Constructing State-by-Industry Capital Stocks: Measurement, Productivity, and Regional Economic Inference

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Abstract

Production-function estimates of productivity depend on the measurement of factor inputs, yet capital stocks are often unavailable at the geographic and industry level at which productivity is studied. We examine the consequences of this measurement problem by constructing four alternative state-by-industry capital-stock series for U.S. manufacturing from 1998–2021. The measures preserve national industry controls but differ in the geographic information used to allocate capital and in whether investment is accumulated through a perpetual-inventory method. We then carry these alternative capital measures through a common production-accounting framework to assess how capital measurement affects estimated productivity and subsequent economic inference. Capital and productivity levels are highly correlated across methods, and industry composition consistently explains most cross-state variation in manufacturing productivity. However, annual capital growth, within-industry productivity rankings, productivity-growth decompositions, and changes in cross-state productivity dispersion are substantially more sensitive to the capital construction. All four measures suggest conditional beta convergence, while conclusions about sigma convergence differ across methods. Associations between measured geographic productivity and subsequent employment, investment, and output growth also show mostly similar signs in state-industry specifications, though magnitudes differ. The results show that production-function conclusions about productivity levels can be robust even when the underlying capital input is uncertain, while inference about productivity dynamics and relative geographic performance can depend significantly on how capital is measured.

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1 Introduction

Capital is a central input into the measurement of productivity, the study of economic growth, and the comparison of economic performance across places. At the national level, the Bureau of Economic Analysis (BEA) publishes detailed estimates of fixed assets by industry and asset type. At the subnational level, BEA publishes extensive measures of output, income, compensation, and employment by state and industry. What is missing is a comparable official measure of the capital stock employed by each industry within each state.

This missing quantity creates a practical measurement problem for regional economic analysis. Studies of state productivity, capital intensity, returns to investment, regional convergence, and adjustment to local shocks require some measure of the capital input. In its absence, researchers must either omit capital, substitute a proxy, or construct state capital stocks by allocating national quantities across places. Each approach embeds assumptions about where productive capital is located and how that location evolves over time.

The problem is not simply that a desired statistic has not been published. Capital stocks themselves are generally constructed rather than directly observed. The conventional perpetual inventory approach accumulates investment flows while accounting for depreciation and an initial capital stock (Christensen and Jorgenson, 1969). At the regional level, an additional problem arises: national investment, depreciation, and capital totals must also be assigned geographically. Different observable state-level signals can therefore imply different geographic distributions of an otherwise identical national capital stock.

There is also no observed state-by-industry capital stock against which a constructed measure can be comprehensively validated. Consequently, alternative methods may satisfy the same national accounting constraints while assigning meaningfully different quantities of capital to individual states and industries. The relevant question is therefore not only how state capital stocks can be constructed, but how much economic inference depends on the construction used.

We address this problem by constructing four alternative state-by-industry capital-stock series for U.S. manufacturing over 1998–2021. Each method preserves national industry capital totals but uses different state-level information or accumulation logic to recover their geographic distribution. One method allocates national capital directly using state-industry value-added shares. A second allocates investment using value-added shares and accumulates capital recursively through a perpetual inventory method. A third retains the perpetual-inventory structure but uses observed manufacturing capital-expenditure shares to allocate non-R&D investment. A fourth uses state investment shares to implement a benchmark-

consistent reduced form of the Steady Growth approach associated with Blades (2015).

Each construction imposes national accounting consistency with its own source controls. The empirical comparison retains the available source vintages, whose national totals differ slightly after industry harmonization; Section 3 reports those differences.

Rather than selecting one preferred series and proceeding as if the regional capital stock were observed, we treat the alternative constructions as a structured measurement experiment. We first evaluate their internal accounting properties, cross-method agreement, and relationships with observable economic quantities. We then ask how uncertainty in the geographic allocation of capital propagates into measured productivity and subsequent economic inference.

This approach allows us to distinguish conclusions that are robust to reasonable alternative capital constructions from those that are sensitive to the measurement system itself. We compare productivity levels and growth rates, state rankings, decompositions of state productivity into industry-composition and within-industry geographic components, cross-state dispersion and convergence, and relationships between measured productivity and subsequent economic outcomes. In this sense, the alternative capital-stock measures are not merely competing datasets. They provide a sensitivity analysis of regional economic inference when the underlying capital stock is unobserved.

Our measurement strategy builds on a long literature on constructing capital stocks from incomplete information. At the national level, perpetual inventory methods infer capital from histories of investment, depreciation, and initial conditions (Christensen and Jorgenson, 1969). Although this approach has become standard in productivity and growth accounting, it can be demanding when sufficiently long and consistent investment histories are unavailable.

Alternative methods have therefore been developed for settings in which the information required for a full perpetual inventory calculation is limited. Blades (2015), for example, develops a Steady Growth Model that approximates capital stocks using current investment, past investment growth, and depreciation assumptions. Comparing the resulting estimates with official perpetual-inventory stocks, he shows that the simpler method can generate plausible estimates under appropriate conditions. This illustrates a broader feature of capital measurement: the preferred construction depends in part on the information available and on the assumptions one is willing to impose.

The geographic allocation of capital creates an additional set of choices. A direct antecedent to the present paper is Garofalo and Yamarik (2002), who construct state capital-stock and gross-investment estimates by apportioning national capital among the states. Their resulting series is then used to study regional convergence within a neoclassical growth

framework. Yamarik (2013) extends this approach, constructing state capital and investment estimates for 1990–2007 by allocating national stocks using state-level industry income data and evaluating the resulting series using production-function and growth regressions. More recently, El-Shagi and Yamarik (2019) refine and extend this lineage, including changes to the treatment of mining and the derivation of investment.

A distinct approach is taken by Turner, Tamura, Schoellman, et al. (2011), who assemble historical estimates of physical capital and land for U.S. states and sectors covering as much as 150 years. Their construction draws on historical source data for agriculture, manufacturing, and other sectors and uses sensitivity analysis and comparisons with alternative benchmarks where possible. Related work uses these estimates to conduct long-run growth accounting across states (Turner, Tamura, and Mulholland, 2013). This historical reconstruction illustrates that regional capital measurement need not take the form of a single allocation rule; the appropriate method can vary with sector, period, and the underlying source information.

These measurement exercises matter because state capital stocks are not only objects of interest in their own right. They also serve as inputs into substantive economic analysis. Growth and convergence are an early and natural example. Barro and Sala-i-Martin (1991) use the long history of U.S. states to study regional convergence, migration, and sectoral productivity, while the construction of explicit state capital measures by Garofalo and Yamarik (2002) allows such questions to be recast more directly in a Solow growth-accounting framework. Similarly, Turner, Tamura, and Mulholland (2013) combine long-run state measures of physical capital, human capital, land, and output to decompose state economic growth and compare the resulting patterns with those found in cross-country growth accounting.

State capital is also relevant when the economic question originates outside the growth literature. Peri (2012), for example, studies immigration, employment, and productivity across U.S. states. In that setting, accounting for physical capital is important because an increase in labor supply may be accompanied by capital adjustment; observed changes in output or wages need not reflect labor-supply effects alone. Capital measurement is therefore an intermediate input into the interpretation of a substantive labor-market and productivity question rather than the principal object of the analysis.

More broadly, U.S. states and regions are frequently used as a laboratory for macroeconomic questions because they combine substantial cross-sectional variation with a shared national currency, monetary policy, federal institutional environment, and broadly comparable statistical system. Nakamura and Steinsson (2014), for example, exploit regional variation in military procurement to estimate the effects of government spending while taking advantage of the common national policy environment. Their analysis does not require a state capital-

stock series, but it illustrates why regional data are useful for macroeconomic identification. Improved measures of regional factor inputs can expand the set of mechanisms that can be distinguished in such settings, including whether adjustment to local shocks occurs through employment, capital accumulation, productivity, or some combination of the three.

Similar considerations arise in quantitative spatial economics. Desmet and Rossi-Hansberg (2014) model the spatial evolution of U.S. manufacturing and services through productivity, innovation, trade costs, and the geographic distribution of economic activity. Models of this type do not necessarily require observed state capital stocks because factor allocations can be endogenous objects of the model. Direct measures of regional capital could nevertheless provide additional moments for calibration, validation, and decomposition, and could help distinguish high productivity that attracts capital from high measured output associated with greater capital intensity.

State-level capital measures are therefore potentially useful across several downstream applications: productivity accounting, growth and convergence, labor-market adjustment, public-finance and macroeconomic shocks, structural transformation, and quantitative spatial analysis. The common issue is that capital provides a link between observed output and the productive capacity available in a location. Omitting or poorly measuring that input can change the interpretation assigned to observed geographic differences.

The need for multiple constructions arises because geographic capital is not uniquely mapped by any single observable state-level signal. Structures are physically fixed, but many other productive assets are mobile across locations or are used by firms operating in multiple states. Machinery and vehicles may move across plants or jurisdictions, while software, research and development, and other intellectual property products need not have an economically meaningful location that coincides with the place where the resulting production is recorded. Multistate firms further complicate the assignment of assets to individual establishments or states.

Different observable state-level quantities can therefore support different allocation rules. Value added may proxy for the scale of productive activity in a location; investment flows provide information about where capital is being accumulated; and observed capital expenditures provide a more direct signal for some industries and asset categories. None of these quantities, however, is itself the capital stock. Moreover, because capital is durable, differences in initial conditions, investment allocation, depreciation, and accumulation can propagate through time.

These issues make it difficult to identify a uniquely correct regional capital measure even when national capital totals are well measured. Comparing alternative constructions that share the same national controls but differ in their geographic signals therefore provides

information about which empirical conclusions reflect common underlying variation and which depend on the measurement assumptions used to recover the regional stock.

The comparison produces a consistent distinction between capital levels and capital dynamics. Alternative methods generate very similar broad cross-sectional distributions of state-industry capital. Pairwise correlations of log capital levels range from approximately 0.95 to 0.99, and state rankings within industry-year cells are also highly correlated. Agreement is substantially weaker for annual capital growth, where pairwise correlations range from approximately 0.10 to 0.54. Thus, the methods agree much more strongly about where capital is located than about how rapidly it is changing.

This distinction carries into measured productivity. Productivity levels remain highly correlated across capital-stock methods, but productivity growth and within-industry relative performance are more sensitive to the construction used. Depending on the method pair, a nontrivial share of state-industry-year observations changes productivity quartile. The broad geographic productivity landscape is therefore relatively stable, even while the interpretation of particular states, industries, and periods can change.

The productivity decompositions reveal a sharp contrast. For productivity levels, industry composition accounts for the large majority of cross-state variation under every capital-stock method. The estimated within-industry geographic contribution is comparatively small, although its magnitude varies across methods. For productivity growth, however, the importance assigned to state-specific variation is substantially more measurement-sensitive. Methods that produce relatively similar productivity levels can therefore imply different accounts of the dynamics underlying regional productivity growth.

Some longer-run conclusions are nevertheless robust. All four capital measures imply conditional beta convergence in state-industry productivity over the horizons considered, although the estimated rate varies across methods. Conclusions about sigma convergence are less uniform. More generally, the results suggest that capital measurement uncertainty does not mechanically invalidate regional productivity analysis. Instead, its importance depends on the object being studied: broad level differences and some long-run patterns are comparatively robust, whereas short-run changes, geographic residuals, and some decompositions are more sensitive to how capital is measured.

The remainder of the paper proceeds as follows. Section 2 describes the four capital-stock construction methods. Section 3 evaluates their accounting properties, cross-method agreement, and relationships with observable economic quantities. Section 4 introduces the productivity-accounting framework and compares measured productivity across capital methods. Section 5 examines the decomposition of productivity levels and growth. Section 6 studies productivity dispersion and convergence, and Section 7 examines associations

with subsequent economic outcomes. Section 8 discusses the implications, and Section 9 concludes.

2 Measurement Framework and Construction of Capital Stocks

For method m, state s, industry j, and year t, the estimates satisfy

$$\sum_{s} K^{m}_{sjt} = K^{N}_{jt}. \tag{1}$$

2.1 Method 1: Value-added allocation method

The value-added share allocation, also referred to as "Method 1" is a top-down approach for estimating the capital stock. This method creates state-level capital stock measures by constructing state-industry-level value-added series, distributing national industry-level capital stocks based on the value-added of each industry in each state, and aggregating these values across industries for each state. We create the dataset for the years between 1998 and 2021. This method is similar to that employed by Garofalo and Yamarik, 2002.

The implementation is performed separately for two capital buckets: (i) research and development (RD), and (ii) non-RD capital, consisting of equipment, structures, and intellectual property products excluding research and development.

For industry i, state s, and year t, define the non-RD capital stock as

$$K_i^{WO}(t) = K_i^{E}(t) + K_i^{S}(t) + \left[K_i^{IPP}(t) - K_i^{RD}(t)\right],$$

where $K_i^{E}(t)$, $K_i^{S}(t)$, and $K_i^{IPP}(t)$ denote national capital stocks of equipment, structures, and total intellectual property products, respectively. Thus, the non-RD intellectual property products component is obtained as the residual after research and development capital is deducted from total intellectual property products.

