A Macroeconomic Model of the Cross-Section of Currencies*

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Abstract

We study the cross-section of currencies using a macroeconomic model with heterogeneous countries, segmented asset markets, and risk-averse financial intermediaries. The model admits a two-factor structure of bilateral exchange rates. The factors combine global business cycle shocks and financial shocks that reprice the business cycle risk. Countries' exposures to these factors are heterogeneous and depend on their reliance on commodity exports and dollar asset holdings. We estimate the resulting factor structure empirically and use it to quantify the model. Our results indicate substantial heterogeneity in the drivers of dollar exchange rates across countries. Economies with high commodity exposure or low dollar holdings are risky, and their exchange rates fluctuate in response to global output and risk shocks. The exchange rates of countries with low commodity incidence and high dollar assets are less volatile, and their fluctuations are mostly determined by idiosyncratic local shocks.

Key Words: currencies, exchange rates, risk, macroeconomy

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1 Introduction

Bilateral exchange rates exhibit significant fluctuations that follow systemic patterns, characterized by differential sensitivities across countries. A notable example is the episode of the 2008 global financial crisis, where most currencies depreciated against the US dollar, with large variation in magnitudes. For instance, the Australian dollar depreciated by 39%, the Hungarian forint by 25%, and the Thai baht by 15%. A core question in international macroeconomics and finance is what drives these currency movements in the cross-section. This topic has primarily been analyzed in the international finance literature, which documents that most of the variation in bilateral exchange rates can be accounted for by two factors to which countries have heterogeneous exposures (Verdelhan, 2018; Lustig and Richmond, 2020).

In this paper, we develop a macroeconomic model of the cross-section of currencies that provides a structural interpretation of the factor structure of exchange rates. The framework features heterogeneous countries, segmented asset markets, and global financial intermediaries. In the model, exchange rates are a function of a "dollar" and a "commodity" factor, with country-specific loadings. The dollar factor reflects a combination of shocks to US non-traded output and to the risk-bearing capacity of intermediaries, and the commodity factor reflects a combination of the same risk-taking shocks and shocks to global traded output. Countries' loadings on the factors depend on their dollar asset holdings and on the incidence of commodities in their production.

We use empirical estimates of this factor structure to discipline and validate the model. Our quantitative analysis indicates substantial heterogeneity in exchange rate drivers across countries. The relative importance of global macroeconomic and financial shocks and idiosyncratic local shocks depends on countries' dollar assets and commodity exposure. In the case of exchange rates relative to the US dollar, the relevance of risk shocks stands out because they move relative prices in the US and in the other countries in the opposite directions.

We start by developing a macroeconomic model of the global economy. Our model introduces heterogeneous countries into a world economy with segmented asset markets and financial intermediaries with limited risk-bearing capacity (as in, e.g., Gabaix and Maggiori, 2015; Itskhoki and Mukhin, 2021), embedded in a canonical structure with tradable and non-tradable goods (as in, e.g., Schmitt-Grohé and Uribe, 2017). Aggregate shocks affect global output and the risk-taking capacity of intermediaries, broadly capturing the global business and financial cycle. Countries

feature two dimensions of ex-ante heterogeneity: they differ in their endowments of commodities and in their holdings of dollar assets.

Commodity exporters are more exposed to commodity-biased fluctuations in global tradable output. An expansion in global tradable output increases tradable consumption and the relative price of non-tradable goods in all countries, with larger increases in commodity-intensive ones. Countries are also affected by US-specific fluctuations, with exposures determined by their dollar asset holdings. Contractions in US non-tradable output increase the price of non-tradables in the US and affect other countries' non-tradable prices through a realignment of capital flows. Countries with large dollar assets increase their consumption of tradable goods because they benefit from capital gains on their dollar holdings. This increase in consumption is financed by capital inflows from countries with low dollar assets, whose tradable consumption falls to clear the goods market. Relative prices of non-traded goods increase in countries with large dollar assets.

Global financial intermediaries facilitate capital flows, and the global financial cycle in our model is generated by shocks to their risk-bearing capacity, which leads to repricing of risk generated by output shocks. In equilibrium, intermediaries absorb the global demand for dollar assets by issuing dollar bonds and invest the proceeds in the bonds of all other currencies. Currencies endogenously have different risk profiles, which determine the intermediaries' portfolio and the response of capital flows to risk shocks. Intermediaries operate with a stochastic discount factor that combines marginal utility of tradable consumption of all countries. Commodity exporters are a poor hedge because their currencies depreciate during periods of low global output, which is when marginal utility of consumption is high. Countries with low dollar assets are a poor hedge because their currencies depreciate more following contractions in US non-tradable output, which is also when global marginal utility is high. On the other hand, the US, other dollarized countries, and non-commodity producers are a good hedge because their currencies appreciate during these episodes. Following shocks that increases risk aversion, intermediaries rebalance away from risky to safe countries, causing the currencies of risky countries to depreciate through capital flight, and the dollar and other safe currencies to appreciate due to a flight to safety.

We analytically characterize equilibrium exchange rates and allocations under a particularly tractable parameterization. The real exchange rate between the currencies of countries i and j is given by the ratio of their non-tradable goods prices. It depends on the three aggregate shocks and the idiosyncratic shocks of both countries. An expansion in global tradable output

appreciates currency i relative to j if i has more commodities than j. Similarly, an expansion of US non-tradable output appreciates the currency i relative to j if i has more dollar assets than j. Finally, the exposure of i and j's exchange rate to the global risk shock is a weighted average of their relative commodity exposure and their dollar assets.

In the case of exchange rates relative to the US dollar, we show that they admit a two-factor structure, with a dollar and a commodity factor. The dollar factor is the average depreciation of all currencies relative to the dollar, and it structurally reflects a combination of the shock to US non-tradable output and part of the risk shock that interacts with dollar assets. The commodity factor is the average depreciation of currencies with above-median commodity incidence relative to those below the median. It structurally reflects a combination of the shock to global tradable output and the other part of the risk shock that interacts with commodity incidence. The countries' loadings on the dollar and commodity factors depend on their dollar assets and commodity incidence, respectively. Finally, the unexplained innovations to exchange rates in the factor structure map to the countries' idiosyncratic shocks to non-traded output.

In the second part of the paper, we estimate the factor structure implied by the model using data on bilateral exchange rates for 32 countries. The two-factor structure empirically explains approximately half of the variation of monthly changes in bilateral exchange rates relative to the US dollar, and its explanatory power is almost as good as that of the first two principal components — an upper bound on how much any two global factors can explain. We show that, consistent with the model's predictions, countries' loadings on the dollar factor are negatively related to their share of dollar assets in the data, and their loadings on the commodity factor are positively related to the share of commodities in their total exports. These results are largely in line with the findings of the literature (e.g., Ready et al., 2017; Verdelhan, 2018; Richmond, 2019). In particular, the commodity factor captures similar cross-sectional variation as alternative factors that exploit cross-country heterogeneity in interest rate differentials or trade network centrality.

The empirical counterparts of the factors contain useful information that we use to discipline our model quantitatively. Because the dollar factor reflects US non-traded output and global risk premium shocks, and the commodity factor follows global tradable output and global risk premia, the variances of the three aggregate shocks can be read off the factors' covariance matrix. In particular, in the data the dollar factor is more volatile than the commodity factor, which points to a higher relevance of US non-traded output shocks relative to global tradable output. The

factors are also positively correlated in the data, which points to the importance of risk premium shocks. We calibrate our model by targeting moments recovered from the empirical factor structure of exchange rates and the cross-section of commodity intensity and asset dollarization.

We then use the model to study the drivers of currencies in the cross-section. Following prior literature, we focus on bilateral exchange rates relative to the US dollar. We highlight two key takeaways from our quantitative analysis. First, we estimate substantial heterogeneity in exchange rate drivers across countries. On the one hand, countries with either low commodity incidence or high dollar assets have a similar exposure to global shocks as the US and, hence, their bilateral exchange rates are mostly driven by idiosyncratic shocks and less so by global shocks. On the other hand, the exchange rates of commodity-intensive countries are strongly influenced by global tradable output and risk shocks, and the exchange rates of countries with low dollar assets are mostly influenced by US non-traded output and risk shocks. Second, risk shocks stand out as a relevant driver of exchange rates relative to the US dollar because these realign capital flows between the US and the majority of the other countries, moving non-traded prices in opposite directions and significantly affecting bilateral exchange rates.

We analyze in retrospect the opening examples. The relatively mild depreciation of the Thai baht during the global financial crisis is congruent with its ample dollar assets and low commodity exposure, which implies that its dollar exchange rate should exhibit moderate responses to global shocks. On the other hand, the forint's large depreciation is consistent with low asset dollarization in Hungary, which exposes its currency to US fluctuations and risk shocks. Finally, the sharp depreciation of the Australian dollar is reconciled with the large incidence of commodities and relatively low dollar assets. This exposes the Australian currency to both main sources of global risk and leads financial shocks to dominate its exchange rate against the dollar.

Related Literature Our paper lies at the intersection of the international finance literature that studies the cross section of currencies and the international macroeconomics literature that develops quantitative macroeconomic models of the exchange rate.

An extensive literature in international finance studies the cross-section of exchange rates. A strand of this literature identifies factors that explain the cross-section of excess returns in currency markets and studies the pricing of currency risk (see e.g., Lustig and Verdelhan, 2007; Lustig et al., 2011; Menkhoff et al., 2012; Lustig et al., 2014; Kalemli-Özcan, 2019; Kalemli-Özcan

and Varela, 2021; Chernov et al., 2023; Hassan et al., 2024; Chernov et al., 2025). Other papers have documented that the variation in bilateral exchange rates exhibits strong common components with heterogeneous exposures across countries (Verdelhan, 2018; Lustig and Richmond, 2020; Lilley et al., 2022). Motivated by this empirical literature, a body of work develops theories that explain the origins of currency risk and how countries differ in their exposure to risk (Hassan, 2013; Maggiori, 2017; Colacito et al., 2018; Richmond, 2019). Hassan and Zhang (2021) survey advances in this literature. Our paper relates to Ready et al. (2017), who develop a model of international trade and currency pricing in which currencies of commodity producers are more exposed to global risk. Our model builds on this idea by introducing country heterogeneity in the incidence of commodity production, which gives rise to a commodity factor, and expands on it by modeling a dollar factor that emerges because of country heterogeneity in dollar asset holdings. Closest to our work, Verdelhan (2018) documents that most of the observed monthly variation in bilateral exchange rates can be explained with a two-factor empirical model and proposes a model of the stochastic discount factors that can replicate this structure. We contribute to this literature by developing a quantitative macroeconomic model that admits a two-factor structure tightly linked to that in Verdelhan (2018), and proposes a structural interpretation of the drivers of factors and exchange rates in the cross-section.

Second, our paper builds on the growing literature that develops quantitative macroeconomic models of the exchange rate. A strand of this literature has studied the quantitative drivers of exchange rates (see, e.g., Stockman and Tesar, 1995; Engel and West, 2005; Itskhoki and Mukhin, 2021, 2025; Chahrour et al., 2024; Engel and Wu, 2024; Kekre and Lenel, 2024a; Bodenstein et al., 2024). Another body of work studies the flight to safety properties of the US dollar (see, e.g., Bodenstein et al., 2023; Jiang et al., 2024; Kekre and Lenel, 2024b; Akinci et al., 2026). We contribute to this literature by introducing country heterogeneity aimed at providing a structural explanation of the dynamics of the cross-section of exchange rates. In this sense, our paper also relates to recent work by Kekre and Lenel (2025), who study the heterogeneous spillovers of US monetary policy across countries, and to the literature that studies the heterogeneous effects of the global financial cycle (see, e.g., Morelli et al., 2022; Oskolkov, 2026).

Finally, our paper is related to the literature that studies the relationship between exchange rates and international asset markets imperfections (see Maggiori, 2022, for a recent review). Following the original work of Backus and Smith (1993), a set of papers analyzes how segmentation

in international asset markets shapes the equilibrium properties of exchange rates (see, e.g., Lustig and Verdelhan, 2019; Chernov et al., 2024; Jiang et al., 2023). A related body of work studies the role of imperfect financial intermediation. Gabaix and Maggiori (2015) develop a theory of exchange rate dynamics and frictional financial intermediaries that can account for various properties of exchange rates. Lettau et al. (2014) study the role of financial intermediaries in the pricing of cross-sectional currency risk. Our paper builds on the insights from this literature by incorporating global financial intermediaries with fluctuating risk-bearing capacity that are central to explaining exchange rate movements in the cross section.

Outline. Section 2 describes the model and characterizes the equilibrium. Section 3 presents our empirical analysis. Section 4 conducts the quantitative analysis and decomposes exchange rates into fundamental drivers in cross-section. We conclude in Section 5.

2 A Model of the Cross-Section of Exchange Rates

In this section, we develop a macroeconomic model of the cross-section of currencies. We model the global economy composed of heterogeneous countries and a representative global financial intermediary embedded in a canonical structure with tradable and non-tradable goods. Asset markets are segmented, following modern macroeconomic models of the exchange rate. We consider fluctuations in countries' economic activity, broadly capturing the global business cycle, and fluctuations in the risk-bearing capacity of intermediaries, capturing the global financial cycle.

2.1 Environment

Heterogeneous economies. There is a continuum of small open economies indexed by $i \in [0, 1]$, and the US, indexed by u, which is a mass point. Each economy is a representative household and a central bank. Households maximize

$$\mathbb{E}_0 \sum_{t=0}^{t=\infty} \beta^t \frac{C_{it}^{1-\rho}}{1-\rho}, \tag{1}$$

where $\beta \in (0,1)$ is the discount factor, ρ is the inverse elasticity of intertemporal substitution, and C_{it} is an aggregator that uses C_{it}^T units of tradable and C_{it}^N units of non-tradable goods:

$$C_{it} = \left(\frac{C_{it}^N}{\alpha}\right)^{\alpha} \left(\frac{C_{it}^T}{1-\alpha}\right)^{1-\alpha},$$

where $\alpha \in (0,1)$ is the weight on local non-tradables. The price of tradables is normalized to one, the price of non-tradable is P_{it}^N, and $Q_{it} = (P_{it}^N)^{\alpha}$ is the ideal price index of the consumption aggregator, in terms of tradable goods. For brevity, we will refer to countries' consumption baskets as "local currency". The relative price of consumption baskets between any two countries is their bilateral real exchange rate. For example, $S_{iut} = Q_{ut}/Q_{it}$ is the bilateral real exchange rate of country i against the dollar (i.e., the price of the US consumption basket in terms of the consumption basket in country i). Whenever S_{iut} increases, country's i currency depreciates against the dollar.

Households are endowed with both goods. The non-traded endowment is $Y_{it}^N = N(1 + x_{it})$, where x_{it} is a mean-zero idiosyncratic shock process. The endowment of the undifferentiated tradable goods is $Y_{it}^T = 1 + e_i z_t$, where z_t is an aggregate disturbance in tradable endowments and e_i is country i's exposure to this shock, which is a source of permanent heterogeneity across countries. Appendix A.2 provides a micro-foundation for this formulation of the traded endowment. It shows that this functional form of Y_{it}^T arises in a setup with two types of tradables: final goods and primary inputs (commodities). Final tradable goods are produced using labor and commodities, and z_t is a commodity-biased productivity shock. Exposures e_i map one-to-one to countries' commodity endowments: when input productivity increases, the demand for commodities and their prices rise, and this raises countries' tradable income in proportion to their commodity endowments. The micro-foundation also maps e_i into the share of commodities in country i's exports, a moment we use in the next sections for the empirical and quantitative analysis.

Households can save and borrow in one-period bonds denominated in local currency. We denote their bond purchases at time t by $B_{i,t+1}$. The bonds pay an interest rate $R_{i,t+1}$, known at time t, at maturity at time $t+1$. The household's budget constraint, expressed in tradable goods, is

$$C_{it}^T + P_{it}^N C_{it}^N = Y_{it}^T + P_{it}^N Y_{it}^N + R_{it} Q_{it} B_{it} - Q_{it} B_{i,t+1} + T_{it} + \Pi_{it}, \tag{2}$$

where T_{it} are transfers from the central bank and Π_{it} profit rebates from financial intermediaries. Households choose sequences $\left\{C_{it}^T, C_{it}^N, B_{i,t+1}\right\}_{t \geq 0}$ to maximize equation (1) subject to equation (2).

The optimality conditions are $\alpha C_{it}^{T} = (1-\alpha) P_{it}^{N} C_{it}^{N}$ and $C_{it}^{-\rho} = \beta R_{i,t+1} \mathbb{E}_t C_{i,t+1}^{-\rho}$.

The central bank in each small open economy accumulates dollar assets and transfers returns to the households. At time t, the central bank of country i buys $M_{i,t+1}$ dollar-denominated bonds. Its rebate to households is

$$T_{it} = R_{ut} Q_{ut} M_{it} - Q_{ut} M_{i,t+1}.$$

We assume the following policy for dollar asset accumulation:

$$Q_{ut} M_{i,t+1} = \tau \cdot Q_t M_i + (1-\tau) \cdot Q_{ut} M_i,$$

where $Q_t \equiv \int Q_{it} \mathrm{d}i$ is the global average of consumption basket prices. The policy is a linear combination between targeting a constant value of dollar assets expressed in the global basket of currencies, with a weight $\tau \in (0,1)$, and a constant value in dollars, with the remaining weight. M_i is the target level of dollar assets, and is the second and last source of permanent heterogeneity across countries.

Two points are worth discussing regarding the dollar asset accumulation policy. First, it postulates a simple policy rule where τ governs the degree to which the central banks stabilize the value of their assets stemming from fluctuations in the broad dollar: when the dollar is strong against other currencies, a positive τ prescribes selling dollars bonds to realize capital gains. In Appendix C we provide empirical support for this policy rule showing that it accounts for an important fraction of the variation in dollar assets in the data. Second, we treat the target level of dollar assets, M_i, as a primitive of the model and interpret it as a measure of incidence of dollar assets in each country, above and beyond the central bank's holdings. In reality, asset dollarization emerges endogenously across countries in response to various underlying country-characteristics. In our analysis we remain agnostic about this and refer the reader to the literature that studies the determinants of asset dollarization, which include varying levels of domestic policy risk, exposure to roll-over risk and the depth of domestic financial markets (see, e.g., Bianchi et al., 2018; Drenik et al., 2022; Coppola et al., 2023).

Finally, the problem of US households is symmetric to that of small open economies. We abstract from the US central bank, since its dollar asset accumulation policy is irrelevant because households choose their bond holdings to be on the Euler equation at all times.

Global financial intermediaries. A representative global intermediary trades bonds in all currencies. Let $A_{i,t+1}$ and $A_{u,t+1}$ be its purchases of currency-i and dollar bonds at t. The intermediary starts with no initial capital, so its balance sheet is given by

$$\int Q_{it} A_{i,t+1} \mathrm{d}i + Q_{ut} A_{u,t+1} = 0. \tag{3}$$

The intermediary's preferences are defined over $t+1$ risk-adjusted returns from investing in t. Let $X_{i,t+1}$ be the realized excess returns of investing in i's bonds relative to dollar bonds:

$$X_{i,t+1} \equiv \frac{R_{i,t+1} Q_{i,t+1}}{Q_{it}} - \frac{R_{u,t+1} Q_{u,t+1}}{Q_{ut}}.$$

We can express total realized profits, expressed in tradable goods, as $\int Q_{it} A_{i,t+1} X_{i,t+1} \mathrm{d}i$, where we have substituted out the dollar position using the balance-sheet equation (3). The intermediary chooses $\{A_{i,t+1}\}$ for all i and t to maximize

$$\mathbb{E}_t \left[\Lambda_{t+1} \int Q_{it} A_{i,t+1} X_{i,t+1} \mathrm{d}i \right] - \frac{1}{2\chi} \int \left(Q_{it} A_{i,t+1} - L_{t+1} \right)^2 \mathrm{d}i. \tag{4}$$

We assume that the intermediary's stochastic discount factor is linked to the average stochastic discount factor of all households. Define $\Lambda^{\text{ave}}_{t+1} = \int \Lambda_{i,t+1} \mathrm{d}i + \Lambda_{u,t+1}$ as the average discount factor across all households, where $\Lambda_{i,t+1}$ is country-i households' stochastic discount factor:

$$\Lambda_{i,t+1} = \beta \frac{\partial u(C_{i,t+1})}{\partial C^T_{i,t+1}} \left(\frac{\partial u(C_{it})}{\partial C^T_{it}} \right)^{-1},$$

and $\Lambda_{u,t+1}$ is similarly defined for US households, all in terms of tradable goods. The intermediary's stochastic discount factor is

$$\Lambda_{t+1} = \Gamma_t \cdot \Lambda^{\text{ave}}_{t+1} + (1 - \Gamma_t) \cdot \mathbb{E}_t \left[\Lambda^{\text{ave}}_{t+1} \right],$$

which is a weighted average of a stochastic component, the average discount factor across all households, and a deterministic component, the expected value of the average discount factor across all households, $\mathbb{E}_t \left[\Lambda^{\text{ave}}_{t+1} \right]$. The weight Γ_t governs how much of household's risk aversion is passed to the intermediary. We assume $\Gamma_t = \Gamma(1 + \gamma_t)$, where γ_t is an aggregate shock to the risk-bearing capacity. In Appendix A.3, we study a variant of the intermediary problem with mean-variance preferences, and show that it gives rise to a similar pricing of risks and equilibrium properties of exchange rates.

The intermediary faces portfolio management costs, the second term in equation (4). These

costs capture frictions that make capital flows slow-moving and less elastic to risk-adjusted returns than predicted by frictionless models (as estimated in, e.g., Koijen and Yogo, 2020). The cost decreases in χ and penalizes deviations from a portfolio that issues dollar bonds to satisfy the demand for dollar assets by all countries and invests the proceeds uniformly in all other countries. That is, the reference level of liabilities is $L_{t+1} = \int Q_{ut} M_{i,t+1} \mathrm{d}i$. The optimal portfolio is

$$Q_{it} A_{i,t+1} = L_{t+1} + \chi \left(\mathbb{E}_t \left[\Lambda^{\text{ave}}_{t+1} \right] \mathbb{E}_t [X_{i,t+1}] + \Gamma_t \mathbb{C}_t \left[\Lambda^{\text{ave}}_{t+1}, X_{it+1} \right] \right).$$

The intermediary allocates more to currencies with higher expected returns and higher covariance with the global marginal utility of tradable consumption. Its dollar position is given by

$$Q_{ut} A_{u,t+1} = -L_{t+1} - \chi \left(\Gamma_t \mathbb{C}_t \left[X_{t+1}, \Lambda^{\text{ave}}_{t+1} \right] + \mathbb{E}_t \left[\Lambda^{\text{ave}}_{t+1} \right] \mathbb{E}_t \left[X_{t+1} \right] \right),$$

where $X_{t+1} = \int X_{i,t+1} \mathrm{d}i$. The optimal intermediary's demand for dollar bonds in excess of total dollar asset demand is proportional the average excess returns in foreign currencies and to the covariance between the average foreign currency return and the global marginal utility of tradables.

Finally, we assume that profits are rebated back to each country according to

$$\Pi_{it} = Q_{i,t-1} A_{it} \left(R_{it} \frac{Q_{it}}{Q_{i,t-1}} - \frac{Q_{ut}}{Q_{u,t-1}} \right) - Q_{ut} M_{it} (R_{ut} - 1).$$

Each country receives returns that the intermediary realizes on its bond, including interest income and exchange rate appreciation, net of interest payments on their dollar assets. The US only receives the first part, since it does not maintain reserves.

Aggregate and idiosyncratic shocks. The model features three aggregate shocks: the global tradable endowment shock z_t, the US non-tradable endowment shock x_{ut}, and the risk aversion shock γ_t. The US non-tradable endowment is an aggregate shock because US is economically large and all countries accumulate dollar assets. As we further discuss below, the two endowment shocks are necessary to give rise to the two-factor structure, and the risk shock helps account for its covariance structure as well as for other properties related to exchange rates and risk premia.

In addition to these aggregate shocks, the model also features idiosyncratic shocks to the non-traded endowments of individual countries, x_{it}. We assume that all shocks follow the following

processes with independent innovations:

$$z_{t+1} = \rho_z z_t + \sigma_z \epsilon_{zt+1},$$
$$\gamma_{t+1} = \rho_\gamma \gamma_t + \sigma_\gamma \epsilon_{\gamma t+1},$$
$$x_{ut+1} = \rho_u x_{ut} + \sigma_u \epsilon_{ut+1},$$
$$x_{it+1} = \rho_x x_{it} + \sigma_x \epsilon_{it+1}.$$

Equilibrium. An equilibrium is a set of processes for prices $\{P_{it}^N, R_{it}, Q_{it}, P_{ut}^N, R_{ut}, Q_{ut}\}_{i\in[0,1],t\geq 0}$ and quantities $\{C_{it}^N, C_{it}^T, B_{i,t+1}, A_{i,t+1}, C_{ut}^N, C_{ut}^T, B_{u,t+1}, A_{u,t+1}\}_{i\in[0,1],t\geq 0}$ such that, given dollar asset policies $\{M_{i,t+1}\}_{i\in[0,1],t\geq 0}$, profits, and transfers, all quantities are chosen optimally given prices, and the following market clearing conditions hold:

non-tradable good for all i:	$C_{it}^N = Y_{it}^N,$
tradable good:	$\int C_{it}^T \mathrm{d}i + C_{ut}^T = \int Y_{it}^T \mathrm{d}i + Y_{ut}^T,$
local currency bonds for all i:	$A_{i,t+1} + B_{i,t+1} = 0,$
dollar bonds:	$\int M_{it+1} \mathrm{d}i + A_{u,t+1} + B_{u,t+1} = 0.$

2.2 Equilibrium Characterization

We analyze a first order log-linear approximation of the model around a deterministic steady state. To preserve the relevance of risk shocks in the first order approximation, we follow prior literature and analyze the equilibrium when we jointly set the variance of shocks to zero and scale up intermediaries' risk aversion so that non-zero risk premia persist even in the first order approximation. Specifically, we consider the following double limit $(\sigma_z, \sigma_\gamma, \sigma_u) \longrightarrow 0$ and $\Gamma \longrightarrow \infty$, where $(\Gamma\sigma_z^2, \Gamma\sigma_u^2, \Gamma\sigma_\gamma^2) \longrightarrow (\Gamma_z, \Gamma_u, \Gamma_\gamma)$, which is akin to one used by Borovicka and Hansen (2013) and commonly used in the international macroeconomics literature. In Appendix A.1 we describe the steady state of the model, which features symmetric consumption allocations and heterogeneous net foreign asset positions across countries that reflect their differential exposures to risk.

We analyze the equilibrium in three steps. First, we characterize allocations and prices of individual countries given capital inflows; second, we study capital inflows given prices; and third, we finish by characterizing the equilibrium bilateral exchange rates.

Prices and allocations. The following lemma specifies how the prices of the consumption basket of each country relate to their exogenous shocks and the capital inflows. For convenience, we will work with dollar value of capital inflows: $L_{i,t+1} \equiv Q_{it}A_{i,t+1}/Q_{ut}$ is the dollar value of the position that the intermediary takes in country i.

LEMMA 1. *In equilibrium, the price of country i's consumption basket, $q_{it} \equiv \log Q_{it}$, is given by*

$$q_{it} = \alpha \left(e_i z_t + \Delta l_{i,t+1} - M_i \Delta m_{i,t+1} - x_{it} \right), \tag{5}$$

where $\Delta l_{i,t+1} \equiv L_{i,t+1} - L_{it}$ are country i's financial inflows at time t measured in dollars and $\Delta m_{i,t+1} = \log(M_{i,t+1}) - \log(M_{it})$ is the log change of country i's dollar assets. The log change in dollar assets is common across countries and given by

$$\Delta m_{t+1} = \tau(\Delta q_t - \Delta q_{ut}), \tag{6}$$

where $q_t \equiv \log(Q_t)$ is the log price of the average consumption basket.

