How Small is Small? Nonlinearities in Heterogeneous-Agent Models *

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July 22, 2026

Abstract

In plausibly calibrated heterogeneous-agent models, marginal propensities to consume (MPCs) are highly nonlinear in wealth, declining sharply as households move away from borrowing constraints. As a result, the aggregate consumption response to a fiscal transfer is strongly concave in the transfer's size: larger transfers shift households out of high-MPC regions, dampening the consumption response. We decompose this concavity into two channels—households' movement off the borrowing constraint and the curvature of individual consumption policy functions—and show that the former dominates quantitatively. At empirically relevant transfer sizes, linear methods substantially overstate the consumption effects of fiscal stimulus.

Keywords: Heterogeneous agents, nonlinearities, HANK

JEL classification: D15, D31, D52, E21, E62, G51, C63

*We thank Manuel Amador, Adrien Auclert, Florin Bilbiie, Luigi Bocola, João Guerreiro, Alisdair McKay, Ben Moll, Matt Rognlie, and Ludwig Straub for helpful conversations and Xing Xu for excellent research assistance. We also thank conference participants at Crete Workshop, NBER Summer Institute Workshop on Methods and Applications for Dynamic Equilibrium Models, and Society of Economic Dynamics. The views expressed herein are those of the authors and not necessarily those of the Federal Reserve Bank of Minneapolis or the Federal Reserve System.

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1 Introduction

There has been an explosion of work studying the effects of fiscal and monetary interventions in settings with heterogeneous agents (HA) and incomplete markets. These frameworks are useful because they do a good job of capturing relevant features of household-level spending and savings behavior. In particular, suitably calibrated versions can match both the level and distribution of marginal propensities to consume (MPCs).

These aspects of consumption behavior are especially relevant for studying fiscal interventions, because a failure of Ricardian equivalence lies at the heart of the aggregate and distributional effects of such policies. The size and pattern of MPCs underscore these departures from Ricardian equivalence and drive much of the difference between heterogeneous-agent and representative-agent models. Indeed, the effects of fiscal stimulus programs funded by future taxes are driven entirely by such departures.

A large part of the recent literature has analyzed these settings using various forms of first-order methods.[1] The most common approach, labeled "sequence-space Jacobian" (SSJ) by Auclert, Bardóczy, Rognlie and Straub (2021), constructs an aggregate consumption function by aggregating household consumption policies, and computes equilibrium transition paths using a linear approximation of this mapping in a suitably chosen vector of aggregate prices and policy variables, evaluated around the steady state.[2] It is well understood that such methods approximate nonlinear equilibria well when the policy innovations and exogenous disturbances being studied are small. But how small is small?

We argue that for commonly studied fiscal interventions, small is indeed very small—much smaller than many of the aggregate shocks and interventions that macroeconomists are motivated to explore based on real-world experience.

Our starting point is the observation that for a funded fiscal stimulus program, the bulk of the aggregate effects are driven by the strength of departures from Ricardian equivalence, which in turn are driven by the extent to which households' MPCs exceed those implied by the permanent income hypothesis (PIH).

[1] Examples include Aggarwal, Auclert, Rognlie and Straub (2023), Bardóczy, Sim and Tischbirek (2024), Bayer, Born, Luetticke and Müller (2023), Rachel and Summers (2019), Hänsel (2024), Campos, Fernández-Villaverde, Nuño and Paz (2024), Eichenbaum, Guerreiro and Obradovic (2026), Angeletos, Lian and Wolf (2024a), Angeletos, Lian and Wolf (2024b). Studies that allow for nonlinear effects of fiscal policy in incomplete markets are Carroll, Crawley, Slacalek and White (2021), Hagedorn, Manovskii and Mitman (2019), Brinca et al. (2025) and Mongey and Waugh (2025). Relative to this work, we systematically evaluate the degree of nonlinearity in a canonical heterogeneous-agent New Keynesian (HANK) model and assess the accuracy of linearization and higher-order local methods.

[2] See also Boppart, Krusell and Mitman (2018) for a related approach in the sequence space using finite differences, and Reiter (2009) for a projection-based method.

In plausibly calibrated heterogeneous-agent models, MPCs decline sharply with wealth and may fall discontinuously when a borrowing constraint ceases to bind. Larger transfers therefore move households out of high-MPC regions, making the aggregate consumption response highly concave in the size of the transfer. Linear methods extrapolate the average MPC at steady state and thus overstate the effects of fiscal stimulus. Moreover, we show that the gap between the linear and the nonlinear solution is substantial at empirically relevant transfer sizes.

We provide an analytical decomposition of the two sources of concavity and quantify their importance. One source is the familiar curvature of the individual consumption function (e.g., Carroll and Kimball, 1996). The second source we highlight is that a transfer moves households off the borrowing constraint—a feature that the first-order approximation holds effectively constant. Because a lower share of constrained households replaces an MPC of one with a strictly lower unconstrained MPC, this shift lowers the average MPC, and thus moderates the response of aggregate consumption relative to the linear solution. Our quantitative results show that for fiscal transfers the concavity of the aggregate consumption function is dominated by the movement off the constraint.

We explore the extent to which higher-order local approximations can remedy the inaccuracy of the first-order approximation. We find that for small shocks, second and third-order approximations improve significantly, but for medium-sized shocks, the approximation worsens substantially and performs worse than the linear approximation. A second-order approximation correctly predicts a decline of the share of constrained households but projects a decline at a constant rate. However, the share of constrained households is a highly nonlinear object that flattens out at zero at a modestly large size of transfers. As a result, a second-order approximation ends up understating the aggregate consumption response. The third-order approximation takes into account the curvature in the decline of the share of constrained households and improves for small shocks, but we find it performs poorly for larger shocks.[3]

These features lead us to conjecture that inferring the aggregate effects of a fiscal intervention on the basis of the aggregate consumption response to very small disturbances may be misleading. In the remainder of the paper, we investigate when this is and is not the case. Overall, our findings do not bode well for using first-order methods, such as SSJ, to study the effects of fiscal interventions.

[3]Bhandari, Bourany, Evans and Golosov (2023) and Bilal (2023) develop methods to solve heterogeneous agent models with perturbation. See also Auclert, Rigato, Rognlie and Straub (2026) for another recent contribution. Motivated by the findings of this paper, Bayer, Briglia, Luetticke, Weiss and Winkelmann (2026) develop a perturbation algorithm in the state space that is highly accurate to first-order once the share of constrained households in response to the shock is solved exactly.

We start in Section 2 with a partial equilibrium exercise that highlights the key mechanism behind the nonlinearity and illustrates its quantitative significance. In Section 3, we then analyze various general equilibrium models with different assumptions about production, nominal rigidities, fiscal financing and monetary policy. In Section 4, we derive a general expression for the curvature of the aggregate consumption function and give examples of shocks and other policy interventions where linear solutions approximate nonlinear solutions much better. Our simulations suggest that the nonlinearities we emphasize are significant when a failure of Ricardian equivalence is an important component of the transmission mechanism. Consequently, those shocks and policies for which local methods are accurate in HA models tend to be those where aggregate responses are similar in RA and HA economies.

2 Partial Equilibrium

2.1 Model

Demographics Time is discrete and infinite: $t = 0, 1, \ldots$. The economy is populated by a measure-one continuum of households that face uninsurable idiosyncratic shocks. We assume the path for aggregate disturbances is known at $t = 0$, so there is no aggregate uncertainty.

Preferences Households derive utility from consumption, with $u(c_{it})$ strictly increasing and concave, and discount future utility by β; discount-factor heterogeneity is introduced in the calibration.

Income and assets Each period households receive earnings y_{it} whose realizations are independent across households. We let $F(y)$ denote the C.D.F. of the income shock. We describe in detail the stochastic process below. Households pay a proportional tax at rate τ on their earnings and receive a lump-sum transfer T_t from the government. They can save and borrow in a one-period risk-free real asset that pays an interest rate r.

Household problem Households enter period t with assets $a_{i,t-1}$ and choose next-period assets $a_{i,t}$. They face the borrowing constraint $a_{i,t} \geq 0$. The household problem is

$$\max_{\{c_{i,t}, a_{i,t}\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_{i,t})$$

subject to

$$c_{i,t} + a_{i,t} \leq (1+r)a_{i,t-1} + (1-\tau)y_{i,t} + T_t \quad \text{and} \quad a_{i,t} \geq 0.$$

The first-order condition of this problem yields the standard intertemporal Euler equation:

$$u'(c_{i,t}) \geq \beta(1+r)\mathbb{E}_t u'(c_{i,t+1})$$

with equality if $a_{i,t} > 0$.

Policy experiment. Starting from the steady-state distribution, we study an unexpected one-time increase in the lump-sum transfer to all households at $t = 0$. We implement this by setting the transfer to $T_0 = T + \Delta$ and keeping $T_t = T$ for $t > 0$. We denote aggregate consumption at time t by $\mathcal{C}_t(\Delta)$:

$$\mathcal{C}_0(\Delta) = \int_Y \int_{-\infty}^{\infty} c(\tilde{x} + \Delta, y)\, \mathrm{d}G(\tilde{x} \mid y)\, \mathrm{d}F(y)$$

where $G(\tilde{x} \mid y)$ denotes the distribution of cash on hand conditional on income y and $g(\tilde{x} \mid y)$ denotes its density. In addition, we let $\tilde{x} = (1+r)a_{i,t-1} + (1-\tau)y_{i,t} + T$ be cash-on-hand before the transfer Δ and $c(\tilde{x}, y)$ is the individual consumption policy.

2.2 Inspecting Concavity

The aggregate consumption response is concave for two reasons. The first is familiar: unconstrained household consumption policies are concave in cash on hand (e.g., Carroll and Kimball, 1996). The second—and central to our analysis—is that a transfer moves households off the borrowing constraint, replacing an MPC of one with a lower unconstrained MPC.

We denote by $\bar{x}(y)$ the level of cash on hand at which the borrowing constraint stops binding. We can then express aggregate consumption as

$$\mathcal{C}_0(\Delta) = \int_Y \Bigg[\underbrace{\int_{-\infty}^{\bar{x}(y)-\Delta} c(\tilde{x} + \Delta, y)\, \mathrm{d}G(\tilde{x} \mid y)}_{\text{constrained}} + \underbrace{\int_{\bar{x}(y)-\Delta}^{\infty} c(\tilde{x} + \Delta, y)\, \mathrm{d}G(\tilde{x} \mid y)}_{\text{unconstrained}} \Bigg] \mathrm{d}F(y). \quad (1)$$

Here, a household remains constrained after the transfer if and only if $\tilde{x} + \Delta \leq \bar{x}(y)$. Thus, as Δ rises, the mass of households for which the constraint binds falls.

A constrained household consumes its cash-on-hand and thus its MPC equals one. We

have

$$c_x(x,y) = \begin{cases} 1, & x < \bar{x}(y), \\ c_x^+(x,y), & x > \bar{x}(y), \end{cases}$$

where $c_x^+(x,y) < 1$ is the unconstrained MPC, derived in equation (B.6) of Appendix B. The MPC therefore drops discontinuously at the kink: below $\bar{x}(y)$, constrained households consume one-for-one out of additional cash on hand; above $\bar{x}(y)$, their MPC falls to $c_x^+ < 1$. This discontinuity is visible in the orange dashed low-income MPC line in Figure 1.[4]

Let $\Lambda(\Delta) \equiv \int_Y G(\bar{x}(y) - \Delta \mid y)\,\mathrm{d}F(y)$ denote the share of households that remain constrained under a transfer Δ, and let $\bar{\Lambda} \equiv \Lambda(0)$ be the share at steady state. Differentiating (1) at $\Delta = 0$ gives

$$\mathcal{C}_0'(0) = \bar{\Lambda} \times \underbrace{1}_{\text{constrained}} + (1-\bar{\Lambda})\,\underbrace{\mathbb{E}\big[c_x^+ \mid \tilde{x} > \bar{x}(y)\big]}_{\text{unconstrained}}. \tag{2}$$

Here, the expectation is taken over the joint stationary distribution of cash on hand and income, conditional on the household being unconstrained. The aggregate MPC is thus a weighted average of the MPCs of constrained and unconstrained households, with weights given by their population shares. Because consumption is continuous at the kink and the kink carries no mass, $\mathcal{C}_0'(\Delta)$ is obtained by differentiating directly under the integral sign; so, to first order, movement across the kink leaves the aggregate consumption response unaffected.

Consider now the second-order effects from transfers. We have

$$\mathcal{C}_0''(0) = \underbrace{\Lambda'(0)\,\mathbb{E}\big[(1-c_x^+) \mid \tilde{x} = \bar{x}(y)\big]}_{\text{movement off constraint}} + \underbrace{(1-\bar{\Lambda})\,\mathbb{E}\big[c_{xx}^+ \mid \tilde{x} > \bar{x}(y)\big]}_{\text{individual-policy concavity}}. \tag{3}$$

Here $\Lambda'(0) = -\int_Y g(\bar{x}(y) \mid y)\,\mathrm{d}F(y) < 0$ is the marginal change in the constrained share: its magnitude is the density of households at the kink, aggregated across income states.

The first term is the movement-off-constraint channel. A marginally larger transfer carries households located just below $\bar{x}(y)$ into the unconstrained region, where the MPC drops from one to $c_x^+(\bar{x}(y), y) < 1$; the conditional expectation $\mathbb{E}[(1-c_x^+) \mid \tilde{x} = \bar{x}(y)]$ averages this drop over the crossing households. The second term is the familiar concavity of individual consumption policies in incomplete-markets models for unconstrained households. Thus, in line with Carroll and Kimball (1996), higher transfers reduce the MPC for unconstrained households. We therefore have $\mathcal{C}_0''(0) < 0$.

[4] See Figure E.1 for the corresponding consumption function and Appendix B for a proof that c_x is discontinuous at the boundary of the linear segment.

2.3 Steady-State Calibration

Demographics A period represents a quarter. The numeraire in the model is quarterly GDP per household, which was around \$166,000/4 = \$41,500 in 2019.

Income and assets We set the interest rate to 2% p.a. and the borrowing limit to $\underline{a} = 0$. We model the process for the log of earnings, $\log y_{it}$, as the sum of two orthogonal components, an AR(1) component and an IID component. Following Kaplan and Violante (2022), we assume a quarterly arrival probability of 1/4 for shocks to both components, so that shocks are received on average once a year. We set the parameters of the earnings process to match moments of the household labor income distribution from the Panel Study of Income Dynamics. See Appendix A for details.

Preferences We assume $u(c) = \log(c)$ and introduce discount factor heterogeneity. We assume the discount factor β_i varies across households and remains fixed over their lifetime, and can take five values on an equally spaced grid. We calibrate the mean so that the ratio of mean wealth to annual GDP is 100% (\$166,000), and the range so that the average quarterly MPC out of a one-time 1,000 windfall is 0.25. The resulting discount factors range from 0.949 to 0.993.

Tax and transfers The tax system consists of a proportional tax rate and lump-sum transfer, whose values in steady state we denote by τ and T, respectively. Disposable labor income is $(1 - \tau)y_{it} + T$. We set $\tau = 0.274$ to match total government revenue of 27.4% of GDP in 2019. We set the steady-state lump-sum transfer $T = 15\%$ of GDP, which is around \$25,000 per household per year.

Wealth and MPC distributions Figure 1a shows the stationary wealth distribution overlaid with the MPC as a function of assets, for households with different income levels.[5] Figure 1b shows a histogram of the MPCs out of \$1,000 windfall. Two features of MPCs in this class of models that drive our aggregate findings are apparent. First, the average MPC is declining in wealth and approaches the PIH MPC as assets rise. This decline reflects two forces: (i) at each income level the MPC is weakly declining in wealth; and (ii) very low-wealth households are disproportionately those with low income, who are borrowing

[5]We plot the MPC out of an infinitesimal windfall because this is the MPC that is relevant for the linear vs nonlinear comparisons that are the focus of this paper. We calibrate to the MPC out of a \$1,000 windfall because that is more consistent with the empirical evidence we target.

