Agglomeration, Productivity, and Exports: A Unified Framework with Spillovers*

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Abstract

Firm productivity and export participation reinforce each other over time, and both respond to the productivity and export experience of nearby firms. This paper develops a unified framework in which learning by exporting, productivity-based selection into exporting, local productivity spillovers, and export-network spillovers operate jointly within a dynamic productivity–export network. The two cross-firm channels generate transmission paths that cross from one network to the other—neighbors' export experience raises own productivity, and neighbors' productivity raises own export participation—which are absent by construction in frameworks that study either channel alone. We derive the first-round, finite-horizon, and long-run effects implied by the complete system and decompose its difference from partial models into the mechanical loss of transmission paths and the reallocation of parameters under re-estimation. We embed the network in a structural model of production and export decisions and develop a proxy-variable identification strategy for a multi-market revenue production function that requires neither physical output nor foreign prices: exporters' domestic output shares are recovered from the domestic marginal-revenue condition and the flexible-input first-order conditions, and the productivity process with endogenous exporting and network spillovers is estimated from quasi-differenced moment conditions. Using panel data on Chinese manufacturers over 1998–2007, we find that [to be finished: xxx].

Keywords: learning by exporting; productivity spillovers; export spillovers; agglomeration; production function estimation; spatial networks.

JEL Classification: C23, D24, F14, L25, R12.

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1 Introduction

Productivity and export participation are jointly determined at the firm level. More productive firms are more likely to enter foreign markets, consistent with models of self-selection into exporting (Roberts & Tybout, 1997; Melitz, 2003; Das et al., 2007; Aw et al., 2011), and firms often become more productive after they begin to export, a mechanism commonly referred to as learning by exporting (Clerides et al., 1998; Blalock & Gertler, 2004; Salomon & Shaver, 2005; De Loecker, 2007; De Loecker, 2013; Garcia-Marin & Voigtländer, 2019). Neither relationship is confined to the firm itself. Firms located in dense industrial clusters raise their productivity by learning from nearby peers (Greenstone et al., 2010; Combes et al., 2012; Malikov & Zhao, 2023; Baum-Snow et al., 2024; Giroud et al., 2024), and firms surrounded by experienced exporters face lower informational and entry costs and are more likely to export (Koenig et al., 2010; Fernandes & Tang, 2014). Each of these four mechanisms—learning by exporting, productivity-based selection into exporting, productivity spillovers, and export-network spillovers—has been studied extensively, but almost always in isolation.

This separation matters because the four mechanisms are not additive. Combining productivity spillovers with learning by exporting, or export-network spillovers with productivity-based selection, generates transmission paths that cross from one network to the other. A neighbor's export experience raises a firm's own productivity through two routes: the neighbor first becomes more productive and that gain spills over, or the firm itself is induced to export and then learns from doing so. Symmetrically, a neighbor's productivity raises a firm's own export participation, either through the productivity spillover or through the neighbor's induced entry into exporting. Such paths are undefined in a framework that contains only one of the two networks, and a framework that estimates both networks but never combines them does not quantify these paths. Because productivity and export participation are persistent and feed back on each other within the firm, cross-network paths are repeated and amplified over time. Estimates obtained from partial frameworks may therefore understate, or misattribute across channels, the overall effects of exporting and agglomeration.

This paper develops a unified framework in which the four mechanisms operate jointly within a dynamic productivity–export network, and it provides an identification and estimation strategy for the structural parameters that govern them. We proceed in three steps.

First, we characterize the interdependent network analytically. A parsimonious linear system consisting of a productivity law of motion and an export-participation equation, each containing an own lag, a within-firm cross term, and a peer-weighted network term, yields closed-form expressions for the shortest cross-network transmission paths, for finite-horizon responses to an export-market-access shock, and, under dynamic stability, for infinite-horizon cumulative and steady-state multipliers. The complete-system productivity multiplier factorizes into the conditional multiplier of a productivity-only model that treats the change in exporting as given and the endogenous export

response of the complete system, which makes precise what a one-equation framework captures and what it leaves unspecified. We further decompose the difference between the responses of the complete system and of a restricted model that omits a spillover channel into two components: the mechanical loss of transmission paths, evaluated at the complete-system parameters, and the reallocation of the remaining parameters when the restricted model is re-estimated.

Second, we embed this network in a structural model of firm production and export decisions. Firms produce a single output with a common technology, sell it in a domestic market with constant-elasticity demand and in an aggregate foreign market whose demand is left unrestricted, and choose flexible inputs, investment, and export status subject to fixed and sunk export costs. Productivity follows a controlled first-order Markov process in which the firm's own lagged export status and the lagged productivity of its neighbors enter as predetermined shifters (De Loecker, 2013; Doraszelski & Jaumandreu, 2013; Malikov & Zhao, 2023). The export decision depends on current productivity and on the lagged export participation of neighbors, through which firms with incomplete information about foreign demand learn from local peers.

Third, we develop a proxy-variable identification strategy for this multi-market environment that requires neither physical output nor foreign prices. Exporters' revenue combines domestic and foreign sales, so deflating total revenue by an industry price index recovers neither physical output nor a valid revenue production function. We instead work with domestic revenue alone, in which the only latent objects are productivity and the share of output that the firm allocates to the domestic market. The flexible-input revenue shares of non-exporters identify the scaled labor and material elasticities as in Gandhi et al. (2020); a control function in the spirit of Ackerberg et al. (2015) recovers the domestic demand shock of exporters; and the domestic marginal-revenue condition, combined with the flexible-input first-order conditions, then delivers each exporter's domestic output share. The capital elasticity, the demand parameter that separates production elasticities from their revenue-scaled counterparts, and the productivity process—including the learning-by-exporting and productivity-spillover coefficients—are identified from quasi-differenced moment conditions on the productivity innovation, with region fixed effects absorbing correlated local effects that would otherwise contaminate the spillover coefficient. Restricting attention to non-exporters would avoid the output-allocation problem but would select the sample on latent productivity; our strategy retains exporting observations and recovers their domestic shares explicitly. The export equation is estimated as a dynamic linear probability model with firm and year effects, first differenced and instrumented with twice-lagged export status, productivity, and peer exposure. Because the steps are sequential and productivity enters the export equation as a generated regressor, inference is based on a cluster bootstrap over neighborhoods that re-runs the entire procedure.

The framework does not require that the productivity and export equations be estimated simultaneously, and its empirical value does not hinge on large differences in primitive coefficients across specifications. Its value lies in combining consistently estimated productivity and export

processes into a single dynamic system, and thereby quantifying transmission and feedback paths that are undefined or mechanically absent in partial models. Re-estimating the restricted specifications reveals, in addition, whether the remaining coefficients absorb part of the omitted mechanism. These two exercises—deleting a channel at the complete-system estimates and comparing with the re-estimated partial model—are the empirical counterparts of the analytical decomposition.

We apply the framework to Chinese manufacturers over 1998–2007. Neighborhoods are defined by three-digit postal codes, and in the baseline network a firm learns only from neighbors that are more profitable than itself. We find xxx

Our contributions relate to three literatures. The first is the structural estimation of production functions and productivity (Olley & Pakes, 1996; Levinsohn & Petrin, 2003; Ackerberg et al., 2015; Gandhi et al., 2020), and in particular revenue-based estimation under imperfect competition (De Loecker, 2011; Grieco et al., 2016) and estimation with multiple outputs or markets (Orr, 2022; Valmari, 2023). We show that a revenue production function remains identifiable when firms sell in a foreign market with unobserved prices, by recovering the domestic output share from optimality conditions rather than by assuming it away, and we extend the controlled Markov process of Doraszelski & Jaumandreu (2013) and De Loecker (2013) to include both endogenous export status and network spillovers (Malikov & Zhao, 2023; Li et al., 2022). The second is the literature on learning by exporting and export dynamics (Clerides et al., 1998; De Loecker, 2007; Aw et al., 2011; De Loecker & Warzynski, 2012; De Loecker, 2013; Garcia-Marin & Voigtländer, 2019) and on local export spillovers (Koenig et al., 2010; Fernandes & Tang, 2014). Relative to this work, we allow the productivity gains from exporting to propagate to neighbors and feed back into export participation, and we quantify the resulting cumulative effects rather than the within-firm effect alone. The third is the literature on industrial agglomeration and spatial productivity spillovers (Greenstone et al., 2010; Kantor & Whalley, 2014; Hou et al., 2020; Malikov & Zhao, 2023; Baum-Snow et al., 2024; Giroud et al., 2024). Whereas Giroud et al. (2024) study the propagation of local productivity spillovers through firms' internal networks, we study propagation through the interaction between a productivity network and an export network, and we show that the cross-network component is exactly what single-network studies omit.

The remainder of the paper proceeds as follows. Section 2 develops a parsimonious system of interdependent productivity and export networks and characterizes the cross-network transmission and feedback effects omitted when the two processes are analyzed separately. Section 3 presents the structural model of firm production and export participation, incorporating learning-by-exporting, productivity spillovers, and export-network spillovers. Section 4 establishes identification of the model, and Section 5 details the estimation procedures. Section 6 describes the data. Section 7 presents the empirical estimates and comparisons between the complete system and the corresponding partial models. Section 8 concludes. Additional derivations, robustness checks, and results are provided in the appendices.

2 Productivity and Export Network

This section introduces a parsimonious linear benchmark that characterize the interdependent network linking firm productivity and export participation. The benchmark keeps the key features but is not yet the structural production model used for identification and estimation in the later sections. Its purpose is to isolate the mechanisms through which learning by exporting, productivity-based selection into exporting, productivity spillovers, and export-network spillovers jointly generate within-firm feedback, cross-firm transmission, and dynamic propagation. It thereby clarifies the conceptual limitations of analyzing productivity and export spillovers separately. We first characterize the shortest cross-network transmission paths and then derive finite- and infinite-horizon system multipliers. Section 3 embeds these mechanisms in a structural model of firm production and export participation.

We assume that firm $i \in \{1, \ldots, n\}$ in period $t \in \{1, \ldots, T\}$ has the following productivity law of motion:

$$\omega_{it} = \alpha_0 + \alpha_1 \, \omega_{i,t-1} + \alpha_2 \, e_{i,t-1} + \alpha_3 \sum_{j \in \mathcal{N}(i)} s_{ij,t-1} \omega_{j,t-1} + v_{it}, \tag{2.1}$$

which allows the firm to improve its productivity ω_{it} through both its own activities and learning from neighboring firms. Here, $e_{i,t-1}$ is an indicator for firm i's export participation in period $t-1$, $\mathcal{N}(i)$ denotes the set of neighborhoods for firm i, and $\{s_{ij,t-1}, j \neq i\}$ are peer weights. The inclusion of $e_{i,t-1}$ in the productivity process reflects the idea that firms enhance productivity through export experience—a mechanism commonly referred to as "learning-by-exporting," which has been widely studied by policymakers and researchers (see the review by De Loecker, 2013).

The weighted average of neighboring firms' productivity, $\sum_{j \in \mathcal{N}(i)} s_{ij,t-1} \omega_{j,t-1}$, captures local productivity spillovers and induces cross-sectional dependence in ω_{it}. This formulation follows Malikov & Zhao (2023). Neighborhoods $\mathcal{N}(i)$ and peer weights $\{s_{ij,t-1}\}$ can be defined in several ways. When neighbors are defined in terms of geographical proximity, this weighted average reflects agglomeration effects, whereby firms improve productivity by learning from nearby peers. Detailed definitions of $\mathcal{N}(i)$ and $\{s_{ij,t-1}\}$ in our application are provided in the estimation section 5.3.

Following a similar logic, we summarize the firm's export decision using the following linear network representation:

$$e_{it} = \gamma_0 + \gamma_1 \, e_{i,t-1} + \gamma_2 \, \omega_{it} + \gamma_3 \sum_{j \in \mathcal{N}(i)} s_{ij,t-1} e_{j,t-1} + u_{it}. \tag{2.2}$$

Equation (2.2) can be interpreted as a linear probability model for the conditional probability of exporting. The coefficient γ_1 captures persistence in export participation, which may arise from sunk entry costs, accumulated export experience, and persistent firm-level characteristics. The inclusion of current productivity, ω_{it}, captures productivity-based selection into exporting: more productive firms are better able to cover the fixed and sunk costs associated with serving foreign

markets. This timing assumes that firm i observes its period-t productivity before making its export-participation decision.

The weighted average of neighboring firms' lagged export status, $\sum_{j\in\mathcal{N}(i)} s_{ij,t-1}e_{j,t-1}$, captures local export-network spillovers.[1] When the peer weights are normalized to sum to one, this term can be interpreted as the weighted share of firm i's neighbors that exported in the preceding period. Nearby exporters may transmit information about foreign demand, market regulations, distribution channels, logistics, and potential trading partners. Such information reduces uncertainty and lowers the informational and entry costs faced by prospective exporters, thereby increasing their probability of entering foreign markets (Koenig et al., 2010; Fernandes & Tang, 2014). The disturbance u_{it} captures other unobserved determinants of firm i's export decision.

2.1 First-round Interactions Between the Productivity and Export Networks

Equations (2.1) and (2.2) jointly describe an interconnected dynamic system in which productivity and export participation interact both within and across firms. The coefficients α_2 and γ_2 govern the within-firm *direct* effects between exporting and productivity. In particular, α_2 captures learning by exporting, whereas γ_2 captures productivity-based selection into exporting. The coefficients α_3 and γ_3 govern the two cross-firm channels: productivity spillovers and export-network spillovers, respectively.

Combining these four mechanisms generates *indirect* cross-firm effects that operate through both networks. Because export participation is binary, the following derivatives with respect to an export indicator should be interpreted as discrete changes from non-exporting to exporting.

We first consider how firm j's export participation affects firm i's subsequent productivity. A change in $e_{j,t-2}$ can affect ω_{it} through two distinct pathways:

(1) $e_{j,t-2} \longrightarrow e_{i,t-1} \longrightarrow \omega_{it}$: Firm j's export experience raises firm i's probability of exporting through the export network, after which firm i benefits from its own export experience;

(2) $e_{j,t-2} \longrightarrow \omega_{j,t-1} \longrightarrow \omega_{it}$: Firm j first improves its own productivity through exporting, and the resulting productivity gain subsequently spills over to firm i.

Putting them together,

$$\frac{\partial \mathbb{E}[\omega_{it} \mid \cdot]}{\partial e_{j,t-2}} = \underbrace{\gamma_3\ \alpha_2\ s_{ij,t-2}}_{\substack{\text{export-network spillover} \\ \text{followed by learning by exporting}}} + \underbrace{\alpha_2\ \alpha_3\ s_{ij,t-1}}_{\substack{\text{learning by exporting} \\ \text{followed by a productivity spillover}}} . \tag{2.3}$$

[1] In principle, productivity spillovers and export-information spillovers may operate through distinct networks. We use common peer weights $\{s_{ij,t-1}\}$ for expositional parsimony and because both mechanisms are defined over the same geographically determined peer set $\mathcal{N}(i)$ in our empirical application. Allowing the networks to differ leaves the logic of the following analysis unchanged.

If the peer weights are row normalized in each period, so that $\sum_{j\in\mathcal{N}(i)} s_{ij,t-2} = \sum_{j\in\mathcal{N}(i)} s_{ij,t-1} = 1$, the aggregate first-round effect of neighboring firms' exports on firm i's productivity is

$$\mathcal{T}^{e\to\omega} \equiv \sum_{j\in\mathcal{N}(i)} \frac{\partial \mathbb{E}[\omega_{it} \mid \cdot]}{\partial e_{j,t-2}} = \gamma_3\,\alpha_2 + \alpha_2\,\alpha_3 = \alpha_2\,(\gamma_3 + \alpha_3). \tag{2.4}$$

It is the first-round effect in the sense that it captures the shortest paths through which a change at one firm affects another firm across both networks. It does not yet incorporate repeated propagation through productivity persistence, export persistence, or higher-order network connections, which we will discuss in detail in the later subsection.

We next consider how firm j's productivity affects firm i's export participation. Again, two pathways arise:

(1) $\omega_{j,t-1} \longrightarrow \omega_{it} \longrightarrow e_{it}$: Higher productivity of firm j raises firm i's productivity through the productivity network, thereby increasing firm i's propensity to export.

(2) $\omega_{j,t-1} \longrightarrow e_{j,t-1} \longrightarrow e_{it}$: Higher productivity makes firm j more likely to export, and firm j's export experience subsequently raises firm i's export propensity through the export network.

