Macroprudential Policy with Firm Heterogeneity

Emilio Zaratiegui*

Bank of England

September 8, 2026

Click here for the current version

Abstract

Macroprudential policies that restrict foreign borrowing, such as capital controls, have become an accepted component of policy toolkits, despite evidence that they can depress investment and affect firms heterogenously. I introduce a tractable way of modeling misallocation that makes this cost measurable, and estimate it for 18 European economies. Unless the policymaker can separately subsidize investment, financial stability and productivity conflict. In a model matching both sudden-stop and misallocation moments imposing a conventional capital control costs an order of magnitude more than optimal policy gains.

JEL Classification: E13, E44, F41, G01, O16

*Email: emilio.zaratiegui@bankofengland.co.uk. I would like to thank Martin Uribe, Jesse Schreger, and Stephanie Schmitt-Grohé for their continuous guidance and support. I am grateful to Ozge Akinci, Hassan Afrouzi, Javier Bianchi, Fernando Cirelli, Matthieu Gomez, Émilien Gouin-Bonenfant, Amy Handlan, Sebnem Kalemli-Ozkan Jennifer La'O, Carlos Madeira, Kosha Modi, Chris Moser, Noémie Pinardon-Touati, Tommaso Porzio, Ricardo Reis, Pierre Yared and seminar participants at Columbia University, SED 2024 Winter Meeting,Universidad de San Andrés Graduate Conference, the New York Federal Reserve, the Bank of England, UT Austin, Brown University, Texas A&M, EEA 2025 Congress and MMF 2025 Meeting for their thoughtful comments. I am also particularly grateful to Federico Kochen, Razvan Popescu, and the rest of the Orbis initiative at CBS GRID for making the Orbis-Amadeus data available. The views represented here do not necessarily represent the views of the Bank of England.

1 Introduction

Macroprudential policies have become a key policy tool in recent years, as shown by the introduction of Basel III and their recognition as a valid policy instrument by the International Monetary Fund (2022).[1] They are supported by an extensive literature (Bianchi and Mendoza, 2020), which focuses on their role in ensuring financial stability. For small open economies, in which foreign borrowing is a significant source of credit, these policies usually take the form of capital controls.

At the same time, there has been increasing attention to the role of international capital flows in alleviating (Forbes, 2005; Larrain and Stumpner, 2017; Varela, 2017; Bau and Matray, 2023) or exacerbating misallocation (Gopinath et al., 2017; Benigno and Fornaro, 2014). This role introduces an additional consideration to the study of macroprudential policy.

In this paper, I examine how optimal macroprudential policy changes when these two forces are present. To do so, I build on the seminal open economy model with borrowing constraints developed by Bianchi (2011), to which I introduce investment and capital misallocation across firms. I introduce a tractable way of modeling misallocation that generates a link between investment and productivity and has a clear mapping to the data. I leverage this feature to estimate the elasticity of productivity to investment for a sample of European countries using firm-level microdata.

Using this model, I first study the determinants of optimal macroprudential policy. On the one hand, pecuniary externalities mean that households do not fully internalize the costs of their consumption, which leads them to overborrow. Capital controls can correct this, as is standard in the literature. On the other hand, by increasing borrowing costs, they reduce investment and productivity. When the government can separately tax or subsidize borrowing and investment, these forces generate no trade-off. I show that the planner implements capital controls to curb borrowing and subsidizes investment to ensure that households invest the optimal amount.

I then show that a trade-off exists when the government can control the total amount of borrowing using capital controls but cannot alter its composition between investment and consumption. In this second-best scenario, I show that the policymaker faces a trade-off between restricting borrowing to reduce consumption and incentivizing borrowing to increase investment.

To assess the quantitative relevance of the trade-off, I solve the model numerically and show that it can match both sudden stop moments and realistic levels of misallocation.

[1] See Cerutti, Claessens, and Laeven (2017) for a survey of their prevalence across countries.

Focusing on a constant capital control, I find that misallocation results in a subsidy to borrowing being optimal, rather than a tax. Moreover, imposing a tax has sizable welfare consequences.

My starting point is the canonical model of macroprudential policy developed by Bianchi (2011). Households derive utility from consuming tradable and non-tradable goods. The former can be traded with the rest of the world, along with non-contingent financial claims. Households own an endowment of non-tradable goods and supply labor and capital to tradable goods producers that produce individual varieties which are aggregated by competitive firms.

Households are subject to a borrowing constraint that depends on their income measured in units of tradable goods. When the constraint binds, households are forced to deleverage by reducing their consumption of tradable goods. This also reduces demand for non-tradable goods, pushing their price down and reducing the market value of the household's income. This further tightens the borrowing constraint, which gives way to another round of deleveraging in a vicious cycle. These dynamics capture the stylized facts of sudden stops, as financial crises with large current account reversals are known.

The pecuniary externality arises because households do not internalize the fact that increasing the consumption of tradable goods increases the value of non-tradable goods, and therefore their ability to borrow when the borrowing constraint binds. By increasing consumption during a crisis, macroprudential policies that reduce borrowing during good times can be welfare improving. An example of these policies are capital controls, which increase the cost of foreign borrowing.

Because the model studied by Bianchi (2011) and much of the literature does not consider production economies, they do not capture the effects of macroprudential policy on capital accumulation and productivity. To study the qualitative and quantitative implications of this cost, I add the production of tradable goods using labor and capital. To further examine the productivity implications, I consider a setting in which firms produce individual varieties that are aggregated with a CES technology. As shown by Hsieh and Klenow (2009), this allows a clear mapping from the distribution of distortions at firm level to total factor productivity.

I model misallocation by introducing firm-level frictions in the rental market for capital. Domestic banks intermediate between households and firms. The amount of capital firms can rent from a bank is subject to distortions, which leads to dispersion in the marginal return to capital across firms. I assume that these distortions do not grow proportionally with capital, which implies that an increase in the aggregate amount of capital reduces misallocation in the economy. This is in line with the literature on the link between financial development and growth (Rajan and Zingales, 1998; Levine, 2005) and with the experience of countries

that lifted capital controls (Larrain and Stumpner, 2017; Varela, 2017) or restrictions on investment (Bau and Matray, 2023).

I then derive a closed-form expression for the link between total factor productivity and aggregate capital, which resembles a production externality. In this setting, capital controls have two adverse effects on output. First, the increase in borrowing cost raises the required rate of return on capital, which leads to a decrease in investment. Second, reduced investment, through increased misallocation, negatively affects productivity, which means that the same amount of capital produces fewer goods.

As a benchmark, I first study the problem of a policymaker who is not constrained with respect to available instruments. Because they can tax borrowing and investment separately, they face no trade-off. Capital controls address the pecuniary externality and reduce consumption when the economy is not in a crisis, while a subsidy to investment addresses the productivity externality.

I then turn to a more realistic setting, in which the policymaker can only control the total level of debt but not its composition between investment and consumption. In this scenario, I find that the planner faces a trade-off between the two objectives: financial stability and productivity growth. The former requires a capital control to reduce consumption during good times, while the latter needs a subsidy on foreign borrowing to encourage investment and productivity growth.

To gain more intuition, I explore the main determinants of the second-best capital controls: the productivity externality, the probability of a sudden stop, the marginal propensity to invest, and the elasticity of substitution between tradable and non-tradable goods.

The productivity externality is one of the main drivers of the policy trade-off. In one of the key contributions of the paper, I leverage its closed-form expression and estimate a range of plausible values using microdata. Using firm-level balance sheet data from Orbis-Amadeus, which has excellent coverage of private European firms, I measure the dispersion in the marginal returns to capital across firms, which directly informs my estimates. I estimate an elasticity of total factor productivity to investment that ranges between 0.05 and 0.13 for a sample of European economies.

To assess the quantitative importance of these factors, I numerically solve the model, which is able to both match the dispersion of capital ratios across firms and the frequency and intensity of sudden stops. I first find that the constrained efficient allocation delivers significant welfare gains by both imposing capital controls and an investment subsidy. When I restrict the planner to constant capital controls and no investment subsidy, it is optimal to subsidize borrowing. While the welfare gains relative to the decentralized economy are small, I show that implementing usually prescribed capital controls can lead to sizable welfare

losses.

These findings are robust across the range of estimates for misallocation across my sample. I show that borrowing subsidies are optimal across all of them. There is, however, substantial heterogeneity. For countries with the most misallocation, the subsidy is almost 4 times as large than for countries at the lower end of the estimates. In all cases, the welfare losses of ignoring this channel are sizable.

Related literature. This paper connects two strands of the literature. The first studies optimal macroprudential policy in a context of pecuniary externalities and financial frictions that render the decentralized equilibrium inefficient. The second strand studies how frictions in factor markets, through misallocation of resources across firms, lead to reduced aggregate productivity.

Most of the macro-finance literature on macroprudential policy builds on the insights of Bernanke and Gertler (1989) and Kiyotaki and Moore (1997), who demonstrate how financial frictions can amplify the effect of shocks, formalizing an old intuition (Fisher, 1933). The combination of incomplete markets and the role prices play in them lead to the emergence of pecuniary externalities, as studied by Clayton and Schaab (2022), Dávila and Korinek (2017), Gromb and Vayanos (2002) and Lorenzoni (2008). In many cases, these externalities lead to inefficiencies, which motivate the study of policy interventions.[2] This normative literature, starting with Jeanne and Korinek (2010) and Bianchi (2011), has mostly focused on small open economies.[3]

The main contribution of this paper is to build upon the seminal setup of Bianchi (2011), which has been extensively used (Benigno, Chen, et al., 2013; Arce, Bengui, and Bianchi, 2019; Flemming, L'Huillier, and Piguillem, 2019; Seoane and Yurdagul, 2019; Schmitt-Grohé and Uribe, 2020; Bengui and Bianchi, 2022) to study how effects on productivity affect optimal macroprudential policy.

I make two contributions to this literature. The first is to consider both capital accumulation and endogenous productivity, which increases the costs of macroprudential policy in a second-best setting. In this regard, the model I study resembles the one in Bianchi and Mendoza (2020), which also features capital accumulation but has exogenous productivity. By analytically characterizing second-best policy in this setting, I show that considering these additional features affects optimal macroprudential policy.

This also demonstrates the paper's second contribution to this literature, which is to

[2]Schmitt-Grohé and Uribe (2016) study macroprudential policy in the context of wage rigidities and currency pegs.

[3]Noteworthy exceptions are Farhi and Werning (2016) and Elenev, Landvoigt, and Van Nieuwerburgh (2021), who study the effects of macroprudential policies in a quantitative model of a closed economy.

characterize both the constrained-efficient and second-best policy and leverage this characterization to provide a quantification of the trade-off as a function of measurable sufficient statistics. This approach is similar to that of Dávila and Korinek (2017), who also characterize macroprudential policy using sufficient statistics. Compared with their work, I find a direct mapping to the data which allows me to quantify the trade-off.

I also draw on the findings of a recent literature that examines the effects of capital controls on productivity. Larrain and Stumpner (2017) and Varela (2017) study episodes in which restrictions on capital mobility were lifted in Eastern European countries, while Bau and Matray (2023) study the easing of restrictions on foreign equity investments in India. Alfaro, Chari, and Kanczuk (2017) use Brazilian stock market data to analyze the effect of capital controls on firms. All find that easing controls reduces the cost of capital more for constrained firms, which results in increased aggregate productivity through improved allocation of capital. These results are also in line with an earlier literature surveyed by Forbes (2005).[4]

In studying the credit boom in Southern Europe during the introduction of the euro, Gopinath et al. (2017) find a decrease in productivity in the manufacturing sector through increased misallocation.[5] This would suggest that capital inflows can have negative effects on productivity, in line with a theoretical literature on credit booms (Asriyan et al., 2024; Benigno, Chen, et al., 2016; Reis, 2013). Studying a large panel of countries, Monacelli, Sala, and Siena (2023) find that increases in interest rates lead to reductions in aggregate productivity in emerging economies, while the opposite is true for advanced economies. Although in the quantitative exercise I consider a positive effect of capital inflows on productivity, this paper still provides a framework to consider these effects when studying optimal macroprudential policy.

These papers are part of a broader literature[6] on how the misallocation of factors across firms reduces aggregate productivity. I follow the indirect measuring framework introduced by Hsieh and Klenow (2009), which allows me to link model and data in a tractable manner. This approach was also used by Midrigan and Xu (2014), David and Venkateswaran (2019) and David, Venkateswaran, et al. (2021) to decompose the sources of misallocation, and find that technological frictions can explain only a small share of total misallocation between firms. I contribute to this literature by embedding this setup in an optimal policy analysis that takes these empirical results into account.

[4]This is also consistent with Madeira (2024), who examines the effects of macroprudential policies on industry growth and finds that financial frictions play a key role in understanding the real effects of macroprudential policy.

[5]See also Calligaris et al. (2018) and Dias, Robalo Marques, and Richmond (2016)

[6]See, e.g., Restuccia and Rogerson (2013) and Hopenhayn (2014).

In that regard, the paper is related to a growing literature that examines the effects on productivity through misallocation of different policies. Baqaee, Farhi, and Sangani (2024) and González et al. (2023) study how misallocation affects the transmission of monetary policy, and Kurtzman and Zeke (2020) show that quantitative easing can increase misallocation. David and Zeke (2024) study how the monetary policy regime shapes the dynamics of TFP and how optimal monetary policy is affected by this mechanism. This paper is related to theirs with respect to letting the data guide the quantification of the effects of policy on productivity.

Lastly, Andreasen et al. (2023) use a quantitative model to study the effects of the Chilean "encaje", a form of capital control, on productivity. While the specifics of their model differ, they share the conclusion that capital controls decrease TFP. Calibrated to the Chilean example, they find welfare losses of up to 0.61% in consumption-equivalent units. In this paper, I measure how this cost stands up to the macroprudential benefits of capital controls in a quantitative setting.

Layout The remainder of the paper is organized as follows. Section 2 introduces the model that will guide my analysis while Section 3 defines and characterizes the equilibrium and discusses the inefficiencies that motivate studying optimal policy. Section 4 sets up and characterizes optimal policy, and Section 5 studies its determinants. In Section 6, I leverage the results from Section 4 to present quantitative results. Section 8 concludes.

2 Model

To study the policy trade-off, I build on the canonical model presented in Bianchi (2011) to which I add production in the tradable sector along with financial frictions that differ across firms. Aggregate risk is given by the realization of tradable productivity and a stochastic tightening of the borrowing constraint.

2.1 Environment

I study a small open economy composed of households, firms that produce final and tradable goods and domestic banks that intermediate between households and firms. Households trade bonds and goods with the rest of the world, while firms can export tradable goods.

Households. This block follows closely the setup in Bianchi (2011) with the addition of capital goods. Households have preferences over a final consumption good, c_t, described by

$$\mathbb{E}_0 \left[\sum_t \beta^t \frac{{c_t}^{1-\sigma} - 1}{1-\sigma} \right] \tag{1}$$

Households are able to borrow, or save, through a one period, risk-free bond d_{t+1} denominated in units of tradable goods traded with the rest of the world at an exogenous rate r. They also accumulate physical capital k_t, which is rented to tradable firms. Lastly, households receive a fixed endowment of non-tradable goods, y^N, and are the owners of the tradable firms.

I assume that the tradable good is the numéraire, so that p_t is the relative price of final goods, and p_t^N that of non-tradable goods. The budget constraint of the households, omitting references to the state of the economy, is then given by

$$p_t c_t + p_t(k_{t+1} - (1-\delta)k_t) = w_t + (R_t + \tau_t^k)k_t + p_t^N y^N - (1+r)(1+\tau_t^d)d_t + d_{t+1} + \pi_t + T_t \tag{2}$$

where π_t are profits from firms,w_t is the wage rate and R_t is the rental rate of capital. There are two policy instruments: a tax on debt, or capital control, τ_t^d and an investment subsidy or industrial policy τ_t^k. I assume the government sets a lump-sum T_t to balance its budget.

As is common in the macroprudential literature, households also face a borrowing constraint that depends on the market value of their income.

$$d_{t+1} \leq \kappa \left(y_t^T + p_t^N y^N \right), \tag{3}$$

where $\kappa < 1$ is a constant.

The Euler equation for debt d_{t+1}, is given by

$$\frac{c_t^{-\sigma}}{p_t} = \frac{1}{1-\mu_t} \beta(1+r)(1+\tau_t^d)\mathbb{E}\left[\frac{c_{t+1}^{-\sigma}}{p_{t+1}} \right], \tag{4}$$

where μ_t is the Lagrange multiplier on the borrowing constraint (3)[7]. As usual, the households seek to smooth consumption between periods. The capital control τ_t^d shows up as a wedge in this condition, altering the interest rate faced by the households. When the constraint binds, $\mu_t > 0$ acts like a wedge in this Euler equation. To see this, re-arrange (4)

[7]Normalized by the Lagrange multiplier on the budget constraint

as

$$\mu_t = 1 - \beta(1+r)(1+\tau_t^d)\mathbb{E}\left[\left(\frac{c_{t+1}}{c_t}\right)^{-\sigma}\frac{p_t}{p_{t+1}}\right] \tag{5}$$

Expressed in this way, μ_t measures the distance from the desired consumption smoothing when the borrowing constraint binds. Within the model, I consider periods where the borrowing constraint binds to be a crisis or sudden stop. In this way, μ_t serves both as an indicator of whether a crisis is happening, $\mu_t > 0$, and its severity, measured as the drop in consumption relative to a non-crisis state.

Lastly, the Euler equation determining investment is given by

$$c_t^{-\sigma} = \beta\mathbb{E}\left[c_{t+1}^{-\sigma}\left(\frac{R_{t+1}}{p_{t+1}}(1+\kappa\mu_{t+1}) + \frac{\tau_{t+1}^k}{p_{t+1}} + 1 - \delta\right)\right] \tag{6}$$

Ignoring μ_t, this is the standard condition for optimal investment. The presence of crises has two effects on the capital accumulation decision. First, when the constraint is presently binding, $\mu_t > 0$, this raises the effective interest faced by the household, increasing the required rate of return on capital, lowering investment. When the constraint is expected to bind in the next period, $\mu_{t+1} > 0$ the effect is the opposite. Because households understand that more income will increase their borrowing ability, they derive an additional benefit from investing an extra unit of capital, given by $\kappa\mu_{t+1}$.

Conditions (10), (4), (6), along with the budget constraint (2), the borrowing constraint (3), and transversality and complementary slackness conditions characterize the solution to the household's problem.

Final goods producers A representative firm combines tradable and non-tradable inputs to produce final goods x_t to sell to households according to:

$$x_t = \left[\omega {x_{T,t}}^{\frac{\xi-1}{\xi}} + (1-\omega){x_{N,t}}^{\frac{\xi-1}{\xi}}\right]^{\frac{\xi}{\xi-1}} \tag{7}$$

where $x_{T,t}$ and $x_{N,t}$ denote tradable and non-tradable inputs, respectively.

The problem of this firm is given by

$$\max_{x_t^T, x_t^N} p_t x_t - x_t^T - p_t^N x_t^N \tag{8}$$

The first order condition with respect to x_t^T pins down the relative price of final goods

$$p_t = \frac{1}{\omega}\left(\frac{x_t^T}{x_t}\right)^{\frac{1}{\xi}} \tag{9}$$

Combining this condition with the first order condition with respect to x_t^N yields the intratemporal condition:

$$p_t^N = \frac{1-\omega}{\omega}\left(\frac{x_t^T}{x_t^N}\right)^{\frac{1}{\xi}}, \tag{10}$$

which pins down the relative price of non-tradables. Keeping x_t^N fixed, p_t^N is increasing in x_t^T with constant elasticity determined by ξ. Intuitively, the more complementary both goods are, which translates to lower ξ, the higher the price response to changes in the use of tradable inputs will be.

Final tradable goods producers. A competitive firm aggregates differentiated varieties $y_t^T(i)$ to produce a final tradable good according to:

$$y_t^T = \left[\int_0^1 y_t^T(i)^{\frac{\eta-1}{\eta}} \mathrm{di}\right]^{\frac{\eta}{\eta-1}} \tag{11}$$

Solving their cost minimization problem yields a demand for variety i

$$p(i) = \left(\frac{y_t^T}{y_t^T(i)}\right)^{\frac{1}{\eta}}, \tag{12}$$

which depends on total production y_t^T and the price of variety i, $p(i)$. The elasticity of substitution between varieties, η, determines the elasticity of demand.

Intermediate goods firms. Firm i uses capital and labor h to produce its variety according to the technology

$$y(i) = A(i)k(i)^{\alpha}h(i)^{1-\alpha} \tag{13}$$

Productivity is determined by an aggregate component A and an invariant firm-specific component $a(i)$ such that, in logs,

$$\log A(i) = \log A + \log a(i), \tag{14}$$

where, across firms, $\log a(i) \sim \mathcal{N}(0, \sigma_a^2)$. While all firms face the same wage w, I assume that they face different costs of capital $R(i)$, which for now I take as given. Internalizing demand for their variety, their problem is given by

$$\max_{k_t(i),h_t(i)} (1+s)\left(A(i)k(i)^{\alpha}h(i)^{1-\alpha}\right)^{\frac{\eta-1}{\eta}} {y_t^T}^{\frac{1}{\eta}} - R_t(i)k_t(i) - w_t h_t(i), \tag{15}$$

where $1+s=\frac{\eta}{\eta-1}$ is a production subsidy financed with a lump-sum tax that corrects the inefficiency arising from the markup.[8]

The following standard result from the misallocation literature allows me to aggregate firms into a representative firm.

Lemma 1 (Firm Aggregation)**.** *Let σ_2^R be the variance of the cost of capital, $R(i)$. Up to second order, tradable production can be represented by an aggregate firm with production technology*

$$y_t^T = \text{TFP}_t \; k_t^{\alpha} h_t^{1-\alpha}, \tag{16}$$

where

$$\log \text{TFP}_t \; = \log A_t + \frac{1}{2}(\eta-1)\sigma_a^2 - \frac{1}{2}\;\alpha(1+\alpha(\eta-1))\sigma_R^2 \tag{17}$$

and the solution to its problem is characterized by,

$$w_t = (1-\alpha)\,\text{TFP}_t \left(\frac{k_t}{h_t}\right)^{\alpha} \tag{18}$$

$$R_t = \alpha\,\text{TFP}_t \left(\frac{h_t}{k_t}\right)^{1-\alpha} \tag{19}$$

This approximation is exact if $R(i)$ is log-normally distributed.

