reproduces the independently solved model with one mobile domestic capital stock to within 2.3×10^{-7} across all unknowns.

A.7.5 Derivation of the Welfare Decomposition

Write the household budget as

$$P_{n,t} C^R_{n,t} = w_{n,t} L_n + \sum_j \left(r^{K,j}_{n,t} K^j_{n,t} + \Pi^j_{n,t} \right) + \mathcal{R}_t B_{n,t} - B_{n,t+1} - P_{n,t} \sum_j \left(I^j_{n,t} + \chi^j_{n,t} \right).$$

Totally differentiating this identity and applying Shephard's lemma to final and intermediate expenditure gives the price changes evaluated at the corresponding quantities. Cost minimization replaces changes in variable inputs by marginal-cost changes and the direct productivity saving $V^j_{n,t}\, d \log A^j_{n,t}$. Labor-market and intermediate-goods-market clearing

cancel payments and expenditures within a country.

For an international flow from origin o to destination d, let $\bar{p}^j_{o,t} = \mu^j \mathrm{mc}^j_{o,t}$ be the producer price before trade costs. The price paid in the destination is $p^j_{o,d,t} = \tau^j_{o,d,t} \bar{p}^j_{o,t}$, so the destination's expenditure derivative contains

$$-X^j_{o,d,t} \left(d \log \mathrm{mc}^j_{o,t} + d \log \tau^j_{o,d,t} \right).$$

The origin receives $X^j_{o,d,t}\, d \log \mathrm{mc}^j_{o,t}$ through the price received on its exports. Aggregating these two terms across international flows gives equations (27) and (32). Terms involving producer prices cancel when summed across countries, while changes in trade barriers need not cancel because they change delivery resources.

Sectoral operating profits satisfy equation (28). Their total derivative is

$$d\Pi^j_{o,t} = \varrho^j \sum_d dX^j_{o,d,t}.$$

Following Ossa (2014), profit shifting subtracts the change in profits caused by the change in the producer price. The fixed sectoral markup implies $d \log \bar{p}^j_{o,t} = d \log \mathrm{mc}^j_{o,t}$. Adding and subtracting $\Pi^j_{o,t}\, d \log \bar{p}^j_{o,t} = \Pi^j_{o,t}\, d \log \mathrm{mc}^j_{o,t}$ gives equation (29). Substituting $dX = X(\, d \log \mathrm{mc} + d \log \tau) + p\, dq$ gives equation (30). The marginal-cost term subtracts the change in the producer price before trade costs, while the trade-cost term records the change in the destination price caused by trade costs.

The pricing equation gives $\varrho^j p^j_{o,d,t} = p^j_{o,d,t} - \tau^j_{o,d,t} \mathrm{mc}^j_{o,t}$. With fixed trade barriers, substituting this equality into equation (30) and summing across origins and sectors gives equation (31). Demand equates the marginal value of a variety with its price, so the right-hand side values each quantity change at its price-cost gap.

When trade barriers change, the corresponding world sum is

$$\begin{aligned} \sum_{o,j} d\mathcal{W}^{\Pi,j}_{o,t} = & \sum_{o,j,d} \left(p^j_{o,d,t} - \tau^j_{o,d,t} \mathrm{mc}^j_{o,t} \right) dq^j_{o,d,t} \\ & + \sum_{o,j,d} \varrho^j X^j_{o,d,t}\, d \log \tau^j_{o,d,t}. \end{aligned}$$

The first line values quantity changes at their price-cost gaps. The second records the change in profit income associated with the change in trade barriers, while equation (32) records the resources required for delivery. Applying $\varrho^j = \bar{\varrho} + (\varrho^j - \bar{\varrho})$ to the first line gives the common-markup and markup-dispersion terms in equation (33). The dispersion

term captures the effect of reallocating quantities across activities with different markups in Holmes et al. (2014). Together, the two terms correspond to the general expression for welfare changes from reallocation in Baqaee and Farhi (2020), while the country terms show which country's households receive the profit income.

The remaining terms are the change in net bond receipts, the rental value of installed capital, and the resources used for investment and capital adjustment. Collecting all terms and dividing by nominal consumption gives equation (36).

A.7.6 Decomposition Computation

Let $g^c_{n,t}(\zeta)$ denote the local contribution of component c at the equilibrium indexed by ζ, where $\zeta = 0$ is the reference path and $\zeta = 1$ is the path with the specified change. The reported welfare comparisons set the intertemporal preference wedges to zero. The discounted average change in log real consumption is

$$\overline{\Delta W}_n = \sum_c \int_0^1 \left[\sum_{t=0}^{T-1} \frac{\beta^t}{\sum_{s=0}^{T-1} \beta^s} g^c_{n,t}(\zeta) \right] d\zeta.$$

We evaluate the path integral numerically, using each equilibrium to initialize the next. The model comparisons use nine equally spaced interpolation points. For the larger changes inferred from annual data over 2000–2014, Richardson extrapolation combines decompositions computed with five and nine points. Across the reported dynamic and static comparisons, the largest difference between directly computed welfare and the sum of the components is less than 6×10^{-6} in the underlying welfare units.

The tables express the exact additive decomposition in consumption-equivalent units. Let $\overline{\Delta W}^c_n$ denote component c's contribution and let $\lambda_n = 100[\exp(\overline{\Delta W}_n) - 1]$ denote total consumption-equivalent welfare. The reported contribution is $\lambda_n \overline{\Delta W}^c_n / \overline{\Delta W}_n$. The contributions therefore add to λ_n, and dividing a contribution by total consumption-equivalent welfare gives the same component share as the underlying exact decomposition.

For a comparison between two model specifications, a and b, the reported total is the direct difference in consumption-equivalent welfare, $\lambda^a_n - \lambda^b_n$. The contribution of component c is

$$(\lambda^a_n - \lambda^b_n) \frac{\overline{\Delta W}^{c,a}_n - \overline{\Delta W}^{c,b}_n}{\overline{\Delta W}^a_n - \overline{\Delta W}^b_n}.$$

These contributions add to the consumption-equivalent percentage-point difference and preserve the component shares of the exact decomposition.