NBER WORKING PAPER SERIES

DEMOCRATIZING PRIVATE MARKETS: EQUILIBRIUM PREDICTIONS

Lubos Pastor
Robert F. Stambaugh
Lucian A. Taylor

Working Paper 35665
http://www.nber.org/papers/w35665

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
August 2026

Lubos Pastor also serves as an independent director and trustee of Vanguard. The views expressed here do not necessarily reflect those of Vanguard, its funds, or the National Bureau of Economic Research. We thank Yuran Wang and Chris Zhu for excellent research assistance.

NBER working papers are circulated for discussion and comment purposes. They have not been peer-reviewed or been subject to the review by the NBER Board of Directors that accompanies official NBER publications.

© 2026 by Lubos Pastor, Robert F. Stambaugh, and Lucian A. Taylor. All rights reserved. Short sections of text, not to exceed two paragraphs, may be quoted without explicit permission provided that full credit, including © notice, is given to the source.

Democratizing Private Markets: Equilibrium Predictions
Lubos Pastor, Robert F. Stambaugh, and Lucian A. Taylor
NBER Working Paper No. 35665
August 2026
JEL No. G11, G12, G18, G23

ABSTRACT

We study how opening private markets to retail investors affects investors and firms. Calibrating a production-based asset-pricing model, we find that democratization raises retail investors' welfare, but barely. Retail investors benefit little from democratization even if they face no extra cost or other disadvantage relative to institutional investors. The private market is simply too small for improved risk sharing to matter much. Whether or not private-market retail investing is costly, democratization substantially reshapes private firms' ownership and materially reduces their cost of capital, leading them to grow. Private equity earns a positive CAPM alpha, which shrinks after democratization.

Lubos Pastor
The University of Chicago
Booth School of Business
and NBER
lubos.pastor@chicagobooth.edu

Robert F. Stambaugh
University of Pennsylvania
The Wharton School
Finance Department
and NBER
stambaugh@wharton.upenn.edu

Lucian A. Taylor
University of Pennsylvania
Wharton School
Finance Department
and NBER
luket@wharton.upenn.edu

1. Introduction

Private markets have long been open only to institutional and high-net-worth investors. Retail investors enjoy ready access to public markets but have been largely shut out of private markets, which include private equity (e.g., buyouts and venture capital) as well as private credit, real estate, and infrastructure funds. This exclusion of retail investors is especially striking against the backdrop of private markets' expanding presence. For example, private equity funds' share of total U.S. equity more than doubled from 2000 to 2025, rising from less than 3% to more than 7%, as illustrated in Figure 1.

Retail access to private markets has begun to open, moving toward "democratization." In August 2025, the SEC changed its practice of confining closed-end funds' holdings of private funds to accredited investors, and its 2026 regulatory agenda includes a proposal to facilitate retail participation further through registered funds. An executive order issued in the same month directed the Department of Labor and the SEC to examine opening defined-contribution plans to private assets, and in March 2026 the Department proposed a safe harbor for fiduciaries who add such assets to plan menus.[1] New investment products such as semi-liquid funds offer retail investors private-market exposure at greater liquidity, though typically also a higher cost, than traditional draw-down structures.[2]

Will democratizing private markets make retail investors better off? Access to private markets allows retail investors to diversify beyond publicly traded assets. Against this benefit stand frictions often invoked to question the value of retail access: retail investors face high access costs, adverse selection, and complicated redemptions due to the difficulty of valuing private assets.[3] We find that the benefit will be small—even absent these frictions, democratization will raise retail investors' welfare by only a few basis points of wealth.

That small welfare gain does not mean democratization is inconsequential. For example, it could reshape firms' ownership, assets' expected returns, or firms' investments. To analyze these effects, we build a model in which portfolio choices, asset prices, and capital investment

[1] SEC Division of Investment Management, Accounting and Disclosure Information 2025-16, "Registered Closed-End Funds of Private Funds," August 15, 2025. See also "Enhancing Retail Exposure to Private Markets," SEC Spring 2026 Unified Agenda, with a proposal targeted for October 2026; Executive Order 14330, "Democratizing Access to Alternative Assets for 401(k) Investors," August 7, 2025; and Department of Labor, "Fiduciary Duties in Selecting Designated Investment Alternatives," proposed March 31, 2026.

[2] Retail-oriented private-market products are now offered by institutions such as BlackRock, Vanguard/Blackstone, and T. Rowe Price/Goldman Sachs joint ventures. Morningstar's *The State of Semiliquid Funds 2026* reports that assets in such vehicles grew 120% over four years, to nearly $600 billion.

[3] See, for example, Balloch, Mainardi, Oh, and Vokata (2026), Ewens and Faber (2026), Fang, Goldstein, and Zeng (2026), Miller, Yimfor, and Zhang (2026), Pegoraro, Shive, and Zambrana (2026), and Robinson and Wallskog (2026).

are jointly determined in equilibrium. The model has two investors, institutional and retail, and two firms, public and private, whose payoffs are positively but imperfectly correlated. We compare two economies: pre-democratization, which prohibits retail investing in the private firm, and post-democratization, which allows it at a cost.

In a frictionless setting where all investors can access private markets costlessly, the standard CAPM obtains, and each investor's fractional ownership of a firm equals the investor's share of total wealth. Pre-democratization, when retail investors are excluded from private markets, they overweight the public firm relative to their wealth share. The overweight is an imperfect proxy for the private-firm exposure they would choose in the frictionless setting: it captures the part of the private payoff that public equity spans, but none of the rest. Institutions absorb the private risk that retail cannot bear, and they must be paid to do so, which lowers the private firm's price and raises its expected return above its CAPM value. The private firm thus has a positive alpha, whereas the public firm has an offsetting negative alpha. These pricing effects are weaker post-democratization but still present: direct private exposure is costly for the retail investor, so they use it to replace only some of their pre-democratization overweight in the public firm.

The weakening of the pricing effects post-democratization means that the private firm's cost of capital becomes lower, absent any capital-investment responses by either firm. That lower cost of capital induces the private firm to expand its capital, by an amount that is optimal under a convex investment cost. The larger private sector in turn adds to the aggregate risk that investors must absorb, raising the public firm's cost of capital and inducing it to contract slightly. The private firm's capital thus becomes a larger fraction of the economy's total capital post-democratization.

The relative size of the private firm bears importantly on the retail investor's welfare gain from democratization. The gain is smaller when the pre-democratization portfolio better mimics the post-democratization portfolio. Formally, the welfare gain increases with the variance of the private firm's payoff conditional on the public firm's, which we term unspanned private risk. This risk falls with the public-private correlation, but its key determinant is the private firm's relative size, in which it rises quadratically. A larger private firm therefore delivers a disproportionately larger welfare gain to retail investors.

While retail investors gain, institutions lose, as democratization erodes the rent they earn pre-democratization for bearing private risk alone. Aggregate investor welfare generally falls as well, even though democratization improves risk sharing.

We obtain closed-form solutions for all of the model's implications. To assess these

implications quantitatively, we conduct numerous calibrations of the model, each satisfying a set of empirical criteria. For some of the model's implications, we find that democratization produces substantial effects. On average across calibrations, the private firm's annual CAPM alpha falls from 1% pre-democratization to 0.4% post. The firm's cost of capital drops by the same amount, about 60 basis points (bps), spurring the firm to expand its capital by nearly 5%. Retail investors end up owning 40% of the private firm post-democratization, on average, up from zero but still short of the 70% or so that their wealth share would dictate absent frictions.

Despite that substantial retail participation in the private market, the welfare gain to retail investors is very small, as noted earlier. We measure this gain as the change in an investor's certainty equivalent as a fraction of their wealth. Absent changes in firms' capital, the retail investor's welfare gain is at most 3 bps per year across all calibrations and just 1 bp on average. The resulting expansion in the private firm's capital raises the firm's relative size, making the welfare gain higher, but the average is still only 2 bps, and the gain reaches only 7 bps at the 95th percentile of calibrations and 24 bps at most. Institutions' losses are larger, though also small: 11 bps on average and 67 bps at most.

A key reason for retail investors' tiny welfare gain from democratization is the small size of the private market. In applying our theory, we interpret the private market as current holdings of private equity funds, the largest segment of private markets.[4] Democratization makes these holdings available to retail investors. Of course, these holdings omit many private businesses that are inaccessible to outside investors, even institutions, and thus are excluded from democratization.

Despite its high growth, the private market represents only about 7% of the total value of public and private markets. Given this low share, the private market's unspanned risk is simply too small to make capturing it via democratization of much value to the retail investor. If the private market's share were twice as large, at 16%, the welfare gain would jump to 11 bps when accounting for the resulting increase in private-market capital. At a share of 30%, the gain would be 47 bps. Continued expansion of the private market could therefore yield nontrivial future welfare gains under democratization.

Related literature.

The growing importance of private markets is well documented (e.g., Ewens and Farre-

[4]According to Preqin (2026), the 2025 assets under management (AUM) of North American private equity funds (including venture capital) is $5.6 trillion, which is 66% of total private market AUM. The next two largest segments, private real estate and private credit funds, account for 14% and 13% of total private market AUM, respectively.

Mensa, 2020, 2022). We show that as private markets grow, so does the incentive to open them to retail investors. A larger private market generates more unspanned private risk, increasing the risk-sharing benefits of democratization.

The literature on the democratization of private markets is nascent. Miller, Xing, Yimfor, and Zhang (2026) study democratization of venture capital as a mechanism for discovering managerial skill. In their model, retail investors finance unproven managers, allowing talented ones to establish track records and graduate to institutional capital. The benefit of democratization comes from skill discovery. We instead study democratization as an expansion of the investor base, which improves risk sharing, lowers private firms' cost of capital, and raises private-market investment. Miller, Yimfor, and Zhang (2026) provide experimental evidence that individuals and institutions evaluate private investments differently, with individuals placing less weight on past performance. Balloch, Mainardi, Oh, and Vokata (2026) study the distributional consequences of private-market access. They find similar average private-equity performance for individuals and institutions, but larger benefits for wealthier individuals, who use better-performing advisors and face lower intermediation costs. While they study who benefits from access and why, we study how expanding access affects equilibrium asset prices, capital allocation, and firm behavior.

Lindsey and Stein (2026) exploit a Dodd–Frank regulatory change that removed the value of an investor's primary residence from the net-worth calculation used to determine accredited-investor status. As a result, some households previously permitted to invest in private securities became ineligible. The authors show that areas more exposed to this contraction in the investor pool experienced declines in angel investment, firm entry, and employment at young firms. Their evidence provides empirical support for our real-investment channel: whereas they study the effects of restricting individual access to private markets, we study the reverse experiment of expanding retail access.

Retail investors increasingly access private markets through semi-liquid vehicles, such as interval funds, that offer periodic, limited liquidity. Pegoraro, Shive, and Zambrana (2026) find that interval funds outperform ETFs in illiquid, information-insensitive asset classes, such as nontraditional fixed income, but underperform in liquid, information-sensitive markets, such as equity. Ewens and Faber (2026) find that interval and tender-offer funds underperform institutional drawdown funds because of fees, liquidity buffers, and imperfect replication of private-market investments. Fang, Goldstein, and Zeng (2026) show that semi-liquid private credit funds, which hold illiquid loans while offering periodic redemptions, are vulnerable to run-like dynamics.[5] Together, these papers highlight the frictions associated

[5]Related work studies the performance of private credit funds. For example, Erel, Flanagan, and Weisbach

with retail access to illiquid private assets. We take these frictions as given and study their effects on risk sharing, asset prices, firm investment, and investor welfare.

A large literature compares the performance of private-market investments with that of public securities.[6] Estimates vary across studies, in part because measuring private-market returns and adjusting them for risk and illiquidity is difficult. Nonetheless, the evidence is favorable on balance, especially for buyouts: leading surveys by Kaplan and Sensoy (2015) and Korteweg (2019) both conclude that buyout investments outperform public-market benchmarks. We complement this empirical literature by providing an equilibrium interpretation of private-market CAPM alphas. In our model, private assets earn a positive alpha: before democratization, the alpha arises from limited risk sharing, while afterward it reflects retail investors' costs of holding private assets.

Several theoretical studies have implications for expected returns on private investments. Berk and DeMarzo (2026) develop a theory of delegated capital management encompassing both public and private markets. In their model, private-market illiquidity makes information acquisition valuable, and informed investors earn positive alpha as compensation for the resources spent identifying talented private-market managers. In our model, private-market alpha arises without information acquisition or managerial skill. Gârleanu and Panageas (2026) study how disruptive innovation redistributes wealth between incumbent owners and new entrepreneurs. Private equity hedges disruption risk, which lowers expected returns, but also exposes investors to nondiversifiable idiosyncratic risk, which raises them. While they focus on disruption, we study how expanding private-market access changes firm investment and the equilibrium supply of private risk. Ewens, Jones, and Rhodes-Kropf (2013) show that idiosyncratic risk can be priced for a different reason: in their principal-agent model, incentivizing venture capitalists requires them to bear some investment risk.

Our paper is also related to the longstanding literature on international market segmentation. Black (1974) and Stulz (1981) model barriers to international investment as proportional taxes on holdings of foreign assets, much like the retail cost of holding private assets in our post-democratization economy. Errunza and Losq (1985) and Eun and Janakiramanan (1986) instead model legal restrictions on foreign ownership, paralleling the exclusion of retail investors from private markets in our pre-democratization economy. A large empirical literature shows that greater market integration and broader investor access raise valuations

(2026) find insignificant abnormal returns net of fees after risk adjustment, and Robinson and Wallskog (2026) find that publicly traded business development companies have high risk-adjusted returns before fees, but also that fees are higher among funds with larger non-institutional investor bases.

[6]Prominent contributions include Cochrane (2005), Kaplan and Schoar (2005), Phalippou and Gottschalg (2009), Franzoni, Nowak, and Phalippou (2012), and Harris, Jenkinson, and Kaplan (2014), among others.

and lower expected returns.[7] Related evidence shows that market liberalizations stimulate investment and growth.[8] Related work shows that inaccessible foreign assets can sometimes be spanned by accessible securities (Errunza, Hogan, and Hung, 1999), paralleling the spanning mechanism in our model. We apply similar ideas to the segmentation between public and private markets. In our model, retail exclusion forces institutions to bear unspanned private risk, generating a segmentation premium. Democratization broadens participation, lowers this premium, and expands private investment.

Related mechanisms also appear in the broader literature on portfolio choice with frictions. Merton (1987) studies an equilibrium with incomplete investor recognition, in which investors hold only securities they know about. A smaller investor base raises expected returns by concentrating risk among fewer investors, much like limited participation in our model. Basak and Cuoco (1998) show that excluding investors from the stock market can similarly raise the compensation for bearing stock-market risk. They analyze limited participation in public equity; we study its counterpart in private markets and the effects of relaxing it. Heaton and Lucas (2000) show that households facing greater nontradable entrepreneurial income risk hold less public equity, whereas households facing too little private risk in our model compensate by holding more public equity. Ang, Papanikolaou, and Westerfield (2014) analyze portfolio choice with intermittently tradable illiquid assets, while Gourier, Phalippou, and Westerfield (2024) study how capital commitments affect private-market portfolio choice.[9] Whereas this literature takes portfolio frictions and participation limits as given, we study the equilibrium effects of expanding private-market access to retail investors.

This paper is organized as follows. Section 2 explains the model setup. Sections 3 through 6 discuss the model's theoretical implications for investor holdings, asset pricing, investor welfare, and firm investment. Section 7 quantifies these implications by calibrating the model to empirical evidence. Section 8 concludes.

[7] See, for example, Bekaert and Harvey (1995, 2000), Foerster and Karolyi (1999), Errunza and Miller (2000), Henry (2000b), and de Jong and de Roon (2005).

[8] See Henry (2000a), Bekaert, Harvey, and Lundblad (2005), Mitton (2006), and Gupta and Yuan (2009).

[9] Also related are Lerner and Schoar (2004), whose theory explains why private-equity funds may optimally choose illiquidity to attract investors less likely to face liquidity shocks; Gârleanu, Panageas, and Yu (2015), who study incomplete market integration arising from endogenous participation costs; and Greenwood, Hanson, and Liao (2018), who study partially segmented markets in which specialized investors and slow-moving capital allow prices of risk to differ across asset classes.

2. Model

Time is discrete, with two dates $t \in \{0, 1\}$. Investors form portfolios at $t = 0$ and all uncertainty is resolved at $t = 1$. We take the firms' capital stocks as given until we endogenize them in Section 6.

2.1 Firms

There are two firms, indexed by $j \in \{1, 2\}$. Firm 1 is public and firm 2 private. Each firm enters the model with a capital stock K_j, collected in the vector $K \equiv (K_1, K_2)$. Uncertainty enters through a vector of fundamental shocks,

$$\varepsilon = (\varepsilon_1, \varepsilon_2)^\top \sim \mathcal{N}(0, v), \qquad v = \mathrm{Var}(\varepsilon) = \begin{pmatrix} v_{11} & v_{12} \\ v_{12} & v_{22} \end{pmatrix} \succ 0. \tag{1}$$

Firm j's payoff at $t = 1$, denoted by d_j, is affine in its scale and shock:

$$d_j = K_j(\mu_j + \varepsilon_j), \qquad j \in \{1, 2\}, \tag{2}$$

so that $\mathrm{E}(d_j) = \mu_j K_j$, $\mathrm{Var}(d_j) = K_j^2 v_{jj}$, and $\mathrm{Cov}(d_1, d_2) = K_1 K_2 v_{12}$. Here $\mu_j > 0$ is firm j's expected payoff per unit of capital and ε_j is its payoff shock. We collect the public and private payoffs in the vector $d \equiv (d_1, d_2)^\top$ and assume that they are positively correlated:

$$\rho_{12} \equiv \mathrm{Corr}(\varepsilon_1, \varepsilon_2) = \frac{v_{12}}{\sqrt{v_{11} v_{22}}} > 0\,. \tag{3}$$

A central object in what follows is the component of private risk that cannot be replicated using the public asset. We measure it by the conditional variance of the private payoff given the public payoff. From equations (1) and (2), this quantity equals

$$\mathrm{Var}(d_2 \mid d_1) = K_2^2\, v_{2|1}, \qquad v_{2|1} \equiv \mathrm{Var}(\varepsilon_2 \mid \varepsilon_1) = v_{22} - \frac{v_{12}^2}{v_{11}} = v_{22}\left(1 - \rho_{12}^2\right). \tag{4}$$

We refer to $\mathrm{Var}(d_2 \mid d_1)$ as *unspanned private risk*. It is the variance of the private payoff that remains after projecting it on the public payoff, and it governs the strength of the model's central mechanisms.[10] Unspanned private risk is larger when the private firm has more capital (higher K_2) and when the firm's return on that capital is more volatile (higher v_{22}) and less correlated with the public firm's (lower ρ_{12}).

[10] Although we assume normality of ε in equation (1), our results depend on ε only through its first two moments, because wherever $\mathrm{Var}(d_2 \mid d_1)$ appears it is the variance of the residual from the projection in equation (4). Normality makes that residual variance equal to the conditional variance.

Each firm has one share outstanding, with equilibrium price $p_j > 0$ at $t = 0$. There is also a risk-free asset in perfectly elastic supply: one dollar invested at $t = 0$ returns $R_f > 1$ at $t = 1$. We measure returns in excess of the risk-free rate. The excess return on firm j is

$$\widetilde{r}_j \equiv \frac{d_j}{p_j} - R_f. \tag{5}$$

2.2 Investors

There are two investors, indexed by $i \in \{A, B\}$ and interpreted as institutional and retail investors, respectively. Investor i enters with initial wealth W_{0i}. We normalize total initial wealth to one, so that W_{0i} represents investor i's share of aggregate wealth:

$$W_{0A} + W_{0B} = 1\,. \tag{6}$$

At time $t = 0$, investor i chooses holdings $\theta_i = (\theta_{i1}, \theta_{i2})^\top$ of the two firms, where θ_{ij} is the fraction of firm j's shares that investor i holds, and a position b_i in the risk-free asset. Because each firm's price is p_j, the investor's budget constraint at $t = 0$ is

$$p^\top \theta_i + b_i = W_{0i}\,, \tag{7}$$

where $p \equiv (p_1, p_2)^\top$. The institutional investor's terminal wealth at time $t = 1$ is

$$W_{1A} = R_f b_A + \theta_A^\top d\,. \tag{8}$$

The retail investor faces a proportional cost $c \geq 0$ of holding the private asset. As a result, this investor's terminal wealth is

$$W_{1B} = R_f b_B + \theta_B^\top d - c\,\theta_{B2} K_2\,. \tag{9}$$

Investors' preferences are mean–variance over the excess return on initial wealth. Both investors have the same relative risk aversion a. Writing investor i's excess portfolio return as $\widetilde{r}_i^p \equiv W_{1i}/W_{0i} - R_f$, investor i chooses (θ_i, b_i) to maximize

$$\max_{\theta_i, b_i} \; \mathrm{E}(\widetilde{r}_i^p) - \frac{a}{2}\,\mathrm{Var}(\widetilde{r}_i^p)\,. \tag{10}$$

These preferences are equivalent to constant-absolute-risk-aversion utility over terminal wealth, given the normality in equation (1). Using equations (7) through (9), the excess portfolio returns of the two investors are

$$\widetilde{r}_A^p = \frac{\theta_A^\top (d - R_f p)}{W_{0A}} \tag{11}$$

$$\widetilde{r}_B^p = \frac{\theta_B^\top (d - R_f p) - c\,\theta_{B2} K_2}{W_{0B}}\,. \tag{12}$$

2.3 Interpreting the model

Our model's objects have natural real-world analogues. The public firm represents an index fund holding all public stocks. The private firm represents a fund of private-market funds, in which general partners (GPs) play the role of the firm's management and limited partners (LPs) are its investors. Behind these funds lies productive capital. We interpret K_1 as the capital stock of public companies and K_2 as that of the portfolio companies owned by private funds. New capital deployed into those companies corresponds to the capital calls and fund raises through which investors commit capital at fair value.

Institutions invest in private funds directly as LPs. Retail investors reach the same funds indirectly, either through feeder funds that pool retail capital into traditional drawdown funds, or through semi-liquid vehicles, which repurchase shares periodically, such as interval and tender-offer funds. The cost, c, in equation (9) captures frictions borne by retail but not institutional investors, such as compensation to intermediaries, lower net returns reflecting larger cash buffers, or other costs associated with private assets' illiquidity. We model this cost as a deterministic deadweight loss from the investors' perspective.

We interpret the private firm's price, p_2, as the value of investors' claims on K_2: private funds' net asset value. The payoff d_2 corresponds to investors' subsequent cash flows: the distributions they receive, net of the fees and carried interest that constitute the managers' compensation. Investors' outlay at $t = 0$ accrues to founders and other selling shareholders, whom we do not model.

Neither firm is mispriced in our model. For the private firm, fair pricing means investors enter private-market funds on fair terms, even though fund managers may be skilled at identifying, financing, and cultivating their portfolio companies. If managers capture all the rents from that skill through their own compensation, skill raises that compensation without raising investors' net returns: none of that skill survives in the pair (d_2, p_2) that investors face.[11] Fair pricing does not mean zero alpha, however. The private firm earns a positive CAPM alpha, as we show later, but that alpha reflects neither manager skill nor mispricing.

[11] Having fund managers fully capture the rents from their skill is similar to Berk and Green (2004). Fair pricing requires that investors correctly assess the firms' model parameters, as we assume. For private-market funds, acquiring that information can be costly, as modeled by Berk and DeMarzo (2026). One might view d_2 as also net of information-acquisition costs borne by the firm's investors.

2.4 Two economies

Our model describes two economies that are identical in every respect but one: whether retail investors may hold private assets. In the "pre-democratization" economy, retail investors are prohibited from holding private assets; they face the constraint $\theta_{B2} = 0$. In the "post-democratization" economy, that constraint is relaxed to $\theta_{B2} \geq 0$. Retail investors are then permitted to hold private assets, but they bear the cost c on those holdings.[12] We compare various outcomes in these two economies. This comparison should not be read as tracking a single economy before and after a democratization event. If retail investors choose $\theta_{B2} = 0$ in the post-democratization economy, the two economies are the same in all respects.

An equilibrium consists of prices p and portfolios (θ_i, b_i) for $i \in \{A, B\}$, taking K and R_f as given, such that: (i) each investor maximizes the mean–variance objective (10) subject to the budget constraint (7) and either the restriction $\theta_{B2} = 0$ or $\theta_{B2} \geq 0$, depending on which of the two economies applies; and (ii) markets for both assets clear, so that

$$\theta_{A1} + \theta_{B1} = 1 \tag{13}$$

$$\theta_{A2} + \theta_{B2} = 1. \tag{14}$$

3. Investor holdings

In this section, we characterize investors' optimal holdings of the public and private firms' shares, taking as given firms' capital stocks in each regime, K^{pre} and K^{post}. We present the results below and provide proofs in the Appendix.

Proposition 1. *Investors' optimal share holdings pre-democratization are given by*

$$\theta_{A1}^{\text{pre}} = W_{0A} - \delta^{\text{pre}} W_{0B} \tag{15}$$

$$\theta_{B1}^{\text{pre}} = W_{0B} + \delta^{\text{pre}} W_{0B} \tag{16}$$

$$\theta_{A2}^{\text{pre}} = 1 \tag{17}$$

$$\theta_{B2}^{\text{pre}} = 0\,, \tag{18}$$

[12] Because c enters retail's terminal wealth as $-c\,\theta_{B2} K_2$, a negative private position would turn the cost into a subsidy. Restricting $\theta_{B2} \geq 0$ ensures that c represents a genuine cost of holding the private asset.

and their optimal holdings post-democratization are given by

$$\theta_{A1}^{\text{post}} = W_{0A} - (1-\gamma)\delta^{\text{post}} W_{0B} \tag{19}$$

$$\theta_{B1}^{\text{post}} = W_{0B} + (1-\gamma)\delta^{\text{post}} W_{0B} \tag{20}$$

$$\theta_{A2}^{\text{post}} = 1 - \gamma W_{0B} \tag{21}$$

$$\theta_{B2}^{\text{post}} = \gamma W_{0B}, \tag{22}$$

with

$$\delta^{\text{k}} = \frac{\text{Cov}(d_1^{\text{k}}, d_2^{\text{k}})}{\text{Var}(d_1^{\text{k}})}, \quad \text{k} \in \{\text{pre}, \text{post}\}, \tag{23}$$

and

$$\gamma = \max\left(0,\ 1 - \frac{cW_{0A}}{av_{2|1}K_2^{\text{post}}}\right). \tag{24}$$

The natural benchmark for all eight of the θ_{ij} expressions is the frictionless allocation in which share holdings are proportional to investors' wealth shares: $\theta_{ij} = W_{0i}$. This proportional rule is dictated by efficient risk sharing: absent any friction, both investors hold the same (market) portfolio. Every term that departs from W_{0i} reflects a friction—either retail's pre-democratization exclusion from private markets or its post-democratization holding cost.

In the pre-democratization economy, retail investors are barred from the private asset ($\theta_{B2}^{\text{pre}} = 0$), so institutions absorb the entire private firm ($\theta_{A2}^{\text{pre}} = 1$). Unable to obtain private-risk exposure directly, retail investors use public equity as an imperfect substitute. Because public and private payoffs covary positively, retail investors invest more than their wealth share, W_{0B}, in public equity. Specifically, as shown in equation (16), they overweight public equity by $\delta^{\text{pre}} W_{0B}$. To understand that amount, note that δ^{pre} is the slope in a regression of d_2^{pre} on d_1^{pre},

$$d_2^{\text{pre}} = \delta_0 + \delta^{\text{pre}} d_1^{\text{pre}} + \eta. \tag{25}$$

Therefore, for each desired share of d_2^{pre} exposure, δ^{pre} shares of d_1^{pre} exposure mimics that desired d_2^{pre} exposure as closely as possible, in leaving η uncorrelated with d_1^{pre}. The variance of direct d_2^{pre} exposure that is missed by this optimally chosen indirect exposure is $\text{Var}(\eta) = \text{Var}(d_2^{\text{pre}}|d_1^{\text{pre}})$, i.e., unspanned private risk. Retail investors would expose W_{0B} directly to private when unconstrained. When shut out of private, the best they can do is to substitute W_{0B} times the mimicking ratio, δ^{pre}, as an overweight exposure to public.[13] Institutions take the other side, underweighting public equity by that amount. Since institutions are already

[13] Since $\delta^{\text{pre}} = \frac{\text{Cov}(d_1^{\text{pre}}, d_2^{\text{pre}})}{\text{Var}(d_1^{\text{pre}})} = \frac{K_2^{\text{pre}}}{K_1^{\text{pre}}} \rho_{12} \sqrt{\frac{v_{22}}{v_{11}}}$, retail overweights public equity more when the private sector is large relative to the public sector ($K_2^{\text{pre}}/K_1^{\text{pre}}$), when the two sectors are more correlated (ρ_{12}), and when the payoff per dollar of private-firm capital is more volatile relative to public (v_{22}/v_{11}).

saturated with private risk (they hold all of it), they willingly cede this additional indirect exposure via public equity to retail investors hungry for it.

