The Surprising Force of the Accelerator*

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Abstract

We combine a new measure of business investment across U.S. regions with two distinct fiscal policy shocks to study the response of investment to changes in local demand. We find that business investment responds positively to transitory demand shocks, with a preferred estimate of 16 cents per dollar of government spending. This finding suggests a relatively strong accelerator effect from non-investment to investment spending. We contrast our estimates with the predictions of a standard monetary-union model with investment and capital accumulation. For shocks with persistence similar to the empirical shocks, the model generates negative or near-zero investment effects across a range of plausible parameter values. Evidence comparing small and large firms suggests that financial constraints are unlikely to resolve this tension.

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1 Introduction

Co-movement of quantities—consumption, investment, employment—is a dominant feature of business cycles. The accelerator principle, going back to Hansen and Samuelson (Samuelson, 1939), offers a potential explanation on the investment side: investment spending responds to high anticipated demand, so investment endogenously increases during booms, even when it is not the driving force. In this paper we try to measure the strength of this feedback mechanism and to compare it with the predictions of current standard models.

Time-series analyses based on aggregate data face the familiar problem that multiple macroeconomic shocks and forces move together, making causal interpretation difficult. Monetary policy shocks, even if convincingly identified, do not provide a good laboratory for this question because they conflate the effect of changes in the cost of capital and changes in aggregate demand. Firm-level studies on investment have focused on the relation between investment and sales or cash flow at the firm level.[1] This relation is open to multiple interpretations given that sales and cash flow may reflect common shocks and broader macroeconomic forces. [MORE ON LITERATURE]

This paper focuses on variation in aggregate demand determined by fiscal policy shocks at the regional level. The first source is the ARRA stimulus program of 2009, where much of the relevant variation comes from statutory formulas that allocate funds across states. The second is military spending, where variation arises from large differences in DOD spending across metropolitan areas over time.

The regional level and the federal financing of the shocks analyzed imply that we should not capture effects that travel through changes in the cost of capital. In the case of ARRA, the spending was mostly in the form of transfers so we do not expect direct effects on investment. Therefore, we believe our design tends to isolate effects that travel through increases in local demand. In the case of military spending, some direct effects are necessarily present for businesses that procure directly for the federal government. However, by focusing our attention on the effects on non-tradable sectors, we aim to capture pure demand-driven effects.

The two fiscal policy shocks used here have been proposed and studied in previous work on the fiscal policy multiplier (Chodorow-Reich et al., 2012; Wilson, 2012; Nakamura and Steinsson, 2014; Auerbach et al., 2023). The novelty of our paper is to focus on investment responses, using a new measure of capital expenditures constructed from confidential Census

[1] A vast literature that includes Fazzari et al. (1988), Gilchrist and Himmelberg (1995), Kaplan and Zingales (1997), Clearly (1999), Baker et al. (2003), and Lewellen and Lewellen (2016).

Bureau microdata, developed by one of the authors in previous work (Carreño, 2026). This measure gives us investment spending at the regional level and allows us to use the geographic variation to estimate local open-economy investment responses.

The two exercises rely on different sources of variation and therefore provide independent evidence on the response of investment to local demand shocks.

Our main finding is that we observe significantly positive effects on business investment, despite the transitory nature of the demand shock. In our preferred specification for nontradable industries, an additional percentage point of cumulative ARRA payments relative to annual state GDP is associated with an increase of about 0.81% in median capital expenditures. This estimate implies that investment rises by about 16 cents for every additional dollar of ARRA payments.[2]

To interpret these findings, we study two relatively standard models of a monetary union with capital accumulation and investment adjustment costs. Our main model distinguishes tradable and nontradable production, with a separate capital stock for each sector and different compositions of consumption and investment goods. We use this structure to study investment in nontradable industries in the ARRA exercise. A one-sector Galí–Monacelli model with investment provides the framework for the military-spending exercise, which pools investment across industries. Within each framework, we consider complete markets with local fiscal financing and incomplete markets with federal fiscal financing.

We compare the models with the evidence using investment semi-elasticities. Under the benchmark calibrations, both specifications of each model generate negative annual investment responses, well below the positive empirical estimates. At the benchmark fiscal persistence, chosen to capture the transitory nature of the shocks motivating our exercises, annual responses remain well below the empirical magnitudes across the structural parameter variations considered, including cases with small positive responses in TNT. With the benchmark structural parameters, matching the empirical magnitudes requires substantially more persistent fiscal processes than the short-lived shocks motivating our empirical exercises.

This contrast points to a tension between the data and the predictions of these standard benchmarks. Transitory local demand shocks appear to generate investment responses that are substantially stronger than the models imply.

A related question is whether the estimated multipliers reflect genuinely new investment plans or instead the relaxation of financing constraints that allow firms to implement

[2]Because our investment data are annual, we cannot observe quarterly investment responses. We nevertheless examine multi-year dynamics below.

projects that had already been planned. Our evidence is more consistent with the former interpretation. In particular, heterogeneity by firm size shows that the estimated effects are not concentrated among smaller firms, which are more likely to face liquidity constraints (Chodorow-Reich et al., 2022).

Overall, our findings suggest that business investment responds endogenously to temporary demand shocks and that the accelerator mechanism may be quantitatively more important than standard investment models allow.

This paper contributes to several strands of the literature. First, it relates to the literature cited in footnote 1, which studies the empirical relationship between investment and output at the macro level and between investment and sales or cash flow at the firm level. Our contribution is to bring plausibly exogenous demand variation to this question and to quantify the causal response of business investment. In that sense, the paper provides a direct empirical test of the accelerator mechanism.

Second, the paper contributes to the empirical literature on fiscal multipliers. That literature has primarily focused on the responses of output and employment to government spending, where the output multiplier reflects the sum of the responses of each GDP component.[3] Ramey (2011a) reviews the time-series literature, while Chodorow-Reich (2019) reviews the geographic cross-sectional literature. Several papers have used the ARRA (Chodorow-Reich et al., 2012; Wilson, 2012; Conley and Dupor, 2013; Dupor and Mehkari, 2016) and military spending (Ramey, 2011b; Nakamura and Steinsson, 2014; Demyanyk et al., 2019; Auerbach et al., 2023) to study fiscal multipliers. Our contribution is to provide, to our knowledge, the first direct empirical measure of the investment multiplier.

Finally, our theoretical analysis builds on the monetary-union framework of Galí and Monacelli (2005); Galí and Monacelli (2008). We incorporate capital accumulation with adjustment costs and allow consumption and investment to differ in their composition. The tradable–nontradable structure connects the framework directly to the ARRA evidence on investment in nontradable industries, while the one-sector model provides the framework for the military-spending exercise. Within each model, we also compare alternative financial and fiscal arrangements while holding the production and trade structure fixed. This comparison assesses the robustness of the quantitative findings to the joint change in financial markets and fiscal financing.

[3] In particular,

$$\frac{\partial Y_t}{\partial G_t} = 1 + \frac{\partial C_t}{\partial G_t} + \frac{\partial I_t}{\partial G_t} + \frac{\partial X_t}{\partial G_t} - \frac{\partial M_t}{\partial G_t}.$$

The rest of the paper is organized as follows. Section 2 presents empirical evidence from the ARRA stimulus program. Section 3 presents evidence from military spending shocks. Section 4 introduces the two frameworks and their financial and fiscal specifications, and Section 5 compares their investment responses with the empirical evidence. Section 6 concludes.

2 Evidence from the ARRA

The American Recovery and Reinvestment Act (ARRA) was enacted in February 2009, during the Great Recession. At approximately $800 billion, it was one of the largest fiscal stimulus programs in U.S. history.

This episode provides a useful setting for our purposes for two main reasons. First, the ARRA was a one-time fiscal stimulus program, making it a natural source of temporary variation in local demand. Second, a large share of ARRA spending was allocated across states according to statutory formulas. This cross-state variation allows us to study the effect of ARRA disbursements on investment while differencing out aggregate shocks common to all states, such as monetary policy or aggregate expectations. We implement an instrumental variables design across U.S. states, relating cumulative ARRA payments to subsequent state-level investment outcomes. For institutional details on the ARRA and its allocation formulas, see Wilson (2012) and Chodorow-Reich et al. (2012).

The first- and second-stage regressions in our cross-state instrumental variables analysis are:

$$S_i = \delta + \mathbf{X}_{i,0}\Theta + \mathbf{Z}_{i,0}\Phi + \nu_i, \tag{1}$$

and

$$\Delta I_{i,T} = \alpha + \beta \widehat{S}_i + \mathbf{X}_{i,0}\Gamma + \epsilon_{i,T}. \tag{2}$$

The unit of observation is the state. $\Delta I_{i,T}$ denotes investment growth in state i over a window ending in year T. In the baseline specification, $T = 2010$, so the outcome is investment growth between 2008 and 2010. We also consider later windows to study the dynamic effects of the ARRA. S_i denotes cumulative ARRA spending as a share of 2008 GDP in state i, while $\mathbf{X}_{i,0}$ and $\mathbf{Z}_{i,0}$ denote vectors of pre-ARRA controls and instruments, respectively.

Dependent Variable

Our baseline dependent variable is investment growth between 2008 and 2010. Given that our data are annual and the ARRA was enacted in 2009, this window allows us to compare pre- and post-ARRA investment while controlling for heterogeneous pre-trends through the inclusion of 2006–2008 investment growth and the 2008 investment level as covariates.

We construct state-level investment using confidential Census Bureau microdata from the Annual Capital Expenditures Survey (see appendix A for details). The measure captures corporate capital expenditures at the state level and excludes the public sector. Let $I_{i,T}$ denote the weighted median of corporate capital expenditures in state i in year T.[4]

Because our outcome is a state-level median rather than total investment, scaling it by lagged GDP does not have a natural interpretation: the median is already comparable across states, whereas dividing it by state GDP would mechanically mix a firm-level object with a state-level scale variable. Accordingly, our estimates should be interpreted as elasticities: they measure the percentage change in median corporate capital expenditures associated with a one-percentage-point increase in ARRA spending as a share of GDP. Below, we provide a simple back-of-the-envelope exercise to relate these elasticities to dollar-for-dollar magnitudes.

Controls

We include five control variables in each regression. First, following Blanchard and Katz (1992), we control for lagged investment growth and the initial level of investment. Specifically, we include investment growth between 2006 and 2008 and the level of investment in 2008, both measured prior to the ARRA. These two controls absorb pre-existing cross-state differences in investment dynamics and investment levels, including differential exposure to the downturn before the ARRA.

Given the rapid run-up in housing prices before the recession, we also control for the 2003–2007 change in state-level house prices, using the House Price Index from the Federal Housing Finance Agency (FHFA).

The final two controls capture features directly relevant to the ARRA's implementation. First, we include the 2005–2006 change in a three-year trailing average of personal income

[4]Because state-level investment is constructed from survey data rather than from the universe of establishments, we do not directly observe total investment for each state. We therefore aggregate establishment-level capital expenditures using the weighted median, which reduces the influence of outliers and potential reporting or imputation errors.

per capita. This variable directly enters the statutory formula used to determine state allocations of ARRA "Fiscal Relief Funds" from the Department of Health and Human Services (HHS), which were designed to help states finance Medicaid expenses. Finally, we control for estimated ARRA tax benefits received by state residents. This measure sums the benefits from the ARRA's "Making Work Pay" payroll tax cut and the increase in the income threshold for the Alternative Minimum Tax. We also experimented with including state population as a control variable, but it was statistically insignificant and had virtually no effect on the results.

Stimulus Spending Measures

Data on ARRA spending by state come from Wilson (2012), who compiled information from the Recovery.gov website and Funding Notification Reports. Over the August 2009–February 2010 window, roughly two quarters, we construct three measures of stimulus spending: funding announcements, funding obligations, and final payments. Funds are initially authorized to federal agencies. Based on those authorizations, agencies announce how much funding is available for eligible recipients in each state. The funds are then awarded (or "obligated") to recipients and, finally, disbursed once recipients fulfill the terms of their contracts. We report results for all three measures—including announcements, which are closest to the timing of agents' expectations[5]—but we consider payments the preferred measure of stimulus spending, as they are most directly tied to realized demand shocks.

Instruments

There are two main sources of endogeneity. First, some components of S_i are explicit functions of economic conditions and may therefore correlate with post-ARRA outcomes. We address this concern by controlling directly for those factors in the regressions.

Second, although funding announcements are determined mechanically by statutory formulas, the allocation of obligations and payments involves discretion by federal agencies. Some states may have been more effective than others at securing funds and starting projects. If those same states also differed systematically in ways that affected investment outcomes, ordinary least squares estimates would be biased. Following Wilson (2012), we address this concern using instrumental variables.

Most ARRA funds were distributed across states according to statutory formulas, some

[5]See Ramey (2011b), Leeper et al. (2010), and Mertens and Ravn (2010).

specific to the ARRA and others inherited from pre-existing non-ARRA programs. We construct three instruments for S_i—whether measured using announcements, obligations, or payments—based on predetermined allocation factors used by the three federal agencies responsible for most ARRA spending: the Departments of Transportation (DOT), Education (ED), and Health and Human Services (HHS).

1. Department of Transportation Instrument

Roughly three-quarters of DOT's $40 billion in ARRA funds were allocated through the Federal Highway Administration (FHWA), which finances the construction and repair of highways and bridges. The ARRA specified that:

- 50% of these funds be distributed according to the pre-existing Surface Transportation Program (STP) formula. This formula is a weighted average of three factors, each measured with a three-year lag: the state's lane miles of federal-aid highways, vehicle miles traveled on federal-aid highways, and estimated payments into the federal highway trust fund, primarily from fuel excise taxes. As a result, this portion of 2009 ARRA funds was allocated using 2006 data.
- The remaining 50% be distributed in the same proportions as FHWA's 2008 non-ARRA funds (obligation limitations), for which the STP is a major component.

We therefore construct a DOT instrument as the fitted value from a regression of 2009 ARRA DOT obligations on total federal-aid highway miles in 2006, total vehicle miles traveled in 2006, estimated payments into the federal highway trust fund in 2006, and FHWA obligation limitations in 2008.

2. Department of Education Instrument

More than half of ED's ARRA spending came from the $54 billion State Fiscal Stabilization Fund (SFSF). The ARRA specified that 61% of these funds be allocated in proportion to the state's school-aged population (ages 5–24), with the remainder allocated in proportion to total population. We therefore use the state's share of school-aged population, a predetermined demographic variable, as an instrument for ARRA education spending per capita.

3. Health and Human Services Instrument

The ARRA also provided $87 billion to help states finance Medicaid expenditures through

the State Fiscal Relief Fund. In particular, the ARRA temporarily increased state-specific Medicaid reimbursement rates (FMAP) by 6.2 percentage points for all states. As a result, states with higher pre-ARRA Medicaid expenditures per capita received more ARRA HHS funds per capita. We therefore use pre-ARRA Medicaid expenditures as an instrument for ARRA HHS funding.

Baseline Results

Table 1 reports the second-stage estimates. Each column corresponds to a separate regression. Although our preferred measure of stimulus spending is actual payments (column 1), we also report results for obligations and announcements (columns 2 and 3). The estimated coefficient is positive in all three cases and statistically significant. In the payments specification, the estimate implies that cumulative ARRA spending equal to 1% of GDP is associated with a 0.1395 percentage point increase in median investment. This calculation is based on the full corporate sector, excluding only the public sector. We return below to heterogeneity across tradable and non-tradable industries.

The coefficients for obligations and announcements are smaller, but still positive and statistically significant. This pattern is consistent with some anticipation effects in investment responses.

Table 1: Baseline Results

	2008-2010 Investment Growth		
	(1)	(2)	(3)
Payments over GDP	0.1394**		
	(0.06689)		
Obligations over GDP		0.06914**	
		(0.02794)	
Announcements over GDP			0.06970***
			(0.02013)
Include baseline controls	Yes	Yes	Yes
First State F-statistic	10.03	22.5	13.74
FS (F-test) p-value	0	0	0
N (Number of states)	50	50	50

NOTE.—Robust standard errors in parentheses. GDP is lagged to 2008. Baseline controls are enumerated in the text. Following CES guidelines, all numbers are rounded to four significant digits except N, which is rounded to the nearest 10. Significance levels: * 10%, ** 5%, *** 1%.

Tradable vs. Non-Tradable Industries

As Adao et al. (2019) emphasize, a region is affected not only by its own local shocks but also indirectly by shocks in other regions through trade linkages. Our baseline regressions, however, are designed to capture the direct effects of local shocks on local firms. Firms producing purely tradable goods are much less directly exposed to local demand fluctuations, while firms in non-tradable sectors are more directly tied to local spending conditions. This does not imply that tradable producers are unaffected by demand shocks, only that an important share of their response operates through interregional trade and therefore lies outside the scope of our local regressions.

A natural way to sharpen the demand interpretation is therefore to focus on non-tradable industries, which are the sectors most directly exposed to local demand shocks. If the effects indeed concentrate among non-tradables, the estimates in Table 1 can be interpreted as a lower bound relative to an ideal specification that would fully account for trade linkages and capture all demand-based effects. The magnitude of this lower bound depends on the relative importance of tradable and non-tradable industries in the economy. The non-tradable sector should also provide a useful approximation to that ideal estimate as long as demand elasticities are not too different across sectors.[6]

This reasoning also implies that, if the estimated effects operate through a local demand channel, they should be concentrated in non-tradable industries in our regressions. Purely tradable industries should display weaker direct responses, since much of their demand adjustment dissipates geographically. By contrast, a supply-based confounding mechanism would have to affect non-tradables systematically more than tradables in order to generate the same pattern.

The distinction between tradable and non-tradable activities is not perfectly sharp in practice, and different empirical approaches exist in the literature. We follow Mian and Sufi (2014)'s industry-based classification, which assigns industries to tradable and non-tradable groups on the basis of sectoral characteristics and has become standard in this line of work. We prefer this approach because it is transparent, easy to implement in our data, and well suited to our goal of isolating the direct local component of demand shocks. Using that classification, we construct state-level measures of capital expenditures restricted to non-tradable and tradable industries, respectively, and replicate the baseline analysis for each

[6] A rigorous approach would involve constructing a demand shock measure that explicitly incorporates both local and trade-mediated effects, with the aim of capturing all domestic demand responses. This could potentially be implemented using the U.S. Census Commodity Flow Survey microdata. Following much of the literature, we instead focus on non-tradables, as such an extension lies beyond the scope of this paper.

group separately.

Table 2 reports the results, including the baseline estimates from Table 1 for comparison. Relative to the baseline results in columns 1–3, the estimated coefficients are substantially larger when the sample is restricted to non-tradable industries (columns 4–6). In the payments specification, cumulative ARRA transfers equal to 1% of GDP are associated with a 0.8055 percentage point increase in median investment in non-tradable industries. By contrast, the estimated coefficients for tradable industries are statistically insignificant in all three specifications.

To relate this elasticity to a dollar-for-dollar magnitude, we perform a simple back-of-the-envelope conversion. Under the assumptions that (i) the growth rate of weighted median investment approximates the growth rate of total state-level investment, and (ii) state investment is approximately 20% of state GDP, the estimate in column 1 corresponds to an investment response of about 16 cents per dollar of ARRA payments.[7]

The concentration of the effect in non-tradable industries, together with its absence among tradables, is consistent with a demand-based interpretation of our baseline results and strengthens the credibility of the identification strategy. The non-tradable elasticity also provides a useful measure of the direct effect of the demand shock on corporate investment. If one were willing to extrapolate this elasticity to the broader economy—implicitly assuming similar demand elasticities across sectors—a simple back-of-the-envelope calculation would imply a sizable aggregate response. Given that the average state received cumulative ARRA transfers equal to 9.49% of GDP, the implied increase in investment would be $9.49 \times 0.8055 = 7.64$ percentage points. For comparison, average state-level investment growth between 2008 and 2010 was -21.29%.

Small vs. Large Firms

The baseline results are consistent with a demand-based effect on investment, but they do not by themselves distinguish between two possible mechanisms. One possibility is that higher demand relaxes financing constraints by increasing firms' cash flow, allowing constrained

[7] Ideally, we would estimate the regression:

$$\frac{\Delta I}{GDP} = \alpha + \beta S + X\Gamma + \epsilon,$$

but we estimate:

$$\frac{\Delta I}{I} = \tilde{\alpha} + \tilde{\beta} S + X\tilde{\Gamma} + \tilde{\epsilon}.$$

Under these assumptions, the dollar-for-dollar multiplier is approximately $\beta \approx \tilde{\beta} \cdot \frac{I}{GDP}$.

Table 2: Tradable vs Non-Tradable Industries

	Baseline Results			Non-tradable Industries			Tradable Industries		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Payments over GDP	0.1394**			0.8055**			-0.2235		
	(0.06689)			(0.3870)			(0.1728)		
Obligations over GDP		0.06914**			0.5025***			-0.09181	
		(0.02794)			(0.1834)			(0.09375)	
Announcements over GDP			0.06970***			0.4638**			-0.05333
			(0.02013)			(0.2164)			(0.08481)
Include baseline controls	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
First State F-statistic	10.03	22.5	13.74	3.32	13.3	3.84	3.18	9.29	5.77
FS (F-test) p-value	0	0	0	0.013	0	0.0064	0.0145	0	0.0005
N (Number of states)	50	50	50	30	30	30	30	30	30

NOTE.—Dependent variable is 2008-2010 investment growth. Robust standard errors in parentheses. GDP is lagged to 2008. Baseline controls are enumerated in the text. The sample is smaller in the industry splits because state-level cells must satisfy the minimum disclosure threshold within each subgroup. Following CES guidelines, all numbers are rounded to the nearest four significant digits except N, which is rounded to the nearest 10. Significance levels: * 10%, ** 5%, *** 1%.

firms to undertake investment that had already been planned. Another possibility is that the demand shock directly induces new investment plans by raising expected future profitability. Unlike the former channel, these are investment plans that would not have taken place otherwise, regardless of financing conditions.

To shed light on this distinction, we compare the responses of small and large firms. If the estimated effects primarily reflected the relaxation of liquidity constraints, one would expect stronger responses among smaller firms, which are more likely to face such constraints (Chodorow-Reich et al., 2022). Conversely, larger firms should exhibit weaker responses, as they generally have easier access to external financing. If instead the main channel operates through new investment plans triggered by demand, the effect need not be concentrated among small firms.

Table 3 reports the results. We split firms according to whether employment is below or above the median number of employees in the sample and then replicate the baseline analysis for each group.[8] The estimated effects are not concentrated among small firms. Relative to the baseline estimates in columns 1–3, the coefficients for small firms in columns 4–6 are similar but somewhat less precisely estimated.[9] The corresponding estimates for large firms, shown in columns 7–9, are also similar and, if anything, slightly larger in the payments specification.

Overall, these results are difficult to reconcile with an explanation based primarily on financing constraints and are more consistent with the view that demand shocks induce new investment plans.

Dynamic Effects

We next examine the dynamic pattern of the estimates across annual horizons. Because our investment data are annual, we cannot trace a quarterly impulse response. Instead, we estimate a sequence of specifications that keep the right-hand side of equation 2 fixed while shifting the investment-growth window forward.

Table 4 reports the results. Relative to the baseline specification, the estimated coefficient remains of similar magnitude and statistically significant when the dependent variable

[8] We perform the split at the firm level, rather than the establishment level, because financing constraints are more naturally defined at the firm level. Results are similar under alternative splits.

[9] Part of the imprecision for small firms stems from losing some states from the sample, as larger firms naturally operate more establishments. Some states therefore lack a sufficient number of establishments from smaller firms to be included. Additionally, the states that remain tend to have smaller cell sizes, introducing more noise.

Table 3: Small vs Large Firms

	Baseline Results			Small Firms			Large Firms		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Payments over GDP	0.1394**			0.1264			0.1656**		
	(0.06689)			(0.09671)			(0.08277)		
Obligations over GDP		0.06914**			0.04471			0.06633**	
		(0.02794)			(0.0477)			(0.03356)	
Announcements over GDP			0.06970***			0.04595			0.05021**
			(0.02013)			(0.05366)			(0.02368)
Include baseline controls	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
First State F-statistic	10.03	22.5	13.74	4.21	11.75	9.29	7.19	17.16	14.26
FS (F-test) p-value	0	0	0	0.0015	0	0	0	0	0
N (Number of states)	50	50	50	40	40	40	50	50	50

NOTE.—Dependent variable is 2008-2010 investment growth. Robust standard errors in parentheses. GDP is lagged to 2008. Baseline controls are enumerated in the text. The sample is smaller in the size splits because state-level cells must satisfy the minimum disclosure threshold within each subgroup. Following CES guidelines, all numbers are rounded to the nearest four significant digits except N, which is rounded to the nearest 10. Significance levels: * 10%, ** 5%, *** 1%.

Table 4: Some Dynamics

	t	$t+1$	$t+2$	$t+3$
	(1)	(2)	(3)	(4)
Payments over GDP	0.1394**	0.1649***	0.1249	-0.03941
	(0.06689)	(0.05745)	(0.08029)	(0.067)
Include baseline controls	Yes	Yes	Yes	Yes
First State F-statistic	10.03	10.03	10.03	10.03
FS (F-test) p-value	0	0	0	0
N (Number of states)	50	50	50	50

NOTE.—The dependent variables are 2008-2010 investment growth in column (1), 2009-2011 investment growth in column (2), 2010-2012 investment growth in column (3), and 2011-2013 investment growth in column (4). The right-hand side of the regression is unchanged relative to the baseline specification. GDP is lagged to 2008. Robust standard errors in parentheses. Baseline controls are enumerated in the text. Following CES guidelines, all numbers are rounded to the nearest four significant digits except N, which is rounded to the nearest 10. Significance levels: * 10%, ** 5%, *** 1%.

is shifted forward by one year. It declines in the following window and becomes statistically indistinguishable from zero thereafter.

3 Evidence from Military Spending Shocks

Our second empirical exercise studies the effects of military government spending on investment using a panel of U.S. metropolitan statistical areas (MSAs). This exercise is distinct from the ARRA analysis in both the underlying source of variation and the empirical design, and therefore provides an independent test of whether local demand shocks affect corporate investment. We focus on annual changes in DOD spending and, in our preferred specification, restrict attention to contracts with durations shorter than one year.

Working at the MSA level offers several advantages. First, Auerbach et al. (2023) shows that DOD spending exhibits much greater cross-sectional variation across cities than across states, and even less at the national level. This rich local variation allows us to identify the effects of military spending shocks without relying on rare aggregate episodes such as major wars. Second, military spending is often concentrated in a small number of locations within a state rather than spread evenly across the state as a whole. Using MSAs therefore provides a more precise measure of the local incidence of military spending shocks. Third, the combination of substantial cross-sectional variation and panel structure yields considerably greater statistical power than in the cross-state ARRA exercise, including for multi-year

horizon specifications.

The first- and second-stage regressions in our panel instrumental variables analysis are:

$$\frac{G_{i,t} - G_{i,t-1}}{GDP_{i,t-1}} = \delta_i + \zeta_t + \omega Z_{i,t} + \nu_{i,t} \tag{3}$$

and

$$\Delta I_{i,t} = \alpha_i + \gamma_t + \beta \widehat{\left(\frac{G_{i,t} - G_{i,t-1}}{GDP_{i,t-1}}\right)} + \epsilon_{i,t}. \tag{4}$$

The unit of observation is the MSA-year. $\Delta I_{i,t}$ denotes annual investment growth in MSA i in year t. The term $\frac{G_{i,t} - G_{i,t-1}}{GDP_{i,t-1}}$ denotes the change in military government spending as a share of lagged GDP between years $t-1$ and t, where $G_{i,t}$ is military government spending in MSA i in year t and $GDP_{i,t}$ is nominal GDP in MSA i in year t.[10] $Z_{i,t}$ is the instrument for military spending in MSA i and year t. The terms α_i and γ_t denote MSA and year fixed effects, respectively.

MSA fixed effects absorb time-invariant differences across local areas, including persistent differences in industrial composition and other local characteristics that may shape both investment dynamics and military spending exposure. Year fixed effects absorb aggregate shocks and forces common across MSAs, such as nationwide shifts in federal spending, monetary policy, tax policy, uncertainty and expectations shocks, stock market movements, and broader cyclical and structural macroeconomic conditions that could jointly influence investment and military spending.

Our panel spans the years 2001–2016. Given the panel structure of the data, we cluster standard errors at the MSA level.

Dependent Variable

Our dependent variable is annual investment growth at the MSA level. We construct MSA-level investment using confidential Census Bureau microdata from the Annual Capital Expenditures Survey (see appendix A for details). The measure captures corporate capital expenditures at the MSA level and excludes the public sector. Let $I_{i,t}$ denote the weighted median of corporate capital expenditures in MSA i in year t.

As in the ARRA analysis, our outcome is a local median rather than total investment. For that reason, scaling investment by lagged GDP does not have a natural interpretation. Our estimates should therefore be interpreted as elasticities: they measure the percentage

[10] All variables are deflated using the national CPI. Deflating with regional CPIs yields virtually identical results.

change in median corporate capital expenditures associated with a one-percentage-point increase in military spending as a share of GDP.

Military Government Spending

Our local measure of military government spending comes from Demyanyk et al. (2019), who construct it from USAspending.gov. This official government source reports detailed information on Department of Defense (DOD) contracts since 2000, based on DD-350 and DD-1057 military procurement forms. The data cover a wide range of purchases, from small $25 disbursements to procurements exceeding $32 billion.

Because we observe the duration of each contract, our preferred measure restricts the analysis to contracts with durations shorter than one year. The distribution is heavily concentrated at short durations: 67.74% of contracts last one quarter or less, and 79.15% last at most two quarters. For completeness, we also report results using a broader definition that includes contracts with durations shorter than two years. In addition to actual spending, we report results based on obligations, defined as the total value of new contracts signed in a given period minus the amount associated with terminated contracts (deobligations). Obligations are useful because they capture anticipation effects, but they abstract from the timing of actual DOD disbursements,[11] which is especially relevant in a panel setting.

Instrument

Military spending is often used in the fiscal policy literature because aggregate buildups are typically associated with geopolitical developments rather than with local business-cycle conditions. At the local level, however, the allocation of DOD spending may still be endogenous to local conditions. In particular, DOD spending is politically sensitive, so local areas disproportionately affected by a given economic shock may also be more likely to receive military contracts. To address this concern, we follow Nakamura and Steinsson (2014) and use a Bartik-style instrument:

$$Z_{i,t} = \frac{Average(G_{i,t})}{Average(G_t^N)} \times \frac{G_t^N - G_{t-1}^N}{GDP_{t-1}^N}. \tag{5}$$

[11] This means not only that some contracts may be signed in a period different from when disbursements occur, but also that a $100,000 two-year contract signed in 2010 with payments distributed as $50,000 in 2010 and $50,000 in 2011 would appear as a $100,000 obligation in 2010 and $0 in 2011.

Here, $Average(G_{i,t})$ denotes average military spending in MSA i over the sample, $Average(G_t^N)$ is average national defense spending over the sample,[12] G_t^N denotes national defense spending in year t, and GDP_t^N is national GDP in year t.

The instrument combines two components: an MSA-specific exposure term, given by the average share of national defense spending received by the MSA, and a time-varying aggregate term, given by the national change in defense spending as a share of lagged national GDP. This construction isolates variation coming from aggregate military buildups while assigning larger predicted changes to MSAs that are more exposed to national defense spending on average. The identifying assumption is that aggregate changes in U.S. military spending are not driven by economic conditions in the particular MSAs that happen to receive a disproportionate share of DOD contracts.

The instrument is also useful for a second reason. As Auerbach et al. (2020) and Auerbach et al. (2023) note, observed DOD contracts may partly reflect spending on production that would have occurred anyway, either because some contracts were anticipated or because firms smooth production across lumpy contracts. The Bartik-style instrument helps isolate the component associated with aggregate shifts in new military spending, since changes in national defense spending are more closely tied to changes in current production as measured in the NIPA. As discussed below, we also report results based on obligations.

Baseline Results

Table 5 reports the main results. Our preferred specification is column (1), which restricts the sample to contracts with durations shorter than one year. The estimated coefficients are positive and statistically significant across all three specifications. In column (1), an increase in DOD spending equal to 1% of GDP is associated with a 0.1604 percentage point increase in the growth of median capital expenditures.

The coefficients in columns (2) and (3) are somewhat smaller but remain positive and statistically significant. The obligations specification is also consistent with some anticipation effects. We focus on actual spending because it is more directly linked to realized local demand.

Unlike in the ARRA analysis, we do not pursue heterogeneity exercises here. The military-spending regressions are estimated at the MSA×year level, where cell sizes are much smaller than in the state-level ARRA setting. Splitting the sample into subsamples would therefore sharply reduce the number of MSAs with sufficient observations for reliable

[12] We obtain virtually identical results when using $\frac{G_{i,0}}{G_0^N}$ at the start of the sample instead of time averages.

Table 5: Baseline Results

	Investment Growth		
	(1)	(2)	(3)
Δ DOD Spending (one year)	0.1604**		
	(0.06471)		
Δ DOD Spending (two years)		0.1331**	
		(0.06155)	
Δ Dollars Obligated (one year)			0.1124**
			(0.04836)
MSA fixed effects	Yes	Yes	Yes
Year fixed effects	Yes	Yes	Yes
First State F-statistic	17.41	31.92	10.54
FS (F-test) p-value	0.0001	0	0.0016
N (MSA x Year)	1000	1000	1000

NOTE.—Standard errors are clustered at the MSA level. Following CES guidelines, all numbers are rounded to the nearest four significant digits except N, which is rounded to the nearest 100 if larger than 1000, and to the nearest 50 if smaller than 1000. Significance levels: * 10%, ** 5%, *** 1%.

estimation, even under more permissive disclosure thresholds. For that reason, we rely on the ARRA analysis for evidence on heterogeneity.

It is also informative that the estimated elasticities in this section are similar to those in the pooled ARRA specifications, despite relying on a different shock and a different empirical design. This similarity is consistent with the view that the two exercises are capturing a common demand-based mechanism rather than an idiosyncratic feature of either setting.

Dynamic Effects

We next examine the dynamic pattern of the estimates across annual horizons. As in the ARRA exercise, we keep the right-hand side of equation 4 fixed and shift the investment-growth window forward. Because this analysis exploits a panel of MSAs over time rather than a cross-section of states, the estimates are more precisely estimated than in the ARRA case.

Table 6 reports the results. Relative to the baseline specification, the estimated coefficient remains of similar magnitude and statistically significant when the dependent variable is shifted forward by one year. It declines in the following window but remains statistically significant, and becomes statistically indistinguishable from zero thereafter.

