Optimal Tariffs with Trade Dynamics *

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Abstract

We study how the dynamics in trade adjustment influence the optimal unilateral tariff. The optimal unilateral tariff leverages a tariff-imposing country's monopsony power on foreign exports. In a static model, that monopsony power is reflected in the inverse of the foreign country's export supply elasticity. But, when trade responds gradually to tariff changes, that export supply elasticity—and the imposing country's monopsony power—also change over time as firms change the export entry and exit decisions. In a general equilibrium macro model with trade dynamics arising from sunk firm-level investments in market access, we show that the optimal one-time tariff chosen by a government with full commitment depends on a weighted sum of the dynamics of the export supply elasticity. The weights are determined by the evolution of the aggregate economy, but can be well-approximated by a sufficient statistic based on the interest rate and the dynamics of the export supply elasticity. If the government can choose a full path of tariffs with commitment, they choose tariffs that start higher and phase out over time to a lower level to exploit the relative fixity of trade in the short run and lessen the long-run distortion. Finally, without commitment, the government chooses substantially higher tariffs, reflecting and incentive for the government to always surprise households and firms with higher tariffs to exploit short-run market power.

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1. Introduction

This paper contains a re-examination from a dynamic point of view of the familiar problem of a benevolent government unilaterally choosing an optimal tariff. In particular, it is concerned with the interplay between the import tariff chosen by the government and the decisions of firms at home and abroad to make investments in home and foreign market access and to accumulate inputs in the betterment of their investors' welfare. The novelty of our analysis lies in our consideration of the relative fixity of trade and production in the short-run owing to the past firm-level investments in exporting and production infrastructure and the current and future investments firms can make to bring the economy to a new long-run equilibrium aligned with the government's chosen tariffs.

The optimal tariff exploits the monopsony market power on foreign exports of the tariff-imposing country. A tariff reduces demand for the foreign country's exports, leading to a decrease in the foreign country's price that depends on its reliance on the tariff-imposing country. This terms of trade gain for the home country can offset the distortionary effect of the tariff, such that the optimal unilateral tariff is positive. In static trade models, the optimal tariff is closely related to the inverse of the foreign country's export supply elasticity, as this equals the home country's monopsony power (Mill, 1848; Bickerdike, 1906; Johnson, 1953; Gros, 1987; Ossa, 2016). This usual intuition fails, however, when we consider the dynamic adjustments of trade to changes in tariffs. In this case, the monopsony power of the home country changes over time as foreign exporters reduce their investments in foreign market access leading households increasingly to substitute away from the foreign country's exports over time. Put simply, the dynamics of export supply transform optimal tariffs from a static market power problem into an intertemporal market power problem.

We investigate the role of dynamic trade adjustment for the optimal tariff in the general equilibrium macro model of Alessandria et al. (2025a), where heterogeneous firms make sunk

investments in domestic and foreign market access. The intensive and extensive margin of trade determine the level of trade. But, the extensive margin of trade in this model is sticky, it is a stock variable, such that trade, and therefore the export supply elasticity, responds gradually to changes in tariffs. We use the model to derive the optimal tariffs chosen by the government under a variety of constraints. Indeed, with dynamics in trade, there are many notions of an optimal tariff. We consider, for example, the optimal tariff defined as a once-and-for-all change in the tariff level that maximizes welfare, along with an optimal tariff path over time. We also consider the problem of the government when it is unable to fully commit to a given tariff policy. Each of these cases strengthens the intuition discussed above: with dynamic trade adjustment, the optimal tariff must account for how monopsony power of the home country changes over time.

We first consider the optimal "once-and-for-all" tariff chosen by the government to maximize welfare. In a simplified version of our model that abstracts from macro features such as endogenous labor and capital accumulation, we show analytically that the optimal tariff depends on the relative dynamically-weighted responsiveness of imports, the terms of trade, and firm creation, where weights depend on the subjective discount factor, marginal utility of consumption, and level of imports over time. We call this object the Dynamic Export Supply Elasticity or DESE, as it is the dynamic analog of the inverse export supply elasticity formula from static models. Indeed, if the mass of firms is held fixed and the responses of imports and the terms of trade are constant over time, the DESE reduces to the inverse export supply elasticity from the static literature. Although the DESE is calculated in a simplified version of the model, we show in our full model that the DESE is an excellent approximation of the numerical optimal tariff, suggesting that dynamic trade adjustment is the key element of our model that matters for optimal tariff policy. We also show that the inverse export supply elasticity measured in the short- or long-run are poor approximations of the optimal tariff. While a static trade model calculated to have an inverse export supply

elasticity equal to the DESE is a reasonable approximation of the tariff, but not the evolution of the economy.

Even in models with trade adjustment dynamics, it is common for researchers to view policy from the lens of maximizing steady state utility, rather than welfare over the entire transition.[1] Consistent with prior work showing the steady state of a dynamic trade model provides a poor approximation of the welfare gain including the transition,[2] we show that the tariff that maximizes steady state utility is a poor approximation of the optimal tariff in a fully dynamic model.

After considering the once-and-for-all optimal tariff, we turn to deriving the optimal path of tariffs chosen by government to optimize welfare with full commitment. Here, we find that the government would choose higher tariffs relative to the once-and-for-all optimal tariff on impact, and then reduce tariffs over time. Long-run tariffs on the optimal path are a bit smaller than the once-and-for-all optimal tariff. In other words, we find that tariff phaseouts are optimal. This finding is not surprising when we consider that the home country has more monopsony power in the short run when trade is sticky and less in the long run after full substitution. The government chooses to exploit these market power dynamics by charging higher tariffs when the home country has the most monopsony power.

The key to the dynamics of monopsony power is the dynamic export decision of firms as this directly affects the dynamics of foreign export supply. Other margins such as endogenous labor or capital accumulation also affect the foreign export supply elasticity but only indirectly through the relative size of the two countries and these effects are relatively small. One sign of this is that path of the optimal tariff barely changes when we shut down trade dynamics.

The formula we propose for the optimal tariff is exact in an efficient variation of our

[1] Implicitly this is the approach of the quantitative trade literature that describes static models as long-run models and uses these to make policy proscriptions. For example see Costinot and Rodríguez-Clare (2014).

[2] See Alessandria and Choi (2014), Alessandria et al. (2021b), and Mix (2023).

dynamic model. With distortions from taxes and markups, the success of the formula in approximating the optimal tariff depends on the use of tariff revenue. It is an excellent approximation when the tariff revenue is used to finance an investment subsidy. When the revenue is used to offset a labor tax, the formula overstates the optimal tariff by a few percentage points. And when the revenue is rebated lump-sum the formula is unrelated to the optimal tariff.

The change in monopsony power over time highlights a commitment problem for the government. As trade adjusts, the home country loses monopsony power and tariffs become less effective. But trade would adjust by less if firms expect tariffs to be temporary, as sunk investments in market access make them hesitant to overreact. As such, the government has an incentive to promise temporary tariffs to keep trade adjustment limited but to continually exploit the short-term fixity of trade by delivering higher-than-announced tariffs. With this incentive in mind, we solve for the home country's optimal tariff without commitment. We find that the "no commitment" optimal tariff is much higher than with commitment, as firms don't trust the government to stick to a lower tariff level given incentives to exploit high monopsony power on impact. Indeed, we find that the optimal tariff without commitment is closely related to the inverse of the short-run export supply elasticity.

Our paper relates to a large literature on optimal tariffs. Most of this work is done in static trade models (see the review by Caliendo and Parro, 2022) and clarifies the role of various important channels. Some work has been done in static trade models with shocks and assets (see Beshkar and Shourideh, 2020; Dávila et al., 2025). Our analysis builds on a relatively small literature that studies the microeconomic determinants of trade dynamics that grew out of the work of Baldwin and Krugman (1989) and Das et al. (2007) (reviewed by Alessandria et al., 2021a) extended to general equilibrium. Prior work with dynamic models show that the welfare and long-run effects of tariffs depend on the features giving

rise to those dynamics.[3] Our paper extends the analysis of the interaction of tariffs and fiscal policy in Alessandria et al. (2025a) by considering how the dynamics of the export supply elasticity determine the optimal policy and welfare gains. It is the first paper to explicitly connect the dynamics of the export supply elasticity to the optimal tariff. It is also the first paper to solve for the optimal path of tariffs in a model with micro-founded trade dynamics from heterogeneous firms. The issue of time consistency in tariffs in the presence of firm investment decisions appears in the work by Staiger and Tabellini (1987); Lapan (1988), and McLaren (1997). Relative to that work, we provide a full characterization of tariffs in an infinite horizon model with endogenous substitution dynamics.

The next section describes a heterogeneous firm dynamic model with a fiscal tradeoff. Section 3 discusses the problem of a credible benevolent government along with our main results. Section 4 relates the optimal tariff to the DESE. In section 5 we consider the choices of a government without commitment and relate the steady state time consistent tariff to that with commitment. Section 6 concludes.

2. Model

We study the role of tariffs in the competitive equilibrium of the multi-country general equilibrium model of Mix (2023). This model builds on the international real business cycle models of Backus et al. (1994) and Alessandria and Choi (2007) to include firm and exporter dynamics, a roundabout production structure, a realistic composition of trade flows across good types, and multiple countries.[4] Alessandria and Mix (2021) use a version of the model to study the effects of tariff phaseouts in the three GATT rounds. The model has the

[3]For instance, Alessandria and Choi (2014); Alessandria et al. (2021b); Mix (2023) focus on how trade dynamics from firm-level decisions influence the transition to a tariff change while Baldwin (1992); Alessandria and Choi (2014); Ravikumar et al. (2019) study how capital accumulation influences the path of the economy to a tariff change.

[4]One might find it convenient to work with an ad-hoc model of trade adjustment based on habit or adjustment costs, but Alessandria et al. (2021b) show that the aggregate implications are quite different with these reduced form frictions than the micro founded firm level frictions emphasized here.

main features used in tax policy analysis in the closed economy (Prescott, 2004; House and Shapiro, 2006; Trabandt and Uhlig, 2011) and the open economy (Mendoza and Tesar, 1998, 2005; Alessandria et al., 2025a).

The world economy has $\mathcal{I}$ regions, indexed by i and j. We describe the environment in each region, starting with households before introducing intermediate-goods producers and competitive aggregators of consumption, investment, and material inputs composites.

2.1. Households

Each region i contains a representative household with time endowment $\bar{L}_i$ that chooses how much time to work, L_i. The household also chooses quantities to consume C_i and to invest in either physical capital K_i (with investments in physical capital denoted by X_i) or a risk-free internationally traded bond B_i, which is denominated in the consumption good of country 1. The household's budget constraint is

$$\begin{aligned} P_{Ci}C_i + (1-s_i^X)P_{Xi}X_i + \frac{P_{C1}B_i{}'}{R^B} \leq{}& (1-\tau_i^L)W_iL_i + (1-\tau_i^K)(R_iK_i + \Pi_i) \\ &+ \tau_i^K \delta P_{Ii}K_i + P_{C1}B_i - P_{C1}\frac{\phi}{2}(\frac{B_i}{Y_i^N} - \frac{\bar{B}_i}{\bar{Y}_i^N})^2 + T_i \end{aligned} \tag{1}$$

and capital evolves according to

$$K_i' = (1-\delta)K_i + X_i\left[1 - \frac{\psi}{2}\left(\frac{X_i}{X_{i,-1}} - 1\right)^2\right], \tag{2}$$

where the "prime" indicator on K_i and B_i denote quantities taken into the subsequent period; a subscript of -1 denotes the value of a variable in the previous period; P_{Ci} and P_{Ii} denote the region i prices of consumption and physical capital investment; s_i^I is an investment subsidy; R_B is the gross return on the risk-free bond; W_i is the nominal wage; R_i is the rental rate of physical capital; Π_i denotes profits earned by firms in the economy and distributed to households; T_i is a lump sum transfer to households from the government; τ_i^K

is the tax on capital income, including profits; and τ_i^L denotes the labor income tax. In this framework, δ is the depreciation rate of physical capital; and ϕ and ψ determine the convex costs of adjustment in risk-free bond holdings and physical capital investment, respectively. The adjustment cost on risk-free bonds is included to include stationarity but has minimal effects on the optimal tariff. The cost is evaluated based on the ratio of the value of bonds to nominal GDP Y_i^N relative to the same ratio in the initial equilibrium, denoted by $\bar{B}_i/\bar{Y}_i^N$.

Each period, the household in region i chooses consumption, leisure, and investment in capital and bonds to maximize its expected discounted lifetime utility. The household's problem is expressed recursively as

$$V_i^H(K_i, B_i, S) = \max_{C_i, L_i, X_i, B_i'} \frac{\left[C_i^{\mu}(\bar{L}_i - L_i)^{1-\mu}\right]^{1-\sigma} - 1}{1-\sigma} + \beta \mathbb{E} V_i^H(K_i', B_i', S')$$

subject to 1 and 2, where S is the aggregate state variable; β is the discount rate; $1/\sigma$ is the intertemporal elasticity of substitution; and μ dictates the relative value of consumption and leisure in utility.

2.2. Intermediate Goods Producers

In each region i, a mass N_i of monopolistically competitive firms produce differentiated goods using labor, capital, and material inputs. Firms in region i pay a sunk cost $f_{e,i}$ denominated in local labor to start producing in the following period. These firms survive each period with probability n_s. Firms in region i have the same productivity. Conditional on survival, firms may produce for the domestic market at zero cost, but exports to foreign markets are subject to both fixed and variable costs, both of which are heterogeneous across firms and depend on both the firm's export history and stochastic shocks. In particular, non-exporters pay higher fixed costs on average than exporters, leading to a delayed response of the extensive margin of trade to changes in trade costs. Furthermore, new exporters pay higher variable costs

than experienced exporters on average. These features of the model parsimoniously capture features of the exporter life cycle from the data: that exporting exhibits strong hysteresis, and that exports grow with export tenure. These features imply a slower response of trade to changes in trade costs.

Export decisions and export costs across destinations j are determined independently. Each period, a firm in region i with export status $e_{ij} \in \{0, 1\}$ in destination j draws a fixed cost κ_{ij} that must be paid (in labor terms) for the firm to export to j in the subsequent period. The fixed cost is drawn from a Generalized Pareto distribution

$$F_{i,e_{ij}}(\kappa) = 1 - \left(1 + \frac{\upsilon\kappa}{f_{e_{ij}}}\right)^{-\frac{1}{\upsilon}}.$$

With $\upsilon < 1$, the expected fixed cost draw of a firm with export status e_{ij} is $f_{e_{ij}}/(1-\upsilon)$. When matching moments related to exporter entry in calibration, we get $f_0 > f_1$, which implies that non-exporters ($e_{ij} = 0$) draw higher fixed costs on average than exporters ($e_{ij} = 1$).

Variable costs of exporting to destination j also depend on both export history and stochastic shocks. Exports from i to j of intermediates that will be used in the production of a final good G are subject to either a low iceberg cost ξ_{GLij} or a high iceberg cost ξ_{GHij}. All new exporters (defined as firms that export in the current period but not the previous period) face the high iceberg cost. Conditional on continued export participation, firms have a constant probability $1 - \rho$ of switching to the other cost state. As such, new exporters export less than incumbent exporters on average.

As noted above, the iceberg costs on exports depend on the end-use of the export, whether it's used by final goods producers to produce consumption goods $G = C$, investment goods $G = X$, or material goods $G = M$. Heterogeneity in iceberg costs across good types allows the model to capture the relatively high intensity of trade in capital and material goods. Our calibration will map this composition into lower iceberg costs for capital and material

goods and higher iceberg costs for consumption goods.

Because export decisions are made independently and we assume that production is constant returns to scale, we can consider the maximization problem of an intermediate goods producer for each destination separately. Let l_{ij}, k_{ij}, and m_{ij} be the inputs of labor, capital, and materials that a firm uses in the production of goods sold to country j, and let p_{Gij} be the price the firm charges for its good, conditional on the buyer's end use of the good G. Let $\xi_{ij} \in \{\infty, H, L\}$ denote the current cost-state of the firm: $\xi_{ij} = \infty$ denotes a non-exporter facing infinite iceberg costs, $\xi_{ij} = H$ denotes an exporter facing a high iceberg cost, and $\xi_{ij} = L$ denotes an exporter facing a low iceberg cost. As discussed previously, the draw of the fixed cost κ_{ij} depends on the firm's current export status e_{ij} where $e_{ij} = 0$ corresponds to non-exporters with cost-state $\xi_{ij} = \infty$, and $e_{ij} = 1$ corresponds to exporters with cost-state $\xi_{ij} \in \{H, L\}$. The recursive maximization problem of this firm in aggregate state S is

$$V_{ij}(\xi_{ij}, \kappa_{ij}; F_{i,e_{ij}}, S) = \max_{e'_{ij}, p_{Gij}, l_{ij}, k_{ij}, m_{ij}} e_j \left[(1 + s_{Ri}) \sum_G p_{Gj} G_{ij}(f) - W_i l_{ij} - R_i k_{ij} - P_{Mi} m_{ij} \right]$$
$$- W_i e'_{ij} \kappa_{ij} + n_s \mathbb{E} D_i V_{ij}(\xi'_{ij}, \kappa'_{ij}; F_{i,e'_{ij}}, S')$$

subject to the demand functions that arise from CES aggregation of intermediates by final goods producers, which will be discussed in detail in later, and the production function

$$\sum_G \xi_{G\xi_{ij}ij} G_{ij}(f) = z_i (k_{ij}^{\alpha} l_{ij}^{1-\alpha})^{1-\alpha_m} m_{ij}^{\alpha_m},$$

where

$$D_i = \beta \frac{U'_{C_i}}{U_{C_i}} \frac{P_{Ci}}{P'_{Ci}}$$

is the subjective discount factor, $G_{ij}(f)$ is the firm's exports of goods used for end-use G,

P_{Mi} is the price of materials, z_i is the common total factor productivity among all firms in region i, α is capital's share of value added in production, α_m is the material share in total production, and s_{Ri} is a revenue subsidy. This revenue subsidy can be used to restore efficiency by offsetting the markup.

The intermediate goods producer's problem can be broken into a static problem—choosing inputs and prices—and a dynamic problem—choosing tomorrow's export status. The static problem's solution is straightforward and common in the literature, as final goods producers' technology is CES. In particular, optimal prices are the familiar constant markup over marginal costs

$$p_{Gj} = \frac{\theta}{\theta - 1}\xi_{G\xi_j ij}\frac{MC_i}{1 + s_{Ri}} = \frac{\theta}{\theta - 1}\xi_{G\xi_j ij}\frac{1}{z_i(1 + s_{Ri})}\left(\frac{P_{Mi}}{\alpha_m}\right)^{\alpha_m}\left[\frac{\left(\frac{R_i}{\alpha}\right)^{\alpha}\left(\frac{W_i}{1-\alpha}\right)^{1-\alpha}}{1 - \alpha_m}\right]^{1-\alpha_m}. \quad (3)$$

The parameter θ that determines the size of the markup is the elasticity of substitution across varieties from the same source, which enters the intermediate goods producer's problem through the demand functions described later. An exporter's profits, π_j, depend on an aggregate component and its idiosyncratic exporting technology.