In the post-democratization economy, retail investors can trade the private asset. While they can still use the public asset to mimic private exposure, they can also obtain private exposure directly via $\theta_{B2}^{\text{post}} > 0$. The latter exposure, however, entails a holding cost if $c > 0$. Retail investors face a choice: costlessly overweight public as an imperfect proxy for private, versus directly hold private at a cost. They generally combine both options. To see this, first observe in equation (20) that $(1-\gamma)\delta^{\text{post}}W_{0B}$ is retail's overweighting of public relative to the frictionless allocation, W_{0B}. Since δ^{post} shares of public mimic one share of private, that overweighting supplies $(1-\gamma)W_{0B}$ of indirect private exposure. Retail's direct exposure, from equation (22), is γW_{0B}. The two sum to W_{0B}, matching the desired allocation to private in the frictionless case. Of that total, the fraction γ is exposed directly and $1-\gamma$ is exposed indirectly, through the mimicking position that misses unspanned private risk.

We can interpret γ as the degree of retail's participation in private. The higher is γ, the greater is retail investors' direct exposure to private, and the closer are their public and private holdings to the frictionless allocations, $\theta_{ij} = W_{0i}$. The lower is γ, the more retail investors use public exposure to mimic private, thereby investing less in private directly, so $\theta_{B2}^{\text{post}}$ falls short of W_{0B} by a greater amount. Institutions absorb this shortfall, so $\theta_{A2}^{\text{post}}$ exceeds W_{0A} by the offsetting quantity. The value of γ in equation (24) is clearly bounded between 0 and 1, because c, W_{0A}, a, $v_{2|1}$, and K_2^{post} are all non-negative. Retail investors have strictly positive direct exposure to the private asset if and only if

$$\gamma > 0\,, \tag{26}$$

which is the *retail participation condition* that follows directly from equation (22). When $\gamma > 0$, equations (4) and (24) imply

$$\gamma = 1 \;-\; \frac{c\,K_2^{\text{post}}}{\text{Var}(d_2^{\text{post}}|d_1^{\text{post}})a(1/W_{0A})}\,. \tag{27}$$

This equation shows that a higher γ corresponds to a greater added benefit to retail investors of direct private exposure, relative to the cost of that exposure. The numerator of the second term on the right-hand side is the per-share holding cost of direct private exposure, as in equation (9). The denominator rises with the benefit of direct exposure, in three ways. First, it contains $\text{Var}(d_2^{\text{post}}|d_1^{\text{post}})$, the unspanned private risk that is captured by direct exposure but missed by the optimally chosen indirect exposure, as explained earlier. Second, this added risk-sharing benefit increases in risk aversion, a. Finally, the benefit increases in $1/W_{0A}$, because a smaller institutional share means retail would hold more of the private asset even

absent the cost, implying larger risk-sharing gains from access. In sum, γ increases as the benefit of direct private exposure increases relative to the cost.

Observe from equation (24) that $\gamma = 1$ if $c = 0$. In that costless setting, the post-democratization holdings in equations (19) through (22) coincide with the frictionless allocations, $\theta_{ij} = W_{0i}$. Risk sharing is then fully efficient, and retail's private holding, $\theta_{B2}^{\text{post}}$, rises all the way to its wealth share, W_{0B}. In contrast, if c is high enough to make $\gamma = 0$, then retail has no private holding. Equation (24) implies the following important condition:

Corollary 1. Retail-participation condition for *c*. *Post-democratization, retail investors choose to participate in private markets (i.e., $\theta_{B2}^{post} > 0$) if and only if*

$$c < \frac{aK_2^{\text{post}} v_{2|1}}{W_{0A}}\,. \tag{28}$$

The left-hand side represents the cost of retail investors' access to private markets. The right-hand side captures the additional risk-sharing benefit of direct exposure to the private asset. That benefit grows with risk aversion, a, the private sector's size, K_2^{post}, the amount of private risk that public equity cannot hedge, $v_{2|1}$, and the inverse of the institutions' wealth share, W_{0A}, as explained earlier.

Democratization generally reallocates private risk from institutions to retail investors. The reallocation is especially transparent if firms' capital stocks are the same pre- and post-democratization, as shown by the following corollary.

Corollary 2. Effects of democratization on investors' holdings. *If capital stocks are held fixed, i.e., $K^{\text{post}} = K^{\text{pre}} = K$ (and thus $\delta^{\text{pre}} = \delta^{\text{post}} = \delta$), then the post-democratization holdings equal $1 - \gamma$ times the pre-democratization holdings plus γ times the frictionless allocations. Also,*

$$\theta_{A1}^{post} - \theta_{A1}^{pre} = \gamma\delta W_{0B} \tag{29}$$

$$\theta_{A2}^{post} - \theta_{A2}^{pre} = -\gamma W_{0B} \tag{30}$$

$$\theta_{B1}^{post} - \theta_{B1}^{pre} = -\gamma\delta W_{0B} \tag{31}$$

$$\theta_{B2}^{post} - \theta_{B2}^{pre} = \gamma W_{0B}\,. \tag{32}$$

When the retail-participation condition holds ($\gamma > 0$), retail's private holding rises from 0 to γW_{0B}. Market clearing requires institutions to sell down their private position by the same amount. Public holdings migrate in the opposite direction. Since public and private equity are partial substitutes ($\delta > 0$), retail sells public shares to institutions. The magnitude

by which institutional and retail investors exchange their public holdings is increasing in the degree of retail's participation in private, γ, as well as the mimicking ratio, δ. Overall, democratization moves private risk toward retail and public risk toward institutions, reducing the pre-democratization imbalance.

4. Asset prices and expected returns

In this section, we solve for asset prices, taking the capital vector $K = (K_1, K_2)$ as given. Throughout this section, we assume that the parameters are such that the retail-participation condition (26) holds, so that $\theta_{B2}^{\text{post}} > 0$. If the condition does not hold, then the pre- and post-democratization economies coincide and their comparison is meaningless.

The realized excess return on asset j in regime k is

$$\tilde{r}_j^k = \frac{d_j^k}{p_j^k} - R_f\,, \qquad j \in \{1,2\},\ k \in \{\text{pre}, \text{post}\}\,, \tag{33}$$

where p_j^k is the time-0 price of firm j's share in regime k, and d_j^k is the time-1 payoff in that regime. Below, we characterize prices and expected excess returns, $\mathrm{E}\left(\widetilde{r}_j^k\right)$. Our solutions invoke the market portfolio, whose payoff, price, and excess return are

$$d_M^k \;\equiv\; d_1^k + d_2^k\,, \qquad p_M^k \;\equiv\; p_1^k + p_2^k\,, \qquad \widetilde{r}_M^k \;\equiv\; \frac{d_M^k}{p_M^k} - R_f\,, \tag{34}$$

so that $\widetilde{r}_M^k$ is the value-weighted average of the two firms' excess returns.

Proposition 2. *The prices of both assets in each regime are*

$$p_1^{\text{pre}} = \left[\mathrm{E}\left(d_1^{\text{pre}}\right) - a\,\mathrm{Cov}(d_1^{\text{pre}}, d_M^{\text{pre}})\right]/R_f \tag{35}$$

$$p_1^{\text{post}} = \left[\mathrm{E}\left(d_1^{\text{post}}\right) - a\,\mathrm{Cov}(d_1^{\text{post}}, d_M^{\text{post}})\right]/R_f \tag{36}$$

$$p_2^{\text{pre}} = \left[\mathrm{E}\left(d_2^{\text{pre}}\right) - a\,\mathrm{Cov}(d_2^{\text{pre}}, d_M^{\text{pre}}) - a(W_{0B}/W_{0A})\,\mathrm{Var}(d_2^{\text{pre}}|d_1^{\text{pre}})\right]/R_f \tag{37}$$

$$p_2^{\text{post}} = \left[\mathrm{E}\left(d_2^{\text{post}}\right) - a\,\mathrm{Cov}(d_2^{\text{post}}, d_M^{\text{post}}) - a(W_{0B}/W_{0A})\,\mathrm{Var}(d_2^{\text{post}}|d_1^{\text{post}})(1-\gamma)\right]/R_f \tag{38}$$

$$= \left[\mathrm{E}\left(d_2^{\text{post}}\right) - a\,\mathrm{Cov}(d_2^{\text{post}}, d_M^{\text{post}}) - cK_2^{\text{post}}W_{0B}\right]/R_f\,. \tag{39}$$

Each price discounts the asset's certainty-equivalent payoff, in brackets, by the riskless rate. The asset's certainty equivalent equals its expected payoff minus a wedge. Each wedge includes the covariance of the asset's payoff with the market payoff, multiplied by risk aversion. This covariance component is the compensation investors require for the asset's

contribution to aggregate risk. For the public asset, this risk compensation is the only wedge, because public equity is freely tradable by both investors, in both regimes. The public asset's pricing in equations (35) and (36) takes a familiar form, dating to Lintner (1965).

The private asset is different. In the pre-democratization economy, the asset's certainty equivalent subtracts an additional wedge, $a\,(W_{0B}/W_{0A})\,\mathrm{Var}(d_2^{\mathrm{pre}}|d_1^{\mathrm{pre}})$, a segmentation discount. Retail investors are excluded, so institutions must hold the entire private firm and bear the component of its risk that public equity cannot hedge, $\mathrm{Var}(d_2^{\mathrm{pre}}|d_1^{\mathrm{pre}})$. Their required compensation for this undiversified exposure increases with risk aversion and the share of the risk they must carry, W_{0B}/W_{0A}. Post-democratization, they carry only the fraction $1-\gamma$ of that risk borne pre-democratization, because retail investors participate to the degree γ. Democratization shrinks the segmentation discount by the extent to which retail investors participate and take on some of the unspanned risk previously carried by institutions.

Equation (39) shows that the shrunk segmentation discount can also be expressed as a holding-cost discount:

$$cK_2^{\mathrm{post}}W_{0B} \;=\; (1-\gamma)\,a\frac{W_{0B}}{W_{0A}}\,\mathrm{Var}\big(d_2^{\mathrm{post}} \mid d_1^{\mathrm{post}}\big)\;, \tag{40}$$

which follows from equation (24). Intuitively, once retail can participate, the private asset must compensate retail investors for the cost of holding private equity. There is therefore a duality between the risk-based and cost-based interpretations of the pricing discount; they are not competing readings.

Corollary 3. Effects of democratization on prices and expected returns. *Assume capital stocks are held fixed, $K^{\mathrm{post}} = K^{\mathrm{pre}} = K$. Then*

$$p_1^{\mathrm{post}} \;=\; p_1^{\mathrm{pre}} \tag{41}$$

$$p_2^{\mathrm{post}} \;>\; p_2^{\mathrm{pre}} \tag{42}$$

$$p_M^{\mathrm{post}} \;>\; p_M^{\mathrm{pre}} \tag{43}$$

and

$$\mathrm{E}\left(\widetilde{r}_1^{\mathrm{post}}\right) \;=\; \mathrm{E}\left(\widetilde{r}_1^{\mathrm{pre}}\right) \tag{44}$$

$$\mathrm{E}\left(\widetilde{r}_2^{\mathrm{post}}\right) \;<\; \mathrm{E}\left(\widetilde{r}_2^{\mathrm{pre}}\right). \tag{45}$$

$$\mathrm{E}\left(\widetilde{r}_M^{\mathrm{post}}\right) \;<\; \mathrm{E}\left(\widetilde{r}_M^{\mathrm{pre}}\right). \tag{46}$$

By holding capital stocks fixed, this corollary isolates the asset-pricing effects of democratization. For the public asset, we obtain equations (41) and (44) immediately from

Proposition 2 and the following expressions implied by equations (1), (2), and (34):

$$\mathrm{E}(d_j^k) = K_j^k \mu_j, \quad j \in \{1, 2\} \tag{47}$$

$$\mathrm{Cov}(d_1^k, d_M^k) = K_1^k \left(K_1^k v_{11} + K_2^k v_{12}\right) \tag{48}$$

$$\mathrm{Cov}(d_2^k, d_M^k) = K_2^k \left(K_1^k v_{12} + K_2^k v_{22}\right) . \tag{49}$$

Across the two regimes, the public asset's payoff, d_1, has the same expected value and covariance with the market, because K_j^k is fixed across regimes. Therefore, the prices in equations (35) and (36) are the same across regimes. The expected returns must then also be the same across regimes, because the expected payoffs are the same. The public asset's expected excess return reflects compensation only for covariance risk, so it is unaffected by democratization.

The private asset's payoff, d_2, also has the same expectation and covariance with the market across regimes. Therefore, subtracting equation (37) from (39) implies $p_2^{\text{post}} - p_2^{\text{pre}}$ is equal to W_{0B}/R_f times $\left[a(1/W_{0A})\,\mathrm{Var}(d_2^{\text{pre}}|d_1^{\text{pre}}) - cK_2^{\text{post}}\right]$, which is positive, given the participation condition in (26). Thus (42) immediately follows, as then does (45), because the asset's expected payoffs are the same across regimes. Post-democratization, the segmentation discount shrinks, resulting in a higher price and thus lower expected excess return post-democratization. Intuitively, whenever retail investors find it worthwhile to participate, their new risk-sharing benefit outweighs the holding cost, leading to a lower cost of capital for the private firm. Notably, the very condition that induces retail investors to enter is the condition under which their entry lowers the private asset's expected return. This decline in the private firm's cost of capital is the central asset-pricing effect of democratization.

The market portfolio's higher post-democratization price follows simply by summing (41) and (42). As before, the lower post-democratization expected return then follows by noting that the expected market payoff is the same across regimes.

The pricing effects of democratization become less transparent when allowing firms' capital to change. Risk effects then shift, so even the public asset is no longer insulated. For example, the public asset's expected gross return, $\mathrm{E}(d_1^k)/p_1^k$, can be expressed, using equations (35), (36), (47), and (48), as

$$\frac{\mathrm{E}(d_1^k)}{p_1^k} = R_f \frac{\mu_1 K_1^k}{\mu_1 K_1^k - a K_1^k \left(K_1^k v_{11} + K_2^k v_{12}\right)} = R_f \frac{\mu_1}{\mu_1 - a\left(K_1^k v_{11} + K_2^k v_{12}\right)} . \tag{50}$$

The expected return changes across regimes if $\left(K_1^k v_{11} + K_2^k v_{12}\right)$ does. That quantity's change equals $(\Delta K_1 v_{11} + \Delta K_2 v_{12})$, where $\Delta K_j \equiv K_j^{\text{post}} - K_j^{\text{pre}}$. Depending on ΔK, the expected return can change in either direction. We analyze firms' investment responses in Section 6.

Closed-form expressions for assets' expected excess return are provided by Lemma 2 in the Appendix. Here we analyze expected returns in a CAPM context, isolating the deviation of each asset's expected return from that standard pricing benchmark.

Proposition 3. *Denote the CAPM alpha of asset j in regime k:*

$$\alpha_j^k \equiv \mathrm{E}\left(\widetilde{r}_j^k\right) - \beta_j^k \,\mathrm{E}\left(\widetilde{r}_M^k\right), \qquad j \in \{1,2\},\ k \in \{\text{pre}, \text{post}\}, \tag{51}$$

where β_j^k is the market beta, $\beta_j^k \equiv \mathrm{Cov}\left(\widetilde{r}_j^k, \widetilde{r}_M^k\right) / \mathrm{Var}\left(\widetilde{r}_M^k\right)$. These alphas are given by

$$\alpha_1^k = -\,h^k\,\beta_1^k \tag{52}$$

$$\alpha_2^k = +\,h^k\,\beta_1^k\,\frac{p_1^k}{p_2^k}, \tag{53}$$

where the regime-specific wedge, h^k, is

$$h^{\text{pre}} = a\,\frac{W_{0B}}{W_{0A}}\,\frac{(K_2^{\text{pre}})^2}{p_M^{\text{pre}}}\,v_{2|1} \tag{54}$$

$$h^{\text{post}} = a\,\frac{W_{0B}}{W_{0A}}\,\frac{(K_2^{\text{post}})^2}{p_M^{\text{post}}}\,v_{2|1}\,(1-\gamma) \tag{55}$$

$$= c\,W_{0B}\,\frac{K_2^{\text{post}}}{p_M^{\text{post}}}. \tag{56}$$

The alphas in equations (52) and (53) are proportional to h^k. In the pre-democratization economy, h^{pre} is a segmentation wedge: because institutions bear the entire unspanned component of private risk, they demand compensation for it. This compensation scales with risk aversion a, the unspanned risk $v_{2|1}$, and the share of it institutions are forced to carry, W_{0B}/W_{0A}. In the post-democratization economy, h^{post} can be interpreted as either a shrunk segmentation wedge (equation (55)) or a holding-cost wedge (equation (56)). Under the latter interpretation, the private asset must compensate retail investors for the per-dollar cost c. Each wedge vanishes when its underlying friction does: $h^{\text{pre}} = 0$ when private risk is fully spanned by public equity ($v_{2|1} = 0$), and $h^{\text{post}} = 0$ when private equity is costless to hold ($c = 0$), in which cases both alphas are zero and the CAPM holds exactly.

Corollary 4. Signs of the CAPM alphas. *Suppose frictions are present in both economies: $v_{2|1} > 0$ pre-democratization and $c > 0$ post-democratization. Then the private asset carries a positive alpha and the public asset a negative alpha in both economies:*

$$\alpha_2^k > 0 > \alpha_1^k, \qquad k \in \{\text{pre}, \text{post}\}. \tag{57}$$

This corollary follows directly from equations (52) and (53) after recognizing that $h^k > 0$, $p_1^k > 0$, $p_2^k > 0$, and $\beta_1^k > 0$.[14] Intuitively, the friction—segmentation or holding cost—attaches to the private asset alone, so it is the private asset whose expected return is pushed above the CAPM value, giving $\alpha_2^k > 0$. The public asset's negative alpha is simply the mirror-image adjustment required to keep the value-weighted market priced correctly. (Equations (52) and (53) imply $(p_1^k/p_M^k)\alpha_1^k + (p_2^k/p_M^k)\alpha_2^k = 0$.)

Corollary 5. Effects of democratization on the CAPM alphas. *Assume capital stocks are held fixed, $K^{\text{post}} = K^{\text{pre}} = K$. Then*

$$\alpha_1^{\text{pre}} < \alpha_1^{\text{post}} \tag{58}$$

$$\alpha_2^{\text{pre}} > \alpha_2^{\text{post}} \tag{59}$$

Democratization compresses both alphas toward zero: α_1 rises toward zero and α_2 falls toward zero, reaching zero when private equity is costless to hold ($c = 0$). To gain more insight into this compression, we substitute β_1^k from footnote 14 into (52) and (53), obtaining

$$\alpha_1^k = -\frac{\text{Cov}(d_1^k, d_M^k)}{\text{Var}(d_M^k)} \frac{w^k}{p_1^k} \tag{60}$$

$$\alpha_2^k = +\frac{\text{Cov}(d_1^k, d_M^k)}{\text{Var}(d_M^k)} \frac{w^k}{p_2^k}\,, \tag{61}$$

where $w^k \equiv h^k p_M^k$ is the wedge measured in payoff units rather than as a return. With capital fixed, $\text{Cov}(d_1, d_M)$, $\text{Var}(d_M)$, and p_1 are the same in both economies, and the wedge is smaller post-democratization, $w^{\text{post}} < w^{\text{pre}}$, because the retail-participation condition holds. The public alpha therefore compresses in proportion to the wedge,

$$\frac{\alpha_1^{\text{post}}}{\alpha_1^{\text{pre}}} = \frac{w^{\text{post}}}{w^{\text{pre}}}\,. \tag{62}$$

The private alpha compresses by more, due to an additional force: the private firm's price is higher post-democratization once retail investors share the firm's risk. Its compression factor is that of the public alpha scaled by the price ratio,

$$\frac{\alpha_2^{\text{post}}}{\alpha_2^{\text{pre}}} = \frac{w^{\text{post}}}{w^{\text{pre}}} \cdot \frac{p_2^{\text{pre}}}{p_2^{\text{post}}}\,, \qquad \frac{p_2^{\text{pre}}}{p_2^{\text{post}}} < 1\,. \tag{63}$$

[14] Note that $\beta_j^k = (p_M^k/p_j^k)\text{Cov}(d_j^k, d_M^k)/\text{Var}(d_M^k)$, where $\text{Cov}(d_1^k, d_M^k) = K_1^k(K_1^k v_{11} + K_2^k v_{12})$ and $\text{Var}(d_M^k) = (K_1^k)^2 v_{11} + 2K_1^k K_2^k v_{12} + (K_2^k)^2 v_{22}$. Therefore, β_1^k inherits its sign from $\text{Cov}(d_1^k, d_M^k)$, which is positive because we assume $\rho_{12} > 0$, which in turn implies $v_{12} > 0$.

5. Investor welfare

In this section, we examine how investor welfare varies across the pre- and post-democratization economies. We measure an investor's welfare using certainty equivalents (CEs) expressed as a fraction of the investor's initial wealth. Following the definition of investor preferences in Section 2, we define investor i's certainty equivalent as[15]

$$\mathrm{CE}_i(K, \theta_i) = R_f + \mathrm{E}\left(\widetilde{r}_i^p\right) - \frac{a}{2}\,\mathrm{Var}\left(\widetilde{r}_i^p\right). \tag{64}$$

Into this definition, we substitute the equilibrium holdings from Proposition 1 and the equilibrium payoffs underlying Proposition 2, yielding the following results. Just like in Section 4, we assume throughout this section that the retail-participation condition (28) holds.

Proposition 4. *In the pre-democratization economy, the investors' certainty equivalents are*

$$\mathrm{CE}_A^{\mathrm{pre}} = R_f + \frac{a}{2}\left(\mathrm{Var}(d_M|K^{\mathrm{pre}}) + \frac{W_{0B}(2-W_{0B})}{W_{0A}^2}\,\mathrm{Var}(d_2^{\mathrm{pre}}|d_1^{\mathrm{pre}})\right) \tag{65}$$

$$\mathrm{CE}_B^{\mathrm{pre}} = R_f + \frac{a}{2}\Big(\mathrm{Var}(d_M|K^{\mathrm{pre}}) - \mathrm{Var}(d_2^{\mathrm{pre}}|d_1^{\mathrm{pre}})\Big). \tag{66}$$

In the post-democratization economy, they are

$$\mathrm{CE}_A^{\mathrm{post}} = R_f + \frac{a}{2}\,\mathrm{Var}(d_M|K^{\mathrm{post}}) + c\,W_{0B}\,K_2^{\mathrm{post}} + c^2\,\frac{W_{0B}^2}{2a\,v_{2|1}} \tag{67}$$

$$\mathrm{CE}_B^{\mathrm{post}} = R_f + \frac{a}{2}\,\mathrm{Var}(d_M|K^{\mathrm{post}}) - c\,W_{0A}\,K_2^{\mathrm{post}} + c^2\,\frac{W_{0A}^2}{2a\,v_{2|1}}. \tag{68}$$

Each certainty equivalent shares a common term, $\frac{a}{2}\,\mathrm{Var}(d_M)$, which reflects compensation for bearing aggregate market risk. This compensation is identical across investors within a given regime, but it varies across regimes because of differences in the amount of capital at which the variance of market payoffs, $\mathrm{Var}(d_M|K) = v_{11}K_1^2 + 2v_{12}K_1K_2 + v_{22}K_2^2$, is evaluated.

Before democratization, retail investors cannot share unspanned private risk, so their welfare is reduced by a term proportional to $\mathrm{Var}(d_2^{\mathrm{pre}}|d_1^{\mathrm{pre}})$. Institutions bear that risk alone and are paid to do so, so their welfare is increased by a closely related term. Institutions earn this segmentation rent because retail is excluded from holding the private firm.

[15] An equivalent representation can be obtained in wealth levels:

$$\mathrm{CE}_i(K, \theta_i) = \frac{1}{W_{0i}}\left(\mathrm{E}(W_{1i}) - \frac{A_i}{2}\,\mathrm{Var}(W_{1i})\right), \qquad \text{where } A_i = a/W_{0i} \text{ is absolute risk aversion}.$$

Post-democratization, the certainty equivalents include terms involving the holding cost c. The linear terms, $-c\,W_{0A}K_2^{\text{post}}$ for retail and $+c\,W_{0B}K_2^{\text{post}}$ for institutions, are a pure transfer. Recall that only retail investors pay the holding cost c. Institutions gain through the higher equilibrium return the private asset must offer to compensate retail, and the linear terms offset in the aggregate. The quadratic terms, proportional to $c^2/v_{2|1}$, are positive for both investors. They reflect optimal portfolio adjustment: as the cost rises, each investor rebalances away from the frictionless benchmark—retail by underweighting private equity, institutions by overweighting it—to limit its bite. This second-order gain is larger when private risk is more spanned (small $v_{2|1}$), because the cost then looms larger and the incentive to rebalance is stronger.

Corollary 6. Welfare effects of democratization. *Assume capital stocks are held fixed,* $K^{\text{post}} = K^{\text{pre}} = K$. *Then*

$$\text{CE}_A^{\text{post}} - \text{CE}_A^{\text{pre}} \;=\; -\frac{a}{2}\frac{W_{0B}}{W_{0A}^2}\,\gamma\,(2 - W_{0B}\,\gamma)\;\text{Var}(d_2 \mid d_1) \;<\; 0 \tag{69}$$

$$\text{CE}_B^{\text{post}} - \text{CE}_B^{\text{pre}} \;=\; \frac{a}{2}\,\gamma^2\;\text{Var}(d_2 \mid d_1) \;>\; 0 \tag{70}$$

Define aggregate investor welfare as $\text{CE}_{\text{agg}}^k \equiv \big(W_{0A}\text{CE}_A^k + W_{0B}\text{CE}_B^k\big)$. *Then*

$$\text{CE}_{\text{agg}}^{\text{post}} - \text{CE}_{\text{agg}}^{\text{pre}} \;=\; -\frac{a}{2}\frac{W_{0B}}{W_{0A}}\,\gamma\,(2 - \gamma)\;\text{Var}(d_2 \mid d_1) \;<\; 0\,. \tag{71}$$

This corollary shows that institutions lose from democratization ($\text{CE}_A^{\text{post}} < \text{CE}_A^{\text{pre}}$), retail investors gain ($\text{CE}_B^{\text{post}} > \text{CE}_B^{\text{pre}}$), and aggregate investor welfare falls ($\text{CE}_{\text{agg}}^{\text{post}} < \text{CE}_{\text{agg}}^{\text{pre}}$). Institutions lose because democratization erodes the segmentation rent they earned as the sole bearers of private risk—they now pay a higher price for private investments. In contrast, retail investors gain from democratization, because under their participation condition, the diversification benefit from sharing unspanned private risk dominates the holding cost.

The fall in aggregate investor welfare is a net outcome of three forces apparent from Proposition 4. The linear holding-cost terms cancel in the aggregate. Democratization reduces the segmentation rent institutions earn for bearing unspanned private risk, lowering aggregate investor welfare, while investors' rebalancing in response to the holding cost raises welfare through the terms involving c^2. The rent reduction dominates the rebalancing gain precisely when the retail-participation condition holds.

Interestingly, aggregate investor welfare falls even though democratization improves risk sharing. The two facts are connected: the aggregate certainty equivalent is increasing in the

departure from efficient risk sharing. To see this, combine Proposition 4 with $W_{0A}+W_{0B}=1$:

$$\mathrm{CE}^k_{\mathrm{agg}} = R_f + \frac{a}{2}\left[\mathrm{Var}(d_M|K^k) + \frac{\left(\theta^k_{A2}-W_{0A}\right)^2}{W_{0A}W_{0B}}\mathrm{Var}(d^k_2 \mid d^k_1)\right], \tag{72}$$

which is smallest, at given capital, when institutions hold their wealth share of the private firm ($\theta^k_{A2}=W_{0A}$). Each investor's certainty equivalent in excess of R_f equals half the risk premium the investor earns (see the proof of Proposition 4), so spreading unspanned private risk more evenly means less risk borne in total and thus less compensation earned in total. That compensation is delivered through prices: democratization raises p_M, so investors pay more at $t=0$ for the same aggregate payoff. Because investors buy the firms' shares rather than being endowed with them, the extra payment exits the investor sector, accruing to agents outside the investor sector who sold them the shares. Aggregate investor welfare measures investors' surplus, not total surplus, and its decline reflects a transfer out of the investor sector rather than a loss of resources.

Democratization does consume real resources, but that is a separate matter: retail investors bear the deadweight cost $c\,\theta^{\mathrm{post}}_{B2}K^{\mathrm{post}}_2$. This cost is not what drives the sign in (71). Setting $c=0$ eliminates the deadweight loss altogether and delivers fully efficient risk sharing, yet it makes the decline in equation (71) as large as it can be, $-\frac{a}{2}(W_{0B}/W_{0A})\mathrm{Var}(d_2 \mid d_1)$. Whether democratization raises or lowers *social* welfare therefore cannot be read off the sign of the aggregate-CE change; it depends on how one values the transfer between investors and the initial sellers of the firms' shares, and on the real resource cost c.

Corollary 6 delivers crisp implications for investor welfare changes when capital stocks are held fixed across regimes. When capital varies, retail welfare always rises if retail investors choose to participate in private markets. The change in institutional welfare, however, can either rise or fall depending on the extent of capital changes, which impact the second and third terms in equation (67). We examine these changes in our calibration.

