Priority Options in Industrial Organization

Evangelos Rouskas*

Abstract

I introduce the concept of *priority options* in oligopolistic markets with homogeneous products, price-setting firms, and capacity constraints. The *priority options* are consumer options and give to a consumer the right, but are not associated with an obligation, to purchase the underlying product from the firm that sold the option. The firms announce the prices of options and the consumers decide whether they will purchase options before firms choose the prices in the product market. In my baseline model, I examine a duopoly with exogenous symmetric capacities and nonatomic consumers who have common valuations. I find that when only one firm offers options, then this firm can set a generalized price for the underlying product—which includes the option price—equal to the monopoly price. As a result, the profits may increase tremendously. This is the "good" equilibrium. I also find another equilibrium in which the adopter commits to a low price in the underlying product market when they announce the prices of options. In this equilibrium, the adopter wins but to a smaller extent. This is the "bad" equilibrium. The firm that does not offer options is never harmed. When both firms offer options, again no firm is harmed and there are two classes of equilibria in which only one firm wins. I discuss how the analysis is affected when the consumers are atomic.

JEL Classification: C72; C73; D14; D43; D45; G31; G50; L13; M31.

Keywords: priority options; capacity constraints; oligopoly; price commitment.

*Section Quantitative Economics, Department of Statistics and Actuarial-Financial Mathematics, University of the Aegean, Vourlioti Building, Karlovasi, Samos 83200, Greece. Email: erouskas@aegean.gr

1 Introduction

Firms with capacity constraints are relevant in a variety of real-world settings, such as transportation networks, lodging, sporting events, cloud computing, and so on. This has led to a rich academic literature on pricing under capacity constraints that started off with Edgeworth (1925) and is known as the Bertrand-Edgeworth approach. Seminal work in this tradition includes Shubik (1959), Beckmann (1965), Levitan and Shubik (1972), Kreps and Scheinkman (1983), and Davidson and Deneckere (1986).

In this paper, I propose that capacity-constrained firms launch a new offering targeting consumers. I call the new offering *priority option*. This idea, in the form presented here, escapes existing research and practice to my knowledge.

In my baseline model, I follow the canonical duopoly version of Acemoglu *et al.* (2009) with exogenous symmetric capacities and zero production costs. I maintain the assumptions of price-setting firms, nonatomic consumers, common valuations, and homogeneous products. The firms announce the prices of options and the consumers decide whether they will purchase options before firms choose the prices in the product market.

The key to understand *priority options* is that there is no equilibrium in pure strategies for a large set of parameters. Each firm has incentives to set a low price in order to utilize the full capacity, but these incentives are not very strong. Even if a firm sets a price equal to the cost, a strictly positive mass of consumers would be forced to purchase at a high price. In the mixed-strategy equilibria, price dispersion is established with probability one and there exists uncertainty about which firm will set the lowest price in the market. I argue that capacity-constrained firms can take advantage of this uncertainty and increase their profits significantly. The rationale behind this argument is straightforward. There is positive probability that a consumer cannot purchase the product from a capacity-constrained firm only when this firm sets the lowest price in the market.

My first step is to provide a definition: A *priority option* sold by a firm prior to price competition in the underlying product market gives to a consumer the right, but is not associated with an obligation, to purchase the underlying product from this firm. Priority means that if the firm, which sold the option, sets the lowest price in the market, the consumer who purchased the option will be guaranteed one unit of the underlying product at this price, i.e., the option holder will not

be subject to rationing.

Introducing options trasforms the static game of Acemoglu *et al.* (2009) into a dynamic one. In the first stage of the game, the firms simultaneously set the option prices and announce the maximum mass of consumers who can purchase options. In the second stage, the firms simultaneously engage in price competition in the underlying product market. I call the prices of the second stage *spot* prices.

Overview of the Results—Baseline Model

Exogenous adoption of options by one firm (the "good" equilibrium): When only one firm offers options, I find one equilibrium—the "good" equilibrium—in which this firm achieves profits equal to the monopoly profits given the capacity. This result is not based on collusion. The firm that does not sell options is unaffected as compared to the environment in which no firm sells options, and the monopoly outcome is sensitive to the assumption of nonatomic consumers. The reasoning is as follows. The option price is high and the option seller sets a low *spot* price which may be lower than the option price, while the mass of consumers who purchase options equals the capacity. The sum of the option price and the *spot* price equals the common valuation for the underlying product, and the firm that does not sell options also sets the monopoly price. If one of the consumers who purchase the options deviates to not purchasing the option, the said consumer faces zero probability for purchasing at the low *spot* price. This consumer ends up purchasing the product at the monopoly price from the firm that does not sell options. So, there is no incentive for deviation. This equilibrium harms both the consumers who purchase options and the consumers who do not purchase options.

Exogenous adoption of options by one firm (the "bad" equilibrium): When only one firm offers options, I find another equilibrium—the "bad" equilibrium—in which this firm achieves higher profits as compared to the environment in which no firm sells options but lower than the monopoly profits given the capacity. Again, the firm that does not offer options is unaffected. The justification for the "bad" equilibrium is that the "good" equilibrium is based on a high option price and generally for high option prices it is possible that no consumers purchase options. Consequently, the option seller has to set a low option price to ensure that the mass of consumers who purchase

options is positive. The "bad" equilibrium is based on the commitment strategy on behalf of the option seller. According to this strategy, the option seller publicly commits to a low *spot* price when the option price is announced. The commitment strategy is reminiscent of the approach developed by Myatt and Ronayne (2026).[1]

Exogenous adoption of options by both firms: When both firms offer options, then in candidate equilibria in which both firms sell options to a positive mass of consumers there is a commitment strategy out of the candidate equilibrium path that yields for the firm that deviates the aforementioned "bad" equilibrium profits. These profits are stictly higher than the per-firm candidate equilibria profits in which both firms sell options to a positive mass of consumers. So, on any equilibrium path, only one firm commits to a low *spot* price when the option prices are announced and the rival firm sells no options.

Endogenous adoption of options by firms: It is straightforward that at least one firm will offer options if firms can choose whether they will offer options or not. Notably, the outcome that no firm is harmed by options comes in contrast to the quantity-competition literature with homogeneous products initiated by Brander and Lewis (1986) and Fershtman and Judd (1987). According to this literature, in a duopoly, we get two results: When a firm is the only one to adopt a strategy, namely, issuing debt or instructing the manager to maximize revenues, then the adopter benefits and the rival firm is harmed. Additionally, when both firms adopt the strategy in question, then both firms become worse off. Next, I provide a broad literature review.

Literature

Finance: The well-developed concepts of financial call options (Hull 2022; Turner and Zeindler 2023) and real options (Trigeorgis 1996; Smit and Trigeorgis 2004; Chevalier-Roignant and Trigeorgis 2011) provide the inspiration for my *priority options*. The main feature that generates the

[1]The commitment strategy is credible in the sense that the firm finds it difficult or impossible to increase the *spot* price in the future due, for example, to fairness concerns (Kahneman, Knetsch and Thaler 1986) or governmental regulations (Sorensen 2000; Obradovits 2014). The low *spot* price of the commitment strategy is so low that the firm has no incentive to decrease that price. Myatt and Ronayne (2026) are influenced by Varian's (1980) dichotomy in the market, where some consumers are loyal and others are price-sensitive. Such settings can be analyzed with the additional assumption of capacity constraints as Ilin and Shi (2022) demonstrate.

need for options is uncertainty. In financial options, there is uncertainty about the price of a financial asset, such as a stock. In real options, the underlying asset is not financial but real (yet not necessarily tangible) and there is uncertainty about its price or potential. Real options have been analyzed in the literature both under exclusivity and competition. Essentially, my *priority options* are consumer options. I believe that consumer options belong to the field of household finance (Gomes *et al.* 2021; Madura 2025) and can play a role analogous to the role that real options play in the field of corporate finance (Ross *et al.* 2021). However, the consumer options are not covered in the field of household finance; instead, they have been introduced into the field of marketing by Sainam *et al.* (2010).

Marketing: Sainam *et al.* (2010) motivate consumer options in the context of sporting events.[2] In this context, there is one seller of options, the sporting event organizer. The fans are uncertain about their valuation for the final games because, when the tickets become available by the organizer, the fans do not know whether their favorite team will be playing in the finals. Consumer heterogeneity is crucial: Some consumers want to attend the final games irrespective of the finalists, whereas other consumers want to attend the finals only if their favorite team participates. The latter consumers can purchase an option, which gives them the right but is not associated with an obligation to purchase the ticket of the final game, and will exercise the option if their favorite team advances. My contribution is that I generalize the analysis of consumer options to an oligopoly, i.e., two or more firms sell options. In my framework, the valuations of consumers are not uncertain and the consumers are not necessarily heterogeneous.

