Incomplete Markets and International Risk-Sharing*

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Abstract

We study international risk-sharing and international transmission in a model with incomplete markets (uninsurable risk and borrowing constraints) featuring one or two assets. We show analytically that incomplete markets create a wedge in the generalised Euler conditions and in the aggregate risk-sharing condition, thereby affecting international portfolio allocation and the transmission mechanism, even beyond a counterfactual economy with intermediation frictions. When income risk is countercyclical this risk-wedge accounts for the international risk-sharing facts: high consumption growth leads to expected appreciations, the correlation between the marginal utilities (or consumption) across countries and the real exchange rate is negative and in the same magnitude as in the data, cross-country consumption correlation is lower than that of output, the exchange rate is highly persistent (while other facts, such as net export countercyclicaity remain unaltered). This is so since heterogenous exposure induces wealth effects that endogenously produce home consumption and asset biases, leading to exchange rate depreciations even as consumption grows. Heterogenous wealth exposure induces a risk-misallocation wedge that acts as a time-varying disconnect that reconciles the statistics related to international risk-sharing.

Keywords: *Incomplete Markets, International Transmission of Shocks, International Risk Sharing Condition, Arbitrage Wedges, Counter-cyclical Income Risk.*

JEL classification: *E, F.*

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1. Introduction

Incomplete markets are increasingly incorporated into macro models, alongside nominal rigidities, to capture micro elasticities and examine the distributional effects of policies. In this paper, we ask whether and under which conditions incomplete markets, in the form of uninsurable risk and borrowing constraints, can significantly alter the international transmission mechanism and reconcile the Backus and Smith (1993) puzzle (see below for a large literature review on recent papers related to this puzzle). The latter relates to inconsistencies in the international risk-sharing condition, which, in models with complete markets and contrary to the data, imply large, positive correlations of consumption across countries and between the ratio of consumption across countries (or the ratio of marginal utilities across countries) and the real exchange rate. This also implies that the real exchange rate exhibits little persistence in the complete markets model, relative to the data. First, we derive generalized aggregate Euler and uncovered interest parity conditions and show that uninsurable risk and the tightness of the borrowing constraint introduce a wedge in the international arbitrage condition. When income risk is countercyclical, the dynamics of this wedge create a large disconnect between countries' fundamentals and their exchange rates that tilts the international portfolio allocation. In turn, we show that, under countercyclical income risk, consumption growth leads to expected exchange rate appreciation: large wealth effects from heterogeneous exposure raise domestic demand for goods and reduce savings in foreign assets by more, leading to currency appreciation. The opposite is true under recessions. Furthermore, we show that in this case, we can also match well the statistics related to the international risk-sharing and shock transmission: the cross-country consumption correlation is significantly lower than that of output, the ratio of marginal utilities across countries is highly volatile, the correlation between the ratio of marginal utilities and the real exchange rate turns negative, and the real exchange rate is persistent.

First, we show analytically in a simpler one asset endowment economy that uninsurable risk and the Lagrange multipliers on the borrowing constraints introduce a wedge into the aggregate risk-sharing condition. Furthermore, we show that if fluctuations in those wedges are countercyclical the dynamic of the stochastic discount factor induces a disconnect between

the countries' fundamentals and the real exchange rate that tilts the relative demand of domestic and foreign assets. The wedge acts as a discount factor shock (see also Kekre and Lenel (2024)), but the incomplete market features also valuation effects stemming from market clearing among heterogenous agents and the dependence of the stochastic discount factor on the wealth distribution (see Constantinides (1982) for discussing the role of this dependence). Next, we examine whether those frictions have the potential to reconcile also quantitatively the moments that characterize the international risk-sharing and shock transmission.

We then build a structural model by introducing uninsurable risk, borrowing constraints and two assets (see Kaplan and Violante (2018)) into an international business cycle open-economy model with nominal rigidities, capital adjustment costs, and an Armington aggregator for demand components. In line with a long tradition in international macroeconomics[1], we include nominal rigidities on prices: those also grant a meaningful role to policy shocks and allow us to benchmark our results to past studies on international risk-sharing. To study the international risk-sharing condition, we introduce a micro-founded rest of the world that features optimizing consumers and nominal rigidities. For robustness, we study and solve the model under two alternative set-ups for households' portfolio decisions, the first featuring one asset and the second featuring liquid and non-liquid. In an open-economy context, the presence of non-liquid assets also heightens the valuation effects.[2]

We start by examining the international transmission mechanism through impulse response functions and comparing the model with complete and incomplete markets. We focus on productivity shocks as the benchmark case used in past literature to study international risk-sharing. In line with the literature (Obstfeld and Rogoff (1995)) arguing that monetary shocks with sticky prices can account for the exchange rate persistence, we also analyse the transmission mechanisms of monetary shocks. While we do not detect large differences between the model with and without complete markets, we find that in the case with countercyclical income risk a large growth in domestic consumption (relatively to foreign),

1 A long tradition, dating back to Mussa (1982) and Obstfeld and Rogoff (1995), argues that nominal rigidities reconcile several regularities in the dynamics of international asset prices and the relative behavior of exchange rate regimes, both of which are a focus of our study.

2 We also entertain robustness with respect to passive or active fiscal policy stance and alternative international market segmentation.

following a positive productivity shock, leads to an expected appreciation, a fact in line with the empirical evidence documenting a negative correlation between cross-country consumption ratios and the real exchange rate. Under incomplete markets the heterogenous exposure to shocks induces wealth effects, which are larger under countercyclical risk. Those wealth effects drive domestic demand for home goods higher and demand of foreign assets lower, leading to a real exchange rate appreciation even as consumption grows. By decomposing the transmission mechanism in partial and general equilibrium effects we show that the latter account for the largest part of those wealth effects. Hence, it is the combination between the wedge in the stochastic discount factor and the general equilibrium changes in relative asset values, induced by the dynamic of the wealth distribution, that jointly explain the transmission mechanism. Those valuation effects are also pervasive in open economy as noted in past literature.[3]

We then examine a large set of business cycle statistics related to international risk-sharing and the international and domestic transmission mechanisms. The model with incomplete markets and countercyclical income risk can reconcile the negative correlation between the ratio of consumption across countries and the real exchange rate, a correlation of consumption across countries which is positive and lower than that of output, and a high persistence of the real exchange rate. It does so while generating business cycle moments for other domestic and international variables that are in line with the data, as well as a countercyclical dynamic in net exports. Under the two-asset model, the matching of some statistics, such as the volatility of the nominal exchange rate, is further improved due to the valuation effects introduced by the model.

Finally, we provide a counterfactual to validate the role of incomplete markets further. In the structural model, an intermediary channels the savings of heterogeneous households into domestic and foreign assets, according to households' discount factors, which, under incomplete markets, feature the risk wedges highlighted so far. We show that replacing the intermediary's discount factor with the representative-agent equivalent (even leaving other intermediation frictions) would impair the model's ability to account for international risk-sharing facts, even as other frictions may remain active for the intermediary.

3 See Lane and Milesi-Ferretti (2007).

Related Literature. Our work is related to the literature on the international transmission mechanism (e.g., Backus et al. (1992) and Obstfeld and Rogoff (2000a)) and international risk-sharing. Backus and Smith (1993) highlights the discrepancy between the statistics, related to international risk-sharing, of the complete market models and the empirical ones, as well as the excess volatility of the real exchange rate (noted earlier in) Mussa (1986)). Several papers have examined this puzzle in models with a representative agent and nominal rigidities: Chari et al. (2002), Benigno and Thoenissen (2003), or Corsetti et al. (2008). Colacito and Croce (2013) examined the role of long-run risk and Corsetti et al. (2025) that of trade elasticity. Recently, Itskhoki and Mukhin (2021) show that financial shocks, disconnected from fundamentals, are needed to account for most of the statistics related to the above puzzles, while Eichenbaum et al. (2021) highlights the role disturbances to foreign demand for dollar-denominated bonds in accounting for part of the dynamics of the nominal and real exchange rates. Closer to our work is a recent paper by Kekre and Lenel (2024), in which they show that discount factor shocks provide a more complete resolution of the exchange rate puzzles than other UIP deviations: our consumption wedges are akin to discount factor shocks, though as we explain below much of the transmission in models with uninsurable risk goes through general equilibrium effects, which in open economy account for valuation channels. At last, there is a host of open economy models with representative agents featuring portfolio adjustment costs, infrequent portfolio adjustments, agency problems in intermediation or beliefs heterogeneity (see, for instance, Schmitt-Grohé and Uribe (2003), Bacchetta and Van Wincoop (2006), Faia (2007), Bacchetta et al. (2023) or Faia et al. (2025) among others): those various incomplete market specifications all play a role in explaining deviation from international arbitrage (uncovered or covered), but not necessarily on risk-sharing.

The wedge that, in our model, reconciles the international risk-sharing statistics relates to a recent literature that derives consumption wedges consistent with micro elasticities (see, for instance, Berger et al. (2023) or Debortoli and Galí (2024)). Through a counterfactual in our structural model, we show that those wedges are crucial beyond other intermediation wedges: they enter the price of risk (or the SDF) that intermediaries internalize when serving

their clientele in investing across domestic and foreign assets. This connects our paper to a recent empirical literature in asset pricing (see Koijen and Yogo (2019)) showing that latent investors' demand, clientele discount factors, and mandates play a significant role in explaining international arbitrage (see Faia et al. (2025)) or the exchange rate disconnect (see Koijen and Yogo (2026)). Besides the above, consumption wedges have the advantage of reconciling micro- and macro-elasticities. At last, a significant part of the transmission mechanism in our model is tied to general equilibrium effects triggered by changes in the wealth distribution that arise only under uninsurable risk and are particularly meaningful when valuation changes are large, as is the case in the international transmission of shocks.

Several recent studies have introduced uninsurable risk or incomplete markets in small open economy models[4] with one asset (see Guo et al. (2020), Cugat et al. (2019), De Ferra et al. (2020), Giagheddu (2020), Auclert et al. (2021a)), Zhou (2022), Acharya and Challe (2025), Corsetti et al. (2026) or two assets (see Shabalina (2023)). With various goals and model specifications, all examine the distributional consequences of external shocks: under an incomplete market, consumers are non-Ricardian, hence, external shocks, by modifying the terms of trade, induce income and asset valuation effects and affect inter-temporal substitution (see also Kaplan and Violante (2018) for the muted inter-temporal substitutions characterizing those models). A paper by Bacchetta et al. (2026) also examines an open economy with heterogenous agents to show that the demand for safety, arising under uninsurable risk, can affect the international transmission mechanism. Our focus is complementary: we aim to assess if incomplete markets change or improve the models' ability to match the statistics that characterize the international business cycle and international risk-sharing. Finally, recent work by Acharya et al. (2025) studies the international transmission of shocks in an RBC model with incomplete markets and also find that countercyclical risk can account for a large set of facts of the international transmission mechanism.

4 This literature extends to an open economy context the macro models with incomplete markets and nominal rigidities: see for instance Kaplan et al. (2018) or Auclert and Rognlie (2018) or Auclert et al. (2019) among many others.

2. Consumption Wedges, Risk Mis-allocation and International Risk-Sharing

We start by analyzing a simple endowment economy with one asset to show analytically that incomplete markets lead to consumption wedges that affect international risk-sharing conditions and, hence, have the potential to resolve the Backus and Smith (1993) puzzle. In this economy, we also show why countercyclical income risk is needed to resolve the puzzle. In the next section we will assess the implications of those wedges through simulations of a larger structural model. In this simplified world, households are heterogeneous due to uninsurable risk and have the same time discount rate, β. There is no aggregate risk in the economy and the path for interest rates, $\{R_t\}_{t=0}^{\infty}$, is known. Furthermore, in the baseline case, we keep the wealth distribution fixed ($\mu(e^t) = 0\ \forall t$, $\mu(e^t)$ defined below, later in the section we relax this assumption); hence, we abstract from the role of occasionally binding constraints and focus entirely on the consumption wedge. In this economy, the individual consumption Euler equation contingent on the history of the idiosyncratic states $e^t = [e_0, ..., e_t]$ comes from the optimization problem

$$\begin{aligned} &\max_{\{c_t\}_{t=0}^{\infty}} \mathrm{E}_0 \left[\sum_{t=0}^{\infty} \beta^t u(c_t, n_t) \right] \\ &\text{s.t.} \quad c_t + a_t = e_t w_t n_t + R_{t-1} a_{t-1}, \quad a_t \geq 0 \ \ \forall t \end{aligned} \tag{1}$$

and reads as follows:

$$c(e^t)^{-\sigma} - \mu(e^t) = \beta R_t \sum_{e_{t+1}} \pi(e^{t+1} \mid e^t)[c(e^t, e_{t+1})]^{-\sigma} \tag{2}$$

where $\mu(e^t)$ is the shadow price associated with the borrowing constraint, c_t, n_t, a_t stand for household's consumption, hours worked and savings, w_t is the real wage, $u : \mathcal{R}_+^2 \to \mathcal{R}$ is a continuous and differentiable function, increasing in the first argument and decreasing in the second argument. The operator E_t is the expectation conditional on information at time t about the whole state space, and $a_t \geq 0$ is an occasionally binding borrowing constraint. In

Appendix A, we show that, upon aggregation, one obtains the following generalized Euler equation:

$$c_t = (\beta R_t)^{-\frac{1}{\sigma}} \mathrm{E}_t[c_{t+1} V_t^{-1}] \tag{3}$$

where E_t is the expectation operator with respect to the idiosyncratic shocks, $V_t = \int_e \frac{c(e^t)}{c_t} V(e^t, e_{t+1}) dj$ with a continuum of agents, or $V_t = \sum_{e_{t+1}} \pi(e_{t+1} \mid e^t) \frac{c(e^t)}{c_t} V(e^t, e_{t+1})$ with a discrete set of agents, and $V(e^t, e_{t+1}) = [\frac{\mathrm{E}_t[c(e^t, e_{t+1})^{-\sigma}]}{[\mathrm{E}_t\{c(e^t, e_{t+1})\}]^{-\sigma}}]^{\frac{1}{\sigma}} \geq 1$.[5] The wedge V_t is akin to a discount factor shock (see Constantinides (1982)) and increases with the variance of the idiosyncratic shock. From now on, we will often refer to this term as a discount factor or risk factor. We can equally derive the generalized aggregate Euler for the rest of the world:

$$c_t^* = (\beta R_t)^{-\frac{1}{\sigma}} \mathrm{E}_t[c_{t+1}^* (\frac{q_t}{q_{t+1}})^{-\frac{1}{\sigma}} (V_t^*)^{-1}] \tag{4}$$

where we used the no-arbitrage condition between domestic and foreign interest rates: $R_t^* \mathrm{E}_t \frac{q_{t+1}}{q_t} = R_t$. The difference with the representative household in the rest of the world is that $V_t^* = 1 \ \forall t$ in this case.

Lemma 1. *Under incomplete markets and different income processes across countries, the international risk-sharing condition reads as follows:*

$$\kappa (\frac{c_t^*}{c_t})^{-\sigma} \prod_{i=0}^{t-1} (\frac{V_i}{V_i^*})^{-\sigma} = q_t \tag{5}$$

where q_t is the real exchange rate and where $\kappa = (\frac{c_0}{c_0^*})^{-\sigma} q_0$. If the countries face the same process for uninsurable risk (so that $V_t = V_t^* \ \forall t$, the condition is equivalent to the one derived in Backus and Smith (1993). Expectation operator is omitted for brevity.

Proof. *See Appendix A.*

The lemma above states that under incomplete markets, and if countries do not experience the same sequence of shocks, the ratio of the marginal utilities across countries is equal to the real exchange rate but rescaled by the ratio between the earning risk variance, $\frac{V_t}{V_t^*}$. The wedge $\frac{V_t}{V_t^*}$ is the relative price of future risk across countries. Under complete markets, the

5 Those derivations are under no aggregate uncertainty, hence the pattern of interest rates is predictable under perfect foresight.

ratio of marginal utilities equals the real exchange rate. Moreover, since under complete markets agents can perfectly smooth consumption over time, the ratio of marginal utilities is constant, and so is the real exchange rate.[6] In the data, both implications of complete market models are violated: consumption correlation across countries is significantly different from one, and lower than the output correlation, and the real exchange rate is highly volatile and persistent. This is the essence of the Backus and Smith (1993) puzzle. Equation 5 shows that market incompleteness introduces a wedge proportional to the difference in income risk processes across countries. The dynamic of this wedge depends on the process of uninsurable income risk, but to the extent that states in which domestic consumption is high (expansion) are preceded by periods in which idiosyncratic risk is high (high V, recession in the case of countercyclical risk), the correlation between consumption ratio and real exchange rate can be negative. See Lemma 2 below for more details.

Corollary 1. *The consumption wedge induced by uninsurable risk induces risk and saving mis-allocation.* To the extent that the consumption wedge in 5 prevents the marginal utilities to be equalized across countries risk and savings are mis-allocated.

Why countercyclical income risk creates scope for the disconnect. The wedge $\prod_{i=0}^{t-1}(\frac{V_i}{V_i^*})^{-\sigma}$ generates the disconnect between the relative consumption ratios and the real exchange rate. We now show that this is the case only under certain income processes.

