Financial Regulation with Imperfect Instruments*

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Abstract

Financial regulation in practice is imperfect: regulators operate under constraints, and their interventions generate unintended consequences. In this paper, we develop a general approach to analyze imperfect financial regulation. We present a model that provides insights into seemingly distinct problems — including shadow banking, risk-shifting or asset substitution, and political constraints on intervention — and show that Pigouvian wedges and leakage elasticities are sufficient to determine second-best financial regulation. Reverse leakages determine the value of reforms that relax constraints on financial regulation. We further demonstrate how our results can provide new insights into the complex issue of financial regulation with environmental externalities.

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1 Introduction

Financial regulation has become more complex since the crisis of 2008, but is also notoriously imperfect. The current regime is designed to address market failures such as fire sales and distortive government bailouts (e.g., Lorenzoni, 2008; Bianchi, 2011, 2016; Dávila and Korinek, 2018). However, shadow banks contribute to these externalities but remain unregulated (e.g., Adrian and Ashcraft, 2016; Hachem, 2018; Irani et al., 2021). Another source of imperfection is that banks are subject to coarse capital requirements, which may incentivize gaming or risk-shifting (e.g., Jiménez et al., 2017; Behn, Haselmann and Vig, 2022). Uniform regulations are often applied to heterogeneous banks (e.g., Corbae and D'Erasmo, 2021). Finally, a recent literature emphasizes additional challenges associated with "climate finance" and environmental externalities (e.g., Gasparini et al., 2024).

In this paper, we identify general principles for optimal financial policy with imperfect instruments. We analyze a canonical economy with financial frictions and its applications. Heterogeneous intermediaries make investment and financing decisions that may generate externalities. A planner sets corrective taxes or regulations, but the instruments are constrained. For example, the planner may i) be unable to regulate certain agents or decisions, ii) be forced to set uniform regulation across agents or decisions, or iii) face costs that rise with the size of regulation. As we illustrate below, this setup is tractable, but also general enough to capture many practical regulatory imperfections, as well as the most commonly studied market failures/externalities.

Our core insight is that all of these imperfections — typically treated as distinct phenomena — follow the same simple economics. We first show that the marginal welfare effect of any policy change depends only on *Pigouvian wedges* and *policy elasticities*. Pigouvian wedges measure the gap between existing corrective regulations and marginal externalities — a positive wedge, for instance, suggests overregulation relative to the Pigouvian "polluter pays" principle. Policy elasticities measure the equilibrium responses of different decisions to changes in regulation. Intuitively, our characterization shows that policy changes that discourage underregulated decisions or encourage overregulated decisions raise welfare. A less intuitive implication is that nothing else additionally matters for optimal policy, despite the fact that our model allows for market incompleteness, financial frictions, and default.

Three theoretical results follow from this characterization. First, even *perfectly regulated*

decisions — which are not associated with any binding constraint faced by the planner — are not optimally regulated at the Pigouvian level, but are subject to a second-best adjustment. The adjustment is positive, meaning stricter regulation than the Pigouvian level, if imperfectly regulated decisions are underregulated complements or overregulated substitutes; sub-Pigouvian regulation (less strict than the Pigouvian level) arises for underregulated substitutes or overregulated complements. *Leakage elasticities* measure how imperfectly regulated decisions respond to policy changes and therefore play a key role in determining second-best regulation. These leakages are causal responses to regulation that can, in principle, be estimated using the same quasi-experimental variation used in the empirical banking literature (Acharya, Schnabl and Suarez, 2013; Buchak et al., 2018; Irani et al., 2021; Plosser and Santos, 2024). Our results provide an auxiliary economic interpretation for these estimates, and show that they are directly relevant for welfare.

Second, we characterize the optimal second-best regulation of *imperfectly regulated* decisions. By definition, these regulations are subject to binding constraints, but the planner may nonetheless have degrees of freedom in choosing them — for instance, if a constraint dictates that two decisions must be taxed at a uniform rate, while the level of the uniform tax can be chosen freely. In general, the optimal second-best regulation depends on leakage and *reverse leakage* elasticities; the latter capture how perfectly regulated decisions adjust in response to the regulation of imperfectly regulated ones.

Building on these insights, we derive optimal second-best regulation of imperfectly regulated decisions in two common scenarios. If taxes must be *uniform* across heterogeneous agents or decisions, then the optimal regulation is a weighted average of distortions, where the appropriate weights are augmented to incorporate (reverse) leakage elasticities. If a subset of regulations is subject to convex costs, then the optimal regulation is given by an attenuated version (or "shrinkage") of the first-best policy. Reverse leakage further attenuates the optimal regulation.

Third, we characterize the social value of relaxing the constraints faced by a planner who is implementing the optimal second-best policy. This is useful, for instance, to assess the marginal value of reforms that bring shadow banks under the regulatory perimeter. Whenever perfectly and imperfectly regulated decisions are *either* complements or substitutes, reverse leakage *attenuates* the welfare effect of regulating imperfectly regulated decisions, a result that connects to the Le Chatelier principle. As a result, regulatory reform may be desirable, but the quantitative case in favor of reform is weaker

because general equilibrium adjustment offsets part of the direct gain.

With these results in hand, we demonstrate their applied usefulness. We first develop four minimal examples that connect our results to practical problems of financial regulation. Example 1 studies a model with traditional banks and unregulated shadow banks, which compete for funding from outside creditors and may receive government bailouts. Example 2 considers imperfections that arise when policy targets leverage-based metrics — e.g., the ratio of equity capital to risky assets, as in practical capital requirements — but cannot control the overall scale of risky investment. Example 3 considers the classical asset substitution or risk-shifting problem in banking (e.g., Jensen and Meckling, 1976; Allen and Gale, 2000), which arises in practice when imperfect regulations apply the same tax or risk weight to different assets $within$ one type of investor. Example 4 studies a model with uniform regulations $across$ heterogeneous types, who have different marginal effects on excessive credit booms in the sense of Lorenzoni (2008).

All of these seemingly distinct problems fit into our model, and therefore the optimal second-best policy follows from the same set of formulae in each case. Our examples also give practical insights into different kinds of imperfection. For instance, bank leverage should be underregulated in Example 1, because regulated and unregulated leverage choices are substitutes, but overregulated in Example 2, because risky investment and leverage are complements within banks. Moreover, in Examples 3 and 4, the optimal (uniform) second-best regulation is a weighted average of the distortions imposed by different assets (Example 3) or investor types (Example 4), and one can derive the weights from more fundamental objects, such as the policy elasticity of bailout probabilities.

Next, we present a richer application to financial regulation with environmental externalities, which is a complex problem and the subject of a growing literature. Indeed, this application highlights a key advantage of our approach: our formulae indicate the economic forces behind second-best regulation — even in models with many moving parts and several market failures. We develop a model in which investors choose the scale of their risky investment, the composition of their portfolios, and their leverage. The planner controls a risk-weighted capital requirement. This is an imperfect instrument, which effectively regulates investors' leverage and portfolio composition. However, this requirement does not regulate the total scale of investment in different technologies, which is particularly important for environmental concerns. Since leverage and the investment scale are gross complements, we find that overregulating leverage is optimal, consistent with

our general results.

Following the current policy debate on climate finance, we compare optimal policy under a narrow/financial mandate that only considers externalities related to financial stability, and a broad mandate that considers the impact of financial regulation on environmental externalities. We demonstrate that the nature of optimal regulation is substantially different once we account for the imperfections inherent in current regulatory regimes. One implication of our approach is that it is natural to adjust risk weights, as opposed to leverage caps, when regulators become concerned with broader environmental mandates.

Related Literature. A growing literature studies financial regulation with imperfections, mostly in the form of regulatory arbitrage. Theoretical work on shadow banking notes the unintended consequences of liquidity policies (Hachem and Song, 2021; Grochulski and Zhang, 2019; Hachem and Kuncl, 2025) and capital requirements (Plantin, 2015; Huang, 2018; Martinez-Miera and Repullo, 2019; Farhi and Tirole, 2021; Clayton and Schaab, 2021), as well as the general costs and benefits of shadow banks (e.g., Gennaioli, Shleifer and Vishny, 2013; Moreira and Savov, 2017; Ordoñez, 2018). Another strand of work focuses on imperfect regulation of financiers' portfolio choices, with foundational work emphasizing risk-shifting (Kim and Santomero, 1988; Allen and Gale, 2000; Hellmann, Murdock and Stiglitz, 2000), and more recent research emphasizing carbon emissions (Oehmke and Opp, 2026). Other imperfections arise from conflicts between financial regulators, as well as technological innovations (Korinek, 2017; Korinek, Montecino and Stiglitz, 2022). We emphasize the common economics behind second-best regulation in these seemingly distinct scenarios. Quantitative models (Bengui and Bianchi, 2022; Begenau and Landvoigt, 2022; Dempsey, 2026) provide general equilibrium assessments of bank leverage regulation with shadow banks.

A literature in public economics studies second-best corrective taxes (e.g., Salanié, 2011), environmental policy (e.g., Bovenberg and Goulder, 2002) and rent-seeking (e.g., Rothschild and Scheuer, 2016), but does not consider financial frictions or financial regulation. We demonstrate a novel common thread: for instance, the optimal tax formulae in Diamond (1973), the second-best principles in Lipsey and Lancaster (1956), and the Tinbergen (1952) rule can all be derived as corollaries of our general results.

2 General Framework

This section presents our general framework. In Section 3, we use it to derive the principles that determine optimal regulation with imperfect instruments, and illustrate it in a series of examples, before moving on to a richer application in Section 4.

2.1 Environment

We consider an economy with two dates, $t \in \{0, 1\}$, and a single consumption good. At date 1, there is a continuum of possible states $s \in S$, with a cumulative distribution function $F(s)$. There are two groups of agents: investors and creditors.

Investors. There is a finite number of investor types indexed by $i, j \in \mathcal{I} = \{1, 2, \ldots, I\}$. One can think of these types, for instance, as different types of financial intermediaries, such as traditional banks and shadow banks.

Each investor type makes $N \geq 1$ investment/financing decisions, which we collect in a vector $\boldsymbol{x}^i \in \mathbb{R}^N$.[1] We use this abstract notation for financing/investment decisions, but it maps directly into applied settings. Concretely, one can collect each investor's liabilities — such as promised repayments on debt, bonds, or deposits, or the amount of equity sold to outsiders — in a vector of financing decisions $\boldsymbol{b}^i$. Assets — such as a portfolio of securities, lending opportunities, or projects — can be collected in a vector of investment decisions $\boldsymbol{k}^i$. The overall vector of decisions is then simply $\boldsymbol{x}^i = \left(\boldsymbol{b}^i, \boldsymbol{k}^i\right)$, representing both sides of investors' balance sheets.

Investor i's preferences are represented by

$$u^i\left(c_0^i, \left\{c_1^i(s)\right\}_{s\in S}, \overline{\boldsymbol{x}}\right), \tag{1}$$

where c_0^i and $c_1^i(s)$ denote investor i's consumption at date 0 and date 1 in state s, while $\overline{\boldsymbol{x}} = \{\overline{\boldsymbol{x}}^j\}_{j\in\mathcal{I}}$ denotes the aggregate decisions made by all investor types in equilibrium. Since each individual investor is atomistic, her decisions $\boldsymbol{x}^i$ do not affect the aggregate decisions $\overline{\boldsymbol{x}}^i$ of her own type, nor the aggregate decisions $\overline{\boldsymbol{x}}^j$ of other types. The dependence of utility on $\overline{\boldsymbol{x}}$ captures externalities across agents. In financial regulation, this formalism can be used to study bailout externalities, fire-sale pecuniary externalities, and "internalities" from

[1] Section A of the Online Appendix includes explicit definitions of all vectors and matrices used.

distorted beliefs, all of which we demonstrate in Sections 3.3 and 4.

Investor i faces the following budget constraints:

$$c_0^i = n_0^i + Q^i\left(\boldsymbol{x}^i\right) - \mathsf{Y}^i\left(\boldsymbol{x}^i\right) - \boldsymbol{\tau}^i \cdot \boldsymbol{x}^i + T^i \tag{2}$$

$$c_1^i(s) = n_1^i(s) + \rho^i\left(\boldsymbol{x}^i, s\right), \quad \forall s, \tag{3}$$

where n_0^i and $n_1^i(s)$ denote investor endowments. At date 0, investor i raises financing according to the pricing schedule $Q^i(\boldsymbol{x}^i)$, which is determined endogenously in equilibrium, as we detail below. Investor i's decisions also induce a cost $\mathsf{Y}^i(\boldsymbol{x}^i) \geq 0$. This term can capture technological adjustment costs for investment decisions $\boldsymbol{k}^i$, or financing frictions for financing decisions $\boldsymbol{b}^i$. The term $\boldsymbol{\tau}^i \cdot \boldsymbol{x}^i$ represents corrective taxes, subsidies, or regulations, where $\boldsymbol{\tau}^i \in \mathbb{R}^N$, and $T^i \in \mathbb{R}$ denotes the lump-sum transfer or tax that investor i receives. At date 1, the investor enjoys a return $\rho^i(\boldsymbol{x}^i, s)$ from her investments. This function can be interpreted as the investor's residual claim on her investments, after paying back all liabilities to creditors.

It should be clear from this description that our model allows for a wide range of financial frictions and forms of market incompleteness. There is no requirement that the available financing choices $\boldsymbol{x}^i$ span the possible states of the world, for example (but note that complete Arrow-Debreu markets are a special case of this framework). Moreover, the flexible specification of investment returns $\rho^i(\boldsymbol{x}^i, s)$ — along with the analogous returns to creditors below — can be used to represent investor default in certain states s.

Creditors. A unit mass of creditors has preferences

$$u^C\left(c_0^C, \left\{c_1^C(s)\right\}_{s\in S}, \overline{\boldsymbol{x}}\right), \tag{4}$$

where c_0^C and $c_1^C(s)$ denote creditors' consumption at date 0 and at date 1 in state s. As above, $\overline{\boldsymbol{x}}$ denotes the collection of decisions made by all investors. As a concrete interpretation, one can think of creditors as households or other end investors that provide financing to investors/intermediaries.[2]

[2]In earlier versions of this paper, we allowed for investors to invest in each other's liabilities and for creditors' decisions to also be associated with welfare-relevant externalities. Since the main insights are identical in both formulations, we adopt the current formulation, which substantially simplifies the notation.

Creditors face the following budget constraints:

$$c_0^C = n_0^C - \sum_i h_i^C Q^i\left(\overline{\boldsymbol{x}}^i\right) \tag{5}$$

$$c_1^C(s) = n_1^C(s) + \sum_i h_i^C \rho_i^C\left(\overline{\boldsymbol{x}}^i, s\right), \quad \forall s, \tag{6}$$

where n_0^C and $n_1^C(s)$ denote creditors' endowments. At date 0, creditors choose to fund a share h_i^C of each investor i's financing needs $Q^i(\cdot)$. At date 1, creditors receive repayments $\rho_i^C(\overline{\boldsymbol{x}}^i, s)$ from investor i. Notice that the general formulation of investor returns $\rho^i(\cdot)$ and creditor repayments $\rho_i^C(\cdot)$ can accommodate deadweight losses associated with investors' default. By suitably interpreting creditors' preferences, our model can capture nonpecuniary benefits/convenience yields from particular forms of financing, as in, e.g., Stein (2012) or Sunderam (2015).

Equilibrium. For given corrective taxes/subsidies/regulations $\{\boldsymbol{\tau}^i\}_i$ and lump-sum transfers $\{T^i\}_i$, an *equilibrium* consists of consumption allocations $\{c_0^i, c_1^i(s)\}_i$ and $\{c_0^C, c_1^C(s)\}$, investors' decisions $\{\overline{\boldsymbol{x}}^i\}_i$, creditors' funding decisions $\{h_i^C\}_i$, financing schedules $\{Q^i(\cdot)\}_i$, investors' net investment returns $\{\rho^i(\cdot)\}_i$, and creditors' repayments $\{\rho_i^C(\cdot)\}_i$ such that i) investors maximize utility, (1), subject to budget constraints (2) and (3); ii) creditors maximize utility, (4), subject to budget constraints (5) and (6); iii) the planner's budget is balanced, so that $\sum_i T^i = \sum_i \boldsymbol{\tau}^i \cdot \boldsymbol{x}^i$; iv) investors' decisions are consistent in the aggregate, that is, $\boldsymbol{x}^i = \overline{\boldsymbol{x}}^i$, $\forall i$; v) financing decisions satisfy market clearing, that is, $h_i^C = 1$, $\forall i$.

Note that individual investors are price-takers in the sense that they take the pricing schedule $Q^i(\cdot)$ as given. Nonetheless, each individual internalizes that her investment/financing decisions $\boldsymbol{x}^i$ a□ect the amount of financing $Q^i(\boldsymbol{x}^i)$ that she can raise. This equilibrium notion is standard in models that allow for default (e.g., Dubey, Geanakoplos and Shubik, 2005). We proceed as if the model is well-behaved, discussing the necessary regularity conditions in each application. All of our results go through if one imposes type-by-type budget balance $T^i = \boldsymbol{\tau}^i \cdot \boldsymbol{x}^i$, $\forall i$ — instead of the less restrictive $\sum_i T^i = \sum_i \boldsymbol{\tau}^i \cdot \boldsymbol{x}^i$ — in which case Pigouvian taxes are equivalent to quantity regulation.

2.2 Imperfect Policy Instruments

The planner in this model has $I \cdot N$ corrective policy instruments: one for each investor decision, collected in $\boldsymbol{\tau} = \{\boldsymbol{\tau}^i\}_{i \in \mathcal{I}}$. We formalize *imperfect instruments* by assuming that the planner chooses regulations subject to a vector-valued constraint

$$\boldsymbol{\Phi}(\boldsymbol{\tau}) \leq 0. \tag{7}$$

The function $\boldsymbol{\Phi} : \mathbb{R}^{IN} \to \mathbb{R}^M$, where M is the number of constraints, defines feasible regulations.

Our main results — Propositions 1 and 2 — are valid for a general well-behaved $\boldsymbol{\Phi}(\cdot)$. Several familiar imperfections are nested in this formalism. We explore these in detail in our examples below, but briefly review the most important cases here:

1. *Unregulated investors:* The function $\boldsymbol{\Phi}(\cdot)$ can mandate that $\boldsymbol{\tau}^i \equiv 0$ for an "unregulated" type of investor. Unregulated shadow banks are a widely modeled example of this type of imperfection.

2. *Unregulated decisions:* The function $\boldsymbol{\Phi}(\cdot)$ can mandate that $\tau_n^i \equiv 0$ for all elements n of the vector $\boldsymbol{\tau}^i$ that correspond to an unregulated decision *within* a type of investor, even if that type is not entirely unregulated. For instance, financial regulations typically constrain ratios (e.g., of risk-weighted assets to liabilities), leaving the overall scale of the balance sheet as a free — and hence unregulated — decision.

3. *Uniformity:* One can specify a linear function $\boldsymbol{\Phi}(\cdot)$ that imposes uniform regulation on different decisions. This can occur both within and across institutions. *Within* institutions, financial regulations tend to attach the same risk weights to certain buckets of asset classes (e.g., prime mortgages with credit characteristics in a given bucket), so that portfolio choices *within* those buckets (e.g., mortgages with different risk profiles within the prime bucket) are uniformly regulated. *Across* institutions, banks are heterogeneous, for example in size, but capital and liquidity requirements generally do not depend on their observable characteristics.

4. *Costs of regulation/political economy:* Appealing to the duality between constraints and costs, we can also interpret $\boldsymbol{\Phi}(\boldsymbol{\tau})$ as defining the cost of setting regulations. A common observation is that industry lobbying and other political constraints make

tougher financial regulation costly. Formally, a quadratic constraint $\Phi(\boldsymbol{\tau}) \equiv \frac{1}{2}\boldsymbol{\tau}'\boldsymbol{B}\boldsymbol{\tau}$ for a given matrix $\boldsymbol{B}$ can represent convex costs of regulation to capture such phenomena. In addition, if the costs of regulation induce sparsity (Tibshirani, 1996; Gabaix, 2014) — for instance, when based on the L^1 norm of $\boldsymbol{\tau}$ — the set of unregulated agents or commodities arises endogenously.

Most of these examples are associated with relatively simple (i.e., linear or quadratic) constraints $\Phi(\boldsymbol{\tau})$. One could also imagine more complex scenarios, such as dynamic regulatory arbitrage or financial innovation, that would give rise to nonlinear constraints. Our theoretical results can, in principle, be extended to apply to these settings.

In the next section, we characterize optimal second-best regulation in this general class of economies. Our general results do not depend on the financing/investment model studied here, and would also apply to price- or game-theoretic environments — see Section E of the Online Appendix.

3 Optimal Corrective Policy with Imperfect Instruments

We now study the problem of a planner who optimally chooses imperfect corrective regulation. We abstract from considerations about redistribution, and focus on the corrective nature of the regulation. Therefore, we assess the aggregate gains/losses of a marginal policy change by aggregating money-metric welfare changes across agents.[3] That is, the planner evaluates the desirability of a marginal change in a given variable (or vector) z, denoted by $\frac{dW}{dz}$, according to

$$\frac{dW}{dz} \equiv \sum_{i \in \mathcal{I}} \frac{\frac{dV^i}{dz}}{\lambda_0^i} + \frac{\frac{dV^C}{dz}}{\lambda_0^C},$$

where $\frac{dV^i}{dz}$ ($\frac{dV^C}{dz}$) denotes the change in investor i's (creditor's) indirect utility in equilibrium, and $\lambda_0^i > 0$ ($\lambda_0^C > 0$) denotes investor i's (creditor's) date-0 marginal value of consumption. Our definition of welfare effects divides by these marginal values to convert welfare changes

[3] This approach is equivalent to maximizing Kaldor-Hicks efficiency — see Dávila and Schaab (2025) — or to imposing equal generalized social marginal welfare weights, in the sense of Saez and Stantcheva (2016). Section E.3 of the Online Appendix explains how to consider traditional social welfare functions and redistributional considerations.

for different agents into the same dollar units.

To characterize the marginal welfare effect of adjusting regulations, it is useful to first define the *marginal distortions/externalities* associated with the vector $\overline{\boldsymbol{x}}$ of decisions:

$$\boldsymbol{\delta} = -\left(\sum_{j\in\mathcal{I}} \frac{1}{\lambda_0^j}\frac{\partial u^j}{\partial \overline{\boldsymbol{x}}} + \frac{1}{\lambda_0^C}\frac{\partial u^C}{\partial \overline{\boldsymbol{x}}}\right). \qquad \text{(Marginal Distortion)}$$

The vector $\boldsymbol{\delta}$ contains one distortion for each investor decision (of which there are $I \cdot N$ in the economy). We define the distortion as the negative of marginal utility, so that $\boldsymbol{\delta}$ stands for "damage" or negative externalities. In addition, we define *Pigouvian wedges* as the difference between regulations and marginal distortions:

$$\boldsymbol{\omega} = \boldsymbol{\tau} - \boldsymbol{\delta}. \qquad \text{(Pigouvian Wedge)}$$

The well-known "polluter pays" principle of corrective regulation (Pigou, 1920) suggests that all wedges should be set to zero. The first-best Pigouvian tax is given by $\boldsymbol{\tau}^\star = \boldsymbol{\delta}$.[4]

We focus instead on situations where regulatory imperfections prevent the first-best allocation. In these settings, positive Pigouvian wedges represent overregulation, and negative ones underregulation, relative to the Pigou principle (of course, our notation also allows for some decisions to be underregulated while others are overregulated; this would simply lead to different signs for different elements of $\boldsymbol{\omega}$).

Lemma 1 highlights the role of Pigouvian wedges and policy elasticities when evaluating arbitrary policy changes:

Lemma 1. *(Marginal Welfare Effects of Regulation) The marginal welfare effects of varying regulations $\boldsymbol{\tau}$, $\frac{dW}{d\boldsymbol{\tau}}$, are given by*

$$\frac{dW}{d\boldsymbol{\tau}} = \frac{d\boldsymbol{x}}{d\boldsymbol{\tau}}(\boldsymbol{\tau} - \boldsymbol{\delta}) = \frac{d\boldsymbol{x}}{d\boldsymbol{\tau}}\boldsymbol{\omega}, \tag{8}$$

where $\frac{d\boldsymbol{x}}{d\boldsymbol{\tau}}$ is the Jacobian matrix of policy elasticities, of dimension $IN \times IN$.

Lemma 1 shows that the welfare impact of regulation changes depends only on policy

[4] Since our model features incomplete markets and default, the first-best regulation is interpreted as the one that achieves constrained efficiency. Beyond this benchmark, a planner could raise welfare further if she were able to implement better risk sharing, or lower incidence of costly default, than is achievable with the financing instruments that investors use. Our focus on constrained efficiency is shared with the majority of the literature on regulation with financial frictions.

elasticities and Pigouvian wedges. The matrix $\frac{d\boldsymbol{x}}{d\boldsymbol{\tau}}$ of *policy elasticities* — borrowing the terminology of Hendren (2016) — captures the full equilibrium response that a particular change in regulation has on all agents' decisions. The vector $\boldsymbol{\omega}$ of Pigouvian wedges summarizes the extent to which an agent's decision is underregulated or overregulated. The overall welfare effect of varying a particular regulation corresponds to the sum of products of the relevant policy elasticities and Pigouvian wedges, where regulations that i) decrease underregulated decisions (with a negative wedge) or ii) increase overregulated decisions (with a positive wedge) in equilibrium are welfare-improving.

3.1 Second-Best Policy: Perfectly Regulated Decisions

To characterize the optimal second-best policy, we let $\boldsymbol{x}^R$ denote all *perfectly regulated* decisions made by investors. These are decisions for which the planner can choose the corresponding tax or regulation without a binding constraint. We let $\boldsymbol{x}^U$ denote the remaining, *imperfectly regulated* decisions.[5] We use the same partition for all objects in the model: for instance, $\boldsymbol{\tau}^R$ denotes the tax/regulation on perfectly regulated decisions, and $\boldsymbol{\omega}^U$ the Pigouvian wedge on imperfectly regulated decisions. We also apply it to derivatives: $\frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^R}$ is the matrix of *leakage elasticities*, which capture how imperfectly regulated decisions respond to changes in the regulations that can be freely adjusted.[6]

Proposition 1 shows how leakage elasticities determine the second-best policy.

Proposition 1. *(Second-Best Policy: Perfectly Regulated Decisions) The optimal second-best regulation of perfectly regulated decisions satisfies*

$$\boldsymbol{\tau}^R = \boldsymbol{\delta}^R + \left(-\frac{d\boldsymbol{x}^R}{d\boldsymbol{\tau}^R}\right)^{-1} \frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^R}\boldsymbol{\omega}^U, \tag{9}$$

where $\boldsymbol{\delta}^R$ is a vector of marginal distortions, $\boldsymbol{\omega}^U = \boldsymbol{\tau}^U - \boldsymbol{\delta}^U$ is a vector of Pigouvian wedges, $\frac{d\boldsymbol{x}^R}{d\boldsymbol{\tau}^R}$ is a matrix of own perfectly regulated policy elasticities, and $\frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^R}$ is a matrix of leakage elasticities.

