ENDOGENOUS INSTITUTIONAL STABILITY*

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Abstract. Members of an organization must be assigned to positions—legislators to committees, civil servants to postings, workers to teams. This choice over allocations—and more broadly, the assignment procedure that produces them—constitutes an institutional choice that is agreed upon by the very members being assigned. If a coalition can arise endogenously and vote for some alternative allocation, the institution is destabilized. I explore this notion of endogenous institutional stability by bringing together matching theory and social choice. Correlated preferences over positions generate envy and facilitate the formation of large reformist coalitions. I show that institutional stability under majority rule exists robustly even under significant preference correlation, whereas complete popular matching—the plurality rule analog—collapses even under slight correlation. Given the prevalence of (super-)majority rules for internal governance in organizations worldwide, this helps explain why institutional stability persists in practice even when members' preferences are far from idiosyncratic.

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1. **Introduction.**

Institutions are important and enduring, yet they are not unchanging. As North (1990) emphasizes, "institutions are created by human beings. They evolve and are altered by human beings."

Much scholarly attention has focused on institutional change driven by external actors who upend fundamental principles through revolutionary overthrow, elite replacement, or exogenous shocks that disrupt prevailing arrangements. Yet institutions are also reshaped from within, by the very members whose behavior they govern. This paper studies that internal process.

In this paper, I study the assignment of members to positions within an organization, a key institutional choice faced by many organizations. For example, political parties must decide how to assign their politicians to legislative committees, the bureaucracy must structure internal labor markets for the assignment of civil servants to various postings, and firms must organize personnel across various roles and teams. The choice of allocation and, more broadly, the rules and processes by which internal assignments are made, represent a particular choice of institution. But to have effect, these allocation choices and rules require the organization's members to adhere to them. If members are dissatisfied with their allocation, endogenous coalitions can mobilize internally and lobby for change. Depending on which coalition prevails, such institutional reform can generate both winners and losers. Endogenous institutional stability exists when an organization's internal allocations of members to positions are robust to coalitional challenges from within.

Understanding endogenous institutional stability requires a merging of two fields: matching theory and social choice. The assignment of people to positions is formally a matching, generated by an underlying matching mechanism. Social choice studies how groups make collective choices. While typically applied to the choice of a single outcome, I apply it to the choice of a matching, a vector of assignments where each agent gets a different position within the organization over which they might have varied preferences. Studying this *social allocation choice* problem provides insights into when organizational assignments persist and when they

are vulnerable to coalitional reform. Which coalitions arise endogenously to change the allocation in their favor? Are certain assignment procedures more robust to reform? What preference environments give rise to pervasive instability?

Institutional stability is undermined if there exists a coalition of members that can implement an alternative allocation by voting in its favor, *potentially at the expense of those not in the coalition*.[1] Each voting rule used by such a self-governing organization to adjudicate between allocations—majority, plurality, or unanimity—therefore corresponds to a different version of institutional stability: majority stability, popular matching, and Pareto efficiency, respectively.

The canonical notion of stability in one-sided matching, the core or Pareto efficiency, corresponds to unanimity rule. Pareto efficiency ensures that no unanimously preferred reallocation remains unexploited: no group of agents can reshuffle assignments among themselves in a way that makes all of them weakly better off and at least one strictly better off, without affecting others. However, in many organizations, change does not require unanimity; coalitions smaller than the full membership can successfully implement reforms.

I therefore study two additional voting rules—majority and plurality—that permit reallocations generating both winners and losers. I introduce *majority stability*: an allocation is majority stable if no alternative allocation makes a majority of agents strictly better off. Plurality rule corresponds to the notion of "popular matching" in the computer science literature (Abraham et al., 2007). Namely, an allocation is *popular* if there exists no other alternative allocation such that more people strictly prefer the alternative allocation compared to the number of people who strictly prefer the original allocation.