We include all proofs in Appendix A.1. The log price of country i's basket increases with tradable consumption, given by the first three terms of equation (5), and decreases with non-tradable consumption, the last term. A high non-tradable output shock x_{it} increases non-tradable supply and reduces consumption prices in equilibrium. A high global tradable endowment shock increases tradable consumption and local basket prices, with larger increases in countries with high exposures e_i. Inflows $\Delta l_{i,t+1}$ lead to increases in tradable consumption, which again increases the prices of local baskets. Finally, dollar asset accumulation Δm_{t+1}, scaled by the steady state position M_i, lowers local basket prices by decreasing tradable consumption. The dollar asset policy, given by equation (6), generates outward flows when the broad dollar depreciates and inward flows when it appreciates, counteracting its movement and stabilizing the value of the domestic currency.

Capital inflows. The next two lemmas characterize the capital inflows to country i given a change in its consumption prices.

LEMMA 2. *The intermediary's investment in i is determined by*

$$l_{i,t+1} = \chi\varphi \mathbb{E}_t[\Delta q_{i,t+1} - \Delta q_{u,t+1}] + \chi\rho \mathbb{E}_t[\Delta x_{i,t+1} - \Delta x_{u,t+1}] + \chi \mathcal{C}_i \gamma_t + L m_{t+1}, \tag{7}$$

where $\mathcal{C}_i = \lim_{(\sigma_z,\sigma_\gamma,\sigma_u)\to 0,\ \Gamma\to\infty} \Gamma \cdot \mathbb{C}_t[\lambda^{ave}_{t+1}, \Delta q_{i,t+1} - \Delta q_{u,t+1}]$ is country i's limiting risk premium, $\lambda^{ave}_{t+1} \equiv \log(\Lambda^{ave}_{t+1})$ is the log deviation of the average household stochastic discount factor, and

$\varphi = 1 + \rho(1-\alpha)/\alpha$.

Equilibrium capital inflows are given by the intermediary's investments in currency i. These are affected by expected excess returns, and are higher when the currency is expected to appreciate against the dollar, as captured by first and second terms in equation (7), which uses the household's Euler equation to substitute out interest rates. Inflows are also affected by shocks to the intermediary's risk aversion, and their sign depends on the country's riskiness, as measured by $\mathcal{C}_i$. When intermediaries' risk aversion increases, they rebalance portfolios away from countries with a negative covariance $\mathcal{C}_i$ between their exchange rate and the average stochastic discount factor. These countries are risky, and their currencies depreciate in times of high risk aversion through capital outflows. Determining the risk profile of countries requires characterizing the properties of the stochastic discount factor of the average household, which we do next.

LEMMA 3. *In the first order, the average stochastic discount factor is given by*

$$\lambda_{t+1}^{ave} = -\alpha\varphi(e + e_u)\Delta z_{t+1} - (\rho - 1)\alpha\Delta x_{u,t+1}. \tag{8}$$

The average stochastic discount factor captures the average growth of marginal utility of tradable consumption. The first term in equation (8) shows that marginal utility of tradable consumption decreases in the global tradable output shock. An implication of this property is that a currency is a bad hedge if it appreciates against the dollar when global tradable output is high. This makes countries with high exposure to tradable output shocks e_i risky.

The second term shows how marginal utility of tradable consumption is affected by shocks to non-tradable output. This term has two properties. First, non-tradable output of small individual countries integrates out on average, and only US non-tradable output remains because of its large size. Second, the marginal utility of tradable consumption can increase or decrease with non-tradable output depending on how the inverse elasticity of intertemporal substitution, ρ, compares with the elasticity of substitution between the two goods, which is one given the Cobb-Douglas consumption aggregator. When the intertemporal elasticity is lower than one, $\rho > 1$, which is a common parameterization in the literature, the marginal utility of tradables decreases with nontradable output, implying that states of low non-tradable output in the US are states with a high stochastic discount factor for the intermediary. An implication of this property is that a currency is a bad hedge if it depreciates against the dollar when US nontradable output is low. As we further explain below, this makes countries with low dollar assets risky.

Finally, note that the intermediary's risk aversion shock does not affect the average stochastic discount factor across households. The reason is that this shock is redistributive: it reshuffles capital flows but does not affect aggregate consumption. This implies that this shock is not priced as a source of risk.

Bilateral exchange rates against the dollar. We next turn to bilateral exchange rates and their factor structure. We start by analyzing exchange rates relative to the dollar, $s_{it} \equiv \log S_{iut}$, where we drop the u subscript for convenience. To derive analytical results, we make a set of simplifying parametric assumptions. We consider permanent aggregate output shocks by setting $\rho_z = \rho_u = 1$, and transitory shocks to risk aversion by setting $\rho_\gamma = 0$. We also consider the limiting case where intermediaries' portfolio management costs approximate zero, $\chi \longrightarrow \infty$. This makes the expressions for exchange rates tractable by making their dependence on the endogenous states l_{it} and m_{it} arbitrarily small. At the same time, this limit eliminates exchange rate movements resulting from capital gains on reserves. To maintain the reserve management channel operational, we proportionally increase the stock of reserves at an appropriate rate $M_i/\sqrt{\chi} \to m_i$. With these simplifying assumptions, we can characterize the change in the bilateral exchange rate of country i against the dollar, $\Delta s_{i,t+1} = \Delta q_{u,t+1} - \Delta q_{i,t+1}$, in closed form.

PROPOSITION 1. *The change in the bilateral exchange rate of country i against the dollar is*

$$\Delta s_{i,t+1} = (e_u - e_i) \cdot \alpha \Delta z_{t+1} + (\tau\phi(m_i - m) - 1) \cdot \alpha \Delta x_{u,t+1} + \kappa_i \cdot \Delta\gamma_{t+1} + \alpha \Delta x_{i,t+1}.$$

The loading κ_i on the risk aversion shock is given by

$$\kappa_i = (e_i - e_u) \cdot (e + e_u)\alpha^2 \Gamma_z + (1 - \tau\phi(m_i - m)) \cdot \alpha\phi^2 \Gamma_u(\rho - 1),$$

where $m = \int m_i \mathrm{d}i$ is the cross-sectional average level of dollar assets, and $\phi = \sqrt{\alpha/\varphi}$.

The equilibrium changes in the dollar exchange rate of country i depends on the three aggregate shocks and country i's idiosyncratic shock. An increase in country i's idiosyncratic nontradable output depreciates its currency by decreasing its consumption prices. The coefficient of $\Delta s_{i,t+1}$ on Δz_{t+1} reflects the relative exposure of country i and the US to the tradable output shocks. An expansion in global tradable output will appreciate i's currency if i is more exposed to this shock than the US, $e_i > e_u$. The coefficient on $\Delta x_{u,t+1}$ has two parts. First, the -1 reflects the direct effect of an increase in US non-tradable output on US consumption prices: when US nontradable

output increases, its supply becomes more abundant and the relative price of the US consumption falls. The part $\tau\phi(m_i - m)$ reflects reserve management. When an increase in x_{ut} makes the dollar depreciate against the average country, countries increase their reserve holdings, and their currencies depreciate against the dollar in proportion to m_i. At the same time, the global rise in demand for dollar bonds leads to inflows into all countries when the intermediary reshuffles this demand into local currency bonds. These inflows offset reserve purchases on average, making the net foreign assets change in proportion to $m_i - m$.

The coefficient κ_i on the risk aversion shock $\Delta\gamma_{t+1}$ reflects the limiting covariance of $\Delta s_{i,t+1}$ and the average stochastic discount factor $\lambda_{t+1}^{\text{ave}}$. As we show in Lemma 3, the average stochastic discount factor loads on the two aggregate output shocks, Δz_{t+1} and $\Delta x_{u,t+1}$, which are the two sources of priced risk. The first term of κ_i reflects the relative exposure to global tradable output shocks. Countries are risky with respect to the tradable output shock if their tradable output is more procyclical than that in the US, $e_i - e_u > 0$, which implies that their bilateral exchange rate against the dollar appreciates in times of high global tradable output. The second term of κ_i reflects the relative exposure to US non-tradable output shocks, and its sign is determined by the sign of $1 - \tau\phi(m_i - m)$. For the majority of countries, this sign is positive, which implies that they are risky with respect to the US nontradable output shock because their currencies depreciate against the dollar in times of low US nontradable output. The degree of exposure is decreasing in countries' reserves, as these attenuate the depreciation of their currencies in response to a contraction in US nontradable output. The overall impact of risk shocks on currency i depends on the combination of the exposures to these two sources of risk.

The property that the coefficient κ_i inherits the shape of the loadings of $\Delta s_{i,t+1}$ on Δz_{t+1} and $\Delta x_{u,t+1}$, helps illustrate how the two permanent sources of cross-sectional heterogeneity affect the exposure to global shocks. Combining the coefficients in Proposition 1, we obtain:

$$\begin{aligned}\Delta s_{i,t+1} = {} & (\alpha\Delta z_{t+1} - \alpha^2(e + e_u)\Gamma_z\Delta\gamma_{t+1}) \cdot (e_u - e_i) \\ & + (\alpha\Delta x_{u,t+1} - \alpha\phi^2(\rho - 1)\Gamma_u\Delta\gamma_{t+1}) \cdot (\tau\phi(m_i - m) - 1) \\ & + \alpha\Delta x_{i,t+1}.\end{aligned} \tag{9}$$

The first term in equation (9) captures the heterogeneity along the commodity exposure dimension. The commodity exposure e_i determines the response of the exchange rate to the tradable output shocks Δz_{t+1}, and to the part of the risk aversion shocks $\Delta\gamma_{t+1}$ that lead to a repricing of the

commodity risk emanating from Δz_{t+1}. The second captures the heterogeneity along the dollar assets dimension. The level of dollar assets m_i determines the exchange rate response to US non-tradable output shocks $\Delta x_{u,t+1}$, and the part of the risk aversion shocks $\Delta\gamma_{t+1}$ that lead to a repricing of the dollar risk emanating from $\Delta x_{u,t+1}$.

As we show next, the two common components in the first two terms of equation (9) have natural interpretations as a commodity and a dollar factor, respectively. In particular, each term can be isolated by integrating in the cross section under mild assumptions regarding the cross-sectional distribution of (e_i, m_i).

DEFINITION 1. *The commodity and dollar factors are*

$$\begin{aligned}\text{comm}_{t+1} &= 2\int_{i:\ e_i>\text{med}\{e\}} \Delta s_{i,t+1}\mathrm{d}i - 2\int_{i:\ e_i<\text{med}\{e\}} \Delta s_{i,t+1}\mathrm{d}i,\\ \text{doll}_{t+1} &= \int \Delta s_{i,t+1}\mathrm{d}i.\end{aligned}$$

These theoretical definitions map to the empirical definitions of factors used in prior literature. The commodity factor reflects the average change in bilateral exchange rates of countries with above-median commodities against those with below-median commodities. Similarly, the dollar factor reflects the average change in bilateral exchange rates of all countries against the dollar. With these definitions, we can formulate our main result. Denote by e the unconditional average value of e_i in the cross-section, and e_H and e_L the average values of e_i conditional on being above and below the median, respectively.

PROPOSITION 2. *Suppose that e_i and m_i are uncorrelated in the cross-section, and that the US has an average commodity exposure, $e_u = e$. Then, the changes in bilateral exchange rates against the dollar have the following factor structure:*

$$\Delta s_{i,t+1} = \beta_{i,\text{comm}} \cdot \text{comm}_{t+1} + \beta_{i,\text{doll}} \cdot \text{doll}_{t+1} + \varepsilon_{i,t+1}. \tag{10}$$

The factors and residuals are given by

$$\begin{aligned}\text{comm}_{t+1} &= 2\alpha^2 e(e_H - e_L)\Gamma_z \Delta\gamma_{t+1} - (e_H - e_L)\alpha\Delta z_{t+1},\\ \text{doll}_{t+1} &= \alpha\phi^2(\rho - 1)\Gamma_u \Delta\gamma_{t+1} - \alpha\Delta x_{u,t+1},\\ \varepsilon_{i,t+1} &= \alpha\Delta x_{i,t+1}.\end{aligned}$$

The factor loadings are

$$\beta_{i,\text{comm}} = \frac{e_i - e}{e_H - e_L},$$
$$\beta_{i,\text{doll}} = 1 - \tau\phi(m_i - m).$$

This proposition highlights that the dollar and the commodity factors are two aggregations that effectively integrate out one of the three aggregate shocks. By computing the average change in all the exchange rates against the dollar, the dollar factor is effectively abstracting from the shock to global tradable output, when the commodity exposure of the US is the world's average exposure. By computing the average change in the exchange rate of high-minus-low commodity producers, the commodity factor is effectively abstracting from the shock to the US non-tradable output. The risk aversion shock γ_t is reflected in both factors to the extent that both z_t and x_{ut} are priced risks, and movements in γ_t lead to risk repricing. The factor loadings, in turn, reflect the exposure of each currency to the underlying real shocks.

Bilateral exchange rates between i and j. We now study equilibrium changes in the bilateral exchange rates between any pair of countries. As the next proposition shows, for any arbitrary base currency j, the changes in bilateral exchange rates also display a factor structure. Let $\Delta s_{ij,t+1} = \Delta q_{i,t+1} - \Delta q_{j,t+1}$ be the exchange rate depreciation of currency i relative to j, and define the base currency factor $\text{base}_{j,t+1}$ as

$$\text{base}_{j,t+1} = \int \Delta s_{ij,t+1} \text{d}i.$$

PROPOSITION 3. *The changes in bilateral exchange rates relative to a currency j have the following factor structure:*

$$\Delta s_{ij,t+1} = \frac{e_i - e}{e_H - e_L} \cdot \text{comm}_{t+1} - \tau\phi(m_i - m) \cdot \text{doll}_{t+1} + 1 \cdot \text{base}_{j,t+1} + \alpha \Delta x_{i,t+1}. \tag{11}$$

The base currency factor $\text{base}_{j,t+1}$ *is given by*

$$base_{j,t+1} = \frac{e - e_j}{e_H - e_L} \cdot \text{comm}_{t+1} + \tau\phi(m_j - m) \cdot \text{doll}_{t+1} - \alpha \Delta x_{j,t+1}. \tag{12}$$

When we express exchange rates relative to a currency j, the base j currency factor appears

as an additional factor in equation (11). This new factor captures country j's idiosyncratic shock, combined with the dollar and commodity factors to the extent that country j's exposure differs from the average. Additionally, the loadings on the dollar factor in equation (11) differ from those in the factor structure of dollar exchange rates by one. This difference captures the direct effect of US non-tradable output shocks on US consumption prices, which is present in exchange rates against the dollar but not against currency j. Finally, the loadings on the commodity factor are the same as in the factor structure of dollar exchange rates, and the unexplained residual maps into country i's idiosyncratic nontradable output shock.

2.3 Model Identification

In this section we analyze identification in our model. We argue that the estimation of the factor structure of exchange rates provides valuable information regarding the relevance of each of the aggregate shocks and the sources of country heterogeneity. We use the results in this section to conduct the calibration in Section 4.

We start by studying how the time-series properties of the factors are informative about the volatility of the aggregate shocks. The results are summarized in the following proposition.

PROPOSITION 4. *The covariance matrix of the factors is given by*

$$\begin{aligned}
\mathbb{V}[\text{comm}_t] &= \alpha^2(e_H - e_L)^2\sigma_z^2 + 8\alpha^4 e^2(e_H - e_L)^2\sigma_z^4 \cdot (\Gamma\sigma_\gamma)^2, \\
\mathbb{V}[\text{doll}_t] &= \alpha^2\sigma_u^2 + 2\alpha^2\phi^4(\rho - 1)^2\sigma_u^4 \cdot (\Gamma\sigma_\gamma)^2, \\
\mathbb{C}[\text{comm}_t, \text{doll}_t] &= 4\alpha^3\phi^2 e(e_H - e_L)(\rho - 1)\sigma_u^2\sigma_z^2 \cdot (\Gamma\sigma_\gamma)^2.
\end{aligned}$$

The mapping $(\sigma_z^2, \sigma_u^2, (\Gamma\sigma_\gamma)^2) \mapsto (\mathbb{V}[\text{comm}_t], \mathbb{V}[\text{doll}_t], \mathbb{C}[\text{comm}_t, \text{doll}_t])$ *is invertible. The variance of residuals in the factor regression in equation (10) is*

$$\mathbb{V}[\varepsilon_{it}] = \alpha^2\sigma_x^2.$$

This proposition shows that we can recover $(\sigma_z, \sigma_u, \Gamma\sigma_\gamma)$ from the covariance matrix of the factors. The closed form expressions for each volatility are large, and relegated to the proof.However, there are two corollaries from Proposition 4. First, the covariance between the two factors is only due to risk shocks. The size of the correlation is hence informative about their importance relative to the output shocks. A positive correlation would also be indicative of $\rho > 1$, validating the cor-

respondence between strong dollar and high global marginal utility, which is necessary to get the sign of the dollar risk correctly. Second, the variance of the factors provides a comparison between the variance of the two aggregate output shocks. A relatively large variance of the dollar factor compared to the commodity factor is informative of a higher importance of the US non-tradable shock relative to the global tradable output shock.

We next relate the distribution of factor loadings to the cross-sectional distribution of country characteristics. By construction, the first moments of the dollar and commodity loadings are $\mathbb{E}[\beta_{i,\text{doll}}] = 1$ and $\mathbb{E}[\beta_{i,\text{comm}}] = 0$. As the next proposition illustrates, their second moments are related to the second moments of dollar holdings $\{m_i\}$ and commodity exposures $\{e_i\}$.

PROPOSITION 5. *In the cross-section,*

$$\mathbb{V}[\beta_{i,\text{doll}}] = \phi^2\tau^2\mathbb{V}[m_i],$$
$$\mathbb{V}[\beta_{i,\text{comm}}] = \frac{\mathbb{V}[e_i]}{(e_H - e_L)^2}.$$

The variance of m_i is positively related to the cross-sectional variance of the loadings on the dollar factor. In the case of e_i, its variance directly affects the variance of the commodity loadings through the numerator and indirectly by affecting the denominator.

2.4 Excess Returns and Risk Premia

In this section, we analyze how risk premia affect excess returns and interest rate differentials, distinguishing between unconditional and conditional effects. On the one hand, the heterogeneity in countries' exposure to risk is not reflected in unconditional average excess returns or interest rate differentials. This is because, in the steady state, expected consumption growth is zero in all countries, which implies that there are no interest rate differentials, and because exchange rates are also constant in the steady state, this implies that excess returns are zero. Countries' differences in risk premia, in turn, affect intermediaries' portfolio positions and countries' net foreign assets, as detailed in Appendix A. On the other hand, shocks to the risk aversion affect expected excess returns and interest rate differentials, as described in the following proposition.

PROPOSITION 6. *Conditional expected returns, defined as* $\mathbb{E}_t[rx_{i,t+1}] \equiv r_{i,t+1} - r_{u,t+1} - \mathbb{E}_t[\Delta s_{i,t+1}]$,

and interest rate differentials are given by

$$\mathbb{E}_t[rx_{i,t+1}] = [\beta_{i,\text{comm}} \cdot (e + e_u)^2 \varphi \Gamma_z + \beta_{i,\text{doll}} \cdot (\rho - 1)\Gamma_u] \cdot \alpha^2 \gamma_t,$$
$$r_{i,t+1} - r_{u,t+1} = \left[\beta_{i,\text{comm}} \cdot (e + e_u)^2 \rho(1-\alpha)\Gamma_z + \beta_{i,\text{doll}} \cdot \rho(1-\alpha)(\rho-1)\Gamma_u\right] \cdot \alpha\gamma_t.$$

This proposition highlights the relevance of the risk aversion shocks as the sole drivers of variation in excess returns and interest rate differentials. Conditional on a positive risk aversion shock, countries with high commodity exposure e_i and/or low dollar assets m_i, will exhibit excess returns and positive interest rate differentials that reflect a compensation for their exposure to risk. In the following section we empirically test these model predictions.

3 Empirical Analysis

In the first part of this section, we estimate the factor structure implied by the model and document the relationship between the factors and empirical proxies of the model's structural shocks. We then discuss how the factors are related to other factors estimated in the literature. In the second part, we relate the estimated loadings on the two factors to country characteristics in the cross-section, as suggested by the model.

3.1 Factor Structure of Exchange Rates

We use monthly data on bilateral exchange rates vis-a-vis the US dollar for a sample of $N = 32$ countries from 2000 to 2019.[1] Exchange rates are expressed in units of local currency per US dollar, unless noted otherwise. A rising exchange rate means dollar appreciation. For each currency i, we estimate the empirical model analog of equation (10), given by

$$\Delta s_{i,t+1} = \alpha_i + \beta_{i,\text{comm}} \text{comm}_{i,t+1} + \beta_{i,\text{doll}} \text{doll}_{i,t+1} + \epsilon_{i,t+1}, \tag{13}$$

where $\Delta s_{i,t+1}$ is the monthly change in the log exchange rate of currency i per dollar in period $t+1$, $\text{doll}_{i,t+1}$ is the dollar factor, and $\text{comm}_{i,t+1}$ is the commodity factor. The factors are indexed

[1] The sample of countries includes Albania, Australia, Brazil, Canada, Switzerland, Chile, China, Colombia, Costa Rica, Czech Republic, Euro Area, United Kingdom, Hungary, Indonesia, Israel, India, Iceland, Japan, South Korea, Mexico, Malaysia, Norway, Peru, Philippines, Poland, Russia, Sweden, Singapore, Thailand, Turkey, Uruguay, and South Africa. These countries had floating exchange rates and low and stable inflation rates throughout the sample period, which imply a strong comovement between their nominal and real exchange rates. This makes them adequate countries for testing the theory about real exchange rates.

by currency i because we exclude currency i when constructing them to avoid a mechanical relationship between the left-hand side and the right-hand side of equation (13). Paralleling its model counterpart, we construct the commodity factor as the average change in the log exchange rate of high-versus-low commodity producers

$$\text{comm}_{i,t+1} \equiv \frac{2}{N-1} \sum_{j \in \mathcal{I}_{h,i}} \Delta s_{j,t+1} - \frac{2}{N-1} \sum_{j \in \mathcal{I}_{l,i}} \Delta s_{j,t+1}, \tag{14}$$

where $\mathcal{I}_{h,i}$ and $\mathcal{I}_{l,i}$ are the sets of countries in which the time-average share of commodity exports in total exports is above and below the median across countries excluding country i, respectively. We construct the dollar factor as the average change in the log exchange rates of all currencies against the dollar, excluding currency i:

$$\text{doll}_{i,t+1} \equiv \frac{1}{N-1} \sum_{j \neq i} \Delta s_{j,t+1}. \tag{15}$$

We provide further details on the construction of these variables in Appendix B.

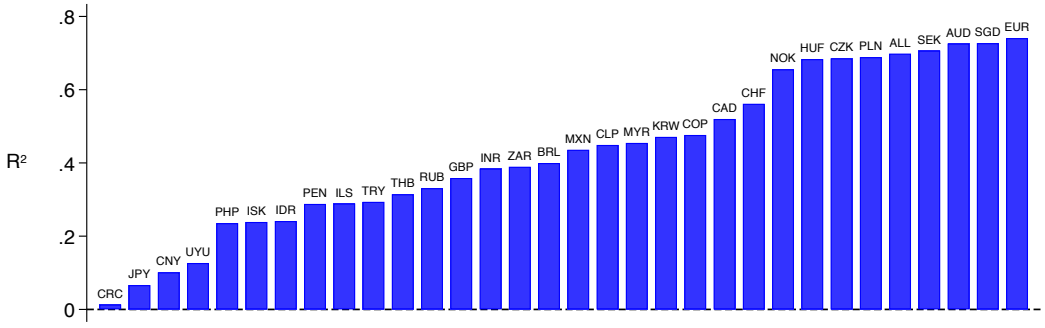
Panel (a) of Figure 1 reports the R^2 from the estimation of equation (13) for all currencies. The factor regressions have high explanatory power, with an average R^2 of 43% and as high as 74% for some countries. As a benchmark for comparison, Appendix Figure B.1 shows that the R^2 are close to those obtained from regressing changes in log exchange rates on their first two principal components, which is an upper bound on how much variation can be explained by any two variables. The average R^2 from the two-principal-components regressions is 51%.

Panel (b) of Figure 1 reports the loadings on the commodity factor by currency, which exhibit significant cross-country heterogeneity, with roughly half of the countries loading positively and the other half negatively. Panel (c) reports the loadings on the dollar factor, which are all positive and also exhibit significant heterogeneity across countries. Tables B.1 and B.2 report these regression estimates along with their Newey-West adjusted standard errors.

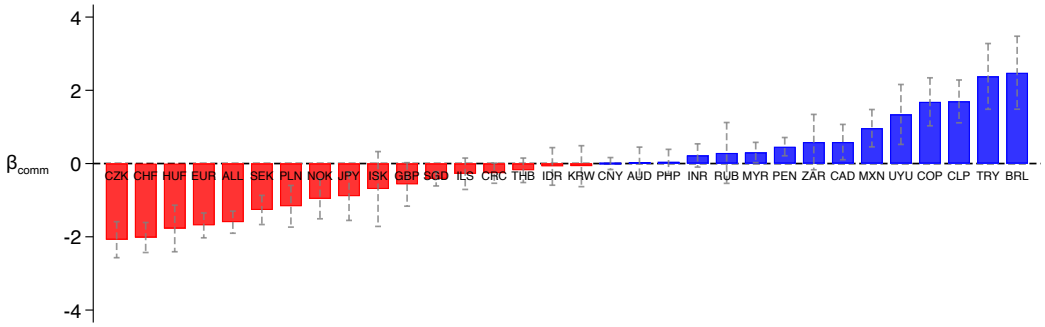
Macroeconomic determinants of factors. Motivated by the model's structural interpretation of the factors summarized in Proposition 2, we estimate the relationship between the factors and empirical proxies of the structural shocks that determine them. We first regress the commodity factor on the changes of a global commodity price index (as a proxy for the commodity-driven global tradable output shocks) and empirical measures of global risk, namely, the VIX, the excess bond premium from Gilchrist and Zakrajšek (2012), and the (negative of) global intermediaries' equity

Figure 1: Factor Structure of Exchange Rates: Country Regressions

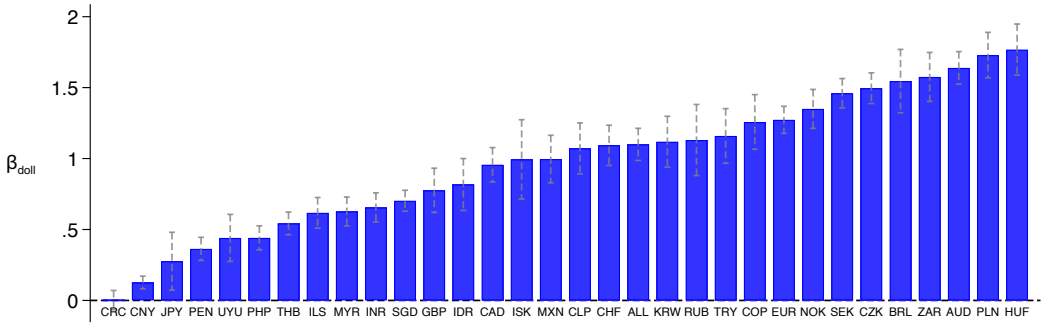
(a) Explanatory power R_i^2



(b) Commodity-factor loadings $\beta_{i,\text{comm}}$



(c) Dollar-factor loadings $\beta_{i,\text{doll}}$



Notes: Panel (a) shows the R^2, representing the fraction of variation in $\Delta s_{i,t+1}$ explained by our empirical model in equation (13) for each currency i. Panel (b) shows the estimated loading of the commodity factor $\text{comm}_{i,t+1}$ for each currency i from our empirical model in equation (13), with 90% confidence interval in gray, calculated using Newey-West standard errors. Panel (c) shows estimated loading of the dollar factor $\text{doll}_{i,t+1}$ for each currency i from our empirical model in equation (13), with 90% confidence interval in gray, calculated using Newey-West standard errors.

capital ratio from He et al. (2017). We provide further details on the description of variables used in the regression analysis in Appendix B. Panel (b) of Appendix Table B.3 shows that, consistent with the model's prediction, the commodity factor is negatively related to the global commodity price index and positively related to all the empirical measures of global risk. We then regress the dollar factor on the changes in US employment in the service sector (as a proxy for US non-tradable output shocks) and measures of global risk. Panel (a) of Appendix Table B.3 reports that the dollar factor is negatively related to changes in the employment rate in the service sector and positively related to changes in the measures of global risk.

