Equilibrium Bidding in Package Markets

Luke Lindsay*

August 2026

Abstract

I compare three package market mechanisms—Shapley value-based pricing, linear pricing, and linear pricing without rationing—across six scenarios ranging from bilateral trade to settings with non-convex costs and an empty core. Using homotopy continuation methods, I compute Bayesian Nash equilibria for each mechanism–scenario combination. A laboratory experiment tests the mechanisms with human bidders. Shapley value-based pricing achieves the highest efficiency both in equilibrium and in the lab, with the largest advantages where competitive prices fail to exist. I also compute optimal BIC and DSIC benchmarks; realised efficiency under Shapley value-based pricing lies within a few percentage points of the BIC bound.

Keywords: Package markets, computing Nash equilibrium, market design, mechanism design, laboratory experiments
JEL codes: C63 (Computational techniques), C92 (Laboratory, group behavior), D4 (Market structure and pricing), D47 (Market design), D82 (Mechanism design)

*Department of Economics, University of Exeter, Streatham Court, Rennes Drive, Exeter EX4 4PU, UK. Email: L.Lindsay@exeter.ac.uk. This research was funded by the University of Exeter. The author declares that he has no relevant or material financial interests or other potential conflicts of interest that relate to the research described in this paper.

I study equilibria in package markets by using a computational approach to find Nash equilibria and by conducting laboratory experiments. I use these methods to compare three different package market mechanisms across a range of scenarios.

Package markets allow traders to bid on combinations of items and prevent them from ending up with undesired combinations. This is important in settings where a buyer's value for an item depends on the other items the buyer holds, or where a seller's cost of providing an item depends on what other items the seller supplies. Package markets have many practical applications, ranging from trading environmental credits and fishing rights to trading radio spectrum and electricity.

Package markets are more complex than markets for divisible goods because scenarios can arise where market-clearing prices do not exist. For example, suppose a buyer is willing to pay 32 for a package of two units of a good, and a seller is willing to accept 30 for a package of three units of the same good. A Pareto-improving trade is possible, but there is no price where supply equals demand. At a per-unit price of 16 or less, the buyer's demand is two units. At a per-unit price of 10 or more, the seller's supply is three units.

A range of package market mechanisms has been proposed in the literature and used in practice. However, without knowing what equilibrium bidding would be in different mechanisms and without knowing how humans would actually bid in these mechanisms, it is difficult to evaluate when package markets would lead to welfare gains and which mechanism would produce the largest gains.

Finding equilibria in trading mechanisms analytically typically involves solving a system of differential equations. However, even in bilateral settings, finding solutions is often challenging. For example, Chatterjee and Samuelson (1983) write, 'the linked differential equations of Theorem 2 resist analytic solutions for all but the most elementary distribution functions.' Standard computational approaches are also not well suited to finding equilibria in trading games. For example, the documentation for the Gambit software (Savani and Turocy, 2025) notes 'Analyzing large games may become infeasible surprisingly quickly.' As well as the difficulty in finding equilibria, trading games typically have multiple equilibria. This means that to evaluate a mechanism based on equilibrium outcomes, some equilibrium selection rule is needed.

This paper compares three package market mechanisms—Shapley value-based pricing, linear pricing, and linear pricing without rationing—across six market scenarios, ranging from simple bilateral trade to settings with non-convex costs and an empty core. It makes three main contributions. First, it provides equilibrium predictions for each mechanism in each scenario. Equilibria are computed using a logit quantal response equilibrium homotopy continuation approach that exploits the structure of market games to handle larger strategy spaces than standard methods allow. Second, it reports a laboratory experiment with 167 participants that tests the three mechanisms across all six scenarios. Third, it benchmarks the mechanisms against optimal Bayesian incentive compatible (BIC) and dominant-strategy incentive compatible (DSIC)

mechanisms for each scenario. The main finding is that Shapley value-based pricing outperforms linear pricing both in equilibrium and in the lab, and that realised efficiency under this mechanism lies within a few percentage points of the BIC bound.

