What does Maximum Employment Mean?

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Abstract

We formalize the maximum, optimal, and equilibrium levels of employment in a frictional model of unemployment and characterize the conditions under which it is desirable to actively pursue employment objectives via monetary policy. Maximum employment corresponds to the lowest feasible unemployment rate subject to market participation by firms and workers, achieved when employers capture the entire surplus to maximize job creation, resulting in low-paying jobs. Inflation is neutral when wage contracts are unrestricted, but can raise employment by eroding real wages when firms are constrained in their ability to frontload compensation. Implementing maximum employment, however, requires prohibitively high inflation. Output-maximizing employment balances the benefits of job creation against recruiting costs and differs from both maximum and equilibrium employment. A welfare-maximizing monetary authority further balances the gains from moving employment toward this level against the welfare costs of inflation. Optimal monetary policy is therefore asymmetric: it focuses solely on price stability in some labor market environments, while raising inflation to increase employment in others. Whether an active employment objective is desirable depends on the source of unemployment rather than its level.

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1 Introduction

The Federal Reserve's congressional mandate to promote maximum employment and price stability has generated extensive research on optimal monetary policy design. While there is a vast literature on appropriate price stability goals, the employment component of the mandate has received less systematic treatment. The current approach models the employment objective indirectly through output gap stabilization or as activity levels consistent with long-run price stability, effectively subordinating employment considerations to inflation targets. This approach reflects the lack of a clear definition of the concept of "maximum" employment in existing work, where employment outcomes are primarily viewed through the lens of their inflationary consequences.

The resulting gap in the literature is particularly problematic given the Fed's explicit statutory requirement to pursue maximum employment as a co-equal objective with price stability. The theory of labor search and matching provide the analytical tools necessary to address this deficiency by offering rigorous definitions of employment concepts within frictional frameworks. However, the connection between these microeconomic foundations and macroeconomic policy targets remains underexplored. Specifically, the literature has not adequately addressed how different employment objectives – whether maximum, optimal, or equilibrium employment – translate into specific inflation rate targets that would implement these goals.

This paper fills this gap by developing a tractable framework that derives inflation targets directly from employment objectives within a frictional labor market model. We define maximum employment as the level associated with the lowest sustainable rate of unemployment that still ensures voluntary participation in the labor market by employers and households. This level is independent of monetary policy and the inflation rate in the economy. To achieve maximum employment, the planner has to incentivize employers to maximize job creation, which requires maximizing their share of the production surplus, while ensuring that the unemployed are not altogether discouraged from searching for a job. This is tantamount to a labor market allocation where employers are provided with the entirety of the surplus while workers are paid their reservation value, leaving them indifferent between unemployment and employment. The key insight is that maximum employment can only be attained by creating many low-paying jobs.

In a general equilibrium setting, workers recoup some of the surplus as dividends, because households are also the owners of firms. Nonetheless, maximum employment level is not optimal because the job creation is costly in a frictional model. The recruiting costs associated with constantly trying to fill a large number of vacant positions so as to maintain the lowest feasible level of unemployment exceeds the additional value created by those jobs. The optimal level of employment, which maximizes the welfare, better balances the costs and benefits of job creation. Equilibrium employment is distinct from the optimal and maximal levels of employment except in special cases, e.g., equilibrium aligns with maximal employment when employees have no bargaining power against employers, and, hence would have been paid their reservation wage anyways.

We then introduce trend inflation in our model and analyze its effects on equilibrium employment. To that end, we restrict the frequency of wage re-negotiations for ongoing employee-employer pairs, which puts deflationary pressure on real wages in an inflationary

environment. Importantly, we allow for wage negotiations when the worker is hired and re-negotiations at random intervals thereafter, or if the real wage is deflated to a point where the workers would prefer unemployment to their job. Under these assumptions, the labor market clears at the hiring margin, since the starting salaries are flexible, and the wage rigidity never results in inefficient separations.[1] We then show that if the space of wage contracts is unrestricted, then (long-run) inflation is neutral, i.e., it does not change equilibrium employment. This is because any real effect that might arise from (anticipated) deterioration of real pay is fully negotiated away at the time of hire by frontloading wage payments.

At sufficiently high inflation levels, however, frontloading of pay requires the employer to compensate the worker more than their marginal product early in their tenure to balance anticipated losses in real wages. Depending on the rate of inflation, the required early compensation can be substantial, straining firms' finances. We therefore consider credit frictions that limit the extent of frontloading that can be financed, thereby, allowing inflation to raise equilibrium employment by eroding worker's share of the surplus.[2] Because such constraints do not bind at low levels of inflation, where, for instance, required frontloading in wage contracts can be handled by firms' internal sources, the model features a non-linear Phillips Curve with inflation raising employment only if it is sufficiently high.

We then ask what levels of inflation are generally consistent with maximal and optimal employment levels. We first show that trying to implement the maximal employment level is prohibitively costly. By contrast, implementing optimal employment is feasible, with the required inflation rate depending on the particular nature of the labor market and the job matching process. The inflation target that implements optimal employment is lower when the marginal value of an additional vacancy in job creation is low, or, equivalently, that of an unemployed worker is high; when the job separation rate from employment to unemployment is low; or, when the real interest rate is low. A number of policy implications emerge. For instance, when the number of job seekers and recruiters are gross complements in job creation and the market is short on unemployed workers relative to vacant positions, the model calls for a lower inflation target. Higher rates of job separation or real interest rates call for higher inflation targets, though this concerns the rate that would implement a given employment objective and not whether pursuing it is worthwhile, a distinction we return to in Section 5.

The preceding analysis takes the employment target as given and derives the inflation rate needed to implement it. This prioritizes the employment objective over the inflation objective. A complete characterization of optimal policy requires the monetary authority to jointly determine both targets by balancing the welfare costs of inflation against the benefits of achieving an employment level that internalizes the costs and benefits of job creation.

To that end, we extend the frictional labor market framework by incorporating a money demand structure based on Sidrauski [1967]. In this environment, inflation creates a welfare cost by taxing real money balances, generating a meaningful trade-off between fully

[1] Therefore, Barro's 1977 criticism of models where wage stickiness disallows opportunities for a mutually beneficial arrangement does not apply in our model.

[2] Lack of commitment to future employment by the worker could also render such supernumerary pay infeasible. Alternative models of monetary non-neutrality we considered do not change the nature of our conclusions although they might differ in the particular mechanism with which inflation affects real allocations.

implementing the employment objective and minimizing the distortions from inflation. The optimal policy balances the marginal welfare loss from inflation against the marginal gain in output from reducing unemployment. Where interior, this solution involves an inflation rate above the Friedman rule [Friedman, 1969] and an unemployment rate below what the decentralized equilibrium alone would deliver. However, this interior solution may not emerge when labor market fundamentals imply a naturally low unemployment rate, or when the long-run Phillips curve is steep such that achieving meaningful employment gains requires prohibitively high inflation. In such cases, optimal policy prescribes singular focus on price stability, which in the Sidrauski framework corresponds to the Friedman rule; because that benchmark is specific to this framework, we also evaluate policy against a zero-inflation target, which is the comparison a central bank actually faces. More generally, an employment shortfall creates scope for intervention but does not by itself imply that intervention is welfare improving.

A key implication of our framework is that optimal policy design and the associated targets depend on labor market fundamentals. When the welfare gains from raising employment are small relative to the required inflation costs, policy should focus exclusively on price stability, effectively operating as a single mandate. In other environments, moderate inflation can raise employment enough to justify those costs, implementing a genuine dual mandate with asymmetric policy responses. An employment shortfall is therefore not sufficient for intervention. Whether active employment policy is desirable depends on the fundamentals generating the shortfall and on how costly inflation is.

Our framework can be viewed as providing a theoretical foundation for the Fed's shortfall approach: policy should "judiciously raise inflation targets when employment falls short," but need not respond symmetrically when unemployment is low, unless inflation itself becomes problematic. Unlike the Fed's 2020 framework, however, which treats the shortfall approach primarily as a response to ZLB risks and asymmetric costs of policy errors, as articulated in Bernanke et al. [2019], our model derives this asymmetry from the fundamental structure of the labor market. As a result, whether policy should focus solely on an inflation target or jointly target both inflation and employment levels evolves with labor market conditions. Secular changes in such characteristics as matching efficiency, separation rates, or the real interest rate, for instance, necessitate corresponding adjustments to the long-run policy stance and whether policy should focus solely on an inflation target or jointly target both inflation and employment levels.

Related Literature. The classic Friedman rule, which prescribes a zero nominal interest rate to eliminate deadweight losses from economizing on money balances, provides the earliest theoretical benchmark for monetary policy targets [Friedman, 1969]. The benchmark New Keynesian models, exemplified by King and Wolman [1996] and Schmitt-Grohe and Uribe [2004], demonstrate that optimal inflation is typically zero or slightly negative when price stickiness is the primary friction, as any positive inflation creates menu costs and distortionary price dispersion. In the presence of distortionary taxation, Phelps [1973] shows that seigniorage revenue from positive inflation should be part of an optimal tax mix. Summers [1991] and, more recently Coibion et al. [2012] and Bernanke et al. [2019] among several others, have argued for higher inflation targets to provide a buffer against the zero lower bound

(ZLB) during recessions. The treatment of the employment goal in this literature is typically guided by what Blanchard and Galí [2007] term the "divine coincidence": in standard New Keynesian models with only nominal rigidities, stabilizing inflation automatically stabilizes the welfare-relevant output gap. This property motivated the focus on inflation targeting, as there is no independent role for the employment mandate.[3]

Earliest engagement with long-run employment effects of inflation comes from Tobin [1972], who argued that moderate inflation "greases the wheels" of the labor market by facilitating real wage adjustments in the presence of downward nominal wage rigidity. Formalizing Tobin's argument, Akerlof et al. [1996] shows that moderate inflation can permanently reduce the natural rate of unemployment. More recently, Hobijn and Şahin [2021] and Eusepi and Şahin [2025] study the interaction between labor force participation and employment levels in a search and matching framework, and find that monetary policy focused on stabilizing jobs inherently promotes maximum employment by strengthening the long-term attachment of the workforce. The employment objective in these studies is centered around the natural unemployment rate. Michaillat and Saez [2024] propose a "full-employment" objective, where resources dedicated to job creation are minimized. Adopting a frictional model of unemployment as the recent literature, we show that maximum employment exceeds both the natural rate of unemployment and Michaillat and Saez's (2024) full-employment and jointly study both the inflation and employment objectives.

A related literature integrates the labor search framework with frictional goods markets and examine the long-run relationship between inflation and the labor market focusing on optimal inflation policy [Berentsen et al., 2011, Bethune et al., 2024, Gu et al., 2025]. Berentsen et al. [2007] examine when both optimal inflation (Friedman rule) and optimal employment (Hosios condition) can be jointly achieved, finding they coincide only under restrictive conditions on the distribution of money holdings across agents. While they characterize when both objectives align, we analyze how to implement employment targets when they do not. Furthermore, our mechanism, based on restricted wage setting, potentially allows inflation to raise employment even at higher levels, while Berentsen et al. [2007] features a negative effect of inflation on economic activity above moderate levels.

Our paper makes three key contributions relative to the literature. First, we provide the first rigorous definition of "maximum employment" within a frictional labor market model, showing it corresponds to the lowest feasible unemployment rate consistent with voluntary participation—a concept distinct from both efficient employment (the Hosios benchmark) and equilibrium employment. Second, we demonstrate that achieving maximum employment requires prohibitively high inflation, as it necessitates creating many low-paying jobs by maximizing employers' surplus. Third, we characterize the states of the economy where it is desirable to pursue an active dual-mandate policy and derive the associated inflation and employment targets.

Our paper also relates to recent work on how inflation affects labor market outcomes and worker well-being through real wage erosion. Afrouzi et al. [2026] document that the wage growth gap between job stayers and job switchers widens substantially during inflationary

[3]When Blanchard and Galí [2007] introduce real wage rigidities, the divine coincidence disappears and a meaningful trade-off emerges, though their focus remains on stabilization rather than on defining employment targets.

periods, with stayers' wages lagging behind. Exploring the behavioral responses workers undertake to mitigate wage erosion, Hajdini et al. [2023] and Pilossoph and Ryngaert [2024] use survey evidence to show that workers with higher inflation expectations increase job search effort, while Pilossoph et al. [2026] estimate the welfare costs of this intensified search activity. Guerreiro et al. [2024] find that approximately one-fifth of workers engaged in costly actions—-requesting raises, participating in union activity, or soliciting outside offers—-to restore their real wages during the recent inflation episode, documenting costs associated with wage renegotiation. These papers collectively demonstrate that wage catch-up with the same employer is costly and often incomplete, leading to persistent real wage erosion for job stayers. Our contribution is to show how this wage rigidity can create profit opportunities that firms exploit through vacancy creation, linking inflation to labor market tightness.

Roadmap. In the next section, we introduce our model environment, describe the decentralized equilibrium. Section 3 discusses the interaction between inflation and economic activity. Section 4 discusses the various policy objectives and presents our key results. Section 5 conducts a quantitative analysis that demonstrates our results by calibrating key model parameters. The quantitative analysis captures the essential features of the U.S. Beveridge Curve and Phillips Curve relationships. Moreover, the quantitative analysis allows us to compute optimal employment rates and illustrate how the associated inflation targets would vary across different labor market environments. Section 6 concludes.

2 Model

This section introduces a monetary economy with frictional labor markets that combines the Sidrauski [1967, 1969] model of money demand with the canonical model of random job search [Diamond, 1982, Mortensen, 1982, Pissarides, 1985], extended to include nominal wage rigidities for ongoing employment relationships.

2.1 Household Problem

Time is continuous and the horizon is infinite. The economy is populated by a representative household with a continuum of members of measure one. Following Lucas [2000] and Sidrauski [1967], households derive direct utility from holding real balances. This reduced-form specification captures in a tractable way the transaction services provided by money, which facilitate consumption purchases and economize on the time and effort required for market transactions.

The household maximizes lifetime utility derived from consumption and real money balances:

$$\max_{\{c,n\}} \int_0^\infty e^{-\rho t} U(c,n) dt, \tag{1}$$

where c denotes consumption, n denotes real money balances, and $\rho > 0$ is the subjective discount rate. We assume the utility function $U(c,n)$ is strictly increasing and concave in both arguments.

The household's budget constraint is:

$$\dot{a} = I(u, d) + T - c - \pi n, \tag{2}$$

where $a \equiv b + n$ denotes total real assets (the sum of real bond holdings b and real money balances n), T represents lump-sum government transfers, and π is the inflation rate. Real bonds earn the real interest rate r, while money is dominated in rate of return, losing value at rate π through inflation.

Household members pool their resources and perfectly insure each other against idiosyncratic employment risk [Andolfatto, 1996]. The household's real income $I(u, d)$ depends on labor market outcomes. Each representative household has a fraction $(1-u)$ of their members employed, earning wage $\bar{w}$ on average, and a fraction u unemployed, receiving flow utility z in consumption equivalent terms from home production and unemployment benefits. The household also receives dividend income d as firm owners. Thus:

$$I(u, d) = (1 - u)\overline{w} + uz + d$$

Because labor market dynamics affect the household's optimization problem only through the income flow $I(u, d)$, we can decompose the household's decision problem. In particular, we can separate the analysis of the labor market equilibrium, which determines $I(u, d)$ as a function of labor market conditions, and solve the household's consumption-savings problem taking the income process as given.

2.2 Labor Market

The labor market is characterized by search and matching frictions in the spirit of the Diamond-Mortensen-Pissarides framework [Diamond, 1982, Mortensen, 1982, Pissarides, 1985]. Unemployed workers search for job opportunities while firms with vacant positions recruit employees. The bilateral nature of search creates frictions: it takes time for unemployed workers to locate suitable vacancies, and it takes time for firms to fill open positions.

We denote by u the measure of unemployed workers and by v the measure of vacant positions. The flow of new matches formed per unit of time is determined by a matching function $m(u, v)$, which depends on the number of searchers on both sides of the market. We assume $m(u, v)$ exhibits constant returns to scale and is strictly increasing and concave in both arguments: $m_u, m_v > 0$, $m_{uu}, m_{vv} < 0$, and $m_{uv} > 0$. These properties capture the congestion effects inherent in the matching process. η_u^m denotes the unemployment elasticity of the matching function, and $\eta_v^m = 1 - \eta_u^m$ is the corresponding vacancy elasticity.

