One Money, Many Debts: Monetary Credibility and Self-Fulfilling Sovereign Risk in a Monetary Union*

Javier Bianchi
Federal Reserve Bank of Minneapolis

Carlos Bolivar
University of Minnesota

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Abstract

We examine self-fulfilling debt crises in a model of a monetary union with nominal rigidities. In the absence of policy commitment, the model features multiple equilibria—one with low interest rates and another with high rates—echoing the insights of Calvo (1988). We show that a credible commitment to an *expansionary* monetary policy can eliminate the high-rate equilibrium and thereby avert self-fulfilling crises. Finally, the risk of confidence crises can give rise to endogenous fragmentation within the union, as interest rate spreads diverge across countries. This asymmetry creates a trade-off for monetary policy, which must balance the differing conditions faced by high- and low-spread member states.

Keywords: Self-fulfilling debt crisis, monetary policy credibility, monetary unions

JEL classification: E52, E58, E62, F33, F34, F41, F45.

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1 Introduction

Conventional wisdom implies that appointing a conservative central banker remedies the inflationary bias of monetary policy. Absent commitment, a central bank has an incentive to stabilize output and tolerate higher inflation; anticipating this, the private sector expects more inflation, and the stabilization gains disappear (Kydland and Prescott, 1977; Barro and Gordon, 1983). Delegating policy to a banker who places a high weight on inflation anchors expectations and improves welfare (Rogoff, 1985).

We show that the opposite prescription holds when governments are exposed to sovereign risk. In a monetary union whose members borrow in a common foreign currency, a government can be pushed into a self-fulfilling debt crisis: when investors expect default, they demand a high interest rate, the heavier repayment burden makes default more attractive, and the pessimistic expectation is validated (Calvo, 1988). A central bank that commits to a more expansionary policy raises the value of repayment and rules out this bad equilibrium. This motivates our central question: how should the central bank of a monetary union set monetary policy to reduce its members' vulnerability to confidence-driven crises?

We answer this question in a two-period model of self-fulfilling debt crises with nominal rigidities. A continuum of ex ante identical countries borrow from foreign lenders and lack commitment to repay. The prices of nontradable goods are sticky, so the nominal exchange rate set by the common central bank determines employment, and the bank trades off the labor wedges of its members. Countries are ex post heterogeneous in the interest rates they face. For intermediate levels of debt, they may end up in either a low- or a high-interest-rate equilibrium, with the particular equilibrium selected by country-specific sunspots, as in Calvo (1988) and Ayres et al. (2018).

The model contains two nested coordination problems. The first is the familiar country-level feedback between borrowing rates and default incentives. The second is a union-level feedback: the cross-country distribution of spreads changes the common central bank's optimal stance, which in turn determines whether that spread distribution can be sustained.

Our first result is that a credible commitment to an expansionary monetary policy eliminates the high-interest-rate equilibrium. By promising to run the economy hot, the central bank lowers the cost of repayment, removes the government's incentive to default, and leaves the risk-free rate as the only equilibrium interest rate. This policy is

time-inconsistent: ex ante the bank wants to ease in order to compress spreads, but ex post, once the interest rate is set, it would prefer not to. The mechanism differs from the familiar one in which inflation erodes the real value of nominal debt. Here commitment works through aggregate-demand stabilization and the labor wedge. Because the commitment policy runs the economy hot, delegating to a more dovish central banker—one that weights employment stabilization more heavily—pushes the discretionary policy toward the commitment allocation, inverting the classic prescription to appoint a conservative banker. When the additional stabilization distortion is small, such a banker attains most of the gains from commitment.

Our second result is that confidence risk gives rise to endogenous fragmentation. When some members face high spreads, the common central bank can no longer close every member's labor wedge with a single exchange rate, and full employment becomes infeasible for the union. This trade-off opens a second, union-wide source of multiplicity. Alongside a safe equilibrium in which all members borrow at the risk-free rate, there is a fragmented equilibrium in which a fraction of ex ante identical countries face high interest rates and a positive probability of default. Fragmentation is self-fulfilling: the heterogeneity it induces is precisely what prevents the central bank from stabilizing the distressed members. A commitment to expansionary policy operates as a selection device, implementing the safe equilibrium for all members.

Finally, we use the framework to evaluate whether a central bank can act as a lender of last resort can rule out fragmentation when it commands enough resources to backstop distressed members, in line with the logic of the ECB's interventions.

Related literature Our paper relates to the literature on self-fulfilling sovereign debt crises. Two mechanisms organize this work. In the first, due to Calvo (1988), pessimistic lenders charge high interest rates that inflate the future repayment burden, raise the incentive to default, and thereby validate those rates; in the second, due to Cole and Kehoe (2000), a coordinated refusal to roll over debt forces default through a liquidity channel. Lorenzoni and Werning (2019) show that the Calvo channel can unfold gradually, and Ayres et al. (2018) formalize the role of expectations, while Ayres et al. (2025) establish that multiplicity is state-dependent and concentrated in stagnations, a feature we capture through a bimodal default-cost shock. A recent quantitative literature disciplines how much of observed spreads reflect such expectations rather than fundamentals (Bocola and Dovis, 2019), and studies how this risk interacts with gambling for redemption and

bailouts (Conesa and Kehoe, 2017; Roch and Uhlig, 2018); Aguiar et al. (2019) examine the role of debt maturity in equilibrium selection. We differ from this literature, which takes monetary policy as fixed or absent, by characterizing how the central bank's stabilization choices reshape the boundaries of the multiplicity region.

A second strand asks whether monetary policy can defuse self-fulfilling crises. Corsetti and Dedola (2016) show that a central bank able to print money provides a backstop, but that the inflation it requires is costly and not always credible (Bacchetta et al., 2018; Camous and Cooper, 2019). Aguiar et al. (2013) emphasize that a central bank unable to commit not to inflate raises borrowing costs in tranquil times, and that joining a monetary union, by making nominal debt effectively real, removes the inflation safety valve. This logic inherits the classic time-inconsistency of monetary policy (Kydland and Prescott, 1977; Barro and Gordon, 1983), whose textbook remedy is to delegate to a conservative, inflation-averse central banker (Rogoff, 1985). We invert this prescription: because the commitment policy runs the economy hot, delegating to a more dovish central banker—one placing greater weight on stabilization—moves the discretionary outcome toward commitment and improves debt sustainability. Our mechanism operates through aggregate-demand stabilization and the labor wedge rather than through a surprise revaluation of nominal debt, complementing work on fiscal commitment and default risk (Liu and Shen, 2022). Relatedly, Schreger et al. (2024) study monetary-fiscal interactions in which hawkish central-bank credibility disciplines borrowing and can expand real debt capacity; our notion of credibility works in the opposite direction, as a commitment to an *expansionary* stance that raises the value of repayment and rules out the high-spread equilibrium.

Finally, our environment connects to work on nominal rigidities and the loss of monetary autonomy in a currency union. Schmitt-Grohé and Uribe (2016) show that downward nominal wage rigidity under a fixed exchange rate generates involuntary unemployment, and Na et al. (2018) cast default and devaluation as substitute instruments for relieving the same real burden. Aguiar et al. (2015) show that the inability to inflate makes union members prone to coordination crises, and Bianchi and Mondragon (2022) that losing monetary independence raises rollover-crisis risk because a government defending the peg cannot stabilize employment; De Grauwe (2013) argues that a credible lender of last resort removes the self-fulfilling bad equilibrium. Most closely related, Fornaro and Grosse-Steffen (2024) show that fragmentation across union members can arise endogenously. We share their focus on endogenous fragmentation, but in our model it stems from the common central bank's stabilization trade-off across heterogeneous members rather than

from equilibrium effects on the world real rate.

2 Model

We consider an economy that lasts for two periods, indexed by $t \in \{0, 1\}$. The economy is populated by a continuum of countries with fiscal authorities (governments subsequently) that borrow from foreign investors. The union is a small open economy in relation to the rest of the world, so there is an exogenous risk-free interest rate denoted by R^*.

Countries belong to a monetary union with a single central bank (CB hereafter). The CB is a benevolent monetary authority that maximizes the welfare of all member countries using equal Pareto weights. Two currencies serve as units of account—the union's domestic currency and the foreign currency—and each country consumes tradable and nontradable goods, with the price of nontradables rigid.

Countries are ex ante identical. In the second period they face a common default-cost shock ϕ and country-specific sunspot realizations ξ_i that select among interest-rate equilibria. The endowment of tradable goods is homogeneous across countries and constant over time. Each country produces a nontradable good using labor.

Fiscal authorities lack commitment, so in every period they choose whether to repay or default. The timing is as follows. In period zero, each government chooses the borrowing ($b_{i,1}$), and the CB chooses the nominal exchange rate (e_0). At the end of period zero, lenders bid an interest rate for each government bond (R_i). In period one, the default cost ϕ is realized; given ϕ, the CB chooses a level of the nominal exchange rate (e_1), and each government then chooses whether to default on the debt.

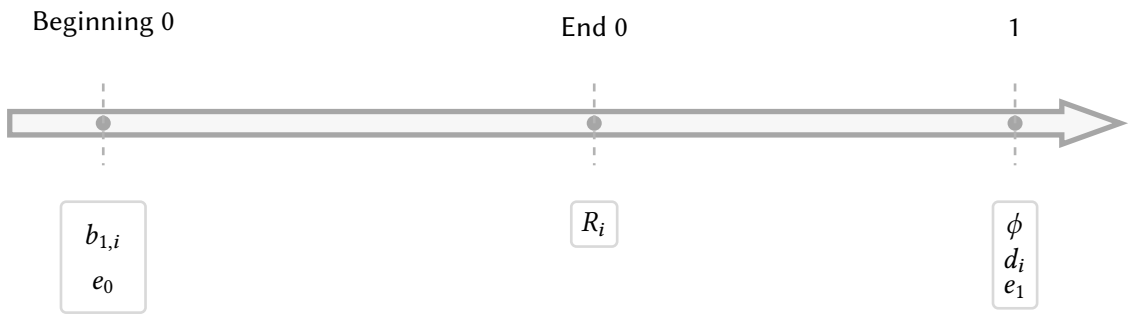


Figure 1: Timeline of events.

2.1 Exogenous Process

There are two sources of uncertainty in the model. First, there is a stochastic cost of default in the second period (ϕ). For simplicity, we assume that all countries face the same default cost. Inspired by Ayres et al. (2025), we consider a distribution with risk of disaster events where the default cost takes two values $\{\phi_L, \phi_H\}$; with probabilities ρ and $1-\rho$, respectively.

The second source is a sunspot ξ_i that selects among equilibria when the interest rate is indeterminate. Suppose that there is more than one interest rate consistent with equilibrium conditions in period zero. In that case, governments face a sunspot (ξ_i) that selects whether the lender's expectations trigger a high or a low interest rate. We also assume that a measure π of countries gets a high interest rate and a measure $(1-\pi)$ of countries gets a low interest rate. We summarize the aggregate stochastic state by defining $s = \{\phi, \{\xi_i\}_i\}$.

2.2 Households

To avoid clutter in the notation, we do not index the variables in each country by i. When it does not cause confusion, we will use x_t to refer to $x_{i,t}$ for some variable x at time t and country i. Each economy is populated by a representative household with preferences given by

$$u(c_0) - v(h_0) + \beta \mathbb{E}\left[u(c_1(s)) - v(h_1(s))\right] \tag{1}$$

where $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$ for $\sigma \neq 1$ (with $u(c) = \log(c)$ when $\sigma = 1$) and $\sigma > 0$, and c_t is a composite good with the CES form

$$c = \left[\omega \left(c^T\right)^{-\mu} + (1-\omega)\left(c^N\right)^{-\mu}\right]^{\frac{1}{-\mu}},$$

where $\mu > 0$ and $\omega \in (0,1)$, so that the elasticity of substitution between tradable and nontradable goods is $1/(1+\mu)$. The disutility of labor is linear, $v(h) = h$. When convenient, we write $u(c^T, c^N)$ for utility as a function of the two consumption goods, and denote its partial derivatives by $\{u_T, u_N\}$.

In each period, households receive a constant endowment of tradable goods (y^T), a lump sum transfer from the government (T_t) expressed in units of foreign currency, profits

from the ownership of firms producing nontradable goods (Π_t) expressed in domestic currency, and labor income for working in the nontradable sector, $W_t h_t$, where W_t is the nominal wage in terms of domestic currency. Furthermore, as is usual in the literature, the representative household does not have access to the international market.

The law of one price applies in the tradable sector, so $P_t^T = e_t P^*$ where e_t is the nominal exchange rate and $P^* = 1$ stands for the international price of tradable goods. Also, we assume that there is nominal rigidity, so the price of nontradable goods is fixed at a value $\overline{P}^N$. The household budget constraint expressed in domestic currency is given by

$$e_0 c_0^T + P^N c_0^N = e_0 y^T + \Pi_0 + W_0 h_0 + e_0 T_0 \tag{2}$$

$$e_1(s) c_1^T(s) + P^N c_1^N(s) = e_1(s) y^T + \Pi_1(s) + W_1(s) h_1(s) + e_1(s) T_1(s) \tag{3}$$

Given the budget constraint, the problem of the household is as follows. Given a sequence of prices, the household maximizes (1) by choosing a sequence of $c_0^N, c_0^T, h_0, \{c_1^N(s), c_1^T(s), h_1(s)\}_s$, subject to (2) and (3). The optimality conditions of this problem are:

$$\frac{P^N}{e_t} = \frac{u_N(t)}{u_T(t)} \tag{4}$$

$$\frac{W_t}{P^N} = \frac{v'(h_t)}{u_N(t)} \tag{5}$$

Condition (4) equates the marginal rate of substitution between tradable and nontradable goods to the relative price. Also, (5) is the standard condition equating the marginal rate of substitution between labor and nontradable consumption with the real wage in terms of nontradable goods.

2.3 Firms

Each country has a representative firm that uses labor to produce nontradable goods. The firm's production function is $y^N = n$, where n stands for the amount of labor hired by the firm. Given that prices are sticky, firms produce to satisfy demand, and thus the firm's labor demand is $n_t = c_t^N$.

2.4 Government

In each country, there is a government that issues one-period bonds (b_1) denominated in foreign currency and provides lump sum transfers to households. The government enters with an initial face value of debt $R_0 b_0$. Following Calvo (1988), we assume that the government collects b_1 from the borrowing in the first period and pays Rb_1 in the second period. Its budget constraint in period zero is the following:

$$R_0 b_0 + T_0 = b_1 \tag{6}$$

In period one, the budget constraint becomes:

$$(1 - d(s))R(b_1, \xi)b_1 = -T_1(s) \tag{7}$$

Here, $d \in \{0, 1\}$ takes the value of 1 if the government defaults or zero otherwise.

2.5 International Lenders

There is a continuum of identical, risk-neutral lenders. They have access to a risk-free asset with a gross interest rate R^* and charge each government an interest rate R. We define the excess return for investing in government bonds as follows.

$$M(R) = \mathbb{E}[R(1 - d(s)) - R^*] \tag{8}$$

The no-arbitrage condition requires the expected excess return on each government bond to be zero in equilibrium, $M(R_i) = 0$.