The rest of the paper is organised as follows. Section 1 describes the environment and the six market scenarios. Section 2 introduces the three mechanisms and compares their efficiency under sincere bidding. Section 3 outlines the computational approach used to find Nash equilibria. Section 4 reports the laboratory experiment and its results. Section 5 discusses the optimal BIC and DSIC mechanisms. Section 6 concludes.

1 Environment

I study six market scenarios; Table 1 summarizes the details. In each scenario, costs and values are integers uniformly drawn from a specified range.

The first three scenarios admit a competitive equilibrium. **Bilateral** has one seller and one buyer, each trading a single unit; it is the setting studied by Myerson and Satterthwaite (1983) and Chatterjee and Samuelson (1983). **Competition** has two sellers and two buyers, each offering or demanding one unit. **Monopoly** has one seller supplying one or two units and two buyers each demanding one unit. The remaining three scenarios do not have a competitive equilibrium. In **Mismatched q**, sellers offer packages of 4 and 5 units while buyers demand 2 and 6; for any per-unit price, total quantity demanded is not equal to total quantity offered. In **Non-convex costs**, one seller supplies three units and two buyers demand two units and one unit respectively; the seller's lowest cost per unit exceeds the highest value per unit of the larger buyer, so efficient trade may require price discrimination.

In **Empty core**, two sellers each supply one unit and two buyers demand one unit and a package of two respectively. Figure 1 shows the structure of possible trades and surplus in this scenario. There are three possible coalitions that can trade: (i) B_1 and S_1; (ii) B_1 and S_2; and (iii) B_2, S_1 and S_2. Given the distribution of costs and values, the maximum possible payment a seller can receive in a coalition with B_1 is 7, since otherwise B_1 and the other seller would form a blocking coalition. However, if a seller is paid 7 by B_1, the outcome is blocked by a coalition of B_2, S_1 and S_2, with each seller receiving a payment of 7.5. Similarly, a coalition of B_2, S_1 and S_2 can be blocked. B_1 can offer a seller 10 but the most B_2 can pay both sellers is $19/2 = 9.5$.

2 Mechanisms

I compare three market mechanisms.

Shapley value-based prices: This is a combinatorial market mechanism introduced by Lindsay (2018). A surplus-maximizing allocation based on the

Bilateral		
seller	supplies 1	$c_1 \sim \text{Uniform}(\{1, 2, \ldots, 19\})$
buyer	demands 1	$v_1 \sim \text{Uniform}(\{1, 2, \ldots, 19\})$
Competition		
seller 1	supplies 1	$c_1 \sim \text{Uniform}(\{1, 2, \ldots, 19\})$
seller 2	supplies 1	$c_2 \sim \text{Uniform}(\{1, 2, \ldots, 19\})$
buyer 1	demands 1	$v_1 \sim \text{Uniform}(\{1, 2, \ldots, 19\})$
buyer 2	demands 1	$v_2 \sim \text{Uniform}(\{1, 2, \ldots, 19\})$
Monopoly		
seller	supplies 1 or 2 units	$c_1 \sim \text{Uniform}(\{1, 2, \ldots, 9\})$
buyer 1	demands 1	$v_1 \sim \text{Uniform}(\{1, 2, \ldots, 9\})$
buyer 2	demands 1	$v_2 \sim \text{Uniform}(\{1, 2, \ldots, 9\})$
Mismatched q		
seller 1	supplies 4	$c_1 \sim \text{Uniform}(\{1, 2, \ldots, 19\})$
seller 2	supplies 5	$c_2 \sim \text{Uniform}(\{1, 2, \ldots, 19\})$
buyer 1	demands 2	$v_1 \sim \text{Uniform}(\{10, 11, \ldots, 19\})$
buyer 2	demands 6	$v_2 \sim \text{Uniform}(\{1, 2, \ldots, 19\})$
Non-convex costs		
seller	supplies 3	$c_1 \sim \text{Uniform}(\{10, 11, \ldots, 19\})$
buyer 1	demands 2	$v_1 \sim \text{Uniform}(\{4, 5, 6\})$
buyer 2	demands 1	$v_2 \sim \text{Uniform}(\{7, 8, \ldots, 14\})$
Empty core		
seller 1	supplies 1	$c_1 \sim \text{Uniform}(\{1, 2, \ldots, 7\})$
seller 2	supplies 1	$c_2 \sim \text{Uniform}(\{1, 2, \ldots, 7\})$
buyer 1	demands 1	$v_1 \sim \text{Uniform}(\{10, 11, \ldots, 14\})$
buyer 2	demands 2	$v_2 \sim \text{Uniform}(\{15, 16, \ldots, 19\})$