6. Firm investment

In prior sections, we take firms' capital stocks, K^{pre}_j and K^{post}_j, as given. The results in those sections are general in that they hold for any positive capital stocks. For example, one could specify K^{pre}_j and K^{post}_j exogenously. In this section, we take a different approach, solving for firms' optimally chosen capital stocks. This approach allows us to study how real investment responds to changes in firms' costs of capital. With this extra modeling step, our model becomes a general-equilibrium production-based asset pricing model.

6.1. Additional assumptions

We add the following assumptions to both economies. Firm j enters with legacy capital $K_j^0 > 0$, an exogenous primitive that is identical across the pre- and post-democratization economies. The firm adjusts its capital from K_j^0 to K_j by investing

$$I_j \equiv K_j - K_j^0 \tag{73}$$

at a cost

$$C_j(I_j) = I_j + \frac{\phi}{2}\frac{I_j^2}{K_j^0}\,, \qquad \phi \geq 0\,. \tag{74}$$

The parameter ϕ is a reduced-form way to capture convex investment costs. Its purpose is to generate empirically plausible investment responses to changes in the cost of capital. The firm is allowed to sell capital ($I_j < 0$), but we require $K_j > 0$, to preclude corner solutions.

The firm finances the investment cost $C_j(I_j)$ through a frictionless seasoned equity offering. The offering raises $C_j(I_j)$ from new shareholders, who receive equity of equal value. After the investment cost is sunk, the firm's total equity is worth $p_j(K_j) = p_j(K_j^0 + I_j)$, with shares outstanding normalized to one, as in earlier sections. Existing shareholders own what remains after the new claim, and they choose I_j to maximize the value of their stake:

$$\max_{I_j}\; p_j(K_j^0 + I_j) - C_j(I_j)\,. \tag{75}$$

We model each firm as a sectoral aggregate, with I_j chosen by a representative sectoral decision maker who internalizes how sectoral scale affects asset prices. Specifically, the firm takes as given the equilibrium price schedule $p_j(K)$ derived earlier and chooses K_j along it. Investors continue to choose their portfolios while taking prices as given, as before.

6.2 Implications

We also assume throughout this section that retail investors would participate in private markets already at the pre-democratization scale:

$$c \;<\; \frac{aK_2^{\text{pre}} v_{2|1}}{W_{0A}}\,. \tag{76}$$

Condition (76) is the retail-participation condition (28) evaluated at K_2^{pre} rather than at K_2^{post}. Imposing it at K_2^{pre} keeps it a restriction on quantities determined before firms invest. We show in the Appendix that (76) implies (28); in other words, if retail is willing to

participate at K_2^{pre} then it also participates at K_2^{post}. Under (76), the first-order conditions characterize each firm's unique optimal capital stock. Two forces make the firm's problem well behaved: the convex adjustment cost, and the firm's price impact, since a larger sector depresses the price at which its output clears.[16]

Proposition 5. *Let $\Delta K_j \equiv K_j^{\text{post}} - K_j^{\text{pre}}$ for $j \in \{1, 2\}$. Then democratization-induced changes in the two firms' capital stocks are given by*

$$\Delta K_1 = -\frac{av_{12}}{2u_1}\,\Delta K_2 \tag{77}$$

$$\Delta K_2 = \frac{2u_1 W_{0B}}{4u_1u_2 - a^2v_{12}^2}\left(\frac{2aK_2^{\text{pre}}v_{2|1}}{W_{0A}} - c\right), \tag{78}$$

where $u_j \equiv av_{jj} + R_f\phi/(2K_j^0)$.

The private firm's investment response reflects a tension between two familiar forces. Democratization reduces the segmentation premium, which lowers the private firm's required payoff and encourages it to expand, but it also introduces the holding cost c, which has the opposite effect. Both forces appear in the factored term of equation (78). Condition (76) signs that term regardless of ϕ, so the adjustment cost does not affect the direction of the response. It affects only the magnitude: a larger ϕ dampens $|\Delta K_2|$, holding K_2^{pre} fixed. Retail's wealth share W_{0B} also affects the magnitude: when the private firm expands, it expands more the larger is W_{0B}, since a greater retail presence means more risk-sharing capacity to unlock.

The public firm faces neither force, but its capital moves in response to the private firm's, as shown in equation (77). The sign of ΔK_1 is the opposite of that of ΔK_2, indicating that a private-firm expansion crowds out the public firm. Specifically, a larger private sector raises the covariance of the public firm's payoff with the market payoff through the term $av_{12}K_2$ in equation (48). Because the two sectors' payoffs are positively correlated ($v_{12} > 0$), additional private capital adds to the aggregate risk that investors must help absorb, raising the return investors demand on it and discouraging public investment.

Corollary 7. *Democratization expands the private firm's capital and contracts the public firm's capital:*

$$\Delta K_2 > 0 > \Delta K_1\,. \tag{79}$$

The private firm expands because the risk-sharing gain from democratization outweighs the holding cost, and condition (76) is what ensures that it does. That condition puts c

[16] We need condition (76) because the private firm's price schedule changes form at the capital stock above which retail participates. Without (76), the switch point lies above K_2^{pre} and the firm's post-democratization objective can have a local optimum on either side of it. The proof of Proposition 5 gives details.

below $aK_2^{\text{pre}} v_{2|1}/W_{0A}$, whereas the factored term in equation (78) stays positive up to twice that quantity. Retail participation is therefore sufficient, with room to spare, for the private firm to expand.

7. Calibrations

This section uses the model to explore quantitatively the effects of democratization on asset holdings, expected returns, investor welfare, and firm investment. We calibrate the model's parameters to satisfy various empirically motivated criteria. A given calibration n of the model requires a value for each of 12 parameters, collected in the vector

$$s_n = [W_{0A}, c, \rho_{12}, \mu_1, \mu_2, R_f, a, v_{11}, v_{22}, \phi, K_1^0, K_2^0]. \tag{80}$$

As in the previous sections, we hold s_n constant across the pre- and post-democratization regimes in order to keep the analysis manageable. A more complicated analysis could allow democratization to affect s_n. We use a set-identification approach to construct a set, $\mathcal{S}$, containing N values of s_n that each satisfies a set of criteria, $\mathcal{R}$:

$$\mathcal{S} = \{s_n | s_n \text{ satisfies } \mathcal{R}\}. \tag{81}$$

The set $\mathcal{R}$ imposes restrictions on some elements of s_n directly and on model-implied quantities that restrict s_n implicitly.

Section 7.1 explains the criteria in $\mathcal{R}$. Section 7.2 examines the model's implications across $s_n \in \mathcal{S}$. Section 7.3 analyzes the role of the private market's size in determining welfare effects and asset prices. Section 7.4 analyzes how the holding cost paid by retail investors impacts their private-market participation and ownership as well as asset prices.

7.1. Calibration criteria and parameter values

Of the model's 12 parameters, which constitute s_n in equation (80), we directly impose criteria for the first six. Additional criteria implicitly restrict parameters by restricting various quantities implied by the model. After explaining all of these criteria, which constitute the set $\mathcal{R}$, we examine the parameter values that set $\mathcal{S}$ comprises.

7.1.1. Criteria that directly restrict parameters

Parameter W_{0A} is institutions' share of total wealth. To follow the model's structure, we classify asset allocators in the data as either institutional investors (i.e., those with access to private-market funds) or retail investors (i.e., those without). We classify high-net-worth households, public and private defined-benefit pensions, and institutional nonprofits (e.g., large foundations and endowments) as institutional investors. We classify non-high-net-worth households, defined-contribution pensions, and non-institutional nonprofits as retail investors. We estimate each group's wealth using data from the Federal Reserve, and we compute W_{0A} as institutional wealth divided by the sum of institutional and retail wealth. Our estimates of W_{0A} vary over a range, 0.219 to 0.410, because we apply thresholds ranging from \$5 million to \$20 million for classifying households as high net worth, and also because we vary the fraction of nonprofit wealth that is "institutional" between zero and one. Additional details are in Appendix A.2.

Parameter c is the proportional cost retail investors pay when investing in private assets. Institutional investors in our model pay no cost to access private investments. The value of c relevant to our calibrations is therefore not the absolute fee level in private funds, but the difference in fees paid by retail and institutional investors for comparable private-fund exposure. Appendix A.3 reviews evidence suggesting c is currently around 0.9% per year, but we consider a range around that estimate, varying c between 0 and 2%.

Parameter ρ_{12} is the correlation between public and private payoffs. If market-to-book ratios are stable over time, then ρ_{12} also equals the correlation between public and private returns, so we consider evidence on the latter. The MSCI (2025) Private Equity Factor, which is estimated from the Burgiss data used in many academic studies, de-smooths reported private-market returns. MSCI (2025) reports a 0.65 correlation between aggregate U.S. public and private equity returns. Couts, Gonçalves, and Loudis (2026) report an average correlation of 0.76 between U.S. large-cap equities and private equity returns, where the average is computed in a sample of Capital Market Assumptions (CMAs) published by 22 asset managers and 23 investment consultants. Our chosen range for ρ_{12}, from 0.5 to 0.8, comfortably contains both estimates.

We do not individually restrict the model parameters μ_1 and μ_2, the expected return on assets (ROA) of public and private firms. We do, however, constrain their difference, as it would be unrealistic to expect private and public firms to have greatly different profitability. Specifically, we restrict $|\mu_1 - \mu_2|$ to be no greater than 10% annually. This bound seems reasonable given the comparisons of private vs. public ROA by Asker, Farre-Mensa, and

Ljungqvist (2015) and Acharya and Xu (2017), which we summarize in Appendix A.4.

Finally, we set the gross risk-free rate to $R_f = 1.0425$, corresponding to the 2025 value of the annual risk-free rate on Kenneth French's website.

7.1.2. Criteria that implicitly restrict parameters

We map the model's implied price of the public firm, p_1^{pre}, to the total market value of public U.S. stocks at the end of 2025. We estimate p_1^{pre} as \$70.6 trillion by summing the market capitalization across all common-stock firms in CRSP that also have valid Compustat accounting data. We map the implied price of the private firm, p_2^{pre}, to the total value of U.S. private equity and venture capital funds at the end of 2025. We approximate that object using Preqin's (2026) reported total AUM in December 2025 of North American PE funds (including VC), \$5.6 trillion.[17] We estimate total wealth, W_0, to be \$82.7 trillion, the sum of p_1^{pre}, p_2^{pre}, and \$6.5 trillion, where the last quantity is our proxy for the amount of the risk-free asset, namely, the amount outstanding of U.S. Treasury securities with duration one year or less in December 2025. In the model, W_0 is normalized to 1, so we target $p_1^{\text{pre}} = 70.6/82.7 = 0.8537$ and $p_2^{\text{pre}} = 5.6/82.7 = 0.0677$. These values help identify firms' legacy capital stocks, K_1^0 and K_2^0.

We equate the volatility of the public firm's return, $\text{Stdev}(\tilde{r}_1^{\text{pre}})$, to 0.2013, the standard deviation of annual excess U.S. market returns from 1927 to 2025.[18] We allow the public firm's annual risk premium, $\text{E}(\tilde{r}_1^{\text{pre}})$, to lie between 4% and 10%. The wide range recognizes the risk premium's inherent imprecision, in part reflecting uncertainty about its dependence on predictive variables such as the dividend-price ratio. These values help identify the public firm's ROA variance (v_{11}) and investors' risk aversion (a).

We restrict the model-implied volatility of the private firm's return, $\text{Stdev}(\tilde{r}_2^{\text{pre}})$, to a range instead of a specific value. Given the shorter history and less precise return data for private equity, as compared to public, private equity's volatility is more difficult to pin down. Using the MSCI (2025) Private Equity Factor, we estimate the volatility of aggregate private equity returns to be 22%, to which we then add plus or minus 5%, restricting $\text{Stdev}(\tilde{r}_2^{\text{pre}})$ to lie between 17% and 27%. These values help pin down the private ROA variance (v_{22}).

For the private firm's model-implied beta, β_2^{pre}, we allow a fairly wide range, between 0.5 and 1.5. This range nests beta estimates across U.S. venture, buyout, distressed, and

[17] We obtain similar estimates using data from Pitchbook and MSCI/Burgiss.

[18] We obtain the annual returns data from Ken French's website.

mezzanine funds, computed from the MSCI (2025) Private Equity Factor. For comparison, Korteweg and Nagel (2025) estimate a VC beta of 2.373 and a buyout beta of 0.908. Using Preqin's VC and PE AUM weights, the combined VC+PE beta implied by the Korteweg and Nagel (2025) estimates is 1.25, which is well inside our range. This beta is useful in identifying v_{11}, v_{22}, and ρ_{12}.

To identify the convex investment cost parameter (ϕ), we bound the private firm's sensitivity of investment to its cost of capital. Specifically, we define

$$\mathcal{E} \equiv \frac{\Delta(I/K)}{\Delta \mathrm{E}(\tilde{r}_2)}, \tag{82}$$

with

$$\Delta(I/K) \equiv \frac{I_2^{\text{post}}}{K_2^0} - \frac{I_2^{\text{pre}}}{K_2^0} \tag{83}$$

$$\Delta \mathrm{E}(\tilde{r}_2) \equiv \mathrm{E}(\tilde{r}_2^{\text{post}}) - \mathrm{E}(\tilde{r}_2^{\text{pre}}). \tag{84}$$

We restrict $\mathcal{E}$ to lie between -13.35 and -1.05, a range that nests existing empirical estimates of investment's response to the cost of capital. Those estimates come from Zwick and Mahon (2017) and five related studies. Details are in Appendix A.5.

We identify firms' expected ROAs (μ_1 and μ_2) by setting firms' optimal investments in the pre-democratization regime, I_1^{pre} and I_2^{pre}, to zero. Therefore, firms' pre-democratization capital stocks satisfy $K_1^{\text{pre}} = K_1^0$ and $K_2^{\text{pre}} = K_2^0$. This restriction views the pre-democratization economy as each firm's undisturbed starting point: absent any repricing-induced investment, a firm's optimal capital stock equals the legacy capital it inherits.

Finally, since our goal is to study retail investors' participation in private markets, we eliminate values of s_n that imply no participation according to Corollary 1. This last restriction aligns the calibration with previous sections' assumed participation, and it also guarantees a nonzero denominator in equation (82)'s definition of $\mathcal{E}$. Many values of s_n satisfy the calibration criteria above and yet imply no retail participation. This participation is not guaranteed, even at empirically plausible parameter values. Dropping this restriction would produce even smaller average retail welfare gains than we report below, since some parameter vectors would imply non-participation and hence zero welfare gains.

7.1.3. Parameter values

Table 1 summarizes the above calibration criteria, which constitute the set $\mathcal{R}$. We apply those criteria to a multi-dimensional parameter grid and construct a set $\mathcal{S}$ containing $N = 16{,}870$

values of s_n that satisfy the criteria in $\mathcal{R}$. Specifically, we first create a six-dimensional, evenly spaced grid for $\{W_{0A}, c, \rho_{12}, \mathrm{E}(\tilde{r}_1), \beta_2^{\mathrm{pre}}, \mathcal{E}\}$, using the intervals specified in Table 1. We use seven grid points for each dimension, producing 7^6 grid points. At each grid point, we solve for the remaining model parameters to match the restrictions in $\mathcal{R}$. If it is impossible to meet all the restrictions, we discard the parameter vector. This process produces a large set $\mathcal{S}$ of parameter combinations satisfying all of the criteria in $\mathcal{R}$. Table 2 summarizes each parameter's distribution across all $s_n \in \mathcal{S}$.

7.2. Calibration results

Table 3 summarizes the differences between post- and pre-democratization values of various model-implied quantities. We consider two cases for firms' capital stocks in the post-democratization regime. The first one, in Panel A, sets $K^{\mathrm{post}} = K^{\mathrm{pre}}$, thereby restricting both I_1^{post} and I_2^{post} to zero. The second one, in Panel B, allows $K^{\mathrm{post}} \neq K^{\mathrm{pre}}$, with I_1^{post} and I_2^{post} chosen optimally. In both cases, Table 3 reports the distribution of the implied post-vs-pre change across the $N = 16{,}870$ values of the parameter vector, s_n, that satisfy the calibration criteria.

Democratization can produce substantial changes. For example, when allowed to invest in the private firm, retail investors give it a portfolio weight of 4% on average, and they typically end up owning over 40% of the firm. This participation by retail investors reduces the private firm's cost of capital by more than 60 basis points (bps) on average, with the firm's market-to-book ratio increasing by roughly that same amount. All of these changes are similar whether or not capital stocks are fixed (Panel A) or adjusted optimally (Panel B).

Democratization's effects on expected returns are explored further in Table 4, which reports CAPM alphas on firms' equity and investors' portfolios. Consistent with Corollary 4, alphas are positive for the private firm and negative for the public firm in both regimes. Pre-democratization, alphas are negative for retail and positive for institutions, which hold all of the positive-alpha asset. Those signs continue post-democratization, because the holding cost causes retail to underweight private and further reduces their net alpha. In both regimes, the magnitude of the private-firm's alpha is more than ten times the public's. Democratization shrinks the alphas toward zero, again consistent with Corollary 4. Pre-democratization, the private firm's alpha is 1% on average across calibrations, compared to 0.4% post-democratization. This difference echoes the average 60 bps drop in the firm's cost of capital noted earlier in Table 3.

The drop in the private firm's cost of capital spurs the firm to invest. As shown in Panel B of Table 3, private-firm capital expands by 4.8% on average, while public-firm capital gets reallocated to private but drops by only 5 bps. These effects translate to about $260 billion of added private capital and a $30 billion reduction in public capital, netting to a $230 billion expansion of the economy's total capital.[19] Moreover, some of the calibrations produce considerably larger impacts than the calibration averages above. Overall, democratization of private markets generally fosters substantial participation by retail investors that has nontrivial impacts on the private firm's cost of capital and investment.

Democratization's effect on retail investors' welfare is quite modest, despite retail's substantial private-market participation. We measure the welfare effect as the change in an investor's certainty equivalent (CE) as a fraction of their wealth, denoted as the CE gain. When capital is held fixed (Panel A), democratization produces a positive CE gain for retail investors, as implied by Corollary 6. The size of the gain, however, is at most 3 bps, and it's just 1 bp on average across the admissible parameter vectors. Accounting for the resulting expansion of private-firm capital, in Panel B, raises retail-investor welfare relative to the fixed-capital case. While retail's mean CE gain in Panel B is still only 2 bps, the gain is somewhat higher among a small fraction of the admissible parameter vectors: the gain's 95th percentile is 7 bps, and the maximum is 24 bps. For most of the admissible parameter space, however, retail's CE gain is just a few basis points.

Democratization generally lowers institutional investors' welfare, opposite the direction for retail. In Panel A, where the flip in direction follows from Corollary 6, institutions' CE loss has a mean of 11 bps and a maximum of 67 bps. Institutions thus generally experience a certainty-equivalent loss, relative to their wealth, that exceeds the corresponding gain experienced by retail investors. Accounting for capital changes, in Panel B, has only a negligible effect on the distribution of institutions' CE losses.

Underlying the above welfare changes are welfare levels, which Table 5 reports for institutions and retail investors. Both pre- and post-democratization, institutions enjoy a higher level of welfare, measured as their certainty-equivalent return in excess of the riskless rate. This makes sense, as having unfettered access to both markets cannot make an investor worse off compared to one whose access is restricted or costly. The dispersion in welfare levels across calibrations essentially reflects that of risk aversion, a, which ranges from 1.07 to 2.75 in Table 2.

[19] Multiplying total wealth ($82.7 trillion) by median K_2^0 (0.067, from Table 2, measured as a fraction of wealth) implies $5.5 trillion in initial private capital; multiplying by 4.8% yields $264 billion. Multiplying total wealth by median K_1^0 (0.799) implies $66.1 trillion in initial public capital; multiplying by −5 bps yields −$33 billion.

Democratization's rather modest welfare effects are consistent with its small effects on investors' portfolio returns. Table 6 reports portfolio correlations between investors as well as across regimes for a given investor. All of the correlations are fairly high, with the lowest occurring between investors pre-democratization, but even those range from 0.929 to 0.994. Especially telling in understanding the tiny change in retail investors' welfare is the correlation between their portfolio returns pre- versus post-democratization. That correlation ranges from 0.992 to 1.000, with a mean and median of 0.999. In other words, although democratization improves risk sharing, that effect is quite small.

7.3. Role of the private market's size

Why does democratization deliver such modest welfare gains for retail investors? A key reason is that the private market is relatively small, so the additional risk being shared post-democratization—the source of the welfare gain—is also small. If the private market were larger, democratization could be worth substantially more to retail investors.

Figure 2 illustrates this point using our model. The horizontal axis is the pre-democratization scale of the private firm relative to the sum of public and private, i.e., $K_2^0/(K_1^0 + K_2^0)$. We increase K_2^0 while holding $K_1^0 + K_2^0$ constant at the sum of the median values of K_1^0 and K_2^0 in Table 2. The vertical axis is the implied certainty equivalent gain for the retail investor, $\mathrm{CE}_B^{\mathrm{post}} - \mathrm{CE}_B^{\mathrm{pre}}$, which we compute using equations (66) and (68), setting the model's parameters (other than K_1^0 and K_2^0) equal to their median values in Table 2. As in Table 3, we consider two cases. The first holds capital fixed, setting $K_1^{\mathrm{post}} = K_1^{\mathrm{pre}} = K_1^0$ and $K_2^{\mathrm{post}} = K_2^{\mathrm{pre}} = K_2^0$. The second case has post-democratization capital optimally adjust away from those pre-democratization levels, with K_1^{post} and K_2^{post} determined by adding the capital changes in equations (77) and (78) to K_1^0 and K_2^0.

At the median values in Table 2, K_2^0 is slightly less than 8% of $K_1^0 + K_2^0$. At that value for the private firm's pre-democratization share of total capital, the CE gain for the retail investor, plotted in Figure 2, is 0.5 bps when capital is held fixed and 2.0 bps when post-democratization capital adjusts optimally. These very modest retail gains from democratization are consistent with those observed in Table 3. If the private firm's pre-democratization capital share is instead twice as high, at 16%, then the CE gains in Figure 2 rise more than fivefold, to 4.0 bps with capital fixed and to 11.2 bps when capital optimally adjusts. If private's capital share is instead 30%, then those CE gains jump to 18.0 bps and 47.0 bps.

Figure 2 illustrates how retail investors' welfare gain from private-market democratiza-

tion increases sharply in that market's relative size, even when that size is held fixed at its pre-democratization level. The reason, discussed earlier, is that the larger size implies a larger amount of unspanned private risk that democratization allows retail investors to share. Formally, recall equation (70), which presents retail investors' welfare gain from democratization in the fixed-capital case. This gain is directly proportional to unspanned private risk, $\mathrm{Var}(d_2 \mid d_1)$, which is sharply (quadratically) increasing in K_2 (see equation (4)).

As discussed in Section 5, democratization lowers the welfare of institutional investors, absent changes in firms' capital (Corollary 6). At the median values in Panel A of Table 3, the CE loss is 8 bps. This loss increases sharply as the private firm's capital share increases, as illustrated by Panel A of Figure 3. When capital adjusts, a CE loss is not guaranteed, but the losses in Panel B of Table 3 shrink only slightly relative to the fixed-capital case. That minimal difference is echoed by the corresponding changing-capital case plotted in Panel A of Figure 3. Panel B of Figure 3 plots the aggregate CE gain, computed as the wealth-weighted average of the retail and institutional effects. The aggregate CE gain is virtually always negative, and the welfare loss grows in magnitude as private's share increases. Intuition behind the aggregate welfare loss comes easily from the fixed-capital case: democratization does not change the market's expected future payoff but does raise its current price, by equation (43). That pricing effect increases in the private market's size.

Magnitudes of pricing effects are often gauged by alphas. Figure 4 plots the model's implied annual CAPM alpha for the public and private firm, pre- and post-democratization. The alphas are plotted versus the private-firm's pre-democratization relative size, using the same horizontal axis as in Figures 2 and 3. As in those figures, the model's other parameters are held constant at the median values in Table 2. When the private firm's relative size also takes its median value, slightly below 8%, the pre-democratization alpha of the firm's equity is just over 1% per year, reading from Panel A of Figure 4. Post-democratization, we see the alpha drops below 0.5%. This drop in alpha is close to the 0.6% average drop in the firm's cost of capital noted earlier in Table 3.

Panel A of Figure 4 also shows that the private firm's pre-democratization alpha increases nearly in proportion to the firm's relative size, roughly doubling to 2% if size doubles to 16%. This strong relation makes sense: the larger is the private firm, the greater is its unspanned payoff risk, so the higher is the compensation to institutions for entirely bearing that risk. The post-democratization alpha decreases in the firm's size, because retail investors are willing to share a greater fraction of the risk the more of it there is. That willingness is tempered by the holding cost, which matters for every size of the private firm, weakening the latter's role in the post-democratization alpha. All of these effects are mirrored in the

opposite direction for the public firm's alpha in Panel B; the magnitudes are smaller, because the weighted-average alpha is zero and the public firm is larger.

Yet another effect of democratization works via the private market's size: democratization spurs private-firm capital investment, which also plays a role in the welfare gain to retail investors. The capital investment expands the private market's relative size, increasing unspanned private risk and thereby amplifying retail investors' welfare gain from sharing that risk. This capital-expansion effect on retail investors' welfare gain can be substantial, as revealed by comparing the two curves plotted in Figure 2.

The relative size of the private market has risen strongly during this century. Figure 1 plots, from 2000 through 2025, the private market's size divided by the size of the public plus private markets. During that period, private's share more than doubled, from below 3% to recent highs between 7% and 9%. Figure 2 suggests retail investors will benefit non-trivially if the trend in Figure 1 continues, or even accelerates due, say, to private's capital expansion spurred by democratization.[20]

Another potential source of growth in private equity's market share is greater penetration by private markets into private (non-publicly-traded) business. Our specification of p_2^{pre}, as explained in section 7.1, reflects the total value of private-market funds. Based on that value, the private market represents 7.4% of public plus private. Our modeling and calibration treat democratization as making this existing private market accessible to retail investors in addition to institutions. Private-market funds, however, hold only part of the business sector that is not publicly traded. Based on U.S. Federal Reserve estimates for year-end 2025, the fraction of corporate equity's market value that is not publicly traded equals 16.1% for nonfinancial business and 12.5% for all corporate business.[21] Part of the gap between these numbers and the 7.4% noted above may represent an opportunity for private-market investing to penetrate further into private business.

7.4. Role of the retail holding cost

We assume that for retail investors to hold equity in the private firm, they bear a proportional cost, c, not borne by institutions. As discussed earlier, the retail investor's implied CE gain in Table 3 is generally quite small across the calibrations, in which c ranges between 0

[20] Computing the ratio in Figure 1 using estimates of market values (p's) as opposed to capital levels (K's) used to construct Figure 2 seems rather inconsequential. For example, private's market share is 7.7% when using the medians of K_1^0 and K_2^0 in Table 2, while the share is 7.4% when using p_1^{pre} and p_2^{pre} in Table 1.

[21] These fractions are computed using the market values reported in Table F51.1.s of the Federal Reserve's release, *Financial Accounts of the United States-Z.1.*

and 2%. In other words, our conclusion that retail investors do not benefit much from democratization does not really hinge on the presence of a holding cost. In fact, even in calibrations with $c = 0$, retail's average CE gain remains tiny: 0.01% and 0.03% in Panels A and B, respectively. That observation does not mean the cost is irrelevant to all effects of democratization. The cost matters strongly for other effects, as we show next.

First, the cost is central to whether democratization matters at all to retail investors. Panel A of Figure 5 illustrates the retail-participation condition in equation (28). The vertical axis is c, and the horizontal axis is the same as in Figures 2 through 4, the private firm's relative size. As before, the model's other parameters are set to their medians in Table 2. The participation boundary extends linearly from the origin and has a slope of roughly 0.20. If the firm represents somewhat less than 8% of total capital (the median across calibrations), investors stay out if c exceeds 1.6%. If the firm's relative size is roughly twice that, at 15%, investors abstain if c exceeds 3%.

Second, the cost affects the degree to which investors participate, γ in equation (24). This degree depends strongly on both c and the private firm's size. For a range of different c values, Panel B of Figure 5 plots γ versus the private firm's relative size. As before, the model's other parameters are fixed at their medians. When c is 0.5%, participation rises sharply as the private firm's size increases, quickly nearing 100% participation. As c takes higher values, participation drops sharply at a given size of the private firm, such that even if 40% of total capital is private, a c of 4% limits retail participation to just over 50%.

Figure 6 illustrates how post-democratization retail ownership of the private firm depends strongly on where c falls within the 0 to 2% range we consider in the calibrations. As noted earlier, on average across the calibrations, retail investors end up holding slightly more than 40% of the private firm post-democratization. We see in Table 3, however, that retail's ownership fraction of private varies between 0 and 78% across calibrations. The value of c plays a key role in this variation. The horizontal axis in Figure 6 is the value of c. The vertical axis is the fraction of the private firm held post-democratization by the retail investor. Recall from equation (22) that this ownership fraction equals γW_{0B}, where W_{0B} is the retail investor's wealth share and γ is the degree of retail participation defined in equation (24). To compute γW_{0B}, we hold the model parameters other than c constant at their median values in Table 2. As earlier, we consider two cases; one holds capital fixed and the other allows post-democratization capital to adjust optimally.