Commodity factor and other factors. In this subsection, we study how the commodity factor relates to two alternative factors previously studied in the literature. One is the carry factor proposed by Verdelhan (2018), computed in a similar way to the commodity factor but sorting countries by their local-currency interest rates; the other is the trade-centrality factor proposed by Richmond (2019), also computed similarly to the commodity factor but sorting countries according to their trade centrality (see Appendix B for further details on the construction of these factors). Panel (a) of Appendix Figures B.5 and B.6 show that the explanatory power of our baseline two-factor specification is very similar to alternative two-factor specifications in which we replace the commodity factor with the carry factor or the trade centrality factor. Additionally, Panel (b) of Appendix Figures B.5 and B.6 shows that the country-loadings on the commodity factor are tightly correlated with the loadings on the carry and the trade-centrality factors. This analysis suggests that the commodity factor captures similar variation as the carry and trade-centrality factors, consistent with the findings of Ready et al. (2017).

The role of dollar basis. We also argue that the dollar factor captures a global factor and not a local US factor that arises simply because we express currencies relative to the US dollar. We perform two related exercises to support this conclusion. First, consider expressing exchange rates relative to a base currency j instead of the dollar. In the model, these exchange rates admit a three-factor structure in which, in addition to the dollar and commodity factor, there is a third "$j-$base" factor, defined in the same way as the dollar factor but expressing all exchange rates relative base currency j. This new factor captures the idiosyncratic shocks to non-traded output of country j, which affect non-traded prices in that country and all bilateral exchange rates. Proposition 3

specifies the exact factor structure and the structural definition of the factors. According to this result, switching to base currency j should shift the dollar factor loadings by 1 for all currencies, the commodity factor loadings should be unchanged, and the base currency factor should have a loading of 1 on all currencies.

We first focus on the British pound as an example of another base. Appendix Figure B.2 shows the three factor loadings of regressing the changes in bilateral exchange rates vis-a-vis the pound on the commodity factor, the dollar factor, and a British pound factor as independent variables, and compares them to the same regression in which we express the exchange rates vis-a-vis the dollar. Consistent with the model, the dollar factor loadings display similar heterogeneity across countries when estimated with exchange rates vis-a-vis the British pound and vis-a-vis the dollar. The dollar loadings remain statistically significant for most countries when estimated on exchange rates vis-a-vis the pound, and are shifted by approximately 1 relative to the specification with USD as the base currency. In contrast, the loadings on the pound factor are positive and fluctuate around 1 in all countries when estimated on exchange rates vis-a-vis the pound, but are close to zero when estimated on exchange rates vis-a-vis the dollar. The commodity loadings are similar when estimated using both bases.

We next scale the analysis and estimate the same regressions for all possible currency bases. Appendix Table B.4 reports in how many regressions in a given currency base the dollar factor is statistically significant. When we pool all the currency bases, the dollar factor remains a statistically significant factor in 682 out of 992 country-currency-base regressions, suggesting that the dollar factor plays a global role over and above the mechanical effect of using it as a currency base.

Second, we show that the reverse is not true. Other currency bases do not have the strong explanatory power of the dollar factor. We illustrate this by estimating a similar specification to the baseline with dollar exchange rates as dependent variables, in which we add an additional base-currency factor, for all possible currency bases. As reported in the last column of Appendix Table B.4, other currency bases are statistically significant in only 268 out of the 992 regressions.

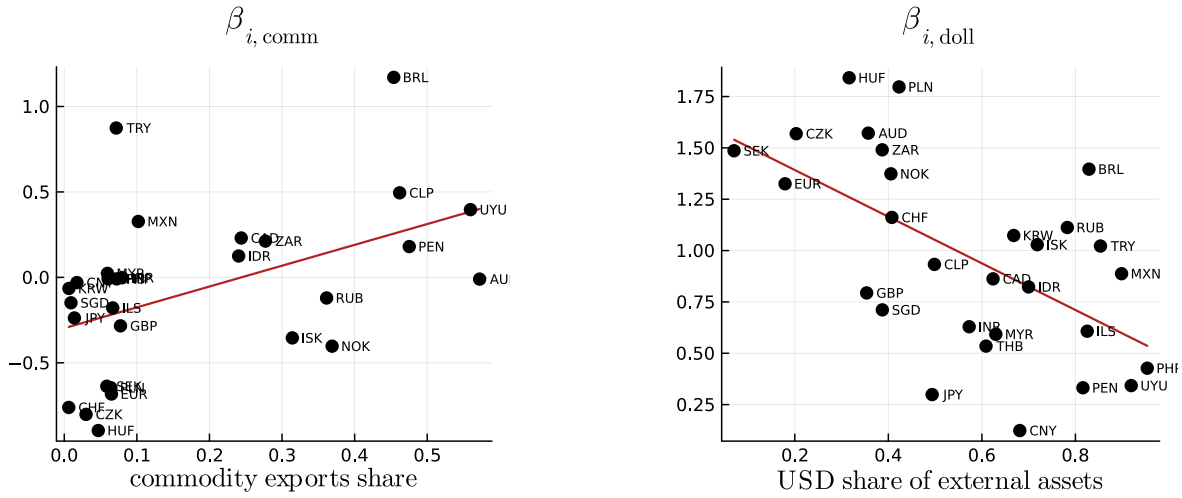
3.2 Determinants of Country-Factor Loadings

We now analyze how the estimated country-loadings from equation (13) are related to certain observable country-characteristics, as predicted by the model. According to Proposition 2, the

loadings on the commodity factor should be positively related to the commodity incidence of a country, and the loadings on the dollar factor should be negatively related to the incidence of dollar assets of a country. Panel (a) of Figure 2 shows that the estimated country loadings on the commodity factor are positively related to the country shares of commodity exports in total exports. Panel (b) shows that the estimated loadings on the dollar factor are negatively related to the share of dollar-denominated portfolio assets in total portfolio assets. Appendix B provides further details on the construction of these variables. Appendix Table B.5 shows the univariate regression estimates associated with both scatter graphs. The coefficient of the share of commodity exports on the commodity loadings is positive and statistically significant, and the R^2 from the regression is 29%. The coefficient of the share of dollar assets on the dollar loadings is negative and statistically significant, and the R^2 from the regression is 29%.

Figure 2: Determinants of Factor Loadings: Baseline Regressions

(a) Commodity loadings on commodity incidence (b) Dollar loadings on dollar assets



Notes: Panel (a) plots the commodity factor loadings for each currency i (y-axis), against the commodity incidence, measured by the share of commodity exports (x-axis). Panel (b) plots the dollar factor loadings for each currency i (y-axis), against the share of dollar-denominated assets (x-axis). Each scatter point represents a currency, and the red line denotes the line of best fit.

Robustness analysis. We show that the relationships between the factor loadings and country-characteristics are robust to several alternative specifications. We summarize the main takeaways from this analysis below and provide further details in Appendix B.

1. *Additional controls.* Appendix Table B.5 shows that the empirical relationships estimated in

the regressions are robust to including various additional controls considered in the literature (e.g., Hassan, 2013; Richmond, 2019; Lustig and Richmond, 2020): trade centrality, trade openness, the economic size of the country, and a set of gravity variables relative to the US (bilateral distance, and dummies for common borders, language, and legal origins).

2. *Alternative measures of commodity incidence and dollar assets.* Appendix Table B.6 shows that the main results are robust to alternative ways of measuring of commodity incidence and dollar assets. For commodity incidence we consider the share of commodity exports in total output, instead of the baseline measure of the share of commodity exports in total exports. For dollar assets, we consider the ratio of dollar assets to total trade (exports plus imports), instead of the ratio of USD assets to total portfolio assets.

3.3 Additional Supportive Evidence

In this section we provide supporting evidence on the model mechanisms and test additional model predictions. First, we show that countries' commodity incidence is tightly related to their sensitivity of tradable output to commodity fluctuations. Second, we provide evidence on how capital gains on dollar assets affect aggregate consumption at the country level. Finally, we test the predictions of the model regarding variation in excess returns.

Tradable output sensitivity. In the model, the commodity incidence of a country affects its exchange rate properties because it determines the sensitivity of tradable output to commodity-biased productivity fluctuations. We show that we obtain the same empirical results on the factor structure of exchange rates and on the determinants of country loadings if we use estimates of the sensitivity of tradable output to commodity fluctuations as the country measures of commodity incidence.

For this, we first obtain country-specific measures of sensitivity to a global commodity price index, $\beta_{i,\text{tos}}$ by estimating the following regression:

$$\Delta y^T_{i,t+1} = \alpha_i + \beta_{i,\text{tos}} \Delta p_{\text{comm},t+1} + \upsilon_{i,t}, \tag{16}$$

where $\Delta y^T_{i,t+1}$ is the annual log change in tradable output from country i in period $t+1$, and $\Delta p_{\text{comm},t+1}$ is the annual log change in the global commodity price index. We then construct the

tradable-output-sensitivity factor as the average change in the log exchange rate of above-median relative to below-median tradable-output-sensitivity-countries (see Appendix B for further details). Appendix Figure B.7 shows that the regression in which we substitute the commodity factor for the tradable-output-sensitivity factor has a similar explanatory power to the baseline regression. It also shows that the tradable-output-sensitivity factor captures similar country variation as the baseline commodity factor.

Appendix Figure B.8 further shows that the country estimates of tradable output sensitivity are tightly related to the country loadings of the different measures commodity factor, including the baseline measure of the factor based on commodity export shares or the one described above based on tradable output sensitivity estimates.

Dollar assets and capital gains. In the model, the countries' dollar assets determine their loadings on the dollar factor because countries with more dollar assets realize larger capital gains in times of broad dollar appreciation, which increases their tradable consumption and attenuates the bilateral depreciation of the local currency. In Appendix Table B.7, we provide supportive evidence of the model's mechanism. The first column shows that the magnitudes of changes in dollar assets are relevant when compared to variation in consumption: for the average and median country, the standard deviation of changes in dollar assets is 33% and 24% of the standard deviation of consumption, respectively. The last column shows that changes in dollar assets are negatively correlated with changes in consumption for roughly 80% of the countries, implying that periods of dollar asset accumulation are associated with periods of contraction in consumption, as predicted by the model.

Excess returns and risk shocks. Finally, we provide evidence supporting the model's predictions regarding how risk shocks affect excess returns across countries. According to Proposition 6, positive risk aversion shocks increase risk premia more in countries with larger loadings on the commodity and dollar factors. In Appendix B.3 we estimate country loadings of excess returns on a measure of global risk shocks, and show that these loadings are positively related to the country loadings on the commodity and dollar factors. This evidence provides additional support to the importance of including risk shocks in our analysis.

4 Quantitative Analysis

We perform the quantitative analysis of the model in three steps. First, we describe the calibration strategy. Second, we analyze the drivers of the relative prices of non-tradables in individual countries. Third, we analyze the drivers of bilateral exchange rates against the dollar in the cross-section and the factor structure of exchange rates.

4.1 Calibration and Validation

Calibration. We calibrate the model to a monthly frequency. We parameterize the cross-sectional distributions of e_i and m_i as two independent log-normal distributions, where we normalize their unconditional means, $e = m = 1$, and calibrate their standard deviations, σ_e and σ_m.[2] Our calibration proceeds in two steps. At the first step, we externally set a subset of parameters. We set $\beta = 0.98$ and the inverse elasticity of intertemporal substitution $\rho = 2$, which are typical values in the literature. We also set the share of non-tradable goods in the consumption aggregator at $\alpha = 0.64$, in line with Burstein et al. (2005), and normalize the average size of the non-tradable endowment $N = \alpha/(1-\alpha)$, so that the local basket price in the steady state is one in all countries: $Q_i = 1$. Panel (a) of Table 1 reports the values of the parameters.

At the second step, we estimate and calibrate the remaining parameters. First, we estimate the dollar asset management parameter, τ, using panel data on countries' international reserves, and estimate the cross-sectional standard deviation of exposures to the global tradable shock, σ_e, using data on commodity exposures. Appendix C provides details on the estimation of these parameters. Second, we internally calibrate the standard deviations of the shocks $(\sigma_z, \sigma_u, \sigma_\gamma, \sigma_x)$ and the cross-sectional standard deviations of exposures to dollar assets σ_m. We use the identification results of Proposition 4 and Proposition 5 and target the second moments of the factors and the residual of the factor regression $(\mathbb{V}[\text{comm}_t], \mathbb{V}[\text{doll}_t], \mathbb{C}[\text{comm}_t, \text{doll}_t], \mathbb{V}[\varepsilon_{it}])$, and the cross sectional standard deviation of the dollar loadings $\text{std}(\beta_{i,\text{doll}})$. Panel (b) of Table 1 reports the estimated and calibrated

[2] The empirical cross-sectional correlation between the share of commodity exports in total exports and the share of dollar assets in total external assets is not statistically significant, which justifies the assumption of independent distributions. Additionally, we can normalize the unconditional means of e_i and m_i because they are not separately identified form other parameters. In the case of e_i, its mean cannot be separately identified from σ_z, given that endowments are $e_i z_t$. In the case of m_i, its average level only affects the steady state net foreign asset position of the US through market clearing of dollar bonds. This position is also determined by the steady state position of intermediaries on dollar bonds, which is determined by the average level of risk aversion, Γ.

Table 1: Model parameters.

Parameter	Value	Role
(A) Externally set		
ρ	2.0	inverse of EIS
β	0.98	discount factor
α	0.64	share of non-tradables
ρ_z, ρ_x, ρ_u	1.0	persistence of shocks to z_t, x_{ut} and x_{it}
ρ_γ	0.0	persistence of shocks to γ_t
(B) Estimated and calibrated		
σ_γ	0.69	standard deviation of innovations to γ_t
σ_u	2.64	standard deviation of innovations to x_{ut}
σ_x	3.88	standard deviation of innovations to x_{it}
σ_z	0.90	standard deviation of innovations to z_t
σ_m	1.04	cross-sectional standard deviation of m_i
σ_e	1.29	cross-sectional standard deviation of e_i
Γ	0.73	SDF weight on stochastic component
τ	0.83	reserve stabilization elasticity

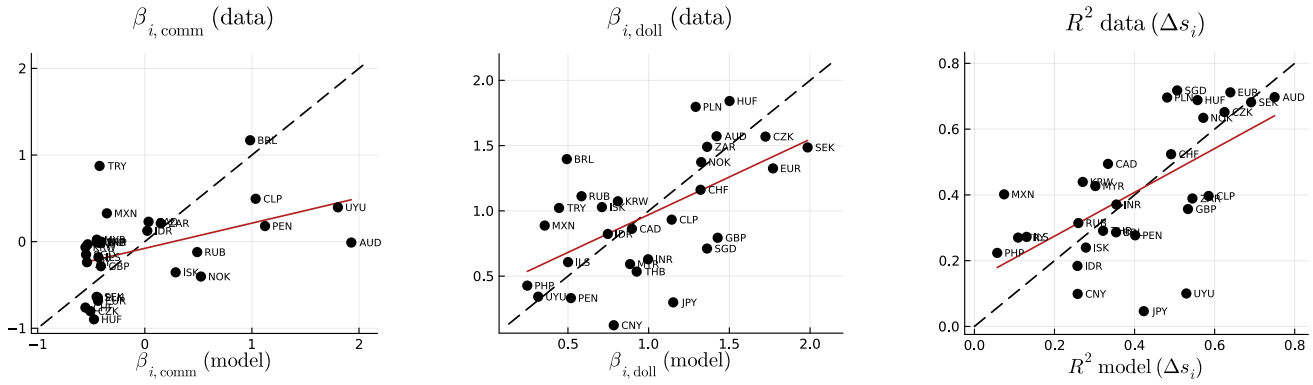
parameter values, and Table 2 reports the fit on the targeted moments. In the data, the dollar factor is more volatile than the commodity factor, which indicates a larger relevance of US non-tradable output shocks relative to global tradable output shocks. Additionally, the two factors are positively correlated in the data, suggesting a relevant role for risk shocks driving this correlation. Finally, we separately calibrate Γ and σ_γ by targeting the steady state net foreign asset position in the US, which only depends on the former.

Table 2: Model fit on targeted and untargeted moments.

Moment	Data	Model
std(doll_t)	1.94pp	1.94pp
std(comm_t)	1.32pp	1.32pp
corr(doll_t,comm_t)	0.31	0.31
std(ε_{it})	2.48pp	2.48pp
std(Δs_{it})	3.24pp	3.34pp
avg R^2 of factors	41%	40%
std($\beta_{i,\text{doll}}$)	0.47	0.47
std($\beta_{i,\text{comm}}$)	0.48	0.73

Model validation. We validate the model by analyzing untargeted moments related to the factor structure regressions of exchange rates. Table 2 shows that the model replicates well the

Figure 3: Factor Regressions in Model and Data



(a) commodity loadings (b) dollar loadings (c) explanatory power

Notes: Panel (a) plots the commodity factor loadings estimated in the two-factor regressions against those implied by the model, $\beta_{i,\text{comm}} = (e_i - 1)/(e_H - e_L)$. Panel(b) plots the same for the dollar factor loadings $\beta_{i,\text{doll}} = 1 - \phi\tau(m_i - 1)$. Panel (c) plots the R-squared of the two-factor regression in the data against that implied by the model. The model mapping from observables to (e_i, m_i) and the expressions for the R-squared of the two-factor regressions are detailed in Appendix C.

unconditional standard deviation of changes in exchange rates, the average R^2 of the factor regressions, and the cross-sectional deviation of the loadings on the commodity factor.

We also compare the fit of factor regressions in different countries in the data with their model counterparts. We identify countries in our model by assigning a value of e_i that matches their place in the empirical distribution of commodity export shares, and a value of m_i that matches their place in the empirical distribution of dollar asset shares. We provide more details on this procedure in Appendix C. Figure 3 shows that the cross-sectional loadings and explanatory power of the factor structure in the model are tightly linked to those in the data. Appendix Table C.1 shows the fit of regressing each data moment on their counterpart in the model.

Alternative calibration relaxing parametric assumptions. In Appendix C.4 we complement the baseline quantitative analysis by studying an additional calibration in which we relax the assumptions that granted tractability and calibrate the persistence of the aggregate shocks and the portfolio management costs. We highlight two takeaways from the analysis of this model parameterization. First, we show that the interpretations of factors as the particular combination of the model's exogenous shocks stated in Proposition 2 still holds quantitatively. The explanatory power of regressing the model's commodity factor on the tradable output shock and the risk shock, and regressing the model's dollar factor on the US nontradable output shock and the risk

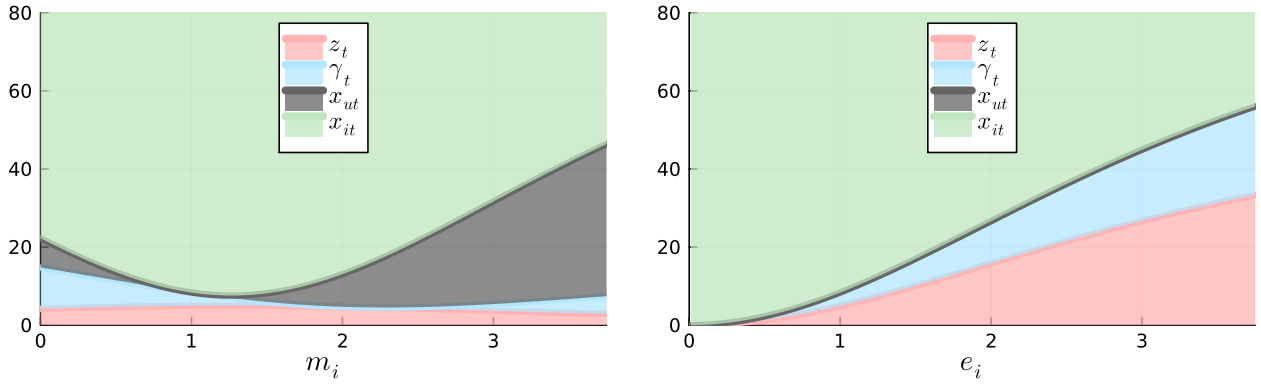
shock exceed 95% in both cases. Second, we show the quantitative results from the baseline model reported in the next sections are preserved under this alternative parameterization.

4.2 Drivers of Exchange Rates in the Cross-Section

In this section we analyze the drivers of country-level prices and bilateral exchange rates in the cross-section.

Drivers of country-level prices. We start our analysis by studying the drivers of individual consumption basket prices q_{it}, which are one component of bilateral exchange rates. We perform a variance decomposition of q_{it} for generic (e_i, m_i), and analyze how each dimension affects country-level prices in Figure 4. Appendix C provides details on the variance decompositions, which are available in closed form, and Appendix Figure C.1, Figure C.2, and Figure C.3 complement this analysis by showing the peak response of key macroeconomic variables of countries with different pairs (e_i, m_i) to the three aggregate shocks.

Figure 4: Variance Decomposition of Country-Level Prices



(a) Countries with different dollar assets (b) Countries with different commodity exposure

Notes: Panel (a) plots the shares of the three global shocks (z_t, γ_t, x_{ut}) and the local shock x_{it} in the variance decomposition of the local basket price changes Δq_{it} as a function of m_i, where each country's e_i is evaluated at the mean $e = 1$. Panel (b) plots the same variance decomposition as a function of e_i, with each country's m_i evaluated at the mean $m = 1$. The model mapping from observables to (e_i, m_i) and the closed-form expressions for the variance decompositions are detailed in Appendix C.

As shown in Figure 4, both global and idiosyncratic shocks are relevant drivers of country-level prices. Focusing on the role of global shocks, in countries with low dollar assets, Figure 4a shows that most of the systemic variation in country-level prices is driven by risk shocks and shocks to US non-tradable output. Increases in risk and decreases in US non-tradable output trigger

capital outflows in these countries and decrease their consumption prices (see Appendix C). As we consider countries with higher dollar assets that are still below the mean, the relevance of these global shocks decreases as higher stocks of dollar assets attenuate the capital flight in response to these shocks. A country with average dollar assets is not exposed to the US non-tradable shock because the capital inflows from selling dollar assets are exactly offset by capital outflows from their local currency bonds. Similarly, there is another threshold of dollar assets (slightly higher than the mean), at which a country is not exposed to the risk shock. To the right of these thresholds, the relevance of risk and US non-tradable shocks increases with dollar assets because the prices in these countries increase in response to increases in risk and decreases in US non-tradable output as they receive more capital flows. Finally, the relevance of global tradable output shocks is minor for countries with average exposure to this shock, regardless of their levels of dollar assets.

As shown in Figure 4b, as we consider countries with higher commodity exposure, the relevance of global tradable shocks and risk shocks driving country-level prices increases monotonically, explaining a negligible fraction when e is close to zero, to more than half when e is three times the average level. The rising relevance of risk shocks is because higher e increases how their excess returns covary with the intermediaries stochastic discount factor.

Bilateral exchange rates. Next, we analyze the drivers of global factors and bilateral exchange rates relative to the dollar. Table 3 shows the variance decomposition of the two global factors and US prices. In the case of the commodity factor, both the global tradable output shocks and risk shocks are comparable drivers, with the former explaining 60% of its variation. In the case of the dollar factor, US non-tradable shocks is the predominant driver accounting for around 75% of its variation, with the reminder explained by risk shocks. The relevance of risk shocks explaining the dollar factor is significantly higher than that explaining US prices because these shocks reshuffle capital flows and affect tradable consumption and country prices in opposite directions.

Table 3: Variance decomposition of q_{ut}, doll_t, and comm_t.

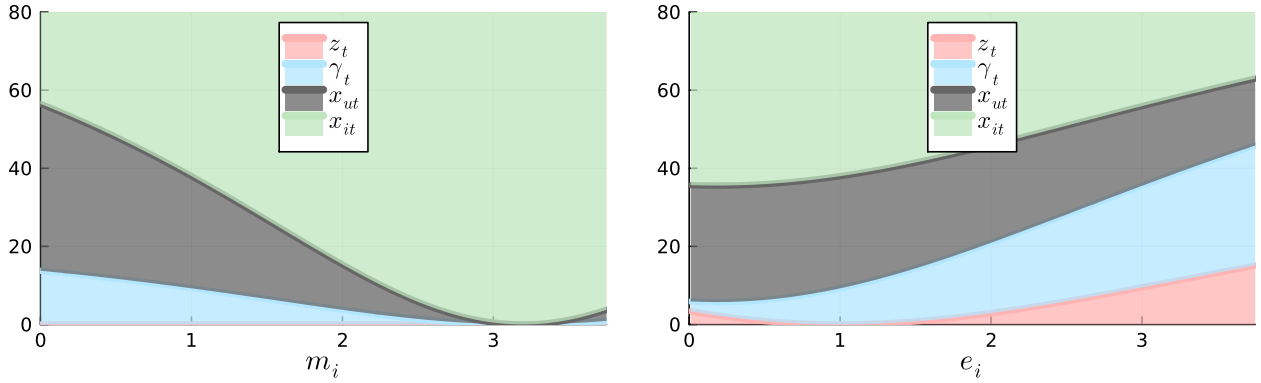
	Δq_{ut}	doll_t	comm_t
$\Delta\gamma_t$	7%	24%	40%
Δz_t	10%		60%
Δx_{ut}	84%	76%	

Figure 5 shows the variance decomposition of bilateral exchange rates relative to the US along the dollar assets and commodity dimensions. For most countries in the distribution, the global shocks, particularly the US non-tradable shocks and the risk shocks, are more relevant explaining bilateral dollar exchange rates than they are explaining country-level prices. This is again because US non-tradable shocks and risk shocks affect country level prices in most countries in a different direction than how they affect US prices. Turning to countries with different dollar assets, Figure 5a shows that, in most of the dollar asset distribution, the relevance of US non-tradable shocks and risk shocks is decreasing in dollar assets. This is because most countries depreciate against the dollar in response to decreases in US non-tradable output and increases in risk, and a larger stock of dollar assets attenuates this depreciation by realizing larger capital gains. A corollary of this is that in countries with significantly high dollar assets, dollar exchange rates are mostly explained by idiosyncratic shocks because their exposure to global shocks is similar to that of the US.

Turning to countries with different commodity exposure, Figure 5b shows that, for countries with commodity exposure above average, the relevance of the global tradable shock and risk shocks increases with their commodity exposure. These countries are more exposed to the global tradable output shock than the US and, hence, their exchange rates appreciate relative to the dollar in response to an expansionary tradable output shock. As the commodity exposure increases, these countries become riskier and their dollar exchange rate depreciates in response to increases in risk. A corollary of this is that idiosyncratic shocks lose relevance in explaining dollar exchange rates as countries increase their commodity exposure.

We now analyze how the model sheds light on the exchange rate dynamics of the three countries cited in the introduction: Thailand, Hungary, and Australia. These countries are illustrative examples of how the characteristics of their economy shape the properties of their exchange rates, summarized in Table 4. On one extreme, the Thai baht is a currency that is relatively stable against the US dollar, and the empirical factor regression only explains a small fraction of the variation in its exchange rate. The model is able to quantitatively explain these properties because Thailand is a country with low exposure to commodities and large dollar assets, which makes its dollar exchange rate largely insulated from global factors. On the other extreme, the Hungarian forint and the Australian dollar exhibit large volatility against the dollar, and the factor regression explains a significant fraction of their fluctuations. Through the lens of the model, these properties arise for different reasons. Hungary is a country with limited dollar assets, which makes its currency exposed

Figure 5: Variance Decomposition of Bilateral Exchange Rates



(a) Countries with different dollar assets (b) Countries with different commodity exposure

Notes: Panel (a) plots the shares of the three global shocks (z_t, γ_t, x_{ut}) and the local shock x_{it} in the variance decomposition of the bilateral exchange rate changes Δs_{it} as a function of m_i, where each country's e_i is evaluated at the mean $e = 1$. Panel (b) plots the same variance decomposition as a function of e_i, with each country's m_i evaluated at the mean $m = 1$. The model mapping from observables to (e_i, m_i) and the closed-form expressions for the variance decompositions are detailed in Appendix C.

to US non-tradable shocks and global risk shocks. Australia is a country with high commodity incidence, and its currency is significantly exposed to global fluctuations in tradable output and global risk shocks.

Table 4: Exchange Rate Properties in Model and Data for Selected Countries.