Let:

- $K_i^{RD}(t)$ and $K_i^{WO}(t)$ denote national capital totals in RD and non-RD;
- $y_{i,s}^{RD}(t)$ denote state-industry RD value added;
- $y_{i,s}^{WO}(t)$ denote state-industry non-RD value added.

State-industry capital is allocated as

$$K_{i,s}^{RD}(t) = K_i^{RD}(t)\frac{y_{i,s}^{RD}(t)}{\sum_{s'} y_{i,s'}^{RD}(t)}, \qquad K_{i,s}^{WO}(t) = K_i^{WO}(t)\frac{y_{i,s}^{WO}(t)}{\sum_{s'} y_{i,s'}^{WO}(t)}.$$

Total state-industry capital is

$$K_{i,s}(t) = K_{i,s}^{RD}(t) + K_{i,s}^{WO}(t),$$

and state totals are

$$K_s(t) = \sum_i K_{i,s}(t).$$

Depreciation Allocation Consistent with Method 1

Since Method 1 is a top-down approach, depreciation is not used to construct the state-industry capital stocks. The accompanying state–industry depreciation series is allocated after the stocks have been constructed. Depreciation is therefore constructed after the capital allocation, separately for RD and non-RD capital, while preserving the corresponding national industry-year depreciation totals.

Let national depreciation by capital bucket be

$$Depr_i^{RD}(t), \qquad Depr_i^{WO}(t).$$

Define national depreciation rates as

$$r_i^{RD}(t) = \frac{Depr_i^{RD}(t)}{K_i^{RD}(t)}, \qquad r_i^{WO}(t) = \frac{Depr_i^{WO}(t)}{K_i^{WO}(t)}.$$

Using the Method 1 capital estimates, we construct a mid-year capital proxy and a first-pass depreciation estimate:

$$\widehat{Depr}_{i,s}^{\,b}(t) = r_i^b(t)\frac{K_{i,s}^b(t) + K_{i,s}^b(t+1)}{2}, \qquad b \in \{RD, WO\}.$$

The first-pass estimates are converted into within-industry-year allocation weights:

$$\omega_{i,s}^b(t) = \frac{\widehat{Depr}_{i,s}^{\,b}(t)}{\sum_{s'} \widehat{Depr}_{i,s'}^{\,b}(t)}.$$

Final state-industry depreciation is then allocated as

$$Depr_{i,s}^{b}(t) = \omega_{i,s}^{b}(t) Depr_{i}^{b}(t).$$

The allocation is rescaled, where necessary, to enforce

$$\sum_{s} Depr_{i,s}^{b}(t) = Depr_{i}^{b}(t), \qquad b \in \{RD, WO\}.$$

Fallback rules are used only in degenerate zero-sum cases. Therefore, national BEA depreciation totals are preserved exactly, while state-industry depreciation reflects the capital allocation implied by Method 1.

2.2 Method 2: Value-Added Investment Allocation and Perpetual Inventory Method

Method 2 differs from Method 1 by allocating national *investment* and *depreciation* flows to states each year, then updating state-industry capital recursively with the perpetual inventory method (PIM). The sample covers 1998–2021.

As in Method 1, implementation is conducted separately for two capital buckets: research and development (RD) and non-RD capital (WO). For each industry i and year t, non-RD national quantities are defined as

$$K_{i}^{WO}(t) = K_{i}^{E}(t) + K_{i}^{S}(t) + \left[K_{i}^{IPP}(t) - K_{i}^{RD}(t)\right],$$

with analogous definitions for $Inv_{i}^{WO}(t)$ and $Depr_{i}^{WO}(t)$.

For each bucket $b \in \{RD, WO\}$, state s, and year t, capital evolves as

$$K_{i,s}^{b}(t+1) = K_{i,s}^{b}(t) + Inv_{i,s}^{b}(t+1) - Depr_{i,s}^{b}(t+1).$$

Base-Year Initialization (1998)

Initial state-industry capital is allocated from national stocks using the same value-added-share principle as Method 1:

$$K_{i,s}^{RD}(1998) = K_{i}^{RD}(1998) \frac{y_{i,s}^{RD}(1998)}{\sum_{s'} y_{i,s'}^{RD}(1998)},$$

$$K_{i,s}^{WO}(1998) = K_{i}^{WO}(1998) \frac{y_{i,s}^{WO}(1998)}{\sum_{s'} y_{i,s'}^{WO}(1998)}.$$

Investment Allocation

For $t = 1999, \ldots, 2021$, national industry investment is distributed to states by contemporaneous value-added shares:

$$Inv_{i,s}^{RD}(t) = Inv_i^{RD}(t) \frac{y_{i,s}^{RD}(t)}{\sum_{s'} y_{i,s'}^{RD}(t)},$$

$$Inv_{i,s}^{WO}(t) = Inv_i^{WO}(t) \frac{y_{i,s}^{WO}(t)}{\sum_{s'} y_{i,s'}^{WO}(t)}.$$

Depreciation Allocation

Depreciation is allocated using a mid-year capital proxy consistent with the recursive PIM implementation:

$$d_{i,s}^{b}(t+1) = K_{i,s}^{b}(t) + \frac{1}{2} Inv_{i,s}^{b}(t+1), \qquad b \in \{RD, WO\}.$$

National depreciation is then distributed proportionally:

$$Depr_{i,s}^{b}(t+1) = Depr_i^{b}(t+1) \left[\frac{d_{i,s}^{b}(t+1)}{\sum_{s'} d_{i,s'}^{b}(t+1)} \right].$$

Perpetual Inventory Update and National Reconciliation

Let the unscaled forward capital be

$$\widetilde{K}_{i,s}^{b}(t+1) = K_{i,s}^{b}(t) + Inv_{i,s}^{b}(t+1) - Depr_{i,s}^{b}(t+1).$$

To enforce exact consistency with national industry totals each year, capital is rescaled:

$$K_{i,s}^{b}(t+1) = K_i^{b}(t+1) \frac{\widetilde{K}_{i,s}^{b}(t+1)}{\sum_{s'} \widetilde{K}_{i,s'}^{b}(t+1)}, \qquad b \in \{RD, WO\}.$$

Hence, by construction,

$$\sum_{s} K_{i,s}^{b}(t+1) = K_i^{b}(t+1), \qquad b \in \{RD, WO\}.$$

Method 2 therefore preserves the Method 1 value-added allocation logic for the initial level and for investment shares, while introducing dynamic capital accumulation through recursive PIM updates and depreciation tied to the capital base.

2.3 Method 3: Capital-Expenditure-Based Investment Allocation with Perpetual Inventory Dynamics

Method 3 applies a hybrid perpetual inventory method (PIM) for manufacturing industries over 1998–2021. As in Methods 1 and 2, the implementation is carried out separately for two capital buckets:

$$b \in \{RD,\ WO\},$$

where WO denotes non-RD capital (equipment + structures + non-RD intellectual property).

State-industry capital evolves recursively as

$$K^b_{i,s}(t+1) = K^b_{i,s}(t) + Inv^b_{i,s}(t+1) - Depr^b_{i,s}(t+1).$$

Base-Year Initialization (1998)

Initial capital stocks are allocated from national benchmarks using value-added shares (same principle as Methods 1 and 2):

$$K^{RD}_{i,s}(1998) = K^{RD}_i(1998)\frac{y^{RD}_{i,s}(1998)}{\sum_{s'} y^{RD}_{i,s'}(1998)},$$

$$K^{WO}_{i,s}(1998) = K^{WO}_i(1998)\frac{y^{WO}_{i,s}(1998)}{\sum_{s'} y^{WO}_{i,s'}(1998)}.$$

Investment Allocation

Method 3 is hybrid in investment allocation.

For RD, investment uses value-added shares:

$$Inv^{RD}_{i,s}(t) = Inv^{RD}_i(t)\frac{y^{RD}_{i,s}(t)}{\sum_{s'} y^{RD}_{i,s'}(t)}.$$

For WO, investment uses ASM capital-expenditure shares:

$$Inv^{WO}_{i,s}(t) = Inv^{WO}_i(t)\,\phi_{i,s}(t), \qquad \phi_{i,s}(t) = \frac{CapExp^{ASM}_{i,s}(t)}{\sum_{s'} CapExp^{ASM}_{i,s'}(t)}.$$

Thus, unlike Method 2, non-RD investment is anchored to observed state-industry capital-expenditure patterns rather than value-added shares.

Depreciation Allocation

Depreciation is allocated using a mid-year capital proxy (as in Method 2):

$$d^b_{i,s}(t+1) = K^b_{i,s}(t) + \frac{1}{2} Inv^b_{i,s}(t+1), \qquad b \in \{RD, WO\}.$$

National depreciation is distributed proportionally:

$$Depr^b_{i,s}(t+1) = Depr^b_i(t+1) \left[\frac{d^b_{i,s}(t+1)}{\sum_{s'} d^b_{i,s'}(t+1)} \right].$$

PIM Update and National Reconciliation

Define unscaled forward capital:

$$\widetilde{K}^b_{i,s}(t+1) = K^b_{i,s}(t) + Inv^b_{i,s}(t+1) - Depr^b_{i,s}(t+1).$$

To enforce exact reconciliation with national industry totals, rescale:

$$K^b_{i,s}(t+1) = K^b_i(t+1) \frac{\widetilde{K}^b_{i,s}(t+1)}{\sum_{s'} \widetilde{K}^b_{i,s'}(t+1)}, \qquad b \in \{RD, WO\}.$$

Hence, for each industry-year and bucket,

$$\sum_s K^b_{i,s}(t+1) = K^b_i(t+1).$$

Overall, Method 3 keeps the Method 2 recursive PIM structure and depreciation logic, but replaces the non-RD investment allocator with ASM-based capital-expenditure shares.

2.4 Method 4: Investment-Share Allocation

Method 4 allocates national industry capital stocks across states using contemporaneous investment shares. The allocation is performed separately for research and development (RD) and non-RD capital (WO). It can be derived by benchmarking a steady-growth capital estimate to the corresponding national stock.

For state s, industry j, year t, and capital category $b \in \{RD, WO\}$, the provisional steady-growth estimate is

$$\widetilde{K}^b_{sjt} = \frac{I^{b,+}_{sjt}}{\delta^b_{jt} + g^b_j}, \tag{2}$$

where $I^{b,+}_{sjt} = \max\{I^b_{sjt}, 0\}$, δ^b_{jt} is the national depreciation rate, and g^b_j is the long-run investment growth rate. The depreciation and growth rates are common across states, and their sum is assumed to be positive.

Benchmarking these estimates to national capital gives

$$K^{b,4}_{sjt} = K^{b,N}_{jt} \frac{\widetilde{K}^b_{sjt}}{\sum_r \widetilde{K}^b_{rjt}} = K^{b,N}_{jt} \frac{I^{b,+}_{sjt}}{\sum_r I^{b,+}_{rjt}}. \tag{3}$$

The common depreciation and growth terms cancel during benchmarking. Method 4 therefore calculates capital directly from investment shares, without estimating these parameters or recursively accumulating past investment.

The allocation requires a positive sum of investment within each industry-year-category cell. Observations with nonpositive investment receive zero category capital. For valid cells, the estimates satisfy

$$\sum_s K^{b,4}_{sjt} = K^{b,N}_{jt}. \tag{4}$$

Total state–industry capital is the sum of the two categories:

$$K^4_{sjt} = K^{RD,4}_{sjt} + K^{WO,4}_{sjt}. \tag{5}$$

The depreciation and growth parameters, δ^b_{jt} and g^b_j, therefore do not enter the final Method 4 capital formula. The accompanying depreciation series is allocated after capital has been constructed. For each category, define the allocation base as

$$B^{b,4}_{sjt} = K^{b,4}_{sjt} + \frac{1}{2} I^b_{sjt}. \tag{6}$$

State–industry depreciation is then

$$\text{Depr}^{b,4}_{sjt} = \text{Depr}^{b,N}_{jt} \frac{B^{b,4}_{sjt}}{\sum_r B^{b,4}_{rjt}}. \tag{7}$$

This allocation preserves the national depreciation total and does not alter the Method 4 capital estimates.

Table 1: State Capital-Stock Methods at a Glance

Method	Method and state signal	Capital-stock construction	Interpretation and tradeoff
Method 1	**Value-added allocation** State RD and non-RD value-added shares	Allocates each national RD and non-RD capital stock directly across states using category-specific value-added shares.	Static and transparent, but assumes value added reveals the geographic location of capital.
Method 2	**Value-added PIM** State RD and non-RD value-added component shares	Initializes 1998 from value-added shares, applies the PIM, and reconciles the resulting state stocks to national controls.	Captures accumulation over time, but initialization and flow-allocation errors can persist through the recursion.
Method 3	**Capital-expenditure PIM** RD value-added shares; non-RD ASM CAPEX shares	Applies the PIM; RD investment follows value added and non-RD investment follows ASM capital expenditure.	Uses observed non-RD spending, but depends on ASM coverage and industry mapping.
Method 4	**Investment-share allocation** Positive state investment shares, separately by category	Multiplies national category capital by each state's positive investment share, separately for RD and non-RD.	Direct and additive; nonpositive investment receives zero capital, and depreciation does not enter the stock formula.

