Retail Capital as a Stepping Stone in Venture Capital: Theory and Empirics *

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Abstract

What are the equilibrium allocation and welfare effects of retail investor entry into private markets? The conventional debate focuses on direct effects: potential benefits (portfolio diversification, access to high-return assets) versus costs (excessive fees, inefficient capital allocation by retail investors). We identify a countervailing mechanism: retail investors fund experimentation with unproven general partners (GPs), enabling institutions to free-ride by poaching proven talent. We document that 34% of value created by institutional investments in private markets comes from GPs who started with retail capital. We formalize this *stepping-stone* mechanism by extending Berk and Green (2004) to venture capital with heterogeneous investors and uncertain manager skill. New GPs with moderate perceived ability initially raise retail capital to reveal their true skill; after observing performance, skilled GPs graduate to institutional capital while unsuccessful GPs exit. Retail capital in our setting plays the role of experimental capital: it funds learning that later, better-informed institutions can exploit. Our calibrated model shows that retail investor entry increases aggregate welfare when stepping-stone benefits exceed inefficient allocation costs.

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1 Introduction

What are the equilibrium allocation and welfare effects of retail investor entry into private markets? Historically, such investment opportunities have long been closed to retail investors. The JOBS Act (2012) and the SEC's 2020 accredited-investor amendments in the United States, MiFID II (2018) in Europe, and the revised ELTIF framework (2023) have begun to permit retail participation.[1]

The conventional debate about retail access focuses on direct effects weighing potential benefits against costs (Cynthia Balloch, Federico Mainardi, Sangmin Oh and Petra Vokata, 2025; Shane Miller, Emmanuel Yimfor and Ye Zhang, 2025). On the benefit side, retail investors could diversify their portfolios and gain access to high-return assets, contributing to financial democratization. On the cost side, retail investors may face excessive fees, more liquidity constraints, lack the sophistication to evaluate investments properly, and allocate capital inefficiently, potentially leading to net wealth destruction. This traditional view evaluates retail access primarily through its impact on retail investors' own returns.

This perspective misses a first-order indirect effect: retail capital produces a skill signal that the rest of the private market consumes. To test this mechanism, we use venture capital (VC) as a laboratory setting. In this market, GP skill is difficult to observe ex ante and takes time to reveal, while investment outcomes are noisy and partly driven by luck, making talent discovery a persistent challenge. In addition, new GPs face limited access to institutional capital due to track record requirements, creating a natural role for retail investors in early-stage funding.

We begin by documenting the pattern. Figure 1 shows that most new general partners (GPs) raise their first funds from retail (individual) investors and transition to institutional

[1] In this paper, we use individual investors and retail investors interchangeably.

capital as they establish track records. Substantial numbers of first- and second-sequence funds rely on retail capital, and institutional capital dominates later sequences.

This pattern has major economic implications. We show that 34% of the value created by institutional VC investments comes from GPs who initially raised retail capital. This fraction is even higher for established funds: among GPs managing their ninth fund or later, those who started with retail capital now account for 67% of institutional assets under management and 44% of institutional value creation. Since institutional capital dominates overall value creation in VC, retail investors' role in talent discovery generates substantial positive externalities for the entire ecosystem. Rather than serving merely as a temporary funding source, retail capital functions as a crucial pathway for discovering talented managers.

Retail investors play a critical role in information production. By funding unproven GPs, retail investors generate performance signals that reveal manager skill. Institutional investors then benefit by selectively investing in GPs who have demonstrated talent with retail capital. We formalize this stepping-stone mechanism through a model of VC fundraising with heterogeneous investors and uncertain GP skill.

Building on the skill-learning framework of Berk and Green (2004), we incorporate key institutional features of VC. Unlike mutual funds, VC involves sequential fundraising with infrequent learning signals over long fund cycles (typically 7–10 years), limiting opportunities to learn about GP skill. Investor types are heterogeneous: institutional investors have lower administrative costs per dollar but require larger minimum fund sizes, while retail investors accept lower expected returns due to access constraints, portfolio benefits, or behavioral factors. Finally, VC creates real value through startup funding, contrasting with the near zero-sum redistribution in mutual fund settings. These features generate market segmentation and information production dynamics absent from the original Berk-Green

framework.

In equilibrium, this segmentation takes the following form. GPs with low perceived skill cannot raise capital from any investors. Those with moderate perceived skill can only access retail investors, whose lower return requirements and smaller minimum fund sizes make them accessible to unproven GPs. Those with high perceived skill prefer institutional capital due to its lower costs and larger scale economies.

We depart from Berk and Green (2004) in ways that are not ad hoc. In Berk-Green, skilled managers absorb inflows by parking excess capital in a passive index. The active sleeve stays at its optimal size, fees are charged on total assets under management, and investors' expected net-of-fee returns fall to zero each period. VC has no such margin. The fund is fully committed to startups, which face decreasing returns to scale, and fundraising imposes a binding minimum size. Moreover, VC compensation consists of both management fees and performance-based carry, with the latter tied directly to investment profits. Together, these leave highly skilled GP's interior optimum strictly below the scale at which the investor participation constraint binds, so LPs of highly skilled GPs earn positive expected net-of-fee returns.

Segmentation and infrequent learning reinforce this wedge and generate performance persistence by slowing the reallocation of capital across GPs. Retail LPs accept lower net returns and fund smaller vehicles, while institutions require larger scale but impose lower operating costs on the GP. Skill signals arrive only at fund maturity, so capital cannot continuously re-equilibrate between fundraises. As a result, capital cannot flow quickly enough toward high-skill managers to arbitrage away excess returns. GPs who succeed with retail capital graduate to institutional funds, and positive selection on posterior skill delivers rents that persist across subsequent funds. This is consistent with the documented

persistence of net-of-fee returns in VC (Kaplan and Schoar, 2005; Harris, Jenkinson, Kaplan and Stucke, 2023; Nanda, Samila and Sorenson, 2020).

The information production role we identify for retail investors is the upstream complement to the role of costly learning by informed investors in Berk and DeMarzo (2026) (BDM hereafter). BDM use an optimal contracting framework to show how fees and fundraising limits can implement constrained first-best allocations for managers of illiquid funds. Their approach focuses on how a single manager can incentivize optimal costly information production by informed investors through their fee structure. In contrast, we embed the problem of learning GP skill into a Berk and Green (2004) environment in which skill is revealed primarily through realized performance, and highlight how retail investor participation allows the market as a whole to benefit from sampling more frequently from the distribution of manager skill. In sum, BDM analyze how sophisticated investors optimally produce and certify information about managers whose viability is already established, while our stepping-stone mechanism shows how retail investors enable managers to generate the track records that make such certification possible in the first place.

Our model delivers four testable predictions, which we take to the data using a sample of 2,212 VC funds matched between SEC Form ADV filings and PitchBook. First, GPs sort by perceived skill into no-fund, retail, and institutional regions, so retail capital should be more prevalent at lower fund sequences. The data confirm this directly: the institutional share of fund counts rises monotonically from approximately 30% at fund 1 to over 60% by fund 5. Second, stronger first-fund performance predicts whether retail-backed GPs later raise institutional capital. Among first-time retail GPs, those in the top quartile of first-fund exit rates are roughly 36 percentage points more likely to raise a later fund from institutional investors than those in the bottom quartile.

Third, the model predicts that GPs should raise larger funds when they move from retail to institutional capital, unless retail-backed GPs were already raising unusually large funds by extracting rents from retail investors. In the data, fund size rises sharply at this transition: graduating GPs raise funds that are $72.8 million larger on average, with a confidence interval well above zero. This evidence supports the stepping-stone mechanism at the point of transition. Fourth, retail-backed GPs account for a large share of the value later created in the institutional VC market. Following Braun, Dorau, Jenkinson and Urban (2025), we measure value added as the dollar value created by VC investments relative to a public-market benchmark. We find that GPs who began with retail capital account for 34% of value added by institutional VC funds in our sample, or roughly 21% of total VC value creation. This share rises steadily across fund sequences. Together with the positive fund-size jump at the transition to institutional capital, this evidence suggests that retail capital helps discover GPs who later create substantial value for institutional investors.

Our paper contributes to four strands of the literature. First, we add to the policy debate on retail access to private markets. Prior work focuses on investor protection (Miller, Yimfor and Zhang, 2025), micro-level regulatory outcomes (Howell, Parker and Xu, 2025), and institutional structures and sophistication in private markets (Rosenberg, 2002; Sensoy, Wang and Weisbach, 2014; Miller, Yimfor and Zhang, 2025). We develop a theoretical framework, supported by empirical evidence, to examine the macro welfare effects of expanding retail access. We identify a positive externality: retail participation generates information about GP skill, improves talent discovery, and shifts the institutional segment toward more efficient capital allocation.

We also contribute to the literature on learning about GP skill in delegated asset management by showing how investor heterogeneity shapes information production and, in turn,

capital allocation dynamics. We adapt the framework of Berk and Green (2004) to private markets, where features such as illiquidity, long investment horizons, and sequential fundraising alter learning and equilibrium outcomes (Metrick and Yasuda, 2010; Cochrane, 2005). In this setting, retail capital serves as a stepping stone by generating information about GP skill and facilitating talent discovery. Our approach complements Berk and DeMarzo (2026), who emphasize a due diligence channel based on ex ante information acquisition. In contrast, we focus on a performance-based learning channel, where skill is revealed through realized performance over time, and show how investor heterogeneity and sequential fundraising shape the dynamics of skill revelation and capital allocation.

We further contribute to the literature on performance persistence in VC and private equity. A large empirical literature documents persistent performance in VC and PE (Kaplan and Schoar, 2005; Nanda, Samila and Sorenson, 2020; Harris, Jenkinson, Kaplan and Stucke, 2023), in contrast to the weak persistence observed in mutual funds (Carhart, 1997). Existing explanations primarily emphasize supply-side factors, including differences in GP skill (Korteweg and Sorensen, 2017), privileged access to deal flow (Nanda, Samila and Sorenson, 2020), network effects that create barriers to entry (Hochberg, Ljungqvist and Lu, 2007), bargaining advantages of high-quality GPs in deal making (Ebrahimian and Zhang, 2025), and market segmentation that limits competitive arbitrage (Lerner, Schoar and Wongsunwai, 2007). We complement this literature by introducing a demand-side mechanism: the interaction between skill uncertainty and investor heterogeneity generates structural persistence that is not eliminated by capital flows, even as investors learn about GP skill over time. This perspective highlights how investor segmentation shapes equilibrium outcomes and helps explain why performance differences persist even as the VC market matures.

Fourth, we contribute to the literature on entry and stepping-stone effects. Work across

industrial organization, labor economics, and information theory shows that market entry and segmentation can generate value through learning and knowledge spillovers (Arrow, 1962; Carroll, 1985; Bourreau and Drouard, 2010; Glaeser and Kerr, 2010). That literature is largely operational and offers limited guidance for financial markets. We extend it to VC by showing that entry by retail capital allows less established GPs to generate skill signals that institutional investors subsequently act on.

The remainder of the paper is organized as follows. Section 2 presents the model. Section 2.5.1 characterizes optimal fund size and investor choice. Section 3 studies the stepping-stone mechanism and its welfare implications. Section 4 describes the data. Section 5 presents quantitative and empirical evidence. Section 6 concludes.

2 Model Setup

We consider a two-period model where GPs and investors share uncertainty about the GP's skill. GPs raise capital at times $t = 0$ and $t = 1$, invest, observe returns at times $t = 1$ and $t = 2$, and all market participants update beliefs about skill symmetrically based on realized investment performance. GPs can raise capital from a single, deep-pocketed investor of type $j \in \{I, R\}$ with I for institutional investors and R for retail investors. We assume investment performance is risk-adjusted throughout, so all agents are risk neutral and discount future payoffs at a common per-period factor β.

We present a simplified version of the model in the following sections, and discuss the full version in Appendix A. Throughout, our calibrated examples and proofs come from the full version of the model to confirm that the intuition of the simple model we present below extends to the general case. We summarize parameters of the model and the values we use

for a typical calibration of the full model in Table 1.

[Table 1 About Here]

2.1 Investment Technology and Payoffs of Market Participants

Any GP that successfully raises a fund at time t realizes a return on their fund investments at time $t+1$ of:

$$r_{j,t+1} = \theta - bA_{j,t} + \epsilon_{t+1}. \tag{1}$$

Where θ is the skill of the GP and $A_{j,t}$ is the size of the fund raised from investor type j. The parameter $b > 0$ enforces decreasing returns to the scale of the fund similar to its role in Berk and Green (2004). This ensures a finite interior optimum fund size for the manager, since their ability to generate risk-adjusted performance declines with the scale of the fund (Bhardwaj, Gupta, Howell and Zimmerschied, 2025). $\epsilon_{t+1} \sim N(0, \sigma_\epsilon^2)$ represents residual performance uncertainty that limits the ability of market participants to learn skill from performance at time 1.

The realized return is divided among the GP and investors based on a standard management and performance-based fee structure. GPs charge a management fee of m on assets under management, generating $m \cdot A_{j,t}$ for the GP in each period the GP is funded. GPs also charge a performance fee of p on realized returns in excess of the institutional investor's outside option, which we normalize to 0. To focus the simplified model narrowly on new mechanisms, we omit manager limited liability and the resulting option value of performance fees, including these forces in the full model in Appendix A. The manager therefore generates performance fees of $p(r_{j,t+1} - m)$. Thus, the GP's payoff or profit $\Pi_{j,t+1}$ after successfully raising a fund of size $A_{j,t}$ at time t from investors of type j is the fees earned net of operating

costs paid by the GP, c_j, which depend on investor type, per dollar raised:

$$\Pi_{j,t+1} = A_{j,t}(pr_{j,t+1} + (1-p)m - c_j). \tag{2}$$

The investor's payoff $\Psi_{j,t+1}$ is the fund return net of fees charged scaled to the fund assets:

$$\Psi_{j,t+1} = A_{j,t}(1-p)(r_{j,t+1} - m). \tag{3}$$

Finally, we assume that a GP must raise a fund larger than some lower bound $\underline{A}_j > 0$ for investors of a given type to be willing to participate. This is consistent with the existence of failed fundraises and observed minimum professional VC fund sizes in practice—the GP must be able to raise at least some minimum amount of capital to have a sufficiently diversified fund capable of funding necessary follow-on rounds for its successful portfolio companies.

2.2 Institutional Versus Retail Capital

Equation (2) and (3) highlight the fact that in our simplified baseline model, institutional and retail investors differ along two dimensions. First, managers pay an operating cost c_j per dollar of funds raised $A_{j,t}$ in each period, which is higher when capital is raised from retail investors, $c_R > c_I$. This parameter captures the benefits that accrue to a manager from raising institutional capital, which could include both direct reductions in costs associated with having a more concentrated or sophisticated investor base as well as benefits offsetting these costs accruing to the manager, like improved capital access or stability, that raising institutional capital provides. The difference in costs and benefits of raising capital from different investor types is critical to our mechanism; lower relative costs for raising capital from institutional investors may motivate GP's to prioritize institutional capital in fundraising.

The second critical difference for our mechanism is the difference in minimum fund sizes necessary for a successful fund raise, $\underline{A}_j$. When raising capital from retail investors, this minimum is assumed to be lower throughout, $\underline{A}_I > \underline{A}_R \geq 0$, consistent with retail investors having greater willingness to bear costs associated with insufficient minimum fund scale or smaller investment sizes per fund. The nonnegative minimum scale of funds raised from each investor type introduces the possibility that the maximum amount of capital that can be raised given investor participation constraints might be smaller than the minimum required fund size. In such cases, the GP cannot raise a fund with that investor type.

In the full model we also allow for differences in the relative perceived quality of investors' outside options, parameterized by a positive per-dollar utility adjustment to investor payoffs in units of expected risk-adjusted performance of δ_j, with higher values representing inferior outside options. Thus, investor realized profits net of the outside option are:

$$\Psi_{j,t+1} = A_{j,t}(1-p)(r_{j,t+1} - m) + \delta_j. \tag{4}$$

We assume that $\delta \equiv \delta_R \geq \delta_I = 0$ throughout, normalizing institutional outside options to 0 and denoting retail outside options by δ when it would not cause confusion. The assumption $\delta_R \geq \delta_I$ represents the inferiority of perceived outside options for retail investors, which raises their willingness to invest with a GP of fixed skill relative to institutions. Several well-documented factors contribute to the likelihood that $\delta_R > \delta_I$ in practice, among them access constraints, portfolio fit, financial constraints, and behavioral and informational factors.[2]

In summary, the stepping-stone mechanism we introduce in our model and document

[2]The parameter δ captures the net effect of the above factors in reduced form. It can be interpreted either as reflecting inherent differences between investor types or as a characteristic that can be influenced by policy choices regarding disclosure, education, and access restrictions. This difference is not critical to the mechanism, but helps in explaining the empirical distribution of fund sizes in the model and introduces novel welfare considerations.

empirically below relies on two key differences between institutional and retail investors. Raising capital from institutions provides a relative benefit to GPs, and retail investors are more willing to supply capital to small funds. We discuss empirical support for each mechanism in light of our model conclusions further in Section 4. Next, we lay out the symmetric learning environment that underpins the stepping-stone mechanism.

2.3 Symmetric Learning About Skill

The second component of our model is symmetric learning about GP skill by all market participants. In contrast to the private signals generated by informed investors in Berk and DeMarzo (2026), all market participants in our model learn symmetrically from observable manager performance about the skill level of managers. The result is that when a manager is successfully funded at time 0, their performance generates a public signal about their skill, which can then be exploited by any investor and the GP when fundraising at time 1. A time 0 investor's successful capital investment thus generates a public good in the form of information on the skill of the manager.

We parameterize this learning as follows. The GP's true skill, θ, is unknown to all participants in the market, however they have a shared normal prior on GP skill $\theta \sim N(\mu_0, \sigma_0^2)$. The expected return given an expected skill belief of μ_t is straightforward:

$$E[r_{j,t+1}|\mu_t] = \mu_t - bA_{j,t} \tag{5}$$

After observing the return of a fund at time 1, all market participants update their priors to a time 1 posterior expected skill of μ_1. A standard result from Bayesian learning with normal priors and signals guarantees a normal posterior distribution at time 1: $\theta \sim N(\mu_1, \sigma_1^2)$

with:

$$\begin{aligned} \mu_1 &= \tfrac{\sigma_0^2(r_{j,1}+bA_{j,0})+\sigma_\epsilon^2\mu_0}{\sigma_0^2+\sigma_\epsilon^2} \\ \sigma_1^2 &= \tfrac{\sigma_0^2\sigma_\epsilon^2}{\sigma_0^2+\sigma_\epsilon^2} \end{aligned} \tag{6}$$

The first line highlights how the fund's realized return generates a signal on skill: after adjusting for decreasing returns to scale of the fund, all market participants update their skill beliefs in the direction of the realized return.

2.4 Expected Payoffs and the GP's Problem

The GP's expected profit for a given fund size is:

$$E[\Pi_{j,t+1}|\mu_t] = A_{j,t}\big(p(\mu_t - bA_{j,t}) + (1-p)m - c_j\big) \tag{7}$$

Denote the manager's policy function for fund size as $A^*_{j,t}(\mu_t)$. The investor's expected payoff if chosen to supply capital to a fund of size $A^*_{j,t}(\mu_t)$ is:

$$U_{j,t}(\mu_t) = E[\Psi_{j,t+1}|\mu_t] = A^*_{j,t}(\mu_t)(1-p)\left(\mu_t - bA^*_{j,t}(\mu_t) - m\right) \tag{8}$$

Where $A^*_{j,t}(\mu_t) = 0$ if the investor type is not chosen by the manager. Because they are deep-pocketed and risk-neutral, investors' willingness to participate in a fund is characterized by conditions ensuring nonnegative expected profits and sufficient minimum scale:

$$A_{j,t}(1-p)\left(\mu_t - bA_{j,t} - m\right) + \beta E[U_{j,t+1}(\mu_{t+1})|\mu_t] \geq 0, \qquad A_{j,t} \geq \underline{A}_j \text{ or } A_{j,t} = 0 \tag{9}$$

where $E[U_{j,t+1}(\mu_{t+1})|\mu_t]$ is the nonnegative continuation value that accrues to an investor at time 0 because of the potential to reinvest with a manager at time 1, which is 0 at time 1 because $t = 2$ is the terminal period.[3]

The GP's problem is the choice of investor type and fund size maximizing expected profit subject to these constraints. At time 1, the GP optimizes:

$$V_1(\mu_1) = \max_{j \in \{I,R,\emptyset\},\ A_{j,1}} \{E[\Pi_{I,2}|\mu_1], E[\Pi_{R,2}|\mu_1], 0\} \tag{10}$$

At time 0, anticipating future learning:

$$V_0(\mu_0) = \max_{j \in \{I,R,\emptyset\},\ A_{j,0}} \{E[\Pi_{I,1}|\mu_0] + \beta E[V_1(\mu_1)|\mu_0], E[\Pi_{R,1}|\mu_0] + \beta E[V_1(\mu_1)|\mu_0], 0\} \tag{11}$$

The GP's continuation value $\beta E[V_1(\mu_1)|\mu_0]$ is positive, weakly monotone increasing in μ_0, and independent of investor type or fund size chosen at 0 as long as a fund is raised. This is because the GP can re-optimize investor choice and fund size decisions in the next fundraising round as long as it successfully raises a fund today.[4] At worst, the GP shuts down tomorrow, the third term in (10), but depending on its updated skill μ_1, the GP may choose to raise a larger or smaller fund or from institutions or retail. Raising a fund at $t = 0$ gives the GP an opportunity to generate a performance signal. Because all market participants observe this

[3]We implicitly assume that managers attempt to raise funds again from the same investors, not just investor types, if they do not change their fundraising strategy, which is consistent with GP behavior in practice. Note that investors may benefit from the manager being able to raise a fund from either type at 0 even if they are not chosen to fund it because the manager may choose the other investor type next period. To simplify the analysis, we assume neither investor type internalizes the strategic implications of attempting to force the other group to bear search costs. We also show that sufficiently large differences in minimum fund scale also rule out its relevance in the full model. This strategic motive does not exist at all in the other market structures we consider below.

[4]As we show below, fundraising constraints are weakly tighter at $t = 1$, so a manager that fails to raise any fund at $t = 0$ and generates no performance signal will also be unable to raise a fund at $t = 1$.

signal and adjust for fund scale, the signal is the same object regardless of whether the fund was raised from retail or institutional investors: $r_{j,1} + bA_{j,0} = \theta + \epsilon_1$. As a result, any GP that raises a fund at $t = 0$ enters the next period with the same set of options conditional on the updated belief: raise retail capital, raise institutional capital, or shut down. The continuation value therefore captures the GP's option value from learning about skill before the next fundraising round.

We simplify the statement of the GP's problem by noting that there is a maximum fund size, $\bar{A}_{j,t}$, that can be raised from each investor given their first participation constraint. This arises because decreasing returns to scale $b > 0$ attenuate the per-dollar investor payoff as fund size increases, eventually resulting in negative expected performance per dollar for investors for sufficiently large funds. This means the constrained maximum fund scale is defined by a fund size sufficiently large that it sets the investor's expected payoff to 0; for lager sizes the investor would not participate in the fund and select their outside option:

$$\bar{A}_{j,t}(1-p)\left(\mu_t - b\bar{A}_{j,t} - m\right) + \beta E[U_{j,t+1}(\mu_{t+1})|\mu_t] = 0 \tag{12}$$

At time 1 this upper bound has a simple form because the continuation value is 0, given $t = 2$ is the terminal date:

$$\bar{A}_1(\mu_1) = \bar{A}_{I,1} = \bar{A}_{R,1} = \frac{\mu_1 - m}{b} \tag{13}$$

So, the maximum fund scale increases at rate $1/b$ as skill rises. At time 0 types with large continuation values tend to have larger constrained maximum scales, as the GP can continue to expand their time 0 fund until this investor benefit is exhausted by diminishing returns to scale as well.

Further, there is a minimum expected skill $\mu^f_{j,t}$ below which funds cannot be raised from a given investor type given their second participation constraint. This is the feasibility bound below which even a fund raised at the minimum scale $\underline{A}_j$ would not be profitable for investors—at larger fund scales diminishing returns to scale guarantee the fund would also be unprofitable to investors. Below $\mu^f_{j,t}$, the manager does not have sufficient expected skill to raise a fund from that investor type and guarantee nonnegative expected payoffs, so the investor type prefers their outside option to any fund satisfying their minimum fund size constraint. Thus, the minimum feasible expected skill is defined by the skill level that sets the investor's expected payoff to 0 at the minimum fund scale:

$$\underline{A}_j(1-p)\left(\underline{\mu}_{j,t} - b\underline{A}_j - m\right) + \beta E[U_{j,t+1}(\mu_{t+1})|\underline{\mu}_{j,t}] = 0 \tag{14}$$

Again, at time 1 this is simplified due to the lack of a continuation value because $t = 2$ is the terminal date:

$$\mu^f_I = b\underline{A}_I + m, \qquad \mu^f_R = b\underline{A}_R + m \tag{15}$$

So retail investors are willing to fund managers with lower expected skill because $\underline{A}_R < \underline{A}_I$, a fact which carries through to time 0 under our parameter restrictions. Since the minimum feasible skill and maximum feasible size constraints come from different investor participation constraints, it is possible that the maximum feasible fund size is smaller than the minimum feasible fund scale — in this case the GP fails to raise a fund from that investor type.

The final element of the GP solution is the unconstrained optimal fund scale and expected profits by investor type. This is the fund scale at which the manager's expected profit is maximized irrespective of the investor's participation constraints, which arises because of

the fund's decreasing returns to scale.[5] This scale is independent of investor or manager continuation values because fund scale choice does not affect the performance signal, and so does not change the options the GP will face next period as long as it raises a fund. Thus, the unconstrained optimal fund scale and profits are given by the first order condition of (7):

$$A^u_{j,t} = \frac{(1-p)m + p\mu_t - c_j}{2pb} \qquad E[\Pi^u_{j,t+1}|\mu_t] = \frac{((1-p)m + p\mu_t - c_j)^2}{4pb} \tag{16}$$

Since $c_I < c_R$, a fund raised from institutions at the unconstrained optimal scale is larger and strictly more profitable than a fund similarly raised from retail for any fixed μ_t. When comparing across investors, the GP evaluates whether they would raise a fund at constrained maximum scale $\bar{A}_{j,t}$ or unconstrained optimal scale $A^u_{j,t}$ given their investor choice, so the range of expected skill at which, given j, $\bar{A}_{j,t}$ or $A^u_{j,t}$ is the optimal fund scale choice is critical. Since $\partial A^u_{j,t}/\partial\mu_t = 1/(2b) < 1/b = \partial\bar{A}_{j,t}/\partial\mu_t$, there is at most a single crossing as skill rises for each type above which unconstrained scale is optimal relative to constrained maximum scale. This crossing, $\mu^h_{j,t}$, is the high level of skill above which the manager would raise a fund at unconstrained optimal scale given it chooses investor type j. This crossing occurs at the expected skill where maximum scale and unconstrained optimal scale are equal, which has a simple form at time 1:

$$\mu^h_{j,t} = \frac{(1+p)m - c_j}{p} \tag{17}$$

Since $c_R > c_I$, $\mu^h_{I,1} > \mu^h_{R,1}$ and raising retail funds at unconstrained optimal scale rather than constrained maximum scale becomes optimal at lower levels of expected skill than for institutional funds. That is, the skill level at which a GP's optimal size choice given investor choice switches from the constrained maximum to the unconstrained optimal scale is lower

[5] In the full model an additional technical constraint is needed to guarantee decreasing returns to scale holds for all $A_{j,t} \in [\underline{A}_j, \bar{A}_{j,t}]$ because of the option value of fees, which we detail in Appendix A.

for retail due to its operational cost disadvantage. This fact is potentially consequential because when both types are feasible and the GP size choice given investor choice is the unconstrained optimal scale rather than constrained maximum scale for both, institutions' cost advantage guarantees that raising an institutional fund is more profitable from (16).

2.5 Investor Choice

GP investor choice is determined by tracing out how investor type feasibility and optimal fund scale and profits given investor choice change with expected skill. Our first main result for the full model summarizes the conclusions for investor choice at both fundraising periods $t \in \{0, 1\}$:

Proposition 1 (Positive Assortative Matching)**.**

If the difference in minimum fund scales $\underline{A}_I - \underline{A}_R$ is sufficiently large, there exist two thresholds at each $t \in \{0, 1\}$, $\mu^f_{R,t} < \mu^*_t$, where the optimal GP choice for $\mu_t < \mu^f_{R,t}$ is $j = \emptyset$, for $\mu_t \in [\mu^f_{R,t}, \mu^*_t)$ is $j = R$, and for $\mu_t \geq \mu^*_t$ is $j = I$.

Appendix A shows this formally and introduces additional technical assumptions, where the proof is essentially an exercise in accounting for the possible configurations of the thresholds of skill that determine size and investor choice to confirm the above logic extends to the general case. The result is identical at times 0 and 1, save that the thresholds $\mu^f_{R,t}$ and μ^*_t are more complex at 0 due to the investor continuation values, which weakly lower the threshold skill at which funds of either type can be raised and weakly raise the maximum fund scale for both. We provide visual and mathematical intuition for this result next.

2.5.1 Size and Investor Choice: General Case Visual Intuition

The majority of the major relations from the simplified model carry through to the general case presented in Appendix A. The intuition for the general solution is highlighted visually for our baseline calibration from Table 1 in Figure 2. Figure 2 shows how optimal fund size and investor choice evolves with rising skill for a typical calibration of the general case at $t = 1$. On the X-axis, we record various levels of expected GP skill μ_t, on the left Y-axis we record the fund scale chosen by the GP, and on the right Y-axis we record the frequency of managers falling at each skill level given an initial distribution of prior skill $\mu_0 \sim \mathcal{N}(0, 0.05^2)$. Throughout, solid curves, orange for institutions and blue for retail, depict the optimal fund scale chosen when an investor type is optimally chosen by the GP. Dashed curves depict the counterfactual: the optimal fund scale that would have been chosen for the other type when choosing that type is feasible but not optimal for the GP. Blue shading represents regions where retail is chosen by the GP and orange shading represents where institutional capital is chosen. Background shading highlights the optimal investor choice and frequency of funds at each skill level.

[Figure 2 About Here]

The intuition for the model solution can be gleaned by analyzing Figure 2 from left to right, i.e. as skill increases. For low levels of skill the manager cannot raise a fund, the blank region below the feasibility threshold for both investor types at the far left of the Figure. As skill rises, retail becomes feasible first as highlighted by the vertical dotted blue line showing the retail feasibility threshold for expected skill $\mu^f_{R,t}$. This threshold is lower for retail because these investors are willing to fund lower minimum fund scales $\underline{A}_R < \underline{A}_I$ and have a potentially worse relative outside option $\delta \geq 0$. Recall that for the general model δ

is a per-dollar relative outside option benefit measured in risk-adjusted return units, with higher δ representing *inferior* outside options. Both forces tend to allow managers to raise funds at minimum scale $\underline{A}_R$ at lower skill levels than for $\underline{A}_I$, as in (15). Lower minimum scale tends to lower the minimum feasible skill because diminishing returns to scale $b > 0$ are less consequential at smaller fund scales and thus minimum scale retail funds are relatively more profitable per dollar, all else equal. Inferior outside options tend to lower the minimum feasible skill because they make any VC fund relatively more attractive than the outside option for any fixed expected skill or fund scale. Net of diminishing returns, a fund raised at minimum scale is relatively more attractive than the outside option as δ rises, so the minimum feasible skill $\mu^f_{R,t}$ at which any fund can be raised falls.

As expected skill rises, retail funds are initially raised at constrained maximum scale, depicted by the upward-sloping solid blue line and blue shading starting at $\mu^f_{R,t}$. Eventually, one of two boundaries is crossed; either institutional funds become feasible at minimum scale and $\mu^f_{I,t}$, depicted by a dotted, orange, vertical line, is crossed, or retail funds raised at unconstrained optimal scale rather than constrained maximum scale become optimal as the high-skill crossing point for retail, $\mu^h_{R,t}$, depicted by a vertical, blue, dashed line, is crossed. For our baseline calibration, $\mu^f_{I,t}$ is crossed first, and the manager has a genuine choice of investor types for all higher levels of skill.

When $\delta = 0$ investor choice when both types are feasible is trivial: the per-dollar profitability of institutions from $c_R > c_I$ dominates investor choice because their constrained maximum scale is the same, $\bar{A}_{R,t} = \bar{A}_{I,t}$, and there is no potential advantage to selecting retail capital.[6] For $\delta > 0$, retail funds can have a size advantage at constrained maximum

[6] At $t = 0$ $\bar{A}_{R,t} \neq \bar{A}_{I,t}$ due to investor continuation values. Investor welfare is only created in funds raised at unconstrained optimal scale, since constrained maximum scale is defined by the investor 0 profit condition. The sign of the difference in maximum scales is determined by how likely a GP is to raise a fund at unconstrained optimal scale from each investor type at time 1 given their initial skill, and is formally

scale because their 0 payoff boundary in (12) is higher as δ increases: GP's can extract the value of retail's inferior outside options by raising larger funds and attenuating per-dollar performance through diminishing returns to scale. Thus, for the baseline general model with $\delta = 0.02$, retail funds can initially be raised at much larger scales than institutional funds. So despite their per-dollar profitability being lower, their total expected profits for the GP are higher when scaled to the larger maximum size of the fund. This is the region between the dotted orange vertical line and the gray vertical line where the blue curve is solid and above the orange curve, which is dashed: the GP is choosing to raise retail funds for their size advantage because they can extract the value of retail's inferior outside option. This region collapses when $\delta = 0$ because there is no retail size advantage.

As expected skill continues to rise, institutional capital eventually becomes optimal, depicted by the vertical gray line at μ_t^* in Figure 2. The order in which $\mu_{I,t}^f$, $\mu_{R,t}^h$, $\mu_{I,t}^h$, and μ_t^* are crossed as skill rises is ambiguous. Critically, however, the fact that $\mu_{I,t}^h > \mu_{R,t}^h$ because of institutions' operational cost advantage, $c_R > c_I$, carries through to the general model. The Figure shows this with the vertical blue dashed line at $\mu_{R,t}^h$, above which raising a retail fund at unconstrained optimal scale given retail is chosen by the GP, being to the left of the vertical orange dashed line at $\mu_{I,t}^h$, the same threshold for institutions.

In our baseline calibration, $\mu_{I,t}^f < \mu_t^* < \mu_{R,t}^h < \mu_{I,t}^h$, so initially there is a region between the gray line μ_t^* and the blue, dashed line $\mu_{R,t}^h$ where the optimal choice given investor type is to raise a fund at constrained maximum scale for both types, but the per-dollar cost advantage of institutions dominates the size advantage of retail and institutional capital is chosen, as depicted by the solid orange curve below the dashed blue curve. Despite crossing $\mu_{R,t}^h$ first, institutions' cost advantage continues to dominate, depicted by the region between

ambiguous.

the vertical, dashed, blue line at $\mu^h_{R,t}$ and the vertical, dashed, orange line at $\mu^h_{I,t}$ where the size choice curve is solid orange despite the counterfactual size choice for retail, the blue dashed curve, being higher for part of the range. In this region, the cost advantage of institutions dominates, and the size advantage of retail reverses as skill rises because unconstrained optimal scale increases more slowly than constrained maximal scale with skill for both types, as in (13) and (16). Finally, above $\mu^h_{I,t}$, unconstrained optimal scale would be chosen for both investor types, and $c_R > c_I$ guarantees the optimal investor type choice in this region is institutional capital, as in (16), so the solid orange curve sits above the dashed blue curve: larger institutional funds with greater per-dollar profitability are the optimal choice for the highest skill managers.

The intuition that feasibility and size (and therefore retail) advantage dominates at low skill and per-dollar profitability (and therefore institutional) advantage dominates at high skill flows through to all parameterizations of the model for which $\delta \geq 0$, $c_R > c_I$, and for which the gap $\underline{A}_R < \underline{A}_I$ is sufficiently large. This implies our main result in Proposition 1. All valid orderings of the thresholds $\mu^f_{R,t}$, $\mu^f_{I,t}$, μ^*_t, $\mu^h_{R,t}$, and $\mu^h_{I,t}$ under these three constraints result in optimal investor type choice by the GP progressing as in the proposition: $\emptyset \to R \to I$. Intuitively, this is because once the per dollar cost advantage of institutions dominates, it dominates for all higher levels of skill. Together, the five thresholds $\mu^f_{R,t} < \mu^f_{I,t} < \mu^*_t < \mu^h_{R,t} < \mu^h_{I,t}$ partition the skill axis into five regions in which the GP's optimization problem takes a qualitatively different form.

Figure 3 makes the size-versus-margin trade-off explicit. Each panel fixes μ at a representative level inside one of the five regions and plots manager profit as a function of fund size for both investor types. The constraints that bind (and hence which type wins) differ across panels, generating the kinks in Figure 2. Again, retail fund expected profits and limits

are plotted in blue and institutional fund expected profits and limits are plotted in orange, with a star representing the optimal size choice given investor choice for each type. The minimum and constrained maximum fund scale at each skill level are plotted as vertical dotted and dashed lines, respectively. In the first region, only retail capital is feasible and the optimal fund scale choice is the constrained maximum scale (Panel A). Once institutional funds become feasible, initially the size advantage of retail dominates (Panel B), but eventually per-dollar profitability dominates and institutional capital is chosen; first at constrained maximum scale (Panels C and D), then at unconstrained optimal scale for very high skill levels (Panel E). Figure 3 highlights the fact that the skill levels at which the constrained maximum scale for each type cross the minimum feasible scale and unconstrained optimal scale are critical to determining investor choice for the GP.

[Figure 3 About Here]

The optimization illustrated in Figure 3 also shows why the GP's value function operates like an option portfolio on posterior expected skill. A GP that successfully raises a fund at time 0 and generates a performance signal could arrive at any of the 6 skill regions after the skill update, the five depicted in the Figure or the low skill region where no fundraising is feasible and the GP shuts down. The option of investor type choice creates distinct kinks in the GP's expected profits at skill levels where the crossover of type choice occurs.

2.5.2 Speculative Investment

Since some GP's choose to raise funds at time 1 at unconstrained optimal scale, and funds raised at this scale generate expected investor welfare, the investor's continuation value from raising a fund with the same manager may be strictly positive for either type at time 0. The

fact that continuation value lowers the skill thresholds at which fundraising is feasible also raises the issue that the option value of fund continuation to the investor at time 1 may incentivize both managers and investors to start a fund unprofitable for investors at time 0, a region of low skill we call the speculative band. In this range of skill investors voluntarily engage in costly search for skill by funding managers they expect to be unprofitable today, because GP's may choose to raise a more profitable fund from the same investors in the next round after a positive skill update.

Institutional speculation is the economically countervailing case to our stepping-stone mechanism. Instead of retail investors funding the lowest skill managers and generating a benefit to overall welfare through the public skill signal, institutions could speculate due to the internal option value of discovering high skill managers because they capture the highest skill portion of the market where funds profitable to investors are raised at unconstrained optimal scale. A sufficiently large difference in minimum fund scales rules out this purely speculative fundraising by making such search too costly for institutions since any loss-making fund is scaled to the minimum fund scale. Alternatively, if institutions could credibly commit to refuse funding a GP at time 0, they could effectively offload these search costs on retail investors since they get the majority of the continuation value for the highest skill managers regardless of which investor funds the GP at time 0, and moderate skill managers generate 0 investor welfare for institutions because they raise funds at constrained maximum scale.[7]

[7]Either mechanism also eliminates the possibility that an institution would fund a loss-making fund that would not be loss-making for retail and eliminates the strategic concerns for investor behavior due to continuation value, since fundraising from institutions is never optimal when they would be engaging in speculative investment.

3 The Stepping Stone Mechanism and Welfare

3.1 Manager Transitions

The fact that very high skill managers in either period prefer to start funds with institutional investors is a direct result of the assumption that the operational benefits of raising institutional capital are positive, $c_I < c_R$. There is, however, a range of positive return realizations for which a manager who would or could only raise funds from retail investors at time 0, $\mu_0 \in [\mu^f_{R,0}, \mu^*_0)$, has a sufficiently large update to their expected skill due to the return signal that they will subsequently raise institutional capital at time 1. This occurs when the unexpected component of returns satisfy:

$$r_{j,1} + bA_{j,0} = \theta + \epsilon_1 > w_0^{-1}(\mu^*_1 - \mu_0)$$

with $w_0 \equiv \frac{\sigma_0^2}{\sigma_0^2+\sigma_\epsilon^2}$. The important characteristic here is that the probability of achieving this level of unexpected return is not specific to the investor type from which funds are raised at time 0 or the scale of the fund: all terms related to j and $A_{j,t}$ cancel from the signal. Only the act of raising a fund at all determines whether the signal is sent, and its quality determines the investor types the manager would fundraise from going forward.