[5] Formally, let the vector $\boldsymbol{\mu} \in \mathbb{R}^M$ denote the vector of Lagrange multipliers associated with the regulatory constraints in Equation (7). If an element of $\frac{d\boldsymbol{\Phi}}{d\boldsymbol{\tau}}\boldsymbol{\mu}$ is zero (not zero), then the corresponding element of decisions $\boldsymbol{x}$ is a perfectly (imperfectly) regulated decision.

[6] All matrices are rigorously defined in Section A of the Online Appendix. When there are R perfectly regulated decisions and U imperfectly regulated ones — where $R + U = I \cdot N$ — then the leakage matrix $\frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^R}$ has dimension $R \times U$.

Proposition 1 shows that optimal regulation has two parts: the marginal distortion $\boldsymbol{\delta}^R$ (following the Pigou principle), and a second-best correction driven by leakage elasticities and Pigouvian wedges. A useful heuristic for interpreting this expression is that $\left(-\frac{d\boldsymbol{x}^R}{d\boldsymbol{\tau}^R}\right)^{-1}\frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^R} \equiv -\frac{d\boldsymbol{x}^U}{d\boldsymbol{x}^R}$ is positive (negative) if regulated and unregulated decisions are gross substitutes (complements). Whenever imperfectly regulated decisions are i) underregulated and ii) complements to regulated ones, the second-best regulation $\boldsymbol{\tau}^R$ switches to overregulation relative to Pigou. Our examples below show that both substitutes and complements are common in financial regulation, depending on the nature of regulatory imperfections.

What emerges from this discussion is that there are forces pushing for both overregulation and underregulation in imperfect financial regulation. For instance, one cannot easily conclude whether one should optimally over- or underregulate bank capital in the presence of shadow banks. However, it would also be too pessimistic to conclude that "anything goes". Indeed, our results provide a clear guide to interpreting the economic forces at play: underregulated substitutes (and, conversely, overregulated complements) generate optimal regulations below the Pigouvian benchmark; while underregulated complements (and, conversely, overregulated substitutes) generate optimal regulations above the Pigouvian benchmark. The nature of the second-best policy then becomes the empirical question of disciplining — using either theory or data — the relevant leakages and wedges.

Note that the distortions $\boldsymbol{\delta}^R$ and $\boldsymbol{\delta}^U$ are, in general, endogenous to the regulatory regime. For instance, if a regulator underregulates banks' leverage to prevent leakage to shadow banks (substitutes), it could still be the case that the marginal social distortion $\boldsymbol{\delta}^R$ grows as a result of underregulation. In our simulations below, we solve for endogenous distortions given common market failures in financial regulation — e.g., fire sales, bank bailouts, and distorted beliefs — and take this mechanism into account.

An interesting alternative interpretation of Equation (9) is that a second-best planner uses the R free instruments $\boldsymbol{\tau}^R$ (on the left-hand side) to target the $R + U$ distortions contained in $\boldsymbol{\delta}^R$ and $\boldsymbol{\omega}^U$ (on the right-hand side). Only when $\boldsymbol{\omega}^U = 0$ does a first-best outcome obtain, consistent with the Tinbergen (1952) rule. More generally, Equation (9) refines the logic: with insu□cient policy instruments, the optimal regulation tries to set a weighted sum of all wedges to zero, with weights determined by leakage elasticities.[7]

[7]Equation (9) immediately implies that $\boldsymbol{\omega}^R \neq 0$ as long as $\frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^R}\boldsymbol{\omega}^U \neq 0$. This result can be related to the discussion of second-best policy in Lipsey and Lancaster (1956), who show that distortions in one market generally call for optimal distortions in others. In contrast to the anything-goes message of that paper, our

Proposition 1 is still an incomplete characterization of second-best financial regulation. To see this, consider uniform regulation. If the same regulations have to be applied to a number of heterogeneous investor types, then all decisions of those types would be classified as imperfectly regulated by our terminology, because they are associated with a binding constraint faced by the regulator. However, the regulator still has degrees of freedom, namely the level of the uniform tax or regulation $\boldsymbol{\tau}^U$. Similarly, in the fourth type of imperfection described above — where policy is costly due to political economy — all decisions are imperfectly regulated, but the regulator still needs to choose the level of the costly tax/regulation $\boldsymbol{\tau}^U$. Proposition 1 is silent on these choices, to which we turn in the next section.

3.2 Second-Best Policy: Imperfectly Regulated Decisions

Proposition 2 characterizes the second-best regulation of imperfectly regulated decisions:

Proposition 2. *(Second-Best Policy: Imperfectly Regulated Decisions) The optimal second-best regulation of imperfectly regulated decisions satisfies* $\frac{dW}{d\boldsymbol{\tau}^U} = \frac{d\boldsymbol{\Phi}}{d\boldsymbol{\tau}^U}\boldsymbol{\mu}$*, where* $\boldsymbol{\mu}$ *is the vector of Lagrange multipliers associated with the constraints on policy instruments, and*

$$\frac{dW}{d\boldsymbol{\tau}^U} = \frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^U}\left(\boldsymbol{I} - \boldsymbol{L}\right)\boldsymbol{\omega}^U, \tag{10}$$

where $\boldsymbol{I}$ *denotes an identity matrix,* $\frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^U}$ *is a matrix of own imperfectly regulated policy elasticities,* $\boldsymbol{\omega}^U$ *is the vector of Pigouvian wedges associated with imperfectly regulated decisions, and where we define the matrix* $\boldsymbol{L} = \left(\frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^U}\right)^{-1}\frac{d\boldsymbol{x}^R}{d\boldsymbol{\tau}^U}\left(\frac{d\boldsymbol{x}^R}{d\boldsymbol{\tau}^R}\right)^{-1}\frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^R}$.

Equation (10) contains the key economics in this proposition, and shows the marginal welfare effect of tightening the regulation $\boldsymbol{\tau}^U$ on imperfectly regulated decisions. We assess this effect while setting the regulation $\boldsymbol{\tau}^R$ to its optimal second-best value. To interpret it, consider the case where there are *only* imperfectly regulated decisions. Then, by Lemma 1, the welfare effect of raising $\boldsymbol{\tau}^U$ would be simply $\frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^U}\boldsymbol{\omega}^U$ — the product of policy elasticities and Pigouvian wedges. Equation (10) suggests that this welfare effect is attenuated by the matrix $\boldsymbol{L}$. Heuristically, one can write $\boldsymbol{L} \equiv \frac{d\boldsymbol{x}^R}{d\boldsymbol{x}^U}\frac{d\boldsymbol{x}^U}{d\boldsymbol{x}^R}$. This is the product of leakages $\frac{d\boldsymbol{x}^U}{d\boldsymbol{x}^R}$, which determine the optimal regulations given in Proposition 1, and *reverse leakages* $\frac{d\boldsymbol{x}^R}{d\boldsymbol{x}^U}$,

characterization shows that the second-best policy has substantial structure: optimal regulation depends systematically on leakage elasticities and patterns of gross complementarity and substitutability.

which are novel to this part of the analysis. Proposition 2 does not yield a closed-form expression for the second-best value of $\boldsymbol{\tau}^U$ in general. However, it can be used to support two novel economic insights.

First, Equation (10) provides the marginal welfare gain of relaxing constraints on regulation for a planner who is implementing the optimal second-best policy. A novel insight is that reverse leakage tends to attenuate the welfare benefit of relaxing constraints, because the matrix $\boldsymbol{L}$ is (heuristically) positive whenever perfectly and imperfectly regulated decisions are either complements or substitutes. The economic intuition when imperfectly regulated decisions are underregulated ($\boldsymbol{\omega}^U < 0$) is as follows: If perfectly and imperfectly regulated decisions are substitutes, tightening the regulation on imperfectly regulated decisions increases perfectly regulated decisions through reverse leakage. But Proposition 1 shows that perfectly regulated decisions are underregulated at the second-best, with $\boldsymbol{\omega}^R < 0$, so this increase reduces welfare. Conversely, if perfectly and imperfectly regulated decisions are complements, tightening the regulation on imperfectly regulated decisions reduces perfectly regulated decisions through reverse leakage. But Proposition 1 shows that perfectly regulated decisions are overregulated at the second-best, with $\boldsymbol{\omega}^R > 0$, so this decrease also reduces welfare. Intuitively, optimally regulating perfectly regulated decisions reduces the gains from further regulation.

We label $\boldsymbol{L}$ the attenuation matrix because this result is reminiscent of the Le Chatelier principle in classical economics. As noted by Milgrom (2006), the Le Chatelier principle more generally explains how the direct effect of a parameter change is augmented by feedback in a system. While existing versions of the principle point towards $amplification$ by feedback in a system, we find the opposite implication, namely $attenuation$, in terms of welfare effects.

Second, Proposition 2 gives a practical recipe for deriving second-best policies once the constraint function $\boldsymbol{\Phi}(\cdot)$ is specified. We focus on two cases from the list of imperfections in Section 2.2: uniform regulation and convex costs.

Uniform Regulation. Suppose that all elements of $\boldsymbol{\tau}^U$ have to be equal to a uniform regulation, which we denote by $\overline{\tau}^U$, while all other regulations $\boldsymbol{\tau}^R$ can be chosen freely. For a concrete example, which we explore in detail below, the elements of $\boldsymbol{\tau}^U$ could be leverage or investment regulations on heterogeneous financial institutions. In this case, Proposition 2 implies that the optimal second-best regulation of imperfectly regulated decisions specializes

to

$$\overline{\tau}^U = \frac{\boldsymbol{\iota}' \frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^U} (\boldsymbol{I} - \boldsymbol{L}) \boldsymbol{\delta}^U}{\boldsymbol{\iota}' \frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^U} (\boldsymbol{I} - \boldsymbol{L}) \boldsymbol{\iota}}, \tag{11}$$

where $\boldsymbol{\iota}$ denotes a vector of ones. Equation (11) shows that the optimal second-best uniform regulation $\overline{\tau}^U$ over a subset of decisions is simply a weighted sum/average of their marginal distortions $\boldsymbol{\delta}^U$. More responsive decisions carry higher weights. As expected, when marginal distortions are symmetric (so $\boldsymbol{\delta}^U = \boldsymbol{\iota}\overline{\delta}$), Equation (11) implies that the first-best regulation $\overline{\tau}^U = \overline{\delta}$ is optimal. However, when marginal distortions are heterogeneous, the first-best cannot be achieved with uniform regulation. Once again, all regulations are adjusted for the matrix $\boldsymbol{L}$ that measures the attenuation of welfare effects in general equilibrium.[8]

Convex Costs of Regulation. Another common imperfection is costly regulation. Formally, we consider a planner who faces quadratic costs of regulation over a subset of decisions, perhaps driven by political economy barriers to financial regulation, with $\boldsymbol{\Phi}(\boldsymbol{\tau}) = \frac{1}{2}\boldsymbol{\tau}^{U\prime}\boldsymbol{B}\boldsymbol{\tau}^U$, for some positive definite matrix $\boldsymbol{B}$. In this case, Proposition 2 implies that the optimal second-best regulation of imperfectly regulated decisions specializes to

$$\boldsymbol{\tau}^U = (\boldsymbol{B} + \boldsymbol{K})^{-1} \boldsymbol{K} \boldsymbol{\delta}^U, \tag{12}$$

where $\boldsymbol{K} = \left(-\frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^U}\right)(\boldsymbol{I} - \boldsymbol{L})$. Equation (12) shows that the optimal policy with quadratic adjustment costs is an attenuated version of the first-best policy. As expected, as costs vanish and $\boldsymbol{B} \to 0$, the optimal policy approaches the first-best, so $\boldsymbol{\tau}^U \to \boldsymbol{\delta}^U$. But as costs grow relative to $\boldsymbol{K}$, the optimal policy approaches zero, so $\boldsymbol{\tau}^U \to 0$. The reverse leakage effects discussed above, which make $\boldsymbol{L}$ positive and $\boldsymbol{K}$ smaller, further attenuate the optimal choice of $\boldsymbol{\tau}^U$.

In summary, the theory clarifies second-best corrective regulation by characterizing the optimal regulation of perfectly regulated decisions (Proposition 1) and imperfectly regulated decisions (Proposition 2), for a general class of regulatory imperfections. These results support practical analysis and unify classical insights often viewed as distinct. We now apply them to concrete problems in financial regulation.

[8] The insight that uniform regulation of heterogeneous externalities is characterized by a weighted sum/average of distortions can be traced back to Diamond (1973). Indeed, when *all* decisions are uniformly regulated, Equation (11) corresponds to Diamond's result. Equation (11) generalizes this result by allowing for a subset of decisions to be perfectly regulated.

3.3 Examples

In this section, we develop a series of examples that illustrate how our results apply to i) various problems in financial regulation, and ii) several widely studied rationales for regulation, including bailouts, pecuniary externalities, and behavioral distortions. Moreover, by studying specific applications, we can connect leakage (and policy) elasticities and Pigouvian wedges to model primitives. In the next section, we move beyond these minimal examples and provide a richer, novel application.

We give a brief exposition of each example in this section. Rigorous definitions and analysis are in Section C of the Online Appendix.

Example 1: Shadow Banking/Unregulated Investors There are two types of investors $i \in \{1, 2\}$ and creditors. Investors of type $i = 1$ are regulated traditional banks, and $i = 2$ are unregulated shadow banks. Regulation is motivated by the presence of government bailouts that benefit both banks and shadow banks. Investors are risk-neutral, with discount factor β^i. Creditors are risk-averse, with preferences $u\left(c_0^C\right) + \beta^C \int u\left(c_1^C(s)\right) dF(s)$.

Investors have a risky project/asset with fixed scale. At date 0, their (only) financing choice is the face value of debt b^i that they issue to creditors. At date 1 in state s, investor i receives the returns to her project, which are s, as well as a bailout transfer $t^i(b^i, s)$, which we assume is increasing in leverage, with $\frac{\partial t^i}{\partial b^i} > 0$. After this, she decides whether to repay b^i to creditors or default.

Creditors purchase banks' and shadow banks' debt for $Q^i(b^i)$ (determined in equilibrium by creditors) at date 0. At date 1, creditors are taxed $(1 + \kappa) \sum_{i=1}^{2} t^i(b^i, s)$ to pay for investor bailouts, where $\kappa > 0$ denotes the deadweight cost of fiscal intervention. Moreover, they receive a repayment $\mathcal{P}^i(b^i, s)$ from investors, which is equal to b^i in states where the investor does not default, and flexibly captures the deadweight costs of default otherwise.

A planner can impose leverage taxes/regulations τ_b^1 on traditional banks, but is forced to set $\tau_b^2 = 0$ for shadow banks. Applying Proposition 1, the marginal welfare effect of increasing the regulation on traditional banks' leverage is

$$\frac{dW}{d\tau_b^1} = \frac{db^1}{d\tau_b^1}\left(\tau_b^1 - \delta_b^1\right) - \frac{db^2}{d\tau_b^1}\delta_b^2, \tag{13}$$

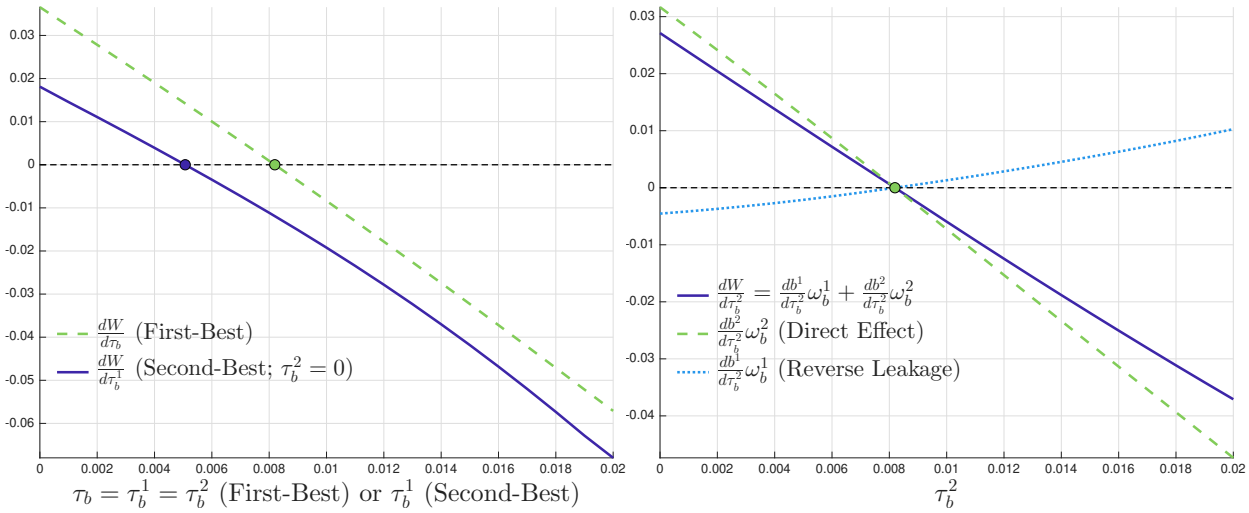


Figure 1: Shadow Banking/Unregulated Investors (Example 1)

Note: The left panel compares the marginal welfare effects of varying corrective regulation in two scenarios. The green dashed line corresponds to the first-best, with $\tau_b = \tau_b^1 = \tau_b^2$ on the horizontal axis. The solid blue line corresponds to the second-best in which $\tau_b^2 = 0$, with τ_b^1 on the horizontal axis. Since both types of investors are symmetric, the value of τ_b at which the first-best marginal welfare effect is zero defines the first-best regulation, and the value of τ_b^1 at which the second-best marginal welfare effect is zero defines the second-best regulation. The right panel illustrates Proposition 2 by showing the marginal value of regulating the shadow sector. The solid dark blue line is the total marginal welfare gain of increasing τ_b^2, with τ_b^1 continually adjusted to its optimal second-best value given τ_b^2. Following Equation (10), this gain decomposes into a direct effect, $\frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^U}\boldsymbol{\omega}^U$ (green dashed line), and a reverse leakage effect, $-\frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^U}\boldsymbol{L}\boldsymbol{\omega}^U$ (light blue dotted line). Both effects are zero at the first-best, $\tau_b = \tau_b^1 = \tau_b^2 = 0.82\%$, and have opposite signs otherwise. We assume that the bailout policy is linearly separable, $t^i\left(b^i, s\right) = \alpha_0^i - \alpha_s^i s + \alpha_b^i b^i$, and that creditors' utility is isoelastic, $u\left(c\right) = c^{1-\gamma}/\left(1-\gamma\right)$. The parameters are $\beta^i = 0.7$, $\phi^i = 0.75$ (the recovery rate in default), $\alpha_0^i = \alpha_s^i = 0$, and $\alpha_b^i = 0.01$ for $i \in \{1, 2\}$; $\kappa = 0.13$, $\gamma = 6$, $\beta^C = 0.98$, $n_0^C = 50$, and $n_1^C\left(s\right) = 50 + 0.1s$, where s is normally distributed with mean 1.3 and standard deviation 0.3, truncated to $[0, 3]$. The optimal first-best regulation is $\tau_b^1 = \tau_b^2 = 0.82\%$, while the optimal second-best regulation when the second type of investors cannot be regulated is $\tau_b^1 = 0.51\%$. Since borrowing decisions are gross substitutes in this example, the second-best policy is *sub-Pigouvian* ($\tau_b^1 < \delta_b^1$).

and one can show that the marginal bailout distortions δ_b^i are equal to

$$\delta_b^i = (1 + \kappa) \int m^C\left(s\right) \frac{\partial t^i\left(b^i, s\right)}{\partial b^i} dF\left(s\right). \tag{14}$$

Thus, the key distortions are the marginal e□ects of more leverage — either by banks or shadow banks — on bailouts in various states of the world at date 1. Because creditors pay for bailouts, the weighting of states is dictated by creditors' stochastic discount factor

(SDF) $m^C(s) = \frac{\beta^C u'(c_1^C(s))}{u'(c_0^C)}$. The optimal second-best policy is then given by

$$\tau_b^1 = \delta_b^1 - \left(-\frac{db^1}{d\tau_b^1}\right)^{-1} \frac{db^2}{d\tau_b^1}\delta_b^2. \tag{15}$$

Equation (15) makes clear that the key objects for its determination are the bailout distortions δ_b^i for both banks and shadow banks, and the leakage elasticity $\frac{db^2}{d\tau_b^1}$. This elasticity arises from the fact that leverage by banks changes creditors' SDF and, hence, the cost of funding for the other investor type; it does not generally have a closed-form solution. Intuitively, if banks take less leverage due to stricter regulation, creditors are exposed to less default risk (as long as bailouts are not generous enough to always rule out default), and therefore require a smaller risk premium when lending to shadow banks. All this implies stronger incentives for shadow banks to borrow, so that we would expect $\frac{db^2}{d\tau_b^1} > 0$. Existing empirical work, while not exactly equivalent to this formula, also shows that bank and shadow bank activities tend to be substitutes (e.g., Irani et al., 2021). In that context, Equation (15) says that the second-best policy τ_b^1 ought to $underregulate$ traditional banks relative to the Pigouvian benchmark, with $\tau_b^1 < \delta_b^1$.[9]

The left panel of Figure 1 compares the first-best and second-best policies when simulating this model. Here, we make banks and shadow banks ex-ante identical, so the first-best regulation always sets $\tau_b = \tau_b^1 = \tau_b^2$. The dashed line shows the marginal welfare effect of tightening first-best regulation. In contrast, the solid line shows the marginal value of varying τ_b^1 (traditional bank regulation) when shadow banks are unregulated ($\tau_b^2 = 0$). In line with the intuition presented above, we find that the optimal second-best policy is sub-Pigouvian, so the regulation that investor 1 faces is lower than the first-best regulation.

The right panel of Figure 1 illustrates Proposition 2 by showing the marginal value of being able to regulate the shadow sector. This panel shows the reverse leakage adjustment discussed above. Regardless of whether the shadow sector is underregulated (when τ_b^2 is below first-best) or overregulated (when τ_b^2 is above first-best), the reverse leakage effect has the opposite sign of the direct effect of adjusting the regulation of the shadow sector, attenuating welfare gains/losses. This illustrates how the presence of perfectly regulated decisions contributes to attenuating the welfare gains of relaxing constraints on regulation.

[9] This is because $\frac{db^1}{d\tau_b^1} < 0$, $\frac{db^2}{d\tau_b^1} > 0$, and $\delta_b^2 > 0$ imply that $\tau_b^1 < \delta_b^1$.

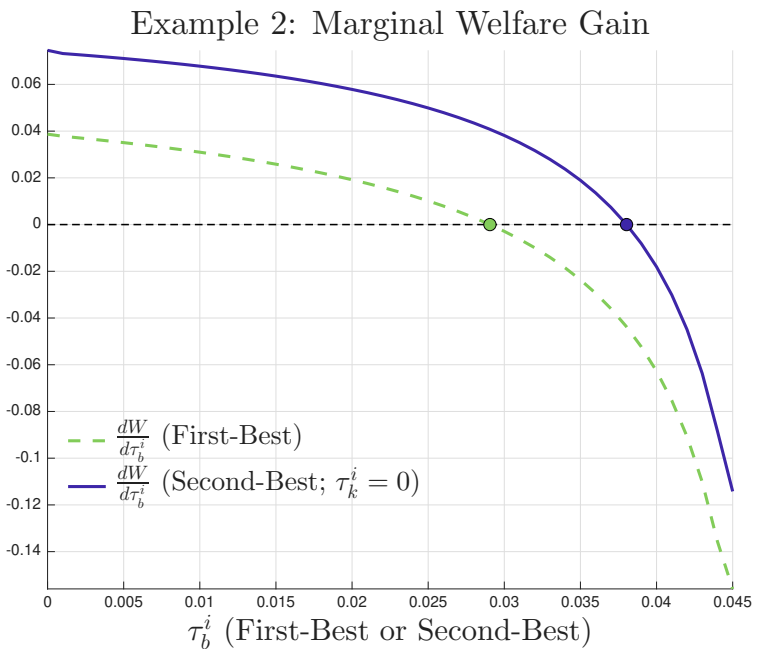


Figure 2: Behavioral Distortions/Unregulated Decisions (Example 2)

Note: Marginal welfare effects under first-best and second-best regulation. The green dashed line corresponds to a scenario in which τ_k^i is set at the first-best level. The solid blue line corresponds to a second-best scenario in which $\tau_k^i = 0$. The value of τ_b^i that makes the first-best marginal welfare effect zero defines the first-best leverage regulation, since τ_k^i is already set at the first-best level. The value of τ_b^i that makes the second-best marginal welfare effect zero defines the second-best regulation. To generate this figure, we assume that the adjustment cost is quadratic: $\Upsilon\left(k^i\right) = k^i + \frac{a}{2}\left(k^i\right)^2$. The parameters are $\beta^i = 0.9$, $\beta^C = 0.95$, $\phi = 0.8$ (the recovery rate in default), and $a = 1$. We assume that investors and creditors perceive s to be normally distributed with mean 1.5 and standard deviation 0.4, truncated to the interval $[0, 4.5]$, and the planner perceives the mean to be 1.3 instead. For reference, the optimal first-best regulation is given by $\tau_b^i = 2.91\%$ and $\tau_k^i = 18.45\%$, while the second-best regulation, when investment cannot be regulated, is $\tau_b^i = 3.80\%$. Since leverage and investment decisions are gross complements in this example, the optimal second-best policy is *super-Pigouvian* ($\tau_b^i > \delta_b^i$).

Example 2: Behavioral Distortions/Unregulated Decisions There is a single type of investor i, and a unit measure of creditors C. Their beliefs about the aggregate state s are captured by the cumulative distribution functions $F^i(s)$ and $F^C(s)$. A planner has beliefs $F^P(s)$ about the aggregate state, which may di□er from those of investors/creditors and provide a rationale for (paternalistic) regulation even in the absence of bailouts. Moreover, investors and creditors are risk-neutral, with discount factors β^i and β^C, respectively.

Investors generate k^i units of productive capital at date 0, with adjustment costs $\Upsilon(k^i)$, and issue debt with face value $b^i k^i$ to creditors — the debt-asset ratio b^i is a measure of leverage. At date 1 in state s, investor i receives returns $s \cdot k^i$ to investment. After this, she decides whether to repay to creditors or default. Creditors purchase banks' debt for

$Q^i(b^i)\,k^i$ at date 0. At date 1, they receive a repayment $\mathcal{P}^i(b^i,s)\,k^i$, which can capture default costs, as described in Example 1.

The planner imposes a capital/leverage requirement $b^i \leq \bar{b}$ on investors. We show in Section C.2 of the Online Appendix that the leverage cap $\bar{b}$, when binding, is equivalent to a corrective tax τ_b^i on investors' leverage choices b^i. However, it leaves the scale k^i of risky investment unregulated, so that the implicit corrective tax on scale is $\tau_k^i = 0$ — this is a common regulatory imperfection in reality, and the one that we focus on in this example.