Common design: All methods cover 1998–2021, 51 states, and 19 harmonized manufacturing industries. RD and non-RD capital are constructed separately and then summed.
Depreciation in Method 4: Depreciation is attached after capital construction for subsequent analysis and does not alter the capital stock.

3 Evaluation of the Capital Measures

3.1 Evaluation without observed state capital stocks

The absence of an observed state–industry capital stock makes evaluation a comparison of accounting properties and economic implications. National consistency is necessary for the allocation exercise, but it does not identify the geographic distribution of capital. Conversely, a close relationship with an observable state quantity may partly reflect the use of that quantity in construction. We therefore examine national aggregation, agreement among the four measures, and their associations with investment, employment, compensation, and output. Throughout this section, Methods 1–4 denote value-added allocation, value-added PIM, capital-expenditure PIM, and investment-share allocation, respectively.

3.2 Internal accounting validity

The comparison covers 23,064 state–industry–year observations over 1998–2021, spanning 19 manufacturing industry groups and the 50 states plus the District of Columbia. Total capital is finite and strictly positive for all four methods in this analysis sample. This property of the comparison sample should be distinguished from the construction rule for Method 4: nonpositive investment can produce zero capital at the category level. Such an allocation is not assigned an arbitrary positive value to permit logarithmic analysis.

We compare the state sums with the national industry-year totals represented by Method 4, which is benchmarked to its national source controls. Method 4 therefore has zero discrepancy in this comparison. The largest relative discrepancies for Methods 1, 2, and 3 are 0.391, 0.407, and 0.321 percent, respectively. These comparisons use the harmonized industry classification and retain differences among the underlying source vintages. They are distinct from the national reconciliation imposed within each method's construction. The estimates are compared as constructed rather than rescaled to eliminate these differences before evaluation.

3.3 Cross-method agreement

3.3.1 Capital levels

Panel A of Table 2 shows close agreement in the broad distribution of capital. Pairwise Pearson correlations of log capital range from 0.951 to 0.988. The two PIM measures have a correlation of 0.988, as do value-added allocation and value-added PIM to the reported

precision. The lowest correlation, 0.951, is between value-added PIM and investment-share allocation.

High correlation nevertheless leaves room for economically substantial differences in individual observations. The median absolute capital difference is 9.4 percent for the two PIM methods and 28.3 percent for value-added PIM versus investment-share allocation, measured relative to the second method in each comparison. Thus, agreement about the broad ordering of capital stocks does not imply agreement about their magnitudes. Figure 1 complements the table with pairwise log-capital comparisons and distributions of log differences. Its 45-degree lines mark equality, while the difference distributions show the extent of disagreement around that benchmark. The table's level and growth correlations use 20,366 observations with the requisite current and lagged capital values; the figure uses its common positive level sample, as described in its notes.

3.3.2 Capital growth

Agreement is much weaker for annual changes in log capital. The correlations range from 0.102 for value-added PIM and investment-share allocation to 0.544 for capital-expenditure PIM and investment-share allocation. Even the two PIM methods, whose capital levels are highly correlated, have a growth correlation of only 0.393. Value-added allocation and value-added PIM have a growth correlation of 0.479.

This distinction is consistent with the different roles of contemporaneous allocation shares and accumulated investment. Direct allocation makes the geographic distribution of the current stock depend on current state shares. PIM carries past allocations forward through the surviving capital stock. The observed growth differences therefore concern the timing of capital adjustment as well as its geographic distribution. The relatively high growth correlation between Methods 3 and 4 is also consistent with their shared use of capital-expenditure information for non-R&D investment.

3.3.3 State rankings within industries

Within-industry-year state rankings are strongly correlated across methods, with average rank correlations between 0.938 and 0.988. The mean absolute rank change ranges from 1.355 places for the two PIM methods to 3.496 places for value-added PIM and investment-share allocation. These statistics compare states producing within the same industry and year, so the agreement is not solely a consequence of differences in the scale of capital across industries. The average changes also conceal larger movements in particular cells: the largest observed rank change is 38 places for the comparison between Methods 2 and 4.

3.3.4 Cross-sectional dispersion

The methods also differ in the dispersion of log capital across states within an industry-year. The mean absolute difference between these standard deviations is 0.047 for value-added allocation and value-added PIM, compared with 0.145 for capital-expenditure PIM and investment-share allocation. These are differences in log-capital standard deviations rather than percentage changes in capital. Because the table reports absolute differences, this column measures disagreement about dispersion without asserting that one method is more dispersed in every industry-year.

3.3.5 Industries and states with the largest disagreement

To identify states with the greatest disagreement in capital magnitudes, we calculate the cross-method log range within each state–industry–year:

$$R^K_{sjt} = \max_m \ln K^m_{sjt} - \min_m \ln K^m_{sjt}. \tag{8}$$

We then average this range over each state's retained industry-year observations, giving each observation equal weight. Figure 2 displays the ten jurisdictions with the largest mean ranges, and Table A5 reports their medians, 90th percentiles, and capital-rank ranges. These are comparisons of state–industry stocks, rather than differences between aggregate state capital totals. The 21,240 observations require positive, finite output, employment, and capital under all four methods; lagged observations are not required for these level comparisons. The common sample spans 18 of the 19 source industry groups. The remaining transportation-equipment group (NAICS 3364–3366 and 3369) has no employment observations in the analysis panel and therefore does not enter this common-input comparison.

The District of Columbia has the largest mean log-capital range, 1.223, followed by Alaska at 1.142, Wyoming at 1.031, and Hawaii at 1.011. The corresponding medians are 1.030, 0.974, 0.960, and 0.728, respectively. The ordering therefore reflects substantial differences across retained observations, rather than just the single largest discrepancy in each state. Delaware and New Mexico also have large mean ranges, 0.907 and 0.903. State coverage differs, as the observation counts in the table show; the comparisons describe each state's available manufacturing industries rather than imposing an identical industry mix across states.

Disagreement in capital magnitudes is distinct from disagreement in ranks. For each observation, we also calculate the range across its four capital ranks, ranking states within the same industry-year and the same common sample. The District of Columbia's mean rank range is 1.82 places, whereas Delaware's is 5.47 and New Mexico's is 5.17. Capital estimates

can therefore differ substantially in proportional terms while leaving a state's position among other states relatively unchanged. These four-method ranges differ from the pairwise average rank changes in Table 2.

Averaging the mean absolute capital-rank changes across the six method pairs and the sample years, the largest industry differences occur in apparel and leather products (NAICS 315–316), petroleum and coal products (324), and motor vehicles, bodies, trailers, and parts (3361–3363). Their average changes are approximately 3.72, 3.38, and 3.29 rank positions, respectively. These comparisons locate disagreement in the within-industry allocation; they do not identify which allocation is closest to the unobserved stock.

The rank and dispersion comparisons show why aggregate agreement should be supplemented by comparisons within industries. A method pair can have a log-level correlation above 0.95 while placing particular states quite differently in the industry distribution. We return to the geographic consequences in Section 5, where Figure 3 identifies the states whose aggregate productivity rankings vary most across capital measures. Those productivity rankings incorporate both capital differences and the state's industry composition; they are distinct from the capital rankings reported here.

Table 2: Internal Validity: Cross-Method Agreement of State Capital Stocks

Panel A: Cross-method agreement

Method A	Method B	Log-level Pearson	Growth correlation	Rank correlation	Median absolute percentage difference	Mean absolute rank change	Average dispersion difference
Value-added allocation	Value-added PIM	0.988	0.479	0.982	0.127	1.686	0.047
Value-added allocation	Capital-expenditure PIM	0.974	0.155	0.974	0.174	2.158	0.128
Value-added allocation	Investment-share allocation	0.956	0.133	0.945	0.262	3.247	0.123
Value-added PIM	Capital-expenditure PIM	0.988	0.393	0.988	0.094	1.355	0.110
Value-added PIM	Investment-share allocation	0.951	0.102	0.938	0.283	3.496	0.120
Capital-expenditure PIM	Investment-share allocation	0.964	0.544	0.954	0.237	3.018	0.145

Notes: Statistics use 20,366 common positive observations. Log-level Pearson and growth correlation are calculated over the common sample. Rank correlation and mean absolute rank change are calculated within industry-year cells and then averaged. Median absolute percentage difference is measured relative to Method B. Average dispersion difference is the mean absolute difference between the methods' within-industry-year standard deviations of log capital.

3.4 External economic validity

Panel B of Table 2 relates contemporaneous outcome growth to capital growth. Each entry is the coefficient from a separate regression of an annual log change in the indicated outcome on the annual log change in capital, with industry fixed effects and standard errors clustered by state–industry. These associations describe how each capital series covaries with observed economic activity. They do not estimate the causal effect of capital accumulation. In particular, raw coefficient magnitudes should not be used to rank methods because the variance of capital growth differs across constructions.

3.4.1 Investment

Investment growth is positively associated with capital growth under every method. The coefficients are 0.671 for value-added allocation, 2.730 for value-added PIM, 3.607 for capital-expenditure PIM, and 0.885 for investment-share allocation. The investment outcome is the

common investment series used in the investment-share construction. Its close association with Method 4 is consequently partly mechanical: investment helps determine the state allocation of that method's capital. This comparison is informative about the relationship between stocks and flows, but it is not an independent validation of Method 4.

3.4.2 Employment growth

The employment-growth coefficients are positive for all four measures, ranging from 0.012 for investment-share allocation to 0.695 for value-added PIM. Value-added allocation and capital-expenditure PIM have coefficients of 0.146 and 0.101. Relative to their reported clustered standard errors, each estimate is distinguishable from zero. The marked variation in magnitude reinforces the distinction between a common positive association and a common quantitative account of capital and employment adjustment.

3.4.3 Real compensation growth

The corresponding coefficients for real compensation growth are 0.198, 0.809, 0.148, and 0.015 in method order. As with employment, all are positive and precisely estimated relative to their reported standard errors. Real compensation is total compensation adjusted by the regional price measure; its growth can reflect changes in employment as well as compensation per worker. It should therefore not be interpreted as a wage-growth measure.

3.4.4 Output growth

Output growth is also positively associated with capital growth under every method. The coefficient is 0.920 for value-added allocation, 2.335 for value-added PIM, 0.277 for capital-expenditure PIM, and 0.048 for investment-share allocation. The use of value-added shares in Methods 1 and 2 makes the output relationship particularly relevant to interpreting those constructions. Strong output covariation is consistent with their allocation rules and cannot, by itself, establish that their geographic capital stocks are more accurate. Taken together, the external comparisons show that all four measures contain economically related variation, while differing in the annual movements they assign to capital.

Table 2: External Economic Validity of State Capital-Stock Methods

Panel B: Contemporaneous economic associations

Method	Investment growth	Employment growth	Compensation growth	Output growth
Method 1: Value-added allocation	0.671 (0.060)	0.146 (0.012)	0.198 (0.013)	0.920 (0.011)
Method 2: Value-added PIM	2.730 (0.109)	0.695 (0.051)	0.809 (0.058)	2.335 (0.065)
Method 3: Capital-expenditure PIM	3.607 (0.093)	0.101 (0.016)	0.148 (0.023)	0.277 (0.040)
Method 4: Investment-share allocation	0.885 (0.004)	0.012 (0.003)	0.015 (0.003)	0.048 (0.010)

Notes: Each entry reports the coefficient on contemporaneous capital growth, with its clustered standard error in parentheses. Regressions include industry fixed effects and cluster standard errors by state-industry. Observations are 22,099 for investment, 20,254 for employment, 20,067 for compensation, and 22,103 for output. Coefficients are descriptive; because capital-growth dispersion differs across methods, magnitudes should not be interpreted as a method ranking without standardization. The Method 4 investment relationship is partly mechanical because investment is its state allocator.