Labor market conditions are summarized by ratio of vacancies per unemployed $\theta \equiv \frac{v}{u}$, which determines two essential transition rates: the job-finding rate $p(\theta) = m(u, v)/u \equiv m(1, \theta)$ for workers, and the vacancy-filling rate $q(\theta) = m(u, v)/v \equiv m(1, \theta)/\theta$ for firms. These transition rates satisfy $p'(\theta) > 0$ and $q'(\theta) < 0$: in a tighter labor market (higher θ), workers find jobs more quickly but firms find it harder to fill vacancies.

Employment relationships are subject to exogenous separation shocks that arrive at rate $\delta \in (0, 1)$. When a separation occurs, the worker returns to unemployment and the firm's position becomes vacant. In steady state, flows into and out of unemployment balance: $p(\theta)u = \delta(1 - u)$, implying the following steady-state unemployment rate:

$$u(\theta) = \frac{\delta}{\delta + p(\theta)}. \tag{3}$$

This relationship defines the Beveridge curve: a negative relationship between vacancies and unemployment, with tighter labor markets (higher θ) associated with lower unemployment.

We now characterize the value of employment and unemployment to workers and firms, taking into account how inflation affects the real value of ongoing employment relationships through nominal wage rigidity.

2.2.1 Worker Value Functions

The value function for an unemployed worker, denoted U, satisfies the Bellman equation:

$$rU = z + p(\theta)[W_0 - U], \tag{4}$$

which states that the flow return to unemployment equals the utility from being unemployed plus the expected capital gain from finding employment. An unemployed worker receives immediate utility z and transitions to employment at rate $p(\theta)$, experiencing a discrete jump in value equal to $W_0 - U$ upon successful matching, where W_0 is the value of a new job to the worker.

The value function for a newly employed worker, W_0, satisfies:

$$rW_0 = w + \delta(U - W_0) + \lambda(\pi)[W - W_0]. \tag{5}$$

The flow return to employment consists of a negotiated wage payment w minus the expected capital loss from job destruction and from inflation. With probability δ per unit time, the employment relationship dissolves and the worker returns to unemployment, experiencing a discrete drop in value equal to $W_0 - U$. At the rate $\lambda(\pi)$, where π is the inflation rate , the value of a job to the worker changes from W_0, resulting in a drop in the job's value to the worker by $W_0 - W \geq 0$. Function $\lambda(\cdot)$ captures the likelihood that inflation triggers wage depreciation, potentially allowing for the possibility of wage indexing. We assume that $\lambda(0) = 0$ and $\lambda'(\pi) > 0$.

The value of an ongoing job to a worker is given by:

$$rW = \underline{w} + \delta[U - W] + \phi[W_0 - W]. \tag{6}$$

The flow return to the worker from an ongoing employment relationship consists of a lower wage payment $\underline{w} \leq w$, which is not negotiated, minus the expected capital loss from job destruction plus the expected capital gain from renegotiating the wage rate. At rate ϕ, workers renegotiate their wage, which results in a capital gain of $W_0 - W$.

2.2.2 Firm Value Functions

The value of maintaining a vacant position, V, is characterized by:

$$rV = -\kappa + q(\theta)[J_0 - V] \tag{7}$$

A firm with a vacancy incurs the flow cost κ and successfully fills the position at rate $q(\theta)$. Upon matching with a worker, the firm experiences a discrete increase in value equal to $J_0 - V$, where J_0 represents the value of a newly filled position and satisfies the following Bellman equation:

$$rJ_0 = y - w + \delta(V - J_0) + \lambda(\pi)[J - J_0] \tag{8}$$

The firm's flow return to a newly filled position equals the worker's productivity less the negotiated wage payment, adjusted for the expected losses from job destruction and expected gains from real wage deflation. When the job is destroyed at rate δ, the firm loses $J_0 - V$. At the rate $\lambda(\pi)$, the firm's gains from the position rise from J_0 to $J \geq J_0$.

The value of an ongoing employment relationship to the firm is:

$$rJ = y - \underline{w} + \delta(V - J) + \phi[J_0 - J] \tag{9}$$

The flow return consists of a higher ongoing profit margin $y - \underline{w}$ minus the expected losses from job destruction and wage renegotiation, which happens at rate ϕ and reduces the value of the job to the firm back to J_0.

2.2.3 Wage Determination and Labor Market Equilibrium

The model has nine endogenous variables: six values in Equations 4-9, the two wage rates w and $\underline{w}$, and the labor market tightness θ. To the six value equations above, we add three conditions to define the labor market equilibrium.

First, we assume that the negotiated wage payment w is determined through Nash bargaining between the worker and firm over the total match surplus $S = (W_0 - U) + (J_0 - V)$. The Nash bargaining protocol maximizes the weighted product of the parties' gains from the match: $w = \arg\max (W_0 - U)^{\beta}(J_0 - V)^{1-\beta}$, where $\beta \in [0, 1]$ represents the worker's relative bargaining power. We restrain the set of possible wages so that at any point in time the wage rate paid may not exceed a worker's instantaneous productivity: $w \leq y$.[4] The first-order condition for this optimization problem yields the surplus-sharing rule:

$$J_0 - V = (1 - \beta)S + \mu_w \tag{10}$$

where $S = J_0 - V + W_0 - U$ is the total match surplus at entry, and $\mu_w \geq 0$ is the Lagrange multiplier associated with the credit constraint. If the credit constraint does not bind ($\mu_w = 0$), Equation (10) states that the worker's bargaining power determines the division of the total match surplus. Otherwise, $\mu_w > 0$, and $w = y$.

The ongoing wage rate $\underline{w} \leq w$ is not negotiated. Without loss of generality we assume that $\underline{w}$ leaves the worker indifferent between unemployment and employment:

$$W = U. \tag{11}$$

This assumption ensures that transitions to the low-wage regime, W, from inflation does not result in a negative surplus share to the worker.

[4]The no-credit requirement could be extended to allow for some borrowing $w \leq \gamma y$ with $\gamma > 1$. The results would be qualitatively similar.

Finally, the equilibrium is characterized by free entry of firms into the vacancy posting process. This condition drives the value of maintaining a vacancy to zero in equilibrium:

$$V = 0. \tag{12}$$

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Equations (10), (11), and (12) along with the value functions 4-9 determine the labor market tightness in equilibrium and, thereby, household income. In equilibrium, total dividends consists of profits from filled positions net of maintaining vacant positions.

$$d = (y - \bar{w})(1 - u) - v\kappa,$$

where $\bar{w}$ is the average wage between workers that are in the two wage regimes $\underline{w}$ and w. Substituting the dividends gives household's income as a function of the unemployment rate and tightness in equilibrium:

$$I(u, \theta) = y(1 - u) + uz - u\theta\kappa.$$

At the steady state, unemployment must lie on the Beveridge Curve: $u(\theta) = \delta/(\delta + p(\theta))$, which implies that household's income is given by $I(\theta) = I(u(\theta), \theta)$.

2.3 Steady-State Consumption and Real Money Balances

Given our focus in long-run policy objectives rather than short-run policy responses to economy's deviations from those objectives, we now briefly review the household's optimality conditions in steady state.[5] The marginal rate of substitution between consumption and money balances equals the nominal interest rate:

$$\frac{U_n(c, n)}{U_c(c, n)} = i = r + \pi, \tag{13}$$

where the second equality follows from the Fisher equation. This condition equates the marginal benefit of holding an additional unit of real balances (the marginal utility U_n relative to the marginal utility of consumption) to its opportunity cost, which is the forgone nominal return i from holding interest-bearing bonds instead.

With government budget balance ($T = \pi n$) and goods market clearing, consumption equals aggregate income:

$$c = I(\theta) \tag{14}$$

Given the income process $I(\theta)$ and inflation π, equations (13) and (14) jointly determine the household's optimal choices of consumption and money balances (c^*, n^*) in steady state.

The household problem is thus characterized by two key margins. First, the consumption-money margin determines the composition of household spending between goods and liquidity services, governed by condition (13). Second, labor market equilibrium determines the

[5]We present the full set of dynamic equilibrium conditions in the Appendix where we construct the monetary authority's planning problem.

level of income available to the household. By solving the labor market equilibrium first, we can treat $I(u, \theta)$ as given when analyzing the household's financial decisions, simplifying the overall characterization of equilibrium.

The complete characterization of the household's optimal choices, combined with the labor market equilibrium, defines a mapping from inflation rates to employment outcomes. This mapping, subject to credit constraints, generates the Phillips Curve relationship that is central to our policy analysis in the next section.

3 Inflation and Employment in Equilibrium

This section characterizes the relationship between inflation and economic activity in the decentralized labor market equilibrium. We establish conditions under which monetary policy affects real allocations – specifically, the level of employment and unemployment – through the labor market.

We begin with a striking neutrality result: when wage contracts are unrestricted, inflation has no effect on equilibrium employment despite the presence of nominal wage rigidities. The key insight is that forward-looking workers and firms fully internalize the effects of anticipated inflation through wage negotiations at the time of hire. Workers demand and receive compensation for future real wage erosion by negotiating higher starting wages, which exactly offsets the redistribution of surplus that would otherwise occur through inflation.

Then, we consider limitations to firms' ability to frontload wages beyond current productivity. Under these constraints, sufficiently high inflation cannot be fully neutralized through upfront compensation, allowing monetary policy to affect real allocations. The result is a nonlinear Phillips Curve relationship. At low inflation rates (below a threshold $\underline{\pi}$), credit constraints do not bind and inflation remains neutral: the equilibrium unemployment rate is independent of the inflation rate. At high inflation rates (above $\underline{\pi}$), credit constraints bind and inflation reduces unemployment by eroding workers' share of the match surplus.

We begin our analysis of the labor market equilibrium by deriving the job creation condition that determines labor market tightness as a function of model primitives.

Substituting $V = 0$ from the free-entry condition into the vacancy value function yields $\kappa = q(\theta)J_0$, which states that the flow cost of maintaining a vacancy equals the expected benefit from filling it. From equation (8), the latter is equivalent to:

$$J_0 = \frac{y - w + \lambda(\pi)[J - J_0]}{r + \delta}.$$

The job creation condition emerges from combining the free entry condition with the value function for filled positions, which yields:

$$\frac{y - w + \lambda(\pi)[J - J_0]}{r + \delta} = \frac{\kappa}{q(\theta)}, \tag{15}$$

indicating that the expected cost of hiring a worker equals the present discounted value of the profit stream from the employment relationship. When $\pi = 0$, equation (15) gives the job creation condition in the canonical random search model. When $\pi > 0$, inflation raises profits, ceteris paribus, and encourages firms to post more vacancies since $J \geq J_0$.

As we show next, however, the effect of inflation on equilibrium vacancy creation, all things considered, depends on the outcome of wage negotiations.

Consider first an economy without any borrowing constraints on firms: $\mu = 0$. The equilibrium wage rate in that case is:

$$w = \beta y + (1 - \beta) rU + \lambda(\pi)(W_0 - W). \tag{16}$$

The negotiated wage rate is a convex combination of productivity and the flow reservation value of an unemployed worker, adjusted for future deflation of the real wage in an inflationary environment. Given our assumptions on $\lambda(\cdot)$, when $\pi = 0$, the model replicates the standard wage setting process in the canonical model. $\pi > 0$ implies higher wages when the worker starts their job, implying that workers recapture at least some of the anticipated loss in value from wage deflation. In fact, they capture the entirety of it if starting salaries are unbounded. This can be established by first showing that a worker's loss from deflation is equal to the firm's gain from wage deflation.

Lemma 1. $J - J_0 = W_0 - W$

Because the inflationary process effectively transfers resources from the worker to the firm, we have $J - J_0 = W_0 - W \geq 0$. Therefore the additional gains in firm's flow profits, $\lambda(\pi)(J - J_0)$ in equation (15), is paid out entirely to the worker by raising starting salaries by the same amount $\lambda(\pi)(W_0 - W)$. Substituting the wage rate in equation (16) into the job creation condition therefore gives the canonical equilibrium condition for labor market tightness in the random search model.

Proposition 1. *If wage payments are unbounded, then the equilibrium wage rate is given by:*

$$w = \beta y + (1 - \beta) z + \beta \kappa \theta + \lambda(\pi) \frac{\beta}{1 - \beta} \frac{\kappa}{q(\theta)}. \tag{17}$$

Considering the equilibrium valuations of jobs and unemployment to a worker, the negotiated wage reflects a combination of productivity, value of leisure, and compensation for the deflationary effect of price inflation on real wages. That compensation depends on the strength of the worker's bargaining power in wage negotiations and increases with the rate of inflation in the economy. Note that wages can easily rise above current productivity level if $\beta > 0$ and the inflation rate is high. The faster the real wage deflation, the higher the upfront negotiated wage has to be to compensate the worker.

Proposition 2. *Equilibrium level of labor market tightness is given by:*

$$\frac{y - z}{r + \delta} = \frac{\kappa}{q(\theta^*)} \times \frac{r + \delta + \beta p(\theta^*)}{(1 - \beta)(r + \delta)}. \tag{18}$$

Equation (18) is the standard equilibrium condition in the canonical model and states that, in equilibrium, the discounted value of the flow surplus associated with a job equals the effective cost of creating that job to the firm. That is higher than the firm's expected cost of filling a vacancy. Because such filling costs are sunk by the firm prior to wage negotiations, workers capture some of those during negotiations as long as they have some

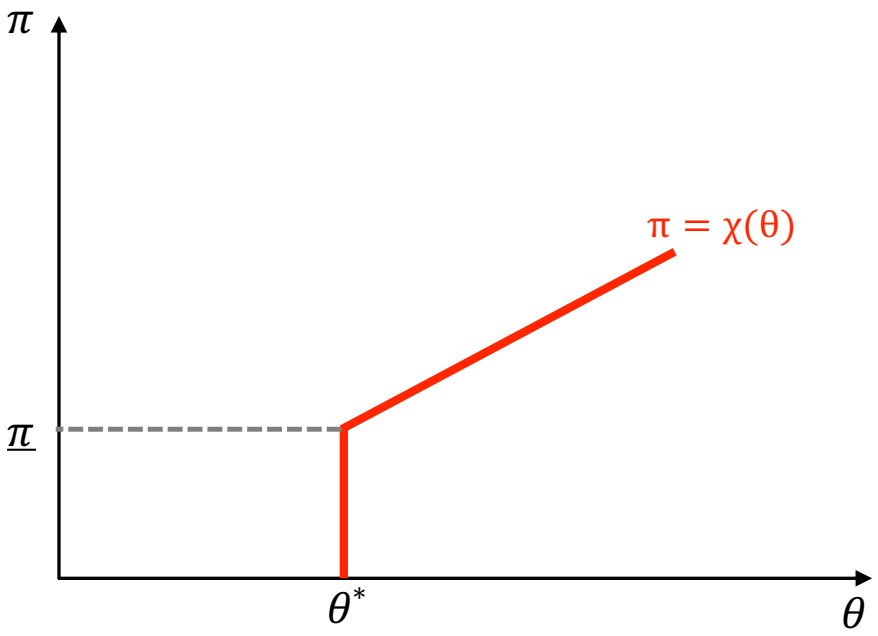


Figure 1: Inflation and Labor Market Tightness

bargaining power, $\beta > 0$, which raises the effective cost of posting vacancies in equilibrium and discourages firm entry.[6] That in turn results in lower job creation than would occur if workers had no bargaining power.

Importantly, π does not appear in the equilibrium condition above, giving our first main result on monetary neutrality.

Corollary 1. *Suppose that w is unbounded. Then the inflation rate has no real effect on equilibrium allocations.*

The model without credit constraints does not feature a long-run Phillips Curve, because the deflationary pressure on the real wage rate is negotiated away. The firm agrees to frontload pay in order to compensate the worker for future loss. Setting $w = y$ gives the inflation level above which the negotiated wage exceeds the contemporaneous productivity level:

$$\lambda(\underline{\pi}) = \frac{1-\beta}{\beta}(r+\delta). \tag{19}$$

Higher levels of inflation require the firm to borrow against future profits to pay its workers. Note that the threshold inflation rate above which this becomes necessary is lower the higher the worker's bargaining power, which determines the amount of inflation-related compensation. Furthermore, the higher the effective discount rate for the match, $r + \delta$, the lower is the required upfront pay to compensate future losses, and, hence, higher is $\underline{\pi}$.