Lemma 2. *If uninsurable risk is countercyclical enough the consumption wedged associated with it can bring the correlation between the consumption ratio and the real exchange rate into a negative zone.*

In Appendix A, we show the proof of Lemma 2. Intuitively, under complete markets, a negative shock to the domestic economy leads output and consumption to decline, while prices and interest rates increase. This leads to an appreciation of the real exchange rate and a positive correlation between the domestic-to-foreign consumption ratio and the real exchange rate. With incomplete markets, when the domestic economy enters a recession ($\hat{y}_{t-1} < 0$), the uninsurable income risk jumps ($\hat{V}_{t-1} \uparrow$), where $\hat{}$ denotes log-deviations from the steady state value and y_t aggregate output. To self-insure households cut consumption,

6 Under aggregate shocks, the same statements hold in approximation.

and engage in precautionary savings. This heightened demand for assets (including foreign) depreciates the domestic currency, causing a real exchange rate depreciation in subsequent periods ($\hat{q}_t \uparrow$, meaning $\Delta\hat{q}_t > 0$).

An alternative way to depict this channel is by considering the term $\prod_{i=0}^{t-1}(\frac{V_i}{V_i^*})^{-\sigma}$ as a risk premium that domestic households pay to foreign investors to hold the assets denominated in the domestic currency. Under countercyclical risk, domestic households exhibit high demand for liquidity; they are willing to pay large premia to foreign investors. In sum, under cyclical risk, the term $\prod_{i=0}^{t-1}(\frac{V_i}{V_i^*})^{-\sigma}$ acts as a time-varying premium, which rises in recessions and declines in booms, when it may even turn into a convenience yield.

Changes in the Wealth Distribution. A key aspect of incomplete-market models is the dynamics of wealth distribution, which affects asset prices through the market-clearing condition. This creates an additional channel separate, but interacting with the consumption wedge described above. We now extend our derivations to account for this.

Lemma 3. *Conditional on the dynamic of the wealth distribution the aggregate generalized Euler equation is:*

$$c_t = E_t[(\beta R_t)^{-\frac{1}{\sigma}} c_{t+1} V_t^{-1}(a_t)] + \Delta(a_t) \tag{6}$$

where $V_t(a_t) = \int \frac{c(e^t,a_t)}{c_t} V(e^t, e_{t+1}, a_t) dD_j(e^t, a(e^t))$ *and* $\Delta(a_t) = (V_t(a_t))^{-1} \int \tilde{v}(e^t, e_{t+1}, a_t) dD_j(e^t, a(e^t))$. *In this case too, if income risk is countercyclical the wedge can induce a negative correlation between marginal utilities across countries and the real exchange rate.*

Derivations of 6 are reported in Appendix A. Here, the wedge $V_t(a_t)$ also depends on the dynamics of the wealth distribution. The aggregate Euler includes an additional term, $\Delta(a_t)$, which captures the tightness of the borrowing constraint, $\tilde{v}$, weighted by the wealth distribution. Changes in the wealth distribution alter the share of borrowing-constrained households, thereby influencing consumption and international portfolio allocation. The corresponding generalized Euler condition for the rest of the world is as follows: $c_t^* = E_t[(\beta R_t^*)^{-\frac{1}{\sigma}} c_{t+1}^* (V_t^*(a_t))^{-1}] + \Delta^*(a_t)$. Imposing no arbitrage between domestic and foreign assets, hence equalizing the real rates in equation 6 and the foreign equivalent, delivers:

$$E_t[\frac{c_{t+1}}{c_t}(V_t(a_t))^{-1}] = E_t[\frac{c_{t+1}^*}{c_t^*}(V_t^*(a_t^*))^{-1}(\frac{q_{t+1}}{q_t})^{\frac{1}{\sigma}}] + \xi_t(a_t) \tag{7}$$

where $\xi_t(a_t) = f(\Delta^*(a_t) - \Delta(a_t))$ is a function of the difference in wealth distribution between the open economy ($\Delta(a_t)$) and the rest of the world ($\Delta^*(a_t)$), which is influenced by the endogenous tightness of borrowing limits, which varies across households. Deviations from the 7 are driven by two factors: (1) the relative income risk across countries, specifically V_t (home) versus V_t^* (foreign), and (2) the relative changes in their wealth distributions.

We will now derive the international risk-sharing condition under incomplete markets.

Theorem 1. *The relation between the real exchange rate and the ratio of marginal utilities across countries under a fixed asset distribution reads as follows:*

$$\frac{c_t}{c_t^*} = (\frac{\kappa}{q})^{-1/\sigma} \prod_{i=0}^{t-1} (\frac{V_i}{V_i^*}) \tag{8}$$

under a changing asset distribution reads as follows:

$$\kappa(\frac{c_t^*}{c_t})^{-\sigma} \prod_{i=0}^{t-1} (\frac{V_i}{V_i^*})^{-\sigma} \left(1 + \frac{\Gamma_t - \Gamma_t^*}{c_0 - \Gamma_t}\right)^{\sigma} = q_t \tag{9}$$

where $\Gamma_t = \sum_{i=0}^{t-1} \Delta_i \left(\prod_{j=0}^{i-1} (\beta R_j)^{-\frac{1}{\sigma}} V_j^{-1}\right)$ *and* $\Gamma_t^* = \sum_{i=0}^{t-1} \Delta_i^* \left(\frac{c_0}{c_0^*} (\frac{q_0}{q_i})^{-\frac{1}{\sigma}} \prod_{j=0}^{i-1} (\beta R_j)^{-\frac{1}{\sigma}} (V_j^*)^{-1}\right)$, *and* $\kappa = (\frac{c_0}{c_0^*})^{-\sigma} q_0$, *and expectation operator is omitted for brevity (see Appendix A for details).*

Proof. *The proof of Theorem 1 is in Appendix A.*[7]

Theorem 1 shows that the relationship between the consumption ratio across countries and the real exchange rate depends on the wedges induced by incomplete markets. If the countries have identical income processes, the wedge would disappear. But to the extent that they are different, the dynamic of the relative wedge depends on the dynamic of the income risk. Equation 8 shows this under the assumption that the wealth distribution is fixed, and it can be used as a useful benchmark of comparison relative to the complete market model. Equation 9 generalizes this by taking into account how the wedge changes with changes in the wealth evolution. Intuitively, under incomplete markets, country-specific, uninsurable shocks permanently affect the wealth distribution, setting consumption across countries on diverging paths. This, in turn, is reflected in the relative prices or the real exchange rate. In

7 For the sake of exposition in this case, we skip the dependence of the discount factor term and the distributions upon assets.

section 4.2, we evaluate numerically the role of those wedges and their ability to reconcile the Backus and Smith (1993) puzzles.

3. General Equilibrium Model with Incomplete Markets

We now examine international transmission and risk-sharing quantitatively through a larger structural general equilibrium model with incomplete markets and nominal rigidities. The environment is an open-economy model with trade in goods and assets, households that face uninsurable risk, and occasionally binding borrowing constraints. Firms in the production sector produce with capital and labor and face price rigidities and capital adjustment costs. We include nominal rigidities in the model, as they contribute to the persistence observed in international relative prices and play a meaningful role in the response to policy shocks: one of our goals is also to characterize the international transmission mechanism in response to various shocks. The rest of the world is modeled micro-founded with optimizing households and firms: this allows us to obtain international risk-sharing conditions from arbitrage between asset returns, or from the aggregate stochastic discount factors across countries.

Trade Structure. Each good demanded in the economy is traded internationally and within each country domestic and foreign produced goods are aggregated according to an Armington aggregator: $y_t = \mathcal{Y}_t(y_{H,t}, y_{F,t}) = \left[(1-\gamma_{hb})^{\frac{1}{\eta_h}}(y_{H,t})^{1-\frac{1}{\eta_h}} + \gamma_{hb}^{\frac{1}{\eta_h}}(y_{F,t})^{1-\frac{1}{\eta_h}}\right]^{\frac{\eta_h}{\eta_h-1}}$, where η_h is the elasticity of substitution between domestically and foreign produced consumption goods, γ_{hb} is the home bias in consumption and $p_{H,t}$ and $p_{F,t}$ are the prices of domestically and foreign produced consumption. The associated price index is: $P_t = \left((1-\gamma_{hb})p_{H,t}^{1-\eta_h} + \gamma_{hb}p_{F,t}^{1-\eta_h}\right)^{\frac{1}{1-\eta_h}}$. The optimal allocation of expenditures yields the following share of domestically produced goods and imports:

$$y_{H,t} = (1-\gamma_{hb})\left(\frac{p_{H,t}}{P_t}\right)^{-\eta_h} y_t; \quad y_{F,t} = \gamma_{hb}\left(\frac{p_{F,t}}{P_t}\right)^{-\eta_h} y_t \tag{10}$$

Foreign households' consumption is aggregated with the same Armington index and the

optimal allocation of expenditure between domestic and foreign produced goods leads to the following export function: $y^*_{H,t} = \gamma^f \left(\frac{p_{H,t}}{p_{F,t}}\right)^{-\eta_f} y^*_t$, where y^* is foreign output and the star index indicates all foreign variables. Since goods are produced by monopolistic firms, each demand component is, in turn, a Dixit-Stiglitz aggregator of different varieties. In each country there is a linear allocation of production to the optimal demands: $Y_t = y_{H,t} + y^*_{H,t}$, $Y^*_t = y_{F,t} + y^*_{F,t}$. In accounting terms total demand of domestic and imported goods should equalize the sums of all components in each of the countries: $c_t + I_t + \psi_p = \mathcal{Y}_t(y_{H,t}, y_{F,t})$ and $c^*_t + \psi^*_p = \mathcal{Y}^*_t(y^*_{H,t}, y^*_{F,t})$, where c_t, c^*_t denote domestic and foreign consumption, I_t denotes domestic investment, ψ_p, ψ^*_p denote domestic and foreign price adjustment costs. In this context we also define the nominal exchange rate $\mathcal{E}_t$ and the real exchange rate $q_t = \frac{\mathcal{E}_t P^*_t}{P_t}$. The wage-based exchange rate, for which we also compute moments, is defined as $q^w_t = \frac{\mathcal{E}_t W^*_t}{W_t}$, where W^*_t and W_t are nominal wages abroad and at home. Furthermore, the law of one price holds: $\frac{p_{F,t}}{P_t} = q_t$.

3.1. Households Optimization

Households have preferences over consumption and labor hours given by the following lifetime expected utility: $\mathrm{E}_0\left[\sum_{t=0}^{\infty} \beta^t u(c_t, n_t)\right]$, where c_t and n_t denote consumption and labor hours and where $u : \mathcal{R}^2_+ \to \mathcal{R}$ is a continuous and differentiable function, increasing in the first argument and decreasing in the second argument. The parameter $\beta \in (0, 1)$ denotes the subjective discount factor and follows a Markov process as in Krusell and Smith (1998).[8] The operator E_t is the expectation conditional on information at time t about the whole state space, which we define below. At the beginning of each period t, households also experience an idiosyncratic productivity shock e_t, which is an n_e-state Markov process with transition matrix $P^e(e_{t+1}|e_t)$ and a stationary distribution $\pi(e)$. Real labor income $\chi_{j,t}$ of individual j at time t is a function of idiosyncratic productivity state, e_t, real wage $w_t = \frac{W_t}{P_t}$, and labor hours n_t: $\chi_{j,t} = e_t w_t n_t$. In our baseline economy, households invest in an asset, a_t, that pays a return, $(1 + r^a_t)$. We will also extend the model and examine the implications of international risk-sharing in a two-asset economy. An intermediary, whose optimization

8 Ferriere and Navarro (2024), also note that in one-asset economies discount factor shocks contribute to the matching of high marginal propensities to consume.

problem we formalize below, channels households' savings to investment in different assets, domestic and international. Households' optimization problem reads as follows:

$$\begin{aligned} V_j(e_t, \beta_t, a_{t-1}) &= \max_{c_t, a_t} u(c_t, n_t) + \beta \mathrm{E}_t V_j(e_{t+1}, \beta_{t+1}, a_t) \\ \text{s.t.} \quad c_t + a_t &= e_t w_t n_t + (1 + r_t^a) a_{t-1}, \qquad\qquad a_t \geq 0, \end{aligned} \tag{11}$$

where E_t is the expectation operator with respect to the entire state-space of the economy, with idiosyncratic and aggregate shocks, $V_t(e_t, \beta_t, a_{t-1})$ is the value function in time t and $a_t \geq 0$ is an occasionally binding borrowing constraint. The first order condition leads to the individual Euler:

$$u_c(c_t, n_t) = \mu_t^1 + \beta \mathrm{E}_t \partial_a V_{t+1}(e_{t+1}, \beta_{t+1}, a_t) \tag{12}$$

where $u_c(c_t, n_t)$ is the marginal utility of consumption and μ_t^1 is the Lagrange multiplier on the borrowing constraint. The envelope condition implies: $\partial_a V_t(e_t, \beta_t, a_{t-1}) = u_c(c_t)(1 + r_t^a)$. The utility is a CRRA function, separable in consumption and labor hours: $u(c_t, n_t) = \frac{c^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} - \varphi \frac{n_t^{1+\rho}}{1+\rho}$. The supply of labor hours is chosen independently of the income realization by a representative member of the household:

$$\varphi n_t^{\rho} = \int u_c(c_t, n_t) \frac{\partial \chi_t}{\partial n_t} dD(e_t, \beta_t, a_{t-1}) \tag{13}$$

where $dD(e_t, \beta_t, a_{t-1})$ is the density of households in each grid point of the endogenous state space $[e_t, \beta_t, a_t]$. Aggregate effective labor supply is given by per capita hours, weighted by the shares of agents in each income realization: $l_t = n_t \int e_t dD(e_t, \beta_t, a_{t-1})$.

3.2. The Production Sector

The production block in each country consists of a final-good and an intermediate-good sector. In the latter, firms use capital and labor to produce varieties, $y_{i,t}$, on which they hold monopoly power. They face price and capital adjustment costs. In the final good sector, a representative firm aggregates varieties through a CES bundle. The optimal demand for each variety, obtained from maximizing the Dixit-Stiglitz aggregator, is $p_{i,t} = \left(\frac{Y_t}{y_{i,t}}\right)^{\frac{1}{\eta}} p_{H,t}$, where η is the elasticity of substitution across varieties. Firms in the domestic economy

choose capital, k_t, labor, l_t, investment, I_t, and prices $p_{i,t}$, to solve the following optimization problem:

$$J_i\left(k_{t-1}\ ,\mathrm{p}_{i,t-1}\right) = \max_{p_{i,t},k_t,I_t,l_t} \left\{ \frac{p_{i,t}}{p_{H,t}}\frac{p_{H,t}}{P_t}y_{i,t} - w_t l_t - I_t - \frac{\eta}{2\kappa}\ln(1+\pi_{H,t})^2 Y_t + \mathrm{E}_t[\frac{J_{i,t+1}(k_t,p_{i,t})}{1+r_{t+1}}] \right\}$$

$$\text{s.t. } k_t = (1-\delta)k_{t-1} + I_t - \frac{1}{2\varkappa\varepsilon_I}\left(\frac{k_t-k_{t-1}}{k_{t-1}}\right)^2 k_{t-1};\ \ y_{i,t} = z_t k_{t-1}^{\nu} l_t^{1-\nu};\ \ p_{i,t} = \left(\frac{Y_t}{y_{i,t}}\right)^{\frac{1}{\eta}} p_{H,t} \tag{14}$$

where r_t is the real rate with which the firms discount the future and z_t is the total factor productivity that follows a VAR(1) process: $z_t = \rho_1 z_{t-1} + \rho_2 Z^*_{t-1} + \varepsilon^z_t$; $Z^*_t = \rho_2 z_{t-1} + \rho_1 Z^*_{t-1} + \varepsilon^{zf}_t$ following Backus et al. (1992), where Z^*_t is the foreign total factor productivity. δ, ε_I, κ and ν are depreciation rate, investment adjustment cost parameter, Phillips curve slope and capital share in production repsectively. Upon imposing symmetry across firms, the first order condition for labor demand is: $l_t = \frac{mc_t(1-\nu)}{w_t}Y_t$. Defining ξ^Q_t as the Lagrange multiplier on the capital accumulation and, hence, as the shadow price of capital, we can derive the firm's first order condition for the capital stock:

$$\begin{aligned}(1+r_{t+1})\xi^Q_t = \nu z_{t+1}\left(\frac{l_{t+1}}{k_t}\right)^{1-\nu} mc_{t+1} - \\ - \left[\frac{k_{t+1}}{k_t} - (1-\delta) + \frac{1}{2\varkappa\varepsilon_I}\left(\frac{k_{t+1}-k_t}{k_t}\right)^2\right] + \frac{k_{t+1}}{k_t}\xi^Q_{t+1}\end{aligned} \tag{15}$$

where mc_{t+1} is the Lagrange multiplier on the production constraint, hence the real marginal cost. The first order condition with respect to investment delivers the price of capital: $\xi^Q_t = 1 + \frac{1}{\varkappa\varepsilon_I}\left(\frac{k_t-k_{t-1}}{k_{t-1}}\right)$. The first order condition with respect to prices leads to the Phillips curve:

$$\log(1+\pi_{H,t}) = \kappa(mc_t - \frac{1}{\mu_p}\frac{p_{H,t}}{P_t}) + E_t[\frac{Y_{t+1}}{Y_t}\log(1+\pi_{H,t+1})\Psi_{t,t+1}] \tag{16}$$

where $\pi_{H,t}$ is domestic inflation such that $\frac{p_{H,t}}{p_{H,t-1}} = 1+\pi_{H,t}$, $\mu_p = \frac{\eta}{\eta-1}$ and $\Psi_{t,t+1} = \frac{1}{1+r_{t+1}}$ is the stochastic discount factor.