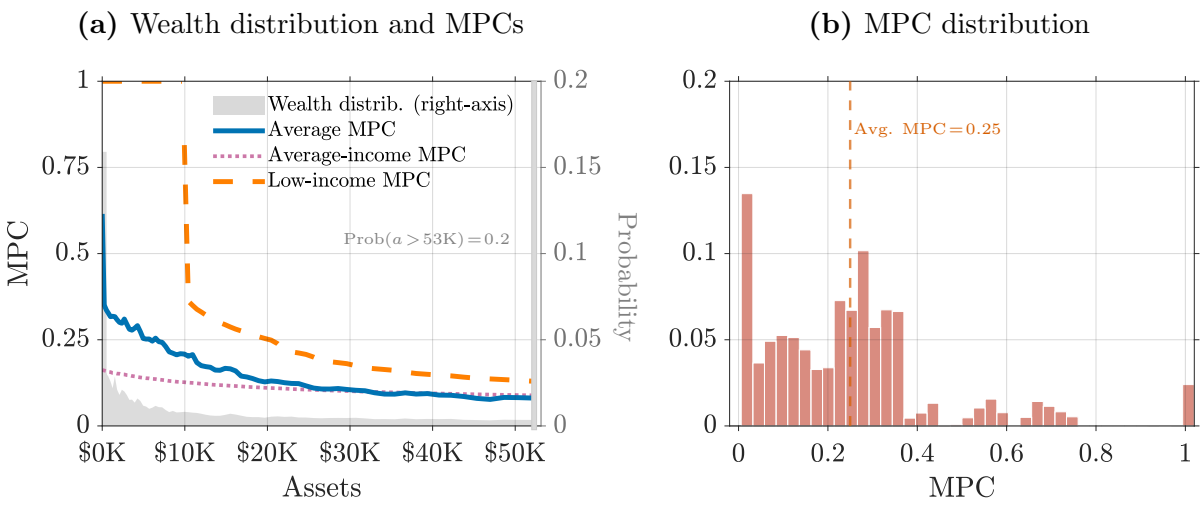


Figure 1: Steady-state wealth distribution and MPCs

Note: The left axis of panel (a) plots, as functions of assets, the average marginal MPC and the marginal MPCs of average- and low-income households with the lowest β_i. The right axis of panel (a) presents the wealth distribution up to \$50K. Panel (b) presents the distribution of MPCs out of \$1,000.

constrained at low asset levels and have an MPC of 100%. As wealth increases, the mix shifts toward high-income households who have much lower MPCs.

Second, the distribution of MPCs is highly dispersed and approximately bimodal. A subset of households with both low assets and low income has very high MPCs, close to 100%. For these households, the MPC function is flat at 100% in a small region above the borrowing constraint, after which it drops discontinuously. The next section explores the implications of these patterns for the shape of the aggregate consumption function.

2.4 Aggregate Transfers in Partial Equilibrium

Computation We compute steady-state decision rules using the endogenous gridpoint method. Since we are studying a one-time transfer at $t = 0$ in partial equilibrium, we can compute the nonlinear response to the transfer shock exactly using the steady-state decision rules and propagating forward the household wealth distribution.

We compute the linear response using a finite difference in Δ to approximate the derivative of the aggregate consumption function $\frac{\partial \mathcal{C}_t(\Delta)}{\partial \Delta}$. In partial equilibrium this is equivalent to how the transfer is treated in the SSJ method that we use for general equilibrium experiments in Section 3. We have also verified that computing $\frac{\partial \mathcal{C}_t(\Delta)}{\partial \Delta}$ using automatic differentiation yields the same result as our central finite-difference approximation, as well as the analytical

expression (2), with differences of order 10^{-7}. See Appendix C for computational details.

Linear vs. nonlinear responses Figure 2 contains the main results of the partial equilibrium exercise. Figure 2a shows the first-quarter increase in aggregate consumption $\mathcal{C}_0(\Delta) - \mathcal{C}_0(0)$, expressed as a percentage of annual GDP per household, as a function of the size of the transfer Δ. The blue solid line labeled "Nonlinear" shows the results computed using the nonlinear solution and the red dashed line labeled "Linear" shows the results from the linear solution. The aggregate consumption response is concave in the size of the transfer. For very small transfers, the linear and nonlinear solutions produce similar aggregate consumption responses. But for larger transfers, the linear solution substantially overstates the consumption response. For example, for a transfer that is 5% of annual GDP per household (around \$8,300), the linear solution overstates the nonlinear solution by around 42%. Even for a transfer of 0.5% of annual GDP (around \$830), the linear solution overstates the nonlinear solution by more than 17%.

Decomposition. Figure 3 quantifies these two forces. The exercise applies the exact nonlinear consumption policy for unconstrained households, while holding fixed the share of constrained households at its steady-state level and assigning constrained households an MPC of one.[6] This simulation is represented by the purple line in Figure 3a. As the figure shows, this economy is substantially closer to the linear simulation than to the nonlinear one. Quantitatively, the movement-off-constraint channel dominates: it accounts for about 93% of the gap between the linear and nonlinear impact responses at a transfer of 1% of annual GDP, and about 82% for a transfer equal to 5% of annual GDP.[7]

Alternative calibrations Figure 2b shows that the concavity of $\mathcal{C}_0(\Delta)$ is a robust feature of this class of models and is not specific to our particular calibration and choice of model features. The figure shows the ratio of the linear consumption response to the nonlinear response for transfers of different sizes, for a range of alternative calibrations and versions of the model. We report the results in terms of the ratio of the linear to the nonlinear consumption responses because the size of the response differs across models.

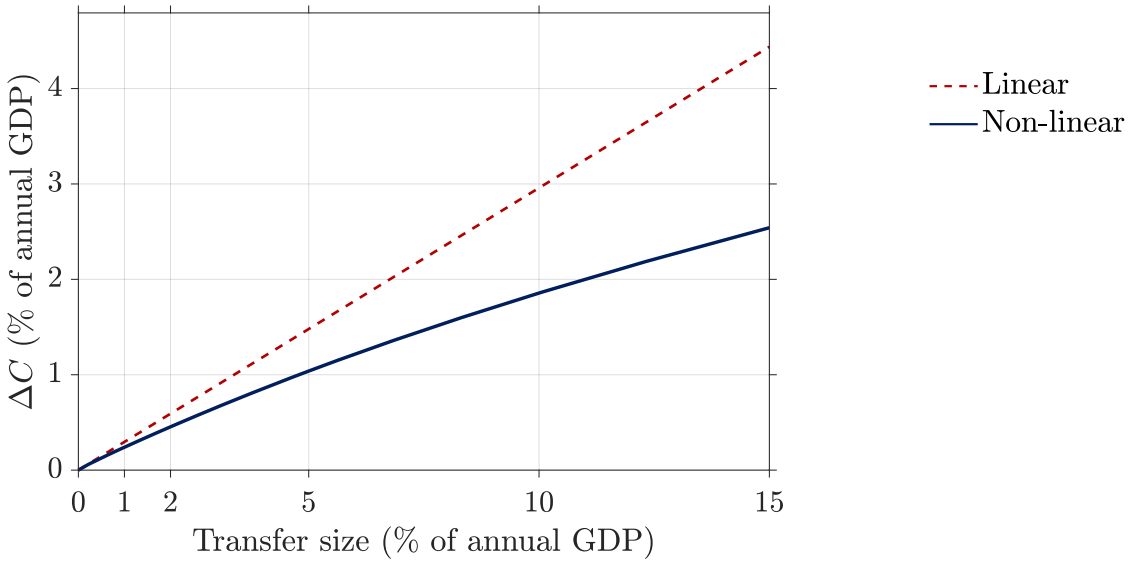
The figure includes the following versions of the model: those without discount-factor heterogeneity, with stochastic discount factors, with different levels of aggregate wealth, a life-

[6] Notice the parallel between an economy that holds the constrained share fixed and a TANK model.

[7] Figure E.5 presents an alternative decomposition where the share of constrained households is solved exactly and we approximate the response of unconstrained households to a first-order. This decomposition delivers almost identical results.

cycle version with finite lives, with lower lump-sum transfers, with a natural borrowing limit, with endogenous labor supply, and with the idiosyncratic earnings process from Auclert et al. (2024). In all these versions, and many others we have experimented with, the linear solution overstates the aggregate consumption response by an economically meaningful magnitude for transfer sizes larger than 2% of annual GDP. In many cases, including our baseline, the errors appear at much smaller transfer sizes.

(a) Aggregate effects of a one-time transfer in partial equilibrium



(b) Linear to nonlinear aggregate consumption response

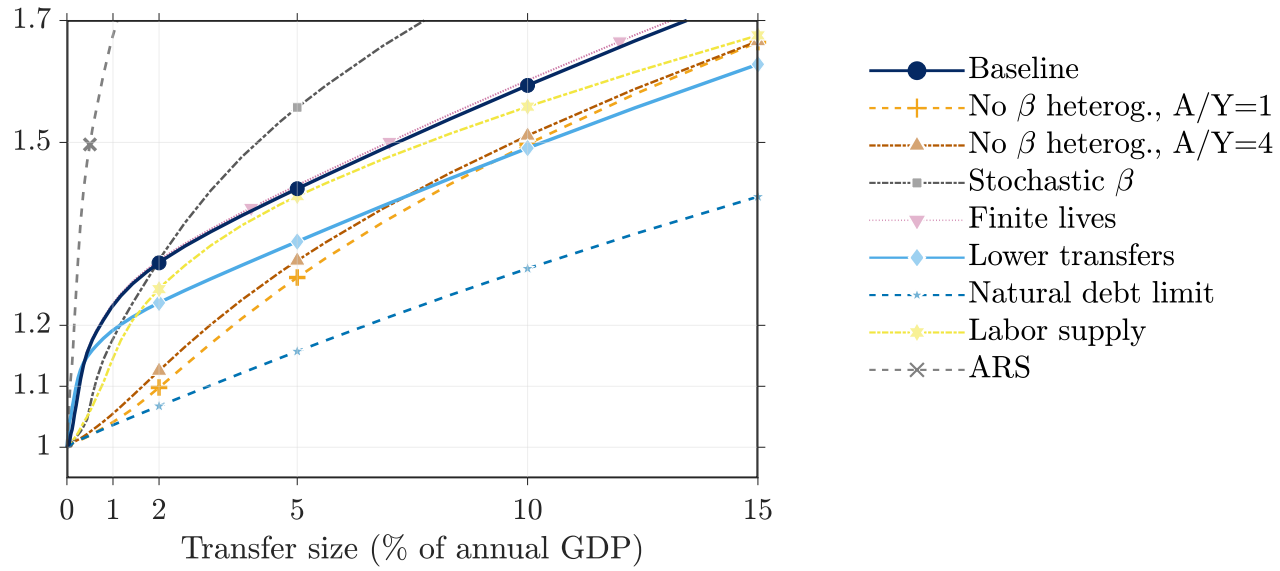
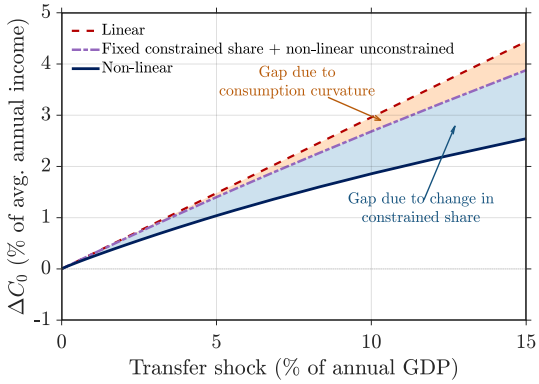


Figure 2: Partial equilibrium effects of a one-time transfer

Notes: Panel (a) expresses ΔC as a percentage of annual GDP. Panel (b) presents the ratio of the linear to the nonlinear consumption response for different model specifications. See Table A.3 in Appendix A.2 for more details.

(a) Decomposition of the linear–nonlinear gap



(b) Change in the constrained share at $t = 0$

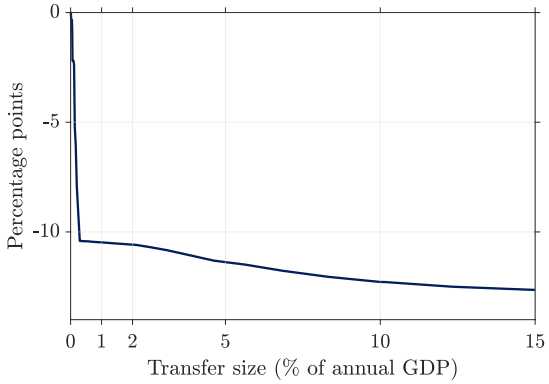


Figure 3: Decomposition of Curvature

Note: In panel (a), the fixed-constrained-share series combines the fully nonlinear response of unconstrained households with constrained-household consumption, holding population shares fixed. Panel (b) presents the change in the share of constrained households as a function of Δ.

2.5 Higher-Order Approximations

It is natural to conjecture that a more accurate solution could be obtained by taking a higher-order expansion around $\Delta = 0$. Figure 4a shows that second- and third-order approximations to $\mathcal{C}_0(\Delta)$ deliver an improvement for shocks below 1%.[8] However, Figure 4b shows that this improvement disappears for moderately larger shocks, as the higher-order approximations become less accurate than the linear approximation.

These results suggest that the neighborhood over which low-order local approximations are quantitatively accurate is limited. The second-order expansion effectively represents the constrained share as declining at a constant rate. In the exact solution, however, the constrained share is highly nonlinear: it falls as transfers relax borrowing constraints, but eventually flattens at zero once all agents become unconstrained. As a result, the local approximation extrapolates poorly and can worsen relative to the linear solution even for relatively small shocks. The third-order term brings in the slope of the density at the kink, $g'(\bar{x}(y) \mid y)$, together with the curvature of the consumption policy at the kink and an interior

[8] We construct the second-order Taylor expansion of $\mathcal{C}_0(\Delta)$ using (3). The third derivative admits an analogous decomposition in terms of the constrained-share function Λ,

$$\mathcal{C}_0'''(0) = \int_Y g'(\bar{x}(y) \mid y)\left[1 - c_x^+(\bar{x}(y), y)\right] \mathrm{d}F(y) + \int_Y g(\bar{x}(y) \mid y)\, c_{xx}^+(\bar{x}(y), y)\, \mathrm{d}F(y) + (1-\bar{\Lambda})\, \mathbb{E}\left[c_{xxx}^+ \mid \tilde{x} > \bar{x}(y)\right],$$

where $\Lambda''(0) = \int_Y g'(\bar{x}(y) \mid y)\, \mathrm{d}F(y)$. Appendix B derives analytically the underlying individual-policy derivatives c_{xx} and c_{xxx}.

third-derivative term. This improves accuracy only in a narrow neighborhood of $\Delta = 0$. Beyond that neighborhood, the implied approximation to the constrained share deteriorates quickly, and the third-order approximation becomes less accurate than the second-order approximation once transfers exceed about 1.5% of annual GDP.

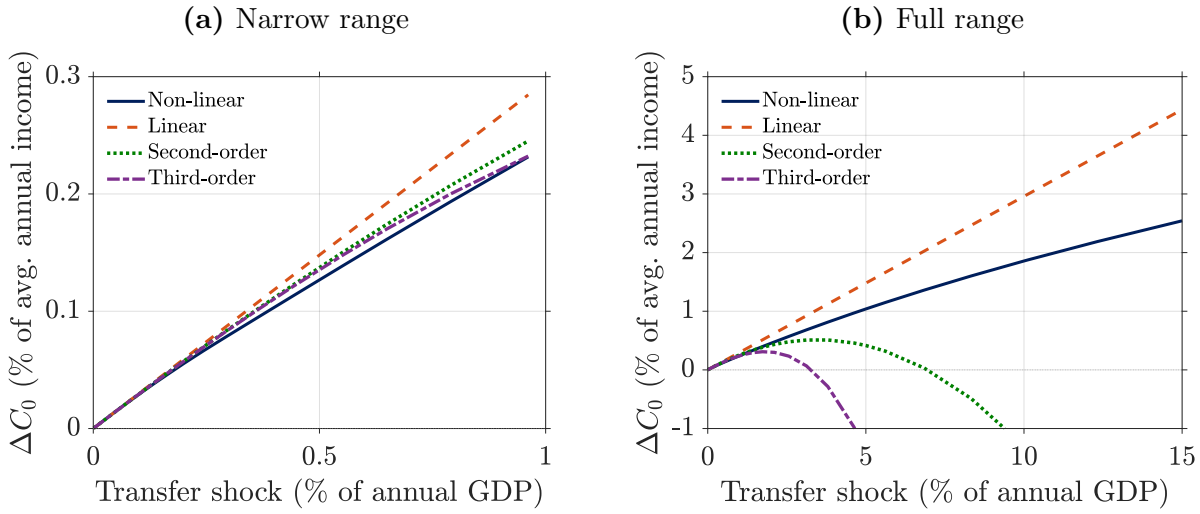


Figure 4: Local higher-order approximations to the impact response of aggregate consumption.