Industrial Organization: Pricing with capacity constraints is a classic and particularly active area of research in the field of industrial organization. Two papers are especially relevant to my work. Parakhonyak *et al.* (2025) find that in a duopoly if one firm can cover the entire market and the other firm is capacity constrained, the equilibrium result may be that both firms set the monopoly price. The additional assumptions on which this result is based are that consumers search sequentially and the search cost equals zero. My monopoly-pricing result arises not only when one firm can cover the entire market but also when both firms are capacity constrained. Somogyi *et al.* (2023) examine pricing with uncertain capacities and generate a Perfect Bayesian equilibrium

[2]See also the relevant work by Balseiro *et al.* (2011).

in pure strategies with every type charging a different price. The possibility of offering *priority options* could allow firms with private information about capacities to reveal this information to rival firms.

Operations Research: Real options have been studied extensively by the operations research literature. The reader is referred to the survey by Trigeorgis and Tsekrekos (2018) for details. One aspect of the operations research literature that is particularly relevant to my work is the focus on queueing systems. The point is that queueing systems generate delay for users and delay can be costly.[3] Therefore, obtaining priority is desirable. Seminal studies in this spirit include Balachandran (1972), Adiri and Yechiali (1974) who also examine nonmonopoly queues, Mendelson and Whang (1990), and Hassin and Haviv (1997). Recently, Gershkov and Winter (2023) studied queueing systems and the effects of priority services on consumer surplus both under monopoly and duopoly.[4] Under duopoly, when customers are homogeneous, all customers purchase the priority service and each provider obtains the monopoly profits from half of the market, i.e., the customers become worse off as compared to the setting without any priority service. This shares some similarity to my monopoly-pricing result, but I obtain asymmetry in profits. On top of that, my "bad" equilibrium benefits the majority of consumers.

Applications to Real-World Markets

My idea could potentially be valuable, for example, to airline companies, hotels, car rental companies, and cloud computing marketplaces. Many of these potential adopters are discussed in the revenue management literature (Gallego and Topaloglu 2019). My model can be viewed as complementary to this strand of literature which tackles problems from real-world settings. In the real-world, there is some uncertainty about demand over time or valuations. This is a type of "out-

[3] Chamberlain *et al.* (2021) analyze queueing systems with advance reservations. My model is very close to the concept of advance reservations, and the motivation for advance reservations may be based on the cloud computing economics literature. In this literature, for example, the paper by Li *et al.* (2014) uses my set of assumptions with two differences. First, they do not examine options. Second, the users of cloud services experience disutility from the mass of other users who purchase from the same firm and the disutility is higher the higher this mass.

[4] Cui *et al.* (2020) study a model "in which customers seeking service can hire others (line-sitters) to wait in line on behalf of them".

side" uncertainty, whereas my model is based on "inside" uncertainty, i.e., uncertainty generated by the randomization of prices.[5]

My target is to highlight a clean industrial-organization result about commitment and rationing. That said, in my view, companies could use some form of *priority options* in the context of established practices. For example, it is the practice of airline companies to offer significant discounts on a limited number of tickets and to consumers who are willing to book tickets in advance that could deliver immediate additional benefit with the introduction of *priority options*. This marketing strategy could be advertised as follows: "Purchase a *priority option* and do not miss out on bargains."

The rest of the paper is organized into four Sections. Section 2 presents the duopoly model with symmetric capacities and nonatomic common-valuation consumers. Section 3 provides the analysis, and Section 4 discusses how the analysis is affected when the consumers are atomic. Section 5 concludes. Proofs and additional material are relegated to Appendix A.

2 The Model with Symmetric Capacities

I analyze the market for a homogeneous product. All agents in the market are risk neutral. On the demand side, there are nonatomic consumers with total mass equal to one. The consumers have unit demand at common valuation $v > 0$ and choose the firm with the lowest price.[6] On the supply side, there are two firms, firm i and firm j, where $i \neq j$. The firms (i) produce and sell the product at zero cost; (ii) set prices simultaneously in the product market; and (iii) are

[5]Note that airline companies already offer options that allow travellers to reserve a seat for a fixed duration before making a final decision on whether to purchase the flight ticket (Jain and Cox 2011; Aydin *et al.* 2017; Sumida and Topaloglu 2019; Chen and Chen 2019). There is also research proposing callable products that could be offered by airline companies (Gallego *et al.* 2008; Gallego and Lee 2020). Gallego and Lee (2022) state: "Low-fare consumers may grant call options to the capacity provider that give it the right to recall capacity at the prespecified recall price. The model is win-win-win to the airline as it makes higher profits, to low-fare consumers who find the recall price attractive, and to high-fare consumers that see improved availability. Airlines have been slow to adapt callable products."

[6]In case all firms set the same price, then each consumer chooses each firm with the same probability. If the firm with the lowest price cannot cover the entire demand, then this firm serves each consumer with the same probability.

capacity constrained. Denote k_i and k_j the capacity of firm i and j respectively. Prior to price competition in the product market, the firms may offer *priority options*. I call the prices in the product market *spot prices*. Denote p^i_{spot} and p^j_{spot} the *spot* prices of firm i and j respectively. More generally, denote $F^i_{spot}(p)$ and $F^j_{spot}(p)$ the degenerate or nondegenerate price distribution of firm i and j respectively in the *spot* market. The *priority options* grant to consumers the right but not the obligation to purchase the product from the firm that sold the option, i.e., the consumer will exercise the option and will not be rationed in case the firm that sold the option sets the lowest price in the *spot* market. The firms set the option prices and announce the maximum mass of consumers who can purchase options simultaneously prior to price competition in the *spot* market. Denote p^i_o and p^j_o the option prices of firm i and j respectively. Denote x^i_o and x^j_o the maximum mass of consumers who can purchase options from firm i and j respectively. The firms choose x^i_o and x^j_o subject to the constraints $0 \leq x^i_o \leq k_i$ and $0 \leq x^j_o \leq k_j$. In sum, the timing is as follows:

Stage 0: The firms simultaneously announce p^i_o and p^j_o as well as x^i_o and x^j_o.

Stage 1: The consumers decide whether they will purchase options.[7]

Stage 2: The firms simultaneously announce p^i_{spot} and p^j_{spot}.

The solution concept is the Subgame Perfect Nash Equilibrium (SPNE). I focus on the equilibria that arise when Assumption 1 holds:[8]

Assumption 1: $k_i = k_j = k \in \left(\frac{1}{2}, 1\right)$

Last but not least, I allow firms to publicly commit to the *spot* price at Stage 0. I call this the commitment strategy.

[7] I assume that there are no speculators in the market who are not interested in purchasing the underlying product and would set as a target the resale of the option.

[8] There is no role for *priority options* when $k_i = k_j = k \in \left(0, \frac{1}{2}\right]$ and when $k_i = k_j = k \geq 1$. Indeed (i) when $k_i = k_j = k \in \left(0, \frac{1}{2}\right]$, then both firms set the monopoly price with probability one; and (ii) when $k_i = k_j = k \geq 1$, then the Bertrand Paradox applies.

3 Analysis

I start with the analysis of the game in which no firm can offer options (Subsection 3.1). This serves as a benchmark. I continue with the analysis of the games in which only firm i can offer options (Subsection 3.2) and both firms can offer options (Subsection 3.3).

3.1 No Firm Can Offer Priority Options

When no firm can offer *priority options*, the equilibrium of the static price-competition game is known in the literature and presented for completeness in Proposition S1.

Proposition S1 (Replicating Acemoglu *et al.* 2009) *Under Assumption 1, when no firm can offer options, there exists a unique symmetric Nash equilibrium in which both firms set prices in the spot market according to the atomless cumulative price distribution function* $F^i_{spot}(p) = F^j_{spot}(p) = \frac{1}{2k-1}\left(k - \frac{(1-k)v}{p}\right)$ *where* $p \in \left[\left(\frac{1-k}{k}\right)v, v\right]$. *Each firm obtains profits equal to* $(1-k)v$. *Each consumer achieves net surplus equal to* $v - kE(p_{min}|no\ options) - (1-k)E(p_{max}|no\ options)$, *where* $E(p_{min}|no\ options)$ *and* $E(p_{max}|no\ options)$ *are the expected minimum price and the expected maximum price in the spot market respectively.*

It holds that:

$$E(p_{min}|no\ options) = \frac{2v(1-k)}{2k-1}\left(1 - \frac{1-k}{2k-1}\ln\left(\frac{k}{1-k}\right)\right)$$

$$E(p_{max}|no\ options) = \frac{2v(1-k)}{2k-1}\left(\frac{k}{2k-1}\ln\left(\frac{k}{1-k}\right) - 1\right)$$

$$kE(p_{min}|no\ options) + (1-k)E(p_{max}|no\ options) = 2(1-k)v$$

It is clear that the profits of both firms are decreasing in k, while each consumer achieves strictly positive net surplus. These implications do not necessarily carry over to the environment in which only one firm can offer *priority options*.

3.2 Only One Firm Can Offer Priority Options

My first result, presented in Proposition 1, demonstrates an increase in profits for the firm that offers *priority options* by $\frac{2k-1}{1-k}$ as compared to Proposition S1, i.e., a larger capacity corresponds to a larger increase in profits. At the same time, the firm that does not offer options is not harmed.