Proof. See B.1 □

This result is very useful to keep the analysis tractable because it shows that it is not necessary to keep track of the entire distribution of $R(i)$, just its variance σ_R^2. As a corollary, it follows that for any variable to have an impact on productivity, it must affect the variance of $R(i)$.

Market clearing. Demand for non-tradables must equal its endowment:

$$x_t^N = y^N \tag{20}$$

In the same way, the consumption of final goods must equal their production:

$$c_t + k_{t+1} - (1-\delta)k_t = x_t \quad \forall\; t \tag{21}$$

[8] This subsidy, which is analogous to the commonly used subsidy in New-Keynesian models, ensures that production is at its socially optimal level. Removing this distortion simplifies the analysis of optimal policy by reducing the scope of intervention for the planner with no qualitative implications.

Similarly, the factor markets clear:

$$h_t = 1 \quad \forall\, t \tag{22}$$

$$k_t = k_t \quad \forall\, t \tag{23}$$

I now explain in detail the determination of σ_R^2, which will lead to the policy trade-off.

2.2 Capital Misallocation

In this section I introduce the key departure in this model from the existing small open economy models in order to model σ_R^2 and its dynamics.

Capital is owned by households, who decide on $t-1$ how much capital to supply in t. Domestic banks intermediate between the firms and the households. Credit markets are segmented, with each market corresponding to a single firm and bank.[9] Each bank takes deposits $\hat{k}(i)$ from the households, which they use to supply the firm in their market and cover intermediation costs, given by the realization of $f(i)$. As such, capital available to firm i is given by

$$k_t(i) = \hat{k}_t(i) - f(i)$$

where $f(i)$ stands in for both overhead costs such as setting up a branch or a relationship with a firm as well as variable costs.[10]

Given market clearing, this determines the cost of capital faced by the firm, and charged by the bank, which is described by

$$R(i) \propto \left(\hat{k}_t(i) - f(i)\right)^{-\frac{1}{1+\alpha(\eta-1)}}$$

Banks choose $\hat{k}_t(i)$ to maximize their expected profits $R(i)\left(\hat{k}_t(i) - f(i)\right) - R\hat{k}_t(i)$, where R is the rate paid to households. To keep this problem tractable, I make the following two assumptions:

Assumption 1 (Segmented Markets)**.** *After shocks $f(i)$ are realized, banks cannot transfer capital between markets.*

Assumption 2 (Shock Scaling)**.** *Shocks are given by $f(i) = A(i)^{\nu(\eta-1)}\hat{k}(i)^{1-\nu}F(i)$, where $F(i)$ is a random variable and $\nu \geq 0$.*

[9] For simplicity, I assume that these banks are perfectly competitive so that there's no interest rate spread. As with the assumption on the subsidy that eliminates the intermediate good markup, I introduce this solely to reduce the number of dimensions on which the competitive equilibrium differs from the efficient allocation.

[10] $f(i)$ could also stand for unexplained credit demand such as government borrowing.

Assumption 2 provides structure on the dependence on the intermediation costs on $\hat{k}(i)$, which is given by ν. For $\nu < 1$, costs scale less than one by one with supplied capital, which would occur if a part of them are fixed. For $\nu = 1$, costs are fixed and do not depend on intermediated capital. Lastly, $\nu > 1$ would imply that total intermediation costs decrease as capital supply increases. In this paper, I will focus on the case where $\nu \leq 1$.

The bank's problem is to choose the amount of deposits $\hat{k}(i)$ from households. Assumption 1 and 2 ensure that the problem will be symmetric across banks up to the productivity of the firm, as shown by the following Lemma.

Lemma 2. *If costs $F(i)$ are scaled by $A(i)^{\nu(\eta-1)}$, then, in equilibrium, each individual bank takes deposits*

$$\hat{k}(i) = \frac{A(i)^{\eta-1}}{\mathbb{E}\left[A(i)\right]^{\eta-1}} k \tag{24}$$

Proof. See B.2 □

As a result, the market clearing rental rate of capital $R(i)$ faced by firm i is

$$R(i) \propto \left(k_t \left(1 - k_t^{-\nu} F(i)\right)\right)^{-\frac{1}{1+\alpha(\eta-1)}} \tag{25}$$

In broad terms, this formulation captures two empirical regularities: first, firms with the same fundamentals use different amounts of capital; second, these distortions are decreasing in the aggregate amount of capital in the economy. I now go over these two points in more detail.

Firms with positive (negative) draws of $F(i)$ will use more (less) capital and pay lower (higher) rental rates. It follows that the larger the variance of $F(i)$ is, the larger σ_R^2 will be, as the following Lemma shows.

Lemma 3 (Rental Rates Dispersion)**.** *For small distortions $F(i)$, the variance of $\log R(i)$, σ_R^2, is given by*

$$\sigma_R^2 = \left(\frac{1}{1+\alpha(\eta-1)} \frac{1}{k_t^{\nu}}\right)^2 \mathbb{V}ar\left[F(i)\right], \tag{26}$$

Proof. See B.3 □

This closed form solution is an important feature, as it makes very clear the link between the aggregate capital stock and σ_R^2 and allows for an analytical characterization of the elasticity of TFP to aggregate capital, defined as

$$\gamma_t \equiv \frac{\partial \log \text{TFP}_t}{\partial \log k_t}$$

Combining Lemmas 1 and 3, it is easy to see that

$$\gamma_t = \alpha(1 + \alpha(\eta - 1))\nu\,\sigma_R^2, \tag{27}$$

which captures how an increase in the amount of capital improves its allocation across firms. γ_t is increasing in both the initial level of misallocation in the economy, given by σ_R^2, and its elasticity with respect to k_t, ν. The latter captures how the frictions in the economy, given by the intermediation costs $F(i)$, scale with k_t. For low values of ν, these frictions are almost proportional to k_t, which means that increases in investment do not lead to significant improvements in capital allocation. On the other hand, if ν is high, small increases in k_t greatly decrease misallocation.

Discussion of formulation. Equation (25) introduces capital misallocation as the result of reduced form intermediation costs. This has the main benefit of being very tractable, as Lemma 3 shows, a feature I will leverage in the quantification section of this article. I argue that this specification is also in line with both empirical facts and the theoretical literature on misallocation.

Regarding the empirical facts, this specification has two main predictions. First, the model predicts that economies with a higher investment rate will experience higher productivity growth. This is consistent with the literature on the deepness of financial markets and growth (Rajan and Zingales, 1998; Levine, 2005).

Second, misallocation of capital across firms increases (decreases) with decreases (increases) in credit. This is line with the literature that looked at the effect of capital controls (Bau and Matray, 2023; Larrain and Stumpner, 2017; Varela, 2017), macroprudential policies (Jiménez et al., 2017; Blattner, Farinha, and Rebelo, 2023), and government spending (Pinardon-Touati, 2024). This qualitative implications are also the same as in the models used by Larrain and Stumpner (2017) and Varela (2017) to interpret their findings.

On the theoretical side, I show in Online Appendix C.1 that equation (25) can be derived from two alternative models that embody commonly used assumptions when studying misallocation: borrowing constraints arising from limited commitment, and heterogeneity in access to equity across firms. In this way, the simplicity of the model is also a feature, as it accommodates a wide class of frictions that can lead to misallocation of capital.

While firms in this model only face one distortion, I show in Online Appendix C.2 that the model can accommodate other distortions, and show that γ retains a closed form expression when including labor misallocation.

3 Decentralized Equilibrium

Having described the environment, I now characterize the main elements of the equilibrium, relegating the definition and its full characterization to Online Appendix A.

Factor prices are pinned down by the combination of Lemma 1 and market clearing,

$$w_t = (1-\alpha)\,\mathrm{TFP}_t\,k_t^{\alpha} \tag{28}$$

$$R_t = \alpha\,\mathrm{TFP}_t\,k_t^{\alpha-1} \tag{29}$$

Combining factor prices, the budget constraint of the household, non-tradable market clearing, the government's budget constraint and the firm's problem yields the tradable resource constraint,

$$x_t^T = \mathrm{TFP}_t k_t^{\alpha} - (1+r)d_t + d_{t+1} \tag{30}$$

A resource constraint for final goods can be derived from market clearing and firms conditions

$$c_t + k_{t+1} = (1-\delta)k_t + \left[\omega x_{T,t}^{\frac{\xi-1}{\xi}} + (1-\omega)y^{N\,\frac{\xi-1}{\xi}}\right]^{\frac{\xi}{\xi-1}} \tag{31}$$

Market clearing and (10) pin down the relative price of non-tradables p_t^N as a function of x_t^T:

$$p_t^N = \frac{1-\omega}{\omega}\left(\frac{x_t^T}{y^N}\right)^{\frac{1}{\xi}} \tag{32}$$

An increase in tradable demand pushes up demand for non-tradable goods. Given that supply is perfectly inelastic, this leads to an increase in the price p_t^N. Because this is a market clearing condition, it depends on aggregate tradable demand x_t^T and households do not internalize this effect of their consumption choices.

To obtain a borrowing constraint that depends on allocations, I use (28), (29) and (32) in (3), which yields

$$d_{t+1} \leq \kappa\left(\mathrm{TFP}_t k_t^{\alpha} + \frac{1-\omega}{\omega}\left(\frac{x_t^T}{y^N}\right)^{\frac{1}{\xi}} y^N\right). \tag{33}$$

Households can borrow up to a fraction κ of their income, which is the sum of tradable output and the value in tradable goods of their non-tradable endowment. Aggregate tradable consumption enters this constraint through its effect on the price of non-tradable goods.

I re-state the Euler equation (4) for debt, which remains unchanged, for convenience

below,

$$\frac{c_t^{-\sigma}}{p_t} = \frac{1}{1-\mu_t}\beta(1+r)(1+\tau_t^d)\mathbb{E}\left[\frac{c_{t+1}^{-\sigma}}{p_{t+1}}\right],$$

Combining the Euler equation for capital (6) and (29) yields

$$c_t^{-\sigma} = \beta\mathbb{E}\left[c_{t+1}^{-\sigma}\left(\frac{\alpha \text{TFP}_{t+1}k_{t+1}^{\alpha-1}}{p_{t+1}}(1+\kappa\mu_{t+1}) + \frac{\tau_{t+1}^k}{p_{t+1}} + 1 - \delta\right)\right] \tag{34}$$

Lastly, given Lemmas 1-3, TFP_t can be written as

$$\text{TFP}_t = \text{TFP}(A_t, k_t) \tag{35}$$

Sources of inefficiency. Before moving on to analyzing optimal policy, I discuss the two features of this economy that make this equilibrium inefficient. First, as it is common in this class of models, the presence of the price of non-tradable goods in the borrowing constraint (33) introduces a pecuniary externality. This is because p_t^N depends on aggregate tradable consumption, an effect not internalized by households. Suppose that the economy hits the borrowing constraint in period t. Because households are not able to borrow as much as desired, they reduce tradable consumption, as shown by condition (4) when $\mu_t > 0$. Lower tradable consumption decreases the value of the non-tradable endowment, lowering the maximum amount households can borrow.

A reduction in d_{t+1} reduces resources available to households in (30), further reducing consumption and investment. As I will discuss in the next section, a planner that internalizes this will place a higher value on consumption relative to households in periods where the borrowing constraint binds. In this setting, macroprudential policy can be welfare improving by increasing consumption during these episodes, which is achieved by reduced borrowing in previous periods.

The second source of inefficiency is the misallocation of capital across firms, which reduces the productivity of the economy, as indicated by Lemma 1. Ideally, the government would like to set the distortions to zero, increasing productivity. Even if it cannot do so, as I will assume, there's still scope for intervention. As shown by Lemma 3, an increase in investment reduces the weight of these distortions, leading to a better allocation of capital across firms and a lower σ_R^2. The sufficient statistic for this channel will be the elasticity of productivity to capital, γ_t.

While the pecuniary externality calls for restrictions on consumption, the imperfections on the capital market mean that increases in investment can be beneficial. Interestingly, both have opposite effects on borrowing. Given sufficient policy instruments, this poses no

problem. If the government, however, is limited to controlling only gross borrowing, then a conflict between objectives arises. In the following section, I discuss optimal policy and this potential trade-off in mode detail.

4 Optimal Policy

In this section I show how a benevolent planner can address the inefficiencies in this economy. I am interested in the problem of a government that internalizes the effect of aggregates but is still subject to the borrowing constraint and cannot control the allocation of capital between firms.[11] I start by considering the solution to this problem when the government can set both the capital control τ_t^d and the investment subsidy τ_t^k, which I call the constrained efficient allocation. This is a useful benchmark and is illustrative of the mechanisms at play. To simplify the analysis I assume throughout the section that the borrowing constraint does not bind at t.[12]

4.1 Constrained Efficient Allocation

I define this allocation as the solution to the problem of a planner that chooses allocations $\{x_t^T, d_{t+1}, k_{t+1}\}$ subject to resource constraints (30,31), the borrowing constraint (33) and the process for TFP.

Along with the constraints, the solution is characterized by,

$$\frac{c_t^{-\sigma}}{p_t} = \beta(1+r)\mathbb{E}_t\left[\frac{c_{t+1}^{-\sigma}}{p_{t+1}}\left(1+\kappa\mu_{t+1}\frac{1}{\xi}\frac{p_{t+1}^N y^N}{x_{t+1}^T}\right)\right], \tag{36}$$

$$c_t^{-\sigma} = \beta\mathbb{E}_t\left[c_{t+1}^{-\sigma}\left((\alpha+\gamma_{t+1})\frac{\text{TFP}_{t+1}k_{t+1}^{\alpha-1}}{p_{t+1}}\left(1+\kappa\mu_{t+1}\left(1+\frac{1}{\xi}\frac{p_{t+1}^N y^N}{x_{t+1}^T}\right)\right)\right.\right. \\ \left.\left.+1-\delta\right)\right], \tag{37}$$

where p_t and p_{t+1}^N are given by (9) and (10) respectively.

Condition (36) pins down optimal borrowing. Compared to the household's condition (4), note that the planner internalizes the effect of its choices on prices when there's a sudden stop. An extra dollar of debt in t reduces available resources in $t+1$ by $1+r$. In turn, the share of non-tradable to tradable consumption times the price elasticity predicts the resulting drop in the value of the non-tradable endowment. Lastly, any drops in the value of income reduces borrowing by κ.

[11] In other words, the government takes the financial system as a technological constraint.

[12] This does not alter the results significantly. Given the structure of the model, the capital control has no effect on borrowing when the constraint binds.

The choice of capital is pinned down by (37). Comparing to its decentralized equilibrium analog (34), the planner internalizes two external effects of investment. First, a higher stock of capital mitigates misallocation across firms, raising TFP_{t+1}. This is shown by the presence of γ_{t+1}, which measures the improvement in productivity that derives from increased investment.

Second, by increasing output in $t+1$, investment increases available resources to the household. This will, ceteris paribus, increase consumption by the marginal productivity of capital. In turn, this increases the value of the non-tradable endowment and the borrowing capacity of the households.

What are the policy implications of this analysis? Proposition 1 derives the optimal taxes that capture the discussed channels.

Proposition 1 (Constrained Efficient Allocation Implementation)**.** *The constrained efficient allocation can be implemented as a decentralized equilibrium with a tax on debt τ_t^d and a subsidy on investment τ_t^k that take the following values,*

$$\tau_t^d = \mathbb{E}_t\left[\frac{c_{t+1}^{-\sigma}}{p_{t+1}}\right]^{-1}\mathbb{E}_t\left[\frac{c_{t+1}^{-\sigma}}{p_{t+1}}\kappa\mu_{t+1}\frac{1}{\xi}\frac{p_{t+1}^N y^N}{x_{t+1}^T}\right] \tag{38}$$

$$\tau^k = \mathbb{E}_t\left[\frac{c_{t+1}^{-\sigma}}{p_{t+1}}\right]^{-1} \mathbb{E}_t\left[\frac{c_{t+1}^{-\sigma}}{p_{t+1}}\text{TFP}_{t+1}k_{t+1}^{\alpha-1}\left((1+\kappa\mu_{t+1})\gamma_{t+1}+(\alpha+\gamma_{t+1})\mu_{t+1}\frac{\kappa}{\xi}\frac{p_{t+1}^N y^N}{x_{t+1}^T}\right)\right] \tag{39}$$

Proof. See B.4 □

The tax on debt, τ_t^d, captures the difference between the social and the private value of a consumption. As previously discussed, because households do not internalize the aggregate effects of their choices, they consume less than it is socially optimal during a sudden stop. The tax then increases the cost of consumption in t so that households internalize this. Note that the tax also depends on the severity and likelihood of a crisis, as reflected by the expectation of the multiplier on the constraint. If $\mu_{t+1}=0$ for all possible states in $t+1$, then the social and private values of consumption are aligned. Likewise, if the sudden stops do not feature large drops in consumption, it follows from (5) that μ_{t+1} will be small, pushing down the tax on debt.

The investment subsidy, τ_t^k, reflects the two sources of inefficiency in this model. The first term captures the additional social value of investment over its private returns, due to its positive effects on productivity, reflected by γ_{t+1}. Additional investment also increases the resources available to the household, which increases consumption and thus borrowing

capacity. This is captured by the second term.

Because it has two available instruments, the policymaker faces no trade-off in its objectives. This is clear in Proposition 1; The capital control τ_t^d does not depend on the productivity externality γ_{t+1}. Likewise, the presence of sudden stops does not reduce the investment subsidy.[13] Thus, in contrast with the literature, this economy does not strictly feature overborrowing; rather, it features overconsumption and underinvestment.

4.2 Second-Best Capital Controls

The previous result depends on the ability of the government to control the composition of credit between consumption and investment. In practice, however, governments may have trouble setting restrictions on each kind of credit. In this section, I present one of the main results of the paper: governments that can only control aggregate borrowing face a trade-off between financial stability and productivity growth.

This constrained government can set capital controls but cannot subsidy investment. Formally, this is equivalent to adding a constraint that sets $\tau^k = 0$ to the planner's problem. It is clear from inspecting the optimality conditions of the competitive equilibrium (4) and (34) that the planner cannot independently implement k_{t+1} and c_t anymore. Reducing consumption requires increasing τ_t^d to raise the interest rate paid by the households, which disincentivizes investment. A reduction in τ_t^d would increase investment but lead to an increase in consumption and debt.

More formally, solving for the optimal policy in this setting requires adding the Euler equation for capital as an implementability constraint. Because this condition depends in part on expectations about future policy, the solution to the Ramsey problem will be time-inconsistent.[14]

To solve for the time-consistent path of capital controls, I follow Klein, Krusell, and Ríos-Rull (2008) and solve for Markov-stationary policy rules that depend only on the current state. The planner takes the state (k_t, d_t, A_t) as given and chooses allocations $\{x_t^T, d_{t+1}, k_{t+1}\}$ subject to the resource constraints, the borrowing constraint (33), and the household's capital Euler (34) as an implementability constraint, taking next-period policy rules as fixed.

To make the problem more intuitive, using the implicit function theorem on (34), we can define k_{t+1} as a function of the state, d_{t+1} and future policies. Substituting this function for k_{t+1} in the problem of the household allows dropping the constraint from the planner's

[13] In fact, the subsidy is increasing in μ_{t+1}.

[14] Capital controls have opposite effects on previous and current periods. In expectation of high capital controls, households invest to increase available resources. When facing high capital controls, however, they reduce investment.

problem.

The key object is then the response of investment to increases in d_{t+1}, which I call the marginal propensity to invest, mpi, given by

$$\text{mpi}_t \equiv \frac{\partial k_{t+1}}{\partial d_{t+1}}, \tag{40}$$

which summarizes the response of investment to changes in borrowing driven by changes in policy, already taking into account intertemporal effects.[15] Along with the constraints, the planner's solution is characterized by the Generalized Euler Equation,

$$\begin{aligned}\frac{c_t^{-\sigma}}{p_t} =&\beta(1+r)\mathbb{E}_t\left[\frac{c_{t+1}^{-\sigma}}{p_{t+1}}\left(1+\kappa\mu_{t+1}\frac{1}{\xi}\frac{p_{t+1}^N y^N}{x_{t+1}^T}\right)\right]\\ &-\text{mpi}_t\beta\mathbb{E}_t\left[\frac{c_{t+1}^{-\sigma}}{p_{t+1}}A_{t+1}k_{t+1}^{\alpha+\gamma_{t+1}-1}\left((1+\kappa\mu_{t+1})\gamma_{t+1}+(\alpha+\gamma_{t+1})\mu_{t+1}\frac{\kappa}{\xi}\frac{p_{t+1}^N y^N}{x_{t+1}^T}\right)\right]\\ &-\Omega_{t+2},\end{aligned} \tag{41}$$

where Ω_{t+2} collects terms representing the impact on $t+2$ onwards.[16] Condition (41) encodes the policy trade-off. Compared to the CE condition (36), there is an additional negative term proportional to mpi_t. Increases in borrowing allow for additional investment, which raises productivity. At the same time, this increased leverage is costly during a sudden stop. These two effects have opposing implications for welfare.

The negative term captures the productivity benefit. To the extent that borrowing finances investment ($\text{mpi}_t > 0$), it raises productivity through γ_{t+1} and output in $t+1$. The first term in the square bracket captures this direct productivity gain and is weighted by $(1+\kappa\mu_{t+1})$ because the resulting increase in resources also relaxes the collateral constraint during a sudden stop. The second term captures the additional collateral value that investment creates through its effect on tradable output. Together, these effects reduce the cost of borrowing relative to the CE, lowering the optimal tax.