When retail investors participate in private ($\gamma > 0$), equation (24) implies that, when K_2^{post} is held fixed, γ declines linearly in c and thus so does γW_{0B}. In the frictionless setting

of $c = 0$, implying $\gamma = 1$, retail's ownership fraction of private in Figure 6 is 72%, retail's wealth share. The ownership fraction declines linearly with c, hitting zero when c reaches about 1.6%. Whether or not capital is held fixed matters little for retail's ownership fraction, which is just slightly higher when the private firm optimally expands its capital.

Figure 6 illustrates again how strongly the private-market participation by retail investors depends on the cost of participating. At an annual cost of 1.6% or more, retail investors stay out of the private market. At half that rate, 0.8%, retail buys nearly 40% of the private firm. As noted earlier, evidence suggests c is in the neighborhood of 0.9% currently. In Figure 6, that cost implies retail investors end up owning nearly a third of the private firm.

The retail holding cost, c, also strongly affects the private firm's alpha. Figure 7 plots, pre- and post-democratization, the CAPM alpha versus c for both the private firm (Panel A) and the public firm (Panel B). As usual, the model's other parameters are fixed at their medians. Because c matters only post-democratization, the pre-democratization alpha plots as a constant value. For the private firm that alpha slightly exceeds 1%, the same pre-democratization value noted earlier in discussing Panel A of Figure 4. Post-democratization, the private firm's alpha depends strongly on c. When c takes its median value of 0.7%, the private-firm alpha is about 0.5%, again the same value noted in the earlier discussion. As noted from Figure 6, the participation threshold for c in this setting is slightly more than 1.6%. As c approaches that threshold in Panel A of Figure 7, and thus as participation declines, the private-firm's post-democratization alpha approaches its pre-democratization alpha. Those two alphas coincide for higher c values. As before in Figure 4, the public-firm alphas in Panel B are simply shrunk mirror-opposites of those in Panel A.

8. Conclusion

Democratizing private markets gives retail investors access to investment opportunities previously open only to institutions and high-net-worth investors. The equilibrium model we develop and calibrate quantifies the effects of democratization on investors' portfolio choices and welfare, assets' expected returns, and firms' investment decisions.

Our main result is that democratizing private markets barely raises retail investors' welfare. This result is not driven by frictions often raised in debates about democratization, such as adverse selection or costlier access. The only disadvantage retail investors face in our model is a cost of holding private assets, but their welfare gain is small even when this cost is zero. Instead, retail investors benefit little because the private market is small, only 7%

of the public-plus-private total, so there is little unspanned private risk to share. Democratization could produce larger future gains if the private market's share grows substantially, perhaps through greater private-fund penetration into non-publicly-traded U.S. businesses.

The welfare gain to retail investors is typically more than offset by a welfare loss for institutions, who will face a higher price of private investment. This result deserves two qualifications. First, our welfare measure is narrower than social welfare. We omit the welfare of unmodeled agents outside the investor sector: the original owners who sell shares to investors, the intermediaries who may collect part of c, and the workers and consumers who may benefit from additional capital. We therefore do not read the aggregate certainty-equivalent decline as a verdict against democratization.

Second, we may slightly overstate institutions' welfare loss. Democratization hurts institutions by raising the prices they pay for new private-market investments. Our two-world comparison computes that welfare loss for a given post-democratization period. We do not model a transition between worlds, during which institutions could sell existing private positions to retail investors at the higher post-democratization prices. These capital gains could dampen institutions' welfare loss, but only slightly, given the gains are one-time and the loss is ongoing post-democratization.

Despite its modest effects on investor welfare, democratization substantially affects asset prices, firm ownership, and capital investment. Pre-democratization, private-firm equity earns a CAPM alpha of about 1% per year, compensating institutions for bearing all of the private market's risk not spanned by the public market. Post-democratization, the alpha falls to 0.4% as retail investors take on some of the unspanned risk, trading off that risk-sharing benefit against the cost of holding private assets. This cost shapes retail's participation in the private market. When it is 0.8% per year, retail ends up owning about 40% of the private firm. Relative to pre-democratization, this additional demand lowers the private firm's cost of capital by 0.6% per year and spurs it to expand its capital by nearly 5%. These effects disappear if the holding cost doubles to 1.6%, because retail investors then invest nothing in the private firm. With no holding cost, retail investors' ownership share of private equals their share of total wealth, over 70%, and private earns no alpha.

Our model could be extended in various ways. First, moving to a multi-period setting would allow a fuller treatment of the private asset's illiquidity, which we capture only indirectly, via a holding cost. Second, one could introduce information asymmetry between retail investors and institutions. Third, considering multiple retail investors could allow variation in access costs across investors. Fourth, multiple private assets would permit dispersion in

private-fund performance. We leave these extensions to future work.

Our model delivers a number of testable predictions. Pre-democratization, private equity earns a positive CAPM alpha that increases in the amount of unspanned private risk. Democratization reduces that alpha, lowers the private firm's cost of capital, raises the firm's market value, and spurs private investment. Post-democratization retail participation in private markets increases in the size of those markets and in the variance of private payoffs, and decreases in the public-private payoff correlation and in retail's cost of holding private assets. Future research can test these predictions, especially after democratization takes place. The next chapter on democratization waits to be written.

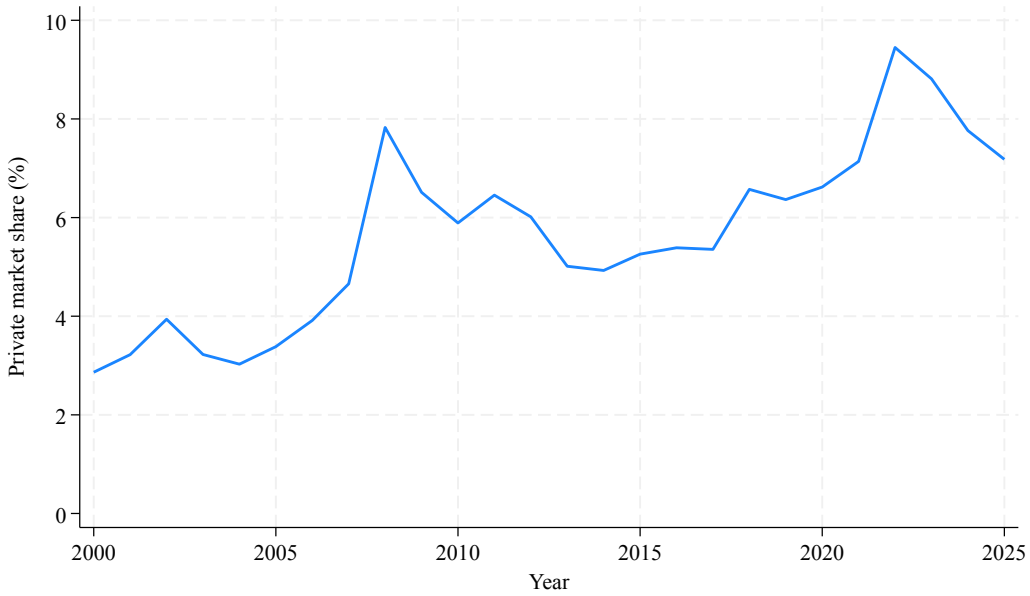


Figure 1. Private market share. This figure plots the annual private market share, defined as $p_{2,t}/(p_{1,t} + p_{2,t})$. Public equity market capitalization, $p_{1,t}$, is the total market capitalization of publicly listed U.S. firms, calculated from the CRSP monthly stock file using the sample of common-stock firms in the CRSP-Compustat universe. Private market assets, $p_{2,t}$, are obtained from the Preqin Pro Dry Powder & AUM tool and equal the sum of dry powder and unrealized value across Buyout, Venture Capital, Growth, Co-investment, and Balanced funds whose Primary Region Focus is North America. The geographic filter is based on each fund's investment focus rather than the location of its manager or headquarters. Funds of funds and other intermediary vehicles are excluded to limit double counting. Both $p_{1,t}$ and $p_{2,t}$ are measured at calendar year-end.

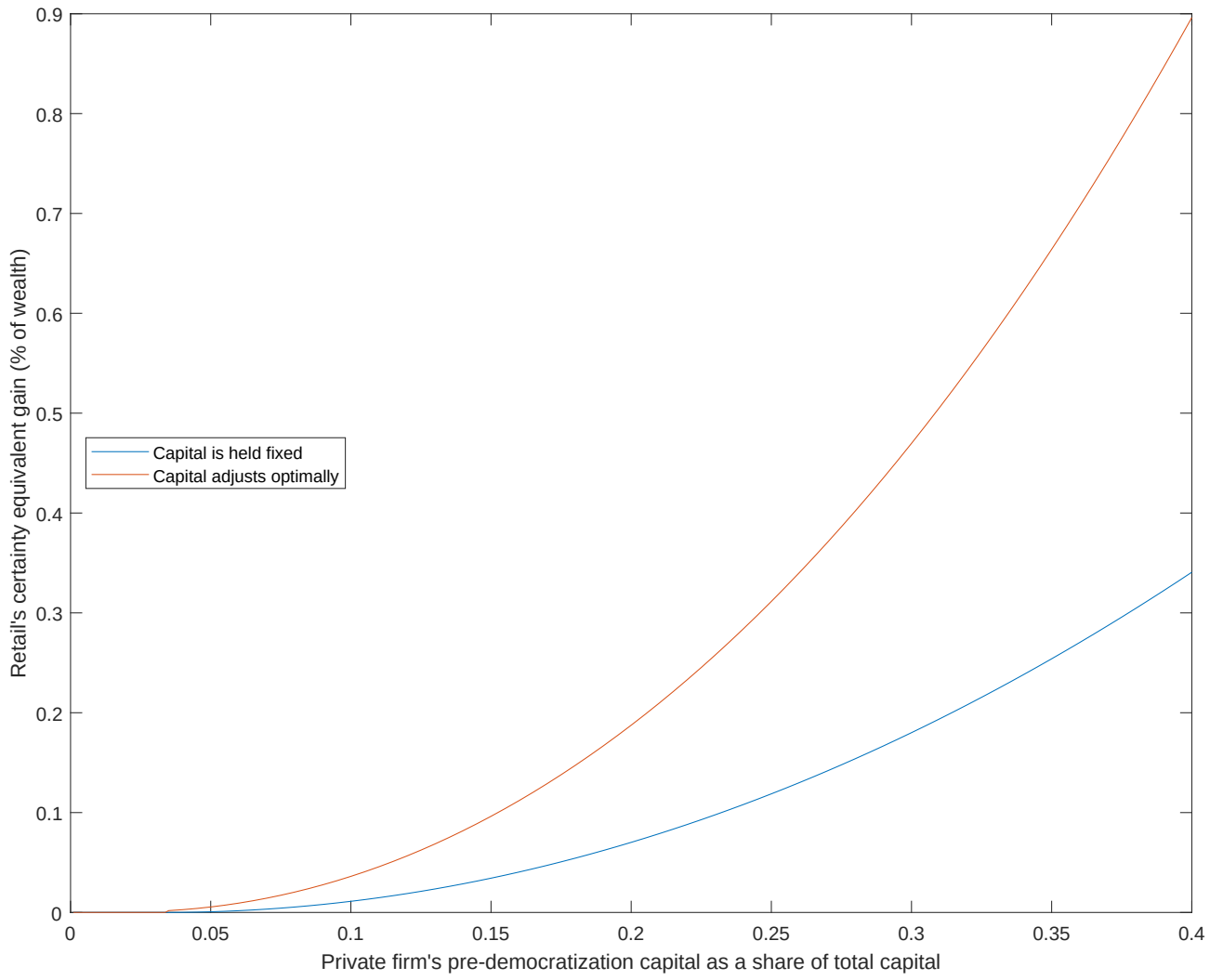


Figure 2. Certainty equivalent gain for retail investors. The vertical axis is the difference between post- and pre-democratization certainty equivalents (as percent of wealth) for the retail investor. The horizontal axis is the private firm's share of total public and private capital, $K_2^0/(K_1^0+K_2^0)$. Two scenarios are plotted: One sets firms' post-democratization capital equal to pre-democratization levels. The other allows post-democratization capital to adjust optimally.

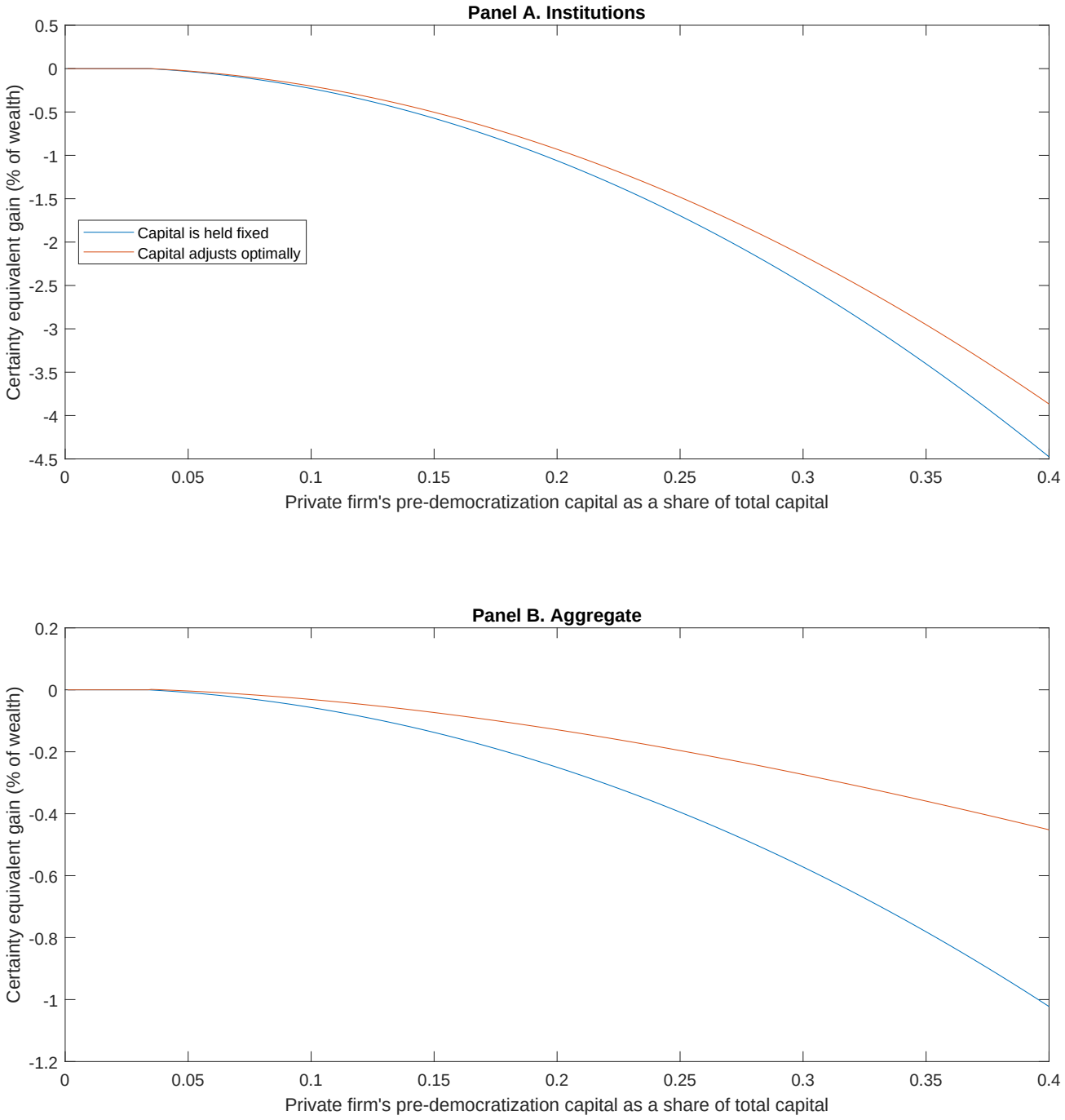


Figure 3. Institutions' and aggregate certainty equivalent gains. The vertical axis is the difference between post- and pre-democratization certainty equivalents (as percent of wealth). The horizontal axis is the private firm's share of total public and private capital, $K_2^0/(K_1^0+K_2^0)$. Two scenarios are plotted: One sets firms' post-democratization capital equal to pre-democratization levels. The other allows post-democratization capital to adjust optimally. Panel A plots certainty equivalent gains for institutional investors; Panel B plots aggregate certainty equivalent gains.

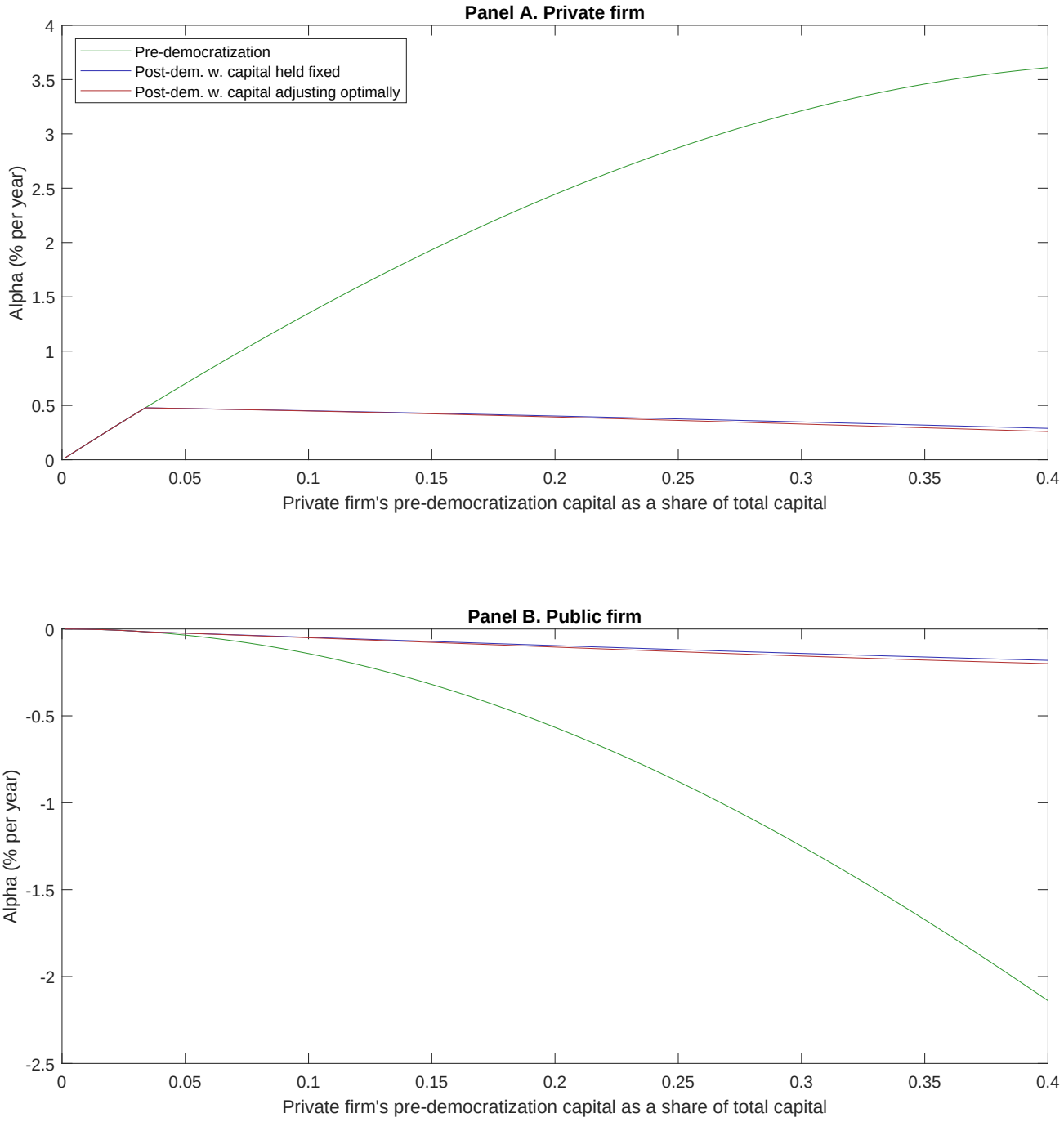


Figure 4. Effect of private-firm capital on alphas. The vertical axis is the annual CAPM alpha (in percent). The horizontal axis is the private firm's share of total public and private capital, $K_2^0/(K_1^0 + K_2^0)$. Three scenarios are plotted: (i) pre-democratization, (ii) post-democratization with firms' capital equal to pre-democratization levels, and (iii) post-democratization with firms' capital adjusting optimally. Panel A plots the alpha on the private firm's equity; Panel B plots the alpha on the public firm's equity.

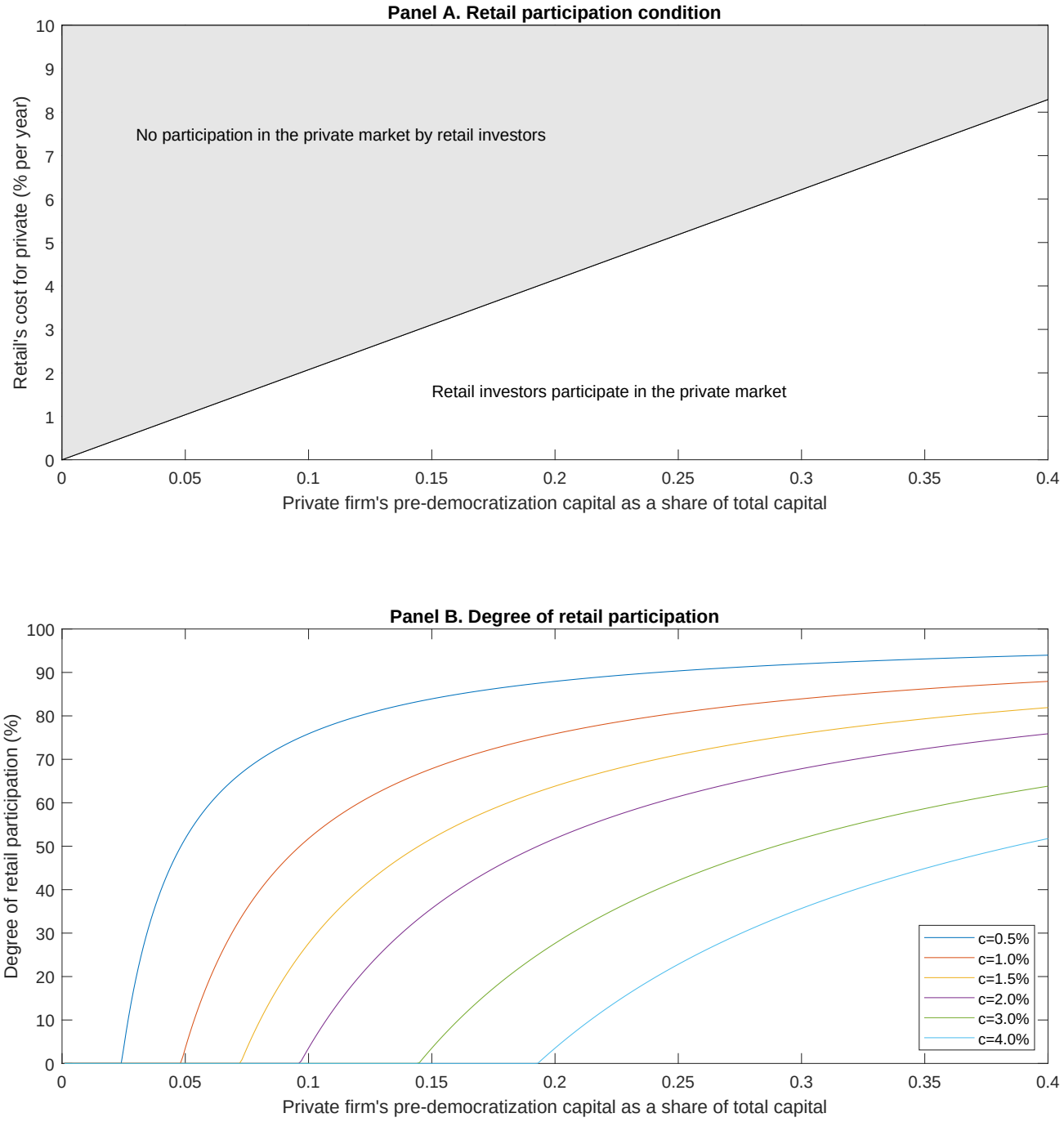


Figure 5. Participation by retail investors in the private market. The horizontal axis in both panels is the private firm's share of total public and private capital, $K_2^0/(K_1^0 + K_2^0)$. In Panel A, which illustrates the retail participation condition, the vertical axis is the proportional cost (c) charged to the retail investor on their private-firm holding. In Panel B, the vertical axis is the degree of retail participation (γ), plotted for various c values. Both panels treat firms' capital as fixed.

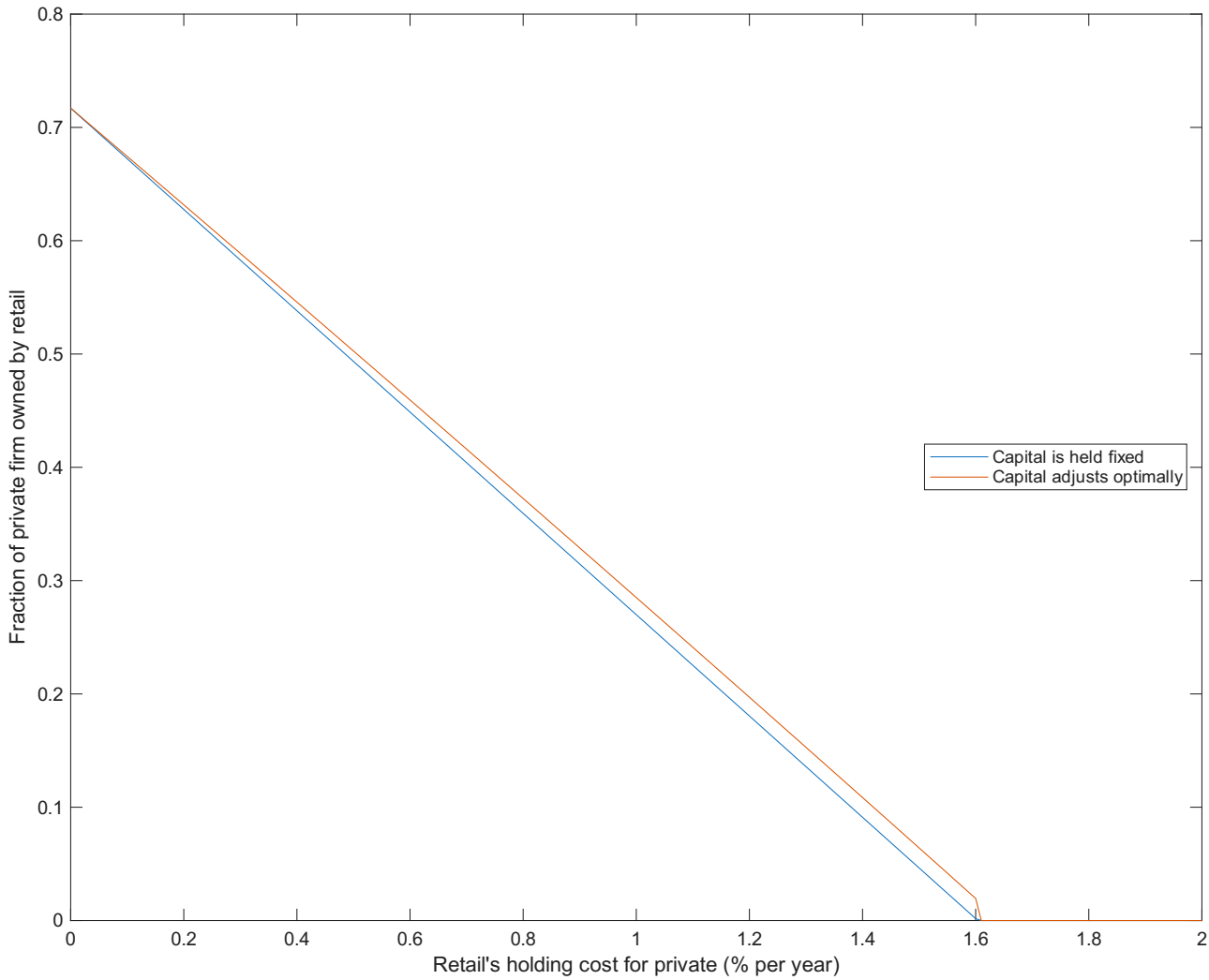


Figure 6. Effect of the holding cost on retail ownership of the private firm. The vertical axis is the fraction of the private firm owned post-democratization by the retail investor. The horizontal axis is the proportional cost (c) charged to the retail investor on their private-firm holding. Two scenarios are plotted: One sets firms' post-democratization capital equal to pre-democratization levels. The other allows post-democratization capital to adjust optimally.

