When Stable Layers Become Unstable: Role Switching in International Finance–Trade Networks

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Abstract

International financial integration can diversify risk but also create feedback that undermines stability from within. We study this duality in a two-layer network linking international portfolio investment and trade. Using bilateral portfolio and trade data for 52 economies over 2015–2024, we construct an integrated financial–trade system and identify the CLiFF, the critical boundary at which local stability is lost. We separate the effects of coupling strength from those of the country-pair composition of cross-layer feedback. Although the financial and trade layers are stable in isolation, feedback between them can destabilize the integrated system. In a conditional simulation with the same initial relative stability margin and adjustment rule, Debt–Trade remains stable, whereas Equity–Trade crosses into the unstable region. This divergence cannot be explained by the scale of cross-layer interaction alone. The destabilizing function can persist even as the country pairs carrying it change over time. We call this reallocation *Role Switching*, and the separation between a persistent function and changing carriers the *Dionysian mechanism*. International portfolios and trade thus allocate not only wealth and risk, but also the functions of shock absorption and amplification. Instability can arise not only from larger shocks or stronger aggregate connections, but also from a reorganization of the paths through which disturbances circulate and return.

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JEL classification: F36, G15, D85

1. Introduction

International financial integration can distribute risk and generate fragility at the same time. In normal times, cross-border financial and trade linkages support liquidity, risk sharing, and the allocation of capital and goods. Under stress, however, the same linkages can propagate disturbances and undermine stability.

The failure of Lehman Brothers in 2008 illustrates the broader logic. Connections among financial institutions that had supported liquidity and risk sharing in normal times also propagated the effects of liquidity hoarding and asset sales during the crisis. The crisis did not necessarily create an entirely new network. Rather, the existing connections acquired different functions under stressed conditions. Financial-network research has described this duality as the robust-yet-fragile property (Haldane, 2013; Acemoglu et al., 2015).

This paper examines a related mechanism operating between international finance and trade. An effect originating in the financial layer may pass through the trade layer and return to the financial layer. Conversely, an effect originating in the trade layer may pass through the financial layer and return to the trade layer. We refer to these recursive cross-layer paths as Finance–Trade–Finance (FTF) and Trade–Finance–Trade (TFT) LOOPs.

These LOOPs do more than transmit an initial disturbance. By feeding the disturbance back into the originating layer, they can either dampen or amplify the response of the integrated system. Their contribution depends on both the strength of cross-layer coupling and the country-pair composition of the resulting feedback.

Our central question is whether an integrated finance–trade system can lose local stability even when its financial and trade layers are each stable in isolation. More specifically, we examine whether changes in the country-pair composition of FTF and TFT LOOPs can alter system stability without a comparable change in the overall scale of cross-layer interaction.

The distinction matters because stabilizing and destabilizing feedback paths may coexist in a multilayer network. So long as their effects offset one another, the system may appear robust in aggregate. Yet that balance can be disturbed when the country pairs carrying these paths change, even if the aggregate scale of the LOOPs remains broadly unchanged. We call this reallocation of feedback functions across country pairs *Role Switching*.

Role Switching does not require a fixed set of intrinsically fragile coun-

tries or links. A bilateral relationship that forms part of an absorptive path in one network structure may form part of an amplifying path in another. The source of fragility is therefore not necessarily the emergence of a new connection or a large increase in aggregate exposure. It may instead be a change in which relationships carry stabilizing and destabilizing feedback.

We refer to this possibility as the *Dionysian mechanism*: a system of relationships that supports stability can, through the reallocation of its feedback functions, generate the conditions for its own instability.[1]

Using bilateral portfolio investment and trade data for 52 economies from 2015 to 2024, we construct separate Debt–Trade and Equity–Trade systems. We evaluate local stability using the spectral abscissa of an integrated Jacobian that combines within-layer dynamics with bidirectional cross-layer effects. For each network, we identify the critical level of cross-layer coupling at which local stability is lost. We call this boundary the CLiFF and use the relative distance between prevailing coupling and the CLiFF to measure the system's remaining stability margin.

The analysis yields three main findings. First, individually stable financial and trade layers can form a locally unstable integrated system once cross-layer feedback is introduced. Stability at the layer level therefore does not guarantee stability of the system as a whole.

Second, current stability need not imply a substantial margin of safety. In a conditional simulation that assigns the two asset networks the same initial relative stability margin and adjustment rule, Debt–Trade remains within its annual stability boundary. Equity–Trade, by contrast, crosses its boundary in 2017 and remains in the locally unstable region thereafter. This divergence cannot be explained by the level of cross-layer coupling alone. The stability boundary itself moves as the observed network structure changes.

Third, Role Switching is quantitatively important. It accounts for 36.8 percent of gross stability variation in Debt–Trade and 24.1 percent in Equity–Trade. Its effect is not fixed in sign: reallocation across country pairs can stabilize or destabilize the system, depending on the sensitivity of the paths gaining or losing weight. In the Equity–Trade TFT LOOP, the aggregate contribution of the high-sensitivity region is destabilizing in all nine annual

[1]The term draws on the Dionysian duality between forces that sustain collective vitality and those that loosen the restraints preserving order. In the present setting, the same system of economic relationships can support stability under one allocation of feedback and undermine it under another.

changes, even though the leading country pairs vary over time. The destabilizing function can therefore persist without remaining attached to the same bilateral relationships.

The evidence on the global financial cycle is consistent with this distinction, although it is exploratory. In Equity–Trade, larger deviations of the global financial factor from its sample mean are associated with a larger cross-layer response before the FTF and TFT contributions offset one another. They are not clearly associated with the direction of the net stability effect. The global financial environment may therefore intensify activity across feedback paths without determining whether the resulting effect is stabilizing or destabilizing.

These findings make three contributions. First, the paper extends the robust-yet-fragile argument from propagation within a single network to recursive interaction between economically distinct networks. Existing research shows that connectivity can support risk sharing while also creating channels of contagion (Elliott et al., 2014; Acemoglu et al., 2015). We show that an additional source of fragility arises when an effect leaves one layer, passes through another, and returns. This cross-layer mechanism cannot be recovered by analyzing finance and trade separately.

Second, the paper shifts attention from current stability to distance from instability. The spectral abscissa determines whether the integrated system is locally stable, while the CLiFF identifies the cross-layer coupling at which that stability is lost. The method draws on the linearization logic of network stability analysis (Pecora and Carroll, 1998), but does not test the stability of a synchronized state. It evaluates local stability around a reference state constructed from observed financial and trade networks. The resulting stability margin is a structural measure of proximity to a moving boundary, not a forecast of crisis timing.

Third, the paper connects aggregate stability to the country-pair composition of cross-layer feedback. Role Switching is not a change in centrality rankings. It is a change in the allocation of stabilizing and destabilizing functions across recursive bilateral paths. This distinction makes it possible to separate persistence in a network function from persistence in the relationships carrying that function.

The paper consequently redirects the analysis of international financial fragility from a fixed set of vulnerable countries toward the changing organization of feedback. The relevant question is not only which countries are central or which connections are large. It is also which country-pair reallo-

cations can transform relationships that supported stability into paths that amplify disturbances.

2. Related Literature on the Stability of International Financial and Trade Networks

2.1. International Portfolios, Trade, and the Global Financial Cycle

Research on international capital flows has moved from external financial conditions toward the composition and adjustment of gross foreign positions. Calvo et al. (1993) established the importance of global factors, including world interest rates, while Lane and Milesi-Ferretti (2007) constructed internationally comparable data on gross foreign assets and liabilities. Obstfeld (2012) emphasized their composition, valuation effects, and liquidity risks. Subsequent studies showed that large adjustments in gross capital flows can occur behind relatively modest changes in net flows (Forbes and Warnock, 2012; Broner et al., 2013).

International portfolios are also closely connected to trade. Information frictions and geographic distance affect cross-border equity flows (Portes and Rey, 2005), and bilateral trade and financial asset holdings reinforce one another (Aviat and Coeurdacier, 2007). Global financial conditions may also affect exports through firms' balance sheets (Bruno and Shin, 2023). These findings suggest that financial and trade relationships should not be treated as independent channels of international transmission.

Theoretical work has clarified how international portfolios adjust. Devereux et al. (2020) showed how risk-premium adjustment can ensure the stationarity of net foreign asset positions and the global distribution of wealth. Davis and van Wincoop (2022) demonstrated that portfolio heterogeneity generates adjustments in gross flows and asset-specific net flows. Together, these studies shift attention from aggregate net positions to the composition of international balance sheets.

A common global component is also central to international financial adjustment. Rey (2013) documented a global financial cycle in capital flows, asset prices, and financial intermediation. Miranda-Agrippino and Rey (2020) extracted common factors from international asset prices and capital flows and showed how U.S. monetary policy is transmitted through global financial conditions. Exposure to this common environment, however, need not produce uniform responses across countries, asset classes, or investors.

What remains less understood is how such heterogeneous responses are organized across country pairs connecting financial and trade networks, and how this organization affects the stability of the integrated system. Existing studies establish important links among global financial conditions, portfolio adjustment, and trade, but do not characterize the recursive paths through which an effect passes from finance to trade and back to finance, or in the opposite direction.

This paper treats the global financial cycle as a common external financial environment, rather than as a factor that mechanically determines whether the system becomes more or less stable. We combine international portfolio and trade relationships in a multilayer network and examine how the country-pair configurations of FTF and TFT loops shape the system's response. A supplementary analysis then relates deviations in the observed global financial cycle from its sample mean to the magnitude and net direction of contributions from changes in LOOP configuration.

2.2. Financial Intermediation, Leverage, and Real-Economy Transmission

Financial intermediaries' balance sheets are central to the international transmission of shocks. Asset prices, leverage, and credit supply can reinforce one another (Adrian and Shin, 2010, 2014). Bruno and Shin (2015) extended this mechanism to international finance, showing that global funding conditions affect banks' risk-taking and cross-border credit supply. Cetorelli and Goldberg (2012) further documented how internal capital markets within global banks transmit financial shocks to lending across countries.

Recent work emphasizes the roles of intermediary heterogeneity and firm-to-firm linkages. Coimbra and Rey (2024) showed that asset reallocation among intermediaries with different risk-taking incentives can produce a nonlinear increase in leverage and systemic risk when interest rates are low. Alfaro et al. (2021) found that bank credit-supply shocks affect not only the investment and production of borrowing firms but also other firms connected through supplier–customer relationships. Financial shocks therefore extend beyond the financial sector through credit, firm activity, and production networks.

This literature primarily studies transmission from financial intermediaries to firms or from firms to their trading partners. Less is known about the feedback that arises when financial effects pass through trade relationships and subsequently return to financial relationships. We represent finance

and trade as a two-layer international network and examine whether this recursive interaction dampens or amplifies the local dynamics of the integrated system.

2.3. Robustness and Fragility in Financial Networks

Financial-network research has shown that connectivity can both diversify risk and amplify shocks. Concentration and complexity can intensify liquidity crises (Gai et al., 2011), while integration and diversification can have non-monotonic effects on financial cascades (Elliott et al., 2014). Acemoglu et al. (2015) showed that the same network may absorb small shocks but amplify sufficiently large ones. This dual role of connectivity is central to the Robust-yet-Fragile (RyF) property.

More recent work has relaxed the assumption that the network is exogenous. Jackson and Pernoud (2021) reviewed mechanisms involving defaults, common asset holdings, fire sales, credit freezes, expectations, and endogenous counterparty choice. Erol and Vohra (2022) showed that a higher probability of favorable shocks can encourage network formation and ultimately raise systemic risk. In related work on supply networks, Elliott et al. (2022) demonstrated that firms' choices of trading partners and resilience investments may generate equilibrium networks that remain vulnerable to small common shocks.

These studies primarily examine propagation within a single network or link formation before a shock. Less is known about how feedback between observed international financial and trade networks is reconfigured by changes in their country-pair composition. In particular, it remains unclear whether such reconfiguration can alter the stability of the integrated system even when each layer is stable in isolation and the aggregate strength of cross-layer connections changes little.

We address this question using a Jacobian that combines financial and trade networks in a two-layer system. Our focus is not only on how far a shock travels along existing links. We examine how changes in the country-pair composition of recursive finance–trade paths reallocate the network's stabilizing and destabilizing functions. This extends the RyF argument from thresholds associated with connectivity and shock size to stability changes associated with the configuration of cross-layer feedback.

2.4. Cross-Layer Interdependence and System-Wide Instability

The stability of a multilayer system cannot generally be inferred from the stability of its individual layers. Buldyrev et al. (2010) showed that a failure can pass from one layer to another and return to the originating layer, generating cascades that would not arise in either layer alone. The general representation of such multilayer interdependence is developed in Kivelä et al. (2014) and Bianconi (2018). In financial networks, aggregating different types of exposures into a single layer can substantially misstate systemic risk (Poledna et al., 2015; Silva et al., 2017). Related work also studies international propagation through multiple financial exposures, trade, and cross-border investment (del Rio-Chanona et al., 2020; Starnini et al., 2019).

Existing studies largely ask how an exogenous failure or shock propagates across interconnected layers. Our question is different: can cross-layer interaction change the local stability of the system even when each layer is stable in isolation? In our setting, an effect originating in finance passes through trade and returns to finance, or travels in the opposite direction. Repetition of this feedback can destabilize the joint system without destabilizing either layer separately.

We examine this mechanism using observed international financial and trade networks. We distinguish changes in the overall strength of cross-layer coupling from changes in the country-pair configuration carrying the feedback. This distinction allows us to test whether a previously stable system can cross its stability boundary because recursive paths are reallocated across country pairs, even when their aggregate scale does not increase.

2.5. Connection to Network Macroeconomics

Network macroeconomics shows that the aggregate effects of shocks depend not only on their size but also on the configuration of relationships among economic agents (Baqaee and Farhi, 2019; Baqaee and Rubbo, 2023). Positions within international production networks are likewise not fixed. Cingolani et al. (2017) showed that countries' centralities vary across stages of global value chains. Using industry-level multilayer trade networks, Alves et al. (2022) documented substantial changes in the relative positions of major economies over time, including a pronounced reorganization in 2007, immediately before the global financial crisis.

Countries' positions in trade and finance also need not coincide. Recent evidence on the international monetary system indicates an increasingly

asymmetric specialization of countries in global trade and finance (International Monetary Fund, 2025). This literature documents changes in network position, but does not establish whether those changes stabilize or destabilize an integrated financial and real system.

We connect changes in observed financial and trade relationships to their stability contributions through cross-layer feedback. This allows us to examine whether the stability of the integrated system can change because the country pairs carrying recursive effects are reallocated, even when the aggregate scale of the relationships changes little.

2.6. The Open Question and Our Approach

Previous research has examined international portfolio adjustment, the transmission of financial conditions to trade, shock amplification through networks, and interdependence across multiple layers. What remains unresolved is how these mechanisms jointly affect stability in observed international financial and trade networks. In particular, can feedback between two individually stable layers destabilize the integrated system, and can its stability change solely because the country-pair configuration carrying that feedback is rearranged?

We address these questions using bilateral securities holdings and trade relationships for 2015–2024. We construct a two-layer finance–trade network and formulate bidirectional feedback through FTF and TFT loops. We then identify the critical cross-layer coupling at which the system loses local stability and separate the contributions of coupling strength and country-pair configuration. This approach distinguishes the effect of the aggregate scale of cross-layer feedback from that of its internal allocation across country pairs.

3. Multilayer Networks and Stability

3.1. Cross-Layer Mappings between Financial and Trade Networks

We combine financial and trade layers covering the same 52 countries. The financial layer records bilateral holdings of equity or debt securities, while the trade layer records bilateral trade in goods. Each country therefore has one state variable in each layer.

Multilayer-network analysis represents different types of relationships as separate layers and incorporates within- and cross-layer propagation into a single dynamic system (De Domenico et al., 2013; Kivelä et al., 2014; Boccaletti et al., 2014). We follow this general representation.

Let A_t^F and A_t^T denote the adjacency matrices of the financial and trade layers. We construct directional cross-layer mappings from ordered products of these two observed matrices:

$$C_t^{FT} = \text{offdiag}\big(A_t^T A_t^F\big)\,, \qquad C_t^{TF} = \text{offdiag}\big(A_t^F A_t^T\big)\,. \tag{1}$$

Here, offdiag$(\cdot)$ sets the diagonal elements of a matrix to zero. Columns index sending countries and rows index receiving countries. Thus, C_t^{FT} describes the configuration of paths from the financial layer to the trade layer, whereas C_t^{TF} describes paths in the opposite direction.

These matrices are not additional observed links. They are composite propagation structures obtained by combining observed financial and trade relationships in a specified order. The general matrix representation follows the multilayer-network literature; using the ordered products of the two observed networks as effective directional mappings is a modeling choice specific to this paper. Because matrix multiplication is generally noncommutative, we do not assume that the two mappings are equal or that one is the transpose of the other.

When the directional effects operate in succession, they form the return paths illustrated in Figure 1. For example, an effect originating in the United States in the financial layer may pass through direct and indirect responses within the entire trade network and return to Japan in the financial layer. The intermediate response is not restricted to a single country. A corresponding path may originate in trade, pass through the financial network, and return to the trade layer.

For the country-pair analysis, we classify a return to a country other than the origin as Role Switching and separate a return to the country of origin as a Self Return. Setting the diagonal elements of each one-directional mapping to zero does not eliminate all Self Returns. An effect may still pass through other countries and through the intermediate layer before returning to its country of origin. Self Return is therefore determined by comparing the origin and final destination of the complete round-trip path.