2.6 Central Bank

The last agent in the economy is the CB. At the beginning of each period, the CB chooses the nominal exchange rate in the union (e_t).

2.7 Competitive Equilibrium

The labor and nontradable goods markets clear in every country. That is,

$$\begin{aligned} c^N_{i,0} &= n_{i,0} \quad \text{for all } i \\ c^N_{i,1}(s) &= n_{i,1}(s) \quad \text{for all } i \end{aligned} \tag{9}$$

We are ready to define the competitive equilibrium of the monetary union.

Definition 1. (Competitive Equilibrium) Given an initial condition $\{R_0 b_0\}$, government policies $\{b_{i,1}, d_i, T_{i,0}, T_{i,1}\}_i$ and the policy of the central bank $\{e_t\}_{0,1}$; the equilibrium consists of a sequence of prices $\{P^N_t, W_{i,t}\}_{t=0,1}$ and $\{R_i\}_i$; and households allocations $\{\{c^T_{i,t}, c^N_{i,t}, h_{i,t}\}_i\}_{t=0,1}$ such that:

i Given prices and government policies: $\{\{c^T_{i,t}, c^N_{i,t}, h_{i,t}\}_i\}_{t=0,1}$ maximizes (1) subject to (2) and (3) in all countries.

ii Given prices and government policies: the non-arbitrage condition holds. That is, $M(R_i) = 0$ for all countries.

iii Given prices and government policies: the labor market clears $h_{i,t} = n_{i,t}$ for all countries.

iv Given prices and households policies: $\{\{b_{i,1}, d_i(s), T_{i,0}, T_{i,1}(s)\}_i\}$ are consistent with government budget constraints (6) and (7) in all countries.

Before analyzing optimal policy, we characterize the equilibrium conditions. Combining the household and government budget constraints, the firm's zero-profit condition, and (9), we obtain:

$$\begin{aligned} c^T_0 &= y^T + b_1 - R_0 b_0 \\ c^T_1(s) &= y^T - R(b_1, \xi) b_1 (1 - d(s)) \end{aligned} \tag{10}$$

We impose the feasibility restriction $y^T - R(b_1, \xi) b_1 > 0$, so that tradable consumption is positive whenever the government repays.

Labor wedge Next, we define the wedge in the labor market due to nominal rigidities. First, we combine (4) and (9) to obtain the equilibrium employment as a function of tradable consumption and the nominal exchange rate, as in Bianchi and Mondragon (2022):

$$\mathcal{H}(c_t^T, e_t) = \left[\frac{1-\omega}{\omega}\frac{e_t}{\overline{P}^N}\right]^{\frac{1}{1+\mu}} c_t^T \tag{11}$$

This expression indicates that employment in equilibrium is an increasing function of e and c^T. Depreciation reduces the relative price and increases the demand for nontradables, leading to higher employment. Moreover, when the two goods are complements, a higher c^T raises demand for nontradables at a given relative price, increasing employment. Additionally, for any level of c^T, we can define the flexible-price level of employment as

$$\overline{h}(c^T) = \{h : u_N(c^T, h) = v'(h)\} \tag{12}$$

Define the labor wedge as follows:

$$\tau_t(c_t^T, e_t) = u_N\left(c^T, \mathcal{H}\left(c_t^T, e_t\right)\right) - v'\left(\mathcal{H}\left(c_t^T, e_t\right)\right) \tag{13}$$

The labor wedge would be zero under flexible prices. In contrast, with nominal rigidities, depending on the consumption of tradable goods and the nominal exchange rate, the labor wedge could be positive or negative. We say that the economy is running hot when the labor wedge is negative.

In this environment, the government could manage the labor wedge by changing the level of transfers to households. The mechanism works as follows. Higher (or lower) transfers increase (decrease) the consumption of tradable goods, increase (decrease) the demand for nontradable goods boosting (depressing) labor, and reducing the labor wedge if employment is lower (higher) than the optimal level under flexible prices.

Alternatively, the CB can change e_t to reduce the labor wedge. The complementarity of the two policies in stabilizing employment is the interaction between fiscal and monetary policy that we study in this paper. To focus on these mechanisms, in most of the paper we assume that the CB budget and the budget of each government are independent, with no transfers between the central bank and member governments or across member governments.[1]

[1]See Bassetto et al. (2021) for an example of models exploring the relationship of fiscal and monetary

Interest Rate We close this section by describing the interest rate at equilibrium. Given the bimodal nature of the uncertainty in the model, three possible values of R could solve the break-even condition of international lenders depending on the probability of default.

In the first case, for a given Rb_1 the government repays regardless of the realization of the default cost, the probability of default is zero, and the equilibrium interest rate equals the risk-free rate, so $R = R^*$. In the second case, given Rb_1, the government repays only when the default cost is high, and the equilibrium interest rate is $R = \frac{R^*}{1-\rho}$. In the last case, given Rb_1 the government defaults regardless of the realization of the default cost, and $R = \infty$.

3 Optimal Policy Without Commitment

We characterize the optimal policy of governments and the CB. Because both lack commitment, we study the Markov perfect equilibrium of the Nash game between the individual governments and the central bank, restricting attention to symmetric policy functions across member countries.

Government objective The preferences of the government are described by:

$$U = u(c_0) - v(h_0) + \beta \mathbb{E}\left[u\left(c_1(s)\right) - v\left(h_1(s)\right) - \phi d(s)\right] \tag{14}$$

where ϕ follows the bimodal distribution introduced in Section 2.

Central Bank objective The preferences of the CB are described by:

$$W = \int_i [u(c_{i,0}) - v(h_{i,0}) + \beta \mathbb{E}(u(c_{i,1}(s)) - v(h_{i,1}(s)) - \phi d_i(s))] di \tag{15}$$

3.1 Optimal Policy in Second Period

In the second period, countries receive heterogeneous interest rates. Recall that a fraction π of countries receives the high interest rate $\left(\frac{R^\star}{1-\rho}\right)$, while a fraction $1-\pi$ receives the low interest rate $R^\star$. The aggregate payoff-relevant state variables are the outstanding debt

authorities through a common budget constraint.

issued by member countries (B_1), the fraction π receiving the high interest rate, and the default cost ϕ, which we summarize by $\Omega_u = \{B_1, \pi, \phi\}$.

Problem of Governments The relevant state in the second period for each government is $\{R_1 b_1, \phi, e_1\}$, and the value function is the following:

$$V_1(R_1 b_1, \phi, e_1) = \max_d (1-d) V^R(R_1 b_1, e_1) + d V^D(\phi, e_1) \tag{16}$$

where V^R and V^D represent the value of repaying and defaulting. Under repayment, the government value is:

$$V^R(R_1 b_1, e_1) = u(y^T - b_1 R_1, \mathcal{H}(y^T - b_1 R_1, e_1)) - v(\mathcal{H}(y^T - b_1 R_1, e_1)) \tag{17}$$

On the other hand, if the government defaults, the value is as follows:

$$V^D(\phi, e_1) = u(y^T, \mathcal{H}(y^T, e_1)) - v(\mathcal{H}(y^T, e_1)) - \phi \tag{18}$$

We characterize the government's default decision with the following threshold:

$$V^D(\bar{\phi}(R_1 b_1, e_1), e_1) = V^R(R_1 b_1, e_1) \tag{19}$$

The government defaults whenever the value of doing so is greater than the value of repayment. Because the value of default is strictly decreasing in the cost of default, the default function is:

$$d(R_1 b_1, e_1) = \begin{cases} 1 & \text{if} \quad \phi < \bar{\phi}(R_1 b_1, e_1), \\ 0 & \text{otherwise.} \end{cases} \tag{20}$$

Consequently, the probability of default is:

$$F(R_1 b_1, e_1) = Pr[\phi < \bar{\phi}(R_1 b_1, e_1)] \tag{21}$$

Problem of the Central Bank In the second period, the central bank solves, in each state:

$$W_1(\Omega_u) = \max_{e_1} \int_i \left[V_1 \left(R_{1,i} b_{1,i}, \phi, e_1 \right) \right] di \tag{22}$$

where $R_{1,i}b_{1,i}$ is the debt payment of country i. The first-order condition for the nominal exchange rate is:

$$0 = \int_i \left[(1-d_i)\,\tau_1\left(y^T - b_{1,i}R_{1,i}, e_1\right)\mathcal{H}_e\left(y^T - b_{1,i}R_{1,i}, e_1\right) + d_i\,\tau_1\left(y^T, e_1\right)\mathcal{H}_e\left(y^T, e_1\right)\right] \mathrm{d}i \tag{23}$$

The optimal choice of the central bank is to minimize the average wedge in the union. The condition (23) captures the average effect of a change in the nominal exchange rate on the labor wedge in each economy (τ_1)[2]. If all countries face the same debt payments, it is feasible for the central bank to choose e_1 so that the labor wedge in all economies is zero. On the other hand, as heterogeneity increases and governments face different interest rates, a trade-off in monetary policy emerges. While some countries will experience a negative labor wedge, others will have a positive one. The labor wedge affects countries' default decisions.

3.2 Optimal Policy in the First Period

Next, we analyze the optimal policy in period zero.

International Investors Using the second-period default probability, the break-even condition for international investors is:

$$R^* = \left[1 - F\left(R_1 b_1, e_1\right)\right] R_1 \tag{24}$$

Problem of Governments Given the policies of the governments and the central bank in the second period, governments in the first period solve the following.

$$V_0(e_0) = \max_{b_1} u\left(c_0^T, \mathcal{H}\left(c_0^T, e_0\right)\right) - v\left(\mathcal{H}\left(c_0^T, e_0\right)\right) + \beta\mathbb{E}\left[V_1\left(R(b_1,\xi)b_1, \phi, e_1\right)\right] \tag{25}$$

subject to

$$c_0^T = y^T + b_1 - R_0 b_0$$
$$R^* = [1 - F(R_1 b_1, e_1)]R_1$$

[2]Note the impact of the monetary policy in the default decision of the government is canceled out in the FOC. This is because, evaluated at the threshold $\bar{\phi}(R_1 b_1, e_1)$, the value of the government is the same under repayment and default.

All countries have the same initial debt ($R_0 B_0$). As a result, all countries will choose the same policy, so $b_{i,1} = B_1$.

Problem of the Central Bank Given the policies of the governments and the central bank in the second period, CB in the first period solves the following.

$$W_0 = \max_{e_0} \int_i [V_0(e_0)] di \tag{26}$$

It is feasible for the central bank to implement a zero-labor wedge for all countries in the first period. So, the central bank chooses e_0 for which the labor wedge is zero. That is, given the optimal policy of the government B_1, e_0 solves the following:

$$0 = \tau_0(y^T + B_1 - R_0 B_0, e_0) \tag{27}$$

3.3 Markov Equilibrium

Now, we define the Markov equilibrium in this environment. The symmetric Markov equilibrium of the global economy is as follows.

Definition 2. (Symmetric Markov Equilibrium) Given initial conditions $\{R_0 b_0\}$; a Markov perfect equilibrium is defined by a set of strategies $\{\mathcal{E}_0, \mathcal{E}_1(\Omega_u), \{\mathcal{B}_{i,1}, \mathcal{D}_{i,1}(R_{i,1} b_{i,1}, e_1, \phi)\}_i\}$; value functions $\{W_0, W_1(\Omega_u), \{V_0(e_0), V_1(R_{i,1} b_{i,1}, e_1, \phi)\}_i\}$ and a set of interest-rate schedules $\{\mathcal{R}_i(b_{i,1}, e_1, \xi)\}$ such that:

i Given the interest rate schedule and the policy function of the central bank: $\{\mathcal{B}_{i,1}\}$ solves the problem of each government in period zero, and $\{V_{i,0}\}_i$ attains the maximum.

ii Given a face value of debt and the policy function of the central bank: $\{\mathcal{D}_{i,1}\}_i$ solves the problem of each government in every state in period one, and $\{V_{i,1}\}_i$ achieves the maximum.

iii Given the policy functions of the governments and the interest rate schedules: $\{\mathcal{E}_0, \mathcal{E}_1\}$ solves the central bank problem, and W_0, W_1 attain the maximum.

iv Given the policy functions of the government and the central bank, $\mathcal{R}_i$ satisfies each country's break-even condition of international investors.

3.4 Multiplicity

Next, we study the multiplicity in period one. We use government policies to solve the equilibrium interest rate. Similar to Calvo (1988) and Ayres et al. (2018), we find more than one interest rate consistent with equilibrium conditions for intermediate debt levels.

We partition the space of debt and nominal exchange rate into three regions: a high-rate region $\mathcal{I}^{\mathcal{H}}$, a risky region $\mathcal{I}^{\mathcal{R}}$, and a low-rate region $\mathcal{I}^{\mathcal{L}}$. Formally, we define:

Definition 3. (Debt Space Regions) It is possible to break the space of $\{b_1, e_1\}$ as follows:

$$\begin{aligned}
\mathcal{I}^{H} &= \left\{b_{i,1}, e_1 : \bar{\phi}(R^* b_{i,1}, e_1) > \phi_L\right\} \\
\mathcal{I}^{L} &= \left\{b_{i,1}, e_1 : \bar{\phi}\left(\frac{R^*}{1-\rho} b_{i,1}, e_1\right) \leq \phi_L\right\} \\
\mathcal{I}^{R} &= \left\{b_{i,1}, e_1 : \bar{\phi}\left(\frac{R^*}{1-\rho} b_{i,1}, e_1\right) > \phi_L \geq \bar{\phi}(R^* b_{i,1}, e_1)\right\}
\end{aligned}$$

For a given monetary policy e_1 and a low default cost ($\phi = \phi_L$), the debt in $\mathcal{I}^{\mathcal{H}}$ is such that the government defaults even if it can borrow at the risk-free rate. In $\mathcal{I}^{\mathcal{L}}$, the government always repays even if faced with a high interest rate. In $\mathcal{I}^{\mathcal{R}}$, the debt is such that the government repays if facing a risk-free rate and defaults if it faces a high interest rate. In this region, therefore, we have multiple equilibria. If investors expect the government to repay its debt even when the default cost is low, they will charge a low interest rate, and the government indeed repays in every state. Conversely, if investors believe that low default costs will lead the government to default, they will charge a high interest rate. This higher rate makes default more attractive, reinforcing the investors' expectation and justifying the high interest rate.

We use the fact that the value function of the repayment is continuous and strictly decreasing in R—which holds under the maintained condition $u_T + \tau \mathcal{H}_{c^T} > 0$, so that the resource loss from a heavier repayment burden outweighs the fall in labor disutility—to characterize the boundaries of these regions in the following lemma:

Lemma 1. *There exist two thresholds of debt* $\underline{b}(e_1)$, $\overline{b}(e_1)$ *such that* $b_{i,1} \in \mathcal{I}^R$ *if and only if* $\overline{b}(e_1) \geq b_{i,1} > \underline{b}(e_1)$.