The term Ω_{t+2} captures the effect that current borrowing has on future investment decisions. This object, which is a result of the incomplete set of instruments available to the planner, introduces an additional intertemporal perspective to the policy problem. If cur-

[15] Formally, mpi_t is computed via the implicit function theorem applied to (34); Appendix B.5 provides the derivation, including an equivalent formulation based on lump-sum transfers.

[16] More precisely,

$$\Omega_{t+2} \equiv \sum_{s\geq t+2}\beta^{s-t}\mathbb{E}_t\left[\frac{c_s^{-\sigma}}{p_s}A_s k_s^{\alpha+\gamma_s-1}\left((1+\kappa\mu_s)\gamma_s+(\alpha+\gamma_s)\mu_s\frac{\kappa}{\xi}\frac{p_s^N y^N}{x_s^T}\right)\frac{\partial k_s}{\partial d_{t+1}}\right]$$

rent borrowing crowds out future investment, this would dampen the benefits of increased investment today.

The following proposition characterizes the second-best capital control $\tau_t^{d,sb}$ that balances this trade-off.

Proposition 2 (Implementing the Second-Best Allocation)**.** *The tax on debt $\tau_t^{d,sb}$ that implements the second-best allocation is*

$$\tau_t^{d,sb} = \tau_t^d - \frac{\text{mpi}_t}{1+r} \times \tau_{t+1}^k - \frac{\Omega_{t+2}}{\beta(1+r)\,\mathbb{E}_t\left[c_{t+1}^{-\sigma}/p_{t+1}\right]}, \tag{42}$$

where τ_t^d and τ_{t+1}^k, defined in Proposition 1, are the instruments that implement the constrained efficient allocation.

Proof. See Appendix B.5. □

The last term in (42) is proportional to Ω_{t+2} and corrects for the effect of current borrowing on future investment. In the rest of the theoretical analysis in this paper I abstract from this dimension by assuming that from $t+2$ onwards either frictions vanish or the government can use both instruments, so that $\Omega_{t+2} = 0$.[17]

Setting this term aside, Proposition 2 shows that the constrained planner sets a capital control that is a linear combination of the constrained efficient instruments. The weight on the investment subsidy τ^k is given by the marginal propensity to invest, mpi_t, which determines how much additional borrowing is channelled into investment.

5 Determinants of Optimal Policy

In this section I explore the main determinants of second-best capital controls. To fix ideas, I focus on each source of inefficiency to study how it affects optimal capital controls.

5.1 Pecuniary Externality

Perfect foresight and no investment. To help with the intuition, I first consider a case where the borrowing constraint binds with probability equal to 1 in the next period, there are no other sources of uncertainty and tradable production is an endowment $y_{t+1}^T = y^T$.

[17] This assumption makes exposition simpler at little analytical cost. I have verified quantitatively that this term tends to be small.

Under this scenario, the optimal tax is

$$\tau_t^{d,sb} = \tau_t^d = \kappa\mu_{t+1} \times \frac{1}{\xi}\frac{p_{t+1}^N y^N}{x_{t+1}^T}$$

Without investment, this solution matches the constrained efficient allocation, as there's no trade-off for the planner. As all terms are strictly positive, this implies a positive tax on debt that reduces borrowing and, as a result, tradable consumption. Moreover, the tax is increasing in both of the terms.

The second term gives the response of the value of the non-tradable endowment to changes in x_{t+1}^T. It depends on the price elasticity $\frac{1}{\xi}$, which is inversely related to the substitutability between goods, and the share of non-tradable to tradable consumption. The higher this response is, the more effective macroprudential will be, as any increase in x_{t+1}^T will lead to larger increases in the value of the non-tradable endowment. The amount of income that can be pledged as collateral, given by κ, has a similar effect, as it translates the changes in income to changes in borrowing.

The tax also depends on the severity of the sudden stop, which is given by μ_{t+1}. Recall from (5) that μ_{t+1} reflects the consumption drop during a sudden stop. It follows that capital controls should be higher for countries where crises are more harmful in terms of consumption.

Perfect foresight and investment. How does the introduction of investment affect the results? I now bring back tradable production with exogenous productivity (i.e. $\gamma_{t+1} = 0$), so that the pecuniary externality remains the only source of inefficiency. The second-best capital control is now given by

$$\tau_t^{d,sb} = \kappa\mu_{t+1}\left(1 - \text{mpi} \times \frac{\alpha \text{TFP}_{t+1} k_{t+1}^{\alpha-1}}{1+r}\right) \times \frac{1}{\xi}\frac{p_{t+1}^N y^N}{x_{t+1}^T}$$

The introduction of capital means that households now allocate resources between consumption and investment and these choices are an important input for policy. Note that, an extra dollar in borrowing during t leads to mpi additional cents in investment. This has the effect of increasing production, and thus available resources, during $t+1$. Because macroprudential policy seeks to control consumption, it is possible to think of the mpi as measuring an additional cost of capital controls. Thus, the higher the propensity to invest is, the lower the tax will be.

Within the model studied in this paper, mpi is characterized by the optimality conditions and is therefore implicitly determined by the primitives of the model. Framing the discussion

in terms of the former, however, allows for more generality. Indeed, one could think of more complex settings, such as models with rich heterogeneity within households, a more detailed production structure or financial system, in which this sufficient statistic still applies.

5.2 Misallocation

Perfect foresight and no borrowing constraint. As previously discussed, the frictions that lead to misallocation result in a positive externality of investment, as a higher capital stock increases total factor productivity. To make the intuition clear, I will keep the perfect foresight assumption for the time being while assuming that there's no borrowing constraint (i.e., $\mu_{t+1} = 0$). In that economy, the second-best capital control would be

$$\tau_t^{d,sb} = -\text{mpi} \times \gamma_{t+1} \times \frac{\text{TFP}_{t+1} k_{t+1}^{\alpha-1}}{1+r}$$

The second term is the constrained efficient subsidy, that depends on γ_{t+1}. Recall from (27), that γ_{t+1} depends in part on the existing level of misallocation in the economy. This has two important implications; the first is that the subsidy will vary across countries. It is an empirical regularity[18] that misallocation is decreasing in the level of development, implying that the subsidy would be higher for less developed economies. The second is that within a country, as the capital stock increases, misallocation will fall and γ_{t+1} will decrease. In this way, the externality resembles a dynamic externality that shrinks as the economy develops (Redding, 1999; Melitz, 2005; Itskhoki and Moll, 2019; Ottonello, Perez, and Witheridge, 2024).

The size of the subsidy also depends on the efficiency of the government in increasing investment, given by mpi. If the economy mostly redirects borrowing to consumption, that would make the subsidy relatively more expensive, and its optimal level would be higher. An interesting implication is that the subsidy will more effective for economies that either have a naturally higher mpi or can control it in some way. This is consistent, for instance, with the experience of South Korea (Noland, 2007).

So far, I have considered a perfect foresight scenario, where the sudden stop is all but certain. In reality, sudden stops are low probability events[19]. This is important because it introduces a third factor into the trade-off which has asymmetric effects on the two forces at play. While increases in productivity will occur in all states of the world, the borrowing constraint only binds in a few of them.

[18] (See, e.g., Hsieh and Klenow, 2009; Buera, Kaboski, and Shin, 2015; Restuccia and Rogerson, 2017)

[19] Bianchi and Mendoza (2020) report a frequency of 2.4% with significant heterogeneity across development levels

Uncertainty. First, assume that $\gamma_{t+1} = 0$. Then, the second-best capital control is

$$\tau_t^{d,sb} = \mathbb{E}\left[\frac{c_{t+1}^{-\sigma}}{p_{t+1}}\right]^{-1} \mathbb{E}\left[\frac{c_{t+1}^{-\sigma}}{p_{t+1}}\kappa\mu_{t+1} \times \frac{1}{\xi}\frac{p_{t+1}^N y^N}{x_{t+1}^T}\right]$$

To unpack the role the probability of a sudden stop plays, let π be the probability that $\mu_{t+1} > 0$. It is possible to write

$$\tau_t^{d,sb} = \mathbb{E}\left[\frac{c_{t+1}^{-\sigma}}{p_{t+1}}\right]^{-1} \times \pi \times \mathbb{E}\left[\left(\frac{c_{t+1}^{-\sigma}}{p_{t+1}}\right)\kappa\mu_{t+1} \times \frac{1}{\xi}\frac{p_{t+1}^N y^N}{x_{t+1}^T} \,\middle|\, \mu_{t+1} > 0\right].$$

As long as $\pi < 1$, the introduction of uncertainty reduces the optimal capital control. This is intuitive, because its benefits only manifest during a sudden stop, while its costs occur in every state of the world.

These implications also carry over to the case where $\gamma_{t+1} > 0$. Because the benefits to productivity happen in all states of the world, a lower probability of a sudden stop reduces the benefit to capital controls while keeping their costs intact.

What are the overall implications of each of the factors discussed above? The following corollary summarizes the contribution of each.

Corollary. *The second-best capital controls, $\tau_t^{d,sb}$ are*

1. *Increasing in the non-tradable share of consumption $\frac{p_{t+1}^N y^N}{x_{t+1}^T}$, non-tradable price elasticity, $\frac{1}{\xi}$, pledgeable fraction of income κ, sudden stop severity, μ_{t+1} and sudden stop probability, π*

2. *Decreasing in misallocation, σ_R^2, its elasticity with respect to capital, ν, and the marginal propensity to invest in tradables* mpi

The benefits of capital controls are predominantly given by the strength of the pecuniary externality, which leads the planner to restrict consumption using capital controls. The introduction of investment reduces the effectiveness of capital controls because restricting borrowing leads to decreases in investment, which are costly. The marginal to propensity to invest, mpi, summarizes this channel.

While investment dampens optimal capital controls, the presence of frictions in the allocation of capital further increase the costs of capital controls. Because these frictions diminish as the stock of capital increases, capital controls reduce productivity. The strength of this channel is summarized by γ_{t+1}, which in turn depends on the level of misallocation in the economy, σ_R^2 and the elasticity of misallocation with respect to k_{t+1}, ν.

6 Quantitative Results

To assess whether the trade-off identified in the previous section is quantitatively relevant I numerically solve the model and evaluate different policies, and validate the sufficient-statistic formula of Proposition 2 against the model's quantitative prescription. In this section I detail the implementation of the model.

6.1 Calibration

The model is set to annual frequency. I calibrate some parameters to target long-run moments as well as the frequency and severity of sudden stops, and set the rest of them externally. The only departure from the model in the previous section is the introduction of capital adjustment costs[20], which help match the long-run volatility of investment as well as its dynamics around sudden stops.

6.1.1 Externally Set Parameters.

Table 1 summarizes the parameters set according to the literature. The tightness of the collateral constraint, given by κ, and the elasticity of substitution between tradable and non-tradable goods, determine the strength of the pecuniary externality. I follow Bianchi (2011) and set $\kappa = 0.32$ and $\xi = 0.83$. I also set $\sigma = 2$, which plays an important role by driving the desire of households to smooth consumption and also their risk aversion. Lastly, I set $r = 0.04$.

For the parameters governing tradable production, I set $\alpha = 0.35$ and $\delta = 0.1$. The elasticity of substitution between varieties, η, determines the effect of misallocation on productivity, as can be seen in Lemma 1. I follow Hsieh and Klenow (2009) and make the conservative choice of $\eta = 3$.

6.1.2 Calibrated Parameters

Table 2 lists the parameters calibrated to match empirical moments. I calibrate β so that the simulated economy experiences a sudden stop around 2.5% of the time.[21] I assume that aggregate TFP A_t follows an AR(1) process with persistence $\rho = 0.85$ and standard deviation of innovations $\sigma_A = 0.0125$ to target standard deviation of tradable output of around 3%. I set A to match a ratio of non-tradable to tradable output of 2. Capital adjustment costs,

[20] I use quadratic adjustment costs of the form $\frac{\phi}{2}(k_{t+1} - k_t)^2$ and set ϕ to match a desired volatility of investment.

[21] I define a sudden stop as an episode in which the current account over GDP is over the 95th percentile of its ergodic distribution and the borrowing constraint binds.

Table 1: Externally set Parameters

Parameter	Value	Source
σ	2	Bianchi (2011)
ξ	0.83	Bianchi (2011)
κ	0.32	Bianchi (2011)
r	0.04	Bianchi (2011)
α	0.35	Bianchi and Mendoza (2020)
δ	0.1	Bianchi and Mendoza (2020)
η	3	Hsieh and Klenow (2009)

Notes: Parameters set externally following the literature. σ is the coefficient of relative risk aversion; ξ the elasticity of substitution between tradable and non-tradable goods; κ the tightness of the collateral constraint; r the world interest rate; α the capital share in tradable production; δ the depreciation rate; and η the elasticity of substitution across tradable varieties. The Source column reports the reference each value is taken from.

determined by ϕ, drive both the long-run volatility of investment as well as its dynamics during sudden stops. In their absence, the former is too volatile compared to the data, while the opposite is true for the latter. I set $\phi = 3$, which delivers a reasonable compromise between both targets.

Table 2: Calibrated Parameters

Parameter	Value	Target
β	0.94	Sudden stop frequency $= 2.8\%$
ρ	0.85	$\text{corr}\left(y_t^T, y_{t-1}^T\right) = 0.85$
σ_A	0.0125	$\mathbb{S}td\left[y_t^T\right] = 3\%$
A	1.75	$\mathbb{E}\left[p_t^N y_t^N / y_t^T\right] = 2$
ϕ	3	$\mathbb{S}td\left[i_t\right] / \mathbb{S}td\left[y_t\right] = 3$

Notes: Parameters calibrated within the model to match the targets in the last column. A sudden stop is defined as an episode in which the current-account-to-GDP ratio exceeds the 95th percentile of its ergodic distribution while the borrowing constraint binds. Log TFP follows an AR(1) process, and ϕ governs quadratic capital adjustment costs.

6.1.3 Productivity Externality

One key element for the quantitative relevance of the trade-off is the effect of investment on TFP through reduced misallocation, given by γ_{t+1}. Recall from (27) that it is given by,

$$\gamma_t = \alpha(1 + \alpha(\eta - 1))\nu\,\sigma_R^2,$$

The magnitude of γ_{t+1} will depend, then, on ν and the average σ_R^2 in the economy, which is partially a function of σ_F^2. I jointly calibrate them in the following way: Given an estimate for σ_R^2, I set ν to target a value of $\gamma_t = 0.1$ for Spain, which is at the lower end of the estimates of the effects of investment into TFP surveyed by González et al. (2024)[22] as it is not possible to separate how much of the reported effects are due to reduced misallocation as opposed to other channels such as innovation.

Given ν, I set σ_F^2 to match a targeted σ_R^2. I estimate this moment for 18 countries and target the average level of misallocation in Spain, which is close to the median estimate.

To estimate σ_R^2, I follow the procedure used by Hsieh and Klenow (2009). Recall that σ_R^2 is the variance of the log of the rental rates, $R(i)$, paid by the firms producing tradable goods varieties. It follows from profit maximization that

$$R(i) = \alpha\frac{p(i)y^T(i)}{k(i)}, \tag{43}$$

Where the denominator is the value added of the firm and the numerator the stock of capital it possesses. Thus, estimating σ_R^2 requires firm-level data that allows to estimate the ratio of these two numbers. The Orbis-Amadeus historical product, which has extensive coverage of private firms in Europe (Kalemli-Özcan et al., 2024), provides exactly that. I now detail the steps I take to measure σ_R^2.

Data cleaning. To clean the dataset I follow the procedure by Kalemli-Özcan et al. (2024) as closely as possible. The input for the Orbis dataset is income and balance sheet statements submitted annually by firms. To have consistent units, I keep only unconsolidated statements that cover 12 months.

I drop spells with errors in the following way. First, I tag unrealistic changes in assets or sales[23], or negative values for assets, sales, employment or liabilities. I also tag observations that do not report employment or report a number larger than 2 million, or where balance

[22] González et al. (2024) report that a 1 p.p drop in interest rates leads to an increase in TFP between 0.42 and 1.72. Assuming a semi-elasticity of investment of 4 (Wroblewski, 2025) implies $\gamma \in (0.1, 0.43)$

[23] In the order of 10^3

sheet identities don't hold. I drop all tagged observations and only keep the spell after the last identified error.

As it is common in the literature that measures misallocation, I focus on the manufacturing sector[24], defined using the 4-digit NACE classification. Section D.2 presents some summary statistics of the resulting sample which spans 18 countries[25] over the period 1996-2016, covering 1,050,610 unique firms for a total of 9,143,358 observations. Employment by these firms adds up to 11,559,081 employees per year, with the firm size distribution resembling that reported by Eurostat.[26]

Variable construction. I construct $p(i)y^T(i)$ by subtracting the cost of materials from the operating revenue of the firm and the capital stock $k(i)$ as the sum of tangible and intangible fixed assets. To mitigate potential measurement error, I winsorize both variables at the bottom and top 1% levels. I then estimate $R(i)$ using equation (43) and its log variance at the 4-digit industry level. Lastly, I compute the weighted average across industries, where I use total value added as the weight for each industry.

The second column of Table 3 show the average value of σ_R^2 between 2012 and 2016[27] for each country in the sample. The estimates range between 1.33 and 2.86, with a mean of 2.15, indicating a not negligible degree of dispersion. Given $\sigma_R^2 = 2.34$ for Spain, I set $\nu = 0.07$ to target $\gamma_{t+1} = 0.1$. Table 4 summarizes the calibration of ν and σ^F.

Combining the country-level estimates of σ_R^2 and ν, it is possible to estimate γ_{t+1} for all the countries in the sample. The third column of Table 3 shows the estimated γ_{t+1} using this estimate for ν. A 1% increase in capital leads to increases in TFP between 0.06% and 0.12%.

In Online Appendix D.3 I show that, consistent with Lemma 3, these estimates of γ_{t+1} are decreasing in both the capital stock and income across countries.

Robustness As shown by equation (27), the estimation of γ depends on properly choosing ν and estimating σ_R^2. In Online Appendix F.1.1, I estimate alternative values of ν using identified effects from Albrizio, Beatriz, and Khametsin (2024), Bau and Matray (2023) and Pinardon-Touati (2024) and show that they yield considerably higher estimates of ν. As such, I see the value of ν used in this paper as a conservative estimate.

In Online Appendix F.1.2, I show alternative estimates of γ using different ways of

[24] Defined as firms that report NACE codes between 1010 and 3320

[25] I use the same sample as Kalemli-Özcan et al. (2024) and drop the United Kingdom and Greece as some variables required for the analysis are not populated.

[26] See Section D.2 in the Online Appendix.

[27] I focus on the latter part of the sample to maximize coverage of manufacturing activity.

Table 3: Misallocation Estimates

Country	σ_R^2	$\tilde{\gamma}$
Austria	1.52	0.06
Belgium	1.54	0.07
Czech Rep.	2.43	0.10
Estonia	1.93	0.08
Finland	1.58	0.07
France	1.33	0.06
Germany	1.58	0.07
Hungary	2.69	0.12
Italy	2.13	0.09
Latvia	2.86	0.12
Norway	1.99	0.09
Poland	2.35	0.10
Portugal	1.99	0.09
Romania	2.72	0.12
Slovakia	2.68	0.11
Slovenia	2.73	0.12
Spain	2.34	0.10
Sweden	2.32	0.10
Mean	2.15	0.09

Notes: σ_R^2 is the value-added-weighted average across 4-digit manufacturing industries of the within-industry variance of log marginal revenue products of capital, computed from Orbis-Amadeus firm-level data over 2012–2016 following Hsieh and Klenow (2009). $\tilde{\gamma}$ is the implied productivity externality γ_{t+1} obtained by combining each country's σ_R^2 with the calibrated value of ν.

measuring σ_R^2. I find that the main results are not driven by small firms, measured either by capital or number of workers, nor by young firms. Results are also robust to potential misreporting by firms, which I address by using averages within firm to estimate added value and the capital stock. Lastly, in Online Appendix F.2, I show that the estimates are also similar when allowing for labour misallocation.

6.2 Solution

Because the model features an occasionally binding constraint, a global solution method is needed. Moreover, the presence of externalities means that value function iteration cannot be used to solve for the decentralized equilibrium.

Instead, I iterate on the Euler equations of the decentralized equilibrium, adapting the algorithm in Bianchi (2011) to accommodate two state variables. To solve for the planner's

Table 4: Misallocation Parameters

Parameter	Value	Target
ν	0.072	$\gamma_{t+1} = 0.1$
σ_F^2	6.365	$\mathbb{E}\left[\sigma_R^2\right] = 2.34$

Notes: ν is the elasticity of aggregate TFP to reductions in capital misallocation and σ_F^2 the dispersion of firm-level financial frictions. The two are jointly calibrated: given $\sigma_R^2 = 2.34$ for Spain, ν targets $\gamma_{t+1} = 0.1$, and σ_F^2 is set so the model reproduces the targeted σ_R^2.

allocation I use value function iteration. Additional details on the solution method as well as its implementation are provided in Online Appendix G.

7 Quantitative Analysis

Having solved for policy functions over a grid of values for the state variables of the model I simulate paths of exogenous productivity to find the ergodic distribution of the economy.

The second column of Table 5 shows the frequency of sudden stops as well as the behavior of consumption, investment and the current account during these episodes. Comparing to the first column, which displays the moments estimated by Bianchi and Mendoza (2020), the model does a good job in matching the frequency of sudden stops as well as the average current account reversal. Under the decentralized equilibrium, the economy experiences a crisis almost 2.4% of the time. In these episodes, there's a 3.7 p.p. reversal of the current account, as the ability of the economy to borrow is sharply diminished. The model does not quite match the composition of the adjustment between consumption and investment when compared to the data. It predicts that households cut consumption by 2.5%, compared to the observed 4%, and investment by 10 p.p. more than the almost 20% reported by Bianchi and Mendoza (2020).[28]

Constrained efficient allocation. The third column of Table 5 shows the corresponding moments under the constrained efficient allocation. In line with the literature, it achieves a large reduction in the probability of crises, down to 0.35%. Conditional on a sudden stop, it adjusts investment by less than the decentralized equilibrium, reflecting the positive effects of investment than the planner internalizes.