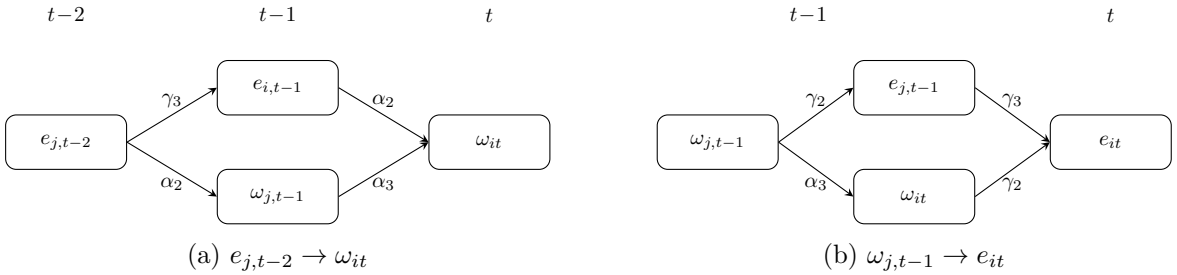
Jointly, we have

$$\frac{\partial \mathbb{E}[e_{it} \mid \cdot]}{\partial \omega_{j,t-1}} = \underbrace{\alpha_3\,\gamma_2\,s_{ij,t-1}}_{\substack{\text{productivity spillover} \\ \text{followed by productivity-based export entry}}} + \underbrace{\gamma_2\,\gamma_3\,s_{ij,t-1}}_{\substack{\text{productivity-based export entry} \\ \text{followed by an export-network spillover}}}. \tag{2.5}$$

Under row-normalized peer weights, the corresponding aggregate first-round effect is

$$\mathcal{T}^{\omega\to e} \equiv \sum_{j\in\mathcal{N}(i)} \frac{\partial \mathbb{E}[e_{it} \mid \cdot]}{\partial \omega_{j,t-1}} = \alpha_3\,\gamma_2 + \gamma_2\,\gamma_3 = \gamma_2\,(\alpha_3 + \gamma_3). \tag{2.6}$$

Figure 1 summarizes these four transmission pathways.



(a) $e_{j,t-2} \to \omega_{it}$ (b) $\omega_{j,t-1} \to e_{it}$

Figure 1: First-round transmission channels between productivity and export participation

2.1.1 Consequences of omitting a spillover channel.

The joint system makes clear what is lost when either cross-firm channel is omitted from considering a single equation instead of the system. Table 1 reports the aggregate first-round effects implied by the complete system and by two restricted partial specifications.

Table 1: First-Round Transmission Effects under Partial Models

Transmission effect	Full model ($\mathcal{M}_F$)	No export network ($\mathcal{M}_{R_\gamma} : \gamma_3 = 0$)	No productivity spillovers ($\mathcal{M}_{R_\alpha} : \alpha_3 = 0$)
Neighbors' exports → own productivity	$\alpha_2\,(\gamma_3 + \alpha_3)$	$\alpha_2\,\alpha_3$	$\alpha_2\,\gamma_3$
Neighbors' productivity → own exporting	$\gamma_2\,(\alpha_3 + \gamma_3)$	$\gamma_2\,\alpha_3$	$\gamma_2\,\gamma_3$

Notes: The entries report aggregate first-round effects under row-normalized peer weights. Let $\mathcal{M}_F$ denote the complete system and $\mathcal{M}_{R_\theta}$ a restricted model with restriction on parameter θ.

A specification without export-network spillovers retains the pathway operating through neighboring firms' productivity but omits the pathway operating through neighboring firms' export experience. Conversely, a specification without productivity spillovers retains the export-network pathway but omits the pathway operating through cross-firm productivity transmission. Therefore, when the four underlying effects are positive, either restricted model understates the total cross-firm transmission implied by the complete system.

To make this distinction concrete, consider a policy-relevant question: how does an expansion in exporting induced by an external shock, such as a tariff reduction, affect firm productivity? Suppose that the shock is represented by an additive common shifter entering the export equation (2.2), and let d_h^e denote its value at horizon h, with $h = 0$ corresponding to the impact period. As shown in Appendix B, for our complete system, the productivity response at horizon $h = 2$, two periods after the shock, can be written as

$$\Delta\omega = \underbrace{\alpha_2\,(\alpha_1 + \gamma_1 + \gamma_2\alpha_2)\,d_0^e}_{\text{within-firm persistence and feedback}} + \underbrace{\alpha_2\,(\alpha_3 + \gamma_3)\,d_0^e}_{\text{first-round cross-firm transmission}} + \underbrace{\alpha_2\,d_1^e}_{\text{response to a renewed intervention}}, \tag{2.7}$$

where the meaning of each component is labeled explicitly. Thus, the first-round effect reported in Table 1 is the cross-firm component of the horizon-two productivity response, rather than the complete response at that horizon. Equation (2.7) provides an exact illustration of the mechanical loss caused by deleting a spillover channel: imposing $\gamma_3 = 0$ removes $\alpha_2\gamma_3 d_0^e$ from the horizon-two productivity response, whereas imposing $\alpha_3 = 0$ removes $\alpha_2\alpha_3 d_0^e$. At this short horizon, the terms capturing firm-level persistence and feedback, as well as the direct effect, remain unchanged.

More generally, there are two conceptually distinct consequences of using a partial framework. First, holding the parameters of the complete system fixed, deleting a spillover channel mechanically eliminates all transmission paths that require that channel. Second, when the restricted model is re-

estimated, the remaining parameters may absorb part of the omitted mechanism, and the recovered productivity and export paths may also change. These two consequences can be separated formally in the following decomposition.

For example, let

$$\Delta\omega_{i,h}(\mathcal{M},\boldsymbol{\theta};\boldsymbol{d}) \equiv \omega_{i,t+h}(\mathcal{M},\boldsymbol{\theta};\boldsymbol{d}) - \omega_{i,t+h}(\mathcal{M},\boldsymbol{\theta};\mathbf{0})$$

denote firm i's productivity response at horizon h to an external shock path $\boldsymbol{d}$ under model $\mathcal{M}$ and parameter vector $\boldsymbol{\theta}$. Let $\mathcal{M}_F$ denote the full/complete system and $\mathcal{M}_R$ a restricted model, with estimated parameter vectors $\widehat{\boldsymbol{\theta}}_F$ and $\widehat{\boldsymbol{\theta}}_R$, respectively. Finally, let $\widehat{\boldsymbol{\theta}}_{F\to R}$ denote the parameter vector obtained by imposing the restriction associated with $\mathcal{M}_R$ on the complete-system estimates while holding all remaining complete-system parameters fixed. The difference between the complete-system response and the response generated by the re-estimated restricted model can then be decomposed as

$$\begin{aligned}
&\Delta\omega_{i,h}\big(\mathcal{M}_F,\widehat{\boldsymbol{\theta}}_F;\boldsymbol{d}\big) - \Delta\omega_{i,h}\big(\mathcal{M}_R,\widehat{\boldsymbol{\theta}}_R;\boldsymbol{d}\big) \\
&= \underbrace{\Delta\omega_{i,h}\big(\mathcal{M}_F,\widehat{\boldsymbol{\theta}}_F;\boldsymbol{d}\big) - \Delta\omega_{i,h}\big(\mathcal{M}_R,\widehat{\boldsymbol{\theta}}_{F\to R};\boldsymbol{d}\big)}_{\text{mechanical loss of transmission pathways}} + \underbrace{\Delta\omega_{i,h}\big(\mathcal{M}_R,\widehat{\boldsymbol{\theta}}_{F\to R};\boldsymbol{d}\big) - \Delta\omega_{i,h}\big(\mathcal{M}_R,\widehat{\boldsymbol{\theta}}_R;\boldsymbol{d}\big)}_{\text{effect of re-estimating the restricted model}}. \quad (2.8)
\end{aligned}$$

The first component isolates the dynamic consequences of removing the channel while retaining the remaining complete-system estimates. The second component captures parameter reallocation induced by estimating the restricted specification. Its sign is not determined in general because it depends on the relationship between the omitted network variable and the included regressors, as well as on how changes in the estimated parameters propagate through the dynamic system.

The horizon-two expressions above are special cases of the mechanical component in Equation (2.8). The entries in Table 1 characterize the mechanical loss of the shortest transmission paths evaluated at the complete-system parameters; they should not be interpreted as effects evaluated at the probability limits of separately re-estimated partial models. At longer horizons, deleting either spillover channel also removes higher-order paths generated by productivity persistence, export persistence, repeated within-firm feedback, and further rounds of cross-firm propagation. The exact finite-horizon responses are therefore obtained by recursively iterating the productivity and export equations, as shown in Appendix B. In the empirical analysis, we implement the decomposition in Equation (2.8) for a short finite horizon alongside the comparison of the shortest-path effects reported in Table 1, thereby complementing the infinite-horizon analysis developed in the next subsection.

In addition, this comparison concerns the omission of an economic channel rather than whether the productivity and export equations are estimated simultaneously or sequentially. A sequential procedure can preserve all four transmission paths, provided that both equations are correctly specified and consistently estimated and that their parameters are subsequently combined within

the complete dynamic system. The value of our unified framework therefore lies not in simultaneous estimation per se, but in its explicit representation and quantification of the full set of transmission and feedback mechanisms. This structure also allows us to distinguish the mechanical loss of pathways caused by deleting a channel from the changes in the remaining primitive parameters that arise when the corresponding restricted model is re-estimated.

2.2 Dynamic Propagation and Long-Run Effects

The preceding subsection characterizes the shortest transmission paths linking firms' productivity and export participation. These first-round effects do not capture the repeated propagation of a shock through productivity persistence, export persistence, and higher-order network connections. We now characterize these dynamic effects by iterating the complete productivity–export system.

For this analysis, we hold the network fixed and collect the peer weights $\{s_{ij}\}$ in the $n \times n$ matrix S.[2] Define

$$B_\omega \equiv \alpha_1 I_n + \alpha_3 S, \qquad B_e \equiv \gamma_1 I_n + \gamma_3 S,$$

where I_n is an $n \times n$ identity matrix. Let $\boldsymbol{x}_t$ denote an $n \times 1$ vector of exogenous shifters of firms' export decisions, with coefficient δ_x. Here, $\boldsymbol{x}_t$ may represent a policy intervention of interest which induces, for example, an exogenous change in foreign demand or export costs.[3] We focus on deviations/changes from a baseline path, so that constants and mean-zero innovations are suppressed. The dynamic system can be written in a vector form as

$$\Delta\boldsymbol{\omega}_t = B_\omega \Delta\boldsymbol{\omega}_{t-1} + \alpha_2 \Delta\boldsymbol{e}_{t-1}, \tag{2.9}$$

$$\Delta\boldsymbol{e}_t = B_e \Delta\boldsymbol{e}_{t-1} + \gamma_2 \Delta\boldsymbol{\omega}_t + \delta_x \Delta\boldsymbol{x}_t, \tag{2.10}$$

where Δ is a difference operator, and $\boldsymbol{\omega}_t$ and $\boldsymbol{e}_t$ are vectors stacked across n firms. Let $\Delta\boldsymbol{z}_t \equiv [\Delta\boldsymbol{\omega}_t', \Delta\boldsymbol{e}_t']'$. Substituting Equation (2.9) into Equation (2.10) gives

$$\Delta\boldsymbol{z}_t = A\Delta\boldsymbol{z}_{t-1} + G\Delta\boldsymbol{x}_t, \tag{2.11}$$

where

$$A = \begin{bmatrix} B_\omega & \alpha_2 I_n \\ \gamma_2 B_\omega & B_e + \gamma_2 \alpha_2 I_n \end{bmatrix}, \qquad G = \begin{bmatrix} 0_{n\times n} \\ \delta_x I_n \end{bmatrix}. \tag{2.12}$$

Accordingly, successive powers of A encode progressively longer transmission paths through the interconnected productivity and export networks, as detailed below. A derivation of Equations (2.11)

[2] With a time-varying network, the h-period response is governed by ordered products of period-specific transition matrices rather than by a single matrix power. A closed-form steady-state multiplier therefore generally requires a fixed or convergent network.

[3] Here, we introduce an exogenous shock to the export equation and trace its effects on productivity and export participation. An analogous exercise can be conducted for a shock to the productivity equation; because the derivation is similar, we omit it for brevity.

and (2.12) is provided in Appendix A.

2.2.1 Propagation through networks.

Consider first a one-time shock $\Delta \boldsymbol{x}_t = \boldsymbol{d}_x$, followed by $\Delta \boldsymbol{x}_{t+h} = \mathbf{0}$ for all $h \geq 1$. Its period-$(t+h)$ effect is

$$\Delta \boldsymbol{z}_{t+h} = A^h G \boldsymbol{d}_x, \qquad h = 0, 1, 2, \ldots. \tag{2.13}$$

Provided that the system is stable, the long-run level response to this one-time shock converges to zero: $\lim_{h\to\infty} \Delta \boldsymbol{z}_{t+h} = \mathbf{0}$.

Nevertheless, the shock may affect productivity and export participation for several periods before the system returns to its baseline. The cumulative response summarizes this transitional impact:

$$\sum_{h=0}^{\infty} \Delta \boldsymbol{z}_{t+h} = (I_{2n} - A)^{-1} G \boldsymbol{d}_x. \tag{2.14}$$

Now suppose that the same shock is permanent, so that $\Delta \boldsymbol{x}_{t+h} = \boldsymbol{d}_x$ for every $h \geq 0$. The response at horizon h is

$$\Delta \boldsymbol{z}_{t+h} = \sum_{r=0}^{h} A^r G \boldsymbol{d}_x,$$

which converges to the *steady-state* response

$$\Delta \boldsymbol{z}^* = (I_{2n} - A)^{-1} G \boldsymbol{d}_x. \tag{2.15}$$

Thus, the cumulative response to a one-time shock and the steady-state response to a permanent shock contain the same matrix multiplier, but they represent different economic experiments. The former sums a sequence of transitory responses, whereas the latter measures a permanent change in long-run levels.[4]

Under additional simplifying assumptions, the matrix representation reduces to a more transparent scalar form. Consider a permanent export shifter that is common across firms, $\boldsymbol{x} = x\mathbf{1}_n$, and suppose that the network matrix is row normalized, so that $S\mathbf{1}_n = \mathbf{1}_n$. Provided that the joint system is dynamically stable, the common steady-state responses are

$$\frac{\partial \omega^*}{\partial x} = \frac{\alpha_2 \delta_x}{(1-\alpha_1-\alpha_3)(1-\gamma_1-\gamma_3) - \alpha_2\gamma_2}, \tag{2.16}$$

$$\frac{\partial e^*}{\partial x} = \frac{(1-\alpha_1-\alpha_3)\delta_x}{(1-\alpha_1-\alpha_3)(1-\gamma_1-\gamma_3) - \alpha_2\gamma_2}. \tag{2.17}$$

[4]The matrix formulation also accommodates firm-pairwise interventions. If only firm j experiences the shock, the effect on firm i is determined by the (i, j) element of the relevant matrix multiplier. The explicit firm-level expressions are reported in Appendix A.

These closed-form expressions make transparent how parameters related to productivity persistence, export persistence, the two spillover channels, and the within-firm productivity–export feedback jointly determine the long-run responses. They also provide a natural benchmark for comparison with the more restrictive partial models considered next.

2.2.2 From a Conditional Productivity-Network Multiplier to the Complete-System Multiplier

Having characterized the long-run response in the complete system, we next consider how a more restrictive partial framework yields a different multiplier. To conserve space, we focus on the productivity response to an externally imposed change in export participation; the reverse exercise can be derived analogously.

Consider a partial model that retains only the productivity network in Equation (2.1). Suppose that export participation increases permanently by the vector $\boldsymbol{d}_e$, with the magnitude of this change treated as exogenously given rather than determined by the export equation. The productivity equation alone implies

$$\Delta\boldsymbol{\omega}^*_{\text{cond}} = [I_n - B_\omega]^{-1}\,\alpha_2\boldsymbol{d}_e. \tag{2.18}$$

According to the definition of B_ω, this conditional multiplier captures learning by exporting, own-productivity persistence, and repeated propagation through the productivity network. It does not, however, determine how export participation responds endogenously to the underlying economic shock.

Suppose, in addition, that S is row normalized and that all firms experience the same permanent change in export participation, $\boldsymbol{d}_e = d_e\mathbf{1}_n$. Equation (2.18) then reduces to

$$\Delta\omega^*_{i,\text{cond}} = \frac{\alpha_2}{1-\alpha_1-\alpha_3}d_e, \qquad i = 1,\ldots,n. \tag{2.19}$$

Thus, $\alpha_2/(1-\alpha_1-\alpha_3)$ measures the steady-state productivity response per unit of an imposed permanent common change in export participation. This expression is not the long-run multiplier of the complete productivity–export system; rather, it is one component of that multiplier.

To see the connection, the complete-system productivity response in Equation (2.16) can be factorized as

$$\frac{\partial\omega^*}{\partial x} = \underbrace{\frac{\alpha_2}{1-\alpha_1-\alpha_3}}_{\substack{\text{productivity response per unit}\\ \text{of an imposed export change}}} \times \underbrace{\frac{\partial e^*}{\partial x}}_{\substack{\text{endogenous export response}\\ \text{in the complete system}}}. \tag{2.20}$$

The first component is the conditional productivity-network multiplier: it determines how a given change in export participation propagates through learning by exporting, own-productivity persistence, and productivity spillovers. The second component determines how the underlying exogenous shifter changes export participation through export persistence, export-network spillovers,

and feedback from productivity. A framework based only on the productivity equation captures the first propagation margin but leaves the endogenous export-response margin unspecified. The complete productivity response therefore requires both network equations.