Proposition 1 (the "good" equilibrium without commitment)[9] *Under Assumption 1, when only firm i can offer options, there exists a SPNE as follows:*
Option Market: Firm i sets $p_o^i = \left(\frac{2k-1}{k}\right) v$ with probability one and announces that the maximum mass of consumers who can purchase options equals $x_o^i = k$. A mass of consumers equal to k purchase the options of firm i.
Spot Market: Firm i sets $p_{spot}^i = \left(\frac{1-k}{k}\right) v$ with probability one and firm j sets $p_{spot}^j = v$ with probability one. Each consumer who purchased an option from firm i exercises the option with probability one.
The profits of firm i are equal to kv. The profits of firm j are equal to $(1-k)v$. Each consumer achieves net surplus equal to zero.

[INSERT TABLE 1 HERE]

In Table 1, I compare the profits of firms between Proposition S1 and Proposition 1. In Proposition 1, for firm i, the generalized price, i.e., the option price and the spot price, add up to v. So, both firms set the monopoly price, and the firm that offers the options captures demand equal to the capacity. This strong result holds because the consumers are nonatomic. To understand this, note that if one of the consumers who purchase the options deviates to not purchasing the option, the said consumer faces zero probability for purchasing at $\left(\frac{1-k}{k}\right) v$ and purchases the product from firm j at a *spot* price equal to v.

However, the firm that offers options may be forced to set an option price strictly lower than $\left(\frac{2k-1}{k}\right) v$. The reason is that when the option price is high, it is possible that no consumers purchase options. In case no consumers purchase options, in the *spot* market, we get the result presented in Proposition S1. The firm that offers options can achieve higher profits than $(1-k)v$ and lower than

[9] The "good" equilibrium is valid with commitment as well.

kv by simply setting a low option price. To understand how the low option price is determined, consider the following reasoning. When no consumers purchase options, as explained in Proposition S1, the expected payoff for a representative consumer is:

$$v - kE(p_{min}|no\ options) - (1-k)E(p_{max}|no\ options)$$

Now suppose that no consumers purchase options and a representative consumer deviates to purchasing an option. The expected payoff for the consumer who deviates is:

$$-p_o^i + prob(both\ prices < v - p_o^i)$$
$$\left(v - \frac{1+k}{2}E(p_{min}|both\ prices < v - p_o^i) - \frac{1-k}{2}E(p_{max}|both\ prices < v - p_o^i)\right)$$
$$+prob(one\ price < v - p_o^i)prob(one\ price > v - p_o^i)(1+k)\left(v - E(p|one\ price < v - p_o^i)\right)$$

where $E(p_{min}|both\ prices < v - p_o^i)$ and $E(p_{max}|both\ prices < v - p_o^i)$ are the expected minimum price and the expected maximum price in the *spot* market when both prices in the *spot* market are lower than $v - p_o^i$ respectively, while $E(p|one\ price < v - p_o^i)$ is the expected price in the *spot* market when the firm in question sets a spot price lower than $v - p_o^i$. It holds that:

$$prob(both\ prices < v - p_o^i) = \frac{1}{(2k-1)^2}\left(\frac{(2k-1)v - kp_o^i}{v - p_o^i}\right)^2$$

$$prob(one\ price < v - p_o^i)prob(one\ price > v - p_o^i) = \frac{(1-k)p_o^i((2k-1)v - kp_o^i)}{(2k-1)^2(v-p_o^i)^2}$$

$$E(p_{min}|both\ prices < v - p_o^i) = \frac{2v(1-k)(v-p_o^i)}{(2k-1)v - kp_o^i}\left(1 - \frac{v(1-k)}{(2k-1)v - kp_o^i}\ln\left(\frac{k(v-p_o^i)}{(1-k)v}\right)\right)$$

$$E(p|one\ price < v - p_o^i) = \frac{v(1-k)(v-p_o^i)}{(2k-1)v - kp_o^i}\ln\left(\frac{k(v-p_o^i)}{(1-k)v}\right)$$

The firm that offers options can set an option price lower than or equal to the price that satisfies Equation (1) and this way avoids a situation in which no consumers purchase options:

$$v - kE(p_{min}|no\ options) - (1-k)E(p_{max}|no\ options) = -p_o^i + prob(both\ prices < v - p_o^i)$$
$$\left(v - \frac{1+k}{2}E(p_{min}|both\ prices < v - p_o^i) - \frac{1-k}{2}E(p_{max}|both\ prices < v - p_o^i)\right)$$
$$+prob(one\ price < v - p_o^i)prob(one\ price > v - p_o^i)(1+k)\left(v - E(p|one\ price < v - p_o^i)\right) \quad (1)$$

Denote $p_o^{threshold}$ the solution to Equation (1). In Table 2, I provide simulations for $p_o^{threshold}$.

[INSERT TABLE 2 HERE]

By the definition of $p_o^{threshold}$, when firm i sets $p_o^i \in \left(0, p_o^{threshold}\right)$, then there is no equilibrium in which no consumers purchase options. Importantly, though, when firm i sets $p_o^i \in \left(0, p_o^{threshold}\right)$, then there is also no equilibrium in which the mass of consumers who purchase options belongs to the interval $(0, k]$. The reason is technical. To be precise, in the *spot* market, if some consumers have purchased options, both firms play atoms and each firm's support does not contain only one *spot* price. This turns out to be unacceptable, as a firm can unilaterally deviate and increase profits. Therefore, in anticipation of consumers rejecting an option price weakly higher than $p_o^{threshold}$, it seems that the firm cannot set option prices in the interval $\left(0, p_o^{threshold}\right)$. Fortunately, this is not the whole story. I prove that tractability is restored in case the firm that offers options publicly commits to the *spot* price at Stage 0. In such cases, the firm sets an option price slightly lower than $p_o^{threshold}$ and sells options to a mass equal to their capacity.[10] The seller of options achieves higher profits than the per-firm profits in Proposition S1 but lower than the monopoly profits given the capacity. I present my second result in Proposition 2 and provide relevant simulations in Tables 3-4.

Proposition 2 (the "bad" equilibrium with commitment) *Under Assumption 1, when only firm i can offer options, there exists a SPNE as follows:*

Option Market: Firm i chooses the option price $p_o^i = p_o^{threshold}$ *with probability one, announces that the maximum mass of consumers who can purchase options equals* $x_o^i = k$*, and publicly commits that the spot price* p_{spot}^i *will be equal to* $\left(\frac{1-k}{k}\right) v$ *with probability one. A mass of consumers equal to* k *purchase the options of firm* i*.*

Spot Market: Firm j sets a spot price $p_{spot}^j = v$ *with probability one. Each consumer who purchased an option from firm i exercises the option with probability one.*

Firm i obtains profits equal to $(1-k)v + kp_o^{threshold}$*. Firm j obtains profits equal to* $(1-k)v$*. Each consumer who purchases options achieves a net surpus equal to* $\left(\frac{2k-1}{k}\right) v - p_o^{threshold} > 0$ *and each consumer who does not purchase options achieves a net surplus equal to zero.*

[INSERT TABLES 3-4 HERE]

[10] The firm sets an option price equal to $p_o^{threshold} - \epsilon$ where ϵ tends to zero from above. Such an ϵ does not exist.

3.3 Both Firms Can Offer Priority Options

To facilitate the expression of my arguments, I introduce new notation. Denote z_i the mass of consumers who purchase only the option of firm i, z_j the mass of consumers who purchase only the option of firm j, and z_{i+j} the mass of consumers who purchase the options of both firms. I restrict attention to equilibria with commitment, and I state two Lemmas which considerably simplify the analysis.[11]

Lemma 1 *Under Assumption 1, when only one firm commits, then the configurations (i) $z_{i+j} > 0$; and (ii) $z_i > 0$ and $z_j > 0$ are not possible.*

Lemma 2 *Under Assumption 1, it is not possible that both firms commit.*

I present two classes of equilibria on analogy with Propositions 1-2. The class of "good" equilibria presented in Proposition 3 is the counterpart of Proposition 1. The main difference with Proposition 1 is that the equilibria cannot be sustained without commitment (see Lemma A1 in the Appendix). The class of "bad" equilibria presented in Proposition 4 is the counterpart of Proposition 2. These equilibria are valid only when the firm that does not commit sets an option price strictly higher than $p_o^{threshold}$. The firm following the commitment strategy understands that if both option prices are strictly higher than $p_o^{threshold}$, this can lead to an outcome in which no consumers purchase options.

Proposition 3 (the "good" equilibria with commitment) *Under Assumption 1, when both firms can offer options, there exists a class of SPNE as follows:*
Option Market: One firm sets an option price equal to $\left(\frac{2k-1}{k}\right) v$ with probability one, announces that the maximum mass of consumers who can purchase options equals k, and publicly commits that the spot price will be equal to $\left(\frac{1-k}{k}\right) v$ with probability one. The other firm sets an option price weakly higher than zero and does not commit. A mass of consumers equal to k purchase the options of the firm that commits.

[11] In the Appendix, I show that candidate equilibria with no firm committing can be discarded.

Spot Market: The firm that does not commit sets an option price equal to v with probability one. Each consumer who purchased an option from the firm that commits exercises the option with probability one.