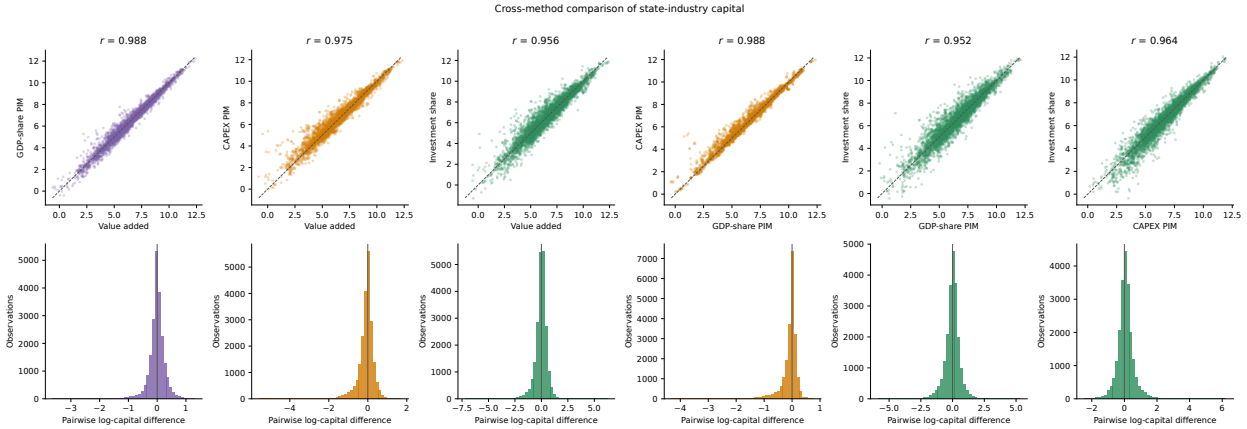


Figure 1: Pairwise Comparison of State–Industry Capital Stocks Across Methods

Notes: The upper panels display a fixed random sample of at most 5,000 common-sample observations per method pair, drawn without replacement using seed 1729, to reduce overplotting. The Pearson correlations reported above the upper panels and the lower-panel distributions use all available observations in the strictly positive common comparison sample. The dashed 45-degree line indicates equal capital estimates. In each lower panel, a positive log-capital difference means that the first method named in the corresponding upper panel produces the larger estimate.

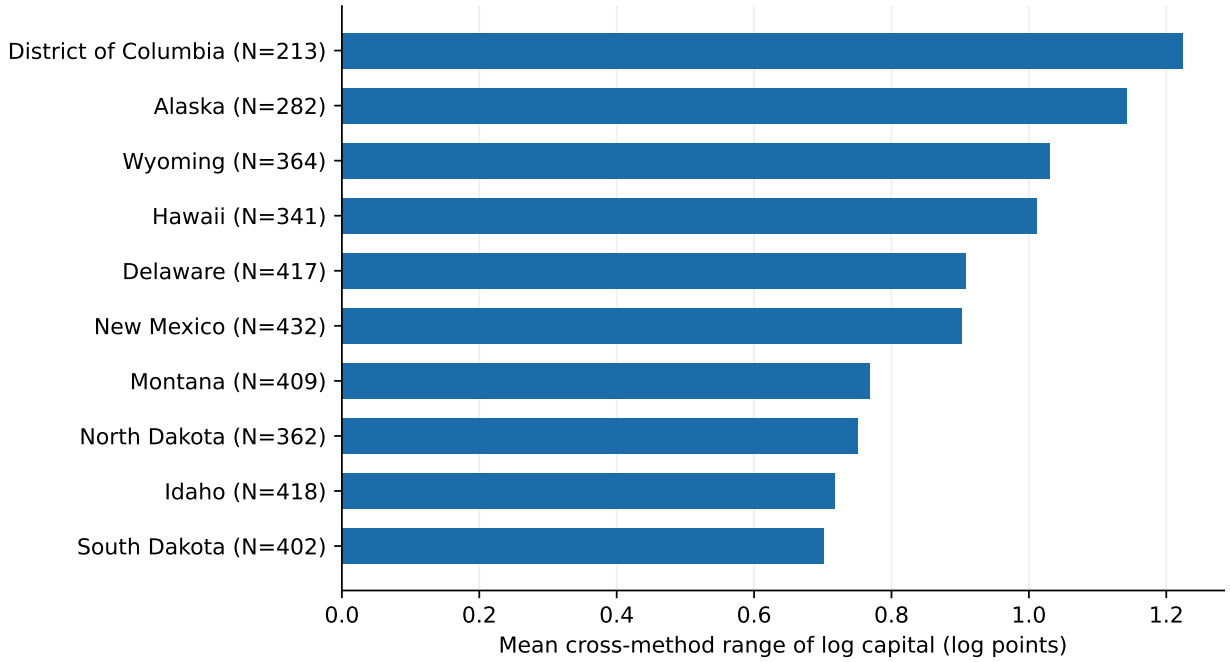


Figure 2: States with the Largest Cross-Method Differences in Capital Stocks

Notes: For each state–industry–year observation, the capital range is the largest minus the smallest log capital stock across the four methods. Bars show the ten jurisdictions with the highest mean ranges over their retained industry-year observations. Each observation receives equal weight; the statistic does not compare aggregate state capital totals. The common sample requires finite, strictly positive output, employment, and capital under all four methods. Parentheses report the number of retained observations. Table A5 also reports medians, 90th percentiles, and within-industry-year capital-rank ranges.

4 Productivity Accounting Framework

We next examine how the capital differences documented in Section 3 affect measured productivity. Output and employment are held fixed across capital methods. Within a capital-share specification, differences in the resulting residual therefore arise from the capital measure rather than from a change in the observed output or labor input.

4.1 Measured state-industry productivity

For capital-stock method m, measured productivity is defined by

$$\ln A^m_{sjt} = \ln Y_{sjt} - \alpha_{jt} \ln K^m_{sjt} - (1 - \alpha_{jt}) \ln L_{sjt}. \tag{9}$$

Here Y_{sjt} is value added, L_{sjt} is employment, and K^m_{sjt} is the capital stock for each state s and industry j under method m. The residual is an accounting measure of productivity and measures output relative to the contribution attributed to the two observed inputs under constant returns to scale. For two methods evaluated with the same capital share,

$$\ln A^m_{sjt} - \ln A^{m'}_{sjt} = -\alpha_{jt} \left(\ln K^m_{sjt} - \ln K^{m'}_{sjt} \right). \tag{10}$$

Thus, assigning more capital to a given state–industry observation reduces its measured productivity, holding output and employment fixed. The capital share determines how much of a log-capital difference is transmitted to the productivity residual.

It should be noted that the productivity residual captures variation in value added that is not accounted for by measured capital and employment under the assumed input elasticities. In addition to differences in productive efficiency, it can reflect variation in capacity utilization, hours worked, labor quality, and other factors omitted from the accounting framework. Its interpretation therefore depends on both the production assumptions and the measurement of inputs.

This paper focuses on the role of capital measurement. Therefore, holding output, employment, and capital shares fixed across methods isolates how alternative capital allocations affect measured productivity. The resulting differences reveal how strongly conclusions about relative productivity depend on the construction of state-by-industry capital stocks.

4.1.1 Capital-share specification

The main results allow the capital share to differ by industry and year while imposing the same share across states in a given industry-year. Therefore, for each industry and year:

$$\alpha_{jt} = \min\left\{0.95, \max\left\{0.05, \frac{\sum_s \text{GOS}_{sjt}}{\sum_s Y_{sjt}}\right\}\right\}. \tag{11}$$

Aggregating the numerator and denominator before taking their ratio gives a common national industry-year share. This differs from averaging the state-specific ratios and avoids making a state's own income share an additional source of geographic variation in its accounting weight.[1]

4.2 Sensitivity of measured productivity

Table 3 reports the sensitivity results of measured productivity. Log-productivity levels are highly correlated across capital measures, with correlations from 0.928 to 0.979. The two PIM methods have a level correlation of 0.978 and a median absolute difference of 0.030 log points. Value-added PIM and investment-share allocation produce the lowest level correlation, 0.928, and the largest median absolute difference, 0.097 log points.

Within-industry rankings are less stable than the level correlations alone would suggest. Rank correlations range from 0.743 to 0.929, and between 22.3 and 44.0 percent of observations change productivity quartile when the capital measure changes. The smallest quartile-switching share occurs between the two PIM methods; the largest occurs between value-added PIM and investment-share allocation. These are changes in a state's position among states in the same industry and year, not changes in its rank across the entire manufacturing sample.

Annual productivity growth is especially close between the two PIM measures, with a correlation of 0.988. Comparisons involving investment-share allocation are weaker, ranging from 0.649 to 0.715. The distinction is consistent with the capital-growth comparisons in Table 2: methods that imply similar capital levels can still assign different annual changes to the capital input. Because output and labor growth are common to the productivity calculations, productivity-growth agreement can nevertheless exceed capital-growth agreement.

The fixed-share results confirm that rank sensitivity is not confined to the preferred

[1] Alternative specifications separate sensitivity to the capital allocation from sensitivity to the share used to value its contribution. Table A1 reports the fixed share, the constrained national industry share, the preferred constrained national industry-year share, and the unconstrained industry-year share. The constrained and unconstrained industry-year results are very similar: their level correlations, median absolute differences, and quartile-switching shares coincide to the displayed precision.

income-share specification. Under $\alpha = 0.33$, the share changing quartile ranges from 19.7 to 41.6 percent (Table A1, Panel A). The evidence therefore supports a distinction between a broadly similar distribution of measured productivity and materially different classifications of particular state–industry observations.

Table 3: Sensitivity of Measured Productivity to the Capital-Stock Method

Method A	Method B	Level correlation	Growth correlation	Rank correlation	Median absolute productivity difference	Share changing productivity quartile
Value added	Value-added PIM	0.979	0.927	0.917	0.041	0.262
Value added	CAPEX PIM	0.956	0.909	0.885	0.056	0.296
Value added	Investment share	0.932	0.649	0.780	0.090	0.417
Value-added PIM	CAPEX PIM	0.978	0.988	0.929	0.030	0.223
Value-added PIM	Investment share	0.928	0.663	0.743	0.097	0.440
CAPEX PIM	Investment share	0.949	0.715	0.814	0.082	0.381

Notes: Results use the preferred constrained national industry-year capital share, $\alpha_{jt} = \min\{0.95, \max\{0.05, \text{GOS}_{jt}/\text{VA}_{jt}\}\}$. Level, rank, and quartile comparisons each use 21,240 common state–industry–year observations. Growth correlations use 20,254 valid annual productivity changes. Level and growth columns report Pearson correlations of log productivity and its annual change. Rank correlation is based on within-industry-year state ranks. Median absolute differences are in log points. The quartile-change measure is the share of observations assigned to different within-industry-year productivity quartiles. Productivity is an accounting residual, not a technology shock.

5 Decomposing Productivity at the State–Industry and Aggregate State Levels

We examine how capital measurement affects the sources of measured productivity variation at two levels. The state–industry decomposition separates variation across state–industry–year observations into a common time component, national industry differences, and states' deviations from their national industry benchmarks. The aggregate state decomposition first combines industry productivity residuals using each industry's share of state manufacturing value added, then separates the resulting index into industry composition and within-industry geographic performance. Together, these decompositions show how alternative capital measures affect both productivity differences within industries and the contributions of industry mix and relative industry performance to aggregate state manufacturing productivity.

5.1 State–industry variance decomposition

The state–industry decomposition separates measured productivity into a common time component, a national industry component, and a component capturing each state's productivity

relative to its national industry benchmark. We define $a^m_{sjt} = \ln A^m_{sjt}$ for the log-productivity accounting residual under capital method m. Next, for each industry-year, we define the employment-weighted national benchmark

$$\overline{a}^m_{jt} = \sum_s \omega_{sjt} a^m_{sjt}, \qquad \omega_{sjt} = \frac{L_{sjt}}{\sum_r L_{rjt}}. \tag{12}$$

Let $\overline{a}^m_t$ be the unweighted mean of a^m_{sjt} over the common state–industry observations in year t. The observation-level identity is

$$a^m_{sjt} = \underbrace{\overline{a}^m_t}_{\text{aggregate-time component}} + \underbrace{\left(\overline{a}^m_{jt} - \overline{a}^m_t\right)}_{\text{national-industry component}} + \underbrace{\left(a^m_{sjt} - \overline{a}^m_{jt}\right)}_{\text{state-within-industry component}}. \tag{13}$$

The aggregate-time component captures movements shared by observations in a year. The national-industry component captures the position of an industry's national benchmark relative to that annual mean. The final component is the state's deviation from its employment-weighted national industry-year benchmark. This is a three-way decomposition of variation among state–industry observations. It answers a different question from the aggregate-state decomposition below, which first combines industries within a state using value-added shares.

Pooling the common state–industry–year observations gives

$$\begin{aligned} \operatorname{Var}(a^m_{sjt}) = {} & \operatorname{Var}(\overline{a}^m_t) + \operatorname{Var}(\overline{a}^m_{jt} - \overline{a}^m_t) + \operatorname{Var}(a^m_{sjt} - \overline{a}^m_{jt}) \\ & + 2\operatorname{Cov}\left(\overline{a}^m_t, \overline{a}^m_{jt} - \overline{a}^m_t\right) + 2\operatorname{Cov}\left(\overline{a}^m_t, a^m_{sjt} - \overline{a}^m_{jt}\right) \\ & + 2\operatorname{Cov}\left(\overline{a}^m_{jt} - \overline{a}^m_t, a^m_{sjt} - \overline{a}^m_{jt}\right). \end{aligned} \tag{14}$$

Table 4 shows that national industry differences account for the largest share of pooled variation under every capital method. Their share declines from 0.799 under value-added allocation to 0.753 under value-added PIM, 0.740 under capital-expenditure PIM, and 0.701 under investment-share allocation. The aggregate-time share is smaller and stable, ranging from 0.067 to 0.083.