2.2.1 Dynamic exporting decision

A firm that decides to export in the next period not only receives revenue from foreign sales, but also a higher probability of lower fixed and variable costs going forward. Thus, part of the fixed cost paid to export is an investment in further market access. For ease of exposition, let $V_\xi(\kappa) = V_{ij}(\xi_{ij}, \kappa_{ij}; F_{i,e_{ij}}, S)$ for destination j for a firm in cost-state ξ with

fixed cost κ. The dynamic problems for firms of each type can be written as

$$\begin{aligned}
V_\infty(\kappa) &= \pi_\infty + \max\{-W\kappa + n_s\mathbb{E}DV_H(\kappa'), n_s\mathbb{E}DV_\infty(\kappa')\},\\
V_H(\kappa) &= \pi_L + \max\{-W\kappa + n_s\mathbb{E}D[(1-\rho)V_L(\kappa') + \rho V_H(\kappa')], n_s\mathbb{E}DV_\infty(\kappa')\},\\
\text{and } V_L(\kappa) &= \pi_H + \max\{-W\kappa + n_s\mathbb{E}D[(1-\rho)V_H(\kappa') + \rho V_L(\kappa')], n_s\mathbb{E}DV_\infty(\kappa')\}.
\end{aligned}$$

The decision to export for a firm in cost-state ξ reduces to an evaluation of the fixed cost κ relative to a threshold κ^*_ξ, defined for each cost-state as

$$W\kappa^*_\infty = n_s\mathbb{E}D[V_H(\kappa') - V_\infty(\kappa')], \tag{4}$$

$$W\kappa^*_H = n_s\mathbb{E}D[(1-\rho)(V_L(\kappa') - V_\infty(\kappa')) + \rho(V_H(\kappa') - V_\infty(\kappa')], \tag{5}$$

$$\text{and } W\kappa^*_L = n_s\mathbb{E}D[(1-\rho)(V_H(\kappa') - V_\infty(\kappa')) + \rho(V_L(\kappa') - V_\infty(\kappa')]. \tag{6}$$

To solve for the expectation terms on the right-hand side of equations 4–6, we take the expected difference between the value of exporters of each type and the value of non-exporters. To be specific, let $\pi^*(\xi)$ be the expected profits of a firm with export cost state ξ where $\pi^*(\infty) = 0$. Let $dV_\xi \equiv \mathbb{E}(V_\xi(\kappa) - V_\infty(\kappa))$. Using the threshold values above, we can write these terms dV_ξ, which show up in the threshold fixed cost equations, as

$$\begin{aligned}
dV_H &= \pi^*(H) - W\left[\int_0^{\kappa^*_H} \kappa dF_1(\kappa) - \int_0^{\kappa^*_0} dF_0(\kappa)\right]\\
&\quad + n_s\mathbb{E}D\left[[\rho F_1(\kappa^*_H) - F_0(\kappa^*_\infty)]\, dV'_H + (1-\rho)F_1(\kappa^*_H)dV'_L\right]\\
dV_L &= \pi^*(L) - W\left[\int_0^{\kappa^*_L} \kappa dF_1(\kappa) - \int_0^{\kappa^*_\infty} dF_0(\kappa)\right]\\
&\quad + n_s\mathbb{E}D\left[[(1-\rho)F_1(\kappa^*_L) - F_0(\kappa^*_0)]\, dV'_H + \rho F_1(\kappa^*_L)dV'_L\right].
\end{aligned}$$

Conditional on being in the same cost state, firms make identical decisions over inputs,

prices, and future export status. This model outcome greatly simplifies the state space of the problem as we only need to keep track of the number of firms in each cost-state for each destination, which can be easily derived given the fixed cost thresholds.

2.2.2 Free entry

Intermediate goods producers exogenously exit the market with probability $1 - n_s$. Each period, there is an unbounded mass of potential firms that may begin producing in the subsequent period by paying fixed cost $f_{e,i}$, denominated in labor. In the first year of production, firms only serve the domestic market and begin drawing export fixed costs for all destinations. Potential firms enter until the cost of entry $W_i f_{e,i}$ equates to their expected lifetime profits:

$$W_i f_e = n_s \mathbb{E} D_i \left[V'_{dom,i} + \sum_{j \neq i} V'_{\infty,ij} \right],$$

where

$$V_{dom,i} = \pi_{ii} + n_s \mathbb{E} D'_i V'_{dom,i},$$

and π_{ii} is the optimal static profits from operating in the domestic region.

2.3. Final Goods Producers

Final goods—including consumption goods C_i, investment goods X_i and material inputs M_i—are produced in each country using domestic and imported intermediate goods. As explained in the exposition of the intermediate goods producers' problem, the iceberg cost, and therefore the price of imported intermediates, depends on the type of final good being produced. Each type of final good $G \in \{C, X, M\}$ is produced by a continuum of competitive

firms. The aggregation technology for final good G is CES

$$G_i = \left[\sum_{j \in \mathcal{I}} \left[\int_{f \in \Omega_{ji}} G_{ji}(f)^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}\frac{\gamma-1}{\gamma}} \right]^{\frac{\gamma}{\gamma-1}}$$

where Ω_{ji} is the set of firms in region j that export to region i, θ is the elasticity of substitution across varieties of a common source region, and $\gamma < \theta$ is the elasticity of substitution across varieties from different source regions.

We represent the problem of final goods producers of good G with a representative firm that chooses inputs to maximize profits

$$\pi_{Gi} = \max P_{Gi} G_i - \sum_{j \in \mathcal{I}} \tau_{ji} \int_{f \in \Omega_{ji}} P_{Gji}(f) G_{ji}(f) df$$

subject to the aggregation technology described above, with P_{Gi} the familiar price index that arises with CES aggregation

$$P_{Gi} = \left[\sum_{j \in \mathcal{I}} (\tau_{ji} P_{Gji})^{1-\gamma} \right]^{\frac{1}{1-\gamma}} \qquad \text{where} \qquad P_{Gji}^{1-\theta} = \int_{f \in \Omega_{ji}} P_{Gji}(f)^{1-\theta} df.$$

Tariffs imposed by region i on imports from j (τ_{ji}) in our simulations will be the same for all good types, though the model easily allows for tariffs to differ by good type (Hoang and Mix, 2026).

2.4. Government Budget Constraint and Fiscal Policy

The government collects tax revenue on imports. Tariff revenue TR_i is measured as

$$TR_i = \sum_{G \in \{C,X,M\}} \sum_{j \neq i} \tau_{ji} \int_{f \in \Omega_{ji}} P_{Gji}(f) G_{ji}(f) df$$

We assume that the government balances its budget each period, so the lump sum transfer is equal to the sum over all sources of government revenue

$$T_i = \tau_i^L W_i L_i + \tau_i^K (R_i K_i - \delta P_{Ii} K_i + \Pi_i) + TR_i - s_{Ri} Rev_i - s_i^I P_{Ii} I_i.$$

2.5. Aggregation and Market Clearing

While a model with heterogeneous firms would normally entail a very large state space, two assumptions make aggregation in our model quite simple. First, we assume that export and production decisions are made independently across destinations. Second, we assume that fixed cost draws are independently and identically distributed (iid) for all firms and periods conditional on having the same cost state. As such, the problem of keeping track of the fixed and iceberg costs of an individual firm reduce to keeping track of the number of firms in each cost state. Using the cost-state specific export thresholds, one can easily derive laws of motion for the number of firms in each state, as detailed in Mix (2023).

In equilibrium, the capital and materials market must clear such that the sum of capital demanded by intermediate producers is equal to the capital stock held by households and the sum of materials demanded by producers is equal to the materials sold by final goods producers. Similarly, the labor market clears. Labor supplied by households must be equal to the sum of labor demanded for production *and* payment of exporting and entry fixed costs by intermediate goods producers. Finally, bonds are in zero net supply, $\sum_i B_i = 0$.

2.6. Calibration

The model is calibrated to two symmetric countries. The macro side of the model is calibrated to be consistent with typical international macro models and models of fiscal policy. The trade side is calibrated to be consistent with several aspects of firm and exporter dynamics and largely follows Mix (2023) and Alessandria and Mix (2021).

The labor endowment $\bar{L}$ and productivity z in each country are normalized to 1. A period is a year. We set the discount factor $\beta = 0.96$ and the capital depreciation rate $\delta = 0.10$. The intertemporal elasticity of substitution $1/\sigma = 1$. The Frisch elasticity is set to 2.

With the use of materials in production, our model differentiates between gross output and value added in the economy. The materials share in intermediate goods production, α_m, is chosen so that the ratio of gross output to value added is 2, consistent with the US.

We assume the elasticity of substitution between varieties from the same source is $\theta = 6$. This implies a markup of 20 percent and is in line with most macro analysis in monetary models. The elasticity of substitution between different sources is set to $\gamma = 4$ and generates annual trade elasticity of 3. Together, γ and θ (along with calibrated exporter behavior, discussed below) yield a long-run trade elasticity of about 8.0 to a permanent global change in tariffs.[5] We also use the same shape parameter $v = 0.65$ for the generalized Pareto distribution as calibrated in Mix (2023). The survival rate of intermediate goods producers is set to n_s =0.98 to match the labor share of exiting firms in the data.

There is disagreement on the dynamics of the trade elasticity. Following Head and Mayer (2014) and Simonovska and Waugh (2014), much quantitative work in static trade models targets a trade elasticity of 4 to 5. Recent work by Boehm et al. (2023) finds a 1 year trade elasticity of less than 1 that rises to about 2 after ten years. In contrast, Alessandria et al. (2025b) recover a short-run elasticity of about 3 and long-run elasticity of 14 with about half the gap between short-run and long-run closed every five years for large permanent changes in US trade policy on China and Vietnam. Yi (2003) points out that the relationship between the growth in US imports and fall in tariffs points to a long-run elasticity that is between ten to fifteen. We show the trade elasticity influences our findings.

Similar to House and Shapiro (2006), we use the estimates of marginal tax rates from the

[5]The long-run trade elasticity to a global permanent reform is not constant but depends on the initial and final tariff, although the curvature in the trade elasticity is modest.

Congressional Budget Office (US Congress, CBO 2025) in 2025. Thus, we set the marginal labor tax to 27.2 percent and the marginal capital tax to 14.7 percent. Initial tariffs are set to 4 percent in line with recent tariffs before 2025.[6] We assume these tax policies are expected to be constant. All told, government revenue (and spending) amounts to 22.3 percent of GDP in our initial steady state, with 18.0 percentage points of the revenue from labor taxes, 3.7 percentage points from capital taxes, and 0.6 percentage points from tariff revenue. All revenue is rebated lump sum. Our initial equilibrium sets investment subsidies $s_i^I = 0$ and revenue subsidies $s_i^R = 0$.

We calibrate the model to be consistent with aspects of the US economy from 2019-2024. Other parameters that still need to be calibrated include iceberg costs $\xi_{G\xi ij}$, scale parameters f_{ei}, the probability of changing shipping cost states $1-\rho_i$, the relative preference for consumption and leisure μ, and the labor share of value added $1-\alpha$. With our two-region symmetric model (so that the parameters are the same for each region), this list includes 11 parameters, 6 of which are iceberg costs. As in Mix (2023), we simplify the iceberg cost structure and impose the assumption that low-cost-state iceberg costs are proportional to high-cost-state iceberg costs $\xi_{GLij} = \zeta\xi_{GHij}$ with $\zeta < 1$. This reduces the parameters related to iceberg costs to 4 and the total parameters to be assigned to 9. We choose the parameters to jointly match the following moments in steady state:

1. Imports-to-GDP ratio = 0.15
2. Capital goods share of imports = 0.2
3. Material goods share of imports = 0.6
4. New exporter share of total exports = 0.05
5. Export share of exporters with five or less years of tenure = 0.4
6. Export participation rate = 0.2
7. Share of exporters that are new = 0.15
8. Frisch elasticity = 2
9. Labor share of value added = 0.66

These moments are consistent with the current structure of the US economy. The import

[6]This is close to the current tariff given 2011-2015 import weights. The optimal tariffs are largely unaffected by our initial tariffs.

share of 15 percent, for example, roughly matches the US import share in recent years, as does the split of imports between capital, material, and consumption goods. Moments related to the importance of new exporters are consistent with average export churning statistics in the World Bank Exporter Dynamics Database (see Mix, 2023 and Steinberg, 2023).

In our baseline exercises, we assume that initial and long run bonds are equal to 0. That is, we assume $\bar{B}_i = 0$ for all countries i both in the initial and terminal stationary equilibria. The adjustment costs and solution window are such that the net foreign asset returns to steady state by the end of the transition we consider.[7]

2.7. Two alternative models

As our competitive equilibrium is inefficient owing to the taxes and markups, we find it convenient to also solve two models that bring us in steps closer to the canonical analysis of optimal tariffs.

The first model, which we call the *efficient model*, sets income taxes to zero, maintains the no investment subsidy, and introduces a revenue subsidy to offset the distortion from having a markup on some goods and not others (labor and capital). The revenue subsidy is funded by a lump sum tax. Bilbiie et al. (2019) show that this revenue subsidy can restore efficiency to production and entry. The export supply elasticity here will continue to vary over time, but we will start from an efficient equilibrium.

The second model, we we call the *efficient static model*, eliminates exporter dynamics by assuming all firms start as exporters and have the same variable cost of shipping. Further, we hold the mass of firms constant by eliminating the death shock and free entry decision and normalizing the mass to equal our baseline model.

[7]Solving for a new steady state bond position without adjustment costs has very small effects on our welfare calculations but substantially increases computational time.

3. Optimal Tariff Policy and Results

First, we define the three types of tariff reforms commonly considered. Second, we move on to comparing the effects of these reforms on the economy. Third, we relate our results to the export supply elasticity. Fourth we discuss some sensitivity of our results to the model structure.

3.1. Tariff Policy

Our dynamic model allows us to consider the effect of alternative tariff changes. In dynamic economies, it is common to consider the following three cases:

1. **One-time permanent tariff reform:** Define a one-time policy: set $\tau_t = \tau^{OT}$ for all $t \geq 0$ with no further adjustment.

$$\tau^{OT} = \arg\max_{\tau} \ \mathbb{E}_0 \sum_{t \geq 0} \beta^t u(C_t(\tau), 1 - h_t(\tau))$$

2. **Steady-state optimal tariff:** We compute the stationary optimal tariff τ^{SS} as the tariff that maximizes stationary utility subject to stationary equilibrium.

$$\tau^{SS} = \arg\max_{\tau} \ u(C_\infty(\tau), 1 - h_\infty(\tau))$$

3. **Optimal Path:** The Government chooses $\{\tau_t\}_{t \geq 0}$ to maximize household welfare subject to equilibrium and fiscal rules:

$$\max_{\{\tau_t\}} \mathbb{E}_0 \sum_{t \geq 0} \beta^t u(C_t, 1 - h_t).$$

We solve for the transition of the economy from the tariff reform starting from an initial steady state in which agents expect tariffs to remain constant. As discussed, we assume that

all tariff revenue is used to change the tax on investment. We compute the compensating variation as the percent change in consumption that would leave home agents indifferent between the current steady state forever and the path induced by the policy

$$\frac{u\left(\left(1+\frac{\eta_i}{100}\right)C_i^*, 1-L_i^*\right)}{1-\beta} = \sum_{t=0}^{\infty}\beta^t u\left(C_{it}, 1-L_{it}\right).$$

We also measure the mean discounted tariff as

$$\tau^{PV} = \frac{\sum_{t=0}^{\infty}\beta^t u_{c,t}\tau_t}{\sum_{t=0}^{\infty}\beta^t u_{c,t}}.$$

3.2. The optimal tariff

We report some outcomes for the three tariffs considered in Table 2. The optimal one-time tariff is about 17.34 percent, while the tariff that maximizes steady state utility is 23.26 percent. With the optimal path, the mean discounted expected tariff is 17.55 percent and the long-run tariff is 16.30 percent. The difference in optimal long-run tariff across these cases reflect the object being maximized and the instrument being used.

The optimal path of tariffs is plotted in Figure 1 against a log scale of time (to help see the first few years). The optimal path starts with a tariff of 27.24, or almost about 10 percentage points above the one-time change, but falls nearly linearly to the one-time change in the next five years, before falling another 1 percentage point below the one-time change. Unlike the conventional view since Mussa (1978), there is an optimal phaseout of tariffs as the home country seeks to exploit its market power more aggressively upfront but reduces the long run distortion.

We now turn to the effects of these policies on welfare and the long-run state of the economy, based on a compensating differential. Consistent with prior work with trade dynamics in general equilibrium (Alessandria and Choi, 2014; Alessandria et al., 2021b; Mix, 2023), we

find a large gap between measures of welfare and steady state utility. Our one-time policy yields a welfare gain of about 1.54 percent and a long-run increase in steady state utility of 3.91 percent. That steady state utility is higher than welfare primarily reflects the high capital intensity of traded goods. In contrast, the steady state maximizing tariff yields a smaller welfare gain of 1.45 and a larger gain in steady state utility of 4.05 percent. The optimal path of tariffs yields a small increase in welfare of about 0.018 percent over the constant tariff and it leads to slightly lower long run utility of 3.84 percent.

The transition of the Home economy over the first 100 years to the one-time and path of tariffs is plotted in Figure 2. In all cases for the dynamic model, consumption falls before rising by almost four percent in the long-run. Employment rises and then gradually declines, but ends higher than it starts. There is a substantial increase in physical capital and the stock of firms. Tariff revenue and the subsidy start high and then fall. Trade as a share of GDP falls. The first year has a large drop from the intensive margin and then there is a more gradual transition that is mostly done in ten years. The transition of trade to GDP is actually relatively fast compared to the aggregate economy as only 60 percent of the long-run increase in the capital stock has been achieved by then. The permanent reform induces a persistent trade deficit to finance investment. Importantly, the long-run of the economy is a very poor approximation of the aggregate effects on well-being in the early stages of the reform.

Relative to the one-time tariff change, the path of tariffs induces a slightly larger cut in consumption initially and a trade surplus when tariffs are relatively high. This contrasts with a trade deficit with the one-time tariff change as the country runs a deficit on impact to finance the substantial investment taking place. The path of tariff leads to an almost discrete drop in the trade share of GDP.

3.3. Optimal Path of Tariff

Figure 1 compares the optimal tariff path across three environments that differ in their treatment of distortions and trade dynamics.

We begin with the baseline economy, which features taxes and a roundabout production structure, with tariff revenue used to subsidize investment. In this case, the optimal path deviates substantially from the optimal one-time tariff change discussed above. The government sets a tariff that is significantly higher on impact and then gradually reduces it over time, converging to a level slightly below the one-time change. This pattern reflects the low short-run trade elasticity and the government's incentive to exploit limited substitution possibilities in the short run, followed by a phaseout as trade adjusts.

