Jobless Modernity

Laura Alfaro [*] Marcela Eslava[†] Martí Mestieri[‡] Luis Felipe Sáenz[§]

September 15, 2026

Abstract

Modern firms in developing economies contribute far more to income than to employment. While this gap is often attributed to weak economic activity, high-quality job creation can lag behind value-added growth when the firms driving it face distortions against labor or adopt labor-saving technologies. Using a general-equilibrium model with constrained occupational choice across occupations in traditional and modern production, we assess the importance of these channels. Distortions differ in the strength of their effects across margins: non-labor distortions mainly compress aggregate income, while labor distortions mainly govern access to modern jobs. Because access affects wages and determines who enters modern employment, it governs both employment and the distribution of earnings. We use fifty years of Colombian microdata representative of the universe of private firms and workers, both formal and informal, as well as direct measures of labor distortions. We then estimate modern firms' wedges on revenue and capital, jointly with technology and demand, and embed them in the mode. The gap, measured as the difference between the share of firms with at least ten workers in total labor earnings and their share of job, in Colombia is 51 percentage points. Across these periods, distortions explain 40 to 60 percent of this *jobless modernity*, and labor distortions make up 70 percent of that share.

Keywords: Employment, Modern, Distortions, Distribution, Inequality
JEL Codes: E24, E26, O11, O14, O25.

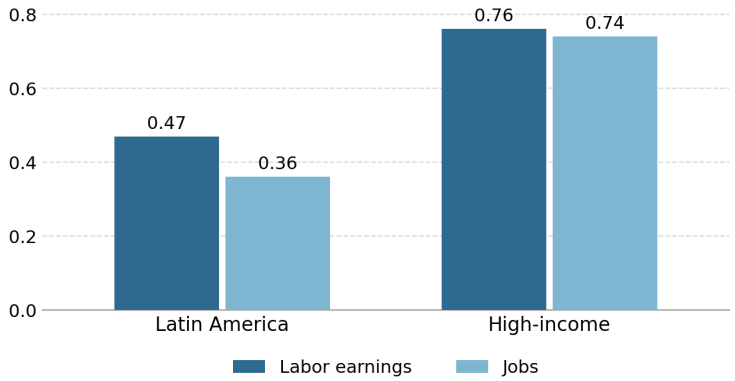
[*]Harvard Business School
[†]Universidad de Los Andes.
[‡]Institut d'Analisi Economica - CSIC, CREI, BSE and CEPR.
[§]University of South Carolina.

We thank the audiences at the ASSA Meetings, CEDE, CEPR-Bread, IEA Conference, LACEA-LAMES Annual Meetings, Oxford-STEG conference, Penn State, RIDGE, SEA, UAB, USC, Tubinguen, the Barcelona Summer Forum, LSE's Institute for Inequality Studies, the Institute for Fiscal Studues, the Alicante Workshop on Macroeconomics and Productivity for their comments and suggestions. Special thanks for very useful comments and dedicated discussions to Dani Rodrik, Federico Huneeus, Hugo Hopenhayn, Paco Buera, Gabriel Ulyssea. Andrés Soto has provided outstanding research assistance.

1 Introduction

Modern firms in developing economies contribute substantially more to aggregate income than to employment. We refer to this divergence or gap between their income and employment shares as *jobless modernity*. Figure 1 documents this pattern: In eleven Latin American economies, firms with at least ten workers, our empirical definition of modern firms, generate 47% of labor earnings but account for only 36% of employment.[1] By comparison, the corresponding shares across thirteen high-income economies are nearly identical, at 76% and 74%. Jobless modernity has a distributional counterpart: because modern jobs are typically higher paying, access to modernity shapes the earnings distribution. In Latin America, the share of workers employed by modern firms goes from only 4% in the bottom earnings decile to 56% in the top decile (Figure 2). In high-income economies, the corresponding shares are 55% and 77%.

Figure 1: Modern Firms: Share of in Labor Earnings and Jobs

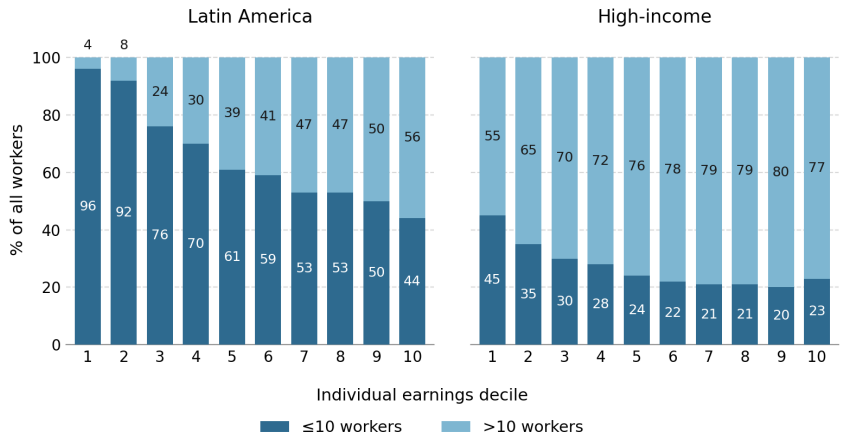


Notes: Modern firms are defined as those with >10 workers. Latin America: Argentina, Bolivia, Brazil, Chile, Colombia, Costa Rica, Dominican Republic, Mexico, Paraguay, Peru, and Uruguay. High-income: countries with per capita income of > 40K US dollars in 2024 present: Austria, Belgium, Denmark, Finland, France, Germany, Ireland, Italy, Luxembourg, Malta, Sweden, Switzerland, and United States. *Source:* Own calculations using household or employment survey data.

The gap captures a central tension for emerging economies: production may shift toward larger, more productive firms without a corresponding transformation of employment. One interpretation of this phenomenon is that developing economies have simply not modernized enough and that employment will eventually follow as modern production expands. Yet the same aggregate gap can arise from fundamentally different forces, some of which may be particularly prevalent in developing economies. Modern firms may adopt capital- or input-intensive technologies that generate substantial value added with little labor; limited worker capabilities may restrict access to modern jobs; scarce

[1] To distinguish formal from informal and modern from traditional firms, the 10-employee threshold is frequently used, partly because of data availability, (Schwartzman, 2025; Hsieh and Klenow, 2009; Hsieh and Olken, 2014)

Figure 2: Individual Earnings across Firm Size



Notes: Modern firms are defined as those with >10 workers. Latin America: Argentina, Bolivia, Brazil, Chile, Colombia, Costa Rica, Dominican Republic, Mexico, Paraguay, Peru, and Uruguay. High-income: countries with per capita income of at least 40K US dollars in 2024 present in the database: Austria, Belgium, Denmark, Finland, France, Germany, Ireland, Italy, Luxembourg, Malta, Sweden, Switzerland, and United States. *Source:* Own calculations using household or employment survey data.

entrepreneurial or managerial capabilities may constrain firm creation and growth; and demand for traditional goods or preferences for self-employment may sustain traditional activity. Distortions to the efficient allocation of productive resources across producers further are prevalent in developing economies (Restuccia and Rogerson, 2008; Hsieh and Klenow, 2009). Distortions to relative firm size may bias growth against job creation, particularly when regulations impose employment costs or distort technology adoption.

Disentangling these forces requires more than aggregate measures of productivity, employment, or factor shares. First, size-distortions whether operating through labor or other margins, vary with firm productivity. Second, distortions differ both between modern, usually formal, firms and traditional producers and across firms within the modern sector. Third, factor intensity in the production function cannot be identified from aggregate variables in the presence of distortions. Identifying these margins therefore requires microdata on both modern firms and traditional, typically informal, producers.

To address these challenges, we first develop and estimate a general-equilibrium model of occupational choice that captures the duality of production, in which productive modern firms coexist with minimum-scale, frequently informal businesses operating traditional technologies (Lewis, 1954; Harris and Todaro, 1970; Tybout, 2000; Engbom et al., 2025). In the model, heterogeneous individuals choose among traditional self- employment, modern wage work, and modern entrepreneurship. Modern firms hire labor and combine it with capital and intermediate inputs, whereas traditional producers rely on their own productive capabilitie. Modern firms also face a minimum-earnings requirement and firm-specific distortions to labor, capital, and revenue, from which traditional producers are exempt. The model jointly determines entry into modern entrepreneur-

ship, firm scale, labor demand, occupational selection, wages, and the allocation of income across occupations. It therefore links firm-level distortions to both the scale of modern production and workers' access to modern jobs. Counterfactuals that remove each distortion separately distinguish barriers to modern production from barriers to modern employment.

The model is informed by fifty years rich of Colombian household, employment, and firm census microdata from 1971 to 2019, covering private-sector firms and workers in formal and informal activities. Through the lens of the model, this information allows us to recover the distribution of productive capabilities across workers in the modern segment. Modern firms' prices and output and input quantities data allow estimating production technologies and demand jointly and recover firm-specific revenue and capital wedges. The data also report mandatory non-salary labor costs separately from wages, allowing us to measure heterogeneous labor distortions directly. Coverage of both modern and traditional producers identifies occupational selection across the two production modes, while the historical span permits comparable decompositions in 1971 and 2019. The data's historical span allows us to conduct comparable decompositions at the two ends of a 50-year period.

Four main findings emerge. First, distortions account for a substantial share of jobless modernity. In 2019, the model-implied gap between the employment and total income shares of modern production is -51 percentage points (-20 between employment and labor earnings, close to the data). Removing distortions reduces it to 31 percentage points, implying that distortions account for approximately 40 percent of the observed gap. Across the periods we study, their contribution ranges from 40 to 60 percent. Barriers that keep highly productive firms from expanding employment thus explain a substantial fraction of the gap.

Second, the barriers to modern employment differ from those to modern production. Labor distortions account for approximately 70 percent of the distortion-induced component of jobless modernity. Non-labor distortions operate primarily through firm scale and the income generated by modern production. Non-salary labor costs have a similar effect, but also increase the cost of labor vs. other factors and exacerbates the effects of the minimum earnings constraint, which resticts low productivity workers' access to modern wage employment. Not all reforms that raise modern are equally effective to produce an expansion of modern employment.

Third, distortions have important distributional consequences. Removing them raises real earnings in every occupation, increases the labor share, and reduces overall earnings inequality. These effects arise in part as workers move from traditional activities into higher-paying modern jobs. Broader entry into modern employment changes the composition of modern workers, while the removal of firm-level wedges raises modern

sector wages. The aggregate gap between modern income and employment is therefore closely connected to the allocation of workers across occupations and the distribution of earnings.

Finally, jobless modernity has persisted despite substantial economic change. Between 1971 and 2019, the employment share of modern firms in Colombia increased by approximately 10 percentage points. Their income share, however, increased by slightly more, leaving the jobless-modernity gap broadly unchanged—47 percentage points in 1971 and 51 percentage points in 2019. The counterfactual decomposition shows that technical change and human capital improvements between the two periods fully explain the increase in the scale of the modern segment, leaving the gap between its employment and income contribution unchanged. Correspondingly, there is little change in the gap induced by changes in distortions. Even though Colombia went through non-trivial reforms over the period, the barriers to modern job creation remained consequential.

To summarize, a central insight is that the barriers to modern production need not be the same as those to modern employment: revenue and capital distortions primarily affect firm scale and modern income, whereas labor distortions operate mainly through firms' demand for workers and occupational selection, shaping who gains access to modern wage employment. Reforms may therefore raise aggregate income and modern production without proportionately broadening access to modern jobs.

We argue that these findings are relevant beyond Colombia, particularly for Latin America and potentially for other emerging and developing economies.[2] They identify a path in which the modernization of production proceeds without the broad incorporation of workers into modern employment. Modern technologies diffuse, capabilities accumulate, and productive firms generate a growing share of aggregate income, yet modern firms do not absorb a broad segment of the labor force because of persistent distortions.

Jobless modernity is therefore not simply a symptom of insufficient modernization that naturally disappears as development proceeds: it can persist because the forces governing modern production and modern employment operate through different margins. Because the gains from modernization remain concentrated among the relatively small group of workers and entrepreneurs connected to modern production, this weakens a central channel through which development has historically broadened economic opportunity: the expansion of stable, higher-paying employment and the emergence of a broad middle class.

Related Literature. First, we study and document a largely overlooked dimension of the role of mdoern firms in developing economies: they employ relatively few workers

[2]For example, India exhibits a similar gap of approximately 10 percentage points.

given the income they generate. A strand of the literature primarily asks why, on average, productive firms fail to attain the scale observed in richer economies (Tybout, 2000; Poschke, 2018; Hopenhayn, 2016; Hsieh and Klenow, 2014; Bento and Restuccia, 2017, 2021; Akcigit et al., 2021; Eslava et al., 2023; Eslava, 2026). We instead ask why modern firms' contributions to income and employment diverge, and why this divergence persists even as modern production expands. By making this gap the central object of analysis, we show that the expansion of modern production and the incorporation of workers into modern employment are governed by distinct forces, with different implications for aggregate income, the labor share, and earnings inequality. Our analysis shows that reforms with have similar implications for firm scale may operate differently on firms' labor demand and workers' occupational choices.

A closely related literature examines the effects of distortions to efficient allocation across heterogeneous firms on the size distribution of firms and aggregate efficiency and income in developing economies (Restuccia and Rogerson, 2008; Hsieh and Klenow, 2009; Alfaro et al., 2009; Midrigan and Xu, 2014). Distortions are estimated as wedges between the data and structural modelling of optimal size, We identify which distortions restrict modern production and which prevent its expansion from translating into modern employment. Separating labor distortions from revenue and capital distortions reveals that the former operate principally through labor demand and occupational selection, whereas the latter principally through firm scale and aggregate income. We separate these mechanisms by combining directly measured regulatory costs, including mandatory employer contributions and minimum-earnings constraints, with remaining estimated revenue and capital wedges.

The analysis therefore extends the misallocation literature to study which distortions prevent increases in modern production from translating into broader access to modern jobs, complementing recent efforts to identify the institutional sources and incidence of firm-level distortions (Atkin et al., 2025; Restuccia, 2025; Garicano et al., 2016). Unlike most of the misallocation literature moreover, our analysis uses data on both formal and informal producers and takes into account the differential bite of regulation-born distortions on the two groups.[3] In so doing, we build upon literature bridging the gap from micro to macro approaches to understanding the economy.[4] We push this attempt to a full coverage of the economy in countries characterized by high informality and presence of traditional producers.

Third, we contribute to research connecting firm organization to the distribution of earn-

[3]D'Erasmo and Moscoso Boedo (2012) also consider informality and costs specific to formality in the context of a quantitative model with misallocation, and seek to match aggregate data on the prevalence of informality. We embed this approach in the context of occupational choice, informed with the full distribution of workers' earnings in formal vs. informal jobs.

[4]Buera et al. (2021); Buera and Moll (2015); Mestieri et al. (2017)

ings. A large literature shows that employer characteristics contribute to wage differences across workers: larger firms pay higher wages, worker and firm effects jointly shape earnings, and the sorting of workers across firms accounts for an important share of inequality (Brown and Medoff, 1989; Abowd et al., 1999; Card et al., 2013; Song et al., 2019). Related work connects firm heterogeneity and the reallocation of economic activity across firms to wage inequality and the aggregate labor share (Helpman et al., 2017; Autor et al., 2020). In developing economies, workers who are self-employed or work at micro-establishments earn substantially less (Eslava et al., 2026); morevoer, this type of work tends to be an absorbing state, especially among workers with few years of education (de Mel et al. (2008); Donovan et al. (2023). We identify a distinct mechanism operating through access to modern employment. Distortions affect whether workers remain in traditional activities or enter higher-paying modern jobs, thereby linking changes in firm organization to the labor share and earnings inequality through both occupational allocation and relative earnings.