Domestic Monetary Policy. Monetary policy follows a Taylor-type rule: $i_t = \rho_r i_{t-1} + (1-\rho_r)(r^* + \phi\pi_t + \phi_y(y_t - \bar{y}) + \phi_{\mathcal{E}}(\mathcal{E}_t - 1)) + \varepsilon^r_t$, where ε^r_t is the monetary policy shock, and these shocks, as opposed to TFP shocks, are uncorrelated across countries.

3.3. The Rest of the World and Net Foreign Asset Position

Rest of the World. The rest of the world is a parsimonious economy that allows us to meaningfully derive an international risk-sharing condition and to retain the elements that most affect the international transmission of shocks. Foreign households' consumption-saving decisions lead to the following Euler condition on aggregate consumption: $u_{c,t}(c_t^*) = \beta E_t[u_{c,t+1}(c_{t+1}^*)(1+r_{t+1}^*)]$. Next, monopolistically competitive firms, which produce with labor $Y_t^* = Z_t^*(N_t^*)^{1-\nu^*}$ and face Rotemberg adjustment costs on prices (same ones as in the domestic economy), generate the following Phillips curve: $\log(1+\pi_t^*) = \kappa(mc_t^* - \frac{1}{\mu_p^*}) + E_t[\frac{Y_{t+1}^*}{Y_t^*}\log(1+\pi_{t+1}^*)\Psi_{t,t+1}^*]$, where $mc_t^* = \frac{\varphi^*}{u_{c,t}(c_t^*)(1-\nu^*)z_t^*}\left(\frac{Y_t^*}{z_t^*}\right)^{(\nu^*+\alpha^*)/(1-\nu^*)}$. Finally, the foreign monetary authority adopts the following Taylor rule: $i_t^* = \rho_r^* i_{t-1}^* + (1-\rho_r^*)(r^* + \phi^*\pi_t^* + \phi_y^*(y_t^* - \bar{y}^*)) + \varepsilon_r^*$.

In the domestic economy residents can invest either in domestic equities or in domestic and internationally traded bonds. They do so through an intermediary, whose optimization problem we describe below. Given this allocation of savings, the net foreign asset position of foreign security holdings is given by the total demand of assets minus the domestic value of equity issuance: $NFA_t = \mathcal{A}_t - B_t - v_t$.[9] The aggregate net foreign asset position is determined by the period t net export and the accumulated returns on past saving or borrowing in foreign assets:

$$NFA_t = (\frac{p_{H,t}}{P_t}y_{H,t}^* - \frac{p_{F,t}}{P_t}y_{F,t}) + (1+\tilde{r}_t)NFA_{t-1} \tag{17}$$

where the total notional return accumulated on the net foreign position is given by: $1+\tilde{r}_t = \frac{q_t}{q_{t-1}}(1+r_t^*)$.

3.4. The Intermediary and International Arbitrage Deviations

In the domestic economy, a competitive intermediary collects savings from the heterogenous households in the form of deposits, or short-term liabilities, $D_t = \mathcal{A}_t$ and invests them either in domestic equities, x_t, or in domestic bonds, B_t, and international bonds, B_t^*, by maximizing the sum of future discounted profits. The return on domestic bonds is, $(1+r_t)$, the dollar return on international bonds is $(1+r_t^*)$. Each stock pays in every period a dividend, d_t, and

9 In the baseline economy we assume that the domestic government supplies bonds in zero net supply, hence $NFA_t = \mathcal{A}_t - v_t$.

carries a capital gain (or loss) of $\frac{v_t}{v_{t-1}}$. The intermediary rebates to households a contractually agreed (and known at time t) return on savings $(1 + r_t^a)$.

Each equity share pays a dividend, d_t, and the shares may experience a capital gain, $\frac{v_t}{v_{t-1}}$. Hence, the gross domestic currency return accruing from each equity share is $(\frac{d_t+v_t}{v_{t-1}})$. Markets are competitive, so the intermediary competes and aims to achieve zero profit in the long run. The aggregate amount of international bonds at time t is equivalent to the net foreign asset position: $B_t^* = NFA_t$ and the total value of stocks plus the total value of international bonds equal total savings, $\mathcal{A}_t = B_t^* + x_t v_t = NFA_t + x_t v_t$. When intermediating foreign bonds, which are harder to monitor and deploy, the intermediary faces a portfolio management cost, $\psi_{B^*}(-B_t^* - exp(-B_t^*))$. The intermediary solves the following optimization problem:

$$V_t^I(\mathcal{A}_t, B_t, B_t^*, x_t) = Max_{[\mathcal{A}_{t+1}, B_{t+1}, B_{t+1}^*, x_{t+1}]} \mathrm{E}_t[\pi_t^I + \frac{1}{1+r_{t+1}} V_{t+1}^I(\mathcal{A}_{t+1}, B_{t+1}, B_{t+1}^*, x_{t+1}))\} \tag{18}$$

subject to the net wealth accumulation:

$$\pi_t^I + v_t x_{t+1} + B_{t+1}^* + B_{t+1} + (1 + r_t^a)\mathcal{A}_t = \tag{19}$$

$$= \mathcal{A}_{t+1} + (1 + r_t)B_t + (1 + r_t^*)\frac{q_t}{q_{t-1}} B_t^* + (d_t + v_t)x_t + \psi_{B^*}(-B_t^* - exp(-B_t^*)) \tag{20}$$

Taking first order conditions with respect to $[\mathcal{A}_t, B_t, B_t^*, x_t]_{t=0}^{\infty}$ read as follows:

$$1 + \mathrm{E}_t \frac{V_{A,t+1}}{1+r_{t+1}} = 0;\ -1 + \mathrm{E}_t \frac{V_{B,t+1}}{1+r_{t+1}} = 0;\ -1 + \mathrm{E}_t \frac{V_{B^*,t+1}}{1+r_{t+1}} = 0;\ v_t + \mathrm{E}_t \frac{V_{x,t+1}}{1+r_{t+1}} = 0 \tag{21}$$

$$V_{A,t} = (1 + r_t^a);\ V_{B,t} = (1 + r_t);\ V_{B^*,t} = (1 + r_t^*)\frac{q_t}{q_{t-1}} + \psi_{B^*}(exp(-B_t^*) - 1);\ V_{x,t} = d_t + v_t \tag{22}$$

Substitution of partial derivatives implies:

$$\mathrm{E}_t(1+r_{t+1}) = \mathrm{E}_t(1+r_{t+1}^a) = \frac{\mathrm{E}_t(v_{t+1} + d_{t+1})}{v_t} = \mathrm{E}_t(1+r_{t+1}^*)\frac{q_{t+1}}{q_t} + \psi_{B^*}(exp(-B_{t+1}^*) - 1) \tag{23}$$

Equation 23 imposes no arbitrage conditions between domestic equities and domestic and foreign government bonds. Furthermore, the equality between the returns on invested assets

and the return rebated to households results from competitive markets, implying that the intermediary earns zero profit per unit of intermediated assets.[10] The third equation delivers the uncovered interest rate parity.[11]

A Counterfactual Investors' Mandate for Validation. In the structural model the intermediary channels the savings of heterogenous agents into portfolio allocation, across domestic and foreign asset demand. In line with recent empirical evidence by Koijen and Yogo (2019) or Koijen and Yogo (2026) the intermediary portfolio allocation, as we all the wedges in the international excess returns (see Faia et al. (2025)) depend on the stochastic discount factor with which the intermediary's clientele discount future portfolio returns and the portfolio adjustment costs described above. Both are part of the investment mandates. In the model the discount factor used in the financial intermediaries' problem is the real interest rate in the economy that changes with the precautionary savings and the tightness of borrowing constraints - wedges discussed in Section 2. One may therefore wonder what is the relative role of the SDF, which depends on the incomplete market wedge, and other adjustment costs. In Appendix C.1 we re-derive the intermediary's problem by replacing the discount factor with the equivalent under representative agent. We rely on this counterfactual case to assess the role of the incomplete markets discount factor, on top and above other intermediation frictions.

Aggregation, Equilibrium Conditions. Individual policy functions are aggregated according to the joint distribution of income and wealth states: $D(e_t, \beta_t, A_-,) = Pr(e = e_t, \beta = \beta_t, a_{t-1} \in A_-)$. The aggregate policy functions for asset savings and consumption for every period $i \geq t$ are: $\mathcal{A}_t(r_i^a, n_i) = \left(\int a(e_t, \beta_t, a_{t-1}) dD(e_t, \beta_t, a_{t-1}) \right)$ and $\mathcal{C}_t(r_i^a, n_i) = \left(\int c(e_t, \beta_t, a_{t-1}) dD(e_t, \beta_t, a_{t-1}) \right)$. Market clearing in the asset market implies: $\mathcal{A}_t = x_t v_t + NFA_t$. Finally, dividends on firms' equities read as follows: $d_t = \frac{p_{H,t}}{P_t} Y_t - w_t l_t - I_t - \psi_p$, with ψ_p being price adjustment costs. The nominal interest rate is linked to the real interest rate by the Fisher rule: $1 + i_{t-1} = (1 + r_t)(1 + \pi_t)$.

10 In the case of zero net supply of domestic government bonds, arbitrage implies that portfolio shares shall satisfy: $r_t^a = \frac{v_{t-1}}{\mathcal{A}_{t-1}} \frac{d_t + v_t}{v_{t-1}} + \frac{NFA_{t-1}}{\mathcal{A}_{t-1}} \frac{q_t}{q_{t-1}} (1 + r_t^*) + \left(1 - \frac{v_{t-1} + NFA_{t-1}}{\mathcal{A}_{t-1}}\right) (1 + r_t) - 1$.

11 The wedge implied by the portfolio adjustment costs, $\psi_{B^*}(exp(-B_{t+1}^*) - 1)$, is akin to the UIP deviation proposed in Schmitt-Grohé and Uribe (2003). They show that incomplete markets induce a non-stationary consumption path due to wealth transfers across countries and that the UIP wedge enables the recovery of stationarity.

Definition of Competitive Equilibrium. *A competitive equilibrium is a sequence* $[c_t, y_{H,t}, y_{F,t}, c_t^*, y_{H,t}^*, a_t, n_t, k_t, i_t, \pi_t, \pi_t^*, l_t, l_t^*, d_t, mc_t, mc_t^*]_{t=0}^{\infty}$*, that solves, for given sequence of market clearing prices* $[p_t, r_t, r_t^a, r_t^*, w_t, w_t^*, q_t, v_t, \chi_t]_{t=0}^{\infty}$ *the households' Euler conditions, 12, the firms' optimization problem, 14, the foreign households aggregate Euler condition, Phillips curve and resource constraint, the net foreign asset position, 17, arbitrage among assets hold, intermediaries make zero profits on intermediation, the UIP holds up to a wedge due to intermediation costs, hence* $\mathrm{E}_t(1 + r_{t+1}) = \mathrm{E}_t(1 + r_{t+1}^*)\frac{q_{t+1}}{q_t} + \psi_{B^*}(exp(-NFA_{t+1}) - 1)$*, domestic and foreign monetary policy follow Taylor rules.*

4. Quantitative Results

We now solve the model numerically and present quantitative results. We first assess the international business cycle relative to the open-economy representative-agent model by examining impulse responses. Second, we simulate and examine the moments of several domestic and international variables as well as the statistics related to international risk-sharing conditions and the Backus and Smith (1993) puzzle. In doing so, we compare the same statistics across the model with complete and incomplete markets, in the latter case distinguishing between acyclical and cyclical risk. The model-based statistics are also compared to the data equivalents and the statistics from past studies.

Solution Method and Calibration. The steady state of the model is solved through the endogenous grid point method a' la Carroll (2006). The stochastic steady state involves solving for a fixed point between the individual asset demand and the interest rate that clears the market. The dynamic response to shocks is obtained through an approximation method, the sequence space Jacobian (see Boppart et al. (2018) and Auclert et al. (2019)). The method consists of obtaining a solution to the individual policy functions via a Jacobian approximation, evaluated at each grid point in response to a path for the aggregate shock. This delivers a sequence of policy functions. Those are then aggregated, so that the shares characterizing the evolution of the distribution solve the set of macro equilibrium and market-clearing conditions.

Concerning calibration, time is in quarters. Most of the parameters are set based on the relevant literature or to achieve specific asset distribution targets for the US. The steady state real interest rate is calibrated to 0.0125, close to the 5-year average of the Effective Federal Funds Rate (see Auclert et al. (2019)). The time discount factor, β_{high}, is then obtained from the asset market clearing condition. As in Ferriere and Navarro (2024) we set β to take three values: $\beta \in \beta_{low}, \beta_{mid}, \beta_{high}$. We follow Krusell and Smith (1998) and set the persistence of $\pi_\beta(\beta, \beta) = 0.995$, corresponding to an average duration of 50 years. ν, the capital share in production, is set to match the capital to output ratio of 10. The response to inflation in the Taylor rule is set to 2.0, the response to output is set to 0.08, and following Smets and Wouters (2007), the interest rate smoothing parameter is set to 0.8. Following past literature in open economy a small weight on the exchange rate is applied to mimic the reaction to international conditions and to target the volatility of the nominal exchange rate. The slope of the Phillips curve and the capital depreciation rate are calibrated to 0.1, 0.02 following Auclert et al. (2019). The parameter for the capital adjustment costs is chosen to match the ratio of 2.5 of investment standard deviation to the GDP standard deviation.[12] Inverse intertemporal elasticity of substitution and inverse Frisch elasticity are set to 0.5 and 1, respectively. The parameters that characterize the autoregressive idiosyncratic process are $\rho_e = 0.966$ and $\sigma_e = 0.92$ following Auclert et al. (2019) (the unconditional standard deviation of idiosyncratic income shocks is $0.92\sqrt{1-\rho_e^2} = 0.238$). The parameters that determine the price markup, the capital share, the labor dis-utility, and the Frisch elasticity in the foreign economy follow Smets and Wouters (2007). The parameter for home-bias in demand is from Gali and Monacelli (2005). The elasticity of substitution between domestically and foreign-produced goods is set to 1.5, equally in both countries. The parameter that characterizes the premium, $\psi_{B^*}(exp(-NFA) - 1)$, is set to 0.001 following Schmitt-Grohé and Uribe (2003). Table 1 summarizes the values chosen for calibration.

Calibration of shocks is taken from Smets and Wouters (2007): monetary policy shocks have persistence of 0.15 and standard deviation of 0.25 and are uncorrelated; total factor productivity shocks are correlated between the countries with 0.91 persistence (taken from

12 The parameter is the same for HANK and RANK versions, but differs for the HANK model with countercyclical risk.

Backus et al. (1992)) and a cross-correlation of TFP shocks chosen to match cross-correlation of domestic and foreign GDPs of 0.6.[13] Standard deviation of 0.46 for the VAR residuals is taken from Smets and Wouters (2007). We also entertain other calibrations; the results remain robust. All parameters are the same across model versions, except cross-correlation between TFP processes and capital adjustment cost parameter are set differently in HANK with countercyclical risk (but are the same in HANK and RANK), since those two are picked to match investment volatility and cross-country correlation of outputs.

In the version of the model with countercyclical risk, the standard deviation of idiosyncratic income shocks (i.e., σ_t^e is the standard deviation for the process that e_t follows) is time-varying: $\sigma_t^y = \bar{\sigma}^y e^{-\zeta(y_t - \bar{y})}$, where $\sigma_t^y = w_t n_t \sigma_t^e$. This specification is used, for example, in Acharya and Challe (2025). In the baseline specification, we chose $\zeta = 8.6$, which is in the ballpark of the values used for this parameter in Acharya and Challe (2025) and Acharya et al. (2023) and which delivers Backus-Smith correlation of -0.35. The correlation monotonically decreases with the increase in ζ.

4.1. International Transmission Mechanism

We begin by examining the international transmission mechanism via impulse responses to standard macro shocks. The discussion of the international transmission of shocks enables us to understand the channels operating in the model and to discuss the patterns compatible with stylized facts on the international risk-sharing condition, in particular, the negative correlation between the ratio of consumption across countries and the real exchange rate. Our main focus is on productivity shocks, whose responses have been discussed in greater detail in past studies on international risk-sharing conditions. We then also present the response to monetary policy shocks to examine a case of demand shocks.

Figures 1 and 2 show impulse responses to TFP and monetary policy shocks for a selected set of macro and open-economy variables, and compare the models with complete (yellow line) and incomplete markets (black line), and in the latter case also consider the case with cyclical risk (green dashed line). The direction of the aggregate transmission mechanism

13 The cross-correlation is close to the one originally used in Backus et al. (1992) and is equal to 0.0771 for the HANK with countercyclical risk (in Backus et al. (1992) it is 0.088).