The results presented above are for a discrete income process. In the appendix, we show that the quality of the second-order local approximation is very similar when the transitory income component is continuous.[9]

3 General Equilibrium Model

In this section, we study the extent to which the nonlinearity of the aggregate consumption function impacts the general equilibrium effects of one-time fiscal stimulus programs. We extend the model from the previous section to include production, labor supply, asset market clearing and nominal rigidities, as in canonical HANK models (e.g., Kaplan and Violante, 2014; Auclert et al., 2023).

[9]In both cases we compute the second-order Taylor expansion from equation (3). In the continuous case the aggregate density at the kink, $-\Lambda'(0)$, can be obtained directly; in the discrete case we estimate it with a kernel density estimator. On the other hand, computing instead C'' with finite differences and a discrete income process is extremely sensitive to the choice of step size in the numerical derivative. Auclert et al. (2026) compute higher-order perturbations with finite differences and emphasize the importance of introducing a continuous shock.

3.1 Model

Preferences Households have utility over real consumption c_{it} and hours worked n_{it}:

$$\mathbb{E}_0 \sum_{t \geq 0} \beta_i^t \left[u(c_{it}) - v(n_{it}) \right]$$

where $v(\cdot)$ is a strictly increasing, convex function of hours worked.

Income and assets Real household income consists of earnings $e_{it} \frac{W_t}{P_t} n_{it}$ and dividend income d_{it}, where e_{it} is an idiosyncratic productivity shock that follows a process analogous to the one for y_{it} in Section 2. In this section, we denote total non-asset income as $y_{it} = e_{it} \frac{W_t}{P_t} n_{it} + d_{it}$. With this notation, the borrowing and budget constraints are the same as in Section 2, with the return on inherited assets given by r_{t-1}.

Labor supply In our baseline model we assume flexible prices and sticky wages. The aggregate effective labor input $N_t = \int_i n_{it} e_{it} di$ is chosen by a labor union and is allocated across households so that each household works the same number of hours, implying that $n_{it} = N_t$, where we normalize the cross-sectional mean of efficiency units to one, $\int_i e_{it}\, di = 1$. In Section 3.4 we also report results from a version of the model with sticky prices, flexible wages, and a labor supply choice at the household level.

Production A representative firm produces the final good Y_t with a linear production function that uses effective labor N_t as its only factor of production. Equilibrium therefore implies a real wage of $\frac{W_t}{P_t} = 1$ per effective unit of labor and zero profits so that $d_{it} = 0$.

Wage Phillips curve We adopt the wage-setting model in Auclert et al. (2024), in which monopolistically competitive labor unions set nominal wages subject to quadratic adjustment costs to maximize the welfare of a household with aggregate consumption $C_t = \int_i c_{it} di$ and hours N_t. This leads to the wage Phillips curve

$$\pi_t = \kappa \left(v'(N_t) - \frac{1 - \tau}{C_t} \right) + \bar{\beta}\, \pi_{t+1}, \tag{4}$$

where π_t denotes wage (and price) inflation, κ depends on the wage adjustment cost and $\bar{\beta}$ is the average discount factor. This is the same linearized wage Phillips curve that arises in an analogous representative-agent economy (e.g., Erceg et al., 2000). We choose to work

directly with the linearized Phillips curve so as to focus entirely on the nonlinearity of the consumption function stemming from the borrowing constraint.[10]

Fiscal policy The government issues real bonds B_t subject to the budget constraint[11]

$$B_t - B_{t-1} = G + T_t - \tau Y_t + r_{t-1} B_{t-1}, \tag{5}$$

where G is a fixed level of government consumption. Outside the steady state, the government adjusts the lump-sum transfer component of the tax-and-transfer schedule according to the rule

$$T_t = T - \phi_B \bar{r}(B_{t-1} - \bar{B}) \tag{6}$$

where $\phi_B > 1$ and $\bar{r}$ and $\bar{B}$ are the steady-state levels of the interest rate and government debt. This restriction implies that transfers are set so as to ensure that debt always returns to its steady-state level. Hence, fiscal policy is passive in the language of Leeper (1991).

Monetary policy The central bank sets the nominal interest rate according to the Taylor rule

$$i_t = \bar{i} + \phi_\pi(\pi_t - \bar{\pi}), \tag{7}$$

where $\bar{i}$ is the steady-state nominal rate, $\bar{\pi}$ is steady-state inflation and $\phi_\pi > 1$. Hence, monetary policy is active in the language of Leeper (1991). A standard Fisher equation connects real and nominal rates, $1 + i_t = (1 + r_t)(1 + \pi_{t+1})$ for $t \geq 0$.

Equilibrium Given fiscal and monetary policy rules (6) and (7), and an initial distribution of households over assets, productivity and discount factors, Γ_0, an equilibrium is a sequence of household policy functions $\{c_t(a, e, \beta), a'_t(a, e, \beta)\}_{t=0}^{\infty}$ and associated distributions $\{\Gamma_t(a, e, \beta)\}_{t=1}^{\infty}$, aggregate quantities $\{Y_t, C_t, B_t, N_t, d_t\}_{t=0}^{\infty}$ and prices $\{P_t, W_t, i_t, r_t\}_{t=0}^{\infty}$ such that (i) households and firms optimize; (ii) the wage Phillips curve (4) is satisfied, $P_t = W_t$ and $d_t = 0$; (iii) the government budget constraint (5) holds; (iv) the asset market clears

$$B_t = \int a'_t(a, e, \beta)\, \mathrm{d}\Gamma_t(a, e, \beta) \quad \text{for } t \geq 0;$$

[10] Eggertsson and Singh (2019) examine this type of nonlinearity in a representative agent New Keynesian model at the zero lower bound and find that these nonlinearities play a very modest role. Other examples of studies on the importance of nonlinearities in different contexts with incomplete markets include Cao and Nie (2017) and de Groot, Durdu and Mendoza (2025).

[11] We present results for an economy in which the government issues nominal bonds in Section 3.4.

and (v) aggregate consumption is given by

$$C_t = \int c_t(a, e, \beta)\, \mathrm{d}\Gamma_t(a, e, \beta) \quad \text{for } t \geq 0.$$

Walras' law implies that the goods-market-clearing condition $C_t + G = Y_t$ holds.

3.2 Calibration

The calibration follows closely the parameterization of the partial equilibrium model in Section 2. Below we describe the features that are specific to the general equilibrium version. See Table A.4 in Appendix A.2 for further details.

Preferences We assume the following functional form for the disutility of labor

$$v(n) = \chi \frac{n^{1+\frac{1}{\nu}}}{1+\frac{1}{\nu}},$$

where ν is the Frisch elasticity of labor supply, which we set equal to 1. We normalize χ so that quarterly output equals one in steady state. As in Section 2, we choose the set of discount factors to match a ratio of average wealth to annual GDP of 100% and an average quarterly MPC of 25% out of a one-time \$1,000 windfall.

Idiosyncratic income process The stochastic process for individual efficiency units of labor e_{it} is the same as the process for household labor income in Section 2.

Wage setting We set the slope of the wage Phillips curve $\kappa = 0.01$, as in Auclert et al. (2024).

Fiscal policy We set government consumption G to 10.4% of annual GDP. The steady-state tax rate and lump-sum transfer are the same as in Section 2. Hence the primary surplus as a percentage of annual GDP is $\tau - T - G = (0.274 - 0.15) - 0.104 = 2\%$. With a debt-to-GDP ratio of 100%, this implies a steady-state interest rate of 2% p.a. We set the parameter governing the speed of repayment outside the steady state to $\phi_B = 8$. This implies that the half-life of an increase in government debt is roughly 5 years.

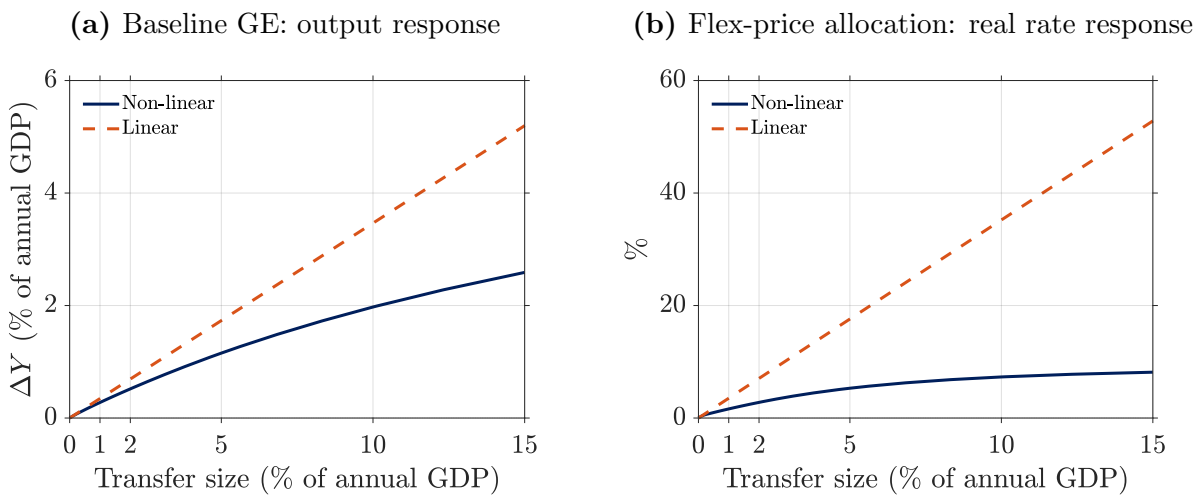


Figure 5: Aggregate effects of a one-time transfer in general equilibrium

Notes: The left panel presents the first-quarter output response with sticky wages. The right panel presents the first-quarter real interest-rate response with flexible wages. Rate deviations are annualized and expressed in percentage points.

Monetary policy We set steady-state inflation to $\bar{\pi} = 0$, so that the steady-state nominal rate is $\bar{\imath} = 2\%$. In our baseline experiments we set the Taylor rule coefficient $\phi_\pi = 1.5$.

3.3 Aggregate Transfers in General Equilibrium

Experiment As in Section 2, we study an unexpected one-time increase in the lump-sum transfer to all households at $t = 0$, which we implement by setting $T_0 = T + \Delta$. In our baseline experiment we assume that the stimulus is financed by lowering future lump-sum transfers for $t > 0$ according to the fiscal rule (6).

Computation We obtain the linear solutions using the SSJ method, following Auclert et al. (2021). We compute nonlinear solutions by finding the path of aggregate variables that satisfy the market-clearing conditions for each stimulus size Δ.

Baseline HANK results Figure 5a shows the aggregate output response for different values of Δ for the linear and nonlinear solution methods. The nonlinear solution is concave in the size of the shock. For a stimulus that is 2% of annual steady-state GDP (around \$3,320 per household), the first-quarter output response is around 0.53% of GDP according to the nonlinear solution and around 0.71% according to the linear solution. The error in the linear

solution is thus around 34%. For a stimulus of size 5% of annual steady-state GDP (around $8,300), the first-quarter error in the linear solution is around 50%.

For reference, the fiscal transfers deployed during the COVID-19 pandemic were far larger—on the order of 12.5% of annual GDP, measured by transfers to households and firms together with the increase in government debt.[12] Figure F.5 in Appendix F presents the impulse responses of output and other macroeconomic variables. As the figure shows, the linear solution understates the persistence of the shock. Because the nonlinear solution takes into account that some initially constrained households save a fraction of the transfers, this implies a higher level of aggregate demand going forward. Figure F.4 reports the analogous four-quarter responses. Naturally, because the response is averaged over more periods and the linear solution understates persistence, the gap between the linear and nonlinear simulations is smaller for the four-quarter response.[13] For purposes of estimation and inference, it is the entire path of the aggregate response that is arguably the most relevant one.

To understand where the errors in the linear solution stem from, it is useful to recap the mechanism by which a one-time transfer raises output in this model. Because of a failure of Ricardian equivalence, the combination of a higher transfer at $t = 0$ and lower future transfers leads households to want to increase their consumption, which leads to a rise in the natural interest rate. In the presence of nominal rigidities, this leads to an inflationary output boom as long as the central bank does not fully accommodate the rise in the natural rate by raising the nominal rate.[14]

The strength of this mechanism depends on the initial upward pressure on household consumption, which is determined entirely by the departure from Ricardian equivalence, as reflected in the subset of households with MPCs above the PIH. Recall that in a representative-agent version of this experiment there would be no effect on output for any size of stimulus. But as we saw in Figure 1, the high-MPC households are those who are on or close to the borrowing constraint and so have a consumption response that is highly nonlinear in the size of the transfer. Thus the same forces that drive the concave response of consumption in the partial equilibrium experiments in Section 2 also lead to concavity in the output response in

[12] In the United States the main programs were the CARES Act (March 2020, ≈ $2.2 trillion: direct payments to households, expanded unemployment insurance, and forgivable loans to firms under the Paycheck Protection Program), the December 2020 relief package (≈ $0.9 trillion), and the American Rescue Plan (March 2021, ≈ $1.9 trillion). In practice, however, transfers are directed toward lower-income households. We analyze this in Section 4.

[13] Auclert et al. (2026) also emphasize that the nonlinear solution delays the response.

[14] The precise mechanism by which output rises depends on the particular model of nominal rigidities. In this example, which features a monopolistic labor union, nominal wage rigidity, and a fixed real wage, the union trades off the cost of wage adjustments with the utility cost of supplying labor in excess of the utility-maximizing level.

general equilibrium.

Interest rate response with flexible wages Before investigating the robustness of these findings to alternative model specifications, Figure 5b shows the interest rate response when there are no wage adjustment costs. The red dashed and blue solid lines show the linear and nonlinear first-quarter responses when unions continue to choose the hours on behalf of households. In this economy, the fiscal stimulus policy has no effect on employment, aggregate output, or aggregate consumption. However, it does lead to movements in the real interest rate. The funded stimulus program is a mechanism to relieve the effects of borrowing constraints. Constrained households with high MPCs increase their consumption in response to the shock. Since output is unaffected, the interest rate must rise to induce higher-wealth households to cut their consumption. Once again, because the strength of these effects is tied to the shape of the consumption function for constrained households, the effect is weaker for larger values of Δ.

Figure F.3 in Appendix F shows the interest rate response in an analogous version of the model without a labor union, in which individual households can choose labor supply freely. In this model there are small output effects because labor-supply incentives differ across households with different productivity and asset levels. But since the extent of nonlinearity is ultimately tied to the failure of Ricardian equivalence, the aggregate effect looks similar to the version with a union.

3.4 Other Versions of the HANK Model

The baseline general equilibrium model in the previous section features sticky wages, flexible prices, real government debt and an active-monetary/passive-fiscal policy configuration. In this section we investigate whether the extent of nonlinearity in the effects of fiscal stimulus is affected by changes in these model features. The findings are summarized in Figure 6.

Faster fiscal adjustment The green dash-dot line labeled "Faster fiscal adjustment" in Figure 6 shows that the nonlinearity is more pronounced when the fiscal authority stabilizes debt more rapidly with a fiscal rule that sets $\phi_B = 16$ rather than $\phi_B = 8$ in the baseline. Since future transfers are lowered sooner, consumption is affected primarily among the most constrained households, whose departure from Ricardian equivalence is most severe. As we have seen, these households have the most nonlinear consumption functions.