It is immediate that we should expect the mass of surviving managers currently raising retail capital to shift towards institutional capital as a fraction after their next return realization, where the switchers would tend to have greater realized performance and initial skill. Since skill is conditionally normally distributed in the model $\mu_1 \sim \mathcal{N}\left(\mu_0, \sigma^2_{\mu,0} = \frac{\sigma_0^4}{\sigma_0^2+\sigma_\epsilon^2}\right)$, the probability of "graduating" to institutional capital given μ_0 is:

$$P(Graduate|\mu_0) = \Phi\left(\frac{\mu_0 - \mu_1^*}{\sigma_{\mu,0}}\right)$$

This is the sense in which institutional investors poach successful management talent from the retail section of the market. Panels A (our general baseline) and B (a market structure eliminating retail) of Figure 4 show this dynamic for a typical calibration of the model when prior skill is distributed $\mu_0 \sim \mathcal{N}\left(\mu_p, \sigma_p^2\right)$ in the population. Figure 4 depicts the frequency with which managers raise capital from institutions (orange area) and retail (blue area) at times 0 (left plots) and 1 (right plots) under three market structures; the baseline with the stepping stone mechanism and retail participation (Panel A), a case without retail participation (Panel B), and a case without investor type switching between fundraising periods (Panel C). Vertical lines depict the key cutoff thresholds for feasibility and optimality of investor types and fund scales as in Figure 2. In the first period, some funds that might not have been raised otherwise are raised with retail capital. This is the blue area below the orange vertical line at $\tilde{\mu}_{I,0}^f$ in the left plots Panel A that is absent in Panel B, with $\tilde{\mu}_{I,0}^f$ the institutional feasibility threshold adjusted for time 1 continuation value. In the second period surviving funds with the highest realized returns shift towards institutional capital on average, the red-dashed mass in the right plot of Panel A that is no longer present in Panel B when retail is excluded. Panels B and C also highlight how the institutional capital and the speculative funding band partially replace retail funding if retail investors are excluded from the market (Panel B) or type switching is restricted (Panel C), but to the extent minimum scale differences are large or learning is slow this replacement is relatively small.

[Figure 4 About Here]

This is the core of the stepping stone mechanism we document empirically in Section 4.

Institutional capital is more desirable due to its operational benefits to the manager, but retail is better equipped to supply capital to small funds of managers with low to moderate skill. At sufficiently high expected skill, the profitability effect dominates the GP's decision making on investor choice. Thus, we should expect to see a mass of managers raising capital from retail investors before they generate performance signals, who then are more likely to raise capital from institutions after exceptional returns and more likely to exit after inferior returns.

3.2 Welfare Impact of the Stepping Stone Mechanism

We consider the change in investor, GP, and total welfare expected under three scenarios. The first is our baseline scenario as described above and in Appendix A. The second case compares these with a market structure in which retail investors are excluded entirely, $\not{R}$. The final case compares these to a market structure that prohibits type switching $\not{T}$: a GP's investor type is fixed at time 0, so at time 1 they either raise from j again or raise no fund. We evaluate welfare under two treatments of δ. Under Convention W, δ is a genuine outside-option benefit: retail investors who accept a contract with a negative hard-dollar return are compensated by escaping an even worse outside option, and that compensation enters welfare. Under Convention W^0, δ shapes investors' acceptance decisions but generates no welfare, so the same contract is a rent transfer from retail to the GP. The empirically plausible case in which δ is partly genuine and partly behavioral falls between the two.

Our analysis of the welfare impacts of retail participation in private markets and the stepping stone mechanism itself is simplified by a critical conclusion from Section 2.4: investor welfare is exactly 0 when a fund is raised at the maximum scale, since this scale is defined by minimizing the benefit to investors of the fund net of their outside option. Thus, when

perceived outside options δ are not treated as a genuine welfare benefit, all funds raised at maximum scale from retail destroy investor value through excessive scale, and when it is, or in the simplified case where outside options are equal to 0, these funds create no investor welfare.

To formally derive the expected welfare effects of different market structures, we integrate expected GP and investor welfare over the population distribution of μ_0. For visual intuition, we assume the distribution to be $\mu_0 \sim \mathcal{N}(\mu_p, \sigma_p^2)$, but the main tradeoffs and welfare ambiguity results are not specific to this distribution. The welfare calculation is again an exercise in carefully tracking the GP's optimal choice when expected skill falls in various ranges and measuring the total measure of active managers for each period in each range. For investor welfare, these are only potentially positive when the GP selects a type that motivates an unconstrained optimum fund size rather than constrained maximum scale. The tradeoff for retail access when δ is not valued is that welfare is destroyed by excessive fund scale and this destruction is worse when retail funds are permitted, but the stepping-stone mechanism increases the supply of high-skilled managers that create welfare overall, primarily for institutions and GP's. The tradeoff for allowing the graduation/demotion mechanism is similar: type switching reduces the population of high skill GP's matching with retail, reducing the value created by the retail outside option and reducing the incentive of retail investors to speculate on low expected skill GP's at time 0 for the chance of finding a high-skill GP who will invest with them at time 1. So the magnitude of retail search is smaller under the baseline than when type switching is allowed because retail cannot capture as much of the value of search. This delivers our second main result:

Proposition 2 (Welfare Ambiguity)**.**

1. *Managers always benefit from retail access, and benefit from type switching for suffi-*

ciently large operational cost and outside option differences[8]

2. *Retail access is socially valuable when δ is valued, and ambiguous when it is not*

3. *The social value of type switching is ambiguous both when δ is valued and when it is not*

This highlights the two fundamental welfare tradeoffs of the stepping-stone mechanism: welfare is created through the mass of graduating funds and the relative operational efficiency of institutional funds, but welfare is destroyed by inefficiently large retail funds and the loss of value created by retail's outside option. The balance of the latter two losses is controlled by the extent to which retail's relative outside option δ is a genuine benefit or a behavioral distortion, which shifts the relative efficiency of retail access and type switching. The overall welfare tradeoffs are determined by the masses of agents in three groups. Funds raised at constrained maximum scale destroy welfare when δ is not valued. Institutional funds raised at unconstrained optimal scale create welfare through cost efficiency. Retail funds raised at unconstrained optimal scale create welfare through the retail outside option and search incentives.

A comparison of Panels A, B and C of Figure 4 highlights these shifting masses of agents for a typical calibration of the general model. In the left plot of each figure, relevant for time 0, the identity of the investor type chosen in the speculative band changes when retail investors are restricted (moving from Panel A to B): fewer low-skill managers are funded but speculative funding by institutions replaces some retail funding. Moving from Panel A to C (restricting type switching) we see that speculative funding again plays a role, albeit in this

[8]The tradeoff is that when type switching is prohibited and learning is strong, the retail speculative band is wide, see Figure 4 left Panel A vs Panel C. Thus type-locking creates a within-retail stepping-stone mechanism, the welfare benefits of which dominate the cross-type stepping-stone mechanism for small operational cost and outside option differences.

case for both investor types and much more significantly for retail. When type switching is restricted (Panel C), retail funds can capture more of the value of speculative funding, since institutions can no longer poach successful managers. This incentivizes more retail search for skill among low-skill managers, the light blue area in Panel C, creating welfare for retail and managers that is sacrificed in the baseline market structure.

We depict the changes in welfare when δ is and is not valued by the social planner for each agent group in Figure 5 in Panels A and B, respectively. We depict welfare differences for a range of typical calibrations near our baseline, varying the strength of skill learning on the X-axis and recording the difference in welfare in the baseline case less the two alternative market structures on the Y-axis, no retail on the left of each Panel, $B - \not{R}$, and no investor type transitions on the right, $B - \not{T}$. Since the variance of posterior skill $\sigma^2_{\mu,0}$ is a derived parameter, we vary it by fixing σ_0 and varying σ_ϵ.[9] Total welfare differences and differences for each investor type appear as a green line (total), orange line (institutional investors), blue line (retail investors), or purple line (managers).

[Figure 5 About Here]

The four plots in Figure 5 highlight our core conclusions for total welfare ambiguity by showing that total welfare in the baseline less welfare in the no retail or no investor type switching cases, the green line, crosses 0 as the strength of learning changes. When δ is valued, retail access benefits managers and potentially investors of both types, as displayed by the positive total, manager, and institutional welfare in left Panel A.[10] When δ is not valued

[9] Similar results obtain when $\sigma_{r,0}$ is fixed rather than σ_0, with the exception of the $B - \not{R}$ comparison when δ is not valued. Figures A.5 and A.6 depict changes in welfare for a range of calibrations near the baseline as the three major parameter differences in the model vary, ΔA, Δc, and $\Delta\delta$, for the case where δ is valued by the social planner and the case where it is not, respectively. These show the same welfare ambiguity, demonstrating that this is not unique to our specific choice of how to vary σ_μ.

[10] Retail investors can benefit when there is a range of expected skill for which retail funds are raised at

by the social planner, access benefits are ambiguous as well, because the welfare destroyed by inefficiently large retail funds can outweigh the welfare created by the stepping-stone, especially when learning is weaker, as displayed in left Panel B. Permitting investor type switching benefits managers and institutions at the expense of retail, and the total welfare impact of switching can be negative regardless of whether δ is a genuine benefit as displayed in right Panel A and right Panel B. The cost of switching to retail is potentially larger in right Panel A than right Panel B because allowing switching sacrifices the value of retail's genuinely inferior outside options in addition to reducing retail incentives to search for GP skill.[11]

In summary, we identify a potential positive effect of retail entry into venture capital markets, a stepping stone mechanism through which managers prove their skill after raising from retail then graduate to institutional funding. The welfare effects of introducing this mechanism, or introducing retail in general, are ambiguous because while increased talent discovery creates value for both investors and managers, funds raised at inefficient large size from retail when their outside option is behavioral and denying retail valuable access when their outside option is genuine can potentially destroy more than this value in general. This raises several empirical questions of interest, most notably the magnitude of value creation among retail and graduating funds, which we turn to next.

unconstrained optimal scale, which does not occur near our baseline calibration but holds whenever $\mu_t^* > \mu_{R,t}^h$ at either date for a given calibration. Figures A.5 and A.6 highlight that this can occur when other elements of the calibration change.

[11] Although GP's always benefit individually from the switching option by revealed preference, the increased search by retail when switching is prohibited can actually benefit the GP population by including more managers in the market. This fact creates manager welfare ambiguity in $B-\mathcal{T}$, and can be seen in Figures A.5 and A.6.

4 Data and Measurement

This section describes the data sources, sample construction, and key variable definitions used to test our model's predictions and quantify the economic impact of retail investor access to venture capital.

4.1 Data Sources

Our empirical analysis combines three primary data sources to construct a comprehensive dataset of venture capital funds, their investors, and portfolio investments.

4.1.1 SEC Form ADV Filings

To identify investor type, we use SEC Form ADV filings required under the Dodd-Frank Act. Since 2012, investment advisers managing private funds with at least $150 million in assets must file Form ADV, which records fund structure and investor-eligibility requirements.[12]

Form ADV requires funds to disclose which Investment Company Act exclusions they rely on. Section 3(c)(1) funds target retail (individual) investors. Section 3(c)(7) funds target institutional investors. We use this regulatory classification as our primary measure of investor type, because it reflects legally binding restrictions on eligibility and fund structure that map directly onto the investor heterogeneity in our model. Appendix A describes the exclusions in more detail.

[12]The reporting requirement began in 2012 for all funds active at that time, providing a comprehensive snapshot of the industry from that point forward.

4.1.2 Data of Venture Capital Funds in PitchBook

We obtain fund-level data from PitchBook, which provides detailed information on venture capital funds, including fund names, vintage years, total commitments (Assets Under Management), and geographic locations. Fund sequence number is calculated based on vintage years of funds in the same GP company. The database covers a large universe of venture capital funds globally, with particularly comprehensive coverage of U.S. funds.

4.1.3 Deal-Level Investment Data

We obtain deal-level investment information from PitchBook, including deal dates, deal sizes, pre-money and post-money valuations, and exit outcomes. These data let us build bottom-up measures of fund performance from realized cash flows, complementing the fund-level returns. We also use Preqin cash-flow data for robustness.

4.2 Sample Construction

We match SEC Form ADV filings to PitchBook funds using fund names and adviser (GP) identities, achieving a match rate of approximately 8.4% (2,212 funds) of funds with ADV filings through an exact matching of keywords and websites.

Our primary sample consists of venture capital funds with identifiable investor eligibility classifications from Form ADV matched to PitchBook fund characteristics and deal-level investments. We apply the following filters to the data:

1. **Fund Type:** We focus on venture capital funds, excluding buyout, growth equity, and other private equity strategies to maintain sample homogeneity.

2. **Vintage Year Restrictions:** For our main analyses, we include all 2,212 matched

funds. We conduct robustness checks using funds with vintage years from 2003 onward (2,065 out of 2,212), ensuring that all funds in the sample had the opportunity to be captured in Form ADV filings initiated in 2012, given typical fund lives of 10-12 years. We conduct robustness checks using funds with vintage years from 2012 onward (1,705 funds), and funds with vintage years before 2015 to ensure sufficient time for performance observation (919 funds). Results are similar.

4.3 Key Variable Definitions

4.3.1 Fund Sequence and GP Career Progression

Fund sequence represents the ordinal number of a fund raised by the same general partner (GP) or fund management company. We track fund sequences from 1 (first-time fund) through 9+ to analyze career progression and the stepping stone mechanism. Fund sequence is determined at the GP entity level, linking all funds raised by the same management team.

Importantly, we track whether GPs switch between retail 3(c)(1) and 3(c)(7) structures across their fund sequence. A transition from 3(c)(1) in early funds to 3(c)(7) in later funds provides direct evidence of the stepping stone mechanism; GPs demonstrating skill with retail capital and graduating to institutional capital.

4.3.2 Value Added Measurement

We construct the measure of economic value added (VA) to capture the value creation gross of fees and costs:

Deal-Level Value Added (Pre-Fee):

Following the methodology of Braun, Dorau, Jenkinson and Urban (2025), which adapts

the value-added framework of Berk and Van Binsbergen (2015) to the venture capital and private equity setting, we compute the value added (net present value, NPV) at the deal level using investment cash flows and exit outcomes. For each portfolio company investment, we calculate

$$VA_{deal} = \sum_{t} \frac{CF_t}{(1 + r_{benchmark})^t}, \tag{18}$$

where CF_t represents cash inflows (exits, dividends) minus cash outflows (investments) at time t. Following the Public Market Equivalent (PME) approach in Kaplan and Schoar (2005), we use the return on the S&P 500 index as the benchmark return $r_{benchmark}$ for discounting. We then aggregate deal-level value added to the fund level to obtain a measure of gross value creation before management fees and carried interest.

4.4 Summary Statistics

Table 2 presents summary statistics for our main sample. Panel A reports the distribution of our sample across Investment Company Act exclusion types. Of the 2,212 funds raised by 810 GPs, 57.1% rely on Section 3(c)(1) (Retail Funds) while 31.6% use Section 3(c)(7) (Institutional Funds), with the remaining 11.4% employing both exemptions. Panel B reveals substantial economic differences between these structures, consistent with the fact that retail and institutional capital sources impose different constraints. Section 3(c)(1) funds average \$89.3 million in size compared to \$503.9 million for Section 3(c)(7) funds, a 5.6-fold difference. This size gap reflects the targeting of different LP bases. Institutional funds require higher median minimum investments (\$0.25 million versus \$0.10 million) but attract modestly larger investor bases (69.1 LPs versus 52.9 LPs), consistent with qualified purchasers writing larger

checks despite the higher entry barrier.

[Table 2 About Here]

Panel C shows the evolution of fund characteristics across the GP lifecycle. First-time funds average $108.3 million and attract 47.2 investors with a median minimum investment of $0.10 million. By the sixth fund, the mean fund size is $548.2 million with 85.1 investors, a nearly fivefold increase consistent with reputation building and performance signaling. The sample contains 810 first-time funds; attrition across sequences (503 second funds, 601 in sequences 3–5, 298 in sequence 6+) reflects natural exit and the selection process through which only successful GPs raise follow-on funds. Cross-sectional dispersion also widens with sequence: fund-size standard deviations rise from $198.0 million at Fund 1 to $1,173.5 million at Fund 6+, consistent with the model's prediction that performance signals resolve uncertainty about GP quality over time.

5 Empirical Evidence

In this section, we state and test four crucial model predictions empirically. Section 5.1 summarizes the predictions, referring back to the equations, propositions, and figures in the model section that justify them.

5.1 Model Predictions for Empirical Tests

The predictions follow from the equilibrium characterized in Section 2.5.1 and the stepping-stone mechanism in Section 3.

Empirical Prediction 1 (Investor-type sorting)**.** *GPs sort by perceived skill into three regions (no fund, retail, and institutional), so retail capital is more prevalent at lower fund sequences and institutional capital at higher sequences.*

This restates the $\emptyset \to R \to I$ ordering of Proposition 1, with fund sequence serving as the empirical proxy for the unobserved prior μ_0.

Empirical Prediction 2 (Graduation rises with realized performance)**.** *Among first-time GPs raising retail capital, the probability of graduating to institutional capital in their next fund is increasing in fund-1 realized performance.*

This follows from the closed-form graduation probability derived in Section 3, $P(\text{Graduate} \mid \mu_0) = \Phi\big((\mu_0 - \mu_1^*)/\sigma_{\mu,0}\big)$, combined with the Bayesian update in equation (6): because the posterior μ_1 is monotone in the realized performance signal, graduation probability rises with fund-1 realized return after adjusting for fund scale.

Empirical Prediction 3 (Sign of the fund-size jump at graduation)**.** *The average change in fund size at graduation, $E[A_{N+1} - A_N \mid R \to I]$, is unambiguously positive under the stepping-stone mechanism alone (when $\delta_R = 0$). When $\delta_R > 0$, the jump can become negative if the inflated size of retail funds from rent extraction dominates the stepping-stone graduation effect.*

When $\delta_R = 0$, the constrained-maximum fund scale in equation (13) is identical across types, so retail funds cannot exceed institutional funds through size advantage. Combined with $\underline{A}_R < \underline{A}_I$ and positive selection on fund-1 performance (Prediction 2), pre-graduation retail funds operate below $\underline{A}_I$ while post-graduation institutional funds operate at or above $\underline{A}_I$. The graduation jump is strictly positive on average.

When $\delta_R > 0$, retail funds at constrained-maximum scale can exceed the institutional minimum because the inferior retail outside option lets GPs extract rents through larger funds. Figure 2 shows this rent-extraction region: between $\mu^f_{I,t}$ and μ^*_t, retail fund size (solid blue) exceeds the counterfactual institutional fund size (dashed orange); Panels B and C of Figure 3 visualize the same configuration in profit space. When this rent-extraction inflation is large relative to the stepping-stone graduation effect, the fund-size jump can turn negative, and the empirical sign identifies which force dominates.

Empirical Prediction 4 (Economic importance of the stepping-stone channel)**.** *The stepping-stone share, measured by the fraction of institutional value added (and assets under management) attributable to GPs whose first fund raised retail capital, represents a substantial fraction of total institutional value added (and AUM) in venture capital.*

To formalize this measure, let $g_I(\mu_1)$ denote an institutional-segment outcome at posterior skill μ_1: either expected value added $v_I(\mu_1) \equiv (\mu_1 - bA^*_{I,1}(\mu_1) - c_I) \cdot A^*_{I,1}(\mu_1)$ or expected fund size $A^*_{I,1}(\mu_1)$. With $\omega(\mu_0)$ the cross-sectional density of initial perceived skill and $f(\mu_1 \mid \mu_0)$ the conditional density of the posterior under the Bayesian update in (6), the model-implied stepping-stone share is

$$\pi_{SS}(g_I) \equiv \frac{\displaystyle\int_{\mu^f_{R,0}}^{\mu^*_0} \omega(\mu_0) \int_{\mu^*_1}^{\infty} g_I(\mu_1)\, f(\mu_1 \mid \mu_0)\, d\mu_1\, d\mu_0}{\displaystyle\int_{\mu^f_{R,0}}^{\infty} \omega(\mu_0) \int_{\mu^*_1}^{\infty} g_I(\mu_1)\, f(\mu_1 \mid \mu_0)\, d\mu_1\, d\mu_0}. \tag{19}$$

Setting $g_I = v_I$ yields the stepping-stone share of institutional value added; setting $g_I = A^*_{I,1}$ yields the stepping-stone share of institutional AUM. In both cases, the numerator integrates over GPs who started in the retail-eligible range $[\mu^f_{R,0}, \mu^*_0)$ and whose posterior graduated them to the institutional region, while the denominator integrates over all institutional-

segment GPs.

5.2 Test of Prediction 1: Investor-Type Sorting Across Fund Sequence

Figure 1 plots the share of venture capital funds raised from retail and institutional investors by fund sequence. Consistent with Prediction 1 and Proposition 1, retail capital dominates among first- and second-sequence funds, while institutional capital dominates from the fifth fund onward. The institutional share rises monotonically from approximately 30% at fund 1 to over 60% by fund 5. This pattern matches the model's prediction that less-established GPs, namely those whose initial perceived skill μ_0 has not yet been verified through performance signals, are concentrated in the retail-eligible region $[\mu^f_{R,0}, \mu^*_0)$ and therefore raise predominantly retail funds, while GPs whose skill posterior has been updated above μ^*_0 graduate to institutional capital.

These shares capture only the extensive margin of investor-type sorting. Two further implications of the model are visible in the joint distribution of investor type and fund size within each sequence. First, the scale ordering in Proposition 1 implies that retail funds should be smaller than institutional funds at every sequence, with the gap widening as institutional capital progressively relaxes the scale constraint. Second, the graduation channel implies that the retail-graduated subset of institutional funds, defined as institutional funds whose family launched as retail and later transitioned, should arrive at a discounted scale relative to always-institutional peers and converge to them over successive vintages. To examine these intensive-margin predictions, Figure 6 reports kernel-smoothed densities of log fund size separately for each fund sequence within the family. Each panel displays three groups: retail funds (solid), institutional funds (dashed), and the subset of institutional

funds whose family started with a retail fund (dotted). Each density is scaled by its share of the panel sample, so that the retail and institutional densities sum to one within each panel.

Three patterns emerge. First, within every sequence, the retail distribution peaks well to the left of the institutional distribution, consistent with the scale ordering implied by Proposition 1. The upper-tail overlap is consistent with the rent-extraction region described in Prediction 3, in which retail funds can be raised at substantial scale when $\delta_R > 0$. Second, mass shifts from retail to institutional capital as families accumulate vintages, tracing the graduation channel depicted in Figure 4, and within the institutional group the retail-graduated subset grows steadily, accounting for roughly half of all institutional funds by sequence 6. Third, the two distributions shift to the right at very different rates: the retail peak stays in the tens of millions throughout, whereas the institutional peak moves from about \$90M at sequence 1 to roughly \$300M by sequence 6. This widening gap reinforces the scale ordering as institutional capital progressively relaxes the constraint that keeps retail funds small. The retail-graduated subset itself traces this trajectory: its peak sits well below the always-institutional peak at sequence 2, when these funds are newly transitioned, and meets or slightly exceeds it by sequence 6, consistent with retail-graduated GPs reaching a skill level at least comparable to their always-institutional peers.

[Figure 6 About Here]

5.3 Test of Prediction 2: Graduation by Realized Performance

We proxy the performance signal s_1 with the fund-1 realized exit rate, defined as the share of the fund's portfolio companies that reached IPO or acquisition by the end of our sample. Figure A.2 plots empirical graduation likelihood by exit-rate quartile for the 2010–2021 vintages of first-time 3(c)(1) funds. Managers in the top exit-rate quartile graduate at

roughly 36 percentage points higher rates than those in the bottom quartile, tracing out the shape of the graduation probability stated in Prediction 2.

Table 3 confirms the pattern parametrically. In a linear probability model with vintage fixed effects (column 3), a one-unit rise in fund-1 exit rate is associated with a 20.9 percentage-point increase in graduation probability. The coefficient falls from 0.41 in the cross-sectional specification (column 2) to 0.21 with vintage fixed effects, suggesting that within-vintage variation drives roughly half of the raw correlation. Column 4 reports the intensive-margin specification: conditional on graduating, a one-unit rise in fund-1 exit rate predicts a 52.3 log-point increase in fund-2 size, and log fund-1 size enters with an elasticity of 0.72. These two results together match the prediction that the posterior update driving graduation also scales up the follow-on fund, since $A^*_{I,1}$ is monotone in μ_t from (16).

[Table 3 About Here]

5.4 Test of Prediction 3: Fund-Size Jump at Graduation

Prediction 3 states that the average fund-size change at graduation is positive under the stepping-stone mechanism alone and can turn negative only if rent-extraction inflation generated by $\delta_R > 0$ overturns the stepping-stone graduation effect on average. The empirical sign of the graduation jump therefore identifies the dominant force at the boundary.

Figure 7 reports the model prediction (Panel A) and the empirical estimate (Panel B) of average fund-size changes by transition type. The empirical pattern is unambiguous: the graduation jump ($R \to I$) is +\$72.8M with a 95% confidence interval that excludes zero by a substantial margin. The institutional-to-retail demotion ($I \to R$) shows the symmetric pattern at −\$74.0M. Within-type stayers show small positive changes consistent with within-segment growth: +\$19.7M for retail and +\$45.9M for institutional.

[Figure 7 About Here]

The strictly positive sign rules out the alternative regime in which rent-extraction inflation under $\delta_R > 0$ drives a net negative jump as the dominant force. Graduating GPs move from pre-graduation retail funds operating below the institutional minimum, where retail capital fills a niche that institutional capital structurally cannot serve at low skill, to post-graduation institutional funds operating at scales the institutional segment uniquely supports.

5.5 Test of Prediction 4: Stepping-Stone Share of Institutional Value Added and AUM

Prediction 4 states that π_{SS}, the fraction of institutional value added and AUM attributable to GPs whose first fund raised retail capital, represents a substantial fraction of the institutional total. We measure both shares directly in the matched ADV-PitchBook sample.

Aggregating total value added across funds by current investor type and the GP's original funding source, 37% of total VC value added comes from currently retail-oriented funds, while 63% comes from currently institutional funds. Within the institutional segment, 34% of the value added is attributable to GPs whose first fund raised retail capital. Roughly 21% of total VC value creation, 0.63×0.34, is therefore attributable to GPs whose track record began in the retail segment. The corresponding decomposition for assets under management (AUM) is similar in structure but with somewhat larger stepping-stone shares: 35% of total VC AUM comes from currently retail-oriented funds, while 65% comes from currently institutional funds, and within the institutional segment 39% of AUM is attributable to GPs whose first fund raised retail capital. Roughly 25% of total VC AUM, 0.65×0.39, is therefore

attributable to GPs whose track record began in the retail segment.

Figure 8 reports this share by fund sequence, separately for AUM (left panel) and value added (right panel). The fraction attributable to retail-origin GPs rises monotonically from 18% at sequence 2 to between 29% and 67% at sequences 7–9+. The pattern indicates that the stepping-stone channel becomes more, not less, central to value creation as fund families mature: GPs whose first fund raised retail capital and whose performance signal supported graduation continue to scale up institutional funds across subsequent vintages.

[Figure 8 About Here]

It is worth noting that our measure π_{SS} is an upper bound on the strictly stepping-stone-attributable share, since some retail-origin GPs may have followed a rent-extract-then-graduate path under $\delta_R > 0$ rather than the stepping-stone channel as described in Prediction 3. The strictly positive average graduation jump documented in Section 5.4 bounds how large that contamination can be on average: rent-extraction inflation in pre-graduation retail funds does not overturn the stepping-stone graduation effect at the boundary, so the bulk of the share captured by π_{SS} is attributable to the stepping-stone channel.

Supporting evidence: the 2020 retail-access expansion. The model's mechanism for how retail access feeds the stepping-stone channel turns on the supply of eligible retail LPs. The SEC's December 2020 accredited-investor amendment provides a direct test of whether expansions in retail access operate through this channel. The amendment extended qualified-purchaser status to holders of specified professional licenses (Series 7, 65, and 82) and to knowledgeable employees of private funds, raising the supply of investors legally eligible for 3(c)(1) offerings. Table 4 reports a difference-in-differences specification on Form D filings for 2016–2022. The coefficient on 3(c)(1) $\times$ Post-2020 is 13.26 in the tightest specification

with state-by-year fixed effects (column 4), corresponding to roughly 13 additional LPs per 3(c)(1) offering after the shock. The pre-shock 3(c)(1) mean is 29.6 LPs per offering, so the treatment effect is roughly a 45% increase. Figure 9 shows the event-study dynamics: pre-treatment coefficients are indistinguishable from zero through 2019, the treatment effect emerges in 2020, and persists through 2022.

[Table 4 About Here]

[Figure 9 About Here]

This evidence confirms that exogenous expansions in retail access operate through the channel the model assumes: the supply of retail LPs available to first-time GPs rises after the shock. Whether the post-shock fund-2 cohort delivers a measurable shift in π_{SS} requires fund-2 outcomes for first-time funds raised after 2020, which are mostly not yet observable in our sample. We leave that direct vintage-level test for future work as the data mature.

5.6 Quantitative Magnitudes: Value-Added Dynamics

Sections 5.2–5.5 test the four predictions of the model. This subsection reports calibrated value-added magnitudes alongside their empirical counterparts as a consistency check. Statistical inference on value-added changes is constrained by the smaller sample of funds with realized cash flows in our matched data, so we treat these comparisons as a magnitude check that complements the size-jump and stepping-stone-share evidence rather than as a sharp hypothesis test on its own.

Quantitative magnitudes for value-added dynamics across fund sequences and transition types reflect the joint behavior of two channels: positive selection on the performance signal raises posterior skill among graduates, and fund scale responds to posterior skill and is

discontinuous across the type boundary. Figures 10 and 11 place model-implied magnitudes from the calibration in Table 1 alongside their empirical counterparts.

Panel A of Figure 10 reports the model's prediction for value added by capital source across fundraising periods, and Panel B reports the empirical analog by fund sequence. Both show that institutional value added rises substantially across periods (sequences) while retail value added remains modest. The model produces this pattern through the joint operation of the two channels above: graduates with high posterior skill enter the institutional segment, and optimal fund scale responds to posterior skill. The empirical magnitudes are consistent with the calibration.

[Figure 10 About Here]

Figure 11 reports the model's prediction (Panel A) and the empirical estimate (Panel B) of average value-added changes by transition type. Both show that $R \to I$ transitions generate the largest positive change in value added, $I \to R$ transitions generate the largest negative change, and within-type stayers ($R \to R$ and $I \to I$) experience small changes. The directional pattern is consistent with the regime identified in Section 5.4, where the graduation jump in fund size is statistically significant.

[Figure 11 About Here]

6 Conclusion

This paper develops a framework for the role of retail investors in venture capital. We extend the Berk-Green learning model with three institutional features of venture capital: minimum fund sizes, investor heterogeneity, and discrete fundraising. In the resulting equilibrium, retail capital is a stepping stone for emerging managers.

Our contributions are threefold. First, we show that retail access creates economic value through a learning channel that enables talent discovery and reallocates capital to higher-skilled GPs over time. Second, we characterize the conditions under which market segmentation between retail and institutional investors raises welfare. Third, we derive testable predictions that map onto observable variation in fund sequences, investor type, and realized performance.

Our findings imply that policies expanding retail access to venture capital can generate aggregate welfare gains, provided they preserve the segmentation that makes the stepping-stone mechanism work and protect retail investors from the largest downside outcomes. The policy implication is symmetric: restricting retail access cuts off the main supply of information about unproven managers, with possible downstream consequences for institutional selection and capital allocation.

The stepping-stone mechanism extends beyond the venture capital setting we study. The general principle is that one form of capital can finance opaque early-stage projects or managers when more efficient investors are unwilling to commit because information is scarce, and that the resulting investments produce information allowing higher-quality projects to graduate to more efficient sources of capital. Angel investors and government limited partners may play comparable roles in other private markets, financing entrants that institutional capital initially avoids and producing track records that support subsequent selection (Brander, Du and Hellmann, 2015; Howell, 2017; Lerner, 1999). The same logic points to an additional margin in the literature on early-stage subsidies, including angel investment tax credits (Denes, Howell, Mezzanotti, Wang and Xu, 2023): even when such policies do not change the average quality of firms at entry, they may affect the transition of higher-quality firms into more efficient capital markets. Our findings suggest that, in opaque markets where

information accrues only after capital is committed, the value of nontraditional early financing may operate through the information it generates for subsequent allocation rather than through direct selection at entry.

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Figures

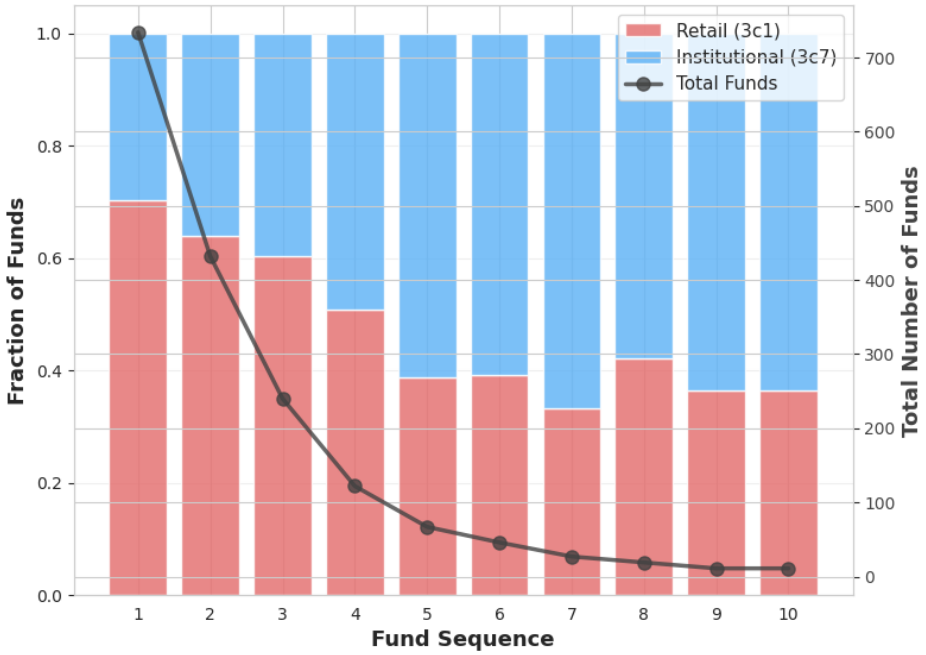


Figure 1: **Distribution of Funds by Capital Source and Fund Sequence.** This figure shows the fraction of venture capital funds by fund sequence (the Nth fund raised by each GP), separately for retail and institutional investors. The classification follows Form ADV filings: funds relying on Section 3(c)(1) of the Investment Company Act are retail, and those relying on Section 3(c)(7) are institutional. The gray line, using the right axis, reports the total number of funds.

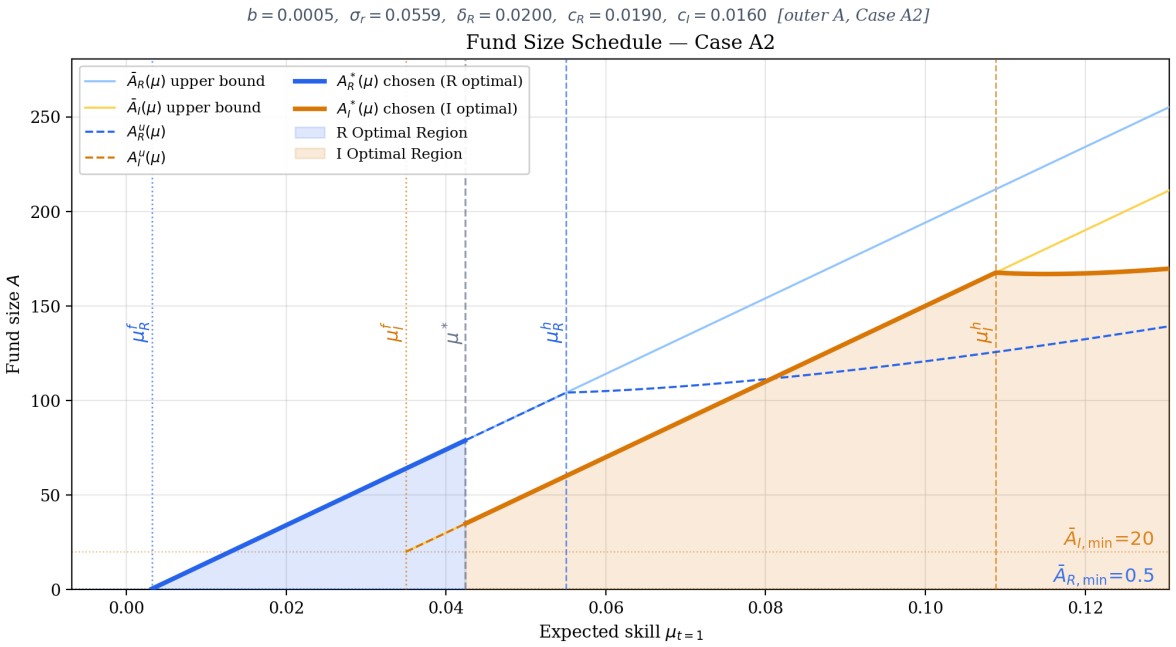


Figure 2: **Expected Skill and Fund Size** This figure shows how the GP optimization problem changes with expected skill by tracing the fund size an investor type chosen by the GP as a function of skill for typical model calibration at time $t = 1$. The X-axis is expected skill μ_t, and the Y-axis is fund scale. The main panel shows the optimal size if retail is chosen as a solid blue curve, and if institutions are chosen as solid orange. The counterfactual size for the other type, when feasible, is depicted as a dashed curve of the opposite color. Vertical blue lines are used for the feasibility cutoff and cutoff for unconstrained fundraising for retail (dotted and dashed, respectively) and vertical orange lines are used for institutions, with the cutoff for switching from retail to institutions μ_t^* as a vertical gray line. Lighter solid lines highlight the constrained maximum fund scale for retail (blue) and institutions (orange) when it does not bind.