A key insight that emerges from my analysis is that the existence of institutional stability depends on how correlated individuals' preferences are over positions. Greater correlation intensifies competition and generates envy over each other's assignments, making it easier to assemble coalitions that benefit from a change to the matching.

[1]Although I define institutional stability as a property of the matching with respect to coalitional deviations, my analysis also speaks to the stability of the broader institutional structure—the matching mechanism—that generated this matching. I highlight properties of matching mechanisms that strengthen or weaken institutional stability.

However, I show that the effect of preference correlation depends sharply on the voting rule, which determines which coalitions are decisive.

While Pareto efficient allocations under unanimity rule always exist and are easily attained by mechanisms such as Serial Dictatorship, they set a relatively weak benchmark for institutional stability, as they do not protect against reforms that create both winners and losers.

For majority rule, I establish a Chaos Theorem—in the spirit of McKelvey (1976), but adapted to the discrete structure of one-to-one matching using group-theoretic techniques. I show that with perfectly or near-perfectly correlated preferences, a sequence of majority-supported reassignments can move the organization from any allocation to any other.

However, this fragility does not persist at intermediate levels of correlation. For positively but imperfectly correlated preferences, majority stability is robust across a wide range of environments. The key constraint restricting majority overrules is a *packing problem*: a majority cannot be reassigned into a minority of positions. Thus, majority overrules require a majority to be moved into a new set of seats using some *chains of envy*: i's seat given to j, j's seat given to k, etc. While correlation increases the prevalence of envy, it does not guarantee that envy aligns into such chains. Undermining majority stability requires not only sufficient envy, but also the right configuration of envy relationships.

By contrast, I show that complete (i.e., every agent is assigned to a distinct seat and no agent/seat is left unassigned) popular matchings are extremely fragile. Although their existence can be cleanly characterized, even the slightest correlation in preferences is sufficient to eliminate them. I demonstrate this both theoretically and empirically, and show that when they do exist, they impose stringent constraints on mechanism design, such as restrictions on priority orderings in Serial Dictatorship.

This comparison highlights an important distinction: plurality-based notions of stability are too fragile to account for observed institutional persistence, whereas majority and supermajority rules—commonly used in practice—are far more robust to correlated preferences. This robustness may explain why many organizations exhibit stable assignment structures despite substantial alignment in members' preferences over positions.

In the main analysis, I assume selfish preferences, whereby agents care only about their own assignments from the vector of assignments. Relaxing the assumption to allow selfish but interdependent preferences—where agents may have arbitrary preferences over allocations that leave their own assignment unchanged—weakly reduces the set of majority stable allocations. However, no analogous monotonicity result holds for popular matchings.

I identify preference environments that support institutional stability. Determining the existence of majority stable allocations is computationally challenging, as changes in the status quo allocation alter the entire structure of envy and coalition incentives. I show that existence of majority stable allocations is not monotonic in preference correlation. However, I provide an integer linear programming formulation—computationally tractable in practice—to test whether a given allocation is majority stable.

Standard preference restrictions from social choice, such as weak single-peakedness or order restrictions, offer little traction in this setting. However, I provide computationally simple algorithms—based on minimizing average rank and greedy rank minimization—that perform extremely well in identifying majority stable allocations, albeit very rarely, failing to detect the existence of majority stable allocations given a set of preferences.

Finally, using simulations, I compare the performance of Serial Dictatorship and the Boston mechanism (as used respectively, for example, by Senate Republicans and Democrats in committee assignments) in generating majority stable and popular outcomes. The Boston mechanism's tendency to prioritize higher-ranked choices (under truthful reporting) reduces envy and enhances institutional stability relative to Serial Dictatorship. This comparison illustrates how institutional design—the choice of matching mechanism—shapes the resilience of institutions to endogenous coalitional reform.