Table 1: Six market scenarios.

Notes: Each row describes one trader: role (seller/buyer), quantity supplied or demanded, and distribution of types. Here c_i denotes seller cost and v_j denotes buyer value. All distributions are discrete uniform over the stated set.

values reported in bids is implemented, and the surplus is divided among the traders with winning bids.

Linear prices: This is a combinatorial market mechanism based on Bichler et al. (2018). An allocation is implemented that maximizes reported surplus subject to the constraint that all buyers pay p_b per unit and all sellers receive p_s per unit with $p_b \geq p_s$.

Linear prices, no rationing: This is a combinatorial market mechanism based on Milgrom and Watt (2025). It works like the linear prices mechanism but adds one constraint: the prices are chosen so that there are no losing

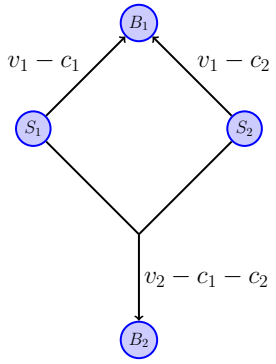


Figure 1: Empty core scenario: structure of possible trades and surplus.

Notes: B_1 can trade with either seller; B_2 demands a package of two units from both sellers. The ranges of costs and values are as follows: $c_1, c_2 \in \{1, \ldots, 7\}$, $v_1 \in \{10, \ldots, 14\}$, $v_2 \in \{15, \ldots, 19\}$.

buy bids with a bid per unit above p_b and there are no losing asks with an ask per unit below p_s. That is, there is no rationing.

The simplest way to compare the mechanisms is to assume sincere bidding. That is, traders bid their true cost or value. Panel A of Table 2 shows the predicted efficiency of the mechanisms under sincere bidding across the six scenarios. Efficiency is defined as the fraction of the potential gains from trade that are realized. For all scenarios, the Shapley value-based mechanism achieves 100% efficiency. The linear prices mechanisms achieve full efficiency in the scenarios where competitive prices exist (bilateral, competition and monopoly), but only achieve a fraction of the potential gains in the mismatched q and non-convex costs scenarios. In the empty core scenario, the linear prices, no rationing mechanism achieves 0.37 efficiency, while the other two mechanisms achieve 100% efficiency.

Panels B, C, and D of Table 2 are described in the following sections.

3 Computing equilibrium

A computational approach is used to find Bayesian Nash equilibria in each of the scenario-mechanism combinations. The approach uses logit quantal response equilibrium (LQRE) (McKelvey and Palfrey, 1995, 1998), applied to the *agent-normal form* of the Bayesian game: each (player, type) pair is treated as a separate agent who chooses an action. Let I denote the set of agents (player-type pairs) and J_i the number of actions available to agent i. Let s_{ij} denote agent i's jth action and π_{ij} the probability that i plays s_{ij}. Agent i's expected

