Equipment, Structures, and the Limits of Investment-Specific Technological Change*

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Abstract

Models of investment-specific technological change treat the falling relative price of equipment as a source of growth. The relative price of structures has moved the other way. It has risen in rich countries, and in the United States it is 80 percent higher than in 1970. We document this divergence across countries, show that the U.S. rise covers nearly every type of structure, and attribute about half of the post-1996 rise in construction prices to falling construction productivity. To quantify its cost, we build an endogenous growth model in which equipment and structures enter both final production and R&D. In the U.S. calibration, rising structures prices subtract 0.63 percentage points a year from per-capita growth, half of the 1.24 points that cheaper equipment adds, with 30 percent of the drag operating through R&D. A nested CES extension finds that structures substitute for unskilled labor and that the two capital-labor bundles substitute for each other. These margins lower the drag to about 0.4 points. Rising structures prices have pushed firms toward unskilled labor and slowed the rise in the skill premium. Without this margin, the model overstates the 1963-2019 rise by 30 percent.

Keywords: Equipment, Structures, Investment-Specific Technological Change, Endogenous Growth
JEL Codes: O30, O40, E22

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1 Introduction

Between 1970 and 2020, the price of equipment in the United States fell by 70 percent relative to the GDP deflator. The price of structures rose by 80 percent. The first trend is well known. Since Greenwood, Hercowitz and Krusell (1997), economists have interpreted the falling relative price of equipment as the signature of investment-specific technological change (ISTC), and models built on this mechanism attribute much of postwar growth in output per hour to cheaper equipment. The second trend is absent from the canonical model. Structures are produced one-for-one from final output, so their relative price is constant by construction. This assumption was consistent with the data through the 1980s. It is not consistent with the data since then. Production requires both equipment and the structures that house it. Equipment has become cheaper while the buildings needed to hold it have become more expensive. We ask how much of the growth gain from cheaper equipment has been offset by more expensive structures.

We begin with three facts about the two capital prices, using data from the Penn World Table (PWT), the International Comparison Program (ICP), and the U.S. Bureau of Economic Analysis (BEA).

First, the divergence is concentrated in high-income countries. Equipment prices have fallen faster in the top tercile of countries by 1970 income than in the bottom tercile, and structures prices have risen in the top tercile but not in the bottom tercile. The United States is at the extreme of the structures trend.[1] Second, within the United States, the two trends have different compositions. The decline in equipment prices is concentrated in computers and communication equipment, whereas the rise in structures prices after 1990 appears in nearly every category of structures, residential and nonresidential alike. Third, across 19 U.S. sectors, those whose capital relied more on structures in 1970 experienced slower TFP growth over 1970-2016 and slower real investment growth over 1970–2020. A 10 percentage point higher structures share of the capital stock is associated with 0.1 percentage points lower TFP growth per year. We interpret this as a correlation that is consistent with our mechanism, not as a causal estimate.

To see why structures prices have risen, we decompose the construction sector's output price using KLEMS data. The sign of this relationship follows from accounting. Under the unit-cost identity, slower TFP growth passes into faster output-price growth for given input prices. The decomposition tells us the magnitude. From 1996 to 2020, about half of the cumulative rise in the relative price of U.S. construction output is accounted for by falling construction TFP. In the aggregate economy, TFP growth offsets input-cost pressure. In construction, it adds to it. The same pattern holds in other countries. Across thirteen KLEMS economies, falling construction

[1] Sorting countries by their 1970–2020 growth rather than by their 1970 income leaves structures prices flat in both the fastest- and slowest-growing terciles, so the rise in structures prices is tied to income levels rather than to growth.

TFP accounts on average for 59 percent of the rise in relative construction prices.

To quantify the growth cost of these trends, we develop an endogenous growth model with two capital stocks. The model nests Greenwood et al. (1997). As in their setup, equipment and structures are separate stocks with their own depreciation rates and production elasticities, and investment-specific technological change lowers the price of equipment. We depart in two ways. We allow the efficiency of structures investment to decline, so that the relative price of structures rises as it has since 1990, and we let both capital types enter the production of ideas as well as goods. The R&D block follows the "lab-equipment" formulation of Rivera-Batiz and Romer (1991). Along the balanced growth path, per-capita growth is a linear combination of population growth and the two ISTC rates. The weights on the two rates, which we call growth multipliers, are closed-form functions of the factor shares and the returns to knowledge. Under the conditions stated in Section 3.5, the model is equivalent to one with an explicit construction sector whose TFP growth is the structures ISTC rate, which connects the calibration to the KLEMS evidence.

In such a model, a rising price of structures necessarily lowers growth. The sign is not the question. The questions are how large the effect is relative to the gains from equipment, and through which channels it operates. We identify three channels. The first is reduced capital deepening: when structures prices rise, the steady-state ratio of structures to output falls. The second and third operate through R&D. Structures enter knowledge production directly, as laboratory and office space, so their rising price raises the user cost of research capital. They also enter indirectly through the aggregate resource constraint, since slower growth in the structures stock tightens the resources available to fund innovation. The R&D channels are what allow structures prices to affect the growth rate of ideas, and not only the level of output.

We calibrate the model to the United States over 1970–2020. The model generates per-capita growth of 1.43 percent a year, against 1.69 percent in the data. Population growth accounts for 0.82 percentage points. Equipment ISTC adds 1.24 points. Structures ISTC subtracts 0.63 points, so the net contribution of investment-specific change is 0.61 points, about half of the equipment contribution. Of the 0.63 point drag, 0.44 points come from reduced capital deepening and 0.19 points from the R&D sector, split roughly evenly between the direct input channel and the indirect resource channel. The drag remains sizable under alternative parameter values.

The size of the drag depends on a measured price trend, so we ask how much of that trend could be measurement error. We first document how the BEA structures deflator is constructed. About 36 percent of structures investment is deflated by the Census constant-quality indexes for new houses, 12 percent by construction cost indexes, and the remainder by fixed-specification producer price indexes and by components to which structure quality does not apply. Only the first group is exposed to the unobserved-quality critique in its usual form. The cost indexes price inputs rather than finished buildings, so if construction productivity has fallen, as the KLEMS

evidence indicates, they understate rather than overstate the price of structures. Applying the most generous of the bounds in Garcia and Molloy (2025) to each component gives an upper bound of 0.34 percentage points a year on the bias in the structures price trend. At that bound the drag falls from 0.63 to 0.45 points. Using the price of nonresidential structures alone, which rose faster than the price of all structures, raises it to 0.73 points. We also ask whether research facilities in particular have escaped the trend. The closest direct evidence, a unit-value series from the NCSES survey of academic research space, shows the real cost per square foot of new research space rising about 3 percent a year since 2002, at least as fast as nonresidential structures as a whole (Section 4.6).

We then ask how the United States would have fared with other countries' price trends. Feeding the high-income median's ISTC rates into the U.S. calibration raises the ISTC contribution to growth by about 0.4 percentage points, and more than four fifths of that gap comes from the structures margin. The gap between the United States and other rich countries is a structures gap. The comparison with low-income countries is different. Relative to the low-income median, the U.S. contribution is only slightly higher, because the large equipment advantage of the United States is mostly offset by its larger drag. The two capital prices therefore have opposite implications for convergence. Falling equipment prices favor the rich countries, where they have fallen faster. Rising structures prices work against them. Capital price trends are a weaker source of divergence than the equipment price alone would suggest.[2]

Beyond Cobb-Douglas, we extend the model to a nested CES specification with an equipment-skilled-labor bundle and a structures-unskilled-labor bundle, and estimate the substitution elasticities by GMM on the dataset of Ohanian, Orak and Shen (2023). The data reject Cobb-Douglas in every nest. Structures and unskilled labor are substitutes ($\hat{\sigma}_S = 2.24$), and the two bundles are substitutes in the outer nest ($\hat{\sigma}_0 = 1.42$), while equipment and skilled labor remain complements ($\hat{\sigma}_M = 0.72$), as in Krusell, Ohanian, Ríos-Rull and Violante (2000). The outer nest also allows the equipment-skilled-labor bundle to substitute for the structures-unskilled-labor bundle, a margin that the Cobb-Douglas baseline fixes at a unit elasticity. With these estimates the drag falls from 0.63 to about 0.4 percentage points, and it declines slowly over time, to 0.36 by the end of the sample, as the structures share of output falls in response to its own price. The CES estimates also identify a channel through which structures prices affect wage inequality. Because structures and unskilled labor are substitutes, a rising price of structures raises the relative demand for unskilled labor and dampens the rise in the skill premium. The canonical four-factor spec-

[2]This result has implications for development accounting. Hsieh and Klenow (2007) showed that poor countries face a high relative price of investment and invest less in real terms as a result. Our data add the time dimension. Over time, equipment prices have fallen faster in rich countries, while structures prices have risen there and stayed flat in poor countries. Structures, which are built locally, are the component of investment that poor countries obtain relatively cheaply, and that advantage has grown since 1970.

ification of Krusell et al. (2000) embeds structures as a Cobb-Douglas factor separable from the skill-complementary capital stock and rules out this channel by construction. In a counterfactual in the spirit of their exercise, removing this margin from our estimated model raises the implied 1963–2019 rise in the U.S. skill premium by 30 percent.

Our work connects to four areas of research. First, we build upon the foundational literature on investment-specific technological change (ISTC). Greenwood et al. (1997) measure investment-specific technological change by the decline in the relative price of equipment and attribute the bulk of postwar U.S. growth to it. Their model already distinguishes equipment from structures, but technological change is confined to equipment. Structures are produced one-for-one from final output, so their relative price is constant and they play no independent role in long-run growth, an assumption consistent with the roughly stationary structures prices of their sample period.[3] We depart from this framework in three ways. First, we allow the relative price of structures to rise, as it has since 1990. Second, we embed both capital types in an endogenous growth model in which structures enter the production of ideas as well as goods. Third, we quantify the resulting structural drag and its distributional consequences. While we confirm their finding regarding the efficiency gains in equipment, we document that the rising relative price of structures acts as a significant offsetting force.[4]

Second, our framework builds on the "lab-equipment" formulation of endogenous growth (Rivera-Batiz and Romer, 1991), in which capital goods are direct inputs into R&D. In this class of models, investment-specific technological change lowers the user cost of research capital, so cheaper equipment raises the rate of innovation as well as the level of output. We add a second capital good to the research technology. Because structures also enter R&D, their rising relative price works against this force in two ways: it raises the cost of research facilities directly, and it tightens the resources available for innovation through the aggregate resource constraint. In our calibration these two channels account for about 30 percent of the structural drag.[5]

[3]The investment-specific technological change (ISTC) has also been used as an explanation for a wide range of macro trends, including the increasing skill premium (Krusell et al., 2000) and the falling labor share (Karabarbounis and Neiman, 2014).

[4]Our framework also engages with the cost-disease literature (Baumol, 1967). The ISTC literature has focused on falling equipment prices as a positive force on aggregate growth; the cost-disease literature has emphasized that rising relative prices in low-productivity sectors slow it. We connect the two by showing that structures play a cost-disease role on the capital side, raising the relative price of an essential input into both production and innovation. Our CES estimates further connect to this literature in a precise sense: the magnitude of the drag, and the rate at which it attenuates over time, depend on how easily the economy substitutes across input bundles with differential productivity growth.

[5]Our approach is also related to Foerster et al. (2022), who quantify how sectoral trends in TFP map into trend GDP growth in a sixteen-sector model with input-output and investment linkages. Construction emerges as an investment hub whose multiplier is three times its value-added share, and it accounts for 0.75 percentage points of the postwar decline in trend growth. They describe their sectoral multipliers as generalizing the investment-specific channel of Greenwood et al. (1997) to many interconnected capital-producing sectors. Our static capital-deepening term is that channel with two capital types and a single user sector. We differ in two respects. We identify structures-specific

Third, we contribute to the literature on relative prices and development accounting. Jones (1994) documents a negative relationship between the relative price of machinery and the rate of economic growth. Hsieh and Klenow (2007) argue that low real investment rates in poor countries stem from a high relative price of investment, and they already separate producer durables from structures: in the 1996 ICP benchmark, the price of durables is unrelated to income while the price of structures rises with it. Our cross-section extends their finding to later ICP rounds, in which the structures gradient strengthens. Our main addition is the time dimension. Over 1970–2020, equipment prices fell faster in rich countries, while structures prices rose in rich countries and stayed flat in poor ones. The counterfactual in Section 4 quantifies what this divergence implies for ISTC-driven growth in the two groups.

Finally, we provide a macroeconomic transmission mechanism for recent micro-founded evidence on construction sector productivity. Goolsbee and Syverson (2023) document a secular decline in physical value added per worker in the U.S. construction sector and call it a "strange and awful path." Using KLEMS data, we show that this decline accounts for about half of the rise in the relative price of construction output in the United States, and for 59 percent on average across thirteen KLEMS economies. Our model then quantifies what the resulting rise in structures prices costs the aggregate economy, in output and in ideas.

2 Data and Stylized Facts

2.1 Data

Our primary panel dataset for cross-country analysis is constructed by merging two sources: the International Comparison Program (ICP) and the Penn World Table (PWT). We work at the country-year level and focus on the 1950–2020 period, which provides the broadest coverage of the disaggregated investment price series required for our analysis. For the analysis of the United States, we use the Fixed Assets Accounts tables from the Bureau of Economic Analysis (BEA).[6]

The core explanatory variables are the prices of investment goods by asset type. We obtain these from the World Bank's ICP, which reports Purchasing Power Parities (PPPs) for detailed expenditure categories. Rather than relying on aggregate investment price indices, we extract PPP series for two distinct capital categories: equipment[7] and structures.[8] This disaggregation is

technological change from relative prices rather than from sectoral TFP, which lets the analysis travel to the many countries for which investment prices exist but industry-level productivity accounts do not. And we let structures enter R&D, so that their price affects the rate of innovation and not only the level of the capital stock.

[6]The definition of capital and the coverage of each dataset used in our analysis are discussed in Appendix A.

[7]We use the term 'equipment' consistently throughout. In the underlying data, related categories appear under varying labels (e.g., 'Machinery and Equipment', 'equipment'); we harmonize these into 'equipment'.

[8]We mainly focus on equipment and structures, which together account for about 80% of total investment and

central to our analysis because it allows us to trace heterogeneous movements in relative prices across capital types and to link them to differential ISTC.

Measures of aggregate economic performance and productivity are taken from PWT version 11.0 (Feenstra et al., 2015). We use output-side real GDP at current PPPs (cgdpo) as our main measure of aggregate output and the TFP series (ctfp) to characterize productivity. To account for cross-country differences in labor quality, we include the human capital index (hc) as a control.[9] Finally, we use total population (pop) to control for scale.

2.2 Stylized Facts

Fact 1: High-income countries benefit from falling equipment prices but face sharply rising structures prices relative to low-income countries.

We begin by examining trends in the relative prices of equipment and structures, utilizing data from the PWT. All price series are deflated by the GDP price index and normalized to 1 in 1970. We divide countries into three groups based on their 1970 GDP per capita: the top and bottom terciles are defined as the high-income group and low-income group, respectively.

As shown in Figure 1, the relative price of aggregate investment declines after 1970 for the United States (solid black line) and for both income groups (dashed medians). The low-income median (green dashed line) is above the high-income median (red dashed line) for much of the post-1980 period, indicating a modest but visible divergence.

However, disaggregating investment into equipment and structures reveals striking differences.[10] The left panel of Figure 2 plots the evolution of equipment prices. Equipment prices decline rapidly in both the United States and other high-income economies. Notably, in contrast to the aggregate investment price, the median equipment price for the high-income group falls significantly faster than that of the low-income group, which aligns with Lian et al. (2020).

The right panel of Figure 2 shows the evolution of relative prices for structures. Here, we observe a trend distinct from that of equipment. In this category, the United States exhibits a particularly steep increase, outpacing the general upward trend observed across the high-income group. By contrast, the low-income group displays a flat path.[11][12]

over 90% of the capital stock. As shown in Appendix B.1.1, their investment shares are stable over time and across regions.

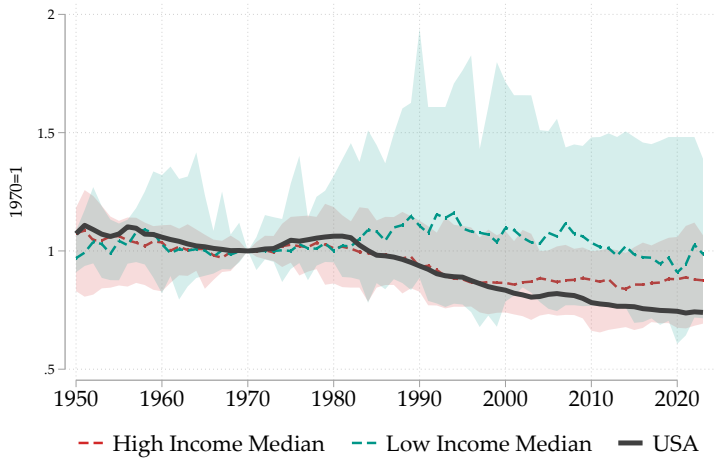
[9] We complement this measure with the PWT Labor Detail file, which provides more granular information on educational attainment. In particular, we use average years of schooling (yr_sch) primarily from Barro and Lee (2013).

[10] PWT Capital Detail reports four different types of capital: Structures, Machinery and Equipment, Transport Equipment, and Other.

[11] Trends for the remaining asset categories (transport equipment and other assets) are reported in Appendix B.1.2.

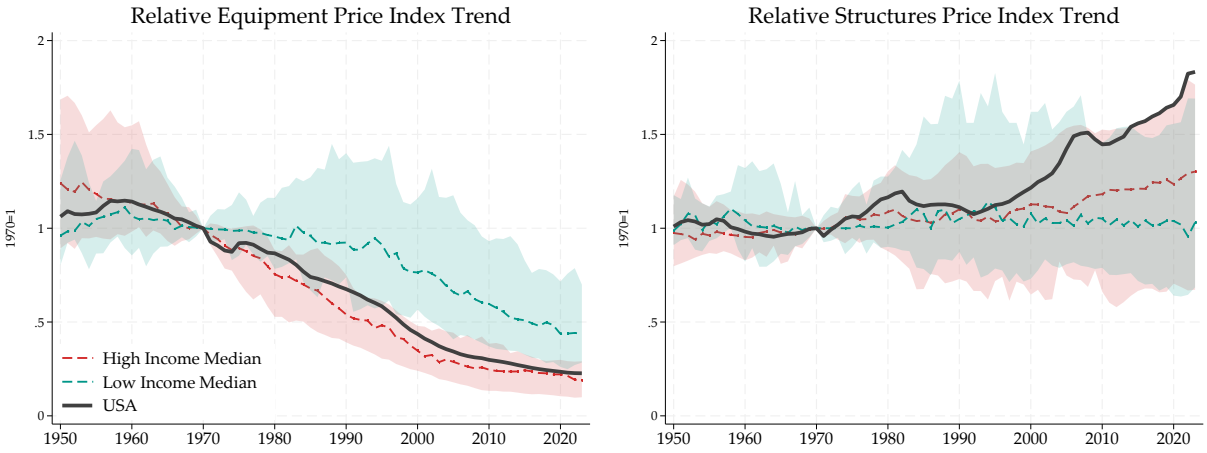
[12] We obtain similar patterns when using capital-stock-based price indices for both asset types, as reported in Appendix B.1.3.

Figure 1: **Relative Investment Price Index Trend (1950–2020)**



Notes. The relative price is calculated by dividing the investment price index by the GDP price index, normalized to 1 in 1970. High and low-income groups are defined based on the top and bottom terciles of real GDP per capita in 1970. A total of 120 countries with reported prices in 1970 are included in the analysis. The low-income group (indicated in green) and the high-income group (indicated in red) represent the bottom and top terciles, respectively. Shaded areas represent the 15th to 85th percentile range. Dashed lines indicate the median for each group, and the solid black line denotes the United States. *Sources:* Penn World Table 11.0; PWT Capital Detail; and authors' calculations.

Figure 2: **Relative Investment Price Index Trend by Asset Type (1950–2020)**



Notes. The relative price for each asset type is calculated by dividing each type's investment price index by the GDP price index, normalized to 1 in 1970. High and low-income groups are defined based on the top and bottom terciles of real GDP per capita in 1970. A total of 120 countries with reported prices in 1970 are included in the analysis. The low-income group (indicated in green) and the high-income group (indicated in red) represent the bottom and top terciles, respectively. Shaded areas represent the 15th to 85th percentile range. Dashed lines indicate the median for each group, and the solid black line denotes the United States as a benchmark. *Sources:* Penn World Table 11.0; PWT Capital Detail; and authors' calculations.

Because both series are deflated by the GDP deflator, a Balassa-Samuelson effect could appear here. In faster-growing economies, the price of nontradable structures would rise relative to the

price of tradable equipment. Sorting countries by their 1970–2020 growth rate rather than by their 1970 income confirms a mild version of this pattern, but it works entirely through equipment, whose relative price falls slightly faster in the fastest-growing tercile. The relative price of structures stays near its 1970 level in both the fastest- and slowest-growing terciles. The U.S. increase to 1.6 by 2020 is therefore not a growth effect (see Appendix Figure B.6).

In addition to the PWT, we also examine price trends using the ICP and confirm that the patterns we document in the PWT are also present in the ICP data. Following Hsieh and Klenow (2007), we normalize U.S. GDP per capita (measured in PPP terms) to one in each ICP survey year, and we likewise normalize each country's relative equipment price and relative structures price by the corresponding U.S. values to enable cross-country comparisons. Examining changes from 1996 to 2021, we find that countries' equipment prices relative to the United States increase on average, whereas relative structures prices decline steadily. Over the same period, the correlation between relative equipment prices and GDP per capita decreases from 0.02 to -0.08, while the correlation between relative structures price and GDP per capita increases from 0.26 to 0.66. Appendix B.1.5 reports the corresponding plots as well as the estimated coefficient changes in the cross-sectional regressions across survey years.

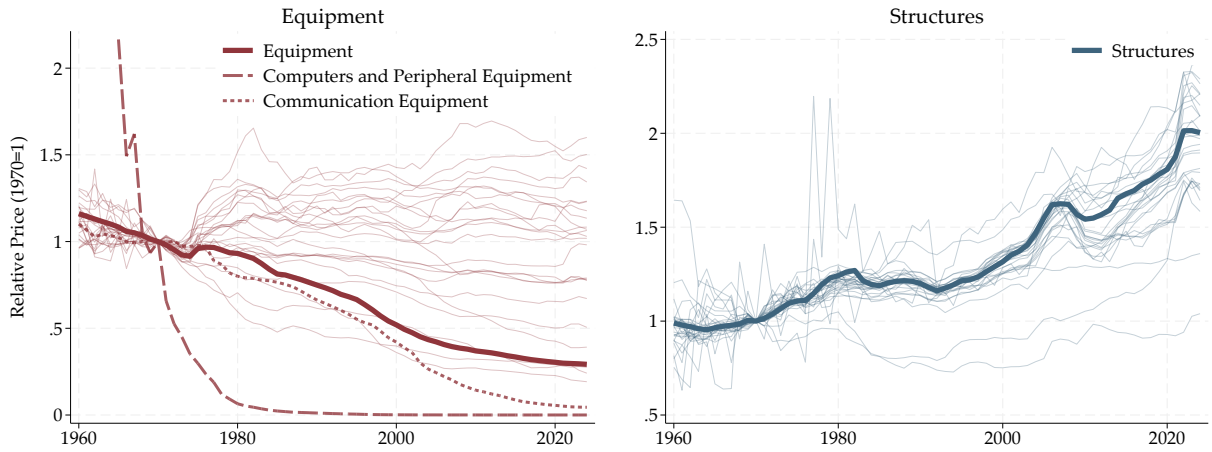
Finally, we provide additional evidence from cross-country regressions on the relationship between relative investment prices and GDP per capita. We show that lower equipment prices are strongly associated with higher GDP per capita, whereas structures prices exhibit a much weaker association with GDP per capita. Appendix B.1.6 provides details on the regression specifications and results.

Fact 2: In the U.S., the post-1970 decline in equipment prices was largely driven by computers and related items, whereas the post-1990 rise in structures prices was broad-based across nearly all types of structures.

We now narrow our focus to the United States and document the evolution of investment prices at the level of BEA asset types.[13] Figure 3 highlights two contrasting patterns in relative investment prices across BEA asset categories. First, the relative price of equipment exhibits a sharp and persistent decline after 1970. This decline is not uniform across equipment categories; rather, it is concentrated in information-processing assets. In particular, computers and peripheral equipment display the steepest and most sustained price declines, while communication equipment shows similarly large reductions. Together, these two categories account for about 20% of total equipment investment, and the broader category of information-processing

[13]BEA provides detailed asset-type price indexes, whereas the PWT Capital Detail provides only four categories: equipment, structures, transportation, and other assets. In Appendix A, we describe the coverage of each dataset in detail. In addition, Figure A.1 shows that the relative prices of equipment and structures reported by the BEA are broadly consistent with those in the PWT, which supports the use of both datasets.

Figure 3: **Relative Investment Price Index Trend by Asset Type (1960–2020)**



Notes. The figure plots trends in relative investment prices by BEA asset type. For each asset category, an implicit price index is recovered from BEA data as nominal investment divided by the corresponding chain-type quantity index. The resulting series is deflated using the GDP Deflator and then normalized to 1 in 1970 for each category. The sample is restricted to 1960—2020. Thin lines show disaggregated categories, while thicker highlighted lines emphasize the aggregate equipment series and selected subcomponents (information processing equipment, computers and peripheral equipment, and communication equipment). Details of the asset classes are provided in Table A.I. The petroleum and natural gas structures category is omitted from the right panel. *Sources:* U.S. Bureau of Economic Analysis (BEA), BEA Fixed Assets Accounts Tables 2.7 and 2.8; Gross Domestic Product: Implicit Price Deflator (GDPDEF) from Federal Reserve Bank of St. Louis (FRED); and authors' calculations.

equipment represents roughly 35 to 40%.[14] This concentration is consistent with rapid improvements in embodied technology and the associated quality-adjusted price declines emphasized in the investment-specific technological change literature (Jones, 1994; Byrne and Pinto, 2015; Lian et al., 2020).

Second, in sharp contrast, the relative price of structures rises markedly after 1990, as shown in the right panel of Figure 3. This increase is not driven by a single structure type: the disaggregated series move upward together, indicating a broad-based rise across nearly all categories. In addition, the trend is not driven by either residential or nonresidential structures. Figure B.9 in Appendix B.2 compares the trends in residential structures, nonresidential structures, and other related items and shows that the rise in relative structure prices occurs broadly across these categories. This pattern suggests that post-1990 structures price dynamics reflect an economy-wide shift in the structures component of investment rather than compositional change within structures.[15]

Table 1 summarizes, for 2020, the ten BEA asset categories with the highest investment shares

[14] Refer to Appendix Table A.I for each asset's relative price and its 2020 investment share within the corresponding capital segment, measured as the asset's investment divided by total asset-class investment.

[15] These results are robust to using BEA price indices constructed from the current-cost net stock series, as reported in Appendix B.2.

in each asset class and shows how total investment is allocated across these categories within the broad groups of equipment and structures. It also reports their relative prices in 2020, where each asset's real price in 1970 is normalized to 1. The table shows that equipment prices are largely driven by the rapid price declines of information and communication technology-related equipment, whereas structures exhibit similar patterns across sub-asset classes and are not concentrated in residential housing. All sub-asset classes are reported in Appendix Table A.I.

Table 1: **Investment Shares by BEA Asset Category and Relative Prices in 2020**

Asset Class	**Share**	**Price**	**Asset Class**	**Share**	**Price**
Equipment	100%	0.304	Structures	100%	1.808
Communication Equipment	10.96%	0.06	Housing (1 To 4 Unit)	21.47%	1.72
Computers (Peripheral Equip.)	10.92%	<0.01	Improvements	19.78%	1.55
Medical Equip.	9.57%	0.43	Brokers' Commission	12.90%	1.87
Light Trucks	9.45%	0.76	Office	6.17%	1.91
General Industrial Machinery	8.26%	1.06	Electric	5.66%	1.72
Other Nonresidential Equip.	6.45%	0.77	Housing (5 Or More)	5.54%	2.14
Electrical Transmissions	4.27%	0.76	Manufacturing	5.00%	1.74
Furniture And Fixtures	4.09%	1.01	Petroleum / Natural Gas	3.97%	4.40
Other Trucks, Buses, etc	3.80%	1.21	Warehouses	2.85%	1.87
Special Industry Machinery	3.79%	1.22	Lodging	2.28%	1.77

Notes: **Share** denotes each detailed BEA asset category's share of total 2020 investment within the corresponding asset type, Equipment or Structures. **Price** is the ratio of the asset's real price in 2020 to its real price in 1970, where real prices are nominal prices deflated by the GDP deflator and the 1970 level is normalized to 1. Within each broad group, categories are ordered in descending order of **Share**.

Fact 3: U.S. industries whose capital relied more on equipment than structures in 1970 experienced faster TFP and real investment growth in the following decades.

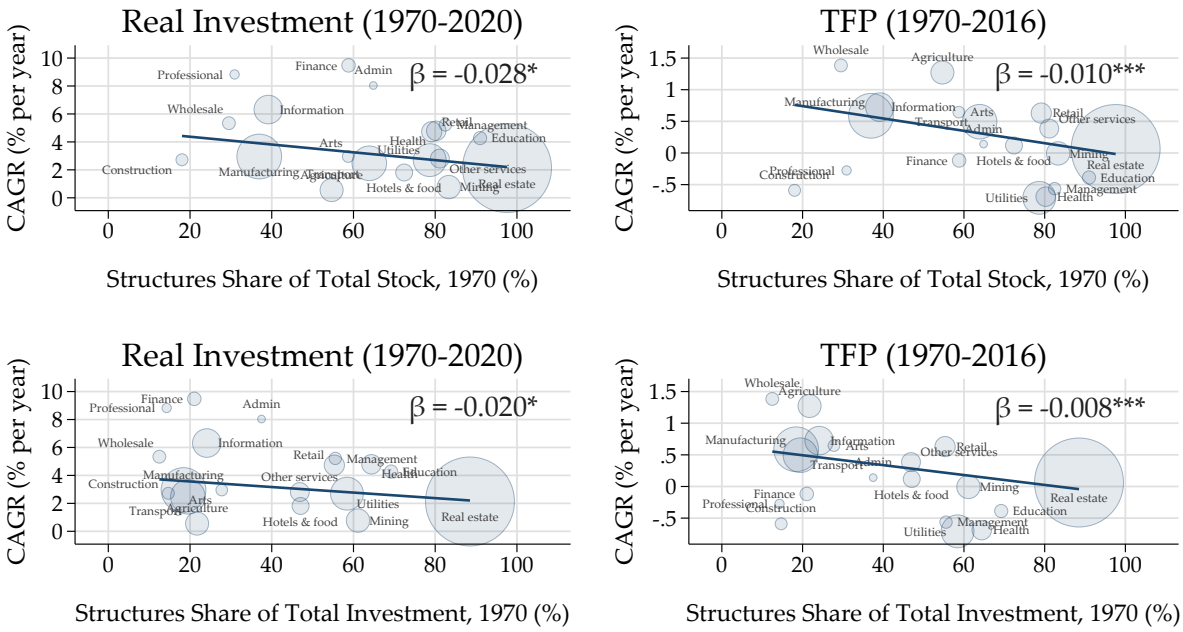
Given the fact that the price dynamics of equipment and structures diverge, we conduct a cross-sector analysis to examine whether sectors that relied more heavily on structures experienced lower TFP and investment growth. To do so, we use an approximately two-digit NAICS classification that provides us with 19 sectors. We first measure sector-level reliance on structures using two measures, the share of structures in total fixed capital stock and the share of structures in total fixed investment. In the baseline, both shares are measured in 1970.