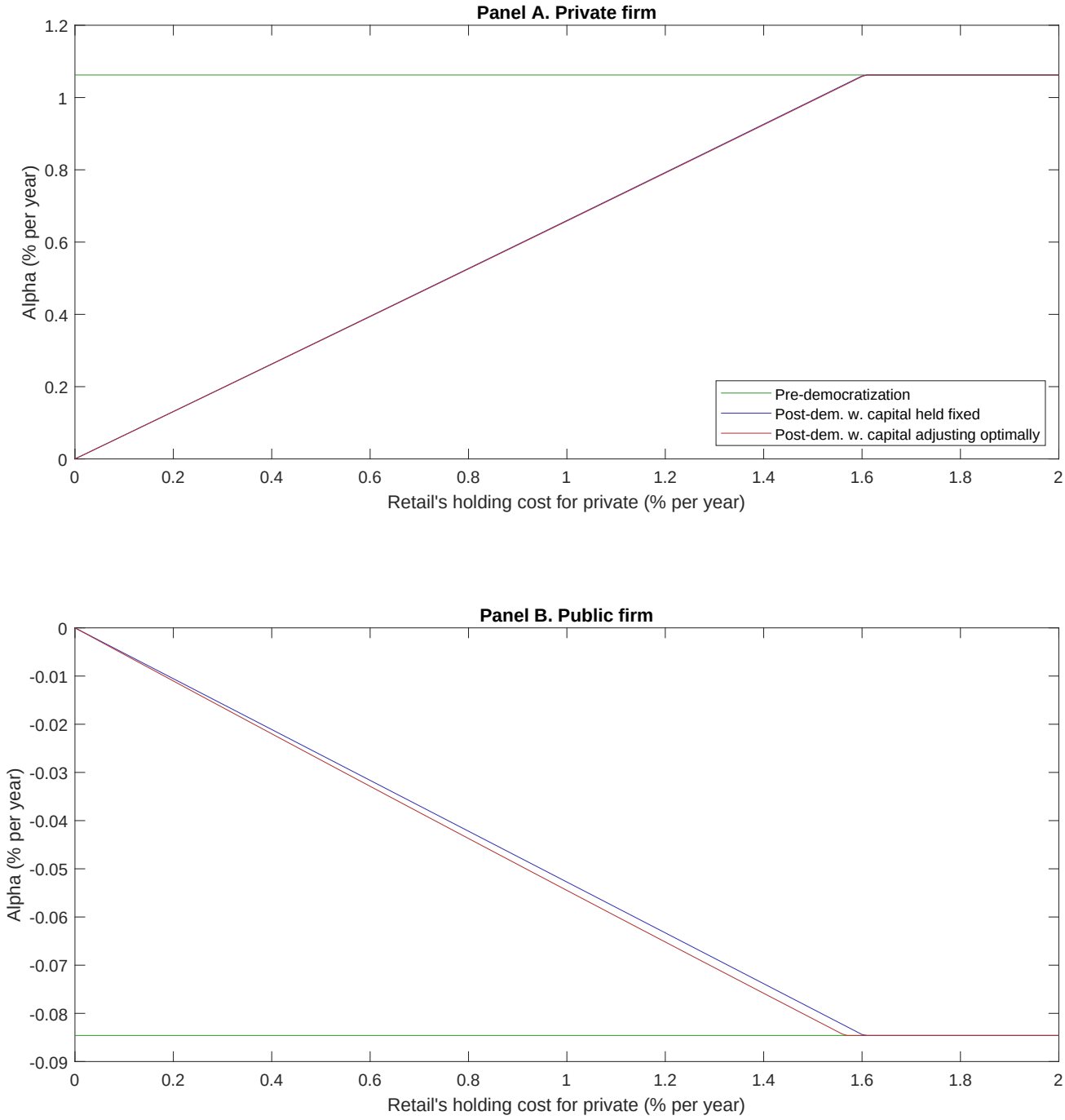


Figure 7. Effect of the retail holding cost on alphas. The vertical axis is the annual CAPM alpha (in percent). The horizontal axis is the proportional cost (c) charged to the retail investor on their private-firm holding. Three scenarios are plotted: (i) pre-democratization, (ii) post-democratization with firms' capital equal to pre-democratization levels, and (iii) post-democratization with firms' capital adjusting optimally. Panel A plots the alpha on the private firm's equity; Panel B plots the alpha on the public firm's equity.

Table 1
Calibration Criteria

This table lists the criteria that restrict the model's parameters in the calibrations. Panel A lists the criteria that directly restrict parameters of the model. Panel B lists criteria imposed on implied model quantities, implicitly restricting the model parameters.

Description	Quantity	Restriction
Panel A. Criteria that directly restrict parameters		
Institutions' wealth share	W_{0A}	$\in [0.219, 0.410]$
Retail holding cost of private	c	$\in [0, 0.02]$
Correlation of private w. public	ρ_{12}	$\in [0.5, 0.8]$
Private vs. public expected ROA difference	$\lvert\mu_1 - \mu_2\rvert$	≤ 0.10
Gross risk-free rate	R_f	$= 1.0425$
Panel B. Criteria that implicitly restrict parameters		
Public equity's share price	p_1^{pre}	$= 70.6/82.7$
Private equity's share price	p_2^{pre}	$= 5.6/82.7$
Public equity's return volatility	$\text{Stdev}(\tilde{r}_1^{\text{pre}})$	$= 0.2013$
Public equity's expected excess return	$\text{E}(\tilde{r}_1^{\text{pre}})$	$\in [0.04, 0.10]$
Private equity's return volatility	$\text{Stdev}(\tilde{r}_2^{\text{pre}})$	$\in [0.17, 0.27]$
Private equity's return beta	β_2^{pre}	$\in [0.5, 1.5]$
Investment response to the cost of capital	$\mathcal{E}$	$\in [-13.35, -1.05]$
Public firm's pre-dem. investment	I_1^{pre}	$= 0$
Private firm's pre-dem. investment	I_2^{pre}	$= 0$
Retail participation in private market	γ	> 0

Table 2
Parameter Values

For each of the model's parameters, this table summarizes the distribution of the parameter's values used in the calibrations.

Description	Parameter	Mean	Min	p5	p25	p50	p75	p95	Max
Institutions' wealth share	W_{0A}	0.302	0.219	0.219	0.251	0.283	0.346	0.410	0.410
Retail holding cost of private	c	0.006	0.000	0.000	0.003	0.007	0.010	0.017	0.020
Correlation of private w. public	ρ_{12}	0.646	0.500	0.500	0.550	0.650	0.750	0.800	0.800
Public firm's expected ROA	μ_1	1.191	1.122	1.123	1.165	1.188	1.231	1.255	1.257
Private firm's expected ROA	μ_2	1.131	1.072	1.086	1.106	1.129	1.156	1.184	1.212
Gross risk-free rate	R_f	1.043	1.043	1.043	1.043	1.043	1.043	1.043	1.043
Risk aversion	a	1.945	1.072	1.099	1.608	1.923	2.411	2.713	2.747
Public firm's ROA variance	v_{11}	0.046	0.044	0.044	0.045	0.046	0.048	0.049	0.049
Private firm's ROA variance	v_{22}	0.058	0.032	0.034	0.045	0.061	0.070	0.076	0.079
Investment-cost convexity	ϕ	1.843	0.110	0.141	0.235	0.419	1.046	4.264	957.03
Public firm's endowed capital	K_1^0	0.799	0.776	0.777	0.785	0.799	0.808	0.823	0.823
Private firm's endowed capital	K_2^0	0.067	0.065	0.066	0.066	0.067	0.067	0.067	0.067

Table 3
Effects of Democratization

This table summarizes post- versus pre-democratization differences between various model-implied quantities. For each quantity, the table reports the distribution of the implied post-pre change across all values of the parameter vector (s_n) satisfying the calibration criteria.

Description	Mean	Min	p5	p25	p50	p75	p95	Max
Panel A. When capital is held fixed ($K^{\text{post}} = K^{\text{pre}}$)								
Change in private-firm cost of capital (%)	−0.66	−3.84	−1.76	−0.92	−0.52	−0.26	−0.05	0.00
Change in private-firm market/book (%)	0.60	0.00	0.05	0.23	0.47	0.83	1.57	3.44
Post-dem. fraction of private held by retail	0.41	0.00	0.05	0.22	0.41	0.61	0.75	0.78
Retail's post-dem portfolio weight on private	0.04	0.00	0.00	0.02	0.04	0.06	0.07	0.07
Institutions' CE change (% of wealth)	−0.11	−0.68	−0.31	−0.15	−0.08	−0.04	−0.01	0.00
Retail investors' CE change (% of wealth)	0.01	0.00	0.00	0.00	0.00	0.01	0.02	0.03
Aggregate CE change (% of wealth)	−0.03	−0.12	−0.06	−0.04	−0.02	−0.01	0.00	0.00
Panel B. When capital adjusts optimally ($K^{\text{post}} \neq K^{\text{pre}}$)								
Percentage change in private-firm capital	4.76	0.00	0.19	1.07	2.97	6.54	15.48	51.22
Percentage change in public-firm capital	−0.05	−0.86	−0.23	−0.07	−0.02	0.00	0.00	0.00
Change in private-firm cost of capital (%)	−0.62	−3.78	−1.63	−0.86	−0.50	−0.25	−0.05	0.00
Change in private-firm market/book (%)	0.56	0.00	0.04	0.22	0.45	0.78	1.46	3.39
Post-dem. fraction of private held by retail	0.42	0.00	0.05	0.23	0.42	0.62	0.75	0.78
Retail's post-dem portfolio weight on private	0.04	0.00	0.01	0.02	0.04	0.07	0.08	0.11
Institutions' CE change (% of wealth)	−0.09	−0.66	−0.26	−0.13	−0.07	−0.04	−0.01	0.00
Retail investors' CE change (% of wealth)	0.02	0.00	0.00	0.01	0.01	0.03	0.07	0.24
Aggregate CE change (% of wealth)	−0.01	−0.10	−0.03	−0.02	−0.01	0.00	0.01	0.08

Table 4
Alphas for Firms and Investors

This table summarizes the distribution of model-implied CAPM alphas across all values of the parameter vector (s_n) satisfying the calibration criteria. Each panel shows alphas for the two firms (private and public) and two investor portfolios (retail and institutional). An investor's portfolio alpha is the weighted sum of portfolio weights times asset alphas; retail investors' portfolio alpha is net of the cost they pay to hold the private asset. All alphas are in percent per year.

Description	Mean	Min	p5	p25	p50	p75	p95	Max
Panel A. Pre-democratization								
Private firm	0.99	0.14	0.33	0.58	0.88	1.28	2.01	3.38
Public firm	−0.08	−0.27	−0.16	−0.10	−0.07	−0.05	−0.03	−0.01
Retail's portfolio	−0.07	−0.24	−0.14	−0.09	−0.06	−0.04	−0.02	−0.01
Institutions' portfolio	0.19	0.01	0.04	0.09	0.15	0.25	0.46	0.86
Panel B. Post-democratization, when capital is held fixed ($K^{\text{post}} = K^{\text{pre}}$)								
Private firm	0.40	0.00	0.00	0.18	0.36	0.65	1.14	1.46
Public firm	−0.03	−0.12	−0.09	−0.05	−0.03	−0.01	0.00	0.00
Retail's portfolio	−0.03	−0.12	−0.09	−0.05	−0.03	−0.01	0.00	0.00
Institutions' portfolio	0.05	0.00	0.00	0.00	0.02	0.07	0.19	0.37
Panel C. Post-democratization, when capital adjusts optimally ($K^{\text{post}} \neq K^{\text{pre}}$)								
Private firm	0.40	0.00	0.00	0.18	0.36	0.65	1.14	1.46
Public firm	−0.03	−0.14	−0.09	−0.05	−0.03	−0.01	0.00	0.00
Retail's portfolio	−0.04	−0.16	−0.09	−0.05	−0.03	−0.01	0.00	0.00
Institutions' portfolio	0.05	0.00	0.00	0.00	0.02	0.07	0.19	0.37

Table 5
Investor Welfare

This table summarizes the distribution of model-implied investor welfare across all values of the parameter vector (s_n) satisfying the calibration criteria. We tabulate investor i's certainty equivalent in excess of the risk-free rate, per unit of the investor's initial wealth: $\text{CE}_i - R_f = E(\widetilde{r}_i^p) - \frac{a}{2}\text{Var}(\widetilde{r}_i^p)$, where $\widetilde{r}_i^p$ is the investor's excess portfolio return and a is risk aversion. Retail investors' portfolio return is net of the cost they pay to hold the private asset. All values are in percentage points of wealth per year.

Description	Mean	Min	p5	p25	p50	p75	p95	Max
Panel A. Pre-democratization								
Retail	3.227	1.775	1.798	2.663	3.185	4.045	4.551	4.607
Institution	3.408	1.811	1.893	2.762	3.372	4.221	4.829	5.203
Panel B. Post-democratization, when capital is held fixed ($K^{\text{post}} = K^{\text{pre}}$)								
Retail	3.234	1.775	1.804	2.672	3.188	4.050	4.560	4.626
Institution	3.295	1.781	1.831	2.704	3.254	4.106	4.648	4.864
Panel C. Post-democratization, when capital adjusts optimally ($K^{\text{post}} \neq K^{\text{pre}}$)								
Retail	3.250	1.775	1.810	2.680	3.200	4.067	4.603	4.779
Institution	3.313	1.781	1.839	2.712	3.271	4.132	4.687	4.918

Table 6
Portfolio Return Correlations

This table summarizes the distribution of model-implied correlations between investors' excess portfolio returns across all values of the parameter vector (s_n) satisfying the calibration criteria. Each row reports the correlation between the two portfolio returns named, where pre and post refer to the pre- and post-democratization economies.

Correlation pair	Mean	Min	p5	p25	p50	p75	p95	Max
Panel A. Pre-economy								
Institution (pre), retail (pre)	0.974	0.929	0.945	0.966	0.977	0.984	0.990	0.994
Panel B. Post- (and pre-) economy, when capital is held fixed ($K^{\text{post}} = K^{\text{pre}}$)								
Institution (pre), institution (post)	0.994	0.958	0.980	0.992	0.996	0.999	1.000	1.000
Retail (pre), retail (post)	0.999	0.996	0.997	0.998	0.999	1.000	1.000	1.000
Institution (post), retail (post)	0.993	0.943	0.976	0.989	0.996	1.000	1.000	1.000
Panel C. Post- (and pre-) economy, when capital adjusts optimally ($K^{\text{post}} \neq K^{\text{pre}}$)								
Institution (pre), institution (post)	0.994	0.958	0.981	0.992	0.997	0.999	1.000	1.000
Retail (pre), retail (post)	0.999	0.992	0.997	0.998	0.999	1.000	1.000	1.000
Institution (post), retail (post)	0.993	0.943	0.976	0.989	0.996	1.000	1.000	1.000

References

Acharya, Viral V., and Zhaoxia Xu, 2017, Financial dependence and innovation: The case of public versus private firms, *Journal of Financial Economics* 124, 223–243.

Ang, Andrew, Dimitris Papanikolaou, and Mark M. Westerfield, 2014, Portfolio choice with illiquid assets, *Management Science* 60, 2737–2761.

Asker, John, Joan Farre-Mensa, and Alexander Ljungqvist, 2015, Corporate investment and stock market listing: A puzzle?, *Review of Financial Studies* 28, 342–390.

Balloch, Cynthia, Federico Mainardi, Sangmin Oh, and Petra Vokata, 2026, Democratizing private markets? Private equity performance of individual investors, Working paper.

Basak, Suleyman, and Domenico Cuoco, 1998, An equilibrium model with restricted stock market participation, *Review of Financial Studies* 11, 309–341.

Begenau, Juliane, and Emil N. Siriwardane, 2024, Fee variation in private equity, *Journal of Finance* 79, 1199–1247.

Bekaert, Geert, and Campbell R. Harvey, 1995, Time-varying world market integration, *Journal of Finance* 50, 403–444.

——— , 2000, Foreign speculators and emerging equity markets, *Journal of Finance* 55, 565–613.

——— , and Christian Lundblad, 2005, Does financial liberalization spur growth?, *Journal of Financial Economics* 77, 3–55.

Berk, Jonathan B., and Peter M. DeMarzo, 2026, A unified theory of delegated capital management, Working paper.

Berk, Jonathan B., and Richard C. Green, 2004, Mutual fund flows and performance in rational markets, *Journal of Political Economy* 112, 1269–1295.

Black, Fischer, 1974, International capital market equilibrium with investment barriers, *Journal of Financial Economics* 1, 337–352.

Cochrane, John H., 2005, The risk and return of venture capital, *Journal of Financial Economics* 75, 3–52.

Couts, Spencer J., Andrei S. Gonçalves, and Johnathan Loudis, 2026, The subjective risk and return expectations of institutional investors, Working paper.

Cummins, Jason G., Kevin A. Hassett, and R. Glenn Hubbard, 1996, Tax reforms and investment: A cross-country comparison, *Journal of Public Economics* 62, 237–273.

——— , Robert E. Hall, and Ricardo J. Caballero, 1994, A reconsideration of investment behavior using tax reforms as natural experiments, *Brookings Papers on Economic Activity* 1994, 1–59.

de Jong, Frank, and Frans A. de Roon, 2005, Time-varying market integration and expected returns in emerging markets, *Journal of Financial Economics* 78, 583–613.

Desai, Mihir A., and Austan D. Goolsbee, 2004, Investment, overhang, and tax policy, *Brookings Papers on Economic Activity* 2004, 285–338.

Edgerton, Jesse, 2010, Investment incentives and corporate tax asymmetries, *Journal of Public Economics* 94, 936–952.

Erel, Isil, Thomas Flanagan, and Michael S. Weisbach, 2026, Risk-adjusting the returns to private debt funds, Working paper.

Errunza, Vihang, Ked Hogan, and Mao-Wei Hung, 1999, Can the gains from international diversification be achieved without trading abroad?, *Journal of Finance* 54, 2075–2107.

Errunza, Vihang, and Etienne Losq, 1985, International asset pricing under mild segmentation: Theory and test, *Journal of Finance* 40, 105–124.

Errunza, Vihang R., and Darius P. Miller, 2000, Market segmentation and the cost of capital in international equity markets, *Journal of Financial and Quantitative Analysis* 35, 577–600.

Eun, Cheol S., and S. Janakiramanan, 1986, A model of international asset pricing with a constraint on the foreign equity ownership, *Journal of Finance* 41, 897–914.

Ewens, Michael, and Jacob Faber, 2026, Liquid claims on illiquid assets: The economics of retail access to private markets, Working paper.

Ewens, Michael, and Joan Farre-Mensa, 2020, The deregulation of the private equity markets and the decline in IPOs, *Review of Financial Studies* 33, 5463–5509.

——— , 2022, Private or public equity? The evolving entrepreneurial finance landscape, *Annual Review of Financial Economics* 14, 271–293.

Ewens, Michael, Charles M. Jones, and Matthew Rhodes-Kropf, 2013, The price of diversifiable risk in venture capital and private equity, *Review of Financial Studies* 26, 1854–1889.

Fang, Chuck, Itay Goldstein, and Yao Zeng, 2026, The fragility of semi-liquid private credit funds, Working paper.

Foerster, Stephen R., and G. Andrew Karolyi, 1999, The effects of market segmentation and investor recognition on asset prices: Evidence from foreign stocks listing in the United States, *Journal of Finance* 54, 981–1013.

Franzoni, Francesco, Eric Nowak, and Ludovic Phalippou, 2012, Private equity performance and liquidity risk, *Journal of Finance* 67, 2341–2373.

Gârleanu, Nicolae, Stavros Panageas, and Jianfeng Yu, 2015, Financial entanglement: A theory of incomplete integration, leverage, crashes, and contagion, *American Economic Review* 105, 1979–2010.

Gârleanu, Nicolae B., and Stavros Panageas, 2026, Finance in a time of disruptive growth, Working paper.

Gourier, Elise, Ludovic Phalippou, and Mark M. Westerfield, 2024, Capital commitment, *Journal of Finance* 79, 3407–3457.

Greenwood, Robin, Samuel G. Hanson, and Gordon Y. Liao, 2018, Asset price dynamics in partially segmented markets, *Review of Financial Studies* 31, 3307–3340.

Gupta, Nandini, and Kathy Yuan, 2009, On the growth effect of stock market liberalizations, *Review of Financial Studies* 22, 4715–4752.

Harris, Robert S., Tim Jenkinson, and Steven N. Kaplan, 2014, Private equity performance: What do we know?, *Journal of Finance* 69, 1851–1882.

Hassett, Kevin A., and R. Glenn Hubbard, 2002, Tax policy and business investment, in Alan J. Auerbach, and Martin Feldstein, ed.: *Handbook of Public Economics*, vol. 3 . chap. 20, pp. 1293–1343 (Elsevier: Amsterdam).

Heaton, John, and Deborah Lucas, 2000, Portfolio choice and asset prices: The importance of entrepreneurial risk, *Journal of Finance* 55, 1163–1198.

Henry, Peter Blair, 2000a, Do stock market liberalizations cause investment booms?, *Journal of Financial Economics* 58, 301–334.

——— , 2000b, Stock market liberalization, economic reform, and emerging market equity prices, *Journal of Finance* 55, 529–564.

Kaplan, Steven N., and Antoinette Schoar, 2005, Private equity performance: Returns, persistence, and capital flows, *Journal of Finance* 60, 1791–1823.

Kaplan, Steven N., and Berk A. Sensoy, 2015, Private equity performance: A survey, *Annual Review of Financial Economics* 7, 597–614.

Kaplan, Steven N., and Per Strömberg, 2009, Leveraged buyouts and private equity, *Journal of Economic Perspectives* 23, 121–46.

Korteweg, Arthur, 2019, Risk adjustment in private equity returns, *Annual Review of Financial Economics* 11, 131–152.

——— , and Stefan Nagel, 2025, Risk-adjusted returns of private equity funds: A new approach, *Review of Financial Studies* 38, 2557–2601.

Lerner, Josh, and Antoinette Schoar, 2004, The illiquidity puzzle: Theory and evidence from private equity, *Journal of Financial Economics* 72, 3–40.

Lindsey, Laura Anne, and Luke C. D. Stein, 2026, Angels, entrepreneurship, and employment dynamics: Evidence from investor accreditation rules, Working paper.

Lintner, John, 1965, The valuation of risk assets and the selection of risky investments in stock portfolios and capital budgets, *Review of Economics and Statistics* 47.

Merton, Robert C., 1987, A simple model of capital market equilibrium with incomplete information, *Journal of Finance* 42, 483–510.

Metrick, Andrew, and Ayako Yasuda, 2010, The economics of private equity funds, *The Review of Financial Studies* 23, 2303–2341.

Miller, Shane, Ran Xing, Emmanuel Yimfor, and Ye Zhang, 2026, Retail capital as a stepping stone in venture capital: Theory and empirics, Working paper.

Miller, Shane, Emmanuel Yimfor, and Ye Zhang, 2026, Investor expertise and private investment selection, Working paper.

Mitton, Todd, 2006, Stock market liberalization and operating performance at the firm level, *Journal of Financial Economics* 81, 625–647.

MSCI, 2025, The MSCI private equity factor model, Research paper, November 2025.

Pegoraro, Stefano, Sophie Shive, and Rafael Zambrana, 2026, Democratizing illiquid assets: Liquidity transformation and performance in interval funds, Working paper.

Phalippou, Ludovic, and Oliver Gottschalg, 2009, The performance of private equity funds, *Review of Financial Studies* 22, 1747–1776.

Phalippou, Ludovic, Christian Rauch, and Marc Umber, 2018, Private equity portfolio company fees, *Journal of Financial Economics* 129, 559–585.

Preqin, 2026, North american private markets in 2026, *Regional Insights*.

Robinson, David T., and Berk A. Sensoy, 2013, Do private equity fund managers earn their fees? compensation, ownership, and cash flow performance, *The Review of Financial Studies* 26, 2760–2797.

Robinson, David T., and Melanie Wallskog, 2026, The growth of private lending and retail access to alternative investments, Working paper.

Stulz, René M., 1981, On the effects of barriers to international investment, *Journal of Finance* 36, 923–934.

Zwick, Eric, and James Mahon, 2017, Tax policy and heterogeneous investment behavior, *American Economic Review* 107, 217–248.

Appendix

A.1. Proofs

Throughout the proofs we work with the payoff moments introduced in Section 2.1: $\mathrm{Var}(d_j) = K_j^2 v_{jj}$, $\mathrm{Cov}(d_1, d_2) = K_1 K_2 v_{12}$, and unspanned private risk $\mathrm{Var}(d_2 \mid d_1) = K_2^2 v_{2|1}$ from (4). We also abbreviate the expected excess payoff on firm j's equity as

$$z_j \;\equiv\; \mathrm{E}(d_j) - R_f p_j \;=\; \mu_j K_j - R_f p_j \,. \tag{A.1}$$

Within each regime we suppress the superscript k, writing K_j, p_j, and δ for K_j^k, p_j^k, and δ^k.

Proof of Proposition 1

First, we express the institution's objective function in terms of the institution's holdings. Substituting the budget constraint (7) into (8) eliminates b_A and yields the excess portfolio return (11), which we can write as

$$\widetilde{r}_A^{\,p} = \frac{\theta_{A1}\,(d_1 - R_f p_1) + \theta_{A2}\,(d_2 - R_f p_2)}{W_{0A}} \,. \tag{A.2}$$

We now compute the mean and variance of this return. From (A.1), the mean is

$$\mathrm{E}(\widetilde{r}_A^{\,p}) = \frac{\theta_{A1} z_1 + \theta_{A2} z_2}{W_{0A}} \,. \tag{A.3}$$

The prices p_1, p_2 and the rate R_f are known at $t = 0$, so the variance is

$$\mathrm{Var}(\widetilde{r}_A^{\,p}) = \frac{\mathrm{Var}(\theta_{A1} d_1 + \theta_{A2} d_2)}{W_{0A}^2} = \frac{\theta_{A1}^2 \,\mathrm{Var}(d_1) + 2\,\theta_{A1}\theta_{A2}\,\mathrm{Cov}(d_1, d_2) + \theta_{A2}^2 \,\mathrm{Var}(d_2)}{W_{0A}^2} \,. \tag{A.4}$$

Substituting (A.3) and (A.4) into (10), the institution chooses $(\theta_{A1}, \theta_{A2})$ to maximize

$$G_A(\theta_{A1}, \theta_{A2}) \;\equiv\; \frac{\theta_{A1} z_1 + \theta_{A2} z_2}{W_{0A}} \;-\; \frac{a}{2 W_{0A}^2} \Big[\theta_{A1}^2 \,\mathrm{Var}(d_1) + 2\,\theta_{A1}\theta_{A2}\,\mathrm{Cov}(d_1, d_2) + \theta_{A2}^2 \,\mathrm{Var}(d_2)\Big] \,. \tag{A.5}$$

Following the same steps for the retail investor, equations (7) and (9) give

$$\widetilde{r}_B^{\,p} = \frac{\theta_{B1}\,(d_1 - R_f p_1) + \theta_{B2}\,(d_2 - R_f p_2) - c\,\theta_{B2} K_2}{W_{0B}} \,. \tag{A.6}$$

The holding cost $c\,\theta_{B2} K_2$ is deterministic. It therefore enters the mean,

$$\mathrm{E}(\widetilde{r}_B^{\,p}) = \frac{\theta_{B1} z_1 + \theta_{B2}\,(z_2 - c K_2)}{W_{0B}} \,, \tag{A.7}$$

but not the variance, which is (A.4) with A replaced by B throughout. Hence the retail investor's objective G_B is (A.5) with θ_A replaced by θ_B, with W_{0A} replaced by W_{0B}, and

with z_2 replaced by $z_2 - cK_2$. The cost lowers the retail investor's excess payoff and changes nothing else. This is the only difference between the two investors' problems.

Having established the objective functions, we now turn to the first-order conditions. For the institution, we differentiate (A.5) with respect to θ_{A1} and set the result to zero,

$$\frac{z_1}{W_{0A}} - \frac{a}{2W_{0A}^2}\Big[2\,\theta_{A1}\,\mathrm{Var}(d_1) + 2\,\theta_{A2}\,\mathrm{Cov}(d_1, d_2)\Big] = 0\,, \tag{A.8}$$

which gives the first condition below. Differentiating with respect to θ_{A2} gives the second:

$$z_1 = \frac{a}{W_{0A}}\Big[\theta_{A1}\mathrm{Var}(d_1) + \theta_{A2}\mathrm{Cov}(d_1, d_2)\Big] \tag{A.9}$$

$$z_2 = \frac{a}{W_{0A}}\Big[\theta_{A1}\mathrm{Cov}(d_1, d_2) + \theta_{A2}\mathrm{Var}(d_2)\Big]\,. \tag{A.10}$$

For the retail investor, the first-order conditions are the same two equations with B in place of A and with $z_2 - cK_2$ in place of z_2:

$$z_1 = \frac{a}{W_{0B}}\Big[\theta_{B1}\mathrm{Var}(d_1) + \theta_{B2}\mathrm{Cov}(d_1, d_2)\Big] \tag{A.11}$$

$$z_2 - cK_2 = \frac{a}{W_{0B}}\Big[\theta_{B1}\mathrm{Cov}(d_1, d_2) + \theta_{B2}\mathrm{Var}(d_2)\Big]\,. \tag{A.12}$$

Each bracketed expression equals $\mathrm{Cov}(d_j,\, \theta_{i1}d_1 + \theta_{i2}d_2)$, the covariance between the payoff of firm j's share and the payoff of investor i's portfolio. Every condition therefore equates the expected excess payoff on a share to a/W_{0i} times that share's marginal contribution to the investor's portfolio risk.