Capital measurement matters most for the state-within-industry component. Its share rises from 0.143 under value-added allocation to 0.214 under value-added PIM, 0.288 under capital-expenditure PIM, and 0.312 under investment-share allocation. The cross-method range is 16.9 percentage points, and Methods 3 and 4 assign about twice the share obtained with Method 1. The covariance contribution is negative for all four methods, from −0.025 to

Table 4: State–Industry Productivity Variance Decomposition

Method	Total variance	Aggregate time share	National industry share	State within industry share	Covariance share
Value added	0.364	0.083	0.799	0.143	-0.025
Value-added PIM	0.379	0.079	0.753	0.214	-0.047
CAPEX PIM	0.383	0.067	0.740	0.288	-0.095
Investment share	0.416	0.074	0.701	0.312	-0.086

Notes: Log productivity is decomposed into an aggregate-time mean, an employment-weighted national-industry deviation, and a state-within-industry residual. Variances are pooled over 21,240 common state–industry–year observations using the preferred constrained national industry-year capital share. The covariance entry is twice the sum of all pairwise component covariances. The four shares sum to one; the cross-method range in the state-within-industry share is 0.169.

-0.095, so the three separate variance shares can sum to more than one before the covariance term is included. The result shows that the inferred source of state–industry productivity dispersion is sensitive to capital construction; it does not determine which capital measure is more accurate or give the components a causal interpretation.

5.2 Aggregate-state productivity level decomposition

The aggregate state decomposition combines industry productivity residuals using their shares of state manufacturing value added. It separates the resulting index into an industry-composition component and a within-industry geographic component.

Let $a^m_{sjt} = \ln A^m_{sjt}$ denote the log-productivity accounting residual for state s, industry j, year t, and capital method m. Define $\theta_{sjt} = Y_{sjt} / \sum_\ell Y_{s\ell t}$ as industry j's share of total reported manufacturing value added in state s and year t. As above, $\overline{a}^m_{jt}$ denotes the employment-weighted national industry-year benchmark, and $\widetilde{u}^m_{sjt} = a^m_{sjt} - \overline{a}^m_{jt}$ denotes the state's deviation from that benchmark.

Let $\mathcal{J}_{st}$ denote the industries with valid productivity residuals under all capital methods in state s and year t. The aggregate state manufacturing productivity index is

$$a^m_{st} \equiv \sum_{j \in \mathcal{J}_{st}} \theta_{sjt} a^m_{sjt} = \underbrace{\sum_{j \in \mathcal{J}_{st}} \theta_{sjt} \overline{a}^m_{jt}}_{C^m_{st}\text{: industry composition}} + \underbrace{\sum_{j \in \mathcal{J}_{st}} \theta_{sjt} \widetilde{u}^m_{sjt}}_{G^m_{st}\text{: within-industry geography}} . \tag{15}$$

The industry-composition component, C^m_{st}, assigns each industry its national productivity benchmark while retaining the state's observed industry shares. The within-industry geo-

graphic component, G^m_{st}, uses the same shares to combine the state's deviations from those benchmarks. Thus, a^m_{st} is an index constructed from industry log-productivity residuals. The national benchmarks are likewise averages of state–industry log residuals.

The weights retain total reported state manufacturing value added as their denominator. Industries without valid productivity residuals are excluded from the sums, and the remaining weights are not rescaled. Where coverage is incomplete, a^m_{st} is therefore a weighted sum whose weights add to less than one.

For each year and capital method, cross-state variation in the index satisfies

$$\mathrm{Var}_s(a^m_{st}) = \mathrm{Var}_s(C^m_{st}) + \mathrm{Var}_s(G^m_{st}) + 2\,\mathrm{Cov}_s(C^m_{st}, G^m_{st}). \tag{16}$$

The variance shares are calculated by dividing each term on the right-hand side by $\mathrm{Var}_s(a^m_{st})$ within each year. The table reports their averages over 1998–2021.

Table 5: Aggregate-State Productivity Decomposition of Productivity Levels

Method	State-productivity variance	Industry-composition share	Within-industry geographic share	Covariance share
Value added	0.415	0.932	0.024	0.044
Value-added PIM	0.425	0.905	0.040	0.055
CAPEX PIM	0.431	0.891	0.058	0.051
Investment share	0.437	0.892	0.068	0.040

Notes: Entries are means across 24 annual, value-added-weighted decompositions using the preferred constrained national industry-year capital share and 51 states. Component shares include the covariance term and therefore sum to one within each year. Across methods, the mean within-industry geographic share ranges from 0.024 to 0.068, a difference of 0.044.

Table 5 shows that the industry-composition component, C^m_{st}, accounts for most cross-state variation in the manufacturing productivity index under every capital method.[2] Its average variance share is 93.2 percent under value-added allocation, 90.5 percent under value-added PIM, 89.1 percent under capital-expenditure PIM, and 89.2 percent under investment-

[2]Table A2 also reports the component variances, allowing the share changes to be assessed alongside the change in total variance.

share allocation. Industry composition therefore remains the largest component across all four capital constructions. This contribution reflects states' observed industry shares together with differences in national industry productivity benchmarks; it does not identify the causal effect of changing a state's industry mix.

The within-industry geographic component, G_{st}^m, accounts for a smaller share of cross-state variation. Its average variance share is 2.4 percent under value-added allocation, 4.0 percent under value-added PIM, 5.8 percent under capital-expenditure PIM, and 6.8 percent under investment-share allocation. The share under investment-share allocation is approximately 2.8 times that under value-added allocation, a difference of 4.4 percentage points.

The remaining contribution comes from twice the covariance between C_{st}^m and G_{st}^m. Its average share is 4.4, 5.5, 5.1, and 4.0 percent under value-added allocation, value-added PIM, capital-expenditure PIM, and investment-share allocation, respectively. The positive covariance contributions indicate that states with higher industry-composition components also tend to have higher within-industry geographic components. For each method, the three reported shares sum to 100 percent, apart from rounding.

Value-added allocation ties assigned capital directly to a state's share of industry value added, which can reduce the variation left in the productivity residual. Capital measures based on accumulated investment or current investment shares need not imply the same relationship. The larger geographic variance shares under these methods are consistent with this difference in construction, although they do not establish which method more accurately measures geographic productivity differences.

5.2.1 State-ranking sensitivity

To summarize the geographic consequences, we rank the 51 jurisdictions by their aggregate productivity index in each year, with rank 1 assigned to the highest value. In the following statistic, $Rank_s^m$ is the average of these annual ranks over the sample. It is not the rank of the state's time-averaged productivity.

$$\Delta Rank_s = \max_{m,m'} \left| Rank_s^m - Rank_s^{m'} \right|. \tag{17}$$

Figure 3 displays the 15 jurisdictions with the largest ranges. Virginia has the largest change: its average annual rank is 25.1 under value-added PIM and 16.6 under investment-share allocation, a range of 8.5 places. New Mexico ranges from 21.0 under value-added allocation to 28.5 under capital-expenditure PIM. Idaho ranges from 24.2 under value-added PIM to 31.2 under investment-share allocation. The direction of the ranking change is

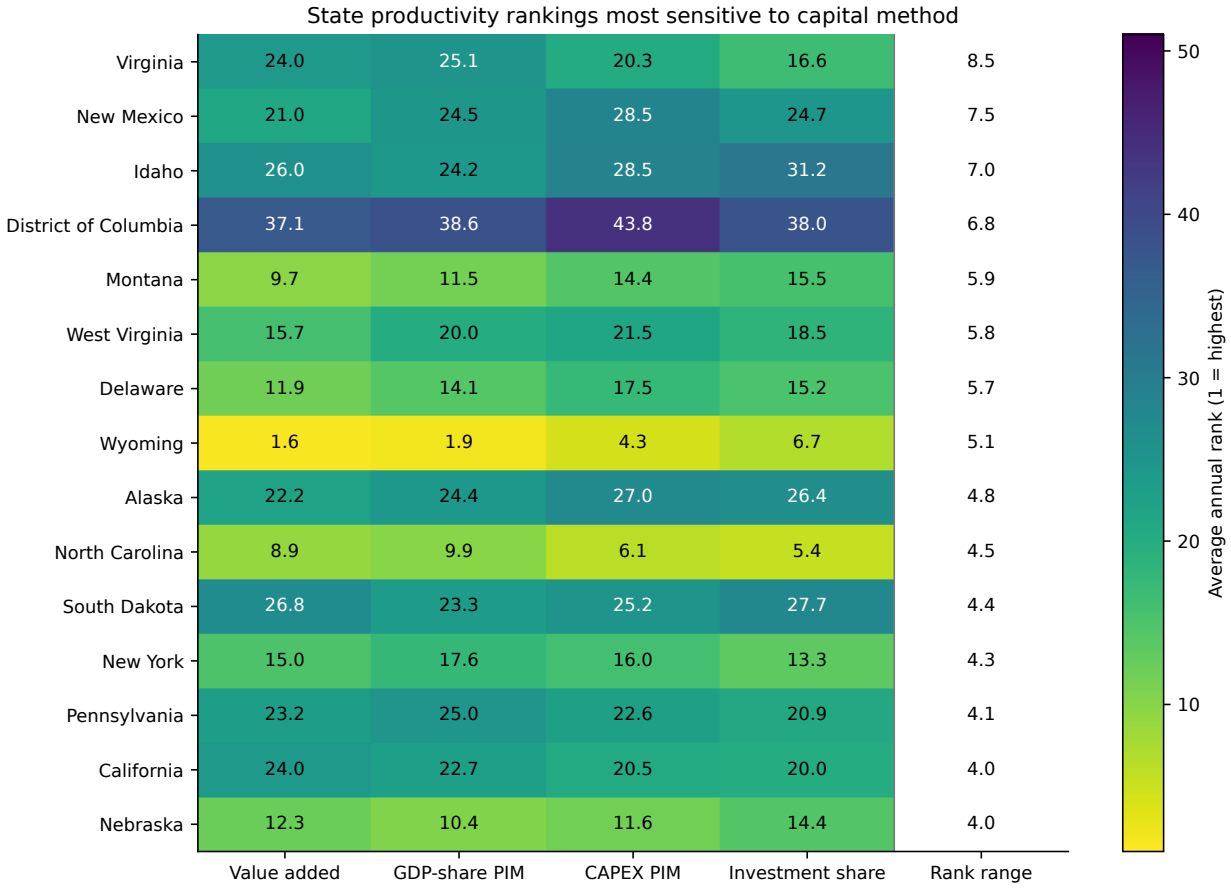


Figure 3: Sensitivity of Average State Productivity Rankings to the Capital-Stock Method

Notes: For each method and year, the 51 states are ranked by value-added-weighted aggregate state productivity, with rank 1 denoting the highest measured productivity. Entries report each state's average annual rank over 1998–2021. Rank range is the maximum minus the minimum average rank across the four methods. The figure shows the 15 states with the largest ranges. No observations are sampled. Productivity is an accounting residual, not a technology shock.

therefore not uniform across states. A capital method that raises one state's measured relative productivity can lower another's.

These aggregate rankings answer a different question from the within-industry-year comparisons in Table 3. Aggregation combines the industry-specific residuals using each state's value-added shares, so within-industry changes can either offset or reinforce one another in the state index.

5.3 Aggregate-state productivity growth decomposition

Similar productivity levels need not imply similar productivity dynamics. As shown in Section 3, the capital measures agree more closely on levels than on annual changes. The productivity residual attributes output changes not accounted for by measured capital and labor to productivity growth. Differences in capital adjustment can therefore alter the amount of growth attributed to productivity even when the underlying output and employment series are identical across methods. There is also an accounting distinction between the two decompositions. The level exercise compares states in each year and averages the resulting variance shares. The growth exercise pools annual state observations and asks how the variation in a lagged-share productivity-growth index divides between national-industry growth and state-specific excess growth.

As in Section 5, the industry productivity terms are log residuals. The left-hand side below denotes the aggregate growth index $\sum_j \theta_{sj,t-1}\Delta a^m_{sjt}$, constructed with lagged value-added shares. With changing industry shares, this index need not equal the first difference of the contemporaneously weighted state productivity index in (15). The use of lagged weights isolates productivity changes at the preceding year's industry composition.

Only observations with valid lagged shares, industry productivity changes, and national benchmark changes enter the sum. As in the level decomposition, the original manufacturing shares are retained rather than rescaled across the industries with valid changes.

$$\Delta a^m_{st} = \sum_j \theta_{sj,t-1}\Delta \bar{a}^m_{jt} + \sum_j \theta_{sj,t-1}\left(\Delta a^m_{sjt} - \Delta \bar{a}^m_{jt}\right). \tag{18}$$

The two components use the same valid observations and weights. The first assigns each industry its national benchmark growth; the second captures the state's excess growth relative to that benchmark. Their sum is the aggregate growth index by construction. Table 6 reports the variance decomposition across 1,173 state-year observations, covering the 23 annual changes from 1999 through 2021 for 51 jurisdictions.

