Half the Signal:
Retail Attention and the Price of Volatility *

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August 2026

Abstract

Public retail attention identifies stocks that become more volatile over the following week, while option traders can observe the same signal before the outcome is realized. The implied volatility surface responds to attention but does not make its volatility forecast redundant. Conditioning on the full surface, including fixed nonlinear representations, still leaves about half of attention's relation with next-week volatility. Nearly all measurable spanning comes from at-the-money volatility across maturities, while call-versus-put discussion predicts the matching option-wing tilt. The remaining relation survives controls for recent price movements and improves recursive volatility forecasts. The results show why price response and predictive encompassing are distinct properties of a market forecast.

Keywords: investor attention, retail options, realized volatility, implied volatility, social media
JEL codes: G12, G14, G40

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1 Introduction

Option prices can move with a public signal without fully incorporating what that signal predicts. The distinction matters because implied volatility is widely treated as the market's forecast of future risk and is often used as a sufficient control for what the option market knows. A visible response is evidence that the signal reaches the market, but it is not evidence that the price makes the signal redundant. When a freely observable signal still forecasts realized volatility after the entire surface is observed, response and predictive encompassing are different properties of the price. This paper asks whether that distinction matters for public retail attention.

Existing research establishes the two sides of this question separately. Retail option demand moves implied volatility and the shape of the option surface (Gârleanu et al., 2009; Bollen and Whaley, 2004; Eaton et al., 2026). Investor attention predicts trading, returns, and volatility (Barber and Odean, 2008; Da et al., 2011; Andrei and Hasler, 2015; Audrino et al., 2020). Social-media activity has also been linked to option prices (Wei et al., 2016; Frino et al., 2022). A separate literature studies how well implied volatility forecasts realized volatility (Canina and Figlewski, 1993; Christensen and Prabhala, 1998; Jiang and Tian, 2005). These findings show that the relevant variables are connected, but they do not reveal whether the full stock-level surface already contains the future-volatility relation associated with a public attention signal. That is the open question here.

The answer is not obvious. Complete encompassing is plausible because options are direct claims on volatility, option traders can observe public discussion in real time, and the surface provides many prices across maturities and moneyness. A sufficiently rich surface could aggregate the signal even if no single volatility level does. Partial encompassing is also plausible. The signal is noisy, the relevant volatility episode may be short lived, and a market maker can adjust average volatility levels without perfectly reallocating prices across names. Demand related to attention can also move quotes without revealing how much of the response reflects information. The same observed price response is therefore consistent with very different degrees of predictive redundancy.

I study the question using daily stock-level attention on r/wallstreetbets. The selected measure is the lagged trailing score of log distinct comment authors who mention a stock. It captures breadth of public participation rather than the number of posts generated by a few users, which is useful when a small number of accounts can dominate message volume. The score is known before the forward volatility window begins, and the main specifications compare stocks on the same day after absorbing persistent differences across names. The active population is

fixed by the archived posts record rather than selected using future outcomes. The measure does not identify investors, trades, or signed option demand. Its role is to provide a public signal whose relation with future realized volatility can be read against contemporaneous option prices.

The empirical object is a reduced-form spanning share. I first estimate the attention slope in a forecast of forward realized volatility. I then estimate the same slope after conditioning on an option-price benchmark, always on identical observations. The share $\phi = 1 - \beta_P/\beta_0$ records how much of the original attention relation disappears when that benchmark enters. A larger share means that the benchmark makes more of the signal predictively redundant. A share below one means that a conditional attention slope remains under the stated representation and sample. It is not a cardinal fraction of information, and it does not identify why the remaining slope survives. This interpretation is especially important when the benchmark is a vector of noisy and interpolated surface nodes.

Four findings organize the evidence. First, public retail attention predicts a stock's firm-specific volatility over the following week. Second, the option surface responds to attention but leaves a positive conditional relation with future volatility. Third, almost all measurable spanning comes from at-the-money volatility across maturities, even though smile shape responds and its call-versus-put tilt lines up with the direction of public option discussion. Finally, the remaining relation survives recent-price controls and adds a small forecast improvement in recursive tests, while higher attention predicts lower subsequent delta-hedged option-buyer gains. Together, these findings describe response without encompassing rather than an identified demand or information mechanism.

The first finding is economically and statistically clear. Across 93,350 active name-days from 2020 through 2024, a one-standard-deviation increase in attention predicts 0.0320 higher annualized realized volatility over the following five trading days after trailing volatility, size, ticker fixed effects, and date fixed effects are included. The relation is primarily firm specific and declines at longer horizons. Replacing date fixed effects with industry-by-date fixed effects changes the comparable active-sample coefficient only from 0.0318 to 0.0307. The result is therefore not an aggregate volatility episode or a same-industry daily shock attached to popular names. It is a cross-sectional relation between public attention and subsequent volatility.

The option surface contains important but incomplete parts of that relation. On the fixed 61,434-row common grid, the thirty-day at-the-money level spans 0.345 of the attention slope. The full standardized twelve-node surface spans 0.466 and leaves a positive coefficient of 0.0140. The three at-the-money maturity levels alone deliver essentially the same share. Adding call-put

asymmetries and the observed wings changes little. Fixed quadratic representations raise the share to 0.488 and 0.497, while a shallow tree leaves it at 0.469. The conditional coefficient and the share remain bounded away from full encompassing under separate date and ticker resampling. The full surface is more informative than a conventional level, but the added information is concentrated in the level term structure and still does not encompass attention.

The response extends to surface shape even though shape adds little spanning. Conditional on the same-maturity at-the-money level, a one-standard-deviation increase in attention is associated with about 0.0008 higher wing convexity at both ten and thirty days. The corresponding call-minus-put wing tilt rises by 0.0030 and 0.0018. Both maturity-specific vectors are distinguishable from zero under date and ticker resampling, while the sixty-day vector remains unresolved. These estimates use the complete surface grid, which covers 538 of the 841 names with a thirty-day node and disproportionately excludes smaller, higher-implied-volatility firms. The result therefore sharpens the central distinction on the measured population. Surface response reaches shape margins that add almost no predictive span beyond the at-the-money curve.

The direction of public option discussion sharpens this shape result. I classify comments that explicitly describe long-call or long-put positions and construct a daily call-minus-put author score, while keeping the amount of directional discussion as a separate control. A one-standard-deviation shift toward call language is associated with a 0.0017 larger average call-minus-put wing tilt across the three maturities, with a double-clustered t-statistic of 2.57. The estimate is positive at every maturity and survives a bounded share score, short discussion windows, signed stock-return controls, and the exclusion of familiar meme and mega-cap names. This same-sample exploratory result concerns stated public positions. It links discussion direction to the matching price coordinate without identifying executed trades or price pressure.

The prices clearly respond. Attention raises at-the-money implied volatility at each maturity, with the largest response at the short end of the curve. The measured response is of a similar average scale to a matched physical benchmark, although the ratio estimates are not precise enough to establish exact calibration. At the same time, the spanning share falls as option maturity lengthens for the short-horizon volatility outcome. The short end therefore contains more of the near-term signal, while the back of the curve contains less. This combination is the paper's central fact. The surface moves in the direction implied by the signal, yet the cross section of those movements does not make the signal redundant. The response also has a payoff consequence. Higher attention predicts lower subsequent delta-hedged straddle gains under conventional option-return controls. The result is not a claim that option prices fail to react.

Recent price history is the strongest alternative explanation, and the evidence contains an adverse fact that should be stated directly. With thirty-day at-the-money implied volatility as the only price control, trailing absolute five-day return and attention each have a spanning share of 0.427. In the fixed joint horse race, attention falls from 0.0151 to 0.0076 after the full surface enters, for a conditional share of 0.500. Absolute return remains independently predictive and has a share of 0.399. Both conditional coefficients remain positive under date and ticker resampling. A strict recursive benchmark that already contains HAR dynamics, size, and all twelve surface nodes gains 0.293 percent in cross-sectional out-of-sample fit when attention is added. Attention is related to recent price history, but its conditional forecast content is not reducible to that history.

Public data on option trades bearing the single-leg auction non-ISO (SLAN) marker, the internalized retail identifier of Bryzgalova et al. (2023), narrow this boundary. Attention predicts greater same-day internalized option activity, but the proxy does not resolve direction or detectably account for the surface-conditioned residual. Because SLAN is not customer-origin signed flow and has no maturity dimension, information aggregation, attention-related demand, omitted public news, and quote noise remain admissible. The short sample also cannot stand in for the missing historical object.

The results have two implications. First, a single implied volatility level can materially understate what the option market contains because the maturity curve carries additional forecast content. Researchers who use implied volatility as a sufficient market forecast should therefore distinguish one node from the broader surface. Second, even the full observed surface can respond to a public signal without encompassing it. That distinction matters for empirical tests of information aggregation, because response, predictive redundancy, forecast improvement, and implementable value are separate objects. It also matters for forecasting practice, where a market price may remain an excellent predictor while an external public signal adds a small conditional correction. The forecast gain here is small and does not establish an option return or a trading opportunity.

The closest paper is Eaton et al. (2026). They use brokerage outages to show that retail option trading affects the level and shape of the implied volatility surface. Their design identifies a contemporaneous retail-demand effect on option prices and provides a benchmark for how retail buying and writing appear across contracts. I begin instead with a public attention signal and ask whether the surface makes its relation with future realized volatility redundant. The contribution is a full-surface encompassing result and its maturity allocation, not another estimate of retail

demand's causal price effect. Without historical signed flow, I cannot attribute the residual to the retail-demand channel they identify.

The paper also relates to research on retail option trading and demand-based option prices. Retail investors account for an important share of option activity, concentrate in short-dated contracts, and can affect both option and underlying markets (Bryzgalova et al., 2023; Eaton et al., 2022; de Silva et al., 2026; Bogousslavsky and Muravyev, 2024). Demand pressure can move implied volatility away from a frictionless information benchmark (Gârleanu et al., 2009; Bollen and Whaley, 2004). This work explains why a price response need not be a pure measure of information. I do not add a new measure of signed retail flow, estimate dealer inventories, or identify price pressure. I show instead that the observed surface's response to a public retail-attention signal is not the same as encompassing that signal's future-volatility relation.

A second strand studies attention and social interaction in asset prices. Search activity and other attention measures predict volatility (Vlastakis and Markellos, 2012; Hamid and Heiden, 2015; Dimpfl and Jank, 2016; Audrino et al., 2020), while retail platforms and social networks shape trading and returns (Barber et al., 2022; Welch, 2022; Cookson et al., 2023; Bradley et al., 2024; Pedersen, 2022; Hu et al., 2025). Earlier work also links social-media activity to option prices (Wei et al., 2016; Frino et al., 2022). I do not claim the first relation between attention and volatility or the first response of options to social media. The contribution is to use the attention-volatility relation as a public signal and measure how it is distributed across the full surface, how much remains after the surface, and whether the residual survives a direct recent-price comparison.