Table 6: Some Dynamics

	t	$t+1$	$t+2$	$t+3$
	(1)	(2)	(3)	(4)
Δ DOD Spending (one year)	0.1604**	0.1532**	0.09447*	-0.01175
	(0.06471)	(0.06469)	(0.05435)	(0.02695)
MSA fixed effects	Yes	Yes	Yes	Yes
Year fixed effects	Yes	Yes	Yes	Yes
First State F-statistic	17.41	71.64	99.24	110.1
FS (F-test) p-value	0.0001	0	0	0
N (MSA x Year)	1000	950	850	800

NOTE.—The dependent variable is investment growth in column (1), and the corresponding outcome shifted forward by one, two, and three years in columns (2), (3), and (4), respectively. Standard errors are clustered at the MSA level. Following CES guidelines, all numbers are rounded to the nearest four significant digits except N, which is rounded to the nearest 100 if larger than 1000, and to the nearest 50 if smaller than 1000. Significance levels: * 10%, ** 5%, *** 1%.

4 Theoretical Framework

To interpret the investment responses documented above, we consider two monetary-union frameworks with a continuum of regions, building on Galí and Monacelli (2005); Galí and Monacelli (2008). Both incorporate capital accumulation with convex adjustment costs and allow consumption and investment to differ in their composition. The central question is whether these relatively standard frameworks can reproduce the large, positive investment semi-elasticities documented above when government-spending shocks are transitory and calibrations are economically reasonable.

We use a tradable–nontradable model with a separate capital stock for each sector (TNT) to study the ARRA evidence. Its sectoral structure captures differences in exposure to local demand and allows us to focus on investment in nontradable production. For the military spending exercise, which pools investment across industries, we use a one-sector Galí–Monacelli model augmented with investment (GM). This framework provides an aggregate investment benchmark for that application.

Within each framework, we consider two financial and fiscal specifications. The first combines complete markets with local fiscal financing: households trade state-contingent claims, and regional governments finance their purchases through local lump-sum taxes. The second combines incomplete markets with federal fiscal financing: interregional asset trade is restricted to a one-period noncontingent nominal bond, and a balanced federal system

finances regional purchases through uniform lump-sum taxes. Within each application, the comparison holds the production and trade structure fixed and changes financial markets and fiscal financing jointly. The following subsections introduce each framework and its two specifications; the corresponding appendices provide the full derivations and equilibrium conditions.

4.1 A Tradable-Nontradable Model with Investment

For the ARRA exercise, we consider two financial and fiscal specifications of a common tradable–nontradable economy: complete markets with local fiscal financing, and incomplete markets with federal fiscal financing. Both share the preferences, production technologies, goods baskets, and monetary policy described below.

The monetary union consists of a continuum of regions indexed by $i \in [0, 1]$. Each region is infinitesimal and takes union-wide conditions as given. Time is discrete. Every region contains a representative household, competitive final-good assemblers, and two continua of monopolistically competitive firms, indexed by $j \in [0, 1]$. Firms belong to either the tradable sector, T, or the nontradable sector, N. Tradable goods are sold throughout the union, while nontradable goods are sold only in their region of production. Labor is mobile between sectors within a region and earns a common nominal wage $W_{i,t}$. Installed capital is specific to each sector.

Consumption and investment goods. Consumption and investment are composites of tradable and nontradable goods. For each use $X \in \{C, I\}$, let ν_X denote the expenditure share on tradables. Within the tradable component, a share $1 - \alpha_X$ is allocated to locally produced goods and a share α_X to a basket of tradable goods from across the union. These aggregates are Cobb–Douglas, with unit elasticity of substitution across regional origins within the union basket. Varieties within each sector and region are combined with elasticity $\epsilon > 1$.

Let $P_{T,i,t}$ and $P_{N,i,t}$ denote the prices of regional tradable and nontradable output, and let $P^*_{T,t}$ be the geometric union average of tradable prices. The resulting consumption and investment price indices are

$$P_{X,i,t} = \left[P_{T,i,t}^{1-\alpha_X} (P^*_{T,t})^{\alpha_X}\right]^{\nu_X} P_{N,i,t}^{1-\nu_X}, \qquad X \in \{C, I\}. \tag{6}$$

The parameters (ν_C, α_C) and (ν_I, α_I) allow consumption and investment to differ both in

their tradable content and in their geographical sourcing.

The investment basket is used to install capital in either sector. We denote investment destined for sector $s \in \{T, N\}$ by $\mathcal{I}_{s,i,t}$, with total investment $\mathcal{I}_{i,t} = \mathcal{I}_{T,i,t} + \mathcal{I}_{N,i,t}$. Both destinations use the same basket and therefore face the same purchase price $P_{I,i,t}$. The composition of investment goods is governed by (ν_I, α_I) regardless of the sector in which the resulting capital will be employed.

Households and investment. Households have identical preferences, with discount factor $\beta \in (0,1)$, logarithmic utility from private and public consumption, and isoelastic disutility of labor with inverse Frisch elasticity φ. Public consumption is taken as given. The household owns local firms and both sectoral capital stocks and receives their profits and rental income. Wages are flexible, and financial trading opportunities are specified below.

At date t, the predetermined stock $K_{s,i,t}$ is used in production. Current investment determines the stock available at $t+1$ according to

$$K_{s,i,t+1} = \left[1 - \delta + \Phi\left(\frac{\mathcal{I}_{s,i,t}}{K_{s,i,t}}\right)\right] K_{s,i,t}, \qquad s \in \{T, N\}, \tag{7}$$

where $\delta \in (0,1)$ is depreciation and $\Phi(x) = x - \frac{\tau}{2}(x-\delta)^2$, with $\tau > 0$. The installation technology is identical across sectors, but adjustment costs depend separately on investment relative to the capital stock of each sector.

Let $x_{s,i,t} \equiv \mathcal{I}_{s,i,t}/K_{s,i,t}$, and let $q_{s,i,t}$ denote the value, in date-t regional consumption units, of marginal capital installed for period $t+1$. Optimal investment satisfies

$$\frac{P_{I,i,t}}{P_{C,i,t}} = q_{s,i,t}\Phi'(x_{s,i,t}) = q_{s,i,t}\left[1 - \tau(x_{s,i,t} - \delta)\right]. \tag{8}$$

Define $r^K_{s,i,t}$ as the rental return on sectoral capital in consumption units. The household's real stochastic discount factor is $M^C_{i,t,t+1} = \beta C_{i,t}/C_{i,t+1}$. The value of installed capital satisfies

$$q_{s,i,t} = \mathbb{E}_t\Big\{ M^C_{i,t,t+1}\Big[r^K_{s,i,t+1} + q_{s,i,t+1}\big(1 - \delta + \Phi(x_{s,i,t+1}) - x_{s,i,t+1}\Phi'(x_{s,i,t+1})\big)\Big]\Big\}. \tag{9}$$

The installation condition equates the purchase price of investment goods to the value of the additional capital they create. The value equation incorporates future rental income and the continuation value of capital, including its effect on subsequent installation costs. Within

a region, both destinations face the same investment price and discount factor, while their capital values can differ with expected sectoral rental returns. These optimality conditions hold in both financial specifications; equilibrium consumption, rental returns, and capital values depend on the financial and fiscal arrangement.

Firms and price setting. Firms combine capital and labor using the same technology in both sectors:

$$Y_{s,i,t}(j) = K_{s,i,t}(j)^{\xi} L_{s,i,t}(j)^{1-\xi}, \qquad \xi \in (0,1), \tag{10}$$

where productivity is normalized to one. Firms rent capital in sector-specific markets and hire labor at the common regional wage. Cost minimization determines factor demands and sectoral marginal costs.

Prices are sticky à la Calvo. In each sector, a firm can reset its price with probability $1-\theta$ and otherwise keeps its nominal price unchanged. Firms value future profits using the nominal discount factor of their local owner. Tradable firms charge the same price at every destination. The common parameters ϵ, ξ, and θ imply that the two sectors differ in market reach rather than in their production technology, desired markup, or frequency of price adjustment.

Public purchases and monetary policy. Government purchases use the same basket as private consumption. Let $\mathcal{G}_{i,t}$ denote exogenous public expenditure in units of the geometric union consumption price index $P^*_{C,t}$. In both specifications, $P_{C,i,t}G_{i,t} = P^*_{C,t}\mathcal{G}_{i,t}$. The physical quantity of public consumption therefore adjusts to its regional price for a given expenditure budget in common units. The allocation of the tax burden differs across the two specifications.

The central bank sets a common gross nominal interest rate according to $R^*_t = \beta^{-1}(\Pi^*_{C,t})^{\phi_\pi}$, where $\Pi^*_{C,t} = P^*_{C,t}/P^*_{C,t-1}$ is union consumption-price inflation and $\phi_\pi > 1$.

Nontradable output supplies private consumption, public purchases, and investment within the producing region. Tradable output supplies local demand and demand from the union basket. Goods, labor, and capital markets clear in both specifications.

Complete markets and local fiscal financing. Households trade a complete set of state-contingent nominal claims. With equal Pareto weights and symmetric initial wealth, complete markets imply

$$P_{C,i,t}C_{i,t} = P^*_{C,t}C^*_t, \tag{11}$$

where C_t^* is geometric union consumption. Regional consumption therefore depends on the price of the local consumption basket relative to that of the union. Each regional government finances its purchases with contemporaneous lump-sum taxes on the local household:

$$\mathcal{T}_{i,t} = P_{C,i,t} G_{i,t} = P_{C,t}^* \mathcal{G}_{i,t}. \tag{12}$$

State-contingent claims are in zero net supply. Regional trade balances may differ from zero, with contingent transfers supporting the risk-sharing allocation. Appendix B presents this specification in full.

Incomplete markets and federal fiscal financing. The alternative specification combines federal financing with a one-period noncontingent nominal bond. Households retain ownership of local firms and capital; the bond is the only financial asset traded across regions. Its payoff $B_{i,t}$ is inherited, and $B_{i,t+1}$ is purchased at date t for $B_{i,t+1}/R_t^*$. The nominal payoff is chosen before next-period news arrives. The bond carries no holding cost or premium and is in zero net supply.

The federal system balances its budget each period and levies the same lump-sum tax on every household. Writing $\mathcal{G}_t^U = \int_0^1 \mathcal{G}_{i,t}\,\mathrm{d}i$, total taxes and the net federal receipt of region i are

$$\mathcal{T}_{i,t} = P_{C,t}^* \mathcal{G}_t^U, \qquad F_{i,t} = P_{C,t}^* (\mathcal{G}_{i,t} - \mathcal{G}_t^U). \tag{13}$$

Thus a region with above-average expenditure receives a positive net fiscal inflow. These receipts sum to zero across regions. Combining the household and public budgets gives

$$\frac{B_{i,t+1}}{R_t^*} - B_{i,t} = NX_{i,t} + F_{i,t}, \tag{14}$$

where $NX_{i,t}$ is the nominal trade balance, defined as regional production revenue less private and public absorption. Federal financing changes the regional tax burden, while the bond permits intertemporal saving and borrowing.

The bond Euler equation replaces complete risk sharing. Define relative nominal consumption expenditure by

$$\zeta_{i,t} = \log\left(\frac{P_{C,i,t} C_{i,t}}{P_{C,t}^* C_t^*}\right). \tag{15}$$

Complete risk sharing would set $\zeta_{i,t} = 0$ in every region. With the bond, the first-order Euler condition instead implies $\mathbb{E}_t \zeta_{i,t+1} = \zeta_{i,t}$. Its level is selected jointly with investment by the household's present-value budget. Fiscal news can change this expenditure allocation

without changing the inherited bond payoff, and a transitory fiscal expansion can leave persistent changes in relative financial positions and capital. The bond equates expected nominal discount factors across regions, while state-contingent valuations remain those of local owners. Appendix C derives the equilibrium and its solution, including the financial transversality condition and the permanent component.

Solution and comparison. We solve both specifications to first order around the same symmetric zero-inflation steady state. Their linear equilibria separate into a union block and regional deviations from union averages. The union block is common at first order, while the regional allocation depends on the financial and fiscal specification. This comparison assesses complete markets with local financing against incomplete markets with federal financing; it does not separately identify the contribution of each change. Section 5.1 sets out the calibration, and Section 5.2 presents the ARRA experiment and compares the investment responses of the two specifications with the evidence on nontradable industries.

4.2 A One-Sector Galí–Monacelli Benchmark

For the military spending exercise, we consider a one-sector Galí–Monacelli model with investment. Relative to Galí and Monacelli (2008), we introduce capital accumulation with convex adjustment costs and allow consumption and investment to differ in their degree of openness. The union consists of a continuum of small open economies indexed by $i \in [0, 1]$, each with a single production sector. Each region is of measure zero, so developments in an individual region do not affect union-wide aggregates. As in TNT, we consider complete markets with local fiscal financing and incomplete markets with federal fiscal financing, within the same real economic environment.

The common economic environment. Households have the same preferences as in TNT. They consume, invest, and supply labor. Consumption and investment combine locally produced and imported goods according to

$$C_{i,t} = \frac{C_{H,i,t}^{1-\alpha_C} C_{F,i,t}^{\alpha_C}}{(1-\alpha_C)^{1-\alpha_C} \alpha_C^{\alpha_C}}, \qquad \mathcal{I}_{i,t} = \frac{\mathcal{I}_{H,i,t}^{1-\alpha_I} \mathcal{I}_{F,i,t}^{\alpha_I}}{(1-\alpha_I)^{1-\alpha_I} \alpha_I^{\alpha_I}}, \tag{16}$$

where H and F denote domestic and imported goods, respectively, and α_C and α_I are their import expenditure shares. As in GM, substitution across regions has unit elasticity, while substitution across varieties within each region has constant elasticity $\epsilon > 1$. Let $P_{i,t}$ denote

the domestic producer price index and P_t^* its geometric union average. The associated price indices are

$$P_{C,i,t} = P_{i,t}^{1-\alpha_C}(P_t^*)^{\alpha_C}, \qquad P_{I,i,t} = P_{i,t}^{1-\alpha_I}(P_t^*)^{\alpha_I}. \tag{17}$$

Their geometric union averages both equal P_t^*. Allowing α_I to differ from α_C permits the relative price of investment goods to respond to regional relative prices.

Households own domestic firms and capital. They rent capital to local firms and receive rental income and profits. Capital productive at t is predetermined, while current investment determines the stock available at $t+1$. Accumulation is subject to convex adjustment costs (Hayashi, 1982):

$$K_{i,t+1} = \mathcal{I}_{i,t} - \frac{\tau}{2}\left(\frac{\mathcal{I}_{i,t}}{K_{i,t}} - \delta\right)^2 K_{i,t} + (1-\delta)K_{i,t}, \tag{18}$$

where $\tau \geq 0$ governs adjustment costs and $\delta \in (0,1)$ is the depreciation rate. This is the same investment technology used in TNT. The installation and capital-valuation conditions in equations (8)–(9) apply with a single capital stock and the local household's consumption discount factor. Thus investment depends on expected rental returns, the continuation value of capital, and its purchase price relative to consumption.

Firms combine capital and labor using a Cobb–Douglas technology. With productivity normalized to one, firm j produces

$$Y_{i,t}(j) = K_{i,t}(j)^{\xi} L_{i,t}(j)^{1-\xi}, \tag{19}$$

where $\xi \in (0,1)$ is the capital share. Capital and labor are allocated across firms through competitive regional factor markets. Prices are sticky à la Calvo and wages are flexible. Firms value future profits using the nominal discount factor of their local owner. The common monetary authority responds to union inflation according to $R_t^* = \beta^{-1}(P_t^*/P_{t-1}^*)^{\phi_\pi}$, with $\phi_\pi > 1$.

Government purchases fall entirely on domestically produced goods. Let $\mathcal{G}_{i,t}$ denote public expenditure in units of P_t^*. In both specifications, $P_{i,t}G_{i,t} = P_t^*\mathcal{G}_{i,t}$, where $G_{i,t}$ is the quantity purchased. For a given expenditure budget, an increase in domestic prices relative to union prices reduces that quantity. Public purchases have no direct import component; private consumption and investment determine the response of imports.

Complete markets and local fiscal financing. Households trade a complete set of state-contingent nominal claims. With symmetric initial wealth and equal Pareto weights, risk sharing implies $P_{C,i,t}C_{i,t} = P_t^*C_t^*$, where C_t^* is geometric union consumption. Each

regional government finances its purchases with contemporaneous lump-sum taxes on the local household:

$$P_{i,t}G_{i,t} = P_t^*\mathcal{G}_{i,t} = \mathcal{T}_{i,t}, \tag{20}$$

where $\mathcal{T}_{i,t}$ is nominal tax revenue. Contingent claims clear across regions and support the risk-sharing allocation. Appendix D presents the household optimality conditions and the remaining equilibrium relations.

Incomplete markets and federal fiscal financing. Households retain ownership of local firms and capital and trade only a one-period noncontingent nominal bond across regions. As in the incomplete-markets TNT model, $B_{i,t}$ is the inherited payoff and $B_{i,t+1}$ is purchased for $B_{i,t+1}/R_t^*$ before next-period news arrives. The bond carries no holding cost or premium and is in zero net supply.

A balanced federal system finances public purchases through uniform lump-sum taxes. With $\mathcal{G}_t^U = \int_0^1 \mathcal{G}_{i,t}\,\mathrm{d}i$, total taxes and the net federal inflow are

$$\mathcal{T}_{i,t} = P_t^*\mathcal{G}_t^U, \qquad \mathcal{F}_{i,t} = P_t^*(\mathcal{G}_{i,t} - \mathcal{G}_t^U). \tag{21}$$

Net bond purchases equal regional net exports plus $\mathcal{F}_{i,t}$. The regional tax burden therefore depends on aggregate expenditure, while private saving and borrowing respond to regional income and absorption.

Relative nominal consumption expenditure, $\zeta_{i,t} = \log[P_{C,i,t}C_{i,t}/(P_t^*C_t^*)]$, is now endogenous. The first-order bond Euler condition gives $\mathbb{E}_t\zeta_{i,t+1} = \zeta_{i,t}$, and the household's present-value budget selects its level jointly with investment. Fiscal news can therefore alter regional consumption expenditure and leave persistent financial and capital adjustments. Nominal discount factors have a common conditional expectation but can differ across regions state by state. Appendix E derives the goods market without imposing complete risk sharing and presents the full financial closure and solution.

Both specifications are solved to first order around the same symmetric zero-inflation steady state. Their union blocks coincide at this order, while regional expenditure and investment respond to the joint change in financing and financial markets. Section 5.1 sets out the calibration, and Section 5.3 presents the military spending experiment and compares the annual investment responses of the two specifications with the empirical evidence.

5 Quantitative Analysis

This section compares the investment responses generated by the two models with the ARRA and military-spending evidence. We use TNT to study investment in nontradable production in the ARRA exercise and GM to study aggregate investment in the military-spending exercise. Within each application, we compare complete markets with local fiscal financing and incomplete markets with federal fiscal financing, using the same benchmark structural parameters and fiscal innovations across the two specifications.

We express the responses as semi-elasticities with respect to fiscal expenditure measured relative to annual GDP.[13] We first present the calibration and then describe the fiscal experiments, benchmark responses, investment mechanisms, and sensitivity to structural parameters and fiscal persistence.

5.1 Calibration

Both models are calibrated at a quarterly frequency, with TNT used for ARRA and GM for military spending. Within each model, the two fiscal and financial specifications share the same benchmark calibration. Table 7 reports the parameters common to the two frameworks and the composition parameters used in each application. Following Galí and Monacelli (2008), we set $\beta = 0.99$, $\varphi = 3$, $\epsilon = 6$, and $\theta = 0.75$. These values imply a steady-state annual real interest rate of approximately 4%, a Frisch elasticity of labor supply of 1/3, a price markup of 20%, and an average duration of prices of four quarters.

We set the capital exponent to $\xi = 0.35$, implying that capital accounts for 35% of total payments to capital and labor. Quarterly depreciation is $\delta = 0.025$, corresponding to an annual depreciation rate of approximately 10%. We set the adjustment-cost parameter to $\tau = 4$. At the steady state, the resulting elasticity of the investment–capital ratio with respect to Tobin's q divided by the relative price of investment is $1/(\tau\delta) = 10$. The steady-state share of government purchases in output is $\gamma = 0.20$, following Nakamura and Steinsson (2014). We normalize the utility weight on public consumption to $\chi_G = \gamma$ and set the monetary-policy coefficient to $\phi_\pi = 1.5$. These parameters are common to both models and, in TNT, to both production sectors.

[13] The empirical outcomes measure growth in regional weighted medians of capital expenditures, while model outcomes measure deviations of aggregate investment quantities from steady state. Differences in aggregation, timing, and regression specifications imply that the exercises provide a comparison in common units rather than an exact replication of the empirical estimators.

In the TNT model, the tradable shares of consumption and investment are $\nu_C = 0.30$ and $\nu_I = 0.42$. These values are rounded estimates of the U.S. tradable input shares reported by Lombardo and Ravenna (2012), which account for tradable inputs embodied in final expenditures. To calibrate sourcing within the tradable component, we use the approximately 83% interregional share in the manufacturing-shipment calculation of Nakamura and Steinsson (2014). We set $\alpha_C = 0.85$ and assume the same conditional sourcing pattern for investment, $\alpha_I = 0.85$.[14] The higher tradable share of investment consequently makes its overall composition more open than that of consumption.

For the GM benchmark, we set $\alpha_C = 0.40$, following Galí and Monacelli (2005), and $\alpha_I = 0.35$. The latter matches the rounded share of investment expenditure sourced from other regions in the TNT benchmark.

In both exercises, log deviations of government expenditure in common-price units follow a first-order autoregressive process with coefficient ρ_G. The empirical shocks motivating our exercises are concentrated over a short horizon: the ARRA payments measure covers approximately two quarters, while military contract durations are concentrated within one or two quarters. These short horizons motivate our benchmark choice of $\rho_G = 0.25$. Following an isolated innovation, this value implies that 93.75% of the cumulative response of the fiscal process occurs within the first two quarters. We examine more persistent processes in the quantitative exercises below. Sections 5.2 and 5.3 specify the fiscal innovations and the measurement of investment responses in each exercise.

5.2 ARRA Shocks

Fiscal experiment and measurement. As discussed in section 2, the ARRA was a one-time fiscal stimulus whose allocation across states was largely governed by heterogeneous statutory formulas. We use the observed distribution of ARRA payments to construct the fiscal experiment. Let S_i denote cumulative payments in state i, divided by its annual GDP in 2008. We assign the observed exposures for the 50 states to model regions, using the payments data described in Wilson (2012). The payment measure covers roughly two quarters. We compare the two TNT specifications, using the same structural calibration and fiscal innovations under complete markets with local fiscal financing and incomplete markets with federal fiscal financing.

[14] With $N = 50$ equally weighted regions, the union basket includes the home region. The share sourced from other regions within the tradable component is therefore $\alpha_C(N-1)/N = 0.833$. The corresponding share in the full investment basket is $\nu_I\alpha_I(N-1)/N = 0.34986$.

Table 7: Benchmark Calibration

Panel A. Common parameters

Parameter	Description	Value
β	Discount factor	0.99
φ	Inverse Frisch elasticity	3
ϵ	Elasticity across varieties	6
θ	Calvo probability	0.75
ξ	Capital exponent	0.35
δ	Depreciation rate	0.025
τ	Investment adjustment costs	4
γ	Government purchases / output	0.20
χ_G	Public consumption utility weight	0.20
ϕ_π	Monetary-policy coefficient	1.5
ρ_G	Fiscal persistence	0.25

Panel B. Consumption and investment composition

Parameter	Description	TNT ARRA	GM Military
ν_C	Tradable share of consumption	0.30	—
ν_I	Tradable share of investment	0.42	—
α_C	Consumption openness	0.85	0.40
α_I	Investment openness	0.85	0.35

NOTE.—Periods are quarters. Panel A reports parameters shared by both models. Within each model, the values in both panels apply to complete markets with local fiscal financing and incomplete markets with federal fiscal financing. In TNT, α_C and α_I are the weights on the union basket within the tradable components of consumption and investment. In GM, they are the imported shares of the corresponding complete baskets.

Starting from the symmetric steady state, we introduce an unanticipated fiscal innovation at date 0. In both specifications, the log deviation of government expenditure measured at common prices follows

$$\widehat{\mathcal{G}}_{i,t} = \rho_G \widehat{\mathcal{G}}_{i,t-1} + \varepsilon^G_{i,t}, \qquad \widehat{\mathcal{G}}_{i,-1} = 0. \tag{22}$$

We choose the initial innovation in each region so that expenditure over the first two quarters matches its observed exposure:

$$S_i = \frac{\gamma}{4}\left(\widehat{\mathcal{G}}_{i,0} + \widehat{\mathcal{G}}_{i,1}\right), \qquad \varepsilon^G_{i,0} = \frac{4S_i}{\gamma(1+\rho_G)}. \tag{23}$$

The factor of four converts quarterly steady-state output into annual GDP. Subsequent innovations are zero along the impulse-response path, while the initial shock decays at rate $\rho_G = 0.25$. We retain the positive mean of observed exposures, so the experiment includes both a common fiscal expansion and differences in regional exposure.

To measure investment responses, we estimate the cross-sectional regression

$$v_{i,h} = a_h + b_h S_i + u_{i,h}, \tag{24}$$

with an intercept and equal weights across states. In both specifications, the dependent variable is the log deviation of investment in the nontradable sector, measured either on impact or as an average over the first model year:

$$v_{i,\text{impact}} = \widehat{\imath}_{N,i,0}, \qquad v_{i,\text{year}} = \frac{1}{4}\sum_{t=0}^{3} \widehat{\imath}_{N,i,t}. \tag{25}$$

The coefficient b_h is a semi-elasticity: an additional percentage point of payments relative to annual GDP is associated with a b_h percent change in investment. The intercept absorbs the common response, leaving the slope to measure differences across regions. We emphasize the first-year average and report the impact response alongside it to describe the initial adjustment.

Benchmark responses. Table 8 compares the model responses with the preferred nontradables estimate from section 2. The empirical semi-elasticity is 0.8055, whereas the first-year semi-elasticities are -1.553 under complete markets with local financing and -1.278 under incomplete markets with federal financing. The impact responses are -2.874 and -2.373, respectively. In both benchmark specifications, regions with greater fiscal exposure reduce nontradable investment relative to less exposed regions.

Table 8: ARRA Shocks: Empirical and Model Investment Responses

	Impact	First-year average
Complete markets and local fiscal financing	−2.874	−1.553
Incomplete markets and federal fiscal financing	−2.373	−1.278
Empirical nontradables estimate	—	0.8055

NOTE.—Model entries are semi-elasticities of investment in nontradable production in TNT with respect to cumulative ARRA payments divided by annual GDP. Both specifications use equation (24); the first-year average covers quarters 0–3. The empirical estimate is from section 2. Both specifications use the TNT parameters in table 7 and the same fiscal innovations.

Incomplete markets with federal financing attenuate the contraction, but the difference between the two specifications is small relative to their distance from the empirical estimate. This comparison changes financial markets and fiscal financing jointly. The following exercises examine whether alternative structural parameters or greater fiscal persistence can bring either specification closer to the empirical magnitude.

Investment mechanism. The investment condition in equation (8) provides a common accounting of the response. Its linear approximation for the nontradable sector is

$$\widehat{\iota}_{N,i,t} = \widehat{k}_{N,i,t} + \frac{\widehat{q}_{N,i,t}}{\tau\delta} - \frac{\widehat{p}_{I/C,i,t}}{\tau\delta}, \tag{26}$$

where $\widehat{p}_{I/C,i,t}$ is the log deviation of the investment price relative to the consumption price. Applying the same regression and time aggregation to each term yields the contributions shown in appendix figure F.1. Over the first year, the value of installed capital contributes −1.802 under complete markets with local financing and −1.637 under incomplete markets with federal financing. The current relative investment price partly offsets these contributions by 0.334 and 0.429, respectively. Productive capital contributes a further −0.084 and −0.070; this component is zero on impact because capital is predetermined.

The fall in q_N reflects the distinction between demand for current production and the value of additional productive capacity. With sticky prices, the fiscal expansion initially raises production and factor demand while the capital stock is fixed. Rental income on existing capital rises on impact. Investment undertaken at that date, however, only becomes productive in the following period. Since the fiscal shock decays rapidly, higher current rental income provides a limited incentive to install new capital. In both specifications, the present-value contribution of subsequent rental returns is small.

Intertemporal valuation also depends on regional consumption dynamics. Under complete markets, the risk-sharing condition in equation (11) implies that the initial increase in regional consumption prices lowers consumption relative to the union. As relative prices revert and consumption recovers, the regional real discount factor, $M^C_{i,t,t+1} = \beta C_{i,t}/C_{i,t+1}$, is lower relative to the rest of the union. This reduces the current value of future capital payoffs.

Under incomplete markets, regional consumption expenditure is instead selected by the bond Euler equation and the household's intertemporal budget constraint. Federal financing and the associated wealth adjustment change consumption expenditure and the equilibrium paths of prices and rental returns. The present-value accounting in appendix F.1.2 nevertheless shows that the contribution of the real discount factor remains the largest negative component of q_N in both specifications. A common nominal interest rate does not eliminate regional differences in real intertemporal valuation.

The current relative investment price supports investment in both specifications. Investment has a larger tradable component than consumption, so its price rises less than the regional consumption price. Investment goods consequently become cheaper in consumption units. With incomplete markets and federal financing, this contribution is larger and the decline in q_N is smaller. Together, these changes attenuate the investment contraction without closing the gap with the empirical response.

Parameter sensitivity. Table 9 compares first-year semi-elasticities under alternative parameter values, holding fiscal persistence and price rigidity at their benchmark values. Each row applies the same parameter changes to both specifications. Panel A varies labor supply, depreciation, and investment adjustment costs individually. More elastic labor supply and faster depreciation attenuate the contraction in both specifications. Higher adjustment costs also reduce the contraction by slowing the adjustment of capital.

Panel B changes the composition of investment goods. A larger tradable share, together with greater sourcing through the union basket, reduces the dependence of investment on locally produced goods. With nearly all investment goods tradable, the first-year semi-elasticities are -0.593 under complete markets with local financing and -0.275 under incomplete markets with federal financing. Increasing the weight on the union basket within tradables to 0.95 as well raises these responses to -0.389 and -0.076, respectively.

Panel C combines changes in labor supply, capital adjustment, and investment composition. These cases assess how far the responses move under joint parameter changes, including investment baskets with very large tradable shares. All first-year responses under complete

Table 9: ARRA Shocks: Parameter Sensitivity in TNT

Specification	Parameter values	Complete markets Local financing	Incomplete markets Federal financing
Panel A. Benchmark and individual parameters			
Benchmark		−1.553	−1.278
More elastic labor supply	$\varphi = 1$	−0.973	−0.649
Higher depreciation	$\delta = 0.04$	−1.109	−0.905
Higher adjustment costs	$\tau = 8$	−1.013	−0.821
Panel B. Investment composition			
Nearly all investment goods tradable	$\nu_I = 0.9999$	−0.593	−0.275
More tradable investment and union sourcing	$\nu_I = 0.75$, $\alpha_I = 0.95$	−1.019	−0.704
Nearly all investment goods tradable and union sourcing	$\nu_I = 0.9999$, $\alpha_I = 0.95$	−0.389	−0.076
Panel C. Joint parameter changes			
Labor, capital, and investment composition	$\varphi = 1$, $\delta = 0.04$, $\tau = 8$, $\nu_I = 0.75$, $\alpha_I = 0.95$	−0.212	−0.044
Labor, capital, and investment composition	$\varphi = 1$, $\delta = 0.04$, $\tau = 8$, $\nu_I = 0.9999$, $\alpha_I = 0.95$	−0.051	0.125
Labor supply and investment composition	$\varphi = 0.5$, $\nu_I = 0.9$	−0.236	0.101

NOTE.—Entries are first-year semi-elasticities of nontradable investment, averaging quarters 0–3. Columns compare complete markets with local fiscal financing and incomplete markets with federal fiscal financing. All rows retain $\rho_G = 0.25$ and $\theta = 0.75$; unlisted parameters take their benchmark values. The parameter ν_I is the tradable share of investment goods; α_I is the weight on the union basket within that component. Reported responses refer to investment destined for nontradable production. A value of $\nu_I = 0.9999$ approximates fully tradable investment goods. Panel C assesses joint sensitivity and does not represent a preferred recalibration. The empirical semi-elasticity is 0.8055.

markets with local financing remain negative, with a maximum of -0.051. Two cases under incomplete markets with federal financing generate positive first-year responses, 0.125 and 0.101. Thus positive investment responses are possible within the reported calibrations, but even the largest remains well below the empirical semi-elasticity of 0.8055.

Fiscal persistence. We next vary ρ_G while holding the structural parameters at their benchmark values. For each value of persistence, we rescale the initial innovations according to equation (23), preserving each state's observed two-quarter expenditure exposure. Greater persistence places more spending beyond that initial window and extends the expected increase in demand. This normalization holds the initial payment exposure fixed, while allowing total spending over the full response to increase.

Figure 1 compares the first-year investment semi-elasticities. In both specifications, the response initially becomes somewhat more negative and then rises as persistence increases further. Under complete markets with local financing, the first-year response crosses zero near $\rho_G = 0.901$ and reaches the empirical semi-elasticity near 0.930. Under incomplete markets with federal financing, the corresponding crossings occur near 0.845 and 0.886.[15]

More persistent spending raises the value of productive capacity because additional capital can serve demand over a longer horizon. Incomplete markets with federal financing lower the persistence needed to reproduce the empirical magnitude. Even so, under the benchmark structural calibration, both specifications require a substantially more persistent fiscal expansion than the short-lived stimulus motivating the exercise.

5.3 Military Spending Shocks

Fiscal experiment and measurement. As discussed in section 3, the military-spending exercise uses annual variation in DOD spending across MSAs, focusing on contracts with durations shorter than one year. We study this variation using the two GM specifications, whose aggregate investment responses provide a comparison for the empirical estimate pooling all industries in the sample. We compare complete markets with local fiscal financing and incomplete markets with federal fiscal financing, using the same structural parameters and fiscal innovations. Parameters follow the military calibration in table 7, with $\alpha_C = 0.40$ and $\alpha_I = 0.35$.

[15]These values are approximate crossings obtained by linear interpolation between adjacent values in the persistence grid. Under complete markets with local financing, the first-year response crosses zero between 0.90 and 0.925 and reaches 0.8055 between 0.925 and 0.95. Under incomplete markets with federal financing, the corresponding brackets are $[0.825, 0.85]$ and $[0.875, 0.90]$.