This factorization also clarifies the consequences of omitting a spillover channel. Setting $\alpha_3 = 0$ eliminates repeated propagation through the productivity network, whereas setting $\gamma_3 = 0$ eliminates propagation through the export network. Either restriction changes not only the first-round transmission effects discussed in the preceding subsection but also the cumulative and long-run effects generated by the dynamic system.

Finally, the existence of these long-run multipliers requires dynamic stability. For the conditional productivity model, the relevant condition is $\rho(B_\omega) < 1$. For the complete productivity–export system, the corresponding condition is $\rho(A) < 1$, where $\rho(\cdot)$ denotes the spectral radius, defined as the largest absolute value of a matrix's eigenvalues. If the relevant stability condition fails, finite-horizon responses remain well defined, but the infinite-horizon cumulative and steady-state effects do not. In our empirical application, where the observed time horizon is finite, we therefore complement the theoretical long-run multipliers with a finite-horizon analysis. This exercise examines how jointly modeling the complete productivity–export network alters the evolution of productivity and export participation relative to more restrictive partial specifications.

3 A Structural Model of Firm Production and Exporting

The preceding section characterizes the interdependence between firm productivity and export participation within a networked system. We now develop a structural model of firm production and export decisions that embeds these productivity–export linkages and provides the foundation for the identification and estimation strategy developed below.

3.1 Production Technology

We assume that, in period t, firm i combines capital (K), labor (L), and materials (M) to produce a single output (Q) according to the following gross-output production function:

$$Q_{it} = F(K_{it}, L_{it}, M_{it}) \exp\{\omega_{it}\}, \tag{3.1}$$

where ω_{it} denotes firm-specific persistent productivity.[5] Following the proxy-variable literature on production-function identification, we assume that productivity evolves according to a first-order Markov process (Olley & Pakes, 1996; Levinsohn & Petrin, 2003; Ackerberg et al., 2015; Gandhi et al., 2020), as specified in Equation (2.1). Relative to the conventional exogenous Markov specification, we allow the firm's lagged export participation and the lagged productivity of neighboring firms to enter as predetermined productivity shifters (De Loecker, 2013; Doraszelski & Jaumandreu, 2013; Li et al., 2022). Let $\mathcal{I}_{i,t-1}$ denote the information available for firm i in period $t-1$ when it makes its production and export decisions, before the realization of the period-t productivity innovation. The Markov assumption then implies

$$\mathbb{E}[v_{it} \mid \mathcal{I}_{i,t-1}] = 0. \tag{3.2}$$

Accordingly, v_{it} captures an unanticipated productivity innovation with zero conditional mean given all predetermined information.

Although ω_{it} is unobserved by the econometrician, it is observed by firm i when the firm makes its input decisions. Productivity is therefore generally correlated with the firm's choices of (K_{it}, L_{it}, M_{it}), giving rise to the standard simultaneity problem in production-function estimation.

We adopt the widely used Cobb–Douglas specification for $F(\cdot)$ (see, e.g., Olley & Pakes, 1996; Levinsohn & Petrin, 2003; Doraszelski & Jaumandreu, 2013; Ackerberg et al., 2015; Malikov & Zhao, 2023), which yields the following log-linear production function:

$$q_{it} = \beta_k k_{it} + \beta_l l_{it} + \beta_m m_{it} + \omega_{it}, \tag{3.3}$$

where lowercase letters denote the logarithms of their uppercase counterparts.

3.2 Domestic and International Markets

In each period t, firm i sells its output in the domestic market and, if it exports, in one or more foreign markets. For the domestic market, we assume that the firm faces monopolistic competition and have the following constant-elasticity demand curve :

$$Q_{it}^D = Q_{st}^D \left(\frac{P_{it}^D}{P_{st}^D} \right)^{\eta^D} \exp\{\xi_{it}^D\}, \tag{3.4}$$

[5] Following Grieco et al. (2016), Orr (2022), and Valmari (2023), we abstract from measurement error in physical output. In a multi-market setting, where Q_{it} aggregates quantities sold across markets, allowing for such error may require additional assumptions about how measurement errors in market-specific quantities arise and aggregate. This simplification is also motivated based on our identification strategy, which is based on a revenue production function: output measurement error would enter the revenue equation jointly with the demand disturbance and would generally be difficult to distinguish from the demand shock without further restrictions. Because our analysis focuses on the dynamic linkages between productivity and export participation rather than on markup measurement, for which output measurement error is especially consequential, we maintain this simplifying assumption.

where the superscript D denotes the domestic market, and s indexes the firm's industry or sector. The variables P_{st}^D and Q_{st}^D denote the corresponding aggregate price and quantity indices, respectively, with Q_{st}^D serving as an aggregate demand shifter. The term ξ_{it}^D denotes an idiosyncratic domestic-demand innovation that is unobserved by the econometrician and unanticipated when the firm makes its period-t production decisions. It is assumed independent of the firm's information set: $\mathbb{P}_\xi(\xi_{it} \mid \mathcal{I}_{it}) = \mathbb{P}_\xi(\xi_{it})$, and $\eta \leq -1$ is the price elasticity of demand.

The above Dixit–Stiglitz demand system is widely used in studies that relax the assumption of perfect competition when firm-level output prices are unavailable (see, e.g., Aw et al., 2011; De Loecker, 2011; Grieco et al., 2016). In the domestic market, the industry-level price index P_{st}^D is often observable, whereas Q_{st}^D can be constructed, for example, through a market-share-weighted average of firms' deflated output. The assumption that ξ_{it}^D is independent of the firm's information set is stronger than necessary for identification. More generally, demand may vary systematically with observable nonproduction determinants, such as firm characteristics, which can be included directly as demand shifters. Identification then requires a slightly modified conditional-mean restriction on the remaining demand shocks.

Characterizing foreign demand presents additional challenges. Firms often export to multiple destinations, making it difficult for researchers to observe the complete set of destination-specific demand variables $\{P_{st}^{F_c}, Q_{st}^{F_c}, \eta^{F_c}\}$ for each destination $c \in \mathcal{C}$. Moreover, firms may have incomplete information about foreign-market conditions before entering export markets. Because our following identification strategy does not require a parametric specification of foreign demand, we leave foreign-market demand unrestricted. For notation simplicity, throughout the remainder of the paper, we treat all non-domestic destinations as a single aggregate foreign market, denoted by F.

From the perspective of firm i, we assume that the domestic-market variables $\{P_{st}^D, Q_{st}^D, \eta^D\}$ are contained in its period-t information set, $\mathcal{I}_{it}$, whereas the corresponding foreign-market conditions $\{P_{st}^F, Q_{st}^F, \eta^F\}$ are not fully observed when the firm makes its export decision. The firm may nevertheless form expectations about foreign demand using the information available to it, including its own past export experience and that of neighboring firms. .

3.3 Production and Export Decisions

In the model, firms solve a sequence of static and dynamic optimization problems. At the beginning of each period, a firm observes its state variables and chooses whether to export (e_{it}). Conditional on its export status, it determines its domestic and foreign sales by choosing its flexible labor and material inputs to maximize contemporaneous operating profits. The firm also chooses capital investment to determine the further capital stock and to maximize the discounted value of expected future profits. Thus, the current capital (K_{it}) is predetermined within the period, whereas labor (L_{it}) and materials (M_{it}) are flexible inputs.

Let $\mathbb{E}_t[\cdot] \equiv \mathbb{E}[\cdot \mid \mathcal{I}_{it}]$, and recall that $\mathcal{I}_{it}$ is the information available to firm i when its period-t export and production decisions are made. For compactness, define the firm's local network state variable set as

$$\mathcal{G}_{i,t-1} \equiv \left\{ \left(s_{ij,t-1}, e_{j,t-1}, \omega_{j,t-1} \right) : j \in \mathcal{N}(i) \right\}.$$

The firm's state vector can then be written as

$$S_{it} = \left(K_{it}, \omega_{it}, e_{i,t-1}, P^D_{st}, Q^D_{st}, P^L_{it}, P^M_{it}, \mathcal{G}_{i,t-1}, t \right),$$

where P^L_{it} and P^M_{it} are labor and material input prices, respectively. Firms do not fully observe foreign-market conditions before entering export markets. They nevertheless form expectations about foreign demand using the information contained in S_{it}, including their own past export experience and that of neighboring firms. The weights $s_{ij,t-1}$ allow the informativeness of neighboring firms to vary across peers. For example, the export experience of firms that are more similar to firm i may provide more relevant information about foreign-market opportunities. The time index t captures common aggregate conditions not otherwise included explicitly in the state vector.

Let Q^D_{it} and Q^F_{it} denote the quantities sold in the domestic and aggregate foreign markets, respectively. Consistent with the production function in Equation (3.1), total output is $Q_{it} = Q^D_{it} + e_{it} Q^F_{it}$, where $e_{it} \in \{0, 1\}$ denotes firm i's export status. Conditional on e_{it}, the firm's expected static operating profit is

$$\begin{aligned} \Pi(S_{it}, e_{it}) \equiv \max_{Q^D_{it}, Q^F_{it}, L_{it}, M_{it} \geq 0} \quad & \mathbb{E}_t \left[P^D_{it} Q^D_{it} + e_{it} P^F_{it} Q^F_{it} - P^L_{it} L_{it} - P^M_{it} M_{it} \right] \\ \text{s.t.} \quad & Q^D_{it} + e_{it} Q^F_{it} = F(K_{it}, L_{it}, M_{it}) \exp\{\omega_{it}\}. \end{aligned} \tag{3.5}$$

Here, P^D_{it} and P^F_{it} are the market-specific output prices implied by the corresponding demand conditions. The expectation is taken over uncertainty in domestic and foreign market conditions that remains unresolved given $\mathcal{I}_{it}$. The input prices P^L_{it} and P^M_{it} are taken as given by the firm and may vary across firms and over time because of non-monopsony reasons such as differences in suppliers or local input-market conditions.

The flexible-input choice embedded in Equation (3.5) can equivalently be represented by the following dual conditional variable-cost function:

$$\begin{aligned} C(S_{it}, e_{it}, Q^D_{it}, Q^F_{it}) \equiv \min_{L_{it}, M_{it} \geq 0} \quad & P^L_{it} L_{it} + P^M_{it} M_{it} \\ \text{s.t.} \quad & Q^D_{it} + e_{it} Q^F_{it} = F(K_{it}, L_{it}, M_{it}) \exp\{\omega_{it}\}. \end{aligned} \tag{3.6}$$

Thus, the static problem can be viewed as a two-stage optimization: for any feasible allocation of output across the domestic and foreign markets, the firm first minimizes the conditional variable cost and then chooses the output allocation that maximizes expected revenue net of this minimized cost.

In our model, we assume that output sold in the domestic and foreign markets is produced using the same technology. Market-specific differences enter through demand conditions and export costs rather than through separate production technologies. Although restrictive, this common-technology assumption is standard in the literature on firm production and exporting.

We next characterize the firm's dynamic export decision. Its value function is

$$V_{it}(S_{it}) = \max_{e_{it}\in\{0,1\}} \Big\{ \Pi(S_{it}, e_{it}) - e_{it}\big[C_{it}^F + (1 - e_{i,t-1})C_{it}^S\big] + e_{it}V_{it}^1(S_{it}) + (1 - e_{it})V_{it}^0(S_{it}) \Big\}, \quad (3.7)$$

where C_{it}^F denotes the fixed cost of exporting and C_{it}^S denotes the sunk entry cost, which is incurred only when a firm that did not export in period $t-1$ enters the foreign market in period t. The terms $V_{it}^1(S_{it})$ and $V_{it}^0(S_{it})$ are the continuation values conditional on exporting and serving only the domestic market, respectively. For $a \in \{0, 1\}$, they are defined in the following way:

$$\begin{aligned} V_{it}^a(S_{it}) \equiv \max_{I_{it}} & \Big\{ \delta\, \mathbb{E}_t \left[V_{i,t+1}(S_{i,t+1}) \mid S_{it}, e_{it} = a, I_{it} \right] - C^K(K_{it}, I_{it}) \Big\}, \quad (3.8) \\ \text{s.t.} \qquad & K_{i,t+1} = (1 - d)K_{it} + I_{it}, \end{aligned}$$

where δ is the one-period discount factor, d is the capital depreciation rate, I_{it} is investment, and $C^K(K_{it}, I_{it})$ is the capital adjustment-cost function. Because export participation affects both current operating profits and future productivity, the firm's optimal investment and hence its next-period capital stock generally depend on its current export decision.

Define the expected net gain from exporting as

$$\Delta_{it}^E(S_{it}) \equiv \Pi(S_{it}, 1) - \Pi(S_{it}, 0) + V_{it}^X(S_{it}) - V_{it}^D(S_{it}) - C_{it}^F - (1 - e_{i,t-1})C_{it}^S. \quad (3.9)$$

The optimal export rule is therefore

$$e_{it} = \begin{cases} 1, & \text{if } \Delta_{it}^E(S_{it}) \geq 0, \\ 0, & \text{otherwise.} \end{cases} \quad (3.10)$$

The above export decision rule is largely drawn from previous works of Roberts & Tybout (1997), Das et al. (2007), and Aw et al. (2011), with two extensions. First, it is integrated with firms' production decisions, following the tradition in production economics and empirical industrial organization (e.g., Olley & Pakes, 1996; Levinsohn & Petrin, 2003; Ackerberg et al., 2015; Gandhi et al., 2020). Second, as will become clearer in the next section, we allow for cross-sectional dependence in export decisions, i.e., local export spillovers, by assuming that potential exporters have incomplete information about foreign markets prior to entry and learn about foreign demand through both their own past exporting experience and that of neighboring firms.

4 Identification

In this section, we develop the identification strategy for the production and export model presented above. The main challenge is that firm productivity, export participation, and local spillovers are jointly determined. Our identification strategy exploits firms' optimal input choices, the dynamic evolution of productivity, and the export participation decisions to recover the structural parameters governing the productivity-export network system in (2.1) and (2.2). The resulting estimates allow us to quantify both direct effects and the dynamic propagation of productivity and export spillovers.

4.1 Production Function and Productivity

Identification of the production function (3.1) presents several challenges. First, a simultaneity problem arises because firms' input choices are endogenously determined with respect to latent productivity ω_{it}. Second, in most manufacturing datasets, physical output quantities are unobserved, and researchers typically observe only sales revenue. When output prices vary across firms, physical output cannot be directly recovered from observed sales by deflating revenue with an industry-level price index. Third, for exporting firms, observed revenue combines sales from domestic and foreign markets. Because demand conditions may differ across markets, using aggregate revenue as a measure of output can introduce additional biases in production function estimation.

To formalize the output measurement challenge in our multi-market setting, define the conventional log-deflated revenue measure as $\tilde{r}_{it} \equiv \ln\left(\frac{P_{it}^D Q_{it}^D + P_{it}^F Q_{it}^F}{P_{st}^D}\right)$. Even with the domestic demand structure in (3.4), identification cannot proceed directly from $\tilde{r}_{it}$ because the firm-specific foreign price P_{it}^F is unobserved. Therefore, unlike the single-market setting, industry-level deflation of total revenue does not recover physical output or a valid revenue-based production function and, consequently, does not identify the underlying production technology in our multi-market environment. Rather than attempting to recover physical output from observed revenue, we address this challenge by combining firms' optimality conditions for flexible inputs with the domestic-market expected-marginal-revenue condition. The latter links domestic sales to the marginal cost of total production, which may be allocated across domestic and foreign markets.

Formally, consider the domestic market. Using the demand system in (3.4), the firm's log-deflated domestic revenue can be written as

$$\begin{aligned}\tilde{r}_{it}^D \equiv \ln\left(\frac{P_{it}^D Q_{it}^D}{P_{st}^D}\right) &= \rho^D \ln Q_{it}^D - \frac{1}{\eta^D}\ln Q_{st}^D - \frac{1}{\eta^D}\xi_{it}^D \\ &= \rho^D\left(\ln \tau_{it}^D + \ln Q_{it}\right) - \frac{1}{\eta^D}\ln Q_{st}^D - \frac{1}{\eta^D}\xi_{it}^D, \end{aligned} \tag{4.1}$$

where $\rho^D \equiv (1+\eta^D)/\eta^D$ and

$$\tau_{it}^D \equiv \frac{Q_{it}^D}{Q_{it}} \tag{4.2}$$

denotes the share of total output allocated to the domestic market. The second equality of (4.1) follows from substituting the inverse demand function, $P_{it}^D = P_{st}^D \left(\frac{Q_{it}^D}{Q_{st}^D}\right)^{1/\eta^D} \exp\left\{-\frac{1}{\eta^D}\xi_{it}^D\right\}$.

