Green Coins

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Abstract

We propose a novel macrofinance model for the EU area comprising climate damages, an Emission Trading System, and credit markets with (1) a bias for 'net-zero' brown firms, and (2) opaque voluntary carbon credit (VCC) markets. Our model suggests that the implied allocation is far from the first-best. Relevant welfare gains can be obtained in a setting in which a Green Coin Central Bank (GCCB) runs a transparent blockchain in which coins are mined after a decentralized verification of private offsets, and the GCCB manages green coin prices through open market operations.

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1 Introduction

Emission regulation is a priority that poses many challenges and prompts several key macroeconomic questions. How can we let emissions regulation coexist with decentralized private markets? How can we prevent fraudulent emissions miscounting due to asymmetric information? In addition, how can we design policy tools that promptly react to economic- and climate-related conditions? To address these questions, we need a model of the interactions of public emission trading systems (ETS), private carbon credit markets, and carbon-sensitive credit markets. In addition, we advocate for a novel regulation setting in which high-integrity privately issued carbon offsets are traded in the same market and at the same price as emission permits.

In what follows, we argue that a 'measure-and-issue' mechanism paired with an externally verifiable carbon ledger can provide the level of integrity needed in private carbon credit markets. In addition, a digital green coin market could easily enable a benevolent authority to alter the cost of emissions in real time via open market operations. More broadly, this study intends to promote a novel *"Green MacroFinTech"* approach to emissions regulation. We see technology as a powerful tool to mitigate information frictions in carbon markets via decentralized verification. Green coins are one possible implementation of this concept and they can enhance policy making.

Specifically, we take seriously corporate frictions and study an economy in which a brown sector finances its investments with a mix of equity and corporate debt. Corporate debt is preferred to equity because it benefits from a tax shield, but borrowing is limited by a constraint linked to reported net emissions. Firms with lower net emissions therefore obtain greater access to credit. Net emissions are computed as gross emissions minus declared offsets. In addition, following the European ETS, we assume that the brown sector faces an emission constraint, i.e., by regulation, corporate net emissions must be matched by emissions allowances whose supply is decided by a central authority that allocates them once a year.

In this setting, the price of allowances reflects the marginal loss in cashflows that firms would incur if additional production-related emissions could not be covered by allowances. Note that the brown sector has a twofold incentive to buy privately issued carbon credits. First, these credits reduce net emissions and enable firms to get more credit by appearing as 'net-zero' firms. Second, according to pre-2021 ETS rules, these credits can lower the need of allowances and relax the regulation constraint.

The economy comprises a second sector that uses green assets, i.e., all assets that contribute to either offsetting or sequestering CO_2 from the atmosphere. For the sake of simplicity, the green sector does not produce a final good. Rather, it provides a valuable service to the brown sector by reducing CO_2 and issuing private carbon credits accordingly. In order to capture the opaqueness of this market, we assume the existence of a non-negative wedge between declared offsets and real offsets. This is a key concern identified by consumer advocates, specialized press, and regulators. For an empirical assessment of this wedge, see Probst et al. (2024).

Indeed, in the US, the White House has highlighted this issue in the White House statement about voluntary carbon markets joint policy. In the EU, the ETS regulation has been modified several times in the last few years in order to avoid an inappropriate use of private carbon credits. Moreover, in the aftermath of the European 2019 'Green Deal', the European Commission has approved a very relevant Corporate Sustainability Reporting Directive (CSRD, 2022/2464/EU). Because carbon accounting remains an evolving field (Reichelstein 2025), the treatment of opaque offsets in corporate reporting remains a first-order concern (see, for example, Bloomberg).

We compile a novel dataset that combines EU macroeconomic aggregates with prices and quantities of emission allowances, futures on private carbon credits, and green investments. Using these data, we calibrate the model and show that it captures important features of macro quantities both in the brown and green sector as well as carbon prices and asset returns in these two sectors. In a second step, we consider a second setting with a radically

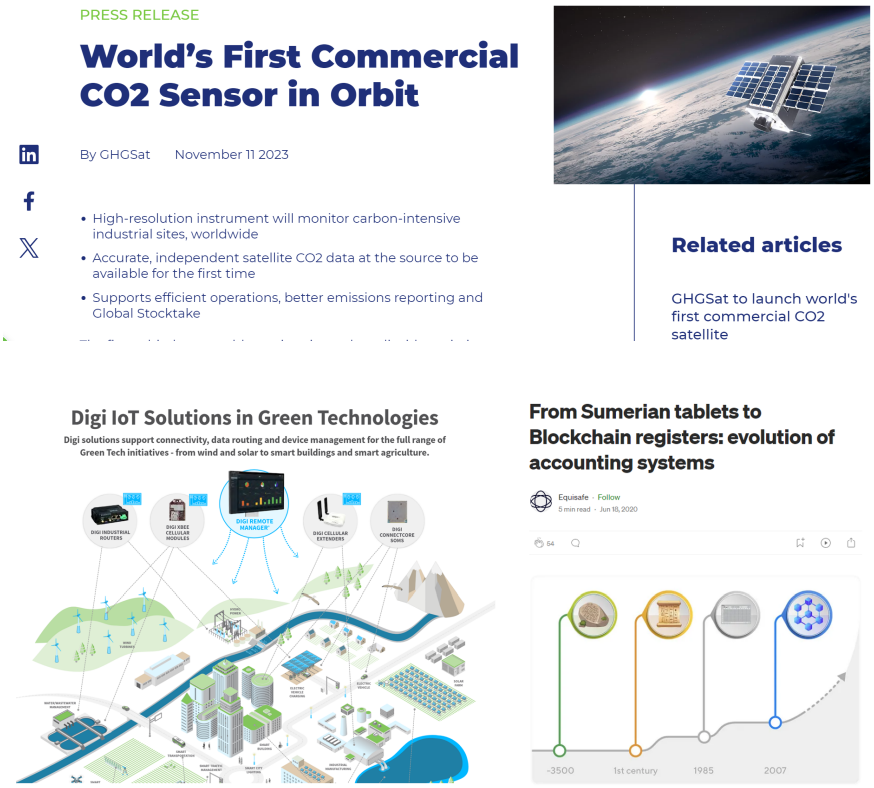


Fig. 1: New Monitoring Technologies

Notes: The top panel refers to new satellite technology that identifies CO_2 emission in great detail offered by GHGSat. The bottom panel refers to the interplay of IoT and Blockchain.

different way to deal with emissions based on the decentralized monetization of emissions. Our results suggest that this setting could produce sizable welfare gains.

Specifically, we propose a simplified structure in which neither self-certified carbon credits nor auctions of emission allowances exist. We assume that the following protocol is in place. There is a blockchain that is run and monitored by a Green Coin Central Bank (GCCB, henceforth). A coin is issued to the private sector every time a unit of carbon offset is produced, and it is retired at the end of the period. Thanks to the new technological paradigm called 'Internet of Things' (IoT), these actions can be measured in real time using geolocalized sensors able to transmit data. Periodically, data can be independently verified and coins can be issued proportionally to realized offsets. As a result, the supply of green coins tracks in real-time the supply of effective offsets. In addition, the GCCB is assumed

to have access to a GHG Satellite technology that enables superior monitoring on ex-post realized offsets. In short, this is a setting with real-time full information on emissions and offsets (see, for example, figure 1).

The GCCB controls the supply of green coins available to the public through regular open market operations. In this setting, green coins are valuable to the private sector because they are the only assets that provide certified offsets and help firms meet regulatory emission limits. By tightening or loosening the green coin policy, the GCCB can better steer the economy toward the first best. Most importantly, in a setting in which the GCCB commits to an implementable green policy rule that responds to both economic shocks and state variables, we find that the welfare losses from the first-best are greatly reduced. In the next section, we discuss more the feasibility of this platform.

Interestingly, the damage originated by emissions acts as a distortional taxation margin since it is proportional to output. We find that the GCCB finds it optimal to implement an 'emission smoothing' policy, in the spirit of what is observed in the fiscal policy literature. Exogenous shocks that immediately increase emissions are partially accommodated by the GCCB, meaning that this entity increases the supply of green coins available to the public by buying a smaller share of outstanding coins. As the stock of emissions increases over time, the GCCB slowly tightens the green coin market and increases its holdings of green coins. The welfare benefits can be as large as 5.3% of time-zero first-best consumption.

Further, according to our calibration, the social cost of carbon (SCC) is \$79 per tCO_2, using 2021 as base year. This estimate falls within the range reported in the literature, although it lies toward the lower end because our macro-finance model discounts future climate damages at a relatively high rate. Importantly, the GCCB targets an average price of \$107 in the decentralized economy with green coins, and it obtains a SCC of \$78, very close to the value observed at the first-best. The discrepancy between the price of the green coins and the SCC is due to the presence of distortions in the financing market (deviation from Modigliani-Miller).

Additional results. We run sensitivity analysis to 'inspect the mechanism' and obtain additional insights. First, the welfare results that we describe continue to hold whether we assume that GCCB has full or limited information about all fundamental shocks. Through the lens of the model, as long as the GCCB commits to keep the average price of the green coins high enough, the economy can get closer to the first-best. In simple terms, by promising a high average price for offsets and high penalty for emissions, the GCCB can correct the relative size of brown and green capital.

We also show that providing very high subsidies to green investment in the ETS system would not produce substantial welfare benefits. To the extent to which VCCs are perceived as opaque, their demand is subdued, their price is depressed and investing further in green assets is not optimal. More broadly, the ETS setting cannot get close to the first-best just by adjusting a subsidy lever.

Last but not least, we study the ETS setting assuming that only a portion of purchased VCCs can be used for compliance. The spirit of this experiment is to capture the additional constraints introduced in the EU ETS post-2021. We find that this move increases substantially the cost of allowances, consistent with what is observed in the data. Welfare increases by 1% because this restrictive measure reduces the accumulation of CO_2 and hence climate damages. The remaining welfare loss is substantial.

Related literature. Because our policy focus is novel, only a limited number of studies are directly related to our work. What follows is not a comprehensive review of the literature; rather, it highlights a subset of key contributions. Bustamante and Zucchi (2022) focuses on a firm subject to emission constraint and enabled to buy private VCCs. Our research differs from this work in many crucial-and-novel dimensions. In particular, we (i) account for the opaqueness of VCCs by considering a wedge between declared emission reductions and effective ones; (ii) have a quantitative focus; (iii) use new *EU* data; (iii) price carbon emissions in a macrofinance integrated assessment model, and (iv) propose a novel, policy-

relevant, and implementable framework based on green coins. We believe that tokenization can substantially improve emission management and resolve some of the frictions highlighted in Borri, Liu, Tsyvinski, and Wu (2024). Our model is consistent with the findings of Hengge et al. (2023), implying that the ETS affects the cost of capital of our brown firms.

Several modeling assumptions come from the Financial Economics of Climate literature (among others, see Giglio, Kelly, and Stroebel 2021, Ľuboš Pástor et al. 2021, BOLTON and KACPERCZYK 2023). We differ for our "Green MacroFinTech" approach that focuses on regulation in models that replicate both quantities and asset prices. Like us, Chittaro et al. (2026) focus on optimal taxation of emissions. We differ for our focus on monetization of offsets.

The interplay between macroeconomic aggregates and asset prices has been the main object of interest of the macro-finance literature (see, among many others, Ai (2007); Belo, Bazdresch, and Lin (2014); Corhay, Kung, and Schmid (2020)). The aggregate implications of emissions regulation and its interaction with FinTech in a macro-finance model have yet to be assessed. Our research aims to fill this gap in the literature.

FinTech has recently attracted many contributions (see, among others, Biais, Capponi, Cong, Gaur, and Giesecke (2023), Niepelt (2024), Brunnermeier and Payne (2022)). Our contribution differs from this literature by studying how FinTech can be used at the macroeconomic level to improve climate policy. To the best of our knowledge, nobody has focused on the benefits that FinTech can produce in the context of the green transition.

2 Institutional Background and Green Tech

A reader familiar with the EU ETS system evolution, the integrity initiatives in the voluntary carbon market, and tokenization may skip this section. The reader interested in both our modeling choices and our postulated digital coin platform may instead pay attention to this section.

First of all, we started this study in the aftermath of phase 4 of the EU ETS (see, for example, EP), namely after the use of VCCs was banned in 2021. Our goal is to compare the pre-2021 ETS setting with a counterfactual in which there is (i) integrity in the VCC market (see, for example, ICVCM), and (ii) perfect integration of VCCs and emission allowances. At the moment, the EU is working on an EU certification protocol, but it is not considering the use of certificates on the EU ETS for compliance (see EU 2025/2358). This study argues that such segmentation could be very inefficient.