Legislative organization provides the principal application. A large literature treats committee assignments as equilibrium outcomes of legislators' demands and the organizational objectives of parties and chambers (Shepsle and Weingast, 1987; Krehbiel, 1991; Cox and McCubbins, 2005; Thakur, 2025). I complement this literature by asking when a given party-specific allocation of legislators to committees—and the assignment procedure that produces it—is stable against subsequent challenges from within the party. Online Appendix A calibrates

the framework to the U.S. Senate committee-assignment problem and uses simulations to illustrate the sharp difference across notions of institutional stability: complete popular matchings are highly fragile to even modest preference correlation, whereas majority stable allocations continue to exist across a much wider range of preference environments. The framework, however, extends well beyond the U.S. Congress, connecting to a broader empirical literature on positional allocation within organizations across legislative systems.[2]

Analogous allocation problems arise in cabinet and shadow-cabinet appointments, civil-service postings, and the assignment of workers to roles and teams within firms. Across these settings, members' preferences are often substantially aligned: U.S. Senators consistently rank prestigious committees highly (Thakur, 2025), Indian civil servants tend to rank distressed and underdeveloped states near the bottom of their preferences (Thakur, 2018), and workers may compete for high-profile teams within firms (Cowgill and Koning, 2018). Yet the resulting assignments and the procedures that generate them frequently persist without successful internal reform. The analysis explains how such persistence can coexist with widespread competition and envy, while also identifying the conditions under which it may break down. Shocks that increase preference alignment—such as electoral changes that alter the composition of a legislature or crises that increase the jurisdiction, power, or credit-claiming opportunities attached to particular committees—can create the chains of envy required to turn a previously majority stable allocation into one vulnerable to internal reorganization.

2. **Related Literature.**

Addressing these new questions of institutional stability brings together three large but, until now, largely distinct literatures: matching theory, social choice, and the political economy of

[2]In parliamentary systems with strong party discipline, the internal caucus or parliamentary group often plays the role analogous to that of the full chamber in the U.S. Senate: it is the arena in which assignments are contested and coalitions form (Martin and Mickler, 2019). Parliamentary groups in the European Parliament, Bundestag, and many other legislatures distribute committee seats and leadership positions through procedures that are formally or informally majoritarian (Laver and Shepsle 1990, 1996). Westminster systems have analogous internal allocation problems: cabinet and shadow-cabinet portfolio assignments, and–since the 2010 Wright Reforms–select committee chair elections via secret ballots. The endogenous institutional stability analysis I develop applies directly to these settings, even though formal committee power varies considerably across institutional contexts.

endogenous institutional change; extending each and unifying them in a particularly important context.[3]

Political Economy of Endogenous Institutional Change: Within political economy, the study of endogenous institutional change examines how the rules governing collective life are themselves shaped by political processes. The classical treatment by North (1990) emphasizes that institutions evolve through both incremental adaptation and punctuated change, driven by shifting relative prices and the interests of organized actors. Building on a game-theoretic foundation, Greif and Laitin (2004) develop a dynamic account of how institutions persist in changing environments and how the very processes they unleash can lead to their own demise—introducing the concept of "quasi-parameters," features of the environment that an institution shapes endogenously over time and that can ultimately render it self-undermining. This paper contributes a formal mechanism of analogous character: changes in the correlation structure of members' preferences can render previously stable allocations vulnerable to majority challenge, even without any exogenous shock to the external environment. More broadly, the paper speaks to a large literature on institutional change—sharing its concern for the interaction between formal rules and underlying power distributions (Acemoglu and Robinson, 2006), focusing on the dynamic stability of majority-chosen outcomes (Acemoglu et al., 2021), and highlighting how environmental shocks (e.g., shifts in preference correlations across members as a result of entry/exit) can trigger institutional change (Mahoney and Thelen, 2010).