Next, we consider the efficient economy. To recover efficiency, we set all income taxes to zero and set the revenue subsidy to offset the markup. We rebate tariff revenue lump sum. We think of this model as closest in spirit to a competitive model. In this environment, the optimal one-time tariff is slightly below that in the baseline economy (16.68 vs 17.34). However, the optimal path is markedly steeper. Tariffs rise more sharply on impact to 35.5 percent and decline more quickly, settling at a lower long-run level of 15.1 percent. The present value path of tariff is 15.37 percent, only slightly lower than our baseline of 17.55.

While the efficient economy yields tariffs that are qualitatively similar to the baseline model, the effects on welfare and steady state utility are quite different. The gain from the one-time tariff is only 0.62 percent and the benefit of the optimal path is only 0.67 percent (compared to 1.54. and 1.55 percent for the baseline model). The gap with steady state utility is even larger as both paths lead to lower long run utility. Indeed, the steady state utility maximizing tariff is almost half the one-time tariff (8.58 percent vs 16.68 percent) and only yields an increase in utility of 0.05 percent. That welfare exceeds steady state utility springs from the fact that the tariff raise the relative price of investment and materials to

consumption leading to sharp decline in employment, consumption, and the capital stock.

Finally, we compare these results to a model with the same fiscal and production structure as the baseline model but without trade dynamics. In this case, which we call the static trade model, the optimal policy is a nearly constant tariff. We calibrate the trade elasticity in the static model to match the present-value trade elasticity in the dynamic model. Under this mapping, the static optimal tariff coincides closely with the one-time optimal tariff in the baseline dynamic economy. This highlights that static models can approximate the level of the optimal tariff when appropriately calibrated, but they miss the intertemporal dimension of policy. In particular, they cannot capture the incentive to set higher tariffs initially when the short-run trade elasticity is low and to reduce them as adjustment occurs.

Taken together, these results show that trade dynamics—specifically, the gap between short-run and long-run trade responses—are central for understanding the optimal path of tariffs. The government exploits the relative fixity of trade in the short run by front-loading tariffs and then reduces them as firms adjust through entry, exit, and investment.

Figure 2 illustrates how these differences in policy translate into macroeconomic dynamics by comparing the optimal path to the one-time tariff change in the baseline economy. Under both policies, consumption initially declines as resources are reallocated toward investment and export participation, before rising in the long run. However, the optimal path induces a larger initial decline in consumption due to the higher front-loaded tariffs. This is accompanied by a trade surplus on impact, reflecting reduced imports and stronger domestic production incentives. In contrast, the one-time tariff change leads to a trade deficit initially, as the economy borrows to finance investment during the transition.

Over time, both policies generate increases in capital, firm entry, and trade volumes, but the timing differs. The optimal path accelerates adjustment early on due to the stronger initial distortion, while the one-time change produces a smoother transition. Despite these differences, long-run outcomes are similar, consistent with the small welfare differences across

the two policies. The key distinction lies in the transition: the optimal path reallocates resources more aggressively in the short run to take advantage of low trade elasticities, while the one-time policy spreads these adjustments more evenly over time.

When we eliminate the trade dynamics, but retain physical capital investment, we find the optimal tariff changes very little along the transition path. It actually starts out a little lower than its long-run level and rises. These effects are related to differential incentives to accumulate capital.

4. Trade dynamics and the optimal tariff

We now discuss how trade dynamics influence the optimal tariff and the welfare gain from trade. We relate these trade dynamics to the dynamics of the export supply elasticity and propose a new formula that accounts for the transition. We show this formula holds across alternative calibrations of our model that generate a gap between the short and long-run trade response. We also compare our formula to other approximations that abstract from some margins.

4.1. The Dynamic Export Supply Elasticity

It is well known that the export supply elasticity determines the optimal tariff. In a static model of monopolistic competition in which all firms export, Gros (1987) shows the inverse of this elasticity yields the optimal tariff for a small country. As country size changes there are well-known additional considerations. Indeed, the classic result for an Armington model in with countries are specialized in their own variety, is that the optimal tariff is equal to the inverse of the export supply elasticity,

$$\tau - 1 = \frac{\dfrac{d \ln q_t^*}{d \ln \tau}}{\dfrac{d \ln M_t}{d \ln \tau}} = \frac{1}{\epsilon_x} = \frac{1}{\gamma - 1} \frac{1}{\lambda_F},$$

where λ_F denotes the foreign expenditure share on its own goods.

With trade dynamics, we show that the one-time tariff is closely related to a dynamic counterpart to this equation. In the appendix, we derive a general formula for the one-time optimal tariff as

$$\tau^{OT} - 1 = \frac{\sum_t \omega_t^M \dfrac{d \ln q_t^*}{d \ln \tau}}{\sum_t \omega_t^M \dfrac{d \ln M_t}{d \ln \tau}} \qquad \omega_t^M \equiv \beta^t \frac{u_C(C_t)}{P_t} p_{FH,t}^{B,*} M_t,$$

This is our discounted export supply elasticity (DESE) result. There are several parts to the formula. At its most basic we want to equate the one-time tariff to the inverse of the appropriately weighted present value export supply elasticity. The numerator captures the discounted sum of changes in the terms of trade from the tariffs while the denominator captures the discounted sum of real imports from the change in tariffs. The weights depend on the discounted marginal value of consumption deflated by the price of consumption, which is closely related to the inverse of the interest rate, times the value of imports. Note that if the responses of the terms of trade and real imports to a change in tariffs is constant over time, this formula reduces to the simple inverse export supply elasticity from the static model.

In Figure 3 we plot the key components determining the optimal one-time tariff for 100 years. The upper left panel plots the dynamics of the terms of trade (price of imports to exports measured in foreign markets). The terms of trade slightly overshoots its long run value as imports fall in value. The second panel plots the path of real imports, which adjust immediately through the intensive margin and then continues to fall. It is useful to note that the decline in real imports on impact is about 40 percent of the long run change. The third panel plots the derivative of the elasticity of the terms of trade to tariffs and real imports to tariffs, evaluated at the optimal tariff. Notice that the marginal effect of tariffs on the terms

of trade is quite stable at a half while the marginal effect of tariffs on real imports drops discretely and then continues to fall. The fourth panel plots the path of the ratio of these two derivatives along with the path from the finite differences from our movement to the optimal. The derivative at the optimal varies slightly less than the finite difference to the optimal. The fifth panel plots the path of three weights as share of their sum: 1) the theoretical weight, 2) the SDF alone, and 3) the constant beta weight. In the constant β case the first period will account for 4 percent of the value of the transition ($\frac{1}{1-\beta}$. Subsequent periods fall in value by β^t. Given the relatively low trade elasticity on impact, trade is relatively high upfront pushing the first period weight above 4 percent. Moving to our theoretical weight we put more weight on the first few periods as the marginal utility is relatively high owing the strong returns to investment from the policy. The final panel reports how applying our formula to a finite number of periods would lead one to overstate the optimal tariff.

4.2. Varying Trade Dynamics

We now show that the formula holds broadly for a range of parameterizations with different short- and long-run elasticities. Specifically, we vary the Armington elasticity, γ, and solve for the optimal one-time tariff change. Recall, with our fixed extensive margins of exporting and production, the Armington elasticity is also the short-run trade elasticity. The long-run trade elasticity is determined by the entry and exit decisions of firms. When we increase the short-run elasticity, we also increase the long-run elasticity and the gap between the two. We then compare to the optimal tariff to our formula and others using different export supply elasticities. We do this for our efficient and baseline models.

In Figure 4 we explore the relationship between the one-time unilateral optimal tariff and that based on different notions of the export supply elasticity, conveniently plotted as the 100 times the inverse of the export supply elasticity. We start with our efficient dynamic model. We find that the inverse of the DESE equals the optimal tariff.

When we use alternative export supply elasticities to approximate the optimal tariff, we fare worse. Using the short-run export supply elasticity leads us to overstate the optimal tariff by about 20-25 percentage points. Proportionally, the optimal tariff from the short-run export supply elasticity is quite large with a higher Armington elasticity. For instance, when the Armington elasticity is 5 and the long-run trade elasticity is 60, the optimal tariff is about 8 percent but the short-run elasticity points to an optimal tariff of 25 percent. We also consider the relationship between the optimal one-time tariff and the long-run trade elasticity (the triangles). We find this fares better, although it leads to a lower optimal tariff by 2-3 percentage points. The absolute gap is smallest the larger the long-run response.

When we return to our baseline inefficient model in Figure 5, we find the DESE continues to be an excellent approximation to the optimal tariff. As before using the short-run or long-run export supply elasticity yields a worse approximation.

We now show that the DESE does not always recover the optimal tariff. Indeed, it works best for the efficient model and with the best tax tradeoff. To see this, we solve for the optimal tariff when the tariff revenue is either 1) rebated lump sum or 2) used to reduce the labor tax. Figure 6 plots these two additional cases along with our investment subsidy offset. With the labor tax, the DESE formula would lead to an optimal tariff that is a bit higher than optimal. When the tariff revenue is rebated lump sum, the DESE does not work at all. Given the distortions in the model, the optimal tariff is generally negative and varies very little with the DESE.

4.3. Approximations of the DESE

We recover the DESE by solving an infinite horizon model with several endogenous state variables. Work in the sufficient statistic vein (Arkolakis et al., 2012) suggests using observed elasticities and trade shares in place of a macro model. Here we explore how alternative approximations of our formula fare for a range of elasticities recovered from our baseline

model. We find the export supply elasticity in these alternative formulas to overweight the long-run trade response, but to still yield an approximation that is within 1 percentage point of the optimal tariff.

We compare the optimal tariff from our model to the two approximations that only require knowing the discount factor and the dynamics of the export elasticity. The first measure is the present value of the inverse of the export supply elasticities, that is

$$PV(\frac{1}{\epsilon_t^X}) = (1-\beta)\sum_{t=0}^{\infty}\frac{\beta^t}{\epsilon_t^X}.$$

The second one is the inverse of present value of export supply elasticities, that is

$$\frac{1}{PV(\epsilon_t^X)} = \frac{1}{(1-\beta)\sum_{t=0}^{\infty}\beta^t\epsilon_t^X},$$

Figure 7 plots gap between the optimal tariff recovered from these formulas and the optimal one-shot tariff. We find that these two approximations tend to understate the optimal tariff by 0.5 to 1.25 percentage points. The gap is smaller (larger) in absolute (proportional) sense with a smaller optimal tariff. The present value of the inverse export supply elasticity is a slightly better fit than the inverse of the present value of the export supply elasticity.

5. Time Consistent Tariff

We now shift our analysis to consider a government that lacks commitment on trade policy. As in Kydland and Prescott (1977) and Barro and Gordon (1983), a policy is time consistent if it is optimal to implement in every period given that future governments will re-optimize. The resulting equilibrium is Markov: the government takes future policy rules as given and therefore does not internalize how current actions influence future policy choices. In trade

environments, this constraint is quantitatively important. In stylized models, Staiger and Tabellini (1987); Lapan (1988) and McLaren (1997) show that lack of commitment limits the government's ability to manipulate intertemporal distortions (e.g., entry or investment), leading time-consistent tariffs to differ systematically from their Ramsey counterparts. In our framework, we recover a similar result with the time consistent tariff more closely related to the short-run trade elasticity than the long-run trade elasticity.

Government Problem. The economy is summarized by a vector of state variables $s \in \mathcal{S}$, which includes the endogenous objects that evolve over time (e.g. firms, exporters, and capital in each country plus net foreign assets). Given a tariff τ and state s, a competitive equilibrium determines per-period utility $U(s, \tau)$ and a law of motion

$$s' = \Gamma(s, \tau).$$

The government chooses tariffs to maximize the present discounted value of utility,

$$\sum_{t=0}^{\infty} \beta^t U(s_t, \tau_t),$$

subject to the equilibrium mapping $s_{t+1} = \Gamma(s_t, \tau_t)$ and the constraint that policy must be time consistent. As a result, the problem is recursive and the relevant solution concept is a Markov equilibrium in which the tariff is a function of the current state, $\tau = T(s)$.

Time-Consistent Steady State. We restrict attention to stationary Markov equilibria and characterize only the associated steady state. A time-consistent steady state is a pair $(\bar{s}, \bar{\tau})$ and value function V such that

$$V(\bar{s}) = U(\bar{s}, \bar{\tau}) + \beta V(\bar{s}), \quad \bar{s} = \Gamma(\bar{s}, \bar{\tau}),$$

and

$$\bar{\tau} \in \arg\max_{\tau} \left\{ U(\bar{s}, \tau) + \beta V\big(\Gamma(\bar{s}, \tau)\big) \right\}.$$

If the solution is interior, $\bar{\tau}$ satisfies

$$\frac{\partial U(\bar{s}, \tau)}{\partial \tau} + \beta V_s(\bar{s}) \cdot \frac{\partial \Gamma(\bar{s}, \tau)}{\partial \tau} = 0.$$

This formulation imposes time consistency at the steady state while abstracting from the full policy function $T(s)$ and transition dynamics.[8]

In Figure 8 we plot the time consistent tariff in our model for a range of different short-run trade elasticities represented as the inverse of the short-run trade elasticity, γ-1. The time consistent tariff is a bit below this value but rises slightly less than linearly with it.

The time consistency problem is evident from the credible transition path as the government has an incentive to raise tariffs in the short-run more than the long-run. This tariff path yields a trade surplus followed by a trade deficit. As we noted, in financial autarky we get less overshooting of the tariff. With this in mind, we also plot the time consistent tariff when we shut down international borrowing and lending (financial autarky). Removing the option to borrow and save in international markets alters the time consistent tariff a bit. Relative to our baseline, financial autarky lowers the optimal tariff when the trade elasticity is low and raises it when it is low.

In Figure 9 we plot the time consistent tariff against the credible one-time tariff change along with a regression line and its coefficient. In this case, we are using the same structural parameters but a very different solution concept. There is a very tight linear relationship in this space. The time consistent tariff is about 15 percentage points higher and grows 12 percent faster than the one-time tariff.

[8] We solve for the time consistent tariff by searching for the tariff that the government would not want to deviate from for one period.

6. Conclusion

The central insight of the optimal tariff literature is that a country exploits its monopsony power on foreign countries through the foreign export supply elasticity it faces. Yet export supply decisions are inherently dynamic. Investments in firm creation, market access, and capital accumulation determine the responsiveness of trade flows over time, causing monopsony power itself to evolve. We show that the welfare consequences of tariffs are governed not simply by the current degree of monopsony power, but by its entire future path. These dynamics determine the path of optimal tariff policy.

Our main contribution is to derive the dynamic analogue to the Mill-Bickerdike-Johnson export supply elasticity tariff formula. We show that the optimal tariff is determined by a dynamic export supply elasticity. This dynamic elasticity can differ substantially from both the short-run and long-run elasticity. It also depends on the evolution of the aggregate economy through its effect on the stochastic discount factor. Despite the complexity of the dynamic environment, optimal tariff policy remains closely linked to a single sufficient statistic. The challenge for future work is therefore not simply to measure these elasticities at various horizons, but to recover the dynamic elasticity that governs the welfare effects of tariff policy.

A second implication is that the tariff that maximizes discounted lifetime welfare can differ substantially from the tariff that maximizes steady-state utility. The source of this difference is familiar from many dynamic economic environments: policies affect both long-run allocations and the transition path used to reach them. In the context of trade policy, these transition effects operate through market-access investments and capital accumulation. They have effects on the steady level of integration and allocations through firm creation and investment. As a result, optimal tariff calculations based on the steady state can provide a poor guide to policy even when they accurately describe long-run outcomes. Understanding

optimal trade policy therefore requires understanding the dynamics of trade adjustment.

The dynamics of monopsony power create an incentive for governments to exploit the relative fixity of trade and production structures in the short run while preserving future market power. Under commitment, the optimal policy is therefore front-loaded. Governments initially set tariffs above their long-run level and then gradually reduce them over time. This declining tariff path balances the gains from exploiting current monopsony power against the costs of discouraging future investments that make foreign supply more elastic.

The ability to commit to future policy is quantitatively important when the foreign export supply is dynamic. When governments cannot commit and instead reoptimize each period, the resulting stationary time-consistent tariff is substantially larger than the welfare-maximizing tariff. In this case, tariff policy becomes more closely tied to short-run foreign export supply elasticities because governments fail to internalize the future effects of current policy choices. These findings suggest that credibility and commitment are central determinants of both the level and persistence of trade barriers.

More broadly, our results suggest that the key objects governing optimal trade policy are fundamentally dynamic. The relevant elasticity is not the short-run elasticity, the long-run elasticity, or the elasticity observed in a static equilibrium. Rather, it is a dynamic object that reflects the entire path of trade adjustment. Recovering this object—and understanding how policy affects it appears essential for measuring monopsony power and designing optimal trade policy in a dynamic world.