Fourth, our analysis relates to the literature on dual economies, informality, and occupational allocation. The coexistence of productive modern firms and a large traditional sector has been central to development economics since Lewis (1954) and Harris and Todaro (1970). Subsequent work studies informality as a competitive fringe, an alternative for lower-productivity workers, or an equilibrium response to regulation and enforcement (Rauch, 1991; La Porta and Shleifer, 2014; Ulyssea, 2018; Dix-Carneiro et al., 2021). Our framework emphasizes self-employment and informal work in the traditional sector as the occupational margin that absorbs workers who do not enter modern wage employment. Because the distortions we study apply to modern firms but largely spare traditional producers, they affect not only the allocation of resources across firms but also the allocation of workers across occupations. Unlike work that takes formal–informal sorting as the principal outcome, we use this margin to explain the divergence between modern income and modern employment and its consequences for the labor share and earnings inequality.

Finally, we rely on empirical work that uses firm-level prices and quantities to distinguish efficiency from demand/quality/appeal and other determinants of firm size (Foster et al., 2008; De Loecker et al., 2016; Hottman et al., 2016). We extend this work to link it to occupational choice. Because product differentiation is a differentiating characteristic of modern production technologies, the demand/efficiency distinction allows us to identify the distribution of entrepreneurial talent. We build most directly on Eslava et al. (2023), who decompose differences in plant size and growth into productivity, demand, and wedges. We extend this approach in other two directions.

First, we combine firm data with household microdata covering the entire economy, including self-employment and small traditional producers, to jointly characterize the en-

tire distribution of worker and entrepreneurial capabilities and occupational selection. Second, we use firms' reported non-salary labor costs to distinguish this margin of distortion from residual revenue and capital wedges, permitting us to identify the distortions governing employment separately from those governing production. Third, we implement the analysis at two points nearly five decades apart. This allows us to estimate (potentially labor-biased) technological changes over this long period, as well as changes in the distributions of human capital and distortions over this long period. The distincition determines whether the forces separating modern production from modern employment recede with development. These extensions connect the decomposition of firm size to the allocation of workers across occupations and, ultimately, to aggregate income, the labor share, and earnings inequality.

The remainder of the paper is organized as follows. Section 2 develops the model of occupational choice with dual production and characterizes its main mechanisms. Section 3 describes the historical microdata, estimation, and model quantification. Section 4 presents the counterfactual policy experiments, decomposes the sources of jobless modernity in 2019, and compares their roles in 2019 and 1971. The last section concludes.

2 An Occupational Choice Model with Dual Production

To study how jobless modernity arises in equilibrium, we develop a general-equilibrium model of occupational choice with traditional and modern production. Traditional producers are self-employed, use only their own productive ability, and face no production distortions. Modern entrepreneurs combine hired labor, capital, and intermediate inputs, but face firm-level distortions. Heterogeneous agents choose among traditional self-employment, wage work in a modern firm, and modern entrepreneurship. The interaction of occupational choice, firm heterogeneity, and distortions determines both the scale of modern production and the extent to which it creates modern jobs.

2.1 Production Technologies

Modern production is organized by entrepreneurs. Each modern entrepreneur operates one firm that produces one differentiated variety. Let $\mathcal{M}$ denote the endogenous set of modern entrepreneurs. Because each entrepreneur operates one firm and produces one variety, $\mathcal{M}$ also indexes modern firms and varieties. A modern firm $i \in \mathcal{M}$ hires labor, rents capital, and purchases intermediate inputs, producing according to

$$q_i = a_i l_i^{\gamma_l} k_i^{\gamma_k} m_i^{\gamma_m}, \qquad \gamma_l + \gamma_k + \gamma_m = 1, \tag{1}$$

where l_i denotes labor measured in efficiency units, k_i denotes capital, and m_i denotes intermediate inputs.[5] The entrepreneur's productive ability a_i shifts the productivity of the modern technology. These firms, unlike traditional producers, can expand beyond the entrepreneur's own labor by hiring workers and combining multiple factors of production.

Modern firms produce differentiated varieties under monopolistic competition and sell their output to a competitive aggregator. The aggregator combines these varieties into a modern composite that households consume and modern firms use as an intermediate input. We denote the price of this composite by P_M. Firm i faces the inverse demand schedule $p_i = p_i(q_i; d_i)$, where d_i captures the appeal of its variety and raises demand at a given price. The firm also faces wedges on revenue, labor, and capital. These wedges may be positive or negative, so they can act as taxes or subsidies. We collect them in

$$\Gamma_i = \left(\tau_i^R, \tau_i^l, \tau_i^k\right). \tag{2}$$

In both the model and its empirical counterpart, the wedges depend on the firm's attributes (a_i, d_i).

Given factor prices and the demand schedule for its variety, a modern firm chooses labor, capital, and intermediate inputs to maximize

$$\pi_i \equiv \max_{l_i, k_i, m_i} \left\{(1-\tau_i^R)p_i(q_i; d_i)q_i - (1+\tau_i^l)wl_i - (1+\tau_i^k)rk_i - P_M m_i\right\}, \tag{3}$$

subject to equation (1). The revenue wedge changes the return to sales, while the labor and capital wedges change the effective prices of the corresponding inputs.

Traditional production is carried out by self-employed individuals. Because they use only their own productive ability, agent i's output is

$$q_i^T = a_i. \tag{4}$$

Traditional output is homogeneous and used only for final consumption. Traditional producers do not hire workers, use capital or intermediate inputs, or face the distortions applied to modern firms.

[5] We assume constant returns to scale because we cannot reject them in the estimation.

2.2 Endowments and Individual Attributes

The economy contains a unit mass of heterogeneous agents. Each agent is endowed with one unit of time, which is supplied inelastically to a single occupation, and draws an attribute vector (a_i, d_i, s_i) from the joint distribution $F(a, d, s)$. Agents observe all three attributes before choosing an occupation.

The first attribute, a_i, is productive ability. A wage worker supplies a_i efficiency units of labor to a modern firm. A self-employed producer generates a_i units of traditional output. For a modern entrepreneur, a_i raises firm productivity in equation (1). Ability therefore affects returns in all three occupations, though its effect depends on the occupation selected.

The second attribute, d_i, captures the entrepreneur's ability to create demand for the firm's differentiated variety.[6] It matters only when the agent becomes an entrepreneur. A higher d_i shifts demand outward and raises the return to modern entrepreneurship. Entrepreneurial success therefore depends on production efficiency, captured by a_i, and demand appeal, captured by d_i.

The third attribute, $s_i > 0$, captures an idiosyncratic amenity or cost of self-employment. The normalization $s_i = 1$ represents neither an amenity nor a cost. Values above one increase the occupational value of self-employment, while values below one reduce it. The shifter affects occupational choice but not traditional output.

2.3 Occupational Choice

Each agent chooses one of three mutually exclusive occupations: traditional self- employment, wage work in a modern firm, or modern entrepreneurship. Traditional self-employment generates income a_i and occupational value $s_i a_i$. A wage worker supplies a_i efficiency units of labor and earns wa_i. A modern entrepreneur receives maximized profits π_i from the firm problem in equation (3).

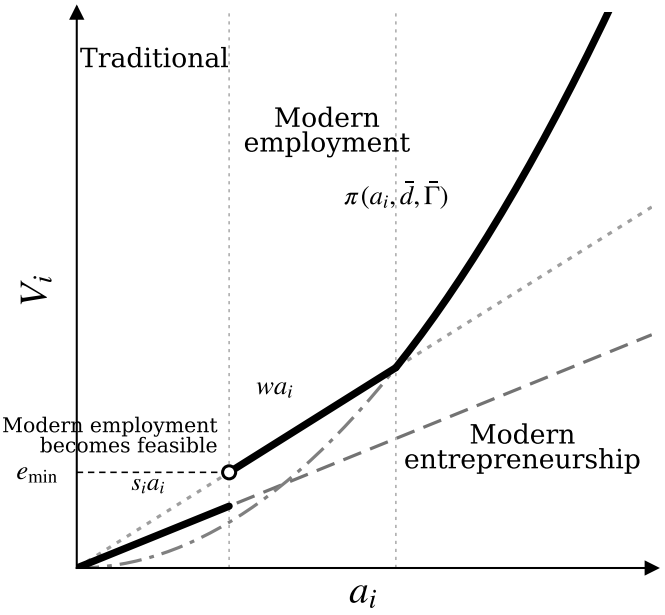
Access to modern wage employment is subject to a minimum-earnings requirement. An agent is eligible only if:

$$wa_i \geq e_{\min}, \qquad \text{or equivalently} \qquad a_i \geq a_{\min} \equiv \frac{e_{\min}}{w}. \tag{5}$$

Here $e_{\min}$ is the minimum earnings a modern employer must pay. Agents below the ability cutoff $a_{\min}$ cannot be hired by modern firms.

[6] Demand appeal captures product attributes such as quality and consumer awareness that raise sales at a given price.

Figure 3: Occupational Sorting along Productive Ability, a_i, Case $s_i < 1$.



Let V_i^T, V_i^W, and V_i^E denote the values of traditional self-employment, modern wage employment, and modern entrepreneurship. These values are:

$$V_i^T = s_i a_i, \qquad V_i^W = \begin{cases} wa_i, & \text{if } a_i \geq a_{\min}, \\ -\infty, & \text{if } a_i < a_{\min}, \end{cases} \qquad V_i^E = \pi_i. \tag{6}$$

Agent i chooses:

$$o_i \in \arg\max \left\{ V_i^T, V_i^W, V_i^E \right\}. \tag{7}$$

Assigning $-\infty$ to wage work below $a_{\min}$ embeds the eligibility restriction directly in occupational choice.

Figure 3 illustrates occupational selection as productive ability varies, holding d_i, s_i, prices, and wedges fixed. Traditional self-employment and potential wage earnings are linear in ability. For $a_i < a_{\min}$, modern wage work is infeasible. In the configuration shown, wage employment becomes optimal once the ability cutoff is reached and remains optimal at intermediate levels of ability. At sufficiently high a_i, entrepreneurial profits exceed wage earnings, and the agent operates a modern firm. We turn to discuss occupational choice in more detail in Section 2.6.2.

2.4 Preferences and Demand

Consumers value the modern composite and the homogeneous traditional good. Aggregate consumption combines the amount of the modern composite used for final consumption, C_M, and traditional consumption, C_T, according to

$$U = \left[\phi_M^{1/\rho} C_M^{(\rho-1)/\rho} + (1-\phi_M)^{1/\rho} C_T^{(\rho-1)/\rho}\right]^{\rho/(\rho-1)}. \tag{8}$$

The elasticity of substitution between the modern and traditional goods exceeds one, $\rho > 1$, and $\phi_M \in (0,1)$.

A competitive aggregator purchases x_i units of each modern variety $i \in \mathcal{M}$ and produces the modern composite according to:

$$Q_M = \left(\int_{i\in\mathcal{M}} d_i^{1/\sigma} x_i^{(\sigma-1)/\sigma}\, di\right)^{\sigma/(\sigma-1)}, \tag{9}$$

where x_i denotes the total amount of variety i absorbed by the aggregator. The elasticity of substitution across modern varieties is σ. We assume $\sigma > \rho > 1$, so modern varieties are more substitutable with one another than the modern and traditional goods are with each other.

The aggregator operates under perfect competition. Cost minimization gives the price index:

$$P_M = \left(\int_{i\in\mathcal{M}} d_i p_i^{1-\sigma}\, di\right)^{1/(1-\sigma)}. \tag{10}$$

and demand for each variety:

$$x_i = d_i \left(\frac{p_i}{P_M}\right)^{-\sigma} Q_M. \tag{11}$$

The modern composite has two uses. Households consume C_M, while modern firms use $\mathrm{Y}_m \equiv \int_{j\in\mathcal{M}} m_j\, dj$ as an intermediate input. Market clearing requires $Q_M = C_M + \mathrm{Y}_m$. The same price P_M applies to both uses.

Let E denote aggregate nominal expenditure and P the corresponding consumption price index. With the price of the traditional good normalized to one,

$$P = \left[\phi_M P_M^{1-\rho} + (1-\phi_M)\right]^{1/(1-\rho)}. \tag{12}$$

In equilibrium, $q_i = x_i$. The demand for each variety therefore implies the inverse de-

mand schedule:

$$p_i = P_M d_i^{1/\sigma} \left(\frac{Q_M}{q_i} \right)^{1/\sigma} = D_M^{1/\sigma} d_i^{1/\sigma} q_i^{-1/\sigma}, \qquad D_M \equiv P_M^{\sigma} Q_M. \tag{13}$$

The shifter D_M captures total demand for modern varieties from final and intermediate users.

2.5 Equilibrium

We now close the model. Given the joint distribution $F(a, d, s)$ and the schedule of firm-level wedges Γ_i, equilibrium prices determine demand, firm decisions, and occupational sorting. Let $\mathcal{T}$, $\mathcal{W}$, and $\mathcal{M}$ denote the sets of agents who choose traditional self-employment, modern wage work, and modern entrepreneurship, respectively.

$$\mathcal{T} = \{i : o_i = T\}, \qquad \mathcal{W} = \{i : o_i = W\}, \qquad \mathcal{M} = \{i : o_i = E\}. \tag{14}$$

These sets are endogenous because the occupational values in equation (7) depend on the wage, demand for modern goods, and entrepreneurial profits. They determine the mass and composition of traditional producers, modern workers, and modern firms.

Households choose consumption to maximize equation (8) subject to their budget constraint and choose occupations according to equation (7). The competitive aggregator minimizes the cost of producing the modern composite, and modern firms solve equation (3). We normalize the price of the traditional good to one.

Goods markets clear when $q_i = x_i$ for every $i \in \mathcal{M}$, $Q_M = C_M + Y_m$, and $C_T = \int_{i \in \mathcal{T}} a_i \, di$.

The market for efficiency units of labor clears when:

$$\int_{i \in \mathcal{W}} a_i \, di = \int_{i \in \mathcal{M}} l_i \, di. \tag{15}$$

Capital services are available at the rental rate r, and aggregate capital employed satisfies:

$$K = \int_{i \in \mathcal{M}} k_i \, di. \tag{16}$$

Capital ownership is common to all agents and therefore does not affect occupational choice.

Net revenues from the three wedges are rebated to households through the lump-sum

transfer T. The government budget satisfies:

$$T = \int_{i \in \mathcal{M}} \left[\tau_i^R p_i q_i + \tau_i^l w l_i + \tau_i^k r k_i \right] di. \tag{17}$$

Household demand follows from utility maximization subject to the aggregate budget constraint. Aggregate expenditure satisfies

$$E = w \int_{i \in \mathcal{W}} a_i \, di + \int_{i \in \mathcal{M}} \pi_i \, di + \int_{i \in \mathcal{T}} a_i \, di + rK + T. \tag{18}$$

Income consists of wage earnings, entrepreneurial profits, income from traditional production, capital income, and government transfers.

Equilibrium Definition. Given the joint distribution $F(a, d, s)$, the minimum-earnings requirement $e_{\min}$, and a schedule of firm-level wedges $\{\Gamma_i\}_{i \in [0,1]}$, a competitive equilibrium consists of prices, consumption allocations, aggregator and firm decisions, occupational choices, aggregate capital, and transfers such that households, the modern aggregator, and firms optimize, occupational choices satisfy equations (5) and (7), goods and factor markets clear, and the government and aggregate household budget constraints hold.