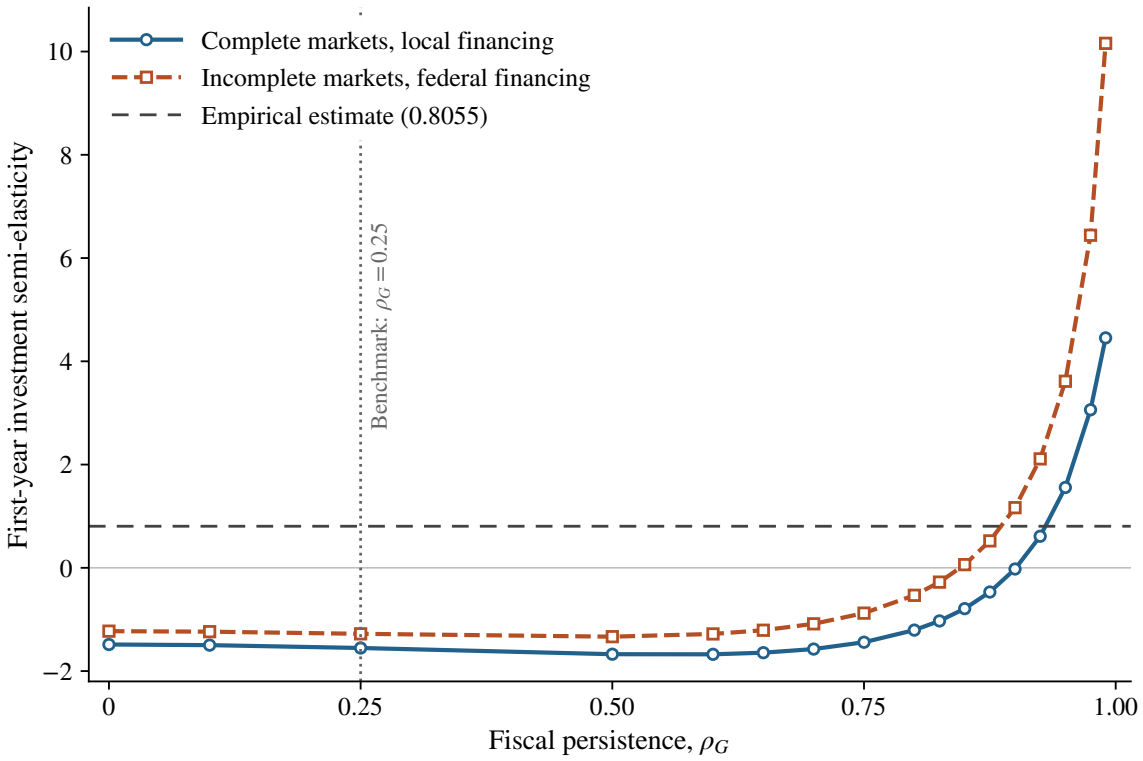


Figure 1: ARRA Shocks: Fiscal Persistence and Investment in TNT

NOTE.—Both curves report first-year semi-elasticities of nontradable investment, averaging quarters 0–3. Structural parameters remain at their benchmark values, and initial innovations are rescaled to preserve expenditure exposure over the first two quarters. The horizontal dashed line is the empirical semi-elasticity, 0.8055. The vertical dotted line marks $\rho_G = 0.25$. Markers show the evaluated persistence values; lines connect adjacent points.

We introduce recurrent fiscal innovations. Government expenditure measured at common prices follows

$$\widehat{\mathcal{G}}_{i,t} = \rho_G \widehat{\mathcal{G}}_{i,t-1} + \varepsilon^G_{i,t}, \qquad \varepsilon^G_{i,t} \overset{\text{iid}}{\sim} \mathcal{N}(0, \sigma^2_G), \tag{27}$$

where innovations are independent across regions and quarters, and $\rho_G = 0.25$. The innovation standard deviation, $\sigma_G = 0.08670$, is calibrated to the dispersion of annual military expenditure relative to a fixed regional GDP reference, after removing region and year effects. We use this same fiscal scale in both specifications. Appendix F.2.1 describes the fiscal inputs and calibration details.

For each specification, we simulate 1,000 panels of $N = 100$ regions observed for 1,000 quarters, giving 250 years per region. Every panel starts at the symmetric steady state, with zero inherited bond positions, and retains all periods from the initial date. Each replication uses the same fiscal innovations in the two specifications. This common initialization defines a finite-panel comparison: under incomplete markets, regional wealth has a permanent component, so its distribution depends on the initial condition and the elapsed horizon.

Before estimating the investment response, we aggregate the quarterly series to annual frequency. Let $\mathcal{T}_a$ denote the four quarters in year a, and define

$$x_{i,a} = \frac{\gamma}{4} \sum_{t \in \mathcal{T}_a} \widehat{\mathcal{G}}_{i,t}, \qquad v_{i,a} = \frac{1}{4} \sum_{t \in \mathcal{T}_a} \widehat{\iota}_{i,t}. \tag{28}$$

Here, $x_{i,a}$ measures the deviation of annual government expenditure as a share of annual steady-state output. To first order, $v_{i,a}$ is the proportional deviation of annual physical investment from its steady-state level. For each panel, we estimate

$$v_{i,a} = \mu_i + \mu_a + b\, x_{i,a} + u_{i,a}, \tag{29}$$

with region and year fixed effects and equal observation weights. The coefficient b measures the percent deviation of investment associated with an additional percentage point of government expenditure relative to annual output, conditional on these fixed effects. We report the mean of the 1,000 estimated coefficients for each specification.

Benchmark responses. Table 10 compares the model responses with the preferred empirical estimate from section 3. The empirical semi-elasticity is 0.1604, while the mean annual semi-elasticities are -1.757 under complete markets with local financing and -1.654 under incomplete markets with federal financing. The joint change in financial markets and fiscal financing attenuates the contraction by 0.103, but both responses remain far below the

positive empirical reference.

Table 10: Military Spending Shocks: Empirical and Model Investment Responses

	Annual semi-elasticity
Complete markets and local fiscal financing	−1.757
Incomplete markets and federal fiscal financing	−1.654
Empirical estimate: all industries	0.1604

NOTE.—GM entries are mean coefficients from 1,000 paired panels, each containing 100 regions and 250 years, initialized at the symmetric steady state and using the same fiscal innovations in both specifications. Each coefficient uses the annual measures in equation (28) and the fixed-effects regression in equation (29). Parameters are given in table 7. The empirical estimate is from column (1) of table 5. It measures growth in regional median capital expenditures; model responses measure deviations of aggregate physical investment from steady state.

Appendix F.2 reports the investment decomposition and the sensitivity of both specifications to structural parameters and fiscal persistence. The structural variations considered at $\rho_G = 0.25$ can bring the annual responses close to zero, but all remain negative. Under the benchmark structural calibration, reaching the empirical reference requires a substantially more persistent fiscal process than the transitory shocks motivating the exercise.

6 Concluding Remarks

How do demand shocks affect investment? And how strong is the accelerator mechanism? These questions are central to understanding business cycle fluctuations and to distinguishing among competing macroeconomic frameworks. In standard investment models, firms compare the current cost of acquiring and installing capital with its expected future returns. In that environment, a transitory demand shock generates limited incentives to increase investment.

Previous empirical work has documented co-movement between investment and output at the aggregate level, and between investment and sales at the firm level. However, such co-movement is open to multiple interpretations and therefore provides only limited evidence on the strength of the accelerator mechanism. This paper contributes by combining a new measure of business investment across U.S. regions, constructed from confidential Census Bureau microdata, with two distinct sources of local demand variation: the ARRA stimulus program of 2009 and military spending shocks. The first is allocated across states largely through statutory formulas, while the second exhibits substantial variation across cities. We

use this variation to estimate local investment responses, differencing out aggregate shocks and broader macroeconomic forces. Our preferred estimate implies that investment rises by about 16 cents for every additional dollar of ARRA payments. The ARRA evidence also shows that investment responses are not concentrated among smaller firms, making an explanation based primarily on liquidity constraints less plausible.

To interpret these findings, we study two standard models of a monetary union with capital accumulation and investment adjustment costs. Our main model distinguishes tradable and nontradable production, allowing us to study investment in nontradable industries in the ARRA exercise. A one-sector Galí–Monacelli model provides the framework for studying aggregate investment responses to military spending. Within each framework, we consider complete markets with local fiscal financing and incomplete markets with federal fiscal financing. Under the benchmark calibrations, both specifications of each model generate negative annual investment responses. At the benchmark fiscal persistence, annual responses remain well below the positive empirical estimates across the structural parameter variations considered, including cases with small positive responses in TNT. With the benchmark structural parameters, reproducing the empirical magnitudes requires substantially more persistent fiscal processes than the transitory shocks motivating the exercises.

The evidence therefore points to a tension between the data and the predictions of these standard benchmarks. Transitory local demand shocks appear to generate investment responses that are substantially stronger than the models imply. More broadly, the results suggest that the accelerator mechanism may be quantitatively more important than standard frameworks allow.

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Appendices

A Data

No publicly available dataset provides comprehensive annual capital expenditure data below the national level. The closest public source is the Census of Manufacturers, but it is limited to the manufacturing sector and to Economic Census years. To overcome these limitations, we adapt the methodology in Carreño (2026) and use restricted-access Annual Capital Expenditures Survey (ACES) microdata from the Census Bureau to construct new measures of capital expenditures at the MSA and state levels.

Our main data source is the Census Bureau's Annual Capital Expenditures Survey, an annual survey of approximately 50,000 firms. The survey covers all domestic non-farm businesses and reports detailed information on capital expenditures by type, including new and used equipment and structures. A subset of firms also reports additional information on intangibles and other items, as well as industry-level breakdowns.

Because ACES is observed at the firm level, we next allocate capital expenditures to establishments so that they can be assigned to the corresponding MSA or state and then aggregated geographically. To do so, we merge ACES to the Longitudinal Business Database (LBD), which covers the universe of establishments and reports the parent firm, establishment location, industry classification, and variables such as employment and payroll. After matching firms to establishments and assigning each establishment to its corresponding MSA or state, we distinguish between two cases: (i) single-unit firms,[16] or firms whose establishments are all located within a single MSA or state; and (ii) firms with establishments spread across multiple MSAs or states. The first group can be assigned directly to a single geography. The second requires an allocation procedure, which we describe next.

To allocate capital expenditures from the firm level to the establishment level for multi-establishment firms operating across different MSAs or states, we proceed as follows. First, we restrict the sample to:

- Firms for which at least 95% of business activity is concentrated in a single two-digit NAICS industry.[17] This excludes firms with highly diversified business lines that are

[16] Single-unit firms are firms with only one establishment. In 2015, they accounted for 96.07% of all firms and 48.15% of employment. Multi-unit firms represented 3.93% of firms and 51.85% of employment.

[17] We use a 95% threshold rather than 100% because multi-establishment firms often have nearly all establishments in one NAICS industry except for one or two small outliers accounting for a negligible share of activity. Results are virtually unchanged when the threshold is set to 100% or 90%.

difficult to compare across establishments.

- Firms with fewer than 50 establishments.[18] This excludes very large firms with extensive establishment networks.

These restrictions yield a sample covering 99.49% of all firms in the LBD universe of non-farm businesses with employees and 65.54% of employment in 2015. Within this sample, we retain 86.94% of multi-unit firms, representing 33.54% of employment among such firms.

We then allocate each firm's total capital expenditures across its establishments in proportion to each establishment's share of the firm's total economic activity. Ideally, this allocation would be based on value added or revenue, but such measures are available only at the firm level in the revenue-enhanced LBD and not at the establishment level. We therefore use total payroll as our establishment-level proxy.[19] We aggregate the resulting establishment-level capital expenditures to the MSA or state level using the weighted median. We also impose a minimum cell size of 200 observations per MSA or state-year, which covers all states and most MSAs with populations above 150,000.[20] Nominal values are deflated using the national CPI,[21] and growth rates are constructed as log differences.

Two concerns are worth noting. First, some capital expenditures may be associated with the firm as a whole and may therefore benefit all establishments simultaneously. Even if such expenditures are not physically located at a single site, they contribute economically to all establishments and are therefore still relevant for a spatial analysis. Second, payroll may not perfectly capture the allocation of capital expenditures within the firm, especially when establishments differ in their capital-labor ratios. This concern is unavoidable in principle, but it is likely to be less severe when comparing establishments within the same firm and within the same NAICS classification.

Columns (1) and (2) of Table A.1 compare our sample with the full LBD universe of non-farm businesses with employees. The sample covers 99.49% of firms and 65.54% of employment. As expected, the average firm in the sample is smaller than the average firm in

[18] Tighter restrictions, such as limiting the sample to firms with fewer than 20 establishments, yield virtually identical results.

[19] Allocations based on employment yield a similar measure. Payroll has two advantages: economically, it better captures heterogeneity in labor costs across workers; and technically, it is sourced from IRS data and therefore has fewer missing values.

[20] In Table 2, columns (4)–(9), and Table 3, columns (4)–(9), the subgroup exercises substantially reduce cell sizes. To preserve coverage, we relax the threshold to 100 observations in those cases. Results remain broadly similar when retaining the 200-observation threshold, although the number of states in the sample falls sharply. Results are also very similar when varying the threshold to 150 or 50 observations.

[21] Deflating with regional CPIs, or even at the establishment level using industry-disaggregated PPIs, yields a very similar measure and virtually identical results.

the full universe, reflecting the exclusion of very large firms. At the same time, the employment percentiles are very similar, suggesting that the sample remains broadly representative aside from the extreme upper tail. The lower panel reports payroll per employee, a proxy for relative labor productivity, and these moments are also very similar in the sample and in the full universe.

Columns (3) and (4) compare the subset of multi-unit firms in our sample with the full universe of multi-unit firms in the LBD. This comparison is useful because all single-unit firms are included in our sample, whereas the sample restrictions operate primarily through the selection of multi-unit firms. The same pattern emerges. The average multi-unit firm in the sample is smaller than in the full universe, again because the restrictions exclude the largest firms. However, the employment percentiles remain close, and payroll per employee is also very similar across the two groups, supporting the representativeness of the multi-unit subsample.

Table A.2 summarizes the properties of our main dependent variable, investment growth.[22] The top panel reports separate regressions of investment growth on three MSA-level macroeconomic variables: real GDP growth, employment growth, and personal income growth. These regressions are intended only to characterize the joint behavior of investment growth with standard macroeconomic indicators, not to identify causal effects.

Two facts emerge. First, investment growth is positively correlated with all three macroeconomic variables and the estimated relationships are statistically significant. A one-percentage-point increase in real GDP growth is associated with a 2.8 percentage point increase in investment growth; the corresponding coefficients are 6.2 for employment growth and 3.6 for personal income growth. Second, investment growth is substantially more volatile than these macroeconomic indicators. The bottom panel shows that its distribution is centered close to zero, with a 25th percentile of about -50%, a 75th percentile of about $+50\%$, and a standard deviation of roughly 110%.

Other Data. We use state-level ARRA spending data constructed from Recovery.gov by Wilson (2012). We also use military government spending data from USAspending.gov, taken from Demyanyk et al. (2019) through 2013 and extended by the authors using the same methodology for subsequent years. Regional GDP data are from the BEA.

[22] Given the substantial volatility of investment, we follow an approach inspired by Tornqvist et al. (1985), which has become standard in parts of the investment literature following Doms and Dunne (1998) and in related areas such as firm dynamics after Davis et al. (1998). Specifically, we measure investment growth relative to the average level of investment in the previous two periods. This reduces noise and improves statistical power.

Table A.1: Sample Statistics

	Overall Statistics		Within multi-unit firms	
	Sample	Universe	Sample	Universe
Mean employment	18.87	28.65	145.8	378
s.d. employment	206.4	963.5	843.9	4801
p-25 employment	1	1	14	15
Median employment	4	4	36	44
p-75 employment	10	10	101	135
Mean payroll	867.4	1416	8316	21310
s.d. payroll	13690	46350	64650	230600
p-25 payroll	37	37	458	535
Median payroll	112	113	1452	1780
p-75 payroll	349	356	4453	6000
Mean pay/emp	40080	40150	52010	52240
s.d. pay/emp	76980	76930	72180	71340
p-25 pay/emp	16000	16000	24170	24630
Median pay/emp	29000	29100	41400	42000
p-75 pay/emp	48000	48000	62550	63220
% firms	99.49	100	86.94	100
% employment	65.54	100	33.54	100

NOTE.—The table reports means, standard deviations, 25th percentiles, medians, and 75th percentiles for employment (March 12 employment), annual payroll (in $1,000), and annual payroll per employee (dollars per employee). Columns (1) and (2) compare the sample with the full LBD universe. Columns (3) and (4) compare the multi-unit firms in the sample with the full universe of multi-unit firms in the LBD. The LBD universe includes all non-farm businesses with employees. % firms denotes the sample as a percentage of the universe of firms, and % employment denotes the sample's employment as a percentage of total employment in the universe. All figures refer to 2015. Following CES guidelines, all numbers are rounded to the nearest four significant digits, and reported percentiles are pseudo-percentiles.

Table A.2: Exploring the Dependent Variable

	Investment growth		
	(1)	(2)	(3)
Real GDP growth	2.821***		
	(0.9776)		
Employment growth		6.243***	
		(1.781)	
Personal income growth			3.568***
			(1.273)
p-25 dependent variable	-0.4972	-0.4972	-0.4972
Median dependent variable	-0.0372	-0.0372	-0.0372
p-75 dependent variable	0.4645	0.4645	0.4645
N	1000	1000	1000

NOTE.—The table reports three separate regressions of investment growth on real GDP growth, employment growth, and personal income growth at the MSA level. Standard errors are clustered at the MSA level. Following CES guidelines, all numbers are rounded to the nearest four significant digits except N, which is rounded to the nearest 100 if larger than 1,000 and to the nearest 50 otherwise. Reported percentiles are pseudo-percentiles. Significance levels: * 10%, ** 5%, *** 1%.

B The Tradable–Nontradable Model: Complete Markets and Local Fiscal Financing

This appendix presents the tradable–nontradable model with complete markets and local fiscal financing introduced in Section 4.1. Households trade a complete set of state-contingent nominal claims across regions, and each regional government finances its purchases with contemporaneous lump-sum taxes on the representative household in its region.

Regions are indexed by $i \in [0,1]$, sectors by $s \in \{T, N\}$, and firms within each sector by $j \in [0,1]$. Time is discrete. Capital $K_{s,i,t}$ is available for production at t, and investment $\mathcal{I}_{s,i,t}$ determines the stock available at $t+1$. Labor is mobile between sectors within a region, while installed capital is sector-specific. Productivity is normalized to one.

B.1 Final-Good Aggregation and Demand

Competitive assemblers combine varieties within each sector according to

$$Y_{s,i,t} = \left[\int_0^1 Y_{s,i,t}(j)^{(\epsilon-1)/\epsilon}\,\mathrm{d}j\right]^{\epsilon/(\epsilon-1)}, \qquad \epsilon > 1. \tag{B.1}$$

Cost minimization gives the sectoral price index and variety demand:

$$P_{s,i,t} = \left[\int_0^1 P_{s,i,t}(j)^{1-\epsilon}\,\mathrm{d}j\right]^{1/(1-\epsilon)}, \tag{B.2}$$

$$Y_{s,i,t}(j) = \left(\frac{P_{s,i,t}(j)}{P_{s,i,t}}\right)^{-\epsilon} Y_{s,i,t}. \tag{B.3}$$

Tradable varieties obey the law of one price across destinations. A star on a price denotes its geometric union average; in particular,

$$P^*_{s,t} = \exp\left\{\int_0^1 \log P_{s,i,t}\,\mathrm{d}i\right\}. \tag{B.4}$$

For a final use $a \in \{C, I\}$, denote the final quantity by $X^a_{i,t}$, with $X^C_{i,t} = C_{i,t}$ and $X^I_{i,t} = \mathcal{I}_{i,t}$. Define the normalized Cobb–Douglas aggregator

$$\mathcal{A}_\omega(x_1, x_2) = \frac{x_1^\omega x_2^{1-\omega}}{\omega^\omega (1-\omega)^{1-\omega}}, \qquad \omega \in (0,1). \tag{B.5}$$

Its unit cost is $p_1^{\omega} p_2^{1-\omega}$. The final and tradable baskets are

$$X^a_{i,t} = \mathcal{A}_{\nu_a}(X^a_{T,i,t}, X^a_{N,i,t}), \tag{B.6}$$

$$X^a_{T,i,t} = \mathcal{A}_{1-\alpha_a}(X^a_{H,i,t}, X^a_{F,i,t}), \tag{B.7}$$

$$X^a_{F,i,t} = \exp\left\{\int_0^1 \log X^a_{h\to i,t}\,\mathrm{d}h\right\}. \tag{B.8}$$

Here H denotes local tradable production, F the union basket, and $X^a_{h\to i,t}$ purchases from origin h through that basket. The parameters $\nu_a, \alpha_a \in (0,1)$ are, respectively, the tradable expenditure share and the union-basket share conditional on tradables. Including the home region in the union basket is immaterial in the continuum economy.

The tradable and final-use price indices are

$$P^a_{T,i,t} = P^{1-\alpha_a}_{T,i,t}(P^*_{T,t})^{\alpha_a}, \tag{B.9}$$

$$P_{a,i,t} = (P^a_{T,i,t})^{\nu_a} P^{1-\nu_a}_{N,i,t}. \tag{B.10}$$

The associated expenditure demands satisfy

$$P^a_{T,i,t}X^a_{T,i,t} = \nu_a P_{a,i,t}X^a_{i,t}, \qquad P_{N,i,t}X^a_{N,i,t} = (1-\nu_a)P_{a,i,t}X^a_{i,t}, \tag{B.11}$$

$$P_{T,i,t}X^a_{H,i,t} = \nu_a(1-\alpha_a)P_{a,i,t}X^a_{i,t}, \qquad P^*_{T,t}X^a_{F,i,t} = \nu_a\alpha_a P_{a,i,t}X^a_{i,t}. \tag{B.12}$$

Within the union basket, $P_{T,h,t}X^a_{h\to i,t} = P^*_{T,t}X^a_{F,i,t}$ in expenditure-density terms.

Public consumption uses the same aggregator as private consumption, so $P_{G,i,t} = P_{C,i,t}$. Both destinations of installed investment use the investment basket, with

$$\mathcal{I}_{i,t} = \mathcal{I}_{T,i,t} + \mathcal{I}_{N,i,t}. \tag{B.13}$$

The quantities $X^I_{T,i,t}$ and $X^I_{N,i,t}$ are inputs into this basket; $\mathcal{I}_{T,i,t}$ and $\mathcal{I}_{N,i,t}$ identify the sectors in which capital is installed.

It is useful to define

$$S_{i,t} = \frac{P^*_{T,t}}{P_{T,i,t}}, \qquad Z_{i,t} = \frac{P_{N,i,t}}{P^*_{T,t}}, \qquad h_a = \nu_a(1-\alpha_a). \tag{B.14}$$

The price indices imply the exact relations

$$\frac{P_{a,i,t}}{P^*_{T,t}} = Z_{i,t}^{1-\nu_a} S_{i,t}^{-h_a}, \tag{B.15}$$

$$\frac{P_{I,i,t}}{P_{C,i,t}} = Z_{i,t}^{\nu_C-\nu_I} S_{i,t}^{h_C-h_I}, \tag{B.16}$$

$$P^*_{a,t} = (P^*_{T,t})^{\nu_a}(P^*_{N,t})^{1-\nu_a}. \tag{B.17}$$

B.2 Households and Capital Accumulation

The representative household in region i maximizes

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[(1-\chi_G)\log C_{i,t} + \chi_G \log G_{i,t} - \frac{L_{i,t}^{1+\varphi}}{1+\varphi} \right], \tag{B.18}$$

where $\beta \in (0,1)$, $\chi_G \in [0,1)$, and $\varphi \geq 0$. Public consumption is taken as given. The household owns local firms and the two capital stocks. Its nominal budget constraint is

$$\begin{aligned} P_{C,i,t}C_{i,t} + P_{I,i,t}\mathcal{I}_{i,t} + \mathbb{E}_t[\mathcal{Q}_{t,t+1}D_{i,t+1}] \\ \leq D_{i,t} + W_{i,t}L_{i,t} + \sum_{s\in\{T,N\}} \mathcal{R}^K_{s,i,t}K_{s,i,t} + \mathcal{D}_{i,t} - \mathcal{T}_{i,t}. \end{aligned} \tag{B.19}$$

Here $D_{i,t+1}$ is a state-contingent nominal payoff purchased at t, $\mathcal{Q}_{t,t+1}$ is the nominal stochastic discount factor, $\mathcal{R}^K_{s,i,t}$ is the nominal rental payment per unit of capital, $\mathcal{D}_{i,t}$ denotes dividends, and $\mathcal{T}_{i,t}$ denotes lump-sum taxes.

Define $w_{i,t} = W_{i,t}/P_{C,i,t}$. Optimal labor supply and asset purchases imply

$$C_{i,t}L_{i,t}^{\varphi} = (1-\chi_G)w_{i,t}, \tag{B.20}$$

$$\mathcal{Q}_{t,t+1} = \beta \frac{C_{i,t}}{C_{i,t+1}} \frac{P_{C,i,t}}{P_{C,i,t+1}}. \tag{B.21}$$

The nominal discount factor is common across regions. The gross nominal interest rate satisfies $1/R^*_t = \mathbb{E}_t\mathcal{Q}_{t,t+1}$. Thus the consumption Euler equation is

$$1 = R^*_t \mathbb{E}_t \left[\beta \frac{C_{i,t}}{C_{i,t+1}} \frac{P_{C,i,t}}{P_{C,i,t+1}} \right]. \tag{B.22}$$

With equal Pareto weights and symmetric initial wealth, complete markets yield

$$P_{C,i,t}C_{i,t} = P^*_{C,t}C^*_t, \qquad C^*_t = \exp\left\{\int_0^1 \log C_{i,t}\,\mathrm{d}i\right\}. \tag{B.23}$$

The real discount factor in regional consumption units is

$$M^C_{i,t,t+1} = \mathcal{Q}_{t,t+1}\frac{P_{C,i,t+1}}{P_{C,i,t}} = \beta\frac{C_{i,t}}{C_{i,t+1}}. \tag{B.24}$$

For each sector, capital evolves according to

$$K_{s,i,t+1} = [1-\delta+\Phi(x_{s,i,t})]K_{s,i,t}, \qquad x_{s,i,t} = \frac{\mathcal{I}_{s,i,t}}{K_{s,i,t}}, \tag{B.25}$$

where $\delta \in (0,1)$ and

$$\Phi(x) = x - \frac{\tau}{2}(x-\delta)^2, \qquad \tau > 0. \tag{B.26}$$

Let $q_{s,i,t}$ be the value of capital installed for $t+1$ in date-t consumption units, and define $r^K_{s,i,t} = \mathcal{R}^K_{s,i,t}/P_{C,i,t}$. The investment and capital optimality conditions are

$$\frac{P_{I,i,t}}{P_{C,i,t}} = q_{s,i,t}\Phi'(x_{s,i,t}), \tag{B.27}$$

and

$$q_{s,i,t} = \mathbb{E}_t\Big\{M^C_{i,t,t+1}\Big[r^K_{s,i,t+1} + q_{s,i,t+1}\big(1-\delta + \Phi(x_{s,i,t+1}) - x_{s,i,t+1}\Phi'(x_{s,i,t+1})\big)\Big]\Big\}. \tag{B.28}$$

The term $1-\delta+\Phi(x)-x\Phi'(x)$ is the derivative of next-period capital with respect to current capital, holding investment fixed. The household's plan satisfies the no-Ponzi condition on financial claims and, for each sector,

$$\lim_{T\to\infty}\mathbb{E}_t\left[M^C_{i,t,T}q_{s,i,T}K_{s,i,T+1}\right] = 0, \qquad M^C_{i,t,T} = \beta^{T-t}\frac{C_{i,t}}{C_{i,T}}. \tag{B.29}$$

B.3 Firms and Price Setting

Firm j in sector s produces

$$Y_{s,i,t}(j) = K_{s,i,t}(j)^{\xi}L_{s,i,t}(j)^{1-\xi}, \qquad \xi \in (0,1). \tag{B.30}$$

Firms rent capital within their sector and hire labor at the common regional wage. Nominal marginal cost is

$$\mathrm{MC}_{s,i,t} = \frac{(\mathcal{R}^K_{s,i,t})^{\xi} W_{i,t}^{1-\xi}}{\xi^{\xi}(1-\xi)^{1-\xi}}. \tag{B.31}$$

The factor-demand conditions are

$$\mathcal{R}^K_{s,i,t} K_{s,i,t}(j) = \xi\, \mathrm{MC}_{s,i,t} Y_{s,i,t}(j), \tag{B.32}$$

$$W_{i,t} L_{s,i,t}(j) = (1-\xi) \mathrm{MC}_{s,i,t} Y_{s,i,t}(j). \tag{B.33}$$

Real marginal cost for price setting is $mc_{s,i,t} = \mathrm{MC}_{s,i,t}/P_{s,i,t}$. Rental returns $r^K_{s,i,t}$ are instead measured in consumption units, as in the household's capital Euler equation.

In each sector, a fraction $1-\theta$ of firms can reset prices each period, with $\theta \in (0,1)$, while the remaining fraction keeps its previous nominal price. There is no indexation. A firm that resets at t chooses $P^{\#}_{s,i,t}$ to maximize

$$\mathbb{E}_t \sum_{k=0}^{\infty} \theta^k \mathcal{Q}_{t,t+k} \left[P^{\#}_{s,i,t} - \mathrm{MC}_{s,i,t+k} \right] \left(\frac{P^{\#}_{s,i,t}}{P_{s,i,t+k}} \right)^{-\epsilon} Y_{s,i,t+k}, \tag{B.34}$$

where $\mathcal{Q}_{t,t} = 1$ and $\mathcal{Q}_{t,t+k}$ is the product of the intervening one-period nominal discount factors. A tradable firm sets one price for all destinations. The optimal reset price satisfies

$$P^{\#}_{s,i,t} = \mu \frac{F_{s,i,t}}{H_{s,i,t}}, \qquad \mu = \frac{\epsilon}{\epsilon - 1}, \tag{B.35}$$

with

$$F_{s,i,t} = \mathrm{MC}_{s,i,t} Y_{s,i,t} P^{\epsilon}_{s,i,t} + \theta \mathbb{E}_t[\mathcal{Q}_{t,t+1} F_{s,i,t+1}], \tag{B.36}$$

$$H_{s,i,t} = Y_{s,i,t} P^{\epsilon}_{s,i,t} + \theta \mathbb{E}_t[\mathcal{Q}_{t,t+1} H_{s,i,t+1}]. \tag{B.37}$$

The sectoral price index evolves according to

$$P^{1-\epsilon}_{s,i,t} = \theta P^{1-\epsilon}_{s,i,t-1} + (1-\theta)(P^{\#}_{s,i,t})^{1-\epsilon}. \tag{B.38}$$

Define price dispersion and gross sectoral inflation by

$$\Delta_{s,i,t} = \int_0^1 \left(\frac{P_{s,i,t}(j)}{P_{s,i,t}} \right)^{-\epsilon} \mathrm{d}j, \qquad \Pi_{s,i,t} = \frac{P_{s,i,t}}{P_{s,i,t-1}}. \tag{B.39}$$

Since firms within a sector face the same factor prices, aggregate production and the law of

motion for dispersion are

$$Y_{s,i,t} = \frac{K_{s,i,t}^{\xi} L_{s,i,t}^{1-\xi}}{\Delta_{s,i,t}}, \tag{B.40}$$

$$\Delta_{s,i,t} = \theta \Pi_{s,i,t}^{\epsilon} \Delta_{s,i,t-1} + (1-\theta) \left(\frac{P_{s,i,t}^{\#}}{P_{s,i,t}} \right)^{-\epsilon}. \tag{B.41}$$

There is no production subsidy. Using variety demand, aggregate dividends can be written as

$$\mathcal{D}_{i,t} = \sum_{s \in \{T,N\}} \left[P_{s,i,t} Y_{s,i,t} - W_{i,t} L_{s,i,t} - \mathcal{R}_{s,i,t}^{K} K_{s,i,t} \right]. \tag{B.42}$$

B.4 Policy and Equilibrium

The government purchases the consumption basket and finances its expenditure with contemporaneous lump-sum taxes. Let $\mathcal{G}_{i,t}$ denote public expenditure in units of the union consumption price index. Then

$$P_{C,i,t} G_{i,t} = P_{C,t}^{*} \mathcal{G}_{i,t} = \mathcal{T}_{i,t}. \tag{B.43}$$

The stochastic specification of $\mathcal{G}_{i,t}$ is given in Section 5. Union public expenditure in the same units is the arithmetic aggregate $\mathcal{G}_t^U = \int_0^1 \mathcal{G}_{i,t} \, \mathrm{d}i$.

The central bank follows

$$R_t^{*} = \beta^{-1} (\Pi_{C,t}^{*})^{\phi_\pi}, \qquad \Pi_{C,t}^{*} = \frac{P_{C,t}^{*}}{P_{C,t-1}^{*}}, \qquad \phi_\pi > 1. \tag{B.44}$$

Let $A_{i,t}^{C} = C_{i,t} + G_{i,t}$ denote consumption-type absorption. The nontradable goods market clears when

$$P_{N,i,t} Y_{N,i,t} = (1-\nu_C) P_{C,i,t} A_{i,t}^{C} + (1-\nu_I) P_{I,i,t} \mathcal{I}_{i,t}. \tag{B.45}$$

Tradable output supplies local home-biased demand and purchases through the union basket:

$$\begin{aligned} P_{T,i,t} Y_{T,i,t} = {} & h_C P_{C,i,t} A_{i,t}^{C} + h_I P_{I,i,t} \mathcal{I}_{i,t} \\ & + \int_0^1 \left[\nu_C \alpha_C P_{C,h,t} A_{h,t}^{C} + \nu_I \alpha_I P_{I,h,t} \mathcal{I}_{h,t} \right] \mathrm{d}h. \end{aligned} \tag{B.46}$$

Risk sharing and the public budget imply

$$P_{C,i,t}A^C_{i,t} = P^*_{C,t}(C^*_t + \mathcal{G}_{i,t}). \tag{B.47}$$

Factor markets and the market for contingent claims satisfy

$$K_{s,i,t} = \int_0^1 K_{s,i,t}(j)\,\mathrm{d}j, \qquad L_{s,i,t} = \int_0^1 L_{s,i,t}(j)\,\mathrm{d}j, \tag{B.48}$$

$$L_{i,t} = L_{T,i,t} + L_{N,i,t}, \qquad \int_0^1 D_{i,t}\,\mathrm{d}i = 0. \tag{B.49}$$

Asset-market clearing holds state by state. Regional nominal output and the trade balance are

$$\mathrm{GDP}^{nom}_{i,t} = P_{T,i,t}Y_{T,i,t} + P_{N,i,t}Y_{N,i,t}, \tag{B.50}$$

$$NX_{i,t} = \mathrm{GDP}^{nom}_{i,t} - P_{C,i,t}A^C_{i,t} - P_{I,i,t}\mathcal{I}_{i,t}. \tag{B.51}$$

The goods markets imply $\int_0^1 NX_{i,t}\,\mathrm{d}i = 0$. Individual regional balances may be nonzero and are supported by contingent transfers.