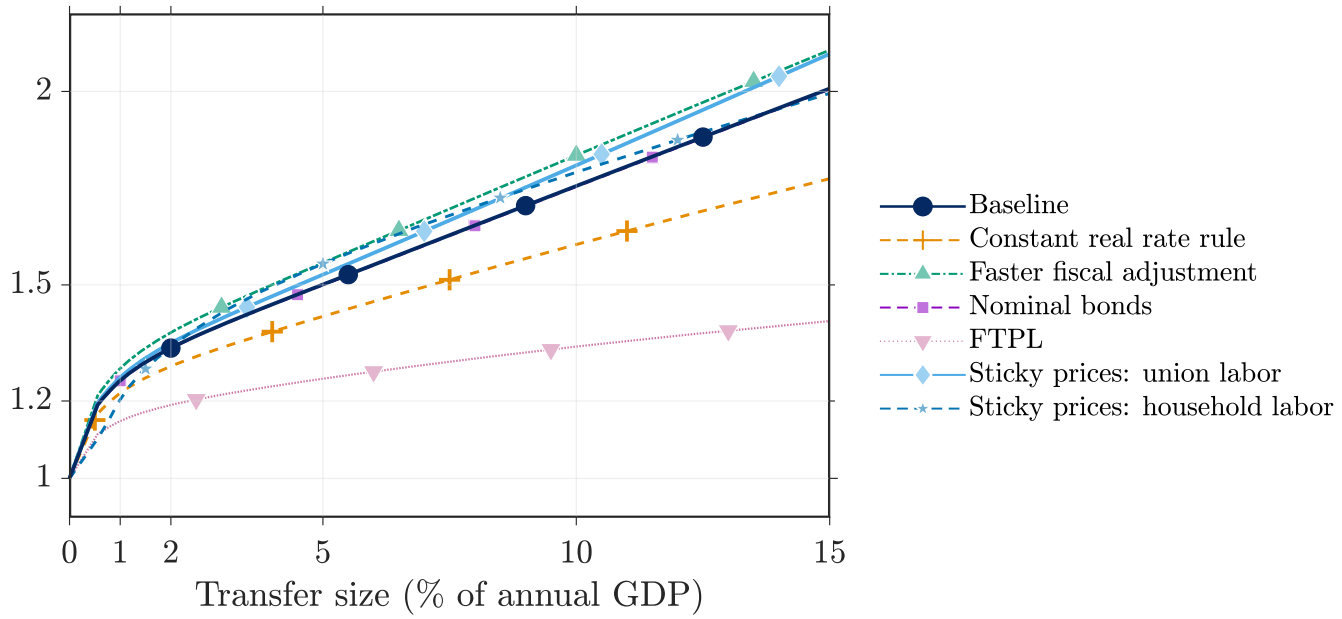


Figure 6: Sensitivity in HANK

Notes: The figure presents the ratio of the linear to the nonlinear first-quarter output response for different model specifications.

Fixed real rate The orange dashed line labeled "Constant real rate rule" in Figure 6 shows results from an economy where the central bank keeps the real rate constant. This economy has slightly less nonlinearity than the baseline economy.

Sticky prices and flexible wages An alternative model of nominal rigidities is one with sticky prices and flexible wages. We model sticky prices by introducing an intermediate-goods sector with a CES aggregator, monopolistic competition, and quadratic price adjustment costs as in Rotemberg (1982). This version of the model features a standard New Keynesian Phillips curve of the form

$$\pi_t = \kappa \left(\frac{W_t}{P_t} - 1 \right) + \bar{\beta} \pi_{t+1}$$

Figure 6 displays results for two versions, one in which the union chooses a common number of hours for all households (light-blue solid line), and one in which households freely choose how much labor to supply (blue dashed line).[15] Both versions exhibit a degree of nonlinearity similar to that in the baseline model.

[15]When households choose labor supply, they set n_{it} so that $\frac{1}{c_{it}} e_{it}(1-\tau)\frac{W_t}{P_t} = v'(n_{it})$. When the labor union chooses labor supply, it sets N_t so that $\frac{1}{C_t}\frac{W_t}{P_t}(1-\tau) = v'(N_t)$.

Unfunded fiscal stimulus We also consider a version of the model in which future inflation, rather than future primary surpluses, pays for the stimulus, as in the Fiscal Theory of the Price Level. To implement this version, we make two changes to the baseline model.

First, we assume that the government issues nominal rather than real debt. This, on its own, has a negligible effect on the extent of nonlinearity, as illustrated by the purple dashed line labeled "Nominal bonds" in Figure 6. The only effect of this change is that the unexpected inflation in the first period following the stimulus lowers the real value of debt that must be repaid with lower future surpluses.

Second, we alter the fiscal and monetary rules so that $\phi_B = \phi_\pi = 0$. This has the effect of pegging the nominal rate at $i_t = \bar{i}$ and holding real transfers fixed at their steady-state level $T_t = T$ for $t > 0$. Hence, in the language of Leeper (1991), fiscal policy is active and monetary policy is passive.

The ratio of the linear to nonlinear first-quarter output response is shown with the pink dotted line labeled "FTPL" in Figure 6. The figure shows that for an unfunded stimulus the extent of nonlinearity is much less severe than for the other versions of the model.[16] The reason is that a large part of the output effect of an unfunded stimulus does not rely on a failure of Ricardian equivalence or on households having MPCs above the PIH. Even in a representative-agent model, an unfunded stimulus of this type has an expansionary effect on output. The upward pressure on consumption arises because households seek to exchange nominal government debt for real consumption, as explained in Cochrane (2023). Redistribution toward high-MPC households plays a role as well, as explained by Kaplan, Nikolakoudis and Violante (2024), but it is a small part of the overall effect on output.

4 Extensions

We have focused on a particular policy experiment—a one-time lump-sum fiscal transfer—in which a failure of Ricardian equivalence is crucial for the aggregate effects. This section shows that the concavity result of Section 2.2 is a special case of a general decomposition, and uses that decomposition to organize a range of other shocks and policies.[17] The common thread is that the strength of the nonlinearity is governed by how much a shock moves borrowing-constrained, high-MPC households across the kink in their consumption policy.

[16] Figure F.6 reports the aggregate output effect of the unfunded stimulus, which is larger than for the funded stimulus.

[17] As in Section 2.2, we maintain the partial-equilibrium framework for the analytical decomposition, which is useful to illustrate the quantitative exercises we then conduct in general equilibrium.

4.1 General Formulation of Nonlinearities

Let ω denote an arbitrary shock, assumed to be a scalar, and suppose individual consumption, as a function of cash-on-hand (before transfers), x, and income, y, has a kink at a shock-dependent threshold $\bar{x}(y;\omega)$:

$$c(x,y;\omega) = \begin{cases} c^-(x,y;\omega), & x < \bar{x}(y;\omega), \\ c^+(x,y;\omega), & x > \bar{x}(y;\omega), \end{cases}$$

with consumption continuous at the threshold satisfying $c^-(\bar{x},y;\omega) = c^+(\bar{x},y;\omega)$. In our baseline the constrained branch has consumption equal to cash-on-hand but here we keep it generic, so that the same formulas cover soft borrowing limits and other kinks in the policy function. Holding the impact distribution $G(x \mid y)$ of cash on hand fixed, aggregate consumption is

$$C(\omega) = \int_Y \left[\int_{-\infty}^{\bar{x}(y;\omega)} c^-(x,y;\omega)\, \mathrm{d}G(x \mid y) + \int_{\bar{x}(y;\omega)}^{\infty} c^+(x,y;\omega)\, \mathrm{d}G(x \mid y) \right] \mathrm{d}F(y). \tag{8}$$

Differentiating (8) and using continuity at the kink, the first derivative is

$$C'(\omega) = \int_Y \left[\int_{-\infty}^{\bar{x}(y;\omega)} c^-_\omega(x,y;\omega)\, \mathrm{d}G(x \mid y) + \int_{\bar{x}(y;\omega)}^{\infty} c^+_\omega(x,y;\omega)\, \mathrm{d}G(x \mid y) \right] \mathrm{d}F(y). \tag{9}$$

As in the benchmark transfer of Section 2.2, movement across the threshold has *no* first-order aggregate effect: because consumption is continuous at the kink, the boundary terms cancel and $C'(\omega)$ is obtained by differentiating directly under the integral sign.

Let $\Lambda(\omega) \equiv \int_Y G(\bar{x}(y;\omega) \mid y)\, \mathrm{d}F(y)$ denote the share of households on the constrained branch. Differentiating (9) then yields the second derivative in the same form as the benchmark decomposition of Section 2.2,

$$C''(\omega) = \underbrace{\Lambda'(\omega)\, \frac{\displaystyle\int_Y g(\bar{x} \mid y)\, \bar{x}_\omega(y;\omega) \left[c^-_\omega(\bar{x},y;\omega) - c^+_\omega(\bar{x},y;\omega) \right] \mathrm{d}F(y)}{\displaystyle\int_Y g(\bar{x} \mid y)\, \bar{x}_\omega(y;\omega)\, \mathrm{d}F(y)}}_{\text{movement across the kink}}$$
$$+ \underbrace{\Lambda(\omega)\, \mathbb{E}\left[c^-_{\omega\omega}(x,y;\omega) \mid x < \bar{x}(y;\omega) \right] + [1-\Lambda(\omega)]\, \mathbb{E}\left[c^+_{\omega\omega}(x,y;\omega) \mid x > \bar{x}(y;\omega) \right]}_{\text{individual policy curvature}}. \tag{10}$$

The fraction multiplying $\Lambda'(\omega)$ is the average of $c^-_\omega - c^+_\omega$ across the households crossing the threshold, weighted by the density $g(\bar{x} \mid y)$ and the threshold movement $\bar{x}_\omega$, given by

$$\bar{x}_\omega(y;\omega) = -\frac{c^-_\omega - c^+_\omega}{c^-_x - c^+_x}, \tag{11}$$

which we obtain by totally differentiating $c^-(\bar{x}, y; \omega) = c^+(\bar{x}, y; \omega)$ with respect to ω and $\bar{x}$.

It is therefore the change in the constrained share times the change in the marginal shock response among crossing households, while the second line is the weighted average of individual policy curvature within each regime.

Remark 1 (Curvature of aggregate consumption function)**.** *Equation* (10) *highlights that the curvature of the aggregate consumption function is determined by two components: (i) movement across the kink, multiplied by the change in the marginal shock response at the kink; and (ii) and the weighted average of the second derivative of the individual consumption function with respect to the shock.*

4.2 Special Cases

We now analyze how a range of shocks fit as special cases of (10). For simplicity, we consider the baseline hard borrowing constraint, which implies $c^- = x + \omega$, $c^-_\omega = 1$, and $c^-_{\omega\omega} = 0$.

Baseline lump-sum transfers. The benchmark is recovered by setting $\omega = \Delta$, a current lump-sum transfer: the constrained branch is $c^-(x, y, \omega) = x + \Delta$, so $c^-_\Delta = 1$, $c^+_\Delta = c^+_x$, and the threshold shifts one-for-one, $\bar{x}_\Delta = -1$.

Targeted transfers. A targeted transfer delivers $\delta(x, y)\,\omega$ to each household with a state (x, y), where $\int \delta \,\mathrm{d}G\,\mathrm{d}F = 1$, so it costs the same as the uniform lump-sum. The uniform lump-sum of the previous case is $\delta = 1$. Constrained households spend it fully, $c^-_\omega = \delta$, and unconstrained households spend a fraction, $c^+_\omega = c^+_x\,\delta$, so the response gap at the kink is $\delta(\bar{x})(1 - c^+_x)$. Moreover, from (11), $\bar{x}_\omega = -\delta(\bar{x})$, the threshold moves by the amount of resources delivered to the households at the kink. When transfers are targeted to the low-income households, we have $\delta(\bar{x}) > 1$, and so the threshold moves *more* of them off the constraint per dollar than a uniform transfer, so both its first-order effect and its concavity are stronger. Targeting low-income households amplifies precisely the movement-off-constraint channel that generates the nonlinearity.

TFP shocks. A positive TFP shock raises each household's income in proportion to its productivity, so it hands the low-income households at the kink less resources than those unconstrained, $\delta(\bar{x}) < 1$. Both the response gap at the kink and the threshold movement scale with $\delta(\bar{x}, y)$. Consequently, the movement-off-constraint component scales pointwise with $\delta(\bar{x}, y)^2$. Because low-income households at the kink receive less than the average exposure under a proportional shock, this component is much smaller than under an equal-outlay lump-sum transfer. The response is still concave, but far less than the lump-sum, as shown in Figure G.1b.[18]

Tax cuts. A cut in the proportional labor-income tax works by the same logic. It raises after-tax income in proportion to earnings, $\delta(x, y) \propto y$, so like a TFP shock it reaches the low-income households at the kink only weakly ($\delta(\bar{x}) < 1$) and generates far less curvature than an equal-outlay lump-sum. See Figure G.1d.[19]

Borrowing constraint. We specify the constraint as $a' \geq -\underline{a}$ and consider a shock to $\underline{a}$. Relaxing the limit (increasing $\underline{a}$) raises the consumption of initially constrained households one-for-one, $c^-_{\underline{a}} = 1$, while leaving unconstrained households unaffected, $c^+_{\underline{a}} = 0$. Hence $c^-_{\underline{a}} - c^+_{\underline{a}} = 1$ and, from (11), $\bar{x}_{\underline{a}} = -1/(1 - c^+_x) < 0$, so $\Lambda'(\underline{a}) < 0$: a relaxation moves households *off* the constraint. Relative to a lump-sum transfer, a relaxation of the borrowing limit exhibits stronger concavity because the marginal response of a crossing household falls all the way from one (when constrained) to zero (once unconstrained), rather than from one to c^+_x.

Anticipated transfers. A transfer ω_1 that is received next period but anticipated today does not change current cash on hand, so constrained households are unaffected, $c^-_{\omega_1} = 0$, while unconstrained households raise current consumption, $c^+_{\omega_1} > 0$. This case is instructive because *both* ingredients of the composition term reverse sign relative to a current transfer. First, the constrained share now *rises* rather than falls: the threshold moves up, $\bar{x}_{\omega_1} = c^+_{\omega_1}/(1 - c^+_x) > 0$, so $\Lambda'(\omega_1) > 0$—anticipating future income, unconstrained households spend more and save less, and some are pushed *onto* the current constraint. Second, the response gap at the kink now runs the other way, $c^-_{\omega_1} - c^+_{\omega_1} = -c^+_{\omega_1} < 0$, whereas for a current transfer it is $1 - c^+_x > 0$.

[18] The constrained households hold only 3.9% of aggregate earnings against a 12.9% population share, so a proportional shock delivers less than a third of resources for those households crossing the kink compared to a lump-sum transfer.

[19] Note that in general equilibrium, there are additional effects on labor supply. In their calibration, Auclert et al. (2025) find that the aggregate response to a tax cut is slightly convex rather than concave.

Because the two signs change, their product $\Lambda'(\omega_1)\left(c^-_{\omega_1} - c^+_{\omega_1}\right)$ is again negative and the aggregate consumption function remains concave to future transfers.

Discount-factor shocks. A temporary rise in patience β lowers the current consumption of unconstrained households, $c^+_\beta < 0$, but leaves constrained households unchanged, $c^-_\beta = 0$, since they already consume all cash on hand. The response gap at the kink is $-c^+_\beta > 0$ and the threshold falls, $\bar{x}_\beta = c^+_\beta/(1 - c^+_x) < 0$, so $\Lambda'(\beta) < 0$. The key point is that the shock acts *only* through the unconstrained branch: it delivers no additional resources to the high-MPC households at the kink. Both the direct effect and the movement term are therefore muted, so discount-factor shocks generate far weaker nonlinearities than lump-sum transfers.

Interest rate. A change in the real interest rate r changes the intertemporal price of consumption. For those constrained, there are no effects, $c^-_r = 0$ and the unconstrained substitute toward the future, $c^+_r < 0$, so the movement channel is concave—as for every shock—but quantitatively small. As a result, Figure G.1a shows weak nonlinearities, consistent with the simulations in Auclert et al. (2021), which show that the SSJ tracks the nonlinear response very accurately. These results are also consistent with Werning (2015) and Kaplan et al. (2018), who show that heterogeneity has only a small impact on the aggregate effects of monetary policy shocks.[20]

5 Conclusions

Heterogeneous-agent models are useful for studying the effects of fiscal stimulus because the interaction between borrowing constraints and incomplete markets for idiosyncratic risk leads to a failure of Ricardian equivalence. However, the resulting MPCs are highly nonlinear in household wealth. As a result, the marginal aggregate effects of fiscal transfers to high-MPC households decline rapidly with transfer size. First-order methods in this class of models effectively extrapolate the average MPC and do not take into account the movement of households on and off the borrowing constraint. For some shocks and policy experiments, this approximation is accurate. But if local linear methods such as SSJ are to yield reliable quantitative results when studying fiscal stimulus—particularly redistributive stimulus targeted to high-MPC households—then "small" must be very small indeed.