Traditionally, VCCs are issued at the onset of a project and are based on estimated future offsets. Third-party verification is costly and often unreliable because of distorted incentives. Unfortunately, this market design generates opacity. Instead of certifying expected long-term offsets streams, one could use existing technology to measure offsets and sell them after they are effectively realized. Like for other commodities, this process would make offsets comparable across projects.[1]

Measurements must be externally validated. While we acknowledge that there are several ways to operationalize verification, we think of tokenization as an interesting tool. Periodically tokenizing offset data on a decentralized blockchain would grant both transparency and scalability (Swinkels 2024). In China, for example, tokenization is already being applied to emissions (Cong et al. 2025). In what follows, we envision a similar concept adopted for offsets. The institution that we denote as GCCB, could be either a domestic institution (for example, a division of the local central bank) or an international reputable entity, such as a development bank. In what follows, we do not take a specific stand on this institutional aspect.

Lastly, we note that a blockchain-based platform could minimize costs and environmental impact by adopting a 2-layer structure. A layer 2 blockchain could verify intra-period transactions and send a bulk of data to the underlying blockchain only once a period for

[1]For carbon farming, sensors based on the measurement of the eddy covariance could detect realized net emission exchanges between the soil and the atmosphere. For afforestation on large areas, one could use satellite technology. For renewable energy projects, KWs produced could be easily converted into tons of CO_2 by using emission data from brown energy plants.

broader verification (lower energy consumption and verification fees). In short, we think of green coins as cost-effective devices to digitalize the verification of offsets. Such a cheaper and credible technology could enable a wide range of private actors to be included in the green market.

3 The Models

In what follows, we describe the settings that we intend to focus on. We start by describing the planner's problem. We then turn our attention to the setting that should be calibrated using EU data, i.e., a model with an ETS and an opaque market for VCCs. In the last step, we explore a setting with green coins.

3.1 The planner's problem

The household. We assume that the representative agent has Epstein and Zin (1989) recursive preferences so that she prices long-run risks, including those related to climate change:

$$U_t = \left[(1-\beta) C_t^{1-\frac{1}{\psi}} + \beta (\mathbb{E}_t[U_{t+1}^{1-\gamma}])^{\frac{1-\frac{1}{\psi}}{1-\gamma}} \right]^{\frac{1}{1-\frac{1}{\psi}}},$$

where C_t is the amount of output consumed. We do not include leisure in the utility function for the sake of parsimony. As a result, the supply of labor is fixed and normalized to one in both sectors. In this setting, the stochastic discount factor (SDF) is

$$M_{t+1} = \beta \left(\frac{C_{t+1}}{C_t} \right)^{-\frac{1}{\psi}} \left(\frac{U_{t+1}}{\left[\mathbb{E}_t U_{t+1}^{1-\gamma} \right]^{\frac{1}{1-\gamma}}} \right)^{\frac{1}{\psi}-\gamma}.$$

The brown sector. The brown sector produces output, Y_t, according to the following production function:

$$Y_t = (Z_t L_{bt})^{1-\alpha} K_{bt-1}^{\alpha}, \tag{1}$$

where Z, L_b, K_b, denote productivity, labor, and capital deployed in this sector, respectively. Capital evolves as follows,

$$K_{bt} = (1-\delta)K_{bt-1} + \Psi(I_{bt}/K_{bt-1})K_{bt-1}, \tag{2}$$

where Ψ denotes Jermann (1998) convex adjustment costs.

Similar to Shapiro and Metcalf (2023), the fraction of output that is not disrupted by climate damage is determined by the following function

$$\chi(s_t) = \exp(-\chi_0 \cdot s_t), \tag{3}$$

where s_t captures carbon dioxide concentration and accumulates depending on net emissions in the economy. In what follows, output net of climate-related damages is denoted by Y_t^n and determined as follows:

$$Y_t^n = \chi\left(s_t\right) \cdot Y_t. \tag{4}$$

Gross emissions produced by the brown sector is assumed to be proportional to gross output:

$$E_t = \lambda_t \cdot Y_t, \tag{5}$$

where the process that determines emissions per unit of added value is subject to the external

shocks,

$$\log \lambda_t \quad = (1-\rho_\lambda)\mu_\lambda + \rho_\lambda \log \lambda_{t-1} + \sigma_\lambda \epsilon_{\lambda,t}. \tag{6}$$

We specify this law of motion in log units so that λ_t is always non negative. As in the data, we model λ_t as a persistent process. Among other channels, external shocks capture unmodeled technology improvements as well as natural disasters (e.g., fires) and changes in energy sources policy (e.g., shifts from renewable to coal-based energy).

In addition, we allow for the negative feedback between CO_2 and output by defining aggregate productivity growth, Δz, as follows:

$$\begin{aligned} \Delta z_{t+1} &= \mu_z + x_t + \sigma_z \epsilon_{z,t+1} \\ x_t &= \rho_x x_{t-1} - \phi_s \cdot (\log(s_{t-1}) - \log(\overline{s})) + \sigma_x \epsilon_{x,t}. \end{aligned}$$

The specification above is common in the long-run risk literature, except for the fact that it accounts for an endogenous disruptive effect of CO_2 on expected long-run growth ($\phi_s > 0$). Thus, green assets are beneficial because they not only they help relax the emission constraint but also indirectly improve productivity by reducing CO_2 concentration. In what follows, we denote ϵ_z and ϵ_x as short- and long-run productivity shocks, respectively.

The green sector. The green sector does not participate to the goods market. It solely produces CO_2 offsets according to the following production function:

$$G_t = (Z_{gt} L_{gt})^{1-\kappa} K_{g,t-1}^{\kappa}, \tag{7}$$

where capital evolves as follows,

$$K_{gt} = (1-\delta_g) K_{g,t-1} + H(I_{gt}/K_{g,t-1}) K_{g,t-1}, \tag{8}$$

and H captures convex adjustment costs as in Jermann (1998). Along the balanced growth path, productivity in this sector is cointegrated with that of the brown sector:

$$\log \frac{Z_{g,t}}{Z_t} - \overline{zc} = (1-\phi_g)\left(\log \frac{Z_{g,t-1}}{Z_{t-1}} - \overline{zc}\right) - (1-\phi_g)(\Delta z_t - \mu). \tag{9}$$

We model the cointegration residual so that the green sector productivity adapts slowly to innovations in the brown sector (small ϕ_g), consistent with the fact that green technology innovations arrive at a slower pace compared to innovations in the traditional brown sector (e.g., slow improvement in solar panels efficiency, slow improvement in the efficiency-emissions ratio for the production of batteries). The parameter $\overline{zc}$ accounts for differences in the levels of productivity which are to be expected given that this sector specializes on offsets, whereas the brown sector focuses on goods and services.

The climate feedback. Let NE_t denote net emissions in the economy:

$$NE_t := E_t - G_t, \tag{10}$$

we model the accumulation of CO_2 as follows:

$$s_t = (1-\delta_s)s_{t-1} + NE_t/Z_t, \tag{11}$$

where net emissions are standardized by productivity and hence must be interpreted as relative to the size of the economy and δ_s captures the fraction of CO_2 that is naturally depleted in the atmosphere.

Key optimality conditions. The full solution of the planner's problem can be found in the Appendix. Here we report the returns of the three endogenous state variables in the

model, namely a proxy for the stock of CO_2, brown capital, and green capital, respectively:

$$R^P_{s,t} = \frac{\chi_0 Y^n_t + (1-\delta_s) q^P_{s,t}}{q^P_{s,t-1}} \tag{12}$$

$$R^P_{b,t} = \left(q^P_{bt} \cdot \left[(1-\delta) + \Psi_t - \Psi'_t \cdot \frac{I_{bt}}{K_{bt-1}} \right] + \left(1 - q^P_{st} \frac{\lambda_t}{\chi_t} \right) \alpha \frac{Y^n_t}{K_{bt-1}} \right) / q^P_{bt-1} \tag{13}$$

$$R^P_{g,t} = \left(q^P_{gt} \cdot \left[(1-\delta_g) + H_t - H'_t \cdot \frac{I_{gt}}{K_{g,t-1}} \right] + q^P_{st} \kappa \frac{\tilde{G}_t}{K_{g,t-1}} \right) / q^P_{g,t-1}. \tag{14}$$

Variables denoted as q^P_{jt} refer to marginal-q, i.e., the marginal cost of an additional unit of capital of type j. The return associated to the stock of CO_2 reflects variations in the present value of future damages, $q^P_{s,t}$. The return of brown capital, $R^P_{b,t}$, differs from that in a neoclassical model because the marginal product of capital is reduced by the associated loss in value due to the additional emissions, $q^P_{st} \frac{\lambda_t}{\chi_t}$. The return of the green sector is driven by the marginal productivity of green capital, i.e., by the market value of the marginal offsets, $q^P_{st} \kappa \frac{\tilde{G}_t}{K_{g,t-1}}$.

3.2 A model with an ETS and opaque VCCs

In this section, we focus on a decentralized economy with key features that resemble the EU ETS system. We calibrate this setting to EU data and then run a counterfactual analysis of the benefits of a green coin-based system.

Green authority. The green authority observes declared net emission, $\widetilde{NE}$,

$$\widetilde{NE}_t = \lambda_t Y_t - \widetilde{G}_t,$$

where $\widetilde{G}$ refers to reported emission offsets which can differ from the true offsets, G. A penalty is applied when net emissions exceed the emission allowances, EA:

$$C^{EA}_t = \phi^{EA}_0 \cdot \exp\left(\phi^{EA}_1 \left\{ \frac{\lambda_t Y_t - \widetilde{G}_t}{EA_t} - 1 \right\} \right) \cdot Z_{t-1}.$$

In practice, this cost function mimics (i.e., it convexifies) the emission constraint

$$\lambda_t Y_t - \widetilde{G}_t - EA_t \leq 0. \tag{15}$$

The authority commits to the following rule for the supply of emission allowances to be auctioned:

$$\begin{aligned} EA_t &= \exp(a_t) \cdot Z_{g,t} > 0 \\ a_t &= (1-\rho_a)(\mu_a - \phi_a \cdot (s_{t-1} - \overline{s})) + \rho_a a_{t-1} + \sigma_a \epsilon_{a,t-j}. \end{aligned}$$

Along the balanced growth path, allowances must grow with the economy, and a_t must be interpreted as a cointegration process expressed in log units. We think of this process as an AR(1) with an endogenously time-varying level,

$$\bar{a}_t := \mu_a - \phi_a \cdot (s_{t-1} - \overline{s}),$$

that is reduced when the endogenous stock of emission increases above steady state ($\phi_a > 0$). When ϕ_a is large enough, this policy implies that allowances can be kept constant or even reduced for periods when GHG emissions grow excessively.

We note three additional important points: (i) the stock of accumulated CO_2 depends on true offsets, G, not on declared offsets, $\widetilde{G}$; (ii) ϵ_a captures emission policy shocks; and (iii) we assume that the allowances supply is decided j-period ahead in order to capture the fact in the EU ETS system, the quantity of allowances to be auctioned is chosen ex-ante. In our calibration, j is one year. This exogenous policy rule can be calibrated by jointly studying emission allowances auctioned in the ETS (available at low frequency) as well as the behavior of emission prices (available at high frequency).

As discussed in what follows, we have a well-defined capital structure in the brown sector because the government offers a tax shield, τ, on corporate interest rates, $r_t^B B_{t-1}$. In

addition, in the spirit of what observed in Europe, we assume that the cost of new green assets, I_{gt}, is subsidized at a rate τ_g. We assume that these payments are managed by the green authority and, for the sake of simplicity and to keep our analysis transparent, this authority has a balanced budget:

$$p_t \cdot EA_t + T_t = \tau_g I_{gt} + \tau r_t^B B_{t-1},$$

where $p_t \cdot EA_t$ is the amount collected by selling emission allowances, and T_t is a lump sum transfer from or to the household that bridges the gap between cash inflows and outflows.

Brown firm. The brown firm represents all assets and economic activities that produce positive gross emissions and faces the following problem:

$$\begin{aligned} V_{b,t} &= \max_{B_t, I_{bt}, L_{bt}, \widetilde{G}_t, EA_t} D_{b,t} + \mathbb{E}_t(M_{t+1} V_{t+1}) \\ D_{b,t} &= \underbrace{Y_t^n - W_{bt} L_{bt} - I_{bt}}_{\text{neoclassical}} \underbrace{-(1-\tau) r_t^B B_{t-1} + \Delta B_t - C_t^E - C_t^B}_{\text{deviation from MM}} \\ &\quad - \underbrace{(p_t \cdot EA_t + C_t^{EA})}_{\text{cost of EA}} - \underbrace{(p_{gt} \widetilde{G}_t + C_t^C)}_{\text{cost of VCCs}} \end{aligned}$$

where Y_t^n, and K_t evolve as described in the previous section. The following cost functions,

$$\begin{aligned} C_t^E &= \phi_0 \cdot \exp\left(\phi_1 \left\{1 - \frac{\theta_t K_t}{B_t}\right\}\right) \cdot Z_{t-1} \quad (\text{mimics } B_t \leq \theta_t \cdot K_t), \\ \theta_t &\equiv \frac{\theta_0}{1 + \exp(-\widetilde{G}_t / \lambda_t Y_t)}, \\ C_t^B &= \eta Z_{t-1} \left(\frac{B_t}{Y_t} - \frac{B_{ss}}{Y_{ss}}\right)^2, \\ C_t^C &= \frac{\zeta}{2} \frac{(\widetilde{G}_t - G_t)^2}{Z_{t-1}}, \end{aligned}$$

capture the cost of financial distress, debt issuance, and VCC-related reputation, respectively. As in Croce et al. (2012), the tax shield on corporate interest makes this firm favor debt-financing (deviation from Modigliani-Miller). This aspect is important because it allows us to capture the attention of credit markets for 'net-zero' firms.