Given initial capital stocks, initial price distributions, the fiscal process, and initial financial wealth consistent with the risk-sharing allocation, an equilibrium is a collection of household and firm allocations, asset payoffs, prices, and taxes satisfying household optimality and transversality, assembler cost minimization, firm factor demand and optimal price setting, the policy rules, and market clearing. One initial nominal price normalization completes the description. The aggregate household budget and union resource constraint follow from these conditions and Walras' law.

B.5 Steady State

Consider a deterministic symmetric steady state with zero inflation. Normalize all sectoral and final-good prices to one. Price dispersion is one, and reset prices equal prevailing prices. Define

$$\bar{Y} = \bar{Y}_T + \bar{Y}_N, \qquad \bar{K} = \bar{K}_T + \bar{K}_N, \qquad \bar{\mathcal{I}} = \bar{\mathcal{I}}_T + \bar{\mathcal{I}}_N. \tag{B.52}$$

The steady-state public-expenditure share is $\gamma = \bar{G}/\bar{Y}$, so $\bar{\mathcal{G}}_i = \bar{\mathcal{G}}^U = \bar{G} = \gamma\bar{Y}$. The parameters γ and χ_G govern, respectively, the fiscal level and preferences.

Let $m = 1/\mu = (\epsilon - 1)/\epsilon$. Since $\Phi(\delta) = \delta$ and $\Phi'(\delta) = 1$, the investment conditions give

$$\bar{q}_T = \bar{q}_N = 1, \qquad \bar{r}_K = \beta^{-1} - 1 + \delta. \tag{B.53}$$

Both sectors have marginal cost m, and factor demand implies

$$\bar{r}_K = m\xi \frac{\bar{Y}_s}{\bar{K}_s}, \qquad \bar{w} = m(1-\xi)\frac{\bar{Y}_s}{\bar{L}_s}. \tag{B.54}$$

Thus

$$\frac{\bar{K}}{\bar{Y}} = \frac{m\xi}{\bar{r}_K}, \qquad \kappa_{KL} \equiv \frac{\bar{K}_s}{\bar{L}_s} = \left(\frac{m\xi}{\bar{r}_K}\right)^{1/(1-\xi)}. \tag{B.55}$$

The investment and private-consumption shares are

$$\iota \equiv \frac{\bar{\mathcal{I}}}{\bar{Y}} = \delta \frac{m\xi}{\bar{r}_K}, \qquad c_Y \equiv \frac{\bar{C}}{\bar{Y}} = 1 - \gamma - \iota > 0. \tag{B.56}$$

The sectoral goods markets determine the output shares:

$$\omega_T \equiv \frac{\bar{Y}_T}{\bar{Y}} = \nu_C(1-\iota) + \nu_I \iota, \tag{B.57}$$

$$\omega_N \equiv \frac{\bar{Y}_N}{\bar{Y}} = (1-\nu_C)(1-\iota) + (1-\nu_I)\iota. \tag{B.58}$$

Because technologies and factor ratios are identical across sectors,

$$\frac{\bar{Y}_s}{\bar{Y}} = \frac{\bar{K}_s}{\bar{K}} = \frac{\bar{L}_s}{\bar{L}} = \frac{\bar{\mathcal{I}}_s}{\bar{\mathcal{I}}} = \omega_s, \qquad s \in \{T, N\}. \tag{B.59}$$

Labor supply determines the scale of the economy:

$$\bar{L} = \left[\frac{(1-\chi_G)m(1-\xi)}{c_Y}\right]^{1/(1+\varphi)}, \tag{B.60}$$

$$\bar{Y} = \kappa_{KL}^{\xi}\bar{L}, \qquad \bar{K} = \kappa_{KL}\bar{L}, \tag{B.61}$$

$$\bar{\mathcal{I}} = \delta\bar{K}, \qquad \bar{C} = c_Y\bar{Y}, \tag{B.62}$$

$$\bar{G} = \gamma\bar{Y}, \qquad \bar{w} = m(1-\xi)\kappa_{KL}^{\xi}. \tag{B.63}$$

Aggregate dividends equal $(1-m)\bar{Y}$, taxes equal $\bar{G}$, and $\bar{R}^* = 1/\beta$. Symmetric net asset positions and trade balances are zero. The Calvo recursions satisfy

$$\bar{F}_s = \frac{m\bar{Y}_s}{1-\beta\theta}, \qquad \bar{H}_s = \frac{\bar{Y}_s}{1-\beta\theta}, \qquad \mu\frac{\bar{F}_s}{\bar{H}_s} = 1. \tag{B.64}$$

B.6 Log-Linear Equilibrium

For a positive variable $X_{i,t}$, let $\widehat{x}_{i,t} = \log(X_{i,t}/\bar{X})$. In particular, $\widehat{\iota}_{s,i,t}$ denotes the log deviation of $\mathcal{I}_{s,i,t}$; the unhatted ι remains the steady-state investment share. For regional log deviations, define

$$\widehat{x}^*_t = \int_0^1 \widehat{x}_{i,t}\,\mathrm{d}i, \qquad \widetilde{x}_{i,t} = \widehat{x}_{i,t} - \widehat{x}^*_t. \tag{B.65}$$

The following equilibrium conditions are first-order approximations unless identified as exact log identities. An equation indexed by s applies to both sectors.

Prices, expenditure, and risk sharing. Let lower-case p denote a log price and define

$$s_{i,t} = p^*_{T,t} - p_{T,i,t}, \qquad z_{i,t} = p_{N,i,t} - p^*_{T,t}, \qquad \widetilde{z}_{i,t} = z_{i,t} - z^*_t. \tag{B.66}$$

Here $z^*_t = p^*_{N,t} - p^*_{T,t}$ and $\int_0^1 s_{i,t}\,\mathrm{d}i = 0$. The regional consumption-price deviation from the union index and the relative investment price satisfy

$$\psi^C_{i,t} \equiv p_{C,i,t} - p^*_{C,t} = (1-\nu_C)\widetilde{z}_{i,t} - h_C s_{i,t}, \tag{B.67}$$

$$\widehat{p}_{I/C,i,t} \equiv p_{I,i,t} - p_{C,i,t} = (\nu_C - \nu_I) z_{i,t} + (h_C - h_I) s_{i,t}. \tag{B.68}$$

Both are exact log identities. Their union and relative investment-price components are

$$\widehat{p}^*_{I/C,t} = (\nu_C - \nu_I) z^*_t, \qquad \widetilde{p}_{I/C,i,t} = (\nu_C - \nu_I)\widetilde{z}_{i,t} + (h_C - h_I) s_{i,t}. \tag{B.69}$$

Write the first-order fiscal decomposition as

$$\widehat{\mathcal{G}}_{i,t} = \widehat{\mathcal{G}}^U_t + u^G_{i,t}, \qquad \int_0^1 u^G_{i,t}\,\mathrm{d}i = 0. \tag{B.70}$$

At first order, the log deviation of the arithmetic aggregate $\mathcal{G}^U_t$ equals the average regional log deviation. Risk sharing and the public budget imply

$$\widehat{c}_{i,t} = \widehat{c}^*_t - \psi^C_{i,t}, \qquad \widehat{g}_{i,t} = \widehat{\mathcal{G}}^U_t + u^G_{i,t} - \psi^C_{i,t}, \tag{B.71}$$

where $\widehat{g}_{i,t}$ is the log deviation of physical public consumption. In consumption units, the log deviation of the stochastic discount factor is

$$\widehat{m}^C_{i,t,t+1} = \widehat{c}^*_t - \widehat{c}^*_{t+1} + \psi^C_{i,t+1} - \psi^C_{i,t}. \tag{B.72}$$

Define expenditure in units of the union tradable price by

$$e^A_{i,t} = \frac{P_{C,i,t}(C_{i,t} + G_{i,t})}{P^*_{T,t}}, \qquad e^I_{i,t} = \frac{P_{I,i,t}\mathcal{I}_{i,t}}{P^*_{T,t}}, \tag{B.73}$$

and let $e^{A,U}_t$ and $e^{I,U}_t$ be their arithmetic union aggregates. With

$$\lambda_C = \frac{c_Y}{1-\iota}, \qquad \lambda_G = \frac{\gamma}{1-\iota}, \qquad \widehat{\iota}_{i,t} = \omega_T \widehat{\iota}_{T,i,t} + \omega_N \widehat{\iota}_{N,i,t}, \tag{B.74}$$

the union expenditures and regional deviations satisfy

$$\widehat{e}^{A,U}_t = (1-\nu_C) z^*_t + \lambda_C \widehat{c}^*_t + \lambda_G \widehat{\mathcal{G}}^U_t\,, \tag{B.75}$$

$$\widehat{e}^{I,U}_t = (1-\nu_I) z^*_t + \widehat{\iota}^*_t, \tag{B.76}$$

$$\widetilde{e}^A_{i,t} = \lambda_G u^G_{i,t}, \tag{B.77}$$

$$\widetilde{e}^I_{i,t} = (1-\nu_I)\widetilde{z}_{i,t} - h_I s_{i,t} + \widetilde{\iota}_{i,t}. \tag{B.78}$$

The union and relative investment aggregates use the same weights ω_T and ω_N as in (B.74).

Capital valuation and price setting. Linearizing the capital law and installation condition gives

$$\widehat{k}_{s,i,t+1} = (1-\delta)\widehat{k}_{s,i,t} + \delta \widehat{\iota}_{s,i,t}, \tag{B.79}$$

$$\widehat{p}_{I/C,i,t} = \widehat{q}_{s,i,t} - \tau\delta(\widehat{\iota}_{s,i,t} - \widehat{k}_{s,i,t}). \tag{B.80}$$

To obtain the linear capital Euler equation, define $B(x) = 1 - \delta + \Phi(x) - x\Phi'(x)$. Since $B(\delta) = 1-\delta$, $B'(\delta) = \tau\delta$, and $x_{s,i,t} - \delta$ equals $\delta(\widehat{\iota}_{s,i,t} - \widehat{k}_{s,i,t})$ to first order,

$$\begin{aligned}\widehat{q}_{s,i,t} = \mathbb{E}_t\Big[&\widehat{m}^C_{i,t,t+1} + \beta\bar{r}_K \widehat{r}^K_{s,i,t+1} + \beta(1-\delta)\widehat{q}_{s,i,t+1} \\ &+ \beta\tau\delta^2(\widehat{\iota}_{s,i,t+1} - \widehat{k}_{s,i,t+1})\Big].\end{aligned} \tag{B.81}$$

Substituting the next-period installation condition yields the equivalent reduced form

$$\widehat{q}_{s,i,t} = \mathbb{E}_t\left[\widehat{m}^C_{i,t,t+1} + \beta\bar{r}_K \widehat{r}^K_{s,i,t+1} + \beta\widehat{q}_{s,i,t+1} - \beta\delta\widehat{p}_{I/C,i,t+1}\right]. \tag{B.82}$$

Let $\pi_{s,i,t} = p_{s,i,t} - p_{s,i,t-1}$. Linearization of the Calvo recursions and price index gives

$$\pi_{s,i,t} = \beta\mathbb{E}_t\pi_{s,i,t+1} + \kappa\widehat{mc}_{s,i,t}, \qquad \kappa = \frac{(1-\theta)(1-\beta\theta)}{\theta}. \tag{B.83}$$

The output and nominal discount-factor terms cancel between the linearized numerator and denominator of the reset price. Price dispersion has no first-order variation around the symmetric zero-inflation steady state, so

$$\widehat{y}_{s,i,t} = \xi \widehat{k}_{s,i,t} + (1-\xi)\widehat{l}_{s,i,t}. \tag{B.84}$$

Union equilibrium. Define the steady-state expenditure weights

$$\lambda_{NA} = \frac{(1-\nu_C)(1-\iota)}{\omega_N}, \qquad \lambda_{NI} = \frac{(1-\nu_I)\iota}{\omega_N}, \tag{B.85}$$

$$\lambda_{TA} = \frac{\nu_C(1-\iota)}{\omega_T}, \qquad \lambda_{TI} = \frac{\nu_I \iota}{\omega_T}. \tag{B.86}$$

Each pair sums to one. Factor demand implies the union rental returns and marginal costs

$$\widehat{r}^{K,*}_{s,t} = \widehat{w}^*_t + \frac{\widehat{y}^*_{s,t} - \widehat{k}^*_{s,t}}{1-\xi}, \tag{B.87}$$

$$\widehat{mc}^*_{T,t} = \widehat{w}^*_t + \frac{\xi}{1-\xi}(\widehat{y}^*_{T,t} - \widehat{k}^*_{T,t}) + (1-\nu_C)z^*_t, \tag{B.88}$$

$$\widehat{mc}^*_{N,t} = \widehat{w}^*_t + \frac{\xi}{1-\xi}(\widehat{y}^*_{N,t} - \widehat{k}^*_{N,t}) - \nu_C z^*_t. \tag{B.89}$$

Using these definitions, the union equilibrium is determined by

$$\widehat{c}^*_t = \mathbb{E}_t \widehat{c}^*_{t+1} - \left(\phi_\pi \pi^*_{C,t} - \mathbb{E}_t \pi^*_{C,t+1}\right), \tag{B.90}$$

$$\varphi(\omega_T \widehat{l}^*_{T,t} + \omega_N \widehat{l}^*_{N,t}) = \widehat{w}^*_t - \widehat{c}^*_t, \tag{B.91}$$

$$\widehat{y}^*_{s,t} = \xi \widehat{k}^*_{s,t} + (1-\xi)\widehat{l}^*_{s,t}, \tag{B.92}$$

$$\widehat{y}^*_{N,t} + z^*_t = \lambda_{NA}\widehat{e}^{A,U}_t + \lambda_{NI}\widehat{e}^{I,U}_t, \tag{B.93}$$

$$\widehat{y}^*_{T,t} = \lambda_{TA}\widehat{e}^{A,U}_t + \lambda_{TI}\widehat{e}^{I,U}_t, \tag{B.94}$$

$$\widehat{k}^*_{s,t+1} = (1-\delta)\widehat{k}^*_{s,t} + \delta \widehat{\iota}^*_{s,t}, \tag{B.95}$$

$$\widehat{p}^*_{I/C,t} = \widehat{q}^*_{s,t} - \tau\delta(\widehat{\iota}^*_{s,t} - \widehat{k}^*_{s,t}), \tag{B.96}$$

$$\widehat{q}^*_{s,t} = \mathbb{E}_t\Big[\widehat{c}^*_t - \widehat{c}^*_{t+1} + \beta \bar{r}_K \widehat{r}^{K,*}_{s,t+1} + \beta \widehat{q}^*_{s,t+1} - \beta\delta \widehat{p}^*_{I/C,t+1}\Big], \tag{B.97}$$

$$\pi^*_{C,t} = \beta \mathbb{E}_t \pi^*_{C,t+1} + \kappa\left[(1-\nu_C)\widehat{mc}^*_{N,t} + \nu_C \widehat{mc}^*_{T,t}\right], \tag{B.98}$$

$$z^*_t - z^*_{t-1} = \beta \mathbb{E}_t(z^*_{t+1} - z^*_t) + \kappa(\widehat{mc}^*_{N,t} - \widehat{mc}^*_{T,t}). \tag{B.99}$$

The final two equations are independent combinations of the sectoral Phillips curves. Home-bias parameters allocate tradable expenditure across origins and therefore do not enter the union block.

Regional equilibrium. Define

$$b_N^G = \frac{(1-\nu_C)\gamma}{\omega_N}, \qquad b_T^G = \frac{h_C\gamma}{\omega_T}, \tag{B.100}$$

$$\eta_N^I = \frac{(1-\nu_I)\iota}{\omega_N}, \qquad \eta_T^I = \frac{h_I\iota}{\omega_T}. \tag{B.101}$$

Subtracting the union equations from the regional conditions gives

$$\begin{aligned}
\widetilde{y}_{N,i,t} &= b_N^G u_{i,t}^G - [1-\eta_N^I(1-\nu_I)]\widetilde{z}_{i,t} \\
&\quad - \eta_N^I h_I s_{i,t} + \eta_N^I \widetilde{\iota}_{i,t}, && \text{(B.102)}\\
\widetilde{y}_{T,i,t} &= b_T^G u_{i,t}^G + \eta_T^I(1-\nu_I)\widetilde{z}_{i,t} \\
&\quad + [1-\eta_T^I h_I]s_{i,t} + \eta_T^I \widetilde{\iota}_{i,t}, && \text{(B.103)}\\
\widetilde{y}_{s,i,t} &= \xi\widetilde{k}_{s,i,t} + (1-\xi)\widetilde{l}_{s,i,t}, && \text{(B.104)}\\
\widetilde{l}_{i,t} &= \omega_T\widetilde{l}_{T,i,t} + \omega_N\widetilde{l}_{N,i,t}, && \text{(B.105)}\\
\varphi\widetilde{l}_{i,t} &= \widetilde{w}_{i,t} + \psi_{i,t}^C, && \text{(B.106)}\\
\widetilde{r}_{s,i,t}^K &= \widetilde{w}_{i,t} + \frac{\widetilde{y}_{s,i,t} - \widetilde{k}_{s,i,t}}{1-\xi}, && \text{(B.107)}\\
\widetilde{mc}_{T,i,t} &= \widetilde{w}_{i,t} + \frac{\xi}{1-\xi}(\widetilde{y}_{T,i,t} - \widetilde{k}_{T,i,t}) \\
&\quad + (1-\nu_C)\widetilde{z}_{i,t} + (1-h_C)s_{i,t}, && \text{(B.108)}\\
\widetilde{mc}_{N,i,t} &= \widetilde{w}_{i,t} + \frac{\xi}{1-\xi}(\widetilde{y}_{N,i,t} - \widetilde{k}_{N,i,t}) \\
&\quad - \nu_C\widetilde{z}_{i,t} - h_C s_{i,t}, && \text{(B.109)}\\
\widetilde{p}_{I/C,i,t} &= \widetilde{q}_{s,i,t} - \tau\delta(\widetilde{\iota}_{s,i,t} - \widetilde{k}_{s,i,t}), && \text{(B.110)}\\
\widetilde{k}_{s,i,t+1} &= (1-\delta)\widetilde{k}_{s,i,t} + \delta\widetilde{\iota}_{s,i,t}, && \text{(B.111)}\\
\widetilde{q}_{s,i,t} &= \mathbb{E}_t\Big[\psi_{i,t+1}^C - \psi_{i,t}^C + \beta\bar{r}_K\widetilde{r}_{s,i,t+1}^K \\
&\quad + \beta\widetilde{q}_{s,i,t+1} - \beta\delta\widetilde{p}_{I/C,i,t+1}\Big], && \text{(B.112)}\\
s_{i,t} - s_{i,t-1} &= \beta\mathbb{E}_t(s_{i,t+1} - s_{i,t}) - \kappa\widetilde{mc}_{T,i,t}, && \text{(B.113)}\\
\widetilde{z}_{i,t} - \widetilde{z}_{i,t-1} &= \beta\mathbb{E}_t(\widetilde{z}_{i,t+1} - \widetilde{z}_{i,t}) + \kappa\widetilde{mc}_{N,i,t}. && \text{(B.114)}
\end{aligned}$$

The common nominal policy rate drops out of the relative block. The regional real discount-factor term $\psi^C_{i,t+1} - \psi^C_{i,t}$ remains in the capital valuation equations.

Recovery of regional quantities and inflation. Regional log deviations are recovered by adding union and relative components. Consumption and public quantities follow from (B.71). Let $\Delta x_t = x_t - x_{t-1}$. Sectoral and final-use inflation are recovered from

$$\pi^*_{T,t} = \pi^*_{C,t} - (1-\nu_C)\Delta z^*_t, \qquad \pi^*_{N,t} = \pi^*_{C,t} + \nu_C \Delta z^*_t, \tag{B.115}$$

$$\pi_{T,i,t} = \pi^*_{T,t} - \Delta s_{i,t}, \qquad \pi_{N,i,t} = \pi^*_{N,t} + \Delta \widetilde{z}_{i,t}, \tag{B.116}$$

$$\pi_{a,i,t} = (1-\nu_a)\pi_{N,i,t} + h_a \pi_{T,i,t} + \nu_a \alpha_a \pi^*_{T,t}, \qquad a \in \{C, I\}. \tag{B.117}$$

In particular, $\pi_{C,i,t} - \pi^*_{C,t} = \Delta \psi^C_{i,t}$. Capital $\widehat{k}_{s,i,t}$ in all production and rental-return equations is the stock productive at t.

Implied identities. The two union goods markets imply the resource constraint

$$\omega_T \widehat{y}^*_{T,t} + \omega_N \widehat{y}^*_{N,t} = c_Y \widehat{c}^*_t + \gamma \widehat{\mathcal{G}}^U_t + \iota \widehat{\iota}^*_t. \tag{B.118}$$

Subtracting the two sectoral installation conditions and capital valuation equations gives

$$(\widetilde{\iota}_{N,i,t} - \widetilde{\iota}_{T,i,t}) - (\widetilde{k}_{N,i,t} - \widetilde{k}_{T,i,t}) = \frac{\widetilde{q}_{N,i,t} - \widetilde{q}_{T,i,t}}{\tau\delta}, \tag{B.119}$$

$$\widetilde{q}_{N,i,t} - \widetilde{q}_{T,i,t} = \beta \mathbb{E}_t \Big[\widetilde{q}_{N,i,t+1} - \widetilde{q}_{T,i,t+1} + \bar{r}_K (\widetilde{r}^K_{N,i,t+1} - \widetilde{r}^K_{T,i,t+1}) \Big]. \tag{B.120}$$

The common investment-price and regional discount-factor terms cancel from these sectoral comparisons. These identities, the union resource constraint, and the alternative forms of the capital Euler equation are implied by the equilibrium system.

Fiscal states and the finite-region representation. The fiscal process specified in Section 5 completes the system. For its first-order AR(1) specification, with persistence $\rho_G \in [0, 1)$, averaging and subtracting the average yield

$$\widehat{\mathcal{G}}^U_t = \rho_G \widehat{\mathcal{G}}^U_{t-1} + \bar{\varepsilon}^G_t, \tag{B.121}$$

$$u^G_{i,t} = \rho_G u^G_{i,t-1} + \varepsilon^G_{i,t} - \bar{\varepsilon}^G_t, \qquad \bar{\varepsilon}^G_t = \int_0^1 \varepsilon^G_{i,t} \, \mathrm{d}i. \tag{B.122}$$

The common innovation and regional deviations are obtained from the same regional innovations.

In the finite-region implementation, integrals across regions are replaced by equally weighted averages. If H_N has orthonormal columns orthogonal to the vector of ones $\mathbf{1}_N$, a vector of regional log deviations can be represented as

$$\mathbf{x}_t = \mathbf{1}_N \bar{x}_t + H_N \mathbf{x}_t^R, \qquad \bar{x}_t = \frac{1}{N} \mathbf{1}_N' \mathbf{x}_t, \qquad \mathbf{x}_t^R = H_N' \mathbf{x}_t. \tag{B.123}$$

The same transformation is applied to fiscal innovations and initial conditions. The union equations determine the common component, and the regional equations determine the $N-1$ contrast coordinates. Physical regional quantities follow by reconstruction.

The tradable union basket includes all origins. Consequently, the effective local expenditure share within its tradable component is $1-\alpha_a+\alpha_a/N$; the share purchased from other regions in the full final-use basket is $\nu_a\alpha_a(1-1/N)$. This convention preserves a common union tradable price index and converges to the continuum sourcing shares as N increases.

C The Tradable–Nontradable Model: Incomplete Markets and Federal Fiscal Financing

This appendix presents the tradable–nontradable model with incomplete markets and federal fiscal financing introduced in Section 4.1. Households trade a one-period noncontingent nominal bond, and regional public expenditure is financed through a balanced federal system with uniform lump-sum taxes. We describe the economic environment, derive its first-order equilibrium, and explain how the household budget and a present-value condition jointly determine regional expenditure and asset positions.

Regions are indexed by $i \in [0,1]$, sectors by $s \in \{T, N\}$, and firms within each sector by $j \in [0,1]$. Time is discrete. Capital $K_{s,i,t}$ is available for production at t, and investment $\mathcal{I}_{s,i,t}$ determines the stock available at $t+1$. Labor is mobile between sectors within a region, while installed capital is sector-specific. Productivity is normalized to one.

C.1 Final-Good Aggregation and Demand

Competitive assemblers combine varieties within each sector according to

$$Y_{s,i,t} = \left[\int_0^1 Y_{s,i,t}(j)^{(\epsilon-1)/\epsilon}\,\mathrm{d}j\right]^{\epsilon/(\epsilon-1)}, \qquad \epsilon > 1. \tag{C.1}$$

Cost minimization gives the sectoral price index and variety demand:

$$P_{s,i,t} = \left[\int_0^1 P_{s,i,t}(j)^{1-\epsilon}\,\mathrm{d}j\right]^{1/(1-\epsilon)}, \tag{C.2}$$

$$Y_{s,i,t}(j) = \left(\frac{P_{s,i,t}(j)}{P_{s,i,t}}\right)^{-\epsilon} Y_{s,i,t}. \tag{C.3}$$

Tradable varieties obey the law of one price across destinations. A star on a price denotes its geometric union average; in particular,

$$P^*_{s,t} = \exp\left\{\int_0^1 \log P_{s,i,t}\,\mathrm{d}i\right\}. \tag{C.4}$$

For a final use $a \in \{C, I\}$, denote the final quantity by $X^a_{i,t}$, with $X^C_{i,t} = C_{i,t}$ and

$X^I_{i,t} = \mathcal{I}_{i,t}$. Define the normalized Cobb–Douglas aggregator

$$\mathcal{A}_\omega(x_1, x_2) = \frac{x_1^\omega x_2^{1-\omega}}{\omega^\omega (1-\omega)^{1-\omega}}, \qquad \omega \in (0,1). \tag{C.5}$$

Its unit cost is $p_1^\omega p_2^{1-\omega}$. The final and tradable baskets are

$$X^a_{i,t} = \mathcal{A}_{\nu_a}(X^a_{T,i,t}, X^a_{N,i,t}), \tag{C.6}$$

$$X^a_{T,i,t} = \mathcal{A}_{1-\alpha_a}(X^a_{H,i,t}, X^a_{F,i,t}), \tag{C.7}$$

$$X^a_{F,i,t} = \exp\left\{ \int_0^1 \log X^a_{h\to i,t}\, \mathrm{d}h \right\}. \tag{C.8}$$

Here H denotes local tradable production, F the union basket, and $X^a_{h\to i,t}$ purchases from origin h through that basket. The parameters $\nu_a, \alpha_a \in (0,1)$ are, respectively, the tradable expenditure share and the union-basket share conditional on tradables. Including the home region in the union basket is immaterial in the continuum economy.

The tradable and final-use price indices are

$$P^a_{T,i,t} = P^{1-\alpha_a}_{T,i,t} (P^*_{T,t})^{\alpha_a}, \tag{C.9}$$

$$P_{a,i,t} = (P^a_{T,i,t})^{\nu_a} P^{1-\nu_a}_{N,i,t}. \tag{C.10}$$

The associated expenditure demands satisfy

$$P^a_{T,i,t} X^a_{T,i,t} = \nu_a P_{a,i,t} X^a_{i,t}, \qquad P_{N,i,t} X^a_{N,i,t} = (1-\nu_a) P_{a,i,t} X^a_{i,t}, \tag{C.11}$$

$$P_{T,i,t} X^a_{H,i,t} = \nu_a (1-\alpha_a) P_{a,i,t} X^a_{i,t}, \qquad P^*_{T,t} X^a_{F,i,t} = \nu_a \alpha_a P_{a,i,t} X^a_{i,t}. \tag{C.12}$$

Within the union basket, $P_{T,h,t} X^a_{h\to i,t} = P^*_{T,t} X^a_{F,i,t}$ in expenditure-density terms.

Public consumption uses the same aggregator as private consumption, so $P_{G,i,t} = P_{C,i,t}$. Both destinations of installed investment use the investment basket, with

$$\mathcal{I}_{i,t} = \mathcal{I}_{T,i,t} + \mathcal{I}_{N,i,t}. \tag{C.13}$$

The quantities $X^I_{T,i,t}$ and $X^I_{N,i,t}$ are inputs into this basket; $\mathcal{I}_{T,i,t}$ and $\mathcal{I}_{N,i,t}$ identify the sectors in which capital is installed.

It is useful to define

$$S_{i,t} = \frac{P^*_{T,t}}{P_{T,i,t}}, \qquad Z_{i,t} = \frac{P_{N,i,t}}{P^*_{T,t}}, \qquad h_a = \nu_a(1-\alpha_a). \tag{C.14}$$

We also write $a_a = \nu_a \alpha_a$ for the union-basket expenditure share in final use $a \in \{C, I\}$. The price indices imply the exact relations

$$\frac{P_{a,i,t}}{P^*_{T,t}} = Z_{i,t}^{1-\nu_a} S_{i,t}^{-h_a}, \tag{C.15}$$

$$\frac{P_{I,i,t}}{P_{C,i,t}} = Z_{i,t}^{\nu_C - \nu_I} S_{i,t}^{h_C - h_I}, \tag{C.16}$$

$$P^*_{a,t} = (P^*_{T,t})^{\nu_a} (P^*_{N,t})^{1-\nu_a}. \tag{C.17}$$

C.2 Households and Capital Accumulation

The representative household in region i maximizes

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[(1 - \chi_G) \log C_{i,t} + \chi_G \log G_{i,t} - \frac{L_{i,t}^{1+\varphi}}{1+\varphi} \right], \tag{C.18}$$

where $\beta \in (0,1)$, $\chi_G \in [0,1)$, and $\varphi \geq 0$. Public consumption is taken as given. The household owns local firms and the two capital stocks. Its nominal budget constraint is

$$\begin{aligned} P_{C,i,t} C_{i,t} + P_{I,i,t} \mathcal{I}_{i,t} + \frac{B_{i,t+1}}{R^*_t} \\ = B_{i,t} + W_{i,t} L_{i,t} + \sum_{s \in \{T,N\}} \mathcal{R}^K_{s,i,t} K_{s,i,t} + \mathcal{D}_{i,t} - \mathcal{T}_{i,t}. \end{aligned} \tag{C.19}$$

Here $B_{i,t}$ is the nominal bond payoff received at the beginning of t, positive for a creditor. The payoff $B_{i,t+1}$ is chosen at t and is not contingent on the innovation realized at $t+1$; its date-t price is $1/R^*_t$. The bond is in zero net supply and carries no holding or transaction cost. The remaining terms are nominal capital rentals $\mathcal{R}^K_{s,i,t}$, dividends $\mathcal{D}_{i,t}$, and the household's total lump-sum tax liability $\mathcal{T}_{i,t}$.

Define $w_{i,t} = W_{i,t}/P_{C,i,t}$. Optimal labor supply gives

$$C_{i,t} L_{i,t}^{\varphi} = (1 - \chi_G) w_{i,t}. \tag{C.20}$$

The household's marginal valuation of nominal payoffs is

$$\mathcal{Q}_{i,t,t+1} = \beta \frac{C_{i,t}}{C_{i,t+1}} \frac{P_{C,i,t}}{P_{C,i,t+1}}, \tag{C.21}$$

and the bond Euler equation is

$$1 = R_t^* \mathbb{E}_t \mathcal{Q}_{i,t,t+1}. \tag{C.22}$$

The conditional expectation of $\mathcal{Q}_{i,t,t+1}$ is common across regions, but the valuation kernels need not coincide state by state. Firms use the kernel of their local owner. In regional consumption units, this kernel is

$$M^C_{i,t,t+1} = \mathcal{Q}_{i,t,t+1} \frac{P_{C,i,t+1}}{P_{C,i,t}} = \beta \frac{C_{i,t}}{C_{i,t+1}}. \tag{C.23}$$

Define geometric union consumption and relative nominal private expenditure by

$$C_t^* = \exp\left\{ \int_0^1 \log C_{i,t} \, \mathrm{d}i \right\}, \tag{C.24}$$

$$\zeta_{i,t} = \log\left(\frac{P_{C,i,t} C_{i,t}}{P^*_{C,t} C^*_t} \right), \qquad \int_0^1 \zeta_{i,t} \, \mathrm{d}i = 0. \tag{C.25}$$

The zero mean of $\zeta_{i,t}$ follows from the definitions of the geometric indices. Its regional value is endogenous. In particular,

$$C_{i,t} = C_t^* \frac{P^*_{C,t}}{P_{C,i,t}} e^{\zeta_{i,t}}, \tag{C.26}$$

whereas complete risk sharing with equal Pareto weights would impose $\zeta_{i,t} = 0$ in every region. Aggregate nominal private expenditure satisfies the exact identity

$$\int_0^1 P_{C,i,t} C_{i,t} \, \mathrm{d}i = P^*_{C,t} C^*_t \int_0^1 e^{\zeta_{i,t}} \, \mathrm{d}i. \tag{C.27}$$

Its equality to $P^*_{C,t} C^*_t$ is a first-order result around symmetry, not an exact aggregation property of the incomplete-markets economy.