[28]Higher adjustment costs would allow the model to match these two moments at the cost of too little investment volatility at business cycle frequency. Matching conditional and unconditional volatilities could be achieved by having either a higher curvature of adjustment costs than the quadratic form allows or a borrowing constraint that includes capital.

Table 5: Sudden Stop Moments

	Data	D.E.	C.E.	Optimal Constant Tax
SS frequency (%)	2.80	2.74	0.25	8.05
Consumption (% change)	-4.00	-2.81	-3.50	-2.87
Investment (% change)	-19.00	-26.35	-25.77	-27.86
CA/GDP (pp change)	2.80	3.69	4.23	4.01

Notes: Moments computed over sudden stop episodes. The Data column reports the empirical moments from Bianchi and Mendoza (2020). D.E. is the decentralized equilibrium, C.E. the constrained efficient allocation, and the last column the equilibrium under the welfare-maximizing constant debt tax. Consumption and investment are average percent deviations during sudden stops; CA/GDP is the average change in the current-account-to-GDP ratio, in percentage points.

Measured in consumption equivalent units, this allocation delivers welfare gains of 0.35%, which are sizable when compared to usual estimates of the cost of business cycles in representative household models. Proposition 1 can be used to back-out the capital control τ_t^d and investment subsidy τ_t^k that implement this allocation in a competitive equilibrium. On average, the resulting capital control is almost 2%, while the investment subsidy is approximately 3.5%.

Simple interventions. I now focus on the second best case, only allowing the planner to set the optimal control. Moreover, in line with the literature, I also require the planner to set a constant tax over the cycle.

Under these conditions, I find that the welfare maximizing tax is effectively a 2.5% subsidy on borrowing ($\tau^{d,sb} = -0.025$). This delivers modest welfare gains (0.03%) compared to the constrained efficient allocation, which is to be expected given the restrictions imposed. The fourth column of table 5 shows that under this borrowing subsidy, sudden stops are more than twice as frequent, which is an expected consequence of the increased borrowing. This increased financial instability is compensated by an average increase in consumption of around 1.5%.

Figure 1 plots welfare gains as a function of the constant tax. For negative values (subsidy) the welfare gains are relatively small, and the curve is relatively flat. For positive values, however, welfare losses become steep. A capital control of 2.5%, which is the average of the first-best value, leads to welfare losses of around 0.4%. Given that this in the ballpark of capital controls suggested by the literature, this result shows that ignoring effects on investment can be costly.

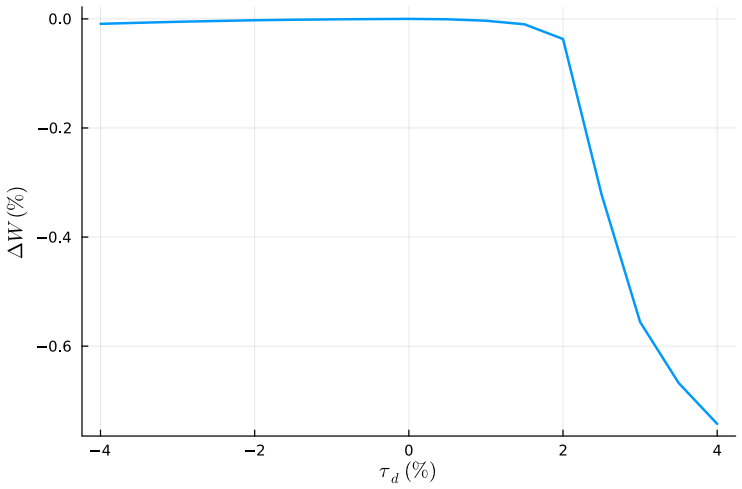


Figure 1: Welfare gains as a function of the constant capital control.

Notes: The figure plots the welfare gains, in consumption-equivalent units relative to the decentralized equilibrium, as a function of a constant debt tax τ^d held.

The role of misallocation. As suggested by Proposition 2, the second-best capital control grows with γ and hence with the existing level of misallocation σ_R^2. As shown by Table 3 the countries in my sample greatly differ in their levels of misallocation. To assess how this variation could affect policy prescription, I repeat the previous exercise for each country in the sample, targeting their σ_R^2 and keeping the rest of the targets as in Table 2.[29]

Figure 2 displays the second-best capital control as a function of σ_R^2 over the range of values in Table 3. As expected, the second-best capital control declines in misallocation, from a 1.2% tax on borrowing for a counterfactual economy with no misallocation to a 3.7% subsidy at the highest estimated dispersion.

There's two important takeaways from Figure 2. The first is that even for low levels of misallocation, such as those experimented by France, the introduction of misallocation leads to a reversal in the sign of the tax on borrowings. The second is that there's significant heterogeneity in the optimal policy across the sample. The implied subsidy for a country such as Latvia is around 4 times that of France.

What are the welfare implications of this heterogeneity across countries? Figure 3 plots the welfare effect of imposing a 2.5% capital control for each of these economies. Note that it is negative even when there is no misallocation. This is because the optimal capital control would be around 1% in this scenario, as shown by Figure 2. In this economy, a constant 2.5% tax would lead to welfare losses of around 0.3% percent. The losses are greater when

[29] In this sense, the exercise is better understood as a way of quantifying the importance of misallocation for macroprudential policy and not a prescription for each particular country.

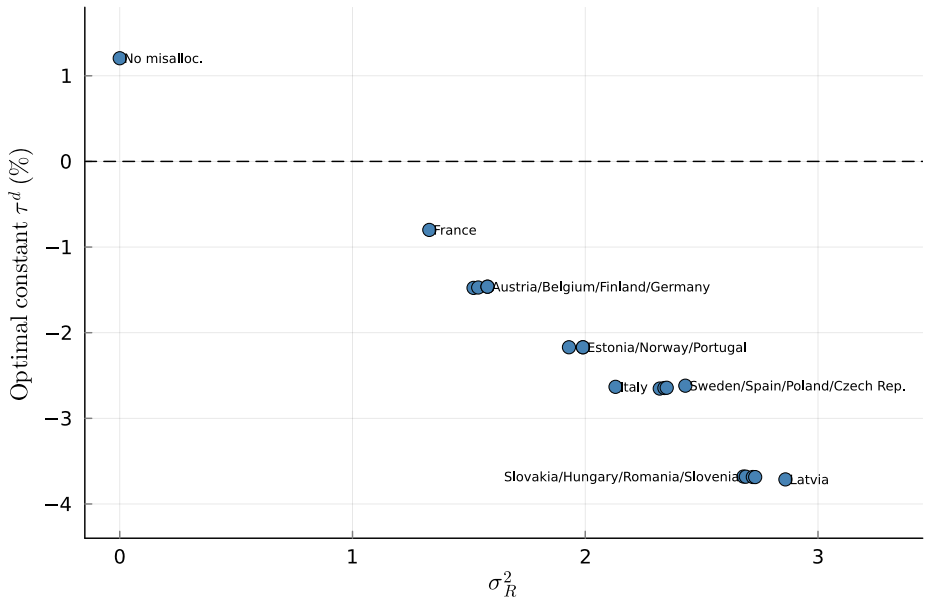


Figure 2: Optimal constant intervention as a function of misallocation

Notes: The figure plots the optimal constant debt tax τ^d (in percent; negative values are borrowing subsidies) when the model is calibrated to each country's estimated σ_R^2 from Table 3. The dashed line marks $\tau^d = 0$: above it the optimal intervention is a conventional capital control, below it a borrowing subsidy.

there's misallocation, getting to almost 0.5% for Latvia.

8 Conclusion

In this paper, I study optimal macroprudential policy when its effects on investment and capital misallocation are considered. To accomplish this, I extend the standard model of macroprudential policy to incorporate production by firms that face heterogeneous frictions in capital markets.

I propose a tractable and flexible way of modeling these frictions, which result in capital being misallocated and, in turn, lower aggregate productivity as a result. Macroprudential policies, such as capital controls, that reduce investment also exacerbate misallocation, in line with the empirical literature.

Turning to optimal policy, I first study the constrained efficient solution, in which the policymaker has enough available instruments. In this scenario, there's no trade-off, since the policymaker can tax borrowing to address the pecuniary externality and subsidize investment to improve productivity.

I then study the second-best problem, in which the policymaker can only control total

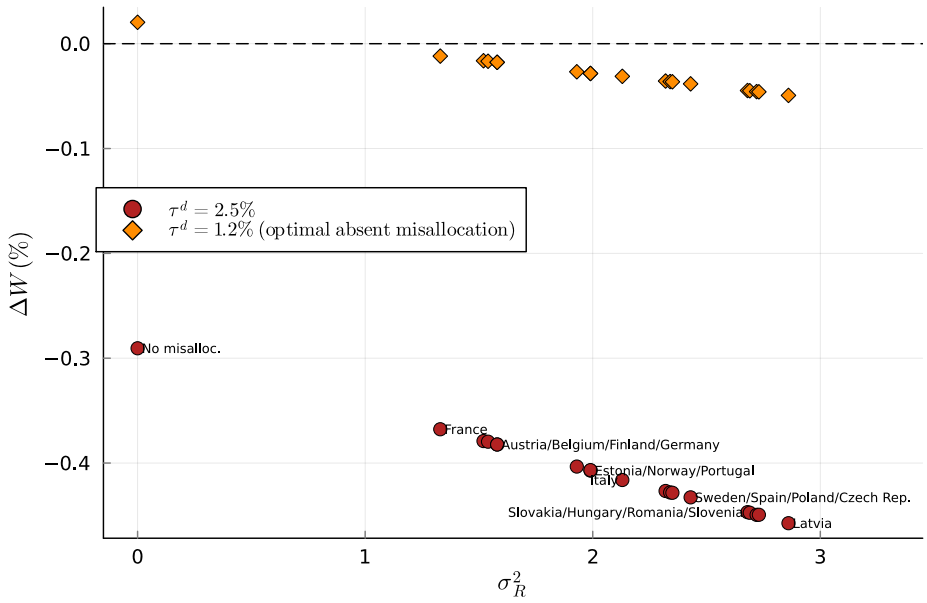


Figure 3: Welfare cost of a conventional capital control

Notes: Welfare change, in consumption-equivalent units relative to the decentralized equilibrium, from imposing a constant 2.5% debt tax in each economy of Figure 2. Each economy is re-calibrated to its estimated σ_R^2 as described there. The dashed line marks zero: points below it indicate that the conventional policy reduces welfare relative to no intervention.

borrowing. I characterize the solution to this problem and show that a trade-off exists. An increase in borrowing reduces welfare if a sudden stop occurs, but also leads to increased investment and productivity. I characterize this trade-off using several sufficient statistics, which allows me to provide intuition for the result and explain the qualitative effects of each of them.

To explore the quantitative relevance of this trade-off, I numerically solve the model. I leverage the tractable specification for misallocation, along with firm-level microdata, to calibrate the relevant parameters. The model is able to replicate the empirical facts of sudden stops, thus providing a good setup to study welfare consequences of different policies.

When restricting the planner to only constant capital controls, it is optimal to subsidize foreign borrowing rather than restricting, as a result of the adverse effects of capital controls on productivity through misallocation. The model also allows me to show that ignoring this channel can lead to severe welfare equivalents, comparable to the cost of business cycles.

I leverage the 18 countries in my sample to evaluate the sensitivity of my findings to different levels of misallocation. Across a wide range of estimates, I find that all countries would find it optimal to subsidy borrowing if investment subsidies are not allowed. There is still substantial heterogeneity across countries: Optimal subsidies are almost 4 times larger

for the countries with the most misallocation compared to those at the lower end. I also find that welfare losses are substantial across all of them when introducing capital controls. This suggests that this channel is quantitatively relevant and must be taken into account when designing macroprudential policies.

References

Albrizio, Silvia, Gonzalez Beatriz, and Dmitry Khametsin (July 2024). *A Tale of Two Margins: Monetary Policy and Capital Misallocation*. Working Paper 2024/121. International Monetary Fund.

Alfaro, Laura, Anusha Chari, and Fabio Kanczuk (2017). "The real effects of capital controls: Firm-level evidence from a policy experiment". *Journal of International Economics* 108: 191–210.

Andreasen, Eugenia et al. (Feb. 2023). *Beware the Side Effects: Capital Controls, Trade, Misallocation and Welfare*. Working Paper 30963. National Bureau of Economic Research.

Arce, Fernando, Julien Bengui, and Javier Bianchi (Sept. 2019). *A Macroprudential Theory of Foreign Reserve Accumulation*. Working Paper 26236. National Bureau of Economic Research.

Asriyan, Vladimir et al. (June 2024). "Falling Interest Rates and Credit Reallocation: Lessons from General Equilibrium". *The Review of Economic Studies*: rdae065.

Baqaee, David R., Emmanuel Farhi, and Kunal Sangani (2024). "The Supply-Side Effects of Monetary Policy". *Journal of Political Economy* 132 (4): 1065–1112.

Bau, Natalie and Adrien Matray (2023). "Misallocation and Capital Market Integration: Evidence From India". *Econometrica* 91 (1): 67–106.

Bengui, Julien and Javier Bianchi (2022). "Macroprudential policy with leakages". *Journal of International Economics* 139: 103659.

Benigno, Gianluca, Huigang Chen, et al. (2013). "Financial crises and macro-prudential policies". *Journal of International Economics* 89 (2): 453–470.

— (May 2016). *Optimal Capital Controls and Real Exchange Rate Policies: A Pecuniary Externality Perspective*. Working Paper 22224. National Bureau of Economic Research.

Benigno, Gianluca and Luca Fornaro (2014). "The Financial Resource Curse". *The Scandinavian Journal of Economics* 116 (1): 58–86.

Bernanke, Ben and Mark Gertler (1989). "Agency Costs, Net Worth, and Business Fluctuations". *The American Economic Review* 79 (1): 14–31. (Visited on 08/31/2024).

Bianchi, Javier (Dec. 2011). "Overborrowing and Systemic Externalities in the Business Cycle". *American Economic Review* 101 (7): 3400–3426.

Bianchi, Javier and Enrique G Mendoza (2020). "A Fisherian Approach to Financial Crises: Lessons from the Sudden Stops Literature". *Review of Economic Dynamics* 37: S254–S283.

— (2018). "Optimal Time-Consistent Macroprudential Policy". *Journal of Political Economy* 126 (2): 588–634.

Blattner, Laura, Luisa Farinha, and Francisca Rebelo (June 2023). "When Losses Turn into Loans: The Cost of Weak Banks". *American Economic Review* 113 (6): 1600–1641.

Buera, Francisco J., Joseph P. Kaboski, and Yongseok Shin (2015). "Entrepreneurship and Financial Frictions: A Macrodevelopment Perspective". *Annual Review of Economics* 7 (Volume 7, 2015): 409–436.

Calligaris, Sara et al. (Sept. 2018). "The productivity puzzle and misallocation: an Italian perspective". *Economic Policy* 33 (96): 635–684.

Cerutti, Eugenio, Stijn Claessens, and Luc Laeven (2017). "The use and effectiveness of macroprudential policies: New evidence". *Journal of Financial Stability* 28: 203–224.

Chodorow-Reich, Gabriel (Oct. 2013). " The Employment Effects of Credit Market Disruptions: Firm-level Evidence from the 2008-9 Financial Crisis". *The Quarterly Journal of Economics* 129 (1): 1–59.

Clayton, Christopher and Andreas Schaab (Jan. 2022). "Multinational Banks and Financial Stability". *The Quarterly Journal of Economics* 137 (3): 1681–1736.

David, Joel M, Venky Venkateswaran, et al. (2021). "Capital allocation in developing countries". *The World Bank Economic Review* 35 (4): 1102–1121.

David, Joel M. and Venky Venkateswaran (July 2019). "The Sources of Capital Misallocation". *American Economic Review* 109 (7): 2531–67.

David, Joel M. and David Zeke (2024). *Risk-Taking, Capital Allocation and Monetary Policy.* Tech. rep.

Dávila, Eduardo and Anton Korinek (Feb. 2017). "Pecuniary Externalities in Economies with Financial Frictions". *The Review of Economic Studies* 85 (1): 352–395.

Dias, Daniel A., Carlos Robalo Marques, and Christine Richmond (2016). "Misallocation and productivity in the lead up to the Eurozone crisis". *Journal of Macroeconomics* 49: 46–70.

Elenev, Vadim, Tim Landvoigt, and Stijn Van Nieuwerburgh (2021). "A Macroeconomic Model With Financially Constrained Producers and Intermediaries". *Econometrica* 89 (3): 1361–1418.

Farhi, Emmanuel and Iván Werning (2016). "A Theory of Macroprudential Policies in the Presence of Nominal Rigidities". *Econometrica* 84 (5): 1645–1704.

Feenstra, Robert C, Robert Inklaar, and Marcel P Timmer (2015). "The next generation of the Penn World Table". *American Economic Review* 105 (10): 3150–82.

Fisher, Irving (1933). "The Debt-Deflation Theory of Great Depressions". *Econometrica* 1 (4): 337–357. (Visited on 08/31/2024).

Flemming, Jean, Jean-Paul L'Huillier, and Facundo Piguillem (2019). "Macro-prudential taxation in good times". *Journal of International Economics* 121: 103251.

Forbes, Kristin J (May 2005). *The Microeconomic Evidence on Capital Controls: No Free Lunch*. Working Paper 11372. National Bureau of Economic Research.

González, Beatriz et al. (2023). *Firm Heterogeneity, Capital Misallocation and Optimal Monetary Policy*. Working Paper 2890. European Central Bank.

— (2024). "Firm heterogeneity, capital misallocation and optimal monetary policy".

Gopinath, Gita et al. (June 2017). "Capital Allocation and Productivity in South Europe". *The Quarterly Journal of Economics* 132 (4): 1915–1967.

Gromb, Denis and Dimitri Vayanos (2002). "Equilibrium and welfare in markets with financially constrained arbitrageurs". *Journal of Financial Economics* 66 (2). Limits on Arbitrage: 361–407.

Herreño, Juan (2023). *Aggregating the Effect of Bank Credit Supply Shocks on Firms*. Tech. rep.

Hopenhayn, Hugo A. (2014). "Firms, Misallocation, and Aggregate Productivity: A Review". *Annual Review of Economics* 6 (Volume 6, 2014): 735–770.

Hsieh, Chang-Tai and Peter J. Klenow (Nov. 2009). "Misallocation and Manufacturing TFP in China and India". *The Quarterly Journal of Economics* 124 (4): 1403–1448.

International Monetary Fund (2022). *Review of The Institutional View on The Liberalization and Management of Capital Flows*. Tech. rep.

Itskhoki, Oleg and Benjamin Moll (2019). "Optimal Development Policies With Financial Frictions". *Econometrica* 87 (1): 139–173.

Jeanne, Olivier and Anton Korinek (May 2010). "Excessive Volatility in Capital Flows: A Pigouvian Taxation Approach". *American Economic Review* 100 (2): 403–07.

Jiménez, Gabriel et al. (2017). "Macroprudential Policy, Countercyclical Bank Capital Buffers, and Credit Supply: Evidence from the Spanish Dynamic Provisioning Experiments". *Journal of Political Economy* 125 (6): 2126–2177.

Kalemli-Özcan, Şebnem et al. (Apr. 2024). "How to Construct Nationally Representative Firm-Level Data from the Orbis Global Database: New Facts on SMEs and Aggregate Implications for Industry Concentration". *American Economic Journal: Macroeconomics* 16 (2): 353–74.

Kiyotaki, Nobuhiro and John Moore (1997). "Credit Cycles". *Journal of Political Economy* 105 (2): 211–248.

Klein, Paul, Per Krusell, and José-Víctor Ríos-Rull (2008). "Time-Consistent Public Policy". *The Review of Economic Studies* 75 (3): 789–808.

Kochen, Federico (2022). *Finance over the life cycle of firms*. Tech. rep.

Korinek, Anton and Enrique G. Mendoza (2014). "From Sudden Stops to Fisherian Deflation: Quantitative Theory and Policy". *Annual Review of Economics* 6 (Volume 6, 2014): 299–332.

Kurtzman, Robert and David Zeke (2020). "Misallocation costs of digging deeper into the central bank toolkit". *Review of Economic Dynamics* 38: 94–126.

Larrain, Mauricio and Sebastian Stumpner (2017). "Capital Account Liberalization and Aggregate Productivity: The Role of Firm Capital Allocation". *The Journal of Finance* 72 (4): 1825–1858.

Levine, Ross (2005). "Chapter 12 Finance and Growth: Theory and Evidence". In: *Handbook of Economic Growth*. Ed. by Philippe Aghion and Steven N. Durlauf. Vol. 1. Handbook of Economic Growth. Elsevier: 865–934. URL: https://www.sciencedirect.com/science/article/pii/S1574068405010129.

Lorenzoni, Guido (July 2008). "Inefficient Credit Booms". *The Review of Economic Studies* 75 (3): 809–833.

Madeira, Carlos (2024). "The impact of macroprudential policies on industrial growth". *Journal of International Money and Finance* 145: 103106.

Melitz, Marc J. (2005). "When and how should infant industries be protected?" *Journal of International Economics* 66 (1): 177–196.

Midrigan, Virgiliu and Daniel Yi Xu (Feb. 2014). "Finance and Misallocation: Evidence from Plant-Level Data". *American Economic Review* 104 (2): 422–58.

Monacelli, Tommaso, Luca Sala, and Daniele Siena (2023). "Real interest rates and productivity in small open economies". *Journal of International Economics* 142: 103746.

Noland, Marcus (May 2007). "481South Korea's Experience with International Capital Flows". In: *Capital Controls and Capital Flows in Emerging Economies: Policies, Practices, and Consequences*. University of Chicago Press. ISBN: 9780226184975. URL: https://doi.org/10.7208/chicago/9780226184999.003.0011.

Ottonello, Pablo, Diego J Perez, and William Witheridge (May 2024). *The Exchange Rate as an Industrial Policy*. Working Paper 32522. National Bureau of Economic Research.