Preferences and the set of active modern firms determine demand for modern goods. Demand, firm attributes, and wedges determine each firm's scale, input choices, and profits. Profits and the wage shape occupational choices, which determine the mass of modern entrepreneurs, the supply of labor to modern firms, and traditional output. The model therefore jointly determines how much income modern firms generate and how many workers they employ.

2.6 Equilibrium Characterization and Model Mechanisms

This section uses the model to characterize jobless modernity and the mechanisms through which distortions shape it. We first examine occupational selection, then separate the minimum-earnings requirement, which restricts access to modern wage work, from firm-level wedges that reduce labor demand and firm scale. Finally, we trace these mechanisms into the earnings gap between modern workers and the self-employed and into inequality across percentiles of the earnings distribution.

2.6.1 Jobless Modernity through the Lens of the Model

Modern occupational income and employment. We use the equilibrium allocation to construct the model counterpart of jobless modernity. Let,

$$n_T \equiv \int_{\mathcal{T}} di, \qquad n_W \equiv \int_{\mathcal{W}} di, \qquad n_E \equiv \int_{\mathcal{M}} di$$

denote the masses of traditional self-employed workers, modern wage workers, and modern entrepreneurs. Because the economy contains a unit mass of agents,

$$n_T + n_W + n_E = 1.$$

Modern employment includes wage workers employed by modern firms and the entrepreneurs who operate those firms. Its share of total employment is therefore,

$$S_M^N \equiv n_W + n_E. \tag{19}$$

Define occupational income generated in each activity as:

$$Y_T \equiv \int_{\mathcal{T}} a_i \, di, \qquad Y_W \equiv w \int_{\mathcal{W}} a_i \, di, \qquad Y_E \equiv \int_{\mathcal{M}} \pi_i \, di.$$

Modern occupational income is $Y_M \equiv Y_W + Y_E$, and its share of total occupational income is:

$$S_M^Y \equiv \frac{Y_M}{Y_T + Y_M}. \tag{20}$$

This measure matches the occupational earnings object used in the quantitative analysis. It excludes capital income and lump-sum transfers. Entrepreneurial profits are the occupational earnings of modern entrepreneurs.

We define the jobless-modernity gap as:

$$\mathcal{J} \equiv S_M^Y - S_M^N. \tag{21}$$

A positive value of $\mathcal{J}$ means that modern activities account for a larger share of occupational income than of employment.

The equilibrium also determines the distribution of individual earnings. Excluding cap-

ital income and lump-sum transfers, realized occupational earnings are:

$$y_i = \begin{cases} a_i, & i \in \mathcal{T}, \\ wa_i, & i \in \mathcal{W}, \\ \pi(a_i, d_i, \Gamma_i), & i \in \mathcal{M}. \end{cases} \tag{22}$$

The self-employment shifter s_i does not enter equation (22). It affects the value of self-employment and hence occupational choice, but not the income generated by traditional production. Real earnings are given by y_i/P.

The gap therefore captures the extent to which modern production generates income without incorporating a commensurate share of the workforce. The jobless-modernity gap has a direct interpretation in terms of average occupational income. Let:

$$n_M \equiv S_M^N, \qquad \bar{y}_M \equiv \frac{Y_M}{n_M}, \qquad \bar{y}_T \equiv \frac{Y_T}{1 - n_M}$$

denote the modern employment share and average occupational income in modern and traditional activities.

Lemma 1 (The jobless-modernity gap) *The jobless-modernity gap satisfies:*

$$\mathcal{J} = \frac{n_M(1 - n_M)(\bar{y}_M - \bar{y}_T)}{n_M \bar{y}_M + (1 - n_M)\bar{y}_T}. \tag{23}$$

Consequently, $\mathcal{J} > 0$ if and only if $\bar{y}_M > \bar{y}_T$.

The proof follows from: using

$$S_M^Y = \frac{n_M \bar{y}_M}{n_M \bar{y}_M + (1 - n_M)\bar{y}_T} \qquad \text{and} \qquad S_M^N = n_M.$$

We obtain equation (23) by subtracting S_M^N from S_M^Y. To characterize the magnitude of the gap, define

$$R \equiv \frac{\bar{y}_M}{\bar{y}_T}.$$

This ratio compares average realized occupational income across all agents in modern and traditional activities. We now can characterize when the jobless modernity gap becomes largest.

Corollary 1 (When Jobless Modernity is Largest) *The gap can be written as:*

$$\mathcal{J} = \frac{n_M(1 - n_M)(R - 1)}{1 + n_M(R - 1)}. \tag{24}$$

For a given n_M, the gap increases strictly with R. For a given $R > 1$, the gap is hump-shaped in n_M and reaches its maximum at:

$$n_M^* = \frac{1}{1+\sqrt{R}}, \qquad \mathcal{J}_{\max} = \frac{\sqrt{R}-1}{\sqrt{R}+1}.$$

A large gap therefore requires a sizable difference in average occupational income together with an incomplete transition into modern employment. A decline in modern employment does not mechanically increase the gap. It does so only when the economy begins above n_M^*.

Empirical interpretation. The income and employment shares jointly determine R. Rearranging equation (24) gives:

$$R = \frac{S_M^Y}{1-S_M^Y}\frac{1-n_M}{n_M}. \tag{25}$$

The household-survey evidence in the next section provides an empirical analogue based on labor earnings.[7] In Colombia around 1970, firms with at least ten workers accounted for 43 percent of labor earnings but only 25 percent of employment. Average labor earnings per person were therefore 2.26 times higher in modern than in traditional activities. The corresponding ratio was 1.87 in Colombia in 2019 and 1.13 in the United States.

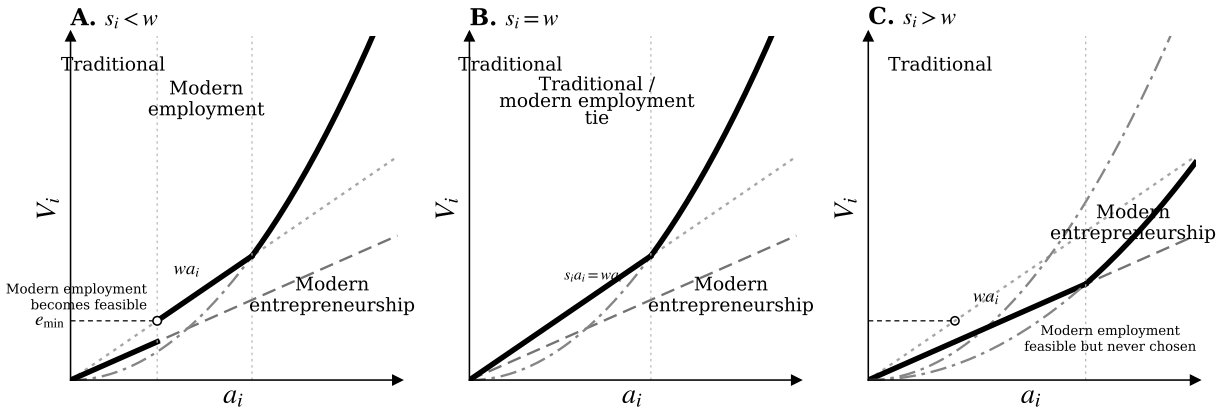
The corresponding values of n_M^* are 40 percent around 1970 and 42 percent in 2019, above Colombia's observed employment shares of 25 and 33 percent. Colombia therefore lies below n_M^* in both periods. Over this range, an expansion of modern employment can initially widen the gap if R remains unchanged. The persistence of a large Colombian gap reflects both limited modern employment and a substantial difference in average earnings across activities. The United States combines broad modern employment with near parity in average earnings across the two groups, producing a small gap.

2.6.2 Selection Across Occupations

Occupational selection depends on how productive ability a_i affects the three occupational values and on how s_i and d_i shift their relative positions. Figure 3 introduced the case $s_i < w$, which is reproduced in Panel A of Figure 4. Figure 4 extends that analysis to other values of s_i. Figures 5 and 6 then introduce variation in entrepreneurial capability d_i. Prices, aggregate demand, and firm-level wedges are held fixed throughout to isolate

[7] Figure 7 uses labor earnings from household surveys. The quantitative measure used in Section 3 also includes entrepreneurial profits.

Figure 4: Occupational Choice for Different Self-Employment Values



occupational selection.

Panel A of Figure 4 reproduces the case discussed in Figure 3. Below $a_{\min}$, modern wage employment is infeasible and the agent selects traditional self-employment. Once wage work becomes feasible, it dominates traditional self-employment because $w > s_i$. At sufficiently high ability, the return to entrepreneurship overtakes the two linear occupational values. Panel B considers $s_i = w$. Traditional self-employment and wage work then provide the same occupational value whenever wage work is feasible. Panel C considers $s_i > w$. Traditional self-employment dominates wage work, so modern employment can be feasible without being selected. Only sufficiently high-ability agents enter modern entrepreneurship.

Figure 5: Occupational Choice in (a_i, d_i) Space

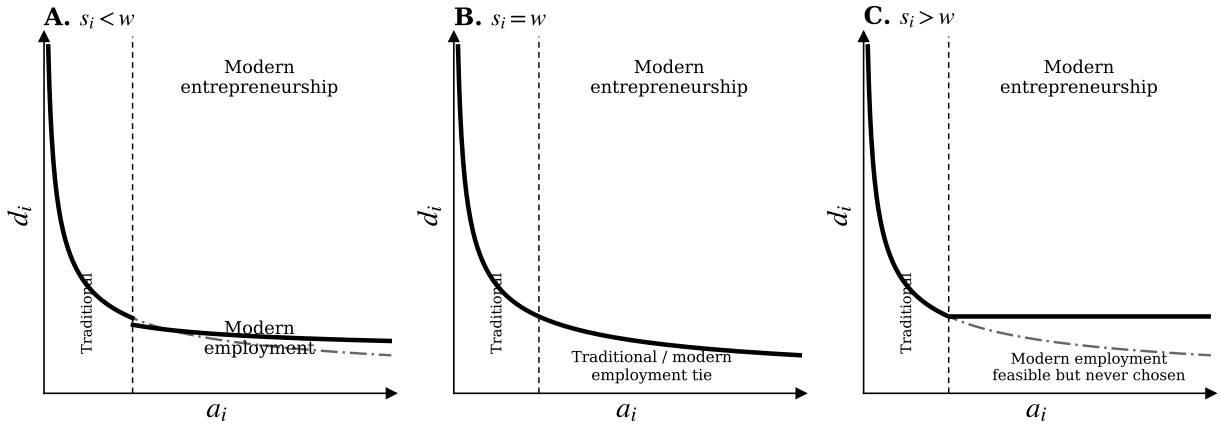
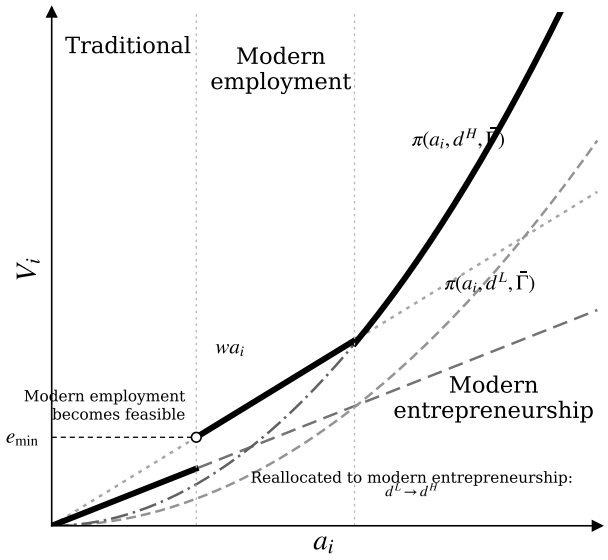


Figure 5 represents the same three cases in (a_i, d_i) space. A higher d_i raises entrepreneurial profits and reduces the level of productive ability required to operate a modern firm. When $s_i < w$, modern wage employment occupies the region above the minimum-earnings cutoff and below the entrepreneurship boundary. When $s_i = w$, wage work

Figure 6: Entrepreneurial Capability and Occupational Choice



and traditional self-employment provide the same value in this region. When $s_i > w$, traditional self-employment dominates wage work wherever entrepreneurship is not selected.

Figure 6 isolates the role of entrepreneurial capability in the occupational-value diagram. A higher d_i shifts the entrepreneurial-profit schedule upward without changing the value of wage work, the value of self-employment, or the minimum-earnings cutoff. It therefore lowers the ability threshold for entrepreneurship and reallocates some agents from modern wage work into modern firm ownership. Entrepreneurial capability changes the composition of modern employment without directly changing access to wage work.

2.6.3 Labor and Non-Labor Distortions

The model distinguishes barriers to modern production from barriers to modern employment. All firm-level wedges can reduce entrepreneurial returns and firm scale. Labor-market distortions have additional effects on employment. The minimum-earnings requirement changes who can enter modern wage work, while the labor wedge changes how much labor modern firms demand.

To make these channels precise, define $\mu \equiv \sigma/(\sigma-1)$ and the common pre-distortion unit-cost index

$$\kappa = \left(\frac{w}{\gamma_l}\right)^{\gamma_l} \left(\frac{r}{\gamma_k}\right)^{\gamma_k} \left(\frac{P_M}{\gamma_m}\right)^{\gamma_m}. \tag{26}$$

Let $\chi_i \equiv a_i d_i^{1/(\sigma-1)}$ denote firm capability. Under monopolistic competition with constant

markups, entrepreneurial profits can be written as

$$\pi_i = \left(1 - \frac{1}{\mu}\right) D_M \left[\frac{\chi_i}{\mu\kappa} \underbrace{\frac{(1-\tau_i^R)^{\sigma/(\sigma-1)}}{(1+\tau_i^l)^{\gamma_l}(1+\tau_i^k)^{\gamma_k}}}_{\Omega(\Gamma_i)} \right]^{\sigma-1}. \tag{27}$$

Equation (27) displays the common scale channel. Holding aggregate demand and prices fixed, a more adverse realization of any component of Γ_i lowers profits. It shifts the entrepreneurial-value schedule downward, discourages entry into modern entrepreneurship, and compresses the scale of firms that continue to operate.

The labor wedge has an additional effect on employment. The first-order condition for labor implies

$$l_i = \frac{\gamma_l}{\mu w(1+\tau_i^l)}(1-\tau_i^R)p_i q_i. \tag{28}$$

Thus, τ_i^l lowers labor demand conditional on the firm's net revenue and also reduces firm scale through $\Omega(\Gamma_i)$. It raises the cost of labor relative to other inputs. Revenue and capital wedges affect modern employment through firm entry, scale, and input choices, but do not directly determine which workers are eligible for modern wage employment.

The minimum-earnings requirement operates on this eligibility margin. As equation (5) shows, an agent can become a modern wage worker only if $a_i \geq a_{\min} = e_{\min}/w$. Unlike the firm-level wedges, $e_{\min}$ does not enter a firm's input demand conditional on prices. It changes occupational selection by excluding workers below the threshold from modern wage employment. A tighter threshold can therefore raise average earnings among modern wage workers through selection even as it reduces the number of modern jobs.