For each sector, capital evolves according to

$$K_{s,i,t+1} = [1 - \delta + \Phi(x_{s,i,t})] K_{s,i,t}, \qquad x_{s,i,t} = \frac{\mathcal{I}_{s,i,t}}{K_{s,i,t}}, \tag{C.28}$$

where $\delta \in (0,1)$ and

$$\Phi(x) = x - \frac{\tau}{2}(x - \delta)^2, \qquad \tau > 0. \tag{C.29}$$

Let $q_{s,i,t}$ be the value of capital installed for $t+1$ in date-t consumption units, and define

$r^K_{s,i,t} = \mathcal{R}^K_{s,i,t}/P_{C,i,t}$. The investment and capital optimality conditions are

$$\frac{P_{I,i,t}}{P_{C,i,t}} = q_{s,i,t}\Phi'(x_{s,i,t}), \tag{C.30}$$

and

$$q_{s,i,t} = \mathbb{E}_t\Big\{ M^C_{i,t,t+1}\Big[r^K_{s,i,t+1} + q_{s,i,t+1}\big(1-\delta + \Phi(x_{s,i,t+1}) - x_{s,i,t+1}\Phi'(x_{s,i,t+1})\big)\Big]\Big\}. \tag{C.31}$$

The term $1-\delta+\Phi(x)-x\Phi'(x)$ is the derivative of next-period capital with respect to current capital, holding investment fixed. The household's plan satisfies the no-Ponzi condition on bond positions and, for each sector, the capital transversality condition

$$\lim_{T\to\infty} \mathbb{E}_t\left[M^C_{i,t,T} q_{s,i,T} K_{s,i,T+1}\right] = 0, \qquad M^C_{i,t,T} = \beta^{T-t}\frac{C_{i,t}}{C_{i,T}}. \tag{C.32}$$

The financial transversality restriction used in the first-order solution is stated in Section C.7.

C.3 Firms and Price Setting

Firm j in sector s produces

$$Y_{s,i,t}(j) = K_{s,i,t}(j)^{\xi} L_{s,i,t}(j)^{1-\xi}, \qquad \xi \in (0,1). \tag{C.33}$$

Firms rent capital within their sector and hire labor at the common regional wage. Nominal marginal cost is

$$\mathrm{MC}_{s,i,t} = \frac{(\mathcal{R}^K_{s,i,t})^{\xi} W_{i,t}^{1-\xi}}{\xi^{\xi}(1-\xi)^{1-\xi}}. \tag{C.34}$$

The factor-demand conditions are

$$\mathcal{R}^K_{s,i,t} K_{s,i,t}(j) = \xi\, \mathrm{MC}_{s,i,t} Y_{s,i,t}(j), \tag{C.35}$$

$$W_{i,t} L_{s,i,t}(j) = (1-\xi)\mathrm{MC}_{s,i,t} Y_{s,i,t}(j). \tag{C.36}$$

Real marginal cost for price setting is $mc_{s,i,t} = \mathrm{MC}_{s,i,t}/P_{s,i,t}$. Rental returns $r^K_{s,i,t}$ are instead measured in consumption units, as in the household's capital Euler equation.

In each sector, a fraction $1-\theta$ of firms can reset prices each period, with $\theta \in (0,1)$, while the remaining fraction keeps its previous nominal price. There is no indexation. A firm that

resets at t chooses $P^{\#}_{s,i,t}$ to maximize

$$\mathbb{E}_t \sum_{k=0}^{\infty} \theta^k \mathcal{Q}_{i,t,t+k} \left[P^{\#}_{s,i,t} - \mathrm{MC}_{s,i,t+k} \right] \left(\frac{P^{\#}_{s,i,t}}{P_{s,i,t+k}} \right)^{-\epsilon} Y_{s,i,t+k}, \tag{C.37}$$

where $\mathcal{Q}_{i,t,t} = 1$ and $\mathcal{Q}_{i,t,t+k}$ is the product of the intervening one-period nominal discount factors. A tradable firm sets one price for all destinations. The optimal reset price satisfies

$$P^{\#}_{s,i,t} = \mu \frac{F_{s,i,t}}{H_{s,i,t}}, \qquad \mu = \frac{\epsilon}{\epsilon - 1}, \tag{C.38}$$

with

$$F_{s,i,t} = \mathrm{MC}_{s,i,t} Y_{s,i,t} P^{\epsilon}_{s,i,t} + \theta \mathbb{E}_t[\mathcal{Q}_{i,t,t+1} F_{s,i,t+1}], \tag{C.39}$$

$$H_{s,i,t} = Y_{s,i,t} P^{\epsilon}_{s,i,t} + \theta \mathbb{E}_t[\mathcal{Q}_{i,t,t+1} H_{s,i,t+1}]. \tag{C.40}$$

The sectoral price index evolves according to

$$P^{1-\epsilon}_{s,i,t} = \theta P^{1-\epsilon}_{s,i,t-1} + (1-\theta)(P^{\#}_{s,i,t})^{1-\epsilon}. \tag{C.41}$$

Define price dispersion and gross sectoral inflation by

$$\Delta_{s,i,t} = \int_0^1 \left(\frac{P_{s,i,t}(j)}{P_{s,i,t}} \right)^{-\epsilon} \mathrm{d}j, \qquad \Pi_{s,i,t} = \frac{P_{s,i,t}}{P_{s,i,t-1}}. \tag{C.42}$$

Since firms within a sector face the same factor prices, aggregate production and the law of motion for dispersion are

$$Y_{s,i,t} = \frac{K^{\xi}_{s,i,t} L^{1-\xi}_{s,i,t}}{\Delta_{s,i,t}}, \tag{C.43}$$

$$\Delta_{s,i,t} = \theta \Pi^{\epsilon}_{s,i,t} \Delta_{s,i,t-1} + (1-\theta) \left(\frac{P^{\#}_{s,i,t}}{P_{s,i,t}} \right)^{-\epsilon}. \tag{C.44}$$

There is no production subsidy. Using variety demand, aggregate dividends can be written as

$$\mathcal{D}_{i,t} = \sum_{s \in \{T,N\}} \left[P_{s,i,t} Y_{s,i,t} - W_{i,t} L_{s,i,t} - \mathcal{R}^{K}_{s,i,t} K_{s,i,t} \right]. \tag{C.45}$$

C.4 Policy and Equilibrium

Public purchases use the consumption basket. Let $E^G_{i,t}$ denote nominal public expenditure and $\mathcal{G}_{i,t}$ the same expenditure in units of the union consumption price index:

$$E^G_{i,t} = P_{C,i,t} G_{i,t} = P^*_{C,t} \mathcal{G}_{i,t}, \qquad \mathcal{G}^U_t = \int_0^1 \mathcal{G}_{i,t} \, \mathrm{d}i. \tag{C.46}$$

The fiscal process is specified below; its calibration and the mapping to observed spending are described in Section 5. Total taxes are uniform across households, and the consolidated public budget balances each period:

$$\mathcal{T}_{i,t} = \mathcal{T}_t = P^*_{C,t} \mathcal{G}^U_t = \int_0^1 E^G_{h,t} \, \mathrm{d}h. \tag{C.47}$$

The net federal receipt of region i is

$$F_{i,t} = E^G_{i,t} - \mathcal{T}_t = P^*_{C,t} (\mathcal{G}_{i,t} - \mathcal{G}^U_t), \qquad \int_0^1 F_{i,t} \, \mathrm{d}i = 0. \tag{C.48}$$

There is no public debt. An equivalent accounting convention retains a uniform local tax at its steady-state level, $\bar{\mathcal{T}}^L = \bar{P}^*_C \bar{G}$, and uses federal transfers to finance changes in regional expenditure:

$$TR_{i,t} = E^G_{i,t} - \bar{\mathcal{T}}^L, \qquad \mathcal{T}^F_t = \int_0^1 TR_{h,t} \, \mathrm{d}h, \tag{C.49}$$

$$\mathcal{T}_{i,t} = \bar{\mathcal{T}}^L + \mathcal{T}^F_t, \qquad F_{i,t} = TR_{i,t} - \mathcal{T}^F_t. \tag{C.50}$$

The transfer $TR_{i,t}$ finances the regional government's purchases. It is not an additional payment in the household budget. Both representations imply the same uniform total taxes and the same net federal receipt.

The central bank follows

$$R^*_t = \beta^{-1} (\Pi^*_{C,t})^{\phi_\pi}, \qquad \Pi^*_{C,t} = \frac{P^*_{C,t}}{P^*_{C,t-1}}, \qquad \phi_\pi > 1. \tag{C.51}$$

Let $A^C_{i,t} = C_{i,t} + G_{i,t}$ denote consumption-type absorption. The nontradable goods market clears when

$$P_{N,i,t} Y_{N,i,t} = (1 - \nu_C) P_{C,i,t} A^C_{i,t} + (1 - \nu_I) P_{I,i,t} \mathcal{I}_{i,t}. \tag{C.52}$$

Tradable output supplies local home-biased demand and purchases through the union basket:

$$P_{T,i,t}Y_{T,i,t} = h_C P_{C,i,t}A^C_{i,t} + h_I P_{I,i,t}\mathcal{I}_{i,t} + \int_0^1 \left[\nu_C \alpha_C P_{C,h,t}A^C_{h,t} + \nu_I \alpha_I P_{I,h,t}\mathcal{I}_{h,t}\right] \mathrm{d}h. \tag{C.53}$$

The definitions of private and public expenditure imply

$$P_{C,i,t}A^C_{i,t} = P^*_{C,t}(C^*_t e^{\zeta_{i,t}} + \mathcal{G}_{i,t}). \tag{C.54}$$

Factor markets and the bond market satisfy

$$K_{s,i,t} = \int_0^1 K_{s,i,t}(j)\,\mathrm{d}j, \qquad L_{s,i,t} = \int_0^1 L_{s,i,t}(j)\,\mathrm{d}j, \tag{C.55}$$

$$L_{i,t} = L_{T,i,t} + L_{N,i,t}, \qquad \int_0^1 B_{i,t}\,\mathrm{d}i = 0. \tag{C.56}$$

Bond-market clearing holds at every date and after every history. Regional nominal output and the trade balance are

$$\mathrm{GDP}^{nom}_{i,t} = P_{T,i,t}Y_{T,i,t} + P_{N,i,t}Y_{N,i,t}, \tag{C.57}$$

$$NX_{i,t} = \mathrm{GDP}^{nom}_{i,t} - P_{C,i,t}A^C_{i,t} - P_{I,i,t}\mathcal{I}_{i,t}. \tag{C.58}$$

The goods markets imply $\int_0^1 NX_{i,t}\,\mathrm{d}i = 0$. Combining the household budget, dividends, and the public budget yields the regional financial identity

$$\frac{B_{i,t+1}}{R^*_t} - B_{i,t} = NX_{i,t} + F_{i,t}. \tag{C.59}$$

Thus bond purchases reflect the trade balance plus the net federal receipt. Integrating this identity is consistent with zero net bond supply.

Given initial capital stocks, initial price distributions, the fiscal process, and initial bond positions summing to zero, an equilibrium is a collection of household and firm allocations, bond holdings, prices, taxes, and federal transfers satisfying household optimality and transversality, assembler cost minimization, firm factor demand and optimal price setting, the policy rules, and market clearing. One initial nominal price normalization completes the description. The aggregate household budget and union resource constraint follow from these conditions and Walras' law.

C.5 Steady State

Consider a deterministic symmetric steady state with zero inflation. Normalize all sectoral and final-good prices to one. Price dispersion is one, and reset prices equal prevailing prices. Define

$$\bar{Y} = \bar{Y}_T + \bar{Y}_N, \qquad \bar{K} = \bar{K}_T + \bar{K}_N, \qquad \bar{\mathcal{I}} = \bar{\mathcal{I}}_T + \bar{\mathcal{I}}_N. \tag{C.60}$$

The steady-state public-expenditure share is $\gamma = \bar{G}/\bar{Y}$, so $\bar{\mathcal{G}}_i = \bar{\mathcal{G}}^U = \bar{G} = \gamma\bar{Y}$. The parameters γ and χ_G govern, respectively, the fiscal level and preferences.

Let $m = 1/\mu = (\epsilon - 1)/\epsilon$. Since $\Phi(\delta) = \delta$ and $\Phi'(\delta) = 1$, the investment conditions give

$$\bar{q}_T = \bar{q}_N = 1, \qquad \bar{r}_K = \beta^{-1} - 1 + \delta. \tag{C.61}$$

Both sectors have marginal cost m, and factor demand implies

$$\bar{r}_K = m\xi\frac{\bar{Y}_s}{\bar{K}_s}, \qquad \bar{w} = m(1-\xi)\frac{\bar{Y}_s}{\bar{L}_s}. \tag{C.62}$$

Thus

$$\frac{\bar{K}}{\bar{Y}} = \frac{m\xi}{\bar{r}_K}, \qquad \kappa_{KL} \equiv \frac{\bar{K}_s}{\bar{L}_s} = \left(\frac{m\xi}{\bar{r}_K}\right)^{1/(1-\xi)}. \tag{C.63}$$

The investment and private-consumption shares are

$$\iota \equiv \frac{\bar{\mathcal{I}}}{\bar{Y}} = \delta\frac{m\xi}{\bar{r}_K}, \qquad c_Y \equiv \frac{\bar{C}}{\bar{Y}} = 1 - \gamma - \iota > 0. \tag{C.64}$$

The sectoral goods markets determine the output shares:

$$\omega_T \equiv \frac{\bar{Y}_T}{\bar{Y}} = \nu_C(1-\iota) + \nu_I\iota, \tag{C.65}$$

$$\omega_N \equiv \frac{\bar{Y}_N}{\bar{Y}} = (1-\nu_C)(1-\iota) + (1-\nu_I)\iota. \tag{C.66}$$

Because technologies and factor ratios are identical across sectors,

$$\frac{\bar{Y}_s}{\bar{Y}} = \frac{\bar{K}_s}{\bar{K}} = \frac{\bar{L}_s}{\bar{L}} = \frac{\bar{\mathcal{I}}_s}{\bar{\mathcal{I}}} = \omega_s, \qquad s \in \{T, N\}. \tag{C.67}$$

Labor supply determines the scale of the economy:

$$\bar{L} = \left[\frac{(1-\chi_G)m(1-\xi)}{c_Y}\right]^{1/(1+\varphi)}, \tag{C.68}$$

$$\bar{Y} = \kappa_{KL}^{\xi}\bar{L}, \qquad \bar{K} = \kappa_{KL}\bar{L}, \tag{C.69}$$

$$\bar{\mathcal{I}} = \delta\bar{K}, \qquad \bar{C} = c_Y\bar{Y}, \tag{C.70}$$

$$\bar{G} = \gamma\bar{Y}, \qquad \bar{w} = m(1-\xi)\kappa_{KL}^{\xi}. \tag{C.71}$$

Aggregate dividends equal $(1-m)\bar{Y}$, taxes equal $\bar{G}$, and $\bar{R}^* = 1/\beta$. Symmetric net asset positions and trade balances are zero. The Calvo recursions satisfy

$$\bar{F}_s = \frac{m\bar{Y}_s}{1-\beta\theta}, \qquad \bar{H}_s = \frac{\bar{Y}_s}{1-\beta\theta}, \qquad \mu\frac{\bar{F}_s}{\bar{H}_s} = 1. \tag{C.72}$$

C.6 Log-Linear Equilibrium

For a positive variable $X_{i,t}$, let $\widehat{x}_{i,t} = \log(X_{i,t}/\bar{X})$. In particular, $\widehat{\iota}_{s,i,t}$ denotes the log deviation of $\mathcal{I}_{s,i,t}$; the unhatted ι remains the steady-state investment share. For regional log deviations, define

$$\widehat{x}_t^* = \int_0^1 \widehat{x}_{i,t}\,\mathrm{d}i, \qquad \widetilde{x}_{i,t} = \widehat{x}_{i,t} - \widehat{x}_t^*. \tag{C.73}$$

The following equilibrium conditions are first-order approximations unless identified as exact log identities. An equation indexed by s applies to both sectors.

Prices, expenditure, and the bond Euler equation. Let lower-case p denote a log price and define

$$s_{i,t} = p_{T,t}^* - p_{T,i,t}, \qquad z_{i,t} = p_{N,i,t} - p_{T,t}^*, \qquad \widetilde{z}_{i,t} = z_{i,t} - z_t^*. \tag{C.74}$$

Here $z_t^* = p_{N,t}^* - p_{T,t}^*$ and $\int_0^1 s_{i,t}\,\mathrm{d}i = 0$. The regional consumption-price deviation from the union index and the relative investment price satisfy

$$\psi_{i,t}^C \equiv p_{C,i,t} - p_{C,t}^* = (1-\nu_C)\widetilde{z}_{i,t} - h_C s_{i,t}, \tag{C.75}$$

$$\widehat{p}_{I/C,i,t} \equiv p_{I,i,t} - p_{C,i,t} = (\nu_C - \nu_I)z_{i,t} + (h_C - h_I)s_{i,t}. \tag{C.76}$$

Both are exact log identities. Their union and relative investment-price components are

$$\widehat{p}^{*}_{I/C,t} = (\nu_C - \nu_I) z^{*}_t, \qquad \widetilde{p}_{I/C,i,t} = (\nu_C - \nu_I)\widetilde{z}_{i,t} + (h_C - h_I) s_{i,t}. \tag{C.77}$$

Write the first-order fiscal decomposition as

$$\widehat{\mathcal{G}}_{i,t} = \widehat{\mathcal{G}}^{U}_t + u^{G}_{i,t}, \qquad \int_0^1 u^{G}_{i,t}\,\mathrm{d}i = 0. \tag{C.78}$$

At first order, the log deviation of the arithmetic aggregate $\mathcal{G}^{U}_t$ equals the average regional log deviation. The private-expenditure definition and the public spending rule imply

$$\widehat{c}_{i,t} = \widehat{c}^{*}_t + \zeta_{i,t} - \psi^{C}_{i,t}, \qquad \widehat{g}_{i,t} = \widehat{\mathcal{G}}^{U}_t + u^{G}_{i,t} - \psi^{C}_{i,t}, \tag{C.79}$$

where $\widehat{g}_{i,t}$ is the log deviation of physical public consumption. In consumption units, the log deviation of the stochastic discount factor is

$$\widehat{m}^{C}_{i,t,t+1} = \widehat{c}^{*}_t - \widehat{c}^{*}_{t+1} + \psi^{C}_{i,t+1} - \psi^{C}_{i,t} - (\zeta_{i,t+1} - \zeta_{i,t}). \tag{C.80}$$

Linearizing the nominal bond Euler equation and subtracting its union average gives

$$\zeta_{i,t} = \mathbb{E}_t \zeta_{i,t+1}. \tag{C.81}$$

Relative nominal private expenditure is therefore a martingale at first order. It may jump when information arrives. Its expected change is zero, but its realized change need not be zero.

Define expenditure in units of the union tradable price by

$$e^{A}_{i,t} = \frac{P_{C,i,t}(C_{i,t} + G_{i,t})}{P^{*}_{T,t}}, \qquad e^{I}_{i,t} = \frac{P_{I,i,t}\mathcal{I}_{i,t}}{P^{*}_{T,t}}, \tag{C.82}$$

and let $e^{A,U}_t$ and $e^{I,U}_t$ be their arithmetic union aggregates. With

$$\lambda_C = \frac{c_Y}{1-\iota}, \qquad \lambda_G = \frac{\gamma}{1-\iota}, \qquad \widehat{\iota}_{i,t} = \omega_T \widehat{\iota}_{T,i,t} + \omega_N \widehat{\iota}_{N,i,t}, \tag{C.83}$$

the union expenditures and regional deviations satisfy

$$\widehat{e}_t^{A,U} = (1-\nu_C)z_t^* + \lambda_C \widehat{c}_t^* + \lambda_G \widehat{\mathcal{G}}_t^U, \tag{C.84}$$

$$\widehat{e}_t^{I,U} = (1-\nu_I)z_t^* + \widehat{\iota}_t^*, \tag{C.85}$$

$$\widetilde{e}_{i,t}^{A} = \lambda_C \zeta_{i,t} + \lambda_G u_{i,t}^G, \tag{C.86}$$

$$\widetilde{e}_{i,t}^{I} = (1-\nu_I)\widetilde{z}_{i,t} - h_I s_{i,t} + \widetilde{\iota}_{i,t}. \tag{C.87}$$

The union and relative investment aggregates use the same weights ω_T and ω_N as in (C.83).

Capital valuation and price setting. Linearizing the capital law and installation condition gives

$$\widehat{k}_{s,i,t+1} = (1-\delta)\widehat{k}_{s,i,t} + \delta\widehat{\iota}_{s,i,t}, \tag{C.88}$$

$$\widehat{p}_{I/C,i,t} = \widehat{q}_{s,i,t} - \tau\delta(\widehat{\iota}_{s,i,t} - \widehat{k}_{s,i,t}). \tag{C.89}$$

To obtain the linear capital Euler equation, define $B(x) = 1 - \delta + \Phi(x) - x\Phi'(x)$. Since $B(\delta) = 1-\delta$, $B'(\delta) = \tau\delta$, and $x_{s,i,t} - \delta$ equals $\delta(\widehat{\iota}_{s,i,t} - \widehat{k}_{s,i,t})$ to first order,

$$\widehat{q}_{s,i,t} = \mathbb{E}_t\Big[\widehat{m}_{i,t,t+1}^C + \beta\bar{r}_K\widehat{r}_{s,i,t+1}^K + \beta(1-\delta)\widehat{q}_{s,i,t+1} + \beta\tau\delta^2(\widehat{\iota}_{s,i,t+1} - \widehat{k}_{s,i,t+1})\Big]. \tag{C.90}$$

Substituting the next-period installation condition yields the equivalent reduced form

$$\widehat{q}_{s,i,t} = \mathbb{E}_t\left[\widehat{m}_{i,t,t+1}^C + \beta\bar{r}_K\widehat{r}_{s,i,t+1}^K + \beta\widehat{q}_{s,i,t+1} - \beta\delta\widehat{p}_{I/C,i,t+1}\right]. \tag{C.91}$$

Let $\pi_{s,i,t} = p_{s,i,t} - p_{s,i,t-1}$. Linearization of the Calvo recursions and price index gives

$$\pi_{s,i,t} = \beta\mathbb{E}_t\pi_{s,i,t+1} + \kappa\widehat{mc}_{s,i,t}, \qquad \kappa = \frac{(1-\theta)(1-\beta\theta)}{\theta}. \tag{C.92}$$

The output and local nominal valuation-kernel terms cancel between the linearized numerator and denominator of the reset price. Price dispersion has no first-order variation around the symmetric zero-inflation steady state, so

$$\widehat{y}_{s,i,t} = \xi\widehat{k}_{s,i,t} + (1-\xi)\widehat{l}_{s,i,t}. \tag{C.93}$$

Union equilibrium. The zero means of $\zeta_{i,t}$ and the bond positions, together with the balanced aggregate public budget, preserve the union block at first order around the symmetric steady state. This equivalence does not impose complete risk sharing on regional allocations or assert exact aggregation away from the reference state.

Define the steady-state expenditure weights

$$\lambda_{NA} = \frac{(1-\nu_C)(1-\iota)}{\omega_N}, \qquad \lambda_{NI} = \frac{(1-\nu_I)\iota}{\omega_N}, \tag{C.94}$$

$$\lambda_{TA} = \frac{\nu_C(1-\iota)}{\omega_T}, \qquad \lambda_{TI} = \frac{\nu_I\iota}{\omega_T}. \tag{C.95}$$

Each pair sums to one. Factor demand implies the union rental returns and marginal costs

$$\widehat{r}^{K,*}_{s,t} = \widehat{w}^*_t + \frac{\widehat{y}^*_{s,t} - \widehat{k}^*_{s,t}}{1-\xi}, \tag{C.96}$$

$$\widehat{mc}^*_{T,t} = \widehat{w}^*_t + \frac{\xi}{1-\xi}(\widehat{y}^*_{T,t} - \widehat{k}^*_{T,t}) + (1-\nu_C)z^*_t, \tag{C.97}$$

$$\widehat{mc}^*_{N,t} = \widehat{w}^*_t + \frac{\xi}{1-\xi}(\widehat{y}^*_{N,t} - \widehat{k}^*_{N,t}) - \nu_C z^*_t. \tag{C.98}$$

Using these definitions, the union equilibrium is determined by

$$\widehat{c}^*_t = \mathbb{E}_t\widehat{c}^*_{t+1} - \left(\phi_\pi\pi^*_{C,t} - \mathbb{E}_t\pi^*_{C,t+1}\right), \tag{C.99}$$

$$\varphi(\omega_T\widehat{l}^*_{T,t} + \omega_N\widehat{l}^*_{N,t}) = \widehat{w}^*_t - \widehat{c}^*_t, \tag{C.100}$$

$$\widehat{y}^*_{s,t} = \xi\widehat{k}^*_{s,t} + (1-\xi)\widehat{l}^*_{s,t}, \tag{C.101}$$

$$\widehat{y}^*_{N,t} + z^*_t = \lambda_{NA}\widehat{e}^{A,U}_t + \lambda_{NI}\widehat{e}^{I,U}_t, \tag{C.102}$$

$$\widehat{y}^*_{T,t} = \lambda_{TA}\widehat{e}^{A,U}_t + \lambda_{TI}\widehat{e}^{I,U}_t, \tag{C.103}$$

$$\widehat{k}^*_{s,t+1} = (1-\delta)\widehat{k}^*_{s,t} + \delta\widehat{\iota}^*_{s,t}, \tag{C.104}$$

$$\widehat{p}^*_{I/C,t} = \widehat{q}^*_{s,t} - \tau\delta(\widehat{\iota}^*_{s,t} - \widehat{k}^*_{s,t}), \tag{C.105}$$

$$\widehat{q}^*_{s,t} = \mathbb{E}_t\Big[\widehat{c}^*_t - \widehat{c}^*_{t+1} + \beta\bar{r}_K\widehat{r}^{K,*}_{s,t+1} + \beta\widehat{q}^*_{s,t+1} - \beta\delta\widehat{p}^*_{I/C,t+1}\Big], \tag{C.106}$$

$$\pi^*_{C,t} = \beta\mathbb{E}_t\pi^*_{C,t+1} + \kappa\left[(1-\nu_C)\widehat{mc}^*_{N,t} + \nu_C\widehat{mc}^*_{T,t}\right], \tag{C.107}$$

$$z^*_t - z^*_{t-1} = \beta\mathbb{E}_t(z^*_{t+1} - z^*_t) + \kappa(\widehat{mc}^*_{N,t} - \widehat{mc}^*_{T,t}). \tag{C.108}$$

The final two equations are independent combinations of the sectoral Phillips curves. Home-bias parameters allocate tradable expenditure across origins and therefore do not enter the union block.

Regional equilibrium. The relative investment-expenditure term is $\widetilde{e}^I_{i,t} = \widetilde{\iota}_{i,t} + \psi^I_{i,t}$, where $\psi^I_{i,t} = (1-\nu_I)\widetilde{z}_{i,t} - h_I s_{i,t}$. Define the signed bond position in units of steady-state nominal output by

$$b_{i,t} = \frac{B_{i,t}}{\bar{P}^*_C \bar{Y}}. \tag{C.109}$$

The denominator is constant. Since the steady-state bond position is zero, $b_{i,t}$ is a level deviation, not a log deviation. Interest-rate and price deviations multiplying bond holdings are of second order around this reference state.

Subtracting the union equations from the regional conditions gives

$$\begin{aligned}
\omega_N(\widetilde{y}_{N,i,t} + \widetilde{z}_{i,t}) &= (1-\nu_C)(c_Y\zeta_{i,t} + \gamma u^G_{i,t}) \\
&\quad + (1-\nu_I)\iota\widetilde{e}^I_{i,t}, && \text{(C.110)} \\
\omega_T(\widetilde{y}_{T,i,t} - s_{i,t}) &= h_C(c_Y\zeta_{i,t} + \gamma u^G_{i,t}) \\
&\quad + h_I\iota\widetilde{e}^I_{i,t}, && \text{(C.111)} \\
\widetilde{y}_{s,i,t} &= \xi\widetilde{k}_{s,i,t} + (1-\xi)\widetilde{l}_{s,i,t}, && \text{(C.112)} \\
\widetilde{l}_{i,t} &= \omega_T\widetilde{l}_{T,i,t} + \omega_N\widetilde{l}_{N,i,t}, && \text{(C.113)} \\
\varphi\widetilde{l}_{i,t} &= \widetilde{w}_{i,t} + \psi^C_{i,t} - \zeta_{i,t}, && \text{(C.114)} \\
\widetilde{r}^K_{s,i,t} &= \widetilde{w}_{i,t} + \frac{\widetilde{y}_{s,i,t} - \widetilde{k}_{s,i,t}}{1-\xi}, && \text{(C.115)} \\
\widetilde{mc}_{T,i,t} &= \widetilde{w}_{i,t} + \frac{\xi}{1-\xi}(\widetilde{y}_{T,i,t} - \widetilde{k}_{T,i,t}) \\
&\quad + (1-\nu_C)\widetilde{z}_{i,t} + (1-h_C)s_{i,t}, && \text{(C.116)} \\
\widetilde{mc}_{N,i,t} &= \widetilde{w}_{i,t} + \frac{\xi}{1-\xi}(\widetilde{y}_{N,i,t} - \widetilde{k}_{N,i,t}) \\
&\quad - \nu_C\widetilde{z}_{i,t} - h_C s_{i,t}, && \text{(C.117)} \\
\widetilde{p}_{I/C,i,t} &= \widetilde{q}_{s,i,t} - \tau\delta(\widetilde{\iota}_{s,i,t} - \widetilde{k}_{s,i,t}), && \text{(C.118)} \\
\widetilde{k}_{s,i,t+1} &= (1-\delta)\widetilde{k}_{s,i,t} + \delta\widetilde{\iota}_{s,i,t}, && \text{(C.119)} \\
\widetilde{q}_{s,i,t} &= \mathbb{E}_t\Big[\psi^C_{i,t+1} - \psi^C_{i,t} + \beta\bar{r}_K\widetilde{r}^K_{s,i,t+1} \\
&\quad + \beta\widetilde{q}_{s,i,t+1} - \beta\delta\widetilde{p}_{I/C,i,t+1}\Big], && \text{(C.120)} \\
s_{i,t} - s_{i,t-1} &= \beta\mathbb{E}_t(s_{i,t+1} - s_{i,t}) - \kappa\widetilde{mc}_{T,i,t}, && \text{(C.121)} \\
\widetilde{z}_{i,t} - \widetilde{z}_{i,t-1} &= \beta\mathbb{E}_t(\widetilde{z}_{i,t+1} - \widetilde{z}_{i,t}) + \kappa\widetilde{mc}_{N,i,t}, && \text{(C.122)}
\end{aligned}$$

$$\zeta_{i,t} = \mathbb{E}_t \zeta_{i,t+1}, \tag{C.123}$$

$$\begin{aligned}\beta b_{i,t+1} - b_{i,t} = (1 - a_C)\gamma u^G_{i,t} - a_C c_Y \zeta_{i,t} \\ - a_I \iota \widetilde{e}^I_{i,t}.\end{aligned} \tag{C.124}$$

The common nominal policy rate drops out of the relative block. Equation (C.120) uses the bond Euler equation to eliminate the conditional expectation of $\zeta_{i,t+1} - \zeta_{i,t}$. Before this substitution, the regional discount-factor term is $\psi^C_{i,t+1} - \psi^C_{i,t} - (\zeta_{i,t+1} - \zeta_{i,t})$. Thus the reduced capital-valuation equations retain their familiar form, while expenditure, labor supply, and the financial constraint determine a different regional allocation.

To derive the financial equation, the goods markets give

$$\frac{NX_{i,t}}{\bar{P}^*_C \bar{Y}} = -a_C(c_Y \zeta_{i,t} + \gamma u^G_{i,t}) - a_I \iota \widetilde{e}^I_{i,t}, \tag{C.125}$$

at first order. Similarly, $F_{i,t}/(\bar{P}^*_C \bar{Y}) = \gamma u^G_{i,t}$. Substitution into (C.59) yields (C.124), where β is the steady-state price of the one-period bond. These accounting identities are implied by the household budget and goods markets; they are not additional independent restrictions.

Recovery of regional quantities and inflation. Regional log deviations are recovered by adding union and relative components. Consumption and public quantities follow from (C.79). Let $\Delta x_t = x_t - x_{t-1}$. Sectoral and final-use inflation are recovered from

$$\pi^*_{T,t} = \pi^*_{C,t} - (1 - \nu_C)\Delta z^*_t, \qquad \pi^*_{N,t} = \pi^*_{C,t} + \nu_C \Delta z^*_t, \tag{C.126}$$

$$\pi_{T,i,t} = \pi^*_{T,t} - \Delta s_{i,t}, \qquad \pi_{N,i,t} = \pi^*_{N,t} + \Delta \widetilde{z}_{i,t}, \tag{C.127}$$

$$\pi_{a,i,t} = (1 - \nu_a)\pi_{N,i,t} + h_a \pi_{T,i,t} + \nu_a \alpha_a \pi^*_{T,t}, \qquad a \in \{C, I\}. \tag{C.128}$$

In particular, $\pi_{C,i,t} - \pi^*_{C,t} = \Delta \psi^C_{i,t}$. Capital $\widehat{k}_{s,i,t}$ in all production and rental-return equations is the stock productive at t.

Implied identities. The two union goods markets imply the resource constraint

$$\omega_T \widehat{y}^*_{T,t} + \omega_N \widehat{y}^*_{N,t} = c_Y \widehat{c}^*_t + \gamma \widehat{\mathcal{G}}^U_t + \iota \widehat{\iota}^*_t. \tag{C.129}$$

Subtracting the two sectoral installation conditions and capital valuation equations gives

$$(\widetilde{\iota}_{N,i,t} - \widetilde{\iota}_{T,i,t}) - (\widetilde{k}_{N,i,t} - \widetilde{k}_{T,i,t}) = \frac{\widetilde{q}_{N,i,t} - \widetilde{q}_{T,i,t}}{\tau\delta}, \tag{C.130}$$

$$\widetilde{q}_{N,i,t} - \widetilde{q}_{T,i,t} = \beta\mathbb{E}_t\Big[\widetilde{q}_{N,i,t+1} - \widetilde{q}_{T,i,t+1} + \bar{r}_K(\widetilde{r}^K_{N,i,t+1} - \widetilde{r}^K_{T,i,t+1})\Big]. \tag{C.131}$$

The common investment-price and regional discount-factor terms cancel from these sectoral comparisons. These identities, the union resource constraint, and the alternative forms of the capital Euler equation are implied by the equilibrium system.

Fiscal states and the finite-region representation. The fiscal process specified in Section 5 completes the system. For its first-order AR(1) specification, with persistence $\rho_G \in [0, 1)$, averaging and subtracting the average yield

$$\widehat{\mathcal{G}}^U_t = \rho_G \widehat{\mathcal{G}}^U_{t-1} + \bar{\varepsilon}^G_t, \tag{C.132}$$

$$u^G_{i,t} = \rho_G u^G_{i,t-1} + \varepsilon^G_{i,t} - \bar{\varepsilon}^G_t, \qquad \bar{\varepsilon}^G_t = \int_0^1 \varepsilon^G_{i,t}\,\mathrm{d}i. \tag{C.133}$$

The regional innovations have zero conditional mean. The common innovation and regional deviations are obtained from the same regional innovations.