Specifically, we differ from Croce et al. (2012) by allowing the borrowing constraint tightness, θ, to be a decreasing function of declared net emissions. Banks offer more favorable credit terms to firms with lower emissions or credible decarbonization targets, while more carbon-intensive borrowers face relatively tighter credit supply, especially when lenders are themselves climate-committed (Altavilla et al., 2024; Degryse et al., 2023; Kacperczyk and Peydró, 2024). Hence, this firm has an incentive to buy VCCs even in the absence of an ETS, similar to what we observe in the data. As in Croce et al. (2012), altering the mix of debt and equity comes at a cost, C^B, and hence the borrowing constraint has significant implications for investment.

In the ETS system, this firm must balance gross emissions by buying either VCCs or emission allowances to minimize the penalty cost C^{EA}. Both in the data and in the model, VCCs are on average cheaper than EU allowances, $\mathbb{E}[p_{gt}] < \mathbb{E}[p_t]$. However, VCCs carry an intrinsic cost C^C related to their opaqueness, i.e., the gap between declared and effective offsets. The firm understands that acquiring overstated offsets could lead to reputational damage and legal consequences (see, for example, the case of Delta Airlines).

The first order conditions imply that investment is distorted by the emission constraint,

$$\begin{aligned} q_{bt} &= \frac{1}{\Psi_t'} = \mathbb{E}_t\left[M_{t+1}\frac{\partial V_{bt+1}}{\partial K_{bt}}\right] - \frac{\partial C_t^E}{\partial K_t} \\ \frac{\partial V_{bt}}{\partial K_{bt-1}} &= \underbrace{\frac{\partial Y_t^n}{\partial K_{bt-1}} + q_{bt}\left(1-\delta-\frac{\Psi_t' I_{bt}}{K_{bt-1}}+\Psi_t\right)}_{\text{neoclassical}} - \underbrace{\left(\frac{\partial C_t^E}{\partial K_{bt-1}} + \frac{\partial C_t^B}{\partial K_{bt-1}}\right)}_{\text{deviation from MM}} - \underbrace{\frac{\partial C_t^{EA}}{\partial K_{bt-1}}}_{\text{EA distress}}. \end{aligned}$$

In addition, the demand of emissions is related to the emission distress cost, or, equivalently,

the multiplier assigned to the emission constraint,

$$p_t = \phi_1^{EA} \cdot \frac{C_t^{EA}}{EA_t} \cdot \frac{(\lambda_t Y_t - \widetilde{G}_t)}{EA_t}.$$

Most importantly, the price of VCC is anchored to that of allowances traded on the ETS,

$$p_{gt} = \underbrace{p_t \cdot \frac{EA_t}{\lambda_t Y_t - \widetilde{G}_t}}_{\text{link with } p_t} + \underbrace{\phi_1 \cdot \theta_t' \cdot \frac{C_t^E}{B_t} \cdot \frac{K_{bt}}{\lambda_t Y_t}}_{\text{savings on borrowing}} \underbrace{-\zeta \cdot \frac{\widetilde{G}_t - G_t}{Z_{t-1}}}_{<0, \text{ opaque}},$$

and it features two additional margins going in opposite directions. On the one hand, VCCs are more valuable than allowances because they help firms reduce their net emissions and secure better financing terms. On the other hand, because of opaqueness, VCCs are traded at a discount.

Green firm. The green sector is defined as the set of assets that delivers CO_2 offsets and sequestration. This sector determines the endogenous supply of both effective and declared offset. Specifically, the firm faces the following neoclassical problem:

$$\begin{aligned} V_{gt} &= \max_{I_{gt}, L_{gt}} D_{gt} + \mathbb{E}_t(M_{t+1} V_{g,t+1}) \\ D_{gt} &= p_{gt} \widetilde{G}_t^s - (1 - \tau_{gt}) I_{gt} - W_{gt} L_{gt} \\ \widetilde{G}_t^s &= \xi_t G_t \\ \xi_t &= 1 + b_0 \exp(b_1 \epsilon_{\xi,t}), \end{aligned}$$

where $\epsilon_{\xi,t}$ is independent and identically distributed $\mathcal{N}(0,1)$, and K_{gt} and G_t are defined as in the previous section. This shock is important for (i) making this market opaque, and (ii) reducing the correlation between emission price, p_t, and VCC price, p_{gt}, as in the data. In equilibrium, the amount of offsets supplied by the green sector equals to the amount demanded by the brown sector, $\widetilde{G}_t^s = \widetilde{G}_t$.

We note that the positive part of the cash-flow, D_g, originates from (i) selling VCCs to the brown sector and (ii) receiving a subsidy proportional to green investment. In this formulation of the model, we think of the gap between reported and realized offsets as driven by an exogenous process, ξ_t, which is always greater than or equal to one and that captures both changes in moral hazard, ex-post lower efficiency of green assets, and changes in monitoring ability. The financing of this firm is frictionless, in order to both (i) capture the 'benevolence' that capital markets are giving to green projects and (ii) isolate the role of financing frictions only on brown firms.

At the optimum, we have

$$\begin{aligned} q_{gt} &:= \frac{1-\tau_{gt}}{H_t'} = \mathbb{E}_t\left(M_{t+1}\frac{\partial V_{g,t+1}}{\partial K_{gt}}\right) \\ \frac{\partial V_{gt}}{\partial K_{g,t-1}} &= p_{g,t}\frac{\partial \widetilde{G}_t^s}{\partial K_{g,t-1}} + q_{gt}\{1-\delta_g + H_t - H_t'\cdot(I_{gt}/K_{g,t-1})\}, \end{aligned}$$

implying that green investment is driven by the present value of the future offset flows. This flow is crucially driven by the price of VCCs which, in turn, is anchored to the price of allowances on the ETS.

Aggregate resource constraint and consumer budget constraint. Let V_{jt}^{ex} denote the ex-dividend price of asset j at time t, i.e., $V_{jt}^{ex} = V_{jt} - D_{jt}$. The household budget constraint is defined as follows:

$$C_t + V_{bt}^{ex} + V_{gt}^{ex} = V_{bt} + V_{gt} + W_{bt} + W_{g,t} - T_t$$

Since offsets are sold to the brown firm, they represent intermediate goods that do not show up in GDP. Therefore, we have

$$Y_t^n = C_t + I_{bt} + I_{gt} + C_t^{EA} + C_t^B + C_t^E + C_t^C.$$

3.3 A Model with Green Coins

The economy described in the previous section features many sources of inefficiencies, and it is far away from the first-best. In what follows, we consider a setting in which there is scope for a central authority to improve welfare. Specifically, we think of an authority called Green Coins Central Bank (GCCB) that runs and monitors the only authorized system in which offsets are recorded, verified, and monetized or, equivalently, tokenized. The GCCB can alter the price of the green coins with open market operations to bring the economy closer to the first best.

In this setting, offsets are properly measured, but allocations can still be inefficient due to the presence of financial frictions. We will show that the GCCB can improve welfare by responding to exogenous shocks in ways that make investment activities in both the brown and green sector to be more efficient from a social perspective.

Unmodeled frictions. In the context of our model, all we need is to assume that the GCCB has a verification technology and an exchange in which to trade offset units. The model is silent on whether the verification of the transaction should be centralized or decentralized. In what follows, we assume that the system runs on a decentralized blockchain and use blockchain-related terminology. We believe that this technology is superior in at least three dimensions. First, it is very scalable and can be adopted by large, medium, and small businesses. Equivalently, it features smaller adoption fixed costs. Second, in an economy with multiple agents, a decentralized verification of the offsets transactions minimizes the incentives for misreporting. Third, since offsets and emissions may be localized in very different countries, a common blockchain can (i) homogenize offsets accounting and (ii) minimize country-level incentives to overstate offsets and under-report emissions.

The green blockchain. We assume that the GCCB runs a blockchain that mines new coins every period as soon as an offset is verified. Coins are fully retired at the end of the

period. As a result, the total supply of green coins in each period equals the total mass of true offsets in every period and every state of the world, G_t. The per-period cost of running the blockchain is $\omega_g G_t$, i.e., it is proportional to all transactions which are verified by the platform. In our model, the green firm is the only miner of green coins, and it has an incentive to sell them to the brown firms who can claim 1 unit of offsets for each coin. The brown firm buys green coins at the spot price p_g. In this setting, all emissions and offsets are verified over the platform, and hence there is no VCC.

The role of the GCCB. In each period, the green authority conducts open market operations in order to buy a share $S_{g,t}$ of the total mass of available green coins, G_t. Thus the total supply of offsets accessible to the brown sector is

$$G_t^P := G_t \cdot (1 - S_{g,t}),$$

and the private sector is now subject to the following constraint:

$$NE_t = \underbrace{\lambda_t Y_t}_{\text{gross emissions}} - \underbrace{G_t^P}_{\text{private offsets}} \leq \underbrace{\bar{A} \cdot Z_{g,t-1}}_{\text{emissions allowed}} . \tag{16}$$

By altering the supply of green coins available to the public, the GCCB can manipulate in real time the price of green coins, i.e., the only assets that count for emission regulation. This formulation resembles what central banks do with bonds in order to alter yields. Note that thanks to the monitoring implemented by the blockchain, there is no longer a distinction between true offsets, G, and declared offset ($G_t = \tilde{G}_t \quad \forall t$).

The authority is assumed to follow this Taylor-style green rule,

$$S_{gt} = \frac{\exp(sg_t)}{1 + \exp(sg_t)} \in (0,\, 1),$$

where

$$sg_t = A_0 + [A_s\, A_k\, A_{lr}] \cdot \begin{bmatrix} \hat{s}_{t-1} \\ \widehat{\frac{K_{gt-1}}{K_{bt-1}}} \\ x_{t-1} \end{bmatrix} + [A_z\, A_\lambda\, A_x] \cdot \begin{bmatrix} \epsilon_{z,t} \\ \epsilon_{\lambda,t} \\ \epsilon_{x,t} \end{bmatrix}, \tag{17}$$

so that the holdings of green coins are both history dependent (it depends of the demeaned state variables, $\hat{s}_{t-1}$, $\widehat{\frac{K_{gt-1}}{K_{t-1}}}$, and x_{t-1}) and sensitive to all contemporaneous shocks in the economy ($\epsilon_{z,t}$, $\epsilon_{\lambda,t}$, and $\epsilon_{x,t}$). The coefficients of this rule cannot be estimated in the data since this is a counterfactual setting. In our analysis, we will explore the welfare implications of allowing the GCCB to respond to economic states by changing these parameters.

The authority net payout to the household is:

$$\Pi_t^{GB} = -(p_{g,t} \cdot G_t \cdot S_{g,t} + \omega_g \cdot G_t + \tau_g I_{gt} + \tau r_t^B B_{t-1}),$$

which embodies the resources spent on buying new coins and running the blockchain.

Brown Firm. The new problem faced by the brown firm is:

$$\begin{aligned} V_{b,t}^{GC} &= \max_{B_t, I_{bt}, L_{bt}, G_t} D_{b,t}^{GC} + \mathbb{E}_t(M_{t+1} V_{t+1}) \\ D_{b,t}^{GC} &= \underbrace{Y_t^n - W_{bt} L_{bt} - I_{bt}}_{\text{neoclassical}} \underbrace{-(1-\tau) r_t^B B_{t-1} + \Delta B_t - C_t^E - C_t^B}_{\text{deviation from MM}} \\ &\quad - \underbrace{C_t^{EA}}_{\text{cost of net emissions}} - \underbrace{p_{gt} G^P{}_t}_{\text{cost of coins}}, \end{aligned}$$

in which there are no longer allowances traded on the ETS and VCCs, only green coins. The variables Y_t^n, K_{bt}, C_t^E, C_t^B, are defined as in the setting with the ETS system. The C_t^{EA} cost function is formulated to reflect the emission constraint in equation (16) as opposed to equation (15):

$$C_t^{EA} = \phi_0^{EA} \cdot \exp\left(\phi_1^{EA} \left\{ \frac{(\lambda_t Y_t - G_t^P)}{\bar{A} Z_{g,t-1}} - 1 \right\}\right) \cdot Z_{t-1}.$$

At the optimum, the price of green coins is

$$p_{gt} = \underbrace{\phi_1^{EA} \cdot \frac{C_t^{EA}}{\bar{A} Z_{g,t-1}}}_{\text{savings on the cost of NE>0}} + \underbrace{\phi_1 \cdot \theta_t' \cdot \frac{C_t^E}{B_t} \cdot \frac{K_{bt}}{\lambda_t Y_t}}_{\text{savings on borrowing}}.$$

Importantly, this price still reflects both the benefits related to cheaper financing of low net-emission firms and the tightness of the emission constraint. In contrast to the previous setting, now the GCCB can directly control the emission constraint tightness by altering its holding of green coins.