Each objective is strictly concave, because its matrix of second derivatives is $-a/W_{0i}^2$ times the covariance matrix of (d_1, d_2), and $\mathrm{Var}(\theta_1 d_1 + \theta_2 d_2) > 0$ for every $(\theta_1, \theta_2) \neq (0, 0)$ by $v \succ 0$ in (1) and $K_j > 0$. Each investor's optimum is therefore unique, and the conditions above are sufficient as well as necessary. Two qualifications apply to the retail investor. Pre-democratization, $\theta_{B2} = 0$ is imposed, so only (A.11) applies. Post-democratization the constraint is $\theta_{B2} \geq 0$, so the optimum either satisfies both (A.11)–(A.12) or sets $\theta_{B2} = 0$. We handle this case at the end of the proof.

Note that the four conditions (A.9)–(A.12) involve prices only through z_1 and z_2. The remaining steps eliminate these two unknowns rather than solve for them, so holdings can be obtained without computing prices.

Measure the institution's holdings as deviations from the frictionless allocation W_{0A}:

$$x_j \;\equiv\; \theta_{Aj} - W_{0A}\,, \qquad j \in \{1, 2\}\,. \tag{A.13}$$

Market clearing (13)–(14) then gives $\theta_{Bj} = 1 - \theta_{Aj}$. Substituting from (6), we obtain

$$\theta_{Bj} = (W_{0A} + W_{0B}) - (W_{0A} + x_j) = W_{0B} - x_j\,. \tag{A.14}$$

The single number x_j thus records both the institution's overweight of firm j and the retail's underweight of it. The four unknown holdings reduce to the two unknowns x_1 and x_2.

Equating the right-hand sides of (A.9) and (A.11),

$$\frac{\theta_{A1}\mathrm{Var}(d_1) + \theta_{A2}\mathrm{Cov}(d_1, d_2)}{W_{0A}} = \frac{\theta_{B1}\mathrm{Var}(d_1) + \theta_{B2}\mathrm{Cov}(d_1, d_2)}{W_{0B}}\,. \tag{A.15}$$

Substituting $\theta_{Aj} = W_{0A} + x_j$ and $\theta_{Bj} = W_{0B} - x_j$ from (A.13) and (A.14) and rearranging,

$$\begin{aligned} \text{left-hand side} &= \frac{W_{0A}\big[\mathrm{Var}(d_1) + \mathrm{Cov}(d_1, d_2)\big] + \big[x_1\mathrm{Var}(d_1) + x_2\mathrm{Cov}(d_1, d_2)\big]}{W_{0A}} \\ &= \mathrm{Var}(d_1) + \mathrm{Cov}(d_1, d_2) + \frac{x_1\mathrm{Var}(d_1) + x_2\mathrm{Cov}(d_1, d_2)}{W_{0A}} \qquad \text{(A.16)} \\ \text{right-hand side} &= \frac{W_{0B}\big[\mathrm{Var}(d_1) + \mathrm{Cov}(d_1, d_2)\big] - \big[x_1\mathrm{Var}(d_1) + x_2\mathrm{Cov}(d_1, d_2)\big]}{W_{0B}} \\ &= \mathrm{Var}(d_1) + \mathrm{Cov}(d_1, d_2) - \frac{x_1\mathrm{Var}(d_1) + x_2\mathrm{Cov}(d_1, d_2)}{W_{0B}}\,. \qquad \text{(A.17)} \end{aligned}$$

The term $\mathrm{Var}(d_1) + \mathrm{Cov}(d_1, d_2)$ appears on both sides and cancels, leaving

$$\Big[x_1\mathrm{Var}(d_1) + x_2\,\mathrm{Cov}(d_1, d_2)\Big]\left(\frac{1}{W_{0A}} + \frac{1}{W_{0B}}\right) = 0\,. \tag{A.18}$$

Both wealth shares are positive, so the expression inside the bracket must be zero. Dividing it by $\mathrm{Var}(d_1) > 0$ and using $\delta = \mathrm{Cov}(d_1, d_2)/\mathrm{Var}(d_1)$ from (23),

$$x_1 = -\,\delta\, x_2\,. \tag{A.19}$$

The institution's public-equity underweight is δ times its private-equity overweight. Equation (A.19) holds in both economies, because pre-democratization the retail investor still chooses θ_{B1} freely, so (A.11) continues to apply, with $\theta_{B2} = 0$.

Pre-democratization, $\theta_{B2} = 0$, so market clearing (14) gives $\theta_{A2} = 1$, and (A.13) gives

$$x_2 = 1 - W_{0A} = W_{0B}\,, \tag{A.20}$$

using (6). Substituting into (A.19) gives $x_1 = -\delta W_{0B}$, so $\theta_{A1} = W_{0A} - \delta W_{0B}$ and, by (A.14), $\theta_{B1} = W_{0B} + \delta W_{0B}$. Together with $\theta_{A2} = 1$ and $\theta_{B2} = 0$, these are (15)–(18). This establishes the first half of the proposition.

Post-democratization, we first consider the case when $\theta_{B2} > 0$. Now (A.12) applies as well. Solving (A.10) and (A.12) for z_2 and equating the results,

$$\frac{a}{W_{0A}}\Big[\theta_{A1}\mathrm{Cov}(d_1, d_2) + \theta_{A2}\mathrm{Var}(d_2)\Big] = cK_2 + \frac{a}{W_{0B}}\Big[\theta_{B1}\mathrm{Cov}(d_1, d_2) + \theta_{B2}\mathrm{Var}(d_2)\Big]\,. \tag{A.21}$$

Substituting the deviations and grouping exactly as in (A.16)–(A.17), with $\mathrm{Cov}(d_1, d_2)$ and $\mathrm{Var}(d_2)$ now in the roles played there by $\mathrm{Var}(d_1)$ and $\mathrm{Cov}(d_1, d_2)$, the two sides become

$$\text{left-hand side} = a\Big[\mathrm{Cov}(d_1, d_2) + \mathrm{Var}(d_2)\Big] + \frac{a}{W_{0A}}\Big[x_1\mathrm{Cov}(d_1, d_2) + x_2\mathrm{Var}(d_2)\Big] \tag{A.22}$$

$$\text{right-hand side} = cK_2 + a\Big[\mathrm{Cov}(d_1, d_2) + \mathrm{Var}(d_2)\Big] - \frac{a}{W_{0B}}\Big[x_1\mathrm{Cov}(d_1, d_2) + x_2\mathrm{Var}(d_2)\Big]\,. \tag{A.23}$$

The term $a[\mathrm{Cov}(d_1, d_2) + \mathrm{Var}(d_2)]$ cancels, and collecting the deviation terms leaves

$$a\Big[x_1\mathrm{Cov}(d_1, d_2) + x_2\mathrm{Var}(d_2)\Big]\left(\frac{1}{W_{0A}} + \frac{1}{W_{0B}}\right) = cK_2\,. \tag{A.24}$$

The second factor equals $(W_{0A} + W_{0B})/(W_{0A}W_{0B}) = 1/(W_{0A}W_{0B})$, so (A.24) becomes

$$x_1\,\mathrm{Cov}(d_1, d_2) + x_2\,\mathrm{Var}(d_2) = \frac{cK_2\,W_{0A}W_{0B}}{a}\,. \tag{A.25}$$

This is one equation in the two unknowns x_1 and x_2; (A.19) is the other. Eliminating x_1 by substituting $x_1 = -\delta x_2 = -x_2\,\mathrm{Cov}(d_1, d_2)/\mathrm{Var}(d_1)$,

$$x_2\left[\mathrm{Var}(d_2) - \frac{\mathrm{Cov}(d_1, d_2)^2}{\mathrm{Var}(d_1)}\right] = \frac{cK_2\,W_{0A}W_{0B}}{a}\,. \tag{A.26}$$

The bracket is unspanned private risk. Substituting the payoff moments from Section 2.1,

$$\mathrm{Var}(d_2) - \frac{\mathrm{Cov}(d_1, d_2)^2}{\mathrm{Var}(d_1)} = K_2^2 v_{22} - \frac{(K_1K_2v_{12})^2}{K_1^2v_{11}} = K_2^2\left(v_{22} - \frac{v_{12}^2}{v_{11}}\right) = K_2^2\,v_{2|1} = \mathrm{Var}(d_2 \mid d_1)\,, \tag{A.27}$$

where the last two equalities are (4). Dividing (A.26) by $K_2^2 v_{2|1} > 0$,

$$x_2 = \frac{c\,W_{0A}W_{0B}}{a\,v_{2|1}K_2} = (1-\gamma)\,W_{0B}\,, \tag{A.28}$$

where the second equality uses the definition of γ in (24) on the branch where $\gamma > 0$. Substituting (A.28) into (A.19) gives $x_1 = -(1-\gamma)\delta W_{0B}$, and then (A.13) and (A.14) deliver (19)–(22).

Finally, if the post-democratization optimum has $\theta_{B2} = 0$, the holdings are the pre-democratization ones, (15)–(18). What remains is to determine when this case arises, since (24) claims it arises when $1 - cW_{0A}/(av_{2|1}K_2^{\mathrm{post}}) \le 0$. The relevant object is already in hand. At any holdings satisfying market clearing and the institution's condition (A.10), the latter determines z_2. Substituting it into retail's derivative with respect to θ_{B2} shows that $W_{0B}\,\partial G_B/\partial\theta_{B2}$ equals the difference between the two sides of (A.21). That difference is computed in (A.22)–(A.23) and simplified in (A.25):

$$W_{0B}\,\frac{\partial G_B}{\partial\theta_{B2}} = \frac{a\Big[x_1\,\mathrm{Cov}(d_1, d_2) + x_2\,\mathrm{Var}(d_2)\Big]}{W_{0A}W_{0B}} - cK_2\,. \tag{A.29}$$

Evaluating this expression at $x_2 = W_{0B}$ and $x_1 = -\delta W_{0B}$, its numerator becomes

$$x_1 \operatorname{Cov}(d_1, d_2) + x_2 \operatorname{Var}(d_2) = W_{0B}\Big[\operatorname{Var}(d_2) - \delta \operatorname{Cov}(d_1, d_2)\Big] = W_{0B} \operatorname{Var}(d_2 \mid d_1) = W_{0B}\, K_2^2\, v_{2|1}\,, \tag{A.30}$$

where the second equality substitutes $\delta = \operatorname{Cov}(d_1, d_2)/\operatorname{Var}(d_1)$ and applies (A.27). Substituting (A.30) into (A.29), dividing by W_{0B}, and factoring out $aK_2^2 v_{2|1}/W_{0A}$,

$$\frac{\partial G_B}{\partial \theta_{B2}} = \frac{a\, K_2^2\, v_{2|1}}{W_{0A} W_{0B}} \left[1 - \frac{c\, W_{0A}}{a\, v_{2|1} K_2}\right]. \tag{A.31}$$

The bracket in (A.31) is the second argument of the maximum in (24), and the factor multiplying it is strictly positive. The retail investor's marginal valuation of the first unit of private equity therefore carries the sign of that argument. It is nonpositive exactly when the argument is, so the corner allocation is an equilibrium precisely on the branch where $\gamma = 0$; and it is strictly positive otherwise, so the corner is not an equilibrium when $\gamma > 0$. The two branches thus partition the parameter space, which is what the truncation in (24) asserts. Setting $\gamma = 0$ in (19)–(22) reproduces (15)–(18), so those four equations describe the post-democratization equilibrium on both branches. ■

Proof of Corollary 1

From equation (22), the retail investor's post-democratization private holding is $\theta_{B2}^{\text{post}} = \gamma W_{0B}$. Since $W_{0B} > 0$, we have $\theta_{B2}^{\text{post}} > 0$ if and only if $\gamma > 0$, which is condition (26). From equation (24), it follows immediately that

$$\gamma > 0 \quad \Longleftrightarrow \quad 1 - \frac{cW_{0A}}{a\, v_{2|1} K_2^{\text{post}}} > 0 \quad \Longleftrightarrow \quad c < \frac{aK_2^{\text{post}} v_{2|1}}{W_{0A}}\,, \tag{A.32}$$

which is condition (28). In addition, when this condition fails, $\gamma = 0$ and $\theta_{B2}^{\text{post}} = 0$. ■

Proof of Corollary 2

With $K^{\text{post}} = K^{\text{pre}} = K$, definition (23) depends on the regime only through K, so $\delta^{\text{pre}} = \delta^{\text{post}} = \delta$, and γ in (24) is evaluated at the common K_2. Proposition 1 then gives the eight holdings in terms of the two numbers δ and γ.

We first prove the claim that the post-democratization holdings are a convex combination of the pre-democratization holdings and the frictionless allocations. Taking each of the four holdings in turn and using $W_{0A} + W_{0B} = 1$ in the third line,

$$(1-\gamma)\big(W_{0A} - \delta W_{0B}\big) + \gamma W_{0A} = W_{0A} - (1-\gamma)\delta W_{0B} = \theta_{A1}^{\text{post}} \tag{A.33}$$

$$(1-\gamma)\big(W_{0B} + \delta W_{0B}\big) + \gamma W_{0B} = W_{0B} + (1-\gamma)\delta W_{0B} = \theta_{B1}^{\text{post}} \tag{A.34}$$

$$(1-\gamma)\cdot 1 + \gamma W_{0A} = W_{0A} + (1-\gamma) W_{0B} = \theta_{A2}^{\text{post}} \tag{A.35}$$

$$(1-\gamma)\cdot 0 + \gamma W_{0B} = \gamma W_{0B} = \theta_{B2}^{\text{post}}\,, \tag{A.36}$$

where the left-hand sides are $(1-\gamma)$ times the pre-democratization holdings (15)–(18) plus γ times the frictionless allocations, and the right-hand sides are the post-democratization holdings (19)–(22). Since $\gamma \in [0,1]$, this is a convex combination.

Subtracting equations (15)–(18) from equations (19)–(22) term by term,

$$\theta_{A1}^{\text{post}} - \theta_{A1}^{\text{pre}} = \big[W_{0A} - (1-\gamma)\delta W_{0B}\big] - \big[W_{0A} - \delta W_{0B}\big] = \gamma\delta W_{0B} \tag{A.37}$$
$$\theta_{A2}^{\text{post}} - \theta_{A2}^{\text{pre}} = \big[W_{0A} + (1-\gamma)W_{0B}\big] - 1 = -\gamma W_{0B} \tag{A.38}$$
$$\theta_{B1}^{\text{post}} - \theta_{B1}^{\text{pre}} = \big[W_{0B} + (1-\gamma)\delta W_{0B}\big] - \big[W_{0B} + \delta W_{0B}\big] = -\gamma\delta W_{0B} \tag{A.39}$$
$$\theta_{B2}^{\text{post}} - \theta_{B2}^{\text{pre}} = \gamma W_{0B} - 0 = \gamma W_{0B}\,, \tag{A.40}$$

where (A.38) uses $W_{0A} + W_{0B} = 1$. These are equations (29)–(32). ■

Lemma 1. *Assume the retail-participation condition* (28) *holds. Then the equilibrium expected excess payoffs* (A.1) *are*

$$z_1^k = a\,\mathrm{Cov}\big(d_1^k, d_M^k\big)\,, \qquad k \in \{\text{pre}, \text{post}\} \tag{A.41}$$
$$z_2^{\text{pre}} = a\,\mathrm{Cov}(d_2^{\text{pre}}, d_M^{\text{pre}}) + a\,\frac{W_{0B}}{W_{0A}}\,\mathrm{Var}(d_2^{\text{pre}} \mid d_1^{\text{pre}}) \tag{A.42}$$
$$z_2^{\text{post}} = a\,\mathrm{Cov}\big(d_2^{\text{post}}, d_M^{\text{post}}\big) + c\,W_{0B}K_2^{\text{post}}\,, \tag{A.43}$$

and the equilibrium share prices are

$$p_1^k = \frac{K_1^k}{R_f}\Big[\mu_1 - a\left(K_1^k v_{11} + K_2^k v_{12}\right)\Big], \qquad k \in \{\text{pre}, \text{post}\} \tag{A.44}$$
$$p_2^{\text{pre}} = \frac{K_2^{\text{pre}}}{R_f}\Big[\mu_2 - a\,(K_1^{\text{pre}} v_{12} + K_2^{\text{pre}} v_{22}) - a\,\frac{W_{0B}}{W_{0A}} K_2^{\text{pre}} v_{2|1}\Big] \tag{A.45}$$
$$p_2^{\text{post}} = \frac{K_2^{\text{post}}}{R_f}\Big[\mu_2 - a\left(K_1^{\text{post}} v_{12} + K_2^{\text{post}} v_{22}\right) - c\,W_{0B}\Big]\,. \tag{A.46}$$

The market price $p_M^k = p_1^k + p_2^k$ satisfies

$$R_f\,p_M^{\text{pre}} = \mathrm{E}(d_M^{\text{pre}}) - a\,\mathrm{Var}(d_M \mid K^{\text{pre}}) - a\,\frac{W_{0B}}{W_{0A}}\,\mathrm{Var}(d_2^{\text{pre}} \mid d_1^{\text{pre}}) \tag{A.47}$$
$$R_f\,p_M^{\text{post}} = \mathrm{E}\big(d_M^{\text{post}}\big) - a\,\mathrm{Var}\big(d_M \mid K^{\text{post}}\big) - c\,W_{0B}K_2^{\text{post}}\,, \tag{A.48}$$

where $\mathrm{E}(d_M^k) = \mu_1 K_1^k + \mu_2 K_2^k$.

Proof of Lemma 1

The institution faces no constraint, so (A.9) and (A.10) hold in both economies. The optimal holdings from Proposition 1 determine z_1 and z_2, and (A.1) inverts to

$$p_j^k = \frac{\mu_j K_j^k - z_j^k}{R_f}\,. \tag{A.49}$$

Pre-democratization, we substitute $\theta_{A1} = W_{0A} - \delta W_{0B}$ and $\theta_{A2} = 1$ into (A.9) and use $\delta \operatorname{Var}(d_1) = \operatorname{Cov}(d_1, d_2)$ from (23):

$$\begin{aligned} z_1^{\text{pre}} &= a\left[\operatorname{Var}(d_1) - \frac{\delta W_{0B} \operatorname{Var}(d_1)}{W_{0A}} + \frac{\operatorname{Cov}(d_1, d_2)}{W_{0A}}\right] \\ &= a\left[\operatorname{Var}(d_1) + \frac{\operatorname{Cov}(d_1, d_2)\,(1 - W_{0B})}{W_{0A}}\right] = a\Big[\operatorname{Var}(d_1) + \operatorname{Cov}(d_1, d_2)\Big] \\ &= a \operatorname{Cov}(d_1, d_M)\ , \end{aligned} \tag{A.50}$$

where the third equality uses $1 - W_{0B} = W_{0A}$ from (6) and the last uses $d_M = d_1 + d_2$ from (34). The same substitution in (A.10) gives

$$\begin{aligned} z_2^{\text{pre}} &= a\left[\operatorname{Cov}(d_1, d_2) - \frac{\delta W_{0B} \operatorname{Cov}(d_1, d_2)}{W_{0A}} + \frac{\operatorname{Var}(d_2)}{W_{0A}}\right] \\ &= a\Big[\operatorname{Cov}(d_1, d_2) + \operatorname{Var}(d_2)\Big] + a\,\frac{W_{0B}}{W_{0A}}\Big[\operatorname{Var}(d_2) - \delta \operatorname{Cov}(d_1, d_2)\Big] \\ &= a \operatorname{Cov}(d_2, d_M) + a\,\frac{W_{0B}}{W_{0A}} \operatorname{Var}(d_2 \mid d_1)\,, \end{aligned} \tag{A.51}$$

where we used $\operatorname{Var}(d_2)/W_{0A} - \operatorname{Var}(d_2) = \operatorname{Var}(d_2) W_{0B}/W_{0A}$, and (A.27). Equations (A.50) and (A.51) are (A.41) and (A.42)—the first two results of the lemma.

Post-democratization, we substitute $\theta_{A1} = W_{0A} - (1-\gamma)\delta W_{0B}$ and $\theta_{A2} = W_{0A} + (1-\gamma) W_{0B}$ into (A.9) and group the $(1-\gamma)$ terms,

$$\begin{aligned} z_1^{\text{post}} &= a\Big[\operatorname{Var}(d_1) + \operatorname{Cov}(d_1, d_2)\Big] + a\,(1-\gamma)\,\frac{W_{0B}}{W_{0A}}\Big[\operatorname{Cov}(d_1, d_2) - \delta \operatorname{Var}(d_1)\Big] \\ &= a \operatorname{Cov}(d_1, d_M)\,, \end{aligned} \tag{A.52}$$

because the second bracket vanishes by (23). This is the post-democratization version of (A.41). The public asset's expected excess payoff is thus the same function of capital in both regimes, whatever the value of γ. The same substitution in (A.10) gives

$$\begin{aligned} z_2^{\text{post}} &= a\Big[\operatorname{Cov}(d_1, d_2) + \operatorname{Var}(d_2)\Big] + a\,(1-\gamma)\,\frac{W_{0B}}{W_{0A}}\Big[\operatorname{Var}(d_2) - \delta \operatorname{Cov}(d_1, d_2)\Big] \\ &= a \operatorname{Cov}(d_2, d_M) + a\,(1-\gamma)\,\frac{W_{0B}}{W_{0A}} \operatorname{Var}(d_2 \mid d_1)\,, \end{aligned} \tag{A.53}$$

which differs from (A.42) only in the factor $1 - \gamma$. Condition (28) places us on the branch $\gamma > 0$, where (24) gives $1 - \gamma = cW_{0A}/(a v_{2|1} K_2^{\text{post}})$. Substituting that expression and $\operatorname{Var}(d_2 \mid d_1) = (K_2^{\text{post}})^2 v_{2|1}$ into (A.53),

$$a\,(1-\gamma)\,\frac{W_{0B}}{W_{0A}} \operatorname{Var}(d_2 \mid d_1) = a\,\frac{c\,W_{0A}}{a\,v_{2|1} K_2^{\text{post}}}\,\frac{W_{0B}}{W_{0A}}\left(K_2^{\text{post}}\right)^2 v_{2|1} = c\,W_{0B} K_2^{\text{post}}\,, \tag{A.54}$$

which gives (A.43).

Next, we solve for asset prices. Substituting (A.41)–(A.43) into (A.49) and expanding the covariances as $\mathrm{Cov}(d_1, d_M) = K_1(K_1 v_{11} + K_2 v_{12})$ and $\mathrm{Cov}(d_2, d_M) = K_2(K_1 v_{12} + K_2 v_{22})$, obtained from (2) and $d_M = d_1 + d_2$, yields (A.44)–(A.46) after factoring K_j^k out of each bracket. In (A.45) the factored form uses $\mathrm{Var}(d_2^{\mathrm{pre}} \mid d_1^{\mathrm{pre}}) = (K_2^{\mathrm{pre}})^2 v_{2|1}$.

For the market price, add the two expected excess payoffs. Since $d_M = d_1 + d_2$,

$$\mathrm{Cov}(d_1, d_M) + \mathrm{Cov}(d_2, d_M) = \mathrm{Var}(d_M)\,, \tag{A.55}$$

so (A.41)–(A.43) give $z_1^k + z_2^k = a\,\mathrm{Var}(d_M \mid K^k)$ plus the regime's wedge. Substituting into (A.49) summed over j delivers (A.47)–(A.48). ■

Proof of Proposition 2

Rewriting equations (A.44) through (A.46) in Lemma 1, making use of equations (27) and (47) through (49) in the text, gives equations (35) through (39) in Proposition 2. ■

Lemma 2. *The expected excess return of asset j in regime k is*

$$\mathrm{E}\left(\widetilde{r}_j^k\right) = R_f \frac{q_j^k}{\mu_j - q_j^k}\,, \tag{A.56}$$

where $q_j^k \equiv (\mu_j K_j^k - R_f p_j^k)/K_j^k = z_j^k/K_j^k$ is the expected excess payoff on firm j's equity per unit of capital, $\mu_j > q_j^k$,

$$q_1^{\mathrm{pre}} = \frac{a}{K_1^{\mathrm{pre}}}\,\mathrm{Cov}(d_1^{\mathrm{pre}}, d_M^{\mathrm{pre}}) \tag{A.57}$$

$$q_2^{\mathrm{pre}} = \frac{a}{K_2^{\mathrm{pre}}}\,\mathrm{Cov}(d_2^{\mathrm{pre}}, d_M^{\mathrm{pre}}) + a\frac{W_{0B}}{W_{0A}} K_2^{\mathrm{pre}} v_{2|1} \tag{A.58}$$

$$q_1^{\mathrm{post}} = \frac{a}{K_1^{\mathrm{post}}}\,\mathrm{Cov}(d_1^{\mathrm{post}}, d_M^{\mathrm{post}}) \tag{A.59}$$

$$q_2^{\mathrm{post}} = \frac{a}{K_2^{\mathrm{post}}}\,\mathrm{Cov}(d_2^{\mathrm{post}}, d_M^{\mathrm{post}}) + cW_{0B}\,, \tag{A.60}$$

and $\mathrm{E}\left(\widetilde{r}_j^k\right)$ is increasing in q_j^k.

Proof of Lemma 2

The definition of q_j^k in Lemma 2 rearranges to

$$R_f\, p_j^k = K_j^k \left(\mu_j - q_j^k\right)\,. \tag{A.61}$$

Taking expectations in (33), using $\mathrm{E}(d_j) = \mu_j K_j^k$ from (2), and substituting (A.61),

$$\begin{aligned} \mathrm{E}\big(\widetilde{r}_j^k\big) &= \frac{\mu_j K_j^k}{p_j^k} - R_f \;=\; \frac{R_f\,\mu_j K_j^k}{K_j^k\left(\mu_j - q_j^k\right)} - R_f \\ &= R_f\left[\frac{\mu_j}{\mu_j - q_j^k} - 1\right] \;=\; R_f\,\frac{q_j^k}{\mu_j - q_j^k}\,. \end{aligned} \tag{A.62}$$

We just proved (A.56). Dividing (A.41)–(A.43) by K_j^k and using $q_j^k = z_j^k/K_j^k$,

$$q_1^k = \frac{a}{K_1^k}\,\mathrm{Cov}\big(d_1^k, d_M^k\big)\,, \qquad k \in \{\mathrm{pre}, \mathrm{post}\} \tag{A.63}$$

$$q_2^{\mathrm{pre}} = \frac{a}{K_2^{\mathrm{pre}}}\,\mathrm{Cov}(d_2^{\mathrm{pre}}, d_M^{\mathrm{pre}}) + \frac{a}{K_2^{\mathrm{pre}}}\,\frac{W_{0B}}{W_{0A}}\,\mathrm{Var}(d_2^{\mathrm{pre}} \mid d_1^{\mathrm{pre}}) \tag{A.64}$$

$$q_2^{\mathrm{post}} = \frac{a}{K_2^{\mathrm{post}}}\,\mathrm{Cov}\big(d_2^{\mathrm{post}}, d_M^{\mathrm{post}}\big) + c\,W_{0B}\,. \tag{A.65}$$

Equation (A.63) gives (A.57) and (A.59), and (A.65) gives (A.60). Substituting $\mathrm{Var}(d_2^{\mathrm{pre}} \mid d_1^{\mathrm{pre}}) = (K_2^{\mathrm{pre}})^2 v_{2|1}$ from (4) into the second term of (A.64) gives $a(W_{0B}/W_{0A})K_2^{\mathrm{pre}} v_{2|1}$, which is (A.58).