The key object in equilibrium — as highlighted in Dávila and Walther (2023) — is an agent's valuation $M(b^i)$ for a unit of investment, which represents a levered version of Tobin's q, and is defined by

$$M\left(b^i\right) = \beta^i \int \max\left\{s - b^i, 0\right\} dF^i(s) + \beta^C \int \mathcal{P}^i\left(b^i, s\right) dF^C(s). \tag{16}$$

By contrast, the paternalist planner values a unit of investment according to $M^P(b^i)$, which replaces agents' beliefs in Equation (16) with the planner's beliefs $F^P(s)$.

Applying Proposition 1, the marginal welfare effect of increasing the regulation of investors' leverage is

$$\frac{dW}{d\tau_b^i} = \frac{db^i}{d\tau_b^i}\left(\tau_b^i - \delta_b^i\right)k^i - \frac{dk^i}{d\tau_b^i}\delta_k^i, \tag{17}$$

where the marginal distortions in this example are defined by

$$\delta_b^i = \frac{dM(b^i)}{db^i} - \frac{dM^P(b^i)}{db^i} \quad \text{and} \quad \delta_k^i = M\left(b^i\right) - M^P\left(b^i\right).$$

The optimal second-best regulation satisfies

$$\tau_b^i = \delta_b^i - \left(-\frac{db^i}{d\tau_b^i}\right)^{-1} \frac{d\log k^i}{d\tau_b^i}\delta_k^i. \tag{18}$$

Equation (18) summarizes the second-best adjustment. In this example, one can show analytically that unregulated and regulated decisions are complements: the leakage elasticity from leverage to investment is negative ($\frac{dk^i}{d\tau_b^i} < 0$). Since agents are more optimistic than the planner, the scale distortion is positive, $\delta_k^i > 0$. As implied by our results in Section 3, the optimal second-best regulation on leverage is then greater than the Pigouvian benchmark ($\tau_b^i > \delta_b^i$). A comparison between this example and the previous one (shadow banking) highlights that both leakage elasticities featuring substitutes and those featuring

complements matter in common regulatory scenarios. A number of recent empirical studies confirm that the leakage elasticity from leverage to risky investments is negative, in the sense that banks with lower capital ratios originate a larger volume of risky loans (e.g., Jiménez et al., 2014; Dell'Ariccia, Laeven and Suarez, 2017; Acharya et al., 2018; Plosser and Santos, 2024).

Example 3: Asset Substitution/Uniform Decision Regulation There is one type of investor i, with discount factor β^i, and creditors, who do not discount; all agents are risk-neutral. Investors choose the scale of two risky capital investments k_1^i and k_2^i, which are subject to an adjustment cost of $Y\left(k_1^i, k_2^i\right)$. For each unit of investment, investors raise one dollar in insured deposits from creditors, so total deposits are $b^i = k_1^i + k_2^i$. At date 1 in state s, investor i receives the payoff $d_1(s) k_1^i + d_2(s) k_2^i$. Notice that the risk/return profiles of the two assets can be different when $d_1(s) \neq d_2(s)$. If payoffs are insufficient to repay creditors' deposits, the government pays the remainder, financed by a tax on creditors with deadweight cost κ as in Example 1, while investors consume zero.[10]

A planner can regulate investments to combat the moral hazard associated with deposit insurance, but is subject to a uniformity constraint:

$$\overline{\tau}_k = \tau_k^1 = \tau_k^2.$$

Practically, this would occur if capital types 1 and 2 are different loans made by banks, which nevertheless attract the same regulatory risk weight. It could also occur if capital of type 2, say, is riskier, but a bank uses internal models to "game" risk weighting and ensures that it gets treated the same as type 1 by the regulator. Applying our general results, the marginal welfare effect of increasing investment regulation is

$$\frac{dW}{d\overline{\tau}_k} = \frac{dk_1^i}{d\overline{\tau}_k}\left(\overline{\tau}_k - \delta_1\right) + \frac{dk_2^i}{d\overline{\tau}_k}\left(\overline{\tau}_k - \delta_2\right), \tag{19}$$

where the marginal distortions are

$$\delta_j = (1+\kappa)\int_{\underline{s}}^{s^\star\left(k_1^i, k_2^i\right)} \left[1 - d_j(s)\right] dF(s), \qquad j \in \{1,2\}, \tag{20}$$

with $s^\star\left(k_1^i, k_2^i\right)$ denoting the threshold state below which bailouts become necessary.

[10] Formally, the bailout function is $t\left(k_1^i, k_2^i, s\right) = \max\left\{k_1^i + k_2^i - d_1(s) k_1^i - d_2(s) k_2^i, 0\right\}$.

The distortion in Equation (20) captures standard concerns about risk-shifting or asset substitution: investors do not internalize that an additional dollar invested in asset j raises expected bailout costs whenever $d_j(s) < 1$ in bailout states. This externality can generate excessive investment in risky assets. In particular, if asset 2 is riskier in that it has lower payoffs than asset 1 in states with bailouts, then $\delta_2 > \delta_1$.

The corresponding second-best regulation satisfies

$$\overline{\tau}_k = \frac{\frac{dk_1^i}{d\overline{\tau}_k}\delta_1 + \frac{dk_2^i}{d\overline{\tau}_k}\delta_2}{\frac{dk_1^i}{d\overline{\tau}_k} + \frac{dk_2^i}{d\overline{\tau}_k}}. \tag{21}$$

Equation (21) shows that the optimal uniform regulation is a weighted average of the distortions imposed by both types of investment. The appropriate weight on each distortion is determined by the sensitivity of the corresponding investment to the uniform regulation. Thus, when both investment responses have the same sign and $\delta_2 > \delta_1$, the common risk weight lies between the two first-best regulations: it overregulates the safer asset and underregulates the riskier one.

Assuming separable quadratic investment costs, it is possible to provide further intuition on the weights in Equation (21) — see Section C.3 of the Online Appendix. The weights depend on the curvature of investment costs, the sensitivity of the bailout probability to uniform regulation, and each asset's net payoff $d_j(s^\star) - 1$ at the bailout boundary. These objects determine how strongly each investment responds when the same regulatory wedge is applied to both assets.

Example 4: Pecuniary Externalities/Uniform Investor Regulation In this example, we study the form of the second-best policy in an environment with fire-sale externalities. We focus on a case in which it would be optimal to set investor-specific regulations, but the planner is constrained to set the same corrective regulation for all investors. There are two types of investors $i \in \{1, 2\}$, and households H. There are three dates, $t \in \{0, 1, 2\}$, and no uncertainty. Investors are risk-neutral and do not discount the future.

Investors generate k_0^i units of capital at date 0, with adjustment cost $\Upsilon^i(k_0^i)$, financed from their own endowment. At date 1, they face a reinvestment need $\xi^i k_0^i$, which they meet by selling $k_0^i - k_1^i$ units of capital to households at an endogenous price q. At date 2, their remaining capital yields $z^i k_1^i$. Households are also risk-neutral, and can turn k_1^H units of

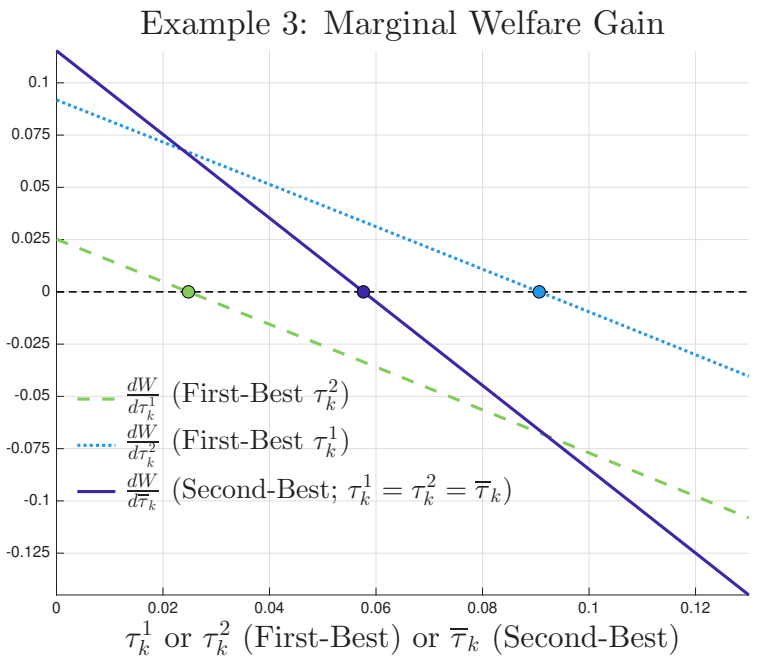


Figure 3: Asset Substitution/Uniform Decision Regulation (Example 3)

Note: This figure compares the marginal welfare effects of varying corrective regulation under first-best asset-specific regulation and second-best uniform regulation. The dashed and dotted lines vary one asset-specific tax while holding the other at its first-best level; the solid line varies the common tax $\overline{\tau}_k = \tau_k^1 = \tau_k^2$. We assume separable quadratic adjustment costs, $\Upsilon\left(k_1^i, k_2^i\right) = \frac{1}{2}\left(k_1^i\right)^2 + \frac{1}{2}\left(k_2^i\right)^2$, and returns $d_j(s) = \bar{d} + \sigma_j s$, where s is standard normal. Parameters are $\beta^i = 0.8$, $\kappa = 0.1$, $\bar{d} = 1.3$, $\sigma_1 = 0.3$, and $\sigma_2 = 0.5$. The first-best regulations are $\tau_k^1 = 2.48\%$ and $\tau_k^2 = 9.06\%$, while the optimal uniform regulation is $\overline{\tau}_k = 5.76\%$.

purchased capital into $G\left(k_1^H\right)$ units of consumption, where $G(\cdot)$ is concave. This setup defines a downward-sloping demand curve for sold capital at date 1.

As in Lorenzoni (2008), the failure to internalize fire-sale prices q can generate socially excessive investment/credit booms among investors. A planner can regulate investment but is subject to a uniformity constraint across investor types:

$$\overline{\tau}_k = \tau_k^1 = \tau_k^2,$$

which makes the problem of choosing the optimal $\overline{\tau}_k$ a second-best problem.

While this environment is richer than our baseline model, we show in Section C.4 of the Online Appendix that it can be mapped to our general results. The marginal welfare effect of varying the uniform corrective regulation of investments, $\overline{\tau}_k$, is given by

$$\frac{dW}{d\overline{\tau}_k} = \frac{dk_0^1}{d\overline{\tau}_k}\left(\overline{\tau}_k - \delta_k^1\right) + \frac{dk_0^2}{d\overline{\tau}_k}\left(\overline{\tau}_k - \delta_k^2\right), \tag{22}$$

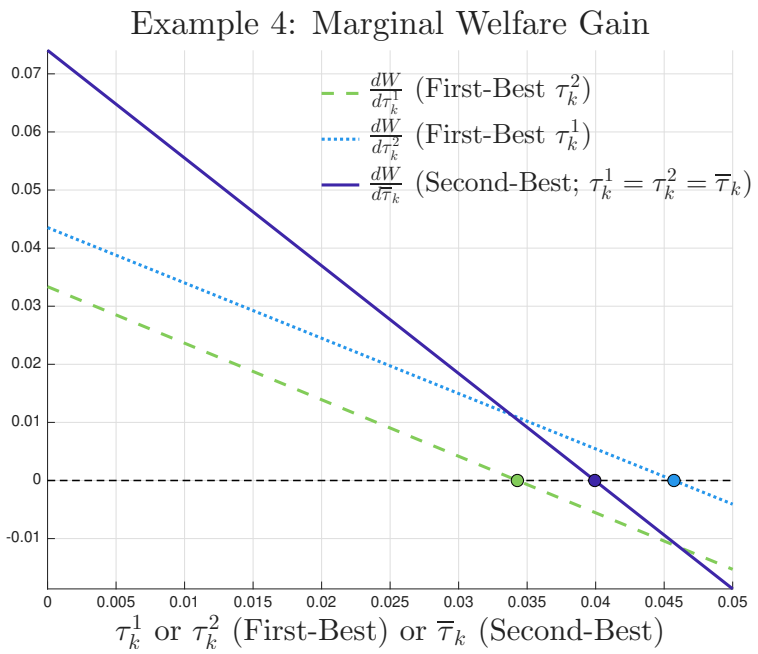


Figure 4: Pecuniary Externalities/Uniform Investor Regulation (Example 4)

Note: This figure compares the marginal welfare effects of varying the corrective regulation in two different scenarios. The green dashed line and the light blue dotted line illustrate the first-best regulation. The green dashed line corresponds to a scenario in which τ_k^2 is set at the first-best level (previously computed), while the light blue dotted line corresponds to a scenario in which τ_k^1 is set at the first-best level. Therefore, the values of τ_k^1 and τ_k^2 that respectively make each line zero define the first-best regulation. The solid dark blue line corresponds to a second-best scenario in which $\tau_k^1 = \tau_k^2 = \overline{\tau}_k$, so its zero defines the second-best regulation. To generate this figure, we assume that the adjustment cost of investment is quadratic: $\Upsilon^i\left(k_0^i\right) = \frac{a^i}{2}\left(k_0^i\right)^2$, and that the technology of households is isoelastic: $G\left(k_1^H\right) = \left(k_1^H\right)^\alpha/\alpha$. The parameters are $\alpha = 0.5$, $a^1 = a^2 = 1$, $z^1 = z^2 = 1.5$, $\xi^1 = 0.3$, and $\xi^2 = 0.4$. For reference, the optimal first-best regulation is given by $\tau_k^1 = 3.43\%$ and $\tau_k^2 = 4.57\%$, while the second-best regulation, when the regulation is uniform, is $\overline{\tau}_k = 3.99\%$.

where

$$\delta_k^i = -\frac{\partial q}{\partial k_0^i}\sum_{\ell=1}^{2}\left(\frac{z^\ell}{q} - 1\right)\left(k_0^\ell - k_1^\ell\right). \tag{23}$$

This is a distributive pecuniary externality in the sense of Dávila and Korinek (2018): the distortion combines the sensitivity of the fire-sale price to investment, differences in marginal valuations, and investors' net trade positions.

The corresponding second-best regulation satisfies

$$\overline{\tau}_k = \frac{\frac{dk_0^1}{d\overline{\tau}_k}\delta_k^1 + \frac{dk_0^2}{d\overline{\tau}_k}\delta_k^2}{\frac{dk_0^1}{d\overline{\tau}_k} + \frac{dk_0^2}{d\overline{\tau}_k}}. \tag{24}$$

Equation (24) weights each distortion by the sensitivity of the corresponding capital decision

to the uniform regulation. Figure 4 compares the resulting first- and second-best marginal welfare effects.

In the next section, we present an application to financial regulation with environmental externalities. This application is significantly richer than the minimal examples above. It will also combine several of the channels that the examples studied in isolation; in that sense, we aim to illustrate that our framework can be employed in a transparent analysis of second-best policy problems with many moving parts.

4 Financial Regulation with Environmental Externalities

Central banks and macro-prudential regulators have become increasingly interested in accounting for environmental concerns. There are two possible motivations for this. First, there are links between the financial system, a primary target of macro-prudential regulation, and climate-related risks, as evidenced by a growing literature on climate finance (e.g., Giglio, Kelly and Stroebel, 2021). For instance, financial institutions may be exposed to climate-related or transition risks. Second, some believe that prudential regulation should take into account its effect on the broader societal goal of sustainable investment. For instance, the European Central Bank's bond purchase program has taken the latter motivation into account by introducing preferential treatment for bonds associated with "green" technologies (Papoutsi, Piazzesi and Schneider, 2021).

A nascent academic literature studies the welfare implications of financial regulation with environmental concerns (e.g., Oehmke and Opp, 2026; Döttling and Rola-Janicka, 2025). We now use our general results to characterize optimal policy with environmental externalities and *imperfect macro-prudential regulation*. We analyze both unintended consequences of regulations and the appropriate second-best responses.[11]

This application features financial regulation when investors choose a portfolio of investments with different environmental effects. Regulation is conducted via risk-weighted

[11] For example, Andrew Bailey, the Governor of the Bank of England, has stated that

> "*any incorporation of climate change into regulatory capital requirements would need to be grounded in robust data and be designed to support safety and soundness while avoiding unintended consequences or compromising our other objectives*".

See https://www.bankofengland.co.uk/speech/2021/june/andrew-bailey-reuters-events-global-responsible-business-2021.

leverage constraints. As discussed briefly in Example 2 above, these requirements constrain only relative quantities on institutions' balance sheets but leave the overall scale of investment as a free variable. This imperfection is especially relevant in the context of climate finance: ultimately, investment scale is what drives environmental impacts. To capture the two motivations for policy discussed above, we pay special attention to contrasting the role of climate-related risks when regulation has a narrow/financial mandate versus a broad/environmental mandate. Our results, which directly apply the formulae from the general model, yield new insights into the differences between these two cases, and into the way in which climate-conscious regulation should be adjusted for imperfections. Finally, we characterize the value of extending the set of policy tools in the face of environmental externalities, which relates to the Le Chatelier/reverse leakage adjustments that we have characterized in the general case.

Environment. We assume that there is a single type of investor, in unit measure and indexed by i, and a unit measure of creditors, indexed by C. Both investors and creditors have risk-neutral preferences given by

$$c_0^i + \beta^i \int c_1^i(s)\, dF(s) \quad \text{and} \quad c_0^C + \beta^C \int c_1^C(s)\, dF(s) - \Psi\left(\theta^i\right) k^i,$$

where the term $\Psi(\theta^i) k^i$ introduces an environmental externality, as described below.[12] The budget constraints of investors at date 0 and date 1 are given by

$$\begin{aligned} c_0^i &= n_0^i + Q^i\left(b^i, \theta^i\right) k^i - k^i - \Upsilon\left(k^i\right) - \Omega\left(\theta^i\right) k^i, \\ c_1^i(s) &= k^i \max\left\{d_1(s)\,\theta^i + d_2(s)\left(1-\theta^i\right) + t\left(b^i, \theta^i, s\right) - b^i, 0\right\}, \quad \forall s. \end{aligned}$$

At date 0, investors, endowed with n_0^i, make capital investments k^i in two sectors. A fraction θ^i is invested in sector 1, and the remaining $1-\theta^i$ in sector 2. Investors issue debt with face value $b^i k^i$ to creditors, so that b^i measures investors' leverage. The equilibrium price of debt can be written as $Q^i(b^i, \theta^i) k^i$, where $Q^i(b^i, \theta^i)$ denotes the market value of debt per unit of capital. A unit of capital costs one unit of the consumption good. Capital investments are also subject to an adjustment cost $\Upsilon(k^i)$ and an additional cost $\Omega(\theta^i) k^i$ of adjusting the sectoral portfolio composition. At date 1, once a state s is realized, investor i receives

[12] The assumption that this distortion only impacts creditors and is linear in capital simplifies the exposition, but does not affect the qualitative insights of our analysis.

$d_j(s)$ per unit of investment in sector $j \in \{1,2\}$, and a bailout transfer $t(b^i, \theta^i, s)$ per unit of capital that may depend on the amount of debt issued and portfolio weights. If the sum of these revenues is larger than the face value of debt, investors repay their debt and consume the residual claim. Otherwise, they optimally choose to default.[13]

In this application, motivated by the existing regulatory instruments, we assume that investors are subject to a *risk-weighted capital requirement*:[14]

$$b^i + \varphi\theta^i \leq \bar{b}. \tag{25}$$

Intuitively, as in Example 2 above, the requirement in Equation (25) places an upper bound $\bar{b}$ on investors' leverage but, extending the example, the leverage cap is adjusted in proportion to the share θ^i invested in sector 1. In the case with $\varphi > 0$, on which we will focus without loss of generality, the relative risk weight φ on sector 1 is positive, and the leverage cap becomes tighter when investors increase θ^i. Imposing this constraint on investors is equivalent to imposing corrective taxes, as we will discuss below.

The budget constraints of creditors at date 0 and date 1 are given by

$$\begin{aligned} c_0^C &= n_0^C - h^i Q^i\left(b^i, \theta^i\right) k^i, \\ c_1^C(s) &= n_1^C(s) - (1+\kappa)\, t\left(b^i, \theta^i, s\right) k^i + h^i \mathcal{P}^i\left(b^i, \theta^i, s\right) k^i. \end{aligned}$$

At date 1, creditors are taxed $(1+\kappa)$ times the government bailout, where $\kappa > 0$ denotes the deadweight cost of fiscal intervention. Moreover, creditors who buy a fraction h^i of investors' debt pay the market price at date 0, and receive a payment $\mathcal{P}^i(b^i, \theta^i, s)\, k^i$ at date 1. This payment, which incorporates investors' optimal default decision, is defined as

[13] This specification of bailouts corresponds to a model where the government has limited commitment, which connects our work to the treatment of bailouts in Farhi and Tirole (2012), Bianchi (2016), Chari and Kehoe (2016), Keister (2016), Gourinchas, Martin and Messer (2025), Cordella, Dell'Ariccia and Marquez (2018), Dávila and Walther (2020), and Dovis and Kirpalani (2020), among others.

[14] Risk-weighted capital requirements under the Basel Accords ensure that the ratio of equity to risk-weighted assets in leveraged institutions (e.g., banks) is at least equal to a constant fraction $\bar{c}$. In our context, equity is $\left(1-b^i\right)k^i$ and risk-weighted assets can be represented as $\left[w_1\theta^i + w_2\left(1-\theta^i\right)\right]k^i$, where w_j is the risk weight on sector j investments. Thus, we can express a risk-weighted capital requirement as

$$1 - b^i \geq \bar{c}\left[w_1\theta^i + w_2\left(1-\theta^i\right)\right] \iff b^i + \underbrace{\bar{c}(w_1 - w_2)}_{\equiv\varphi}\theta^i \leq \underbrace{1 - \bar{c}w_2}_{\equiv\bar{b}},$$

which is equivalent to our formulation in (25), with φ denoting the relative risk weight on sector 1 investments, scaled by the required ratio $\bar{c}$.

follows:

$$\mathcal{P}^i\left(b^i,\theta^i,s\right)=\begin{cases} b^i, & d_1(s)\,\theta^i+d_2(s)\,(1-\theta^i)+t\left(b^i,\theta^i,s\right)\geq b^i \\ \phi\left[d_1(s)\,\theta^i+d_2(s)\,(1-\theta^i)\right. & \\ \qquad \left.+t\left(b^i,\theta^i,s\right)\right], & \text{otherwise.}\end{cases}$$

Investors default when their assets are worth less than the promised repayment b^i per unit of capital, and repay b^i in full otherwise. In default, creditors recover a fraction $\phi<1$ of their assets, so that $1-\phi$ can be interpreted as the deadweight cost of default. For simplicity, we assume that primitives are such that there exists a default threshold $s^\star\left(b^i,\theta^i\right)$, so that investors default when $s<s^\star\left(b^i,\theta^i\right)$ and repay otherwise.[15]

Finally, recall that creditors' preferences include a utility loss of $\Psi\left(\theta^i\right)k^i$ as a result of investors' choices. This term reflects an environmental externality. Investors' portfolio choices θ^i can affect this loss. For example, if $\Psi'\left(\theta^i\right)>0$, then the environmental externality is increasing in the investment share in sector 1, meaning that sector 1 is associated with more pollution than sector 2.

Equilibrium. For given regulatory parameters $\left\{\overline{b},\varphi\right\}$ defining the constraint (25) and a given bailout policy $t\left(b^i,\theta^i,s\right)$, an *equilibrium* is defined by leverage, portfolio, and investment decisions $\{b^i,\theta^i,k^i\}$, a default decision rule, and a pricing schedule $Q^i\left(b^i,\theta^i\right)$ such that investors and creditors maximize their utility and the market for debt clears, i.e., $h^i=1$.

We rely on the fact that equilibrium choices $\{b^i,\theta^i,k^i\}$ are given by the solution to the following reformulation of the problem faced by investors:

$$\max_{\{b^i,\theta^i,k^i\}}\left[M\left(b^i,\theta^i\right)-\Omega\left(\theta^i\right)-1\right]k^i-\Upsilon\left(k^i\right)\qquad\text{subject to}\qquad k^i\left(b^i+\varphi\theta^i\right)\leq k^i\overline{b},\quad(26)$$

[15] The threshold $s^\star\left(b^i,\theta^i\right)$ is unique under the standard assumptions that i) $d_j(s)$, $j\in\{1,2\}$, is increasing in s (i.e., higher asset returns in good states), and ii) the bailout transfer $t\left(b^i,\theta^i,s\right)$ is decreasing in s and increasing in b^i (i.e., larger bailouts in bad states/for more levered investors).

where $M\left(b^i, \theta^i\right)$ is given by

$$M\left(b^i, \theta^i\right) = \underbrace{\beta^i \int_{s^\star\left(b^i,\theta^i\right)}^{\bar{s}} \left(d_1(s)\,\theta^i + d_2(s)\left(1-\theta^i\right) + t\left(b^i, \theta^i, s\right) - b^i\right) dF(s)}_{\text{equity valuation}}$$
$$+ \underbrace{\beta^C \left(\int_{s^\star\left(b^i,\theta^i\right)}^{\bar{s}} b^i dF(s) + \phi \int_{\underline{s}}^{s^\star\left(b^i,\theta^i\right)} \left[d_1(s)\,\theta^i + d_2(s)\left(1-\theta^i\right) + t\left(b^i, \theta^i, s\right)\right] dF(s) \right)}_{\text{debt valuation} \quad = Q^i\left(b^i,\theta^i\right)}. \tag{27}$$

Intuitively, the function $M\left(b^i, \theta^i\right)$ can be interpreted as the sum of the market values of equity (owned by investors) and debt (owned by creditors) per unit of investment. The second term in Equation (27) corresponds to the equilibrium price of debt $Q^i\left(b^i, \theta^i\right)$, which incorporates the fact that investors default in states $s < s^\star\left(b^i, \theta^i\right)$ in which the value of their assets is less than the promised repayment b^i. In problem (26), investors maximize the market value of investment net of costs. Without loss of generality, we scale the regulatory constraint by total investment $k^i \geq 0$.

An important aspect of this application is that the planner's instruments, and the equivalent corrective taxes, are imperfect. This can be seen by writing investors' first-order conditions as follows:

$$\frac{\partial M\left(b^i, \theta^i\right)}{\partial b^i} = \upsilon \equiv \tau_b \tag{28}$$

$$\frac{\partial M\left(b^i, \theta^i\right)}{\partial \theta^i} - \Omega'\left(\theta^i\right) = \upsilon\varphi \equiv \tau_\theta \tag{29}$$

$$M\left(b^i, \theta^i\right) - \Omega\left(\theta^i\right) - 1 - \Upsilon'\left(k^i\right) = 0, \tag{30}$$

where $\upsilon \geq 0$ is the Lagrange multiplier on the regulatory constraint. The first two conditions, which define optimal leverage and portfolio weights, show that the constraint in Equation (25) implies effective corrective taxes τ_b on leverage b^i and τ_θ on portfolios θ^i. The third condition, which defines optimal total investment k^i, does not contain a corrective tax. Intuitively, the capital requirement (25) constrains ratios but leaves the overall scale k^i of investors' balance sheet as a free, unregulated variable. By contrast, in a world with perfect instruments, the planner would be able to set a corrective tax τ_k on k^i in addition to τ_b and τ_θ. We return to the value of introducing such a tax below.