		Thailand	Hungary	Australia
(a) Data				
std(Δs_{it})		2.0%	3.9%	3.6%
factor R^2		29%	69%	70%
e_i		low	low	high
m_i		high	low	low
(b) Model				
std(Δs_{it})		3.0%	3.7%	5.0%
factor R^2		32%	56%	75%
Δs_{it} variance shares	$\Delta\gamma_t$	9%	14%	48%
	Δz_t	7%	3%	21%
	Δx_{ut}	84%	83%	31%

5 Conclusion

We develop a model of the global economy to study the drivers of exchange rates in the cross-section. The model fulfills two goals. First, it provides an equilibrium interpretation to the

empirical factor structure of exchange rates, mapping fundamental shocks to factors and fundamental properties of countries to their factor loadings. Second, it provides a mapping that allows us to recover the parameters of fundamental shocks from this factor structure.

Returning to the motivating question of the paper, our analysis indicates that there is no single answer to what drives exchange rates across countries, because the drivers of exchange rates critically depend on which exchange rate we refer to. Focusing on bilateral exchange rates against the dollar, the currencies of high-commodity countries are driven by a combination of financial and global output shocks. The currencies of countries with low dollar assets are affected by financial shocks and US-specific output shocks. Low-dollar and high-commodity countries are the ones with most volatile price levels and dollar exchange rates, and the contribution of financial shocks to their exchange rates is the largest. These countries' exchange rates also feature the highest explanatory power of global factors. In contrast, the exchange rates of high-dollar and low-commodity countries feature the lowest volatility and lowest explanatory power of global shocks, with their currencies are mostly driven by local shocks.

Finally, in the case of exchange rates relative to the US dollar, the relevance of risk shocks stands out because they affect relative prices in the US and in most of the other countries in different directions. This is because the US is a particularly safe country that receives capital flows from other countries in times of high global risk aversion.

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A Theoretical Appendix

A.1 Proofs and Additional Theoretical Results

Proof of Lemma 1. Plugging profit and fiscal rebates into the budget constraint, we get country i's resource constraint:

$$C_{it}^T + P_{it}^N C_{it}^N = Y_{it}^T + P_{it}^N Y_{it}^N + R_{it} Q_{it} B_{it} - Q_{it} B_{i,t+1} + R_{ut} Q_{ut} M_{it} - Q_{ut} M_{i,t+1} + Q_{i,t-1} A_{it} \left(R_{it} \frac{Q_{it}}{Q_{i,t-1}} - \frac{Q_{ut}}{Q_{u,t-1}} \right) - Q_{ut} M_{it} (R_{ut} - 1) \tag{A.1}$$

Use the market clearing for non-tradable goods to cancel $P_{it}^N C_{it}^N$ on the left and $P_{it}^N Y_{it}^N$ on the right. Use the market clearing for country i's bonds to replace B_{it} with $-A_{it}$:

$$C_{it}^T = Y_{it}^T + Q_{it} A_{i,t+1} - \frac{Q_{ut}}{Q_{u,t-1}} Q_{i,t-1} A_{it} + Q_{ut} M_{it} - Q_{ut} M_{i,t+1} \tag{A.2}$$

Next, denoting by L_{it} the dollar value of country i's bonds purchased by the intermediary and using $Q_{ut} L_{i,t+1} = Q_{it} A_{i,t+1}$,

$$C_{it}^T = Y_{it}^T + Q_{ut} L_{i,t+1} - Q_{ut} L_{it} + Q_{ut} M_{it} - Q_{ut} M_{i,t+1} \tag{A.3}$$

The analogous constraint for the US is

$$C_{ut}^T = Y_{ut}^T + Q_{ut} L_{u,t+1} - Q_{ut} L_{ut} \tag{A.4}$$

Next, linearize the dollar asset policy rule $Q_{ut} M_{i,t+1} = \tau \cdot Q_t M_i + (1 - \tau) \cdot Q_{ut} M_i$ and observe that first-order deviations of reserves are the same in all countries:

$$m_{t+1} = \tau (q_t - q_{ut}) \tag{A.5}$$

Log-linearizing the resource constraints (linearizing with respect to L_{it} and L_{ut}),

$$q_{it} = \alpha e_i z_t - \alpha x_{it} + \alpha \Delta l_{i,t+1} - \alpha M_i \Delta m_{t+1} \tag{A.6}$$

$$q_{ut} = \alpha e_u z_t - \alpha x_{ut} + \alpha \Delta l_{u,t+1} \tag{A.7}$$

This completes the proof. □

Proof of Lemma 2. Let $\Lambda_{t+1}^{\text{ave}} = \int \Lambda_{i,t+1} \mathrm{d}i + \Lambda_{u,t+1}$. The intermediaries' optimal portfolio is

$$Q_{ut} L_{i,t+1} = L_{t+1} + \chi (\mathbb{E}_t[\Lambda_{t+1}^{\text{ave}}] \mathbb{E}_t[X_{i,t+1}] + \Gamma_t \mathbb{C}_t[\Lambda_{t+1}^{\text{ave}}, X_{i,t+1}]) \tag{A.8}$$

Here L_{t+1} is the benchmark level of assets given in tradable goods:

$$L_{t+1} \equiv Q_{ut} \int M_{i,t+1} di \tag{A.9}$$

Rewrite the optimal choice as

$$L_{i,t+1} = \int M_{i,t+1} di + \chi(\mathbb{E}_t[\Lambda^{\text{ave}}_{t+1}]\mathbb{E}_t[Q^{-1}_{ut} X_{i,t+1}] + \Gamma_t \mathbb{C}_t[\Lambda^{\text{ave}}_{t+1}, Q^{-1}_{ut} X_{i,t+1}]) \tag{A.10}$$

Now use the fact that $X_{i,t+1}$ expands as

$$X_{i,t+1} \approx \beta^{-1}(\Delta q_{i,t+1} - \Delta q_{u,t+1} + r_{i,t+1} - r_{u,t+1}) \tag{A.11}$$

This implies that $Q^{-1}_{ut} X_{i,t+1} \approx \beta^{-1}(\Delta q_{i,t+1} - \Delta q_{u,t+1} + r_{i,t+1} - r_{u,t+1})$, since $Q_u = 1$ in the steady state. Taking the double limit $(\sigma_z, \sigma_u, \sigma_\gamma) \longrightarrow (0,0,0)$ and $(\Gamma\sigma_z^2, \Gamma\sigma_u^2, \Gamma\sigma_\gamma^2) \longrightarrow (\Gamma_z, \Gamma_u, \Gamma_\gamma)$ and using $\Lambda^{\text{ave}}_{t+1} \approx \beta(1 + \lambda^{\text{ave}}_{t+1})$, rewrite the optimal choice as

$$\begin{aligned} L_i + l_{i,t+1} \approx{}& L(1 + m_{t+1}) + \chi \mathbb{E}_t[\Delta q_{i,t+1} - \Delta q_{u,t+1} + r_{it} - r_{ut}] \\ &+ (1 + \gamma_t)\chi \cdot \lim_{\Gamma \to \infty,\ (\sigma_u, \sigma_z, \sigma_\gamma) \to 0} \Gamma \mathbb{C}_t[\lambda^{\text{ave}}_{t+1}, \Delta q_{i,t+1} - \Delta q_{u,t+1}] \end{aligned} \tag{A.12}$$

Here we use the fact that

$$L = \int M_i di \tag{A.13}$$

and the fact that $m_{i,t+1} \equiv m_{t+1}$: all countries have the same log-deviations of dollar assets. In the zeroth order we have

$$L_i = L + \chi \mathcal{C}_i \tag{A.14}$$

Here

$$\mathcal{C}_i \equiv \lim_{\Gamma \to \infty,\ (\sigma_u, \sigma_z, \sigma_\gamma) \to 0} \Gamma \mathbb{C}_t[\lambda^{\text{ave}}_{t+1}, \Delta q_{i,t+1} - \Delta q_{u,t+1}] \tag{A.15}$$

In the first order, we have

$$l_{i,t+1} = L m_{t+1} + \chi \mathbb{E}_t[\Delta q_{i,t+1} - \Delta q_{u,t+1} + r_{i,t+1} - r_{u,t+1}] + \gamma_t \chi \mathcal{C}_i \tag{A.16}$$

To eliminate the interest rates, observe that Euler equations linearize into

$$r_{i,t+1} = \rho \mathbb{E}_t[\Delta c_{i,t+1}] \tag{A.17}$$

From the condition that $\alpha Q_{it} C_{it} = P_{it} N_{it}$ and $P_{it} = Q_{it}^{\frac{1}{\alpha}}$, we get

$$c_{it} = \frac{1 - \alpha}{\alpha} q_{it} + x_{it} \tag{A.18}$$

This implies

$$l_{i,t+1} = L m_{t+1} + \chi \varphi \mathbb{E}_t[\Delta q_{i,t+1} - \Delta q_{u,t+1}] + \chi \rho \mathbb{E}_t[\Delta x_{i,t+1} - \Delta x_{u,t+1}] + \chi \mathcal{C}_i \gamma_t \tag{A.19}$$

Here the composite parameter φ is

$$\varphi = 1 + \frac{\rho(1-\alpha)}{\alpha} \tag{A.20}$$

This completes the proof. □

Proof of Lemma 3. Consider the within-period optimality conditions:

$$C_{it}^T = (1-\alpha) Q_{it} C_{it} \tag{A.21}$$

$$P_{it}^N C_{it}^N = \alpha Q_{it} C_{it} \tag{A.22}$$

Using $Q_{it} = (P_{it}^N)^\alpha$ and log-linearizing,

$$c_{it}^T = q_{it} + c_{it} \tag{A.23}$$

$$c_{it}^N = \frac{\alpha - 1}{\alpha} q_{it} + c_{it} \tag{A.24}$$

From this and $c_{it}^N = x_{it}$, it follows that

$$c_{it} = \frac{1-\alpha}{\alpha} q_{it} + x_{it} \tag{A.25}$$

$$c_{it}^T = \frac{1}{\alpha} q_{it} + x_{it} \tag{A.26}$$

Now consider the log-linearization of $\Lambda_{i,t+1}$ and $\Lambda_{u,t+1}$:

$$\lambda_{i,t+1} = (1-\rho)\Delta c_{i,t+1} - \Delta c_{i,t+1}^T = -\left(1 + \frac{\rho(1-\alpha)}{\alpha}\right) \Delta q_{i,t+1} - \rho \Delta x_{i,t+1} \tag{A.27}$$

$$\lambda_{u,t+1} = (1-\rho)\Delta c_{u,t+1} - \Delta c_{u,t+1}^T = -\left(1 + \frac{\rho(1-\alpha)}{\alpha}\right) \Delta q_{u,t+1} - \rho \Delta x_{u,t+1} \tag{A.28}$$

Denote $\varphi = 1 + (1-\alpha)\rho/\alpha$. Integrating,

$$\begin{aligned} \lambda_{t+1}^{\text{ave}} &= -\varphi \left(\Delta q_{u,t+1} + \int \Delta q_{i,t+1} \mathrm{d}i \right) - \rho \Delta x_{u,t+1} - \rho \int \Delta x_{i,t+1} \mathrm{d}i \\ &= -\varphi((e+e_u)\alpha \Delta z_{t+1} - \alpha \Delta x_{u,t+1}) - \rho \Delta x_{u,t+1} = -(e+e_u)\alpha\varphi \Delta z_{t+1} - (\rho - 1)\alpha \Delta x_{u,t+1} \end{aligned} \tag{A.29}$$

This uses the fact that $q_{ut} + \int q_{it} \mathrm{d}i = (e+e_u)\alpha z_t - \alpha x_{ut}$, which follows from Lemma 1. This completes the proof. □

Auxiliary results. We next provide some auxiliary results that completely characterize equilibrium. We solve for a linear equilibrium of the form

$$q_{ut} = \theta_u z_t + \xi_u x_{ut} + \mu_u \gamma_t + \omega_u l_{ut} + \zeta_u m_t \tag{A.30}$$
$$q_{it} = \theta_i z_t + \xi_i x_{ut} + \mu_i \gamma_t + \omega_i l_{it} + \zeta_i m_t + \eta_i x_{it} + \delta_i l_{ut} \tag{A.31}$$
$$l_{u,t+1} = D_{l,z} z_t + D_{l,x} x_{ut} + D_{l,\gamma} \gamma_t + D_{l,l} l_{ut} + D_{l,m} m_t \tag{A.32}$$
$$m_{t+1} = D_{m,z} z_t + D_{m,x} x_{ut} + D_{m,\gamma} \gamma_t + D_{m,l} l_{ut} + D_{m,m} m_t \tag{A.33}$$
$$l_{i,t+1} = D_{l,i} l_{it} + D_{l,x,i} x_{it} + D_{l,z,i} z_t + D_{l,u,i} x_{ut} + D_{l,\gamma,i} \gamma_t + D_{l,l,i} l_{ut} + D_{l,m,i} m_t \tag{A.34}$$
$$\tag{A.35}$$

These conjectures imply

$$\mathcal{C}_i = -(\theta_i - \theta_u)(e + e_u)\alpha\varphi\Gamma_z - (\xi_i - \xi_u)(\rho - 1)\alpha\Gamma_u \tag{A.36}$$

Here the notation is

$$\Gamma_z \equiv \lim_{\Gamma \to \infty, \sigma_z \to 0} \Gamma \sigma_z^2 \tag{A.37}$$
$$\Gamma_u \equiv \lim_{\Gamma \to \infty, \sigma_u \to 0} \Gamma \sigma_u^2 \tag{A.38}$$

Lemma 4 first describes the globally important variables: the US price level, the evolution of the US debt, and the evolution of reserves. Lemma 5 then describes the dynamics of country-specific price levels q_{it} and external debt l_{it}. Proposition 7 shows the stationarity properties of exchange rates. After solving the general case, we consider specific parts of the parameter space that deliver tractable results.

LEMMA 4. *The US price level coefficients in the linear equilibrium are*

$$\theta_u = \alpha \cdot \frac{e_u - \alpha\chi\varphi\tilde{\omega}(1 - \rho_z)(e + e_u)}{1 - 2\alpha\chi\varphi\tilde{\omega}(1 - \rho_z)} \tag{A.39}$$
$$\xi_u = -\alpha \cdot \frac{1 - \chi\tilde{\omega}(\rho + \alpha\varphi)(1 - \rho_u)}{1 - 2\chi\tilde{\omega}\alpha\varphi(1 - \rho_u)} \tag{A.40}$$
$$\mu_u = -\frac{\alpha L_u \tilde{\omega}}{1 - 2\alpha\chi\varphi\tilde{\omega}(1 - \rho_\gamma)} \tag{A.41}$$
$$\omega_u = \alpha\tilde{\omega} \tag{A.42}$$
$$\zeta_u = 0 \tag{A.43}$$

Here $\tilde{\omega}$ is the negative root of $2\alpha\chi\varphi\tilde{\omega}^2 - \tilde{\omega} - 1 = 0$. The evolution of the US debt follows

$$D_{l,z} = (e_u - e) \cdot \frac{\alpha\chi\varphi\tilde{\omega}(1-\rho_z)}{1 - 2\alpha\chi\varphi\tilde{\omega}(1-\rho_z)} \tag{A.44}$$

$$D_{l,x} = \frac{\chi\tilde{\omega}(\rho - \alpha\varphi)(1-\rho_u)}{1 - 2\alpha\chi\varphi\tilde{\omega}(1-\rho_u)} \tag{A.45}$$

$$D_{l,l} = \tilde{\omega} + 1 \tag{A.46}$$

$$D_{l,\gamma} = \mu_u/\alpha \tag{A.47}$$

$$D_{l,m} = 0 \tag{A.48}$$

The coefficients in the evolution of reserves are

$$D_{m,z} = \frac{\alpha\tau(e - e_u)}{1 - 2\alpha\chi\varphi\tilde{\omega}(1-\rho_z)} \tag{A.49}$$

$$D_{m,x} = \alpha\tau \cdot \frac{1 - 2\rho\chi\tilde{\omega}(1-\rho_u)}{1 - 2\alpha\varphi\chi\tilde{\omega}(1-\rho_u)} \tag{A.50}$$

$$D_{m,l} = -2\tau\omega_u \tag{A.51}$$

$$D_{m,\gamma} = -2\tau\mu_u \tag{A.52}$$

$$D_{m,m} = 0 \tag{A.53}$$

Proof of Lemma 4. Integrating equation (A.19) and using $q_{ut} + q_t = (e + e_u)\alpha z_t - \alpha x_{ut}$,

$$\begin{aligned} l_{u,t+1} &= \chi\varphi\mathbb{E}_t[\Delta q_{u,t+1} - \Delta q_{t+1}] + \chi\rho\mathbb{E}_t[\Delta x_{u,t+1}] + L_u\gamma_t \\ &= \chi\varphi\mathbb{E}_t[2\Delta q_{u,t+1} - (e+e_u)\alpha\Delta z_{t+1}] + \chi(\rho+\alpha\varphi)\mathbb{E}_t[\Delta x_{u,t+1}] + L_u\gamma_t \\ &= \chi\varphi(2\theta_u - \alpha(e+e_u))\mathbb{E}_t[\Delta z_{t+1}] + \chi(2\varphi\xi_u + \rho + \alpha\varphi)\mathbb{E}_t[\Delta x_{u,t+1}] + 2\chi\varphi\mu_u\mathbb{E}_t[\Delta\gamma_{t+1}] \\ &+ L_u\gamma_t + 2\chi\varphi\omega_u\Delta l_{u,t+1} + 2\chi\varphi\zeta_u\Delta m_{t+1} \end{aligned} \tag{A.54}$$

This uses the fact that integrating equation (A.14) leads to $L_u = -\chi\int \mathcal{C}_i \mathrm{d}i$. Reorganizing,

$$\begin{aligned} (\alpha - 2\alpha\chi\varphi\omega_u)\Delta l_{u,t+1} &= 2\alpha\chi\varphi\zeta_u\Delta m_{t+1} - \alpha l_{ut} + (\alpha L_u + 2\chi\varphi\alpha\mu_u(\rho_\gamma - 1))\gamma_t \\ &+ \alpha\chi\varphi(\rho_z - 1)(2\theta_u - \alpha(e+e_u))z_t + \alpha\chi(\rho_u - 1)(2\varphi\xi_u + \rho + \alpha\varphi)x_{ut} \end{aligned} \tag{A.55}$$

Plugging this into equation (A.7), we can see that $\zeta_u = 0$, and ω_u satisfies

$$\omega_u = -\frac{\alpha^2}{\alpha - 2\alpha\chi\varphi\omega_u} \tag{A.56}$$

Define $\tilde{\omega}$ by $\omega_u = \alpha\tilde{\omega}$. This solves

$$2\alpha\chi\varphi\tilde{\omega}^2 - \tilde{\omega} - 1 = 0 \tag{A.57}$$

This implies

$$\alpha - 2\alpha\chi\varphi\omega_u = -\frac{\alpha}{\tilde{\omega}} \tag{A.58}$$

Using this to to reorganize equation (A.55),

$$\begin{aligned}\alpha\Delta l_{u,t+1} = \omega_u l_{ut} - (\alpha L_u + 2\chi\varphi\alpha\mu_u(\rho_\gamma - 1))\tilde{\omega}\gamma_t \\ - \alpha\chi\varphi\tilde{\omega}(\rho_z - 1)(2\theta_u - \alpha(e + e_u))z_t - \alpha\chi\tilde{\omega}(\rho_u - 1)(2\varphi\xi_u + \rho + \alpha\varphi)x_{ut}\end{aligned} \tag{A.59}$$

This implies

$$\theta_u = \alpha \cdot \frac{e_u - \alpha\chi\varphi\tilde{\omega}(1-\rho_z)(e+e_u)}{1 - 2\alpha\chi\varphi\tilde{\omega}(1-\rho_z)} \tag{A.60}$$

$$\xi_u = -\alpha \cdot \frac{1 - \chi\tilde{\omega}(\rho + \alpha\varphi)(1-\rho_u)}{1 - 2\chi\tilde{\omega}\alpha\varphi(1-\rho_u)} \tag{A.61}$$

$$\mu_u = -\frac{\alpha L_u \tilde{\omega}}{1 - 2\alpha\chi\varphi\tilde{\omega}(1-\rho_\gamma)} \tag{A.62}$$

For $\alpha\Delta l_{u,t+1}$, we now have

$$\alpha\Delta l_{u,t+1} = \omega_u l_{ut} + \mu_u\gamma_t + \frac{\alpha^2\chi\varphi\tilde{\omega}(e_u - e)(1-\rho_z)}{1 - 2\alpha\chi\varphi\tilde{\omega}(1-\rho_z)} z_t + \frac{\alpha\chi\tilde{\omega}(\rho - \alpha\varphi)(1-\rho_u)}{1 - 2\alpha\chi\varphi\tilde{\omega}(1-\rho_u)} x_{ut} \tag{A.63}$$

This implies

$$D_{l,z} = (e_u - e) \cdot \frac{\alpha\chi\varphi\tilde{\omega}(1-\rho_z)}{1 - 2\alpha\chi\varphi\tilde{\omega}(1-\rho_z)} \tag{A.64}$$

$$D_{l,x} = \frac{\chi\tilde{\omega}(\rho - \alpha\varphi)(1-\rho_u)}{1 - 2\alpha\chi\varphi\tilde{\omega}(1-\rho_u)} \tag{A.65}$$

$$D_{l,l} = 1 + \tilde{\omega} \tag{A.66}$$

$$D_{l,\gamma} = \frac{\mu_u}{\alpha} \tag{A.67}$$

Recall that

$$m_{t+1} = \tau(q_t - q_{ut}) = (\alpha(e + e_u) - 2\theta_u)\tau z_t - (\alpha + 2\xi_u)\tau x_{ut} - 2\tau\mu_u\gamma_t - 2\tau\omega_u l_{ut} \tag{A.68}$$

The implication is that

$$D_{m,z} = \frac{\alpha\tau(e - e_u)}{1 - 2\alpha\chi\varphi\tilde{\omega}(1-\rho_z)} \tag{A.69}$$

$$D_{m,x} = \alpha\tau \cdot \frac{1 - 2\rho\chi\tilde{\omega}(1-\rho_u)}{1 - 2\alpha\varphi\chi\tilde{\omega}(1-\rho_u)} \tag{A.70}$$

$$D_{m,l} = -2\tau\omega_u \tag{A.71}$$

$$D_{m,\gamma} = -2\tau\mu_u \tag{A.72}$$

This completes the proof. □

LEMMA 5. *Country-specific price level coefficients in the linear equilibrium are*

$$\theta_i = \frac{\alpha e_i - \alpha\varphi\chi\omega\theta_u(1-\rho_z) + \alpha\omega D_{m,z}(M_i - L) - \alpha\varphi\chi\omega D_{l,z}(\delta_i - \omega_u)}{1 - \alpha\varphi\chi\omega(1-\rho_z)} \tag{A.73}$$

$$\xi_i = \frac{\alpha\omega D_{m,x}(M_i - L) - \alpha\varphi\chi\omega D_{l,x}(\delta_i - \omega_u) - \alpha\chi\omega(\rho + \varphi\xi_u)(1-\rho_u)}{1 - \alpha\varphi\chi\omega(1-\rho_u)} \tag{A.74}$$

$$\mu_i = -\frac{\alpha\omega\chi\mathcal{C}_i + \mu_u\alpha\varphi\chi\omega(1-\rho_\gamma) + 2\alpha\omega\tau\mu_u(M_i - L) + \alpha\varphi\chi\omega\tilde{\mu}(\delta_i - \omega_u)}{1 - \alpha\varphi\chi\omega(1-\rho_\gamma)} \tag{A.75}$$

$$\omega_i = \alpha\omega \tag{A.76}$$

$$\zeta_i = -\alpha\omega M_i \tag{A.77}$$

$$\eta_i = -\alpha\frac{1 - \chi\omega\rho(1-\rho_x)}{1 - \alpha\chi\varphi\omega(1-\rho_x)} \tag{A.78}$$

$$\delta_i = \omega_u \cdot \frac{\alpha\varphi\chi\omega\tilde{\omega} - 2\alpha\omega\tau(M_i - L)}{\alpha\varphi\chi\omega\tilde{\omega} + 1} \tag{A.79}$$

Here ω is the negative root of $\alpha\chi\varphi\omega^2 - \omega - 1 = 0$ and $\tilde{\mu}$ is defined as $\mu_u = \alpha\tilde{\mu}$. The coefficients in the evolution of country-specific debt are

$$D_{l,i} = 1 + \omega \tag{A.80}$$

$$D_{l,x,i} = -\frac{\chi\omega(\rho - \alpha\varphi)(1-\rho_x)}{1 - \alpha\varphi\chi\omega(1-\rho_x)} \tag{A.81}$$

$$D_{l,z,i} = \frac{\theta_i - \alpha e_i + \alpha M_i D_{m,z}}{\alpha} \tag{A.82}$$

$$D_{l,u,i} = \frac{\xi_i + \alpha M_i D_{m,x}}{\alpha} \tag{A.83}$$

$$D_{l,\gamma,i} = \frac{\mu_i + \alpha M_i D_{m,\gamma}}{\alpha} \tag{A.84}$$

$$D_{l,l,i} = \frac{\delta_i + \alpha M_i D_{m,l}}{\alpha} \tag{A.85}$$

$$D_{l,m,i} = -(\omega + 1)M_i \tag{A.86}$$

Proof of Lemma 5. Plugging the linear conjecture into equation (A.19),

$$\begin{aligned}(\alpha - \alpha\chi\varphi\omega_i)\Delta l_{i,t+1} = {} & \alpha L m_{t+1} + \alpha\chi\varphi(\theta_i - \theta_u)(\rho_z - 1)z_t - \alpha\chi(\rho - \varphi(\xi_i - \xi_u))(\rho_u - 1)x_{ut} \\ & + \alpha\chi(\varphi\eta_i + \rho)(\rho_x - 1)x_{it} + \alpha\chi\varphi\zeta_i\Delta m_{t+1} + \alpha\chi\varphi(\delta_i - \omega_u)\Delta l_{u,t+1} \\ & + (\alpha\chi\varphi(\mu_i - \mu_u)(\rho_\gamma - 1) + \alpha(L_i - L))\gamma_t - \alpha l_{it}\end{aligned} \tag{A.87}$$

Plugging this back into the linear conjecture, we see that ω_i satisfies

$$\omega_i = -\frac{\alpha^2}{\alpha - \alpha\chi\varphi\omega_i} \tag{A.88}$$

Define ω by $\omega_i = \alpha\omega$. This parameter solves

$$\alpha\chi\varphi\omega^2 - \omega - 1 = 0 \tag{A.89}$$

Using this, we can reorganize equation (A.87):

$$\begin{aligned}\alpha\Delta l_{i,t+1} = {} & \omega_i l_{it} - \alpha\omega L m_{t+1} - \alpha\chi\varphi\omega\zeta_i\Delta m_{t+1} - \alpha\chi\varphi\omega(\delta_i - \omega_u)\Delta l_{u,t+1} \\ & + \alpha\chi\varphi\omega(\theta_i - \theta_u)(1-\rho_z)z_t + \alpha\chi\omega(\varphi(\xi_i - \xi_u) - \rho)(1-\rho_u)x_{ut} + \alpha\chi\omega(\varphi\eta_i + \rho)(1-\rho_x)x_{it} \\ & + (\alpha\chi\varphi\omega(1-\rho_\gamma)(\mu_i - \mu_u) - \alpha\omega\chi\mathcal{C}_i)\gamma_t \end{aligned} \tag{A.90}$$

Collecting the coefficients on m_t in the linear conjecture,

$$\zeta_i = \alpha\chi\varphi\omega\zeta_i + \alpha M_i \tag{A.91}$$

This means $\zeta_i = -\alpha\omega M_i$. Collecting the coefficients on m_{t+1} in the linear conjecture,

$$-\alpha\omega L - \alpha\chi\varphi\omega\zeta_i - \alpha M_i = \alpha\omega(M_i - L) \tag{A.92}$$

Collecting the terms on x_{it},

$$\eta_i = -\alpha\frac{1 - \chi\omega\rho(1-\rho_x)}{1 - \alpha\chi\varphi\omega(1-\rho_x)} \tag{A.93}$$