Then, we calculate the TFP for each sector from KLEMS data for each year. In the Historical BEA–BLS KLEMS data[16], MFP corresponds to the TFP measure used in our analysis. Each sector's TFP is calculated by residualizing real gross-output growth with respect to the growth of capital, labor, and intermediate inputs using two-period average nominal cost shares, following

[16]See https://www.bea.gov/data/special-topics/integrated-industry-level-production-account-klems

the methodology of the dataset. We then aggregate the underlying output and inputs into the 19 sectors and recalculate the TFP residual at the sector level.[17] Finally, we accumulate annual TFP log growth and calculate its CAGR from 1970 to 2016, which is the most recent year available in the Historical KLEMS data. Additionally, we calculate each sector's 1970–2020 compound annual growth rate of real total fixed investment.

Figure 4: **Structures Reliance, TFP Growth, and Real Investment Growth**



Notes. The figure shows the cross-sector relationship between reliance on structures in 1970 and subsequent TFP and real fixed-investment growth across 19 two-digit NAICS sectors in the U.S. The upper panels measure reliance on structures as the structures share of total fixed capital stock, and the lower panels use the structures share of total fixed investment. The left panels report the CAGR of real fixed investment from 1970 to 2020, and the right panels report the CAGR of TFP from 1970 to 2016. Total fixed capital and investment include equipment, structures, and intellectual property products. Circle size represents total fixed capital stock in 1970. The fitted lines and reported β coefficients are estimated using OLS weighted by the same 1970 capital stock, with heteroskedasticity-robust standard errors. ***, **, and * denote statistical significance at the 1, 5, and 10 percent levels, respectively. *Source.* BEA–BLS Integrated Industry-Level Production Account, BEA Fixed Assets Accounts, and authors' calculations.

Figure 4 shows the association between a sector's reliance on structures in 1970 and its subsequent TFP growth over 1970–2016 and real fixed investment growth over 1970–2020. The upper panels measure structure dependence using the total stock, while the lower panels measure it

[17] We map the 61 KLEMS industries into 19 NAICS sectors and sum nominal gross output and input values across the member industries of each sector. We aggregate their log growth rates using two-period average nominal shares, construct the sector-level quantity indexes, and then recalculate the TFP residual from the aggregated sector production account. Thus, the 19-sector TFP measure is recalculated after aggregation. This is the same method we apply to replicate the average MFP growth reported in BEA–BLS Table 11.1 for the 61 underlying KLEMS industries over 1963–2016.

using total fixed investment. The right panels focus on TFP growth, and the left panels display real investment growth. The size of each circle represents the total stock of capital in 1970, and the coefficients are the OLS estimates for the two measures, with the 1970 capital stock used as a weight.

A 10 percentage point higher structures share of capital stock is associated with 0.1 percentage points lower annual TFP growth and 0.28 percentage points lower annual real investment growth. When reliance on structures is measured using the investment share, the corresponding estimates are 0.08 percentage points for TFP growth and 0.2 percentage points for real investment growth. The key message is that if a sector relied more on structures in either its total stock or investment, the sector experienced lower growth rates of TFP and real investment.

For robustness checks, we use the average of the annual shares from 1965 to 1975, and the 1970 shares calculated using Equipment plus Structures as the denominator. Appendix B.3 reports the results and shows that the relationship between structure reliance and subsequent TFP and investment growth is robust across these alternative measures.

2.3 Why Have Structures Prices Increased? Construction Productivity

A rapid increase in the price of structures after 1990 is consistent with a decline in construction productivity in the U.S. during the same period. Goolsbee and Syverson (2023) document that, while U.S. aggregate productivity (measured in value-added per worker) doubled between 1970 and 2020, it simultaneously fell by 40 percent in the construction sector. We find that the Bureau of Labor Statistics (BLS) productivity growth estimates for the construction sector are negative on average, −0.55% per year over the 1987 to 2024 period, while productivity growth averaged 0.89% per year for the private business sector.[18]

Several factors may have contributed to stagnant construction productivity. For example, Goolsbee and Syverson (2023) showed that construction firms' abilities to turn materials into output have deteriorated and there is little reallocation from low-productivity places to high-productivity places. D'Amico et al. (2024) find that the post-1970 productivity decline coincides with increases in proxies for land-use regulation. Local land-use controls limit the size of building projects, reducing both scale economies and incentives to invest in innovation.[19]

To assess the extent to which the rise in construction-sector prices reflects productivity decline, we use the U.S. data from the EU KLEMS dataset to decompose the relative output price

[18]The annual average productivity growth from 1987 to 2024 is 1.18% for Agriculture, 1.51% for Mining, 0.67% for Utilities, 1.15% for Durable manufacturing sector, 0.09% for Nondurable manufacturing sector, 1.10% for Trade, 0.58% for Transportation and warehousing, 1.01% for Information, -0.03% for Finance, insurance, real estate, and leasing, and 0.23% for Services.

[19]The productivity problem also appears in the public sector. Brooks and Liscow (2023) find that spending per mile in the Interstate Highway System increased more than threefold from the 1960s to the 1980s.

of construction into input cost and TFP components. EU KLEMS also offers harmonized cross-country coverage under a common industrial classification, making it well suited to a comparative analysis of construction-sector price dynamics within a unified framework.

We decompose the construction sector output price using the Törnqvist approach within a duality framework.[20] This decomposition follows from the duality between the production function and the cost function under constant returns to scale and competitive factor markets. The primal identity states that output growth equals the share-weighted sum of input growth and TFP growth. Its dual counterpart expresses output price growth as the share weighted sum of input price growth minus TFP growth. Accordingly, under constant returns to scale and competitive factor markets, the gross-output price of sector i, P_i, satisfies the following unit-cost identity:

$$\Delta \ln P_i = \bar{s}_{L,i} \Delta \ln P_{L,i} + \bar{s}_{K,i} \Delta \ln P_{K,i} + \bar{s}_{II,i} \Delta \ln P_{II,i} - \Delta \ln A_i, \tag{1}$$

where $\bar{s}_{j,i}$ denotes the Törnqvist two-period average cost share of input j in gross output, $P_{j,i}$ denotes the price of input j, and $\Delta \ln A_i$ denotes TFP growth in sector i. Subtracting $\Delta \ln P_{agg}$ from both sides yields the following expression

$$\Delta \ln \left(\frac{P_i}{P_{agg}} \right) = \text{rel}_{L,i} + \text{rel}_{K,i} + \text{rel}_{II,i} - \Delta \ln A_i,$$

where each relative input cost term is defined as $\text{rel}_{j,i} \equiv \bar{s}_{j,i} \left(\Delta \ln P_{j,i} - \Delta \ln P_{agg} \right)$. Accordingly, each relative cost term measures the contribution of sector specific input price movements, relative to the aggregate price level, to the sector's relative output price change.[21]

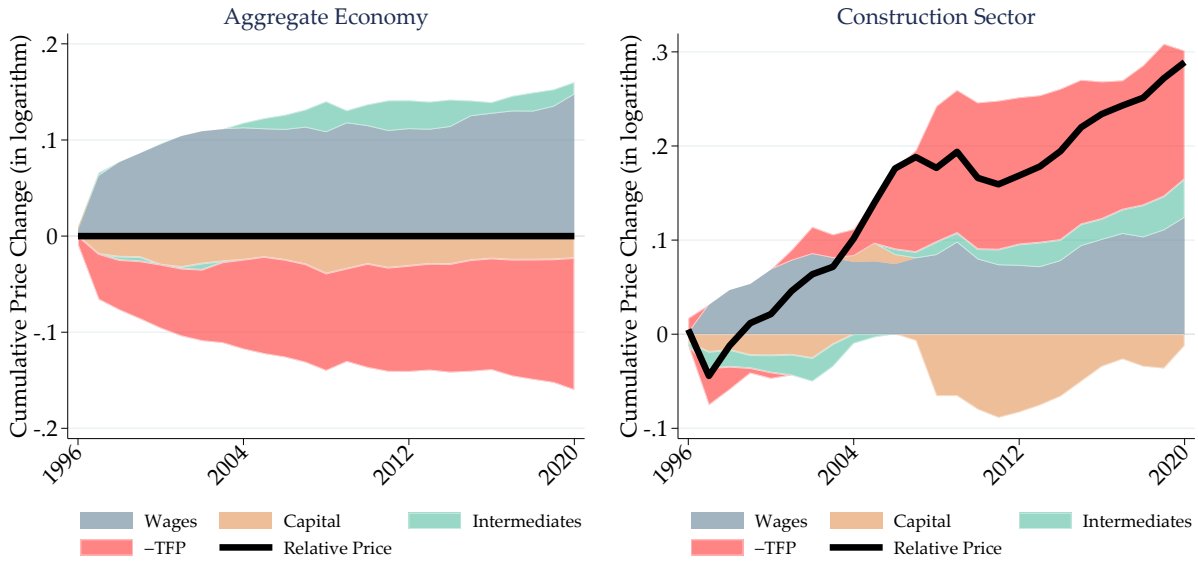
Figure 5 presents a decomposition of the cumulative change in output prices, $\sum \Delta \ln(P_i/P_{agg})$, using 1996 as the base year and cumulating annual changes through 2020, into relative input-cost components and TFP for both the aggregate economy and the construction sector. The aggregate level decomposition is included for comparison with the construction sector. Note that, by construction, the decomposition in the aggregate output-price panel, $\sum \Delta \ln(P_{agg}/P_{agg})$, sums to zero.

The aggregate panel on the left provides a useful benchmark. Although labor costs make a positive contribution at the aggregate level, this upward pressure is offset by TFP growth and

[20] Additional details on the Törnqvist approach are provided in Appendix C.1. Because the Törnqvist index has a duality foundation, we use the dual approach for two reasons. First, our question is price based, since it asks why construction prices have risen faster than the general price level. The dual formulation maps this directly into cost drivers such as labor, capital, intermediate inputs, and productivity. Second, industry output quantities are often measured with greater error or constructed by deflating nominal values, while input prices and cost shares are generally more reliable in the national accounts.

[21] Labor contribution ($rel_{L,i}$) reflects sector specific labor cost pressure relative to the aggregate price level, capital contribution ($rel_{K,i}$) reflects relative capital service cost pressure, and intermediate input contribution ($rel_{II,i}$) reflects relative price pressure from purchased materials and services. Positive values indicate that the corresponding input cost rises faster than the aggregate benchmark and therefore pushes the sector's relative output price upward.

Figure 5: **Decomposition of Relative Prices: Aggregate vs. Construction Sector**



Notes. The figure presents a cumulative decomposition of sectoral gross-output prices relative to the aggregate economy in the United States within the Törnqvist dual framework. The left panel reports the aggregate economy, while the right panel presents the construction sector. Each shaded area denotes the cumulative log-point contribution of a given channel to the change in a sector's gross-output price relative to the aggregate economy since the beginning of the sample. The productivity component, $(-\Delta \ln A)$, is defined as the negative of the dual total factor productivity residual. A negative value indicates that productivity growth offsets input-cost pressures and lowers the relative price. Accordingly, the −TFP area captures the contribution of productivity shortfalls rather than productivity gains. The black line, $\sum \Delta \ln(P_i/P_{agg})$, traces the cumulative change in the sector's gross-output price relative to the aggregate economy. *Sources:* EU KLEMS 2024 and authors' calculations.

movements in capital costs. The construction sector presents a clear contrast. In the construction sector, the positive contribution of labor costs is not accompanied by an offsetting productivity component. Instead, the contribution of negative TFP is itself strongly positive, indicating that weak productivity growth amplified rather than absorbed cost pressures. Intermediate input costs also push relative prices upward, while the capital component is negative over part of the sample and therefore provides only limited relief.[22]

Cross-country Evidence We repeat the same exercise for other countries using KLEMS data to examine whether the role of TFP in driving construction price increases is specific to the United States. Figure 6 plots each country's cumulative TFP growth against its cumulative change in the relative construction price, accumulated from 1996 through 2021 (2020 for the United States, the United Kingdom, and Japan; 1999–2021 for Belgium), and provides a cross-sectional sum-

[22]This pattern is also consistent with the value-added growth decomposition reported in Appendix C.2. At the aggregate level, value added growth is driven largely by TFP growth, whereas in construction TFP acts as a drag on value added growth.

mary of their correlation.[23] The figure shows that cumulative construction-sector TFP growth is negatively correlated with cumulative changes in relative construction prices across countries. This negative relationship is expected under the unit-cost identity above: for given input-price growth, slower TFP growth passes one for one into faster output-price growth. Because TFP is a residual, any secular change in construction markups is absorbed into measured TFP, so the figure decomposes price growth rather than testing a hypothesis. Most countries experienced both an increase in relative construction prices and a decline in cumulative construction-sector TFP; Belgium is the one exception, with both magnitudes close to zero. On average across those thirteen economies, the decline in TFP accounts for 59% of the increase in relative construction prices, while the corresponding share for the United States is 47%.[24]

These results suggest that the rise in relative prices in the construction sector cannot be understood simply as an increase in input costs. Rather, the key distinction from the aggregate economy is the absence of offsetting productivity growth in construction. In this sense, the results are consistent with the view that the increase in construction prices after 1990 is closely linked to the productivity slowdown in the U.S. construction sector (Goolsbee and Syverson, 2023), which raises the effective cost of accumulating structures.

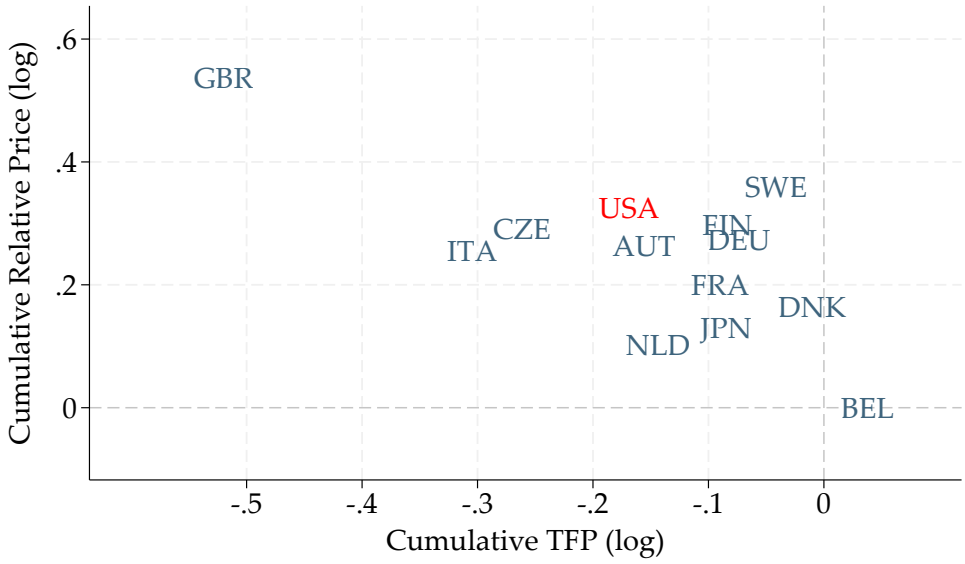
3 The Model

Motivated by these findings, we develop an endogenous growth framework that disaggregates capital into equipment and structures. Building on Romer (1990), we adopt the Rivera-Batiz and Romer (1991) "lab-equipment" specification, in which capital is not only essential for final-goods production but also enters as an input in the R&D sector. Within this framework, we characterize the balanced growth path.

[23] The cross section includes 13 economies, the United States, Japan, Germany, France, the United Kingdom, Italy, Austria, Belgium, the Czech Republic, Denmark, Finland, the Netherlands, and Sweden. The cumulative window is 1996–2021, except for the United States, the United Kingdom, and Japan, which end in 2020, and Belgium, which begins in 1999 because its construction labor quantity index is unavailable earlier. We use gross-output decompositions except for the United Kingdom and Italy, for which EU KLEMS 2024 does not provide continuous gross-output deflators, so we retain these reference economies with the value-added analogue. Bulgaria, Cyprus, Ireland, Lithuania, Malta, Slovakia, Slovenia, and Spain are excluded for the same gross-output limitation. Croatia, Hungary, Poland, Portugal, and Romania lack construction labor and capital quantity indexes, while Estonia, Greece, Latvia, and Luxembourg begin only in 2008 for construction labor quantity.

[24] Appendix C.3 further decomposes and plots cumulative changes in relative construction prices into cost-pressure and TFP contributions for the United States, Japan, Germany, France, Italy, and the United Kingdom. The results show that, regardless of the accumulation period considered, declines in construction-sector TFP consistently contribute to increases in relative construction prices across major economies. This pattern is therefore not confined to a particular year, but reflects a persistent cross-country feature.

Figure 6: **Cross-Country Construction Relative Prices vs. Productivity**



Notes. The figure plots each country's cumulative TFP growth against its cumulative change in the relative construction price, accumulated from 1996 through 2021 (2020 for the United States, the United Kingdom, and Japan; 1999–2021 for Belgium). The cross section covers the United States, Japan, Germany, France, the United Kingdom, Italy, Austria, Belgium, the Czech Republic, Denmark, Finland, the Netherlands, and Sweden. *Source.* EU KLEMS 2024 and authors' calculations.

3.1 Capital Accumulation and ISTC

The economy features two distinct capital stocks, equipment K_E and structures K_S. Investment expenditures $I_E(t)$ and $I_S(t)$ are measured in units of the final good, which is the numeraire. Following Greenwood et al. (1997), $q_E(t)$ and $q_S(t)$ denote the efficiency with which one unit of final output is transformed into physical capital. The relative prices of the two capital goods are

$$P_E(t) \equiv \frac{1}{q_E(t)}, \qquad P_S(t) \equiv \frac{1}{q_S(t)}. \tag{2}$$

Capital accumulation satisfies

$$\dot{K}_E(t) = q_E(t) I_E(t) - \delta_E K_E(t), \tag{3}$$

$$\dot{K}_S(t) = q_S(t) I_S(t) - \delta_S K_S(t). \tag{4}$$

Investment efficiency evolves exogenously at constant rates

$$\frac{\dot{q}_E(t)}{q_E(t)} = \gamma_E, \qquad \frac{\dot{q}_S(t)}{q_S(t)} = \gamma_S. \tag{5}$$

Equipment and structures are allocated between final production and R&D

$$K_E(t) = K_E^Y(t) + K_E^R(t), \qquad K_S(t) = K_S^Y(t) + K_S^R(t). \tag{6}$$

3.2 Firms

3.2.1 Final-Goods Sector

A representative competitive final-good firm produces

$$Y(t) = A(t)\big(K_E^Y(t)\big)^{\alpha_E}\big(K_S^Y(t)\big)^{\alpha_S}\big(L_u(t)\big)^{1-\alpha_E-\alpha_S} \tag{7}$$

by choosing equipment, structures, and unskilled labor. Aggregate knowledge $A(t)$ is a nonrival stock of productive knowledge generated by past R&D. The final-good firm takes $A(t)$ as given when choosing current inputs.

The firm solves

$$\max_{K_E^Y, K_S^Y, L_u} \left\{ Y(t) - R_E(t)K_E^Y(t) - R_S(t)K_S^Y(t) - w_u(t)L_u(t) \right\}. \tag{8}$$

The first-order conditions are

$$R_E(t) = \alpha_E \frac{Y(t)}{K_E^Y(t)}, \qquad R_S(t) = \alpha_S \frac{Y(t)}{K_S^Y(t)}, \qquad w_u(t) = \big(1-\alpha_E-\alpha_S\big)\frac{Y(t)}{L_u(t)}. \tag{9}$$

3.2.2 R&D Sector

The competitive R&D sector produces new aggregate knowledge using equipment, structures, and skilled labor according to

$$\dot{A}(t) = A(t)^{\phi}\big(K_E^R(t)\big)^{\eta_E}\big(K_S^R(t)\big)^{\eta_S}\big(L_s(t)\big)^{1-\eta_E-\eta_S}. \tag{10}$$

The existing stock $A(t)$ affects current research productivity and, under the baseline calibration, lowers research output for fixed research inputs and is taken as given when current R&D inputs are chosen, as in Jones (1995).

Following Akcigit et al. (2022), the government rewards innovation directly and this policy is captured by $p_A(t)$ paid for each verified unit of new aggregate knowledge.[25] The reward $p_A(t)$

[25] The baseline formulation abstracts from an explicit monopoly markup to keep the decentralized implementation parsimonious, while Appendix D.6 provides a rigorous treatment. Introducing a standard patent-based intermediate-goods sector would not change the balanced-growth results as long as the associated static monopoly distortion is offset so that final-good producers face the competitive user price. In that case, the markup affects patent rents and the level of the innovation reward, but it does not alter the growth rates of factor prices or the constant sectoral

compensates the R&D sector for the value created by innovation while the accumulated stock $A(t)$ remains freely available in final production.

Taking $A(t)$, $p_A(t)$, and factor prices as given, the competitive R&D sector solves

$$\max_{K_E^R, K_S^R, L_s} \left\{ p_A(t)\dot{A}(t) - R_E(t)K_E^R(t) - R_S(t)K_S^R(t) - w_s(t)L_s(t) \right\}. \tag{11}$$

The first-order conditions are

$$R_E(t) = \eta_E \frac{p_A(t)\dot{A}(t)}{K_E^R(t)}, \qquad R_S(t) = \eta_S \frac{p_A(t)\dot{A}(t)}{K_S^R(t)}, \qquad w_s(t) = \left(1 - \eta_E - \eta_S\right) \frac{p_A(t)\dot{A}(t)}{L_s(t)}. \tag{12}$$

Constant returns in equipment, structures, and skilled labor imply

$$p_A(t)\dot{A}(t) = R_E(t)K_E^R(t) + R_S(t)K_S^R(t) + w_s(t)L_s(t). \tag{13}$$

The equilibrium reward schedule is characterized in Appendix D.

3.3 Households, Government, and Market Clearing

Population grows at the constant rate n

$$\dot{N}(t) = nN(t). \tag{14}$$

Households. A fixed fraction $s \in (0,1)$ of the population supplies skilled labor to the R&D sector, while the remaining fraction supplies unskilled labor to final production

$$L_s(t) = sN(t), \qquad L_u(t) = (1-s)N(t). \tag{15}$$

Households own the physical capital stocks, supply skilled and unskilled labor, and pay a lump-sum tax $T_A(t)$. The representative household maximizes

$$U_0 = \int_0^\infty e^{-\rho t} N(t) \frac{c(t)^{1-\sigma}}{1-\sigma}\, dt, \qquad c(t) = \frac{C(t)}{N(t)}. \tag{16}$$

subject to

$$C(t) + I_E(t) + I_S(t) + T_A(t) = w_u(t)L_u(t) + w_s(t)L_s(t) + R_E(t)K_E(t) + R_S(t)K_S(t). \tag{17}$$

allocation ratios that determine the BGP.

The household Euler equation is

$$\frac{\dot{c}(t)}{c(t)} = \frac{1}{\sigma}\big(r(t) - \rho\big). \tag{18}$$

No arbitrage between financial wealth and physical capital implies

$$r(t) = q_E(t)R_E(t) - \gamma_E - \delta_E, \tag{19}$$

$$r(t) = q_S(t)R_S(t) - \gamma_S - \delta_S. \tag{20}$$

Government. Let $G_A(t)$ denote aggregate innovation-reward payments to the R&D sector. The government finances the reward payments with the lump-sum tax collected from households

$$T_A(t) = G_A(t). \tag{21}$$

The aggregate reward payment is

$$G_A(t) = p_A(t)\dot{A}(t). \tag{22}$$

Market Clearing. Together with capital-market clearing and equation (17), the market clearing condition can be derived as

$$C(t) + I_E(t) + I_S(t) = Y(t). \tag{23}$$

3.4 Balanced Growth Path (BGP)

To obtain a regular solution to the balanced-growth equations, we impose the standard semi-endogenous restriction (see Appendix D for the full derivation):

$$\Delta \equiv (1-\phi)(1-\alpha_E-\alpha_S) - (\eta_E+\eta_S) > 0. \tag{24}$$

On a BGP, aggregate quantities $(Y, C, I_E, I_S, K_E, K_S)$ grow at constant rates, and allocation shares remain constant. In particular, capital growth rates satisfy

$$g_{K_E} = g_Y + \gamma_E, \qquad g_{K_S} = g_Y + \gamma_S,$$

given constant r and, given the fixed labor allocation rule, labor in both sectors grows at the population rate,

$$\frac{\dot{L}_s}{L_s} = \frac{\dot{L}_u}{L_u} = n.$$

Log-differentiating the final-goods and innovation technologies yields the BGP system:

$$g_Y = g_A + \alpha_E(g_Y + \gamma_E) + \alpha_S(g_Y + \gamma_S) + (1 - \alpha_E - \alpha_S)n, \tag{25}$$

$$(1 - \phi)g_A = (\eta_E + \eta_S)g_Y + \eta_E\gamma_E + \eta_S\gamma_S + (1 - \eta_E - \eta_S)n. \tag{26}$$

Solving (25) for g_Y delivers a decomposition:

$$g_Y = n + \underbrace{\frac{1}{1 - \alpha_E - \alpha_S} g_A}_{\text{TFP contribution}} + \underbrace{\frac{\alpha_E\gamma_E + \alpha_S\gamma_S}{1 - \alpha_E - \alpha_S}}_{\text{ISTC-induced deepening}}. \tag{27}$$

Substituting (27) into (26) yields the endogenous TFP growth rate:

$$g_A = \frac{(1 - \alpha_E - \alpha_S)n + \big(\eta_E(1 - \alpha_S) + \eta_S\alpha_E\big)\gamma_E + \big(\eta_S(1 - \alpha_E) + \eta_E\alpha_S\big)\gamma_S}{(1 - \phi)(1 - \alpha_E - \alpha_S) - (\eta_E + \eta_S)}. \tag{28}$$

3.5 Equivalence to an Explicit Construction Sector

A natural question is whether our treatment of structures-specific technological change misses an important margin that would arise if an explicit construction sector instead produced structures. We show that the baseline results are unchanged when structures are produced by a competitive construction sector that converts the final good into new structures using construction productivity. Under these conditions, the explicit construction-sector economy is exactly equivalent to the baseline model. It generates the same relative price of structures, capital accumulation, equilibrium allocations, and balanced-growth path. Consequently, all subsequent quantitative implications of the baseline model, including the structural drag from rising structures prices, are unchanged. Appendix D.7 establishes this equivalence formally and relates the construction-sector representation to the KLEMS decomposition.

4 Quantitative Analysis

4.1 Calibration

We calibrate the model to match U.S. long-run moments and parameter estimates from the literature, using transparent mappings from data to model objects. In particular, we construct the exogenous trends that pin down the BGP by targeting population growth and ISTC.

The population growth rate n is set to the average growth rate of total population in the PWT from 1970 to 2020. We proxy asset-type ISTC with relative-price trends in BEA. The equipment ISTC rate γ_E is set to the average annual decline in the relative price of equipment, while the

structures ISTC rate γ_S is set analogously using the relative price of structures. Over our sample, from 1970 to 2020, this implies around $\gamma_E = 0.024$ and $\gamma_S = -0.012$, consistent with a rapidly falling relative price of equipment and a rising relative price of structures.

On the production side, we calibrate the Cobb-Douglas factor elasticities by combining information on the aggregate labor income share and the composition of investment. Using the PWT Labor Detail report, we set the labor income share to 0.62, which implies an aggregate capital share of 0.38, reported in 2015. We then split this aggregate capital share between equipment and structures using the U.S. average equipment share within the sum of equipment and structures investment from the PWT Capital Detail data. Specifically, the equipment share is 0.39, so that the output elasticity of equipment is calibrated to $\alpha_E = 0.15$, while the output elasticity of structures is calibrated to $\alpha_S = 0.23$. We use this investment composition as a simple empirical proxy to split the aggregate capital income share across asset types.[26]

Turning to the innovation block, we set the intertemporal knowledge-spillover parameter to $\phi = -1.45$ to match the TFP growth measured by the Bureau of Labor Statistics.[27] We discipline the Cobb-Douglas input elasticities in the R&D production function using factor-share evidence from the National Center for Science and Engineering Statistics (NCSES) Business Enterprise Research and Development (BERD) data (Égert, 2016; Moretti et al., 2025). Using BERD data from 2015, we set the labor share in R&D to 0.67, which implies an R&D capital share of 0.33. We allocate this R&D capital share across equipment and structures using the observed equipment share in R&D capital spending, 0.79, which implies that equipment accounts for $\eta_E = 0.26$ of the R&D technology, while structures account for the remaining $\eta_S = 0.07$. Importantly, the BERD capital-expenditure measure pertains specifically to assets used in R&D operations. Its structures category covers purchased or improved buildings and other fixed facilities used for R&D, such as signal towers and windmills, and is therefore unlikely to include residential housing. Appendix A.4 provides further details on the BERD capital-expenditure classification.[28] Table 2

[26] In the United States, the equipment share is about 38.8% in 2015, which pins down α_E at 0.1469. Figure B.2 shows that the equipment share is stable across countries despite variation in income, implying that the allocation of investment between equipment and structures is relatively stable across countries as well. Finally, Figure B.3 shows the U.S. trend in the share of equipment investment out of total investment in equipment and structures, which remains stable.

[27] Bloom et al. (2020) estimate ϕ to be around -2 using a semi-endogenous growth model with U.S. aggregate data from 1930 to 2015, where TFP growth measures idea output and wage-deflated NIPA intellectual property investment measures research input. We do not directly adopt this estimate because their measures of research input and aggregate TFP do not map directly into our model. Instead, we calibrate ϕ to match TFP within our framework. As a robustness check, setting ϕ to -2 rather than our baseline value of -1.45 leaves the results largely unchanged, with the structural drag changing from 0.63 to 0.59 percentage points. See Table 5.

[28] We use the observed composition of R&D capital spending as a proxy to allocate the aggregate R&D capital share across equipment and structures. Figure A.2 shows the time series of the equipment investment share in the R&D sector, and the share has remained around 80% since 2015. Figure A.3 illustrates that the composition of R&D capital expenditures is stable across countries as well, with equipment accounting for roughly 80% of total equipment-plus-structures spending over a wide range of income levels.

reports all the calibrated parameters used in the numerical analysis.

Table 2: **Calibration of Parameters for the U.S. Economy**

Symbol	Description	Value	Source
	Demographics & Labor		
n	Population growth rate	0.0098	PWT (1970–2020)
	ISTC		
γ_E	Inverse of relative Equipment price growth (TFP)	0.0238	BEA (1970–2020)
γ_S	Inverse of relative Structures price growth (TFP)	−0.0118	BEA (1970–2020)
δ_E	Depreciation Rate of Equipment	0.130	BEA (1970–2020)
δ_S	Depreciation Rate of Structures	0.030	BEA (1970–2020)
	Production Technology		
α_E	Share of Equipment in Output	0.1469	PWT (2015)
α_S	Share of Structures in Output	0.2312	PWT (2015)
	R&D Sector Parameters		
ϕ	Knowledge spillover parameter	−1.4535	Target TFP
η_E	Share of Equipment in R&D	0.2607	BERD (2015)
η_S	Share of Structures in R&D	0.0699	BERD (2015)

Notes. The model is calibrated at annual frequency. The table summarizes the baseline parameterization for the U.S. economy. Together, these values discipline the balanced-growth implications used in the quantitative exercises.