Green firm. The green sector determines the endogenous supply of green coins. Specifically, the firm faces the following neoclassical problem:

$$\begin{aligned} V_{gt}^{GC} &= \max_{I_{gt}, L_{gt}} D_{gt} + \mathbb{E}_t(M_{t+1} V_{g,t+1}) \\ D_{gt} &= p_{gt} G_t - (1 - \tau_{gt}) I_{gt} - W_{gt} L_{gt}, \end{aligned}$$

where all variables are defined as before and there is no longer any wedge between declared and actual offsets.

Aggregate resource constraint and consumer budget constraint. In this setting, the household budget constraint is defined as follows:

$$C_t + V_{bt}^{ex,GC} + V_{gt}^{ex,GC} = V_{bt}^{GC} + V_{gt}^{GC} + W_{bt} + W_{g,t} + \Pi_t.$$

Since offsets are sold to the brown firm, they represent intermediate goods that do not show up in GDP. Therefore, we have

$$Y_t^n = C_t + I_{bt} + I_{gt} + C_t^{EA} + C_t^B + C_t^E + \omega_g G_t.$$

4 Data and Calibration

In what follows, we describe the data that we use in order to calibrate our main specification to European data. We then illustrate our calibration and solution methods.

4.1 Data

We focus on the EA19, i.e., the 19 countries in the Euro Area as of 2015, in order to maximize our sample length across time series. All macroeconomic series on national aggregates and price indices are from Eurostat. Data on allocated emission allowances are obtained from the European Environment Agency (EU ETS data viewer). We use the annual aggregate of total allocated allowances (free plus auctioned, EUA) for the 2005–2022 period.

Data on green investments and prices for futures related to CO_2 offsets are from Bloomberg. Specifically, with regard to the CME commodity market exchange, we consider daily prices for "XGB1 Comdty: CBL CORE GLOBAL EMISSIONS OFFSET (C-GEO) FUTURES - TR", "LGO1 Comdty: Generic 1st CBL Nature-Based Global Emissions Offset Futures", and "EEXX04EA Index: EEXX European Emission Allowances EUA Spot (Phase 4)" over the sample 2021-2023. In addition, we recover green investments by country in billions of Euros at the yearly frequency over the 2004-2023 sample.

Haver Analytics enables us to obtain daily observations over the sample 2005-2024 for futures and emission allowances "ICE ECX EUA Futures: 1st Position: Settlement Price (Euro/Metric Tonne)", "CME Reg Sess:Carbon Offset Futures: 1st Position: Settlement Price (US/Offset)" and "NYMEX RSes:CBL Nat-Based Glob Emiss Offset".

Emissions as percent of GDP by country at the annual frequency for the 2010-2020 sample are from The United Nations Economic Commission for Europe (UNECE). This source provides kilograms per unit of 2021 real PPP-adjusted GDP (expressed in USD). We complement these data with those from the `Clean Investment Monitor`. This dataset

Table 1: Calibration

Parameter	Value	Parameter	Value
Preferences		***Climate***	
Discount factor (β)	0.97	GHG depletion rate % (δ_s)	0.55
Risk aversion (γ)	10.00	Mean of log emission rate (μ_λ)	-1.86
IES (ψ)	1.20	Persistent of log emission rate (ρ_λ)	0.50
		Log emission rate volatility (σ_λ)	0.02
		Damage function parameters % (χ_0)	0.19
Brown sector		***Green sector***	
Capital share (α)	0.35	Capital share (κ)	0.65
Capital depreciation rate % (δ)	7.60	Capital depreciation rate % (δ_g)	7.60
Elasticity of investment adj. costs (ω)	1.20	Elasticity of investment adj. costs (ν)	20.00
Intensity of debt adjustment costs (η)	0.40	Green-brown differences in	
Debt-to-book avg (θ_0)	0.50	levels of TFP ($\overline{zc}$)	-0.11
Intensity of distress costs (ϕ_1)	20.00		
Intensity of emission violation costs (ϕ_1^{EA})	20.00		
Emission violation costs parameter % (ϕ_0^{EA})	0.07		
Productivity		***ETS Authority***	
Average productivity growth (μ_z)	0.01	Brown-sector debt tax shield (τ)	0.20
Short-run productivity volatility (σ_z)	0.05	Green-sector investment subsidy (τ_g)	0.30
Long-run productivity persistence (ρ_x)	0.94	Mean emission allowances (μ_a)	-1.92
Long-run productivity volatility % (σ_x)	0.50	Persistent of emission allowances (ρ_a)	0.27
Long-run productivity exposure		Allowance shock volatility (σ_a)	0.12
to GHG emissions ($\phi_s, \times 1000$)	0.02	Allowance sensitivity to emissions % (ϕ_a)	8.71
Cointegration parameter (ϕ_g)	0.20		
GCCB		***VCCs***	
Cost of running blockchain (ω_g)	0.10	Reporting gap avg. % (b_0)	2.21
Average emissions allowance ($\bar{A} = e^{\mu_a}$)	0.15	Reporting gap volatility (b_1)	0.50
		Reporting costs intensity (ζ)	15.44

Notes: This table reports our benchmark annual calibration. See section 4.2 for a detailed discussion.

refers mostly to green investments done in the US energy and manufacturing sectors and can be interpreted as providing a lower bound on investment over the sample 2018:Q1-2024:Q1.

Verified carbon credits data are obtained mostly from the Goldstandard Registry of projects that offset emissions. We complement this source with the data on sustainable projects reported by both the International Renewable Energy Agency (IRENA) and the International Energy Agency (IEA).

4.2 Calibration and Solution Method

Calibration. We report our benchmark calibration for the model with the ETS system in table 1. The main moments that we target are reported in table 2. The EA19 economy is

not too dissimilar from the US one in terms of basic macroeconomic statistics. As a result, our calibration is similar to the one in prior work (see, among others, Croce 2014 and Croce et al. 2012).

Preferences are calibrated in the spirit of the long-run risk literature, with $\gamma > \psi^{-1}$ so that investors dislike long-term climate uncertainty. The *brown sector* is calibrated in the spirit of the business cycle literature to replicate key moments for our EU aggregate. In the decentralized economy, financial leverage matters. In the absence of emission concerns, or, equivalently, when the brown firm's net emissions are zero, setting $\theta_0 = .5$ implies a book debt-to-assets ratio of 50%. At the steady state, net emissions are positive and we have a debt-to-asset ratio of 33%. The coefficients ϕ_0^{EA} and ϕ_1^{EA} are calibrated to a small and large figure, respectively. This strategy enables us to convexify the emission constraint. the same applies to the parameter ϕ_1, which is important to convexify the leverage constraint. The parameter ϕ_0 is chosen so that the constraint is binding at the deterministic steady state, as in Croce et al. (2012).

Productivity parameters are obtained from EU productivity data. Both the cointegration parameter, ϕ_g, and the exposure of long-run productivity to emissions, ϕ_s, are calibrated to be small and undetectable in small sample. This choice makes our calibration conservative in two dimensions. First, it is well-known that the market price of risk of long-run productivity is substantial and hence having a large ϕ_s would enhance our results. Second, data on green investments are recent and hence to assess cointegration in small sample is empirically very challenging. Our calibration enforces balanced-growth without relying on strong empirical results.

The parameters group under *"Climate"* refer to the interaction between the production side of the economy and the concentration of CO_2. The parameters that govern the evolution of λ_t mimic the properties of emissions-per-GDP obtained from UNECE. The damage specification is in the spirit of Shapiro and Metcalf (2023). We target an average damage of about 4% of output. This number is broadly consistent with macroeconomic estimates, and

it is set to a conservative value relative to new estimates (for a comprehensive review, see, among others, NGFS).

The parameters referring to the *green sector* are chosen to reproduce three current empirical facts. First, this sector is more capital-intensive than the brown sector (higher capital share). Second, this sector features safer returns, which require us to calibrate the adjustment costs to a mild level in order to match our observed greenium. The greenium is computed by sorting European public firms according to their direct emission-intensity reported in Trucost. We use the returns of the top (bottom) quintile to measure the return of brown (green) firms in our model. Returns are equally weighted. Looking at equally-weighted returns of emission intensity-sorted firms mitigates bias concerns related to size. The parameter $\overline{zc}$ determines the average productivity gap across sectors and it is chosen to match the relative size of capital in the two sectors according to our available data.

The parameters group under *"ETS Authority"* are obtained mostly by looking at historical data on emission allowances from the EU ETS system. The debt tax shield is set to 20%, to account for country heterogeneity and specific upper limits on deductibility (among others, see Fatica et al. 2012 and Castillo et al. 2017). The green sector subsidy is set to 30% of investment to account for the several subsidies provided in the EU. We run sensitivity analysis with respect to this parameter in section 6.

Reliable data on the opaqueness of the VCC market are not available. Many specialized outlets suggest that it is substantial (for a recent example, see Bloomberg). In practice, we set the reporting gap parameter $b_0 = 2.2\%$, an intentionally small number to show how powerful opaqueness is in altering the allocation of capital across brown and green assets. This is a conservative choice, our data suggest that we could also consider a distortion fifty times larger and hence strengthen further our results. The report cost intensity ($zeta$) is set to target a conservative $E(p)/E(p_g)$ ratio of just 1.5 (in the data, we have 4.15).

Under the Green Coins setting, we calibrate ω_g so that the cost of running the blockchain is equal to 1.55% of output. We think of this target as a reasonable upperbound since (i)

Table 2: Simulated Moments

	Data		Model		
	Point Est.	S.Err.	ETS	GCCB	Planner
Calibrated Emission Allowances					
$\mathbb{E}(a)$	-1.92	0.12	-1.92		
$\sigma(a)$	0.12	0.02	0.12		
$ACF1(a)$	0.27	0.20	0.20		
Calibrated Emissions per Unit of Output					
$\mathbb{E}(\log(\lambda))$	-1.98	0.07	-1.86	-1.86	-1.86
$\sigma(\log(\lambda))$	0.02	0.22%	0.02	0.02	0.02
$ACF1(\log(\lambda))$	0.29	0.17	0.40	0.40	0.40
Standard Macro Moments					
$\mathbb{E}(\Delta c)$ (%)	0.96	0.40	1.00	1.00	1.00
$\sigma(\Delta c)$ (%)	2.22	0.57	2.80	2.80	3.20
$ACF1(\Delta c)$	0.03	0.23	0.03	0.01	-0.02
$\sigma(\Delta c)/\sigma(\Delta gdp)$	0.73	0.12	0.87	0.86	0.99
$\sigma(\Delta i)/\sigma(\Delta gdp)$	1.98	0.28	1.60	1.44	1.04
$\mathbb{E}(C)/E(GDP)$	0.72	0.43%	0.75	0.75	0.74
Asset Prices					
$\mathbb{E}(r_f)$ (%)	3.39	2.09	3.01	3.07	2.92
$\mathbb{E}(r^B)$ (%)	6.27	6.52	4.65	4.46	4.45
$\mathbb{E}(r^B - r^G)$ (%)	3.34	1.98	2.26	1.82	1.68
Carbon Prices: ETS Allowances, VCC, and SCC					
$\mathbb{E}(p)/\mathbb{E}(p_g)$	4.15	1.87	1.50	1	–
$\mathbb{E}(p_g)$, 2021 USD, tCO_2			37	107	–
$E[SCC]$, 2021 USD, tCO_2			73	78	79

Notes: Empirical moments are computed using annual data. All data sources are discussed in 4.1. Standard errors are calculated using HAC-adjusted standard errors. The entries for the model are obtained by averaging the results across simulated small samples. Our baseline calibration is detailed in table 1. ACF(1) stands for autocorrelation function with a lag equal to 1. SCC stands social cost of carbon and is computed as detailed in the Appendix. Emissions λ_t are expressed in kilograms per unit of real 2021 output (PPP, USD). Carbon prices are obtained by multiplying model-implied carbon prices by 1000, so that they are expressed in 2021-USD per tCO_2.

recent state-of-the-art verification protocols are less expensive; (ii) tokenizing offsets involves a moderate number of transactions to be verified, and (iii) the GCCB could run most of its verification on a layer 2 blockchain that submits proofs to the base chain in bulk once a month. In this case, we assume that the emission target, $\overline{A}$, is constant. For comparability, the target is equal to the average one estimated from the ETS setting.