[20] Rather, the nature of MPCs affects the transmission mechanism of monetary policy and the decomposition of the response into direct and indirect effects.

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Online Appendix to "How Small is Small? Nonlinearities in Heterogeneous-Agent Models"

Javier Bianchi and Greg Kaplan

A Calibration

A.1 Income Process

This appendix describes the estimation of the income process used in our quantitative analysis. We follow closely the approach of Kaplan and Violante (2022) (hereafter KV).

Panel Study of Income Dynamics

We use data from the Panel Study of Income Dynamics (PSID) on total annual household labor income for households whose heads were aged 25 to 65 from 1968 to 2008, following KV. We drop households with annual labor income of less than \$7,250 in 2016 dollars, which corresponds to 1,000 hours per year at \$7.25 per hour (part-time employment at the minimum wage). We remove age and year effects in a first stage by regressing household labor income on a full set of year and age dummies. We then construct the empirical counterparts to $m_{j,d}$ using the residuals from this regression.

Let $\log y_t^{ann}$ denote the log of annual labor income in year t, and let annual income growth at lag d be

$$\Delta_d \log y_t^{ann} = \begin{cases} \log y_t^{ann} & \text{if } d = 0, \\ \log y_{t+d}^{ann} - \log y_t^{ann} & \text{if } d > 0. \end{cases}$$

Define cross-sectional moments of annual income growth of order j at lag d as

$$m_{j,d} = \mathbb{E}\Big[\big(\Delta_d \log y_t^{ann}\big)^j\Big]$$

and the kurtosis of income growth at lag d as

$$\kappa_d = \frac{m_{4,d}}{(m_{2,d})^2}.$$

Table A.1 reports the empirical estimates for the cross-sectional moments, which coincide with those in KV. In estimation we target $m_{2,0}$, $m_{2,1}$, and $m_{2,5}$; the remaining moments serve

as untargeted validation.

Table A.1: Empirical moments of annual income growth at different lags.
Source: PSID 1968–2008.

Lag (d)	$m_{2,d}$	$m_{4,d}$	κ_d
0	0.504	0.930	3.65
1	0.142	0.220	10.90
2	0.207	0.369	8.57
3	0.235	0.410	7.42
4	0.280	0.544	6.96
5	0.295	0.557	6.39

Estimated Income Process

We model the quarterly income process as in KV:

$$\log y_t = \begin{cases} z_t + \varepsilon_t & \text{with probability } \lambda_\varepsilon, \qquad \varepsilon_t \sim \mathcal{N}\left(-\frac{\sigma_\varepsilon^2}{2},\, \sigma_\varepsilon^2\right), \\ z_t & \text{with probability } 1-\lambda_\varepsilon, \end{cases}$$

$$z_t = \begin{cases} \phi_z\, z_{t-1} + \eta_t & \text{with probability } \lambda_\eta, \qquad \eta_t \sim \mathcal{N}\left(-\frac{\sigma_\eta^2}{2},\, \sigma_\eta^2\right), \\ \phi_z\, z_{t-1} & \text{with probability } 1-\lambda_\eta. \end{cases}$$

We define annual income y^{ann} as the sum of the four quarterly incomes within the year. Based on this definition, we construct the model counterparts of all the empirical moments in Table A.1.

With $\lambda_\varepsilon = \lambda_\eta = 1/4$ fixed—so that each type of shock arrives on average once per year—we have three free parameters $(\phi_z,\, \sigma_\eta^2,\, \sigma_\varepsilon^2)$. We estimate them by minimizing the sum of squared deviations between the model-implied and empirical second moments $(m_{2,0},\, m_{2,1},\, m_{2,5})$.

We discretize the shock process using $n_z = 11$ Rouwenhorst states for the persistent component and $n_\varepsilon = 5$ points for the transitory component, for a total of 55 income states.

Table A.2 reports the estimated parameters. The persistent component has quarterly autocorrelation $\phi_z = 0.9878$ and innovation variance $\sigma_\eta^2 = 0.0439$; the transitory shock has variance $\sigma_\varepsilon^2 = 0.6376$, implying large but infrequent income fluctuations.

Specifically, the transitory mixture distribution $(1-\lambda_\varepsilon)\,\delta_0 + \lambda_\varepsilon\,\mathcal{N}(-\sigma_\varepsilon^2/2,\, \sigma_\varepsilon^2)$ is approxi-

mated with $n_\varepsilon = 5$ points by solving for the grid and probability weights that match its second and fourth central moments. The resulting grid spans $\log y^\varepsilon \in \{-2.85,\ -1.42,\ 0,\ 1.42,\ 2.85\}$ with probability weights $\{\approx 0,\ 0.042,\ 0.917,\ 0.042,\ \approx 0\}$, placing 91.7% of the mass at the no-shock point, consistent with the 75% no-shock probability from $\lambda_\varepsilon = 0.25$ plus the mass contributed by the discretized normal at zero.

Table A.2: Parameter estimates for the quarterly income process.

Parameter	Description	Symbol	Value
A. Structural parameters			
AR(1) persistence	Persistent component	ϕ_z	0.9878
Persistent shock variance	Innovation variance	σ^2_η	0.0439
Transitory shock variance	Transitory variance	σ^2_ε	0.6376
Persistent arrival rate	Quarterly probability	λ_η	0.250
Transitory arrival rate	Quarterly probability	λ_ε	0.250
B. Discretization			
Persistent states	Rouwenhorst grid	n_z	11
Transitory states	Mixture approximation	n_ε	5
Total income states		$n_z \times n_\varepsilon$	55
C. Targeted moments			
Variance, lag 0		$m_{2,0}$	0.504
Variance, lag 1		$m_{2,1}$	0.142
Variance, lag 5		$m_{2,5}$	0.295

A.2 Further Details on Calibration

Table A.3: Calibration: parameter values and steady-state moments across PE scenarios

	Parameters		Moments	
Scenario	$\bar{\beta}$	σ_β	Avg. MPC	A/Y
Baseline	0.970	0.011	0.25	1.00
No β heterog., $A/Y = 1$	0.991	0	0.09	1.00
No β heterog., $A/Y = 4$	0.993	0	0.05	4.00
Stochastic β	0.993	0.066	0.25	1.00
Finite lives	0.978	0.011	0.25	1.00
Lower transfers	0.966	0.013	0.25	1.00
Natural debt limit	0.961	0.016	0.25	1.00
Endogenous labor supply	0.956	0.018	0.32	1.00
ARS income process	0.976	0.006	0.25	1.00

Note: $\bar{\beta}$ and σ_β denote the mean and cross-sectional standard deviation of the discount factor across households; for the five-point Rouwenhorst grid σ_β also equals the spacing between adjacent support points, so $\beta_j = \bar{\beta} + (j-3)\sigma_\beta$ with population weights $\{1, 4, 6, 4, 1\}/16$. Avg. MPC is the average quarterly MPC out of a one-time \$1,000 windfall. The versions of the model are as follows: *No β heterog. ($A/Y = 1$)* and *No β heterog. ($A/Y = 4$)* are versions without discount-factor heterogeneity, calibrated to match aggregate wealth-to-annual-GDP ratios of 1 and 4, respectively. *Stochastic β*: discount factors follow an idiosyncratic AR(1) process with autocorrelation $\rho_\beta = 0.990$. *Finite lives*: a life-cycle version with overlapping generations and an annual mortality probability of 2%. *Lower transfers*: steady-state lump-sum transfers are set to half of the baseline value. *Natural debt limit*: households face a 0.1% probability of transitioning to a zero-income state, and their lump-sum transfers are set to zero. *Endogenous labor supply*: households choose hours worked endogenously with isoelastic disutility from labor and unitary Frisch elasticity. *ARS*: replaces the baseline idiosyncratic earnings process with the one used in Auclert et al. (2024).

Table A.4: Summary of Parameters for the General Equilibrium Model

Parameter	Value	Description
$\bar{B}$	4	Steady-state government debt
$\bar{r}$	0.02	Steady-state real rate (annualized)
$\bar{\pi}$	0	Steady-state inflation
κ	0.01	Phillips curve slope
ν	1	Frisch elasticity of labor supply
ϕ_π	1.5	Taylor coefficient
ϕ_B	8	Fiscal rule
G	0.104	Government spending
T	0.15	Government transfers
χ	0.82	Labor disutility

B Analytical Characterization of MPCs

This appendix derives analytical first-order and higher-order derivatives of the individual consumption function.

We denote by x cash on hand after transfers and write the MPC as $c_x(x, y)$.[21] Next-period cash on hand is

$$x' = (1 + r)[x - c(x, y)] + y',$$

where y' is next-period non-asset income.

Let $\bar{x}(y)$ denote a borrowing-constraint boundary point: locally, the constraint binds for $x < \bar{x}(y)$ and is slack for $x > \bar{x}(y)$. Define the one-sided MPCs at the boundary by

$$c_x^-(\bar{x}(y), y) \equiv \lim_{\varepsilon \downarrow 0} c_x(\bar{x}(y) - \varepsilon, y), \qquad c_x^+(\bar{x}(y), y) \equiv \lim_{\varepsilon \downarrow 0} c_x(\bar{x}(y) + \varepsilon, y).$$

Let $V(a, y)$ denote the beginning-of-period value function, with asset state a. Given cash on hand x, the household solves

$$\max_{a' \geq \underline{a}} \{u(x - a') + \beta \mathbb{E}[V(a', y') \mid y]\}.$$

The consumption policy is

$$c(x, y) = x - a'(x, y).$$

For the slack branch, use the notation

$$c \equiv c(x, y), \qquad c' \equiv c(x', y'), \qquad c_x \equiv c_x(x, y), \qquad (c_x)' \equiv c_x(x', y'),$$

and

$$(c_{xx})' \equiv c_{xx}(x', y'), \qquad (c_{xxx})' \equiv c_{xxx}(x', y').$$

Define

$$\Omega \equiv \beta(1 + r)^2 \mathbb{E}\left[u''(c')(c_x)' \mid y\right],$$

$$\Psi \equiv \beta(1 + r)\mathbb{E}\left[u'''(c')((c_x)')^2 + u''(c')(c_{xx})' \mid y\right],$$

$$\Theta \equiv \beta(1 + r)\mathbb{E}\left[u''''(c')((c_x)')^3 + 3u'''(c')(c_x)'(c_{xx})' + u''(c')(c_{xxx})' \mid y\right].$$

[21] The consumption policy varies across households with different values of β; we do not index it to simplify notation.

Constrained branch. First consider the constrained branch. If the borrowing constraint binds, then

$$a'(x, y) = 0$$

Therefore

$$c(x, y) = x$$

Differentiating with respect to cash on hand gives

$$c_x(x, y) = 1, \qquad c_{xx}(x, y) = 0, \qquad c_{xxx}(x, y) = 0. \tag{B.1}$$

Thus

$$c_x^-(\bar{x}(y), y) = 1.$$

Slack branch: first derivative. Now consider the slack branch, where $a'(x, y) > \underline{a}$. The first-order condition is

$$u'(c(x, y)) = \beta \mathbb{E}\left[V_a(a'(x, y), y') \mid y\right]. \tag{B.2}$$

At smooth continuation points, differentiating (B.2) with respect to x gives

$$u''(c)c_x = \beta \mathbb{E}\left[V_{aa}(a'(x, y), y') \mid y\right] a'_x.$$

Since

$$a'_x = 1 - c_x(x, y),$$

we obtain

$$u''(c)c_x = \widetilde{\Omega}(1 - c_x), \tag{B.3}$$

where

$$\widetilde{\Omega} \equiv \beta \mathbb{E}\left[V_{aa}(a'(x, y), y') \mid y\right].$$

If the continuation value is strictly concave at the smooth continuation points reached from the slack branch, then

$$\widetilde{\Omega} < 0.$$

Since $u''(c) < 0$, equation (B.3) implies

$$c_x(x, y) = \frac{\widetilde{\Omega}}{u''(c) + \widetilde{\Omega}}, \qquad 0 < c_x(x, y) < 1. \tag{B.4}$$

Thus the slack-side MPC is strictly below the constrained-side MPC.

The expression above can be written in terms of the Euler equation. At a smooth continuation point, the envelope condition gives

$$V_a(a', y') = (1+r)u'(c(x', y')), \qquad x' = (1+r)a' + y'. \tag{B.5}$$

Differentiating (B.5) with respect to a' gives

$$V_{aa}(a', y') = (1+r)^2 u''(c')(c_x)'.$$

Therefore

$$\widetilde{\Omega} = \beta(1+r)^2 \mathbb{E}\left[u''(c')(c_x)' \mid y\right] \equiv \Omega.$$

Substituting this into (B.4) gives

$$c_x(x, y) = \frac{\Omega}{u''(c) + \Omega}. \tag{B.6}$$

Slack branch: second derivative. The Euler equation on the slack branch is

$$u'(c) = \beta(1+r)\mathbb{E}\left[u'(c') \mid y\right]. \tag{B.7}$$

Define

$$F(z, y') \equiv u'(c(z, y')).$$

Then, evaluated at $z = x'$,

$$F_z = u''(c')(c_x)', \tag{B.8}$$

$$F_{zz} = u'''(c')((c_x)')^2 + u''(c')(c_{xx})', \tag{B.9}$$

$$F_{zzz} = u''''(c')((c_x)')^3 + 3u'''(c')(c_x)'(c_{xx})' + u''(c')(c_{xxx})'. \tag{B.10}$$

Also,

$$x'_x = (1+r)(1-c_x), \qquad x'_{xx} = -(1+r)c_{xx}, \qquad x'_{xxx} = -(1+r)c_{xxx}. \tag{B.11}$$

Differentiating (B.7) twice with respect to x gives

$$u'''(c)c_x^2 + u''(c)c_{xx} = \beta(1+r)\mathbb{E}\left[F_{zz}(x'_x)^2 + F_z x'_{xx} \mid y\right].$$

Using (B.8), (B.9), and (B.11), this becomes

$$u'''(c)c_x^2 + u''(c)c_{xx} = [(1+r)(1-c_x)]^2\Psi - \Omega c_{xx}.$$

Rearranging yields

$$c_{xx}(x,y) = \frac{[(1+r)(1-c_x)]^2\Psi - u'''(c)c_x^2}{u''(c)+\Omega}. \tag{B.12}$$

Slack branch: third derivative. Differentiating (B.7) three times with respect to x gives

$$u''''(c)c_x^3 + 3u'''(c)c_xc_{xx} + u''(c)c_{xxx} = \beta(1+r)\mathbb{E}\left[F_{zzz}(x'_x)^3 + 3F_{zz}x'_xx'_{xx} + F_zx'_{xxx} \mid y\right].$$

Using (B.8), (B.9), (B.10), and (B.11), the right-hand side is

$$[(1+r)(1-c_x)]^3\Theta - 3(1+r)^2(1-c_x)c_{xx}\Psi - \Omega c_{xxx}.$$

Therefore

$$u''''(c)c_x^3 + 3u'''(c)c_xc_{xx} + u''(c)c_{xxx} = [(1+r)(1-c_x)]^3\Theta - 3(1+r)^2(1-c_x)c_{xx}\Psi - \Omega c_{xxx}.$$

Rearranging gives

$$c_{xxx}(x,y) = \frac{[(1+r)(1-c_x)]^3\Theta - 3(1+r)^2(1-c_x)c_{xx}\Psi - u''''(c)c_x^3 - 3u'''(c)c_xc_{xx}}{u''(c)+\Omega}. \tag{B.13}$$

Summary. Combining the constrained-branch derivatives in (B.1) with the slack-branch formulas in (B.6), (B.12), and (B.13), we obtain

$$c_x(x,y) = \begin{cases} 1, & x < \bar{x}(y), \\ \dfrac{\Omega}{u''(c)+\Omega}, & x > \bar{x}(y), \end{cases}$$

$$c_{xx}(x,y) = \begin{cases} 0, & x < \bar{x}(y), \\ \dfrac{[(1+r)(1-c_x)]^2\Psi - u'''(c)c_x^2}{u''(c)+\Omega}, & x > \bar{x}(y), \end{cases}$$

$$c_{xxx}(x,y) = \begin{cases} 0, & x < \bar{x}(y), \\ \dfrac{[(1+r)(1-c_x)]^3\Theta - 3(1+r)^2(1-c_x)c_{xx}\Psi - u''''(c)c_x^3 - 3u'''(c)c_x c_{xx}}{u''(c)+\Omega}, & x > \bar{x}(y). \end{cases}$$

In particular,

$$c_x^-(\bar{x}(y),y) = 1, \qquad 0 < c_x^+(\bar{x}(y),y) < 1,$$

so the MPC is discontinuous at the point where the borrowing constraint becomes slack.