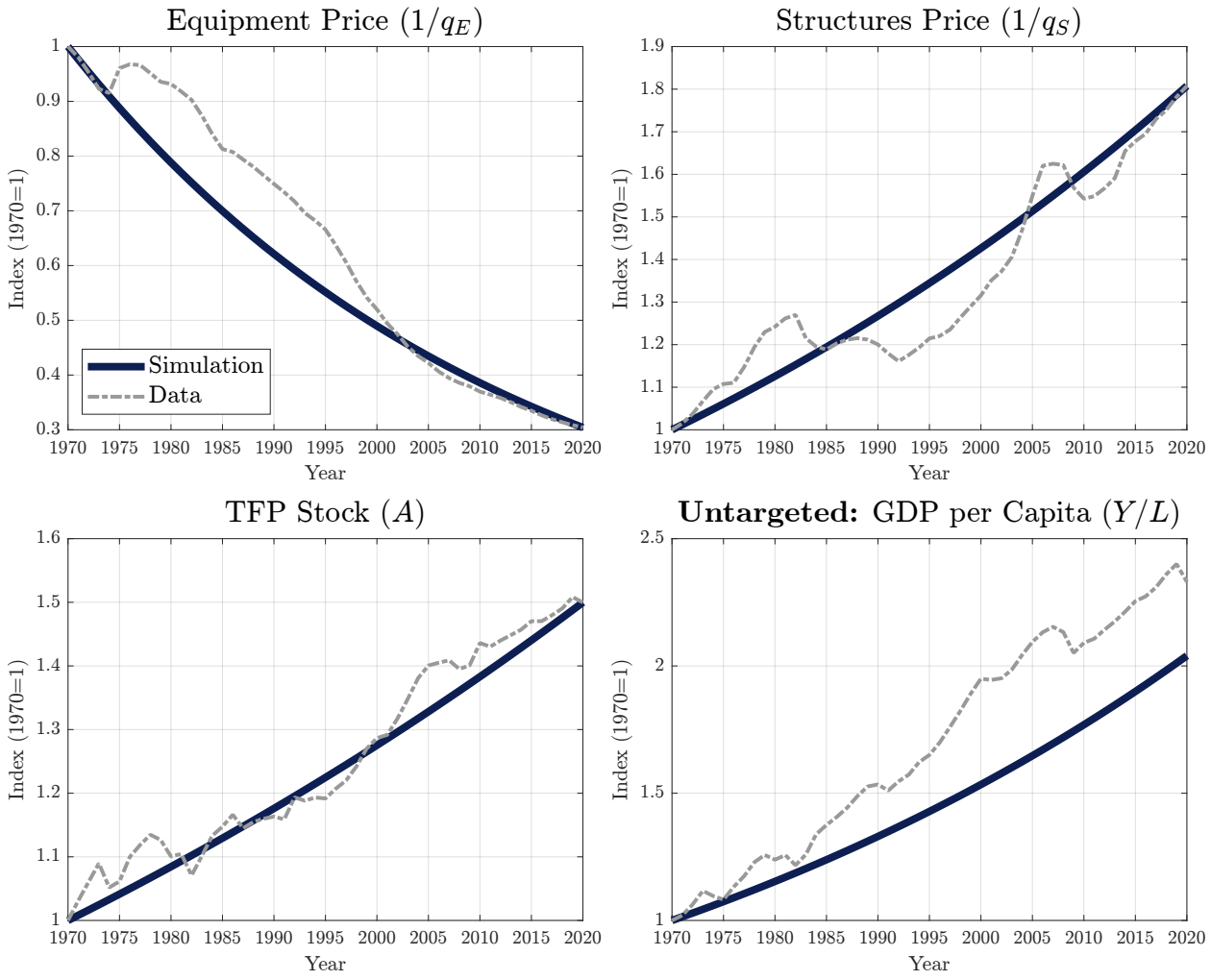
4.2 Model Performance

We first assess the calibrated model's fit to U.S. growth from 1970 to 2020, then quantify the impact of structural drag on long-run growth. Figure 7 compares the simulated series with the empirical data.

Targeted Moments: Asset-Type Prices and TFP The top panels of Figure 7 show the relative prices of equipment ($1/q_E$) and structures ($1/q_S$). These series are the primary exogenous drivers of the model, with γ_E and γ_S calibrated to match the empirical trends. The simulation closely tracks the steady decline in equipment costs and the secular rise in structures prices, reflecting the ISTC imbalance observed in the U.S. data. Given the other structural parameters, including γ_E and γ_S, we calibrate ϕ to match the TFP stock calculated by the BLS.

Untargeted Moments: GDP per capita The bottom-right panel presents the untargeted moment, GDP per capita (Y/N). The empirical benchmarks are sourced from the PWT. The calibrated

Figure 7: **Baseline Simulation. Asset-Type Prices and Macroeconomic Aggregates**



Notes. Each panel overlays the data and simulated series. The top panels show the targeted relative prices of equipment $(1/q_E)$ and structures $(1/q_S)$, while the bottom-left panel shows the targeted TFP stock (A). The bottom-right panel shows GDP per capita (Y/N), which is not targeted in the calibration. The asset-type relative price series are taken from BEA and GDP per capita is taken from the Penn World Table (PWT), while the TFP stock is obtained from the Bureau of Labor Statistics (BLS) (Private Business Sector. Total Factor Productivity MFPPBS in FRED). All series are normalized to 1 in 1970.

model closely matches the observed growth paths of GDP per capita.

4.2.1 Benchmark Comparisons

Our model introduces two new margins relative to standard growth frameworks: capital heterogeneity (equipment versus structures) and the use of capital in the R&D sector. To assess the quantitative importance of these margins, we compare our baseline specification to commonly used benchmarks, including a neoclassical one-capital growth model and a Romer-type model.

We show that our model delivers a GDP per capita growth rate that is close to the U.S. data and compares favorably with the alternative specifications. Additional details and model comparisons are provided in Appendix E.

4.3 Structural Drag Effect on Growth

To quantify the long-run impact of structural drag, we decompose balanced-growth per-capita output growth. Combining (27) with the closed-form expression for g_A in (28), and defining per-capita output growth as $g_y \equiv g_Y - n$, yields the linear representation

$$g_y = \mu_n^y \cdot n + \mathcal{M}_E \cdot \gamma_E + \mathcal{M}_S \cdot \gamma_S. \tag{29}$$

The coefficients in (29) are interpretable multipliers that map observed trend components into long-run per-capita growth. Grouping terms by n, γ_E, and γ_S implies

$$\mu_n^y \equiv \frac{1}{\Delta}, \qquad \mathcal{M}_E \equiv \frac{\alpha_E}{\Psi} + \frac{\eta_E(1-\alpha_S)+\eta_S\alpha_E}{\Psi\,\Delta}, \qquad \mathcal{M}_S \equiv \frac{\alpha_S}{\Psi} + \frac{\eta_S(1-\alpha_E)+\eta_E\alpha_S}{\Psi\,\Delta}, \tag{30}$$

where $\Psi \equiv 1-\alpha_E-\alpha_S$ and $\Delta \equiv (1-\phi)\Psi - (\eta_E+\eta_S)$ as in (24). We maintain the regularity condition $\Delta > 0$.

Per-capita output growth can be decomposed into three parts. The population growth multiplier $\mu_n^y \equiv 1/\Delta$ captures a pure R&D scale (induced-innovation) effect (Romer, 1990; Jones, 1995). The equipment ISTC term, $\mathcal{M}_E\gamma_E$, captures the equipment contribution. When $\gamma_E > 0$, equipment efficiency growth raises per-capita output growth through capital deepening and through induced innovation. The structures term, $\mathcal{M}_S\gamma_S$, captures the structures contribution. When structures ISTC is negative, as in the U.S. case, this term turns negative and depresses long-run per-capita growth.

Each multiplier $\mathcal{M}_k$ has a transparent two-part structure. The terms α_k/Ψ capture the static capital-deepening channel through the final-goods technology. The remaining terms, proportional to $1/(\Psi\Delta)$, capture the induced-innovation channel that operates through the equilibrium idea-growth response embedded in (28).

Focusing on the structures contribution, we can rewrite $\mathcal{M}_S$ as the sum of three interpretable components: (i) capital deepening in final-goods production, (ii) the direct cost of structures used as R&D facilities, and (iii) an indirect resource-allocation channel that operates through the

economy-wide constraint on innovation:

$$\mathcal{M}_S = \underbrace{\frac{\alpha_S}{\Psi}}_{\text{Static Deepening}} + \underbrace{\underbrace{\frac{\eta_S(1-\alpha_E)}{\Psi\Delta}}_{\text{Direct Input Channel}} + \underbrace{\frac{\eta_E\alpha_S}{\Psi\Delta}}_{\text{Indirect Resource Channel}}}_{\text{R\&D-related Structural Drag}} \tag{31}$$

A decline in structures ISTC, i.e., a lower γ_S, reduces growth not only by lowering output and weakening incentives for capital accumulation in the final-goods sector (the static-deepening component), but also by increasing R&D-related structural drag through two mechanisms: it raises the effective user cost of structures required for R&D activity (the direct input channel) and, through the general-equilibrium resource constraint, tightens the economy-wide availability of resources for innovation (the indirect resource channel). Equation (31) quantifies the contribution of each channel to the overall drag. In particular, the term $\eta_S(1-\alpha_E)/(\Psi\Delta)$ captures the direct input channel because it is proportional to η_S and therefore arises only when structures enter the R&D technology as an input (e.g., facilities). The term $\eta_E\alpha_S/(\Psi\Delta)$ captures the indirect resource channel because it loads on α_S: changes in structures affect aggregate production incentives and, via the final-goods resource constraint, the equilibrium scale of research inputs.

Under the U.S. calibration, we decompose balanced-growth per-capita output growth by applying (29) together with the structures-channel split in (31). Table 3 reports the resulting contributions in percentage points (pp) per year.

Table 3: **U.S. Per-Capita Output Growth g_y Decomposition**

Component	Growth rate (pp/year)	Share of g_y (%)
Population ($\mu_n^y n$)	0.82	57.5
Equipment Boost ($\mathcal{M}_E\gamma_E$)	1.24	86.8
Structural Drag ($\mathcal{M}_S\gamma_S$)	-0.63	-44.3
Total g_y	**1.43**	**100.0**

Notes. This table decomposes U.S. per-capita output growth, g_y, into three additive components. The population term, $\mu_n^y n$, captures the contribution of population growth scaled by the model-implied multiplier. The equipment component, $\mathcal{M}_E\gamma_E$, measures the positive contribution from equipment-specific investment technology improvement, while the structures component, $\mathcal{M}_S\gamma_S$, captures the effect of structures-specific price changes, which is negative in the U.S. calibration. "Share of g_y" reports each component divided by total g_y; because the structures term is negative, shares can exceed 100 percent in absolute value.

Per-capita output grows by 1.43 pp per year, driven by population growth (0.82 pp) and equipment-specific ISTC (1.24 pp), while structures subtract 0.63 pp, offsetting about 51% of the equipment boost.

Table 4: **U.S. Structural Drag: Channel Decomposition**

Component	Growth rate (pp/year)	Share of Structural Drag (%)	Share of R&D Drag (%)
Structural Drag $\mathcal{M}_S\gamma_S$	-0.63	100	—
Static Deepening $\frac{\alpha_S}{\Psi}\gamma_S$	-0.44	69.7	—
R&D-related Structural Drag	-0.19	30.3	100
Direct Channel $\frac{\eta_S(1-\alpha_E)}{\Psi\Delta}\gamma_S$	-0.09	15.1	49.7
Indirect Channel $\frac{\eta_E\alpha_S}{\Psi\Delta}\gamma_S$	-0.10	15.2	50.3

Notes. Shares in the third column are computed relative to the total structures contribution, $\mathcal{M}_S\gamma_S$. Shares in the last column are computed relative to the R&D-related structural drag component. The *direct input channel* corresponds to the term $\eta_S(1-\alpha_E)/(\Psi\Delta)\gamma_S$ and captures the increase in the effective user cost of structures used as an input into R&D activity (e.g., research facilities) when structures ISTC is weak. The *indirect resource channel* corresponds to the term $\eta_E\alpha_S/(\Psi\Delta)\gamma_S$ and captures the general-equilibrium effect whereby weak structures efficiency growth tightens the final-goods resource constraint and reduces the equilibrium scale of research inputs. Negative growth rates indicate a drag on per-capita output growth.

As shown in Table 4, the structural drag decomposes into a static-deepening component of −0.44 pp and R&D-related structural drag of −0.19 pp. This R&D-related structural drag is split almost evenly across the two channels implied by equation (31): the direct input channel accounts for 49.7% (−0.09 pp) and the indirect resource channel accounts for 50.3% (−0.10 pp).

4.4 Sensitivity Analysis on Structural Drag

We conduct a sensitivity analysis by varying three structural parameters and the exogenous structures ISTC trend. Specifically, we vary the knowledge-spillover parameter, ϕ, the output elasticity of structures, α_S, the R&D elasticity of structures, η_S, and the structures ISTC rate, γ_S.[29] When varying α_S and η_S, we hold fixed the total capital share, $\alpha_E+\alpha_S$, and the total R&D input elasticity, $\eta_E+\eta_S$, respectively.

Table 5 examines how the quantitative importance of structural drag changes under alternative values of key parameters. The sensitivity analysis shows that structural drag remains quantitatively important across all specifications. In the baseline calibration, structural drag reduces per-capita output growth by 0.63 percentage points per year, while the equipment boost raises

[29]The candidates for ϕ values are drawn from Jones (2022). We impose a technical restriction on the parameter range to ensure the model's structural integrity. Specifically, following Jones (2022), we can match ϕ with β in Jones (2022) by $\phi \equiv 1-\beta$. In the Jones (2022) specification $\dot{A}/A = A^{-\beta}$, β measures the strength of diminishing research opportunities: when $\beta > 0$, a higher technology level A reduces the proportional growth rate. In our specification $\dot{A} = A^{\phi}$, ϕ governs the curvature (returns to scale) of knowledge production and $\phi < 1$ implies diminishing returns. Therefore, the higher β is, the lower the ϕ is. We note that the case where $\beta = 0.2$ (which implies $\phi = 0.8$) is excluded from our analysis as it violates the fundamental regularity condition of our model, $\Delta > 0$ (equation (24)).

per-capita output growth by 1.24 percentage points per year. Thus, the decline in structures-specific investment technology offsets about 51.02% of the positive growth contribution from equipment-specific investment technology. This result is not driven by a narrow parameter choice. Across the alternatives reported in the table, structural drag remains negative and offsets a sizable fraction of the equipment boost.

Table 5: **Sensitivity of Structural Drag and Equipment Boost to Key Parameters**

	Structural Drag (pp/year)	Equipment Boost (pp/year)	|Drag|/Boost (%)	g_y (pp/year)
Panel A. Knowledge spillover ϕ				
$\phi = 0.0$	-1.22	3.33	36.74	5.47
$\phi = -1.0$	-0.69	1.45	47.74	1.83
$\phi = -1.4535^*$	-0.63	1.24	51.02	1.43
$\phi = -2.0$ (Bloom et al., 2020)	-0.59	1.09	54.14	1.14
$\phi = -4.0$	-0.52	0.85	61.26	0.68
Panel B. Output elasticity of structures α_S				
$\alpha_S = 0.17$	-0.48	1.54	31.41	1.87
$\alpha_S = 0.20$	-0.56	1.39	39.97	1.65
$\alpha_S = 0.2312^*$	-0.63	1.24	51.02	1.43
$\alpha_S = 0.26$	-0.70	1.10	63.96	1.22
$\alpha_S = 0.29$	-0.77	0.95	81.51	1.00
Panel C. R&D elasticity of structures η_S				
$\eta_S = 0.02$	-0.58	1.34	43.53	1.58
$\eta_S = 0.05$	-0.61	1.29	47.14	1.50
$\eta_S = 0.0699^*$	-0.63	1.24	51.02	1.43
$\eta_S = 0.10$	-0.66	1.19	55.26	1.35
$\eta_S = 0.12$	-0.68	1.14	59.86	1.28
Panel D. Structures price trend γ_S				
$\gamma_S = -0.0101$ (PWT)	-0.54	1.24	43.52	1.52
$\gamma_S = -0.0108$ (BEA Residential)	-0.58	1.24	46.54	1.48
$\gamma_S = -0.0118^*$ (BEA All)	-0.63	1.24	51.02	1.43
$\gamma_S = -0.0136$ (BEA Non-Residential)	-0.73	1.24	58.60	1.33

Notes. Structural drag, equipment boost, and g_y are reported in percentage points per year. The ratio |Drag|/Boost reports the share of the equipment boost offset by structural drag. Panel (A) varies the knowledge-spillover parameter, ϕ. Panel (B) varies the output elasticity of structures, α_S, while holding $\alpha_E + \alpha_S = 0.378$ fixed, so the output elasticity of equipment, α_E, adjusts accordingly. Panel (C) varies the R&D elasticity of structures, η_S, while holding $\eta_E + \eta_S = 0.331$ fixed, so the R&D elasticity of equipment, η_E, adjusts accordingly. The asterisk marks the baseline calibration. For Panel (D), γ_S is drawn from PWT, BEA structures, BEA residential structures only, and BEA nonresidential structures only, ordered from the highest to the lowest value.

Panel (A) shows that as ϕ becomes more negative, induced innovation weakens and both structural drag and the equipment boost shrink in absolute value. Structural drag remains important, however, with the offset ratio rising from 36.74% at $\phi = 0$ to 61.26% at $\phi = -4$, including

the value $\phi = -2$ from Bloom et al. (2020). In addition, the per-capita output growth rate g_y is highly sensitive to the value of ϕ. As ϕ increases from -4 toward 0, we observe a significant acceleration in growth, from 0.68 pp per year to 5.47 pp per year.

Panel (B) shows that increasing α_S strengthens structural drag and reduces the equipment boost because the total capital share is fixed. As α_S rises from 0.17 to 0.29, structural drag increases from 0.48 to 0.77 pp per year, the equipment boost falls from 1.54 to 0.95 pp per year, and the offset ratio rises from 31.41% to 81.51%.

Panel (C) shows that increasing η_S has similar effects. As η_S rises from 0.02 to 0.12, structural drag increases from 0.58 to 0.68 pp per year, the equipment boost falls from 1.34 to 1.14 pp per year, and the offset ratio rises from 43.53% to 59.86%. Overall, the structural-drag mechanism remains robust across the parameter values considered.

Panel (D) reports the results when the structures ISTC rate is measured using alternative datasets. We measure γ_S using BEA Fixed Assets Accounts Tables 2.7 and 2.8, considering nonresidential structures and residential structures separately, with each series deflated by the GDP deflator. The corresponding estimates of γ_S are -0.0136 and -0.0108, respectively. In addition, we consider the measure from PWT, which yields -0.0101. The nonresidential-structures measure shows a more negative γ_S, increasing the structural drag from 0.63 percentage points to 0.73 percentage points, whereas using residential structures and the PWT measure, the structural drag declines to 0.58 and 0.54 percentage points, respectively.

4.5 Measurement Error and Bounds on the Structural Drag

Section 2.3 discussed the decline in construction-sector productivity as one driver of rising structures prices. A concern remains that part of the secular increase in the relative price of structures reflects deflator mismeasurement rather than a genuine productivity decline.[30] Goolsbee and Syverson (2023) conclude that the weak performance of the construction sector is not merely a figment of measurement error. Even so, the structures price index may miss improvements in the unobserved quality of structures.[31] In this subsection we ask how large that bias could possibly be and how much of the structural drag survives under the most generous correction. We proceed

[30] Several authors have documented potential biases in the International Comparison Program. See, e.g., Deaton and Heston (2010) and Argente et al. (2023).

[31] Gort et al. (1999) argue that such improvements are substantial. Inferring embodied technological progress from the decline of office rents with building age, in a panel of about 200 U.S. office buildings observed over 1988–96, they estimate that new structures gain about 1 percent per year in efficiency units, one third of the rate they assign to equipment. Their buildings were built before 1988 and their calibration matches U.S. aggregates over 1959–96, the postwar decades in which the relative price of structures was roughly stationary. The estimate therefore describes the period before the divergence that drives our results, not the rise in structures prices after 1990. We take it as a reminder that any bias runs toward overstating the price trend, and we turn to direct evidence on the deflators themselves.

in three steps. We first document what the structures investment deflator is made of. We then aggregate index-by-index upper bounds on unobserved-quality bias into an upper bound for the structures price trend. Finally, we map that bound into the drag.

What the structures price is made of. The price of structures that disciplines γ_S in our calibration is the deflator for BEA private fixed investment in structures. Table 6 lists the price index BEA uses to deflate each asset category. Three features matter for the measurement question. First, the constant-quality indexes based on sales prices of new homes deflate only about 36 percent of structures investment. These are the Census Bureau's hedonic price indexes for single-family and multifamily houses under construction. Only this part of the deflator is exposed to the unobserved-quality critique in its standard form. Here alone, prices come from transactions in heterogeneous units whose quality must be controlled econometrically. Second, roughly 12 percent rests on construction cost indexes such as the Turner, Handy–Whitman, and AUS indexes. These indexes price a basket of inputs rather than a finished structure. By construction, they cannot register changes in construction-sector productivity at all.[32] Third, the remainder is deflated by fixed-specification producer price indexes for new buildings, by input-price and labor-cost indexes with no structure-quality channel, or by components to which structure quality does not apply at all. The largest such component is brokers' commissions and other ownership transfer costs (12.9 percent), which are priced by a services PPI.

An upper bound on quality bias in the structures price. Garcia and Molloy (2025) assess how far the growth of measured construction prices could be biased upward by unobserved improvements in structure quality. For the single-family index, their three approaches bracket the bias between roughly zero and 0.8 percentage points per year. A comparison with R.S. Means construction-cost estimates that hold detailed characteristics fixed implies a negligible bias. Observable-but-unused quality measures imply about 0.5. The econometric bounding technique of Oster (2019) yields 0.8 under deliberately permissive assumptions about unobservables. Throughout this subsection we use the top of that range. Every number that follows is therefore an upper bound on the bias, not an estimate of it. For the other deflator components we adopt Garcia and Molloy's own assignments.[33]

[32] An input-cost deflator imputes the growth of input prices to the price of the finished structure. It therefore forces measured sectoral productivity toward zero (Pieper, 1991). The direction matters here. If construction productivity declined, as the evidence in Section 2.3 indicates, an input-cost index *understates* the true increase in the price of structures. We do not credit this offsetting force in the bound below.

[33] The multifamily index is built by the same hedonic method and receives the same 0.8 bound. The construction cost indexes are based on input costs rather than property sales prices and receive half the single-family bound, 0.4. The fixed-specification building PPIs, the input-price and labor-cost indexes, and the non-construction components receive zero.

Table 6: **Deflator Sources for BEA Private Structures Investment, by Asset Category**

Asset class	Share	Deflator (NIPA Handbook, December 2024)
Housing (1–4 unit)	21.47%	Census constant-quality (hedonic) index for new one-family houses under construction
Improvements	19.78%	Unweighted avg. of the Census one-family index, the PPI for home maintenance and repair construction, and the BLS ECI for construction
Brokers' commissions	12.90%	PPI for residential real estate brokerage
Office	6.17%	PPI for office building construction[a]
Electric	5.66%	Handy–Whitman construction cost indexes for electric plants and utility buildings
Housing (5 or more)	5.54%	Census constant-quality (hedonic) index for new multifamily houses under construction
Manufacturing	5.00%	PPI for industrial buildings[a]
Warehouse group[b]	4.18%	PPI for warehouse construction[a]
Petroleum and natural gas	3.97%	PPIs for oil and gas wells and for oil and gas field services
Health care[b]	2.96%	PPI for health care building construction[a]
Small C–T group[b]	2.81%	C–T average
Lodging	2.28%	C–T average
Communication	1.60%	AUS telephone plant cost index
Other power	1.49%	Handy–Whitman gas index of public utility construction costs
Educational and vocational	1.21%	PPI for new school construction[a]
Other commercial	1.00%	PPI for warehouse construction[a]
Manufactured homes	0.84%	PPI for mobile homes
Land transportation	0.76%	Weighted avg. of the BLS ECI, Bureau of Reclamation construction cost trends, and PPIs for construction inputs
Other residential	0.29%	Census one-family index
Other	0.12%	FHWA highway and Handy–Whitman water utility construction cost indexes

Notes. Shares are 2020 nominal investment shares within BEA private structures investment (Fixed Assets Table 2.7). Deflators follow Table 6.A of the BEA NIPA Handbook, Chapter 6 (December 2024). Minor components of a category's deflator are omitted. "C–T average" is the unweighted average of the Census one-family index and the Turner building-cost index. BEA itself notes that quality-adjusted output price measures are unavailable for some nonresidential components, so it combines input-cost and output-price measures (Ch. 6, p. 6-13).

[a] BLS prices fixed model buildings, combining quarterly cost estimates for specified building assemblies with contractor overhead-and-profit markups surveyed monthly (Bureau of Labor Statistics, 2025).

[b] Merged rows, with 2020 shares in parentheses. The warehouse group consists of warehouses (2.85), multimerchandise shopping (0.73), and food and beverage (0.59). Health care consists of hospitals (1.54), medical buildings (1.01), and special care (0.41). The small C–T group consists of amusement and recreation (1.10), mining (0.61), farm (0.57), air (0.30), and religious (0.24).

Table 7 aggregates these index-level bounds using the deflator weights of our structures investment price. The implied upper bound on the quality bias in the growth of the structures price is

$$\bar{b} \;=\; 0.8 \times 0.36 + 0.4 \times 0.12 + 0 \times 0.52 \;\approx\; 0.34 \text{ percentage points per year.}$$

Table 7: **An Upper Bound on Quality Bias in the Structures Investment Price**

Deflator component	Upper bound (pp/yr)	Weight (%)	Contribution (pp/yr)
Sales-price-based constant-quality indexes (Census single-family and multifamily)	0.8	36.4	0.29
Construction cost indexes (Turner, Handy–Whitman, AUS, and similar)	0.4	11.8	0.05
Fixed-specification building PPIs, input-price and labor-cost indexes, and non-construction components	0	51.8	0
Upper bound on bias in the structures price trend, $\bar{b}$		100	0.34

Notes. Weights are 2020 investment shares (BEA Fixed Assets Table 2.7) under the deflators of Table 6. Averaged deflators are attributed to their underlying indexes. Upper bounds follow Garcia and Molloy (2025), Section 3.4. The 0.8 is the most generous of their three estimates of the single-family bias, obtained with the Oster (2019) bounding method, so every entry is an upper bound rather than an estimate.

Implications for the structural drag. In our calibration, the implied structural drag is 0.63 percentage points per year. At the lower end of the range the bias is negligible and the drag stays at 0.63 percentage points. At the upper bound $\bar{b} = 0.34$, more than two thirds of the measured trend is genuine and the drag is about 0.45 percentage points per year. Even when every index is pushed to the top of Garcia and Molloy's range and the offsetting understatement in the input-cost components is ignored, two thirds of the structural drag survives. In addition, when γ_S is measured from nonresidential structures alone, the drag becomes larger, at 0.73 percentage points, as nonresidential structures exhibit faster price growth than residential structures and other items, as Figure B.9 in Appendix B.2 shows.

4.6 Structures Prices in the R&D Sector

The model assigns one investment efficiency q_S to structures in both uses. Research facilities therefore face the same price trend as the structures used in final-goods production. The assumption matters for the R&D-related part of the drag. If laboratories and research offices had escaped the cost increases documented for structures as a whole, the direct input channel in (31) would

overstate the burden that rising structures prices place on innovation. This subsection asks what is known about the price of research facilities.

The short answer is that no official statistic measures it. The NCSES Business Enterprise Research and Development survey reports how much firms spend on R&D structures and equipment, which is the source of η_S in Section 4, but it collects no prices. When BEA capitalized R&D in the national accounts, it deflated R&D investment with an input-cost index. The capital component of that index is priced with the economy-wide NIPA investment deflators for nonresidential structures and for equipment rather than with R&D-specific asset prices (Bureau of Economic Analysis, 2024; Copeland et al., 2007). The national accounts therefore impose the same assumption that we make. Nothing in the official price data can say whether research facilities are special.

The closest direct evidence comes from the NCSES Survey of Science and Engineering Research Facilities. The survey is congressionally mandated and biennial, and it has covered research space at U.S. colleges and universities since 1986. For each cohort of construction projects started in a two-year window, institutions report the new research space to be created, in net assignable square feet, and the estimated cost to complete the projects. Table 8 reports the cost per square foot for every cohort since FY 2002–03, the first under the survey's current design (National Center for Science and Engineering Statistics, 2025).[34] The cost per square foot of new academic research space rose from $456 in the FY 2002–03 cohort to $1,320 in the FY 2022–23 cohort, or about 5.5 percent per year. The GDP deflator rose 2.3 percent per year over the same window, so the real cost of research space grew about 3 percent per year between endpoints. Over the same twenty years the BEA price index for private nonresidential structures rose 2.0 percent per year relative to the GDP deflator. Research space has therefore become more expensive at least as fast as nonresidential structures in general.[35]

The asset-level BEA prices in Table A.I point the same way. Among the categories closest to research facilities, office and educational buildings rose faster than structures as a whole over 1970–2020, and manufacturing plants nearly as fast. A final consideration reinforces the direction. Frontier R&D increasingly runs on data centers, whose cost is set by power, land, and permitting rather than by the falling price of the equipment they house. The Census Bureau classifies data centers within office construction and began reporting them separately in 2024, so the office price index in Table A.I already reflects part of this demand. To the extent research facilities differ from

[34] Before FY 2003 the survey sampled the roughly 525 largest research performers and recorded projects costing more than $100,000. Since FY 2003 it covers every institution with at least $1 million of R&D expenditures and records projects costing more than $250,000.

[35] A caveat applies. The series is a unit value rather than a constant-quality price index. What the series can support is a direction. Nothing in it suggests that research facilities have escaped the cost pressures documented for structures in Section 2.3.

Table 8: **Cost of New Academic Research Space per Square Foot, FY 2002–2023**

Project start	Cost ($M)	Space (M NASF)	$ per NASF	2017 $ per NASF
FY 2002–03	7,389	16.2	456	598
FY 2004–05	6,030	10.2	591	736
FY 2006–07	5,924	8.8	673	790
FY 2008–09	7,336	9.9	741	839
FY 2010–11	6,396	8.1	790	872
FY 2012–13	5,433	6.6	823	876
FY 2014–15	5,680	5.1	1,114	1,150
FY 2016–17	5,929	6.7	885	893
FY 2018–19	5,139	5.6	918	890
FY 2020–21	4,493	4.0	1,123	1,042
FY 2022–23	7,261	5.5	1,320	1,098

Notes. Each row is the cohort of new construction projects for science and engineering research space that U.S. academic institutions started in the two fiscal years shown. Cost is the estimated completion cost of the cohort in current dollars. It includes planning, site preparation, construction, fixed equipment, building infrastructure, and nonfixed equipment costing over $1 million. Space is net assignable square feet (NASF). The last column deflates cost per NASF by the GDP implicit price deflator averaged over the two fiscal years (2017 = 100). *Sources:* NCSES Survey of Science and Engineering Research Facilities (NSF 25-319, Tables 9, 12, and 26; Science and Engineering Indicators 2010, Table 5-5), BEA, and authors' calculations.

the aggregate, the difference is more likely to tighten the structures constraint on innovation than to relax it.