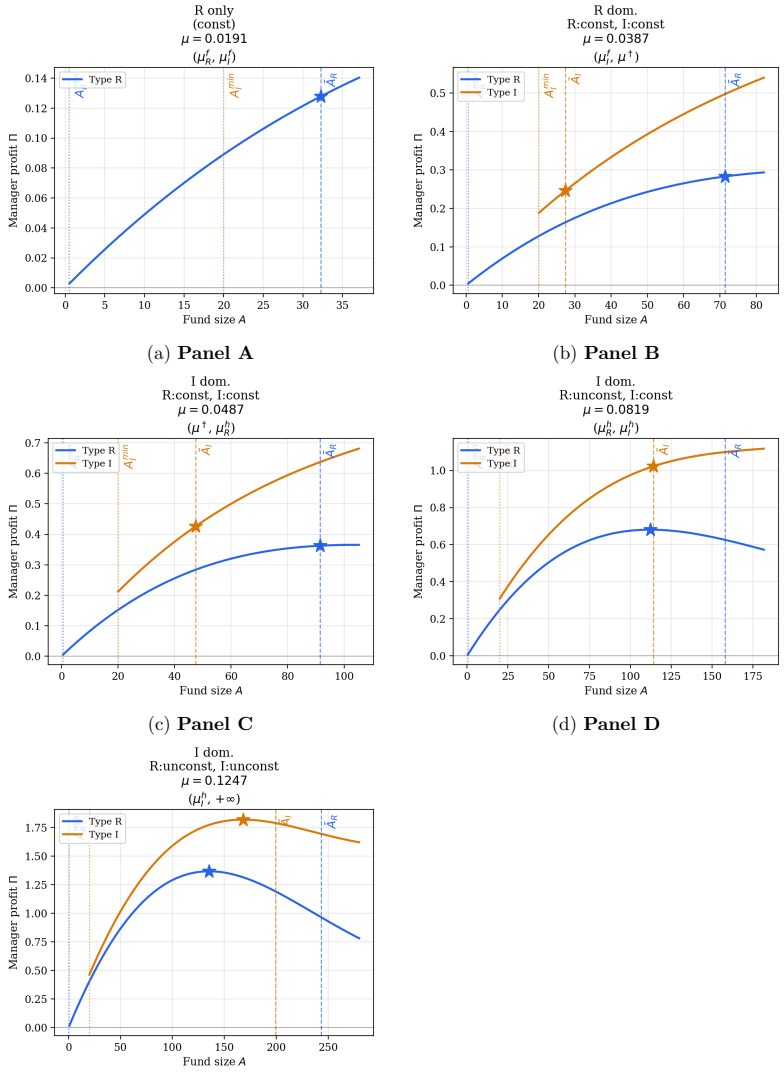
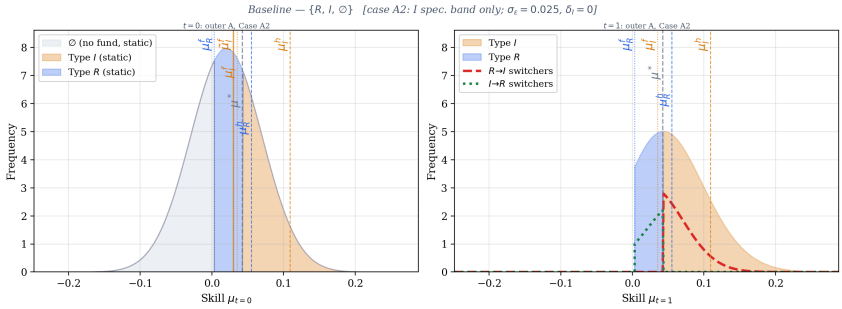
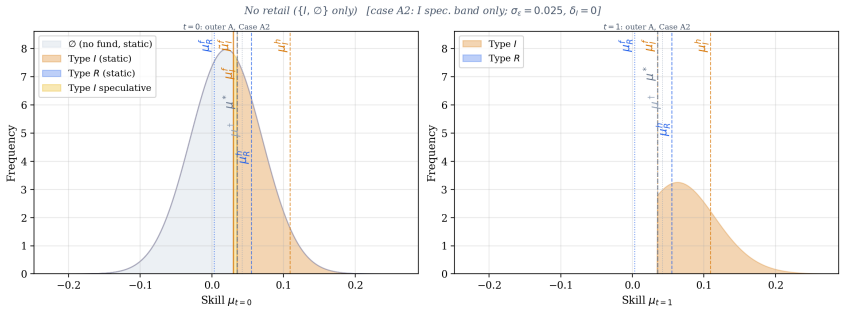


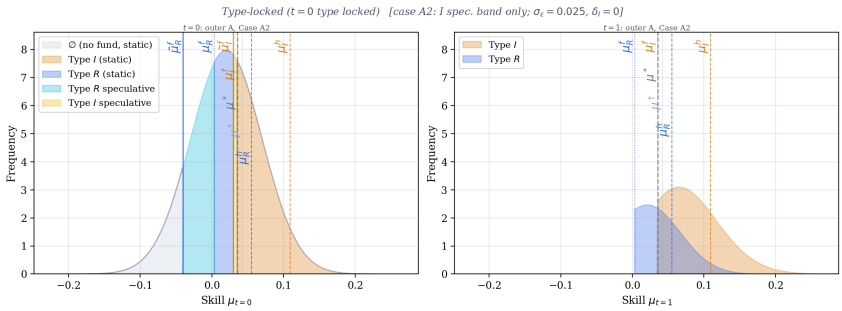
Figure 3: **Expected Skill and Fund Size Optimization** This figure shows how the GP optimization problem changes with expected skill for a typical model calibration. All panels depict the fund scale on the X-axis and the expected manager profit on the Y-axis, where panels vary across rising levels of expected manager skill. Vertical lines depict the minimum and maximum fund scales (dotted and dashed, respectively) for each investor type (blue for retail, orange for institutional) at each skill level. Each Panel shows a different region of optimality for the GP's sizing and fund choice in increasing order of μ_t: Only R Feasible (Panel A), Both Feasible, Constrained R Optimal (Panel B), Both Feasible, Constrained I Optimal (Panels C and D), and Both Feasible, Unconstrained I Optimal (Panel E).



(a) **Panel A: Baseline**

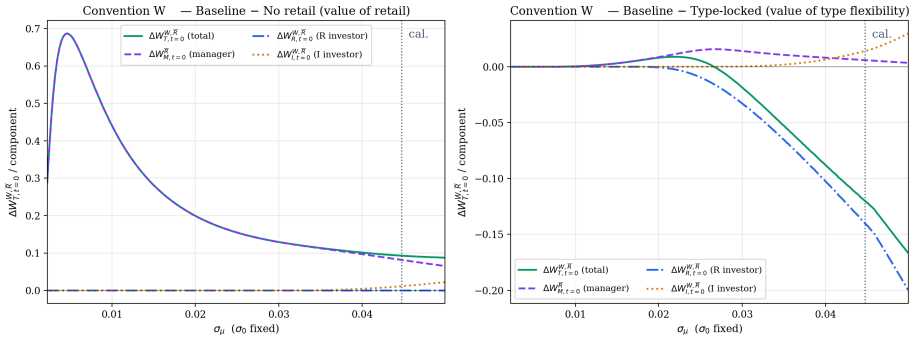


(b) **Panel B: Case $\not{R}$**

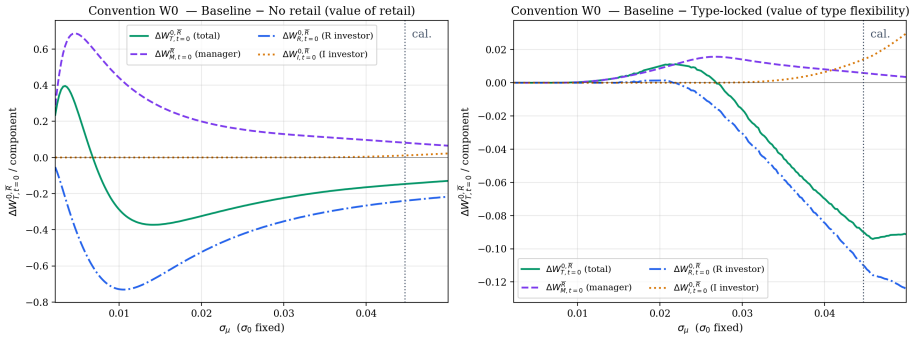


(c) **Panel C: Case $\not{I}$**

Figure 4: **Distribution of Funds by Capital Source and Market Structure** This figure shows the distribution of fund outcomes in the first and second fundraising periods by expected skill, choice of investor type, and market structure for a typical model calibration. Panel A shows how investor type switching shifts the distribution of active funds in the baseline case, with the blue area representing retail funds, the orange area representing institutional funds, the red dashed area of the right hand plot showing the stepping-stone funds, and the green dashed area showing demoted funds. Panel B shows how this restricts manager choice and shrinks the distribution of successful active managers when retail is excluded. Panel C shows how restricting type switching locks some high return managers into retail fundraising, while excluding some low realization managers originally raising from institutions. In the first fundraising period this type-locking creates increased search incentives for retail, depicted by the large light blue speculative band in left Panel C that is partially absent in left Panel A.



(a) **Panel A: Variation in Learning Standard Deviation (δ Valued)**



(b) **Panel B: Variation in Learning Standard Deviation (δ Not Valued)**

Figure 5: **Welfare Outcomes by Market Structure** This figure shows welfare differences between market structures as the strength of learning, the variance of posterior skill $\sigma_{\mu,0}$, changes by fixing σ_0 to the baseline calibration value and varying σ_ϵ. Both panels show differences in total welfare in green, manager welfare in purple, and investor welfare in blue (retail) and orange (institutions). The left plot of each panel is the welfare difference between baseline welfare and a market structure excluding retail ($\not{\mathcal{R}}$) and the right plot is the welfare difference between the baseline and a market structure preventing graduation and demotion ($\not{\mathcal{T}}$). Panel A depicts the case where outside options δ are a genuine benefit valued by the social planner and Panel B depicts the case where δ affects behavior but is not included in welfare or valued by the social planner. All panels show the baseline calibration as a vertical gray dotted line.

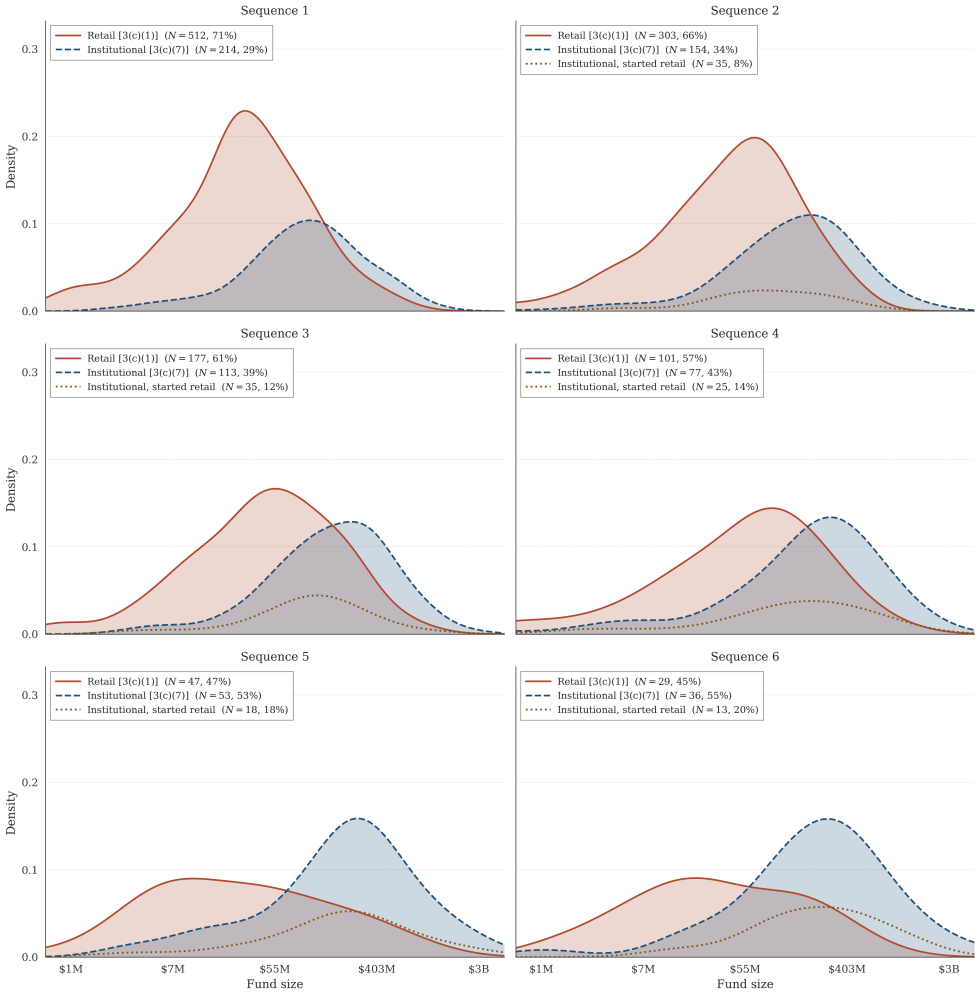
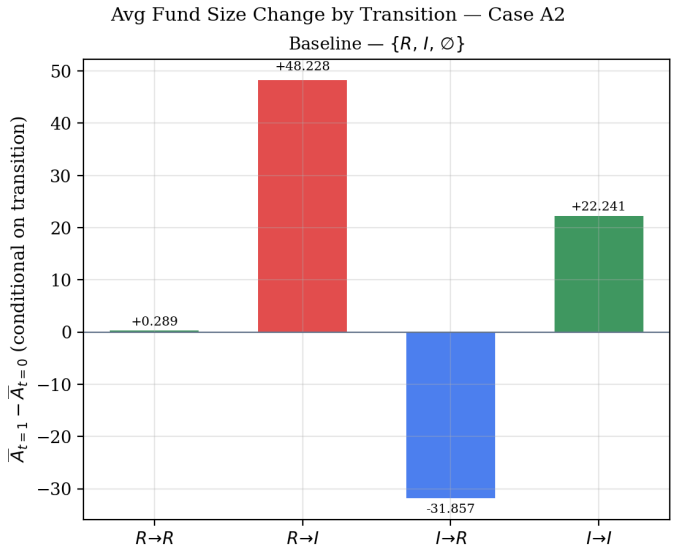


Figure 6: **Fund size distributions by fund sequence: retail, institutional, and graduated funds**

Each panel plots Gaussian kernel density estimates of log fund size (in $M) at a given fund sequence within the family, for the merged PitchBook–Form ADV sample of U.S. venture capital funds. Within each panel, three groups are shown: *retail* funds claiming only the 3(c)(1) exemption (solid), *institutional* funds claiming only the 3(c)(7) exemption (dashed), and *institutional funds that started as retail* (dotted), defined as institutional funds whose family's first observed fund was 3(c)(1). The first two groups are mutually exclusive and partition the panel sample; the third is a subset of the institutional group and is shown to isolate the graduation channel. Densities are scaled to absolute fund counts: each within-group density is multiplied by that group's share of the panel sample. Fund size is from PitchBook; the 3(c)(1)/3(c)(7) classification is taken from each fund's exemption claim on SEC Form ADV. Funds claiming both exemptions are excluded throughout. Group sample sizes and panel shares are reported in each panel's legend.

Panel A: Theory



Panel B: Data

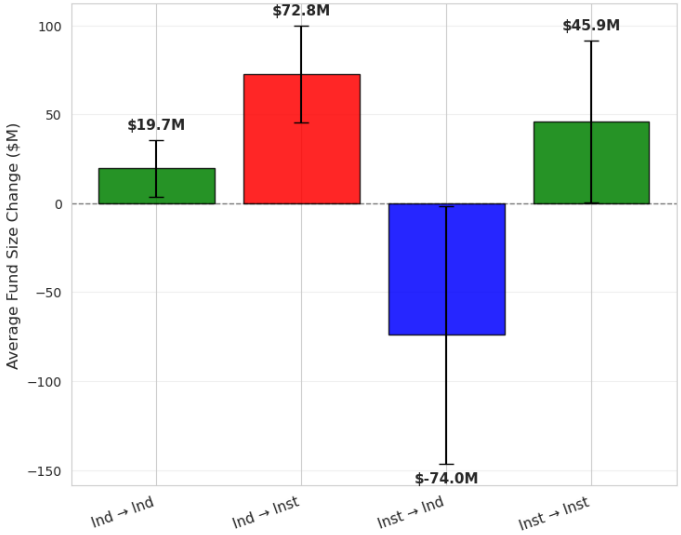


Figure 7: **Change of Fund Size by Switching Type** This figure plots the average change in fund size between consecutive funds for GPs categorized by the investor-type transition: Individual-Individual, Individual-Institutional (R→I, graduation), Institutional-Individual (demotion), and Institutional-Institutional. Panel A is from our model, and Panel B is from the data. Investor type in the data is from Form ADV filings. Fund size in the data is the committed capital reported in PitchBook. Each bar reports the average change in fund size of fund N relative to fund N-1, conditional on the transition between fund N-1 and fund N.

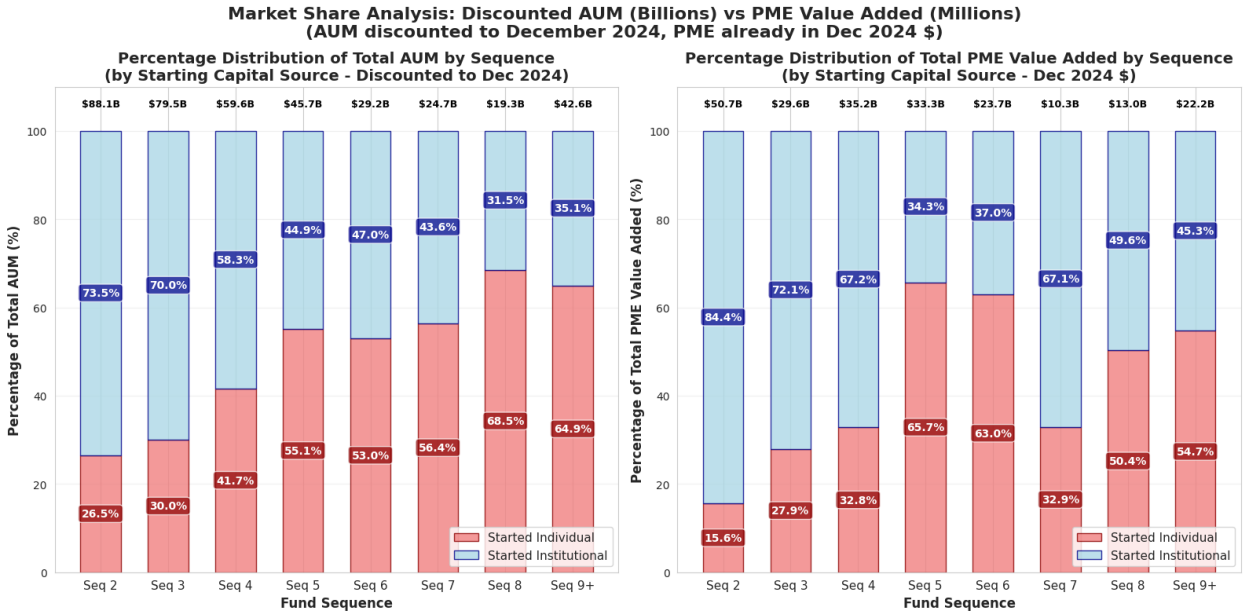


Figure 8: *Distribution of Institutional Funds' Value by Starting Capital Source.* This figure plots, for each institutional fund sequence from 2 through 9+, the share of total institutional AUM (left panel) and value added (right panel) from GPs whose first fund raised from retail investors. Institutional funds are Section 3(c)(7) funds; GP origin is the investor type of the first fund. AUM is from PitchBook. Value added is computed at the deal level following Braun, Dorau, Jenkinson and Urban (2025), using the PME methodology of Kaplan and Schoar (2005) with the S&P 500 benchmark, and aggregated to the fund level. The sample consists of 2,212 venture capital funds matched between SEC Form ADV filings and PitchBook, vintages 1973–2023. All values are in millions of December 2024 dollars. Dollar totals are shown above each bar.

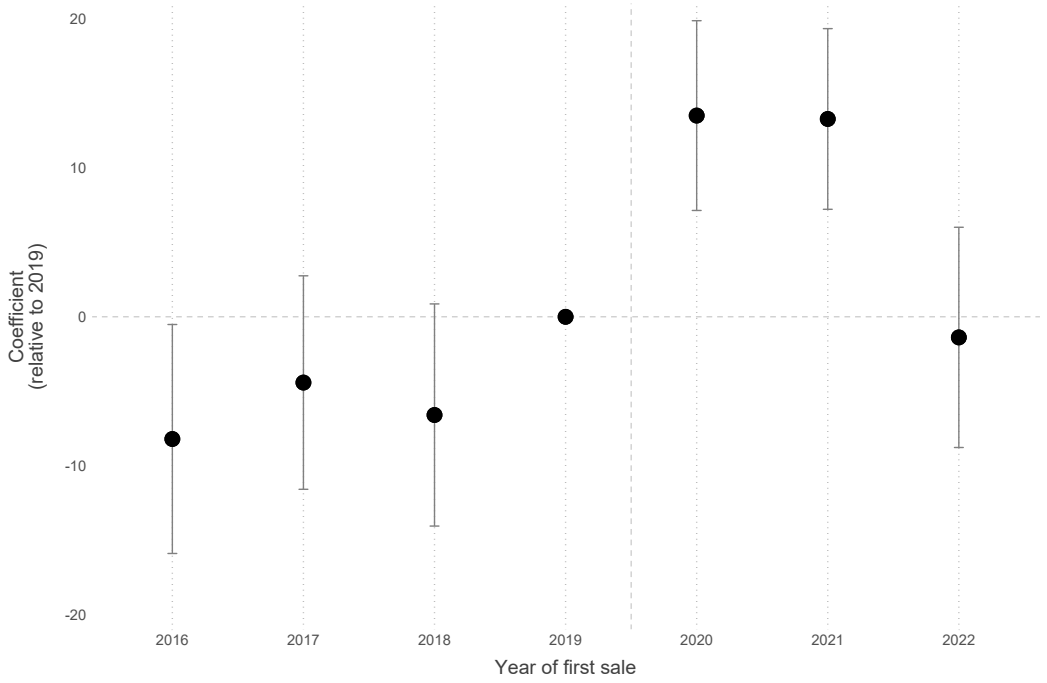
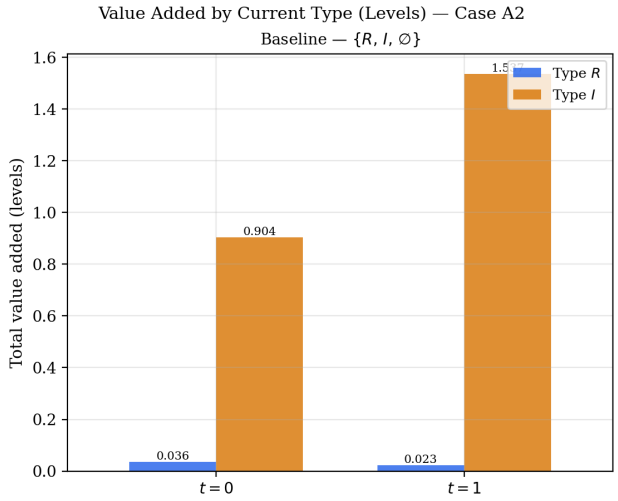


Figure 9: **Accredited investor expansion and 3(c)(1) fund investor counts (Form D)**

This figure plots event study coefficients from a regression of the number of investors (winsorized at the 1st and 99th percentiles) on year × 3(c)(1) interaction terms, with year and state fixed effects. The sample consists of SEC Form D filings classified as "Venture Capital Fund" with 3C exemptions, matched to Form ADV via Schedule D Section 7.B(1) Form D file numbers. The 3(c)(1)/3(c)(7) classification is from Form ADV. The sample uses a symmetric window around the SEC's expansion of the accredited investor definition, effective December 8, 2020, to include holders of Series 7, 65, and 82 licenses and knowledgeable employees of private funds (sample: 2016–2022, reference year: 2019, $N = 20{,}434$). Funds relying on both 3(c)(1) and 3(c)(7) are assigned to 3(c)(1) because they are also affected by the expansion of the accredited investor definition. Robust standard errors. Whiskers denote 95% confidence intervals. The dashed vertical line marks the treatment year.

Panel A: Theory



Panel B: Data

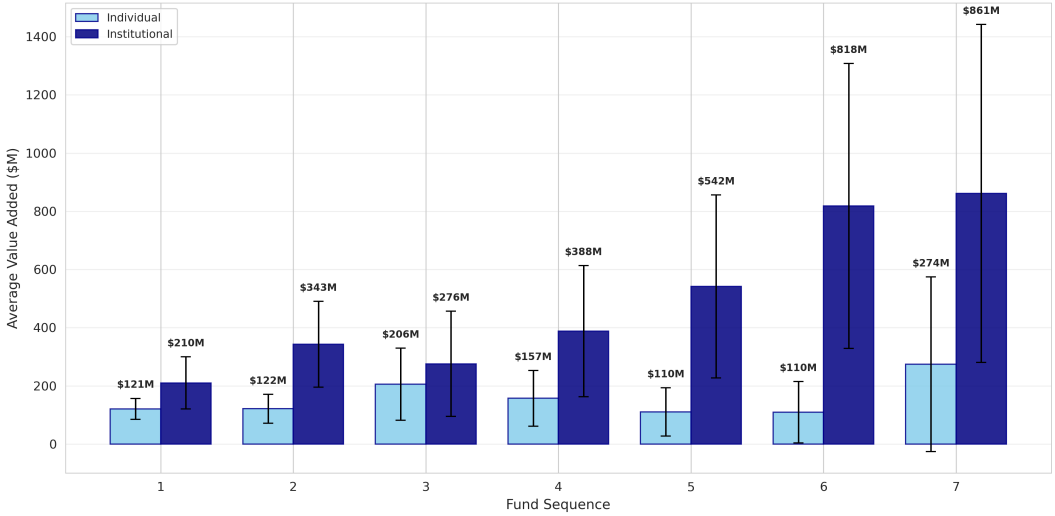
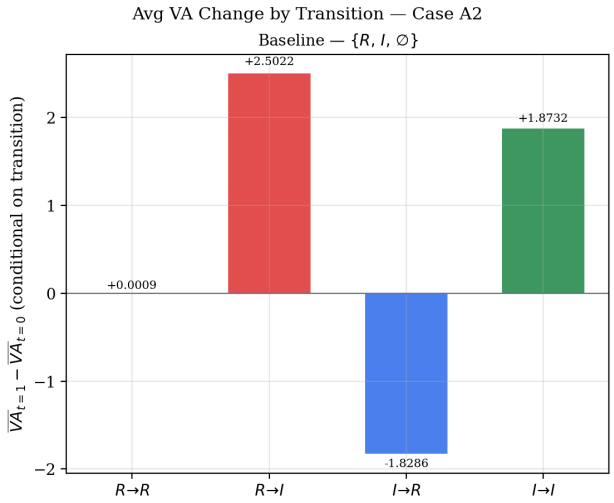


Figure 10: **Value Added by Fund Sequence and Investor Type** This figure plots the average fund-level value added by fund sequence and investor type. Panel A is from our model, and Panel B is from the data. Investor type in the data is classified from Form ADV filings, with Section 3(c)(1) as retail and Section 3(c)(7) as institutional. Value added is computed at the deal level following Braun, Dorau, Jenkinson and Urban (2025), using the PME methodology of Kaplan and Schoar (2005) with the S&P 500 benchmark, and aggregated to the fund level. The sample consists of 2,212 venture capital funds matched between SEC Form ADV filings and PitchBook, vintages 1973–2025. Values are in millions of December 2024 dollars.

Panel A: Theory



Panel B: Data



Figure 11: **Change of Value Added by Switching Type** This figure plots average fund-level value added for GPs categorized by the investor-type transition between consecutive funds: Individual-Individual, Individual-Institutional (R→I, graduation), Institutional-Individual (demotion), and Institutional-Institutional. Panel A is from our model, and Panel B is from the data. Investor type in the data is from Form ADV filings. Value added is computed as in Figure 10. Each bar reports the average value added of fund N conditional on the transition between fund N-1 and fund N.

Tables

Table 1: Summary of Model Parameters

This table summarizes the model parameters of the baseline model and any restrictions imposed throughout the model analysis.

Parameter	Description	Calibration
θ	GP's true skill, symmetric uncertainty/learning	
μ_0	Prior mean of GP skill at time 0	
μ_1	Posterior mean of GP skill at time 1	
σ_0^2	Prior variance of GP skill at time 0	0.050^2
σ_1^2	Posterior variance of GP skill at time 1	
σ_ϵ^2	Return noise	0.025^2
$A_{j,t}$	Fund size chosen at time t for investor type j	
$r_{j,t+1}$	Realized gross fund return at time t	
c_j	Operational cost per dollar for investor type j	$c_R = 0.019 > c_I = 0.016$
m	Management fee charged per dollar of funds raised	0.02
p	Performance fee charged on returns above the outside option	0.2
b	Diseconomy of scale parameter, $b > 0$	0.0005
δ_j	Investor type j's utility adjustment parameter	$\delta = \delta_R > \delta_I = 0$ $\delta_R = 0.03$
β	Discount factor for future payoffs	$1 > \beta = 0.9 > 0$
$\underline{A}_j$	Minimum fund size when funds raised from investor type j	$20 = \underline{A}_I > \underline{A}_R = 0.5 > 0$
μ_p	Population mean expected skill at $t = 0$	0.00
σ_p	Population standard deviation expected skill at $t = 0$	0.05

Table 2: Summary Statistics

This table reports summary statistics for the VC funds in our sample. Panel A presents the full sample of 2,212 funds, Panel B breaks down the statistics by LP type, and Panel C by fund sequence. Funds are classified into three categories based on their Form ADV exclusions: (i) Retail Funds relying on Section 3(c)(1), (ii) Institutional Funds relying on Section 3(c)(7), and (iii) Mixed Funds using both exclusions. Fund sequence denotes the Nth fund established by a GP based on vintage year.

Panel A: Overall summary statistics	
Characteristic	Value
Total Funds	2212
Total GPs	810
Vintage Years	1973-2023
Mean Vintage Year	2015
Retail Funds (3c1)	1262
Institutional Funds (3c7)	698
Mixed Funds	252
% Retail	57.1
% Institutional	31.6
% Mixed	11.4

Panel B: Summary statistics by LP type					
		Fund Size($M)		Number	Minimum Investment
LP Type	N	Mean	SD	of LPs	Median($M)
Retail (3c1)	1262	89.3	218.4	52.9	0.10
Institutional (3c7)	698	503.9	933.2	69.1	0.25
Panel C: Summary statistics by fund sequence					
1	810	108.3	198.0	47.2	0.10
2	503	171.6	355.4	52.6	0.10
3-5	601	319.5	625.7	58.3	0.13
6+	298	548.2	1173.5	85.1	0.05

Table 3: **Fund 1 exit rate and graduation from 3(c)(1) to 3(c)(7)**

This table examines whether a GP's first fund performance predicts graduation from 3(c)(1) to 3(c)(7). Graduation is defined as a GP whose first fund relies on the 3(c)(1) exemption subsequently raising a fund that relies on the 3(c)(7) exemption; the 3(c)(1)/3(c)(7) classification is from SEC Form ADV filings. Funds relying on both exemptions are assigned to 3(c)(7). The exit rate is the fraction of a fund's unique portfolio company investments that result in an IPO or acquisition, computed from PitchBook deal-level data. In columns (1)–(3), the dependent variable is an indicator for whether the GP graduated. In column (4), the dependent variable is the log size of the GP's second fund. The sample consists of GPs whose first fund is 3(c)(1) with 2010–2021 vintages and available PitchBook exit data. ***, **, and * indicate significance at the 1%, 5%, and 10% level, respectively. Robust standard errors in parentheses.

	I(Graduated)			**ln(Fund 2 Size)**
	(1)	(2)	(3)	(4)
Fund 1 Exit Rate	0.3910***	0.4137***	0.2091*	0.5233*
	(0.0953)	(0.0940)	(0.1254)	(0.2977)
ln(Fund 1 Size)		0.0287**	0.0352**	0.7187***
		(0.0133)	(0.0137)	(0.0454)
Vintage FE	No	No	Yes	Yes
Observations	386	385	385	214
R^2	0.053	0.063	0.109	0.596

Table 4: **Accredited investor expansion and 3(c)(1) fund investor counts (Form D)**

This table examines the effect of the SEC's expansion of the accredited investor definition (effective December 8, 2020) on the number of investors in 3(c)(1) venture capital funds. The dependent variable is the number of investors from SEC Form D filings (winsorized at the 1st and 99th percentiles). The sample consists of Form D filings classified as "Venture Capital Fund" with 3C exemptions, matched to Form ADV via Schedule D Section 7.B(1) Form D file numbers. The 3(c)(1)/3(c)(7) classification is from Form ADV. Funds relying on both 3(c)(1) and 3(c)(7) exemptions are assigned to 3(c)(1) because they are also affected by the expansion of the accredited investor definition. The sample covers 2016–2022, a symmetric window around the treatment. *3(c)(1) × Post-2020* is the interaction of an indicator for funds classified as 3(c)(1) with an indicator for sale years 2020 and later. ***, **, and * indicate significance at the 1%, 5%, and 10% level, respectively. Robust standard errors in parentheses.

	Number of Investors			
	(1)	(2)	(3)	(4)
3(c)(1) × Post-2020	12.74***	13.87***	13.42***	13.26***
	(1.95)	(1.97)	(1.98)	(1.99)
3(c)(1) Fund	-3.09**	-3.09**	-2.45*	-2.35
	(1.41)	(1.41)	(1.42)	(1.43)
Year FE	No	Yes	Yes	–
State FE	No	No	Yes	–
State × Year FE	No	No	No	Yes
Observations	20,434	20,434	20,434	20,434
R^2	0.006	0.022	0.028	0.030

Appendix A: Proofs of Core Model Results

A Proofs

[Preliminary and Incomplete Beyond Appendix A.6]

A.1 Model Setup

A.1.1 Return Process

Let $r_{j,t+1} = \theta - bA_{j,t} + \epsilon_{t+1}$, where $\theta \sim \mathcal{N}(\mu_t, \sigma_t^2)$ is the manager's skill and $\epsilon_{t+1} \sim \mathcal{N}(0, \sigma_\epsilon^2)$ is i.i.d. noise. Conditional on $\mathcal{F}_t$:

$$r_{j,t+1} \mid \mathcal{F}_t \sim \mathcal{N}(\mu_t - bA_{j,t},\ \sigma_{r,t}^2), \qquad \sigma_{r,t}^2 \equiv \sigma_t^2 + \sigma_\epsilon^2 \tag{A.1}$$

Here $b > 0$ is the capacity penalty, $A_{j,t}$ is fund size, $m > 0$ is the base management fee rate, $p \in (0, 1)$ is the performance fee rate, and $c_j > 0$ is the manager's per-unit operating cost.

A.1.2 Belief Updating

The return $r_{j,t+1} = \theta - bA_{j,t} + \epsilon_{t+1}$ already contains the capacity cost $-bA_{j,t}$. The *raw innovation* $r_{j,t+1} - \mu_t$ therefore has a non-zero mean:

$$r_{j,t+1} - \mu_t = (\theta - \mu_t) - bA_{j,t} + \epsilon_{t+1} \ \sim\ \mathcal{N}(-bA_{j,t},\ \sigma_{r,t}^2) \tag{A.2}$$

The centred quantity that isolates the skill signal is obtained by adding back $bA_{j,t}$:

$$r_{j,t+1} - (\mu_t - bA_{j,t}) = r_{j,t+1} + bA_{j,t} - \mu_t = (\theta - \mu_t) + \epsilon_{t+1} \ \sim\ \mathcal{N}(0,\ \sigma_{r,t}^2) \tag{A.3}$$

The sufficient statistic for updating beliefs about θ is therefore the *adjusted return*:

$$s_{t+1} \ \equiv\ r_{j,t+1} + bA_{j,t} \ =\ \theta + \epsilon_{t+1} \ \sim\ \mathcal{N}(\theta,\ \sigma_\epsilon^2) \tag{A.4}$$

which eliminates the capacity term entirely. With Gaussian prior $\theta \mid \mathcal{F}_t \sim \mathcal{N}(\mu_t, \sigma_t^2)$, Bayesian updating on s_{t+1} gives:

$$\sigma_{t+1}^2 = \frac{\sigma_t^2\,\sigma_\epsilon^2}{\sigma_t^2 + \sigma_\epsilon^2}, \qquad \mu_{t+1} = \mu_t + w_t\,(s_{t+1} - \mu_t) = \mu_t + w_t\,(r_{j,t+1} + bA_{j,t} - \mu_t), \qquad w_t \equiv \frac{\sigma_t^2}{\sigma_t^2 + \sigma_\epsilon^2} \tag{A.5}$$

The $+bA_{j,t}$ sign in the update follows from (A.3): the Kalman innovation is $r_{j,t+1} + bA_{j,t} - \mu_t$, not the raw $r_{j,t+1} - \mu_t$. The manager must therefore disclose fund size so that investors can

reconstruct s_{t+1} from the observed $r_{j,t+1}$. The *learning variance* $\sigma^2_{\mu,t-1}$ measures how much the posterior mean shifts in the step $t-1 \to t$:

$$\sigma^2_{\mu,t-1} \;\equiv\; \mathrm{Var}(\mu_t \mid \mu_{t-1}) = \sigma^2_{t-1} - \sigma^2_t = \frac{\sigma^2_{t-1}\,\sigma^2_t}{\sigma^2_\epsilon}, \qquad \mu_t \mid \mu_{t-1} \;\sim\; \mathcal{N}(\mu_{t-1},\, \sigma^2_{\mu,t-1}) \tag{A.6}$$

with variance decomposition $\sigma^2_{r,t-1} = \sigma^2_{\mu,t-1} + \sigma^2_{r,t}$: the return variance at $t-1$ equals the learning variance plus the residual return variance at t.

A.1.3 Manager and Investor Payoffs

Per-unit fees are $f_{t+1}/A_{j,t} = p\max\{r_{j,t+1} - bA_{j,t} - m, 0\} + m$. Taking expectations:

Definition 1 (Core payoff quantities)**.** *Let $\Pi_{j,t+1}$ denote the managerś realized profit and $\Psi_{j,t+1}$ the investorś realized payoff (Convention W, with δ_j valued; Convention W0 uses $\Psi^0_{j,t+1}$ with $\delta_j = 0$). For expected quantities conditional on μ_t we write $E[\Pi_{j,t+1}|\mu_t]$, $E[\Psi_{j,t+1}|\mu_t]$, etc. For manager of type j with fund size A at time t, define:*

$$z_{j,t} \equiv \frac{\mu_t - bA_{j,t} - m}{\sigma_{r,t}} \tag{A.7}$$

$$\tilde{\mu}_{j,t} \equiv \mathbb{E}_t[\max\{r_{j,t+1} - bA_{j,t} - m,\, 0\}] = (\mu_t - bA_{j,t} - m)\Phi(z_{j,t}) + \sigma_{r,t}\phi(z_{j,t}) \tag{A.8}$$

$$E[\Pi_{j,t+1}|\mu_t] = A_{j,t}(p\tilde{\mu}_{j,t} + m - c_j) \tag{A.9}$$

$$E[\Psi_{j,t+1}|\mu_t] = A_{j,t}(\mu_t - bA_{j,t} - p\tilde{\mu}_{j,t} - m + \delta_j) \quad \textit{(Convention W)} \tag{A.10}$$

$$E[\Psi^0_{j,t+1}|\mu_t] = A_{j,t}(\mu_t - bA_{j,t} - p\tilde{\mu}_{j,t} - m) \quad \textit{(Convention W0, } \delta_j = 0\textit{)} \quad \textit{(Convention W0)} \tag{A.11}$$

$$E[\Pi^{tot}_{j,t+1}|\mu_t] = E[\Pi_{j,t+1}|\mu_t] + E[\Psi_{j,t+1}|\mu_t] = A_{j,t}(\mu_t - bA_{j,t} - c_j + \delta_j) \quad \textit{(Convention W)} \tag{A.12}$$

$$E[\Pi^{tot,0}_{j,t+1}|\mu_t] = E[\Pi_{j,t+1}|\mu_t] + E[\Psi^0_{j,t+1}|\mu_t] = A_{j,t}(\mu_t - bA_{j,t} - c_j) \quad \textit{(Convention W0)} \tag{A.13}$$

where $\delta_j \geq 0$ is the investor's outside-option value (included under Convention W, excluded under Convention W0), and $\lambda(z) \equiv \phi(z)/\Phi(z)$ is the inverse Mills ratio .

Remark 1 (Cancellation of m)**.** *Adding expected payoffs: $E[\Pi_{j,t+1}|\mu_t] + E[\Psi_{j,t+1}|\mu_t] = A_{j,t}(\mu_t - bA_{j,t} - c_j + \delta_j)$. The management fee m cancels exactly, so total welfare is independent of m under Convention W (and likewise under W0).*

The key derivative is:

$$\frac{\partial \tilde{\mu}_{j,t}}{\partial A_{j,t}} = -b\Phi(z_{j,t}) \tag{A.14}$$

Derivation. By the Leibniz rule, $\tilde{\mu} = (z\sigma_r)\Phi(z) + \sigma_r\phi(z)$ where $z = (mu - bA - m)/\sigma_r$. Then $\partial z/\partial A = -b/\sigma_r$ and $\partial\tilde{\mu}/\partial A = (\partial z/\partial A)\sigma_r\Phi(z) + (z\sigma_r)\phi(z)(\partial z/\partial A) + \sigma_r(-z\phi(z))(\partial z/\partial A)$. The last two terms cancel, leaving $\partial\tilde{\mu}/\partial A = -b\Phi(z)$. □

A.1.4 Upper Bound on Fund Size

At any period $\tau \in \{t-1, t\}$, investors participate if and only if their two-period payoff is non-negative:

$$E[\Psi_{j,\tau+1}(A)|\mu_\tau] + \beta\, E[U_{j,\tau+1}(\mu_{\tau+1})|\mu_\tau] \geq 0 \tag{A.15}$$

where $E[U_{j,\tau+1}(\mu_{\tau+1})|\mu_\tau]$ is the type-j investor continuation value (Definition 7), with $E[U_{j,t+1}(\mu_{t+1})|\mu_t] \equiv 0$ at the terminal period t. The *upper bound* $\bar{A}_{j,\tau}(\mu_\tau)$ is the largest A satisfying (A.15) with equality.