C Numerical Algorithm

C.1 Partial Equilibrium Algorithm

In partial equilibrium, the path for prices is exogenous, so the problem reduces to solving the household block and aggregating over the induced distribution. The algorithm is as follows.

1. Discretize idiosyncratic income using a Markov chain (see Appendix A), choose an asset grid $\mathcal{A}$ and set a horizon $\mathbb{T}$.

2. For a given transfer path, solve the household problem using the endogenous-grid method (Carroll, 2006), interpolating linearly for assets not on the grid.

3. Given the policy rules, propagate the cross-sectional distribution forward until it converges to the ergodic distribution $\Gamma(a, y, \beta)$.

4. Compute the linear response by finite differences. We solve the full household transition under a small transfer ε at $t = 0$. At $t = 0$,

$$s_0 \;=\; \frac{C_0^{nl}(+\varepsilon) - C_0^{nl}(-\varepsilon)}{2\varepsilon},$$

   where

$$C_0^{nl}(\varepsilon) \equiv \int c_i((1+r)a + y_i + \varepsilon,\, y_i;\, \beta_i)\, \mathrm{d}\Gamma_0(a,i),$$

   The first argument of the policy is period-0 cash on hand: the transfer ε augments cash on hand $(1+r)a+y_i$, not beginning-of-period assets. Here β_i is the household's discount factor and Γ_0 is the (given) steady-state distribution over assets a and types i. We use $\varepsilon = 1.0 \times 10^{-5}$. The derivatives obtained using automatic differentiation and central

finite differences differ by approximately 10^{-7}. When the perturbed cash-on-hand argument lies off the policy grid, we interpolate $c(\cdot)$ linearly.

Then, using the first-order derivative s_0, we compute the linear impulse response to a transfer of size Δ as

$$\Delta C_0^{lin} \;=\; \Delta \cdot s_0.$$

For $t \geq 1$, the transfer has already been paid at $t = 0$, so it propagates only through the distribution, which is advanced from the period-0 perturbed distribution under the (time-invariant) steady-state policies:

$$C_t^{nl}(\varepsilon) \equiv \int c_i((1+r)a + y_i,\, y_i;\, \beta_i)\, \mathrm{d}\Gamma_t(a, i;\, \varepsilon),$$

where $\Gamma_t(\cdot\,;\varepsilon)$ is the date-t distribution along the transition induced by the $t = 0$ transfer ε. The horizon-t linear response is then

$$s_t \;=\; \frac{C_t^{nl}(+\varepsilon) - C_t^{nl}(-\varepsilon)}{2\varepsilon}, \qquad \Delta C_t^{lin} \;=\; \Delta \cdot s_t.$$

5. Compute the nonlinear response by evaluating the individual consumption policies at the transfer and aggregating using the distribution, which is propagated forward. That is,

$$C_t^{nl}(\Delta) \equiv \int c_i\big((1+r)a + y_i + \Delta\, \mathbf{1}\{t=0\},\, y_i;\, \beta_i\big)\, \mathrm{d}\Gamma_t(a, i;\, \Delta),$$

where in partial equilibrium the policy functions are time-invariant (prices are fixed): the transfer enters period-0 cash on hand directly and, for $t \geq 1$, only through the propagated distribution $\Gamma_t(\cdot\,;\Delta)$.

For the asset grid, we use an exponential grid with $n_a = 200$ points on $[0, \bar{a}]$, where $\bar{a} = 4{,}000$. We use $\mathbb{T} = 32$.

D General Equilibrium

Let the unknown transition path be

$$X \equiv \{C_t, N_t, Y_t, \pi_t, i_t, r_t, B_t\}_{t=0}^{\mathbb{T}-1}.$$

We compute the first-order transition using the sequence-space Jacobian method of Auclert et al. (2021). We use their Python toolkit to compute it with $\mathbb{T} = 150$.

To solve the household problem nonlinearly, we first conjecture a vector of sequences X, and then solve backward for policy functions using EGM and propagate the distribution forward. This yields household asset demand $A_t(X;\Delta)$ at each date. We then evaluate the residuals from the equilibrium conditions and iterate with a modified Newton method.

Specifically, we conjecture $\{Y_t, \pi_t\}_{t=0}^{\mathbb{T}-1}$ and then obtain $\{C_t, N_t, i_t, B_t, r_t\}_{t=0}^{\mathbb{T}-1}$ using $Y_t = N_t = C_t + G$ and

$$
\begin{aligned}
i_t &= \bar{i} + \phi_\pi(\pi_t - \bar{\pi}), \\
1 + r_t &= \frac{1 + i_t}{1 + \pi_{t+1}}. \\
B_t &= (1 + r_{t-1})B_{t-1} + G + T_t - \tau Y_t.
\end{aligned}
$$

The nonlinear equilibrium residuals for all $t \geq 0$ are

$$
\begin{aligned}
F_t^A(X;\Delta) &= A_t(X;\Delta) - B_t(X;\Delta), \\
F_t^\pi(X;\Delta) &= \kappa \left(v'(N_t) - \frac{1-\tau}{C_t} \right) + \bar{\beta}\, \pi_{t+1} - \pi_t,
\end{aligned}
$$

Given the conjectured X, we solve backward for policy functions using EGM, propagate the distribution forward, compute asset demands and evaluate residuals. We update the conjectured sequences by a modified Newton method.

E Partial Equilibrium: Additional Results

E.1 Additional Figures

Figure E.1: Consumption Function for Low-Income Households

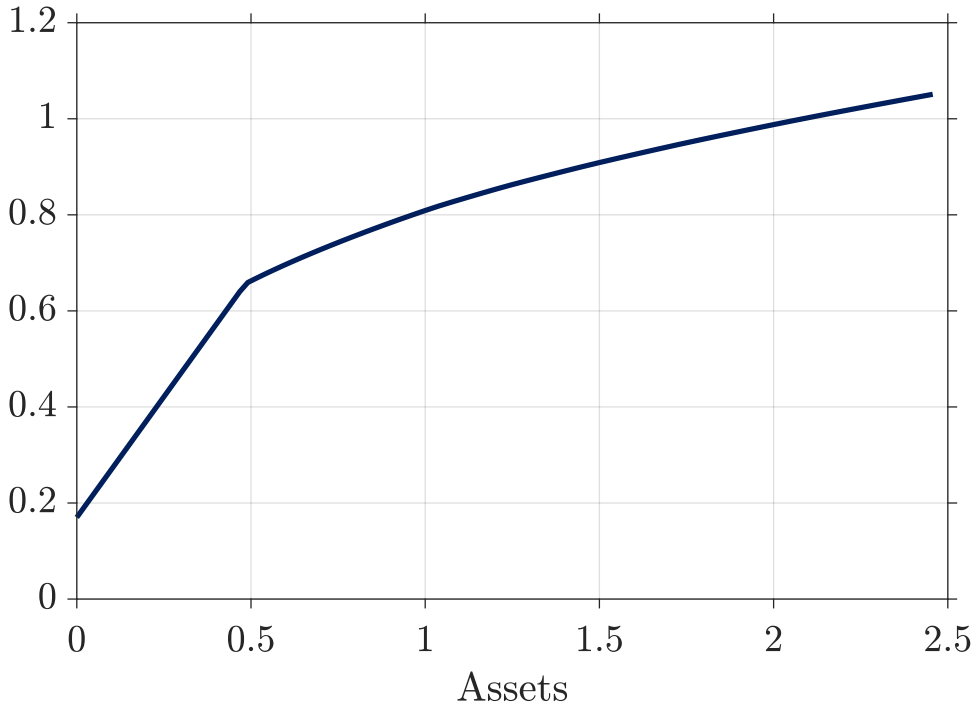
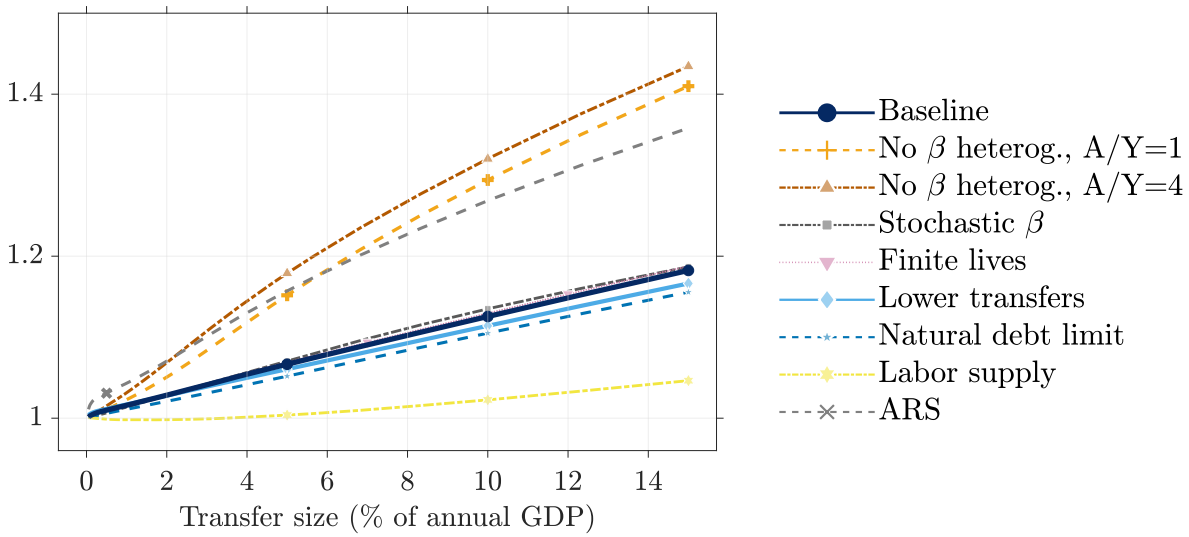
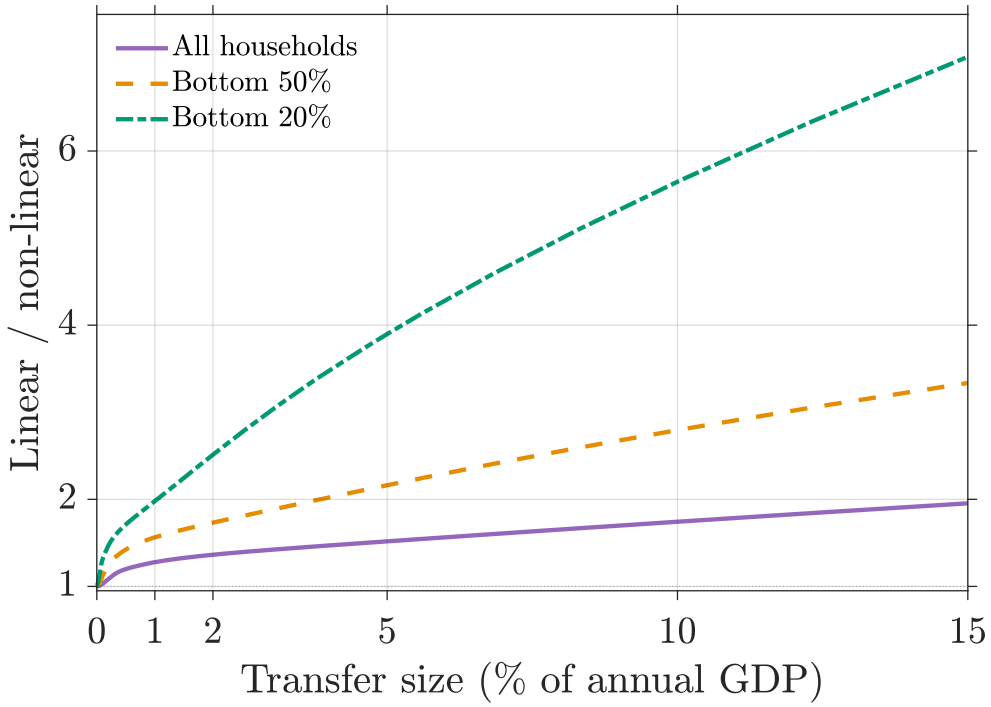


Figure E.2: Ratio of Linear to Nonlinear First-Year Aggregate Consumption Responses



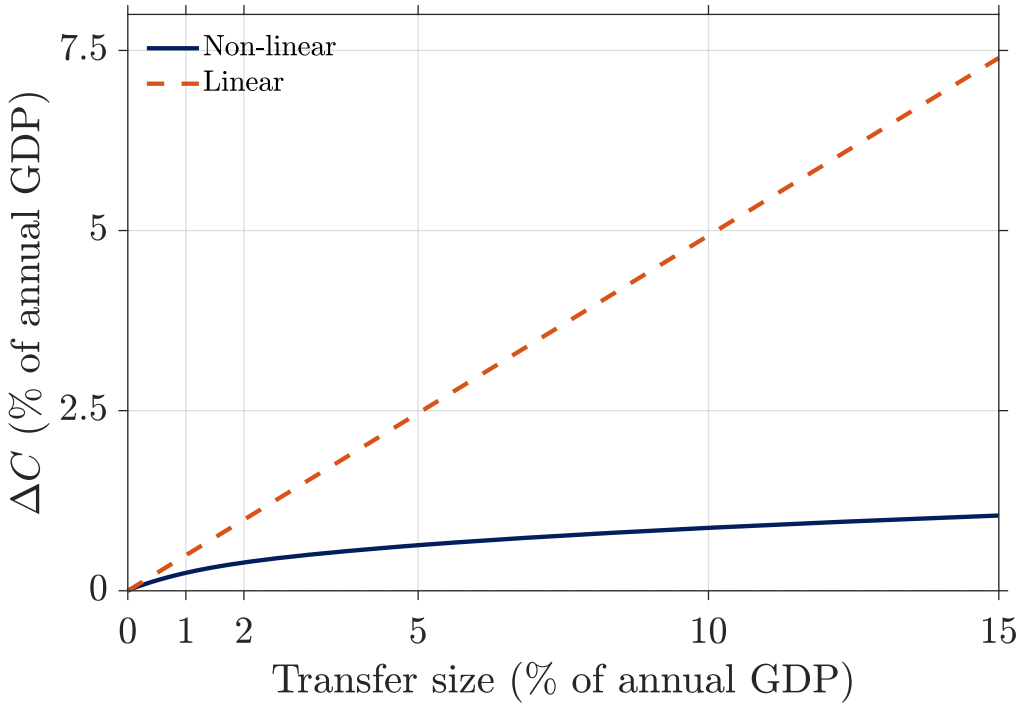
Note: The figure presents the ratio of the 12-month linear to nonlinear consumption response in the partial equilibrium model.

Figure E.3: Targeted Transfers



Note: Ratio of the linear to the nonlinear partial-equilibrium impact consumption response to a one-time transfer, as a function of transfer size, when the period-0 transfer is directed to all households, the bottom 50%, or the bottom 20% of the income distribution.