Optimal Corrective Policy. In this environment, we can express the marginal externalities $\{\delta_b, \delta_\theta, \delta_k\}$ associated with investors' choices and decompose them into a financial (i.e., bailout-related) and an environmental component as follows:

$$\delta_b = \underbrace{(1+\kappa)\,\beta^C \int_{\underline{s}}^{\bar{s}} \frac{\partial t\left(b^i, \theta^i, s\right)}{\partial b^i} dF(s)}_{\equiv \chi_b} \tag{31}$$

$$\delta_\theta = \underbrace{(1+\kappa)\,\beta^C \int_{\underline{s}}^{\bar{s}} \frac{\partial t\left(b^i, \theta^i, s\right)}{\partial \theta^i} dF(s)}_{\equiv \chi_\theta} + \underbrace{\Psi'\left(\theta^i\right)}_{\equiv \psi_\theta} \tag{32}$$

$$\delta_k = \underbrace{(1+\kappa)\,\beta^C \int_{\underline{s}}^{\bar{s}} t\left(b^i, \theta^i, s\right) dF(s)}_{\equiv \chi_k} + \underbrace{\Psi\left(\theta^i\right)}_{\equiv \psi_k}. \tag{33}$$

The term χ_k in Equation (33) measures the marginal distortion in capital choices due to bailouts, while ψ_k is the distortion due to environmental externalities. Equations (31) and (32) define the distortions associated with leverage and portfolio choices per unit of capital. An important point is that leverage induces only a financial distortion, since environmental damage is determined by the technologies that are operated in this economy, and is independent of how these technologies are financed.

Applying our general results, the welfare effects of varying the leverage cap $\bar{b}$ and the risk weight φ, respectively, are given by

$$\frac{dW}{db} = \frac{db^i}{db}\left(\tau_b - \delta_b\right) k^i + \frac{d\theta^i}{db}\left(\tau_\theta - \delta_\theta\right) k^i - \frac{dk^i}{db}\delta_k, \tag{34}$$

$$\frac{dW}{d\varphi} = \frac{db^i}{d\varphi}\left(\tau_b - \delta_b\right) k^i + \frac{d\theta^i}{d\varphi}\left(\tau_\theta - \delta_\theta\right) k^i - \frac{dk^i}{d\varphi}\delta_k. \tag{35}$$

Notice that, even though we are working in terms of a quantity constraint, our general characterization of welfare effects from Lemma 1 applies, after suitably adjusting for k^i. This feature highlights the usefulness of our approach for analyzing quantity-based regulation.

Specifically, Equations (34) and (35) show that marginal welfare effects depend on Pigouvian wedges — defined in terms of the equivalent taxes $\{\tau_b, \tau_\theta\}$ in Equations (28) and (29) — as well as policy elasticities. First-best regulation is prevented by the fact that the unregulated scale decision k^i introduces an additional distortion δ_k.

In the remainder of this section, we use this characterization to derive concrete insights into optimal regulation with environmental externalities. First, we analyze the distinction

between optimal policy motivated by a narrow financial stability mandate and a broader mandate that accounts for environmental externalities. Importantly, we provide a novel treatment of these questions accounting for imperfections in policy instruments. Finally, we characterize the value of relaxing constraints on regulation by imposing corrective regulation on the total scale of investment.

4.1 Imperfect Regulation with Narrow/Financial Mandates

We first consider a financial regulator who has a narrow mandate and is only concerned with financial externalities. In terms of our decomposition of distortions, we interpret a narrow mandate as meaning that the regulator acts as if the climate-related distortions $\{\psi_\theta, \psi_k\}$ are both equal to zero. In the background, one can interpret that the distribution of states, $F(s)$, and the payo□s of the di□erent investments, $d_1(s)$ and $d_2(s)$, account for climate risks. Applying our general results yields the optimal policy in this case:

Corollary 1. *(Imperfect Regulation with Narrow/Financial Mandates) The optimal policy of a regulator with a narrow/financial mandate is given by*

$$\begin{pmatrix} \tau_b \\ \tau_\theta \end{pmatrix} = \begin{pmatrix} \chi_b \\ \chi_\theta \end{pmatrix} + \begin{pmatrix} \frac{db^i}{d\bar{b}} & \frac{d\theta^i}{d\bar{b}} \\ \frac{db^i}{d\varphi} & \frac{d\theta^i}{d\varphi} \end{pmatrix}^{-1} \begin{pmatrix} \frac{d\log k^i}{d\bar{b}} \\ \frac{d\log k^i}{d\varphi} \end{pmatrix} \chi_k, \tag{36}$$

where χ_b, χ_θ, and χ_k denote the financial component of the respective externalities, defined in Equations (31) *through* (33)*.*

Equation (36) shows that the optimal leverage cap — represented by τ_b — and the optimal risk weight — represented by τ_θ — are set in response to two terms.[16] The first term captures the marginal externality associated with a change in b^i or θ^i, which in this case corresponds to the marginal response of expected bailouts to more leverage or more investment in sector 1.

The second term, which arises only with imperfect instruments, is proportional to the leakage elasticities $\frac{d\log k^i}{d\bar{b}}$ and $\frac{d\log k^i}{d\varphi}$, and scales with the total expected bailout, via χ_k.[17]

[16] Corollaries 1 and 2 express the leakage and policy elasticities with respect to the regulatory parameters $(\bar{b}, \varphi)$ rather than the equivalent taxes (τ_b, τ_θ). Since the inverse matrix of policy elasticities multiplies the leakage elasticities, the chain rule implies that the reparametrization leaves the product unchanged whenever the map from $(\bar{b}, \varphi)$ to (τ_b, τ_θ) is locally invertible.

[17] The appropriate leakage elasticities in this application are semi-elasticities, i.e., responses of log investment to policy reforms. This occurs because we have expressed leverage and portfolio choices per unit of capital.

Both these elasticities fall into the "complements" case: Stricter leverage regulation ($\downarrow \bar{b}$) or a stricter relative risk weight ($\uparrow \varphi$) leads to decreases in k^i in equilibrium. Therefore, Equation (36) generally calls for *overregulation* of leverage and risk. Finally, notice that the relevant leakage elasticities are modulated by an inverse matrix of policy elasticities between b^i and θ^i.

The implication for financial regulation with environmental externalities is that any adjustment for climate-related risk should be determined only by its impact on financial externalities (in this particular case, this emerges from the presence of bailouts). For instance, the risk weight equivalent tax τ_θ should be increased if sector 1 is associated with climate-related tail risk that makes large bailouts more likely (i.e., if $\mathbb{E}_s\left[\frac{\partial t\left(b^i,\theta^i,s\right)}{\partial \theta^i}\right] > 0$). In addition, the setting with imperfect instruments implies that taxes on *both* leverage and portfolio weights should increase if climate-related risk increases the magnitude of the total expected bailout.

4.2 Imperfect Regulation with Broad/Environmental Mandates

We now consider a financial regulator with a broad mandate who cares directly about mitigating environmental distortions. We focus on the case where the environmental distortions satisfy $\psi_\theta > 0$ and $\psi_k > 0$. In this case, increases in overall scale as well as concentrated investments in sector 1 are associated with greater environmental damage.

Corollary 2. *(Imperfect Regulation with Broad/Environmental Mandates) a) When policy has been set optimally according to a narrow/financial mandate, the welfare benefits of marginal policy changes are given by*

$$\frac{dW}{db} = -k^i \frac{d\theta^i}{db}\psi_\theta - \frac{dk^i}{db}\psi_k \tag{37}$$

$$\frac{dW}{d\varphi} = -k^i \frac{d\theta^i}{d\varphi}\psi_\theta - \frac{dk^i}{d\varphi}\psi_k, \tag{38}$$

where ψ_θ and ψ_k denote the environmental component of the respective externalities, defined in Equations (32) *and* (33).

b) The optimal policy of a regulator with a broad/environmental mandate is given by

$$\begin{pmatrix} \tau_b \\ \tau_\theta \end{pmatrix} = \begin{pmatrix} \chi_b \\ \chi_\theta + \psi_\theta \end{pmatrix} + \begin{pmatrix} \frac{db^i}{db} & \frac{d\theta^i}{db} \\ \frac{db^i}{d\varphi} & \frac{d\theta^i}{d\varphi} \end{pmatrix}^{-1} \begin{pmatrix} \frac{d\log k^i}{db} \\ \frac{d\log k^i}{d\varphi} \end{pmatrix} (\chi_k + \psi_k), \tag{39}$$

where χ_b, χ_θ, and χ_k denote the financial component and ψ_θ and ψ_k denote the environmental component of the respective externalities, defined in Equations (31) *through* (33).

Corollary 2 develops two insights into the distinction between narrow/financial and broad/environmental mandates. First, Equations (37) and (38) highlight the additional welfare effects, from the perspective of a broad mandate, of adjusting either the leverage cap or risk weights, when policy has previously been optimized according to a narrow mandate. These equations are useful for deciding whether policy should be adjusted at the margin once a regulator decides to take environmental outcomes into account. The relevant marginal welfare effects are determined by the environmental distortions ψ_θ and ψ_k and the associated leakage elasticities. Note that the policy elasticities of leverage (i.e., $\frac{db^i}{d\bar{b}}$ and $\frac{db^i}{d\varphi}$) are irrelevant here, because the mode of financing has no marginal impact on environmental concerns.

Equation (37) shows that a regulator who adjusts the leverage cap $\bar{b}$ in response to environmental concerns faces a trade-off. Indeed, while it is natural that scale and leverage are generally complements, implying $\frac{dk^i}{d\bar{b}} > 0$,[18] the response of optimal portfolio choices $\frac{d\theta^i}{d\bar{b}}$ is ambiguous in theory, and depends on the functional form of returns to investment in each sector. Since the environmental distortions ψ_θ and ψ_k are assumed positive, the two terms in Equation (37) may have opposite signs. As a result, it is unclear whether leverage requirements should be relaxed or tightened in response to environmental concerns, and their impact on welfare may be offset by portfolio adjustments.

By contrast, Equation (38) demonstrates that risk weights are a natural tool for addressing environmental concerns. The portfolio share θ^i responds to its own regulation in the regular way, $\frac{d\theta^i}{d\varphi} < 0$, and total capital k^i is a complement, $\frac{dk^i}{d\varphi} < 0$. Therefore, it is clear that risk weights ought to be tightened when regulators account for environmental externalities.

The second insight emerging from Corollary 2 is the characterization of optimal policy in Equation (39). There are two differences from the equivalent characterization with a narrow mandate in Equation (36). First, the marginal distortion on portfolio choices is augmented, which calls for greater relative risk weights on the polluting sector (sector 1). Second, the scale distortion is augmented by ψ_k. The latter point is particularly important for

[18] Recall that $d\bar{b} > 0$ stands for a *looser* leverage cap, that is, a lower effective tax on leverage. Hence, $\frac{dk^i}{d\bar{b}} > 0$ is equivalent to $\frac{dk^i}{d\tau_b} < 0$, which corresponds to the complements case in our general, tax-based notation.

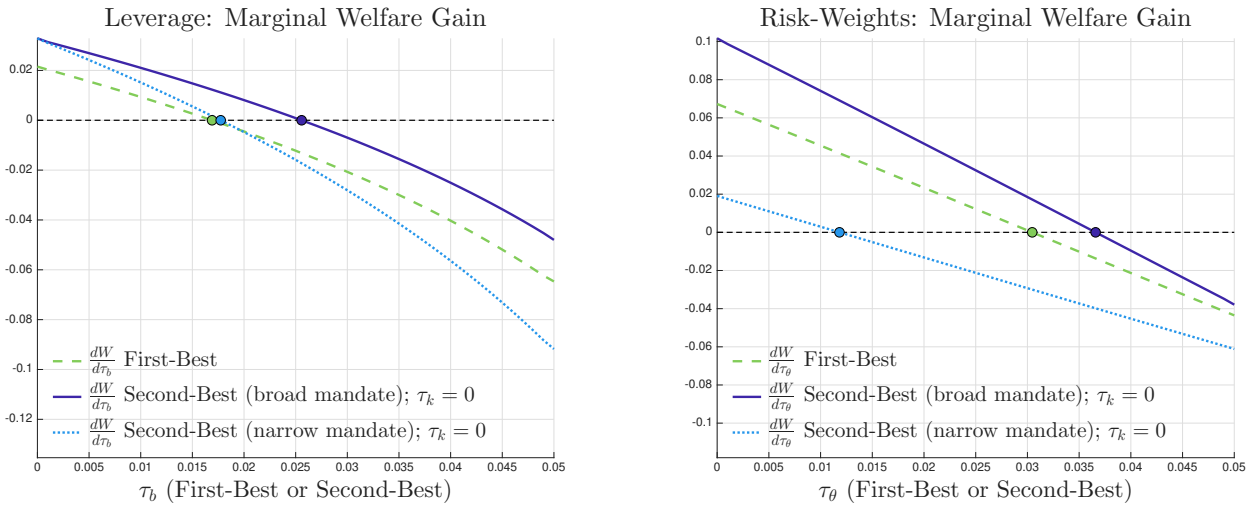


Figure 5: Financial Regulation with Environmental Externalities

Note: The left panel of Figure 5 compares the marginal welfare effects of varying corrective leverage regulation (τ_b) in three different scenarios. The green dashed line corresponds to the first-best scenario, in which τ_θ and τ_k are held fixed at their first-best levels (previously computed). The solid dark blue line corresponds to a second-best scenario in which the regulator has a broad mandate and cares about financial and environmental distortions. In this case, we compute welfare gains setting $\tau_k = 0$ and holding τ_θ fixed at the optimal second-best level for a broad mandate. The light blue dotted line corresponds to a second-best scenario in which the regulator has a narrow mandate and cares exclusively about financial distortions. In this case, we compute the welfare gains perceived by the narrow regulator (with $\psi_\theta = \psi_k = 0$), setting $\tau_k = 0$ and holding τ_θ fixed at the optimal second-best level for a narrow mandate. The right panel of Figure 5 compares the analogous marginal welfare effects of varying corrective risk weight regulation (τ_θ) in the same three scenarios, now holding τ_b and τ_k fixed. This figure assumes that the bailout policy is linearly separable, $t\left(b^i, \theta^i, s\right) = \alpha_0^i + \alpha_b^i b^i + \alpha_\theta^i \theta^i - \alpha_s^i s$, that the adjustment cost is quadratic, $\Upsilon\left(k^i\right) = (a/2)\left(k^i\right)^2$, and that the functions $\Omega\left(\theta^i\right)$ and $\Psi\left(\theta^i\right)$ are of the constant elasticity of substitution (CES) form in terms of θ^i and $1-\theta^i$, so $\Omega\left(\theta^i\right) = z_\Omega\left(a_\Omega\left(\theta^i\right)^{\eta_\Omega} + (1-a_\Omega)\left(1-\theta^i\right)^{\eta_\Omega}\right)^{\frac{1}{\eta_\Omega}}$ and $\Psi\left(\theta^i\right) = z_\Psi\left(a_\Psi\left(\theta^i\right)^{\eta_\Psi} + (1-a_\Psi)\left(1-\theta^i\right)^{\eta_\Psi}\right)^{\frac{1}{\eta_\Psi}}$. The parameters are $\beta^i = 0.9$, $\beta^C = 0.98$, $\phi = 0.7$, $a = 1$, $\alpha_0^i = \alpha_s^i = 0$, $\alpha_b^i = 0.015$, $\alpha_\theta^i = 0.01$, $\kappa = 0.15$, $d_1(s) = \bar{d}_1 s$ with $\bar{d}_1 = 1.01$, $d_2(s) = \bar{d}_2 s$ with $\bar{d}_2 = 1$, $z_\Omega = 0.25$, $a_\Omega = 0.5$, $\eta_\Omega = 1.5$, $z_\Psi = 0.25$, $a_\Psi = 0.55$, and $\eta_\Psi = 1.5$, where s is normally distributed with mean 1.7 and standard deviation 0.8, truncated to the interval $[0, 3]$. For reference, the optimal first-best regulation is $\tau_b = 1.69\%$, $\tau_\theta = 3.05\%$, and $\tau_k = 14.37\%$, the optimal second-best regulation with a broad mandate is $\tau_b = 2.56\%$, $\tau_\theta = 3.66\%$, and $\tau_k = 0$, while the optimal second-best regulation with a narrow mandate is $\tau_b = 1.77\%$, $\tau_\theta = 1.18\%$, and $\tau_k = 0$.

our analysis. The scale distortion matters purely due to imperfect regulation and leakage elasticities. Equation (39) demonstrates that adjustments for leakage elasticities become *more* important once the regulator cares about environmental effects.

Figure 5 illustrates the relation between the first-best and second-best solutions in both the narrow and the broad mandate cases. In particular, the left panel shows the marginal welfare effect of varying leverage regulation (in terms of τ_b), while the right panel shows the marginal welfare effect of varying risk weights (in terms of τ_θ). Consistent with Corollary 2, Figure 5 shows that the optimal second-best policy under a broad mandate overregulates both leverage and portfolio weights relative to the first-best. However, consistent with the insights discussed above, the relation between the first-best regulation and the second-best regulation for a regulator with a narrow mandate is less clear-cut. In the case we illustrate, it turns out that a narrow regulator overregulates leverage relative to the first-best, but not portfolio weights. This is mainly due to the fact that the leakage elasticity with respect to capital is greater in magnitude for leverage. By contrast, a broad regulator overregulates both leverage and portfolio weights relative to the first-best, because it places a greater weight on all leakage elasticities to capital.

4.3 The Value of Regulating Scale

To close the analysis of this application, we consider a regulator who is able to impose a corrective tax $\tau_k k^i$ on investors to correct for the (previously unregulated) externalities associated with the scale decision k^i. The key economic insights can be obtained by considering the marginal welfare effect of increasing τ_k.

Corollary 3. *(Environmental Externalities/Regulating Unregulated Decisions) When the planner can impose a corrective tax τ_k on the total scale of investment k^i, the marginal welfare effect of varying τ_k is given by*

$$\frac{dW}{d\tau_k} = \underbrace{\frac{db^i}{d\tau_k}}_{=0}(\tau_b - \delta_b)\,k^i + \underbrace{\frac{d\theta^i}{d\tau_k}}_{=0}(\tau_\theta - \delta_\theta)\,k^i + \frac{dk^i}{d\tau_k}(\tau_k - \delta_k) = \frac{dk^i}{d\tau_k}\omega_k. \tag{40}$$

In this environment there are no reverse leakage effects from regulating scale onto leverage and portfolio decisions. Intuitively, the investors' problem in (26) can be approached in two steps. First, investors choose leverage and portfolios to maximize market values $M(b^i, \theta^i)$ per unit of total capital. Second, they set the marginal cost of capital equal to its

maximized market value. Since the first step does not depend on the cost/tax of capital, b^i and θ^i are independent of τ_k in equilibrium.

This fact has two novel economic implications. First, we note that the case for regulating scale here is much stronger than in settings with reverse leakage, such as the shadow-banking case of Example 1. In particular, the capital-specific elements of the Le Chatelier/reverse leakage adjustment matrix $\boldsymbol{L}$, which usually dampens the welfare impact of regulating unregulated decisions, are zero. Moreover, the case for regulating scale is clearly $stronger$ when the regulator has a $broad/environmental\ mandate$, other things equal, since this mandate takes into account the full marginal distortion $\delta_k = \chi_k + \psi_k$.

Second, we see from Equation (40) that the optimal level of the tax on capital is always given by $\tau_k = \delta_k$, which corresponds to the first-best or Pigouvian correction. The absence of reverse leakage implies that there is no incentive to over- or underregulate scale, once the regulator is allowed to do so. This is true even when the regulation of leverage and portfolio decisions is imperfect (with $\tau_b \neq \delta_b$ and/or $\tau_\theta \neq \delta_\theta$).

5 Conclusion

This paper has systematically studied financial regulation in the face of various imperfections and "unintended consequences". In a general model with financial frictions, we have shown that Pigouvian wedges and leakage elasticities determine the optimal second-best policy. This approach leads to intuitive characterizations of second-best financial regulation. For instance, we have provided insights on whether one should over- or underregulate in the presence of imperfections. Indeed, we have found that both cases are possible in standard problems of financial regulation.

Our general model provides insights into many known cases of regulatory imperfections, such as shadow banking, risk-shifting, uniform regulation, or costly regulation. It can also deal with a wide variety of market failures, such as fire-sale externalities, bailouts, or behavioral distortions. In an application to the recent debate on financial regulation with environmental concerns, we have demonstrated that our approach can shed light on the key economic forces and trade-o□s in highly complex policy problems. We hope that it can also inspire research on new types of imperfections that will inevitably arise from future waves of financial innovation.

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Online Appendix

Section A of this Online Appendix provides explicit definitions of the vectors and matrices used in the paper and in this appendix. Sections B, C, and D provide proofs and derivations for Sections 3 and 4 of the paper. Section E presents additional proofs and derivations.

A Matrix Definitions

For each investor type $i \in \mathcal{I}$, the $N \times 1$ vectors of decisions, regulations, marginal distortions, and Pigouvian wedges are

$$\boldsymbol{x}^i = \begin{pmatrix} x_1^i \\ \vdots \\ x_N^i \end{pmatrix}, \quad \boldsymbol{\tau}^i = \begin{pmatrix} \tau_1^i \\ \vdots \\ \tau_N^i \end{pmatrix}, \quad \boldsymbol{\delta}^i = \begin{pmatrix} \delta_1^i \\ \vdots \\ \delta_N^i \end{pmatrix}, \quad \boldsymbol{\omega}^i \equiv \boldsymbol{\tau}^i - \boldsymbol{\delta}^i.$$

Here x_n^i denotes decision n by investor type i; this notation is the same as in the main text and need not correspond to a consumption good.

Stacking investor types gives the $IN \times 1$ vectors

$$\boldsymbol{x} = \begin{pmatrix} \boldsymbol{x}^1 \\ \vdots \\ \boldsymbol{x}^I \end{pmatrix}, \quad \boldsymbol{\tau} = \begin{pmatrix} \boldsymbol{\tau}^1 \\ \vdots \\ \boldsymbol{\tau}^I \end{pmatrix}, \quad \boldsymbol{\delta} = \begin{pmatrix} \boldsymbol{\delta}^1 \\ \vdots \\ \boldsymbol{\delta}^I \end{pmatrix}, \quad \boldsymbol{\omega} = \boldsymbol{\tau} - \boldsymbol{\delta}.$$

The paper uses the convention that rows of the policy-elasticity matrix index regulations and columns index decisions. Thus

$$\frac{d\boldsymbol{x}}{d\boldsymbol{\tau}} = \begin{pmatrix} \frac{d\boldsymbol{x}^1}{d\boldsymbol{\tau}^1} & \cdots & \frac{d\boldsymbol{x}^I}{d\boldsymbol{\tau}^1} \\ \vdots & \frac{d\boldsymbol{x}^i}{d\boldsymbol{\tau}^j} & \vdots \\ \frac{d\boldsymbol{x}^1}{d\boldsymbol{\tau}^I} & \cdots & \frac{d\boldsymbol{x}^I}{d\boldsymbol{\tau}^I} \end{pmatrix}, \qquad \frac{d\boldsymbol{x}^i}{d\boldsymbol{\tau}^j} = \left(\frac{dx_n^i}{d\tau_m^j} \right)_{m,n=1}^{N}. \tag{OA-1}$$

When decisions and regulations are partitioned into perfectly regulated (R) and imperfectly regulated (U) components, the same Jacobian is partitioned as

$$\frac{d\boldsymbol{x}}{d\boldsymbol{\tau}} = \begin{pmatrix} \frac{d\boldsymbol{x}^R}{d\boldsymbol{\tau}^R} & \frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^R} \\ \frac{d\boldsymbol{x}^R}{d\boldsymbol{\tau}^U} & \frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^U} \end{pmatrix},$$

where the four blocks have dimensions $R \times R$, $R \times U$, $U \times R$, and $U \times U$, respectively. For M regulatory constraints $\boldsymbol{\Phi}(\boldsymbol{\tau}) = 0$, $\frac{d\boldsymbol{\Phi}}{d\boldsymbol{\tau}}$ denotes the corresponding $IN \times M$ Jacobian under the same

row convention, and $\boldsymbol{\mu} \in \mathbb{R}^M$ denotes the vector of constraint multipliers.

B Proofs and Derivations: Sections 3.1 and 3.2

Investors' Problem. The problem solved by investor i in Lagrangian form is

$$\max_{c_0^i, \left\{c_1^i(s)\right\}, \boldsymbol{x}^i} \mathcal{L}^i,$$

where $\mathcal{L}^i$ is given by

$$\begin{aligned}\mathcal{L}^i = & u^i\left(c_0^i, \left\{c_1^i(s)\right\}_{s\in S}, \left\{\overline{\boldsymbol{x}}^j\right\}_{j\in\mathcal{I}}\right) \\ & - \lambda_0^i\left(c_0^i - n_0^i - Q^i\left(\boldsymbol{x}^i\right) + \Upsilon^i\left(\boldsymbol{x}^i\right) + \boldsymbol{\tau}^i \cdot \boldsymbol{x}^i - T^i\right) \\ & - \int \lambda_1^i(s)\left(c_1^i(s) - n_1^i(s) - \rho^i\left(\boldsymbol{x}^i, s\right)\right) dF(s),\end{aligned}$$

where λ_0^i and $\lambda_1^i(s)$ denote the Lagrange multipliers that correspond to investor i's budget constraints.[19] The consumption optimality conditions imply that $\lambda_0^i = \frac{\partial u^i}{\partial c_0^i}$ and $\lambda_1^i(s)\, dF(s) = \frac{\partial u^i}{\partial c_1^i(s)}$. The optimality conditions for investor i are given by

$$-\lambda_0^i\left(-\frac{\partial Q^i\left(\boldsymbol{x}^i\right)}{\partial \boldsymbol{x}^i} + \frac{\partial \Upsilon^i\left(\boldsymbol{x}^i\right)}{\partial \boldsymbol{x}^i} + \boldsymbol{\tau}^i\right) + \int \lambda_1^i(s)\frac{\partial \rho^i\left(\boldsymbol{x}^i, s\right)}{\partial \boldsymbol{x}^i} dF(s) = 0, \ \forall i, \tag{OA-2}$$

where Equation (OA-2) can be expressed as

$$-\frac{\partial Q^i\left(\boldsymbol{x}^i\right)}{\partial \boldsymbol{x}^i} + \frac{\partial \Upsilon^i\left(\boldsymbol{x}^i\right)}{\partial \boldsymbol{x}^i} + \boldsymbol{\tau}^i = \int m^i(s)\frac{\partial \rho^i\left(\boldsymbol{x}^i, s\right)}{\partial \boldsymbol{x}^i} dF(s), \ \forall i, \tag{OA-3}$$

where $m^i(s) = \frac{\lambda_1^i(s)}{\lambda_0^i}$ denotes the stochastic discount factor of investor i.[20] These conditions are Euler equations for both financing and investment.