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Figure 1: Optimal Tariff Path and Model Features

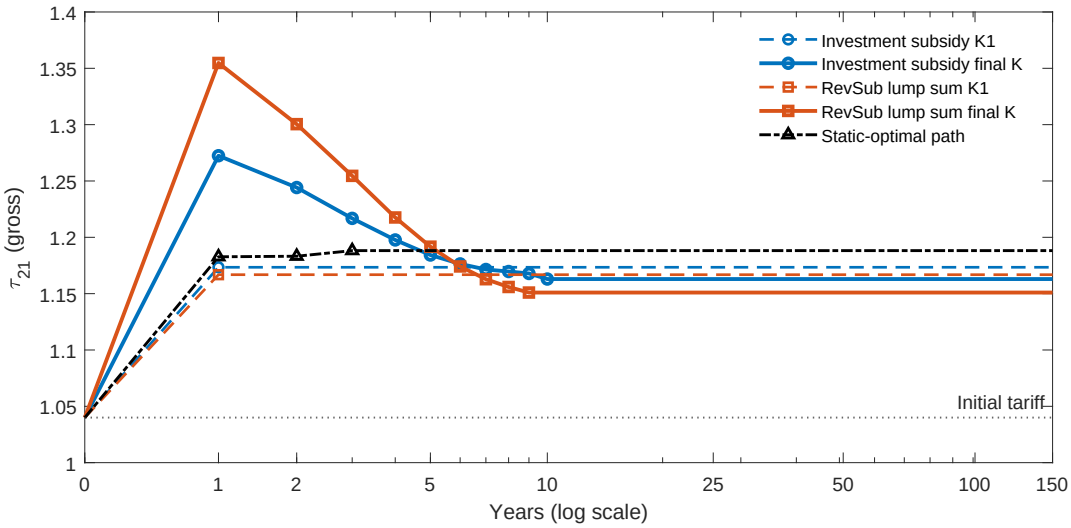


Figure 2: Macro Aggregates with Optimal Tariff Path

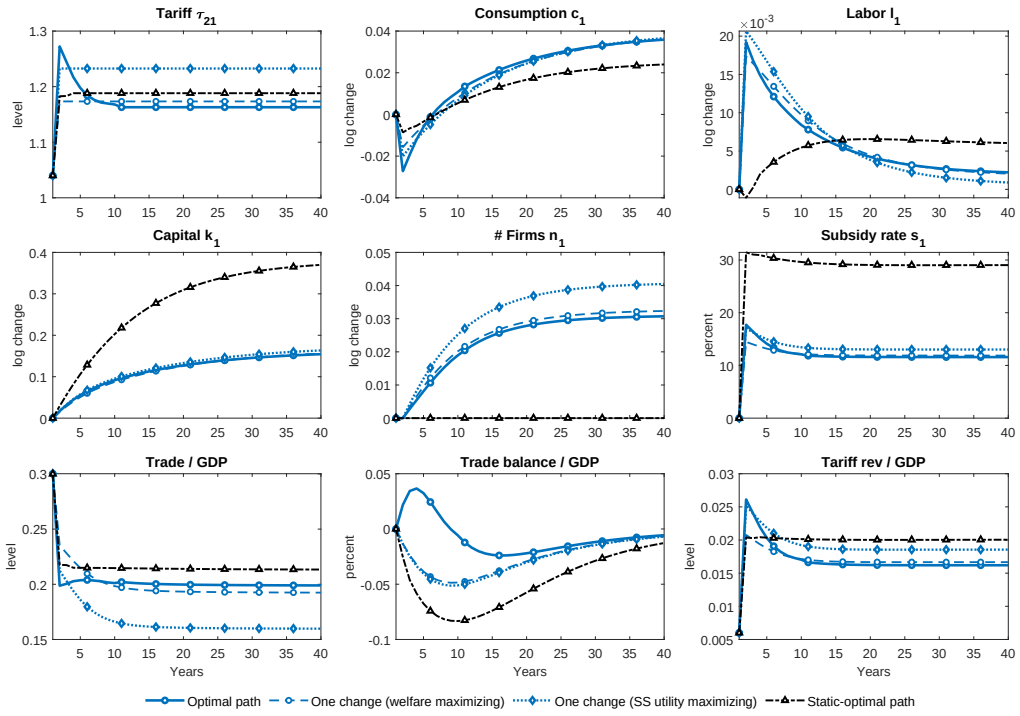


Figure 3: Export Supply Elasticity with Optimal One-time Tariff Change

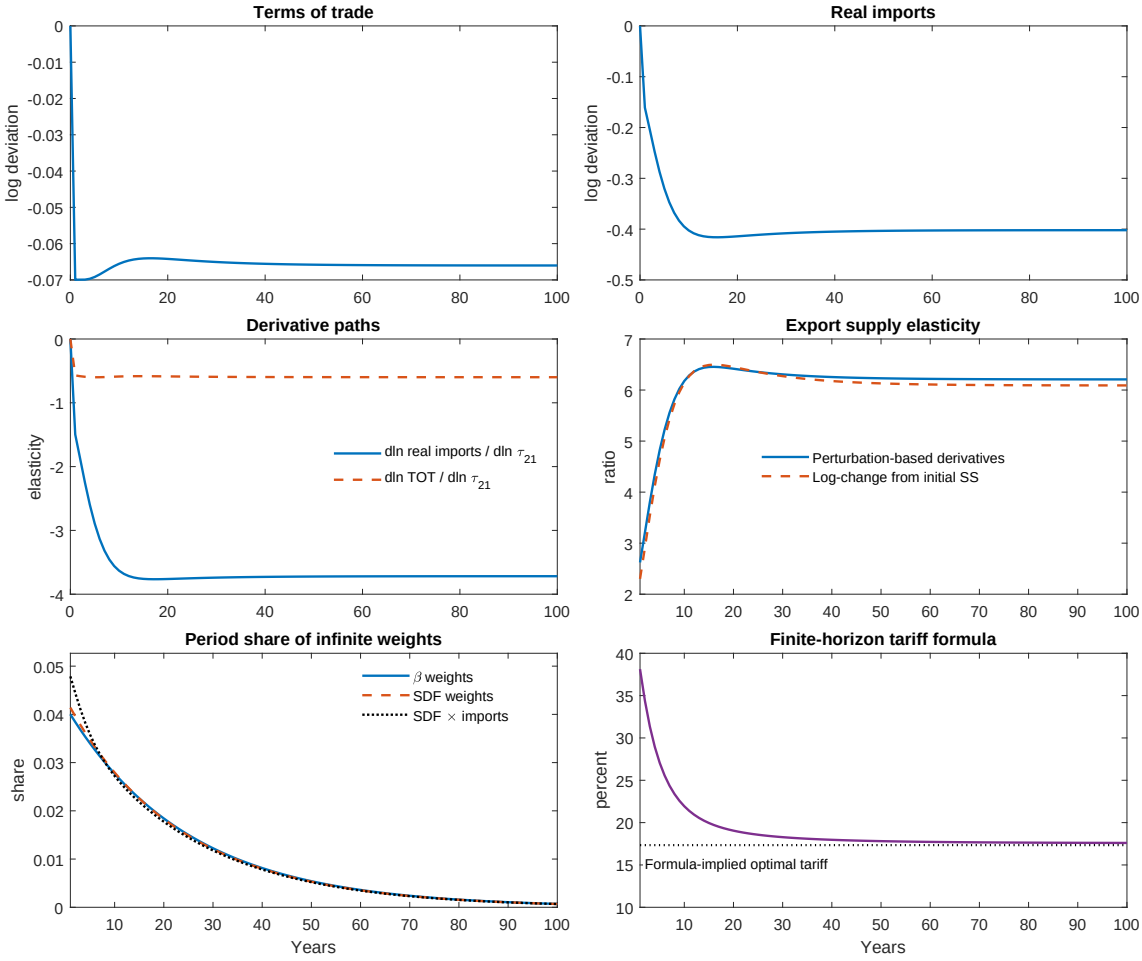


Figure 4: Optimal Tariff and Trade Elasticity — Efficient Economy

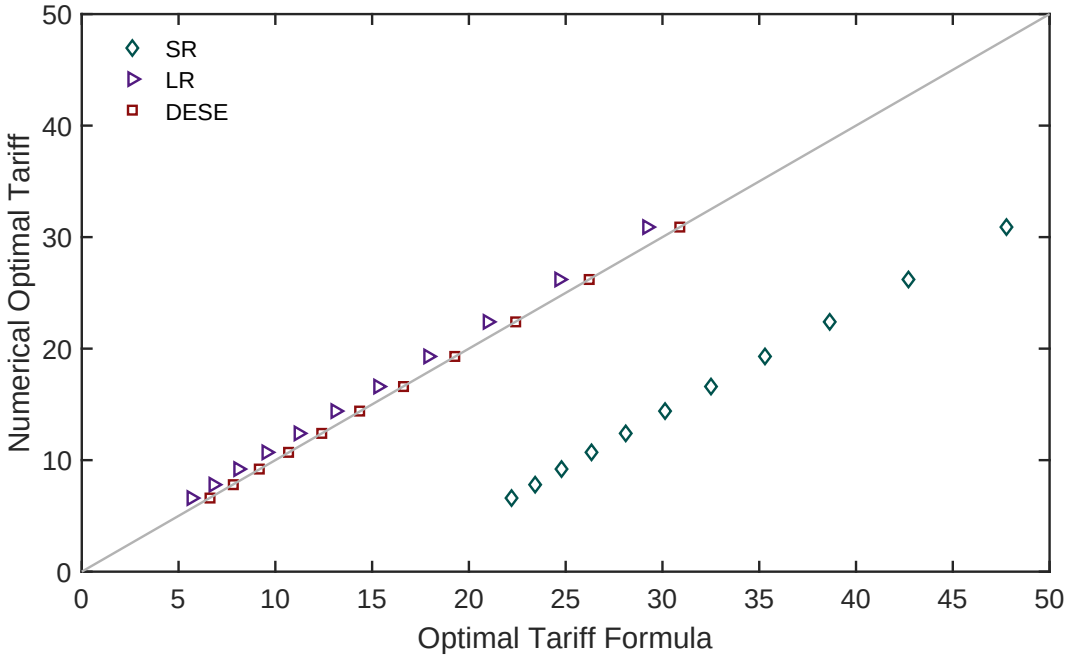


Figure 5: Optimal Tariff and Trade Elasticity — Baseline Economy

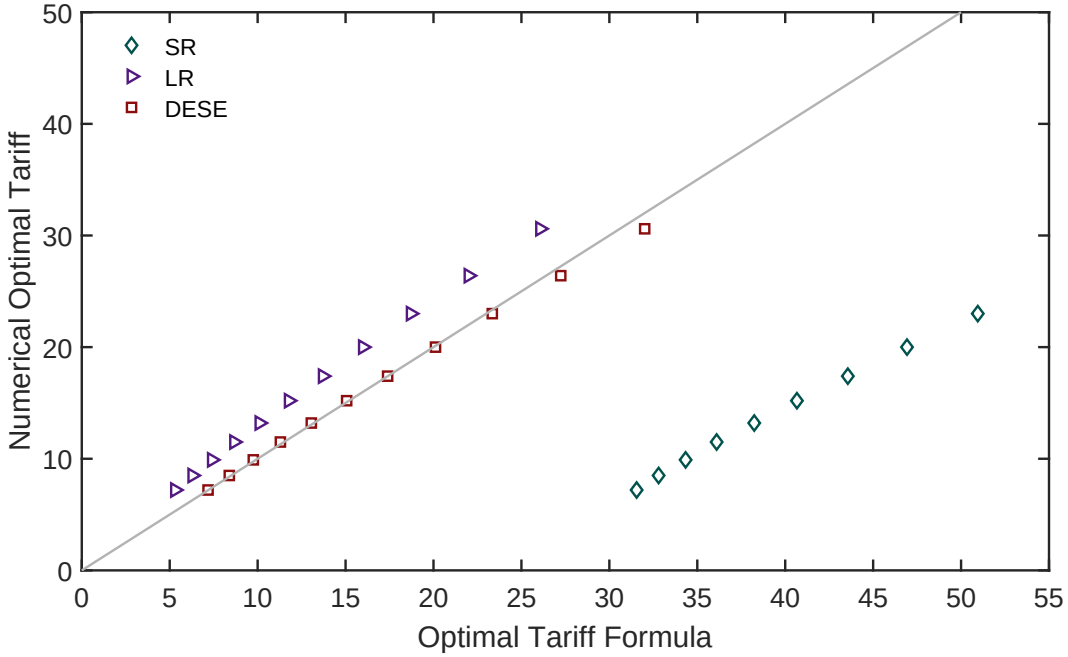


Figure 6: Optimal Tariff and DESE — Alternative Tax Uses

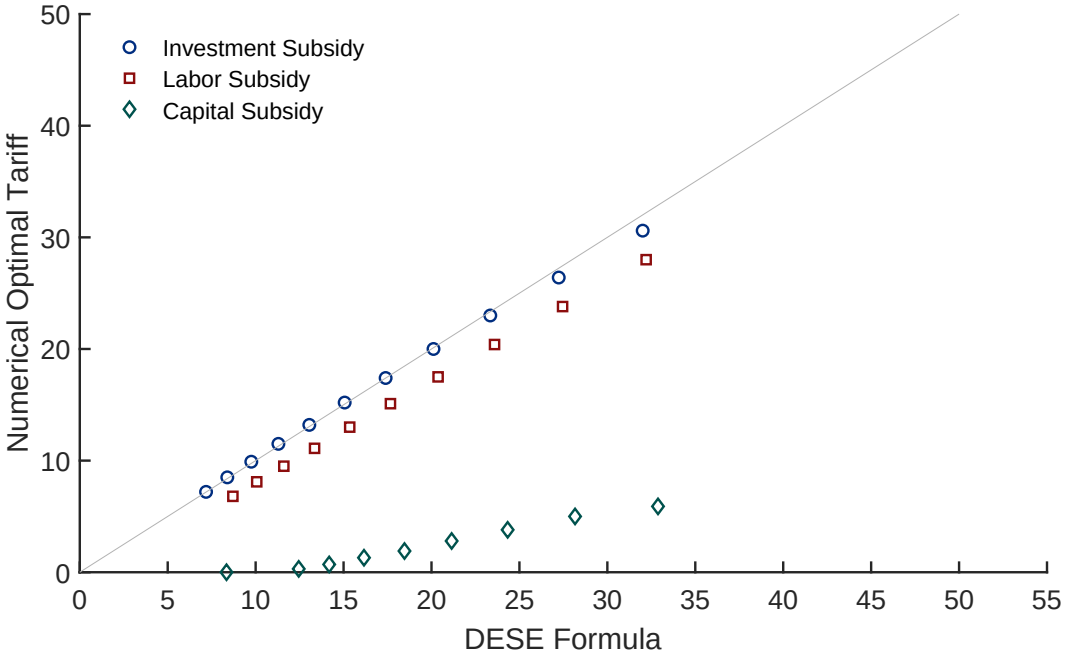


Figure 7: Formula Approximation

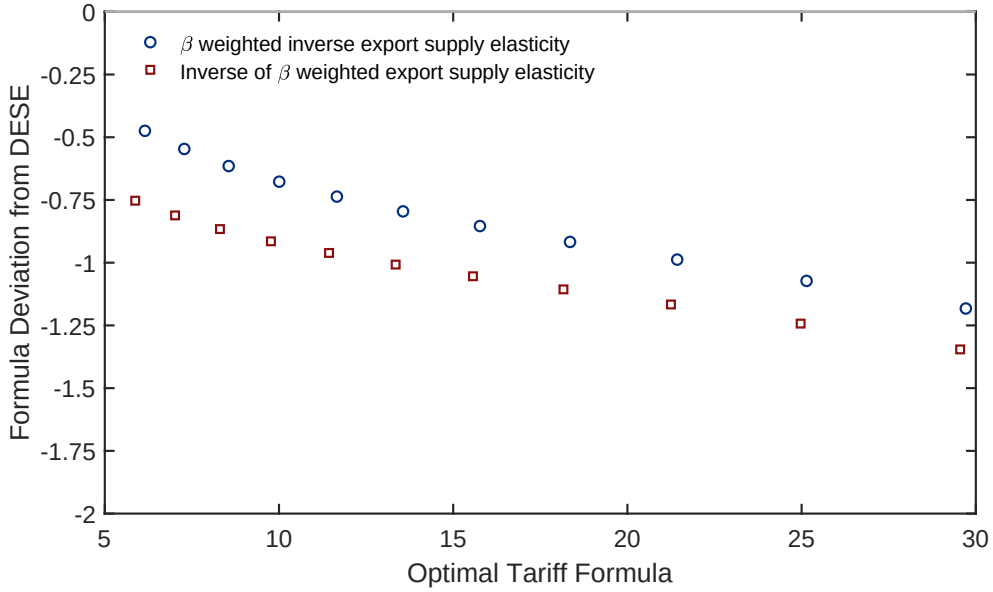


Figure 8: Optimal Tariff (Credibility and SR elasticity)

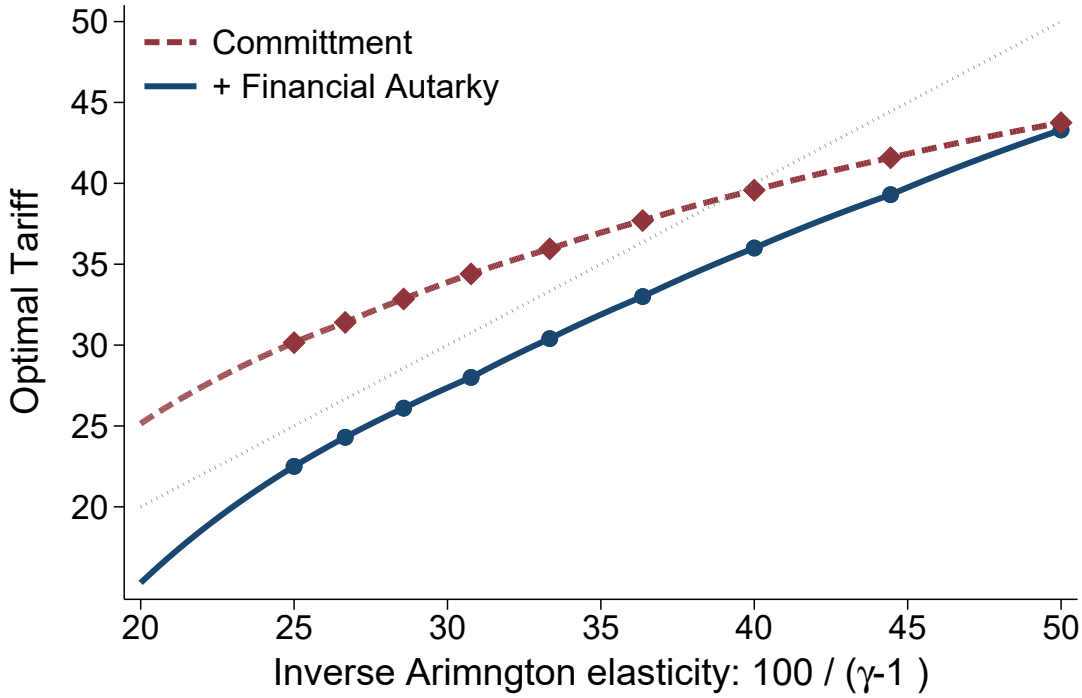


Figure 9: Optimal Tariff (Credibility vs Commitment)