A third strand studies volatility forecasts and the information content of options. Realized-volatility dynamics provide demanding forecasting benchmarks (Andersen et al., 2003; Corsi, 2009; Bollerslev et al., 2016), and implied volatility often adds substantial predictive content (Kambouroudis et al., 2021). Classic studies evaluate the relation between implied and realized volatility (Canina and Figlewski, 1993; Christensen and Prabhala, 1998; Jiang and Tian, 2005). Related work studies option-price underreaction and return predictability from volatility signals (Poteshman, 2001; Goyal and Saretto, 2009; Heston et al., 2026). My result is narrower than a structural efficiency or return-predictability claim. It shows that a fixed linear and nonlinear reading of the equity option surface does not make a public attention signal redundant for forecasting the cross section of near-term realized volatility.

The broader stake is how market prices are interpreted. A price can acknowledge a public signal, move by an economically plausible amount, and still leave the signal useful for predicting

which assets experience the outcome. Reading response as full incorporation would miss that distinction, while reading any residual as mispricing would go too far. The equity option surface provides an unusually rich setting in which to measure both objects because it offers prices across names, maturities, and moneyness before realized volatility is observed. For retail attention, the maturity curve contains much more than a conventional volatility level, but the full observed surface still stops short of encompassing the public forecast.

2 Measuring response scale and spanning

A single response coefficient cannot say whether a price vector encompasses a public signal. Response scale asks how much the price moves with the signal on average. Spanning asks whether the signal retains conditional predictive content after the price vector is observed. These objects can differ. A price can respond at a scale close to a physical benchmark while leaving a positive conditional forecasting coefficient. The distinction is about linear redundancy, not necessarily a change in the daily rank ordering of securities.

The central object is the spanning share. Write β_0 for the slope of future realized volatility on attention and β_P for the same slope after a vector of option prices P enters. Then $\phi(P) = 1 - \beta_P/\beta_0$. By Frisch–Waugh–Lovell, $\beta_0 - \beta_P$ is the portion of the original linear attention slope spanned by P. This identity is unconditional on a structural model. In a general covariance system, $\phi(P)$ need not lie between zero and one, and adding prices need not move it monotonically. I therefore report the remaining coefficient, the absorbed slope, and the share, without clipping the ratio or calling it a literal fraction of information.

A vector benchmark supplies a precision comparison under explicit assumptions. After removing the paper's controls and fixed effects, let attention and the realized outcome be $z_i = S_i + \eta_i$ and $RV_i = S_i + u_i$. Collect K observed option nodes in

$$P_i = gS_i + \nu_i, \tag{1}$$

where g contains the nodes' loadings on the attention-linked variance state and ν_i contains option-specific components that do not forecast the realized outcome. If ν_i is orthogonal to (S_i, η_i, u_i) with covariance Σ_ν, the entire surface is equivalent for forecasting S_i to the scalar $g'\Sigma_\nu^{-1}P_i$. Its precision about the state is $\mathcal{I}_P = g'\Sigma_\nu^{-1}g$ (Vanden, 2008). Under this benchmark, $\phi(P)$ rises with $\mathcal{I}_P$ and is invariant to any invertible rotation or rescaling of the price vector. It is still a spanning share, not a cardinal fraction of information, because its level also depends on

noise in the attention proxy. Outside the benchmark, only the nested-projection identity applies.

For maturity m, write the observed quote as $P_{m,i} = a_m Q_{m,i}$, where $Q_{m,i} = G_m S_i + B_{m,i} + \widetilde{\xi}_{m,i}$ and $a_m \neq 0$ rescales the entire observed quote. This pure rescaling divides out of ϕ. An additive error left outside a_m would not. The loading G_m does not divide out relative to background variation. A smaller long-maturity response is therefore not by itself lower spanning, and the full curve can recover content that no single node contains. The realized-horizon by option-maturity matrix is an empirical fact that generally requires multiple variance states. The scalar benchmark does not derive its shape.

The benchmark yields three empirical restrictions.

1. Response scale can be near its physical benchmark while the spanning share remains below one.

2. Under orthogonal node noise, the joint curve spans weakly more than any one node. Outside that benchmark, nesting alone does not guarantee a monotone ratio.

3. If surface shape identifies which attention episodes become extreme moves, tail-sensitive nodes should add to the tail-outcome spanning share.

Attention-related option demand can either raise the loading on S_i or add price noise. The same observed share can therefore reflect incomplete information aggregation or demand-contaminated prices. Signed maturity-specific flow would be needed to separate them. The benchmark organizes the evidence, not its cause. Appendix A gives the vector derivation and the demand extension.

Two qualifications keep the benchmark in its proper register. The observed vector is an interpolated representation of option quotes rather than the latent market state, so correlated quote noise, risk-premium variation, or demand can break the precision ordering. The attention score is also measured with error. A common rescaling of the score cancels from the ratio, but a change in measurement precision generally does not. Under the orthogonal benchmark, noisier attention can even raise the measured share because conditioning on prices increases attenuation of the residual signal. The selected author construction is therefore chosen for a cleaner first stage and stable inference, not because the share is invariant to proxy quality. These restrictions make the framework an organizing map rather than a structural information decomposition.

3 Data and measurement

The sample comprises U.S. stocks matched to Reddit attention between January 2020 and December 2024 at the daily frequency. The active sample contains 93,350 name-days across 938 CRSP-matched tickers. The dense covered universe, which retains quiet dates after a ticker enters the observable risk set, contains 1,046,140 ticker-days across 946 names. The latter is not the full CRSP universe because the source panel uses a lifetime mention screen.[1]

Attention. The source archive covers r/wallstreetbets posts and comments from 2020 through 2025. For each ticker and day, I count distinct comment authors, apply $\log(1 + x)$, and compare the result with the ticker's trailing 30-day history. I lag the score by one calendar day, so attention dated t uses comments only through $t-1$. The lagged posts-mention count, rather than the comment count, continues to determine whether a name-day enters the active sample. This population-fixed design changes the signal without changing the paper's 93,350-row estimation population. I refer to the lagged log-author score as attention throughout. Tables label whether its one-standard-deviation scaling is inherited from the fixed dense covered panel or recomputed for an explicit common-population comparison.

The source corpus contains 86.7 million comments. Ticker extraction admits both plain strings and dollar-prefixed cashtags after removing common-word aliases and requiring a valid security mapping. Distinct authors are platform accounts, not verified individuals, so the score measures participation breadth on Reddit rather than a count of unique investors. A cashtag-only version produces a similar first stage and spanning pattern. The online supplement compares the posts score, comment volume, author breadth, and intensity-given-breadth variants. The selected construction combines a strong response with lower tail leverage while preserving the fixed active population inherited from the posts count.

Sample construction proceeds in layers so that changing a price control does not silently change the attention population. The 93,350-row active panel carries the headline volatility regression. The 81,809-row paired-ATM panel asks what a conventional 30-day level spans. The 61,434-row grid requires all twelve standardized nodes and is the common population for every linear and nonlinear surface comparison. The grid retains 538 names and 65.8 percent of active name-days. Relative to excluded rows, it is larger and less volatile. I therefore treat the surface result as a statement about Reddit-active, option-covered names with complete standardized grids, not about the full stock market. Forecast models and paired spanning comparisons never

[1]The established risk set begins strictly after a ticker's first positive lagged mention. The first active date is excluded from dense active-quiet regressions but retained in the separate all-active confirmation.

mix these populations.

Prices and volatility targets. CRSP daily returns for every attention ticker mapping to a common share produce forward and trailing realized volatility. Forward five-day realized volatility for name i at day t is the annualized standard deviation of daily log returns over $t+1$ to $t+5$. Trailing 21-day volatility and log market capitalization are the baseline controls. The panel mean of forward five-day volatility is 66.7 percent annualized with median market capitalization of \$7.9 billion. This is a large-cap-tilted but high-volatility universe.[2]

Options and earnings. ATM implied volatility is the arithmetic mean of exact OptionMetrics standardized thirty-day call-delta 50 and put-delta minus-50 nodes, with both sides required on the same effective ticker-security-date link. The corrected active regression sample contains 81,809 name-days across 841 names. For the term structure I add the standardized ten- and sixty-day nodes and the 25-delta wings on the same names, so that the same signal can be read against the front, middle, and back of the maturity curve. The ten-day node exists for 538 of the covered names, and the rebuilt thirty-day node reproduces the paired series exactly. A separate strike-level cache is used only for validation. Earnings announcement dates are Compustat quarterly report dates, covering 90.3 percent of active name-days. Appendix C gives the exact timing and matching rules.

Auxiliary sources. A few further sources support single exhibits and enter nowhere else. To check that the forecast is not specific to one community, I rebuild attention from the r/stocks and r/options posting archives using the identical construction. To ask whether attention recruits option demand or supply, I use Cboe open-close data, which reports customer opening volume split into buys and sells, over its 2025-03 to 2025-08 public window. That exhibit relates attention to order flow rather than to realized or implied volatility, and so sits outside the main sample. For the full-surface tail exhibit I draw the per-contract OptionMetrics option chain near the ten-day maturity, from which a model-free tail variance is integrated across all out-of-the-money strikes. It serves the single tail-spanning table and enters nowhere else.

Standard errors are double-clustered by ticker and day in all panel specifications. The inference leans on neither dimension and on no influential unit. A cluster bootstrap over tickers alone returns narrower bands on every headline object, and deleting any single name or any single month leaves the spanning share inside its reported band in every case.[3] Fama–MacBeth slopes carry Newey–West standard errors with ten lags. In the two-part models, attention intensity is

[2]The tail indicator equals one when the absolute cumulative return over the same forward window reaches ten percent. Appendix C gives the timing and units of every variable.

[3]For the full grid, deleting any one ticker produces shares between 0.456 and 0.477. Deleting any one month produces shares between 0.450 and 0.508. Both ranges lie inside the conservative $[0.393, 0.581]$ band.

standardized among active observations in the relevant estimation sample and set to zero on quiet observations. Table A2 reports summary statistics.

The appendix records sample, timing, and inference conventions and reports the 2025 evaluation separately.

Appendix Table A2 reports summary statistics and sample coverage.

4 Attention and future realized volatility

The central fact is cross-sectional. After persistent name differences and common daily shocks are absorbed, attention predicts next week's realized volatility across names on the same day. I estimate

$$RV_{i,t}^{t+1,t+5} \;=\; \beta\, z_{i,t} \;+\; \kappa\, RV_{i,t}^{\text{trail}} \;+\; \delta \log MktCap_{i,t} \;+\; \alpha_i \;+\; \lambda_t \;+\; \varepsilon_{i,t}, \tag{2}$$

where $z_{i,t}$ is attention as of $t-1$, $RV_{i,t}^{\text{trail}}$ is trailing 21-day realized volatility, α_i and λ_t are ticker and day fixed effects, and $\varepsilon_{i,t}$ is the residual. The attention slope β is estimated on the full active sample. Its option-covered counterpart, written β_0 in the share exercise of Section 5, is the same slope on the names that carry an implied-volatility node. Table 1 reports the specification path.

Table 1. Public retail attention and forward realized volatility. The dependent variable is annualized realized volatility over days $t+1$ to $t+5$. Attention is the lagged trailing z-score of log distinct comment authors, standardized on the fixed dense covered panel. Controls are trailing 21-day realized volatility and log market capitalization. Standard errors are double-clustered by ticker and day. The sample contains 93,350 active name-days across 938 names from 2020 to 2024.