The profits of the firm that commits are equal to kv. The profits of the other firm are equal to $(1-k)v$. Each consumer achieves net surplus equal to zero.

Proposition 4 (the "bad" equilibria with commitment) *Under Assumption 1, when both firms can offer options, there exists a class of SPNE as follows:*

Option Market: One firm sets an option price equal to $p_o^{threshold}$ with probability one, announces that the maximum mass of consumers who can purchase options equals k, and publicly commits that the spot price will be equal to $\left(\frac{1-k}{k}\right)v$ with probability one. The other firm sets an option price strictly higher than $p_o^{threshold}$ and does not commit. A mass of consumers equal to k purchase the options of the firm that commits.

Spot Market: The firm that does not commit sets an option price equal to v with probability one. Each consumer who purchased an option from the firm that commits exercises the option with probability one.

The profits of the firm that commits are equal to $(1-k)v + kp_o^{threshold}$. The profits of the other firm are equal to $(1-k)v$. Each consumer who purchases options achieves a net surpus equal to $\left(\frac{2k-1}{k}\right)v - p_o^{threshold} > 0$ and each consumer who does not purchase options achieves a net surplus equal to zero.

4 Discussion

In this Section, I assume that consumers are atomic and explain how the analysis is affected. The total number of consumers is denoted by M_a and each firm has capacity $k_i = k_j = k_a$. All other assumptions of Section 2 are maintained. To make the model with atomic consumers directly comparable to the model with nonatomic consumers, I focus on Assumption 2:

Assumption 2:[12] $M_a = 10$ and $k_a \in \{6, 7, 8, 9\}$

[12] I could also focus on the assumption $M_a = 100$ and $k_a \in \{51, 52, 53, ..., 60, ..., 70, ..., 80, ..., 90, ..., 97, 98, 99\}$ and

4.1 No Firm Can Offer Priority Options

The equilibrium derived from the static game without options is described in Proposition S2. In Corollary 1, I highlight the equivalence between Propositions S1 and S2.

Proposition S2 (benchmark model with atomic consumers and without options) *Under Assumption 2, when no firm can offer options, there exists a unique symmetric Nash equilibrium in which both firms set prices in the spot market according to the atomless cumulative price distribution function* $F^i_{spot}(p) = F^j_{spot}(p) = \frac{1}{2k_a-10}\left(k_a - \frac{(10-k_a)v}{p}\right)$ *where* $p \in \left[\left(\frac{10-k_a}{k_a}\right)v, v\right]$. *Each firm obtains profits equal to* $(10-k_a)v$. *Each consumer achieves net surplus equal to* $v - \frac{k_a}{10}E(p_{min}|no\ options\ atomic) - \frac{10-k_a}{10}E(p_{max}|no\ options\ atomic)$, *where* $E(p_{min}|no\ options\ atomic)$ *and* $E(p_{max}|no\ options\ atomic)$ *are the expected minimum price and the expected maximum price in the spot market respectively.*

It holds that:

$$E(p_{min}|no\ options\ atomic) = \frac{2v(10-k_a)}{2k_a-10}\left(1 - \frac{10-k_a}{2k_a-10}\ln\left(\frac{k_a}{10-k_a}\right)\right)$$

$$E(p_{max}|no\ options\ atomic) = \frac{2v(10-k_a)}{2k_a-10}\left(\frac{k_a}{2k_a-10}\ln\left(\frac{k_a}{10-k_a}\right) - 1\right)$$

$$\frac{k_a}{10}E(p_{min}|no\ options\ atomic) + \frac{10-k_a}{10}E(p_{max}|no\ options\ atomic) = \left(\frac{2(10-k_a)}{10}\right)v$$

Corollary 1 (comparison between S1 and S2) *The cumulative price distribution function described in Proposition S1 is identical to the cumulative price distribution function described in Proposition S2 when: (i)* $k = 0.6$ *and* $k_a = 6$*; (ii)* $k = 0.7$ *and* $k_a = 7$*; (iii)* $k = 0.8$ *and* $k_a = 8$*; (iv)* $k = 0.9$ *and* $k_a = 9$.

4.2 Only One Firm Can Offer Priority Options

Denote $p^{threshold}_{o,a}$ the option price above which the atomic consumers may not purchase options. It follows from Corollary 1 that when $k_a = 6$, $k_a = 7$, $k_a = 8$, and $k_a = 9$, the value of $p^{threshold}_{o,a}$ is identical to the value of $p^{threshold}_{o}$ computed by Equation (1) when $k = 0.6$, $k = 0.7$, $k = 0.8$, and

other analogous assumptions.

$k = 0.9$ respectively. The "bad" equilibrium is known in these cases. I will emphasize here the "good" equilibrium because it differs as compared to the environment with nonatomic consumers. To see this, consider that the option seller sets an option price equal to $\frac{2k_a-10}{k_a}$ and k_a consumers purchase options. Then, one of the consumers who purchase options unilaterally has an incentive to switch to not purchasing an option because they face positive probability for purchasing from the low *spot* price. This problem is resolved by an option price strictly lower than $\frac{2k_a-10}{k_a}$ and strictly higher than $p_{o,a}^{threshold}$. Proposition 5 and Table 5 can be interpreted as the counterparts to Proposition 1 and Table 1 respectively.

Proposition 5 (the "good" equilibrium with atomic consumers and commitment)[13] *Under Assumption 2, when only firm i can offer options, there exists a SPNE as follows:*
Option Market: Firm i sets $p_o^i = \left(\frac{10-k_a}{11-k_a}\right)\left(\frac{2k_a-10}{k_a}\right)v$ with probability one, announces that the maximum number of consumers who can purchase options equals $x_o^i = k_a$, and publicly commits that the spot price will be equal to $\left(\frac{10-k_a}{k_a}\right)v$ with probability one. A number of consumers equal to k_a purchase the options of firm i.
Spot Market: Firm j sets $p_{spot}^j = v$ with probability one. Each consumer who purchased an option from firm i exercises the option with probability one.
The profits of firm i are equal to $\left(\frac{(10-k_a)(k_a+1)}{11-k_a}\right)v$. The profits of firm j are equal to $(10-k_a)v$. Each consumer who purchases options achieves net surplus $\left(\frac{2k_a-10}{(11-k_a)k_a}\right)v$ and each consumer who does not purchase options achieves a net suplus equal to zero.

[INSERT TABLE 5 HERE]

5 Conclusion

I have introduced the concept of *priority options* in price-setting oligopolistic markets with exogenous symmetric capacity constraints. The canonical model with capacity constraints by Acemoglu *et al.* (2009) represents the benchmark model in my setup. The predictions with *priority options* are very different as compared to Acemoglu *et al.* (2009).

[13] Contrary to Proposition 1, the "good" equilibrium here is not valid without commitment.

My baseline duopoly model leads to a large or small increase in profits for one of the adopters of options. I derive asymmetry in profits with one firm winning and the other firm remaining unaffected, although my model is symmetric. It deserves to be emphasized that the large increase in profits can be very large, while the small increase in profits can be nonnegligible.

The *priority options* do not generate losses for firms in my market with two competitors. This can be contrasted to the quantity-competition literature initiated by Brander and Lewis (1986) and Fershtman and Judd (1987). In this sense, my model can be viewed as a contribution to the long-standing comparison between price and quantity competition. I also proved that my main predictions carry over to markets with atomic consumers with some small adjustments. These adjustments violate some type of equivalence that emerges from the existing literature between markets with atomic and nonatomic consumers.

In follow-up research, I study the *priority options* (i) with asymmetric and endogenous capacities; (ii) with more than two firms; (iii) with heterogeneous consumers, where the heterogeneity reflects either differences in the valuations for the underlying product (affine demand) or differences in loyalty (Ilin and Shi 2022); and (iv) in the presence of speculators who are not interested in purchasing the underlying product but want to resell the options in order to capture profits.