3.2. Two-Layer Dynamics and Local Stability

We evaluate local stability by linearizing the network dynamics around a reference state and examining the resulting eigenvalues. The Master Stability Function (MSF) similarly uses linearized variational equations to study the stability of network dynamics (Pecora and Carroll, 1998). The relationship

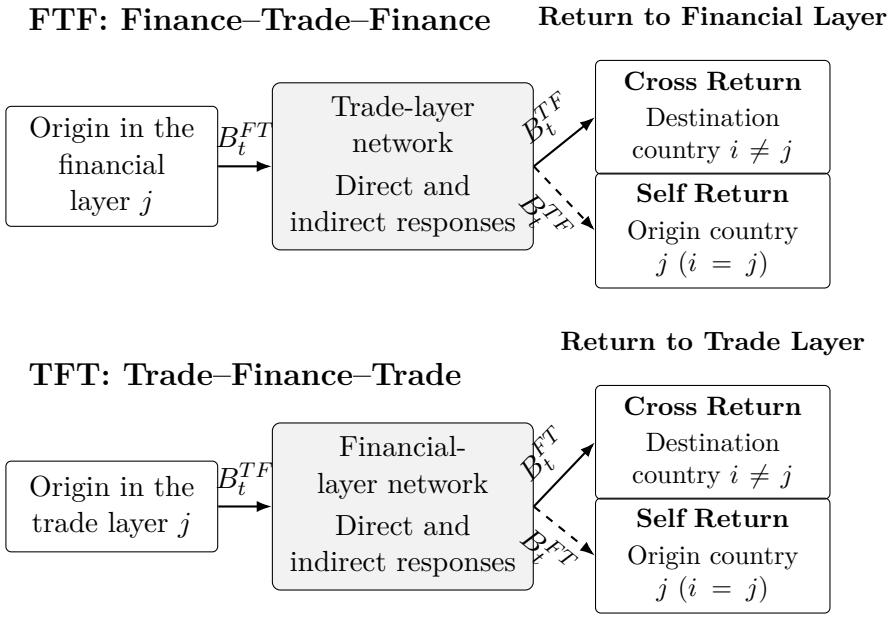


Figure 1: Directional cross-layer feedback and return destinations. The upper panel shows Finance–Trade–Finance (FTF) feedback: an effect originating in the financial layer passes through the trade layer and returns to the financial layer. The lower panel shows Trade–Finance–Trade (TFT) feedback in the opposite direction. The central boxes aggregate direct and indirect responses within the intermediate layer. A Cross Return reaches a country other than the origin ($i \neq j$), whereas a Self Return returns to the country of origin ($i = j$). Dashed arrows distinguish Self Returns visually and do not indicate their exclusion from the model.

between network structure and dynamics, including extensions to multilayer systems, is reviewed in Kivelä et al. (2014), Boccaletti et al. (2014), and Bianconi (2018).

We borrow the linearization and spectral logic of the MSF, but do not impose or test a synchronized state in which all countries have identical dynamics. Instead, we evaluate local stability around a reference state of an asymmetric two-layer system comprising financial and trade networks.

Each country has one state variable in each layer. With 52 countries, the joint state is therefore 104-dimensional. Linearizing the financial and trade dynamics around the reference state yields the following 104×104 Jacobian. The underlying dynamic equations and the derivation of its four blocks are provided in Appendix Appendix A:

$$J_t = \begin{pmatrix} D_t^F & B_t^{TF} \\ B_t^{FT} & D_t^T \end{pmatrix}. \tag{2}$$

The diagonal blocks D_t^F and D_t^T describe country-specific adjustment and propagation within the financial and trade layers, respectively. The off-diagonal blocks B_t^{FT} and B_t^{TF} describe the effective cross-layer effects from finance to trade and from trade to finance.

More specifically, B_t^{FT} and B_t^{TF} are constructed by normalizing the directional mappings C_t^{FT} and C_t^{TF} defined in the preceding subsection and incorporating cross-layer coupling, direction-specific coefficients, and the signs implied by the dynamics. The C matrices describe the configuration of composite financial–trade paths, whereas the B matrices describe how those paths enter the linearized dynamics.

The within-layer coupling parameter γ_t governs the strength of propagation through the diagonal blocks. The cross-layer coupling parameter σ_t governs the strength of the off-diagonal effects and hence the feedback between finance and trade. The Jacobian J_t combines these within-layer dynamics and bidirectional cross-layer effects in a single linear operator.

Consequently, stability of the individual layers does not guarantee stability of the integrated system. Cross-layer feedback operating through the off-diagonal blocks can render the two-layer system locally unstable even when each diagonal block is stable in isolation.

3.3. From the Network to the Jacobian

An element $J_{ij,t}$ of the Jacobian measures how a small change at node j affects the rate of change at node i. A nonzero element therefore represents a local dynamic channel from j to i.

Whereas an adjacency matrix records observed connections, such as bilateral trade or securities holdings, the Jacobian describes how perturbations are transmitted through those connections—in which direction, with which sign, and at what strength. It can therefore be interpreted as the dynamic network induced by the observed financial and trade relationships.[2]

[2]We distinguish a *dynamic network*, whose links and weights change over time, from *dynamics on a network*, in which perturbations are dampened or amplified through a given set of links. For each year, the Jacobian describes the evolution of small perturbations while holding that year's network structure and coupling parameters fixed.

A perturbation may travel beyond directly connected countries, pass through multiple countries and layers, and eventually return to its originating layer. The eigenvalues of the Jacobian summarize the combined effect of these direct, indirect, and recursive paths. We evaluate stability using the spectral abscissa,

$$\alpha_t = \max_k \operatorname{Re}\left\{\lambda_k(J_t)\right\}. \tag{3}$$

If $\alpha_t < 0$, all sufficiently small perturbations decay over time and the reference state is locally stable. The condition $\alpha_t = 0$ defines the stability boundary, which we call the CLiFF. If $\alpha_t > 0$, a perturbation grows along at least one direction and the system is locally unstable.

This spectral approach shares the basic logic of the Master Stability Function, which relates network structure and coupling strength to stability (Pecora and Carroll, 1998; Bianconi, 2018). We do not, however, apply the Master Stability Function directly. Its standard formulation evaluates deviations from a synchronized state using Lyapunov exponents. We impose no synchronization condition and instead evaluate the reference state using the largest real part of the eigenvalues of a Jacobian constructed from observed financial and trade networks. The Master Stability Function thus provides a methodological point of reference, not the stability measure used in our analysis.

3.4. Cross-Layer LOOPs

A LOOP is a cross-layer feedback process in which an effect leaves one layer, propagates through the other layer, and returns to the layer of origin. Figure 2 represents this process in three stages: (1) the effect leaves the originating layer, (2) it propagates within the intermediate layer, and (3) it returns to the originating layer.

In an FTF LOOP, an effect originating in the financial layer moves to the trade layer, propagates through direct and indirect trade relationships, and returns to the financial layer. A TFT LOOP operates in the opposite direction: an effect originating in trade passes through the financial layer before returning to trade.

Stage (2) is not restricted to a single country or a single link. An FTF LOOP, for example, may pass through several countries within the trade network before returning to the financial layer. A LOOP is therefore not a

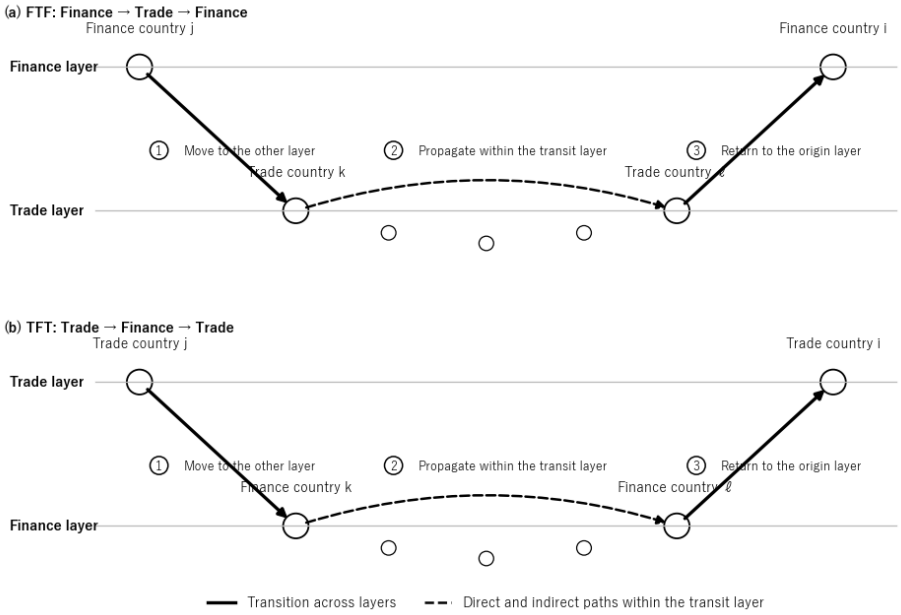


Figure 2: Cross-layer LOOPs between the financial and trade layers

Note: The upper panel shows Finance–Trade–Finance (FTF) feedback, with the financial layer serving as both the originating and return layer. The lower panel shows Trade–Finance–Trade (TFT) feedback. The thick solid arrows in stages (1) and (3) represent movement across layers. The dashed path in stage (2) represents direct and indirect propagation within the intermediate layer. The numbers indicate the order of transmission.

simple three-edge path. It represents cross-layer feedback that aggregates direct and indirect responses within the intermediate layer. A formal definition is provided in Appendix Appendix A.3.

FTF and TFT are not two separate networks. They describe feedback within the same financial–trade system, viewed from different originating layers. We therefore do not treat them as independent risks to be added mechanically. Both directions enter the integrated Jacobian and jointly determine the local stability of the system.

The stability effect of a LOOP depends not only on its aggregate scale but also on the country pairs carrying the return paths. Two LOOP configurations of equal scale may have different effects if one allocates feedback to country pairs that dampen perturbations and the other allocates it to pairs that amplify them. The next subsection illustrates this distinction with a

three-country, two-layer numerical example in which within-layer structure and aggregate cross-layer coupling are held fixed, but a change in country-pair configuration reverses the stability of the system.

3.5. A Numerical Example: Stability under LOOP Reallocation

We use a minimal three-country, two-layer example to show how the allocation of LOOPs across country pairs can change stability. Suppose that countries A, B, and C appear in both the financial and trade layers. Let the within-layer blocks governing the decay of small perturbations be

$$D^F = \mathrm{diag}(-0.2, -1.0, -1.0), \qquad D^T = \mathrm{diag}(-1.0, -0.2, -1.0). \tag{4}$$

Perturbations decay more slowly in financial country A and trade country B than at the other nodes. These within-layer dynamics remain fixed throughout the example.

Let P describe connections from financial to trade countries. Columns index sending countries in the financial layer, and rows index receiving countries in the trade layer. With cross-layer coupling $q = 0.3$, the joint Jacobian is

$$J(P) = \begin{pmatrix} D^F & -qP^{\mathsf{T}} \\ -qP & D^T \end{pmatrix}. \tag{5}$$

The lower-left block $-qP$ maps effects from finance to trade, while the upper-right block $-qP^{\mathsf{T}}$ returns them from trade to finance. Together, these two effects form an FTF LOOP.

Consider first the country-pair configuration

$$P_0 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}. \tag{6}$$

For example, the unit entry in row 3 and column 1 represents a connection from financial country A to trade country C. Under this configuration, the weakly damped financial country A is paired with the more strongly damped trade country C. The spectral abscissa is

$$\max \mathrm{Re}\,\{\lambda[J(P_0)]\} = -0.1. \tag{7}$$

Because this value is negative, small perturbations decay and the integrated system is locally stable.

Now hold the within-layer dynamics and coupling strength fixed, but change the country-pair configuration to

$$P_1 = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}. \tag{8}$$

Financial country A is now paired with trade country B. Because both nodes have relatively weak damping, the feedback between them is more amplifying.

The matrices P_0 and P_1 contain the same number of cross-layer connections, and every connection has the same strength $q = 0.3$. Their diagonal elements are also zero, so neither configuration adds a direct same-country link between the financial and trade layers. This does not rule out a Self Return through the complete round trip: an effect can pass through another country's node in the intermediate layer and still return to its country of origin. The only change in the example is which financial and trade countries are connected.

Despite the unchanged scale of cross-layer coupling, the new configuration yields

$$\max \mathrm{Re}\left\{\lambda[J(P_1)]\right\} = +00.1. \tag{9}$$

The spectral abscissa is now positive, so small perturbations grow and the integrated system is locally unstable.

This example shows that stability can reverse even when within-layer structure, the number of cross-layer links, and their strength all remain unchanged. What changes is the destination of the cross-layer connections. This illustrative reallocation should be distinguished from Role Switching in the empirical decomposition below, which is defined specifically as a change in the off-diagonal country-pair composition of completed LOOPs.

Economically, international relationships of the same aggregate scale can have different stability effects depending on the countries through which feedback circulates. Routing an effect through countries with strong local damping is not equivalent to routing it between countries in which perturbations decay slowly. Diversification in the volume of international transactions therefore need not imply diversification of dynamic risk.

3.6. Beyond Scale: Role Switching

The numerical example raises a simple question. If the number and strength of cross-layer connections are fixed, what causes the spectral abscissa to change?

The answer may lie not in the aggregate scale of the LOOPs, but in the country pairs carrying them. A reallocation across country pairs would have no stability effect if all countries had identical adjustment properties and sensitivities. When these characteristics differ, however, LOOPs of equal scale can have different effects depending on where their recursive paths are located.

We call this reallocation *Role Switching*. It is not merely a change in the country associated with the largest LOOP. Role Switching refers to a change in the country-pair composition of off-diagonal return paths, distinct from a change in the aggregate scale of the LOOPs. It therefore describes who carries dynamic risk rather than how much cross-layer connectivity exists.

This distinction motivates the empirical decomposition. For each annual transition, we first decompose the change in the spectral abscissa into four upper-level factors: within-layer structure, within-layer coupling γ, cross-layer coupling Ω, and LOOP configuration. Each factor is switched from its previous-year value to its current-year value. For a given ordering, its marginal contribution is the resulting change in the spectral abscissa, conditional on the factors already switched. We average these contributions over all $4! == 24$ possible orderings.

We then allocate the contribution of LOOP configuration between Role Switching, which consists of off-diagonal paths returning to a country other than the origin, and Self Return, which consists of diagonal paths returning to the country of origin. The reported five-component decomposition is therefore hierarchical: it combines an order-averaged decomposition over four upper-level factors with a within-factor allocation of LOOP configuration between Role Switching and Self Return.

A positive Role Switching contribution means that the reallocation of country-pair paths raises the spectral abscissa and moves the system toward instability. A negative contribution means that the reallocation is stabilizing. The decomposition allows us to determine whether an observed stability change reflects stronger aggregate feedback or a shift of existing feedback toward country pairs with more amplifying effects.

4. Empirical Strategy and Results

The empirical analysis proceeds in four steps. First, we harmonize the financial and trade data and construct two-layer networks combining either debt or equity positions with bilateral trade. Second, we parameterize within-layer and cross-layer interactions using the coupling parameters γ and σ and describe the resulting network structures. Third, we evaluate local stability from the integrated Jacobian and quantify the contributions of FTF and TFT feedback. Finally, we decompose annual stability changes into their underlying components and conduct statistical and robustness analyses of the country-pair patterns.

4.1. Data Sources and Offshore Reallocation

Our financial data come from the International Monetary Fund's (IMF) Portfolio Investment Positions by Counterpart Economy (PIP; formerly the Coordinated Portfolio Investment Survey, CPIS) (International Monetary Fund, 2026b). Bilateral trade data come from the IMF's International Trade in Goods by Partner Country (IMTS; formerly the Direction of Trade Statistics, DOTS) (International Monetary Fund, 2026a). The sample covers 2015–2024.

When a bilateral trade observation in IMTS is missing or reported as zero, we supplement it with the corresponding mirror observation from UN Comtrade. We use FOB values from export reports and CIF values from import reports, selecting the larger value when both are available. The trade layer is therefore based primarily on IMTS, with UN Comtrade used to prevent missing observations from being treated mechanically as absent network links.

PIP is generally compiled on a residence basis. Securities issued through subsidiaries in offshore financial centers, such as the Cayman Islands or Luxembourg, are therefore assigned to the location of the immediate issuer rather than to the country of the ultimate parent. The recorded investment destination may consequently differ from the country to which the underlying corporate exposure is ultimately associated.

To address this issue, we use reallocation information from the Global Capital Allocation Project (GCAP) of Coppola et al. (2021). We reassign PIP positions from the residence of the immediate issuer to the nationality of its ultimate parent. In addition to its global restatement matrices, GCAP provides European reallocation data based on Securities Holdings Statistics.

We use the reallocation information corresponding to the relevant asset category and geographic coverage.

A directed link from country i to country j in the financial layer therefore measures securities held by residents of country i whose ultimate parent is assigned to country j. The resulting financial network is not fully nationality-based: it is a bilateral exposure matrix defined by

$$\text{investor residence} \times \text{ultimate-parent nationality}.$$

The financial and trade layers consequently use different attribution rules. This is an intentional measurement choice rather than a statistical inconsistency. The trade layer records where goods move from and to, and therefore retains the residence-based locations of exporters and their trading partners. The financial layer retains the residence of the investor but reallocates the destination from the immediate issuer's residence to the nationality of the ultimate parent. As emphasized by Coppola et al. (2021), residence-based securities statistics may assign exposures routed through offshore subsidiaries to financial-intermediation centers. McGuire et al. (2024) similarly distinguishes the residence principle, which is useful for locating economic activity, from the nationality principle, which is useful for analyzing corporate control and financial risk. The two concepts serve complementary analytical purposes.

We therefore retain finance and trade as separate layers rather than combining them into a common measure or mechanically imposing the same country-attribution rule. Each layer uses the economic unit appropriate to the transmission channel it represents.

The final sample contains 52 countries and economies.[3] After reallocation, we exclude the Cayman Islands, Bermuda, the Bahamas, Luxembourg, Ireland, Cyprus, Mauritius, and the Netherlands. In our data, these economies appear primarily as locations of offshore issuance or financial intermediation rather than as the final destinations of the underlying financial exposures.