	Bilateral	Competition	Monopoly	Mismatched q	Non-convex costs	Empty core
Panel A: Efficiency with sincere bidding						
Shapley value-based prices	1.00	1.00	1.00	1.00	1.00	1.00
Linear prices	1.00	1.00	1.00	0.95	0.12	1.00
Linear prices, no rationing	1.00	1.00	1.00	0.95	0.12	0.37
Panel B: Efficiency with computed equilibria						
Shapley value-based prices	0.72	0.93	0.80	0.86	0.64	0.83
Linear prices	0.72	0.94	0.74	0.83	0.08	0.90
Linear prices, no rationing	0.72	0.94	0.74	0.83	0.08	0.65
Panel C: Efficiency in lab experiments						
Shapley value-based prices	0.86	0.91	0.80	0.91	0.79	0.89
Linear prices	0.85	0.92	0.75	0.83	0.07	0.82
Linear prices, no rationing	0.85	0.91	0.87	0.81	0.02	0.62
Panel D: Efficiency with optimal mechanisms						
BIC	0.88	0.97	0.93	0.97	0.87	1.00
DSIC	0.79	0.91	0.84	0.90	0.75	1.00

Table 2: Efficiency in the six scenarios under different mechanisms and bidding strategies

Notes: Efficiency is the proportion of potential gains from trade realised. Panel A: efficiency under sincere bidding. Panel B: predicted efficiency at the computed Nash equilibrium. Panel C: realised efficiency in the lab. Panel D: efficiency of optimal Bayesian incentive compatible (BIC) and dominant-strategy incentive compatible (DSIC) mechanisms.

payoff from playing s_{ij} when the other agents play the mixed strategy π_{-i} is

$$\bar{u}_{ij}(\pi) = u_i(s_{ij}, \pi_{-i}),$$

where the expectation is taken over the types of the other players according to the prior distribution. A logit quantal response equilibrium is a profile π satisfying

$$\pi_{ij} = \frac{e^{\lambda \bar{u}_{ij}(\pi)}}{\sum_{k=1}^{J_i} e^{\lambda \bar{u}_{ik}(\pi)}}.$$

When $\lambda = 0$, all actions are played with equal probability; the limit points as $\lambda \to \infty$ are Nash equilibria.

Nash equilibria are found using homotopy continuation methods similar to Turocy (2005). The parameter is initialised at $\lambda = 0$ and gradually increased. I exploit features of market games that allow this approach to find equilibria in larger games than standard methods would allow.

The efficiency predictions are reported in Panel B of Table 2. In nearly all cases, the equilibrium efficiency levels are below the sincere bidding predictions. This can be attributed to strategic bidding, that is sellers marking up their costs and buyers shading their values.

Figure 2 shows an example of the computed equilibrium bidding strategies for the bilateral trade scenario. In this scenario, all three mechanisms are equivalent. The plots are heat maps showing the distribution of bids for different private-value draws. The filled gray circles indicate the cells where private value and bid are equal. In this equilibrium, trade occurs only when both the buyer and seller bid exactly 10. Buyers with a private value above 10 bid 10 with certainty, since trading at price 10 yields a strictly positive payoff. Buyers with a value of 10 are indifferent between trading at price 10 and not trading, so all bids in $\{0, \ldots, 10\}$ yield zero payoff and are played with equal probability. Buyers with a value below 10 would make a loss by trading at price 10, so all bids in $\{0, \ldots, 9\}$ yield zero payoff and are played with equal probability. The seller side is analogous. It can be readily verified that this is indeed a Nash equilibrium.