Solution method and simulation. We solve our models using the perturbation method. Specifically, we use `Dynare++` to implement a second-order log approximation. We find the performance of our model under the ETS setting very satisfactory, given that it reproduces several dimensions of both the macro and the emission data (see table 2).[2]

Importantly, our model replicates the cross section of returns observed in the EU. The average annual equity log return for brown stocks under the ETS system is equal to 4.65%, a substantial number given our conservative calibration of corporate leverage (returns are levered by a coefficient of approximately 1.5). The greenium is equal to 2.26%, i.e., 66% of the one observed in our small sample. These dimensions are important for the discussion of the social cost of carbon (SCC), i.e., the expected present value of future damages due to additional emissions (for a formal derivation, see equation (A9) in the Appendix). It is well known that this estimate is very sensitive to the discount rate adopted. In our case, we use the same equilibrium stochastic discount factor that we use to price the cross section of brown and green equities.

The ratio $\mathbb{E}(p)/\mathbb{E}(p_g)$ considers the average cost of emissions on the European ETS versus the average cost of emissions on the private VCC market. On average, ETS allowances have been four times more expensive than VCCs. Given our short timeseries, however, we face substantial uncertainty on this moment and hence we target a ratio of 1.5. As a result, in the model, VCCs trade at \$37 per ton, a price observed in 2021 for high-integrity credits. Simultaneously, the model-implied price of emission allowances is about € 47, a value consistent with the range observed in 2021.

The model-implied SCC is \$73, a plausible number according to Moore et al. (2024). Since recent studies argue for a higher SCC, in what follows we note four things. First, our discount rate accounts for the pro-cyclicality of emissions and hence it discounts future damages more strongly. Second, our model intentionally abstracts away from rare events and tipping points. Third, accounting for inflation, this price becomes \$91 in 2025, or,

[2]Long-sample simulations produce similar results, see table C1 in the Appendix.

equivalently, about € 80. Finally, we note that SCC is not a good proxy for welfare. In the next section, we show that the model with the ETS produces a substantial welfare loss even though it implies a SCC lower than the SCC obtained under the planner's allocation.

5 Main Results

In this section, we share results relevant to show how our model could be used for policy analysis.

5.1 Optimal GCCB Policy Rule.

Given the encouraging results obtained from calibrating the model with the ETS setting, we now proceed with the calibration of our counterfactual economy with green coins. In order to do so, we need to calibrate the parameters in equation (17). Since there is no data counterpart, we collect all of these parameters in a vector θ and choose them in order to solve the following problem:

$$\theta^* := \arg\min_{\theta} \max\{\mathbb{E}\left[(R^P_{bt} - R^{GC}_{bt}(\theta))^2\right], \mathbb{E}\left[(R^P_{gt} - R^{GC}_{gt}(\theta))^2\right]\}, \tag{18}$$

where R^P_{bt} and R^P_{gt} are defined in equations (13)–(14) and refer to the return of brown and green capital chosen by planner, respectively. The returns R^{GC}_{bt} and R^{GC}_{gt}, defined in the Appendix, equation (A36) and (A44), respectively, are obtained by solving the setting with green coins given a specific value of θ.

In practice, we simulate both the planner's problem and the green coin economy using the same set of common exogenous shocks and minimize the maximum average discrepancy in the returns of brown and green capital across states and histories. By doing so, we identify a green coin policy that mimics as much as possible the dynamics of the returns and capital stocks that we would observe at the first-best. As shown in our figures (C1)–(C2) in the appendix, the biggest average discrepancy is driven by the returns of green capital. As a

Table 3: Optimal GCCB Policy Parameters

	A_0	A_s	A_k	A_{lr}	A_λ	A_x	A_z
θ^*	-0.111	2000	-1055	-837	-4.46	-0.89	-2.99
$\frac{\partial u}{\partial(\cdot)}(\times 100)$	37.34	0.000	0.000	0.000	0.002	0.000	0.021

Notes: This table reports the values of the parameters determined as in equation (18). In the second row, we report the partial marginal variation of the welfare function with respect to each parameter.

result, the optimal parameters are mainly driven by the green sector. All parameters belong to the interior, with the exception of A_0. In our model, we require total GHGs concentration to be non-negative, and thus the number of green coins the authority holds cannot be too large. That explains the corner solution with respect to A_0.

Policy parameters and implications. We report our parameter values in the first row of table 3. First of all, the value of A_0 implies that it is optimal for the GCCB to hold on average 47% of the total coins, i.e., the institution must commit to a quite 'hawkish' target. The GCCB increases this share with the stock of CO_2 ($A_s > 0$), but it also accommodates long-run growth ($A_{lr} < 0$) in the overall economy. In addition, the GCCB reduces its holdings of coins when the green sector has become particularly large compared to the brown one ($A_k < 0$).

The rightmost three coefficients refer to the contemporaneous response to the three unexpected shocks in the economy. In all cases, the GCCB responds by increasing the supply of green coins available to the private sector, i.e, by reducing its share of coins. We note that this behavior corresponds to implementing an 'emission-smoothing policy'.

Specifically, the emission constraint produces costs proportional to output, and hence it has effects similar to those of distortionary taxes. Under the optimal policy, it is optimal to accommodate expansionary productivity shocks by leaving more green coins to the private sector and letting their price decline. Note that accommodating these shocks in the short-run causes an increase in the future stock of CO_2, s. In addition, to the extent that this shock diffuses slowly to the green sector, green capital decreases relative to brown capital. These

two channels reinforce each other, and they imply that the short-run accommodation slowly turns into a more stringent emission constraint for the long-run.

Similarly, if the economy is affected by an adverse shock to the emission-to-GDP intensity, $\epsilon_\lambda > 0$, the GCCB responds by partially offset this shock by leaving more coins to the private sector. As the stock of CO_2 increases, the GCCB increases its purchases of coins in the long-run making them more expensive for the private sector.

Welfare analysis. It is important to assess the convenience of the token-based system compared to the ETS setting. First of all, we must point out that in the setting with the ETS, the welfare costs are equal to 5.92% of time-0 first-best consumption. Our green coin setting overcomes most of this loss as it features a welfare loss of just 0.57% of first-best time-0 consumption. In what follows, we 'inspect the mechanism' in order to detail where these welfare gains come from. We continue to refer to the simulated moments reported in table 2 and complete them with additional results.

The second row of table 3 shows the sensitivity of our welfare function in the settings with green coins with respect to each parameter evaluated at θ^*. Looking at the magnitudes of these derivatives, the most important parameters that we discuss are A_0 and A_z. In figure 2, we plot average moments as we vary these two parameters. Specifically, we focus on how these parameters affect the relative size of the green and brown sector, the average damage, and the risk premia for the green and brown sectors.

In each panel of figure 2 we plot two lines. The dotted (solid) lines refer to difference between the moment obtained in our green coin setting and the corresponding value at the first-best (under the ETS setting). Hence when the red (blue) line gets close to zero, it means that the green coin-based setting mimics the planner (ETS setting) stochastic steady state.

A_0 is a key parameter that allows the green coin setting to mimic the same relative size of the green sector obtained at the first-best. As a result, the implied average damage in

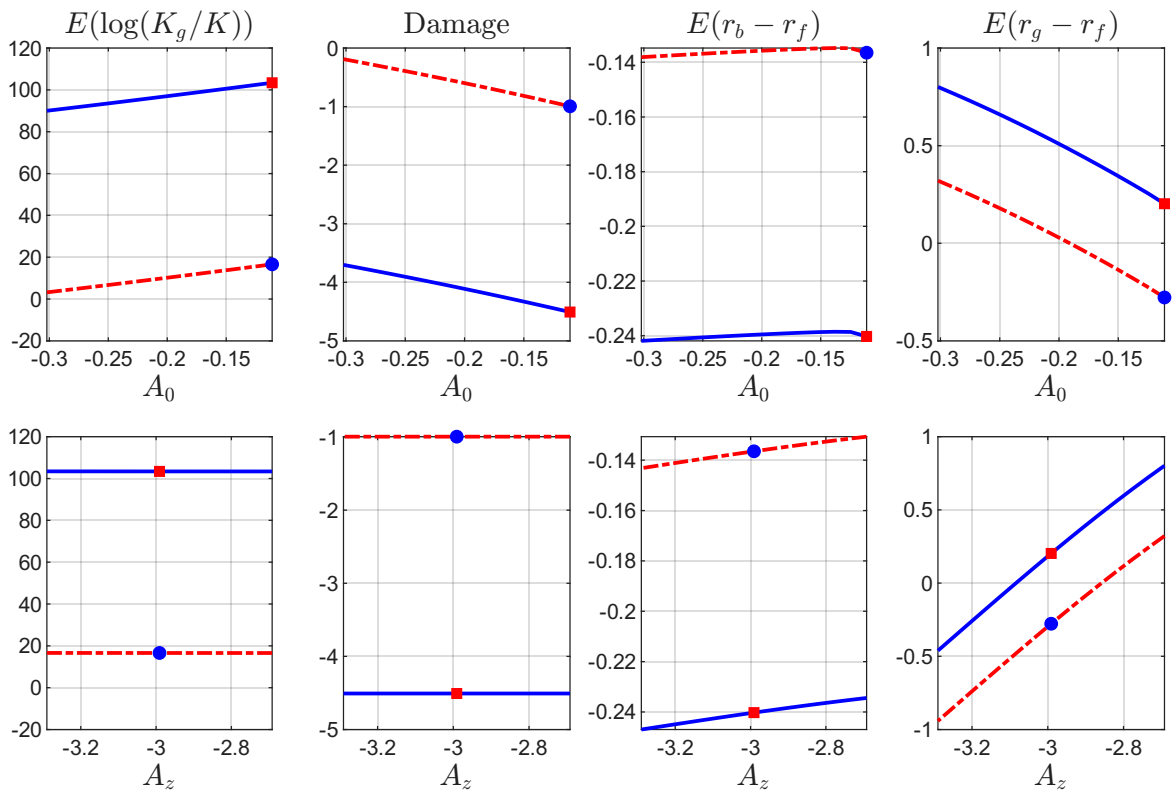


Fig. 2: Policy Parameters and Stochastic Steady State Values

Notes: In each panel, the blue (red) line shows the difference between the reported moment computed under the green coin setting and the ETS (first-best) setting. All variables are multiplied by 100.

this economy is very similar to the first-best level. The ETS setting, instead, features a much smaller green sector and a bigger damage. Interestingly, by committing ex-ante to buy a higher average share of green coins, the GCCB sustains the green sector and reduces its cost of capital while keeping the cost of equity for the brown sector almost constant. At the optimum, the cost of equity in the brown sector is very similar to the level obtained under the ETS system, whereas the cost of capital of the green sector is reduced even below its first-best level. We note that our setting with green coins is still affected by financing frictions, and hence it is not supposed to perfectly mimic the first-best. Given the implicit subsidy to brown capital through the tax shield on debt, the GCCB subsidizes the green sector through a policy that makes the cashflow associated to the offsets safer. By no arbitrage, a lower risk premium stimulates green investment. As reported in table 2, the GCCB finds it optimal to

have a market price for carbon nearly 40% higher than the SCC. This is required in order to stimulate enough investment in the green sector and align the SCC value across the market economy ($78) and the first-best ($79).

Finally, we note that the riskiness of both the green and the brown sector is sensitive to A_z, i.e., the contemporaneous response of the GCCB to short-run growth shocks. If this parameter is not negative enough, i.e., if the GCCB does not accommodate positive shocks, the risk premium increases. Simultaneously, in the brown sector a less accommodating policy results in cashflows that respond less to long-run news. This is because of the hedging effect coming from the cost of offsetting emissions. As a result, as A_z becomes less negative, the risk premium in the brown sector declines. These effects on the cost of capital ultimately affect the relative size of the green and brown sectors. Specifically, a less accommodating policy response to an emission intensity shock reduces the relative share of green capital in the economy. The latter effect is not easily visible in our panel because it is second-order compared to the difference in our steady state moments across the ETS and the first-best settings.

5.2 Dynamic Responses

Analyzing the dynamic responses to shocks across the three settings help us further in understanding in which dimensions that green coin setting dominates the ETS. In what follows, we discuss the responses depicted in figure 3.