Specification	Coef (annualized RV / 1σ)	t (two-way cl.)	N
Entity FE only	0.0380	11.29	93,350
+ Trailing RV + size	0.0329	10.77	93,350
+ Date FE (absorbs market-wide vol)	**0.0320**	**11.48**	93,350

Entity fixed effects alone give 0.0380. With trailing volatility and size, the coefficient is 0.0329. Adding date fixed effects leaves it at 0.0320 ($t = 11.48$). The near invariance is the relevant comparison. The relation is cross-sectional within day rather than an artifact of the day's aggregate volatility environment. On a given day, the names receiving more attention realize more volatility over the following week. Per standard deviation of attention on the fixed dense covered panel, the coefficient is 0.0320 higher annualized realized volatility, or 3.20 annualized-volatility percentage points. On the same 80,218 FF12-classifiable rows, replacing date fixed effects with industry-by-date fixed effects changes the coefficient from 0.0318 to 0.0307

($t = 10.19$). On classifiable full-grid rows, the corresponding spanning share changes from 0.461 to 0.482, and the surface-conditioned attention coefficient remains 0.0134 ($t = 5.16$). Appendix Table A3 reports the detailed comparisons. Table 2 collects the corresponding estimates discussed in the main text.

Table 2. Industry controls and recent price history. The table reports the industry-by-date comparison, the full-grid industry-date conditional estimate, and a joint regression of attention and recent absolute return on common samples.

Comparison	Specification	Coef.	t	ϕ	N
Industry controls	Active, ticker + date	0.0318	10.79	–	80,218
Industry controls	Active, ticker + FF12×date	0.0307	10.19	–	80,218
Industry controls	Full grid, ticker + FF12×date	0.0134	5.16	0.482	55,251
Recent prices	Attention + full grid	0.0076	3.82	0.500	61,429
Recent prices	|5-day return| + full grid	0.0419	3.74	0.399	61,429

Notes: Industry rows use identical FF12-classifiable rows within each sample. The full-grid industry row reports the coefficient after the complete 12-node surface enters and its reduced-form spanning share. Freshness rows enter attention and trailing absolute five-day return jointly with standardized 1-, 5-, and 21-day realized volatility, log market capitalization, and the full 12-node surface. All freshness variables end at $t - 1$. Standard errors are clustered by ticker and original calendar date. Paired date-block and ticker-pairs 95% bands are, respectively: active FF12×date β, [0.0231, 0.0366] / [0.0255, 0.0365]; full-grid FF12×date conditioned β, [0.0064, 0.0208] / [0.0093, 0.0183], and ϕ, [0.393, 0.635] / [0.404, 0.568]; freshness attention β, [0.0035, 0.0116] / [0.0042, 0.0112], and ϕ, [0.381, 0.678] / [0.375, 0.655]; |5-day return| β, [0.0119, 0.0751] / [0.0217, 0.0605], and ϕ, [0.288, 0.667] / [0.309, 0.564].

Breadth and intensity are not interchangeable. An intensity-given-breadth score also forecasts volatility, while the log-author score remains the headline because it combines a strong first stage with stability rather than maximizing the spanning share.

The relation is not confined to entry into discussion. Under the posts-based attention measure, both entry and intensity among already discussed names predict forward volatility, and the intensity estimate remains positive after current ATM implied volatility enters. Independent StockTwits attention also predicts volatility and is only partially spanned on matched option-covered rows. These exercises support breadth across discussion states and platforms. The online supplement reports these results separately because they use posts rather than distinct comment authors.

4.1 Freshness and trading-session timing

The signal's response level is sensitive to freshness, while its spanning share is not. On the paper panel, replacing lagged intraday authors with same-day intraday authors raises the first-stage coefficient from 0.0448 to 0.0678, about one half, but leaves the full-grid share at 0.456

versus 0.457. The paired same-day-minus-lagged share is -0.002 with a band of $[-0.066, 0.034]$. Overnight and all-day comparisons are similarly unresolved. Repeating the paired tests on the separately admitted 210,228-row comment panel yields tighter bands that also span zero. Thus freshness changes how much subsequent volatility the score predicts, not the fraction spanned by the contemporaneous surface.

Intraday shares exceed overnight shares in all four author/mention by lagged/same-day cells and in both panels. Three of the four individual bands include zero, so no cell is singled out. The pooled contrast places all eight signals on the same date-block draw and in one fixed-effect demeaning. It is 0.031 on the paper panel, with a 95 percent band of [0.003, 0.063], and 0.025 on the extended panel, with a band of [0.0003, 0.056]. The surface therefore spans a modestly larger share of attention that arrives while the market is open than while it is shut. The result is a pooled session-timing difference, distinct from the unresolved freshness comparisons.

The online supplement reports the full timing and session comparisons.

The broader attention object also runs ahead of the volatility it forecasts. The online supplement reports a distinct pre-open posts-count measure, fixed before any of the day's trading, that forecasts same-day open-to-close and high-low volatility. This supports timing for public retail attention generally. It is not a direct timing test of the selected score.

The volatility attention forecasts is the name's own, and it is transient. Decomposing daily returns with a trailing sixty-day market beta, attention forecasts the idiosyncratic component at 0.0322, while the systematic coefficient is small and unresolved. The forecast declines along the comparable horizons, from 0.0320 over five days to 0.0137 over 21 days, with a one-day absolute-return analogue of 0.0382, the decay of a transient episode whose volatility footprint fades as opinions settle (Pedersen, 2022). It remains 0.0304 outside observed earnings windows and 0.0167 inside them, so observed earnings timing does not subsume it (de Silva et al., 2026). These are properties of the signal itself, established before any option price enters (Table 3). Appendix Figure A1 plots the four coefficients and their date-block intervals.

Table 3. Anatomy of the volatility relation. Panel A reports the horizon profile, Panel B decomposes forward five-day volatility using a trailing market beta, and Panel C splits observations by whether the forecast window contains an observed earnings announcement. All specifications include ticker and date fixed effects. Standard errors are double-clustered by ticker and day.

Outcome	Coef / 1σ	t	N
Panel A. Forward volatility by horizon:			
1-day absolute return	0.0382	13.02	93,350
5 days (headline)	0.0320	11.48	93,350
10 days	0.0221	10.22	92,930
21 days	0.0137	8.07	92,067
Panel B. Forward 5d RV decomposition:			
Idiosyncratic	0.0322	12.43	92,310
Systematic	-0.0007	-1.03	92,310
Panel C. Observed earnings window:			
No observed earnings in fwd window	0.0304	9.61	74,852
Observed earnings in fwd window	0.0167	2.37	9,479

5 What the option surface encompasses

The forecast is established. The question is how much of it option prices already hold. On stocks with option coverage, the attention slope on forward realized volatility falls from 0.0297 to 0.0195 once thirty-day at-the-money implied volatility enters the regression, so implied volatility already carries a share $\phi = 1 - 0.0195/0.0297 = 0.342$ of the attention relation, with a date-block bootstrap band of $[0.306, 0.380]$. The score here is standardized on the fixed dense covered panel. A positive conditional relation remains after the thirty-day price is observed.

The share is a third for a specific reason, and the reason matters for how to read it. The drop from β_0 to β_{IV} equals $\gamma_{IV}\,\pi_{IV}$, where π_{IV} is how much the implied-volatility level moves with attention and γ_{IV} is how well that level forecasts realized volatility. Here γ_{IV} is 0.749, so the level strongly forecasts realized volatility. What is smaller is π_{IV}. The thirty-day level moves with attention by 0.0136 in volatility units. The price responds to attention, but the response does not make attention redundant.

A share of one-third needs a yardstick. Applying the same measurement to signals already embedded in the option surface puts a stock's trailing realized volatility at 0.85 and its size at 0.91, while retail attention sits at 0.43. The appendix reports this benchmark ladder. Recent absolute return is the adverse exception, and Section 6 treats it directly rather than using the simple ladder to claim that attention is uniquely unspanned.

The missing content belongs partly to maturity. On the roughly five hundred names for which a ten-day node exists, implied volatility responds most at the front and least at the back, and the spanning share is higher at ten days than at thirty days. The price therefore responds in the direction of the near-term signal, while longer maturities contain less of it. The detailed response-scale and price-unit translations remain in the submission appendix.

The maturity pattern raises the encompassing question directly. Table 4 places the 30-day node, the ATM curve, and the full standardized grid on the same 61,434 name-days. Requiring all twelve nodes cuts the panel to 61,434 name-days across 538 names, 65.81 percent of the 93,350-row active panel, and tilts it toward larger and calmer names, with a median log market capitalization of 23.92 against 21.14 among dropped rows and a median trailing realized volatility of 0.52 against 0.73. The 30-day share on this population is 0.345. The 0.342 of Section 5 is the same object measured on the 81,809 option-covered rows. Combining the three ATM maturities raises the share to 0.466. The full grid leaves a coefficient of 0.0140, with a conservative date-and-ticker band of $[0.0083, 0.0199]$ and a null-imposed ticker wild p-value of 0.0001. Relative to 30-day ATM alone, it adds 0.122 to the share, with conservative paired bounds of $[0.083, 0.181]$. Every ticker and month deletion preserves the positive residual. The three ATM levels already produce a share of 0.466. Adding their call-minus-put differences contributes 0.007, a small increment whose separate date and ticker bands exclude zero. Adding the wings then contributes -0.007, with both bands spanning zero. Thus the level term structure, not the wings, accounts for essentially all of the full surface's span. The framework permits small nonmonotonic steps because the share is a covariance ratio rather than a variance decomposition. The observed grid contains more of the relation than a single conventional node, but it does not encompass attention.

Table 4. Joint spanning by the standardized option grid. Panel A adds the 30-day ATM level, the 10/30/60-day ATM curve, ATM call-put asymmetries, and the full 12-node invertible basis in a fixed order. Panel B compares the full grid with 30-day ATM. Every specification uses the same 61,434 name-days, 538 names, controls, and stock and day fixed effects. Analytic standard errors are double-clustered. Date bands use 2,000 paired 10-day-block draws. Ticker bands use 2,000 paired ticker draws. All 9,999 null-imposed ticker wild draws are valid. No ratio is clipped. Separate ticker and date bands and full single-ticker and single-month deletion censuses accompany the analytic inference.

Specification or object	Estimate	Analytic SE	Date band	Ticker band
Panel A: Remaining attention coefficient and spanning share				
Controls only, β_0	0.02632	0.00320		
+ 30-day ATM, β_1	0.01725	0.00241		
Spanning share, ϕ_1	0.3447			
+ 10/30/60-day ATM curve, β_2	0.01405	0.00232		
Spanning share, ϕ_2	0.4661			
+ ATM call-put asymmetry, β_3	0.01386	0.00229		
Spanning share, ϕ_3	0.4734			
Full 12-node grid, β_4	0.01405	0.00227	[0.00825, 0.01991]	[0.01066, 0.01806]
Spanning share, ϕ_4	0.4663		[0.3935, 0.5811]	[0.4035, 0.5377]
Panel B: Full grid relative to 30-day ATM				
Additional absorbed coefficient, $\beta_1 - \beta_4$	0.00320		[0.00235, 0.00399]	[0.00225, 0.00411]
Additional spanning share, $\phi_4 - \phi_1$	0.1216		[0.0830, 0.1808]	[0.0864, 0.1561]

Little added span from the wings does not mean that smile shape is unresponsive. On the same complete grid, attention is positively associated with wing convexity and call-minus-put tilt at ten and thirty days after conditioning on the corresponding at-the-money level. Both two-dimensional vectors are distinguishable from zero under date and ticker resampling. The sixty-day vector remains unresolved. The complete grid covers 538 of the 841 names with a thirty-day node and disproportionately excludes smaller, higher-implied-volatility firms. Appendix Table A10 reports the six estimates and simultaneous bands. The pattern does not identify why the smile changes, but it establishes that response is broader than encompassing.