Table 6: Aggregate-State Productivity Decomposition, Panel B: Growth

Method	Aggregate-growth variance	National-industry share	State-specific share	Covariance share
Value added	0.003	0.640	0.424	-0.065
Value-added PIM	0.007	0.293	0.756	-0.049
CAPEX PIM	0.007	0.282	0.770	-0.052
Investment share	0.010	0.188	0.894	-0.082

Notes: Lagged state-industry shares decompose aggregate state productivity growth into national-industry growth and state-specific excess growth. Variance shares use 1,173 common state-year observations and include the covariance term, so the three shares sum to one. The maximum numerical error in the additive growth identity is below the displayed precision.

Under value-added allocation, the national-industry component accounts for 64.0 percent of aggregate-growth variance. Its share falls to 29.3 percent under value-added PIM and 28.2 percent under capital-expenditure PIM, and to 18.8 percent under investment-share allocation. Thus, the direct value-added allocation gives national industry movements a substantially larger role in the variation of state productivity growth than do the other three measures.

The close agreement between the two PIM shares is consistent with their high productivity-growth correlation in Table 3. Their different investment allocators matter for some comparisons, but they imply a similar broad division of growth variation in this decomposition.

The state-specific component accounts for 42.4 percent of growth variance under value-added allocation, compared with 75.6 percent under value-added PIM, 77.0 percent under capital-expenditure PIM, and 89.4 percent under investment-share allocation. The range is 47.0 percentage points. These shares concern variation in growth around national industry movements; they are not shares of the average growth rate and do not measure the fraction of growth caused by state-specific technology shocks.

The covariance contribution is negative under every method, with shares of -0.065, -0.049, -0.052, and -0.082, respectively. National-industry and state-specific components thus partly offset each other in the pooled growth observations. The two separate variance shares can sum to more than one because twice their covariance is negative. Retaining this term reconciles the component shares with total growth variance.

5.3.1 Sensitivity of levels versus growth

The comparison of Table 6 with Table 5 shows pronounced differences between level versus growth decomposition. Industry composition accounts for roughly nine-tenths of mean annual level variance under every method, while the growth decomposition assigns a much wider range of importance to the state-specific component. The smaller geographic share in levels therefore does not establish a small geographic role in annual productivity dynamics.

Total aggregate-growth variance also rises across the measures, from approximately 0.003 under value-added allocation to 0.007 under each PIM method and 0.010 under investment-share allocation. Capital measurement changes both the amount of variation attributed to productivity growth and its division between the two components. Table A2 reports the underlying component variances for both exercises. Since the level and growth shares use different variance comparisons, their contrast should be interpreted as sensitivity of two distinct economic objects, rather than a decomposition of one common variance.

6 Productivity Dispersion and Convergence

Convergence provides a further application of the measurement comparison. The question is whether the same observed output and employment histories imply a similar relationship between initial productivity and subsequent growth when capital is measured differently. We distinguish beta convergence, in which initially less productive observations grow faster, from sigma convergence, in which productivity dispersion declines. The two need not coincide: a negative relationship between initial productivity and growth does not rule out other changes that widen the distribution.

6.1 Beta convergence

The industry-specific formulation below relates annualized productivity growth to its initial log level. Here A again denotes productivity before taking logarithms, as in (9); the regressor and the growth calculation use its log residual.

For industry j, method m, and horizon τ,

$$g_{sj}^{A,m} = a_j^m + \beta_j^m \ln A_{sj,t_0}^m + \varepsilon_{sj}^m, \tag{19}$$

where

$$g_{sj}^{A,m} = \frac{\ln A_{sj,t_T}^m - \ln A_{sj,t_0}^m}{t_T - t_0}. \tag{20}$$

The main estimates in the left panel of Table 7 pool industries and impose a common slope within each method and horizon, while allowing industry-specific intercepts. Thus, the reported coefficient is β_τ^m from the pooled industry-fixed-effect counterpart of (19), rather than an average of separately estimated industry slopes. Standard errors are HC1 heteroskedasticity-robust.

Every reported pooled beta coefficient is negative and statistically significant at the 1 percent level. At five years, the estimates range from -0.093 under capital-expenditure PIM to -0.111 under investment-share allocation. At ten years, they range from -0.066 under capital-expenditure PIM to -0.074 under value-added PIM. Over the full 23-year interval, the estimates are -0.029, -0.035, -0.032, and -0.033 for Methods 1–4, respectively. Conditional on industry, initially lower measured productivity is therefore associated with faster subsequent growth under each construction.

6.2 Sigma convergence

The sigma comparison instead follows dispersion across states within an industry, defined as

$$\sigma_{jt}^{A,m} = SD_s\left(\ln A_{sjt}^m\right). \tag{21}$$

For each industry and eligible starting year, we compare the initial and terminal standard deviations on the observations available at both ends of the horizon, divide their difference by the horizon length, and average across these comparisons. A negative entry in the right panel Table 7 indicates declining dispersion. Reported significance for this mean change is based on a test of whether the industry–starting-year changes average to zero.

The sigma results are less uniform than the beta results. Under value-added allocation, dispersion declines at all three horizons, with mean annual changes of -0.0009, -0.0011, and -0.0012. The five-year change is significant at the 10 percent level; the ten- and 23-year changes are significant at the 1 and 5 percent levels, respectively. Capital-expenditure PIM instead implies increasing dispersion at each horizon, with changes of 0.0029, 0.0022, and 0.0036, all significant at the 1 percent level. Value-added PIM also has positive changes, although only the 23-year estimate, 0.0017, is significant at the 5 percent level. None of the investment-share sigma estimates is statistically distinguishable from zero at the reported significance levels.

Table 7: Convergence Sensitivity Across Capital-Stock Methods

	Beta convergence: $\widehat{\beta}$			**Sigma convergence: annual $\Delta\sigma$**		
Method	**5-year**	**10-year**	**23-year**	**5-year**	**10-year**	**23-year**
Value added	-0.097*** (0.001)	-0.069*** (0.001)	-0.029*** (0.002)	-0.0009*	-0.0011***	-0.0012**
Value-added PIM	-0.106*** (0.002)	-0.074*** (0.001)	-0.035*** (0.003)	0.0011	0.0004	0.0017**
CAPEX PIM	-0.093*** (0.002)	-0.066*** (0.001)	-0.032*** (0.003)	0.0029***	0.0022***	0.0036***
Investment share	-0.111*** (0.002)	-0.072*** (0.001)	-0.033*** (0.002)	-0.0002	0.0003	0.0006

Notes: Beta entries report coefficients from annualized future productivity growth on initial log productivity with industry fixed effects; HC1 robust standard errors appear in parentheses. Sigma entries report the mean annual change in within-industry state dispersion. The beta samples contain 16,659, 12,216, and 864 observations at the 5-, 10-, and 23-year horizons, respectively; every sample covers 51 states. The cross-method beta ranges are 0.018, 0.008, and 0.006. Statistical significance: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

7 Economic Relevance of Measured Geographic Productivity

We examine whether measured geographic productivity is associated with subsequent growth in employment, real compensation, output, and investment, and whether these relationships depend on the capital measure. Output is measured by value added, and real compensation is total compensation adjusted by the regional price measure. Each outcome is measured as an annualized log change over three years.

The analysis considers two levels of aggregation. The state–industry specification relates subsequent growth to productivity relative to the national industry benchmark. The aggregate-state specification relates growth in state manufacturing totals to the within-industry geographic component of state productivity. The specifications use different controls and productivity scales, so their coefficients are not directly comparable.

7.1 State–industry specification

The explanatory variable is the state–industry productivity gap relative to its employment-weighted national industry-year benchmark. For each capital method, the gap is standardized within the outcome's estimation sample to have mean zero and standard deviation one.

Denoting this standardized gap by $\widetilde{u}^{m,*}_{sjt}$, we estimate

$$\frac{\ln Z_{sj,t+3} - \ln Z_{sjt}}{3} = \gamma^m \widetilde{u}^{m,*}_{sjt} + \rho \ln Z_{sjt} + \mu_{sj} + \lambda_t + \varepsilon^m_{sjt}, \tag{22}$$

where Z is employment, real compensation, output, or investment. The coefficient γ^m measures the association between a one-standard-deviation increase in the productivity gap and subsequent annualized growth.

The regressions control for the initial outcome level, state–industry fixed effects, and year fixed effects. Standard errors are clustered by state. For each outcome, the estimation sample is held constant across capital methods and requires an observation exactly three years after the starting date. Samples range from 17,577 to 18,588 observations across 51 jurisdictions. Table 8 reports the estimated productivity coefficients.

The estimates describe conditional associations. Capital allocation and measured productivity may both reflect economic conditions, so the coefficients do not identify causal effects. The exercise also does not evaluate out-of-sample forecasting performance.

Table 8: Economic Relevance of Measured Geographic Productivity, Panel A: State–Industry Specification

Three-year outcome	**Value added**	**Value-added PIM**	**CAPEX PIM**	**Investment share**
Output growth	-0.024***	-0.022***	-0.025***	-0.016***
	(0.003)	(0.003)	(0.004)	(0.002)
Employment growth	0.014***	0.010***	0.010***	0.006***
	(0.003)	(0.003)	(0.003)	(0.002)
Real compensation growth	0.008**	0.006**	0.005*	0.003
	(0.003)	(0.003)	(0.003)	(0.002)
Investment growth	0.018***	0.021***	0.030***	0.029***
	(0.004)	(0.003)	(0.003)	(0.003)

Notes: Each cell reports the coefficient on a one-standard-deviation increase in the method-specific state–industry productivity gap; state-clustered standard errors appear in parentheses. Outcomes are annualized three-year log changes at the state–industry level. All models control for the log initial outcome and include state–industry and year fixed effects. Samples are held constant across methods within each outcome and contain 17,577–18,588 observations across 51 states. Estimates are predictive associations, not causal effects. Statistical significance: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

7.2 Aggregate-state specification

The aggregate-state specification uses the within-industry geographic component, G^m_{st}, defined in Section 5. This component enters in its original log-index units, without standardization. For each outcome, growth is calculated from state manufacturing totals:

$$\frac{\ln Z_{s,t+3} - \ln Z_{st}}{3} = \beta^m G^m_{st} + \lambda_t + \varepsilon^m_{st}. \tag{23}$$

The specification includes year fixed effects but no state fixed effects or initial-outcome control. It uses 1,071 state-year observations, with standard errors clustered by state. Table 8 reports the coefficients, and Table A4 provides confidence intervals and additional regression statistics. Differences in controls and productivity scaling mean that larger coefficients here do not necessarily indicate stronger relationships than those in the state–industry specification.

Table 8: Economic Relevance of Measured Geographic Productivity, Panel B: Aggregate-State Specification

Three-year outcome	**Value added**	**Value-added PIM**	**CAPEX PIM**	**Investment share**
Output growth	-0.090***	-0.077***	-0.060***	-0.054***
	(0.021)	(0.016)	(0.014)	(0.012)
Employment growth	-0.004	0.015	0.016	0.002
	(0.011)	(0.012)	(0.014)	(0.009)
Real compensation growth	0.006	0.025**	0.021*	0.003
	(0.013)	(0.011)	(0.012)	(0.008)
Investment growth	-0.046	-0.032*	0.008	0.061***
	(0.031)	(0.019)	(0.017)	(0.019)

Notes: Each cell reports the coefficient on the method-specific state-year within-industry geographic-productivity component; state-clustered standard errors appear in parentheses. Outcomes are annualized three-year changes in log state totals. Models include year fixed effects, contain no additional controls, and use 1,071 observations. Estimates are predictive associations, not causal effects. Statistical significance: * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$.

7.3 Regression Results

7.3.1 Employment growth

At the state–industry level, higher measured geographic productivity is associated with faster subsequent employment growth under all four capital methods. The coefficients are 0.014 under value-added allocation, 0.010 under each PIM method, and 0.006 under investment-share allocation, all significant at the 1 percent level. A one-standard-deviation increase in the productivity gap is thus associated with approximately 0.6–1.4 percentage points higher annual employment growth over the following three years.

At the aggregate-state level, employment coefficients range from -0.004 to 0.016, and none is statistically significant at conventional levels. The positive association within state–industry pairs therefore does not extend to a precisely estimated relationship for total state manufacturing employment.

7.3.2 Real compensation growth

The state–industry coefficients for real compensation growth are positive under every method, although their magnitude and precision vary. The estimates are 0.008 under value-added allocation and 0.006 under value-added PIM, both significant at the 5 percent level. The capital-expenditure PIM estimate is 0.005 and significant at the 10 percent level. The investment-share estimate is 0.003 and is not statistically significant.