A Appendix: Proofs and Additional Material

Proof of Proposition S1: Suppose that firm j sets a price equal to the marginal cost. The best response of firm i is to set a price equal to v and attract the consumers who could not purchase at the low price. That is, the minmax profits for each firm are equal to $(1-k)v$. Due to the symmetry, one determines the common cumulative distribution function ($F_{spot}(p)$) as follows:

$$(1-k)pF_{spot}(p) + kp(1-F_{spot}(p)) = (1-k)v$$

or

$$F_{spot}(p) = \frac{1}{2k-1}\left(k - \frac{(1-k)v}{p}\right)$$

where $p \in \left[\left(\frac{1-k}{k}\right)v, v\right]$. The firms play no atoms in equilibrium. If both firms play a price strictly higher than the lower bound with positive probability, then one of the firms unilaterally has the incentive to charge a slightly lower price with this probability and increase the profits. Both firms playing the lower bound of the support with positive probability cannot be sustained in equilibrium because a firm has a unilateral incentive to set v with the same probability. **Q.E.D.**

Proof of Proposition 1: It holds that $p_o^i + p_{spot}^i = v$, i.e., the consumers who purchase the options exercise the options and obtain zero payoff. Should a consumer unilaterally decide not to purchase the option, this consumer purchases the product at p_{spot}^i with zero probability (because the mass of consumers who purchase the options equals k) and ends up purchasing the product at $p_{spot}^j = v$ with probability one, which results in a payoff equal to zero. The consumers who do not purchase options cannot switch to purchasing options because all options are sold. The firm that does not offer options has minmax profits equal to $(1-k)v$ and cannot increase the equilibrium profits by choosing a different *spot* price because this firm is not willing to set a *spot* price strictly lower than $\left(\frac{1-k}{k}\right)v$. The firm that offers options could set a *spot* price equal to $v-\epsilon$, where ϵ tends to zero from above, and capture profits $kp_o^i + (1-k)v$. These profits are equal to kv when $p_o^i = \left(\frac{2k-1}{k}\right)v$. Hence, the firm that offers options cannot increase the profits and utilizes the full capacity by charging a generalized monopoly price in equilibrium. **Q.E.D.**

Proof of Proposition 2: The left-hand side of Equation (1) is lower than the right-hand side of Equation (1) when p_o^i tends to zero from above. Additionally, the right-hand side of Equation (1) is strictly decreasing in p_o^i and, in particular, when p_o^i is equal to or higher than $\left(\frac{2k-1}{k}\right)v$, it takes negative values. When firm i sets $p_o^i = p_o^{threshold}$ with probability one and announces that a mass of consumers equal to $x_o^i \in [0,k]$ can purchase options, then the minmax profits for both firms in the *spot* market are equal to $(1-k)v$. To understand this, note that firm j achieves minmax profits $(1-k)v$ and therefore does not choose prices lower than $\left(\frac{1-k}{k}\right)v$. Consequently, firm i has minmax profits equal to $\max\left\{k\left(\frac{1-k}{k}\right)v, (1-k)(v-p_o^i), (1-k-y(1-k))v\right\} = (1-k)v$, where $y \in [0,k]$ represents the mass of consumers who purchase options. Provided that $p_o^i \in \left(0, p_o^{threshold}\right)$, I will prove in four steps that—without public commitment for the *spot* price at Stage 0—there is no equilibrium in which $y \in [0,k]$.

First Step—It is obvious by Equation (1) that should $y = 0$, then a representative consumer can deviate to buying an option and increase their payoff.

Second Step—Suppose that $y \in (0, 1-k)$. Then, if the *spot* price of the firm that sold the options is strictly higher than $v - p_o^{threshold}$, the other firm will never set a *spot* price weakly lower than $v - p_o^{threshold}$. In the same spirit, if the *spot* price of the firm that sold the options is weakly lower than $v - p_o^{threshold}$, the other firm will never set a *spot* price weakly higher than $v - p_o^{threshold}$. These arguments mean that one determines the common cumulative distribution function $(F_{spot}^H(p))$ for prices strictly higher than $v - p_o^{threshold}$ as follows:

$$(1-k-y)pF_{spot}^H(p) + kp(1-F_{spot}^H(p)) = (1-k)v$$

or

$$F_{spot}^H(p) = \frac{1}{2k-1+y}\left(k - \frac{(1-k)v}{p}\right)$$

This distribution function places an atom at v, so there is positive probability that both firms in the *spot* market set the monopoly price. In this case, one of the firms unilaterally can set a slightly lower price than v with the probability that this firm plays v and increase their payoff.

Third Step—Suppose that $y \in [1-k, k)$. Additionally, suppose that $(1-y)v > k\left(v - p_o^{threshold}\right)$ or $y < 1 - k + k\frac{p_o^{threshold}}{v}$ and $k < 1 - k + k\frac{p_o^{threshold}}{v}$. Then, when $y \in [1-k, k)$ and the *spot* price of the firm that sold the options is strictly higher than $v - p_o^{threshold}$, the other firm will never set

a *spot* price weakly lower than $v - p_o^{threshold}$. In the same spirit, if the *spot* price of the firm that sold the options is weakly lower than $v - p_o^{threshold}$, the other firm will never set a *spot* price weakly higher than $v - p_o^{threshold}$. The common cumulative distribution function ($F_{spot}^H(p)$) for prices in the interval $\left[\frac{k}{1-y}\left(v - p_o^{threshold}\right), v\right]$ is determined as follows:

$$(1-y)p\left(1 - F_{spot}^H(p)\right) = (1-k)v$$

or

$$F_{spot}^H(p) = 1 - \frac{(1-k)v}{(1-y)p}$$

This distribution function places an atom at v, so there is positive probability that both firms in the *spot* market set the monopoly price. Again, this cannot be an equilibrium distribution. Now, suppose that $k > 1 - k + k\frac{p_o^{threshold}}{v}$. Then, when $y \in \left[1-k, 1-k+k\frac{p_o^{threshold}}{v}\right)$, the arguments are the same as above. At the same time, when $y \in \left[1-k+k\frac{p_o^{threshold}}{v}, k\right)$, the firm that does not sell options does not choose prices in the interval $\left(v - p_o^{threshold}, v\right)$, i.e., this firm has a gap in the *spot* price distribution. Therefore, in the *spot* market, both firms play atoms and unilateral deviations are profitable.

Fourth Step—Suppose that $y = k$. It holds that $(1-k)v < k\left(v - p_o^{threshold}\right)$. Again, the firm that does not sell options does not choose prices in the interval $\left(v - p_o^{threshold}, v\right)$ and both firms play atoms in the *spot* market. Hence, there exist profitable unilateral deviations.

Next, I argue that with commitment the profits from the *spot* market equal $(1-k)v$ for the firm that commits. This occurs because the firm commits to a *spot* price equal to $\left(\frac{1-k}{k}\right)v$. The best response from the rival firm in the *spot* market is the monopoly price. Accordingly, both firms in the *spot* market obtain their minmax profits. Suppose that the mass of consumers who purchase options equals y where $0 \leq y < k$. Then, the payoff of each consumer who does not purchase options equals $v - \left(\frac{k-y}{1-y}\right)\left(\frac{1-k}{k}\right)v - \left(\frac{1-k}{1-y}\right)v$. This payoff is weakly lower than $v - 2(1-k)v$. It holds that $v - p_o^{threshold} - \left(\frac{1-k}{k}\right)v$ is strictly higher than the right-hand side of Equation (1). In turn, the right-hand side of Equation (1) is equal to $v - kE(p_{min}|no\ options) - (1-k)E(p_{max}|no\ options)$ when the options price equals $p_o^{threshold}$. But $v - kE(p_{min}|no\ options) - (1-k)E(p_{max}|no\ options) = v - 2(1-k)v$. I have proved that when $0 \leq y < k$, the consumers who do not purchase options have an incentive to switch to purchasing an option. Consider that $y = k$. The consumers who

do not purchase options achieve zero net surplus and the consumers who purchase options achieve a strictly positive net surplus. However, now the consumers who do not purchase options cannot switch to purchasing an option because there are no options available. The Proof is now complete. **Q.E.D.**

Proof of Lemma 1: When only one firm commits, then this firm commits to a *spot* price equal to $\left(\frac{1-k}{k}\right) v$ and the rival firm sets a *spot* price equal to the monopoly price. Therefore, it is not optimal for consumers to purchase the options of both firms or to purchase only the options of the firm that does not commit. **Q.E.D.**

Proof of Lemma 2: Suppose that one firm commits to a *spot* price equal to $\left(\frac{1-k}{k}\right) v$. If the rival firm does not commit, the rival firm captures profits $(1-k)v$. If the rival firm commits to a *spot* price equal to $\widehat{p}_{spot}$ and sets an option price equal to $\widehat{p}_o$, the following argumentation holds: First, $\widehat{p}_{spot}$ cannot be equal to $\left(\frac{1-k}{k}\right) v$ because in this scenario there is no dispersion in the *spot* market and therefore no consumers will purchase options. Second, $\widehat{p}_{spot}$ cannot be strictly higher than $\left(\frac{1-k}{k}\right) v$ because in this scenario the consumers will not purchase options at $\widehat{p}_o$, since $\widehat{p}_{spot}$ is the highest *spot* price. Third, the case with $\widehat{p}_{spot} < \left(\frac{1-k}{k}\right) v$ could be the only relevant case. When the mass of consumers who purchase options at $\widehat{p}_o$ is equal to $y \in (0, k)$, the following inequalities must be satisfied simultaneously:

$$v - \widehat{p}_{spot} - \widehat{p}_o \geq v - \left(\frac{k-y}{1-y}\right) \widehat{p}_{spot} - \left(\frac{1-k}{1-y}\right) \left(\frac{1-k}{k}\right) v$$

or

$$\widehat{p}_{spot} \leq \left(\frac{1-k}{k}\right) v - \left(\frac{1-y}{1-k}\right) \widehat{p}_o$$

and

$$k\widehat{p}_{spot} + y\widehat{p}_o \geq (1-k)v$$

These two inequalities cannot be satisfied simultanesouly. Therefore, I consider $y = k$. The inequalities above are modified as follows:

$$k\left(\widehat{p}_{spot} + \widehat{p}_o\right) \leq (1-k)v$$

and

$$k\left(\widehat{p}_{spot}+\widehat{p}_{o}\right)\geq(1-k)v$$

So, provided that one firm commits to a *spot* price equal to $\left(\frac{1-k}{k}\right)v$, then when $\widehat{p}_{spot}+\widehat{p}_{o}=\left(\frac{1-k}{k}\right)v$, the rival firm is indifferent between committing with $\widehat{p}_{spot}+\widehat{p}_{o}=\left(\frac{1-k}{k}\right)v$ and $\widehat{p}_{spot}<\left(\frac{1-k}{k}\right)v$ and not committing. This means that one firm committing and the other firm not committing constitutes an equilibrium. To see that this is the unique configuration in equilibrium, consider that one firm commits with $\widehat{p}_{spot}+\widehat{p}_{o}=\left(\frac{1-k}{k}\right)v$ and $\widehat{p}_{spot}<\left(\frac{1-k}{k}\right)v$. Then, the rival firm cannot commit with $\widetilde{p}_{spot}$ and $\widetilde{p}_{o}$ and a mass equal to k purchasing at $\widetilde{p}_{o}$.[14] The reason is that $\widetilde{p}_{spot}$ and $\widetilde{p}_{o}$ must satisfy the following inequalities simultaneously:

$$v-\widetilde{p}_{spot}-\widetilde{p}_{o}\geq v-\widehat{p}_{spot}$$

or

$$\widetilde{p}_{spot}+\widetilde{p}_{o}\leq\widehat{p}_{spot}$$

and

$$k\left(\widetilde{p}_{spot}+\widetilde{p}_{o}\right)\geq(1-k)v$$

It is true that:

$$\widehat{p}_{spot}-\widetilde{p}_{o}<\left(\frac{1-k}{k}\right)v-\widetilde{p}_{o}$$

or

$$\left(\frac{1-k}{k}\right)v-\widehat{p}_{spot}>0$$

because by assumption $\widehat{p}_{spot}<\left(\frac{1-k}{k}\right)v$. In sum, if one firms commits with $\widehat{p}_{spot}+\widehat{p}_{o}=\left(\frac{1-k}{k}\right)v$ and $\widehat{p}_{spot}<\left(\frac{1-k}{k}\right)v$, the best response of the rival firm is no commitment. When one firm does not commit, the best response of the rival firm is to commit at a *spot* price equal to $\left(\frac{1-k}{k}\right)v$ and so on.

[14] A mass equal to $y\in(0,k)$ is again not relevant because it is impossible that the following inequalities are satisfied simultaneously

$$\widetilde{p}_{spot}\leq\widehat{p}_{spot}-\left(\frac{1-y}{1-k}\right)\widetilde{p}_{o}$$

and

$$k\widetilde{p}_{spot}+y\widetilde{p}_{o}\geq(1-k)v$$

I have proved that either only firm j commits or only firm i commits. **Q.E.D.**

Proofs of Propositions 3-4: The Proofs are based on the arguments presented in the previous results. **Q.E.D.**

Next, I prove that when both firms can offer options, then there is no equilibrium in which no firm commits. I start with a series of Lemmas (Lemmas A1-A3) and conclude with Proposition A1 accompanied with Tables A1-A2.

Lemma A1 *When no firm commits, then the following configurations are not possible: (i)* $z_{i+j} = z_i = 0$ *and* $z_j > 0$*; or (ii)* $z_{i+j} = z_j = 0$ *and* $z_i > 0$.

Proof of Lemma A1: The firm that sells no options has an incentive to commit to a *spot* price equal to $\left(\frac{1-k}{k}\right) v$ and sell options to a strictly positive mass. **Q.E.D.**

Lemma A2 *When no firm commits, then it is not possible that* $z_{i+j} > 0$.

Lemma A3 *When no firm commits, then it is not possible that* $p_o^i \neq p_o^j$ *and* $z_i > 0$ *and* $z_j > 0$.

Proofs of Lemmas A2-A3: The Proof is based on the Proof of Proposition 2. In Lemma A2, both firms would set *spot* prices according to a distribution with the upper bound of support higher than $v - p_o^i - p_o^j$ which would lead both firms to atoms. In Lemma A3, both firms would set *spot* prices according to a distribution with the upper bound of support higher than $v - \max\{p_o^i, p_o^j\}$ which would again lead both firms to atoms. **Q.E.D.**

From Lemmas A1-A3, it follows that I need to check whether the so-called Type 1 and Type 2 symmetric candidate equilibria without commitment constitute equilibria of the dynamic game. Below, I provide the definition of the Type 1 symmetric candidate equilibria without commitment.

Type 1 symmetric candidate equilibria:

Stage 0

$$p_o^i = p_o^j = p_o^{T1} > p_o^{threshold},\ x_o^i = x_o^j = k$$

Stage 1

$$z_{i+j} = 0,\ 0 < z_i \leq k,\ 0 < z_j \leq k,\ z_i + z_j = 1$$

Stage 2

$$F_{spot}^i(p) = F_{spot}^j(p) = F_{spot}^{T1}(p)$$

The following explanations will lead to a more precise Definition of the Type 1 symmetric candidate equilibria. First, note that as $z_{i+j} = 0,\ 0 < z_i \leq k,\ 0 < z_j \leq k,\ z_i + z_j = 1$, in the *spot* market, $F_{spot}^{T1}(p) = \frac{1}{2k-1}\left(k - \frac{(1-k)\left(v-p_o^{T1}\right)}{p}\right)$ with $p \in [\left(\frac{1-k}{k}\right)\left(v - p_o^{T1}\right), v - p_o^{T1}]$. Second, z_i and z_j have to satisfy $z_i = z_j = \frac{1}{2}$. This is necessary for the payoff from purchasing only the option of firm i to be equal to the payoff from purchasing only the option of firm j, given that $p_o^i = p_o^j$. Observe that the payoff from purchasing only the option of firm i equals

$$v - p_o^{T1} - \frac{1}{2}\left(\frac{v - p_o^{T1}}{v}\right) E(p_{min}|no\ options)\left(1 + \frac{k - z_j}{1 - z_j}\right)$$

$$-\frac{1}{2}\left(1 - \frac{k - z_j}{1 - z_j}\right)\left(\frac{v - p_o^{T1}}{v}\right) E(p_{max}|no\ options)$$

whereas the payoff from purchasing only the option of firm j equals

$$v - p_o^{T1} - \frac{1}{2}\left(\frac{v - p_o^{T1}}{v}\right) E(p_{min}|no\ options)\left(1 + \frac{k - z_i}{1 - z_i}\right)$$

$$-\frac{1}{2}\left(1 - \frac{k - z_i}{1 - z_i}\right)\left(\frac{v - p_o^{T1}}{v}\right) E(p_{max}|no\ options)$$

Third, the payoff from purchasing only one option must be weakly higher than the payoff from not purchasing options. The payoff from not purchasing options equals

$$v - \left(\frac{k - \frac{1}{2}}{\frac{1}{2}}\right)\left(\frac{v - p_o^{T1}}{v}\right) E(p_{min}|no\ options) - \left(1 - \frac{k - \frac{1}{2}}{\frac{1}{2}}\right)\left(\frac{v - p_o^{T1}}{v}\right) E(p_{max}|no\ options)$$

so the relevant requirement can be stated as follows:

$$p_o^{T1} \leq \overline{p}_o^{T1}$$

where $\overline{p}_o^{T1}$ is the common option price which sets the payoff from only one option equal to the payoff from not purchasing options. Fourth, the payoff from purchasing only one option must be weakly higher than the payoff from purchasing two options. The payoff from purchasing two options equals

$$-2p_o^{T1} + prob\left(both\ prices < v - 2p_o^{T1}\ and\ upper\ bound\ v - p_o^{T1}\right)$$

$$\left(v - E\left(p_{min}|both\ prices < v - 2p_o^{T1}\ and\ upper\ bound\ v - p_o^{T1}\right)\right)$$

$$+2prob\left(one\ price > v - 2p_o^{T1}\ and\ upper\ bound\ v - p_o^{T1}\right)$$

$$prob\left(one\ price < v - 2p_o^{T1}\ and\ upper\ bound\ v - p_o^{T1}\right)$$

$$\left(v - E\left(p|one\ price < v - 2p_o^{T1}\ and\ upper\ bound\ v - p_o^{T1}\right)\right)$$

where

$$prob\left(both\ prices < v - 2p_o^{T1}\ and\ upper\ bound\ v - p_o^{T1}\right) = \frac{1}{(2k-1)^2}\left(\frac{(2k-1)v-(3k-1)p_o^{T1}}{v-2p_o^{T1}}\right)^2$$

$$prob\left(one\ price < v - 2p_o^{T1}\ and\ upper\ bound\ v - p_o^{T1}\right) = \frac{1}{2k-1}\left(\frac{(2k-1)v-(3k-1)p_o^{T1}}{v-2p_o^{T1}}\right)$$

$$prob\left(one\ price > v - 2p_o^{T1}\ and\ upper\ bound\ v - p_o^{T1}\right) = \frac{(1-k)p_o^{T1}}{(2k-1)(v-2p_o^{T1})}$$

$$E\left(p_{min}|both\ prices < v - 2p_o^{T1}\ and\ upper\ bound\ v - p_o^{T1}\right) =$$

$$\frac{2(1-k)\left(v-p_o^{T1}\right)\left(v-2p_o^{T1}\right)}{(2k-1)v-(3k-1)p_o^{T1}} - \frac{2(1-k)^2\left(v-p_o^{T1}\right)^2\left(v-2p_o^{T1}\right)}{\left((2k-1)v-(3k-1)p_o^{T1}\right)^2}\ln\left(\frac{k\left(v-2p_o^{T1}\right)}{(1-k)\left(v-p_o^{T1}\right)}\right)$$

$$E\left(p|one\ price < v - 2p_o^{T1}\ and\ upper\ bound\ v - p_o^{T1}\right) = \frac{(1-k)\left(v-p_o^{T1}\right)\left(v-2p_o^{T1}\right)}{(2k-1)v-(3k-1)p_o^{T1}}\ln\left(\frac{k\left(v-2p_o^{T1}\right)}{(1-k)\left(v-p_o^{T1}\right)}\right)$$

and the additional relevant requirement can be stated as follows:

$$p_o^{T1} \geq \underline{p}_o^{T1}$$

where $\underline{p}_o^{T1}$ is the common option price which sets the payoff from only one option equal to the payoff from two options.