The direction of option discussion matches this price response. I classify explicit descriptions of long-call and long-put positions and define direction as log call-author breadth minus log put-author breadth. Gross directional breadth enters separately, and all zero-direction rows remain in the sample. A one-standard-deviation shift toward calls predicts a 0.0019 larger ten-day call-minus-put tilt and a 0.0017 larger average tilt across maturities. The estimates are positive at all three maturities under ticker and date fixed effects and standard errors clustered by ticker and day. Bounded share scores, three- and seven-day text windows, signed-return controls, and broader ticker attachment preserve the relation. The separate-wing estimates locate the direct-text result primarily in a rising call wing. Table 5 reports these comparisons. The measure captures disclosed discussion rather than executed orders, so the result links discussion direction

to smile shape but does not establish an option-flow mechanism.

Table 5. Directional option discussion and call-minus-put wing tilt

	10 day	30 day	60 day	Average
Panel A. Direct long-call versus long-put language				
Directional score	0.0019	0.0014	0.0017	0.0017
	(2.97)	(2.20)	(2.11)	(2.57)
Panel B. Alternative constructions, average tilt				
Bounded call-put share				0.0016 (3.00)
Prior and same-day signed returns				0.0018 (2.86)
Three-day discussion window				0.0028 (3.10)
Seven-day discussion window				0.0033 (2.65)
Direct plus root-confirmed tickers				0.0027 (3.14)
Including short-option language				0.0017 (2.72)
Panel C. Separate wing responses				
Call wing relative to ATM	0.0014 (2.77)	0.0012 (2.37)	0.0014 (2.82)	
Put wing relative to ATM	-0.0006 (-1.62)	-0.0003 (-0.76)	-0.0003 (-0.72)	

Notes: The sample contains 61,434 ticker days from 2020 through 2024. The directional score is positive for explicit long-call language and negative for explicit long-put language, and is standardized over the estimation sample. All specifications retain zero-direction days and include ticker and date fixed effects, selected unsigned attention, trailing realized volatility, log market capitalization, same-maturity ATM implied volatility, and gross directional breadth. The signed-return row also controls for the prior one- and five-day returns and the market-day return. Parentheses report t-statistics from standard errors clustered by ticker and date. The evidence concerns stated public positions, not executed order flow.

The linear result does not depend on treating the surface as an additive set of nodes. I consider three nonlinear readings. The first adds all unique squares and interactions of the three ATM maturity levels. The second adds quadratic terms in five principal components of the standardized surface. The third compresses the surface with a depth-two boosted tree whose tuning is confined to 2020–2021. Table 6 compares each reading with the same linear grid on the same observations. The ATM quadratic raises the share from 0.466 to 0.488, and the five-component quadratic raises it to 0.497. For both, the paired increment bands exclude zero by date and by ticker. The tree adds little and leaves the share at 0.469. More important, every benchmark leaves a positive attention coefficient. It is 0.0140 under the linear grid, 0.0135 under the ATM quadratic, 0.0132 under the five-component quadratic, and 0.0140 under the tree, with each analytic t-statistic above six. These tests rule out the particular concern that the linear grid misses simple curvature or low-order interactions. They do not establish non-encompassing against every measurable function of a latent surface.

Table 6. Linear and nonlinear surface encompassing. The specifications use the full linear 12-node grid, quadratic terms in the three ATM levels, quadratic terms in five surface principal components, or a depth-two boosted-tree index tuned on 2020–2021. Every row uses the same 61,434 name-days, controls, and fixed effects. Date and ticker columns report separate paired-bootstrap bands.

Benchmark	β	t	Spanning share	Date / ticker bands
linear grid	0.0140	6.20	0.466	[0.397,0.574] / [0.401,0.534]
quadratic ATM	0.0135	6.17	0.488	[0.423,0.595] / [0.426,0.559]
quadratic PCs	0.0132	6.04	0.497	[0.431,0.611] / [0.435,0.570]
shallow tree	0.0140	6.21	0.469	[0.401,0.574] / [0.405,0.539]

Figure 1 collects the nested surface readings on the fixed complete grid. The attention coefficient falls as the benchmark expands from the thirty-day level to the maturity curve and then to the full surface, but it remains positive under every fixed linear and nonlinear reading. This visual separation between response and encompassing is the central empirical result.

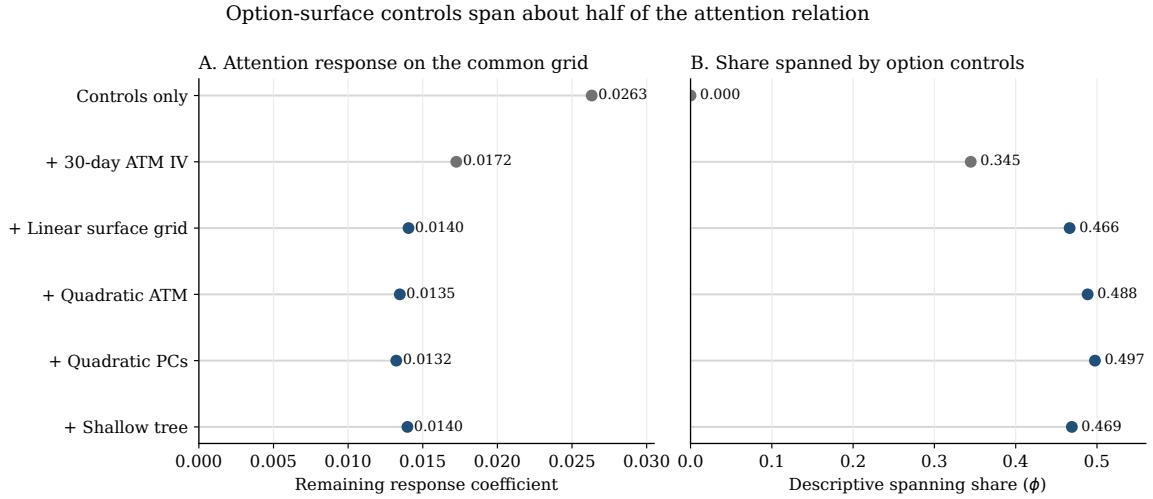
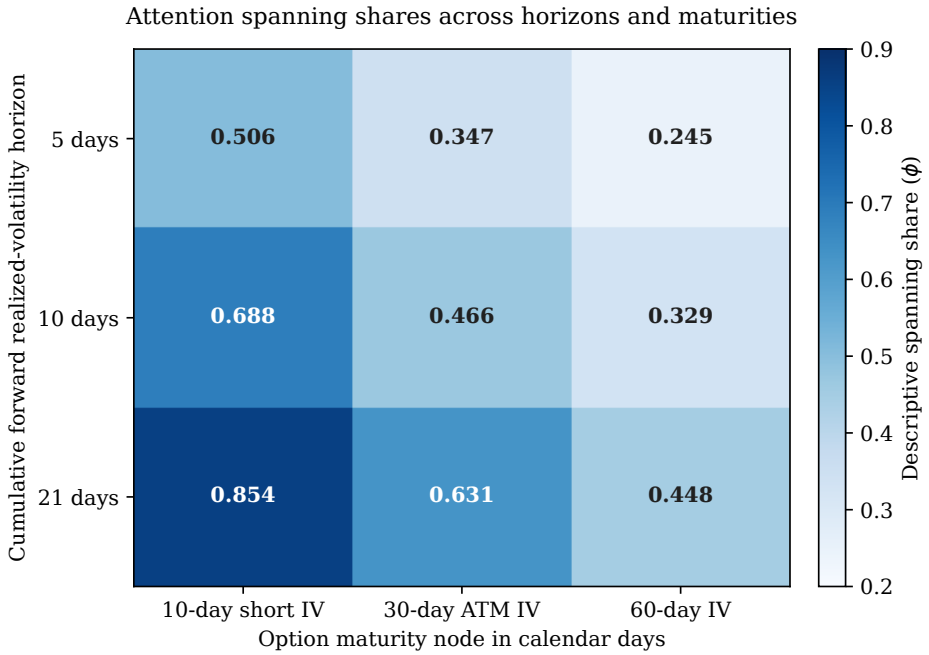


Figure 1. Attention coefficients under increasingly rich option-surface benchmarks. Every point uses the same 61,434 name-days on the complete 12-node grid. The leftmost coefficient is therefore 0.0263, the common-grid baseline, rather than the 0.0320 headline estimate on the broader 93,350-row active sample. Tables 4 and 6 report the corresponding inference.

The realized-horizon by option-maturity matrix gives the sharpest view of where the span sits. The share ranges from 0.245 when sixty-day implied volatility is asked to span five-day realized volatility to 0.854 when ten-day implied volatility is asked to span twenty-one-day realized volatility. Thus the short option node contains more of the attention signal, and more of a longer-horizon volatility outcome is already reflected in the price. The nine cells are not globally monotone toward a matched-horizon diagonal. Under the same procedure, trailing volatility is

almost fully spanned in every cell.



61,434 name-days and 538 names. Rows are trading-day outcome windows; columns are calendar-day IV nodes.

Figure 2. Spanning shares by realized-volatility horizon and option maturity. Each cell reports $1-\beta_{IV}/\beta_0$ on the common 61,434-row surface grid. Darker cells indicate that the option node makes more of the attention forecast redundant. Appendix Table A5 reports the underlying estimates.

Appendix Table A5 reports the complete realized-horizon by option-maturity matrix.

5.1 Response scale, tails, and regime boundaries

A two-thirds shortfall in the thirty-day price invites a reading it does not deserve. Part of the gap is horizon. A thirty-day option should not move much for a burst of volatility that lasts a week. The right comparison holds the horizon fixed. Under a matched physical benchmark, in which a maturity's implied variance equals the expected realized variance over its life, the implied-volatility response that would fully price the signal is computable from how attention forecasts realized volatility at the matching horizon. Table A6 sets the measured response beside that matched physical benchmark.

Appendix Table A6 reports the response-scale and matched-horizon comparisons.

Measured in volatility units and matched to the horizon, the response ratios run from 0.80 at the short node to 1.13 at the long node. The point estimates are consistent with a similar average response, but their intervals also admit meaningful underreaction and overreaction. Variance-

space ratios run from 0.63 to 0.90, with broad and overlapping intervals. Their difference from the volatility-space ratios is descriptive rather than a structural tail test. Even at matched horizons, point spanning shares run from 0.52 to 0.62 and remain below one. The remaining slope is not eliminated by matching horizons and is identified in the within-day, cross-name dimension.[4]

Figure 3 states the two halves of the result together. The measured implied-volatility response tracks its matched-physical-benchmark point estimate across maturities, while the spanning share stays far below the trailing-volatility placebo. The response scale and the cross-name span are distinct empirical objects.