This distinction maps into our empirical strategy. The two labor distortions have direct empirical counterparts. The effective minimum-earnings threshold is disciplined by its position in the observed earnings distribution and by occupational allocation. We measure τ_i^l from mandatory non-salary labor costs reported separately from wage payments. By contrast, τ_i^R and τ_i^k are recovered from firm behavior as residual revenue and capital wedges. The model therefore distinguishes empirically grounded labor-market barriers from other forces that compress modern firm size.

The distinction also clarifies why reforms that generate similar increases in modern production need not create the same number of modern jobs. Reducing adverse non-labor wedges expands firm scale and modern income. Relaxing the minimum-earnings requirement broadens access to modern wage work, while reducing τ_i^l raises firms' demand for labor and reduces their incentive to substitute away from it. Once wages, prices, and entry adjust jointly, the quantitative strength of these channels is determined

by the estimated model.

2.6.4 Effects on Income Distribution

The jobless-modernity gap has a distributional interpretation, but it is not itself an inequality index. It depends on average income in modern and traditional activities and on the share of agents in each group. It does not describe how earnings are distributed within and across occupations. Section 4 therefore focuses on two related distributional objects: the ratio of average earnings between modern wage workers and the self-employed, and earnings ratios across percentiles of the overall distribution.

The worker to self-employed earnings ratio has an exact decomposition in the model. Let $\bar{a}_W$ and $\bar{a}_T$ denote average ability among modern wage workers and traditional self-employed workers. Since the former earn wa_i and the latter earn a_i, their average earnings satisfy

$$\frac{\bar{y}_W}{\bar{y}_T} = \underbrace{w}_{\text{price effect}} \quad \underbrace{\frac{\bar{a}_W}{\bar{a}_T}}_{\text{selection effect}} . \tag{29}$$

The price effect captures the return to an efficiency unit of labor in modern firms. The selection effect captures differences in average ability induced by occupational sorting. The minimum-earnings requirement acts directly on the second term by preventing agents with $a_i < a_{\min}$ from becoming modern wage workers. Firm-level wedges act on the first term through labor demand and the equilibrium wage. They also affect selection indirectly because $a_{\min} = e_{\min}/w$. Section 4 uses equation (29) to separate changes in relative earnings caused by the wage from those caused by the composition of the two groups.

Relaxing the minimum-earnings requirement allows some agents close to the threshold to move from traditional self-employment into modern wage work. When modern wage employment pays more than traditional production for these entrants, their earnings rise. At the same time, average earnings among modern workers can fall because the new workers have lower ability than incumbent modern workers. Broader access to better-paying jobs can therefore reduce the modern-worker to self-employed earnings ratio. This mechanism connects the model to the strong earnings gradient in access to modern jobs documented in Figure 2.

The percentile ratios in Section 4 show where these changes occur in the earnings distribution. The ratios e_{P50}/e_{P25} and e_{P50}/e_{P10} measure dispersion below the median, where exclusion from modern wage work is most relevant. The ratio e_{P90}/e_{P50} measures dispersion above the median, while e_{P95}/e_{P10} captures the distance across the tails. These ratios do not admit the exact multiplicative decomposition in equation (29), but they are governed by the same price and selection channels.

When the minimum-earnings requirement leaves lower-ability agents in traditional self-employment, it can keep the lower percentiles of the earnings distribution low and increase ratios that compare them with the median. Relaxing the requirement can raise those lower percentiles as new workers enter modern employment, compressing the lower half of the distribution even when average earnings among modern workers decline through composition. Changes in w instead rescale modern-worker earnings relative to self-employment. They can shift the median and upper percentiles and can also change the ranking of agents across occupations.

Distortions therefore generate two opposing effects. Exclusion can raise the selection component of the modern-worker to self-employed earnings ratio and widen percentile gaps. Wage compression lowers modern-worker earnings relative to self-employment and partly offsets that effect. The counterfactual decomposition in Section 4 holds either occupational allocation or the wage fixed to distinguish the two forces. In the estimated model, the exclusion effect dominates for both the modern-worker to self-employed earnings ratio and the percentile ratios. Removing distortions consequently lowers these measures even though the equilibrium wage rises.

The theory alone does not determine which force dominates. A lower τ_i^l raises labor demand directly, while lower revenue and capital wedges raise it through firm scale and input choices. All three can change w, occupational sorting, and hence the two sets of distributional measures. The aggregate price index does not introduce a separate channel here because the common deflator P cancels from average-earnings and percentile ratios. Section 4 quantifies whether selection or prices dominate and identifies the part of the earnings distribution in which each reform has its largest effect.

3 Model quantification

Our quantification assigns (a, d, s, τ) draws to each individual in our population of workers. We then compute the occupational choice for each agent conditional on the individual's draws and parameters specific to the context of our data. To solve the general equilibirum, we conjecture values for the aggregate demand shifter D_m, the price of the intermediates, and the wage in the modern manufacturing labor market w and compute a *tâtonnement* process until we find market clearance. The remaining of the section explains the model parameterization.

We begin by presenting the data and follow with the four steps in which our quantification proceeds. First, we use the Household Surveys together with model insights to obstain the distribution of a modern and traditional sector workers (i.e. excluding modern entrepreneurs). Second, we estimate the modern firm's technology and demand

system using establishment-level manufacturing data. Third, we use the resulting firm-level residuals to recover the joint distribution of entrepreneurial ability, product appeal, and distortions in the modern sector. Fourth, we calibrate the remaining preference, traditional-sector, and occupational-choice parameters to match moments from 1970s Colombia. This structure makes transparent which objects are estimated directly from the data and which are pinned down in equilibrium.

3.1 Historical Micro Data

We combine two complementary sources produced by DANE. Household Surveys are representative of the universe of the working population. They provide individual-level information on occupation, sector, firm size, and labor income, and the size of establishment where the person works. We use the 1971 and 2019 waves of the survey. These data allow us to measure the employment and labor earnings shares of modern versus traditional production and to recover the earnings distributions relevant for workers and the self-employed.

Households survey data show that Colombia's modern exhibits a pattern of jobless modernity similar to what we previously documented for its regional neighbors, persistent since at least 1971 (Figure 7). The Figure also provides contrast with the United States, for which there is comparable information for the (end of the) 1970s, although going back to that time in the US the only available establishment size threshold is 25 employees. The figure thus shows comparisons using both thresholds to define modernity, for the years in which each is available. Under both definitions and for both periods, Colombia's modern firms represent a much smaller share of employment than labor earnings, while the same does not hold for the US.

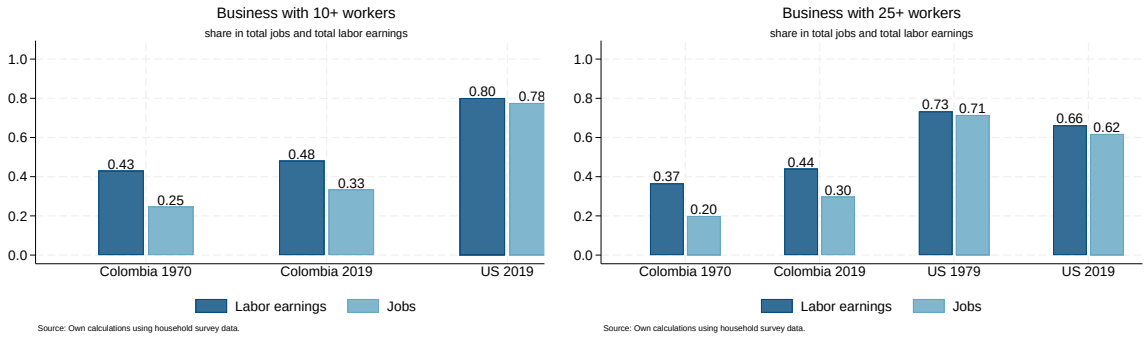


Figure 7

We complement this information with detailed microdata on modern firms. The Annual

Manufacturing Survey covers the universe of establishments with at least ten employees and reports revenue, payroll, capital, material inputs, and product-level information for both outputs and mateial inputs. For each of them, there are separate reports of value and quantity, so that average prices can also be obtained. It is this richness that allows us to estimate both production technology (the γ parameters, and elasticities of substitution across modern products, σ). We have access to data going back to 1974. The richness and wide time coverage of these data is unparalleled for other sectors in the economy (and even for manufacturing most other countries), which forces us to impose the parameters and distributions obtained from these data to modern firms irrespective of the sector they are in. Appendix A.1 provides additional construction details.

3.2 Obtaining a distributions outside modern entrepreneurs

In light of the model, we use the income distributions from the household surveys to estimate the ex-post distribution of a for salaried and own-account workers. We use individual reports of labor earnings of workers in both the modern and the traditional sector.[8] We exclude modern entrepreneurs from this approach, both because the quality of the data is not strong for reports of entrepreneurial returns, and because we can rely on insights from the model to separately estimate the distribution of a and d using rich data on the firms these entrepreneurs run. We explain this alternative approach in the following subsection.

For traditional-sector workers, $a = earnings$, so that we directly observe the distribution of a. For those salaried in the modern sector, meanwhile, $a = earnings/w$, where we estimate w as the earnings premium for workers in modern firms vs. traditional firms, in a Mincer equation setting. After controlling for observables (education, age, sex, sector of activity, occupation and region), we estimate that working in the modern sector carries a premium of 1.5 in 1971 and 1.8 in 2019, relative to being in the traditional sector.

3.3 Firm Level Estimation

For entrepreneurs and their firms, we use the information in the Annual Manufacturing survey to estimate the parameters of technology and demand $\{\gamma_l, \gamma_k, \gamma_m, \sigma\}$, and subsequently obtain measures of a, d for each manufacturing establishment. These correspond to the ex-post distributions of each of these dimensions of talent among those who self-select as entrepreneurs. Using this information, as well as directly reported information

[8] Labor earnings reports are censored at the minimum non-zero value reported. The minimum cutoff is adjusted through calibration in the full general equilibrium model to reproduce moments of the income distribution.

of τ_l in the survey, we also estimate τ_k, τ_r for each firm. We now explain the estimation in detail.

The manufacturing data directly report, for each unit, revenue (pq), total salary payments (wl), capital stocks (k), value of material inputs used ($p_m m$). We build, based on our theory for product demand, plant level deflators for output, p, and materials, p_m, which we use to deflate revenue and value of materials to obtain y and m. In the data, the empirical analog of our theoretical modern firms is the manufacturing establishment, but we will continue to use the language "firm" for consistency.[9] The data allows us to accurately measure, for each estimation period, technological parameters that are crucial for simulations in the model. Per-period estimation for periods several decades appart shedds light on patterns of (potentially factor-biased) technical change over relatively long periods. We later use these different estimates to identify the role of distortions separatelt from that of factor-biased technology changes.

We begin by estimating the production-demand system containing equations 1 and 13. We let both technology and demand parameters vary across 29 manufacturing sectors for two particular periods of interest. Our approach that relies on both cross-sectional and time-series variation, following the literature on proxy estimation of production function and the extension by Eslava et al. (2023) to jointly estimate production and demand functions.[10] We use data from the 2000-2019 waves to estimate the 2019 modern-sector production and demand system (and to measure the joint distribution of firm types and distortions) and the earliest available waves, 1974–1989, to inform the 1971 quantification. Our estimation closely follows that in Eslava et al. (2023) but implements it for two separate periods, including a historical one. Table 1 reports the corresponding estimates, averaged across manufacturing sectors. It is these averages that we feed into the calibration.

Table 1: **Factor and demand elasticities**

Year	γ_l	γ_k	γ_m	σ
1974 - 1989	0.374	0.105	0.542	2.391
2000 - 2019	0.259	0.123	0.675	2.702

Table 1 highlights two patterns that matter for the counterfactuals. First, labor intensity falls sharply between 1974–1989 and 2000–2012, from 0.374 to 0.259 (with growth in the materials elasticity as a counterpart. Second, the elasticity of substitution across modern varieties rises from 2.391 to 2.793.

[9] Over 90% of firms are uni-establishment in the data.

[10] Proxy methods to estimate production functions have been developed by Olley and Pakes (1996); Levinsohn and Petrin (2003); Ackerberg et al. (2015); De Loecker and Warzynski (2012); Gandhi et al. (2020)

Having estimated the parameters of the firm problem, we obtain a for each firm in each period as the idiosyncratic residual of the production function (1) and d as the idiosyncratic residual of demand function (13). We work only with the idiosyncratic part of firm-level a and d by taking the log difference vs. the mean of logs in the respective sector and year.

We subsequently obtain measures of idiosyncratic firm-level distortions. We directly measure $\tau^l = \frac{\text{payroll tax}}{wl}$ for each establishment, taking advantage of reports in the data about the payroll mandatory non-salary labor costs paid by each establishment. Using only the idiosyncratic component of τ_l and of k and l in the capital-labor ratio, we obtain (the idiosyncratic component of) $1+\tau^k = \frac{l}{k}(1+\tau^l)$. We then solve for $(1-\tau^R)$ re-writing equation 27 as:

$$(1-\tau^R) = \frac{\mu(1+\tau^l)^{\gamma_l}(1+\tau^k)^{\gamma_k}(pq)^{\frac{1}{(\sigma-1)}}}{D_{i,M}ad^{\frac{1}{\sigma-1}}} \tag{30}$$

Since all firm-level variables and parameters in the right-hand side of this equation have been measured at this point, we can estimate the idiosyncratic variation in $(1-\tau^R)$ up to $D_{i,M}$.

Table 2 reports correlation coefficients across these moments. Two facts stand out. First, the composite distortion term, $\frac{(1-\tau^R)}{(1+\tau^l)^{\gamma_l}(1+\tau^k)^{\gamma_k}}$, is negatively correlated with quality adjusted productivity $ad^{\frac{1}{\sigma-1}}$, with correlations of -0.26 and -0.32. That is, the firms with the highest underlying productivity do not expand proportionally because they face stronger wedges. Second, this is partly explained by effective non-salary labor costs falling more heavily on more highly productive firms in the early period, when ($corr(ad^{\frac{1}{\sigma-1}}, (1+\tau^l)^{-1}) = 0.51$)). The same does not hold in modern times, when the correlation fell essentially to zero.

Table 2: **Correlation matrix for establishment attributes**

	Productivity $ad^{\frac{1}{\sigma-1}}$	Composite dist. $\frac{(1-\tau^R)}{(1+\tau^l)^{\gamma_l}(1+\tau^k)^{\gamma_k}}$	L-dist. $\frac{1}{(1+\tau^l)}$
Panel A: 1971			
$ad^{\frac{1}{\sigma-1}}$	1.00		
$\frac{(1-\tau^R)}{(1+\tau^l)^{\gamma_l}(1+\tau^k)^{\gamma_k}}$	-0.26	1.00	
$\frac{1}{(1+\tau^l)}$	-0.51	0.05	1.00
Panel B: 2019			
$ad^{\frac{1}{\sigma-1}}$	1.00		
$\frac{(1-\tau^R)}{(1+\tau^l)^{\gamma_l}(1+\tau^k)^{\gamma_k}}$	-0.32	1.00	
$\frac{1}{(1+\tau^l)}$	-0.01	0.01	1.00

3.4 Underlying type distributions f_a, f_d and distributions of $\tau(a,d)$

Our model calibration uses the non-parametric f_a distribution obtained from putting together the ex-post distributions of a for the three occupational groups, estimated as explained in the previous subsections. The distributions for workers in modern salaried employment and the traditional sector are obtained directly from peso-denominated earnings data, and easily put together by adding up the densities at each point in the two distributions. Further adding the a data values and corresponding densities obtained for modern entrepreneurs from the Annual Manufacturing Survey poses a challenge, however. The techniques employed in our firm estimation only allow us to estimate idiosyncratic attributes of firms so that by construction, our firm estimates of a are centered around $a = 1$, while the income reports in the household survey are in monetary units.