Table 1: Calibration Parameters

Parameter	Value	Description
K/Y	10.0	Capital to output ratio, ν is adjusted
B / Y	14	total wealth to output ratio, μ_p is adjusted
r	0.0125	Real interest rate, β is adjusted
$\Delta\beta$	0.028	Change in the discount factor
ρ_ω	0.995	Discount factor persistence
δ	0.02	Depreciation rate
σ	0.5	Inverse intertemporal elasticity of substitution
ρ	1	Inverse Frisch elasticity
ε_I	4.0	Investment adjustment cost parameter
ρ_e	0.966	Autocorrelation of idiosyncratic income shocks
σ_e	0.92	Std of idiosyncratic income shocks
μ_p^*	1.1	Foreign economy price markup
α^*	1.83	Foreign inverse Frisch elasticity
φ^*	1.0	Foreign economy dis-utility of labor supply
γ_{hb}	0.4	Home bias parameter
ν^*	0.19	$1-\nu^*$ is Foreign economy labor share
$\rho_r = \rho_r^*$	0.8	Domestic and Foreign Taylor rule smoothing parameter
$\phi = \phi^*$	2.0	Domestic and Foreign Taylor rule inflation coefficient
$\phi_y = \phi_y^*$	0.08	Domestic and Foreign Taylor rule output gap coefficient
$\kappa_p = \kappa_p^*$	0.1	Domestic and Foreign price Phillips curve slope
η_h	1.5	Elasticity of substitution between foreign and domestic goods
η_f	1.5	Foreign elasticity of substitution between foreign and domestic goods
ψ_{NFA}	0.001	UIP deviation coefficient
ζ	8.6	Cyclicality of idiosyncratic risk
$\phi_{\mathcal{E}}$	0.004	Domestic Taylor rule exchange rate coefficient

is similar across the models with and without incomplete markets with acyclical risk, but exhibits more significant differences under cyclical risk.

First, a positive TFP shock (Figure 1) is expansionary and reduces inflation. In turn, the central bank reduces the interest rate. Qualitatively, the transmission is similar across all models, but the cyclical-risk model exhibits the largest divergence. The decline in interest rates, coupled with sticky prices, induces a real depreciation on impact in the first two models, because domestic assets become less attractive, causing investors to sell the currency. Net exports are countercyclical since the expansion increases the demand for both domestic and foreign goods, more so the latter. Under the incomplete markets two opposing effects emerge.

First, the exchange rate depreciation reduces disposable income, thereby dampening the increase in demand and the decline in the net exports-to-GDP ratio. Second, the wealth effects associated with heterogeneous exposure drive domestic demand for goods higher (under expansionary shocks), leading to real exchange rate appreciation, even as consumption grows. The wealth effect, which can account for the negative correlation between the consumption ratio and the real exchange rate, are more potent under countercyclical risk: since households face less uncertainty in a boom, more exposed households have a higher marginal propensity to consume, leading to a larger increase in consumption demand than in the other models. In turn, the consumption ratio between the two economies increases more under countercyclical risk. The large increase in domestic demand pushes domestic prices upward, leading to an appreciation already in the 5th quarter. In sum, the disposable income effect prevails in an incomplete market model with acyclical risk, and this leads to a marked and sustained real exchange rate depreciation. On the contrary, the wealth effect prevails in the model with countercyclical risk, leading to an association between consumption growth and the expected appreciation.

An alternative way to depict the mechanism is through the lens of equation 5. In it, the term $\prod_{i=0}^{t-1}(\frac{V_i}{V_i^*})^{-\sigma}$ acts as a risk premium, with a persistent dynamic. Under countercyclical income risk, domestic households have a significant appetite for liquidity; they are willing to pay large expected premia to foreign investors in exchange for liquidity. This risk premia creates a disconnect between the fundamentals and the real exchange rate, and, under countercyclical risk, it is time-varying.

This transmission mechanism and the negative association between the consumption ratio and expected movements in real exchange rates are consistent with the negative correlation between the consumption ratio across countries and the real exchange rate, as envisaged in Backus and Smith (1993), a condition that we will examine more in detail in Section 4.2.

We also assess robustness across solution methods. A recent paper by Bianchi and Kaplan (2026) noted that in some cases responses of consumption may be altered under a non-linear

solution. Figure C.2 in Appendix C.5 presents nonlinear impulse responses to a productivity shock, which, except for GDP and net export, are very close to linear ones.[14]

Given extensive literature arguing that monetary shocks and sticky prices can help account for the persistence of real exchange rates (e.g., Obstfeld and Rogoff (1995)), we also examine the transmission mechanism of monetary shocks. In section 4.2, we will examine the international risk-sharing conditions in terms of business cycle moments and correlations under both productivity and monetary shocks. A monetary contraction (Figure 2) reduces GDP and all spending components. In turn, the real exchange rate appreciates during a monetary contraction. Those patterns are well known in the open-economy literature and are similar across complete and incomplete-market models,[15] with the largest deviations appearing under cyclical risk. Here, once again, the largest drop in domestic consumption (relative to foreign consumption) leads to an expected depreciation, already at the first quarter, which is consistent with the negative correlation between the consumption ratio and the real exchange rate.

4.1.1. Decomposing the Channels under Incomplete Markets.

As explained in the analytical section, the consumption wedge alone does not account for the facts of interest; rather, it is the combination of the consumption wedge with the dynamics of wealth and the MPC distribution that does. The latter, in turn, also induces general equilibrium changes in asset values. The transmission mechanism under incomplete markets features direct, or partial-equilibrium, and indirect, or general-equilibrium, channels (see, for instance, Auclert et al. (2019) among others). The first captures the direct responses of the consumption and savings policy functions to the shocks; the second captures the responses to changes in asset prices following the adjustment in asset markets.[16] Since those valuation channels are empirically relevant in an open economy context, other authors have already derived and extended those decompositions also in an open economy context: Auclert et

14 Note that the non-linear solution admits only MIT shocks: since a crucial part of international risk-sharing is the anticipatory effects, we maintain the linear solution to the impulse responses as our baseline.

15 Generally speaking, the dynamic is dampened in the model with incomplete markets and acyclical risk. This is due to the muted inter-temporal substitution that characterizes the behavior of non-Ricardian agents in the incomplete market model (see Kaplan and Violante (2014)).

16 Solution methods to incomplete-market models typically feature a fixed point between the individual consumption policies and market-clearing conditions.

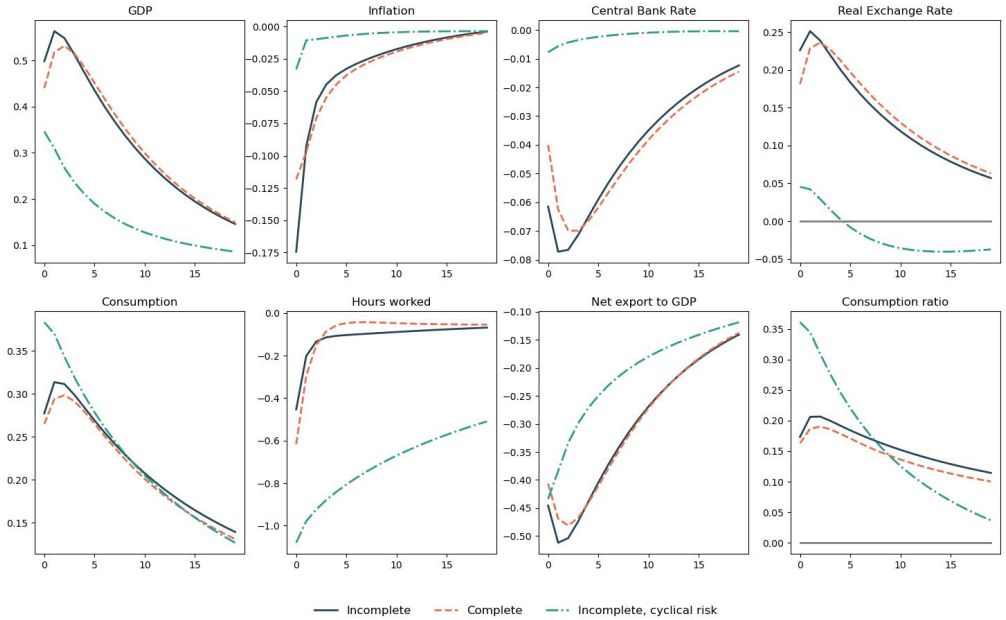


Figure 1: Impulse Response to a Productivity Shock. The panels in the figure present impulse responses of selected macro and open economy variables, as percentage deviations from the steady state, in response to a 1 percent increase in TFP which has persistence of 0.91. Inflation, Central Bank Rate, and Net export are shown in deviations from steady state. X-axis shows the time in quarters. Calibration is described in 4. The case with a cyclical risk includes time-varying risk that follows: $\sigma_t^y = \bar{\sigma}^y e^{-\zeta(y_t - \bar{y})}$.

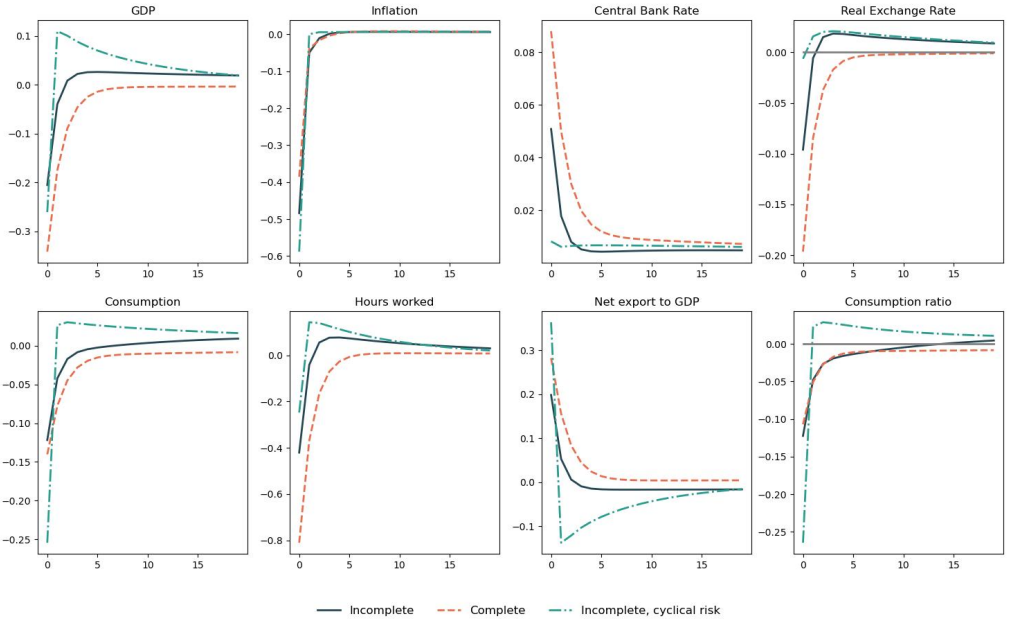


Figure 2: Impulse Response to a Monetary Policy Shock. The panels in the figure present impulse responses of selected macro and open economy variables, as percentage deviations from the steady state, in response to a 25 b.p. contractionary monetary policy shock with zero persistence. Inflation, Central Bank Rate, and Net export are shown in deviations from steady state. X-axis shows the time in quarters. Calibration is described in 4. The case with a cyclical risk includes time-varying risk that follows: $\sigma_t^y = \bar{\sigma}^y e^{-\zeta(y_t - \bar{y})}$.

al. (2021b) derived a decomposition focusing on the response of the consumption policy to external shocks; Corsetti et al. (2026) derived it for a generalized consumption function that responds to changes in all income components (labor and capital income) and interest rates.

We now examine the role of general equilibrium effects arising from the dynamics of the wealth distribution in the transmission of shocks in our model. We rely on a generalized decomposition along the lines of Corsetti et al. (2026):

$$C_t = \mathcal{C}_t(r_t, r_t^a, w_t l_t) = \mathcal{C}_t(r_t, r_t^a, \frac{1}{\mu}\frac{p_{H,t}}{P_t}F_t^{-1}(l_t, k_t)) \tag{24}$$

where the second equality stems from the steady state pricing condition ($w_t l_t = \frac{1}{\mu}\frac{p_{H,t}}{P_t}F_t^{-1}$ with $l_t = n_t \int e_t dD(e_t, \beta_t, a_{t-1})$ being the aggregate labor supply and F_t^{-1} the inverse of the production function. In a nutshell, the individual consumption policy functions depend on interest rates, determined by market clearing, and on all income sources. The second building block is the market-clearing condition:

$$Y_t = (1-\gamma_{hb})(\frac{p_{H,t}}{P_t})^{-\eta} y_t + \gamma_f (\frac{p_{H,t}}{p_{F,t}})^{-\eta_f} y_t^\star \tag{25}$$

where $y_t = c_t + I_t$.[17] The decomposition has several implications, mostly relevant for studies focusing on the role of external shocks for domestic aggregate consumption and GDP. From the perspective of our study, these equilibrium conditions highlight the dependence of consumption on expected changes in the exchange rate and international asset valuations, which would be absent in an economy with complete markets. We are interested in assessing the relative roles of general and partial equilibrium effects, particularly in the model version with countercyclical risk, which, as we show below, accounts for several empirical regularities.[18]

17 For the sake of exposition, we skip the price adjustment costs. Note that by rewriting the market-clearing condition in terms of consumption and substituting it into the policy function 24 yields the analytical solution to the fixed-point problem that characterizes models with heterogeneous agents.

18 Numerically, the decomposition identifies the part of the response due to changes in individual policy function and the response due to changes in the wealth distribution, which in turn triggers changes in the asset market clearing. To obtain the response of the aggregate variables in the economy the sequence space Jacobian relies on computing a first order Taylor approximation or else the Jacobian matrix: $\mathcal{J}_{t,s} = \frac{d\mathcal{Y}_{t,s}}{dx} = \frac{\mathcal{Y}_t^s - \mathcal{Y}_{ss}}{dx}$ where dx represent a small shock. In this context:

$$\mathcal{Y}_t^s = (\mathbf{y}_t^s)' \mathbf{D}_t^s \tag{26}$$

defines the output, or vector of aggregate variables, at time t in response to the shock at time s, and the

Figure 3 shows the decomposition in the aggregate, direct, and indirect effects comparing the incomplete market model with acylical risk (first panel), the representative agent model (second panel), and the incomplete market model with countercylical risk (third panel). In the representative agent model, which does not include indirect effects, consumption declines through the saving substitution channel, as exchange rate appreciation increases the relative cost of domestic goods. In the incomplete market model, with acyclical risk, the consumption response arises from a combination of direct and indirect channels. First, wealthy households hold significant savings, and an interest rate hike is akin to a "windfall" that increases their total lifetime wealth and current purchasing power. Second, in an open economy, a higher interest rate improves the "dynamic terms of trade" for a country with negative net foreign assets. This, in turn, expands the country's future consumption possibilities and current consumption. In an open economy, the real exchange rate appreciation, induced by the interest rate hike, increases the cost of domestic goods and reduces their demand. The indirect effects of the shock lead to a consumption decline, which accounts for a large share of the overall consumption response. An interest rate hike increases the cost of capital for firms which lowers labor demand and households' labor income. Additionally, in the versions of the model with fiscal policy, see Section 4.3 and Appendix C.2, an interest rate hike increases the cost of government borrowing: the higher taxes needed to restore fiscal balance reduce disposable income and consumption.[19] In the cyclical risk model, the decomposition is similar to the acyclical risk case, but the decline in consumption from indirect effects is much stronger. This, in turn, accounts for the stronger decline in the consumption ratio across countries, which leads to the expected real exchange rate appreciation. As explained above, the wealth effects arising under incomplete markets are the main driver of the association between consumption ratios and the real exchange rate observed in the data, but those same wealth effects are largely induced by general equilibrium changes in asset values.

distribution evolves as as follow: $\mathbf{D}^s_{t+1} = (\Lambda^s_t)'\mathbf{D}^s_t$; $\mathbf{y}^s_t$ is the vector of individual policy functions, y. Since y and Λ are both differentiable, for small shocks dx differentiating Equation (26) delivers:

$$d\mathcal{Y}^s_t = \mathbf{y}'_{ss} d\mathbf{D}^{ss}_t + (d\mathbf{y}^s_t)'\mathbf{D}_{ss} \tag{27}$$

Equation 27 shows that the change in the aggregate consumption/assets is driven by the change in policy functions multiplied by the steady state distribution and the change in the distribution times the steady state policy functions.

19 Further indirect effects arise from the changes in the supply and demand of assets.

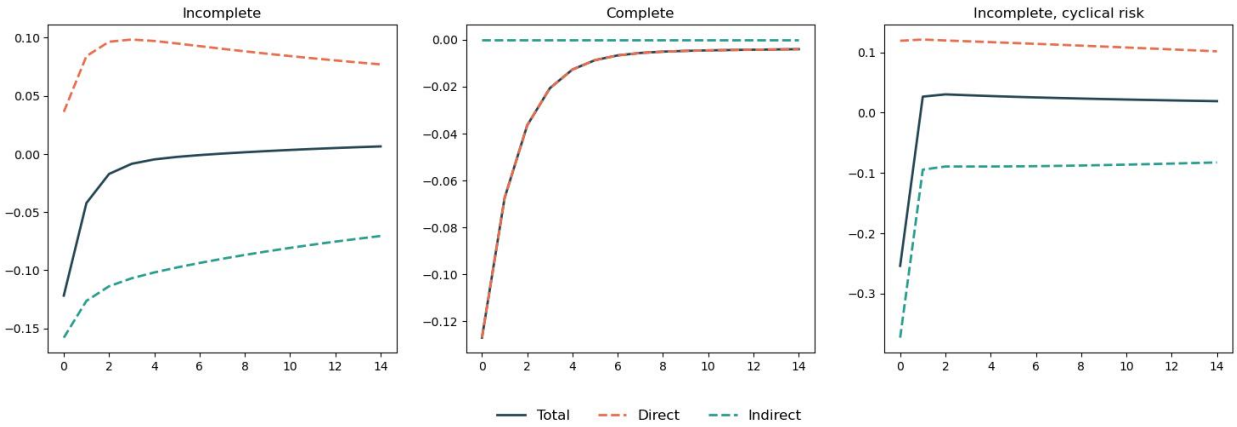


Figure 3: Impulse Response of Domestic Consumption to a Monetary Policy Shock. Dark lines show total effect, dashed orange lines show direct effect, dash-dotted green lines show indirect effect.