Proof. In Appendix A.1 □

The boundaries of the multiplicity region are functions of the CB's monetary policy. In Section 5, we characterize how monetary policy affects governments' exposure to multiplicity. Also, in Section 6, we study the strategic interactions between monetary policy and default policy.

The multiplicity region $\left[\underline{b}(e_1), \overline{b}(e_1)\right]$ is the interval of debt over which the government repays when it faces the risk-free rate but defaults when it faces the high rate. Its boundaries are increasing in the nominal exchange rate: a more expansionary monetary policy shifts both thresholds to the right and reduces the government's exposure to multiplicity. We establish this comparative static formally in the proof of Proposition 1 (Appendix A.2) and illustrate it in Section 5, Figure 3; it relates our mechanism to Bianchi and Mondragon (2022).

3.5 Equilibrium Interest Rate

Multiplicity implies that the government faces an interest rate schedule in the first period that depends on the fundamentals (b_1) and the expectations of investors. We use a sunspot (ξ) to choose the equilibrium in the risk zone as follows:

$$R(b_1, e_1, \xi) = \begin{cases} R^* & b_1 < \underline{b}(e_1) \\ R^* & \overline{b}(e_1) > b_1 \geq \underline{b}(e_1) \text{ and } \xi = 1 \\ \frac{R^*}{1-\rho} & \overline{b}(e_1) > b_1 \geq \underline{b}(e_1) \text{ and } \xi = 0 \\ \frac{R^*}{1-\rho} & b_1 \geq \overline{b}(e_1) \end{cases}$$

We adopt the tie-breaking convention that the government repays whenever it is indifferent, that is, $d = 0$ when $\phi = \bar{\phi}(Rb_1, e_1)$. Under this convention the multiplicity region is the half-open interval $\left(\underline{b}(e_1), \overline{b}(e_1)\right]$ of Lemma 1: at $b_1 = \underline{b}(e_1)$ the country repays at the risk-free rate, so the low interest rate is the unique equilibrium, whereas at $b_1 = \overline{b}(e_1)$ both interest rates remain consistent with equilibrium.

Unlike Ayres et al. (2025), we assume that the government faces uncertainty in the

interest rate when choosing a debt level that lies in the multiplicity region. Ayres et al. (2025) assume instead that there are two interest schedules in period zero and solve the problem of the government assuming that it knows whether investors will charge a high or a low interest rate if they choose a level of debt in the multiplicity region.

4 Optimal Policy with Commitment in Monetary Policy

In this model, the CB has incentives to promise future monetary easing because such promises lower current interest rates and reduce the probability of default. However, once the interest rate is set, the central bank has an incentive to deviate ex post.

To analyze this problem, we describe an equilibrium in which the CB can commit to a monetary policy in the first period. We assume that the commitment is non-state-contingent. The CB promises a fixed e_1 regardless of the realization of the shocks (or the default decision).

4.1 Policy in Second Period

We begin by describing the problem of government in the second period as a function of the relevant state. The value of the government is defined as follows.

$$V_1^C(Rb_1, \phi, e_1) = \max_d (1-d)V^{C,R}(Rb_1, e_1) + dV^{C,D}(\phi, e_1) \tag{28}$$

where $V^{C,R}$ and $V^{C,D}$ are the values of repayment and default, respectively, defined as:

$$\begin{aligned} V^{C,R}(Rb_1, e_1) &= u\left(y^T - b_1 R, \mathcal{H}\left(y^T - b_1 R, e_1\right)\right) - v\left(\mathcal{H}\left(y^T - b_1 R, e_1\right)\right) \\ V^{C,D}(\phi, e_1) &= u\left(y^T, \mathcal{H}\left(y^T, e_1\right)\right) - v\left(\mathcal{H}\left(y^T, e_1\right)\right) - \phi \end{aligned}$$

As in the Markov equilibrium, we can describe the default decisions of the government using the threshold:

$$V^{C,D}\left(\bar{\phi}_C\left(Rb_1, e_1\right), e_1\right) = V^{C,R}\left(Rb_1, e_1\right) \tag{29}$$

In this case, the default function of the government is:

$$d_C\left(Rb_1, e_1\right) = \begin{cases} 1 & \text{if} \quad \phi < \bar{\phi}_C\left(Rb_1, e_1\right), \\ 0 & \text{otherwise.} \end{cases} \tag{30}$$

Here, the probability of default becomes:

$$F_C(Rb_1, e_1) = Pr[\phi < \bar{\phi}_C(Rb_1, e_1)] \tag{31}$$

Also, given the break-even condition of international investors, the equilibrium interest rate of the government is:

$$R^C\left(b_1, \xi, e_1\right) = \begin{cases} R^* & b_1 < \underline{b}_C(e_1) \\ R^* & \overline{b}_C(e_1) > b_1 \geq \underline{b}_C(e_1) \text{ and } \xi = 1 \\ \frac{R^*}{1-\rho} & \overline{b}_C(e_1) > b_1 \geq \underline{b}_C(e_1) \text{ and } \xi = 0 \\ \frac{R^*}{1-\rho} & b_1 \geq \overline{b}_C(e_1) \end{cases}$$

where $\underline{b}_C(e_1), \overline{b}_C(e_1)$ are defined analogously to the Markov equilibrium and take the following form:

$$\begin{aligned} \underline{b}_C(e_1) =& \{b_1 : \bar{\phi}_C(R^\star b_1, e_1) = \phi_L\} \\ \overline{b}_C(e_1) =& \left\{b_1 : \bar{\phi}_C\left(\frac{R^\star}{1-\rho} b_1, e_1\right) = \phi_L\right\} \end{aligned}$$

In the case of commitment, the CB does not choose any policy in period one, so we characterize the optimal policy in period zero.

4.2 Policy in First Period

Problem of the Government The government in the first period solves the following:

$$V_0^C\left(e_0, e_1\right) = \max_{b_1} \; u\left(c_0^T, \mathcal{H}\left(c_0^T, e_0\right)\right) - v\left(\mathcal{H}\left(c_0^T, e_0\right)\right) + \beta\, \mathbb{E}\left[V_1^C\left(R^C\left(b_1, \xi, e_1\right) b_1, \phi, e_1\right)\right] \tag{32}$$

$$\text{s.t.} \quad c_0^T = y^T + b_1 - R_0 b_0$$

Problem of the CB Let us define the problem of a CB with the technology to commit to a monetary policy as follows.

$$W^C = \max_{e_0, e_1} \int_i \left[V_0^C \left(e_0, e_1 \right) \right] \, di \tag{33}$$

The difference in this problem is that the CB internalizes the effect of the nominal exchange rate on interest rates and default decisions in the second period.

4.3 Continuation Equilibrium

We define the continuation equilibrium in which the central bank chooses the optimal monetary policy taking as given the default function in the second period. The continuation equilibrium is defined for a given committed policy e_1: governments and investors take e_1 as given in conditions (i), (ii), and (iv), while the central bank chooses $\mathcal{E}_0$ optimally in condition (iii), taking the committed e_1 as given. The value functions and the interest-rate schedule are therefore written as functions of the committed e_1.

Definition 4. (Symmetric Continuation Equilibrium) Given initial conditions $\{R_0 b_0\}$; a Continuation equilibrium is defined by a set of strategy profile $\{\mathcal{E}_0, \mathcal{B}_C, \mathcal{E}_1, \mathcal{D}_C(Rb_1, \phi, e_1)\}$; value functions $\{W^C, V_0^C(e_0, e_1), V_1^C(Rb_1, \phi, e_1)\}$ and an interest rate schedule $\mathcal{R}(b_1, e_1, \xi)$ such that:

i Given the interest rate schedule and the policy function of the central bank: $\mathcal{B}_C$ solves the problem of each government in period zero, and V_0^C attains the maximum.

ii Given a face value of debt and the policy function of the central bank: $\mathcal{D}_C$ solves the problem of each government in every state and V_1^C achieves the maximum.

iii Given the policy functions of the governments and the interest rate schedules: $\mathcal{E}_0$ solves the CB problem given the committed e_1, and W^C attains the maximum.

iv Given the policy functions of the government and the central bank, $\mathcal{R}^C$ satisfies the break-even condition of international investors for each country.

We characterize this equilibrium in Section 6, where we show that commitment eliminates the fragmented equilibrium and selects the safe allocation for all members of the union.

5 Partial Equilibrium Analysis

We begin by describing how monetary policy affects the government's vulnerability to multiple equilibria. We study the optimal policy of an individual government under two monetary regimes. First, we consider an accommodative monetary policy regime, where the CB adjusts the nominal exchange rate to close the labor wedge in that country. We think of this case as an economy with homogeneity among the union members, so the CB accommodates country-specific conditions in response to domestic conditions.

We also consider a fixed monetary policy, where the CB chooses $e_1 = e_0 = \bar{e}$.[3] We think of this case as an economy with heterogeneity, where the analyzed country is small compared to the union's size. In this case, the CB chooses the nominal exchange rate to stabilize the union's average labor wedge and does not react to domestic conditions.

We establish conditions under which, for a given level of debt, the government is more vulnerable to self-fulfilling crises in an economy with a fixed monetary policy than in an economy with an accommodative monetary policy. We also analyze the optimal debt policy and study how the monetary regime shapes the vulnerability of the economy.

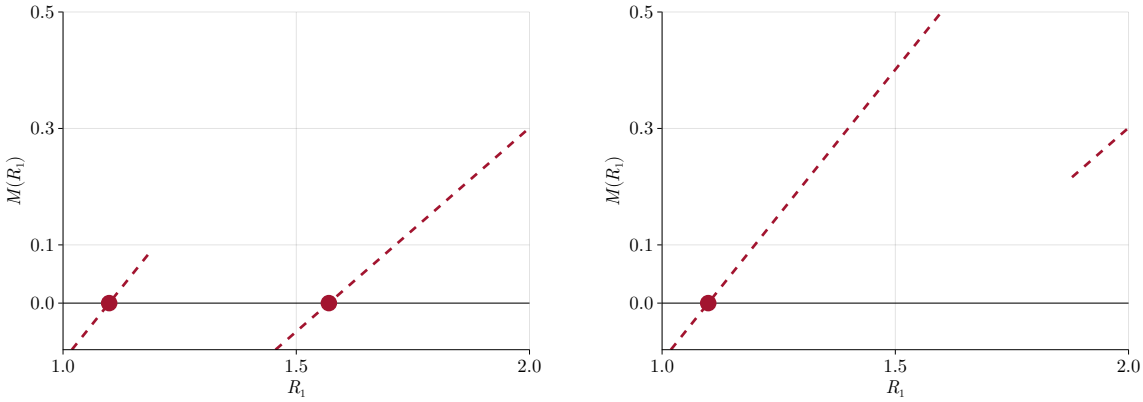
5.1 Exposure in Period One

We first illustrate the mechanism graphically and then provide a formal analysis.

In Figure 2, we show the excess returns of international investors for a given level of debt. In panel 2a, we plot the excess return for an economy with a fixed monetary policy. We plot a combination of nominal exchange rate and debt so that there is a positive labor wedge. In this economy, two interest rates are consistent with zero excess return, so there are multiple equilibria. In contrast, in panel 2b, we plot the equilibrium in an economy with an accommodative monetary policy. For this economy, the equilibrium interest rate

[3] Throughout, "fixed" and "accommodative" (or "flexible") describe whether the central bank's policy responds to an individual member's conditions, rather than a national exchange-rate regime; a member of the monetary union has no independent exchange rate. The small-open-economy analysis in this section serves as a benchmark for the union problem studied below.

for the country is unique.



(a) Fixed Monetary Policy (b) Accommodative Monetary Policy

Figure 2: Investors' Net Revenues

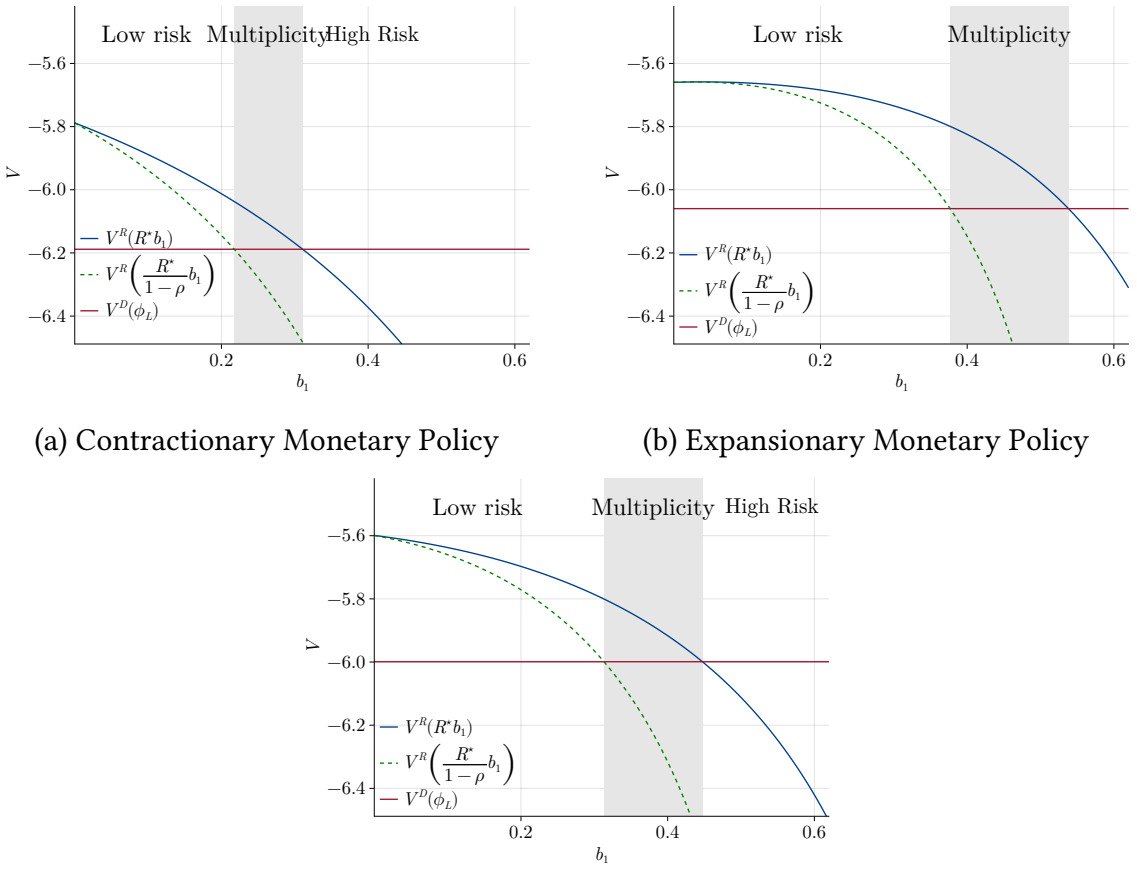
Note: In this figure, we use $y^T = 0.95$, $R^* = 1.04$, $\rho = 0.2$, $\omega = 0.3$ and $\mu = (1/0.83) - 1$

This result can be understood as follows. The difference between these two economies concerns the labor-market wedge. In this example, the labor wedge is positive for a fixed monetary policy, so households work less than they would under flexible prices. As we increase the interest rate R, payments to lenders in the second period are higher, so the government should reduce the consumption of tradable goods. Because monetary policy does not react to domestic conditions, reduced consumption of tradable goods also implies a higher labor wedge. As a result, the probability of default increases faster in an economy with a fixed monetary policy than in an accommodative monetary policy where the CB uses monetary policy to implement a zero wedge in the labor market.