[3]The 52 countries and economies are Australia, Austria, Belgium, Bulgaria, Belarus, Brazil, Canada, Switzerland, Chile, China, Colombia, Costa Rica, Czechia, Germany, Denmark, Spain, Estonia, Finland, France, the United Kingdom, Greece, Hong Kong, Honduras, Hungary, Indonesia, India, Iceland, Israel, Italy, Japan, Kazakhstan, Korea, Kuwait, Lithuania, Mexico, Malta, Mongolia, Malaysia, Norway, New Zealand, Pakistan, the Philippines, Poland, Portugal, Romania, Saudi Arabia, Singapore, Sweden, Thailand, Türkiye, Uruguay, and the United States.

Hong Kong and Singapore remain in the sample because they also have substantial real economic activity and important roles in regional trade.

For example, suppose that a Japanese resident holds a security issued by a Cayman Islands subsidiary whose ultimate parent is a U.S. firm. The GCAP reallocation changes the position from Japan→Cayman Islands to Japan→United States. If the ultimate parent is Japanese, the position is reclassified as Japan→Japan and is excluded from the international country-pair network.[4]

Finally, we normalize bilateral financial positions and trade flows by the nominal GDP of the investing and exporting countries, respectively. GDP is taken from the World Development Indicators series measured in current U.S. dollars. When GDP is unavailable for a country-year, we replace it with the mean GDP of the sample economies observed in that year.[5] This normalization prevents large economies, such as the United States and China, from dominating the network solely because of their absolute size. The resulting networks measure the strength of bilateral financial and trade relationships relative to the scale of the originating economy.

4.2. Empirical Analysis I: Structure of the Financial–Trade Multilayer Network

4.2.1. Mapping the Stability Boundary

Section 3 showed that local stability of the two-layer system can be evaluated from the spectral abscissa of its Jacobian. We now apply this criterion to the observed equity–trade and debt–trade networks and map the boundary separating stable and unstable parameter combinations.

For the network observed in year t, let $J_t(\gamma, \sigma)$ denote the Jacobian associated with within-layer coupling γ and cross-layer coupling σ. Its spectral abscissa is

$$\alpha_t(\gamma, \sigma) = \max_k \operatorname{Re}\left\{\lambda_k[J_t(\gamma, \sigma)]\right\}.$$

[4]We also conducted the stability analysis using the unadjusted, residence-based PIP data. Under that specification, offshore financial centers such as the Cayman Islands and Luxembourg made exceptionally large contributions. The resulting network reflected the location of securities issuance and intermediation more strongly than the ultimate corporate allocation of financial exposure.

[5]GDP observations are imputed for Belarus, Costa Rica, Czechia, Spain, Honduras, Hungary, Malta, Mongolia, Pakistan, Romania, and Uruguay.

We calculate the combinations of (γ, σ) satisfying

$$\alpha_t(\gamma, \sigma) = 0$$

for each year and each asset–trade network. These combinations form the stability boundaries shown in Figure 3. Because the boundary resembles a steep drop around a critical level of cross-layer coupling, we refer to it as the CLiFF.

Both networks display a well-defined boundary between stable and unstable regions. For a given value of γ, increasing σ moves the system toward the boundary. At $\sigma = \sigma_t^*$, the spectral abscissa reaches zero; beyond this point, at least one perturbation direction becomes locally unstable.

The economic interpretation is straightforward. When cross-layer coupling is weak, an effect transmitted to the other layer is sufficiently dampened by within-layer adjustment. As σ rises, feedback between finance and trade becomes stronger, and the effect returns to its originating layer with less attenuation. Stability is lost when this recursive effect is strong enough to offset the damping represented by the diagonal blocks of the Jacobian.

The position of the boundary depends jointly on γ and σ. Figure 4 illustrates how their combination changes the relative contribution of cross-layer paths to movements in the dominant eigenvalue.

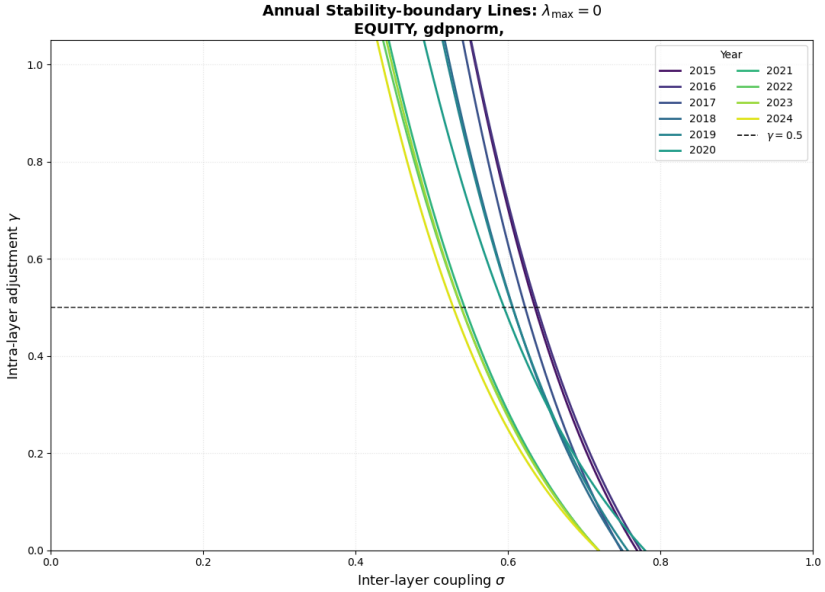
The relative importance of cross-layer transmission rises as within-layer coupling weakens or cross-layer coupling strengthens. For example, in the debt–trade network, holding $\gamma = 1.25$ fixed and increasing σ from 0.05 to 0.10 raises the cross-layer share from 17.7% to 30.4%. Under the same parameter change, the corresponding share in the equity–trade network rises from 16.1% to 28.6%.

These exercises establish the geometry of the stability problem. The Robust-yet-Fragile property is not associated with either coupling parameter alone. It depends on the combination of within-layer damping, cross-layer feedback, and the network structure determining the location of the CLiFF. The following analyses use this boundary to evaluate how observed network changes and country-pair reallocation move the system toward or away from instability.

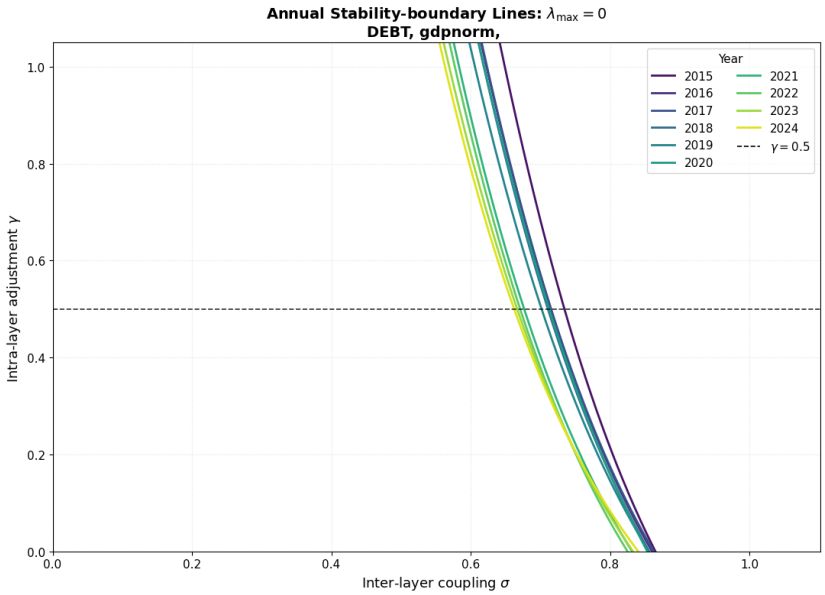
4.3. Structural Composition of the Critical Mode

4.3.1. The CLiFF Index and Its Structural Decomposition

We begin by examining how the stability boundary of the finance–trade system evolved after 2015. For each annual network, we fix within-layer



(a) Equity–Trade network



(b) Debt–Trade network

Figure 3: Annual stability boundaries for the Equity–Trade and Debt–Trade networks

Note: The upper panel presents the Equity–Trade network, and the lower panel presents the Debt–Trade network. Each curve represents the set of (γ, σ) combinations satisfying $\alpha_t(\gamma, \sigma) = 0$ for a given year. The horizontal axis reports interlayer coupling σ, and the vertical axis reports within-layer coupling γ. Points to the left of a given year's boundary satisfy $\alpha_t(\gamma, \sigma) < 0$ and are locally stable, whereas points to the right satisfy $\alpha_t(\gamma, \sigma) > 0$ and are locally unstable. The horizontal dashed line marks the reference slice $\gamma = 0.5$. Its intersection with each annual boundary defines the critical interlayer coupling σ_t^*.

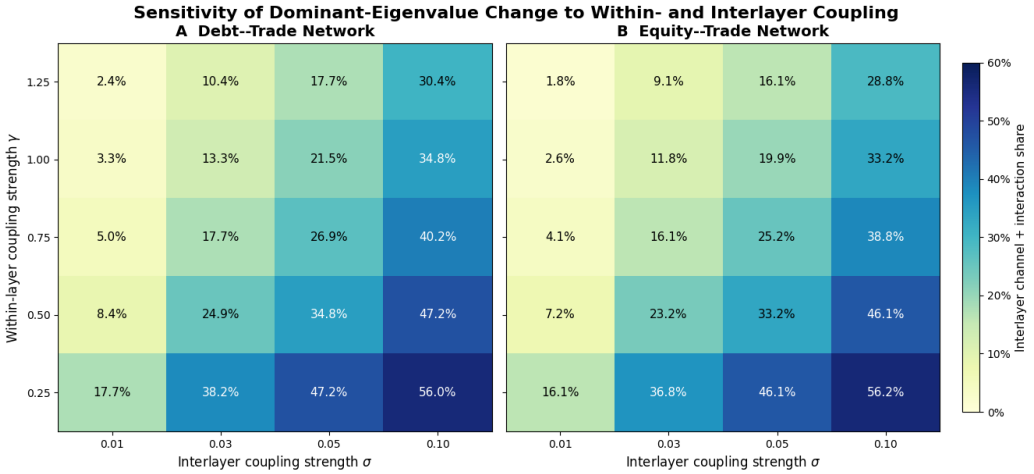


Figure 4: Cross-layer contribution to changes in the dominant eigenvalue

Note: The figure reports the percentage contribution of cross-layer paths to the change in the dominant eigenvalue for different combinations of within-layer coupling γ and cross-layer coupling σ. The left panel presents the debt–trade network, and the right panel presents the equity–trade network. Darker shading indicates a larger contribution from cross-layer interaction.

coupling at $\gamma = 0.50$ and calculate the critical cross-layer coupling σ_t^* at which the spectral abscissa of the Jacobian equals zero. This calculation identifies the intersection of each annual boundary in Figure 3 with the reference slice $\gamma = 0.50$.

To compare the annual stability boundaries with their 2015 levels, we define the *CLiFF index* as

$$I_t^{\mathrm{CLiFF}} = \frac{\sigma_t^*(\gamma = 0.50)}{\sigma_{2015}^*(\gamma = 0.50)}. \tag{10}$$

The index is normalized to one in 2015. A value above one indicates that the system can sustain stronger cross-layer coupling before reaching the stability boundary. A value below one indicates that the boundary is reached at a weaker level of cross-layer coupling than in 2015.

The relative change in the CLiFF is

$$I_t^{\mathrm{CLiFF}} - 1 = \frac{\sigma_t^* - \sigma_{2015}^*}{\sigma_{2015}^*}. \tag{11}$$

To simplify notation, we suppress the common restriction $\gamma = 0.50$ in this

and the following expressions. For example, $I_t^{\mathrm{CLiFF}} - 1 = -0.10$ means that the critical cross-layer coupling is 10 percent below its 2015 level.

We decompose the relative change into contributions from within-layer and cross-layer network structure:

$$\underbrace{I_t^{\mathrm{CLiFF}} - 1}_{\substack{\text{Relative}\\ \text{CLiFF change}}} = \underbrace{\frac{\phi_t^{\mathrm{Within}}}{\sigma_{2015}^*}}_{\substack{\text{Within-layer}\\ \text{contribution}}} + \underbrace{\frac{\phi_t^{\mathrm{Interlayer}}}{\sigma_{2015}^*}}_{\substack{\text{Cross-layer}\\ \text{contribution}}}. \tag{12}$$

Here, ϕ_t^{Within} is the change in the critical coupling associated with changes in the structures of the financial and trade layers. The term $\phi_t^{\mathrm{Interlayer}}$ is the corresponding change associated with the structure connecting the two layers. By construction,

$$\phi_t^{\mathrm{Within}} + \phi_t^{\mathrm{Interlayer}} = \sigma_t^* - \sigma_{2015}^*.$$

The upper panels of Figure 5 show that the critical cross-layer coupling generally declined after 2015 in both networks. Relative to the 2015 benchmark, the systems therefore became less tolerant of cross-layer coupling.

The timing differs across asset classes. In the debt–trade network, the CLiFF index increased temporarily in 2020 before declining again after 2021. In the equity–trade network, the index fell more sharply between 2020 and 2021. These movements coincide with the disruption of trade and financial markets during the COVID-19 period. We interpret this timing as descriptive rather than causal.

The lower panels show which part of the shift originated within the individual layers and which part originated in the structure connecting them. The black line reports the total relative change. The gray and blue bars report the within-layer and cross-layer contributions, respectively. The two contributions sum to the total change in each year.

By 2024, the critical coupling in the debt–trade network was approximately 10 percent below its 2015 level, with most of the decline accounted for by changes in cross-layer structure. In the equity–trade network, the corresponding decline was approximately 17 percent. About 14 percentage points arose from cross-layer structure and 3 percentage points from within-layer structure.

The main result is therefore not simply that the CLiFF shifted inward. Most of the decline in the system's tolerance for cross-layer coupling came

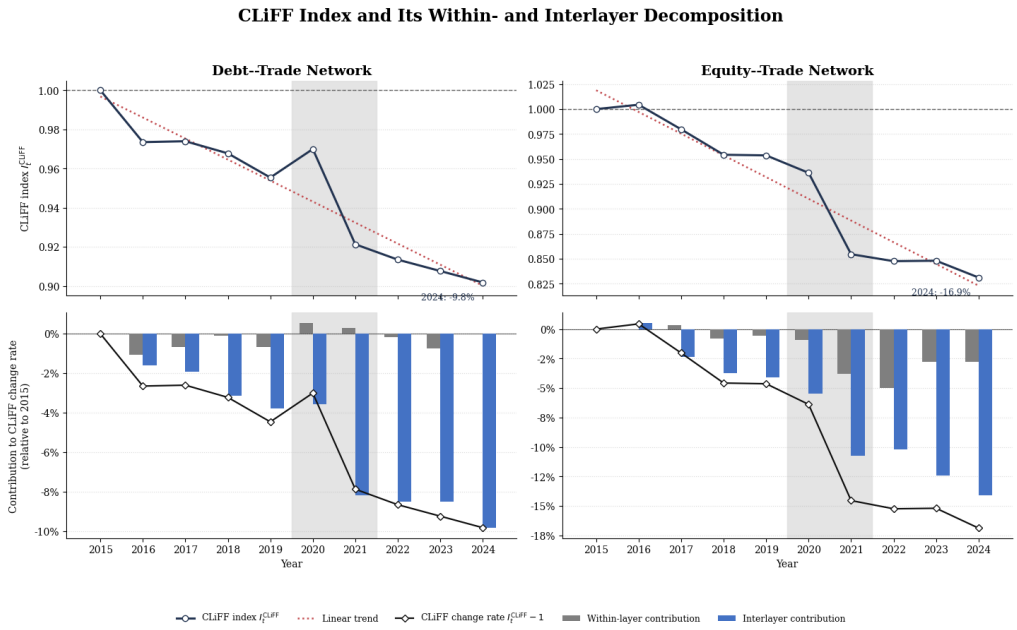


Figure 5: Evolution and Structural Decomposition of the CLiFF Index

Note: The upper panels report the CLiFF index, $I_t^{\mathrm{CLiFF}} = \sigma_t^*(\gamma = 0.50)/\sigma_{2015}^*(\gamma = 0.50)$, for the debt–trade and equity–trade networks. The critical coupling σ_t^* is the value at which the spectral abscissa of the Jacobian equals zero. An index below one indicates that the system reaches its stability boundary at weaker cross-layer coupling than in 2015. The lower panels report the relative change in the CLiFF, $I_t^{\mathrm{CLiFF}} - 1$, and its decomposition into within-layer and cross-layer contributions. Negative contributions shift the critical coupling below its 2015 level.

from changes in the paths connecting finance and trade, rather than from structural changes within either layer alone.

4.3.2. Cross-Layer Return Operators

The distinction between within-layer recursion and cross-layer feedback is central to our analysis. Cycles that remain entirely within finance or trade are already embedded in the diagonal blocks D^F and D^T. We isolate instead a cross-layer round trip: an effect leaves its originating layer, propagates through the other layer, and then returns to the same type of layer.

The two directional return operators are

$$M_F(\lambda) = B^{TF}(\lambda I - D^T)^{-1}B^{FT}, \qquad M_T(\lambda) = B^{FT}(\lambda I - D^F)^{-1}B^{TF}. \quad (13)$$

The first operator describes Finance–Trade–Finance (FTF) feedback. An effect leaves the financial layer through B^{FT}, propagates within the trade layer through $(\lambda I - D^T)^{-1}$, and returns to finance through B^{TF}. The second operator describes Trade–Finance–Trade (TFT) feedback in the opposite direction. The resolvent in each expression summarizes both direct and indirect propagation within the intermediate layer.

Under our convention that columns index origins and rows index destinations, $[M_F(\lambda)]_{ij}$ measures an FTF return from the financial node of country j to that of country i. Similarly, $[M_T(\lambda)]_{ij}$ measures a TFT return from the trade node of country j to that of country i. We refer to one such cross-layer round trip as a LOOP.