Definition 2 (Upper bound quantities)**.** *Write $\bar{z}_{j,\tau} \equiv (\mu_\tau - b\bar{A}_{j,\tau} - m)/\sigma_{r,\tau}$ for the z-score at the upper bound. The binding constraint in (A.15) gives:*

$$\bar{z}_{j,\tau}(1 - p\Phi(\bar{z}_{j,\tau})) - p\phi(\bar{z}_{j,\tau}) + \delta_j/\sigma_{r,\tau} = -\frac{\beta\, E[U_{j,\tau+1}(\mu_{\tau+1})|\mu_\tau](\mu_\tau)}{\bar{A}_{j,\tau}(\mu_\tau)\,\sigma_{r,\tau}} \tag{A.16}$$

$$\bar{A}_{j,\tau}(\mu_\tau) = (\mu_\tau - m - \bar{z}_{j,\tau}\sigma_{r,\tau})/b \tag{A.17}$$

At period t where $E[U_{j,t+1}(\mu_{t+1})|\mu_t] \equiv 0$, the right side of (A.16) vanishes and $\bar{z}_{j,t}$ solves:

$$\bar{z}_{j,t}(1 - p\Phi(\bar{z}_{j,t})) - p\phi(\bar{z}_{j,t}) + \delta_j/\sigma_{r,t} = 0 \tag{A.18}$$

Since (A.18) depends only on $\bar{z}_{j,t}$ and $\delta_j/\sigma_{r,t}$, $\bar{z}_{j,t}$ is constant *in μ_t (for fixed $\sigma_{r,t}$). At period $t-1$ where $E[U_{j,t}(\mu_t)|\mu_{t-1}] \geq 0$, the right side of (A.16) is non-positive. Let $\bar{z}^*_{j,t-1}$ denote the zero of (A.18) at $\sigma_{r,t-1}$ (the static reference z-score). Then $\bar{z}_{j,t-1} \leq \bar{z}^*_{j,t-1}$ with strict inequality when $E[U_{j,t}(\mu_t)|\mu_{t-1}] > 0$, and:*

$$\bar{A}_{j,t-1}(\mu_{t-1}) \;\geq\; (\mu_{t-1} - m - \bar{z}^*_{j,t-1}\sigma_{r,t-1})/b \qquad (\textit{strict when } E[U_{j,t}(\mu_t)|\mu_{t-1}] > 0) \tag{A.19}$$

Define:

$$G_{j,\tau} \equiv \tilde{\mu}_{j,\tau}(\bar{A}_{j,\tau}) = \sigma_{r,\tau}\big(\bar{z}_{j,\tau}\Phi(\bar{z}_{j,\tau}) + \phi(\bar{z}_{j,\tau})\big) \tag{A.20}$$

$$\bar{\pi}_{j,\tau} \equiv p\, G_{j,\tau} + m - c_j \tag{A.21}$$

Key identity*: multiply (A.16) by $-\bar{A}_{j,\tau}\sigma_{r,\tau}$ and use $G_{j,\tau} = \sigma_{r,\tau}(\bar{z}\Phi(\bar{z}) + \phi(\bar{z}))$:*

$$p\, G_{j,\tau} = \bar{z}_{j,\tau}\sigma_{r,\tau} + \delta_j + \frac{\beta\, E[U_{j,\tau+1}(\mu_{\tau+1})|\mu_\tau](\mu_\tau)}{\bar{A}_{j,\tau}(\mu_\tau)} \tag{A.22}$$

Proof. *Multiply (A.16) by $-\bar{A}\sigma_r$: $-\bar{A}\bar{z}\sigma_r(1-p\Phi(\bar{z})) + p\bar{A}\sigma_r\phi(\bar{z}) - \bar{A}\delta_j = \beta\, E[U_{j,\tau}(\mu_{\tau+1})|\mu_\tau]$. The left side equals $p\bar{A}\sigma_r(\bar{z}\Phi(\bar{z}) + \phi(\bar{z})) - \bar{A}(\bar{z}\sigma_r + \delta_j) = p\bar{A}G - \bar{A}(\bar{z}\sigma_r + \delta_j)$. So $p\bar{A}G = \bar{A}(\bar{z}\sigma_r + \delta_j) + \beta\, E[U_{j,\tau}(\mu_{\tau+1})|\mu_\tau]$; divide by $\bar{A}$.* □

At period t (CV=0): $p\, G_{j,t} = \bar{z}_{j,t}\sigma_{r,t} + \delta_j$ and therefore:

$$\bar{\pi}_{j,t} = \bar{z}_{j,t}\sigma_{r,t} + \delta_j + m - c_j \tag{A.23}$$

At period $t-1$: $\bar{\pi}_{j,t-1} = \bar{z}_{j,t-1}\sigma_{r,t-1} + \delta_j + m - c_j + \beta\, E[U_{j,t}(\mu_t)|\mu_{t-1}]/\bar{A}_{j,t-1}$.

The per-unit payoffs at $A = \bar{A}_{j,\tau}$ *(using* $\bar{\psi}_{j,\tau} \equiv E[\Psi_{j,\tau+1}(\bar{A}_{j,\tau})|\mu_\tau]/\bar{A}_{j,\tau}$*):*

$$\bar{\psi}_{j,\tau} = -\frac{\beta\, E[U_{j,\tau+1}(\mu_{\tau+1})|\mu_\tau]}{\bar{A}_{j,\tau}}, \quad \bar{\psi}^0_{j,\tau} = -\delta_j - \frac{\beta\, E[U_{j,\tau+1}(\mu_{\tau+1})|\mu_\tau]}{\bar{A}_{j,\tau}}, \quad \bar{\pi}^{tot}_{j,\tau} = \bar{\pi}_{j,\tau}, \quad \bar{\pi}^{tot,0}_{j,\tau} = \bar{\pi}_{j,\tau} - \delta_j \tag{A.24}$$

At t *(CV=0):* $\bar{\psi}_{j,t} = 0$ *and* $\bar{\psi}^0_{j,t} = -\delta_j$.

Remark 2 (Manager continuation value is independent of investor type and fund size)**.** *As long as a fund is raised at* $t-1$*, the manager's continuation value* $\beta\, E[V_t(\mu_t)|\mu_{t-1}]$ *is independent of which investor type* j *was chosen and of the fund size* $A_{j,t-1}$ *chosen. This follows because the performance signal available to all market participants at* t *is the* adjusted return

$$s_t = r_{j,t} + bA_{j,t-1} = \theta + \epsilon_t,$$

which cancels all terms involving j *and* $A_{j,t-1}$ *(see (A.4)). The posterior* $\mu_t \mid \mu_{t-1} \sim \mathcal{N}(\mu_{t-1}, \sigma^2_{\mu,t-1})$ *therefore depends only on* μ_{t-1}*, and the manager's value function* $V_t(\mu_t)$ *depends only on* μ_t*. Consequently, raising any fund at* $t-1$ *— regardless of type or size — confers the same option value going into* t*: the manager can re-optimise both investor type and fund size in the next round, constrained only by the updated belief* μ_t*.*

Lemma 1 (Derivatives of upper bound)**.** *At period* t*:*

$$\frac{\partial \bar{A}_{j,t}}{\partial \mu_t} = \frac{1}{b}, \qquad \frac{d\bar{z}_{j,t}}{d\delta_j} = \frac{-1}{\sigma_{r,t}(1 - p\Phi(\bar{z}_{j,t}))} < 0, \qquad \frac{\partial \bar{A}_{j,t}}{\partial \delta_j} = \frac{+1}{b(1 - p\Phi(\bar{z}_{j,t}))} > 0.$$

Proof. Since $\bar{z}_{j,t}$ is constant in μ_t (from (A.18)), equation (A.17) gives $\partial \bar{A}_{j,t}/\partial \mu_t = 1/b$. For δ_j: implicit differentiation of (A.18) gives $[1 - p\Phi(\bar{z}_{j,t})]\, d\bar{z}_{j,t}/d\delta_j = -1/\sigma_{r,t}$, hence $d\bar{z}_{j,t}/d\delta_j = -1/[\sigma_{r,t}(1 - p\Phi(\bar{z}_{j,t}))] < 0$. Applying the chain rule to $\bar{A}_{j,t} = (\mu_t - m - \bar{z}_{j,t}\sigma_{r,t})/b$:

$$\frac{\partial \bar{A}_{j,t}}{\partial \delta_j} = \frac{-\sigma_{r,t}}{b} \cdot \frac{d\bar{z}_{j,t}}{d\delta_j} = \frac{+1}{b(1 - p\Phi(\bar{z}_{j,t}))} > 0.$$

A higher outside-option value δ_j tightens the ZPC threshold $\bar{z}_{j,t}$ while relaxing the capacity constraint, allowing the manager to raise a strictly larger fund. □

A.1.5 Feasibility Thresholds

Definition 3 (Static feasibility threshold at t)**.** *The* static feasibility threshold *is the value of* μ_t *at which* $\bar{A}_{j,t} = \underline{A}_j$*:*

$$\mu^f_{j,t} \equiv m + \bar{z}_{j,t}\sigma_{r,t} + b\underline{A}_j \tag{A.25}$$

For $\mu_t \geq \mu^f_{j,t}$*, type* j *is feasible (*$\bar{A}_{j,t} \geq \underline{A}_j$*). For* $\mu_t < \mu^f_{j,t}$*,* $\bar{A}_{j,t} < \underline{A}_j$ *and no fund can be raised.*

Definition 4 (Speculative feasibility threshold at $t-1$)**.** *A type-j investor at $t-1$ who funds the manager anticipates a* type-specific *continuation value: they can reinvest at t only when (a) the manager raises a type-j fund at t (i.e., the manager does not switch type), and (b) that fund operates in the unconstrained region (so the per-unit investor payoff is strictly positive; under Convention W it equals zero at the constrained upper bound $\bar{A}_{j,t}$). Note that type switching between periods — e.g., a manager funded by R at $t-1$ who switches to I at t — generates additional welfare for type-I investors at t that is captured in the aggregate welfare operators (Sections A.6–A.7) but does not enter the type-j participation constraint at $t-1$: once the manager commits to type j at $t-1$, the other type's behavior at $t-1$ is off the equilibrium path for that investor's decision. The type-j continuation value relevant for the $t-1$ participation constraint is therefore*

$$E[U_{j,t}(\mu_t)|\mu_{t-1}] \;=\; \mathbb{E}_{t-1}\big[E[\Psi_{j,t+1}|\mu_t]\big(A^*_{j,t}(\mu_t)\big)\cdot \mathbf{1}\{j^*(\mu_t)=j\}\cdot \mathbf{1}\{\mu_t > \mu^h_{j,t}\}\;\big|\;\mu_{t-1}\big]_+ \tag{A.26}$$

where $j^*(\mu_t)$ is the manager's optimal type at t and the subscript $+$ denotes truncation at zero.

The speculative feasibility threshold $\hat{\mu}^f_{j,t-1}$ *solves:*

$$E[\Psi_{j,t}|\mu_{t-1}](\underline{A}_j,\,\hat{\mu}^f_{j,t-1}) + \beta\, E[U_{j,t}(\mu_t)|\mu_{t-1}](\hat{\mu}^f_{j,t-1}) = 0. \tag{A.27}$$

The region $\hat{\mu}^f_{j,t-1} \le \mu_{t-1} < \mu^f_{j,t-1}$ is the speculative band *for type j: investors fund at $\underline{A}_j$ with a negative static payoff offset by the continuation value.*

Remark 3 (Existence of the speculative band is case-dependent)**.** *Whether $\hat{\mu}^f_{j,t-1} < \mu^f_{j,t-1}$ (a genuine speculative band) depends on whether $E[U_{j,t}(\mu_t)|\mu_{t-1}] > 0$. Under Convention W, $E[\Psi_{j,\tau+1}|\mu_\tau](\bar{A}_{j,\tau}) = 0$ throughout the constrained region, so type j investors receive positive continuation payoffs only when the manager reaches the unconstrained region for type j at t. This in turn requires $\mu_t > \mu^h_{j,t}$ and $j^*(\mu_t) = j$. Section A.5 characterises exactly which cases admit a genuine speculative band for each type.*

A.1.6 Remaining Threshold Definitions

Definition 5 (Unconstrained threshold $\mu^h_{j,\tau}$)**.** *The* unconstrained threshold $\mu^h_{j,\tau}$ *is the value of μ_τ at which the manager's unconstrained optimum $A^u_{j,\tau}(\mu_\tau)$ equals the upper bound $\bar{A}_{j,\tau}(\mu_\tau)$. The manager's FOC for A at any period τ is:*

$$p\big[(\mu_\tau - 2bA - m)\Phi(z) + \sigma_{r,\tau}\phi(z)\big] + m - c_j = 0, \qquad z \equiv \frac{\mu_\tau - bA - m}{\sigma_{r,\tau}} \tag{A.28}$$

Setting $A = \bar{A}_{j,\tau}$, so that $z = \bar{z}_{j,\tau}$ and $\mu_\tau - 2b\bar{A} - m = \bar{z}_{j,\tau}\sigma_{r,\tau} - b\bar{A} = -(\mu_\tau - m - \bar{z}_{j,\tau}\sigma_{r,\tau}) + \bar{z}_{j,\tau}\sigma_{r,\tau}$. Substituting $b\bar{A}_{j,\tau} = \mu_\tau - m - \bar{z}_{j,\tau}\sigma_{r,\tau}$ gives $\mu_\tau - 2b\bar{A} - m = -(b\bar{A}_{j,\tau}) + \bar{z}_{j,\tau}\sigma_{r,\tau} -$

$(m - m) = 2\bar{z}_{j,\tau}\sigma_{r,\tau} - (\mu_\tau - m)$. *Inserting into the FOC and solving for* μ_τ*:*

$$\mu^h_{j,\tau} = m + 2\bar{z}_{j,\tau}\sigma_{r,\tau} + \frac{p\sigma_{r,\tau}\phi(\bar{z}_{j,\tau}) + m - c_j}{p\Phi(\bar{z}_{j,\tau})} \tag{A.29}$$

This formula holds at both periods $\tau \in \{t-1, t\}$*, with* $\bar{z}_{j,\tau}$ *determined by (A.16). At period* t*:* $\bar{z}_{j,t}$ *is constant (from (A.18)), so* $\mu^h_{j,t}$ *is independent of* μ_t*. At period* $t-1$*:* $\bar{z}_{j,t-1} \leq \bar{z}^*_{j,t-1}$ *(the static reference, with strict inequality when* $E[U_{j,t}(\mu_t)|\mu_{t-1}] > 0$*), so* $\mu^h_{j,t-1} \leq \mu^{h,*}_{j,t-1}$*, with strict inequality when* $E[U_{j,t}(\mu_t)|\mu_{t-1}] > 0$*. Since* $\mu^h_{j,\tau}$ *is increasing in* $\bar{z}_{j,\tau}$ *(proved in Lemma 2.5), the continuation value strictly lowers the unconstrained threshold at* $t-1$*.*

For $\mu_\tau < \mu^h_{j,\tau}$ *the FOC has no solution at* $A < \bar{A}_{j,\tau}$*, so the investor ZPC binds:* $A^*_{j,\tau} = \bar{A}_{j,\tau}$*. For* $\mu_\tau \geq \mu^h_{j,\tau}$*, the unconstrained optimum satisfies* $A^u_{j,\tau} \leq \bar{A}_{j,\tau}$*.*

Definition 6 (Profit-crossing point $\mu^\dagger_\tau$ and switching threshold μ^*_τ)**.** $\mu^\dagger_\tau$ *is the value of* μ_τ *at which constrained manager profits of the two types are equal:* $\Pi_{R,\tau+1}(\bar{A}_{R,\tau}) = \Pi_{I,\tau+1}(\bar{A}_{I,\tau})$*. Since* $\Pi_{j,\tau+1}(\bar{A}_{j,\tau}) = \bar{A}_{j,\tau}\bar{\pi}_{j,\tau}$*, this requires:*

$$\bar{A}_{R,\tau}(\mu^\dagger_\tau)\,\bar{\pi}_{R,\tau}(\mu^\dagger_\tau) = \bar{A}_{I,\tau}(\mu^\dagger_\tau)\,\bar{\pi}_{I,\tau}(\mu^\dagger_\tau) \tag{A.30}$$

At period t*, using* $\bar{A}_{j,t} = (\mu_t - m - \bar{z}_{j,t}\sigma_{r,t})/b$ *and* $\bar{\pi}_{j,t} = \bar{z}_{j,t}\sigma_{r,t} + \delta_j + m - c_j$ *(both constant in* μ_t *once the* μ_t *dependence in* $\bar{A}$ *is factored out), equation (A.30) simplifies to:*

$$\mu^\dagger_t = m + \sigma_{r,t}\frac{\bar{z}_{R,t}\bar{\pi}_{R,t} - \bar{z}_{I,t}\bar{\pi}_{I,t}}{\bar{\pi}_{R,t} - \bar{\pi}_{I,t}} \tag{A.31}$$

which is well-defined and finite since $\bar{\pi}_{R,t} \neq \bar{\pi}_{I,t}$ *(Lemma 2 Part 9).*

At period $t-1$*,* $\bar{\pi}_{j,t-1}$ *includes the continuation term* $\beta\, E[U_{j,t}(\mu_t)|\mu_{t-1}]/\bar{A}_{j,t-1}$ *(from (A.22)), which depends on* μ_{t-1} *through both* $E[U_{j,t}(\mu_t)|\mu_{t-1}]$ *and* $\bar{A}_{j,t-1}(\mu_{t-1})$*. The explicit formula (A.31) does not apply;* $\mu^\dagger_{t-1}$ *is defined implicitly by (A.30), and* μ^*_{t-1} *(defined case-by-case per Section A.4 Table 1 at* $t-1$*). Since* $E[U_{I,t}(\mu_t)|\mu_{t-1}] \geq E[U_{R,t}(\mu_t)|\mu_{t-1}]$ *in all cases (with strict inequality in Cases A1/A2), I's constrained profit is higher relative to R's at* $\mu^\dagger_t$*, so* $\mu^\dagger_{t-1} \leq \mu^\dagger_t$ *(proved in Proposition 4).*

The switching threshold μ^*_τ *is the value of* μ_τ *at which the manager actually switches from R to I in equilibrium. Its identity depends on the inner regime (Section A.4):*

- ***Cases A1, B1a****:* $\mu^*_\tau = \mu^f_{I,\tau}$ *(equivalently* $\hat{\mu}^f_{I,\tau}$ *at* $t-1$*); I dominates from first feasibility.*
- ***Case A2****:* $\mu^*_\tau = \mu^\dagger_\tau$ *(CRCI crossing); both types constrained at the switch.*
- ***Cases A3/4, B1b, B2, B3****:* $\mu^*_\tau = \mu^\ddagger_\tau$ *(URCI crossing); R unconstrained and I constrained at the switch.* $\mu^\ddagger_\tau$ *is defined implicitly by* $\Pi_R(A^u_{R,\tau}(\mu^\ddagger_\tau)) = \bar{A}_{I,\tau}(\mu^\ddagger_\tau)\bar{\pi}_{I,\tau}$ *and exists by IVT (Lemma 3, Part 4).*

The old formula $\mu^*_\tau \equiv \max(\mu^\dagger_\tau, \mu^f_{I,\tau})$ *correctly identifies the switch only in Cases A1 and A2; it is superseded for Cases A3/4 and B by the URCI crossing* $\mu^\ddagger_\tau$*.*

A.1.7 Parameter Assumptions

Structural. $b > 0$, $p \in (0,1)$, $\delta_j \geq 0$, $c_j > 0$, $m > c_j$, $j \in \{R, I\}$.

Type comparisons.

$$c_R > c_I, \qquad \delta_R > \delta_I = 0, \qquad \underline{A}_R < \underline{A}_I \tag{A.32}$$

We normalise $\delta_I = 0$ throughout; $\delta_R > 0$ is the retail outside-option advantage. These imply $\bar{z}_{R,t} < \bar{z}_{I,t}$ (from (A.18): larger δ pushes $\bar{z}$ downward).

Regularity. $b\bar{A}_{j,t}\lambda(\bar{z}_{j,t})/\sigma_{r,t} < 2$ (ensures the SOC holds at the unconstrained optimum; see Lemma 2.1).

Scale gap condition (Cases A2 and A3/4). In Cases A1 and A3 (where $\mu_t^\dagger > \mu_{I,t}^f$, so R has a genuine static dominance region), the $\emptyset \to R \to I$ ordering at $t-1$ can be disrupted if the I speculative band extends below $\mu_{R,t-1}^f$. The *scale gap condition* rules this out:

$$b(\underline{A}_I - \underline{A}_R) \;\geq\; \frac{\beta\, E[U_{I,t}(\mu_t)|\mu_{t-1}](\mu_{I,t-1}^f)}{\underline{A}_I\big(1 - p\,\Phi(\bar{z}_{I,t-1})\big)} \;-\; (\bar{z}_{I,t-1} - \bar{z}_{R,t-1})\,\sigma_{r,t-1} \tag{A.33}$$

where $E[U_{I,t}(\mu_t)|\mu_{t-1}](\mu_{I,t-1}^f)$ is defined in (A.26). Condition (A.33) ensures $\hat{\mu}_{I,t-1}^f \geq \mu_{R,t-1}^f$: the I speculative band lies entirely within $(\mu_{R,t-1}^f, \mu_{I,t-1}^f)$, so R retains an uninterrupted lower feasibility window. See Proposition 8 for the proof.

Remark 4. *The right side of* (A.33) *equals the width of the I speculative band* net of *the outside-option contribution* $(\bar{z}_I - \bar{z}_R)\sigma_r > 0$ *already provided by* $\delta_R > \delta_I$. *When* $\sigma_{\mu,t-1} \to 0$ *(no learning between periods),* $E[U_{I,t}(\mu_t)|\mu_{t-1}] \to 0$ *and* (A.33) *is trivially satisfied for any* $\underline{A}_I > \underline{A}_R$.

Remark 5. *In Case A2 (*$\mu_t^\dagger \leq \mu_{I,t}^f$*), R has no genuine static dominance region. If the I speculative band extends below* $\mu_{R,t-1}^f$*, the* $t-1$ *ordering collapses to* $\emptyset \to I \to I$*, which is consistent with the overall conclusion that I is the efficient type at high* μ*. In Case B, the large minimum scale required for* $\mu_{I,t}^f \geq \mu_{R,t}^h$ *implies the static I loss at* $\mu_{R,t-1}^f$ *always dominates any continuation value, so the band cannot reach below* $\mu_{R,t-1}^f$ *(Proposition 9). Condition* (A.33) *is therefore needed only in Cases A2 and A3/4.*

A.2 Lemma and Threshold Ordering

Lemma 2 (Manager's optimisation problem)**.** *Under Section A.1.7:*

1. (Second-order condition.) *The manager's profit* $\Pi_{j,t+1}(A)$ *is strictly concave in* A *on* $[\underline{A}_j, \bar{A}_{j,t}]$ *iff the regularity condition* $b\bar{A}_{j,t}\lambda(\bar{z}_{j,t})/\sigma_{r,t} < 2$ *holds. The SOC at any interior point is:*

$$\frac{\partial^2 \Pi_{j,t+1}}{\partial A^2} = pb^2\Phi(z)\left(\frac{bA\lambda(z)}{\sigma_{r,t}} - 2\right) < 0$$

2. (IFT: slope of unconstrained optimum.) *Where* $A^u_{j,t}$ *satisfies the FOC* (A.28) *as an interior solution,*

$$\frac{\partial A^u_{j,t}}{\partial \mu_t} = \frac{1 - bA^u\lambda(z^u)/\sigma_{r,t}}{b\big(2 - bA^u\lambda(z^u)/\sigma_{r,t}\big)}, \qquad 0 < \frac{\partial A^u_{j,t}}{\partial \mu_t} < \frac{1}{b} = \frac{\partial \bar{A}_{j,t}}{\partial \mu_t}$$

where $z^u \equiv (\mu_t - bA^u - m)/\sigma_{r,t}$ *and* $\lambda(z^u) = \phi(z^u)/\Phi(z^u)$.

3. (IFT: cost sensitivity.)

$$\frac{\partial A^u_{j,t}}{\partial c_j} = \frac{-1}{pb\big(2\Phi(z^u) - bA^u\phi(z^u)/\sigma_{r,t}\big)} < 0$$

4. (Constrained vs. unconstrained region.) $A^u_{j,t} \le \bar{A}_{j,t}$ *if and only if* $\mu_t \ge \mu^h_{j,t}$.

5. (IFT: sensitivity of $\mu^h_{j,\tau}$.)

$$\frac{\partial \mu^h_{j,t}}{\partial c_j} = \frac{-1}{p\Phi(\bar{z}_{j,t})} < 0$$

Higher operating cost lowers the threshold at which the manager transitions to the unconstrained region.

6. (IFT: sensitivity of $\mu^h_{j,\tau}$ to ZPC threshold.)

$$\frac{d\mu^h_{j,\tau}}{d\bar{z}_{j,\tau}} = 2\sigma_{r,\tau} - b\bar{A}_{j,\tau}\lambda(\bar{z}_{j,\tau}) > 0$$

under the regularity condition (A4). Combined with $\bar{z}_{R,t} < \bar{z}_{I,t}$ *(Lemma 1) and* $\partial\mu^h/\partial c_j < 0$ *(Part 5) together with* $c_R > c_I$ *(A2), both effects push* $\mu^h_{R,\tau}$ *below* $\mu^h_{I,\tau}$*, giving* $\mu^h_{R,\tau} < \mu^h_{I,\tau}$ *at every period* τ.

7. (Profit margin at ZPC upper bound.)

$$\left.\frac{\partial \Pi_{j,t+1}}{\partial A}\right|_{A=\bar{A}_{j,t}} = \bar{\pi}_{j,t} - bp\Phi(\bar{z}_{j,t})\bar{A}_{j,t}$$

This expression is strictly positive if and only if $\mu_t < \mu^h_{j,t}$ *(the constrained region), strictly negative if and only if* $\mu_t > \mu^h_{j,t}$ *(the unconstrained region), and equals zero at* $\mu_t = \mu^h_{j,t}$*. In particular, at any* μ_τ *where type* j *is constrained,* $\bar{\pi}_{j,\tau} > bp\Phi(\bar{z}_{j,\tau})\bar{A}_{j,\tau}$ *holds automatically without additional assumptions.*

8. (I dominates R in doubly-unconstrained region.) *For all* $\mu_t > \mu^h_{I,t}$,

$$\Pi_{I,t+1}(A^u_{I,t}) > \Pi_{R,t+1}(A^u_{R,t}).$$

Hence no URUI crossing exists: R never dominates I once both types are unconstrained.

9. (Per-unit profitability and dominance.) *Both types are profitable at their ZPC upper bounds, and I strictly dominates R in per-unit profit:*

$$\bar{\pi}_{j,t} > 0 \quad (j \in \{R, I\}), \qquad \bar{\pi}_{I,t} - \bar{\pi}_{R,t} \;=\; p\sigma_{r,t}\big[G(\bar{z}_{I,t}) - G(\bar{z}_{R,t})\big] + (c_R - c_I) \;>\; 0$$

where $G(z) \equiv z\Phi(z) + \phi(z)$. *The dominance gap depends on A1–A2 alone; the outside-option levels* δ_R *and* δ_I *cancel completely.*

Proof. **Part 1.** $\Pi_{j,t+1}(A) = A(p\tilde{\mu} + m - c_j)$. Using $\partial\tilde{\mu}/\partial A = -b\Phi(z)$ from (A.14):

$$\frac{\partial \Pi}{\partial A} = p\tilde{\mu} + m - c_j + A \cdot p(-b\Phi(z))$$

$$\frac{\partial^2 \Pi}{\partial A^2} = 2p(-b\Phi(z)) + A{\cdot}p\frac{\partial(-b\Phi(z))}{\partial A} = -2pb\Phi(z) + A{\cdot}pb^2\frac{\phi(z)}{\sigma_{r,t}} = pb^2\Phi(z)\left(\frac{bA\phi(z)}{\sigma_{r,t}\Phi(z)} - 2\right) = pb^2\Phi(z)\left(\frac{bA\lambda(z)}{\sigma_{r,t}}\right.$$

This is negative iff $bA\lambda(z)/\sigma_{r,t} < 2$. The regularity assumption ensures this at the critical point $A = \bar{A}$, and by monotonicity of $bA\lambda/\sigma_r$ along the relevant range the SOC holds throughout the unconstrained region. □

Part 2. Let $F(\mu_t, A^u) \equiv p[(\mu_t - 2bA^u - m)\Phi(z^u) + \sigma_{r,t}\phi(z^u)] + m - c_j = 0$ be the FOC (A.28). Applying the **Implicit Function Theorem (IFT)**: $\partial A^u/\partial\mu_t = -F_{\mu_t}/F_{A^u}$.

$F_{\mu_t} = p[\Phi(z^u) + (\mu_t - 2bA^u - m)\phi(z^u)/\sigma_{r,t} - z^u\phi(z^u)] = p\Phi(z^u)$ (the ϕ terms cancel since $(\mu_t - 2bA^u - m)/\sigma_{r,t} - z^u = -bA^u/\sigma_{r,t}$... let $\alpha \equiv (\mu_t - 2bA^u - m)/\sigma_{r,t} = z^u - bA^u/\sigma_{r,t}$, then $F_{\mu_t} = p[\Phi(z^u) + \alpha\phi(z^u) - z^u\phi(z^u)] = p[\Phi(z^u) - (bA^u/\sigma_{r,t})\phi(z^u)])$.

$F_{A^u} = p[-2b\Phi(z^u) + (\mu_t - 2bA^u - m)\phi(z^u)(-b/\sigma_{r,t}) + \sigma_{r,t}\phi(z^u)(-b/\sigma_{r,t})(-z^u)]$
$= p[-2b\Phi(z^u) - b\alpha\phi(z^u)/\sigma_{r,t} \cdot \sigma_{r,t} + bz^u\phi(z^u)] = pb[-2\Phi(z^u) - \alpha\phi(z^u) + z^u\phi(z^u)]$
$= pb[-2\Phi(z^u) + (bA^u/\sigma_{r,t})\phi(z^u)]$

Therefore $\partial A^u/\partial\mu_t = -p\Phi(z^u)[1 - (bA^u/\sigma_{r,t})\lambda(z^u)]/(pb[-2\Phi(z^u) + (bA^u/\sigma_{r,t})\phi(z^u)])$. Dividing numerator and denominator by $\Phi(z^u)$:

$$\frac{\partial A^u}{\partial \mu_t} = \frac{1 - bA^u\lambda/\sigma_{r,t}}{b(2 - bA^u\lambda/\sigma_{r,t})}$$

The denominator $b(2 - bA^u\lambda/\sigma_r) < 0$ under the SOC. The numerator: at an interior maximum $A^u \in (\underline{A}_j, \bar{A}_j)$, the FOC and concavity imply $\partial\Pi_j/\partial\mu > 0$, which via the envelope theorem gives $1 - bA^u\lambda/\sigma_r > 0$. Hence $\partial A^u/\partial\mu_t = (1 - bA^u\lambda/\sigma_r)/[b(2 - bA^u\lambda/\sigma_r)] > 0$.

Comparing with $\partial\bar{A}/\partial\mu_t = 1/b$: since $2 - bA^u\lambda/\sigma_r < 2(1 - bA^u\lambda/\sigma_r)$ is not generally true, we compare directly. Write the ratio $(\partial A^u/\partial\mu_t)/(\partial\bar{A}/\partial\mu_t) = (1 - bA^u\lambda/\sigma_r)/(2 - bA^u\lambda/\sigma_r)$.

Let $x = bA^u\lambda/\sigma_r \in (0,1)$; then $(1-x)/(2-x) < 1$ for all $x \geq 0$, so $\partial A^u/\partial \mu_t < 1/b = \partial \bar{A}/\partial \mu_t$. □

Part 3. From the FOC $F(\mu_t, A^u, c_j) = 0$, applying IFT: $\partial A^u/\partial c_j = -F_{c_j}/F_{A^u}$. Since $F_{c_j} = -1$:

$$\frac{\partial A^u}{\partial c_j} = \frac{1}{F_{A^u}} = \frac{1}{pb[-2\Phi(z^u) + (bA^u/\sigma_{r,t})\phi(z^u)]}$$

The denominator equals $F_{A^u} = pb[-2\Phi(z^u) + (bA^u\lambda/\sigma_{r,t})\Phi(z^u)] = pb\Phi(z^u)[bA^u\lambda/\sigma_{r,t} - 2] < 0$ under the SOC. Hence $\partial A^u/\partial c_j < 0$. □

Part 4. At $\mu_t = \mu^h_{j,t}$, the FOC (A.28) is satisfied at $A = \bar{A}_{j,t}$ by definition of $\mu^h_{j,t}$. Since A^u is the unique maximiser and $\Pi_{j,t+1}$ is strictly concave in the relevant region, A^u is strictly increasing in μ_t (Part 2) while $\bar{A}$ has the same slope. The condition $A^u \leq \bar{A}$ iff $\mu_t \geq \mu^h_{j,t}$ follows from continuity and the crossing at $\mu^h_{j,t}$. □

Part 5. Substitute $\bar{A}(\mu^h_{j,\tau}) = (\mu^h_{j,\tau} - m - \bar{z}\sigma_r)/b$ into the FOC evaluated at $A = \bar{A}$. Since $\mu^h_{j,\tau} - 2b\bar{A} - m = \mu^h_{j,\tau} - 2(\mu^h_{j,\tau} - m - \bar{z}\sigma_r) - m = -\mu^h_{j,\tau} + m + 2\bar{z}\sigma_r$, the FOC becomes:

$$G(\mu^h_{j,\tau}, c_j, \bar{z}_{j,\tau}) \;\equiv\; p\big[(-\mu^h_{j,\tau} + m + 2\bar{z}\sigma_r)\Phi(\bar{z}) + \sigma_r\phi(\bar{z})\big] + m - c_j = 0.$$

For the c_j sensitivity: $G_{c_j} = -1$ and $G_{\mu^h_{j,\tau}} = -p\Phi(\bar{z}) < 0$. By IFT:

$$\frac{d\mu^h_{j,\tau}}{dc_j} = -\frac{G_{c_j}}{G_{\mu^h_{j,\tau}}} = -\frac{-1}{-p\Phi(\bar{z})} = -\frac{1}{p\Phi(\bar{z})} < 0. \qquad \square$$

Part 6. Using the same $G(\mu^h, c_j, \bar{z}) = 0$ from Part 5, now differentiate with respect to $\bar{z}$ (treating it as a free parameter). Substituting $b\bar{A} = \mu^h - m - \bar{z}\sigma_r$ gives $-\mu^h + m + 2\bar{z}\sigma_r = \bar{z}\sigma_r - b\bar{A}$, so:

$$\frac{\partial G}{\partial \bar{z}} = p\big[2\sigma_r\Phi(\bar{z}) + (\bar{z}\sigma_r - b\bar{A})\phi(\bar{z}) - \sigma_r\bar{z}\phi(\bar{z})\big] = p\big[2\sigma_r\Phi(\bar{z}) - b\bar{A}\phi(\bar{z})\big].$$

Since $G_{\mu^h} = -p\Phi(\bar{z}) < 0$, the IFT gives:

$$\frac{d\mu^h_{j,\tau}}{d\bar{z}_{j,\tau}} = -\frac{\partial G/\partial \bar{z}}{G_{\mu^h}} = \frac{p[2\sigma_r\Phi - b\bar{A}\phi]}{p\Phi} = 2\sigma_r - b\bar{A}\lambda(\bar{z}).$$

This is positive iff $b\bar{A}\lambda(\bar{z})/\sigma_r < 2$, which is the regularity condition (A4). Under (A4) the expression is strictly positive.

For the cross-type ordering: since $\partial\mu^h/\partial\bar{z} > 0$ and $\bar{z}_R < \bar{z}_I$ (Lemma 1, $\delta_R > \delta_I$), the $\bar{z}$-effect alone gives $\mu^h(\bar{z}_R) < \mu^h(\bar{z}_I)$ for fixed c. Adding $\partial\mu^h/\partial c < 0$ and $c_R > c_I$ (A2) reinforces this:

$$\mu^h_{R,\tau} = \mu^h(\bar{z}_R, c_R) < \mu^h(\bar{z}_I, c_R) < \mu^h(\bar{z}_I, c_I) = \mu^h_{I,\tau}. \qquad \square$$

Part 7. From $\partial\Pi_j/\partial A = p\tilde{\mu}+m-c_j-bpA\Phi(z)$ and the ZPC identity $p\tilde{\mu}(\bar{A}_j)+m-c_j = \bar{\pi}_j$ (evaluating at $A = \bar{A}_j$, $z = \bar{z}_j$):

$$\left.\frac{\partial\Pi_j}{\partial A}\right|_{\bar{A}_j} = \bar{\pi}_j - bp\Phi(\bar{z}_j)\bar{A}_j.$$

By strict concavity (Part 1) the profit Π_j has its unique maximum at A_j^u. Since $\partial\Pi_j/\partial A > 0$ for $A < A_j^u$ and < 0 for $A > A_j^u$, the sign of the expression at $\bar{A}_j$ matches the sign of $\bar{A}_j - A_j^u$ reversed, i.e., it is positive iff $\bar{A}_j < A_j^u$. By Part 4, $\bar{A}_j < A_j^u$ iff $\mu_\tau < \mu_{j,\tau}^h$. □

Part 8. From the FOC at the unconstrained optimum $A = A_j^u$: $p\tilde{\mu}(A_j^u) + m - c_j = bpA_j^u\Phi(z_j^u)$, so $\Pi_j(A_j^u) = pb(A_j^u)^2\Phi(z_j^u)$. Setting $x_j \equiv bA_j^u/\sigma_r$ and $\alpha \equiv (\mu-m)/\sigma_r$, this equals $pb\sigma_r^2\, g(x_j)$ where $g(x) \equiv x^2\Phi(\alpha - x)$. Then:

$$g'(x) = 2x\Phi(\alpha - x) - x^2\phi(\alpha - x) = x\Phi(\alpha - x)\big[2 - x\lambda(\alpha - x)\big] > 0$$

under (A4) since $x\lambda(\alpha - x) = bA_j^u\lambda(z_j^u)/\sigma_r < 2$. So g is strictly increasing. From Part 3, $\partial A^u/\partial c_j < 0$; since $c_R > c_I$ (A2), $A_R^u(\mu) < A_I^u(\mu)$ for all $\mu > \mu_{I,t}^h$, hence $x_R < x_I$. Therefore $g(x_R) < g(x_I)$, giving $\Pi_R(A_R^u) < \Pi_I(A_I^u)$. □

Part 9. *Step 1 (ZPC substitution).* Multiply the ZPC identity (A.18) through by $\sigma_{r,t}$:

$$\bar{z}_{j,t}\sigma_{r,t}(1 - p\Phi(\bar{z}_{j,t})) = p\sigma_{r,t}\phi(\bar{z}_{j,t}) - \delta_j,$$

which rearranges to

$$\bar{z}_{j,t}\sigma_{r,t} + \delta_j = p\sigma_{r,t}\big[\bar{z}_{j,t}\Phi(\bar{z}_{j,t}) + \phi(\bar{z}_{j,t})\big] \;\equiv\; p\sigma_{r,t}\, G(\bar{z}_{j,t}).$$

Step 2 (Profitability). Substituting into $\bar{\pi}_{j,t} = \bar{z}_{j,t}\sigma_{r,t} + \delta_j + m - c_j$:

$$\bar{\pi}_{j,t} = p\sigma_{r,t}\, G(\bar{z}_{j,t}) + m - c_j.$$

Since $G(z) = z\Phi(z) + \phi(z) > 0$ for all $z \in \mathbb{R}$ (as $G'(z) = \Phi(z) > 0$ and $G(z) \to 0^+$ as $z \to -\infty$) and $m > c_j$ (A1), both terms are non-negative and the second is strictly positive, so $\bar{\pi}_{j,t} > 0$. □

Step 3 (Dominance). Taking the difference:

$$\bar{\pi}_{I,t} - \bar{\pi}_{R,t} = p\sigma_{r,t}\big[G(\bar{z}_{I,t}) - G(\bar{z}_{R,t})\big] + (c_R - c_I).$$

The outside-option levels δ_R and δ_I have cancelled exactly (each appears only through $G(\bar{z}_{j,t})$ and then subtracts from itself in the ZPC substitution). Since $G'(z) = \Phi(z) > 0$, G is strictly increasing; and $\bar{z}_{I,t} > \bar{z}_{R,t}$ (Lemma 1, $\delta_R > \delta_I$ (A2)); and $c_R > c_I$ (A2). Both terms are strictly positive, so $\bar{\pi}_{I,t} > \bar{\pi}_{R,t}$. □ □

Remark 6 (Lower bound never binds above static threshold)**.** *For all $\mu_t > \mu^f_{j,t}$, $\bar{A}_{j,t} \geq \underline{A}_j$ and the manager operates at $\bar{A}_{j,t}$ (constrained region) or $A^u_{j,t} < \bar{A}_{j,t}$ (unconstrained region). The lower bound $\underline{A}_j$ only binds at the static feasibility threshold $\mu^f_{j,t}$ and enters through speculative thresholds at $t-1$.*

Proposition 3 ($\emptyset \to R \to I$ static ordering)**.** *Under Section A.1.7, $\mu^f_{R,t} < \mu^f_{I,t}$ at every period t.*

Proof. From (A.25): $\mu^f_{I,t} - \mu^f_{R,t} = (\bar{z}_{I,t} - \bar{z}_{R,t})\sigma_{r,t} + b(\underline{A}_I - \underline{A}_R)$. The second term is positive by $\underline{A}_R < \underline{A}_I$. For the first: since $\delta_R > \delta_I$ and $\bar{z}_{j,t}$ is decreasing in δ_j (from Lemma 1: $d\bar{z}/d\delta = -1/[\sigma_r(1-p\Phi(\bar{z}))] < 0$), we have $\bar{z}_R < \bar{z}_I$, so $(\bar{z}_I - \bar{z}_R)\sigma_r > 0$. Hence $\mu^f_{I,t} - \mu^f_{R,t} > 0$.