4.7 Counterfactual Analysis

To quantify the extent to which U.S. long-run growth has been hindered by the rising relative cost of structures, we conduct a counterfactual exercise benchmarking the United States against two reference groups: the median high-income and median low-income economies, as classified in Section 2.2.

Table 9 summarizes the observed ISTC profiles. For this exercise, we calculate equipment ISTC and structures ISTC for the United States using PWT data for consistency. As a result, γ_E changes from 2.38% to 2.90%, while γ_S changes from −1.18% to −1.01%. While equipment ISTC, γ_E, remains a robust driver of growth across all groups, structures ISTC, γ_S, exhibits a stark divergence. Notably, the U.S. displays a more adverse structures ISTC (−1.01%) than the high-income median (−0.42%), indicating a distinctive "structural drag" in the U.S. economy.

Table 9: **Observed ISTC Rates by Region (1970–2020)**

Region	Equipment (γ_E)	Structures (γ_S)
Low Income	1.66%	−0.08%
High Income	3.02%	−0.42%
The United States	2.90%	−1.01%

Notes. ISTC rates are measured as the negative of the average growth rate of relative investment prices for equipment and structures over 1970–2020. All rates are computed from PWT relative investment prices. The baseline calibration uses BEA prices for the United States.

What would have happened if the U.S. had faced different ISTC rates? We feed the region-specific ISTC pairs from Table 9 into the U.S.-calibrated model as counterfactual shocks, holding other parameters fixed. Table 10 reports the implied growth contributions from ISTC.

Table 10: **Counterfactual U.S. Growth under Alternative ISTC Shocks**

Shock Scenario	Equipment Boost	Structural Drag	g_y^{ISTC}
LIC Scenario	0.86pp	−0.04pp	0.82pp
HIC Scenario	1.57pp	−0.22pp	1.35pp
The United States	1.50pp	−0.54pp	0.96pp

Notes. Each row reports the implied per-capita output growth contribution from ISTC, g_y^{ISTC}, under alternative relative-price (ISTC) shock profiles. The "Equipment Boost" and "Structural drag" columns decompose g_y^{ISTC} into the contributions associated with equipment and structures ISTC, respectively. LIC (HIC) scenarios apply the observed low-income (high-income) ISTC rates to the U.S. calibration, while the last row reports the PWT-based U.S. benchmark used in this counterfactual exercise.

These counterfactuals deliver two results. First, relative to the high-income median scenario, the U.S. shortfall is almost entirely a structures shortfall. The equipment contribution is only 0.07 percentage points lower in the U.S. (1.50pp vs. 1.57pp), whereas the structural drag is 0.32 percentage points larger in magnitude (−0.54pp vs. −0.22pp), so the structures margin accounts for 0.32 of the 0.39 percentage point gap in the net ISTC contribution (0.96pp vs. 1.35pp). Second, relative to the low-income median scenario, the U.S. has a much larger equipment contribution (1.50pp vs. 0.86pp), but most of this advantage is cancelled by the more negative structures contribution (−0.54pp vs. −0.04pp). The U.S. net ISTC contribution ends up only 0.14 percentage points above the LIC scenario (0.96pp vs. 0.82pp). The two capital prices thus work in opposite directions: falling equipment prices favor the United States over low-income economies, while rising structures prices penalize it, and on net the price of capital separates the two by far less than the equipment series alone would suggest.

5 Extension: CES Output Technology

The baseline model makes two simplifying assumptions: a Cobb–Douglas production function across all factors, and a strict division of labor in which unskilled workers enter only production and skilled workers enter only research. The CES extension relaxes the latter restriction by allowing skilled labor to enter final production as well, and allows the substitution patterns between equipment and skilled labor and between structures and unskilled labor to differ, as emphasized by Krusell et al. (2000).

We therefore generalize the production function to a nested CES structure in which two capital–labor bundles are combined to produce output, $Y = AQ$ where the top nest aggregates an equipment–skilled-labor bundle M and a structures–unskilled-labor bundle S,

$$Q = \left[\omega M^{\rho_0} + (1-\omega)S^{\rho_0}\right]^{1/\rho_0}, \qquad \rho_0 \equiv \frac{\sigma_0 - 1}{\sigma_0}, \tag{32}$$

and

$$M = \left[\theta_M (K_E^Y)^{\rho_M} + (1-\theta_M)(L_s^Y)^{\rho_M}\right]^{1/\rho_M}, \quad S = \left[\theta_S (K_S^Y)^{\rho_S} + (1-\theta_S) L_u^{\rho_S}\right]^{1/\rho_S} \tag{33}$$

where $\rho_j \equiv (\sigma_j - 1)/\sigma_j$ for $j \in \{M, S\}$.

The parameter ω governs the weight on M relative to S in the top nest. Within each bundle, θ_M and θ_S govern the weight on capital relative to labor. The elasticities σ_0, σ_M, and σ_S allow substitution patterns to differ across nests. Although equipment and structures do not enter the same CES nest directly, the nested specification does not impose a unit elasticity of substitution between the two capital types. Their relative use adjusts through substitution between the M and S bundles in the outer nest, together with substitution within each capital-labor bundle.[36]

Note that this differs from Krusell et al. (2000), where structures enter Cobb–Douglas and only the equipment–skilled-labor margin is modeled as CES. Our specification instead places structures inside a CES bundle with unskilled labor, allowing the structures–labor substitution elasticity to be estimated rather than restricted to one.

It is worth noting that this extension preserves the baseline structural-drag channel emphasized in the Cobb–Douglas case, but it no longer restricts the effect of structures ISTC to operate through a fixed loading coefficient. Because the CES structure makes factor shares endogenous, changes in structures ISTC affect output growth not only through the direct drag channel but also through additional reallocation margins across and within bundles. Furthermore, this framework is rich enough to analyze how structures affect the skilled wage premium on top of equipment-

[36] We focus on the output side rather than the R&D production function because evidence on labor–capital interactions in R&D is limited, and detailed input data are scarce: the BERD survey reports capital expenditure breakdowns only from 2015 onward.

skilled-labor complementarity, which we discuss in the following section.

5.1 Estimation Results for the CES Output Function

We begin by estimating the CES output function following Ohanian et al. (2023). We use annual U.S. nonfarm-business-sector data for the period 1963 to 2019, including sector-specific capital, labor hours, and wage data constructed by Ohanian et al. (2023). Because our specification estimates an additional parameter, the elasticity of substitution between structures and unskilled labor, we supplement their data with a structures-price series based on the investment-based structures price constructed from BEA Fixed Assets Accounts Tables and deflated by the GDP deflator.[37] The parameters are estimated by GMM using three sets of model-implied restrictions as in Ohanian et al. (2023). The first restriction equates the return on structures and the return on equipment. The second matches the aggregate labor share, using the gross labor share. The third matches the skilled-to-unskilled wage bill ratio. Details on the data construction and estimation procedure are provided in Appendix F.1.

Table 11 reports the estimates for the CES specification. Panel A of Table 11 presents the parameter estimates with standard errors, and Panel B of Table 11 reports Wald tests of the Cobb–Douglas restriction ($\rho = 0$, i.e., $\sigma = 1$) for each bundle.

The estimates deliver three main results. First, equipment capital and skilled labor are complements ($\hat{\sigma}_M = 0.72$), preserving the central message of Krusell et al. (2000) and Ohanian et al. (2023). In addition, the Cobb–Douglas restriction $\sigma_M = 1$ is rejected, confirming that the complementarity between equipment and skilled labor is significantly different from unitary elasticity. Second, structures and unskilled labor are substitutes ($\hat{\sigma}_S = 2.24$), indicating that the data do not support the implicit Cobb–Douglas treatment. Third, the outer elasticity governing substitution between the two bundles is $\hat{\sigma}_0 = 1.42$. The equality test further sharpens the picture. The two inner elasticities differ significantly, confirming that the equipment-skilled-labor and structures-unskilled-labor bundles exhibit distinct substitution patterns. Taken together, these results highlight a key departure from the Krusell et al. (2000) specification, which effectively treats structures as entering in Cobb–Douglas form with the rest of production.[38]

[37] Ohanian et al. (2023) normalize the structures price to one because structures enter as the Cobb-Douglas counterpart to the equipment-skilled labor CES composite and their prices are treated as being measured in the same numeraire units. In our modified specification, the additional structures-related parameter requires a separate structures-price series. The baseline uses the investment-based structures price constructed from BEA Fixed Assets Accounts Tables 2.7 and 2.8 and deflated by the GDP deflator, combined with the gross labor-share definition in Krusell et al. (2000) and Ohanian et al. (2023). For robustness, we also estimate the model using a stock-based structures price constructed from BEA Fixed Assets Accounts Tables 2.1 and 2.2, again deflated by the GDP deflator, and combine each price measure with both net and gross labor-share definitions.

[38] The results are robust across alternative specifications, including gross and net labor-share measures and stock-based and investment-based structures-price series; see Appendix F.2 for details.

Table 11: **Parameter Estimation Results**

Panel A. Parameter estimates

	$\hat{\rho}$	(SE)	$\hat{\sigma} = \frac{1}{1-\hat{\rho}}$
Outer (σ_0)	0.30	(0.01)	1.42
Equip–skilled (σ_M)	−0.40	(0.03)	0.72
Struct–unskilled (σ_S)	0.55	(0.06)	2.24
	Estimate	(SE)	
ω	0.57	(0.01)	
θ_M	0.25	(0.02)	
θ_S	0.58	(0.04)	

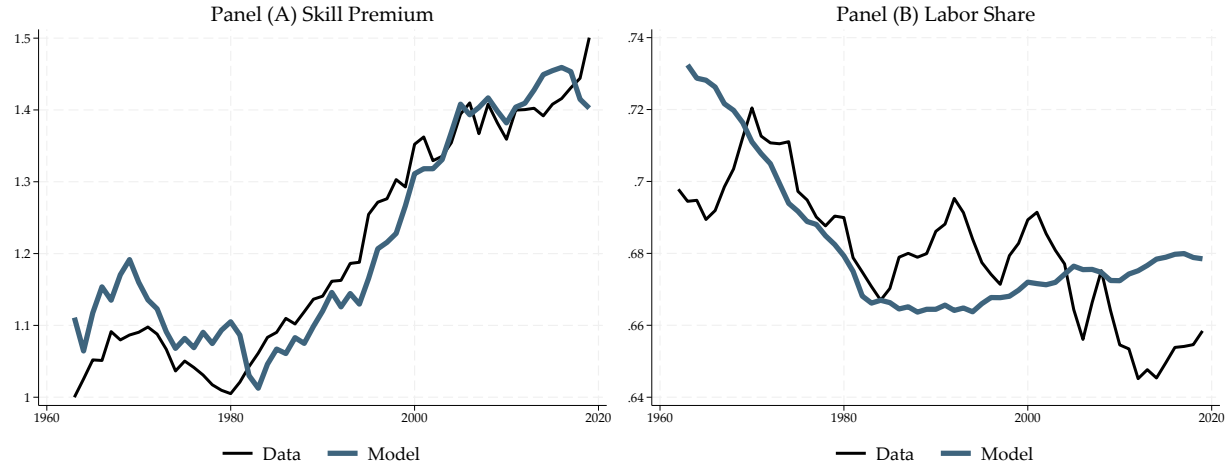
Panel B. Specification Tests

H_0	Wald $\chi^2(1)$	p-value
$\sigma_0 = 1$	528.0	0.000
$\sigma_M = 1$	177.6	0.000
$\sigma_S = 1$	86.2	0.000
$\sigma_M = \sigma_S$	498.1	0.000

Notes. The table reports GMM estimation results using nonfarm business sector data with the investment-based structures price and the gross labor-share specification. Standard errors are reported in parentheses. $\sigma = 1/(1-\rho)$. Wald tests are $\chi^2(1)$. The first three test $\rho = 0$ ($\sigma = 1$), and the last test examines $\rho_M = \rho_S$.

Although we impose a different functional form for aggregating inputs, the model delivers similar performance in matching the skill premium. We evaluate the model's fit for the skill premium and the labor share.

Figure 8: **Model Fit**



Notes. Panel A shows that the model-implied skill premium tracks the empirical trend. Panel B shows that the model does not reproduce the post-1994 labor-share decline, instead implying a stable or rising labor share.

Figure 8 shows that the trend in the skill premium, which is not targeted in the estimation, closely matches the trend observed in the sample. By contrast, the model closely matches the labor share until 1994 but does not capture its subsequent decline, sharing the limitation pointed out in Ohanian et al. (2023).

5.2 Structural Drag under CES Output Function

Having characterized the substitution patterns among inputs both within input bundles and across input bundles, we now turn to their implications for growth through structural drag. Under the CES output function, GDP per capita growth is given by the following analytical expression with the local-stationarity conditions.[39]

$$g_{y,t} = \frac{n + \big((1-\phi)\mathbf{a}_{E,t} + \eta_E\big)\gamma_E + \big((1-\phi)\mathbf{a}_{S,t} + \eta_S\big)\gamma_S}{(1-\phi)(1-\mathbf{a}_{E,t}-\mathbf{a}_{S,t}) - (\eta_E + \eta_S)}, \tag{34}$$

The main difference from the Cobb–Douglas benchmark lies in the factor shares $\mathbf{a}_{E,t}$ and $\mathbf{a}_{S,t}$. Under Cobb–Douglas, these shares are fixed at the primitive values $\alpha_E \equiv \omega\theta_M$ and $\alpha_S \equiv (1-\omega)\theta_S$, so the coefficient associated with structures ISTC is as follows.[40]

$$D^{\mathrm{CD}} = \frac{(1-\phi)\alpha_S + \eta_S}{(1-\phi)(1-\alpha_E-\alpha_S) - (\eta_E+\eta_S)}. \tag{35}$$

Under CES, by contrast, the factor shares $\mathbf{a}_{E,t}$ and $\mathbf{a}_{S,t}$ are endogenous objects that vary with the equilibrium allocation of quantities across the nested production structure. In particular, the corresponding output-level shares are

$$\mathbf{a}_{E,t} = s_{M,t}s_{K_E|M,t}, \qquad \mathbf{a}_{S,t} = (1-s_{M,t})s_{K_S|S,t}. \tag{36}$$

where

$$s_{K_E|M,t} = \frac{\theta_M K_{E,t}^{\rho_M}}{\theta_M K_{E,t}^{\rho_M} + (1-\theta_M)L_{s,t}^{\rho_M}}, \quad s_{K_S|S,t} = \frac{\theta_S K_{S,t}^{\rho_S}}{\theta_S K_{S,t}^{\rho_S} + (1-\theta_S)L_{u,t}^{\rho_S}}, \quad s_{M,t} = \frac{\omega M_t^{\rho_0}}{\omega M_t^{\rho_0} + (1-\omega)S_t^{\rho_0}}.$$

In words, the equipment share in output $\mathbf{a}_{E,t}$ is given by the top-nest share of the M-bundle multiplied by the within-M-bundle share of equipment, while the structures share in output $\mathbf{a}_{S,t}$ is given by the top-nest share of the S-bundle, $1-s_{M,t}$, multiplied by the within-S-bundle share of

[39] To derive the expression in the main text, we close the following margins under the nested CES output function. The first two margins are the changes in equipment investment and structures investment relative to GDP. In addition, we close the reallocations of equipment, structures, and skilled labor between the output sector and the research sector. These assumptions are based on the empirical stability of capital investment shares relative to GDP, while the remaining assumptions are imposed because the data do not separately identify the output-sector and research-sector allocations of equipment, structures, and skilled labor. The derivation, along with the assumptions maintained throughout the analysis, is provided in Appendix F.3.

[40] Note that expression (35) is algebraically identical to the Cobb-Douglas structural-drag coefficient in equation (31). Let $\Psi \equiv 1-\alpha_E-\alpha_S$ and $\Delta \equiv (1-\phi)\Psi - (\eta_E+\eta_S)$. Then

$$\mathcal{M}_S = \frac{\alpha_S}{\Psi} + \frac{\eta_S(1-\alpha_E)+\eta_E\alpha_S}{\Psi\Delta} = \frac{\alpha_S\Delta + \eta_S(1-\alpha_E)+\eta_E\alpha_S}{\Psi\Delta} = \frac{(1-\phi)\alpha_S+\eta_S}{\Delta} \equiv D^{\mathrm{CD}}.$$

structures. This decomposition makes clear that output-level shares are shaped by both between-bundle allocation and within-bundle composition.

Because these shares are endogenous, the effect of structures-price growth is not exhausted by a single term as in the Cobb–Douglas case. Differentiating (34) with respect to γ_S yields the total effect of structures ISTC:

$$\frac{\mathrm{d}g_{y,t}}{\mathrm{d}\gamma_S} = \left.\frac{\partial g_{y,t}}{\partial \gamma_S}\right|_{\mathbf{a}} + \frac{\partial g_{y,t}}{\partial \mathbf{a}_{E,t}}\frac{\mathrm{d}\mathbf{a}_{E,t}}{\mathrm{d}\gamma_S} + \frac{\partial g_{y,t}}{\partial \mathbf{a}_{S,t}}\frac{\mathrm{d}\mathbf{a}_{S,t}}{\mathrm{d}\gamma_S}. \tag{37}$$

The first term is the direct effect:

$$\left.\frac{\partial g_{y,t}}{\partial \gamma_S}\right|_{\mathbf{a}} = \frac{(1-\phi)\mathbf{a}_{S,t} + \eta_S}{(1-\phi)(1-\mathbf{a}_{E,t}-\mathbf{a}_{S,t}) - (\eta_E + \eta_S)}, \tag{38}$$

which is the nested-CES counterpart to the structural drag term in the Cobb–Douglas case, evaluated at the current endogenous shares. The remaining terms capture the additional channels that arise because structural ISTC γ_S also changes the output shares themselves. Under the CES structure, the response of the equipment share, the second term in the decomposition in equation (37), is given by

$$\frac{\mathrm{d}\mathbf{a}_{E,t}}{\mathrm{d}\gamma_S} = (\sigma_0 - 1)\mathbf{a}_{E,t}(1 - s_{M,t})s_{K_S|S,t}\mu_S, \qquad \mu_S \equiv \frac{1}{r + \delta_S + \gamma_S} \tag{39}$$

whereas the response of the structures share can be decomposed into two components

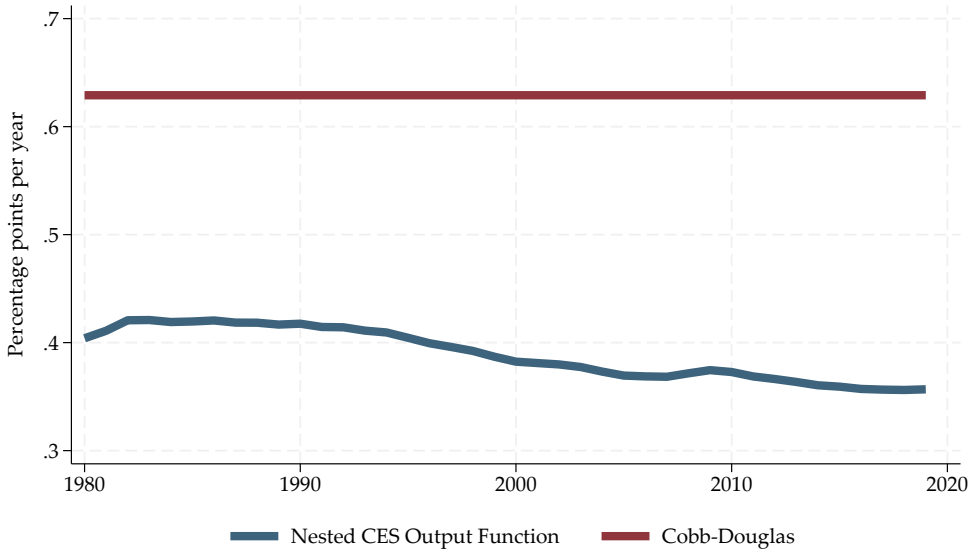
$$\frac{\mathrm{d}\mathbf{a}_{S,t}}{\mathrm{d}\gamma_S} = -(\sigma_0 - 1)s_{M,t}(1 - s_{M,t})s^2_{K_S|S,t}\mu_S - (\sigma_S - 1)(1 - s_{M,t})s_{K_S|S,t}(1 - s_{K_S|S,t})\mu_S. \tag{40}$$

Equations (39) and (40) show that the indirect effect of structures ISTC operates through three distinct margins: an outer-nest reallocation effect through $\mathbf{a}_{E,t}$, an outer-nest reallocation effect through $\mathbf{a}_{S,t}$, and a within-S substitution effect through $\mathbf{a}_{S,t}$. Thus, under CES, structural drag is shaped not only by the direct loading term in (38), but also by the endogenous adjustment of output shares obtained under the stated partial-equilibrium closure.

Based on the estimated CES parameters in Table 11, lower structures ISTC increases the user cost through expected capital gains, increasing the structures share and reducing the equipment share at the estimated elasticities. First, since $\sigma_0 > 1$, the top nest is sufficiently substitutable that an increase in structures-price growth raises the user cost of structures and induces reallocation away from the structures-intensive bundle and toward the M-bundle. Second, since $\sigma_S > 1$, the within-S nest is also substitutable, so the share of structures within that bundle declines as well. These two margins work in the same direction, implying a downward trend in $\mathbf{a}_{S,t}$ and, in turn,

a weakening of the structural-drag force. Finally, the outer-nest reallocation raises $\mathbf{a}_{E,t}$, but only modestly: because equipment and skilled labor are complements within the M-nest ($\sigma_M < 1$), the within-bundle composition cannot adjust freely, dampening the rise in $\mathbf{a}_{E,t}$.

Figure 9: **Structural Drag Comparison, Cobb-Douglas vs. CES**



Notes. The figure compares the model-implied structural drag under different output technologies. The red line plots the implied structural drag under the Cobb–Douglas specification based on the baseline calibration, while the blue line shows the time-varying structural drag implied by the CES production function in equation (37). For the CES case, we compute the model-implied series using the capital-stock and labor-hours data from Ohanian et al. (2023).

Figure 9 shows that the CES local measure is below the Cobb-Douglas BGP contribution and declines over the plotted sample evaluated at observed input paths. Relative to the Cobb–Douglas benchmark, where the drag is at approximately 0.63, the CES specification delivers a smaller drag that also declines over time. In the data, the CES-based drag starts at around 0.4 and falls to around 0.36 in recent periods.

Thus, the main implication of moving beyond Cobb–Douglas is not that structural drag disappears, but that its magnitude is lower and continues to decline as the economy endogenously adjusts its input mix. Under the CES extension, negative structures ISTC still exerts a drag on GDP per capita growth, but its quantitative effect becomes smaller because endogenous share adjustment allows the economy to reallocate away from structures-intensive production margins as relative prices change. In particular, substitution both across bundles and within the structures–unskilled-labor bundle reduces the output share of structures over time, thereby weakening the growth impact of adverse structures ISTC.

5.3 How Do Structures Affect the Skill Premium?

The extended CES specification also allows us to analyze how the presence of structures, together with their rising relative price, affects the skill premium, a question that the Cobb–Douglas specification cannot address.[41]

Following Krusell et al. (2000), the growth rate of the skill premium can be decomposed as follows:[42]

$$g_\pi = \frac{1}{\sigma_0}\left(g_{L_u} - g_{L_s}\right) + \left(\frac{1}{\sigma_M} - \frac{1}{\sigma_0}\right) s_{K_E|M}\left(g_{K_E} - g_{L_s}\right) - \left(\frac{1}{\sigma_S} - \frac{1}{\sigma_0}\right) s_{K_S|S}\left(g_{K_S} - g_{L_u}\right). \tag{41}$$

Equation (41) decomposes skill-premium growth into three components: a relative labor-supply term, an equipment–skilled-labor complementarity term in the spirit of Krusell et al. (2000), and a structures-unskilled-labor term that captures the role of structures under the estimated CES technology.

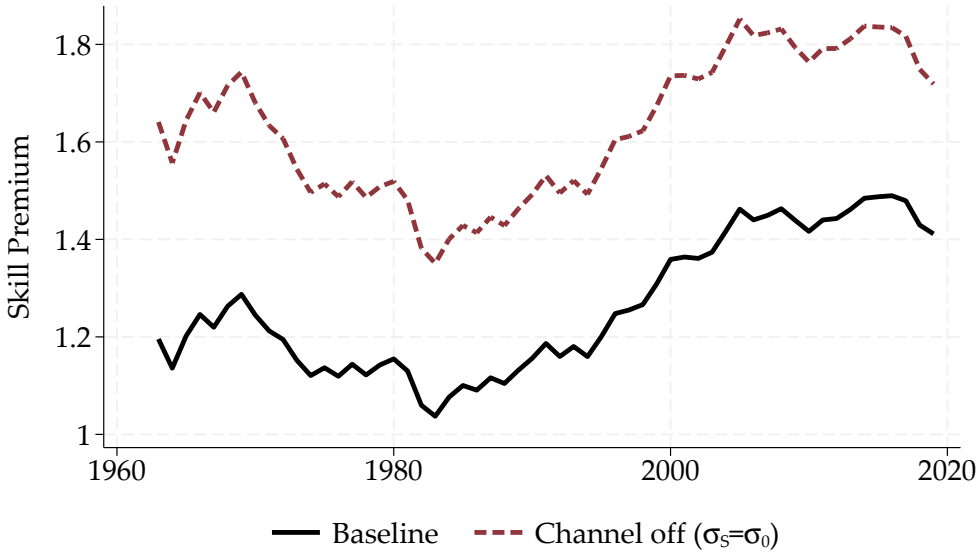
As shown above, structures and unskilled labor are substitutes, suggesting that a decline in structures ISTC reduces the skill premium by shifting demand away from structures and toward unskilled labor. To quantify this channel, we follow the counterfactual exercise in Krusell et al. (2000) and impose that the elasticity of substitution between structures and unskilled labor is equal to the elasticity governing substitution between the M- and S-bundles. This exercise isolates the quantitative role of the structures–unskilled labor margin without changing the broader CES environment. By imposing $\sigma_0 = \sigma_S$, the specification removes the additional structure-specific substitution term in the skill-premium decomposition, so the difference relative to the baseline can be attributed to that channel alone. The rest of the model, including the production structure, the other parameters, and the equilibrium framework, remains unchanged, which makes the comparison internally consistent.

Figure 10 shows that the counterfactual skill premium is consistently higher than the baseline path once the structures-unskilled-labor margin is shut down by imposing $\sigma_0 = \sigma_S$. The intuition is straightforward. When structures and unskilled labor are allowed to substitute within their own bundle, the rising relative price of structures induces firms to lean more heavily on unskilled labor as a partial replacement. This raises the relative demand for unskilled workers and compresses the skilled–unskilled wage gap. Once the margin is removed and structures are forced to substitute with the rest of production only through the outer nest, this offsetting force

[41] Our framework differs from the existing literature in an important respect. Whereas previous studies typically assume a Cobb–Douglas aggregation of structures, a CES bundle of equipment capital and skilled labor, and unskilled labor (Krusell et al., 2000), we instead adopt the CES production structure in (32) and use the estimated substitutability parameters for each margin.

[42] See Appendix F.4 for the derivation.

Figure 10: **Counterfactual Skill Premium**



Notes. The figure plots the actual skill premium in black and the counterfactual skill premium in red under the restriction $\sigma_0 = \sigma_S$ from 1963 to 2019. The model-implied series is computed using capital-stock and labor-hours data from Ohanian et al. (2023).

disappears, and the implied skill premium is higher than in the baseline.

Quantitatively, the gap between the two series is economically meaningful. Over 1963–2019, the counterfactual specification implies a skill premium approximately 30% higher than the baseline. This magnitude carries a direct implication for the canonical capital–skill complementarity framework of Krusell et al. (2000), which embeds structures as a Cobb–Douglas factor separable from the skill-complementary capital stock and therefore rules out this channel by construction. In that specification, the entire rise in the skill premium attributable to capital must be loaded onto the equipment–skilled-labor margin. Our estimates suggest that doing so overstates the role of capital–skill complementarity, because part of what would be assigned to equipment is in fact offset by structures–unskilled-labor substitution. The structures channel is thus not only a force shaping aggregate growth, as emphasized in Section 5.2, but also a previously unrecognized determinant of measured wage inequality.

6 Concluding Remarks

The decline in the relative price of equipment is usually read as a measure of how much technological change is embodied in capital. This paper argues that this reading is incomplete because it ignores structures, which make up more than half of the capital stock. Since 1990 the relative price of structures has risen in rich countries, and in the United States it has risen faster than in

the typical rich country. Falling equipment prices have lowered the cost of producing both output and ideas. Rising structures prices have raised it. In our calibrated model the second effect offsets about half of the first: structures subtract 0.63 percentage points a year from U.S. per-capita growth against a 1.24 point contribution from equipment. About 70 percent of the drag comes from slower capital deepening and 30 percent from the R&D sector, where structures are an input into research and also compete with research for resources. The drag shrinks under the most generous correction for quality mismeasurement in structures prices, to 0.45 points, but it remains.

Allowing inputs to substitute changes the magnitude of the drag but not the conclusion. Structures and unskilled labor are substitutes within their bundle, that bundle is a substitute for the equipment-skilled-labor bundle, and equipment and skilled labor remain complements. These margins reduce the drag by roughly 40 percent, to about 0.4 percentage points, and cause it to decline slowly as the economy shifts away from structures. They also show that structures prices matter for wage inequality. In the estimated model, firms respond to expensive structures by using more unskilled labor, which dampens the rise in the skill premium. Removing this margin, as the canonical capital-skill complementarity framework does by construction, raises the implied 1963–2019 rise in the U.S. skill premium by 30 percent.

The cross-country evidence has implications for development accounting as well. Poor countries have not shared in the fast decline of equipment prices, but they have also been spared the rise in structures prices. In the model the second difference offsets most of the first, so capital price trends are a weaker source of divergence between rich and poor countries than the equipment price alone would suggest.

One limitation deserves mention. The model uses a single price of structures for all uses, so housing, factories, and research facilities are priced the same. We would have preferred to price research facilities separately, but no official statistic measures their cost. The closest direct evidence, the NCSES survey of academic research space, suggests that research facilities have become more expensive at least as fast as nonresidential structures in general, so the single price is more likely to understate than to overstate the cost facing R&D. A model with separate structures stocks, and price data to discipline it, would settle the question.

The findings bear on growth policy and on the study of inequality. Efforts to speed technological progress in equipment-producing industries will deliver less than expected if the buildings that house the equipment keep getting more expensive. Why construction productivity has followed what Goolsbee and Syverson (2023) call a "strange and awful path" remains an open question. Our framework provides a way to evaluate the aggregate consequences of any improvement in construction productivity, and it links construction-sector stagnation to the wage gap between skilled and unskilled workers, so those consequences extend beyond aggregate output.

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ONLINE APPENDIX

A Details on Capital Price and Expenditure Measurement

Our primary analysis examines how heterogeneity in capital relates to development outcomes using three main data sources. For international price and capital measures, we use the International Comparison Program (ICP) and the Penn World Table (PWT) version 11.0, especially the Capital Detail module. For the research and development (R&D) sector, we use the National Center for Science and Engineering Statistics (NCSES) Business Enterprise Research and Development (BERD) survey for U.S. data, complemented by gross domestic expenditure on R&D (GERD) for cross-country comparisons.

We use the PWT Capital Detail data to characterize time-series trends in capital prices, with a focus on equipment and structures. We use ICP price data primarily for cross-sectional regressions that relate asset-type price levels to GDP per capita. We use BERD to measure and decompose R&D-related capital expenditures by asset type, and we use GERD for robustness checks. This section clarifies the interpretation of each variable and describes data coverage.