Next we consider the scenario where the firm is limited in its ability to frontload pay due to credit constraints. Specifically, we assume the highest salary a firm can pay is equal to

[6] This is represented by the $\beta\kappa\theta$ term in the equilibrium wage function.

productivity: $w \leq y$. Therefore, when $\pi \geq \underline{\pi}$, $w = y$, which gives the following job creation condition:

Proposition 3. *In an economy with credit constraints, the equilibrium level of labor market tightness solves equation (18) if $\pi \leq \underline{\pi}$ and solves the following equation if $\pi \geq \underline{\pi}$:*

$$\frac{\lambda(\pi)}{r+\delta+\lambda(\pi)+p(\theta)} \times \frac{(y-z)}{r+\delta} = \frac{\kappa}{q(\theta)}. \tag{20}$$

Proposition 3 describes a positive relationship between the inflation rate and equilibrium labor market tightness, and, hence, a negative relationship between inflation and the unemployment rate.

Formally, we define the long-run Phillips curve as $\pi = \chi(\theta)$, where $\chi(\theta)$ is given by:

$$\chi(\theta) = \begin{cases} [\pi_{min}, \underline{\pi}] & \text{if } \theta = \theta^* \\ \lambda^{-1}\left(\frac{\kappa[r+\delta+p(\theta)](r+\delta)}{(y-z)q(\theta)-\kappa(r+\delta)}\right) & \text{if } \theta > \theta^* \end{cases} \tag{21}$$

where θ^* is implicitly defined by (18) and π_{min} is the minimum feasible inflation the monetary authority could target (we set it at the Friedman rule). Note that when $\theta > \theta^*$, $\chi'(\theta) > 0$, i.e., higher inflation raises equilibrium tightness.[7]

Figure 1 illustrates the non-linearity of this relationship in the model. Inflation has no effect on employment at low levels ($\pi < \underline{\pi}$), but raises employment at sufficiently high inflation rates.[8]

In what follows, we focus on the economy with credit constraints and consider different interpretations of the Fed's employment mandate and examine the levels of inflation that are consistent with each interpretation.

4 Policy Objectives

This section examines different policy objectives and their implementation through inflation policy. We first define two employment objectives based on the wording of the congressional mandate:

> "*The Board of Governors of the Federal Reserve System and the Federal Open Market Committee shall maintain long run growth of the monetary and credit aggregates commensurate with the* ***economy's long run potential to increase production****, so as to promote effectively the goals of* ***maximum employment****, stable prices and moderate long-term interest rates.*" (emphasis added)
>
> Section 2A
> Federal Reserve Act

[7] Note that since λ^{-1} is increasing in its argument, $\chi(\theta) > 0$ if and only if the argument is increasing in θ. Differentiation gives: $\frac{d}{d\theta}\frac{\kappa[r+\delta+p(\theta)](r+\delta)}{(y-z)q(\theta)-\kappa(r+\delta)} > 0 \iff p'(\theta)[(y-zq(\theta)-\kappa(r+\delta)-(y-z)q'(\theta)(r+\delta+p(\theta)) > 0$, which holds because $p'(\theta) > 0$, $q'(\theta) < 0$, and $(y-zq(\theta)-\kappa(r+\delta) > 0$ from equation (18).

[8] To see this, note that the left hand side of equation (20) is increasing in π, which requires a rise in θ to rebalance since $p'(\theta) > 0$ and $q'(\theta) < 0$.

We begin with a definition of "maximum" employment because it is an explicitly stated goal in the mandate. We then consider the employment level that maximizes output level. Established in [Hosios, 1990], output-maximizing unemployment balances the benefits of creating jobs with the cost of maintaining job vacancies, and is a well-known benchmark in frictional models of unemployment. It also connects, in our view, to the congressional mandate through its statement of the economy's long-run output potential as a means to achieve the listed goals.

Because equilibrium employment does not necessarily align with either of these employment objectives in frictional labor markets, we then analyze the problem of a planner that aims to achieve the employment objectives via monetary policy, weighing potential employment gains against welfare costs of inflation.

4.1 Maximum Employment

Suppose the planner wants to maximize steady-state employment, which is equivalent to minimizing unemployment in our setup. In doing so, the planner must ensure market participation by firms and workers: firms are willing to operate and post vacancies, and unemployed workers are interested in seeking employment:

$$\max_{w} n = (1-u) \quad \text{subject to} \quad W_0 \geq U \text{ and } J_0 \geq V$$

At the steady-state equilibrium, where the unemployment rate is stable, hires equal separations: $m(u,v) = \delta(1-u)$, which gives the following unemployment rate:

$$u(\theta) = \frac{\delta}{\delta + p(\theta)}. \tag{22}$$

Equation (22) implicitly describes the Beveridge Curve relationship between vacancies and unemployment. Because $p'(\theta) > 0$, the steady-state unemployment rate is decreasing in labor market tightness. Therefore the planner's problem is equivalent to maximizing $\theta = v/u$, which requires incentivizing vacancy creation as much as possible. That entails delivering the entire surplus from the match to the firm. The lowest value that will not deter unemployed workers from searching for a job in the first place requires $W_0 = U$, which leads to a corner solution to the planner's problem where the workers' participation constraint binds. The level of maximum attainable employment consistent with the planner's solution is, therefore, well-defined as summarized by the following proposition.

Proposition 4. *The maximum attainable labor market tightness solves*

$$\frac{y-z}{r+\delta} = \frac{\kappa}{q(\theta^{max})}, \tag{23}$$

and the corresponding maximum employment rate is:

$$e^{max} = 1 - u^{min} = \frac{p(\theta^{max})}{\delta + p(\theta^{max})} \tag{24}$$

In the planner's allocation, the firm takes all of the surplus from the match. Note that the maximum employment level only depends on the fundamental parameters of the economy. It does not depend on the inflation rate, implying that it is a stable target regardless of the adopted monetary stance.

By definition, equilibrium employment is below the maximal employment level so long as $\beta > 0$, i.e., workers capture some of the match surplus: $\theta^* < \theta^{max} \iff \beta > 0$. Recall from Proposition 3 that equilibrium employment is increasing in the inflation rate. It is natural to ask: What level of the inflation rate implements maximum employment? Comparison of equations (20) and (23) reveals, however, that equilibrium employment approaches maximum employment only as a limiting case, when the inflation rate approaches infinity $\pi \to \infty$.[9]

4.2 Maximum Output

Following Hosios [1990], our definition of the output-maximizing unemployment rate maximizes (steady-state) resources available for consumption:

$$\max_{u,v} \int_0^\infty e^{-rt}[(1-u)y + uz - v\kappa]$$

subject to

$$\dot{u} = (1-u)\delta - m(u,v),$$

where we substituted $r = \rho$ from the steady-state. The solution to this optimal control problem gives the well-known Hosios optimality condition, which is stated below:

Proposition 5 (Hosios, 1990)**.** *The optimal tightness level is given by:*

$$\frac{y-z}{r+\delta} = \frac{\kappa}{q(\theta^H)} \cdot \frac{r+\delta+\eta_u^m p(\theta^H)}{\eta_v^m (r+\delta)}, \tag{25}$$

where $\eta_u^m = m_u(u,v)u/m$ and $\eta_v^m = m_v(u,v)v/m$ denote the elasticities of the matching function with respect to the numbers of unemployed workers and vacancies.

The optimality of the equilibrium is paramount to understanding the recipes for monetary policy for a planner that seeks to maximize welfare. Comparing equations (25) and (18) reveals the well-known condition for equilibrium optimality in the canonical, unconstrained economy: $\beta = \eta_u^m = (1-\eta_v^m)$. This ensures that the social marginal return to an additional unemployed worker and an additional vacant position are aligned by the private incentives of job seekers and firms.[10]

The Hosios condition is met for all values of u and v when $m(u,v)$ is given by a Cobb-Douglas function with the relevant elasticities set to β and $1-\beta$. In that case, all equilibria are efficient in the unconstrained economy. Because credit constraints do not bind at low levels

[9] Maximum employment in equilibrium requires $\frac{\pi}{r+\delta+\pi+p(\theta)} = 1$ which can only hold if $\pi = \infty$.

[10] The marginal value of an unemployed worker is the product of marginal matches created and the surplus each match generates: $m_u(u,v) \times S$, while the private return equals the product of the job-finding probability and the workers' share of the surplus $m(u,v)/u \times \beta S$. Equating the two gives: $\beta = \eta_u^m$. Similarly, $\beta = (1-\eta_v^m)$ follows from the social value of vacancies and firms' posting incentives: $m_v(u,v) \times S = m(u,v)/v \times (1-\beta)S$.

of inflation, all levels of inflation $\pi < \underline{\pi}$ ensure that the equilibrium is optimal. Therefore a natural strategy for a central bank that aims to maximize welfare is to maintain a low inflation rate.

However, the requirement that the matching function be of Cobb-Douglas form with elasticities coinciding with relative bargaining powers in wage negotiations is an extremely special case. More generally, matching elasticities vary with the numbers of unemployed workers and vacancies. In that case the optimality of equilibrium employment depends on where the elasticities stand vis-à-vis the bargaining powers of workers and firms. Moreover, because inflation is non-neutral only above a certain level, the planner's ability to implement optimal employment whenever the equilibrium deviates from it depends on whether that deviation happens at high or low inflation rates. We go over these cases below.

When $\eta_u^m < \beta$, equilibrium employment is suboptimal. In that case, the planner can raise the employment level by targeting an inflation rate above $\underline{\pi}$. Comparing equations (25) and (20) gives the inflation rate that delivers maximum resources as follows.

Proposition 6. *If $\eta_u^m < \beta$, the output maximizing inflation rate is*

$$\pi^* = \lambda^{-1}\left(\frac{1-\eta_u^m}{\eta_u^m}(r+\delta)\right) > \underline{\pi} \tag{26}$$

Resource maximization requires the planner to follow an active monetary policy by raising π above $\underline{\pi}$. The corresponding inflation target is decreasing in the elasticity of the matching function with respect to unemployed workers. If the numbers of unemployed workers and vacancies are gross complements in the matching process, then a low elasticity is associated with a higher unemployment rate relative to vacant positions. In that case the optimal inflation target rises with the unemployment rate suggesting that optimal policy is countercyclical.

When $\eta_u^m > \beta$, workers retain too low a share of the match surplus and employment is super-optimal: $\theta^H < \theta^* < \theta^{max}$. Because the inflation rate is already low in unconstrained equilibria, an active monetary policy which raises the inflation rate high enough to stimulate employment would worsen welfare by raising employment further above optimality. Therefore the optimal policy calls for a low inflation target $\pi^* < \underline{\pi}$. If inflation happens to be high enough that the economy is constrained when $\eta_u^m > \beta$, then policy could improve welfare by lowering the inflation target.

We now turn to the monetary policy and analyze the potential welfare tradeoffs between employment and price stability objectives.

4.3 Monetary Authority's Problem

The objective of the monetary authority is to maximize the representative household's welfare. The set of constraints on policy choices include a government balanced budget and equilibrium conditions for agents' choices. Economically, the most natural policy instrument of the monetary authority in our environment is the growth rate of the nominal money supply, $\tau(t) \equiv \dot{N}(t)/N(t)$, or equivalently the nominal interest rate that implements the same

monetary allocation.[11] If money growth is the instrument, τ is chosen by the monetary authority, while inflation π, nominal interest rate, and the remaining allocation variables are endogenously determined given the firm's and worker's optimal choices and the evolution of real balances, $\dot{n} = (\tau - \pi)n$. That said, we can write the monetary authority's policy problem through its primal representation in which the authority chooses $\{c, n, \pi, \theta\}$ subject to all private-equilibrium and feasibility restrictions. Given the labor market equilibrium conditions and household optimality conditions, π is ultimately the only remaining free choice. Hence, the control vector (c, n, π, θ) in the global Hamiltonian should be viewed as a convenient parameterization of the implementable allocation, while the monetary authority's primitive intervention is the monetary policy required to implement that allocation.

As a result, the global monetary authority problem is:

$$\max_{\{c,n,\pi,\theta\}_{t\geq 0}} \int_0^\infty e^{-\rho t} U(c_t, n_t)\, dt$$

subject to:

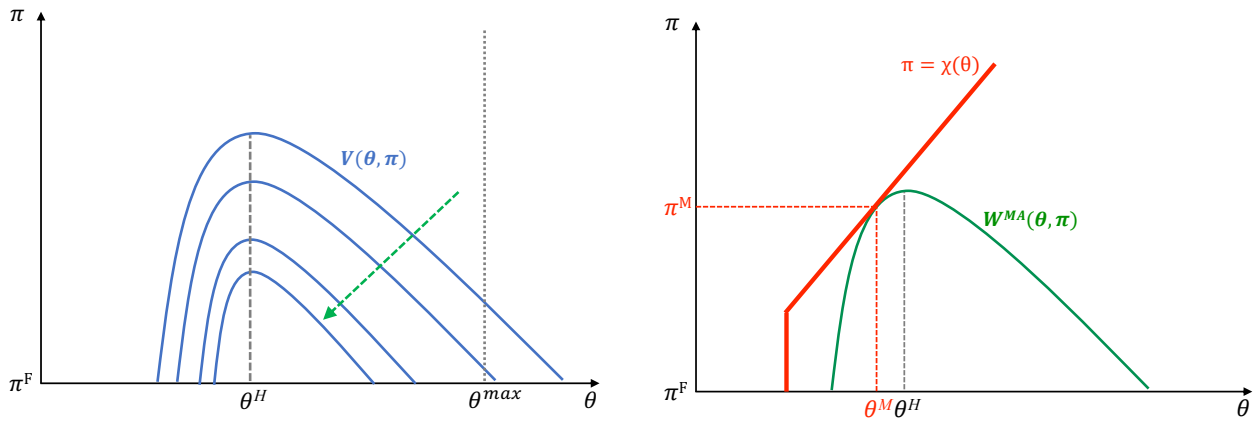
$$\begin{cases} \dot{\vartheta} = (\rho + \pi)\vartheta - U_n & (EQ') \\ \vartheta - U_c = 0 & (SR) \\ y - u(y - z + \kappa\theta) - c = 0 & (RC) \\ \dot{u} = \delta(1-u) - up(\theta) & (LM) \\ \dot{J}_0 = (\widehat{r} + \delta + \lambda(\pi))J_0 - (y - w) - \lambda(\pi)S & (GJ) \\ \dot{S} = [\widehat{r} + \delta + p(\theta)]S - p(\theta)J_0 - (y - z) & (GS) \\ \frac{\kappa}{q(\theta)} - J_0 = 0 & (FE) \\ J_0 - (1-\beta)S - \mu_w = 0 & (NB) \\ \mu_w \geq 0, \qquad (y - w) \geq 0, \qquad \mu_w(y - w) = 0 & (CS) \end{cases} \tag{27}$$

Constraint (EQ') combines household's intertemporal and intra-temporal optimality conditions.[12] To simplify the problem, we augment the state space by introducing $\vartheta(t)$ as an independent state and impose the identity $\vartheta(t) = U_c(c(t), n(t))$ as a path equality constraint (SR).[13] Constraint (RC) is the resource constraint, that shows that consumption equals output, including what is produced by employed workers – $(1-u)y$, what is produced in home production – uz, and net of vacancy costs – $\kappa\theta$. Constraint (LM) imposes the restrictions given labor market's frictional nature and it governs transitional deviations from the steady-state unemployment rate (off the Beveridge Curve). Similarly, (GJ) and (GS) are the dynamic value functions for a newly created job match (J_0) and its match surplus (S). Constraint (FE) imposes free entry condition. Finally, (NB) is the Nash Bargaining

[11] While our formulation is a literal interpretation of the stated policy instrument, *"long run growth of the monetary and credit aggregates"*, the problem can equivalently be stated as a choice of the nominal interest rate, which is closer to Fed's current practice.