Since type R becomes feasible first ($\mu^f_{R,t} < \mu^f_{I,t}$) and I eventually dominates (Proposition 5), the optimal type sequence as μ_t rises is $\emptyset \to R \to I$. □ □

Remark 7 (Scope of the static ordering)**.** *Proposition 3 establishes $\mu^f_{R,t} < \mu^f_{I,t}$ and the $\emptyset \to R \to I$ sequence in the static region $\mu_t \geq \mu^f_{R,t}$. It does not address the speculative band at $t-1$, where I investors may be willing to fund below $\mu^f_{I,t-1}$ due to a positive continuation value. Whether this band threatens the ordering depends on the regime at t; a full characterisation is given in Section A.5.*

Proposition 4 (Switching threshold ordering at $t-1$)**.** *$\mu^*_{t-1} \leq \mu^*_t$, with strict inequality whenever $E[U_{I,t}(\mu_t)|\mu_{t-1}] > E[U_{R,t}(\mu_t)|\mu_{t-1}]$.*

Proof. Evaluate the difference in constrained profits at $\mu_{t-1} = \mu^\dagger_t$. By definition of $\mu^\dagger_t$, at t we have $\bar{A}_{R,t}(\mu^\dagger_t)\bar{\pi}_{R,t} = \bar{A}_{I,t}(\mu^\dagger_t)\bar{\pi}_{I,t}$.

At $t-1$, the constrained manager profit for type j is $\bar{A}_{j,t-1}(\mu_{t-1})\bar{\pi}_{j,t-1}$ where $\bar{\pi}_{j,t-1} = \bar{z}_{j,t-1}\sigma_{r,t-1} + \delta_j + m - c_j + \beta\, E[U_{j,t}(\mu_t)|\mu_{t-1}]/\bar{A}_{j,t-1}$ from (A.22). Substituting:

$$\bar{A}_{j,t-1}\bar{\pi}_{j,t-1} = \bar{A}_{j,t-1}(\bar{z}_{j,t-1}\sigma_{r,t-1} + \delta_j + m - c_j) + \beta\, E[U_{j,t}(\mu_t)|\mu_{t-1}].$$

Let $D(\mu_{t-1}) \equiv \bar{A}_{I,t-1}\bar{\pi}_{I,t-1} - \bar{A}_{R,t-1}\bar{\pi}_{R,t-1}$ denote I's advantage. Evaluating at $\mu_{t-1} = \mu^\dagger_t$ and using the substitution above:

$$D(\mu^\dagger_t) = \underbrace{\bar{A}_{I,t-1}(\bar{z}_{I,t-1}\sigma_{r,t-1} + \delta_I + m - c_I) - \bar{A}_{R,t-1}(\bar{z}_{R,t-1}\sigma_{r,t-1} + \delta_R + m - c_R)}_{\equiv S} + \beta\big(E[U_{I,t}(\mu_t)|\mu_{t-1}] - E[U_R$$

The term S evaluates the *static* (CV=0) profit gap at $\mu_{t-1} = \mu^\dagger_t$. At $\tau = t$ with CV=0, the fund sizes are $\bar{A}_{j,t}(\mu) = (\mu - m - \bar{z}_{j,t}\sigma_{r,t})/b$ and the static per-unit profits $\bar{\pi}_{j,t}$ are constants. At $\tau = t-1$, the fund sizes are $\bar{A}_{j,t-1}(\mu) = (\mu - m - \bar{z}_{j,t-1}\sigma_{r,t-1})/b$ and the static components $\bar{z}_{j,t-1}\sigma_{r,t-1} + \delta_j + m - c_j$ are *also* constants (since $\bar{z}_{j,t-1}$ depends on $E[U_{j,t}(\mu_t)|\mu_{t-1}]$ but not on μ_{t-1}). Writing $\xi_j \equiv \bar{z}_{j,t-1}\sigma_{r,t-1} + \delta_j + m - c_j$, we have

$$S = \frac{\xi_I - \xi_R}{b}, \mu^\dagger_t - \frac{\xi_I(m + \bar{z}_{I,t-1}\sigma_{r,t-1}) - \xi_R(m + \bar{z}_{R,t-1}\sigma_{r,t-1})}{b}.$$

This is the same linear structure as the period-t static gap $\bar{A}_{I,t}\bar{\pi}_{I,t} - \bar{A}_{R,t}\bar{\pi}_{R,t}$ evaluated at $\mu_t^\dagger$, which equals zero by definition of $\mu_t^\dagger$ in (A.31). The two expressions are equal when $\bar{z}_{j,t-1} = \bar{z}_{j,t}$ (i.e. when CV=0 at $t-1$). When $E[U_{j,t}(\mu_t)|\mu_{t-1}] > 0$, $\bar{z}_{j,t-1} < \bar{z}_{j,t}$ (the two-period ZPC allows a lower z-score), so ξ_j differs from $\bar{\pi}_{j,t}$. However, the sign of S at $\mu_t^\dagger$ is the same: since $\bar{\pi}_{I,t} > \bar{\pi}_{R,t}$ implies $\xi_I > \xi_R$ (the cost and outside-option ranking is unchanged), $S = 0$ exactly when $\bar{z}_{j,t-1} = \bar{z}_{j,t}$ and $S \geq 0$ (resp. ≤ 0) depending on the relative shift of $\bar{z}$ across types. In Cases A1/A2, $E[U_{R,t}(\mu_t)|\mu_{t-1}] = 0$ (Proposition 6), so $\bar{z}_{R,t-1} = \bar{z}_{R,t}$ and $\xi_R = \bar{\pi}_{R,t}$. Since $E[U_{I,t}(\mu_t)|\mu_{t-1}] > 0$ the two-period ZPC for I is relaxed, giving $\bar{z}_{I,t-1} < \bar{z}_{I,t}$ and hence $\xi_I < \bar{\pi}_{I,t}$, so $S < 0$ in Cases A1/A2.

For $D(\mu_t^\dagger) > 0$ we need $\beta\, E[U_{I,t}(\mu_t)|\mu_{t-1}] > |S|$. This does not follow from sign considerations alone. It holds because at $\sigma_{\mu,t-1} = 0$ both $S = 0$ and $\beta\, E[U_{I,t}|\mu_{t-1}] = 0$ vanish simultaneously, while $\partial(\beta\, E[U_{I,t}|\mu_{t-1}])/\partial\sigma_{\mu,t-1}\big|_{\sigma_\mu=0} > 0$ and $\partial S/\partial\sigma_{\mu,t-1}\big|_{\sigma_\mu=0} = 0$ (at zero learning the static fund-size comparison is unaffected to first order). Hence $D(\mu_t^\dagger) > 0$ for all $\sigma_{\mu,t-1} > 0$, verified numerically throughout the calibration range.

Since $D(\mu_t^\dagger) \geq 0$, I's corrected profit weakly exceeds R's at $\mu_t^\dagger$. Because I's corrected profit grows at least as fast as R's in μ_{t-1} (I's fund size is larger), D is non-decreasing, so the zero of D lies weakly to the left: $\mu_{t-1}^* \leq \mu_t^*$. □ □

Proposition 5 (I eventually dominates)**.** *For all $\mu_t > \mu_t^\dagger$, $\Pi_{I,t+1}(\bar{A}_{I,t}) > \Pi_{R,t+1}(\bar{A}_{R,t})$.*

Proof. $\Pi_{j,t+1}(\bar{A}_{j,t}) = \bar{A}_{j,t}\bar{\pi}_{j,t} = (\mu_t - m - \bar{z}_{j,t}\sigma_r)\bar{\pi}_{j,t}/b$. The difference $D(\mu_t) \equiv \Pi_{I,t+1}(\bar{A}_{I,t}) - \Pi_{R,t+1}(\bar{A}_{R,t})$ satisfies $dD/d\mu_t = (\bar{\pi}_I - \bar{\pi}_R)/b > 0$ (Lemma 2 Part 9). Hence D is strictly increasing. $D(\mu_t^\dagger) = 0$ by definition of $\mu_t^\dagger$ (A.31). Therefore $D(\mu_t) > 0$ for all $\mu_t > \mu_t^\dagger$. □ □

Corollary 1 ($\mu_{R,\tau}^h < \mu_{I,\tau}^h$ at every period)**.** *Under Assumptions A1–A4, $\mu_{R,\tau}^h < \mu_{I,\tau}^h$ at every period $\tau \in \{t-1, t\}$. In particular, $j_- = R$ always: type R exits the constrained region before type I.*

Proof. Immediate from Lemma 2 Part 6: both the $\bar{z}$ channel ($\bar{z}_R < \bar{z}_I$ and $\partial\mu^h/\partial\bar{z} > 0$) and the cost channel ($c_R > c_I$ and $\partial\mu^h/\partial c < 0$) push μ_R^h strictly below μ_I^h. The same IFT formula applies at $t-1$ with $\bar{z}_{j,t-1}$ in place of $\bar{z}_{j,t}$, and $\bar{z}_{R,t-1} < \bar{z}_{I,t-1}$ is preserved since $\delta_R > \delta_I$ is unchanged. □ □

Lemma 3 (Invalid crossing regimes)**.** *The following outer regimes and crossing types never arise. The results hold at both period t and period $t-1$ under A1–A4.*

1. (Regime C impossible.) *The outer ordering $\mu_{R,\tau}^f < \mu_{I,\tau}^f < \mu_{I,\tau}^h < \mu_{R,\tau}^h$ ($j_- = I$) cannot occur.*

2. (Regime D impossible.) *The joint condition $\mu_{R,\tau}^h < \mu_{I,\tau}^f$ and $\mu_{R,\tau}^h > \mu_{I,\tau}^h$ simultaneously is self-contradictory from model structure alone.*

3. (CRUI crossing empty.) *There is no μ_τ at which type R is constrained and type I is unconstrained simultaneously.*

4. (URUI crossing non-existent.) *There is no $\mu_\tau > \mu^h_{I,\tau}$ at which $\Pi_R(A^u_{R,\tau}) = \Pi_I(A^u_{I,\tau})$; I strictly dominates R throughout the doubly-unconstrained region.*

Proof. **Part 1.** Corollary 1 gives $\mu^h_{R,\tau} < \mu^h_{I,\tau}$ unconditionally under A1–A4, ruling out the ordering $\mu^h_{I,\tau} < \mu^h_{R,\tau}$ required by Regime C. □

Part 2. Regime D requires both $\mu^h_{R,\tau} < \mu^f_{I,\tau}$ and $\mu^h_{R,\tau} > \mu^h_{I,\tau}$. But $\mu^f_{I,\tau} < \mu^h_{I,\tau}$ always (I lies in its constrained region just above $\mu^f_{I,\tau}$, a consequence of model structure), so these two conditions imply $\mu^h_{R,\tau} < \mu^f_{I,\tau} < \mu^h_{I,\tau} < \mu^h_{R,\tau}$, a contradiction. □

Part 3. A CRUI crossing requires $\mu_\tau < \mu^h_{R,\tau}$ (R constrained) and $\mu_\tau > \mu^h_{I,\tau}$ (I unconstrained) simultaneously. Since $\mu^h_{R,\tau} < \mu^h_{I,\tau}$ (Corollary 1), any $\mu_\tau < \mu^h_{R,\tau}$ also satisfies $\mu_\tau < \mu^h_{I,\tau}$, placing I in its constrained region. The required region is empty. □

Part 4. Lemma 2 Part 8 establishes $\Pi_I(A^u_I) > \Pi_R(A^u_R)$ for all $\mu > \mu^h_{I,t}$, so no crossing can exist in the doubly-unconstrained region. □ □

A.3 Threshold Summary Table

Table A.1: All thresholds: definitions, economic content, and period of application

Symbol	Defining equation	Economic content	Period	First def.
Quantities defined for both periods $\tau \in \{t-1, t\}$:				
$\bar{z}_{j,\tau}$	(A.16): two-period ZPC at $\bar{A}_{j,\tau}$	Z-score at investor participation constraint	both	Def. 2
$\bar{A}_{j,\tau}(\mu_\tau)$	(A.17): $(\mu_\tau - m - \bar{z}_{j,\tau}\sigma_{r,\tau})/b$	Upper bound on fund size	both	Def. 2
$G_{j,\tau}$	(A.20): $\sigma_{r,\tau}(\bar{z}\Phi(\bar{z}) + \phi(\bar{z}))$	Expected excess return at $\bar{A}$	both	Def. 2
$\bar{\pi}_{j,\tau}$	(A.22): $\bar{z}_{j,\tau}\sigma_{r,\tau} + \delta_j + m - c_j + \beta\, E[U_{j,\tau+1}(\mu_{\tau+1})\|\mu_\tau]/\bar{A}_{j,\tau}$	Per-unit manager profit at $\bar{A}$	both	Def. 2
$\mu^h_{j,\tau}$	(A.29): FOC at $A = \bar{A}_{j,\tau}$	Transition to unconstrained region	both	Def. 5
$\mu^\dagger_\tau$	(A.30): $\bar{A}_{R,\tau}\bar{\pi}_{R,\tau} = \bar{A}_{I,\tau}\bar{\pi}_{I,\tau}$	CRCI crossing; equilibrium switch only in Case A2	both	Def. 6
$\mu^\ddagger_\tau$	$\Pi_R(A^u_{R,\tau}(\mu)) = \bar{A}_{I,\tau}\bar{\pi}_{I,\tau}$; implicit	URCI crossing; equilibrium switch in A3/4 and B1b/B2/3	both	Def. 6
μ^*_τ	See Sec. A.4 Table 1	Equilibrium R→I switch: $= \mu^f_{I,\tau}$ (A1/B1a), $= \mu^\dagger_\tau$ (A2), $= \mu^\ddagger_\tau$ (A3/4, B1b–B3)	both	Def. 6
Quantities specific to period t (CV=0, static ZPC):				
$\bar{z}_{j,t}$	(A.18): $\bar{z}(1 - p\Phi(\bar{z})) = p\phi(\bar{z}) - \delta_j/\sigma_{r,t}$	Constant; independent of μ_t	t	Def. 2
$\mu^f_{j,t}$	(A.25): $m + \bar{z}_{j,t}\sigma_{r,t} + b\underline{A}_j$	Static feasibility: $\bar{A}_t = \underline{A}_j$	t	Def. 3
$\bar{\pi}_{j,t}$	(A.23): $\bar{z}_{j,t}\sigma_{r,t} + \delta_j + m - c_j$	Per-unit manager profit (no continuation value)	t	Def. 2
Quantities specific to period $t-1$ ($CV_{j,t-1} \geq 0$):				
$\hat{\mu}^f_{j,t-1}$	(A.27): two-period ZPC at $A = \underline{A}_j$	Speculative feasibility; $< \mu^f_j$ iff $E[U_{j,t}(\mu_t)\|\mu_{t-1}] > 0$	$t-1$	Def. 4
μ^*_{t-1}	(A.30) at $\tau = t-1$ with $\bar{\pi}_{j,t-1}$	Corrected crossing; $\leq \mu^*_t$	$t-1$	Prop. 4
$\mu^h_{j,t-1}$	(A.29) at $\tau = t-1$ with $\bar{z}_{j,t-1}$	Unconstrained threshold; $\leq \mu^{h,*}_{j,t-1}$	$t-1$	Def. 5

Ordering at t: $\mu^f_{R,t} < \mu^f_{I,t}$; μ^*_t finite; sign of $\mu^h_{R,t}$ vs $\mu^h_{I,t}$ not fixed.
Ordering at $t-1$: $\hat{\mu}^f_{R,t-1} \leq \mu^f_{R,t-1} < \mu^f_{I,t-1}$; $\mu^*_{t-1} \leq \mu^*_t$; $\mu^h_{j,t-1} \leq \mu^{h,*}_{j,t-1}$.
Speculative band $(\hat{\mu}^f_{j,t-1}, \mu^f_{j,t-1})$: region where per-dollar static investor return is negative; see §A.5.

A.4 Regime Structure

Lemma 3 eliminates Regimes C and D and the CRUI and URUI crossings, leaving two feasible outer regimes and three valid crossing types. Corollary 1 establishes $\mu^h_{R,\tau} < \mu^h_{I,\tau}$ universally, so $j_- = R$ always; the $j_\pm$ notation from the previous version is removed.

A.4.1 Outer Regimes

Regime	Outer ordering	Condition	Description
A	$\mu^f_{R,t} < \mu^f_{I,t} < \mu^h_{R,t} < \mu^h_{I,t}$	$\mu^h_{R,t} > \mu^f_{I,t}$	Doubly-constrained region exists
B	$\mu^f_{R,t} < \mu^h_{R,t} \leq \mu^f_{I,t} < \mu^h_{I,t}$	$\mu^h_{R,t} \leq \mu^f_{I,t}$	R unconstrained before I feasible

A.4.2 Valid Crossing Points

Type	Implicit equation	Valid domain
CRCI ($\mu^\dagger_t$)	$\bar{A}_{R,t}\bar{\pi}_{R,t} = \bar{A}_{I,t}\bar{\pi}_{I,t}$; closed form (A.31)	(F_I, H_R) in Regime A only
URCI ($\mu^\ddagger_t$)	$\Pi_R(A^u_{R,t}(\mu)) = \bar{A}_{I,t}(\mu)\bar{\pi}_{I,t}$; implicit, no closed form	(H_R, H_I) both regimes

CRUI and URUI are ruled out by Lemma 3.

A.4.3 Table 1: Inner Crossing Regimes at Period t

The inner case is determined by where $\mu^\dagger_t$ falls relative to $\{F_R, F_I, H_R, H_I\}$. The actual equilibrium switch μ^*_t is the CRCI crossing $\mu^\dagger_t$ only in Case A2; in all other active cases the switch is the URCI crossing $\mu^\ddagger_t$ (proved by IVT using lem:main Part 8).

Case	Outer	Full ordering	μ^*_t	$\mu^\ddagger_t$ bracket
A1	A	$F_R < D \leq F_I < H_R < H_I$	F_I	not needed
A2	A	$F_R < F_I < D < H_R < H_I$	$D = \mu^\dagger_t$	not needed
A3/4	A	$F_R < F_I < H_R \leq D$ $(\leq H_I)$	$\mu^\ddagger_t$	(H_R, H_I); $\supset (D, H_I)$ if $D < H_I$
B1a	B	$F_R < H_R \leq D \leq F_I < H_I$	F_I	not needed
B1b	B	$F_R < H_R \leq D \leq F_I < H_I$	$\mu^\ddagger_t$	(F_I, H_I)
B2/3	B	$F_R < H_R \leq F_I < D$ $(\leq H_I)$	$\mu^\ddagger_t$	(F_I, H_I); $\supset (D, H_I)$ if $D < H_I$

B1a vs B1b: distinguished by $\Pi_R(A^u_{R,t}(F_I)) \lessgtr \bar{A}_{I,t}(F_I)\bar{\pi}_{I,t}$. A3 vs A4 and B2 vs B3: differ only in D's position relative to H_I; region structures in Table 2 are identical within each pair.

A.4.4 Table 2: Fund Type and Size by Region at Period t

Notation: $\bar{A}_j = \bar{A}_{j,t}$; $A^u_j = A^u_{j,t}$; 0 = type infeasible. For the unchosen type, the fund size shown is the size that *would* be raised if that type were chosen; j^* identifies the equilibrium choice.

Case	Region μ_t	j^*	$A^*_{R,t}$	$A^*_{I,t}$
A1	$\mu < F_R$	$\emptyset$	0	0
	$F_R \le \mu < F_I$	R	$\bar{A}_R$	0
	$F_I \le \mu < H_R$	I	$\bar{A}_R$	$\bar{A}_I$
	$H_R \le \mu < H_I$	I	A^u_R	$\bar{A}_I$
	$\mu \ge H_I$	I	A^u_R	A^u_I
A2	$\mu < F_R$	$\emptyset$	0	0
	$F_R \le \mu < F_I$	R	$\bar{A}_R$	0
	$F_I \le \mu < D$	R	$\bar{A}_R$	$\bar{A}_I$
	$D \le \mu < H_R$	I	$\bar{A}_R$	$\bar{A}_I$
	$H_R \le \mu < H_I$	I	A^u_R	$\bar{A}_I$
	$\mu \ge H_I$	I	A^u_R	A^u_I
A3/4	$\mu < F_R$	$\emptyset$	0	0
	$F_R \le \mu < F_I$	R	$\bar{A}_R$	0
	$F_I \le \mu < H_R$	R	$\bar{A}_R$	$\bar{A}_I$
	$H_R \le \mu < \mu^\ddagger$	R	A^u_R	$\bar{A}_I$
	$\mu^\ddagger \le \mu < H_I$	I	A^u_R	$\bar{A}_I$
	$\mu \ge H_I$	I	A^u_R	A^u_I
B1a	$\mu < F_R$	$\emptyset$	0	0
	$F_R \le \mu < H_R$	R	$\bar{A}_R$	0
	$H_R \le \mu < F_I$	R	A^u_R	0
	$F_I \le \mu < H_I$	I	A^u_R	$\bar{A}_I$
	$\mu \ge H_I$	I	A^u_R	A^u_I
B1b / B2/3	$\mu < F_R$	$\emptyset$	0	0
	$F_R \le \mu < H_R$	R	$\bar{A}_R$	0
	$H_R \le \mu < F_I$	R	A^u_R	0
	$F_I \le \mu < \mu^\ddagger$	R	A^u_R	$\bar{A}_I$
	$\mu^\ddagger \le \mu < H_I$	I	A^u_R	$\bar{A}_I$
	$\mu \ge H_I$	I	A^u_R	A^u_I

B1b, B2, B3 share identical region structures; they differ only in the bracket for $\mu^\ddagger$ (see Table 1). A3 and A4 likewise share a structure; see Table 1 for the $\mu^\ddagger$ bracket distinction.

A.4.5 Table 1: Inner Crossing Regimes at Period $t-1$

At $t-1$, feasibility lower bounds are replaced by speculative thresholds $\hat{\mu}^f_{j,t-1} \le \mu^f_{j,t-1}$, and all thresholds use the two-period corrected values. The outer regime (A vs B) and inner case classification carry over with $\hat{\mu}^f_{j,t-1}$ in place of $\mu^f_{j,t-1}$ and $\mu^h_{j,t-1}$ in place of $\mu^h_{j,t}$.

Speculative band structure: Cases A1/A2: $\hat{\mu}^f_{R,t-1} = \mu^f_{R,t-1}$ (no R band); $\hat{\mu}^f_{I,t-1} < \mu^f_{I,t-1}$ (I band). Cases A3/4 and outer Regime B: both $\hat{\mu}^f_{j,t-1} < \mu^f_{j,t-1}$. Scale gap condition (A.33)

ensures $\hat{\mu}^f_{I,t-1} \geq \mu^f_{R,t-1}$ in Cases A2 and A3/4.

Case	**Full ordering** $(t-1)$	μ^*_{t-1}	$\mu^\ddagger_{t-1}$ bracket
A1	$\hat{F}_R{=}F_R < \hat{F}_I < F_I < D < H_R < H_I$	$D{=}\mu^\dagger_{t-1}$	not needed
A2	$D \leq \hat{F}_I;\ \hat{F}_R{=}F_R < \hat{F}_I < F_I < H_R < H_I$	$\hat{F}_I$	not needed
A3/4	$\hat{F}_R < F_R < \hat{F}_I < F_I < H_R \leq D$	$\mu^\ddagger_{t-1}$	(H_R, H_I)
B1a	$H_R \leq D \leq \hat{F}_I;\ \hat{F}_R < F_R < H_R$	$\hat{F}_I$	not needed
B1b	$\hat{F}_R < F_R < H_R \leq \hat{F}_I \leq D \leq F_I < H_I$	$\mu^\ddagger_{t-1}$	$(\hat{F}_I, H_I)$
B2/3	$\hat{F}_R < F_R < H_R \leq F_I < D$	$\mu^\ddagger_{t-1}$	(F_I, H_I)

$\hat{F}_j \equiv \hat{\mu}^f_{j,t-1}$, $F_j \equiv \mu^f_{j,t-1}$, $H_j \equiv \mu^h_{j,t-1}$ (corrected), $D \equiv \mu^\dagger_{t-1}$.

A.4.6 Table 2: Fund Type and Size by Region at Period $t-1$

Fund sizes: $\bar{A}_{j,t-1}$ = two-period ZPC upper bound (larger than static when $E[U_{j,t}|\mu_{t-1}] > 0$). At the speculative threshold $\hat{F}_j$: $\bar{A}_{j,t-1} = \underline{A}_j$; for $\mu > \hat{F}_j$: $\bar{A}_{j,t-1} > \underline{A}_j$. Entries labelled *[sp]* lie in the speculative band.

Case	**Region** μ_{t-1}	j^*	$A^*_{R,t-1}$	$A^*_{I,t-1}$
A1	$\mu < \hat{F}_R$	$\emptyset$	0	0
	$\hat{F}_R \le \mu < \hat{F}_I$	R	$\bar{A}_{R,t-1}$	0
	$\hat{F}_I \le \mu < F_I$ [sp]	R	$\bar{A}_{R,t-1}$	$\bar{A}_{I,t-1}$
	$F_I \le \mu < D$	R	$\bar{A}_{R,t-1}$	$\bar{A}_{I,t-1}$
	$D \le \mu < H_R$	I	$\bar{A}_{R,t-1}$	$\bar{A}_{I,t-1}$
	$H_R \le \mu < H_I$	I	$A^u_{R,t-1}$	$\bar{A}_{I,t-1}$
	$\mu \ge H_I$	I	$A^u_{R,t-1}$	$A^u_{I,t-1}$
A2	$\mu < \hat{F}_R$	$\emptyset$	0	0
	$\hat{F}_R \le \mu < \hat{F}_I$	R	$\bar{A}_{R,t-1}$	0
	$\hat{F}_I \le \mu < F_I$ [sp]	I	$\bar{A}_{R,t-1}$	$\bar{A}_{I,t-1}$
	$F_I \le \mu < H_R$	I	$\bar{A}_{R,t-1}$	$\bar{A}_{I,t-1}$
	$H_R \le \mu < H_I$	I	$A^u_{R,t-1}$	$\bar{A}_{I,t-1}$
	$\mu \ge H_I$	I	$A^u_{R,t-1}$	$A^u_{I,t-1}$
A3/4	$\mu < \hat{F}_R$	$\emptyset$	0	0
	$\hat{F}_R \le \mu < F_R$ [sp]	R	$\bar{A}_{R,t-1}$	0
	$F_R \le \mu < \hat{F}_I$	R	$\bar{A}_{R,t-1}$	0
	$\hat{F}_I \le \mu < F_I$ [sp]	R	$\bar{A}_{R,t-1}$	$\bar{A}_{I,t-1}$
	$F_I \le \mu < H_R$	R	$\bar{A}_{R,t-1}$	$\bar{A}_{I,t-1}$
	$H_R \le \mu < \mu^\ddagger$	R	$A^u_{R,t-1}$	$\bar{A}_{I,t-1}$
	$\mu^\ddagger \le \mu < H_I$	I	$A^u_{R,t-1}$	$\bar{A}_{I,t-1}$
	$\mu \ge H_I$	I	$A^u_{R,t-1}$	$A^u_{I,t-1}$
B1a	$\mu < \hat{F}_R$	$\emptyset$	0	0
	$\hat{F}_R \le \mu < F_R$ [sp]	R	$\bar{A}_{R,t-1}$	0
	$F_R \le \mu < H_R$	R	$\bar{A}_{R,t-1}$	0
	$H_R \le \mu < \hat{F}_I$	R	$A^u_{R,t-1}$	0
	$\hat{F}_I \le \mu < F_I$ [sp]	I	$A^u_{R,t-1}$	$\bar{A}_{I,t-1}$
	$F_I \le \mu < H_I$	I	$A^u_{R,t-1}$	$\bar{A}_{I,t-1}$
	$\mu \ge H_I$	I	$A^u_{R,t-1}$	$A^u_{I,t-1}$
B1b/B2/3	$\mu < \hat{F}_R$	$\emptyset$	0	0
	$\hat{F}_R \le \mu < F_R$ [sp]	R	$\bar{A}_{R,t-1}$	0
	$F_R \le \mu < H_R$	R	$\bar{A}_{R,t-1}$	0
	$H_R \le \mu < \hat{F}_I$	R	$A^u_{R,t-1}$	0
	$\hat{F}_I \le \mu < \mu^\ddagger$	R	$A^u_{R,t-1}$	$\bar{A}_{I,t-1}$
	$\mu^\ddagger \le \mu < H_I$	I	$A^u_{R,t-1}$	$\bar{A}_{I,t-1}$
	$\mu \ge H_I$	I	$A^u_{R,t-1}$	$A^u_{I,t-1}$

[sp] = speculative band: static investor payoff negative, two-period payoff zero. B1b, B2, B3 differ only in the bracket for $\mu^\ddagger_{t-1}$; see Table 1 $(t-1)$.

A.5 The Speculative Motive

This section characterises when each type has a genuine speculative band at $t-1$ — that is, when $\hat{\mu}^f_{j,t-1} < \mu^f_{j,t-1}$ — and derives the condition under which the I speculative band does not threaten the $\emptyset \to R \to I$ ordering.

Recall from Section A.4.6 that the speculative band is the sub-region of the unified two-period constrained region $[\hat{\mu}^f_{j,t-1}, \mu^h_{j,t-1})$ in which the manager's static per-dollar investor return is negative. Throughout this region — including the speculative sub-band — the manager sets $A^*_{j,t-1} = \bar{A}_{j,t-1}(\mu_{t-1})$ and the two-period investor ZPC binds. The speculative band is therefore not a separate region operationally, but its lower boundary $\hat{\mu}^f_{j,t-1}$ is what may threaten the ordering: if the I lower boundary falls below $\mu^f_{R,t-1}$, I investors fund managers at skill levels where R should be the exclusive type.

A.5.1 When does each type have a speculative band?

Recall $E[U_{j,t}(\mu_t)|\mu_{t-1}]$ from (A.26): under Convention W, $E[\Psi_{j,\tau+1}|\mu_\tau](\bar{A}_{j,\tau}) = 0$ throughout the constrained region, so $E[U_{j,t}(\mu_t)|\mu_{t-1}] > 0$ requires $\mu_t > \mu^h_{j,t}$ *and* $j^*(\mu_t) = j$.

Proposition 6 (Speculative band existence by case)**.**

1. ***Cases A1 and A2*** *($\mu^*_t \le \mu^h_{R,t}$): $E[U_{R,t}(\mu_t)|\mu_{t-1}] \equiv 0$ and $\hat{\mu}^f_{R,t-1} = \mu^f_{R,t-1}$. Type R has no speculative band. Type I has a genuine speculative band: $E[U_{I,t}(\mu_t)|\mu_{t-1}] > 0$ and $\hat{\mu}^f_{I,t-1} < \mu^f_{I,t-1}$.*

2. ***Cases A3/4 and B*** *($\mu^*_t > \mu^h_{R,t}$, equivalently $\mu^\ddagger_t > \mu^h_{R,t}$): Both types have genuine speculative bands: $E[U_{R,t}(\mu_t)|\mu_{t-1}] > 0$, $E[U_{I,t}(\mu_t)|\mu_{t-1}] > 0$, and $\hat{\mu}^f_{R,t-1} < \mu^f_{R,t-1}$, $\hat{\mu}^f_{I,t-1} < \mu^f_{I,t-1}$.*

Proof. **Part 1.** In Cases A1/A2, $\mu^*_t \le \mu^h_{R,t}$ (switch at F_I or $\mu^\dagger$, both below H_R). The manager runs R for $\mu_t \in (\mu^f_{R,t}, \mu^*_t)$. For all such μ_t, $\mu_t < \mu^*_t \le \mu^h_{R,t}$, so R is in its constrained region with $E[\Psi_{R,t+1}|\mu_t](\bar{A}_R) = 0$. Hence $E[U_{R,t}(\mu_t)|\mu_{t-1}] = 0$ and $\hat{\mu}^f_{R,t-1} = \mu^f_{R,t-1}$.

For $j = I$: the manager runs I for $\mu_t > \mu^*_t$ and I is unconstrained for $\mu_t > \mu^h_{I,t}$. So $E[\Psi_{I,t+1}|\mu_t](A^u_{I,t}) > 0$ for $\mu_t > \max(\mu^*_t, \mu^h_{I,t})$. Since the prior distribution on μ_t has full support and $\sigma_{\mu,t-1} > 0$, $E[U_{I,t}(\mu_t)|\mu_{t-1}](\mu^f_{I,t-1}) > 0$. The two-period I payoff at $\mu^f_{I,t-1}$ is $0 + \beta\, E[U_{I,t}(\mu_t)|\mu_{t-1}](\mu^f_{I,t-1}) > 0$, so by continuity there exists $\hat{\mu}^f_{I,t-1} < \mu^f_{I,t-1}$ solving (A.27).

Part 2. In Cases A3/4 and B, $\mu^*_t = \mu^\ddagger_t > \mu^h_{R,t}$, so the manager runs R unconstrained on $(\mu^h_{R,t}, \mu^*_t)$ and $E[\Psi_{R,t+1}|\mu_t](A^u_{R,t}) > 0$ there. Hence $E[U_{R,t}(\mu_t)|\mu_{t-1}] > 0$ and $\hat{\mu}^f_{R,t-1} < \mu^f_{R,t-1}$. The I argument carries over identically. □ □

A.5.2 When does the I speculative band threaten the ordering?

The $\emptyset \to R \to I$ ordering at $t-1$ requires $\mu^f_{R,t-1}$ to be an uninterrupted lower bound for R's dominance window. The I speculative band $(\hat{\mu}^f_{I,t-1}, \mu^f_{I,t-1})$ threatens this if $\hat{\mu}^f_{I,t-1} < \mu^f_{R,t-1}$: I investors then fund below R's static threshold, and the manager may prefer I over R in part of what would otherwise be R's exclusive zone.

Proposition 7 (Threat is confined to Cases A2 and A3/4)**.** *In Cases A1 and outer Regime B, the I speculative band does not disrupt the overall ordering conclusion.*

1. ***Case A2****: R has no genuine static dominance region at t (I dominates from its entry at $\mu^f_{I,t}$). If the I speculative band extends below $\mu^f_{R,t-1}$, the $t-1$ ordering becomes $\emptyset \to I \to I$, which is consistent with the efficiency conclusion that I is the high-μ type.*
2. ***Outer Regime B****: $\hat{\mu}^f_{I,t-1} > \mu^f_{R,t-1}$ always, so the band cannot reach below $\mu^f_{R,t-1}$. (Proved in Proposition 9.)*

The threat to the $\emptyset \to R \to I$ ordering therefore arises only in Cases A2 and A3/4, where R has a genuine static dominance region $(\mu^f_{I,t}, \mu^*_t)$ at t (equal to $(\mu^f_{I,t}, \mu^\dagger_t)$ in Case A2 and $(\mu^f_{I,t}, \mu^\ddagger_t)$ in Cases A3/4).

Proof. **Part 1 (Case A2).** In Case A2 at t, $\mu^\dagger_t \le \mu^f_{I,t}$, meaning I dominates R as soon as I becomes feasible. If the I lower boundary at $t-1$ extends below $\mu^f_{R,t-1}$ (i.e. $\hat{\mu}^f_{I,t-1} < \mu^f_{R,t-1}$), then for $\mu_{t-1} \in (\hat{\mu}^f_{I,t-1}, \mu^f_{R,t-1})$ only I's two-period constrained region is active. The manager runs an I fund at $A^*_{I,t-1} = \bar{A}_{I,t-1}(\mu_{t-1})$, while R is infeasible (its lower boundary $\hat{\mu}^f_{R,t-1} = \mu^f_{R,t-1}$ has not yet been reached). The period-$t-1$ ordering in this sub-region is $\emptyset \to I$. Above $\mu^f_{R,t-1}$, the standard $\emptyset \to R \to I$ sequence applies. There is no non-monotone region. The overall conclusion that I is the efficient high-μ type is unaffected. □(for Part 1)

Part 2 (Outer Regime B). Proved as Proposition 9 below. □

A.5.3 The scale gap condition (Cases A2 and A3/4)

In Cases A2 and A3/4, the I speculative band extends from $\hat{\mu}^f_{I,t-1}$ up to $\mu^f_{I,t-1}$. The ordering threat materialises only if this band dips below $\mu^f_{R,t-1}$. The following condition — phrased as a minimum required gap between $\underline{A}_I$ and $\underline{A}_R$ — ensures it does not.

Proposition 8 (Scale gap condition)**.** *In Cases A2 and A3/4, suppose Condition (A.33) holds. Then $\hat{\mu}^f_{I,t-1} \ge \mu^f_{R,t-1}$: the I speculative band lies entirely within $(\mu^f_{R,t-1}, \mu^f_{I,t-1})$ and the $\emptyset \to R \to I$ ordering at $t-1$ is preserved.*

Proof. It suffices to show that the two-period I payoff is non-positive at $\mu_{t-1} = \mu^f_{R,t-1}$:

$$\mathrm{tp}_I(\mu^f_{R,t-1}) \;\equiv\; E[\Psi_{I,t+1}|\mu_t](\underline{A}_I, \mu^f_{R,t-1}) + \beta\, E[U_{I,t}(\mu_t)|\mu_{t-1}](\mu^f_{R,t-1}) \;\le\; 0. \tag{A.34}$$

Since the two-period payoff is increasing in μ_{t-1}, (A.34) implies the unique zero $\hat{\mu}^f_{I,t-1}$ lies at or above $\mu^f_{R,t-1}$.

Step 1: linearise the I speculative band width. Near $\mu^f_{I,t-1}$, differentiate $E[\Psi_{I,t+1}|\mu_t](\underline{A}_I, \mu_t)$ with respect to μ:

$$\frac{d}{d\mu} E[\Psi_{I,t+1}|\mu_t](\underline{A}_I, \mu_t) = \underline{A}_I\big(1 - p\Phi(z_I(\mu, \underline{A}_I))\big) > 0$$

where $z_I(\mu, \underline{A}_I) = (\mu - b\underline{A}_I - m)/\sigma_r$. At $\mu = \mu^f_{I,t-1}$, $z_I = \bar{z}_{I,t-1}$ (by definition of $\mu^f_{I,t-1}$). Since $E[\Psi_{I,t+1}|\mu_t](\underline{A}_I, \mu^f_{I,t-1}) = 0$ by the static feasibility definition, a first-order expansion gives:

$$E[\Psi_{I,t+1}|\mu_t](\underline{A}_I, \mu^f_{R,t-1}) \;\approx\; -\underline{A}_I(1 - p\Phi(\bar{z}_{I,t-1})) \cdot (\mu^f_{I,t-1} - \mu^f_{R,t-1}).$$

Expanding $\mu^f_{I,t-1} - \mu^f_{R,t-1} = (\bar{z}_I - \bar{z}_R)\sigma_{r,t-1} + b(\underline{A}_I - \underline{A}_R)$:

$$-E[\Psi_{I,t+1}|\mu_t](\underline{A}_I, \mu^f_{R,t-1}) \;\gtrsim\; \underline{A}_I(1 - p\Phi(\bar{z}_I))\big[(\bar{z}_I - \bar{z}_R)\sigma_r + b(\underline{A}_I - \underline{A}_R)\big]. \tag{A.35}$$

Step 2: bound the continuation value at $\mu^f_{R,t-1}$. Since $E[U_{I,t}(\mu_t)|\mu_{t-1}]$ is increasing in μ_{t-1} (higher current skill raises probability of reaching the I unconstrained region at t), and $\mu^f_{R,t-1} < \mu^f_{I,t-1}$:

$$E[U_{I,t}(\mu_t)|\mu_{t-1}](\mu^f_{R,t-1}) \;\le\; E[U_{I,t}(\mu_t)|\mu_{t-1}](\mu^f_{I,t-1}).$$

Step 3: sufficient condition. Substituting into (A.34) and using (A.35):

$$\mathrm{tp}_I(\mu^f_{R,t-1}) \;\lesssim\; -\underline{A}_I(1 - p\Phi(\bar{z}_I))\big[(\bar{z}_I - \bar{z}_R)\sigma_r + b(\underline{A}_I - \underline{A}_R)\big] + \beta\, E[U_{I,t}(\mu_t)|\mu_{t-1}](\mu^f_{I,t-1}).$$

This is non-positive if and only if

$$(\bar{z}_I - \bar{z}_R)\sigma_r + b(\underline{A}_I - \underline{A}_R) \;\ge\; \frac{\beta\, E[U_{I,t}(\mu_t)|\mu_{t-1}](\mu^f_{I,t-1})}{\underline{A}_I(1 - p\Phi(\bar{z}_I))},$$

i.e. $b(\underline{A}_I - \underline{A}_R) \ge \frac{\beta\, E[U_{I,t}(\mu_t)|\mu_{t-1}](\mu^f_{I,t-1})}{\underline{A}_I(1-p\Phi(\bar{z}_I))} - (\bar{z}_I - \bar{z}_R)\sigma_r$, which is exactly Condition (A.33). □ □

Remark 8 (Economic interpretation)**.** *The RHS of* (A.33) *is the I speculative band width — measured in μ units via the slope of the investor ZPC —* net of *the outside-option contribution $(\bar{z}_I - \bar{z}_R)\sigma_r > 0$ already provided for free by $\delta_R > \delta_I$. The LHS is the minimum-scale gap's contribution to the static threshold separation. The condition says the gap in minimum fund sizes must cover whatever band width the outside-option ordering cannot.*

A.5.4 Outer Regime B: the ordering is always preserved

Proposition 9 (Outer Regime B is threat-free)**.** *In outer Regime B ($\mu^f_{I,t} \geq \mu^h_{R,t}$, i.e. Cases B1a–B3), $\hat{\mu}^f_{I,t-1} > \mu^f_{R,t-1}$ for all admissible parameter values. The I speculative band cannot extend below $\mu^f_{R,t-1}$.*

Proof. It suffices to show $\mathrm{tp}_I(\mu^f_{R,t-1}) < 0$.