The main message that arises from this decomposition is that in the incomplete market model with cyclical risk much of the consumption response, which in turn accounts for the real exchange rate response, depends on general equilibrium effects.

4.2. International Risk-Sharing and Business Cycle Moments

We now use the model's simulations to compute statistics on international risk-sharing to assess how the model performs relative to empirical observations. We also have a large set of other business cycle moments for macro and international variables to verify that the model's implications remain consistent with other regularities. Table 2 shows the statistics for the model with incomplete markets and acyclical risk, and for complete markets and incomplete markets with cyclical risk. To highlight the performance of our model, we compare these moments to their counterparts in the data. To organize the discussion, we first review the models' implications for international risk-sharing and compare them with those of the complete market model. This will allow us to understand the mechanism driving the numerical results.

Models' Implications. As noted earlier, the open economy literature has uncovered a number of discrepancies for the transmission mechanism (see Obstfeld and Rogoff (2000b)) and the risk-sharing conditions (Backus and Smith (1993)) between the complete-market open-economy model, RBC or with nominal rigidities, and the data. Under complete markets, and if preferences are iso-elastic (additively separable across time and goods), the risk-sharing

condition entails a monotonic relationship between the real exchange rate and the ratio of home to foreign consumption:

$$\kappa(\frac{c_t^*}{c_t})^{-\sigma} = q_t \tag{28}$$

In the presence of an Arrow-Debreu security, consumption dynamics do not respond to idiosyncratic, individual, or country-specific shocks and are highly correlated across countries. This implies that the correlation between domestic and foreign consumption in a complete-market model is close to 1 and very similar to that of GDP. In the data (see, for instance, the last column in Table 2), both are below 1, and the consumption correlation is lower than that of output. Second, based on 28, the correlation of the ratio of marginal utility and the real exchange rate implied by the model is positive and high, while it is negative in the data. The past literature has consistently reported negative, and often large, values for this statistic across several countries. Benigno and Thoenissen (2003), who computed this correlation across different countries, report numbers ranging from roughly -0.10 to -0.50. Our own computation, shown in the last column of Table 2, provides a value of roughly -0.34. An additional and related implication of the efficient allocation of resources in a complete market economy is that employment is negatively correlated across countries: resources are reallocated to the economy with higher productivity. In the data, however, employment correlation is positive across countries.

To sum up, the complete market model is characterized by high cross-country correlation of consumption, as high as that of cross-country GDP, by a volatility of the ratio of consumption, or of the marginal utilities, across countries which is close to one, and by a tight connection between the volatility of the real exchange rate and the ratio of marginal utilities across countries. In the data, we observe a lower correlation between consumptions than between GDPs, a volatility of the ratio of marginal utilities different from 1, and a negative correlation between this ratio and the real exchange rate.

Let us now examine the implications of our incomplete market model. First, in Lemma 2, we have shown that in the incomplete market model, the following relation holds:

$$E_t[\frac{c_{t+1}}{c_t}(V_t(a_t))^{-1}] = E_t[\frac{c_{t+1}^*}{c_t^*}(V_t^*(a_t^*))^{-1}\frac{q_{t+1}}{q_t}] + \xi_t(a_t) \tag{29}$$

so that the connection between the expected real exchange rate and the ratio of marginal utilities depends on the $\xi_t(a_t) = \Delta^*(a_t) - \Delta(a_t)$, which is the difference in the wealth distribution between the two countries and on V_t and V_t^*, which are the country processes for income risk. Those two elements create a disconnect between the real exchange rate and the marginal-utility ratio, as shown in the last row of the statistics. Hence, potentially uninsurable risk and borrowing constraints may help reconcile some of the facts regarding the dynamics of exchange rates and their relation to consumption growth across countries. Next, in Theorem 1, we have shown that, keeping fixed the asset evolution, the relation between the ratio of consumption across countries and the real exchange rate depends on the wedge $\prod_{i=0}^{t-1}(\frac{V_i}{V_i^*})$. Taking into account changes in the wealth distribution, the relation reads as follows:

$$\kappa(\frac{c_t^*}{c_t})^{-\sigma}\prod_{i=0}^{t-1}(\frac{V_i}{V_i^*})^{-\sigma}\left(1+\frac{\Gamma_t-\Gamma_t^*}{c_0-\Gamma_t}\right)^{\sigma}=q_t \tag{30}$$

where $\Gamma_t = \sum_{i=0}^{t-1} E_0\Delta_i \prod_{j=0}^{i-1}(\beta R_j)^{-\frac{1}{\sigma}} V_j^{-1}$ and $\Gamma_t^* = \frac{c_0}{c_0^*}\sum_{i=0}^{t-1} E_0\Delta_i^*(\frac{q_0}{q_i})^{-\frac{1}{\sigma}} \prod_{j=0}^{i-1}(\beta R_j)^{-\frac{1}{\sigma}}(V_j^*)^{-1}$ The baseline wedge remains $\prod_{i=0}^{t-1}(\frac{V_i}{V_i^*})^{-\sigma}$, with an additional term under a changing asset distribution. In both cases, the wedge depends on the properties of income risk in each country, which are then also transmitted to wealth accumulation. In sum, in the incomplete market model, any disconnect between consumption or marginal utilities across countries, or between their cross-country ratios and the real exchange rate, largely depends on the properties of earnings risk. Countercyclical risk is likely to amplify the fluctuations of this wedge.

In what follows, we compare the statistics shown in Table 2 for the complete and incomplete-market models with acyclical and cyclical risk with the data.

Note that past literature also noted that other regularities characterize the transmission of shocks (see Obstfeld and Rogoff (2000b)). For instance, the correlation between output and net exports is negative in the data: when domestic income rises, domestic demand rises relatively more than foreign demand. This turns the net exports negative.[20] Therefore, to

20 Other channels may reinforce this regularity. If domestic investment rises, this too tends to push net exports into negative territory.

provide a complete assessment, Table 2 also reports those additional statistics, as well as second moments for a large set of macro variables.

Statistic	Incomplete	Complete	Countercyclical Risk	Data
Backus–Smith				
$corr(c_t/c_t^*, q_t)$	0.805	0.777	-0.346	-0.340
Cross-correlations Between Domestic and Foreign				
Consumption, $corr(c_t, c_t^*)$	0.525	0.574	0.564	0.675
Output, $corr(y_t, y_t^*)$	0.634	0.658	0.603	0.673
Hours worked, $corr(n_t, N_t^*)$	0.075	-0.164	0.165	0.697
Correlations with GDP				
Consumption – GDP	0.959	0.963	0.876	0.882
NX – GDP	0.016	0.088	-0.501	-0.212
Autocorrelations				
Real Exchange Rate	0.887	0.878	0.804	0.802
Consumption Ratio	0.959	0.961	0.904	0.721
NX	0.761	0.781	0.762	0.981
Consumption	0.955	0.953	0.895	0.810
Investment	0.890	0.882	0.603	0.947
Hours worked	0.591	0.549	0.967	0.928
Nominal Interest Rate	0.907	0.905	0.859	0.978
Standard Deviations Relative to Output				
Consumption, c	0.690	0.660	1.206	0.832
Investment, I	2.479	2.558	2.453	4.896
Inflation, π	0.363	0.297	0.545	0.721
Real exchange rate, q	0.456	0.471	0.524	4.816
Ratio of marginal utilities, u_{c^*}/u_c	1.177	1.082	2.144	-
Ratio of consumptions, c/c^*	0.377	0.346	0.686	
Nominal exchange rate, $\mathcal{E}$	3.838	3.556	4.728	6.151
Wage-based real exchange rate, q^w	5.557	4.814	7.925	6.322

Table 2: Second Moments, Correlations and Persistences for Baseline Economy. The Table reports second moments for a large set of macro and open economy variables. Parameters (except for shock processes) are calibrated as in 4. Fast Fourier Transform is used to compute second moments in the model. The details of the empirical statistics calculation are presented in Appendix B.

International Risk-Sharing Statistics. We now examine the simulated statistics and the data equivalent, shown in Table 2. Appendix B provides details of the computation of the empirical moments that are based on US and EU data. First and foremost, the first row shows that both the complete and the incomplete-market models with acyclical risk produce positive

correlations between the consumption-to-real-exchange-rate ratio and the real exchange rate, which is negative in the data. On the contrary, the model with countercyclical risk produces a negative correlation that is very close to the data equivalent. As shown in 30, incomplete markets induce a wedge in the international risk-sharing condition, which has the potential of reconciling several of the risk-sharing statistics observed in the data. However, for this to be the case, the wedge has to be disconnected from domestic fundamentals, and this occurs when the process for uninsurable earnings shocks is countercyclical.

Second, the disconnect in consumption dynamics across countries implies that the volatility of the ratio of marginal utilities should differ from one. The Table 2 shows that this is close to one in the complete market model, while it is larger than one in the incomplete-market models, and especially under countercyclical income risk.

Next, perfect risk-sharing implies that consumption correlation is close to one and similar to the output correlation. In the data, the correlation is positive but significantly lower than one and lower than the output correlation. The statistics in Table 2 show that all the models we consider generate a consumption correlation close to the data. This is mainly because, in all our models, international markets are incomplete due to the intermediation costs incurred by the intermediary. That alone cannot explain the correlation between the consumption ratio and the real exchange rate, but it can reduce the consumption correlation relative to the model with full international insurance.

Next, let us examine the simulated statistics for the volatility and persistence of the nominal and the real exchange rate (persistence reported in row 11 and volatilities reported in the last two rows of Table 2). The model with countercyclical risk can generate volatilities and persistence that are higher and more closely aligned with the data for both the nominal and the real exchange rate. It should be noted, however, that even in this model, the volatility of the real exchange rate does not reach the level observed in the data.

Unlike the other models, the countercyclical risk model can generate a negative correlation between GDP and net exports. In this case, indeed, when income rises, risk declines, and households consume significantly more. This, in turn, increases imports by a significantly greater amount than in the other models.

Finally, the last part of Table 2 reports second moments of various macro and open economy variables. All models perform relatively well compared to the data. However, the model with countercyclical risk can also generate inflation volatility that is higher and closer to the data.

Robustness. For robustness, Appendix C.2 reports the same table of statistics for the case with an active fiscal policy issuing positive debt in the domestic economy. Since households are non-Ricardian in the incomplete market model it seems meaningful to verify that results remain unchanged when we introduce positive government debt. They are.

A Counterfactual for Validation. At the end of section 3.4 we discussed the role of intermediation mandates, which materialize in the form of clientele discount factors and other portfolio adjustment costs. In principle both affects the international arbitrage conditions, and in turn the international risk-sharing. In Appendix C.1 we derive the intermediary's problem by replacing the discount factor on future profits with the one relevant under a complete market model. We then simulate and rely on this counterfactual economy to show numerically that when the discount factor is taken from the aggregate Euler equation (i.e. akin a household head ownership), the Backus-Smith correlation is almost exactly 1.

With these conditions the Backus-Smith correlation is almost exactly 1, see Table 3 below.

Backus–Smith		**Correlations with Real Exchange Rate**	
$corr(c_t/c_t^*, q_t)$	0.9989	NX – Real Exchange rate	0.502
Cross-correlations		**Autocorrelations**	
Consumption, $corr(c_t, c_t^*)$	0.908	Real Exchange Rate	0.817
Output, $corr(y_t, y_t^*)$	-0.691	Consumption Ratio	0.813
Hours worked, $corr(n_t, n_t^*)$	-0.167	NX	0.788
Correlations with GDP		Consumption	0.927
Consumption – GDP	-0.454	Investment	0.894
NX – GDP	-0.767	Hours worked	0.786
		Nominal Interest Rate	0.881

Table 3: Second Moments, Correlations and Persistences for the One-Asset Economy, a Counterfactual. The table reports second moments for a large set of macro and open economy variables. Parameters (except for shock processes) are calibrated as in 4. Fast Fourier Transform is used to compute second moments in the model.

4.3. The model with Two Assets

We now extend the model to include two assets. We describe only the optimization problem that differs from our baseline open-economy model.

Households Problem. At the beginning of each period t, households experience an idiosyncratic productivity shock e_t, which follows an n_e-state Markov process with transition matrix $P(e_{t+1}|e_t)$ and a stationary distribution $\pi(e)$. Real after-tax labor income ξ_t, at time t, is a function of idiosyncratic productivity shock e_t, labor hours supplied n_t, real wages w_t, and taxes, τ_t: $\xi_t = (1-\tau_t)e_t w_t n_t$. Labor hours are homogeneous across income realizations and are chosen by the household head. Each household, in period t, chooses consumption, c_t, savings in liquid, b_t, and illiquid, a_t, assets to maximize the following value function:

$$\begin{aligned} V_t(e_t, a_{t-1}, b_{t-1}) = &\max_{c_t, a_t, b_t} u(c_t, n_t) + \beta E_t V_{t+1}(e_{t+1}, a_t, b_t) \\ \text{s.t.} \quad & c_t + a_t + b_t = \xi_t + (1+r_t^a)a_{t-1} + (1+r_t^b)b_{t-1} - \Phi(a_t, a_{t-1}), \qquad a_t \geq 0, \quad b_t \geq \underline{b} \end{aligned} \tag{31}$$

where E_t is the expectation operator with respect to the entire state-space of the economy, with idiosyncratic and aggregate shocks, $V_t(e_t, a_{t-1}, b_{t-1})$ is the value function in time t. Following an established tradition (see Baumol (1952) and Tobin (1956)) we include a portfolio adjustment cost, which encompasses a convex cost components, and reads as follows: $\Phi(a_t, a_{t-1}) = \frac{\chi_1}{\chi_2}\left|\frac{a_t - (1-r_t^a)a_{t-1}}{(1+r_t^a)a_{t-1}+\chi_0}\right|^{\chi_2}[(1+r_t^a)a_{t-1} + \chi_0]$, with $\chi_0 > 0$, $\chi_1 > 0$ and $\chi_2 > 1$. Note that $\Phi(a_t, a_{t-1})$ is bounded, differentiable,[21] and convex in both arguments. The first-order conditions with respect to liquid and non-liquid assets are:

$$u_c(c_t, n_t)[1 + \Phi_{a_t}(a_t, a_{t-1})] = \mu_t^1 + \beta \mathrm{E}_t \partial_a V_{t+1}(e_{t+1}, a_t, b_t) \tag{32}$$

$$u_c(c_t, n_t) = \mu_t^2 + \beta \mathrm{E}_t \partial_b V_{t+1}(e_{t+1}, a_t, b_t) \tag{33}$$

where $u_c(c_t, n_t)$ is the marginal utility of consumption. The envelope condition implies:

21 Except at $a_t = (1-r_t^a)a_{t-1}$.

$\partial_a V_t(e_t, a_{t-1}, b_{t-1}) = u_c(c_t)(1 + r_t^a - \Phi_{a_{t-1}}(a_t, a_{t-1}))$ and $\partial_b V_t(e_t, a_{t-1}, b_{t-1}) = u_c(c_t)(1 + r_t^b)$. labor supply, chosen independently of the income realization, is:

$$\varphi n_t^\rho = \int u_c(e_t, a_{t-1}, b_{t-1}) \frac{\partial \xi_t}{\partial n_t} dD(e_t, a_{t-1}, b_{t-1}) \tag{34}$$

where $dD(e_t, a_{t-1}, b_{t-1})$ is the density of households in each triplet of the endogenous state space $[e_t, a_{t1-1}, b_{t-1}]$. Aggregate effective labor supply is given by per capita hours, weighted by the shares of agents in each income realization: $l_t = n_t \int e_t dD(e_t, a_{t-1}, b_{t-1})$.

Tax rate is set by the fiscal authority following: $\tau_t = \frac{(r_t - \omega_t)B^g + G}{w_t l_t}$. Goods market clearing condition in this version of the model features government expenditures and portfolio adjustment costs: $c_t + I_t + \psi_p + G + \Phi(a_t, a_{t-1}) = y_t$.