The jump in the probability of default from zero to ρ occurs at a lower interest rate in an economy with a fixed monetary policy. This implies that for this parameterization, the economy is more vulnerable to having multiple equilibria under a fixed exchange rate. However, under an accommodative policy the economy can be more vulnerable when it is running hot. In the next example, we explore this case.

In Figure 3, we plot the default and repayment values for the government. There are two candidates for an equilibrium interest rate: a low interest rate ($R^\star$) and a high interest rate ($\frac{R^\star}{1-\rho}$). The solid blue line is the repayment value with a low interest rate as a function of the debt b_1, while the red dashed line shows the repayment value for a high interest rate. We also plot the default value when the realization of the default cost is low (ϕ_L).

In panel 3a, we plot the values for the government under a contractionary monetary policy, which is a fixed monetary policy where the CB chooses the nominal exchange rate so that the labor wedge is positive. The gray area represents the debt in the risk zone. Panel 3b shows the values of the government under an expansionary monetary policy in which the CB chooses the nominal exchange rate so that there is a negative labor wedge in the economy. Finally, in panel 3c, we plot the government's value with an accommodative monetary policy.



(a) Contractionary Monetary Policy

(b) Expansionary Monetary Policy

(c) Accommodative Monetary Policy

Figure 3: Period Two Value Functions

Note: In this figure, we use $y^T = 0.95$, $R^* = 1.04$, $\rho = 0.2$, $\omega = 0.3$ and $\mu = (1/0.83) - 1$

The value of repayment declines faster in an economy with fixed monetary policy when the labor wedge is positive. Intuitively, the repayment of debt becomes costlier in

the presence of a positive labor wedge because the wedge increases as debt rises, due to a reduction in the consumption of tradable goods. Consequently, economies with contractionary monetary policies exhibit multiple equilibria with lower values of b_1 compared to economies with accommodative monetary policies.

However, under an expansionary monetary policy, the value of repayment decreases more slowly for an economy with a fixed monetary policy. In this economy, the labor wedge is negative, so reducing the consumption of tradable goods to repay the debt is less costly for the government because it reduces labor in the nontradable sector and closes the labor wedge. In this case, the economy is less vulnerable under a fixed monetary policy than under an accommodative monetary policy.

In contrast to Bianchi and Mondragon (2022), in our model the economy can be more or less exposed to multiplicity in a union where monetary policy does not react to domestic conditions, depending on whether the economy has a positive or a negative labor wedge. Formally, we establish the following.

Proposition 1. *Assume $\sigma \geq 1$. Let $\underline{b}_{flex}$, $\overline{b}_{flex}$, $\underline{b}_{fix}(\overline{e})$, and $\overline{b}_{fix}(\overline{e})$ be the threshold debt for an economy with a flexible and fixed exchange rate regime, respectively. Define e^+ and e^- as:*

$$V_1^D(\phi_L, e^+) = V_1^R\left(R^\star \underline{b}_{flex}, e^+\right)$$
$$V_1^D(\phi_L, e^-) = V_1^R\left(\frac{R^\star}{1-\rho}\overline{b}_{flex}, e^-\right)$$

Then:

1. $\underline{b}_{flex} > \underline{b}_{fix}(\overline{e})$ *for every* $\overline{e} < e^+$. *And* $\underline{b}_{flex} \leq \underline{b}_{fix}(\overline{e})$ *for every* $\overline{e} \geq e^+$
2. $\overline{b}_{flex} > \overline{b}_{fix}(\overline{e})$ *for every* $\overline{e} < e^-$. *And* $\overline{b}_{flex} \leq \overline{b}_{fix}(\overline{e})$ *for every* $\overline{e} \geq e^-$

Proof. In Appendix A.2 □

Proposition 1 establishes a monotonicity result for the boundaries of the multiplicity region in an economy with a fixed monetary policy. As the CB chooses a more expansionary monetary policy $\overline{e}$, the economy is less exposed to multiple equilibria. This result extends Bianchi and Mondragon (2022), which studies rollover crises in the sense of Cole and Kehoe (2000), to the self-fulfilling debt crises of Calvo (1988).

In addition, Proposition 1 divides the space of monetary policy into two regions. In the first region, where monetary policy is contractionary and the labor wedge is positive as in Bianchi and Mondragon (2022), the economy would be less exposed to multiple equilibria if monetary policy were adjusted in response to domestic conditions.

However, in this environment, there is another case. When the labor wedge is negative, the economy is more exposed to multiple equilibria under an accommodative monetary policy. The result can be understood as follows. If monetary policy is sufficiently expansionary, employment will be higher than the optimal level under flexible prices. In those cases, the government's cost of reducing the consumption of tradable goods is lower because it helps reduce the labor wedge. As a result, the government has stronger incentives to repay than under an accommodative policy, where the labor wedge is zero and this additional benefit of repaying is absent.

5.2 Optimal Debt Policy

In this subsection, we study the optimal debt policy and compare the endogenous exposure to self-fulfilling crises of a government under a fixed monetary-policy regime and a government with an accommodative monetary policy regime. Specifically, we compare an economy in which monetary policy is adjusted to a zero labor wedge in both periods with one in which it is fixed at $\overline{e}$ in both periods.

In Figure 4, we plot the value of the government as a function of debt for both accommodative and fixed monetary policy. The red (dashed) line represents the value of the government when there is an accommodative monetary policy. The red circle represents the optimal level of debt. In addition, we plot the threshold of the risky interest rate region ($\underline{b}$). In this example, for an accommodative monetary policy, the government maximizes its value at a level of debt outside the risk zone.

The blue (solid) line represents the value of the government under a fixed monetary policy. The blue circle is the optimal level of debt. We plot the limits of the risky interest rate for the economy under fixed monetary policy ($\underline{b}(\overline{e})$). In contrast with the economy with an accommodative monetary policy, under a fixed monetary policy, the government finds it optimal to take a level of debt inside the risky zone. The government chooses a level of debt similar to the case of a flexible monetary policy. However, because paying that debt would increase the labor wedge in period one, the interest rate schedule that the government faces implies that the interest rate is higher for similar levels of debt.

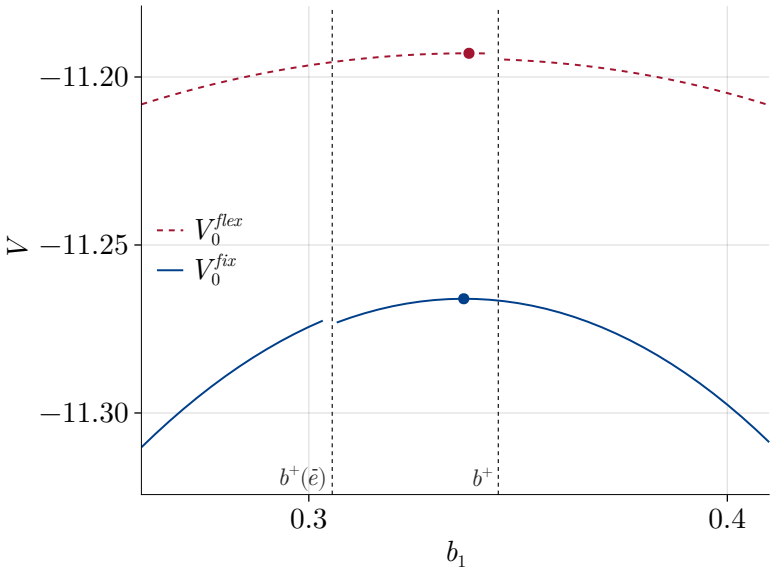


Figure 4: Payoffs of the Government as a Function of Debt

Note: In this figure, we use $y^T = 0.8$, $\beta = 0.91$, $R_0 B_0 = 0.65$, $R^* = 1.04$, $\rho = 0.2$, $\omega = 0.3$, $\mu = (1/0.83) - 1$ and $\pi = 0.05$

6 General Equilibrium in the Monetary Union

Now, we are ready to analyze the equilibrium in the monetary union. We analyze the interactions of optimal monetary policy with the country's risk of default and describe how the interaction of monetary and default policies might expose the union to a self-fulfilling crisis.

At the regional level, there are two possible equilibria. In a good equilibrium, all countries face a low interest rate and no labor wedge. In contrast, in the bad equilibrium, some countries have high interest rates and a positive probability of default in period one, and all countries face a labor wedge.

6.1 Equilibrium in the Second Period

Recall that there is symmetry in period zero, so all countries choose the same level of debt B_1. We characterize the equilibrium in period one by guessing and verifying.

Symmetric Equilibrium. Let us guess that the CB chooses e_s and that the safe zone threshold is $\underline{b}(e_S) = B_s$. By definition, if $B_1 = B_s$, all countries will repay in period one regardless of the default cost and will face a risk-free interest rate.

If countries choose $B_1 = B_s$, it is feasible in period one for the central bank to implement a zero-labor wedge because there is no heterogeneity between countries. Formally, B_s and e_s solve the following system:

$$0 = \tau_1 \left(y^T - R^* B_s, e_S \right) \tag{34}$$

$$\phi_L = \bar{\phi} \left(\frac{R^*}{1-\rho} B_s, e_S \right) \tag{35}$$

In Figure 5, we plot the government value for $B_1 = B_s$ as a function of the nominal exchange rate. The solid red line is the default value for an individual government (under ϕ_L). The green dashed line is the repayment value when the interest rate is high. The solid blue line is the repayment value when the interest rate is low.

Consistent with the definition of B_s, when the CB chooses e_S, individual governments are indifferent between repayment and default if the interest rate is high, so at $B_1 = B_s$, the government will always repay. As a result, there is no support for a high interest rate, and there is a unique equilibrium interest rate for all countries. In this equilibrium, the government value is given by the intersection of the repayment value under a low interest rate (the solid blue line) and the optimal exchange rate (e_S). We call this equilibrium the symmetric equilibrium.

Fragmented Equilibrium. However, in this economy, there is another equilibrium. As before, we construct the second equilibrium by guessing and verifying. Suppose that lenders guess that at B_s, some countries will default when the default cost is low.

In this belief, there will be heterogeneity in the interest rate. A fraction π of countries will get a high interest rate, while a fraction $1 - \pi$ will get a low interest rate. We label this heterogeneity fragmentation. In a fragmented economy, the central bank will face a trade-off in monetary policy. In addition, the optimal policy of the CB will be state-dependent.

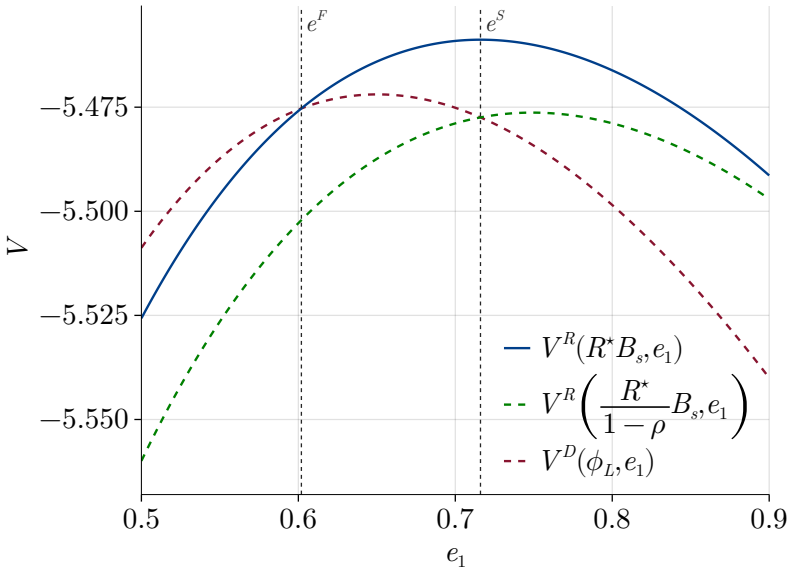


Figure 5: Multiplicity in a Monetary Union

Note: In this figure, we use $y^T = 0.8$, $\beta = 0.91$, $R_0 B_0 = 0.65$, $R^* = 1.04$, $\rho = 0.2$, $\omega = 0.3$, $\mu = (1/0.83) - 1$ and $\pi = 0.05$

We label monetary policy in this equilibrium $e_F^+(\phi)$. When governments choose $B_1 = B_s$, $e_F^+(\phi)$ solves the following:

$$0 = \pi \left[\tau_1\left(y^T, e_F^+(\phi_L)\right)\right] \mathcal{H}_e\left(y^T, e_F^+(\phi_L)\right) + (1-\pi)\left[\tau_1\left(y^T - R^* B_s, e_F^+(\phi_L)\right)\right] \mathcal{H}_e\left(y^T - R^* B_s, e_F^+(\phi_L)\right) \quad (36)$$

$$0 = \pi \left[\tau_1\left(y^T - B_s\frac{R^*}{1-\rho}, e_F^+(\phi_H)\right)\right] \mathcal{H}_e\left(y^T - B_s\frac{R^*}{1-\rho}, e_F^+(\phi_H)\right) + (1-\pi)\left[\tau_1\left(y^T - R^* B_s, e_F^+(\phi_H)\right)\right] \mathcal{H}_e\left(y^T - R^* B_s, e_F^+(\phi_H)\right) \quad (37)$$

The nominal exchange rate is state contingent because governments' default decisions depend on default costs. When default costs are low, governments facing high interest rate default, making transfers to households zero. Without contractionary fiscal policy, the CB implements less expansionary monetary policy. On the other hand, when default costs are high, all governments repay, forcing countries with high interest rates into contractionary

fiscal policy, which incentivizes more expansionary monetary policy from the CB.

Furthermore, $e_F^+(\phi_L) < e_S$ because when default costs are low, some countries default, making their fiscal policy less contractionary than in symmetric equilibrium, so the CB implements less aggressive monetary policy. In contrast, $e_F^+(\phi_H) > e_S$ because when default costs are high, countries with a high interest rate face a more contractionary fiscal policy than in symmetric equilibrium, prompting the CB to implement a more aggressive expansionary policy to reduce their labor wedge.

In Figure 5, we plot the optimal nominal exchange rate under fragmentation when the default cost is low. We can see that when the central bank chooses $e_F^+(\phi_L)$, countries that receive a high interest rate will find it optimal to default (under ϕ_L). As a consequence, there is support for the initial guess that some countries will have default risk, which, in turn, supports high interest rates in the first place.

In summary, if countries choose $B_1 = B_s$ and investors expect the monetary union to be in a fragmented equilibrium where some countries will default, they will coordinate high interest rates for some union members. Since this fragmentation would make it difficult for the CB to implement a zero-labor wedge, those expectations will become self-fulfilling.