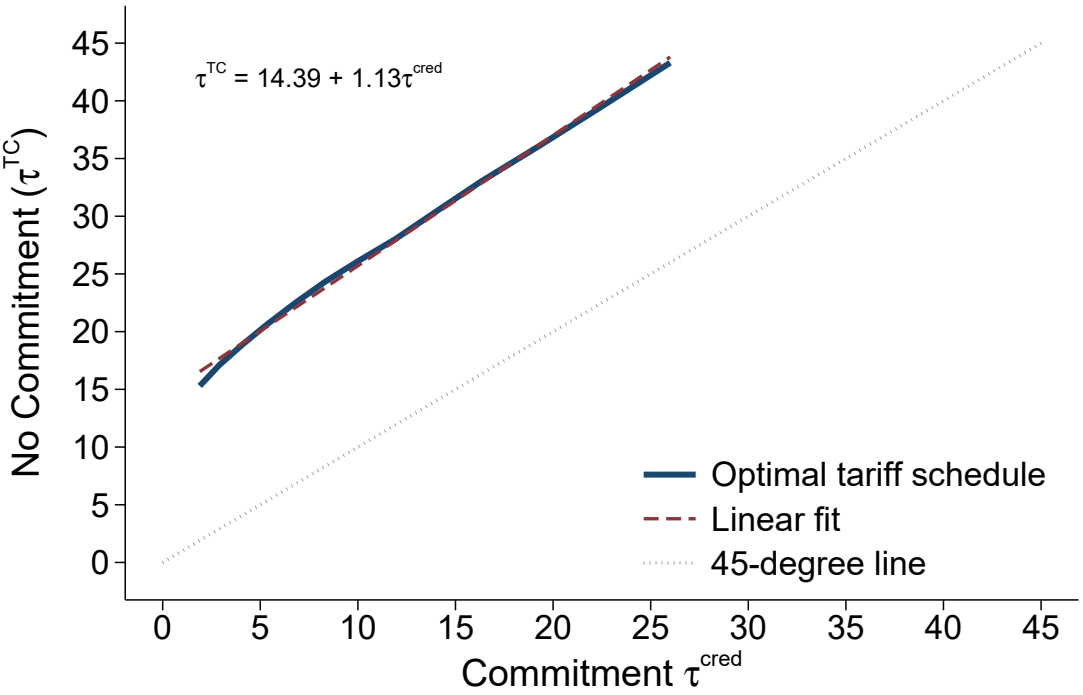


Table 1: annual calibration

Panel A: Externally Calibrated Parameters			
Param	Description	Value	Target
β	discount factor	0.96	
δ	capital depreciation rate	0.1	
σ	CRRA parameter	1	
γ	Armington elasticity	4	annual trade elasticity = 4
θ	Elasticity of substitution between varieties	6	Markup = 20%
n_s	Firm survival rate	0.98	Incumbent firm share(Alessandria and Choi, 2014)
α_m	Share of materials in intermediate production	0.6	Gross output/value added in X & M
α_C	Share of materials in C production	0.99	Gross output/value added in C
$\bar{L}_i$	Labor endowment	1	Normalization
Panel B: Internally Calibrate Parameters			
μ	Relative preference C vs. L	0.33	Average Frisch elasticity=2
f_e	Entry cost	4.16	Average $N_i = \bar{L}_i$
α	Capital share of value added	0.57	Average WL/VA = 66%
f_0	Fixed cost scale (nonexporters)	3.25	Fraction exporters = 0.2
f_1	Fixed cost scale (exporters)	0.05	Incumbent export share = 0.95
ν	Fixed cost shape parameter	0.65	Churning & export volume relationship
ξ_{LH}	Ratio ξ_{GL}/ξ_{GH}	0.69	Export share: firms $\leq$5 years tenure
ξ_{CH}	Consumption-good iceberg cost	2.28	End-use trade flows
ξ_{XH}	Investment-good iceberg cost	1.27	End-use trade flows
ξ_{MH}	Material-good iceberg cost	2.14	End-use trade flows
ρ	Persistency of efficiency state	0.82	Target jointly w/ other parameters
τ_k	Marginal Capital Tax	0.147	OMB (01/25)
τ_l	Marginal Labor Tax	0.272	OMB (01/25)
	Efficient case: $\tau_l = \tau_k = s_I = 0$; $s^R = \theta/(\theta-1)$		
	+ Static $f_0 = f_1 = 0, f_e = \infty, n_s = 1$		

Table 2: Consumption-Equivalent Welfare Gains (%)

	Mean tariff PV	Welfare	Utility	Steady state Consumption	Labor
Panel A: Distorted economy, investment subsidy					
One Time	17.341	1.537	3.906	3.993	0.168
Steady State	23.261	1.453	4.054	4.078	0.047
Optimal Path	17.545	1.554	3.843	3.942	0.191
Optimal Path τ_0	27.242				
Optimal Path τ_∞	16.296				
Panel B: Efficient economy, lump-sum rebate					
One Time	16.683	0.616	−0.057	−1.669	−1.629
Steady State	8.583	0.396	0.051	−0.711	−0.764
Optimal Path	17.370	0.667	−0.022	−1.506	−1.498
Optimal Path τ_0	35.474				
Optimal Path τ_∞	15.098				

* Welfare/Utility based on Compensating Variation.

Appendix

This appendix has two parts. First we describe the solution algorithm for the optimal path of tariffs. Second, we derive the dynamic export supply elasticity that recovers the one-time optimal unilateral tariff for a class of simpler models than our baseline.

A. Solution Method for the Optimal Tariff Path

We solve the Ramsey problem with full commitment by approximating the infinite-dimensional sequence of tariffs with a finite-dimensional parameterization. Specifically, we represent the tariff path using a finite set of evaluation points ("knots") and interpolate between them.

Let the break dates satisfy

$$1 = t_1 < t_2 < \cdots < t_K.$$

For a given set of breaks, the algorithm chooses the gross tariff levels at those dates, $\{\tau_{t_k}\}_{k=1}^{K}$, together with a curvature parameter $\alpha \in [0, 1]$. The full tariff path over a horizon of $T = 150$ periods is then constructed as

$$\tau_t = (1 - \alpha)\tau_t^{\text{lin}} + \alpha\tau_t^{\text{pchip}}, \qquad t = 1, \ldots, T,$$

where τ_t^{lin} is the piecewise-linear interpolant through the knots and τ_t^{pchip} is the shape-preserving cubic Hermite interpolant (PCHIP) through the same knots. Thus, $\alpha = 0$ delivers a piecewise-linear path, $\alpha = 1$ delivers a fully curved PCHIP path, and intermediate values blend the two. After the final break date t_K, the tariff is held constant. This parameterization provides a flexible but low-dimensional approximation to the space of admissible tariff paths, allowing for both smooth and kinked adjustments while keeping the optimization tractable.

In the investment-KAC specification, each knot tariff is constrained to lie in the interval

$$0.95 \leq \tau_{t_k} \leq 1.50,$$

Given a candidate path, we feed the implied sequence $\{\tau_t\}_{t=1}^{T}$ into a Dynare perfect-foresight equilibrium solver, which computes the transition path of the economy given the full sequence of tariffs. Welfare is evaluated using the simulated paths for Home consumption and labor. Let $u(c_t, \ell_t)$ denote period utility, and let (c_0^{ss}, ℓ_0^{ss}) and $(c_{T+1}^{ss}, \ell_{T+1}^{ss})$ denote the initial and terminal steady states. We compute normalized lifetime utility (scaled by $1-\beta$) as

$$W = (1-\beta)\sum_{t=1}^{T} \beta^{t-1} u(c_t, \ell_t) + \beta^T u(c_{T+1}^{ss}, \ell_{T+1}^{ss}),$$

and convert this object into a consumption-equivalent welfare measure relative to the initial steady state,

$$\text{CEV} = 100 \left[\left(\frac{W}{u(c_0^{ss}, \ell_0^{ss})} \right)^{\frac{1}{\mu(1-\sigma)}} - 1 \right].$$

This consumption-equivalent welfare measure is the objective maximized by the algorithm.

For a fixed set of break dates, the inner optimization is solved numerically in three steps. First, we apply a Nelder–Mead algorithm to the vector of knot tariffs and the curvature parameter α. Second, holding the knot tariffs fixed, we perform a one-dimensional polishing step over α. Third, when available, we refine the solution using a quasi-Newton method (`fminunc`). When the dimension of the problem increases, the previous lower-dimensional solution is used as a warm start: the old knot tariffs are interpolated onto the new break grid, and the previously optimal α is used as the initial guess. If Dynare fails at a candidate point, that candidate is assigned a very poor objective value, so infeasible regions are automatically penalized. This multi-step procedure improves convergence in a non-convex objective with occasionally ill-behaved regions due to equilibrium non-existence.

The number of evaluation points is determined endogenously by an expanding-tail procedure. The algorithm starts with a single break at period 1, then successively solves on the grids

$$[1], \quad [1,2,3], \quad [1,2,3,4], \quad \ldots$$

adding one new break at the end after each iteration. After each solve, we check whether the optimized tariff path has become approximately flat in the tail. Specifically, the procedure stops when

$$|\tau_{t_K} - \tau_{t_{K-1}}| \leq \varepsilon \qquad \text{and} \qquad |\tau_{t_{K-1}} - \tau_{t_{K-2}}| \leq \varepsilon,$$

with tolerance $\varepsilon = 0.005$. This stopping rule ensures that the terminal portion of the path is effectively constant, providing a good approximation to the infinite-horizon problem.

We have verified that increasing the horizon T, tightening the tolerance ε, or adding additional knots does not materially affect the resulting tariff paths or welfare outcomes.

B. The Dynamic Export Supply Elasticity and the Optimal One-Time Tariff

The standard way to compute the unilateral optimal tariff in a static trade model is to express the government's first-order condition in terms of the foreign export-supply elasticity facing the tariff-imposing country. This is the Mill–Bickerdike–Johnson logic: a tariff improves the terms of trade, but it also distorts the import margin (Mill, 1848; Bickerdike, 1906; Johnson, 1953). In many quantitative trade models, including Armington and monopolistic-competition environments, this logic implies that the optimal tariff rate is approximately the inverse of a macro export-supply elasticity (Gros, 1987; Costinot and Rodríguez-Clare, 2014; Ossa, 2016; Caliendo and Parro, 2022). The static formula isolates the basic force behind unilateral optimal tariffs: market power over foreign export supply. This force remains

operative in dynamic economies. However, intertemporal adjustment changes the object that measures that market power. When tariffs affect capital accumulation or exporter participation, the relevant inverse elasticity is no longer generally a static derivative evaluated within one allocation, but a present-value ratio of transition derivatives.

This section shows how that benchmark generalizes when the tariff induces transition dynamics. We first derive the formula in a static Armington model and show that an endogenous labor-supply margin does not introduce an additional term. It only changes the relationship between primitives. We then consider two deliberately simple dynamic environments. The first is a two-country two-good Armington model with capital accumulation, in the spirit of the international real business cycle model of Backus et al. (1994). The second removes capital and instead introduces gradual adjustment in the mass of foreign exporters to Home in the spirit of Alessandria and Choi (2007). These two cases isolate two different sources of transition dynamics: capital accumulation and firm-level export participation. Capital accumulation operates indirectly on the foreign export supply elasticity by affecting the capital stocks and thus the relative size of the two economies. Export decisions directly influence the export supply elasticity by changing the incentives to enter and exit the export market.

In the dynamic exercises Home chooses a one-time permanent gross tariff $\tau > 1$ on imports from Foreign, so the tariff rate is $\tau - 1$. Foreign's gross tariff is fixed at one. For different values of the Armington elasticity, γ, we compute the welfare-maximizing one-time tariff along the full transition path and compare it with the tariff implied by the first-order condition. Our solution is a golden search over the one-time tariff and does not use the first order condition to recover the optimal tariff. The central object is a present-value inverse dynamic export-supply elasticity. The dynamic formula can be written as

$$\tau - 1 = \frac{1}{\varepsilon^{q,PV}}, \tag{A.1}$$

where $\varepsilon^{q,PV}$ is computed from general-equilibrium transition derivatives, weighted by the household's marginal value of resources along the transition. The comparison below asks whether (A.1), evaluated at the numerical optimum, reproduces the welfare-maximizing tariff.

We should note that the first two approaches do not influence the trade elasticity, which measures the division of final expenditures between imports and domestic expenditures given the terms of trade, measured as a unit value, while the last measure does lead the trade elasticity to grow over time.

B.1. Static Armington Benchmark

Consider a static two-country Armington economy. Home consumes a CES aggregate of a Home good and imports from Foreign. Let p_H be the producer price of the Home good and p_F^* the Foreign producer price of the imported good. Home imposes a gross tariff τ on imports, so the Home consumer price of imports is τp_F^*. The Home price index is

$$P_H = \left[(1-\alpha_M)p_H^{1-\sigma} + \alpha_M(\tau p_F^*)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}, \tag{A.2}$$

and import demand is

$$M_{FH} = \alpha_M P_H^{\sigma-1}(\tau p_F^*)^{-\sigma} E_H, \tag{A.3}$$

where E_H is Home expenditure. Tariff revenue is rebated lump sum,

$$R = (\tau - 1)p_F^* M_{FH}. \tag{A.4}$$

Let X_{HF} denote Home exports to Foreign and assume balanced trade at producer prices,

$$p_H X_{HF} = p_F^* M_{FH}. \tag{A.5}$$

The terms of trade relevant for Home's import decision are

$$q^* \equiv \frac{p_F^*}{p_H}. \tag{A.6}$$

A reduction in q^* is an improvement in Home's terms of trade.

It is useful to derive the tariff first-order condition in a way that makes clear why endogenous labor supply does not change the formula. Suppose Home output is produced one-for-one with labor, $Y_H = h$, and the representative household has preferences $u(C_H, 1-h)$. The household budget constraint is

$$P_H C_H = p_H h + R. \tag{A.7}$$

The labor-supply first-order condition is

$$\frac{u_{1-h}(C_H, 1-h)}{u_C(C_H, 1-h)} = \frac{p_H}{P_H}. \tag{A.8}$$

Let d denote the total differential induced by a marginal change in the tariff. Using (A.8),

$$\begin{aligned} \frac{P_H}{u_C} dU_H &= P_H dC_H - p_H dh \\ &= d(P_H C_H) - C_H dP_H - p_H dh \\ &= h\, dp_H + dR - C_H dP_H. \end{aligned} \tag{A.9}$$

The term $p_H dh$ cancels because the labor margin is chosen optimally. This is the key envelope result: endogenous labor affects the allocation, but it does not create an additional wedge in the tariff first-order condition.

By Shephard's lemma for the Armington aggregator,

$$C_H dP_H = D_{HH} dp_H + M_{FH} d(\tau p_F^*), \tag{A.10}$$

where D_{HH} is Home absorption of the Home good. Since $Y_H = D_{HH} + X_{HF}$, equations (A.9) and (A.10) imply

$$\begin{aligned} \frac{P_H}{u_C} dU_H &= X_{HF} dp_H + dR - M_{FH} d(\tau p_F^*) \\ &= -p_F^* M_{FH} d\log q^* + (\tau - 1) p_F^* M_{FH} d\log M_{FH}. \end{aligned} \tag{A.11}$$

The first term is the terms-of-trade gain. The second term is the marginal cost of distorting imports. Differentiating with respect to $\log \tau$ gives

$$\frac{\partial U_H}{\partial \log \tau} = \frac{u_C}{P_H} p_F^* M_{FH} \left[(\tau - 1) \frac{\partial \log M_{FH}}{\partial \log \tau} - \frac{\partial \log q^*}{\partial \log \tau} \right]. \tag{A.12}$$

The optimal tariff therefore satisfies

$$\tau^S - 1 = \frac{\dfrac{\partial \log q^*}{\partial \log \tau}}{\dfrac{\partial \log M_{FH}}{\partial \log \tau}} \equiv \frac{1}{\varepsilon^{q,S}}, \tag{A.13}$$

where $\varepsilon^{q,S}$ is the export-supply elasticity facing Home, measured along the equilibrium response to the tariff. The same expression obtains when labor is fixed. With endogenous labor, the labor response changes the equilibrium elasticities in (A.13), but the structure of the formula is unchanged because the labor-supply response is an optimized private margin.

Equation (A.13) is the static benchmark for the dynamic exercises. The issue in a dynamic economy is not whether the terms-of-trade gain and import-distortion cost remain the relevant forces; they do. The issue is that a permanent tariff change generates an entire

transition path of prices, quantities, and marginal utilities. The static inverse elasticity must therefore be replaced by a present-value ratio of transition derivatives.

B.2. Armington Model with Capital

Consider a two-country Armington economy with Home and Foreign goods. Each country $i \in \{H, F\}$ produces a country-specific good using capital and a fixed labor endowment,

$$Y_{i,t} = A_i K_{i,t}^{\alpha} L_i^{1-\alpha}. \tag{A.14}$$

Let $p_{H,t}$ and $p_{F,t}$ denote producer prices. Home labor is the numeraire. Goods are combined into an Armington aggregate as shown in (A.2). The resulting import demand is the same as (A.3). Tariff revenue is rebated as a lump sum to the Home household. The Armington aggregate can be consumed, invested, or used to pay investment adjustment costs,

$$D_{H,t} = C_{H,t} + I_{H,t} + \Phi(I_{H,t}, K_{H,t}), \qquad \Phi(I, K) = \frac{\phi_I}{2}\left(\frac{I}{K} - \delta_K\right)^2 K, \tag{A.15}$$

with capital evolving according to

$$K_{H,t+1} = (1 - \delta_K) K_{H,t} + I_{H,t}. \tag{A.16}$$

The Foreign equations are symmetric, except that Foreign's tariff is fixed at one.

For a permanent tariff τ, Home welfare is

$$W_H(\tau) = \sum_{t=0}^{\infty} \beta^t u(C_{H,t}(\tau)). \tag{A.17}$$

Differentiating with respect to the permanent tariff gives

$$\frac{\partial W_H}{\partial \log \tau} = \sum_{t=0}^{\infty} \beta^t u_C(C_{H,t}) \frac{\partial C_{H,t}}{\partial \log \tau}. \tag{A.18}$$

This expression is not yet useful because a tariff changes consumption both directly, through the import wedge, and indirectly, through the whole transition of wages, prices, investment, and capital.

The key simplification comes from using the household's intertemporal optimality conditions. In the Armington economy with capital, the final good can be consumed, invested, or used to pay adjustment costs. A tariff change therefore affects welfare through the path of final-good resources and through the induced path of capital accumulation. However, along an equilibrium transition, the household's Euler equation and the envelope condition for capital imply that the first-order welfare effect of reallocating final goods across dates through investment is already priced by the household's stochastic discount factor. Hence, after applying the envelope condition, the welfare derivative can be written in terms of the tariff-induced change in the Home final-good budget at each date:

$$\frac{\partial W_H}{\partial \log \tau} = \sum_{t=0}^{\infty} \beta^t \frac{u_C(C_{H,t})}{P_{H,t}} \frac{\partial \mathcal{I}_{H,t}}{\partial \log \tau}, \tag{A.19}$$

where $\mathcal{I}_{H,t}$ denotes Home's nominal income measured in units of the Home final-good price index. Equation (A.19) is the dynamic analogue of the static envelope argument: endogenous choices such as investment and capital do not enter directly once the derivative is evaluated along the household's optimal transition.

It remains to characterize the tariff-induced change in Home income. Let

$$q_t^* \equiv \frac{p_{F,t}}{p_{H,t}} \tag{A.20}$$

denote the producer-price terms of trade relevant for Home's import wedge, and let

$$V_t^M \equiv p_{F,t} M_{FH,t} \tag{A.21}$$

denote the producer-value of Home imports from Foreign. Holding fixed the household's optimal intertemporal allocation, a marginal tariff change affects Home income through two channels.