We deal with this problem by taking advantage of the optimal occupational choice, which implies that the range of a that ex-post selects into entrepreneurship is the full range of the ex-ante distribution f_a. We thus take a_{min} to correspond to the maximum income of workers in the Household Survey and obtain a_{max} by requiring the overall range to replicate the range of a in the manufacturing firm data.[11] In practice, as predicted by the model, ln_a has a wider range for entrepreneurs than for the rest of workers.

The optimal occupational choice also implies that modern entrepreneurs (firms) with large enough a should display a distribution of d similar to that in the population, es-

[11] We estimate the range and distribution in log terms.

pecially at the tail. With this in mind, we estimate parameters for a Pareto distribution for d using the d distribution observed among the top decile of the a distribution among manufacturing firms.

Each agent draws a type a, d. This also implies a $((\tau^R(a,d), \tau^l(a,d), \tau^k(a,d))$ assignment for each individual, since our quantification aligns with the firm-level correlations between some the τ distortions and firms' productivity reported in Table 2. In particular, we compute in the data the average idiosyncratic distortion for each (a, d) type, and use this profile to assign model agents τs that correspond to their (a, d) draws. Because, at this point, we have only considered idiosyncratic variation across firms, we scale each of the distortions (τ^R, τ^l, τ^k) by an intercept. For τ_l, directly measured in the data, the intercept is simply the average level of non-salary labor costs paid per unit of wage, which stands at $\tau_l = 0.46$ in 2019 and a nearly identical $\tau_l = 0.47$ in 1971 (although the overall tau_l profile went through fundamental changes, see Table 2). For τ^R and τ_k, we fix intercepts that approximate tax rates of corporate income and dividends in 1971, respectively. We set $\tau^R = 0.2$ and $\tau^k = 0$, taken from historical accounts on Colombian tax reforms (Cardenas and Perry, 1986).

3.5 Model parameterization

Our baseline targets 2019 Colombia, combining the 2019 Household Survey with manufacturing estimates from 2010–2019. We parameterize the modern production function with the period's estimates from Table 1, impose exact constant returns to scale by adjusting the materials share, and feed the estimated non-parametric objects for (a, d) and the corresponding distortion schedules into the simulated economy.[12] The remaining parameters are internally calibrated to match moments of the data. Table 3 records the method used for the identification of each parameter. The first panel summarizes the micro-estimation explained above, while the second lists the internally calibrated parameters and the targeted moments.

[12] One of our subsequent counterfactual exercises calibrates an alternative baseline for 1971, using the production and demand paramenters for the earlier period from Table 1 and the corresponding a, d distributions for that period.

Table 3: **Model Parameters: Estimated from the Microdata, Internally Calibrated or Exogenously Set**

Estimation from Micro Data / Set in Calibration		
Parameter	Type	Empirical Counterpart
$\gamma_l, \gamma_k, \gamma_m, \sigma$	Point est.	Joint estimates from demand and production function
$(a_j, d_j); j \in M$	Non-param. dist.	Residual production/demand function
$\tau_l(a_j, d_j)$	$0.459 + [\cdot]_{122\times30}$	Reported payroll tax
$\tau^R(a_j, d_j)$	$0.2 + [\cdot]_{122\times30}$	$R_j \propto \pi_j \propto a_j^{\sigma-1} d_j \frac{(1-\tau_j^R)^\sigma}{(1+\tau_j^l)^{\gamma_l(\sigma-1)}}$
$(a_j); j \in W$	Non-param. dist.	Earnings/(W wage premium)
$(a_j); j \in T$	Non-param. dist.	Earnings
Internally calibrated endowing agents with (a,d,s) and solving GE		
Parameters	ρ ϕ_M min_e s_- s^+ s_{tail} $factor_a$	
Targets	$\frac{N_M}{N}$ $\frac{e_{min}}{e_{50}}$ $\frac{\|\|e>e_{min}\|\|}{N}$ w_M $\frac{e_{P50}}{e_{P10}}$ $\frac{e_{P75}}{e_{P50}}$ $\frac{e_{P50}}{e_{P5}}$	

We calibrate the taste shifter and elasticity of substitution that govern modern expenditure (ρ, ϕ_M), the minimum-earnings threshold min_e, and the parameters of the Pareto distribution of self-employment utility shocks, s_-, s^+, s_{tail}. We seek to match the modern employment share, the ratio of the regulatory minimum wage to median earning, the fraction of workers who earn above the regulatory earnings floor, the modern earnings premium (w_M), and moments from the income distribution, in particular the P50/P10 and P75/P50 ratios. All parameters are simultaneously identified by all targeted moments. We subsequently calibrate an adjustment factor for a_{min}, since this value is censored in the individual income data from where we obtain it. This step targets the P50/P5 income ratio. We set externally the user cost of capital $r = 0.2$.

Table 4: Internally calibrated parameters

Parameter	Value	Definition
ϕ_M	0.949	Preference weight modernity
ρ	1.158	Elasticity substitution modern-traditional
e_{min}	0.0008	Minimum earnings constraint
Pareto tail s	0.559	Tail par. self-employment shocks distribution
Lower bound s	0.619	Lower bound self-employment shocks
Upper bound s	34.00	Upper bound self-employment shocks
Factor a_{min}	0.93	a_{min} censoring adjustment

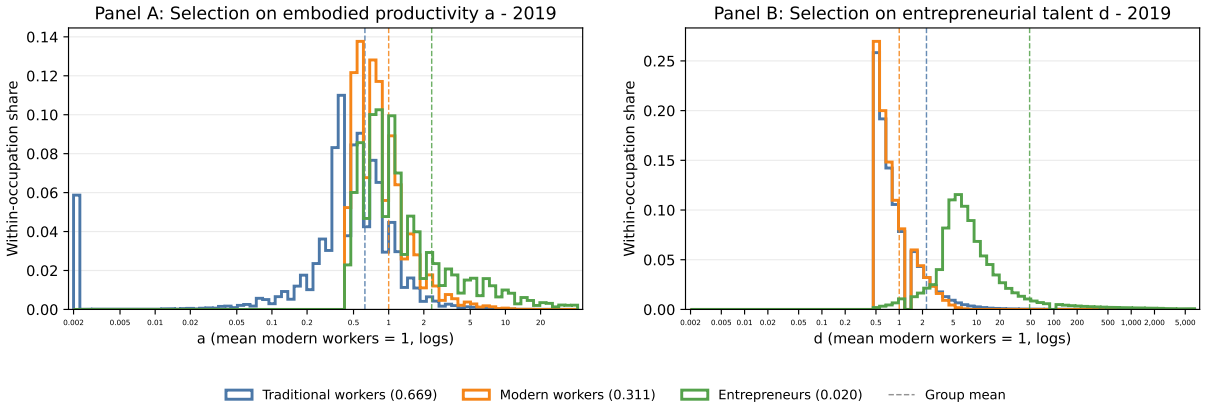
Table 4 reports the calibrated parameter values, while table 5 accounts for the calibration fit, reporting values for the targeted moments in the data and the calibrated model. The model matches the modern employment share and other targeted moments closely. It also reproduces well untargeted moments, such as the entrepreneur share, the modern % of labor earnings, the fraction of workers in the top half of the distribution who engage in traditional employment, and other segments of the income distribution.

Table 5: Internal Calibration: Targets and Fit

Moment	Data	Baseline
Modern % of jobs	33.6	33.1
e_{min}/e_{P50}	0.9	0.9
$\|\|e < e_{min}\|\|/N$	46.0	46.1
Modern wage premium	1.8	1.5
e_{P50}/e_{P5}	10.1	10.1
e_{P50}/e_{P10}	3.3	3.3
e_{P75}/e_{P50}	1.3	1.7
Untargeted moments		
$(N_{modern_{entrep}}/N) \times 100$	1.6	2.0
Modern % of labor earnings	51.1	53.4
e_{P50}/e_{P25}	1.6	1.8
e_{P90}/e_{P50}	2.0	2.6
$\|\|e > e_{p50}\Omega Self\|\|/\|\|e > e_{p50}\|\|$	35.0	36.4

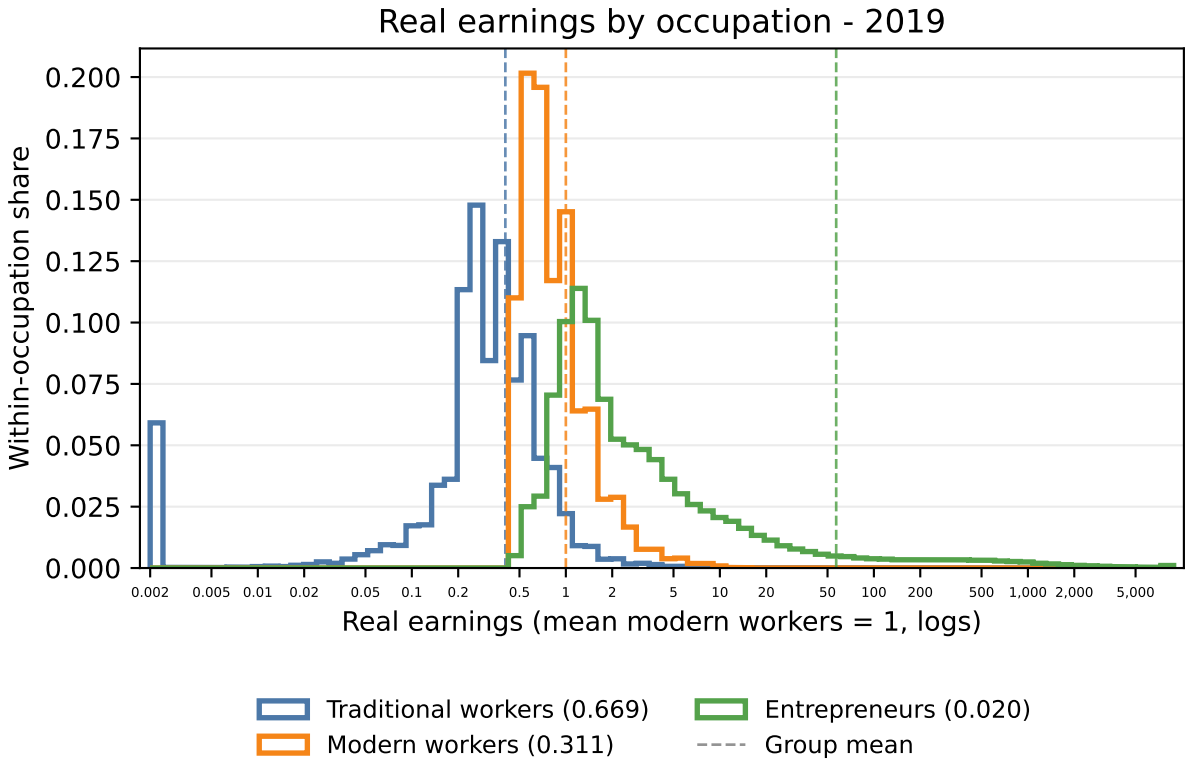
To illustrate the characteristics of the equilibrium occupational choice Figure 8 shows the selection on a and d of workers in different occupations. Modern entrepreneurs are selected from the top of the d distribution. They also display on average higher a, but it is d, talent specific to the producing differentiated goods, that really sets them appart. On average, workers who select into jobs in the traditional sector have lower a than modern salaried workers and modern entrepreneurs, although there is considerable overlap of the two groups in the middle part of the a distribution. Figure 9, in turn, presents the baseline equilibrium distribution of earnings by occupation. This distribution reflects not only the selection of differential talent workers across occupations, but also the fact that working in the modern sector carries a premium. As a result, the distribution of earnings in the traditional sector is shifted to the left compared to workers engaged in modern production. Within modernity, in turn, the income distribution for entrepreneurs is substantially to the right of that for salaried workers. As is the case in the data, the there is substantial overlap also between these three distributions.

Figure 8



Horizontal axis censored at 0.2% and at the top 0.2% of the distribution.

Figure 9



Horizontal axis censored at 0.2% and at the top 0.2% of the distribution.

4 Counterfactual policy experiments

In this section, we use the estimated model to quantify the forces that underlie jobless modernity. As shown in the baseline calibration, our quantitative model economy provides a comprehensive accounting of the relationship between modern production and employment observed in the data. By varying distortions, technologies, and capabilities, one at a time, we can determine how much of the observed gap reflects barriers to modern job creation and how much reflects the characteristics of modern production itself.

We proceed in two steps. We first characterize jobless modernity in Colombia in 2019, decomposing the gap between modern production and modern employment into its underlying components. We then use the same framework to trace the origins of this gap over the previous five decades, asking how changes in technology, human capital, and distortions shaped Colombia's path of modernization.

4.1 The Anatomy of jobless modernity in 2019

We begin by asking how much of the large divergence between the income and employment shares of modern firms can be attributed to distortions, and how much reflects differences in human capital and technology, namely the primitives of the model.

Table 6 provides a first accounting of jobless modernity. Relative to the counterfactual economy in which distortions are absent, distortions reduce the modern share of income from 90.8% to 84.2% and the modern share of employment from 60.0% to 33.1%. As a result, the gap between the two rises from 30.9 to 51.1 percentage points.

The effects of distortions are therefore considerably stronger on employment than on

Table 6: **Accounting for Jobless Modernity: Benchmark and Counterfactual Economies**

	Technology, preferences & distortions	Technology & preferences only
	Baseline	Counterfactual
	(1)	(2)
A. Modern income (%)	84.2	90.8
B. Modern jobs (%)	33.1	60.0
Modern joblessness (A-B)	**51.1**	**30.9**
Aggregate income	0.77	1.00
Average business size	1.45	2.40

production. Even in the distorted economy, modern firms continue to account for more than four-fifths of aggregate income. Their contribution to employment, however, falls to roughly one third of the workforce. Put differently, distortions do not merely reduce the scale of modern activity. They weaken the ability of the modern sector to translate production into jobs.

The same comparison reveals substantial aggregate effects. Removing distortions raises aggregate income from 0.77 to 1.00 and increases average business size from 1.45 to 2.40 workers. Distortions therefore make the economy both poorer and more fragmented. Productive firms operate at smaller scale, fewer workers gain access to modern employment, and the resulting divergence between modern production and modern employment gives rise to jobless modernity.

The results in Table 6 establish that distortions are an important source of jobless modernity. The next question is whether all distortions contribute equally to the gap between modern production and modern employment. The distinction is important because different distortions operate through different margins. Some primarily affect firm scale and aggregate income, while others directly influence firms' demand for labor and workers' access to modern employment. Consequently, policies that expand modern production need not generate comparable increases in modern employment. The next table decomposes the contribution of labor and non-labor distortions to jobless modernity.

Table 7 shows that labor distortions are the dominant source of jobless modernity. Relative to the economy without distortions, labor distortions alone increase the joblessness-modernity gap from 30.9 to 44.4 percentage points, while non-labor distortions increase it to 35.3 percentage points. Although both types of distortions contribute to the divergence between modern production and modern employment, labor distortions have a substantially larger effect.