The critical implication of this analysis is that B_s is the safe zone threshold for an individual country only when investors expect that the monetary union will not have heterogeneity. Instead, if lenders expect the union to be in fragmented equilibrium (with non-zero labor wedges), there is a lower threshold of the safe zone, which we denote $\hat{B}_s$, with $\hat{B}_s < B_s$. We formalize this result in the next proposition.

Proposition 2. *Let B_s, B_h, e_S, and $e_F^-(\phi_H)$ be the thresholds for government debt and the nominal exchange rates for the safe and default zones, such that the debt thresholds solve:*

$$\phi_L = \bar{\phi}\left(\frac{R^*}{1-\rho}B_s, e_S\right)$$

$$\phi_H = \bar{\phi}\left(\frac{R^*}{1-\rho}B_h, e_F^-(\phi_H)\right)$$

And the thresholds for the monetary policy solve:

$$0 = \tau_1\left(y^T - R^*B_s, e_S\right)$$

$$0 = \pi \left[\tau_1 \left(y^T - \frac{R^*}{1-\rho} B_h, e_F^-(\phi_H) \right) \right] \mathcal{H}_e \left(y^T - \frac{R^*}{1-\rho} B_h, e_F^-(\phi_H) \right) + (1-\pi) \left[\tau_1 \left(y^T - R^* B_h, e_F^-(\phi_H) \right) \right] \mathcal{H}_e \left(y^T - R^* B_h, e_F^-(\phi_H) \right)$$

Then, at least another two thresholds exist for the safe and default zone.

Proof. In Appendix A.3 □

Proposition 2 establishes a novel form of multiplicity beyond the one described in Ayres et al. (2018). The multiplicity in our model is different in the following sense. First, the multiplicity in Calvo (1988) arises due to the interaction between the government default policy and the interest rate. Given a debt policy, a high interest rate generates incentives for the government to default, which, in turn, increases the risk and gives support to high interest rates in the first place. This mechanism implies that there are two possible combinations of interest rates and probabilities of default that can be an equilibrium. In equation (3.5) when we define the interest rate schedule, we use a sunspot to restore uniqueness with respect to this type of multiplicity. However, proposition 2 implies that using a sunspot is not enough to have a unique equilibria in this environment.

The second key difference is that this multiplicity stems from the interaction between monetary policy, default policy, and interest rate schedules. If the central bank stabilizes employment in countries facing high interest rates, it eliminates their self-fulfilling crisis risk, making interest rates dependent on monetary policy. However, the central bank's cost of achieving zero labor wedge depends on the interest rate schedule itself. In the bad equilibrium, the interest rate schedule creates heterogeneity among union members, increasing the cost of implementing an expansionary monetary policy. As a consequence, when the union enters a fragmented region, implementing zero labor wedge becomes infeasible, which in turn increases the risk of default. This creates a reinforcement loop: monetary policy affects interest rates which affect the feasibility of monetary policy, generating fragmented equilibria where some members face high default probability.

Third, the multiplicity in Proposition 2 refers to the union rather than to individual countries, having one symmetric equilibrium and one fragmented equilibrium. Here, the risk of default in one country spreads to the other countries through monetary policy, increasing the risk for the other members of the union. When the risk of default increases

in one member of the union the response of the CB increases risk of default of other members of the union.

6.2 Role of Commitment

Interestingly, under the commitment of the CB, the equilibrium is unique. In Figure 6, we plot the default and repayment values for the government at high and low interest rates. In addition, we plot the optimal exchange rate under commitment.

When the CB has the technology to commit to a nominal exchange rate, it chooses the same nominal exchange rate as in the best Markov equilibrium ($e_C = e_S$). Under $e_C = e_S$, the value of repayment is strictly larger than the value of default at the risk-free rate and weakly larger at the high rate, where the government is indifferent. By the tie-breaking rule the government repays, so default occurs with zero probability; consequently, there is no support for a high interest rate, and the equilibrium is unique.

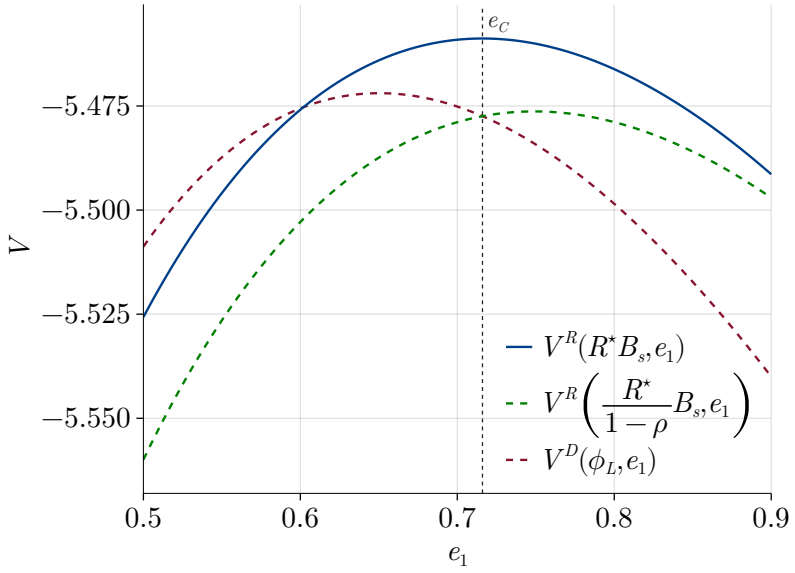


Figure 6: Multiplicity and Commitment

Note: In this figure, we use $y^T = 0.8$, $\beta = 0.91$, $R_0 B_0 = 0.65$, $R^* = 1.04$, $\rho = 0.2$, $\omega = 0.3$, $\mu = (1/0.83) - 1$ and $\pi = 0.05$

In this example, there is no cost to the central bank of committing to a noncontingent nominal exchange rate. We can see this by noting that, in the best Markov equilibrium,

the optimal policy is not contingent on the realization of the default cost and coincides with the optimal policy under commitment.

In economies with a positive risk of default in the best equilibrium, the CB would face the traditional trade-off between commitment and flexibility. However, the insight that commitment helps the CB to eliminate multiplicity will follow, providing an incentive for the CB to prefer commitment over flexibility.

6.3 Optimal Debt in a Monetary Union

The proposition 2 implies that there are at least two equilibrium interest rate schedules. To study the effect of multiplicity in the optimal policy of period zero, we assume that countries will know the interest rate schedules they face (that is, there is no uncertainty about the thresholds) before choosing the level of debt. In particular, we consider the following two interest rate schedules.

$$R(b_1, \xi) = \begin{cases} R^* & b_1 < B_s \\ R^* & B_h > b_1 \geq B_s \text{ and } \xi = 1 \\ \frac{R^*}{1-\rho} & B_h \geq b_1 \geq B_s \text{ and } \xi = 0 \\ \infty & b_1 > B_h \end{cases}$$

Here, B_h and B_s follow the definition in Proposition 2. We call this schedule the *optimistic* schedule. In this schedule, lenders coordinate the higher thresholds in accordance with the equilibrium conditions. We also consider the following interest rate schedule.

$$\hat{R}(b_1, \xi) = \begin{cases} R^* & b_1 < \hat{B}_s \\ R^* & \hat{B}_h > b_1 \geq \hat{B}_s \text{ and } \xi = 1 \\ \frac{R^*}{1-\rho} & \hat{B}_h \geq b_1 \geq \hat{B}_s \text{ and } \xi = 0 \\ \infty & b_1 > \hat{B}_h \end{cases}$$

Here, $e_F^+(\phi_L)$ solves (36) as defined before and e_D solves:

$$0 = \tau_1 \left(y^T, e_D \right) \tag{38}$$

In addition, $\hat{B}_s, \hat{B}_h$ are defined as the levels of debt at which:

$$\phi_L = \bar{\phi}\left(\frac{R^*}{1-\rho}\hat{B}_s, e_F^+(\phi_L)\right) \tag{39}$$

$$\phi_H = \bar{\phi}\left(\frac{R^*}{1-\rho}\hat{B}_h, e_D\right) \tag{40}$$

We label this schedule the *pessimistic* interest rate schedule because lenders coordinate the lower thresholds consistent with equilibrium conditions. This schedule is the fixed point under the belief that the central bank plays the fragmented continuation: $e_F^+(\phi_L)$ and e_D are the central bank's optimal exchange rates under that belief in the ϕ_L (fragmented) and ϕ_H (all-default) states, so $\hat{B}_s$ and $\hat{B}_h$ are the belief-consistent safe- and default-zone thresholds rather than policies imported from other configurations.

In Figure 7, we plot the government value in period zero as a function of debt. We denote $\overline{V}_0$ as the value of the government that faces a *pessimistic* interest rate schedule and $\hat{V}_0$ the value of the government that faces an *optimistic* interest rate schedule.

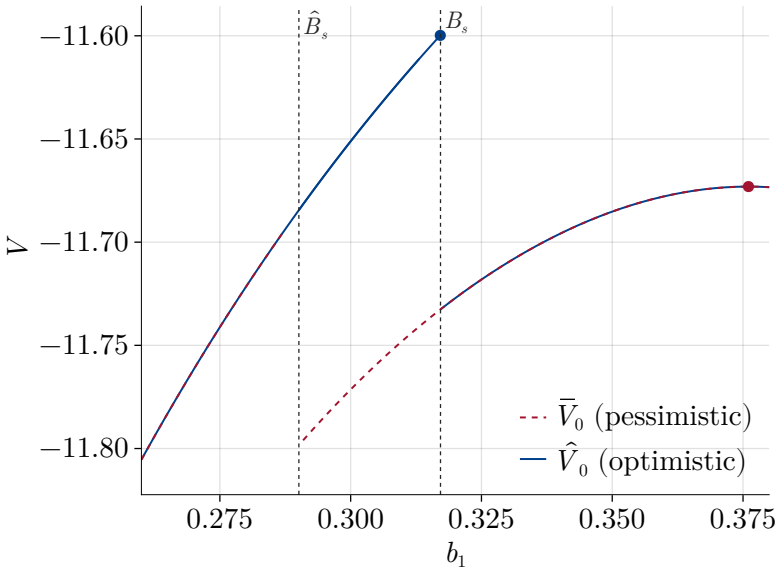


Figure 7: Optimal Debt and Multiplicity

Note: In this figure, we use $y^T = 0.8$, $\beta = 0.91$, $R_0 B_0 = 0.65$, $R^* = 1.04$, $\rho = 0.2$, $\omega = 0.3$, $\mu = (1/0.83) - 1$ and $\pi = 0.05$

In this example, facing a more favorable interest-rate schedule incentivizes the government to reduce consumption and avoid entering the multiplicity region. On the other hand,

a government facing a worse interest rate function finds it costly to restrict consumption to the safe zone, so it chooses to enter the multiplicity region.

6.4 Optimal Monetary Policy in the Monetary Union

We now describe how the central bank's optimal monetary policy depends on the level of union-wide debt B_1 and on which equilibrium prevails. Recall that, by symmetry, all countries choose the same debt B_1 in period zero.

Consider first the good (symmetric) Markov equilibrium. For low debt levels, $B_1 < B_s$, there is no heterogeneity: all countries face the risk-free rate and repay, so the central bank can implement a zero labor wedge for every member. The optimal exchange rate e_1^M solves

$$0 = \tau_1 \left(y^T - R^\star B_1, e_1^M \right). \tag{41}$$

For intermediate debt levels, $B_1 \in (B_s, B_h)$, a fraction π of countries receives a high interest rate and the union fragments, so the optimal policy becomes state contingent. When the default cost is low (ϕ_L), the π high-rate countries default and the optimal exchange rate $e_1^M(\phi_L)$ solves

$$\begin{aligned} 0 = \pi\, \tau_1 \left(y^T, e_1^M(\phi_L) \right) \mathcal{H}_e \left(y^T, e_1^M(\phi_L) \right) \\ + (1-\pi)\, \tau_1 \left(y^T - R^\star B_1, e_1^M(\phi_L) \right) \mathcal{H}_e \left(y^T - R^\star B_1, e_1^M(\phi_L) \right). \end{aligned} \tag{42}$$

When the default cost is high (ϕ_H), all countries repay (the π high-rate countries at the rate $\frac{R^\star}{1-\rho}$), and $e_1^M(\phi_H)$ solves

$$\begin{aligned} 0 = \pi\, \tau_1 \left(y^T - \tfrac{R^\star}{1-\rho} B_1, e_1^M(\phi_H) \right) \mathcal{H}_e \left(y^T - \tfrac{R^\star}{1-\rho} B_1, e_1^M(\phi_H) \right) \\ + (1-\pi)\, \tau_1 \left(y^T - R^\star B_1, e_1^M(\phi_H) \right) \mathcal{H}_e \left(y^T - R^\star B_1, e_1^M(\phi_H) \right). \end{aligned} \tag{43}$$

Because the high-rate countries run a more contractionary fiscal policy under ϕ_H than under ϕ_L, the central bank implements a more expansionary policy in the former state, so that $e_1^M(\phi_L) < e_1^M(\phi_H)$.

Figure 8 shows the optimal exchange rate e_1 as union debt B_1 rises, in the low default-cost state ϕ_L. It plots three policies: the good (optimistic) Markov equilibrium (solid), the bad (pessimistic) Markov equilibrium (dotted), and commitment (dashed). When debt is

low the three coincide: no country defaults, and the central bank keeps every member at full employment. As debt grows, the union becomes vulnerable to a self-fulfilling crisis. The bad equilibrium breaks down first: once debt passes the threshold $\hat{B}_s$, pessimistic investors charge high interest rates, a fraction of countries default, and the exchange rate drops to the crisis level. The good equilibrium holds out longer and only breaks down at the higher threshold B_s. In between, for $\hat{B}_s < B_1 < B_s$, the same level of debt is consistent with either outcome—the safe path or the crisis—depending only on what investors expect.

Commitment changes this. In the region with multiple equilibria, $\hat{B}_s < B_1 < B_s$, commitment works as a selection device: by promising a sufficiently expansionary exchange rate, the central bank makes the crisis equilibrium impossible and keeps the union on the safe path. For higher debt, $B_1 > B_s$, even the good Markov equilibrium falls into crisis and a fraction of countries default; there commitment goes one step further and runs a *more expansionary* policy than the Markov government would. This larger e_1—a more depreciated domestic currency—lowers the interest rate paid by the countries that would otherwise be charged the high rate, compressing their spreads and cushioning the cost of the crisis.