More recent formal work examines the dynamic evolution of organizations with endogenous membership. Gieczewski (2021) studies organizations—such as cities, trade unions, and cooperatives—in which current members choose policy, policy changes attract or repel future members, and the resulting feedback dynamics converge to "slippery slope" equilibria even when agents are forward-looking. In his setting, membership evolves through entry and exit; in contrast, in my setting allocations evolve through internal coalition formation and voting. These mechanisms are complementary: entry-and-exit dynamics determine who participates

[3]For a very different focus and approach in combining matching theory and social choice, see Pycia and Unver (2020), who characterize properties of matching mechanisms satisfying Arrovian efficiency and auditability, and Afacan and Bo (2022), who consider strategy-proof implementations of ω-popular matchings under weighted voting.

in allocation decisions, while institutional stability governs which allocations survive given a fixed membership. Gieczewski and Kosterina (2024) extend this framework to policy experimentation, showing that organizations may persist indefinitely with failing policies because unsuccessful experiments drive out conservative members, thereby shifting the pivotal vote. Their analysis underscores that the composition of the decisive coalition is itself endogenous to institutional design—a theme that is also central to my framework.

In focusing on the stability of institutional choices themselves, this paper shares the underlying spirit of the literature on self-stable voting rules and constitutional choice (Barbera and Jackson, 2004; Messner and Polborn, 2004; Maggi and Morelli, 2006; Koray, 2000). However, rather than choosing a voting rule, agents in my model choose the matching mechanism/allocation itself. In doing so, the analysis highlights the importance of correlation across agents' preferences for the stability of institutional choice. The paper also shares some philosophical underpinnings with the literature on coalition formation and non-transferable utility games (Aumann and Dreze, 1974; Banerjee et al., 2001), as the voting rule (e.g., majority, plurality, unanimity) and agents' preferences over positions jointly determine the set of outcomes that coalitions can achieve.

Social Choice: In social choice, a single outcome is selected for a group of agents, each with potentially distinct preferences on what they prefer. When the outcome of choice is a vector of assignments, as in this setting, an intricate structure of preference restrictions is placed on the social choice problem. This paper analyzes this important sub-case of social choice: *social allocation choice* over a vectors of assignments, where each agent receives their own assignment. Agents might have varied preferences over their own assignment in this vector, leading to intricate preference restrictions within the broader social choice problem. Agents collectively vote on whether to retain the allocation produced by the matching mechanism or to adopt an alternative mechanism/allocation. A central objective of the paper is to distinguish between the institutional stability of certain allocations and that of the mechanisms that generate them.

Depending on the voting rule that determines which coalitions are decisive, institutional stability generalizes canonical notions of stability. In one-sided markets, stability asks whether an allocation admits a coalition that can reassign positions among its members such that all

are weakly better off and at least one is strictly better off. Such coalitional deviations are local, leaving non-members' assignments unchanged. In addition to Pareto efficiency, I analyze broader notions of institutional stability—such as popular matching and majority stability—under which coalitions can implement global deviations that benefit their members, potentially at the expense of others. This tension between coalitions seeking to reorganize allocations in their favor, thereby creating both winners and losers in the process, lies at the heart of institutional change in real organizations. The paper therefore studies endogenous coalition formation and characterizes the environments in which self-governing bodies are susceptible to reorganization, as well as how internal decision rules (e.g., voting thresholds) shape institutional stability and evolution.

Matching: A small but growing literature on popular matching (i.e., institutional stability under plurality rule) and its variants has developed within computer science (see the surveys by Manlove (2013) and Cseh (2017)). This literature studies the existence, characterization, and computational complexity of finding Condorcet winners under plurality rule in one-sided settings (Abraham et al., 2007; Sng and Manlove, 2010; Manlove, 2013; McDermid and Irving, 2011; Kavitha and Nasre, 2009; McCutchen, 2008; Kavitha et al., 2011), as well as in two-sided settings on bipartite graphs (Huang and Kavitha, 2013; Kavitha, 2014; Cseh et al., 2017; Cseh and Kavitha, 2016) and non-bipartite graphs (Chung, 2000; Biro et al., 2010; Huang and Kavitha, 2017). Its emphasis on algorithmic complexity has often rendered it a normatively appealing but relatively abstract solution concept, limiting its uptake in economics and applied market design. This paper shows that the existence of complete popular matchings (i.e., under plurality rule) imposes stringent requirements on the preferences of individuals and the design details of the matching mechanisms. Even slight correlation across agents' preferences renders complete popular matching non-existent. By contrast, many real-world organizations rely on majority or supermajority decision rules. Consistent with this, I show that allocations satisfying majority stability—a new notion of institutional stability introduced here—exist robustly even under substantial preference correlation.