[INSERT TABLE A1 HERE]

In Table A1, I provide simulation results for $\underline{p}_o^{T1}$ and $\overline{p}_o^{T1}$. Consider a Type 1 candidate equilibrium without commitment. The profits for a firm in this candidate equilibrium must be higher than the profits from a unilateral deviation in which the firm in question switches to an option price equal to $p_o^{threshold}$ with commitment. In case the firm that deviates sets an option price strictly higher

than $p_o^{threshold}$, then it is possible that no consumers purchase options. The maximum profits from the Type 1 candidate equilibria equal

$$(1-k)\left(v-\overline{p}_o^{T1}\right)+\frac{1}{2}\overline{p}_o^{T1}$$

or

$$(1-k)v+\left(k-\frac{1}{2}\right)\overline{p}_o^{T1}$$

while the maximum profits from deviation can equal

$$(1-k)v+kp_o^{threshold}$$

Condition A1 is helpful for Proposition A1:

Condition A1: $p_o^{threshold}>\left(1-\frac{1}{2k}\right)\overline{p}_o^{T1}$

Proposition A1 *When Condition A1 holds, then there are no equilibria in which no firm commits.*

[INSERT TABLE A2 HERE]

Proof of Proposition A1: Comparing Table A2 with Table 2, one observes that Condition A1 holds for all parameters examined. However, there is one more type of symmetric candidate equilibria that needs to be checked, namely the Type 2 symmetric candidate equilibria. Below, I define this type.

Type 2 symmetric candidate equilibria:

Stage 0

$$p_o^i=p_o^j=p_o^{T2}>p_o^{threshold},\ x_o^i=x_o^j=k$$

Stage 1

$$z_{i+j}=0,\ 0<z_i\leq k,\ 0<z_j\leq k,\ z_i+z_j<1$$

Stage 2

$$F_{spot}^i(p)=F_{spot}^j(p)=F_{spot}^{T2}(p)$$

In the Type 2 symmetric candidate equilibria without commitment, it must hold that $0<z_i=z_j<\frac{1}{2}$. Based on this requirement, one understands that the following inequalities are valid:

$p_o^{T2} \leq \overline{p}_o^{T2} < \overline{p}_o^{T1}$ where $\overline{p}_o^{T2}$ is the counterpart of $\overline{p}_o^{T1}$, i.e., the highest permissible p_o^{T2}. So, when Condition A1 holds, this implies that the Type 2 symmetric candidate equilibria without commitment do not constitute equilibria of the dynamic game. **Q.E.D.**

Proof of Proposition S2: The Proof is based on the Proof of Proposition S1 with small adjustments. **Q.E.D.**

Proof of Proposition 5: A consumer who purchases an option achieves net surplus

$$v - \left(\frac{10 - k_a}{k_a}\right) v - p_o^i$$

If k_a consumers purchase options, then the payoff for a consumer who switches to not purchasing an option is represented by the expression:

$$v - \left(\frac{10 - k_a}{k_a}\right) v \left(\frac{k_a - (k_a - 1)}{10 - (k_a - 1)}\right) - \left(\frac{10 - k_a}{10 - (k_a - 1)}\right) v$$

It must hold that:

$$v - \left(\frac{10 - k_a}{k_a}\right) v - p_o^i \geq v - \left(\frac{10 - k_a}{k_a}\right) v \left(\frac{k_a - (k_a - 1)}{10 - (k_a - 1)}\right) - \left(\frac{10 - k_a}{10 - (k_a - 1)}\right) v$$

This yields that the optimal option price equals $\left(\frac{10-k_a}{11-k_a}\right)\left(\frac{2k_a-10}{k_a}\right) v$. Simply observe that in case the option seller targets a smaller number of consumers than k_a as the option buyers, then this would require a lower option price than $\left(\frac{10-k_a}{11-k_a}\right)\left(\frac{2k_a-10}{k_a}\right) v$. That is, the option seller would sell less options at a lower option price, which is clearly suboptimal. **Q.E.D.**

References

[1] Acemoglu, Daron, Kostas Bimpikis, and Asuman Ozdaglar. 2009. "Price and Capacity Competition." *Games and Economic Behavior*, 66(1): 1-26.

[2] Adiri, I., and U. Yechiali. 1974. "Optimal Priority-Purchasing and Pricing Decisions in Nonmonopoly and Monopoly Queues." *Operations Research*, 22(5): 1051-1066.

[3] Aydin, Nurşen, Ş. İlker Birbil, and Hüseyin Topaloglu. 2017. "Delayed Purchase Options in Single-leg Revenue Management." *Transportation Science*, 51(4): 1031-1386.

[4] Balachandran, K. R. 1972. "Purchasing Priorities in Queues." *Management Science*, 18(5-Part-1): 319-326.

[5] Balseiro, Santiago, Guillermo Gallego, Caner Göçmen, and Robert Phillips. 2011. "Revenue Management of Consumer Options for Sporting Events." Columbia University, Center for Pricing and Revenue Management, Working Paper Series: No. 2011-1

[6] Beckmann, Martin J. with the assistance of D. Hochstädter. 1965. "Bertrand-Edgeworth Duopoly Revisited." In R. Henn (Ed.) *Operations Research Verfahren* Volume III: 55-68. Meisenheim: Verlag Anton Hain

[7] Brander, James A., and Tracy R. Lewis. 1986. "Oligopoly and Financial Structure: The Limited Liability Effect." *American Economic Review*, 76(5): 956-970.

[8] Chamberlain, Jonathan, Eran Simhon, and David Starobinski. 2021. "Preemptible Queues with Advance Reservations: Strategic Behavior and Revenue Management." *European Journal of Operational Research*, 293(2): 561-578.

[9] Chen, Ming, and Zhi-Long Chen. 2019. "Uncertain about your Travel Plan? Lock it and Decide Later: Dynamic Pricing with a Fare-lock Option." *Transportation Research Part E: Logistics and Transportation Review*, 125: 1-26.

[10] Chevalier-Roignant, Benoit, and Lenos Trigeorgis. 2011. "Competitive Strategy: Options and Games." MIT Press.

[11] Cui, Shiliang, Zhongbin Wang, and Luyi Yang. 2020. "The Economics of Line-Sitting." *Management Science*, 66(1): 227-242.

[12] Davidson, Carl, and Raymond Deneckere. 1986. "Long-Run Competition in Capacity, Short-Run Competition in Price, and the Cournot Model." *RAND Journal of Economics*, 17(3): 404-415.

[13] Edgeworth, Francis Y. 1925. "The Pure Theory of Monopoly." In *Papers Relating to Political Economy* Volume 1 (Chapter E): 111–142. Macmillan.

[14] Fershtman, Chaim, and Kenneth L. Judd. 1987. "Equilibrium Incentives in Oligopoly." *American Economic Review*, 77(5): 927-940.

[15] Gallego, Guillermo, S. G. Kou, and Robert Phillips. 2008. "Revenue Management of Callable Products." *Management Science*, 54(3): 550–564.

[16] Gallego, Guillermo, and Haengju Lee. 2020. "Callable Products with Dependent Demands." *Naval Research Logistics*, 67(3): 185-200.

[17] Gallego, Guillermo, and Haengju Lee. 2022. "Callable Products for an Implicit Time Model with Dependent Demands." *Naval Research Logistics*, 70(1): 121-128.

[18] Gallego, Guillermo, and Huseyin Topaloglu. 2019. *Revenue Management and Pricing Analytics*. Springer.

[19] Gershkov, Alex, and Eyal Winter. 2023. "Gainers and Losers in Priority Services." *Journal of Political Economy*, 131(11): 3103-3155.

[20] Gomes, Francisco, Michael Haliassos, and Tarun Ramadorai. 2021. "Household Finance." *Journal of Economic Literature*, 59(3): 919-1000.