With respect to positive short-run productivity shocks, consumption, brown investment, and output behave as in a standard neoclassical model. Under the first best, there is a reallocation of investment away from the green sector in favor of the brown sector. This is because the productivity gain affects immediately the brown sector, but it propagates slowly over time in the green sector. In the ETS economy, in contrast, the opposite happens. The emission allowances do not adjust promptly to this shock and hence there is a strong demand for VCCs which is followed by more investment in green assets. Also the green coin

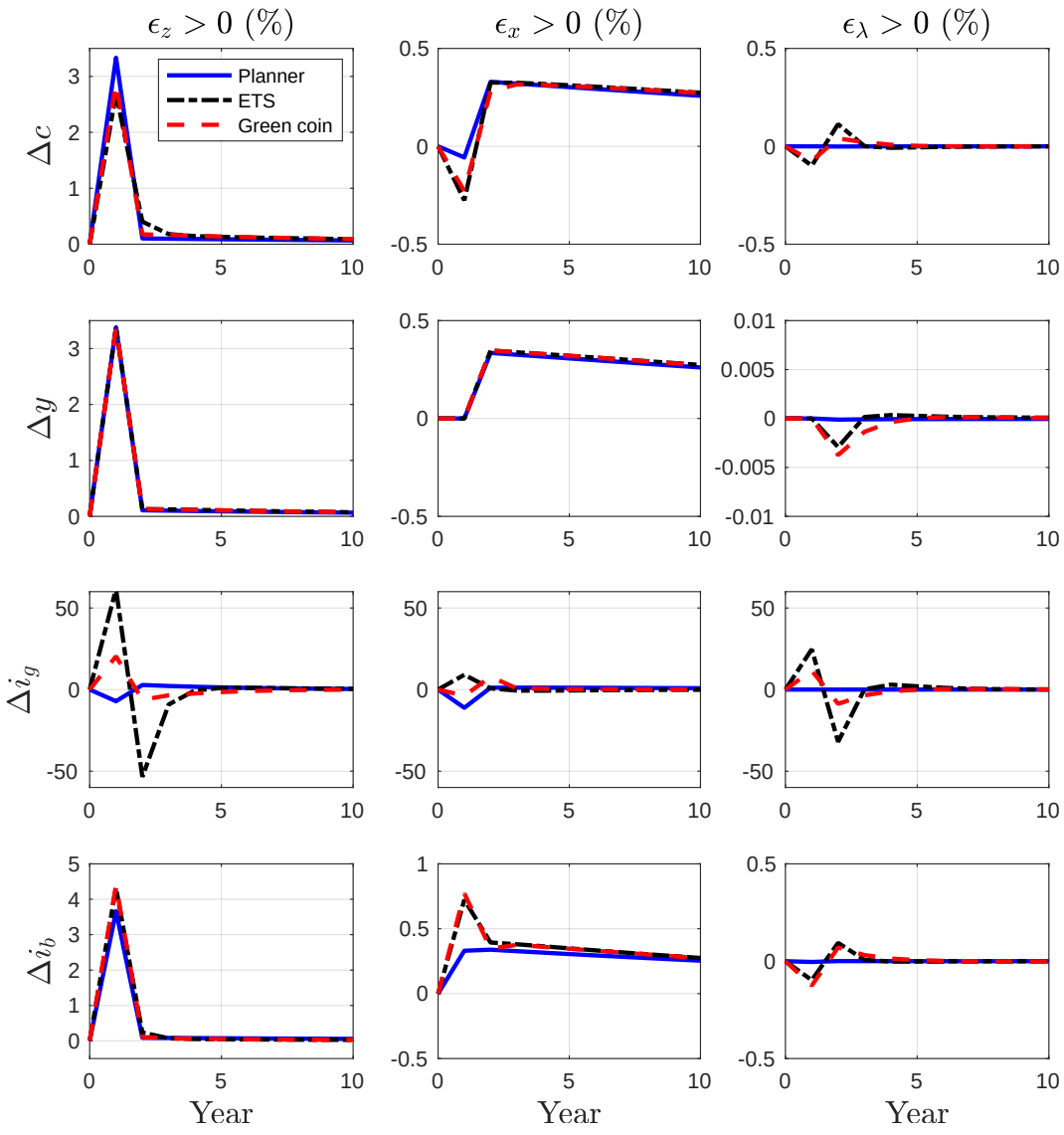


Fig. 3: Impulse Responses: Quantities

Notes: This figure depicts responses to a one-standard deviation shock. All responses are multiplied by 100. The benchmark calibration is detailed in table 1.

setting features an increase in green investment, but this increase is moderate and hence this response is closer to that measured at the first-best.

When we turn our attention to long-run productivity news, the green investment in the green coin setting mimics very closely the planner's solution. Both the ETS and the green coin settings feature excessive increase in investment and this is due to the additional distortion coming from leverage (Croce et al., 2012).

In the ETS system, a positive emission shock, λ, promotes a reallocation of investment resources. A higher λ prompts a higher demand of VCCs, and hence it increases green investment. Simultaneously, by resource constraint, brown investment must decline. Because of convex adjustment costs, these adjustments also imply that green assets appreciate, and thus they provide a hedge against emission shocks. Brown stocks, instead, depreciate in a high marginal utility state, and hence they carry an additional risk-premium. These differences in exposure generate a greenium. Interestingly, at the first-best, the response of macroeconomic quantities with respect to this shock go in the same direction, but they are two orders of magnitude smaller. Our green coin setting promotes a reallocation of investment resources that is in between that observed in the ETS setting and at the first-best. Hence, this setting mitigates distortions also with respect to changes in the emission-to-output ratio.

We now turn our attention to the cost of carbon across our three settings. We report our model implied values in table 4. Note that these values refer to marginal prices rescaled by productivity. In order to better interpret these results, we also report the implied equilibrium discount rate for future damages. The marginal cost of CO_2 declines as we move from the Planner's setting toward the ETS system. At first sight, one may find this result surprising as our welfare results go exactly in the opposite direction. In reality, this result is fully driven by the discount rate. The cost of CO_2 is a present value, and it declines with higher discount rate. In the ETS system, the damage is more procyclical, and thus it carries a higher risk premium. The marginal cost of carbon does not reflect welfare because it is silent on the quantity of emissions. In our model, emissions under the ETS system are 20 times higher than at the first-best and hence the implied damage produces significant welfare losses.

Table 4: Emission Price and Discount Rate

Planner		GCCB		ETS	
q_s	r_s	q_s	r_s	q_s	r_s
0.079	4.81	0.078	4.87	0.073	4.87

Notes: This table reports the implied cost of emissions scaled by productivity and its discount rate across our three settings.

6 Additional Sensitivity Analysis

In this section, we run sensitivity analysis in order to better decompose our main results. We focus on the role of policy and then perturb key parameters.

Timing of information. In our main section, we assume that the GCCB can control the supply of coins simultaneously with the realization of shocks, whereas in the ETS system we assume a one-period delay. In order to understand whether this is the key dimension driving our welfare benefits, we replace equation (17) with the following equation:

$$sg_t = A_0 + [A_s\, A_k\, A_{lr}] \cdot \begin{bmatrix} \hat{s}_{t-1} \\ \widehat{\frac{K_{gt-1}}{K_{bt-1}}} \\ x_{t-1} \end{bmatrix} + [A_z\, A_\lambda\, A_x] \cdot \begin{bmatrix} \epsilon_{z,t-1} \\ \epsilon_{\lambda,t-1} \\ \epsilon_{x,t-1} \end{bmatrix}, \tag{17b}$$

We report the new optimal parameters in table 5, panel A. In this case, the welfare loss is equal to 1.52%, that is, about 1% higher than in the case in which the GCCB responds immediately to shocks. The net welfare benefit with respect to the ETS setting is about 4.5%, a prominent figure.

Flexibility. Thanks to the verification services obtained via tokenization, the GCCB can alter the price of offsets in a flexible way. We assess the value of this flexibility by comparing our previous results to the case in which the GCCB can choose only A_0, while setting all other policy parameters to zero. Equivalently, the GCCB does not dynamically adjust its holdings, it commits to buying a fixed share of offsets. In this case, at the optimum $A_0 = -0.111$, implying that the GCCB on average retains 47% of the tokens. The implied welfare benefits

Table 5: Optimal GCCB Policy Parameters

	A_0	A_s	A_k	A_{lr}	A_λ	A_x	A_z
Panel A: timing of information							
θ^*	-0.178	1209.625	-382.750	-1901.500	-84.282	957.295	-26.534
$\frac{\partial u}{\partial(\cdot)}(\times 100)$	5.672	-0.000	-0.000	0.000	0.000	0.000	0.000
Panel B: less sensitivity to net emissions							
θ^*	-0.108	1987.171	-1058.499	-688.250	-4.492	-1.082	-2.989
$\frac{\partial u}{\partial(\cdot)}(\times 100)$	57.880	-0.000	-0.001	-0.000	0.002	0.000	0.020

Notes: In panel A, we report the values of the parameters determined as in equation (17b). In panel B, we use our benchmark calibration but adopt a leverage function less sensitive to emissions (see equation (19)). Across all panels, in the second row, we report the partial marginal variation of the welfare function with respect to each parameter.

are very similar to those obtained under our benchmark policy. This result is consistent with the small sensitivity of welfare to marginal variations of the other parameters reported in table 3.

The role of long-run productivity risk. We resolve the model by setting $\sigma_x = 0$ so that long-run risk is removed. By comparing the ETS setting with and without long-run risk we obtain a welfare cost of long-run risk of 28% of consumption. The cost is similar also under the GCCB and the planner settings. Interestingly, when comparing the ETS system to the planner's economy without long-run risk, we obtain a slightly higher welfare loss, in the order of 6.4%. The GCCB setting mitigates this loss to just 1.72%, a value higher than under the setting with long-run risk. These figures suggest that actively managing the price of carbon with respect to long-run productivity news is a relevant dimension and it enables the GCCB to get closer to the planner's allocation relative to the setting in which we have only short-run productivity shocks.

Higher subsidies. We question whether most of the inefficiency produced by the ETS setting can be eliminated through a traditional tool, namely, by increasing green investment subsidies. We solve our ETS setting with $\tau_g = 90\%$ and we find small improvements. Despite the cost of this policy more than triples in the economy, the realized net emissions per unit of GDP decline by 0.3‰. This result suggests that the real problem in the green sector

with opaque VCCs is the excessively low price assigned to the offsets. Unless opaqueness is removed, offsets remain underpriced by brown firms and investment in the green sector is subdued.

VCC partial exclusion. We recompute equilibrium quantities when the ETS-based economy allows only a fraction $\iota < 1$ of VCCs to be used for the computation of net emissions, that is, $\widetilde{NE}_t = \lambda_t Y_t - \iota \cdot \widetilde{G}_t$. In what follows, we focus on a partial ban of VCCs and set $\iota = 80\%$. A complete ban of the VCCs like the one implemented in the EU ETS in 2021 (phase 4) would enhance these results. The key insight is that now the demand of VCCs accounts for the fact that VCCs are inferior substitutes of allowances,

$$p_{gt} = \iota \cdot \underbrace{p_t \cdot \frac{EA_t}{\lambda_t Y_t - \widetilde{G}_t}}_{\text{link with } p_t} + \underbrace{\phi_1 \cdot \theta_t' \cdot \frac{C_t^E}{B_t} \cdot \frac{K_{bt}}{\lambda_t Y_t}}_{\text{savings on borrowing}} \underbrace{-\zeta \cdot \frac{\widetilde{G}_t - G_t}{Z_{t-1}}}_{<0,\text{ opaque}},$$

and hence their relative value, $\mathbb{E}(p)/\,\mathbb{E}(p_g)$, must decline.

We find that allowances become 2.2 times more valuable than VCCs, as opposed to the 1.5 ratio obtained under the benchmark calibration with $\iota = 100\%$. In addition, the price of allowances increases to about \$87, an increase of about 85%, qualitatively consistent with the increase seen post-2021 on the ETS market. This intervention produces a welfare benefit of 1% and brings the economy closer to the first-best. The improvement is mainly driven by the fact that the damage related to the stock of CO_2 declines by 1% and hence it improves the average level of consumption.

Less green lending. We question what could happen to the economy if leverage becomes less sensitive to net emissions. We think of it as a change in lending practices away from 'ESG-labeled' products. Specifically, we modify the θ_t function as follows

$$\theta_t = \frac{\theta_0}{1 + \exp(-0.8 \cdot \widetilde{G}_t / \lambda_t Y_t)}, \tag{19}$$

so that the sensitivity to net emissions is reduced by 20%, from 1 to 0.8. We note several results. First, in this setting the optimal policy parameters of the GCCB remain almost unchanged, implying that the main intuitions developed in the previous section do not depend on the sensitivity of leverage to emissions.

Turning our attention to the ETS setting, we note that the price of allowances relative to that of the VCCs increases from 1.5 to 1.65. This result comes from two channels. The price of VCCs declines to \$36, as a reduction in net emission is no longer as advantageous as before for expanding leverage. Simultaneously, the price of allowances increases to about \$60, as the emission gap must be matched by demanding more ETS allowances. In this setting, ultimately the VCC market is even more depressed and there is a higher accumulation of CO_2. As a result, welfare declines by 0.6% relative to the ETS under our benchmark calibration.