[12] The intertemporal optimality condition is given by the Euler equation $\frac{\dot{U}_c}{U_c} = \rho - r$. The intra-temporal optimality condition is given by $\frac{U_n}{U_c} = r + \pi$.

[13] In the augmented Hamiltonian, ϑ is an independent argument, so first-order conditions take partial derivatives holding ϑ fixed; the dependence of ϑ on (c, n) is captured solely through the constraint term $\mu_\vartheta(U_c(c(t), n(t)) - s(t))$, where μ_ϑ is a multiplier. For more details, see Kamien and Schwartz [2012], Part II, Section 2, pages 125–127.



(a) Indifference Curves and Planner's Allocation (b) Monetary Authority's Constrained Allocation

Figure 2: Monetary Authority's Policy Problem

condition, that includes a multiplier for the credit constraints (μ_w). (CS) is the complementary slackness condition on the credit constraint. We present details about the Monetary Authority Problem and optimal solution in Appendix section B.3.

We start by analyzing the first-best allocation, which we dub as the social planner's problem. Differently from the monetary authority, the planner is not constrained by the households' and firms' optimal choices. In other words, the only constraints faced by the planner are (LM) and (RC). As a result, the mechanism through which inflation impacts vacancy creation – by incentivizing firms to post vacancies – is severed. As a result, the planner's optimal choice is to separately optimize π and θ. Recall that the labor market, which effectively determines the resources available for consumption, can be analyzed in isolation of the household's choices money balances. Consequently, the social planner sets $\theta = \theta^H$ to maximize available resources (Hosios condition) and $\pi = \pi_{min}$ to minimize the welfare costs of inflation (Friedman rule).

First-best allocation is illustrated in Figure 2-a. The blue curves show the planner's indifference curves between inflation and activity, as measured by labor market tightness. The preferences are satiated around (π^F, θ^H). Lemma 2 formalizes this result.

Lemma 2. *In this economy, a planner would optimally choose $\theta = \theta^H$ and $\pi = \pi_{min}$, where θ^H is given by the Hosios condition and π_{min} is given by the Friedman rule.*

Moving to the monetary authority problem, let's initially consider the case in which $\chi'(\theta) \leq 0$, where inflation is neutral due to unconstrained wage setting. In this case, because inflationary policy fails to raise economic activity, implementing the Friedman rule $\pi = \pi_{min}$ maximizes welfare. Under the Friedman rule $\pi_{min} < \underline{\pi}$, money is neutral and the monetary authority has no influence on vacancy creation. As a result, $\theta = \theta^*$. The following lemma summarizes this result.

Lemma 3. *If $\chi'(\theta) \leq 0$, then $\pi^M = \pi_{min}$ and $\theta^M = \theta^*$.*

By contrast, in our model, given equation (21), we have $\chi'(\theta) \geq 0$, with strict inequality for $\theta > \theta^*$. This presents the planner with the opportunity to raise labor market tightness, and, thereby, employment and output. The cost of doing so is the higher seignorage tax on real money balances associated with an inflationary environment. Because affecting labor market outcomes requires raising inflation at least above $\underline{\pi} > 0$, any gain in consumption-related welfare has to surpass a fixed minimal amount to cover the cost of raising inflation from the first best, π_F, to $\underline{\pi}$. Whether it does depends on how costly inflation is and how effective inflation is in raising equilibrium employment, and, ultimately, consumption. This suggests that welfare maximizing policy is state dependent, and, therefore, does not necessarily have a dual target.

In economies where a dual-target is welfare maximizing, the optimal inflation rate balances marginal consumption gains from inflationary policy, mediated via higher employment, against its marginal cost associated with higher seignorage tax on money holdings:

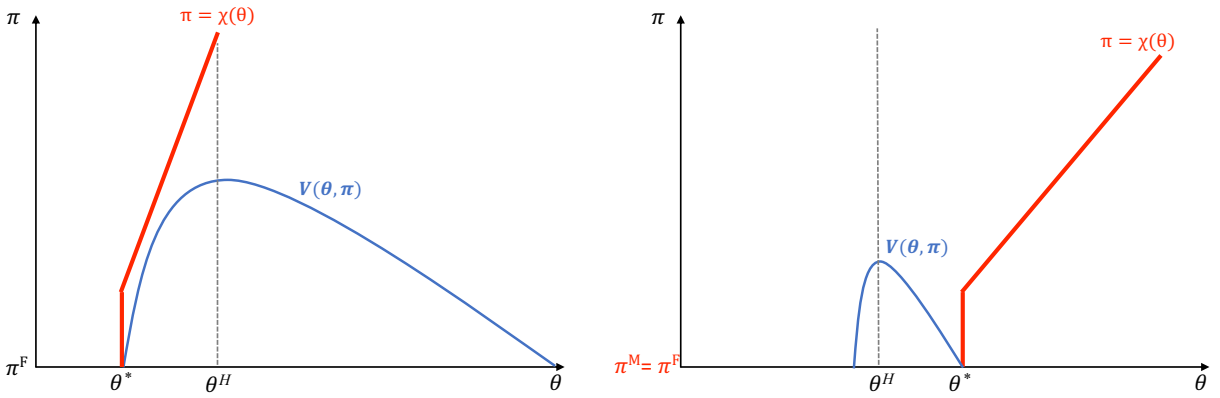
$$\chi'(\theta) = -\frac{W_\theta^{MA}}{W_\pi^{MA}}, \tag{28}$$

where W_θ^{MA}/W_π^{MA} is the planner's marginal rate of substitution between inflation and labor market tightness. Figure 2-b shows such an optimum. The green curve, $W^{MA}(\pi, \theta)$, represents the level set of the authority's welfare criterion over (π, θ) pairs.[14] In the case shown, $\theta^* < \theta^H$, so that raising labor market tightness raises output. With the depicted level curves, the monetary authority finds it desirable to do so, and the optimal policy involves dual policy objectives: $u(\theta^M)$ and $\pi^M > \underline{\pi}$. Note that the optimal target for tightness falls short of the output-maximizing Hosios benchmark: $u(\theta^*) > u(\theta^M) > u(\theta^H)$. This is because the marginal gain from raising employment further does not compensate for the cost of additional inflation.

Figure 3 shows two situations where it is best to focus solely on the inflation objective. Panel (a) depicts a case where θ^* is sub-optimally low $< \theta^H$, but the implementable Phillips curve is too steep for the welfare gains from higher employment to justify the associated increase in inflation. Panel (b) shows a case where the $\theta^* > \theta^H$, so the low-inflation allocation already implies sufficiently strong labor-market performance. In both cases, the authority optimally places no independent weight on raising employment and instead focuses solely on the inflation objective.

These cases suggest that optimal policy does not necessarily have a dual objective at all times. Whether an employment target is active depends on the state of the labor market, in particular where equilibrium unemployment is situated relative to its output-maximizing level at any given time, and on the cost of inflation associated with bringing unemployment closer to that level.

[14] $W^{MA}(\pi, \theta)$ is a graphical summary of the full dynamic Ramsey problem after the remaining controls and states have been solved out. Details presented in Appendix Section B.4.



(a) Steep (or Vertical) Long-run Phillips Curve (b) Low Inflation - Low Unemployment Economy

Figure 3: Cases for Inflation-Focused Monetary Policy

Notes.— Figure displays two situations under which the optimal policy only targets a low level of inflation (single-mandate).

5 Quantitative Analysis

To illustrate the quantitative implications of our findings, we calibrate the model economy to the pre-pandemic U.S. economy. We follow Lucas [2000] in setting the utility function as:

$$U(c,n) = \frac{u_0}{1-\sigma}\left[c\varphi\left(\frac{n}{c}\right)\right]^{1-\sigma}$$

The household's optimality conditions yield the money demand function $n/c = N_0(r+\pi)^{-\eta_m}$. Following Lucas [2000], we set $N_0 = 0.27$ and $\eta_m = 0.5$ to approximate the empirical money demand curve for the US.

We assume that the matching function is given by:

$$m(u,v) = A \times \frac{uv}{u+v},$$

which displays varying elasticities with respect to vacancies and unemployed: when the labor market is tight (v/u high), fewer matches are generated by additional vacancies.[15]

We calibrate the labor market at monthly frequency and set the following parameters externally. Labor productivity is normalized to $y = 1$ and the consumption equivalent value of unemployment is set to $z = 0.7$. The monthly separation rate is $\delta = 0.024$, the average value during our sample period and the real interest rate is calibrated to an annual value of 4 percent. We set the monthly renegotiation rate $\phi = 0.19$, which implies 90 percent of workers renegotiate their contracts with their employer at least once in a given year. Table 1 summarizes the parameters and the corresponding targets or sources.

We calibrate the remaining parameters internally. To position the economy on the empirical Beveridge curve, we target an unemployment rate of 5 percent when $v/u = 0.75$. To

[15] We deviate from the commonly used Cobb-Douglas form, which provides a poor fit to the empirical Beveridge Curve during tight labor markets.

Parameter	Value	Target	Value
Labor productivity (y)	1.0	Normalized	—
Unemp. utility (z)	0.7	Shimer [2005]	0.7
Discount rate (%)	0.34	Real interest rate (% annual)	4.0
Money demand level (A_m)	0.18	Mean M1/GDP [Lucas, 2000]	0.27
Money demand elasticity (η)	0.5	M1/GDP by GDP[Lucas, 2000]	0.5
Renegotiation rate (ϕ)	0.19	Annual probability of ≥ 1 (%)	90.0

Table 1: Externally Calibrated Parameters

capture the empirical relationship between inflation and real wage depreciation, we target a ratio of 0.45. We target an annual inflation rate of 1.7 percent. Finally, we target the slope of the empirical relationship between the inflation rate and labor market tightness.

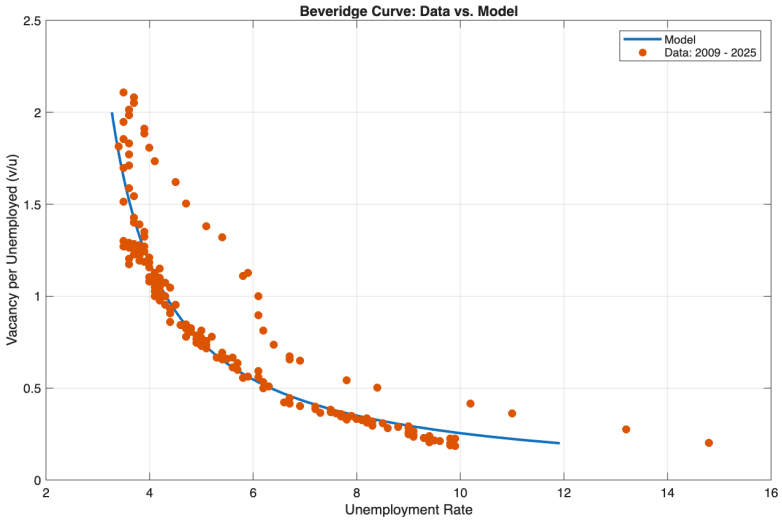


Figure 4: Beveridge Curve

Notes.– The model is calibrated to generate 5 percent unemployment at vacancy-to-unemployment ratio of 0.75. Monthly data values are taken from the BLS.

The calibrated model successfully replicates key features of the U.S. labor market and the empirical relationship between inflation and employment. The model generates a downward-sloping Beveridge curve consistent with the data, and a non-linear Phillips curve that is flat at low inflation rates and negatively sloped at higher rates.

The quantitative results indicate that maximum employment (minimum unemployment rate of approximately 2.2%) would require prohibitively high inflation rates approaching infinity. In contrast, output-maximizing employment level (Hosios condition) corresponds to an unemployment rate of approximately 3.3%, which could be implemented with an inflation rate of 4.9%. When accounting for the welfare costs of inflation through the monetary authority's problem, the optimal (active) policy target involves a somewhat higher unem-

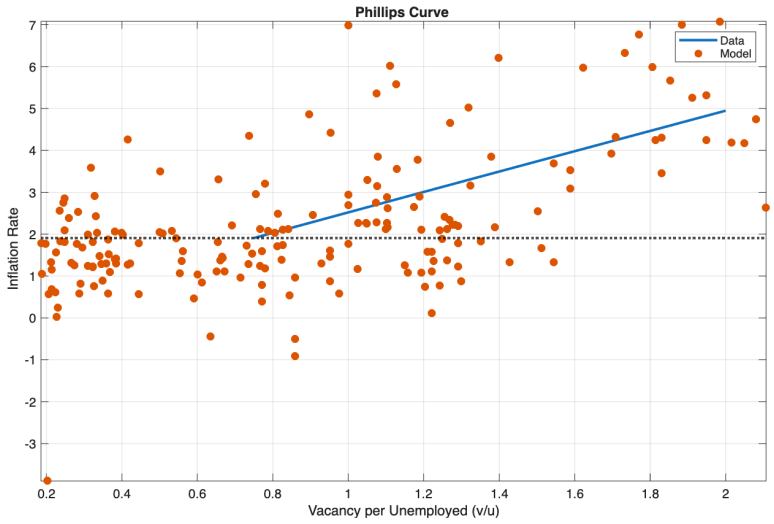


Figure 5: Inflation Rate and Labor Market Tightness

Notes.– Monthly data values for PCE inflation are taken from the FRED.

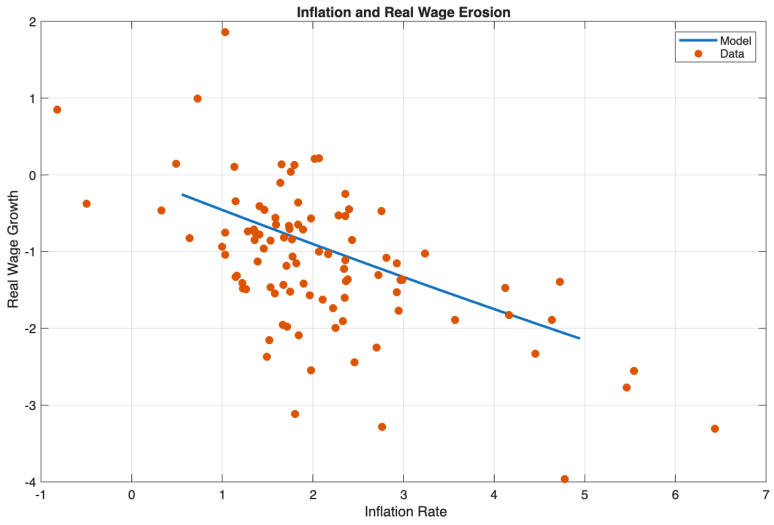


Figure 6: Inflation and Real Wage Growth

Notes.– Real wage growth reflects annualized quarterly growth rate in the employment cost index net of PCE inflation and average productivity growth of 2 percent during the sample period.

ployment rate of 3.9% and a more moderate inflation rate of 3.3%. However, the active target does not provide the maximal welfare in our calibrated benchmark economy. Actively targeting an inflation rate of 3.3% to bring unemployment down to 3.9% yields roughly the same welfare as maintaining an inflation rate of around 1%, which leaves the unemployment rate at its 5% benchmark. Lower inflation rate targets only improve upon that welfare, and

are, therefore, strictly preferable from a welfare perspective.

These findings illustrate the key trade-offs in setting monetary policy under the dual mandate. While it is theoretically possible to push unemployment below its natural rate through higher inflation, the welfare costs of doing so quickly become prohibitive. The optimal policy balances the benefits of lower unemployment against the costs of higher inflation, resulting in targets that are more moderate than those that would emerge from focusing solely on either objective in isolation.

5.1 Shifting Fundamentals and the Duality of Policy Objectives

The preceding analysis establishes that the optimal policy need not carry an active employment objective: whether it does depends on how much employment inflation can buy and on how costly that inflation is. In this subsection, we illustrate how the monetary authority's decision to switch between a single (inflation-focused) mandate and a dual (accommodative) mandate depends on the labor market fundamentals.