Figure E.4: Targeted Transfers in Partial Equilibrium



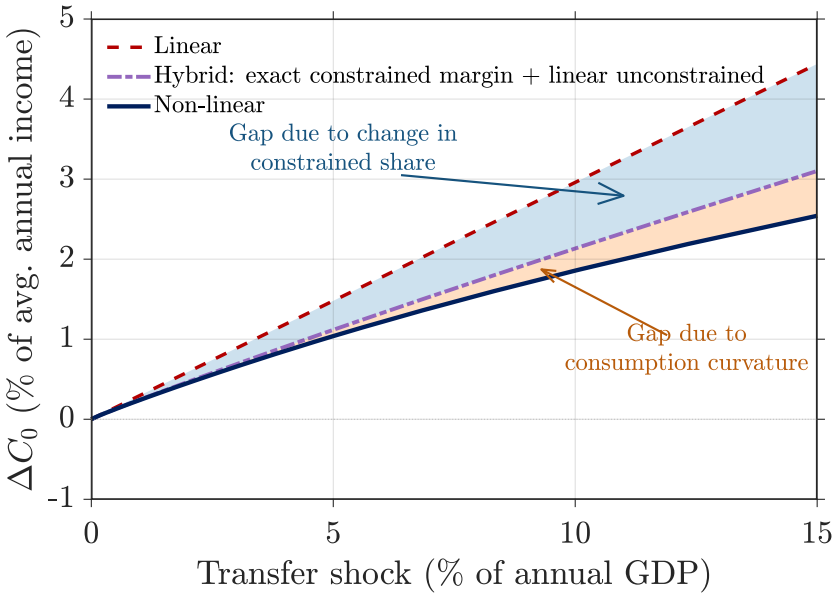
Note: Consumption response in the baseline partial-equilibrium model where transfers in period 0 are directed toward households in the bottom 20% of the income distribution. Partial-equilibrium counterpart of Figure F.1.

E.2 Alternative Decomposition

Figure E.5 presents an alternative decomposition of the sources of concavity. The purple line represents the aggregate consumption response considering the exact share of constrained households and computing the response of unconstrained households using a first-order Taylor approximation. That is,

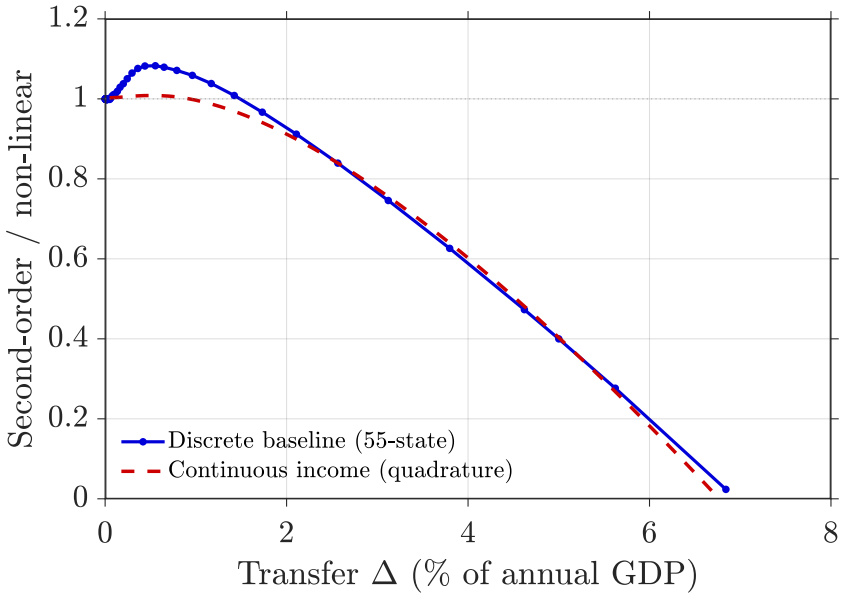
$$\mathcal{C}_0^{\text{hyb}}(\Delta) = \int_Y \left[\underbrace{\int_{-\infty}^{\bar{x}(y)-\Delta} c(\tilde{x}+\Delta, y)\, \mathrm{d}G(\tilde{x} \mid y)}_{\text{exact constrained share, MPC=1}} + \underbrace{\int_{\bar{x}(y)-\Delta}^{\infty} \left[c(\tilde{x}, y) + c_x^+(\tilde{x}, y)\, \Delta \right] \mathrm{d}G(\tilde{x} \mid y)}_{\text{unconstrained: first-order Taylor}} \right] \mathrm{d}F(y)$$

Figure E.5: Alternative decomposition of concavity



E.3 Additional Results on Higher-Order Approximations

Figure E.6: Higher-order approximations: Continuous vs. Discrete Income process



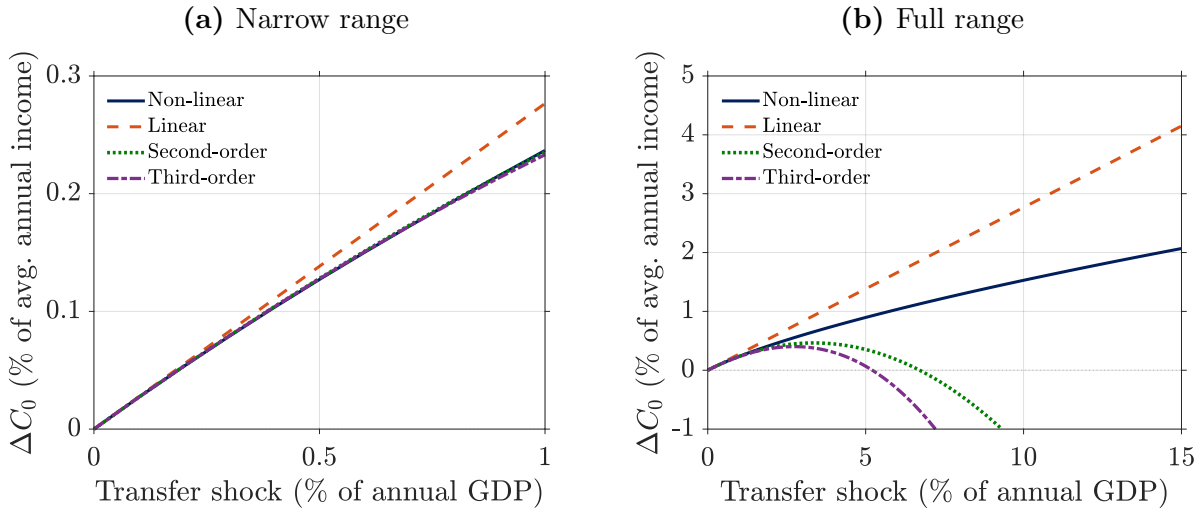


Figure E.7: Local higher-order approximations to the impact response of aggregate consumption, continuous income.

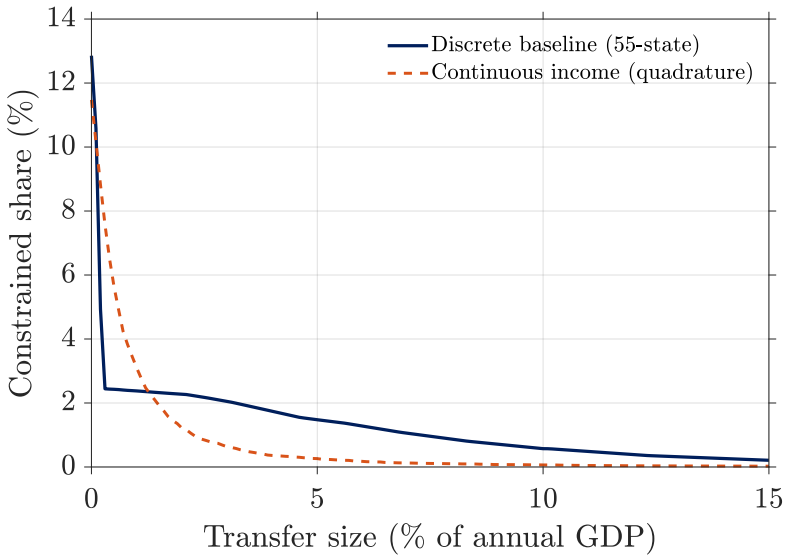
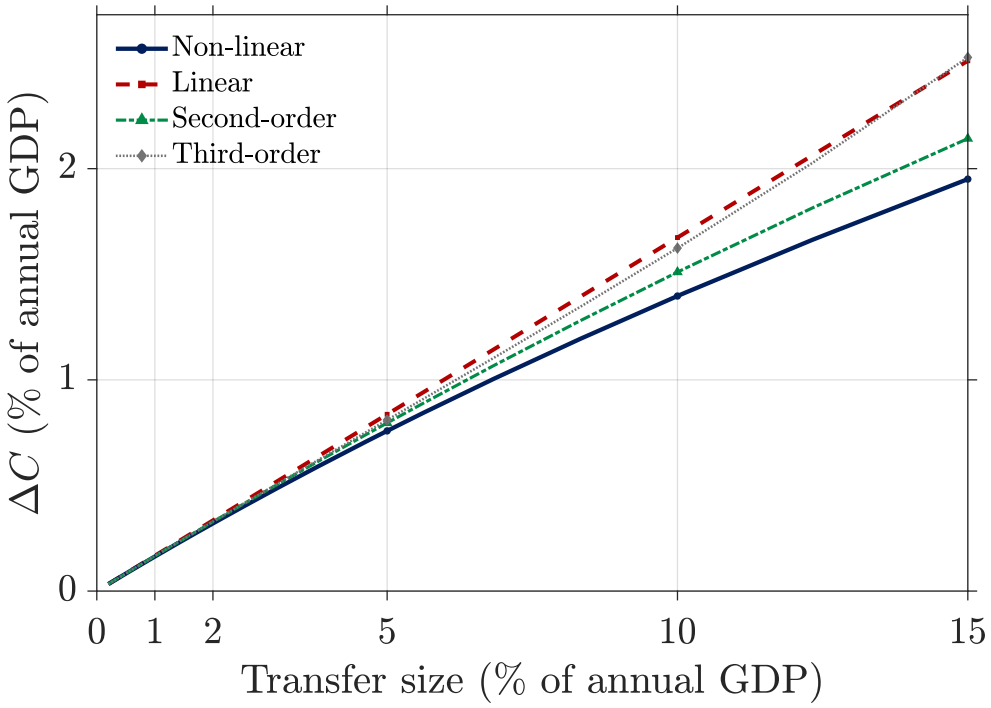


Figure E.8: Constrained share at $t = 0$ as a function of transfer size: discrete vs. continuous income.

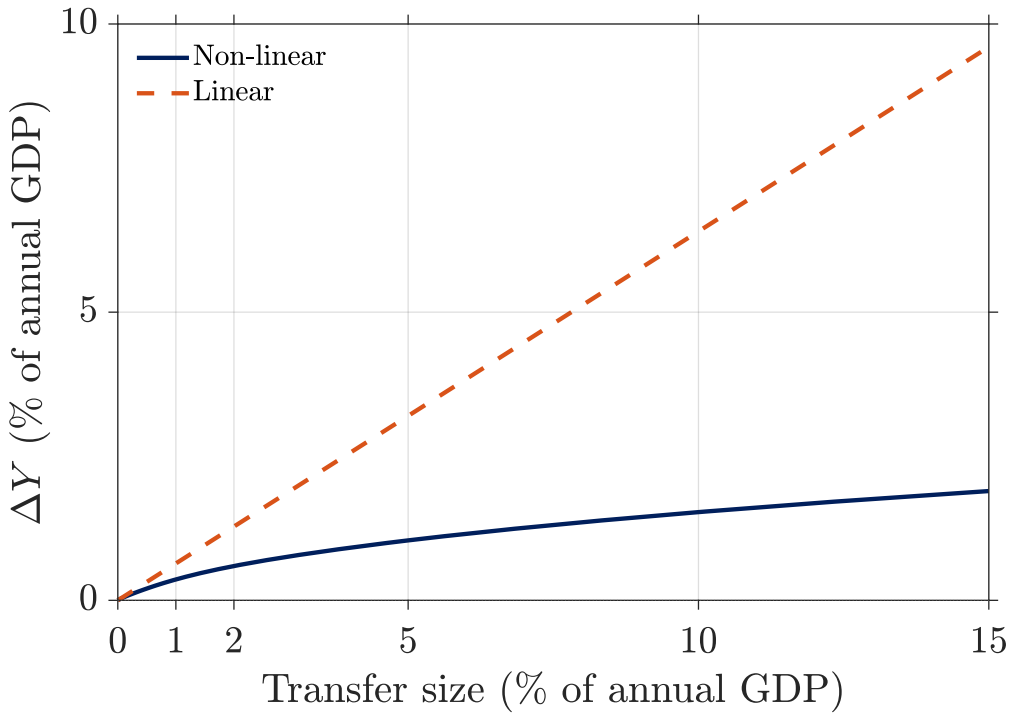
Figure E.9: Consumption Response of Initially Unconstrained Households



Note: Consumption response for initially unconstrained households, as a percentage of annual GDP per household.

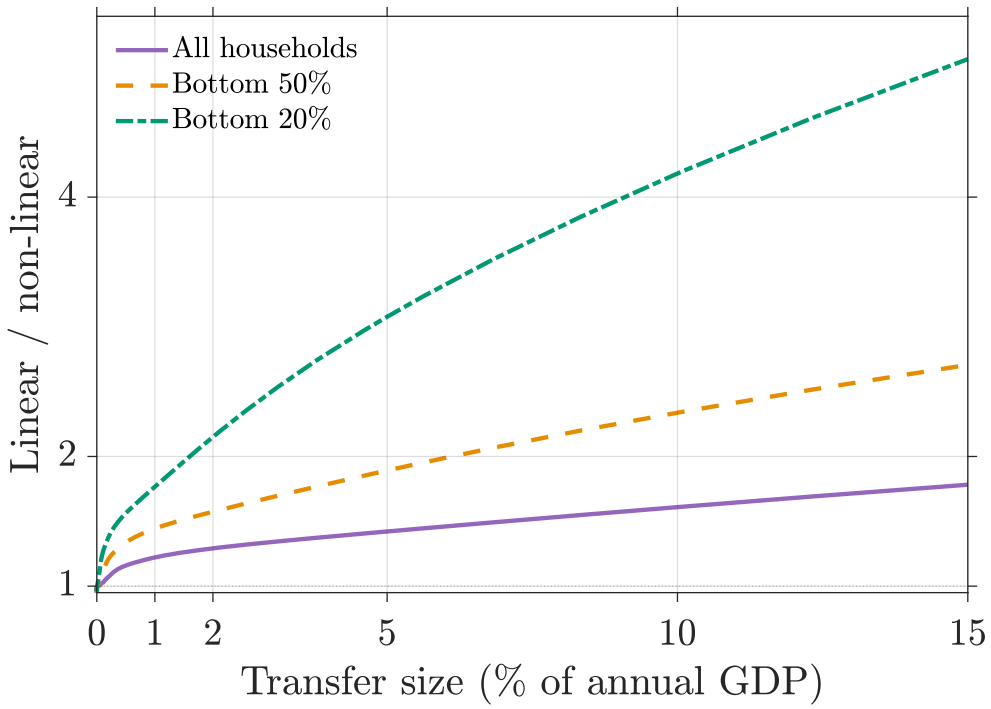
F Additional Figures: General Equilibrium

Figure F.1: Targeted Transfers



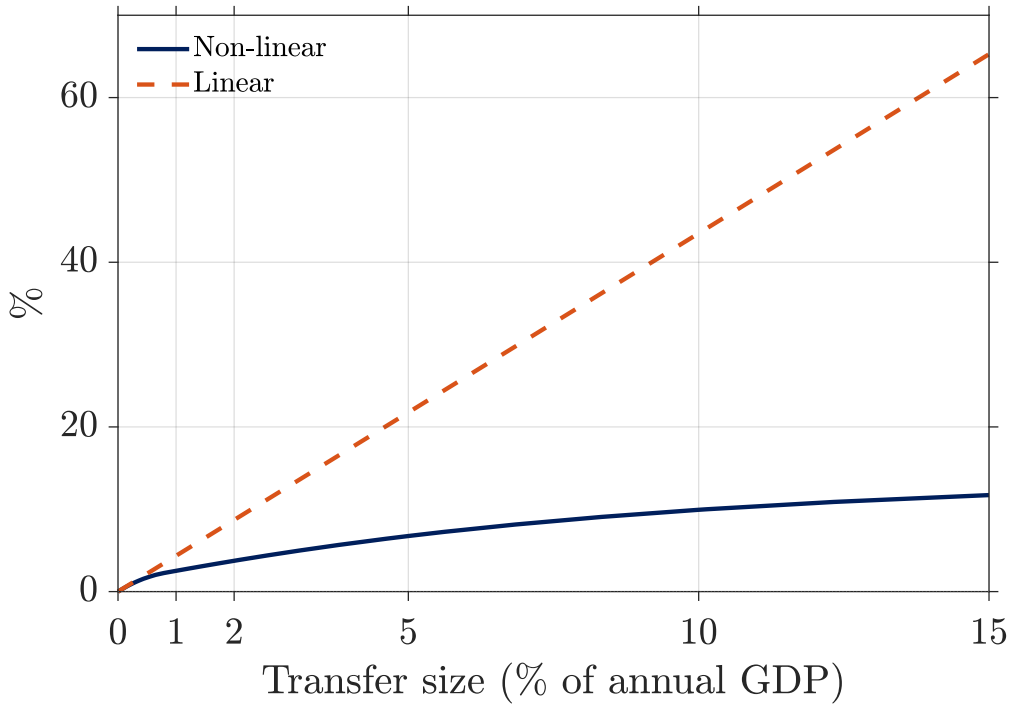
Note: Output response in the baseline HANK model where transfers in period 0 are directed toward households in the bottom 20% of the income distribution.