7 Conclusions

We propose a new model to examine emissions regulation in general equilibrium. We study an inefficient decentralized economy with an ETS, opaque VCCs, and credit markets with a bias for 'net-zero' firms. We show that this setting is far away from the first-best. We then analyze a counterfactual setting in which emission offsets are tokenized, and tokens available to the private sector are managed by a Green Coin Central Bank (GCCB) that controls the cost of emissions with state-contingent open market operations. We show that this setting can produce substantial welfare gains.

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A Planner's problem

The Lagrangian for the planner's problem:

$$\begin{aligned}\mathcal{L} = &\left[(1-\beta)\tilde{C}_t^{1-\frac{1}{\psi}} + \beta(\mathbb{E}_t[U_{t+1}^{1-\gamma}])^{\frac{1-\frac{1}{\psi}}{1-\gamma}}\right]^{\frac{1}{1-\frac{1}{\psi}}} \\ &+ \{(1-\delta)K_{t-1} + \Psi(I_t/K_{t-1})K_{t-1} - K_t\}Q_t \\ &+ \{(1-\delta_g)K_{g,t-1} + H(I_{gt}/K_{g,t-1})K_{g,t-1} - K_{gt}\}Q_{g,t} \\ &+ \{\chi(s_t)(Z_tL_t)^{1-\alpha}K_{t-1}^{\alpha} - (\tilde{C}_t + I_{gt} + I_t)\}\Omega_t \\ &+ \left\{s_t - \left((1-\delta_s)s_{t-1} + \{\lambda_t(Z_tL_t)^{1-\alpha}K_{t-1}^{\alpha} - (Z_{gt}L_{gt})^{1-\kappa}K_{g,t-1}^{\kappa}\}/Z_{t-1}\right)\right\}Q_{st},\end{aligned}$$

where Q_t, $Q_{g,t}$, Ω_t, and $Q_{s,t}$ are multipliers. The first order conditions are

$$0 = U_t^{1/\psi}\beta(\mathbb{E}_t[U_{t+1}^{1-\gamma}])^{\frac{1-\frac{1}{\psi}}{1-\gamma}-1}\mathbb{E}_t\left(U_{t+1}^{-\gamma}\frac{\partial U_{t+1}}{\partial K_t}\right) - Q_t \tag{A1}$$

$$q_t := \frac{Q_t}{\Omega_t} = 1/\Psi'(I_t/K_{t-1}) \tag{A2}$$

$$0 = U_t^{1/\psi}\beta(\mathbb{E}_t[U_{t+1}^{1-\gamma}])^{\frac{1-\frac{1}{\psi}}{1-\gamma}-1}\mathbb{E}_t\left(U_{t+1}^{-\gamma}\frac{\partial U_{t+1}}{\partial K_{gt}}\right) - Q_{gt} \tag{A3}$$

$$q_{gt} := \frac{Q_{g,t}}{\Omega_t} = 1/H'(I_{gt}/K_{g,t-1}) \tag{A4}$$

$$0 = (1-\beta)U_t^{1/\psi}\tilde{C}_t^{-\frac{1}{\psi}} - \Omega_t \tag{A5}$$

$$0 = -\chi_0\cdot\chi_t\cdot Y_t\Omega_t + U_t^{1/\psi}\beta(\mathbb{E}_t[U_{t+1}^{1-\gamma}])^{\frac{1-\frac{1}{\psi}}{1-\gamma}-1}\mathbb{E}_t\left(U_{t+1}^{-\gamma}\frac{\partial U_{t+1}}{\partial s_t}\right) + Q_{st}. \tag{A6}$$

Plug (A5) into (A1), (A3), and (A6), we obtain

$$q_t = \mathbb{E}_t\left(M_{t+1}\frac{1}{\Omega_{t+1}}\frac{\partial U_{t+1}}{\partial K_t}\right) \tag{A7}$$

$$q_{gt} = \mathbb{E}_t\left(M_{t+1}\frac{1}{\Omega_{t+1}}\frac{\partial U_{t+1}}{\partial K_{gt}}\right) \tag{A8}$$

$$\hat{q}_{st} := \frac{Q_{st}}{\Omega_t} = \chi_0\cdot\chi_t\cdot Y_t + \mathbb{E}_t\left(M_{t+1}\frac{1}{\Omega_{t+1}}\cdot\left(-\frac{\partial U_{t+1}}{\partial s_t}\right)\right) \tag{A9}$$

$$\hat{p}_{st} := \mathbb{E}_t \left(M_{t+1} \hat{q}_{s,t+1} \right) \tag{A10}$$

$$\hat{q}_{st} = \chi_0 \cdot \chi_t \cdot Y_t + (1 - \delta_s) \hat{p}_{st}. \tag{A11}$$

The envelop conditions read

$$\frac{\partial U_t}{\partial K_{t-1}} = Q_t \cdot \left[(1 - \delta) + \Psi_t - \Psi_t' \cdot \frac{I_t}{K_{t-1}} \right] + \left(\Omega_t \chi_t - \frac{Q_{st}}{Z_{t-1}} \lambda_t \right) \alpha \frac{Y_t}{K_{t-1}} \tag{A12}$$

$$\frac{\partial U_t}{\partial K_{g,t-1}} = Q_{gt} \cdot \left[(1 - \delta_g) + H_t - H_t' \cdot \frac{I_{gt}}{K_{g,t-1}} \right] + \frac{Q_{st}}{Z_{t-1}} \kappa \frac{\tilde{G}_t}{K_{g,t-1}} \tag{A13}$$

$$\frac{\partial U_t}{\partial s_{t-1}} = -(1 - \delta_s) Q_{st}. \tag{A14}$$

After defining $q_{st} = \hat{q}_{st}/Z_{t-1}$ and $p_{st} = \hat{p}_{st}/Z_{t-1}$, we obtain the returns as reported in the paper

$$R^P_{s,t} = \frac{Z_{t-1}}{Z_{t-2}} \frac{q_{st}}{p_{s,t-1}} \tag{A15}$$

$$R^P_{I,t} = \left(q_t \cdot \left[(1 - \delta) + \Psi_t - \Psi_t' \cdot \frac{I_t}{K_{t-1}} \right] + (\chi_t - q_{st} \lambda_t) \, \alpha \frac{Y_t}{K_{t-1}} \right) / q_{t-1} \tag{A16}$$

$$R^P_{G,t} = \left(q_{gt} \cdot \left[(1 - \delta_g) + H_t - H_t' \cdot \frac{I_{gt}}{K_{g,t-1}} \right] + q_{st} \kappa \frac{\tilde{G}_t}{K_{g,t-1}} \right) / q_{g,t-1}, \tag{A17}$$

and the associated Euler equations:

$$1 = \mathbb{E}_t \left[M_{t+1} R^P_{s,t+1} \right] \tag{A18}$$

$$1 = \mathbb{E}_t \left[M_{t+1} R^P_{I,t+1} \right] \tag{A19}$$

$$1 = \mathbb{E}_t \left[M_{t+1} R^P_{G,t+1} \right] \tag{A20}$$

B ETS with opaque VCCs

B.1 Brown firm

The brown firm solves

$$V_t = \max_{K_t, B_t, I_t, G_t, EA_t} D_t + \mathbb{E}_t(M_{t+1} V_{t+1}) \tag{A21}$$

subject to

$$\begin{aligned} D_t = {} & \chi(s_t) \cdot Y_t - W_t L_t - (1-\tau) r_t^B B_{t-1} + \Delta B_t - I_t - C_t^E - C_t^B \\ & - p_t \cdot EA_t - p_{gt} G_t - C_t^C - C_t^{EA} \end{aligned} \tag{A22}$$

$$K_t = (1-\delta) K_{t-1} + \Psi(I_t / K_{t-1}) K_{t-1} \tag{A23}$$

Optimality with respect to debt. The first order condition with respect to corporate debt and the Envelope theorem jointly imply:

$$\begin{aligned} 1 &= \frac{\partial C_t^B}{\partial B_t} + \frac{\partial C_t^E}{\partial B_t} - \mathbb{E}_t \left[M_{t+1} \frac{\partial V_{t+1}}{\partial B_t} \right] \\ \frac{\partial V_t}{\partial B_{t-1}} &= -(1 + (1-\tau) r_{t-1}^B). \end{aligned}$$

Taking into account the fact that at the equilibrium r^B is equal to the risk-free rate:

$$\frac{\partial C_t^B}{\partial B_t} + \frac{\partial C_t^E}{\partial B_t} = \mathbb{E}_t \left[M_{t+1} \tau \right] r_{f,t},$$

which becomes

$$\frac{\partial C_t^B}{\partial B_t} + \frac{\partial C_t^E}{\partial B_t} = \frac{\tau \cdot r_{f,t}}{1 + r_{f,t}},$$

The left-hand side refers to the total marginal cost of issuing an extra unit of debt. The right-hand side measures the value of the corporate interest tax advantage.

Optimality with respect to EA. The demand of emission allowances is related to the marginal environmental distress cost:

$$p_t = -\frac{\partial C_t^{EA}}{\partial EA_t} = \phi_1^{EA} \cdot \frac{C_t^{EA}}{EA_t} \cdot \frac{\lambda_t Y_t - \widetilde{G}_t}{EA_t}$$

Optimality with respect to declared green credits. The first order condition with respect to G_t imply the following:

$$p_{gt} = \underbrace{p_t \cdot \frac{EA_t}{\lambda_t Y_t - \widetilde{G}_t}}_{\text{savings on EA}} + \underbrace{\phi_1 \cdot \theta_t' \cdot \frac{C_t^E}{B_t} \cdot \frac{K_{bt}}{\lambda_t Y_t}}_{\text{savings on borrowing}} \underbrace{-\zeta \cdot \frac{\widetilde{G}_t - G_t}{Z_{t-1}}}_{\text{<0, opaque}},$$

Optimality with respect to capital. Envelope and first order conditions with respect to investment and capital imply the following:

$$\begin{aligned} q_t &= \frac{1}{\Psi_t'} = \mathbb{E}_t\left[M_{t+1}\frac{\partial V_{t+1}}{\partial K_t}\right] - \frac{\partial C_t^E}{\partial K_t} \\ \frac{\partial V_t}{\partial K_{t-1}} &= \chi(s_t)\frac{\partial Y_t}{\partial K_{t-1}} - \frac{\partial C_t^{EA}}{\partial K_{t-1}} - \frac{\partial C_t^E}{\partial K_{t-1}} - \frac{\partial C_t^B}{\partial K_{t-1}} + q_t\left(1 - \delta - \frac{\Psi_t' I_{bt}}{K_{t-1}} + \Psi_t\right). \end{aligned}$$

B.2 Green firm

$$V_{gt} = \max_{K_{gt}, I_{gt}} D_{gt} + \mathbb{E}_t(M_{t+1} V_{g,t+1}) \tag{A24}$$

$$D_{gt} = p_{gt} G_t^s - (1 - \tau_{gt}) I_{gt} - W_{gt} L_{gt} \tag{A25}$$

$$K_{gt} = (1 - \delta_g) K_{g,t-1} + H(I_{gt}/K_{g,t-1}) K_{g,t-1}, \tag{A26}$$

where $G_t^s = \xi_t \tilde{G}_t$; $\xi_t = 1 + b_0 \exp(b_1 \epsilon_{\xi,t})$; and $\tilde{G}_t = (Z_{g,t} L_{gt})^{1-\kappa} K_{g,t-1}^{\kappa}$.