We consider four labor market fundamentals in the model: the separation rate (δ), workers' bargaining power (β), productivity (y), and the discount rate (ρ). We vary each individually and keep all other parameters at their benchmark values. For each labor market fundamental, we compare three stationary policies. The first is the *accommodative* policy π^M of Section 4, i.e., the rest point of the monetary authority's problem on the credit-constrained branch, with associated tightness θ^M solving (28). The second is the *Friedman rule* $\pi^F = \pi_{min} = -r$, which sets the nominal interest rate to zero. Because the optimality rule of the Friedman rule is specific to the Sidrausky framework, we also consider a *zero-inflation target*, $\pi = 0$. This captures models where the cost of inflation is linked to price dispersion, for instance, and negative inflation rates are therefore just as costly as positive ones. The zero target is arguably the empirically relevant alternative to accommodation since no central bank contemplates the Friedman rule, but many aim to operate near price stability. Because $\underline{\pi} > 0$, a zero target always leaves the credit constraint slack, so it implements the decentralized allocation θ^* while still levying a positive inflation tax at $i = r$.

Ranking the three policies requires only the steady-state consumption-equivalent criterion. Under the homothetic specification of Section 5, substituting money demand into the utility function gives $U = G\big(c\,\Psi(i)\big)$ for an increasing G, where Ψ is decreasing in the nominal rate. Normalizing $\Psi(0) = 1$ and following Lucas [2000], define the welfare loss, $\mathcal{L}(i)$, from a nominal rate i as the proportional consumption compensation that restores $i = 0$ welfare,

$$\mathcal{L}(i) \equiv \frac{1}{\Psi(i)} - 1 \;\geq\; 0, \qquad \mathcal{L}(0) = 0, \quad \mathcal{L}' > 0, \tag{29}$$

which for $\eta_m = \frac{1}{2}$ takes the closed form $\mathcal{L}(i) = N_0\sqrt{i}$. The three policies are then ranked by consumption net of the inflation tax,

$$\tilde{c} \;\equiv\; \frac{c}{1+\mathcal{L}(i)}, \qquad \tilde{c}^{\,M} = \frac{I(\theta^M)}{1+\mathcal{L}(r+\pi^M)}, \quad \tilde{c}^{\,F} = I(\theta^*), \quad \tilde{c}^{\,0} = \frac{I(\theta^*)}{1+\mathcal{L}(r)}. \tag{30}$$

where $\tilde{c}$, $\tilde{c}^M$, and $\tilde{c}^0$ gives us the consumption net of the inflation tax and zero-inflation,

accommodative, and Friedman rule monetary policies, respectively.[16]

The welfare cost of inflation is governed by the scale of money demand: with $\eta_m = \frac{1}{2}$, $\mathcal{L}(i) = N_0\sqrt{i}$ is proportional to N_0 at every nominal rate. We therefore index how expensive inflation is by

$$\zeta \equiv \frac{N_0}{N_0^{\text{bench}}}, \tag{31}$$

so that $\zeta = 1$ is our calibration, $\zeta = \frac{1}{2}$ an economy in which inflation is half as costly, and so on. Since ζ scales $\mathcal{L}(\cdot)$ uniformly, it is equivalent to measuring the cost of inflation at any fixed reference rate relative to its calibrated value; at $\zeta = 1$ the tax levied by a zero-inflation target is $\mathcal{L}(r) = 1.02$ percent of consumption. Setting $\tilde{c}^M = \tilde{c}^F$ and $\tilde{c}^M = \tilde{c}^0$ defines two loci in the plane of ζ against a fundamental. [17]

Figure 7 draws both loci against each of the four fundamentals. Each row is one exercise. The left panel puts the parameter on the vertical axis, whereas the right panel replaces it with the associated steady-state unemployment rate $u(\theta^*)$. In each panel, the shading darkens with the welfare rank of accommodative policy, with the darkest shade indicating its superiority to the Friedman rule, π^F, medium shade its superiority to zero-inflation target, and the lightest showing its sub-optimality to both. The dot shows the benchmark calibration.

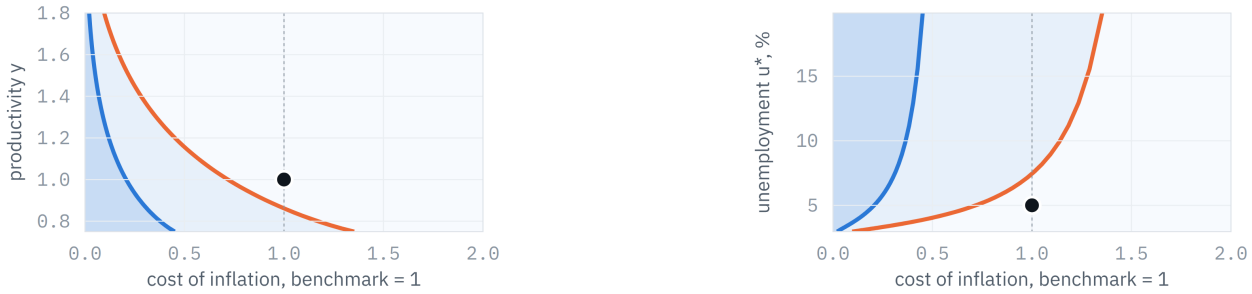
Lower productivity works in the expected direction. With z fixed, a smaller y shrinks the match surplus, lowers θ^* and raises unemployment. If productivity drops sufficiently, then active inflationary policy becomes preferable to the zero-inflation target. At the benchmark cost of inflation, this requires a productivity drop that pushes the unemployment rate above 7.5 percent. Lower the cost of inflation, the sooner the accommodative policy is triggered. The Friedman locus, however, never crosses at the benchmark. Inflation costs would have to be less than half its calibrated value before accommodation dominated price stability in the Friedman sense.

Not all surges in the unemployment rate call for monetary action. Panel (b) compares policies when a higher job separation rate, rather than a drop in productivity, drives the unemployment rate up. Bear in mind that these are permanent (or at least highly persistent) changes. Raising the separation rate from 7 to 57 percent a year drives $u(\theta^*)$ from 1.3 to 14.1 percent, yet maintaining a low inflation target is still preferable to accommodative policy. This reflects two conflicting effects on the policy maker's decision. On the one hand, the higher unemployment rate strengthens the case for monetary intervention. On the other hand, higher δ means jobs are short-lived, reducing the welfare gains to creating them. The positive slope of the indifference loci indicate that a higher δ makes accommodation more attractive on the margin, but because the loci are steeper, it takes a much larger surge in the unemployment rate or a lower inflation cost to justify monetary intervention.

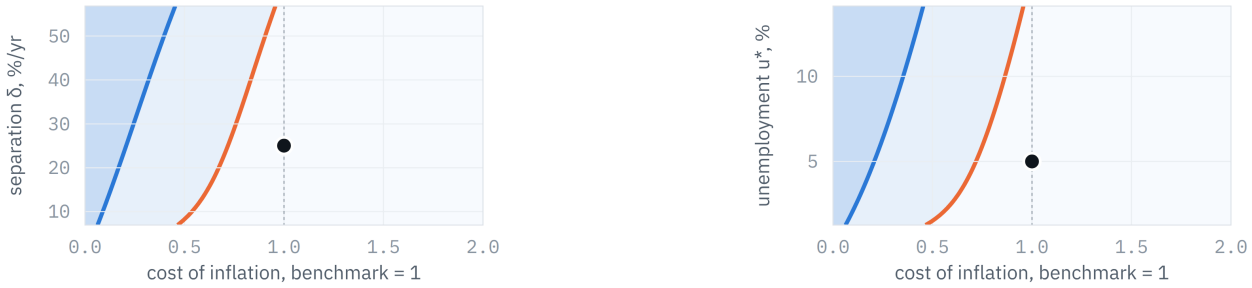
Panel (c) varies workers' bargaining power parameter. A higher β leaves workers with more of the match surplus, depresses θ^* further below the Hosios optimal, θ^H, and simulta-

[16] Because G is monotone, the ranking is invariant to the curvature of U. In particular, the ranking does not depend on σ. Because $\mathcal{L}(0) = 0 < \mathcal{L}(r)$, the Friedman rule always weakly dominates the zero target, so there are exactly three possible rankings, according to whether π^M places first, second or third.

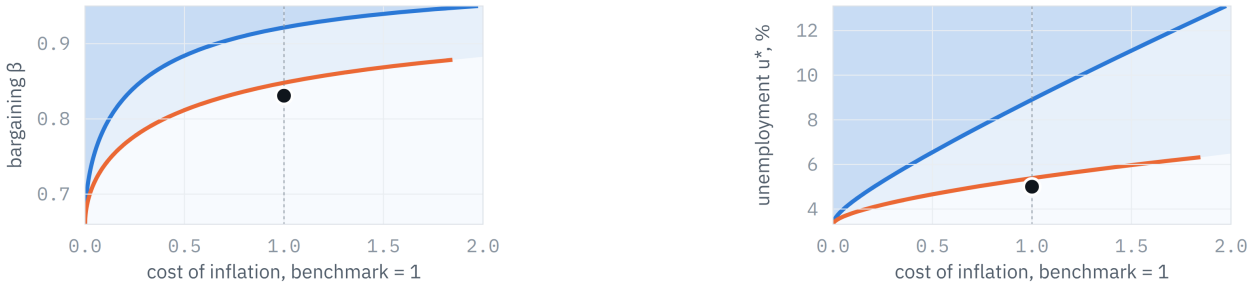
[17] Note that θ^M and π^M are themselves functions of ζ – a cheaper money demand raises the accommodative inflation rate, from $\pi^M = 3.28$ percent at $\zeta = 1$ to 4.97 percent as $\zeta \to 0$ – so each locus is solved jointly with (28) rather than read off a fixed branch optimum.



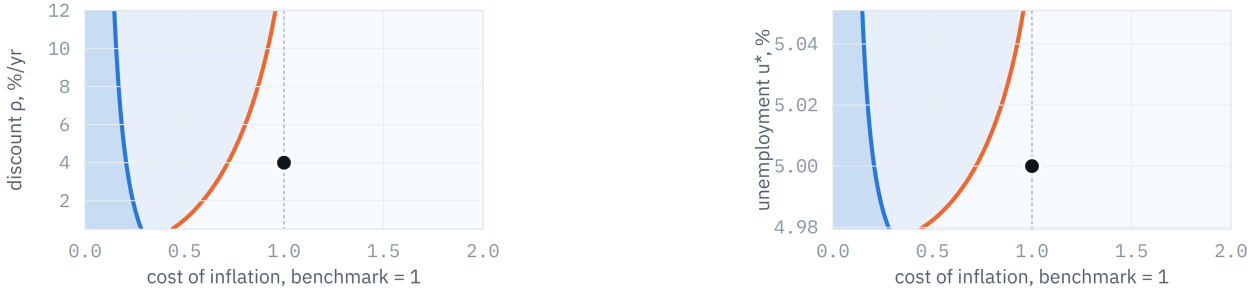
(a) Productivity y



(b) Separation rate δ



(c) Bargaining weight β



(d) Discount rate ρ

Figure 7: Fundamentals and the Duality of Policy Objectives

Notes.— Each row varies one fundamental, holding the others at the benchmark; left panels put the parameter on the vertical axis, right panels the implied $u(\theta^*)$. The horizontal axis is the cost of inflation relative to the benchmark, $\zeta = N_0/N_0^{\text{bench}}$. Blue locus: π^M equivalent to the Friedman rule π^F. Orange locus: π^M equivalent to a zero-inflation target. Shading darkens with the rank of π^M – darkest $\pi^M \succ \pi^F \succ 0\%$, then $\pi^F \succ \pi^M \succ 0\%$, lightest $\pi^F \succ 0\% \succ \pi^M$. Dashed vertical: calibrated cost of inflation; dot: benchmark economy.

neously lowers $\underline{\pi}$, so monetary policy both has more to fix and becomes cheaper to deploy. Because the calibrated economy is close to the boundary at which an active employment objective becomes preferable to zero-inflation policy, even small increases in the unemployment rate call for intervention when they are caused by an increase in β.

Finally, Panel (d) shows changes in the discount rate, representing either shifting time preferences or long-run growth rates.

A higher discount rate raises the nominal interest rate under both accommodation and the zero-inflation target, but leaves the Friedman rule at $i = 0$. Accommodation therefore loses ground to the Friedman rule, whose inflation tax does not rise, while it gains on the zero target, because the concavity of $\mathcal{L}$ shrinks the incremental tax $\mathcal{L}(r + \pi^M) - \mathcal{L}(r)$. This accounts for the opposite slopes of the two loci in Panel (d), though the effect is unlikely to be quantitatively relevant, since the required change in the discount rate is large — in part because the unemployment rate is insensitive to it.

A higher discount rate raises the welfare weight on current consumption and the welfare-maximization is more willing to entertain an accommodative, indicated by the positive slope of the orange loci, which depicts indifference points between zero-inflation target and interventionist approach. Quantitatively, this is unlikely to be relevant, however, since the change in the discount rate to justify intervention is substantial in part because the unemployment rate is not sensitive to discount rate changes. By contrast, Friedman rule becomes even more appealing under higher discount rates, because zero nominal interest rate implies an even more negative inflation rate, $\pi^F = -\rho$. Whether impatience favors a dual mandate therefore depends entirely on which alternative one holds it against.

The right-hand panels illustrate the central point of our exercise: any one measure of labor market conditions is not a sufficient statistic to guide monetary policy. Panels (a) and (b) call for different responses to higher unemployment rates, and Panel (d) shows potentially varying policy approaches even when the unemployment rate is stable. What makes an active employment objective desirable is not the level of unemployment but its source.

It should also be noted that in our calibrated economy, the equilibrium labor market tightness is inferior to its optimal level, which implies that active inflationary policy is an option (see Figure 2b). This remains the case in all parameter combinations shown in Figure 7. This may not generalize to other economies if, for instance, fundamentals already deliver a tight labor market relative to what the planner would choose.

6 Conclusion

This paper develops a framework for the Federal Reserve's employment mandate by formalizing the concepts of maximum, optimal, and equilibrium employment within a frictional labor market model. We then characterize how a welfare-maximizing monetary authority trades off employment gains against the costs of inflation. Our analysis yields several key insights.

First, we show that maximum employment—defined as the lowest feasible unemployment rate consistent with voluntary labor market participation—can only be achieved by creating many low-paying jobs where employers capture the entire match surplus. Implementing this objective requires prohibitively high inflation rates, making it an impractical policy target.

Second, we demonstrate that optimal employment, which balances the costs and benefits of job creation, can be implemented with moderate inflation targets that depend systematically on labor market fundamentals. The optimal inflation rate is lower when matching efficiency is high, separation rates are low, or real interest rates are low, or real interest rates are low. Once the welfare costs of inflation are taken into account, however, the monetary authority may stop short of this employment target or prefer a low-inflation allocation altogether.

Third, our framework provides a theoretical foundation for asymmetric monetary policy responses to labor market conditions. When the welfare gains from raising employment are small relative to the inflation costs, policy should focus on price stability. In other environments, moderate inflation may be desirable because it raises employment enough to justify those costs. This asymmetry arises from labor market frictions rather than from the zero lower bound or asymmetric policy-error costs. In our benchmark calibration, equilibrium employment is below its optimal level, yet low inflation still dominates active employment policy in welfare terms.

Finally, our analysis suggests that optimal policy targets are endogenous to labor market conditions and should be updated as these fundamentals evolve over time. The quantitative exercises reinforce this point. Similar unemployment outcomes can imply different policy prescriptions depending on their source. Unemployment alone is therefore not a sufficient statistic for the appropriate monetary-policy stance.

References

Hassan Afrouzi, Andres Blanco, Andres Drenik, and Erik Hurst. A Theory of How Workers Keep up with Inflation*. *The Quarterly Journal of Economics*, page qjag007, January 2026. ISSN 0033-5533. doi: 10.1093/qje/qjag007. URL https://doi.org/10.1093/qje/qjag007.

George A. Akerlof, William T. Dickens, George L. Perry, Robert J. Gordon, and N. Gregory Mankiw. The Macroeconomics of Low Inflation. *Brookings Papers on Economic Activity*, 1996(1):1–76, 1996. ISSN 0007-2303. doi: 10.2307/2534646. URL https://www.jstor.org/stable/2534646.