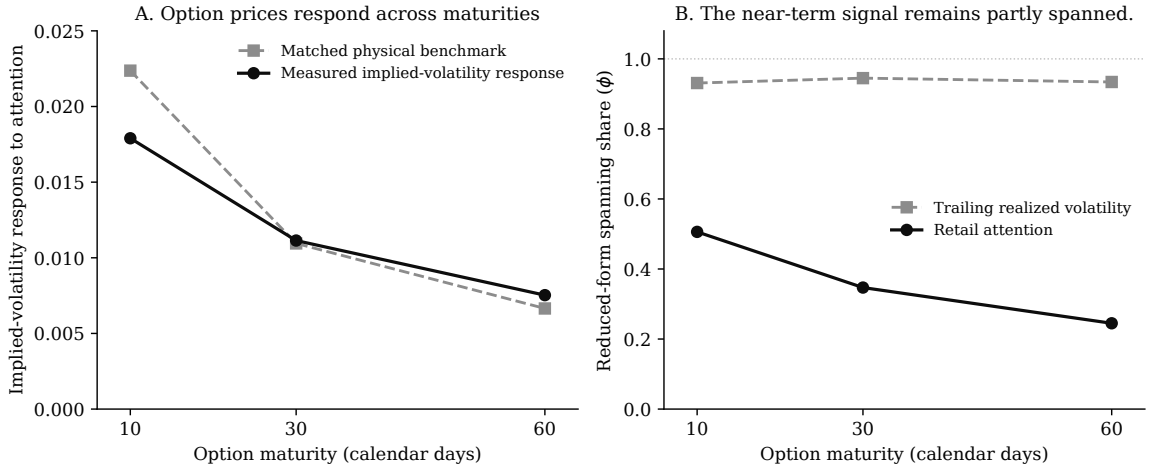


Figure 3. Response scale and cross-name spanning. Panel A plots the measured implied-volatility response to a one standard deviation increase in attention against its matched-physical-benchmark value, across the ten-, thirty-, and sixty-day maturities. The point estimates converge at the back of the curve, although the ratio estimates are imprecise. Panel B plots the spanning share by maturity, which sits far below the same share for a stock's trailing volatility, a signal the market has long known that lies near one throughout. Panel A is in annualized volatility units per one standard deviation of attention, with the measured response and the matched-physical-benchmark value plotted as the two labeled series. Panel B is a unit-free share, with attention and trailing volatility as the two labeled series. Values are the vol-space responses of Table A6 and the five-day row of Table A5.

A separate state split asks where the surface absorbs more. For prior-week moves of average magnitude, adding the full option surface reduces the predicted future-volatility response to a one-standard-deviation attention increase by 0.35 annualized volatility points more after negative weeks than after positive weeks. The date- and ticker-resampled bands both exclude zero, and

[4]The shortfall is not an artifact of scheduled news. Splitting the covered panel on whether an observed earnings announcement falls in the forward five-day window, the share is essentially the same inside and outside, 0.30 against 0.32, so the spanning shortfall is broad-based rather than concentrated on unscheduled episodes. It is therefore not the scheduled earnings event risk the option surface is known to price through pre-announcement concavity (Alexiou et al., 2025).

every ticker and month deletion retains the sign (Appendix Table A11, Panel A).

The appendix asks whether the remaining span sits in features omitted by the at-the-money curve. The observed wings add little in the fixed standardized grid and only a small increment in the broader tail sample. Call-put differences are jointly detectable but remain secondary to the common maturity-level response. These decompositions support the main allocation result without identifying whether the remaining relation reflects information or demand.

6 What remains after the surface

A demanding alternative is that attention merely repackages recent price history. The adverse fact is that trailing absolute five-day return and attention each have a standalone spanning share of 0.427 when thirty-day at-the-money implied volatility is the only price control. The joint test therefore places both signals in the same fixed basis of recent returns, realized-volatility histories, and size, with every price-history window ending at $t-1$. Attention falls from 0.0151 to 0.0076 after the full twelve-node grid enters, for a conditional spanning share of 0.500. Absolute return remains separately predictive and has a share of 0.399. Both conditional coefficients remain positive under date and ticker resampling. Table 2 reports this horse race together with the industry-by-date comparison.

The result does not make attention uniquely unspanned. It shows instead that the attention relation is not reducible to recent price history. The separate forecast test asks whether this conditional relation survives a stricter recursive setting rather than only an in-sample projection.

The stricter forecast check conditions on the full grid rather than one implied-volatility level. Table 7 reports 36 monthly expanding fits from 2022 through 2024. Attention raises cross-sectional out-of-sample R^2 by 0.293 percent, and the loss improvement is positive in every year. The daily rank-IC increment is 0.0009. The Clark–West improvement is also positive in every residual-spread tercile, so the gain is not confined to illiquid names and is difficult to attribute solely to stale quotes. A distinct-author measure therefore identifies forecasting value in the unspanned residual. The gain is statistically unambiguous but economically small, and it supports rather than replaces the paper's spanning result.

The same forecast protocol applied to the nonlinear benchmarks leaves the result intact. Across the linear grid, ATM quadratic, principal-component quadratic, and shallow-tree specifications, attention raises cross-sectional out-of-sample R^2 by 0.293 to 0.312 percent, with rank increments from 0.0008 to 0.0014. Table 7 therefore evaluates the residual against the same fixed linear, quadratic, principal-component, and tree surface readings used in sample. The forecast evidence

shows that the residual contains incremental predictive content under these representations. It does not reveal whether the content reflects a noisy public state, omitted public news, attention-related demand, or some mixture of them.

Table 7. Recursive forecasts after linear and nonlinear surface benchmarks. Models are estimated at 36 monthly origins over 2022–2024. Benchmark and attention-augmented models use identical observations, and all training outcomes end before the forecast month. Evaluation removes date means from outcomes and forecasts and gives dates equal weight.

Benchmark	CS OOS R^2 (%)	Loss t	CW t	Δ rank IC	Rank t
Linear grid	0.293	4.66	7.41	0.0009	3.50
Quadratic ATM	0.312	4.73	7.66	0.0009	3.11
Quadratic PCs	0.301	4.49	7.49	0.0008	3.13
Shallow tree	0.303	3.86	6.24	0.0014	4.35

The residual also has an option-payoff counterpart. I follow a same-strike, same-expiration at-the-money straddle for five trading days, rebalance the stock hedge daily, and scale its gain by entry vega. With conventional option-return controls, ticker and date fixed effects, and 1/99 winsorization, the attention coefficient is -0.001899 on 57,095 name-days, with a double-clustered t-statistic of -3.35. Higher attention is therefore followed by a lower delta-hedged gain to the option buyer, consistent with options being priced more richly relative to their ensuing hedged payoffs. The raw outcome, Fama–MacBeth estimates, residual-attention portfolios, quote-quality screens, and separate call and put positions preserve the negative relation. Adding the three ATM maturity levels attenuates the coefficient to -0.001208, and the full surface leaves -0.001172. This is a conditional pricing relation rather than evidence that Reddit users made the trades or that an executable arbitrage remains.

Table 8. Retail attention and subsequent delta-hedged option gains. The position combines same-strike, same-expiration at-the-money calls and puts, rebalances the stock hedge daily for five trading days, and scales the midpoint-priced gain by entry vega.

Specification	Estimate	t-statistic	Observations
Panel A. Midpoint-priced delta-hedged straddle gain			
Realized volatility and size	-0.003853	-5.88	57,095
Add 30-day ATM implied volatility	-0.002154	-3.75	57,095
Literature controls	-0.001899	-3.35	57,095
ATM implied-volatility term structure	-0.001208	-2.08	57,095
Full implied-volatility surface	-0.001172	-2.02	57,095
Panel B. Conventional sensitivities with literature controls			
Raw outcome	-0.002018	-2.99	57,095
Full quoted-spread charge	-0.002603	-3.99	57,095
Fama–MacBeth	-0.001456	-2.25	57,095
Residual-attention quintile H–L	-0.007059	-2.44	57,095
Quote-quality proxy	-0.001944	-3.29	47,839
Strict ATM and quote-quality proxy	-0.001013	-2.08	39,576
Panel C. Separate option legs			
Delta-hedged call gain	-0.000412	-3.73	57,095
Delta-hedged put gain	-0.000235	-2.96	57,095

Notes: The sample contains attention-active name-days from January 2020 through December 2024. The selected score dated t uses comment authors through calendar day t-1. The position combines same-strike, same-expiration call and put contracts and rebalances the stock hedge daily over five trading days. Except for the raw row, outcomes are winsorized at the 1st and 99th percentiles. Panel A scales the midpoint-priced gain by entry vega. Literature controls include 30-day ATM implied volatility, the historical-minus-implied volatility gap, log market capitalization, relative quoted spread, log dollar option volume, and an indicator for two-sided quoting. Regressions include ticker and date fixed effects, with standard errors clustered by ticker and date. The Fama–MacBeth row uses Newey–West standard errors with 10 lags. The H–L row is the mean daily difference between the highest and lowest residual-attention quintiles, also with Newey–West standard errors. The full-spread row charges the complete quoted spread at entry and exit. The quote-quality proxy requires positive bids, positive quoted spreads, a minimum \$0.125 midpoint, and fresh entry and exit last-trade dates. The strict ATM band is 0.975 to 1.025. Contract-level volume and risk-free financing are unavailable in the saved pull. Panel C scales each leg by its securities-value denominator. Coefficients describe conditional associations, not identified trade direction or causal effects.

The incremental forecast content above is a week-one object, covering $t + 1$ through $t + 5$, and does not extend to week two, $t + 6$ through $t + 10$, under the appendix design. At that later horizon, the premise attention coefficient is already near zero and negative before the later surface enters, and the estimated difference has the opposite sign from absorption (Appendix Table A12, Panel B).

A signed decomposition of later variation gives a different limit. Attention predicts capped variation on both positive-return and negative-return days after full-surface conditioning, while their difference is unresolved and the raw comparison is tail sensitive (Appendix Table A11, Panel B).

On the public SLAN sample, a one-standard-deviation increase in attention predicts a 0.138-

standard-deviation increase in same-day internalized option activity. Neither calls nor puts show a coherent same-sign directional response. Adding the contract-based signed-level proxy does not detectably attenuate the surface-conditioned attention coefficient at the 0.003 attenuation scale detectable in this sample. SLAN is a public internalized-flow proxy rather than customer-origin signed flow, and its sample runs from January 2020 through June 2021. The public proxy therefore cannot resolve direction, and customer-origin signed flow by maturity remains the binding missing measurement (Appendix Table A12, Panel A, and Appendix Figure A2). Incomplete aggregation, attention-related demand, omitted public news, and quote noise therefore remain admissible. The evidence establishes a small forecast improvement beyond the observed surface, not a mechanism, option return, or implementable trading strategy.

The selected measure is also evaluated on the 2025 calendar window. The response is 0.0315, close to 0.0320 in 2020–2024, and the ten-day less thirty-day spanning difference is 0.156 in both periods. Both forecast comparisons remain positive at smaller magnitudes. The thirty-day share, timing contrast, and rank increment are imprecise. Appendix Table A14 reports all seven quantities.

