# Wealth Inequality and Safe Asset Demand*

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**Abstract**

We study how wealth inequality shapes safe-asset demand in heterogeneous-agent economies. Asset bubbles arise when agents face sufficiently high probabilities of falling into extreme poverty. In such cases, assets insulated from idiosyncratic risk become infinitely large relative to agents' wealth *ex-post*, which makes them infinitely valuable *ex-ante*. This mechanism is fundamentally different from classic rational bubbles, generates bubbles even under stationary wealth distributions, and requires no aggregate uncertainty or growth. Using this insight, we revisit the pricing of aggregate assets in standard heterogeneous-agent economies, and show that prior analyses may be incomplete due to overlooked possibility of transversality condition violation.

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# 1 Introduction

It is well understood that idiosyncratic risk induces precautionary saving, which increases household demand for safe assets and lowers the risk-free interest rate. Likewise, a large literature has examined how different fortunes shape wealth inequality. However, less is known about *how* the shape of the wealth distribution affects the demand for safe assets, and *when* it lowers the discount rate enough to sustain asset bubbles. Clarifying this link is essential for understanding asset valuation in heterogeneous-agent economies.

This paper studies the relationship between wealth inequality and safe asset demand. We characterize conditions on the wealth distribution under which the present discounted value of an asset approaches infinity in the distant future, violating the transversality condition (TVC). In such cases, the asset's price exceeds the present discounted value of the asset's cash flows, which is usually regarded as its fundamental value.

We find that the TVC is closely linked to the wealth distribution, which is shaped by the nature of the idiosyncratic risk. We uncover a new intuition for TVC violation to arise in heterogeneous-agent economies, which is distinct from the classic rational bubble: in our setting, TVC violation requires the wealth of a significant fraction of agents to become sufficiently low relative to the average wealth in the economy. When these agents evaluate an asset that is unaffected by their idiosyncratic risk, they expect the asset to become infinitely large relative to their own wealth *ex-post*. Anticipating this outcome *ex-ante*, they place exceptionally high value on such assets, leading to TVC violation even in the absence of aggregate uncertainty or economic growth.

Our analysis offers a new perspective on how wealth inequality influences asset prices. Through the lens of our characterization, we revisit workhorse heterogeneous-agent models like Constantinides and Duffie (1996), and show that some prior analyses may be incomplete due to the overlooked possibility of TVC violation.

**Analysis of Permanent Idiosyncratic Risk.** We begin with a standard setting in which households face uninsurable idiosyncratic risk that is *permanent* and follows a random walk for each household. As the wealth distribution evolves, the vast majority of households become extremely

poor, while a vanishing share of households accumulates nearly all the wealth. Over time, this divergence leads to an exploding degree of wealth inequality.

To understand the implications for asset pricing, consider how a household in this economy prices an aggregate asset such as government debt. The value of this asset depends on its cash flow and discount rate. In standard representative-agent models, asset cash flows and household consumption are cointegrated, and their covariance capturing risk premium is the primary determinant of valuation. In contrast, with permanent idiosyncratic risk, the household anticipates its wealth to decline far below the population average in almost all states of the world. As a result, the asset's cash flow will become arbitrarily larger than the household's wealth.

This divergence in scale between the household and the asset is the key determinant of asset valuation in this environment: while the household's marginal utility growth correctly prices the asset's return *period by period*, the asset's long-run discounted value tends to infinity *in the limit*, leading to TVC violation for aggregate assets. In other words, the precondition for asset bubbles to arise is that the household anticipates its wealth share to become negligible in the long run with a sufficiently high probability, which requires a sufficiently large degree of wealth inequality.

Importantly, not all permanent idiosyncratic risk leads to bubbles. When its magnitude is sufficiently small, although inequality still grows without bound and most individuals' wealth shares still tend to zero, this process occurs gradually and is outweighed by the subjective discount factor. In this case, the asset's terminal value remains finite and converges to zero in the limit. Therefore, evaluating asset bubbles requires a *quantitative* comparison between the magnitude of idiosyncratic risk and the strength of time discounting, and our result offers a diagnostic tool for this comparison for general processes of idiosyncratic risk and wealth distribution.

**Analysis of Transitory Idiosyncratic Risk.** The above analysis suggests that TVC violation arises when permanent idiosyncratic risk is sufficiently strong, which causes wealth inequality to diverge over time. A natural question is whether TVC violation can also occur when idiosyncratic risk is transitory and the wealth distribution is stationary—a potentially more realistic setting for studying asset prices.

To address this question, we consider two mechanisms to generate a stationary wealth distribution under idiosyncratic risk: wealth reset and death. In the first case, we additionally introduce an idiosyncratic redistribution shock that hits a randomly selected fraction of households every period, and resets their wealth to the average level. This shock generates a stationary wealth distribution in a tractable way, while preserving the total amount of resources in the economy. To our surprise, even in this setting, TVC violation can still occur. As before, the key lies in understanding the properties of the wealth distribution.

Specifically, when the intensity of the wealth reset shock is low, each household's wealth can deviate from the population average for an extended period before returning to the average. While the wealth distribution remains stationary, its left tail can become sufficiently heavy, so that the household anticipates large enough dispersion in its marginal utility growth, which depresses the discount rate and causes bubbles to emerge.

Conversely, when the intensity of the wealth reset shock is sufficiently high, the wealth distribution becomes more concentrated around the mean. In this case, households expect their wealth levels to remain comparable to the size of the economy in most states, raising the discount rate and restoring the TVC.

These results show that TVC violation for aggregate assets can occur when the wealth distribution is stationary but sufficiently left-skewed. Quantitatively, when we calibrate our model based on U.S. post-war Gini coefficient, the model does not rule out such cases. Therefore, the degree of skewness in the wealth distribution required for asset bubbles might be empirically plausible. That said, the Gini coefficient only offers a partial assessment. Evaluating whether the U.S. wealth distribution has the shape required to generate bubbles calls for a more detailed empirical analysis, with particular attention to the left tail of the distribution, which has received less scrutiny compared to the right tail that describes the very rich.

In the second case, we introduce mortality risk to our model. We assume that households die at a constant rate, and new households are born to maintain a constant population size. As a result of this mortality risk, households become more impatient, consuming a larger fraction of their wealth and discounting future cash flows more heavily.

We consider two model variants. First, if each newborn household inherits the wealth of a specific dying household, the wealth distribution diverges as in the permanent risk case. If the magnitude of the idiosyncratic risk is sufficiently large relative to the subjective discount rate, households are concerned about their vanishing wealth shares in scenarios where they do survive, leading to TVC violation. The only difference is that the subjective discount rate needs to be adjusted upwards to account for impatience due to the mortality risk.

Second, if the wealth of dying households is pooled together and distributed equally to each newborn household, the wealth distribution becomes stationary like the case with wealth reset. TVC violation may still occur if wealth inequality is sufficiently large, with death playing a role similar to wealth reset. Therefore, mortality risk does not overturn our core mechanism.

**Implications for Asset Valuation.** Our results highlight the left tail of the wealth distribution as an important determinant of asset valuation. For example, the setting in Constantinides and Duffie (1996) corresponds to the permanent-idiosyncratic-risk case studied in this paper, which leads to a diverging wealth distribution. When such risk is sufficiently large, the marginal utility growth in the lower tail becomes so steep that aggregate assets become infinitely valuable. In this case, the no-trade equilibrium breaks down. Constantinides and Duffie (1996) also consider a model variant in their appendix, which yields a stationary wealth distribution. Our framework implies that, even in this case, TVC violation can still occur if the wealth distribution is sufficiently left-skewed. This, too, precludes the existence of no-trade equilibrium.

These findings suggest that, while idiosyncratic risk offers a promising avenue to generate realistic risk premia, it may also lead to TVC violation. Ignoring this possibility may risk an incomplete analysis of asset valuation. Thus, instead of imposing the aggregate TVC as an exogenous assumption, it needs to be verified as an equilibrium condition. Conversely, our results clarify what is needed for the TVC to hold—agents need to have a finite expected growth rate in marginal utility, which requires mechanisms to induce mean-reversion and limit the left tail of the wealth distribution. Since these mechanisms are also central to determining the finite-period risk-return trade-off, the risk premium question needs to be considered jointly with the infinite-horizon TVC.

Our characterization of the TVC also extends to Bewley-type models with occasionally binding borrowing constraints. These borrowing constraints introduce an important caveat, as the standard present value equation becomes an inequality: $P > PV(CF) + TVC$. Asset prices can exceed discounted cash flows, i.e., $P > PV(CF)$, even when TVC is not violated. This inequality offers a different mechanism that elevates asset value above discounted cash flows. Even in these cases, our results still offer a valid characterization of TVC violation, which remains a sufficient, though no longer necessary condition for $P > PV(CF)$. For example, Aiyagari (1994); Huggett (1993); Krueger and Lustig (2010) consider models in which households receive strictly positive labor incomes, which guarantee lower bounds on their wealth levels. Since no households' wealth will become infinitesimally small relative to the population average, the TVC always holds.

**Comparing Asset Bubbles.** Santos and Woodford (1997) develop a general characterization of asset bubbles, with the main result showing "nonexistence of pricing bubbles in an equilibrium with the property that the economy's aggregate endowment has a finite value." Consistent with their result, the bubble we study arises in an economy in which the aggregate endowment has infinite value. On the other hand, we deviate from their focus on assets with positive cash flows. Our analysis allows for cases with potentially negative cash flows, such as a government running persistent deficits. Relaxing this assumption uncovers a new class of bubbles distinct from the examples studied in Santos and Woodford (1997). Therefore, it is useful to briefly discuss the classic bubble types and how they differ from the bubble we study.

First, consider classic rational bubbles in OLG models. These bubbles arise when agents use assets for intertemporal consumption smoothing, which gives value to otherwise intrinsically worthless assets. A canonical example of such bubbles is fiat money.

Second, Hirano and Toda (2025) show that asset bubbles necessarily emerge in models featuring multiple assets or sectors with unbalanced growth. These bubbles typically apply to settings like land in a growing economy or sectors with divergent growth rates.

The bubble we study comprises a third, distinct type. The key driving force is the dispersion in the agents' marginal utility growth, which is shaped by the idiosyncratic risk they face. This

form of bubble is relevant for studying assets whose cash flows serve as good hedges against idiosyncratic risk, such as government debt.

There is an interesting parallel between the second and third types of bubbles. The former relies on divergent growth rates *across sectors*, whereas latter depends on dispersion in wealth accumulation *across agents*. That said, the case we study does not require permanent divergence in wealth, as bubbles can arise even under a stationary wealth distribution.

Despite these similarities, the bubble we study is fundamentally different from the other two types in both mechanism and implications for asset pricing. Internet Appendix E presents simple models for each bubble type to illustrate the following key differences:

*(1) Cash Flow/Price Ratio:* In the first two types of bubbles, this ratio is either zero or vanishing over time. In fact, when cash flows are non-negative, bubbles can arise only if the cash flow/price ratio is zero or vanishing (Montrucchio, 2004; Hirano and Toda, 2025). For instance, money (Type 1 bubble) has no cash flow, and rents have to diminish relative to land prices (Type 2 bubble) over time. In contrast, the bubble we study sustains a non-zero, non-vanishing cash flow/price ratio, which is made possible by *negative* cash flows. For example, the government can run a persistent deficit with a stationary deficit/debt ratio.

*(2) The Transversality Term:* All three bubble types feature a positive present discounted value of the asset price in distant future, violating the standard TVC. However, this value is finite in Types 1 and 2 bubbles, but infinite in the type we study. Specifically, in the first type of bubble, since the asset has no cash flow, the terminal value equals the current price of the bubble asset, which is finite. In the second type, the terminal value measures the difference between the current asset price and the present value of cash flows, both of which are finite. In contrast, the bubble we study has an infinitely positive value, which is balanced by an infinitely negative present value of cash flows to result in a finite asset price.

*(3) Short-Sale Constraint:* The first two bubble types require short-sale constraints on the bubble asset, so that agents cannot profit from selling the overvalued asset to finance higher consumption (Kocherlakota, 1992). For instance, OLG models inherently impose such constraints since agents must liquidate positions at death. The bubble we study, however, does not rely on

such constraints. In our framework, agents can freely trade the bubble asset like government debt without violating equilibrium conditions.

In sum, by characterizing the conditions under which bubbles emerge in heterogeneous-agent economies, our analysis broadens the scope of asset pricing behaviors that can be rationalized by a bubbly economy, and offers a clear framework for future research to assess its empirical relevance.

**Literature Review.** Our paper contributes to the large literature on heterogeneous-agent asset pricing (Grossman and Shiller, 1982; Constantinides and Duffie, 1996; Angeletos, 2007; Toda, 2014; Kogan, Papanikolaou, and Stoffman, 2020; Panageas, 2020; Schmidt, 2025; Gomez, 2025; Marin and Singh, 2025). While this literature primarily examines how idiosyncratic risk shapes finite-horizon risk-return trade-offs, we study its implications for the TVC. Our primary contribution is to develop a *sufficient-statistic* characterization of the TVC in terms of the wealth distribution, applicable to a broad class of heterogeneous-agent models. We show that wealth inequality, which is also central to generating realistic risk premia in these models, can lead to TVC violation. Surprisingly, with a sufficient amount of wealth inequality, TVC violation can even arise under stationary wealth distributions.

Our paper is not about introducing a new modeling ingredient to derive results different from prior analyses. The TVC violation we study arises in standard settings with idiosyncratic risk such as Constantinides and Duffie (1996). In prior works, Brunnermeier et al. (2020, 2022) study such environments and acknowledge the possibility of TVC violation. Their analysis adopts a *dynamic trading perspective*, which focuses on how asset valuation reflects the cash flows accruing to individual investors. Due to individual trading and rebalancing activities, these cash flows do not coincide with the aggregate cash flows paid out by the asset issuer. Using this perspective, they offer important insights into why individuals value government debt. Our approach complements theirs by focusing on a *buy-and-hold perspective*, which prices assets based on the asset issuer's *aggregate cash flows*. We characterize the conditions under which this issuer-level TVC fails, which is central to the present value approach commonly used in financial valuation.

Our analysis parallels the literature on asset bubbles in overlapping-generation (OLG) mod-

els (Samuelson, 1958; Diamond, 1965; Blanchard and Watson, 1982), as well as in more general frameworks (Santos and Woodford, 1997; Hirano and Toda, 2024). Just as Tirole (1985) characterizes conditions under which bubbly equilibrium exists in OLG economies, we characterize conditions under which TVC fails in heterogeneous-agent economies. Our results can be applied to any aggregate assets including government debt, which has been studied by Blanchard (2019); Abel and Panageas (2022); Gârleanu and Panageas (2023); Panageas (2024) in OLG models, and by Jiang et al. (2020); Reis (2021); Mian et al. (2022); Kocherlakota (2025) in models with long-lived agents. Our results also offer a perspective which helps interpret the infinite-horizon buy-and-hold valuation exercise in Jiang et al. (2024b); Chen et al. (2022).

Moreover, Lucas (1992); Atkeson and Lucas (1992) study resource allocation under idiosyncratic shocks. Lucas (1992) notes that permanent idiosyncratic risk generates a diverging wealth distribution, an insight on which our analysis builds. Our contribution is to derive the valuation implications of this divergence, and characterize exactly when and how TVC violation arises. We show that this permanent divergence in wealth distribution is neither sufficient nor necessary to generate TVC violation. What matters instead is sufficient dispersion in marginal utility growth, which can happen even when the wealth distribution is stationary.

Our paper proceeds as follows. Section 2 analyzes the baseline model with permanent idiosyncratic risk. Section 3 introduces transitory idiosyncratic risk to study settings with stationary wealth distributions, and discusses the model's empirical relevance. Section 4 discusses our results' implications for workhorse heterogeneous-agent models. Section 5 concludes.

# 2 Transversality Condition under Permanent Idiosyncratic Risk

## 2.1 Model Set-Up

We study a standard heterogeneous-agent economy with idiosyncratic risk, which nests a broad class of models including Constantinides and Duffie (1996); Brunnermeier, Merkel, and Sannikov (2022). The insight also applies to a wider range of heterogeneous-agent models, such as Grossman and Shiller (1982); Angeletos (2007); Toda (2014); Schmidt (2025), which feature agents facing

more general types of aggregate and idiosyncratic shocks.

We begin by outlining the key equilibrium conditions of the model. The economy consists of a continuum of infinitely lived households indexed by $i \in [0, 1]$, each maximizing expected lifetime utility:

$$\mathbb{E}_0 \left[ \int_0^\infty e^{-\rho t} \log(c_t^i) dt \right].$$

For expositional simplicity, we assume log preference. We allow for more general risk aversion in our calibration and present details in Internet Appendix B.

In equilibrium, individual consumption is subject to both aggregate and idiosyncratic shocks:

$$\frac{dc_t^i}{c_t^i} = g_t dt + \sigma_t dZ_t + \tilde{\sigma}_t d\tilde{Z}_t^i, \tag{1}$$

where $g_t$ denotes the common growth rate, $\sigma_t$ captures the volatility of the aggregate shock $Z_t$, and $\tilde{\sigma}_t$ captures the volatility of the idiosyncratic shock $\tilde{Z}_t^i$.

This consumption process naturally arises from numerous heterogeneous-agent settings. In Internet Appendix A.1, we follow Brunnermeier et al. (2022) to derive it as the endogenous outcome of a model with capital accumulation. Alternatively, it can be obtained from Constantinides and Duffie (1996), which we discuss in Section 4. Our analysis focuses on the implications of this consumption process for asset valuation, which does not rely on specific model details.

Under these assumptions, the marginal utility can be expressed as

$$m_t^i = e^{-\rho t} (c_t^i)^{-1}.$$

Substituting in the law of motion for consumption yields the following dynamics of marginal utility:

$$\frac{dm_t^i}{m_t^i} = -r_t^f dt - \sigma_t dZ_t - \tilde{\sigma}_t d\tilde{Z}_t^i, \tag{2}$$

which implies that the risk-free rate equals

$$r_t^f = \rho + g_t - \sigma_t^2 - \tilde{\sigma}_t^2. \tag{3}$$

The risk-free rate $r_t^f$ reflects the expected growth rate $g_t$ and the variances of aggregate and idiosyncratic risks. Higher volatility strengthens the precautionary saving motive, exerting a downward pressure on the risk-free rate.

## 2.2 Transversality Condition for Individual Wealth

Let $n_t^i$ denote household wealth, which is the market value of its asset holdings. Let $r_t^{w,i}$ denote the return on the household's wealth portfolio. In the full model presented in Appendix A.1, this return is endogenously determined by the household's optimal portfolio choice over financial assets such as equities and bonds. Here, we focus on the more general relationship between consumption, wealth, and asset return:

$$dn_t^i = -c_t^i dt + n_t^i dr_t^{w,i}.$$

Iterating this equation forward, we obtain that the household's wealth is equal to the present discounted value of its future consumption plus a transversality term:

$$m_t^i n_t^i = \mathbb{E}_t \left[ \int_0^\infty m_{t+j}^i c_{t+j}^i dj \right] + \lim_{T \to \infty} \mathbb{E}_t \left[ m_T^i n_T^i \right]. \tag{4}$$

The transversality condition (TVC) for individual wealth is

$$\lim_{T \to \infty} \mathbb{E}_t \left[ m_T^i n_T^i \right] = 0. \tag{5}$$

When this condition holds, wealth can be expressed as the present discounted value of future consumption.

In our setting, this individual-level TVC holds for all households. Given the log preference,

the envelope condition implies a constant consumption-to-wealth ratio $c_t^i/n_t^i = \rho$. The pricing kernel can be expressed as

$$m_t^i = e^{-\rho t}\rho^{-1}(n_t^i)^{-1}. \tag{6}$$

From (2) and (6), we obtain

$$\frac{d(m_t^i n_t^i)}{m_t^i n_t^i} = -[\underbrace{r_t^f}_{\text{risk-free rate}} + \underbrace{\sigma_t^2}_{\text{risk premium for aggregate risk}} + \underbrace{\tilde{\sigma}_t^2}_{\text{risk premium for idiosyncratic risk}} - \underbrace{g_t}_{\text{growth rate}}]dt, \tag{7}$$

which expresses the dynamics of the transversality term $m_t^i n_t^i$ in terms of the discount rate and the growth rate. Plugging in the risk-free rate in (3) yields

$$(r_t^f + \sigma_t^2 + \tilde{\sigma}_t^2) - g_t = \rho > 0. \tag{8}$$

Thus, the appropriate discount rate for individual wealth, which accounts for both aggregate and idiosyncratic risk premia, exceeds the growth rate. As the discounted individual wealth $m_t^i n_t^i$ converges to zero, the household perceives finite wealth, so that it will not demand more consumption goods than available in the economy.

## 2.3 Transversality Condition for Aggregate Wealth

We now turn to the TVC associated with the aggregate wealth, defined as $\bar{n}_t = \int_0^1 n_t^i di$. Its law of motion is given by

$$\frac{d\bar{n}_t}{\bar{n}_t} = g_t dt + \sigma_t dZ_t,$$

which, compared to an individual household's wealth $n_t^i$, averages out the idiosyncratic risk.

Consider a claim that pays out the aggregate wealth $\bar{n}_T$ at some distant future date $T$. Its time-0 valuation from the perspective of household $i$ is $\mathbb{E}_0\left[m_T^i \bar{n}_T\right]$, which captures how much the household is willing to pay in exchange for receiving the aggregate wealth in the future date.

The corresponding aggregate TVC is given by

$$\lim_{T\to\infty} \mathbb{E}_0\left[m_T^i \bar{n}_T\right] = 0. \tag{9}$$

Unlike the individual-level TVC (5), this aggregate-level TVC is not part of any household's optimality conditions. As we will see below, it is instead a condition on the valuation of aggregate assets. We still refer to it as the aggregate-level TVC to highlight its relationship to and difference from the individual-level TVC.

To motivate this condition, consider, for example, how households price government debt. Similar to the individual household's budget constraint (4), the government faces the following standard intertemporal budget condition:

$$m_t^i b_t = \mathbb{E}_t\left[\int_0^\infty m_{t+j}^i s_{t+j} dj\right] + \lim_{T\to\infty} \mathbb{E}_t\left[m_T^i b_T\right], \tag{10}$$

where $b_t$ denotes the market value of government debt and $s_t$ denotes the primary surplus. Let $\vartheta_t = b_t/\bar{n}_t$ denote the government debt-to-aggregate wealth ratio. The government issuer's transversality condition is given by

$$\lim_{T\to\infty} \mathbb{E}_t\left[m_T^i \vartheta_T \bar{n}_T\right].$$

Since $\vartheta_t \in (0,1)$, $0 < \mathbb{E}_t\left[m_T^i \vartheta_T \bar{n}_T\right] < \mathbb{E}_t\left[m_T^i \bar{n}_T\right]$ for all time $T$. So, the TVC (9) for aggregate wealth is a sufficient condition for the TVC for government debt to hold. More generally, for any asset whose cash flow is cointegrated with the size of the economy, the TVC (9) for aggregate wealth provides a key condition for ruling out explosive valuation paths.[1]

Equivalently, we can analyze this TVC from the perspective of the average household. We define the equal-weighted average marginal utility growth as $\bar{m}_t = \int_0^1 (m_t^i/m_0^i) di$, which evolves

[1] As a special case, if the government maintains a constant debt/wealth ratio $\vartheta_t$, then, Eq. (9) is also a necessary condition.

according to

$$\frac{d\bar{m}_t}{\bar{m}_t} = -r_t^f dt - \sigma_t dZ_t.$$

Compared with the individual household's stochastic discount factor (SDF) $m_t^i/m_0^i$, the average SDF $\bar{m}_t$ has the same drift, but does not load on any idiosyncratic risk. Since the aggregate wealth $\bar{n}_T$ is also free of idiosyncratic risk, the two TVCs are identical:

$$\lim_{T\to\infty} \mathbb{E}_0\left[m_T^i \bar{n}_T\right] = 0 \quad \Leftrightarrow \quad \lim_{T\to\infty} \mathbb{E}_0\left[\bar{m}_T \bar{n}_T\right] = 0.$$

For analytical convenience, we focus on the TVC evaluated using the average SDF $\bar{m}_t$.

Unlike the individual-level TVC, this aggregate-level TVC does not necessarily hold. We provide two complementary perspectives to characterize this condition, one based on the degree of inequality in the wealth distribution and the other based on the comparison between the discount rate ($r$) and the growth rate ($g$).

We begin with the wealth distribution perspective. Let $\eta_t^i = n_t^i/\bar{n}_t$ denote the wealth share of individual $i$. By construction, the wealth share averages to 1: $\int_0^1 \eta_t^i di = 1$. The following proposition characterizes the relationship between the aggregate TVC and the wealth share distribution.