Step 1: lower bound on the static I loss at $\mu^f_{R,t-1}$. Dropping the non-negative $p\tilde{\mu}$ term from $E[\Psi_{I,t+1}|\mu_t]$:

$$\frac{E[\Psi_{I,t+1}|\mu_t](\underline{A}_I, \mu^f_{R,t-1})}{\underline{A}_I} \;\leq\; \mu^f_{R,t-1} - b\underline{A}_I - m + \delta_I = \bar{z}_R\sigma_r - b(\underline{A}_I - \underline{A}_R) + \delta_I.$$

The outer Regime B condition $\mu^f_{I,t} \geq \mu^h_{R,t}$ requires $b\underline{A}_I \geq \mu^h_{R,t} - m - \bar{z}_{I,t}\sigma_r$. Since $\mu^h_{R,t} - m - \bar{z}_I\sigma_r = \mu^h_{R,t} - \mu^f_{R,t-1} + \bar{z}_R\sigma_r - \bar{z}_I\sigma_r + b\underline{A}_R$, we get $b(\underline{A}_I - \underline{A}_R) \geq \mu^h_{R,t} - \mu^f_{R,t-1} + (\bar{z}_R - \bar{z}_I)\sigma_r \geq \mu^h_{R,t} - \mu^f_{R,t-1}$ (using $\bar{z}_R < \bar{z}_I$). Therefore:

$$\frac{E[\Psi_{I,t+1}|\mu_t](\underline{A}_I, \mu^f_{R,t-1})}{\underline{A}_I} \;\leq\; \bar{z}_R\sigma_r - (\mu^h_{R,t} - \mu^f_{R,t-1}) + \delta_I = \mu^f_{R,t-1} - \mu^h_{R,t} + \delta_I.$$

Under $\delta_I < \delta_R$ and the structural assumption $\delta_R < m - c_R + \bar{z}_R\sigma_r = \bar{\pi}_R - (m - c_R) + \ldots$, the term $\mu^f_{R,t-1} - \mu^h_{R,t} + \delta_I$ is strongly negative (as $\mu^f_{R,t-1} \ll \mu^h_{R,t}$ in Case B). So $E[\Psi_{I,t+1}|\mu_t](\underline{A}_I, \mu^f_{R,t-1}) \leq \underline{A}_I(\mu^f_{R,t-1} - \mu^h_{R,t} + \delta_I) \ll 0$.

Step 2: upper bound on the continuation value at $\mu^f_{R,t-1}$. The continuation value requires $\mu_t > \mu^h_{I,t} > \mu^h_{R,t}$. In Case B, $\mu^h_{R,t} \geq \mu^f_{I,t} \gg \mu^f_{R,t-1}$. The transition probability is:

$$E[U_{I,t}(\mu_t)|\mu_{t-1}](\mu^f_{R,t-1}) \;\leq\; \Big[1 - \Phi\Big(\tfrac{\mu^h_{I,t} - \mu^f_{R,t-1}}{\sigma_{\mu,t-1}}\Big)\Big] \cdot \sup_{\mu_t > \mu^h_{I,t}} E[\Psi_{I,t+1}|\mu_t](A^u_{I,t}).$$

Since $\mu^h_{I,t} - \mu^f_{R,t-1} \geq \mu^h_{R,t} - \mu^f_{R,t-1}$ which is at least $O(\mu^h_{R,t})$ and typically much larger than $\sigma_{\mu,t-1}$, this tail probability is exponentially small. The supremum of $E[\Psi_{I,t+1}|\mu_t]$ grows polynomially in μ_t, so their product is dominated by the static loss for any reasonable $\sigma_{\mu,t-1}$.

Step 3. The static loss $|E[\Psi_{I,t+1}|\mu_t]| \sim O(\underline{A}_I \cdot (\mu^h_{R,t} - \mu^f_{R,t-1}))$ grows with $\underline{A}_I$ (which is large in Case B), while the continuation value is exponentially suppressed. Therefore $\mathrm{tp}_I(\mu^f_{R,t-1}) < 0$, giving $\hat{\mu}^f_{I,t-1} > \mu^f_{R,t-1}$. □ □

A.5.5 Summary: ordering at $t-1$

Case	R lower boundary		I lower boundary	$t-1$ ordering conclusion
A1, A2	$\hat{\mu}^f_{R,t-1}$ $\mu^f_{R,t-1}$ $(E[U_{R,t}(\mu_t)\|\mu_{t-1}] = 0)$	$=$	$\hat{\mu}^f_{I,t-1}$ $\mu^f_{I,t-1}$	$<$ A2: I lower boundary may fall below $\mu^f_{R,t-1}$; ordering $\emptyset \to I$ then $\emptyset \to R \to I$ is consistent. A1: scale gap cond. (A.33) needed to ensure $\hat{\mu}^f_{I,t-1} \geq \mu^f_{R,t-1}$.
A3/4	$\hat{\mu}^f_{R,t-1}$ $\mu^f_{R,t-1}$ $(E[U_{R,t}(\mu_t)\|\mu_{t-1}] > 0)$	$<$	$\hat{\mu}^f_{I,t-1}$ $\mu^f_{I,t-1}$	$<$ Scale gap cond. (A.33) ensures $\hat{\mu}^f_{I,t-1} \geq \mu^f_{R,t-1}$; $\emptyset \to R \to I$ preserved with corrected thresholds $\mu^h_{j,t-1} < \mu^{h,*}_{j,t-1}$ and $\mu^*_{t-1} \leq \mu^*_t$.
Reg. B	$\hat{\mu}^f_{R,t-1}$ $\mu^f_{R,t-1}$ $(E[U_{R,t}(\mu_t)\|\mu_{t-1}] > 0)$	$<$	$\hat{\mu}^f_{I,t-1}$ $\mu^f_{I,t-1}$	$<$ $\hat{\mu}^f_{I,t-1} > \mu^f_{R,t-1}$ always (Prop. 9); $\emptyset \to R \to I$ at $t-1$ preserved unconditionally. Outer Regime B: $\mu^f_{I,t-1} \geq \mu^h_{R,t-1}$.

In all cases with $E[U_{j,t}(\mu_t)|\mu_{t-1}] > 0$, the two-period upper bound $\bar{A}_{j,t-1} > \bar{A}^*_{j,t-1}$ (static reference), the unconstrained threshold $\mu^h_{j,t-1} < \mu^{h,*}_{j,t-1}$, and the profit crossing $\mu^*_{t-1} \leq \mu^*_t$. These corrections are derived from the same two-period ZPC and do not require additional assumptions beyond those already in place.

Remark 9 (Graduation probability)**.** *A manager who raises a retail fund at $t-1$ (i.e. $\mu_{t-1} \in [\hat{\mu}^f_{R,t-1}, \mu^*_{t-1})$)* graduates *to institutional capital at t if the updated belief $\mu_t \geq \mu^*_t$. Since $\mu_t \mid \mu_{t-1} \sim \mathcal{N}(\mu_{t-1}, \sigma^2_{\mu,t-1})$, the graduation probability has a closed form derived as follows:*

$$\begin{aligned} P(\textit{Graduate} \mid \mu_{t-1}) &= P(\mu_t \geq \mu^*_t \mid \mu_{t-1}) \\ &= P\left(\frac{\mu_t - \mu_{t-1}}{\sigma_{\mu,t-1}} \geq \frac{\mu^*_t - \mu_{t-1}}{\sigma_{\mu,t-1}} \,\middle|\, \mu_{t-1} \right) \\ &= 1 - \Phi\left(\frac{\mu^*_t - \mu_{t-1}}{\sigma_{\mu,t-1}} \right) \\ &= \Phi\left(\frac{\mu_{t-1} - \mu^*_t}{\sigma_{\mu,t-1}} \right), \end{aligned}$$

where the last step uses the symmetry $1-\Phi(x)=\Phi(-x)$ *of the standard normal CDF. The graduation probability is therefore increasing in* μ_{t-1} *(higher-skilled retail managers are more likely to graduate) and decreasing in* $\sigma_{\mu,t-1}$ *for* $\mu_{t-1}<\mu_t^*$ *(less learning per period reduces the probability of crossing the threshold). Since* $\mu_t^*>\mu_{t-1}$ *for all managers in the retail band, this probability is strictly less than* $\frac{1}{2}$*, confirming that graduation is a positive but improbable event for any given retail manager.*

A.6 Welfare Operators

Let $\omega(\mu)$ denote the cross-sectional density of manager skill in the population. For a Gaussian population, $\omega=\mathcal{N}(\mu_p,\sigma_p^2)$. Aggregate welfare is the integral of individual payoffs over ω. We define operators for the two regions of the skill distribution that arise at each period, together with the continuation value operators that link $t-1$ to t.

A.6.1 Continuation Value Operators

Definition 7 (Type-specific continuation values)**.** *Define the type-*j *investor's* period-t continuation utility *as*

$$U_{j,t}(\mu_t)\;\equiv\;E[\Psi_{j,t+1}|\mu_t]\big(A^*_{j,t}(\mu_t)\big)\cdot\mathbf{1}\{j^*(\mu_t)=j\}\cdot\mathbf{1}\{\mu_t>\mu^h_{j,t}\}$$

so that $U_{j,t}(\mu_t)>0$ *only when type* j *is both chosen and unconstrained at* t*. The* type-j investor continuation value *at* $t-1$ *is then:*

$$E[U_{j,t}(\mu_t)|\mu_{t-1}]\;\equiv\;E[U_{j,t}(\mu_t)|\mu_{t-1}]_+$$

Expanding the expectation explicitly:

$$E[U_{j,t}(\mu_t)|\mu_{t-1}]\;\equiv\;\mathbb{E}_{t-1}\big[E[\Psi_{j,t+1}|\mu_t]\big(A^*_{j,t}(\mu_t)\big)\cdot\mathbf{1}\{j^*(\mu_t)=j\}\cdot\mathbf{1}\{\mu_t>\mu^h_{j,t}\}\;\big|\;\mu_{t-1}\big]_+ \tag{A.36}$$

where $j^*(\mu_t)=\arg\max_j\Pi_{j,t+1}(A^*_{j,t}(\mu_t))$ *is the manager's optimal type at* t*, the subscript* $+$ *denotes truncation at zero, and* $\mu_t\mid\mu_{t-1}\sim\mathcal{N}(\mu_{t-1},\sigma^2_{\mu,t-1})$*.*

For $\not{T}$ *(type-locked managers), the condition* $j^*(\mu_t)=j$ *is dropped:*

$$E[U^{\text{locked}}_{j,t}(\mu_t)|\mu_{t-1}]\;\equiv\;\mathbb{E}_{t-1}\big[E[\Psi_{j,t+1}|\mu_t]\big(A^*_{j,t}(\mu_t)\big)\cdot\mathbf{1}\{\mu_t>\mu^h_{j,t}\}\;\big|\;\mu_{t-1}\big]_+ \tag{A.37}$$

In both cases $E[U_{j,t+1}(\mu_{t+1})|\mu_t]\equiv 0$ *at the terminal period* t*.*

Remark 10 (Aggregated continuation value)**.** *Integrating* $E[U_{j,t}(\mu_t)|\mu_{t-1}]$ *against* $\omega(\mu_{t-1})$ *and applying Fubini's theorem:*

$$\int E[U_{j,t}(\mu_t)|\mu_{t-1}]\,\omega(\mu_{t-1})\,d\mu_{t-1}=\int_{\mu^h_{j,t}}^{\infty}E[\Psi_{j,t+1}|\mu_t](A^u_{j,t}(\mu_t))\cdot\mathbf{1}\{j^*(\mu_t)=j\}\cdot P(\mu_{t-1}\in S\mid\mu_t)\,f(\mu_t)\,d\mu_t \tag{A.38}$$

where S is the set of μ_{t-1} for which j is active, $f(\mu_t) = \mathcal{N}(\mu_p, \sigma_p^2 + \sigma_{\mu,t-1}^2)$ is the marginal prior on μ_t, and $P(\mu_{t-1} \in S \mid \mu_t)$ is the conditional probability of the relevant $t-1$ skill region given μ_t. When ω is Gaussian, $f(\mu_t)$ is Gaussian but $P(\mu_{t-1} \in S \mid \mu_t)$ involves a truncated Gaussian probability, so (A.38) does not reduce to a standard normal integral. Numerical integration is required in all cases.

A.6.2 Operator $\mathcal{C}$: Two-Period Constrained Region

The two-period constrained region at $t-1$ is $[\hat{\mu}_{j,t-1}^f, \mu_{j,t-1}^h)$, in which the manager sets $A_{j,t-1}^* = \bar{A}_{j,t-1}(\mu_{t-1})$ and the investor's two-period ZPC binds. This region includes the speculative band $[\hat{\mu}_{j,t-1}^f, \mu_{j,t-1}^f)$ as its lower sub-region.

Period-t operators

At t there is no continuation value, $\bar{z}_{j,t}$ is constant (from (A.18)), and $\bar{A}_{j,t}(\mu_t) = (\mu_t - m - \bar{z}_{j,t}\sigma_{r,t})/b$ is linear in μ_t.

$$\mathcal{C}_{j,\omega}^m(l,u)\Big|_t \equiv \int_l^u E[\Pi_{j,t+1}(\bar{A}_{j,t})|\mu_t]\,\omega\,d\mu_t = \frac{\bar{\pi}_{j,t}}{b}\int_l^u (\mu_t - m - \bar{z}_{j,t}\sigma_{r,t})\,\omega\,d\mu_t \tag{A.39}$$

$$\mathcal{C}_{j,\omega}^\Psi(l,u)\Big|_t = 0 \tag{A.40}$$

$$\mathcal{C}_{j,\omega}^{\Psi,0}(l,u)\Big|_t = -\delta_j\int_l^u \bar{A}_{j,t}(\mu_t)\,\omega\,d\mu_t \tag{A.41}$$

For $\omega = \mathcal{N}(\mu_p, \sigma_p^2)$ with $h^x \equiv (x-\mu_p)/\sigma_p$, the linearity of $\bar{A}_{j,t}$ yields *closed forms*:

$$\mathcal{C}_{j,\omega}^m(l,u)\Big|_t = \frac{\bar{\pi}_{j,t}}{b}\Big[(\mu_p - m - \bar{z}_{j,t}\sigma_{r,t})\big(\Phi(h^u) - \Phi(h^l)\big) + \sigma_p\big(\phi(h^l) - \phi(h^u)\big)\Big] \tag{A.42}$$

$$\mathcal{C}_{j,\omega}^{\Psi,0}(l,u)\Big|_t = -\frac{\delta_j}{b}\Big[(\mu_p - m - \bar{z}_{j,t}\sigma_{r,t})\big(\Phi(h^u) - \Phi(h^l)\big) + \sigma_p\big(\phi(h^l) - \phi(h^u)\big)\Big] \tag{A.43}$$

obtained by writing $\mu_t = \mu_p + \sigma_p\xi$ and using $\int_a^b \xi\,\phi(\xi)\,d\xi = \phi(a) - \phi(b)$.

Period-$(t-1)$ operators

At $t-1$, $\bar{z}_{j,t-1}(\mu_{t-1})$ satisfies the two-period ZPC (A.16) and depends on μ_{t-1} through $E[U_{j,t}(\mu_t)|\mu_{t-1}]$ and $\bar{A}_{j,t-1}(\mu_{t-1})$. Consequently $\bar{A}_{j,t-1}$ is non-linear in μ_{t-1} and $\bar{\pi}_{j,t-1}$ is non-constant.

$$\mathcal{C}^{m}_{j,\omega}(l,u)\Big|_{t-1} \equiv \int_l^u E[\Pi_{j,t}(\bar{A}_{j,t-1}(\mu))|\mu]\,\omega(\mu)\,d\mu \tag{A.44}$$

$$\mathcal{C}^{\Psi}_{j,\omega}(l,u)\Big|_{t-1} = 0 \tag{A.45}$$

$$\mathcal{C}^{\Psi,0}_{j,\omega}(l,u)\Big|_{t-1} = -\delta_j \int_l^u \bar{A}_{j,t-1}(\mu)\,\omega(\mu)\,d\mu \tag{A.46}$$

The vanishing of $\mathcal{C}^{inv}$ at $t-1$ follows directly from the binding two-period ZPC: $E[\Psi_{j,t}|\mu_{t-1}](\bar{A}_{j,t-1},\mu)+\beta\,E[U_{j,t}(\mu_t)|\mu]=0$ pointwise, so the two-period welfare integrand is identically zero throughout $[\hat{\mu}^f_{j,t-1},\mu^h_{j,t-1})$.

Remark 11 (No closed form for $\mathcal{C}^m$ or $\mathcal{C}^{inv,0}$ at $t-1$)**.** *Even when ω is Gaussian, the $t-1$ operators (A.44) and (A.46) do not reduce to closed forms. The function $\bar{A}_{j,t-1}(\mu_{t-1})$ depends on $\bar{z}_{j,t-1}(\mu_{t-1})$, which is the solution to the implicit equation (A.16) with $E[U_{j,t}(\mu_t)|\mu_{t-1}]$ on the right-hand side. Since $E[U_{j,t}(\mu_t)|\mu_{t-1}]$ is itself a nonlinear integral of μ_{t-1}, the product $\bar{A}_{j,t-1}(\mu_{t-1})\,\bar{\pi}_{j,t-1}(\mu_{t-1})$ is a non-polynomial function of μ_{t-1} that does not simplify under Gaussian integration. Numerical evaluation is required.*

Expanding (A.44) using (A.22):

$$\mathcal{C}^{m}_{j,\omega}(l,u)\Big|_{t-1} = \int_l^u \bar{A}_{j,t-1}(\mu)\,\bar{\pi}^{s}_{j,t-1}(\mu)\,\omega(\mu)\,d\mu + \beta\int_l^u E[U_{j,t}(\mu_t)|\mu]\,\omega(\mu)\,d\mu \tag{A.47}$$

where $\bar{\pi}^s_{j,t-1}(\mu) \equiv \bar{z}_{j,t-1}(\mu)\sigma_{r,t-1}+\delta_j+m-c_j$ is the static component of per-unit profit. The second term is the aggregate continuation value, evaluable via (A.38).

A.6.3 Operator $\mathcal{U}$: Unconstrained Region

In the unconstrained region $(\mu^h_{j,\tau},\infty)$ the manager sets $A=A^u_{j,\tau}(\mu_\tau)$, the interior solution to FOC (A.28). No closed form exists for $A^u_{j,\tau}$, so $\mathcal{U}$ is defined as an integral operator at both periods.

Period-t operators

$$\mathcal{U}^{m}_{j,\omega}(l,u)\Big|_{t} \equiv \int_l^u E[\Pi_{j,t+1}(A^u_{j,t})|\mu_t]\,\omega\,d\mu_t \tag{A.48}$$

$$\mathcal{U}^{\Psi}_{j,\omega}(l,u)\Big|_{t} \equiv \int_l^u E[\Psi_{j,t+1}(A^u_{j,t})|\mu_t](A^u_{j,t})\,\omega\,d\mu_t > 0 \tag{A.49}$$

$$\mathcal{U}^{\Psi,0}_{j,\omega}(l,u)\Big|_{t} \equiv \int_l^u E[\Psi^0_{j,t+1}|\mu_t](A^u_{j,t})\,\omega\,d\mu_t \tag{A.50}$$

The strict positivity of $\mathcal{U}^{inv}$ follows because $A^u_{j,t} < \bar{A}_{j,t}$ in the unconstrained region, and $E[\Psi_{j,t+1}|\mu_t]$ is strictly decreasing in A beyond the static upper bound (the per-unit investor return is negative at $\bar{A}$), so $E[\Psi_{j,t+1}|\mu_t](A^u_{j,t}) > E[\Psi_{j,t+1}|\mu_t](\bar{A}_{j,t}) = 0$.

Period-$(t-1)$ operators

At $t-1$ the two-period investor payoff in the unconstrained region is $E[\Psi_{j,t}|\mu_{t-1}](A^u_{j,t-1}) + \beta\, E[U_{j,t}(\mu_t)|\mu_{t-1}]$:

$$\mathcal{U}^m_{j,\omega}(l,u)\Big|_{t-1} \equiv \int_l^u E[\Pi_{j,t}(A^u_{j,t-1})|\mu_{t-1}]\,\omega\, d\mu_{t-1} \tag{A.51}$$

$$\mathcal{U}^\Psi_{j,\omega}(l,u)\Big|_{t-1} \equiv \int_l^u \Big[E[\Psi_{j,t}|\mu_{t-1}](A^u_{j,t-1}) + \beta\, E[U_{j,t}(\mu_t)|\mu_{t-1}]\Big]\,\omega\, d\mu_{t-1} > 0 \tag{A.52}$$

$$\mathcal{U}^{\Psi,0}_{j,\omega}(l,u)\Big|_{t-1} \equiv \int_l^u \Big[E[\Psi^0_{j,t}|\mu_{t-1}](A^u_{j,t-1}) + \beta\, E[U_{j,t}(\mu_t)|\mu_{t-1}]\Big]\,\omega\, d\mu_{t-1} \tag{A.53}$$

Strict positivity of (A.52): since $A^u_{j,t-1} < \bar{A}_{j,t-1}$ (the ZPC is slack in the unconstrained region), $E[\Psi_{j,t}|\mu_{t-1}](A^u_{j,t-1}) > -\beta\, E[U_{j,t}(\mu_t)|\mu_{t-1}]$, which gives two-period investor payoff > 0.

Remark 12 (No closed form for $\mathcal{U}$ operators at either period)**.** *$A^u_{j,\tau}$ is defined implicitly by FOC (A.28) and has no closed form in μ_τ. The $\mathcal{U}$ operators therefore require numerical integration at both t and $t-1$, regardless of the distributional form of ω. The additional $E[U_{j,t}|\mu_{t-1}]$ term in the $t-1$ operators further involves the truncated Gaussian convolution described in Remark 10 above, confirming that no Gaussian simplification is available.*

A.6.4 Welfare Decomposition at $t-1$

Total welfare for each type j at $t-1$ decomposes over the two-period constrained and unconstrained regions:

$$W_{M_j,t-1} = \mathcal{C}^m_{j,\omega}(\hat{\mu}^f_{j,t-1},\, \mu^h_{j,t-1}) + \mathcal{U}^m_{j,\omega}(\mu^h_{j,t-1},\, \infty) \tag{A.54}$$

$$W^W_{j,t-1} = \underbrace{\mathcal{C}^\Psi_{j,\omega}(\hat{\mu}^f_{j,t-1},\, \mu^h_{j,t-1})}_{=0} + \underbrace{\mathcal{U}^\Psi_{j,\omega}(\mu^h_{j,t-1},\, \infty)}_{>0} = \mathcal{U}^\Psi_{j,\omega}(\mu^h_{j,t-1},\, \infty) > 0 \tag{A.55}$$

$$W^0_{j,t-1} = \underbrace{\mathcal{C}^{\Psi,0}_{j,\omega}(\hat{\mu}^f_{j,t-1},\, \mu^h_{j,t-1})}_{<0} + \mathcal{U}^{\Psi,0}_{j,\omega}(\mu^h_{j,t-1},\, \infty) \tag{A.56}$$

The vanishing of $\mathcal{C}^{inv}$ in (A.55) is the key result: under Convention W, two-period investor welfare in the constrained region (including the speculative band) is identically zero because

the investor's two-period ZPC binds pointwise. Investor welfare comes entirely from the unconstrained region.

Remark 13 (Period-t welfare needed for continuation values)**.** *The period-t operators $\mathcal{C}^m$, $\mathcal{U}^m$, and $\mathcal{U}^{inv}$ evaluated at period t enter the continuation values $E[U_{j,t}(\mu_t)|\mu_{t-1}]$ through (A.36). They are not welfare objectives in themselves (the welfare analysis is conducted at $t-1$) but are necessary inputs for computing the $t-1$ operators numerically.*

A.7 Investor Welfare

A.7.1 Welfare Conventions

Convention W includes $\delta_j \geq 0$ in the investor payoff. *Convention W0* sets $\delta_j = 0$ for welfare accounting; equilibrium behaviour is unchanged under both. The participation constraint and all thresholds are determined under Convention W and carry over unchanged to W0.

Remark 14 (Interpretation and empirical relevance of δ)**.** *Convention W treats the entire outside-option differential δ_R as genuine welfare: retail investors truly value VC access at δ_R per dollar above what institutions require. Convention W0 treats it as entirely behavioural: δ_R distorts retail behaviour but creates no social value, so the planner ignores it. The truth is almost certainly between these extremes — part of the retail outside-option disadvantage reflects genuine access constraints and portfolio-fit considerations (welfare-relevant), while part reflects behavioural biases or information frictions (welfare-irrelevant). In the absence of direct evidence on the decomposition, neither extreme is more defensible than the other: W and W0 are both knife-edge cases, and the empirically relevant welfare impact lies in between. We therefore present results under both conventions throughout, so that readers can calibrate conclusions against their own priors on the magnitude of the behavioural component.*

A.7.2 Two-Period Welfare Operators at $t-1$

Let $\omega = \mathcal{N}(\mu_p, \sigma_p^2)$ be the cross-sectional skill density. A fund raised at $t-1$ generates a performance signal, inducing Bayesian updating $\mu_t \mid \mu_{t-1} \sim \mathcal{N}(\mu_{t-1}, \sigma^2_{\mu,t-1})$. Managers who raise no fund generate no signal and remain at $\mu_t = \mu_{t-1}$. Define

$$\mathcal{A}_{t-1}(\text{sc}) \equiv \{\mu_{t-1} : \text{some fund raised at } t-1 \text{ under scenario sc}\} \tag{A.57}$$

Explicitly: $\mathcal{A}_{t-1}(\not{R}) = [\hat{\mu}^f_{I,t-1}, \infty)$. For the baseline and $\not{T}$, the active sets depend on the scenario-specific speculative thresholds and type-choice rules: $\mathcal{A}_{t-1}(\text{baseline}) = [\hat{\mu}^{f,\text{base}}_{R,t-1}, \infty)$ and $\mathcal{A}_{t-1}(\not{T}) = [\hat{\mu}^{f,\not{T}}_{R,t-1}, \infty)$, where $\hat{\mu}^{f,\not{T}}_{R,t-1} \leq \hat{\mu}^{f,\text{base}}_{R,t-1}$ because locking into R is attractive at low skill (the locked R continuation value is large relative to the static loss). The type-switching threshold also differs: $\mu^*_{t-1}(\not{T}) \neq \mu^*_{t-1}(\text{base})$ because in $\not{T}$ the manager compares

$E[\Pi_{j,t-1}|\mu_{t-1}] + \beta\, V^{\mathscr{F}}_{j,t-1}(\mu_{t-1})$, where $V^{\mathscr{F}}_{j,t-1}(\mu_{t-1}) \equiv \mathbb{E}_{t-1}[\max(\Pi_{j,t}(A^*_{j,t}, \mu_t), 0)]$ is the manager's own locked continuation value, which differs by type.

The *full two-period* investor welfare for type j is:

$$W_{j,t-1}(\text{sc}) = \underbrace{\int_{-\infty}^{\infty} E[\Psi_{j,t}|\mu_{t-1}]\big(A^*_{j,t-1}(\mu_{t-1})\big) \cdot \mathbf{1}\big\{j^*_{t-1}(\text{sc}) = j\big\} \cdot \omega(\mu_{t-1})\, d\mu_{t-1}}_{w_j^{t-1}(\text{sc})}$$
$$+\ \beta \underbrace{\int_{\mathcal{A}_{t-1}(\text{sc})} \omega(\mu_{t-1}) \int_{-\infty}^{\infty} E[\Psi_{j,t+1}|\mu_t]\big(A^*_{j,t}(\mu_t)\big) \cdot \mathbf{1}\big\{j^*_t(\text{sc}) = j\big\} \cdot f(\mu_t|\mu_{t-1})\, d\mu_t\, d\mu_{t-1}}_{w_j^t(\text{sc})} \tag{A.58}$$

where $j^*_{t-1}(\text{sc})$ and $j^*_t(\text{sc})$ encode the scenario's type-choice rule, and $E[\Psi_{j,t+1}|\mu_t]$ is the Convention W payoff (replace with $E[\Psi^0_{j,t+1}|\mu_t]$ for W0). The period-t integral is over *all* managers who raised a fund at $t-1$ — not just type-j managers — because any manager who learned at $t-1$ may transition to an investable type-j skill at t.

Fubini form. Switching integration order in $w_j^t(\text{sc})$:

$$w_j^t(\text{sc}) = \int_{-\infty}^{\infty} E[\Psi_{j,t+1}|\mu_t](A^*_{j,t}) \cdot \mathbf{1}\{j^*_t(\text{sc}) = j\} \cdot p^{\text{sc}}(\mu_t)\, d\mu_t, \quad p^{\text{sc}}(\mu_t) \equiv \int_{\mathcal{A}_{t-1}(\text{sc})} f(\mu_t|\mu_{t-1})\, \omega(\mu_{t-1})\, d\mu_{t-1} \tag{A.59}$$

$p^{\text{sc}}(\mu_t)$ is the scenario-dependent predictive density of μ_t. With $\mathcal{A}_{t-1} = [a, \infty)$, $s^2 = \sigma_p^2 + \sigma^2_{\mu,t-1}$, and $\rho = \sigma_p^2/s^2$:

$$p^{\text{sc}}(\mu_t;\, a) = \Phi\left(\frac{\rho\, \mu_t + (1-\rho)\mu_p - a}{\sigma_p\sqrt{1-\rho}}\right) \cdot \frac{1}{s}\phi\left(\frac{\mu_t - \mu_p}{s}\right) \tag{A.60}$$

This is a product of Φ and ϕ; no further simplification is available under Gaussian ω, and numerical integration is required.

Welfare decomposition. Using (A.45) (two-period ZPC binds in constrained region):

$$W^W_{j,t-1} = \underbrace{w_j^{t-1,W}}_{=\,0 \text{ in constrained region}} + \beta\, w_j^{t,W} \;=\; \mathcal{U}^{\Psi}_{j,\omega}(\mu^h_{j,t-1}, \infty) + \beta\, w_j^{t,W} \tag{A.61}$$

$$W^0_{j,t-1} = w_j^{t-1,0} + \beta\, w_j^{t,0} \tag{A.62}$$

Under Convention W, $w_j^{t-1,W}$ is zero in the constrained region (ZPC) and positive in the unconstrained region $(\mu^h_{j,t-1}, \infty)$. Under Convention W0, the constrained contribution $-\delta_j \int \bar{A}_{j,t-1}\, \omega < 0$ does not vanish, so $W^0_{j,t-1}$ depends on the sign balance between the constrained loss and the unconstrained gain.

A.7.3 Welfare Comparisons at $t-1$

Define the *shift band* as $\mathcal{S} \equiv (\mu^*_{t-1}(\not{\mathcal{I}}),\, \mu^*_{t-1}(\text{base}))$: the interval of skill where managers choose R under the baseline but choose I under $\not{\mathcal{I}}$ because the locked I continuation value outweighs the locked R continuation value at those skill levels. Since $V^{\not{\mathcal{I}}}_{I,t-1}(\mu_{t-1}) > V^{\not{\mathcal{I}}}_{R,t-1}(\mu_{t-1})$ at high μ_{t-1} (institutional funds are more profitable at high skill), $\mu^*_{t-1}(\not{\mathcal{I}}) < \mu^*_{t-1}(\text{base})$ and $\mathcal{S}$ is non-empty. The *R-window* is $\mathcal{R} \equiv [\hat{\mu}^{f,\text{base}}_{R,t-1},\, \hat{\mu}^{f}_{I,t-1})$: the interval where R funds are raised under the baseline but no fund is raised under $\not{\mathcal{R}}$. Note that $\not{\mathcal{I}}$ also raises R funds throughout $\mathcal{R}$, since R dominates below $\mu^*_{t-1}(\not{\mathcal{I}})$.

Proposition 10 (Baseline vs Case 1)**.** $\Delta W^{\not{\mathcal{R}}}_{M,t-1} \equiv W^{\text{base}}_{M,t-1} - W^{\not{\mathcal{R}}}_{M,t-1} \geq 0$. *Under Convention W:*

1. $\Delta W^{W,\not{\mathcal{R}}}_{R,t-1} = 0$ *in Cases A1/A2;* $\Delta W^{W,\not{\mathcal{R}}}_{R,t-1} > 0$ *in Cases A3/4 and outer Regime B.*

2. $\Delta W^{W,\not{\mathcal{R}}}_{I,t-1} > 0$ *in all cases.*

Under Convention W0, since $\delta_I = 0$ *is assumed throughout:*

3. $\Delta W^{0,\not{\mathcal{R}}}_{I,t-1} > 0$ *in all cases.*

4. $\Delta W^{0,\not{\mathcal{R}}}_{R,t-1} < 0$ *in Cases A1/A2; ambiguous in Cases A3/4 and outer Regime B.*

Consequently $\Delta W^{W,\not{\mathcal{R}}}_{T,t-1} \equiv \Delta W^{\not{\mathcal{R}}}_{M,t-1} + \Delta W^{W,\not{\mathcal{R}}}_{R,t-1} + \Delta W^{W,\not{\mathcal{R}}}_{I,t-1} > 0$ *in all cases, while* $\Delta W^{0,\not{\mathcal{R}}}_{T,t-1} \equiv \Delta W^{\not{\mathcal{R}}}_{M,t-1} + \Delta W^{0,\not{\mathcal{R}}}_{R,t-1} + \Delta W^{0,\not{\mathcal{R}}}_{I,t-1}$ *is ambiguous in Cases A1/A2 (the positive learning term may be outweighed by the outside-option cost* $|\Delta W^{0,\not{\mathcal{R}}}_{R,t-1}| = \delta_R \int_{\mathcal{R}} \bar{A}_R\, \omega\, d\mu_{t-1}$*).*

Proof. **Manager welfare.** Baseline has a strictly wider choice set, so $\Delta W^{\not{\mathcal{R}}}_{M,t-1} \geq 0$.

R investors, Convention W. In Cases A1/A2, $E[U_{R,t}(\mu_t)|\mu_{t-1}] = 0$ and R investors get zero two-period welfare in the constrained region and zero in the unconstrained region (R never chosen when $\mu_t > \mu^h_{R,t}$ in A1/A2). Under $\not{\mathcal{R}}$ there are no R funds. $\Delta W^{W,\not{\mathcal{R}}}_{R,t-1} = 0$. In Cases A3/4 and outer Regime B, $E[U_{R,t}(\mu_t)|\mu_{t-1}] > 0$, so baseline gives positive R investor welfare from the R unconstrained region; $\not{\mathcal{R}}$ gives zero. $\Delta W^{W,\not{\mathcal{R}}}_{R,t-1} > 0$.

I investors, Convention W. The difference $\Delta W^{W,\not{\mathcal{R}}}_{I,t-1}$ has two components.

Component 1 (R-window $\mathcal{R}$): Managers in $\mathcal{R}$ raise R funds under baseline, generating learning. Some transition to $\mu_t > \mu^h_{I,t}$ (with $j^*_t = I$) and I investors capture positive welfare $E[\Psi_{I,t+1}|\mu_t](A^u_{I,t}) > 0$. Under $\not{\mathcal{R}}$ these managers raise no fund, $\mu_t = \mu_{t-1} < \mu^f_{I,t} < \mu^h_{I,t}$, and I investors get zero. Contribution: $\beta \int_{\mathcal{R}} \omega \int_{\mu^h_{I,t}}^{\infty} E[\Psi_{I,t+1}|\mu_t](A^u_{I,t}) \cdot f(\mu_t|\mu_{t-1})\, d\mu_t\, d\mu_{t-1} > 0$.

Component 2 ($[\hat{\mu}^f_{I,t-1}, \mu^*_{t-1,\text{b}})$): In this band, managers run R under baseline and I under $\not{\mathcal{R}}$ at $t-1$. At $t-1$: baseline gives $w_I^{t-1,W} = 0$ (no I fund); $\not{\mathcal{R}}$ gives $E[\Psi_{I,t}|\mu_{t-1}](\bar{A}_{I,t-1}) =$

$-\beta\, E[U_{I,t}(\mu_t)|\mu_{t-1}] \le 0$. At t: since $\mu^h_{I,t} > \mu^*_t$ (in A1/A2), $\mu_t > \mu^h_{I,t}$ implies $j^*_t = I$ under both scenarios, so the t-period I welfare is $E[U_{I,t}(\mu_t)|\mu_{t-1}]$ under both. Two-period totals: baseline $= \beta\, E[U_{I,t}(\mu_t)|\mu_{t-1}] > 0$; $\not{R} = -\beta\, E[U_{I,t}(\mu_t)|\mu_{t-1}] + \beta\, E[U_{I,t}(\mu_t)|\mu_{t-1}] = 0$. Contribution from Component 2: $\beta \int E[U_{I,t}(\mu_t)|\mu_{t-1}]\, \omega\, d\mu_{t-1} > 0$.

Combining: $\Delta W^{W,\not{R}}_{I,t-1} > 0$ in all cases.

Convention W0 — I investors ($\Delta W^{0,\not{R}}_{I,t-1} > 0$).

Step 1. Key identity. By assumption $\delta_I = 0$, so the W0 investor payoff $E[\Psi^0_{I,\tau+1}(A)|\mu_\tau] \equiv A\big(\mu_\tau - bA - p\tilde{\mu}(\mu_\tau, A) - m\big)$ equals the Convention W payoff $E[\Psi_{I,\tau+1}(A)|\mu_\tau]$ at every (A, μ_τ, τ).

Step 2. Implication for continuation values. Since the payoffs are identical, $E[U^0_{I,\tau+1}(\mu_{\tau+1})|\mu_\tau](\mu_\tau) = E[U_{I,\tau+1}(\mu_{\tau+1})|\mu_\tau](\mu_\tau)$ for every μ_τ, τ.

Step 3. Implication for welfare. The full two-period welfare operator (A.58) depends on investor payoffs only through the integrand. Because $E[\Psi^0_I|\mu_\tau] = E[\Psi_I|\mu_\tau]$ pointwise (since $\delta_I = 0$), $W^0_{I,t-1}(\text{sc}) = W^W_{I,t-1}(\text{sc})$ for every scenario sc.

Step 4. Conclusion. $\Delta W^{0,\not{R}}_{I,t-1} = W^0_{I,t-1}(\text{base}) - W^0_{I,t-1}(\not{R}) = W^W_{I,t-1}(\text{base}) - W^W_{I,t-1}(\not{R}) = \Delta W^{W,\not{R}}_{I,t-1} > 0$ by the Convention W argument above.

Convention W0 — R investors ($\Delta W^{0,\not{R}}_{R,t-1}$).