The model therefore identifies a meaningful distinction between barriers to modern production and barriers to modern employment. The former operate primarily through firm scale and aggregate income, whereas the latter operate through labor demand and occupational sorting. The evidence in Table 7 suggests that, in Colombia, the dominant

Table 7: **Which Distortions Generate Jobless Modernity?**

	Baseline	Labor distortions	Non-labor distortions	No distortions
A. Modern income (%)	84.2	89.8	85.9	90.8
B. Modern jobs (%)	33.1	45.4	50.6	60.0
Modern joblessness (A-B)	**51.1**	**44.4**	**35.3**	**30.9**

barriers to modern job creation are labor-market distortions rather than constraints on production per se.

The importance of this distinction extends beyond aggregate efficiency. By affecting who gains access to modern jobs, distortions also shape the distribution of earnings and the division of income between labor and entrepreneurs. We now turn to these distributional consequences.

4.2 Distortions and joblessness: 2019 vs. 1971

Having established the anatomy of jobless modernity in 2019, Table 8 traces how changes in fundamentals and distortions shaped its evolution since 1971. Starting from the 2019 equilibrium, we sequentially replace the economy's fundamentals and distortions with their 1971 counterparts. This exercise allows us to distinguish the contribution of technological and human-capital change from that of distortions in shaping the long-run evolution of modern production and modern employment.

The first column reports the benchmark 2019 economy. The second replaces contemporary fundamentals with those estimated for the 1970s while holding distortions fixed at their 2019 values. Modern production falls substantially, with the modern share of income declining from 84.2% to 74.6%, while the modern employment share falls from 33.1% to 23.6%. Yet the gap between the two remains essentially unchanged. In other words, lower levels of human capital and more labor-intensive technologies reduce the scale of modernization but do not, by themselves, account for jobless modernity.

The third column introduces the distortion environment characteristic of the early 1970s. Modern income and modern employment decline further, to 69.9% and 23.0%, respectively. Distortions therefore contributed not only to a smaller modern sector, but also to the inability of that sector to absorb labor. Their effects were particularly severe because they operated in an economy where modern production was considerably more labor intensive and human capital was scarcer than today.

As a whole, these results suggest a distinctive path of development. Modernization did

Table 8: Genesis of Jobless Modernity

	2019 baseline (1)	+ 1971 fundamentals (2)	+ 1971 distortions (3)
A. Modern share of income	84.2	74.6	69.9
B. Modern share of jobs	33.1	23.6	23.0
Modern joblessness (A-B)	51.1	51.1	46.9

occur between 1971 and 2019: technologies improved, human capital accumulated, and modern firms came to account for a larger share of economic activity. Yet the relationship between modern production and modern employment changed remarkably little. The modern sector generated more output, but not a proportionate increase in modern jobs. We interpret this pattern as a form of distorted modernization, or what we term *growing like Latin America*: a process in which the productive possibilities created by technological progress and human-capital accumulation are only partially realized because distortions persistently limit the expansion of modern employment.

5 Conclusions

This paper studies why many developing economies modernize in value added without modernizing in employment. Using Colombian historical microdata and a general-equilibrium model of occupational choice, we show that jobless modernity is generated by two distinct mechanisms. The first is segmentation: a minimum-earnings constraint excludes low-ability workers from salaried modern jobs and pushes them into self-employment. The second is within-modern misallocation: firm-level wedges compress the scale of productive firms and limit their labor demand.

Quantitatively, the decomposition is clear. Relative to the undistorted allocation, the gap between the modern share of value added and the modern share of employment rises from 30.9 to 51.1 percentage points. Averaging over the order in which distortions are introduced, 77.5% of this increase in joblessness is due to the minimum-earnings constraint and 22.5% to firm-level wedges. Measured instead as the shortfall in modern jobs relative to the undistorted benchmark, 58.3% of missing modern jobs is due to the minimum-earnings constraint and 41.7% to the wedge bundle. Thus, all distortions matter, but they do not matter through the same margin, and the main source of jobless modernity is labor-market segmentation.

The decomposition within firm-level wedges is equally sharp. Revenue distortions account for most of the aggregate income loss, while labor distortions are the most important firm-level force depressing modern job creation. Capital distortions, by contrast, do little to expand inclusive modern employment. Distortions therefore do not simply reduce efficiency in a generic sense: some mainly reduce output, others mainly reduce jobs, and those differences are central for understanding the development path of emerging economies.

The counterfactuals reinforce this message. Labor-saving technological change can sharply raise output while leaving modern employment unchanged. Broad improvements in embodied human capital, especially among workers near the margin of entering mod-

ern wage work, are more effective at expanding modern jobs and compressing inequality. The broader implication is that employment-rich development depends not only on raising productivity, but on relaxing the barriers that keep workers out of modern employment in the first place.

References

ABOWD, J. M., F. KRAMARZ, AND D. N. MARGOLIS (1999): "High Wage Workers and High Wage Firms," *Econometrica*, 67, 251–333.

ACKERBERG, D. A., K. CAVES, AND G. FRAZER (2015): "Identification Properties of Recent Production Function Estimators," *Econometrica*, 83, 2411–2451.

AKCIGIT, U., H. ALP, AND M. PETERS (2021): "Lack of Selection and Limits to Delegation: Firm Dynamics in Developing Countries," *American Economic Review*, 111, 231–275.

ALFARO, L., A. CHARLTON, AND F. KANCZUK (2009): *Plant Size Distribution and Cross-Country Income Differences*, University of Chicago Press, 243–272.

ATKIN, D., B. BERNADAC, D. DONALDSON, T. GARG, AND F. HUNEEUS (2025): "The Incidence of Distortions," Working Papers Central Bank of Chile 1026, Central Bank of Chile.

AUTOR, D., D. DORN, L. F. KATZ, C. PATTERSON, AND J. VAN REENEN (2020): "The Fall of the Labor Share and the Rise of Superstar Firms," *The Quarterly Journal of Economics*, 135, 645–709.

BENTO, P. AND D. RESTUCCIA (2017): "Misallocation, Establishment Size, and Productivity," *American Economic Journal: Macroeconomics*, 9, 267–303.

——— (2021): "On average establishment size across sectors and countries," *Journal of Monetary Economics*, 117, 220–242.

BROWN, C. AND J. MEDOFF (1989): "The Employer Size-Wage Effect," *Journal of Political Economy*, 97, 1027–1059.

BUERA, F. J., J. P. KABOSKI, AND R. M. TOWNSEND (2021): "From Micro to Macro Development," NBER Working Papers 28423, National Bureau of Economic Research, Inc.

BUERA, F. J. AND B. MOLL (2015): "Aggregate Implications of a Credit Crunch: The Importance of Heterogeneity," *American Economic Journal: Macroeconomics*, 7, 1–42.

CARD, D., J. HEINING, AND P. KLINE (2013): "Workplace Heterogeneity and the Rise of West German Wage Inequality," *The Quarterly Journal of Economics*, 128, 967–1015.

CARDENAS, M. AND G. PERRY (1986): "Diez años de reformas tributarias en Colombia," Tech. rep., Fedesarrollo.

De Loecker, J., P. K. Goldberg, A. K. Khandelwal, and N. Pavcnik (2016): "Prices, Markups, and Trade Reform," *Econometrica*, 84, 445–510.

De Loecker, J. and F. Warzynski (2012): "Markups and Firm-Level Export Status," *American Economic Review*, 102, 2437–71.

de Mel, S., D. McKenzie, and C. Woodruff (2008): "Who Are the Microenterprise Owners? Evidence from Sri Lanka on Tokman v. de Soto," IZA Discussion Papers 3511, Institute of Labor Economics (IZA).

D'Erasmo, P. N. and H. J. Moscoso Boedo (2012): "Financial Structure, Informality and Development," *Journal of Monetary Economics*, 59, 286–302.

Dix-Carneiro, R., P. K. Goldberg, C. Meghir, and G. Ulyssea (2021): "Trade and Informality in the Presence of Labor Market Frictions and Regulations," Working Paper 28391, National Bureau of Economic Research.

Donovan, K., W. J. Lu, and T. Schoellman (2023): "Labor Market Dynamics and Development," *The Quarterly Journal of Economics*, 138, 2287–2325.

Engbom, N., H. Malmberg, T. Porzio, F. Rossi, and T. Schoellman (2025): "Economic Development According to Chandler," Working Paper 34483, National Bureau of Economic Research.

Eslava, M. (2026): "Firm size and dynamics in less-developed economies," *Prepared for Annual Review of Economics*.

Eslava, M., J. C. Haltiwanger, and N. Urdaneta (2023): "The Size and Life-cycle Growth of Plants: The Role of Productivity, Demand and Wedges," *The Review of Economic Studies*, Forthcoming.

Eslava, M., M. Melendez Arjona, L. D. Tenjo Galvis, and N. Urdaneta (2026): "Business Size, Development, and Inequality in Latin America : A Tale of One Tail," *Econometric Society monographs 2025 World Congress volumes*, 3.

Foster, L., J. Haltiwanger, and C. Syverson (2008): "Reallocation, Firm Turnover, and Efficiency: Selection on Productivity or Profitability?" *American Economic Review*, 98, 394–425.

Gandhi, A., S. Navarro, and D. A. Rivers (2020): "On the Identification of Gross Output Production Functions," *Journal of Political Economy*, 128, pp. 2973–3016.

Garicano, L., C. Lelarge, and J. Van Reenen (2016): "Firm Size Distortions and the Productivity Distribution: Evidence from France," *American Economic Review*, 106, 3439–3479.

HARRIS, J. R. AND M. P. TODARO (1970): "Migration, Unemployment and Development: A Two-Sector Analysis," *The American Economic Review*, 60, 126–142.

HELPMAN, E., O. ITSKHOKI, M.-A. MUENDLER, AND S. J. REDDING (2017): "Trade and Inequality: From Theory to Estimation," *The Review of Economic Studies*, 84, 357–405.

HOPENHAYN, H. (2016): "Firm Size and Development," *Economía Journal*, 17, 27–49.

HOTTMAN, C. J., S. J. REDDING, AND D. E. WEINSTEIN (2016): "Quantifying the Sources of Firm Heterogeneity," *The Quarterly Journal of Economics*, 131, 1291–1364.

HSIEH, C.-T. AND P. J. KLENOW (2009): "Misallocation and Manufacturing TFP in China and India," *The Quarterly Journal of Economics*, 124, 1403–1448.

——— (2014): "The Life Cycle of Plants in India and Mexico," *The Quarterly Journal of Economics*, 129, 1035–1084.

HSIEH, C.-T. AND B. A. OLKEN (2014): "The Missing "Missing Middle"," *Journal of Economic Perspectives*, 28, 89–108.

LA PORTA, R. AND A. SHLEIFER (2014): "Informality and Development," *Journal of Economic Perspectives*, 28, 109–26.

LEVINSOHN, J. AND A. PETRIN (2003): "Estimating Production Functions Using Inputs to Control for Unobservables," *The Review of Economic Studies*, 70, 317–341.

LEWIS, W. A. (1954): "Economic Development with Unlimited Supplies of Labour," *The Manchester School*, 22, 139–191.

MESTIERI, M., J. SCHAUER, AND R. TOWNSEND (2017): "Human Capital Acquisition and Occupational Choice: Implications for Economic Development," *Review of Economic Dynamics*, 25, 151–186.

MIDRIGAN, V. AND D. Y. XU (2014): "Finance and Misallocation: Evidence from Plant-Level Data," *American Economic Review*, 104, 422–58.

OLLEY, G. S. AND A. PAKES (1996): "The Dynamics of Productivity in the Telecommunications Equipment Industry," *Econometrica*, 64, 1263–1297.

POSCHKE, M. (2018): "The firm size distribution across countries and skill-biased change in entrepreneurial technology," *American Economic Journal: Macroeconomics*, 10, 1–41.

RAUCH, J. E. (1991): "Modelling the informal sector formally," *Journal of Development Economics*, 35, 33–47.

RESTUCCIA, D. (2025): "On Productivity and Distortions across Countries," NBER Working Papers 34573, National Bureau of Economic Research, Inc.

RESTUCCIA, D. AND R. ROGERSON (2008): "Policy Distortions and Aggregate Productivity with Heterogeneous Plants," *Review of Economic Dynamics*, 11, 707–720.

SCHWARTZMAN, M. (2025): "From Street Markets to Shopping Malls: The Modern Service Multiplier," Steg working paper, Structural Transformation and Economic Growth (STEG).

SONG, J., D. J. PRICE, F. GUVENEN, N. BLOOM, AND T. VON WACHTER (2019): "Firming Up Inequality," *The Quarterly Journal of Economics*, 134, 1–50.

TYBOUT, J. (2000): "Manufacturing Firms in Developing Countries: How Well Do They Do, and Why?" *Journal of Economic Literature*, 38, 11–44.

ULYSSEA, G. (2018): "Firms, Informality, and Development: Theory and Evidence from Brazil," *American Economic Review*, 108, 2015–47.

APPENDIX

A Data Construction

A.1 Modern firm's problem

The modern firm solves

$$Max_{\{k,l\}} \left(1-\tau^R\right) py - \left(1+\tau^l\right) wl - \left(1+\tau^K\right) rk - p_m m \tag{31}$$

subject to 46. Revenue $R = py$ is given by,

$$R = D^{\frac{1}{\sigma}} d^{\frac{1}{\sigma}} y^{1-\frac{1}{\sigma}} \tag{32}$$

The first order condition for any factor $x = \{k, l\}$ with price p^x is thus given by

$$\begin{aligned} \left(1+\tau^x\right) p^x &= \left(1-\tau^R\right)\left[p\frac{\partial y}{\partial x} + y\left(\frac{\partial p}{\partial y} + \frac{\partial p}{\partial P}\frac{\partial P}{\partial y}\right)\frac{\partial y}{\partial x}\right] \\ &= \left(1-\tau^R\right) p\left(\frac{\sigma-1}{\sigma}\right)(1-s)\frac{\partial y}{\partial x} \end{aligned} \tag{33}$$

where s is the firm's revenue share in the market, and the last line uses Shepard's lemma ($\frac{\partial P}{\partial y}\frac{y}{P} = -s$), and the facts that $\frac{\partial p}{\partial y}\frac{y}{p} = -\frac{1}{\sigma}$ and $\frac{\partial p}{\partial P}\frac{P}{p} = \frac{\sigma-1}{\sigma}$.

Moreover, the markup is given by:[1]

$$\mu^{-1} = \left(\frac{\sigma-1}{\sigma}\right)(1-s) \tag{34}$$

Thus, optimal k is given by $\left(1+\tau^k\right) rk = \left(1-\tau^R\right)\mu^{-1}\alpha\gamma R$, and the optimal capital-labor ratio is given by

$$l = \frac{(1-\alpha-\phi)\, r\left(1+\tau^k\right)}{\alpha w\left(1+\tau^l\right)} k \equiv \theta k \tag{35}$$

whilst the optimal capital-materials ratio is

$$m = \frac{\phi r\left(1+\tau^k\right)}{\alpha p_m} k \equiv \theta_m k \tag{36}$$

Using these FOCs, profits are given by:

$$\begin{aligned} \pi &= \left(1-\tau^R\right) R - \left[\left(1+\tau^l\right) w\theta k + \left(1+\tau^k\right) rk + p_m\theta_m k\right] \\ &= \left(1-\tau^R\right) R\left(1-\mu^{-1}\gamma\right) \end{aligned} \tag{37}$$

Further, plugging (35), (36) and (34) into the FOC for capital and replacing p using (46), we obtain optimal capital

$$k^{1-\gamma\left(1-\frac{1}{\sigma}\right)} = \frac{\left(1-\tau^R\right)\mu^{-1}\widetilde{\chi}^{\frac{\sigma-1}{\sigma}}\alpha\gamma\theta^{(1-\alpha-\phi)\gamma\frac{\sigma-1}{\sigma}}\theta_m^{\phi\gamma\frac{\sigma-1}{\sigma}}}{\left(1+\tau^k\right) r} \tag{38}$$

$$= \left(1-\tau^R\right)\mu^{-1}\widetilde{\chi}^{\frac{\sigma-1}{\sigma}}\gamma\left(\frac{\alpha}{r(1+\tau^k)}\right)^{1-(1-\alpha)\gamma\frac{\sigma-1}{\sigma}}\left(\frac{1-\alpha-\phi}{w(1+\tau^l)}\right)^{(1-\alpha-\phi)\gamma\frac{\sigma-1}{\sigma}}\left(\frac{\phi}{p_m}\right)^{\phi\gamma\frac{\sigma-1}{\sigma}} \tag{39}$$

where $\widetilde{\chi} = D^{\frac{1}{\sigma-1}} d^{\frac{1}{\sigma-1}} a = D^{\frac{1}{\sigma-1}}\chi$ combines firm capability and market demand.