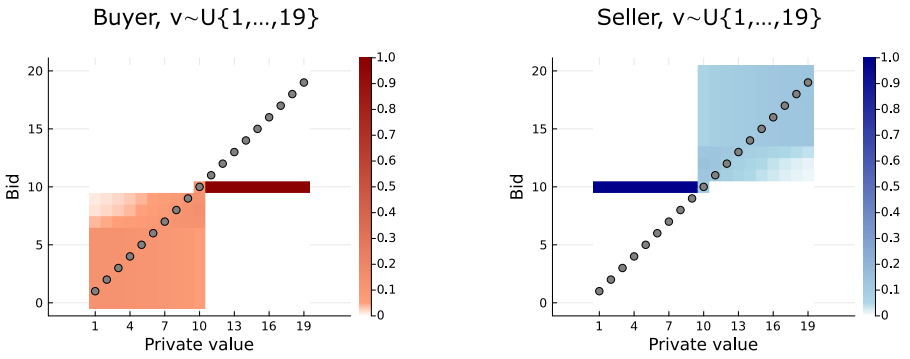


Figure 2: Equilibrium bidding in the bilateral trade scenario

4 Lab experiments

To test how the mechanisms perform with human traders, I ran a laboratory experiment. It had three treatments (one for each mechanism). There were 167 participants (53-58 in each treatment). Each participant completed up to 30 market rounds. Each market round was one of the six scenarios. Subjects were re-matched and received new cost/value draws each round. Not every participant was active every round, since the number of subjects in the session was not necessarily divisible by the number of traders in the scenario. Participants were paid based on their earnings in a randomly selected round in which they were active.

The realised efficiency levels are reported in Panel C of Table 2. The largest differences between treatments occur in the non-convex costs and empty core scenarios. This is consistent with the theoretical predictions. The Shapley value-based mechanism achieves the highest realised efficiency overall. Notice that for some scenario-mechanism combinations, the lab efficiency values exceed the computed equilibrium efficiency values.

Figure 3 illustrates bidding behaviour using the Mismatched q scenario under Shapley value-based pricing. Each filled circle represents an observed bid; the shading shows the predicted bid distribution from the LQRE model with precision parameter $\lambda = 1.6$ (the maximum likelihood estimate).

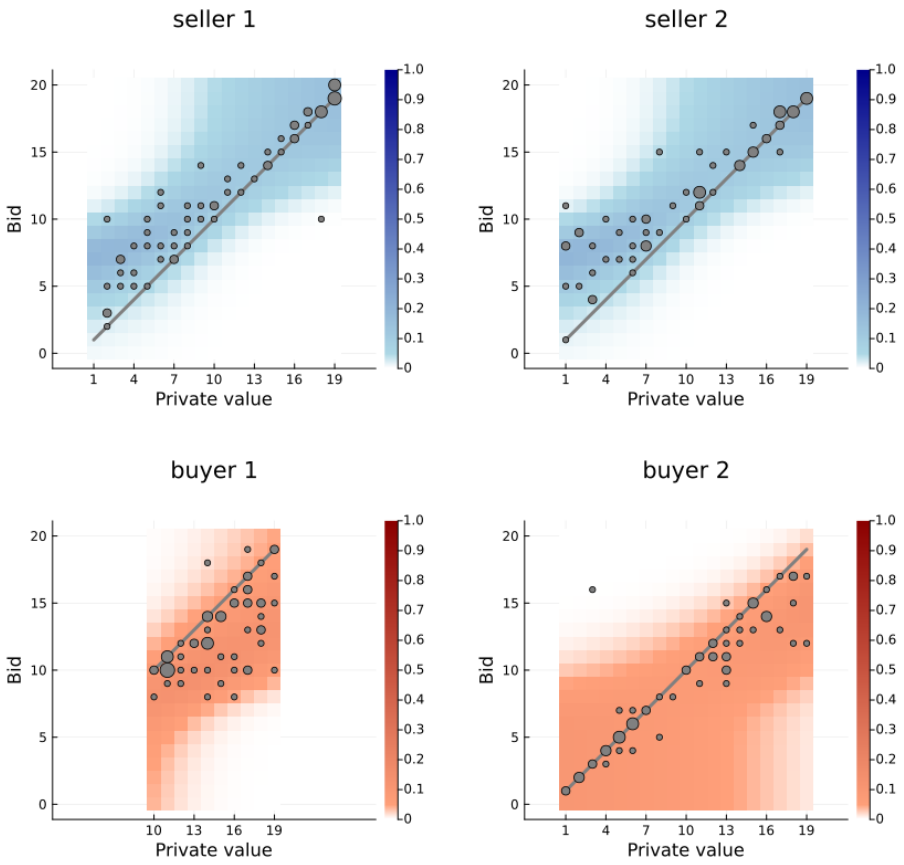


Figure 3: Mismatched q: observed and predicted bids ($\lambda = 1.6$).