Financial Intermediary. Households save in liquid and illiquid assets. The aggregate supply of households' savings in liquid assets is $\mathcal{B}_t$ and in non-liquid assets is $\mathcal{A}_t$. The financial intermediary receives a mandate to invest in both liquid (indexed S) and non-liquid (indexed L) assets. For each of these asset classes, the intermediary invests both domestically and abroad. The intermediary can invest in government bonds, B^g and B^*, which can be liquid or non-liquid, and in equities, which are illiquid. In Appendix C, we present results for an alternative version of the model that features non-liquid bonds. The return to liquid government bonds carries a convenience yield, ω. Any international asset gives the net foreign asset position in liquid (labeled with an index S) and non-liquid assets (labeled with an index L), hence: $NFA_t^S = B_t^{S,*} = \mathcal{B}_t - B^g$ and $NFA_t^L = \mathcal{A}_t - x_t v_t$, so that the asset market clearing condition modifies to $x_t v_t + B^g + NFA_t^L + NFA_t^S = x_t v_t + B^g + NFA_t = \mathcal{B}_t + \mathcal{A}_t$. The intermediary faces an intermediation cost. The intermediary objective function now reads as follows:

$$V_t^I(\mathcal{A}_t, \mathcal{B}_t, B_t^S, B_t^{S,*}, B_t^L, B_t^{L,*}, x_t) = \tag{35}$$

$$max_{[\mathcal{A}_{t+1}, \mathcal{B}_{t+1}, B_{t+1}^S, B_{t+1}^{S,*}, B_{t+1}^L, B_{t+1}^{L,*}, x_{t+1}]} \mathrm{E}_t[\pi_t^I + \frac{1}{1 + r_{t+1}} V_{t+1}^I(\mathcal{A}_{t+1}, \mathcal{B}_{t+1}, B_{t+1}^S, B_{t+1}^{S,*}, B_{t+1}^L, B_{t+1}^{L,*}, x_{t+1}))$$

(36)

The first order conditions imply no arbitrage among the returns of all the assets the intermediary[22] can invest and also imply the following UIPs:

$$\mathrm{E}_t(1+r_{t+1}) = \mathrm{E}_t(1+r^*_{t+1})\frac{q_{t+1}}{q_t} + \psi_{B^*}(exp(-NFA_{t+1})-1) \tag{37}$$

$$\mathrm{E}_t(1+r_{t+1}-\omega_{t+1}) = \mathrm{E}_t(1+r^*_{t+1}-\omega^*_{t+1})\frac{q_{t+1}}{q_t} + \psi_{B^*}(exp(-NFA_{t+1})-1) \tag{38}$$

We can also define the return, accruing to intermediary on foreign investment, as $1+\tilde{r}_t = \frac{NFA^L_{t-1}}{NFA_{t-1}}\frac{q_t}{q_{t-1}}(1+r^*_t) + \frac{NFA^S_{t-1}}{NFA_{t-1}}\frac{q_t}{q_{t-1}}(1+r^*_t-\omega^*)$.

Calibration. Calibration of the two-asset model follows calibration of the one-asset model, except for the introduction of additional parameters, which are stated in the table below. The investment adjustment cost parameter is set to match the investment volatility of 2.5 following Itskhoki and Mukhin (2021).[23] Total wealth to GDP ratio ($(\mathcal{A}+\mathcal{B})/Y$), similarly to the one asset version, is set to 14.[24]

Table 4: Additional/Different Calibration Parameters

Parameter	Value	Description
χ_1	4.89	Targets liquid assets to GDP ratio: $B/Y = 1.04$
χ_0	0.25	Portfolio adjustment cost parameter, pivot
χ_2	2	Portfolio adjustment cost parameter, curvature
ω	0.005	Liquidity premium
B^g/Y	2.8	Government debt to GDP ratio
G/Y	0.2	Government expenditures to GDP ratio
ε_I	2.6	Investment adjustment cost parameter
ζ	8.5	Cyclicality of idiosyncratic risk
$\phi_{\mathcal{E}}$	0.0025	Domestic Taylor rule exchange rate coefficient
$\underline{b}$	0.0	Borrowing constrained for liquid assets

Impulse Response Functions. Before discussing the statistics on international risk-sharing, we briefly review the main features of shocks' transmission and the business-cycle properties of this model. Figures 4 and 5 present impulse response functions for monetary

22 The portfolio shares of each of the asset class shall satisfy the following returns rebated to households: $1+r^b_t = \frac{NFA^S_{t-1}}{B_{t-1}}\frac{q_t}{q_{t-1}}(1+r^*_t-\omega^*) + \left(1-\frac{NFA^S_{t-1}}{B_{t-1}}\right)(1+r_t-\omega_t)$, and $1+r^a_t = \frac{v_{t-1}}{A_{t-1}}\frac{div_t+v_t}{v_{t-1}} + \frac{NFA^L_{t-1}}{A_{t-1}}\frac{q_t}{q_{t-1}}(1+r^*_t) + \left(1-\frac{v_{t-1}+NFA^L_{t-1}}{A_{t-1}}\right)(1+r_t)$.

23 This parameter is set to 4.9 in the model with incomplete markets.

24 Cross-country correlation between TFP shocks is 0.082 for the model with countercyclical risk. Similarly to the one-asset version, it is set to match the cross-country correlation of GDPs at 0.6.

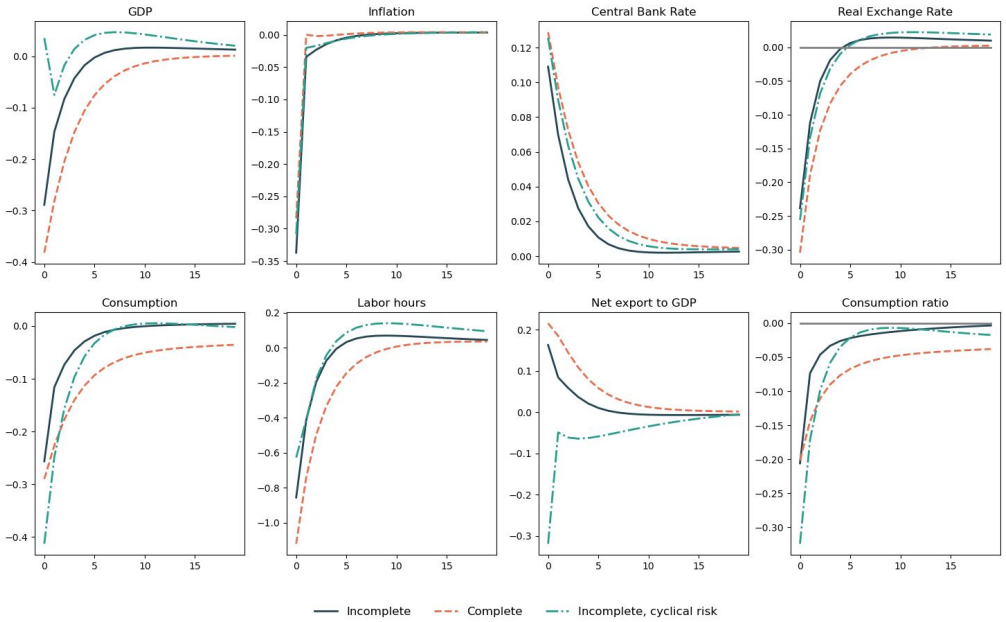


Figure 4: Impulse Response to Monetary Policy Shock: Two Assets. The panels in the figure present impulse responses of selected macro and open economy variables, as percentage deviations from the steady state, in response to a 25 b.p. contractionary monetary policy shock with zero persistence. Inflation, Central Bank Rate, and Net export are shown in deviations from the steady state. The X-axis shows the time in quarters. Calibration is described in 4. The case with a cyclical risk includes time-varying risk that follows: $\sigma_t^y = \bar{\sigma}^y e^{-\zeta(y_t - \bar{y})}$. Two assets are introduced according to Section 4.3.

policy shocks and productivity shocks, respectively. The qualitative patterns are very similar to the ones observed in the other models considered above, with the only exception that they are somewhat more persistent and exhibit larger fluctuations on impact. Most variables exhibit hump-shaped dynamics. Appendix C.4 also presents the direct and indirect effects decomposition for this model. The wealth effects, to which we attributed the association between the dynamic of the consumption ratio and that of the real exchange rate, are even larger in this model due to the larger response of the non-liquid asset values. As in the one-asset model, the indirect, or general equilibrium, effect again accounts for the bulk of the transmission mechanism.

Business Cycle Moments and International Risk Sharing. We present results by focusing on the international risk-sharing statistics. In Table 5 we present international risk-sharing statistics, including the case with cyclical risk. In this case, we detect no large differences relative to the economy with one asset. The main results are very similar to the case with one asset, with some improvement in the matching of other statistics. For instance,

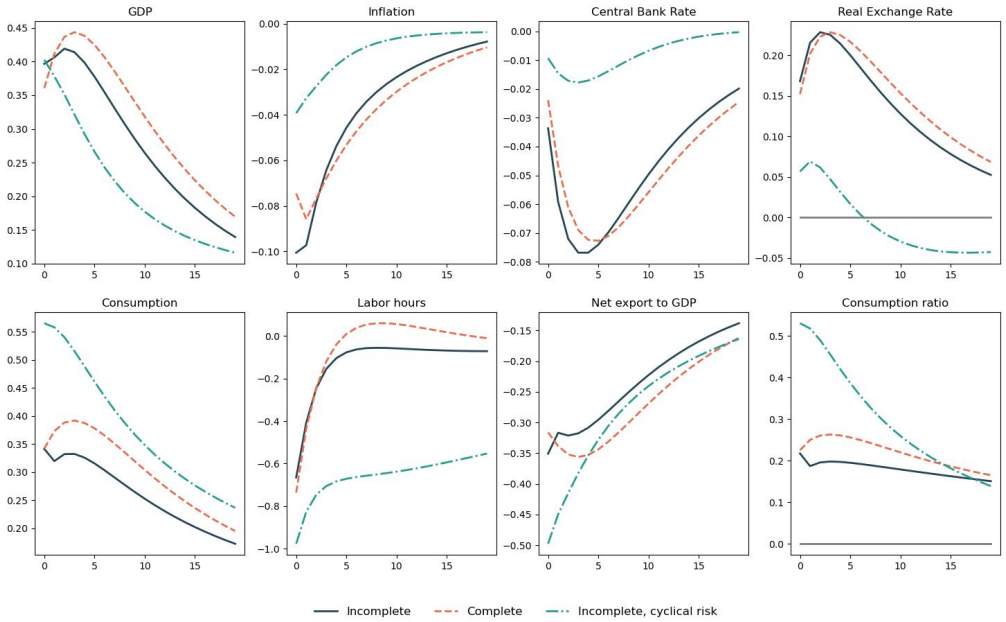


Figure 5: Impulse Response to Productivity Shock: Two Assets. The panels in the figure present impulse responses of selected macro and open economy variables, as percentage deviations from the steady state, in response to a 1 percent increase in TFP, which has persistence of 0.91. Inflation, Central Bank Rate, and Net export are shown in deviations from the steady state. The X-axis shows the time in quarters. Calibration is described in 4. The case with a cyclical risk includes time-varying risk that follows: $\sigma_t^y = \bar{\sigma}^y e^{-\zeta(y_t - \bar{y})}$. See Section 4.3 for equilibrium conditions.

the model with two assets does a much better job at matching the correlation of employment across countries.

Robustness. In the model with two assets it is possible to conceive that only a subset of assets is traded and ask what this would imply for the facts of interest. In Appendix C.3 we show that the results for the model with two assets, of which only one is traded. Results are similar.

Statistic	Incomplete	Complete	Countercyclical Risk	Data
Backus–Smith				
$corr(c_t/c_t^*, q_t)$	0.692	0.783	-0.353	-0.340
Cross-correlations Between Domestic and Foreign				
Consumption, $corr(c_t, c_t^*)$	0.465	0.636	0.613	0.675
Output, $corr(y_t, y_t^*)$	0.617	0.647	0.619	0.673
Hours worked, $corr(n_t, N_t^*)$	-0.254	-0.412	0.244	0.697
Correlations with GDP				
Consumption – GDP	0.964	0.935	0.887	0.882
NX – GDP	0.291	0.391	-0.371	-0.212
Autocorrelations				
Real Exchange Rate	0.846	0.855	0.829	0.802
Consumption Ratio	0.952	0.964	0.935	0.721
NX	0.805	0.796	0.807	0.981
Consumption	0.943	0.954	0.939	0.810
Investment	0.902	0.901	0.717	0.947
Hours worked	0.600	0.675	0.965	0.928
Nominal Interest Rate	0.909	0.913	0.835	0.978
Standard Deviations Relative to Output				
Consumption, c	0.933	0.985	1.661	0.832
Investment, I	2.522	2.449	2.494	4.896
Inflation, π	0.323	0.246	0.275	0.721
Real exchange rate, q	0.552	0.535	0.490	4.816
Ratio of marginal utilities, u_{c^*}/u_c	1.661	1.553	2.634	-
Nominal exchange rate, $\mathcal{E}$	5.466	4.589	5.091	6.151
Wage-based real exchange rate, q^w	6.576	5.568	5.542	6.322

Table 5: Second Moments, Correlations and Persistences for the Two-Asset Economy. The Table reports second moments for a large set of macro and open economy variables. Parameters (except for shock processes) are calibrated as in 4. Fast Fourier Transform is used to compute second moments in the model. The details of the empirical statistics calculation are presented in Appendix B.

5. Conclusions

Nominal rigidities and incomplete markets are two pillars of the Keynesian doctrine. The first has been included in macro and open-economy models for a long time. We build and examine the international transmission mechanism in an open economy model with nominal rigidities and incomplete markets, in the form of uninsurable risk and borrowing constraints.

We analytically prove that those elements induce a discount factor wedge in international risk-sharing conditions and uncovered interest rate parities, potentially reconciling numerous stylized facts with the data. The wealth effects, induced by differential household exposure to shocks and by general equilibrium adjustment, are the key drivers of the association between the consumption ratio and the real exchange rate that matches the data. Those wealth effects are particularly pronounced under countercyclical risk.

We then show numerically that key statistics characterizing the risk-sharing conditions and the behavior of the real exchange rate in the data can be quantitatively reconciled with our model only under the countercyclical risk case. In this case, the aforementioned wedge is highly volatile and also disconnected from the domestic fundamentals.

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Online Appendix (For Online Publication Only)

Incomplete Markets and International Risk-Sharing

Faia and Shabalina

September 4, 2026

A. Analytical Derivations

Derivations of the Generalized Euler condition. Multiplying, and dividing, the right hand side by $[\sum_{e_{t+1}} \pi(e^{t+1} \mid e^t) c(e^t, e_{t+1})]^{-\sigma} = [\mathrm{E}_t\{c(e^t, e_{t+1})\}]^{-\sigma}$, where E_t is the expectation operator with respect to the idiosyncratic shock e_t, we obtain:

$$c(e^t)^{-\sigma} - \mu(e^t) = \beta R_t [E_t\{c(e^t, e_{t+1})\}]^{-\sigma} \frac{\mathrm{E}_t[c(e^t, e_{t+1})^{-\sigma}]}{[\mathrm{E}_t\{c(e^t, e_{t+1})\}]^{-\sigma}} \tag{A.1}$$

or equivalently:

$$c(e^t) V(e^t, e_{t+1}) - \tilde{v}(e^t) = [\beta R_t]^{-\frac{1}{\sigma}} \mathrm{E}_t[c(e^t, e_{t+1})] \tag{A.2}$$

where $V(e^t, e_{t+1}) = [\frac{\mathrm{E}_t[c(e^t, e_{t+1})^{-\sigma}]}{[\mathrm{E}_t\{c(e^t, e_{t+1})\}]^{-\sigma}}]^{\frac{1}{\sigma}} \geq 1$ captures the effects of individual consumption risk on individual consumption choices. This is also the main wedge that distinguishes the models with ex post heterogeneity from models with a representative agent. It captures the Jensen's inequality distance between expected consumption and the consumption policy function, evaluated at the expected income. The term $\tilde{v}(e^t) = V(e^t, e_{t+1})\{c(e^t) - [(c(e^t))^{-\sigma} - \mu(e^t)]^{-\frac{1}{\sigma}}\}$ captures instead the interactions between individual consumption risk and the borrowing constraint, whose shadow value is given by $\mu(e^t)$. Intuitively, as the constraint tightens, consumption is less likely to be determined by the marginal utility. Dividing and multiplying the left hand side of A.2 by aggregate consumption, and aggregating across households, being continuous types in this case, delivers:

$$c_t \int \frac{c(e^t)}{c_t} V(e^t, e_{t+1}) dj = (\beta R_t)^{-\frac{1}{\sigma}} \int \mathrm{E}_t\{c(e^t, e_{t+1})\} dj \tag{A.3}$$

where the term $\sum_{e_t} \pi(e^t)\tilde{v}(e^t) \to 0$ as we assumed not binding borrowing contraints. With a discrete set of individual types, A.3 reads as: $c_t \sum_{e_{t+1}} \pi(e_{t+1} \mid e^t) \frac{c(e^t)}{c_t} V(e^t, e_{t+1}) = (\beta R_t)^{-\frac{1}{\sigma}} \sum_{e_{t+1}} \pi(e_{t+1} \mid e^t) \mathrm{E}_t\{c(e^t, e_{t+1})\}$. As a result:

$$c_t = (\beta R_t)^{-\frac{1}{\sigma}} E_t[c_{t+1}] V_t^{-1} \tag{A.4}$$

where E_t is the expectation operator with respect to the idiosyncratic risk, $V_t = \int \frac{c(e^t)}{c_t} V(e^t, e_{t+1}) dj$ with a continuum of agents, or $V_t = \sum_{e_{t+1}} \pi(e_{t+1} \mid e^t) \frac{c(e^t)}{c_t} V(e^t, e_{t+1})$ with a discrete set of agents.