Formally, the $N \times 1$ vectors $\frac{\partial Q^i\left(\boldsymbol{x}^i\right)}{\partial \boldsymbol{x}^i}$, $\frac{\partial \Upsilon^i\left(\boldsymbol{x}^i\right)}{\partial \boldsymbol{x}^i}$, and $\boldsymbol{\tau}^i$ are given by:

$$\frac{\partial Q^i}{\partial \boldsymbol{x}^i} = \begin{pmatrix} \frac{\partial Q^i}{\partial x_1^i} \\ \vdots \\ \frac{\partial Q^i}{\partial x_N^i} \end{pmatrix}, \quad \frac{\partial \Upsilon^i}{\partial \boldsymbol{x}^i} = \begin{pmatrix} \frac{\partial \Upsilon^i}{\partial x_1^i} \\ \vdots \\ \frac{\partial \Upsilon^i}{\partial x_N^i} \end{pmatrix}, \quad \text{and} \quad \boldsymbol{\tau}^i = \begin{pmatrix} \tau_1^i \\ \vdots \\ \tau_N^i \end{pmatrix}.$$

[19] Without loss of generality, we define the state s multipliers $\lambda_1^i(s)$ inside the expectation.

[20] Note that a sufficient regularity condition for the second term of Equation (OA-2) to be valid is that $\rho^i\left(\boldsymbol{x}^i, s\right)$ is continuous. Otherwise, all results follow when the second term is $\frac{\partial}{\partial \boldsymbol{x}^i}\left[\int \lambda_1^i(s)\rho^i\left(\boldsymbol{x}^i, s\right) dF(s)\right]$.

Similarly, we define the $N \times 1$ vector $\int \lambda_1^i(s) \frac{\partial \rho^i(\boldsymbol{x}^i, s)}{\partial \boldsymbol{x}^i} dF(s)$ as follows:

$$\int \lambda_1^i(s) \frac{\partial \rho^i(\boldsymbol{x}^i, s)}{\partial \boldsymbol{x}^i} dF(s) = \begin{pmatrix} \int \lambda_1^i(s) \frac{\partial \rho^i(\boldsymbol{x}^i, s)}{\partial x_1^i} dF(s) \\ \vdots \\ \int \lambda_1^i(s) \frac{\partial \rho^i(\boldsymbol{x}^i, s)}{\partial x_N^i} dF(s) \end{pmatrix}.$$

Creditors' Problem. The problem solved by creditors in Lagrangian form is

$$\max_{c_0^C, \{c_1^C(s)\}, \{h_i^C\}} \mathcal{L}^C,$$

where $\mathcal{L}^C$ is given by

$$\begin{aligned} \mathcal{L}^C = & u^C\left(c_0^C, \left\{c_1^C(s)\right\}_{s\in S}, \left\{\overline{\boldsymbol{x}}^j\right\}_{j\in\mathcal{I}}\right) - \lambda_0^C\left(c_0^C - n_0^C + \sum_{i\in\mathcal{I}} h_i^C Q^i\left(\overline{\boldsymbol{x}}^i\right)\right) \\ & - \int \lambda_1^C(s)\left(c_1^C(s) - n_1^C(s) - \sum_{i\in\mathcal{I}} h_i^C \rho_i^C\left(\overline{\boldsymbol{x}}^i, s\right)\right) dF(s), \end{aligned}$$

where λ_0^C and $\lambda_1^C(s)$ denote the Lagrange multipliers that correspond to the creditors' budget constraints. The consumption optimality conditions imply that $\lambda_0^C = \frac{\partial u^C}{\partial c_0^C}$ and $\lambda_1^C(s) dF(s) = \frac{\partial u^C}{\partial c_1^C(s)}$. The optimality conditions for creditors regarding $\left\{h_i^C\right\}$ are

$$-\lambda_0^C Q^i\left(\boldsymbol{x}^i\right) + \int \lambda_1^C(s) \rho_i^C\left(\boldsymbol{x}^i, s\right) dF(s) = 0, \ \forall i, \tag{OA-4}$$

where we use the fact that $\boldsymbol{x}^i = \overline{\boldsymbol{x}}^i$ in equilibrium. Equation (OA-4) can be expressed as

$$Q^i\left(\boldsymbol{x}^i\right) = \int m^C(s) \rho_i^C\left(\boldsymbol{x}^i, s\right) dF(s), \ \forall i, \tag{OA-5}$$

where we define $m^C(s) = \frac{\lambda_1^C(s)}{\lambda_0^C}$, characterizing the financing schedules $Q^i\left(\boldsymbol{x}^i\right)$ that investors face. Note that the stochastic discount factor $m^C(s)$ is an equilibrium object, which depends on the choices of all investors in the model and the policy.

Proof of Lemma 1. (Marginal Welfare Effects of Regulation) We initially characterize the $N \times 1$ vectors $\frac{\frac{dV^i}{d\tau^j}}{\lambda_0^i}$ and $\frac{\frac{dV^C}{d\tau^j}}{\lambda_0^C}$, which correspond to the money-metric welfare changes of type i investors and creditors when $\boldsymbol{\tau}^j$ changes.

Investors. We express the financing schedules faced by investors as a function of the creditors' stochastic discount factor $m^C(s)$, which is in turn in equilibrium a function of the consumption of creditors in all dates and states. This allows us to separately account for any general equilibrium pecuniary effects. Formally, we represent the equilibrium financing schedules in Equation (OA-5) for an investor i as follows:

$$Q^i\left(\boldsymbol{x}^i; m^C(s)\right) = \int m^C(s)\, \rho_i^C\left(\boldsymbol{x}^i, s\right) dF(s),$$

where we make explicit the dependence on $m^C(s)$. The money-metric change in indirect utility for investor i when varying the regulation that investor j faces is given by the following $N \times 1$ vector:

$$\begin{aligned}\frac{\frac{dV^i}{d\boldsymbol{\tau}^j}}{\lambda_0^i} = \frac{dc_0^i}{d\boldsymbol{\tau}^j}\overbrace{\left(\frac{\frac{\partial u^i}{\partial c_0^i} - \lambda_0^i}{\lambda_0^i}\right)}^{=0} + \int \frac{dc_1^i(s)}{d\boldsymbol{\tau}^j}\overbrace{\left(\frac{\frac{\frac{\partial u^i}{\partial c_1^i(s)}}{dF(s)} - \lambda_1^i(s)}{\lambda_0^i}\right)}^{=0} dF(s) \\ + \frac{d\boldsymbol{x}^i}{d\boldsymbol{\tau}^j}\overbrace{\left(\frac{\partial Q^i\left(\boldsymbol{x}^i; m^C(s)\right)}{\partial \boldsymbol{x}^i} - \frac{\partial \Upsilon^i(\boldsymbol{x}^i)}{\partial \boldsymbol{x}^i} - \boldsymbol{\tau}^i + \int m^i(s)\frac{\partial \rho^i(\boldsymbol{x}^i, s)}{\partial \boldsymbol{x}^i} dF(s)\right)}^{=0} \\ + \frac{dT^i}{d\boldsymbol{\tau}^j} - \frac{d\boldsymbol{\tau}^i}{d\boldsymbol{\tau}^j}\boldsymbol{x}^i + \frac{\partial Q^i\left(\boldsymbol{x}^i; m^C(s)\right)}{\partial m^C(s)}\frac{dm^C(s)}{d\boldsymbol{\tau}^j} + \sum_{\ell \in \mathcal{I}} \frac{d\boldsymbol{x}^\ell}{d\boldsymbol{\tau}^j}\frac{1}{\lambda_0^i}\frac{\partial u^i}{\partial \overline{\boldsymbol{x}}^\ell},\end{aligned} \tag{OA-6}$$

where the $N \times 1$ vectors $\frac{dT^i}{d\boldsymbol{\tau}^j}$ and $\boldsymbol{x}^i$ are given by

$$\frac{dT^i}{d\boldsymbol{\tau}^j} = \begin{pmatrix} \frac{dT^i}{d\tau_1^j} \\ \vdots \\ \frac{dT^i}{d\tau_N^j} \end{pmatrix} \qquad \text{and} \qquad \boldsymbol{x}^i = \begin{pmatrix} x_1^i \\ \vdots \\ x_n^i \\ \vdots \\ x_N^i \end{pmatrix},$$

and where the matrix $\frac{d\boldsymbol{\tau}^i}{d\boldsymbol{\tau}^j}$, of dimension $N \times N$, is given by

$$\frac{d\boldsymbol{\tau}^i}{d\boldsymbol{\tau}^j} = \begin{cases} \boldsymbol{I}_N, & \text{if } i = j \\ \boldsymbol{0}_N, & \text{if } i \neq j, \end{cases}$$

which is either an N-dimensional identity matrix, $\boldsymbol{I}_N$, when $i = j$, or an $N \times N$ matrix of zeros, $\mathbf{0}_N$, when $i \neq j$. We also define the $N \times 1$ vector $\frac{\partial Q^i\left(\boldsymbol{x}^i; m^C(s)\right)}{\partial m^C(s)} \frac{dm^C(s)}{d\boldsymbol{\tau}^j}$ as

$$\frac{\partial Q^i\left(\boldsymbol{x}^i; m^C\left(s\right)\right)}{\partial m^C\left(s\right)} \frac{dm^C\left(s\right)}{d\boldsymbol{\tau}^j} = \begin{pmatrix} \int \frac{dm^C(s)}{d\tau_1^j} \rho_i^C\left(\boldsymbol{x}^i, s\right) dF\left(s\right) \\ \vdots \\ \int \frac{dm^C(s)}{d\tau_N^j} \rho_i^C\left(\boldsymbol{x}^i, s\right) dF\left(s\right) \end{pmatrix}.$$

Note that we use the fact that

$$\frac{dQ^i\left(\boldsymbol{x}^i; m^C\left(s\right)\right)}{d\boldsymbol{\tau}^j} = \frac{d\boldsymbol{x}^i}{d\boldsymbol{\tau}^j} \frac{\partial Q^i}{\partial \boldsymbol{x}^i} + \frac{\partial Q^i}{\partial m^C\left(s\right)} \frac{dm^C\left(s\right)}{d\boldsymbol{\tau}^j},$$

as well as

$$\frac{d\left(\boldsymbol{\tau}^i \cdot \boldsymbol{x}^i - T^i\right)}{d\boldsymbol{\tau}^j} = \frac{d\boldsymbol{\tau}^i}{d\boldsymbol{\tau}^j} \boldsymbol{x}^i + \frac{d\boldsymbol{x}^i}{d\boldsymbol{\tau}^j} \boldsymbol{\tau}^i - \frac{dT^i}{d\boldsymbol{\tau}^j}.$$

Note that $\frac{\partial u^i}{\partial \overline{\boldsymbol{x}}^\ell}$ denotes an $N \times 1$ gradient vector.

Creditors. In the case of creditors, we can express the $N \times 1$ vector $\frac{\frac{dV^C}{d\boldsymbol{\tau}^j}}{\lambda_0^C}$ as follows:

$$\begin{aligned}
\frac{\frac{dV^C}{d\boldsymbol{\tau}^j}}{\lambda_0^C} &= \frac{dc_0^C}{d\boldsymbol{\tau}^j} \overbrace{\left(\frac{\frac{\partial u^C}{\partial c_0^C} - \lambda_0^C}{\lambda_0^C}\right)}^{=0} + \int \overbrace{\left(\frac{\frac{\frac{\partial u^C}{\partial c_1^C(s)}}{\frac{dF(s)}{\lambda_0^C}} - \lambda_1^C\left(s\right)}{\lambda_0^C}\right)}^{=0} \frac{dc_1^C\left(s\right)}{d\boldsymbol{\tau}^j} dF\left(s\right) \\
&\quad - \sum_{i \in \mathcal{I}} \frac{dh_i^C}{d\boldsymbol{\tau}^j} \overbrace{\left(Q^i\left(\boldsymbol{x}^i; m^C\left(s\right)\right) - \int m^C\left(s\right) \rho_i^C\left(\boldsymbol{x}^i, s\right) dF\left(s\right)\right)}^{=0} \\
&\quad - \sum_{i \in \mathcal{I}} h_i^C \left(\frac{dQ^i\left(\boldsymbol{x}^i; m^C\left(s\right)\right)}{d\boldsymbol{\tau}^j} - \int m^C\left(s\right) \frac{d\rho_i^C\left(\boldsymbol{x}^i, s\right)}{d\boldsymbol{\tau}^j} dF\left(s\right)\right) + \sum_{\ell \in \mathcal{I}} \frac{d\boldsymbol{x}^\ell}{d\boldsymbol{\tau}^j} \frac{1}{\lambda_0^C} \frac{\partial u^C}{\partial \overline{\boldsymbol{x}}^\ell} \\
&= - \sum_{i \in \mathcal{I}} \frac{d\boldsymbol{x}^i}{d\boldsymbol{\tau}^j} \underbrace{\left(\frac{\partial Q^i}{\partial \boldsymbol{x}^i} - \int m^C\left(s\right) \frac{\partial \rho_i^C\left(\boldsymbol{x}^i, s\right)}{\partial \boldsymbol{x}^i} dF\left(s\right)\right)}_{=0} - \sum_{i \in \mathcal{I}} \frac{\partial Q^i}{\partial m^C\left(s\right)} \frac{dm^C\left(s\right)}{d\boldsymbol{\tau}^j} + \sum_{\ell \in \mathcal{I}} \frac{d\boldsymbol{x}^\ell}{d\boldsymbol{\tau}^j} \frac{1}{\lambda_0^C} \frac{\partial u^C}{\partial \overline{\boldsymbol{x}}^\ell},
\end{aligned} \tag{OA-7}$$

where in the second equality we use the fact that $h_i^C = 1$ and the fact that $\frac{d\rho_i^C(\boldsymbol{x}^i,s)}{d\boldsymbol{\tau}^j} = \frac{d\boldsymbol{x}^i}{d\boldsymbol{\tau}^j}\frac{\partial \rho_i^C}{\partial \boldsymbol{x}^i}$, and where the $N \times 1$ vector $\frac{\partial \rho_i^C}{\partial \boldsymbol{x}^i}$ is given by

$$\frac{\partial \rho_i^C}{\partial \boldsymbol{x}^i} = \begin{pmatrix} \frac{\partial \rho_i^C}{\partial x_1^i} \\ \vdots \\ \frac{\partial \rho_i^C}{\partial x_N^i} \end{pmatrix}.$$

Note that $\frac{d\boldsymbol{x}^\ell}{d\boldsymbol{\tau}^j}$ is defined as in Equation (OA-1), and that $\frac{\partial u^C}{\partial \overline{\boldsymbol{x}}^\ell}$ denotes an $N \times 1$ gradient vector.

Aggregation. First, we can express the sum across investors of the change in money-metric indirect utilities as follows:

$$\begin{aligned}\sum_{i\in\mathcal{I}} \frac{\frac{dV^i}{d\boldsymbol{\tau}^j}}{\lambda_0^i} &= \sum_{i\in\mathcal{I}} \left(\frac{dT^i}{d\boldsymbol{\tau}^j} - \frac{d\boldsymbol{\tau}^i}{d\boldsymbol{\tau}^j}\boldsymbol{x}^i\right) + \sum_{i\in\mathcal{I}} \frac{\partial Q^i}{\partial m^C(s)}\frac{dm^C(s)}{d\boldsymbol{\tau}^j} + \sum_{i\in\mathcal{I}}\sum_{\ell\in\mathcal{I}} \frac{d\boldsymbol{x}^\ell}{d\boldsymbol{\tau}^j}\frac{1}{\lambda_0^i}\frac{\partial u^i}{\partial \overline{\boldsymbol{x}}^\ell} \\ &= \sum_{i\in\mathcal{I}} \frac{d\boldsymbol{x}^i}{d\boldsymbol{\tau}^j}\left(\boldsymbol{\tau}^i + \sum_{\ell\in\mathcal{I}} \frac{1}{\lambda_0^\ell}\frac{\partial u^\ell}{\partial \overline{\boldsymbol{x}}^i}\right) + \sum_{i\in\mathcal{I}} \frac{\partial Q^i}{\partial m^C(s)}\frac{dm^C(s)}{d\boldsymbol{\tau}^j},\end{aligned}$$

where we use the fact that

$$\sum_{i\in\mathcal{I}} \left(\frac{dT^i}{d\boldsymbol{\tau}^j} - \frac{d\boldsymbol{\tau}^i}{d\boldsymbol{\tau}^j}\boldsymbol{x}^i\right) = \sum_{i\in\mathcal{I}} \frac{d\boldsymbol{x}^i}{d\boldsymbol{\tau}^j}\boldsymbol{\tau}^i,$$

as well as the following identity:

$$\sum_{i\in\mathcal{I}}\sum_{\ell\in\mathcal{I}} \frac{d\boldsymbol{x}^\ell}{d\boldsymbol{\tau}^j}\frac{1}{\lambda_0^i}\frac{\partial u^i}{\partial \overline{\boldsymbol{x}}^\ell} = \sum_{i\in\mathcal{I}}\sum_{\ell\in\mathcal{I}} \frac{d\boldsymbol{x}^i}{d\boldsymbol{\tau}^j}\frac{1}{\lambda_0^\ell}\frac{\partial u^\ell}{\partial \overline{\boldsymbol{x}}^i}.$$

Therefore, we can express $\frac{dW}{d\boldsymbol{\tau}^j}$ as follows:

$$\begin{aligned}\frac{dW}{d\boldsymbol{\tau}^j} &= \sum_{i\in\mathcal{I}} \frac{\frac{dV^i}{d\boldsymbol{\tau}^j}}{\lambda_0^i} + \frac{\frac{dV^C}{d\boldsymbol{\tau}^j}}{\lambda_0^C} \\ &= \sum_{i\in\mathcal{I}} \frac{d\boldsymbol{x}^i}{d\boldsymbol{\tau}^j}\left(\boldsymbol{\tau}^i + \sum_{\ell\in\mathcal{I}} \frac{1}{\lambda_0^\ell}\frac{\partial u^\ell}{\partial \overline{\boldsymbol{x}}^i}\right) + \sum_{i\in\mathcal{I}} \frac{d\boldsymbol{x}^i}{d\boldsymbol{\tau}^j}\frac{1}{\lambda_0^C}\frac{\partial u^C}{\partial \overline{\boldsymbol{x}}^i} \\ &= \sum_{i\in\mathcal{I}} \frac{d\boldsymbol{x}^i}{d\boldsymbol{\tau}^j}\left(\boldsymbol{\tau}^i - \underbrace{\left(-\left(\sum_{\ell\in\mathcal{I}} \frac{1}{\lambda_0^\ell}\frac{\partial u^\ell}{\partial \overline{\boldsymbol{x}}^i} + \frac{1}{\lambda_0^C}\frac{\partial u^C}{\partial \overline{\boldsymbol{x}}^i}\right)\right)}_{=\boldsymbol{\delta}^i}\right).\end{aligned}$$

Hence, by switching indices and stacking we find that

$$\frac{dW}{d\boldsymbol{\tau}} = \frac{d\boldsymbol{x}}{d\boldsymbol{\tau}}\left(\boldsymbol{\tau} - \boldsymbol{\delta}\right) = \frac{d\boldsymbol{x}}{d\boldsymbol{\tau}}\boldsymbol{\omega},$$

as in Equation (8) in the text.

Proof of Proposition 1. (Second-Best Policy: Perfectly Regulated Decisions) At the second-best optimum, it must be that $\frac{dW}{d\boldsymbol{\tau}^R} = 0$. Applying Lemma 1, we can express these optimality conditions as

$$\frac{dW}{d\boldsymbol{\tau}^R} = \frac{d\boldsymbol{x}}{d\boldsymbol{\tau}^R}\left(\boldsymbol{\tau} - \boldsymbol{\delta}\right) = \frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^R}\left(\boldsymbol{\tau}^U - \boldsymbol{\delta}^U\right) + \frac{d\boldsymbol{x}^R}{d\boldsymbol{\tau}^R}\left(\boldsymbol{\tau}^R - \boldsymbol{\delta}^R\right) = 0.$$

Assuming that the matrix $\frac{d\boldsymbol{x}^R}{d\boldsymbol{\tau}^R}$ is invertible, we rearrange this expression as follows:

$$\frac{d\boldsymbol{x}^R}{d\boldsymbol{\tau}^R}\left(\boldsymbol{\tau}^R - \boldsymbol{\delta}^R\right) = -\frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^R}\left(\boldsymbol{\tau}^U - \boldsymbol{\delta}^U\right) \iff \boldsymbol{\tau}^R = \boldsymbol{\delta}^R - \left(\frac{d\boldsymbol{x}^R}{d\boldsymbol{\tau}^R}\right)^{-1}\frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^R}\left(\boldsymbol{\tau}^U - \boldsymbol{\delta}^U\right),$$

which corresponds to Equation (9) in the text.

Proof of Proposition 2. (Second-Best Policy: Imperfectly Regulated Decisions) Applying Lemma 1, we can express the marginal welfare effects of adjusting the regulation of imperfectly regulated decisions (those for which the planning constraints bind) as

$$\frac{dW}{d\boldsymbol{\tau}^U} = \frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^U}\left(\boldsymbol{\tau}^U - \boldsymbol{\delta}^U\right) + \frac{d\boldsymbol{x}^R}{d\boldsymbol{\tau}^U}\left(\boldsymbol{\tau}^R - \boldsymbol{\delta}^R\right).$$

So defining the vector of Lagrange multipliers associated with the regulatory constraints by $\boldsymbol{\mu}$, the optimality conditions for such decisions are given by $\frac{dW}{d\boldsymbol{\tau}^U} = \frac{d\boldsymbol{\Phi}}{d\boldsymbol{\tau}^U}\boldsymbol{\mu}$, along with $\boldsymbol{\Phi}\left(\boldsymbol{\tau}^U\right) = 0$ when $\boldsymbol{\Phi}\left(\cdot\right)$ captures constraints. From Proposition 1, we have that

$$\boldsymbol{\tau}^R - \boldsymbol{\delta}^R = \left(-\frac{d\boldsymbol{x}^R}{d\boldsymbol{\tau}^R}\right)^{-1}\frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^R}\left(\boldsymbol{\tau}^U - \boldsymbol{\delta}^U\right),$$

so combining the last two equations we obtain

$$\begin{aligned}\frac{dW}{d\boldsymbol{\tau}^U} &= \frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^U}\left(\boldsymbol{\tau}^U - \boldsymbol{\delta}^U\right) - \frac{d\boldsymbol{x}^R}{d\boldsymbol{\tau}^U}\left(\frac{d\boldsymbol{x}^R}{d\boldsymbol{\tau}^R}\right)^{-1}\frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^R}\left(\boldsymbol{\tau}^U - \boldsymbol{\delta}^U\right) \\ &= \frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^U}\left(\boldsymbol{I} - \underbrace{\left(\frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^U}\right)^{-1}\frac{d\boldsymbol{x}^R}{d\boldsymbol{\tau}^U}\left(\frac{d\boldsymbol{x}^R}{d\boldsymbol{\tau}^R}\right)^{-1}\frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^R}}_{\equiv \boldsymbol{L}}\right)\left(\boldsymbol{\tau}^U - \boldsymbol{\delta}^U\right) = \frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^U}\left(\boldsymbol{I} - \boldsymbol{L}\right)\boldsymbol{\omega}^U,\end{aligned}$$

which corresponds to Equation (10) in the text.

C Proofs and Derivations: Section 3.3

C.1 Example 1

Environment. Investors and creditors have preferences

$$c_0^i + \beta^i \int c_1^i(s)\, dF(s) \qquad \text{and} \qquad u\left(c_0^C\right) + \beta^C \int u\left(c_1^C(s)\right) dF(s),$$

respectively. Investors face the following budget constraints:

$$\begin{aligned} c_0^i &= n_0^i + Q^i\left(b^i\right) - \tau_b^i b^i + T_0^i \\ c_1^i(s) &= n_1^i(s) + \max\left\{s + t^i\left(b^i, s\right) - b^i, 0\right\}, \quad \forall s. \end{aligned}$$

Investor i optimally defaults at date 1 if $s + t^i\left(b^i, s\right) - b^i < 0$.[21] Assuming that $1 + \frac{\partial t^i\left(b^i,s\right)}{\partial s} > 0$, there exists a unique threshold $s^{i\star}\left(b^i\right)$ such that default occurs if and only if $s < s^{i\star}\left(b^i\right)$. We assume that in this case creditors can recover a fraction ϕ^i of asset payoffs, while bailouts are recovered in full, with $1 - \phi^i$ capturing deadweight losses from default. Therefore, the definition of the repayment eventually received by creditors, $\mathcal{P}^i\left(b^i, s\right)$, is

$$\mathcal{P}^i\left(b^i, s\right) = \begin{cases} \phi^i s + t^i\left(b^i, s\right) & s \in \left[\underline{s}, s^{i\star}\left(b^i\right)\right) \\ b^i & s \in \left[s^{i\star}\left(b^i\right), \bar{s}\right]. \end{cases}$$

[21] It is straightforward to make bailouts depend on the decisions of all investors.

Creditors face the following budget constraints:

$$c_0^C = n_0^C - \sum_{i\in\mathcal{I}} h^i Q^i\left(b^i\right),$$
$$c_1^C(s) = n_1^C(s) + \sum_{i\in\mathcal{I}} h^i \mathcal{P}^i\left(b^i, s\right) - (1+\kappa)\sum_{i\in\mathcal{I}} t^i\left(b^i, s\right), \quad \forall s.$$

For given corrective taxes/subsidies $\{\tau_b^1, \tau_b^2\}$, lump-sum transfers $\{T_0^1, T_0^2\}$, and bailout transfers $\{t^1\left(b^1, s\right), t^2\left(b^2, s\right)\}$, an *equilibrium* is fully determined by investors' borrowing decisions, $\{b^1, b^2\}$, and financing schedules, $\{Q^1\left(b^1\right), Q^2\left(b^2\right)\}$, such that investors maximize their utility given the financing schedules and creditors set the schedules optimally, so that $h^1 = h^2 = 1$.