To explore bidding behaviour, I estimated the following regression model. For each scenario s and role r (seller or buyer), the bid is regressed on the trader's type (cost for sellers, value for buyers) and treatment dummies for the

	Bilateral				Competition				Monopoly					
	Sell		Buy		Sell		Buy		Sell item 1		Sell item 2		Buy	
Type	0.78	**	0.77	**	0.82	**	0.84	**	0.74	**	0.73	**	0.83	**
	(0.03)		(0.03)		(0.02)		(0.02)		(0.05)		(0.06)		(0.02)	
Linear prices	-0.62		0.08		-0.19		0.20		0.23		0.27		-0.10	
	(0.37)		(0.47)		(0.38)		(0.36)		(0.29)		(0.39)		(0.24)	
LP, no rationing	-0.36		0.27		-0.46		0.29		0.42		0.38		-0.12	
	(0.36)		(0.31)		(0.31)		(0.30)		(0.33)		(0.39)		(0.19)	
Intercept	3.80	**	0.82	*	3.17	**	0.24		2.10	**	2.48	**	0.37	*
	(0.43)		(0.35)		(0.33)		(0.26)		(0.30)		(0.36)		(0.18)	
n obs.	410		410		380		380		195		195		389	

Table 3: Bidding regressions: Bilateral, Competition and Monopoly scenarios.

Notes: Dependent variable: bid. Type is cost (sellers) or value (buyers). Baseline mechanism: Shapley value-based prices; Linear prices and LP no rationing are treatment dummies. Standard errors in parentheses. ** $p < 0.01$, * $p < 0.05$.

non-baseline mechanisms:

$$\text{bid}_{i,s,r} = \alpha_{s,r} + \beta_{s,r}\,\text{Type}_{i,s,r} + \delta^{\text{LP}}_{s,r}\,D^{\text{LP}}_{i} + \delta^{\text{LPNR}}_{s,r}\,D^{\text{LPNR}}_{i} + \varepsilon_{i,s,r}, \qquad (1)$$

where i indexes the observation, Type is cost or value, D^{LP} and D^{LPNR} are dummies for Linear prices and Linear prices (no rationing), and the baseline mechanism is Shapley value-based prices. Coefficients are estimated separately by scenario and role.

The results are reported in Tables 3 and 4. Several patterns are apparent. First, there is evidence of strategic bidding. Sincere bidding would produce a type coefficient of one and an intercept of zero. Instead, we observe type coefficients are positive but less than one. The intercept is also positive, and greater for sellers than for buyers. Second, moving from bilateral trade to competition, the type coefficients get closer to one and intercepts closer to zero, suggesting bidders respond to the level of competition. Third, there is no obvious pattern in the dummy coefficients for the non-baseline mechanisms. This is consistent with bidders behaving similarly across mechanisms.

5 Optimal Mechanisms

To put the mechanisms in a broader context, I compute optimal mechanisms for each scenario using linear programming. First, I consider Bayesian Incentive Compatible (BIC) mechanisms. The linear program chooses an allocation rule and a payment rule to maximise ex ante expected social surplus, where the expectation is taken over the type distribution. The following constraints are imposed:

Ex-post individual rationality (IR): If players truthfully report their types, they receive a non-negative payoff.

Ex-post budget balance (BB): The total amount of money paid by buyers is equal to the total amount of money received by sellers.

	Mismatched Q				Non-convex cost				Empty core			
	Sell		Buy		Sell		Buy		Sell		Buy	
Type	0.76	**	0.77	**	0.82	**	0.81	**	0.61	**	0.66	**
	(0.02)		(0.03)		(0.05)		(0.04)		(0.07)		(0.06)	
Linear prices	0.03		-0.63		-0.06		-0.27		0.86		-0.26	
	(0.37)		(0.57)		(0.47)		(0.33)		(0.43)		(0.63)	
LP, no rationing	-0.01		-0.26		-0.33		-0.27		0.32		0.38	
	(0.34)		(0.39)		(0.38)		(0.32)		(0.48)		(0.51)	
Intercept	4.07	**	1.46	**	3.10	**	0.97	*	3.80	**	2.21	**
	(0.36)		(0.43)		(0.80)		(0.42)		(0.41)		(0.76)	
n obs.	379		380		195		390		379		379	

Table 4: Bidding regressions: Mismatched q, non-convex costs and empty core scenarios.