At the aggregate-state level, the value-added PIM and capital-expenditure PIM coefficients are 0.025 and 0.021, significant at the 5 and 10 percent levels, respectively. The value-added allocation and investment-share allocation coefficients are not statistically significant. Because real compensation is measured as a total, these associations reflect changes in both employment and compensation per worker.

7.3.3 Output and investment growth

Higher measured geographic productivity is associated with lower subsequent output growth at the state–industry level. The coefficients are -0.024 under value-added allocation, -0.022 under value-added PIM, -0.025 under capital-expenditure PIM, and -0.016 under investment-share allocation. All are significant at the 1 percent level.

These negative associations may reflect subsequent adjustment following relatively high measured productivity. Their interpretation also requires attention to the accounting relationship: current output enters both the productivity residual and the starting value of subsequent output growth. Controlling for initial output does not make these estimates evidence that higher productivity causes output to decline.

Investment-growth coefficients are positive under all four methods: 0.018 under value-added allocation, 0.021 under value-added PIM, 0.030 under capital-expenditure PIM, and 0.029 under investment-share allocation. All are significant at the 1 percent level. Together with the employment results, these estimates suggest that higher measured productivity gaps are associated with subsequent input expansion, even as output growth is lower. The regressions do not identify the mechanism underlying these relationships. Investment also supplies allocation information for the investment-share capital measure, linking this outcome to that method's construction.

At the aggregate-state level, output coefficients remain negative and significant under every method, ranging from -0.090 to -0.054. Investment results vary more substantially. The coefficient is -0.046 and insignificant under value-added allocation, -0.032 and significant at the 10 percent level under value-added PIM, 0.008 and insignificant under capital-expenditure PIM, and 0.061 and significant at the 1 percent level under investment-share allocation. Thus, the consistently positive investment association in the state–industry specification does not persist in the aggregate-state specification, which uses fewer controls.

7.3.4 Stability across capital methods

The main state–industry results have a stable pattern of signs: geographic productivity is negatively related to subsequent output growth and positively related to employment, investment, and real compensation growth. The first three associations are significant under every method; the precision of the compensation association depends on the capital measure. The magnitudes also vary, even after standardizing the geographic gap. We interpret that variation as sensitivity of the reported associations, rather than as a formal test that the coefficients differ across methods.

The aggregate-state results are less uniform for employment and investment. Taken with the decomposition results, this distinction shows why the economic interpretation should specify both the capital measure and the level of aggregation. The evidence supports several common conditional relationships within state–industry pairs, while the strength and, for aggregate investment, the direction of an association can depend on how geographic productivity is measured.

8 Discussion

8.1 Capital measurement and regional inference

The absence of directly observed state–industry capital stocks limits the conclusions that can be drawn from national accounting consistency and comparisons with regional economic activity. National benchmarks constrain aggregate stocks but leave their geographic distribution undetermined. Comparing four capital constructions within a common analytical framework identifies which regional conclusions persist across allocation choices and which depend on them. It does not establish which construction most closely approximates the underlying capital stock.

The validation exercises should be interpreted accordingly. Output and investment help determine the allocations, so their association with estimated capital partly reflects construction. These relationships demonstrate correspondence with observed economic activity but provide limited independent evidence for selecting a preferred measure. Moreover, the four methods may share errors in locating capital. Their range characterizes sensitivity to the allocation signals and accumulation assumptions considered here; it is not a confidence interval for capital or productivity.

8.2 Robust findings and measurement sensitivity

Several conclusions persist across methods. Capital levels are highly correlated, and industry composition accounts for 89.1–93.2 percent of the mean annual cross-state variance in the manufacturing productivity index (Table 5). The pooled industry-conditional beta-convergence coefficient is also negative and statistically significant at every reported horizon under all four methods. In the state–industry regressions, the positive associations of measured geographic productivity with subsequent employment and investment growth, and its negative association with output growth, survive the choice of capital measure. These findings establish robustness within the reported samples and specifications.

Annual dynamics and relative geographic performance are more sensitive. Capital-growth correlations are weaker than capital-level correlations, and the share of productivity-growth variance attributed to the state-specific component ranges from 42.4 to 89.4 percent. Agreement about broad capital levels therefore provides limited assurance about the sources of annual productivity growth.

The same distinction applies to productivity rankings. Between 22.3 and 44.0 percent of observations change within-industry productivity quartile across method pairs. The median cross-method range in the geographic productivity gap is 0.135 log points, rising to 0.433 at

the 90th percentile (Table A6). Thus, high correlations in productivity levels coexist with substantial disagreement about the relative performance of individual locations.

Sensitivity also depends on the measure being compared. The District of Columbia and Alaska exhibit large proportional differences in capital without equally large differences in capital rankings (Table A5). Virginia, New Mexico, and Idaho are among the states with the most sensitive aggregate productivity rankings (Figure 3). Proportional capital differences and aggregate productivity rankings summarize different features of the data: the latter also incorporate industry shares and the productivity accounting weights.

Measurement sensitivity extends beyond annual fluctuations. Over the full sample period, productivity dispersion declines under value-added allocation and increases under the PIM methods; the change under investment-share allocation is statistically inconclusive. The direction of sigma convergence is therefore less robust than the negative pooled beta-convergence relationship. Aggregation matters as well: the aggregate-state investment-growth coefficient changes sign across capital methods, although the state–industry coefficient is positive under every method.

8.3 Implications for empirical applications

Regional growth accounting and convergence analyses depend on how observed output is attributed to capital and productivity. An allocation rule can alter that attribution while leaving output and employment unchanged. Capital construction should therefore be treated as a substantive measurement choice and evaluated within the economic application of interest.

The appropriate comparison depends on the conclusion being assessed. For productivity rankings and geographic gaps, reporting the distribution of changes across observations is more informative than an overall correlation. For convergence, both the initial-productivity coefficient and the change in dispersion should be examined across capital measures. For subsequent economic growth, comparisons should hold the sample and regression specification fixed. Coefficients from the state–industry and aggregate-state models are not directly comparable because their controls and productivity scales differ. Cross-method differences in point estimates also do not constitute a formal test of coefficient equality.

Coverage and admissibility require similar attention. Under investment-share allocation, a nonpositive category-specific allocator produces zero capital for that category; this is a consequence of the allocation rule and need not indicate an absence of productive assets. Total capital is positive under all four methods in the analysis panel. Logarithmic comparisons require positive, finite inputs, and their coverage should be reported explicitly. The

common 18-industry sample excludes the remaining transportation-equipment group because employment is unavailable, rather than because capital is zero.

8.4 Limitations and further research

All four constructions face common limitations in assigning the assets of multistate firms, mobile equipment, and intellectual property to locations. They also do not identify state-specific depreciation or utilization. These factors can enter the productivity residual alongside unmeasured hours, labor quality, markups, and departures from the assumed production relationship.

Aggregate productivity comparisons additionally depend on price measures, industry weights, and coverage. The reported indices retain original value-added shares when industries with invalid productivity observations are omitted. Differences in available coverage can therefore affect state comparisons. The findings apply to manufacturing over 1998–2021; their relevance to other sectors and periods remains an empirical question.

The regression evidence is descriptive. Measured productivity shares accounting components with some outcomes, and the estimates do not identify exogenous changes in productivity. In addition, the shorter convergence horizons contain overlapping intervals, while the reported HC1 standard errors do not account for the resulting dependence. Statistical significance in those regressions should be interpreted with this limitation in view.

Stronger validation requires independent information on the geographic distribution of productive assets. Establishment-level asset records, additional investment data, and improved information on the location and use of intangible assets would help distinguish competing allocations. For the normalized steady-growth construction, depreciation and growth parameters common across states cancel during national reconciliation. Refining those common parameters alone therefore cannot resolve geographic allocation uncertainty; progress requires better evidence on the allocation itself.

9 Conclusion

The absence of an observed state–industry capital stock leaves regional economic analysis with a consequential measurement choice. National capital totals can be assigned across states using different geographic signals and accumulation rules, each of which changes the capital input against which output is evaluated. This paper examines that choice for U.S. manufacturing over 1998–2021 by comparing four constructions: value-added allocation, value-added PIM, capital-expenditure PIM, and investment-share allocation.

The contribution is threefold. We set out the construction and national accounting implications of the four measures, including the investment-share reduced form of the benchmarked steady-growth method. We evaluate their agreement and relationships with observable economic activity without treating any one measure as observed regional capital. We then trace the differences through productivity accounting, geographic decompositions, convergence, and subsequent economic outcomes. This sequence addresses the question raised in the introduction: how much regional inference depends on the construction used to recover the geographic capital stock.

The answer depends on the object of interest. Broad capital and productivity levels are closely related across methods, industry composition accounts for most mean annual cross-state productivity variance, and the pooled beta-convergence relationship is negative under every construction and horizon considered. Yet these common findings coexist with substantial changes in within-industry productivity classifications, geographic gaps, and the interpretation of growth. The state-specific share of productivity-growth variance ranges from 42.4 to 89.4 percent, and the methods do not agree on whether cross-state productivity dispersion declines over the full period.

The economic-relevance comparisons reinforce this qualified conclusion. The main state–industry specification yields common signs for the associations of measured geographic productivity with subsequent output, employment, and investment growth. Their magnitudes vary, compensation results differ in precision, and the aggregate-state investment association is sensitive to the capital measure. Robustness to capital construction is therefore a property of a particular result and specification, rather than a general property of a preferred dataset.

For researchers using regional capital estimates, the practical implication is to carry the measurement comparison through to the economic conclusion. National accounting consistency should be reported alongside geographic disagreement, and a stable overall correlation should be supplemented by comparisons of the rankings, gaps, or growth relationships on which the application relies. The range across these four methods provides evidence about sensitivity to their assumptions; it does not identify the true regional stock or bound all possible measurement error.

Independent information on where assets are held and used would allow a stronger assessment of the alternative allocations. Until such evidence is available, transparent construction and explicit sensitivity analysis can make regional capital estimates more useful for the productivity and growth questions that motivate them. The results here show both the economic conclusions that survive this comparison and the geographic inferences for which the capital measure remains consequential.

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A Additional Productivity-Sensitivity Results

A.1 Alternative Capital-Share Specifications

A.1.1 Fixed capital share

$$\alpha = 0.33. \tag{24}$$

The fixed-share case applies the same input elasticities to every observation. It provides a common comparison in which differences across capital methods enter the residual with an identical weight.

A.1.2 National industry capital shares

The industry-specific alternative takes the ratio of gross operating surplus to value added, summed across states and years within each industry. This ratio is held constant over time and bounded between 0.05 and 0.95. It allows persistent differences in capital income shares across industries without allowing annual share movements to change the accounting weights.

A.1.3 National industry-year capital shares

Our preferred specification calculates the share separately for each industry and year:

$$\alpha_{jt} = \min\left\{0.95, \max\left\{0.05, \frac{\sum_s \text{GOS}_{sjt}}{\sum_s Y_{sjt}}\right\}\right\}. \tag{25}$$

Aggregating the numerator and denominator before taking their ratio gives a common national industry-year share. This differs from averaging the state-specific ratios and avoids making a state's own income share an additional source of geographic variation in its accounting weight.

A.1.4 Constrained and unconstrained specifications

Table A1 reports the fixed share, the constrained national industry share, the preferred constrained national industry-year share, and the unconstrained industry-year share. The latter uses the gross-operating-surplus ratio without bounding it. The constrained and unconstrained industry-year results are very similar: their level correlations, median absolute differences, and quartile-switching shares coincide to the displayed precision. The largest displayed change in a growth correlation is 0.005. The principal cross-method differences therefore persist when the bounds are relaxed.

The following tables provide additional detail for the main productivity comparisons. Table A1 varies the capital share while holding the four capital constructions fixed. In addition to correlations and quartile changes, it reports the states with the largest average absolute changes in within-industry-year productivity rank. Those rank changes differ from the aggregate-state ranks in Figure 3.