David Andolfatto. Business Cycles and Labor-Market Search. *The American Economic Review*, 86(1):112–132, 1996. ISSN 0002-8282. URL https://www.jstor.org/stable/2118258.

Robert J. Barro. Long-term contracting, sticky prices, and monetary policy. *Journal of Monetary Economics*, 3(3):305–316, 1977. URL https://ideas.repec.org//a/eee/moneco/v3y1977i3p305-316.html.

Aleksander Berentsen, Guillaume Rocheteau, and Shouyong Shi. Friedman Meets Hosios: Efficiency in Search Models of Money. *The Economic Journal*, 117(516):174–195, 2007. ISSN 0013-0133. URL https://www.jstor.org/stable/4121913.

Aleksander Berentsen, Guido Menzio, and Randall Wright. Inflation and Unemployment in the Long Run. *American Economic Review*, 101(1):371–398, February 2011. ISSN 0002-8282. doi: 10.1257/aer.101.1.371. URL https://www.aeaweb.org/articles?id=10.1257/aer.101.1.371.

Ben S. Bernanke, Michael T. Kiley, and John M. Roberts. Monetary Policy Strategies for a Low-Rate Environment. *AEA Papers and Proceedings*, 109:421–426, May 2019. ISSN 2574-0768. doi: 10.1257/pandp.20191082. URL https://www.aeaweb.org/articles?id=10.1257/pandp.20191082.

Zachary Bethune, Michael Choi, Guillaume Rocheteau, and Sébastien Lotz. Inflation and Unemployment in the Long Run Revisited, June 2024. URL https://papers.ssrn.com/abstract=4873898.

Olivier Blanchard and Jordi Galí. Real Wage Rigidities and the New Keynesian Model. *Journal of Money, Credit and Banking*, 39:35–65, 2007. ISSN 0022-2879. URL https://www.jstor.org/stable/4123055.

Olivier Coibion, Yuriy Gorodnichenko, and Johannes Wieland. The Optimal Inflation Rate in New Keynesian Models: Should Central Banks Raise Their Inflation Targets in Light of the Zero Lower Bound? *The Review of Economic Studies*, 79(4):1371–1406, 2012. URL https://ideas.repec.org//a/oup/restud/v79y2012i4p1371-1406.html.

Peter A. Diamond. Wage Determination and Efficiency in Search Equilibrium. *The Review of Economic Studies*, 49(2):217–227, April 1982. ISSN 0034-6527. doi: 10.2307/2297271. URL https://doi.org/10.2307/2297271.

Stefano Eusepi and Ayşegül Şahin. Assessing Maximum Employment: A Flow-Based Approach. *NBER Working Papers*, May 2025. URL https://ideas.repec.org//p/nbr/nberwo/33878.html. Number: 33878.

Milton Friedman. *The Optimum Quantity of Money, and Other Essays.* Aldine De Gruyter, Chicago, 1969. ISBN 978-0-202-06030-9.

Chao Gu, Janet Hua Jiang, and Liang Wang. Credit conditions, inflation, and unemployment. *Journal of Economic Theory*, 230:106081, December 2025. ISSN 0022-0531. doi: 10.1016/j.jet.2025.106081. URL https://www.sciencedirect.com/science/article/pii/S0022053125001279.

Joao Guerreiro, Jonathon Hazell, Chen Lian, and Christina Patterson. Why Do Workers Dislike Inflation? Wage Erosion and Conflict Costs, September 2024. URL https://www.nber.org/papers/w32956.

Ina Hajdini, Edward S. Knotek, John Leer, Mathieu Pedemonte, Robert W. Rich, and Raphael Schoenle. Low Passthrough from Inflation Expectations to Income Growth Expectations: Why People Dislike Inflation. Technical Report 22-21R, Federal Reserve Bank of Cleveland, March 2023. URL https://fedinprint.org/item/fedcwq/94384/95864.

Bart Hobijn and Ayşegül Şahin. Maximum Employment and the Participation Cycle. *NBER Working Papers*, September 2021. URL https://ideas.repec.org//p/nbr/nberwo/29222.html. Number: 29222.

Arthur J. Hosios. On The Efficiency of Matching and Related Models of Search and Unemployment. *The Review of Economic Studies*, 57(2):279–298, April 1990. ISSN 0034-6527. doi: 10.2307/2297382. URL https://doi.org/10.2307/2297382.

Morton I. Kamien and Nancy Lou Schwartz. *Dynamic Optimization: The Calculus of Variations and Optimal Control in Economics and Management*. Courier Corporation, November 2012. ISBN 978-0-486-48856-1. Google-Books-ID: 0IoGUn8wjDQC.

Robert G. King and Alexander L. Wolman. Inflation Targeting in a St. Louis Model of the 21st Century, March 1996. URL https://www.nber.org/papers/w5507.

Robert E. Lucas. Inflation and Welfare. *Econometrica*, 68(2):247–274, 2000. ISSN 0012-9682. URL https://www.jstor.org/stable/2999427.

Pascal Michaillat and Emmanuel Saez. u* = sqrt(uv): The Full-Employment Rate of Unemployment in the United States. *Brookings Papers on Economic Activity*, 55(2 (Fall)):323–424, 2024. URL https://ideas.repec.org//a/bin/bpeajo/v55y2024i2024-02p323-424.html.

Dale T. Mortensen. The Matching Process as a Noncooperative Bargaining Game. *NBER Chapters*, pages 233–258, 1982. URL https://ideas.repec.org//h/nbr/nberch/4437.html.

Edmund S. Phelps. Inflation in the Theory of Public Finance. *The Swedish Journal of Economics*, 75(1):67–82, 1973. ISSN 0039-7318. doi: 10.2307/3439275. URL https://www.jstor.org/stable/3439275.

Laura Pilossoph and Jane M. Ryngaert. Job Search, Wages, and Inflation, October 2024. URL https://www.nber.org/papers/w33042.

Laura Pilossoph, Jane M. Ryngaert, and Jesse J. Wedewer. The Search Costs of Inflation in the Labor Market, June 2026. URL https://www.nber.org/papers/w35369.

Christopher A. Pissarides. Short-Run Equilibrium Dynamics of Unemployment, Vacancies, and Real Wages. *The American Economic Review*, 75(4):676–690, 1985. ISSN 0002-8282. URL https://www.jstor.org/stable/1821347.

Christopher A. Pissarides. *Equilibrium Unemployment Theory, second edition*. MIT Press, March 2000. ISBN 978-0-262-26406-8.

Stephanie Schmitt-Grohe and Martin Uribe. Optimal fiscal and monetary policy under sticky prices. *Journal of Economic Theory*, 114(2):198–230, February 2004. ISSN 0022-0531. doi: 10.1016/S0022-0531(03)00111-X. URL https://www.sciencedirect.com/science/article/pii/S002205310300111X.

Robert Shimer. The Cyclical Behavior of Equilibrium Unemployment and Vacancies. *American Economic Review*, 95(1):25–49, March 2005. ISSN 0002-8282. doi: 10.1257/0002828053828572. URL https://www.aeaweb.org/articles?id=10.1257/0002828053828572.

Miguel Sidrauski. Inflation and Economic Growth. *Journal of Political Economy*, 75(6): 796–810, 1967. ISSN 0022-3808. URL https://www.jstor.org/stable/1829572.

Miguel Sidrauski. Rational Choice and Patterns of Growth. *Journal of Political Economy*, 77(4):575–585, 1969. ISSN 0022-3808. URL https://www.jstor.org/stable/1829321.

Lawrence Summers. Panel Discussion: Price Stability: How Should Long-Term Monetary Policy Be Determined? *Journal of Money, Credit and Banking*, 23(3):625–631, 1991. ISSN 0022-2879. doi: 10.2307/1992697. URL https://www.jstor.org/stable/1992697.

James Tobin. Friedman's Theoretical Framework. *Journal of Political Economy*, 80(5): 852–863, 1972. ISSN 0022-3808. URL https://www.jstor.org/stable/1830415.

A Mathematical Appendix

This appendix contains the proofs for the propositions and derivations for some of the equations in the main text. We first list the equations that describe the model for convenience. The model is summarized by the following value functions:

$$\begin{aligned} rV &= -\kappa + q(\theta)(J_0 - V) \\ rJ_0 &= y - w + \delta(V - J_0) + \lambda(\pi)(J - J_0) \\ rJ &= y - \underline{\mathrm{w}} + \delta(V - J) + \phi(J_0 - J) \\ rU &= z + p(\theta)(W_0 - U) \\ rW_0 &= w + \delta(U - W_0) + \lambda(\pi)(W - W_0) \\ rW &= \underline{\mathrm{w}} + \delta(U - W) + \phi(W_0 - W) \end{aligned}$$

and the following equilibrium conditions and assumptions:

$$\begin{aligned} V &= 0 \\ W &= U \\ \beta(J_0 - V) &= (1 - \beta)(W_0 - U) + \mu, \end{aligned}$$

where $\mu \geq 0$ is the multiplier on the firm's credit constraint. Corresponding to these 9 equations, the steady state consists of 9 unknowns: 6 values, two wage rates, w and $\underline{\mathrm{w}}$, and labor market tightness, θ.

A.1 Proof of Lemma 1

Proof. The difference between equations (8) and (9) and the difference between (6) and (5) gives:

$$J - J_0 = \frac{w - \underline{\mathrm{w}}}{r + \delta + \phi + \lambda(\pi)} = W_0 - W. \tag{A.1}$$

□

A.2 Proof of Proposition 1

Proof. First note that:

$$W_0 - W = W_0 - U = \frac{\beta}{1 - \beta} \frac{\kappa}{q(\theta)}.$$

The first equality is by assumption. The second equality uses the Nash bargaining equation along with the free entry condition that implies $V = 0$ and $J_0 = \kappa / q(\theta)$:

$$W_0 - U = \frac{\beta}{1 - \beta}(J_0 - V) = \frac{\beta}{1 - \beta} J_0 = \frac{\beta}{1 - \beta} \frac{\kappa}{q(\theta)}$$

Substituting for $W_0 - U$ from the above in equation (4) gives:

$$rU = z + p(\theta) \frac{\beta}{1 - \beta} \frac{\kappa}{q(\theta)}$$

Then, substituting for rU and $W_0 - W$ from the equations above gives the following representation for the wage equation:

$$w = \beta y + (1-\beta)z + \beta\kappa\theta + \lambda(\pi)\frac{\beta}{1-\beta}\frac{\kappa}{q(\theta)}$$

□

A.3 Proof of Proposition 2

Proof. Simply substitute the solution for the wage rate from (17) and $J - J_0 = W_0 - W = \frac{\beta}{1-\beta}\frac{\kappa}{q(\theta)}$ in the job creation condition:

$$\begin{aligned}\frac{y - w + \lambda(\pi)(J - J_0)}{r+\delta} &= \frac{y - \beta y - (1-\beta)rU - \lambda(\pi)(W_0 - W) + \lambda(\pi)(J - J_0)}{r+\delta} \\ &= \frac{(1-\beta)(y - rU)}{r+\delta} \\ &= \frac{(1-\beta)(y-z)}{r+\delta} - \frac{\beta(p(\theta))}{r+\delta}\frac{\kappa}{q(\theta)} = \frac{\kappa}{q(\theta)},\end{aligned}$$

where the last equality follows from the job creation condition. Rearranging terms gives:

$$\frac{\kappa}{q(\theta)}\left[1 + \frac{\beta(p(\theta))}{r+\delta}\right] = \frac{\kappa}{q(\theta^*)} \times \frac{r+\delta+\beta p(\theta^*)}{(1-\beta)(r+\delta)} = \frac{y-z}{r+\delta}$$

□

A.4 Proof of Proposition 3

Proof. Setting $w = y$ in the job creation condition (15) gives:

$$\frac{y - y + \lambda(\pi)(J - J_0)}{r+\delta} = \frac{\kappa}{q(\theta)}$$

Subtracting J_0 from J:

$$J - J_0 = \frac{w - \underline{w}}{r + \phi + \lambda(\pi) + \delta}$$

Setting $W = U$ gives $\underline{w}$:

$$\underline{w} = z + (p(\theta) - \phi)(W_0 - U) = z + (p(\theta) - \phi)\frac{w - z}{r + \delta + \lambda(\pi) + \phi}$$

where the last equality follows from subtracting rU from rW_0. Subtracting both sides from w gives:

$$w - \underline{w} = (w - z)\frac{r + \delta + \lambda(\pi) + \phi}{r + \delta + \lambda(\pi) + p(\theta)}$$

Note, when $\phi = p(\theta)$, then $\underline{w} = z$ as expected.

Setting $w = y$ and plugging this back in for $J - J_0$ in the job creation condition gives:

$$\frac{\lambda(\pi)}{r + \delta + \lambda(\pi) + p(\theta)}\frac{(y-z)}{r+\delta} = \frac{\kappa}{q(\theta)},$$

which holds for $\pi > \underline{\pi} > 0$. □

A.5 Proof of Proposition 4

Proof. The lowest value that will not deter unemployed workers from searching for a job in the first place requires $W_0 = U$. Using equations (5) and (4), the added value to a worker from a job is:

$$W_0 - U = \frac{w - z - \lambda(\pi)(W_0 - W)}{r + \delta + p(\theta)}$$

Setting that to zero gives the lowest acceptable wage to the worker: $w = z + \lambda(\pi)(W_0 - W)$. Substituting it back in the job creation condition in (15) gives the maximum attainable labor market tightness:

$$\frac{y - z + \lambda(\pi)(J - J_0 - W_0 + W)}{r + \delta} = \frac{\kappa}{q(\theta)},$$

where $J - J_0 - W_0 + W = 0$ by Lemma 1. □

A.6 Proof of Proposition 6

Proof. First note that $\pi^* > \underline{\pi}$ whenever $\eta_u^m < \beta$. Therefore the equilibrium is constrained and $w = y$. Comparing equations (20) and (25), labor market tightness is optimal if and only if:

$$\frac{r + \delta + \lambda(\pi) + p(\theta^H)}{\lambda(\pi)} = \frac{r + \delta + \eta_u^m p(\theta^H)}{\eta_v^m (r + \delta)}$$

Solving for $\lambda(\pi)$ and considering that $\lambda(\cdot)$ is monotonic, gives the condition in the proposition. □

B Additions to the Appendix

B.1 Additional Result – Phillips Curve

Lemma B.1. *The long-run Phillips Curve is always satisfied*

Proof. Consider the case in which $\pi > \underline{\pi}$. Then, the out of steady state system of Bellman equations is given by:

$$\begin{cases} rV = -\kappa + q(\theta)[J_0 - V] + \dot{V} & (V) \\ rJ_0 = \delta(V - J_0) + \lambda(\pi)(J - J_0) + \dot{J}_0 & (J_0) \\ rJ = y - \underline{w} + \delta(V - J) + \phi(J_0 - J) + \dot{J} & (J) \\ rU = z + p(\theta)(W_0 - U) + \dot{U} & (U) \\ rW_0 = y + \delta(U - W_0) + \lambda(\pi)(W - W_0) + \dot{W}_0 & (W_0) \\ rW = \underline{w} + \delta(U - W) + \phi(W_0 - W) + \dot{W} & (W) \end{cases}$$

as pointed out by Pissarides [2000], since firms can open and close vacancies instantaneously, we must always have that $V = \dot{V} = 0$. Hence, from (V), we have:

$$J_0 = \frac{\kappa}{q(\theta)} \tag{A.2}$$

while from (J_0) and (J), we have:

$$\dot{J}_0 = -\lambda(\pi)J + (r + \delta + \lambda(\pi))J_0 \tag{A.3}$$

and

$$\dot{J} = -(y - \underline{w}) + (r + \delta + \phi)J - \phi J_0 \tag{A.4}$$

substituting (A.2) and (A.4), we have:

$$\dot{J} = -(y - \underline{w}) - \phi\frac{\kappa}{q(\theta)} + (r + \delta + \phi)J \tag{A.5}$$

Moreover, since $W = U$, from $(W) - (U)$, we have:

$$0 = \underline{w} - z - (p(\theta) - \phi)(W_0 - U)$$

therefore:

$$\underline{w} = z + (p(\theta) - \phi)(W_0 - U) \tag{A.6}$$

Similarly, from $(W_0) - (U)$, we have:

$$r(W_0 - U) = y - z - \delta(W_0 - U) - p(\theta)(W_0 - U) - \lambda(\pi)(W_0 - U) + \dot{W}_0 - \dot{U}$$

Rearranging it:

$$\dot{W}_0 - \dot{U} = -(y - z) + (r + \delta + \lambda(\pi) + p(\theta))(W_0 - U) \tag{A.7}$$

From (A.5) − (A.3), we have:

$$\dot{J} - \dot{J}_0 = -(y - \underline{w}) + (r + \delta + \lambda(\pi) + \phi)(J - J_0) \tag{A.8}$$

substituting (A.6), we have:

$$\dot{J} - \dot{J}_0 = -y + z + (p(\theta) - \phi)(W_0 - U) + (r + \delta + \lambda(\pi) + \phi)(J - J_0) \tag{A.9}$$

From (A.9) − (A.7), we have:

$$\dot{J} - \dot{J}_0 - \dot{W}_0 + \dot{U} = (r + \delta + \lambda(\pi) + \phi)(J - J_0 - W_0 + U) \tag{A.10}$$

Define $J = S$ and $S_0 = J_0 + W_0 - U$, we have:

$$\dot{S} - \dot{S}_0 = (r + \delta + \lambda(\pi) + \phi)(S - S_0) \tag{A.11}$$

Notice that the surpluses S and S_0 represent the match surplus in States 1 and 0. However, the only difference between the two is how the surplus is distributed and not its value. Hence $S = S_0$, $\forall t$. Therefore, $\dot{S} - \dot{S}_0 = 0$, $\forall t$.