Next, substituting the log Cobb-Douglas production function in Equation (3.3) into the above expression yields

$$\tilde{r}_{it}^D = \rho^D \beta_k k_{it} + \rho^D \beta_l l_{it} + \rho^D \beta_m m_{it} + \rho^D \omega_{it} + \rho^D \ln \tau_{it}^D + (1-\rho^D)\, q_{st}^D + \epsilon_{it}^D, \tag{4.3}$$

where $\epsilon_{it}^D \equiv -\frac{1}{\eta^D}\xi_{it}^D$ denotes the transformed domestic demand shock.

Consistent estimation of the above revenue production function recovers the underlying production technology parameters. However, the equation contains two latent components, ω_{it} and τ_{it}^D. The former represents persistent firm productivity and can be addressed using the conventional proxy-variable approaches in the production function literature. To identify the latter, we exploit the domestic-market optimality condition, under which expected marginal revenue in the domestic market equals the marginal cost of production, where the marginal cost is derived from the firm's cost-minimization problem over total output allocated across domestic and foreign markets.

Remark 1 *A seemingly attractive alternative is to estimate Equation* (4.3) *using only non-exporting observations. For these observations,* $\tau_{it}^D = 1$*, so the output-allocation term* $\ln \tau_{it}^D$ *drops out. This simplification, however, creates two difficulties. First, because export participation is endogenous to productivity, the non-exporter sample is selected on latent productivity. Unless this selection is explicitly accommodated, moments constructed from this subsample need not identify the production technology. Second, identifying* ρ^D*, and hence separating the underlying elasticities* β_j *from the scaled production elasticities* $\rho^D \beta_j$*, relies on variation in the aggregate demand shifter* q_{st}^D*. When the time dimension is short and industries are defined at a coarse level, this variation may be limited, potentially weakening finite-sample identification and reducing the precision of the resulting estimates. For these reasons, our identification strategy retains exporting observations and explicitly recovers their domestic output shares.*

From the profit maximization problem (3.5), the first-order condition with respect to domestic output implies

$$\mathbb{E}_t\left[\frac{\partial\left(P_{it}^D Q_{it}^D\right)}{\partial Q_{it}^D}\right] = MC_{it},$$

where MC_{it} does not have a market-specific superscript because the production process is common across domestic and foreign markets. Under the CES demand system in (3.4), this condition becomes

$$\rho^D P_{st}^D \left(\frac{Q_{it}^D}{Q_{st}^D}\right)^{1/\eta^D} \Lambda^D = MC_{it} \qquad \text{or} \qquad \rho^D \frac{R_{it}^D}{Q_{it}^D} \frac{\Lambda^D}{\exp\left\{\epsilon_{it}^D\right\}} = MC_{it} \tag{4.4}$$

where $\Lambda^D = \mathbb{E}_t\left[\exp\left\{\epsilon_{it}^D\right\}\right]$, $R_{it}^D \equiv P_{it}^D Q_{it}^D$, and the inverse-demand function is used to get the second equality.

To characterize MC_{it}, consider the conditional variable-cost problem in (3.6). Because domestic and foreign quantities enter the production constraint only through total output $Q_{it} = Q_{it}^D + e_{it}Q_{it}^F$, the associated conditional cost function depends on the market-specific quantities only through Q_{it}. Let ν_{it} denote the Lagrange multiplier on the production constraint. Under the Cobb–Douglas technology, the first-order conditions for the two flexible inputs are

$$P_{it}^L = \nu_{it}\beta_l \frac{Q_{it}}{L_{it}} \qquad \text{and} \qquad P_{it}^M = \nu_{it}\beta_m \frac{Q_{it}}{M_{it}}.$$

By the envelope theorem, the multiplier equals the marginal cost of expanding total output, i.e., $\nu_{it} = MC_{it}$. Consequently,

$$MC_{it} = \frac{P_{it}^L L_{it}}{\beta_l Q_{it}} = \frac{P_{it}^M M_{it}}{\beta_m Q_{it}} = \frac{P_{it}^L L_{it} + P_{it}^M M_{it}}{(\beta_l + \beta_m) Q_{it}}. \tag{4.5}$$

The final equality follows by summing the two flexible-input first-order conditions, and it is preferred since it uses information of both.

Substituting (4.5) into the revenue-based representation of (4.4), and using $Q_{it}^D = \tau_{it}^D Q_{it}$, yields

$$\tau_{it}^D = \rho^D(\beta_l + \beta_m) \frac{R_{it}^D}{P_{it}^L L_{it} + P_{it}^M M_{it}} \frac{\Lambda^D}{\exp\{\epsilon_{it}^D\}}. \tag{4.6}$$

Once consistent estimates of $\rho^D(\beta_l + \beta_m)$ and the demand-shock adjustment factor $\Lambda^D / \exp\{\epsilon_{it}^D\}$ have been obtained, Equation (4.6) becomes a feasible proxy of τ_{it}^D. The estimated τ_{it}^D can then be substituted as a generated regressor into the revenue production function (4.3) in the subsequent estimation step.

We now propose a three-step procedure to identify the revenue production function (4.3).

Step One: Identification of the scaled labor and material coefficients.

We first identify the scaled flexible-input coefficients, $\beta_l^* \equiv \rho^D \beta_l$ and $\beta_m^* \equiv \rho^D \beta_m$. These coefficients appear in both the revenue production function and the above proxy of τ_{it}^D. Under the maintained assumption that exporters and non-exporters share the same production technology, they can be identified directly using non-exporting observations. For such observations, $e_{it} = 0$, so that $Q_{it}^D = Q_{it}$ and $\tau_{it}^D = 1$. Combining the optimality conditions in (4.4) and (4.5), we have the following input-to-revenue share equations:

$$\ln S_{it}^{L,D} \equiv \ln\left(\frac{P_{it}^L L_{it}}{R_{it}^D}\right) = \ln\big(\beta_l^* \, \Lambda^D\big) - \epsilon_{it}^D, \tag{4.7}$$

$$\ln S_{it}^{M,D} \equiv \ln\left(\frac{P_{it}^M M_{it}}{R_{it}^D}\right) = \ln\big(\beta_m^* \, \Lambda^D\big) - \epsilon_{it}^D. \tag{4.8}$$

Because export status is chosen before the realization of the domestic-demand innovation and ϵ_{it}^D

is independent of the firm's information set, $\mathbb{E}[\epsilon_{it}^D \mid e_{it} = 0] = 0$. The OLS estimator can be applied to these two equations based on non-exporters, and coefficients β_l^* and β_m^* are identified from this conditional mean equality.

Specifically, let $\mathbb{E}_0[\cdot] \equiv \mathbb{E}[\cdot \mid e_{it} = 0]$. For $X \in \{L, M\}$, with the corresponding production coefficient indexed by $x \in \{l, m\}$, it follows that $\Lambda^D = \mathbb{E}_0[\exp\{\mathbb{E}_0[\ln S_{it}^{X,D}] - \ln S_{it}^{X,D}\}]$ according to the definition of Λ^D. Hence, the scaled input coefficients are identified as

$$\beta_x^* = \exp\left\{\mathbb{E}_0[\ln S_{it}^{X,D}]\right\} \Big/ \mathbb{E}_0\left[\exp\left\{\mathbb{E}_0[\ln S_{it}^{X,D}] - \ln S_{it}^{X,D}\right\}\right], \tag{4.9}$$

for $(X, x) \in \{(L, l), (M, m)\}$. This step adapts the flexible-input-share identification argument of Gandhi et al. (2020). More broadly, the use of firms' behavioral assumptions, such as profit maximization, to identify production-function parameters is well established in the literature; see, for example, Kumbhakar (2013) and Kumbhakar et al. (2021).

Step Two: Identification of the domestic output share τ_{it}^D.

Step One partially identifies the revenue production function (4.3). To recover its remaining parameters, we next need to estimate the domestic output share τ_{it}^D. For non-exporting observations, this share is a known constant, i.e., $\tau_{it}^D = 1$. For exporting observations, however, τ_{it}^D is latent, and estimating it from (4.6) requires an estimate of the domestic demand innovation ϵ_{it}^D. In this step, we recover ϵ_{it}^D following a similar procedure to the first stage of the Ackerberg et al. (2015) estimator.

Define total flexible-input expenditure as $V_{it} \equiv P_{it}^L L_{it} + P_{it}^M M_{it}$ and let $\beta_v \equiv \beta_l + \beta_m$. Using $R_{it}^D = P_{it}^D Q_{it}^D$ and the inverse domestic-demand function, (4.6) can be rewritten as

$$\tau_{it}^D = Q_{st}^D V_{it}^{\eta^D} \left(\rho^D \beta_v P_{st}^D \Lambda^D\right)^{-\eta^D} Q_{it}^{-(1+\eta^D)}. \tag{4.10}$$

Substituting (4.10) into (4.1) eliminates the latent domestic output share and yields the following implicit representation:

$$\tilde{r}_{it}^D = \Psi_t\left(k_{it}, l_{it}, m_{it}, \omega_{it}, V_{it}, P_{st}^D, Q_{st}^D\right) + \epsilon_{it}^D, \tag{4.11}$$

where $\Psi_t(\cdot)$ is an unknown composite function implied jointly by the production technology, the domestic demand structure, and the firm's optimal domestic output allocation. We don't track its specific functional form since nonparametric estimation will be applied later. Persistent productivity ω_{it} is the only unobserved argument of this function.

To control for ω_{it} among exporters, we follow the first-stage control-function argument of Ackerberg et al. (2015). Define $\mathbf{x}_{it}^M \equiv \left(e_{i,t-1}, \sum_{j \in \mathcal{N}(i)} s_{ij,t-1} e_{j,t-1}, P_{st}^D, Q_{st}^D, P_{it}^L, P_{it}^M, t\right)$. The vector $\mathbf{x}_{it}^M$ collects the observed state variables, excluding K_{it} and ω_{it}, that shift the exporter's current flexible-input choices through domestic demand, input costs, and information about foreign-market condi-

tions. Conditional on $e_{it} = 1$, optimal material demand can be written as

$$m_{it} = \mathcal{M}\left(k_{it}, \omega_{it}, \mathbf{x}_{it}^{M}\right).$$

Assuming that $\mathcal{M}_t(\cdot)$ is strictly increasing in ω_{it} conditional on its remaining arguments—that is, more productive firms use more materials—the material-demand function can be inverted to obtain

$$\omega_{it} = h\left(k_{it}, m_{it}, \mathbf{x}_{it}^{M}\right). \tag{4.12}$$

Substituting (4.12) into (4.11) produces the reduced-form representation

$$\tilde{r}_{it}^{D} = \phi\left(k_{it}, l_{it}, m_{it}, \mathbf{x}_{it}^{M}\right) + \epsilon_{it}^{D}, \qquad e_{it} = 1, \tag{4.13}$$

where $\phi(\cdot)$ is an unknown composite function. Because export participation, output allocation, and flexible-input choices are determined before ϵ_{it}^{D} is realized,

$$\mathbb{E}\left[\epsilon_{it}^{D} \,\middle|\, k_{it}, l_{it}, m_{it}, \mathbf{x}_{it}^{M}, e_{it} = 1\right] = 0. \tag{4.14}$$

Equation (4.13) therefore identifies $\phi_t(\cdot)$ nonparametrically, and its residual identifies ϵ_{it}^{D} for exporting observations.[6]

Finally, combining these residuals with the quantities identified in Step One makes the domestic output share representation (4.6) feasible:

$$\widehat{\tau}_{it}^{D} = (1 - e_{it}) + e_{it}\left[\left(\widehat{\beta}_{l}^{*} + \widehat{\beta}_{m}^{*}\right)\frac{R_{it}^{D}}{V_{it}}\frac{\widehat{\Lambda}^{D}}{\exp\{\widehat{\epsilon}_{it}^{D}\}}\right]. \tag{4.15}$$

Thus, $\widehat{\tau}_{it}^{D} = 1$ for non-exporters, whereas the expression in brackets recovers the domestic output share for exporters.

Step Three: Identification of the remaining production and productivity parameters.

Steps One and Two identify the scaled flexible-input coefficients $\beta_{l}^{*} \equiv \rho^{D}\beta_{l}$ and $\beta_{m}^{*} \equiv \rho^{D}\beta_{m}$, the domestic-demand innovation ϵ_{it}^{D}, and the domestic output share τ_{it}^{D} for the full sample. It remains to identify the scaled capital coefficient $\beta_{k}^{*} \equiv \rho^{D}\beta_{k}$, the demand parameter ρ^{D}, and the parameters governing the productivity process.

Define $\chi_{it} \equiv \tilde{r}_{it}^{D} - \beta_{l}^{*} l_{it} - \beta_{m}^{*} m_{it} - \epsilon_{it}^{D}$, which nets out the identified components of $\tilde{r}_{it}^{D}$ and can therefore be treated as a generated observable in the subsequent analysis. Using (4.3), we then have

$$\omega_{it} = \frac{1}{\rho^{D}}\left(\chi_{it} - \beta_{k}^{*} k_{it} - \rho^{D}\ln\tau_{it}^{D} - (1 - \rho^{D})\, q_{st}^{D}\right), \tag{4.16}$$

where $\beta_{k}^{*} \equiv \rho^{D}\beta_{k}$. Conditional on β_{k}^{*} and ρ^{D}, Equation (4.16) provides a parameter-indexed

[6] For non-exporting observations, ϵ_{it}^{D} is identified from the residual of the first step.

analytical proxy of productivity.

To identify the remaining parameters, we exploit the dynamic productivity process in (2.1). For any variable x, define its weighted peer average as $\bar{x}_{-i,t} \equiv \sum_{j \in \mathcal{N}(i)} s_{ij,t} x_{jt}$. Substituting (4.16) for the firm's own and its peers' productivity into (2.1) yields the following estimable nonlinear model

$$\begin{aligned}\chi_{it} =& \alpha_0^* + \alpha_1 \chi_{i,t-1} + \alpha_2^* e_{i,t-1} + \alpha_3 \bar{\chi}_{-i,t-1} + \beta_k^* \left(k_{it} - \alpha_1 k_{i,t-1} - \alpha_3 \bar{k}_{-i,t-1}\right) + \\ & \rho^D \left(\ln \tau_{it}^D - \alpha_1 \ln \tau_{i,t-1}^D - \alpha_3 \overline{\ln \tau^D}_{-i,t-1}\right) + (1-\rho^D)\left(q_{st}^D - \alpha_1 q_{s,t-1}^D - \alpha_3 \bar{q}_{-i,t-1}^D\right) + v_{it}^*, \quad (4.17)\end{aligned}$$

where $\alpha_0^* \equiv \rho^D \alpha_0$, $\alpha_2^* \equiv \rho^D \alpha_2$, $v_{it}^* \equiv \rho^D v_{it}$, and $\bar{q}_{-i,t-1}^D$ is the peer-weighted log aggregate demand shifter.[7]

Let $\boldsymbol{\theta} \equiv \left(\beta_k^*, \rho^D, \alpha_0^*, \alpha_1, \alpha_2^*, \alpha_3\right)'$, and denote the productivity innovation implied by (4.17) as $v_{it}^*(\boldsymbol{\theta})$. Since $v_{it}^* = \rho^D v_{it}$ and $\mathbb{E}[v_{it} \mid \mathcal{I}_{i,t-1}] = 0$, the remaining parameters are identified from the moment conditions

$$\mathbb{E}[v_{it}^*(\boldsymbol{\theta})\, \mathbf{z}_{it}] = \mathbf{0}, \quad (4.18)$$

where $\mathbf{z}_{it}$ is a vector of observed variables that are predetermined with respect to the period-t productivity innovation. For example, the instrument vector can be specified as

$$\mathbf{z}_{it} = \left(1,\, e_{i,t-1},\, k_{it},\, k_{i,t-1},\, \bar{k}_{-i,t-1},\, l_{i,t-1},\, m_{i,t-1},\, \chi_{i,t-1},\, \bar{\chi}_{-i,t-1},\, \ln \tau_{i,t-1}^D,\, \overline{\ln \tau^D}_{-i,t-1},\, q_{s,t-1}^D,\, \bar{q}_{-i,t-1}^D\right)'. \quad (4.19)$$

All variables in (4.19) are observed or recovered from previous steps.

Once $\boldsymbol{\theta}$ is identified, the underlying input elasticities and unscaled productivity parameters are recovered as

$$\beta_j = \frac{\beta_j^*}{\rho^D},\ j \in \{k, l, m\}, \qquad \alpha_0 = \frac{\alpha_0^*}{\rho^D}, \qquad \alpha_2 = \frac{\alpha_2^*}{\rho^D}, \quad (4.20)$$

while α_1 and α_3 are identified directly from (4.17). Finally, firm productivity is recovered from (4.16) with the unknown replaced with their estimates.