A.1 The International Comparison Program

We focus on ICP price measures for two capital categories, Machinery and Equipment (item 1501100) and Structures (item 1501200). In the ICP classification, these categories correspond to expenditure items under gross fixed capital formation.

Machinery and Equipment. This category corresponds to Machinery and Equipment (1501100) under Gross Fixed Capital Formation (1501000). It covers Metal products and equipment (1501110), Fabricated metal products (1501111), Electrical and optical equipment (1501112), General purpose equipment (1501115), Special purpose equipment (1501116), and Transport equipment (1501120).

Structures. This category corresponds to Structures (1501200) under Gross Fixed Capital Formation (1501000). It covers Residential buildings (1501210), Non-residential buildings (1501220), and Civil engineering works (1501230).

Other ICP capital formation components. In addition to Machinery and Equipment (1501100) and Structures (1501200), the ICP capital formation block includes Other Products (1501300) within Gross Fixed Capital Formation (1501000), Changes in Inventories (1502000) within Gross Capital Formation (1500000), and Acquisitions Less Disposals of Valuables (1503000) within Gross Capital Formation (1500000). We report these categories for completeness.

A.2 Penn World Table Capital Detail

PWT capital measures, including capital services and relative prices of capital, are constructed from investment data by asset.[43] Because investment is originally estimated at a more disaggregated level, the PWT Capital Detail file provides a convenient aggregation into four asset groups. **Machinery and Equipment** includes computers, communication equipment, and other equipment. **Structures** include residential and non-residential structures. **Transport equipment** is reported as a separate asset group. **Other assets** include software, other intellectual property products, and cultivated assets.

A.3 U.S. Bureau of Economic Analysis Fixed Assets / Industry Accounts

For the U.S.-focused analyses in the main text, especially the analyses underlying Facts 2 and 3, we use detailed data from the U.S. BEA Fixed Assets Accounts. These data allow us to measure asset-specific investment prices, capital stocks, and industry-level exposure to structures.

Asset categories. The BEA asset classification[44] allows us to distinguish between equipment and structures. We define equipment as information-processing equipment, industrial equipment, transportation equipment, and other equipment assets.[45] We define structures as residential and nonresidential structures, together with associated nonbuilding structures.[46] The post-1990 increase in relative structures prices documented in the main text is broad based across these subcategories rather than driven by a single component.

BEA Fixed Assets Accounts. Based on the above definitions, the BEA Fixed Assets Accounts provide detailed information on investment flows, capital stocks, and implicit price indices by asset type and by industry. We use both investment-based and stock-based measures to characterize the evolution of relative prices and capital composition.

For investment in private fixed assets by asset type, we use BEA Fixed Assets Accounts Tables 2.7 and 2.8, which report nominal investment and chain-type quantity indexes for private fixed

[43] https://www.rug.nl/ggdc/docs/userguidetopwt90datafiles.pdf

[44] BEA NIPA Handbook, Ch. 6, Dec 2024 update, defines PFI fixed assets as structures, equipment, and intellectual property products. https://www.bea.gov/resources/methodologies/nipa-handbook/pdf/chapter-06.pdf

[45] BEA capitalization guidance treats an item as equipment if it has a useful life of more than 1 year, is not an integral part of a structure and is not included in the value of that structure, and would normally be charged to a capital account in business accounting records.

[46] BEA includes in structures the structures of new nonresidential and residential buildings, improvements to buildings, and certain types of equipment that are considered an integral part of the structure. It also includes nonbuilding structures, mobile structures, manufactured homes, well drilling and exploration, mine development, brokers' commissions and ownership transfer costs, and net purchases of existing structures from governments.

assets by detailed asset category. For each asset type, we convert the chain-type quantity index into a real investment series using the corresponding base-year nominal value, and then construct an implicit investment price index as nominal investment divided by real investment. For capital stocks by asset type, we use BEA Fixed Assets Tables 2.1 and 2.2, which report current-cost net capital stocks and the corresponding chain-type quantity indexes by detailed asset category, to verify that the observed investment price dynamics extend to the capital stock itself.

BEA Fixed Assets Accounts by Industry. To study cross-industry heterogeneity in exposure to structures, we use BEA Fixed Assets tables that report investment and capital stocks by industry. Structures intensity at the industry level is measured using BEA Table 3.7ESI, Investment in Private Fixed Assets by Industry, and Table 3.7S, Investment in Private Structures by Industry. For each industry, we compute the structures investment share in 1970 as structures investment divided by total private fixed investment.

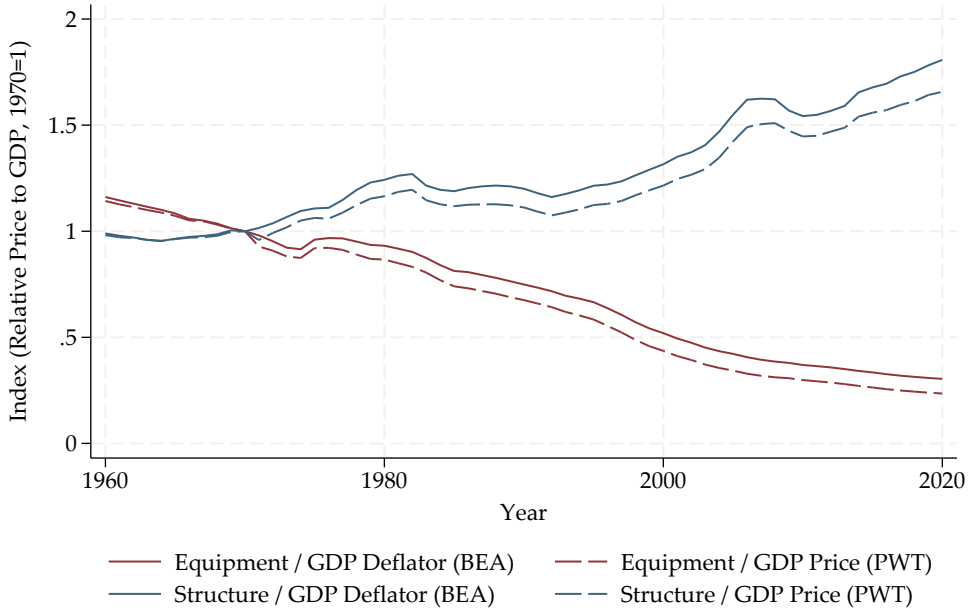
As an alternative measure of exposure, we compute the structures share in the capital stock using Table 3.1ESI, Current-Cost Net Stock of Private Fixed Assets, Equipment, Software, and Intellectual Property Products by Industry, and Table 3.1S, Current-Cost Net Stock of Private Structures by Industry. This stock-based measure captures longer-run technological and cost exposure beyond contemporaneous investment flows.

Comparison with Penn World Table Capital. To ensure that the BEA and PWT Capital Detail exhibit consistent trends, we compare the relative prices of equipment and structures across the two sources, where relative prices are measured relative to the GDP price.

Figure A.1 compares the U.S. relative prices of equipment and structures from the BEA and the PWT Capital Detail over 1960–2020. For the BEA, we normalize the price indexes for these two asset types by the GDP deflator. For the PWT Capital Detail, we normalize the corresponding price indexes by the GDP price in the national accounts, constructed by dividing nominal GDP in local currency units by real GDP.

BEA asset composition of asset-class investment and relative prices. Table A.I reports, for the year 2020, how total investment in each asset class is allocated across detailed BEA asset categories within the broad groups of equipment and structures. The table also reports relative prices in 2020, where each asset's real price in 1970 is normalized to 1.

Figure A.1: **Relative Investment Price Trends for Two Asset Types from BEA and PWT**



Notes. The figure compares U.S. asset price indices for equipment and structures constructed from the BEA and the PWT Capital Detail over 1960–2020, with all series normalized to 1970=1. For the BEA series, an asset-type price index is constructed as the ratio of nominal investment to real investment, where real investment is obtained by converting the BEA quantity index, 2017=100, using the 2017-dollar level. The BEA relative price is computed by dividing the normalized asset price index by the GDP deflator, GDPDEF. For the PWT series, asset price indices for machinery and equipment and structures are normalized to 1970=1. The GDP price in national accounts is computed as $p_{gdp} = v_{gdp}/q_{gdp}$, normalized to 1970=1, and the PWT relative price is defined as the normalized asset price index divided by the normalized p_{gdp}. Solid lines denote BEA series and long-dashed lines denote PWT series. Equipment and structures are distinguished by color as indicated in the legend. *Source.* Bureau of Economic Analysis, Fixed Assets, Penn World Table 11.0, PWT Capital Detail, and authors' calculations.

A.4 NCSES Business Enterprise Research and Development (BERD) Capital Expenditures Classification

We utilize the National Center for Science and Engineering Statistics (NCSES) Business Enterprise Research and Development (BERD) survey to calibrate the model.

BERD defines capital expenditures as payments for assets that typically have a useful life of more than one year and are recorded on a company's balance sheet. Long-lived assets used in R&D operations are not counted as current R&D expense. Instead, any depreciation recorded for those assets would be included in R&D expense.[47]

For the 2015 BERD survey instrument, the relevant items request domestic capital expenditures for R&D operations in Question 2.32 and a required breakdown of those domestic R&D capital expenditures in Question 2.33. The categories are Structures, Equipment, Capitalized soft-

[47] https://ncses.nsf.gov/surveys/business-enterprise-research-development

Table A.I: **Investment Shares by BEA Asset Category and Relative Prices in 2020**

Asset Class	Share	Price	Asset Class	Share	Price
Equipment	100%	0.304	Structures	100%	1.808
Communication Equipment	10.961%	0.056	Housing (1 To 4 Unit)	21.473%	1.719
Computers And Peripheral Equipment	10.916%	<0.001	Improvements	19.777%	1.545
Medical Equipment And Instruments	9.573%	0.426	Brokers' Commission	12.897%	1.866
Light Trucks	9.447%	0.763	Office	6.173%	1.907
General Industrial Machinery	8.257%	1.062	Electric	5.661%	1.715
Other Nonresidential Equipment	6.454%	0.765	Housing (5 Or More)	5.540%	2.135
Electrical Transmissions	4.273%	0.760	Manufacturing	5.001%	1.735
Furniture And Fixtures	4.092%	1.008	Petroleum And Natural Gas	3.972%	4.399
Other Trucks, Buses, And Truck Trailers	3.795%	1.206	Warehouses	2.854%	1.869
Special Industry Machinery	3.786%	1.219	Lodging	2.282%	1.769
Aircraft	3.597%	1.524	Communication	1.595%	0.929
Nonmedical Instruments	3.074%	0.795	Hospitals	1.535%	1.524
Construction Machinery	3.065%	1.368	Other Power	1.488%	2.042
Agricultural Machinery	2.948%	1.306	Educational And Vocational	1.205%	2.020
Service Industry Machinery	2.875%	0.857	Amusement And Recreation	1.097%	1.682
Metalworking Machinery	2.794%	1.045	Medical Buildings	1.010%	1.526
Mining And Oilfield Machinery	2.299%	1.549	Other Commercial	0.996%	1.729
Fabricated Metal Products	1.803%	1.117	Manufactured Homes	0.841%	1.324
Residential Equipment	1.550%	0.279	Land Transportation	0.761%	1.333
Engines And Turbines	0.874%	1.094	Multimerchandise Shopping	0.734%	1.768
Electrical Equipment	0.820%	0.655	Mining	0.606%	1.797
Railroad Equipment	0.703%	1.203	Food And Beverage Establishments	0.592%	1.830
Photocopy And Related Equipment	0.658%	0.228	Farm	0.565%	1.753
Ships And Boats	0.550%	1.252	Special Care	0.411%	1.532
Autos	0.451%	0.502	Air	0.303%	1.685
Office And Accounting Equipment	0.388%	0.289	Other Residential	0.289%	2.021
			Religious	0.236%	1.866
			Other	0.121%	1.516

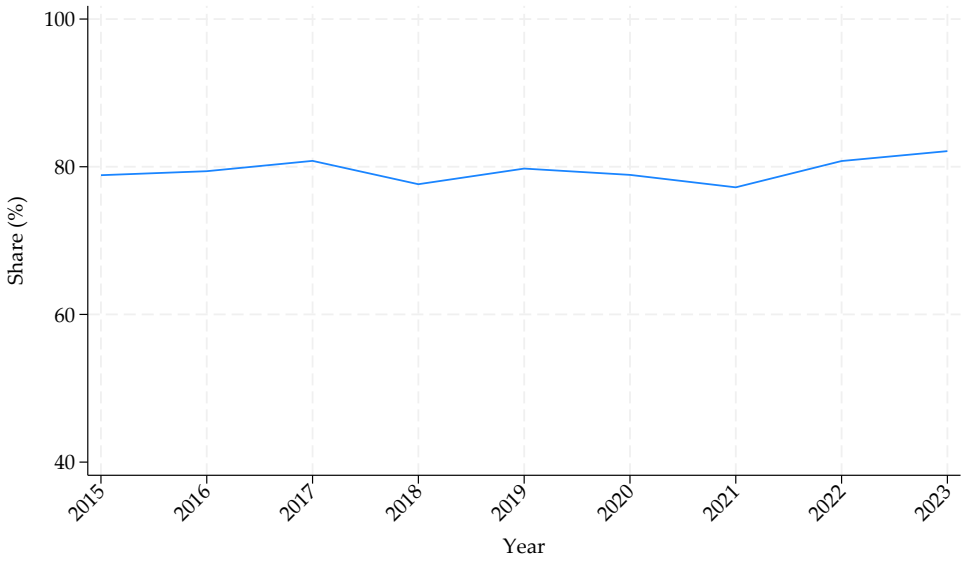
Notes. Share denotes each detailed BEA asset category's share of total 2020 investment within the corresponding asset type, Equipment or Structures. **Price** is the ratio of the asset's real price in 2020 to its real price in 1970, where real prices are nominal prices deflated by the GDP deflator and the 1970 level is normalized to 1. Within each broad group, categories are ordered in descending order of Share.

ware, and All other expenditures. Structures include the costs of purchased or improved buildings and other facilities such as signal towers or windmills that are fixed to the land. All other expenditures include the costs of purchased patents or other intangible assets.[48] The instrument also notes that assets acquired through merger and acquisition activities should be excluded, and that capital expenditures benefiting both R&D and non-R&D operations should be allocated on a reasonable basis.

Figure A.2 plots a time series of equipment investment as a share of total investment in equipment and structures. The series is constructed using total expenditures across all industries. Since 2015, the share has remained stable at around 80%.

[48] https://www2.census.gov/programs-surveys/brdis/information/qbyqbrdi.pdf

Figure A.2: **Equipment Investment Share in R&D Sector in the U.S.**



Notes. The figure shows a time series of equipment investment as a share of the sum of equipment and structures investment. The time series is constructed using total expenditures across all industries. *Source.* BERD and authors' calculations.

A.5 Gross Domestic Expenditure on R&D (GERD)

Gross domestic expenditure on research and development (GERD) measures the total intramural spending on R&D performed within a country during a given reference year. GERD is constructed as the aggregate of R&D expenditures reported by all performing sectors under the OECD reporting framework, which distinguishes both performing sectors and sources of funds. In this paper, we focus on the business enterprise performing sector and exclude government, higher education, private nonprofit, and foreign-funded performing sectors from our baseline construction.

Capital types are classified based on The Frascati Manual.[49] We use two capital expenditure categories. Land and Buildings include land acquired for R&D use, such as testing grounds and sites for laboratories or pilot plants, and buildings constructed or purchased for R&D use, including major improvements, modifications, and repairs. This corresponds to Sections 4.48 to 4.50. Machinery and equipment include major capitalized machinery and equipment acquired for use in the performance of R&D. This corresponds to Section 4.51.

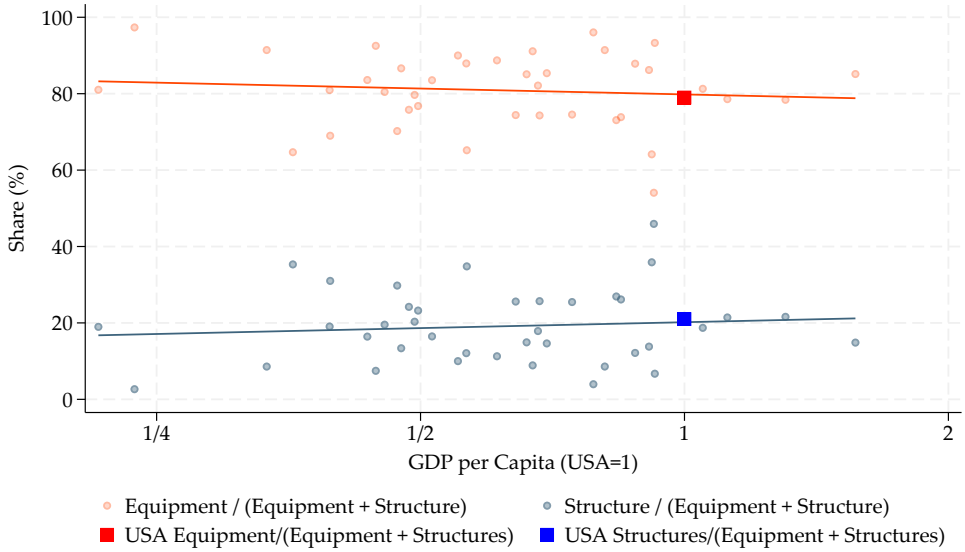
A practical issue for empirical work is that land and buildings are frequently reported jointly, and separate identification of land is not consistently available. While land is conceptually part of the land and buildings category, when the category is reported as a single total, the reported value is typically dominated by buildings and related construction and improvements rather than

[49] https://www.oecd.org/content/dam/oecd/en/publications/reports/2015/10/frascati-manual-2015g1g57dcb/9789264239012-en.pdf

land acquisition. For this reason, we use land and buildings as the structures measure.

We calculate the allocation of capital expenditures between equipment, measured as machinery and equipment, and structures, measured as land and buildings, using GERD data for 45 countries based on 2014 to 2019 averages. We then compare these patterns with U.S. evidence from BERD, yielding a total sample of 46 countries.

Figure A.3: **R&D Capital Allocation, 2014–2019 Average, versus GDP per Capita, 2017**



Notes. The orange series reports the 2014–2019 average equipment share and the blue series reports the corresponding average structures share. For countries other than the United States, we use GERD and define the equipment share as machinery and equipment divided by land and buildings plus machinery and equipment, with the structures share defined as land and buildings divided by land and buildings plus machinery and equipment. For the United States, we use BERD and define the equipment share as equipment divided by structures plus equipment, with the structures share defined as structures divided by structures plus equipment. GDP per capita in PPP terms on the horizontal axis is measured in 2017 and expressed relative to the United States, so that the U.S. equals one. *Source.* BERD, GERD, PWT 11.0, and authors' calculations.

Figure A.3 plots the 2014–2019 average structures share and equipment share in total equipment-plus-structures investment against relative GDP per capita in 2017, normalized to one for the United States. Across the income distribution, the equipment share in R&D capital expenditure is close to 80%, consistent with the 79% estimate for the United States from BERD. Correspondingly, the structures share is close to 20%. Overall, these results indicate that the equipment intensity of R&D capital spending is broadly stable across countries.

A.6 EU KLEMS Database

The EU KLEMS database provides harmonized growth and productivity accounts for major countries, including the U.S. It offers detailed industry-level data within a consistent accounting framework across countries. In this paper, we use the 2024 vintage, which starts in 1996. From the

national accounts module, we draw gross output at current prices (GO_CP), intermediate inputs (II_CP), value added (VA_CP), labor compensation (COMP), and the corresponding price indexes (GO_PI, II_PI, and VA_PI) for each NACE/ISIC sector. From the growth accounts module, we use quantity indexes for labor input (LAB_QI) and capital services (CAP_QI). We compute nominal labor compensation as COMP and nominal capital compensation as VA_CP-COMP, and derive implicit factor price indexes by dividing each nominal compensation measure by the corresponding quantity index. These variables allow us to construct industry-level cost shares and input price changes for each country-year, which constitute the building blocks of the dual price decomposition.

B Robustness Checks on Stylized Facts

B.1 Additional Analyses for Fact 1

B.1.1 Investment Share Trend

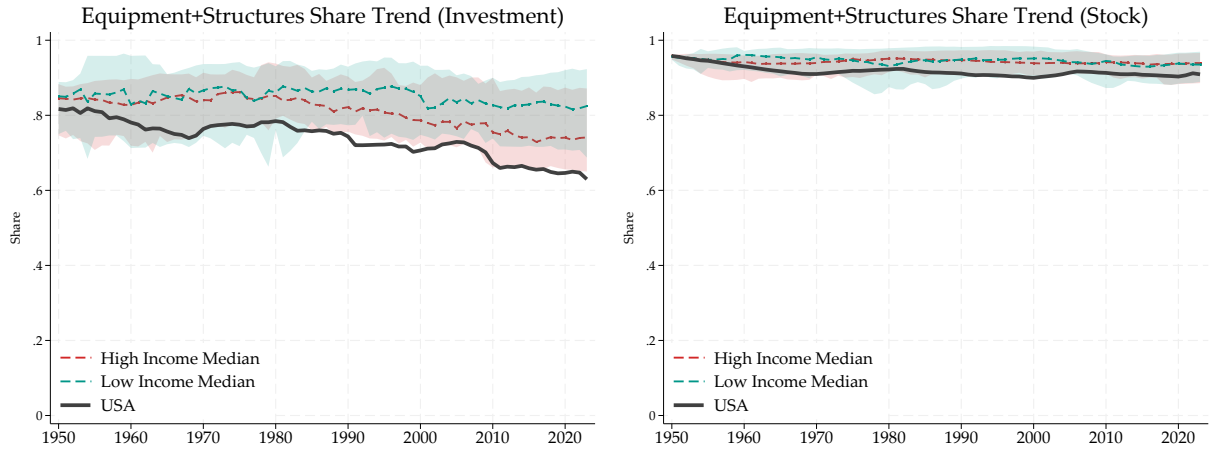
The components we emphasize in the main text, which are equipment and structures, constitute a large and stable fraction of aggregate investment and capital. Figure B.1 shows that the combined equipment and structures share is high and broadly stable over time in both flows and stocks.

In the left panel (investment), the group medians remain in a relatively narrow band throughout the sample, indicating that cross-country differences in the evolution of disaggregated relative prices documented in the main text are not driven by large shifts in compositional weights between equipment and structures. In the right panel (capital stock), the combined share is even more stable, with medians close to unity and limited dispersion across groups, consistent with equipment and structures accounting for the overwhelming majority of the capital stock over time.

Additionally, we plot how the equipment and structures investment shares vary across countries. Figure B.2 plots, for 2021, each country's equipment and structures shares within equipment-plus-structures investment against relative GDP per capita (USA = 1), with the United States highlighted and fitted trend lines shown.

Finally, we plot the time series for the share of equipment investment in total investment in equipment and structures in Figure B.3. This series is used to calibrate the model parameter that pins down the output share of equipment relative to structures. As the figure shows, the equipment share increased by roughly 5 percentage points from the 1970s to the 1980s and remained broadly stable for the subsequent two decades. The series displays a transitory fluctuation around the 2008 Great Financial Crisis, with a mean of approximately 40%.

Figure B.1: **Shares of Structures and Equipment in Total Investment and Capital Stock**



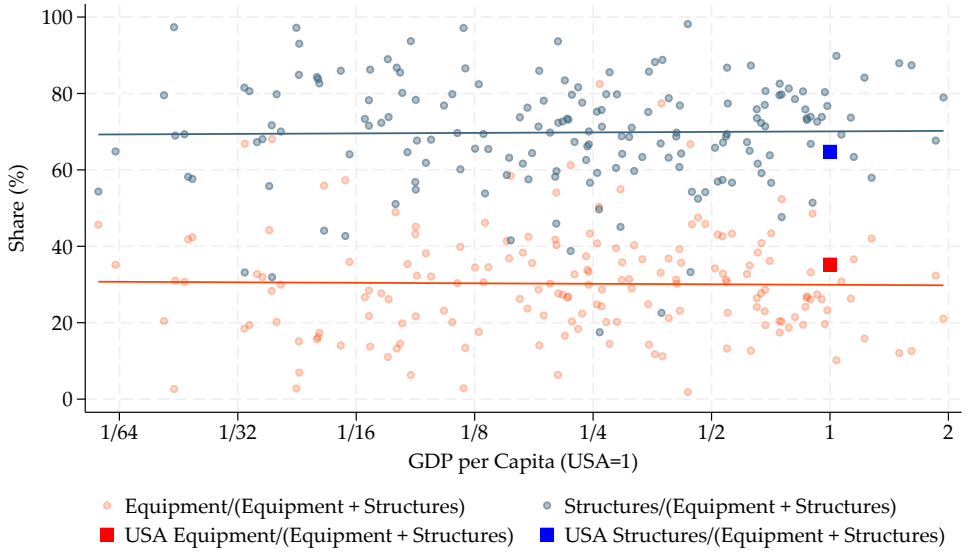
Notes. The figure plots time series of investment and capital-stock shares constructed from Penn World Table capital-detail categories. The share is computed as the ratio of a component to the sum of four components: Structures, Machinery and Equipment, Transport Equipment, and Other assets. The left panel is based on investment (*Ic*) and the right panel is measured based on capital stock (*Nc*). High and low-income groups are defined based on the top and bottom terciles of real GDP per capita in 1970. A total of 120 countries with reported prices in 1970 are included in the analysis. The low-income group (indicated in green) and the high-income group (indicated in red) represent the bottom and top terciles, respectively. Shaded areas represent the 15th to 85th percentile range. Dashed lines indicate the median for each group, and the solid black line denotes the United States as a benchmark. *Source.* Penn World Table 11.0; PWT Capital Detail; and authors' calculations.

B.1.2 Relative Price Trend of Other Capital Asset Types

PWT's equipment measure does not include transport capital, whereas the ICP Machinery and Equipment category includes transport equipment. In addition, the category of other assets may also be subject to compositional mixing. Motivated by these concerns, we replicate the same exercise using price data for transport equipment and other assets.

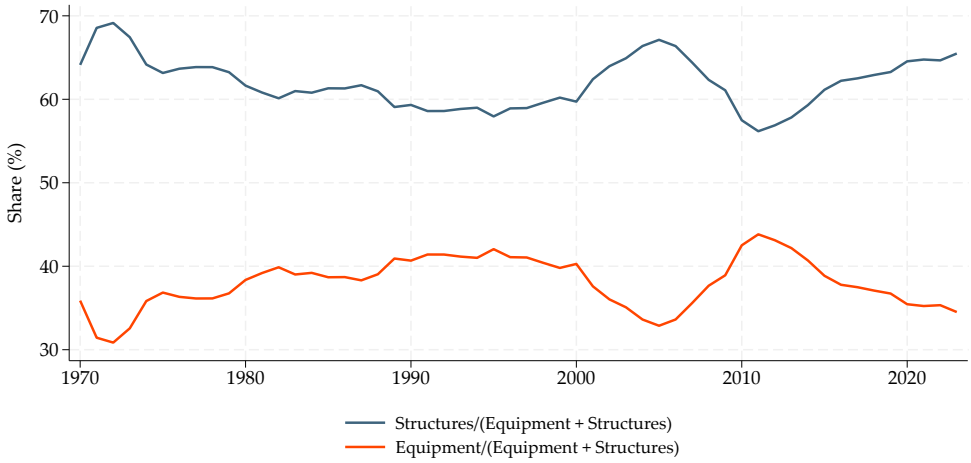
Figure B.4 shows that for these two capital categories it is difficult to identify heterogeneity across the United States, the high-income group, and the low-income group that is as pronounced as what we observe for equipment or structures. This finding mitigates our concern that the cross-group patterns documented for equipment and structures are driven mechanically by category definitions or compositional mixing in the underlying price data. If the inclusion of transport equipment in ICP's equipment, or the potential mixture embedded in other assets, were a first-order source of the observed heterogeneity, we would expect to see similarly large and systematic differences across income groups for transport equipment and other assets themselves. Instead, the absence of comparable divergence suggests that our main results for equipment and structures are unlikely to be an artifact of these classification issues.

Figure B.2: **Equipment and Structures Investment Allocation**



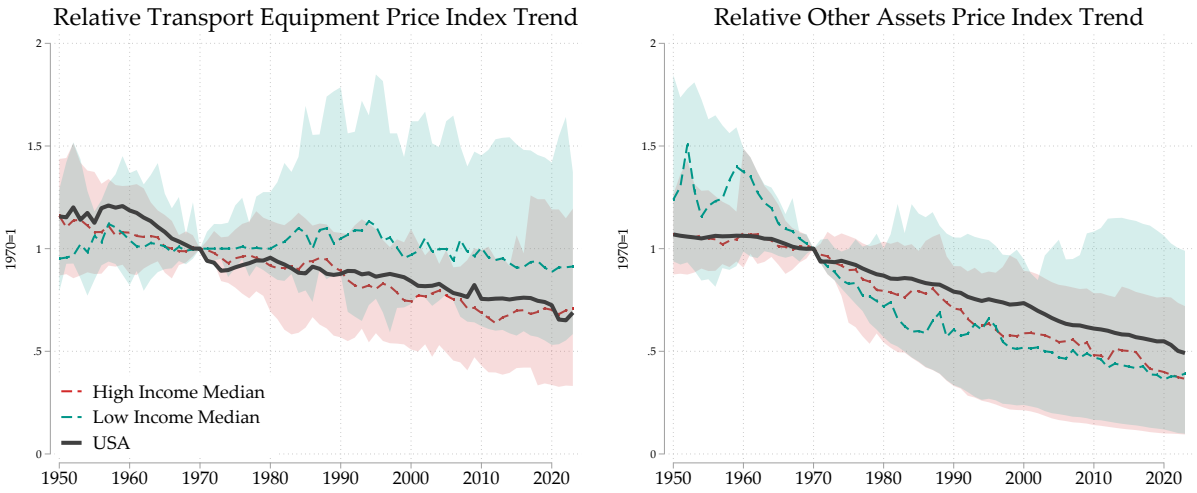
Notes. The figure plots, for 2021, each country's investment allocation between equipment and structures against relative GDP per capita (USA=1). Circles show country observations; the U.S. observation is highlighted. Solid lines are separate linear fits of the equipment share and the structures share on relative GDP per capita. The equipment share is defined as $E/(E+S)$ and the structures share as $S/(E+S)$, where E and S correspond to equipment and structures components of investment, respectively. GDP per capita (in PPP terms) is expressed relative to the United States so that the U.S. equals one. *Source*: PWT 11.0 Capital Detail

Figure B.3: **Equipment and Structures Investment Allocation Time Series in the U.S.**



Notes. The figure plots the allocation of investment between equipment and structures in the United States from 1970 to 2024. The equipment share is defined as $E/(E+S)$ and the structures share as $S/(E+S)$, where E and S denote the equipment and structures components of investment, respectively. *Source*: PWT 11.0 Capital Detail

Figure B.4: **Relative Investment Price Index Trend of Other Capital Assets**



Notes. The relative price for each asset type is calculated by dividing each asset type's investment price index by the GDP price index, normalized to 1 in 1970. High and low-income groups are defined based on the top and bottom terciles of real GDP per capita in 1970. A total of 120 countries with reported prices in 1970 are included in the analysis. The low-income group indicated in green and the high-income group indicated in red represent the bottom and top terciles, respectively. Shaded areas represent the 15th to 85th percentile range. Dashed lines indicate the median for each group, and the solid black line denotes the United States as a benchmark. *Source.* Penn World Table 11.0, PWT Capital Detail, and authors' calculations.