This means

$$\begin{aligned} q_{it} = {} & \omega_i l_{it} + \zeta_i m_t + \eta_i x_{it} + \alpha\omega(M_i - L)m_{t+1} - \alpha\chi\varphi\omega(\delta_i - \omega_u)\Delta l_{u,t+1} \\ & + [\alpha e_i + \alpha\chi\varphi\omega(\theta_i - \theta_u)(1-\rho_z)]z_t + \alpha\chi\omega(\varphi(\xi_i - \xi_u) - \rho)(1-\rho_u)x_{ut} \\ & + (\alpha\chi\varphi\omega(1-\rho_\gamma)(\mu_i - \mu_u) - \alpha\omega\chi\mathcal{C}_i)\gamma_t \end{aligned} \tag{A.94}$$

For δ_i, we have

$$\delta_i = \omega_u \cdot \frac{\alpha\varphi\chi\omega\tilde{\omega} - 2\alpha\omega\tau(M_i - L)}{\alpha\varphi\chi\omega\tilde{\omega} + 1} \tag{A.95}$$

For θ_i and ξ_i, we have

$$\theta_i = \frac{\alpha e_i - \alpha\varphi\chi\omega\theta_u(1-\rho_z) + \alpha\omega D_{m,z}(M_i - L) - \alpha\varphi\chi\omega D_{l,z}(\delta_i - \omega_u)}{1 - \alpha\varphi\chi\omega(1-\rho_z)} \tag{A.96}$$

$$\xi_i = \frac{\alpha\omega D_{m,x}(M_i - L) - \alpha\varphi\chi\omega D_{l,x}(\delta_i - \omega_u) - \alpha\chi\omega(\rho + \varphi\xi_u)(1-\rho_u)}{1 - \alpha\varphi\chi\omega(1-\rho_u)} \tag{A.97}$$

Finally, for μ_i, we have

$$\begin{aligned} \mu_i &= -\frac{\alpha\omega\chi\mathcal{C}_i + \mu_u\alpha\varphi\chi\omega(1-\rho_\gamma) + 2\alpha\omega\tau\mu_u(M_i - L) + \alpha\varphi\chi\omega\tilde{\mu}(\delta_i - \omega_u)}{1 - \alpha\varphi\chi\omega(1-\rho_\gamma)} \\ &= \mu_u - \frac{\alpha\omega\chi\mathcal{C}_i + \mu_u(1 + 2\alpha\omega\tau(M_i - L)) + \alpha\varphi\chi\omega\tilde{\mu}(\delta_i - \omega_u)}{1 - \alpha\varphi\chi\omega(1-\rho_\gamma)} \\ &= \mu_u - \frac{\alpha\omega\chi\mathcal{C}_i}{1 - \alpha\varphi\chi\omega(1-\rho_\gamma)} - \frac{\mu_u}{1 - \alpha\varphi\chi\omega(1-\rho_\gamma)} \cdot \frac{1 + 2\alpha\omega\tau(M_i - L)}{1 + \alpha\varphi\omega\chi\tilde{\omega}} \end{aligned} \tag{A.98}$$

To characterize the evolution of l_{it}, use equation (A.6):

$$\alpha l_{i,t+1} = \alpha l_{it} + q_{it} - \alpha e_i z_t + \alpha x_{it} + \alpha M_i m_{t+1} - \alpha M_i m_t \tag{A.99}$$

This implies

$$D_{l,z,i} = \frac{\theta_i - \alpha e_i + \alpha M_i D_{m,z}}{\alpha} \tag{A.100}$$
$$D_{l,u,i} = \frac{\xi_i + \alpha M_i D_{m,x}}{\alpha} \tag{A.101}$$
$$D_{l,\gamma,i} = \frac{\mu_i + \alpha M_i D_{m,\gamma}}{\alpha} \tag{A.102}$$
$$D_{l,i} = \omega + 1 \tag{A.103}$$
$$D_{l,m,i} = -(\omega + 1) M_i \tag{A.104}$$
$$D_{l,l,i} = \frac{\delta_i + \alpha M_i D_{m,l}}{\alpha} \tag{A.105}$$
$$D_{l,x,i} = \frac{\alpha + \eta_i}{\alpha} = -\frac{\chi\omega(\rho - \alpha\varphi)(1 - \rho_x)}{1 - \alpha\varphi\chi\omega(1 - \rho_x)} \tag{A.106}$$

This completes the proof. □

PROPOSITION 7. *Endogenous states (l_{ut}, l_{it}, m_t) evolve as*

$$\begin{aligned} l_{u,t+1} &= (1 + \tilde{\omega}) l_{ut} + F_u(z_t, \gamma_t, x_{ut}) \\ l_{i,t+1} &= (1 + \omega) l_{it} + F_i(z_t, \gamma_t, x_{ut}, x_{it}, l_{ut}, m_t) \\ m_{t+1} &= F_m(z_t, \gamma_t, x_{ut}, l_{ut}) \end{aligned}$$

Here the coefficients $\tilde{\omega}$ and ω satisfy $-1 < \omega < \tilde{\omega} < 0$ and both converge to zero if $\chi \longrightarrow \infty$.

Proof of Proposition 7. Directly follows from the statements of Lemma 4 and Lemma 5. □

If exogenous processes are stationary, endogenous processes, including exchange rates, are stationary too. The reason is the portfolio management cost. The intermediary's portfolio gravitates towards a fixed benchmark, which makes the economy return to a stable distribution of investment positions in the long run. The US position has a persistence coefficient $1 + \tilde{\omega} \in (0, 1)$, every other country's position has persistence $1 + \omega \in (0, 1)$, and the global reserve position does not depend on its own past values. Inflows into the US are more persistent than inflows into other countries due to their feedback: they affect the US dollar exchange rate, which further drives financial flows. Both persistence coefficients become a unit root if portfolio frictions disappear. This relates our portfolio costs to stationarity-inducing devices in the spirit of Schmitt-Grohé and Uribe (2003).

Limiting case. We next characterize the limit of small portfolio costs and large reserve assets under $e_u = e$ and a different assumption on the persistence of risk aversion shocks: $\rho_\gamma = 0$. We assume $\rho_z = \rho_x = \rho_u = 1$ for tractability and introduce new notation for auxiliary variables:

$$\phi \equiv \sqrt{\alpha/\varphi} \tag{A.107}$$
$$\psi_i \equiv \phi\tau(m_i - m) - 1 \tag{A.108}$$

We prove the results that characterize bilateral exchange rate depreciation against the US for all countries, our currency factors, and loadings, in the limit $\chi \longrightarrow \infty$ when $M_i = \sqrt{\chi} m_i$.

Proof of Proposition 1. US coefficients are

$$\theta_u = \alpha e_u \tag{A.109}$$
$$\xi_u = -\alpha \tag{A.110}$$
$$\mu_u = \tilde{\omega}\omega_u L_u \tag{A.111}$$

Coefficients of individual countries are

$$\theta_i = \alpha e_i \tag{A.112}$$
$$\xi_i = \alpha^2 \omega\tau(M_i - L) \tag{A.113}$$
$$\eta_i = -\alpha \tag{A.114}$$
$$\zeta_i = -\alpha\omega M_i \tag{A.115}$$
$$\delta_i = \omega_u - \omega_u \cdot \frac{1 + 2\alpha\omega\tau(M_i - L)}{1 + \alpha\varphi\chi\omega\tilde{\omega}} \tag{A.116}$$
$$\mu_i = \mu_u - \frac{\mu_u}{1 - \alpha\varphi\chi\omega} \cdot \frac{1 + 2\alpha\omega\tau(M_i - L)}{1 + \alpha\varphi\chi\omega\tilde{\omega}} - \frac{\alpha\omega\chi\mathcal{C}_i}{1 - \alpha\varphi\chi\omega} \tag{A.117}$$

US debt coefficients are

$$D_{l,z} = 0 \tag{A.118}$$
$$D_{l,x} = 0 \tag{A.119}$$
$$D_{l,l} = 1 + \tilde{\omega} \tag{A.120}$$
$$D_{l,\gamma} = \tilde{\omega}^2 \tag{A.121}$$

Total reserve demand coefficients are

$$D_{m,z} = 0 \tag{A.122}$$
$$D_{m,x} = \alpha\tau \tag{A.123}$$
$$D_{m,l} = -2\tau\omega_u \tag{A.124}$$
$$D_{m,\gamma} = -2\tau\mu_u \tag{A.125}$$

Assume $\chi \longrightarrow \infty$ and $\chi^{-\frac{1}{2}} M_i \longrightarrow m_i$ for all i. In this limit,

$$\tilde{\omega} \sim -\frac{1}{\sqrt{2\alpha\varphi\chi}} \tag{A.126}$$
$$\omega \sim -\frac{1}{\sqrt{\alpha\varphi\chi}} \tag{A.127}$$

According to Lemma 3,

$$\mathcal{C}_i \longrightarrow \Gamma_z \alpha^2 \varphi (e + e_u)(e_u - e_i) + \Gamma_u \alpha^2 (\rho - 1)\psi_i \equiv c_i \tag{A.128}$$

$$\frac{L_u}{\chi} \longrightarrow \Gamma_u \alpha^2 (\rho - 1) \equiv l_u \tag{A.129}$$

The limits of the coefficients are

$$\chi^{-\frac{1}{2}} \omega_u \longrightarrow -\frac{\phi}{\sqrt{2}} \tag{A.130}$$

$$\mu_u \longrightarrow \frac{l_u}{2\varphi} \tag{A.131}$$

$$\theta_i \longrightarrow \alpha e_i \tag{A.132}$$

$$\xi_i \longrightarrow -\alpha \tau \phi (m_i - m) \tag{A.133}$$

$$\zeta_i \longrightarrow \phi m_i \tag{A.134}$$

$$\chi^{-\frac{1}{2}} \omega_i \longrightarrow -\phi \tag{A.135}$$

$$\chi^{-\frac{1}{2}} \delta_i \longrightarrow -\frac{\phi}{\sqrt{2}} \cdot \frac{1 + 2\sqrt{2}\tau\phi(m_i - m)}{1 + \sqrt{2}} \tag{A.136}$$

$$\mu_i \longrightarrow \mu_u + \frac{\mathcal{C}_i}{\varphi} \tag{A.137}$$

It follows that $\Delta l_{u,t+1} = O(1)$. Now consider $\alpha \Delta l_{i,t+1}$ and $\alpha M_i \Delta m_{t+1}$:

$$\begin{aligned} \alpha l_{i,t+1} = {} & \alpha(1+\omega) l_{it} + (\alpha^2 \tau M_i + \alpha^2 \omega \tau (M_i - L)) x_{ut} - \alpha(\omega + 1) M_i m_t + (\mu_i - 2\alpha\tau\mu_u M_i)\gamma_t \\ & + (\delta_i - 2\tau\omega_u M_i) l_{ut} \end{aligned} \tag{A.138}$$

$$\alpha M_i \Delta m_{t+1} = \alpha^2 \tau M_i x_{ut} - 2\alpha\tau\mu_u M_i \gamma_t - 2\alpha\tau\omega_u M_i l_{ut} - \alpha M_i m_t \tag{A.139}$$

From this, it follows that

$$\alpha \Delta l_{i,t+1} - \alpha M_i \Delta m_{t+1} = \alpha^2 \omega \tau (M_i - L) x_{ut} + \mu_i \gamma_t + \delta_i l_{ut} + \omega(\alpha l_{it} - \alpha M_i m_t) = O(1) \tag{A.140}$$

Plug this into q_{it}:

$$\begin{aligned} q_{it} &= \alpha e_i z_t - \alpha x_{it} + \alpha \Delta l_{i,t+1} - \alpha M_i \Delta m_{t+1} \\ &= \alpha e_i z_t - \alpha x_{it} + \alpha^2 \omega \tau (M_i - L) x_{ut} + \mu_i \gamma_t + \delta_i l_{ut} + \omega(\alpha l_{it} - \alpha M_i m_t) \qquad \text{(A.141)} \\ &= \alpha e_i z_t - \alpha x_{it} - \alpha \phi \tau (m_i - m) x_{ut} + \left(\frac{l_u}{2\varphi} + \frac{c_i}{\varphi} \right) \gamma_t + O(\chi^{-\frac{1}{2}}) \qquad \text{(A.142)} \end{aligned}$$

Now consider q_{ut}:

$$q_{ut} = \alpha e z_t - \alpha x_{ut} + \mu_u \gamma_t + \omega_u l_{ut} = \alpha e z_t - \alpha x_{ut} + \frac{l_u}{2\varphi} \gamma_t + O(\chi^{-\frac{1}{2}}) \tag{A.143}$$

It follows that

$$q_{it} - q_{ut} = (e_i - e_u)\alpha z_t - \alpha\psi_i x_{ut} - \alpha x_{it} + \frac{c_i}{\varphi}\gamma_t + O(\chi^{-\frac{1}{2}}) \tag{A.144}$$

Defining the depreciation of currency i against the dollar as $\Delta s_{i,t+1} = \Delta q_{u,t+1} - \Delta q_{i,t+1}$,

$$\Delta s_{i,t+1} \longrightarrow (e_u - e_i)\alpha\Delta z_{t+1} + \alpha\psi_i\Delta x_{u,t+1} + \alpha\Delta x_{i,t+1} + \kappa_i\Delta\gamma_{t+1} \tag{A.145}$$

Here κ_i is

$$\kappa_i \equiv -\frac{c_i}{\varphi} = \Gamma_z\alpha^2(e + e_u)(e_i - e_u) + \Gamma_u\alpha\phi^2(\rho - 1)(1 - \tau\phi(m_i - m)) \tag{A.146}$$

This completes the proof. □

Proof of Proposition 2. Assume that $e_u = e$ and that (e_i, m_i) are independent in the cross-section. Then, integrating $\Delta s_{i,t+1}$,

$$\text{doll}_{t+1} = -\alpha\Delta x_{u,t+1} + \Gamma_u\alpha\phi^2(\rho - 1)\Delta\gamma_{t+1} \tag{A.147}$$

Integrating over $e > \text{med}\{e\}$ and $e < \text{med}\{e\}$ and subtracting,

$$\text{comm}_{t+1} = (e_L - e_H)\alpha\Delta z_{t+1} - 2\alpha^2\Gamma_z(e_L - e_H)e\Delta\gamma_{t+1} \tag{A.148}$$

The expression for $\Delta s_{i,t+1}$ then follows from plugging these into Proposition 1. □

Proof of Proposition 3. Take $\Delta s_{ij,t+1} \equiv \Delta s_{i,t+1} - \Delta s_{j,t+1}$. According to Proposition 2,

$$\Delta s_{ij,t+1} = \frac{e_i - e_j}{e_H - e_L} \cdot \text{comm}_{t+1} + \tau\phi(m_j - m_i) \cdot \text{doll}_{t+1} + \alpha\Delta x_{i,t+1} - \alpha\Delta x_{j,t+1} \tag{A.149}$$

Integrating this over i,

$$\text{base}_{j,t+1} = \frac{e - e_j}{e_H - e_L} \cdot \text{comm}_{t+1} + \tau\phi(m_j - m) \cdot \text{doll}_{t+1} - \alpha\Delta x_{j,t+1} \tag{A.150}$$

Plugging this back,

$$\Delta s_{ij,t+1} = \text{base}_{j,t+1} + \frac{e_i - e}{e_H - e_L} \cdot \text{comm}_{t+1} + \tau\phi(m - m_i) \cdot \text{doll}_{t+1} + \alpha\Delta x_{i,t+1} \tag{A.151}$$

This completes the proof. □

Proof of Proposition 4. This functional form of the factor moments follows directly from Proposition 2 that expresses the factors in terms of the shocks. The mapping from $(\sigma_u^2, \sigma_z^2, (\Gamma\sigma_\gamma)^2)$ to the second moments is not trivially invertible because it is not linear. However, we can still

express these parameters:

$$\sigma_z^2 = \frac{V_c V_d - C_{cd}^2}{\alpha^2(e_H - e_L)^2 V_d + 2\alpha^2\varphi e(e_H - e_L)(\rho-1)^{-1}C_{cd}} \tag{A.152}$$

$$\sigma_u^2 = \frac{V_c V_d - C_{cd}^2}{\alpha^2 V_c + \alpha\phi^2(e_H - e_L)(\rho-1)(2e)^{-1}C_{cd}} \tag{A.153}$$

$$(\Gamma\sigma_\gamma)^2 = C_{cd} \cdot \frac{(\alpha V_c + \phi^2(e_H - e_L)(\rho-1)(2e)^{-1}C_{cd}) \cdot ((e_H - e_L)V_d + 2e\varphi(\rho-1)^{-1}C_{cd})}{4\phi^2 e(\rho-1)(V_c V_d - C_{cd}^2)^2} \tag{A.154}$$

Here the notation is $V_d = \mathbb{V}[\text{doll}_t]$, $V_c = \mathbb{V}[\text{comm}_t]$, and $C_{cd} = \mathbb{C}[\text{doll}_t, \text{comm}_t]$. □

Proof of Proposition 5. The proposition follows directly from Proposition 2. □

Proof of Proposition 6. Conditional expected excess returns on currency i and interest rate differentials are:

$$\begin{aligned}
\mathbb{E}_t[rx_{i,t+1}] &\equiv r_{i,t+1} - r_{u,t+1} - \mathbb{E}_t[\Delta s_{i,t+1}] = \rho\mathbb{E}_t[\Delta c_{i,t+1} - \Delta c_{u,t+1}] + \mathbb{E}_t[\Delta q_{i,t+1} - \Delta q_{u,t+1}] \\
&= \frac{\rho(1-\alpha)}{\alpha}\mathbb{E}_t[\Delta q_{i,t+1} - \Delta q_{u,t+1}] + \mathbb{E}_t[\Delta q_{i,t+1} - \Delta q_{u,t+1}] = \varphi\mathbb{E}_t[\Delta q_{i,t+1} - \Delta q_{u,t+1}] \\
&= -\varphi\mathbb{E}_t[\Delta s_{i,t+1}], \qquad \text{(A.155)} \\
r_{i,t+1} - r_{u,t+1} &= \rho\mathbb{E}_t[\Delta c_{i,t+1} - \Delta c_{u,t+1}] \\
&= -\frac{\rho(1-\alpha)}{\alpha}\mathbb{E}_t[\Delta s_{i,t+1}] \qquad \text{(A.156)}
\end{aligned}$$

Using the expression for the exchange rate depreciation,

$$\begin{aligned}
\mathbb{E}_t[rx_{i,t+1}] &= -\varphi \cdot \mathbb{E}_t[(e_u - e_i) \cdot \alpha\Delta z_{t+1} + (\tau\phi(m_i - m) - 1) \cdot \alpha\Delta x_{u,t+1} + \kappa_i \cdot \Delta\gamma_{t+1} + \alpha\Delta x_{i,t+1}] \\
&= \varphi\kappa_i\gamma_t, \qquad \text{(A.157)} \\
r_{i,t+1} - r_{u,t+1} &= \frac{\rho(1-\alpha)}{\alpha}\kappa_i\gamma_t. \qquad \text{(A.158)}
\end{aligned}$$

Here $\kappa_i = (e_i - e_u) \cdot (e + e_u)\alpha^2\Gamma_z + (1 - \tau\phi(m_i - m)) \cdot \alpha\phi^2\Gamma_u(\rho - 1)$, so

$$\mathbb{E}_t[rx_{i,t+1}] = [(e_i - e_u) \cdot (e + e_u)\varphi\alpha^2\Gamma_z + (1 - \tau\phi(m_i - m)) \cdot \alpha^2\Gamma_u(\rho-1)] \cdot \gamma_t \tag{A.159}$$

$$r_{i,t+1} - r_{u,t+1} = [(e_i - e_u) \cdot (e + e_u)\rho(1-\alpha)\alpha\Gamma_z + (1 - \tau\phi(m_i - m)) \cdot \rho(1-\alpha)\phi^2\Gamma_u(\rho-1)] \cdot \gamma_t \tag{A.160}$$

This completes the proof. □

A.2 A Commodity-Based Economy

In this section, we present a simple model of a production economy with countries that differ in their endowments of commodities. We show that this economy provides a microfoundation for the tradable endowment formulation we assume in the baseline model. In particular, we show that the country-specific endowments of commodities map into the tradable output exposure parameter e_i in our baseline model. Additionally, we show that the global tradable output fluctuations, z_t, are generated by commodity-biased productivity shocks. Finally, we also show the mapping between e_i

and the steady-state export share of commodities, a result we lever from in the empirical analysis.

Each country produces two final goods: tradable and non-tradable. It also exports non-differentiated commodities and uses labor. Non-tradable goods are simply endowed at the level Y_{it}^N. Tradable goods are produced by a representative firm using labor H_{it} and commodities X_{it}:

$$Y_{it}^T = z_i H_{it} + Z_t X_{it} \tag{A.161}$$

Here Z_t is the global productivity shock with $\mathbb{E}[Z_t] = 1$, and z_i is the country-specific labor productivity. The price of the commodity input is M_t, the wage is W_{it}. The firm solves

$$\max_{H_{it}, X_{it}} \; P_{it} Y_{it}^T - W_{it} H_{it} - M_t X_{it} \tag{A.162}$$

An interior solution implies that, for all i,

$$P_{it} Z_t = M_t \tag{A.163}$$

$$P_{it} z_i = W_{it} \tag{A.164}$$

Countries consume a Cobb-Douglas aggregator of their own non-tradable good and all tradable goods, putting a mass point on their own good:

$$\ln(C_{it}) = \theta \log(C_{it}^N) + (1-\theta)\alpha \log(C_{iit}^T) + (1-\theta)(1-\alpha) \int \log(C_{ijt}^T) dj - \nu \tag{A.165}$$

Here ν is a normalization constant:

$$\nu = \theta \log(\theta) + (1-\theta)\alpha \log((1-\theta)\alpha) + (1-\theta)(1-\alpha) \log((1-\theta)(1-\alpha)) \tag{A.166}$$

This leads to the following demand:

$$C_{it}^N = \theta \frac{Q_{it} C_{it}}{P_{it}^N} \tag{A.167}$$

$$C_{iit}^T = (1-\theta)\alpha \frac{Q_{it} C_{it}}{P_{it}^T} \tag{A.168}$$

$$C_{ijt}^T = (1-\theta)(1-\alpha) \frac{Q_{it} C_{it}}{P_{jt}^T} \tag{A.169}$$

Here the price index is

$$\log(Q_{it}) = \theta \log(P_{it}^N) + (1-\theta)\alpha \log(P_{it}^T) + (1-\theta)(1-\alpha) \int \log(P_{jt}^T) dj \tag{A.170}$$

Normalize

$$\int \log(P_{jt}^T) dj = 0 \tag{A.171}$$

Since we have $P_{it} Z_t = M_t$ for all i, which means P_{it} are the same for all i, we have $P_{it} = 1$ for all i. The individual country's price index is then $Q_{it} = (P_{it}^N)^\theta$. Moreover, $M_t = Z_t$.

Country i has a commodity endowment e_i. We parameterize it so that $e_i + z_i = 1$ for all i:

natural advantage in commodities in some countries is balanced out by a natural advantage in producing final goods in others. This will allow us to arrive at a symmetric steady state. Country i's consolidated budget constraint is

$$\begin{aligned} Q_{it}C_{it} = P_{it}^N Y_{it}^N + P_{it}^T Y_{it}^T + M_t(e_i - X_{it}) \\ + R_{it}Q_{it}B_{it} - Q_{it}B_{i,t+1} + T_{it} + \Pi_{it} \end{aligned} \tag{A.172}$$

Here $B_{i,t+1}$ is borrowing in local currency bonds for the next period, B_{it} is the debt at the start of this period, T_{it} are net transfers from the government, and Π_{it} are the profit rebates from the intermediaries. Using the fact that $P_{it}^N Y_{it}^N = \theta Q_{it}C_{it}$, the fact that $Q_{it} = (P_{it}^N)^\theta$, assuming $Y_{it}^N = \theta/(1-\theta)$, the fact that $P_{it}^T Y_{it}^T = z_i H_{it} + Z_t X_{it}$, and the fact that $M_t = Z_t$,

$$(Q_{it})^\theta = z_i + Z_t e_i + R_{it}Q_{it}B_{it} - Q_{it}B_{i,t+1} + T_{it} + \Pi_{it} \tag{A.173}$$

In the steady state, $Z_t = 1$. The profit rebate rule and reserve management rules from the baseline model imply $R_{it}Q_{it}B_{it} - Q_{it}B_{i,t+1} + T_{it} + \Pi_{it} = 0$. We also have $z_i + e_i = 1$. All of this implies $Q_i = 1$. To arrive at the steady-state export shares in commodities, use the market clearing for traded goods:

$$Y_{it}^T = C_{iit}^T + \int C_{jit}^T dj \tag{A.174}$$

This leads to the following expression:

$$Y_{it}^T = \alpha(1-\theta)Q_{it}C_{it} + (1-\alpha)(1-\theta)\int Q_{jt}C_{jt}dj \tag{A.175}$$

Since $Q_i = P_i^N = 1$ in the steady state, using $P_{it}^N Y_{it}^N = \theta Q_{it}C_{it}$ leads to $C_i = 1/(1-\theta)$, so $Y_i^T = 1$ for all i. The steady-state export share of commodities is then

$$S_i = \frac{e_i}{1+e_i} \tag{A.176}$$

Denoting $z_t = Z_t - 1$ and using $e_i + z_i = 1$, we can transform the budget constraint in equation (A.172) into

$$C_{it}^T + P_{it}^N C_{it}^N = 1 + e_i z_t + R_{it}Q_{it}B_{it} - Q_{it}B_{i,t+1} + T_{it} + \Pi_{it} \tag{A.177}$$

A.3 A Model with Mean-Variance Intermediaries

In this section we consider a variant of the baseline model in which global financial intermediaries have mean-variance preferences over returns. We show that this specification differs from the baseline in the implicit stochastic discount factor used by intermediaries, and gives rise to similar pricing of risks as the baseline model under certain parametric assumptions.

Suppose the global financial intermediary's objective function is given by

$$\mathbb{E}_t[\Pi_{t+1}] - \frac{\beta\Gamma_t}{2}\mathbb{V}_t[\Pi_{t+1}] - \frac{\beta^{-1}}{2\chi}\int (Q_{it}A_{i,t+1} - L_{t+1})^2 di \tag{A.178}$$

Here the profits Π_{t+1} are given by

$$\Pi_{t+1} = \int Q_{it} A_{i,t+1} X_{i,t+1} di \tag{A.179}$$

Excess returns are defined as before:

$$X_{i,t+1} = \frac{R_{i,t+1} Q_{i,t+1}}{Q_{it}} - \frac{R_{u,t+1} Q_{u,t+1}}{Q_{ut}} \tag{A.180}$$

The optimal choice of the intermediary is

$$Q_{it} A_{i,t+1} = L_{t+1} + \chi(\mathbb{E}_t[\beta X_{i,t+1}] - \Gamma_t \mathbb{C}_t[\beta \Pi_{t+1}, \beta X_{i,t+1}]) \tag{A.181}$$

This is reminiscent of the optimal choice in our baseline model:

$$Q_{it} A_{i,t+1} = L_{t+1} + \chi(\mathbb{E}_t[\Lambda^{\text{ave}}_{t+1}] \mathbb{E}_t[X_{i,t+1}] + \Gamma_t \mathbb{C}_t[\Lambda^{\text{ave}}_{t+1}, X_{i,t+1}]) \tag{A.182}$$

With mean-variance preferences, profits act as the opposite of the stochastic discount factor: individual returns are more valuable in states where the total profits are low.