Pinardon-Touati, Noémie (2024). *The crowding out effect of local government debt: Micro- and macro-estimates*. Tech. rep.

Rajan, Raghuram G. and Luigi Zingales (1998). "Financial Dependence and Growth". *American Economic Review* 88 (3): 559–586. (Visited on 08/07/2024).

Redding, S (Jan. 1999). "Dynamic comparative advantage and the welfare effects of trade". *Oxford Economic Papers* 51 (1): 15–39.

Reis, Ricardo (Aug. 2013). *The Portuguese Slump and Crash and the Euro Crisis*. Working Paper 19288. National Bureau of Economic Research.

Restuccia, Diego and Richard Rogerson (2013). "Misallocation and productivity". *Review of Economic Dynamics* 16 (1). Special issue: Misallocation and Productivity: 1–10.

— (Aug. 2017). "The Causes and Costs of Misallocation". *Journal of Economic Perspectives* 31 (3): 151–74.

Schmitt-Grohé, Stephanie and Martín Uribe (2016). "Downward Nominal Wage Rigidity, Currency Pegs, and Involuntary Unemployment". *Journal of Political Economy* 124 (5): 1466–1514.

— (May 2020). "Multiple Equilibria in Open Economies with Collateral Constraints". *The Review of Economic Studies* 88 (2): 969–1001.

Seoane, Hernán D. and Emircan Yurdagul (2019). "Trend shocks and sudden stops". *Journal of International Economics* 121: 103252.

Tauchen, George (1986). "Finite State Markov-Chain Approximations to Univariate and Vector Autoregressions". *Economics Letters* 20 (2): 177–181.

Varela, Liliana (Aug. 2017). "Reallocation, Competition, and Productivity: Evidence from a Financial Liberalization Episode". *The Review of Economic Studies* 85 (2): 1279–1313.

World Bank (2021). *World Development Indicators*. URL: `https://databank.worldbank.org/home` (visited on 08/19/2024).

Wroblewski, Caleb (2025). *The interest rate elasticity of investment: micro estimates and macro implications*. Tech. rep. Mimeo, UC Berkeley.

Online Appendix

Contents

A Equilibrium Definition and Characterization **43**

B Proofs **46**

B.1 Proof of Lemma 1 . . . 46

B.2 Proof of Lemma 2 . . . 48

B.3 Proof of Lemma 3 . . . 51

B.4 Proof of Proposition 1 . . . 51

B.5 Proof of Proposition 2 . . . 52

C Capital Misallocation Block **54**

C.1 Possible Microfoundations . . . 54

C.1.1 Credit shocks . . . 54

C.1.2 Borrowing Constraints . . . 54

C.1.3 Internal and External Finance . . . 55

C.2 Including Additional Sources of Misallocation . . . 56

D Data Appendix **58**

D.1 Country Panel Data . . . 58

D.2 Orbis . . . 58

D.3 The Productivity Externality and Development . . . 58

E Long-Run Moments **62**

F Robustness **63**

F.1 TFP Elasticity with respect to Investment . . . 63

F.1.1 Frictions Elasticity ν . . . 63

F.1.2 Capital Misallocation σ_R^2 . . . 66

F.2 Model With Additional Distortions . . . 67

G Computational Appendix **72**

G.1 State-Space Discretization . . . 72

G.2 Decentralized Equilibrium: Euler-Equation Iteration . . . 72

G.3 Constrained-Efficient Planner: Value-Function Iteration . . . 73

G.4 Simulation . . . 74

G.5 Welfare Calculation 74
G.6 Optimal Tax Computation 75

A Equilibrium Definition and Characterization

Definition (Decentralized Equilibrium)**.** *A Decentralized equilibrium is a set of (state-contingent)*

1. *household allocations* $\{c_t, d_{t+1}, k_{t+1}\}$
2. *firm allocations* $\{x_t, x_t^T, x_t^N, k_t, h_t\}$
3. *rental rates dispersion* σ_R^2
4. *prices* $\{w_t, p_t, p_t^N, R_t\}$
5. *Government policy* $\{\tau_t^d, \tau_t^k, T_t\}$

such that, given initial allocations $\{k_0, d_0\}_t$*, the non-tradable endowment* y^N*, world interest rate* r*,* A_t*,and* σ_a^2*,*

1. *Given prices and government policy, the allocations solve the household problem.*
2. *Given prices, the allocations solve the firm's problem.*
3. *The rental rate dispersion is consistent with profit maximizing by firms.*
4. *Markets clear*
5. *The government budget is balanced*

Characterization. By Lemma 1 and factor market clearing, the tradable sector collapses into aggregate output $y_t^T = \text{TFP}_t k_t^\alpha$, with $h_t = 1$, and the factor prices (28)-(29). Combining the household budget constraint (2), firm profits, non-tradable market clearing, and the government budget constraint delivers the tradable and final-good resource constraints (30) and (31), which replace the budget constraint together with the goods-market clearing conditions. The relative prices p_t^N and p_t then follow residually from (32) and (9). What is left is a system in $\{c_t, d_{t+1}, k_{t+1}, x_t^T, \mu_t\}$ and the prices and productivity terms, summarized below.

Productivity and prices.

$$\sigma_R^2 = \left(\frac{1}{1+\alpha(\eta-1)}\frac{1}{k_t^{\nu}}\right)^2 \mathbb{V}ar\left[F(i)\right], \tag{A.1}$$

$$\log \mathrm{TFP}_t = \log A_t + \frac{1}{2}(\eta-1)\sigma_a^2 - \frac{1}{2}\,\alpha\big(1+\alpha(\eta-1)\big)\sigma_R^2, \tag{A.2}$$

$$w_t = (1-\alpha)\,\mathrm{TFP}_t\, k_t^{\alpha}, \tag{A.3}$$

$$R_t = \alpha\,\mathrm{TFP}_t\, k_t^{\alpha-1}, \tag{A.4}$$

$$p_t^N = \frac{1-\omega}{\omega}\left(\frac{x_t^T}{y^N}\right)^{\frac{1}{\xi}}, \tag{A.5}$$

$$p_t = \frac{1}{\omega}\left(\frac{x_t^T}{x_t}\right)^{\frac{1}{\xi}}, \tag{A.6}$$

where $x_t^N = y^N$ and $x_t = \left[\omega {x_t^T}^{\frac{\xi-1}{\xi}} + (1-\omega){y^N}^{\frac{\xi-1}{\xi}}\right]^{\frac{\xi}{\xi-1}}$.

Resource constraints.

$$x_t^T = \mathrm{TFP}_t k_t^{\alpha} - (1+r)d_t + d_{t+1}, \tag{A.7}$$

$$c_t + k_{t+1} = (1-\delta)k_t + x_t. \tag{A.8}$$

Household optimality.

$$\frac{c_t^{-\sigma}}{p_t} = \frac{1}{1-\mu_t}\beta(1+r)(1+\tau_t^d)\mathbb{E}\left[\frac{c_{t+1}^{-\sigma}}{p_{t+1}}\right], \tag{A.9}$$

$$c_t^{-\sigma} = \beta\mathbb{E}\left[c_{t+1}^{-\sigma}\left(\frac{\alpha\mathrm{TFP}_{t+1}k_{t+1}^{\alpha-1}}{p_{t+1}}\left(1+\kappa\mu_{t+1}\right) + \frac{\tau_{t+1}^k}{p_{t+1}} + 1 - \delta\right)\right], \tag{A.10}$$

$$d_{t+1} \leq \kappa\left(\mathrm{TFP}_t k_t^{\alpha} + p_t^N y^N\right), \tag{A.11}$$

$$\mu_t \geq 0, \tag{A.12}$$

$$\mu_t\left[\kappa\left(\mathrm{TFP}_t k_t^{\alpha} + p_t^N y^N\right) - d_{t+1}\right] = 0, \tag{A.13}$$

together with the transversality conditions

$$\lim_{T\to\infty}\mathbb{E}_0\left[\beta^T\frac{c_T^{-\sigma}}{p_T}\,d_{T+1}\right] = 0, \tag{A.14}$$

$$\lim_{T\to\infty}\mathbb{E}_0\left[\beta^T c_T^{-\sigma}\,k_{T+1}\right] = 0. \tag{A.15}$$

Government.

$$\tau_t^d(1+r)d_t = \tau_t^k k_t + T_t, \tag{A.16}$$

so that the revenue raised by the capital control finances the investment subsidy and the lump-sum rebate.

Given the initial conditions $\{k_0, d_0\}$, a policy $\{\tau_t^d, \tau_t^k\}$ with T_t set residually by (A.16), and the exogenous tradable productivity A_t following a Markov process, conditions (A.1)-(A.16) evaluated at every history characterize a decentralized equilibrium. Because $\text{TFP}_t = \text{TFP}(A_t, k_t)$ and all conditions depend on the past only through (k_t, d_t), the equilibrium is recursive in the state (A_t, k_t, d_t), with the endogenous objects expressed as time-invariant policy functions of that state.

B Proofs

B.1 Proof of Lemma 1

Proof. I prove this Lemma in two parts. First, I derive the aggregate production function and TFP. Lastly, I obtain the first order conditions of the representative firm.

Aggregate production function. Let MC be the marginal cost of the firm, derived from solving the cost minimization problem of the firm, with first order conditions:

$$R(i) = \alpha MC(i)\frac{y(i)}{k(i)} \tag{B.1}$$

$$w = (1-\alpha)MC(i)\frac{y(i)}{h(i)} \tag{B.2}$$

Combining into the production function yields the marginal cost of the firm:

$$MC(i) = A(i)^{-1}\left(\frac{R(i)}{\alpha}\right)^{\alpha}\left(\frac{w}{1-\alpha}\right)^{1-\alpha} \tag{B.3}$$

We can now solve for the problem of the firm as:

$$\max_{y(i)} \frac{\eta}{\eta-1} y(i)^{\frac{\eta-1}{\eta}} y^{\frac{1}{\eta}} - y(i)MC(i) \tag{B.4}$$

with solution:

$$y(i) = (MC(i))^{-\eta} y \tag{B.5}$$

Plugging back into the factor demands, we get:

$$k(i) = A(i)^{\eta-1} y \left(\frac{\alpha}{R(i)}\right)^{1+\alpha(\eta-1)} \left(\frac{1-\alpha}{w}\right)^{(1-\alpha)(\eta-1)} \tag{B.6}$$

$$h(i) = A(i)^{\eta-1} y \left(\frac{\alpha}{R(i)}\right)^{\alpha(\eta-1)} \left(\frac{1-\alpha}{w}\right)^{\alpha+(1-\alpha)\eta} \tag{B.7}$$

Then, we can obtain aggregate production as

$$y = \left[\int y(i)^{\frac{\eta-1}{\eta}} di\right]^{\frac{\eta}{\eta-1}} \tag{B.8}$$

$$= \frac{\left[\int y(i)^{\frac{\eta-1}{\eta}} di\right]^{\frac{\eta}{\eta-1}}}{\left(\int k_t(i)di\right)^{\alpha}\left(\int h_t(i)di\right)^{1-\alpha}} {k_t}^{\alpha} {h_t}^{1-\alpha} \tag{B.9}$$

$$= \frac{\left[\int A(i)^{\eta-1} R(i)^{-\alpha(\eta-1)} di\right]^{\frac{\eta}{\eta-1}}}{\left(\int (A(i))^{\eta-1} (R(i)^{-1})^{1+\alpha(\eta-1)} di\right)^{\alpha} \left(\int (A(i))^{\eta-1} (R(i)^{-1})^{\alpha(\eta-1)} di\right)^{1-\alpha}} {k_t}^{\alpha} {h_t}^{1-\alpha} \quad \text{(B.10)}$$

Assuming either log-normality or up to second order, we can write:

$$y = TFP k^{\alpha} h^{1-\alpha} \quad \text{(B.11)}$$

where[30]

$$TFP = \left(\int A(i)^{\eta-1}\right)^{\frac{1}{\eta-1}} \exp\left[-\frac{1}{2}\alpha(1+\alpha(\eta-1))\sigma_R^2\right] \quad \text{(B.12)}$$

taking logs concludes the first part of the proof.

First order conditions. Aggregate labor and capital demand at the firm level,

$$k = y\alpha^{1+\alpha(\eta-1)} \left(\frac{1-\alpha}{w}\right)^{(1-\alpha)(\eta-1)} \int A(i)^{\eta-1} R(i)^{-(1+\alpha(\eta-1))} \quad \text{(B.13)}$$

$$h = y\alpha^{\alpha(\eta-1)} \left(\frac{1-\alpha}{w}\right)^{\alpha+(1-\alpha)\eta} \int A(i)^{\eta-1} R(i)^{-\alpha(\eta-1)} \quad \text{(B.14)}$$

divide by each other,

$$\frac{k}{h} = \alpha \frac{w}{1-\alpha} \exp\left(-\mathbb{E}\left[\log R(i)\right] + \frac{1}{2}(1+2\alpha(\eta-1))\sigma_R^2\right) \quad \text{(B.15)}$$

plug back into (B.14),

$$1 = TFP^{\eta} \left(\frac{k}{h}\right)^{\alpha\eta} \left(\frac{1-\alpha}{w}\right)^{\eta}, \quad \text{(B.16)}$$

solving for w,

$$w = (1-\alpha) TFP \left(\frac{k}{h}\right)^{\alpha} \quad \text{(B.17)}$$

In the same way, substitute $\frac{1-\alpha}{w}$ in (B.13)

$$\exp\left(\mathbb{E}\left[\log R(i)\right]\right) = \alpha \mathbb{E}\left[A(i)^{\eta-1}\right]^{\frac{1}{\eta-1}} \left(\frac{h}{k}\right)^{(1-\alpha)} \exp\left(\frac{1}{2}(1-3\alpha+\alpha^2+2\alpha\eta-\alpha^2\eta)\sigma_R^2\right) \quad \text{(B.18)}$$

[30] Note that the covariance between $A(i)$ and $R(i)$ drops out due to properties of the log-normal distribution between (B.10)and (B.12)

Let $R \equiv \frac{\int R(i)k(i)}{\int k(i)}$,

$$R = \frac{\int A(i)^{\eta-1} y \alpha^{1+\alpha(\eta-1)} R(i)^{-\alpha(\eta-1)} \left(\frac{1-\alpha}{w}\right)^{(1-\alpha)(\eta-1)}}{\int A(i)^{\eta-1} y \left(\frac{\alpha}{R(i)}\right)^{1+\alpha(\eta-1)} \left(\frac{1-\alpha}{w}\right)^{(1-\alpha)(\eta-1)}} \tag{B.19}$$

$$R = \exp\left(\mathbb{E}\left[\log R(i)\right] - \frac{1}{2}\left(1 + 2\alpha(\eta-1)\right)\sigma_R^2\right) \tag{B.20}$$

$$R = \frac{\exp\left(\mathbb{E}\left[\log R(i)\right]\right)}{\exp\left(\frac{1}{2}\left(1 + 2\alpha(\eta-1)\right)\sigma_R^2\right)} \tag{B.21}$$

$$R = \frac{\alpha \mathbb{E}\left[A(i)^{\eta-1}\right]^{\frac{1}{\eta-1}} \left(\frac{h}{k}\right)^{(1-\alpha)} \exp\left(\frac{1}{2}(1 - 3\alpha + \alpha^2 + 2\alpha\eta - \alpha^2\eta)\sigma_R^2\right)}{\exp\left(\frac{1}{2}\left(1 + 2\alpha(\eta-1)\right)\sigma_R^2\right)} \tag{B.22}$$

$$R = \alpha \mathbb{E}\left[A(i)^{\eta-1}\right]^{\frac{1}{\eta-1}} \left(\frac{h}{k}\right)^{(1-\alpha)} \exp\left(\frac{1}{2}\alpha(1 + \alpha(\eta-1))\sigma_R^2\right) \tag{B.23}$$

$$R = \alpha TFP \left(\frac{h}{k}\right)^{(1-\alpha)} \tag{B.24}$$

□

B.2 Proof of Lemma 2

Proof. To simplify the proof, I will solve the problem of a benevolent planner that must allocate capital to firms. This is analogous to the structure described in the main body because banks are assumed to be competitive. I solve this problem by backward induction. First, firms choose labor given capital.

$$\max_{h(i)} \left(A(i)k(i)^{\alpha} h(i)^{1-\alpha}\right)^{\frac{\eta-1}{\eta}} y^{\frac{1}{\eta}} - wh(i) \tag{B.25}$$

with first order condition,

$$(1-\alpha)\frac{\eta-1}{\eta}\left(A(i)k(i)^{\alpha}\right)^{\frac{\eta-1}{\eta}} h(i)^{\frac{\alpha-\alpha\eta-1}{\eta}} = w \tag{B.26}$$

$$h(i) = \left(\frac{(1-\alpha)^{\frac{\eta-1}{\eta}}\left(A(i)k(i)^{\alpha}\right)^{\frac{\eta-1}{\eta}}}{w}\right)^{\frac{\eta}{1+\alpha(\eta-1)}} \tag{B.27}$$

Which means the value of the firm $V(A(i),k(i))$, taking capital and productivity as given, is

$$\begin{aligned}
V(A(i),k(i)) =& \left(A(i)k(i)^{\alpha}\right)^{\frac{\eta-1}{\eta}} y^{\frac{1}{\eta}} \left(\left((1-\alpha)\frac{\frac{\eta-1}{\eta}\left(A(i)k(i)^{\alpha}\right)^{\frac{\eta-1}{\eta}}}{w} \right)^{\frac{\eta}{1+\alpha(\eta-1)}} \right)^{(1-\alpha)\frac{\eta-1}{\eta}} \\
& - w \left(\frac{(1-\alpha)^{\frac{\eta-1}{\eta}}\left(A(i)k(i)^{\alpha}\right)^{\frac{\eta-1}{\eta}}}{w} \right)^{\frac{\eta}{1+\alpha(\eta-1)}} \\
=& \left(\left(A(i)k(i)^{\alpha}\right)^{\frac{\eta-1}{\eta}} y^{\frac{1}{\eta}}\right)^{\frac{\eta}{1+\alpha(\eta-1)}} w^{-\frac{(1-\alpha)(\eta-1)}{1+\alpha(\eta-1)}} \left((1-\alpha)\frac{\eta-1}{\eta}\right)^{\frac{(1-\alpha)(\eta-1)}{1+\alpha(\eta-1)}} \\
& - w^{-\frac{(1-\alpha)(\eta-1)}{1+\alpha(\eta-1)}} \left((1-\alpha)\frac{\eta-1}{\eta}\left(A(i)k(i)^{\alpha}\right)^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{1+\alpha(\eta-1)}} \\
=& \left(\left(A(i)k(i)^{\alpha}\right)^{\frac{\eta-1}{\eta}} y^{\frac{1}{\eta}}\right)^{\frac{\eta}{1+\alpha(\eta-1)}} w^{-\frac{(1-\alpha)(\eta-1)}{1+\alpha(\eta-1)}} \\
& \left(\left((1-\alpha)\frac{\eta-1}{\eta}\right)^{\frac{(1-\alpha)(\eta-1)}{1+\alpha(\eta-1)}} - \left((1-\alpha)\frac{\eta-1}{\eta}\right)^{\frac{\eta}{1+\alpha(\eta-1)}} \right) \\
=& \left(\left(A(i)k(i)^{\alpha}\right)^{\frac{\eta-1}{\eta}} y^{\frac{1}{\eta}}\right)^{\frac{\eta}{1+\alpha(\eta-1)}} \left(\frac{(1-\alpha)^{\frac{\eta-1}{\eta}}}{w}\right)^{\frac{(1-\alpha)(\eta-1)}{1+\alpha(\eta-1)}} \frac{1+\alpha(\eta-1)}{\eta} \\
V(A(i),k(i)) =& \, k(i)^{\frac{\alpha(\eta-1)}{1+\alpha(\eta-1)}} A(i)^{\frac{\eta-1}{1+\alpha(\eta-1)}} \Gamma,
\end{aligned}$$

where Γ collects all the terms that do not depend on $k(i)$ or $A(i)$.