Table A1: Productivity Sensitivity Across Capital-Share Specifications

Method A	Method B	Level corr.	Growth corr.	Rank corr.	Mean abs. difference	Median abs. difference	Share changing quartile	Largest avg. rank change	State	N
Panel A: Fixed capital share ($\alpha = 0.33$)										
Value added	Value-added PIM	0.984	0.972	0.938	0.062	0.039	0.233	5.592	South Dakota	21,240
Value added	CAPEX PIM	0.967	0.963	0.909	0.087	0.054	0.271	10.582	District of Columbia	21,240
Value added	Investment share	0.944	0.692	0.814	0.125	0.086	0.391	7.507	Alaska	21,240
Value-added PIM	CAPEX PIM	0.984	0.992	0.946	0.056	0.029	0.197	7.150	District of Columbia	21,240
Value-added PIM	Investment share	0.937	0.695	0.782	0.134	0.094	0.416	8.674	Alaska	21,240
CAPEX PIM	Investment share	0.954	0.739	0.832	0.116	0.079	0.363	10.662	District of Columbia	21,240
Panel B: Constrained national industry capital share										
Value added	Value-added PIM	0.978	0.908	0.916	0.072	0.042	0.264	6.458	South Dakota	21,240
Value added	CAPEX PIM	0.954	0.896	0.884	0.103	0.058	0.298	12.178	District of Columbia	21,240
Value added	Investment share	0.925	0.577	0.778	0.147	0.093	0.421	8.653	Louisiana	21,240
Value-added PIM	CAPEX PIM	0.977	0.989	0.929	0.067	0.031	0.221	8.000	District of Columbia	21,240
Value-added PIM	Investment share	0.921	0.599	0.741	0.156	0.100	0.446	9.521	Alaska	21,240
CAPEX PIM	Investment share	0.943	0.658	0.811	0.133	0.085	0.386	12.291	District of Columbia	21,240
Panel C: Constrained national industry-year capital share										
Value added	Value-added PIM	0.979	0.927	0.917	0.072	0.041	0.262	6.241	South Dakota	21,240
Value added	CAPEX PIM	0.956	0.909	0.885	0.104	0.056	0.296	12.183	District of Columbia	21,240
Value added	Investment share	0.932	0.649	0.780	0.146	0.090	0.417	8.616	Louisiana	21,240
Value-added PIM	CAPEX PIM	0.978	0.988	0.929	0.068	0.030	0.223	7.798	District of Columbia	21,240
Value-added PIM	Investment share	0.928	0.663	0.743	0.154	0.097	0.440	9.418	Alaska	21,240
CAPEX PIM	Investment share	0.949	0.715	0.814	0.132	0.082	0.381	12.272	District of Columbia	21,240
Panel D: Unconstrained national industry-year capital share										
Value added	Value-added PIM	0.979	0.928	0.917	0.072	0.041	0.262	6.241	South Dakota	21,240
Value added	CAPEX PIM	0.956	0.909	0.885	0.103	0.056	0.296	12.183	District of Columbia	21,240
Value added	Investment share	0.932	0.654	0.780	0.146	0.090	0.417	8.616	Louisiana	21,240
Value-added PIM	CAPEX PIM	0.978	0.988	0.929	0.068	0.030	0.223	7.798	District of Columbia	21,240
Value-added PIM	Investment share	0.928	0.668	0.743	0.154	0.097	0.440	9.418	Alaska	21,240
CAPEX PIM	Investment share	0.949	0.719	0.814	0.132	0.082	0.381	12.272	District of Columbia	21,240

Notes: The table reports all productivity-agreement measures across the four capital-share specifications. Mean and media differences are in log points. The state column identifies the state associated with the largest average absolute change in industry-year productivity rank. Constrained shares are clipped to the interval $[0.05, 0.95]$; the unconstrained specification us national industry-year gross-operating-surplus share. Productivity is an accounting residual, not a technology shock.

Table A2 reports the component variances underlying the level and growth shares in Section 5 and Section 5.3. Panel A gives the pooled three-way state–industry decomposition. Panel B averages annual cross-state level decompositions, and Panel C uses the pooled annual state growth observations. The covariance contribution in Panel A is twice the sum of the three pairwise component covariances; Panels B and C each contain two components and one corresponding covariance term.

Table A3 supplies the standard errors, significance levels, and sample sizes for the horizon comparisons in Section 6. The sigma results summarize annualized changes in dispersion across the eligible industry–starting-year comparisons.

Table A2: Detailed Productivity-Decomposition Results

Panel A: State–industry productivity levels

Method	Total variance	Aggregate-time variance	National-industry variance	State-within variance	Covariance term	Aggregate-time share	National-industry share	State-within share	Covariance share
Value added	0.3644	0.0303	0.2910	0.0522	-0.0091	0.083	0.799	0.143	-0.025
Value-added PIM	0.3787	0.0301	0.2854	0.0811	-0.0178	0.079	0.753	0.214	-0.047
CAPEX PIM	0.3830	0.0256	0.2833	0.1103	-0.0363	0.067	0.740	0.288	-0.095
Investment share	0.4164	0.0308	0.2917	0.1299	-0.0360	0.074	0.701	0.312	-0.086

Panel B: Aggregate-state productivity levels

Method	Total variance	Composition variance	Geographic variance	Covariance term	Composition share	Geographic share	Covariance share
Value added	0.4153	0.3856	0.0094	0.0203	0.932	0.024	0.044
Value-added PIM	0.4252	0.3835	0.0161	0.0256	0.905	0.040	0.055
CAPEX PIM	0.4310	0.3826	0.0233	0.0251	0.891	0.058	0.051
Investment share	0.4372	0.3868	0.0277	0.0227	0.892	0.068	0.040

Panel C: Aggregate-state productivity growth

Method	Total variance	National variance	State-specific variance	Covariance term	National share	State-specific share	Covariance share
Value added	0.0030	0.0019	0.0013	-0.0002	0.640	0.424	-0.065
Value-added PIM	0.0066	0.0019	0.0050	-0.0003	0.293	0.756	-0.049
CAPEX PIM	0.0069	0.0019	0.0053	-0.0004	0.282	0.770	-0.052
Investment share	0.0103	0.0019	0.0092	-0.0008	0.188	0.894	-0.082

Notes: Panel A reports the pooled state-industry variance decomposition. Panel B reports means across annual aggregate-state level decompositions. Panel C reports the pooled aggregate-state growth decomposition. In Panel A, the covariance term is twice the sum of all three pairwise component covariances; in Panels B and C, it is twice the covariance between the two components. Shares include the covariance term and sum to one, subject only to rounding.

Table A3: Detailed Convergence Results Across Capital-Stock Methods

Horizon	Method	$\widehat{\beta}$	Robust SE	Beta p-value	Initial σ	Terminal σ	Annual $\Delta\sigma$	$\Delta\sigma$ p-value	Observations
5	Value added	-0.097	0.001	< 0.001	0.211	0.207	-0.0009	0.072	16,659
5	Value-added PIM	-0.106	0.002	< 0.001	0.261	0.266	0.0011	0.149	16,659
5	CAPEX PIM	-0.093	0.002	< 0.001	0.293	0.307	0.0029	0.001	16,659
5	Investment share	-0.111	0.002	< 0.001	0.323	0.322	-0.0002	0.857	16,659
10	Value added	-0.069	0.001	< 0.001	0.214	0.203	-0.0011	0.000	12,216
10	Value-added PIM	-0.074	0.001	< 0.001	0.260	0.264	0.0004	0.363	12,216
10	CAPEX PIM	-0.066	0.001	< 0.001	0.286	0.308	0.0022	0.000	12,216
10	Investment share	-0.072	0.001	< 0.001	0.320	0.323	0.0003	0.650	12,216
23	Value added	-0.029	0.002	< 0.001	0.218	0.191	-0.0012	0.012	864
23	Value-added PIM	-0.035	0.003	< 0.001	0.218	0.257	0.0017	0.024	864
23	CAPEX PIM	-0.032	0.003	< 0.001	0.218	0.301	0.0036	0.001	864
23	Investment share	-0.033	0.002	< 0.001	0.313	0.328	0.0006	0.424	864

Notes: Beta models use annualized future productivity growth, industry fixed effects, and HC1 robust standard errors. Sigma statistics average industry-start-year cells on the same horizon. All beta estimates indicate statistically significant conditional convergence. Each horizon covers 51 states.

Table A4 provides additional detail for the aggregate-state regressions in Section 7, including confidence intervals, goodness of fit, and sample sizes. These models use the unstandardized state-year geographic component, year fixed effects, and 1,071 observations. Their coefficients therefore have a different scale and conditioning interpretation from the standardized state–industry estimates in Panel A of Table 8.

Table A4: Aggregate State-Level Economic-Relevance Regressions

Outcome	**Method**	**Coefficient**	**SE**	p**-value**	**95% CI**	R^2	**Observations**
Output growth	Value added	-0.090	0.021	< 0.001	[-0.132, -0.049]	0.225	1,071
Output growth	Value-added PIM	-0.077	0.016	< 0.001	[-0.109, -0.045]	0.235	1,071
Output growth	CAPEX PIM	-0.060	0.014	< 0.001	[-0.088, -0.032]	0.228	1,071
Output growth	Investment share	-0.054	0.012	< 0.001	[-0.077, -0.030]	0.227	1,071
Employment growth	Value added	-0.004	0.011	0.743	[-0.025, 0.018]	0.549	1,071
Employment growth	Value-added PIM	0.015	0.012	0.199	[-0.008, 0.039]	0.553	1,071
Employment growth	CAPEX PIM	0.016	0.014	0.262	[-0.012, 0.044]	0.554	1,071
Employment growth	Investment share	0.002	0.009	0.850	[-0.015, 0.019]	0.549	1,071
Real compensation growth	Value added	0.006	0.013	0.647	[-0.020, 0.032]	0.335	1,071
Real compensation growth	Value-added PIM	0.025	0.011	0.027	[0.003, 0.048]	0.341	1,071
Real compensation growth	CAPEX PIM	0.021	0.012	0.086	[-0.003, 0.045]	0.341	1,071
Real compensation growth	Investment share	0.003	0.008	0.709	[-0.013, 0.020]	0.335	1,071
Investment growth	Value added	-0.046	0.031	0.133	[-0.107, 0.014]	0.189	1,071
Investment growth	Value-added PIM	-0.032	0.019	0.099	[-0.070, 0.006]	0.189	1,071
Investment growth	CAPEX PIM	0.008	0.017	0.654	[-0.025, 0.040]	0.187	1,071
Investment growth	Investment share	0.061	0.019	0.002	[0.022, 0.099]	0.198	1,071

Notes: This appendix reports the aggregate-state robustness specification that previously appeared in the main table. Outcomes are annualized three-year changes in log state totals. Models include year fixed effects, no additional controls, and state-clustered standard errors. Confidence intervals use the normal 1.96 critical value. Estimates are predictive associations rather than causal effects.

Table A5 identifies the jurisdictions with the largest differences in state–industry capital across methods. Log-capital ranges compare magnitudes within an observation; rank ranges compare the state's position among the common observations within an industry-year. The state summaries weight retained industry-year observations equally, rather than weighting by the size of their capital stocks.

Table A6 reports the distribution of the cross-method geographic-productivity-gap range defined in Section 5. Panel A summarizes all retained observations, and Panel B identifies the jurisdictions with the largest mean ranges. The sample is common across methods, while each method's national benchmark retains the employment-weighted definition used in the main decomposition.

Table A5: States with the largest disagreement in capital stocks

State	N	Mean	Median	90th pct.	Mean rank range
District of Columbia	213	1.223	1.030	2.262	1.82
Alaska	282	1.142	0.974	2.282	2.29
Wyoming	364	1.031	0.960	1.831	2.73
Hawaii	341	1.011	0.728	1.795	2.65
Delaware	417	0.907	0.703	1.807	5.47
New Mexico	432	0.903	0.774	1.709	5.17
Montana	409	0.768	0.637	1.459	3.15
North Dakota	362	0.751	0.665	1.393	4.14
Idaho	418	0.718	0.597	1.305	4.66
South Dakota	402	0.701	0.570	1.239	4.59

Notes: States are ordered by the mean range of log capital across the four methods. Mean, median, and 90th percentile refer to this range, in log points. The final column reports the mean cross-method range of a state's descending capital rank within industry and year, using average ranks for ties. All four ranks use the same states within each industry-year. Each retained state–industry–year observation receives equal weight. Only observations with positive finite output, employment, and capital under all four methods enter; the full sample contains 21,240 observations. The common comparison covers 18 industry groups in 51 jurisdictions. N is the number of retained observations in each state.

Table A6: Sensitivity of geographic productivity gaps to capital measurement

State or sample	N	Mean	Median	90th pct.
Panel A: Distribution across all observations				
All observations	21,240	0.203	0.135	0.433
Panel B: Ten states with the largest mean range				
District of Columbia	213	0.457	0.390	0.911
Alaska	282	0.451	0.309	1.112
Hawaii	341	0.374	0.238	0.756
Wyoming	364	0.367	0.315	0.708
New Mexico	432	0.361	0.268	0.728
Delaware	417	0.333	0.231	0.680
Montana	409	0.286	0.220	0.578
Idaho	418	0.272	0.206	0.576
Maine	432	0.271	0.175	0.630
South Dakota	402	0.261	0.205	0.469

Notes: Entries summarize $\Delta Gap_{sjt} = \max_m Gap_{sjt}^{(m)} - \min_m Gap_{sjt}^{(m)}$, in log points, using the constrained national industry-year capital share. Each method's gap subtracts its employment-weighted industry-year mean log productivity, calculated using that method's valid observations. The range is evaluated on the common sample with positive finite output, employment, and all four capital stocks, and finite productivity under all four methods. Each retained state–industry–year observation receives equal weight. The sample spans 1998–2021, 51 jurisdictions, and 18 industry groups. The overall 95th percentile is 0.612; the maximum is 3.649. N denotes observations. Panel B is ordered by the state mean range.