Then, from $S - S_0 = 0$, we can show that:

$$(r + \delta)\kappa(r + \delta + \lambda(\pi) + p(\theta)) = \lambda(\pi)(y - z)q(\theta)$$

so the long-run Phillips curve is always satisfied. □

B.2 Derivations – Implementability Restrictions

In order to simplify notation and our calculations, let's define assets at time t as $a(t) \equiv b(t) + n(t)$. As a result, the household problem becomes:

$$\max_{\{\theta,c,n\}} \int_0^\infty e^{-\rho t} U(c,n) dt$$

subject to:

$$\begin{cases} \dot{a} = y - u(y - z + \kappa\theta) + T - c + ra - (r+\pi)n \\ \dot{u} = (1-u)\delta - up(\theta) \end{cases}$$

Then, the F.O.C.s are:

$$\begin{cases} U_c - \mu_a = 0 & (\mathcal{H}_c) \\ U_n - \mu_a(r+\pi) = 0 & (\mathcal{H}_n) \\ r\mu_a = -\dot{\mu}_a + \rho\mu_a & (\mathcal{H}_a) \\ -\mu_a(y - z + \theta\kappa) - \mu_u(\delta + p(\theta)) = -\dot{\mu}_u + \rho\mu_u & (\mathcal{H}_u) \end{cases}$$

and the transversality conditions:

$$\lim_{t\to\infty} e^{-\rho t}\mu_a(t)a(t) = 0 \tag{A.12}$$

and

$$\lim_{t\to\infty} e^{-\rho t}\mu_u(t)u(t) = 0 \tag{A.13}$$

Now, let's take the asset law of motion and rearrange:

$$c + (r+\pi)n - y + u(y - z + \kappa\theta) - T = ra - \dot{a}$$

to simplify notation, define $Y(\theta, u) \equiv y - u(y - z + \kappa\theta)$. Then, substituting $Y(\theta, u)$, multiplying both sides by $e^{-\rho t}\mu_a$ and integrating over $[0, \bar{T}]$, we have:

$$\int_0^{\bar{T}} e^{-\rho t}\mu_a \left[c + (r+\pi)n - Y(\theta,u) - T\right] dt = \int_0^{\bar{T}} e^{-\rho t}\mu_a \left[ra - \dot{a}\right] dt$$

substituting $(\mathcal{H}_c)$ and $(\mathcal{H}_n)$ on the left hand side, we have:

$$\int_0^{\bar{T}} e^{-\rho t} \left[U_c c + U_n n - U_c Y(\theta,u) - U_c T\right] dt = \int_0^{\bar{T}} e^{-\rho t}\mu_a \left[ra - \dot{a}\right] dt \tag{A.14}$$

Then, notice that:

$$\frac{d}{dt} e^{-\rho t}\mu_a a = e^{-\rho t}\left\{\dot{a}\mu_a + \dot{\mu}_a a - \rho\mu_a a\right\}$$

substituting $\dot{\mu}_a = (\rho - r)\mu_a$ and rearranging, we have:

$$\frac{d}{dt} e^{-\rho t}\mu_a a = -e^{-\rho t}\mu_a(ra - \dot{a})$$

Therefore, substituting it in the right hand side of (A.14), we have:

$$\int_0^{\bar{T}} e^{-\rho t}\mu_a \left[ra - \dot{a}\right] dt = \mu_a(0)a_0 - e^{-\rho\bar{T}}\mu_a(\bar{T})a(\bar{T})$$

from the transversality condition (A.12), we have that $\lim_{\bar{T}\to\infty} e^{-\rho\bar{T}}\mu_a(\bar{T})a(\bar{T}) = 0$. Hence, the implementability constraint becomes:

$$\int_0^{\infty} e^{-\rho t}\left[U_c c + U_n n - U_c Y(\theta, u) - U_c T\right] dt = U_c(c(0), n(0))a_0. \tag{A.15}$$

Finally, let's transform the implementability constraint into an auxiliary state variable $\mathcal{A}(t)$. Define $\Psi(t)$ as:

$$\Psi(t) \equiv U_c c + U_n n - U_c Y(\theta, u) - U_c T \tag{A.16}$$

then, we define $\mathcal{A}(t) \equiv \int_t^{\infty} e^{-\rho(s-t)}\Psi(s)ds$. As a result, the implementability constraint can be rewritten as:

$$\dot{\mathcal{A}} = \rho\mathcal{A} - \Psi \tag{A.17}$$

and $\lim_{t\to\infty} \mathcal{A}(t) = 0$.

Finally, from $(\mathcal{H}_c)$ and $(\mathcal{H}_a)$, we have:

$$\frac{dU_c}{dt} = (\rho - r)U_c \tag{A.18}$$

which is the Euler equation that highlights the intertemporal substitutability by households.

B.3 Derivations – Monetary Authority Problem

In this section, we present in more details the problem of the monetary authority. As mentioned in more details in the main text, we can write the monetary authority's policy problem through its primal representation in which the authority chooses $\{c, n, \pi, \theta\}$ subject to all private-equilibrium and feasibility restrictions. Given the labor market equilibrium conditions and household optimality conditions, π is ultimately the only remaining free choice. Hence, the control vector (c, n, π, θ) in the global Hamiltonian should be viewed as a convenient parameterization of the implementable allocation, while the monetary authority's primitive intervention is the monetary policy required to implement that allocation.

As a result, the global monetary authority problem is:

$$\max_{\{c,n,\pi,\theta\}_{t\geq 0}} \int_0^{\infty} e^{-\rho t}U(c_t, n_t)\ dt$$

subject to:

$$\begin{cases} \dot{\vartheta} = (\rho + \pi)\vartheta - U_n & (EQ') \\ \vartheta - U_c = 0 & (SR) \\ \dot{u} = \delta(1-u) - up(\theta) & (LM) \\ y - u(y - z + \kappa\theta) - c = 0 & (RC) \\ \dot{J}_0 = (\widehat{r} + \delta + \lambda(\pi))J_0 - (y - w) - \lambda(\pi)S & (GJ) \\ \dot{S} = [\widehat{r} + \delta + p(\theta)]S - p(\theta)J_0 - (y - z) & (GS) \\ \frac{\kappa}{q(\theta)} - J_0 = 0 & (FE) \\ \pi \geq \pi_{min} & (FR) \\ J_0 - (1-\beta)S - \mu_w = 0 & (NB) \\ \mu_w \geq 0, \qquad (y - w) \geq 0, \qquad \mu_w(y - w) = 0 & (CS) \\ \mu_\pi \geq 0, \qquad (\pi - \pi_{min}) \geq 0, \qquad \mu_w(\pi - \pi_{min}) = 0 & (CS1) \end{cases} \tag{A.19}$$

where (CS) and $(CS1)$ are the complementary slackness conditions. Furthermore, we have that u, ϑ, J_0, and S are states. Moreover, from the intratemporal substitution of consumption and real balances from the household problem, we have:

$$\widehat{r}(c, n, \vartheta, \pi) = \frac{U_n(c, n)}{\vartheta} - \pi \tag{A.20}$$

Then, combining this equation with the Euler equation provides us with (EQ'). Moreover, S is the matching surplus at creation, i.e., $S = J_0 + W_0 - U$.

Mathematically, this is an optimal-control problem with complementarity constraints, or equivalently a hybrid optimal-control problem.

Let μ_u, μ_ϑ, μ_J, μ_S be the current-value costates. Let ν, ξ ,μ_R be the multipliers on free entry, $\vartheta = U_c$, and resource clearing. μ_w and μ_π are the multipliers for credit constraints and inflation lower bound, respectively. The Hamiltonian underlying both regimes is:

$$\mathcal{H}^G = \left\{ \begin{array}{l} \lambda_0 U(c, n) + \mu_u[\delta(1-u) - up(\theta)] + \mu_\vartheta[(\rho + \pi)\vartheta - U_n] \\ + \mu_J\left[(\widehat{r} + \delta + \lambda(\pi))J_0 - (y - w) - \lambda(\pi)S\right] \\ + \mu_S\left[(\widehat{r} + \delta + p(\theta))S - (y - z) - p(\theta)J_0\right] \\ + \nu\left[\dfrac{\kappa}{q(\theta)} - J_0\right] + \xi[\vartheta - U_c] + \mu_R[Y(\theta, u) - c] + \mu_\pi[\pi - \pi_{min}] \end{array} \right\}$$

As a result, the global maximum condition is:

$$a_t^* \in \arg\max_{a \in \mathcal{A}(x_t)} \mathcal{H}^G(x_t, \mu_t, a),$$

where $\mathcal{A}(x_t)$ contains the Nash and wage-cap complementarity restrictions. The admissible set is the union $\mathcal{A} = \mathcal{A}^N \cup \mathcal{A}^C$. In other words,

$$\max_{\mathcal{A}} \mathcal{H}^G = \max\left\{\max_{\mathcal{A}^N} \mathcal{H}^G, \max_{\mathcal{A}^C} \mathcal{H}^G\right\}$$

In other words, we solve $\mathcal{H}^G$ on each complementarity regime, then join the solutions through switching conditions.

B.4 Solving the credit-constrained branch

On the strictly credit-constrained branch, we have that $\mu_w > 0$ and $w = y$. as a result, (GJ) becomes:

$$\dot{J}_0 = (\widehat{r} + \delta + \lambda(\pi))J_0 - \lambda(\pi)S \quad (GJ')$$

In this section, we focus on an interior monetary-policy solution on the credit-constrained branch, so the lower monetary bound is slack and its multiplier is zero. In steady state, $\pi > \underline{\pi} > \pi_{\min}$ is satisfied.

since $\pi > \underline{\pi}$, constraint (FR) is trivialy satisfied since $\pi_{min} < \underline{\pi}$. Hence, $\mu_\pi = 0$. As a result, the constrained Hamiltonian is:

$$\mathcal{H}^C = \left\{ \begin{array}{l} U + \mu_u[\delta(1-u) - up(\theta)] + \mu_\vartheta[(\rho+\pi)\vartheta - U_n] \\ + \mu_J[(\widehat{r} + \delta + \lambda(\pi))J_0 - \lambda(\pi)S] \\ + \mu_S\{[\widehat{r} + \delta + p(\theta)]S - (y-z) - p(\theta)J_0\} \\ + \nu\left[\dfrac{\kappa}{q(\theta)} - J_0\right] + \xi(\vartheta - U_c) + \mu_R(Y - c) \end{array} \right\}.$$

where we restrict attention to the normal case and normalize $\lambda_0 = 1$. Then, the optimality conditions are:

$$\left\{ \begin{array}{ll} U_n - \mu_\vartheta U_{nn} + \frac{\mu_J J_0 + \mu_S S}{\vartheta} U_{nn} - \xi U_{cn} = 0 & (\mathcal{H}_n) \\ U_c - \mu_\vartheta U_{cn} + \frac{\mu_J J_0 + \mu_S S}{\vartheta} U_{cn} - \xi U_{cc} - \mu_R = 0 & (\mathcal{H}_c) \\ -\mu_u u p'(\theta) + \mu_S p'(\theta)(S - J_0) - \nu\kappa\frac{q'(\theta)}{q(\theta)^2} - \mu_R\kappa u = 0 & (\mathcal{H}_\theta) \\ \mu_\vartheta\vartheta + \mu_J[-J_0 - \lambda'(\pi)(S - J_0)] - \mu_S S = 0 & (\mathcal{H}_\pi) \\ -[\delta + p(\theta)]\mu_u - \mu_R(y - z + \kappa\theta) = -\dot{\mu}_u + \rho\mu_u & (\mathcal{H}_u) \\ \mu_\vartheta(\rho + \pi) - (\mu_J J_0 + \mu_S S)\frac{U_n}{\vartheta^2} + \xi = -\dot{\mu}_\vartheta + \rho\mu_\vartheta & (\mathcal{H}_\vartheta) \\ \mu_J(\widehat{r} + \delta + \lambda(\pi)) - \mu_S p(\theta) - \nu = -\dot{\mu}_J + \rho\mu_J & (\mathcal{H}_{J_0}) \\ -\mu_J\lambda(\pi) + \mu_S[\widehat{r} + \delta + p(\theta)] = -\dot{\mu}_S + \rho\mu_S & (\mathcal{H}_S) \end{array} \right.$$

Notice that ξ, μ_R, and ν can be eliminated algebraically. Moreover, from (FE) we can write θ as a function of J_0 and parameters, since $q(\theta)$ is a strictly decreasing function. From (RC), we can rewrite c as a function of J_0 and u. Assuming $U_{cn} \neq 0$, we have that (SR) allows us to pin down n as a function of ϑ, J_0, and u.[18] Finally, from $(\mathcal{H}_\pi)$, we are able to pin down π as a function of J_0, S, ϑ, μ_ϑ, and μ_J. Consequently, we end up with a differential system for endogenous variables (u,ϑ,J_0,S,μ_u,μ_ϑ,μ_J,μ_S). The system is pin down by the constraints

$$\begin{array}{ll} \dot{u} = \delta(1-u) - up(\theta) & (LM) \\ \dot{\vartheta} = (\rho + \pi)\vartheta - U_n & (EQ') \\ \dot{J}_0 = (\widehat{r} + \delta + \lambda(\pi))J_0 - \lambda(\pi)S & (GJ') \\ \dot{S} = [\widehat{r} + \delta + p(\theta)]S - p(\theta)J_0 - (y - z) & (GS) \end{array}$$

[18] By assuming $U_{cn} \neq 0$ for any values, we rule out not only a constant bu also non-monotonic utility functions in n. As a result, we are able to invert the U_c function.

and the current-value costate equations:

$$\begin{aligned}
&-[\delta + p(\theta)]\mu_u - \mu_R(y - z + \kappa\theta) = -\dot{\mu}_u + \rho\mu_u && (\mathcal{H}_u)\\
&\mu_\vartheta(\rho + \pi) - (\mu_J J_0 + \mu_S S)\tfrac{U_n}{\vartheta^2} + \xi = -\dot{\mu}_\vartheta + \rho\mu_\vartheta && (\mathcal{H}_\vartheta)\\
&\mu_J(\widehat{r} + \delta + \lambda(\pi)) - \mu_S p(\theta) - \nu = -\dot{\mu}_J + \rho\mu_J && (\mathcal{H}_{J_0})\\
&-\mu_J\lambda(\pi) + \mu_S[\widehat{r} + \delta + p(\theta)] = -\dot{\mu}_S + \rho\mu_S && (\mathcal{H}_S)
\end{aligned}$$

while ξ, μ_R, ν, θ, c, n, and π are pinned down as previously described.