Now make the substitution to the dollar value of debt: $Q_{ut} L_{i,t+1} = Q_{it} A_{i,t+1}$. In the first order, the intermediary's profits are

$$\Pi_{t+1} \approx \beta^{-1} \int L_i (\Delta q_{i,t+1} - \Delta q_{u,t+1} + r_{i,t+1} - r_{u,t+1}) di \tag{A.183}$$

Rewrite the optimal choice in terms of $L_{i,t+1}$:

$$L_{i,t+1} = \int M_{i,t+1} di + \chi(\mathbb{E}_t[\beta Q^{-1}_{ut} X_{i,t+1}] - \Gamma_t \mathbb{C}_t[\beta \Pi_{t+1}, \beta Q^{-1}_{ut} X_{i,t+1}]) \tag{A.184}$$

Use the fact that $Q^{-1}_{ut} X_{i,t+1} \approx \beta^{-1}(\Delta q_{i,t+1} - \Delta q_{u,t+1} + r_{i,t+1} - r_{u,t+1})$. Assuming that the intermediary uses the first-order approximation for profits Π_{t+1}, its optimal choice in the first order is

$$\begin{aligned} L_i + l_{i,t+1} = {} & L(1 + m_{t+1}) + \chi \mathbb{E}_t[\Delta q_{i,t+1} - \Delta q_{u,t+1} + r_{i,t+1} - r_{u,t+1}] \\ & - \chi \Gamma (1 + \gamma_t) \mathbb{C}_t \left[\int L_i (\Delta q_{i,t+1} - \Delta q_{u,t+1}) di, \Delta q_{i,t+1} - \Delta q_{u,t+1} \right] \end{aligned} \tag{A.185}$$

Here we use the fact that $m_{i,t+1} \equiv m_{t+1}$: every country chooses the same log-deviation of dollar assets. In the zeroth order,

$$L_i = L - \chi \hat{\mathcal{C}}_i \tag{A.186}$$

Here the object $\hat{\mathcal{C}}_i$ is the limiting risk premium on country i's currency:

$$\hat{\mathcal{C}}_i = \lim_{(\sigma_z, \sigma_u, \sigma_\gamma) \to 0, \Gamma \to \infty} \Gamma \mathbb{C}_t \left[\int L_i (\Delta q_{i,t+1} - \Delta q_{u,t+1}) di, \Delta q_{i,t+1} - \Delta q_{u,t+1} \right] \tag{A.187}$$

It is the direct analog of the limiting risk premium $\mathcal{C}_i$ in the baseline model. In the first order,

$$l_{i,t+1} = \chi\varphi\mathbb{E}_t[\Delta q_{i,t+1} - \Delta q_{u,t+1}] + \chi\rho\mathbb{E}_t[\Delta x_{i,t+1} - \Delta x_{u,t+1}] - \chi\hat{\mathcal{C}}_i\gamma_t + Lm_{t+1} \tag{A.188}$$

This result is the direct analog of Lemma 2. The only difference between this expression and that in the baseline model is the composition of the limiting risk premium $\hat{\mathcal{C}}_i$: using profits instead of the consumers' stochastic discount factors changes the pricing of risk.

Denote by β_i the vector of exposures of country i's exchange rate against the dollar to the three global shocks:

$$\beta_i = \begin{pmatrix} \theta_i - \theta_u \\ \xi_i - \xi_u \\ \mu_i - \mu_u \end{pmatrix} \tag{A.189}$$

Let $(\Gamma_z, \Gamma_u, \Gamma_\gamma)$ be the vector of limits of $(\Gamma\sigma_z^2, \Gamma\sigma_u^2, \Gamma\sigma_\gamma^2)$ and let $D_\Gamma = \text{diag}\{\Gamma_z, \Gamma_u, \Gamma_\gamma\}$. Use this notation to make progress on $\hat{\mathcal{C}}_i$:

$$\hat{\mathcal{C}}_i = \int L_j\beta_j' D_\Gamma \beta_i dj = \int (L - \chi\hat{\mathcal{C}}_j)\beta_j' D_\Gamma \beta_i dj = L\int \beta_j' dj D_\Gamma \beta_i - \chi\int \hat{\mathcal{C}}_j\beta_j' dj D_\Gamma \beta_i \tag{A.190}$$

This implies

$$\begin{aligned} \int \hat{\mathcal{C}}_j\beta_j' dj &= L\int \beta_j' dj D_\Gamma \int \beta_i\beta_i' di - \chi\int \hat{\mathcal{C}}_j\beta_j' dj D_\Gamma \int \beta_i\beta_i' di \\ &= L\int \beta_j' dj D_\Gamma \int \beta_i\beta_i' di \left(I + \chi D_\Gamma \int \beta_i\beta_i' di\right)^{-1} \end{aligned} \tag{A.191}$$

Plugging this back,

$$\begin{aligned} \hat{\mathcal{C}}_i &= L\int \beta_j' dj \left(I - \chi D_\Gamma \int \beta_j\beta_j' dj \left(I + \chi D_\Gamma \int \beta_i\beta_i' di\right)^{-1}\right) D_\Gamma\beta_i \\ &= L\int \beta_j' dj \left(I + \chi D_\Gamma \int \beta_i\beta_i' di\right)^{-1} D_\Gamma\beta_i \end{aligned} \tag{A.192}$$

Using $\mathbb{E}[\cdot]$ to denote the cross-sectional expectation,

$$\hat{\mathcal{C}}_i = L\cdot\mathbb{E}[\beta'](I + \chi D_\Gamma\mathbb{E}[\beta\beta'])^{-1} D_\Gamma\beta_i \tag{A.193}$$

This expression is reminiscent of the limiting risk premium in the baseline model, except instead of the exposures of the average SDF to the global shocks in Lemma 3, each currency's exposure β_i is weighted by $L\cdot\mathbb{E}[\beta'](I + \chi D_\Gamma\mathbb{E}[\beta\beta'])^{-1}$, which represents the exposure of the intermediary's profits. In fact, the mean-variance intermediary's SDF linearizes as follows:

$$\begin{aligned} \Pi_{t+1} &\approx \int L_i(\Delta q_{i,t+1} - \Delta q_{u,t+1})di = L\cdot\mathbb{E}[\beta'](I - (I + \chi D_\Gamma\mathbb{E}[\beta\beta'])^{-1}\chi D_\Gamma\mathbb{E}[\beta\beta'])\Delta\boldsymbol{y}_{t+1} \\ &= L\cdot\mathbb{E}[\beta'](I + \chi D_\Gamma\mathbb{E}[\beta\beta'])^{-1}\Delta\boldsymbol{y}_{t+1} \end{aligned} \tag{A.194}$$

Here $\Delta \boldsymbol{y}_{t+1} = (\Delta z_{t+1}\ \Delta x_{u,t+1}\ \Delta \gamma_{t+1})'$ is the vector of global shocks.

This SDF, in principle, loads on the financial shock γ_t, which is not the case in our baseline model, where the financial shock is purely redistributive. However, if the double limit $\Gamma \longrightarrow \infty$ and $(\sigma_z, \sigma_u, \sigma_\gamma) \longrightarrow (0,0,0)$ with $(\Gamma\sigma_z^2, \Gamma\sigma_u^2, \Gamma\sigma_\gamma^2) \longrightarrow (\Gamma_z, \Gamma_u, \Gamma_\gamma)$ features small financial shocks ($\Gamma_\gamma = 0$), then only $\theta_i - \theta_u$ and $\xi_i - \xi_u$ enter the expression for $\hat{\mathcal{C}}_i$. If $\rho_z = \rho_u = 1$, for these coefficients we have

$$\theta_i - \theta_u = \alpha(e_i - e_u) \tag{A.195}$$
$$\xi_i - \xi_u = \alpha + \alpha^2 \omega \tau (M_i - L) \tag{A.196}$$

For the cross-sectional expectations, we have

$$\mathbb{E}[\theta_i - \theta_u] = \alpha(e - e_u) \tag{A.197}$$
$$\mathbb{E}[\xi_i - \xi_u] = \alpha \tag{A.198}$$

If M_i and e_i are uncorrelated,

$$\mathbb{E}[(\theta_i - \theta_u)(\xi_i - \xi_u)] = \alpha^2(e - e_u) \tag{A.199}$$

With these moments, we can easily invert $I + \chi D_\Gamma \mathbb{E}[\beta\beta']$ and compute $L \cdot \mathbb{E}[\beta'](I + \chi D_\Gamma \mathbb{E}[\beta\beta'])^{-1}$. The linearized profits become

$$\begin{aligned}\Pi_{t+1} &\approx \frac{\alpha(e - e_u) \cdot L(1 + \chi\alpha^4\omega^2\tau^2\Gamma_u \mathbb{V}[M])}{\Delta}\Delta z_{t+1} + \frac{\alpha \cdot L(1 + \chi\alpha^2\Gamma_z \mathbb{V}[e])}{\Delta}\Delta x_{u,t+1} \\ &\equiv (e - e_u)\zeta_z \Delta z_{t+1} + \zeta_u \Delta x_{u,t+1}\end{aligned} \tag{A.200}$$

Here $\Delta > 0$ is

$$\begin{aligned}\Delta = 1 &+ \alpha^2\chi[\Gamma_u(1 + \alpha^2\omega^2\tau^2\mathbb{V}[M]) + \Gamma_z(\mathbb{V}[e] + (e - e_u)^2)] \\ &+ \alpha^4\chi^2\Gamma_z\Gamma_u(\mathbb{V}[e](1 + \alpha^2\omega^2\tau^2\mathbb{V}[M]) + (e - e_u)^2\alpha^2\omega^2\tau^2\mathbb{V}[M])\end{aligned} \tag{A.201}$$

This result is the direct analog of Lemma 3. Written side-by-side, the linearized average household SDF for Lemma 3 and the profits-based SDF of the mean-variance intermediary are

$$\lambda_{t+1}^{\text{ave}} = -(e + e_u)\alpha\varphi\Delta z_{t+1} - (\rho - 1)\alpha\Delta x_{u,t+1} \tag{A.202}$$
$$\lambda_{t+1}^{\text{profit}} = -(e - e_u)\zeta_z\Delta z_{t+1} - \zeta_u\Delta x_{u,t+1} \tag{A.203}$$

Here ζ_z and ζ_u are the positive coefficients from equation (A.200).

Two important observations are in order. First, the mean-variance intermediary is exposed to dollar risk in equilibrium. It has to absorb the demand for dollar assets from all regular countries and only partly passes it through to the US households. Risk aversion pushes the intermediary to match its dollar liabilities with US-originated dollar assets, but the portfolio cost pushes it to invest in regular countries. The outcome of this tradeoff is partial insurance against the dollar risk. Second, the intermediary's exposure to the commodity risk depends on the commodity intensity of the US relative to the world average. The intermediary's profit is only positively exposed to the global tradable output shock of $e_u < e$, which makes its dollar liabilities appreciate less in times of high z_t that its predominantly non-dollar assets. If $e_u = e$, the intermediary has no exposure

to the commodity risk in equilibrium. This is because it chooses a uniform portfolio of regular countries' bonds, since this portfolio carries no exposure to z_t, and hence choosing it fulfills both of the intermediary's objectives: minimize exposure to risk and approximate uniform asset weights.

B Empirical Appendix

B.1 Data description

- *Exchange rate*: The dependent variable in our empirical model equation (13) is constructed based on exchange rate contracts data from the London Stock Exchange (LSEG). We construct end-of-month bilateral exchange rates vis-a-vis the US dollar from daily to-USD spot exchange rate contracts, and then take the first difference of log exchange rate of local currency i per dollar.

- *Commodity factor*: We define high and low commodity producers based on countries' time-averaged annual share of raw materials in merchandise exports over 2015-2019, using data from the World Integrated Trade Solution (WITS). A country is categorized as a high commodity producer $i \in \mathcal{I}_{h,i}$, if its time-averaged share of raw materials in merchandise exports is above the median across countries in our sample, excluding the country itself; and as a low commodity producer $i \in \mathcal{I}_{l,i}$ if it is below the median. We then construct the country-specific commodity factor by differencing the average change in the log exchange rate of high commodity producers and that of low commodity producers as in equation (14).

- *Dollar factor*: We construct the currency-specific dollar factor as the cross-sectional-average of monthly change in the log exchange rates of all currencies against the dollar, excluding the currency itself as in equation (15).

- *Carry factor*: Following Verdelhan (2018), we first sort currencies in our sample, excluding the currency itself, into six portfolios based on their local-currency interest rates, which are approximated using the one-month forward discount rate. We then construct the currency-specific carry factor by taking the difference between the average change in the log exchange rate of the 6th portfolio (the one with the highest interest rates), and that of the first portfolio (the one with the lowest interest rates).

- *Trade-centrality factor*: We first construct the Richmond (2019) trade centrality measure as in equation (B.1), using bilateral trade data from the International Monetary Fund's (IMF) International Trade in Goods dataset (IMTS).

$$v_{it} = \sum_{j=1}^{N} \left(\frac{\widetilde{X}_{ijt} + \widetilde{X}_{jit}}{GDP_{it} + GDP_{jt}} \right) \tilde{s}_{jt} \tag{B.1}$$

  where v_{it} is the trade centrality of country i at time t; $\frac{\widetilde{X}_{ijt}+\widetilde{X}_{jit}}{GDP_{it}+GDP_{jt}}$ is the bilateral trade intensity between country i and j at time t, measured as the total bilateral exports scaled by the combined GDP; and $\tilde{s}_{jt}$ is country j's exports as a share of total trade across our sample countries at time t. We then construct the country-specific trade-centrality factor in a manner similar to the commodity factor, categorizing countries into high- or low-trade-centrality groups based on whether a country's 2015–2019 average trade centrality is above or below the median of the sample (excluding the country itself).

- *Tradable-output-sensitivity factor*: The dependent variable in the regression model equation (16) is based on data from the Organization for Economic Co-operation and Develop-

ment's (OECD) Inter-Country Input-Output tables (ICIO) (Note that the ICIO database does not include data for Albania or Uruguay from our sample). We categorize agriculture, mining, and manufacturing as the tradable sectors and deflate each country's annual tradable output (measured in USD) by U.S. CPI index. The independent variable is based on monthly commodity prices from the World Bank Commodity Price Data and we use the end-of-year total index. We estimate the empirical model in equation (16) over the 2000-2019 period to obtain $\beta_{i,\text{tos}}$ for each country i, representing its tradable output sensitivity. Finally, we construct the tradable-output-sensitivity factor in a manner similar to the commodity factor, categorizing countries into high- or low-sensitivity groups based on whether a country's β_{tos} is above or below the median of the sample (excluding the country itself).

- *Share of dollar-denominated assets*: We primarily use data from the IMF's Portfolio Investment Positions by Counterpart Economy (formerly CPIS) to construct each country's share of dollar-denominated assets. We calculate this share by dividing a country's USD-denominated portfolio assets by its total portfolio assets and using the 2015-2019 average. For countries not covered by the CPIS, we impute the share using the ratio of a country's holding of US Treasuries (from the Treasury International Capital (TIC) System) to its total international investment assets (from IMF's International Investment Position (IIP)).

- *Commodity incidence (or share of commodity exports)*: We use 2015-2019 average of a country's annual share of raw materials in merchandise exports.

- *U.S. non-tradable employment*: For the regressions in panel A of Table B.3, we use the first difference of HP-filtered log U.S. monthly non-tradable sector employment. We construct this series by subtracting total employees in goods-producing sectors from the total private-sector nonfarm employees, downloaded from the FRED.

- *Commodity price index*: For the regressions in panel B of Table B.3, we use log difference of the month Commodities for Index: All excluding Gold from the IMF Primary Commodity Price System.

- *VIX*: We use the end-of-month Cboe Volatility Index in Table B.3, constructed from daily closing values of the index from the Cboe.

- *Excess bond premium*: Following Gilchrist and Zakrajšek (2012), we use the first difference of monthly excess bond premium in Table B.3, provided by the Board of Governors of the Federal Reserve System.

- *Intermediary capital risk factor*: Following He et al. (2017), we use monthly intermediary capital risk factor in Table B.3, downloaded from Zhiguo He's website.

- *Trade centrality*: Following Richmond (2019), we use 2015-2019 average of a country's trade centrality measure as an additional control variable for specification (2) of Table B.5.

- *Trade openness*: We use 2015-2019 average of a country's total trade (imports and exports) scaled by its GDP as an additional control variable for specification (3) of Table B.5.

- *Country size*: Following Hassan (2013), we use 2015-2019 average of a country's log share of GDP in our sample as an additional control variable for specification (4) of Table B.5.

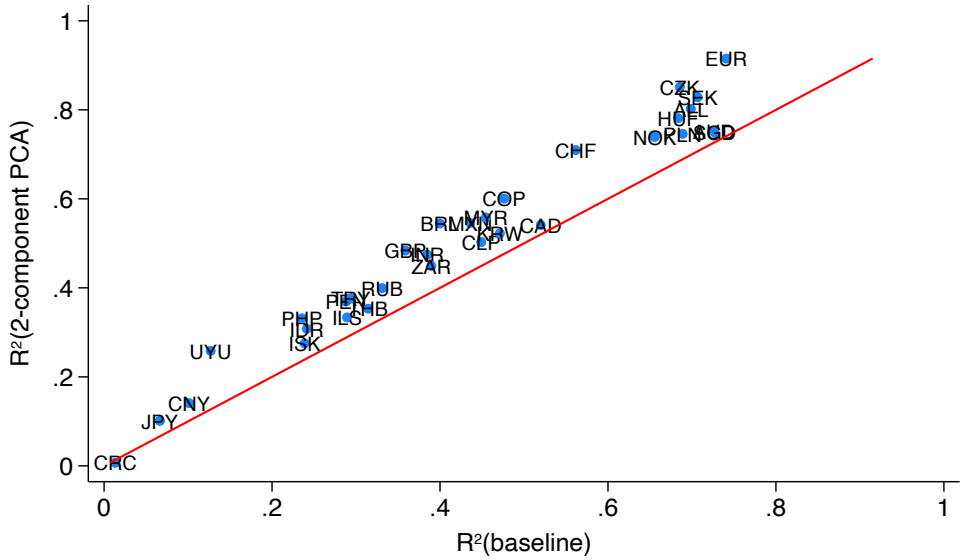
- *Distance*: we include log harmonic geographic distance to U.S. from the World Bank's World Development Indicators (WDI) as an additional control variable for specification (5) of Table B.5.

- *Gravity variables*: Following Lustig and Richmond (2020), we include a set of other gravity variables relative to U.S. as additional control variables in specification (6) of Table B.5 using data from the Research and Expertise on the World Economy (CEPII) Gravity dataset. These include: a dummy variable indicating whether country i shares a common language with the U.S.; a dummy variable for whether it shares a common legal origin; and a dummy variable for whether country i is contiguous with the U.S.

- *Net acquisition of USD assets*: We construct each country's net acquisition of USD-denominated assets using data from the IMF's Balance of Payments (BOP). We multiply each country's net financial assets (defined as the net acquisition of financial assets less net incurrence of liabilities) by our previously imputed share of dollar-denominated assets, and detrend the annual series over 2000-2019.

B.2 Additional figures and tables

Figure B.1: Explanatory Power of Factor Regression and Principal Components



Notes: This figure plots the R^2 from a two-principal-component model (y-axis) against the R^2 from the empirical model in equation (13) (x-axis) for each currency i. Both measures represent the fraction of variation in log exchange rate changes $\Delta s_{i,t+1}$ explained by the respective models. Each scatter point represents a currency, and the red line denotes the 45-degree line.

Table B.1: Factor Structure of Exchange Rates: Regression Estimates

Country	α	β_{comm}	β_{doll}	R^2	N
Albania	-0.001	-1.601***	1.100***	0.699	240
	(0.001)	(0.183)	(0.069)		
Australia	-0.002*	0.038	1.639***	0.726	240
	(0.001)	(0.250)	(0.070)		
Brazil	0.000	2.480***	1.546***	0.400	240
	(0.002)	(0.606)	(0.136)		
Canada	-0.002*	0.586**	0.956***	0.520	240
	(0.001)	(0.293)	(0.074)		
Switzerland	-0.002*	-2.020***	1.093***	0.562	240
	(0.001)	(0.249)	(0.086)		
Chile	-0.001	1.698***	1.072***	0.449	240
	(0.002)	(0.355)	(0.109)		
China	-0.001*	0.001	0.126***	0.101	240
	(0.000)	(0.099)	(0.027)		
Colombia	-0.000	1.686***	1.258***	0.476	240
	(0.002)	(0.399)	(0.117)		
Costa Rica	0.003***	-0.264	0.003	0.014	240
	(0.001)	(0.167)	(0.041)		
Czech Republic	-0.002**	-2.079***	1.496***	0.686	240
	(0.001)	(0.299)	(0.066)		
Euro Area	-0.001	-1.688***	1.273***	0.741	240
	(0.001)	(0.206)	(0.058)		
United Kingdom	0.000	-0.570	0.776***	0.359	240
	(0.001)	(0.361)	(0.095)		
Hungary	-0.000	-1.772***	1.768***	0.684	240
	(0.001)	(0.386)	(0.110)		
Indonesia	0.002	-0.078	0.817***	0.241	240
	(0.002)	(0.312)	(0.111)		
Israel	-0.001	-0.279	0.617***	0.289	240
	(0.001)	(0.261)	(0.066)		
India	0.001	0.221	0.655***	0.385	240
	(0.001)	(0.193)	(0.063)		

Notes: This table reports regression estimates for constant α, the commodity factor loading β_{comm}, and the dollar factor loading β_{doll} for each currency i (listed in the first column), as specified in equation (13). Newey-West adjusted standard errors are reported in brackets. The table also includes the R^2, representing the fraction of variation in Δs_{t+1} explained by our empirical model for each currency's regression.

Table B.2: Factor Structure of Exchange Rates: Regression Estimates

Country	α	β_{comm}	β_{doll}	R^2	N
Iceland	0.002	-0.695	0.995***	0.239	240
	(0.002)	(0.621)	(0.170)		
Japan	0.001	-0.892**	0.276**	0.066	240
	(0.002)	(0.402)	(0.124)		
South Korea	-0.001	-0.071	1.119***	0.471	240
	(0.001)	(0.340)	(0.109)		
Mexico	0.001	0.965***	0.996***	0.436	240
	(0.001)	(0.310)	(0.102)		
Malaysia	-0.001	0.302*	0.627***	0.455	240
	(0.001)	(0.169)	(0.062)		
Norway	-0.000	-0.964***	1.350***	0.656	240
	(0.001)	(0.331)	(0.083)		
Peru	-0.001	0.461***	0.363***	0.288	240
	(0.001)	(0.152)	(0.049)		
Philippines	0.000	0.052	0.441***	0.236	240
	(0.001)	(0.204)	(0.052)		
Poland	-0.002	-1.167***	1.729***	0.689	240
	(0.001)	(0.345)	(0.097)		
Russia	0.002	0.290	1.131***	0.331	240
	(0.002)	(0.505)	(0.152)		
Sweden	-0.000	-1.268***	1.460***	0.707	240
	(0.001)	(0.242)	(0.063)		
Singapore	-0.001***	-0.417***	0.703***	0.727	240
	(0.001)	(0.118)	(0.045)		
Thailand	-0.001	-0.184	0.543***	0.315	240
	(0.001)	(0.203)	(0.049)		
Turkey	0.007**	2.380***	1.159***	0.294	240
	(0.003)	(0.546)	(0.117)		
Uruguay	0.004*	1.341***	0.441***	0.127	240
	(0.002)	(0.497)	(0.101)		
South Africa	0.001	0.585	1.575***	0.390	240
	(0.002)	(0.460)	(0.105)		

Notes: This table reports regression estimates for constant α, the commodity factor loading β_{comm}, and the dollar factor loading β_{doll} for each currency i (listed in the first column), as specified in equation (13). Newey-West adjusted standard errors are reported in brackets. The table also includes the R^2, representing the fraction of variation in Δs_{t+1} explained by our empirical model for each currency's regression.

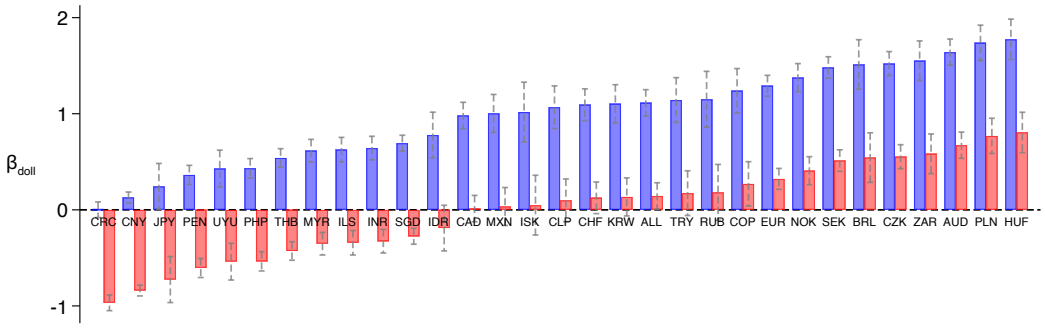
Table B.3: Macroeconomic Determinants of the Dollar and Commodity Factor

	(1)	(2)	(3)
Panel A: Dollar Factor			
US Nontradable Employment	-2.698**	-2.770**	-2.376**
	(1.088)	(1.319)	(1.132)
VIX	0.002***		
	(0.000)		
EBP		0.025***	
		(0.007)	
Intermediary Capital			-0.136***
			(0.022)
R^2	0.284	0.141	0.218
Observations	240	240	240
Panel B: Commodity Factor			
Commodity Price Index	-0.0181**	-0.0146*	-0.0166
	(0.009)	(0.008)	(0.010)
VIX	0.0004***		
	(0.000)		
EBP		0.0068***	
		(0.002)	
Intermediary Capital			-0.0223***
			(0.006)
R^2	0.138	0.136	0.097
Observations	240	240	240

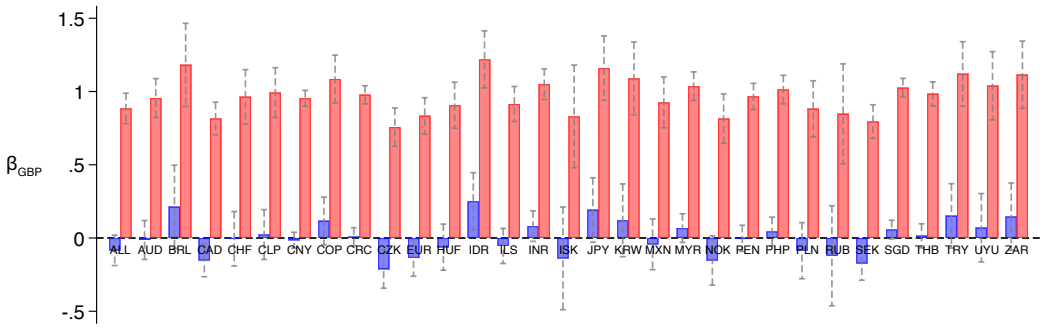
Notes: Panel A reports time-series regressions of the monthly dollar factor (constructed similarly to equation (15), but including all currencies) on the first difference of HP-filtered log U.S. non-tradable sector employment and various global risk measures. Column (1) uses the first difference of the VIX, Column (2) uses the first difference of the excess bond premium from Gilchrist and Zakrajšek (2012), and Column (3) uses the intermediary capital risk factor from He et al. (2017). Panel B reports time-series regressions of the monthly commodity factor (similar to equation (14), but including all currencies) on the log difference of the commodity price index (excluding gold) and the same measures of global risk used in Panel A. Appendix B provides more details on variables used in this table.

Figure B.2: Factor Structure Regressions: Comparing Dollar and British Pound Bases

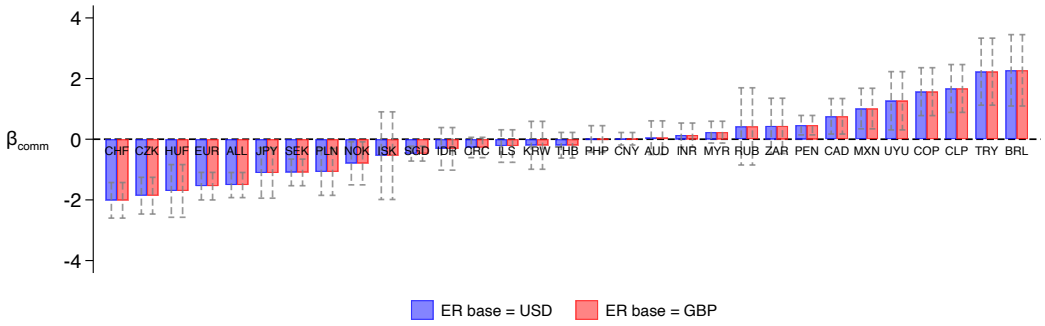
(a) Dollar-factor loadings $\beta_{i,\text{doll}}$



(b) British pound-factor loadings $\beta_{i,\text{gbp}}$



(c) Commodity-factor loadings $\beta_{i,\text{comm}}$



Notes: Each panel shows the estimated loadings in two regressions that only differ on the base of the exchange rates used as dependent variables. These regressions include the dollar factor, the British pound factor and the commodity factor as independent variables. Panel (a) shows estimated loading of the dollar factor $\text{doll}_{i,t+1}$ for each currency i. Panel (b) shows estimated loading of the British pound factor $\text{gbp}_{i,t+1}$ for each currency i. Panel (c) shows the estimated loading of the commodity factor $\text{comm}_{i,t+1}$ for each currency i. Each bar includes 90% confidence interval in gray, calculated using Newey-West standard errors.