For the country-pair decomposition, we distinguish between the off-diagonal and diagonal elements of these operators. An off-diagonal element ($i \neq j$) represents a *Cross Return*: the effect returns to a country other than its origin. A diagonal element ($i = j$) represents a *Self Return*. This classification depends on the endpoints of the round trip, not on the countries traversed in the intermediate layer. A path may pass through several foreign countries and still constitute a Self Return if it ultimately returns to its country of origin.

A single LOOP is the basic unit of the country-pair decomposition. Repeated movements between finance and trade form higher-order paths through compositions of these return operators. Such paths are not discarded: together with within-layer propagation, they remain embedded in the eigenvalues of the full Jacobian. The decomposition isolates the contribution of a

single round trip while the stability criterion continues to reflect the dynamics of the integrated system.

4.4. Stability-Boundary Crossings and Their Decomposition

To identify the channels through which annual network reconfiguration erodes stability, we construct a near-boundary counterfactual for each pair of consecutive years. For the network observed in year $t-1$, we first calculate the critical cross-layer coupling σ^*_{t-1} at which the spectral abscissa equals zero. We then set the coupling parameter to

$$\bar{\sigma}_t = 0.95\sigma^*_{t-1}. \tag{14}$$

This places the initial network close to, but strictly inside, the stable region.

We next trace the effect of replacing the previous-year network with the current-year network. Holding $\bar{\sigma}_t$ fixed, we define the linear path

$$J_t(s;\bar{\sigma}_t) = (1-s)J_{t-1}(\bar{\sigma}_t) + sJ_t(\bar{\sigma}_t), \qquad 0 \le s \le 1. \tag{15}$$

The endpoints $s = 0$ and $s = 1$ correspond to the networks observed in years $t-1$ and t, respectively. Intermediate values are computational interpolations between the two observed networks; they are not estimates of the economy's within-year adjustment path. This construction preserves the full observed annual change rather than selecting particular links or searching for a network configuration that generates instability.

Let

$$s^*_t = \inf\{s \in [0,1] : \alpha[J_t(s;\bar{\sigma}_t)] \ge 0\} \tag{16}$$

denote the first point at which the interpolated system reaches the stability boundary. For transitions in which such a crossing occurs, we decompose the change in the spectral abscissa from $s = 0$ to $s = s^*_t$ into four components: within-layer structure, the aggregate scale of cross-layer LOOPs, the country-pair allocation of Cross Returns, and Self Returns.

Table 1 reveals substantial variation across assets and feedback directions. In the FTF decomposition of the debt–trade network, Scale provides the largest destabilizing contribution, 0.0814, while Role offsets more than half of this effect with a contribution of -0.0453. In the corresponding TFT decomposition, Role is instead the largest destabilizing component, at 0.0306.

Table 1: Decomposition of the Stability-Boundary Crossing

Network	Direction	Within	Scale	Role	Self	Total
Debt–Trade	FTF	+0.0023	+0.0814	−0.0453	+0.0049	+0.0434
Debt–Trade	TFT	+0.0023	+0.0040	+0.0306	+0.0064	+0.0434
Equity–Trade	FTF	+0.0142	+0.0160	+0.0053	+0.0061	+0.0415
Equity–Trade	TFT	+0.0142	−0.0089	+0.0313	+0.0049	+0.0415

Note: FTF denotes feedback that originates in the financial layer, passes through the trade layer, and returns to finance. TFT denotes feedback in the opposite direction. Within is the contribution of changes in within-layer structure. Scale is the contribution of changes in the aggregate magnitude of cross-layer LOOPs. Role is the contribution of reallocating Cross Returns across country pairs ($i \neq j$). Self is the contribution of changes in Self Returns ($i = j$). Total is the change in the spectral abscissa between the initial state and the stability-boundary crossing. Positive values move the system toward instability; negative values are stabilizing. For each network, the FTF and TFT rows provide alternative direction-specific decompositions of the same total crossing and should not be added together. Entries are rounded to four decimal places.

Country-pair reallocation therefore operates in opposite directions across the two feedback orientations of the same network.

Role is destabilizing in both decompositions of the equity–trade network. In the FTF direction, Within and Scale make the largest positive contributions, while Role provides a smaller additional increase. In the TFT direction, Scale is stabilizing (-0.0089), but this effect is more than offset by the positive Role contribution (0.0313). The boundary crossing in this direction is therefore associated primarily with a reallocation of Cross Returns rather than an expansion in their aggregate scale.

The results establish that the stability effect of cross-layer interaction cannot be inferred from LOOP size alone. Changes in Scale and Role may reinforce one another, as in the equity–trade FTF decomposition, or operate in opposite directions, as in the debt–trade FTF and equity–trade TFT decompositions. Role Switching is consequently not intrinsically stabilizing or destabilizing. Its effect depends on which country pairs acquire greater weight and on the sensitivity of the stability boundary to those paths.

5. Simulation with State-Dependent Coupling

5.1. A Conditional Simulation with State-Dependent Coupling

The preceding analysis held within-layer coupling γ and cross-layer coupling σ fixed in order to map the stability boundary. We now allow both forms of coupling to evolve with the state of the system. Specifically, effective cross-layer coupling Ω responds to global financial conditions, its own lagged change, and the previous year's stability margin. Within-layer coupling γ then adjusts partway toward the level required to restore the target margin relative to the CLiFF.

The simulation begins in 2015 with Ω set at 95 percent of its critical value. The system therefore starts with a 5 percent stability margin. This Near-CLiFF initialization is not intended to predict the timing of a crisis. It provides a common conditional benchmark from which to examine whether state-dependent adjustment can move the system across its stability boundary as the observed financial and trade networks evolve.

The simulation combines the spectral stability framework developed above with state-dependent updating of the coupling parameters (Pecora and Carroll, 1998). It is not a fully endogenous model of network formation. The global financial cycle enters as an external input, while the annual financial and trade structures are taken from the data. Conditional on these inputs, Ω and γ are updated recursively from the preceding state. The global financial cycle is measured following the international financial-factor literature (Miranda-Agrippino and Rey, 2020).

In the theoretical analysis, σ denotes a generic cross-layer coupling parameter. In the simulation, it is replaced by the asset- and time-specific effective coupling $\Omega_{a,t}$, where $a \in \{\text{Debt}, \text{Equity}\}$ and t indexes years. This change in notation distinguishes the generic parameter used to map the stability boundary from the time-varying coupling generated by the conditional simulation.

The parameter $\Omega_{a,t}$ does not create new cross-layer links. It scales the aggregate strength with which effects propagate through the financial–trade paths already present in the observed network. An increase in $\Omega_{a,t}$ strengthens transmission through these paths, whereas a decrease weakens it. The paths themselves remain determined by the observed financial and trade structures in year t.

For each asset and year, local stability is measured by

$$\alpha_{a,t} = \max_k \operatorname{Re}\left\{\lambda_k\left[J_{a,t}\left(\gamma_{a,t}, \Omega_{a,t}\right)\right]\right\}. \tag{17}$$

The system is locally stable when $\alpha_{a,t} < 0$ and locally unstable when $\alpha_{a,t} > 0$. The condition $\alpha_{a,t} = 0$ defines the CLiFF. The simulation therefore traces how changes in the observed network structure, external global financial conditions, and state-dependent coupling jointly move the Debt–Trade and Equity–Trade systems toward or away from their respective stability boundaries.

5.1.1. Initialization and the Cross-Layer Stability Margin

The simulation is designed to examine whether state-dependent changes in within- and cross-layer coupling can move a system from stability to instability when it begins near the CLiFF. We therefore initialize the 2015 effective cross-layer coupling at 95 percent of its critical value:

$$\Omega_{a,2015} = 0.95\,\Omega^*_{a,2015}\left(\gamma_{a,2015}\right). \tag{18}$$

Here, $\Omega^*_{a,t}(\gamma_{a,t})$ denotes the critical cross-layer coupling at which the spectral abscissa equals zero. This initialization does not assume that either observed network was actually close to its CLiFF in 2015. It is a conditional experiment that places the Debt–Trade and Equity–Trade systems at the same relative distance from their respective stability boundaries. The initial values of Ω need not be equal because the two networks have different critical coupling levels.

We measure the remaining distance to the boundary using the *cross-layer coupling stability margin*,

$$R_{a,t} = 1 - \frac{\Omega_{a,t}}{\Omega^*_{a,t}\left(\gamma_{a,t}\right)}. \tag{19}$$

A value of $R_{a,t} = 0.05$ means that current cross-layer coupling is 5 percent below its critical value. The condition $R_{a,t} = 0$ corresponds to the CLiFF, while $R_{a,t} = -0.02$ means that current coupling exceeds the critical value by 2 percent. Thus,

$$R_{a,t} > 0 \quad \text{indicates a positive stability margin,}$$

$$R_{a,t} = 0 \quad \text{identifies the CLiFF,}$$

and

$R_{a,t} < 0$ indicates that the critical coupling has been exceeded.

The margin $R_{a,t}$ is a relative-distance measure, not an independent stability criterion. Final stability classification is based on the sign of the spectral abscissa $\alpha_{a,t}$.

5.1.2. Nonlinear Updating of Cross-Layer Coupling

The effective cross-layer coupling $\Omega_{a,t}$ evolves with the global financial environment and the state inherited from the preceding year. The simulation combines the financial and trade networks observed over 2015–2024 with an observed global financial factor (GFC). We introduce no additional stochastic shocks. Conditional on the initial state, the path of $\Omega_{a,t}$ is determined by the observed GFC and lagged system variables, while annual stability is evaluated using the network structure observed in each year.

The updating equation uses the signed standardized factor GFC_t^z. This variable differs from $|GFC_t^z|$, the absolute deviation from the sample mean used in the supplementary analysis in Appendix Appendix B. The former retains the direction of global financial conditions; the latter measures only the magnitude of the deviation from the sample mean.

Cross-layer coupling evolves according to

$$\begin{aligned}\Delta \log \Omega_{a,t} = \widehat{\alpha}_a + \widehat{\rho}\, \Delta \log \Omega_{a,t-1} + \widehat{\beta}_G GFC_t^z \\ + \beta_A [R_{a,t-1}]_+ + \delta H(R_{a,t-1}) - \chi [-R_{a,t-1}]_+ .\end{aligned} \tag{20}$$

where

$$[x]_+ \equiv \max\{x, 0\}. \tag{21}$$

The term $[R_{a,t-1}]_+$ is positive only when the system retains a positive stability margin. By contrast, $[-R_{a,t-1}]_+$ is positive only after cross-layer coupling has exceeded its critical value.

The nonlinear activation function is

$$H(R) = \frac{1}{1 + \exp[k(R - \tau_R)]}. \tag{22}$$

Because $H(R)$ is decreasing in R, it becomes larger as the stability margin narrows and the system approaches the CLiFF. The parameter τ_R determines where this nonlinear response becomes active, while k governs its steepness.

The level of effective cross-layer coupling is updated recursively:

$$\Omega_{a,t} = \Omega_{a,t-1} \exp(\Delta \log \Omega_{a,t}) \,. \tag{23}$$

Equation (20) combines an empirical component with three state-dependent terms. The first line captures asset-specific average growth, persistence in coupling growth, and the signed global financial factor. The second line introduces state dependence. When the system remains below the CLiFF, $\beta_A[R_{a,t-1}]_+$ allows coupling to expand with the available margin. The term $\delta H(R_{a,t-1})$ introduces an additional nonlinear response as that margin narrows. Once the critical value has been exceeded, $-\chi[-R_{a,t-1}]_+$ provides a countervailing contraction.

The coefficients $\widehat{\alpha}_a$, $\widehat{\rho}$, and $\widehat{\beta}_G$ are estimated from the observed data. In contrast, β_A, δ, τ_R, k, and χ are simulation parameters governing adjustment near and beyond the CLiFF; they are not estimated behavioral coefficients. Their values, the estimation procedure for the empirical coefficients, and the parameter ranges used in the sensitivity analysis are reported in Table Appendix A.1 in the Appendix.

5.1.3. State-Dependent Adjustment of Within-Layer Coupling

As cross-layer coupling $\Omega_{a,t}$ changes, so does the within-layer coupling required to preserve a given stability margin. We set the target margin to $r^* = 0.05$. For a given value of $\Omega_{a,t}$, the critical cross-layer coupling consistent with this target is

$$\widetilde{\Omega}^*_{a,t} = \frac{\Omega_{a,t}}{1 - r^*} = \frac{\Omega_{a,t}}{0.95}. \tag{24}$$

The intuition is straightforward. If current coupling is $\Omega_{a,t} = 0.95$, the target CLiFF is $0.95/0.95 = 1.00$. Current coupling then lies 5 percent below its critical value.

We next solve for the target within-layer coupling $\widetilde{\gamma}_{a,t}$ that places the CLiFF at $\widetilde{\Omega}^*_{a,t}$:

$$\alpha_{a,t}\left(\widetilde{\gamma}_{a,t}, \widetilde{\Omega}^*_{a,t}\right) = \max_k \mathrm{Re}\left\{\lambda_k\left[J_{a,t}\left(\widetilde{\gamma}_{a,t}, \frac{\Omega_{a,t}}{0.95}\right)\right]\right\} = 0. \tag{25}$$

Adjustment is partial rather than instantaneous. In each year, realized within-layer coupling closes 25 percent of the gap between its lagged value and its target:

$$\gamma_{a,t} = \gamma_{a,t-1} + 0.25\,(\widetilde{\gamma}_{a,t} - \gamma_{a,t-1})\,. \tag{26}$$

If no target value exists within the prespecified search range for γ, we retain its lagged value. The search range and numerical procedure are reported in Table Appendix A.1 in the Appendix.

The sequence of updates within each year is therefore

$$\boxed{R_{a,t-1} \longrightarrow \Omega_{a,t} \longrightarrow \widetilde{\gamma}_{a,t} \longrightarrow \gamma_{a,t} \longrightarrow R_{a,t}}. \tag{27}$$

The previous year's stability margin affects current cross-layer coupling. Given that coupling, the model computes the within-layer coupling required to restore the target margin. Partial adjustment then determines realized $\gamma_{a,t}$, which in turn determines the current CLiFF and the resulting stability margin.

The Equity–Trade network illustrates how adjustment can fall behind. In 2016, effective cross-layer coupling was

$$\Omega_{\text{Equity},2016} = 0.63222.$$

Maintaining a 5 percent margin would have required a CLiFF of

$$\widetilde{\Omega}^{*}_{\text{Equity},2016} = \frac{0.63222}{0.95} = 0.66550.$$

Because γ adjusted only partially, the realized CLiFF reached only 0.64464. The remaining stability margin was therefore

$$R_{\text{Equity},2016} = 1 - \frac{0.63222}{0.64464} = 0.0193. \tag{28}$$

Thus, the margin fell from its 5 percent target to 1.93 percent. In 2017, cross-layer coupling increased to 0.65513, while the realized CLiFF was 0.64204:

$$R_{\text{Equity},2017} = 1 - \frac{0.65513}{0.64204} = -0.0204. \tag{29}$$

Within-layer adjustment was no longer sufficient to preserve stability. The Equity–Trade system exceeded its critical coupling by approximately 2.04 percent and entered the locally unstable region.

5.2. State-Dependent Coupling Adjustment and Network Stability

We now examine whether state-dependent adjustment is sufficient to preserve stability as the observed financial and trade networks evolve. In each year, interlayer coupling $\Omega_{a,t}$ is first updated using the global financial environment, its own lagged change, and the previous year's stability margin. Within-layer coupling $\gamma_{a,t}$ then adjusts partially toward the level required to restore the target margin of 5 percent. Iterating these two steps produces annual stability paths for the Debt–Trade and Equity–Trade networks from 2015 through 2024.

The two coupling parameters do not evolve independently. An increase in $\Omega_{a,t}$ changes the adjustment in $\gamma_{a,t}$ required to restore the target margin. Realized $\gamma_{a,t}$, together with the network observed in year t, then determines the critical interlayer coupling

$$\Omega^*_{a,t} \equiv \Omega^*_{a,t}(\gamma_{a,t}).$$

The CLiFF therefore varies with realized within-layer coupling and the underlying network structure, while the system's position relative to it also depends on realized interlayer coupling.

The exercise produces a clear contrast between the two asset networks. As Figure 6 shows, Debt–Trade remains locally stable throughout the sample, whereas Equity–Trade crosses the stability boundary in 2017 and remains in the unstable region thereafter.

The middle panels compare realized interlayer coupling with the critical value implied by the contemporaneous network and realized within-layer coupling. If

$$\Omega_{a,t} < \Omega^*_{a,t},$$

the system remains on the stable side of the CLiFF. If

$$\Omega_{a,t} > \Omega^*_{a,t},$$

realized coupling exceeds the stability limit. The sign of the spectral abscissa in the lower panels confirms this classification.

For Debt–Trade, $\Omega_{a,t}$ remains below its critical value in every year. The gap narrows substantially in 2021, but $\alpha_{\mathrm{Debt},t}$ remains negative. Under the specified adjustment rule, within-layer adjustment is therefore sufficient to preserve local stability.