Since $K_j^k > 0$ and $R_f > 1$, equation (A.61) shows that the maintained assumption $p_j^k > 0$ is equivalent to $\mu_j > q_j^k$, as stated in the proposition. Finally, differentiating (A.56),

$$\frac{\partial\,\mathrm{E}\big(\widetilde{r}_j^k\big)}{\partial q_j^k} = R_f\,\frac{\mu_j}{\big(\mu_j - q_j^k\big)^2} \;>\; 0\,, \tag{A.66}$$

so $\mathrm{E}\big(\widetilde{r}_j^k\big)$ is increasing in q_j^k. ■

Proof of Corollary 3: Supplied in the text. ■

Proof of Proposition 3

From (33) and (34), each excess return is an affine function of a payoff, so

$$\mathrm{E}\big(\widetilde{r}_j^k\big) = \frac{z_j^k}{p_j^k}\,, \qquad \mathrm{E}\big(\widetilde{r}_M^k\big) = \frac{z_1^k + z_2^k}{p_M^k}\,, \tag{A.67}$$

using (A.1) and $p_M^k = p_1^k + p_2^k$. Likewise $\mathrm{Cov}(\widetilde{r}_j^k, \widetilde{r}_M^k) = \mathrm{Cov}(d_j^k, d_M^k)/(p_j^k p_M^k)$ and $\mathrm{Var}(\widetilde{r}_M^k) = \mathrm{Var}(d_M^k)/(p_M^k)^2$, so the market beta defined in Proposition 3 is as stated in footnote 14:

$$\beta_j^k = \frac{p_M^k}{p_j^k}\,\frac{\mathrm{Cov}\big(d_j^k, d_M^k\big)}{\mathrm{Var}\big(d_M^k\big)}\,. \tag{A.68}$$

Substituting (A.67) and (A.68) into the definition of alpha in (51), we obtain

$$\alpha_j^k = \frac{z_j^k}{p_j^k} - \frac{p_M^k}{p_j^k}\,\frac{\mathrm{Cov}\big(d_j^k, d_M^k\big)}{\mathrm{Var}\big(d_M^k\big)}\cdot\frac{z_1^k + z_2^k}{p_M^k} = \frac{1}{p_j^k}\left[z_j^k - \frac{\mathrm{Cov}\big(d_j^k, d_M^k\big)}{\mathrm{Var}\big(d_M^k\big)}\,\big(z_1^k + z_2^k\big)\right]. \tag{A.69}$$

Let w^k denote the private asset's wedge measured in payoff units, that is, the second term of z_2^k in Lemma 1:

$$w^{\mathrm{pre}} \equiv a\,\frac{W_{0B}}{W_{0A}}\,\mathrm{Var}(d_2^{\mathrm{pre}} \mid d_1^{\mathrm{pre}}) = a\,\frac{W_{0B}}{W_{0A}}\,(K_2^{\mathrm{pre}})^2\,v_{2|1}\,, \qquad w^{\mathrm{post}} \equiv c\,W_{0B}K_2^{\mathrm{post}}\,. \tag{A.70}$$

The wedge w^{post} can also be written in the same form as its pre-democratization counterpart. Substituting equation (27) into (A.70) gives

$$w^{\text{post}} = (1-\gamma)\, a\, \frac{W_{0B}}{W_{0A}}\, \text{Var}\big(d_2^{\text{post}} \mid d_1^{\text{post}}\big) = (1-\gamma)\, a\, \frac{W_{0B}}{W_{0A}}\, \big(K_2^{\text{post}}\big)^2\, v_{2|1}\,. \tag{A.71}$$

Lemma 1 then reads $z_1^k = a\,\text{Cov}(d_1^k, d_M^k)$ and $z_2^k = a\,\text{Cov}(d_2^k, d_M^k) + w^k$. Adding these two quantities, with $\text{Cov}(d_1^k, d_M^k) + \text{Cov}(d_2^k, d_M^k) = \text{Var}(d_M^k)$ from (A.55), yields

$$z_1^k + z_2^k = a\,\text{Var}\big(d_M^k\big) + w^k\,. \tag{A.72}$$

For the public asset, substituting z_1^k and (A.72) into (A.69) leads to

$$\alpha_1^k = \frac{1}{p_1^k}\,\text{Cov}\big(d_1^k, d_M^k\big)\left[a - \frac{a\,\text{Var}\big(d_M^k\big) + w^k}{\text{Var}\big(d_M^k\big)}\right] = -\,\frac{\text{Cov}\big(d_1^k, d_M^k\big)}{\text{Var}\big(d_M^k\big)}\,\frac{w^k}{p_1^k}\,. \tag{A.73}$$

For the private asset, the same substitution gives

$$\begin{aligned}\alpha_2^k &= \frac{1}{p_2^k}\left[a\,\text{Cov}\big(d_2^k, d_M^k\big) + w^k - \frac{\text{Cov}\big(d_2^k, d_M^k\big)}{\text{Var}\big(d_M^k\big)}\,\big(a\,\text{Var}\big(d_M^k\big) + w^k\big)\right] \\ &= \frac{w^k}{p_2^k}\left[1 - \frac{\text{Cov}\big(d_2^k, d_M^k\big)}{\text{Var}\big(d_M^k\big)}\right] = \frac{\text{Cov}\big(d_1^k, d_M^k\big)}{\text{Var}\big(d_M^k\big)}\,\frac{w^k}{p_2^k}\,.\end{aligned} \tag{A.74}$$

The two alphas thus share a numerator and differ only in which price divides it.

Back to betas, we rearrange (A.68) at $j = 1$, obtaining

$$\frac{\text{Cov}\big(d_1^k, d_M^k\big)}{\text{Var}\big(d_M^k\big)} = \beta_1^k\,\frac{p_1^k}{p_M^k}\,. \tag{A.75}$$

Substituting (A.75) into (A.73) and (A.74),

$$\alpha_1^k = -\,\frac{w^k}{p_M^k}\,\beta_1^k\,, \qquad \alpha_2^k = +\,\frac{w^k}{p_M^k}\,\beta_1^k\,\frac{p_1^k}{p_2^k}\,, \tag{A.76}$$

which are (52) and (53) with

$$h^k \equiv \frac{w^k}{p_M^k}\,. \tag{A.77}$$

Substituting (A.70) and (A.71) into (A.77) gives (54), (55), and (56). Each wedge vanishes with its friction: $w^{\text{pre}} = 0$ when $v_{2|1} = 0$ and $w^{\text{post}} = 0$ when $c = 0$ (equivalently, when $\gamma = 1$), in which cases $\alpha_1^k = \alpha_2^k = 0$. ■

Proof of Corollary 4: Immediate from Proposition 3. ■

Proof of Corollary 5

First, note that with $K^{\text{post}} = K^{\text{pre}} = K$, (A.70) gives

$$w^{\text{post}} - w^{\text{pre}} = c\, W_{0B} K_2 - a\, \frac{W_{0B}}{W_{0A}} K_2^2 v_{2|1} = W_{0B} K_2 \left(c - \frac{a K_2 v_{2|1}}{W_{0A}} \right) \; < \; 0\,, \tag{A.78}$$

where the inequality is the retail-participation condition (28) at the common K_2. Combining this inequality with equation (A.73), the public alpha changes as follows:

$$\alpha_1^{\text{post}} - \alpha_1^{\text{pre}} = \frac{\text{Cov}(d_1, d_M)}{\text{Var}(d_M)} \cdot \frac{w^{\text{pre}} - w^{\text{post}}}{p_1} \; > \; 0\,, \tag{A.79}$$

which is (58). The private alpha in (A.74) depends also on p_2^k, which is equal to $p_2^k = (\mu_2 K_2 - z_2^k)/R_f$ based on (A.61), and for which

$$p_2^{\text{post}} - p_2^{\text{pre}} = \frac{w^{\text{pre}} - w^{\text{post}}}{R_f} \; > \; 0\,. \tag{A.80}$$

Therefore, (A.74) implies

$$\frac{\alpha_2^{\text{post}}}{\alpha_2^{\text{pre}}} = \frac{w^{\text{post}}}{w^{\text{pre}}} \cdot \frac{p_2^{\text{pre}}}{p_2^{\text{post}}} \; < \; 1\,. \tag{A.81}$$

Since both alphas are positive by Corollary 4, we obtain $\alpha_2^{\text{post}} < \alpha_2^{\text{pre}}$, which is (59). ■

Proof of Proposition 4

First, we establish a simple baseline expression for certainty equivalents. Multiplying (A.9) by θ_{A1}, multiplying (A.10) by θ_{A2}, and adding the two terms yields

$$\theta_{A1} z_1 + \theta_{A2} z_2 = \frac{a}{W_{0A}} \Big[\theta_{A1}^2 \,\text{Var}(d_1) + 2\,\theta_{A1}\theta_{A2}\,\text{Cov}(d_1, d_2) + \theta_{A2}^2 \,\text{Var}(d_2) \Big]\,. \tag{A.82}$$

Dividing both sides by W_{0A}, the left-hand side is $\text{E}(\widetilde{r}_A^p)$ by (A.3) and the right-hand side is $a\,\text{Var}(\widetilde{r}_A^p)$ by (A.4), so

$$\text{E}(\widetilde{r}_A^p) = a\,\text{Var}(\widetilde{r}_A^p)\,. \tag{A.83}$$

The same operation on (A.11) and (A.12) gives $\theta_{B1} z_1 + \theta_{B2}(z_2 - cK_2)$ on the left, which is $W_{0B}\,\text{E}(\widetilde{r}_B^p)$ by (A.7), so $\text{E}(\widetilde{r}_B^p) = a\,\text{Var}(\widetilde{r}_B^p)$ as well. Pre-democratization only (A.11) is available, but $\theta_{B2} = 0$ there, so the omitted terms are zero on both sides and the conclusion is unchanged. Substituting from (A.83) into (64), we obtain

$$\text{CE}_i = R_f + a\,\text{Var}(\widetilde{r}_i^p) - \frac{a}{2}\text{Var}(\widetilde{r}_i^p) = R_f + \frac{a}{2}\,\text{Var}(\widetilde{r}_i^p) \tag{A.84}$$

for both investors in both economies. Welfare in excess of the risk-free rate is thus equal to the variance penalty, $\frac{a}{2}\text{Var}(\widetilde{r}_i^p)$, that the investor optimally chooses to bear or, alternatively, to half of the risk premium the investor earns.

Denote $\eta \equiv d_2 - \delta\, d_1$, which is the residual in (25) up to an additive constant. Using $\delta = \mathrm{Cov}(d_1, d_2)/\mathrm{Var}(d_1)$ from (23) in the first line and (A.27) in the second and third,

$$\mathrm{Cov}(d_1, \eta) = \mathrm{Cov}(d_1, d_2) - \delta\, \mathrm{Var}(d_1) = 0 \tag{A.85}$$

$$\mathrm{Var}(\eta) = \mathrm{Var}(d_2) - 2\delta\, \mathrm{Cov}(d_1, d_2) + \delta^2 \mathrm{Var}(d_1) = \mathrm{Var}(d_2) - \delta\, \mathrm{Cov}(d_1, d_2) = \mathrm{Var}(d_2 \mid d_1) \tag{A.86}$$

$$\mathrm{Cov}(d_M, \eta) = \mathrm{Cov}(d_1, \eta) + \mathrm{Cov}(d_2, \eta) = \mathrm{Var}(d_2) - \delta\, \mathrm{Cov}(d_1, d_2) = \mathrm{Var}(d_2 \mid d_1)\ . \tag{A.87}$$

Now write the holdings as deviations from frictionless allocations (as in the proof of Proposition 1) and impose $x_1 = -\delta x_2$ from (A.19):

$$\theta_{A1} d_1 + \theta_{A2} d_2 = W_{0A}\,(d_1 + d_2) + x_1 d_1 + x_2 d_2 = W_{0A}\, d_M + x_2\, \eta \tag{A.88}$$

$$\theta_{B1} d_1 + \theta_{B2} d_2 = W_{0B}\,(d_1 + d_2) - x_1 d_1 - x_2 d_2 = W_{0B}\, d_M - x_2\, \eta\,. \tag{A.89}$$

Both investors hold the market portfolio, and x_2 measures how many units of the residual the institution holds and the retail investor gives up. Taking variances in (A.88)–(A.89), applying (A.86)–(A.87), and dividing by W_{0i}^2 as in (A.4),

$$\mathrm{Var}(\widetilde{r}_A^{\,p}) = \mathrm{Var}(d_M) + \left(\frac{2x_2}{W_{0A}} + \frac{x_2^2}{W_{0A}^2}\right) \mathrm{Var}(d_2 \mid d_1) \tag{A.90}$$

$$\mathrm{Var}(\widetilde{r}_B^{\,p}) = \mathrm{Var}(d_M) + \left(-\frac{2x_2}{W_{0B}} + \frac{x_2^2}{W_{0B}^2}\right) \mathrm{Var}(d_2 \mid d_1)\,. \tag{A.91}$$

Substituting (A.90)–(A.91) into (A.84) expresses both certainty equivalents, in both economies, in terms of x_2. It remains to insert its two equilibrium values.

Pre-democratization, $x_2 = W_{0B}$ by (A.20). The coefficient in (A.90) becomes

$$\frac{2W_{0B}}{W_{0A}} + \frac{W_{0B}^2}{W_{0A}^2} = \frac{W_{0B}\,(2W_{0A} + W_{0B})}{W_{0A}^2} = \frac{W_{0B}\,(2 - W_{0B})}{W_{0A}^2}\,, \tag{A.92}$$

using $W_{0A} + W_{0B} = 1$ from (6). This gives (65). The coefficient in (A.91) becomes

$$-\frac{2W_{0B}}{W_{0B}} + \frac{W_{0B}^2}{W_{0B}^2} = -2 + 1 = -1\,, \tag{A.93}$$

which gives (66).

Post-democratization, $x_2 = (1-\gamma) W_{0B}$ by (A.28). From (24) and (28),

$$1 - \gamma = \frac{c\, W_{0A}}{a\, v_{2|1} K_2^{\mathrm{post}}}\,. \tag{A.94}$$

Two identities follow, both obtained by multiplying (A.94) by $\mathrm{Var}(d_2 \mid d_1) = (K_2^{\mathrm{post}})^2 v_{2|1}$, once as written and once squared:

$$(1-\gamma)\, \mathrm{Var}(d_2 \mid d_1) = \frac{c\, W_{0A}}{a\, v_{2|1} K_2^{\mathrm{post}}} \left(K_2^{\mathrm{post}}\right)^2 v_{2|1} \;=\; \frac{c\, W_{0A} K_2^{\mathrm{post}}}{a} \tag{A.95}$$

$$(1-\gamma)^2\, \mathrm{Var}(d_2 \mid d_1) = \frac{c^2 W_{0A}^2}{a^2 v_{2|1}^2 \left(K_2^{\mathrm{post}}\right)^2} \left(K_2^{\mathrm{post}}\right)^2 v_{2|1} \;=\; \frac{c^2 W_{0A}^2}{a^2 v_{2|1}}\,. \tag{A.96}$$

Now substitute $x_2 = (1-\gamma)W_{0B}$ into the four terms of (A.90)–(A.91), each premultiplied by $a/2$ as in (A.84). The institution's two terms are

$$\frac{a}{2}\cdot\frac{2x_2}{W_{0A}}\operatorname{Var}(d_2 \mid d_1) = \frac{a\,W_{0B}}{W_{0A}}(1-\gamma)\operatorname{Var}(d_2 \mid d_1) \;=\; \frac{a\,W_{0B}}{W_{0A}}\cdot\frac{c\,W_{0A}K_2^{\text{post}}}{a} \;=\; c\,W_{0B}K_2^{\text{post}} \tag{A.97}$$

$$\frac{a}{2}\cdot\frac{x_2^2}{W_{0A}^2}\operatorname{Var}(d_2 \mid d_1) = \frac{a}{2}\frac{W_{0B}^2}{W_{0A}^2}(1-\gamma)^2\operatorname{Var}(d_2 \mid d_1) \;=\; \frac{a}{2}\frac{W_{0B}^2}{W_{0A}^2}\cdot\frac{c^2W_{0A}^2}{a^2 v_{2|1}} \;=\; \frac{c^2W_{0B}^2}{2a\,v_{2|1}}, \tag{A.98}$$

using (A.95) and (A.96) respectively. Substituting these into (A.84) gives (67). For the retail investor the factors of W_{0B} cancel, leaving

$$-\frac{a}{2}\cdot\frac{2x_2}{W_{0B}}\operatorname{Var}(d_2 \mid d_1) = -\,a\,(1-\gamma)\operatorname{Var}(d_2 \mid d_1) \;=\; -\,c\,W_{0A}K_2^{\text{post}} \tag{A.99}$$

$$\frac{a}{2}\cdot\frac{x_2^2}{W_{0B}^2}\operatorname{Var}(d_2 \mid d_1) = \frac{a}{2}(1-\gamma)^2\operatorname{Var}(d_2 \mid d_1) \;=\; \frac{c^2W_{0A}^2}{2a\,v_{2|1}}, \tag{A.100}$$

again by (A.95) and (A.96). Substituting into (A.84) gives (68). ■

Proof of Corollary 6

First, we establish a convenient form for the certainty equivalents. Recalling from (A.13) that $\theta_{A2} = W_{0A} + x_2$ and $\theta_{B2} = W_{0B} - x_2$, we can complete the square in (A.90)–(A.91),

$$\frac{2x_2}{W_{0A}} + \frac{x_2^2}{W_{0A}^2} = \left(\frac{\theta_{A2}}{W_{0A}}\right)^2 - 1, \qquad -\frac{2x_2}{W_{0B}} + \frac{x_2^2}{W_{0B}^2} = \left(\frac{\theta_{B2}}{W_{0B}}\right)^2 - 1, \tag{A.101}$$

so that both investors' certainty equivalents take the single form

$$\text{CE}_i = R_f + \frac{a}{2}\operatorname{Var}(d_M) + \frac{a}{2}\left[\left(\frac{\theta_{i2}}{W_{0i}}\right)^2 - 1\right]\operatorname{Var}(d_2 \mid d_1), \qquad i \in \{A, B\}. \tag{A.102}$$

Welfare depends on θ_{i2}/W_{0i}, the investor's private holding relative to the frictionless benchmark. At that benchmark, $\theta_{i2} = W_{0i}$, and CE_i is simply equal to $R_f + \frac{a}{2}\text{Var}(d_M)$.

With $K^{\text{post}} = K^{\text{pre}} = K$, neither $\operatorname{Var}(d_M)$ nor $\operatorname{Var}(d_2 \mid d_1)$ vary across the two economies. Therefore, differencing (A.102) implies

$$\text{CE}_i^{\text{post}} - \text{CE}_i^{\text{pre}} = \frac{a}{2}\left[\left(\frac{\theta_{i2}^{\text{post}}}{W_{0i}}\right)^2 - \left(\frac{\theta_{i2}^{\text{pre}}}{W_{0i}}\right)^2\right]\operatorname{Var}(d_2 \mid d_1). \tag{A.103}$$

Proposition 1 supplies the four holdings. For the institution, substituting $\theta_{A2}^{\text{pre}} = 1$ from (17) and $\theta_{A2}^{\text{post}} = 1 - \gamma W_{0B}$ from (21), we obtain

$$\left(\frac{1-\gamma W_{0B}}{W_{0A}}\right)^2 - \left(\frac{1}{W_{0A}}\right)^2 = \frac{(1-\gamma W_{0B})^2 - 1}{W_{0A}^2} = -\,\frac{\gamma W_{0B}\,(2-\gamma W_{0B})}{W_{0A}^2}. \tag{A.104}$$

Substituting (A.104) into (A.103) gives (69). For the retail investor, (18) and (22) give $\theta_{B2}^{\text{pre}} = 0$ and $\theta_{B2}^{\text{post}} = \gamma W_{0B}$, so the bracket in (A.103) is simply γ^2, which gives (70).

Weighting the two changes by wealth shares, we obtain

$$\begin{aligned} \text{CE}_{\text{agg}}^{\text{post}} - \text{CE}_{\text{agg}}^{\text{pre}} &= W_{0A}\left(-\frac{a}{2}\frac{W_{0B}}{W_{0A}^2}\gamma\,(2-\gamma W_{0B})\right)\text{Var}(d_2 \mid d_1) + W_{0B}\,\frac{a}{2}\gamma^2\,\text{Var}(d_2 \mid d_1) \\ &= \frac{a}{2}\,\frac{W_{0B}\gamma}{W_{0A}}\Big[-(2-\gamma W_{0B}) + \gamma W_{0A}\Big]\text{Var}(d_2 \mid d_1) \\ &= -\frac{a}{2}\,\frac{W_{0B}}{W_{0A}}\,\gamma\,(2-\gamma)\,\text{Var}(d_2 \mid d_1)\,, \end{aligned} \tag{A.105}$$

which is (71). The last equality relies on $W_{0A} + W_{0B} = 1$. ■

Proof of Proposition 5

Since the map from I_j to K_j is one-to-one (see equation (73)), we can treat K_j as the choice variable. Substituting (74) into (75), firm j chooses K_j to maximize

$$V_j(K_j) \;\equiv\; p_j(K) - \big(K_j - K_j^0\big) - \frac{\phi}{2}\frac{\big(K_j - K_j^0\big)^2}{K_j^0}\,, \tag{A.106}$$

taking the other firm's capital as given and $p_j(K)$ in Lemma 1 as the equilibrium price schedule. Equation (A.44) shows that firm 1 faces the same schedule in both economies. Firm 2 does not: it faces (A.46) where retail participates and (A.45) where it does not.

To show that the firm's objective function in (A.106) is concave, we differentiate it twice. For firm 1, we take p_1 from (A.44):

$$\frac{\partial^2 V_1}{\partial K_1^2} = -\frac{2a\,v_{11}}{R_f} - \frac{\phi}{K_1^0} \;<\; 0\,. \tag{A.107}$$

For firm 2, using prices from (A.46) and then (A.45),

$$\frac{\partial^2 V_2}{\partial K_2^2} = -\frac{2a\,v_{22}}{R_f} - \frac{\phi}{K_2^0} \quad\text{and}\quad \frac{\partial^2 V_2}{\partial K_2^2} = -\frac{2a}{R_f}\left(v_{22} + \frac{W_{0B}}{W_{0A}}v_{2|1}\right) - \frac{\phi}{K_2^0}\,, \tag{A.108}$$

both strictly negative. In each expression the first term is the firm's price impact, since a larger sector depresses the price at which its output clears, and the second is the convex adjustment cost. Either alone suffices, so concavity survives also at $\phi = 0$.

Firm 2's objective function has a kink in it. Define

$$\bar{K}_2 \;\equiv\; \frac{c\,W_{0A}}{a\,v_{2|1}}\,, \tag{A.109}$$

so that (24) can be written $\gamma = (K_2 - \bar{K}_2)/K_2$ on the branch where $\gamma > 0$, and thus

$$\gamma > 0 \quad\Longleftrightarrow\quad K_2 > \bar{K}_2\,. \tag{A.110}$$

For $K_2 > \bar{K}_2$ firm 2 faces the price schedule in (A.46); for $K_2 \leq \bar{K}_2$ it faces (A.45) because retail does not participate (so the pre- and post-democratization economies coincide). The two schedules agree at the kink, since substituting (A.109) into the segmentation wedge gives

$$a\,\frac{W_{0B}}{W_{0A}}\bar{K}_2 v_{2|1} = a\,\frac{W_{0B}}{W_{0A}}\cdot\frac{c\,W_{0A}}{a\,v_{2|1}}\cdot v_{2|1} = c\,W_{0B}\,, \tag{A.111}$$

so p_2 is continuous. Its slope is not. In $R_f\,p_2$ the holding-cost wedge enters as $-c\,W_{0B}K_2$ and the segmentation wedge as $-a(W_{0B}/W_{0A})K_2^2 v_{2|1}$, so their derivatives are $-c\,W_{0B}$ and $-2a(W_{0B}/W_{0A})K_2 v_{2|1}$, the latter equal to $-2c\,W_{0B}$ at $\bar{K}_2$ by (A.111). Hence

$$\left.\frac{\partial p_2}{\partial K_2}\right|_{\bar{K}_2^+} - \left.\frac{\partial p_2}{\partial K_2}\right|_{\bar{K}_2^-} = \frac{c\,W_{0B}}{R_f} \;>\; 0\,. \tag{A.112}$$

Firm 2's objective is thus concave on each side of $\bar{K}_2$ but has a convex kink there. Essentially, the marginal value of capital jumps up as K_2 crosses the point at which retail begins to participate. The objective is thus not globally concave, and a local optimum could in principle sit on either side of $\bar{K}_2$. This is what condition (76) rules out.

Next, we show that the post-democratization optimal capital stock lies above the kink. Rearranging condition (76) using (A.109), we obtain

$$\frac{aK_2^{\text{pre}}v_{2|1}}{W_{0A}} - c = \frac{a\,v_{2|1}}{W_{0A}}\left(K_2^{\text{pre}} - \bar{K}_2\right), \tag{A.113}$$

so (76) is equivalent to $\bar{K}_2 < K_2^{\text{pre}}$: retail would already participate at the pre-democratization scale. What can we say about K_2^{post}? First, it cannot be true that $K_2^{\text{post}} < \bar{K}_2$. Suppose it were true. In a neighborhood of that point firm 2 faces the segmented schedule, so both firms' first-order conditions are the pre-democratization ones, so $K_2^{\text{post}} = K_2^{\text{pre}} > \bar{K}_2$, contradicting $K_2^{\text{post}} < \bar{K}_2$. Suppose instead $K_2^{\text{post}} = \bar{K}_2$. Optimality at a kink requires the left derivative to be nonnegative and the right derivative nonpositive, but by (A.112) the right derivative exceeds the left, so both cannot hold. Since Section 6.2 assumes $K_2^{\text{post}} > 0$, ruling out a corner, we conclude

$$K_2^{\text{post}} \;>\; \bar{K}_2\,. \tag{A.114}$$

Two conclusions follow. First, by (A.110), retail participates at the post-democratization capital stock. Condition (76), which is imposed at K_2^{pre}, thus implies the retail-participation condition (28) at the equilibrium K_2^{post}, as claimed in the text. Second, since the optimum lies strictly above $\bar{K}_2$, the schedule firm 2 faces there is (A.46), and its objective on that region is strictly concave by (A.108). Firm 2's optimum is thus the stationary point of the participating branch, which is the first-order condition derived below.

No such argument is needed pre-democratization. There retail is barred from the private asset by assumption, so firm 2 faces (A.45) at every K_2, its objective has no kink, and it is globally strictly concave by (A.108).

On to the first-order conditions. Differentiating (A.106) and setting the result to zero, the marginal value of capital equals its marginal cost. For firm 1, using (A.44),

$$\frac{\mu_1 - 2aK_1v_{11} - aK_2v_{12}}{R_f} = 1 + \phi\,\frac{K_1 - K_1^0}{K_1^0}\,, \tag{A.115}$$

which holds in both economies. For firm 2 on the participating branch, using (A.46),

$$\frac{\mu_2 - aK_1v_{12} - 2aK_2v_{22} - c\,W_{0B}}{R_f} = 1 + \phi\,\frac{K_2 - K_2^0}{K_2^0}\,, \tag{A.116}$$

and on the segmented branch, using (A.45),

$$\frac{\mu_2 - aK_1v_{12} - 2aK_2v_{22} - 2a\,(W_{0B}/W_{0A})\,K_2v_{2|1}}{R_f} = 1 + \phi\,\frac{K_2 - K_2^0}{K_2^0}\,. \tag{A.117}$$

Each wedge enters these conditions differently from how it enters the required payoff. By (A.58) the segmentation wedge per unit of capital is $a\,(W_{0B}/W_{0A})\,K_2v_{2|1}$, yet it appears in (A.117) doubled, whereas by (A.60) the holding-cost wedge is $c\,W_{0B}$ and appears in (A.116) unchanged. The reason is that the total deduction from $R_f\,p_2$ is the per-unit wedge times K_2, which is proportional to K_2^2 in the segmented case and to K_2 in the participating case, so differentiating with respect to K_2 doubles the first and leaves the second:

$$\frac{\partial}{\partial K_2}\left[a\,\frac{W_{0B}}{W_{0A}}K_2^2v_{2|1}\right] = 2\,a\,\frac{W_{0B}}{W_{0A}}K_2v_{2|1}\,, \qquad \frac{\partial}{\partial K_2}\Big[c\,W_{0B}K_2\Big] = c\,W_{0B}\,. \tag{A.118}$$

Corollary 7 turns on this asymmetry.

Next, we show that each economy's equilibrium solves a linear system with a unique solution. Multiply (A.115) through by R_f and collect the terms in K_1, using $2av_{11} + R_f\phi/K_1^0 = 2u_1$ with u_j as defined in Proposition 5:

$$2u_1K_1 + av_{12}K_2 = \mu_1 - R_f + R_f\phi\,. \tag{A.119}$$

Treating (A.116) and (A.117) the same way, and writing $\widehat{u}_2 \equiv u_2 + a\,(W_{0B}/W_{0A})\,v_{2|1}$ for the segmented branch, we have

$$av_{12}K_1 + 2u_2K_2 = \mu_2 - c\,W_{0B} - R_f + R_f\phi \tag{A.120}$$

$$av_{12}K_1 + 2\widehat{u}_2K_2 = \mu_2 - R_f + R_f\phi\,. \tag{A.121}$$

Each economy's equilibrium therefore solves a linear system in (K_1, K_2): (A.119) with (A.120) post-democratization, and (A.119) with (A.121) pre-democratization. The post-democratization determinant is $4u_1u_2 - a^2v_{12}^2$, and expanding u_1 and u_2,

$$4u_1u_2 - a^2v_{12}^2 = a^2\left(4\det v + 3v_{12}^2\right) + 2aR_f\phi\left(\frac{v_{11}}{K_2^0} + \frac{v_{22}}{K_1^0}\right) + \frac{R_f^2\phi^2}{K_1^0K_2^0} \;>\; 0\,, \tag{A.122}$$

where positivity follows because $\det v > 0$ by $v \succ 0$ in (1) and $\phi \geq 0$. Since $\widehat{u}_2 > u_2$, the pre-democratization determinant $4u_1\widehat{u}_2 - a^2v_{12}^2$ is larger still, hence also positive. Each system has a unique solution, and each economy a unique pair of optimal capital stocks.

First, we solve for the public firm's response to democratization. Equation (A.119) holds in both economies with the same right-hand side, so subtracting the pre-democratization version from the post-democratization one, we have

$$2u_1\Delta K_1 + av_{12}\Delta K_2 = 0 \qquad \Longrightarrow \qquad \Delta K_1 = -\frac{av_{12}}{2u_1}\,\Delta K_2\,, \tag{A.123}$$

which is (77).