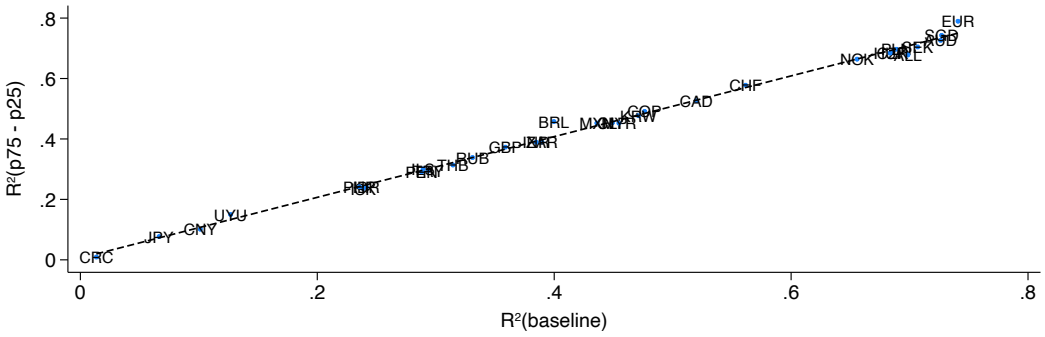
Table B.4: Factor Structure of Exchange Rates: The Role of the Dollar Basis

	# of Regressions with significant	
Currency Base	Dollar Factor	Currency-Base Factor
Albania	22	14
Australia	23	5
Brazil	19	4
Canada	22	6
Switzerland	23	9
Chile	22	1
China	17	6
Colombia	20	9
Costa Rica	19	3
Czech Republic	22	15
Euro Area	23	17
United Kingdom	22	5
Hungary	20	10
Indonesia	22	5
Israel	20	2
India	22	10
Iceland	22	5
Japan	22	9
South Korea	22	9
Mexico	22	7
Malaysia	22	14
Norway	22	12
Peru	21	6
Philippines	23	13
Poland	22	9
Russia	22	5
Sweden	19	14
Singapore	23	14
Thailand	22	11
Turkey	23	15
Uruguay	21	1
South Africa	21	3
Dependent variable relative to	currency base	USD
All currency bases	687	268

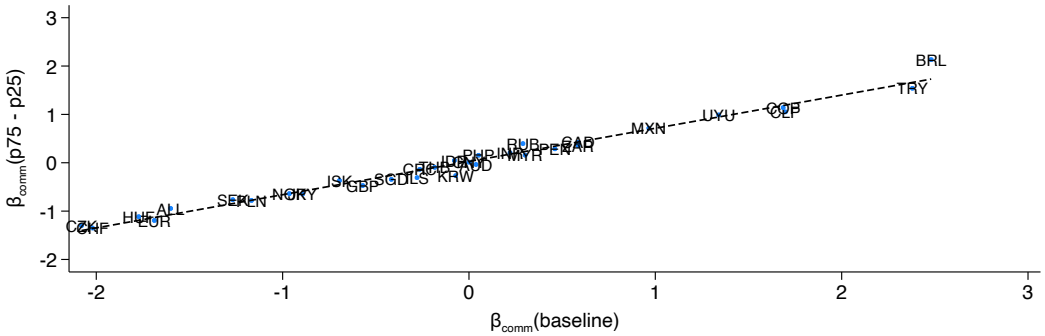
Notes: This table reports the counts of statistically significant dollar and currency-base factor loadings for each base currency i, evaluated at the 5% significance level. Appendix B provides details on the construction of the currency-base factor. The second column reports the number of significant dollar factor loadings obtained from three-factor regressions, where the dependent variable is the change in log exchange rate vis-à-vis base currency i, and the independent variables are dollar, commodity, and base-currency factors. For each base currency i, the count is out of 32; in total, 687 out of 992 regressions yield a statistically significant dollar factor loading. The last column reports the number of significant base-currency factor loadings obtained from the same three-factor regressions, but the dependent variable is relative to USD.

Figure B.3: Exchange Rate Regressions: Baseline and Alternative Commodity Factor

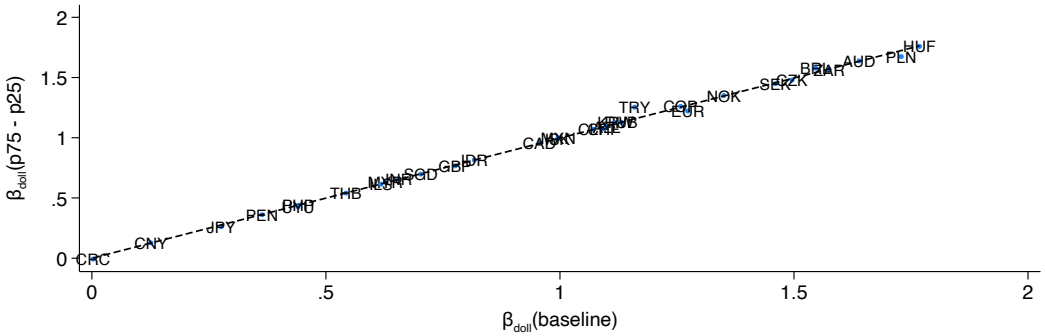
(a) Explanatory power R_i^2



(b) Commodity-factor loadings $\beta_{i,\text{comm}}$



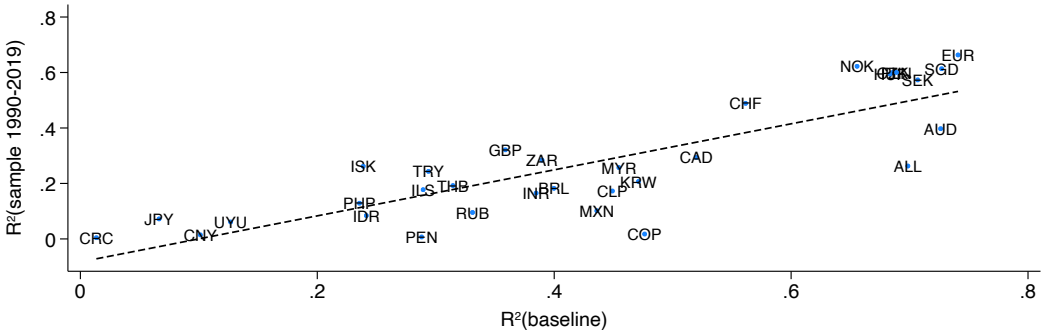
(c) Dollar-factor loadings $\beta_{i,\text{doll}}$



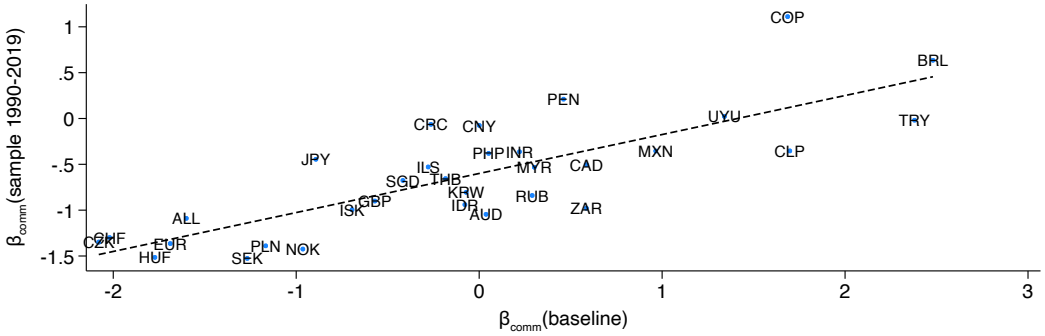
*Notes:*Panel (a) plots the R^2 for each currency i, from the modified empirical model (y-axis) - where the commodity factor is constructed using the 75th and 25th percentiles of the commodity export share - against the R^2 from the baseline model in equation (13) (x-axis). Panel (b) plots the estimated commodity factor loadings from the modified empirical model against those from the baseline model in equation (13). Panel (c) plots the estimated dollar factor loadings from the modified empirical model against those from the baseline model in equation (13). Each scatter point represents a currency, and the gray dashed line denotes the line of best fit.

Figure B.4: Exchange Rate Regressions: Baseline and 1990-2019 Sample

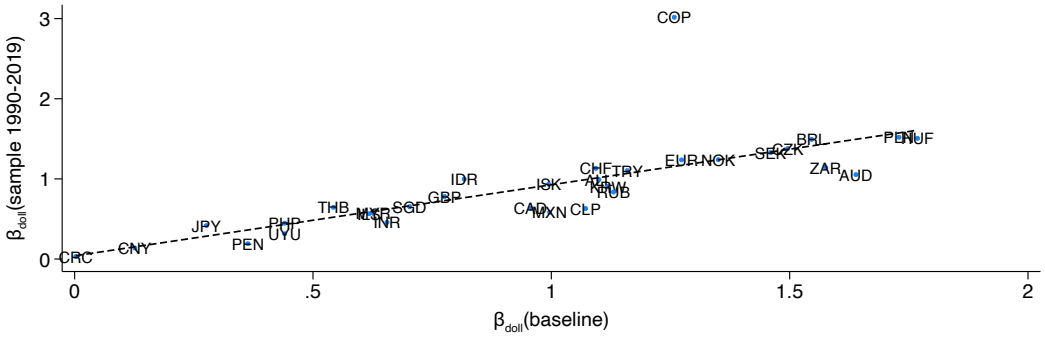
(a) Explanatory power R_i^2



(b) Commodity-factor loadings $\beta_{i,\text{comm}}$



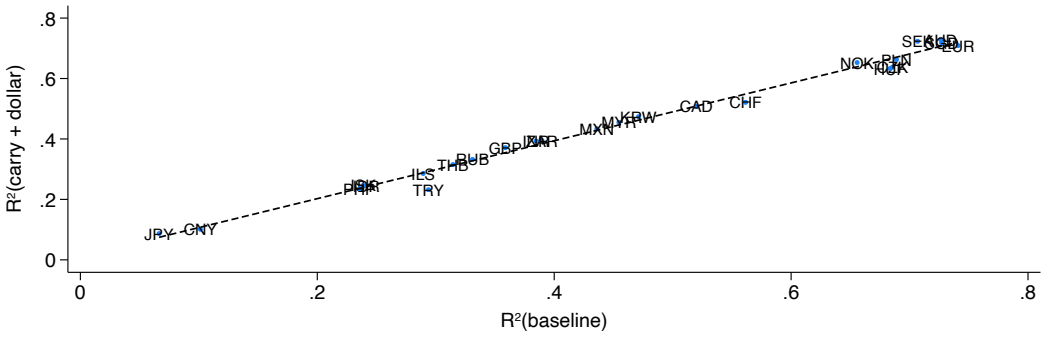
(c) Dollar-factor loadings $\beta_{i,\text{doll}}$



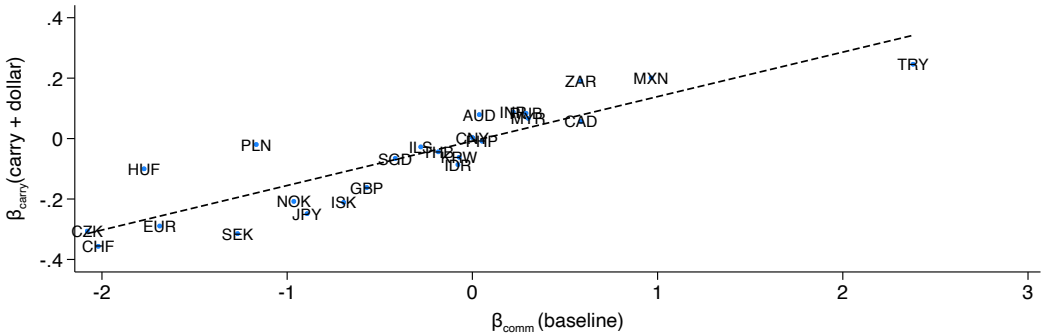
*Notes:*Panel (a) plots the R^2 for each currency i, from the 1990-2019 extended sample (y-axis), against the R^2 from the baseline model in equation (13) (x-axis), which uses the 2000-2019 sample. Panel (b) plots the estimated commodity factor loadings from the 1990-2019 extended sample against those from the baseline. Panel (c) plots the estimated dollar factor loadings from the 1990-2019 extended sample against those from the baseline. Each scatter point represents a currency, and the gray dashed line denotes the line of best fit.

Figure B.5: Exchange Rate Regressions: Baseline and Carry-Factor

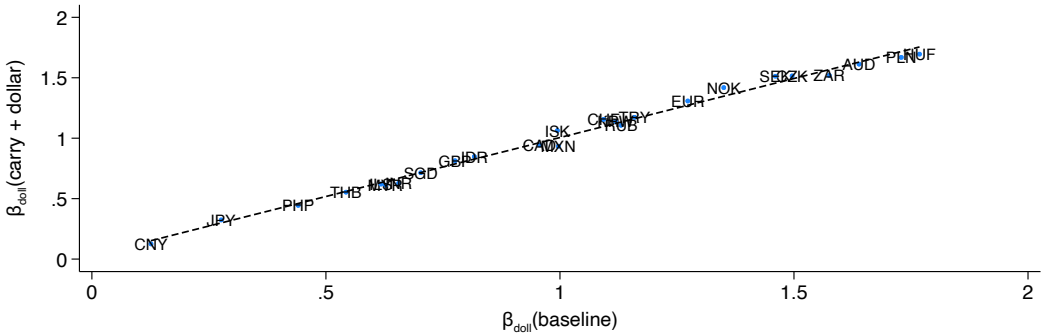
(a) Explanatory power R_i^2



(b) Commodity- and carry-factor loadings $\beta_{i,\text{comm}}$



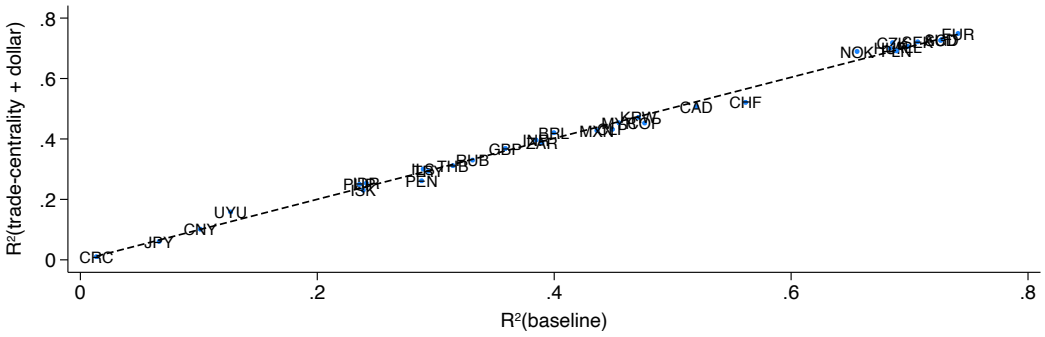
(c) Dollar-factor loadings $\beta_{i,\text{doll}}$



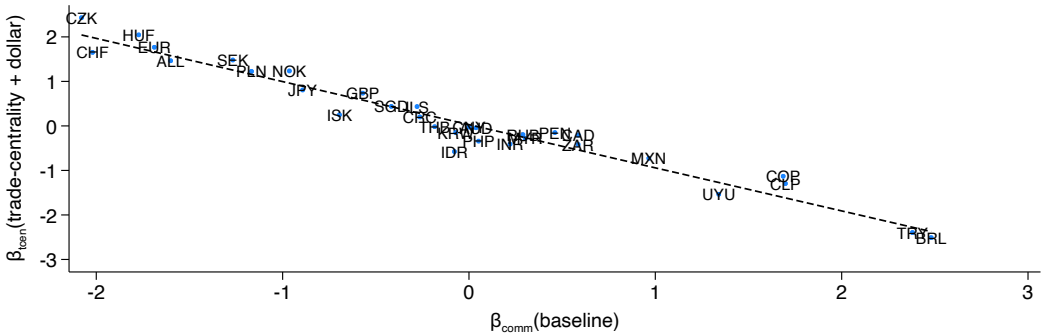
*Notes:*Appendix B provides details on the construction of carry factor proposed by Verdelhan (2018). Panel (a) plots the R^2 for each currency i, from the modified empirical model (y-axis) - where the commodity factor is replaced by the carry factor - against the R^2 from the baseline model in equation (13) (x-axis). Panel (b) plots the estimated carry factor loadings from the modified empirical model against the estimated commodity factor loadings from our baseline model. Panel (c) plots the estimated dollar factor loadings from the modified empirical model against those from the baseline model. Each scatter point represents a currency, and the gray dashed line denotes the line of best fit.

Figure B.6: Exchange Rate Regressions: Baseline and Trade-Centrality Factor

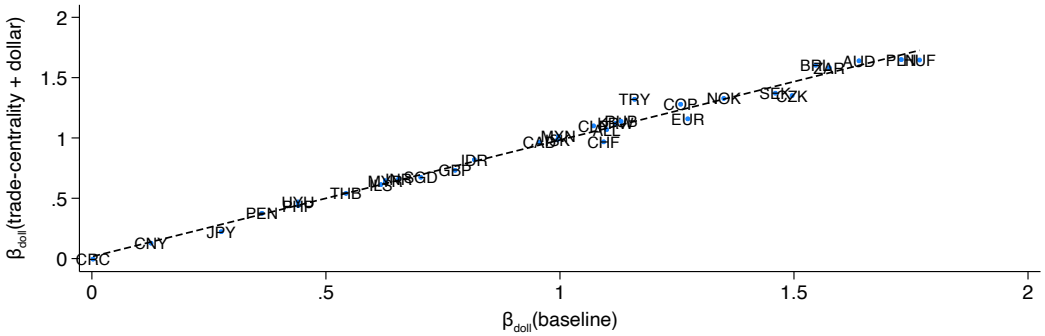
(a) Explanatory power R_i^2



(b) Commodity- and trade-centrality-factor loadings $\beta_{i,\text{comm}}$



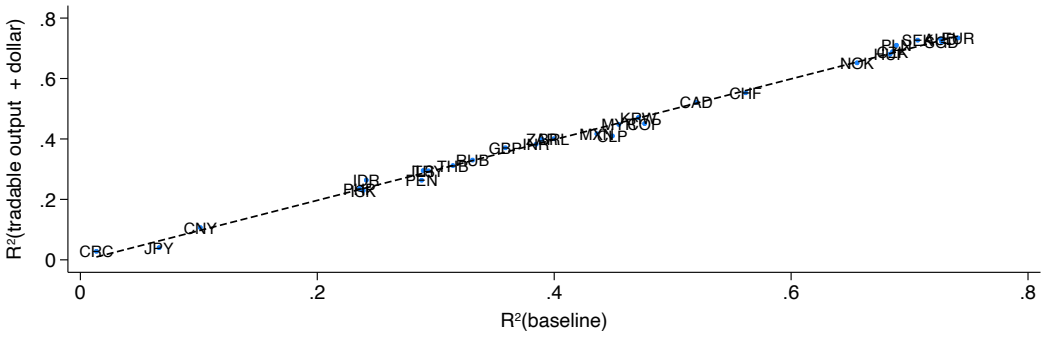
(c) Dollar-factor loadings $\beta_{i,\text{doll}}$



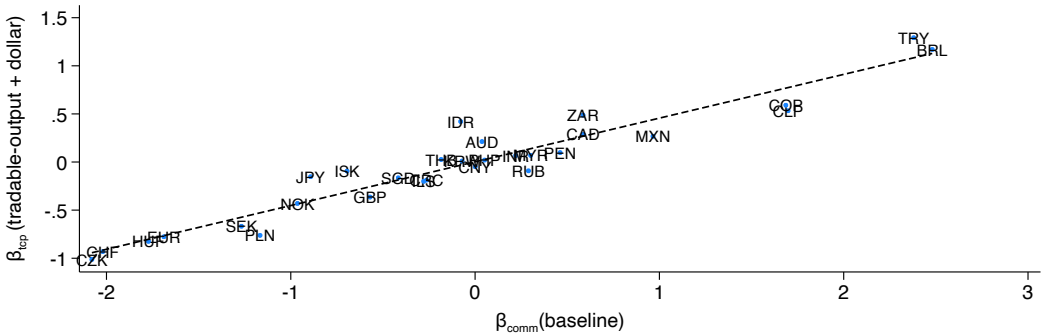
Notes: Panel (a) plots the R^2 for each currency i, from the modified empirical model (y-axis) - where the commodity factor is replaced by the trade-centrality factor - against the R^2 from the baseline model in equation (13) (x-axis). Panel (b) plots the estimated trade-centrality factor loadings from the modified empirical model against the estimated commodity factor loadings from our baseline model. Panel (c) plots the estimated dollar factor loadings from the modified empirical model against those from the baseline model. Each scatter point represents a currency, and the gray dashed line denotes the line of best fit. Appendix B provides details on the construction of the trade-centrality factor proposed by Richmond (2019).

Figure B.7: Exchange Rate Regressions: Baseline and Tradable-Output Sensitivity Factor

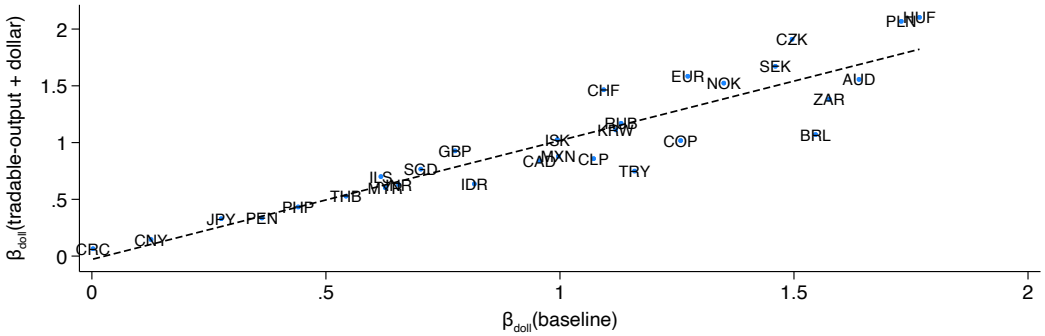
(a) Explanatory power R_i^2



(b) Commodity- and tradable-output-sensitivity-factor loadings $\beta_{i,\text{comm}}$



(c) Dollar-factor loadings $\beta_{i,\text{doll}}$



Notes: Panel (a) plots the R^2 for each currency i, from the modified empirical model (y-axis) - where the commodity factor is replaced by the tradable-output-sensitivity factor - against the R^2 from the baseline model in equation (13) (x-axis). Panel (b) plots the estimated tradable-output-sensitivity factor loadings from the modified empirical model against the estimated commodity factor loadings from our baseline model. Panel (c) plots the estimated dollar factor loadings from the modified empirical model against those from the baseline model. Each scatter point represents a currency, and the gray dashed line denotes the line of best fit. Appendix B provides details on the construction of the tradable-output-sensitivity factor.

Table B.5: Cross-Sectional Regressions of Factor Loadings on USD Assets and Commodity Exports Shares

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Panel A: Commodity factor loading β_{comm}							
Comm share	3.371***	2.252***	2.797***	2.649***	2.441***	2.442**	2.687**
	(0.904)	(0.794)	(0.887)	(0.942)	(0.878)	(0.932)	(1.211)
Controls		(1) + USD assets	(2) + trade cent	(3) + trade open	(4) + country size	(5) + distance	(6) + gravity
R^2	0.291	0.558	0.574	0.577	0.639	0.650	0.680
Observations	32	32	32	32	32	32	32
Panel B: Dollar factor loading β_{doll}							
USD assets	-1.030***	-1.234***	-1.265***	-1.266***	-1.271***	-1.282***	-1.277***
	(0.265)	(0.265)	(0.289)	(0.290)	(0.290)	(0.295)	(0.328)
Controls		(1) + Comm share	(2) + trade cent	(3) + trade open	(4) + country size	(5) + distance	(6) + gravity
R^2	0.285	0.378	0.380	0.380	0.380	0.387	0.461
Observations	32	32	32	32	32	32	32

Notes: This table reports the results of cross-sectional regressions of factor loadings $\beta_{i,\text{comm}}$ and $\beta_{i,\text{doll}}$ on the share of dollar-denominated assets, the share of commodity exports, and various controls. Appendix B provides further details on the construction of these variables. Panel A presents results for the commodity factor loading. Column (1) reports the estimated coefficient from the univariate regression, with Newey-West adjusted standard errors reported in brackets. Columns (2) through (7) sequentially add the share of dollar-denominated assets, trade centrality, trade openness, log GDP share, log harmonic geographical distance to the U.S., and a set of gravity variables as controls. Panel B estimates similar specifications for the dollar factor loading.

Table B.6: Factor Loadings Regressions: Alternative Measures of Commodities and Dollar Assets

Comm share	3.371***			
	(0.904)			
alternative Comm share		12.778**		
		(5.296)		
USD assets			-1.125***	
			(0.289)	
alternative USD assets				-0.442*
				(0.241)
R^2	0.291	0.132	0.287	0.077
Observations	32	31	31	25

Notes: First two columns of this table report the results of cross-sectional univariate regressions of commodity factor loadings $\beta_{i,\text{comm}}$ on the baseline and alternative measures of commodity shares respectively. The last two columns report similar results of dollar factor loadings $\beta_{i,\text{doll}}$ on the baseline and alternative measures of dollar assets shares respectively. For the alternative commodity share, we use the share of commodity exports in total output. For the alternative dollar assets share, we consider the ratio of dollar assets to total trade (exports plus imports).

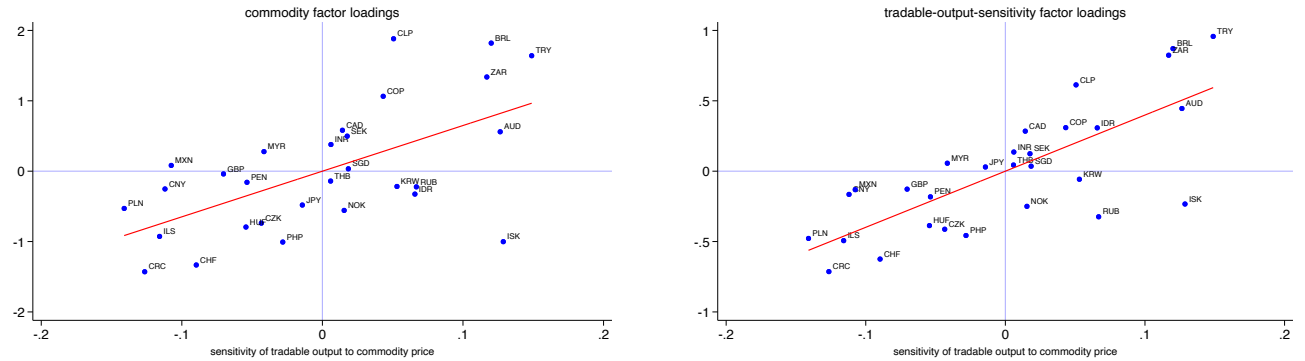
Table B.7: Evidence on Model's Mechanism: Net Acquisition of USD Assets and Consumption

	std(Δ USD net assets)/std(Consumption)	corr(Δ USD net assets, Consumption)
Albania	0.086	-0.603
Australia	0.097	-0.424
Brazil	0.109	-0.773
Canada	0.181	0.018
Switzerland	0.660	-0.346
Chile	0.284	0.104
China	0.834	-0.235
Colombia	0.110	-0.234
Costa Rica	0.429	-0.291
Czech Republic	0.143	0.076
Euro Area	0.043	-0.423
United Kingdom	0.346	-0.170
Hungary	0.222	-0.672
Indonesia	0.121	-0.248
Israel	0.574	-0.124
India	0.239	-0.409
Iceland	1.534	-0.565
Japan	0.127	-0.034
South Korea	0.382	-0.135
Mexico	0.209	-0.595
Malaysia	0.370	-0.104
Norway	0.382	0.611
Peru	0.301	-0.463
Philippines	0.485	-0.273
Poland	0.173	-0.570
Russia	0.175	0.040
Sweden	0.039	0.163
Singapore	1.071	0.297
Thailand	0.444	-0.564
Turkey	0.242	-0.769
Uruguay	0.267	-0.515
South Africa	0.074	-0.459
average	0.336	-0.272
median	0.240	-0.282

Notes: First column of this table reports the ratio of standard deviations of net acquisition of USD assets (net of USD liabilities) and consumption. Second column of this table reports the correlation between these two variables. Both variables are detrended at the annual frequency over the sample period. Appendix B provides further details on the construction of these variables..