Investment are determined by

$$q_{gt} = (1 - \tau_{gt})/H'_t = \mathbb{E}_t \left(M_{t+1} \frac{\partial V_{g,t+1}}{\partial K_{gt}} \right) \tag{A27}$$

$$\frac{\partial V_{gt}}{\partial K_{g,t-1}} = p_{g,t} \frac{\partial G^s_t}{\partial K_{g,t-1}} + q_{gt}\{1 - \delta_g + H_t - H'_t \cdot (I_{gt}/K_{g,t-1})\} \tag{A28}$$

C Green Coin Setting

C.1 Brown firm

The brown sector faces the following problem

$$V_t = \max_{K_t, B_t, I_t, G^{PD}_{gt}} D_t + \mathbb{E}_t(M_{t+1} V_{t+1}) \tag{A29}$$

$$D_t = \chi(s_t) \cdot Y_t - W_t L_t - (1 - \tau) r^B_t B_{t-1} + \Delta B_t - I_t - C^E_t - C^B_t \tag{A30}$$
$$- p_{gt} G^{PD}_{gt} - C^{EA}_t$$

$$K_t = (1 - \delta) K_{t-1} + \Psi(I_t/K_{t-1}) K_{t-1} \tag{A31}$$

where

$$C^E_t = \phi_0 \cdot \exp\left(\phi_1 \left\{ 1 - \frac{\theta_t K_t}{B_t} \right\} \right) \cdot Z_{t-1} \quad (\text{mimics } B_t \leq \theta_t \cdot K_t) \tag{A32}$$

$$C^B_t = \eta Z_{t-1} \left(\frac{B_t}{Y_t} - \frac{B_{ss}}{Y_{ss}} \right)^2 \tag{A33}$$

$$\theta_t \equiv \theta \left(\frac{G^{PD}_t}{\lambda_t Y_t} \right); \quad \theta'(\cdot) > 0 \tag{A34}$$

$$C^{EA}_t = \phi^{EA}_0 \cdot \exp\left(\phi^{EA}_1 \left\{ \frac{(\lambda_t Y_t - G^{PD}_t)}{\bar{A} \cdot Z_{g,t-1}} - 1 \right\} \right) \cdot Z_{t-1} \tag{A35}$$

Optimality with respect to capital. Envelope and first order conditions with respect

to investment and capital imply the following:

$$\begin{aligned} q_t &= \frac{1}{\Psi_t'} = \mathbb{E}_t\left[M_{t+1}\frac{\partial V_{t+1}}{\partial K_t}\right] - \frac{\partial C_t^E}{\partial K_t} \\ \frac{\partial V_t}{\partial K_{t-1}} &= \chi(s_t)\cdot\frac{\partial Y_t}{\partial K_{t-1}} - \frac{\partial C_t^{EA}}{\partial K_{t-1}} - \frac{\partial C_t^E}{\partial K_{t-1}} - \frac{\partial C_t^B}{\partial K_{t-1}} + q_t\left(1-\delta-\frac{\Psi_t' I_{bt}}{K_{t-1}}+\Psi_t\right). \end{aligned}$$

After defining

$$R_{bt}^{GC} = \frac{\frac{\partial V_t}{\partial K_{t-1}}}{q_{t-1} + \frac{\partial C_{t-1}^E}{\partial K_{t-1}}}, \tag{A36}$$

the Euler equation associated with brown investment reads

$$1 = \mathbb{E}_t(M_{t+1}R_{b,t+1}^{GC}).$$

C.2 Green firm

The green sector faces

$$V_{gt} = \max_{K_{gt},I_{gt}} D_{gt} + \mathbb{E}_t(M_{t+1}V_{g,t+1}) \tag{A37}$$

$$D_{gt} = p_{gt}G_{gt} - (1-\tau_{gt})I_{gt} - W_{gt}L_{gt} \tag{A38}$$

$$K_{gt} = (1-\delta_g)K_{g,t-1} + H(I_{gt}/K_{g,t-1})K_{g,t-1} \tag{A39}$$

First-order conditions with respect to capital and investment:

$$q_{gt} = \mathbb{E}_t\left(M_{t+1}\frac{\partial V_{g,t+1}}{\partial K_{gt}}\right) \tag{A40}$$

$$q_{gt} = (1-\tau_{gt})/H_t' \tag{A41}$$

$$\tag{A42}$$

The envelop condition is

$$\frac{\partial V_{gt}}{\partial K_{g,t-1}} = p_{g,t}\frac{\partial G_t^s}{\partial K_{g,t-1}} + q_{gt}\{1 - \delta_g + H_t - H_t' \cdot (I_{gt}/K_{g,t-1})\} \tag{A43}$$

After defining

$$R_{gt}^{GC} = \frac{\partial V_{gt}}{\partial K_{g,t-1}}/q_{g,t}, \tag{A44}$$

we obtain the Euler equation:

$$1 = \mathbb{E}_t(M_{t+1}R_{g,t+1}^{GC}).$$

C.3 Additional results

In table C1, we report simulation results when we simulate the model over a millennium. Moments are averaged across repetitions.

Table C1: Long Sample Simulation Results

Moment	Planner S	Planner L	GCCB S	GCCB L	Dec. S	Dec. L
$\sigma(\Delta c)$	0.032	0.036	0.028	0.034	0.028	0.034
$ACF1(\Delta c)$	−0.017	0.207	0.007	0.266	0.033	0.277
$\sigma(\Delta c)/\sigma(\Delta y)$	0.987	0.987	0.855	0.905	0.868	0.922
$\sigma(\Delta i)/\sigma(\Delta y)$	1.039	1.034	1.441	1.344	1.602	1.477
$\sigma(\log \lambda)$	0.023	0.024	0.023	0.024	0.023	0.024
$ACF1(\log \lambda)$	0.402	0.492	0.402	0.492	0.402	0.492

Notes: In the columns denoted by S (L) The entries for the model are obtained by averaging the results across simulated small (long) samples. Our baseline calibration is detailed in table 1. ACF(1) stands for autocorrelation function with a lag equal to 1.

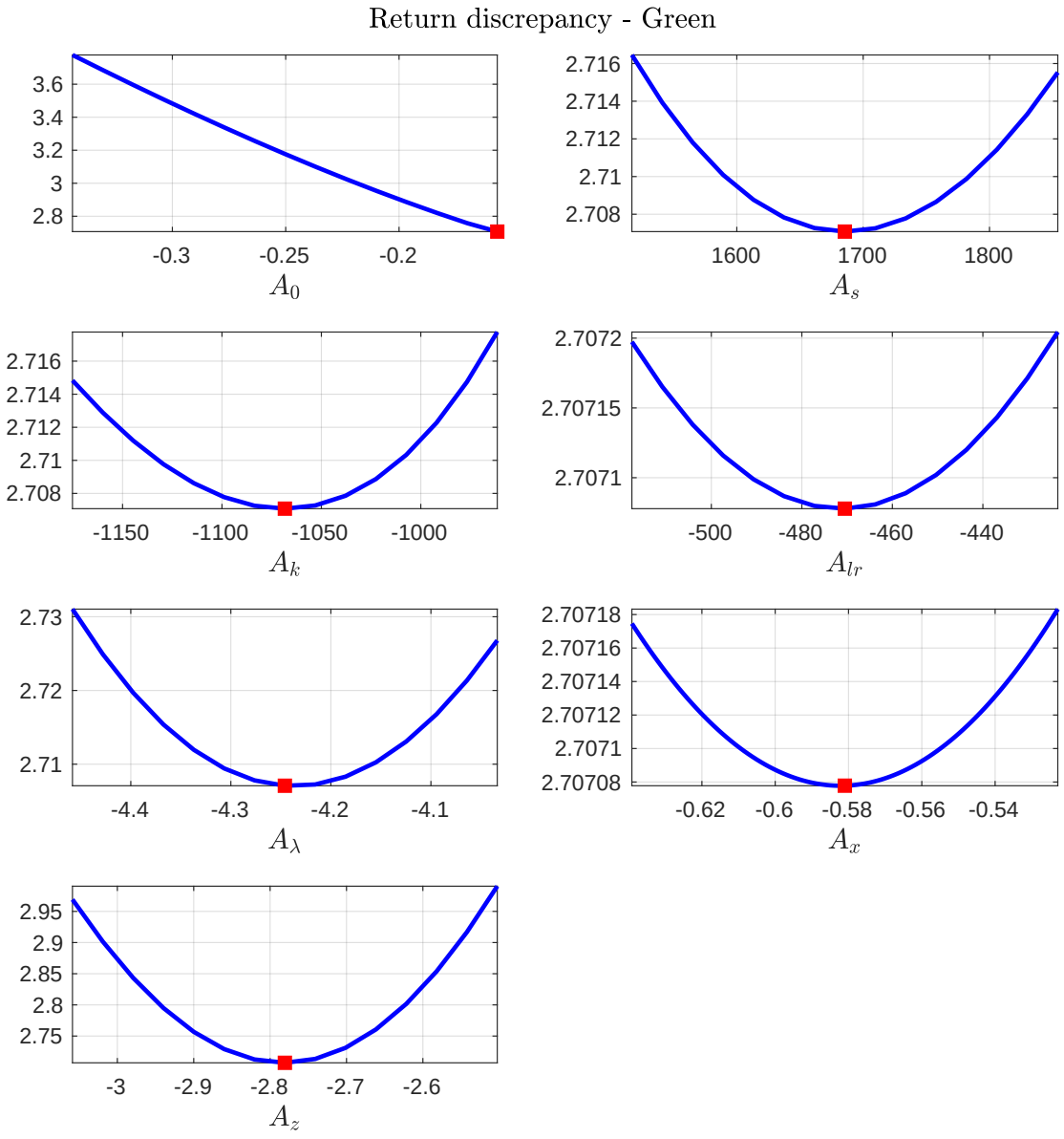


Fig. C1: Green policy (I)

Notes: This figure shows the average squared discrepancy of the returns for the brown sector:

$$\mathbb{E}\left[(R_{gt}^{P} - R_{gt}^{GC}(\theta))^2\right].$$

The markers denote the value of the parameters that minimize the objective function defined in equation (18) in main text.

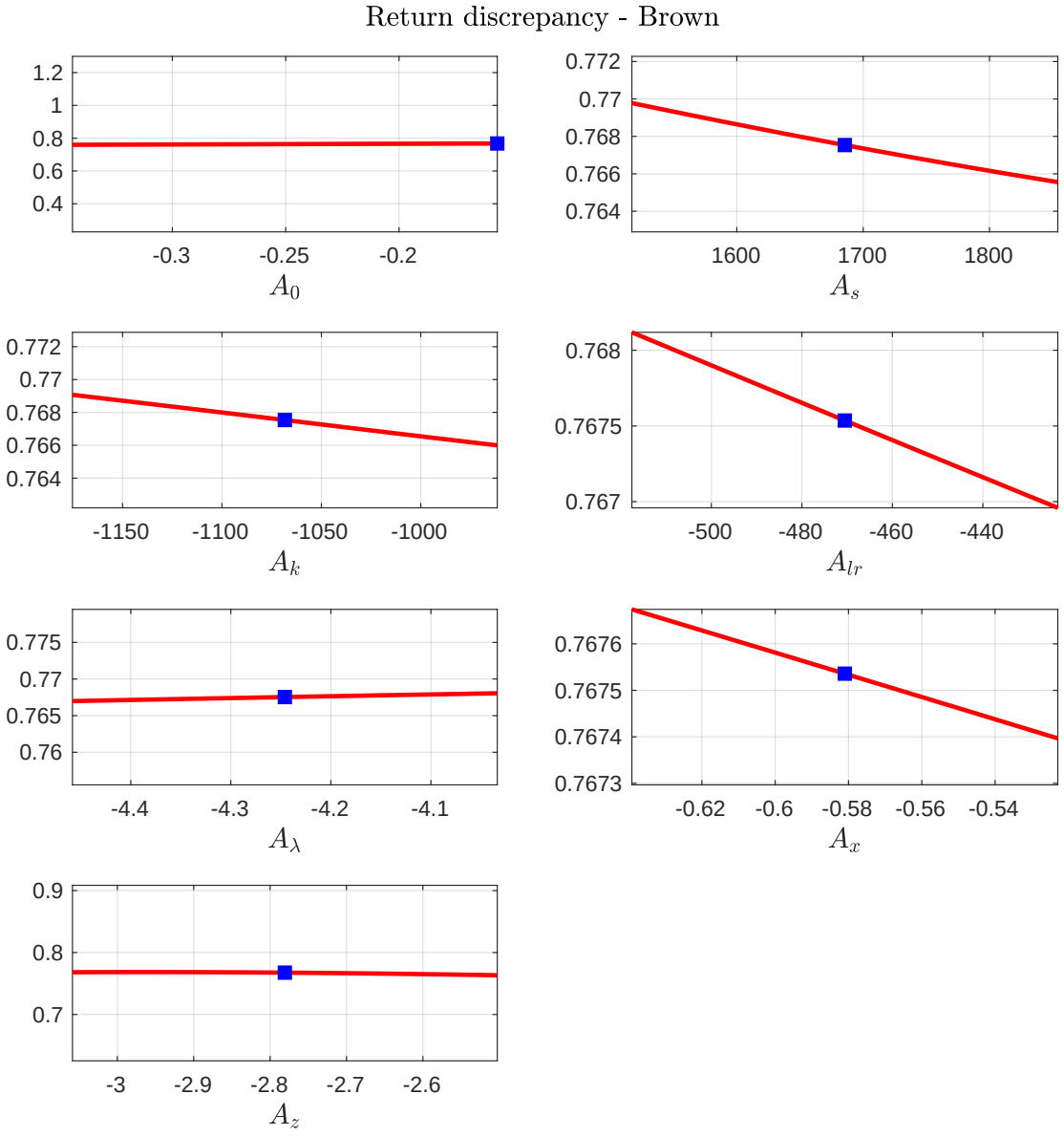


Fig. C2: Green policy (II)

Notes: This figure shows the average squared discrepancy of the returns for the brown sector:

$$\mathbb{E}\left[(R_{bt}^{P} - R_{bt}^{GC}(\theta))^2\right].$$

The markers denote the value of the parameters that minimize the objective function defined in equation (18) in main text.