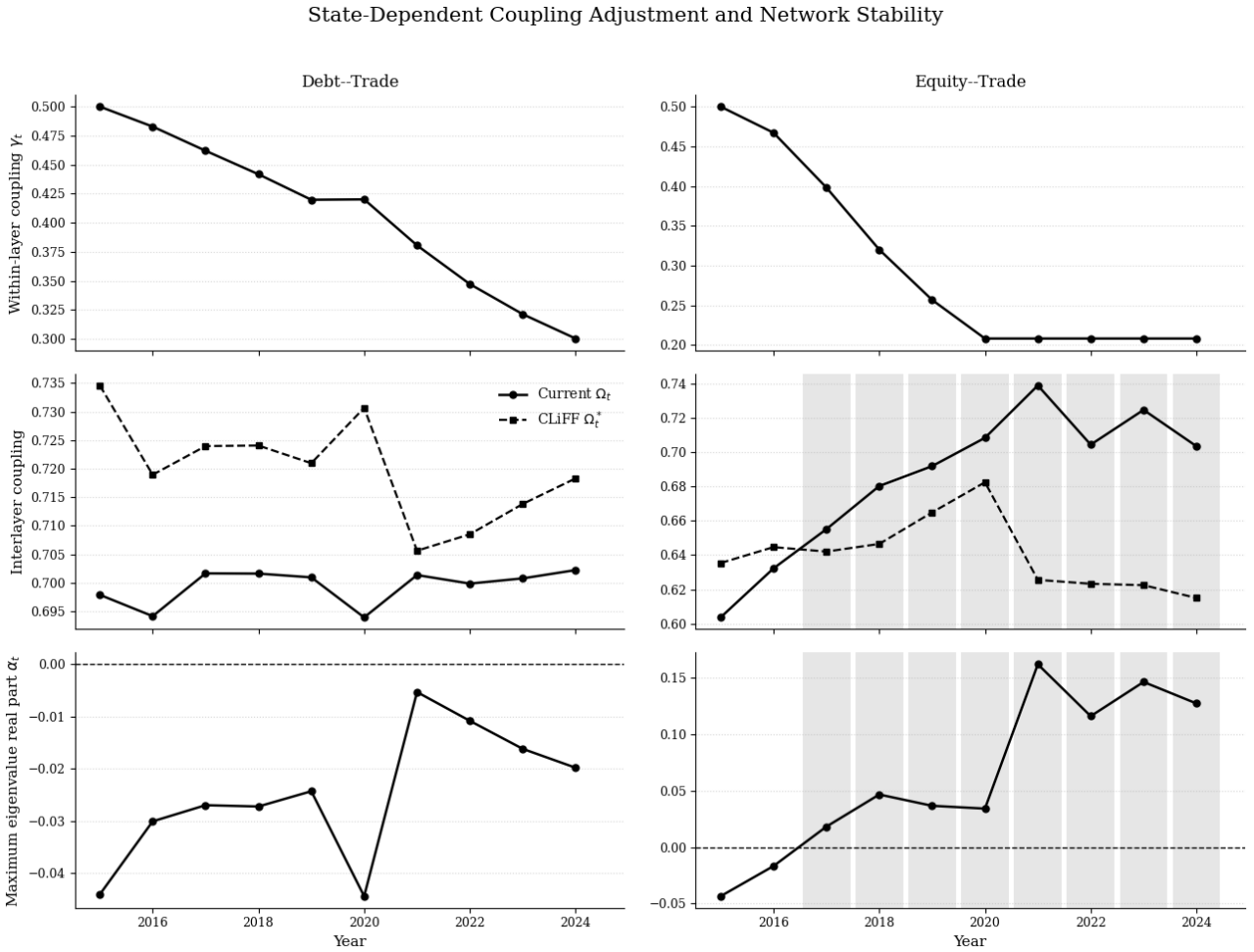


Figure 6: State-dependent coupling adjustment and network stability. The upper panels report adjusted within-layer coupling $\gamma_{a,t}$. In the middle panels, the solid line denotes current interlayer coupling $\Omega_{a,t}$, and the dashed line denotes the critical interlayer coupling $\Omega^*_{a,t}$. The lower panels report the spectral abscissa $\alpha_{a,t}$ of the integrated two-layer system. The system is locally stable when $\alpha_{a,t} < 0$, lies on the CLiFF when $\alpha_{a,t} = 0$, and is locally unstable when $\alpha_{a,t} > 0$. Shaded areas indicate years in which the system lies in the unstable region.

The Equity–Trade network follows a different path. In 2017, realized interlayer coupling rises to

$$\Omega_{\text{Equity},2017} = 0.65513,$$

while the corresponding critical value is

$$\Omega^*_{\text{Equity},2017} = 0.64204.$$

Interlayer coupling therefore exceeds the CLiFF, and the spectral abscissa becomes positive. This is the first simulated transition into the unstable region.

Figure 6 identifies when the two stability paths diverge. Figure 7 shows how the divergence arises by locating each annual outcome in the (Ω, γ) plane.

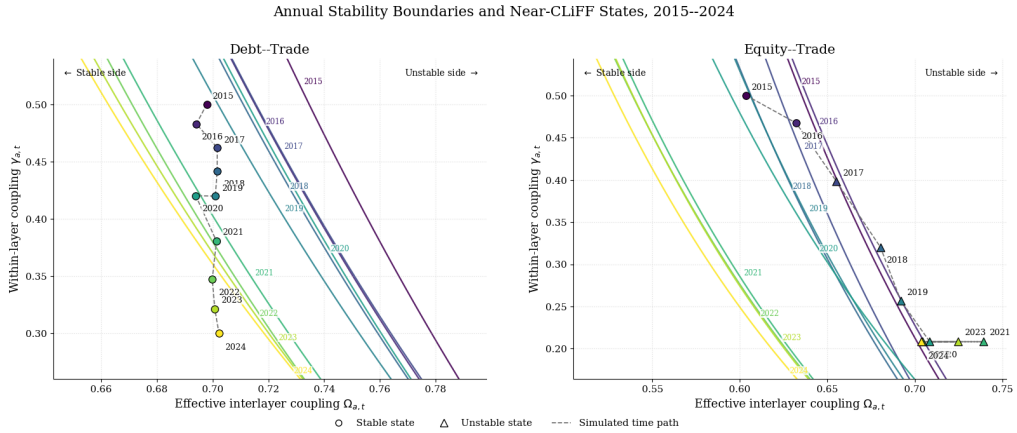


Figure 7: Annual stability boundaries and the path of the state-dependent simulation. The horizontal axis reports interlayer coupling $\Omega_{a,t}$, and the vertical axis reports within-layer coupling $\gamma_{a,t}$. Each curve is the locus at which the spectral abscissa equals zero for the network observed in that year and thus separates the stable and unstable regions. Circles denote stable states, triangles denote unstable states, and the gray dashed line traces the simulated path from 2015 to 2024.

Figure 7 plots each simulated annual state against the stability boundary generated by the network observed in the same year. Each point represents the pair $(\Omega_{a,t}, \gamma_{a,t})$ implied by the state-dependent updating rules. Because the underlying network changes over time, an annual state must be evaluated

against its own contemporaneous boundary rather than against a boundary held fixed across years.

A point to the left of its corresponding boundary satisfies $\Omega_{a,t} < \Omega^*_{a,t}$ and is locally stable. A point to the right satisfies $\Omega_{a,t} > \Omega^*_{a,t}$ and is locally unstable. Every Debt–Trade observation lies on the stable side of its corresponding boundary. Equity–Trade lies on the stable side in 2015 and 2016, crosses to the unstable side in 2017, and remains there through 2024.

The comparison highlights why the level of $\Omega_{a,t}$ alone is not sufficient to characterize stability. The relevant object is the position of realized coupling relative to the boundary generated by the contemporaneous network. As that boundary shifts over time, the same combination of Ω and γ may be stable under one network configuration but unstable under another.

For Equity–Trade, the constraint on within-layer adjustment becomes binding from 2021 onward. No value of $\gamma_{a,t}$ within the prespecified search range restores the target stability margin of 5 percent. The simulation therefore retains the previous year's value of $\gamma_{a,t}$, and the system remains unstable. This result does not imply that the real economy has exhausted all possible forms of adjustment. It means more narrowly that the target margin cannot be restored within the adjustment rule and parameter range imposed in this conditional simulation.

The central finding is that state dependence does not necessarily make the system self-correcting. Starting from the same relative stability margin and applying the same adjustment rule, Debt–Trade remains within its evolving stability boundary, whereas Equity–Trade crosses that boundary. The difference is not driven by coupling strength alone. It emerges from the interaction between coupling adjustment and the changing architecture of the two networks.

5.3. Decomposing the Stability Path

Why do the Debt–Trade and Equity–Trade networks follow such different stability paths? To answer this question, we decompose the annual change in the spectral abscissa into five components:

$$\Delta\alpha_{a,t} = \phi^{\text{Within}}_{a,t} + \phi^{\gamma}_{a,t} + \phi^{\Omega}_{a,t} + \phi^{\text{Role}}_{a,t} + \phi^{\text{Self}}_{a,t}. \tag{30}$$

The first component, $\phi^{\text{Within}}_{a,t}$, captures changes in within-layer network structure. The second, $\phi^{\gamma}_{a,t}$, captures the state-dependent adjustment of within-layer coupling. The third, $\phi^{\Omega}_{a,t}$, measures the contribution of changes

in interlayer coupling. The remaining two components allocate the contribution of changes in LOOP configuration according to their return destination. The term $\phi_{a,t}^{\text{Role}}$ is the contribution assigned to changes in off-diagonal LOOPs that return to a country other than their country of origin, whereas $\phi_{a,t}^{\text{Self}}$ is the contribution assigned to changes in diagonal LOOPs that return to the originating country.

The decomposition proceeds in two stages. In the first stage, we consider four upper-level factors: within-layer structure, within-layer coupling γ, interlayer coupling Ω, and LOOP configuration. For any given ordering, the marginal contribution of a factor is defined as the change in the spectral abscissa when that factor is switched from its previous-year state to its current-year state. Factors already introduced are held at their current-year values, while factors not yet introduced remain at their previous-year values. We calculate these marginal contributions for all $4! = 24$ possible orderings and average them across orderings.

In the second stage, we allocate the contribution of LOOP configuration between Role Switching and Self Return. The former corresponds to off-diagonal return paths, while the latter corresponds to diagonal paths that return to the country of origin. Thus, the five components in Equation (30) do not result from switching five independent factors. They combine an order-averaged decomposition across four upper-level factors with a subsequent allocation of the LOOP-configuration contribution by return destination. Positive values raise the spectral abscissa and therefore contribute to instability; negative values contribute to stability.

Figure 8 shows that the mechanisms underlying the two stability paths differ sharply. In the Debt–Trade network, the adjustment of within-layer coupling γ operates almost consistently in the stabilizing direction. Role Switching, by contrast, raises the spectral abscissa in most years. Its destabilizing contribution is particularly large in 2021 and accounts for much of the increase in the spectral abscissa that year.

Interlayer contributions fluctuate more strongly in the Equity–Trade network. Changes in Ω and Role Switching both make substantial destabilizing contributions in 2021. In 2022, the contribution of Ω reverses sign and becomes stabilizing. Neither interlayer coupling nor LOOP configuration therefore has an inherently stabilizing or destabilizing effect. The contribution of each depends on the network observed in that year and on the direction of the dominant mode.

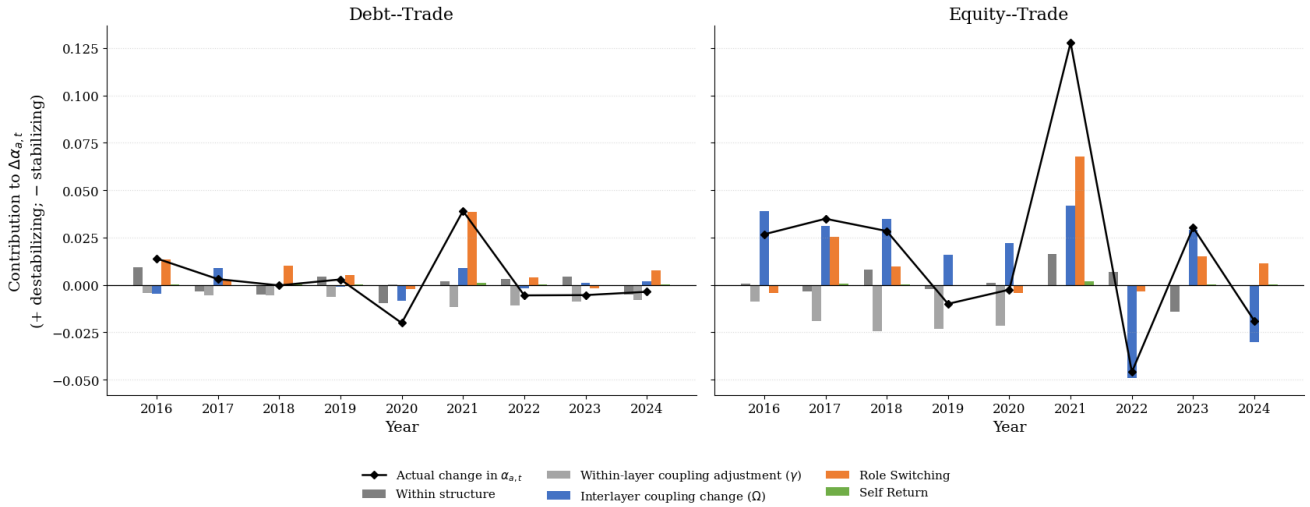


Figure 8: Contributions to annual changes in the spectral abscissa. The black line reports the realized annual change $\Delta\alpha_{a,t}$. The bars report the contributions obtained by averaging marginal effects over all orderings of the four upper-level factors and then allocating the LOOP-configuration component between Role Switching and Self Return.

Table 2: Cumulative contributions and gross shares of the five components, 2015–2024

Asset	Component	Net contribution	Gross share (%)
Debt	Within-layer structure	0.0003	20.0
	Within-layer coupling adjustment	−0.0610	26.4
	Interlayer coupling change	0.0054	15.9
	Role Switching	0.0776	36.8
	Self Return	0.0019	0.9
	Total change	**0.0242**	—
Equity	Within-layer structure	0.0134	9.0
	Within-layer coupling adjustment	−0.0969	16.4
	Interlayer coupling change	0.1343	49.8
	Role Switching	0.1171	24.1
	Self Return	0.0029	0.6
	Total change	**0.1708**	—

Notes: The sample consists of the nine annual transitions observed over 2015–2024. Net contribution is the sum of the signed annual contributions of each component. Total change is the sum of the cumulative contributions of all five components. Gross share is the sum of the absolute annual contributions of a given component divided by the corresponding sum across all five components. It is therefore neither a coefficient of determination nor a causal share of explained variation.
Each upper-level contribution is obtained by averaging its marginal effect over all 24 possible orderings of the four upper-level factors. The contribution of LOOP configuration is then allocated between Role Switching, for paths that return to a country other than their origin, and Self Return, for paths that return to the originating country.

Table 2 summarizes the cumulative contributions and confirms that changes in the stability of the two networks arise through different channels. For Debt–Trade, Role Switching accounts for the largest gross share of total variation, at 36.8 percent, and has a cumulative contribution of 0.0776. Much of this destabilizing effect is offset by the adjustment of within-layer coupling, whose cumulative contribution is −0.0610. The resulting total change in the spectral abscissa is therefore limited to 0.0242.

For Equity–Trade, changes in interlayer coupling account for 49.8 percent of gross variation, while Role Switching accounts for a further 24.1 percent. Both components make positive cumulative contributions and together out-

weigh the stabilizing adjustment in γ. As a result, the cumulative increase in the spectral abscissa reaches 0.1708, substantially exceeding that of the Debt–Trade network.

Self Return contributes comparatively little in either network. Its cumulative contribution is 0.0019 for Debt–Trade and 0.0029 for Equity–Trade. Over this period, the principal contribution of changes in LOOP configuration therefore comes not from paths returning to the same country, but from the reallocation of return paths across countries.

The global financial cycle is not introduced as a separate factor in this decomposition. Information from the GFC enters the updating rule for interlayer coupling Ω and is also indirectly reflected in the subsequent state-dependent adjustment of γ. The contribution of Ω therefore cannot be interpreted as the stand-alone effect of the GFC. The association between the GFC and the magnitude of cross-layer responses is examined separately in Section 5.3.4 and Appendix Appendix B, using the combined LOOP-configuration contribution of Role Switching and Self Return.

The decomposition points to a broader conclusion. Stability depends not only on how strongly the two layers are coupled, but also on which country pairs carry the resulting feedback. Even when adjustment within the individual layers is stabilizing, changes in interlayer coupling and the reallocation of LOOPs across country pairs can jointly move the integrated system toward instability.

5.3.1. Country-Pair Contributions of Role Switching

We next identify the country pairs underlying the aggregate contribution of Role Switching. For a LOOP originating in country j and returning to country i, the country-pair contribution is

$$\phi_{ij,t}^{\text{Role}} = \Delta C_{ij,t}\, \bar{g}_{ij,t}, \tag{31}$$

where $\Delta C_{ij,t}$ is the change in the country-pair composition weight, holding the overall scale of the LOOP fixed, and $\bar{g}_{ij,t}$ is the sensitivity of the spectral abscissa to that change. Consistent with the matrix convention used throughout the paper, the index pair ij denotes a path from origin country j to destination country i.

The scalar weight $C_{ij,t}$ used in this decomposition should not be confused with the directional mapping matrices C^{FT} and C^{TF}. The latter describe the full cross-layer mapping between the financial and trade layers, whereas

$C_{ij,t}$ records the relative weight of a particular origin–destination pair within a LOOP configuration.

A positive value, $\phi_{ij,t}^{\text{Role}} > 0$, indicates that the change in the country-pair configuration raises the spectral abscissa and therefore contributes to instability. A negative value indicates a stabilizing contribution. These changes do not represent movements of funds themselves. They measure changes in the relative importance of country pairs in carrying cross-layer feedback.

Tables 3 and 4 report the country pairs that remain under several screening criteria based on contribution size and sensitivity. Specifically, we combine the 25, 50, and 100 largest absolute contributions with sensitivity thresholds corresponding to the top 25 and top 10 percent, producing six screening criteria.

These screens are not tests of statistical significance for individual country pairs. Their purpose is more limited: to verify that the reported patterns do not depend on a single display threshold and to identify, descriptively, the country pairs carrying the largest aggregate Role contributions.

Table 3: Major destabilizing Role Switching pairs

Network and path	Year	Country pair	Role contribution
Debt–Trade FTF	2017	China → France	+0.001174
Debt–Trade FTF	2017	Germany → United States	+0.000937
Debt–Trade FTF	2021	Belgium → Japan	+0.000518
Debt–Trade FTF	2019	Belgium → United Kingdom	+0.000207
Equity–Trade TFT	2018	United States → Mexico	+0.000541
Equity–Trade TFT	2017	Hong Kong → Malaysia	+0.000330

For these pairs, changes in LOOP composition, combined with high sensitivity, raise the spectral abscissa. China → France and Germany → United States, for example, are among the main destabilizing pairs in the 2017 Debt–Trade FTF LOOP. In the Equity–Trade TFT LOOP, United States → Mexico and Hong Kong → Malaysia emerge as major destabilizing contributors.