**Proposition 1.** *Assuming all households start with equal wealth shares, i.e., $\eta_0^i = 1$, the transversality condition for aggregate wealth can be expressed as*[2]

$$\lim_{T\to\infty} \mathbb{E}_0[\bar{m}_T \bar{n}_T] = \lim_{T\to\infty} e^{-\rho T} \bar{n}_0 \mathbb{E}_0\left[\int_0^1 (\eta_T^i)^{-1} di\right]. \tag{11}$$

The proof is presented in Appendix A.2. Eq. (11) is the central equation of our paper: whether the aggregate TVC holds or not depends on the distribution of the wealth share as summarized by $\int_0^1 (\eta_T^i)^{-1} di$, which is a measure of inequality. A more unequal wealth distribution implies that

[2]More generally, without assuming $\eta_0^i = 1$, the transversality condition is

$$\lim_{T\to\infty} \mathbb{E}_0[\bar{m}_T \bar{n}_T] = \lim_{T\to\infty} e^{-\rho T} \bar{n}_0 \mathbb{E}_0\left[\int_0^1 (\eta_T^i/\eta_0^i)^{-1} di\right].$$

more households expect to have lower wealth shares $\eta_T^i$ and hence higher marginal utilities $(\eta_T^i)^{-1}$.

Consider two extreme cases as examples. If households have identical wealth share, i.e., $\eta_T^i = 1$ for all $i$, then, the TVC simplifies to the representative-agent case: $\lim_{T\to\infty} \mathbb{E}_0[\bar{m}_T \bar{n}_T] = \lim_{T\to\infty} e^{-\rho T}\bar{n}_0$, which always holds as long as the subjective discount rate $\rho$ is positive.

By contrast, if a non-zero mass of households have vanishing wealth shares $\eta_T^i$ as time goes by, then, their marginal utilities $(\eta_T^i)^{-1}$ will become infinitely large. In this case, the right-hand side of Eq. (11) may diverge to infinity.

This integral offers a *diagnostic tool* for general processes of idiosyncratic risk and wealth distribution. In the setting we study here, the idiosyncratic risk is permanent. Each household's wealth share follows a geometric random walk:

$$\frac{d\eta_t^i}{\eta_t^i} = \tilde{\sigma}_t d\tilde{Z}_t^i.$$

As time passes, almost all households become arbitrarily poor, while wealth becomes concentrated in a vanishing measure of rich households. As a result, the wealth distribution becomes increasingly skewed, leading to an exploding amount of inequality.

## 2.4 Calibrated Example

We illustrate this point by simulating the model under a simple calibration. To generate a realistic magnitude of risk premia, we assume that agents have CRRA utility with a risk aversion coefficient of $\gamma = 2$. We also shut down aggregate risk and aggregate growth by setting $\sigma_t = 0$ and $g_t = 0$, allowing us to isolate the role of idiosyncratic risk.[3] Appendix B presents detailed derivations of this extension, and Appendix Table A.1 reports the parameter values used.

Our calibration relies on the following assumptions:

**Assumption 1.** *(a) The idiosyncratic risk has a constant volatility:* $\tilde{\sigma}_t = \tilde{\sigma}$.

[3]While this assumption is not necessary, it implies a constant aggregate wealth $\bar{n}_T$ which further simplifies Eq. (11), so that we can focus on the distribution term. With a general risk aversion coefficient $\gamma$, a non-zero growth rate generates a time-varying $\bar{n}_T^{1-\gamma}$ term. While this term does not affect the distribution of household wealth, it influences the growth rate of marginal utility in the same way as the time discount factor $\rho$.

*(b) All households start with equal wealth shares:* $\eta_0^i = 1$.

These assumptions simplify our analysis but are not essential to our results. The $r - g$ perspective developed in the next subsection characterizes the TVC with general $\tilde{\sigma}_t$ and $\eta_0^i$.

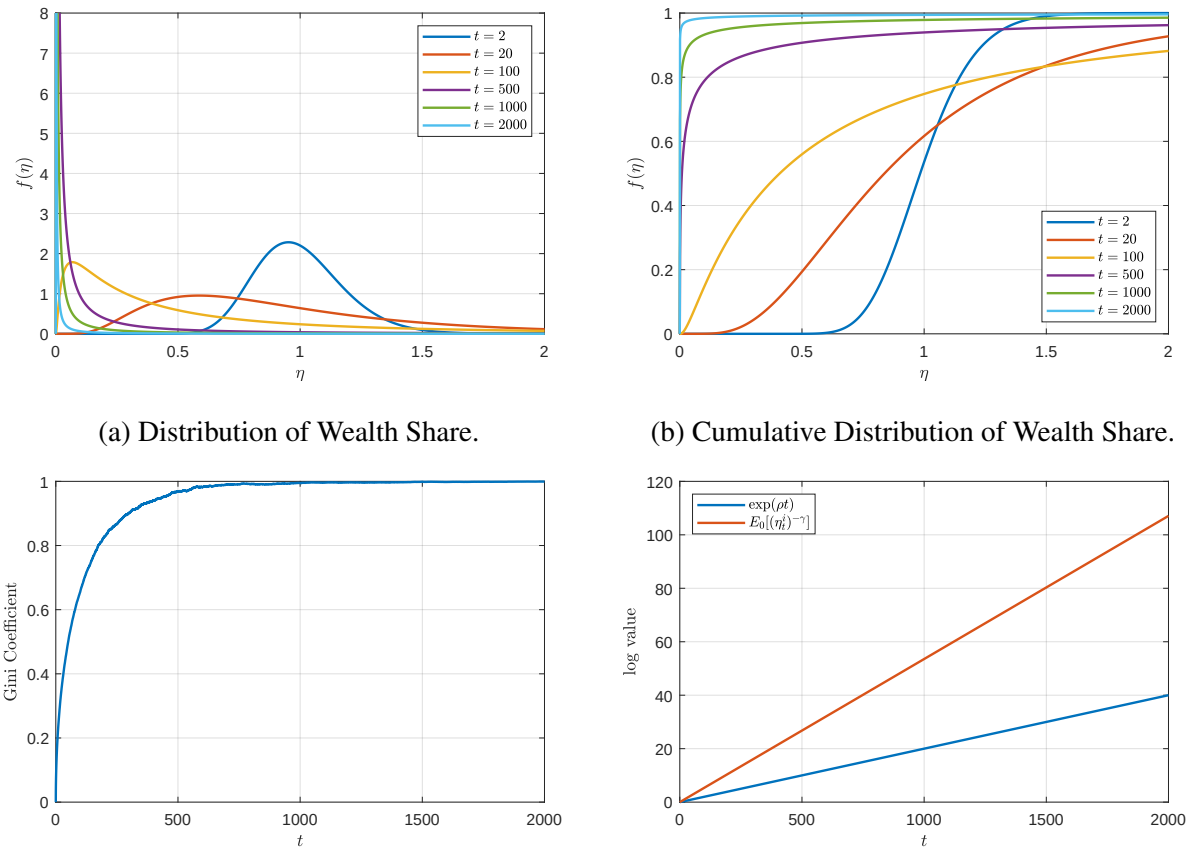
Figure 1 illustrates the evolution of the wealth distribution under these assumptions. Panel (a) plots the cross-sectional distribution of wealth shares $\eta_t^i$ for different time $t$. As time passes, while the average wealth share remains equal to 1, its distribution becomes increasingly skewed. The median and the mode of the wealth distribution shift to the left, indicating that most households' wealth share $\eta_t^i$ approaches zero, while a vanishing share of households accumulate a disproportionate share of wealth. Panel (b) plots the cumulative distribution of the wealth share, highlighting the same pattern of increasing concentration.

Panel (c) plots the Gini coefficient of the wealth distribution, a standard measure of inequality. As the distribution becomes increasingly dispersed, the Gini coefficient approaches one, indicating extreme concentration of wealth.

As almost all households become negligibly small relative to the aggregate wealth $\bar{n}_T$, they assign an infinite valuation to claims on aggregate wealth, or, more generally, any asset whose payoff scales with the size of the economy. As (11) shows, the TVC depends on the relative strength of the dispersion in marginal utility $\int_0^1 (\eta_T^i)^{-1} di$ and the time discounting $\exp(-\rho T)$. If the expected marginal utility grows to infinity faster than the subjective discount factor approaching zero, this transversality term diverges to infinity.

Panel (d) plots these two competing forces. Both the inequality term $\int_0^1 (\eta_T^i)^{-\gamma} di$ and the inverse subjective discount factor $\exp(\rho T)$ approach infinity in the limit. Under our calibration, the magnitude of the idiosyncratic risk is sufficiently high, so that the rise in inequality—reflected in the inequality term—dominates the effect of discounting. This results in an infinite value of the discounted marginal utility growth in the limit.

It is worth noting that this result is not driven by a force which, as a household becomes poorer, increases the magnitude of its idiosyncratic risk and strengthens the precautionary saving motive. In our setting, poor and rich households hold the same portfolio, and agree on the market price of risk. Instead, their difference lies in the size of their wealth. The TVC is violated when




(a) Distribution of Wealth Share.

(b) Cumulative Distribution of Wealth Share.

(c) Gini Coefficient.

(d) TVC for Aggregate Wealth.

**Figure 1:** Dynamics with Permanent Idiosyncratic Risk.

some households become infinitesimally small relative to the size of the aggregate asset, which is different from a mechanism driven by an exploding amount of risk premium.

## 2.5 The $r$ Minus $g$ Perspective

We can also interpret our results through the lens of the familiar "$r$ minus $g$" condition. The discounted aggregate wealth has the following dynamics:

$$\frac{d(\bar{m}_t\bar{n}_t)}{\bar{m}_t\bar{n}_t} = [-\underbrace{r_t^f}_{\text{risk-free rate}} + \underbrace{g_t}_{\text{growth rate}} - \underbrace{\sigma_t^2}_{\text{agg. risk premium}}]dt.$$

In contrast to the discounted individual wealth in (7), the discounted aggregate wealth does not contain the risk premium for idiosyncratic risk, because it is not exposed to such risk. The relevant

$r - g$ condition becomes

$$(r_t^f + \sigma_t^2) - g_t = \rho - \tilde{\sigma}_t^2,$$

which, compared to the condition (8) for individual wealth, is no longer guaranteed to be positive. This condition governs the aggregate TVC, which holds if and only if

$$\lim_{T\to\infty} \mathbb{E}_0 \left[\bar{m}_T \bar{n}_T\right] = \lim_{T\to\infty} \bar{n}_0 \mathbb{E}_0 \left[\exp\left(-\int_0^T \left[\rho - \tilde{\sigma}_t^2\right] dt\right)\right] = 0. \tag{12}$$

Thus, the aggregate TVC is violated when the magnitude of the idiosyncratic risk $\tilde{\sigma}_t^2$ is sufficiently large relative to the subjective discount rate $\rho$. In this case, because inequality explodes fast enough, households lower their discount rate $r$ on the aggregate wealth below its growth rate $g$.

Conversely, if the variance of the permanent idiosyncratic risk is sufficiently low, i.e., $\tilde{\sigma}_t^2 < \rho$, the discount rate $r$ exceeds the growth rate $g$, and the aggregate TVC holds: $\lim_{T\to\infty} \mathbb{E}_0 \left[\bar{m}_T \bar{n}_T\right] = 0$. This conclusion is consistent with the wealth distribution perspective above: in this case, although wealth inequality continues to rise due to the permanent idiosyncratic risk, it does so slowly that the growth in marginal utility is dominated by time discounting.

As a result, the TVC for aggregate wealth is violated only when wealth inequality explodes sufficiently quickly, which drives the discount rate below the growth rate, i.e., $r < g$. Conversely, what the $r - g$ condition captures in this setting is how fast permanent idiosyncratic risk drives the wealth inequality to explode.

## 2.6 What Does It Mean to Violate the TVC for Aggregate Wealth?

When the TVC for aggregate wealth is violated, the government intertemporal budget condition, reproduced below,

$$m_t^i b_t = \mathbb{E}_t \left[\int_0^\infty m_{t+j}^i s_{t+j} dj\right] + \lim_{T\to\infty} \mathbb{E}_t \left[m_T^i b_T\right],$$

has an infinite TVC term $\lim_{T\to\infty} \mathbb{E}_t \left[m_T^i b_T\right] = \infty$. This result has different implications in different settings.

As Brunnermeier et al. (2022) show, the asset value can remain finite when the government runs enough deficits to generate an infinitely negative present value of surpluses, which is just enough to offset the infinitely positive TVC term. In this case, the asset bubble benefits the asset issuer, allowing it to run deficits in perpetuity. In contrast, when the TVC holds and there is no asset bubble, the government issuer must run surpluses to back up a positive value of debt.

In economies like Constantinides and Duffie (1996), when the TVC is violated, the equilibrium commonly studied in that literature does not exist. As a result, finite-period Euler equations and positive risk-free interest rates might not be enough to guarantee agents' optimality conditions. We will discuss the implications of our results for this class of models in Section 4.

# 3 Transversality Condition under Transitory Idiosyncratic Risk

So far, our analysis has focused on the case of *permanent* idiosyncratic risk, and shows it can lead to TVC violation for aggregate assets. As shown in Proposition 1, the key mechanism operates through the left tail of the wealth distribution: permanent idiosyncratic shocks generate an increasingly heavy left tail, causing expected marginal utility growth to diverge over time.

In practice, however, it is more realistic to consider environments where the wealth distribution remains stationary, which requires the idiosyncratic risk to be *transitory*. In this section, we study the implications of transitory idiosyncratic risk for the TVC. Surprisingly, TVC violations can still occur. In fact, standard mechanisms that produce stationary wealth distributions can generate sufficiently heavy left tails. As a result, expected marginal utility can diverge despite the absence of permanent idiosyncratic shocks.

## 3.1 Model Set-Up

We introduce a simple modification to our baseline model to make the idiosyncratic risk transitory. Specifically, each household's wealth $n_t^i$ is reset at random intervals governed by a Poisson process

with intensity $\zeta$. Let $\tilde{J}_t^i$ denote this jump process, which is i.i.d. across households.[4] For simplicity, we assume each household's wealth is always reset to the average level $\bar{n}_t$ upon the shock, which preserves the total amount of wealth in this economy and can thus be interpreted as a form of redistribution. As we discuss below, the key results are robust to more general specifications of post-reset wealth distribution.

In this setting, individual consumption evolves according to

$$\frac{dc_t^i}{c_t^i} = g_t dt + \sigma_t dZ_t + \tilde{\sigma}_t d\tilde{Z}_t^i + \frac{\bar{n}_t - n_t^i}{n_t^i} d\tilde{J}_t^i,$$

which endogenously arises from the household's optimization problem in the presence of wealth reset shocks. We present detailed derivations of this model variant in Appendix A.1.

This consumption process differs from (1) in the baseline model with no wealth reset in three ways. First, exposure to the redistribution risk induces mean reversion in the wealth share $n_t^i$: when the jump $d\tilde{J}_t^i$ occurs, wealth is reset to the average level $\bar{n}_t$.

Second, the household internalizes the redistribution risk in its consumption decision. Since a realization of the reset shock erases accumulated wealth above the average level, the household becomes effectively more impatient and consumes a higher share of its wealth: the consumption-wealth ratio $c_t^i/n_t^i = \rho + \zeta$ is increasing in $\zeta$. Since the household consumes more and saves less, the growth rate in this economy can be different from the baseline model under the same calibration.

Third, the risk-free rate is given by

$$r_t^f = \rho + \zeta + g_t - \sigma_t^2 - \tilde{\sigma}_t^2.$$

Compared to (3) in the baseline model, the risk-free rate increases with the jump intensity $\zeta$, which captures the additional impatience induced by the wealth reset mechanism.

Throughout this section, we focus on the parameter region in which the magnitude of id-

[4]Modeling mean reversion in this way provides a tractable mechanism for generating a stationary wealth distribution. Similarly, Irie (2024) proposes a setting where agents face Poisson shocks that switch their types. By contrast, a proportional wealth tax leads to substantially more complex dynamics, even under log utility.

iosyncratic risk is sufficiently large such that the TVC fails in the absence of any jump risk. This restriction is without loss of generality: if the TVC holds in the absence of jump risk, it will continue to hold for any level of jump intensity, since the reset mechanism can only dampen inequality and compress the dispersion in marginal utility growth.

## 3.2 Transversality Condition for Aggregate Wealth

Next, we revisit the TVC for aggregate assets in the presence of wealth reset. As in Section 2, the TVC can be characterized using either an individual household's SDF or the average SDF. We find it convenient to work with the average SDF, defined as $\bar{m}_t = \int_0^1 (m_t^i/m_0^i)di$. Its law of motion is given by

$$d\bar{m}_t = -[\rho + g_t - \sigma_t^2 - \tilde{\sigma}^2]\bar{m}_t dt - \sigma_t \bar{m}_t dZ_t + \int_0^1 e^{-\rho t}\hat{\rho}^{-1}\left(\bar{n}_t^{-1} - (n_{t-}^i)^{-1}\right)\frac{1}{m_0^i}d\tilde{J}_t^i di.$$

On the right-hand side, the drift term and the $dZ_t$ term are standard as in the baseline model, whereas the last term captures the effect of the wealth reset shock. While this shock is purely redistributive and leaves the aggregate wealth $\bar{n}_t$ unchanged, it affects the average marginal utility $\bar{m}_t$ since individual marginal utility is a non-linear function of wealth.

Our analysis applies to any aggregate assets whose cash flows are unaffected by idiosyncratic risk. Here, we take the government debt as a concrete example. Let $b_t$ denote the aggregate market value of government debt in real terms. We obtain the following result which generalizes the government budget constraint (10):

**Proposition 2.** *In the presence of wealth reset, the intertemporal government budget condition is given by*

$$b_0 = \mathbb{E}_0\left[\int_0^T \bar{m}_t s_t dt\right] + \mathbb{E}_0\left[\bar{m}_T b_T\right] - \zeta\mathbb{E}_0\left[\int_0^T e^{-\rho t}\left(\frac{\bar{n}_t}{\bar{n}_0}\right)^{-1} b_t dt\right].$$

The proof is presented in Appendix A.5. This expression has a natural interpretation from the perspective of the average household: government debt is valued based on (i) future primary

surpluses (the first term on the right-hand side), (ii) the standard transversality term (the second term), and (iii) an additional term capturing the effect of wealth reset.

To see why, note that when a reset occurs, a household's wealth is replaced by the average wealth level regardless of its prior holdings. Therefore, the asset value does not fully accrue to the current holder's benefit. The final term captures this expected loss, which equals the jump intensity $\zeta$ times the present value of asset holdings. This loss of value reduces the asset value below the sum of the present value of cash flows and the terminal value.

Since the market value of government debt is bounded by the aggregate wealth, i.e., $b_t < \bar{n}_t$, the last term is finite. Proposition 2 implies

$$b_0 \leq \mathbb{E}_0 \left[ \int_0^T \bar{m}_t s_t dt \right] + \mathbb{E}_0 \left[ \bar{m}_T \bar{n}_T \right],$$

which shows that the discounted aggregate wealth $\bar{m}_t \bar{n}_t$ again arises as the key object governing the TVC for aggregate assets. In what follows, we characterize its behavior from the perspectives of wealth distribution and the $r - g$ condition.

We begin with the wealth distribution perspective. The following proposition characterizes the stationary wealth distribution that the wealth distribution tends to in the limit.

**Proposition 3.** *Starting from any initial distribution, the wealth distribution converges to the distribution with density function $f$ that satisfies*

$$f(\eta) = \begin{cases} C\eta^{-\frac{3}{2}+\frac{\sqrt{1+\frac{8\zeta}{\tilde{\sigma}^2}}}{2}}, & 0 < \eta < 1, \\ C\eta^{-\frac{3}{2}-\frac{\sqrt{1+\frac{8\zeta}{\tilde{\sigma}^2}}}{2}}, & \eta \geq 1, \end{cases} \tag{13}$$

*where $C$ is a constant that ensures that the distribution integrates to one.*

The proof is presented in the Appendix A.6. Even though this distribution is obtained by introducing wealth reset to an expanding log-normal process, the limiting stationary distribution is not log-normal, but rather follows a power-law distribution.[5] Eq. (13) shows that a higher

[5]This feature is consistent with the more general cases studied by Reed (2001); Toda (2014). The "power law on both tails" is a common feature in models with a constant probability of birth and death.

idiosyncratic volatility $\tilde{\sigma}^2$ leads to heavier tails, while a higher jump intensity $\zeta$ leads to lighter tails.

We again consider a general risk aversion coefficient $\gamma$. Then, the stationary density function (13) has two distinct regimes depending on the relative strength of mean reversion and idiosyncratic risk. If

$$\zeta > \frac{\gamma(1+\gamma)}{2}\tilde{\sigma}^2, \tag{14}$$

i.e., the jump intensity is sufficiently high relative to the magnitude of the idiosyncratic risk, then, the left tail of the wealth distribution is light enough to ensure a finite transversality term:

$$\lim_{T\to\infty} \mathbb{E}_0 \left[ \int_0^1 (\eta_T^i)^{-\gamma} di \right] < \infty.$$

In this case, the TVC for aggregate wealth holds.

Conversely, when (14) fails, i.e., $\zeta \leq \frac{\gamma(1+\gamma)}{2}\tilde{\sigma}^2$, the jump intensity is overshadowed by the magnitude of the idiosyncratic risk. Although the limiting wealth distribution remains stationary, its left tail is sufficiently heavy to generate an infinite $\lim_{T\to\infty} \mathbb{E}_0 \left[ \int_0^1 (\eta_T^i)^{-\gamma} di \right]$, which, as we show below, may lead to TVC violation.

For simplicity, we maintain the assumption that households begin with identical wealth shares: $\eta_0^i = 1$. Then, the TVC (11) in Proposition 1, reproduced below with a general risk aversion coefficient $\gamma$, also holds in this setting:

$$\lim_{T\to\infty} \mathbb{E}_0[\bar{m}_T \bar{n}_T] = \lim_{T\to\infty} e^{-\rho T} \left( \frac{\bar{n}_T}{\bar{n}_0} \right)^{1-\gamma} \mathbb{E}_0 \left[ \int_0^1 (\eta_T^i)^{-\gamma} di \right]. \tag{15}$$

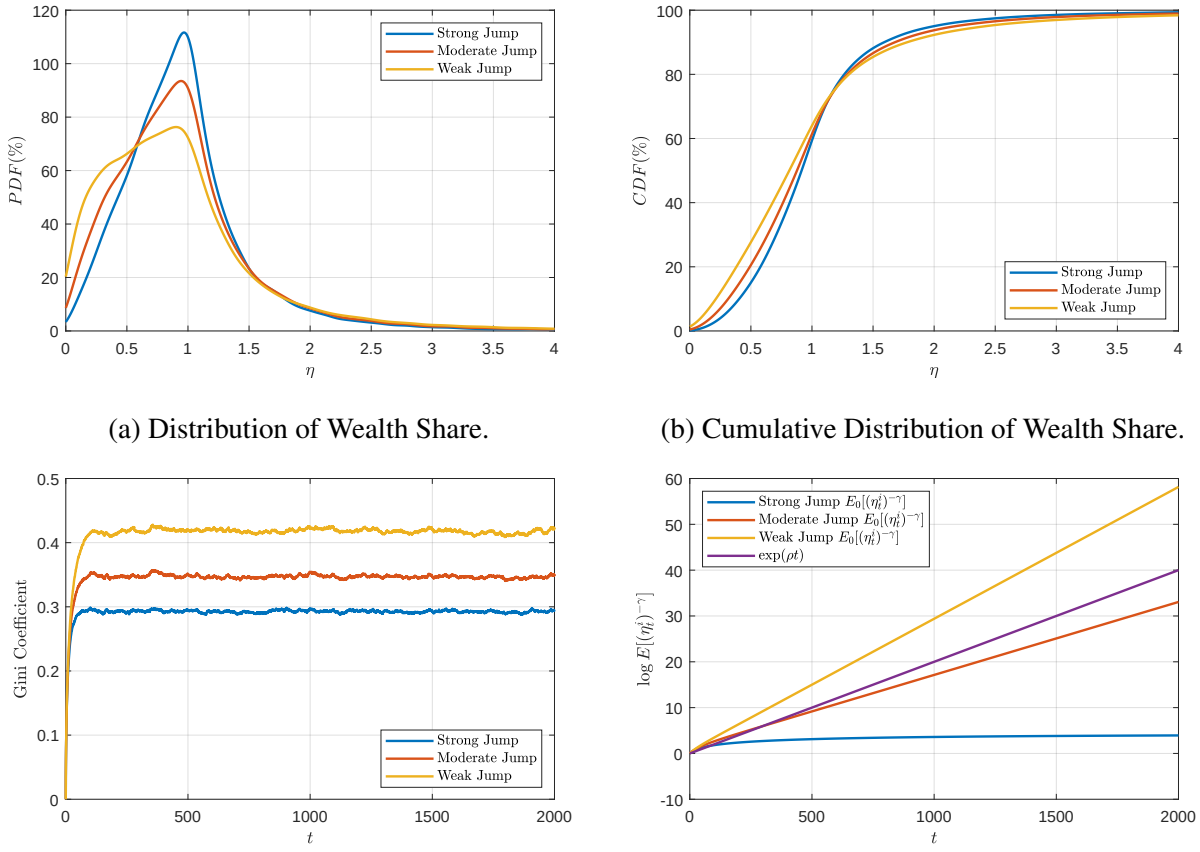
As in the baseline case with permanent idiosyncratic risk, whether the TVC holds for the aggregate wealth depends on how the integral of inverse wealth share $\int_0^1 (\eta_T^i)^{-\gamma} di$ evolves over time and compares to the decay of time discount $e^{-\rho T}$. The key difference is that transitory idiosyncratic risk induces mean-reverting wealth dynamics, whereas permanent idiosyncratic risk generates ever-increasing dispersion in the wealth distribution $\eta_T^i$.