In the finite-region implementation, integrals across regions are replaced by equally weighted averages. If H_N has orthonormal columns orthogonal to the vector of ones $\mathbf{1}_N$, a vector of regional log deviations can be represented as

$$\mathbf{x}_t = \mathbf{1}_N \bar{x}_t + H_N \mathbf{x}^R_t, \qquad \bar{x}_t = \frac{1}{N}\mathbf{1}'_N \mathbf{x}_t, \qquad \mathbf{x}^R_t = H'_N \mathbf{x}_t. \tag{C.134}$$

The same transformation is applied to fiscal innovations and initial conditions. The union equations determine the common component, and the regional equations determine the $N-1$ contrast coordinates. Physical regional quantities follow by reconstruction. Bond positions and relative nominal expenditure are represented entirely in the contrast space:

$$\mathbf{b}_t = H_N \mathbf{b}^R_t, \qquad \boldsymbol{\zeta}_t = H_N \boldsymbol{\zeta}^R_t. \tag{C.135}$$

This enforces their zero means. Including the fiscal states, the system has 15 union equations and 21 equations per contrast, for a total of $15+21(N-1)$. The bond Euler equation repeated in the regional system is the same condition derived above; implied resource and accounting

identities are not counted as extra equations.

The tradable union basket includes all origins. Consequently, the effective local expenditure share within its tradable component is $1-\alpha_a+\alpha_a/N$; the share purchased from other regions in the full final-use basket is $\nu_a\alpha_a(1-1/N)$. This convention preserves a common union tradable price index and converges to the continuum sourcing shares as N increases. Regions retain the price-taking behavior of the continuum economy; the finite representation does not introduce strategic effects on union aggregates. Uniform federal taxation also means that each of the N observations bears $1/N$ of the aggregate tax increase generated by its own fiscal innovation.

C.7 Initial Conditions and Financial Solvency

The quantitative responses start from the symmetric steady state before the initial fiscal innovation. In deviations,

$$\widehat{k}^*_{s,0}=0, \qquad \widetilde{k}_{s,i,0}=0, \qquad b_{i,0}=0, \tag{C.136}$$

$$z^*_{-1}=0, \qquad s_{i,-1}=\widetilde{z}_{i,-1}=0, \qquad \widehat{\mathcal{G}}^U_{-1}=u^G_{i,-1}=0. \tag{C.137}$$

Capital productive at date zero and the bond payoff received at that date are inherited. Current reset prices, investment, capital valuations, and $\zeta_{i,0}$ are determined after observing the innovation. In particular, $b_{i,0}=0$ does not imply $\zeta_{i,0}=0$. Expectations are conditional on information available at the decision date. Setting subsequent realized innovations to zero when constructing an impulse response does not give households advance information about future surprises.

In the first-order approximation around zero bond holdings, the financial transversality condition is

$$\lim_{H\to\infty}\beta^{H+1}\mathbb{E}_t b_{i,t+H+1}=0. \tag{C.138}$$

The capital transversality conditions in (C.32) continue to apply. The financial condition permits a finite nonzero limiting bond position. It excludes the homogeneous explosive solution $b_{i,t+1}=b_{i,t}/\beta$ with nonzero amplitude.

Iterating the financial constraint and using (C.138) gives

$$0 = b_{i,t} + \sum_{h=0}^{\infty} \beta^h \mathbb{E}_t \Big[(1 - a_C)\gamma u^G_{i,t+h} - a_C c_Y \zeta_{i,t+h} - a_I \iota \widetilde{e}^I_{i,t+h} \Big]. \tag{C.139}$$

Since $\mathbb{E}_t \zeta_{i,t+h} = \zeta_{i,t}$ and $\mathbb{E}_t u^G_{i,t+h} = \rho_G^h u^G_{i,t}$, this becomes

$$\frac{a_C c_Y}{1-\beta} \zeta_{i,t} = b_{i,t} + \frac{(1 - a_C)\gamma}{1 - \beta\rho_G} u^G_{i,t} - a_I \iota \sum_{h=0}^{\infty} \beta^h \mathbb{E}_t \widetilde{e}^I_{i,t+h}. \tag{C.140}$$

The expenditure adjustment is determined jointly with investment: the last term depends on the real allocation induced by $\zeta_{i,t}$. The following subsections characterize the possible permanent component and then solve this condition, including the entire future transition.

C.8 The Regional Stationary Family

Consider the conditional long-run position after a transitory fiscal innovation, once the union real deviations, the relative fiscal state, and inflation have returned to zero. The regional stationary family is a first-order characterization around the symmetric reference state. The Phillips curves imply zero deviations of marginal costs, while accumulation and installation imply $\widetilde{\iota}_{s,i,\infty} = \widetilde{k}_{s,i,\infty}$ and $\widetilde{q}_{s,i,\infty} = \widetilde{p}_{I/C,i,\infty}$. Capital valuation then implies $\widetilde{r}^K_{s,i,\infty} = \widetilde{p}_{I/C,i,\infty}$.

Equal technologies and factor prices across sectors imply equality of the two sectoral producer prices within each region in this limit. At first order,

$$\widetilde{z}_{i,\infty} = -s_{i,\infty}. \tag{C.141}$$

Producer prices can still differ across regions, and the sectoral equality is not imposed during the transition.

Define the coefficients

$$\Omega_I = \frac{(1-a_C)c_Y}{1-(1-a_I)\iota}, \tag{C.142}$$

$$\Omega_P = \frac{1+\varphi\Omega_I}{(1+\varphi)\left[1+\dfrac{\xi a_I}{1-\xi}\right]}, \tag{C.143}$$

$$H_b = \frac{a_C c_Y + a_I \iota \Omega_I}{1-\beta}. \tag{C.144}$$

These are derived from the existing parameters. The stationary family is parametrized by $\zeta_{i,\infty}$:

$$\widetilde{e}^I_{i,\infty} = \Omega_I \zeta_{i,\infty}, \tag{C.145}$$

$$s_{i,\infty} = -\Omega_P \zeta_{i,\infty}, \qquad \widetilde{z}_{i,\infty} = \Omega_P \zeta_{i,\infty}, \tag{C.146}$$

$$\widetilde{c}_{i,\infty} = \left[1-(1-a_C)\Omega_P\right]\zeta_{i,\infty}, \tag{C.147}$$

$$b_{i,\infty} = H_b \zeta_{i,\infty}. \tag{C.148}$$

The total capital and investment deviations, aggregated with ω_T and ω_N, satisfy

$$\widetilde{k}_{i,\infty} = \widetilde{\iota}_{i,\infty} = \left[\Omega_I - (1-a_I)\Omega_P\right]\zeta_{i,\infty}. \tag{C.149}$$

The sectoral capital stocks are

$$\begin{aligned}\widetilde{k}_{N,i,\infty} = (1-a_I)s_{i,\infty} &+ \frac{(1-\nu_C)c_Y}{\omega_N}\zeta_{i,\infty} \\ &+ \frac{(1-\nu_I)\iota}{\omega_N}\widetilde{e}^I_{i,\infty},\end{aligned} \tag{C.150}$$

$$\begin{aligned}\widetilde{k}_{T,i,\infty} = (1-a_I)s_{i,\infty} &+ \frac{h_C c_Y}{\omega_T}\zeta_{i,\infty} \\ &+ \frac{h_I \iota}{\omega_T}\widetilde{e}^I_{i,\infty}.\end{aligned} \tag{C.151}$$

For each sector, the remaining quantities and valuations follow from

$$\widetilde{\iota}_{s,i,\infty} = \widetilde{k}_{s,i,\infty}, \tag{C.152}$$

$$\widetilde{y}_{s,i,\infty} = \widetilde{k}_{s,i,\infty} + a_I s_{i,\infty}, \tag{C.153}$$

$$\widetilde{l}_{s,i,\infty} = \widetilde{k}_{s,i,\infty} + \frac{a_I}{1-\xi} s_{i,\infty}, \tag{C.154}$$

$$\widetilde{w}_{i,\infty} = -\left(a_C + \frac{\xi a_I}{1-\xi}\right) s_{i,\infty}, \tag{C.155}$$

$$\widetilde{q}_{s,i,\infty} = \widetilde{r}^K_{s,i,\infty} = \widetilde{p}_{I/C,i,\infty} = (a_I - a_C) s_{i,\infty}. \tag{C.156}$$

To obtain Ω_I, summing the two regional goods markets at the stationary allocation gives

$$\widetilde{e}^I_{i,\infty} = (1-a_C) c_Y \zeta_{i,\infty} + (1-a_I) \iota \widetilde{e}^I_{i,\infty}. \tag{C.157}$$

Substitution into labor supply and the factor-price conditions yields Ω_P. The stationary financial equation gives H_b. These relations characterize the direction of the permanent adjustment; its amplitude is selected by the initial state and the present-value budget.

The regional positions sum to zero, so the persistent component represents a redistribution of financial claims within the union. It does not create net financial assets. Public expenditure in union-consumption units returns to its reference level, while regional physical public consumption can retain a relative deviation through the local price index.

C.9 Solution and Conditional Dynamics

For this subsection, suppress the region index and the tildes on regional contrasts. Let x_t collect the 18 real variables in the regional system: the two relative prices, the two capital stocks, investment by destination, the two capital valuations, sectoral output and labor, total labor, the wage, rental returns, and marginal costs. The real state at the decision date is

$$d_t = \begin{pmatrix} k_{T,t} & k_{N,t} & s_{t-1} & z_{t-1} & u_t \end{pmatrix}', \qquad f' = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \tag{C.158}$$

where u_t denotes the relative fiscal state after its current innovation. The bond payoff b_t is inherited separately. The first four entries of d_t are also inherited; current relative prices remain endogenous.

The conditional real subsystem. The coefficients on the real variables in the regional equations are those of the complete-markets real subsystem. Relative private expenditure

shifts its goods-market and labor-supply equations. Conditional on $\zeta = 0$, write the stable solution as

$$x_t = Ld_t, \qquad d_{t+1} = Ad_t + J\varepsilon^R_{t+1}, \qquad J = f, \tag{C.159}$$

where ε^R_{t+1} is the fiscal innovation of the contrast. Here L is 18×5, A is 5×5, and $f'A = \rho_G f'$. This representation requires a determined stable solution of the conditional real subsystem. It does not impose $\zeta = 0$ in the incomplete-markets equilibrium.

Let v_x collect the coefficients of the stationary family per unit of ζ, as derived in Section C.8, and let

$$v = \begin{pmatrix} v_{k_T} & v_{k_N} & v_s & v_z & 0 \end{pmatrix}'. \tag{C.160}$$

Thus $v_s = -\Omega_P$, $v_z = \Omega_P$, and $f'v = 0$. If ℓ is the row that recovers relative investment expenditure from the conditional real solution, then $e_{I,t} = \ell d_t$ when $\zeta = 0$. For the full equilibrium, define $\eta_t = d_t - v\zeta_t$. The real allocation and state transition can be written as

$$x_t = v_x\zeta_t + L\eta_t, \tag{C.161}$$

$$e_{I,t} = \Omega_I\zeta_t + \ell\eta_t, \tag{C.162}$$

$$d_{t+1} = v\zeta_t + A\eta_t + J\varepsilon^R_{t+1}. \tag{C.163}$$

Here $e_{I,t}$ is the same contrast denoted by $\widetilde{e}^I_{i,t}$ above. The shift applies to the initial conditions as well: if inherited capital is zero but its selected stationary deviation is nonzero, the corresponding entry of η_0 is the negative of that stationary deviation.

Present-value selection. For a stable A, define the row

$$\mathbf{r} = \ell(I - \beta A)^{-1}, \qquad \mathbf{r} - \beta\mathbf{r}A = \ell, \tag{C.164}$$

where I is the 5×5 identity matrix. The conditional investment-expenditure path then satisfies

$$\mathbb{E}_t e_{I,t+h} = \Omega_I\zeta_t + \ell A^h(d_t - v\zeta_t), \tag{C.165}$$

$$\sum_{h=0}^{\infty} \beta^h \mathbb{E}_t e_{I,t+h} = \frac{\Omega_I}{1-\beta}\zeta_t + \mathbf{r}(d_t - v\zeta_t). \tag{C.166}$$

This expression retains both the permanent component and the full stable transition. Define

$$\mathbf{n} = \frac{(1 - a_C)\gamma}{1 - \beta\rho_G} f' - a_I \iota \mathbf{r}, \tag{C.167}$$

$$D_\zeta = H_b - a_I \iota \mathbf{r} v. \tag{C.168}$$

The scalar D_ζ is a derived coefficient, distinct from dividends. Substituting the infinite sum into (C.140) gives

$$\zeta_t = \frac{b_t + \mathbf{n} d_t}{D_\zeta}, \qquad D_\zeta \neq 0. \tag{C.169}$$

Consequently, a fiscal innovation from the symmetric initial state implies

$$d_0 = J\varepsilon_0^R, \qquad b_0 = 0, \qquad \zeta_0 = \frac{\mathbf{n}J}{D_\zeta}\varepsilon_0^R. \tag{C.170}$$

The sign and magnitude of this adjustment follow from the equilibrium coefficients. Conditional on a determined stable real subsystem and $D_\zeta \neq 0$, the present-value condition selects a unique allocation of this form for a given state (d_t, b_t).

News and the permanent component. The coefficients satisfy

$$\mathbf{n} - \beta \mathbf{n} A = (1 - a_C)\gamma f' - a_I \iota \ell, \qquad D_\zeta = H_b + \mathbf{n} v. \tag{C.171}$$

Using these identities, the state transition, and the financial constraint gives

$$\zeta_{t+1} = \zeta_t + \frac{\mathbf{n}J}{D_\zeta}\varepsilon_{t+1}^R, \tag{C.172}$$

$$\eta_{t+1} = A\eta_t + \left(J - v\frac{\mathbf{n}J}{D_\zeta}\right)\varepsilon_{t+1}^R. \tag{C.173}$$

Since the innovation has zero conditional mean, the expenditure policy satisfies the bond Euler equation. No separate wealth innovation is introduced. Along a realized path with no further innovations, ζ_t stays at its post-news value and η_t converges to zero under A.

The financial position can be recovered from

$$b_t = D_\zeta \zeta_t - \mathbf{n} d_t = H_b \zeta_t - \mathbf{n}\eta_t. \tag{C.174}$$

It converges conditionally to $H_b\zeta_t$, satisfying financial transversality. This recovery also avoids propagating numerical errors through the explosive homogeneous component of the forward bond recursion. The current-account equation remains an equilibrium identity at

every date.

For any linear regional observable $o_t = o_\infty \zeta_t + \ell_o \eta_t$, the discounted tail from date $t + H$ is

$$\sum_{h=H}^{\infty} \beta^h \mathbb{E}_t o_{t+h} = \beta^H \left[\frac{o_\infty}{1-\beta} \zeta_t + \ell_o (I - \beta A)^{-1} A^H \eta_t \right]. \tag{C.175}$$

Thus the solution retains the infinite horizon analytically rather than selecting expenditure by fixing assets or real quantities at a finite terminal date. The admissible unit root in relative expenditure is distinct from the excluded financial root $1/\beta$.

The solution is local and of first order. Under recurrent innovations, the permanent component is updated by each new realization and need not converge along a realized path. The conditional stationary family and the first-order transversality condition do not establish a stationary distribution of wealth or global nonlinear solvency

D The Galí–Monacelli Model with Investment: Complete Markets and Local Fiscal Financing

This appendix presents the Galí–Monacelli model with investment, complete markets, and local fiscal financing introduced in Section 4.2. We follow Galí and Monacelli (2008) for the standard aggregation and price-setting relations, allowing for capital accumulation and different consumption and investment baskets. Households trade a complete set of state-contingent nominal claims, and each regional government finances its purchases of domestic goods with contemporaneous lump-sum taxes.

The union consists of a continuum of small regions indexed by $i \in [0,1]$. Capital $K_{i,t}$ is available for production at t, and investment $\mathcal{I}_{i,t}$ determines the stock available at $t+1$. Productivity is normalized to one. Hats denote log deviations from a symmetric steady state with zero inflation and unit relative prices; lower-case price symbols denote logarithms. Starred log deviations denote union averages.

D.1 Households

The representative household owns domestic firms and capital and maximizes

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[(1-\chi_G) \log C_{i,t} + \chi_G \log G_{i,t} - \frac{L_{i,t}^{1+\varphi}}{1+\varphi} \right], \tag{D.1}$$

where $\beta \in (0,1)$, $\chi_G \in [0,1)$, and $\varphi \geq 0$. Public consumption is taken as given. The nominal budget constraint is

$$\begin{aligned} P_{C,i,t} C_{i,t} + P_{I,i,t} \mathcal{I}_{i,t} + \mathbb{E}_t[\mathcal{Q}_{t,t+1} D_{i,t+1}] \\ \leq D_{i,t} + W_{i,t} L_{i,t} + P_{C,i,t} r_{i,t}^K K_{i,t} + \mathcal{D}_{i,t} - \mathcal{T}_{i,t}. \end{aligned} \tag{D.2}$$

Here $D_{i,t+1}$ is a state-contingent nominal payoff, $\mathcal{Q}_{t,t+1}$ is its common nominal pricing kernel, $r_{i,t}^K$ is the rental rate of capital in consumption units, and $\mathcal{D}_{i,t}$ denotes dividends, accounted for separately from $D_{i,t}$. Taxes are lump sum. The usual solvency and transversality conditions apply.

The consumption and investment baskets combine domestic and imported goods with unit elasticity of substitution. Their import expenditure shares are α_C and α_I, respectively.

The corresponding price indices are

$$P_{C,i,t} = P_{i,t}^{1-\alpha_C}(P_t^*)^{\alpha_C}, \qquad P_{I,i,t} = P_{i,t}^{1-\alpha_I}(P_t^*)^{\alpha_I}, \tag{D.3}$$

where P_t^* is the geometric union average of domestic producer prices. Writing $w_{i,t} = W_{i,t}/P_{C,i,t}$ and

$$M_{i,t,t+1}^C = \beta \frac{C_{i,t}}{C_{i,t+1}}, \qquad \mathcal{Q}_{t,t+1} = M_{i,t,t+1}^C \frac{P_{C,i,t}}{P_{C,i,t+1}}, \tag{D.4}$$

labor supply and the consumption Euler equation are

$$C_{i,t} L_{i,t}^{\varphi} = (1-\chi_G) w_{i,t}, \qquad 1 = R_t^* \mathbb{E}_t \left[M_{i,t,t+1}^C \frac{P_{C,i,t}}{P_{C,i,t+1}} \right], \tag{D.5}$$

where R_t^* is the common gross nominal interest rate.

Capital accumulation follows

$$K_{i,t+1} = (1-\delta) K_{i,t} + \mathcal{I}_{i,t} - \frac{\tau}{2} \left(\frac{\mathcal{I}_{i,t}}{K_{i,t}} - \delta \right)^2 K_{i,t}, \tag{D.6}$$

where $\delta \in (0,1)$ is the depreciation rate and $\tau \geq 0$ governs installation costs. Let $q_{i,t}$ be the value, in date-t consumption units, of capital installed for $t+1$, and define $x_{i,t} = \mathcal{I}_{i,t}/K_{i,t}$. Optimal investment and capital accumulation satisfy

$$\frac{P_{I,i,t}}{P_{C,i,t}} = q_{i,t} \left[1 - \tau(x_{i,t} - \delta) \right], \tag{D.7}$$

$$q_{i,t} = \mathbb{E}_t \Bigg[M_{i,t,t+1}^C \Bigg\{ r_{i,t+1}^K + q_{i,t+1} \Bigg[1 - \delta + \tau(x_{i,t+1} - \delta) x_{i,t+1} - \frac{\tau}{2} (x_{i,t+1} - \delta)^2 \Bigg] \Bigg\} \Bigg]. \tag{D.8}$$

D.2 Risk Sharing and Relative Prices

Define the terms of trade by $s_{i,t} = p_t^* - p_{i,t}$. Complete financial markets, with symmetric initial wealth and equal Pareto weights, imply

$$\widehat{c}_{i,t} = \widehat{c}_t^* + (1-\alpha_C) s_{i,t}. \tag{D.9}$$

The price indices give

$$p_{C,i,t} = p_{i,t} + \alpha_C s_{i,t}, \qquad p_{I,i,t} - p_{C,i,t} = (\alpha_I - \alpha_C) s_{i,t}. \tag{D.10}$$

Let $\pi_{i,t} = \Delta p_{i,t}$, $\pi_{C,i,t} = \Delta p_{C,i,t}$, and $\pi_t^* = \Delta p_t^*$. Then

$$\pi_{C,i,t} = \pi_{i,t} + \alpha_C \Delta s_{i,t}, \qquad \Delta s_{i,t} = \pi_t^* - \pi_{i,t}. \tag{D.11}$$

The geometric union averages of the consumption and investment price indices both equal P_t^*.

D.3 Firms

Firm $j \in [0,1]$ in region i produces

$$Y_{i,t}(j) = K_{i,t}(j)^{\xi} L_{i,t}(j)^{1-\xi}, \qquad \xi \in (0,1). \tag{D.12}$$

Firms rent capital and hire labor in competitive regional markets. Let $mc_{i,t}$ denote nominal marginal cost divided by the domestic producer price index. Cost minimization yields a common capital-labor ratio across firms and

$$w_{i,t} = mc_{i,t} \frac{P_{i,t}}{P_{C,i,t}} (1-\xi) \left(\frac{K_{i,t}}{L_{i,t}} \right)^{\xi}, \tag{D.13}$$

$$r_{i,t}^K = mc_{i,t} \frac{P_{i,t}}{P_{C,i,t}} \xi \left(\frac{K_{i,t}}{L_{i,t}} \right)^{\xi-1}. \tag{D.14}$$

The elasticity of substitution across varieties within each region is $\epsilon > 1$. There is no production subsidy. A fraction $\theta \in (0,1)$ of firms keeps its price unchanged each period. The first-order price-setting condition is

$$\pi_{i,t} = \beta \mathbb{E}_t \pi_{i,t+1} + \lambda \widehat{mc}_{i,t}, \qquad \lambda = \frac{(1-\theta)(1-\beta\theta)}{\theta}. \tag{D.15}$$

Marginal cost in this equation is measured in producer-price units; factor payments above are measured in consumption units.

D.4 Government and Monetary Policy

The regional government purchases domestic goods and finances them with contemporaneous lump-sum taxes on the local household:

$$P_{i,t}G_{i,t} = P_t^*\mathcal{G}_{i,t} = \mathcal{T}_{i,t}. \tag{D.16}$$

Hence public quantities and expenditure satisfy

$$\widehat{g}_{i,t} = \widehat{\mathcal{G}}_{i,t} + s_{i,t}, \qquad \widehat{g}_t^* = \widehat{\mathcal{G}}_t^*. \tag{D.17}$$

Here $\widehat{\mathcal{G}}_t^* = \int_0^1 \widehat{\mathcal{G}}_{i,t}\,\mathrm{d}i$. Section 5 specifies the fiscal processes and the information available to households in each experiment.

Monetary policy follows

$$r_t^* = \phi_\pi \pi_t^*, \qquad r_t^* \equiv \log R_t^* - \log(1/\beta). \tag{D.18}$$

D.5 Log-Linear Equilibrium

Let $\gamma = \bar{G}/\bar{Y}$ denote the steady-state public expenditure share. The symmetric steady state satisfies

$$\overline{mc} = \frac{\epsilon - 1}{\epsilon}, \qquad \bar{q} = 1, \qquad \bar{r}^K = \frac{1}{\beta} - 1 + \delta. \tag{D.19}$$

Define the investment share and the consumption share of private absorption by

$$\iota \equiv \frac{\bar{\mathcal{I}}}{\bar{Y}} = \frac{\delta\xi\,\overline{mc}}{\bar{r}^K}, \qquad \gamma_C \equiv \frac{\bar{C}}{\bar{C} + \bar{\mathcal{I}}} = \frac{1-\gamma-\iota}{1-\gamma}. \tag{D.20}$$

Parameters satisfy $1 - \gamma - \iota > 0$. The remaining steady-state quantities follow from production, household labor supply, $\bar{\mathcal{I}} = \delta\bar{K}$, and $\bar{C} = (1-\gamma-\iota)\bar{Y}$. In particular, dividends equal $(1 - \overline{mc})\bar{Y}$. The parameter γ fixes the steady-state share, while public expenditure away from steady state follows the processes specified above.

Write $\widehat{\iota}_{i,t} = \log(\mathcal{I}_{i,t}/\bar{\mathcal{I}})$, reserving the unhatted ι for its steady-state output share. Pro-

duction and factor-market equilibrium imply

$$\widehat{y}_{i,t} = \xi\widehat{k}_{i,t} + (1-\xi)\widehat{\ell}_{i,t}, \tag{D.21}$$

$$\widehat{mc}_{i,t} = \widehat{c}_{i,t} + (1+\varphi)\widehat{\ell}_{i,t} - \widehat{y}_{i,t} + \alpha_C s_{i,t}, \tag{D.22}$$

$$\widehat{r}^K_{i,t} = \widehat{mc}_{i,t} - \alpha_C s_{i,t} + \widehat{y}_{i,t} - \widehat{k}_{i,t}. \tag{D.23}$$

Using the next-period installation condition to simplify the linearized capital Euler equation, the investment block is

$$\widehat{q}_{i,t} = (\alpha_I - \alpha_C)s_{i,t} + \tau\delta(\widehat{\iota}_{i,t} - \widehat{k}_{i,t}), \tag{D.24}$$

$$\widehat{k}_{i,t+1} = (1-\delta)\widehat{k}_{i,t} + \delta\widehat{\iota}_{i,t}, \tag{D.25}$$

$$\begin{aligned}\widehat{q}_{i,t} = {} & \widehat{c}_{i,t} - \mathbb{E}_t\widehat{c}_{i,t+1} + \beta\bar{r}^K\mathbb{E}_t\widehat{r}^K_{i,t+1} \\ & + \beta\mathbb{E}_t\widehat{q}_{i,t+1} - \beta\delta(\alpha_I - \alpha_C)\mathbb{E}_t s_{i,t+1}.\end{aligned} \tag{D.26}$$

Goods-market clearing requires

$$\begin{aligned}\widehat{y}_{i,t} = (1-\gamma)\Big\{ & \gamma_C\widehat{c}_{i,t} + (1-\gamma_C)\big[(1-\alpha_I)\widehat{\iota}_{i,t} + \alpha_I\widehat{\iota}^*_t\big] \\ & + \big[\alpha_C\gamma_C + \alpha_I(2-\alpha_I)(1-\gamma_C)\big]s_{i,t}\Big\} + \gamma\widehat{g}_{i,t}.\end{aligned} \tag{D.27}$$

Together with risk sharing, the inflation identities, the Phillips curve, and the fiscal quantity equation, these conditions determine each region's allocation given union-wide aggregates.

The union satisfies the starred counterparts of equations (D.21)–(D.26) and the Phillips curve (D.15), with the terms-of-trade terms set to zero. Union-wide goods-market clearing and the consumption Euler equation are

$$\widehat{y}^*_t = (1-\gamma)\big[\gamma_C\widehat{c}^*_t + (1-\gamma_C)\widehat{\iota}^*_t\big] + \gamma\widehat{g}^*_t, \tag{D.28}$$

$$\widehat{c}^*_t = \mathbb{E}_t\widehat{c}^*_{t+1} - \big(r^*_t - \mathbb{E}_t\pi^*_{t+1}\big). \tag{D.29}$$

These equations, the monetary rule, and the common fiscal process close the union block. The regional consumption Euler equation follows from the union Euler equation, risk sharing, and the inflation identities. Initial capital stocks and relative prices, together with the transversality conditions, complete the equilibrium specification.

E The Galí–Monacelli Model with Investment: Incomplete Markets and Federal Fiscal Financing

This appendix presents the Galí–Monacelli model with investment, incomplete markets, and federal fiscal financing introduced in Section 4.2. We retain the production and trade structure of Galí and Monacelli (2008), augmented with capital accumulation and distinct consumption and investment baskets. Households own domestic firms and capital and trade a one-period noncontingent nominal bond. A balanced federal system finances regional public purchases through uniform lump-sum taxes. We derive the demand system without imposing complete risk sharing and explain how the household budget and a present-value condition jointly determine relative expenditure and financial positions.

The union consists of a continuum of small regions indexed by $i \in [0,1]$, each of which takes union aggregates as given. Varieties within a region are indexed by $j \in [0,1]$. Capital $K_{i,t}$ is available for production at t, and investment $\mathcal{I}_{i,t}$ determines the stock available at $t+1$. Productivity is normalized to one. Hats denote log deviations from a symmetric steady state with zero inflation and unit prices; lower-case price symbols denote logarithms. The comparison with Appendix D changes fiscal financing and financial markets jointly.

E.1 Final-Good Aggregation and Demand

Competitive assemblers combine varieties from region i according to

$$Y_{i,t} = \left[\int_0^1 Y_{i,t}(j)^{(\epsilon-1)/\epsilon}\,\mathrm{d}j\right]^{\epsilon/(\epsilon-1)}, \qquad \epsilon > 1. \tag{E.1}$$

The corresponding price index and variety demand are

$$P_{i,t} = \left[\int_0^1 P_{i,t}(j)^{1-\epsilon}\,\mathrm{d}j\right]^{1/(1-\epsilon)}, \tag{E.2}$$

$$Y_{i,t}(j) = \left(\frac{P_{i,t}(j)}{P_{i,t}}\right)^{-\epsilon} Y_{i,t}. \tag{E.3}$$

The law of one price holds across destinations. Define the geometric union price index

$$P_t^* = \exp\left\{\int_0^1 \log P_{i,t}\,\mathrm{d}i\right\}. \tag{E.4}$$

For final use $a \in \{C, I\}$, write $X^C_{i,t} = C_{i,t}$ and $X^I_{i,t} = \mathcal{I}_{i,t}$. The domestic and imported composites enter the normalized Cobb–Douglas basket

$$X^a_{i,t} = \frac{(X^a_{H,i,t})^{1-\alpha_a}(X^a_{F,i,t})^{\alpha_a}}{(1-\alpha_a)^{1-\alpha_a}\alpha_a^{\alpha_a}}, \qquad X^a_{F,i,t} = \exp\left\{\int_0^1 \log X^a_{h\to i,t}\,\mathrm{d}h\right\}. \tag{E.5}$$

Here H denotes domestic goods and $X^a_{h\to i,t}$ denotes purchases from origin h through the imported basket. Each origin's composite has the same within-region elasticity ϵ. The import expenditure shares satisfy $\alpha_C \in (0,1)$ and $\alpha_I \in [0,1]$, with boundary baskets interpreted by continuity. Including the home region in the imported basket is immaterial in the continuum economy.

Cost minimization implies

$$P_{a,i,t} = P_{i,t}^{1-\alpha_a}(P^*_t)^{\alpha_a}, \tag{E.6}$$

$$P_{i,t}X^a_{H,i,t} = (1-\alpha_a)P_{a,i,t}X^a_{i,t}, \qquad P^*_t X^a_{F,i,t} = \alpha_a P_{a,i,t}X^a_{i,t}. \tag{E.7}$$

Within the imported basket, $P_{h,t}X^a_{h\to i,t} = P^*_t X^a_{F,i,t}$ in expenditure-density terms. Public purchases consist entirely of domestic goods, so their price is $P_{i,t}$.

Define nominal private expenditures and their arithmetic union aggregates by

$$E^C_{i,t} = P_{C,i,t}C_{i,t}, \qquad E^I_{i,t} = P_{I,i,t}\mathcal{I}_{i,t}, \qquad E^{a,U}_t = \int_0^1 E^a_{i,t}\,\mathrm{d}i. \tag{E.8}$$

Summing domestic and imported demand for region i gives its goods-market condition in levels:

$$\begin{aligned} P_{i,t}Y_{i,t} = {} & (1-\alpha_C)E^C_{i,t} + (1-\alpha_I)E^I_{i,t} \\ & + \alpha_C E^{C,U}_t + \alpha_I E^{I,U}_t + P_{i,t}G_{i,t}. \end{aligned} \tag{E.9}$$

This equation holds without complete risk sharing. Net exports, defined as nominal production less domestic absorption, therefore satisfy

$$\begin{aligned} NX_{i,t} &= P_{i,t}Y_{i,t} - E^C_{i,t} - E^I_{i,t} - P_{i,t}G_{i,t} \\ &= -\alpha_C(E^C_{i,t} - E^{C,U}_t) - \alpha_I(E^I_{i,t} - E^{I,U}_t). \end{aligned} \tag{E.10}$$

Public purchases enter both local sales and local absorption, so they cancel directly in this identity. They affect net exports through equilibrium private expenditure. Integrating over regions gives zero union net exports.

E.2 Households and Capital Accumulation

The representative household maximizes

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[(1-\chi_G) \log C_{i,t} + \chi_G \log G_{i,t} - \frac{L_{i,t}^{1+\varphi}}{1+\varphi} \right], \tag{E.11}$$

where $\beta \in (0,1)$, $\chi_G \in [0,1)$, and $\varphi \geq 0$. Public consumption is taken as given. The household owns domestic firms and capital; ownership claims are not traded across regions. Its nominal budget constraint is

$$\begin{aligned} P_{C,i,t} C_{i,t} + P_{I,i,t} \mathcal{I}_{i,t} + \frac{B_{i,t+1}}{R_t^*} \\ = B_{i,t} + W_{i,t} L_{i,t} + P_{C,i,t} r_{i,t}^K K_{i,t} + \mathcal{D}_{i,t} - \mathcal{T}_{i,t}. \end{aligned} \tag{E.12}$$

Here $r_{i,t}^K$ is the rental rate in consumption units, $\mathcal{D}_{i,t}$ denotes dividends after factor payments, and $\mathcal{T}_{i,t}$ is the total lump-sum tax liability. The bond payoff $B_{i,t}$ is inherited, with positive values denoting creditors. The household purchases $B_{i,t+1}$ at price $1/R_t^*$ after observing date-t information. Its nominal payoff cannot depend on the innovation realized at $t+1$. The bond carries no holding cost or premium and is the only financial asset traded across regions.

Writing $w_{i,t} = W_{i,t}/P_{C,i,t}$, labor supply is

$$C_{i,t} L_{i,t}^{\varphi} = (1-\chi_G) w_{i,t}. \tag{E.13}$$

The local owner's consumption and nominal valuation kernels are

$$M_{i,t,t+1}^C = \beta \frac{C_{i,t}}{C_{i,t+1}}, \qquad \mathcal{Q}_{i,t,t+1} = M_{i,t,t+1}^C \frac{P_{C,i,t}}{P_{C,i,t+1}}. \tag{E.14}$$

The bond Euler equation requires

$$1 = R_t^* \mathbb{E}_t \mathcal{Q}_{i,t,t+1}. \tag{E.15}$$

Thus the conditional expectation of the nominal kernel is common, while its realization need not coincide across regions. Firms use the valuation kernel of their local owner.