First, the tariff distorts the import margin. Since Home pays the consumer price $\tau p_{F,t}$ for imports but the Foreign producer receives $p_{F,t}$, the marginal deadweight-loss term is proportional to the tariff wedge, $\tau - 1$, times the change in import demand:

$$d\mathcal{I}_{H,t}^{DW} = (\tau - 1) V_t^M \, d \log M_{FH,t}. \tag{A.22}$$

Second, the tariff changes the producer price of imports relative to Home goods. A fall in the Foreign producer price, holding import quantities fixed, is a terms-of-trade gain for Home. In log form, this contribution is

$$d\mathcal{I}_{H,t}^{TOT} = -V_t^M \, d \log q_t^*. \tag{A.23}$$

Combining (A.22) and (A.23), the date-t income effect of a permanent tariff perturbation is

$$\frac{\partial \mathcal{I}_{H,t}}{\partial \log \tau} = V_t^M \left[(\tau - 1) \frac{\partial \log M_{FH,t}}{\partial \log \tau} - \frac{\partial \log q_t^*}{\partial \log \tau} \right]. \tag{A.24}$$

Substituting (A.24) into (A.19) gives

$$\frac{\partial W_H}{\partial \log \tau} = \sum_{t=0}^{\infty} \beta^t \frac{u_C(C_{H,t})}{P_{H,t}} V_t^M \left[(\tau - 1) \frac{\partial \log M_{FH,t}}{\partial \log \tau} - \frac{\partial \log q_t^*}{\partial \log \tau} \right]. \tag{A.25}$$

Define the present-value weight

$$\omega_t^K \equiv \beta^t \frac{u_C(C_{H,t})}{P_{H,t}} V_t^M. \tag{A.26}$$

Then (A.25) becomes

$$\frac{\partial W_H}{\partial \log \tau} = \sum_{t=0}^{\infty} \omega_t^K \left[(\tau - 1) \frac{\partial \log M_{FH,t}}{\partial \log \tau} - \frac{\partial \log q_t^*}{\partial \log \tau} \right]. \tag{A.27}$$

The first term inside brackets is the marginal cost of increasing the import distortion along the transition. The second term is the marginal terms-of-trade gain from reducing the producer price of imports relative to Home goods.

At an interior optimum, (A.27) equals zero. Rearranging yields

$$\tau_K^{FOC} - 1 = \frac{1}{\varepsilon_K^{q,PV}} \equiv \frac{\sum_{t=0}^{\infty} \omega_t^K \frac{\partial \log q_t^*}{\partial \log \tau}}{\sum_{t=0}^{\infty} \omega_t^K \frac{\partial \log M_{FH,t}}{\partial \log \tau}}. \tag{A.28}$$

Thus, capital accumulation does not add a separate first-order term to the optimal tariff formula. Its effect is embedded in the transition derivatives of imports and the terms of trade, and in the stochastic-discount-factor weights ω_t^K. The resulting object is a ratio of present-value-weighted derivatives. It is not generally equal to the present-value average of period-by-period inverse elasticities, because the numerator and denominator must be aggregated before taking the ratio.

Figure **??** compares the tariff from (A.28) with the tariff obtained by directly maximizing (A.17). The points lie close to the 45-degree line, showing that the present-value formula recovers the numerical optimum. Figures **??** and **??** show that capital accumulation generates only modest differences between short-run and long-run elasticities in this simple Armington environment. Thus, in the capital-only model, the formula works because the transition

derivatives correctly price the gradual adjustment of production and absorption, even though the elasticity dynamics are quantitatively small.

B.3. Firm dynamics without capital: calibration and formula validation

This appendix isolates the role of exporter dynamics in the optimal-tariff formula. The full quantitative model combines capital accumulation, materials, fiscal policy, and firm-level export dynamics. The simplified exercise below removes capital and materials and keeps only the slow-moving export-participation margin. The purpose is threefold. First, we document why the calibration implies economically important differences between short-run and long-run elasticities. Second, we show that the optimal-tariff formula is a present-value object, not a static elasticity formula. Third, we clarify the accounting of the terms-of-trade price ratio: if the numerator uses the import-bundle price but the denominator uses a domestic variety price, the formula needs an extra numerator term; if instead the terms of trade are written as a ratio of ideal bundle price indices, that term is absorbed into the price-ratio derivative.

B.3.1 Calibration and the size of elasticity dynamics

The baseline quantitative model is calibrated so that the short-run quarterly trade elasticity is about 3.75, while exporter dynamics raise the long-run elasticity to about 7 for a permanent tariff change. Thus, even at the baseline calibration, the long-run trade response is almost twice the impact response. This gap is central for policy because a one-time permanent tariff affects welfare along the entire transition, not only in the impact period or in the terminal steady state.

The fixed-mass diagnostic model below uses a deliberately sparse calibration that preserves this core mechanism. The code calls the Home Armington elasticity σ; in the full model, the analogous cross-source elasticity is denoted by γ. In the diagnostic runs,

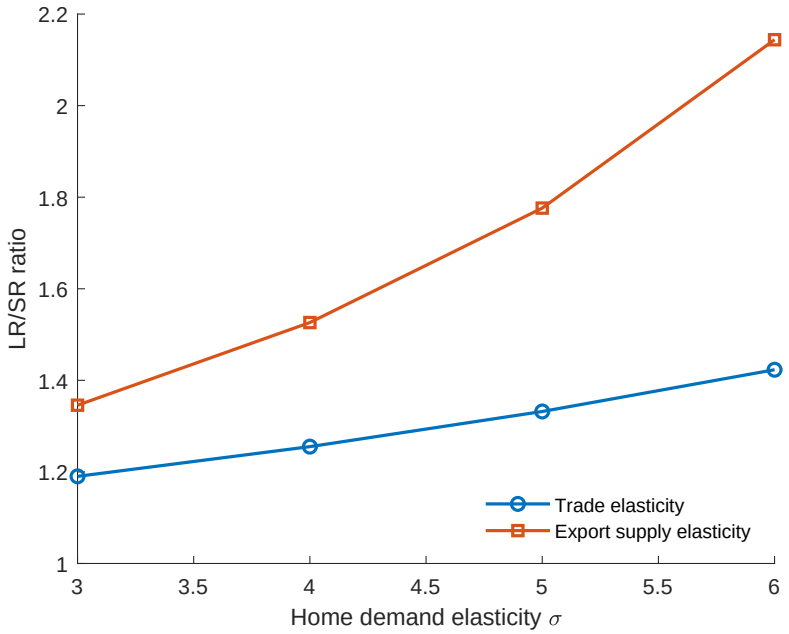


Figure A.1: Long-run to short-run elasticity ratios in the fixed-mass exporter-dynamics diagnostic. The figure plots the LR/SR ratio for the trade elasticity and the export-supply elasticity across the Armington elasticity grid. The dashed line is the static benchmark with no horizon dependence.

$\sigma \in \{3, 4, 5, 6\}$ and the exporter exit probability is $\delta_x = 0.12$. Figure A.1 plots the long-run elasticity relative to the short-run elasticity. A ratio equal to one is the static benchmark. Ratios above one indicate that exporter entry and exit amplify the trade response over time. This is why the optimal tariff is disciplined by a present-value elasticity object rather than by either the impact elasticity or the terminal long-run elasticity alone.

Table A.1 reports the parameter values used in the fixed-mass validation figures. The diagnostic strips away capital accumulation, materials, input-output linkages, financial dynamics, and variation in the total mass of domestic firms, so the only dynamic state is the mass of active exporters.

We report the endogenous-labor cases. The exogenous-labor cases produce the same accounting message and are omitted to avoid repetition. We also report two markup/fiscal closures: one without the sales revenue subsidy and one with the sales revenue subsidy that

Table A.1: Calibration of the fixed-mass firm-dynamics validation exercise

Object	Value	Role
β	0.96	Annual discount factor in the diagnostic model
σ	$\{3, 4, 5, 6\}$	Home Armington demand elasticity grid
α_M	0.25	Import weight in the Armington aggregator
θ	10	Elasticity across exporters within an origin bundle
N_H, N_F	$1, 1$	Fixed total potential firm masses
δ_x	0.12	Exit probability of active exporters
δ_n	0.02	Operating-firm death parameter retained in the code
$\bar{f}$	16	Upper support of export fixed-cost draws
T	60	Transition horizon used in the diagnostic runs
Frisch elasticity	1	Endogenous labor supply elasticity
Initial tariff	$\tau_0 = 1$	Symmetric free-trade steady state

offsets the markup wedge in buyer prices. The label "revenue subsidy" in the figure file names refers to this sales subsidy, not to tariff-revenue recycling. When active, the subsidy makes buyer prices equal marginal cost while firms still receive markup revenue, and it is financed by a lump-sum tax on the domestic household.

B.3.2 Fixed-mass exporter dynamics

The total mass of potential firms is fixed at one in each country. This closure keeps the Home domestic bundle price fixed and leaves export participation as the only slow-moving state. Let $m_{FH,t}$ denote the mass of Foreign firms exporting to Home. Conditional on a Foreign variety producer price $p^*_{FH,t}$, the producer price index for the Foreign bundle sold in Home is

$$\mathcal{P}^*_{FH,t} = m^{1/(1-\theta)}_{FH,t} p^*_{FH,t}, \tag{A.29}$$

where $\theta > 1$ is the elasticity across varieties within an origin. A Home tariff increase raises the buyer price of this bundle directly, through $\tau \mathcal{P}^*_{FH,t}$, and indirectly over time, by reducing

the value of serving Home and hence reducing future Foreign exporter entry.

Let $\Delta_{FH,t}$ be the value surplus from being an active Foreign exporter to Home, net of the value of not serving the market. With fixed costs distributed uniformly on $[0, \bar{f}]$, the interior entry probability is

$$e_{FH,t} = \frac{\beta \Delta_{FH,t+1}}{w_t^* \bar{f}}, \tag{A.30}$$

and the exporter law of motion is

$$m_{FH,t+1} = (1 - \delta_x) m_{FH,t} + \left[1 - (1 - \delta_x) m_{FH,t}\right] e_{FH,t}. \tag{A.31}$$

The surplus satisfies

$$\Delta_{FH,t} = \pi_{FH,t} + \beta(1 - \delta_x)\Delta_{FH,t+1} - \frac{\left(\beta \Delta_{FH,t+1}\right)^2}{2 w_t^* \bar{f}}. \tag{A.32}$$

A tariff increase lowers exporter profitability, reduces entry, and gradually contracts the mass of Foreign exporters serving Home. On impact, $m_{FH,t}$ is predetermined, so the import response is governed mostly by substitution across existing varieties. In the long run, entry and exit also change the set of available Foreign varieties, amplifying the trade response.

Home welfare along the transition induced by a permanent tariff τ is

$$W_H(\tau) = \sum_{t=0}^{\infty} \beta^t \left[u(C_{H,t}) - v(L_{H,t})\right], \qquad C_{H,t} = \frac{E_{H,t}}{P_{H,t}}. \tag{A.33}$$

Endogenous labor changes the equilibrium path and the welfare weights, but it does not add a separate term to the optimal-tariff formula. The household labor first-order condition,

$$v_L(L_{H,t}) = u_C(C_{H,t}) \frac{w_{H,t}}{P_{H,t}}, \tag{A.34}$$

makes the direct utility cost of a tariff-induced labor adjustment cancel the corresponding

labor-income term in the budget derivative.

B.3.3 Present-value first-order condition

Define the producer value of Home imports and the welfare weight

$$V_t^M \equiv \mathcal{P}_{FH,t}^* M_{FH,t}, \qquad \omega_t \equiv \beta^t \frac{u_C(C_{H,t})}{P_{H,t}} V_t^M. \tag{A.35}$$

The signed present-value import-demand component is

$$\mathcal{D}_M \equiv \sum_{t=0}^{\infty} \omega_t \frac{\partial \log M_{FH,t}}{\partial \log \tau}. \tag{A.36}$$

We distinguish two terms-of-trade price ratios. The first uses the Foreign import-bundle price in Home relative to the Home variety producer price in Foreign,

$$q_t^v \equiv \frac{\mathcal{P}_{FH,t}^*}{p_{HF,t}^*}, \tag{A.37}$$

where $p_{HF,t}^*$ is the Home variety producer price for sales to Foreign. This is the object labelled $q^* = p_{bundle}^{FH}/p^{HF,*}$ in the diagnostic figures. The second uses a ratio of ideal CES price indices,

$$q_t^I \equiv \frac{\mathcal{P}_{FH,t}^*}{\mathcal{P}_{HF,t}^*}, \qquad \mathcal{P}_{HF,t}^* = m_{HF,t}^{1/(1-\theta)} p_{HF,t}^*. \tag{A.38}$$

This object compares the Foreign bundle sold to Home with the Home bundle sold to Foreign. It is therefore the natural terms-of-trade object when the trade quantities are source-level bundles rather than individual varieties.

Let

$$N_q^v \equiv \sum_{t=0}^{\infty} \omega_t \frac{\partial \log q_t^v}{\partial \log \tau}, \tag{A.39}$$

$$N_q^I \equiv \sum_{t=0}^{\infty} \omega_t \frac{\partial \log q_t^I}{\partial \log \tau}. \tag{A.40}$$

When firm profits are rebated to the Home household, the welfare derivative also contains the effect of the tariff on Home net firm income. Let $\Pi_{H,t}$ denote Home firm profits net of fixed costs and define

$$N_\Pi \equiv \sum_{t=0}^{\infty} \beta^t \frac{u_C(C_{H,t})}{P_{H,t}} \frac{\partial \Pi_{H,t}}{\partial \log \tau}. \tag{A.41}$$

If the sales revenue subsidy is active, let $T_{H,t}^{rs}$ denote the lump-sum tax used to finance it, and define

$$N_{rs} \equiv \sum_{t=0}^{\infty} \beta^t \frac{u_C(C_{H,t})}{P_{H,t}} \frac{\partial T_{H,t}^{rs}}{\partial \log \tau}. \tag{A.42}$$

For the fixed-total-mass model, the domestic absorption bundle price is fixed, so there is no separate domestic-price correction term. If the terms of trade are measured with the variety-price denominator q_t^v, the welfare derivative is

$$\frac{\partial W_H}{\partial \log \tau} = (\tau - 1)\mathcal{D}_M - N_q^v + N_\Pi - \mathbf{1}_{rs} N_{rs}, \tag{A.43}$$

where $\mathbf{1}_{rs} = 1$ when the revenue subsidy is active and zero otherwise. The corresponding FOC-implied tariff is

$$\tau^{FOC} - 1 = \frac{N_q^v - N_\Pi + \mathbf{1}_{rs} N_{rs}}{\mathcal{D}_M}. \tag{A.44}$$

The terms $-N_\Pi + \mathbf{1}_{rs} N_{rs}$ are not additional economic wedges created by exporter dynamics. They are the income-accounting terms that are needed because q_t^v compares an import bundle price with a single Home variety price.

The ideal-price-index ratio absorbs this correction. Since

$$\log q_t^I = \log q_t^v - \log\left(\frac{\mathcal{P}_{HF,t}^*}{p_{HF,t}^*}\right), \tag{A.45}$$

the derivative of the Home export bundle price relative to the Home variety price is already inside $\partial \log q_t^I / \partial \log \tau$. In the fixed-mass diagnostic, the envelope accounting implies

$$N_q^I = N_q^v - N_\Pi + \mathbf{1}_{rs} N_{rs}. \tag{A.46}$$

Thus the same optimal-tariff formula can be written more compactly as

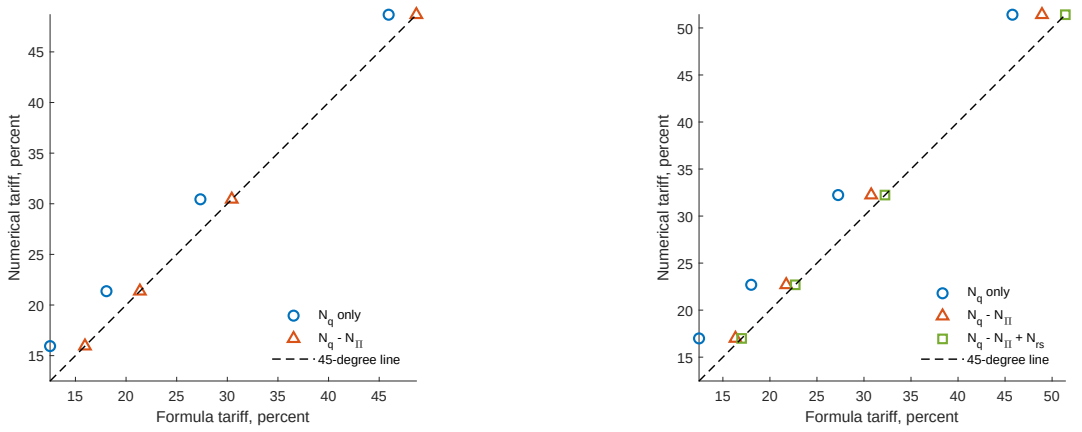
$$\tau^{FOC} - 1 = \frac{N_q^I}{\mathcal{D}_M}. \tag{A.47}$$

This is the cleanest implementation when imports and exports are measured as ideal bundles. If one instead uses the import-bundle-to-domestic-variety price ratio, the numerator must include the correction in (A.44). In a variable-total-mass version, a separate domestic absorption price-index correction can remain; the point here is that the export-bundle correction is eliminated by using the ideal import-bundle/export-bundle price ratio.

B.3.4 Numerical validation and the price-ratio diagnostic

Figure A.2 compares the FOC-implied tariff with the numerical welfare-maximizing tariff for $\sigma \in \{3, 4, 5, 6\}$. The left panel reports the endogenous-labor fixed-mass model without the sales revenue subsidy. The code's baseline terms-of-trade numerator uses q_t^v, so the formula that uses only N_q^v misses the numerical optimum. Adding the profit correction, $N_q^v - N_\Pi$, brings the formula close to the 45-degree line. The right panel repeats the exercise when the sales revenue subsidy is active. In that case the full expression is $N_q^v - N_\Pi + N_{rs}$, which again matches the numerical optimum. Thus the validation exercise confirms that

exporter dynamics require present-value transition derivatives, while the extra accounting terms depend on how the terms-of-trade price ratio is written.

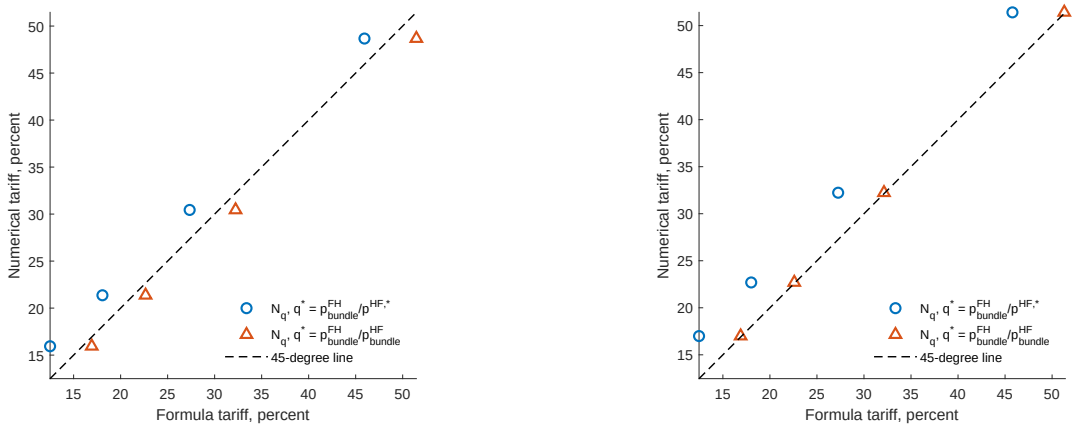


(a) No sales revenue subsidy

(b) With sales revenue subsidy

Figure A.2: Firm-dynamics validation without capital. The figure compares the numerical welfare-maximizing one-time tariff with the FOC-implied tariff in the fixed-total-mass model with endogenous labor. The figure labels use the variety-denominator terms-of-trade numerator N_q^v. The full formula includes the corresponding income-accounting terms: rebated profits in both panels and the subsidy-financing tax in panel (b).