### 3.3 Calibrated Example

We consider three calibrations to illustrate these cases: a case with a high jump intensity $\zeta = 5.4\%$, a case with an intermediate jump intensity $\zeta = 3.8\%$, and a case with a low jump intensity $\zeta = 2.5\%$. The expected marginal utility growth $\lim_{T\to\infty} \mathbb{E}_0 \left[\int_0^1 (\eta_T^i)^{-\gamma} di\right]$ is finite in the first case, and infinite in the second and third cases. All other variables are identical to the previous baseline case. If we set the jump intensity $\zeta$ to zero, we would recover the case illustrated in Figure 1.

In all three cases, households start with identical wealth shares at $t = 0$. The wealth distribution quickly converges to the stationary distribution given by (13). Figure 2 illustrates the properties of these three cases.

Panel (a) plots the limiting stationary distribution of wealth share $\eta_t^i$. The case with a higher




(a) Distribution of Wealth Share.

(b) Cumulative Distribution of Wealth Share.

(c) Gini Coefficient.

(d) TVC for Aggregate Wealth.

**Figure 2:** Dynamics with Transitory Idiosyncratic Risk

jump intensity has a greater mass around the mean and a thinner left tail, while the case with a lower jump intensity has a heavier left tail. In all three cases, the left tail of the wealth distribution declines with a power function. This tail is heavier than the one implied by a log-normal distribution which guarantees a finite $\mathbb{E}_0\left[\int_0^1(\eta_T^i)^{-\gamma}di\right]$.

Panel (b) plots the cumulative distribution of the wealth share $\eta_t^i$. Both panels show that there is no qualitative difference between the distributions obtained in these three cases, as they belong to the same family of distributions with slightly different jump intensity $\zeta$.

Panel (c) plots the evolution of the Gini coefficient. Consistent with the convergent behavior of the wealth distribution, the Gini coefficient also stabilizes quickly and takes similar values across the three cases, ranging from 0.3 to 0.4. Thus, standard inequality measures do not distinguish sharply between these cases.

Finally, panel (d) reports the average path of marginal utility growth in Eq. (15), which determines the TVC for aggregate wealth. We break down this path into two components: the integral of inverse wealth share $\mathbb{E}_0\left[\int_0^1(\eta_T^i)^{-\gamma}di\right]$ and the subjective discount factor $\exp(\rho T)$. The discounted marginal utility growth is given by the ratio between these two components. In contrast to the previous panels in this figure, this decomposition reveals substantial differences across the three cases.

With high jump intensity (blue), the integral of inverse wealth share $\mathbb{E}_0\left[\int_0^1(\eta_T^i)^{-\gamma}di\right]$ converges to a finite value. By contrast, with lower (yellow) or intermediate (red) jump intensity, this integral diverges to infinity, reflecting a sufficiently heavy left tail and large downside risk in individual consumption paths. This leads to an infinite expected value of the marginal utility growth in the limit.

Comparing these values of expected marginal utility growth to the subjective discount factor $\exp(\rho T)$ yields further differentiation. With a high jump intensity (blue), the expected value of marginal utility growth is finite while the subjective discount factor grows exponentially, which guarantees a vanishing TVC term. With intermediate jump intensity (red), while the expected marginal utility growth diverges to infinity, it does so at a slower rate than the subjective discount factor. As a result, the TVC still holds. With a low jump intensity (yellow), the expected marginal

utility growth diverges to infinity at a faster rate than the subjective discount factor, leading to a violation of the TVC for aggregate wealth.

Table 1(a) summarizes these different cases. With high jump intensity, the TVC holds and $\mathbb{E}_0\left[\int_0^1(\eta_T^i)^{-\gamma}di\right]$ converges to a finite value. With low jump intensity, the TVC is violated and $\mathbb{E}_0\left[\int_0^1(\eta_T^i)^{-\gamma}di\right]$ diverges to infinity. With an intermediate jump intensity, while $\mathbb{E}_0\left[\int_0^1(\eta_T^i)^{-\gamma}di\right]$ diverges to infinity, the TVC still holds.

These results underscore a key insight: despite the visual similarities of the wealth distribution in these three cases, small differences in the thickness of the left tail have first-order implications for asset valuation. According to Eq. (15), the TVC for aggregate wealth is violated only when the left tail is heavy enough to make the expected marginal utility growth diverge faster than the rate of time discounting.

Table 1(b) reports the corresponding characterizations for the model without wealth reset, which we studied in the previous section. In these cases, idiosyncratic risk is permanent and $\mathbb{E}_0\left[\int_0^1(\eta_T^i)^{-\gamma}di\right]$ always diverges to infinity. However, the TVC still holds if the idiosyncratic risk is sufficiently small, so that time discounting dominates the growth in marginal utility.

**Table 1:** Summary of the TVC Results for Aggregate Wealth.

| *(a) With Wealth Reset, Transitory Idiosyncratic Risk* | | | | |
|---|---|---|---|---|
| Jump Intensity | Parameter Region | TVC holds? | $\mathbb{E}[\int(\eta_\infty^i)^{-\gamma}di]=\infty$? | $r>g$? |
| High | $\hat{c}>\gamma\tilde{\sigma}^2+\rho+(\gamma-1)g$ | Yes | No | Yes |
| Moderate | $\gamma\tilde{\sigma}^2<\hat{c}\leq\bar{\gamma}\bar{\sigma}^2+\rho+(\gamma-1)g$ | Yes | Yes | Yes |
| Low | $0<\hat{c}\leq\gamma\tilde{\sigma}^2$ | No | Yes | No |
| *(b) Without Wealth Reset, Permanent Idiosyncratic Risk* | | | | |
| Idiosyncratic Volatility | Parameter Region | TVC holds? | $\mathbb{E}[\int(\eta_\infty^i)^{-\gamma}di]=\infty$? | $r>g$? |
| Low | $\hat{c}>\gamma\tilde{\sigma}^2$ | Yes | Yes | Yes |
| High | $\hat{c}\leq\gamma\tilde{\sigma}^2$ | No | Yes | No |

*Notes:* $\hat{c}$ is the consumption-wealth ratio for the general case with wealth reset and power utility. Under log utility, $\hat{c}=\rho+\zeta$.

## 3.4 The $r$ Minus $g$ Perspective

Next, we turn to the $r-g$ perspective, which allows us to obtain more general results characterizing the dynamics of the TVC term $\bar{m}_t\bar{n}_t$ without relying on Assumption 1 and the specific wealth distribution.

**Proposition 4.** *The TVC term for aggregate wealth has the following dynamics:*

$$d[\bar{m}_t\bar{n}_t] = -\left[\rho + \zeta - \tilde{\sigma}_t^2\right]\bar{m}_t\bar{n}_t dt + \zeta e^{-\rho t}\bar{n}_0 dt.$$

*If $\tilde{\sigma}_t = \tilde{\sigma}$ is constant, the aggregate TVC holds if and only if*

$$\rho + \zeta > \tilde{\sigma}^2. \tag{16}$$

The proof is presented in Appendix A.7. This proposition shows that the sign of $\rho + \zeta - \tilde{\sigma}^2$ plays a crucial role in determining the TVC for aggregate wealth. In fact, as

$$(r_t^f + \sigma^2) - g_t = \rho + \zeta - \tilde{\sigma}^2,$$

this proposition shows that the relevant condition for the TVC is again "$r - g > 0$." To satisfy the TVC for aggregate wealth, we need either a small enough volatility $\tilde{\sigma}$ of the idiosyncratic risk, or a high enough jump risk $\zeta$ so that the households' wealth frequently reverts to the mean, or a high enough subjective discount rate $\rho$.

Table 1(a) also reports the $r - g$ condition in the last column. Eq. (16) is satisfied when the jump intensity is high or moderate, which ensures that $r > g$ and the TVC holds. In contrast, with a low jump intensity, the marginal utility growth explodes fast enough to dominate the subjective discount rate in (15), which leads to TVC violation.

The results we obtain from the $r - g$ perspective are more general than the previous results from the wealth distribution perspective, which assumes a specific post-reset wealth distribution. In Internet Appendix B.3, we impose an arbitrary wealth distribution for households after experiencing the wealth reset, including a log-normal one. In this case, while the stationary wealth

distribution no longer takes the form of Eq. (13), Proposition 4 from the $r - g$ perspective still holds.

### 3.5 Comparison to U.S. Inequality Data

The results in this section show that a stationary wealth distribution does not guarantee the TVC for aggregate assets. TVC violation can still occur when the left tail of the stationary wealth distribution is heavy enough, so that the households expect to experience a very high marginal utility upon negative realizations of idiosyncratic shock. In this case, the integral of inverse wealth share, which measures the degree of inequality in the wealth distribution, can diverge to infinity just like the case with permanent idiosyncratic risk.

A natural question is whether the wealth distribution implied by cases with TVC violation is empirically plausible. If the wealth distribution needs to be incredibly skewed to generate TVC violation, then TVC violation is merely a theoretical curiosity.

Figure 3 compares our model variants with the U.S. data. We use the Gini coefficient based on income data to measure the degree of inequality in the U.S.[6] From 1963 to 2022, the Gini coefficient has remained stable, fluctuating around 0.4. In the baseline model with permanent idiosyncratic risk, if we start with the same degree of inequality, the wealth distribution will diverge over time. Under our calibration, which is motivated by the empirical moments considered in Brunnermeier et al. (2022), we expect the Gini coefficient to rise to 0.6 around 2020 and approach 0.7 around 2050.

In contrast, models with transitory idiosyncratic risk imply a stable Gini coefficient. We again consider the three cases with different values of jump intensity $\zeta$. Both the case with moderate jump intensity and the case with low jump intensity yield a Gini coefficient around 0.4, even though the TVC holds in the former case and fails in the latter.

Thus, we obtain a possibility result: we do not need an extremely skewed wealth distribution

[6]The wealth in our model encompasses not only financial wealth but also human capital. Greenwald et al. (2021)'s estimates show that the Gini for financial wealth is approximately 0.74 in the U.S., whereas the adjusted total wealth Gini is 0.41, which aligns closely with the figures used here. 0.4 is also close to the Gini index for consumption, which is relevant in marginal utility dispersion.

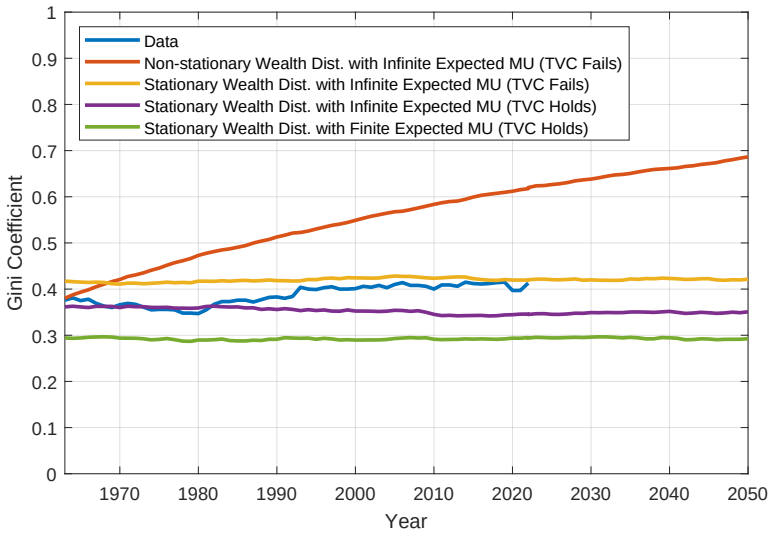



**Figure 3:** Gini Coefficient of Consumption Inequality.

to generate TVC violation, as long as households have reasonably high risk aversion ($\gamma = 2$ in our calibration). In this case, the market value of long-lived assets may exceed the discounted present value of its cash flows.

That said, two caveats must be noted. First, the Gini coefficient is not the only relevant metric for evaluating the degree of inequality in the wealth distribution. TVC violation relies on households expecting sufficiently large dispersions in marginal utility growth, which makes it important to examine household beliefs about wealth dynamics. We leave this analysis to future work.

Second, and relatedly, TVC violation in our setting relies on households' marginal utility being unbounded—that is, the Inada condition plays an important role. If, for example, the households' marginal utility remains finite when their consumption approaches zero, the TVC holds even when the wealth distribution has similar statistics.

## 3.6 Summary

Our analysis of permanent and transitory idiosyncratic risk yields several insights. First, a non-stationary wealth distribution may generate a violation of TVC for aggregate assets, but it depends on the rate of divergence. The TVC can still hold if the wealth distribution diverges more slowly than the subjective discount factor. Only with sufficiently rapid divergence is the TVC violated.

Second, a stationary wealth distribution is not sufficient to ensure the aggregate TVC. The left tail is critical: if it is sufficiently heavy, the TVC can still be violated. In such cases, households anticipate their wealth shrinking to a negligible fraction of the economy for long enough time, and therefore perceive aggregate assets to be infinitely valuable.

Third, the condition $r - g > 0$ remains both necessary and sufficient for the TVC to hold. In our setting, this gap reflects the relative magnitude of the idiosyncratic risk and the subjective discount factor.

Fourth, with a plausible level of risk aversion, the left-tail risk required for TVC violations may be consistent with the observed Gini coefficient in post-war U.S. data. To evaluate the possibility of asset bubbles, we need to look further into the left tail of the wealth distribution.

## 3.7 Transversality Condition under Stochastic Death

Our characterizations of the TVC continue to hold in a model where households face a constant probability of death, denoted by $\zeta$. The mortality risk increases households' effective discount rate, making them behave more impatiently. Aside from this adjustment, asset valuation continues to depend on the distribution of marginal utility growth conditional on survival. If the wealth and consumption distributions are sufficiently left-skewed, the TVC can still fail. Thus, even in the presence of mortality risk, our main result holds: the validity of the aggregate TVC hinges on the thickness of the left tail of the wealth distribution.

Internet Appendix B.4 formalizes this argument in a continuous-time setting that extends our baseline model. Households face a constant death rate $\zeta$, while a fraction $\zeta$ of new households are born and receive the wealth of the deceased households. This ensures that aggregate wealth and financial assets are preserved. We consider two plausible cases.

First, suppose each newborn household directly inherits the wealth of a specific deceased household. In this case, the model is equivalent to the baseline setting with permanent idiosyncratic risk, as discussed in Section 2. The wealth distribution has an explosive degree of inequality, and the aggregate TVC may be violated if idiosyncratic risk is sufficiently large. The only difference is that the subjective discount rate $\rho$ needs to be augmented by the death rate $\zeta$ to reflect

households' impatience. The necessary and sufficient condition for the aggregate TVC to hold is

$$\rho + \zeta - \tilde{\sigma}_t^2 > 0, \tag{17}$$

which compares the effective discount rate $\rho + \zeta$ to the magnitude of idiosyncratic risk $\tilde{\sigma}_t^2$.

Second, suppose the wealth of deceased households is pooled and redistributed equally among the newborns. This setup corresponds to the model with wealth reset shocks described in Section 3. The wealth distribution becomes stationary, but the aggregate TVC remains sensitive to the left tail of the wealth distribution. The TVC condition is again given by Eq. (17), which ensures that the left tail of the stationary wealth distribution is not too heavy. Conversely, if this condition fails, TVC violation occurs even when the wealth distribution is stationary.

This discussion shows that introducing stochastic death does not simply fix TVC violation for aggregate assets. Whether the TVC holds or not continues to hinge on the degree of wealth inequality, or, more precisely, the balance between the effective subjective discount factor and the magnitude of idiosyncratic risk. While death makes households more impatient and thus makes TVC violations less likely to happen, it does not fundamentally alter the mechanism by which left-tail risk generates such violations.

# 4 Implications for Theories of Heterogeneous-Agent Economies

Our results identify wealth inequality as a central determinant of TVC in heterogeneous-agent economies. In this section, we apply this insight to workhorse asset pricing models, which provide good examples to illustrate the implications of our analysis for asset valuation.

## 4.1 Constantinides-Duffie

We first consider the class of idiosyncratic risk models pioneered by Constantinides and Duffie (1996) and extended by many others. This economy features heterogeneous households facing idiosyncratic risk and admits a no-trade equilibrium. In this framework, while households can

freely trade financial assets such as equities and bonds, their labor income is subject to permanent idiosyncratic shocks. These shocks generate a gap between individual and aggregate consumption growth. In equilibrium, the consumption growth of household $i$ is given by

$$\frac{c_{t+1}^i}{c_t^i} = \frac{c_{t+1}}{c_t} \exp\left(-\frac{1}{2}\tilde{\sigma}_{t+1}^2 + \tilde{\sigma}_{t+1}\tilde{\varepsilon}_{t+1}^i\right),$$

where $c_{t+1}/c_t$ is the aggregate consumption growth, and $\varepsilon_{t+1}^i$ is an i.i.d. idiosyncratic shock. Full model details are presented in Appendix C.

Let $\eta_t^i = c_t^i/c_t$ denote the household's consumption share. Assuming CRRA utility, the household's one-period pricing kernel is

$$m_{t,t+1}^i = e^{-\rho}\left(\frac{c_{t+1}^i}{c_t^i}\right)^{-\gamma} = e^{-\rho}\left(\frac{c_{t+1}}{c_t}\right)^{-\gamma}\left(\frac{\eta_{t+1}^i}{\eta_t^i}\right)^{-\gamma}.$$

Relative to the representative-agent benchmark, the additional term $\left(\frac{\eta_{t+1}^i}{\eta_t^i}\right)^{-\gamma}$ introduces extra variation in the pricing kernel, which can help generate more realistic risk premia. A large subsequent literature builds on this mechanism by specifying idiosyncratic risk processes that better match key asset pricing moments.

Since the idiosyncratic shocks are permanent, the consumption share $\eta_t^i$ follows a random walk, leading to an exploding degree of inequality in the limit. This setting closely parallels our baseline model in Section 2, in which permanent idiosyncratic shocks drive most households' consumption shares to zero in the limit. If the magnitude of permanent idiosyncratic risk is sufficiently large relative to time discount, the TVC can fail. Thus, instead of imposing the TVC as an assumption, it is better to understand it as an endogenous outcome, which ultimately depends on the degree of idiosyncratic risk and the resulting wealth distribution.

**Asset Pricing Example.** To illustrate this point, consider any long-lived asset that pays a non-negative dividend stream $d_t$. We would like to construct a case where the one-period Euler equation

holds,

$$\mathbb{E}_t[m^i_{t,t+1} r_{t+1}] = 1,$$

yet the TVC fails, so that the discounted value of the dividend stream approaches infinity:

$$\sum_{t=1}^{\infty} \mathbb{E}_0 \left[ m^i_{0,t} d_t \right] = \infty.$$

For concreteness, suppose this asset is the claim to the aggregate consumption $c_t$. The argument readily extends to assets whose cash flows are not subject to idiosyncratic risk. For simplicity, assume constant aggregate growth $g_t = g > 0$, constant idiosyncratic risk volatility $\tilde{\sigma}_t = \tilde{\sigma} > 0$, and assume households have log utility: $\gamma = 1$. The pricing kernel from time $0$ to $t$ is

$$m^i_{0,t} = \exp(-\rho t) \left( \frac{c^i_t}{c^i_0} \right)^{-1} = \exp(-\rho t) \left( \frac{c_t}{c_0} \right)^{-1} \exp\left( \frac{t}{2} \tilde{\sigma}^2 - \tilde{\sigma} \sum_{k=1}^{t} \tilde{\varepsilon}^i_k \right).$$

The present value of the consumption strip at horizon $t$ is

$$\mathbb{E}_0[m^i_{0,t} c_t] = \mathbb{E}_0 \left[ \exp(-\rho t) \left( \frac{c_t}{c_0} \right)^{-1} \exp\left( \frac{t}{2} \tilde{\sigma}^2 - \tilde{\sigma} \sum_{k=1}^{t} \tilde{\varepsilon}^i_k \right) c_t \right] = c_0 \exp((\tilde{\sigma}^2 - \rho) t).$$

When the idiosyncratic risk is sufficiently large such that $\tilde{\sigma}^2 > \rho$, the present value of the consumption strip grows exponentially, leading to an infinite sum of discounted cash flows, i.e., $\sum_{t=1}^{\infty} \mathbb{E}_0[m^i_{0,t} c_t] = \infty$. Therefore, the consumption claim will be infinitely valuable, and the no-trade equilibrium breaks down.[7]

This example illustrates why the one-period risk-return relationship and the TVC contain distinct information about asset valuation. Correctly pricing one-period returns alone is not sufficient to ensure well-defined asset prices in heterogeneous-agent models. A complete characterization

[7] Constantinides and Duffie (1996) impose the condition $\lim_{T\to\infty} \mathbb{E}[m^i_{0,T}] = 0$ (their Eq. (5)). This condition is not sufficient to rule out TVC violation. In our example, if $\rho < \tilde{\sigma}^2 < \rho + g$, the pricing kernel satisfies

$$\lim_{T\to\infty} \mathbb{E}_0[m^i_{0,T}] = \lim_{T\to\infty} \exp((-\rho - g + \tilde{\sigma}^2) t) = 0,$$

so that while the risk-free rate is positive, the TVC still fails.

requires verifying the TVC in the presence of persistent heterogeneity and idiosyncratic risk.

**Transitory Idiosyncratic Risk.** In their appendix, Constantinides and Duffie (1996) also explore a model variant that incorporates wealth reset, which resembles our model variant with a stationary wealth distribution. As we show in Section 3, the TVC can still fail if the left tail of the stationary wealth distribution is sufficiently heavy. Therefore, the TVC cannot be taken for granted even in stationary economies.

## 4.2 Aiyagari-Bewley-Huggett Models

Since Bewley (1986); Huggett (1993); Aiyagari (1994), it has been well understood that idiosyncratic risk can depress risk-free discount rates. Given this intuition, it may seem unsurprising that risk-free rates can be sufficiently low to support asset bubbles.

However, the settings that give rise to our results differ from these classic environments in two important ways. The first difference concerns the income process, and the second difference is related to the household borrowing constraint.

**Lower Bound on Income.** In Huggett (1993); Aiyagari (1994) and Krueger and Lustig (2010), the household income is specified by a Markov chain process, which is bounded from below by a strictly positive value. In Internet Appendix D.2, we show that, if the household endowment $y_t^i$ is bounded away from zero, i.e., $y_t^i \geq \underline{y} > 0$ for all households and states, the household's consumption share is also above a strictly positive value. This, in turn, implies that the marginal utility of consumption cannot become infinite, even in the limit.

Our Proposition 1 shows that asset bubbles are driven by a large degree of dispersion in the distribution of the consumption share, i.e., $\mathbb{E}_0\left[\int_0^1 (\eta_T^i)^{-1} di\right]$. When the consumption share has a strictly positive lower bound, the integral of the inverse consumption share $\int_0^1 (\eta_T^i)^{-1} di$ is finite, which prevents the marginal utility from exploding.

This property rules out the possibility of TVC violation. As a result, these classic settings with idiosyncratic risk represent a specific family of wealth distributions, in which the left tail is

truncated away from zero by a strictly positive lower bound on income. In comparison, asset pricing settings such as Constantinides and Duffie (1996); Brunnermeier et al. (2022) allow household consumption share to become arbitrarily small. It is this distinction that underlies the possibility of asset bubbles in the latter class of models that we study in this paper.

**Borrowing Constraint.** In addition, each household's asset position $b_t^i$ is usually subject to a lower bound:

$$b_t^i \geq q_t^i.$$

When $q_t^i = 0$, this corresponds to a no-short-selling constraint. More generally, $q_t^i < 0$ captures borrowing limits that allow households to take on debt up to a certain leverage level.

The borrowing constraint turns the asset issuer's intertemporal budget constraint into an inequality rather than an equality. When the constraint binds, households would like to shift consumption from the future to the present, but are unable to do so through borrowing. In the standard discrete-time formulation, the Euler equation becomes

$$m_t^i \geq \mathbb{E}_t \left[ m_{t+1}^i (1+r) \right],$$

with strict inequality when the constraint is binding.