Define geometric union consumption and relative nominal consumption expenditure by

$$C_t^* = \exp\left\{\int_0^1 \log C_{i,t}\,\mathrm{d}i\right\}, \qquad \zeta_{i,t} = \log\left(\frac{P_{C,i,t}C_{i,t}}{P_t^* C_t^*}\right). \tag{E.16}$$

The geometric union average of $P_{C,i,t}$ equals P_t^*, so $\int_0^1 \zeta_{i,t}\,\mathrm{d}i = 0$. With $s_{i,t} = \log(P_t^*/P_{i,t})$, the exact consumption allocation and expenditure aggregate are

$$C_{i,t} = C_t^* \exp\{\zeta_{i,t} + (1-\alpha_C)s_{i,t}\}, \tag{E.17}$$

$$E_t^{C,U} = P_t^* C_t^* \int_0^1 \exp(\zeta_{i,t})\,\mathrm{d}i. \tag{E.18}$$

Equality of $E_t^{C,U}$ and $P_t^* C_t^*$ holds to first order around symmetry. Complete risk sharing with equal Pareto weights would impose $\zeta_{i,t} = 0$ in every region. Here it is determined by the household's intertemporal budget.

Capital accumulation follows

$$K_{i,t+1} = [1 - \delta + \Phi(x_{i,t})]K_{i,t}, \qquad x_{i,t} = \frac{\mathcal{I}_{i,t}}{K_{i,t}}, \qquad \Phi(x) = x - \frac{\tau}{2}(x-\delta)^2, \tag{E.19}$$

with $\delta \in (0,1)$ and $\tau > 0$. Let $q_{i,t}$ value capital installed for $t+1$ in date-t consumption units. The investment and capital conditions are

$$\frac{P_{I,i,t}}{P_{C,i,t}} = q_{i,t}\Phi'(x_{i,t}), \tag{E.20}$$

$$q_{i,t} = \mathbb{E}_t\Big\{M_{i,t,t+1}^C\Big[r_{i,t+1}^K + q_{i,t+1}\big(1-\delta + \Phi(x_{i,t+1}) - x_{i,t+1}\Phi'(x_{i,t+1})\big)\Big]\Big\}. \tag{E.21}$$

The term $1-\delta+\Phi(x)-x\Phi'(x)$ is the derivative of next-period capital with respect to current capital, holding investment fixed. The household's plan satisfies the no-Ponzi condition on bond positions and the capital transversality condition

$$\lim_{T\to\infty} \mathbb{E}_t[M_{i,t,T}^C q_{i,T} K_{i,T+1}] = 0, \qquad M_{i,t,T}^C = \beta^{T-t}\frac{C_{i,t}}{C_{i,T}}. \tag{E.22}$$

Section E.7 states the financial transversality restriction used in the first-order solution.

E.3 Firms and Price Setting

Firm j in region i produces

$$Y_{i,t}(j) = K_{i,t}(j)^{\xi} L_{i,t}(j)^{1-\xi}, \qquad \xi \in (0,1). \tag{E.23}$$

Capital and labor are allocated through competitive regional factor markets. Nominal marginal cost $\mathrm{MC}_{i,t}$ is common across firms. Writing $mc_{i,t} = \mathrm{MC}_{i,t}/P_{i,t}$, cost minimization implies

$$w_{i,t} = mc_{i,t} \frac{P_{i,t}}{P_{C,i,t}} (1-\xi) \left(\frac{K_{i,t}}{L_{i,t}} \right)^{\xi}, \tag{E.24}$$

$$r^K_{i,t} = mc_{i,t} \frac{P_{i,t}}{P_{C,i,t}} \xi \left(\frac{K_{i,t}}{L_{i,t}} \right)^{\xi-1}. \tag{E.25}$$

There is no production subsidy. Aggregate dividends satisfy

$$\mathcal{D}_{i,t} = P_{i,t} Y_{i,t} - W_{i,t} L_{i,t} - P_{C,i,t} r^K_{i,t} K_{i,t}. \tag{E.26}$$

A fraction $\theta \in (0,1)$ of firms retains its price each period. If $P^{\#}_{i,t}$ is the reset price, demand while that price remains in place is

$$Y_{i,t+h|t} = \left(\frac{P^{\#}_{i,t}}{P_{i,t+h}} \right)^{-\epsilon} Y_{i,t+h}. \tag{E.27}$$

The reset-price condition is

$$\sum_{h=0}^{\infty} \theta^h \mathbb{E}_t \left[\mathcal{Q}_{i,t,t+h} Y_{i,t+h|t} \left(P^{\#}_{i,t} - \frac{\epsilon}{\epsilon - 1} \mathrm{MC}_{i,t+h} \right) \right] = 0, \tag{E.28}$$

where $\mathcal{Q}_{i,t,t+h} = \beta^h P_{C,i,t} C_{i,t} / (P_{C,i,t+h} C_{i,t+h})$ and $\mathcal{Q}_{i,t,t} = 1$. The producer price index obeys

$$P^{1-\epsilon}_{i,t} = \theta P^{1-\epsilon}_{i,t-1} + (1-\theta)(P^{\#}_{i,t})^{1-\epsilon}. \tag{E.29}$$

With price dispersion $\Delta_{i,t} = \int_0^1 (P_{i,t}(j)/P_{i,t})^{-\epsilon} \, \mathrm{d}j$, aggregate production satisfies

$$\Delta_{i,t} Y_{i,t} = K^{\xi}_{i,t} L^{1-\xi}_{i,t}. \tag{E.30}$$

Dispersion has no first-order perturbation around the symmetric zero-inflation allocation

with an initially symmetric price distribution. Writing $\pi_{i,t} = \log P_{i,t} - \log P_{i,t-1}$, linearizing price setting yields

$$\pi_{i,t} = \beta \mathbb{E}_t \pi_{i,t+1} + \lambda \widehat{mc}_{i,t}, \qquad \lambda = \frac{(1-\theta)(1-\beta\theta)}{\theta}. \tag{E.31}$$

The direct perturbation of the local valuation kernel multiplies a zero reset-price residual at the steady state and therefore drops out of this first-order equation. This does not impose a common state-contingent kernel. Marginal cost is measured in producer-price units, while wages and capital rentals above are measured in consumption units.

E.4 Fiscal Financing, Monetary Policy, and Equilibrium

Public expenditure in units of the union price index is denoted by $\mathcal{G}_{i,t}$:

$$P_{i,t} G_{i,t} = P_t^* \mathcal{G}_{i,t}, \qquad \mathcal{G}_t^U = \int_0^1 \mathcal{G}_{i,t} \, \mathrm{d}i. \tag{E.32}$$

To describe federal financing of changes in nominal expenditure, retain the constant reference local tax $\bar{\mathcal{T}}^L = \bar{P}^* \bar{\mathcal{G}}$. The gross transfer to the regional government and the uniform federal household tax are

$$TR_{i,t} = P_t^* \mathcal{G}_{i,t} - \bar{\mathcal{T}}^L, \qquad \mathcal{T}_t^F = \int_0^1 TR_{i,t} \, \mathrm{d}i. \tag{E.33}$$

The regional budget is $P_{i,t} G_{i,t} = \bar{\mathcal{T}}^L + TR_{i,t}$, and the federal budget balances each period. Total taxes and the net federal inflow to region i satisfy

$$\mathcal{T}_{i,t} = \bar{\mathcal{T}}^L + \mathcal{T}_t^F = P_t^* \mathcal{G}_t^U, \tag{E.34}$$

$$\mathcal{F}_{i,t} = TR_{i,t} - \mathcal{T}_t^F = P_t^* (\mathcal{G}_{i,t} - \mathcal{G}_t^U), \qquad \int_0^1 \mathcal{F}_{i,t} \, \mathrm{d}i = 0. \tag{E.35}$$

Transfers may be signed. Each household pays the same nominal tax, including changes in the common price level. An individual infinitesimal region takes that tax as given. The gross transfer finances the regional government and is not added again to the private budget.

Using the dividend identity and the goods market, the household budget becomes

$$\frac{B_{i,t+1}}{R_t^*} - B_{i,t} = NX_{i,t} + \mathcal{F}_{i,t}. \tag{E.36}$$

This is a consolidated form of the same private budget, not an additional independent

restriction. Bond-market clearing requires

$$\int_0^1 B_{i,t}\,\mathrm{d}i = 0. \tag{E.37}$$

There is no public debt or external net asset position for the union.

The central bank follows

$$R_t^* = \beta^{-1}\left(\frac{P_t^*}{P_{t-1}^*}\right)^{\phi_\pi}, \qquad \phi_\pi > 1. \tag{E.38}$$

The fiscal process used in the first-order equilibrium is

$$\widehat{\mathcal{G}}_{i,t} = \rho_G \widehat{\mathcal{G}}_{i,t-1} + \varepsilon_{i,t}^G, \qquad \rho_G \in [0,1), \qquad \mathbb{E}_t \varepsilon_{i,t+1}^G = 0. \tag{E.39}$$

Section 5 specifies the innovations, calibration, and measurement in the military-spending exercise. Innovations arrive as unanticipated news; independence of innovations does not imply zero persistence of expenditure.

Given initial productive capital, bond payoffs summing to zero, inherited prices, the fiscal state, and a common nominal price normalization, equilibrium satisfies household optimality and transversality, firm optimality, assembler demand, the policy rules, and goods, factor, and bond-market clearing. Integrating the regional budgets recovers the union budget. Initial price distributions are symmetric for the first-order experiments.

E.5 The Symmetric Steady State

Normalize steady-state prices to one and set $\bar{B}_i = 0$. Let $\gamma = \bar{G}/\bar{Y} \in (0,1)$, and define

$$m = \overline{mc} = \frac{\epsilon - 1}{\epsilon}, \qquad \bar{r}^K = \frac{1}{\beta} - 1 + \delta, \qquad \bar{q} = 1, \tag{E.40}$$

$$\iota = \frac{\bar{\mathcal{I}}}{\bar{Y}} = \frac{\delta m \xi}{\bar{r}^K}, \qquad c_Y = \frac{\bar{C}}{\bar{Y}} = 1 - \gamma - \iota > 0. \tag{E.41}$$

The investment share ι is an unhatted constant; the log investment deviation below is denoted by $\widehat{\iota}_{i,t}$. The private-consumption share used in Appendix D is $\gamma_C = c_Y/(1-\gamma)$.

The factor-price and labor-supply conditions give

$$\kappa_{KL} = \left(\frac{m\xi}{\bar{r}^K}\right)^{1/(1-\xi)}, \qquad \bar{L} = \left[\frac{(1-\chi_G)m(1-\xi)}{c_Y}\right]^{1/(1+\varphi)}. \tag{E.42}$$

The remaining quantities are

$$\bar{K} = \kappa_{KL}\bar{L}, \qquad \bar{Y} = \kappa_{KL}^{\xi}\bar{L}, \qquad \bar{\mathcal{I}} = \delta\bar{K}, \tag{E.43}$$

$$\bar{C} = c_Y\bar{Y}, \qquad \bar{G} = \bar{\mathcal{G}} = \gamma\bar{Y}, \qquad \bar{\mathcal{D}} = (1-m)\bar{Y}. \tag{E.44}$$

Also $\bar{R}^* = 1/\beta$, $\bar{\Delta} = 1$, and $\bar{\mathcal{T}} = \bar{\mathcal{T}}^L = \gamma\bar{Y}$. Gross federal transfers and the federal tax increment are zero at this reference. These are the same symmetric real quantities as in the complete-markets specification; the financial closure determines how regions depart from that reference.

E.6 Log-Linear Equilibrium

For any positive quantity or real valuation, a star on its log deviation denotes its union average, and a tilde denotes its deviation from that average. For example, $\widehat{k}_{i,t} = \widehat{k}^*_t + \widetilde{k}_{i,t}$. In particular,

$$\widehat{\iota}_{i,t} = \log(\mathcal{I}_{i,t}/\bar{\mathcal{I}}), \qquad u_{i,t} = \widehat{\mathcal{G}}_{i,t} - \widehat{\mathcal{G}}^U_t, \qquad b_{i,t} = \frac{B_{i,t}}{\bar{P}^*\bar{Y}}. \tag{E.45}$$

Here $\widehat{\mathcal{G}}^U_t = \int_0^1 \widehat{\mathcal{G}}_{i,t}\,\mathrm{d}i$ is the first-order deviation of the arithmetic fiscal aggregate. Assets are scaled in levels, since their reference position is zero.

The price indices imply the exact log-price relations

$$p_{C,i,t} - p^*_t = -(1-\alpha_C)s_{i,t}, \qquad p_{I,i,t} - p^*_t = -(1-\alpha_I)s_{i,t}, \tag{E.46}$$

$$p_{IC,i,t} \equiv p_{I,i,t} - p_{C,i,t} = (\alpha_I - \alpha_C)s_{i,t}, \tag{E.47}$$

$$\pi_{i,t} = \pi^*_t - \Delta s_{i,t}, \qquad \pi_{C,i,t} = \pi^*_t - (1-\alpha_C)\Delta s_{i,t}, \tag{E.48}$$

where $\pi^*_t = \Delta p^*_t$ and $\pi_{C,i,t} = \Delta p_{C,i,t}$. Consumption, public quantities, and investment expenditure satisfy

$$\widehat{c}_{i,t} = \widehat{c}^*_t + \zeta_{i,t} + (1-\alpha_C)s_{i,t}, \tag{E.49}$$

$$\widehat{g}_{i,t} = \widehat{\mathcal{G}}^U_t + u_{i,t} + s_{i,t}, \tag{E.50}$$

$$\widetilde{e}^I_{i,t} = \widetilde{\iota}_{i,t} - (1-\alpha_I)s_{i,t}. \tag{E.51}$$

The last variable is the regional contrast of log investment expenditure in P^*_t units, distinct from physical investment.

Demand and the financial constraint. Linearizing the primitive goods market (E.9) gives

$$\begin{aligned}\widehat{y}_{i,t} = c_Y\big[(1-\alpha_C)\widehat{c}_{i,t} + \alpha_C\widehat{c}^*_t + \alpha_C(2-\alpha_C)s_{i,t}\big] \\ + \iota\big[(1-\alpha_I)\widehat{\iota}_{i,t} + \alpha_I\widehat{\iota}^*_t + \alpha_I(2-\alpha_I)s_{i,t}\big] + \gamma\widehat{g}_{i,t}.\end{aligned} \tag{E.52}$$

When $\zeta_{i,t} = 0$, the consumption term reduces to $c_Y(\widehat{c}_{i,t} + \alpha_C s_{i,t})$, as in the complete-markets appendix. With incomplete markets, that substitution would assign coefficient c_Y to $\zeta_{i,t}$ instead of the correct $(1-\alpha_C)c_Y$. The primitive demand relation must therefore be retained before forming regional contrasts.

Subtracting the union average, and using the expenditure definitions, gives

$$\widetilde{y}_{i,t} - s_{i,t} = (1-\alpha_C)c_Y\zeta_{i,t} + (1-\alpha_I)\iota\widetilde{e}^I_{i,t} + \gamma u_{i,t}. \tag{E.53}$$

The first-order net-export and financial equations are

$$\frac{NX_{i,t}}{\bar{P}^*\bar{Y}} = -\alpha_C c_Y\zeta_{i,t} - \alpha_I\iota\widetilde{e}^I_{i,t}, \tag{E.54}$$

$$\beta b_{i,t+1} - b_{i,t} = \gamma u_{i,t} - \alpha_C c_Y\zeta_{i,t} - \alpha_I\iota\widetilde{e}^I_{i,t}. \tag{E.55}$$

The net federal inflow is $\mathcal{F}_{i,t}/(\bar{P}^*\bar{Y}) = \gamma u_{i,t}$ to first order. Its coefficient in the current account is also γ, since public purchases contain no direct imports. Fluctuations in interest rates multiplying bond holdings are second order around $\bar{B}_i = 0$. Common nominal-price changes enter gross taxes and transfers, but cancel in their regional difference.

Linearizing the bond Euler equation gives

$$r^*_t = \mathbb{E}_t\big[\pi^*_{t+1} + \widehat{c}^*_{t+1} - \widehat{c}^*_t + \zeta_{i,t+1} - \zeta_{i,t}\big], \qquad r^*_t = \log R^*_t - \log(1/\beta). \tag{E.56}$$

Averaging determines the union consumption Euler equation. Subtracting that average yields

$$\zeta_{i,t} = \mathbb{E}_t\zeta_{i,t+1}. \tag{E.57}$$

Relative nominal consumption expenditure is a martingale at first order. Its expected change is zero, while new information can change its realized value.

The union block. The union has ten endogenous real and inflation variables and the fiscal state. Its equilibrium is

$$\widehat{y}_t^* = c_Y \widehat{c}_t^* + \iota \widehat{\iota}_t^* + \gamma \widehat{\mathcal{G}}_t^U, \tag{E.58}$$

$$\widehat{y}_t^* = \xi \widehat{k}_t^* + (1-\xi)\widehat{\ell}_t^*, \tag{E.59}$$

$$\widehat{w}_t^* = \widehat{c}_t^* + \varphi \widehat{\ell}_t^*, \tag{E.60}$$

$$\widehat{w}_t^* = \widehat{mc}_t^* + \xi(\widehat{k}_t^* - \widehat{\ell}_t^*), \tag{E.61}$$

$$\widehat{r}_t^{K,*} = \widehat{mc}_t^* + (\xi - 1)(\widehat{k}_t^* - \widehat{\ell}_t^*), \tag{E.62}$$

$$\widehat{q}_t^* = \tau\delta(\widehat{\iota}_t^* - \widehat{k}_t^*), \tag{E.63}$$

$$\widehat{k}_{t+1}^* = (1-\delta)\widehat{k}_t^* + \delta\widehat{\iota}_t^*, \tag{E.64}$$

$$\widehat{q}_t^* = \widehat{c}_t^* - \mathbb{E}_t\widehat{c}_{t+1}^* + \beta\bar{r}^K\mathbb{E}_t\widehat{r}_{t+1}^{K,*} + \beta\mathbb{E}_t\widehat{q}_{t+1}^*, \tag{E.65}$$

$$\pi_t^* = \beta\mathbb{E}_t\pi_{t+1}^* + \lambda\widehat{mc}_t^*, \tag{E.66}$$

$$\widehat{c}_t^* = \mathbb{E}_t\widehat{c}_{t+1}^* - \big(\phi_\pi\pi_t^* - \mathbb{E}_t\pi_{t+1}^*\big), \tag{E.67}$$

$$\widehat{\mathcal{G}}_t^U = \rho_G\widehat{\mathcal{G}}_{t-1}^U + \bar{\varepsilon}_t^G, \qquad \bar{\varepsilon}_t^G = \int_0^1 \varepsilon_{i,t}^G \,\mathrm{d}i. \tag{E.68}$$

Zero mean regional expenditure and financial contrasts, together with balanced aggregate fiscal financing, imply the same first-order union block as under complete markets. This is an equivalence of the linearized aggregate equations around symmetry.

The regional real block. The real conditions comprise the goods market (E.53) and

$$\widetilde{y}_{i,t} = \xi\widetilde{k}_{i,t} + (1-\xi)\widetilde{\ell}_{i,t}, \tag{E.69}$$

$$\widetilde{w}_{i,t} = \zeta_{i,t} + (1-\alpha_C)s_{i,t} + \varphi\widetilde{\ell}_{i,t}, \tag{E.70}$$

$$\widetilde{w}_{i,t} = \widetilde{mc}_{i,t} - \alpha_C s_{i,t} + \xi(\widetilde{k}_{i,t} - \widetilde{\ell}_{i,t}), \tag{E.71}$$

$$\widetilde{r}_{i,t}^K = \widetilde{mc}_{i,t} - \alpha_C s_{i,t} + (\xi - 1)(\widetilde{k}_{i,t} - \widetilde{\ell}_{i,t}), \tag{E.72}$$

$$\widetilde{q}_{i,t} = p_{IC,i,t} + \tau\delta(\widetilde{\iota}_{i,t} - \widetilde{k}_{i,t}), \tag{E.73}$$

$$\widetilde{k}_{i,t+1} = (1-\delta)\widetilde{k}_{i,t} + \delta\widetilde{\iota}_{i,t}, \tag{E.74}$$

$$\begin{aligned}\widetilde{q}_{i,t} = {} & (1-\alpha_C)(s_{i,t} - \mathbb{E}_t s_{i,t+1}) + \beta\bar{r}^K\mathbb{E}_t\widetilde{r}_{i,t+1}^K \\ & + \beta\mathbb{E}_t\widetilde{q}_{i,t+1} - \beta\delta\mathbb{E}_t p_{IC,i,t+1},\end{aligned} \tag{E.75}$$

$$\Delta s_{i,t} = \beta\mathbb{E}_t\Delta s_{i,t+1} - \lambda\widetilde{mc}_{i,t}. \tag{E.76}$$

The bond Euler equation (E.57), the current account (E.55), and

$$u_{i,t} = \rho_G u_{i,t-1} + \varepsilon^R_{i,t}, \qquad \varepsilon^R_{i,t} = \varepsilon^G_{i,t} - \bar{\varepsilon}^G_t \tag{E.77}$$

complete the twelve-variable regional system. Consumption and inflation are recovered from their identities rather than introduced as independent restrictions.

To obtain the capital-valuation equations, linearizing (E.21) first gives

$$\begin{aligned}\widehat{q}_{i,t} = \widehat{c}_{i,t} - \mathbb{E}_t\widehat{c}_{i,t+1} + \beta\bar{r}^K\mathbb{E}_t\widehat{r}^K_{i,t+1} + \beta(1-\delta)\mathbb{E}_t\widehat{q}_{i,t+1} \\ + \beta\tau\delta^2\mathbb{E}_t(\widehat{\iota}_{i,t+1} - \widehat{k}_{i,t+1}).\end{aligned} \tag{E.78}$$

Next-period installation replaces the last term by $\beta\delta\mathbb{E}_t(\widehat{q}_{i,t+1} - p_{IC,i,t+1})$. Subtracting the union equation and using $\mathbb{E}_t\zeta_{i,t+1} = \zeta_{i,t}$ yields (E.75). The substitution eliminates expected expenditure growth, not its realized innovation. Similarly, the relative Phillips equation follows from $\pi_{i,t} - \pi^*_t = -\Delta s_{i,t}$.

Regional reconstruction. Full regional quantities are obtained by adding the union component and the regional contrast. The means of $s_{i,t}$, $\zeta_{i,t}$, $u_{i,t}$, and $b_{i,t}$ are zero. In the quantitative representation with N equally weighted observations, the common innovation is the mean of the same N regional draws, and the regional deviations are represented in $N-1$ orthonormal contrast coordinates. No separate common shock is introduced. The observations take union aggregates as given; N controls the representation and sampling and does not change the import shares or introduce regional market power.

The union and contrast systems can therefore be solved separately and reconstructed in regional coordinates. The resulting inventory has $11 + 12(N-1)$ variables and equations. The redundant mean financial equation follows from zero net bond supply and balanced federal transfers.

E.7 Initial Conditions and Financial Solvency

The military exercise starts from the symmetric reference before the first innovation:

$$\widehat{k}_{i,0} = 0, \qquad b_{i,0} = 0, \qquad s_{i,-1} = 0, \qquad \widehat{\mathcal{G}}_{i,-1} = 0. \tag{E.79}$$

Capital productive at date zero and the bond payoff received then are inherited. Current prices, investment, capital valuation, and $\zeta_{i,0}$ adjust after the fiscal innovation is observed.

Thus $b_{i,0} = 0$ does not imply $\zeta_{i,0} = 0$. More generally, the solution below applies to given small inherited states with zero mean relative prices and bond positions. Each future innovation is observed only when its date arrives.

Around zero bond holdings, the first-order financial transversality condition is

$$\lim_{H\to\infty} \beta^{H+1}\mathbb{E}_t b_{i,t+H+1} = 0. \tag{E.80}$$

It permits a finite nonzero limiting asset position but excludes the homogeneous explosive solution $b_{i,t+1} = b_{i,t}/\beta$ with nonzero amplitude. The capital transversality condition (E.22) continues to apply.

Iterating the current account and imposing (E.80) gives

$$0 = b_{i,t} + \sum_{h=0}^{\infty} \beta^h \mathbb{E}_t \left[\gamma u_{i,t+h} - \alpha_C c_Y \zeta_{i,t+h} - \alpha_I \iota \widetilde{e}^I_{i,t+h}\right]. \tag{E.81}$$

Using $\mathbb{E}_t\zeta_{i,t+h} = \zeta_{i,t}$ and $\mathbb{E}_t u_{i,t+h} = \rho_G^h u_{i,t}$, this becomes

$$\frac{\alpha_C c_Y}{1-\beta}\zeta_{i,t} = b_{i,t} + \frac{\gamma}{1-\beta\rho_G}u_{i,t} - \alpha_I\iota\sum_{h=0}^{\infty}\beta^h\mathbb{E}_t\widetilde{e}^I_{i,t+h}. \tag{E.82}$$

Future investment expenditure depends on the allocation induced by $\zeta_{i,t}$. This condition therefore selects expenditure jointly with the real equilibrium; it is not a consumption rule based solely on the present value of the fiscal inflow.

E.8 The Regional Stationary Family

Consider the conditional long-run regional position after a transitory fiscal innovation, once union real deviations, the relative fiscal state, and inflation have returned to zero. The following family is a first-order characterization near the symmetric reference. The Phillips curve implies $\widetilde{mc}_{i,\infty} = 0$, while accumulation, installation, and capital valuation imply

$$\widetilde{\iota}_{i,\infty} = \widetilde{k}_{i,\infty}, \qquad \widetilde{q}_{i,\infty} = \widetilde{r}^K_{i,\infty} = p_{IC,i,\infty} = (\alpha_I - \alpha_C)s_{i,\infty}. \tag{E.83}$$

The rental condition then gives $\widetilde{y}_{i,\infty} - \widetilde{k}_{i,\infty} = \alpha_I s_{i,\infty}$. Hence

$$\widetilde{y}_{i,\infty} - s_{i,\infty} = \widetilde{e}^I_{i,\infty} = (1-\alpha_C)c_Y\zeta_{i,\infty} + (1-\alpha_I)\iota\widetilde{e}^I_{i,\infty}. \tag{E.84}$$

Define

$$\Omega_I = \frac{(1-\alpha_C)c_Y}{1-(1-\alpha_I)\iota}, \tag{E.85}$$

$$\Omega_P = \frac{1+\varphi\Omega_I}{(1+\varphi)\left[1+\dfrac{\xi\alpha_I}{1-\xi}\right]}, \tag{E.86}$$

$$H_b = \frac{\alpha_C c_Y + \alpha_I \iota \Omega_I}{1-\beta}. \tag{E.87}$$

The first coefficient follows from (E.84). Combining labor supply and factor demand gives

$$(1+\varphi\Omega_I)\zeta_{i,\infty} + (1+\varphi)\left[1+\frac{\xi\alpha_I}{1-\xi}\right] s_{i,\infty} = 0, \tag{E.88}$$

which yields Ω_P. The stationary current account gives H_b.

The family is parametrized by $\zeta_{i,\infty}$:

$$\widetilde{e}^I_{i,\infty} = \Omega_I \zeta_{i,\infty}, \tag{E.89}$$

$$s_{i,\infty} = -\Omega_P \zeta_{i,\infty}, \tag{E.90}$$

$$\widetilde{k}_{i,\infty} = \widetilde{\iota}_{i,\infty} = \left[\Omega_I - (1-\alpha_I)\Omega_P\right]\zeta_{i,\infty}, \tag{E.91}$$

$$\widetilde{y}_{i,\infty} = \widetilde{k}_{i,\infty} + \alpha_I s_{i,\infty}, \tag{E.92}$$

$$\widetilde{\ell}_{i,\infty} = \widetilde{k}_{i,\infty} + \frac{\alpha_I}{1-\xi} s_{i,\infty}, \tag{E.93}$$

$$\widetilde{c}_{i,\infty} = \zeta_{i,\infty} + (1-\alpha_C) s_{i,\infty}, \tag{E.94}$$

$$\widetilde{w}_{i,\infty} = -\left(\alpha_C + \frac{\xi\alpha_I}{1-\xi}\right) s_{i,\infty}, \tag{E.95}$$

$$b_{i,\infty} = H_b \zeta_{i,\infty}. \tag{E.96}$$

Together with (E.83) and zero marginal-cost deviation, these expressions specify all regional variables. They determine the direction of a permanent adjustment; the present-value budget determines its amplitude.

The equality of investment and capital log deviations reflects replacement investment at the new capital stock. It does not imply that the deviation of gross investment is zero. Net investment returns to zero in this conditional limit. Likewise, $\widehat{g}_{i,\infty} = s_{i,\infty}$: public expenditure in common-price units returns to its reference, while the quantity purchased can differ because the local price has changed.

The limiting trade balance satisfies

$$\frac{NX_{i,\infty}}{\bar{P}^*\bar{Y}} = (\beta - 1)b_{i,\infty}. \tag{E.97}$$

A creditor can sustain a trade deficit through financial income. Bond positions sum to zero, so this component represents redistribution within the union rather than creation of aggregate net financial assets.

E.9 Solution and Conditional Dynamics

Suppress the region index and the tildes for this subsection. Let x_t collect the nine real regional variables $(s_t, k_t, \iota_t, q_t, y_t, \ell_t, w_t, r_t^K, mc_t)'$. Here ι_t is the regional log investment contrast, while the time-invariant ι remains its steady-state output share. The real state at the decision date is

$$d_t = \begin{pmatrix} k_t & s_{t-1} & u_t \end{pmatrix}', \qquad J = \begin{pmatrix} 0 & 0 & 1 \end{pmatrix}', \qquad f' = J'. \tag{E.98}$$

Capital and the previous relative price are inherited; u_t already contains the current fiscal innovation. The bond payoff b_t is inherited separately, so the full regional economic state has four components.

The conditional real subsystem. Conditional on $\zeta = 0$, the nine real equations and the fiscal law coincide with the relative real subsystem of the complete-markets specification. If that subsystem has a determined stable solution, write

$$x_t = Ld_t, \qquad d_{t+1} = Ad_t + J\varepsilon_{t+1}^R, \tag{E.99}$$

where L is 9×3, A is 3×3, and $f'A = \rho_G f'$. Conditioning this auxiliary subsystem on $\zeta = 0$ does not impose complete risk sharing in the full equilibrium.

Let v_x collect the stationary coefficients of x per unit of ζ, as derived in Section E.8, and define

$$v = \begin{pmatrix} v_k & v_s & 0 \end{pmatrix}', \qquad v_k = \Omega_I - (1 - \alpha_I)\Omega_P, \qquad v_s = -\Omega_P. \tag{E.100}$$

Thus $f'v = 0$. If L_ι and L_s are the investment and relative-price rows of L, define the investment-expenditure row $\ell_I = L_\iota - (1 - \alpha_I)L_s$. For the full equilibrium, set $\eta_t = d_t - v\zeta_t$.

The real allocation and state transition are

$$x_t = v_x \zeta_t + L\eta_t, \tag{E.101}$$

$$e_t^I = \Omega_I \zeta_t + \ell_I \eta_t, \tag{E.102}$$

$$d_{t+1} = v\zeta_t + A\eta_t + J\varepsilon_{t+1}^R. \tag{E.103}$$

The variable e_t^I is the same investment-expenditure contrast denoted by $\widetilde{e}_{i,t}^I$ above. This representation uses $\mathbb{E}_t \zeta_{t+1} = \zeta_t$. The state shift applies to initial conditions as well: zero inherited capital requires $\eta_{k,0} = -v_k \zeta_0$ when the selected stationary capital deviation is nonzero.

Present-value selection. For stable A, define the discounted-resolvent row

$$\mathbf{r} = \ell_I (I - \beta A)^{-1}, \qquad \mathbf{r} - \beta \mathbf{r} A = \ell_I, \tag{E.104}$$

where I is the 3×3 identity matrix. The conditional investment-expenditure path then satisfies

$$\mathbb{E}_t e_{t+h}^I = \Omega_I \zeta_t + \ell_I A^h (d_t - v\zeta_t), \tag{E.105}$$

$$\sum_{h=0}^{\infty} \beta^h \mathbb{E}_t e_{t+h}^I = \frac{\Omega_I}{1-\beta} \zeta_t + \mathbf{r}(d_t - v\zeta_t). \tag{E.106}$$

The expression retains both the permanent component and the entire stable transition. Define

$$\mathbf{n} = \frac{\gamma}{1 - \beta\rho_G} f' - \alpha_I \iota \mathbf{r}, \tag{E.107}$$

$$D_\zeta = H_b - \alpha_I \iota \mathbf{r} v = H_b + \mathbf{n} v. \tag{E.108}$$

The scalar D_ζ is derived from the equilibrium coefficients and is distinct from dividends. Substitution into (E.82) gives

$$\boxed{\zeta_t = \frac{b_t + \mathbf{n} d_t}{D_\zeta}}, \qquad D_\zeta \neq 0. \tag{E.109}$$

Conditional on a determined stable real subsystem and $D_\zeta \neq 0$, the present-value condition selects a unique allocation of this form for each given state (d_t, b_t). In particular, from

symmetric initial conditions,

$$d_0 = J\varepsilon_0^R, \qquad b_0 = 0, \qquad \zeta_0 = \varkappa_G \varepsilon_0^R, \qquad \varkappa_G = \frac{\mathbf{n}.J}{D_\zeta}. \tag{E.110}$$

The sign and magnitude of $\varkappa_G$ depend on the equilibrium investment response as well as fiscal resources.

New information and predetermined assets. The coefficient identity

$$\mathbf{n} - \beta \mathbf{n} A = \gamma f' - \alpha_I \iota \ell_I \tag{E.111}$$

and the current account imply

$$\zeta_{t+1} = \zeta_t + \varkappa_G \varepsilon_{t+1}^R, \tag{E.112}$$

$$\eta_{t+1} = A\eta_t + (J - v\varkappa_G)\varepsilon_{t+1}^R. \tag{E.113}$$

There is no separate financial innovation: the surprise is the relative fiscal innovation already specified. Its zero conditional mean ensures that the expenditure policy satisfies the bond Euler equation. Assets are recovered from

$$b_t = D_\zeta \zeta_t - \mathbf{n} d_t = H_b \zeta_t - \mathbf{n}\eta_t. \tag{E.114}$$

The direct loading of a new innovation on the inherited bond payoff is

$$H_b \varkappa_G - \mathbf{n}(J - v\varkappa_G) = D_\zeta \varkappa_G - \mathbf{n}J = 0. \tag{E.115}$$

Thus the new information changes expenditure without changing the nominal payoff purchased previously. Productive capital is also inherited; only investment chosen after the news affects next-period capital.