Recent work by Aziz et al. (2023) builds on my notion of majority stability to study the computational complexity of verification and existence of k-stable allocations—those for which

no alternative matching benefits at least k out of N agents—in both one-sided and two-sided environments. They show that existence of majority-stable allocations in my one-sided setting is NP-complete. Complementing this, I show that verifying whether a given allocation is majority stable can be done in polynomial time via a linear programming approach (Section 5.2.1), and I develop algorithms that identify majority stable allocations with high reliability in practice. More broadly, the paper provides intuition for these computational findings and situates institutional stability as a practically relevant concept.

Finally, building on Theorem 1, which highlights the potential power of the agenda-setter, Thakur and Bendor (2024) analyze the dynamic path of organizational restructuring when agents value not only their own assigned positions but also organizational output, the welfare of their friends, and the composition of their teams.

3. Voting rules and institutional stability.

The criteria underlying different notions of institutional stability are determined by the voting rule used to choose between allocations. In this section, I introduce three such voting rules—plurality rule, majority rule and unanimity rule—and their corresponding institutional stability counterparts: popular matching, majority stability, and Pareto efficiency.

3.1. The Set-Up.

Consider a set of agents $\mathbb{I}$ and a set of seats $\mathbb{C}$, indexed by i and c, of sizes N and C, respectively. For ease of exposition, assume both N and C are odd. Let $\varnothing$ denote remaining unassigned. Each agent $i \in \mathbb{I}$ has a complete and transitive preference relation $\succeq_i$ over seats to which they may be assigned or remaining unassigned in $\mathbb{C} \cup \{\varnothing\}$ (referred to as i's ***assignment***). Seats themselves have no preference over agents, so the market is one-sided. If an agent i is not indifferent between two assignments ($\sim_i$ denotes indifference), then they have a strict preference ($\succ_i$). If agent i prefers remaining unmatched rather than being assigned to seat c, i.e., if $\varnothing \succ_i c$, then seat c is said to be unacceptable to i.

To measure the extent to which agents' preferences over seats are correlated, I use a ***correlation coefficient*** that generalizes the Spearman rank correlation coefficient to N rank order

preference lists (Thakur, 2018).[4] Formally, for preference rankings r_{ic} of agents $i = 1, ..., N$ over seats $c = 1, ..., C$, the correlation coefficient is given by

$$\rho = 1 - \frac{\sum_{c=1}^{C} \left(\frac{1}{N-1} \sum_{i=1}^{N} (r_{ic} - \overline{r}_c)^2 \right)}{C\left(\frac{(C-1+1)^2-1}{12} \right)}$$

where $\overline{r}_c = \frac{1}{N} \sum_{i=1}^{N} r_{ic}$.

An ***allocation***, or ***matching***, is defined as $M : \mathbb{I} \cup \mathbb{C} \to \mathbb{I} \cup \mathbb{C} \cup \{\varnothing\}$ such that for every $i \in \mathbb{I}$ and $c \in \mathbb{C}$, $i = M(c)$ if and only if $M(i) = c$, $M(i) \in C \cup \{\varnothing\}$, and $M(c) \in \mathbb{I} \cup \{\varnothing\}$. That is, an agent is matched to a seat if and only if the seat is matched to the agent, and every agent and seat is either matched or remains unmatched. Agent i is said to ***envy*** agent j under matching M if $M(j) \succ_i M(i)$.