Uncertainty reveal. After choosing k and before choosing labor, $F(i)$ is revealed, such that

$$k(i) = \hat{k}(i) - A^{\nu(\eta-1)} F(i) \hat{k}(i)^{1-\nu} \tag{B.28}$$

where $\hat{k}(i)$ is the capital chosen by the banks. Then, the value ex-ante V_e is given by

$$V_e(\hat{k}(i), A(i)) = \mathbb{E}\left[\left(\hat{k}(i) - A^{\nu(\eta-1)} F(i) \hat{k}(i)^{1-\nu} \right)^{\frac{\alpha(\eta-1)}{1+\alpha(\eta-1)}} A(i)^{\frac{\eta-1}{1+\alpha(\eta-1)}} \Gamma \right] \tag{B.29}$$

Capital choice. The allocation of capital is a solution to

$$\max_{k(i)} \int V_e(k(i), A(i)) \quad \text{s.t.} \quad \int k(i) = k \tag{B.30}$$

with first order condition:

$$\begin{aligned}&\hat{k}(i)\mathbb{E}\left[\left(1-\frac{A^{\nu(\eta-1)}F(i)}{\hat{k}(i)^{\nu}}\right)^{\frac{1}{1+\alpha(\eta-1)}}\left(1-(1-\nu)\frac{A^{\nu(\eta-1)}F(i)}{\hat{k}(i)^{\nu}}\right)\right]^{-(1+\alpha(\eta-1))}\\&=A(i)^{\eta-1}\left(\frac{1+\alpha(\eta-1)}{\alpha(\eta-1)}\Gamma\lambda\right)^{-(1+\alpha(\eta-1))}\end{aligned}\tag{B.31}$$

Dividing by the same condition for firm j,

$$\begin{aligned}&\frac{\hat{k}(i)}{\hat{k}(j)}\left(\frac{\mathbb{E}\left[\left(1-\frac{A(i)^{\nu(\eta-1)}F(i)}{\hat{k}(i)^{\nu}}\right)^{\frac{1}{1+\alpha(\eta-1)}}\left(1-(1-\nu)\frac{A(i)^{\nu(\eta-1)}F(i)}{\hat{k}(i)^{\nu}}\right)\right]^{-(1+\alpha(\eta-1))}}{\mathbb{E}\left[\left(1-\frac{A(j)^{\nu(\eta-1)}F(j)}{\hat{k}(j)^{\nu}}\right)^{\frac{1}{1+\alpha(\eta-1)}}\left(1-(1-\nu)\frac{A(j)^{\nu(\eta-1)}F(j)}{\hat{k}(j)^{\nu}}\right)\right]^{-(1+\alpha(\eta-1))}}\right)^{-(1+\alpha(\eta-1))}\\&=\left(\frac{A(i)}{A(j)}\right)^{\eta-1}\end{aligned}\tag{B.32}$$

I guess and later verify that $k(i)=\bar{A}A(i)^{\eta-1}k$, where $\bar{A}$ is a constant.

$$\begin{aligned}&\left(\frac{A(i)}{A(j)}\right)^{\eta-1}\left(\frac{\mathbb{E}\left[\left(1-\frac{F(i)}{(\bar{A}k)^{\nu}}\right)^{\frac{1}{1+\alpha(\eta-1)}}\left(1-(1-\nu)\frac{F(i)}{(\bar{A}k)^{\nu}}\right)\right]^{-(1+\alpha(\eta-1))}}{\mathbb{E}\left[\left(1-\frac{F(j)}{(\bar{A}k)^{\nu}}\right)^{\frac{1}{1+\alpha(\eta-1)}}\left(1-(1-\nu)\frac{F(j)}{(\bar{A}k)^{\nu}}\right)\right]^{-(1+\alpha(\eta-1))}}\right)^{-(1+\alpha(\eta-1))}\\&=\left(\frac{A(i)}{A(j)}\right)^{\eta-1}\end{aligned}\tag{B.33}$$

$$1=1\tag{B.34}$$

To find $\bar{A}$, plug into the resource constraint,

$$\int k(i)=k\tag{B.35}$$

$$\int \bar{A}A(i)^{\eta-1}k=k\tag{B.36}$$

$$\bar{A}k\int A(i)^{\eta-1}=k\tag{B.37}$$

$$\bar{A}=\mathbb{E}\left[A(i)^{\eta-1}\right]^{-1}\tag{B.38}$$

□

B.3 Proof of Lemma 3

Proof. Start from capital demand (B.6) and solve for $R(i)$ as a function of $k(i)$,

$$R(i) = \alpha \left(k(i)^{-1} A(i)^{\eta-1} y \left(\frac{1-\alpha}{w} \right)^{(1-\alpha)(\eta-1)} \right)^{\frac{1}{1+\alpha(\eta-1)}} \tag{B.39}$$

Combining with Lemma 2,

$$R(i) = \alpha \left(k^{-1} \left(1 + \frac{F(i)}{k^{\nu}} \right)^{-1} y \left(\frac{1-\alpha}{w} \right)^{(1-\alpha)(\eta-1)} \right)^{\frac{1}{1+\alpha(\eta-1)}}, \tag{B.40}$$

taking logs,

$$\log R(i) = \log \alpha + \frac{1}{1+\alpha(\eta-1)} \left(-\log k - \log \left(1 + \frac{F(i)}{k^{\nu}} \right) + \log y + \log \left(\frac{1-\alpha}{w} \right)^{(1-\alpha)(\eta-1)} \right), \tag{B.41}$$

Lastly, using the assumption of small $F(i)$,

$$\log R(i) = \log \alpha + \frac{1}{1+\alpha(\eta-1)} \left(-\log k - \frac{F(i)}{k^{\nu}} + \log y + \log \left(\frac{1-\alpha}{w} \right)^{(1-\alpha)(\eta-1)} \right). \tag{B.42}$$

It follows that

$$\mathbb{V}ar \left[\log R(i) \right] = \left(\frac{1}{1+\alpha(\eta-1)} \frac{1}{k^{\nu}} \right)^2 \sigma_F^2 \tag{B.43}$$

□

B.4 Proof of Proposition 1

Proof. I need to show that conditions (4),(30),(31),(33) and (34) hold. The constraints hold trivially. Set τ^d so that (4) holds,

$$(1+\tau_t^d)\mathbb{E} \left[\frac{c_{T,t+1}^{-\sigma}}{p_{t+1}} \right] = \mathbb{E}_t \left[\frac{c_{t+1}^{-\sigma}}{p_{t+1}} \left(1 + \kappa \mu_{t+1} \times \frac{1}{\xi} \frac{p_{t+1}^N y^N}{x_{t+1}^T} + \right) \right] \tag{B.44}$$

$$\tau_t^d \mathbb{E} \left[\frac{c_{T,t+1}^{-\sigma}}{p_{t+1}} \right] = \mathbb{E}_t \left[\frac{c_{t+1}^{-\sigma}}{p_{t+1}} \kappa \mu_{t+1} \times \frac{1}{\xi} \frac{p_{t+1}^N y^N}{x_{t+1}^T} \right] \tag{B.45}$$

$$\tau^d = \mathbb{E} \left[\frac{c_{T,t+1}^{-\sigma}}{p_{t+1}} \right]^{-1} \mathbb{E}_t \left[\frac{c_{t+1}^{-\sigma}}{p_{t+1}} \kappa \mu_{t+1} \times \frac{1}{\xi} \frac{p_{t+1}^N y^N}{x_{t+1}^T} \right] \tag{B.46}$$

and τ^k so that (34) holds,

$$\mathbb{E}\left[c_{t+1}^{-\sigma}\left(\frac{\alpha \text{TFP}_{t+1}k_{t+1}^{\alpha-1}}{p_{t+1}}(1+\kappa\mu_{t+1})+\frac{\tau_{t+1}^k}{p_{t+1}}\right)\right]$$
$$=\mathbb{E}\left[c_{t+1}^{-\sigma}\left((\alpha+\gamma_{t+1})\frac{\text{TFP}_{t+1}k_{t+1}^{\alpha-1}}{p_{t+1}}\left(1+\kappa\mu_{t+1}\left(1+\frac{1}{\xi}\frac{p_{t+1}^N y^N}{x_{t+1}^T}\right)\right)+\right.\right. \tag{B.47}$$
$$\left.\left.(1-\delta)(1+\kappa\mu_{t+1}\frac{1}{\xi}\frac{p_{t+1}^N y^N}{x_{t+1}^T})\right)\right]$$

$$\tau_{t+1}^k\mathbb{E}\left[\frac{c_{t+1}^{-\sigma}}{p_{t+1}}\right]$$
$$=\mathbb{E}\left[c_{t+1}^{-\sigma}\left(\frac{\text{TFP}_{t+1}k_{t+1}^{\alpha-1}}{p_{t+1}}\left(\gamma_{t+1}(1+\kappa\mu_{t+1})+(\alpha+\gamma_{t+1})\kappa\mu_{t+1}\frac{1}{\xi}\frac{p_{t+1}^N y^N}{x_{t+1}^T}\right)+\right.\right. \tag{B.48}$$
$$\left.\left.(1-\delta)\frac{1}{\xi}\frac{p_{t+1}^N y^N}{x_{t+1}^T}\right)\right]$$

$$\tau^k=$$
$$\mathbb{E}\left[\frac{c_{t+1}^{-\sigma}}{p_{t+1}}\right]^{-1}\mathbb{E}\left[c_{t+1}^{-\sigma}\left(\left(\alpha+\gamma_{t+1}\frac{\text{TFP}_{t+1}k_{t+1}^{\alpha-1}}{p_{t+1}}+1-\delta\right)\frac{1}{\xi}\frac{p_{t+1}^N y^N}{x_{t+1}^T}+\right.\right. \tag{B.49}$$
$$\left.\left.\kappa\mu_{t+1}\gamma_{t+1}\frac{\text{TFP}_{t+1}k_{t+1}^{\alpha-1}}{p_{t+1}}\right)\right]$$

□

B.5 Proof of Proposition 2

Proof. Marginal propensity to invest. The SB planner takes the household's capital Euler (34), with $\tau^k = 0$, as an implementability constraint. An additional unit of d_{t+1} relaxes the resource constraint (30) by 1 unit. The household distributes this between investment and consumption according to (34), holding next-period policy rules fixed. Applying the implicit function theorem to (34) gives the equilibrium investment response $\text{mpi}_t = \partial k_{t+1}/\partial d_{t+1}$, with $\partial c_t^T/\partial d_{t+1} = 1 - \text{mpi}_t$ from the resource constraint. Substituting these responses into the planner's first-order condition for d_{t+1} yields (41).

Reduction to closed form. Under $\gamma_s = 0$ for all $s \geq t+2$, the term Ω_{t+2} in (41) vanishes. I find the tax $\tau_t^{d,sb}$ such that the household's debt Euler (4) coincides with (41). Setting the right-hand sides equal and using $\text{TFP}_{t+1}k_{t+1}^{\alpha-1} = A_{t+1}k_{t+1}^{\alpha+\gamma_{t+1}-1}$:

$$(1+\tau_t^{d,sb})\mathbb{E}_t\left[\frac{c_{t+1}^{-\sigma}}{p_{t+1}}\right]=\mathbb{E}_t\left[\frac{c_{t+1}^{-\sigma}}{p_{t+1}}\left(1+\kappa\mu_{t+1}\frac{1}{\xi}\frac{p_{t+1}^N y^N}{x_{t+1}^T}\right.\right.$$

$$-\frac{\text{mpi}_t \, \text{TFP}_{t+1} k_{t+1}^{\alpha-1}}{1+r}\left[\gamma_{t+1}(1+\kappa\mu_{t+1})+(\alpha+\gamma_{t+1})\kappa\mu_{t+1}\frac{1}{\xi}\frac{p_{t+1}^N y^N}{x_{t+1}^T}\right]\Bigg)\Bigg].$$

Applying definitions (38) and (39), the right-hand side becomes $\left(1+\tau_t^d-\frac{\text{mpi}_t}{1+r}\,\tau_{t+1}^k\right)\mathbb{E}_t\left[\frac{c_{t+1}^{-\sigma}}{p_{t+1}}\right]$. Dividing both sides by $\mathbb{E}_t\left[\frac{c_{t+1}^{-\sigma}}{p_{t+1}}\right]$ yields (42).

Alternative tractable derivation. Equivalently, consider a formulation in which households receive lump-sum transfers T_t from the government rather than borrowing directly. The planner chooses T_t (equivalently d_{t+1}), taking household policy functions for c_t^T and k_{t+1} as given. The marginal propensity to invest $\text{mpi}_t = \partial k_{t+1}/\partial T_t$ arises as a household policy response, and the planner's first-order condition for T_t is equivalent to (41). Under $\gamma_s = 0$ for $s \geq t+2$, this delivers (42) directly.

□

C Capital Misallocation Block

In this section I explore some alternative microfoundations and show how to generalize the model to accommodate additional sources of misallocation.

C.1 Possible Microfoundations

I know explore three possible microfoundations that span three commonly mentioned sources of misallocation: Credit shocks, borrowing constraints, and equity constraints.

C.1.1 Credit shocks

In this model, the frictions $F(i)$ represent shocks to the credit supply of firms, as in the models of Chodorow-Reich (2013), Herreño (2023), and Pinardon-Touati (2024). These shocks can be overhead costs of the bank, monitoring or operating costs, additional sources of demand that reduce available credit to firms such as government credit demand or changes in the balance sheet of banks.

No matter the source, I assume that these shocks $S(i)$ take the form

$$S(i) = A^{\eta-1}F(i)k^{\gamma} \tag{C.1}$$

Where $F(i)$ is a random variable that represents different realizations in these shocks across firms. For instance, a firm that is more geographically remote or more opaque will imply higher costs for the bank. Likewise, different firms might be served by banks with different balance sheets, or they might be located in regions with different government spending. What is important is that the magnitude of these shocks do not scale perfectly with the total capital available to the bank. This is achieved by $0 < \gamma < 1$.

Let $\nu = 1 - \gamma$, then, as a result, the amount banks can lend is given by

$$k(i) = \frac{A(i)^{\eta-1}}{\mathbb{E}\left[A(i)\right]^{\eta-1}} k \left(1 - \frac{F(i)}{\mathbb{E}\left[A(i)^{\eta-1}\right]^{\nu} k^{\nu}}\right) \tag{C.2}$$

C.1.2 Borrowing Constraints

In this model, I assume that the household in charge of the bank can divert a fraction of the credit. If they do that, the other households can seize a fraction of their capital k. I follow the same timing as Bianchi and Mendoza (2018) and assume that this diversion happens before any other trading occurs. Once the capital is seized, I assume that the household buys it again.

After diverting the credit, the household can hide a fraction $A(i)^{\nu(\eta-1)}F(i)k(i)^{-\nu}$ of its assets, where $F(i)$ is the idiosyncratic component. While in this case it is a characteristic of the household, it can also be interpreted as describing the firm level of opaqueness or perceived riskiness.

It follows that, to avoid any diversion in equilibrium, the following incentive compatibility must hold,

$$k(i) \leq \frac{A(i)^{\eta-1}}{\mathbb{E}\left[A(i)\right]^{\eta-1}} k \left(1 - \frac{F(i)}{\mathbb{E}\left[A(i)^{\eta-1}\right]^{\nu} k^{\nu}}\right) \tag{C.3}$$

In equilibrium, (C.3) holds with equality.

C.1.3 Internal and External Finance

In this setup, firms assemble capital by combining bank credit and equity $E(i)$ in the following way,

$$k(i) = \left(\theta k_b(i)^{\frac{\rho-1}{\rho}} + (1-\theta)(A(i)^{\eta-1}E(i))^{\frac{\rho-1}{\rho}}\right)^{\frac{\rho}{\rho-1}} \tag{C.4}$$

The CES specification is a reduced form way of modelling the imperfect substitutability between both sources of financing, given by ρ in this case.

For simplicity, I assume that Equity is fixed, although it would be possible to have it be a choice of the firm. Then, total capital can be written as

$$k(i) = \theta k_b(i) \left(1 + \frac{1-\theta}{\theta}\left(\frac{A(i)^{\eta-1}E(i)}{k_b(i)}\right)^{\frac{\rho-1}{\rho}}\right)^{\frac{\rho}{\rho-1}} \tag{C.5}$$

Using Lemma 2,

$$k(i) = \theta \frac{A(i)^{\eta-1}}{\mathbb{E}\left[A(i)^{\eta-1}\right]} k \left(1 + \frac{1-\theta}{\theta}\left(\frac{E(i)}{\frac{k}{\mathbb{E}[A(i)^{\eta-1}]}}\right)^{\frac{\rho-1}{\rho}}\right)^{\frac{\rho}{\rho-1}} \tag{C.6}$$

$$\tag{C.7}$$

Let $F(i) \equiv \left(\frac{1-\theta}{\theta}\mathbb{E}\left[A(i)^{\eta-1}\right]E(i)\right)^{\frac{\rho-1}{\rho}}$ and $\nu \equiv \frac{\rho-1}{\rho}$ to write

$$k(i) = \theta \frac{A(i)^{\eta-1}}{\mathbb{E}\left[A(i)^{\eta-1}\right]} k \left(1 + \frac{F(i)}{k^{\nu}}\right)^{\frac{1}{\nu}}, \tag{C.8}$$

which yields the desired result.

C.2 Including Additional Sources of Misallocation

In this section, I show that the model can accommodate other sources of misallocation that are not related to investment. To keep things simple, I consider only one additional distortion: Firms face a labor wedge τ^w.

Taking the stock of capital $k(i)$ as given, the problem of an individual firm is given by

$$\max_{h(i)} y^{\frac{1}{\eta}} \left(A(i)k(i)^{\alpha}h(i)^{1-\alpha}\right)^{\frac{\eta-1}{\eta}} - (1+\tau^w(i))wh(i), \tag{C.9}$$

All other elements of the model remain the same, including the timing: Banks select the supply to the market populated by firm i before knowing the shocks $F(i)$ and the realization of the labor wedge $\tau^w(i)$.

As a result, and following a logic similar to Lemma 2, the firm's cost of capital is described by

$$R(i) \propto \left(\left(1-\frac{F(i)}{k^{\nu}}\right)\left((1+\tau^w(i))w\right)^{(1-\alpha)(\eta-1)}\right)^{\frac{1}{1+\alpha(\eta-1)}} \tag{C.10}$$

Let $\sigma^2_{\tau^w}$ be the variance of $\tau^w(i)$ and σ_{F,τ^w} its covariance with $F(i)$. Then, the variance of $\log R(i)$ is given by

$$\sigma^2_R = \frac{1}{(1+\alpha(\eta-1))^2}\left(\frac{\sigma^2_F}{k^{2\nu}} + ((1-\alpha)(\eta-1))^2\sigma^2_{\tau^w} + 2\frac{(1-\alpha)(\eta-1)}{k^{\nu}}\sigma_{F,\tau^w}\right) \tag{C.11}$$

It follows from (C.11) that the elasticity of TFP with respect to capital is now given by

$$\gamma = \nu\frac{\alpha}{1+\alpha(\eta-1)}\left(\frac{\sigma^2_F}{k^{2\nu}} + \frac{(1-\alpha)(\eta-1)}{k^{\nu}}\sigma_{F,\tau^w}\right)\frac{1}{k} \tag{C.12}$$

Note that now there's not a direct link to an empirical moment. Identification, however, is made possible by considering two additional moments: Dispersion in the marginal revenue productivity of labor (mrpl) and the capital to labor ratio, with the former defined as

$$\text{mrpl} \propto \frac{\left(A(i)k(i)^{\alpha}h(i)^{1-\alpha}\right)^{\frac{\eta-1}{\eta}}}{h(i)} \tag{C.13}$$

Solving for optimal labor in (C.9), it is easy to see that

$$\log \text{mrpl} \propto \log 1 + \tau^w(i) \tag{C.14}$$

And so it follows that the variance of the log mrpl identifies $\sigma^2_{\tau^w}$,

$$\mathbb{V}ar\left[\log \text{mrpl}\right] = \sigma^2_{\tau^w} \tag{C.15}$$

Similarly, the variance of the log capital to labor ratio is given by

$$\mathbb{V}ar\left[\log \frac{k(i)}{h(i)}\right] = \frac{1}{(1+\alpha(\eta-1))^2}\left(\eta^2\sigma^2_{\tau^w} + \frac{F(i)}{k^{2\nu}} - 2\frac{\eta}{k^\nu}\sigma_{F,\tau^w}\right) \tag{C.16}$$

Then, after using (C.15), (C.16) and (C.11) define a system of two equations and two unknowns. Solving for $\frac{\sigma_{F,\tau^w}}{k^\nu}$ yields

$$\frac{\sigma_{F,\tau^w}}{k^\nu} = \frac{(1+\alpha(\eta-1))^2(\mathbb{V}ar\left[mrpk\right] - \mathbb{V}ar\left[\log\frac{k(i)}{h(i)}\right]) + \sigma^2_{\tau^w}\left(\eta^2 - ((1-\alpha)(\eta-1))^2\right)}{2(\eta+(1-\alpha)(\eta-1))} \tag{C.17}$$

and $\frac{\sigma^2_F}{k^{2\nu}}$ is given by

$$\frac{\sigma^2_F}{k^{2\nu}} = (1+\alpha(\eta-1))^2\mathbb{V}ar\left[\log\frac{k(i)}{h(i)}\right] - \eta^2\sigma^2_{\tau^2} + 2\frac{\eta}{k^\nu}\sigma_{F,\tau^w} \tag{C.18}$$

Together, they pin down γ in (C.12).

D Data Appendix

In this section I provide more details into the data work on Section 6.

D.1 Country Panel Data

Table D.1 shows summary stats for the sample of countries used to calculate the time series moments. The sample has 19 middle-income and 17 high-income economies.

D.2 Orbis

I access the Orbis dataset through the Orbis Project at the CBS grid, where it was made available through the efforts of Kochen (2022).

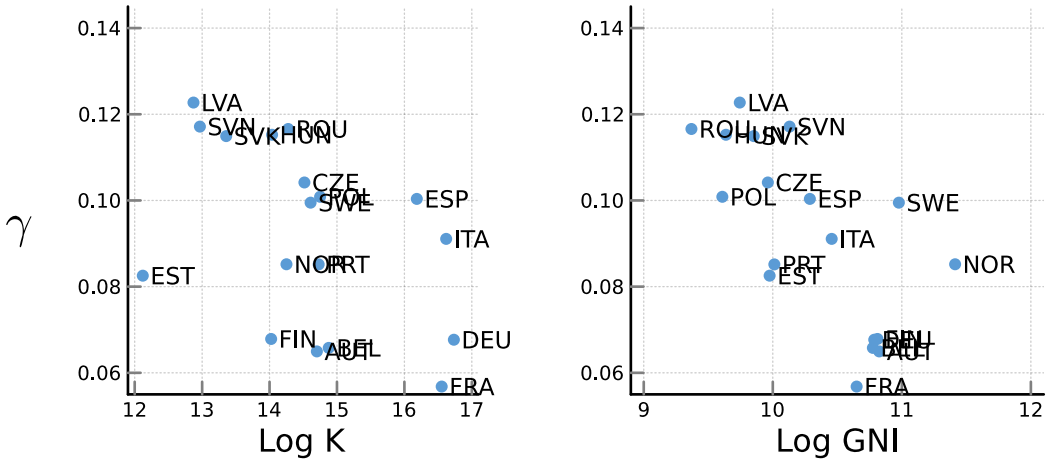
Table D.2 presents summary statistics for the 18 countries selected in the sample, which follow the sample from Kalemli-Özcan et al. (2024). Table D.3 shows the distribution of output and employment across the firm-size distribution. The results are similar to those reported by Kalemli-Özcan et al. (2024)[31], who compare it to aggregate statistics from Eurostat and find that Orbis provided a representative sample.

D.3 The Productivity Externality and Development

Figure D.1 shows the relationship between γ_{t+1} and the stock of capital and Gross National Income (GNI). As predicted by the formulation in Lemma 3, γ_{t+1} is decreasing in the stock of capital. Moreover, it is also decreasing in GNI, suggesting that it is inversely related to the degree of development of an economy.