[21] Hassin, Refael, and Moshe Haviv. 1997. "Equilibrium Threshold Strategies: The Case of Queues with Priorities." *Operations Research*, 45(6): 966-973.

[22] Hull, John C. 2022. *Options, Futures, and Other Derivatives.* 11th edition, Pearson.

[23] Ilin, Cornelia, and Guanming Shi. 2022. "Competition, Price Dispersion and Capacity Constraints: The Case of the U.S. Corn Seed Industry." *European Review of Agricultural Economics*, 49(3): 557-592.

[24] Jain, Adishwar K., and Raymond A. K. Cox. 2011. "Airfare Price Insurance: A Real Option Model." *Journal of Risk Finance*, 12(1): 5-14.

[25] Kahneman, Daniel, Jack L. Knetsch, and Richard H. Thaler. 1986. "Fairness as a Constraint on Profit Seeking: Entitlements in the Market." *American Economic Review*, 76(4): 728-741.

[26] Levitan, Richard, and Martin Shubik. 1972. "Price Duopoly and Capacity Constraints." *International Economic Review*, 13(1): 111-122.

[27] Li, Xianwei, Bo Gu, Cheng Zhang, Kyoko Yamori, and Yoshiaki Tanaka. 2014. "Price Competition in a Duopoly IaaS Cloud Market." In *Proceedings of the 16th Asia-Pacific Network Operations and Management Symposium*, 1-4.

[28] Madura, Jeff. 2025. *Personal Finance.* 8th edition, Pearson.

[29] Mendelson, Haim, and Seungjin Whang. 1990. "Optimal Incentive-Compatible Priority Pricing for the M/M/1 Queue." *Operations Research*, 38(5): 870- 883.

[30] Myatt, David P., and David Ronayne. 2026. "Stable Price Dispersion under Heterogeneous Buyer Consideration." *RAND Journal of Economics*, 57(1): 103-121.

[31] Obradovits, Martin. 2014. "Austrian-Style Gasoline Price Regulation: How It May Backfire." *International Journal of Industrial Organization*, 32(1): 33-45.

[32] Parakhonyak, Alexei, Martin Peitz, and Anton Sobolev. 2025. "Price Competition with Zero Consumer Search Costs and Limited Capacity." Research in Progress, Presented at the 2025 Annual Conference of the European Economic Association, Bordeaux, France

[33] Ross, Stephen A., Randolph W. Westerfield, Jeffrey Jaffe, and Bradford D. Jordan. 2021. *Corporate Finance.* 13th edition, McGraw-Hill.

[34] Sainam, Preethika, Sridhar Balasubramanian, and Barry L. Bayus. 2010. "Consumer Options: Theory and an Empirical Application to a Sports Market." *Journal of Marketing Research*, 47(3): 401-414.

[35] Shubik, Martin. 1959. *Strategy and Market Structure.* Wiley.

[36] Smit, Han T. J., and Lenos Trigeorgis. 2004. *Strategic Investment: Real Options and Games.* Princeton University Press

[37] Somogyi, Robert, Wouter Vergote, and Gabor Virag. 2023. "Price Competition with Capacity Uncertainty—Feasting on Leftovers." *Games and Economic Behavior*, 140: 253-271.

[38] Sorensen, Alan T. 2000. "Equilibrium Price Dispersion in Retail Markets for Prescription Drugs." *Journal of Political Economy*, 108(4): 833-850.

[39] Sumida, Mika, and Huseyin Topaloglu. 2019. "An Approximation Algorithm for Capacity Allocation Over a Single Flight Leg with Fare-Locking." *INFORMS Journal on Computing*, 31(1): 83-99.

[40] Trigeorgis, Lenos, and Andrianos E. Tsekrekos. 2018. "Real Options in Operations Research: A Review." *European Journal of Operational Research*, 270(1): 1-24.

[41] Trigeorgis, Lenos. 1996. *Real Options.* MIT Press.

[42] Turner, Amanda, and Dirk Zeindler. 2023. *Stochastic Finance.* Cambridge University Press.

[43] Varian, Hal R. 1980. "A Model of Sales." *American Economic Review*, 70(4): 651-659.

	$k = 0.51$	$k = 0.6$	$k = 0.7$	$k = 0.8$	$k = 0.9$	$k = 0.99$
Profits Per Firm without Options [Prop. S1]	$0.49v$	$0.4v$	$0.3v$	$0.2v$	$0.1v$	$0.01v$
Profits for Firm j [Prop. 1]	$0.49v$	$0.4v$	$0.3v$	$0.2v$	$0.1v$	$0.01v$
$p_o^i = \left(\frac{2k-1}{k}\right) v$ [Prop. 1]	$\approx 0.039v$	$\approx 0.333v$	$\approx 0.57v$	$0.75v$	$\approx 0.888v$	$\approx 0.9898v$
Profits for Firm i [Prop. 1]	$0.51v$	$0.6v$	$0.7v$	$0.8v$	$0.9v$	$0.99v$
Increase (%) in Profits for Firm i	$\approx 4\%$	50%	$\approx 133.33\%$	300%	800%	$9,800.00\%$

Table 1: Symmetric Capacities—Proposition S1 versus Proposition 1 Parameter: $v > 0$. Only firm i offers options.

	$k = 0.51$	$k = 0.6$	$k = 0.7$	$k = 0.8$	$k = 0.9$	$k = 0.99$
$p_o^{threshold}$	$\approx 0.00313646v$	$\approx 0.02165v$	$\approx 0.0265v$	$\approx 0.020684v$	$\approx 0.009331v$	$\approx 0.000274v$

Table 2: Symmetric Capacities—Equation (1) Parameter: $v > 0$. Only firm i offers options.

	$k = 0.51$	$k = 0.6$	$k = 0.7$
Profits Per Firm without Options [Prop. S1]	$0.49v$	$0.4v$	$0.3v$
Profits for Firm j [Prop. 2]	$0.49v$	$0.4v$	$0.3v$
Profits for Firm i [Prop. 2]	$\approx 0.491599v$	$\approx 0.41299v$	$\approx 0.315155v$
Increase (%) in Profits for Firm i	$\approx 0.3264\%$	$\approx 3.2475\%$	$\approx 5\%$

Table 3: Symmetric Capacities—Proposition S1 versus Proposition 2 Parameter: $v > 0$. Only firm i offers options.

	$k = 0.8$	$k = 0.9$	$k = 0.99$
Profits Per Firm without Options [Prop. S1]	$0.2v$	$0.1v$	$0.01v$
Profits for Firm j [Prop. 2]	$0.2v$	$0.1v$	$0.01v$
Profits for Firm i [Prop. 2]	$\approx 0.2165v$	$\approx 0.1083979v$	$\approx 0.010271v$
Increase (%) in Profits for Firm i	$\approx 8.27\%$	$\approx 8.3979\%$	$\approx 2.71\%$

Table 4: Symmetric Capacities—Proposition S1 versus Proposition 2 Parameter: $v > 0$. Only firm i offers options.

	$k_a = 6$	$k_a = 7$	$k_a = 8$	$k_a = 9$
Profits Per Firm without Options [Prop. S2]	$4v$	$3v$	$2v$	v
Profits for Firm j [Prop. 5]	$4v$	$3v$	$2v$	v
$p_o^i = \left(\frac{10-k_a}{11-k_a}\right)\left(\frac{2k_a-10}{k_a}\right) v$ [Prop. 5]	$\approx 0.266v$	$\approx 0.42857v$	$0.5v$	$\approx 0.444v$
Profits for Firm i [Prop. 5]	$\approx 5.6v$	$\approx 6v$	$6v$	$\approx 4.9999v$
Increase (%) in Profits for Firm i	$\approx 40\%$	$\approx 100\%$	200%	$\approx 399.99\%$

Table 5: Symmetric Capacities—Proposition S2 versus Proposition 5 Parameter: $v > 0$. Only firm i offers options.

	$k = 0.51$	$k = 0.6$	$k = 0.7$	$k = 0.8$	$k = 0.9$	$k = 0.99$
$\overline{p}_o^{T1}$	$\approx 0.00636345v$	$\approx 0.0418894v$	$\approx 0.0505218v$	$\approx 0.039753v$	$\approx 0.01832133v$	$\approx 0.0005484537v$
$\underline{p}_o^{T1}$	$\approx 0.0061v$	$\approx 0.04135v$	$\approx 0.05038v$	$\approx 0.0397397958v$	$\approx 0.01832116v$	$\approx 0.0005484537v$

Table A1: Symmetric Capacities—Type 1 Symmetric Candidate Equilibria Parameter: $v > 0$. Both firms offer options.

	$k = 0.51$	$k = 0.6$	$k = 0.7$	$k = 0.8$	$k = 0.9$	$k = 0.99$
$\left(1 - \frac{1}{2k}\right) \overline{p}_o^{T1}$	$\approx 0.00012v$	$\approx 0.00698v$	$\approx 0.0144v$	$\approx 0.0149v$	$\approx 0.00814v$	$\approx 0.000271v$

Table A2: Symmetric Capacities—Condition A1 Parameter: $v > 0$. Both firms offer options.