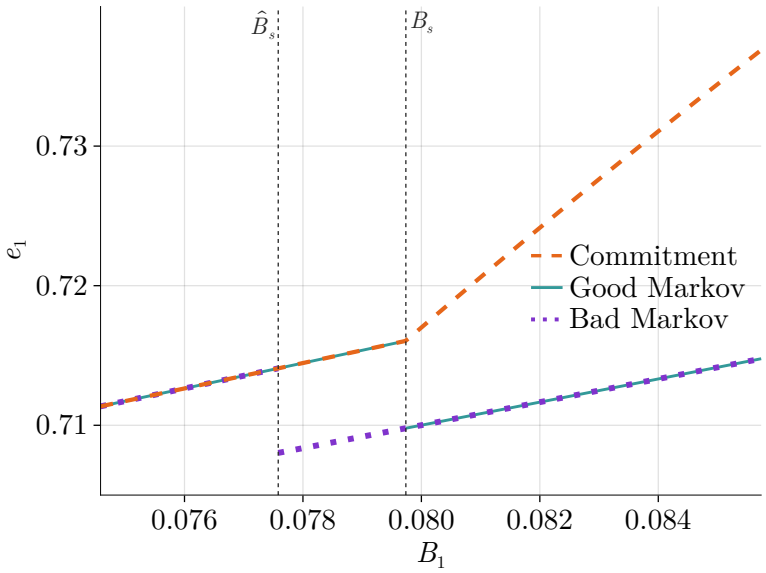


Figure 8: Optimal Monetary Policy in the Monetary Union

Note: Optimal union exchange rate e_1 against union debt B_1 in the low default-cost state ϕ_L: good Markov (solid), bad Markov (dotted), and commitment (dashed). The thresholds $\hat{B}_s < B_s$ delimit the bad- and good-Markov safe zones. Parameters: $y^T = 0.8$, $\beta = 0.91$, $R_0 B_0 = 0.65$, $R^* = 1.04$, $\rho = 0.2$, $\omega = 0.3$, $\mu = (1/0.83) - 1$ and $\pi = 0.05$.

7 Extensions

This section discusses two extensions to the baseline model. First, we allow the CB to act as a lender of last resort, and second, we assume there is a cost for the CB to change the nominal exchange rate.

7.1 Anti-Fragmentation Policies

The baseline model assumes that the CB cannot intervene in the financial markets by buying government bonds. This assumption helps to focus on the relationship between fiscal and monetary policy through its effect on the labor market. However, in practice, the CB can act as a lender of last resort to align investors' expectations and select the best equilibrium.

In Appendix B, we relax this assumption by allowing the central bank to impose a lump-sum tax over the members of the union that we interpret as seigniorage and use those resources to purchase governments' bonds. With lump-sum transfers, the budget constraint of the CB becomes:

$$\begin{aligned} \int_i [T_0] \; di &= \int_i \left[B_i^{CB} \right] \; di, \\ \int_i [T_1] \; di &= \int_i \left[(1 - d_i) \, R_{1,i} \, B_i^{CB} \right] \; di \end{aligned} \tag{44}$$

Here, B_i^{CB} denotes the principal (proceeds) the CB lends country i when it buys its bonds: at date zero the CB levies the lump-sum seigniorage tax T_0 to finance the purchases, and at date one it collects $R_{1,i} \, B_i^{CB}$ from every government that repays and rebates the proceeds lump-sum through T_1. Because all countries are homogeneous at period zero, the tax is homogeneous. On the no-fragmentation path all members repay ($d_i = 0$) and $R_{1,i} = R^\star$, so the operation breaks even in present value and is costless to the CB. With this tool, the CB can eliminate fragmented equilibria by announcing that it will buy bonds from any government facing a high interest rate. At equilibrium, the policy has no cost to the CB because there would be no purchases.

However, an important constraint of this policy is that the CB should be able to guarantee that if investors refuse to buy government bonds at a low interest rate, the CB can honor its promise to buy those bonds. Because the CB is bounded by the period-zero

resource constraints of the countries, and must be able to purchase the proceeds B_1 of the fraction π of countries that would face a high interest rate, an antifragmentation policy is feasible if:

$$y^T \geq \pi B_1 \tag{45}$$

This constraint reflects a well-studied concern that cap policies on spreads could eliminate the possibility of multiple equilibria as long as the third party can guarantee it would have enough resources to make purchases that move spreads to the promised cap in any out-of-equilibrium path.

Furthermore, in a monetary union, investors' confidence in the policy would depend on the number of countries facing high interest rates. Antifragmentation policies are more likely to be effective as the percentage of countries receiving a high interest rate π is low.

Another assumption is that the fiscal incidence of money creation (seigniorage) is a lump-sum levy on union members. Instead, suppose that there is a cost for the CB to collect resources. In that case, it might not be optimal for the CB to implement antifragmentation policies when the cost of collecting resources is high enough. Because seigniorage is in practice costly and the CB's capacity to finance these operations is limited, antifragmentation policies are likely to complement the expansionary monetary policy analyzed above.

7.2 Cost of Devaluation

Another assumption is that the CB incurs no cost in managing the nominal exchange rate. In Appendix C, we relax this assumption by introducing a cost for the CB of deviating from a target in the nominal exchange rate. This extension considers a CB that targets inflation, or that faces a cost of using monetary policy to stabilize employment.

For concreteness in this extension, we assume that in the case of multiplicity, with probability π, all union members receive a high interest rate. In contrast, with probability $(1 - \pi)$, all members receive a low interest rate. This assumption eliminates heterogeneity among countries, which removes the CB's cost of fragmentation and isolates the effect of an exogenous cost of deviating from the nominal exchange rate target.

Because this assumption removes the cross-country heterogeneity that generates the fragmentation thresholds of Proposition 2, the multiplicity thresholds are unique. The

exogenous devaluation cost, by contrast, leaves monetary policy time-inconsistent, as in the baseline model.

As in the baseline model, the CB has an incentive to promise future devaluation in period zero to reduce interest rates. It also has incentives to deviate in period one to avoid the cost of moving the nominal exchange rate from the target. On the other hand, for some levels of debt in the multiplicity region, the CB could select the preferred equilibrium if it has the technology to commit to a future devaluation.

On the other hand, because the CB's cost of moving the nominal exchange rate is fixed, this extension differs from the baseline model in that there are no complementarities between the monetary policy and the interest rate schedule. As a result, the boundaries of the multiplicity regions are unique.

8 Conclusions

A central bank in a monetary union can shield its members from self-fulfilling debt crises by committing to run the economy hot: by promising an expansionary stance, it raises the value of repayment and removes the high-interest-rate equilibrium. Because it cannot deliver full employment to members facing divergent spreads, confidence risk also produces endogenous fragmentation, and the same commitment selects the safe equilibrium for the union as a whole. Once such risk is present, the classic case for a conservative central banker is inverted: because the commitment policy stabilizes demand, delegating to a more dovish central banker moves the discretionary outcome toward commitment and helps anchor expectations away from default. The framework rationalizes both the divergence of euro-area periphery spreads during stress and the calming effect of credible commitments to intervene.

A natural next step is to embed the mechanism in a dynamic, quantitative model with long-term debt, where the interaction between the maturity structure and the central bank's stabilization motive can be disciplined against the spread dynamics of individual member states.

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A Proofs

A.1 Lemma 1

Lemma 1 There exist two thresholds of debt $\underline{b}(e_1)$, $\overline{b}(e_1)$ such that $b_{i,1} \in \mathcal{I}^R$ if and only if $\overline{b}(e_1) \geq b_{i,1} > \underline{b}(e_1)$.

Proof. We want to prove:

1. $$V^D(e_1) \geq V^R(y_L^T - bR^*, e_1)$$

   if and only if $b \geq \overline{b}(e_1)$.

2. $$V^D(e_1) \geq V^R(y^T - b\frac{R^*}{1-\rho}, e_1)$$

   if and only if $b \geq \underline{b}(e_1)$.

3. And $\overline{b}(e_1) \geq \underline{b}(e_1)$.

Let $\overline{e}$ be the nominal exchange rate. Then it is possible to establish:

1. First, note $V^D(\overline{e})$ is independent of bR^*
2. Second, at $b = 0$: $V^R(y_L^T - bR^*, \overline{e}) > V^D(\overline{e})$
3. Third, because V^R is strictly decreasing in bR^* under the maintained condition $u_T + \tau \mathcal{H}_{c^T} > 0$ (so that the loss of tradable resources dominates the accompanying fall in nontradable labor disutility), both functions cross at most once and the threshold $\overline{b}_{flex}$ is unique.
4. Fourth, note we can use the same argument to establish the uniqueness of $\underline{b}_{flex}$
5. Finally, $u()$ is strictly increasing in c^T. Also:

$$V^R(y^T - \overline{b}_{flex}R^*, \overline{e}) = V^D(\overline{e}) = V^R(y^T - \underline{b}_{flex}\frac{R^*}{1-\rho}, \overline{e})$$

Then it has to be that $\overline{b}_{flex}R^* = \underline{b}_{flex}\frac{R^*}{1-\rho}$ which implies $\overline{b}(e_1) \geq \underline{b}(e_1)$.

□

A.2 Proposition 1

Let $\underline{b}_{flex}$, $\overline{b}_{flex}$, $\underline{b}_{fix}(\overline{e})$ and $\overline{b}_{fix}(\overline{e})$ be the threshold debt for an economy with a flexible and a fixed exchange rate regime, respectively. Define e^+ and e^- as:

$$V_1^D(\phi_L, e^+) = V_1^R\left(R^\star \underline{b}_{flex}, e^+\right)$$
$$V_1^D(\phi_L, e^-) = V_1^R\left(\frac{R^\star}{1-\rho}\overline{b}_{flex}, e^-\right)$$

Then:

1. $\underline{b}_{flex} > \underline{b}_{fix}(\overline{e})$ for every $\overline{e} < e^+$. And $\underline{b}_{flex} \leq \underline{b}_{fix}(\overline{e})$ for every $\overline{e} \geq e^+$
2. $\overline{b}_{flex} > \overline{b}_{fix}(\overline{e})$ for every $\overline{e} < e^-$. And $\overline{b}_{flex} \leq \overline{b}_{fix}(\overline{e})$ for every $\overline{e} \geq e^-$

Proof. By definition, at e^+, e^-, the thresholds of the economy under a fixed and flexible monetary policy of the union coincide. Then, we need to prove that the thresholds are strictly increasing in e_1 to establish the result.

Let $\overline{e} < e^+$. By definition:

$$V_1^D(\phi_L, e^+) = V_1^R\left(R^\star \underline{b}_{flex}, e^+\right)$$

So we want to prove the following:

$$V_1^D(\phi_L, \overline{e}) > V_1^R\left(R^\star \underline{b}_{flex}, \overline{e}\right)$$

We prove that $\underline{b}_{flex}(e)$ is strictly increasing in e, which delivers the result; the argument for $\overline{b}_{flex}(e)$ is identical after replacing $R^\star$ by $\frac{R^\star}{1-\rho}$ (both thresholds are defined at the same default cost ϕ_L; only the interest rate differs). The threshold solves $V_1^R\left(R^\star \underline{b}_{flex}, e\right) = V_1^D(\phi_L, e)$. By the implicit function theorem,

$$\frac{d\underline{b}_{flex}}{de} = \frac{V_{1,e}^D - V_{1,e}^R}{R^\star\, \partial V_1^R/\partial(Rb)},$$

and since $\partial V_1^R/\partial(Rb) < 0$, the threshold is strictly increasing in e if and only if $V_{1,e}^R > V_{1,e}^D$. Differentiating the value functions gives $V_{1,e}^R = \tau\left(c^T, e\right)\mathcal{H}_e\left(c^T, e\right)$ evaluated at $c^T = y^T -$

$\underline{b}_{flex} R^\star$, and $V^D_{1,e} = \tau\left(y^T, e\right)\mathcal{H}_e\left(y^T, e\right)$. Hence the threshold is increasing in e whenever $g\left(c^T\right) \equiv \tau\left(c^T, e\right)\mathcal{H}_e\left(c^T, e\right)$ is decreasing in c^T.

Along the market-clearing locus $\mathcal{H}(c^T, e) = \kappa(e)c^T$, we have $\mathcal{H}_e = \kappa'(e)c^T$ and $\mathcal{H}_{e,c^T} = \kappa'(e) > 0$. With linear disutility ($\nu' = 1$), $\tau = u_N - 1$, and since u_N is homogeneous of degree $-\sigma$ along this locus,

$$\tau_{c^T} = u_{N,T} + u_{N,N}\mathcal{H}_{c^T} = -\sigma\,\frac{u_N}{c^T} < 0.$$

Therefore

$$g_{c^T} = \tau_{c^T}\,\mathcal{H}_e + \tau\,\mathcal{H}_{e,c^T} = \kappa'(e)\left[(1-\sigma)u_N - 1\right],$$

which is negative whenever $\sigma \geq 1$. Under this condition g is decreasing in c^T, so $V^R_{1,e} > V^D_{1,e}$ and $\underline{b}_{flex}(e)$ is strictly increasing in e.

□

A.3 Proposition 2

Proof. We establish existence by construction, under the maintained continuity and monotonicity of the default threshold $\bar{\phi}(\cdot)$ in its arguments. Let us begin with B_s, e_s. Assume B_s, e_s are the unique threshold for the safe zone. Then there does not exist another combination of $B_s, e_s, R^\star$ that configures an equilibrium given $B_1 = B_s$. In the body of the text we provide a counterexample, which leads to a contradiction.

Next, let us consider the combination of $B_h, e^-_F(\phi_H)$. Assume they are the unique combination of debt and the nominal exchange rate that configures the thresholds of the default zone. Then there does not exist any other combination of $B_h, e^-_F(\phi_H), \frac{R^\star}{1-\rho}$ that configures an equilibrium.

We now construct this second combination explicitly. Let the boundary of the high interest rate zone be $B_h = \overline{b}\left(e^-_F(\phi_H)\right)$ and monetary policy when governments choose $B_1 = B_h$ will be given by $e^-_F(\phi)$. We confirm the guess by verifying:

$$0 = \pi\left[\tau_1\left(y^T - \frac{R^*}{1-\rho}B_h, e^-_F(\phi_H)\right)\right]\mathcal{H}_e\left(y^T - \frac{R^*}{1-\rho}B_h, e^-_F(\phi_H)\right) + (1-\pi)\left[\tau_1\left(y^T - R^*B_h, e^-_F(\phi_H)\right)\right]\mathcal{H}_e\left(y^T - R^*B_h, e^-_F(\phi_H)\right) \quad (46)$$

$$\phi_H = \bar{\phi}\left(\frac{R^*}{1-\rho}B_h, e^-_F\left(\phi_H\right)\right) \quad (47)$$

B_h and $e^-_F(\phi)$ are the combination of fiscal and monetary policies in period zero such that if

the cost of default and the interest rate are high, governments would be indifferent between repayment and default. In other words, B_h is the maximum level of debt that the government can support with a finite interest rate. If the government increases the debt beyond B_h, the lenders would anticipate that even if the default cost is high, the government would default. For $B_1 > B_h$, there is no support for a finite interest rate. Also, we highlight that at B_h the government will default if the cost of default is low, so the interest rate in equilibrium is the high interest rate.

Absent heterogeneity, the CB chooses an expansionary monetary policy $e_F^-(\phi_H)$ that closes the output gap. In turn, because governments are facing a finite interest rate, they choose to implement a contractionary fiscal policy and repay, which gives incentives to the CB to implement an expansionary monetary policy. However, it is possible to construct a second equilibrium given $B_1 = B_h$. Before doing it, let us define the optimal monetary policy when all governments default as follows.

$$0 = \tau_1 \left(y^T, e_D \right) \tag{48}$$

If lenders expect the CB to follow a less expansionary policy e_D, they anticipate that the government will default even if the default cost is high. In this case, the interest rate will be infinite for all countries. All countries will default and the government will implement e^D that confirms the initial expectation.