Second, we solve for the private firm's response. Subtract (A.121), evaluated at K^{pre}, from (A.120), evaluated at K^{post}:

$$av_{12}\Delta K_1 + 2u_2K_2^{\text{post}} - 2\widehat{u}_2K_2^{\text{pre}} = -\,c\,W_{0B}\,. \tag{A.124}$$

Substituting $2\widehat{u}_2 = 2u_2 + 2a\,(W_{0B}/W_{0A})\,v_{2|1}$ and collecting the u_2 terms into ΔK_2,

$$av_{12}\Delta K_1 + 2u_2\Delta K_2 = 2a\,\frac{W_{0B}}{W_{0A}}v_{2|1}K_2^{\text{pre}} - c\,W_{0B} = W_{0B}\left(\frac{2aK_2^{\text{pre}}v_{2|1}}{W_{0A}} - c\right). \tag{A.125}$$

Eliminating ΔK_1 with (A.123),

$$\Delta K_2\left(2u_2 - \frac{a^2v_{12}^2}{2u_1}\right) = W_{0B}\left(\frac{2aK_2^{\text{pre}}v_{2|1}}{W_{0A}} - c\right), \tag{A.126}$$

and multiplying both sides by $2u_1 > 0$,

$$\Delta K_2\left(4u_1u_2 - a^2v_{12}^2\right) = 2u_1W_{0B}\left(\frac{2aK_2^{\text{pre}}v_{2|1}}{W_{0A}} - c\right). \tag{A.127}$$

Dividing by the determinant, which is positive by (A.122), gives (78). ■

Because $u_1 > 0$ and the determinant is positive by (A.122), the sign of ΔK_2 is the sign of the final parenthesis, which Corollary 7 evaluates. While ϕ affects that parenthesis through K_2^{pre}, it cannot change the parenthesis's sign, because condition (76) is imposed at whatever K_2^{pre} that system delivers, making the parenthesis exceed $aK_2^{\text{pre}}v_{2|1}/W_{0A} > 0$ for every $\phi \geq 0$.

Proof of Corollary 7

To show $\Delta K_2 > 0$, we just need to show that the term in the parentheses in (78) is positive. Denote that term by T and let B denote the bound appearing in condition (76):

$$T \;\equiv\; \frac{2aK_2^{\text{pre}}v_{2|1}}{W_{0A}} - c\,, \qquad B \;\equiv\; \frac{aK_2^{\text{pre}}v_{2|1}}{W_{0A}}\,, \qquad \text{so} \qquad T = B + (B - c)\,. \tag{A.128}$$

Since $B > 0$ and condition (76) states $c < B$, we have $T > 0$, which then immediately implies $\Delta K_2 > 0$ based on (78). This is half of the corollary: the private firm expands. The other half, that the public firm contracts, follows immediately from (77), which shows that ΔK_1 and ΔK_2 have opposite signs. ■

A.2. Estimating wealth shares (W_{0A})

A.2.1. Overview

The calibration in Section 7 sets a range for W_{0A}, the fraction of total wealth held by institutional investors, meaning investors with access to diversified private-market funds. This appendix explains how we obtain that range.

We interpret an investor as an asset-allocation unit rather than as the ultimate beneficial owner of wealth, because access to private-market funds is determined by the allocator's legal eligibility, scale, sophistication, and institutional infrastructure. Defined-benefit (DB) pension plans are investors, by our definition, because they make allocation decisions on behalf of their beneficiaries. Individual beneficiaries of DB plans, by contrast, are not the relevant allocators of DB pension assets, so we do not treat them as investors. This allocator-level framing also avoids double-counting: pension wealth is counted at the plan-asset level, not at the household-claim level, so each dollar appears exactly once.

We estimate W_{0A} as

$$W_{0A} = \frac{W^{\text{alloc}}_{\text{HH,HNW}} + W_{\text{DB}} + W^{\text{alloc}}_{\text{NP,inst}}}{W^{\text{alloc}}_{\text{HH}} + W_{\text{DB}} + W_{\text{DC}} + W^{\text{alloc}}_{\text{NP}}}. \tag{A.129}$$

The numerator captures wealth controlled by investors with private market access: directly allocable wealth held by high-net worth (HNW) households ($W^{\text{alloc}}_{\text{HH,HNW}}$), public and private DB pension plan assets (W_{DB}), and the portion of allocable nonprofit wealth managed by institutional allocators ($W^{\text{alloc}}_{\text{NP,inst}}$, e.g. large endowments and foundations). The denominator W_0 is total allocator-level wealth, which includes both HNW and non-HNW households' allocable wealth ($W^{\text{alloc}}_{\text{HH}}$), the wealth of both DB and DC (defined contribution) pensions, and allocable nonprofit wealth ($W^{\text{alloc}}_{\text{NP}}$). DC plans enter only the denominator because, under current regulations and except for minor exceptions, DC plan participants lack effective access to diversified private-market funds. Given our focus on the U.S., which does not have a sovereign wealth fund, we exclude such funds. Non-U.S. sovereign wealth funds do invest in U.S. private-market funds, but we treat those foreign investors as outside our model.

Household i's directly allocable wealth, W_i^{alloc}, includes household-controlled financial assets, including IRA balances, business equity, and investment real estate net of associated debts; it excludes primary-residence equity, consumer durables, DB pension claims, DC plan balances, and retirement annuities. Excluding DB claims and DC balances avoids double-counting, given (A.129). Because of these exclusions, W_0 should not be interpreted as total household net worth. Instead, W_0 is the portion of wealth relevant to portfolio allocation between public and private assets.

We classify a household as having effective private-market access if $W_i^{\text{alloc}} \geq \bar{W}$. Below, we show results for three values of the threshold $\bar{W}$: \$5 million, \$10 million, and \$20 million.

The $5 million threshold is motivated by the individual qualified-purchaser standard under U.S. securities law, which requires at least $5 million of investments. The higher thresholds reflect the additional capital and operational demands of diversified private-market access, including minimum commitments, illiquidity management, and due-diligence capacity across multiple managers, strategies, and vintage years. Additional details are in the next subsection.

Household wealth data come from the Federal Reserve's Distributional Financial Accounts (DFA, 2022Q3), which combine Survey of Consumer Finances microdata with the Financial Accounts to produce well-calibrated estimates at the top of the wealth distribution. We use DFA data from 2022Q3 because it is the latest quarter for which the DFA wealth-cutoff series are available. We convert total-net-worth amounts to allocable wealth by subtracting primary-residence equity, consumer durables, DB pension claims, DC plan balances, and retirement annuities using Federal Reserve Z.1 data for 2022Q4.

W_{DB} is the aggregate value of funded assets held by U.S. private, state-and-local, and federal DB pension plans (Federal Reserve Z.1, 2022Q4), excluding unfunded claims on plan sponsors. DC plan assets W_{DC} are similarly from the Z.1 accounts. Nonprofit net worth W_{NP} is from the Z.1 nonprofit balance sheet. We estimate allocable nonprofit wealth as $W_{\mathrm{NP}}^{\mathrm{alloc}} = \lambda_{\mathrm{NP}} W_{\mathrm{NP}}$, where λ_{NP} is the Z.1 ratio of nonprofit financial assets to total nonprofit assets. Because institutional and non-institutional nonprofit allocators cannot be cleanly separated in the data, we report two bounds: a conservative case ($W_{\mathrm{NP,inst}}^{\mathrm{alloc}} = 0$) and a broad case ($W_{\mathrm{NP,inst}}^{\mathrm{alloc}} = W_{\mathrm{NP}}^{\mathrm{alloc}}$).

Table A.1 reports W_{0A} for each HNW threshold and NP assumption. We use the table's minimum value, 0.219, and its maximum value, 0.410, to create our range for W_{0A}.

Table A.1: Calibrated values of W_{0A}

HNW threshold $\bar{W}$	Conservative ($W_{\mathrm{NP,inst}}^{\mathrm{alloc}} = 0$)	Broad ($W_{\mathrm{NP,inst}}^{\mathrm{alloc}} = W_{\mathrm{NP}}^{\mathrm{alloc}}$)
$\bar{W}$ =5 M	0.351	0.410
$\bar{W}$ =10 M	0.274	0.333
$\bar{W}$ =20 M	0.219	0.278

A.2.2. Details

Household allocable wealth. Conceptually, household i's directly allocable wealth is the portion of household wealth controlled by the household and available for portfolio allocation, excluding primary-residence equity, consumer durables, DB pension claims, DC plan balances, and retirement annuities. The aggregate objects are

$$W_{\mathrm{HH}}^{\mathrm{alloc}} = \sum_i \omega_i W_i^{\mathrm{alloc}}, \qquad W_{\mathrm{HH,HNW}}^{\mathrm{alloc}}(\bar{W}) = \sum_i \omega_i \mathbf{1}\{W_i^{\mathrm{alloc}} \geq \bar{W}\} W_i^{\mathrm{alloc}}, \tag{A.130}$$

where ω_i is the SCF survey weight.

HNW threshold justification. Under Regulation D, natural persons qualify as accredited investors if they have net worth exceeding $1 million (excluding the primary residence) or income exceeding $200,000 individually ($300,000 jointly) in each of the two prior years. Many private funds, especially those relying on Investment Company Act exemptions, instead require *qualified-purchaser* status. For an individual, qualified-purchaser status requires at least $5 million of "investments," a narrower concept than total net worth that includes securities, investment real estate, commodity interests, and cash equivalents held for investment. Our baseline threshold of $5 million applies this qualified-purchaser standard to household allocable wealth, which closely resembles the standard's notion of investment. The $10 million and $20 million thresholds reflect the additional practical requirements of diversified private-market access, such as meeting multiple managers' minimum commitments simultaneously and maintaining a staff capable of ongoing due diligence.

Analysis of DFA data. We implement the household aggregates using the Federal Reserve's Distributional Financial Accounts (DFA), accessed through FRED, because the DFA is specifically designed to produce well-calibrated wealth-distribution estimates at the top of the distribution by combining SCF microdata with Financial Accounts aggregates.

The seven FRED series used are: WFRBLT01026 (top-1% HH net worth level, $39.7 trillion), WFRBST01134 (top-1% share, 29.50%), WFRBLN09053 (90th–99th pct. HH net worth, $49.0 trillion), WFRBLTP1246 (top-0.1% HH net worth, $17.7 trillion), WFRBLN09304 (minimum net worth for 90th–99th pct., $2,148,192), WFRBL99T999309 (minimum for 99th–99.9th pct., $11,146,773), and WFRBLTP1311 (minimum for top 0.1%, $46,365,277). Total DFA household net worth is inferred as $39.7 T/0.2950 = $134.4 trillion. The top-10% and top-0.1% wealth shares are ($39.7 + $49.0)/$134.4 = 65.97% and $17.7/$134.4 = 13.19%.

Since the DFA does not use the exact thresholds $\bar{W}$ we do, we estimate wealths about those thresholds by log-linear Pareto interpolation. Within each DFA interval we fit $S(> X) = S_l(X/X_l)^{-\gamma}$, where S_l and X_l are the cumulative wealth share and minimum-wealth cutoff at the lower endpoint, and γ is calibrated to match the upper endpoint. For $X \in$ [$2,148,192, $11,146,773]:

$$\gamma_1 = \frac{\ln(0.6597/0.2950)}{\ln(11{,}146{,}773/2{,}148{,}192)} = 0.489, \quad S(> X) = 0.6597\Big(\tfrac{X}{2{,}148{,}192}\Big)^{-0.489}.$$

For $X \in$ [$11,146,773, $46,365,277]:

$$\gamma_2 = \frac{\ln(0.2950/0.1319)}{\ln(46{,}365{,}277/11{,}146{,}773)} = 0.564, \quad S(> X) = 0.2950\Big(\tfrac{X}{11{,}146{,}773}\Big)^{-0.564}.$$

Applied directly to total-net-worth thresholds of $5 million, $10 million, and $20 million, this interpolation yields shares of 43.65%, 31.11%, and 21.21%, corresponding to $58.7, $41.8, and $28.5 trillion. For the calibration, however, the thresholds apply to allocable wealth rather than total net worth, so the next subsection adjusts these dollar cutoffs using the estimated allocable fraction.

Non-allocable components and allocable fraction. We compute allocable household net worth by subtracting non-allocable or separately counted components from DFA total HH net worth ($134.4 trillion), using Z.1 data for 2022Q4: (i) primary-residence equity of $23.3 trillion; (ii) consumer durable goods of $7.6 trillion; (iii) household DB pension claims of $15.7 trillion; (iv) DC plan balances of $9.1 trillion; and (v) retirement annuities of $3.1 trillion. This gives $W_{\text{HH}}^{\text{alloc}} = \$75.7$ trillion (56.3% of DFA total HH net worth).

Because the public DFA cutoff data are for total net worth rather than allocable wealth, we approximate allocable wealth as a constant fraction $\lambda = 0.563$ of total net worth within the relevant top-wealth intervals. For an allocable-wealth threshold $\bar{W}$, we therefore apply the Pareto interpolation to the total-net-worth cutoff $\bar{W}/\lambda$, then multiply the resulting total-net-worth amount by λ. The implied total-net-worth cutoffs are $8.9 million, $17.8 million, and $35.5 million for allocable thresholds of $5 million, $10 million, and $20 million. This yields HNW allocable wealth of $24.9 trillion, $17.2 trillion, and $11.6 trillion, respectively.

Pension and nonprofit components. All series are from FRED (Z.1 release, 2022Q4). The funded defined-benefit pension components are:

- Private DB funded assets (Z.1 Table L.115.a, series BOGZ1FL572000075Q): $2.9 trillion.
- State and local government DB funded assets (Z.1 Table L.120, series BOGZ1FL222-000075Q): $5.3 trillion.
- Federal DB funded assets (Z.1 Table L.119, series BOGZ1FL342000075Q): $2.4 trillion.

These sum to $W_{\text{DB}} = \$10.6$ trillion. DC plan assets are total private, state-and-local, and federal DC pension-fund assets (series BOGZ1FL594090055Q), giving $W_{\text{DC}} = \$9.1$ trillion. Nonprofit net worth is from Z.1 Table B.101.n (series BOGZ1FL162090005A), giving $W_{\text{NP}} = \$10.3$ trillion. The same table reports nonprofit total assets of $12.8 trillion and financial assets of $7.4 trillion, so the nonprofit allocable fraction is $\lambda_{\text{NP}} = 7.4/12.8 = 0.58$. This gives allocable nonprofit wealth of $W_{\text{NP}}^{\text{alloc}} = \$6.0$ trillion.

Total allocator-level wealth is

$$W_0 = W_{\text{HH}}^{\text{alloc}} + W_{\text{DB}} + W_{\text{DC}} + W_{\text{NP}}^{\text{alloc}} = \$101.4 \text{ trillion}. \tag{A.131}$$

Component values are in Table A.2.

Table A.2: Wealth-share calibration: component values

Component	Symbol	Value (tr. USD)	Source
Household wealth			
Allocable HH wealth	$W_{\text{HH}}^{\text{alloc}}$	75.7	DFA 2022Q3 minus non-allocable components
$\bar{W} \geq 5$ M threshold	$W_{\text{HH,HNW}}^{\text{alloc}}$	24.9	DFA Pareto interpolation
$\bar{W} \geq 10$ M threshold		17.2	DFA Pareto interpolation
$\bar{W} \geq 20$ M threshold		11.6	DFA Pareto interpolation
Funded DB pension assets			
Total funded DB assets	W_{DB}	10.6	Z.1 L.115.a/L.119/L.120
Private DB		2.9	BOGZ1FL572000075Q
State/local DB		5.3	BOGZ1FL222000075Q
Federal DB		2.4	BOGZ1FL342000075Q
DC plan assets	W_{DC}	9.1	BOGZ1FL594090055Q
Nonprofit net worth	W_{NP}	10.3	BOGZ1FL162090005A
Allocable nonprofit wealth	$W_{\text{NP}}^{\text{alloc}}$	6.0	Z.1 financial-assets haircut
Total allocator wealth	W_0	101.4	Sum of allocable components

A.3. Retail holding cost

Retail access. Balloch, Mainardi, Oh, and Vokata (2026) identify three main sources of retail-investor-level fee drag in private equity: (1) differences in LP terms across investor tiers, (2) access fees charged by platforms or other intermediaries, and (3) advisory fees.

Using their terminology, differences in LP terms across investor tiers refer to variation in the fees that limited partners pay to general partners, including differences in management fees and carried-interest rates. Begenau and Siriwardane (2024) also study this dimension of fee heterogeneity, though in the context of institutional investors. While this comparison is interesting, it is not essential for our purposes.

While differences in LP terms concern the fees paid within a given private-equity fund, a separate issue is the cost of gaining access to the asset class in the first place. In particular, it is useful to distinguish between access fees and advisory fees. Access fees are the charges paid to intermediaries that provide investors or advisors with access to a broader menu of private equity opportunities. These intermediaries may include platforms, feeder funds, investment consultants, and other specialized advisors. Advisory fees, by contrast, are the fees charged directly by financial advisors for investment advice, and these fees typically decline with client wealth.

Quantitatively, Balloch et al. report that the one-time median access fee is 1.9% of total contributions, and roughly 20% of investments in their cash-flow data incur such charges. Advisory fees are charged *annually* as a percentage of assets under management and range from 1.75% for smaller accounts to 0.50% for the largest accounts, and the fees are applied marginally by AUM tier.[22] If we sum the access and advisory fees in the \$250,000 to \$1 million bucket, and assume a one-period model, the total comes to 2.9% for retail investors.

Ewens and Faber (2026) provide another useful benchmark for calibrating c using registered retail private-market vehicles. In their return decomposition, they set quarterly retail-vehicle fees equal to $f^R = 67$ basis points and quarterly institutional drawdown-fund fees equal to $f^D = 45$ basis points.[23] The implied retail-minus-institutional access-cost wedge is therefore $4 \times (67 - 45) = 88$ basis points, or approximately 0.9% per year.

Industry reports provide a useful comparison. A benchmark comes from Cliffwater's 2024 review of 19 evergreen private-equity funds, which finds an average all-in expense ratio

[22] In one exercise, Balloch et al. assume PE assets from \$0 to \$400,000 are subject to an annual fee of 1.75%. The next \$350,000 of assets (i.e., from \$400,000 to \$750,000) is charged 1.25% annually, followed by 1.00% annually on the next \$250,000 (up to \$1,000,000), 0.75% annually on the next \$2,000,000 (up to \$3,000,000), 0.60% annually on the next \$7,000,000 (up to \$10,000,000), and 0.50% annually on the next \$15,000,000 as well as on all assets above \$25,000,000.

[23] See Ewens and Faber (2026), Section 4.3, p. 18. They state that the decomposition fixes average cash holdings at $w^c = 5.6\%$, quarterly drawdown fees at $f^D = 45$ bps, and quarterly retail fees at $f^R = 67$ bps. They also note that these values are asset-weighted averages and that drawdown fees are from Preqin.

of 2.91%, with a range from 0.96% to 5.49%.[24] Morningstar reports that, as of Q3 2025, semiliquid funds averaged an annual net expense ratio of over 3%.[25]

Specific fund disclosures show a similar pattern. For example, StepStone Private Markets reports total annual fund operating expenses of 3.19% for Class S, 2.59% for Class D, and 2.34% for Class I, alongside a 1.40% management fee, distribution and shareholder servicing fees of up to 0.85%, and, for some share classes, an upfront sales load of 3.50%.[26] Hamilton Lane Private Assets Fund reports expense ratios of 4.08% for Class R, 3.63% for Class D, and 3.38% for Class I, with a 1.50% base management fee and additional distribution or servicing charges for certain classes.[27] Carlyle's retail private markets fund is somewhat lower on the base fee, charging a 1.25% management fee, but still reports total annual expenses of 3.26% to 4.01% across share classes and may also impose sales loads of up to 3.00% and class-specific servicing fees.[28] Overall, these disclosures suggest that a realistic estimate for retail investors' cost of accessing private equity is roughly 3% per year on an all-in basis, with higher costs possible once sales loads, servicing fees, and incentive fees are included.[29]

Institutional access. The classic benchmark in private equity is the familiar "2 and 20" compensation structure, under which general partners charge an annual management fee of roughly 2% of committed or invested capital and retain about 20% of profits as carried interest.[30] More broadly, the literature often describes management fees as falling in the range of about 1% to 2% of committed capital, with carried interest typically set at 20%.[31] In addition to these headline terms, private-equity funds may also generate transaction and monitoring fees at the portfolio-company level, so the full economic cost of private equity can exceed the standard management-fee-and-carry benchmark.[32]

Overall, the evidence suggests that retail investors face higher costs to access private equity than institutional investors. If one takes 2% as a conservative benchmark for the traditional institutional management fee and 2.9% as a benchmark for the average all-in cost of retail evergreen vehicles, the implied difference is roughly 0.9 percentage points, similar to

[24]See Cliffwater, *The Price of Perpetuity: Understanding Fees in Evergreen Private Equity*, August 1, 2024, available here.

[25]Morningstar, "The Rise of Evergreen Funds: A New Way to Access Private Markets," available here.

[26]StepStone Private Markets Prospectus, available here.

[27]Hamilton Lane Private Assets Fund Prospectus, available here.

[28]Carlyle AlpInvest Private Markets Fund Factsheet, available here.

[29]Cliffwater, *The Price of Perpetuity: Understanding Fees in Evergreen Private Equity*, August 1, 2024, available here.

[30]See Metrick and Yasuda (2010), who describe the typical private-equity fund as following a "2/20/1" structure: roughly 2% management fees, 20% carried interest, and 1% GP ownership; and Robinson and Sensoy (2013), who study compensation, ownership, and cash-flow performance in private-equity funds.

[31]Kaplan and Strömberg (2009) note that private-equity partnerships typically compensate general partners through management fees equal to about 1% to 2% of committed capital together with carried interest usually equal to 20% of profits.

[32]See Kaplan and Strömberg (2009); see also Phalippou, Rauch, and Umber (2018), who study private-equity portfolio-company fees.

the value implied by Ewens and Faber (2026). Given that some funds and industry reports place the fees of retail funds above 2.9%, we allow values of the fee gap, c, as high as 2%. We allow values of c as low as zero to reflect a scenario in which democratization reaches a level of scale and efficiency such that retail investors face the same costs as institutional investors.

A.4. Profitability estimates for private vs. public firms

The accounting data for private companies are challenging to obtain. Relatively few studies, including the two described below, assemble private-firm data and report corresponding ROA measures.

Asker, Farre-Mensa, and Ljungqvist (2015) study U.S.-incorporated firms, using Compustat for public companies and Sageworks for private companies.[33] They measure ROA for public and private firms as operating income before depreciation divided by beginning-of-year total assets. They report ROA moments for both the full sample and a matched sample. The matched sample is constructed using nearest-neighbor matching in panel data: for each public firm, they select the private firm in the same four-digit NAICS industry with the closest level of total assets, subject to a caliper requiring the ratio of total assets between the matched pair to be less than 2. Matches are formed with replacement and then preserved in subsequent years to maintain the panel structure. For their full sample, they report a mean ROA of 6.5% for public firms and 7.5% for private firms, while the median ROA is 11.1% for public and 9.5% for private. In their matched sample, the mean ROA is -6.0% for public and 8.4% for private, while the medians are 5.1% and 12.3%.

Acharya and Xu (2017) also report ROAs for public and private firms. Their study focuses on U.S. private and public firms over 1994–2004 and draws financial statement data from S&P Capital IQ, which covers both listed firms and a sample of private firms.[34] They construct an industry-and-size matched sample by pairing each private firm with a public firm in the same three-digit SIC industry that is closest in size. In their study, ROA is defined as EBITDA divided by total assets. In their full sample, they report average ROA of 2.67% for private firms and 3.79% for public firms; in the matched sample, average ROA is 5.78% for private and 4.95% for public.

[33] Sageworks provides income-statement and balance-sheet information for privately held firms. The data are supplied by accounting firms on behalf of their clients rather than by firms directly. Specific sample filters are described in Section 2.1 of Asker, Farre-Mensa, and Ljungqvist (2015).

[34] They note that S&P Capital IQ provides coverage of U.S. private firms with minimum revenues of $5 million or with public debt issuances.

A.5. Sensitivity of investment to the cost of capital

The calibration in Section 7 sets a range for the sensitivity of investment to cost of capital. This appendix explains how we obtain that range, using the results from Zwick and Mahon (2017). The notation in this appendix is self contained and corresponds to their paper.

Zwick and Mahon estimate the effects on corporate investment of plausibly exogenous changes in one component of the user cost of capital. The user cost of capital, UC, can be written as

$$UC = Ax, \tag{A.132}$$

where

$$A = q(r + \delta - \pi) \tag{A.133}$$

and

$$x = \frac{1 - \tau z}{1 - \tau}. \tag{A.134}$$

Here, q is the capital-goods price, which we normalize to 1; r is the required return; δ is the economic depreciation rate; π is expected capital-goods inflation; τ is the corporate tax rate; and z is the present value of depreciation deductions per dollar of investment. The term A is the non-tax user-cost component. The term x is the tax adjustment factor, or tax wedge component of user cost. The factor x affects UC because depreciation deductions reduce the after-tax cost of acquiring capital: a higher z means a larger present value of tax shields, lowering the numerator $1 - \tau z$. The denominator $1 - \tau$ grosses up the required after-tax return into a pre-tax rental cost, so x captures how the corporate tax system scales the non-tax user-cost component $A = q(r + \delta - \pi)$.

Zwick and Mahon's identification strategy exploits plausibly exogenous changes in z (and hence x) coming from temporary changes in bonus depreciation. They compute z in a sample of more than 120,000 firms drawn from roughly two million corporate tax returns. We use Zwick and Mahon's tax-term regressions, which take the form

$$\frac{I_{i,t}}{K_{i,t-1}} = \alpha_i + \delta_t + \beta x_{N,t} + \varepsilon_{i,t}, \tag{A.135}$$

where i indexes firms and N indexes the firms' industries.

We translate the coefficient β into the effect of a given change in UC on $I_{i,t}/K_{i,t-1}$, as follows. Holding A fixed, equation (A.132) implies that a change in x changes the user cost according to

$$\Delta UC = A\Delta x. \tag{A.136}$$

According to the regression (A.135),

$$\frac{\Delta(I/K)}{\Delta x} = \beta. \tag{A.137}$$

Therefore, the object we target equals

$$\frac{\Delta(I/K)}{\Delta UC} = \frac{\beta}{A}. \tag{A.138}$$

We use Zwick and Mahon's baseline value $r = 0.07$. We choose $\delta = 0.10$, a common annual value in RBC and business-cycle calibrations. We also set $\pi = 0$, which is common in calibrations that normalize $q = 1$, as we do here. Substituting these values into (A.133) yields $A = 0.17$. Thus,

$$\frac{\Delta(I/K)}{\Delta UC} = \frac{\beta}{0.17}. \tag{A.139}$$

Zwick and Mahon (2017) report several estimates of β from their own analysis and from previous studies. We construct a range for β that nests all those estimates. Our lower bound on β is -2.27, the lowest out of six β estimates from Panel C of Zwick and Mahon's Table 3. Figure 4 Panel A in Zwick and Mahon (2017) shows that -2.27 is lower than 18 estimates from preexisting studies: Cummins, Hassett, Hubbard, Hall, and Caballero (1994), Hassett and Hubbard (2002), Cummins, Hassett, and Hubbard (1996), Desai and Goolsbee (2004), and Edgerton (2010). Our upper bound for β is -0.178, the maximum of the 18 estimates from those previous studies, as reported in Zwick and Mahon's Figure 4 Panel A. That upper bound exceeds the β values estimated by Zwick and Mahon, including in the largest decile of firms, as shown in their Figure 4 Panel B.

The value of β most relevant to our analysis is likely closer to -0.178 than -2.27. Ideally, we would use β estimates relevant to very large firms. The two firms in our model represent aggregates of firms in the public and private sectors. Large firms dominate in those aggregates. One could even interpret each firm in our model as a single mega-sized firm. Figure 4 Panel B in Zwick and Mahon (2017) shows that their β estimates strongly increase in firm size. Therefore, a high value of β is most relevant to our analysis. Nevertheless, our analysis uses the full range of beta values in the previous paragraph.

Dividing our β range by 0.17 provides a range for the investment sensitivity:

$$\frac{\Delta(I/K)}{\Delta UC} \in [-13.35, -1.05]. \tag{A.140}$$

We map $\Delta(I/K)$ to our model's quantity $\Delta(I/K) = I_2^{\text{post}}/K_2^0 - I_2^{\text{pre}}/K_2^0$, defined in (83). The calibration criteria in Table 1 include the condition that $I_2^{\text{pre}} = 0$, i.e. $K_2^0 = K_2^{\text{pre}}$, so $\Delta(I/K) = I_2^{\text{post}}/K_2^0$, the private firm's investment rate in the post economy. We map ΔUC to $\Delta \text{E}(\tilde{r}_2) \equiv \text{E}(\tilde{r}_2^{\text{post}}) - \text{E}(\tilde{r}_2^{\text{pre}})$, defined in (84), the change in the private firm's expected excess return from the pre to the post economy. We take $\text{E}(\tilde{r}_2^{\text{post}})$ from the partial-equilibrium model to avoid confounding the change in cost of capital with a change in K from pre to post. In summary, using the elasticity $\mathcal{E}$ defined in (82), the bound we impose is

$$\mathcal{E} = \frac{I_2^{\text{post}}/K_2^0}{\text{E}(\tilde{r}_2^{\text{post}}) - \text{E}(\tilde{r}_2^{\text{pre}})} \in [-13.35, -1.05]. \tag{A.141}$$