Figure A.3 shows the same point from the price-index side. The circles use $q_t^v = \mathcal{P}^*_{FH,t}/p^*_{HF,t}$, the import-bundle price relative to the Home variety price. Because this ratio omits the Home export-bundle price-index component, the N_q^v-only formula lies away from the 45-degree line. The triangles use $q_t^I = \mathcal{P}^*_{FH,t}/\mathcal{P}^*_{HF,t}$, the ratio of ideal bundle price indices. This definition incorporates the missing export-bundle correction, so the formula aligns with the numerical optimum without separately adding the profit/subsidy accounting term. Equations (A.44)–(A.47) show that these are two equivalent implementations of the same FOC.



(a) No sales revenue subsidy

(b) With sales revenue subsidy

Figure A.3: Terms-of-trade price-ratio diagnostic. Circles compute the formula using the import-bundle price relative to the Home variety price, $q_t^v = \mathcal{P}^*_{FH,t}/p^*_{HF,t}$. Triangles compute it using the ideal price-index ratio, $q_t^I = \mathcal{P}^*_{FH,t}/\mathcal{P}^*_{HF,t}$. The latter absorbs the export-bundle correction that otherwise appears as a separate numerator term.

C. Sunk-Cost Exporter Dynamics: The One-Time Tariff and the committed Tariff Path

The previous section established that, once a permanent tariff induces a transition path, the static inverse export-supply elasticity is replaced by a present-value ratio of transition derivatives. The fixed-mass diagnostic of Section B.3 isolated the exporter-participation margin while holding the total mass of producers fixed. This section completes the argument by deriving the optimal tariff in the full model, in which the participation margin is driven by a sunk export cost (Alessandria and Choi, 2007), the total mass of producers is endogenous through free entry, labor supply is elastic, and a production subsidy can be used to align private and social incentives.

Two objects are derived. The first is the optimal one-time permanent tariff τ: a single rate chosen at date 0 and maintained forever, internalizing how a constant tariff moves the entire equilibrium path. The second is the optimal committed tariff path $\{\tau_t\}_{t\geq 0}$: the date-

by-date Ramsey tariff that implements the planner's allocation through Foreign's competitive offer surface. Both reduce to the present-value inverse export-supply elasticity of Section B. They differ only in how they aggregate the cross-date elasticities of Foreign export supply: the one-time tariff collapses them into a single scalar FOC, whereas the committed path conditions each τ_t on the date-t terms of trade.

C.1. Environment and Equilibrium Objects

There are two countries, Home H and Foreign F. Home labor is the numeraire, $w_{H,t} = 1$, and $w_{F,t}$ denotes the Foreign wage (written w_t^* in Section B.3). Foreign sells differentiated varieties to Home at the producer price

$$p^*_{FH,t} = \frac{\theta}{\theta - 1}\, w_{F,t}, \tag{A.48}$$

where $\theta > 1$ is the elasticity of substitution across varieties within an origin. Tariff revenue is rebated lump sum to the Home household.

Nested CES demand. Home combines a domestic bundle $D_{HH,t}$ and a Foreign import bundle $M_{FH,t}$ through a CES aggregator G with cross-source (Armington) elasticity γ, $\theta > \gamma > 1$. The Home price index is

$$P_{H,t} = \left[(1-\alpha_M)\big(p^D_{H,t}\big)^{1-\gamma} + \alpha_M\big(\tau\,\mathcal{P}^*_{FH,t}\big)^{1-\gamma}\right]^{\frac{1}{1-\gamma}}, \tag{A.49}$$

where $p^D_{H,t}$ is the domestic-bundle price, α_M is the import weight, and the Foreign import-bundle price in Home is the ideal CES index over the active Foreign varieties,

$$\mathcal{P}^*_{FH,t} = m_{FH,t}^{\frac{1}{1-\theta}}\, p^*_{FH,t} = m_{FH,t}^{-\frac{1}{\theta-1}}\, p^*_{FH,t}, \tag{A.50}$$

with $m_{FH,t}$ the mass of Foreign firms exporting to Home. Import-bundle demand is

$$M_{FH,t} = \alpha_M \, E_{H,t} \, P_{H,t}^{\gamma-1} \left(\tau \, \mathcal{P}^*_{FH,t}\right)^{-\gamma}, \qquad C_{H,t} = \frac{E_{H,t}}{P_{H,t}}, \tag{A.51}$$

where $E_{H,t}$ is Home nominal expenditure and $C_{H,t}$ real consumption. Symmetrically, $X_{HF,t}$ is the Home export bundle to Foreign, priced at the ideal index

$$\mathcal{P}^*_{HF,t} = m_{HF,t}^{-\frac{1}{\theta-1}} \, p^*_{HF,t}, \tag{A.52}$$

with $m_{HF,t}$ the mass of Home exporters and $p^*_{HF,t}$ the Home variety price for sales to Foreign. The terms of trade relevant for Home's import wedge is the ideal price-index ratio of Section B.3,

$$q^I_t \equiv \frac{\mathcal{P}^*_{FH,t}}{\mathcal{P}^*_{HF,t}} \;\equiv\; \pi_t. \tag{A.53}$$

A fall in q^I_t is a Home terms-of-trade improvement. We write $\pi_t \equiv q^I_t$ when this relative price is the planner's manipulated object.

Sunk-cost exporter dynamics. New producers enter as non-exporters. Each period a non-exporter draws a sunk export cost f in labor units, $f \sim U[0, \bar{f}]$ for Foreign firms (and $f \sim U[0, \bar{f}^*]$ for Home firms). A producer that pays the cost begins exporting next period if it survives, and once an exporter it keeps exporting until producer death, which occurs with probability δ each period. Define the survival-adjusted discount factor

$$q \equiv \beta(1-\delta) \qquad \text{(discount factor times the survival probability)}. \tag{A.54}$$

With uniform draws, the share of non-exporters that start exporting at t is

$$e_{FH,t} = \frac{q \, \Delta_{FH,t+1}}{w_{F,t} \, \bar{f}}, \tag{A.55}$$

where $\Delta_{FH,t+1}$ is the value surplus of switching from non-exporter to exporter, evaluated before the sunk cost is paid. Let $Q_{FH,t}$ be total Foreign exports to Home, so each active exporter sells $q_{FH,t} = Q_{FH,t}/m_{FH,t}$ and earns the operating surplus $\pi_{FH,t} = p^*_{FH,t}\, q_{FH,t}$; the producer value of imports is then $\mathcal{P}^*_{FH,t} M_{FH,t} = m_{FH,t}\, \pi_{FH,t}$. The surplus obeys

$$\Delta_{FH,t+1} = q\big(\underbrace{\pi_{FH,t+1} + \Delta_{FH,t+2}}_{V_x}\big) - \frac{\big(q\,\Delta_{FH,t+2}\big)^2}{2\,w_{F,t+1}\,\bar{f}}, \tag{A.56}$$

in which the last term is the option value of already being an exporter at $t+2$ (it equals $\frac{1}{2}\, e_{FH,t+1}\, q\, \Delta_{FH,t+2}$). The mass of Foreign exporters evolves as

$$m_{FH,t+1} = (1-\delta)\big[m_{FH,t} + e_{FH,t}\,(N^F_t - m_{FH,t})\big], \tag{A.57}$$

with $N^F_t - m_{FH,t}$ the mass of potential (currently non-exporting) Foreign producers. Home exporter dynamics are symmetric in $(m_{HF,t}, e_{HF,t}, \bar{f}^*)$.

Firm creation. The total mass of producers is endogenous. Entry costs f_E units of labor and a new producer begins operating subject to the free-entry condition and the producer law of motion

$$f_E\, w_{H,t} = q\, V_{D,t+1}, \qquad N_t = (1-\delta)\big(N_{t-1} + N^e_{t-1}\big), \tag{A.58}$$

where $V_{D,t}$ is the value of a Home producer and N^e_t the mass of Home entrants. The unit labor requirements for the domestic and export bundles fall in the number of varieties through love of variety,

$$a_t \equiv a(N_t) = N_t^{-\frac{1}{\theta-1}}, \qquad b_t \equiv b(m_{HF,t}) = m_{HF,t}^{-\frac{1}{\theta-1}}. \tag{A.59}$$

Labor market, profits, and the production subsidy. With elastic labor, the household chooses L_t subject to disutility $v(L_t)$, and the labor first-order condition is

$$v'(L_t) = u_C(C_{H,t})\,\frac{w_{H,t}}{P_{H,t}} = \frac{u_{C,t}}{P_{H,t}}, \qquad u_{C,t} \equiv u_C(C_{H,t}). \tag{A.60}$$

Labor clears across production, producer entry, and export entry,

$$L_t = L_{p,t} + f_E\,N_t^e + L_t^x, \qquad L_{p,t} = a_t\,D_{HH,t} + b_t\,X_{HF,t}, \qquad L_t^x = \frac{\bar{f}^*}{2}\,(N_t - m_{HF,t})\,e_{HF,t}^2, \tag{A.61}$$

where $L_{p,t}$ is production labor and L_t^x the labor used to pay Home export-entry draws. An ad-valorem production subsidy s on all Home sales, financed by a lump-sum tax $T_t^s = s\,\kappa_s L_{p,t}$, makes prices

$$p_{H,t}^D = \kappa_s\,a_t, \qquad \mathcal{P}_{HF,t}^* = \kappa_s\,b_t, \qquad \kappa_s \equiv \frac{\theta}{(\theta-1)(1+s)}. \tag{A.62}$$

Two values matter: $\kappa_0 = \theta/(\theta-1)$ is laissez-faire markup pricing, while $\kappa_s = 1$ (i.e. $1+s = \theta/(\theta-1)$) implements marginal-cost buyer prices. Per-unit receipts $(1+s)\,p = \frac{\theta}{\theta-1} w_H$ are invariant to s, so firm profits and the entire entry block in (A.55)–(A.58) are unchanged by the subsidy. Throughout, define the household's discounted marginal value of resources

$$\mu_t \equiv \beta^t\,\frac{u_{C,t}}{P_{H,t}}. \tag{A.63}$$

C.2. The Home Government's One-Time Tariff Problem

Home chooses a single permanent gross tariff τ. Home welfare and its tariff derivative are

$$W_0(\tau) = \sum_{t=0}^{\infty} \beta^t \big[u(C_{H,t}) - v(L_t)\big], \qquad \frac{dW_0}{d\ln\tau} = \sum_{t=0}^{\infty} \beta^t \Big[u_{C,t}\frac{dC_{H,t}}{d\ln\tau} - v'(L_t)\frac{dL_t}{d\ln\tau}\Big]. \tag{A.64}$$

This is not yet usable, because a permanent tariff moves consumption through the whole transition of prices, masses, entry, and labor. We make it usable with the household budget identity and Shephard's lemma.

Expenditure identity. Home income equals labor income, profits, the tariff rebate, and minus the subsidy tax,

$$E_{H,t} = w_{H,t}L_t + \Pi_{H,t} + (\tau - 1)\,\mathcal{P}^*_{FH,t}M_{FH,t} - T^s_t, \qquad \Pi_{H,t} = \frac{L_{p,t}}{\theta - 1} - f_E N^e_t - L^x_t. \quad \text{(A.65)}$$

Using $\kappa_s(1+s) = \theta/(\theta-1)$ and labor clearing (A.61), the entry-cost terms and the subsidy bill cancel, leaving a representation that is uniform in s:

$$E_{H,t} = \kappa_s L_{p,t} + (\tau - 1)\,\mathcal{P}^*_{FH,t}M_{FH,t} = p^D_{H,t}D_{HH,t} + \mathcal{P}^*_{HF,t}X_{HF,t} + (\tau - 1)\,\mathcal{P}^*_{FH,t}M_{FH,t}. \quad \text{(A.66)}$$

The bracketed first two terms are total Home sales, $(\text{sales})_t \equiv p^D_{H,t}D_{HH,t} + \mathcal{P}^*_{HF,t}X_{HF,t}$.

Shephard's lemma and $P_{H,t}\,dC_{H,t}$. Differentiating $P_{H,t}C_{H,t} = E_{H,t}$ gives $P_{H,t}\,dC_{H,t} = dE_{H,t} - C_{H,t}\,dP_{H,t}$. Shephard's lemma applied to the Armington index, holding behavioral responses fixed in $dP_{H,t}$, gives

$$C_{H,t}\,dP_{H,t} = D_{HH,t}\,dp^D_{H,t} + M_{FH,t}\,d\big(\tau\,\mathcal{P}^*_{FH,t}\big). \quad \text{(A.67)}$$

Substituting (A.66) and (A.67),

$$\begin{aligned} P_{H,t}\,dC_{H,t} &= d(\text{sales})_t + (\tau-1)\,\mathcal{P}^*_{FH,t}\,dM_{FH,t} - M_{FH,t}\,d\mathcal{P}^*_{FH,t} - D_{HH,t}\,dp^D_{H,t} \\ &= p^D_{H,t}\,dD_{HH,t} + \mathcal{P}^*_{HF,t}\,dX_{HF,t} + X_{HF,t}\,d\mathcal{P}^*_{HF,t} + (\tau-1)\,\mathcal{P}^*_{FH,t}\,dM_{FH,t} - M_{FH,t}\,d\mathcal{P}^*_{FH,t}. \end{aligned} \quad \text{(A.68)}$$

Balanced trade $\mathcal{P}^*_{HF,t}X_{HF,t} = \mathcal{P}^*_{FH,t}M_{FH,t}$ merges the two bundle-price terms into the terms of trade,

$$X_{HF,t}\,d\mathcal{P}^*_{HF,t} - M_{FH,t}\,d\mathcal{P}^*_{FH,t} = -\,\mathcal{P}^*_{FH,t}M_{FH,t}\,d\ln\left(\frac{\mathcal{P}^*_{FH,t}}{\mathcal{P}^*_{HF,t}}\right) = -\,\mathcal{P}^*_{FH,t}M_{FH,t}\,d\ln q^I_t. \quad \text{(A.69)}$$

Hence

$$P_{H,t}\,dC_{H,t} = \mathcal{P}^*_{FH,t}M_{FH,t}\Big[(\tau-1)\,d\ln M_{FH,t} - d\ln q^I_t\Big] + \Big(p^D_{H,t}\,dD_{HH,t} + \mathcal{P}^*_{HF,t}\,dX_{HF,t}\Big). \quad \text{(A.70)}$$

Valuing the labor margin. The labor disutility term in (A.64) is priced with the household labor FOC (A.60):

$$-\beta^t v'(L_t)\frac{dL_t}{d\ln\tau} = -\mu_t\, w_{H,t}\frac{dL_t}{d\ln\tau}. \quad \text{(A.71)}$$

This is the envelope step: at the optimal labor choice the utility cost of marginal work equals its wage value, so labor adds no separate wedge beyond what it does to the equilibrium objects.

The welfare derivative. Collecting (A.70)–(A.71) and recalling $V^M_t \equiv \mathcal{P}^*_{FH,t}M_{FH,t}$, $\omega_t \equiv \mu_t V^M_t$,

$$\boxed{\frac{dW_0}{d\ln\tau} = \sum_{t=0}^{\infty}\mu_t\left\{\mathcal{P}^*_{FH,t}M_{FH,t}\left[(\tau-1)\frac{d\ln M_{FH,t}}{d\ln\tau} - \frac{d\ln q^I_t}{d\ln\tau}\right] + \Theta_t\right\}} \quad \text{(A.72)}$$

with the domestic production block

$$\begin{aligned}\Theta_t &\equiv p^D_{H,t}\frac{dD_{HH,t}}{d\ln\tau} + \mathcal{P}^*_{HF,t}\frac{dX_{HF,t}}{d\ln\tau} - w_{H,t}\frac{dL_t}{d\ln\tau} \\ &= \underbrace{(\kappa_s - 1)\frac{dL_{p,t}}{d\ln\tau}}_{\text{labor wedge}\times\text{empl.}} + \frac{\kappa_s}{\theta-1}\left(a_t D_{HH,t}\frac{d\ln N_t}{d\ln\tau} + b_t X_{HF,t}\frac{d\ln m_{HF,t}}{d\ln\tau}\right) - \underbrace{f_E\frac{dN^e_t}{d\ln\tau} - \frac{dL^x_t}{d\ln\tau}}_{\text{entry outlays}}.\end{aligned} \tag{A.73}$$

The expansion in (A.73) uses $p^D_{H,t} = \kappa_s a_t$, $\mathcal{P}^*_{HF,t} = \kappa_s b_t$, $d(a_t D_{HH,t}) = a_t\, dD_{HH,t} - \frac{1}{\theta-1} a_t D_{HH,t}\, d\ln N_t$, and the analogous expression for $b_t X_{HF,t}$. The terms in (A.72)–(A.73) have transparent readings:

- **Import-volume distortion** $(\tau-1)\, d\ln M_{FH,t}/d\ln\tau$: with $\tau > 1$ imports are already too low, so a further contraction is a first-order loss proportional to the wedge.
- **Terms-of-trade manipulation** $-\,d\ln q^I_t/d\ln\tau$: a pure transfer from Foreign, operating both through the Foreign wage $w_{F,t}$ (the intensive, static channel) and through the Foreign exporter mass $m_{FH,t}$ via love of variety in (A.50) (the extensive, dynamic channel).
- **Labor wedge** $(\kappa_s - 1)\, dL_{p,t}/d\ln\tau$: output is valued at consumer prices (κ_s times labor cost) but labor is valued at the wage; netting leaves the price–marginal-cost wedge, which is $\frac{1}{\theta-1} > 0$ at laissez-faire and collapses date by date under the corrective subsidy $\kappa_s = 1$.
- **Entry–investment flows**: variety payoffs (output-valued, hence κ_s) net of entry outlays (wage-valued). Together with the labor wedge these make up the domestic production block Θ_t, whose discounted sum vanishes under home production efficiency, as shown next.