[31] Table D.2.5

Figure D.1: Correlation of γ_{t+1} with GNI and the capital stock



Notes: The Figure plots the relationship between the estimates of γ_{t+1}, and the log of the capital stock (K) and Gross National Income (GNI), respectively, at the country-year level. The capital stock is obtained from Feenstra, Inklaar, and Timmer (2015) and is measured in PPP US dollars. GNI is sourced from World Bank (2021) and is measured in current US dollars using the Atlas method.

Table D.1: Summary Statistics for Panel of Countries

Country	First Year	Last Year	σ_Y	σ_C/σ_Y	$\sigma_{CA/Y}$	Sudden Stops
Middle-Income						
Argentina	1979	2012	9.75	1.19	3.05	3
Brazil	1979	2012	9.73	1.38	2.01	1
Chile	1979	2012	11.43	1.18	3.37	2
Colombia	1979	2012	5.74	1.48	2.84	1
Czech Republic	1979	2012	5.64	0.95	1.78	1
Dominican Republic	1979	2012	7.40	0.86	3.11	1
Ecuador	1979	2012	7.58	1.37	3.00	2
Egypt	1979	2012	3.98	0.75	3.35	0
El Salvador	1979	2012	8.17	1.64	2.45	1
Hungary	1979	2012	13.95	0.57	3.35	0
Indonesia	1979	2012	7.01	0.81	2.38	1
Malaysia	1979	2012	5.63	1.53	7.59	1
Mexico	1979	2012	6.44	1.14	2.32	2
Morocco	1979	2012	4.19	1.12	2.85	4
Pakistan	1979	2012	4.24	0.85	2.49	1
Peru	1979	2012	13.75	0.84	2.98	2
Philippines	1979	2012	10.13	0.48	2.74	1
Portugal	1979	2012	4.84	1.49	4.86	1
South Africa	1979	2012	9.56	0.91	2.67	2
South Korea	1979	2012	7.18	1.38	3.49	1
Tunisia	1979	2012	6.23	2.10	2.38	1
Ukraine	1979	2012	25.39	1.13	4.88	0
Venezuela	1979	2012	8.96	1.83	6.64	1
High-Income						
Canada	1979	2012	3.99	0.91	2.10	1
Denmark	1979	2012	3.04	0.97	1.54	1
Finland	1979	2012	6.45	0.82	3.44	0
France	1979	2012	2.44	1.09	1.05	1
Germany	1979	2012	2.42	1.00	2.26	0
Iceland	1979	2012	7.32	1.18	5.99	2
Italy	1979	2012	2.58	1.01	1.72	1
Netherlands	1979	2012	4.34	1.39	1.60	1
Norway	1979	2012	3.60	1.36	4.04	2
Spain	1979	2012	5.42	1.22	2.84	0
Sweden	1979	2012	4.48	1.02	2.34	1
Switzerland	1979	2012	2.54	0.58	2.44	0
United Kingdom	1979	2012	5.07	1.17	1.35	1
United States	1979	2012	3.01	1.10	1.30	2

Notes: This table presents summary statistics for the sample of countries used to estimate time-series moments in Section 6. σ_Y, σ_C and $\sigma_{CA/Y}$ are the standard deviations of log GDP per capita, log consumption per capita and the current account over GDP. All three variables are quadratically detrended. Sudden stops refer to the number of Sudden Stops as identified by Korinek and Mendoza (2014).

Table D.2: Summary Statistics for Countries in Orbis Sample

Country	N. Firms	N. Obs	Total Employees
Austria	9,283	88,004	72,401
Belgium	23,074	255,545	424,162
Czech Republic	42,064	314,829	823,799
Estonia	7,278	61,552	60,523
Finland	13,570	122,259	199,776
France	133,919	1,312,707	1,449,405
Germany	69,432	622,434	1,711,145
Hungary	144,485	1,084,401	483,002
Italy	204,584	1,659,439	2,042,621
Latvia	9,467	71,653	92,640
Norway	16,938	116,000	45,704
Poland	29,400	199,855	776,776
Portugal	55,038	462,410	352,406
Romania	62,621	539,698	807,445
Slovakia	22,344	136,706	190,646
Slovenia	20,231	138,929	116,640
Spain	158,426	1,634,164	1,520,580
Sweden	28,456	322,773	389,409

Notes: The first column shows the number of unique firms per country in the data. The second column and third column show the average number across years of observations and total employees respectively.

Table D.3: Size Distribution in the Manufacturing Sector: Total Sample, 2006

Panel A: Gross Output	AT	BE	CZ	DE	EE	ES	FI	FR	HU	IT	LV	NO	PL	PT	RO	SE	SI	SK
1 to 19 employees	0.08	0.05	0.08	0.02	0.12	0.21	0.07	0.09	0.07	0.14	0.12	0.19	0.02	0.20	0.15	0.14	0.11	0.15
20 to 249 employees	0.28	0.43	0.45	0.27	0.65	0.60	0.42	0.46	0.51	0.61	0.63	0.56	0.35	0.63	0.50	0.45	0.44	0.45
250+ employees	0.64	0.52	0.47	0.72	0.23	0.20	0.51	0.45	0.42	0.25	0.25	0.25	0.63	0.17	0.35	0.41	0.45	0.39
Panel B: Employment																		
1 to 19 employees	0.16	0.11	0.05	0.01	0.12	0.23	0.09	0.09	0.03	0.11	0.13	0.23	0.01	0.25	0.11	0.16	0.09	0.08
20 to 249 employees	0.38	0.40	0.39	0.30	0.56	0.47	0.41	0.33	0.26	0.50	0.54	0.48	0.34	0.53	0.35	0.33	0.39	0.38
250+ employees	0.45	0.50	0.55	0.69	0.32	0.30	0.49	0.59	0.72	0.38	0.33	0.29	0.65	0.22	0.54	0.51	0.52	0.53

Notes: This table presents the firm size distribution of output and employment for the countries in the sample. I use total operating revenue as the definition of revenue at the firm level. The year 2006 is chosen for comparability to Kalemli-Özcan et al. (2024). The countries are Austria (AT), Belgium (BE), Czech Republic (CZ), Germany (DE), Estonia (EE), Spain (ES), Finland (FI), France (FR), Hungary (HU), Italy (IT), Latvia (LV), Norway (NO), Poland (PO), Portugal (PT), Romania (RO), Sweden (SE), Slovenia (SI) and (Slovakia).

E Long-Run Moments

Table E.1 reports unconditional means of key aggregates across the three allocations studied in the paper: the decentralized equilibrium (D.E.), the constrained-efficient planner (C.E.), and the second-best allocation that restricts the planner to a constant debt tax (Optimal Constant Tax). All ratios are computed as period-by-period ratios averaged over a long simulation of $T = 1{,}000{,}000$ periods, discarding the first 10% as burn-in.

The D.E. features higher average debt (D/Y) than the C.E., consistent with the overborrowing result. The constrained-efficient planner reduces borrowing and reallocates resources toward investment, as seen in the higher I/Y ratio. The optimal constant-tax economy lies between the two: the subsidy to borrowing ($\tau^d < 0$) keeps debt elevated relative to the C.E. but raises investment relative to the D.E. through the productivity channel.

Table E.1: Long-Run Moments

	D.E.	C.E.	Second Best	Optimal Constant Tax
Means				
C/Y	0.915	0.896	0.916	0.914
I/Y	0.072	0.091	0.072	0.072
D/Y	0.313	0.309	0.313	0.314
TB/Y	0.013	0.012	0.013	0.013
Volatility (CV)				
$\sigma(Y^T)$	2.994	2.769	3.015	3.074
$\sigma(C)/\sigma(Y)$	1.067	1.033	1.060	1.114
$\sigma(I)/\sigma(Y)$	2.281	1.553	2.274	2.659
$\sigma(TB/Y)$	0.010	0.005	0.009	0.015
Correlations with GDP				
$\rho(Y, C)$	0.994	0.997	0.994	0.992
$\rho(Y, I)$	0.925	0.961	0.930	0.931
$\rho(Y, TB/Y)$	-0.628	-0.560	-0.623	-0.729

Notes: Ergodic means from a simulation of $T = 1{,}000{,}000$ periods. C/Y: consumption-to-GDP ratio (nominal). I/Y: investment-to-GDP ratio. D/Y: debt-to-GDP ratio (beginning-of-period debt). TB/Y: trade-balance-to-GDP ratio. D.E.: decentralized equilibrium. C.E.: constrained-efficient planner. Optimal Constant Tax: private equilibrium under the welfare-maximizing constant debt tax.

F Robustness

F.1 TFP Elasticity with respect to Investment

The elasticity of TFP with respect to investment, γ, is a key object for the results of the paper, as it drives the costs of macroprudential policy in the quantitative exercise. In this section, I explore a few different ways of assigning a value to this parameter.

Recall from (27) that γ is given by

$$\alpha(1+\alpha(\eta-1))\nu\sigma_R^2$$

I fix the values of α and η following the literature and estimate σ_R^2 in the data, while I set $\nu = 0.1$ by targeting $\gamma = 0.1$ for Spain. In this section I perform robustness checks for the latter two.

F.1.1 Frictions Elasticity ν

Table F.1 presents estimates for ν using empirical findings from 4 different papers. My baseline estimate, $\nu = 0.1$, is significantly smaller than any of the estimates that find a positive relationship between investment and productivity growth. When using the results

Table F.1: Summary of ν Estimates

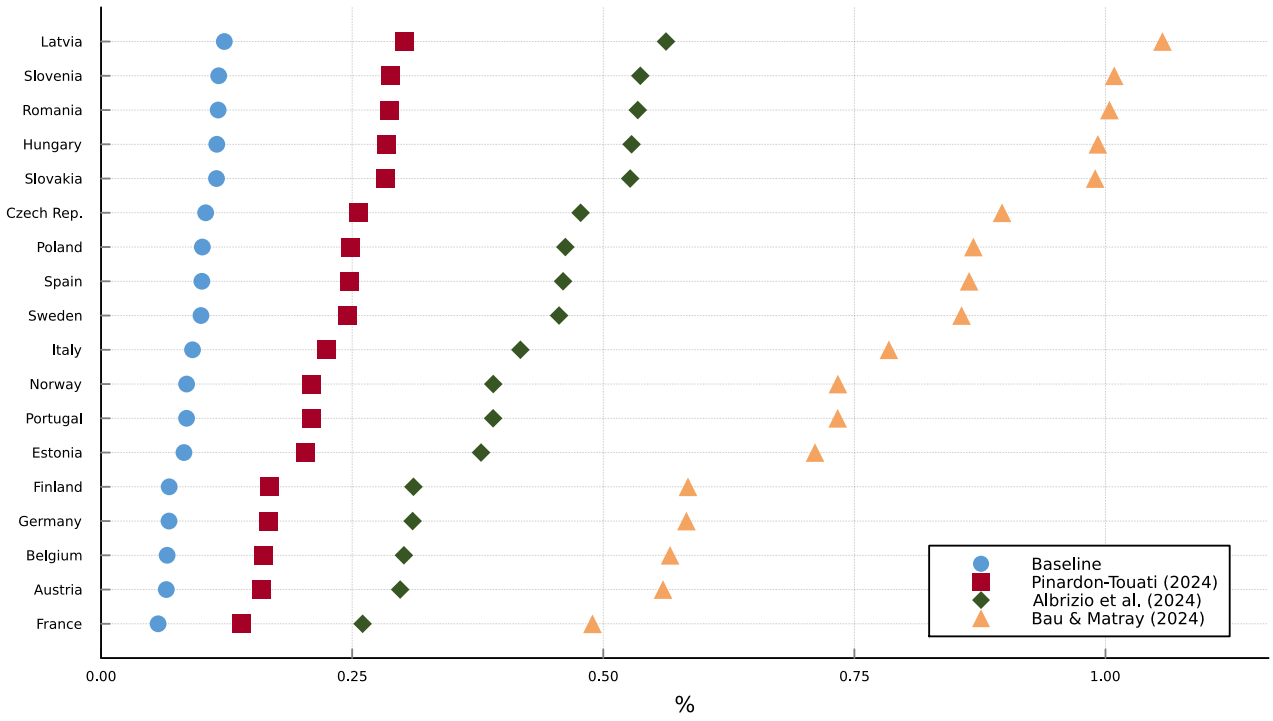
Estimate	Source
0.22	Pinardon-Touati (2024)
0.33	Albrizio, Beatriz, and Khametsin (2024)
0.62	Bau and Matray (2023)
-0.11	Gopinath et al. (2017)

Notes: This table summarizes the different estimates of the parameter ν considered in the analysis. See Section F.1.1 for details on their calculation.

from Gopinath et al. (2017), the estimates flip signs. It is important to keep in mind, that their work captures misallocation dynamics during a credit boom while this paper focuses on the effects of policy changes.

Figure F.1 plots the comparison between my baseline γ estimates (blue) to those obtained using the estimates of Albrizio, Beatriz, and Khametsin (2024) (red) and Bau and Matray (2023) (green).

Figure F.1: γ estimates for different ν.



Notes: The figure plots estimates for γ using the estimates from ν in Table F.1. Section 6 explains how to obtain estimates for γ, while Section F.1.1 details the source of the alternative estimates.

Albrizio, Beatriz, and Khametsin (2024) Using Spanish data with similar coverage to Orbis[32], Albrizio, Beatriz, and Khametsin (2024) study the causal effects of monetary policy shocks on misallocation. In their baseline results, a one standard deviation monetary shock leads to an increase in the capital stock of the average firm of around 1.2%. At the same, they find a reduction in the variance of log MRPK (σ_R^2) of 0.8%. Using Lemma 3 and taking logs, the result can be written as

$$\Delta \log \sigma_R^2 = -2 \times \nu \times \Delta \log k \tag{F.1}$$

Using their results, it follows that $\nu = 0.33$.

Bau and Matray (2023) This article exploits a set of reforms in India that made it easier for foreign capital to access firms. To identify their effects, they leverage the staggered liberalization across industries and also the difference between firms with high and low marginal

[32] Central de Balances Integrada and Directorio Central de Estadistica

revenue productivity of capital (mrpk).

As shown in B.3, it is possible to write the *mrpk* of firm i as

$$\log \text{mrpk}(i) \propto -\frac{1}{1+\alpha(\eta-1)}\left(\log k - \frac{F(i)}{k^{\nu}}\right) \tag{F.2}$$

To obtain the relative change between firms, take the difference for firms i and j,

$$\log \text{mrpk}(i) - \log \text{mrpk}(j) = \frac{1}{1+\alpha(\eta-1)}\frac{F(i)-F(j)}{k^{\nu}} \tag{F.3}$$

and take the derivative with respect to $\log k$,

$$\frac{\partial \log \text{mrpk}(i)}{\partial \log k} - \frac{\partial \log \text{mrpk}(j)}{\partial \log k} = \frac{\nu}{1+\alpha(\eta-1)}\left(\frac{F(j)-F(i)}{k^{\nu}}\right) \tag{F.4}$$

$$\frac{\partial \log \text{mrpk}(i)}{\partial \log k} - \frac{\partial \log \text{mrpk}(j)}{\partial \log k} = -\nu\left(\log \text{mrpk}(i) - \log \text{mrpk}(j)\right) \tag{F.5}$$

which links the relative changes in mrpk to changes in total capital. Then,

$$\Delta \log \text{mrpk}(i) - \Delta \log \text{mrpk}(j) = -\nu\left(\log \text{mrpk}(i) - \log \text{mrpk}(j)\right) \times \Delta \log k \tag{F.6}$$

Bau and Matray (2023) report[33] a 32% increase in capital and that firms with high mrpk reduced their mrpk by 32% more than low mrpk firms. They also mention that the former originally had a mrpk 160% higher than the latter. It follows that $\nu = 0.62$. Using $\nu = 0.22$, equation (F.6) predicts an 11% decrease in the mrpk of high mrpk firms relative to low mrpk firms[34] Of note is also the fact that they identify the effect of foreign capital entering into the market, which potentially improves the ability of the financial system. In the model, this would translate into an increase in ν, which here I take as a primitive.

Pinardon-Touati (2024) Studying the effect of government spending on private borrowing in France, she finds that a 0.28% decrease in capital leads on average to a 0.04% decrease in TFP, which corresponds to $\gamma = 0.14$. Given a $\sigma_R^2 = 1.32$ for France, that yields $\nu = 0.22$.

Gopinath et al. (2017) In their seminal paper, Gopinath et al. (2017) documented an increase in mrpk dispersion occurring at the same time as declines in real interest rate declines in several Southern European economies.

[33] See Table 4 (Bau and Matray, 2023).

[34] This value is slightly larger than the upper endpoint of the 95% confidence interval for their estimate.

In the first column of Table V, they report the change in the standard deviation of mrpk and the total change in the capital stock. Given Lemma 3,

$$\Delta\sigma_R = -\nu \times \sigma_R \times \Delta \log k$$

Gopinath et al. (2017) report $\Delta\sigma_R = 0.034$ and $\Delta \log k = 0.22$. While they do not report the initial value of σ_R, I can use the value reported by Albrizio, Beatriz, and Khametsin (2024), who study the same period, and is approximately 1.6. As a result, $\nu = -0.1$.

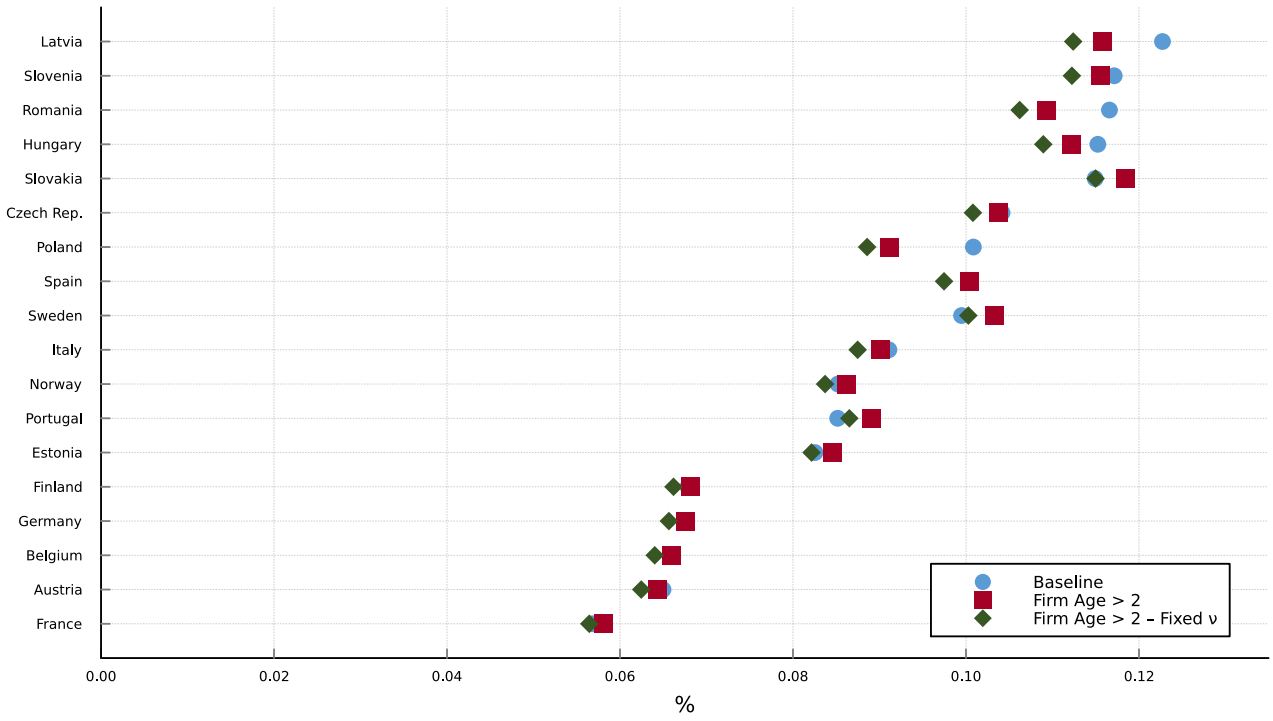
F.1.2 Capital Misallocation σ_R^2

In this section, I perform robustness checks over the estimation of σ_R^2, which is one of the key inputs for estimating γ. First, I consider more restricted samples to ensure that no group particularly drives the results. Second, I use within-firm averages to mitigate potential measurement error.

Figure F.2 plots estimates of γ when considering only firms older than 2 years to estimate capital misallocation. In blue I show the estimates when modifying ν to maintain $\gamma = 0.1$ for Spain. For comparison, I also plot in green the estimates while keeping ν unchanged. Even when keeping ν fixed the results do not change significatively, showing that results are not driven by young firms having high dispersion in returns to capital. Figure F.3 plots estimates of γ for the sample of firms with more than 5 employees. While there's a modest reduction in the estimates for countries in the upper end of the estimates, the overall message remains that these costs are significative and that results are not driven by small firms. Figure F.4 plots estimates of γ when excluding firms which report a stock of capital larger than 10,000 euros, as a way of ensuring that results are not driven by very small firms. Although the estimates change, the picture remains virtually the same when it comes to the magnitude of the results.

To mitigate potential measurement from firms misreporting data, I estimate σ_R^2 by using a within-firm moving average of their added value and capital stock. Figure F.5 plots the implied γ by country (red) compared to the baseline estimates (blue). Figure F.5 shows that applying this procedure modestly shrinks the estimates. For the median country, the resulting γ is 12% smaller than the baseline estimate. The main result that borrowing subsidies are optimal when the probability of a crisis is low still apply for these estimates.