For future reference, plugging (38) into (35), labor demand is

[1] See Hottman, Redding and Weinstein, 2016, and Eslava, Haltiwanger and Urdaneta, 2022.

$$l^{1-\gamma\left(1-\frac{1}{\sigma}\right)} = \left(1-\tau^R\right)\mu^{-1}\widetilde{\chi}^{\frac{\sigma-1}{\sigma}}\gamma\left(\frac{\alpha}{r\left(1+\tau^k\right)}\right)^{\alpha\gamma\left(\frac{\sigma-1}{\sigma}\right)}\left(\frac{1-\alpha-\phi}{w\left(1+\tau^l\right)}\right)^{\left(1-(\alpha+\phi)\gamma\left(\frac{\sigma-1}{\sigma}\right)\right)}\left(\frac{\phi}{p_m}\right)^{\phi\gamma\frac{\sigma-1}{\sigma}} \quad (40)$$

Because $y = a(k\theta^{(1-\alpha-\phi)}\theta_m^{\phi})^{\gamma}$ and $R = py = D^{\frac{1}{\sigma}}d^{\frac{1}{\sigma}}y^{\frac{\sigma-1}{\sigma}}$, we obtain

$$R = \left[\widetilde{\chi}^{\frac{1}{\gamma}}\frac{\left(1-\tau^R\right)}{\left(1+\tau^k\right)^{\alpha}\left(1+\tau^l\right)^{1-\alpha-\phi}}\mu^{-1}\gamma\left(\frac{\alpha}{r}\right)^{\alpha}\left(\frac{1-\alpha-\phi}{w}\right)^{1-\alpha-\phi}\left(\frac{\phi}{p_m}\right)^{\phi}\right]^{\frac{\gamma(\sigma-1)}{\sigma-\gamma(\sigma-1)}} \quad (41)$$

One could thus write revenue (and profits, using (37)) as a the function of quality-adjusted quantity productivity, distortions in product and labor markets (τ^R, τ^l and τ^k), markups and a set of parameters and equilibrium input prices that are constant across firms (of the same sector).

$$R = \left(\widetilde{\chi}\mu^{-\gamma}\frac{\left(1-\tau^R\right)^{\gamma}}{\left(1+\tau^k\right)^{\alpha\gamma}\left(1+\tau^l\right)^{\gamma(1-\alpha-\phi)}}\Theta\right)^{\frac{(\sigma-1)}{\sigma-\gamma(\sigma-1)}} \quad (42)$$

$$\pi = \left(\widetilde{\chi}\left(1-\mu^{-1}\gamma\right)^{\frac{\sigma-\gamma(\sigma-1)}{(\sigma-1)}}\mu^{-\gamma}\frac{\left(1-\tau^R\right)^{\frac{\sigma}{\sigma-1}}}{\left(1+\tau^k\right)^{\alpha\gamma}\left(1+\tau^l\right)^{\gamma(1-\alpha-\phi)}}\Theta\right)^{\frac{(\sigma-1)}{\sigma-\gamma(\sigma-1)}} \quad (43)$$

where $\Theta = \gamma^{\gamma}(\frac{\alpha}{r})^{\gamma\alpha}\left(\frac{1-\alpha-\phi}{w}\right)^{\gamma(1-\alpha-\phi)}(\frac{\phi}{p_m})^{\phi\gamma}$. Revenue and profits are convex in a and in quality-adjusted quantity productivity $\widetilde{\chi}$ whenever $\frac{(\sigma-1)}{\sigma-\gamma(\sigma-1)} > 1$ and concave otherwise.

Aggregation of modern sector As shown in Appendix C, modern manufacturing productivity is given by

$$TFP_{MF} = \frac{C_{MF}}{K^{\alpha}L^{(1-\alpha)}} = D^{-\frac{1}{\sigma-1}}\left(\sum_I\left[\left(\frac{\widetilde{\chi}}{tfpr^{\gamma}}\right)^{\frac{1}{\sigma\left(1-\gamma\left(1-\frac{1}{\sigma}\right)\right)}}TFPR_{MF}\right]^{\sigma-1}\right)^{\frac{1}{\sigma-1}} \quad (44)$$

where $tfpr \equiv \frac{py}{k^{\alpha}l^{(1-\alpha)}} = \frac{\left(\frac{\alpha}{\left(1+\tau^k\right)r}\right)^{\alpha}\left(\frac{1-\alpha-\phi}{w\left(1+\tau^l\right)}\right)^{1-\alpha-\phi}(\frac{\phi}{p_m})^{\phi}\mu}{\gamma(1-\tau^R)}$ is a modern firm's revenue productivity (see Appendix C) and $TFPR_{MF} \equiv \frac{E_{MF}}{K^{\alpha}L^{1-\alpha}}$ is aggregate revenue productivity in

manufacturing.

A.2 One-sector derivations

Let E denote aggregate expenditure, P the aggregate price index, and P_M the modern price index defined in equation (10). Utility maximization yields

$$E_M = \phi_M \left(\frac{P_M}{P}\right)^{1-\rho} E, \qquad E_M = P_M C_M.$$

The resource constraint for the modern composite then gives

$$Q_M = \frac{E_M}{P_M} + \Upsilon_m.$$

Combining this expression with the aggregator's demand for variety i gives

$$q_i = D_M d_i p_i^{-\sigma}, \qquad D_M \equiv P_M^{\sigma} Q_M = \frac{E_M}{P_M^{1-\sigma}} + P_M^{\sigma} \Upsilon_m.$$

Inverting variety demand yields equation (13).

A.3 Additional derivations for the three-sector framework

The nested CES structure in Section 2 implies the expenditure shares and price indexes

$$\begin{aligned} &\frac{E_i}{E} = \lambda_i \left(\frac{P_i}{P}\right)^{1-\epsilon}, \qquad \frac{E_{i,M}}{E_i} = \phi_{i,M} \left(\frac{P_{i,M}}{P_i}\right)^{1-\rho}, \qquad \frac{E_{i,T}}{E_i} = (1-\phi_{i,M}) \left(\frac{P_{i,T}}{P_i}\right)^{1-\rho}, \\ &P_i = \left(\phi_{i,M} P_{i,M}^{1-\rho} + (1-\phi_{i,M}) P_{i,T}^{1-\rho}\right)^{\frac{1}{1-\rho}}, \qquad P = \left(\sum_{i \in A,I,S} \lambda_i P_i^{1-\epsilon}\right)^{\frac{1}{1-\epsilon}}, \end{aligned} \tag{45}$$

where $P_{i,T}$ denotes the price of traditional goods in sector i, and expenditures satisfy $E_i = P_i C_i$, $E_{i,M} = P_{i,M} C_{i,M}$, and $E_{i,T} = P_{i,T} C_{i,T}$.

Combining final demand (**??**) and intermediate demand (**??**) yields the firm-level demand shifter

$$D_{i,M} \equiv \frac{E_{i,M}}{P_{i,M}^{1-\sigma}} + \frac{p_m \Upsilon_m}{p_m^{1-\sigma}}, \qquad p_{i,j} = D_{i,M}^{1/\sigma} d_{i,j}^{1/\sigma} q_{i,j}^{-1/\sigma}. \tag{46}$$

The modern firm's first-order conditions are

$$(1-\tau_j^R)\frac{p_{i,j}q_{i,j}}{l_{i,j}}\gamma_l\frac{\sigma-1}{\sigma} = w(1+\tau_j^l), \tag{47}$$

$$(1-\tau_j^R)\frac{p_{i,j}q_{i,j}}{k_{i,j}}\gamma_k\frac{\sigma-1}{\sigma} = (1+\tau_j^k)r, \tag{48}$$

$$(1-\tau_j^R)\frac{p_{i,j}q_{i,j}}{m_{i,j}}\gamma_m\frac{\sigma-1}{\sigma} = p_m. \tag{49}$$

These conditions imply that profits are proportional to after-tax revenue:

$$\pi_{i,j} = (1-\tau_j^R)p_{i,j}q_{i,j}\left(1-\frac{1}{\mu}\right). \tag{50}$$

These conditions imply the input ratios

$$\frac{k_{i,j}}{l_{i,j}} = \frac{\gamma_k}{\gamma_l}\frac{w(1+\tau_j^l)}{r(1+\tau_j^k)}, \qquad \frac{m_{i,j}}{l_{i,j}} = \frac{\gamma_m}{\gamma_l}\frac{w(1+\tau_j^l)}{p_m}. \tag{51}$$

Substituting these ratios into (47), together with (**??**) and (46), yields optimal labor demand:

$$l_{i,j} = \frac{D_{i,M}}{\mu^\sigma}\left(\frac{(a_j d_j^{1/(\sigma-1)})^{\sigma-1}(1-\tau_j^R)}{c^{\sigma-1}\hat{\tau}_j^{\sigma-1}}\right)\left(\frac{\gamma_l}{w(1+\tau_j^l)}\right). \tag{52}$$

Define the common unit-cost index and the composite distortion by

$$c = \left(\frac{w}{\gamma_l}\right)^{\gamma_l}\left(\frac{r}{\gamma_k}\right)^{\gamma_k}\left(\frac{p_m}{\gamma_m}\right)^{\gamma_m}, \qquad \hat{\tau}_j = \frac{(1+\tau_j^l)^{\gamma_l}(1+\tau_j^k)^{\gamma_k}}{1-\tau_j^R}. \tag{53}$$

Using optimal labor demand, pre-tax revenue can be written as

$$p_{i,j}q_{i,j} = D_{i,M}\left(\frac{a_j d_j^{1/(\sigma-1)}}{\mu c\hat{\tau}_j}\right)^{\sigma-1}. \tag{54}$$

Combining (54) with (50) delivers the profit expression reported in Section 2.6.3.

Because entrepreneurs are indifferent across sectors and the distribution of (a, d, s, Γ) is

common across sectors, equilibrium pricing implies

$$p_{i,j} = \frac{\mu c \hat{\tau}_j}{a_j}. \tag{55}$$

Therefore,

$$P_{i,M}^{1-\sigma} = \sum_{j \in N_{i,M}} d_j p_j^{1-\sigma} = \frac{N_{i,M}}{N_M} \sum_{i \in \{A,I,S\}} \sum_{j \in N_{i,M}} d_j p_j^{1-\sigma} = \frac{N_{i,M}}{N_M} \sum_{i \in \{A,I,S\}} P_{i,M}^{1-\sigma}, \tag{56}$$

which implies that sectoral expenditure shares, modern employment shares, and entrepreneur shares satisfy

$$\alpha_{i,M} \equiv \frac{E_{i,M}}{E_M} = \frac{P_{i,M}^{1-\sigma}}{\sum_{i \in A,I,S} P_{i,M}^{1-\sigma}} = \frac{L_{i,M}}{L_M} = \frac{N_{i,M}}{N_M}. \tag{57}$$

An analogous argument applies to labor demand because firm-level labor demand depends on firm characteristics, whose equilibrium distribution is the same across sectors.

The equilibrium conditions summarized in Section 2.5 can be written as follows. Given prices $(w, \{p_{i,j}\}_{i,j}, \{P_{i,M}\}_{i \in \{A,I,S\}})$, agents choose occupations to maximize (**??**), the labor market clears,

$$\sum_{i \in A,I,S} \sum_{j \in \mathcal{W}_i} a_{i,j} = \sum_{i \in A,I,S} \sum_{j \in N_{i,M}} l_{i,j}, \tag{58}$$

goods markets satisfy

$$\begin{aligned} E &= \sum_{i \in A,I,S} \left(\sum_{j \in N_{i,T}} q_{i,j} + \sum_{j \in N_{i,M}} c_{i,j} \right) \\ &= \sum_{i \in A,I,S} \left(\sum_{j \in N_{i,T}} a_j + \sum_{j \in N_{i,M}} (\pi_{i,j} + w l_{i,j} + r k_{i,j}) \right) + \hat{T}, \end{aligned} \tag{59}$$

and total distortion revenues rebated to households are

$$\hat{T} = \sum_{i \in A,I,S} \sum_{j \in N_{i,M}} \left(\tau_j^R p_{i,j} q_{i,j} + \tau_j^l w l_{i,j} + \tau_j^k r k_{i,j} \right). \tag{60}$$

B Estimation of parameters of production and demand functions in modern manufacturing

We follow the structural value-added approach to the estimation of the establishment's production function (Ackerberg et al., 2015, Gandhi et al. 2017). An establishment indexed by i produces y_{it} using labor l_{it} and capital k_{it}, with technology

$$y_{it} = a_{it} l_{it}^{1-\alpha} k_{it}^{\alpha} \tag{61}$$

The underlying assumption is that production is Leontieff, $y_{it} = \min(l_{it}^{1-\alpha} k_{it}^{\alpha} a_{it}, m_{it})$. At the optimum, thus, (61) is satisfied and $m_{it} = l_{it}^{1-\alpha} k_{it}^{\alpha} a_{it}$.

Following Ackerberg, Caves, and Frazer (2015, ACF henceforth), we use material inputs to proxy for productivity. The Leontieff assumption implies the optimal choice of material inputs can be written as:

$$\ln m_{it} = h\left(\ln a_{it}, \ln k_{it}, \ln l_{it}\right)$$

Inverting, we obtain

$$\ln a_{it} = h^{-1}\left(\ln m_{it}, \ln l_{it}, \ln k_{it}\right) \equiv h^{-1}\left(\overrightarrow{W_{it}}\right)$$

Incorporating this expression, recognizing that y_{it} is subject to measurement error and other shocks not observed by either the econometrician or the firm at the time of making input choices, and denoting by $\widehat{y_{it}} = y_{it} e^{\varepsilon_{it}}$ measured y_{ft}, we write:

$$\ln \widehat{y_{it}} = \alpha\gamma \ln k_{it} + (1-\alpha)\gamma \ln l_{it} + h^{-1}\left(\overrightarrow{W_{it}}\right) + \varepsilon_{it} \tag{62}$$

y_{it} is a CES composite of varieties produced by i. We follow Eslava et al. (2022) to jointly estimate 62 and the demand function implied by preferences in our setting:

$$R_{it} = D_t + d_{it} + p_{it}^{1-\sigma} \tag{63}$$

where $R_{it} = p_{it} y_{it}$ is $i's$ revenue. CES preferences across varieties produced within i imply (see Eslava et al. (2023)) $p_{it} = \overline{P_{ft}^{*}} P_{fB} \left(\Lambda_{ft}^{Q}\right)^{\frac{1}{\sigma-1}}$ where $s_{ijt} = \frac{p_{ijt} y_{ijt}}{R_{it}}$; $\Omega_{t,t-1}^{i}$ is the set of products that i produces in both t and $t-1$ and s_{ijt}^{*} is the share that product j represents in the revenue that i obtains in t from products in $\Omega_{t,t-1}^{i}$; $\lambda_{it}^{QF} = \frac{\sum_{\Omega_{t,t-1}^{i}} s_{ijt}}{\sum_{\Omega_{t,t-1}^{i}} s_{ijt-1}}$;

and $\lambda_{it}^{QRW} = \prod_{\Omega_{t,t-1}^{i}} \left(\frac{s_{ijt}^{*}}{s_{ijt-1,\Omega_{t,t-1}^{i}}^{*}} \right)^{\frac{1}{\|\Omega_{t,t-1}^{i}\|}}$.