Equilibrium Characterization. We conjecture and verify that the price $Q^i\left(b^i; m^C(s)\right)$ of investors' debt is a function of b^i and creditors' stochastic discount factor $m^C(s) = \beta^C \frac{u'\left(c_1^C(s)\right)}{u'\left(c_0^C\right)}$. Substituting creditors' budget constraints into their objective, we obtain the simplified version of their maximization problem:

$$V^C\left(b^i, m^C(s)\right) = \max_{\{h^i\}_{i\in\mathcal{I}}} u\left(n_0^C - \sum_{i\in\mathcal{I}} h^i Q^i\left(b^i; m^C(s)\right)\right) + \beta^C \int u\left(n_1^C(s) + \sum_{i\in\mathcal{I}} h^i \mathcal{P}^i\left(b^i, s\right) - (1+\kappa)\sum_{i\in\mathcal{I}} t^i\left(b^i, s\right)\right) dF(s),$$

where $V^C(\cdot)$ denotes creditors' indirect utility as a function of investors' debt choice and market prices. The first-order conditions for this problem, combined with market clearing ($h^i = 1$), yield the following debt-pricing equation:

$$Q^i\left(b^i; m^C(s)\right) = \int_{\underline{s}}^{s^{i\star}\left(b^i\right)} m^C(s)\left(\phi^i s + t^i\left(b^i, s\right)\right) dF(s) + \int_{s^{i\star}(b^i)}^{\bar{s}} m^C(s)\, b^i dF(s).$$

Note that the stochastic discount factor in equilibrium must satisfy the fixed-point equation

$$m^C(s) = \beta^C \frac{u'\left(n_1^C(s) + \sum_{i\in\mathcal{I}} \mathcal{P}^i\left(b^i, s\right) - (1+\kappa)\sum_{i\in\mathcal{I}} t^i\left(b^i, s\right)\right)}{u'\left(n_0^C - \sum_{i\in\mathcal{I}} Q^i\left(b^i; m^C(s)\right)\right)}.$$

Substituting investors' budget constraints into their objective, and ignoring exogenous endowments, we obtain the simplified version of their maximization problem:

$$V^i\left(\tau_b^i, T_0^i, m^C(s)\right) = \max_{b^i} \beta^i \int_{s^{i\star}(b^i)}^{\bar{s}} \left(s + t^i\left(b^i, s\right) - b^i\right) dF(s) + Q^i\left(b^i; m^C(s)\right) - \tau_b^i b^i + T_0^i,$$

where $V^i(\cdot)$ denotes investors' indirect utility as a function of regulation and market prices. The first-order condition determining the optimal b^i is

$$-\beta^i \int_{s^{i\star}(b^i)}^{\bar{s}} \left(1 - \frac{\partial t^i\left(b^i, s\right)}{\partial b^i}\right) dF(s) + \frac{\partial Q^i\left(b^i; m^C(s)\right)}{\partial b^i} = \tau_b^i,$$

where

$$\begin{aligned}\frac{\partial Q^i\left(b^i; m^C(s)\right)}{\partial b^i} = &\int_{s^{i\star}(b^i)}^{\bar{s}} m^C(s)\, dF(s) + \int_{\underline{s}}^{s^{i\star}(b^i)} \frac{\partial t^i\left(b^i, s\right)}{\partial b^i} m^C(s)\, dF(s) \\ &- \left(1 - \phi^i\right) m^C\left(s^{i\star}\left(b^i\right)\right) s^{i\star}\left(b^i\right) f\left(s^{i\star}\left(b^i\right)\right) \frac{ds^{i\star}\left(b^i\right)}{db^i}.\end{aligned}$$

Marginal Welfare Effects. The money-metric marginal welfare effects of changing the regulation τ_b^j of investor type $j \in \{1, 2\}$ are given by

$$\frac{dW}{d\tau_b^j} = \frac{1}{\lambda_0^C} \frac{dV^C}{d\tau_b^j} + \sum_{i \in \mathcal{I}} \frac{dV^i}{d\tau_b^j},$$

where $\lambda_0^C = u'\left(c_0^C\right)$, since $\lambda_0^i = 1$. Using an envelope argument parallel to our general results, we obtain, abstracting from pecuniary effects that cancel after aggregating,

$$\frac{dV^C}{d\tau_b^j} = -(1+\kappa)\beta^C \int u'\left(c_1^C(s)\right) \sum_{i \in \mathcal{I}} \frac{\partial t^i\left(b^i, s\right)}{\partial b^i} \frac{db^i}{d\tau_b^j} dF(s),$$

and

$$\frac{dV^i}{d\tau_b^j} = \tau_b^i \frac{db^i}{d\tau_b^j},$$

where we have used the assumption that $T_0^i = \tau_b^i b^i$. Thus, we obtain

$$\frac{dW}{d\tau_b^j} = \sum_{i \in \mathcal{I}} \frac{db^i}{d\tau_b^j} \left(\tau_b^i - \underbrace{(1+\kappa) \int m^C(s) \frac{\partial t^i\left(b^i, s\right)}{\partial b^i} dF(s)}_{=\delta_b^i} \right). \tag{OA-8}$$

It follows that the first-best policy must satisfy $\tau_b^i = \delta_b^i$, $i \in \{1, 2\}$.

Simulation. In our simulation, we let $t^i\left(b^i, s\right) = \alpha_0^i - \alpha_s^i s + \alpha_b^i b^i$, with $\alpha_s^i < 1$, so that we can solve explicitly for the default threshold

$$s^{i\star}\left(b^i\right) = \left(\frac{1 - \alpha_b^i}{1 - \alpha_s^i}\right) b^i - \frac{1}{1 - \alpha_s^i} \alpha_0^i.$$

We further assume that creditors have constant relative risk aversion with coefficient γ. Figure OA-1 illustrates comparative statics of the model in the context of the second-best policy, in which $\tau_b^2 = 0$.

C.2 Example 2

Environment. Investors and creditors have risk-neutral preferences

$$c_0^i + \beta^i \int c_1^i(s)\, dF^i(s) \qquad \text{and} \qquad c_0^C + \beta^C \int c_1^C(s)\, dF^C(s).$$

Investors face the following budget constraints:

$$\begin{aligned} c_0^i &= n_0^i + Q^i\left(b^i\right) k^i - \Upsilon\left(k^i\right) \\ c_1^i(s) &= n_1^i(s) + \max\left\{s - b^i, 0\right\} k^i, \quad \forall s, \end{aligned}$$

as well as the leverage cap $b^i \leq \bar{b}$ which, for convenience, we scale by capital, and write as $b^i k^i \leq \bar{b} k^i$. At date 1, investors optimally decide to default when $s < b^i$, and to repay otherwise. Recovery in default is as in Example 1. Therefore, the definition of the repayment eventually received by creditors per unit of capital k^i, $\mathcal{P}^i\left(b^i, s\right)$, is

$$\mathcal{P}^i\left(b^i, s\right) = \begin{cases} \phi s & s \in \left[\underline{s}, b^i\right) \\ b^i & s \in \left[b^i, \bar{s}\right]. \end{cases}$$

Creditors face the following budget constraints:

$$\begin{aligned} c_0^C &= n_0^C - h^i Q^i\left(b^i\right) k^i \\ c_1^C(s) &= n_1^C(s) + h^i \mathcal{P}^i\left(b^i, s\right) k^i, \quad \forall s. \end{aligned}$$

For a given leverage cap $\bar{b}$, an *equilibrium* is defined by an investment decision k^i, a leverage decision b^i, and a default decision rule such that i) investors maximize their utility given $Q^i(\cdot)$, and ii) creditors set the schedule $Q^i(\cdot)$ optimally, so that $h^i = 1$.

Equilibrium Characterization. Since creditors are risk-neutral, they must be indifferent between all quantities of debt purchased in equilibrium. Hence, the valuation of debt *per unit of*

capital in equilibrium satisfies

$$Q^i\left(b^i\right) = \beta^C\left(\int_{b^i}^{\bar{s}} b^i dF^C\left(s\right) + \phi\int_{\underline{s}}^{b^i} s dF^C\left(s\right)\right).$$

Substituting the valuation of debt and the budget constraints into investors' objective function, and ignoring exogenous endowments, we obtain the simplified version of their maximization problem:

$$\max_{b^i,k^i} M\left(b^i\right)k^i - \Upsilon\left(k^i\right) - \tau_b^i\left(b^i k^i - \bar{b}k^i\right),$$

where τ_b^i is the shadow price of the leverage cap, and $M\left(b^i\right)$ is given by

$$M\left(b^i\right) = \beta^i\int_{b^i}^{\bar{s}}\left(s - b^i\right)dF^i\left(s\right) + Q^i\left(b^i\right),$$

which corresponds to the definition given in the text after substituting $Q^i\left(b^i\right)$. The first-order conditions in this problem are

$$\frac{dM\left(b^i\right)}{db^i} - \tau_b^i = 0 \tag{OA-9}$$

$$M\left(b^i\right) - \Upsilon'\left(k^i\right) = 0. \tag{OA-10}$$

Notice that no corrective tax/shadow price appears in the second condition, so that the scale k^i of capital is effectively unregulated. When considering the value of regulating scale, we change this condition to $M\left(b^i\right) - \Upsilon'\left(k^i\right) = \tau_k^i$.

Marginal Welfare Effects. As shown by Dávila and Walther (2023), social welfare for a planner who computes welfare using beliefs F^P is given by

$$W = M^P\left(b^i\right)k^i - \Upsilon\left(k^i\right),$$

where $M^P\left(b^i\right)$ denotes the present value of payoffs under the planner's beliefs:

$$M^P\left(b^i\right) = \beta^i\int_{b^i}^{\bar{s}}\left(s - b^i\right)dF^P\left(s\right) + \beta^C\left(\int_{b^i}^{\bar{s}} b^i dF^P\left(s\right) + \phi\int_{\underline{s}}^{b^i} s dF^P\left(s\right)\right).$$

The marginal welfare effects of varying τ_b^i, after differentiating and substituting investors' first-order conditions, can be written as

$$\begin{aligned}\frac{dW}{d\tau_b^i} &= k^i \frac{dM^P\left(b^i\right)}{db^i}\frac{db^i}{d\tau_b^i} + \left(M^P\left(b^i\right) - \Upsilon'\left(k^i\right)\right)\frac{dk^i}{d\tau_b^i} \\ &= k^i\left(\tau_b^i - \underbrace{\left(\frac{dM\left(b^i\right)}{db^i} - \frac{dM^P\left(b^i\right)}{db^i}\right)}_{\delta_b^i}\right)\frac{db^i}{d\tau_b^i} + \left(\tau_k^i - \underbrace{\left(M\left(b^i\right) - M^P\left(b^i\right)\right)}_{\delta_k^i}\right)\frac{dk^i}{d\tau_b^i}. \qquad \text{(OA-11)}\end{aligned}$$

Simulation. In our simulation, we assume that investment adjustment costs are quadratic, i.e., $\Upsilon\left(k^i\right) = k^i + \frac{a}{2}\left(k^i\right)^2$, in which case Equation (OA-10) takes the form

$$k^i = \frac{1}{a}\left(M\left(b^i\right) - 1 - \tau_k^i\right).$$

Figure OA-2 illustrates comparative statics of the model in the context of the second-best policy, in which $\tau_k^i = 0$.

C.3 Example 3

Environment. Investors and creditors have risk-neutral preferences given, respectively, by:

$$\begin{aligned}c_0^i + \beta^i \int c_1^i\left(s\right) dF\left(s\right) \\ c_0^C + \int c_1^C\left(s\right) dF\left(s\right).\end{aligned}$$

Each unit of investment costs one unit at date 0 and is financed by one unit of insured deposits, so total borrowing is $b^i = k_1^i + k_2^i$. Investors face the budget constraints

$$\begin{aligned}c_0^i &= n_0^i + b^i - k_1^i - k_2^i - \Upsilon\left(k_1^i, k_2^i\right) - \tau_k^1 k_1^i - \tau_k^2 k_2^i + T_0^i, \\ c_1^i\left(s\right) &= \max\left\{d_1\left(s\right)k_1^i + d_2\left(s\right)k_2^i - b^i, 0\right\}, \qquad \forall s.\end{aligned}$$

The bailout transfer is the minimum amount required to repay insured deposits:

$$\begin{aligned}t\left(k_1^i, k_2^i, s\right) &= \max\left\{b^i - d_1\left(s\right)k_1^i - d_2\left(s\right)k_2^i, 0\right\} \\ &= \max\left\{\left[1 - d_1\left(s\right)\right]k_1^i + \left[1 - d_2\left(s\right)\right]k_2^i, 0\right\}. \qquad \text{(OA-12)}\end{aligned}$$

The threshold state below which bailouts are positive, denoted $s^\star\left(k_1^i, k_2^i\right)$, is implicitly defined by

$$\left[d_1\left(s^\star\right) - 1\right] k_1^i + \left[d_2\left(s^\star\right) - 1\right] k_2^i = 0. \tag{OA-13}$$

We assume aggregate portfolio returns are increasing in s, so this threshold is unique. Creditors face the budget constraints

$$\begin{aligned} c_0^C &= n_0^C - b^i, \\ c_1^C\left(s\right) &= n_1^C\left(s\right) + b^i - \left(1+\kappa\right) t\left(k_1^i, k_2^i, s\right), \qquad \forall s. \end{aligned}$$

Thus deposits are repaid in full in every state, but creditors also bear the fiscal cost of bailouts.

For given corrective taxes/subsidies $\{\tau_k^1, \tau_k^2\}$, lump-sum transfers T_0^i, and bailout policy in Equation (OA-12), an *equilibrium* is given by investment choices (k_1^i, k_2^i) and deposits $b^i = k_1^i + k_2^i$ such that investors maximize utility subject to their budget constraints and creditors supply the associated deposits. Under uniform regulation, the planner chooses $\tau_k^1 = \tau_k^2 = \overline{\tau}_k$.

Equilibrium Characterization. Substituting the budget constraints and $b^i = k_1^i + k_2^i$ into investors' objective, and ignoring exogenous endowments, gives

$$\begin{aligned} V^i\left(\tau_k^1, \tau_k^2, T_0^i\right) = \max_{k_1^i, k_2^i} \beta^i \int_{s^\star\left(k_1^i, k_2^i\right)}^{\bar{s}} \left\{\left[d_1\left(s\right) - 1\right] k_1^i + \left[d_2\left(s\right) - 1\right] k_2^i\right\} dF\left(s\right) \\ - \Upsilon\left(k_1^i, k_2^i\right) - \tau_k^1 k_1^i - \tau_k^2 k_2^i + T_0^i. \end{aligned}$$

The boundary term vanishes because equity receives zero at $s = s^\star$. Interior first-order conditions are therefore

$$\begin{aligned} \beta^i \int_{s^\star}^{\bar{s}} \left[d_1\left(s\right) - 1\right] dF\left(s\right) - \frac{\partial \Upsilon}{\partial k_1^i} - \tau_k^1 = 0, \\ \beta^i \int_{s^\star}^{\bar{s}} \left[d_2\left(s\right) - 1\right] dF\left(s\right) - \frac{\partial \Upsilon}{\partial k_2^i} - \tau_k^2 = 0. \end{aligned} \tag{OA-14}$$

Marginal Welfare Effects. Using a parallel argument to our general results, we obtain the marginal welfare effects:

$$\frac{dW}{d\tau_k^\ell} = \sum_{j \in \{1,2\}} \frac{dk_j^i}{d\tau_k^\ell}\left(\tau_k^j - \delta_j\right), \qquad \delta_j = \left(1+\kappa\right) \int_{\underline{s}}^{s^\star} \left[1 - d_j\left(s\right)\right] dF\left(s\right). \tag{OA-15}$$

Thus the first-best regulation of each asset is $\tau_k^j = \delta_j$. If asset 2 has lower payoffs in bailout states, it creates a larger expected deposit shortfall and therefore has a larger Pigouvian regulation.

Derivation of Leakage Elasticities with Separable Costs. Assume $\Upsilon\left(k_1^i, k_2^i\right) = \frac{z_1}{2}\left(k_1^i\right)^2 + \frac{z_2}{2}\left(k_2^i\right)^2$. Investors' first-order conditions become

$$k_1^i = \frac{1}{z_1}\left(\beta^i \int_{s^\star}^{\bar{s}} \left[d_1\left(s\right) - 1\right] dF\left(s\right) - \tau_k^1\right),$$
$$k_2^i = \frac{1}{z_2}\left(\beta^i \int_{s^\star}^{\bar{s}} \left[d_2\left(s\right) - 1\right] dF\left(s\right) - \tau_k^2\right).$$

Applying the implicit function theorem and Leibniz rule, and imposing uniform regulation, gives

$$\frac{dk_j^i}{d\overline{\tau}_k} = \frac{1}{z_j}\left[-\beta^i\left(d_j\left(s^\star\right) - 1\right) f\left(s^\star\right) \frac{ds^\star}{d\overline{\tau}_k} - 1\right]. \tag{OA-16}$$

Let the bailout probability be $\pi = F\left(s^\star\right)$. Since $d\pi/d\overline{\tau}_k = f\left(s^\star\right) ds^\star/d\overline{\tau}_k$, Equation (OA-16) can be written as

$$\frac{dk_j^i}{d\overline{\tau}_k} = \frac{1}{z_j}\left[-\beta^i\left(d_j\left(s^\star\right) - 1\right) \frac{d\pi}{d\overline{\tau}_k} - 1\right].$$

Hence the weights in the optimal uniform regulation depend on i) the curvature z_j of investment costs, ii) the sensitivity of the bailout probability to regulation, and iii) each asset's net payoff $d_j\left(s^\star\right) - 1$ at the bailout boundary. In particular, the weight on δ_1 is

$$\frac{\frac{dk_1^i}{d\overline{\tau}_k}}{\frac{dk_1^i}{d\overline{\tau}_k} + \frac{dk_2^i}{d\overline{\tau}_k}} = \frac{1}{1+\zeta_1},$$

where

$$\zeta_1 = \frac{z_1}{z_2}\frac{1 + \beta^i\left[d_2\left(s^\star\right) - 1\right]\frac{d\pi}{d\overline{\tau}_k}}{1 + \beta^i\left[d_1\left(s^\star\right) - 1\right]\frac{d\pi}{d\overline{\tau}_k}}.$$

Simulation. Figure OA-3 illustrates comparative statics under uniform regulation using the calibration in Figure 3.

C.4 Example 4

Preferences. Investors have utility

$$u^i = c_0^i + c_1^i + c_2^i$$

and face the following budget constraints:

$$\begin{aligned} c_0^i &= n_0^i - \Upsilon^i\left(k_0^i\right) - \tau_k^i k_0^i + T_0^i \\ c_1^i &= q\left(k_0^i - k_1^i\right) - \xi^i k_0^i \\ c_2^i &= z^i k_1^i. \end{aligned}$$

Consumption must be non-negative at all dates.

Households' budget constraint at date 1 is

$$c_1^H = n_1^H + G\left(k_1^H\right) - q k_1^H.$$

For given corrective taxes/subsidies $\{\tau_k^1, \tau_k^2\}$ and lump-sum transfers $\{T_0^1, T_0^2\} = \{\tau_k^1 k_0^1, \tau_k^2 k_0^2\}$, an *equilibrium* is defined as investors' investment decisions $\{k_0^i, k_1^i\}$, households' capital allocation k_1^H, and an equilibrium price q, such that investors' and households' utilities are maximized subject to their constraints and the capital market clears, $\sum_i \left(k_0^i - k_1^i\right) = k_1^H$.

Equilibrium Characterization. Households' optimization problem at date 1 can be expressed as

$$V^H(q) = \max_{k_1^H} G\left(k_1^H\right) - q k_1^H,$$

where the solution to the households' problem is characterized by $q = G'\left(k_1^H\right)$. When combined with market clearing, given by $\sum_i \left(k_0^i - k_1^i\right) = k_1^H$, we find the following equation, which the price q must satisfy:

$$q = G'\left(k_1^H\right) = G'\left(\sum_i \left(k_0^i - k_1^i\right)\right) = G'\left(\frac{1}{q}\sum_i \xi^i k_0^i\right).$$

Notice that this equation defines q as an implicit function of capital investments k_0^i. Below, we derive a solution for the equilibrium value of q in terms of primitives under standard functional forms.

We solve the investors' problem recursively. At date 1, the non-negativity constraint on consumption is necessarily binding. It follows that the investor optimally chooses $c_1^i = 0$ and

$$k_1^i = \left(1 - \frac{\xi^i}{q}\right) k_0^i.$$

Thus, investor i's maximized utility (i.e., value function) from date 1 onwards is

$$v_1^i\left(q, k_0^i\right) = z^i\left(1 - \frac{\xi^i}{q}\right) k_0^i.$$

At date 0, ignoring exogenous endowments, we can express investors' optimization problem as

$$\begin{aligned} V^i\left(\tau_k^i, T_0^i, q\right) &= \max_{k_0^i}\left\{v_1^i\left(q, k_0^i\right) - \Upsilon^i\left(k_0^i\right) - \tau_k^i k_0^i + T_0^i\right\} \\ &= \max_{k_0^i}\left\{z^i\left(1 - \frac{\xi^i}{q}\right)k_0^i - \Upsilon^i\left(k_0^i\right) - \tau_k^i k_0^i + T_0^i\right\}, \end{aligned}$$

where $V^i(\cdot)$ denotes investors' indirect lifetime utility as a function of taxes and market prices. The first-order condition determining optimal investment k_0^i is given by

$$z^i\left(1 - \frac{\xi^i}{q}\right) = \Upsilon^{i\prime}\left(k_0^i\right) + \tau_k^i.$$

Assuming quadratic adjustment costs, we obtain the closed-form solution

$$k_0^i = \frac{1}{a^i}\left(z^i\left(1 - \frac{\xi^i}{q}\right) - \tau_k^i\right).$$

Marginal Welfare Effects. The marginal welfare effect of changing the regulation τ_k^j of investor type j is given by

$$\frac{dW}{d\tau_k^j} = \sum_{\ell\in\mathcal{I}}\frac{dV^\ell}{d\tau_k^j} + \frac{dV^H}{d\tau_k^j}.$$

Using the envelope theorem, parallel to our general results, we obtain

$$\frac{dV^H}{d\tau_k^j} = \frac{\partial V^H}{\partial q}\frac{dq}{d\tau_k^j}.$$

Similarly, we have

$$\begin{aligned} \frac{dV^\ell}{d\tau_k^j} &= \frac{\partial V^\ell}{\partial \tau_k^j} + \frac{\partial V^\ell}{\partial T_0^\ell}\frac{dT_0^\ell}{d\tau_k^j} + \frac{\partial V^\ell}{\partial q}\frac{dq}{d\tau_k^j} \\ &= \tau_k^\ell\frac{dk_0^\ell}{d\tau_k^j} + \frac{\partial v_1^\ell}{\partial q}\frac{dq}{d\tau_k^j}, \end{aligned}$$

where we have used the assumption that $T_0^\ell = \tau_k^\ell k_0^\ell$. Combining, we obtain

$$\begin{aligned} \frac{dW}{d\tau_k^j} &= -k_1^H\frac{dq}{d\tau_k^j} + \sum_{\ell\in\mathcal{I}}\left(\tau_k^\ell\frac{dk_0^\ell}{d\tau_k^j} + \frac{\partial v_1^\ell}{\partial q}\frac{dq}{d\tau_k^j}\right) \\ &= \sum_{i\in\mathcal{I}}\tau_k^i\frac{dk_0^i}{d\tau_k^j} + \left(\sum_{\ell\in\mathcal{I}}\frac{\partial v_1^\ell}{\partial q} - k_1^H\right)\frac{dq}{d\tau_k^j}. \end{aligned} \tag{OA-17}$$

Since q in equilibrium is an implicit function of initial capital investments k_0^i, $i \in \{1,2\}$, we can write

$$\frac{dq}{d\tau_k^j} = \sum_{i\in\mathcal{I}} \frac{\partial q}{\partial k_0^i}\frac{dk_0^i}{d\tau_k^j}.$$

Moreover, notice that

$$\sum_{\ell\in\mathcal{I}} \frac{\partial v_1^\ell}{\partial q} - k_1^H = \sum_{\ell\in\mathcal{I}} \frac{z^\ell}{q}\frac{\xi^\ell}{q} k_0^\ell - k_1^H = \sum_{\ell\in\mathcal{I}} \left(\frac{z^\ell}{q} - 1\right)\left(k_0^\ell - k_1^\ell\right),$$

where the last equality follows from the market clearing condition $k_1^H = \sum_{\ell\in\mathcal{I}}\left(k_0^\ell - k_1^\ell\right)$. Substituting into (OA-17) yields

$$\begin{aligned}\frac{dW}{d\tau_k^j} &= \sum_{i\in\mathcal{I}} \tau_k^i \frac{dk_0^i}{d\tau_k^j} + \sum_{\ell\in\mathcal{I}} \left(\frac{z^\ell}{q} - 1\right)\left(k_0^\ell - k_1^\ell\right)\sum_{i\in\mathcal{I}} \frac{\partial q}{\partial k_0^i}\frac{dk_0^i}{d\tau_k^j} \\ &= \sum_{i\in\mathcal{I}} \left(\tau_k^i - \underbrace{\left(-\frac{\partial q}{\partial k_0^i}\right)\sum_{\ell\in\mathcal{I}} \left(\frac{z^\ell}{q} - 1\right)\left(k_0^\ell - k_1^\ell\right)}_{=\delta_k^i}\right)\frac{dk_0^i}{d\tau_k^j}.\end{aligned}$$

Simulation. For our simulation, we assume that $G\left(k_1^H\right) = \frac{\left(k_1^H\right)^\alpha}{\alpha}$, which implies that $G'\left(k_1^H\right) = \left(k_1^H\right)^{\alpha-1}$. We can express the equilibrium price in closed form as

$$q = \left(\sum_i \xi^i k_0^i\right)^{\frac{\alpha-1}{\alpha}}. \tag{OA-18}$$

With quadratic adjustment costs $\Upsilon^i\left(k_0^i\right) = \frac{a^i}{2}\left(k_0^i\right)^2$, investors' optimal choices at date 0 satisfy

$$k_0^i = \frac{1}{a^i}\left(z^i\left(1 - \frac{\xi^i}{q}\right) - \tau_k^i\right).$$

Note that $\frac{\partial k_0^i}{\partial q} = \frac{z^i}{a^i}\frac{\xi^i}{q^2} > 0$. Note also that $z^i\left(1 - \frac{\xi^i}{q}\right) - \tau_k^i > 0$ is required for $k_0^i > 0$. Combining the optimal choice of k_0^i with the characterization of the price in Equation (OA-18) yields a solution for q in terms of primitives:

$$q = \left(\sum_i \frac{\xi^i}{a^i}\left(z^i\left(1 - \frac{\xi^i}{q}\right) - \tau_k^i\right)\right)^{\frac{\alpha-1}{\alpha}}.$$

As expected, the same change in k_0^i has a stronger impact on the price at date 1 for those investors with a higher ξ^i, who are forced to sell more at date 1. Note that we can write $\frac{\partial q}{\partial k_0^i} = \xi^i \frac{\alpha-1}{\alpha} q^{\frac{1}{1-\alpha}}$, so $\frac{\partial q}{\partial k_0^i}$ is higher in absolute value when q is higher.

Figure OA-4 illustrates comparative statics of the model in the context of the second-best policy, in which $\overline{\tau}_k = \tau_k^1 = \tau_k^2$.

D Proofs and Derivations: Section 4

Since creditors are risk-neutral, they must be indifferent between all quantities of debt purchased in equilibrium. Hence, the valuation of debt *per unit of capital* in equilibrium is

$$Q^i\left(b^i,\theta^i\right) = \beta^C\left(\int_{s^\star\left(b^i,\theta^i\right)}^{\bar{s}} b^i dF\left(s\right) + \phi\int_{\underline{s}}^{s^\star\left(b^i,\theta^i\right)}\left[d_1\left(s\right)\theta^i + d_2\left(s\right)\left(1-\theta^i\right) + t\left(b^i,\theta^i,s\right)\right]dF\left(s\right)\right).$$

Substituting the valuation of debt and the budget constraints into investors' objective function, and ignoring exogenous endowments, we obtain the simplified version of their maximization problem in Equation (26).