As a result, the government's intertemporal budget condition (10) also becomes an inequality:

$$b_t \geq \lim_{T\to\infty} \mathbb{E}_t \left[ \sum_{k=1}^{T} \frac{m_{t+k}^i}{m_t^i} s_{t+k} \right] + \lim_{T\to\infty} \mathbb{E}_t \left[ \frac{m_T^i}{m_t^i} b_T \right]. \tag{18}$$

The proof is presented in Internet Appendix D.1. This inequality implies that the market value of government debt $b_t$ can exceed the present value of expected future surpluses even when the TVC holds. In other words, the present value of surpluses provides only a lower bound on debt valuation. Thus, while TVC violations for long-lived assets are sufficient to generate $P > PV(CF)$, they

are not necessary.[8] That said, our characterizations of the TVC term remain applicable to these settings.

### 4.3 Brunnermeier-Merkel-Sannikov

Our results offer a complementary view to Brunnermeier et al. (2022) on the intertemporal government budget constraint. As discussed in the literature review, they offer a *dynamic trading perspective*, which focuses on individual agents' wealth dynamics and portfolio holdings. Since the TVC for individual wealth always holds, the asset value always coincides with the discounted present value of cash flows that individual agents obtain from their trading activities.

Our analysis does not contradict this view. In contrast, we focus on the *buy-and-hold perspective*, which evaluates the aggregate budget from the asset issuer's stand point. This perspective is commonly used in standard present value approaches, which evaluate assets based on the issuer-level aggregate cash flows, making it essential to understand whether and when the TVC holds at this aggregate level.

Our results provide a general framework to evaluate this aggregate TVC, as well as concrete examples in which this condition fails. We show that the validity of the aggregate TVC hinges on the shape of the wealth distribution. When the left tail is sufficiently heavy, the TVC for aggregate assets can fail, and asset prices may diverge from the present value of expected cash flows.

## 5 Conclusion

This paper characterizes the conditions under which idiosyncratic risk gives rise to asset bubbles for long-lived assets. We find that the degree of wealth inequality is the key to this question. Not only do non-stationary wealth distributions with explosive inequality generate asset bubbles, but stationary wealth distributions with sufficiently heavy left tails can violate TVC as well.

[8]The Euler equation may become an inequality even in the absence of truly idiosyncratic risk, as in Townsend (1980). Internet Appendix D.3 provides a concrete example of this mechanism. Moreover, belief distortion can also lead to $P > PV(CF)$ under econometricians' information set, even when the $TVC$ holds from the households' perspective (Jiang et al., 2024a).

These results offer a new perspective on the valuation of long-lived assets in heterogeneous-agent models. For example, the TVC may fail in settings considered by Constantinides and Duffie (1996) and Brunnermeier et al. (2022), where the wealth distribution explodes over time. In contrast, the TVC holds in settings considered by Aiyagari (1994); Bewley (1986); Huggett (1993), if any agent's wealth share is above a strictly positive value. Further empirical works can help distinguish between these cases in the data.

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# A Model Details

In this Appendix, we provide derivation details and additional explanations for models in Section 3. Note that the permanent risk model in Section 2 can be viewed as a special case of the transitory risk model in Section 3 with wealth reset intensity $\zeta = 0$.

Throughout the appendix, for any jump-diffusion process

$$X_t = X_0 + \int_0^t \mu_{s-}^X ds + \int_0^t \sigma_{s-}^X dZ_s + \sum_{k=1}^{N(t)} \Delta X_k,$$

we use $Z_t$ to denote a standard Brownian motion, $N(t)$ to denote a counting process, $\mathbb{E}_t[dX_t]/dt$ to denote $\mu_{t-}^X$, and $\langle dX_t, dX_t\rangle/dt$ to denote $(\sigma_{t-}^X)^2$.

## A.1 Model Derivation

### A.1.1 Set-up

**Households.** Household's lifetime utility is given by

$$V_0^i = \mathbb{E}\left[\int_0^\infty e^{-\rho t} \log c_t^i dt\right].$$

The capital evolution follows

$$\frac{d\tilde{k}^i}{\tilde{k}^i} = \left(\Phi(\iota_t^i) - \delta\right) dt + \tilde{\nu}_t d\tilde{Z}_t^i + d\Delta_t^{k,i},$$

where $d\Delta_t^{k,i}$ is the physical capital transactions between households, and

$$\Phi(\iota) = \frac{1}{\phi}\log(1 + \phi\iota).$$

The idiosyncratic risk processes $\tilde{Z}_t^i$ are independent across households. Besides, households are subject to wealth reset shock with intensity $\zeta$. In the permanent shock case, $\zeta = 0$. Formally, let

$\tilde{J}_t^i$ be a Poisson process with intensity $\zeta$, the wealth dynamic for household $i$ is given by

$$\frac{dn_t^i}{n_{t-}^i} = -\frac{c_t^i}{n_{t-}^i}dt + dr_t^B + \theta_t^{K,i}\left(dr_t^{K,i}(\iota_t^i) - dr_t^B\right) + \theta^{E,i}\left(dr_t^{E,i} - dr_t^B\right) + \theta_t^{\bar{E},i}\left(dr_t^{\bar{E},i} - dr_t^B\right) + \frac{\bar{n}_t - n_{t-}^i}{n_{t-}^i}d\tilde{J}_t^i,$$

where $\tilde{J}_t^i$ and $\tilde{J}_t^j$ are stochastically independent if $i \neq j$. Households face a skin-in-the-game constraint when issuing equity, i.e.,

$$-\theta_t^{E,i} \leq (1-\bar{\chi})\theta_t^{K,i}.$$

**Government.** The nominal growth of government debt follows

$$dB_t = \mu_t^B B_t dt.$$

Government budget constraint is given by

$$i_t B_t + P_t \mathfrak{g} K_t = \mu_t^B B_t + P_t \tau_t a_t K_t,$$

where nominal rate $i_t$, spending and tax rate $\mathfrak{g}$, productivity $a_t$ are exogenous processes. Assume those processes, together with $\tilde{\nu}_t$, are jointly Markov and are driven by some Brownian motion $Z_t$. This creates aggregate risk in the economy. Tax rate $\tau_t$ is determined endogenous. With $q_t^B$ defined below, the budget constraint can be restated as

$$\check{\mu}_t^B q_t^B + \hat{s}_t = 0, \tag{A.1}$$

where $\check{\mu}_t^B := \mu_t^B - i_t$ is the bond issuance (net of interest payment) rate. $\hat{s}_t := \tau_t a_t - \mathfrak{g}$ is the real surplus-capital ratio.

### A.1.2 Symmetric Equilibrium

**Aggregation.** Under symmetric equilibrium,

$$\forall i \in [0,1], \quad (\theta_t^{K,i}, \theta_t^{E,i}, \theta_t^{\bar{E},i}, \iota_t^i, \hat{c}_t^i) = (\theta_t^K, \theta_t^E, \theta_t^{\bar{E}}, \iota_t, \hat{c}_t),$$

where $\hat{c}_t^i = c_t^i / n_t^i$ is the consumption-wealth ratio.

Under symmetric equilibrium, the returns adjust such that no one trades physical capital with others, i.e., $\Delta_t^{k,i} = 0$. Aggregate capital evolves according to

$$dK_t = (\Phi(\iota_t) - \delta) K_t dt.$$

Define the real price of physical capital as $q_t^K$ and the real bond-capital ratio as $q_t^B = \frac{B_t/P_t}{K_t}$. The state variable $\vartheta_t$ is defined as the share of wealth in government bond, i.e.,

$$\vartheta_t = \frac{q_t^B}{q_t^B + q_t^K}. \tag{A.2}$$

Assume that $q_t^K$, $q_t^B$ and $\vartheta_t$ all follow geometric Brownian motion with drift terms $\mu_t^{q,K}$, $\mu_t^{q,B}$ and $\mu_t^{\vartheta}$ and volatility terms $\sigma_t^{q,K}$, $\sigma_t^{q,B}$ and $\sigma_t^{\vartheta}$.

The difference of volatility $\sigma_t^{q,B} - \sigma_t^{q,K}$ is $\frac{\sigma_t^{\vartheta}}{1-\vartheta_t}$. Indeed, by definition,

$$\vartheta_t = \frac{q_t^B}{q_t^B + q_t^K}.$$

By Itô's lemma,

$$\vartheta_t \sigma_t^{\vartheta} = \frac{q_t^B}{q_t^B + q_t^K} \left( \sigma_t^{q,B} - \frac{\sigma_t^{q,B} q_t^B + \sigma_t^{q,K} q_t^K}{q_t^B + q_t^K} \right) = \frac{q_t^B}{q_t^B + q_t^K} \frac{(\sigma_t^{q,B} - \sigma_t^{q,K}) q_t^K}{q_t^B + q_t^K},$$

i.e.,

$$\sigma_t^{\vartheta} = \sigma_t^{q,B} - \frac{\sigma_t^{q,B} q_t^B + \sigma_t^{q,K} q_t^K}{q_t^B + q_t^K} = (\sigma_t^{q,B} - \sigma_t^{q,K})(1 - \vartheta_t). \tag{A.3}$$

The difference of drift $\mu_t^{q,B} - \mu_t^{q,K}$ can be derived similarly. Indeed, by Itô's lemma,

$$\begin{aligned}\mu_t^{\vartheta}\vartheta_t &= \frac{q_t^B}{q_t^B+q_t^K}\left(\mu_t^{q,B} - \frac{q_t^B\mu_t^{q,B}+q_t^K\mu_t^{q,K}}{q_t^B+q_t^K} + \left(\frac{q_t^B\sigma_t^{q,B}+q_t^K\sigma_t^{q,K}}{q_t^B+q_t^K}\right)^2 - \left(\frac{q_t^B\sigma_t^{q,B}+q_t^K\sigma_t^{q,K}}{q_t^B+q_t^K}\right)\sigma_t^{q,B}\right)\\ &= \frac{q_t^B}{q_t^B+q_t^K}(1-\vartheta_t)\left((\mu_t^{q,B}-\mu_t^{q,K}) - \left(\frac{q_t^B\sigma_t^{q,B}+q_t^K\sigma_t^{q,K}}{q_t^B+q_t^K}\right)(\sigma_t^{q,B}-\sigma_t^{q,K})\right),\end{aligned}$$

i.e.,

$$\frac{\mu_t^{\vartheta}}{1-\vartheta_t} = \mu_t^{q,B} - \mu_t^{q,K} + (\sigma_t^{q,B}-\sigma_t^{\vartheta})(\sigma_t^{q,K}-\sigma_t^{q,B}). \tag{A.4}$$

**Return Processes.** Here, we express the return process with the drift terms of $q_t^B$, $q_t^K$ and $\vartheta_t$.

Inflation:

$$\frac{dP_t}{P_t} = \frac{d(B_t/(q_t^BK_t))}{B_t/(q_t^BK_t)} = \left(\mu_t^B - \mu_t^{q,B} - \Phi(\iota_t) + \delta + (\sigma_t^{q,B})^2\right)dt - \sigma_t^{q,B}dZ_t$$

Bond return:

$$dr_t^B = i_t dt + \frac{d(1/P_t)}{1/P_t} = i_t dt + \frac{d(q_t^BK_t/B_t)}{q_t^BK_t/B_t} = \left(\Phi(\iota_t) - \delta - \breve{\mu}_t^B + \mu_t^{q,B}\right)dt + \sigma_t^{q,B}dZ_t. \tag{A.5}$$

Capital return:

$$\begin{aligned}dr_t^K &= \frac{(1-\tau_t)a_t - \iota_t^i}{q_t^K}dt + \frac{d(q_t^K\tilde{k}_t^i)}{q_t^K\tilde{k}_t^i}\\ &= \left(\frac{(1-\tau_t)a_t-\iota_t^i}{q_t^K} + \mu_t^{q,K} + (\Phi(\iota_t^i)-\delta)\right)dt + \sigma_t^{q,K}dZ_t + \tilde{\nu}_t d\tilde{Z}_t^i.\end{aligned} \tag{A.6}$$

Equity has the same risk loading as capital, but the expected return may be different when skin-in-the-game constraint is binding. Idiosyncratic risk loadings average out for aggregate equity.

**Market Clearing.** Goods market clearing:

$$\hat{c}_t(q_t^B + q_t^K) + \mathfrak{g} + \iota_t = a_t. \tag{A.7}$$

Asset market clearing:

$$\theta_t^K = \frac{q_t^K K_t}{(q_t^B + q_t^K)K_t} = 1 - \vartheta_t, \tag{A.8}$$

$$\theta^E = -\theta^{\bar{E}}. \tag{A.9}$$

### A.1.3 Solution

**Households Problem.** We use dynamic programming to solve the households' portfolio choice problem. Guess that the value function takes the form $V(n_t^i, v_t) = v_t + \frac{1}{\rho+\zeta}\log n_t^i$, where $v_t$ depends on aggregate level variables only. Heuristically,

$$\begin{aligned} V(n_t^i, t) &= \max_{c_t^i, \iota_t^i, \theta_t^i} \mathbb{E}_t \left[\log c_t^i dt + e^{-\rho dt}(1_{\text{jump}} V(n_{t+dt}^i, t+dt) + 1_{\text{nojump}} V(n_{t+dt}^i, t+dt))\right] \\ &= \max_{c_t^i, \iota_t^i, \theta_t^i} \mathbb{E}_t \left[\log c_t^i dt + (1-\rho dt)(\zeta dt V(\bar{n}_t, t+dt) + (1-\zeta dt)V(n_{t+dt}^i, t+dt))\right] \end{aligned}$$

The Hamilton-Jacobi-Bellman equation is

$$\begin{aligned} &(\rho+\zeta)V(n_{t-}^i, v_t) \\ &= \max_{c_t^i, \iota_t^i, \theta_t^i} \mathbb{E}_t \left[\log c_t^i dt + \zeta V(\bar{n}_t, v_t)dt + V_t dt + V_n(n_{t-}^i, v_t)dn_t^i + \frac{1}{2}V_{nn}(n_{t-}^i, v_t)\langle dn_t^i, dn_t^i\rangle\right]/dt \\ &\Rightarrow \rho v_t + \log n_{t-}^i - \frac{\zeta}{\rho+\zeta}\log \bar{n}_t \\ &= \max_{c_t^i, \iota_t^i, \theta_t^i} \mathbb{E}_t \left\{\log c_t^i dt + \dot{v}_t dt + \frac{1}{\rho+\zeta}\frac{dn_t^i}{n_{t-}^i} - \frac{1}{2}\frac{1}{\rho+\zeta}\left\langle \frac{dn_t^i}{n_{t-}^i}, \frac{dn_t^i}{n_{t-}^i}\right\rangle\right\}/dt, \end{aligned}$$

where we use $\mathbb{E}_t[dn_t^i]/dt$ to denote the drift term of $n_t^i$, as we have emphasized at the beginning of the appendix. For simplicity, we denote $\hat{\rho} := \rho + \zeta$. The HJB equation can be restated as

$$
\begin{aligned}
0 = \max_{c_t^i, \iota_t^i, \theta_t^i} \Bigg\{ & -(\hat{\rho} - \zeta) v_t - \log n_{t-}^i + \frac{\zeta}{\hat{\rho}} \log \bar{n}_t + \dot{v}_t + \log c_t^i \qquad \text{(A.10)} \\
& - \frac{1}{\hat{\rho}} \frac{c_t^i}{n_t^i} + \frac{1}{\hat{\rho}} (1 - \theta_t^{K,i} - \theta_t^{E,i} - \theta_t^{\bar{E},i}) \left( \Phi(\iota_t) - \delta - \breve{\mu}_t^B + \mu_t^{q,B} \right) \\
& + \frac{1}{\hat{\rho}} \theta_t^{K,i} \left( \frac{(1 - \tau_t) a_t - \iota_t^i}{q_t^K} + \mu_t^{q,K} + (\Phi(\iota_t^i) - \delta) \right) + \frac{1}{\hat{\rho}} \theta_t^{E,i} \frac{\mathbb{E}_t[dr_t^{E,i}]}{dt} + \frac{1}{\hat{\rho}} \theta_t^{\bar{E},i} \frac{\mathbb{E}_t[dr_t^{\bar{E}}]}{dt} \\
& - \frac{1}{2\hat{\rho}} \left( (1 - \theta_t^{K,i} - \theta_t^{E,i} - \theta_t^{\bar{E},i}) \sigma_t^{q,B} + (\theta_t^{K,i} + \theta_t^{E,i} + \theta_t^{\bar{E},i}) \sigma_t^{q,K} \right)^2 - \frac{1}{2\hat{\rho}} (\theta_t^{K,i} + \theta_t^{E,i})^2 \tilde{\nu}_t^2 \Bigg\}. \qquad \text{(A.11)}
\end{aligned}
$$

s.t.

$$
-\theta_t^{E,i} \leq (1 - \bar{\chi}) \theta_t^{K,i}
$$

The lagrangian is hence given by

$$
\begin{aligned}
\mathcal{L}_t^i = & -(\hat{\rho} - \zeta) v_t - \log n_{t-}^i + \frac{\zeta}{\hat{\rho}} \log \bar{n}_t + \dot{v}_t + \log c_t^i \\
& - \frac{1}{\hat{\rho}} \frac{c_t^i}{n_t^i} + \frac{1}{\hat{\rho}} (1 - \theta_t^{K,i} - \theta_t^{E,i} - \theta_t^{\bar{E},i}) \left( \Phi(\iota_t) - \delta - \breve{\mu}_t^B + \mu_t^{q,B} \right) \\
& + \frac{1}{\hat{\rho}} \theta_t^{K,i} \left( \frac{(1 - \tau_t) a_t - \iota_t^i}{q_t^K} + \mu_t^{q,K} + (\Phi(\iota_t^i) - \delta) \right) + \frac{1}{\hat{\rho}} \theta_t^{E,i} \frac{\mathbb{E}_t[dr_t^{E,i}]}{dt} + \frac{1}{\hat{\rho}} \theta_t^{\bar{E},i} \frac{\mathbb{E}_t[dr_t^{\bar{E}}]}{dt} \\
& - \frac{1}{2\hat{\rho}} \left( (1 - \theta_t^{K,i} - \theta_t^{E,i} - \theta_t^{\bar{E},i}) \sigma_t^{q,B} + (\theta_t^{K,i} + \theta_t^{E,i} + \theta_t^{\bar{E},i}) \sigma_t^{q,K} \right)^2 - \frac{1}{2\hat{\rho}} (\theta_t^{K,i} + \theta_t^{E,i})^2 \tilde{\nu}_t^2 \\
& + \lambda_t^i (\theta_t^{E,i} + (1 - \bar{\chi}) \theta_t^{K,i})
\end{aligned}
$$

The first order conditions w.r.t. $c_t^i$, $\iota_t^i$, $\theta_t^{K,i}$, $\theta_t^{E,i}$ and $\theta_t^{\bar{E},i}$ are

$$-\frac{1}{\hat{\rho} n_{t-}^i} + \frac{1}{c_t^i} = 0, \tag{A.12}$$

$$-\frac{1}{q_t^K} + \frac{1}{1+\phi\iota_t^i} = 0, \tag{A.13}$$

$$\frac{\mathbb{E}_t[dr_t^{K,i}]}{dt} - \frac{\mathbb{E}_t[dr_t^B]}{dt} = \left(\sigma_t^{q,B} - (\theta_t^{K,i} + \theta_t^{E,i} + \theta_t^{\bar{E},i})\frac{\sigma_t^\vartheta}{1-\vartheta_t}\right)(\sigma_t^{q,K} - \sigma_t^{q,B}) + (\theta_t^{K,i} + \theta_t^{E,i})\tilde{\nu}_t^2 - (1-\bar{\chi})\lambda_t^i, \tag{A.14}$$

$$\frac{\mathbb{E}_t[dr_t^{E,i}]}{dt} - \frac{\mathbb{E}_t[dr_t^B]}{dt} = \left(\sigma_t^{q,B} - (\theta_t^{K,i} + \theta_t^{E,i} + \theta_t^{\bar{E},i})\frac{\sigma_t^\vartheta}{1-\vartheta_t}\right)(\sigma_t^{q,K} - \sigma_t^{q,B}) + (\theta_t^{K,i} + \theta_t^{E,i})\tilde{\nu}_t^2 - \lambda_t^i, \tag{A.15}$$

$$\frac{\mathbb{E}_t[dr_t^{\bar{E}}]}{dt} - \frac{\mathbb{E}_t[dr_t^B]}{dt} = \left(\sigma_t^{q,B} - (\theta_t^{K,i} + \theta_t^{E,i} + \theta_t^{\bar{E},i})\frac{\sigma_t^\vartheta}{1-\vartheta_t}\right)(\sigma_t^{q,K} - \sigma_t^{q,B}). \tag{A.16}$$

Note that we have plugged in $\sigma_t^{q,B} - \sigma_t^{q,K} = \frac{\sigma_t^\vartheta}{1-\vartheta_t}$.

**Characterizing Consumption, Investment $q_t^B$ and $q_t^K$.** The first order condition Eq. (A.12) implies constant consumption-wealth ratio

$$c_t^i = \hat{\rho} n_{t-}^i. \tag{A.17}$$

FOC (A.13) reproduces the Tobin's $q$ condition, and confirms that every household has the same investment ratio

$$q_t^K = 1 + \phi\iota_t. \tag{A.18}$$

Now we can (jointly) pin down $\vartheta_t$, $q_t^B$ and $q_t^K$. Recall goods market clearing condition

$$C_t + \mathfrak{g}K_t + \iota_t K_t = a_t K_t.$$

Plug in $C_t = \hat{\rho}\bar{n}_t = \hat{\rho} q_t^K K_t/(1-\vartheta_t)$ to obtain

$$\frac{\hat{\rho}}{1-\vartheta_t} + \frac{\hat{\rho}\phi}{1-\vartheta_t}\iota_t + \mathfrak{g} + \iota_t = a_t,$$

i.e.,

$$\iota_t = \frac{(a_t - \mathfrak{g})(1-\vartheta_t) - \hat{\rho}}{1 + \hat{\rho}\phi - \vartheta_t}. \tag{A.19}$$

Hence, we can express price of capital $q_t^K$ as

$$q_t^K = \frac{1 - \vartheta_t + \phi(a_t - \mathfrak{g})(1-\vartheta_t)}{1 + \hat{\rho}\phi - \vartheta_t} = (1-\vartheta_t)\frac{1 + \phi(a_t - \mathfrak{g})}{1 + \hat{\rho}\phi - \vartheta_t}.$$

Since $q_t^B/q_t^K = \vartheta_t/(1-\vartheta_t)$,

$$q_t^B = \vartheta_t \frac{1 + \phi(a_t - \mathfrak{g})}{1 + \hat{\rho}\phi - \vartheta_t}. \tag{A.20}$$

**Portfolio Weights** Under symmetric equilibrium,

$$\theta_t^{K,i} = \theta_t^K, \quad \theta_t^{E,i} = \theta_t^E, \quad \theta_t^{\bar{E},i} = \theta_t^{\bar{E}}.$$

Asset Market clearing implies

$$1 - \theta_t^K - \theta_t^E - \theta_t^{\bar{E}} = \vartheta_t$$

and

$$\theta_t^E + \theta_t^{\bar{E}} = 0.$$

When the skin-in-the-game constraint is binding,

$$\theta_t^E + (1-\bar{\chi})\theta_t^K = 0. \tag{A.21}$$

The equations above jointly solve

$$\begin{aligned}
\theta_t^K &= 1 - \vartheta_t, \\
\theta_t^E &= -(1-\bar{\chi})(1-\vartheta_t), \\
\theta_t^{\bar{E}} &= (1-\bar{\chi})(1-\vartheta_t).
\end{aligned}$$

**Return Processes** Under symmetric equilibrium, the outside equity has the same expected return as individual equity, i.e.,

$$\mathbb{E}_t[dr_t^{E,i}] = \mathbb{E}_t[dr_t^{\bar{E},i}], \tag{A.22}$$

with which we can pin down the lagrangian multiplier from Eq. (A.15) and (A.16)

$$\lambda_t^i = (\theta_t^{K,i} + \theta_t^{E,i})\tilde{\nu}_t^2 = \bar{\chi}(1-\vartheta_t)\tilde{\nu}_t^2.$$

Note that $\theta_t^{E,i} + \theta_t^{K,i} \geq \bar{\chi}\theta_t^{K,i}$, hence $\lambda_t^i$ is always positive and the skin-in-the-game constraint is always binding.