Step 1. Period-$t-1$ payoff identity. The two-period ZPC under Convention W gives $E[\Psi_{R,t}(\bar{A}_{R,t-1})|\mu_{t-1}] = -\beta\, E[U_{R,t}(\mu_t)|\mu_{t-1}]$ for $\mu_{t-1} \in \mathcal{R}$. Subtracting the δ_R term:

$$E[\Psi^0_{R,t}|\mu_{t-1}](\bar{A}_{R,t-1}) = E[\Psi_{R,t}(\bar{A}_{R,t-1})|\mu_{t-1}] - \delta_R \bar{A}_{R,t-1} = -\beta\, E[U_{R,t}(\mu_t)|\mu_{t-1}] - \delta_R \bar{A}_{R,t-1}.$$

Step 2. Cases A1/A2. In these cases R is never chosen above $\mu^h_{R,t}$ at period t (since $\mu^\dagger_t < \mu^h_{R,t}$ in A1/A2, all $\mu_t > \mu^*_t$ attract I, not R). Therefore the period-t R-investor welfare is zero: $\beta\, w^{t,0}_R(\text{base}) = 0$. Moreover $E[U_{R,t}(\mu_t)|\mu_{t-1}] = 0$ throughout $\mathcal{R}$ in A1/A2, simplifying Step 1 to $E[\Psi^0_{R,t}|\mu_{t-1}](\bar{A}_{R,t-1}) = -\delta_R \bar{A}_{R,t-1} < 0$. Under $\not{R}$ there are no R funds, so $W^0_{R,t-1}(\not{R}) = 0$. Therefore:

$$\Delta W^{0,\not{R}}_{R,t-1} = W^0_{R,t-1}(\text{base}) - 0 = \int_{\mathcal{R}} \big(-\delta_R \bar{A}_{R,t-1}(\mu_{t-1})\big)\, \omega(\mu_{t-1})\, d\mu_{t-1} < 0.$$

Step 3. Cases A3/4 and outer Regime B. Here $E[U_{R,t}(\mu_t)|\mu_{t-1}] > 0$ in the speculative subband, adding a further negative term $-\beta\, E[U_{R,t}(\mu_t)|\mu_{t-1}] < 0$ to the period-$t-1$ integrand. In the unconstrained sub-region $[\mu^h_{R,t-1}, \mu^*_{t-1})$ the W0 payoff $E[\Psi^0_R(A^u_R)|\mu]$ is negative just above $\mu^h_{R,t}$ (where $A^u_R \approx \bar{A}_R$ is large and the outside option cost $\delta_R A^u_R$

is not fully extracted by the ZPC) and positive for $\mu \gg \mu^h_{R,t}$ (where A^u_R is small). The period-t R-investor welfare is similarly mixed in sign. Overall sign of $\Delta W^{0,\mathscr{K}}_{R,t-1}$ depends on calibration: ambiguous.

Total welfare under Convention W ($\Delta W^{W,\mathscr{K}}_{T,t-1} > 0$). $\Delta W^{W,\mathscr{K}}_{T,t-1} = \Delta W^{\mathscr{K}}_{M,t-1} + \Delta W^{W,\mathscr{K}}_{R,t-1} + \Delta W^{W,\mathscr{K}}_{I,t-1}$. In Cases A1/A2: $\Delta W^{\mathscr{K}}_{M,t-1} \geq 0$, $\Delta W^{W,\mathscr{K}}_{R,t-1} = 0$, $\Delta W^{W,\mathscr{K}}_{I,t-1} > 0$, so $\Delta W^{W,\mathscr{K}}_{T,t-1} > 0$. In Cases A3/4 and outer Regime B: all three terms are non-negative and $\Delta W^{W,\mathscr{K}}_{I,t-1} > 0$, so $\Delta W^{W,\mathscr{K}}_{T,t-1} > 0$.

Total welfare under Convention W0 ($\Delta W^{0,\mathscr{K}}_{T,t-1}$ ambiguous in A1/A2). $\Delta W^{0,\mathscr{K}}_{T,t-1} = \Delta W^{\mathscr{K}}_{M,t-1} + \Delta W^{0,\mathscr{K}}_{R,t-1} + \Delta W^{0,\mathscr{K}}_{I,t-1}$. In Cases A1/A2: $\Delta W^{\mathscr{K}}_{M,t-1} \geq 0$, $\Delta W^{0,\mathscr{K}}_{R,t-1} = -\delta_R \int_{\mathcal{R}} \bar{A}_R \, \omega \, d\mu_{t-1} < 0$, $\Delta W^{0,\mathscr{K}}_{I,t-1} > 0$. The sign depends on whether the learning externality $\Delta W^{\mathscr{K}}_{M,t-1} + \Delta W^{0,\mathscr{K}}_{I,t-1}$ exceeds the outside-option cost $\delta_R \int_{\mathcal{R}} \bar{A}_R \, \omega \, d\mu_{t-1}$. For small δ_R the learning term dominates and $\Delta W^{0,\mathscr{K}}_{T,t-1} > 0$; for large δ_R the cost term dominates and $\Delta W^{0,\mathscr{K}}_{T,t-1} < 0$. □ □

Proposition 11 (Baseline vs Case 2)**.** $\Delta W^{\mathscr{T}}_{M,t-1}$ *is ambiguous in sign. Under Convention W and Convention W0:*

1. $\Delta W^{W,\mathscr{T}}_{I,t-1} > 0$ *and* $\Delta W^{0,\mathscr{T}}_{I,t-1} > 0$ *in all cases.*
2. $\Delta W^{W,\mathscr{T}}_{R,t-1} < 0$ *in Cases A1/A2; ambiguous in Cases A3/4 and outer Regime B.*
3. $\Delta W^{0,\mathscr{T}}_{R,t-1}$ *is ambiguous in all cases.*
4. $\Delta W^{W,\mathscr{T}}_{T,t-1}$ *is ambiguous in all cases. In Cases A1/A2,* $\Delta W^{W,\mathscr{T}}_{T,t-1} < 0$ *when* $|\Delta W^{W,\mathscr{T}}_{R,t-1}|$ *dominates* $\Delta W^{\mathscr{T}}_{M,t-1} + \Delta W^{W,\mathscr{T}}_{I,t-1}$ *(type-locking raises total Convention W welfare above baseline).*

Proof. **Manager welfare ($\Delta W^{\mathscr{T}}_{M,t-1}$ ambiguous).** Two opposing forces determine the sign of $\Delta W^{\mathscr{T}}_{M,t-1} \equiv W_{M,t-1}(\text{base}) - W_{M,t-1}(\mathscr{T})$:

(i) Intensive margin (baseline weakly better per incumbent): For each manager already in the active set $[\hat{\mu}^{f,\text{base}}_{R,t-1}, \infty)$, the baseline allows free type choice at t while $\mathscr{T}$ locks to the t-1 type. By revealed preference the baseline gives weakly higher individual manager welfare.

(ii) Extensive margin ($\mathscr{T}$ admits more entrants): Type-locking relaxes the speculative ZPC, widening the R entry band to $\hat{\mu}^{f,\mathscr{T}}_{R,t-1} \leq \hat{\mu}^{f,\text{base}}_{R,t-1}$. Managers in the extra speculative band $[\hat{\mu}^{f,\mathscr{T}}_{R,t-1}, \hat{\mu}^{f,\text{base}}_{R,t-1})$ earn positive $W_{M,t-1}$ under $\mathscr{T}$ but zero under baseline. This effect is large when σ_μ is large relative to δ_R.

When the extensive margin (ii) dominates the intensive margin (i), $\Delta W^{\mathscr{X}}_{M,t-1} < 0$; otherwise $\Delta W^{\mathscr{X}}_{M,t-1} \geq 0$. The sign is ambiguous in general.

I investors, step-by-step proof of $\Delta W^{W,\mathscr{X}}_{I,t-1} > 0$.

Step 1: partition the skill distribution at $t-1$. Define four disjoint regions based on type choice under baseline vs $\mathscr{X}$:

- $\mathcal{E} = [\hat{\mu}^{f,\mathscr{X}}_{R,t-1}, \hat{\mu}^{f,\text{base}}_{R,t-1})$ — extra $\mathscr{X}$ R speculative band: R under $\mathscr{X}$, inactive under baseline.
- $\mathcal{C} = [\hat{\mu}^{f,\text{base}}_{R,t-1}, \mu^*_{t-1,\mathscr{X}})$ — common R region: R under both scenarios.
- $\mathcal{S} = [\mu^*_{t-1,\mathscr{X}}, \mu^*_{t-1,\text{base}})$ — shift band: R under baseline, I under $\mathscr{X}$.
- $\mathcal{I} = [\mu^*_{t-1,\text{base}}, \infty)$ — common I region: I under both scenarios.

Step 2: $\mathcal{I}$ contributes 0. Both scenarios raise I funds at $t-1$. Under Convention W, the two-period ZPC binds throughout the constrained region, giving zero I investor welfare at $t-1$. The manager type at t is I under both scenarios (optimal in baseline, locked in $\mathscr{X}$), and the period-t distribution $\mu_t \mid \mu_{t-1}$ is identical (same adjusted signal, same kernel). Period-t I welfare is therefore the same under both. $\Delta W^{W,\mathscr{X}}_{I,t-1}$ from $\mathcal{I}$: zero.

Step 3: $\mathcal{S}$ contributes $+\beta\, E[U_{I,t}(\mu_t)|\mu_{t-1}]$ per manager. For $\mu_{t-1} \in \mathcal{S}$:

$$\text{Baseline:} \quad \underbrace{0}_{\text{R at } t\text{-1, I gets 0}} + \beta\, E[U_{I,t}(\mu_t)|\mu_{t-1}]$$

$$\mathscr{X}\text{:} \quad \underbrace{E[\Psi_{I,t}(\bar{A}_{I,t-1})|\mu_{t-1}]}_{\text{I at } t\text{-1, ZPC}} + \beta\, E[U_{I,t}(\mu_t)|\mu_{t-1}]$$

The $t-1$ payoff under $\mathscr{X}$: the ZPC gives $E[\Psi_{I,t}(\bar{A}_{I,t-1})|\mu_{t-1}] + \beta\, E[U_{I,t}(\mu_t)|\mu_{t-1}] = 0$, so $E[\Psi_{I,t}|\mu_{t-1}](\bar{A}_{I,t-1}) = -\beta\, E[U_{I,t}(\mu_t)|\mu_{t-1}]$. The period-$t$ I welfare: since $\mu^h_{I,t} > \mu^*_t$, the event $\{\mu_t > \mu^h_{I,t}\}$ implies $j^*_t = I$ under baseline; under $\mathscr{X}$ the manager is locked to I. Both conditions deliver the same period-t I payoff $E[U_{I,t}(\mu_t)|\mu_{t-1}]$ via (A.36) (same kernel, same threshold). Therefore:

$$\text{Baseline total} = \beta\, E[U_{I,t}(\mu_t)|\mu_{t-1}], \qquad \mathscr{X} \text{ total} = -\beta\, E[U_{I,t}(\mu_t)|\mu_{t-1}] + \beta\, E[U_{I,t}(\mu_t)|\mu_{t-1}] = 0.$$

Contribution per manager in $\mathcal{S}$: $+\beta\, E[U_{I,t}(\mu_t)|\mu_{t-1}] > 0$.

Step 4: $\mathcal{C}$ contributes $+\beta\, E[U_{I,t}(\mu_t)|\mu_{t-1}]$ per manager. Both scenarios raise R at $t-1$; I investors get 0 at $t-1$ under both. At t: under baseline, $j^*_t = I$ for $\mu_t > \mu^h_{I,t}$ (since $\mu^h_{I,t} > \mu^*_t$ in A1/A2 implies I is chosen), delivering I investor welfare $E[U_{I,t}(\mu_t)|\mu_{t-1}]$; under $\mathscr{X}$, the manager is locked to R, delivering 0 to I investors. Contribution per manager in $\mathcal{C}$: $+\beta\, E[U_{I,t}(\mu_t)|\mu_{t-1}] > 0$.

Step 5: $\mathcal{E}$ contributes 0. Under $\mathscr{I}$, managers here raise R at $t-1$ and are locked to R at t, so I investors receive 0 at t. Under baseline, these managers raise no fund (below $\hat{\mu}^{f,\text{base}}_{R,t-1}$), so $\mu_t = \mu_{t-1} < \mu^f_{I,t-1} \ll \mu^h_{I,t}$ and I investors receive 0 at t. Both scenarios deliver 0 to I investors from $\mathcal{E}$. Note that although $\mathscr{I}$ generates additional learning in $\mathcal{E}$ via the R fund, the manager is locked to R at t, so I investors cannot capture any of this learning benefit. $\Delta W^{W,\mathscr{I}}_{I,t-1}$ from $\mathcal{E}$: zero.

Step 6: Aggregate. Summing over all regions:

$$\Delta W^{W,\mathscr{I}}_{I,t-1} = \beta \int_{\mathcal{C}\cup\mathcal{S}} E[U_{I,t}(\mu_t)|\mu_{t-1}]\,\omega(\mu_{t-1})\,d\mu_{t-1} = \beta \int_{[\hat{\mu}^{f,\text{base}}_{R,t-1},\,\mu^*_{t-1,\text{base}})} E[U_{I,t}(\mu_t)|\mu_{t-1}]\,\omega(\mu_{t-1})\,d\mu_{t-1} > 0. \tag{A.63}$$

Positivity follows because $E[U_{I,t}(\mu_t)|\mu_{t-1}] > 0$ throughout $[\hat{\mu}^{f,\text{base}}_{R,t-1}, \mu^*_{t-1,\text{base}})$: this interval lies above $\hat{\mu}^f_{I,t-1}$ (the I speculative threshold, below which $E[U_{I,t}(\mu_t)|\mu_{t-1}] = 0$), and $\hat{\mu}^f_{I,t-1} < \hat{\mu}^{f,\text{base}}_{R,t-1} = \mu^f_{R,t-1}$ in Cases A1/A2 (Proposition 6).

Convention W0. Since $\delta_I = 0$, $E[\Psi^0_{I,t}|\mu_\tau] \equiv E[\Psi_{I,t}|\mu_\tau]$ and $E[U_{I,t}(\mu_t)|\mu_{t-1}]^0 = E[U_{I,t}(\mu_t)|\mu_{t-1}]$. Steps 2–6 carry through with W0 payoffs identical to W payoffs throughout. Hence $\Delta W^{0,\mathscr{I}}_{I,t-1} = \Delta W^{W,\mathscr{I}}_{I,t-1} > 0$.

R investors, Convention W. Managers in $[\hat{\mu}^{f,\mathscr{I}}_{R,t-1}, \mu^*_{t-1}(\mathscr{I}))$ choose R under both scenarios. The expanded speculative band $[\hat{\mu}^{f,\mathscr{I}}_{R,t-1}, \hat{\mu}^{f,\text{base}}_{R,t-1})$ (extra $\mathscr{I}$ speculators) contributes additional positive R investor welfare under $\mathscr{I}$ relative to baseline. For the common R region $[\hat{\mu}^{f,\text{base}}_{R,t-1}, \mu^*_{t-1}(\mathscr{I}))$: the $t-1$ welfare is identical under both scenarios. At t, $\mathscr{I}$ locks these managers to R, so R investors capture $E[\Psi_{R,t+1}|\mu_t](A^u_{R,t}) > 0$ for $\mu_t > \mu^h_{R,t}$, whereas baseline switches to I for $\mu_t > \mu^*_t$. Since $\mu^h_{R,t} < \mu^*_t$ in Cases A1/A2 is FALSE (A1/A2 has $\mu^*_t < \mu^h_{R,t}$), baseline gives R investors zero at t when $\mu_t > \mu^*_t$ (I is chosen). $\mathscr{I}$ gives positive R welfare for $\mu_t > \mu^h_{R,t}$. Hence $\Delta W^{W,\mathscr{I}}_{R,t-1} < 0$ in Cases A1/A2.

In Cases A3 and B, both A3 and B have $\mu^\dagger_t \geq \mu^h_{R,t}$, so $E[U_{R,t}(\mu_t)|\mu_{t-1}] > 0$ under the baseline. We group them because the key qualitative feature is identical: R investors receive positive two-period welfare from the unconstrained R region under the baseline, but under $\mathscr{I}$ this R unconstrained welfare is partially reallocated.

Cases A3/4. Under baseline, R investors earn positive welfare for $\mu_t \in (\mu^h_{R,t}, \mu^*_t)$ (R chosen, unconstrained). Under $\mathscr{I}$, managers in the extra speculative band $[\hat{\mu}^{f,\mathscr{I}}_{R,t-1}, \hat{\mu}^{f,\text{base}}_{R,t-1})$ are locked to R, giving R investors additional speculative-band welfare. However, the shift band $\mathcal{S}$ managers — who choose R under baseline but I under $\mathscr{I}$ — now generate R welfare under $\mathscr{I}$ only at $t-1$ (which is negative, binding ZPC), not at t. The net comparison involves the extra-band gain vs shift-band loss vs the regime-A3 unconstrained-R gain. Sign depends on calibration.

Outer Regime B. Here $\mu^f_{I,t} \geq \mu^h_{R,t}$, so R managers are already unconstrained before I becomes feasible. Under baseline, R investors capture welfare from the large unconstrained

R region $(\mu^h_{R,t}, \mu^*_t)$, which is wide in outer Regime B. Under $\mathscr{T}$, the lock-in expands this region further (extending up to $\mu^*_{t,\mathscr{T}} > \mu^*_t$, or removing the upper cutoff entirely when I managers are locked away from high-skill R). The Case-B unconstrained-R welfare under $\mathscr{T}$ therefore weakly exceeds baseline R welfare — suggesting $\Delta W^{W,\mathscr{T}}_{R,t-1} \leq 0$ for outer Regime B specifically — but the speculative-band comparison introduces opposing effects of ambiguous sign. Sign is ambiguous overall in Cases A3/4 and outer Regime B.

Convention W0 — R investors ($\Delta W^{0,\mathscr{T}}_{R,t-1}$, baseline vs $\mathscr{T}$).

Step 1. Decompose the R-active domain. Under baseline, managers in $[\hat{\mu}^{f,\text{base}}_{R,t-1}, \mu^*_{t-1,\text{base}})$ raise R funds. Under $\mathscr{T}$, managers in $[\hat{\mu}^{f,\mathscr{T}}_{R,t-1}, \mu^*_{t-1,\mathscr{T}})$ raise R funds. Because $\hat{\mu}^{f,\mathscr{T}}_{R,t-1} \leq \hat{\mu}^{f,\text{base}}_{R,t-1}$ and $\mu^*_{t-1,\mathscr{T}} \leq \mu^*_{t-1,\text{base}}$, the $\mathscr{T}$ R-band $[\hat{\mu}^{f,\mathscr{T}}_{R,t-1}, \mu^*_{\mathscr{T}})$ extends further left but ends further left than the baseline band. Write the baseline R-band as the disjoint union $[\hat{\mu}^{f,\text{base}}_{R,t-1}, \mu^*_{\mathscr{T}}) \cup [\mu^*_{\mathscr{T}}, \mu^*_{\text{base}})$, and the $\mathscr{T}$ R-band as $[\hat{\mu}^{f,\mathscr{T}}_{R,t-1}, \hat{\mu}^{f,\text{base}}_{R,t-1}) \cup [\hat{\mu}^{f,\text{base}}_{R,t-1}, \mu^*_{\mathscr{T}})$.

Step 2. Common sub-band $[\hat{\mu}^{f,\text{base}}_{R,t-1}, \mu^*_{\mathscr{T}})$. Both scenarios raise R funds here at $t-1$. The period-$t-1$ R-investor payoff is $E[\Psi^0_{R,t}|\mu_{t-1}](\bar{A}_{R,t-1}) = -\beta\, E[U_{R,t}(\mu_t)|\mu_{t-1}] - \delta_R \bar{A}_{R,t-1}$ (identical under both scenarios because the same fund is raised and the same ZPC applies). At period t: baseline allows free type choice, so R investors receive $E[\Psi_{R,t+1}|\mu_t](A^u_{R,t})$ only when $j^*_t = R$ (i.e. $\mu_t \in (\mu^h_{R,t}, \mu^*_t)$ in A1/A2); $\mathscr{T}$ locks to R, giving $E[\Psi^0_{R,t+1}|\mu_t](A^u_{R,t})$ for all $\mu_t > \mu^h_{R,t}$. The period-t difference is non-negative for A1/A2 ($\mathscr{T}$ captures more R welfare), but the W0 payoff $E[\Psi^0_R(A^u_R)|\mu]$ may be negative near $\mu^h_{R,t}$. Sign ambiguous.

Step 3. Extra $\mathscr{T}$ sub-band $[\hat{\mu}^{f,\mathscr{T}}_{R,t-1}, \hat{\mu}^{f,\text{base}}_{R,t-1})$. Only $\mathscr{T}$ raises R funds here. The W0 R-investor payoff is $E[\Psi^0_R(\bar{A}_R)|\mu_{t-1}] = -\beta\, E[U_{R,t}(\mu_t)|\mu_{t-1}] - \delta_R \bar{A}_R < 0$, contributing negatively to $\mathscr{T}$'s R welfare and hence positively to $\Delta W^{0,\mathscr{T}}_{R,t-1}$.

Step 4. Shift band $\mathcal{S} = [\mu^*_{\mathscr{T}}, \mu^*_{\text{base}})$. Baseline raises R funds; $\mathscr{T}$ raises I funds. The period-$t-1$ R-investor payoff under baseline is $-\beta\, E[U_{R,t}(\mu_t)|\mu_{t-1}] - \delta_R \bar{A}_R < 0$; under $\mathscr{T}$ R investors get zero (no R fund). This sub-band contributes negatively to $\Delta W^{0,\mathscr{T}}_{R,t-1}$.

Step 5. Conclusion. Steps 2–4 each contribute terms of mixed or ambiguous sign. Overall $\Delta W^{0,\mathscr{T}}_{R,t-1}$ (baseline vs $\mathscr{T}$) is ambiguous in all cases. □

□

A.7.4 Welfare Comparison Table

Comparison	$\Delta W_{M,t-1}$	$\Delta W^{W}_{R,t-1}$	$\Delta W^{W}_{I,t-1}$	$\Delta W^{0}_{R,t-1}$	$\Delta W^{0}_{I,t-1}$	$\Delta W^{W}_{T,t-1}$	$\Delta W^{0}_{T,t-1}$
Base vs Case 1 (A1/A2)	≥ 0	$= 0$	> 0	< 0	> 0	> 0	ambig.
Base vs Case 1 (A3/4, B)	≥ 0	> 0	> 0	ambig.	> 0	> 0	ambig.
Base vs Case 2 (A1/A2)	ambig.	< 0	> 0	ambig.	> 0	ambig.	ambig.
Base vs Case 2 (A3/4, B)	ambig.	ambig.	> 0	ambig.	> 0	ambig.	ambig.

For baseline vs Case 2, $\Delta W^{\mathscr{I}}_{M,t-1}$ is ambiguous due to the *extensive margin*: type-locking widens the R speculative band to $[\hat{\mu}^{f,\mathscr{I}}_{R,t-1}, \hat{\mu}^{f,\text{base}}_{R,t-1})$. Extra managers in this band earn positive $W_{M,t-1}$ under $\mathscr{I}$ but zero under baseline (the baseline speculative ZPC requires the full locked-R continuation value to be profitable). When σ_μ is large or δ_R is small this extra-band effect dominates, yielding $\Delta W^{\mathscr{I}}_{M,t-1} < 0$. For baseline vs Case 1, no such extra band exists: the R-window $\mathcal{R}$ contributes positively to $W_{M,t-1}$(base) only, so $\Delta W^{\mathscr{I}}_{M,t-1} \geq 0$ always.

The $\Delta W^{W}_{I,t-1} > 0$ result in all rows reflects the I-investor learning externality: R funds expand the set of managers who generate performance signals at $t-1$, increasing the probability that high-skill managers are discovered and channelled into I funds at t.

Since $\delta_I = 0$ by assumption ($\delta_I = 0$), $E[\Psi^0_I|\mu_\tau] \equiv E[\Psi_I|\mu_\tau]$ identically, so $\Delta W^{0}_{I,t-1} = \Delta W^{W}_{I,t-1} > 0$ in *every* row of the table — I-investor welfare is the same under both conventions.

For R investors under W0 (baseline vs Case 1), the two-period ZPC under Convention W extracts δ_R from investors as part of the constrained equilibrium, so under W0 R investors bear a net outside-option cost of $\delta_R \bar{A}_{R,t-1}$ per unit at $t-1$. In Cases A1/A2 this gives $\Delta W^{0,\mathscr{R}}_{R,t-1} = -\delta_R \int_{\mathcal{R}} \bar{A}_R \, \omega \, d\mu_{t-1} < 0$ (strictly negative, not ambiguous). In Cases A3/B the unconstrained R-investor payoff $E[\Psi^0_R(A^u_R)|\mu]$ can be positive or negative depending on calibration, so the sign is genuinely ambiguous.

The ambiguity of $\Delta W^{0,\mathscr{I}}_{T,t-1}$ in Cases A1/A2 (baseline vs Case 1) arises from the balance between the strictly positive learning externality $\Delta W^{\mathscr{I}}_{M,t-1} + \Delta W^{0,\mathscr{I}}_{I,t-1} > 0$ and the strictly negative outside-option cost $\Delta W^{0,\mathscr{I}}_{R,t-1} < 0$. For small δ_R the learning term dominates ($\Delta W^{0,\mathscr{I}}_{T,t-1} > 0$); for large δ_R the cost term dominates ($\Delta W^{0,\mathscr{I}}_{T,t-1} < 0$). For baseline vs Case 1, $\Delta W^{W,\mathscr{I}}_{T,t-1} > 0$ always: since $\Delta W^{W,\mathscr{I}}_{R,t-1} = 0$ in A1/A2 (R investors receive zero at the constrained optimum under both scenarios) and $\Delta W^{\mathscr{I}}_{M,t-1} + \Delta W^{W,\mathscr{I}}_{I,t-1} > 0$, total Convention W welfare is strictly positive. For baseline vs Case 2 (type-locked), $\Delta W^{W,\mathscr{I}}_{T,t-1}$ is *ambiguous*: $\Delta W^{W,\mathscr{I}}_{R,t-1} < 0$ in Cases A1/A2 because type-locking forces managers to raise R funds even when $\mu_t > \mu^h_{R,t}$, generating positive R-investor welfare under $\mathscr{I}$ that is absent under baseline. When $|\Delta W^{W,\mathscr{I}}_{R,t-1}|$ is large relative to $\Delta W^{\mathscr{I}}_{M,t-1} + \Delta W^{W,\mathscr{I}}_{I,t-1}$, type-locking delivers higher total

Convention W welfare than the baseline. Numerically, $\Delta W^{W,\not\mathcal{I}}_{T,t-1} < 0$ (type-locking is better for total welfare under W) across our A1–A3 calibrations.

A.8 Total Welfare and Social Optimum

A.8.1 Social Optima at $t-1$

Maximising two-period expected total welfare $E[\Pi^{tot}_{j,t}|\mu_{t-1}] = A(\mu_{t-1} - bA - c_j + \delta_j)$ over A at $t-1$ (Convention W):

$$A^{social}_{j,t-1} = \frac{\mu_{t-1} - c_j + \delta_j}{2b} \quad \text{(W)}, \qquad A^{social,0}_{j,t-1} = \frac{\mu_{t-1} - c_j}{2b} \quad \text{(W0)} \tag{A.64}$$

Under our parameter assumptions ($m > c_j$, $\delta_j < m$), $A^{social} < (\mu_{t-1} - m)/b < \bar{A}^*_{j,t-1} \leq \bar{A}_{j,t-1}$: equilibrium fund sizes exceed the social optimum throughout the constrained region. Managers oversupply capacity to extract rents.

A.8.2 Total Welfare Decomposition

Total welfare $W^W_{T_j,t-1} \equiv W_{M_j,t-1} + W^W_{j,t-1}$ at $t-1$ under Convention W:

$$W^W_{T_j,t-1} = \underbrace{\mathcal{C}^m_{j,\omega}(\hat{\mu}^f_{j,t-1}, \mu^h_j)}_{\text{constrained region, manager}} + \underbrace{\mathcal{C}^\Psi_{j,\omega}(\hat{\mu}^f_{j,t-1}, \mu^h_j)}_{=0} + \mathcal{U}^m_{j,\omega}(\mu^h_j, \infty) + \mathcal{U}^\Psi_{j,\omega}(\mu^h_j, \infty) + \beta\, w^{t,W}_j \tag{A.65}$$

The $\beta w^{t,W}_j$ term captures the discounted period-t aggregate welfare arising from managers who learned at $t-1$. Under Convention W, investor welfare in the constrained region is zero, so constrained total welfare equals manager profit alone.

A.8.3 Summary of Welfare Properties

Quantity	Convention W	Convention W0
$\bar{\psi}_{j,\tau}$ at $\bar{A}_{j,\tau}$	$-\beta\, E[U_{j,\tau+1}(\mu_{\tau+1})\|\mu_\tau]/\bar{A}_{j,\tau}$	$-\delta_j - \beta\, E[U_{j,\tau+1}(\mu_{\tau+1})\|\mu_\tau]/\bar{A}_{j,\tau}$
At t (CV=0): $\bar{\psi}_{j,t}$	0	$-\delta_j$
$\mathcal{C}^{inv}$ on $[\hat{\mu}^f_{j,t-1}, \mu^h_j]$	0 (two-period ZPC)	< 0
$\mathcal{U}^{inv}$ on (μ^h_j, ∞)	> 0	ambiguous
$W^W_{j,t-1}$	> 0	ambiguous
I-investor learning externality	> 0 (all scenarios)	> 0 (all scenarios; $\delta_I = 0 \Rightarrow E[\Psi^0_I\|\mu] \equiv E[\Psi_I\|\mu]$)
A^{social}_j vs $\bar{A}_{j,t-1}$	$A^{social} < \bar{A}$	$A^{social,0} < \bar{A}$

A.9 Welfare Derivatives under Convention W0

This section derives the signs of $\partial \Delta T^0 / \partial \theta$ for $\theta \in \{\delta_R, \underline{A}_I, c_R\}$ under Convention W0, where $\Delta T^0 \equiv T^0_{\text{base}} - T^0_{\text{sc}}$ for sc $\in \{\text{case}\,1, \text{case}\,2\}$. We fix $\delta_I = 0$, $\underline{A}_R$, and c_I, and assume $\xi_R \equiv m + \bar{z}_R \sigma_r - c_R > 0$. We write $\Delta W^{0,\not R}_{T,t-1} = W^{0,\not R}_{T,t-1} + \beta W^{0,\not R}_{T,t}$, where $\Delta W^{0,\not R}_{T,t} = \Delta W^{\not R}_{M,t} + \Delta W^{0,\not R}_{R,t} + \Delta W^{0,\not R}_{I,t}$ is the full period-t welfare gain. This includes the manager's *continuation value* $\beta \Delta W_{M,t}$ (profit from raising a fund at period t once μ_t is revealed) and, because $\sigma_{\mu,\tau} > 0$, a genuine *investor learning surplus* from managers who reach the unconstrained region at t. We treat both components throughout; in the calibration of this paper, the period-t component is of comparable magnitude to the period-t-1 component.

A.9.1 Setup and Decomposition

W0 payoff identities. At a constrained upper bound $\bar{A}_j$:

$$\Pi^m_j = \bar{A}_j \bar{\pi}_j, \quad E[\Psi^0_j | \mu] = -\delta_j \bar{A}_j, \quad E[\Pi^{tot,0}_j | \mu] = \bar{A}_j \xi_j. \tag{A.66}$$

Since $\delta_I = 0$, $E[\Psi^0_I | \mu] = \bar{A}_j \cdot 0 = 0$ in the constrained region. Hence $\Delta W^{0,\not I}_{I,t-1} = 0$ for baseline vs $\not I$, and the period-t-1 decomposition reduces to:

$$\Delta W^{0,\not I}_{T,t-1} = \Delta W^{\not I}_{M,t-1} + \Delta W^{0,\not I}_{R,t-1}, \tag{A.67}$$

where $\Delta W^{\not I}_{M,t-1}$ is the manager-profit difference and $\Delta W^{0,\not I}_{R,t-1} = -\delta_R \int_{\mathcal{R}} \bar{A}_R \, \omega \, d\mu < 0$ is the R-investor welfare difference.

Period-t welfare: continuation value and learning surplus. The period-t component $\Delta W^{0,\not R}_{T,t} = \Delta W^{\not R}_{M,t} + \Delta W^{0,\not R}_{R,t} + \Delta W^{0,\not R}_{I,t}$ captures all welfare generated at period t from managers who entered the market at t-1. Two forces operate through $\Delta W^{0,\not R}_{T,t}$:

Manager continuation value. Managers who raised funds at t-1 observe their updated belief μ_t — drawn from the predictive distribution $p(\mu_t | \mu_{t-1}) \sim N(\mu_{t-1}, \sigma^2_{\mu,\tau})$ — and, if μ_t exceeds the feasibility threshold, raise a second fund at t. The resulting manager profit $\Pi^m(\mu_t, A^*_t)$ is included in $\Delta W^{\not R}_{M,t}$. Since the baseline's active set $[\mu^f_{R,t}, \infty)$ is broader than $\not R$'s $[\mu^f_{I,t}, \infty)$ at t-1, the baseline generates a strictly larger pool of managers who draw μ_t and potentially operate at t, giving $\Delta W^{\not R}_{M,t} > 0$.

Investor learning surplus. Because $\sigma_{\mu,\tau} > 0$, the signal genuinely updates beliefs: managers from the extra R-window $[\mu^f_{R,t}, \mu^f_{I,t})$ may draw $\mu_t > \mu^h_j$ and operate in the unconstrained region, generating positive investor payoffs $\Delta W^{0,\not R}_{R,t}$ and $\Delta W^{0,\not R}_{I,t}$ (where the ZPC no longer fully extracts investor surplus). With $\sigma_{\mu,\tau}$ calibrated to be economically substantial — comparable to the width of the skill thresholds — this learning component is a first-order

contributor to total welfare. In the calibration of this paper, $\beta\Delta W^{0,\not R}_{T,t}$ is of similar magnitude to $\Delta W^{0,\not R}_{T,t-1}$, so neither period dominates.

For the $\not I$ comparison, $\Delta W^{0,\not I}_{T,t-1} = 0$ identically (baseline and $\not I$ make identical choices at t-1), so $\Delta W^{0,\not I}_{T,t-1} = \beta\Delta W^{0,\not I}_{T,t}$ consists entirely of the period-t continuation and type-flexibility values.

Period-t-1 regime structure.

Case	$\Delta W^{0,\not R}_{T,t-1}$ **(base vs $\not R$)**	**R-active region $\mathcal{R}$**
A1, A3	$\int_{\mu^f_{R,t}}^{\mu^f_{I,t}} \bar{A}_R\xi_R\omega + \int_{\mu^f_{I,t}}^{\mu^*_t}(\bar{A}_R\xi_R - \bar{A}_I\xi_I)\omega$	$[\mu^f_{R,t}, \mu^*_t)$: Region A∪C
A2	$\int_{\mu^f_{R,t}}^{\mu^f_{I,t}} \bar{A}_R\xi_R\,\omega\,d\mu$	$[\mu^f_{R,t}, \mu^f_{I,t})$: Region A only
B	$\int_{\mu^f_{R,t}}^{\mu^h_{R,t}} \bar{A}_R\xi_R\omega + \int_{\mu^h_{R,t}}^{\mu^f_{I,t}} A^u_R\xi^u_R\omega$	$[\mu^f_{R,t}, \mu^f_{I,t})$: constrained + unconstrained

Region C $= [\mu^f_{I,t}, \mu^*_t)$ exists whenever $\mu^*_t > \mu^f_{I,t}$, i.e. in Cases A2 and A3/4 (and Cases B1b–B3 within outer Regime B). In Cases A1 and B1a, $\mu^*_t = \mu^f_{I,t}$ so baseline assigns R throughout $[\mu^f_{R,t}, \mu^f_{I,t})$ with no I competition.

A.9.2 Main Lemma: Component Derivatives

Lemma 4 (Welfare component derivatives)**.** *The following derivatives, used in all parts below, follow directly from the ZPC and Lemma 1: $d\bar{z}_R/d\delta_R = -[\sigma_r(1-p\Phi(\bar{z}_R))]^{-1} < 0$; $d\bar{A}_R/d\delta_R = [b(1-p\Phi(\bar{z}_R))]^{-1} > 0$; $d\bar{z}_R/dc_R = d\bar{A}_R/dc_R = 0$; $d\mu^f_{j,t}/d\underline{A}_j = b$.*

For each parameter, items i–iii derive the period-t-1 component derivatives; item iv derives the period-t component $\beta\Delta W^{0,\not R}_{T,t}$ (which includes the manager continuation value $\beta\Delta W^{\not R}_{M,t}$ and period-t investor surpluses) for both comparisons.

(A) Derivative with respect to δ_R:

i. $\partial(\Delta W^{\not R}_{M,t-1})/\partial\delta_R > 0$ ***in all cases.***

Case A2 (CRCI switch, $\mu^*_t = \mu^\dagger_t$). Differentiate the CRCI implicit equation $G(\mu^\dagger_t, \delta_R) \equiv \bar{A}_R\bar{\pi}_R - \bar{A}_I\bar{\pi}_I = 0$ by the IFT. Both $\bar{A}_R$ and $\bar{\pi}_R$ depend on δ_R through $\bar{z}_R$, giving:

$$\frac{\partial G}{\partial\delta_R} = \underbrace{\bar{A}_R\frac{d\bar{\pi}_R}{d\delta_R}}_{(i)} + \underbrace{\bar{\pi}_R\frac{d\bar{A}_R}{d\delta_R}}_{(ii)} = \frac{\bar{\pi}_R/b - p\Phi(\bar{z}_R)\bar{A}_R(\mu^\dagger_t)}{1-p\Phi(\bar{z}_R)}.$$

Since $\mu_t^\dagger < \mu_{R,t}^h$ in Case A2, type R is constrained at the crossing. By Lemma 2 Part 7, $\bar{\pi}_R > p\Phi(\bar{z}_R)\cdot b\bar{A}_R(\mu_t^\dagger)$ automatically, so $\partial G/\partial\delta_R > 0$. Since $\partial G/\partial\mu_t^\dagger = (\bar{\pi}_R - \bar{\pi}_I)/b < 0$ (Lemma 2 Part 9), IFT gives $d\mu_t^\dagger/d\delta_R > 0$. The R-active interval expands at $\mu_{R,t}^f$ (left) and $\mu_t^* = \mu_t^\dagger$ (right). Hence $\partial(\Delta W_{M,t-1}^{\not R})/\partial\delta_R > 0$.

Cases A3/4 (URCI switch, $\mu_t^* = \mu_t^\ddagger$). Here δ_R does not enter the unconstrained R profit $\Pi_R(A_{R,t}^u)$ directly at period t (Lemma 2, Part 7: $d\mu_t^\ddagger/d\delta_R = 0$). The upper boundary $\mu_t^\ddagger$ therefore does not shift. However, the R-active interval still expands at the left: $\mu_{R,t}^f$ moves left (rate $-[b(1-p\Phi(\bar{z}_R))]^{-1} < 0$), and the fund size at each $\mu \in [\mu_{R,t}^f, \mu_t^\ddagger)$ grows ($d\bar{A}_R/d\delta_R > 0$, $dA_R^u/d\delta_R \geq 0$). Hence $\partial(\Delta W_{M,t-1}^{\not R})/\partial\delta_R > 0$ via the intensive + left-boundary channel.