7 Conclusion

Public retail attention predicts which stocks become more volatile over the following week. The equity option surface responds to that signal but does not make it redundant. A conventional thirty-day level captures only part of the relation, and the full observed surface spans about 0.47 while leaving a positive conditional attention slope. Fixed nonlinear readings yield the same conclusion. Response and predictive encompassing are different empirical objects.

Nearly all additional span comes from the at-the-money maturity curve. Smile shape also responds, and call-versus-put discussion predicts the matching wing tilt, but these shape margins add little span. Attention remains separately predictive after recent price history, modestly improves recursive forecasts, and predicts lower subsequent delta-hedged option-buyer gains under conventional controls. Together, the results make the residual economically relevant without turning it into a large forecasting or trading claim.

The design does not identify why the residual remains. Customer-origin signed flow by maturity is unavailable, leaving incomplete aggregation, attention-related demand, omitted public news, and quote noise admissible. The contribution is that an option surface can visibly adjust to a public signal and still stop short of encompassing what it predicts.

A The vector spanning benchmark

This appendix derives the spanning share in the vector benchmark of Section 2. Let S, attention noise η, outcome noise u, and price noise ν be mutually independent and mean zero. Attention is $z = S + \eta$, the outcome is $RV = S + u$, and the K observed price nodes satisfy $P = gS + \nu$, with $\mathrm{Var}(\nu) = \Sigma_\nu$ positive definite.

The univariate attention slope is

$$\beta_0 \;=\; \frac{\mathrm{Cov}(RV, z)}{\mathrm{Var}(z)} \;=\; \frac{\sigma_S^2}{\sigma_S^2 + \sigma_\eta^2}. \tag{3}$$

The best linear surface signal is proportional to $Q = g'\Sigma_\nu^{-1}P$. After normalization, Q is equivalent to observing $S + w$, where

$$\sigma_w^2 = \left(g'\Sigma_\nu^{-1}g\right)^{-1}. \tag{4}$$

Conditioning on P is therefore equivalent to conditioning on $S + w$. Solving the normal equations gives

$$\beta_P \;=\; \frac{\sigma_S^2\,\sigma_w^2}{\sigma_S^2\,\sigma_w^2 + \sigma_\eta^2\,\sigma_S^2 + \sigma_\eta^2\,\sigma_w^2}, \tag{5}$$

and the share follows,

$$\phi(P) \;=\; 1 - \frac{\beta_P}{\beta_0} \;=\; \frac{\sigma_S^2\,\sigma_\eta^2}{\sigma_S^2\,\sigma_w^2 + \sigma_S^2\,\sigma_\eta^2 + \sigma_\eta^2\,\sigma_w^2}. \tag{6}$$

The surface precision is $\mathcal{I}_P = g'\Sigma_\nu^{-1}g = \sigma_w^{-2}$. Thus $\phi(P)$ rises with precision. Adding an informative node weakly raises precision under the orthogonal-noise benchmark, and an invertible transformation of P leaves both its span and $\phi(P)$ unchanged. A common response scale applied to the entire observed vector also leaves the share unchanged. These precision-monotonicity statements require the benchmark's covariance restrictions. The regression identity defining the spanning share does not. Even here, ϕ is not a cardinal information fraction because it depends on σ_η^2 as well as surface precision.

Maturity coordinates. For each maturity, write $P_m = a_m Q_m$ with $Q_m = G_m S + B_m + \widetilde{\xi}_m$ and $a_m \neq 0$. Because a_m rescales the entire observed quote, it is an invertible transformation and leaves the span unchanged. This conclusion would be false for $P_m = a_m(G_m S + B_m) + \xi_m$ with an additive ξ_m that does not co-scale. The loading G_m matters relative to the covariance of the background components. Stacking maturities can therefore span more than any single maturity, but the scalar state supplies no restriction on the full matrix of realized horizons and

option maturities. The matrix in the paper is evidence for a richer variance-state model, not a prediction derived here.

Attention-related demand. A reduced-form extension writes $P = gS + \Lambda D + \nu$, where $D = bS + e$ is option demand related to attention. The predictable component bS changes the price loading on the state, while e adds price variation. Depending on their covariances, demand can raise or lower the measured spanning share. Call-put responses are consequently descriptive coordinates, not a structural pressure instrument. Signed maturity-specific flow is needed to distinguish information aggregation from demand contamination.

Noise in the signal raises the measured share

A natural reading of a low share is that the signal is simply noisy. The benchmark says the opposite. Write the share as $\phi = \sigma_S^2\sigma_\eta^2/(\sigma_S^2\sigma_w^2 + \sigma_S^2\sigma_\eta^2 + \sigma_\eta^2\sigma_w^2)$. As a function of σ_η^2 this is $cx/(b + ax)$ with $c = \sigma_S^2$, $a = \sigma_S^2 + \sigma_w^2$ and $b = \sigma_S^2\sigma_w^2$, whose derivative $cb/(b + ax)^2$ is positive, so the measured share rises with the signal's own noise. As $\sigma_w \to 0$ the share tends to one at any noise level, and as $\sigma_\eta \to 0$ it tends to zero for any imperfect price. The price's error about the state, σ_w, is what lowers the share. The signal's own noise does not.

The same ordering can hold beyond the parametric benchmark under stable signs. Let the observed signal be $z = z^* + e$ with classical e. In any least-squares regression the probability limit of z's coefficient is its true value times $\lambda = \mathrm{Var}(z_\perp^*)/(\mathrm{Var}(z_\perp^*) + \sigma_e^2)$, where $z_\perp^*$ is the true signal residualized on the remaining regressors. Adding implied volatility, a variable correlated with z^*, weakly reduces $\mathrm{Var}(z_\perp^*)$ and therefore λ, so the controlled slope is attenuated by more than the uncontrolled one, their ratio falls, and the measured share weakly exceeds the true one, provided the uncontrolled and controlled true attention coefficients are both positive and the added projection does not reverse their signs.

Under those conditions, classical signal noise makes attention appear better spanned, not less spanned. The paired selected-measure gaps can then be read as conservative. The bound does not apply when either coefficient changes sign, when measurement error is correlated with the volatility state, or when the added price vector changes the underlying estimand.

A rejected transient-persistent calibration

Table A1 compares the seven selected-measure moments with the specified two-component benchmark. The fitted transient and persistent half-lives are 2.91 and 44.06 trading days, but the model understates the one- and five-day realized-volatility responses and overstates the

short-maturity spanning share. The centered date-block-bootstrap specification p-value is 0.001. The common orthogonal two-state benchmark is therefore rejected and is not used to interpret the spanning share structurally.

Table A1. Transient-persistent calibration. Parameter bands refit the specified minimum-distance model to the paired date-block moment draws. The overidentification p-value uses the centered block bootstrap.

Object	Estimate	95% bootstrap band	Economic unit
Transient loading	0.0195	[0.0156,0.0246]	annualized volatility
Persistent loading	0.0109	[0.0054,0.0139]	annualized volatility
Transient persistence	0.788	[0.644,0.877]	daily AR coefficient
Persistent persistence	0.984	[0.972,1.000]	daily AR coefficient
Overidentification p	0.001		centered block bootstrap

B Supporting empirical evidence

This appendix reports the sample description, robustness comparisons, and the source tables needed to interpret the principal figures. The online supplement reports additional construction, timing, portability, subsample, and estimator checks.

Table A2. Summary statistics. The main sample contains 93,350 active name-days across 938 names from 2020 through 2024. Realized and implied volatility are annualized decimals, returns are decimals, and market capitalization is in billions of dollars. Implied-volatility variables use the OptionMetrics-covered subset.

Variable	N	Mean	SD	P25	Median	P75
Retail attention (lagged z-score)	93,350	0.422	0.869	-0.129	0.320	0.864
Ticker mentions	93,350	6.0	147.9	1.0	1.0	3.0
Forward RV, 5 days	93,350	0.667	0.685	0.275	0.488	0.824
Trailing RV, 21 days	93,350	0.761	0.702	0.345	0.581	0.939
Absolute forward return, 5 days	93,350	0.084	0.198	0.020	0.049	0.103
Tail move indicator	93,350	0.258	0.437	0.000	0.000	1.000
Market capitalization ($ billions)	93,350	131.9	393.5	1.5	7.9	69.5
Positioning intensity	8,114	1.490	1.213	0.527	1.195	2.301
ATM implied volatility	81,809	0.741	0.492	0.366	0.617	0.990
Forward ATM IV change, 5 days	81,245	-0.0141	0.1596	-0.0520	-0.0055	0.0299

Table A3. Industry controls and recent price history. Panel A compares date and FF12-by-date fixed effects on identical classifiable rows. Panel B includes attention and trailing absolute five-day return jointly with the fixed recent-price basis, then adds the complete 12-node option surface.

Panel A. Industry-by-date fixed effects					
Sample	Fixed effects	Attention coefficient		ϕ	N
		Unconditioned	Conditioned		
Active	Ticker + date	0.0318 (10.79)	–	–	80,218
Active	Ticker + FF12×date	0.0307 (10.19)	–	–	80,218
30-day ATM	Ticker + date	0.0301 (10.04)	0.0198 (8.55)	0.343	74,408
30-day ATM	Ticker + FF12×date	0.0293 (9.50)	0.0188 (7.89)	0.359	74,408
Full grid	Ticker + date	0.0268 (8.07)	0.0145 (6.14)	0.461	55,251
Full grid	Ticker + FF12×date	0.0260 (7.36)	0.0134 (5.16)	0.482	55,251

Panel B. Joint freshness horse race					
Model	Conditioning set	Attention	|5-day return|	ϕ_A / $\phi_{\|r\|}$	N
Recent-price basis	Without option surface	0.0151 (7.28)	0.0698 (5.33)	–	61,429
Full-grid model	Add complete option surface	0.0076 (3.82)	0.0419 (3.74)	0.500 / 0.399	61,429

Paired bootstrap 95% bands for decisive rows			
Object	Date block	Ticker pairs	Valid draws (D/T)
Active, industry-date attention coefficient	[0.0231, 0.0366]	[0.0255, 0.0365]	2,000 / 2,000
Full grid, conditioned attention coefficient	[0.0064, 0.0208]	[0.0093, 0.0183]	2,000 / 2,000
Full grid, industry-date ϕ	[0.393, 0.635]	[0.404, 0.568]	2,000 / 2,000
Attention, full-grid model	[0.0035, 0.0116]	[0.0042, 0.0112]	2,000 / 2,000
Attention, ϕ_A	[0.381, 0.678]	[0.375, 0.655]	2,000 / 2,000
|5-day return|, full-grid model	[0.0119, 0.0751]	[0.0217, 0.0605]	2,000 / 2,000
|5-day return|, $\phi_{\|r\|}$	[0.288, 0.667]	[0.309, 0.564]	2,000 / 2,000

Notes: Panel A uses identical FF12-classifiable rows within each sample. The unconditioned specifications include trailing 21-day realized volatility and log market capitalization. The conditioned specifications add the 30-day at-the-money level or the complete 12-node option surface. Panel B jointly includes attention and standardized 1-, 5-, and 21-day realized volatility and absolute five-day return, with log market capitalization. The second specification adds the complete 12-node surface. All freshness windows end at $t-1$. Parentheses report Cameron–Gelbach–Miller t-statistics clustered by ticker and original calendar date. The final block reports paired date-block and ticker-pairs 95% bands. D/T denotes date/ticker valid draws. For each reported ϕ row, all 2,000 date draws and 2,000 ticker draws have positive denominators. The reduced-form spanning share is $\phi = 1 - \beta_{\text{conditioned}}/\beta_{\text{unconditioned}}$.