The expected return of equity and outside equity in excess of real bond return can be written as

$$\frac{\mathbb{E}_t[dr_t^{\bar{E}}]}{dt} - \frac{\mathbb{E}_t[dr_t^B]}{dt} = \left(\sigma_t^{q,B} - \theta_t^K \frac{\sigma_t^\vartheta}{1-\vartheta_t}\right)(\sigma_t^{q,K} - \sigma_t^{q,B}) = \left(\sigma_t^{q,B} - \sigma_t^\vartheta\right)(\sigma_t^{q,K} - \sigma_t^{q,B}).$$

The expected return of capital in excess of equity is

$$\frac{\mathbb{E}_t[dr_t^{K,i}]}{dt} - \frac{\mathbb{E}_t[dr_t^{E,i}]}{dt} = \bar{\chi}\lambda_t^i = (1-\vartheta_t)\bar{\chi}^2\tilde{\nu}_t^2.$$

### A.1.4 Equilibrium: Discipline Drift of the State Variable

We discipline $\mu_t^{\vartheta}$ as Proposition 1. By Eq. (A.6) and (A.5), the expected return of capital in excess of real bond return is

$$\frac{\mathbb{E}_t[dr_t^{K,i}]}{dt} - \frac{\mathbb{E}_t[dr_t^B]}{dt} = \frac{(1-\tau_t)a_t - \iota_t}{q_t^K} + \mu_t^{q,K} + \check{\mu}_t^B - \mu_t^{q,B}.$$

Plugging Eq. (A.1) and (A.4) yields

$$\frac{\mathbb{E}_t[dr_t^{K,i}]}{dt} - \frac{\mathbb{E}_t[dr_t^B]}{dt} = \frac{\check{\mu}_t^B - \mu_t^{\vartheta}}{1-\vartheta_t} + (\sigma_t^{q,B} - \sigma_t^{\vartheta})(\sigma_t^{q,K} - \sigma_t^{q,B}) + \frac{a_t - g - \iota_t}{q_t^K}.$$

By Eq. (A.14) and (A.19),

$$\begin{aligned}(\sigma_t^{q,B} - \sigma_t^{\vartheta})(\sigma_t^{q,K} - \sigma_t^{q,B}) + (1-\vartheta_t)\bar{\chi}^2\tilde{\nu}_t^2 &= \frac{\check{\mu}_t^B - \mu_t^{\vartheta}}{1-\vartheta_t} + (\sigma_t^{q,B} - \sigma_t^{\vartheta})(\sigma_t^{q,K} - \sigma_t^{q,B}) + \frac{a_t - g - \iota_t}{q_t^K} \\ \Rightarrow \mu_t^{\vartheta} &= \check{\mu}_t^B + \hat{\rho} - (1-\vartheta_t)^2\bar{\chi}^2\tilde{\nu}_t^2. \end{aligned} \tag{A.23}$$

Specifically, in the steady state, we have

$$1 - \vartheta^{SS} = \frac{\sqrt{\check{\mu}^B + \rho + \zeta}}{\bar{\chi}\tilde{\nu}}.$$

### A.1.5 Macro Synthesis

Before proceeding, we briefly summarize the equation system here. We suppress the individual superscript for simplicity.

There are 4 exogenous variables:

$$(\mu_t^B, i_t, a_t, \tilde{\nu}_t),$$

5 endogenous variables:

$$\left(\hat{c}_t, \iota_t, q_t^B, q_t^K, \tau_t\right),$$

and an endogenous state variable $\vartheta_t$.

We have 5 equations: 2 households' first-order conditions (A.12) and (A.13):

$$\hat{c}_t = \hat{\rho} = \rho + \zeta,$$
$$q_t^K = 1 + \phi\iota_t,$$

1 goods market clearing condition:

$$\hat{c}_t \frac{q_t^K}{1-\vartheta_t} + g + \iota_t = a_t,$$

1 wealth share definition:

$$\vartheta_t = \frac{q_t^B}{q_t^B + q_t^K},$$

and 1 government budget constraint:

$$(\mu_t^B - i_t) q_t^B + \tau_t a_t = \mathfrak{g}.$$

Given the state variable $\vartheta_t$, these 5 equations allow us to solve for the 5 endogenous variables.

Besides, the dynamics of state variable $\vartheta_t$ are subject to

$$\mu_t^\vartheta = \check{\mu}_t^B + \hat{\rho} - (1-\vartheta_t)^2 \bar{\chi}^2 \tilde{\nu}_t^2.$$

In this equilibrium, the expected returns are

$$\begin{aligned}
\frac{\mathbb{E}[dr_t^B]}{dt} &= \Phi(\iota_t) - \delta - \check{\mu}_t^B + \mu_t^{q,B}, \\
\frac{\mathbb{E}[dr_t^K]}{dt} &= \Phi(\iota_t) - \delta - \check{\mu}_t^B + \mu_t^{q,B} + (\vartheta_t \sigma_t^{q,B} + (1-\vartheta_t)\sigma_t^{q,K})(\sigma_t^{q,K} - \sigma_t^{q,B}) + (1-\vartheta_t)\bar{\chi}^2\tilde{\nu}_t^2, \\
\frac{\mathbb{E}[dr_t^E]}{dt} &= \Phi(\iota_t) - \delta - \check{\mu}_t^B + \mu_t^{q,B} + (\vartheta_t \sigma_t^{q,B} + (1-\vartheta_t)\sigma_t^{q,K})(\sigma_t^{q,K} - \sigma_t^{q,B}), \\
\frac{\mathbb{E}[dr_t^{\bar{E}}]}{dt} &= \Phi(\iota_t) - \delta - \check{\mu}_t^B + \mu_t^{q,B} + (\vartheta_t \sigma_t^{q,B} + (1-\vartheta_t)\sigma_t^{q,K})(\sigma_t^{q,K} - \sigma_t^{q,B}).
\end{aligned}$$

So far, we have written every aggregate level variable as function of $\vartheta_t$ and pinned down its drift term $\mu_t^\vartheta$. We can then pin down the volatility term $\sigma_t^\vartheta$ by specifying the exogenous processes $a_t$, $\tilde{\nu}_t$, $\check{\mu}_t^B = \mu_t^B - i_t$ and apply Itô's lemma. Then we can pin down $\sigma_t^{q,B}$, $\sigma_t^{q,K}$, $\mu_t^{q,B}$ and $\mu_t^{q,K}$ and solve all the expected returns explicitly.

### A.1.6 Verifying Value Function

I verify the value function by plugging endogenous variables into the HJB Equation (A.11). Indeed,

$$\begin{aligned}
0 = &-(\hat{\rho}-\zeta)v_t - \log n_{t-}^i + \frac{\zeta}{\hat{\rho}}\log \bar{n}_t + \dot{v}_t + (\log\hat{\rho} + \log n_{t-}^i) - 1 + \frac{1}{\hat{\rho}}\left(\Phi(\iota_t) - \delta - \check{\mu}_t^B + \mu_t^{q,B}\right) \\
&+ \frac{1}{\hat{\rho}}(1-\vartheta_t)\left[(\sigma_t^{q,B} - \sigma_t^\vartheta)(\sigma_t^{q,K} - \sigma_t^{q,B}) + (1-\vartheta_t)\bar{\chi}^2\tilde{\nu}_t^2\right] \\
&- \frac{1}{2\hat{\rho}}\left(\sigma_t^{q,B} - \sigma_t^\vartheta\right)^2 - \frac{1}{2\hat{\rho}}(1-\vartheta_t)^2\bar{\chi}^2\tilde{\nu}_t^2,
\end{aligned}$$

where the two $\log n_{t-}^i$ terms cancel out. The equation above is an linear ODE for $v_t$. Note that $\vartheta_t, \sigma_t^\vartheta, \mu_t^\vartheta$ and $\bar{n}_t$ depend on aggregate shock only, so does $v_t$.

### A.1.7 SDF and Wealth Processes

Household $i$'s marginal utility is

$$m_t^i = e^{-\rho t}\frac{1}{c_t^i} = e^{-\rho t}(\rho+\zeta)^{-1}\frac{1}{n_t^i}.$$

Plug in the expected return processes to obtain her wealth process as

$$dn_t^i = g_t n_{t-}^i dt + \sigma_t n_{t-}^i dZ_t + \bar{\chi}(1-\vartheta_t)\tilde{\nu}_t n_{t-}^i d\tilde{Z}_t^i + (\bar{n}_t - n_{t-}^i) d\tilde{J}_t^i,$$

which implies

$$\frac{dm_t^i}{m_{t-}^i} = -[\rho + g_t - \sigma_t^2 - \tilde{\sigma}_t^2]dt - \sigma_t dZ_t - \tilde{\sigma}_t d\tilde{Z}_t^i + \frac{1}{m_{t-}^i}\left(e^{-\rho t}\frac{1}{(\rho+\zeta)\bar{n}_t} - e^{-\rho t}\frac{1}{(\rho+\zeta)\bar{n}_{t-}^i}\right) d\tilde{J}_t^i$$

where

$$\begin{aligned}
\sigma_t &= \sigma_t^{q,B} - \sigma_t^{\vartheta} = (1-\vartheta_t)\sigma_t^{q,K} + \vartheta_t \sigma_t^{q,B}, \\
\tilde{\sigma}_t &= \bar{\chi}(1-\vartheta_t)\tilde{\nu}_t.
\end{aligned}$$

which is consistent with the investors' portfolio which is $\vartheta_t$ in bonds and $1-\vartheta_t$ in capital. Moreover, the idiosyncratic risk loading is $\bar{\chi}$ times the equity allocation $(1-\vartheta_t)$.

## A.2 Proof for Proposition 1

In the permanent shock case, $\zeta = 0$, hence

$$m_t^i = e^{-\rho t}\rho^{-1}(n_t^i)^{-1}.$$

We obtain

$$\begin{aligned}
\lim_{T\to\infty} \mathbb{E}_0\left[\bar{m}_T \bar{n}_T\right] &= \lim_{T\to\infty} \mathbb{E}_0\left[\left(\int_0^1 e^{-\rho T}\left(\frac{n_T^i}{n_0^i}\right)^{-1} di\right)\bar{n}_T\right] \\
&= \lim_{T\to\infty} \mathbb{E}_0\left[e^{-\rho T}\bar{n}_0\left(\int_0^1 \left(\frac{\eta_T^i}{\eta_0^i}\right)^{-1} di\right)\right].
\end{aligned}$$

Normalizing $\eta_0^i = 1$ and $\bar{n}_0 = 1$ yields

$$\lim_{T\to\infty} \mathbb{E}_0 \left[\bar{m}_T \bar{n}_T\right] = \lim_{T\to\infty} \mathbb{E}_0 \left[ e^{-\rho T} \left( \int_0^1 \left(\eta_T^i\right)^{-1} \, di \right) \right].$$

## A.3 Transversality Conditions with Permanent Idiosyncratic Risk

### A.3.1 Individual Level TVC

**The r Minus g Perspective.** Individual TVC is satisfied if $\mathbb{E}_t[d(m_t^i n_t^i)] < 0$, i.e.,

$$-\underbrace{[\rho + g_t - \sigma_t^2 - \tilde{\sigma}_t^2]}_{\text{drift of } m_t^i \text{ (risk free rate)}} + \underbrace{g_t}_{\text{drift of } n_t^i} - \underbrace{[\sigma_t^2 + \tilde{\sigma}_t^2]}_{\text{cross variation (risk premium)}} < 0,$$

i.e., individual TVC is satisfied if

$$\rho > 0. \tag{A.24}$$

As a result, individual TVC always hold. The intuition here is that the growth rate of individual wealth equals the return of individual wealth portfolio minus consumption rate. As long as consumption rate is positive, the growth rate of individual wealth is always smaller than the discount rate.

### A.3.2 Aggregate Level TVC

**The r Minus g Perspective.** The aggregate TVC is $\lim_{T\to\infty} \mathbb{E}_0[m_T^i \bar{n}_T] = 0$. Note that

$$\begin{aligned}
\frac{dm_t^i \bar{n}_t}{m_t^i \bar{n}_t} &= \{-[\rho + g_t - \sigma_t^2 - \tilde{\sigma}^2] + g_t - \sigma_t^2\} dt + (\sigma_t - \sigma_t) dZ_t - \tilde{\sigma}_t d\tilde{Z}_t^i \\
&= -[\rho - \tilde{\sigma}_t^2] dt - \tilde{\sigma}_t d\tilde{Z}_t^i,
\end{aligned}$$

Hence

$$\begin{aligned} d\log(m_t^i \bar{n}_t) &= \frac{dm_t^i \bar{n}_t}{m_t^i \bar{n}_t} - \frac{1}{2}\frac{\langle dm_t^i \bar{n}_t, dm_t^i \bar{n}_t \rangle}{(m_t^i \bar{n}_t)^2} \\ &= -[\rho - \frac{1}{2}\tilde{\sigma}_t^2]dt - \tilde{\sigma}_t d\tilde{Z}_t^i, \end{aligned}$$

which implies

$$m_T^i \bar{n}_T = m_0^i \bar{n}_0 \exp\left(\int_0^T -[\rho - \tilde{\sigma}_t^2]dt\right) \exp\left(\int_0^T -\frac{1}{2}\tilde{\sigma}_t^2 dt - \int_0^T \tilde{\sigma}_t d\tilde{Z}_t^i\right).$$

For simplicity, let idiosyncratic risk be constant over time, i.e., $\tilde{\sigma}_t = \tilde{\sigma}$. Then $\exp\left(\int_0^T -\frac{1}{2}\tilde{\sigma}_t^2 dt - \int_0^T \tilde{\sigma}_t d\tilde{Z}_t^i\right)$ is a martingale, and

$$\mathbb{E}_0[m_T^i \bar{n}_T] = m_0^i \bar{n}_0 \exp\left(\int_0^T -[\rho - \tilde{\sigma}^2]dt\right) = m_0^i \bar{n}_0 \exp\left(-[\rho - \tilde{\sigma}^2]T\right).$$

The aggregate transversality condition holds if and only if

$$\rho > \tilde{\sigma}^2.$$

**The Wealth Distribution Perspective.** The transversality condition is given by

$$\lim_{T\to\infty} \mathbb{E}[m_T^i \bar{n}_T] = \lim_{T\to\infty} \rho^{-1}\mathbb{E}\left[e^{-\rho T}(\eta_T^i)^{-1}\right] = \lim_{T\to\infty} (\rho^{-1}e^{-\rho T})\mathbb{E}\left[(\eta_T^i)^{-1}\right].$$

Recall that

$$\frac{d\eta_t^i}{\eta_t^i} = \tilde{\sigma} d\tilde{Z}_t^i,$$

by Itô's lemma,

$$d(\eta_t^i)^{-1} = \tilde{\sigma}^2(\eta_t^i)^{-1}dt - (\eta_t^i)^{-1}d\tilde{Z}_t^i.$$

Following the same method above, we have

$$\mathbb{E}[(\eta_T^i)^{-1}] = (\eta_0^i)^{-1} \exp\left(\tilde{\sigma}^2 T\right).$$

Hence,

$$\lim_{T\to\infty} \mathbb{E}[m_T^i \bar{n}_T] = \lim_{T\to\infty} \rho^{-1}(\eta_0^i)^{-1} \exp\left((-\rho + \tilde{\sigma}^2)T\right).$$

Again, the aggregate transversality condition holds if and only if

$$\rho > \tilde{\sigma}^2.$$

## A.4 Transversality Conditions with Transitory Idiosyncratic Risk

### A.4.1 Aggregate Level TVC

We show the wealth distribution perspective here only, and leave the r minus g perspective in Appendix A.7 which also serves as the proof for Proposition 4.

**The Wealth Distribution Perspective.** Consider the wealth reset shock with intensity $\zeta$. The wealth dynamics become

$$d\eta_t^i = \tilde{\sigma}_t \eta_{t-}^i d\tilde{Z}_t^i + (1 - \eta_{t-}^i) d\tilde{J}_t^i.$$

By Itô's lemma,

$$\begin{aligned} d(\eta_t^i)^{-1} = &\left[\tilde{\sigma}_t^2 (\eta_{t-}^i)^{-1} + \zeta(1 - (\eta_{t-}^i)^{-1})\right] dt \\ &- \tilde{\sigma}_t (\eta_{t-}^i)^{-1} d\tilde{Z}_t^i + (1 - (\eta_{t-}^i)^{-1})(d\tilde{J}_t^i - \zeta dt), \end{aligned}$$

i.e.,

$$(\eta_t^i)^{-1} = (\eta_0^i)^{-1} + \int_0^t \left[\tilde{\sigma}_s^2(\eta_{s-}^i)^{-1} + \zeta(1-(\eta_{s-}^i)^{-1})\right] ds + \int_0^t -\tilde{\sigma}_s(\eta_{s-}^i)^{-1} d\tilde{Z}_s^i + \int_0^t (1-(\eta_{s-}^i)^{-1})(d\tilde{J}_s^i - \zeta ds).$$

Let $\tilde{\sigma}_t = \tilde{\sigma}$, i.e. constant over time. Assume $\int_0^t \tilde{\sigma}^2(\eta_s^i)^{-2} ds < \infty$ and $\int_0^t (1-(\eta_{s-}^i)^{-1})^2 ds < \infty$ for every $t$, then the last two terms are both martingales with zero conditional expectation. Take expectation on both sides yields

$$\mathbb{E}_0[(\eta_t^i)^{-1}] = (\eta_0^i)^{-1} + \mathbb{E}_0\left[\int_0^t \left[\tilde{\sigma}^2(\eta_{s-}^i)^{-1} + \zeta(1-(\eta_{s-}^i)^{-1})\right] ds\right]$$

Note that jumps happen at most countable times from time 0 to $t$, hence $\int_0^t ((\eta_{s-}^i)^{-1} - (\eta_s^i)^{-1}) ds = 0$, and

$$\begin{aligned}\mathbb{E}_0[(\eta_t^i)^{-1}] &= (\eta_0^i)^{-1} + \mathbb{E}_0\left[\int_0^t \left[\tilde{\sigma}^2(\eta_s^i)^{-1} + \zeta(1-(\eta_s^i)^{-1})\right] ds\right] \\ &= (\eta_0^i)^{-1} + \int_0^t \left[\tilde{\sigma}^2 \mathbb{E}_0[(\eta_s^i)^{-1}] + \zeta(1-\mathbb{E}_0[(\eta_s^i)^{-1}])\right] ds,\end{aligned}$$

differentiating both sides yields an ODE w.r.t. $\mathbb{E}[(\eta_t^i)^{-1}]$, i.e.,

$$d\mathbb{E}_0[(\eta_t^i)^{-1}] = \left[\tilde{\sigma}^2 \mathbb{E}_0[(\eta_t^i)^{-1}] + \zeta(1-\mathbb{E}_0[(\eta_t^i)^{-1}])\right] dt.$$

If $\zeta \neq \tilde{\sigma}^2$, the solution to the ODE is

$$\mathbb{E}_0\left[(\eta_t^i)^{-1}\right] = \left(\frac{\zeta}{\tilde{\sigma}^2 - \zeta} + (\eta_0^i)^{-1}\right) e^{[\tilde{\sigma}^2 - \zeta]t} - \frac{\zeta}{\tilde{\sigma}^2 - \zeta}.$$

If $\zeta = \tilde{\sigma}^2$, the solution to the ODE is

$$\mathbb{E}_0\left[(\eta_t^i)^{-1}\right] = (\eta_0^i)^{-1} + \zeta t.$$

When $\tilde{\sigma}^2 \neq \zeta$, the aggregate TVC condition can be stated as

$$\begin{aligned}\lim_{T\to\infty} \mathbb{E}[m_T^i \bar{n}_T] &= \lim_{T\to\infty} (\rho+\zeta)^{-1} e^{-\rho T} \mathbb{E}\left[(\eta_T^i)^{-1}\right] \\ &= \lim_{T\to\infty} (\rho+\zeta)^{-1} \left(\frac{\zeta}{\tilde{\sigma}^2-\zeta} + (\eta_0^i)^{-1}\right) e^{[\tilde{\sigma}^2-\zeta-\rho]T} - \frac{\zeta}{\tilde{\sigma}^2-\zeta}\exp(-\rho T).\end{aligned}$$

For $\zeta = \tilde{\sigma}^2$ case,

$$\begin{aligned}\lim_{T\to\infty} \mathbb{E}[m_T^i \bar{n}_T] &= \lim_{T\to\infty} (\rho+\zeta)^{-1} e^{-\rho T} \mathbb{E}\left[(\eta_T^i)^{-1}\right] \\ &= (\rho+\zeta)^{-1} e^{-\rho T}[(\eta_0^i)^{-1} + \zeta T].\end{aligned}$$

The aggregate TVC holds if and only if

$$\rho + \zeta > \tilde{\sigma}^2.$$

## A.5 Proof for Proposition 2

For expositional purposes, we prove this in the same environment as Appendix A.1. Denote the nominal government bond outstanding as $B_t$ and real government bond as $b_t$. Denote the real surplus as $s_t$. We obtain

$$dB_t = i_t B_t dt - P_t s_t dt.$$

Hence,

$$
\begin{aligned}
d(m_t^i b_t) &= b_t dm_t^i + m_{t-}^i/P_t dB_t + m_{t-}^i B_t d(1/P_t) + B_t dm_t^i d(1/P_t) \\
&= b_t \left( -(r_t^f - \zeta) m_{t-}^i dt - \sigma_t m_{t-}^i dZ_t - \tilde{\sigma}_t m_{t-}^i d\tilde{Z}_t^i + \left( e^{-\rho t}(\rho+\zeta)^{-1} \bar{n}_t^{-1} - m_{t-}^i \right) d\tilde{J}_t^i \right) \\
&+ i_t b_t m_{t-}^i dt - s_t m_t^i dt + m_{t-}^i b_t (dr_t^B - i_t dt) - \sigma_t m_{t-}^i b_t \sigma_t^{q,B} dt \\
&= -m_{t-}^i s_t dt + m_{t-}^i \zeta b_t dt \\
&- m_{t-}^i b_t (\sigma_t - \sigma_t^{q,B}) dZ_t - m_{t-}^i b_t \tilde{\sigma}_t d\tilde{Z}_t^i + b_t \left( e^{-\rho t}(\rho+\zeta)^{-1} \bar{n}_t^{-1} - m_{t-}^i \right) d\tilde{J}_t^i \\
&= -m_{t-}^i s_t dt + \zeta e^{-\rho t}(\rho+\zeta)^{-1} \bar{n}_t^{-1} b_t dt - m_{t-}^i b_t (\sigma_t - \sigma_t^{q,B}) dZ_t - m_{t-}^i b_t \tilde{\sigma}_t d\tilde{Z}_t^i \\
&+ b_t \left( e^{-\rho t}(\rho+\zeta)^{-1} \bar{n}_t^{-1} - m_{t-}^i \right) d(\tilde{J}_t^i - \zeta t),
\end{aligned}
$$

where we plugged in the definition of bond return, i.e.,

$$
dr_t^B = i_t dt + \frac{d(1/P_t)}{1/P_t} = i_t dt + \frac{d(q_t^B K_t/B_t)}{q_t^B K_t/B_t} = (r_t^f + \sigma_t \sigma_t^{q,B}) dt + \sigma_t^{q,B} dZ_t.
$$

Integrate $d(m_t^i b_t)$ from $t = 0$ to $T$ and take expectation to obtain[9]

$$
\mathbb{E}_0[m_T^i b_T] - m_0^i b_0 = -\mathbb{E}_0 \left[ \int_0^T m_t^i s_t \, dt \right] + \zeta \mathbb{E}_0 \left[ \int_0^T e^{-\rho t}(\rho+\zeta)^{-1} \bar{n}_t^{-1} b_t \, dt \right].
$$

Divide both sides by $m_0^i$ to obtain

$$
b_0 = \mathbb{E}_0 \left[ \int_0^T \frac{m_t^i}{m_0^i} s_t \, dt \right] - \zeta \mathbb{E}_0 \left[ \int_0^T e^{-\rho t} \left( \frac{\bar{n}_t}{\bar{n}_0} \right)^{-1} b_t \, dt \right] + \mathbb{E}_0 \left[ \frac{m_T^i}{m_0^i} b_T \right].
$$

[9]Note that there are at most countably many jumps between time 0 and $T$, and we can replace $t-$ by $t$ in the intergrals, as shown in our previous proofs. Heuristically, $\int_0^T m_t^i s_t \, dt = \int_0^T m_{t-}^i s_t \, dt + \int_0^T (m_t^i - m_{t-}^i) s_t \, dt = \int_0^T m_{t-}^i s_t \, dt$.