Remark 2 *Identification of the productivity-spillover parameter α_3 requires that, conditional on the variables included in the productivity process, lagged peer productivity does not proxy for omitted factors that jointly affect firms in the same location. Firms located near one another may share infrastructure, labor and input markets, local demand conditions, and policy environments. If these*

[7] Note that for the weighted peer average productivity, we have

$$\sum_{j \in \mathcal{N}(i)} s_{ij,t-1} \omega_{j,t-1} = \frac{1}{\rho^D}\left(\bar{\chi}_{-i,t-1} - \beta_k^* \bar{k}_{-i,t-1} - \rho^D \overline{\ln \tau^D}_{-i,t-1} - (1-\rho^D)\, \bar{q}_{-i,t-1}^D\right),$$

where the peer-weighting operator applies to each term of (4.16). Because the aggregate demand shifter may vary at the subindustry–year level while neighborhoods are geographic, neighboring firms generally can belong to different subindustries, and the peer-weighted average of the demand shifter can differ from the firm's own. Precisely, let $s(j)$ denote firm j's subindustry, and define the peer-weighted demand shifter as $\bar{q}_{-i,t-1}^D \equiv \sum_{j \in \mathcal{N}(i)} s_{ij,t-1}\, q_{s(j),t-1}^D$, which would not be a function of i if $s(j)$ does not vary with j.

common factors also affect productivity, the estimated coefficient on peer productivity may capture correlated group effects rather than genuine cross-firm learning.

Following Malikov & Zhao (2023), the assumption of no correlated group effects can be relaxed when the relevant local unobservables are time invariant. Let $r(i)$ denote firm i's region. We augment the productivity process with a location fixed effect $\delta_{r(i)}$:

$$\omega_{it} = \alpha_0 + \alpha_1 \omega_{i,t-1} + \alpha_2 e_{i,t-1} + \alpha_3 \bar{\omega}_{-i,t-1} + \delta_{r(i)} + v_{it}. \tag{4.21}$$

Correspondingly, the quasi-differenced estimating equation in (4.17) *includes a set of fixed parameters $\{\delta_{r(i)}\}$, which is estimated jointly with the remaining structural parameters. In a more flexbile specification, sub-industry fixed effects $\delta_{s(i)}$ can also be added in addition to $\delta_{r(i)}$. The spillover coefficient is then identified from variation in lagged peer exposure within locations or location-industry cells, after controlling for the firm's own lagged productivity and export experience. These fixed effects also help account for persistent spatial sorting of firms into systematically more productive locations.*

Time-invariant fixed effects do not, however, absorb local shocks that change over time. We therefore also consider an alternative specification that add region-year two-way fixed effects, $\delta_{r(i)} + \delta_t$. Such a specification removes shocks common to firms in the same area and period, including changes in local infrastructure, labor markets, industrial policy, and demand conditions.

4.2 Firms' Export Decision

We next describe the identification of firms' export decisions. Following Roberts & Tybout (1997), we approximate the expected net gain from exporting in (3.9) using a flexible function of the firm's state variables. Let $\mathbf{x}_{it}^e \equiv \left(k_{it}, P_{st}^D, Q_{st}^D, P_{it}^L, P_{it}^M\right)'$ be a subset of state variables S_{it}, excluding $(e_{i,t-1},\, \omega_{it},\, \sum_{j\in\mathcal{N}(i)} s_{ij,t-1} e_{j,t-1})'$, that are relevant for the firm's export decision. We parameterize the export net gain as

$$\Delta_{it}^E(S_{it}) \approx \gamma_0 + \gamma_1 e_{i,t-1} + \gamma_2 \omega_{it} + \gamma_3 \sum_{j\in\mathcal{N}(i)} s_{ij,t-1} e_{j,t-1} + \Phi_\tau(\mathbf{x}_{it}^e) + \mu_i + \delta_t - u_{it}, \tag{4.22}$$

where $\Phi_\tau(\cdot)$ is a τth-order polynomial, μ_i captures persistent firm-specific heterogeneity, δ_t denotes year fixed effects, and u_{it} is an idiosyncratic export-profitability shock. [8]

The coefficient γ_1 captures persistence in export participation arising from sunk entry costs, accumulated export experience, and other state dependence. The coefficient γ_2 captures productivity-

[8] Note that time-invariant fixed effects, such as regional and region-industry effects, are absorbed by the finer firm fixed effects μ_i: because a firm's location is (essentially) fixed over the sample period, regions are nested within firms. This is why no separate regional level effect appears in the export equation, in contrast to the productivity process. First differencing, which eliminates μ_i in estimation, removes these time-invariant regional effects along with it, so that only the year fixed effects δ_t remain. For notational simplicity, we use the same fixed-effect notation as in the productivity process (4.21), although the fixed effects are estimated separately in the two equations.

based selection into exporting, while γ_3 measures the export-network effect of neighboring firms' prior export participation. Summarizing neighboring export experience by $\sum_{j\in\mathcal{N}(i)} s_{ij,t-1}e_{j,t-1}$ reduces the high and potentially varying dimension of the network component of S_{it} while allowing more informative peers to receive greater weights.[9]

Define $\Pr(u_{it} \leq c \mid S_{it}, \mu_i, \delta_t) = F_u(c)$, where $F_u(\cdot)$ is a strictly increasing distribution function, and the conditional probability of exporting is

$$\Pr(e_{it} = 1 \mid S_{it}, \mu_i, \delta_{r(i)}) = F_u\left(\gamma_0 + \gamma_1 e_{i,t-1} + \gamma_2 \omega_{it} + \gamma_3 \sum_{j\in\mathcal{N}(i)} s_{ij,t-1}e_{j,t-1} + \Phi_\tau(\mathbf{x}^e_{it}) + \mu_i + \delta_t\right). \quad (4.23)$$

In the above specification, ω_{it} can be replaced by its estimate from Step Three. The specification is now ready for estimation, and the estimation details are provided in the following section.

5 Estimation

This section describes the implementation of the estimation procedure based on the identification strategy developed above. We first outline the sequential estimation of the production function, firm productivity, and export decisions. We then discuss alternative definitions of spatial networks, which govern the structure of inter-firm interactions in both productivity spillovers and the diffusion of export experience.

5.1 Productivity Estimation

We implement the production-function estimator sequentially, following the three-step identification strategy developed in Section 4.1. Let $\mathcal{O}_0 \equiv \{(i,t) : e_{it} = 0\}$ and $\mathcal{O}_1 \equiv \{(i,t) : e_{it} = 1\}$ denote the sets of non-exporting and exporting observations, with corresponding sample sizes N_0 and N_1.

The first step is to estimate the two share functions using non-exporting observations. From these equations, we can recover the transformed domestic demand shock for non-exporters, estimate Λ^D, and identify the scaled labor and material coefficients, $\beta_l^* = \rho^D\beta_l$ and $\beta_m^* = \rho^D\beta_m$ using OLS. However, at this stage, we cannot separately identify the underlying β parameters and ρ^D.
In the second step, following the control-function approach of ACF, we recover the transformed domestic demand shock for **exporting observations**. We then use the domestic-market optimality condition, $MR = MC$, to recover the latent domestic output share τ^D_{it} for exporters.
In the third step, we substitute the estimated domestic output shares into the revenue production function and return to the Markov process to recover the remaining production-function parameters, including the capital coefficient and ρ^D, while controlling for fixed effects. Finally, once ρ^D is

[9]The specification follows the network structure in (2.2) by assuming that, conditional on the firm's own productivity and the remaining state variables, neighboring firms' lagged productivity does not directly affect its export decision. Its effects, if it exists, operate through the firm's own productivity process.

recovered, we return to the scaled labor and material coefficients estimated in the first step and obtain the underlying production elasticities as $\beta_l = \beta_l^*/\rho^D$ and $\beta_m = \beta_m^*/\rho^D$.

Step One: Scaled flexible-input coefficients.

We first estimate the scaled labor and material coefficients by replace the population expression (4.9) with its sample counterpart. For $X \in \{L, M\}$, let $\overline{s}_X^0 \equiv \frac{1}{N_0}\sum_{(i,t)\in\mathcal{O}_0} \ln S_{it}^{X,D}$ and it estimates $\ln \widehat{\beta_x^* \Lambda^D}$. We need to separate Λ^D from this product. Because the model implies that the labor- and material-share equations contain the same domestic-demand innovation, we combine the information from both equations and estimate this innovation for non-exporters as

$$\widehat{\epsilon}_{it}^D \equiv \frac{1}{2}\sum_{X\in\{L,M\}} \left(\overline{s}_X^0 - \ln S_{it}^{X,D}\right), \qquad (i,t) \in \mathcal{O}_0. \tag{5.1}$$

We then estimate Λ^D and the scaled flexible-input coefficients by

$$\widehat{\Lambda}^D = \frac{1}{N_0}\sum_{(i,t)\in\mathcal{O}_0} \exp\left\{\widehat{\epsilon}_{it}^D\right\}, \qquad \widehat{\beta}_x^* = \frac{\exp\{\overline{s}_X^0\}}{\widehat{\Lambda}^D}, \qquad (X,x) \in \{(L,l),(M,m)\}. \tag{5.2}$$

Step Two: Domestic-demand innovations and output shares.

For non-exporters, ϵ_{it}^D is estimated from (5.1); we recover ϵ_{it}^D from the reduced-form regression in (4.13) for exporters. We approximate the unknown composite function using a polynomial sieve, $\phi(k_{it}, l_{it}, m_{it}, \mathbf{x}_{it}^M) \approx \mathbf{p}_\kappa(k_{it}, l_{it}, m_{it}, \mathbf{x}_{it}^M)'\,\boldsymbol{\pi}$, where $\mathbf{p}_\kappa(\cdot)$ is a vector of polynomial basis functions of order κ. In our application, we use a second-order polynomial to balance flexibility and dimensionality. The sieve coefficients are estimated from exporting observations by

$$\widehat{\boldsymbol{\pi}} = \underset{\boldsymbol{\pi}}{\operatorname{argmin}} \sum_{(i,t)\in\mathcal{O}_1} \left[\widetilde{r}_{it}^D - \mathbf{p}_\kappa(k_{it}, l_{it}, m_{it}, \mathbf{x}_{it}^M)'\,\boldsymbol{\pi}\right]^2.$$

The domestic-demand innovation for an exporting observation is then estimated as $\widehat{\epsilon}_{it}^D = \widetilde{r}_{it}^D - \mathbf{p}_\kappa(k_{it}, l_{it}, m_{it}, \mathbf{x}_{it}^M)'\,\widehat{\boldsymbol{\pi}}$. Then, the domestic output share is estimated from (4.15).

Step Three: Remaining production and productivity parameters.

Equation (4.17) is better estimated using a profiled nonlinear IV/GMM estimator based on moment conditions (4.18) when a large number of fixed effects are included. Using the quantities obtained in the first two steps, define the generated observable $\widehat{\chi}_{it} \equiv \widetilde{r}_{it}^D - \widehat{\beta}_l^* l_{it} - \widehat{\beta}_m^* m_{it} - \widehat{\epsilon}_{it}^D$ and the network quasi-difference $\Delta_{\alpha_1,\alpha_3}^N x_{it} \equiv x_{it} - \alpha_1 x_{i,t-1} - \alpha_3 \bar{x}_{-i,t-1}$ for any variable x_{it}. Let $\boldsymbol{a} \equiv (\alpha_1, \alpha_3)'$ collect the two parameters that enter the estimating equation nonlinearly, and let $\mathbf{d}_{it}$ denote the regional, region-industry, or region-time fixed-effect indicators. Rearranging (4.17),

conditional on $\boldsymbol{a}$ we obtain the linear IV model

$$\widehat{y}_{it}(\boldsymbol{a}) = \widehat{\mathbf{x}}_{it}(\boldsymbol{a})'\boldsymbol{b} + v^*_{it}, \tag{5.3}$$

where

$$\begin{aligned}
\widehat{y}_{it}(\boldsymbol{a}) &\equiv \Delta^N_{\alpha_1,\alpha_3}\widehat{\chi}_{it} - \Delta^N_{\alpha_1,\alpha_3} q^D_{st}, \\
\widehat{\mathbf{x}}_{it}(\boldsymbol{a}) &\equiv \left(1,\, e_{i,t-1},\, \Delta^N_{\alpha_1,\alpha_3} k_{it},\, \Delta^N_{\alpha_1,\alpha_3} \ln \widehat{\tau}^D_{it} - \Delta^N_{\alpha_1,\alpha_3} q^D_{st},\, \mathbf{d}'_{it}\right)',
\end{aligned}$$

and $\boldsymbol{b} \equiv (\alpha^*_0,\, \alpha^*_2,\, \beta^*_k,\, \rho^D,\, \boldsymbol{\delta}')'$ collects the parameters that enter linearly. To separately identify the common intercept α^*_0 from the fixed effects, we impose the normalization $\sum_g \delta_g = 0$. The contemporaneous domestic output share is endogenous because it responds to current productivity innovation v_{it}; the instrument vector $\mathbf{z}_{it}$ defined in (4.19) provides the excluded and predetermined variation used in the conditional 2SLS estimation.

Here, $\widehat{\mathbf{x}}^*_{it}(\boldsymbol{a})$ includes the estimated domestic output share $\widehat{\tau}^D_{it}$ from Step Two. This means that, after step two and before proceeding to the step three 2SLS estimation, we need to construct the weighted average of the lagged log domestic output shares, $\overline{\ln \widehat{\tau}^D}_{-i,t-1}$, using the network weights.

To summarize, in the first step, we can directly use OLS on the sample of non-exporters. In the second step, we use the sieve estimator **on the exporter sample** to recover the domestic demand shocks for exporters. We then use the $MR = MC$ condition to recover the domestic output shares for exporters, while for non-exporters the domestic output share is directly equal to one. At the end of Step Two, we combine the demand shocks for non-exporters obtained in Step One with the demand shocks for exporters obtained in Step Two, so that we have the demand shock and domestic output share for the full sample. Then, in the third step, we can use `ivreg`. In my understanding, we do not need to use GMM.

Quick question about Step Two: we will only use the exporter sample to estimate the domestic demand shock, correct?The demand shock for the nonexporters is estimated from the first step from the residuals. Thank you, Shunan. I understand.

Stacking the observations, let $\widehat{\mathbf{y}}(\boldsymbol{a})$ and $\widehat{\mathbf{X}}(\boldsymbol{a})$ denote the dependent-variable vector and regressor matrix implied by (5.3), and let $\mathbf{Z}$ stack the instrument vectors $\mathbf{z}_{it}$, expanded with fixed-effect indicators. For each trial value of $\boldsymbol{a}$, the conditional 2SLS estimator is

$$\widehat{\boldsymbol{b}}(\boldsymbol{a}) = \left[\widehat{\mathbf{X}}(\boldsymbol{a})'\mathbf{P}_Z\widehat{\mathbf{X}}(\boldsymbol{a})\right]^{-1}\widehat{\mathbf{X}}(\boldsymbol{a})'\mathbf{P}_Z\widehat{\mathbf{y}}(\boldsymbol{a}), \qquad \mathbf{P}_Z \equiv \mathbf{Z}(\mathbf{Z}'\mathbf{Z})^{-1}\mathbf{Z}'. \tag{5.4}$$

Let $\widehat{\mathbf{v}}(\boldsymbol{a}) \equiv \widehat{\mathbf{y}}(\boldsymbol{a}) - \widehat{\mathbf{X}}(\boldsymbol{a})\widehat{\boldsymbol{b}}(\boldsymbol{a})$. We estimate the nonlinear parameters by minimizing the concentrated 2SLS criterion:

$$\widehat{\boldsymbol{a}} = \operatorname*{argmin}_{\boldsymbol{a}\in\mathcal{A}} \frac{1}{N}\widehat{\mathbf{v}}(\boldsymbol{a})'\mathbf{P}_Z\widehat{\mathbf{v}}(\boldsymbol{a}), \tag{5.5}$$

where $\mathcal{A}$ imposes the stability restrictions on the productivity process and $N = N_0 + N_1$ is the total number of observations. The remaining parameter estimates are then obtained as $\widehat{\boldsymbol{b}} = \widehat{\boldsymbol{b}}(\widehat{\boldsymbol{a}})$. Finally, the underlying structural parameters are subsequently recovered following (4.20).

Remark 3 *Regarding the detailed implementation of the above profiled estimator when $\mathbf{d}_{it}$ has a large dimension, for each trial value $\boldsymbol{a} = (\alpha_1, \alpha_3)'$, we estimate the conditional linear model* (5.3) *by 2SLS, including the fixed effects, and obtain $\widehat{\boldsymbol{b}}(\boldsymbol{a})$ and the corresponding structural residuals $\widehat{v}_{it}(\boldsymbol{a})$. We then project these residuals on the complete instrument set used in the conditional 2SLS estimation, including the fixed-effect indicators, and denote the fitted values by $\widehat{v}_{it}^{P}(\boldsymbol{a})$. The nonlinear parameters are estimated using the profiled criterion*

$$\widehat{\boldsymbol{a}} = \underset{\boldsymbol{a}\in\mathcal{A}}{\operatorname{argmin}} \frac{1}{N} \sum_{i,t} \left[\widehat{v}_{it}^{P}(\boldsymbol{a})\right]^2 . \tag{5.6}$$

This auxiliary-regression formulation is algebraically equivalent to minimizing the conventional 2SLS criterion $\widehat{\mathbf{v}}(\boldsymbol{a})'\mathbf{P}_Z\widehat{\mathbf{v}}(\boldsymbol{a})$, but avoids explicitly constructing the high-dimensional instrument projection matrix.