□

B Anti-Fragmentation Policies

This Appendix considers the case where the central bank can perform financial operations to stabilize spreads and avoid fragmentation. In particular, we assume that the central bank can tax countries (a seigniorage tax) and buy bonds at the fundamental price.

Problem in Period One Then, the problem of each country in period one becomes:

$$V_1(R_{1,i}b_{1,i}, e_1(\Omega_u), T_1) = \max_d (1-d)V^R(R_{1,i}b_{1,i}, \Omega_u) + d(V^D(\phi, \Omega_u)) \tag{49}$$

where V^R and V^D are

$$V^R(R_{1,i}b_{1,i}, e_1(\Omega_u), T_1) = u(y^T - b_{1,i}R_{1,i} + T_1, \mathcal{H}(y^T - b_{1,i}R_{1,i} + T_1, e_1(\Omega_u))) - v(\mathcal{H}(y^T - b_{1,i}R_{1,i} + T_1, e_1(\Omega_u)))$$

$$V^D(\phi, e_1(\Omega_u)) = u(y^T + T_1, \mathcal{H}(y^T + T_1, e_1(\Omega_u))) - v(\mathcal{H}(y^T + T_1, e_1(\Omega_u))) - \phi$$

And the problem of the central bank becomes:

$$W(\Omega_u) = \max_{e_1, T_1} \int_i [V_1(R_{1,i}b_{1,i}, e_1(\Omega_u))]di \tag{50}$$

$$\text{s.t} \int_i [T_1]di = \int_i [(1-d_i)R_{1,i}B_i^{CB}]di \tag{51}$$

Note that on the no-fragmentation equilibrium path all members repay, so the central bank's bond purchases carry no risk.

Problem in Period Zero In period zero, the problem of the government becomes:

$$V_0(e_0, e_1(\Omega_u), T_0) = \max_{b_{1,i}} u(c_{0,i}^T, \mathcal{H}(c_{0,i}^T, e_0)) - v(\mathcal{H}(c_{0,i}^T, e_0)) + \beta\mathbb{E}[V_1(R(b_{1,i}, \xi_i)b_{1,i}, e_1(\Omega_u))] \tag{52}$$

subject to

$$c_{0,i}^{T} = y^{T} + b_1 - R_0 b_0 - T_0$$
$$R^{*} = [1 - F(R_{1,i} b_{1,i}, e_1(\Omega_u))] R_{1,i}$$

The problem of the central bank becomes:

$$W_0 = \max_{e_0, T_0, B_i^{CB}} \int_i [V_0(e_0, e_1(\Omega_u))] di \tag{53}$$
$$\text{s.t} \quad \int_i [T_0] di = \int_i [B_i^{CB}] di$$

Here B_i^{CB} denotes the principal (proceeds) the CB lends country i: at date zero the CB finances the purchase with T_0, and at date one it collects $R_{1,i} B_i^{CB}$ from each repaying government and rebates it through T_1. If the CB has enough resources to buy b_i from a member of the union at a rate $R^{\star}$ when $b_i \in \mathcal{I}^R$, it can select the best equilibrium, with no need to perform or promise a monetary easing. Because the purchase is costless when the central bank has sufficient resources, executing it on the pessimistic path—after lenders coordinate on the high rate—is sequentially optimal, so the policy is credible even without commitment.

The ability of the central bank to select the best equilibrium in the union depends on its capacity to absorb large stocks of debt and on the size of π.

C Monetary Policy with Exogenous Cost of Devaluation

First, note that because there is no heterogeneity, the problem of the Monetary Union can be written as the problem of a single Small Open Economy. For simplicity, we also combine the monetary and the fiscal authority and call it the government. We will characterize the government's optimal policy. Since the government lacks commitment, we will focus on the Markov equilibrium.

Government objective The preferences of the government are described by:

$$U = u(c_0) - v(h_0) - \gamma\Phi(e_0 - \bar{e}) + \beta\mathbb{E}[u(c_1(s)) - v(h_1(s)) - \gamma\Phi(e_1(s) - \bar{e}) - \phi d(s)] \tag{54}$$

where ϕ follows a bimodal distribution as discussed before, and Φ is a loss function for the government from setting a nominal exchange rate different from the target $\bar{e}$.

C.1 Optimal Policy without Commitment in Monetary Policy

Roadmap We solve the model by backward induction. So, the government in the second period decides its optimal policy based on the relevant state $\{R_1 b_1, \phi\}$. Also, given the risk neutrality of international investors, we replace their problem with a break-even condition. Finally, in the first period, the government decides the optimal debt taking as given the policy functions of the government in the second period and the interest rate function.

Second Period Problem The government value function in the second period is the following:

$$V_1(R_1 b_1, \phi) = \max_d (1-d)V^R(R_1 b_1) + d(V^D - \phi) \tag{55}$$

where V^R and V^D represent the value functions of repaying and defaulting. Under repayment, the government value is:

$$V^R(R_1 b_1) = \max_{e_1^R} \; u(y^T - b_1 R_1, \mathcal{H}(y^T - b_1 R_1, e_1^R)) - v(\mathcal{H}(y^T - b_1 R_1, e_1^R)) - \gamma\Phi(e_1^R - \bar{e}) \tag{56}$$

Where the nominal exchange rate e_1^R solves:

$$0 = \tau_1(y^T - b_1 R_1, e_1^R)\mathcal{H}_e(y^T - b_1 R_1, e_1^R) - \gamma\Phi'(e_1^R - \bar{e}) \tag{57}$$

Condition (57) is the optimality condition that equates the marginal cost of devaluation to the marginal benefit. The latter is given by the marginal benefit from reducing the labor wedge.

On the other hand, if the government defaults, the value is:

$$V^D = \max_{e_1^D} \; u(y^T, \mathcal{H}(y^T, e_1^D)) - v(\mathcal{H}(y^T, e_1^D)) - \gamma\Phi(e_1^D - \bar{e}) \tag{58}$$

where the nominal exchange rate e_1^D solves:

$$0 = \tau_1(y^T, e_1^D)\mathcal{H}_e(y^T, e_1^D) - \gamma\Phi'(e_1^D - \bar{e}) \tag{59}$$

Condition (59) is the counterpart of (57) for the case that the government defaults. The difference between both conditions is that the marginal benefit of the government of using monetary policy under default is different because the consumption of tradable goods is higher.

It is possible to characterize the default decision of the government in the second period by defining the following threshold:

$$\bar{\phi}(R_1 b_1) = V^D - V^R(R_1 b_1) \tag{60}$$

This is the surplus for the government in utility units from increasing consumption by default. The government defaults whenever the surplus of doing so is higher than the cost of default, so the default function is:

$$d(R_1 b_1) = \begin{cases} 1 & \text{if} \quad \phi < \bar{\phi}(R_1 b_1), \\ 0 & \text{otherwise.} \end{cases} \tag{61}$$

The probability of default is:

$$F(R_1 b_1) = Pr[\phi < \bar{\phi}(R_1 b_1)] \tag{62}$$

International Investors Using the probability of default of the government in the second period. The break-even condition of international investors is as follows:

$$R^* = [1 - F(R_1 b_1)]R_1 \tag{63}$$

First Period Problem Given the government policies in the second period V_1, the government in the first period solves:

$$V_0 = \max_{b_1, e_0} u(c_0^T, \mathcal{H}(c_0^T, e_0)) - v(\mathcal{H}(c_0^T, e_0)) - \gamma\Phi(e_0 - \bar{e}) + \beta\mathbb{E}[V_1(R(b_1, \xi)b_1, \phi)] \tag{64}$$

subject to

$$c_0^T = y^T + b_1 - R_0 b_0$$
$$R^* = [1 - F(R_1 b_1)]R_1$$

The FOC of the nominal exchange rate is:

$$0 = \tau_0(c_0^T, e_0)\mathcal{H}_e(c_0^T, e_0) - \gamma\Phi'(e_0 - \bar{e}) \tag{65}$$

This condition equates the marginal benefit of employment by changing e_0 to the marginal cost of devaluing from the target in the first period. Unlike the FOC in the second period, the marginal benefit of devaluation depended on the level of government borrowing. When the government chooses a high b_1, it increases transfers and employment, which changes the marginal benefit of using monetary policy to increase employment.

Markov Equilibrium Now, we are ready to define the Markov equilibrium in this environment.

Definition 5. (Markov Equilibrium) Given initial conditions $\{R_0 b_0\}$; a Markov perfect equilibrium is defined by a set of strategy profile $\{\mathcal{E}_0, \mathcal{B}_1, \mathcal{E}_1(Rb_1, \phi), \mathcal{D}_1(Rb_1, \phi)\}$; value functions $\{V_0, V_1(Rb_1, \phi)\}$ and an interest rate schedule $\mathcal{R}(b_1)$ such that:

i Given the interest rate schedule: $\{\mathcal{E}_0, \mathcal{B}_1\}$ solves the government problem in period zero, and V_0 attains the maximum.

ii Given a face value of debt; $\{\mathcal{E}_1(Rb_1, \phi), \mathcal{D}_1(Rb_1, \phi)\}$ solves the government problem at every state in period one, and $V_1(Rb_1, \phi)$ attains the maximum.

iii Given the policy functions of the government: $\mathcal{R}(b_1)$ satisfies the break-even condition of international investors.

C.2 Optimal Policy with Commitment in Monetary Policy

We close the description of the model by describing the government's problem when it can commit to monetary policy in the second period. We assume that the commitment is non-state-contingent. The government promises a given e_1 regardless of the shocks (or the default decision).

We begin by describing the problem of the government in the second period as a function of the relevant state, which in this case is $\{R_1 b_1, \phi, e_1\}$. In this case, the value of the government is defined as:

$$W_1(R_1 b_1, \phi, e_1) = \max_d (1-d)\, W^R(R_1 b_1, e_1) + d\, \left(W^D(e_1) - \phi \right) \tag{66}$$

Where W^R and W^D are defined as:

$$\begin{aligned} W^R(R_1 b_1, e_1) &= u(y^T - b_1 R_1, \mathcal{H}(y^T - b_1 R_1, e_1)) - v(\mathcal{H}(y^T - b_1 R_1, e_1)) - \gamma \Phi(e_1 - \bar{e}) \\ W^D(e_1) &= u(y^T, \mathcal{H}(y^T, e_1)) - v(\mathcal{H}(y^T, e_1)) - \gamma \Phi(e_1 - \bar{e}) \end{aligned}$$

We can describe the default decisions of the government using the threshold:

$$\bar{\phi}_C(R_1 b_1, e_1) = W^D(e_1) - W^R(R_1 b_1, e_1) \tag{67}$$

In this case, the default function of the government is:

$$d_C(R_1 b_1, e_1) = \begin{cases} 1 & \text{if} \quad \phi < \bar{\phi}_C(R_1 b_1, e_1), \\ 0 & \text{otherwise.} \end{cases} \tag{68}$$

Here, the probability of default becomes:

$$F_C(R_1 b_1, e_1) = Pr[\phi < \bar{\phi}_C(R_1 b_1, e_1) \mid R_1 b_1] \tag{69}$$

Also, given the break-even condition of international investors, the equilibrium interest rate of the government is:

$$R_{1,C}(b_1, \xi, e_1) = \begin{cases} R^* & b_1 < \underline{b}_C(e_1) \\ R^* & \overline{b}_C(e_1) > b_1 \geq \underline{b}_C(e_1) \text{ and } \xi = 1 \\ \frac{R^*}{1-\rho} & \overline{b}_C(e_1) > b_1 \geq \underline{b}_C(e_1) \text{ and } \xi = 0 \\ \frac{R^*}{1-\rho} & b_1 \geq \overline{b}_C(e_1) \end{cases}$$

Where $\underline{b}_C(e_1), \overline{b}_C(e_1)$ are defined as:

$$\underline{b}_C(e_1) = \{b_1 : \bar{\phi}_C(R^\star b_1, e_1) = \phi_L\}$$
$$\overline{b}_C(e_1) = \left\{b_1 : \bar{\phi}_C\left(\frac{R^\star}{1-\rho} b_1, e_1\right) = \phi_L\right\}$$

Let us define the problem of a government with the technology to commit to a monetary policy. The government in the first period solves:

$$W = \max_{b_1, e_0, e_1} u(c_0^T, \mathcal{H}(c_0^T, e_0)) - v(\mathcal{H}(c_0^T, e_0)) - \gamma\Phi(e_0 - \bar{e}) + \beta\mathbb{E}[W_1(R_{1,C}(b_1, \xi, e_1)b_1, \phi, e_1)] \quad (70)$$
$$\text{s.t } c_0^T = y^T + b_1 - R_0 b_0$$

The difference in this problem is that the government internalizes the effect of the nominal exchange rate on the interest rate and on the default decision in the second period. We analyze how the government, with a commitment to monetary policy, can avoid the multiplicity.

Equilibrium We define the continuation equilibrium in which the central bank chooses the optimal monetary policy taking as given the default function in the second period.

Definition 6. (Continuation Equilibrium) Given initial conditions $\{R_0 b_0\}$; a Continuation equilibrium is defined by a set of strategy profile $\{\mathcal{E}_0, \mathcal{B}_C, \mathcal{E}_1, \mathcal{D}_C(Rb_1, \phi, e_1)\}$; value functions $\{W_0, W_1(Rb_1, \phi, e_1)\}$ and an interest rate schedule $\mathcal{R}(b_1, e_1)$ such that:

i Given the interest rate schedule: $\{\mathcal{E}_0, \mathcal{E}_1, \mathcal{B}_1\}$ solves the government problem in period zero, and W_0 attains the maximum.

ii Given a face value of debt and monetary policy; $\{\mathcal{D}_C(Rb_1, \phi, e_1)\}$ solves the government problem at every state in period one, and $W_1(Rb_1, \phi, e_1)$ attains the maximum.

iii Given the policy functions of the government: $\mathcal{R}(b_1, e_1)$ satisfies the break-even condition of international investors.

C.3 Multiplicity and Commitment

We assume the utility cost of the government from deviating to the target of the nominal exchange rate has an exponential form as follows:

$$\Phi(e_t - \bar{e}) = e^{(e_t - \bar{e})} \tag{71}$$

In Figure 9, we show a level of debt for which there are multiple Markov equilibria. We plot the value of repayment under low and high interest rates and the value of default when the default cost is low (ϕ_L) as a function of the nominal exchange rate.