To derive the bond valuation equation from the aggregate SDF, we start by dividing $d(m_t^i B_t/P_t)$ by $m_0^i$, i.e.,

$$d[(m_t^i/m_0^i)b_t] = -\frac{m_{t-}^i}{m_0^i}s_t dt + \zeta e^{-\rho t}\left(\frac{\bar{n}_t}{\bar{n}_0}\right)^{-1} b_t dt \\ - \frac{m_{t-}^i}{m_0^i}b_t(\sigma_t - \sigma_t^{q,B})dZ_t - \frac{m_{t-}^i}{m_0^i}b_t\tilde{\sigma}_t d\tilde{Z}_t^i + b_t\left(e^{-\rho t}\left(\frac{\bar{n}_t}{\bar{n}_0}\right)^{-1} - \frac{m_{t-}^i}{m_0^i}\right)d(\tilde{J}_t^i - \zeta t).$$

Integrate by $i$ to obtain

$$d[\bar{m}_t b_t] = -\bar{m}_t s_t dt + \zeta e^{-\rho t}\left(\frac{\bar{n}_t}{\bar{n}_0}\right)^{-1} b_t dt - \bar{m}_t b_t(\sigma_t - \sigma_t^{q,B})dZ_t.$$

Integrate over time and take expectation to obtain

$$\mathbb{E}_0[\bar{m}_T b_T] - b_0 = -\mathbb{E}_0\left[\int_0^T \bar{m}_t s_t dt\right] + \mathbb{E}_0\left[\int_0^T \zeta e^{-\rho t}\left(\frac{\bar{n}_t}{\bar{n}_0}\right)^{-1} b_t dt\right],$$

i.e.,

$$b_0 = \mathbb{E}_0\left[\int_0^T \bar{m}_t s_t dt\right] - \mathbb{E}_0\left[\int_0^T \zeta e^{-\rho t}\left(\frac{\bar{n}_t}{\bar{n}_0}\right)^{-1} b_t dt\right] + \mathbb{E}_0\left[\bar{m}_T b_T\right].$$

## A.6 Proof for Proposition 3

We use the same proof strategy in Toda (2014). Recall that the individual wealth satisfies

$$\frac{dn_t^i}{n_t^i} = g_t dt + \sigma_t dZ_t + \tilde{\sigma}_t d\tilde{Z}_t^i + \frac{\bar{n}_t - n_t^i}{n_t^i}d\tilde{J}_t^i.$$

For aggregate wealth,

$$\frac{d\bar{n}_t}{\bar{n}_t} = g_t dt + \sigma_t dZ_t.$$

Hence,

$$\frac{d\eta_t^i}{\eta_t^i} = \frac{d(n_t^i/\bar{n}_t)}{n_t^i/\bar{n}_t} = \tilde{\sigma}_t d\tilde{Z}_t^i + \frac{1-\eta_t^i}{\eta_t^i} d\tilde{J}_t^i. \tag{A.25}$$

In the absence of wealth reset, the log wealth share follows

$$d\log \eta_t^i = -\frac{1}{2}\tilde{\sigma}_t^2 dt + \tilde{\sigma}_t d\tilde{Z}_t^i. \tag{A.26}$$

Below, we use generation $s$ to denote the collection of agents whose last wealth reset shock happened at time $s$. Specifically, generation $0$ denotes the collection of agents who have never had any reset shock. We also use $x$ to denote log wealth share, i.e., $x_t^i = \log \eta_t^i$.

At time $t$, denote the (within-generation) probability density function for the log wealth share of generation $0$ as $g(x,t,0)$. Under Assumption 1, $g(x,t,0)$ is subject to the following Kolmogorov Forward Equation

$$\frac{\partial g(x,t,0)}{\partial t} = \frac{\tilde{\sigma}^2}{2}\left(\frac{\partial}{\partial x} g(x,t,0) + \frac{\partial^2}{\partial x^2} g(x,t,0)\right),$$

Note that $g(x,t,0)$ is always a valid pdf at any time $t$, i.e.,

$$\int_{-\infty}^{\infty} g(x,t,0)\, dx = 1, \quad \forall t > 0.$$

Denote the p.d.f. of the initial wealth distribution as $f(x,0)$. Since $g$ is the p.d.f. of the initial log

wealth share, we have[10]

$$\frac{1}{\eta}g(\log\eta,0,0)=f(\eta,0).$$

Similarly, for every generation $s$ such that $0<s<t$, we can find the within-generation p.d.f. of the log wealth share $g(x,t,s)$ by plugging the initial condition $\delta(x)$, which integrates to $1$ but only has mass at $x=0$. By doing this, we obtain that

$$g(x,t,s)=\frac{1}{\tilde{\sigma}\sqrt{2\pi(t-s)}}e^{-\frac{(x+\tilde{\sigma}^2(t-s)/2)^2}{2\tilde{\sigma}^2(t-s)}}$$

Note that the initial mass of generation $s$ is $\zeta ds$ and only $\zeta e^{-\zeta(t-s)}ds$ are left at time $t$. Hence, the distribution of log wealth share of all agents is a mixture of the within-generation p.d.f.s, i.e.,

$$\begin{aligned}
g(x,t) &= e^{-\zeta t}g(x,t,0)+\int_0^t \zeta e^{-\zeta(t-s)}g(x,t,s)ds \\
&= e^{-\zeta t}g(x,t,0) \\
&+ Ce^{-\frac{1}{2}x}\left[e^{-\frac{\sqrt{1+\frac{8\zeta}{\tilde{\sigma}^2}}|x|}{2}}\Phi\left(-\frac{|x|}{\tilde{\sigma}\sqrt{t}}+\frac{\sqrt{8\zeta+\tilde{\sigma}^2}}{2}\sqrt{t}\right)\right] \\
&- Ce^{-\frac{1}{2}x}\left[e^{-\frac{\sqrt{1+\frac{8\zeta}{\tilde{\sigma}^2}}|x|}{2}}\Phi\left(-\frac{|x|}{\tilde{\sigma}\sqrt{t}}-\frac{\sqrt{8\zeta+\tilde{\sigma}^2}}{2}\sqrt{t}\right)\right]
\end{aligned}$$

where $C=\frac{2\zeta}{\tilde{\sigma}\sqrt{8\zeta+\tilde{\sigma}^2}}$. As $t\to\infty$, the first and third term shrink to zero pointwise, while the second

[10]Note that

$$\begin{aligned}
f(\eta,0) &= \frac{d}{d\eta}\mathbb{P}(\eta_0^i\le\eta) \\
&= \frac{d}{d\eta}\mathbb{P}(\log\eta_0^i\le\log\eta) \\
&= \frac{d}{d\eta}\int_{-\infty}^{\log\eta} g(x,0,0)dx \\
&= \frac{d}{d\log\eta}\int_{-\infty}^{\log\eta} g(x,0,0)dx\frac{d\log\eta}{d\eta} \\
&= \frac{1}{\eta}g(\log\eta,0,0).
\end{aligned}$$

term converges to 1. Hence, $g(x,t)$ converges pointwise to

$$g(x,\infty) = Ce^{-\frac{1}{2}x - \frac{\sqrt{1+\frac{8\zeta}{\tilde{\sigma}^2}}|x|}{2}}.$$

Given the p.d.f. for log wealth share $g$, we obtain the limit wealth distribution as

$$\begin{aligned} f(\eta,\infty) &= \frac{1}{\eta} g(\log\eta,\infty) \\ &= \begin{cases} C\eta^{-\frac{3}{2}+\frac{\sqrt{1+\frac{8\zeta}{\tilde{\sigma}^2}}}{2}}, & 0<\eta<1, \\ C\eta^{-\frac{3}{2}-\frac{\sqrt{1+\frac{8\zeta}{\tilde{\sigma}^2}}}{2}}, & \eta \geq 1. \end{cases} \end{aligned}$$

## A.7 Proof for Proposition 4

Note that

$$\begin{aligned} d(m_t^i \bar{n}_t) = &- \left(\rho - \tilde{\sigma}_t^2\right) m_{t-}^i \bar{n}_t dt \\ &+ \zeta[e^{-\rho t}(\rho+\zeta)^{-1} - m_{t-}^i \bar{n}_t]dt \\ &- \tilde{\sigma}_t m_{t-}^i \bar{n}_t d\tilde{Z}_t^i + [e^{-\rho t}(\rho+\zeta)^{-1} - m_{t-}^i \bar{n}_t] d(\tilde{J}_t^i - \zeta t). \end{aligned}$$

Let $\tilde{\sigma}_t = \tilde{\sigma}$. Divide by $m_0^i$ and integrate across $i$ to obtain

$$d(\bar{m}_t \bar{n}_t) = -(\rho + \zeta - \tilde{\sigma}^2)\bar{m}_t \bar{n}_t dt + \zeta e^{-\rho t}\bar{n}_0 dt.$$

If $\zeta \neq \tilde{\sigma}^2$, the ODE admits the following solution

$$\begin{aligned} \bar{m}_t \bar{n}_t = &\bar{n}_0 \exp(-(\rho+\zeta-\tilde{\sigma}^2)t)\left(\frac{\zeta}{\tilde{\sigma}^2-\zeta}+1\right) \\ &- \bar{n}_0 e^{-\rho t}\frac{\zeta}{\tilde{\sigma}^2-\zeta}. \end{aligned}$$

If $\zeta = \tilde{\sigma}^2$, we have instead $\bar{m}_t \bar{n}_t = \bar{n}_0 e^{-\rho t}(1+\zeta t)$. As a special case, when $t=0$, we recover our initial condition $\bar{m}_0 \bar{n}_0 = \bar{n}_0$.

The aggregate TVC holds if and only if

$$\rho + \zeta - \tilde{\sigma}^2 > 0.$$

## A.8 Calibration

**Table A.1:** Parameter Values.

| Variable | Notation | Value | Notes |
|---|---|---|---|
| *Baseline Model with Permanent Idiosyncratic Risk* | | | |
| risk aversion | $\gamma$ | 2 | |
| subjective discount rate | $\rho$ | 0.02 | |
| capital adjustment cost | $\phi$ | 8.1 | |
| Depreciation rate | $\delta$ | 0.055 | |
| Government spending | $\mathfrak{g}$ | 0.15 | |
| real growth rate | $g$ | 0% | Targeted by choosing $a$ |
| idiosyncratic risk variance | $\tilde{\nu}$ | 0.54 | From Brunnermeier et al. (2022) |
| share of undiversifiable risk | $\bar{\chi}$ | 0.3 | From Brunnermeier et al. (2022) |
| share of debt | $\vartheta$ | 0.18 | From Brunnermeier et al. (2022) |
| *Model Variant with Transitory Idiosyncratic Risk* | | | |
| weak wealth reset intensity | $\zeta$ | 2.5% | |
| moderate wealth reset intensity | $\zeta$ | 3.8% | |
| strong wealth reset intensity | $\zeta$ | 5.4% | |

# B Model with General CRRA Utility

In this section, we consider a model variant where households have constant relative risk aversion $\gamma$. As shown in the log utility case, aggregate risk does not matter for our core intuition. Hence, for tractability, we assume there is no aggregate risk, which implies constant consumption-wealth ratio $\hat{c} := c_t^i/n_t^i$. The derivation is similar to the log case. We provide a proof sketch as follows.

## B.1 Model Derivation

### B.1.1 Household's Problem

Similar to the log case, guess that the value function takes the form $V(n_t^i, t) = \frac{\omega_t}{\rho+\zeta}\frac{(n_t^i)^{1-\gamma}}{1-\gamma} + v_t$, where $\omega_t$ and $v_t$ depends on investment opportunity in aggregate level. The Hamilton-Jacobi-Bellman equation is

$$(\rho+\zeta)V(n_t^i,t) = \max_{c_t^i,\iota_t^i,\theta_t^i} \mathbb{E}_t\left[\frac{(c_t^i)^{1-\gamma}}{1-\gamma}dt + \zeta V(\bar{n}_t,t)dt + V_t(n_t^i,t)dt + V_n(n_t^i,t)dn_t^i + \frac{1}{2}V_{nn}(n_t^i,t)\langle dn_t^i, dn_t^i\rangle\right]/dt$$

$$\rho v_t + \omega_t\frac{(n_t^i)^{1-\gamma}}{1-\gamma} - \frac{\zeta}{\rho+\zeta}\omega_t\frac{(\bar{n}_t)^{1-\gamma}}{1-\gamma}$$

$$= \max_{c_t^i,\iota_t^i,\theta_t^i} \mathbb{E}_t\left\{\frac{(c_t^i)^{1-\gamma}}{1-\gamma}dt + \frac{\dot{w}_t}{\rho+\zeta}\frac{(n_t^i)^{1-\gamma}}{1-\gamma}dt + \dot{v}_t dt + \omega_t\frac{(n_t^i)^{1-\gamma}}{\rho+\zeta}\frac{dn_t^i}{n_t^i} - \omega_t\frac{\gamma}{2}\frac{(n_t^i)^{1-\gamma}}{\rho+\zeta}\left\langle\frac{dn_t^i}{n_t^i},\frac{dn_t^i}{n_t^i}\right\rangle\right\}/dt.$$

For simplicity, we denote $\hat{\rho} := \rho+\zeta$. The first order conditions w.r.t. $c_t^i$, $\iota_t^i$, $\theta_t^{K,i}$, $\theta_t^{E,i}$ and $\theta_t^{\bar{E},i}$ are

$$-\omega_t\frac{(n_t^i)^{1-\gamma}}{\hat{\rho}n_t^i} + (c_t^i)^{-\gamma} = 0, \tag{B.1}$$

$$-\frac{1}{q_t^K} + \frac{1}{1+\phi\iota_t^i} = 0, \tag{B.2}$$

$$\frac{\mathbb{E}_t[dr_t^{K,i}]}{dt} - \frac{\mathbb{E}_t[dr_t^B]}{dt} = \gamma(\theta_t^{K,i}+\theta_t^{E,i})\tilde{\nu}_t^2 - (1-\bar{\chi})\lambda_t^i, \tag{B.3}$$

$$\frac{\mathbb{E}_t[dr_t^{E,i}]}{dt} - \frac{\mathbb{E}_t[dr_t^B]}{dt} = \gamma(\theta_t^{K,i}+\theta_t^{E,i})\tilde{\nu}_t^2 - \lambda_t^i, \tag{B.4}$$

$$\frac{\mathbb{E}_t[dr_t^{\bar{E}}]}{dt} - \frac{\mathbb{E}_t[dr_t^B]}{dt} = 0. \tag{B.5}$$

### B.1.2 Verifying Value Function

We can solve for the symmetric equilibrium and verify the value function following the same steps in the log case. Specifically, plugging endogenous variables into the HJB Equation, the coefficient

of $(n_t^i)^{1-\gamma}$ should be zero, i.e.,

$$\begin{aligned}
0 &= -\omega_t \frac{1}{1-\gamma} + \frac{1}{\hat{\rho}}\dot{\omega}_t \frac{1}{1-\gamma} + \left(\frac{\omega_t}{\hat{\rho}}\right)^{-\frac{1-\gamma}{\gamma}} \frac{1}{1-\gamma} - \omega_t \frac{1}{\hat{\rho}} \left(\frac{\omega_t}{\hat{\rho}}\right)^{-\frac{1}{\gamma}} \\
&\quad + \omega_t \frac{1}{\hat{\rho}} \vartheta_t \mathbb{E}[r_t^B]/dt + \omega_t \frac{1}{\hat{\rho}}(1-\vartheta_t)\mathbb{E}[r_t^{K,i}]/dt - \frac{\gamma}{2}\omega_t \frac{1}{\hat{\rho}} \bar{\chi}^2 (1-\vartheta_t)^2 \tilde{\nu}_t^2, \\
0 &= -\rho v_t + \dot{v}_t + \frac{\zeta}{\hat{\rho}} \omega_t \frac{(\bar{n}_t)^{1-\gamma}}{1-\gamma}.
\end{aligned}$$

which are ODEs for $\omega_t$ and $v_t$. Note that $\vartheta_t, \mu_t^\vartheta$ and $\bar{n}_t$ depend on aggregate level variables only, so do $\omega_t$ and $v_t$. The value function is verified. In the absence of aggregate shocks, $\omega_t = \omega$ is constant if the economy starts at the steady state. Consider the steady state where $\nu_t = \nu, \vartheta_t = \vartheta$. Plug in $\mathbb{E}[r_t^B]/dt = r^f$, $\mathbb{E}[r_t^{K,i}]/dt = r^f + \gamma \bar{\chi}^2 (1-\vartheta)^2 \tilde{\nu}^2$ and $\omega_t = \hat{\rho}\hat{c}^{-\gamma}$ to get

$$0 = -\hat{\rho}\hat{c}^{-\gamma}\frac{1}{1-\gamma} + \hat{c}^{1-\gamma}\frac{1}{1-\gamma} - \hat{c}^{1-\gamma} + \hat{c}^{-\gamma} r^f + \hat{c}^{-\gamma}\gamma\bar{\chi}^2(1-\vartheta)^2\tilde{\sigma}^2 - \hat{c}^{-\gamma}\frac{\gamma}{2}\bar{\chi}^2(1-\vartheta)^2\tilde{\nu}^2,$$

i.e.,

$$r^f = \frac{-\gamma\hat{c} + \hat{\rho}}{1-\gamma} - \frac{\gamma}{2}\bar{\chi}^2(1-\vartheta)^2\tilde{\nu}^2, \tag{B.6}$$

The risk free rate $r^f$ equals $\mathbb{E}_t[r_t^B]$ and $\mathbb{E}_t[r_t^E]$ in the absence of aggregate risk, which can be derived as

$$r^f = \Phi(\iota) - \delta + \hat{c} - \gamma\bar{\chi}^2(1-\vartheta)^2\tilde{\nu}^2.$$

$\hat{c}$ can be solved numerically by

$$\hat{c} - (\gamma-1)\Phi(\iota) = \hat{\rho} - (\gamma-1)\delta - \frac{\gamma(\gamma-1)}{2}\bar{\chi}^2(1-\vartheta)^2\tilde{\nu}^2 \tag{B.7}$$

where

$$\iota = \frac{(a-\mathfrak{g})(1-\vartheta)-\hat{c}}{1+\hat{c}\phi-\vartheta}$$
$$\Phi(\iota) = \frac{1}{\phi}\log(1+\phi\iota).$$

Since

$$\frac{d\iota}{d\hat{c}} = \frac{-(1+\hat{c}\phi-\vartheta)-((a-\mathfrak{g})(1-\vartheta)-\hat{c})\phi}{(1+\hat{c}\phi-\vartheta)^2} < 0,$$

$\iota$ and $\Phi(\iota)$ are decreasing in $\hat{c}$. The LHS is monotonically increasing in $\hat{c}$. There is a unique positive solution of $\hat{c}$ if

$$\frac{\gamma}{2}\bar{\chi}^2(1-\vartheta)^2\tilde{\nu}^2 + \frac{\hat{\rho}}{1-\gamma} + \delta < \Phi(a-\mathfrak{g}).$$

Note that

$$\frac{d\hat{c}}{d\hat{\rho}} = \left(1-(\gamma-1)\Phi'(\iota)\frac{d\iota}{dc}\right)^{-1} > 0.$$

Consumption rate increases when subjective discount rate $\rho$ or wealth reset shock intensity $\zeta$ goes up.

### B.1.3 SDF and Wealth Process

We focus on the steady state where consumption-wealth ratio is constant ($\hat{c}$) in the absence of aggregate risks. Again, denote $\tilde{\sigma} = \bar{\chi}(1-\vartheta)\tilde{\nu}$. We have

$$dn_t^i = g_t n_{t-}^i dt + \tilde{\sigma} n_{t-}^i d\tilde{Z}_t^i + (\bar{n}_t - n_{t-}^i)d\tilde{J}_t^i,$$

which implies

$$\frac{dm_t^i}{m_{t-}^i} = -\left[\rho + \gamma g_t - \frac{\gamma(\gamma+1)}{2}\tilde{\sigma}_t^2\right]dt - \gamma\tilde{\sigma}d\tilde{Z}_t^i + \frac{1}{m_{t-}^i}\left(e^{-\rho t}\hat{c}^{-\gamma}\bar{n}_t^{-\gamma} - e^{-\rho t}\hat{c}^{-\gamma}(n_{t-}^i)^{-\gamma}\right)d\tilde{J}_t^i.$$

## B.2 Transversality Conditions with Transitory Idiosyncratic Risk

In this section, we explicitly characterize and examine the transversality condition terms. We impose an additional assumption that $-\rho + (1-\gamma)g < 0$. That is, the aggregate endowment flow has finite valuation in a representative agent economy with growth rate $g$. This assumption is naturally satisfied in most settings where $\rho > 0, \gamma \geq 1, g \geq 0$.

**The Wealth Distribution Perspective.** Consider the wealth reset shock with intensity $\zeta$. The wealth dynamics become

$$d\eta_t^i = \tilde{\sigma}\eta_{t-}^i d\tilde{Z}_t^i + (1-\eta_{t-}^i)d\tilde{J}_t^i.$$

By Itô's lemma,

$$\begin{aligned} d(\eta_t^i)^{-\gamma} = &\left[\frac{\gamma(\gamma+1)}{2}\tilde{\sigma}^2(\eta_{t-}^i)^{-\gamma} + \zeta(1-(\eta_{t-}^i)^{-\gamma})\right]dt \\ &- \gamma\tilde{\sigma}(\eta_{t-}^i)^{-\gamma}d\tilde{Z}_t^i + (1-(\eta_{t-}^i)^{-\gamma})(d\tilde{J}_t^i - \zeta dt). \end{aligned}$$

Take expectation on both sides yields an ODE w.r.t. $\mathbb{E}[(\eta_t^i)^{-\gamma}]$, i.e.,

$$d\mathbb{E}[(\eta_t^i)^{-\gamma}] = \left[\frac{\gamma(\gamma+1)}{2}\tilde{\sigma}^2\mathbb{E}[(\eta_t^i)^{-\gamma}] + \zeta(1-\mathbb{E}[(\eta_t^i)^{-\gamma}])\right]dt.$$

For $\zeta \neq \frac{\gamma(\gamma+1)}{2}\tilde{\sigma}^2$, the solution to the ODE is

$$\mathbb{E}_0\left[(\eta_t^i)^{-\gamma}\right] = \left(\frac{\zeta}{\frac{\gamma(\gamma+1)}{2}\tilde{\sigma}^2 - \zeta} + (\eta_0^i)^{-\gamma}\right)e^{[\frac{\gamma(\gamma+1)}{2}\tilde{\sigma}^2 - \zeta]t} - \frac{\zeta}{\frac{\gamma(\gamma+1)}{2}\tilde{\sigma}^2 - \zeta}.$$

If $\zeta = \frac{\gamma(\gamma+1)}{2}\tilde{\sigma}^2$, the solution becomes

$$\mathbb{E}_0\left[(\eta_t^i)^{-\gamma}\right] = (\eta_0^i)^{-\gamma} + \zeta t.$$

Recall that growth rate of aggregate wealth and risk-free rate jointly satisfy

$$r^f = \rho + \zeta + \gamma g - \frac{\gamma(\gamma+1)}{2}\tilde{\sigma}^2, \tag{B.8}$$

$$g = -\hat{c} + r^f + \gamma\tilde{\sigma}^2, \tag{B.9}$$

where the first equation is from household's Euler equation (note that household does not effectively get the payoff from the risk free asset when wealth reset event happens, hence the risk-free rate does not equal the negative drift rate of the SDF), and the second equation is from the fact that household's wealth portfolio is tradable to herself, of which the total return is $r^f$ plus idiosyncratic risk premium $\gamma\tilde{\sigma}^2$. The simultaneous equations jointly solve for

$$g = \frac{-\hat{c} + \rho + \zeta}{1-\gamma} + \frac{\gamma}{2}\tilde{\sigma}^2,$$

The aggregate TVC condition can be stated as

$$\begin{aligned}
\lim_{T\to\infty}\mathbb{E}[m_T^i\bar{n}_T] &= \lim_{T\to\infty}(\hat{c}^{-\gamma}e^{-\rho T}\bar{n}_T^{1-\gamma})\mathbb{E}\left[(\eta_T^i)^{-\gamma}\right] \\
&= \lim_{T\to\infty}\hat{c}^{-\gamma}\bar{n}_0^{1-\gamma}e^{-\rho T}\exp\left(\left(-\hat{c}+\rho+\zeta-\frac{\gamma(\gamma-1)}{2}\tilde{\sigma}^2\right)T\right) \\
&\times\exp\left(\left(\frac{\gamma(\gamma+1)}{2}\tilde{\sigma}^2-\zeta\right)T\right)\left(\frac{\zeta}{\frac{\gamma(\gamma+1)}{2}\tilde{\sigma}^2-\zeta}+(\eta_0^i)^{-\gamma}\right) \\
&- \lim_{T\to\infty}\hat{c}^{-\gamma}\bar{n}_0^{1-\gamma}e^{-\rho T}\bar{n}_T^{1-\gamma}\frac{\zeta}{\frac{\gamma(\gamma+1)}{2}\tilde{\sigma}^2-\zeta} \\
&= \lim_{T\to\infty}\hat{c}^{-\gamma}\bar{n}_0^{1-\gamma}\exp(-(\hat{c}-\gamma\tilde{\sigma}^2)T)\left(\frac{\zeta}{\frac{\gamma(\gamma+1)}{2}\tilde{\sigma}^2-\zeta}+(\eta_0^i)^{-\gamma}\right) \\
&- \lim_{T\to\infty}\hat{c}^{-\gamma}\bar{n}_0^{1-\gamma}e^{-\rho T}\bar{n}_T^{1-\gamma}\frac{\zeta}{\frac{\gamma(\gamma+1)}{2}\tilde{\sigma}^2-\zeta}.
\end{aligned}$$

In the $\zeta = \frac{\gamma(\gamma+1)}{2}\tilde{\sigma}^2$ case,

$$\begin{aligned}\lim_{T\to\infty} \mathbb{E}[m_T^i \bar{n}_T] &= \lim_{T\to\infty} (\hat{c}^{-\gamma} e^{-\rho T} \bar{n}_T^{1-\gamma}) \mathbb{E}\left[(\eta_T^i)^{-\gamma}\right] \\ &= \lim_{T\to\infty} (\hat{c}^{-\gamma} e^{[-\rho+(1-\gamma)]gT} \bar{n}_0^{1-\gamma}) [(\eta_0^i)^{-\gamma} + \zeta T].\end{aligned}$$

Hence, the aggregate TVC holds if and only if

$$\hat{c} > \gamma\tilde{\sigma}^2$$

and

$$-\rho + (1-\gamma)g < 0.$$

The first condition states that the wealth reset intensity should be big enough such that the marginal utility dispersion does not explode too fast, hence risk free rate will not be lower than the growth rate. The second condition is imposed as we are evaluating long-lived assets with the realized marginal utility process, we want to rule out the case where the whole economy shrinks and marginal utilities explode on average. In other word, we assume transversality condition hold had the economy been populated by one representative agent instead of heterogeneous agents. In this way, we shed light on the effect of heterogeneity and idiosyncratic risk.