Figure B.8: Determinants of Factor Loadings: Tradable-Output Sensitivity Specification

(a) Comm-loadings on tradable-output sensitivity (b) TOS-loadings on tradable-output sensitivity



Notes: Panel (a) plots the commodity factor loadings for each currency i (y-axis), against country's tradable output sensitivity to the commodity price (x-axis). Panel (b) plots the tradable-output-sensitivity factor loadings for each currency i (y-axis), against country's tradable output sensitivity to the commodity price (x-axis). Each scatter point represents a currency, and the red line denotes the line of best fit.

B.3 Evidence on Risk Premia

In this section, we present an empirical test of Proposition 6. We first regress $rx_{i,t+1} \equiv r_{it} - r_{ut} - \Delta s_{i,t+1}$, realized excess returns on each currency i at time $t+1$, on VIX at time t:

$$rx_{i,t+1} = \kappa_i \cdot VIX_t + \epsilon_{i,t+1} \tag{B.2}$$

We use VIX as a proxy for γ_t. Our frequency is quarterly, and we average VIX over each quarter. Second, collecting country-specific coefficients κ_i, we regress them on the dollar and commodity loadings of country i:

$$\hat{\kappa}_i = \alpha + \alpha_{\text{comm}}\beta_{i,\text{comm}} + \alpha_{\text{doll}}\beta_{i,\text{doll}} + \epsilon_i \tag{B.3}$$

Table B.8 demonstrates the results. Commodity and dollar loadings are both correlated with the responsiveness of conditional risk premia to VIX and jointly explain a third of the cross-sectional variation in this responsiveness.

Table B.8: Responsiveness of conditional risk premia to VIX in the cross-section.

	$\hat{\kappa}_i$
$\beta_{i,\text{comm}}$	0.93^{**}
	(0.47)
$\beta_{i,\text{doll}}$	2.99^{***}
	(1.03)
constant	-0.93
	(1.13)
R^2	0.34
N	22

C Quantitative Appendix

C.1 Calibration and Estimation

Estimation of τ. We estimate τ by running the following regression implied by our model:

$$\Delta m_{t+1} = \alpha_{\text{reserves}} + \tau_{\text{reserves}}(\Delta q_t - \Delta q_{ut}) + \epsilon_{t+1}$$

We use the logarithm of the broad dollar index DXY as a measure of $q_{ut} - q_t$ and the logarithm of total reserves expressed in dollars as m_t. After estimating $\tau_{\text{reserves}} = 0.9$ (standard error of 0.15), we adjust it for the share of USD in reserves reported by IMF (equal to 0.6 on average). For simplicity, we assume that the non-dollar part of reserves is denominated in the broad basket of currencies and is not actively managed in response to the change in the broad dollar. This means that the USD value of the non-dollar part of reserves changes one-for-one with the change in DXY. If the average share of the dollar in reserves is S_{USD}, under this assumption the relationship between the recovered τ_{reserves} and τ in our model is

$$\tau_{\text{reserves}} = (1 - S_{\text{USD}}) \cdot 1 + S_{USD} \cdot \tau$$

This leads to $\tau = 0.83$.

Estimating (e_i, m_i). We use the model of a commodity-based global economy presented in Appendix A.2 to inform the mapping from the share of commodities in exports to e_i. Specifically, if S_i^{comm} is country i's share of primary inputs in exports, we recover its e_i as $e_i = S_i^{\text{comm}}/(1-S_i^{\text{comm}})$. We then re-center $\{e_i\}$ around 1, consistent with our normalization that addresses the fact that e_i and z_t are not separately identified because they enter all expressions as a product $e_i z_t$.

We use the data on the share of USD in countries' external portfolio assets to map countries to their m_i. Specifically, we encode the USD share of portfolio assets as S_i^{doll}. Where the data in CPIS are unavailable, we impute the USD share using predicted values based on treasury holdings in TIC. We then use our internally calibrated variance σ_m to map S_i^{doll} to m_i as

$$m_i - 1 = (S_i^{\text{doll}} - \mathbb{E}[S_i^{\text{doll}}]) \cdot \frac{\sigma_m}{\text{std}(S_i^{\text{doll}})} \tag{C.1}$$

Here 1 is the normalized value of the average m_i.

The estimated (e_i, m_i) are not strongly correlated. We estimate a correlation of 0.23 with a 95% confidence interval of $[-0.15, 0.55]$.

C.2 Variance Decompositions

For local price changes in i and u and for the bilateral depreciation of i's currency relative to u, we have

$$\Delta q_{i,t+1} = \alpha e_i \sigma_z \epsilon_{z,t+1} - \alpha\phi\tau(m_i - m)\sigma_u \epsilon_{u,t+1} + \left(\frac{l_u}{2\varphi} + \frac{c_i}{\varphi}\right)\sigma_\gamma(\epsilon_{\gamma,t+1} - \epsilon_{\gamma t}) - \alpha\sigma_x \epsilon_{i,t+1} \tag{C.2}$$

$$\Delta q_{u,t+1} = \alpha e \sigma_z \epsilon_{z,t+1} - \alpha\sigma_u \epsilon_{u,t+1} + \frac{l_u}{2\varphi}\sigma_\gamma(\epsilon_{\gamma,t+1} - \epsilon_{\gamma t}) \tag{C.3}$$

$$\Delta s_{i,t+1} = (e - e_i)\alpha\sigma_z \epsilon_{z,t+1} + \alpha\psi_i \sigma_u \epsilon_{u,t+1} - \frac{c_i}{\varphi}\sigma_\gamma(\epsilon_{\gamma,t+1} - \epsilon_{\gamma t}) + \alpha\sigma_x \epsilon_{i,t+1} \tag{C.4}$$

Here

$$\psi_i = \phi\tau(m_i - m) - 1 \tag{C.5}$$

$$c_i = 2\Gamma\sigma_z^2\alpha^2\varphi e(e - e_i) + \Gamma\sigma_u^2\alpha^2(\rho - 1)\psi_i \tag{C.6}$$

$$l_u = \Gamma\sigma_u^2\alpha^2(\rho - 1) \tag{C.7}$$

Consumption growth is

$$\begin{aligned}\Delta c_{i,t+1} &= \frac{1-\alpha}{\alpha}\Delta q_{i,t+1} + \Delta x_{i,t+1} \\ &= (1-\alpha)e_i\sigma_z\epsilon_{z,t+1} - (1-\alpha)\phi\tau(m_i - m)\sigma_u\epsilon_{u,t+1} + \frac{1-\alpha}{\alpha}\left(\frac{l_u}{2\varphi} + \frac{c_i}{\varphi}\right)\sigma_\gamma(\epsilon_{\gamma,t+1} - \epsilon_{\gamma t}) \\ &\quad - \alpha\sigma_x\epsilon_{i,t+1}\end{aligned} \tag{C.8}$$

For the relative consumption growth, we have

$$\Delta c_{i,t+1} - \Delta c_{u,t+1} = \Delta x_{i,t+1} - \Delta x_{u,t+1} - \frac{1-\alpha}{\alpha}\Delta s_{i,t+1} \tag{C.9}$$

$$\begin{aligned}&= (1-\alpha)(e_i - e)\sigma_z\epsilon_{z,t+1} - (1 + (1-\alpha)\psi_i)\sigma_u\epsilon_{u,t+1} \\ &\quad + \frac{(1-\alpha)c_i}{\alpha\varphi}\sigma_\gamma(\epsilon_{\gamma,t+1} - \epsilon_{\gamma t}) + \alpha\sigma_x\epsilon_{i,t+1}\end{aligned} \tag{C.10}$$

The variance of consumption growth and exchange rate appreciation are

$$\begin{aligned}\mathbb{V}[\Delta c_{i,t+1}] &= (1-\alpha)^2 e_i^2\sigma_z^2 + \left(\frac{1-\alpha}{\alpha}\right)^2\left(\frac{l_u}{2\varphi} + \frac{c_i}{\varphi}\right)^2 \cdot 2\sigma_\gamma^2 \\ &\quad + \alpha^2\sigma_x^2 + (1-\alpha)^2\phi^2\tau^2(m_i - m)^2\sigma_u^2\end{aligned} \tag{C.11}$$

$$\mathbb{V}[\Delta s_{i,t+1}] = \alpha^2 e_i^2\sigma_z^2 + \left(\frac{c_i}{\varphi}\right)^2 \cdot 2\sigma_\gamma^2 + \alpha^2\sigma_x^2 + \alpha^2\psi_i^2\sigma_u^2 \tag{C.12}$$

The moment of interest is $rat_i \equiv \mathbb{V}[\Delta s_{i,t+1}]/\mathbb{V}[\Delta c_{i,t+1}]$.

The variance contributions to $\Delta q_{i,t+1}$ are

$$V_{q,i,z} = \alpha^2 e_i^2 \sigma_z^2 \tag{C.13}$$
$$V_{q,i,u} = \alpha^2 \phi^2 \tau^2 (m_i - m)^2 \sigma_u^2 \tag{C.14}$$
$$V_{q,i,\gamma} = 2\sigma_\gamma^2 \cdot \left(\frac{l_u}{2\varphi} + \frac{c_i}{\varphi} \right)^2 \tag{C.15}$$
$$V_{q,i,\epsilon} = \alpha^2 \sigma_x^2 \tag{C.16}$$

The variance contributions to $\Delta q_{u,t+1}$ are

$$V_{q,u,z} = \alpha^2 e^2 \sigma_z^2 \tag{C.17}$$
$$V_{q,u,u} = \alpha^2 \sigma_u^2 \tag{C.18}$$
$$V_{q,u,\gamma} = 2\sigma_\gamma^2 \cdot \frac{l_u^2}{4\varphi^2} \tag{C.19}$$
$$\tag{C.20}$$

The variance contributions to $\Delta s_{i,t+1}$ are

$$V_{s,i,z} = \alpha^2 (e_i - e)^2 \sigma_z^2 \tag{C.21}$$
$$V_{s,i,u} = \alpha^2 \psi_i^2 \sigma_u^2 \tag{C.22}$$
$$V_{s,i,\gamma} = 2\sigma_\gamma^2 \cdot \left(\frac{c_i}{\varphi} \right)^2 \tag{C.23}$$
$$V_{s,i,\epsilon} = \alpha^2 \sigma_x^2 \tag{C.24}$$

The variance shares are the contributions scaled by their sum. The standard deviation of currency i's depreciation is

$$std_i = \sqrt{V_{s,i,z} + V_{s,i,u} + V_{s,i,\gamma} + V_{s,i,\epsilon}} \tag{C.25}$$

The factor R-squared for currency i is

$$rsq_i = \frac{V_{s,i,z} + V_{s,i,u} + V_{s,i,\gamma}}{V_{s,i,z} + V_{s,i,u} + V_{s,i,\gamma} + V_{s,i,\epsilon}} \tag{C.26}$$

C.3 Additional Figures and Tables

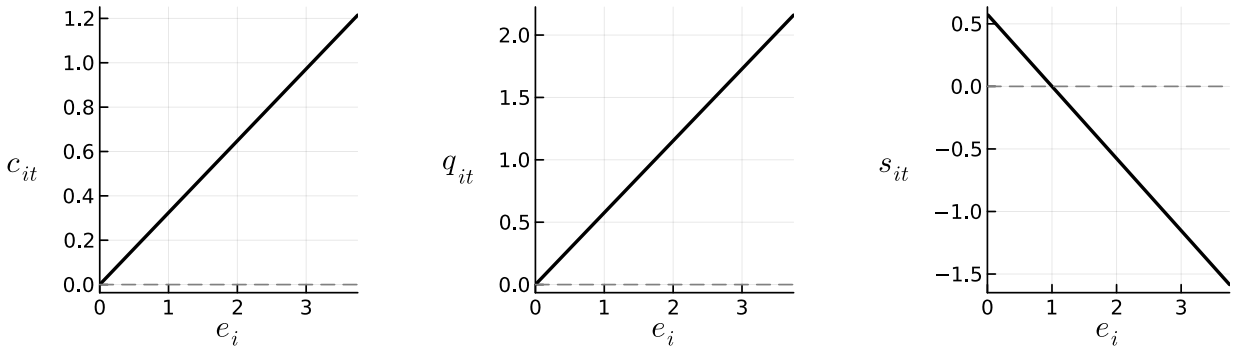
Table C.1 presents the results from regressing factor loadings in the data on those implied by the model. The third column additionally regresses the R^2 from factor regressions in the data on that implied by the model.

Table C.1: Model fit on untargeted moments.

	$\beta_{i,\text{doll}}$	$\beta_{i,\text{comm}}$	R^2
const	0.39	-0.08	0.14
	(0.17)	(0.08)	(0.07)
model	0.58	0.29	0.67
	(0.16)	(0.11)	(0.17)
R^2	0.33	0.20	0.37
N	29	29	29

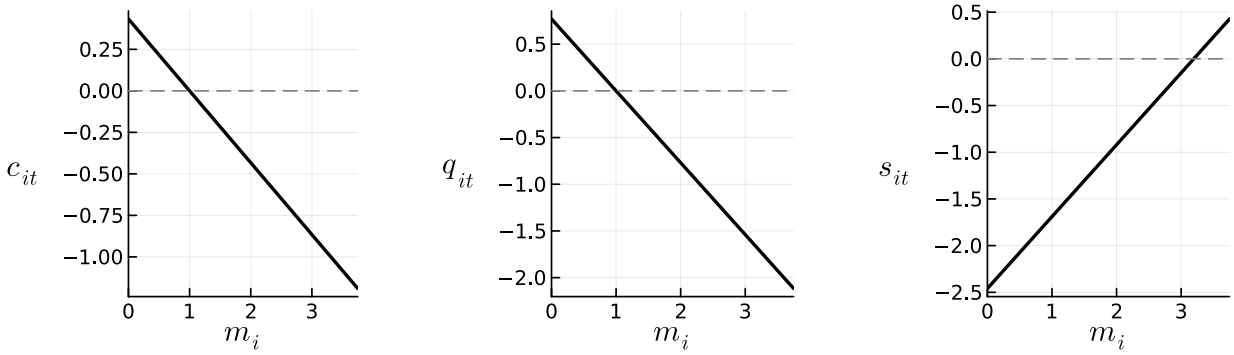
The following figures present peak impulse responses of consumption, prices of local consumption baskets, and exchange rate depreciations to the three global shocks as functions of the relevant cross-sectional variables. First, Figure C.1 shows the peak impulse responses to a standard global tradable endowment shock z_t as a function of the commodity exposure e_i. These impulse responses are constant as a function of the other cross-sectional variable, dollar assets m_i.

Figure C.1: Peak impulse responses of consumption and exchange rate depreciation against the dollar, global tradable endowment shock z_t.



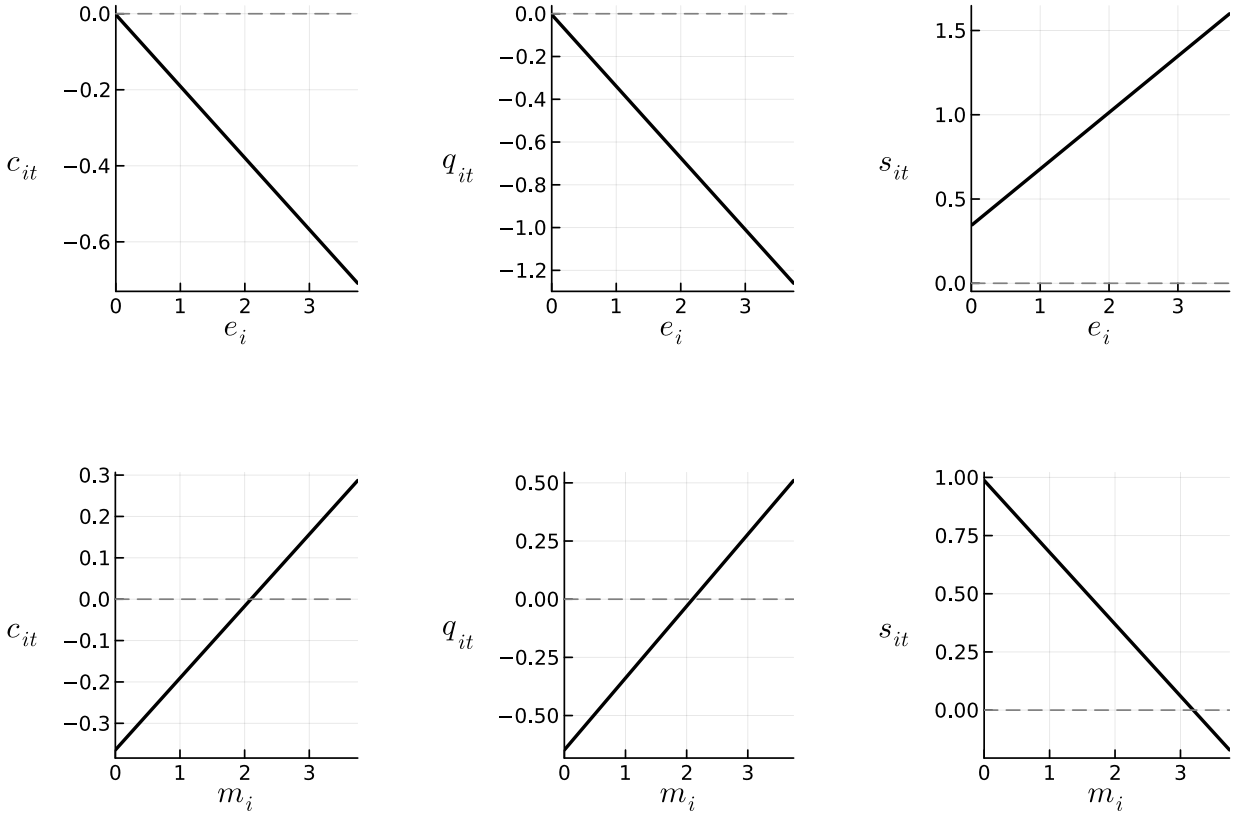
Next, Figure C.2 shows the peak impulse responses to a standard US non-tradable endowment shock x_{ut} as a function of dollar assets m_i. These impulse responses are constant as a function of the other cross-sectional variable, the commodity exposure e_i.

Figure C.2: Peak impulse responses of consumption and exchange rate depreciation against the dollar, US non-tradable endowment shock x_{ut}.



Finally, Figure C.3 shows the peak impulse responses to a standard intermediary's risk-aversion shock γ_t as a function of both commodity exposure e_i and dollar assets m_i.

Figure C.3: Peak impulse responses of consumption and exchange rate depreciation against the dollar, financial shock γ_t.



C.4 Quantitative Model Relaxing Parametric Assumptions

In this section, we relax the parametric assumptions that lead to analytical solutions for the factor structure. Specifically, in the main text we assume $\rho_\gamma = 0$, $\rho_z = \rho_u = \rho_x = 1$, and take the limit of $\chi \longrightarrow \infty$ with $\chi^{-1/2} M_i \longrightarrow m_i$. Without this limit, the linearized solution to the model is still available through Lemma 4 and Lemma 5. This allows us to compute the population moments of factors and factor loadings. These expressions, however, become unwieldy and highly non-linear in parameters, and calibration requires numerical solutions.

Table C.2: Model parameters.

Parameter	Value	Range	Limiting	Role
(A) Externally set				
ρ	2.0	—	2.0	inverse of EIS
β	0.98	—	0.98	discount factor
α	0.64	—	0.64	share of non-tradables
τ	0.83	—	0.83	reserve stabilization elasticity
χ	21	[12, 4546]	∞	intermediaries' portfolio capacity
ρ_z, ρ_x, ρ_u	0.98	—	1.0	persistence of shocks to z_t, x_{ut} and x_{it}
ρ_γ	0.95	—	0.0	persistence of shocks to γ_t
(B) Estimated and calibrated				
σ_γ	0.18	[0.05, 0.23]	0.69	standard deviation of innovations to γ_t
σ_u	2.48	[2.13, 2.50]	2.64	standard deviation of innovations to x_{ut}
σ_x	3.71	[3.04, 3.74]	3.88	standard deviation of innovations to x_{it}
σ_z	1.02	[1.01, 1.68]	0.90	standard deviation of innovations to z_t
σ_M	5.42	[4.05, 177.49]	∞	cross-sectional standard deviation of M_i
L	4.97	[3.72, 162.93]	∞	benchmark intermediated position, $\int M_i \, di$
σ_e	1.29	[1.29, 1.29]	1.29	cross-sectional standard deviation of e_i
Γ	0.79	[0.79, 0.86]	0.73	SDF weight on stochastic component

We set persistence coefficients to $\rho_x = \rho_z = \rho_u = 0.98$ and $\rho_\gamma = 0.95$ to match the quarterly persistence of real and financial shocks in Kekre and Lenel (2024a). For χ, we show two sets of results. First, using the fact that χ affects the persistence of the NFA to GDP ratio in the US, as shown in Proposition 7, we match its annual autocorrelation in the External Wealth of Nations database by Lane and Milesi-Ferretti (2001) and Lane and Milesi-Ferretti (2007). Table C.2 reports parameters for this calibration in the column "Value". Second, we note that χ is the semi-elasticity of the intermediary's asset demand with respect to expected excess returns. Adjusting the semi-elasticity by the total size of the balance sheet allows us to compute the elasticity, which we can map to the estimates of Koijen and Yogo (2020). They estimate the elasticity of around 3 for long-term bonds and around 27 for short-term bonds, which maps into $\chi = 12$ and $\chi = 4546$. We set up a grid of points in this range and calibrate the rest of the model for each value of χ on the grid. Table C.2 reports this range in the column "KY range". The column "Limiting" shows the

baseline results from the main text for comparison.

Table C.3 shows model fit. It is as close as in the limiting calibration. Table C.4 reports the result of the variance decomposition of the US consumption basket price Δq_{ut}, the dollar factor, and the commodity factor. In each cell, we report the value from the headline calibration of χ without brackets. In round brackets, we report the value from the calibration in the main text for comparison. Finally, in the square brackets, we report the range of values corresponding to the values of χ from the Koijen and Yogo (2020) range.

Table C.3: Model fit on targeted and untargeted moments.

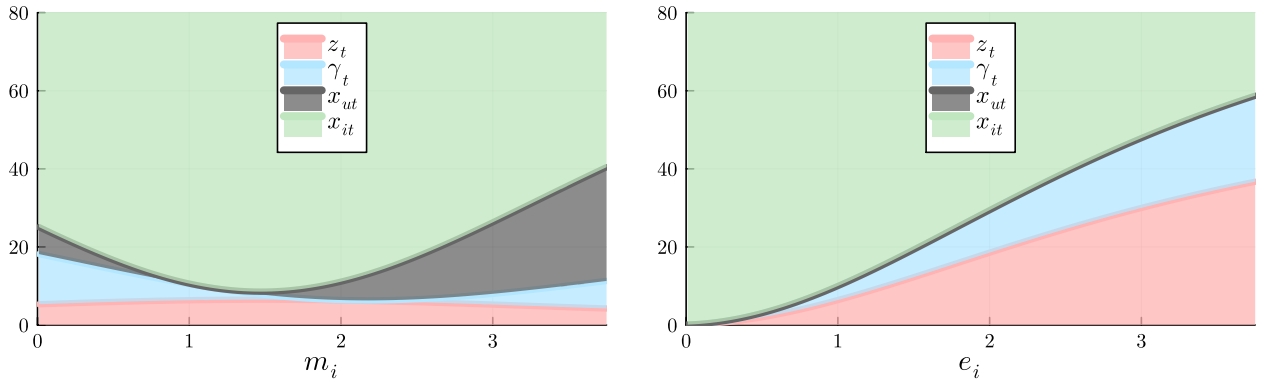
Moment	Data	Model
std(doll_t)	1.94pp	1.94pp
std(comm_t)	1.32pp	1.32pp
corr(doll_t,comm_t)	0.31	0.31
std(ε_{it})	2.48pp	2.48pp
std(Δs_{it})	3.24pp	3.37pp
avg R^2 of factors	41%	40%
std($\beta_{i,\text{doll}}$)	0.47	0.47
std($\beta_{i,\text{comm}}$)	0.48	0.75
NFA/GDP (US)	0.75	0.75
std(M_i)/L	1.09	1.09

Table C.4: Variance decomposition of q_{ut}, doll_t, and comm_t.

	Δq_{ut}	doll_t	comm_t
$\Delta\gamma_t$	7% (7%)	26% (24%)	38% (40%)
	[4%, 7%]	[15%, 26%]	[37%, 64%]
Δz_t	13% (10%)	—	62% (60%)
	[13%, 31%]	—	[36%, 63%]
Δx_{ut}	80% (84%)	74% (76%)	—
	[66%, 80%]	[74%, 85%]	—

Lastly, Figure C.4b and Figure C.5b show the variance decompositions of consumption basket prices and exchange rates in the cross-section, analogously to those on Figure 4b and Figure 5b.

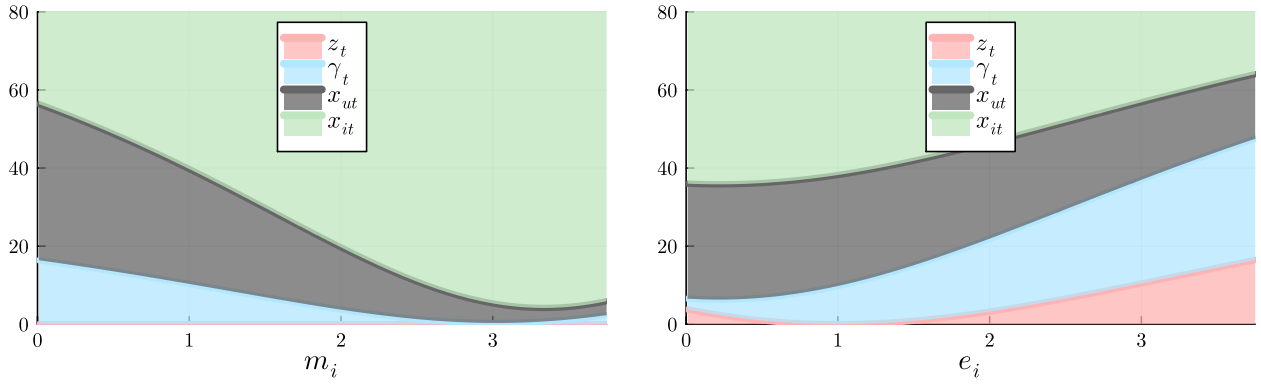
Figure C.4: Variance Decomposition of Country-Level Prices



(a) Countries with different dollar assets (b) Countries with different commodity exposure

Notes: Panel (a) plots the shares of the three global shocks (z_t, γ_t, x_{ut}) and the local shock x_{it} in the variance decomposition of the local basket price changes Δq_{it} as a function of m_i, where each country's e_i is evaluated at the mean $e = 1$. Panel (b) plots the same variance decomposition as a function of e_i, with each country's m_i evaluated at the mean $m = 1$.

Figure C.5: Variance Decomposition of Bilateral Exchange Rates



(a) Countries with different dollar assets (b) Countries with different commodity exposure

Notes: Panel (a) plots the shares of the three global shocks (z_t, γ_t, x_{ut}) and the local shock x_{it} in the variance decomposition of the bilateral exchange rate changes Δs_{it} as a function of m_i, where each country's e_i is evaluated at the mean $e = 1$. Panel (b) plots the same variance decomposition as a function of e_i, with each country's m_i evaluated at the mean $m = 1$.