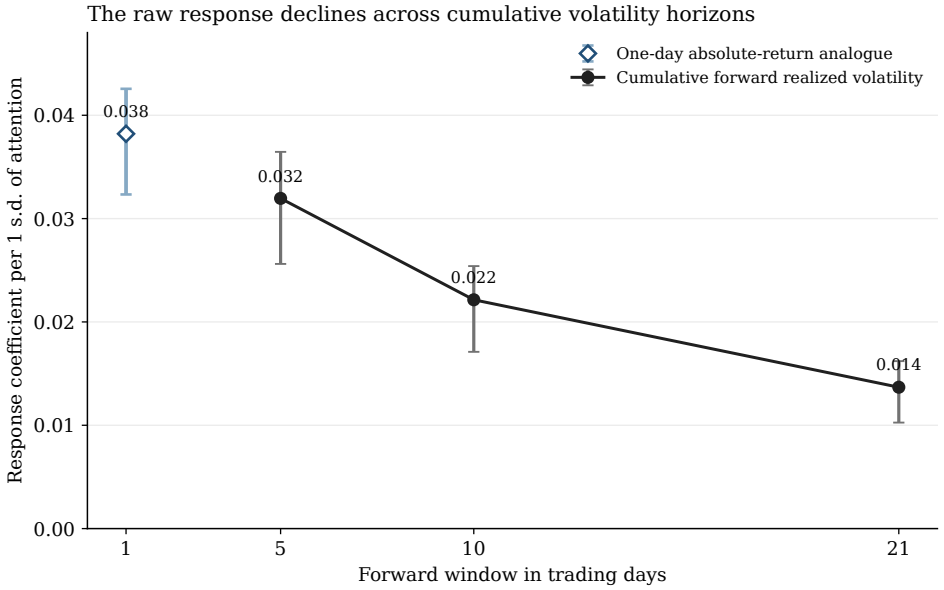


Figure A1. Raw horizon profile. The one-day point uses an absolute-return analogue and is distinct from the cumulative forward realized-volatility outcomes measured over five, ten, and twenty-one trading days. Points are attention coefficients under the paper's controls and fixed effects; bars are paired date-block intervals.

Table A4. The share of a signal already in the price, for retail attention and benchmark signals. The share is $1-\beta_{IV}/\beta_0$ from a forecast of forward five-day realized volatility, with 30-day at-the-money implied volatility as the only control. Bands and paired differences use common ten-day date-block draws. The sample contains 81,809 active option-covered name-days across 841 names from 2020 to 2024.

Signal	Share in price	95% band	Difference from attention	
Retail attention	0.427	[0.374, 0.496]	—	
Trailing |5-day return|	0.427	[0.353, 0.533]	0.001	[−0.067, 0.104]
Trailing realized volatility	0.846	[0.714, 1.015]	0.419*	[0.271, 0.594]
Size (log market cap)	0.911	[0.682, 1.286]	0.484*	[0.238, 0.863]

Table A5. The share of the volatility signal already in the price, by realized-volatility horizon and option maturity. Each entry is $1-\beta_{IV}/\beta_0$ under the paper's controls, within stock and within day. The sample contains 61,434 name-days across 538 names.

Realized-vol horizon	10-day option	30-day option	60-day option
5-day	0.506	0.347	0.245
10-day	0.688	0.466	0.329
21-day	0.854	0.631	0.448

Table A6. Decomposing the 30-day shortfall. The magnitude and tail ratios compare the measured implied-volatility response with its matched physical benchmark. The matched-horizon share is the portion of the corresponding realized-volatility slope spanned by the price. Brackets are 95 percent paired ten-day date-block intervals. The sample contains 61,434 name-days across 538 names.

Option maturity	Magnitude ratio (vol)	Tail ratio (variance)	Matched-horizon share
10-day	0.80 [0.64, 1.05]	0.63 [0.39, 1.25]	0.61 [0.45, 1.06]
30-day	1.02 [0.78, 1.40]	0.84 [0.57, 1.39]	0.62 [0.43, 0.98]
60-day	1.13 [0.74, 1.94]	0.90 [0.57, 1.71]	0.52 [0.33, 0.90]

Table A7. The full out-of-the-money surface and the tail. On a common ten-day-node sample, ϕ^{tail} is the share of a realized five-day tail already in the price under nested conditioning sets that add observed wing measures and model-free tail variance to the at-the-money level. The sample contains 71,760 name-days across 822 names.

Conditioning set	ϕ^{tail}
ATM level only	0.608
ATM + 25-delta shape	0.629
ATM + shape + model-free tail variance	0.632
Addition beyond ATM	+0.024 [+0.014, +0.039]

Table A8. Common and asymmetric at-the-money responses. At maturity m, L_m is the average of the call-delta 50 and put-delta minus-50 implied-volatility nodes and D_m is their difference. $\Delta\phi_D$ is the change in the spanning share when asymmetry enters. All specifications use the same 61,434 name-days.

Maturity	L_m response	D_m response	Wild p_D	$\Delta\phi_D$	Date band	Ticker band
10 days	0.01789	−0.00029	0.666	0.0075	[0.0000, 0.0181]	[−0.0010, 0.0171]
30 days	0.01133	−0.00087	0.207	0.0009	[−0.0014, 0.0061]	[−0.0033, 0.0065]
60 days	0.00750	−0.00095	0.158	0.0009	[−0.0014, 0.0061]	[−0.0030, 0.0068]
Joint ATM curve				0.0074	[0.0020, 0.0156]	[0.0003, 0.0145]

Table A9. Tail and call-put contributions beyond ATM levels. Panel A reports spanning shares for a five-day realized-tail indicator on the broad full-surface sample. Panel B decomposes the historical ATM response into the paired call-put common component and the call-minus-put difference on the fixed common grid. Tail increment bands are 95 percent. The joint call-put increment reports separate date and ticker bands.

	ATM object	Expanded object	Increment	Paired 95% band
Panel A. Full-surface tail spanning				
ATM level	0.608			
ATM plus observed wings		0.629	0.021	
ATM plus full surface		0.632	0.024	[0.014, 0.039]
Panel B. Common and asymmetric ATM responses				
10-day maturity	0.01789	−0.00029	0.0075	individual band includes zero
30-day maturity	0.01133	−0.00087	0.0009	individual band includes zero
60-day maturity	0.00750	−0.00095	0.0009	individual band includes zero
Joint ATM curve			0.0074	date [0.0020, 0.0156]; ticker [0.0003, 0.0145]

Table A10. Attention and smile shape on the complete surface grid. Wing convexity is the average of the call and put wings relative to at-the-money volatility. Call-minus-put tilt is their difference. Each estimate conditions on the same-maturity at-the-money level, the paper's controls, and stock and day fixed effects. Bands control the six reported shape coefficients jointly within each resampling dimension. The sample contains 61,434 name-days across 538 names.

Maturity	Estimate	Date-family 95%	Ticker-family 95%	Inference
Panel A. Wing convexity relative to ATM				
10-day	0.000755	[0.000198, 0.001312]	[0.000067, 0.001443]	Both bands exclude zero
30-day	0.000781	[0.000180, 0.001382]	[0.000184, 0.001379]	Both bands exclude zero
60-day	0.000291	[-0.000183, 0.000765]	[-0.000221, 0.000803]	Bands include zero
Panel B. Call-minus-put wing tilt				
10-day	0.002952	[0.001265, 0.004638]	[0.001129, 0.004774]	Both bands exclude zero
30-day	0.001766	[0.000191, 0.003340]	[0.000174, 0.003357]	Both bands exclude zero
60-day	0.001205	[-0.000437, 0.002848]	[-0.000312, 0.002723]	Bands include zero

Notes: Coefficients are selected-attention slopes in raw implied-volatility units, conditional on same-maturity ATM implied volatility, the paper's controls, ticker effects, and date effects. Date-family and ticker-family bands are simultaneous 95% bands across the six coefficients and retain 2,000 draws each. The common sample contains 61,434 name-days, 538 names, and 1,229 dates from January 2020 through December 2024. Requiring the complete cross-maturity grid excludes 303 of the 841 names with 30-day ATM support. The 60-day margins are unresolved.

Table A11. State-dependent absorption and signed realized variation. Panel A compares the amount of the attention coefficient absorbed by the full option surface after run-ups and drawdowns. Panel B splits five-day realized quadratic variation between nonnegative- and negative-return days under capped and raw constructions.

Outcome	Estimate	Date 95%	Ticker 95%	Inference
Panel A. State-dependent amount absorbed by the option surface				
Run-up amount absorbed	0.003517	—	—	Point estimate
Drawdown amount absorbed	0.007025	—	—	Point estimate
Drawdown minus run-up	0.003508	[0.000521, 0.006257]	[0.000423, 0.006721]	Both bands exclude zero
Spanning-share difference	0.196091	[-0.046155, 0.462489]	[-0.069697, 0.497990]	Bands include zero
Panel B. Signed five-day realized quadratic variation				
Capped positive-return-day variation	0.014430	[0.007055, 0.021805]	[0.007860, 0.021000]	Both bands exclude zero
Capped negative-return-day variation	0.013080	[0.005212, 0.020947]	[0.008050, 0.018109]	Both bands exclude zero
Capped positive minus negative	0.001350	[-0.005245, 0.007945]	[-0.005570, 0.008270]	Bands include zero
Raw positive-return-day variation	0.063867	[-0.020253, 0.147987]	[-0.017768, 0.145502]	Bands include zero
Raw negative-return-day variation	0.014310	[0.000916, 0.027705]	[0.004739, 0.023882]	Both bands exclude zero
Raw positive minus negative	0.049557	[-0.022641, 0.121755]	[-0.027818, 0.126932]	Bands include zero

Notes: The common sample contains 61,434 name-days, 538 names, and 1,229 dates from January 2020 through December 2024. Panel A defines drawdowns and run-ups from the sign of the trailing $t-5$ through $t-1$ return and evaluates state-specific coefficients at mean standardized return magnitude. The amount absorbed is the attention coefficient without the surface minus its coefficient with the full surface. Its date and ticker bands use 2,000 paired draws each. All 538 ticker deletions and all 60 month deletions retain the signs of both Panel A differences. The spanning-share difference is imprecisely estimated. Panel B splits five-day realized quadratic variation between nonnegative- and negative-return days. Capped outcomes use one common 99.5th-percentile cap; raw outcomes are uncapped. Panel B reports attention slopes conditional on the full twelve-node surface, trailing realized volatility, log market capitalization, and ticker and date effects. Date and ticker bands are simultaneous 95 percent bands across the three outcomes and retain 2,000 draws each. The raw signed difference has limited effective-cluster support and is therefore interpreted cautiously. Deleting BBBY changes the capped positive-minus-negative estimate from 0.001350 to -0.000045. No month deletion reverses it, and neither ticker nor month deletion reverses the raw difference. These reduced-form decompositions do not identify trading direction or causality.