B.1.3 Relative Price Trends Based on Capital Stocks

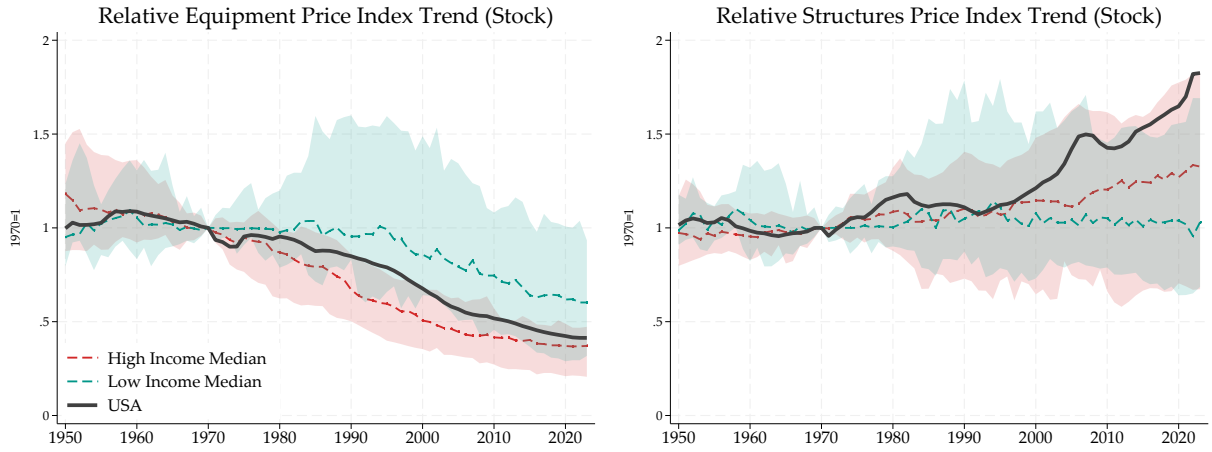
PWT provides price indices for capital based on capital stocks (Capital Stock Price Indices; series codes begin with Np). The stylized facts we report for equipment and structures price trends remain unchanged when we use these stock-based price indices. Figure B.5 shows that equipment prices decline more rapidly in high-income economies, including the United States, than in low-income economies. By contrast, for structures, the capital stock price index rises in high-income economies, especially the United States, while it remains relatively stable in low-income economies.

B.1.4 Relative Price Trends Classified by GDP Growth

We classify countries based on GDP per capita growth rather than their GDP per capita level in 1970. The high-growth group consists of countries in the upper tercile of GDP per capita growth between 1970 and 2020, while the low-growth group consists of countries in the bottom tercile.

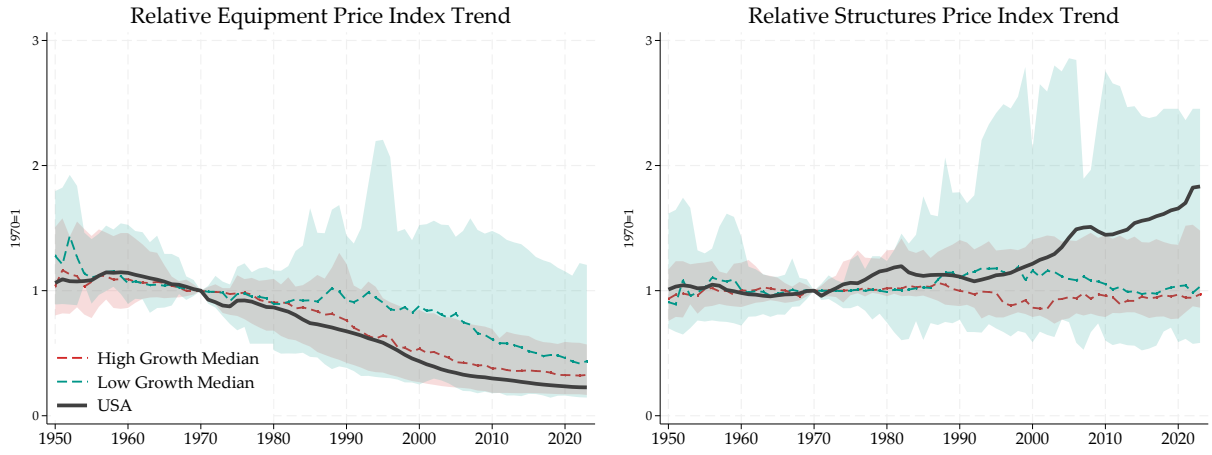
Figure B.6 presents the trends in the relative price of investment for these two groups. For equipment prices, the median in the high-growth group declines more rapidly than that in the low-growth group, while the low-growth group exhibits substantially greater dispersion across countries. For structures prices, the median remains relatively stable in both the high- and low-

Figure B.5: **Relative Capital Stock Price Index Trend**



Notes. The relative price for each asset type is calculated by dividing each asset type's capital stock price index by the GDP price index, normalized to 1 in 1970. High and low-income groups are defined based on the top and bottom terciles of real GDP per capita in 1970. A total of 120 countries with reported prices in 1970 are included in the analysis. The low-income group indicated in green and the high-income group indicated in red represent the bottom and top terciles, respectively. Shaded areas represent the 15th to 85th percentile range. Dashed lines indicate the median for each group, and the solid black line denotes the United States as a benchmark. *Source.* Penn World Table 11.0, PWT Capital Detail, and authors' calculations.

Figure B.6: **Relative Investment Price Index Trends by GDP per Capita Growth Group**



Notes. The relative price for each asset type is calculated by dividing each type's investment price index by the GDP price index, normalized to 1 in 1970. High- and low-growth groups are defined based on the top and bottom terciles of real GDP per capita growth over the 1970–2020 period. A total of 120 countries with reported prices in 1970 are included in the analysis. The low-growth group (indicated in green) and the high-growth group (indicated in red) represent the bottom and top terciles, respectively. Shaded areas represent the 15th to 85th percentile range. Dashed lines indicate the median for each group, and the solid black line denotes the United States as a benchmark. *Sources:* Penn World Table 11.0; PWT Capital Detail; and authors' calculations.

growth groups. However, the low-growth group exhibits substantially greater dispersion, with

the distribution spreading more toward higher relative structures prices.

B.1.5 Price Trends from ICP

We construct ICP-based cross-country relative prices, estimate the evolving cross-sectional relationship between relative prices and income across ICP survey years, and report the corresponding figures (Hsieh and Klenow, 2007).

We use ICP survey-year data on PPP prices for (i) equipment, (ii) structures, and (iii) GDP, together with the associated market exchange rate information to express PPP price measures in a common currency basis. We restrict the sample to countries with non-missing information for the relevant items in each survey year and drop aggregate regions and non-sovereign groupings. We focus on the ICP benchmark years 1996, 2005, 2011, 2017, and 2021 for consistent cross-year comparisons. Following Hsieh and Klenow (2007), we normalize each variable by the United States within each survey year to facilitate cross-country comparisons. Let $P^k_{i,t}$ denote the PPP price of item $k \in \{\text{equipment}, \text{structures}\}$ for country i in survey year t, and let $y_{i,t}$ denote PPP GDP per capita. We construct U.S.-normalized relative prices and income as

$$\tilde{P}^k_{i,t} = \ln\left(\frac{P^k_{i,t}}{P^k_{\text{USA},t}}\right), \qquad \tilde{y}_{i,t} = \ln\left(\frac{y_{i,t}}{y_{\text{USA},t}}\right).$$

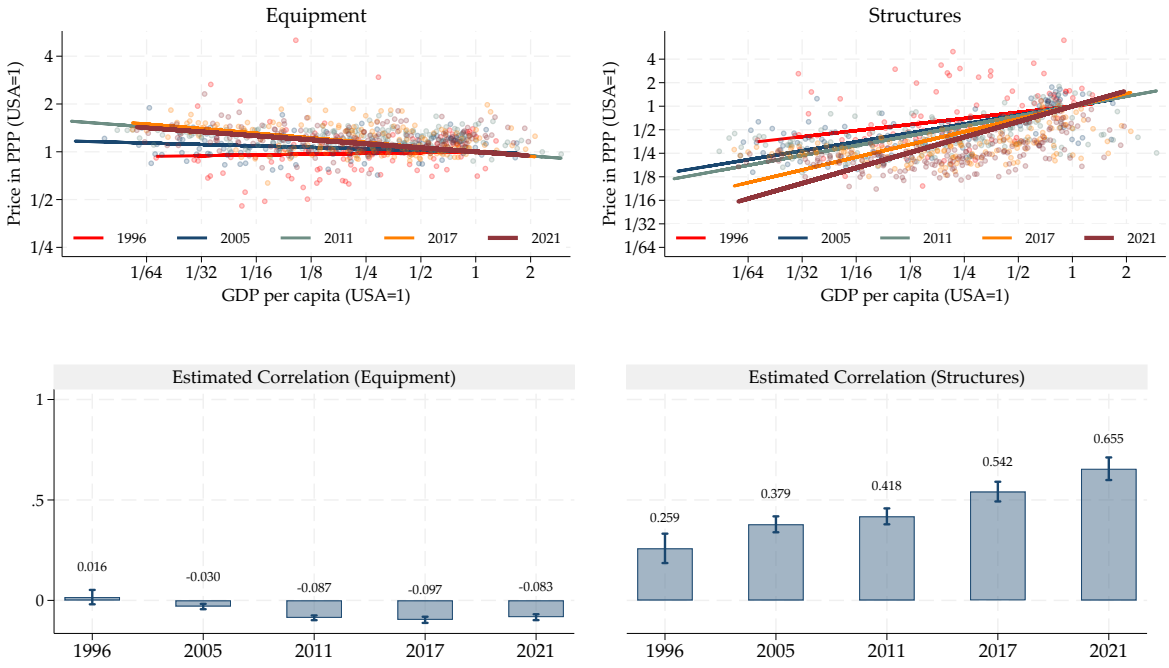
For visualization, we plot U.S.-normalized relative levels, obtained by exponentiating the log ratios above. Thus, the United States equals one in each survey year, and values such as $1/2$ and 2 indicate prices or incomes one half or twice the U.S. level. In addition, for each survey year t, we estimate the income gradient of each relative price using a no-intercept specification:

$$\tilde{P}^k_{i,t} = \beta^k_t \, \tilde{y}_{i,t} + \varepsilon^k_{i,t},$$

where β^k_t captures how relative prices covary with relative income in year t.

Figure B.7 presents ICP-based cross-country evidence on relative prices and their income gradients for equipment and structures across survey years (Hsieh and Klenow, 2007). The top row plots, for each ICP benchmark year (1996, 2005, 2011, 2017, and 2021), the cross-country relationship between U.S.-normalized PPP prices and U.S.-normalized PPP GDP per capita. The fitted lines for equipment are relatively stable across years, although the correlation with GDP per capita turns negative over time, whereas the fitted lines for structures become increasingly steep, indicating a strengthening cross-sectional association between structures prices and income in later survey waves. The bottom row summarizes the corresponding year-by-year regression slopes and their 95% confidence intervals.

Figure B.7: **ICP Cross-Country Relative Prices and Income Gradients by Survey Year**



Notes. Each dot is a country. Both axes report U.S.-normalized relative levels. The United States equals one in each survey year. The underlying regressions use log ratios relative to the United States. The fitted lines in the top row are survey-year-specific cross-sectional regressions with no intercept. The bottom row reports the estimated slopes and 95% confidence intervals for equipment and structures by survey year.

B.1.6 Regression Evidence

Given the differences in price trends across countries, we examine the cross-country association between equipment and structures prices and GDP per capita.

We run the regression below with GDP per capita as the outcome. The key explanatory variables, $\ln(P_{it}^{\text{Equip}})$ and $\ln(P_{it}^{\text{Str}})$, denote the log PPP prices of equipment and structures, respectively, normalized by the exchange rate. The vector $\mathbf{X}_{it}$ includes country-specific controls for schooling, human capital, and population.[50] α_i^h and δ_t^h represent country and year fixed effects, respectively. We consider both contemporaneous income and income five years ahead and estimate the following specification for $h \in \{0, 5\}$.

$$\ln(y_{i,t+h}) = \beta_{\text{Equip}}^h \ln(P_{it}^{\text{Equip}}) + \beta_{\text{Str}}^h \ln(P_{it}^{\text{Str}}) + \mathbf{X}_{it}'\Gamma^h + \lambda^h \ln(y_{i,t-\tau}) + \alpha_i^h + \delta_t^h + \epsilon_{it}^h, \tag{B.1}$$

where τ denotes the lag length for lagged GDP per capita. We report specifications using

[50] Since the PWT human capital index is constructed in part from schooling data and relies on interpolation, the two measures may be mechanically correlated.

$\tau \in \{6, 10\}$ to assess the sensitivity of the conditional correlations to the choice of lagged-income control. Because investment prices are equilibrium objects and may reflect persistent country-level factors that are not fully absorbed by fixed effects and lagged income, we interpret these regressions as descriptive conditional associations rather than as estimates of causal effects.

Table B.I: **Capital Prices and GDP per Capita**

	Control, Lagged GDP $t-6$		Control, Lagged GDP $t-10$	
	$\ln(y_{i,t})$	$\ln(y_{i,t+5})$	$\ln(y_{i,t})$	$\ln(y_{i,t+5})$
	(1)	(2)	(3)	(4)
Log Equipment Price	-0.279***	-0.235**	-0.318***	-0.223*
	(0.097)	(0.113)	(0.114)	(0.117)
Log Structures Price	-0.064	0.019	-0.104*	-0.031
	(0.048)	(0.062)	(0.055)	(0.060)
Years of Schooling	0.310*	0.498**	0.370*	0.405
	(0.165)	(0.247)	(0.207)	(0.262)
Human Capital Index	-1.448*	-2.326*	-1.613*	-1.871
	(0.773)	(1.231)	(0.968)	(1.325)
Log Population	-0.120	-0.161	0.038	-0.040
	(0.113)	(0.135)	(0.125)	(0.137)
$\ln(y_{i,t-6})$	0.386***	0.013		
	(0.048)	(0.068)		
$\ln(y_{i,t-10})$			0.089	-0.082
			(0.072)	(0.083)
Observations	632	494	617	479
R^2	0.973	0.968	0.971	0.971
Country FE	✓	✓	✓	✓
Year FE	✓	✓	✓	✓

Notes. Standard errors are reported in parentheses and are clustered at the country level. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$. Columns (1) and (2) control for GDP per capita at $t-6$, while Columns (3) and (4) control for GDP per capita at $t-10$.

Table B.I shows a consistently negative conditional association between equipment prices and GDP per capita. The coefficient on equipment prices ranges from -0.28 to -0.32 for contemporaneous income and from -0.22 to -0.24 for income five years ahead. The estimates remain negative across both choices of the lagged-income control and are statistically significant at conventional levels, although the estimate for income five years ahead with the $t-10$ control is significant only at the 10 percent level.

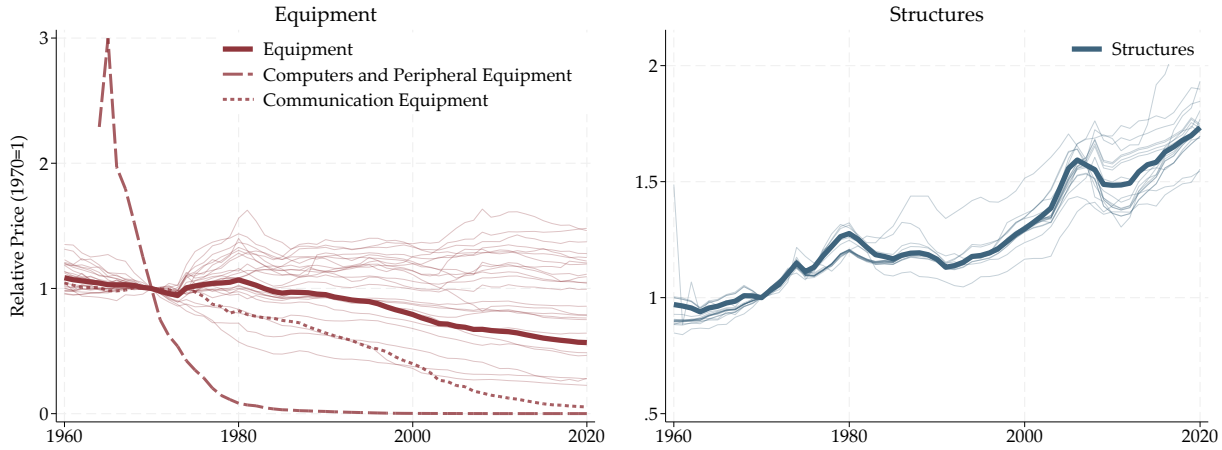
In contrast, the coefficients on structures prices are substantially smaller in magnitude and generally statistically insignificant. The only exception is the contemporaneous specification controlling for GDP per capita at $t-10$, where the coefficient is -0.10 and marginally significant.

Overall, these regressions provide suggestive evidence that equipment prices have a stronger conditional association with income than structures prices in the cross-country data.

B.2 Additional Analyses for Fact 2

We conduct a parallel exercise using BEA data on current-cost net capital stocks in order to assess whether the patterns observed for investment prices extend to capital stock prices. Figure B.8 plots relative net capital stock price indices by BEA asset type over the period 1960—2020. For each asset category, we recover an implicit net-stock price index from current-cost net capital stock values and the corresponding chain-type quantity indexes. We then express each index relative to the GDP deflator and normalize it to unity in 1970.

Figure B.8: **Relative Net Stock Price Index Trend by Asset Type (1960–2020)**



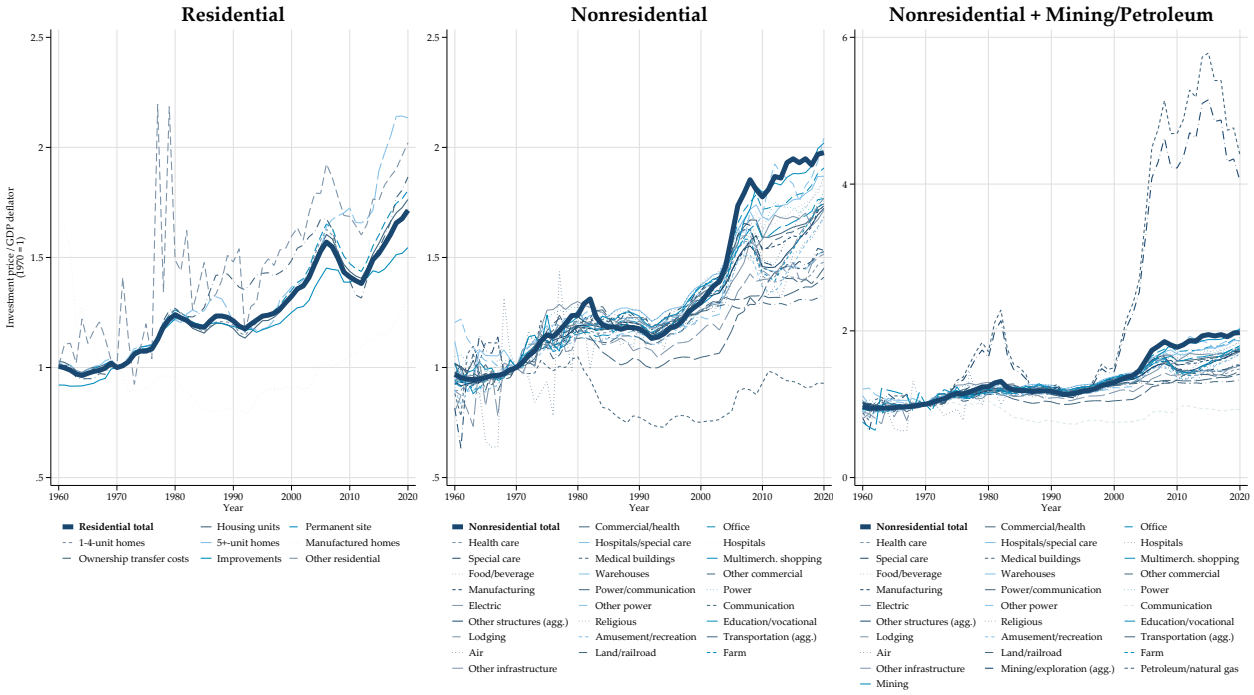
Notes. The figure plots trends in relative net capital stock prices by BEA asset type. For each asset type, we construct an implicit net-stock price index as nominal current-cost net capital stock divided by the corresponding chain-type quantity index for net stocks. The resulting series is deflated by the GDP Deflator and normalized to 1 in 1970 for each category. The sample is restricted to 1960—2020. Thin lines show disaggregated categories, while thicker highlighted lines emphasize the aggregate equipment series and selected subcomponents (computers and peripheral equipment, and communication equipment). *Sources:* U.S. Bureau of Economic Analysis (BEA), BEA Fixed Assets Tables 2.1 and 2.2; Gross Domestic Product: Implicit Price Deflator (GDPDEF) from Federal Reserve Bank of St. Louis (FRED); and authors' calculations.

The resulting trends are highly similar to those for investment prices. Equipment prices fall sharply after 1970, driven primarily by information-processing assets such as computers and peripheral equipment and communication equipment, while structures prices rise broadly after 1990 across disaggregated categories. Overall, the close alignment between investment- and stock-price trends suggests that these divergent dynamics are not unique to investment flows but are also reflected in the evolution of the capital stock itself.

Furthermore, we divide structures into residential and nonresidential. We also plot the aggregate trend together with the trends of each subcategory. Figures B.9 and B.10 show the results,

based on investment and net stock, respectively.

Figure B.9: **Relative Investment Price Index Trend – Structures (1960–2020)**



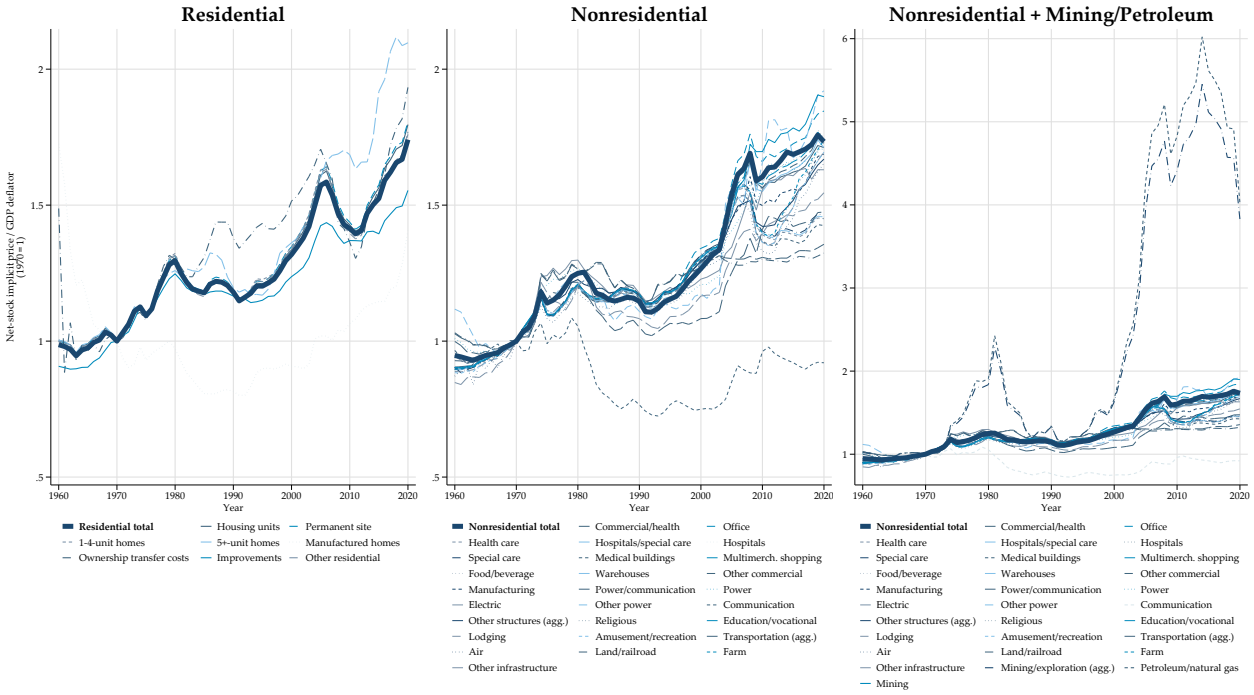
Notes. The figure plots trends in relative investment prices by BEA asset type. For each asset category, an implicit price index is recovered from BEA data as nominal investment divided by the corresponding chain-type quantity index. The resulting series is deflated using the GDP Deflator and then normalized to 1 in 1970 for each category. The sample is restricted to 1960—2020. Thin lines show disaggregated categories, while thicker highlighted lines emphasize the aggregate structures series. Details of the asset classes are provided in Table A.I. The petroleum and natural gas structures category is added to the third panel. *Sources:* U.S. Bureau of Economic Analysis (BEA), BEA Fixed Assets Accounts Tables 2.7 and 2.8; Gross Domestic Product: Implicit Price Deflator (GDPDEF) from Federal Reserve Bank of St. Louis (FRED); and authors' calculations.

B.3 Additional Analyses for Fact 3

For robustness checks, we use the average of the annual shares from 1965 to 1975 and the 1970 shares calculated using equipment plus structures as the denominator as proxies for structure reliance in a given industry.

Figure B.11 shows the relationship between structures dependence, measured by the 1965—1975 average share of stock or investment, and subsequent TFP and real investment growth. A 10 percentage point higher structures share of capital stock is associated with 0.1 percentage points lower annual TFP growth and 0.28 percentage points lower annual real investment growth. Using the structures share of investment, the corresponding estimates are 0.07 and 0.19 percentage

Figure B.10: **Relative Net Stock Price Index Trend – Structures (1960–2020)**

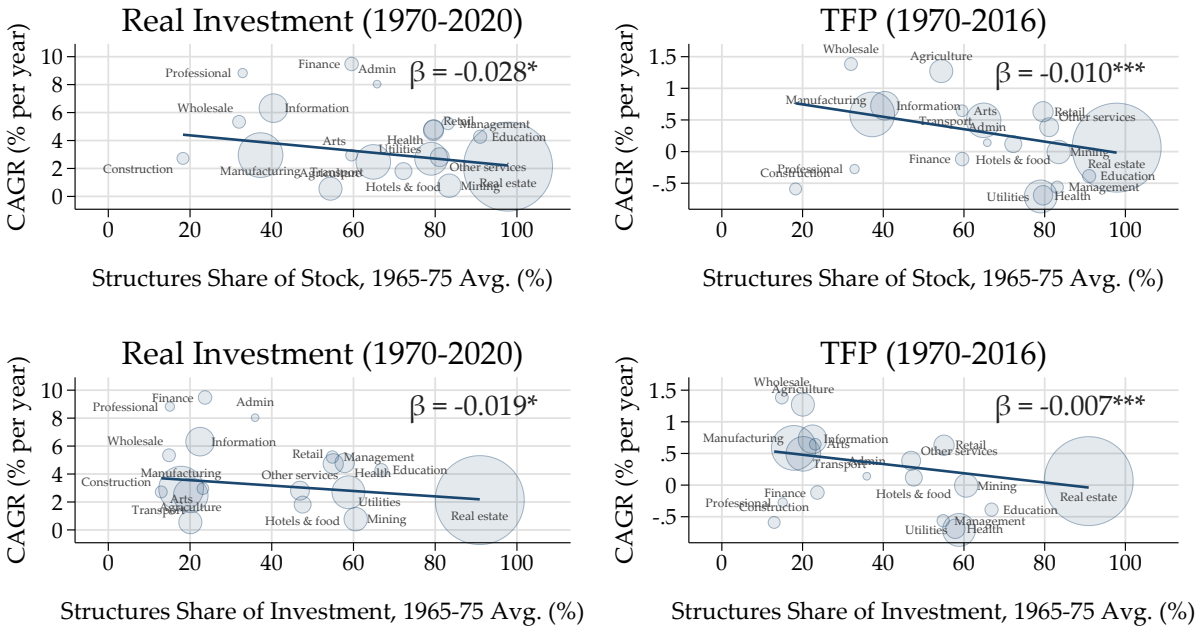


Notes. The figure plots trends in relative net capital stock prices by BEA asset type. For each asset type, we construct an implicit net-stock price index as nominal current-cost net capital stock divided by the corresponding chain-type quantity index for net stocks. The resulting series is deflated using the GDP Deflator and then normalized to 1 in 1970 for each category. The sample is restricted to 1960—2020. Thin lines show disaggregated categories, while thicker highlighted lines emphasize the aggregate structures series. Details of the asset classes are provided in Table A.I. The petroleum and natural gas structures category is added to the third panel. *Sources:* U.S. Bureau of Economic Analysis (BEA), BEA Fixed Assets Accounts Tables 2.1 and 2.2; Gross Domestic Product: Implicit Price Deflator (GDPDEF) from Federal Reserve Bank of St. Louis (FRED); and authors' calculations.

points. Thus, greater reliance on structures is consistently associated with lower TFP and real investment growth.

As a complementary specification, Figure B.12 shows the association between a sector's reliance on structures in 1970 and its subsequent TFP and real investment growth. The upper panels measure structures dependence using the capital stock share, while the lower panels use the investment share. A 10 percentage point higher structures share of capital stock is associated with 0.11 percentage points lower annual TFP growth and 0.28 percentage points lower annual real investment growth. Using the investment share, the corresponding estimates are 0.08 and 0.2 percentage points. Thus, greater reliance on structures is consistently associated with lower TFP and real investment growth.

Figure B.11: **Structures Reliance, 1965–1975 Average**



Notes. The figure shows the cross-sector relationship between reliance on structures and subsequent TFP and real fixed-investment growth across 19 two-digit NAICS sectors in the U.S. The upper panels measure reliance on structures as the average structures share of total fixed capital stock from 1965 to 1975, and the lower panels use the average structures share of total fixed investment over the same period. The left panels report the CAGR of real fixed investment from 1970 to 2020, and the right panels report the CAGR of TFP from 1970 to 2016. Total fixed capital and investment include equipment, structures, and intellectual property products. Circle size represents total fixed capital stock in 1970. The fitted lines and reported β coefficients are estimated using OLS weighted by the same 1970 capital stock, with heteroskedasticity-robust standard errors. ***, **, and * denote statistical significance at the 1, 5, and 10 percent levels, respectively. *Source.* BEA–BLS Integrated Industry-Level Production Account, BEA Fixed Assets Accounts, and authors' calculations.