Along a path with no further realized innovations, ζ remains at its post-news value, η converges to zero under A, and b converges to $H_b\zeta$. The selected path satisfies financial transversality while allowing a permanent regional financial position. Algebraic asset recovery also avoids accumulating numerical errors from forward iteration of the excluded homogeneous financial mode.

Investment dynamics and the infinite horizon. For a relative fiscal innovation of unit amplitude from the symmetric reference, the first-order investment-response coefficients are

$$h_j^I = v_\iota \varkappa_G + L_\iota A^j (J - v\varkappa_G), \qquad v_\iota = v_k, \qquad j \geq 0. \tag{E.116}$$

Unit amplitude normalizes a linear response coefficient. The conditional limit is $v_\iota \varkappa_G$, whereas impact and annual responses also depend on the stable transition. A persistent component of investment therefore does not by itself determine the sign of the measured annual coefficient.

For any linear regional observable $o_t = o_\infty \zeta_t + \ell_o \eta_t$, its discounted tail is

$$\sum_{h=H}^{\infty} \beta^h \mathbb{E}_t o_{t+h} = \beta^H \left[\frac{o_\infty}{1-\beta} \zeta_t + \ell_o (I - \beta A)^{-1} A^H \eta_t \right]. \tag{E.117}$$

The solution retains the infinite horizon analytically, without selecting expenditure through a zero-asset target at a finite date. The admissible unit root in expenditure is distinct from the excluded explosive financial root $1/\beta$.

With repeated fiscal news, the permanent component is updated after each realization. For independent regional innovations with variance σ_G^2 in the N-observation representation, $\mathrm{Var}(\varepsilon_{i,t}^R) = \sigma_G^2(1 - 1/N)$. Conditional on current information,

$$\mathrm{Var}_t(\zeta_{i,t+h} - \zeta_{i,t}) = h \varkappa_G^2 \sigma_G^2 (1 - 1/N). \tag{E.118}$$

When $\varkappa_G \neq 0$, independent innovations and a stationary fiscal state do not imply stationary relative expenditure. These results describe a local first-order solution; the conditional stationary family and financial transversality condition do not establish a stationary wealth distribution or global nonlinear solvency. The quantitative exercise uses a declared finite horizon and initial state, as specified in Section 5.

F Additional Quantitative Results

F.1 ARRA Shocks

F.1.1 Fiscal Inputs and Measurement

The fiscal inputs are the state payment measures used in section 2. The numerator is cumulative ARRA payments excluding the Department of Labor, from the data described in Wilson (2012) and constructed from Recovery.gov and Funding Notification Reports. The payment window extends from August 2009 to February 2010. We divide these payments by nominal state GDP in 2008, obtained from the Bureau of Economic Analysis, to construct S_i. The sample contains the 50 states and excludes the District of Columbia and the territories.

Table F.1: ARRA Shocks: Fiscal Exposure across States

Statistic	Value
Number of states	50
Mean	0.9517
Standard deviation	0.2469
Minimum	0.4997
Maximum	1.6708

NOTE.—Exposure is cumulative ARRA payments excluding the Department of Labor, divided by nominal state GDP in 2008. Distributional statistics refer to $100S_i$: the mean, minimum, and maximum are percentages of annual GDP; the standard deviation is in percentage points and uses the sample denominator $N-1$. States receive equal weights.

Table F.1 summarizes the cross-state distribution. Mean payments amount to 0.952% of annual GDP, with a sample standard deviation of 0.247 percentage points. We assign the full vector of observed exposures in levels to the model regions, preserving its positive mean and cross-state variation. Both TNT specifications use this same vector, with equal region weights and the calibration in table 7. The comparison changes complete markets with local fiscal financing to incomplete markets with federal fiscal financing.

For each value of ρ_G, equation (23) maps the observed exposures into unanticipated innovations at date 0. Subsequent realized innovations are zero along the impulse-response path; expectations and decision rules remain those of the stochastic model. The normalization holds expenditure over quarters 0 and 1 fixed relative to annual steady-state output. Greater persistence changes its allocation between these two quarters and raises spending

beyond that window. In particular, the fiscal process implies

$$\frac{\gamma}{4}\sum_{t=0}^{3}\widehat{\mathcal{G}}_{i,t} = (1+\rho_G^2)S_i. \tag{F.1}$$

At the benchmark persistence, first-year expenditure is therefore 1.0625 times the observed payment exposure. For $0 \leq \rho_G < 1$, the cumulative expenditure response over the entire path satisfies

$$\frac{\gamma}{4}\sum_{t=0}^{\infty}\widehat{\mathcal{G}}_{i,t} = \frac{S_i}{1-\rho_G^2}. \tag{F.2}$$

Both expressions measure additional expenditure in common-price units relative to annual steady-state output, to first order. The second sum is undiscounted. The regressor remains S_i throughout the persistence exercise.

All investment responses use equation (24), with an intercept and equal state weights. Both specifications use log deviations of investment destined for nontradable production, measured on impact or averaged over quarters 0–3, as in equation (25). To first order, this average is also the proportional change in the sum of investment over the first model year, relative to four quarters of steady-state investment. The intercept absorbs the common response to the mean fiscal expansion. The same regression and time aggregation are applied to every component of the decompositions below.

F.1.2 Investment Mechanisms

Investment adjustment. Figure F.1 reports the decomposition of nontradable investment discussed in section 5.2, comparing the two TNT specifications on impact and over the first year. The three contributions follow directly from equation (26): productive capital, the value of installed capital divided by $\tau\delta$, and the negative of the current relative investment price divided by $\tau\delta$.

The contribution of productive capital is zero on impact because the capital stock is predetermined. Over the first year it is negative in both specifications. The value of installed capital contributes -1.802 under complete markets with local financing and -1.637 under incomplete markets with federal financing, while the current relative investment price contributes 0.334 and 0.429, respectively. The latter specification therefore combines a smaller negative valuation contribution with a larger positive price contribution, leaving the total annual response at -1.278, compared with -1.553 under complete markets with local financing.

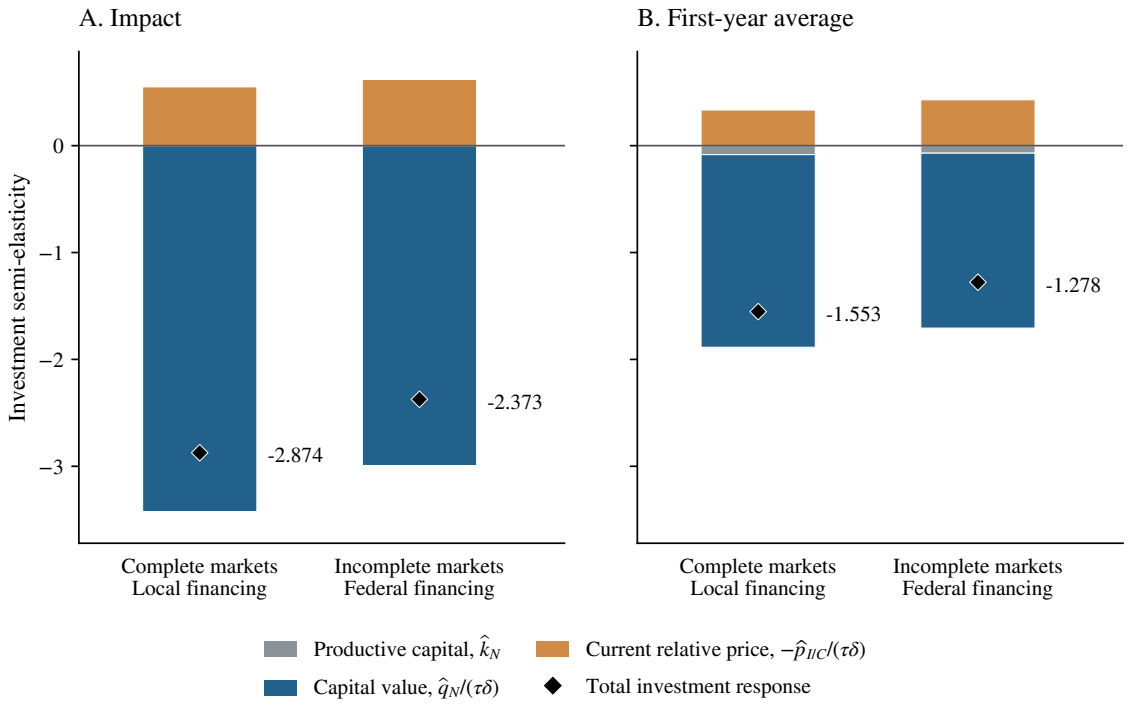


Figure F.1: ARRA Shocks: Investment Decomposition in TNT

NOTE.—Each panel compares complete markets with local fiscal financing and incomplete markets with federal fiscal financing. Bars report the three contributions in equation (26); diamonds indicate their sum, the nontradable investment semi-elasticity. Each component uses the same fiscal exposure and regression as the total response. Panel A reports the impact response; panel B averages quarters 0–3. All parameters take their benchmark values. This is an accounting decomposition of the equilibrium response.

The value of installed capital. The capital valuation equations in appendices B and C permit a further decomposition of q_N. Let $\widehat{r}^K_{N,i,t}$ denote the log deviation of the rental return on nontradable capital in regional consumption units, and define $\widehat{m}^C_{i,t,t+1} = \widehat{c}_{i,t} - \widehat{c}_{i,t+1}$ as the log deviation of the household's real stochastic discount factor. Substituting the next-period installation condition into the linearized capital Euler equation gives

$$\widehat{q}_{N,i,t} = \mathbb{E}_t \left[\widehat{m}^C_{i,t,t+1} + \beta \bar{r}_K \widehat{r}^K_{N,i,t+1} + \beta \widehat{q}_{N,i,t+1} - \beta\delta \widehat{p}_{I/C,i,t+1} \right],$$

where $\bar{r}_K = \beta^{-1} - 1 + \delta$. Iterating this relation to a common reporting date H gives

$$\begin{aligned} \widehat{q}_{N,i,t} = \mathbb{E}_t \sum_{j=0}^{H-t-1} \beta^j \left[\widehat{m}^C_{i,t+j,t+j+1} + \beta \bar{r}_K \widehat{r}^K_{N,i,t+j+1} - \beta\delta \widehat{p}_{I/C,i,t+j+1} \right] \\ + \beta^{H-t} \mathbb{E}_t \widehat{q}_{N,i,H}. \end{aligned} \tag{F.3}$$

We use $H = 20$ for both specifications and for each starting date $t = 0, 1, 2, 3$. The final term retains the continuation value implied by the model solution, including payoffs beyond date H.

Table F.2: ARRA Shocks: Present-Value Accounting for Nontradable Capital

Contribution	Impact	First-year average
Panel A. Complete markets and local fiscal financing		
Discount factor	−0.351497	−0.200461
Subsequent rental returns	−0.003378	0.008635
Future relative investment prices	0.004266	0.003193
Continuation at $H = 20$	0.008275	0.008401
Total response of $\widehat{q}_N$	−0.342334	−0.180232
Panel B. Incomplete markets and federal fiscal financing		
Discount factor	−0.327304	−0.193680
Subsequent rental returns	0.016420	0.019756
Future relative investment prices	0.009028	0.007645
Continuation at $H = 20$	0.002569	0.002608
Total response of $\widehat{q}_N$	−0.299287	−0.163671

NOTE.—Entries are regression slopes for the components of $\widehat{q}_N$ in equation (F.3), using S_i as the regressor. Both panels refer to nontradable capital in TNT and retain the continuation value from the first-order solution at the common reporting date $H = 20$. The first-year average covers starting dates $t = 0, 1, 2, 3$. The contribution of q_N to investment is its response divided by $\tau\delta = 0.10$. All parameters take their benchmark values. Totals may differ from sums of displayed components because of rounding.

Table F.2 applies the fiscal-exposure regression to the four terms in equation (F.3). These entries account for the response of $\widehat{q}_N$ itself; its contribution to investment is obtained by dividing by $\tau\delta = 0.10$. The contribution of the real discount factor is the largest negative component in both specifications. Subsequent rental returns make a relatively small contribution and enter from $t+1$, consistent with the timing of capital accumulation. The future investment-price term belongs to the valuation of installed capital and is distinct from the current investment-price term in figure F.1.

Consumption expenditure and continuation values. The consumption paths entering this accounting depend on the financial and fiscal specification. Complete markets impose the risk-sharing allocation described in appendix B. Under incomplete markets, the household's present-value budget selects relative nominal consumption expenditure, $\zeta_{i,t}$, jointly with investment. Along the ARRA response with no further innovations, $\zeta_{i,t}$ remains at its post-innovation value, while other regional variables converge toward the corresponding stationary allocation. Federal financing and this expenditure adjustment affect the equilibrium paths of consumption prices and rental returns. Their effects are already incorporated in productive capital, q_N, and the relative investment price; they do not constitute an additional term in the installation identity.

The incomplete-markets solution retains possible permanent changes in relative financial positions and capital, as derived in appendix C. Date H is an accounting cutoff, not an imposed return to the symmetric steady state. Bond positions and deviations of q_N at that date are taken from the infinite-horizon solution. At the benchmark, the continuation contribution to the annual response of q_N is 0.002608, compared with a total response of -0.163671. Retaining the future continuation therefore leaves the negative valuation response intact. The finite-date continuation includes both the remaining transition and any permanent component; it is not a measure of the permanent component alone. These decompositions describe additive components of a joint equilibrium, rather than separate causal experiments.

F.1.3 Fiscal Persistence

Table F.3 reports the annual semi-elasticities underlying figure 1, together with the cumulative spending implied by equation (F.2). Both specifications retain their benchmark structural parameters. Initial innovations are rescaled at each persistence value to preserve the same observed payment exposure over quarters 0 and 1.

The spending ratios clarify the counterfactual behind the crossings discussed in section 5.2. Cumulative additional expenditure is 1.067 times the initial payment exposure at the benchmark, 4.267 times at $\rho_G = 0.875$, and 5.263 times at $\rho_G = 0.90$. The latter two points bracket the crossing of the empirical semi-elasticity under incomplete markets with federal financing. Higher persistence extends fiscal demand and increases spending beyond the observed payment window. The crossings are interpolations between evaluated points, conditional on the benchmark structural calibration and the first-order approximation.

Table F.3: ARRA Shocks: Annual Investment and Cumulative Fiscal Spending

	First-year investment semi-elasticity		Cumulative spending
ρ_G	Complete markets Local financing	Incomplete markets Federal financing	Relative to initial payments
0.000	−1.485	−1.227	1.000
0.100	−1.497	−1.237	1.010
0.250	−1.553	−1.278	1.067
0.500	−1.673	−1.334	1.333
0.600	−1.675	−1.281	1.562
0.650	−1.644	−1.209	1.732
0.700	−1.573	−1.085	1.961
0.750	−1.441	−0.878	2.286
0.800	−1.206	−0.532	2.778
0.825	−1.028	−0.277	3.131
0.850	−0.791	0.062	3.604
0.875	−0.468	0.522	4.267
0.900	−0.023	1.165	5.263
0.925	0.611	2.110	6.926
0.950	1.556	3.614	10.256
0.975	3.061	6.442	20.253
0.990	4.453	10.159	50.251

NOTE.—Investment entries are semi-elasticities of nontradable investment in TNT, averaging quarters 0–3. Both specifications retain their benchmark structural parameters. Initial innovations are rescaled at each value of ρ_G to preserve each state's payment exposure S_i over quarters 0 and 1. The last column is the ratio of the undiscounted cumulative expenditure response to that initial exposure, $1/(1-\rho_G^2)$; it is common to both specifications. The empirical reference for the first-year investment response is 0.8055.

F.2 Military Spending Shocks

F.2.1 Fiscal Inputs and Measurement

Fiscal calibration. The fiscal calibration uses the short-contract military-spending series described in section 3. Following the geography of the metropolitan GDP source, we retain areas with observed expenditure and positive GDP throughout 2001–2016. Both series are expressed in 2001 dollars using the national CPI. Zero and negative expenditure observations are retained. The balanced fiscal panel contains 377 areas and 6,032 observations and is distinct from the confidential investment sample.

Let $D_{i,a}^{\text{real}}$ and $Y_{i,a}^{\text{real}}$ denote military expenditure and GDP in this panel. We construct the fiscal measure

$$z_{i,a} = \frac{D_{i,a}^{\text{real}}}{\overline{Y}_i^{\text{real}}}, \qquad \overline{Y}_i^{\text{real}} = \frac{1}{16} \sum_{a=2001}^{2016} Y_{i,a}^{\text{real}}. \tag{F.4}$$

Thus, each area's denominator is fixed at its average real annual GDP over the calibration sample. We remove area and year means, adding back the overall mean, and compute the residual standard deviation using $(377 - 1)(16 - 1)$ degrees of freedom. This yields $s_z = 0.0107135$, or 1.07135 percentage points of annual GDP.

Let $V_A^{\text{S}}(\rho)$ denote the expected residual variance of four-quarter averages of a stationary AR(1) process with unit innovation variance. It incorporates region and year effects and the degrees-of-freedom adjustment for a balanced panel of A years. For the calibration panel, $V_{16}^{\text{S}}(0.25) = 0.3817284$, so

$$\sigma_G = \frac{s_z}{\gamma \sqrt{V_{16}^{\text{S}}(0.25)}} = 0.08670. \tag{F.5}$$

This calculation accounts for both quarterly persistence and annual aggregation.[23] The parameter $\gamma = 0.20$ is the model's steady-state share of total public purchases; the military series calibrates the scale of fiscal fluctuations. The calibration uses fiscal and GDP data, without using investment outcomes or the empirical investment coefficient.

Finite panels and annual measurement. Both GM specifications use equation (27), with union innovations equal to the means of the regional draws. Each specification has 1,000 simulated panels of 100 regions and 250 years. All predetermined states start at

[23] Let $c_h(\rho)$ denote the covariance of annual averages h years apart when quarterly innovation variance is one. Then $V_A^{\text{S}}(\rho) = c_0(\rho) - \frac{2}{A(A-1)} \sum_{h=1}^{A-1} (A-h) c_h(\rho)$, where $c_0(\rho) = [4 + 2\sum_{d=1}^{3} (4-d)\rho^d]/[16(1-\rho^2)]$ and, for $h \geq 1$, $c_h(\rho) = \sum_{j=0}^{3} \sum_{k=0}^{3} \rho^{4h+j-k}/[16(1-\rho^2)]$.

Table F.4: Military Spending Shocks: Fiscal Calibration and Simulation

Statistic	Value
Panel A. Fiscal calibration sample	
Areas in the source panel	377
Years	2001–2016
Area-year observations	6,032
Residual fiscal standard deviation (percentage points of GDP)	1.07135
Panel B. Benchmark simulation, each specification	
Paired replications	1,000
Regions per panel	100
Years per region	250
Region-year observations per panel	25,000
Initial state	Symmetric steady state
Discarded quarters	0
Fiscal persistence, ρ_G	0.25
Quarterly innovation standard deviation, σ_G	0.08670
Root expected residual fiscal variance (percentage points of GDP)	1.07614

NOTE.—Panel A describes the fiscal panel used to calibrate the innovation scale. Its residual standard deviation uses $(377-1)(16-1)$ degrees of freedom. Expenditure is scaled by each area's average real annual GDP in 2001–2016. Panel B describes the paired GM simulations. The root expected residual fiscal variance uses $(100-1)(250-1)$ degrees of freedom. The two panels have different horizons and initializations; the calibrated benchmark innovation scale is preserved.

their symmetric steady-state values, with zero inherited bond positions, and every period is retained. Each replication gives both specifications identical fiscal innovations; all parameter cases share standardized draws.

With the calibrated innovation scale, the square root of the expected residual fiscal variance in this panel is 0.0107614, using $(100-1)(250-1)$ degrees of freedom. Its difference from s_z reflects the different horizon and initialization; the benchmark scale is preserved.

Annual aggregation and the fixed-effects regression follow equations (28) and (29), with equal observation weights. Reported means average 1,000 separately estimated coefficients, with differences computed within each pair. Model outcomes are deviations of annual aggregate physical investment. The empirical estimate concerns growth in regional median capital expenditures and uses spending changes, lagged GDP, and an instrument. The comparison provides a reference in common units rather than a replication of that estimator.

At the benchmark, the fifth and ninety-fifth percentiles are $[-1.7622, -1.7521]$ under complete markets with local financing and $[-1.6591, -1.6489]$ under incomplete markets

with federal financing. Both coefficients are negative and the paired change positive in every replication. The percentiles describe simulation variation, not empirical or parameter uncertainty. A separate covariance check computes $\mathbb{E}[S_{xv}]/\mathbb{E}[S_{xx}]$, using cross-products after both fixed effects; this ratio is distinct from the mean of estimated slopes.

Under incomplete markets, wealth has a permanent component. Financial positions accumulate from the specified initial condition, without imposing a stationary wealth distribution or a return of bonds to zero at the reporting horizon.

F.2.2 Investment Mechanism

The installation condition in appendices D and E gives the same accounting identity in both specifications:

$$\widehat{\iota}_{i,t} = \widehat{k}_{i,t} + \frac{\widehat{q}_{i,t}}{\tau\delta} - \frac{\widehat{p}_{I/C,i,t}}{\tau\delta}, \qquad \widehat{p}_{I/C,i,t} = (\alpha_I - \alpha_C)s_{i,t}, \tag{F.6}$$

where $\widehat{k}_{i,t}$ is capital available for production at t, $q_{i,t}$ is measured in regional consumption units, and $s_{i,t} = \log(P_t^*/P_{i,t})$. Table F.5 applies the annual aggregation and regression to each term, using the same fiscal regressor and sample. The contributions sum to the investment coefficient in every replication and in their means.

Table F.5: Military Spending Shocks: Investment Decomposition

Contribution	Complete markets Local financing	Incomplete markets Federal financing	Change
Productive capital	−0.063455	−0.068258	−0.004803
Value of installed capital	−1.532305	−1.420823	0.111482
Current relative investment price	−0.161313	−0.164799	−0.003486
Total investment response	−1.757073	−1.653880	0.103193

NOTE.—Entries are mean contributions from $\widehat{k}_{i,t}$, $\widehat{q}_{i,t}/(\tau\delta)$, and $-\widehat{p}_{I/C,i,t}/(\tau\delta)$ in equation (F.6), using the 1,000 benchmark paired panels. Each component is aggregated to annual frequency and regressed on the same fiscal measure with region and year fixed effects. Change is incomplete markets with federal financing minus complete markets with local financing, computed within each pair. Totals are calculated before rounding. These are additive components of an equilibrium response.

The value of installed capital accounts for most of the negative coefficient in both specifications. Its contribution changes from −1.532305 to −1.420823, an improvement of 0.111482. Productive capital and the current relative investment price offset part of this improvement, contributing changes of −0.004803 and −0.003486, respectively. Their sum gives the net change of 0.103193 in the annual investment semi-elasticity.

Under the military calibration, $\alpha_I = 0.35$ and $\alpha_C = 0.40$, so investment is somewhat

more intensive in domestically produced goods than consumption. A rise in domestic prices therefore raises the price of investment relative to consumption. The resulting price contribution is negative in both specifications. The split between q and the relative investment price reflects their valuation in consumption units: valuing capital in investment units would combine these two terms, leaving the total unchanged. Regional wealth affects capital, q, and prices through equilibrium; adding wealth as a fourth contribution would count these effects twice. This accounting also does not separately identify the effects of federal financing and incomplete markets.

The productive-capital contribution measures a covariance after removing fixed effects. A permanent capital adjustment following an isolated innovation does not determine its sign in a panel with recurrent news. In the paired comparison, the products of annual observations at the same date contribute 0.004574 to the change in the capital coefficient, while the subtraction of products of regional time means contributes -0.009377. Their sum is -0.004803.[24]

F.2.3 Parameter Sensitivity and Fiscal Persistence

Parameter variation. Table F.6 applies the same structural variations to both specifications while holding consumption openness, price rigidity, and fiscal persistence at their benchmark values. Panel A changes labor supply, depreciation, and adjustment costs individually. Panel B varies the investment import share, including the GM limit of fully imported investment, $\alpha_I = 1$. Panel C combines changes in labor supply, capital adjustment, and investment openness. All twelve cases use the common finite-panel protocol described above.

Several import shares translate the composition of investment goods in TNT into an expenditure share on goods produced outside the home region. With 50 equally weighted regions in the TNT expenditure basket, this mapping is $\alpha_I^{\mathrm{GM}} = \nu_I \alpha_I^{\mathrm{TNT}}(49/50)$, where α_I^{TNT} is the weight on the union basket within tradables. The factor 49/50 belongs to that expenditure-composition mapping; the GM simulations continue to use 100 regions. These cases vary openness within the one-sector GM environment.

Both specifications retain negative annual mean semi-elasticities in all twelve cases. Fully imported investment raises the response to -0.308 under complete markets with local financ-

[24]For annual series $x^{\circ}_{i,a}$ and $y^{\circ}_{i,a}$ after removing year means, let $\bar{x}^{\circ}_i$ and $\bar{y}^{\circ}_i$ denote regional time means. The slope with both fixed effects can be written as $b_y = [\sum_{i,a} x^{\circ}_{i,a} y^{\circ}_{i,a} - A \sum_i \bar{x}^{\circ}_i \bar{y}^{\circ}_i]/S_{xx}$, with $S_{xx} = \sum_{i,a}(x^{\circ}_{i,a} - \bar{x}^{\circ}_i)^2$. Both reported parts use this same denominator; they partition the numerator of the estimated coefficient.

Table F.6: Military Spending Shocks: Parameter Sensitivity in GM

Specification	Parameter values	Complete markets Local financing	Incomplete markets Federal financing
Panel A. Benchmark and individual parameters			
Benchmark		−1.757	−1.654
More elastic labor supply	$\varphi = 1$	−1.006	−0.858
Higher depreciation	$\delta = 0.04$	−1.238	−1.164
Higher adjustment costs	$\tau = 8$	−1.111	−1.047
Panel B. Investment openness			
Investment imports	$\alpha_I = 0.8329167$	−0.796	−0.688
Investment imports	$\alpha_I = 0.69825$	−1.142	−1.033
Investment imports	$\alpha_I = 0.9309069$	−0.515	−0.410
Investment imports	$\alpha_I = 0.80$	−0.885	−0.777
Fully imported investment	$\alpha_I = 1$	−0.308	−0.206
Panel C. Joint parameter changes			
Labor, capital, and investment imports	$\varphi = 1$, $\delta = 0.04$, $\tau = 8$, $\alpha_I = 0.69825$	−0.210	−0.160
Labor, capital, and investment imports	$\varphi = 1$, $\delta = 0.04$, $\tau = 8$, $\alpha_I = 0.9309069$	−0.073	−0.027
Labor supply and investment imports	$\varphi = 0.5$, $\alpha_I = 0.7497$	−0.296	−0.164

NOTE.—Entries are mean annual semi-elasticities across 1,000 paired panels for each row, using equations (28) and (29). All rows retain $\alpha_C = 0.40$, $\theta = 0.75$, $\rho_G = 0.25$, and the benchmark innovation scale; unlisted parameters take their benchmark values. All twelve cases are retained in the prespecified order. The case $\alpha_I = 1$ is the fully imported investment limit. Panel C assesses joint sensitivity and does not represent a preferred recalibration. The empirical reference is 0.1604.

ing and −0.206 under incomplete markets with federal financing. The largest means are −0.073 and −0.027, respectively, obtained with $\varphi = 1$, $\delta = 0.04$, $\tau = 8$, and $\alpha_I = 0.9309069$. These joint variations bring the responses close to zero while leaving them below the empirical reference of 0.1604. They assess sensitivity and do not replace the benchmark calibration. The conclusion concerns the reported finite set of parameter combinations.

Fiscal persistence. We vary ρ_G over the same 17-point grid as in the ARRA exercise, keeping structural parameters at their benchmark values. Innovation scales hold fixed the expected residual variance of the annual fiscal regressor in the zero-initialized 250-year panel.

To make this normalization explicit, let $\Sigma_A^0(\rho)$ be the covariance matrix of A annual averages of an AR(1) process with unit innovation variance and fiscal state equal to zero before the first innovation. Define

$$V_A^0(\rho) = \frac{\mathrm{tr}[H_A \Sigma_A^0(\rho)]}{A-1}, \qquad H_A = I_A - \frac{\mathbf{1}_A \mathbf{1}_A'}{A}. \tag{F.7}$$

Independence across regional innovations implies that the expected sum of squares after removing both fixed effects is $(N-1)(A-1)\gamma^2\sigma_G^2 V_A^0(\rho)$.[25] We therefore set

$$\sigma_G(\rho_G) = \sigma_G(0.25)\left[\frac{V_{250}^0(0.25)}{V_{250}^0(\rho_G)}\right]^{1/2}. \tag{F.8}$$

This calculation uses V_{250}^0, while the original fiscal calibration uses V_{16}^{S}. The normalization preserves the benchmark's expected annual residual variance in the simulated panel, with the benchmark innovation scale unchanged. All persistence values use the same standardized draws in both specifications. Since the model is solved at first order, a common rescaling of innovations changes fiscal and investment deviations proportionally and leaves each regression slope unchanged.

Figure F.2 shows that both annual mean responses remain negative at $\rho_G = 0.95$, where they equal -0.304 under complete markets with local financing and -0.107 under incomplete markets with federal financing. Both become positive between 0.95 and 0.975. Linear interpolation places the zero crossings at approximately 0.9667 and 0.9555, respectively. The empirical reference is reached between 0.975 and 0.99 under complete markets and between 0.95 and 0.975 under incomplete markets, with interpolated crossings of 0.9754 and 0.9637. These are descriptive interpolations between evaluated points.

At $\rho_G = 0.975$, an innovation retains about 90.4% of its initial effect after four quarters and has a half-life of approximately 6.8 years. Thus, although incomplete markets with federal financing lower the persistence needed to approach the empirical magnitude, both specifications require a much longer-lived fiscal process than the transitory shocks motivating the benchmark calibration.

Under incomplete markets, the mean cross-region standard deviation of terminal bond payoffs rises from 18.98% of annual steady-state GDP at $\rho_G = 0.25$ to 57.75% at $\rho_G = 0.99$. Maximum absolute positions across dates, regions, and replications reach 81.65% and 236.09%, respectively. All positions are retained. These magnitudes delimit the interpreta-

[25] For $f_t = \rho f_{t-1} + e_t$ with $f_{-1} = 0$ and unit-variance innovations, $\mathrm{Cov}(f_t, f_s) = \rho^{|t-s|}[1-\rho^{2(\min(t,s)+1)}]/(1-\rho^2)$ for $t, s \geq 0$ and $0 \leq \rho < 1$. The entries of Σ_A^0 average these covariances over the sixteen quarter pairs in each pair of years. This retains the finite initial condition.

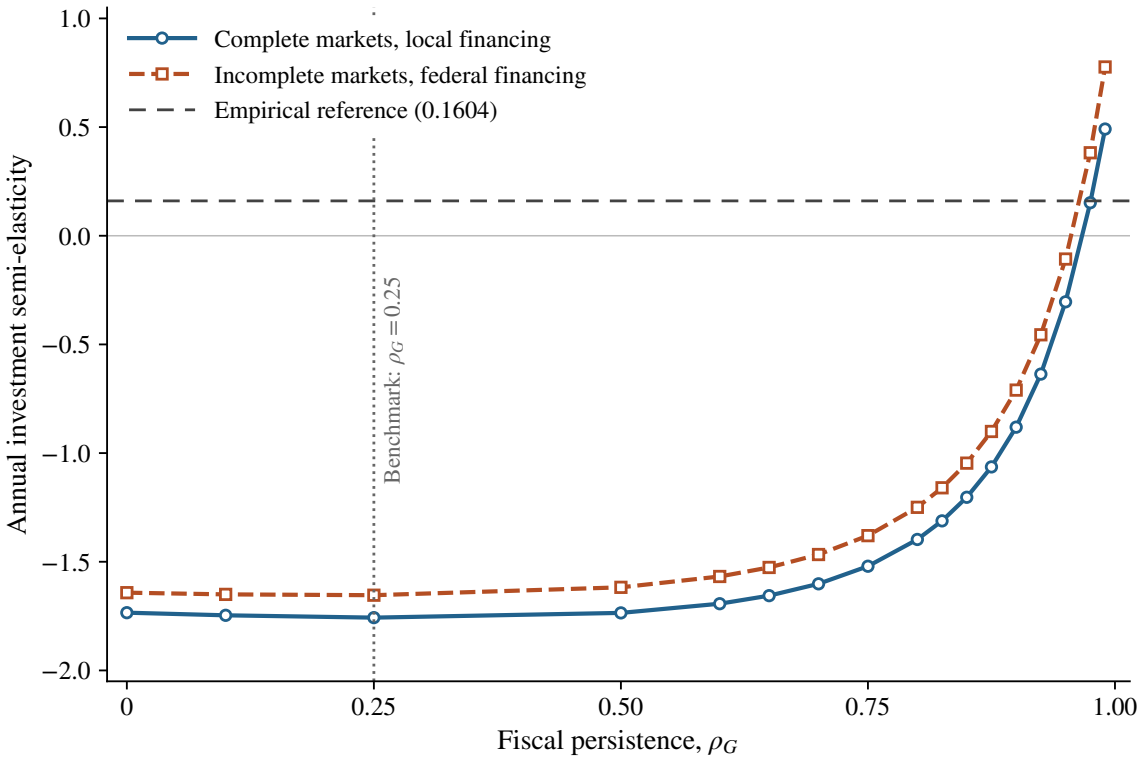


Figure F.2: Military Spending Shocks: Fiscal Persistence and Investment in GM

NOTE.—Lines report mean annual investment semi-elasticities across 1,000 paired panels for each specification. Each panel contains 100 regions and 250 years from the symmetric steady state, retaining all periods. Structural parameters remain at their benchmark values. Innovation scales follow equation (F.8), keeping expected annual fiscal variance after fixed effects constant. All cases share standardized draws. The horizontal dashed line is the empirical reference, 0.1604, and the vertical dotted line marks $\rho_G = 0.25$. Markers identify all 17 evaluated persistence values; lines connect these points.

tion of the persistence exercise under a first-order approximation.