While the second-best policy analysis in the text follows directly from our general results, we also give a self-contained derivation here. Adding the utilities of investors and creditors, imposing market clearing, and ignoring exogenous endowments, we find that maximizing welfare is equivalent to maximizing

$$W\left(b^i,\theta^i,k^i\right) = \left[M\left(b^i,\theta^i\right) - \Omega\left(\theta^i\right) - 1\right]k^i - \Upsilon\left(k^i\right) - \left(1+\kappa\right)\beta^C\int_{\underline{s}}^{\bar{s}} t\left(b^i,\theta^i,s\right)k^i dF\left(s\right) - \Psi\left(\theta^i\right)k^i.$$

We can now write $b^i\left(\bar{b},\varphi\right)$, $\theta^i\left(\bar{b},\varphi\right)$ and $k^i\left(\bar{b},\varphi\right)$ for optimal choices as a function of regulatory parameters, and totally differentiate the welfare function with respect to $\bar{b}$ to obtain

$$\begin{aligned}\frac{dW}{d\bar{b}} &= k^i\left(\frac{\partial M}{\partial b^i} - \delta_b\right)\frac{db^i}{d\bar{b}} + k^i\left(\frac{\partial M}{\partial \theta^i} - \Omega'\left(\theta^i\right) - \delta_\theta\right)\frac{d\theta^i}{d\bar{b}} - \delta_k\frac{dk^i}{d\bar{b}}\\ &= k^i\left(\tau_b - \delta_b\right)\frac{db^i}{d\bar{b}} + k^i\left(\tau_\theta - \delta_\theta\right)\frac{d\theta^i}{d\bar{b}} - \delta_k\frac{dk^i}{d\bar{b}},\end{aligned}$$

where we have substituted the definitions of $\{\delta_b, \delta_\theta, \delta_k\}$ from Equations (31) through (33), as well as the first-order conditions (28), (29) and (30). Similarly, we differentiate with respect to φ to

obtain

$$\begin{aligned}\frac{dW}{d\varphi} &= k^i\left(\frac{\partial M}{\partial b^i}-\delta_b\right)\frac{db^i}{d\varphi}+k^i\left(\frac{\partial M}{\partial \theta^i}-\Omega'\left(\theta^i\right)-\delta_\theta\right)\frac{d\theta^i}{d\varphi}-\delta_k\frac{dk^i}{d\varphi}\\ &= k^i\left(\tau_b-\delta_b\right)\frac{db^i}{d\varphi}+k^i\left(\tau_\theta-\delta_\theta\right)\frac{d\theta^i}{d\varphi}-\delta_k\frac{dk^i}{d\varphi}.\end{aligned}$$

Characterization of Complementarities. Investors' first-order condition for k^i can be written as

$$J\left(\bar{b},\varphi\right)=1+\Upsilon'\left(k^i\right), \tag{OA-19}$$

where

$$J\left(\bar{b},\varphi\right)=\max_{b^i,\theta^i}\left\{M\left(b^i,\theta^i\right)-\Omega\left(\theta^i\right)\text{ subject to }b^i+\varphi\theta^i\leq\bar{b}\right\}. \tag{OA-20}$$

By the envelope theorem, we have

$$\begin{aligned}\frac{\partial J}{\partial \bar{b}} &= \upsilon\geq 0\\ \frac{\partial J}{\partial \varphi} &= -\upsilon\theta^i\leq 0,\end{aligned}$$

where υ denotes the Lagrange multiplier on the constraint in (OA-20). Totally differentiating (OA-19), we now obtain

$$\begin{aligned}\frac{\partial J}{\partial \bar{b}} &= \frac{dk^i}{d\bar{b}}\Upsilon''\left(k^i\right)\\ \frac{\partial J}{\partial \varphi} &= \frac{dk^i}{d\varphi}\Upsilon''\left(k^i\right),\end{aligned}$$

so that the convexity of $\Upsilon\left(\cdot\right)$ immediately implies

$$\frac{dk^i}{d\bar{b}}\geq 0\quad\text{and}\quad\frac{dk^i}{d\varphi}\leq 0,$$

as claimed in the text.

Further Simulation Results. Given our functional form assumptions for the simulation, note that we can express the default threshold $s^\star\left(b^i,\theta^i\right)$ as

$$s^\star\left(b^i,\theta^i\right)=\frac{b^i-\left(\alpha_0^i+\alpha_b^i b^i+\alpha_\theta^i\theta^i\right)}{\bar{d}_1\theta^i+\bar{d}_2\left(1-\theta^i\right)-\alpha_s^i}.$$

Relatedly, note that the marginal distortions in Equations (31) through (33) correspond then to

$$\begin{aligned}
\delta_b &= \underbrace{\beta^C (1+\kappa)\, \alpha_b^i}_{\equiv \chi_b} \\
\delta_\theta &= \underbrace{\beta^C (1+\kappa)\, \alpha_\theta^i}_{\equiv \chi_\theta} + \underbrace{\Psi'\left(\theta^i\right)}_{\equiv \psi_\theta} \\
\delta_k &= \underbrace{\beta^C (1+\kappa) \left(\alpha_0^i + \alpha_b^i b^i + \alpha_\theta^i \theta^i - \alpha_s^i \mathbb{E}\left[s\right]\right)}_{\equiv \chi_k} + \underbrace{\Psi\left(\theta^i\right)}_{\equiv \psi_k}.
\end{aligned}$$

Also, note that

$$\Omega'\left(\theta^i\right) = z_\Omega^{\eta_\Omega} \left(\Omega\left(\theta^i\right)\right)^{1-\eta_\Omega} \left(a_\Omega \left(\theta^i\right)^{\eta_\Omega - 1} - (1-a_\Omega)\left(1-\theta^i\right)^{\eta_\Omega - 1}\right),$$

and similarly for $\Psi'\left(\theta^i\right)$.

E Additional Proofs and Derivations

E.1 Price-Theoretic Formulation

We now show that it is possible to extend our results to a price-theoretic environment as follows. Consider an economy with a finite number $I \geq 1$ of agents (equivalently, agent types in unit measure), indexed by $i, j \in \mathcal{I}$, where $\mathcal{I} = \{1, \ldots, I\}$. There are $N \geq 1$ goods (commodities) indexed by $n \in \mathcal{N}$, where $\mathcal{N} = \{1, \ldots, N\}$. Agent i's preferences are represented by

$$u^i\left(\boldsymbol{x}^i, \bar{\boldsymbol{x}}\right), \tag{OA-21}$$

where $\boldsymbol{x}^i \in \mathbb{R}_+^N$ denotes agent i's consumption bundle, and $\bar{\boldsymbol{x}} = \left\{\bar{\boldsymbol{x}}^j\right\}_{j \in \mathcal{I}}$ denotes the collection of bundles of all agents.[22] Each agent takes $\bar{\boldsymbol{x}}$ as given, so the second argument in $u^i(\cdot)$ captures externalities across agents.

Each agent i faces a budget constraint

$$\boldsymbol{p} \cdot \left(\boldsymbol{x}^i - \boldsymbol{e}^i\right) + \boldsymbol{\tau}^i \cdot \boldsymbol{x}^i = T^i, \tag{OA-22}$$

where $\boldsymbol{e}^i \in \mathbb{R}_+^N$ denotes agent i's endowment of goods; $\boldsymbol{p} \in \mathbb{R}_+^N$ is a price vector; $\boldsymbol{\tau}^i \cdot \boldsymbol{x}^i$ introduces a set of taxes/subsidies (regulations) specific to each agent and commodity, where $\boldsymbol{\tau}^i \in \mathbb{R}^N$; and $T^i \in \mathbb{R}$ denotes the lump-sum transfer or tax that agent i receives or faces to ensure that the

[22] Section A of this Online Appendix defines all vectors and matrices used here.

planner runs a balanced budget. We denote the elements of $\boldsymbol{x}^i$, $\bar{\boldsymbol{x}}^i$, and $\boldsymbol{\tau}^i$ by x_n^i, $\bar{x}_n^i$, and τ_n^i, respectively.

For a given set of regulations $\{\boldsymbol{\tau}^i\}_{i\in\mathcal{I}}$ and transfers $\{T^i\}_{i\in\mathcal{I}}$, an *equilibrium* consists of consumption allocations $\{\boldsymbol{x}^i\}_{i\in\mathcal{I}}$ and a price vector $\boldsymbol{p}$ such that i) agents choose $\boldsymbol{x}^i$ to maximize utility (OA-21) taking $\boldsymbol{p}$ and $\bar{\boldsymbol{x}}$ as given, subject to the budget constraint (OA-22); ii) the planner's budget is balanced, so that $\sum_i T^i = \sum_i \boldsymbol{\tau}^i \cdot \boldsymbol{x}^i$; iii) consumption allocations are consistent in the aggregate, so $\bar{\boldsymbol{x}}^i = \boldsymbol{x}^i$, $\forall i$; and iv) markets clear, so $\sum_i \boldsymbol{x}^i = \sum_i \boldsymbol{e}^i$. We assume at all times that the model is well-behaved.

First, we characterize the change in agent i's indirect utility, denoted by V^i, when varying a specific regulation τ_n^j. Making use of the envelope theorem, we have

$$\frac{dV^i}{d\tau_n^j} = \frac{\partial u^i}{\partial \bar{\boldsymbol{x}}} \cdot \frac{d\bar{\boldsymbol{x}}}{d\tau_n^j} - \lambda^i \left(\frac{d\boldsymbol{p}}{d\tau_n^j} \cdot \left(\boldsymbol{x}^i - \boldsymbol{e}^i \right) + \frac{d\boldsymbol{\tau}^i}{d\tau_n^j} \cdot \boldsymbol{x}^i - \frac{dT^i}{d\tau_n^j} \right),$$

where λ^i denotes the Lagrange multiplier associated with the budget constraint. We use x and $\bar{x}$ equivalently going forward, since they are equal in equilibrium.

Expressing this welfare change in money-metric terms by normalizing by agent i's marginal value of wealth, λ^i, and aggregating across agents, we have

$$\begin{aligned}
\frac{dW}{d\tau_n^j} = \sum_i \frac{\frac{dV^i}{d\tau_n^j}}{\lambda^i} &= \sum_i \frac{1}{\lambda^i} \frac{\partial u^i}{\partial \bar{\boldsymbol{x}}} \cdot \frac{d\boldsymbol{x}}{d\tau_n^j} - \frac{d\boldsymbol{p}}{d\tau_n^j} \cdot \sum_i \left(\boldsymbol{x}^i - \boldsymbol{e}^i \right) - \sum_i \left(\frac{d\boldsymbol{\tau}^i}{d\tau_n^j} \cdot \boldsymbol{x}^i - \frac{dT^i}{d\tau_n^j} \right) \\
&= \sum_i \frac{1}{\lambda^i} \frac{\partial u^i}{\partial \bar{\boldsymbol{x}}} \cdot \frac{d\boldsymbol{x}}{d\tau_n^j} + \sum_i \frac{d\boldsymbol{x}^i}{d\tau_n^j} \cdot \boldsymbol{\tau}^i,
\end{aligned}$$

where the last line follows from i) market clearing, $\sum_i \left(\boldsymbol{x}^i - \boldsymbol{e}^i \right) = 0$, and ii) the fact that the planner's budget is balanced, so

$$\sum_i \frac{dT^i}{d\tau_n^j} = \sum_i \frac{d\boldsymbol{\tau}^i}{d\tau_n^j} \cdot \boldsymbol{x}^i + \sum_i \frac{d\boldsymbol{x}^i}{d\tau_n^j} \cdot \boldsymbol{\tau}^i.$$

Note that $\frac{dW}{d\tau_n^j}$ can be equivalently expressed as

$$\frac{dW}{d\tau_n^j} = \sum_i \frac{1}{\lambda^i} \sum_\ell \frac{\partial u^i}{\partial \bar{\boldsymbol{x}}^\ell} \cdot \frac{d\boldsymbol{x}^\ell}{d\tau_n^j} + \sum_\ell \frac{d\boldsymbol{x}^\ell}{d\tau_n^j} \cdot \boldsymbol{\tau}^\ell = \sum_\ell \frac{d\boldsymbol{x}^\ell}{d\tau_n^j} \left(\boldsymbol{\tau}^\ell - \boldsymbol{\delta}^\ell \right),$$

where $\boldsymbol{\delta}^\ell = -\sum_i \frac{1}{\lambda^i} \frac{\partial u^i}{\partial \bar{\boldsymbol{x}}^\ell}$, so by switching indices and stacking we find that

$$\frac{dW}{d\boldsymbol{\tau}} = \frac{d\boldsymbol{x}}{d\boldsymbol{\tau}} \left(\boldsymbol{\tau} - \boldsymbol{\delta} \right) = \frac{d\boldsymbol{x}}{d\boldsymbol{\tau}} \boldsymbol{\omega},$$

as in Equation (8) in the text.

E.2 Game-Theoretic Formulation

We now show that it is possible to extend our results to a game-theoretic environment as follows. Suppose that agents have preferences of the form

$$u^i\left(\boldsymbol{x}^i, \bar{\boldsymbol{x}}\right)$$

and that they face a constraint given by

$$\Psi^i\left(\boldsymbol{x}^i, \bar{\boldsymbol{x}}; \theta\right) = 0,$$

where θ is a parameter that indexes a general perturbation. In the competitive model in Section 2, the function $\Psi^i\left(\boldsymbol{x}^i, \bar{\boldsymbol{x}}; \theta\right)$ takes the simple form

$$\Psi^i\left(\boldsymbol{x}^i, \bar{\boldsymbol{x}}; \theta\right) = \boldsymbol{p}\left(\bar{x}\right)\left(\boldsymbol{x}^i - \boldsymbol{e}^i\right) + \boldsymbol{\tau}^i\left(\theta\right)\boldsymbol{x}^i - T^i.$$

We consider a Walras-Nash equilibrium with taxes, in which optimality requires that $\frac{\partial u^i}{\partial \boldsymbol{x}^i} = \lambda^i \frac{\partial \Psi^i}{\partial \boldsymbol{x}^i}$. In general, we can express the marginal welfare change in the utility of agent i induced by a general perturbation as

$$\frac{dV^i}{d\theta} = \frac{\partial u^i}{\partial \boldsymbol{x}^i} \cdot \frac{d\boldsymbol{x}^i}{d\theta} + \frac{\partial u^i}{\partial \bar{\boldsymbol{x}}} \cdot \frac{d\bar{\boldsymbol{x}}}{d\theta} - \lambda^i \left(\frac{\partial \Psi^i}{\partial \boldsymbol{x}^i} \cdot \frac{d\boldsymbol{x}^i}{d\theta} + \frac{\partial \Psi^i}{\partial \bar{\boldsymbol{x}}} \cdot \frac{d\bar{\boldsymbol{x}}}{d\theta} + \frac{\partial \Psi^i}{\partial \theta} \right),$$

and normalizing by λ^i to express the marginal welfare change in units of the constraint

$$\frac{dV^i/d\theta}{\lambda^i} = \left(\frac{1}{\lambda^i} \frac{\partial u^i}{\partial \bar{\boldsymbol{x}}} - \frac{\partial \Psi^i}{\partial \bar{\boldsymbol{x}}} \right) \cdot \frac{d\bar{\boldsymbol{x}}}{d\theta} - \frac{\partial \Psi^i}{\partial \theta}.$$

This normalization allows us to express aggregate welfare gains as

$$\frac{dW}{d\theta} = \sum_i \frac{dV^i/d\theta}{\lambda^i} = \sum_i \left(\frac{1}{\lambda^i} \frac{\partial u^i}{\partial \bar{\boldsymbol{x}}} - \frac{\partial \Psi^i}{\partial \bar{\boldsymbol{x}}} \right) \cdot \frac{d\bar{\boldsymbol{x}}}{d\theta} - \sum_i \frac{\partial \Psi^i}{\partial \theta}. \tag{OA-23}$$

This expression is the generalization of Equation (8) in Lemma 1, where the term multiplying $\frac{d\bar{\boldsymbol{x}}}{d\theta}$ defines marginal distortions, and the term $\sum_i \frac{\partial \Psi^i}{\partial \theta}$ captures the direct impact of any policy perturbation. When the policy perturbation takes the form $\boldsymbol{\tau}^i\left(\theta\right)\boldsymbol{x}^i$, then Equation (OA-23) is exactly a generalized version of Lemma 1, with a slightly redefined marginal distortion, which is now augmented to capture interactions among agents via constraints.

E.3 Redistributional Concerns

Given the money-metric marginal welfare effects of varying $\boldsymbol{\tau}^j$, defined in Equations (OA-6) and (OA-7), we can express the marginal welfare effects of varying $\boldsymbol{\tau}^j$, for any set of generalized social marginal welfare weights (Saez and Stantcheva, 2016) or individual weights (Dávila and Schaab, 2025), as follows:

$$\frac{dW}{d\boldsymbol{\tau}^j} = \sum_i \omega^i \frac{\frac{dV^i}{d\boldsymbol{\tau}^j}}{\lambda_0^i} + \omega^C \frac{\frac{dV^C}{d\boldsymbol{\tau}^j}}{\lambda_0^C} = \underbrace{\sum_i \frac{\frac{dV^i}{d\boldsymbol{\tau}^j}}{\lambda_0^i} + \frac{\frac{dV^C}{d\boldsymbol{\tau}^j}}{\lambda_0^C}}_{\text{Efficiency}} + \underbrace{\mathbb{C}ov_{iC}^{\Sigma}\left[\omega^{iC}, \frac{\frac{dV^{iC}}{d\boldsymbol{\tau}^j}}{\lambda_0^{iC}}\right]}_{\text{Redistribution}}, \tag{OA-24}$$

where we assume, without loss of generality, that the weights average one, that is, $\frac{1}{I+1}\left(\sum_i \omega^i + \omega^C\right) = 1$, and where we use the index iC to refer to the set of investors and creditors.[23]

When $\omega^i = \omega^C = 1$, then the redistribution term in Equation (OA-24) is zero. This case is the one studied in the body of the paper. When $\omega^i \neq \omega^C \neq 1$, Equation (OA-24) shows that redistributional concerns enter additively to the marginal welfare effects of varying τ^j. A utilitarian planner simply corresponds to setting marginal welfare weights of the form $\omega^i = \lambda_0^i$, where λ_0^i typically equals marginal utility of consumption. Note that a utilitarian planner with access to lump-sum taxes/transfers finds it optimal to endogenously set $\omega^i = \omega^C = 1$.

E.4 Practical Scenarios

E.4.1 Unregulated Decisions

Equation (9) follows directly from Proposition 1 since at the second-best optimum, the constraints are binding with $\boldsymbol{\tau}^U = 0$. Concretely, we have

$$\boldsymbol{\tau}^R = \boldsymbol{\delta}^R + \left(-\frac{d\boldsymbol{x}^R}{d\boldsymbol{\tau}^R}\right)^{-1} \frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^R}\left(\underbrace{\boldsymbol{\tau}^U}_{=0} - \boldsymbol{\delta}^U\right) = \boldsymbol{\delta}^R - \left(-\frac{d\boldsymbol{x}^R}{d\boldsymbol{\tau}^R}\right)^{-1} \frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^R}\boldsymbol{\delta}^U,$$

as required. It is useful to consider the simple scenario in which there are two agents, $I = 2$, and each agent has a single decision, $N = 1$. Assume that only agent 1 can be regulated, with regulatory constraints dictating that $\tau^2 \equiv 0$. In that case, it follows from (9) that the optimal regulation for the regulated agent is simply given by

$$\tau^1 = \delta^1 - \left(-\frac{dx^1}{d\tau^1}\right)^{-1} \frac{dx^2}{d\tau^1}\delta^2.$$

[23] In Equation (OA-24), the covariance operator, indexed by iC, corresponds to a cross-sectional covariance-sum including investors and creditors.

The optimal regulation on agent 1 is equal to the first-best equivalent δ^1 minus a correction that accounts for the distortion imposed by the other unregulated agent. Assume, for instance, that the distortion by the unregulated agent satisfies $\delta^2 > 0$. The weight on the distortion by the unregulated agent is negative, implying that it pushes τ^1 towards underregulation, whenever i) the regulated agent responds negatively to increased regulation (the "regular" case with $\frac{dx^1}{d\tau^1} < 0$), and ii) the associated leakage elasticity indicates gross substitutes with $\frac{dx^2}{d\tau^1} > 0$.

While this is the simplest case for building intuition, note that the same insight extends to any economy with a single regulated decision and with an arbitrary set of unregulated decisions for which taxes/subsidies are forced to be zero. In this more general case, the optimal policy formula becomes

$$\tau^R = \delta^R - \sum_{(j,n)\in\mathcal{U}} \left(-\frac{dx^R}{d\tau^R}\right)^{-1} \frac{dx_n^j}{d\tau^R}\delta_n^j,$$

where $\mathcal{U}$ denotes the set of imperfectly regulated decisions.

E.4.2 Uniform Regulation

To build intuition for the uniform regulation results, it is useful to first consider the special case where *all* decisions are subject to uniform regulation ($\boldsymbol{x}^U = \boldsymbol{x}$). In that case, it follows from Proposition 2 that the optimal uniform regulation is given by

$$\overline{\tau}^U = \frac{\sum_i \sum_n \frac{dx_n^i}{d\overline{\tau}^U}\delta_n^i}{\sum_i \sum_n \frac{dx_n^i}{d\overline{\tau}^U}} = \frac{\sum_i \sum_n w_n^i \delta_n^i}{\sum_i \sum_n w_n^i}, \tag{OA-25}$$

where we define the policy-response weight $w_n^i \equiv \frac{dx_n^i}{d\overline{\tau}^U}$ and have rewritten the total response of decision x_n^i to the uniform regulation as

$$\frac{dx_n^i}{d\overline{\tau}^U} = \sum_{j\in\mathcal{I}} \sum_{m=1}^{N} \frac{dx_n^i}{d\tau_m^j}.$$

In general, note that the case of uniform regulation is a particular case of linear constraints $\boldsymbol{A}\boldsymbol{\tau}^U = 0$, where

$$\boldsymbol{A} = \begin{pmatrix} 1 & -1 & & \cdots & 0 \\ & 1 & -1 & & \vdots \\ \vdots & & \ddots & \ddots & \\ 0 & \cdots & & 1 & -1 \end{pmatrix},$$

where $\boldsymbol{A}$ has dimensions $(U-1)\times U$. In this case, $\frac{d\boldsymbol{\Phi}}{d\boldsymbol{\tau}^U}=\boldsymbol{A}'$, so Proposition 2 implies that the regulator optimally sets

$$\frac{dW}{d\boldsymbol{\tau}^U}=\frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^U}\left(\boldsymbol{I}-\boldsymbol{L}\right)\boldsymbol{\omega}^U=\boldsymbol{A}'\boldsymbol{\mu},$$

which also implies that

$$\boldsymbol{\iota}'\frac{dW}{d\boldsymbol{\tau}^U}=\boldsymbol{\iota}'\frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^U}\left(\boldsymbol{I}-\boldsymbol{L}\right)\boldsymbol{\omega}^U=\boldsymbol{\iota}'\boldsymbol{A}'\boldsymbol{\mu}=0,$$

since $\boldsymbol{A}\boldsymbol{\iota}=0$, where $\boldsymbol{\iota}$ denotes a vector of ones. Rearranging, we obtain

$$\begin{aligned}0&=\boldsymbol{\iota}'\frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^U}\left(\boldsymbol{I}-\boldsymbol{L}\right)\boldsymbol{\omega}^U\\&=\boldsymbol{\iota}'\frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^U}\left(\boldsymbol{I}-\boldsymbol{L}\right)\left(\boldsymbol{\tau}^U-\boldsymbol{\delta}^U\right)\\&=\boldsymbol{\iota}'\frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^U}\left(\boldsymbol{I}-\boldsymbol{L}\right)\left(\overline{\tau}^U\boldsymbol{\iota}-\boldsymbol{\delta}^U\right),\end{aligned}$$

where the last line uses the fact that all elements of $\boldsymbol{\tau}^U$ must be equal to the same scalar, denoted $\overline{\tau}^U$, at the constrained solution. We solve as follows for the scalar $\overline{\tau}^U$ to complete the derivation of Equation (11):

$$\underbrace{\boldsymbol{\iota}'\frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^U}\left(\boldsymbol{I}-\boldsymbol{L}\right)\boldsymbol{\iota}}_{\text{scalar}}\overline{\tau}^U=\underbrace{\boldsymbol{\iota}'\frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^U}\left(\boldsymbol{I}-\boldsymbol{L}\right)\boldsymbol{\delta}^U}_{\text{scalar}}\iff\overline{\tau}^U=\frac{\boldsymbol{\iota}'\frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^U}\left(\boldsymbol{I}-\boldsymbol{L}\right)\boldsymbol{\delta}^U}{\boldsymbol{\iota}'\frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^U}\left(\boldsymbol{I}-\boldsymbol{L}\right)\boldsymbol{\iota}}.$$

E.4.3 Convex Costs of Regulation

In this case, we have $\frac{d\boldsymbol{\Phi}}{d\boldsymbol{\tau}^U}=\boldsymbol{B}\boldsymbol{\tau}^U$, so Proposition 2 implies that the planner optimally sets

$$\frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^U}\left(\boldsymbol{I}-\boldsymbol{L}\right)\left(\boldsymbol{\tau}^U-\boldsymbol{\delta}^U\right)=\boldsymbol{B}\boldsymbol{\tau}^U.$$

Solving for $\boldsymbol{\tau}^U$ yields

$$\boldsymbol{\tau}^U=\left(\boldsymbol{B}+\left(-\frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^U}\right)\left(\boldsymbol{I}-\boldsymbol{L}\right)\right)^{-1}\left(\left(-\frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^U}\right)\left(\boldsymbol{I}-\boldsymbol{L}\right)\boldsymbol{\delta}^U\right),\tag{OA-26}$$

which establishes Equation (12). In general, the correction relative to the first-best policy is given by $\left(\boldsymbol{B}+\boldsymbol{K}\right)^{-1}\boldsymbol{K}$, which has the interpretation of an attenuation matrix. For instance, in a scenario with a single agent, $I=1$, and a single decision, $N=1$, Equation (OA-26) becomes

$$\tau^U=\frac{\left(-\frac{dx^U}{d\tau^U}\right)}{B+\left(-\frac{dx^U}{d\tau^U}\right)}\delta^U,\tag{OA-27}$$

where B is a non-negative scalar that modulates the cost. In the well-behaved case in which $\frac{dx^U}{d\tau^U} < 0$, it follows that the optimal regulation is simply a scaled-down version of the first-best regulation.[24]

[24]In the general case, note that the perfectly regulated decisions in turn must satisfy:

$$\boldsymbol{\tau}^R = \boldsymbol{\delta}^R + \left(-\frac{d\boldsymbol{x}^R}{d\boldsymbol{\tau}^R}\right)^{-1} \frac{d\boldsymbol{x}^U}{d\boldsymbol{\tau}^R} \left((\boldsymbol{B}+\boldsymbol{K})^{-1}\boldsymbol{K} - \boldsymbol{I}\right)\boldsymbol{\delta}^U.$$