Let $\mathbb{M}$ denote the set of all possible matchings. In addition to preferences over individual seats, agents have preferences over complete matchings. Agent i's preference over two allocations $M, M' \in \mathbb{M}$ is denoted by $M \succeq_i^{\mathbb{M}} M'$. The relationship between preferences over seats and preferences over complete matchings is specified in the next subsection. Let R denote the set of complete and transitive binary preference relations over $\mathbb{M}$ and let $\mathcal{R}$ denote the set of binary relations over $\mathbb{M}$.

Social allocation choice studies the social preference relation $\succeq^S \in \mathcal{R}$ over allocations $M, M' \in \mathbb{M}$ induced by aggregating individual preferences of all agents using some preference aggregation rule $f : R^N \to \mathcal{R}$.

Social allocation choice is thus a special case of social choice in which the outcomes being chosen are *vectors of assignments*, $M \in \mathbb{M}$, assigning each agent a potentially different seat.[5]

[4]This measure is an equally weighted average of Spearman rank correlation coefficients across all distinct pairs of individuals. Specifically, it captures the variance in rankings of each alternative across agents relative to its average ranking, and then aggregates this across all alternatives (numerator). This quantity is then normalized against the variance of a discrete uniform distribution (denominator). If all agents have identical preference rankings, the numerator is 0, and $\rho = 1$ (i.e., perfect positive correlation).
If preference rankings are independently random, each seat's cross-agent rank variance approaches the variance of a discrete uniform distribution over $\{1, ..., C\}$. Summing across the C seats, the numerator approaches $C(C^2 - 1)/12$ and hence $\rho \to 0$ (i.e., no correlation). Note that finite sample values of ρ can be negative, as random independent rankings yield ρ near zero, but zero is not necessarily the lower bound.

[5]In many-to-one settings (e.g., civil servant assignment to states in India (Thakur, 2018)) or many-to-many settings (e.g., U.S. Senate committee assignments (Thakur, 2025)) multiple agents can be assigned the same position.

However, this special case is substantively important across many organizational settings as it captures endogenous envy and distributional conflict over positions within organizations. Moreover, it determines which groups benefit from, oppose, or form coalitions around proposed institutional reforms, and can therefore shape both the evolution and the persistence of assignment institutions.

3.2. **Institutional Stability Concepts & Corresponding Voting Rules.**

I define three notions of institutional stability corresponding to three preference aggregation rules. Let $|M' \succ_i^{\mathbb{M}} M|$ denote the number of agents who strictly prefer matching M' over matching M.

Definition 1. *A matching M is **popular** if $\nexists M'$ such that $|M' \succ_i^{\mathbb{M}} M| > |M \succ_i^{\mathbb{M}} M'|$.*

Definition 2. *A matching M is **majority stable** if $\nexists M'$ such that $|M' \succ_i^{\mathbb{M}} M| \geq \frac{N+1}{2}$.*

Definition 3. *A matching M is **Pareto efficient** if $\nexists M'$ such that $|M \succ_i^{\mathbb{M}} M'| = 0$ and $|M' \succ_i^{\mathbb{M}} M| > 0$.*

Assuming that agents who are indifferent between two allocations abstain from voting, popular matching corresponds to f being plurality rule (or relative pairwise majority rule), majority stability corresponds to a majority rule, [6] and Pareto efficiency corresponds to unanimity rule. I also define a matching M to be *weak Pareto efficient* if $\nexists M'$ such that $|M' \succ_i^{\mathbb{M}} M| = N$.

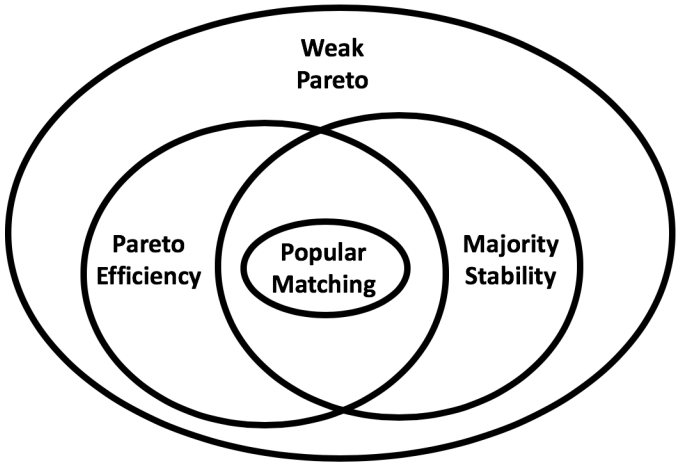
As shown in Figure 1, the set of popular matchings is a subset of both the set of majority stable matchings and the set of Pareto efficient matchings. However, the sets of majority stable and Pareto efficient matchings are not nested within one another. Appendix C provides examples and details.