Figure F.2: γ Estimates for Firms older than 2 years



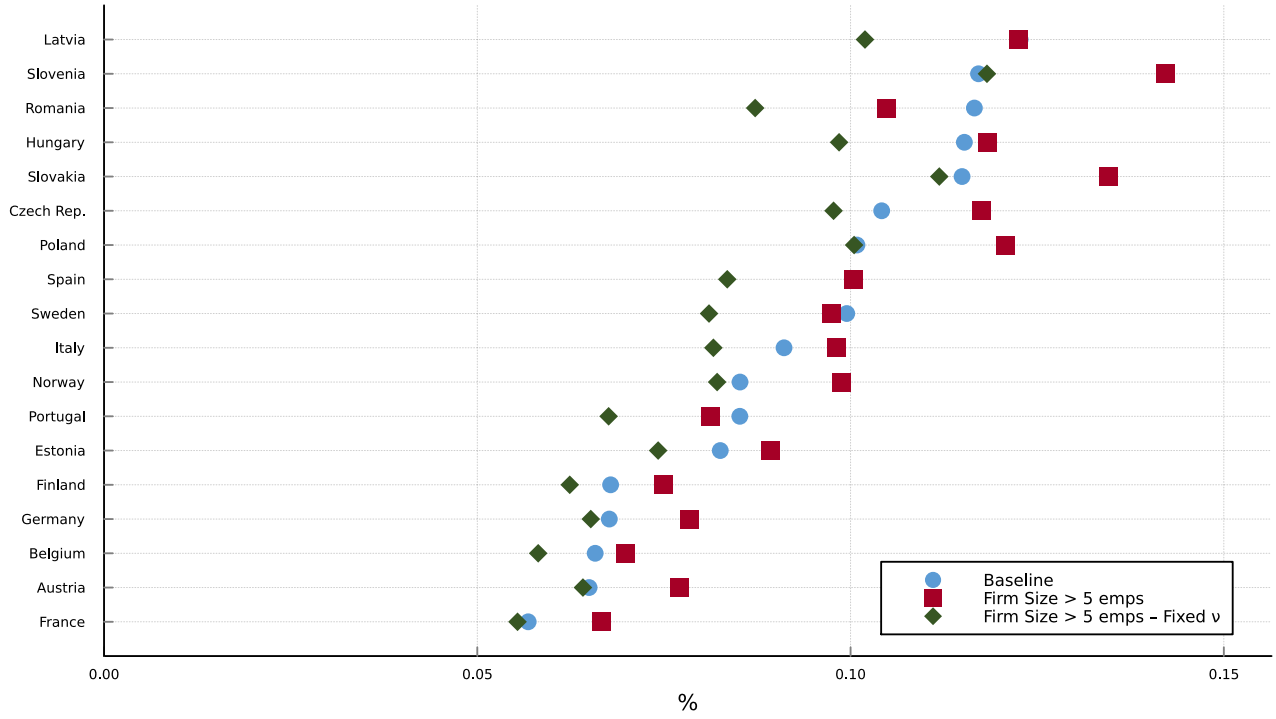
Notes: The figure plots the elasticity of TFP with respect to investment γ for a sample of countries according to (27). The blue bars plot the baseline estimates, explained in Section 6. The red bars plot the estimates using only firms older than 2 years to calculate capital misallocation. The green bar plots the estimates using only firms older than 2 years to calculate capital misallocation and keeping ν fixed. The last two estimates are explained in Section F.1.2

F.2 Model With Additional Distortions

In this section I estimate γ according to the model in Section C.2. Following the identification argument in that section, I estimate $\mathbb{V}ar\left[\log \text{mrpl}(i)\right]$ and $\mathbb{V}ar\left[\log \frac{k(i)}{h(i)}\right]$ similarly to the estimation of $\log \text{mrpk}(i)$. Following Hsieh and Klenow (2009), I use the wage bill to measure labor instead of the number of employees, as a way to account for differences in human capital. Lastly, I follow the same approach as in the baseline exercise and calibrate ν to match $\gamma = 0.1$.

Figure F.6 plots the comparison of the γ estimates from this model with the baseline estimates. For almost all countries in the sample, the estimates are smaller. Except for countries at the upper-end, this discrepancy is small. This is to be expected, as the literature has found that labor misallocation plays only a small role compared to capital misallocation in explaining overall misallocation.

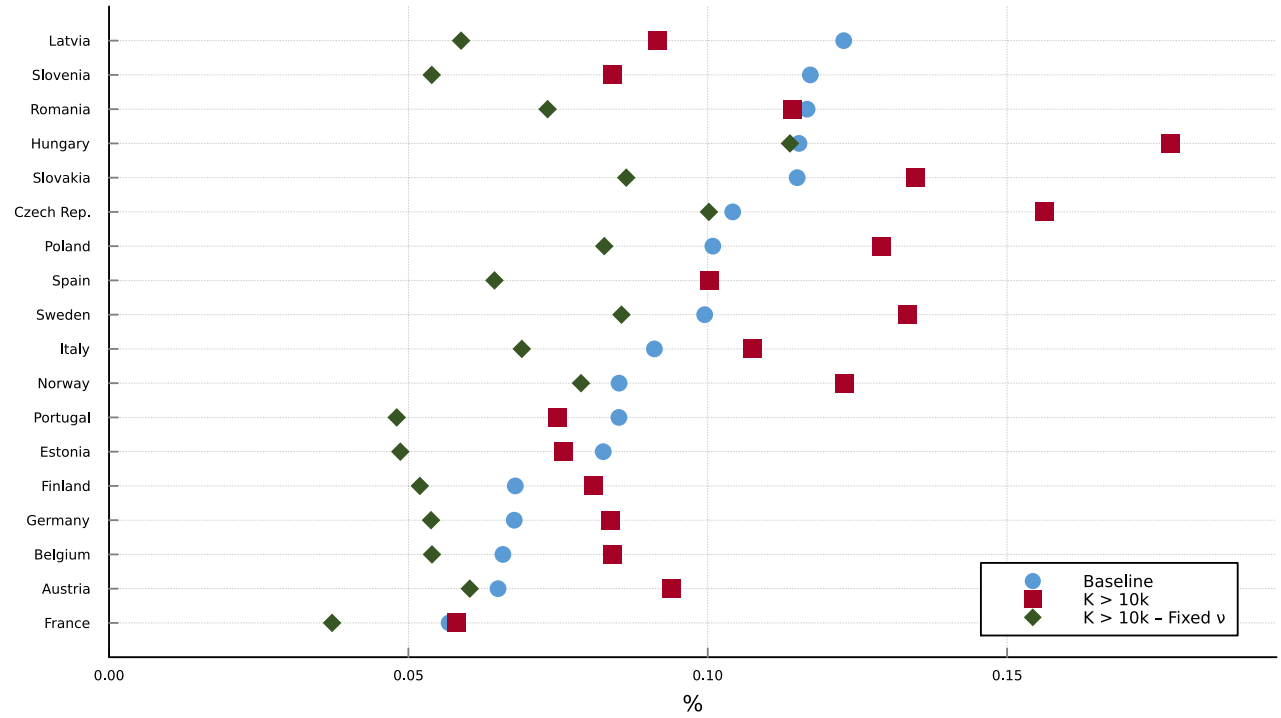
Figure F.3: γ Estimates for Firms with more than 5 employees



Notes: The figure plots the elasticity of TFP with respect to investment γ for a sample of countries according to (27). The blue bars plot the baseline estimates, explained in Section 6. The red bars plot the estimates using only firms with more than 5 employees to calculate capital misallocation. The green bar plots the estimates using only firms with more than 5 employees to calculate capital misallocation and keeping ν fixed. The last two estimates are explained in Section F.1.2

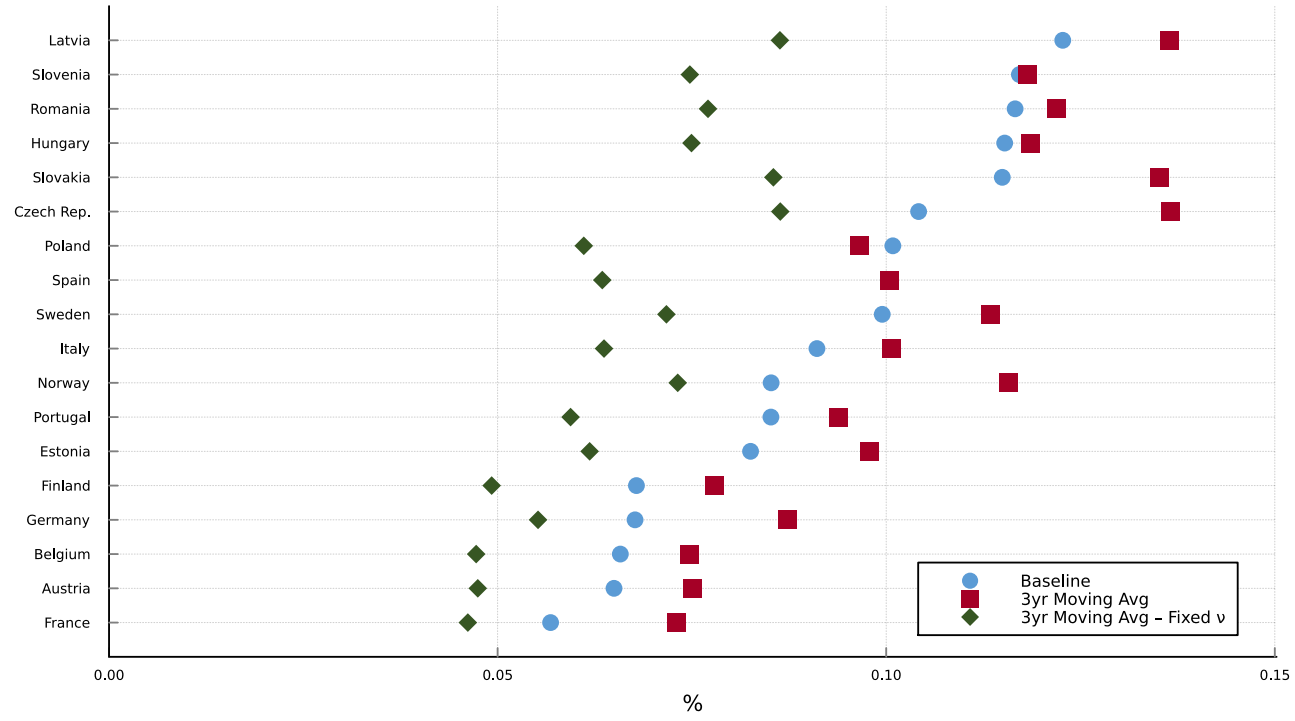
For countries in the upper end of estimates, there's a moderate reduction in the γ estimates. This does not materially affect the results in Section 6.

Figure F.4: γ Estimates Excluding Firms with Low Capital



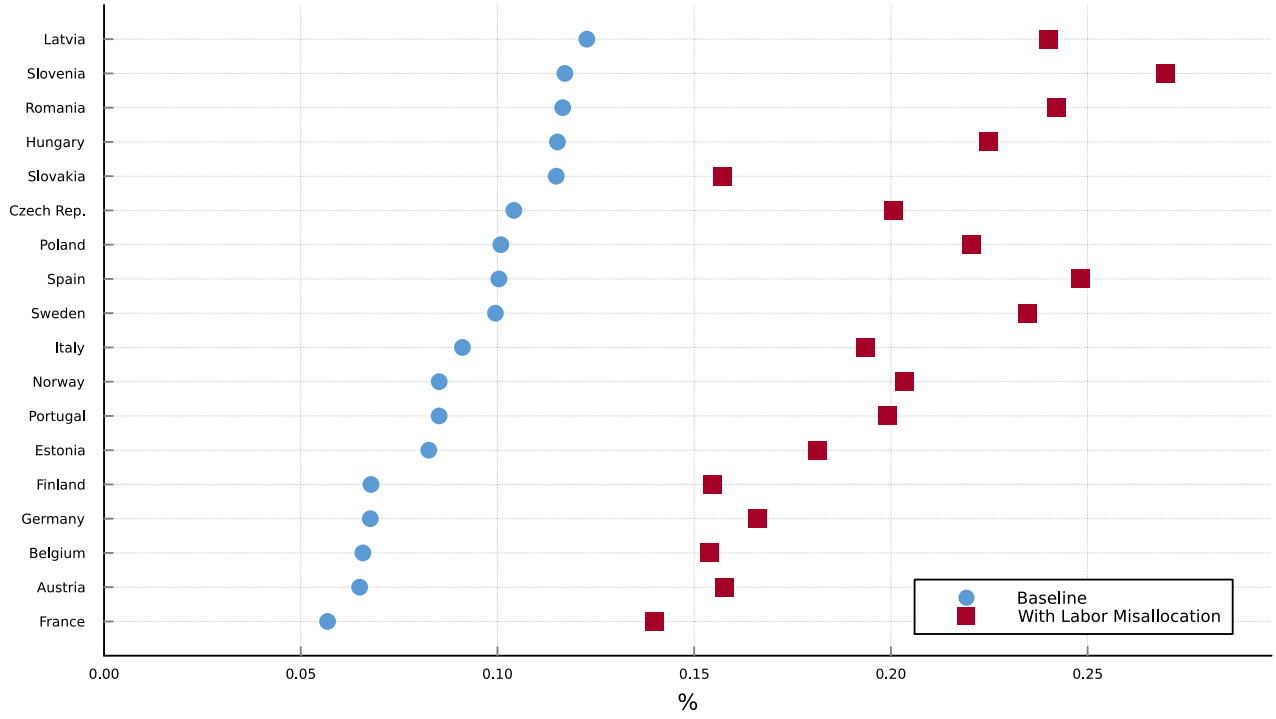
Notes: The figure plots the elasticity of TFP with respect to investment γ for a sample of countries according to (27). The blue bars plot the baseline estimates, explained in Section 6. The red bars plot the estimates using only firms which report a stock of capital larger than 10,000 euros to calculate capital misallocation. The green bar plots the estimates using only firms which report a stock of capital larger than 10,000 euros to calculate capital misallocation and keeping ν fixed. The last two estimates are explained in Section F.1.2

Figure F.5: Using Within-Firm Moving Averages to Measure σ_R^2



Notes: The figure plots the elasticity of TFP with respect to investment γ for a sample of countries according to (27). The blue bars plot the baseline estimates, explained in Section 6. The red bars plot the estimates using within-firm moving averages to calculate capital misallocation. The green bar plots the estimates using within-firm moving averages to calculate capital misallocation and keeping ν fixed. The last two estimates are explained in Section F.1.2

Figure F.6: Model with Labor Misallocation



Notes: The figure plots the elasticity of TFP with respect to investment γ for a sample of countries according to (27). The blue bars plot the baseline estimates, explained in Section 6. The red bars plot the estimates according to the model that allows for labor misallocation, derived in Section C.2.

G Computational Appendix

G.1 State-Space Discretization

The model has three state variables: aggregate TFP A_t, the capital stock k_t, and debt d_t.

TFP process. Log-TFP follows the AR(1) process $\log A_{t+1} = \rho \log A_t + \varepsilon_{t+1}$ with $\varepsilon_{t+1} \sim \mathcal{N}(0, \sigma_\varepsilon^2)$, $\rho = 0.85$, and $\sigma_\varepsilon = 0.0125$. I discretize it via the Tauchen (1986) method into $n_A = 7$ equally-spaced grid points on $[\mu_A - m\,\sigma_A,\ \mu_A + m\,\sigma_A]$ (with $m = 3$ and $\sigma_A^2 = \sigma_\varepsilon^2/(1-\rho^2)$). The transition matrix Π_A is calibrated by simulating 10^6 draws from the AR(1) process and computing empirical transition frequencies.

Endogenous-state grids. Capital and debt are discretized on uniform grids: $n_k = 30$ points on $[k_{\min}, k_{\max}] = [0.4, 0.9]$ and $n_d = 60$ points on $[d_{\min}, d_{\max}] = [0.0, 0.9]$. All policy functions are linearly interpolated between grid nodes, with flat extrapolation at the boundaries.

G.2 Decentralized Equilibrium: Euler-Equation Iteration

Because the collateral constraint is occasionally binding, a global solution method is required. Following Bianchi (2011), I solve the private equilibrium by iterating directly on the households' Euler equations. The key extension relative to Bianchi (2011) is that capital investment is a second endogenous choice, so the iteration must solve for both d' and k' simultaneously.

Define the shadow cost of the debt Euler equation at state $s = (A, k, d)$ and candidate next-period policies (d', k') as

$$\mu_d(d'; k', s) \;=\; \frac{\lambda^c}{p} \;-\; \beta(1+r)\,\mathbb{E}\left[\frac{\lambda^{c\prime}}{p'}\,\middle|\, s\right], \tag{G.1}$$

where λ^c/p is the marginal utility of tradable consumption. Analogously, define the shadow cost of the capital Euler equation as

$$\mu_k(k'; d', s) \;=\; c'_{\text{adj}}(k' - k) \;-\; \beta\,\mathbb{E}\Big[\lambda^{c\prime}\big(F'_k + (1-\delta) + \varphi'(k'' - k')\big)\,\Big|\, s\Big], \tag{G.2}$$

where c'_{adj} denotes the marginal cost of investment and F'_k is the next-period marginal product of capital.

The algorithm proceeds as follows.

1. **Initialization.** Set $d'(s) = d_{\max}$ and $k'(s) = k$ for all grid states.

2. **Iteration.** At each step:

   (a) Compute next-period allocations and marginal utilities implied by the current guesses $\{d'(s), k'(s)\}_s$.

   (b) For each state s, compute the maximum feasible debt $d_{\max}(s) = \min\big(\kappa_t(y^T + \kappa_n p^N y^N),\ d_{\text{ub}}\big)$, i.e. the minimum of the natural debt limit and the collateral ceiling.

   (c) **Joint debt and capital choice.** For any candidate debt level $\tilde{d}$, define $k^*(\tilde{d})$ as the solution to $\mu_k(k; \tilde{d}, s) = 0$ over $[k_{\min}(\tilde{d}, s),\ k_{\max}(\tilde{d}, s)]$ via Brent's method. The debt choice then operates on the composite function $\tilde{d} \mapsto \mu_d(\tilde{d};\ k^*(\tilde{d}),\ s)$:

   - If $\mu_d(d_{\max};\ k^*(d_{\max}),\ s) > 0$: the constraint binds; set $d' = d_{\max}$.
   - If $\mu_d(d_{\min};\ k^*(d_{\min}),\ s) < 0$: corner solution at the lower bound; set $d' = d_{\min}$.
   - Otherwise: solve $\mu_d(\tilde{d};\ k^*(\tilde{d}),\ s) = 0$ via Brent's method.

   The optimal capital is then $k' = k^*(d')$.

3. **Convergence.** Stop when

$$\frac{1}{2}\Big(\max_s |d'_{\text{new}}(s) - d'_{\text{old}}(s)| + \max_s |k'_{\text{new}}(s) - k'_{\text{old}}(s)|\Big) < 10^{-6}, \tag{G.3}$$

4. **Policy evaluation.** Once converged, policy functions are stored as linear interpolants over the state grid and can be evaluated at any point.

G.3 Constrained-Efficient Planner: Value-Function Iteration

I solve the constrained-efficient planner by value-function iteration (VFI).

Let V^n denote the value function at iteration n, and let $\mathbb{E}[V^n|s]$ denote its expectation under the transition matrix Π_A. In practice, $\mathbb{E}[V^n|s]$ is computed by first linearly interpolating V^n over the grid and then multiplying by Π_A.

1. **Initialization.** Set $V^0(s) = 0$ for all s.

2. **Iteration.** At each step n:

   (a) Compute $\mathbb{E}[V^n|s]$ for all s by interpolation and multiplication by Π_A.

(b) **Policy update** (every 100 iterations, and for the first 3 iterations): for each state s, solve

$$(d'(s), k'(s)) \;=\; \arg\max_{(d',k')\in\mathcal{F}(s)} \; u\big(c(d', k', s)\big) + \beta\,\mathbb{E}[V^n|s] \tag{G.4}$$

using Golden Section search over the feasible set $\mathcal{F}(s)$.

(c) Update the value function: $V^{n+1}(s) = u\big(c(d'(s), k'(s), s)\big) + \beta\,\mathbb{E}[V^n|s]$.

3. **Convergence.** Stop when $\max_s |V^{n+1}(s) - V^n(s)| < 10^{-12}$, up to a maximum of 1500 iterations.

The looser policy-update frequency (every 100 iterations) follows the Howard improvement approach: it is faster to iterate on the value function holding policies fixed, updating policies only occasionally. This accelerates convergence relative to solving the optimization at every iteration.

G.4 Simulation

Simulations are run for T periods, discarding the first 10% as a burn-in period. To ensure comparability across regimes, all three allocations (decentralized equilibrium, constrained-efficient planner, and tax-corrected private equilibrium) are evaluated on the same sequence of productivity shocks $\{A_t\}_{t=1}^{T}$ drawn at the outset. Sudden-stop episodes are identified as periods in which the current account reversal exceeds a threshold set equal to one standard deviation of the simulated current account series.

G.5 Welfare Calculation

Given a set of policy functions, I compute the associated value function by running VFI holding policies fixed until convergence (tolerance 10^{-12}, maximum 1500 iterations).

Welfare comparisons are expressed as consumption-equivalent variations (CEV). Let $V^{\mathrm{DE}}(s)$ and $V^{\mathrm{alt}}(s)$ denote the value functions of the decentralized equilibrium and an alternative allocation, respectively. Under CRRA utility with curvature σ, the fraction λ of consumption that must be transferred state-by-state to make the household indifferent satisfies $(1+\lambda)^{1-\sigma} V^{\mathrm{DE}}(s) = V^{\mathrm{alt}}(s)$, so

$$\lambda(s) = \left(\frac{V^{\mathrm{alt}}(s)}{V^{\mathrm{DE}}(s)}\right)^{\frac{1}{1-\sigma}} - 1. \tag{G.5}$$

I evaluate $\lambda(s)$ at each state visited under the ergodic distribution of the decentralized equilibrium and report its mean as the welfare gain.

G.6 Optimal Tax Computation

Constant taxes. To find the optimal constant tax pair (τ_d, τ_k), I search over the grid $(-0.05, 0.05)^2$ and then refine the solution (up to 500 iterations) to minimize the welfare loss of the private equilibrium relative to the constrained-efficient allocation.