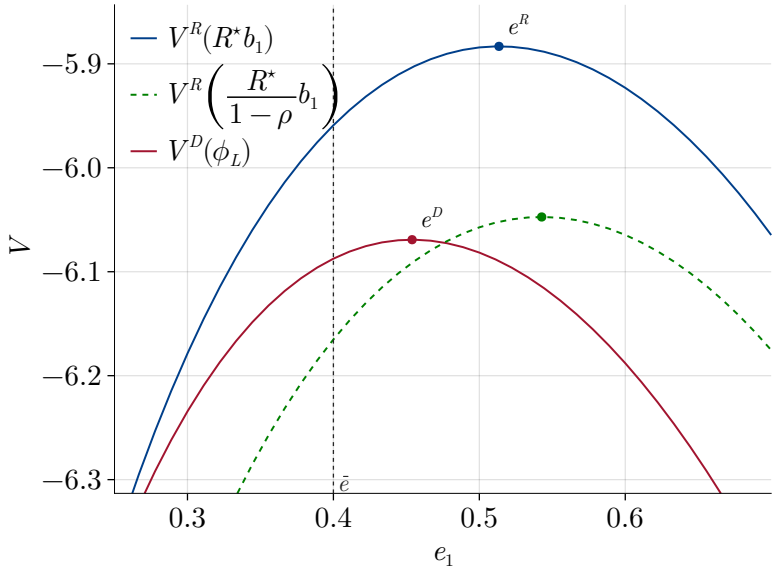


Figure 9: Optimal Monetary Policy

Note: In this figure, we use $y^T = 0.8$, $\beta = 0.91$, $R_0 B_0 = 0.65$, $R^* = 1.04$, $\rho = 0.2$, $\omega = 0.3$, $\mu = (1/0.83) - 1$ and $\pi = 0.05$

Note that the default decision and the nominal exchange rate are closely related in the Markov equilibrium with a cost of devaluation. That is, under repayment, the nominal exchange rate is high because the government wants to reduce the wedge in the labor supply by using monetary policy. On the other hand, the nominal exchange rate is closer to the target under default because the government does not need to reduce the consumption of tradable goods to pay back the debt.

In this economy, there are two Markov equilibria. In the first equilibrium, the interest rate is low. In this case, the government chooses a nominal exchange rate e^R above the target ($\bar{e}$) to reduce the labor wedge and choose to repay.

On the other hand, there is another equilibrium where the interest rate is high. In this case, the required exchange rate to reduce the labor wedge under repayment is far from the target. In this case, when the default cost is low, the government finds it optimal to default and set the nominal

exchange rate to the target. Because the government finds it optimal to default when $\phi = \phi_L$, there is support for a high-interest-rate equilibrium in the Markov economy.

Multiplicity and Commitment Under commitment to monetary policy, the equilibrium is unique. In Figure 10, we plot the value of repayment and default for the government under high and low interest rates. Additionally, we plot the optimal exchange rate under commitment.

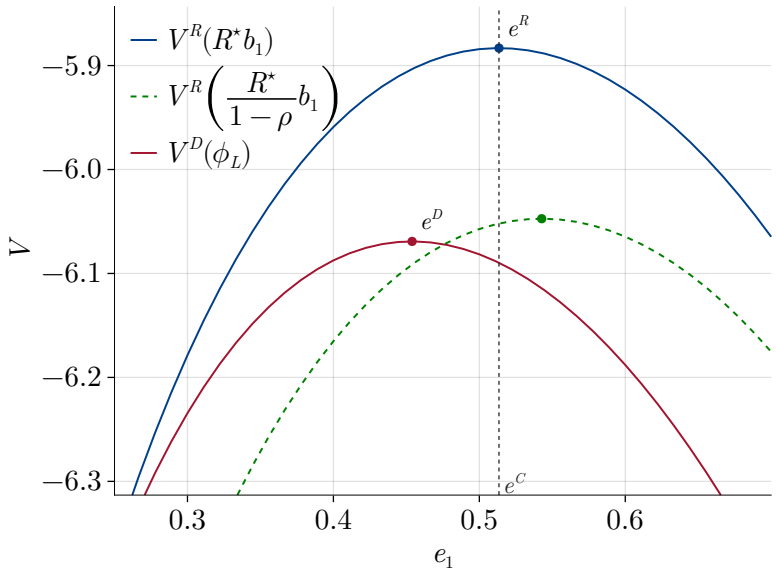


Figure 10: Multiplicity and Commitment

Note: In this figure, we use $y^T = 0.8$, $\beta = 0.91$, $R_0 B_0 = 0.65$, $R^* = 1.04$, $\rho = 0.2$, $\omega = 0.3$, $\mu = (1/0.83) - 1$ and $\pi = 0.05$

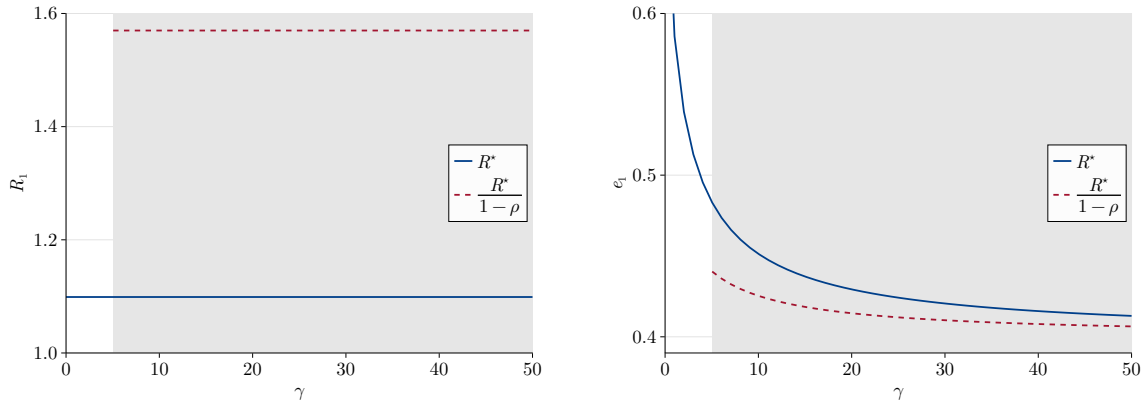
Note that when the government has the technology to commit to a nominal exchange rate, it chooses the same nominal exchange rate in the best Markov equilibrium e^R. Under e^C, the value of repayment is larger than the value of default for both a low and a high interest rate. Consequently, there is no support for a high interest rate under commitment, and the equilibrium is unique.

In this case, the government does not incur a cost by committing to a noncontingent nominal exchange rate. We can see this by noting that in the best Markov equilibrium, the optimal nominal exchange rate is not contingent on the realization of the default cost and coincides with the optimal policy under commitment. In this environment, commitment acts as an equilibrium selection device for the government.

Role of the Cost of Devaluation When devaluation has a cost, the government faces a trade-off between reducing the labor wedge by defaulting, which carries a cost, and deviating

from the nominal exchange rate target. To better understand this trade-off, we will study how equilibrium outcomes change as the cost of deviating from the target increases.

In Figure 11, we plot the equilibrium interest rate and nominal exchange rate as a function of γ, which measures the government's weight on the devaluation cost. In panel 11a, we plot the interest rate. In panel 11b, we plot the nominal exchange rate in equilibrium.



(a) Interest Rate (b) Nominal Exchange Rate

Figure 11: Multiplicity and the Cost of Devaluation

We identify three regions; in the first region, where γ is low, the cost of using monetary policy to reduce the labor wedge is low, so the government always uses the nominal exchange rate to stabilize the economy. This region resembles the case of an economy with a fully flexible exchange rate. The equilibrium is unique, and the nominal exchange rate is high.

For intermediate values of γ, we see that multiplicity in the interest rate affects the nominal exchange rate. In this region, when the interest rate is low, the government uses monetary policy as an active tool to stabilize employment and choose a nominal exchange rate above the target. However, if the interest rate is higher, the cost of using monetary policy is too high, so the government chooses to default and sets the exchange rate at the target.

Finally, for high values of γ, the government never uses monetary policy to stabilize employment. In this region, the nominal exchange rate is always close to the target, and the government faces multiplicity in the interest rate. This region resembles the fixed exchange rate economy, where the monetary policy is unavailable to stabilize employment, and the government faces the risk of a self-fulfilling crisis.

Commitment to monetary policy is particularly helpful for intermediate costs of deviations from the target. If the devaluation cost is too low, the government will always implement a zero

labor wedge under the Markov equilibrium. As a result, the equilibrium is unique regardless of the ability of the government to commit to a monetary policy. On the other hand, if the cost of devaluation is too high, the government would not choose to commit to an expansionary monetary policy. In this case, the cost of performing such devaluation exceeds that of avoiding default if the interest rate were high. As a result, the commitment is irrelevant to the equilibrium. Finally, for the intermediate cost of devaluation, the government could effectively eliminate multiplicity by committing to a nominal exchange rate. In this case, the government can do it without cost because the unique equilibrium under commitment coincides with the "best" Markov equilibrium, as discussed before.

C.4 Leaving a fixed monetary policy

Consider an economy where the government has a fixed monetary policy but can choose to leave the fixed monetary policy by assuming a cost in terms of utility of γ. We find two possible Markov equilibria. One is where the investors expect the government to leave the fixed monetary policy and avoid default and another is where investors expect the government to keep the fixed monetary policy and default if the endowment is low. However, if the government has the technology to commit to leaving the exchange rate in the second period, the equilibrium is unique.

$$\Phi(e_t - \bar{e}) = \begin{cases} 1 & \text{if} \quad e_t \neq \bar{e} \\ 0 & \text{otherwise.} \end{cases} \tag{72}$$

In Figure 12, we plot the value of the government as a function of the exchange rate under repayment, when the interest rate is high and low, and under default. In this case, there is a discrete jump in the utility of the government choosing to set the nominal exchange rate at the target (0.7 in this example).

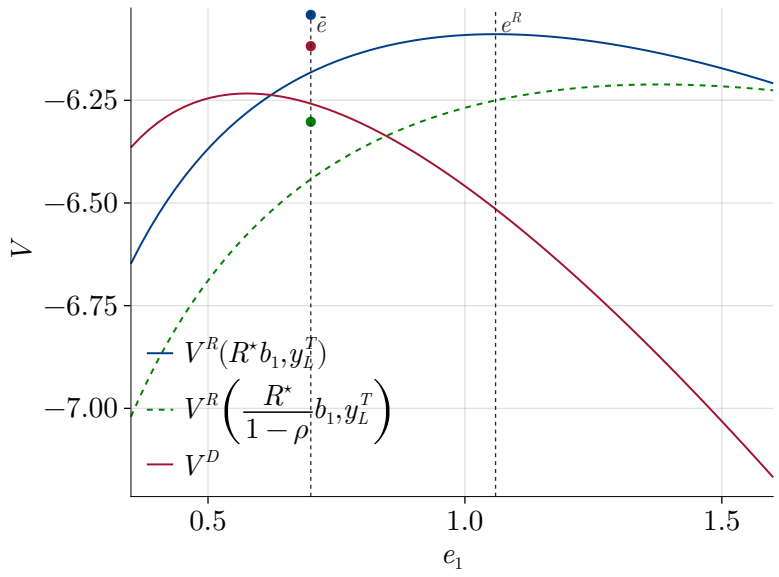


Figure 12: Value of the government as a function of the nominal exchange rate

Note: In this figure, we use $y^T = 0.7$, $\beta = 0.91$, $R_0 B_0 = 0.65$, $R^* = 1.04$, $\rho = 0.2$, $\omega = 0.3$, $\mu = (1/0.83) - 1$ and $\pi = 0.05$

In this case, we can understand multiplicity as follows. If investors expect the government to leave the fixed monetary policy, the utility under repayment for both the high and the low interest rate is higher than the value of default. In this case, the only interest rate consistent with equilibrium conditions is the low interest rate. Critically, if the interest rate is low, the government chooses to leave the fixed monetary policy and repay, which confirms the initial expectation.

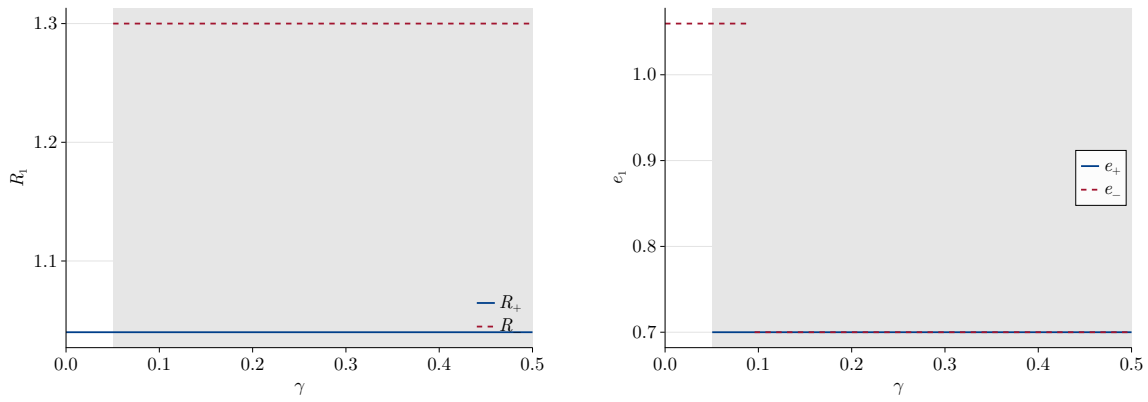
On the other hand, if investors expect the government to defend the fixed monetary policy, they expect the government to default if the endowment is low. They charge a high interest rate, and the government defaults in the second period and keeps the fixed monetary policy, confirming the initial expectation.

In this case, there is no need for investors to coordinate a low interest rate to achieve the "good" equilibrium. If the investors believe that the government will leave the fixed monetary policy, it is enough to kill multiplicity. In this case, the realization of the sunspot might depend on expectations about the interest rate, as in Calvo (1988) or expectations about the monetary policy.

On the other hand, under commitment, the government can avoid multiplicity. If investors fix an expectation that the government will abandon the fixed monetary policy, the value of repayment is above the value of default under both a high and low interest rate. Consequently, the equilibrium is unique, and the interest rate is low. Instead, because the government cannot commit to abandoning

the fixed monetary policy if the endowment is low there is the possibility of multiple equilibria.

In Figure 13 we plot the interest rate and the nominal exchange rate at equilibrium as a function of the cost of abandoning the fixed monetary policy. In panel 13a, we can see that when the cost of leaving the fixed monetary policy is low, the government devalues when the endowment is low, so the probability of default is low, and there is only one equilibrium characterized by a low interest rate. On the other hand, if the cost of leaving the fixed monetary policy is high, there is always the possibility of multiple equilibria.



(a) Interest Rate

(b) Nominal Exchange Rate

Figure 13: Multiplicity in the Nominal Exchange Rate

In panel 13b we see the nominal exchange rate at equilibrium. We can identify three regions. In the first region, when the cost of leaving the fixed monetary policy is low, the nominal exchange rate is unique, and the government always chooses to devalue to implement a zero labor wedge. On the other hand, when the cost of leaving the fixed monetary policy is high, the government always chooses to set the nominal exchange rate at the target. In this region, the interest rate has multiple equilibria, but the nominal exchange rate is unique. Finally, there is an intermediate region of the cost of leaving the fixed monetary policy where there is multiplicity in both the interest rate and the nominal exchange rate.