Unless otherwise stated, I will maintain the following assumptions for ease of exposition and focus on complete, one-to-one matchings, in which every agent is assigned to a distinct seat and no agent or seat remains unmatched:

Assumption 1. *All allocations are feasible and all seats in $\mathbb{C}$ are acceptable to every agent.*

[6] More generally, a super-majority rule with threshold Q can be defined by replacing $\frac{N+1}{2}$ with Q in Definition 2.

Figure 1. Set inclusions for matchings across various institutional stability notions.



Restricting feasibility or acceptability of alternative allocations can only weakly expand institutional stability–unless an institutionally stable allocation itself is ruled out–as it removes potential challengers.

Assumption 2. *Preferences are strict.*

Coarsening preferences–which, after representing each multi-capacity position by identical seat copies, can be used to model many-to-one matching–weakly enlarges the set of majority stable matchings, though no corresponding general monotonicity result holds for popularity or Pareto efficiency.

Assumption 3. *The market is balanced with* $N = C$.

Assumption 4. *Agent* i *cares only about their own assignment. Namely,* $M \succeq_i^{\mathbb{M}} M'$ *if and only if* $M(i) \succeq_i M'(i)$.

Thus, an agent's preference over allocations depends solely on their own assignment. In particular, agent i is indifferent among all allocations that assign them the same seat. This assumption follows the standard matching framework, in which preferences are independent of others' assignments.

In Section 7, I relax this assumption to allow for interdependent preferences. I show that introducing such preferences—provided agents do not vote against their own assignment—leads

to weakly more institutional instability under majority stability. No analogous monotonicity result holds for Pareto efficiency.

Depending on the preference aggregation rule used, different notions of institutional stability protect against different classes of local and global deviations. Under Assumptions 1-4, Pareto efficiency is equivalent to the absence of a "local" blocking coalition that can make all its members weakly better off, and at least one member strictly better off, by permuting only the seats currently assigned to the members of the coalition. That is, instability arises when a coalition can reallocate its own assignments without affecting those of non-members. By contrast, popular matching and majority stability rule out more "global" deviations in which a coalition proposes an alternative that reorganizes the entire allocation in a way that benefits its members and may harm agents not in the coalition who vote against it.

4. Correlated Preferences Undermine Institutional Stability.

As agents' preferences over seats become more correlated, competition over certain positions intensifies, generating greater envy over others' assignments. As a result, higher correlation generally makes it easier to mobilize larger coalitions that can benefit from alternative allocations, thereby undermining institutional stability.

I first highlight the effects of extreme (or near-extreme) preference correlation on institutional stability (Section 4.1) and then discuss the implications of correlated preferences for organizations in practice (Section 4.2).

4.1. At or near the extremes of correlation across agents' preferences.

The impact of correlated preferences on institutional stability is most transparent at the extremes. When preferences are completely uncorrelated (Example 1(a)), the envy-free allocation $(1-A, 2-B, 3-C)$ is majority stable, popular, and Pareto efficient. By contrast, perfectly correlated preferences (Example 1(b))— which correspond to a strict status hierarchy—render any social allocation choice institutionally unstable under both popular matching and majority stability (though any allocation remains Pareto efficient). For instance, regardless of who sits in the top position (seat A), a "crown-prince revolt" by those assigned to seats B and C can destabilize the allocation in such a hierarchical environment.