Figure F.2: Targeted versus Untargeted Transfers



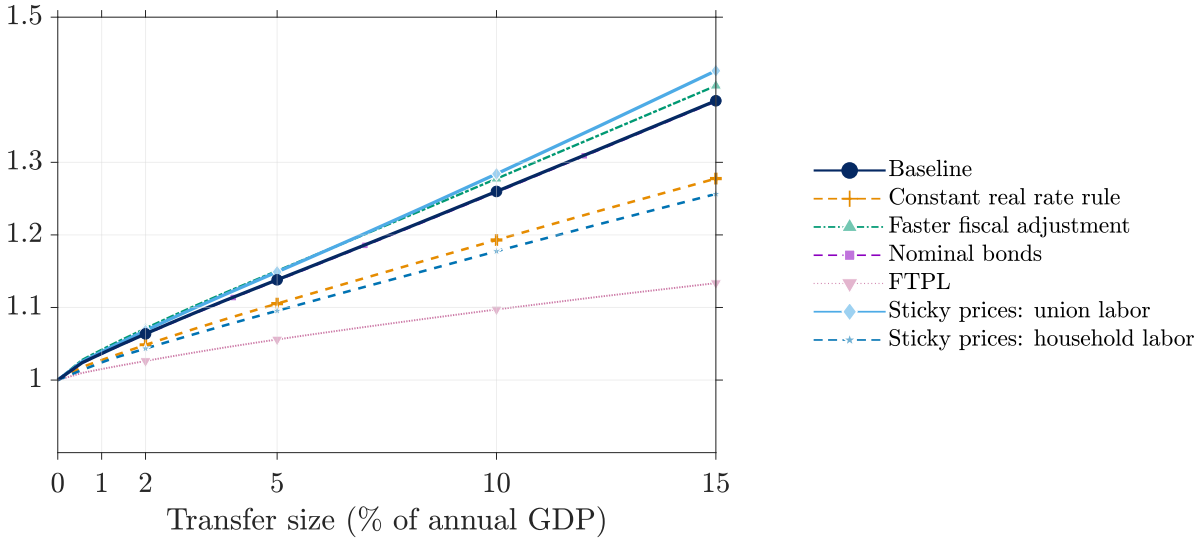
Note: The figure presents the ratio of the linear to the nonlinear general-equilibrium impact output response to a one-time transfer, as a function of transfer size, when the period-0 transfer is directed to all households, the bottom 50%, or the bottom 20% of the income distribution.

Figure F.3: Real Interest Rate Response in Flexible-Price Economy



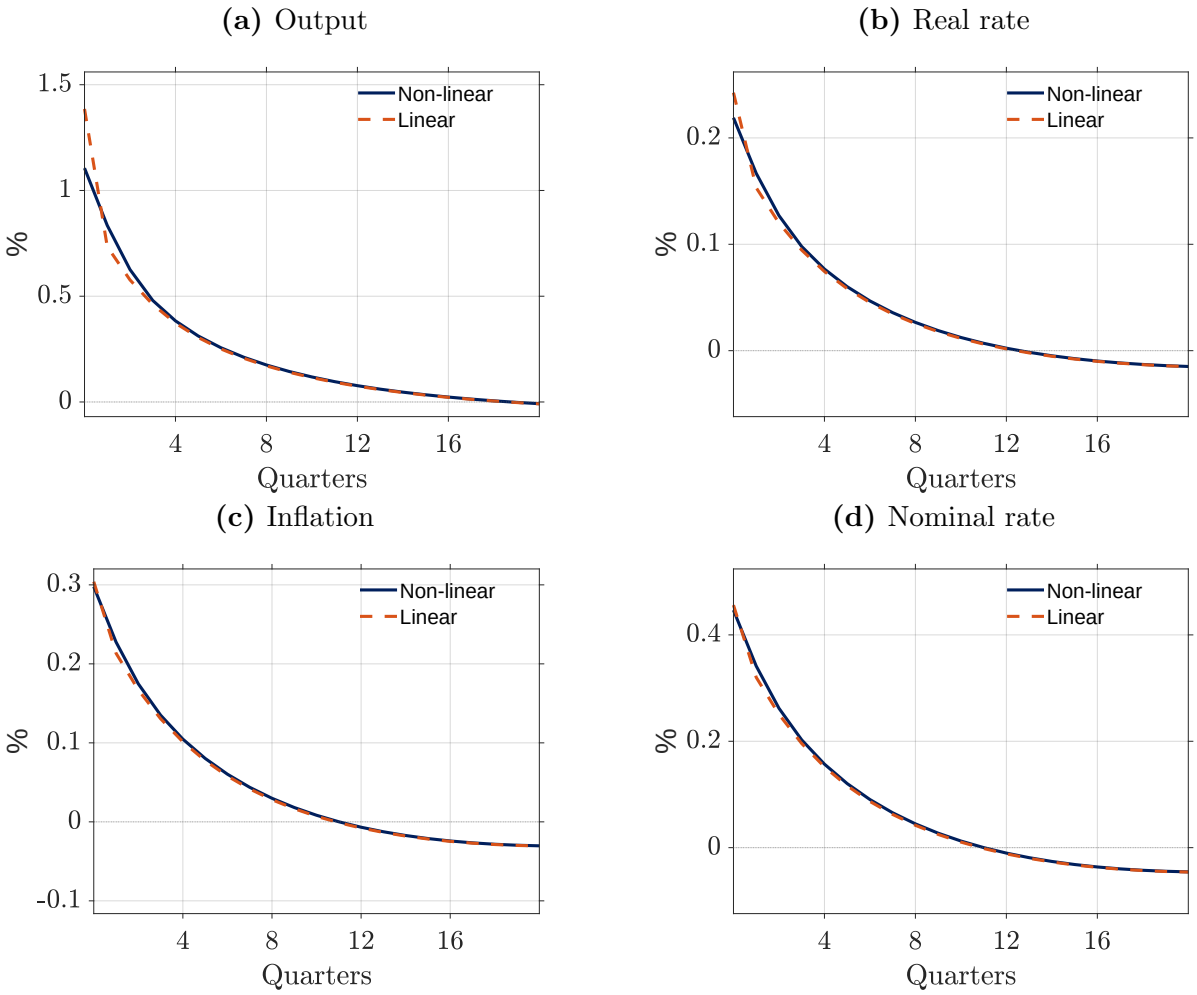
Note: The real interest-rate response in an economy with flexible prices and wages in which households choose labor supply.

Figure F.4: Sensitivity in HANK: Annual Responses



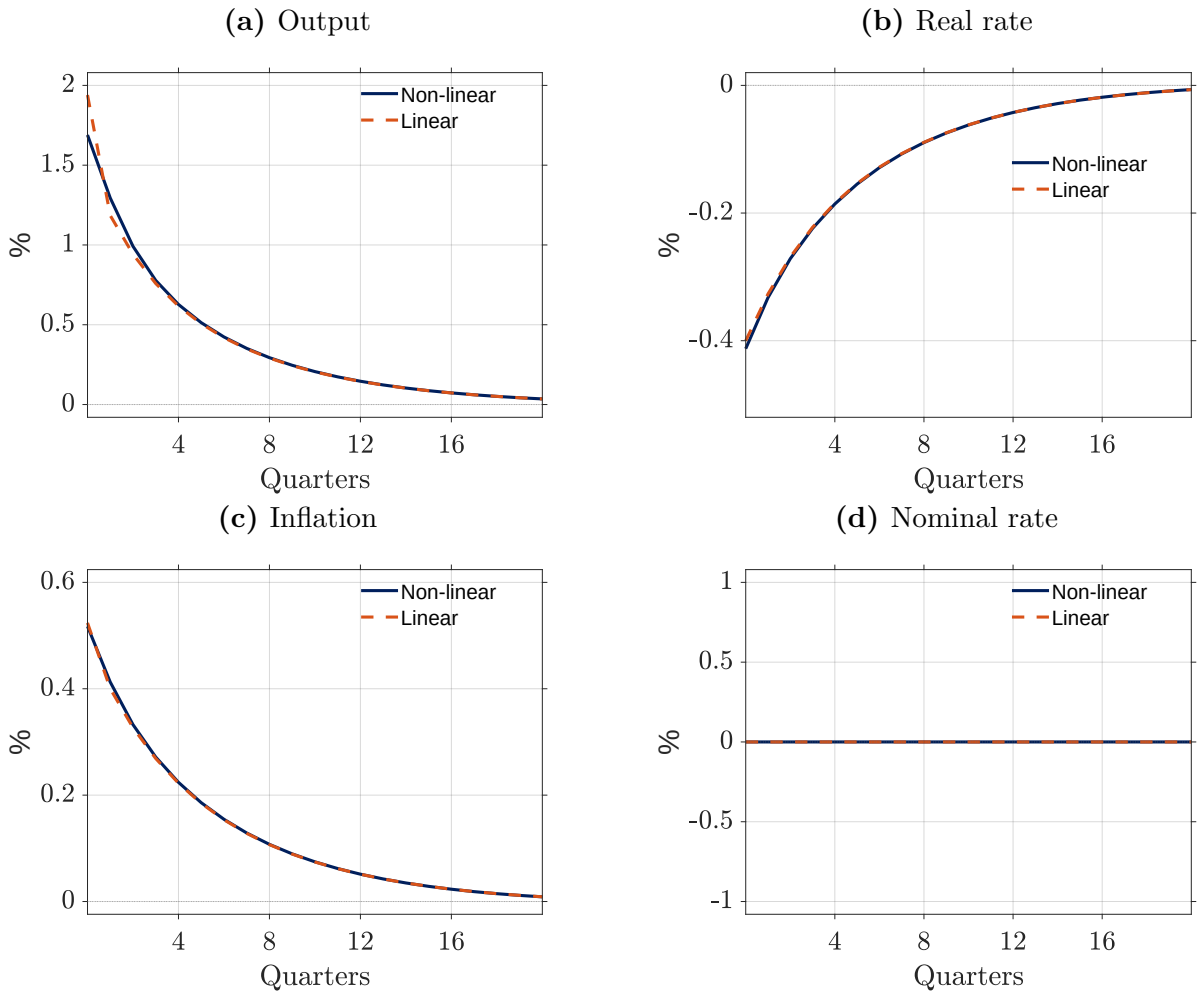
Note: The figure presents the ratio of the 12-month linear to nonlinear output response in the general equilibrium model.

Figure F.5: Impulse responses in the baseline HANK model



Note: Output, real rate, inflation, and nominal rate for a transfer equal to 1% of annual GDP. All variables are expressed as deviations from steady state. Output is expressed in percent. The real rate, inflation, and nominal rate are expressed as annualized percentage-point deviations.

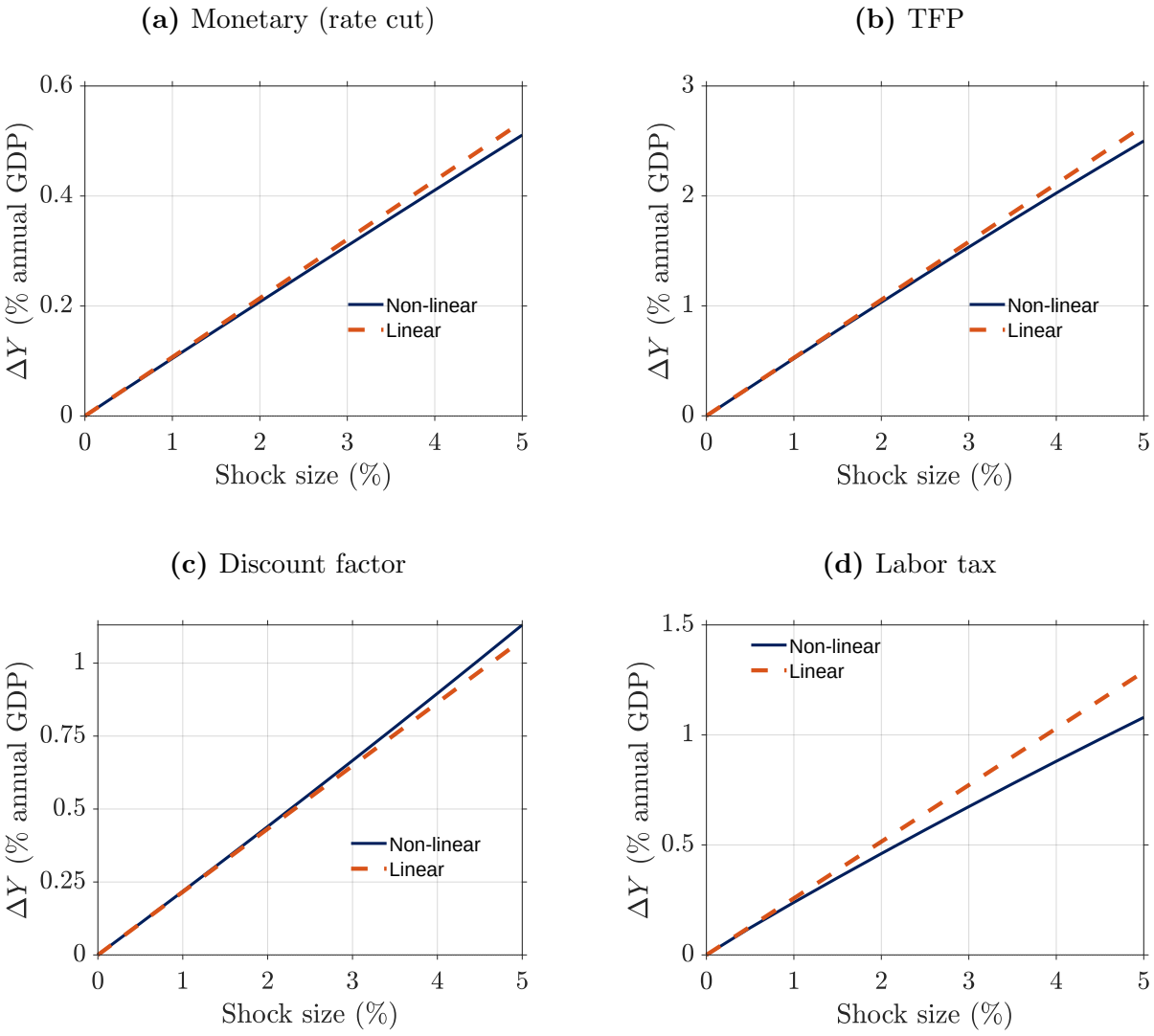
Figure F.6: Impulse responses: FTPL (unfunded fiscal stimulus)



Note: Output, real rate, inflation, and nominal rate for a transfer equal to 1% of annual GDP. All variables are expressed as deviations from steady state. Output is expressed in percent. The real rate, inflation, and nominal rate are expressed as annualized percentage-point deviations.

G Other Shocks

Figure G.1: Nonlinearities in the output response across shocks.



Note: The panels present the first-quarter output response as a function of shock size. Panel (a): monetary shock ε_i (annualized percentage points) such that $i_t = \bar{i} + \phi_\pi(\pi_t - \bar{\pi}) - \varepsilon_i$. Panel (b): productivity shock Z_0. Panel (c): discount factor shock $\tilde{\beta}_{i0} = \beta_{i0}(1 - \varepsilon_\beta)$. Panel (d): labor tax shock $\tau_0 = \tau(1 - \varepsilon_\tau)$.