Cases A1 and outer Regime B. $\mu_t^* = \mu_{I,t}^f$ (A1, B1a) or $\mu_t^\ddagger$ (B1b–B3) is independent of δ_R at period t. The R-active interval expands at $\mu_{R,t}^f$ only, giving an extensive gain. The intensive term $d(\bar{A}_R\bar{\pi}_R)/d\delta_R > 0$ near $\mu_{R,t}^f$. Hence $\partial(\Delta W_{M,t-1}^{\not R})/\partial\delta_R > 0$.

ii. $\partial(\Delta W_{R,t-1}^{0,\not R})/\partial\delta_R < 0$ ***in all cases.***

$\Delta W_{R,t-1}^{0,\not R} = -\delta_R \int_{\mathcal{R}} \bar{A}_R\omega\, d\mu$. *Differentiating:*

$$\frac{\partial(\Delta W_{R,t-1}^{0,\not R})}{\partial\delta_R} = -\underbrace{\int_{\mathcal{R}} \bar{A}_R\omega\, d\mu}_{>0} - \delta_R \underbrace{\frac{\partial}{\partial\delta_R}\int_{\mathcal{R}} \bar{A}_R\omega\, d\mu}_{>0 \text{ (fund sizes grow, interval expands)}}.$$

Both terms are negative, so $\partial(\Delta W_{R,t-1}^{0,\not R})/\partial\delta_R < 0$ in all cases.

iii. $\partial(\Delta W_{I,t-1}^{0,\not R})/\partial\delta_R = 0$ *by the $\delta_I = 0$ identity* (A.66).

iv. ***Period-t externality:*** $\partial(\beta\Delta W_{T,t}^{0,\not R})/\partial\delta_R \leq 0$; $\partial(\Delta W_{T,t-1}^{0,\not T})/\partial\delta_R < 0$ *in all cases.*

For the $\not R$ comparison, two opposing forces operate at period t. Pool expansion *(positive): $\mu_{R,t}^f$ moves left, so a wider set of managers enter at t-1, draw μ_t, and can operate at period t; this raises $\beta\Delta W_{T,t}^{0,\not R}$.* Threshold tightening *(ambiguous): higher δ_R raises μ_t^*, reducing the mass of R-fund managers who switch to I at t and generating additional R-outside-option costs. In the calibration of this paper, the pool-expansion effect dominates: $\partial(\beta\Delta W_{T,t}^{0,\not R})/\partial\delta_R > 0$ numerically, meaning the period-t component* partially offsets *the negative t-1 component. However, the t-1 effect is larger in magnitude, so the total $\partial(\Delta W_{T,t-1}^{0,\not R})/\partial\delta_R < 0$. This is a quantitative calibration conclusion, not a general theoretical result; the sign of $\partial(\beta\Delta W_{T,t}^{0,\not R})/\partial\delta_R$ cannot be signed analytically.*

For the $\not{I}$ comparison ($\Delta W^{0,\not{I}}_{T,t-1} = \beta \Delta W^{0,\not{I}}_{T,t}$ entirely): higher δ_R raises the skill threshold at which $j^*(\mu_t) = I$, reducing the mass of $R \to I$ switchers at t and shrinking the type-flexibility gain. The R-investor W0 cost from being locked to R at t also grows. Hence $\partial(\Delta W^{0,\not{I}}_{T,t-1})/\partial\delta_R < 0$.

(B) Derivative with respect to $\underline{A}_I$:

i. $\partial(\Delta W^{\not{R}}_{M,t-1})/\partial \underline{A}_I > 0$ ***in all cases.***

The only threshold that moves is $\mu^f_{I,t}$ at rate $b > 0$ (ZPC does not involve $\underline{A}_I$).

Cases A2 and A3/4 (Region C exists). Moving $\mu^f_{I,t}$ right shifts a strip from Region C into Region A. The net Leibniz gain:

$$[\bar{A}_R(\mu^f_{I,t})\bar{\pi}_R - (\bar{A}_R(\mu^f_{I,t})\bar{\pi}_R - \bar{A}_I(\mu^f_{I,t})\bar{\pi}_I)]\,\omega(\mu^f_{I,t})\,b = \bar{A}_I(\mu^f_{I,t})\bar{\pi}_I\,\omega(\mu^f_{I,t})\,b > 0.$$

Cases A1 and B1a (no Region C). The upper limit of Region A moves right, adding new territory where baseline has R and $\not{R}$ has $\emptyset$:

$$\frac{\partial(\Delta W^{\not{R}}_{M,t-1})}{\partial \underline{A}_I} = \Pi^m_R(A^*_R(\mu^f_{I,t}), \mu^f_{I,t})\,\omega(\mu^f_{I,t})\,b > 0.$$

ii. $\partial(\Delta W^{0,\not{R}}_{R,t-1})/\partial \underline{A}_I$***: zero in A2/A3/4; negative in A1; positive in outer Regime B.***

Cases A2 and A3/4. $\mathcal{R} = [\mu^f_{R,t}, \mu^*_t)$ ends at μ^*_t, which is independent of $\underline{A}_I$ (μ^*_t is the CRCI or URCI crossing, not a feasibility threshold). No boundary moves, so the derivative is zero.

Case A1. $\mathcal{R} = [\mu^f_{R,t}, \mu^f_{I,t})$ is entirely constrained (since $\mu^f_{I,t} < \mu^h_{R,t}$ in Case A1 and B1a). The upper limit moves right at rate b, adding territory where $E[\Psi^0_{R,t}|\mu](\bar{A}_{R,t}) = -\delta_R \bar{A}_R < 0$:

$$\frac{\partial(\Delta W^{0,\not{R}}_{R,t-1})}{\partial \underline{A}_I} = -\delta_R \bar{A}_R(\mu^f_{I,t})\,\omega(\mu^f_{I,t})\,b < 0.$$

Summing with item (i), the net t-1 total is $\xi_R \bar{A}_R(\mu^f_{I,t})\omega(\mu^f_{I,t})b$, where $\xi_R \equiv m + \bar{z}_R\sigma_r - c_R$. This is positive if and only if $\xi_R > 0$. Since ξ_R is decreasing in δ_R (as $\bar{z}_R$ falls with δ_R), the condition $\xi_R > 0$ can fail for sufficiently large δ_R; at our baseline calibration $\xi_R = 0.0049 > 0$, but $\xi_R < 0$ for $\delta_R \gtrsim 0.008$ in the calibration range. The section header assumption $\xi_R > 0$ must therefore be understood as a local condition on the parameter space being explored, not a universal model implication.

Outer Regime B. Here $\mu^f_{I,t} \geq \mu^h_{R,t}$, so $\mathcal{R}$ contains an unconstrained sub-interval $[\mu^h_{R,t}, \mu^f_{I,t})$ where $A^*_R = A^u_R(\mu)$. In the unconstrained region, the manager does not extract full investor surplus, so $E[\Psi^0_R(A^u_R)|\mu] = A^u_R(\mu - bA^u_R - p\tilde{\mu} - m) > 0$ (investors earn a positive

rent). As $\underline{A}_I$ rises, $\mu^f_{I,t}$ moves right, widening $[\mu^h_{R,t}, \mu^f_{I,t})$ and adding positive $\Delta W^{0,\not R}_{R,t-1}$ territory:

$$\frac{\partial(\Delta W^{0,\not R}_{R,t-1})}{\partial \underline{A}_I} = A^u_R(\mu^f_{I,t})\big(\mu^f_{I,t} - bA^u_R(\mu^f_{I,t}) - p\tilde{\mu}(\mu^f_{I,t}, A^u_R) - m\big)\,\omega(\mu^f_{I,t})\,b > 0.$$

Summing with item (i), $\partial(\Delta W^{0,\not R}_{T,t-1})/\partial \underline{A}_I > 0$ holds for Case B as well (both the V and W0_R components contribute positively).

iii. $\partial(\Delta W^{0,\not R}_{I,t-1})/\partial \underline{A}_I = 0$ *by the $\delta_I = 0$ identity.*

iv. ***Period-t externality:*** $\partial(\beta\Delta W^{0,\not R}_{T,t})/\partial \underline{A}_I > 0$*;* $\partial(\Delta W^{0,\not T}_{T,t-1})/\partial \underline{A}_I \leq 0$ *in all cases.*

For the $\not R$ comparison: raising $\underline{A}_I$ moves $\mu^f_{I,t}$ right, shrinking $\not R$'s t-1 active set $[\mu^f_{I,t}, \infty)$ while leaving the baseline set $[\mu^f_{R,t}, \infty)$ unchanged. The extra managers in the R-window $[\mu^f_{R,t}, \mu^f_{I,t})$ now draw skill signals that $\not R$ never observes, widening the learning gap. Hence $\partial(\beta\Delta W^{0,\not R}_{T,t})/\partial \underline{A}_I > 0$, reinforcing the t-1 result.

For the $\not T$ comparison: higher $\underline{A}_I$ raises $\mu^f_{I,t}$, reducing the set of skill values at which R→I switching at period t is feasible and thus shrinking the type-flexibility value. In Cases A1 and outer Regime B this effect is first-order: the I-threshold directly bounds the switching region. In Cases A2/A3/4 the type-switching threshold μ^*_t is interior to both the R and I feasibility regions, so a change in $\underline{A}_I$ affects only the outer boundary of I feasibility, not the switching threshold itself. The effect on $\partial(\Delta W^{0,\not T}_{T,t-1})/\partial \underline{A}_I$ in A2/A3/4 is non-zero in general but smaller than in A2/B; its sign depends on the balance between the reduced I-feasibility region and the unchanged switching structure. We treat the A2/A3/4 effect as second-order relative to A1 and outer Regime B for the purpose of signing the overall derivative.

(C) Derivative with respect to c_R:

i. $\partial(\Delta W^{\not R}_{M,t-1})/\partial c_R < 0$ ***in all cases (intensive margin).***

Since $d\bar{z}_R/dc_R = 0$, we have $d(\bar{A}_R\bar{\pi}_R)/dc_R = \bar{A}_R(-1) = -\bar{A}_R < 0$ throughout $\mathcal{R}$. Hence $\partial(\Delta W^{\not R}_{M,t-1})/\partial c_R = -\int_{\mathcal{R}} \bar{A}_R\omega\, d\mu < 0$.

ii. $\partial(\Delta W^{0,\not R}_{R,t-1})/\partial c_R$***: positive in A2/A3/4 and outer Regime B; zero in A1.***

Case A2 (CRCI switch, $\mu^*_t = \mu^\dagger_t$) — boundary moves. Differentiating the CRCI equation $G(\mu^\dagger_t, c_R) = \bar{A}_R\bar{\pi}_R(c_R) - \bar{A}_I\bar{\pi}_I = 0$:

$$\frac{d\mu^\dagger_t}{dc_R} = \frac{b\bar{A}_R(\mu^\dagger_t)}{\bar{\pi}_R - \bar{\pi}_I} < 0.$$

The upper limit of $\mathcal{R}$ moves left. The boundary contribution:

$$(-\delta_R \bar{A}_R(\mu_t^\dagger))\,\omega(\mu_t^\dagger)\,\frac{d\mu_t^\dagger}{dc_R} = \frac{\delta_R\, b\, \bar{A}_R(\mu_t^\dagger)^2\,\omega(\mu_t^\dagger)}{|\bar{\pi}_R - \bar{\pi}_I|} > 0.$$

This gain dominates the zero intensive term ($d\bar{A}_R/dc_R = 0$), giving $\partial(\Delta W_{R,t-1}^{0,\not R})/\partial c_R > 0$.

Cases A3/4 (URCI switch, $\mu_t^* = \mu_t^\ddagger$) — boundary moves via $d\mu_t^\ddagger/dc_R < 0$. From the URCI implicit function $H(\mu_t^\ddagger, c_R) = \Pi_R(A_{R,t}^u(\mu_t^\ddagger, c_R)) - \bar{A}_I(\mu_t^\ddagger)\bar{\pi}_I = 0$, applying the IFT with $\partial H/\partial c_R = \partial\Pi_R/\partial c_R|_{A^u} = -A_R^u < 0$ (envelope theorem) and $\partial H/\partial\mu_t^\ddagger < 0$:

$$\frac{d\mu_t^\ddagger}{dc_R} = -\frac{-A_R^u}{\partial H/\partial\mu_t^\ddagger} < 0.$$

The boundary contribution $(-\delta_R \bar{A}_R(\mu_t^\ddagger))\,\omega(\mu_t^\ddagger)\,(d\mu_t^\ddagger/dc_R) > 0$, giving $\partial(\Delta W_{R,t-1}^{0,\not R})/\partial c_R > 0$.

Case A1 (no Region C, $\mu_t^* = \mu_{I,t}^f$). $\mathcal{R} = [\mu_{R,t}^f, \mu_{I,t}^f)$ has a fixed upper limit (both boundaries independent of c_R). Hence $\partial(\Delta W_{R,t-1}^{0,\not R})/\partial c_R = 0$, and the t-1 total $\partial(\Delta W_{T,t-1}^{0,\not R})/\partial c_R < 0$.

Outer Regime B. $\mathcal{R}$ contains an unconstrained sub-interval $[\mu_{R,t}^h, \mu_{I,t}^f)$. Since $d\mu_{R,t}^h/dc_R < 0$ (from Part 5), higher c_R lowers $\mu_{R,t}^h$, widening $[\mu_{R,t}^h, \mu_{I,t}^f)$ and bringing additional high-skill managers into the unconstrained region. The extensive gain dominates the intensive loss, giving $\partial(\Delta W_{R,t-1}^{0,\not R})/\partial c_R > 0$.

iii. $\partial(\Delta W_{I,t-1}^{0,\not R})/\partial c_R = 0$ *by the $\delta_I = 0$ identity.*

iv. ***Period-t externality:*** *$\partial(\beta\Delta W_{T,t}^{0,\not R})/\partial c_R$: non-negative in A2/A3/4 and outer Regime B; near zero in A1. $\partial(\Delta W_{T,t-1}^{0,\not R})/\partial c_R$: positive in A2/A3/4 and outer Regime B; negative in A1.*

For the $\not R$ comparison: at period t, managers who entered via the R-window $[\mu_{R,t}^f, \mu_{I,t}^f)$ at t-1 observe μ_t and choose their type optimally. Since they choose I for high enough μ_t, the period-t I-fund welfare they generate is largely c_R-independent. In Case B, the widening of $[\mu_{R,t}^h, \mu_{I,t}^f)$ under higher c_R (item ii) also expands the pool of managers who enter period t with informative skill signals. The period-t externality therefore reinforces the positive t-1 result in Cases A2/A3/4 and outer Regime B, and is near zero in Case A1 (where $[\mu_{R,t}^f, \mu_{I,t}^f)$ is unchanged). Numerically, in Case B the period-t contribution exceeds the t-1 contribution, making the total $\partial(\Delta W_{T,t-1}^{0,\not R})/\partial c_R$ substantially positive.

For the $\not{I}$ comparison: higher c_R makes R less attractive at t, reducing the threshold at which managers switch to I, and increasing the value of type flexibility in Cases A2/A3/4 and outer Regime B. In Case A1, higher c_R shrinks the t-1 R-active pool, providing fewer high-skill managers for the t-period switch, so $\partial(\Delta W^{0,\not{I}}_{T,t-1})/\partial c_R < 0$.

A.9.3 Propositions for Total Welfare

Summing all components from Lemma 4 gives signed statements for $\Delta W^{0,\not{R}}_{T,t-1} = W^{0,\not{R}}_{T,t-1} + \beta W^{0,\not{R}}_{T,t}$.

Proposition 12 (Derivative w.r.t. δ_R)**.** *$\partial(\Delta W^{0,\not{R}}_{T,t-1})/\partial\delta_R < 0$ and $\partial(\Delta W^{0,\not{I}}_{T,t-1})/\partial\delta_R < 0$ in all cases.*

Proof. For $\Delta W^{0,\not{R}}_{T,t-1}$: the t-1 component satisfies $\partial(\Delta W^{0,\not{R}}_{T,t-1})/\partial\delta_R < 0$ directly, since $d(\bar{A}_R\xi_R)/d\delta_R = (\xi_R/b - \bar{A}_R)/(1 - p\Phi(\bar{z}_R)) < 0$ for all $\mu > \mu^f_{R,t}$ (as $\xi_R/b < \bar{A}_R$ throughout the active region). In Cases A2 and A3/4, the boundary at μ^*_t (Lemma 4(A.i)) adds territory where $\Delta W^{0,\not{R}}_{T,t-1} = -\delta_R\bar{A}_R < 0$, reinforcing the sign. The period-t component $\partial(\beta\Delta W^{0,\not{R}}_{T,t})/\partial\delta_R \leq 0$ by Lemma 4(A.iv). Hence the total is negative. For $\Delta W^{0,\not{I}}_{T,t-1}$: Lemma 4(A.iv) gives $\partial(\Delta W^{0,\not{I}}_{T,t-1})/\partial\delta_R < 0$ directly. □ □

Proposition 13 (Derivative w.r.t. $\underline{A}_I$)**.** *$\partial(\Delta W^{0,\not{R}}_{T,t-1})/\partial\underline{A}_I > 0$ in all cases. For $\Delta W^{0,\not{I}}_{T,t-1}$: the derivative is ≈ 0 in A2/A3/4 and < 0 in Cases A1 and outer Regime B.*

Proof. For $\Delta W^{0,\not{R}}_{T,t-1}$: by Lemma 4(B.i)-(B.ii): in A2 and A3/4 the net t-1 total is $\bar{A}_I(\mu^f_{I,t})\bar{\pi}_I\omega(\mu^f_{I,t})b > 0$; in Case A1 it is $\xi_R\bar{A}_R(\mu^f_{I,t})\omega(\mu^f_{I,t})b > 0$; in B both the V and W0_R components contribute positively so the net is again positive. The period-t component $\partial(\beta\Delta W^{0,\not{R}}_{T,t})/\partial\underline{A}_I > 0$ (Lemma 4(B.iv)) reinforces this, since raising $\underline{A}_I$ widens the learning gap between baseline and $\not{R}$. Hence $\partial(\Delta W^{0,\not{R}}_{T,t-1})/\partial\underline{A}_I > 0$ in all cases.

For $\Delta W^{0,\not{I}}_{T,t-1}$: by Lemma 4(B.iv), higher $\underline{A}_I$ tightens the t-period I-feasibility threshold, reducing type-switching opportunities in Cases A1 and outer Regime B and leaving A2/A3/4 approximately neutral. □ □

Proposition 14 (Derivative w.r.t. c_R)**.**

1. Cases A2, A3/4 and outer Regime B: $\partial(\Delta W^{0,\not{R}}_{T,t-1})/\partial c_R > 0$ *and* $\partial(\Delta W^{0,\not{I}}_{T,t-1})/\partial c_R > 0$.

2. Case A1: $\partial(\Delta W^{0,\not{R}}_{T,t-1})/\partial c_R < 0$*; the sign is ambiguous (negative t-1 component, near-zero t-period component).* $\partial(\Delta W^{0,\not{I}}_{T,t-1})/\partial c_R < 0$.

Proof. **Cases A2/A3/4 — $\Delta W^{0,\not R}_{T,t-1}$.** The t-1 component: the negative intensive term $\partial(\Delta W^{\not R}_{M,t-1})/\partial c_R < 0$ is dominated by the positive boundary term $\partial(\Delta W^{0,\not R}_{R,t-1})/\partial c_R \propto \bar{A}_R(\mu_t^*)^2 > 0$ (Lemma 4(C.i)-(C.ii)). The period-t component $\partial(\beta\Delta W^{0,\not R}_{T,t})/\partial c_R \geq 0$ (Lemma 4(C.iv)) reinforces the positive sign. Total: > 0.

Cases A2/A3/4 — $\Delta W^{0,\not T}_{T,t-1}$. By Lemma 4(C.iv): higher c_R lowers the t-period threshold for R→I switching, widening the type-flexibility gain. $\partial(\Delta W^{0,\not T}_{T,t-1})/\partial c_R > 0$.

Case A1 — $\Delta W^{0,\not R}_{T,t-1}$. No boundary at μ_t^* (Lemma 4(C.ii)): the t-1 total equals $\partial(\Delta W^{\not R}_{M,t-1})/\partial c_R < 0$. The period-$t$ component is near zero in A2 (Lemma 4(C.iv)). Full sign is negative (ambiguous if the period-t contribution is non-trivial, but dominated by the negative t-1 component in the calibration).

Case A1 — $\Delta W^{0,\not T}_{T,t-1}$. By Lemma 4(C.iv): higher c_R shrinks the R-window pool of managers, reducing t-period flexibility value. $\partial(\Delta W^{0,\not T}_{T,t-1})/\partial c_R < 0$.

Outer Regime B. The t-1 component is positive by Lemma 4(C.ii) (widening $[\mu^h_{R,t}, \mu^f_{I,t})$), and the period-$t$ externality reinforces this and is numerically larger than the t-1 contribution (Lemma 4(C.iv)). Both $\Delta W^{0,\not R}_{T,t-1}$ and $\Delta W^{0,\not T}_{T,t-1}$ are positive. □ □

A.9.4 Summary

	$\partial\Delta T^0/\partial(\delta_R - \delta_I)$		$\partial\Delta T^0/\partial(\underline{A}_I - \underline{A}_R)$		$\partial\Delta T^0/\partial(c_R - c_I)$	
Case	**vs $\not R$**	**vs $\not T$**	**vs $\not R$**	**vs $\not T$**	**vs $\not R$**	**vs $\not T$**
A2, A3/4	< 0	< 0	> 0	> 0 (partial offset)	> 0	> 0
A1	< 0	< 0	> 0	< 0	< 0	< 0
Reg. B	< 0	< 0	> 0	< 0	> 0	> 0

The outside-option and institutional-barrier derivatives are unconditional: $\partial\Delta T^0/\partial\delta_R < 0$ and $\partial(\Delta W^{0,\not R}_{T,t-1})/\partial\underline{A}_I > 0$ hold in all cases for both comparisons, with the period-t learning externality reinforcing the t-1 component in each. The retail-cost derivative is case-dependent, reflecting whether the contested zone Region C exists ($\mu_t^* > \mu^f_{I,t}$, i.e. Cases A2/A3/4 and B1b–B3) or not (Cases A1 and B1a) and whether the unconstrained R-manager segment (Case B) contributes a substantial period-t surplus.

Internet Appendix

A Investor Type Classification via Investment Company Act Exclusions

We classify funds based on their reported Investment Company Act exclusions in Form ADV, which provide legally binding frameworks for investor eligibility and fund structure. U.S. venture capital funds typically rely on one of two exclusions from registration as investment companies:

Section 3(c)(7) Funds (Institutional-Oriented):

Funds organized under Section 3(c)(7) of the Investment Company Act (15 U.S.C. § 80a-3(c)(7)) are exempt from investment company registration if:

(i) owned exclusively by "qualified purchasers", which requires substantially higher wealth thresholds: \$5+ million in investments for individuals or \$25+ million for institutional investors.

There is no limit on the number of investors, as long as all are "qualified purchasers". The qualified purchaser requirement aligns with our model's characterization of institutional capital.

Section 3(c)(1) Funds (Retail-Oriented):

Venture captial funds organized under Section 3(c)(1) of the Investment Company Act (15 U.S.C. § 80a-3(c)(1)) are exempt from investment company registration if they satisfy two conditions:

(i) no more than 250 beneficial owners

(ii) no more than \$12 million in aggregate capital contributions and uncalled capital commitments

This type of VC funds may include accredited investors who do not meet the higher "qualified purchaser" standard. Therefore, most of them target accredited individuals with lower wealth levels, aligning with the characterization of retail capital in our model.

Therefore, we classify funds into three categories based on their Form ADV reported exclusions:

- **Retail Funds:** Funds that rely exclusively on the Section 3(c)(1) exclusion.

- **Institutional Funds:** Funds that rely exclusively on the Section 3(c)(7) exclusion.
- **Mixed:** Funds that report using both exclusions. These represent a small fraction of our sample and are excluded from our main analyses but included in robustness checks.

B Value Added Under All Retail Capital Regimes

B.1 Value Added to the Economy from Access to Retail Investors (Complete)

We define the value added of the GP to the real economy (economic value added) as the value created from the investment minus administrative costs, given the GP's choice of investor type j:

$$VA^{j}_{j,t} = (\mu_t - bA_{j,t})A_{j,t} - c_j A_{j,t}. \tag{A.68}$$

The economic value added under the GP's optimal choice of investor type j to maximize her profits given the perceived skill μ_t in period t is denoted by $VA_{j,t}(\mu_t)$. Total value added of the GP conditional on its perceived skill μ_0 in period 0 is:

$$TVA(\mu_0) = VA_0(\mu_0) + \beta E[VA_1(\mu_1)|\mu_0], \tag{A.69}$$

where the expected future value added in period 1 is:

$$E[VA_1(\mu_1)|\mu_0] = \int_{\mu^I_{min}}^{\infty} VA_1(\mu_1) f(\mu_1|\mu_0) d\mu_1 + \int_{\mu^R_{min}}^{\mu^I_{min}} VA_1(\mu_1) f(\mu_1|\mu_0) d\mu_1. \tag{A.70}$$

For GPs raising capital from retail investors in period 0 (i.e., $\mu_0 \in [\mu^R_{min}, \mu^I_{min}]$), the first term captures the value added from the stepping stone (SS) effect:

$$VA_{SS}(\mu_0) = \int_{\mu^I_{min}}^{\infty} VA_1(\mu_1) f(\mu_1|\mu_0) d\mu_1. \tag{A.71}$$

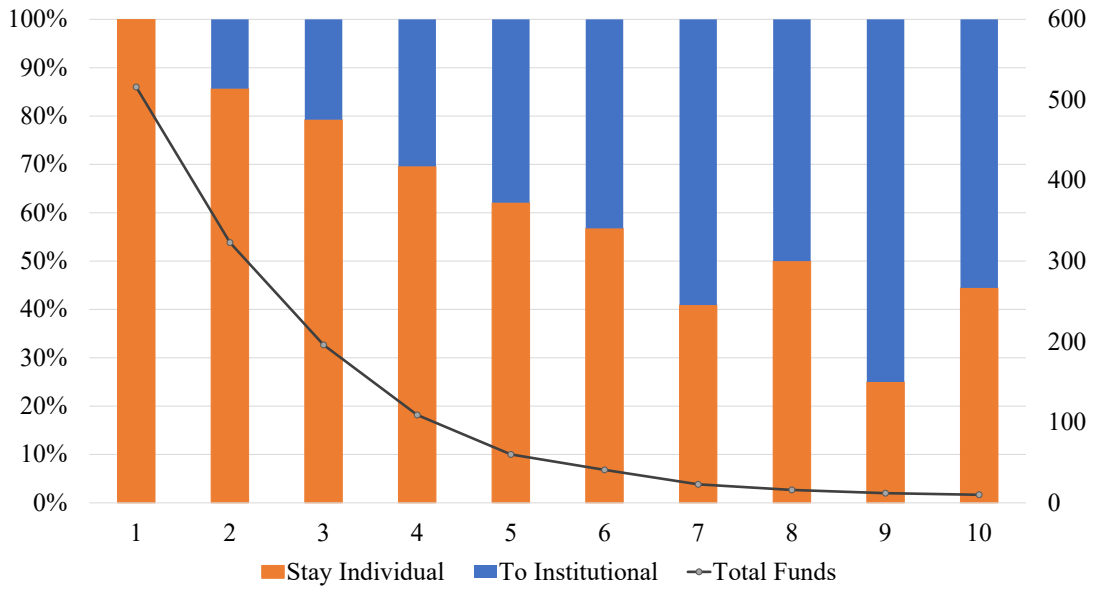


Figure A.1: **Transitions of GPs starting with Retail Capital by Fund Sequence.** This figure shows the fraction of venture capital GPs that start with retail capital (individual investors) by fund sequence (the Nth fund raised by each GP), separately for funds that stay with retail capital and for those that transition to institutional capital. The gray line, using the right axis, reports the total number of GPs that start with retail capital and survive in each fund sequence.

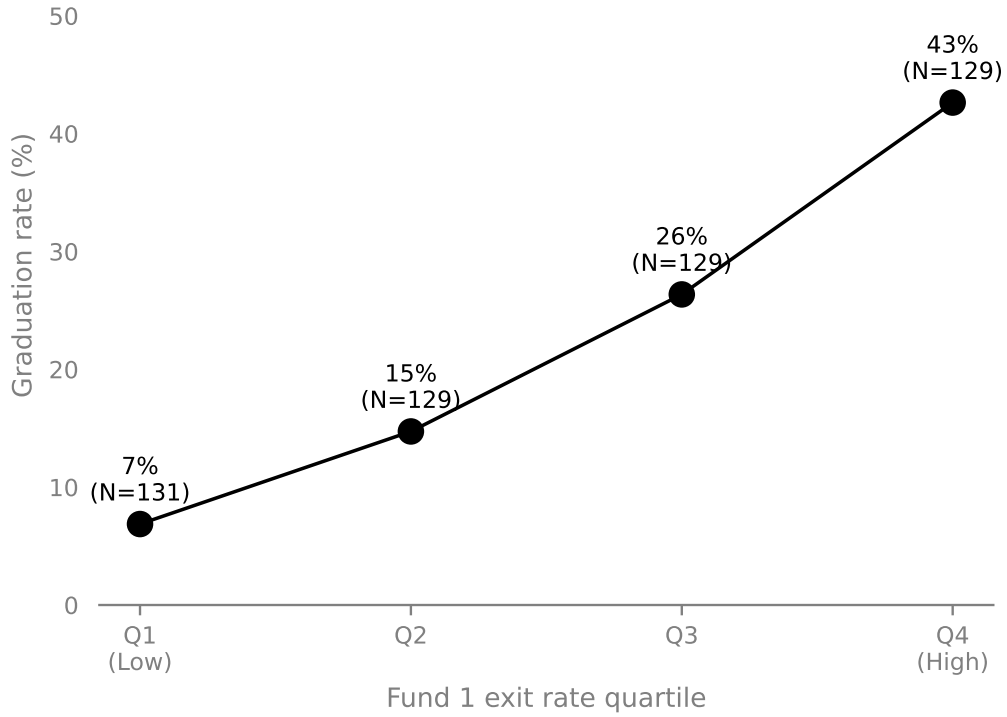


Figure A.2: **Fund 1 exit rate and graduation from 3(c)(1) to 3(c)(7)**

This figure plots the graduation rate from 3(c)(1) to 3(c)(7) by quartile of the GP's first fund exit rate. Graduation is defined as a GP whose first fund relies on the 3(c)(1) exemption subsequently raising a fund that relies on the 3(c)(7) exemption; the 3(c)(1)/3(c)(7) classification is from SEC Form ADV filings. Funds relying on both exemptions are assigned to 3(c)(7). The exit rate is the fraction of a fund's unique portfolio company investments that result in an IPO or acquisition, computed from PitchBook deal-level data. The sample consists of $N = 518$ GPs whose first fund is 3(c)(1) and for which PitchBook exit data are available. Of these, 117 (22.6%) graduate to 3(c)(7). Each dot reports the share of GPs in that quartile who graduated; sample sizes per quartile are shown in parentheses.

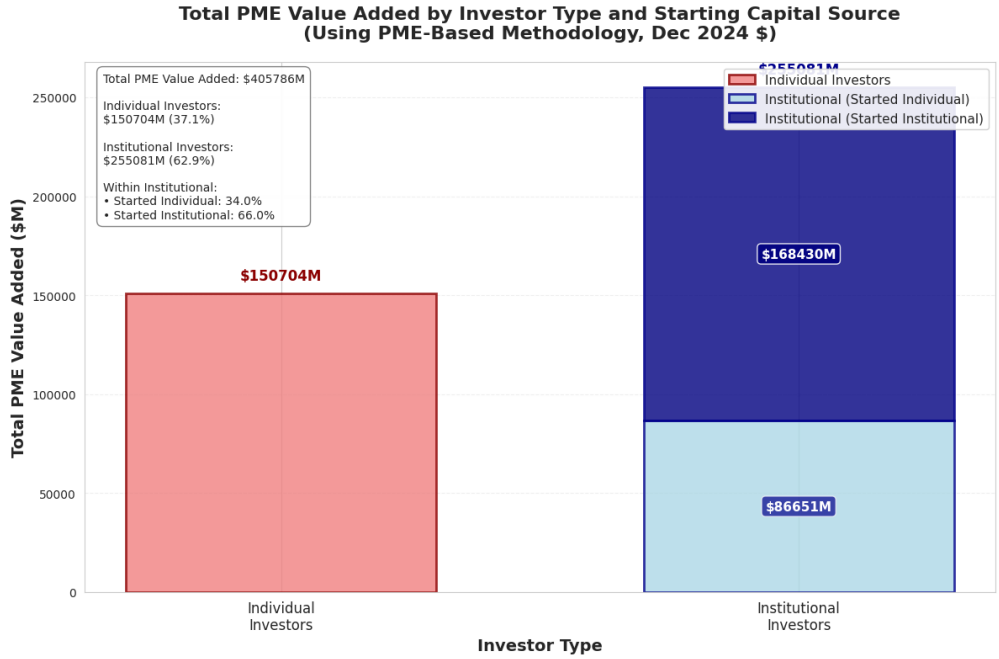


Figure A.3: **Total Value Added by Investor Type**. Value added is measured by the total value added of LPs in dollars using the PME-based methodology and the S&P 500 index as the benchmark.

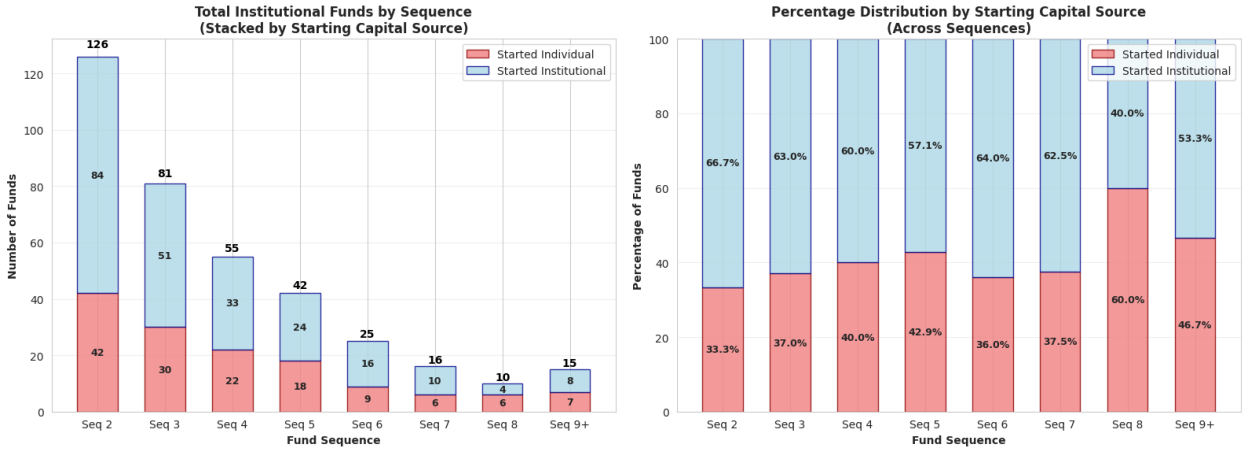
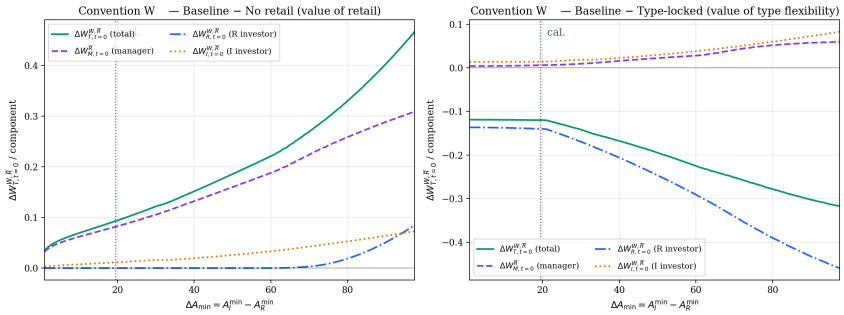
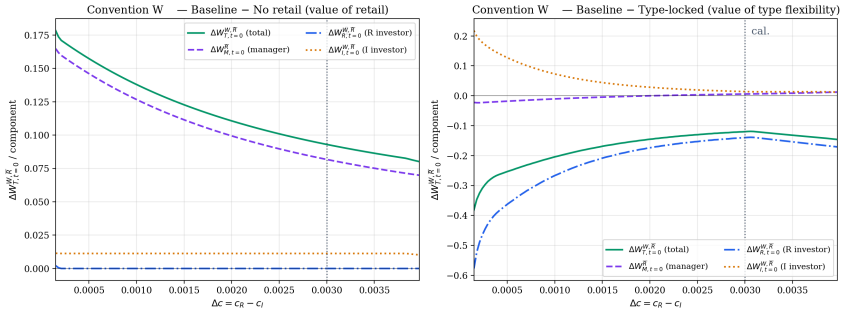


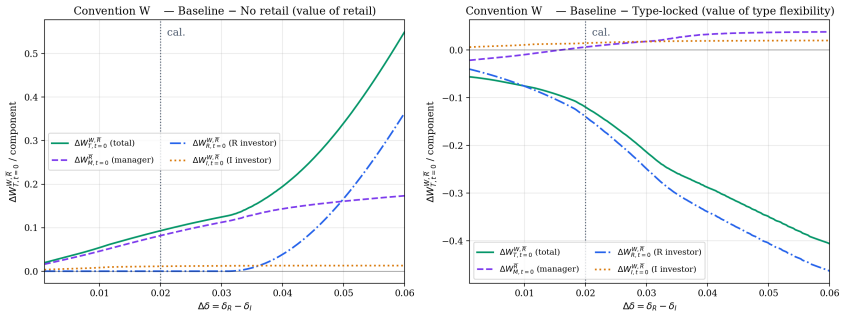
Figure A.4: **Distribution of Institutional Funds by Starting Capital Source**. The fraction of institutional funds that started with retail capital increases substantially over fund sequence, providing strong evidence for the stepping stone mechanism predicted by our model.



(a) **Panel A: Variation in Minimum Scale**

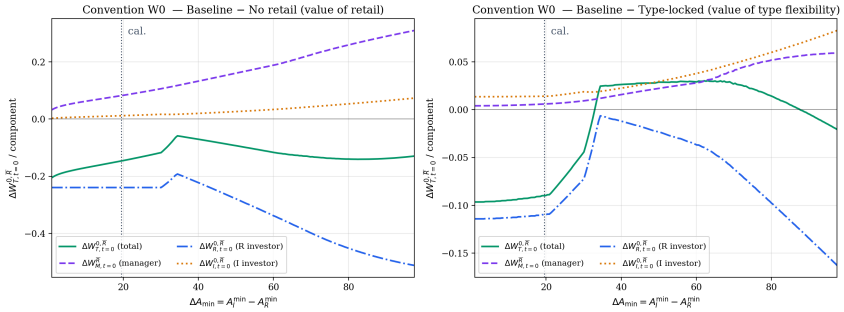


(b) **Panel B: Variation in Operating Costs**

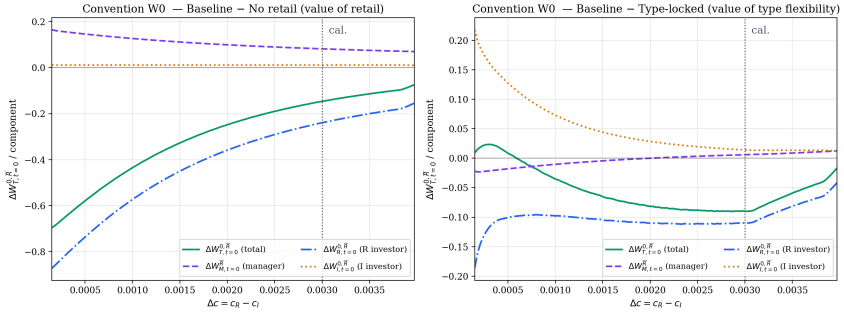


(c) **Panel C: Variation in Outside Options**

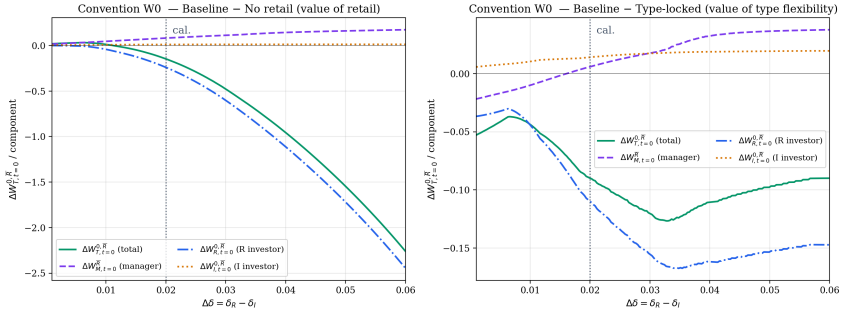
Figure A.5: **Welfare Outcomes by Market Structure (δ Valued)** This figure shows welfare changes across market structures as the relative differences between investor types change for a typical model calibration when δ is valued by the social planner. The left plot of each panel is the welfare difference between baseline welfare and a market structure excluding retail ($\not{R}$) and the right plot is the welfare difference between the baseline and a market structure preventing graduation and demotion. All panels show total welfare in green, manager welfare in blue, and investor welfare in red (retail) and purple (institutions). All panels show the baseline calibration as as a vertical gray dotted line. Panel A varies relative minimum scale, Panel B relative operating costs, and Panel C relative outside options.



(a) **Panel A: Variation in Minimum Scale**



(b) **Panel B: Variation in Operating Costs**



(c) **Panel C: Variation in Outside Options**

Figure A.6: **Welfare Outcomes by Market Structure (δ Distortion)** This figure shows welfare changes across market structures as the relative differences between investor types change for a typical model calibration when δ is not valued by the social planner. The left plot of each panel is the welfare difference between baseline welfare and a market structure excluding retail ($\not{R}$) and the right plot is the welfare difference between the baseline and a market structure preventing graduation and demotion. All panels show total welfare in green, manager welfare in blue, and investor welfare in red (retail) and purple (institutions). All panels show the baseline calibration as as a vertical gray dotted line. Panel A varies relative minimum scale, Panel B relative operating costs, and Panel C relative outside options.