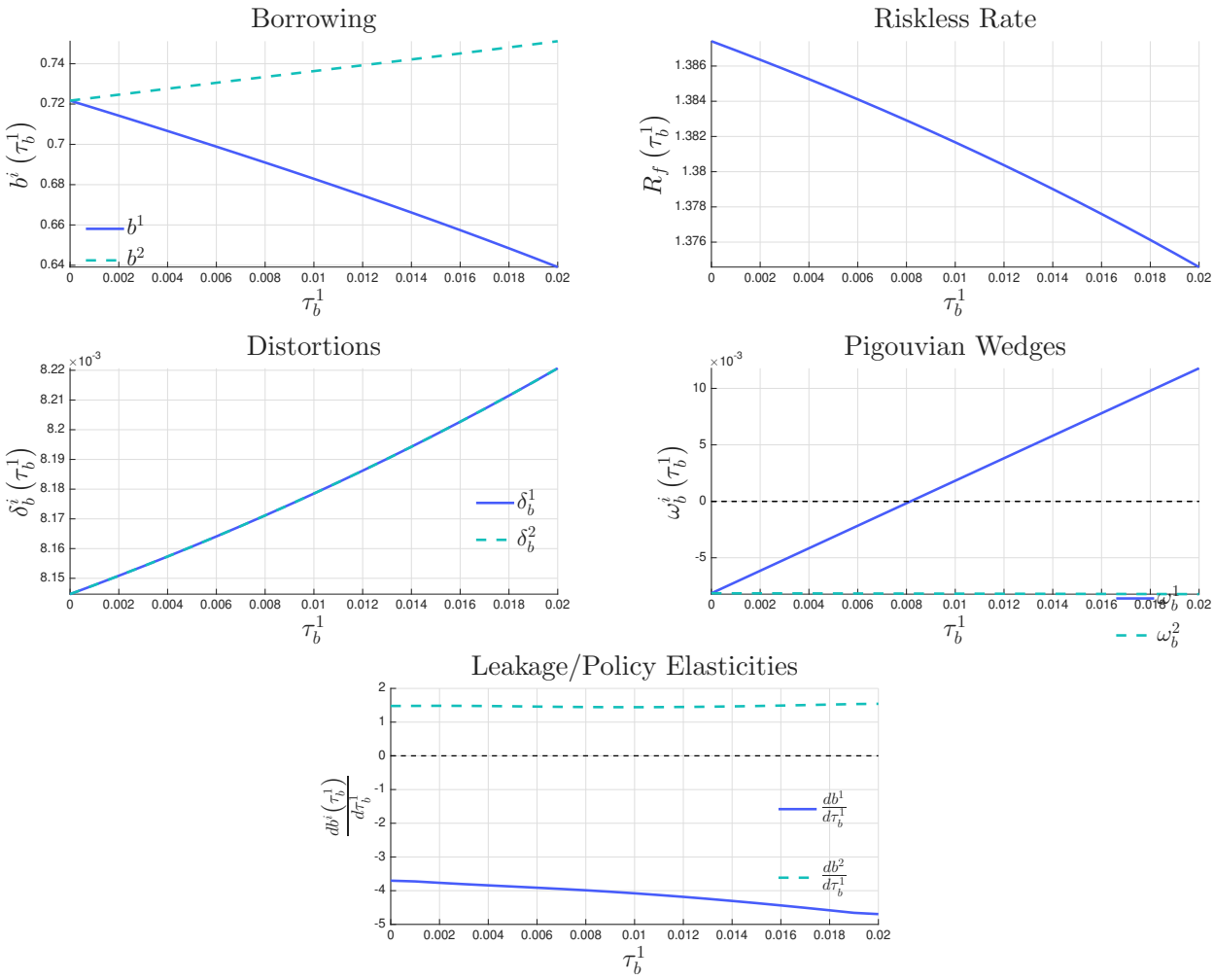


Figure OA-1: Example 1: Second-Best Comparative Statics

Note: Figure OA-1 illustrates relevant comparative statics of Example 1 for different values of τ_b^1, when $\tau_b^2 = 0$. The top left plot shows equilibrium borrowing b^i for both types of investors. The top right plot shows the equilibrium riskless rate. The middle left plot shows the distortion associated with the borrowing choice of each investor, δ_b^1 and δ_b^2, defined in Equation (OA-8) — note that the distortions move inversely with changes in the riskless rate $R^f = \left(\int m^C(s)\, dF(s)\right)^{-1}$, and quantitatively the changes are small. The middle right plot shows the Pigouvian wedge associated with the borrowing decision of each investor, ω_b^1 and ω_b^2. The bottom plot shows the policy elasticity $\frac{db^1}{d\tau_b^1}$ and the critical leakage elasticity $\frac{db^2}{d\tau_b^1} > 0$. The parameters used are described in Figure 1.

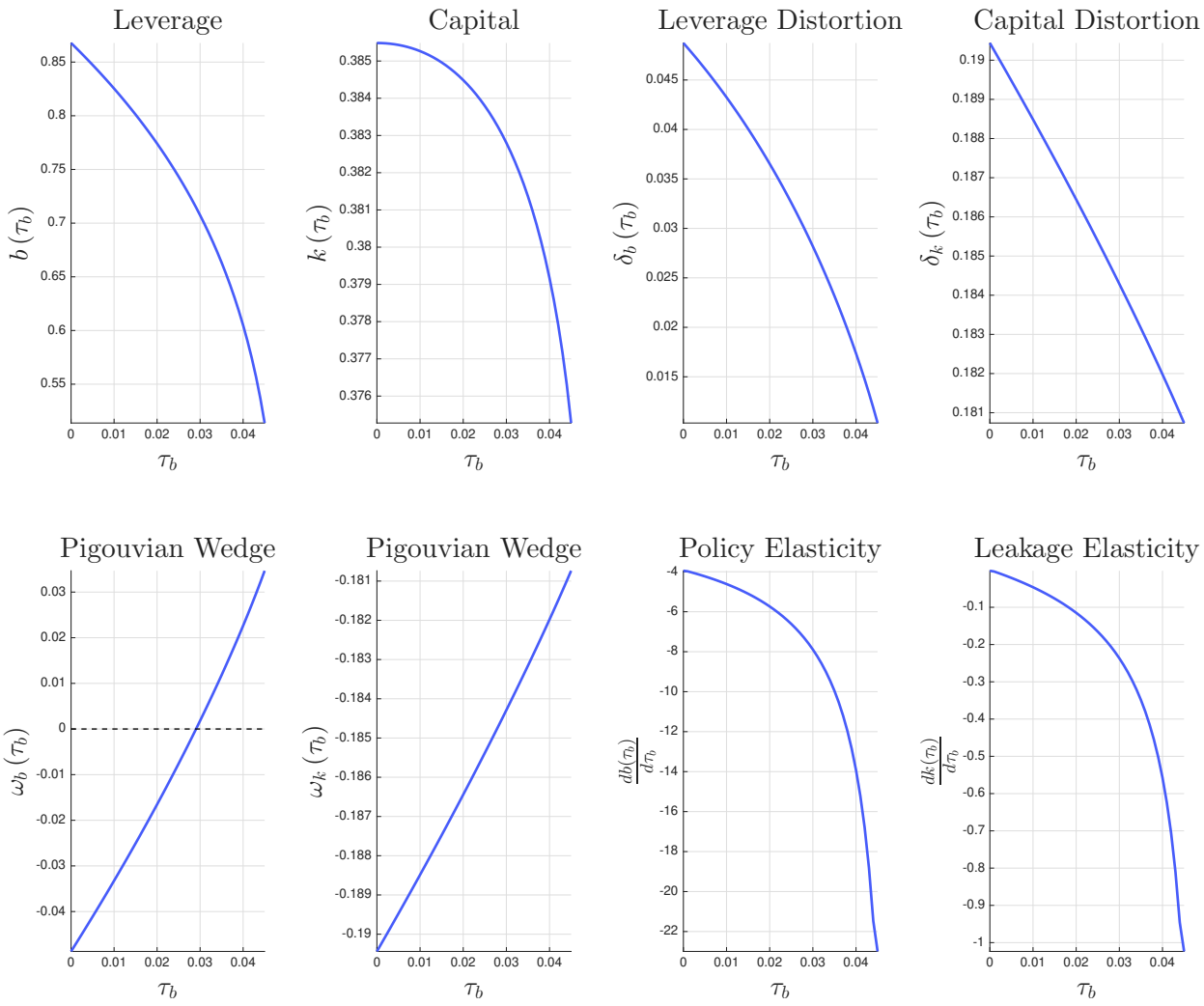


Figure OA-2: Example 2: Second-Best Comparative Statics

Note: Figure OA-2 illustrates relevant comparative statics of Example 2 for different values of τ_b, when $\tau_k = 0$. The top left plot and the top middle-left plot show equilibrium leverage b and investment k. The top middle-right and right plots show the leverage distortion δ_b and the capital distortion δ_k, respectively. The bottom left plot and the bottom middle-left plot show the associated Pigouvian wedges, ω_b and ω_k. The bottom middle-right plot and bottom right plot show the policy elasticity $\frac{db}{d\tau_b}$ and the leakage elasticity $\frac{dk}{d\tau_b}$. The parameters used are described in Figure 2.

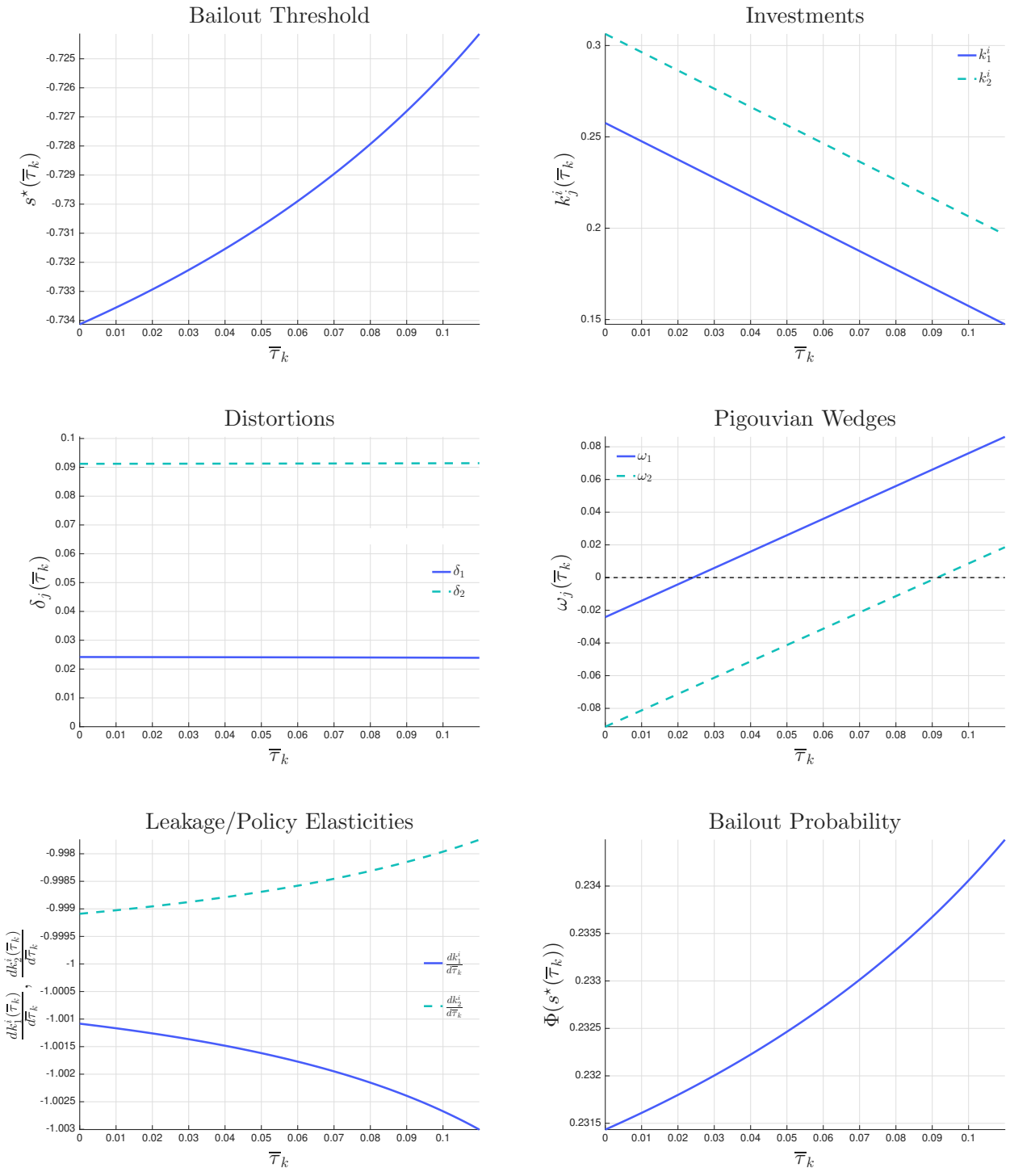


Figure OA-3: Example 3: Second-Best Comparative Statics

Note: The figure reports comparative statics for $\overline{\tau}_k = \tau_k^1 = \tau_k^2$. The top panels show the bailout threshold and the two investment choices. The middle panels show distortions and Pigouvian wedges, and the bottom panels show the investment responses to uniform regulation and the bailout probability. Parameters are described in Figure 3.

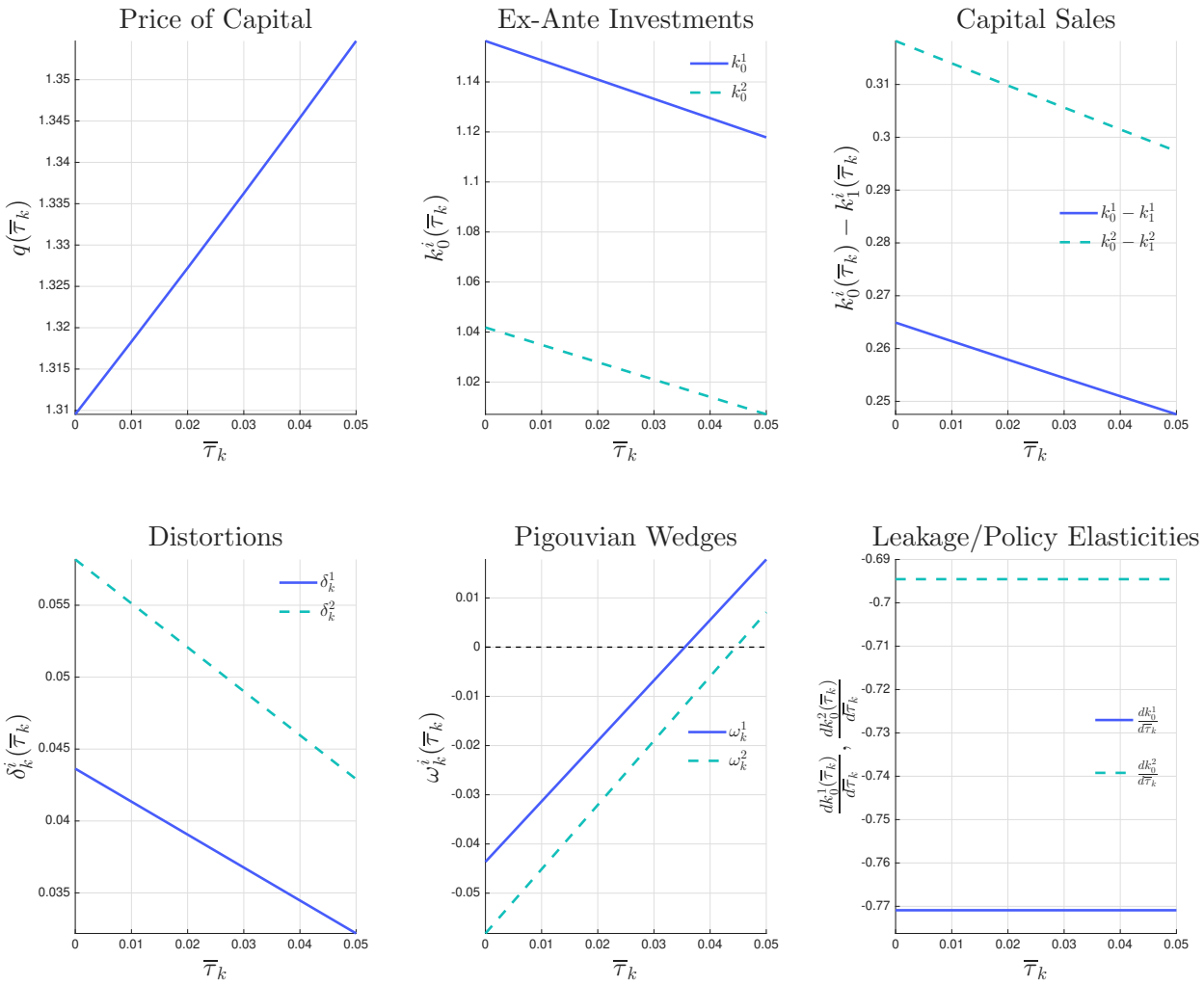


Figure OA-4: Example 4: Second-Best Comparative Statics

Note: Figure OA-4 illustrates relevant comparative statics of Example 4 for different values of $\tau_k^1 = \tau_k^2 = \overline{\tau}_k$. The top left plot shows the price of capital in equilibrium q. The top middle plot shows investment at date 0 for both investor types, k_0^1 and k_0^2. The top right plot shows the amount of capital sold at date 1 for both investor types, $k_0^1 - k_1^1$ and $k_0^2 - k_1^2$. The bottom left plot and the bottom middle plot show the distortions associated with the investment decisions of each investor, δ_k^1 and δ_k^2, and the associated Pigouvian wedges, ω_k^1 and ω_k^2. The bottom right plot shows the leakage/policy elasticities $\frac{dk_0^1}{d\overline{\tau}_k}$ and $\frac{dk_0^2}{d\overline{\tau}_k}$. The parameters used are described in Figure 4.

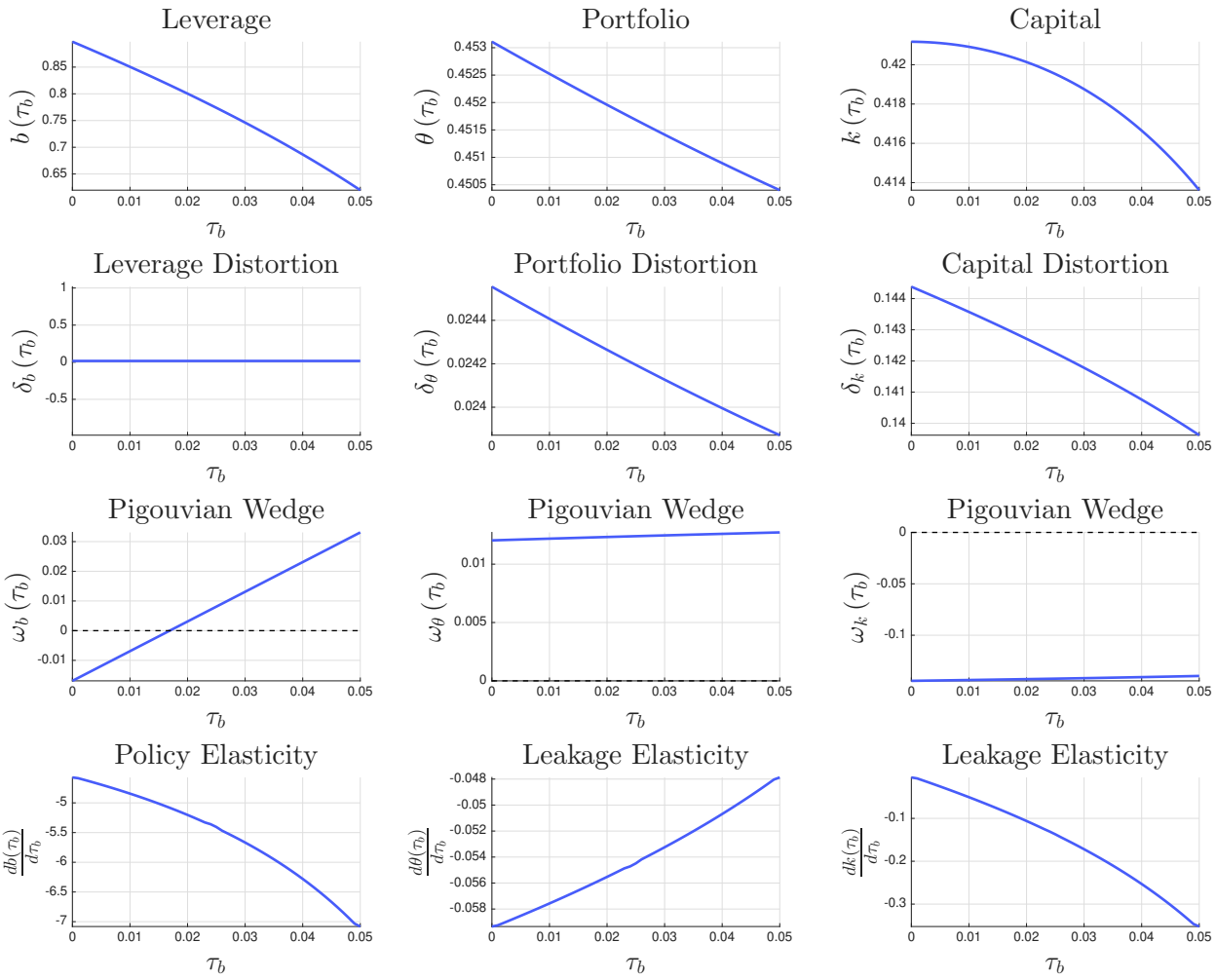


Figure OA-5: Financial Regulation with Environmental Externalities: Second-Best Comparative Statics, Leverage (τ_b)

Note: Figure OA-5 illustrates relevant comparative statics of our application on financial regulation with environmental externalities. In particular, we show how different variables vary with different values of τ_b, when $\tau_k = 0$ and when τ_θ is set at the second-best level under a broad mandate (previously computed). The top row shows equilibrium leverage b^i, portfolio allocations θ^i, and capital k^i. The second row shows leverage, portfolio, and capital distortions, defined in Equations (31) through (33), while the third row shows the associated Pigouvian wedges. The bottom row shows the policy elasticity $\frac{db}{d\tau_b}$ and the two leakage elasticities, $\frac{d\theta}{d\tau_b}$ and $\frac{dk}{d\tau_b}$ (since in this figure we are keeping τ_θ predetermined). The parameters used are described in Figure 5.

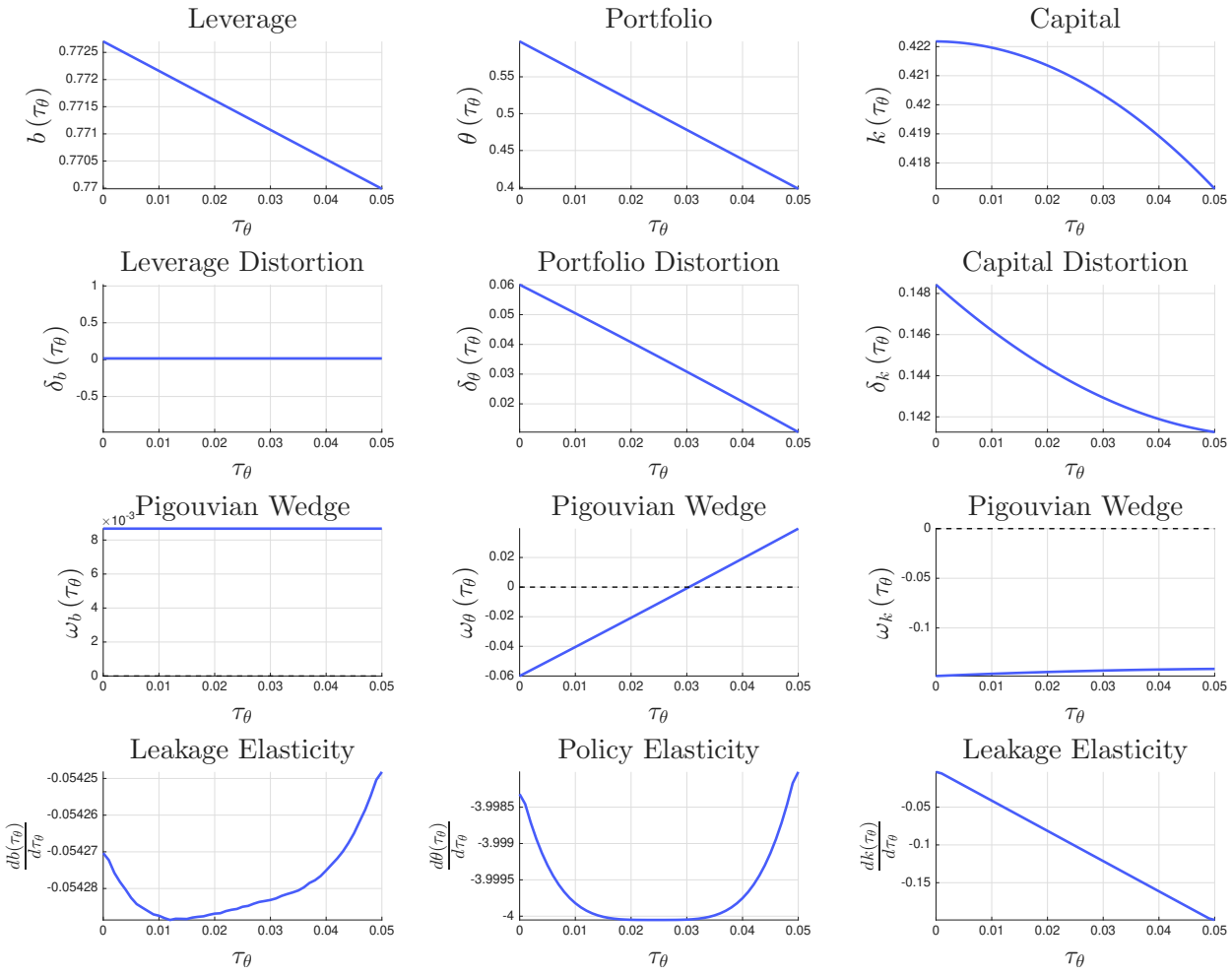


Figure OA-6: Financial Regulation with Environmental Externalities: Second-Best Comparative Statics, Risk Weights (τ_θ)

Note: Figure OA-6 illustrates relevant comparative statics of our application on financial regulation with environmental externalities. In particular, we show how different variables vary with different values of τ_θ, when $\tau_k = 0$ and when τ_b is set at the second-best level under a broad mandate (previously computed). The top row shows equilibrium leverage b^i, portfolio allocations θ^i, and capital k^i. The second row shows leverage, portfolio, and capital distortions, defined in Equations (31) through (33), while the third row shows the associated Pigouvian wedges. The bottom row shows the policy elasticity $\frac{d\theta}{d\tau_\theta}$ and the two leakage elasticities, $\frac{db}{d\tau_\theta}$ and $\frac{dk}{d\tau_\theta}$ (since in this figure we are keeping τ_b predetermined). The parameters used are described in Figure 5.