C.3. Home Production Efficiency: The Domestic Production Block Vanishes

To evaluate $\sum_t \mu_t \Theta_t$ we use the optimality of Home's production block. Holding the trade path $\{M_{FH,t}, X_{HF,t}\}$ fixed, the Home allocation solves an inner planning problem in technology only — it contains no prices and no subsidy, so its constraints, FOCs, and solution are independent of s. Writing $m_t \equiv m_{HF,t}$, $e_t \equiv e_{HF,t}$, the inner problem is

$$\begin{aligned}
\max_{\{D_{HH,t}, L_t, N_t^e, e_t\}} \; & \sum_t \beta^t \big[u\big(G(D_{HH,t}, M_{FH,t})\big) - v(L_t)\big] \\
\text{s.t.} \quad & L_t = a(N_t) D_{HH,t} + b(m_t) X_{HF,t} + f_E N_t^e + L_t^x \quad [\beta^t \lambda_t] \\
& N_{t+1} = (1-\delta)\big(N_t + N_t^e\big) \quad [\beta^t \phi_t] \\
& m_{t+1} = (1-\delta)\big[m_t + e_t(N_t - m_t)\big] \quad [\beta^t \psi_t] \\
& L_t^x = \tfrac{\bar{f}^*}{2}(N_t - m_t)\, e_t^2 \quad [\beta^t \xi_t].
\end{aligned} \tag{A.74}$$

The first-order conditions are

$$\begin{aligned}
&\text{(F1)}\;\; u_{C,t} G_{D,t} = \lambda_t a_t, \quad \text{(F1}')\;\; v'(L_t) = \lambda_t, \qquad \text{(F2)}\;\; \lambda_t f_E = (1-\delta)\phi_t, \\
&\text{(F3)}\;\; \xi_t = -\lambda_t, \qquad \text{(F4)}\;\; (1-\delta)\psi_t = \lambda_t \bar{f}^* e_t, \quad \text{(F5),(F6)}\;\; \phi_t + \beta A_{t+1} = 0, \;\; \psi_t + \beta B_{t+1} = 0,
\end{aligned} \tag{A.75}$$

where (F5) and (F6) are the costate (Bellman) recursions for the producer and exporter masses, with collected coefficients

$$\begin{aligned}
A_t &= \lambda_t a_t' D_{HH,t} + \lambda_t \tfrac{\bar{f}^*}{2} e_t^2 - (1-\delta)\phi_t - (1-\delta) e_t \psi_t, \\
B_t &= \lambda_t b_t' X_{HF,t} - \lambda_t \tfrac{\bar{f}^*}{2} e_t^2 - (1-\delta)(1-e_t)\psi_t.
\end{aligned} \tag{A.76}$$

From (F1) and the household relation $G_{D,t} = p^D_{H,t}/P_{H,t}$,

$$\lambda_t = \frac{u_{C,t} G_{D,t}}{a_t} = \frac{u_{C,t}}{P_{H,t}} \frac{p^D_{H,t}}{a_t} = \kappa_s \frac{u_{C,t}}{P_{H,t}}, \tag{A.77}$$

so the planner's labor shadow value equals the household's marginal value of income, $\beta^t \lambda_t = \mu_t$, only under the corrective subsidy $\kappa_s = 1$; away from it the two differ by the markup factor, $\beta^t \lambda_t = \kappa_s \mu_t$.

A telescoping lemma. The inner problem is solved optimally before and after the perturbation, so its first-order conditions discipline how the home allocation reacts to the tariff. We use them to establish an auxiliary identity stated in terms of the planner's own shadow value of labor λ_t. For any tariff-induced perturbation with $dN_0 = dm_0 = 0$ that satisfies transversality,

$$\mathcal{S}_\lambda \equiv \sum_t \beta^t \lambda_t \big(a_t \, dD_{HH,t} + b_t \, dX_{HF,t} - dL_t \big) = 0. \tag{A.78}$$

This is a statement about the optimality of the home production plan: along the planner's transition, the labor freed up at the margin by adjusting production, exports, and entry is exactly priced by the costate values of the producer and exporter masses. It is not yet a welfare statement — the welfare-relevant object is the domestic production block $\sum_t \mu_t \Theta_t$, weighted by the household value μ_t rather than the planner shadow value λ_t. The two weightings coincide only under home production efficiency, which is the bridge used below.

Proof. **Step 1.** The labor constraint binds before and after the perturbation, so differentiating (A.61),

$$a_t \, dD_{HH,t} + b_t \, dX_{HF,t} - dL_t = -\big(a'_t D_{HH,t} \, dN_t + b'_t X_{HF,t} \, dm_t + f_E \, dN^e_t + dL^x_t \big). \tag{A.79}$$

Step 2. Weight (A.79) by $\beta^t \lambda_t$ and sum over t; eliminate dN^e_t using (F2) together with the

differentiated N-law of motion, and eliminate de_t using (F4) together with the differentiated m-law of motion and the entry-cost technology. **Step 3.** Collecting terms by dN_{t+1} and dm_{t+1} and reindexing,

$$\mathcal{S}_\lambda = -\sum_t \beta^t \Big\{ \big[\phi_t + \beta A_{t+1}\big]\, dN_{t+1} + \big[\psi_t + \beta B_{t+1}\big]\, dm_{t+1} \Big\} = 0, \tag{A.80}$$

which is zero term by term because the bracketed expressions are exactly the costate FOCs (F5) and (F6). ■

The identity holds only after discounting: each bracket combines a current entry or labor outlay with its later payoff, and the present-value sum telescopes through the Bellman recursions (A.75). The lemma is weighted by the planner's shadow value λ_t, not by the household welfare weight μ_t; the two coincide precisely under home production efficiency.

The domestic production block. The welfare-relevant object is $\sum_t \mu_t \Theta_t$, not $\mathcal{S}_\lambda$. Using $p^D_{H,t} = \kappa_s a_t$ and $\mathcal{P}^*_{HF,t} = \kappa_s b_t$, the block in (A.73) is

$$\Theta_t = \kappa_s \big(a_t\, dD_{HH,t} + b_t\, dX_{HF,t}\big) - dL_t \qquad (\text{per } d\ln\tau,\ w_{H,t} = 1). \tag{A.81}$$

Since $\beta^t \lambda_t = \kappa_s \mu_t$, the lemma (A.78) is equivalent to $\sum_t \mu_t \big(a_t\, dD_{HH,t} + b_t\, dX_{HF,t}\big) = \sum_t \mu_t\, dL_t$. Substituting into (A.81),

$$\boxed{\sum_t \mu_t\, \Theta_t = (\kappa_s - 1) \sum_t \mu_t \frac{dL_t}{d\ln\tau}} \qquad \Longrightarrow \qquad \sum_t \mu_t\, \Theta_t = 0 \iff \kappa_s = 1. \tag{A.82}$$

The entire domestic production block — the entry-investment flows and the labor margin together — vanishes only under home production efficiency, namely the corrective subsidy $\kappa_s = 1$ that sets buyer prices equal to marginal cost. At that point the planner's labor shadow value coincides with the household's marginal value of resources, $\beta^t \lambda_t = \mu_t$, so Θ_t is

itself the integrand of the lemma (A.78) and the whole block telescopes to zero. Away from the corrective subsidy the markup wedge $\kappa_s - 1 = \frac{1}{\theta-1} > 0$ survives: because the markup falls on goods and not on leisure, labor supply is distorted, the home production block is not at its efficient configuration, and $\sum_t \mu_t \Theta_t \neq 0$. It is therefore the combined block, not the entry margin in isolation, that home production efficiency neutralizes.

C.4. The One-Time Optimal Tariff Formula

Under home production efficiency ($\kappa_s = 1$), (A.82) gives $\sum_t \mu_t \Theta_t = 0$, so the Ramsey FOC $dW_0/d\ln\tau = 0$ in (A.72) retains only the import-volume and terms-of-trade terms. Solving for the wedge gives the optimal one-time tariff

$$\boxed{\tau - 1 = \frac{\displaystyle\sum_t \mu_t \, \mathcal{P}^*_{FH,t} M_{FH,t} \, \frac{d\ln\left(\mathcal{P}^*_{FH,t}/\mathcal{P}^*_{HF,t}\right)}{d\ln\tau}}{\displaystyle\sum_t \mu_t \, \mathcal{P}^*_{FH,t} M_{FH,t} \, \frac{d\ln M_{FH,t}}{d\ln\tau}} = \frac{N_q^I}{\mathcal{D}_M}} \tag{A.83}$$

where, exactly as in Section B.3.3, $\omega_t = \mu_t \mathcal{P}^*_{FH,t} M_{FH,t}$,

$$\mathcal{D}_M \equiv \sum_t \omega_t \frac{\partial \ln M_{FH,t}}{\partial \ln \tau}, \qquad N_q^I \equiv \sum_t \omega_t \frac{\partial \ln q_t^I}{\partial \ln \tau}. \tag{A.84}$$

Equation (A.83) is the present-value inverse export-supply elasticity $1/\varepsilon^{q,PV}$ of (A.1), now derived in the full model with sunk-cost participation and free entry. Because the ideal terms-of-trade ratio q_t^I is used in the numerator, the export-bundle income-accounting correction of Section B.3.3 is already absorbed and no separate profit/subsidy term is needed. Capital-free exporter dynamics enter only through the transition derivatives and the weights ω_t; they do not add a new first-order force.

C.5. The committed Optimal Tariff Path

We now let Home choose a full path $\{\tau_t\}_{t\geq 0}$. The key observation is that the constraint set is block-separable with the trade path $\{M_{FH,t}, X_{HF,t}\}$ as interface: the Home constraints involve only internal variables and the trade path, while the foreign block involves only the trade path and world prices. This delivers an exact nesting of the planner's problem.

Outer–inner formulation. The inner problem (A.74) returns the value of a given trade path,

$$V\big(\{M_{FH,t}\},\{X_{HF,t}\}\big) = \max_{\{D_{HH,t},L_t,N_t^e,e_t\}} \sum_t \beta^t \big[u\big(G(D_{HH,t},M_{FH,t})\big) - v(L_t)\big] \quad \text{s.t. (A.74)}, \tag{A.85}$$

with the envelope properties $\partial V/\partial M_{FH,t} = \beta^t u_{C,t} G_{M,t} > 0$ (the value of imports) and $\partial V/\partial X_{HF,t} = -\beta^t \lambda_t b(m_t) < 0$ (the labor cost of exports). The outer problem then manipulates the terms of trade along Foreign's offer surface,

$$\max_{\{M_{FH,t},X_{HF,t}\},\pi} V\big(\{M_{FH,t}\},\{X_{HF,t}\}\big) \quad \text{s.t.} \quad \underbrace{M_{FH,t} = \mathcal{X}_t^*(\pi), \quad X_{HF,t} = \pi_t\, \mathcal{X}_t^*(\pi)}_{\text{external implementability}}, \tag{A.86}$$

where $\pi = \{\pi_t\} = \{q_t^I\}$ is the path of relative prices and $\mathcal{X}_t^*(\pi)$ is Foreign export supply to Home as a function of the entire price path. The second constraint is balanced trade, $\mathcal{P}_{HF,t}^* X_{HF,t} = \mathcal{P}_{FH,t}^* M_{FH,t}$, written as $X_{HF,t} = \pi_t M_{FH,t} = \pi_t \mathcal{X}_t^*(\pi)$. Economically: the inner problem is production efficiency given trades, the outer problem is terms-of-trade manipulation along Foreign's offer surface, and $\{M_{FH,t}, X_{HF,t}\}$ are parameters of the inner problem — the nesting is an order of solution, not causality.

Combined planner problem and FOCs. Combining both layers, the planner chooses $\{D_{HH,t}, M_{FH,t}, X_{HF,t}, L_t, e_t, m_t, N_t^e\}$ and the price/tariff path $\{\pi_t, \tau_t\}$ to maximize

$$\begin{aligned}
\max\ & W = \sum_t \beta^t \big[u\big(G(D_{HH,t}, M_{FH,t})\big) - v(L_t)\big] \\
\text{s.t.}\quad & M_{FH,t} = \mathcal{X}_t^*(\pi) && [\beta^t \tilde{\mu}_t] \\
& X_{HF,t} = \pi_t\, \mathcal{X}_t^*(\pi) && [\beta^t \nu_t] \\
& L_t = a(N_t) D_{HH,t} + b(m_t) X_{HF,t} + f_E N_t^e + L_t^x && [\beta^t \lambda_t] \\
& N_{t+1} = (1-\delta)\big(N_t + N_t^e\big) && [\beta^t \phi_t] \\
& m_{t+1} = (1-\delta)\big[m_t + e_t(N_t - m_t)\big] && [\beta^t \psi_t] \\
& L_t^x = \tfrac{\bar{f}^*}{2}(N_t - m_t)\, e_t^2 && [\beta^t \xi_t].
\end{aligned} \tag{A.87}$$

The first-order conditions for the interface and price variables are

$$[\pi_s]: \quad \sum_{t=0}^{\infty} \beta^t \big(\tilde{\mu}_t - \nu_t \pi_t\big) \frac{\partial \mathcal{X}_t^*}{\partial \pi_s} = \beta^s \nu_s\, \mathcal{X}_s^*, \tag{A.88}$$

$$[M_{FH,t}]: \quad u_{C,t} G_{M,t} = \tilde{\mu}_t, \tag{A.89}$$

$$[X_{HF,t}]: \quad \nu_t = \lambda_t\, b(m_t), \tag{A.90}$$

while the inner FOCs are again (A.75). Household optimization supplies the prices in marginal-utility terms,

$$G_{D,t} = \frac{p_{H,t}^D}{P_{H,t}}, \qquad G_{M,t} = \frac{\tau_t\, \mathcal{P}_{FH,t}^*}{P_{H,t}}. \tag{A.91}$$

Organizing the price FOC. Impose production efficiency, $\kappa_s = 1$, so $b(m_t) = \mathcal{P}_{HF,t}^*$ and $\lambda_t = u_{C,t}/P_{H,t}$. Then from (A.89) and (A.91),

$$\tilde{\mu}_t = u_{C,t} \frac{\tau_t\, \mathcal{P}_{FH,t}^*}{P_{H,t}}, \qquad \nu_t = \lambda_t\, \mathcal{P}_{HF,t}^* = \frac{u_{C,t}}{P_{H,t}}\, \mathcal{P}_{HF,t}^*, \qquad \nu_t \pi_t = \frac{u_{C,t}}{P_{H,t}}\, \mathcal{P}_{FH,t}^*, \tag{A.92}$$

where the last equality uses $\pi_t = \mathcal{P}^*_{FH,t}/\mathcal{P}^*_{HF,t}$. Therefore the wedge entering (A.88) is

$$\tilde{\mu}_t - \nu_t \pi_t = \frac{u_{C,t}}{P_{H,t}} \mathcal{P}^*_{FH,t} (\tau_t - 1). \tag{A.93}$$

Writing the cross-date export-supply elasticity

$$\mathcal{E}_{ts} \equiv \frac{\partial \ln \mathcal{X}^*_t(\pi)}{\partial \ln \pi_s} = \frac{\partial \ln M_{FH,t}}{\partial \ln \pi_s}, \qquad \frac{\partial \mathcal{X}^*_t}{\partial \pi_s} = \frac{M_{FH,t}}{\pi_s} \mathcal{E}_{ts}, \tag{A.94}$$

and substituting (A.93)–(A.94) into (A.88), with the right-hand side $\beta^s \nu_s \mathcal{X}^*_s = \beta^s \frac{u_{C,s}}{P_{H,s}} \frac{\mathcal{P}^*_{FH,s}}{\pi_s} M_{FH,s}$, the common factor $1/\pi_s$ cancels and the FOC becomes

$$\sum_{t=0}^{\infty} \beta^t \frac{u_{C,t}}{P_{H,t}} (\tau_t - 1) \mathcal{P}^*_{FH,t} M_{FH,t} \mathcal{E}_{ts} = \beta^s \frac{u_{C,s}}{P_{H,s}} \mathcal{P}^*_{FH,s} M_{FH,s} = \beta^s \frac{u_{C,s}}{P_{H,s}} (\tau_s - 1) \mathcal{P}^*_{FH,s} M_{FH,s} \frac{1}{\tau_s - 1}. \tag{A.95}$$

The committed tariff-path formula. Dividing both sides of (A.95) by $\mu_s(\tau_s - 1)\mathcal{P}^*_{FH,s} M_{FH,s}$ and defining the welfare-and-trade-weight

$$\omega_{ts} \equiv \frac{\mu_t (\tau_t - 1) \mathcal{P}^*_{FH,t} M_{FH,t}}{\mu_s (\tau_s - 1) \mathcal{P}^*_{FH,s} M_{FH,s}}, \qquad \mu_t \equiv \beta^t u_{C,t}/P_{H,t}, \tag{A.96}$$

yields $\sum_t \omega_{ts} \mathcal{E}_{ts} = 1/(\tau_s - 1)$, i.e. for every date s,

$$\boxed{\tau_s - 1 = \frac{1}{\sum_{t=0}^{\infty} \omega_{ts} \mathcal{E}_{ts}}} \tag{A.97}$$

The date-s tariff is the reciprocal of a present-value-weighted sum of the cross-date export-supply elasticities $\mathcal{E}_{ts}$: a committed τ_s accounts for how a change in the date-s terms of trade π_s moves Foreign export supply at every horizon t, weighted by the marginal value of the corresponding import distortions.

C.6. One-Time Tariff versus committed Path

The two formulas share the same primitive — the cross-date elasticity $\mathcal{E}_{ts}$ — but aggregate it differently. Writing them together,

$$\underbrace{\tau - 1 = \frac{\sum_t \mu_t \, \mathcal{P}^*_{FH,t} M_{FH,t} \, \dfrac{d \ln q^I_t}{d \ln \tau}}{\sum_t \mu_t \, \mathcal{P}^*_{FH,t} M_{FH,t} \, \dfrac{d \ln M_{FH,t}}{d \ln \tau}}}_{\text{one-time (best constant tariff)}}, \qquad \underbrace{\tau_s - 1 = \frac{1}{\sum_t \omega_{ts} \, \mathcal{E}_{ts}}}_{\text{committed path (date-by-date)}} . \tag{A.98}$$

The link between them is the chain rule connecting the total tariff derivative of imports to the price-path response: because the tariff acts on Home demand only through the equilibrium terms-of-trade path,

$$\frac{d \ln M_{FH,t}}{d \ln \tau} = \sum_s \underbrace{\frac{\partial \ln M_{FH,t}}{\partial \ln \left(\mathcal{P}^*_{FH,s} / \mathcal{P}^*_{HF,s} \right)}}_{\mathcal{E}_{ts}} \frac{d \ln \left(\mathcal{P}^*_{FH,s} / \mathcal{P}^*_{HF,s} \right)}{d \ln \tau}. \tag{A.99}$$

Substituting (A.99) into the one-time formula shows that the one-time tariff is the single rate that sets a present-value-aggregated version of the committed FOC to zero: it is constrained to be constant across dates and therefore trades off the date-s wedges against one another, whereas the committed path satisfies a separate FOC at each s and so attains a (weakly) higher level of Home welfare. When the elasticity dynamics are flat ($\mathcal{E}_{ts}$ concentrated on the diagonal $t = s$ and horizon-independent), the two coincide and both collapse to the static inverse elasticity (A.13). The gap between them is exactly the part of Foreign's intertemporal offer surface — the off-diagonal $\mathcal{E}_{ts}$ generated by sunk-cost entry and exit — that a single permanent tariff cannot exploit but a time-varying one can.