The presence of highly sensitive pairs on the stabilizing side is equally informative. Belgium → United Kingdom, for example, makes a stabilizing contribution to the Equity–Trade FTF LOOP in 2020. This does not imply that the pair is intrinsically "safe." Rather, its weight within a highly sensitive LOOP path declines in that year, lowering the spectral abscissa.

Table 4: Major stabilizing Role Switching pairs

Network and path	Year	Country pair	Role contribution
Debt–Trade FTF	2017	Singapore → United Kingdom	−0.000796
Debt–Trade FTF	2017	Sweden → United Kingdom	−0.000547
Debt–Trade FTF	2021	United Kingdom → France	−0.000464
Debt–Trade FTF	2018	Belgium → France	−0.000464
Equity–Trade FTF	2020	Belgium → United Kingdom	−0.000665
Equity–Trade FTF	2022	Norway → United States	−0.000357
Equity–Trade FTF	2022	Denmark → United States	−0.000344
Equity–Trade FTF	2019	China → Japan	−0.000236
Equity–Trade TFT	2023	Hong Kong → Mexico	−0.000412
Equity–Trade TFT	2019	United Kingdom → Switzerland	−0.000398
Equity–Trade TFT	2022	Sweden → Mexico	−0.000385
Equity–Trade TFT	2022	Sweden → Germany	−0.000377
Equity–Trade TFT	2019	Singapore → Thailand	−0.000306

Role Switching can therefore operate in either direction. The sign of a country-pair contribution is not an immutable attribute of the countries involved. It is determined jointly by the change in that pair's LOOP weight and the sensitivity of system-wide stability to that change.

5.3.2. Directional Test for High-Sensitivity Paths

The country-pair tables identify individual paths that make sizable Role Switching contributions. We now ask a related aggregate question: does the contribution of the high-sensitivity region exhibit a systematic direction over time?

For each asset a, LOOP direction d, and year t, we sum the Role Switching contributions of country pairs in the top quartile of the sensitivity distribution:

$$\Phi^{H}_{a,d,t} = \sum_{(i,j)\in H_{a,d,t}} \phi^{\text{Role}}_{ij,t}, \tag{32}$$

where $H_{a,d,t}$ denotes the set of country pairs in the top 25 percent of sensitivity for the corresponding asset, direction, and year. A positive value of $\Phi^{H}_{a,d,t}$ indicates that the high-sensitivity region raises the spectral abscissa and

therefore contributes to instability. A negative value indicates a stabilizing contribution.

Let $p_{a,d}$ denote the probability that the aggregate contribution is positive for a given asset and LOOP direction. We test

$$H_0 : p_{a,d} = 0.5, \qquad H_1 : p_{a,d} \neq 0.5. \tag{33}$$

Under the null hypothesis, positive and negative contributions are equally likely; hence, the high-sensitivity region has no systematic direction over time.

We conduct a two-sided exact binomial test using the signs of the nine annual contributions from 2016 through 2024. The test treats the annual signs as independent trials. Because we test four asset-direction combinations—Debt–Trade and Equity–Trade, each for FTF and TFT—we adjust the resulting p-values using the Holm procedure (Holm, 1979). This adjustment controls the family-wise error rate across the four tests at 5 percent.

For Debt–Trade FTF, Debt–Trade TFT, and Equity–Trade FTF, the sign of the high-sensitivity contribution varies across years, and the null hypothesis is not rejected. Equity–Trade TFT is the notable exception: $\Phi^{H}_{a,d,t} > 0$ in all nine years. The unadjusted two-sided p-value is 0.0039, and the conclusion remains significant after the Holm adjustment, with an adjusted p-value of 0.0156.

Thus, under the maintained assumption that annual signs constitute independent trials, the null of equal directional probabilities is rejected for Equity–Trade TFT. The sample nevertheless contains only nine annual observations and does not permit a satisfactory treatment of serial dependence. We therefore interpret the result as exploratory evidence that high-sensitivity Role Switching contributions were systematically tilted toward instability in the Equity–Trade TFT LOOP during the sample period.

This aggregate result is consistent with the individual destabilizing paths reported in Table 3, including United States → Mexico and Hong Kong → Malaysia. It does not, however, imply that these or any other particular country pairs carried the destabilizing function throughout the sample. The direction of the aggregate high-sensitivity contribution may persist even as the country pairs carrying that contribution change over time.

5.3.3. Role Switching: Aggregate Persistence and Country-Pair Reallocation

Three conclusions emerge from these results.

First, Role Switching is not a peripheral source of stability variation. It accounts for 36.8 percent of gross variation in the Debt–Trade network and 24.1 percent in the Equity–Trade network. Understanding stability therefore requires more than measuring the strength of interlayer coupling. It also requires identifying how the country pairs carrying cross-layer feedback are configured.

Second, Role Switching can be either stabilizing or destabilizing, both in principle and in the data. The presence of country pairs on both sides of the decomposition, as reported in Tables 3 and 4, reflects this duality. The sign of a Role contribution is not a permanent attribute of a country or country pair. It depends on the interaction between the change in the pair's LOOP weight and the sensitivity of system-wide stability to that change.

Third, this directional ambiguity has one notable path-specific exception. In the Equity–Trade TFT LOOP, the aggregate contribution of the high-sensitivity region is destabilizing in all nine years. The persistence of this aggregate direction does not, however, establish that the same country pairs carry the destabilizing contribution every year. The identities of the leading pairs reported above vary across years, a pattern that is descriptively consistent with turnover in the paths carrying that contribution.

The common-mode analysis in Appendix Appendix C provides complementary evidence. Years that align more closely with the configuration pattern shared across the sample exhibit greater dispersion in country-pair Role changes and a larger total unoffset Role contribution. The recurring component is therefore associated with stronger reallocation across country pairs and with larger stabilizing and destabilizing contributions before they offset one another. It is not associated with a uniform expansion of all LOOPs or with a systematic increase in country-pair sensitivity.

Taken together, the results distinguish persistence in function from persistence in its carriers. In the Equity–Trade TFT LOOP, the destabilizing function persists at the aggregate level, while the country-pair evidence is consistent with its allocation changing over time. This distinction is central to what we call the *Dionysian mechanism*: a destabilizing function can remain embedded in the network even as the particular relationships through which it operates are reconfigured.

This separation between function and carrier raises a final question: whether the magnitude and direction of the cross-layer response also vary with global financial conditions. We examine this question next by relating the observed global financial cycle to the gross and net responses of the FTF and TFT

channels.

5.3.4. The Global Financial Cycle and Cross-Layer Responses

We finally ask whether cross-layer responses intensify when global financial conditions move further from their sample mean. For each asset, $F_{a,t}$ and $T_{a,t}$ denote the net contributions of LOOP configuration changes through the FTF and TFT channels, including both Role Switching and Self Return. We relate the absolute standardized GFC deviation, $|GFC_t^z|$, to the gross response $|F_{a,t}| + |T_{a,t}|$ and the net response $F_{a,t} + T_{a,t}$. The analysis uses nine annual observations from 2016 through 2024. Appendix Appendix B reports the definitions, permutation procedure, and full results.

For Equity–Trade, larger GFC deviations are associated with stronger cross-layer responses before the FTF and TFT channels offset one another. The rank correlation with the gross response is 0.700 ($p = 0.043$, unadjusted). The association with the net response is not statistically clear, nor do we find clear relationships for Debt–Trade.

The result is therefore about response intensity, not direction. Global financial conditions may activate stronger cross-layer movements without determining whether their combined effect is stabilizing or destabilizing. That outcome depends on how the FTF and TFT contributions offset one another and on the country-pair configuration carrying them.

Given the nine-year sample, the lack of a satisfactory treatment of serial dependence, and the absence of a multiple-testing adjustment, we interpret this evidence as exploratory. It suggests that the GFC covaries with the intensity of Equity–Trade reconfiguration, not that it determines its net stability effect.

6. Discussion

International finance and trade do more than allocate capital and goods across countries. Together, they determine the routes through which a disturbance leaves one market, propagates through another, and returns. Such round trips turn international interdependence into feedback.

The central result of this paper is that a system can lose local stability even when its financial and trade layers are each stable in isolation. The source of this result is not cross-layer interaction alone. Stability also depends on how cross-layer feedback is organized: which country pairs carry

it, through which routes it returns, and how those routes interact with the prevailing network structure.

6.1. Layer Stability Does Not Imply System Stability

Research on financial networks has shown that the same connections that facilitate liquidity provision and risk sharing can also amplify shocks (Haldane, 2013; Elliott et al., 2014; Acemoglu et al., 2015). This paper extends that insight beyond a single financial network to feedback between international finance and trade. An FTF LOOP carries a disturbance from finance through trade and back to finance; a TFT LOOP carries a disturbance from trade through finance and back to trade. These routes can either dampen or amplify the joint response of the two-layer system.

This distinction also matters for the interpretation of international portfolio diversification. A geographically diversified portfolio need not diversify system-wide dynamic risk if the countries in that portfolio are connected through trade in ways that create common feedback routes. The relevant question is therefore not only where capital is invested, but also how financial positions and trade relationships jointly organize the return paths through which disturbances circulate.

The CLiFF makes this issue observable. It identifies, for a given network structure and within-layer coupling, the critical level of cross-layer coupling at which local stability is lost. The distance between prevailing coupling and this boundary measures the remaining stability margin. That margin can shrink because cross-layer coupling increases, but it can also shrink because the network reorganizes and moves the boundary closer to the current state.

The conditional simulation illustrates this point. Starting from the same relative stability margin and applying the same adjustment rule, the Debt–Trade system remains on the stable side of its moving boundary, whereas the Equity–Trade system crosses the boundary in 2017 and remains unstable thereafter. The contrast does not predict the timing of an actual crisis. It shows instead that similar adjustment rules can produce different stability paths when the underlying feedback architecture differs across asset networks.

6.2. Why Roles Switch

Financial and trade relationships evolve under different economic conditions. Financial positions respond to global financing conditions, expected returns, and portfolio decisions. Trade relationships respond to production structures, demand conditions, and changes in supply chains. When these

two layers evolve differently, the cross-layer feedback paths constructed from them are reorganized.

This reorganization is captured by *Role Switching*. The term does not refer simply to changes in a country's centrality within one network. It refers to changes in the country pairs that carry cross-layer feedback and to the direction in which those pairs affect system-wide stability. A country's role in a particular LOOP can change because its relative weight changes, because the relevant return route changes, or because the system becomes more or less sensitive to that route.

The empirical decomposition shows that this mechanism is not peripheral. Role Switching accounts for 36.8 percent of gross stability variation in Debt–Trade and 24.1 percent in Equity–Trade. Its effect is not fixed in sign. Reallocation across country pairs can be stabilizing or destabilizing, depending on the interaction between a change in a pair's LOOP weight and the sensitivity of the system to that change.

There is, however, one notable exception to this directional ambiguity. In the Equity–Trade TFT LOOP, the high-sensitivity region contributes in the destabilizing direction in every year of the sample. The leading country pairs nevertheless change over time. This is the distinction between a persistent function and changing carriers. The destabilizing function remains embedded in the network's feedback architecture, while the country pairs through which it operates are reallocated. This is the mechanism described here as Dionysian.

The common-mode results are consistent with this interpretation. Years that more closely resemble the sample-wide configuration are associated with greater dispersion in country-pair Role changes and with larger gross, pre-offsetting Role contributions. They are not associated with a uniform increase in LOOP size or country-pair sensitivity. What persists is therefore not a fixed set of dangerous links, but a recurring pattern of reallocation in the feedback architecture.

6.3. Implications for Macrofinancial Surveillance

The contrast between Debt–Trade and Equity–Trade cautions against drawing system-wide conclusions from the condition of one market. Policies or market mechanisms that stabilize one asset market may not stabilize the broader finance–trade system. This paper does not estimate the effects of central-bank intervention or other policy actions. It nevertheless shows that

the stability of an individual layer is not sufficient evidence of stability for the integrated system.

The result also points to an extension beyond two layers. Asset sales and common portfolio holdings can connect equity, bond, and trade networks through additional return paths (Duarte and Eisenbach, 2021; Haddad et al., 2021). A disturbance could, for example, travel from equity markets through trade to bond markets and then return through common asset holdings. Such cross-asset LOOPs lie outside the present model, but they follow directly from the logic developed here.

For macrofinancial surveillance, the implication is not to replace conventional indicators of exposures, flows, or centrality. It is to complement them with information on the country pairs carrying cross-layer feedback and on the distance between prevailing coupling and the relevant stability boundary. These measures do not forecast the date of a crisis. They identify where the network architecture is becoming more capable of amplifying disturbances.

6.4. Scope and Limitations

The analysis identifies a structural mechanism under observed annual network configurations and a conditional Near-CLiFF experiment. It does not identify the causal determinants of financial and trade linkages, predict the timing of crises, or estimate the effects of policy intervention. The state-dependent simulation is deliberately conditional: it asks how stability evolves under a specified updating rule, rather than claiming that the rule describes all adjustment in the world economy.

The effective cross-layer maps are constructed from observed financial and trade networks. They summarize possible channels of transmission, rather than directly observed causal coefficients. Moreover, the GFC analysis uses only nine annual changes. The positive association between the absolute GFC deviation and gross, pre-offsetting LOOP responses in Equity–Trade is therefore exploratory. It does not establish that the global financial cycle causes instability, nor does it show that the net response is systematically destabilizing.

These limitations do not alter the paper's central implication. International finance and trade allocate not only capital and goods, but also the capacity to absorb or amplify disturbances. That capacity need not remain attached to particular countries or links. It can move as the organization of cross-layer feedback changes. Understanding macrofinancial stability therefore requires attention not only to the size of connections, but also to the

evolving architecture of the paths through which disturbances circulate and return.

7. Concluding Remarks

This paper examines how changes in the architecture of cross-layer feedback affect macrofinancial stability. Using annual financial and trade networks for 52 economies over 2015–2024, we construct an integrated two-layer Jacobian and identify the CLiFF—the critical boundary at which the system loses local stability. The framework distinguishes changes in the overall strength of cross-layer coupling from changes in the country-pair composition of feedback, which we call Role Switching.

This distinction is empirically consequential. In the conditional simulation, the Debt–Trade system remains within its evolving stability boundary, whereas the Equity–Trade system crosses its boundary in 2017 and remains locally unstable thereafter. Role Switching accounts for 36.8 percent of gross stability variation in Debt–Trade and 24.1 percent in Equity–Trade. Its effect is not fixed in sign: reallocating feedback across country pairs can stabilize or destabilize the system, depending on which pairs carry it. In the Equity–Trade TFT route, the high-sensitivity region contributes in the destabilizing direction in every year of the sample, even though the leading country pairs change over time. We call this separation between a persistent function and changing carriers the Dionysian mechanism. Deviations in the global financial cycle are associated with the magnitude of the Equity–Trade response before the FTF and TFT contributions offset one another, but not with the direction of the net stability effect.

The analysis is conditional: it neither endogenizes the formation of financial and trade linkages nor predicts the timing of crises. It instead identifies a mechanism through which stability can deteriorate without a commensurate increase in aggregate connectivity. Cross-layer feedback can become more destabilizing even when its overall scale changes little. A natural next step is to endogenize how portfolio, financing, and trade decisions reorganize these feedback paths, and how the resulting stability conditions feed back into those decisions. The relevant question is therefore not only which shocks move the economy, but why the same shock can produce a different response today than it did yesterday.

Appendix A. The Financial–Trade Two-Layer System, Cross-Layer LOOPs, and the Jacobian

This appendix connects derivesthe integrated Jacobian of the financial–trade system and explains how how how why connects howWhy howWhyHow it connects the observed network to local stability. The It also shows how how how the Finance–Trade–Finance (FTF) and Trade–Finance–Trade (TFT) LOOPs emerge from the same dynamic system.

The basic distinction is straightforward. The Observed network matrices describe how who who is which countries are connected and how strongly. The Jacobian describes how a small change at one node affects the rate of change at another. The analysis below links the two: the observed financial and trade networks determine the routes through which an effect can travel, while the dynamic model determines how those routes affect stability.

Appendix A.1. From Observed Networks to Directional Cross-Layer Maps

Let $a \in \{\text{debt}, \text{equity}\}$ denote the financial asset and t the observation year. We denote the adjacency matrices of the financial and trade networks by $A^F_{a,t}$ and A^T_t, respectively. Columns identify origin countries and rows identify destination countries. Thus, element ij records a directed relation from country j to country i.

A network matrix becomes economically meaningful for dynamics when it acts on a vector of country-level changes. For example, if $\boldsymbol{x}$ is a vector of small financial disturbances, then $A^F_{a,t}\boldsymbol{x}$ measures how those disturbances are redistributed through the financial network. Applying the trade matrix to the result,

$$A^T_t \left(A^F_{a,t}\boldsymbol{x}\right),$$

passes the redistributed disturbance through the trade network. The ordered product $A^T_t A^F_{a,t}$ can therefore be read as a map: it takes an effect originating in the financial layer and maps it into its destination pattern in the trade layer.

Reversing the order gives a different map. Starting with a disturbance in the trade layer, A^T_t first redistributes it through trade relations, and $A^F_{a,t}$ then maps the resulting exposure into the financial layer. We therefore define the two directional cross-layer maps as

$$C^{FT}_{a,t} = \text{offdiag}\left(A^T_t A^F_{a,t}\right), \qquad C^{TF}_{a,t} = \text{offdiag}\left(A^F_{a,t} A^T_t\right). \tag{A.1}$$

Here, offdiag$(\cdot)$ sets the diagonal elements to zero. The superscript records the direction of the map: $C_{a,t}^{FT}$ maps an effect from the financial layer to the trade layer, while $C_{a,t}^{TF}$ maps an effect from the trade layer to the financial layer.