B.4.1 Credit-constrained branch: steady state

Let's then consider the steady-state equilibrium. From (LM), we pin down $u = \frac{\delta}{\delta + p(\theta)}$. From (EQ'), we have:

$$\vartheta = \frac{U_n}{\rho + \pi} \tag{A.21}$$

since $\vartheta = U_c$, from (A.21) and (A.20), we have that $\widehat{r} = \rho$. From (GJ') and (GS), we have:

$$J_0 = \frac{\lambda(\pi)(y - z)}{(\rho + \delta)[\rho + \delta + \lambda(\pi) + p(\theta)]} \tag{A.22}$$

and

$$S = \frac{[\rho + \delta + \lambda(\pi)](y - z)}{(\rho + \delta)\left[\rho + \delta + \lambda(\pi) + p(\theta)\right]} \tag{A.23}$$

Then, from (FE), $J_0 = \frac{\kappa}{q(\theta)}$,(A.22), (A.22), $(\mathcal{H}_S)$, $(\mathcal{H}_{J_0})$, and $(\mathcal{H}_u)$, we have

$$\mu_S S + \mu_J J_0 = \frac{\kappa}{q(\theta)} \frac{\nu\left[\rho + 2\delta + \lambda(\pi) + p(\theta)\right]}{\delta[\delta + \lambda(\pi) + p(\theta)]} \tag{A.24}$$

Moreover, since $p(\theta) = \theta q(\theta)$, from (A.22) and (FE), we have

$$\lambda(\pi) = \frac{\kappa(\rho + \delta)\theta[\rho + \delta + p(\theta)]}{p(\theta)(y - z) - \kappa(\rho + \delta)\theta} \tag{A.25}$$

After manipulating the system, we are able to eliminate all multipliers, by rewriting them as a function of θ, we obtain

$$\begin{aligned}
0 = \frac{\delta}{\delta + p(\theta)} \left[\frac{p'(\theta)[y - z + \kappa\theta]}{\rho + \delta + p(\theta)} - \kappa\right] \Bigg\{ & \rho[\rho + 2\delta + p(\theta) + \lambda(\pi)][U_c U_{cn} - U_{cc} U_n] \\
& + \frac{\lambda'(\pi)[\delta + p(\theta)](\rho + \delta)}{\lambda(\pi)} \Big[U_c(U_{nn} - \pi U_{cn}) + U_n(\pi U_{cc} - U_{cn})\Big] \Bigg\} \\
+ U_c U_n \left[(\rho + \delta)p'(\theta) - \delta[\delta + \lambda(\pi) + p(\theta)]\frac{q'(\theta)}{q(\theta)}\right].
\end{aligned} \tag{A.26}$$

Notice that (A.26) implicitly pins down steady-state θ as a function of the parameters. All objects in (A.26) are functions of θ alone. In particular, c is given by

$$c(\theta) = y - \frac{\delta}{\delta + p(\theta)}[y - z + \kappa\theta]$$

while π is given by:

$$\pi \equiv \chi(\theta) = \lambda^{-1}\left(\frac{\kappa(\rho+\delta)\theta[\rho+\delta+p(\theta)]}{p(\theta)(y-z)-\kappa(\rho+\delta)\theta}\right) \tag{A.27}$$

and $\lambda(\pi)$ is pinned down by (A.25). Finally, $n(\theta)$ is implicitly pinned down by

$$U_n(c(\theta), n(\theta)) = [\rho + \chi(\theta)]U_c(c(\theta), n(\theta))$$

Hamiltonian representation of the monetary authority's marginal rate of substitution. $W^{MA}(\pi, \theta)$ summarizes the welfare consequences of a combination of inflation and labor-market tightness after all of these equilibrium adjustments have been taken into account.

This interpretation gives a natural economic meaning to its two partial derivatives. Holding inflation fixed, W^{MA}_θ measures the marginal welfare effect of greater labor-market tightness. This effect includes the benefit from faster job finding and lower unemployment, the resource cost of additional vacancy creation, and the continuation-value effects associated with the resulting changes in employment.

Similarly, holding tightness fixed, W^{MA}_π measures the marginal welfare effect of higher inflation. It incorporates the direct distortion to household money holdings as well as the effect of inflation on the rate $\lambda(\pi)$ at which real wages depreciate, which changes the values of employment relationships and therefore matters for the credit-constrained labor market.

The monetary authority must choose π and θ such that $\pi = \chi(\theta)$ is satisfied. We can therefore write the authority's welfare as the one-dimensional function

$$\widetilde{W}^{MA}(\theta) \equiv W^{MA}\big(\chi(\theta), \theta\big). \tag{A.28}$$

At an interior optimum, the marginal welfare gain from additional tightness must exactly offset the marginal welfare cost of the inflation required to implement it. Hence,

$$W^{MA}_\theta + W^{MA}_\pi \chi'(\theta) = 0. \tag{A.29}$$

The global Hamiltonian provides a way of calculating this marginal trade-off even though $W^{MA}(\pi, \theta)$ is not available in closed form. After imposing the stationary equilibrium conditions and eliminating costates, equation (A.26) is the first-order condition for a marginal feasible change in policy. The remaining step is therefore not to *define* W^{MA} from that equation, but rather to express the Hamiltonian first-order condition in the same slope units as equation (A.29).

To do so, differentiate the implementability condition

$$\lambda(\chi(\theta)) = \Lambda(\theta),$$

which gives

$$\Lambda'(\theta) = \lambda'(\pi)\chi'(\theta).$$

Equivalently,

$$\frac{\lambda'(\pi)[\delta + p(\theta)](\rho+\delta)}{\lambda(\pi)} = \frac{1}{\chi'(\theta)}\frac{[\delta + p(\theta)](\rho+\delta)}{\lambda(\pi)}\Lambda'(\theta).$$

This substitution converts the marginal effect operating through $\lambda'(\pi)$ into the corresponding marginal movement along the (π, θ) implementability locus. Multiplying (A.26) by $\chi'(\theta)$ then puts the Hamiltonian condition in the same form as the total welfare derivative above. Collecting terms gives

$$W_\theta^{MA} + W_\pi^{MA}\chi'(\theta) = 0.$$

Importantly, this manipulation does not construct a new welfare function. The reduced welfare function $W^{MA}(\pi, \theta)$ is defined by the underlying Ramsey problem. The Hamiltonian calculation instead provides an *envelope representation* of the local trade-off between its two arguments. In particular, it identifies the marginal-rate-of-substitution ratio as

$$\begin{aligned}\frac{W_\theta^{MA}}{W_\pi^{MA}} = &\frac{\dfrac{[\delta + p(\theta)](\rho+\delta)}{\lambda(\pi)}\Lambda'(\theta)\,u(\theta)\left[\dfrac{p'(\theta)[y - z + \kappa\theta]}{\rho + \delta + p(\theta)} - \kappa\right]}{\rho[\rho + 2\delta + p(\theta) + \lambda(\pi)]\,u(\theta)\left[\dfrac{p'(\theta)[y - z + \kappa\theta]}{\rho + \delta + p(\theta)} - \kappa\right][U_c U_{cn} - U_{cc}U_n]} \\ &\times \Big[U_c(U_{nn} - \pi U_{cn}) + U_n(\pi U_{cc} - U_{cn})\Big] \\ &+ U_c U_n\left[(\rho+\delta)p'(\theta) - \delta[\delta + \lambda(\pi) + p(\theta)]\frac{q'(\theta)}{q(\theta)}\right]\end{aligned}. \tag{A.30}$$

Although equation (A.30) is algebraically cumbersome, its economic interpretation is straightforward. The numerator and denominator summarize, in shadow-value terms, the two sides of the policy trade-off. One side captures the welfare value of changing labor-market tightness, including its effect on employment, vacancy resources, and the continuation values of matches. The other captures the welfare effect of the associated monetary adjustment, including the response of real money balances and the effect of inflation on $\lambda(\pi)$. The curvature terms involving U_{cc}, U_{cn}, and U_{nn} arise because changes in inflation and labor-market conditions alter both consumption and money demand, while the costates translate those induced changes into lifetime rather than merely contemporaneous welfare effects.

Special Case: $U_{cn} = 0$ and linear $\lambda(\pi)$ This special case is relevant due to our choices for the quantitative exercise. In particular, we assume $\lambda(\pi) = \ell\pi$. Then, assuming $U_{nn} \neq 0$, we can solve the system again and obtain a condition similar to (A.26), i.e.

$$\begin{aligned}0 = &\frac{\delta}{\delta + p(\theta)}\left[\frac{p'(\theta)[y - z + \kappa\theta]}{\rho + \delta + p(\theta)} - \kappa\right] \\ &\times \left\{\frac{[\delta + p(\theta)](\rho+\delta)}{\pi}U_{nn}\right. \\ &\qquad \left. + (\rho + \pi)\left[\delta[\delta + p(\theta)] - \rho[\rho + \delta + \ell\pi]\right]U_{cc}\right\} \\ &+ U_c(\rho+\pi)\left[(\rho+\delta)p'(\theta) - \delta[\delta + \ell\pi + p(\theta)]\frac{q'(\theta)}{q(\theta)}\right],\end{aligned} \tag{A.31}$$

B.5 Solving the credit-unconstrained case

On the unconstrained case, $\mu_w = 0$ and $w < y$. Then, from (NB), we have $J_0 = (1-\beta)S \Rightarrow S = \frac{1}{1-\beta}J_0$. As a result, (GJ) becomes:

$$\dot{J}_0 = \left[\widehat{r} + \delta - \frac{\beta}{1-\beta}\lambda(\pi)\right] J_0 - (y - w) \tag{A.32}$$

while (GS) becomes:

$$\frac{\dot{J}_0}{1-\beta} = \left[\widehat{r} + \delta + \beta p(\theta)\right] \frac{J_0}{1-\beta} - (y - z) \tag{A.33}$$

subtracting (A.33) from (A.32), we have:

$$\dot{J}_0 = \left[\widehat{r} + \delta + \lambda(\pi) + p(\theta)\right] J_0 - \left(\frac{1-\beta}{\beta}\right)(w - z) \tag{A.34}$$

given the Nash Bargaining, we have:

$$w = p(\theta)\beta J_0 + \lambda(\pi)\frac{\beta}{1-\beta}J_0 + \beta y + (1-\beta)z \tag{A.35}$$

substituting into (A.34):

$$\dot{J}_0 = \left[\widehat{r} + \delta + \beta p(\theta)\right] J_0 - (1-\beta)(y - z) \tag{A.36}$$

Thus, once Nash bargaining is unconstrained, inflation no longer enters the job-creation equation directly. Monetary policy can affect the labor-market dynamics only thrhough the household intertemporal return $\widehat{r}$. hence, the monetary authority problem becomes:

$$\max_{\{c,n,\pi,\theta\}_{t\geq 0}} \int_0^\infty e^{-\rho t} U(c_t, n_t)\, dt$$

subject to:

$$\begin{cases} \dot{\vartheta} = (\rho + \pi)\vartheta - U_n & (EQ') \\ \vartheta - U_c = 0 & (SR) \\ \dot{u} = \delta(1-u) - up(\theta) & (LM) \\ y - u(y - z + \kappa\theta) - c = 0 & (RC) \\ \dot{J}_0 = \left[\widehat{r} + \delta + \beta p(\theta)\right] J_0 - (1-\beta)(y-z) & (GJ') \\ \frac{\kappa}{q(\theta)} - J_0 = 0 & (FE) \\ \pi \geq \pi_{min} & (FR) \end{cases} \tag{A.37}$$

then, the Hamiltonian becomes:

$$\mathcal{H}^{NC} = \left\{ \begin{array}{l} \lambda_0 U(c,n) + \mu_u[\delta(1-u) - up(\theta)] + \mu_\vartheta[(\rho+\pi)\vartheta - U_n] \\ + \mu_J\left[(\widehat{r} + \delta + \beta p(\theta))J_0 - (1-\beta)(y-z)\right] \\ + \mu_R[y - u(y - z + \kappa\theta) - c] + \xi[\vartheta - U_c] \\ + \nu\left[\frac{\kappa}{q(\theta)} - J_0\right] + \mu_\pi(\pi - \pi_{min}) \end{array} \right\}.$$

Then, the optimality conditions are:

$$\begin{cases} U_n - \mu_\vartheta U_{nn} + \frac{\mu_J J_0}{\vartheta} U_{nn} - \xi U_{cn} = 0 & (\mathcal{H}_n) \\ U_c - \mu_\vartheta U_{cn} + \frac{\mu_J J_0}{\vartheta} U_{cn} - \xi U_{cc} - \mu_R = 0 & (\mathcal{H}_c) \\ -\mu_u u p'(\theta) + \mu_J \beta p'(\theta) J_0 - \nu\kappa \frac{q'(\theta)}{q(\theta)^2} - \mu_R \kappa u = 0 & (\mathcal{H}_\theta) \\ \mu_\vartheta \vartheta - \mu_J J_0 + \mu_\pi = 0 & (\mathcal{H}_\pi) \\ -[\delta + p(\theta)]\mu_u - \mu_R(y - z + \kappa\theta) = -\dot{\mu}_u + \rho\mu_u & (\mathcal{H}_u) \\ \mu_\vartheta(\rho + \pi) - \mu_J J_0 \frac{U_n}{\vartheta^2} + \xi = -\dot{\mu}_\vartheta + \rho\mu_\vartheta & (\mathcal{H}_\vartheta) \\ \mu_J(\widehat{r} + \delta + \beta p(\theta)) - \nu = -\dot{\mu}_J + \rho\mu_J & (\mathcal{H}_{J_0}) \end{cases}$$

From (FE), taking the derivative with respect to time, we have:

$$\dot{J}_0 = -\frac{\kappa}{q(\theta)^2} q'(\theta)\dot{\theta}$$

substituting into (GJ'), we have:

$$\dot{\theta} = \frac{q(\theta)}{q'(\theta)} \left\{ -[\widehat{r} + \delta + \beta p(\theta)] + \frac{(1-\beta)(y-z)q(\theta)}{\kappa} \right\}$$

From (SR), (EQ'), and (A.20), we have:

$$\dot{\vartheta} = (\rho - \widehat{r})\vartheta \tag{EQ}$$

B.5.1 Unconstrained branch: steady state

Let's then consider the steady-state equilibrium. From (LM), we pin down $u = \frac{\delta}{\delta + p(\theta)}$. From (EQ), we have $\widehat{r} = \rho$. From (GJ'), we have:

$$\frac{(1-\beta)(y-z)q(\theta)}{\kappa} = [\widehat{r} + \delta + \beta p(\theta)]$$

which is the equilibrium market tightness as presented by Proposition 2. Therefore $\theta^{ss} = \theta^*$. As a result, $u^{ss} = \frac{\delta}{\delta + p(\theta^*)}$ and $c^{ss} = y - \frac{\delta}{\delta + p(\theta^*)}(y - z + \kappa\theta^*)$. In other words, the steady-state labor allocation and consumption are independent of inflation.

From (A.20) and (EQ), we have:

$$U_n(c^{ss}, n) = (\rho + \pi) U_c(c^{ss}, n) \tag{A.38}$$

As we showed, c^{ss} does not vary with inflation. Then, differentiating (A.38) with respect to π, we have:

$$[U_{nn} - (\rho + \pi)U_{cn}] \frac{dn}{d\pi} - U_c = 0$$

since $U(c, n)$ is concave and strictly increasing in each argument, we must have $U_{nn} < 0$ and $U_c > 0$. Given $U_{cn} \geq 0$, we have that

$$\frac{dn}{d\pi} = \frac{U_c}{[U_{nn} - (\rho + \pi)U_{cn}} < 0$$

Hence, for any $\pi > \pi_{min}$, a reduction in steady-state inflation raises real balances while leaving the steady-state labor allocation and consumption unchanged. Since $U_n > 0$, the monetary authority therefore prefers the lowest feasible long-run rate. Therefore

$$\pi^{ss} = \pi_{min}$$

Assuming the Friedman rule, we have that the minimum nominal interest rate $i_{min} = 0$. From the Fisher equation, we have that $\pi_{min} + \rho = 0$. Henceforth, $\pi_{min} = -\rho$ and

$$\pi^{ss} = -\rho.$$