C Robustness Checks on Construction Sector Productivity

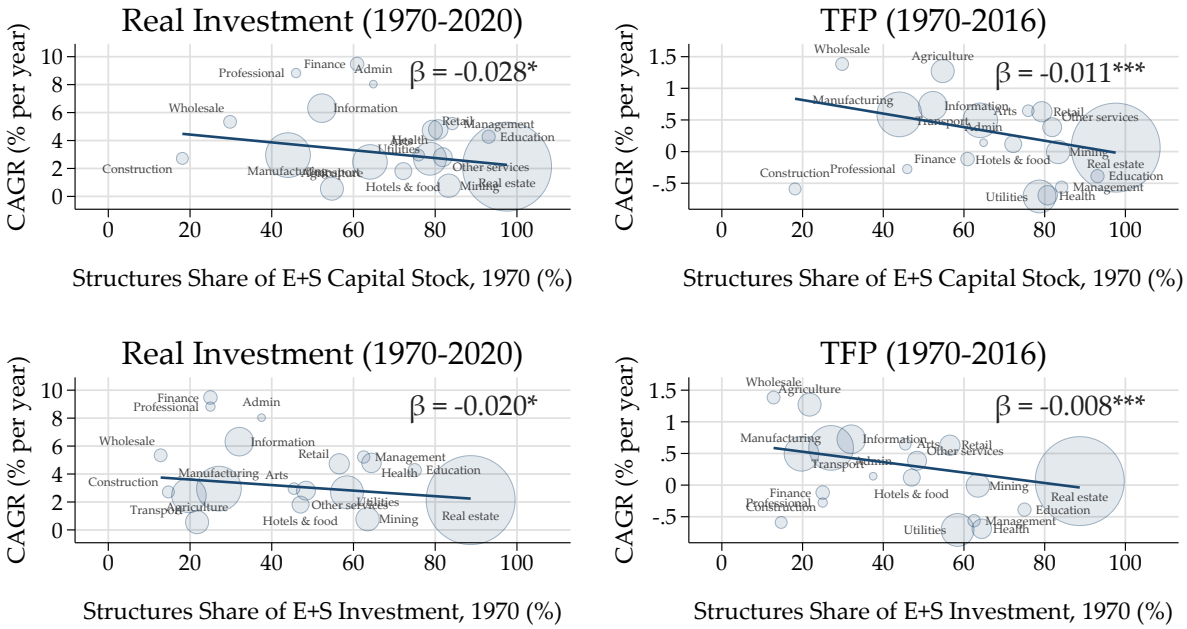
This section uses Törnqvist decompositions, following Diewert (1976), to show that rising structures prices are closely related to weak productivity growth in construction.

C.1 Price Decomposition

Under constant returns and competitive factor markets, zero profit implies that output price equals unit cost. Differentiating this condition gives

$$\Delta \ln P_{i,t} = \bar{s}_{L,i,t} \Delta \ln P^{L}_{i,t} + \bar{s}_{K,i,t} \Delta \ln P^{K}_{i,t} + \bar{s}_{II,i,t} \Delta \ln P^{II}_{i,t} - \Delta \ln A_{i,t}, \tag{C.1}$$

Figure B.12: **Structures Reliance, Equipment and Structures Only**



Notes. The figure shows the cross-sector relationship between reliance on structures in 1970 and subsequent TFP and real fixed-investment growth across 19 two-digit NAICS sectors in the U.S. The upper panels measure reliance on structures as the structures share of equipment plus structures in total fixed capital stock, and the lower panels use the corresponding share in total fixed investment. The left panels report the CAGR of real fixed investment from 1970 to 2020, and the right panels report the CAGR of TFP from 1970 to 2016. Intellectual property products are excluded from the denominator when constructing the structures shares. Circle size represents total fixed capital stock in 1970. The fitted lines and reported β coefficients are estimated using OLS weighted by the same 1970 capital stock, with heteroskedasticity-robust standard errors. ***, **, and * denote statistical significance at the 1, 5, and 10 percent levels, respectively. *Source.* BEA–BLS Integrated Industry-Level Production Account, BEA Fixed Assets Accounts, and authors' calculations.

where P^L, P^K, and P^{II} are the implicit price indexes of labor, capital services, and intermediate inputs. The weights are Törnqvist averages,

$$\bar{s}_{j,i,t} = \frac{1}{2}(s_{j,i,t} + s_{j,i,t-1}).$$

Productivity enters with a negative sign because higher TFP lowers unit cost.

Expressing prices relative to the aggregate gross-output price P_{agg} yields

$$\begin{aligned}\Delta \ln \left(\frac{P_{i,t}}{P_{agg,t}}\right) = \bar{s}_{L,i,t}\left(\Delta \ln P^L_{i,t} - \Delta \ln P_{agg,t}\right) + \bar{s}_{K,i,t}\left(\Delta \ln P^K_{i,t} - \Delta \ln P_{agg,t}\right) \\ + \bar{s}_{II,i,t}\left(\Delta \ln P^{II}_{i,t} - \Delta \ln P_{agg,t}\right) - \Delta \ln A_{i,t}.\end{aligned} \tag{C.2}$$

The first three terms measure cost pressure, while $-\Delta \ln A_{i,t}$ captures the price effect of produc-

tivity growth.

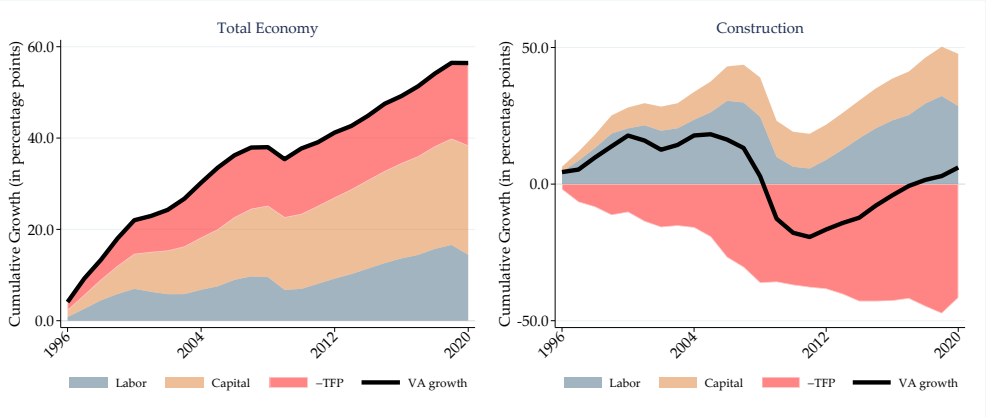
C.2 Value-Added Growth Decomposition

We also decompose real value-added growth into factor contributions. Under constant returns and marginal-product factor pricing,

$$\Delta \ln VA_{i,t} = \bar{s}_{L,i,t} \Delta \ln L_{i,t} + \bar{s}_{K,i,t} \Delta \ln K_{i,t} + \Delta \ln A_{i,t}, \tag{C.3}$$

where L and K are composition-adjusted labor and capital-service indexes, and $\Delta \ln A_{i,t}$ is the Solow residual. EU KLEMS reports these terms as pre-computed contributions, so the identity holds by construction.

Figure C.1: **Cumulative Value-Added Growth Decomposition in the United States**



Notes. The figure decomposes cumulative value-added growth into labor, capital, and productivity components. The left panel reports the total economy, and the right panel reports construction. The black line is cumulative value-added growth. The shaded areas show cumulative factor and productivity contributions. In construction, productivity is negative over much of the sample and offsets gains from labor and capital. *Source.* EU KLEMS 2024 and authors' calculations.

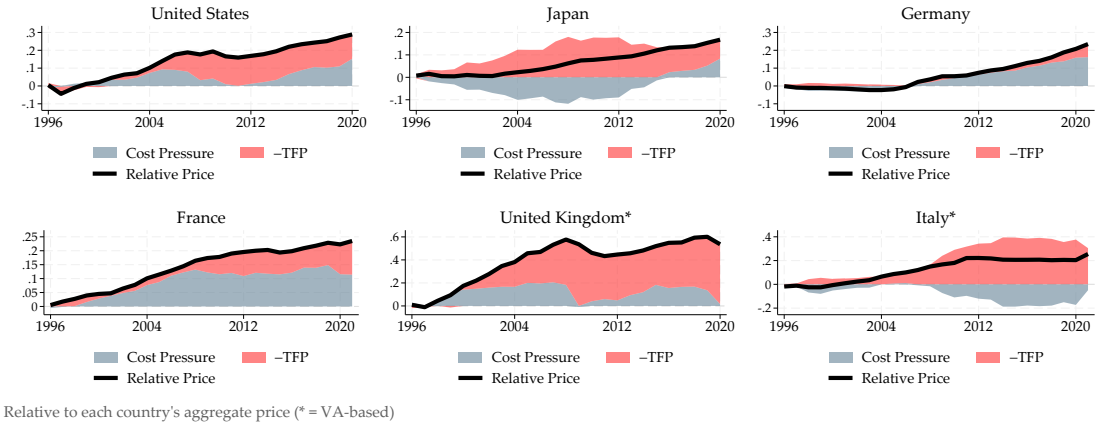
Figure C.1 shows that aggregate value-added growth is supported by both factor accumulation and productivity. In construction, by contrast, productivity contributes negatively for much of the sample, suggesting that the sectoral slowdown reflects weak efficiency growth rather than only weak factor accumulation.

C.3 Cross-Country Evidence

Figure C.2 extends the price decomposition to a cross-country comparison of construction-sector relative prices. It shows that construction prices rise relative to aggregate prices in the United

States, Japan, Germany, Italy, France, and the United Kingdom. These results indicate that the U.S. pattern is part of a broader cross-country trend.

Figure C.2: **Cross-Country Decomposition: Cost Pressure vs. Productivity**



Notes. The figure decomposes the construction sector's relative output price into cost pressure and −TFP. A positive −TFP contribution indicates upward price pressure from weak productivity growth. Cost pressure aggregates input-cost growth relative to the aggregate price level. The decomposition uses gross output except for the United Kingdom and Italy, where value added is used because gross-output price indexes are unavailable in EU KLEMS 2024. *Source.* EU KLEMS 2024 and authors' calculations.

D Details on the model and BGP Derivations

The policy implementation follows Akcigit et al. (2022), who show that when innovation output and technological spillovers are observable, a reward can compensate innovators for the value that users do not pay while a Pigouvian correction can internalize the effect of innovation on other firms' technology. We combine these two corrections in the reward paid for a verified unit of new aggregate knowledge. The semi-endogenous dependence of R&D productivity on the existing knowledge stock follows Jones (1995).

D.1 Final Production

Final output is

$$Y = A(K_E^Y)^{\alpha_E}(K_S^Y)^{\alpha_S}L_u^{1-\alpha_E-\alpha_S}.$$

The knowledge stock A is nonrival and freely available to final-good producers. Taking A and factor prices as given, the competitive final-good sector solves

$$\max_{K_E^Y,K_S^Y,L_u}\left\{A(K_E^Y)^{\alpha_E}(K_S^Y)^{\alpha_S}L_u^{1-\alpha_E-\alpha_S} - R_E K_E^Y - R_S K_S^Y - w_u L_u\right\}.$$

The first-order conditions are

$$R_E = \alpha_E \frac{Y}{K_E^Y}, \qquad R_S = \alpha_S \frac{Y}{K_S^Y},$$
$$w_u = (1 - \alpha_E - \alpha_S)\frac{Y}{L_u}.$$

Hence

$$R_E K_E^Y + R_S K_S^Y + w_u L_u = Y. \tag{D.1}$$

D.2 R&D Production and Innovation Distortions

The competitive R&D sector produces

$$\dot{A} = A^{\phi}(K_E^R)^{\eta_E}(K_S^R)^{\eta_S} L_s^{1-\eta_E-\eta_S}.$$

The current knowledge stock is taken as given when current R&D inputs are chosen.

The first distortion is incomplete appropriability. Since the existing stock A is freely available in final production, final-good firms do not pay innovators for the increase in output created by a marginal addition to knowledge. The current output gain from one additional unit of A is

$$\frac{\partial Y}{\partial A} = \frac{Y}{A}.$$

The second distortion is the intertemporal knowledge spillover. The effect of a larger knowledge stock on future R&D productivity is

$$\frac{\partial \dot{A}}{\partial A} = \phi \frac{\dot{A}}{A}.$$

The R&D sector takes A as given when choosing current inputs and therefore does not internalize this effect.

D.3 Innovation Reward

Let $p_A(t)$ denote the reward for one verified unit of new aggregate knowledge. The government chooses the reward to compensate for both innovation distortions. A marginal unit of knowledge raises current final output by Y/A and affects future R&D productivity, negatively under the baseline calibration. The current-value marginal reward therefore satisfies

$$r(t)p_A(t) = \frac{Y(t)}{A(t)} + \dot{p}_A(t) + \phi \frac{\dot{A}(t)}{A(t)} p_A(t). \tag{D.2}$$

Taking A, p_A, and factor prices as given, the competitive R&D sector solves

$$\max_{K_E^R, K_S^R, L_s} \left\{ p_A A^{\phi} (K_E^R)^{\eta_E} (K_S^R)^{\eta_S} L_s^{1-\eta_E-\eta_S} - R_E K_E^R - R_S K_S^R - w_s L_s \right\}.$$

The first-order conditions are

$$R_E = \eta_E \frac{p_A \dot{A}}{K_E^R}, \qquad R_S = \eta_S \frac{p_A \dot{A}}{K_S^R},$$
$$w_s = (1 - \eta_E - \eta_S) \frac{p_A \dot{A}}{L_s}.$$

Constant returns in equipment, structures, and skilled labor imply

$$p_A \dot{A} = R_E K_E^R + R_S K_S^R + w_s L_s. \tag{D.3}$$

D.4 Households, Government, and Resource Feasibility

Households own the physical capital stocks and supply skilled and unskilled labor. Their aggregate budget is

$$C + I_E + I_S + T_A = w_u L_u + w_s L_s + R_E K_E + R_S K_S.$$

The government pays the aggregate innovation reward

$$G_A = p_A \dot{A}.$$

The reward is financed with the lump-sum tax

$$T_A = G_A.$$

Using equation (D.3)

$$G_A = R_E K_E^R + R_S K_S^R + w_s L_s.$$

Capital-market clearing is

$$K_E = K_E^Y + K_E^R, \qquad K_S = K_S^Y + K_S^R. \tag{D.4}$$

Combining equations (D.1) through (D.4) gives

$$C + I_E + I_S = Y.$$

The household Euler equation is

$$\frac{\dot{c}}{c} = \frac{r - \rho}{\sigma}.$$

For $k \in \{E, S\}$, no arbitrage between financial wealth and physical capital gives

$$r = q_k R_k - \gamma_k - \delta_k.$$

D.5 Balanced Growth Path

On a BGP, $g_C = g_Y$ and $g_c = g_Y - n$. The Euler equation gives

$$r = \rho + \sigma(g_Y - n).$$

Hence r is constant. Dividing the reward equation (D.2) by p_A gives

$$r = \frac{Y}{Ap_A} + g_{p_A} + \phi g_A.$$

On a BGP, r, g_Y, g_A, and g_{p_A} are constant. The equation above therefore requires $Y/(Ap_A)$ to be constant. Since the growth rate of this ratio is $g_Y - g_A - g_{p_A}$, it follows that

$$g_{p_A} = g_Y - g_A.$$

Substituting this relation and $\dot{A}/A = g_A$ back into (D.2) gives

$$p_A = \frac{1}{r - g_Y + (1 - \phi)g_A} \frac{Y}{A},$$

provided

$$r - g_Y + (1 - \phi)g_A > 0.$$

This is the no-bubble condition that ensures a finite positive value of a marginal unit of knowledge. Therefore

$$s_R \equiv \frac{p_A \dot{A}}{Y} = \frac{g_A}{r - g_Y + (1 - \phi)g_A}.$$

The reward-payment share is constant.

Combining the final-good and R&D first-order conditions gives

$$\frac{K_E^R}{K_E^Y} = \frac{\eta_E}{\alpha_E} s_R, \qquad \frac{K_S^R}{K_S^Y} = \frac{\eta_S}{\alpha_S} s_R.$$

Both allocation ratios are constant.

Since r is constant, no arbitrage implies

$$g_{R_E} = -\gamma_E, \qquad g_{R_S} = -\gamma_S.$$

Using the final-good first-order conditions

$$g_{K_E^Y} = g_Y + \gamma_E, \qquad g_{K_S^Y} = g_Y + \gamma_S.$$

The constant allocation ratios imply

$$g_{K_E^R} = g_Y + \gamma_E, \qquad g_{K_S^R} = g_Y + \gamma_S.$$

Capital-market clearing gives

$$g_{K_E} = g_Y + \gamma_E, \qquad g_{K_S} = g_Y + \gamma_S.$$

Log differentiation of final production gives

$$g_Y = g_A + \alpha_E(g_Y + \gamma_E) + \alpha_S(g_Y + \gamma_S) + (1 - \alpha_E - \alpha_S)n.$$

Log differentiation of R&D production gives

$$(1 - \phi)g_A = (\eta_E + \eta_S)g_Y + \eta_E\gamma_E + \eta_S\gamma_S + (1 - \eta_E - \eta_S)n.$$

Define

$$\Psi \equiv 1 - \alpha_E - \alpha_S, \qquad \eta \equiv \eta_E + \eta_S, \qquad \Delta \equiv (1 - \phi)\Psi - \eta.$$

We maintain the standard semi-endogenous-growth regularity condition

$$\Delta > 0.$$

A feasible positive-growth BGP additionally requires

$$\begin{aligned} g_A &> 0, \\ g_Y + \gamma_k + \delta_k &> 0, \qquad r + \gamma_k + \delta_k > 0 \qquad k \in \{E, S\}, \\ r - g_Y + (1 - \phi)g_A &> 0. \end{aligned}$$

Positive consumption and the household transversality condition must also hold. These require-

ments do not establish dynamic stability. The BGP growth rates are

$$g_Y = n + \frac{g_A}{\Psi} + \frac{\alpha_E \gamma_E + \alpha_S \gamma_S}{\Psi},$$

and

$$g_A = \frac{\Psi n + \left[\eta_E(1-\alpha_S) + \eta_S \alpha_E\right]\gamma_E + \left[\eta_S(1-\alpha_E) + \eta_E \alpha_S\right]\gamma_S}{\Delta}.$$

Per-capita output growth is then derived as

$$g_y = \frac{g_A + \alpha_E \gamma_E + \alpha_S \gamma_S}{\Psi}.$$

Define the structural-drag multipliers as the marginal effects of asset-specific ISTC growth on per-capita output growth. For structures,

$$\begin{aligned} \mathcal{M}_S \equiv \frac{\partial g_y}{\partial \gamma_S} &= \frac{\alpha_S}{\Psi} + \frac{\eta_S(1-\alpha_E) + \eta_E \alpha_S}{\Psi \Delta} \\ &= \frac{(1-\phi)\alpha_S + \eta_S}{\Delta}. \end{aligned}$$

Likewise,

$$\begin{aligned} \mathcal{M}_E \equiv \frac{\partial g_y}{\partial \gamma_E} &= \frac{\alpha_E}{\Psi} + \frac{\eta_E(1-\alpha_S) + \eta_S \alpha_E}{\Psi \Delta} \\ &= \frac{(1-\phi)\alpha_E + \eta_E}{\Delta}. \end{aligned}$$

D.6 Patent-Based Implementation with Monopoly Markups

This section provides an alternative decentralized implementation in which new knowledge is embodied in patented producer services. The purpose is to show that explicitly introducing monopoly markups changes static rents and sectoral allocations but does not change the balanced-growth equations derived in the main text.

Define

$$\beta \equiv \alpha_E + \alpha_S.$$

Let $M(t)$ denote the stock of patented designs. Each design $i \in [0, M(t)]$ allows a differentiated producer service $x_i(t)$. Following the standard expanding-variety mechanism of Romer (1990), a competitive final-good producer combines these services according to

$$X(t) = \left[\int_0^{M(t)} x_i(t)^{\frac{\varepsilon-1}{\varepsilon}}\, di\right]^{\frac{\varepsilon}{\varepsilon-1}}, \qquad \varepsilon > 1. \tag{D.5}$$

Final output is

$$Y(t) = X(t)^{\beta} L_u(t)^{1-\beta}.$$

Each producer service is produced with equipment and structures

$$x_i(t) = \left(k^Y_{E,i}(t)\right)^{\alpha_E/\beta} \left(k^Y_{S,i}(t)\right)^{\alpha_S/\beta}. \tag{D.6}$$

Its unit cost is

$$c_x(t) = \frac{R_E(t)^{\alpha_E/\beta} R_S(t)^{\alpha_S/\beta}}{(\alpha_E/\beta)^{\alpha_E/\beta} (\alpha_S/\beta)^{\alpha_S/\beta}}.$$

Cost minimization implies

$$R_E(t) k^Y_{E,i}(t) = \frac{\alpha_E}{\beta} c_x(t) x_i(t), \qquad R_S(t) k^Y_{S,i}(t) = \frac{\alpha_S}{\beta} c_x(t) x_i(t). \tag{D.7}$$

The owner of patent i is the exclusive supplier of x_i. Since the demand elasticity for an individual service is ε, the monopoly price received by the patent owner is

$$\widetilde{p}_i(t) = \mu c_x(t), \qquad \mu \equiv \frac{\varepsilon}{\varepsilon - 1}.$$

Without any correction, the final-good producer pays $\widetilde{p}_i$. Defining

$$\varpi \equiv \frac{1}{\mu} = \frac{\varepsilon - 1}{\varepsilon},$$

the corresponding aggregate factor payments are

$$R_E K^Y_E = \varpi \alpha_E Y, \qquad R_S K^Y_S = \varpi \alpha_S Y.$$

Thus monopoly pricing introduces a static level distortion.

To recover the competitive factor-price conditions used in the main text, suppose that the government subsidizes the use of patented producer services, following the corrective policy logic in Acemoglu et al. (2012). Let the purchaser price be

$$p_i(t) = \left(1 - S_X\right) \widetilde{p}_i(t).$$

Then, setting $S_X = 1/\varepsilon$ implies $p_i(t) = c_x(t)$. The final-good producer therefore pays the production cost of the patented service, while the patent owner continues to receive the monopoly price. The difference is financed by the government.

Homogeneity of final production gives

$$\int_0^M p_i x_i \, di = \beta Y.$$

Using $p_i = c_x$ and equation (D.7) gives

$$R_E K_E^Y = \alpha_E Y, \qquad R_S K_S^Y = \alpha_S Y, \qquad w_u L_u = (1 - \alpha_E - \alpha_S) Y.$$

Hence the aggregate factor-price conditions are exactly

$$R_E = \alpha_E \frac{Y}{K_E^Y}, \qquad R_S = \alpha_S \frac{Y}{K_S^Y}, \qquad w_u = (1 - \alpha_E - \alpha_S) \frac{Y}{L_u}. \tag{D.8}$$

In a symmetric equilibrium,

$$k_{E,i}^Y = \frac{K_E^Y}{M}, \qquad k_{S,i}^Y = \frac{K_S^Y}{M}.$$

Equation (D.6) then gives

$$x_i = \frac{1}{M} (K_E^Y)^{\alpha_E/\beta} (K_S^Y)^{\alpha_S/\beta}.$$

Substitution into equation (D.5) yields

$$Y = M^{\nu} (K_E^Y)^{\alpha_E} (K_S^Y)^{\alpha_S} L_u^{1-\alpha_E-\alpha_S}, \qquad \nu \equiv \frac{\beta}{\varepsilon - 1}.$$

Defining

$$A \equiv M^{\nu} \tag{D.9}$$

therefore recovers exactly the aggregate production function used in the main text

$$Y = A (K_E^Y)^{\alpha_E} (K_S^Y)^{\alpha_S} L_u^{1-\alpha_E-\alpha_S}.$$

The operating profit associated with one patent is

$$\Pi_M = (\widetilde{p}_i - c_x) x_i.$$

Since

$$c_x x_i = \frac{\beta Y}{M},$$

monopoly pricing implies

$$\Pi_M = \frac{\beta}{\varepsilon - 1}\frac{Y}{M} = \nu\frac{Y}{M}. \tag{D.10}$$

The aggregate government payment associated with the use subsidy equals aggregate monopoly profits. Specifically,

$$\int_0^M S_X \widetilde{p}_i x_i \, di = M\Pi_M.$$

The policy therefore removes the static user-price distortion without eliminating the rents that give patents positive value.

Let $V_M(t)$ denote the market value of one patent. No arbitrage implies

$$r(t)V_M(t) = \Pi_M(t) + \dot{V}_M(t). \tag{D.11}$$

Since

$$\dot{A} = \nu M^{\nu-1}\dot{M},$$

a marginal change satisfies

$$dA = \nu M^{\nu-1} dM, \qquad \frac{dM}{dA} = \frac{1}{\nu M^{\nu-1}}.$$

Thus one marginal unit of aggregate knowledge corresponds to $1/(\nu M^{\nu-1})$ additional patents. The market value associated with one marginal unit of aggregate knowledge is therefore

$$p_A \equiv V_M \frac{dM}{dA} = \frac{V_M}{\nu M^{\nu-1}}. \tag{D.12}$$

It follows directly that

$$p_A \dot{A} = V_M \dot{M}.$$

On a balanced growth path,

$$g_M = \frac{g_A}{\nu}.$$

Equation (D.10) implies

$$g_{\Pi_M} = g_Y - g_M.$$

Under the no-bubble valuation, the patent value equals the discounted value of its future operating profits,

$$V_M(t) = \int_t^\infty \exp\left(-\int_t^s r(u)\, du\right) \Pi_M(s)\, ds.$$

On a BGP, r is constant and profits grow at $g_Y - g_M$, so

$$V_M = \frac{\Pi_M}{r - g_Y + g_M},$$

provided

$$r - g_Y + g_M > 0.$$

This transversality condition rules out an explosive patent-value bubble. Using equations (D.9), (D.10), and (D.12) gives

$$p_A = \frac{1}{r - g_Y + g_A/\nu} \frac{Y}{A}.$$

Hence

$$s_R \equiv \frac{p_A \dot{A}}{Y} = \frac{g_A}{r - g_Y + g_A/\nu},$$

which is constant on the BGP. On a positive-growth BGP, this patent value equals the baseline innovation reward only if $\frac{1}{\nu} = 1 - \phi$.

The R&D technology remains

$$\dot{A} = A^{\phi} (K_E^R)^{\eta_E} (K_S^R)^{\eta_S} L_s^{1-\eta_E-\eta_S}.$$

The R&D first-order conditions imply

$$R_E = \eta_E \frac{p_A \dot{A}}{K_E^R}, \qquad R_S = \eta_S \frac{p_A \dot{A}}{K_S^R}.$$

Combining these conditions with equation (D.8) gives

$$\frac{K_E^R}{K_E^Y} = \frac{\eta_E}{\alpha_E} s_R, \qquad \frac{K_S^R}{K_S^Y} = \frac{\eta_S}{\alpha_S} s_R. \tag{D.13}$$

Since s_R is constant, both sectoral allocation ratios are constant.

The household Euler equation implies that r is constant on the BGP. No arbitrage for physical capital then gives

$$g_{R_E} = -\gamma_E, \qquad g_{R_S} = -\gamma_S.$$

Using equation (D.8),

$$g_{K_E^Y} = g_Y + \gamma_E, \qquad g_{K_S^Y} = g_Y + \gamma_S.$$

The constant allocation ratios in equation (D.13) imply

$$g_{K_E^R} = g_Y + \gamma_E, \qquad g_{K_S^R} = g_Y + \gamma_S.$$

Capital-market clearing therefore gives

$$g_{K_E} = g_Y + \gamma_E, \qquad g_{K_S} = g_Y + \gamma_S.$$

Log differentiation of final production consequently gives

$$g_Y = g_A + \alpha_E(g_Y + \gamma_E) + \alpha_S(g_Y + \gamma_S) + (1 - \alpha_E - \alpha_S)n.$$

Log differentiation of R&D production gives

$$(1 - \phi)g_A = (\eta_E + \eta_S)g_Y + \eta_E\gamma_E + \eta_S\gamma_S + (1 - \eta_E - \eta_S)n.$$

These are exactly the two balanced-growth equations obtained in the baseline model.

It follows that

$$g_A = \frac{(1 - \alpha_E - \alpha_S)n + \left[\eta_E(1 - \alpha_S) + \eta_S\alpha_E\right]\gamma_E + \left[\eta_S(1 - \alpha_E) + \eta_E\alpha_S\right]\gamma_S}{(1 - \phi)(1 - \alpha_E - \alpha_S) - (\eta_E + \eta_S)}.$$

The structural-drag multipliers are therefore also unchanged

$$\mathcal{M}_S = \frac{(1 - \phi)\alpha_S + \eta_S}{\Delta}, \qquad \mathcal{M}_E = \frac{(1 - \phi)\alpha_E + \eta_E}{\Delta}.$$

Once s_R is constant, the sectoral capital shares are constant, and the growth rates of equipment and structures are determined by the same no-arbitrage conditions as in the baseline model. The patent parameters therefore affect levels and allocations but disappear from the two equations that determine g_Y and g_A.

D.7 A Construction-Sector Interpretation of Structures ISTC

This section shows that the baseline structures-ISTC formulation is exactly equivalent to an economy with an explicit competitive construction sector under a simple benchmark technology.

Define X_S as the amount of final good used by competitive construction firms to produce new structures. Construction output is given by $J_S = Z_S X_S$, where Z_S denotes exogenous construction productivity. New structures are sold at the relative price P_S, and expenditure on structures is therefore $I_S = P_S J_S$. We show that this construction-sector formulation reproduces the baseline model under the mapping $q_S = Z_S$. The stock evolves according to

$$\dot{K}_S = J_S - \delta_S K_S.$$

All other parts of the economy are unchanged.

A construction firm chooses $X_S \geq 0$ to maximize

$$(P_S Z_S - 1) X_S.$$

Positive finite construction output requires $P_S Z_S = 1$. Competitive pricing therefore implies

$$P_S = \frac{1}{Z_S},$$

and construction profits are zero. Since

$$I_S = P_S J_S = P_S Z_S X_S = X_S,$$

the law of motion becomes

$$\dot{K}_S = Z_S I_S - \delta_S K_S.$$

This is the baseline accumulation equation under the mapping $q_S = Z_S$. Final-good market clearing is also unchanged because the construction sector uses $X_S = I_S$ units of the final good.

The return on one unit of wealth invested in structures is

$$r = \frac{R_S}{P_S} + \frac{\dot{P}_S}{P_S} - \delta_S = Z_S R_S - g_{Z_S} - \delta_S.$$

This coincides with the baseline no-arbitrage condition under

$$q_S = Z_S, \qquad \gamma_S = g_{Z_S}.$$

Hence the explicit construction-sector economy and the baseline economy generate the same equilibrium allocations, prices, and balanced-growth system. The one-for-one structures technology in Greenwood et al. (1997) corresponds to $Z_S \equiv 1$, and therefore to $P_S \equiv 1$ and $\gamma_S = 0$.

Relation to the construction-price decomposition. The equality $\gamma_S = g_{Z_S}$ is specific to the one-input benchmark above. If a richer construction technology uses labor, capital, and intermediate inputs, let c_S denote the unit cost of that input bundle in units of the final good. Competitive pricing then gives

$$P_S^c = \frac{c_S}{Z_S}, \qquad g_{P_S^c} = g_{c_S} - g_{Z_S}.$$

The construction-price decomposition in the empirical section therefore separates the productivity component from relative input-cost components. Consistent with this distinction, declining construction TFP accounts for roughly half of the rise in the U.S. relative construction price, while

the remainder reflects movements in input costs. In this richer setting, the model's q_S should be interpreted as a reduced-form inverse unit cost, or composite investment efficiency, rather than as construction TFP alone. This is an interpretation of the observed structures-price trend, not a claim that the multi-input construction economy is exactly isomorphic to the baseline model.[51]

E Comparison with Standard Growth Models

To highlight the implications of capital heterogeneity, we compare our model with a one-capital benchmark that aggregates equipment and structures into a single stock K.