The element-level representation makes the economic content of the mapping explicit:

$$\left(C_{a,t}^{FT}\right)_{ij} = \mathbf{1}\{i \neq j\} \sum_{k=1}^{N} \left(A_t^T\right)_{ik} \left(A_{a,t}^F\right)_{kj}. \tag{A.2}$$

An effect originating in country j first travels through a financial relation from j to an intermediate country k. It then travels through a trade relation from k to destination country i. Summing over k collects all such two-step routes. The element $\left(C_{a,t}^{FT}\right)_{ij}$ therefore measures the effective financial-to-trade exposure from origin country j to destination country i.

These mappings should not be interpreted as newly observed cross-layer links. The data do not directly report a link from a country's financial state to another country's trade state. Rather, the maps summarize the cross-layer routes implied by the interaction of two observed networks. In this sense, a cross-layer connection is a mapping between state vectors, not an additional class of bilateral transactions.

The order of the two networks matters. In general,

$$A_t^T A_{a,t}^F \neq A_{a,t}^F A_t^T.$$

A disturbance that passes first through financial relations and then through trade relations need not generate the same destination pattern as one that passes through the same two networks in the reverse order. The FTF and TFT channels are therefore not merely two labels for the same set of connections. They represent distinct directional transmission processes.

For the empirical analysis, we normalize each map by its spectral radius:

$$\widetilde{C}_{a,t}^{FT} = \frac{C_{a,t}^{FT}}{\rho(C_{a,t}^{FT})}, \qquad \widetilde{C}_{a,t}^{TF} = \frac{C_{a,t}^{TF}}{\rho(C_{a,t}^{TF})},$$

where $\rho(\cdot)$ denotes the spectral radius. The normalization preserves the relative distribution of the mapping across country pairs while removing differences in its aggregate scale. This allows the model to distinguish where

cross-layer effects are located from how strongly the two layers are coupled overall.

The normalized maps enter the off-diagonal blocks of the Jacobian:

$$B_{a,t}^{FT} = -\sigma_{a,t}\widetilde{C}_{a,t}^{FT}, \qquad B_{a,t}^{TF} = -\sigma_{a,t}\widetilde{C}_{a,t}^{TF}. \tag{A.3}$$

This equation provides connects the network construction directly directly to to the the the dynamic system in the the main text. The The matrices $\widetilde{C}_{a,t}^{FT}$ and $\widetilde{C}_{a,t}^{TF}$ determine determine the direction and relative weight of cross-layer transmission. The scalar $\sigma_{a,t}$ determines its overall strength. The Their Their products form the off-diagonal blocks $B_{a,t}^{FT}$ and $B_{a,t}^{TF}$ in the the integrated Jacobian

$$J_{a,t} = \begin{pmatrix} D_{a,t}^{F} & B_{a,t}^{TF} \\ B_{a,t}^{FT} & D_{a,t}^{T} \end{pmatrix}.$$

The connection between networks and dynamics can now be stated plainly. The observed networks determine who is exposed to whom. The Jacobian translates those exposures into changes in the growth or decay of small disturbances. Stability is therefore not a property of the adjacency matrices alone. It is a property of the dynamic system obtained when the observed network routes are combined with within-layer adjustment and cross-layer coupling.

The matrices $C_{a,t}^{FT}$ and $C_{a,t}^{TF}$ should also be distinguished from the scalar $C_{ij,t}$ used later in the country-pair decomposition. The former are full directional maps between layers. The latter is a composition weight measuring the relative importance of a particular origin–destination pair within a LOOP configuration.

Finally, setting the diagonal of each one-way map to zero removes financial-to-trade or trade-to-financial mappings whose origin and destination are the same country. It does not remove Self Return from the model. A disturbance may leave country j, pass through the other layer and several foreign countries, and eventually return to country j. Such a path appears on the diagonal of the complete round-trip LOOP operator derived below, even though the diagonal elements of the one-way maps are zero.

Appendix A.2. Two-Layer Dynamics and the Integrated Jacobian

The network matrices describe which countries are connected and how strongly. The dynamic equations describe how a small change in one country

affects conditions elsewhere over time. This subsection connects these two objects.

We treat each year as a separate network snapshot. Within year t, the financial and trade networks are held fixed, and we examine how a small disturbance moves through those networks. The networks may change between years. We therefore use t to index the annual network snapshot and τ to represent adjustment time within that snapshot.

Let $\boldsymbol{x}_{a,t} \in \mathbb{R}^N$ collect the financial states of the N countries for asset a, and let $\boldsymbol{y}_t \in \mathbb{R}^N$ collect their trade states. We write the two-layer system as

$$\dot{\boldsymbol{x}}_{a,t} = \boldsymbol{f}^F_{a,t}(\boldsymbol{x}_{a,t}) - \gamma_{a,t}\widetilde{L}^F_{a,t}\boldsymbol{x}_{a,t} - \sigma_{a,t}\widetilde{C}^{TF}_{a,t}\boldsymbol{y}_t, \tag{A.4}$$

$$\dot{\boldsymbol{y}}_t = \boldsymbol{f}^T_{a,t}(\boldsymbol{y}_t) - \gamma_{a,t}\widetilde{L}^T_{a,t}\boldsymbol{y}_t - \sigma_{a,t}\widetilde{C}^{FT}_{a,t}\boldsymbol{x}_{a,t}. \tag{A.5}$$

Each equation contains three parts. The first captures adjustment that would occur without network transmission. The second captures transmission among countries within the same layer. The third captures transmission from the other layer.

The matrices $\widetilde{L}^F_{a,t}$ and $\widetilde{L}^T_{a,t}$ summarize connections within the financial and trade layers. Economically, they allow a country's adjustment to depend not only on its own condition, but also on conditions in the countries to which it is financially or commercially connected. The parameter $\gamma_{a,t}$ determines the overall strength of this within-layer interaction.

The matrices $\widetilde{C}^{TF}_{a,t}$ and $\widetilde{C}^{FT}_{a,t}$ are the directional maps constructed in the previous subsection. The first maps trade-layer conditions into the financial layer; the second maps financial-layer conditions into the trade layer. The parameter $\sigma_{a,t}$ determines the overall strength of transmission between the two layers.

This formulation separates three economically distinct objects. The observed networks determine the routes through which an effect can travel. The directional maps determine how an effect arriving from the other layer is distributed across countries. The coupling parameters determine how strongly those routes affect adjustment.

The minus signs in equations (A.4) and (A.5) are part of the adjustment convention used in the model. They do not imply that every network effect is stabilizing. An effect may pass to the other layer and return before it has been absorbed. Its system-wide effect depends on the complete pattern of within- and cross-layer transmission.

The empirical proxy for interlayer coupling constructed from the observed networks is

$$\Omega_{a,t}^{\text{obs}} = \sigma_{0,a} S_{a,t}, \tag{A.6}$$

where $S_{a,t}$ is an observed interlayer-strength index normalized to one in 2015. Thus, $S_{a,2015} = 1$, but $\Omega_{a,2015}^{\text{obs}}$ need not equal one because its level also depends on the asset-specific scale $\sigma_{0,a}$.

The interlayer coupling $\Omega_{a,t}$ used in the state-dependent simulation is distinct from $\Omega_{a,t}^{\text{obs}}$. It is initialized using either the observed proxy or a value near the CLiFF and is then updated recursively according to equation (20). Thus, σ denotes interlayer coupling in the general model, Ω^{obs} denotes its empirical proxy, and Ω denotes the coupling path generated by the conditional simulation.

We next consider small departures from a benchmark condition $(\boldsymbol{x}_{a,t}^*, \boldsymbol{y}_t^*)$. Define

$$\boldsymbol{u}_{a,t} = \boldsymbol{x}_{a,t} - \boldsymbol{x}_{a,t}^*, \qquad \boldsymbol{v}_t = \boldsymbol{y}_t - \boldsymbol{y}_t^*. \tag{A.7}$$

The benchmark does not require all countries to have identical financial or trade conditions. It is simply the point around which we evaluate the local response of the system.

Absent network transmission, we assume that a small departure moves back toward the benchmark at rate κ:

$$\boldsymbol{f}_{a,t}^F(\boldsymbol{x}_{a,t}) - \boldsymbol{f}_{a,t}^F(\boldsymbol{x}_{a,t}^*) \simeq -\kappa \boldsymbol{u}_{a,t}, \qquad \boldsymbol{f}_{a,t}^T(\boldsymbol{y}_t) - \boldsymbol{f}_{a,t}^T(\boldsymbol{y}_t^*) \simeq -\kappa \boldsymbol{v}_t. \tag{A.8}$$

We set $\kappa = 1$ throughout the analysis. This choice normalizes the unit of adjustment time; it is not intended to explain differences between the Debt–Trade and Equity–Trade networks.

Keeping the local responses around the benchmark gives

$$\frac{d}{d\tau}\begin{pmatrix} \boldsymbol{u}_{a,t} \\ \boldsymbol{v}_t \end{pmatrix} = \underbrace{\begin{pmatrix} -\kappa I - \gamma_{a,t}\widetilde{L}_{a,t}^F & -\sigma_{a,t}\widetilde{C}_{a,t}^{TF} \\ -\sigma_{a,t}\widetilde{C}_{a,t}^{FT} & -\kappa I - \gamma_{a,t}\widetilde{L}_{a,t}^T \end{pmatrix}}_{J_{a,t}(\gamma_{a,t},\sigma_{a,t})} \begin{pmatrix} \boldsymbol{u}_{a,t} \\ \boldsymbol{v}_t \end{pmatrix}. \tag{A.9}$$

The matrix $J_{a,t}$ is the integrated Jacobian used in the main text. Each element records how a small change at one country-layer node affects the

rate of change at another. The diagonal blocks describe transmission within the financial and trade layers. The off-diagonal blocks describe transmission from trade to finance and from finance to trade.

The correspondence with equation (2) in the main text is

$$\begin{aligned} D^F_{a,t} &= -\kappa I - \gamma_{a,t}\widetilde{L}^F_{a,t}, \quad D^T_{a,t} = -\kappa I - \gamma_{a,t}\widetilde{L}^T_{a,t}, \\ B^{TF}_{a,t} &= -\sigma_{a,t}\widetilde{C}^{TF}_{a,t}, \qquad B^{FT}_{a,t} = -\sigma_{a,t}\widetilde{C}^{FT}_{a,t}. \end{aligned} \tag{A.10}$$

This correspondence connects the observed networks directly to the dynamic system used in the main analysis. The financial and trade matrices determine the routes and relative weights of transmission. The four blocks translate those network relationships into responses within and between the two layers, and the integrated Jacobian collects them in a single system. The stability criterion based on this Jacobian is introduced in defined in in the introduced in the in the introduced in the introduced in the introduced in the introduced in the introduced in the introduced inThe the main text.

With $N = 52$ countries, each block is 52×52, and the integrated Jacobian is 104×104.

Appendix A.3. Cross-Layer LOOPs and Return Destinations

We next show how the FTF and TFT LOOPs emerge from the integrated Jacobian. They are not additional structures imposed on the model. They are two ways of tracing the feedback already contained in the off-diagonal blocks of the same two-layer system.

Let λ denote a candidate rate of change for a disturbance in the integrated system. Write the associated financial- and trade-layer components as $\boldsymbol{p}$ and $\boldsymbol{q}$. The block structure of the Jacobian implies

$$\left(\lambda I - D^F_{a,t}\right)\boldsymbol{p} = B^{TF}_{a,t}\boldsymbol{q}, \tag{A.11}$$

$$\left(\lambda I - D^T_{a,t}\right)\boldsymbol{q} = B^{FT}_{a,t}\boldsymbol{p}. \tag{A.12}$$

These two equations describe the same disturbance from the perspective of the two layers. The first relates movements in the financial layer to effects arriving from the trade layer. The second relates movements in the trade layer to effects arriving from the financial layer.

To trace a round trip beginning in finance, solve the second equation for the trade-layer response:

$$\boldsymbol{q} = \left(\lambda I - D^T_{a,t}\right)^{-1} B^{FT}_{a,t} \boldsymbol{p}.$$

Substituting this response into the financial-layer equation gives the Finance–Trade–Finance operator

$$M_F(\lambda) = B^{TF}_{a,t} \left(\lambda I - D^T_{a,t}\right)^{-1} B^{FT}_{a,t}. \tag{A.13}$$

Reading the expression from right to left makes its economic meaning clear. The block $B^{FT}_{a,t}$ carries an effect from the financial layer into the trade layer. The middle term collects the responses generated as that effect passes through the trade network. The block $B^{TF}_{a,t}$ then carries the resulting effect back into the financial layer.

The same argument in the opposite direction gives the Trade–Finance–Trade operator

$$M_T(\lambda) = B^{FT}_{a,t} \left(\lambda I - D^F_{a,t}\right)^{-1} B^{TF}_{a,t}. \tag{A.14}$$

Here, $B^{TF}_{a,t}$ first carries an effect from trade into finance, the middle term collects its direct and indirect responses within the financial network, and $B^{FT}_{a,t}$ returns the resulting effect to the trade layer.

The inverse matrices in the middle should therefore be understood as summaries of what happens inside the intermediate layer. They include not only the immediate response of a directly connected country, but also the responses that continue through other countries within that layer. The LOOP operators compress these intermediate adjustments into a map from the origin layer back to the same type of layer.

Consider the FTF operator. Its element ij measures the effect of a LOOP originating in country j in the financial layer and returning to country i in the financial layer after passing through the trade network. If $i \neq j$, the path is a *Cross Return*. If $i = j$, it is a *Self Return*. The same interpretation applies to the TFT operator, with the trade layer as the origin and return layer.

A Self Return does not mean that the effect remains inside the originating country. It may pass through several foreign countries in the intermediate layer before returning to its country of origin. For this reason, Self Return is identified from the diagonal of the complete round-trip operator $M_F(\lambda)$ or $M_T(\lambda)$, not from the diagonal of either one-way map $C^{FT}_{a,t}$ or $C^{TF}_{a,t}$.

Because each directional cross-layer block contains the factor $-\sigma_{a,t}$, the scalar part of a complete round trip is

$$(-\sigma_{a,t})(-\sigma_{a,t}) = \sigma_{a,t}^2. \tag{A.15}$$

This identity does not mean that every LOOP is destabilizing. It only shows how the overall strength of a round trip depends on interlayer coupling. Whether a particular LOOP raises or lowers the system's spectral abscissa depends on where the effect travels, how it is transmitted within the intermediate layer, and how the resulting country-level responses interact with the rest of the network.

Appendix A.4. Stability, LOOP Magnitude, and Role Switching

Using the notation of the main text, the stability of the integrated system is summarized by the spectral abscissa

$$\alpha_{a,t} = \max_k \operatorname{Re}\left\{\lambda_k\left[J_{a,t}(\gamma_{a,t}, \sigma_{a,t})\right]\right\}. \tag{A.16}$$

As established in the main text, the system is locally stable when $\alpha_{a,t} < 0$, lies on the CLiFF when $\alpha_{a,t} = 0$, and is locally unstable when $\alpha_{a,t} > 0$. The symbol σ denotes interlayer coupling in the general model; the state-dependent simulation uses Ω for the corresponding time-varying coupling.

Equations (A.13) and (A.14) show that the strength of a round trip depends partly on the overall level of interlayer coupling. We refer to the resulting aggregate strength of the return paths as LOOP magnitude. A larger LOOP magnitude means that the financial and trade layers exert a stronger combined influence on one another.

Role Switching captures a different feature of the network. It does not ask how large the LOOP is in aggregate. It asks which origin–destination pairs carry the return paths. A change in Role therefore represents a reallocation of LOOP weight across country pairs rather than an increase or decrease in the total volume of cross-layer feedback.

This distinction matters because countries do not respond identically to a disturbance. A given amount of cross-layer feedback may be absorbed when it passes through one set of country pairs but amplified when it passes through another. As a result, two networks can have similar interlayer coupling and similar aggregate LOOP magnitude yet differ in stability because the return paths are distributed differently across countries.

LOOP magnitude is used here only as a descriptive distinction. It is not introduced as an additional factor in the five-component decomposition. Changes in the overall strength of interlayer coupling are captured by Ω,

while Role Switching captures changes in the country-pair composition of the LOOP after that overall strength is separated out.

The central point is therefore simple: the stability of the two-layer system depends not only on how strongly finance and trade are linked, but also on where their feedback returns. Even if the internal structure of each layer and the aggregate strength of cross-layer interaction change little, reallocating the LOOP across country pairs can move the integrated system from one side of the stability boundary to the other.

Appendix A.5. Order-Averaged Decomposition and the Allocation of LOOP-Composition Effects

For each asset a and annual transition from $t-1$ to t, we seek to identify what moved the spectral abscissa. Four features can change between the two years: within-layer structure, within-layer coupling γ, interlayer coupling Ω, and the country-pair composition of the LOOP.

Their contributions cannot generally be measured by changing each factor in isolation. The effect of a change in Ω, for example, may depend on whether the current-year network structure has already been introduced. Similarly, the effect of a change in LOOP composition may differ depending on whether γ and Ω are held at their previous- or current-year values. The decomposition must therefore account for the order in which the four changes are introduced.

We begin with the complete previous-year system. For a given ordering, we replace one factor at a time with its current-year value. Factors already introduced are held at their current-year values, while those not yet introduced remain at their previous-year values. This produces a sequence of hybrid systems connecting the previous-year network to the current-year network.

Let π denote one ordering of the four factors and let $\alpha_{a,t}^{\pi,m}$ be the spectral abscissa after the first m factors in that ordering have been replaced. The sequence begins and ends at

$$\alpha_{a,t}^{\pi,0} = \alpha_{a,t-1}, \qquad \alpha_{a,t}^{\pi,4} = \alpha_{a,t}.$$

If factor q is introduced at step m, its marginal contribution in ordering π is

$$\delta_{a,t}^{\pi}(q) = \alpha_{a,t}^{\pi,m} - \alpha_{a,t}^{\pi,m-1}. \tag{A.17}$$

Thus, a marginal contribution is simply the change in the spectral abscissa observed when one factor is moved from its previous-year state to its current-year state, given the states of the other three factors at that step. It is an accounting contribution, not a causal estimate.