Table A12. Public option activity and later surface updating. Panel A separates gross activity, direction, and gross-scaled signed levels in the public SLAN sample, then tests whether the signed proxy attenuates the surface-conditioned attention coefficient. Panel B evaluates whether week-one changes in the option surface forecast volatility in the following week.

Object	Estimate	t	Date-block 95% band	Ticker-pair 95% band
Panel A. Public internalized option activity				
Gross contract activity, total	0.13800	27.99	[0.12422, 0.14791]	[0.13012, 0.14584]
Contract imbalance, calls	-0.00547	-1.01	[-0.01578, 0.00397]	[-0.01505, 0.00467]
Gross-scaled signed level, calls	-0.02955	-4.31	[-0.04370, -0.01603]	[-0.04208, -0.01714]
Contract imbalance, puts	0.01882	3.24	[0.00666, 0.02982]	[0.00820, 0.02932]
Gross-scaled signed level, puts	-0.02287	-5.78	[-0.03346, -0.01528]	[-0.02966, -0.01589]
Surface-conditioned forward realized volatility				
Attention coefficient without signed proxy	0.03138	3.66	[0.00948, 0.05314]	[0.01837, 0.04610]
Attention coefficient with signed proxy	0.03079	3.74	[0.00952, 0.05172]	[0.01822, 0.04424]
Attenuation, without minus with signed proxy	0.00059	n.a.	[-0.00018, 0.00161]	[-0.00002, 0.00227]
Panel B. Week-two surface updating				
Attention coefficient, surface at t	-0.00252	-1.49	n.a.	n.a.
Attention coefficient, adding surface change through $t+5$	0.00032	0.19	[-0.00291, 0.00371]	[-0.00237, 0.00329]
Difference, first minus second	-0.00284	-6.08	[-0.00386, -0.00180]	[-0.00367, -0.00204]

Notes: Panel A reports standardized activity outcomes per one-standard-deviation increase in the lagged attention score. Its activity sample contains 37,711 name-days, 747 names, and 370 dates from January 2020 through June 2021. Gross contract activity is log one plus the sum of reported call and put contract volume. Direction is reported contract imbalance. Signed level is volume times imbalance, a gross-scaled proxy rather than signed vega demand. Call direction is unresolved and put direction is positive, while both signed levels are negative, so neither side has a coherent same-sign directional response. The final three Panel A rows use 24,220 name-days, 371 names, and 353 dates and condition on the complete day-t 12-node option surface. Attenuation is the attention coefficient without the signed proxy minus the coefficient with it. The signed-arm null-imposed MDEs are 0.00150 for date blocks and 0.00123 for ticker pairs. A dollar-weighted gross-activity specification gives attenuation 0.00806, with date-block [0.00277, 0.01260] and ticker-pair [0.00445, 0.01217]. Panel B forecasts week-two realized volatility over $t+6$ through $t+10$. The first specification includes the full surface at t; the second adds its change through $t+5$. The sample contains 60,718 name-days, 536 names, and 1,219 origin dates. The paper's headline incremental forecast content concerns week one, $t+1$ through $t+5$. A spanning share is not reported because only 9.70% of date draws and 4.95% of ticker draws have positive first-specification denominators. Panel A uses paired ten-trading-day date blocks; Panel B uses paired 20-origin-day blocks. Ticker-pair bands use common paired draws. Every band retains 2,000 draws. At week two, the premise attention coefficient is near zero and negative before the later surface enters. The estimated difference has the opposite sign from absorption, so there is no continuing relation for the later surface to absorb. SLAN is a public internalized-flow proxy, not customer-origin signed flow by maturity. Neither panel identifies retail demand, information aggregation, or why the surface changes.

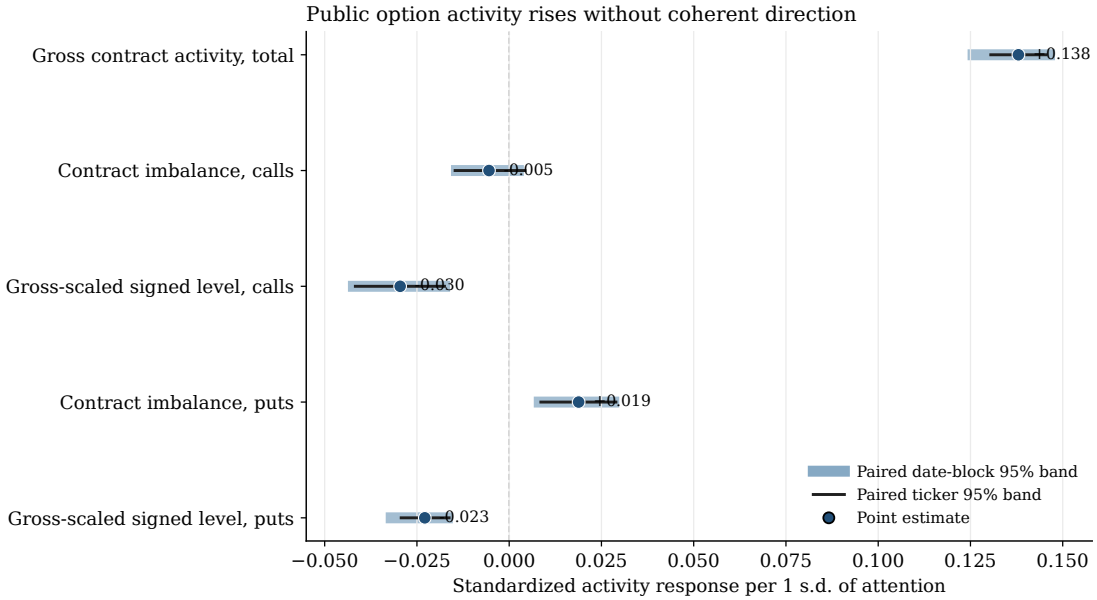


Figure A2. Public internalized-activity proxy and directional measures. The sample covers January 2020 through June 2021. Attention is associated with gross activity, while the call and put directional proxies do not deliver a coherent same-sign pattern. SLAN is not customer-origin signed flow and has no maturity dimension.

Table A13. Economic-scale and payoff boundaries. The reallocation is a mean-preserving forecast-space adjustment to standardized nodes. The half-spread comparison uses a fixed nearest-contract call-put proxy and is noncausal. Payoff rows come from disclosed r/thetagang short-put positions and report controlled attention slopes.

Object	Estimate	t-statistic	Interpretation
Reallocation, 95th percentile	0.261 vol. points		synthetic standardized node
Nearest-contract proxy half-spread, median	0.985 vol. points		noncausal scale comparison
Cells with adjustment no larger than proxy	95.4%		90,390 name-maturity-days
Disclosed short-put gross-loss slope	0.0095	1.37	statistically unresolved
Approximate net-payoff slope	0.0028	0.62	statistically unresolved

Table A14. Retail attention in the 2025 validation sample. The headline panel contains 7,310 name-days; the common-node and full-surface panels contain 6,353. Forecast tests cover 160 dates, except rank IC, which covers 150 dates with at least 20 names. Coefficient and spanning rows report 95 percent intervals or paired bootstrap bands; forecast rows report Clark–West intervals.

Outcome	Estimate	95% interval or band
Headline β	+0.0315 ($t = 4.01$)	[+0.0160, +0.0471]
30-day share ϕ	+0.232	[+0.125, +0.463]
10-day less 30-day ϕ	+0.156	[+0.083, +0.228]
HAR+IV CS OOS R^2	+0.098%	CW [+0.00126, +0.00301]
Pooled timing gap	+0.061	[-0.043, +0.164]
Full-surface rank IC increment	+0.0017	[+0.0002, +0.0032]
Full-panel HAR CS OOS R^2	+0.144%	CW [+0.00153, +0.00983]

C Variable definitions and sample conventions

Table A15 fixes the timing and unit of every empirical object. The selected attention score is built from log distinct comment authors on a dense calendar grid, standardized against the prior 30 calendar days and then lagged one day. The posts count is lagged on the same calendar and defines entry into the fixed active population. The date on an attention observation is therefore the first trading day on which it is usable, and forward outcomes begin after that date.

Table A15. Variable definitions. Volatility variables are annualized decimals. Returns are decimals. All rolling constructions are computed within ticker.

Variable	Definition
Attention, $z_{i,t}$	Distinct r/wallstreetbets comment authors naming ticker i on calendar day $t-1$, transformed by $\log(1+x)$ and standardized against the ticker's trailing 30-day calendar history using at least ten observations. The resulting score is standardized across the fixed dense covered panel before any active, ATM, or full-grid sample restriction. Explicit measure-comparison exhibits instead standardize each score on their stated common population.
Posts-based attention	Daily ticker mentions in r/wallstreetbets posts divided by all collected posts, converted to a trailing 30-day z-score and shifted one day. Used only in supplementary comparisons.
Entry into discussion	Indicator that lagged post mentions are positive. In the posts-based two-part model, it captures entry into discussion at mean active intensity.
Positioning intensity	$\log(1 + \text{sold puts} \times \log(1 + \text{engagement}))$ on r/thetagang disclosure days, standardized before estimation.
Sold-put wall	Modal disclosed sold-put strike for a ticker-day. Used only in the open-interest validation and the pin and breach tests specified in advance.
Forward RV, five days	Annualized standard deviation of daily log returns over trading days $t+1$ through $t+5$.
Other RV horizons	One-day annualized absolute log return, or annualized standard deviation over the next 10 or 21 trading days.
Trailing RV	Annualized standard deviation of daily log returns over the 21 trading days ending at t.
Tail move	Indicator that the absolute cumulative return over $t+1$ through $t+5$ is at least ten percent.
Market capitalization	Absolute CRSP price times shares outstanding. The regressions use its natural logarithm.
ATM implied volatility	Arithmetic mean of the exact OptionMetrics thirty-day call-delta 50 and put-delta minus-50 implied-volatility nodes, with both required on the same effective security-date link.
Forward IV change	ATM implied volatility on the exact fifth next CRSP trading day minus ATM implied volatility at t.
Earnings window	Indicator, on Compustat-covered tickers, that an observed quarterly report date falls from measurement day t through the exact fifth next CRSP trading day. Measurement-day reports are included because Compustat lacks announcement times.

Data availability

The tracked repository reproduces the manuscript and online-supplement PDFs from tracked tables, figures, and result ledgers. Re-estimation requires the restricted CRSP, OptionMetrics, and Compustat inputs and the archived public-platform corpus described in the manifest. Replication code, manuscript sources, generated table fragments, and result records are included in the repository.

CRSP, OptionMetrics, and Compustat data are licensed and cannot be redistributed. Large public platform collections, including Reddit posts and comments and StockTwits records, are not committed because of their size and source constraints. Their provenance and construction are documented. The public SLIM file is acquired from the authors' public data page and is not redistributed. Its source URL, file hash, and path override are documented with the validation and selected-measure decomposition artifacts.

No signed historical customer-flow data covering the 2020–2024 spanning sample are used. A short public 2025 Cboe opening-volume window is retained only as separately labeled supplemental scope evidence. It does not identify the historical information-versus-demand-pressure mechanism.

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