**The r minus g Perspective** Note that Eq. (B.9) implies

$$r^f - g = \hat{c} - \gamma\tilde{\sigma}^2.$$

Note that

$$\begin{aligned} d(m_t^i \bar{n}_t) = &- \left[\rho - \gamma g + \frac{\gamma(\gamma+1)}{2}\tilde{\sigma}^2 + g\right] m_{t-}^i \bar{n}_t dt \\ &+ \zeta[e^{-\rho t}\hat{c}^{-\gamma}\bar{n}_t^{1-\gamma} - m_{t-}^i \bar{n}_t]dt \\ &- \gamma\tilde{\sigma} m_{t-}^i \bar{n}_t d\tilde{Z}_t^i + [e^{-\rho t}\hat{c}^{-\gamma}\bar{n}_t^{1-\gamma} - m_{t-}^i \bar{n}_t]d(\tilde{J}_t^i - \zeta t). \end{aligned}$$

Plug in the expression of $g$ and $r^f - g = \hat{c} - \gamma\tilde{\sigma}^2$ to get

$$\begin{aligned} d(m_t^i \bar{n}_t) = &- \left[r^f - g\right] m_{t-}^i \bar{n}_t dt + \zeta e^{-\rho t}\hat{c}^{-\gamma}\bar{n}_t^{1-\gamma} dt \\ &- \gamma\tilde{\sigma} m_{t-}^i \bar{n}_t d\tilde{Z}_t^i + [e^{-\rho t}\hat{c}^{-\gamma}\bar{n}_t^{1-\gamma} - m_{t-}^i \bar{n}_t]d(\tilde{J}_t^i - \zeta t). \end{aligned}$$

The drift term reflects the discounted growth rate. There is an additional drift adjustment term as wealth jump creates a discrepancy between the SDF and marginal utility (in other word, the wealth portfolio does not pays $n_{t-}^i$ when jump event happens). Divide by $m_0^i$ and integrate across $i$ to obtain

$$d(\bar{m}_t \bar{n}_t) = -(r^f - g)\bar{m}_t \bar{n}_t dt + \zeta e^{-\rho t}\left(\frac{\bar{n}_t}{\bar{n}_0}\right)^{1-\gamma} \bar{n}_0 dt.$$

The ODE admits the following solution[11]

$$\bar{m}_t\bar{n}_t = \bar{n}_0 \exp(-(r^f - g)t)\left(\frac{\zeta}{\frac{\gamma(\gamma+1)}{2}\tilde{\sigma}^2 - \zeta} + 1\right) - \bar{n}_0 e^{(-\rho-(\gamma-1)g)t}\frac{\zeta}{\frac{\gamma(\gamma+1)}{2}\tilde{\sigma}^2 - \zeta}.$$

As a special case, when $t = 0$, we recover our initial condition $\bar{m}_0\bar{n}_0 = \bar{n}_0$. Given $-\rho-(\gamma-1)g < 0$, the aggregate TVC holds if and only if

$$r^f > g, \quad \text{i.e., } \hat{c} > \gamma\tilde{\sigma}^2.$$

## B.3 Arbitrary Wealth Distribution After Wealth Reset

We show that the main results are preserved even if we impose arbitrary wealth distribution after wealth resets. Denote the pdf for wealth resets as $h(\tilde{\eta})$. We require that $\int_0^\infty \tilde{\eta}h(\tilde{\eta})d\tilde{\eta} = 1$, $\int_0^\infty \tilde{\eta}^{-\gamma}h(\tilde{\eta})\ d\tilde{\eta} < \infty$, i.e., the expected marginal utility immediately after the wealth reset is well-defined. The previous specification can be viewed as a special case where $h(\tilde{\eta}) = \delta(\tilde{\eta} - 1)$, where $\delta(.)$ is the dirac function. First, we show that this general case admits the same solution of aggregate variables in the reset-to-average case. Heuristically, household's HJB equation is given

[11] To solve the ODE, note that

$$\begin{aligned} d(\exp((r^f - g)t)\bar{m}_t\bar{n}_t) &= \exp((r^f - g)t)d(\bar{m}_t\bar{n}_t) + (r^f - g)\exp((r^f - g)t)(\bar{m}_t\bar{n}_t)dt \\ &= \zeta \exp((-\rho + (1-\gamma)g + r^f - g)t)\bar{n}_0 dt, \end{aligned}$$

where according to Eq. (B.8),

$$-\rho + (1-\gamma)g + r^f - g = \zeta - \frac{\gamma(\gamma+1)}{2}\tilde{\sigma}^2.$$

Integrate both sides to get

$$\exp((r^f - g)t)\bar{m}_t\bar{n}_t = \bar{m}_0\bar{n}_0 + \zeta\bar{n}_0\frac{1}{\zeta - \frac{\gamma(\gamma+1)}{2}\tilde{\sigma}^2}[\exp((\zeta - \frac{\gamma(\gamma+1)}{2}\tilde{\sigma}^2)t) - 1],$$

note that $\zeta - \frac{\gamma(\gamma+1)}{2}\tilde{\sigma}^2 - (r^f - g) = -\rho + (1-\gamma)g$ and $\bar{m}_0 = 1$. Move $\exp(r^f - g)$ to the RHS to get the solution.

by

$$V(n_t^i,t) = \max_{c_t^i,\iota_t^i,\theta_t^i} \mathbb{E}_t\left[\frac{(c_t^i)^{1-\gamma}}{1-\gamma}dt + e^{-\rho dt}(1_{\text{jump}}V(n_{t+dt}^i,t+dt) + 1_{\text{nojump}}V(n_{t+dt}^i,t+dt))\right]$$
$$= \max_{c_t^i,\iota_t^i,\theta_t^i} \mathbb{E}_t\left[\frac{(c_t^i)^{1-\gamma}}{1-\gamma}dt + (1-\rho dt)(\zeta dt\mathbb{E}^\eta[V(\tilde{\eta}\bar{n}_t,t+dt)] + (1-\zeta dt)V(n_{t+dt}^i,t+dt))\right].$$

$\mathbb{E}^\eta[.]$ denotes taking expectation w.r.t. $\tilde{\eta}$. The only difference is that the after-jump value function changed from $V(\bar{n}_t,t+dt)$ to $V(\tilde{\eta}\bar{n}_t,t+dt)$. Since the assignment of after-jump wealth is independent with agent's pre-jump wealth, the expectation depends on aggregate variables only. As a result, $h(\tilde{\eta})$ only affects $v_t$ in the value function, which does not matter household's consumption or investment choices.

Next, we show that $h(\tilde{\eta})$ does not matter for the transversality condition either. Define the compound Poisson process

$$\tilde{Q}_t^i = \sum_{k=1}^{\tilde{J}_t^i}(\tilde{\eta}_k^i)^{-\gamma},$$

where $\tilde{\eta}_k^i$ is the post-reset wealth share in each reset shock, i.i.d. drawn from the probability density function $h(\tilde{\eta})$. The compensated compound Poisson process is

$$\tilde{Q}_t^i - \mathbb{E}[(\tilde{\eta})^{-\gamma}]\zeta t,$$

which is a martingale. The dynamics of SDF can be written as

$$\begin{aligned}
dm_t^i = &-\left[\rho + \gamma g - \frac{\gamma(\gamma+1)}{2}\bar{\chi}^2(1-\vartheta)^2\tilde{\nu}^2 + \zeta\right]m_{t-}^i dt \\
&+ \zeta e^{-\rho t}\hat{c}^{-\gamma}\bar{n}_t^{-\gamma}\mathbb{E}[(\tilde{\eta})^{-\gamma}]dt \\
&- \gamma\bar{\chi}(1-\vartheta)\tilde{\nu}m_{t-}^i d\tilde{Z}_t^i \\
&+ e^{-\rho t}\hat{c}^{-\gamma}\bar{n}_t^{-\gamma}d(\tilde{Q}_t^i - \mathbb{E}[(\tilde{\eta})^{-\gamma}]\zeta t) \\
&- m_{t-}^i(d\tilde{J}_t^i - \zeta t)
\end{aligned}$$

Hence,

$$\begin{aligned} dm_t^i\bar{n}_t = &-\left[\rho+(\gamma-1)g-\frac{\gamma(\gamma+1)}{2}\bar{\chi}^2(1-\vartheta)^2\tilde{\nu}^2+\zeta\right]m_{t-}^i\bar{n}_t dt \\ &+\zeta e^{-\rho t}\hat{c}^{-\gamma}\bar{n}_t^{1-\gamma}\mathbb{E}[(\tilde{\eta})^{-\gamma}]dt \\ &-\gamma\bar{\chi}(1-\vartheta)\tilde{\nu}m_{t-}^i\bar{n}_t d\tilde{Z}_t^i \\ &+e^{-\rho t}\hat{c}^{-\gamma}\bar{n}_t^{1-\gamma}d(\tilde{Q}_t^i-\mathbb{E}[(\tilde{\eta})^{-\gamma}]\zeta t) \\ &-m_{t-}^i(d\tilde{J}_t^i-\zeta t). \end{aligned}$$

Take expectation to obtain

$$\begin{aligned} d\mathbb{E}\left[m_t^i n_t^i\right] = &-\left[\rho+(\gamma-1)g-\frac{\gamma(\gamma+1)}{2}\bar{\chi}^2(1-\vartheta)^2\tilde{\nu}^2+\zeta\right]\mathbb{E}[m_{t-}^i\bar{n}_t]dt \\ &+\zeta e^{-\rho t}\hat{c}^{-\gamma}\mathbb{E}[(\tilde{\eta})^{-\gamma}]\bar{n}_t^{1-\gamma}dt. \end{aligned}$$

Note that $\rho+(\gamma-1)g-\frac{\gamma(\gamma+1)}{2}\bar{\chi}^2(1-\vartheta)^2\tilde{\nu}^2+\zeta = r^f-g = \hat{c}-\gamma\bar{\chi}^2(1-\vartheta)^2\tilde{\nu}^2$. The solution is

$$e^{(r^f-g)t}\mathbb{E}[m_t^i\bar{n}_t]-m_0^i\bar{n}_0 = \frac{\zeta\hat{c}^{-\gamma}\mathbb{E}[(\tilde{\eta})^{-\gamma}]\bar{n}_0^{1-\gamma}}{\zeta-\frac{\gamma(\gamma+1)}{2}\bar{\chi}^2(1-\vartheta)^2\tilde{\nu}^2}(e^{[\zeta-\frac{\gamma(\gamma+1)}{2}\bar{\chi}^2(1-\vartheta)^2\tilde{\nu}^2]t}-1),$$

i.e.,

$$\begin{aligned} \mathbb{E}[m_t^i\bar{n}_t] = &\left(m_0^i\bar{n}_0-\frac{\zeta\hat{c}^{-\gamma}\mathbb{E}[(\tilde{\eta})^{-\gamma}]\bar{n}_0^{1-\gamma}}{\zeta-\frac{\gamma(\gamma+1)}{2}\bar{\chi}^2(1-\vartheta)^2\tilde{\nu}^2}\right)\exp(-(\hat{c}-\gamma\bar{\chi}^2(1-\vartheta)^2\tilde{\nu}^2)t) \\ &+\frac{\zeta\hat{c}^{-\gamma}\mathbb{E}[(\tilde{\eta})^{-\gamma}]\bar{n}_0^{1-\gamma}}{\zeta-\frac{\gamma(\gamma+1)}{2}\bar{\chi}^2(1-\vartheta)^2\tilde{\nu}^2}\exp(-(\rho+(\gamma-1)g)t). \end{aligned}$$

For all $h(\tilde{\eta})$ such that $\mathbb{E}[(\tilde{\eta})^{-\gamma}]$ exists, aggregate TVC depends on $\hat{c}-\gamma\bar{\chi}^2(1-\vartheta)^2\tilde{\nu}^2$, i.e., $r^f-g$. In the first part we show that $h(\tilde{\eta})$ does not matter for any aggregate variables. Hence, $h(\tilde{\eta})$ does not matter for aggregate TVC.

## B.4 Model with Stochastic Death

We model the stochastic death as follows. Define $\{d_t^i\}$ as a Poisson process with intensity $\zeta$. Death events are independent across agents, i.e., $\{d_t^i\}$ and $\{d_t^j\}$ are stochastically independent for $i \neq j$. Let $d_0^i = 0$. The stochastic death time $\tau^i$ is defined as

$$\tau^i := \inf\{t > 0 \mid d_t^i \geq 1\}$$

The death process is hence defined by

$$death_t^i = d^i_{\{t \wedge \tau^i\}},$$

i.e., when agent $i$ is alive, $death_t^i = 0$. Otherwise, $death_t^i = 1$.

Consider household's problem. Heuristically,

$$\begin{aligned} V(n_t^i, t) &= \max_{c_t^i, \iota_t^i, \theta_t^i} \mathbb{E}_t \left[ \frac{(c_t^i)^{1-\gamma}}{1-\gamma} dt + e^{-\rho dt} (1_{\text{death}} V(n^i_{t+dt}, t+dt) + 1_{\text{nodeath}} V(n^i_{t+dt}, t+dt)) \right] \\ &= \max_{c_t^i, \iota_t^i, \theta_t^i} \mathbb{E}_t \left[ \frac{(c_t^i)^{1-\gamma}}{1-\gamma} dt + (1-\rho dt)(\zeta dt \times 0 + (1-\zeta dt) V(n^i_{t+dt}, t+dt)) \right] \end{aligned}$$

The Hamilton-Jacobi-Bellman equation is

$$\begin{aligned} &(\rho + \zeta) V(n_t^i, t) = \max_{c_t^i, \iota_t^i, \theta_t^i} \mathbb{E}_t \left[ \frac{(c_t^i)^{1-\gamma}}{1-\gamma} dt + \zeta \times 0 + V_t dt + V_n(n_t^i, t) dn_t^i + \frac{1}{2} V_{nn}(n_t^i, t) \langle dn_t^i, dn_t^i \rangle \right] / dt \\ &\rho v_t + \omega_t \frac{(n_t^i)^{1-\gamma}}{1-\gamma} - 0 \\ &= \max_{c_t^i, \iota_t^i, \theta_t^i} \mathbb{E} \left\{ \frac{(c_t^i)^{1-\gamma}}{1-\gamma} dt + \frac{\dot{w}_t}{\rho + \zeta} \frac{(n_t^i)^{1-\gamma}}{1-\gamma} dt + \dot{v}_t dt + \omega_t \frac{(n_t^i)^{1-\gamma}}{\rho + \zeta} \frac{dn_t^i}{n_t^i} - \omega_t \frac{\gamma}{2} \frac{(n_t^i)^{1-\gamma}}{\rho + \zeta} \left\langle \frac{dn_t^i}{n_t^i}, \frac{dn_t^i}{n_t^i} \right\rangle \right\} / dt. \end{aligned}$$

Note that the HJB is essetianlly the same as the wealth reset model, except that the utility after death is zero while the utility after wealth reset is $V(\bar{n}_t, t)$. The first order conditions are the same, though the solution for $v_t$ is different. Hence, we obtain the same equilibrium in the wealth reset model.

Below, we discuss household's decision and the relevant transversality conditions in a different

perspective. Let $n_t^i$ denote the wealth of agent $i$ at time $t$. For $t > \tau^i$, i.e., after the agent's death, we assume that $n_t^i$ denotes the wealth following the agent's consumption and investment plan if it were still alive. We denote $\check{m}_t^i = e^{-\rho t}\hat{c}^{-\gamma}(n_t^i)^{-\gamma}$ as the hypothetical marginal utility if the agent were still alive.

We denote $m_t^i$ as the agent's actual marginal utility, which is zero after death:

$$m_t^i := \begin{cases} \check{m}_t^i = e^{-\rho t}\hat{c}^{-\gamma}(n_t^i)^{-\gamma}, & death_t^i = 0, \\ 0, & death_t^i = 1. \end{cases}$$

Again, denote $\tilde{\sigma}_t = \bar{\chi}(1-\vartheta_t)\tilde{\nu}_t$. We obtain

$$dm_t^i = \begin{cases} -\left[\rho + \gamma g_t - \frac{\gamma(\gamma+1)}{2}\tilde{\sigma}_t^2\right] m_t^i dt - \tilde{\sigma}_t m_t^i \gamma d\tilde{Z}_t^i, & t < \tau^i, \\ -m_{\tau^i-}^i, & t = \tau, \\ 0 & t > \tau^i. \end{cases}$$

**Aggregate TVC.** For long-lived asset, e.g. government bond at $t < \tau^i$,

$$\begin{aligned} d(m_t^i b_t) &= b_t dm_t^i + m_t^i/P_t dB_t + m_t^i B_t d(1/P_t) + B_t dm_t^i d(1/P_t) \\ &= b_t\left(-(r_t^f - \zeta)m_t^i dt - \tilde{\sigma}_t m_t^i \gamma d\tilde{Z}_t^i\right) \\ &+ i_t b_t m_t^i dt - s_t m_t^i dt + m_t^i b_t (dr_t^B - i_t dt) \\ &= -m_t^i s_t dt + m_t^i \zeta b_t dt - m_t^i b_t \tilde{\sigma}_t \gamma d\tilde{Z}_t^i. \end{aligned}$$

At $t = \tau^i$, $d(m_t^i b_t) = -m_{\tau^i-}^i b_{\tau^i}$. At $t \geq \tau^i$, $d(m_t^i b_t) = 0$. Integrate from $t = 0$ to $t = T$ and take expectation. Recall that $\tau^i$ is independent of $n_t^i$ and $b_t$, and

$$\mathbb{E}_0\left[1_{\{\tau^i \leq u\}}\right] = 1 - e^{-\zeta u}.$$

Hence,

$$
\begin{aligned}
m_0^i b_0 &= \mathbb{E}_0\left[\int_0^T -d(m_t^i b_t) + m_T^i b_T\right] \\
&= (1-e^{-\zeta T})\mathbb{E}_0\left[\int_0^T -d(m_t^i b_t) + m_T^i b_T \middle| \tau^i \le T\right] + e^{-\zeta T}\mathbb{E}_0\left[\int_0^T -d(m_t^i b_t) + m_T^i b_T \middle| \tau^i > T\right].
\end{aligned}
$$

The first term is computed as

$$
\begin{aligned}
&\mathbb{E}_0\left[\int_0^T -d(m_t^i b_t) + m_T^i b_T \middle| \tau^i \le T\right] \\
=&\mathbb{E}_0\left[\int_0^{\tau^i} \check{m}_t^i s_t dt \middle| \tau^i \le T\right] - \mathbb{E}_0\left[\int_0^{\tau^i} \zeta \check{m}_t^i b_t dt \middle| \tau^i \le T\right] + \mathbb{E}_0\left[\check{m}_{\tau^i-}^i b_{\tau i} \middle| \tau^i \le T\right],
\end{aligned}
$$

where

$$
\begin{aligned}
\mathbb{E}_0\left[\int_0^{\tau^i} \check{m}_t^i s_t dt \middle| \tau^i \le T\right] &= \mathbb{E}_0\left[\mathbb{E}_0\left[\int_0^{\tau^i} \check{m}_t^i s_t \, dt \middle| \tau^i\right] \middle| \tau^i \le T\right] \\
&= \frac{1}{1-e^{-\zeta T}} \int_0^T \zeta e^{-\zeta u} \mathbb{E}_0\left[\int_0^u \check{m}_t^i s_t \, dt\right] du \\
&= \frac{1}{1-e^{-\zeta T}} \mathbb{E}_0\left[\int_0^T \left(\int_0^u \zeta e^{-\zeta u} \check{m}_t^i s_t \, dt\right) du\right] \\
&= \frac{1}{1-e^{-\zeta T}} \mathbb{E}_0\left[\int_0^T \left(\int_t^T \zeta e^{-\zeta u} \check{m}_t^i s_t \, du\right) dt\right] \\
&= \frac{1}{1-e^{-\zeta T}} \mathbb{E}_0\left[\int_0^T (e^{-\zeta t} - e^{-\zeta T}) \check{m}_t^i s_t dt\right],
\end{aligned}
$$

and similarly

$$
\begin{aligned}
\mathbb{E}_0\left[\int_0^{\tau^i} \zeta \check{m}_t^i b_t dt \middle| \tau^i \le T\right] &= \frac{1}{1-e^{-\zeta T}} \mathbb{E}_0\left[\int_0^T \zeta (e^{-\zeta t} - e^{-\zeta T}) \check{m}_t^i b_t dt\right], \\
\mathbb{E}_0\left[\check{m}_{\tau^i-}^i b_{\tau i} | \tau^i \le T\right] &= \frac{1}{1-e^{-\zeta T}} \mathbb{E}_0\left[\int_0^T \zeta e^{-\zeta u} \check{m}_u^i b_u du\right].
\end{aligned}
$$

The second term is computed as

$$\begin{aligned}&\mathbb{E}_0\left[\int_0^T -d(m_t^i b_t) + m_T^i b_T \middle| \tau^i > T\right]\\ =&\mathbb{E}_0\left[\int_0^T \check{m}_t^i s_t dt\right] - \mathbb{E}_0\left[\int_0^T \zeta \check{m}_t^i b_t dt\right] + \mathbb{E}_0\left[\check{m}_T^i b_T\right].\end{aligned}$$

Finally,

$$\begin{aligned}m_0^i b_0 &= \mathbb{E}_0\left[\int_0^T -d(m_t^i b_t) + m_T^i b_T\right]\\ &= \mathbb{E}_0\left[\int_0^T (e^{-\zeta t} - e^{-\zeta T})\check{m}_t^i s_t dt\right] + \mathbb{E}_0\left[\int_0^T \zeta e^{-\zeta T}\check{m}_t^i b_t dt\right]\\ &+ e^{-\zeta T}\mathbb{E}_0\left[\int_0^T \check{m}_t^i s_t dt\right] - e^{-\zeta T}\mathbb{E}_0\left[\int_0^T \zeta \check{m}_t^i b_t dt\right] + e^{-\zeta T}\mathbb{E}_0\left[\check{m}_T^i b_T\right]\\ &= \mathbb{E}_0\left[\int_0^T (e^{-\zeta t}\check{m}_t^i) s_t dt\right] + \mathbb{E}_0\left[(e^{-\zeta T}\check{m}_T^i) b_T\right].\end{aligned}$$

Recall that $\check{m}_t^i$ is the marginal utility if the agent were alive at time $t$. The TVC term for this aggregate asset is the discounted asset value $m_T^i b_T$ times the probability of surviving until $T$, which is $e^{-\zeta T}$. The aggregate TVC holds if and only if $\hat{c} - \gamma\tilde{\sigma}^2 > 0$.