E.1 BGP Comparison

The one-capital model is obtained by imposing a common ISTC process, $\gamma_S = \gamma_E \equiv \gamma$, aggregating capital shares as $\alpha_{\text{one}} = \alpha_E + \alpha_S$, and collapsing R&D capital intensities to $\eta_{\text{one}} = \eta_E + \eta_S$. The BGP then implies

$$g_A^{\text{one}} = \frac{(1-\alpha_{\text{one}})n + \eta_{\text{one}}\gamma}{(1-\phi)(1-\alpha_{\text{one}}) - \eta_{\text{one}}}. \tag{E.1}$$

We impose the analogous regularity condition

$$(1-\phi)(1-\alpha_{\text{one}}) - \eta_{\text{one}} > 0.$$

Output growth satisfies

$$g_Y^{\text{one}} = n + \frac{g_A^{\text{one}} + \alpha_{\text{one}}\gamma}{1-\alpha_{\text{one}}}. \tag{E.2}$$

Structural Drag in the One-Capital Model. The two-capital numerator can be rewritten as

$$(1-\alpha_E-\alpha_S)n + (\eta_E+\eta_S)\gamma_E - [\eta_S(1-\alpha_E) + \eta_E\alpha_S](\gamma_E - \gamma_S). \tag{E.3}$$

Thus, when $\gamma_E > \gamma_S$, slow structures ISTC generates a negative drag term with weight

$$\kappa_{\text{drag}} \equiv \eta_S(1-\alpha_E) + \eta_E\alpha_S. \tag{E.4}$$

To make the comparison exact, consider the one-capital benchmark that assigns the common ISTC rate $\gamma = \gamma_E$. Since $\alpha_{\text{one}} = \alpha_E + \alpha_S$ and $\eta_{\text{one}} = \eta_E + \eta_S$, the one-capital and two-capital models

[51] The KLEMS object is a construction-sector gross-output price, whereas γ_S is calibrated from a structures investment-price series over a different period. The mapping is therefore interpretive rather than an equality of measured series.

have the same denominator. Their innovation-growth difference is therefore

$$g_A^{\text{one}} - g_A^{\text{two}} = \frac{\kappa_{\text{drag}}(\gamma_E - \gamma_S)}{(1-\phi)(1-\alpha_E-\alpha_S) - (\eta_E+\eta_S)}.$$

Under the maintained regularity condition, this difference is positive whenever $\gamma_E > \gamma_S$. Relative to this equipment-ISTC one-capital benchmark, the innovation gap reflects both the direct use of structures in R&D and the indirect effect of structures in final production on the scale of research. The indirect effect remains when research uses equipment but no structures. The one-capital model therefore creates aggregation bias when it assigns equipment-like efficiency growth to the whole capital stock. The alternative one-capital calibration based on aggregate investment ISTC is evaluated numerically below and is not covered by this analytical ranking.

E.2 Model Comparison

We compare three specifications. The first is a one-capital model with R&D, where $Y = AK^{\alpha_{\text{one}}}L^{1-\alpha_{\text{one}}}$, $\alpha_{\text{one}} = \alpha_E + \alpha_S = 0.38$, and $\eta_{\text{one}} = \eta_E + \eta_S = 0.33$. The second is a no-R&D specification with exogenous TFP growth $g_A = n/(1-\phi) = 0.40\%$. The third is our baseline two-capital model. For the one-capital model, we use two ISTC calibrations. One uses the equipment price decline, following Greenwood et al. (1997); Cummins and Violante (2002). The other uses the aggregate investment price trend, following Justiniano et al. (2011).

Table E.I: **Growth Rates in One-Capital and Two-Capital Models**

Model Specification	**Without R&D**	**With R&D**
One-Capital Model		
Using Equipment ISTC γ_E	2.09%	3.33%
Using Aggregate Investment ISTC γ_I	1.00%	1.44%
Two-Capital Model		
Using Equipment and Structures ISTC γ_E and γ_S	0.76%	**1.43%**
Data		
U.S. GDP per capita growth, 1970–2020	1.69%	

Notes. This table reports steady-state GDP per capita growth across model specifications. The one-capital model uses either $\gamma_E = 0.0238$ or $\gamma_I = 0.0059$. The two-capital model uses $\gamma_E = 0.0238$ and $\gamma_S = -0.0118$. The empirical benchmark is U.S. real GDP per capita growth from PWT.

Table E.I shows that the two-capital model delivers 1.43 percent against 1.69 percent in the data, while the one-capital benchmarks range from 1.00 to 3.33 percent depending on the price series used to measure ISTC.

F Estimation Procedure, Robustness Checks, and Detailed Derivations for the Nested CES Extension

F.1 Details on Estimation Procedure

We estimate the CES production system using annual U.S. data for 1963–2019. The data construction follows Ohanian et al. (2023). Our baseline uses nonfarm business sector (NFBS) data and estimates the CES specification by GMM.

Production structure and normalization. The estimated parameter vector is $\Theta = (\omega, \theta_M, \theta_S, \rho_0, \rho_M, \rho_S)'$. Effective unskilled labor is normalized as $\widetilde{L}_{u,t} = \exp(2)L_{u,t}$, and skilled labor is scaled in the same units, $\widetilde{L}_{s,t} = \exp(2)L_{s,t}$. This fixes the scale of labor units but leaves ρ_S, equivalently σ_S, as a free parameter.

The CES technology is

$$Y_t = A_t \left[\omega \left(\theta_M K_{E,t}^{\rho_M} + (1-\theta_M)\widetilde{L}_{s,t}^{\rho_M} \right)^{\rho_0/\rho_M} + (1-\omega)\left(\theta_S K_{S,t}^{\rho_S} + (1-\theta_S)\widetilde{L}_{u,t}^{\rho_S} \right)^{\rho_0/\rho_S} \right]^{1/\rho_0}. \tag{F.1}$$

Here, A_t denotes Hicks-neutral productivity. Because it enters multiplicatively, it does not affect the model-implied factor shares used in the GMM moments. Let

$$B_{M,t} = \theta_M K_{E,t}^{\rho_M} + (1-\theta_M)\widetilde{L}_{s,t}^{\rho_M}, \qquad B_{S,t} = \theta_S K_{S,t}^{\rho_S} + (1-\theta_S)\widetilde{L}_{u,t}^{\rho_S}, \qquad D_t = \omega B_{M,t}^{\rho_0/\rho_M} + (1-\omega)B_{S,t}^{\rho_0/\rho_S}.$$

The model-implied output elasticities are

$$s_{K_E,t} = \frac{\omega\theta_M K_{E,t}^{\rho_M} B_{M,t}^{\rho_0/\rho_M - 1}}{D_t}, \qquad s_{K_S,t} = \frac{(1-\omega)\theta_S K_{S,t}^{\rho_S} B_{S,t}^{\rho_0/\rho_S - 1}}{D_t},$$

$$s_{L_s,t} = \frac{\omega(1-\theta_M)\widetilde{L}_{s,t}^{\rho_M} B_{M,t}^{\rho_0/\rho_M - 1}}{D_t}, \qquad s_{L_u,t} = \frac{(1-\omega)(1-\theta_S)\widetilde{L}_{u,t}^{\rho_S} B_{S,t}^{\rho_0/\rho_S - 1}}{D_t}.$$

GMM moment conditions. The first moment imposes no-arbitrage between structures and equipment returns,

$$m_{1,t+1}(\Theta) = \left[\frac{s_{K_S,t+1} Y_{t+1}}{P_{S,t} K_{S,t+1}} + (1-\delta_{S,t+1})\frac{P_{S,t+1}}{P_{S,t}} \right] - \left[\frac{s_{K_E,t+1} Y_{t+1}}{P_{E,t} K_{E,t+1}} + (1-\delta_{E,t+1})\frac{P_{E,t+1}}{P_{E,t}} \right]. \tag{F.2}$$

The second moment matches the labor share. For the net-labor-share robustness specification,

$$m_{2,t}^{\text{net}}(\Theta) = \frac{(s_{L_s,t} + s_{L_u,t})Y_t}{Y_t - P_{E,t}\delta_{E,t}K_{E,t} - P_{S,t}\delta_{S,t}K_{S,t}} - LS_t. \tag{F.3}$$

The baseline gross-labor-share moment is

$$m_{2,t}^{\text{gross}}(\Theta) = (s_{L_s,t} + s_{L_u,t}) - LS_t. \tag{F.4}$$

The third moment matches the skilled-to-unskilled wage-bill ratio,

$$m_{3,t}(\Theta) = \frac{s_{L_s,t}}{s_{L_u,t}} - \frac{W_{s,t}L_{s,t}}{W_{u,t}L_{u,t}}. \tag{F.5}$$

For a given labor-share specification, define the stacked moment vector as

$$m_t(\Theta) \equiv \left(m_{1,t+1}(\Theta), m_{2,t}^{j}(\Theta), m_{3,t}(\Theta)\right)', \qquad j \in \{\text{gross}, \text{net}\}.$$

Instruments and objective function. The instrument vector is

$$Z_t = \left(K_{E,t}, K_{S,t}, K_{E,t-1}, K_{S,t-1}, P_{E,t-1}, P_{S,t-1}, t\right)'.$$

We follow Ohanian et al. (2023). The estimator imposes

$$\mathbb{E}[Z_t \otimes m_t(\Theta)] = 0, \qquad g_T(\Theta) = \frac{1}{T}\sum_{t=1}^{T} Z_t \otimes m_t(\Theta). \tag{F.6}$$

The GMM estimator solves

$$\widehat{\Theta} = \arg\min_{\Theta} g_T(\Theta)' W_T g_T(\Theta). \tag{F.7}$$

We initialize the optimizer with a grid search over admissible CES curvature and share parameters, then re-estimate from the best grid point. We report substitution elasticities, share parameters, and Wald tests for Cobb-Douglas and common-elasticity restrictions.

F.2 Robustness Checks on Structural Drag

We estimate four specifications by crossing gross and net labor-share measures with stock-based and investment-based structures-price measures. The baseline is the gross labor-share specification with the investment-based structures price.

Table F.I shows stable qualitative patterns across specifications. The outer elasticity exceeds one, the equipment–skilled elasticity is below one, and the structures–unskilled elasticity is above

Table F.I: **CES Estimates Across Specifications**

Panel A. Elasticity of substitution estimates

	Baseline (Gross, Inv.)	Gross, Stock	Net, Stock	Net, Inv.
Substitution elasticities $\hat{\sigma} = 1/(1-\hat{\rho})$				
σ_0 (outer)	1.418	1.422	1.460	1.443
σ_M (equip–skilled)	0.716	0.698	0.725	0.708
σ_S (struct–unskilled)	2.239	1.819	2.125	1.742
CES exponents $\hat{\rho}$ (SE)				
ρ_0	0.295	0.297	0.315	0.307
	(0.013)	(0.019)	(0.016)	(0.011)
ρ_M	−0.396	−0.432	−0.379	−0.413
	(0.030)	(0.044)	(0.021)	(0.024)
ρ_S	0.553	0.450	0.530	0.426
	(0.060)	(0.090)	(0.049)	(0.072)

Panel B. Distribution parameters

	Baseline (Gross, Inv.)	Gross, Stock	Net, Stock	Net, Inv.
ω	0.570	0.569	0.574	0.580
	(0.005)	(0.006)	(0.008)	(0.006)
θ_M	0.245	0.218	0.251	0.229
	(0.019)	(0.025)	(0.013)	(0.014)
θ_S	0.579	0.513	0.559	0.485
	(0.037)	(0.057)	(0.030)	(0.046)

Notes. GMM estimation on NFBS data. Stock uses the stock-based structures price, while Inv. uses the investment-based structures price. Gross and Net refer to labor-share definitions. Standard errors are in parentheses. The baseline result is shown in the first column.

one. Thus, the estimates imply top-nest substitution, equipment–skilled complementarity, and structures–unskilled substitutability.

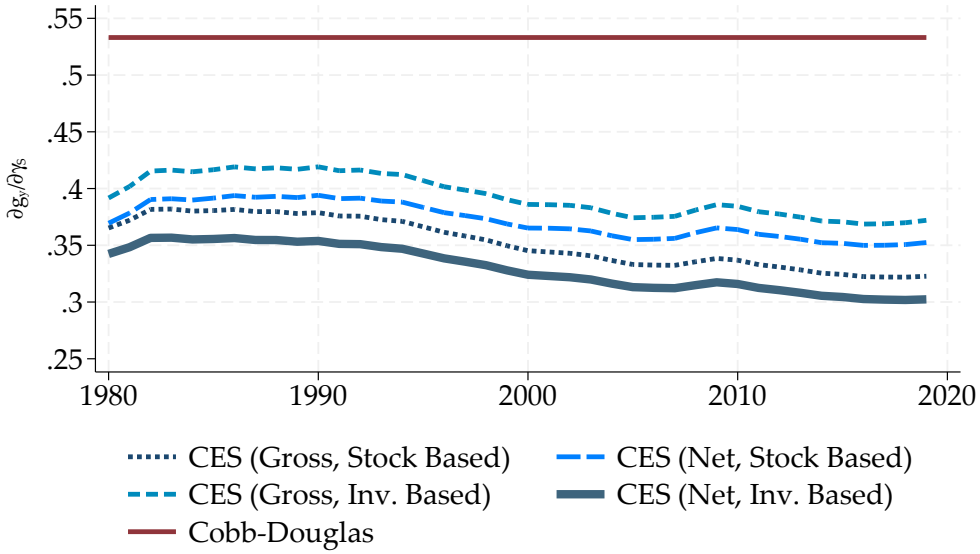
Table F.II: **Specification Tests**

	Baseline (Gross, Inv.)	Gross, Stock	Net, Stock	Net, Inv.
Wald $\chi^2(1)$ *statistics*				
$\sigma_0 = 1$	528.0	242.3	390.3	755.0
$\sigma_M = 1$	177.6	98.6	324.0	305.9
$\sigma_S = 1$	86.2	25.3	115.5	34.8
$\sigma_M = \sigma_S$	498.1	202.9	283.7	199.8

Notes. Reported entries are Wald $\chi^2(1)$ statistics. All tests reject at the 1% level. The baseline result is shown in the first column.

Table F.II confirms that the substitution patterns are statistically distinct. The Cobb-Douglas restrictions are rejected in each nest, and the equality of the two inner elasticities is also rejected. We then compute structural drag under the same four specifications.

Figure F.1: **Structural Drag under Cobb-Douglas and CES**



Notes. The figure compares structural drag under Cobb-Douglas and CES technologies. The red line is the Cobb-Douglas baseline. The blue lines are CES results under Gross/Inv., Gross/Stock, Net/Stock, and Net/Inv. specifications.

Figure F.1 shows that structural drag is smaller under CES than under Cobb-Douglas across all specifications and declines over time.

F.3 Derivation of Structural Drag under Nested CES

The final-goods technology is

$$
\begin{aligned}
Y_t &= A_t \left[\omega M_t^{\rho_0} + (1-\omega) S_t^{\rho_0}\right]^{1/\rho_0}, \\
M_t &= \left[\theta_M (K_{E,t}^Y)^{\rho_M} + (1-\theta_M)(L_{s,t}^Y)^{\rho_M}\right]^{1/\rho_M}, \\
S_t &= \left[\theta_S (K_{S,t}^Y)^{\rho_S} + (1-\theta_S) L_{u,t}^{\rho_S}\right]^{1/\rho_S},
\end{aligned}
$$

where $\rho_j = (\sigma_j - 1)/\sigma_j$ for $j \in \{0, M, S\}$.

Final-goods allocations satisfy

$$
L_{s,t}^Y = (1-\ell_t) L_{s,t}, \qquad K_{E,t}^Y = (1-\chi_{E,t}) K_{E,t}, \qquad K_{S,t}^Y = (1-\chi_{S,t}) K_{S,t}.
$$

For the local derivation, take the allocation rates $\chi_{E,t}$, $\chi_{S,t}$, and ℓ_t as fixed at the evaluation

date. Allocated inputs then differ from aggregate inputs only by multiplicative constants. For example,

$$\theta_M(K_{E,t}^Y)^{\rho_M} = \theta_M(1-\chi_{E,t})^{\rho_M}K_{E,t}^{\rho_M}, \qquad (1-\theta_M)(L_{s,t}^Y)^{\rho_M} = (1-\theta_M)(1-\ell_t)^{\rho_M}L_{s,t}^{\rho_M}.$$

The corresponding constants for the structures bundle are absorbed in the same way. These local multiplicative terms can be absorbed into effective within-nest weights and, because they also rescale the two composite bundles, into the effective top-nest weight. To avoid introducing additional notation, we continue to denote the locally rescaled CES weights by θ_M, θ_S, and ω. Under this local normalization, log differentiation of each CES nest gives share-weighted growth with

$$s_{K_E|M,t} = \frac{\theta_M K_{E,t}^{\rho_M}}{\theta_M K_{E,t}^{\rho_M} + (1-\theta_M)L_{s,t}^{\rho_M}}, \qquad s_{K_S|S,t} = \frac{\theta_S K_{S,t}^{\rho_S}}{\theta_S K_{S,t}^{\rho_S} + (1-\theta_S)L_{u,t}^{\rho_S}},$$

$$s_{M,t} = \frac{\omega M_t^{\rho_0}}{\omega M_t^{\rho_0} + (1-\omega)S_t^{\rho_0}}.$$

Therefore,

$$g_{Y,t} = g_{A,t} + s_{M,t}g_{M,t} + (1-s_{M,t})g_{S,t},$$

where $g_{M,t} = s_{K_E|M,t}g_{K_E^Y,t} + (1-s_{K_E|M,t})g_{L_s^Y,t}$, $g_{S,t} = s_{K_S|S,t}g_{K_S^Y,t} + (1-s_{K_S|S,t})g_{L_u,t}$.

Define $\mathbf{a}_{E,t} = s_{M,t}s_{K_E|M,t}$, $\mathbf{a}_{L_s,t} = s_{M,t}(1-s_{K_E|M,t})$, $\mathbf{a}_{S,t} = (1-s_{M,t})s_{K_S|S,t}$, $\mathbf{a}_{L_u,t} = (1-s_{M,t})(1-s_{K_S|S,t})$. Using

$$g_{K_E^Y,t} = g_{K_E,t} - \frac{\dot{\chi}_{E,t}}{1-\chi_{E,t}}, \qquad g_{K_S^Y,t} = g_{K_S,t} - \frac{\dot{\chi}_{S,t}}{1-\chi_{S,t}}, \qquad g_{L_s^Y,t} = g_{L_s,t} - \frac{\dot{\ell}_t}{1-\ell_t},$$

the exact production-side growth equation is

$$g_{Y,t} = g_{A,t} + \mathbf{a}_{E,t}\left(g_{K_E,t} - \frac{\dot{\chi}_{E,t}}{1-\chi_{E,t}}\right) + \mathbf{a}_{S,t}\left(g_{K_S,t} - \frac{\dot{\chi}_{S,t}}{1-\chi_{S,t}}\right) + \mathbf{a}_{L_s,t}\left(g_{L_s,t} - \frac{\dot{\ell}_t}{1-\ell_t}\right) + \mathbf{a}_{L_u,t}g_{L_u,t}.$$

Since R&D technology is $\dot{A}_t = A_t^{\phi}(K_{E,t}^R)^{\eta_E}(K_{S,t}^R)^{\eta_S}(L_{s,t}^R)^{1-\eta_E-\eta_S}$, and $K_{E,t}^R = \chi_{E,t}K_{E,t}$, $K_{S,t}^R = \chi_{S,t}K_{S,t}$, and $L_{s,t}^R = \ell_t L_{s,t}$,

$$\frac{\dot{g}_{A,t}}{g_{A,t}} = (\phi-1)g_{A,t} + \eta_E\left(g_{K_E,t} + \frac{\dot{\chi}_{E,t}}{\chi_{E,t}}\right) + \eta_S\left(g_{K_S,t} + \frac{\dot{\chi}_{S,t}}{\chi_{S,t}}\right) + (1-\eta_E-\eta_S)\left(g_{L_s,t} + \frac{\dot{\ell}_t}{\ell_t}\right).$$

Let $q_{E,t} = P_{E,t}^{-1}$, $q_{S,t} = P_{S,t}^{-1}$, $\gamma_E = g_{q_E,t}$, and $\gamma_S = g_{q_S,t}$. With $\zeta_{E,t} = K_{E,t}/(q_{E,t}Y_t)$ and $\zeta_{S,t} = K_{S,t}/(q_{S,t}Y_t)$,

$$g_{K_E,t} = g_{Y,t} + \gamma_E + \frac{\dot{\zeta}_{E,t}}{\zeta_{E,t}}, \qquad g_{K_S,t} = g_{Y,t} + \gamma_S + \frac{\dot{\zeta}_{S,t}}{\zeta_{S,t}}.$$

Impose the local closure $\dot{\zeta}_{E,t} = \dot{\zeta}_{S,t} = \dot{\ell}_t = \dot{\chi}_{E,t} = \dot{\chi}_{S,t} = \dot{g}_{A,t} = 0$ and $g_{L_s,t} = g_{L_u,t} = n$ to derive a closed-form solution. These are local stationarity conditions rather than a solution for the full transition path. In particular, $\dot{\chi}_{E,t} = \dot{\chi}_{S,t} = \dot{\ell}_t = 0$ holds the sectoral allocation shares fixed at the evaluation date, while $\dot{g}_{A,t} = 0$ holds the local TFP growth rate fixed. The conditions $\dot{\zeta}_{E,t} = \dot{\zeta}_{S,t} = 0$ imply

$$g_{K_E,t} = g_{Y,t} + \gamma_E, \qquad g_{K_S,t} = g_{Y,t} + \gamma_S.$$

To connect this closure to the investment-share evidence, use the accumulation equation $\dot{K}_{k,t} = q_{k,t} I_{k,t} - \delta_k K_{k,t}$. Since $\zeta_{k,t} = K_{k,t}/(q_{k,t} Y_t)$,

$$\frac{I_{k,t}}{Y_t} = \zeta_{k,t}\big(g_{K_k,t} + \delta_k\big) = \zeta_{k,t}\big(g_{Y,t} + \gamma_k + \delta_k\big), \qquad k \in \{E, S\}.$$

Thus a locally stationary $\zeta_{k,t}$ evaluates the growth system around stable efficiency-adjusted capital-output ratios. When local growth rates and depreciation are treated as given, this provides a local approximation consistent with the slowly moving investment shares used to motivate the closure. Then the production block becomes

$$g_{Y,t} = g_{A,t} + (\mathbf{a}_{E,t} + \mathbf{a}_{S,t}) g_{Y,t} + \mathbf{a}_{E,t}\gamma_E + \mathbf{a}_{S,t}\gamma_S + (1 - \mathbf{a}_{E,t} - \mathbf{a}_{S,t}) n.$$

The knowledge block becomes

$$(1 - \phi) g_{A,t} = (\eta_E + \eta_S) g_{Y,t} + \eta_E \gamma_E + \eta_S \gamma_S + (1 - \eta_E - \eta_S) n.$$

Eliminating $g_{A,t}$ gives

$$\Delta_t g_{Y,t} = (\Delta_t + 1) n + \big((1 - \phi)\mathbf{a}_{E,t} + \eta_E\big)\gamma_E + \big((1 - \phi)\mathbf{a}_{S,t} + \eta_S\big)\gamma_S,$$

where $\Delta_t = (1 - \phi)(1 - \mathbf{a}_{E,t} - \mathbf{a}_{S,t}) - (\eta_E + \eta_S)$. We require $\Delta_t > 0$ at the evaluation date so that the local growth system is well defined with the standard semi-endogenous-growth stability sign. Since $g_{y,t} = g_{Y,t} - n$,

$$g_{y,t} = \frac{n + \big((1 - \phi)\mathbf{a}_{E,t} + \eta_E\big)\gamma_E + \big((1 - \phi)\mathbf{a}_{S,t} + \eta_S\big)\gamma_S}{\Delta_t}.$$

Local comparative statics with respect to structures ISTC. Because $q_{S,t} = P_{S,t}^{-1}$,

$$\gamma_S = g_{q_S,t} = -g_{P_S,t}.$$

Accordingly, the derivatives below are with respect to structures ISTC growth γ_S. A derivative with respect to structures-price growth has the opposite sign.

Define

$$N_t \equiv n + \big((1-\phi)\mathbf{a}_{E,t} + \eta_E\big)\gamma_E + \big((1-\phi)\mathbf{a}_{S,t} + \eta_S\big)\gamma_S, \qquad g_{y,t} = \frac{N_t}{\Delta_t}.$$

Holding the output shares fixed gives the direct effect

$$\left.\frac{\partial g_{y,t}}{\partial \gamma_S}\right|_{\mathbf{a}} = \frac{(1-\phi)\mathbf{a}_{S,t} + \eta_S}{\Delta_t}.$$

The partial derivatives with respect to the endogenous shares are

$$\frac{\partial g_{y,t}}{\partial \mathbf{a}_{E,t}} = \frac{(1-\phi)\big(\gamma_E\Delta_t + N_t\big)}{\Delta_t^2}, \qquad \frac{\partial g_{y,t}}{\partial \mathbf{a}_{S,t}} = \frac{(1-\phi)\big(\gamma_S\Delta_t + N_t\big)}{\Delta_t^2}.$$

To obtain the share responses used in the main text, consider a local comparative static that perturbs γ_S while holding the contemporaneous structures price $P_{S,t}$, the interest rate r_t, and the prices of the other inputs fixed. The no-arbitrage condition

$$r_t = q_{S,t}R_{S,t} - \gamma_S - \delta_S$$

can be written as

$$R_{S,t} = P_{S,t}\big(r_t + \delta_S + \gamma_S\big).$$

Hence

$$\frac{\partial \ln R_{S,t}}{\partial \gamma_S} = \mu_S, \qquad \mu_S \equiv \frac{1}{r_t + \delta_S + \gamma_S}.$$

Under cost minimization in the inner structures-unskilled-labor nest, the structures cost share satisfies the standard CES derivative

$$\frac{\partial s_{K_S|S,t}}{\partial \ln R_{S,t}} = -(\sigma_S - 1)s_{K_S|S,t}\big(1 - s_{K_S|S,t}\big).$$

Therefore

$$\frac{ds_{K_S|S,t}}{d\gamma_S} = -(\sigma_S - 1)s_{K_S|S,t}\big(1 - s_{K_S|S,t}\big)\mu_S.$$

Let $c_{S,t}$ denote the unit cost of the inner S-bundle. By Shephard's lemma,

$$\frac{\partial \ln c_{S,t}}{\partial \ln R_{S,t}} = s_{K_S|S,t},$$

so

$$\frac{d \ln c_{S,t}}{d\gamma_S} = s_{K_S|S,t}\mu_S.$$

At the top nest, the CES share of the M-bundle responds to the relative bundle cost according to

$$\frac{\partial s_{M,t}}{\partial \ln c_{S,t}} = (\sigma_0 - 1)s_{M,t}(1 - s_{M,t}).$$

Thus

$$\frac{ds_{M,t}}{d\gamma_S} = (\sigma_0 - 1)s_{M,t}(1 - s_{M,t})s_{K_S|S,t}\mu_S.$$

Since the perturbation is confined to the structures side, $s_{K_E|M,t}$ is locally unchanged. Using $\mathbf{a}_{E,t} = s_{M,t}s_{K_E|M,t}$,

$$\frac{d\mathbf{a}_{E,t}}{d\gamma_S} = (\sigma_0 - 1)\mathbf{a}_{E,t}(1 - s_{M,t})s_{K_S|S,t}\mu_S.$$

Using $\mathbf{a}_{S,t} = (1 - s_{M,t})s_{K_S|S,t}$ and the product rule,

$$\begin{aligned}\frac{d\mathbf{a}_{S,t}}{d\gamma_S} = & -(\sigma_0 - 1)s_{M,t}(1 - s_{M,t})s^2_{K_S|S,t}\mu_S \\ & -(\sigma_S - 1)(1 - s_{M,t})s_{K_S|S,t}\big(1 - s_{K_S|S,t}\big)\mu_S.\end{aligned}$$

Combining the direct and share-response terms gives

$$\frac{dg_{y,t}}{d\gamma_S} = \frac{\partial g_{y,t}}{\partial \gamma_S}\bigg|_{\mathbf{a}} + \frac{\partial g_{y,t}}{\partial \mathbf{a}_{E,t}}\frac{d\mathbf{a}_{E,t}}{d\gamma_S} + \frac{\partial g_{y,t}}{\partial \mathbf{a}_{S,t}}\frac{d\mathbf{a}_{S,t}}{d\gamma_S}.$$

Outside the fixed-price closure, the structures user-cost response satisfies

$$\frac{d \ln R_{S,t}}{d\gamma_S} = \frac{d \ln P_{S,t}}{d\gamma_S} + \frac{1 + dr_t/d\gamma_S}{r_t + \delta_S + \gamma_S}.$$

F.4 Derivation of Skill Premium under Nested CES

Let $\pi_t = w_{s,t}/w_{u,t}$. Under cost minimization,

$$\pi_t = \frac{\mathbf{a}_{L_s,t}}{\mathbf{a}_{L_u,t}}\frac{L_{u,t}}{L^Y_{s,t}} = \frac{s_{M,t}(1 - s_{K_E|M,t})}{(1 - s_{M,t})(1 - s_{K_S|S,t})}\frac{L_{u,t}}{L^Y_{s,t}}.$$

Under the same local closure used above, $\dot{\ell}_t = \dot{\chi}_{E,t} = \dot{\chi}_{S,t} = 0$, so

$$g_{L^Y_s,t} = g_{L_s,t}, \qquad g_{K^Y_E,t} = g_{K_E,t}, \qquad g_{K^Y_S,t} = g_{K_S,t}.$$

Log differentiation therefore gives

$$g_{\pi,t} = \frac{d}{dt}\ln\left(\frac{s_{M,t}}{1-s_{M,t}}\right) + \frac{d}{dt}\ln(1-s_{K_E|M,t}) - \frac{d}{dt}\ln(1-s_{K_S|S,t}) + g_{L_u,t} - g_{L_s,t}.$$

The required derivatives are

$$\frac{d}{dt}\ln\left(\frac{s_{M,t}}{1-s_{M,t}}\right) = \rho_0(g_{M,t} - g_{S,t}),$$

$$\frac{d}{dt}\ln(1-s_{K_E|M,t}) = -\rho_M s_{K_E|M,t}(g_{K_E,t} - g_{L_s,t}), \quad \frac{d}{dt}\ln(1-s_{K_S|S,t}) = -\rho_S s_{K_S|S,t}(g_{K_S,t} - g_{L_u,t}).$$

Using $g_{M,t} = g_{L_s,t} + s_{K_E|M,t}(g_{K_E,t} - g_{L_s,t})$, $\quad g_{S,t} = g_{L_u,t} + s_{K_S|S,t}(g_{K_S,t} - g_{L_u,t})$, we obtain

$$g_{\pi,t} = (\rho_0 - 1)(g_{L_s,t} - g_{L_u,t}) + (\rho_0 - \rho_M)s_{K_E|M,t}(g_{K_E,t} - g_{L_s,t}) + (\rho_S - \rho_0)s_{K_S|S,t}(g_{K_S,t} - g_{L_u,t}).$$

Since $\rho_j = 1 - 1/\sigma_j$,

$$g_{\pi,t} = \frac{1}{\sigma_0}(g_{L_u,t} - g_{L_s,t}) + \left(\frac{1}{\sigma_M} - \frac{1}{\sigma_0}\right)s_{K_E|M,t}(g_{K_E,t} - g_{L_s,t}) - \left(\frac{1}{\sigma_S} - \frac{1}{\sigma_0}\right)s_{K_S|S,t}(g_{K_S,t} - g_{L_u,t}).$$