Our baseline specification of the productivity process includes region fixed effects $\delta_{r(i)}$, with regions defined at the three-digit-postal-code level—the same level at which the peer network is defined. As discussed in Remark 2, these fixed effects absorb persistent locational advantages, such as shared infrastructure, input markets, and policy environments, as well as the spatial sorting of firms into systematically more productive locations, which are the leading confounders of the productivity-spillover parameter α_3. The spillover effect is then identified from variation in lagged peer exposure within regions, which the profit-based weighting scheme in (5.11) preserves even conditional on location. Estimates under more demanding structures are reported in Section 7 and are broadly consistent with the baseline.

5.2 Export Decision Estimation

The firm's export decision is modeled by Equation (4.23). In principle, assuming a distribution for u_{it}, it could be estimated using a probit or logit specification. However, its dynamic nature and the inclusion of fixed effects make nonlinear estimation challenging. To address this, Bernard & Jensen (2004) use a linear regression to estimate a similar export decision model. Following their approach, we also employ a linear probability model. This specification avoids the computational and incidental-parameter complications associated with nonlinear binary-response models containing fixed effects, while allowing the coefficients on productivity and neighboring export participation to be interpreted directly as marginal effects.

Let $\mathbf{p}_\tau(\mathbf{x}_{it}^e)$ denote the vector of polynomial basis functions used to approximate $\Phi_\tau(\mathbf{x}_{it}^e)$. Re-

placing latent productivity with its estimate, the feasible export equation is

$$e_{it} = \gamma_0 + \gamma_1 e_{i,t-1} + \gamma_2 \widehat{\omega}_{it} + \gamma_3 \bar{e}_{-i,t-1} + \mathbf{p}_\tau(\mathbf{x}^e_{it})'\boldsymbol{\gamma}_x + \mu_i + \delta_t + u_{it}, \tag{5.7}$$

where $\bar{e}_{-i,t-1} \equiv \sum_{j \in \mathcal{N}(i)} s_{ij,t-1} e_{j,t-1}$, μ_i denotes firm fixed effects, and δ_t captures shocks common to firms in the same year. As in the production-function estimation, we use a second-order polynomial approximation, and the intercept γ_0 is separately identified from μ_i and $\delta_{(i),t}$ by letting the sum of the fixed effects to be zero.

Because (5.7) contains both firm fixed effects and lagged export status, estimating it by the conventional within estimator would generate dynamic-panel bias when the time dimension is short. We therefore eliminate μ_i by first differencing:

$$\Delta e_{it} = \gamma_1 \Delta e_{i,t-1} + \gamma_2 \Delta \widehat{\omega}_{it} + \gamma_3 \Delta \bar{e}_{-i,t-1} + \Delta \mathbf{p}_\tau(\mathbf{x}^e_{it})'\boldsymbol{\gamma}_x + \Delta \delta_t + \Delta u_{it}. \tag{5.8}$$

The endogeneity of three variables requires additional discussion. First, the lagged dependent variable $\Delta e_{i,t-1}$ is correlated with Δu_{it} through $e_{i,t-1}$ and is instrumented using $e_{i,t-2}$ under the assumption that u_{it} is serially uncorrelated (Anderson & Hsiao, 1981). Second, the differenced productivity regressor requires an instrument dated $t-2$ as well. Because productivity is recovered in the preceding estimation step, $\widehat{\omega}_{it}$ is measured with a recovery error, which enters the differenced disturbance through the mismeasured regressor $\Delta\widehat{\omega}_{it}$; both the differenced regressor and the differenced disturbance therefore contain the period-$(t-1)$ recovery error. The natural candidate $\widehat{\omega}_{i,t-1}$ contains the period-$(t-1)$ productivity innovation $v_{i,t-1}$ and the period-$(t-1)$ recovery error, so the exclusion restriction fails unless v_{it} and the recovery error are uncorrelated with the contemporaneous export shock u_{it}—a restriction the model does not impose. We therefore instrument $\Delta\widehat{\omega}_{it}$ using $\widehat{\omega}_{i,t-2}$, which is orthogonal to the differenced disturbance under the same serial-uncorrelatedness assumption invoked for $e_{i,t-2}$, provided the observation-specific recovery error is serially independent. Third, an analogous timing concern applies to the differenced peer exposure $\Delta\bar{e}_{-i,t-1}$, which contains neighboring firms' $e_{j,t-1}$ and is therefore correlated with Δu_{it} whenever export shocks are spatially correlated within neighborhoods. We instrument it with the lagged level $\bar{e}_{-i,t-2}$. The export-decision parameters $\boldsymbol{\gamma} \equiv (\gamma_1, \gamma_2, \gamma_3, \boldsymbol{\gamma}_x')'$ are then estimated from the conventional efficient two-step GMM estimator.

5.3 Definitions of Spatial Networks

In our model, both the evolution of firm productivity and export decision are influenced by neighboring firms. We model these spatial interactions by defining peer weights $\{s_{ij,t}, j(\neq i) = 1, \ldots, n\}$. The simplest specification is given by:

$$s_{ij,t} = \frac{\mathbb{1}\{j \in \mathcal{N}(i)\}}{\sum_{j(\neq i)=1}^{n} \mathbb{1}\{j \in \mathcal{N}(i)\}}, \tag{5.9}$$

where $\mathbb{1}\{\cdot\}$ is an indicator function that equals 1 if the condition holds and 0 otherwise. Under this specification, all neighboring firms have the same impact on firm i's productivity and export decisions. Such learning is most likely to occur through unintended channels, such as labor mobility or informal knowledge diffusion, through which both beneficial and detrimental aspects of workplace environments are transmitted across neighboring firms.

Intuitively, firms are more likely to "intentionally" learn from certain peers. For instance, large and small firms often differ in production equipment, management practices, supply chains, and opportunities for access to overseas markets. Therefore, the potential for learning from each other, such as sharing production strategies and export experiences, is limited. Firms are more likely to learn from peers of similar size. To model this, we can define $s_{ij,t}$ as a similarity function:

$$s_{ij,t} = \frac{\exp\{-|a_{it} - a_{jt}|\} \times \mathbb{1}\{j \in \mathcal{N}(i)\}}{\sum_{j(\neq i)=1}^{n} \exp\{-|a_{it} - a_{jt}|\} \times \mathbb{1}\{j \in \mathcal{N}(i)\}}, \tag{5.10}$$

where a_{it} is the measure(s) of firm size, defined as total assets in our application; $|a_{it} - a_{jt}|$ is the L_2 norm of their difference, representing the Euclidean distance between firms i and j.

Another form of "intended learning" is when a firm learns only from more successful neighbors regarding production technologies and export experience. In the business context, success is often measured by profitability, which is most apparent to business owners. In this case, the peer learning weights can be specified as:

$$s_{ij,t} = \frac{\mathbb{1}\{\pi_{jt} > \pi_{it}\} \times \mathbb{1}\{j \in \mathcal{N}(i)\}}{\sum_{j(\neq i)=1}^{n} \mathbb{1}\{\pi_{jt} > \pi_{it}\} \times \mathbb{1}\{j \in \mathcal{N}(i)\}}, \tag{5.11}$$

where π_{it} is firm i's profit in period t. Unlike (5.9) and (5.10), (5.11) restricts learning to one direction, allowing only less profitable firms to learn from more profitable peers. This implies that the most profitable firm in our sample does not learn from its neighbors; only its own past experience matters for improving productivity and making export decisions. We use this network specification as our baseline and examine the sensitivity of the estimates under the other two alternative definitions.

For the definition of $\mathcal{N}(i)$, we use the first three digits of postal codes to characterize a firm's neighborhood. Although the geographical boundaries of neighborhoods remain fixed over time, the number of firms within them may vary across years. For notational simplicity, we ignore the time subscript t in the notation of $\mathcal{N}(i)$.

5.4 Bootstrap Inference

Because our estimation steps are sequential and several regressors are generated in preceding steps, inference must account for uncertainty propagated through the complete estimation procedure. We

use a nonparametric cluster bootstrap in which the sampling units are $\{\mathcal{N}(i)\}$, i.e., the three-digit-postcode neighborhoods defining the baseline peer network.[10] In each bootstrap replication, we draw the same number of neighborhoods as in the original sample, with replacement, and retain all firm-year observations belonging to each selected neighborhood. This preserves both the panel histories of firms and the cross-firm dependence induced by local networks. When a neighborhood is selected more than once, each copy is assigned new neighborhood and firm identifiers and is treated as a separate network. We then re-estimate the entire procedure. Repeating the procedure $B = 500$ times yields the empirical sampling distributions of the estimators, which are used to construct bootstrap standard errors and confidence intervals.

6 Data

The data are drawn from the Chinese Industrial Enterprises Database, a survey conducted by China's National Bureau of Statistics (NBS). The database covers all state-owned firms as well as non-state-owned firms with annual sales exceeding 5 million Chinese yuan (approximately 0.71 million USD). The coverage spans mining, manufacturing, and public utilities. In this paper, we limit our analysis to the electric machinery and equipment manufacturing industry, which encompasses the production of generators and motors, power transmission and distribution equipment, wires and cables, batteries, household electrical and non-electrical appliances, and lighting devices. We choose this industry because it has historically been one of China's most fundamental manufacturing sectors, closely linked to GDP growth and the rising demand for electricity. By its very nature, the industry has played a central role in China's broader industrialization. Additionally, electric machinery and equipment represents China's largest export category, and the industry is characterized by pronounced spatial clustering and industrial agglomeration, typical of technology- and skill-intensive sectors (Malikov & Zhao, 2023). Combined with government policies promoting innovation and technological advancement, these features make the industry an ideal setting for studying the productivity-export network effects. To maintain data quality, we restrict the sample period to 1998–2007. After data cleaning, the final sample consists of a large, unbalanced panel of 96,239 firm-year observations covering 31,266 unique firms.

Question: Do we have information on the number of employees?—It seems that we do. There are two variables: average employment during the year and total employment at the end of the year. We need to decide which one to use and update the data accordingly.We can use the one with fewer missing values. Got it. I checked the the raw data and I think the average labor has fewer missing values. We can use that one.

We construct a firm's domestic revenue (R_{it}^D) as the difference between total sales revenue (R_{it}) and reported export value. The producer price index is used as the domestic output-price index

[10]For specifications using a broader neighborhood definition, the bootstrap clusters are correspondingly defined at the coarsest geographic level containing all peer links.

(P_t^D) and it varies only over time.[11] Capital stock (K_{it}) is measured as net fixed assets deflated by the fixed-asset investment price index. Labor input (L_{it}) is measured by the number of employees. The firm-specific wage (P_{it}^L) is calculated as the total wage bill plus benefits divided by the number of employees. Materials (M_{it}) are measured as the value of intermediate inputs deflated by the industrial input-price index (P_t^M). Revenue, capital, and materials are expressed in thousands of Chinese yuan. The price indices are obtained from the NBS and the World Bank.

Following De Loecker (2011), we construct the log aggregate domestic-demand shifter (q_{st}^D) as the market-share-weighted average of firms' log deflated domestic revenues within each four-digit-subindustry-year cell, where the weights are given by firms' domestic-revenue shares in the corresponding cell. This construction introduces variation across four-digit subindustries and years into the aggregate demand shifter. We drop firm-year observations with missing or nonpositive values for the production variables defined above. We also remove observations for which reported export value exceeds total sales revenue or the input-expenditure-to-revenue share exceeds one. We exclude state-owned enterprises from the estimation sample because their production and input decisions may reflect policy, employment, or other noncommercial objectives, making the maintained profit-maximization assumptions underlying our identification strategy less appropriate for these firms. Finally, we drop observations with the number of employees fewer than 8 since the database is constructed to cover median-to-large firms.

Table 2 reports summary statistics for the key production variables, including the mean, standard deviation, first quartile, median, and third quartile. For all variables, the mean exceeds the median, indicating right-skewed distributions. A firm's export status is measured by the indicator e_{it}, which equals one if firm i reports a positive export value in year t and zero otherwise. As shown in Table 2, 26.7% of firm-year observations correspond to exporters.

Firms are assigned to geographic neighborhoods using the first three digits of their postal codes. Accordingly, $\mathcal{N}(i)$ consists of the other firms located in firm i's three-digit postal-code area, with its membership potentially changing over time as firms enter and exit the sample. The peer-weighting schemes described in Section 5.3 are constructed within these neighborhoods. Table 2 also reports the distributions of firm-level neighbor counts and peer export exposure under different weighting schemes.

7 Results

This section presents the empirical results from applying the proposed model and estimation procedure to Chinese firms in the electric machinery and equipment industry. We first report the estimated production-function parameters, productivity dynamics, and export-decision parameters.

[11] Thus, the general notation P_{st}^D used in the model reduces to P_t^D in the empirical application. The same convention applies to the material input-price index, so that $P_{it}^M = P_t^M$.

Table 2: Data Summary Statistics

Variable	Mean	S.D.	1st Q.	Median	3rd Q.
Domestic revenue (R^D)	54,927	142,223	8,020	16,965	43,797
Total sales revenue (R)	68,952	168,332	10,147	21,370	55,120
Export value (R^F)	14,025	72,280	0	0	560
Capital (K)	14,507	39,129	1,457	3,851	10,935
Labor (L)	206	359	52	100	207
Materials (M)	44,796	107,955	6,735	14,127	36,301
Wage (P^L)	16.28	13.34	9.02	13.00	18.92
Export status (e)	0.27	0.44	0	0	1
Number of neighbors	291	339	48	160	395
Peer export exposure from (5.11)	0.33	0.23	0.15	0.33	0.50
Peer export exposure from (5.10)	0.26	0.19	0.09	0.25	0.39
Peer export exposure from (5.9)	0.27	0.17	0.13	0.27	0.37

Notes: Domestic revenue, total sales revenue, export value, capital, materials, and wages are measured in thousands of Chinese yuan. Labor is measured by the number of employees. Peer export exposure is calculated within three-digit-postcode neighborhoods under the corresponding weighting schemes.

We then use these estimates to quantify the direct and cross-network transmission effects linking productivity and export participation. Next, we compare the complete system with the corresponding restricted partial models and decompose the resulting differences into the mechanical loss of transmission pathways and the effects of re-estimating the restricted specifications. Finally, we assess the robustness of the results to alternative peer-weighting schemes and neighborhood definitions.

12

Results to Be Reported

First get the estimates. The bootstrap inference mentioned below can be done at a latter stage.

1. **Sample composition and identifying variation.**

   - Report the number of observations of exporters and non-exporters.
   - (for our own) when you do the following estimation, for each fixed-effect specification used below, report the within-fixed-effect-cell standard deviations of key independent variables, such as $\bar{\omega}_{-i,t-1}$, $\bar{e}_{-i,t-1}$, and $q^D_{s,t-1}$. They demonstrate whether sufficient identifying variation remains after the fixed effects are absorbed. If the variation is small, it

[12] Because recovered productivity differs across the fixed-effect specifications of the productivity process only through the estimated (β^*_k, ρ^D)—the components χ_{it}, $\ln \tau^D_{it}$, and q^D_{st} in (4.16) being common across specifications—the export-decision estimates are nearly invariant to this choice. We therefore fix $\widehat{\omega}_{it}$ at the baseline specification throughout and report the sensitivity of the export-decision estimates to alternative time-effect structures in Table **??**.

means we need coarser fixed effects. ⇒ The baseline specification should use the most demanding fixed-effect structure that adequately addresses correlated local shocks while retaining sufficient within-cell variation in peer exposure.

2. **Production-function estimates.**

   Report the baseline estimates of the production-function and demand parameters in the main text:

$$\widehat{\beta}_k, \qquad \widehat{\beta}_l, \qquad \widehat{\beta}_m, \qquad \widehat{\mathrm{RTS}} = \widehat{\beta}_k + \widehat{\beta}_l + \widehat{\beta}_m, \qquad \widehat{\rho}^D.$$

   Also report the implied domestic-demand elasticity,

$$\widehat{\eta}^D = \frac{1}{\widehat{\rho}^D - 1}.$$

   The estimates under alternative fixed-effect structures can be reported in the appendix. Because the recovered domestic output share is central to the revised identification strategy, also (for our own) report the following diagnostics:

   - summary statistics (histograms with many bins, such as 50) for $\widehat{\tau}_{it}^D$ among exporters;
   - the proportion of estimated domestic output shares satisfying $0 < \widehat{\tau}_{it}^D \leq 1$; we may need to trim or winsorize some of them if they are not reasonable.
   - summary statistics (histograms with many bins, such as 50) for the estimated domestic-demand innovations, $\widehat{\epsilon}_{it}^D$, separately for exporters and non-exporters. Are there long tails that we need to trim or winsorize?