With four factors, there are $4! = 24$ possible orderings. We calculate the marginal contribution of each factor in every ordering and report its average:

$$\phi^{q}_{a,t} = \frac{1}{24} \sum_{\pi} \delta^{\pi}_{a,t}(q), \qquad q \in \{\text{Within}, \gamma, \Omega, \text{Comp}\}\,. \tag{A.18}$$

Here, Comp denotes LOOP composition. Averaging across all orderings prevents the result from depending on an arbitrary decision to introduce one factor before another. The four upper-level contributions satisfy

$$\Delta\alpha_{a,t} = \phi^{\text{Within}}_{a,t} + \phi^{\gamma}_{a,t} + \phi^{\Omega}_{a,t} + \phi^{\text{Comp}}_{a,t}. \tag{A.19}$$

The contribution of Ω captures the change in the overall strength of interlayer coupling. When Ω is introduced, the other three factors are held at the values implied by that particular ordering. We do not introduce Uniform Scale as an additional factor, because the corresponding change in overall scale is already contained in Ω.

The LOOP-composition component answers a different question. Holding overall interlayer strength apart, it measures the effect of changing which country pairs carry the round-trip response. We divide this component according to the destination of the return path.

Let $C^{d}_{ij,a,t}$ denote the normalized composition weight of a completed LOOP of type $d \in \{\text{FTF}, \text{TFT}\}$ that originates in country j and returns to country i. These scalar composition weights are distinct from the one-way mapping matrices $C^{FT}_{a,t}$ and $C^{TF}_{a,t}$ defined above.

We measure changes in same-country and cross-country return paths by

$$W^{\text{Self}}_{a,t} = \sum_{d} \sum_{i} \left|\Delta C^{d}_{ii,a,t}\right|, \qquad W^{\text{Role}}_{a,t} = \sum_{d} \sum_{i \neq j} \left|\Delta C^{d}_{ij,a,t}\right|. \tag{A.20}$$

The first quantity measures changes in LOOPs that return to their country of origin. The second measures changes in LOOPs that return to a different country. Their shares in the total absolute change are

$$s^{\text{Self}}_{a,t} = \frac{W^{\text{Self}}_{a,t}}{W^{\text{Self}}_{a,t} + W^{\text{Role}}_{a,t}}, \qquad s^{\text{Role}}_{a,t} = \frac{W^{\text{Role}}_{a,t}}{W^{\text{Self}}_{a,t} + W^{\text{Role}}_{a,t}}. \tag{A.21}$$

We use these shares to allocate the LOOP-composition contribution:

$$\phi_{a,t}^{\text{Self}} = s_{a,t}^{\text{Self}} \phi_{a,t}^{\text{Comp}}, \qquad \phi_{a,t}^{\text{Role}} = s_{a,t}^{\text{Role}} \phi_{a,t}^{\text{Comp}}. \tag{A.22}$$

Because the two shares sum to one, the allocation preserves the full LOOP-composition contribution:

$$\phi_{a,t}^{\text{Comp}} = \phi_{a,t}^{\text{Role}} + \phi_{a,t}^{\text{Self}}. \tag{A.23}$$

The resulting five-component decomposition is therefore hierarchical. It does not treat Role Switching and Self Return as two additional factors that are independently switched from one year to the next. Instead, we first decompose the annual change into four upper-level factors and then divide the LOOP-composition component between cross-country and same-country return paths.

The five reported components satisfy

$$\Delta\alpha_{a,t} = \phi_{a,t}^{\text{Within}} + \phi_{a,t}^{\gamma} + \phi_{a,t}^{\Omega} + \phi_{a,t}^{\text{Role}} + \phi_{a,t}^{\text{Self}}. \tag{A.24}$$

Their sum exactly reproduces the observed annual change in the spectral abscissa.

Appendix A.6. State-Dependent Updating Rule and Parameterization

Table Appendix A.1 reports the coefficients and parameter values used in the state-dependent updating rule in equation (20). The estimated coefficients govern the response of cross-layer coupling to its own lagged change and to the observed global financial factor. The remaining parameters govern how the adjustment rule responds as the system approaches or crosses the CLiFF.

Table Appendix A.1: Coefficients and Parameter Values for the State-Dependent Cross-Layer Coupling Rule

Category	Parameter	Value	Determination
Estimated coefficient	$\widehat{\alpha}_{\text{debt}}$	−0.030109	Estimated from observed data
	$\widehat{\alpha}_{\text{equity}}$	0.021590	Estimated from observed data
	$\widehat{\rho}$	−0.598305	Estimated from observed data
	$\widehat{\beta}_G$	0.006845	Estimated from observed data
Simulation parameter	β_A	0.020	Adjustment when the stability margin is positive
	δ	0.050	Additional amplification near the CLiFF
	τ_R	0.050	Threshold defining proximity to the CLiFF
	k	25.0	Steepness of the activation function
	χ	0.50	Contraction after the CLiFF is crossed
Initial condition	$\gamma_{a,2015}$	0.50	Initial within-layer coupling

Notes: The estimated coefficients are obtained by pooling annual changes for the debt and equity networks over 2016–2024. The OLS specification includes asset-specific intercepts, the lagged change in cross-layer coupling, and the observed global financial factor. Standard errors are calculated using HC1, and years are resampled in 2,000 bootstrap replications.

The coefficients $\widehat{\alpha}_{\text{debt}}$ and $\widehat{\alpha}_{\text{equity}}$ are asset-specific intercepts in the coupling update equation; they are distinct from the spectral abscissa $\alpha_{a,t}$ used to assess stability.

The parameters β_A, δ, τ_R, k, and χ are prespecified simulation parameters rather than estimates obtained from the short annual sample. The benchmark simulation initializes within-layer coupling at $\gamma_{a,2015} = 0.50$ and subsequently updates it according to equation (26). Alternative specifications initialize cross-layer coupling either at its observed value or 5 percent below its 2015 critical value.

Appendix B. The Global Financial Cycle and Cross-Layer Responses

This appendix provides the variable definitions, statistical procedure, and full results underlying the analysis reported in Section 5.3.4. The analysis examines the association between the observed global financial cycle factor (GFC) and changes in cross-layer LOOP composition.

We use the annual GFC series observed over 2015–2024. The series is standardized over the full sample, and the absolute standardized value, $|GFC_t^z|$, measures the magnitude of the GFC's deviation from its sample mean. The analysis therefore concerns the magnitude of the deviation, rather than whether the GFC is above or below its sample mean.

For each asset a and year t, let $F_{a,t}$ and $T_{a,t}$ denote the net contributions of changes in LOOP composition along the FTF and TFT routes, respectively. These contributions include both Role Switching, which concerns returns to a country different from the country of origin, and Self Return, which concerns returns to the country of origin.

We define the response before offsetting across the two routes and the net response after combining them as

$$G_{a,t}^{\text{gross}} = |F_{a,t}| + |T_{a,t}|, \qquad G_{a,t}^{\text{net}} = F_{a,t} + T_{a,t}. \tag{B.1}$$

The first measure is the sum of the absolute FTF and TFT contributions after country-pair contributions have been aggregated within each route. It therefore measures the total response before contributions from the two routes offset one another. The second measure is the remaining net response after the two route-level contributions are combined.

To quantify the part of the response concealed by offsetting between FTF and TFT, we define

$$M_{a,t} = G_{a,t}^{\text{gross}} - \left|G_{a,t}^{\text{net}}\right|. \tag{B.2}$$

We refer to $M_{a,t}$ as the cross-route offset and to $M_{a,t}/G_{a,t}^{\text{gross}}$ as the offset rate. The measure $M_{a,t}$ is not an effect attributed to the GFC. It records only the extent to which the FTF and TFT contributions observed in a given year offset one another.

Because changes in LOOP composition are defined relative to the preceding year, the estimation sample consists of the nine annual observations from 2016 through 2024. For each asset and outcome, we calculate its Spearman

rank correlation with $|GFC_t^z|$. Exact two-sided p-values are obtained from all $9! = 362{,}880$ permutations of the nine annual GFC observations.

This procedure provides a descriptive test of association under the assumption that the annual observations are exchangeable. It does not account for serial dependence and does not establish that the GFC causes the observed cross-layer responses.

Table Appendix B.1: Rank Correlations between Deviations in the Global Financial Cycle and Changes in Cross-Layer LOOP Composition

Asset	Outcome	Spearman's ρ	p-value
Debt	Cross-route offset $M_{a,t}$	0.418	0.262
	Offset rate	0.487	0.183
	Gross response $G_{a,t}^{\text{gross}}$	0.383	0.313
	Net response $G_{a,t}^{\text{net}}$	−0.417	0.270
Equity	Cross-route offset $M_{a,t}$	−0.271	0.481
	Offset rate	−0.322	0.400
	Gross response $G_{a,t}^{\text{gross}}$	0.700	0.043
	Net response $G_{a,t}^{\text{net}}$	0.417	0.270

Notes: The sample consists of nine annual observations from 2016 through 2024. The magnitude of the GFC deviation is measured by $|GFC_t^z|$, where the observed GFC series is standardized over 2015–2024. The reported p-values are unadjusted exact two-sided permutation p-values obtained from all $9! = 362{,}880$ permutations of the nine annual GFC observations. The procedure assumes that annual observations are exchangeable and does not account for serial dependence. No adjustment is made for the eight statistical tests reported in the table. The offset rate is $M_{a,t}/G_{a,t}^{\text{gross}}$.

Table Appendix B.1 shows that, for the Equity–Trade network, larger deviations of the GFC from its sample mean are associated with a larger gross response before the FTF and TFT contributions offset one another. The rank correlation is 0.700, with an unadjusted exact permutation p-value of 0.043. By contrast, the correlations with the net response and the cross-route offset are not statistically distinguishable from zero. None of the corresponding associations is statistically clear for the Debt–Trade network.

The Equity–Trade result does not imply that large GFC deviations necessarily destabilize the network. Rather, such deviations are associated with greater activity across the two cross-layer routes before their contributions

are combined. The absence of a clear relationship with the net response indicates that the GFC does not systematically determine whether the resulting contribution is stabilizing or destabilizing.

These results should be interpreted as exploratory evidence. The sample contains only nine annual observations, the permutation procedure does not accommodate serial dependence, and the reported p-values are not adjusted for multiple testing. The evidence therefore points to an association between GFC deviations and the magnitude of the Equity–Trade cross-layer response, not to a causal effect or a systematic directional shift in stability.

Appendix C. A Common Reallocation Pattern in Role Switching

This appendix examines whether country-pair Role Switching contains a common pattern of reallocation that recurs across years. The purpose is not to construct another measure of the global financial cycle. Instead, we ask how closely the annual changes in country-pair LOOP weights align with a pattern shared across the sample, and whether stronger alignment is associated with the magnitude and stability effects of Role Switching.

A common pattern does not imply that all country-pair weights move by the same amount or in the same direction. Some weights may increase while others decrease. What makes the pattern common is that a similar combination of increases and decreases recurs across several years.

Strong alignment with this pattern indicates that an annual reallocation resembles a configuration repeatedly observed elsewhere in the sample. Weak alignment indicates that the year's Role Switching is driven primarily by idiosyncratic changes in particular country pairs. The analysis therefore distinguishes a sample-common component of reallocation from year-specific changes and examines how each is related to network stability.

For each asset and return direction—FTF or TFT—we arrange the changes in the LOOP-composition weights of all directed country pairs in a single vector. To simplify notation, the asset and direction subscripts are suppressed in what follows:

$$\boldsymbol{x}_t = (\Delta C_{ij,t})_{i \neq j}\,. \tag{C.1}$$

Under the matrix convention used throughout the paper, $\Delta C_{ij,t}$ is the annual change in the LOOP-composition weight associated with a return from origin country j to destination country i. Here, $C_{ij,t}$ denotes a country-pair composition weight, not the directional mapping matrix defined in Appendix Appendix A.1. The vector $\boldsymbol{x}_t$ can be read as a map of which country pairs gained or lost importance in Role Switching during year t.

For each evaluation year t, we estimate the first principal component $\boldsymbol{v}_{1,-t}$ using the vectors from the other eight years. This component identifies the most prominent pattern of country-pair reallocation shared by those years. Excluding the evaluation year prevents its own changes from being used both to construct the common pattern and to measure alignment with that pattern.

Let $\bar{\boldsymbol{x}}_{-t}$ denote the mean vector calculated without year t, and normalize $\boldsymbol{v}_{1,-t}$ to have unit length. We measure the alignment of the Role changes in

year t with the common pattern by

$$D_t^{CF} = \frac{\left[\left(\boldsymbol{x}_t - \bar{\boldsymbol{x}}_{-t} \right)^\top \boldsymbol{v}_{1,-t} \right]^2}{\left\| \boldsymbol{x}_t - \bar{\boldsymbol{x}}_{-t} \right\|^2}. \tag{C.2}$$

The index D_t^{CF} lies between zero and one. A value close to one means that the annual reallocation is closely aligned with the pattern identified from the remaining years. A value close to zero means that the annual change is largely orthogonal to that common pattern and is therefore predominantly year-specific. Because the projection is squared, the measure does not depend on the arbitrary sign assigned to the first principal component.

A high value of D_t^{CF} alone does not reveal whether the common pattern makes country-pair changes more uniform or more dispersed. Nor does it establish whether the associated stability effects become larger. We therefore examine whether common-mode alignment is related to the following outcomes:

1. the dispersion of country-pair Role changes, $SD(\Delta C)$;
2. the mean and dispersion of country-pair sensitivities, $\bar{g}_{ij}$;
3. the total Role contribution before positive and negative country-pair contributions offset one another,

$$\sum_{i \neq j} \left| \Delta C_{ij} \bar{g}_{ij} \right|;$$

4. the share of the Role contribution accounted for by the high-sensitivity region; and
5. Scale dominance, a descriptive measure of whether the change in LOOPs is driven more by proportional changes in overall magnitude than by reallocation across country pairs.

Scale dominance is used only as a diagnostic in this appendix. It is not an additional factor in the hierarchical decomposition developed in Appendix Appendix A.5; changes in the overall magnitude of cross-layer coupling are already captured by Ω in that decomposition.

For each outcome, we estimate its association with D_t^{CF} while controlling for asset and return-direction fixed effects. A positive coefficient therefore means that the outcome tends to be larger in years in which Role Switching is more closely aligned with the common reallocation pattern.

Statistical uncertainty is assessed using a year-level bootstrap and an exact permutation test that reassigns the annual correspondence between common-mode alignment and the outcome variables.[6]

Table Appendix C.1: Common-Mode Alignment and Role Switching

Dependent variable	Coefficient on D^{CF}	Exact p-value	Negative share	Positive share
Dispersion in country-pair Role changes: $SD(\Delta C)$	+0.01284	0.025	0.063	0.937
Mean country-pair sensitivity	+0.00019	0.680	0.284	0.716
Dispersion in country-pair sensitivity: $SD(\bar{g})$	−0.00102	0.745	0.636	0.364
Total unoffset Role contribution: $\sum_{i \neq j} \lvert \Delta C_{ij} \bar{g}_{ij} \rvert$	+0.07330	0.014	0.025	0.975
Contribution share of the high-sensitivity region	+0.03735	0.316	0.098	0.902
Scale dominance	+0.98704	0.120	0.137	0.863

Notes: The table reports coefficients on D^{CF} from specifications that control for asset and FTF/TFT fixed effects. Exact two-sided p-values are obtained from all $9! = 362{,}880$ possible reassignments of the nine annual observations under the null hypothesis. The negative and positive shares report the proportions of coefficient estimates below and above zero, respectively, across 5,000 year-level bootstrap samples. The total unoffset Role contribution is the sum of the absolute country-pair contributions before stabilizing and destabilizing contributions offset one another. Scale dominance is calculated as the average of the corresponding FTF and TFT measures. The reported p-values are not adjusted for the six outcomes considered in the table.

Table Appendix C.1 shows that stronger common-mode alignment is associated with greater dispersion in country-pair Role changes. The estimated coefficient on D_t^{CF} is 0.01284, with an exact permutation p-value of 0.025. The coefficient is positive in 93.7 percent of the year-level bootstrap samples.

Common-mode alignment is also positively associated with the total Role contribution before positive and negative country-pair effects offset one an-

[6]The first principal component is estimated using a leave-one-year-out procedure. The bootstrap resamples years 5,000 times while preserving, within each selected year, the corresponding Debt–Trade, Equity–Trade, FTF, and TFT observations. The exact permutation test evaluates all $9! = 362{,}880$ possible reassignments of the nine annual observations. Its validity under the null requires the annual observations to be exchangeable.

other. The estimated coefficient is 0.07330, with an exact permutation p-value of 0.014, and is positive in 97.5 percent of the bootstrap samples. Thus, years that align more closely with the common reallocation pattern exhibit both greater dispersion in country-pair Role changes and greater unoffset Role activity.

This result does not mean that the common pattern is uniformly stabilizing or destabilizing. The dependent variable adds the absolute country-pair contributions before their signs are allowed to offset. A larger value therefore indicates that both stabilizing and destabilizing forces are more active, not that either direction necessarily dominates in the aggregate.

By contrast, common-mode alignment is not clearly associated with the mean or dispersion of country-pair sensitivity, the contribution share of the high-sensitivity region, or Scale dominance. The results therefore provide no clear evidence that alignment with the common pattern systematically raises country-pair sensitivity or produces a proportional expansion of LOOPs as a whole.

Taken together, the evidence is more consistent with reallocation than with uniform expansion. When the common pattern is more strongly expressed, changes in LOOP composition are more dispersed across country pairs, and the stabilizing and destabilizing contributions generated before offsetting are larger. Because the sample contains only nine years and several related outcomes are examined, these findings should be read as exploratory evidence about the structure of Role Switching rather than as a confirmatory test of a stable population relationship.

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