**Individual TVC.** Similarly, for individual wealth, we obtain

$$\begin{aligned}d(m_t^i n_t^i) &= n_t^i dm_t^i + m_t^i dn_t^i + dm_t^i dn_t^i\\ &= n_t^i(-(r_t^f - \zeta)m_t^i dt - \tilde{\sigma}_t \gamma m_t^i d\tilde{Z}_t^i)\\ &+ m_t^i(-c_t^i dt + (r_t^f + \tilde{\sigma}_t^2)n_t^i dt + \tilde{\sigma}_t n_t^i d\tilde{Z}_t^i) - \tilde{\sigma}_t^2 m_t^i n_t^i dt\\ &= -c_t^i m_t^i dt + \zeta m_t^i n_t^i dt + (1-\gamma)\tilde{\sigma}_t m_t^i n_t^i d\tilde{Z}_t^i.\end{aligned}$$

for $t < \tau^i$, i.e., if the agent were still alive. Following the same steps above, we obtain

$$m_0^i n_0^i = \mathbb{E}_0\left[\int_0^T (e^{-\zeta t}\check{m}_t^i) c_t^i dt\right] + \mathbb{E}_0\left[(e^{-\zeta T}\check{m}_T^i)\check{n}_T^i\right].$$

The TVC for individual household's wealth is

$$\lim_{T\to\infty} \mathbb{E}_0 \left[ (e^{-\zeta T} \check{m}_T^i \check{n}_T^i) \right] = 0.$$

Again, the analysis is the same as that in A.1 without wealth reset shock, but the subjective discount rate is augmented by $\zeta$.

# C TVC Violation in the Constantinides-Duffie Economy

**The Economy.** There is a continuum of households indexed by $i \in [0, 1]$. Time is discrete. There is a single nondurable consumption good which serves as the numeraire. Households maximize lifetime expected utility with CRRA $\gamma$. The aggregate dividend is exogenous, denoted as $d_t$. The aggregate labor income process is exogenous, denoted as $\ell_t$. The market clearing condition implies that the aggregate consumption process is $c_t = \ell_t + d_t$.

**Uninsured Idiosyncratic Labor Income Risk.** Consumer $i$ has labor income $\ell_t^i$ defined by

$$\ell_t^i = \delta_t^i c_t - d_t,$$

where

$$\delta_t^i = \exp\left[ \sum_{s=1}^{t} \left( \tilde{\varepsilon}_s^i \tilde{\sigma}_s - \frac{\tilde{\sigma}_s^2}{2} \right) \right].$$

The shocks $\tilde{\varepsilon}_t^i$ are standard normal and i.i.d. across periods and individuals.

**Equilibrium.** We consider the no-trade equilibrium where each household holds one unit of equity. Household $i$'s consumption is $c_t^i = \delta_t^i c_t$. Denote $\mathcal{F}_t$ as the information set available at time

$t$. With household $i$'s SDF given by

$$\begin{aligned} m^i_{t,t+1} &= e^{-\rho}\left(\frac{c^i_{t+1}}{c^i_t}\right)^{-\gamma} \\ &= e^{-\rho}\left(\frac{c_{t+1}}{c_t}\right)^{-\gamma}\exp\left(-\gamma\left(\tilde{\varepsilon}^i_{t+1}\tilde{\sigma}_{t+1}-\frac{\tilde{\sigma}^2_{t+1}}{2}\right)\right) \end{aligned}$$

The Euler equation for households $i$ is

$$\begin{aligned} p_t &= \mathbb{E}\left[e^{-\rho}\left(\frac{c_{t+1}}{c_t}\right)^{-\gamma}\exp\left(-\gamma\left(\tilde{\varepsilon}_{i,t+1}\tilde{\sigma}_{t+1}-\frac{\tilde{\sigma}^2_{t+1}}{2}\right)\right)(p_{t+1}+d_{t+1})\middle|\mathcal{F}_t\right] \\ &= \mathbb{E}\left[\mathbb{E}\left[e^{-\rho}\left(\frac{c_{t+1}}{c_t}\right)^{-\gamma}\exp\left(-\gamma\left(\tilde{\varepsilon}_{i,t+1}\tilde{\sigma}_{t+1}-\frac{\tilde{\sigma}^2_{t+1}}{2}\right)\right)(p_{t+1}+d_{t+1})\middle|\mathcal{F}_t\cup\{\tilde{\sigma}_{t+1}\}\right]\middle|\mathcal{F}_t\right] \\ &= \mathbb{E}\left[e^{-\rho}\left(\frac{c_{t+1}}{c_t}\right)^{-\gamma}\exp\left(\frac{\gamma(\gamma+1)}{2}\tilde{\sigma}^2_{t+1}\right)(p_{t+1}+d_{t+1})\middle|\mathcal{F}_t\right]. \end{aligned}$$

It is verified that no-trade equilibrium is valid, as every household's valuation on the aggregate equity aligns.

The risk-free rate is given by

$$\exp(-r_t)=\mathbb{E}_t\left[e^{-\rho}\left(\frac{c_{t+1}}{c_t}\right)^{-\gamma}\exp\left(\frac{\gamma(\gamma+1)}{2}\tilde{\sigma}^2_{t+1}\right)\right]$$

We consider a special case with constant growth rate $c_{t+1}/c_t=\exp(g)$ and constant intensity of idiosyncratic risk $\tilde{\sigma}_t=\tilde{\sigma}$, which implies

$$r=\rho+\gamma g-\frac{\gamma(\gamma+1)}{2}\tilde{\sigma}^2.$$

**Pricing Long-Lived Asset with Positive Dividend.** Assume that the aggregate dividend is positive and grows at the same speed as consumption. If $\tilde{\sigma}^2$ is large enough, such that $\rho+\gamma g-\frac{\gamma(\gamma+1)}{2}\tilde{\sigma}^2<g$, i.e., $r<g$, there is no no-trade equilibrium. Indeed, suppose there were a no-trade

equilibrium. The present value of dividend flow would be

$$PV_t(d) = \sum_{k=1}^{\infty} d_t \exp(gk - rk) = \infty.$$

The equity would be infinitely valuable. That is, $\forall\ p_t < \infty$, agent can buy and hold an arbitrarily small amount of the asset to obtain an (infinitely) larger life-time utility. Hence, there exists no well-defined price process and no no-trade equilibrium exists.

# D Model Extension with Borrowing Constraints

In this appendix, we provide more details for the heterogeneous-agent models with borrowing constraints, which we discussed in Section 4.2.

## D.1 Intertemporal Government Budget Condition

First, we show that with borrowing constraint, the Euler equation becomes an inequality:

$$\mathbb{E}_t \left[ \frac{m^i_{t+1}}{m^i_t} \exp(r_t) \right] \leq 1.$$

Suppose the household maximizes lifetime utility:

$$\max \sum_{t=0}^{\infty} \mathbb{E}_0 \left[ \exp(-\rho t) u(c_t) \right],$$

such that

$$y_t + b_{t-1} \exp(r_{t-1}) \geq c_t + b_t,$$
$$b_t \geq 0.$$

The Lagrangian is

$$\mathcal{L}_t = \sum_{t=0}^{\infty} \mathbb{E}_0 \left[ \exp(-\rho t) u(c_t) + \zeta_t (y_t + b_{t-1} \exp(r_{t-1}) - c_t - b_t) + \mu_t b_t \right].$$

The first-order conditions are

$$\exp(-\rho t) u'(c_t) = \zeta_t,$$
$$\zeta_t - \mu_t = \mathbb{E}_t[\zeta_{t+1} \exp(r_t)],$$

which implies

$$\mathbb{E}_t \left[ \frac{\exp(-\rho) u'(c_{t+1})}{u'(c_t)} \exp(r_t) \right] = 1 - \frac{\mu_t}{u'(c_t)} \leq 1.$$

with $\mu_t > 0$ if the borrowing constraint is binding.

Next, We evaluate government bond in the presence of borrowing constraint. Recall that

$$b_t \exp(r_t) = s_{t+1} + b_{t+1}.$$

Multiplying both sides by $m^i_{t+1}/m^i_t$ and take expectation to obtain

$$b_t \geq b_t \mathbb{E}_t \left[ \frac{m^i_{t+1}}{m^i_t} \exp(r_t) \right] = \mathbb{E}_t \left[ \frac{m^i_{t+1}}{m^i_t} s_{t+1} \right] + \mathbb{E}_t \left[ \frac{m^i_{t+1}}{m^i_t} b_{t+1} \right].$$

The inequality follows $\mathbb{E}_t \left[ \frac{m^i_{t+1}}{m^i_t} \exp(r_t) \right] \leq 1$, which we proved above. Similarly,

$$b_{t+1} \geq \mathbb{E}_{t+1} \left[ \frac{m^i_{t+2}}{m^i_{t+1}} s_{t+2} \right] + \mathbb{E}_{t+1} \left[ \frac{m^i_{t+2}}{m^i_{t+1}} b_{t+2} \right],$$

which implies

$$b_t \geq \sum_{k=1}^{2} \mathbb{E}_t \left[ \frac{m^i_{t+k}}{m^i_t} s_{t+k} \right] + \mathbb{E}_t \left[ \frac{m^i_{t+2}}{m^i_t} b_{t+2} \right].$$

In this way, we can iterate forward to obtain

$$b_t \geq \sum_{k=1}^{\infty} \mathbb{E}_t \left[ \frac{m_{t+k}^i}{m_t^i} s_{t+k} \right] + \lim_{k \to \infty} \mathbb{E}_t \left[ \frac{m_{t+k}^i}{m_t^i} b_{t+k} \right].$$

## D.2 Consumption Lower Bound in Bewley Model

We consider a simple model without aggregate growth or aggregate uncertainty but with uninsured idiosyncratic risk. There is a continuum of agents on $[0, 1]$. Agent $i$'s endowment process $\{y_t^i\}$ follows a finite space Markov chain, with $y_t^i \in \mathcal{Y} = \{y_1, y_2, \ldots, y_K\}$, with $\underline{y} = \min \mathcal{Y} > 0$. The aggregate (after-tax) endowment is $y = \int_0^1 y_t^i \, di$. Agents can invest into risk-free bonds subject to a borrowing constraint $b_t^i \geq 0$. The supply of bond is fixed, $b = \int_0^1 b_t^i \, di$. In the steady state, the interest rate is constant $r$. Agents maximize lifetime utility

$$\max \sum_{t=0}^{\infty} \mathbb{E}_0 \left[ \exp(-\rho t) u(c_t^i) \right],$$

where the $u(.)$ is assumed to be strictly increasing and concave and satisfies the Inada condition $\lim_{c \to 0} u'(c) = \infty$.

The household budget constraint is

$$c_t^i + b_t^i = b_{t-1}^i \exp(r) + y_t^i.$$

In this setting, we show that consumption has a strictly positive lower bound $\underline{c} > 0$ as well. Let $z_t^i = b_{t-1}^i \exp(r) + y_t^i$ denote the resource available at time $t$. By definition, $z_t^i \geq \underline{y}$. The value function is

$$V(z_t^i, y_t^i) = \max_{c_t^i} \{u(c_t^i) + \exp(-\rho) \mathbb{E}_t [V(z_{t+1}^i, y_{t+1}^i)]\}.$$

If $y_t^i$ is i.i.d., the expectation term on the right-hand side does not depend on $y_t^i$, and we can rewrite value function as $V(z_t^i)$, i.e., a function of available resources. In the case, Aiyagari (1994) proves that the consumption has a positive lower bound, at which household consumes all available

resources. The Euler equation is

$$u'(c_t^i) \geq \exp(r-\rho)\mathbb{E}_t[V_z(z_{t+1}^i, y_{t+1}^i)], \tag{D.1}$$

with equality obtained when $b_t^i > 0$.

Next, we show that the consumption has a positive lower bound in the general case. Note that that the marginal value of wealth $V_z(z_t^i, y_t^i)$ is decreasing in wealth $z_t^i$ for any income state $y_t^i$. Hence

$$V_z(z_t^i, y_t^i) \leq V_z(y_t^i, y_t^i) \leq \overline{V}_z := \sup_{y_t^i \in \mathcal{Y}} V_z(y_t^i, y_t^i).$$

Suppose to the contrary that consumption does not have a positive lower bound. From the Inada condition, on one hand, there exists $0 < c^* < \underline{y}$, such that

$$u'(c_t^i) > \exp(r-\rho)\overline{V}_z \geq \exp(r-\rho)\mathbb{E}_t[V_z(z_{t+1}^i, y_{t+1}^i)], \quad \forall c_t^i < c^*. \tag{D.2}$$

On the other hand, for any such $c_t^i < c^* < \underline{y}$, we must have $b_t^i = y_t^i + b_{t-1}^i \exp(r) - c_t^i \geq \underline{y} - c_t^i > 0$, hence Euler equation (D.1) must hold in equality, i.e.,

$$u'(c_t^i) = \exp(r-\rho)\mathbb{E}_t[V_z(z_{t+1}^i, y_{t+1}^i)],$$

which contradicts (D.2). By contradiction, $c_t^i$ must have a positive lower bound $\underline{c}$.

In this case, the transversality term w.r.t. the aggregate bond always hold, since

$$\lim_{T\to\infty} \mathbb{E}_0[e^{-\rho T}u'(c_T^i)b] \leq \lim_{T\to\infty} \mathbb{E}_0[e^{-\rho T}u'(\underline{c})b] = 0.$$

Following the same steps, we can show that $c_t^i/y_t$ has a positive lower bound if the economy grows at speed $g$, i.e., $y_t = \exp(gt)y$, $b_t = \exp(gt)b$ and the utility function is homothetic, i.e., the utility function has CRRA $\gamma$. It suffices to normalize all variables by $y_t$ and use the normalized subjective discount factor $\exp(-\tilde{\rho}) = \exp(-\rho + (1-\gamma)g)$.

Finally, we show that the TVC holds for aggregate debt with CRRA $\gamma > 1$ and positive growth rate $g > 0$. Denote the aggregate endowment as $y_t = y\exp(gt)$ and aggregate debt as $b_t = b\exp(gt)$ at time t. Note that

$$\lim_{T\to\infty}[e^{-\rho T}y_T^{-\gamma}b_T] = \lim_{T\to\infty}\exp((-\rho+(1-\gamma)g)T) = 0,$$

i.e., the TVC holds if the economy were populated by a hypothetical representative agent. With bounded left tail in consumption distribution, the TVC also holds with heterogeneous agents. Indeed,

$$\lim_{T\to\infty}\mathbb{E}_0[e^{-\rho T}(c_T^i)^{-\gamma}b_T] = \lim_{T\to\infty}\mathbb{E}_0[e^{-\rho T}y_T^{-\gamma}b_T]\mathbb{E}_0\left[\left(\frac{c_T^i}{y_T}\right)^{-\gamma}\right] \leq \lim_{T\to\infty}\mathbb{E}_0[e^{-\rho T}y_T^{-\gamma}b_T]\underline{c}^{-\gamma} = 0.$$

That is, uninsured idiosyncratic risk may lower down real interest rate compared to the representative-agent case, but will not lead to a TVC violation.

## D.3 Townsend and Woodford Model

Townsend (1980) provides another illustration of how borrowing constraints affect asset valuation. We consider a simplified version of Woodford (1990) to highlight this mechanism.

The economy consists of two types of infinitely-lived households. Type A receives endowment $e_1$ in even periods and $e_2$ in odd periods, while type B receives endowment $e_2$ in even periods and $e_1$ in odd periods, with $e_1 > e_2 > 0$. This staggered income pattern generates a motive for intertemporal trade between the two types.

Households can invest in government debt with total supply $d$, which is financed by lump-sum taxes $\tau$ equally split between the two household types. The interest rate is $r$. Households face a borrowing constraint: $b_{i,t} \geq 0$, for $i \in \{A, B\}$. Each household's budget constraint is

$$c_{i,t} + \tau/2 + b_{i,t} = e_{i,t} + (1+r)b_{i,t-1},$$

where the tax equals the interest on the government debt, i.e., $\tau = rd$.

Households maximize lifetime utility $\sum_{t=0}^{\infty} \rho^t u(c_{i,t})$. We assume that the endowment level $e_2$ is sufficiently lower than $e_1$ so that the borrowing constraint is binding in the low-endowment period. By market clearing, the other household must hold all the government debt.

For example, in period 1, household $A$ holds no government, whereas household $B$ holds all the government debt:

$$c_{A,1} = e_2 + (1+r)d - \tau/2 \stackrel{\text{def}}{=} \underline{c},$$
$$c_{B,1} = e_1 - d - \tau/2 \stackrel{\text{def}}{=} \bar{c}.$$

Now, we are in a position to evaluate the government debt. Since the borrowing constraint always binds for the low-endowment type, Eq. (18) applies. For household $A$, we have

$$d > \lim_{T\to\infty} \mathbb{E}_0 \left[ \sum_{t=1}^{T} \rho^t \frac{u'(c_{A,t})}{u'(c_{A,0})} \tau \right] + \lim_{T\to\infty} \mathbb{E}_0 \left[ \rho^T \frac{u'(c_{A,T})}{u'(c_{A,0})} d \right] = \lim_{T\to\infty} \mathbb{E}_0 \left[ \sum_{t=1}^{T} \rho^t \frac{u'(c_{A,t})}{u'(c_{A,0})} \tau \right].$$

The proof is presented below. Because marginal utility is bounded, the transversality term vanishes. This example shows that even when the TVC holds, borrowing constraints can increase the value of government debt above the present value of surpluses.

The government budget constraint is

$$d(1+r) = \tau + d,$$

which implies

$$d = \frac{\tau}{r}$$

for $r > 0$ cases. Since household purchase government bonds only at good times, we obtain the

following Euler equations:

$$\rho \frac{u'(\underline{c})}{u'(\bar{c})} = (1+r)^{-1},$$
$$\rho \frac{u'(\bar{c})}{u'(\underline{c})} \leq (1+r)^{-1},$$

which imply

$$\mathbb{E}_0\left[\sum_{t=1}^{T} \rho^t \frac{u'(c_{A,t})}{u'(c_{A,0})}\tau\right] \leq \mathbb{E}_0\left[\sum_{t=1}^{T}(1+r)^{-t}\tau\right] = \frac{\tau}{r}.$$

We have $\mathbb{E}_0\left[\sum_{t=1}^{T} \rho^t \frac{u'(c_{A,t})}{u'(c_{A,0})}\tau\right] < d$ if borrowing constraint binds infinitely often and $\rho\frac{u'(\bar{c})}{u'(\underline{c})} \leq (1+r)^{-1}$. In the meantime, the transversality condition holds.

$$0 \leq \lim_{T\to\infty} \mathbb{E}_0\left[\rho^T \frac{u'(c_{A,T})}{u'(c_{A,0})} d\right] \leq \lim_{T\to\infty} \mathbb{E}_0\left[\rho^T \frac{u'(\underline{c})}{u'(\bar{c})} d\right] = \lim_{T\to\infty} \rho^{T-1}(1+r)^{-1} d = 0.$$

# E Comparing Different Types of Asset Bubbles

In this section we provide two simple models to illustrate the first and second types of bubbles, which follow Hirano and Toda (2024). For simplicity, we focus on deterministic economies. Denote $q_t$ as the price of the Arrow-Debreu security that delivers one unit of consumption at time $t$. Denote the price and cash flow as $p_t$ and $d_t$. No arbitrage implies

$$q_t p_t = q_{t+1}(p_{t+1} + d_{t+1}).$$

Iterate forward to obtain

$$q_t p_t = \sum_{k=1}^{\infty} q_{t+k} d_{t+k} + \lim_{T\to\infty} q_T p_T.$$

We define the first term as fundamental value, and the second term (the TVC term) as the bubble term.

## E.1 Classic Rational Bubbles

Consider a model of classic rational bubble in Hirano and Toda (2024), which is based on Samuelson (1958). In period $t$, a new agent is born and lives for two periods. Its utility function is given by

$$u(c_t(t), c_{t+1}(t)) = \log c_t(t) + \delta \log c_{t+1}(t).$$

We use $(t)$ to denote the agent's cohort, which is based on when it is born. In any given period, there is a young agent who is just born and an old agent who was born in the previous period.

Each agent receives an endowment of $a$ when young and $b$ when old. There is also an intrinsically useless asset (i.e., cash) that has a unit supply and is initially owned by the old agent in period $0$. Let $p_t$ denote the asset's price and let $x_t(t)$ denote the amount of the asset that is purchase by the young agent. The agent's budget constraints are given by

$$\begin{aligned} c_t(t) + p_t x_t(t) &= a, \\ c_{t+1}(t) &= b + p_{t+1} x_t(t). \end{aligned}$$

We are interested in competitive equilibria in which all agents maximize their utility functions subject to their budget constraints, and the following asset and goods market clearing conditions hold:

$$\begin{aligned} a + b &= c_t(t) + c_t(t-1), \\ x_t(t) + 0 &= 1. \end{aligned}$$

The young agent's Euler equation with respect to the asset is given by

$$\frac{1}{c_t(t)} p_t = \delta \frac{1}{c_{t+1}(t)} p_{t+1}.$$

Unlike earlier models we studied, there are multiple equilibria in this economy. First, there is

a *fundamental equilibrium* in which the asset price is zero, i.e., $p_t = 0$ for all $t$. The asset is fundamentally worthless, and it does not affect agents' autarky consumption patterns.

In addition, there may exist multiple *bubbly equilibria* in which the asset price is positive, i.e., $p_t > 0$ for all $t$. In this case, the Euler equation requires

$$\frac{1}{p_{t+1}} = \frac{a\delta}{b}\frac{1}{p_t} - \frac{\delta+1}{b},$$

If $a\delta \leq b$, the fundamental equilibrium is the only equilibrium. Otherwise, given $a\delta > b$, there exist a continuum of bubbly equilibria indexed by initial asset price $0 < p_0 \leq \frac{a\delta-b}{\delta+1}$, such that

$$\frac{1}{p_t} = \left(\frac{a\delta}{b}\right)^t \left(\frac{1}{p_0} - \frac{\delta+1}{a\delta-b}\right) + \frac{\delta+1}{a\delta-b}.$$

The bubble is stationary if $p_0 = \frac{a\delta-b}{1+\delta}$.

The bubble asset offers a saving vehicle that allows the agents to smooth their consumption across periods. The young agent can purchase the bubble asset from the old agent, with the expectation that it can sell the asset to the next generation when it becomes old. In this way, the agent delays some of its consumption until it becomes old. Note that since the bubble is intrinsically worthless, the fundamental value is zero. The (finite) value of TVC term equals the price of the bubble asset.

## E.2 Unbalanced Growth

Consider another type of bubble of which the cash flow/price ratio is vanishing from Hirano and Toda (2025). The model features similar OLG settings. In period $t$, a new agent is born and lives for two periods. The agent has a unit supply of labor when young and zero when old.

There is only one type of goods and two production sectors in this economy. The two sectors are indexed by $i = 1, 2$. The first sector produces goods using labor $\ell$, while the second sector

produces goods $y$ using labor $\ell$ and land $x$. The production functions are given by

$$f_{1,t}(h) = \exp(g_1 t)\ell,$$
$$f_{2,t}(h) = \exp(g_2 t)\ell^{\alpha} x^{1-\alpha},$$

where the productivity growth in the two sectors satisfies $g_1 > g_2$. Let $w_t$ denote the wage rate, and $h_t$ denote the rental rate of land. In equilibrium, the rental rate of land is

$$h_t = (1-\alpha)\alpha^{\alpha/(1-\alpha)} \exp\left(\left(g_2 + \frac{\alpha}{(1-\alpha)}(g_2 - g_1)\right) t\right),$$

and the land price is

$$p_t = \frac{\delta}{1+\delta} \exp(g_1 t).$$

The cash flow/price ratio is

$$\frac{h_t}{p_t} = \delta^{-1}(1+\delta)(1-\alpha)\alpha^{\alpha/(1-\alpha)} \exp\left(\frac{g_2 - g_1}{1-\alpha} t\right),$$

which decays geometrically, hence

$$\sum_{t=1}^{\infty} \frac{h_t}{p_t} = \delta^{-1}(1+\delta)(1-\alpha)\alpha^{\alpha/(1-\alpha)} \frac{\exp\left(\frac{g_2-g_1}{1-\alpha}\right)}{1 - \exp\left(\frac{g_2-g_1}{1-\alpha}\right)} < \infty$$

By the bubble characterization lemma (Hirano and Toda, 2025; Montrucchio, 2004), the land price exhibits a bubble.

Over time, the cash flow of the land is going to be a vanishing share of its price, generating a classical behavior of asset bubbles. Intuitively, think of the two sectors as tech and traditional industries. The tech industry is growing faster, and its growth ignites a fast growth rate of wage. A fraction of wage income becomes saving and flows into the land, the only saving vehicle in the economy. Since the land supply is fixed, the price have to go up. The gap between the land price and its fundamental value, i.e., the present value of rents, can be viewed as its premium for being

the store of value. As the rent vanishes relative to land price and the tech industry overwhelm the traditional sector, land loses its function as production factor and becomes a pure bubble in the limit.

In this example, there is a unique bubbly equilibrium, and no fundamental equilibrium with no asset bubble. More generally, Hirano and Toda (2025) show that asset bubbles have to arise when two sectors' growth rates diverge while the counterfactual long-run autarky interest rate is below dividend growth, which rules out fundamental equilibria with no asset bubble.