Notes: Dependent variable: bid. Type is cost (sellers) or value (buyers). Baseline mechanism: Shapley value-based prices; Linear prices and LP no rationing are treatment dummies. Standard errors in parentheses. ** $p < 0.01$, * $p < 0.05$.

Bayesian Incentive Compatible (BIC): For all players, if the other players report their true types, the player's expected payoff is maximised by reporting their true type.

Second, I consider Dominant-Strategy Incentive Compatible (DSIC) mechanisms. The same objective is maximised and the constraints are as follows:

Ex-post individual rationality (IR): As above.

Ex-post budget balance (BB): As above.

Dominant-Strategy Incentive Compatible (DSIC): For all players, reporting their true type maximises their expected payoff regardless of the other players' strategies.

The efficiency of the optimal mechanism is reported in Panel D of Table 2. Note that the BIC efficiency values must always be greater than or equal to the DSIC efficiency values. Similarly, by the revelation principle, the BIC efficiency values are greater than or equal to the computed equilibria efficiency values. In some cases, the lab efficiency values exceed the DSIC efficiency values but not the BIC efficiency values.

6 Conclusion

This study compared equilibrium bidding in three package market mechanisms across six market scenarios, ranging from simple bilateral trade to settings with non-convex costs and an empty core.

- I developed computational methods to find Nash equilibria in these markets with private values, using an LQRE homotopy approach that allows larger games to be analysed and selects a single equilibrium.

- I ran laboratory experiments to test how the mechanisms perform with human bidders, with three treatments (one per mechanism) and 167 participants.
- Shapley value-based pricing performs best overall, both in predicted equilibrium efficiency and in realised efficiency in the lab, with the largest advantages in the non-convex costs and empty core scenarios where linear pricing mechanisms perform poorly.
- Shapley value-based pricing has been used in practice to trade environmental offsets, and the results here support its use in package markets where linear prices may fail.
- Finally, I computed the BIC and DSIC optimal mechanisms for each scenario. The BIC efficiency values exceed the lab efficiency values by 2 to 11 percentage points. In contrast, there is no systematic ranking of DSIC and lab efficiency values.

References

Bichler, M., V. Fux, and J. Goeree: 2018, 'A Matter of Equality: Linear Pricing in Combinatorial Exchanges'. *Information Systems Research* **29**(4), 1024–1043.

Chatterjee, K. and W. Samuelson: 1983, 'Bargaining under Incomplete Information'. *Operations Research* **31**(5), 835–851.

Lindsay, L.: 2018, 'Shapley value based pricing for auctions and exchanges'. *Games and Economic Behavior* **108**, 170 – 181. Special Issue in Honor of Lloyd Shapley: Seven Topics in Game Theory.

McKelvey, R. D. and T. R. Palfrey: 1995, 'Quantal Response Equilibria for Normal Form Games'. *Games and Economic Behavior* **10**(1), 6–38.

McKelvey, R. D. and T. R. Palfrey: 1998, 'Quantal Response Equilibria for Extensive Form Games'. *Experimental Economics* **1**, 9–41.

Milgrom, P. and M. Watt: 2025, 'A Walrasian Mechanism with Markups for Nonconvex Economies'. *Review of Economic Studies*.

Myerson, R. B. and M. A. Satterthwaite: 1983, 'Efficient Mechanisms for Bilateral Trading'. *Journal of Economic Theory* **29**(2), 265–281.

Savani, R. and T. L. Turocy: 2025, 'Gambit: The package for computation in game theory, Version 16.3.0'.

Turocy, T. L.: 2005, 'A dynamic homotopy interpretation of the logistic quantal response equilibrium correspondence'. *Games and Economic Behavior* **51**(2), 243–263. Special Issue in Honor of Richard D. McKelvey.