3. **Productivity-process estimates under alternative controls for correlated local effects.**

   Report estimates of $(\alpha_1, \alpha_2, \alpha_3)$ under the following specifications:

   (1) no geographic fixed effects;

   (2) region fixed effects, where regions are defined using the baseline neighborhood level, such as three-digit postal codes;

   (3) region-by-subindustry fixed effects;

   (4) region-by-year fixed effects.

   For each specification, report bootstrap standard errors or confidence intervals for $\widehat{\alpha}_1$, $\widehat{\alpha}_2$, and $\widehat{\alpha}_3$.

   If the region-by-year specification retains sufficient variation in peer productivity, it should be used as the baseline productivity specification.

If the region-by-industry or -year specifications are too demanding, consider using region + industry or year specifications.

For industry FE, they can defined at the 3-digit level.

4. **Export-decision estimates under alternative fixed-effect structures.**

   Report estimates of $(\gamma_1, \gamma_2, \gamma_3)$ under the following specifications:

   (1) firm fixed effects;

   (2) firm fixed effects and year fixed effects;

   (3) firm fixed effects and region-by-year fixed effects.

   When year or region-by-year fixed effects are included, remove the deterministic time trend from $\mathbf{x}_{it}^e$. For each specification, report bootstrap standard errors or confidence intervals.

   The firm-fixed-effect and region-by-year-fixed-effect specification should be used as the baseline if sufficient within-cell variation in $\bar{e}_{-i,t-1}$ remains.

5. **Definition and estimation of the complete and restricted models.**

   Using the preferred fixed-effect structures selected above, estimate the following four models:

   (A) *Complete system*:

   $$F: \qquad \alpha_3 \neq 0, \qquad \gamma_3 \neq 0,$$

   yielding

   $$(\widehat{\boldsymbol{\alpha}}_F, \widehat{\boldsymbol{\gamma}}_F).$$

   (B) *No export-network spillovers*:

   $$R_\gamma: \qquad \alpha_3 \neq 0, \qquad \gamma_3 = 0,$$

   yielding

   $$\left(\widehat{\boldsymbol{\alpha}}_{R_\gamma}, \widehat{\gamma}_{1,R_\gamma}, \widehat{\gamma}_{2,R_\gamma}, \widehat{\gamma}_{3,R_\gamma} = 0\right).$$

   Under the sequential estimation procedure,

   $$\widehat{\boldsymbol{\alpha}}_{R_\gamma} = \widehat{\boldsymbol{\alpha}}_F,$$

   provided that imposing $\gamma_3 = 0$ changes only the structural export equation and leaves the preceding production-function and productivity-estimation steps unchanged.

   (C) *No productivity spillovers*:

   $$R_\alpha: \qquad \alpha_3 = 0, \qquad \gamma_3 \neq 0,$$

yielding

$$\left(\widehat{\alpha}_{1,R_\alpha}, \widehat{\alpha}_{2,R_\alpha}, \widehat{\alpha}_{3,R_\alpha} = 0, \widehat{\boldsymbol{\gamma}}_{R_\alpha}\right).$$

Because imposing $\alpha_3 = 0$ changes the production-function and productivity-process estimation, recover a new productivity series under this restriction and re-estimate the complete export equation using that restricted productivity measure.

(D) *No cross-firm spillovers*:

$$R_{\alpha\gamma}: \qquad \alpha_3 = \gamma_3 = 0,$$

yielding

$$\left(\widehat{\alpha}_{1,R_{\alpha\gamma}}, \widehat{\alpha}_{2,R_{\alpha\gamma}}, \widehat{\alpha}_{3,R_{\alpha\gamma}} = 0, \widehat{\gamma}_{1,R_{\alpha\gamma}}, \widehat{\gamma}_{2,R_{\alpha\gamma}}, \widehat{\gamma}_{3,R_{\alpha\gamma}} = 0\right).$$

Under the sequential procedure,

$$\widehat{\boldsymbol{\alpha}}_{R_{\alpha\gamma}} = \widehat{\boldsymbol{\alpha}}_{R_\alpha},$$

while the export equation must be re-estimated using the productivity measure recovered under $\alpha_3 = 0$ and imposing $\gamma_3 = 0$.

Report the primitive estimates from the four models in a common table. At minimum, the table should contain

$$\widehat{\alpha}_1, \qquad \widehat{\alpha}_2, \qquad \widehat{\alpha}_3, \qquad \widehat{\gamma}_1, \qquad \widehat{\gamma}_2, \qquad \widehat{\gamma}_3.$$

6. **Estimated first-round transmission effects corresponding to Table 1.**

   Report the empirical counterparts of the first-round cross-network effects in Table 1.

   The empirical table should contain two panels:

   - *Panel A: Mechanical channel deletion.* Evaluate all effects at the complete-system estimates, $\widehat{\boldsymbol{\theta}}_F$, and impose the relevant restriction by setting $\widehat{\gamma}_{3,F} = 0$, $\widehat{\alpha}_{3,F} = 0$, or both, while holding all other parameters fixed.
   - *Panel B: Re-estimated restricted models.* Evaluate the same effects using the parameter estimates from R_γ, R_α, and $R_{\alpha\gamma}$.

   Construct bootstrap confidence intervals for these nonlinear combinations and for the differences between the complete and restricted models.

7. **Ten-year finite-horizon responses to a common export-market-access shock.**

   Rather than simulating the unconditional levels of productivity and export participation, compare the finite-horizon responses generated by the complete system and the three re-estimated restricted models under the same normalized common export-market-access shock. This exercise evaluates how the omission of one or both cross-firm spillover channels changes

the propagation of a common external intervention without requiring each model to reproduce the observed levels of productivity and export participation.

Consider the four re-estimated models

$$\mathcal{M}_F, \qquad \mathcal{M}_{R_\gamma}, \qquad \mathcal{M}_{R_\alpha}, \qquad \mathcal{M}_{R_{\alpha\gamma}},$$

with corresponding parameter estimates

$$\widehat{\boldsymbol{\theta}}_F, \qquad \widehat{\boldsymbol{\theta}}_{R_\gamma}, \qquad \widehat{\boldsymbol{\theta}}_{R_\alpha}, \qquad \widehat{\boldsymbol{\theta}}_{R_{\alpha\gamma}}.$$

Let d_h^e denote an additive common shock entering the export equation at horizon h, with $h = 0$ corresponding to the impact period. The main analysis should consider a permanent common shock normalized as

$$d_h^e = 0.10, \qquad h = 0, 1, \ldots, 10,$$

which represents a permanent ten-percentage-point increase in expected export participation at the impact horizon, before subsequent endogenous adjustment.

Under a common shock and row-normalized peer weights, the responses are identical across firms, so the firm index and the peer-aggregation operators can be suppressed. For each model $\mathcal{M}$, initialize

$$\Delta\omega_{-1}^{\mathcal{M}} = 0, \qquad \Delta e_{-1}^{\mathcal{M}} = 0,$$

and recursively calculate

$$\Delta\omega_h^{\mathcal{M}} = (\alpha_{1,\mathcal{M}} + \alpha_{3,\mathcal{M}})\,\Delta\omega_{h-1}^{\mathcal{M}} + \alpha_{2,\mathcal{M}}\Delta e_{h-1}^{\mathcal{M}}, \tag{7.1}$$

$$\Delta e_h^{\mathcal{M}} = (\gamma_{1,\mathcal{M}} + \gamma_{3,\mathcal{M}})\,\Delta e_{h-1}^{\mathcal{M}} + \gamma_{2,\mathcal{M}}\Delta\omega_h^{\mathcal{M}} + d_h^e. \tag{7.2}$$

The productivity response should be calculated first at each horizon because contemporaneous productivity enters the export equation. Since the exercise is conducted in deviations from the corresponding no-shock path, constants, fixed effects, and common exogenous-control paths cancel and need not be simulated explicitly.

For each model, calculate

$$\left\{\Delta\omega_h^{\mathcal{M}}, \Delta e_h^{\mathcal{M}}\right\}_{h=0}^{10}.$$

The main comparison should use the parameter estimates obtained from each re-estimated model. Thus, the complete-system response is evaluated at $\widehat{\boldsymbol{\theta}}_F$, while the restricted-model responses are evaluated at $\widehat{\boldsymbol{\theta}}_{R_\gamma}$, $\widehat{\boldsymbol{\theta}}_{R_\alpha}$, and $\widehat{\boldsymbol{\theta}}_{R_{\alpha\gamma}}$, respectively. See Appendix B for the details of the calculation.

Present the results in a two-panel figure. The first panel should plot the productivity responses

$$\Delta\omega_h^F, \qquad \Delta\omega_h^{R_\gamma}, \qquad \Delta\omega_h^{R_\alpha}, \qquad \Delta\omega_h^{R_{\alpha\gamma}},$$

and the second panel should plot the corresponding expected export-participation responses

$$\Delta e_h^F, \qquad \Delta e_h^{R_\gamma}, \qquad \Delta e_h^{R_\alpha}, \qquad \Delta e_h^{R_{\alpha\gamma}},$$

for every horizon $h = 0, 1, \ldots, 10$.

Construct confidence intervals by repeating the complete estimation and response calculation within each bootstrap replication. The same shock normalization should be used for all models and bootstrap draws.

8. **Decomposition of complete-versus-restricted finite-horizon responses.**

   Implement the decomposition in Equation (2.8) using recursively simulated responses from the productivity and export equations. The main exercise should consider a **permanent common** export-market-access shock and trace its effects on productivity and expected export participation. Because the export equation is estimated as a linear probability model, propagate expected export participation continuously rather than converting fitted values into binary outcomes.

   See Appendix B for the details of this exercise.

9. **Steady-state common-shock multipliers.**

   For each model, first construct the transition matrix

$$\mathcal{A}_c = \begin{pmatrix} \alpha_1 + \alpha_3 & \alpha_2 \\ \gamma_2(\alpha_1 + \alpha_3) & \gamma_1 + \gamma_3 + \gamma_2\alpha_2 \end{pmatrix}$$

   and report its spectral radius,

$$\rho\left(\widehat{\mathcal{A}}_c\right).$$

   If

$$\rho\left(\widehat{\mathcal{A}}_c\right) < 1,$$

   report the estimated steady-state productivity and export responses,

$$\frac{\partial\omega^*}{\partial x} = \frac{\alpha_2}{(1-\alpha_1-\alpha_3)(1-\gamma_1-\gamma_3)-\alpha_2\gamma_2}$$

   and

$$\frac{\partial e^*}{\partial x} = \frac{(1-\alpha_1-\alpha_3)}{(1-\alpha_1-\alpha_3)(1-\gamma_1-\gamma_3)-\alpha_2\gamma_2}.$$

If the estimated system does not satisfy the stability condition, report that the infinite-horizon steady-state multiplier does not exist and focus the economic interpretation on the finite-horizon responses. Bootstrap confidence intervals should be reported only for replications satisfying the stability condition, together with the proportion of stable bootstrap replications.

10. **Robustness and sensitivity analysis.**

    Re-estimate the preferred complete system under the following alternatives:

    - uniform peer weights;
    - peer weights based on similarity in firm size;
    - alternative geographic neighborhood definitions, including two-digit, three-digit, and four-digit postal-code areas; This may change the definition of the regional fixed effects.

    For each robustness specification, report at least

    $$\widehat{\alpha}_1, \qquad \widehat{\alpha}_2, \qquad \widehat{\alpha}_3, \qquad \widehat{\gamma}_1, \qquad \widehat{\gamma}_2, \qquad \widehat{\gamma}_3,$$

    as well as the associated first-round transmission effects and long-run multipliers.

8 Conclusion

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Declaration of Generative AI in Scientific Writing

During the preparation of this work, the authors used ChatGPT and Claude to check grammar, spelling, and restructure sentences to improve the readability. After using the tools, the authors reviewed and edited the content as needed and take full responsibility for the content of the published article.

Other Declaration

The authors declare no competing interests. The data that support the findings of this study are available from the authors upon reasonable request.

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Appendices

A Derivation of the Dynamic Multipliers

A.1 The reduced-form transition system

Let

$$\boldsymbol{\omega}_t = (\omega_{1t}, \ldots, \omega_{nt})', \qquad \boldsymbol{e}_t = (e_{1t}, \ldots, e_{nt})',$$

and define

$$B_\omega \equiv \alpha_1 I_n + \alpha_3 S, \qquad B_e \equiv \gamma_1 I_n + \gamma_3 S.$$

The productivity and export equations are

$$\boldsymbol{\omega}_t = \alpha_0 \mathbf{1}_n + B_\omega \boldsymbol{\omega}_{t-1} + \alpha_2 \boldsymbol{e}_{t-1} + \boldsymbol{v}_t, \tag{A.1}$$

$$\boldsymbol{e}_t = \gamma_0 \mathbf{1}_n + B_e \boldsymbol{e}_{t-1} + \gamma_2 \boldsymbol{\omega}_t + \delta_x \boldsymbol{x}_t + \boldsymbol{u}_t. \tag{A.2}$$

Substituting Equation (A.1) into Equation (A.2) gives

$$\begin{aligned} \boldsymbol{e}_t = {} & (\gamma_0 + \gamma_2 \alpha_0) \mathbf{1}_n + \gamma_2 B_\omega \boldsymbol{\omega}_{t-1} \\ & + (B_e + \gamma_2 \alpha_2 I_n)\, \boldsymbol{e}_{t-1} + \delta_x \boldsymbol{x}_t + \gamma_2 \boldsymbol{v}_t + \boldsymbol{u}_t. \end{aligned} \tag{A.3}$$

Let

$$\boldsymbol{z}_t \equiv \begin{bmatrix} \boldsymbol{\omega}_t \\ \boldsymbol{e}_t \end{bmatrix}.$$

The system can then be written as

$$\boldsymbol{z}_t = \boldsymbol{c} + A \boldsymbol{z}_{t-1} + G \boldsymbol{x}_t + \boldsymbol{\varepsilon}_t, \tag{A.4}$$

where

$$A = \begin{bmatrix} B_\omega & \alpha_2 I_n \\ \gamma_2 B_\omega & B_e + \gamma_2 \alpha_2 I_n \end{bmatrix}, \qquad G = \begin{bmatrix} 0_{n \times n} \\ \delta_x I_n \end{bmatrix},$$

and

$$\boldsymbol{c} = \begin{bmatrix} \alpha_0 \mathbf{1}_n \\ (\gamma_0 + \gamma_2 \alpha_0) \mathbf{1}_n \end{bmatrix}, \qquad \boldsymbol{\varepsilon}_t = \begin{bmatrix} \boldsymbol{v}_t \\ \gamma_2 \boldsymbol{v}_t + \boldsymbol{u}_t \end{bmatrix}.$$

A.2 One-time and permanent shocks

Consider a one-time shock $\Delta \boldsymbol{x}_t = \boldsymbol{d}_x$, with no subsequent shocks. Starting from an unchanged initial state,

$$\Delta \boldsymbol{z}_t = G\boldsymbol{d}_x,$$

and recursive substitution gives

$$\Delta \boldsymbol{z}_{t+h} = A^h G\boldsymbol{d}_x.$$

If $\rho(A) < 1$, then $A^h \to 0$, implying

$$\lim_{h\to\infty} \Delta \boldsymbol{z}_{t+h} = \boldsymbol{0}.$$

Its cumulative response is

$$\sum_{h=0}^{\infty} A^h G\boldsymbol{d}_x = (I_{2n} - A)^{-1} G\boldsymbol{d}_x.$$

For a permanent shock, $\Delta \boldsymbol{x}_{t+h} = \boldsymbol{d}_x$ for every $h \geq 0$, the response at horizon h is

$$\Delta \boldsymbol{z}_{t+h} = \sum_{r=0}^{h} A^r G\boldsymbol{d}_x.$$

Under the same stability condition,

$$\lim_{h\to\infty} \Delta \boldsymbol{z}_{t+h} = (I_{2n} - A)^{-1} G\boldsymbol{d}_x.$$

A.3 General joint-system steady-state multipliers

Define

$$H_\omega \equiv I_n - B_\omega = (1 - \alpha_1) I_n - \alpha_3 S.$$

$$H_e \equiv I_n - B_e = (1 - \gamma_1) I_n - \gamma_3 S.$$

Consider a permanent change in the export shifter $\boldsymbol{x}$ and define

$$\boldsymbol{m}_\omega \equiv \frac{\partial \boldsymbol{\omega}^*}{\partial \boldsymbol{x}'}, \qquad \boldsymbol{m}_e \equiv \frac{\partial \boldsymbol{e}^*}{\partial \boldsymbol{x}'}.$$

Differentiating the steady-state productivity and export equations gives

$$H_\omega \boldsymbol{m}_\omega - \alpha_2 \boldsymbol{m}_e = 0_{n\times n}, \tag{A.5}$$

$$-\gamma_2 \boldsymbol{m}_\omega + H_e \boldsymbol{m}_e = \delta_x I_n. \tag{A.6}$$