Since $y_{it} = \frac{R_{it}}{p_{it}}$ and $p_{it} = \overline{P_{it}^{*}} P_{iB} \left(\Lambda_{it}^{Q} \right)^{\frac{1}{\sigma-1}}$, σ is necessary to measure prices and thus y_{it}. An estimate of σ is not available at the stage of estimating 62 and 63, so we estimate a modified version of 62:

$$\ln \widehat{y_{it}^{*}} = \ln \left(\frac{R_{it}}{\overline{P_{ft}^{*}}} \right) = \alpha\gamma \ln k_{it} + (1-\alpha)\,\gamma \ln l_{it} + h^{-1}\left(\overrightarrow{W_{it}} \right) + \frac{1}{1-\sigma} \Lambda_{ft}^{Q} + \widehat{\varepsilon}_{it} \tag{64}$$

Following Eslava et al. (2023) and the proxy-methods literature on production function estimation, we estimate 64 in two stages. The first stage purges measurement error $\widehat{\varepsilon}_{it}$, while the second stage, jointly estimated with 63 via GMM, estimates (α, γ, σ). Assuming that a_{it} and d_{it} follow flexible AR1 processes:

$$\begin{aligned} \ln a_{it} &= \pi_0^a + \pi_1^a \ln a_{it-1} + \pi_2^A \ln a_{it-1}^2 + \pi_3^A \ln a_{it-1}^3 + \xi_{it}^A \\ \ln a_{it} &= \pi_0^D + \pi_1^D \ln D_{it-1} + \pi_2^A \ln D_{it-1}^2 + \pi_3^A \ln D_{it-1}^3 + \xi_{it}^D \end{aligned}$$

we use the following moment conditions for identification based on those proposed by **?**:

$$E \begin{bmatrix} \ln L_{it} \times \xi_{it}^{A} \\ \ln K_{it} \times \xi_{it}^{A} \\ \ln D_{it-1} \times \xi_{it}^{A} \\ \ln A_{it-1} \times \xi_{it}^{D} \\ \ln A_{it} \\ \ln D_{it} \end{bmatrix} = 0 \tag{65}$$

Table 9 presents the detailed results.

Table 9: Factor and demand elasticities by sector (pooling years)

Sector	$1-\alpha$	α	γ	σ_w	σ	$\gamma\left(\frac{1}{1-\sigma}\right)$
311	0,646	0,354	1,052	8,110	4,073	0,794
313	0,674	0,326	1,042	5,531	4,303	0,799
321	0,546	0,454	0,885	4,776	4,036	0,666
322	0,594	0,406	0,679	6,287	4,019	0,510
323	0,539	0,461	0,946	5,268	4,081	0,714
324	0,640	0,360	0,865	5,975	3,957	0,647
331	0,758	0,242	0,887	4,785	3,828	0,655
332	0,580	0,420	0,884	3,616	4,003	0,663
341	0,588	0,412	0,961	2,570	4,058	0,724
342	0,775	0,225	1,053	2,012	4,057	0,794
351	0,657	0,343	1,127	8,042	4,162	0,857
352	0,563	0,437	0,979	6,171	4,174	0,744
355	0,736	0,264	0,975	4,028	4,176	0,741
356	0,703	0,297	1,050	3,467	4,022	0,789
362	0,685	0,315	1,092	3,745	4,031	0,821
369	0,603	0,397	1,019	6,880	4,078	0,770
371	0,685	0,315	1,249	4,856	4,155	0,949
381	0,659	0,341	0,946	1,912	3,955	0,707
382	0,709	0,291	0,959	3,401	4,011	0,720
383	0,688	0,312	1,056	4,447	4,051	0,795
384	0,748	0,252	1,132	7,526	3,990	0,848
385	0,640	0,360	1,002	4,260	3,901	0,745
390	0,741	0,259	0,971	2,009	4,004	0,729
Average	0,659	0,341	0,992	4,768	4,049	0,747
Min	0,539	0,225	0,679	1,912	3,828	0,510
Max	0,775	0,461	1,249	8,110	4,303	0,949

C TFP

Modern manufacturing productivity is given by

$$TFP_{MF} = \frac{C_{MF}}{K^{\alpha}L^{(1-\alpha)}} = \frac{E_{MF}}{K^{\alpha}L(1-\alpha)}\frac{1}{P_{MF}} = \frac{TFPR_{MF}}{P_{MF}} \tag{66}$$

where E_{MF} is total expenditure in manufacturing, and $TFPR_{MF} \equiv \frac{E_{MF}}{K^{\alpha}L^{1-\alpha}}$ is aggregate revenue productivity in manufacturing.

Define also $tfpr = \frac{py}{k^{\alpha}l^{(1-\alpha)}} = \frac{D^{\frac{1}{\sigma}}d^{\frac{1}{\sigma}}y^{1-\frac{1}{\sigma}}}{\left(k^{\alpha}l^{1-\alpha}\right)} = \frac{D^{\frac{1}{\sigma}}d^{\frac{1}{\sigma}}a^{1-\frac{1}{\sigma}}}{\left(k^{\alpha}l^{1-\alpha}\right)^{1-\gamma\left(1-\frac{1}{\sigma}\right)}}$. Moreover, from the optimal

revenue expression above it is apparent that, at the optimum:

$$k^{\alpha}l^{(1-\alpha)} = \left(\frac{\widetilde{\chi}^{\frac{\sigma-1}{\sigma}}\gamma}{\mu c\left(1-\tau^{R}\right)^{-1}}\right)^{\frac{1}{1-\gamma\left(1-\frac{1}{\sigma}\right)}}. \tag{67}$$

Replacing this into $tfpr$, we obtain:

$$tfpr = \frac{c\mu}{\gamma\left(1-\tau^{R}\right)} \tag{68}$$

where $c = \left(\frac{\alpha}{\left(1+\tau^{k}\right)r}\right)^{\alpha}\left(\frac{1-\alpha}{w\left(1+\tau^{l}\right)}\right)^{1-\alpha}$. As a result, we can also write $k^{\alpha}l^{(1-\alpha)} = \left(\frac{\widetilde{\chi}^{\frac{\sigma-1}{\sigma}}}{tfpr}\right)^{\frac{1}{1-\gamma\left(1-\frac{1}{\sigma}\right)}}$.

Replacing this into $tfpr$ we obtain $tfpr = pa\left(k^{\alpha}l^{1-\alpha}\right)^{\gamma-1} = pa\left(\frac{\widetilde{\chi}^{\frac{\sigma-1}{\sigma}}}{tfpr}\right)^{\frac{\gamma-1}{1-\gamma\left(1-\frac{1}{\sigma}\right)}}$, so that after some manipulation

$$tfpr^{\frac{\gamma}{\sigma\left(1-\gamma\frac{\sigma-1}{\sigma}\right)}} = pa\left(\widetilde{\chi}^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\gamma-1}{1-\gamma\left(1-\frac{1}{\sigma}\right)}} \tag{69}$$

Using 69 in 66

$$\begin{aligned}
TFP_{MF} &= TFPR_{MF}\left(\sum_{I} dp^{1-\sigma}\right)^{\frac{1}{\sigma-1}} \\
&= TFPR_{MF}\left(\sum_{I}\left(d^{\frac{1}{\sigma-1}}\frac{a\left(\widetilde{\chi}^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\gamma-1}{1-\gamma\left(1-\frac{1}{\sigma}\right)}}}{tfpr^{\frac{\gamma}{\sigma\left(1-\gamma\frac{\sigma-1}{\sigma}\right)}}}\right)^{\sigma-1}\right)^{\frac{1}{\sigma-1}} \\
&= D^{-\frac{1}{\sigma-1}}\left(\sum_{I}\left[\left(\frac{\widetilde{\chi}}{tfpr^{\gamma}}\right)^{\frac{1}{\sigma\left(1-\gamma\left(1-\frac{1}{\sigma}\right)\right)}}TFPR_{MF}\right]^{\sigma-1}\right)^{\frac{1}{\sigma-1}}
\end{aligned} \tag{70}$$

where we have used the fact that $\widetilde{\chi} = D^{\frac{1}{\sigma-1}}d^{\frac{1}{\sigma-1}}a$ and $1+\left(\frac{\sigma-1}{\sigma}\right)\frac{(\gamma-1)}{\left(1-\gamma\left(\frac{\sigma-1}{\sigma}\right)\right)} = \frac{1}{\sigma\left(1-\gamma\left(\frac{\sigma-1}{\sigma}\right)\right)}$.

D Data and Additional Results

D.1 Historical Data Sets

The study utilizes long-time series of individual and firm microdata from Colombia. The Colombian data, generally unavailable for other countries, is unparalleled in scope and detail in most of developing countries and provides microdata level information for over 50 years. In particular, we have recovered and use microdata from two official historical sources produced by the Colombian Statistics Bureau, DANE: The Annual Manufacturing Survey available and the Household Survey.

The Household Surveys are the source of official statistics on employment and the distribution of income and are thus representative of the whole economy, formal and informal, in all sectors. For all respondents that take part in production, the survey records whether they are salaried workers, own-account workers without employees or employers, and the size of the business where their activity takes place in broad categories. This provides a comprehensive picture of income and employment in the whole economy, their allocation across sectors and different types of firms, and the distribution of earnings in the economy and within each of these segments. Although there have been methodological changes over time, the Household Surveys go back in time several decades. Unlike other waves for years close to the peak of industrialization, the digital records of the 1971 wave contain the individual responses on the size of the business to which the worker is affiliated, besides income and occupational category. For this reason, we use the household survey information from 1971.

The Annual Manufacturing Survey is a census of all manufacturing establishments with at least 10 employees. We use data for 1974-1989.[2] It records detailed information at the establishment level on the value of output and input use (capital, labor, intermediates consumed), including quantities and values for all individual products and all individual material inputs produced by the establishments plus payroll. A firm in the model corresponds to an establishment (plant) in the data. The within-plant product level information is used to estimate establishment-level prices that are comparable across establishments of the manufacturing sector, even for multiproduct plants. Plant prices are then used to measure plant output for uni- and multi-product plants with differentiated

[2]The data has been collected annually since 1974, but there is a gap in the electronic records in 1977-1982: for this set of years, the information on individual products and material inputs is missing. Thus, although the peak occurred at the end of the 1970s, we use the available data for the full 1974-1989 period to represent the peak of industrialization. We do not use solely 1974-1976 because we have less assurance of the quality of the data for the earliest years, especially at the detailed product-input level. Since the methods we use to separately measure quantities and prices rely on chained firm-level prices, we take advantage of the robust data for the 1980s to improve the quality of our price measurement in the early years.

products and to subsequently estimate the plant production and demand functions **??** and 46 with the methodology proposed by (Eslava et al. (2023)).

In the data, we define the modern segment to correspond to establishments with 10 or more employees. The traditional segment is the complementary set of all establishments with less than 10 employees, including own-account individuals. Since the household surveys classify workers by sector and firm size category and into self-employed/salaried workers/employers, we can construct the share of employment and income (i.e., value-added) represented by modernity in each sector. We also have the detailed distribution of income overall, by sector, and by segment (modern/traditional) within each sector. A common concern when aggregating income data from household surveys is that individuals at the very top of the income distribution are likely under-represented, implying that value-added may be underestimated. We address this concern by scaling income in each sector to reflect the aggregate reported in GGDC data.

The Annual Manufacturing Survey then provides us with a detailed picture of the distribution of productivity and distortions across firms of the modern segment and, consequently, for modern entrepreneurs. Modern manufacturing is the main modern activity during the episode we study. For this reason, the lack of equally detailed data on modern services or modern agriculture, and the difficulties inherent to measuring firm productivity and distortions in services and agriculture in a comparable way to manufacturing, we assume that the distributions we observe in manufacturing also represent the distributions in the modern segments of services and agriculture.

D.2 Additional Data

Using data on establishments (i.e., salaried employment) and focusing on a subset of late industrializers in Latin America, Table 10 shows that micro establishments are much more prevalent in these countries as compared to advanced economies, dominated by early industrializers, especially in terms of their weight in employment. Although this holds in all sectors, the weight of micro establishments is lesser in manufacturing than in basic services (including retail). Moreover, there is a tight link between the prevalence of microestablishments and that of informality in these late industrializers. Even among salaried workers, the vast majority of informal employment, i.e., employment that does not abide by labor regulations, concentrates on micro-employers (Table 11. The proportion grows to almost 100% of informal workers if self-employment is included. The weight of microestablishments in formal employment is much lower, especially in manufacturing, where it does not reach above 10%.

Table 10: **Establishments under 10 Employees as a % of all (Economic Censuses, circa 2005)**

	Unweighted			Employment-weighted		
	Advanced Economies	Latin America	Colombia	Advanced Economies	Latin America	Colombia
Manufacturing	71,1%	91,6%	87,2%	11,5%	30,3%	31,7%
Wholesale-Retail	85,0%	96,9%	96,0%	27,4%	65,8%	61,4%
Accom-Restaur.			95,7%			69,9%
Other Services	88,1%	91,0%	87,8%	25,7%	34,3%	34,0%

Table 11: **Workers in Establishments of at most 10 Employees as a % of all workers in the category: Formal or Informal. Excludes self-employment**

	Latin America		Colombia		
	Formal workers	Informal workers	Formal workers	Informal workers	US
Manufacturing	10%	69%	8%	71%	4%
Wholesale-Retail	28%	82%	24%	88%	11%
Services	15%	60%	11%	73%	13%
Accom-Restaur.	29%	78%	33%	88%	31%

Latin America: average of Argentina, Bolivia, Colombia, El Salvador, Guatemala, Honduras, Mexico, Paraguay, Peru, and Uruguay. Formal workers: contributing to a pension. Pooled data from 2011 to 2015. Latin America from household surveys, the US from Business Dynamics Statistics. Source: Eslava (2018).

Table 12: **Distribution of Colombian workers across employer size classes by sector and formality status: 2012**

Employer size	Manufacturing		Non-manufacturing		All
	Formal workers (45%)	Informal workers (55%)	Formal workers (35%)	Informal workers (65%)	
$\leq$ 10 workers (incl. Self-employed)	0.9%	7.5%	9.1%	53.7%	71.1%
11-50 workers	1.2%	0.6%	4.1%	1.8%	7.6%
51-100 workers	0.6%	0.1%	1.5%	0.2%	2.3%
101+ workers	3.3%	0.2%	14.6%	0.9%	19.0%
Total	6.0%	8.2%	29.2%	56.6%	100%

Source: Colombian Household's Survey. Urban and rural coverage.