Proof of Lemma 1. For brevity we omit expectations operator in the derivation below. Proof of Theorem 1 carries the expectation. Equating the real interest rates from 3 and 4 delivers the following

$$(\frac{c_{t+1}}{c_t})^{\sigma} V_t^{-\sigma} = (\frac{c^*_{t+1}}{c^*_t})^{\sigma} (V^*_t)^{-\sigma} \frac{q_{t+1}}{q_t} \tag{A.5}$$

Using the same equation one period before:

$$(c_t)^{\sigma} = (c_{t-1})^{\sigma} (\frac{c^*_t}{c^*_{t-1}})^{\sigma} \frac{(V^*_{t-1})^{-\sigma}}{(V_{t-1})^{-\sigma}} \frac{q_t}{q_{t-1}} \tag{A.6}$$

From equation A.5 it also follows that:

$$(c_t)^{\sigma} = (c_{t+1})^{\sigma} (\frac{c^*_{t+1}}{c^*_t})^{-\sigma} \frac{(V^*_t)^{\sigma}}{(V_t)^{\sigma}} \frac{q_t}{q_{t+1}} \tag{A.7}$$

Equate the left hand side of A.6 and A.7:

$$(c_{t-1})^{\sigma} (\frac{c^*_t}{c^*_{t-1}})^{\sigma} \frac{(V^*_{t-1})^{-\sigma}}{(V_{t-1})^{-\sigma}} \frac{q_t}{q_{t-1}} = (c_{t+1})^{\sigma} (\frac{c^*_{t+1}}{c^*_t})^{-\sigma} \frac{(V^*_t)^{\sigma}}{(V_t)^{\sigma}} \frac{q_t}{q_{t+1}} \tag{A.8}$$

Substituting $(c_{t-1})^{\sigma}$ from equation A.6 one period backward delivers:

$$(c_{t-2})^{\sigma} (\frac{c^*_{t-1}}{c^*_{t-2}})^{\sigma} \frac{(V^*_{t-2})^{-\sigma}}{(V_{t-2})^{-\sigma}} \frac{q_{t-1}}{q_{t-2}} (\frac{c^*_t}{c^*_{t-1}})^{\sigma} \frac{(V^*_{t-1})^{-\sigma}}{(V_{t-1})^{-\sigma}} \frac{q_t}{q_{t-1}} = (c_{t+1})^{\sigma} (\frac{c^*_{t+1}}{c^*_t})^{-\sigma} \frac{(V^*_t)^{\sigma}}{(V_t)^{\sigma}} \frac{q_t}{q_{t+1}} \tag{A.9}$$

Canceling terms out and grouping others delivers:

$$(\frac{c_{t-2}}{c^*_{t-2}})^{\sigma}\frac{1}{q_{t-2}}\frac{(V^*_{t-2})^{-\sigma}}{V^{-\sigma}_{t-2}}\frac{(V^*_{t-1})^{-\sigma}}{V^{-\sigma}_{t-1}}\frac{(V^*_{t})^{-\sigma}}{V^{-\sigma}_{t}} = (\frac{c_{t+1}}{c^*_{t+1}})^{\sigma}\frac{1}{q_{t+1}} \tag{A.10}$$

Iterating the backward substitution and canceling terms delivers:

$$(\frac{c_0}{c^*_0})^{\sigma}\frac{1}{q_0}\prod_{i=0}^{t-1}(\frac{V_i}{V^*_i})^{\sigma} = (\frac{c_t}{c^*_t})^{\sigma}\frac{1}{q_t} \tag{A.11}$$

Rearranging terms delivers the expression 5 in the main draft.

Proof of Lemma 2. Countercyclical Risk and International Risk-Sharing. Here we provide a formal proof on how countercyclical earning risk affects the consumption wedge so as to generate a negative correlation between the consumption ratio across countries and the real exchange rate. First, we log-linearize 5:

$$\ln\kappa - \sigma(\ln c^*_t - \ln c_t) - \sigma\sum_{i=0}^{t-1}(\ln V_i - \ln V^*_i) = \ln q_t \tag{A.12}$$

Let lowercase variables with hats denote log-deviations from their respective steady states (e.g., $\hat{q}_t = \ln q_t - \ln\bar{q}$, $\hat{c}_t = \ln c_t - \ln\bar{c}$, and $\hat{V}_t = \ln V_t - \ln\bar{V}$). Equation A.12 can then be re-stated as follows (subtracting steady-state version of eq. A.12):

$$-\sigma(\hat{c}^*_t - \hat{c}_t) - \sigma\sum_{i=0}^{t-1}(\hat{V}_i - \hat{V}^*_i) = \hat{q}_t \tag{A.13}$$

Next, we take the first-difference ($\Delta x_t = x_t - x_{t-1}$) of the equation to eliminate the historical summation from period 0 to $t-2$:

$$-\sigma(\Delta\hat{c}^*_t - \Delta\hat{c}_t) - \sigma(\hat{V}_{t-1} - \hat{V}^*_{t-1}) = \Delta\hat{q}_t \tag{A.14}$$

Rearranging A.14 allows us to isolate the relative growth rate of domestic-to-foreign aggregate consumption:

$$\Delta\hat{c}_t - \Delta\hat{c}^*_t = \frac{1}{\sigma}\Delta\hat{q}_t + (\hat{V}_{t-1} - \hat{V}^*_{t-1}) \tag{A.15}$$

As noted in the text, the individual wedge component $V(e^t, e_{t+1}) \geq 1$ increases with the

variance of the uninsurable idiosyncratic shock e. Under aggregate uncertainty, let the conditional variance of this domestic shock at time t be defined as $\sigma^2_{e,t}$ and we assume that this is countercyclical with respect to a domestic aggregate output y_t (also expressed in logarithms):

$$\sigma^2_{e,t} = f(y_t) \quad \text{where} \quad f'(y_t) < 0 \tag{A.16}$$

Because the aggregate wedge V_t scales monotonically with this variance, its log-deviation $\hat{V}_t$ can be linearly approximated as a decreasing function of the aggregate output gap: $\hat{V}_t = -\gamma \hat{y}_t \quad \text{where} \quad \gamma > 0$. Assuming the rest of the world features a symmetric market incompleteness structure, the foreign risk wedge responds to foreign aggregate output fluctuations:$\hat{V}^*_t = -\gamma \hat{y}^*_t$. Subtracting the foreign wedge from the domestic wedge yields the relative market-incompleteness wedge as a function of the cross-country output differential:$\hat{V}_{t-1} - \hat{V}^*_{t-1} = -\gamma(\hat{y}_{t-1} - \hat{y}^*_{t-1})$. We can now substitute this last expression back into the log-linearized risk-sharing relation (Equation A.15). This yields:

$$\Delta \hat{c}_t - \Delta \hat{c}^*_t = \frac{1}{\sigma} \Delta \hat{q}_t - \gamma(\hat{y}_{t-1} - \hat{y}^*_{t-1}) \tag{A.17}$$

To analyze how this property impacts the Backus-Smith correlation, we now take the covariance of both sides with respect to the change in the real exchange rate, $\Delta \hat{q}_t$:

$$\text{Cov}\left(\Delta \hat{c}_t - \Delta \hat{c}^*_t, \Delta \hat{q}_t\right) = \underbrace{\frac{1}{\sigma}\text{Var}(\Delta \hat{q}_t)}_{\text{Standard Transmission (+)}} \underbrace{- \gamma \text{Cov}\left(\hat{y}_{t-1} - \hat{y}^*_{t-1}, \Delta \hat{q}_t\right)}_{\text{Countercyclical Precautionary Wedge (-)}} \tag{A.18}$$

Consider now the case in which the domestic country enters a recession relative to the rest of the world ($\hat{y}_{t-1} - \hat{y}^*_{t-1} < 0$), the domestic uninsurable risk wedge spikes ($\hat{V}_{t-1} \uparrow$). Households increase precautionary asset holdings, reducing aggregate current consumption. This heightened demand for international assets appreciates the domestic currency in real terms, causing a real exchange rate appreciation ($\Delta \hat{q}_t < 0$). Hence their co-movement yields a strongly positive covariance term: $\text{Cov}\left(\hat{y}_{t-1} - \hat{y}^*_{t-1}, \Delta \hat{q}_t\right) > 0$. When the countercyclical sensitivity of the uninsurable shock (γ) is sufficiently large, the second term dominates inducing the negative covariance between the consumption ratio across countries (hence the

marginal utility ratio) and the real exchange rate. In sum, the endogenous volatility of the risk factor wedge $\frac{V_{t-1}}{V^*_{t-1}}$ breaks the standard consumption transmission channel.

Proof of Lemma 3. The Wedge Conditional on the Dynamic of Wealth. Conditional on changes in the dynamic of the wealth distribution, equation A.2 reads as follows:

$$c_t \int \frac{c(e^t, a_t)}{c_t} V(e^t, e_{t+1}, a_t) dD_j(e^t, a(e^t)) - \int \tilde{v}(e^t, e_{t+1}, a_t) dD_j(e^t, a(e^t)) = (\beta R_t)^{-\frac{1}{\sigma}} \int E_t\{c(e^t, e_{t+1}, a_t)\} dD_j(e^t, a(e^t)) \tag{A.19}$$

Rearranging A.19 delivers:

$$c_t = (\beta R_t)^{-\frac{1}{\sigma}} \mathrm{E}_t c_{t+1} (V_t(a_t))^{-1} + \Delta(a_t) \tag{A.20}$$

where $V_t(a_t) = \int \frac{c(e^t, a_t)}{c_t} V(e^t, e_{t+1}, a_t) dD_j(e^t, a(e^t))$ and $\Delta(a_t) = (V_t(a_t))^{-1} \int \tilde{v}(e^t, e_{t+1}, a_t) dD_j(e^t, a(e^t))$. Next, we show that if income risk is countercyclical enough the consumption wedge can reproduce the negative correlation between the ratio of marginal utilities across countries and the real exchange rate. We begin with the wealth-dependent international asset arbitrage condition provided in Equation 7:

$$\mathbb{E}_t \left[\frac{c_{t+1}}{c_t} (V_t(a_t))^{-1} \right] = \mathbb{E}_t \left[\frac{c^*_{t+1}}{c^*_t} (V^*_t(a^*_t))^{-1} (\frac{q_{t+1}}{q_t})^{\frac{1}{\sigma}} \right] + \xi_t(a_t) \tag{A.21}$$

Let lowercase variables with hats denote log-deviations from their steady-state values. In the deterministic steady state, the risk wedges are normalized to unity ($\bar{V} = \bar{V}^* = 1$) and the net cross-country wealth distribution wedge is zero ($\bar{\xi} = 0$). Omitting the expectations operator for analytical tractability yields the following first-order linear approximation around the steady state:

$$\hat{c}_{t+1} - \hat{c}_t - \hat{V}_t(a_t) \approx \hat{c}^*_{t+1} - \hat{c}^*_t - \hat{V}^*_t(a^*_t) + \frac{1}{\sigma}(\hat{q}_{t+1} - \hat{q}_t) + \hat{\xi}_t(a_t) \tag{A.22}$$

Using the first-difference operator ($\Delta x_{t+1} = x_{t+1} - x_t$), we can rewrite A.22 in terms of growth rates:

$$\Delta\hat{c}_{t+1} - \Delta\hat{c}^*_{t+1} - \hat{V}_t(a_t) \approx \frac{1}{\sigma}\Delta\hat{q}_{t+1} - \hat{V}^*_t(a^*_t) + \hat{\xi}_t(a_t) \tag{A.23}$$

Rearranging terms to isolate the growth rate of the real exchange rate yields: $\Delta\hat{q}_{t+1} \approx (\Delta\hat{c}_{t+1} - \Delta\hat{c}^*_{t+1}) + \left[\hat{V}^*_t(a^*_t) - \hat{V}_t(a_t)\right] - \hat{\xi}_t(a_t)$. Lagging the entire equation by one period and moving the cross-country aggregate consumption growth differential on the left yields our baseline estimation equation:

$$\Delta\hat{c}_t - \Delta\hat{c}^*_t \approx \frac{1}{\sigma}\Delta\hat{q}_t + \left[\hat{V}_{t-1}(a_{t-1}) - \hat{V}^*_{t-1}(a^*_{t-1})\right] + \hat{\xi}_{t-1}(a_{t-1}) \tag{A.24}$$

Consider an aggregate shock that induces a domestic recession ($\hat{y}_{t-1} - \hat{y}^*_{t-1} < 0$). Under countercyclical income risk, the individual income risk component increases during a domestic downturn. This increases both precautionary saving motives and increases the likelihood that low-income households become borrowing constrained. Conditioned on the existing asset holdings a_{t-1}, a drop in aggregate output increases the dispersion of individual income shocks. This implies that the cross-country risk wedge differential is negatively related to the the relative output gap:

$$\hat{V}_{t-1}(a_{t-1}) - \hat{V}^*_{t-1}(a^*_{t-1}) = -\gamma(\hat{y}_{t-1} - \hat{y}^*_{t-1}) \quad \text{where} \quad \gamma > 0 \tag{A.25}$$

Let us now examine the terms, $\xi_{t-1}(a_{t-1}) = \Delta^*(a^*_{t-1}) - \Delta(a_{t-1})$, which we define as the liquidity wedge and where Δ represents the shadow price of the borrowing constraint weighted by the wealth distribution. During a domestic recession, aggregate domestic wealth contracts, shifting the distribution function D_j leftward, as a larger mass of households encounter binding borrowing constraints ($\mu(e^t) > 0$). This induced an increase in the domestic wedge $\Delta(a_{t-1})$, which in turn implies that the relative wedge $\hat{\xi}_{t-1}$ falls during domestic recessions. In other words the liquidity wedge positively correlates with the output gap:

$$\hat{\xi}_{t-1}(a_{t-1}) = \phi(\hat{y}_{t-1} - \hat{y}^*_{t-1}) \quad \text{where} \quad \phi > 0 \tag{A.26}$$

Substituting Equation A.25 and Equation A.26 back into the log-linearized risk-sharing relation (Equation A.24) yields:

$$\Delta\hat{c}_t - \Delta\hat{c}_t^* \approx \frac{1}{\sigma}\Delta\hat{q}_t - \gamma(\hat{y}_{t-1} - \hat{y}_{t-1}^*) + \phi(\hat{y}_{t-1} - \hat{y}_{t-1}^*) \tag{A.27}$$

Re-arranging yields: $\Delta\hat{c}_t - \Delta\hat{c}_t^* \approx \Delta\hat{q}_t - (\gamma - \phi)(\hat{y}_{t-1} - \hat{y}_{t-1}^*)$. Next, we take the covariance of both sides of this last expression with respect to changes in the real exchange rate ($\Delta\hat{q}_t$):

$$\text{Cov}\left(\Delta\hat{c}_t - \Delta\hat{c}_t^*, \Delta\hat{q}_t\right) = \underbrace{\frac{1}{\sigma}\text{Var}(\Delta\hat{q}_t)}_{\text{Complete Market Link (+)}} - \underbrace{(\gamma - \phi)\text{Cov}\left(\hat{y}_{t-1} - \hat{y}_{t-1}^*, \Delta\hat{q}_t\right)}_{\text{Dynamic Risk \& Wealth Distribution Wedge (-)}} \tag{A.28}$$

Under a recession, $(\hat{y}_{t-1} - \hat{y}_{t-1}^* < 0)$, households who hold assets increase their demand for internationally traded bonds to smooth consumption against higher uninsurable individual risk. However wealth-poor households hit the borrowing constraint and compress consumption due to an inability to access credit. Both channels depress domestic aggregate demand relatively to foreign demand, inducing a substantial real appreciation of the domestic currency ($\Delta\hat{q}_t < 0$). This guarantees that the output gap and real exchange rate innovations co-move positively: $\text{Cov}\left(\hat{y}_{t-1} - \hat{y}_{t-1}^*, \Delta\hat{q}_t\right) > 0$. If and only if, the precautionary savings effect is large enough and interacts with the borrowing constraint dynamics so that $\gamma > \phi$, the second term in A.28 dominates and the covariance $\text{Cov}\left(\Delta\hat{c}_t - \Delta\hat{c}_t^*, \Delta\hat{q}_t\right)$ is negative.