Equation (A.5) implies

$$\boldsymbol{m}_\omega = H_\omega^{-1} \alpha_2 \boldsymbol{m}_e.$$

Substituting this expression into Equation (A.6) yields

$$\boldsymbol{m}_e = \left[H_e - \alpha_2\gamma_2 H_\omega^{-1}\right]^{-1} \delta_x I_n. \tag{A.7}$$

Consequently,

$$\begin{aligned}\boldsymbol{m}_\omega &= H_\omega^{-1}\alpha_2 \left[H_e - \alpha_2\gamma_2 H_\omega^{-1}\right]^{-1} \delta_x I_n \\ &= \alpha_2 \left[H_e H_\omega - \alpha_2\gamma_2 I_n\right]^{-1} \delta_x I_n. \end{aligned} \tag{A.8}$$

Equation (A.8) is equivalent to the upper $n \times n$ block of

$$(I_{2n} - A)^{-1} G.$$

If only firm j receives a permanent unit export shifter, the resulting effect on firm i is

$$\frac{\partial \omega_i^*}{\partial x_j} = \alpha_2 \delta_x \left[\left(H_e H_\omega - \alpha_2\gamma_2 I_n\right)^{-1}\right]_{ij}. \tag{A.9}$$

A.4 Common-shock simplification

Suppose that S is row normalized, so that

$$S\mathbf{1}_n = \mathbf{1}_n,$$

and that all firms experience the same permanent shifter, $\boldsymbol{x} = x\mathbf{1}_n$. Because $\mathbf{1}_n$ is an eigenvector of S with eigenvalue one,

$$H_\omega \mathbf{1}_n = (1 - \alpha_1 - \alpha_3)\mathbf{1}_n, \qquad H_e \mathbf{1}_n = (1 - \gamma_1 - \gamma_3)\mathbf{1}_n.$$

The common responses therefore satisfy

$$(1 - \alpha_1 - \alpha_3) m_\omega - \alpha_2 m_e = 0$$

and

$$-\gamma_2 m_\omega + (1 - \gamma_1 - \gamma_3) m_e = \delta_x.$$

Solving these two equations gives

$$m_\omega = \frac{\alpha_2 \delta_x}{(1 - \alpha_1 - \alpha_3)(1 - \gamma_1 - \gamma_3) - \alpha_2\gamma_2},$$

and

$$m_e = \frac{(1 - \alpha_1 - \alpha_3)\delta_x}{(1 - \alpha_1 - \alpha_3)(1 - \gamma_1 - \gamma_3) - \alpha_2\gamma_2}.$$

Finally,

$$m_\omega = \frac{\alpha_2}{1-\alpha_1-\alpha_3} m_e,$$

which gives the factorization reported in Equation (2.20).

B Finite-horizon Responses to A Common Export-market-access Shock

This section specializes the response function defined in Equation (2.8) to a common export-market-access shock. Let $\boldsymbol{d}^e \equiv (d_0^e, d_1^e, \ldots)'$ denote the path of additive shocks entering the export equation, where d_h^e incorporates both the change in the underlying export-market-access shifter and its coefficient, and h denotes the horizon relative to the initial intervention, with $h = 0$ corresponding to the impact period. Suppose that all firms receive the same shock at each horizon, so that $d_{i,h}^e = d_h^e$ for every firm i. Starting from common initial responses and under row-normalized peer weights, the productivity and export-partarticipation responses remain identical across firms at every subsequent horizon. In particular,

$$\sum_{j \in \mathcal{N}(i)} s_{ij} \Delta\omega_{j,h}^{\mathcal{M}} = \Delta\omega_h^{\mathcal{M}} \qquad \text{and} \qquad \sum_{j \in \mathcal{N}(i)} s_{ij} \Delta e_{j,h}^{\mathcal{M}} = \Delta e_h^{\mathcal{M}}.$$

We therefore suppress the firm index, the explicit peer-weighting operators, and the function arguments, and write the productivity and export-participation responses under model $\mathcal{M}$ as $\Delta\omega_h^{\mathcal{M}}$ and $\Delta e_h^{\mathcal{M}}$, respectively. Thus, $\Delta\omega_h^{\mathcal{M}}$ is shorthand for $\Delta\omega_{i,h}(\mathcal{M}, \boldsymbol{\theta}_{\mathcal{M}}; \boldsymbol{d}^e)$, with $\Delta e_h^{\mathcal{M}}$ defined analogously. Because the export equation is specified as a linear probability model, $\Delta e_h^{\mathcal{M}}$ should be interpreted as the change in expected export participation.

For a given model $\mathcal{M}$, define

$$a_{\mathcal{M}} \equiv \alpha_{1,\mathcal{M}} + \alpha_{3,\mathcal{M}}, \qquad b_{\mathcal{M}} \equiv \gamma_{1,\mathcal{M}} + \gamma_{3,\mathcal{M}}, \qquad c_{\mathcal{M}} \equiv b_{\mathcal{M}} + \gamma_{2,\mathcal{M}}\alpha_{2,\mathcal{M}}.$$

The coefficient $a_{\mathcal{M}}$ summarizes persistence in productivity through own-productivity persistence and productivity spillovers; $b_{\mathcal{M}}$ summarizes persistence in export participation through own-export persistence and export-network spillovers; and $c_{\mathcal{M}}$ augments this export persistence by the within-firm feedback from exporting to productivity and back to exporting, $\gamma_{2,\mathcal{M}}\alpha_{2,\mathcal{M}}$.

Starting from an unchanged pre-shock path,

$$\Delta\omega_{-1}^{\mathcal{M}} = 0, \qquad \Delta e_{-1}^{\mathcal{M}} = 0,$$

the common responses satisfy

$$\Delta\omega_h^{\mathcal{M}} = a_{\mathcal{M}}\Delta\omega_{h-1}^{\mathcal{M}} + \alpha_{2,\mathcal{M}}\Delta e_{h-1}^{\mathcal{M}}, \tag{B.1}$$

$$\Delta e_h^{\mathcal{M}} = b_{\mathcal{M}}\Delta e_{h-1}^{\mathcal{M}} + \gamma_{2,\mathcal{M}}\Delta\omega_h^{\mathcal{M}} + d_h^e. \tag{B.2}$$

Productivity is calculated first at each horizon because current productivity enters the contemporaneous export equation.

Impact horizon, $h = 0$. At the impact horizon, productivity does not respond because the productivity process depends on lagged export participation:

$$\Delta\omega_0^{\mathcal{M}} = a_{\mathcal{M}}\Delta\omega_{-1}^{\mathcal{M}} + \alpha_{2,\mathcal{M}}\Delta e_{-1}^{\mathcal{M}} = 0.$$

Expected export participation responds immediately to the external shock:

$$\Delta e_0^{\mathcal{M}} = b_{\mathcal{M}}\Delta e_{-1}^{\mathcal{M}} + \gamma_{2,\mathcal{M}}\Delta\omega_0^{\mathcal{M}} + d_0^e = d_0^e.$$

First horizon, $h = 1$. At horizon one, productivity responds to the impact change in export participation:

$$\begin{aligned}\Delta\omega_1^{\mathcal{M}} &= a_{\mathcal{M}}\Delta\omega_0^{\mathcal{M}} + \alpha_{2,\mathcal{M}}\Delta e_0^{\mathcal{M}} \\ &= a_{\mathcal{M}} \times 0 + \alpha_{2,\mathcal{M}} d_0^e \\ &= \alpha_{2,\mathcal{M}} d_0^e. \end{aligned} \tag{B.3}$$

The corresponding export-participation response is

$$\begin{aligned}\Delta e_1^{\mathcal{M}} &= b_{\mathcal{M}}\Delta e_0^{\mathcal{M}} + \gamma_{2,\mathcal{M}}\Delta\omega_1^{\mathcal{M}} + d_1^e \\ &= b_{\mathcal{M}} d_0^e + \gamma_{2,\mathcal{M}}\alpha_{2,\mathcal{M}} d_0^e + d_1^e \\ &= c_{\mathcal{M}} d_0^e + d_1^e. \end{aligned} \tag{B.4}$$

Second horizon, $h = 2$. At horizon two, substitute Equations (B.3) and (B.4) into the productivity recursion:

$$\begin{aligned}\Delta\omega_2^{\mathcal{M}} &= a_{\mathcal{M}}\Delta\omega_1^{\mathcal{M}} + \alpha_{2,\mathcal{M}}\Delta e_1^{\mathcal{M}} \\ &= a_{\mathcal{M}}\left(\alpha_{2,\mathcal{M}} d_0^e\right) + \alpha_{2,\mathcal{M}}\left(c_{\mathcal{M}} d_0^e + d_1^e\right) \\ &= \alpha_{2,\mathcal{M}} a_{\mathcal{M}} d_0^e + \alpha_{2,\mathcal{M}} c_{\mathcal{M}} d_0^e + \alpha_{2,\mathcal{M}} d_1^e \\ &= \alpha_{2,\mathcal{M}}\left(a_{\mathcal{M}} + c_{\mathcal{M}}\right) d_0^e + \alpha_{2,\mathcal{M}} d_1^e. \end{aligned} \tag{B.5}$$

Next, substitute the horizon-one export response and the horizon-two productivity response into the export recursion:

$$\begin{aligned}\Delta e_2^{\mathcal{M}} &= b_{\mathcal{M}}\Delta e_1^{\mathcal{M}} + \gamma_{2,\mathcal{M}}\Delta\omega_2^{\mathcal{M}} + d_2^e \\ &= b_{\mathcal{M}}\left(c_{\mathcal{M}}d_0^e + d_1^e\right) + \gamma_{2,\mathcal{M}}\left[\alpha_{2,\mathcal{M}}(a_{\mathcal{M}} + c_{\mathcal{M}})d_0^e + \alpha_{2,\mathcal{M}}d_1^e\right] + d_2^e \\ &= \left[b_{\mathcal{M}}c_{\mathcal{M}} + \gamma_{2,\mathcal{M}}\alpha_{2,\mathcal{M}}(a_{\mathcal{M}} + c_{\mathcal{M}})\right]d_0^e + \left[b_{\mathcal{M}} + \gamma_{2,\mathcal{M}}\alpha_{2,\mathcal{M}}\right]d_1^e + d_2^e \\ &= \left[b_{\mathcal{M}}c_{\mathcal{M}} + \gamma_{2,\mathcal{M}}\alpha_{2,\mathcal{M}}a_{\mathcal{M}} + \gamma_{2,\mathcal{M}}\alpha_{2,\mathcal{M}}c_{\mathcal{M}}\right]d_0^e + c_{\mathcal{M}}d_1^e + d_2^e \\ &= \left[c_{\mathcal{M}}^2 + \gamma_{2,\mathcal{M}}\alpha_{2,\mathcal{M}}a_{\mathcal{M}}\right]d_0^e + c_{\mathcal{M}}d_1^e + d_2^e, \end{aligned} \tag{B.6}$$

where the final equality uses $c_{\mathcal{M}} = b_{\mathcal{M}} + \gamma_{2,\mathcal{M}}\alpha_{2,\mathcal{M}}$.

One-time shock. For a one-time shock, let

$$d_0^e = d^e, \qquad d_h^e = 0 \quad \text{for every } h \geq 1.$$

The responses through horizon two become

$$\Delta\omega_0^{\mathcal{M}} = 0, \qquad \Delta e_0^{\mathcal{M}} = d^e,$$

$$\Delta\omega_1^{\mathcal{M}} = \alpha_{2,\mathcal{M}}d^e, \qquad \Delta e_1^{\mathcal{M}} = c_{\mathcal{M}}d^e,$$

and

$$\Delta\omega_2^{\mathcal{M}} = \alpha_{2,\mathcal{M}}(a_{\mathcal{M}} + c_{\mathcal{M}})d^e,$$

$$\Delta e_2^{\mathcal{M}} = \left[c_{\mathcal{M}}^2 + \gamma_{2,\mathcal{M}}\alpha_{2,\mathcal{M}}a_{\mathcal{M}}\right]d^e.$$

Expanding the horizon-two productivity response gives

$$\Delta\omega_2^{\mathcal{M}} = \alpha_{2,\mathcal{M}}\left(\alpha_{1,\mathcal{M}} + \gamma_{1,\mathcal{M}} + \gamma_{2,\mathcal{M}}\alpha_{2,\mathcal{M}}\right)d^e + \alpha_{2,\mathcal{M}}\left(\alpha_{3,\mathcal{M}} + \gamma_{3,\mathcal{M}}\right)d^e. \tag{B.7}$$

The first component captures within-firm persistence and feedback. The second component is the first-round cross-firm transmission effect characterized in Table 1.

Permanent shock. For a permanent shock, let

$$d_h^e = d^e \quad \text{for every } h \geq 0.$$

The responses through horizon two become

$$\Delta\omega_0^{\mathcal{M}} = 0, \qquad \Delta e_0^{\mathcal{M}} = d^e,$$

$$\Delta\omega_1^{\mathcal{M}} = \alpha_{2,\mathcal{M}}d^e, \qquad \Delta e_1^{\mathcal{M}} = (1 + c_{\mathcal{M}})d^e,$$

and

$$\Delta\omega_2^{\mathcal{M}} = \alpha_{2,\mathcal{M}}(1 + a_{\mathcal{M}} + c_{\mathcal{M}})d^e,$$

$$\Delta e_2^{\mathcal{M}} = \left[1 + c_{\mathcal{M}} + c_{\mathcal{M}}^2 + \gamma_{2,\mathcal{M}}\alpha_{2,\mathcal{M}}a_{\mathcal{M}}\right] d^e.$$

Relative to the one-time shock, the permanent-shock responses contain additional terms because the intervention is applied again in every subsequent period.

Responses at subsequent horizons. Define the response vector

$$\Delta \boldsymbol{z}_h^{\mathcal{M}} \equiv \begin{pmatrix} \Delta\omega_h^{\mathcal{M}} \\ \Delta e_h^{\mathcal{M}} \end{pmatrix}, \qquad \boldsymbol{g} \equiv \begin{pmatrix} 0 \\ 1 \end{pmatrix},$$

and the transition matrix

$$\boldsymbol{A}_c(\boldsymbol{\theta}_{\mathcal{M}}) \equiv \begin{pmatrix} a_{\mathcal{M}} & \alpha_{2,\mathcal{M}} \\ \gamma_{2,\mathcal{M}}a_{\mathcal{M}} & c_{\mathcal{M}} \end{pmatrix}.$$

Equations (B.1) and (B.2) can then be written as

$$\Delta \boldsymbol{z}_h^{\mathcal{M}} = \boldsymbol{A}_c(\boldsymbol{\theta}_{\mathcal{M}})\Delta \boldsymbol{z}_{h-1}^{\mathcal{M}} + \boldsymbol{g}d_h^e, \qquad \Delta \boldsymbol{z}_{-1}^{\mathcal{M}} = \boldsymbol{0}.$$

For a one-time shock,

$$\Delta \boldsymbol{z}_h^{\mathcal{M}} = \boldsymbol{A}_c(\boldsymbol{\theta}_{\mathcal{M}})^h \boldsymbol{g}d^e,$$

whereas for a permanent shock,

$$\Delta \boldsymbol{z}_h^{\mathcal{M}} = \sum_{\ell=0}^{h} \boldsymbol{A}_c(\boldsymbol{\theta}_{\mathcal{M}})^\ell \boldsymbol{g}d^e.$$

These expressions generate the productivity and export-participation responses at every horizon $h = 1, \ldots, 10$ without requiring construction of the full firm-level network matrix.

Complete-versus-restricted decomposition. For each restricted model $\mathcal{M}_R$, define

$$\Delta y_h^F \equiv \Delta y_h \left(\mathcal{M}_F, \widehat{\boldsymbol{\theta}}_F; \boldsymbol{d}^e\right),$$

$$\Delta y_h^{F\to R} \equiv \Delta y_h \left(\mathcal{M}_R, \widehat{\boldsymbol{\theta}}_{F\to R}; \boldsymbol{d}^e\right),$$

and

$$\Delta y_h^R \equiv \Delta y_h \left(\mathcal{M}_R, \widehat{\boldsymbol{\theta}}_R; \boldsymbol{d}^e\right),$$

where $y \in \{\omega, e\}$. The total complete-versus-restricted difference is

$$D_{y,h}^{\text{total}}(R) \equiv \Delta y_h^F - \Delta y_h^R.$$

Its mechanical component is

$$D_{y,h}^{\text{mech}}(R) \equiv \Delta y_h^F - \Delta y_h^{F \to R},$$

and its re-estimation component is

$$D_{y,h}^{\text{reest}}(R) \equiv \Delta y_h^{F \to R} - \Delta y_h^R.$$

By construction,

$$D_{y,h}^{\text{total}}(R) = D_{y,h}^{\text{mech}}(R) + D_{y,h}^{\text{reest}}(R).$$

To examine the finite-horizon responses to a common shock and distinguish the mechanical loss of spillover channels from the effects of re-estimating the restricted models, we can calculate these quantities at each horizon $h = 1, 2, \ldots$ for a normalized common shock, such as $d^e = 0.10$.