Proof. of Theorem 1. For the sake of exposition we will briefly explain steps that are similar to the proof of Lemma 1. The procedure is similar: we rewrite Equation 6 at period $t = 0$ and iterate one period forward:

$$c_0 = E_0(\beta R_0)^{-\frac{1}{\sigma}} c_1 V_0^{-1} + \Delta_0 = E_0(\beta R_0)^{-\frac{1}{\sigma}}[E_1(\beta R_1)^{-\frac{1}{\sigma}} c_2 V_1^{-1} + \Delta_1]V_0^{-1} + \Delta_0 \tag{A.29}$$

Iterating forward until t delivers:

$$c_0 = E_0 c_t \prod_{i=0}^{t-1}(\beta R_i)^{-\frac{1}{\sigma}} V_i^{-1} + \sum_{i=0}^{t-1} E_0\Delta_i \prod_{j=0}^{i-1}(\beta R_j)^{-\frac{1}{\sigma}} V_j^{-1} \tag{A.30}$$

Similarly for the foreign economy:

$$c_0^* = (\frac{q_0}{q_t})^{-\frac{1}{\sigma}} E_0 c_t^* \prod_{i=0}^{t-1} (\beta R_i)^{-\frac{1}{\sigma}} (V_i^*)^{-1} + \sum_{i=0}^{t-1} E_0 \Delta_i^* (\frac{q_0}{q_i})^{-\frac{1}{\sigma}} \prod_{j=0}^{i-1} (\beta R_j)^{-\frac{1}{\sigma}} (V_j^*)^{-1} \tag{A.31}$$

Combining the two equations above:

$$\frac{c_0}{c_0^*} = \frac{E_0 c_t \prod_{i=0}^{t-1} (\beta R_i)^{-\frac{1}{\sigma}} V_i^{-1} + \sum_{i=0}^{t-1} E_0 \Delta_i \prod_{j=0}^{i-1} (\beta R_j)^{-\frac{1}{\sigma}} V_j^{-1}}{(\frac{q_0}{q_t})^{-\frac{1}{\sigma}} E_0 c_t^* \prod_{i=0}^{t-1} (\beta R_i)^{-\frac{1}{\sigma}} (V_i^*)^{-1} + \sum_{i=0}^{t-1} E_0 \Delta_i^* (\frac{q_0}{q_i})^{-\frac{1}{\sigma}} \prod_{j=0}^{i-1} (\beta R_j)^{-\frac{1}{\sigma}} (V_j^*)^{-1}} \tag{A.32}$$

Defining $\Gamma_t = \sum_{i=0}^{t-1} E_0 \Delta_i \prod_{j=0}^{i-1} (\beta R_j)^{-\frac{1}{\sigma}} V_j^{-1}$ and $\Gamma_t^* = \frac{c_0}{c_0^*} \sum_{i=0}^{t-1} E_0 \Delta_i^* (\frac{q_0}{q_i})^{-\frac{1}{\sigma}} \prod_{j=0}^{i-1} (\beta R_j)^{-\frac{1}{\sigma}} (V_j^*)^{-1}$, multiplying the left-hand side by the denominator, subtracting Γ_t^* from both sides and dividing by $E_0 c_t^* \prod_{i=0}^{t-1} (\beta R_i)^{-\frac{1}{\sigma}} (V_i^*)^{-1}$:

$$(\frac{q_t}{q_0})^{\frac{1}{\sigma}} = \frac{\frac{c_0^*}{c_0} \left(E_0 c_t \prod_{i=0}^{t-1} (\beta R_i)^{-\frac{1}{\sigma}} V_i^{-1} + \Gamma_t - \Gamma_t^* \right)}{E_0 c_t^* \prod_{i=0}^{t-1} (\beta R_i)^{-\frac{1}{\sigma}} (V_i^*)^{-1}} \tag{A.33}$$

Therefore:

$$q_t = q_0 (\frac{c_0}{c_0^*})^{-\sigma} \left(\frac{E_0 c_t^* \prod_{i=0}^{t-1} (\beta R_i)^{-\frac{1}{\sigma}} (V_i^*)^{-1}}{E_0 c_t \prod_{i=0}^{t-1} (\beta R_i)^{-\frac{1}{\sigma}} V_i^{-1} + \Gamma_t - \Gamma_t^*} \right)^{-\sigma} \tag{A.34}$$

Dividing by $\prod_{i=0}^{t-1} (\beta R_i)^{-\frac{1}{\sigma}}$ both numerator and denominator inside the brackets results in:

$$q_t = q_0 (\frac{c_0}{c_0^*})^{-\sigma} \left(\frac{E_0 c_t^* \prod_{i=0}^{t-1} (V_i^*)^{-1}}{E_0 c_t \prod_{i=0}^{t-1} V_i^{-1} + \frac{\Gamma_t - \Gamma_t^*}{\prod_{i=0}^{t-1} (\beta R_i)^{-\frac{1}{\sigma}}}} \right)^{-\sigma} \tag{A.35}$$

Rearranging the expression in A.35 leads to the expression in 9.

B. Empirical Moments

To compute empirical moments for the international and macro variables we used the data for the US and for the European Union (in particular the aggregate series for 27 countries, from 2020). Since harmonized data for the European series is available only from 1999 the

time sample of our quarterly data is from 1999Q1 to 2019Q4. The data is taken from FRED and Eurostat databases. Where possible, seasonally-adjusted series were used.

From the FRED we took GDP, GDP deflator, Investment, Compensation of Employees, Population, Fed Funds rate, Personal Consumption Expenditure, Employment level, Average Weekly hours, CPI index, Export and Imports of Goods and Services and the US exchange rate vis-a-vis the euro. The codes are GDPC1, GDPDEF, FPI, COMPNFB, CNP16OV, FEDFUNDS, PCE, CE16OV, AWHNONAG, CUSR0000SA0L5, EXPGS, IMPGS, EXUSEU. Treatment of the variables is the same as in Smets and Wouters (2007), as such all growth variables are deflated and in per capita terms. Similarly to Itskhoki and Mukhin (2021) for the imports and exports we used total US imports and exports (not only vis-a-vis the European Union). We also apply HP-filter before computing statistics.

From the Eurostat we took GDP, Employment, Population, Consumption, HICP, Total hours, and Total compensation series. We used estat, namq, and pcr tables. From pcr we took HICP, from estat we took Total compensation series, other series are taken from namq (in particular subtables a10_e, fcs, gdp, pe). Differently to the US series we computed hours by dividing total hours by the population (for the US the product of average hours and employment is divided by the population). Wages are computed by dividing Total compensation by the Total hours (for the US average weekly hours series is available).

C. Additional Quantitative Results

C.1. Different Financial Intermediaries' Stochastic Discount Factor

Suppose the intermediaries are owned by the representative head that discounts the future according to the stochastic discount factor $\beta\frac{U'(C_{t+1})}{U'(C_t)}$, then the problem could be rewritten as:

$$V_t^I(\mathcal{A}_t, B_t, B_t^*, x_t) = Max_{[\mathcal{A}_{t+1}, B_{t+1}, B_{t+1}^*, x_{t+1}]} \mathrm{E}_t[\pi_t^I + \beta\frac{U'(C_{t+1})}{U'(C_t)} V_{t+1}^I(\mathcal{A}_{t+1}, B_{t+1}, B_{t+1}^*, x_{t+1}))\} \tag{C.1}$$

subject to the net wealth accumulation:

$$\pi_t^I + v_t x_{t+1} + B_{t+1}^* + B_{t+1} + (1 + r_t^a)\mathcal{A}_t = \tag{C.2}$$

$$= \mathcal{A}_{t+1} + \frac{U'(C_{t-1})}{\beta U'(C_t)} B_t + (1 + r_t^*)\frac{q_t}{q_{t-1}} B_t^* + (d_t + v_t)x_t + \psi_{B^*}(-B_t^* - exp(-B_t^*)) \tag{C.3}$$

Taking first order conditions with respect to $[\mathcal{A}_t, B_t, B_t^*, x_t]_{t=0}^{\infty}$ read as follows:

$$1 + \mathrm{E}_t \beta \frac{U'(C_{t+1})}{U'(C_t)} V_{A,t+1} = 0;\ -1 + \mathrm{E}_t \beta \frac{U'(C_{t+1})}{U'(C_t)} V_{B,t+1} = 0; \tag{C.4}$$

$$-1 + \mathrm{E}_t \beta \frac{U'(C_{t+1})}{U'(C_t)} V_{B^*,t+1} = 0;\ v_t + \mathrm{E}_t \beta \frac{U'(C_{t+1})}{U'(C_t)} V_{x,t+1} = 0 \tag{C.5}$$

$$V_{A,t} = (1 + r_t^a);\ V_{B,t} = \frac{U'(C_{t-1})}{\beta U'(C_t)};\ V_{B^*,t} = (1 + r_t^*)\frac{q_t}{q_{t-1}} + \psi_{B^*}(exp(-B_t^*) - 1);\ V_{x,t} = d_t + v_t \tag{C.6}$$

Substitution of partial derivatives implies:

$$\frac{U'(C_t)}{\beta U'(C_{t+1})} = \frac{\mathrm{E}_t(v_{t+1} + d_{t+1})}{v_t} = \mathrm{E}_t(1 + r_{t+1}^*)\frac{q_{t+1}}{q_t} + \psi_{B^*}(exp(-B_{t+1}^*) - 1) \tag{C.7}$$

C.2. One Asset Model with Fiscal Policy

Since the two-asset model features fiscal policy, below is presented a version of the one asset model with fiscal policy. The latter is introduced by introducing time-varying taxes that adjust according to:

$$\tau_t = \frac{r_t Bg + G}{w_t l_t} \tag{C.8}$$

To calibrate with use the values $G/Y = 0.2$ - a widely used "great ratio" and $Bg/Y = 2.8$, which is taken from Auclert et al. (2019). The results of this version are shown below.

C.3. Two Asset Model with One Traded Asset

In this version of the model only one asset is traded internationally. This modifies the financial intermediary problem, while keeping the problems of all other agents in the economy the same. In particular, financial intermediary trades illiquid asset vis-a-vis the rest of the

Table C.1: Cross-Country Correlations and Autocorrelations

	Incomplete	Complete	Countercyclical Risk
Backus–Smith			
$corr(c_t/c_t^*, q_t)$	0.731	0.717	-0.330
Cross-correlations Between Domestic and Foreign			
Consumption, $corr(c_t, c_t^*)$	0.483	0.580	0.509
Output, $corr(y_t, y_t^*)$	0.630	0.692	0.602
Hours worked, $corr(n_t, N_t^*)$	-0.092	-0.394	0.254
Correlations with GDP			
Consumption – GDP	0.938	0.948	0.917
NX – GDP	0.113	0.256	-0.175
Correlations with Real Exchange Rate			
NX – Real Exchange rate	0.738	0.761	0.767
Autocorrelations			
Real Exchange Rate	0.862	0.869	0.882
Consumption Ratio	0.960	0.969	0.918
NX	0.782	0.814	0.938
Consumption	0.958	0.961	0.888
Investment	0.889	0.888	0.900
Hours worked	0.606	0.672	0.972
Nominal Interest Rate	0.903	0.907	0.865

Table C.2: Second Moments, Correlations and Persistences for the One-Asset Economy with Positive Government Debt. The Table reports second moments for a large set of macro and open economy variables. Parameters (except for shock processes) are calibrated as in 4. Fast Fourier Transform is used to compute second moments in the model.

world and has a technology to transform illiquid asset to liquid with a cost of ω. This is close to the set up in Auclert et al. (2019), where there is a similar transformation technology that the intermediary has. The version where liquid asset is traded and the illiquid return follows $r_t^a = r_t^b + \omega$ delivers similar results with Backus-Smith correlation being negative, but consumption correlations across countries are larger than those of outputs. The returns that are rebated to households, therefore, in this version read as following:

$$r_t^b = r_t^a - \omega \tag{C.9}$$

$$1 + r_t^a = \frac{v_{t-1}}{A_{t-1}} \frac{div_t + v_t}{v_{t-1}} + \frac{NFA_{t-1}}{A_{t-1}} \frac{q_t}{q_{t-1}} (1 + r_t^*) + \left(1 - \frac{v_{t-1} + NFA_{t-1}}{A_{t-1}}\right)(1 + r_t) \tag{C.10}$$

$$\tag{C.11}$$

The return earned on the international net foreign asset position therefore is:

$$1 + \tilde{r}_t = \frac{q_t}{q_{t-1}}(1 + r_t^*) \tag{C.12}$$

And there is one UIP that is the same as Equation 37 or the one used in the one asset version of the model.

C.4. Direct and Indirect Effects Decomposition in Two Asset Model.

Figure C.2 also shows the decomposition of the consumption response to a monetary shock in direct and indirect effects. In line with the equivalence highlighted in the impulse responses, we observe no qualitative change in the decompositions.

C.5. Nonlinear Responses in One Asset Model

Table C.3: Cross-Country Correlations and Autocorrelations

	Incomplete	Complete	Countercyclical Risk
Backus–Smith			
$corr(c_t/c_t^*, q_t)$	0.654	0.589	-0.348
Cross-correlations Between Domestic and Foreign			
Consumption, $corr(c_t, c_t^*)$	0.571	0.608	0.650
Output, $corr(y_t, y_t^*)$	0.600	0.583	0.587
Hours worked, $corr(n_t, N_t^*)$	-0.085	-0.292	0.271
Correlations with GDP			
Consumption – GDP	0.863	0.819	0.809
NX – GDP	0.070	0.057	-0.451
Correlations with Real Exchange Rate			
NX – Real Exchange rate	0.803	0.790	0.854
Autocorrelations			
Real Exchange Rate	0.838	0.860	0.834
Consumption Ratio	0.956	0.971	0.945
NX	0.559	0.569	0.693
Consumption	0.939	0.953	0.951
Investment	0.900	0.907	0.702
Hours worked	0.593	0.682	0.967
Nominal Interest Rate	0.896	0.907	0.824

Table C.4: Second Moments, Correlations and Persistences for the Two-Asset Economy with One Traded Asset. The Table reports second moments for a large set of macro and open economy variables. Parameters (except for shock processes) are calibrated as in 4. Fast Fourier Transform is used to compute second moments in the model.

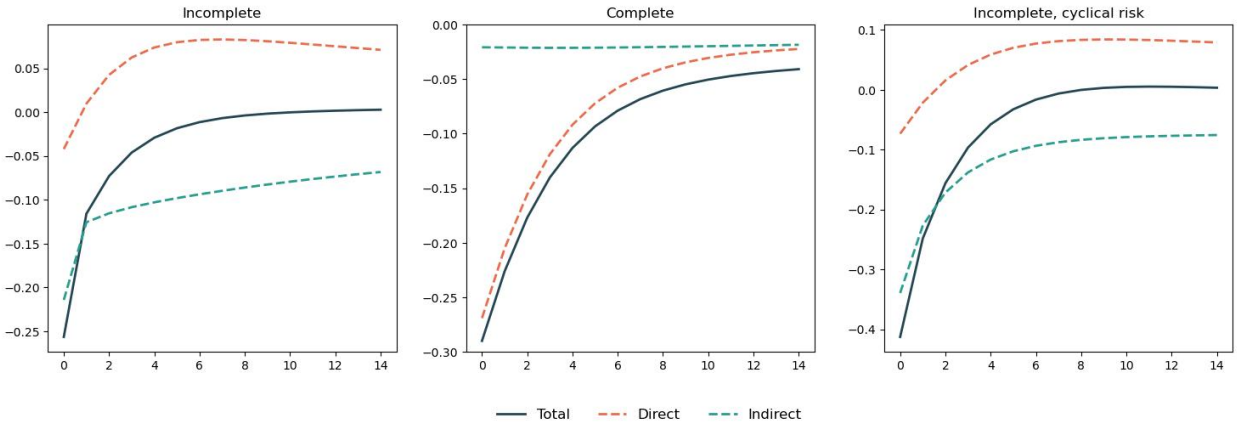


Figure C.1: Impulse Response to Monetary Policy Shock: Two Assets. Two assets are introduced according to Section 4.3. Dark lines show total effect, dashed orange lines show direct effect, dash-dotted green lines show indirect effect.

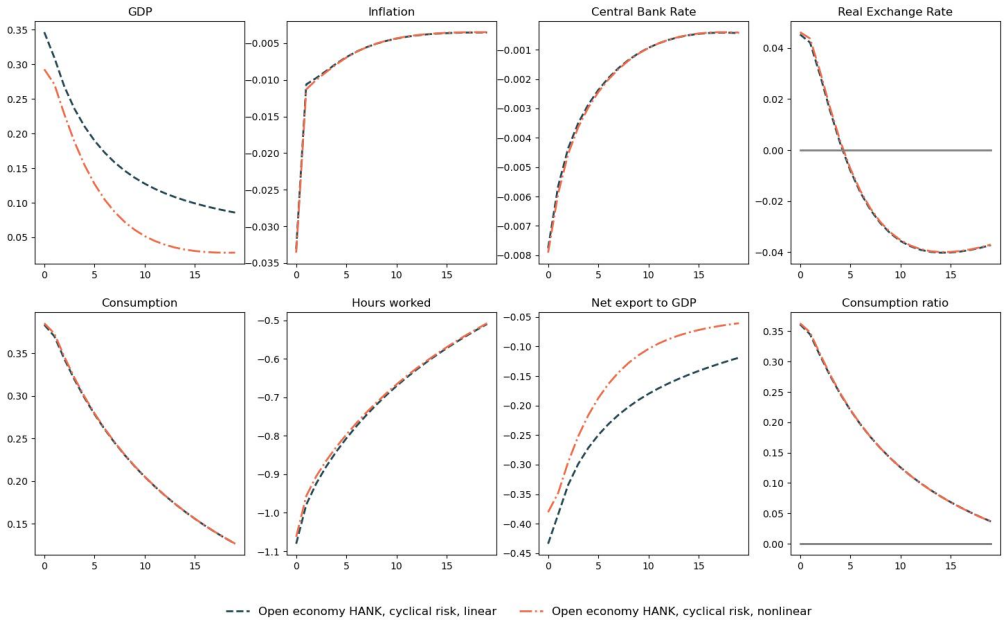


Figure C.2: Impulse Response to Productivity Shock: One Asset, Linear and Nonlinear. The panels in the figure present linear and nonlinear impulse responses of selected macro and open economy variables, as percentage deviations from the steady state, in response to a 1 percent increase in TFP which has persistence of 0.91. Inflation, Central Bank Rate, and Net export are shown in deviations from steady state. X-axis shows the time in quarters. Calibration is described in 4. The case with a cyclical risk includes time-varying risk that follows: $\sigma_t^y = \bar{\sigma}^y e^{-\zeta(y_t - \bar{y})}$.