Example 1. *Consider agents* $\{1,2,3\}$ *ranking seats* $\{A,B,C\}$.

(a) Perfectly uncorrelated preferences: $\rho = 0$.

1	*2*	*3*
A	B	C
B	C	A
C	A	B

(b) Perfectly correlated preferences: $\rho = 1$.

1	*2*	*3*
A	A	A
B	B	B
C	C	C

To further illustrate the role of preference correlation at the extremes, I show that when agents' preferences are perfectly correlated, there exists a majority-approved sequence of reassignments from any allocation to any other allocation, in the spirit of McKelvey's Chaos Theorem (McKelvey, 1976). This result highlights the potentially unrestrained power of the agenda-setter.

Theorem 1. *When agents' preferences are perfectly correlated, there exists a majority-approved sequence of reassignments from any allocation to any other allocation, for* $N \geq 5$.

Proof: There are N seats to be assigned to N agents. Suppose all agents share identical preferences over seats, say $1 \succ 2 \succ ... \succ N$. Fix the order of seats to be $1, ..., N$.[7] An allocation hence defines a permutation of agents over this ordered sequence of seats. For example, an allocation denoted by $5, 2, 9$,... assigns agents $5, 2, 9$,... to seats 1,2,3,..., respectively. The set S of permutations induced by the feasible reassignments clearly forms a subgroup of S_N, the finite symmetric group on N objects.

The N-cycle $(1, 2, ..., N)$ is the movement from $1, 2, ..., N$ to $N, 1, 2, ..., N-1$. The $(1, 2)$ transposition is the movement from $1, 2, ..., N-1, N$ to $2, 1, 3, ..., N$. The well-known theorem for finite symmetric groups[8] states that the set of all possible permutations, S_N, is generated (i.e., a successive composition of these two operators) using these two adjacent movements, stated usually as $(1, 2), (1, 2, ..., N)$ generate the group S_N.

[7]This is key: if instead one fixes the order of agents and tries permutations of seats on this order, it does not form a group, as movements cannot be composed. Namely, the product of two movements which are individually majority-approved path from some initial allocation, cannot necessarily be combined to a majority-approved path (namely, the first movement leads to a new status quo, and hence the second movement might not be majority approved from this new status quo).

[8]Fraleigh, J. B. (2002). A First Course in Abstract Algebra (7th ed.). Pearson. p. 96.

In this setting, the N-cycle $(1, 2, ..., N)$ is allowed by N-1:1 vote. Moreover, the $(1, 2)$ transposition is shown with a sequence of majority-approved movements in Figure 2.[9] (While N-cycles work for $N \geq 3$ by majority rule, for $N = 3$ and $N = 4$, it is not possible to get any consecutive transposition like $(1, 2)$ or $(N, 1)$ by majority rule).

Figure 2. Sequence of allocations for $(1, 2)$ transposition.
Fixing the sequence of seats $1, ..., N$, the figure shows the sequence of majority-approved allocations $A_0, A_1, ..., A_{N-1}$ that lead to the $(1, 2)$ transposition. At each step in the sequence from $A_1, ..., A_{N-1}$, the bottom $N - 1$ agents are moved to the top, which is approved by $N - 1$ agents.

Seat	A_0	A_1	A_2	⋯	A_{N-2}	A_{N-1}
1	1	3	4	…	N	2
2	2	4	5	…	2	1
3	3	…	…	…	1	3
4	4	…	…	…	3	4
…	…	…	…	…	4	…
…	…	…	N-1	…	…	…
…	…	N-1	N	…	…	…
…	…	N	2	…	…	…
N-1	N-1	2	1	…	N-2	N-1
N	N	1	3	…	N-1	N

Thus S is the full group S_N. □

Corollary 2.1, which is rather the corollary of the proof method, illustrates that Theorem 1 is not merely a knife-edge result. It exploits the slack created by super-majority approval under fully correlated preferences.[10]

Corollary 2.1. *If $\left\lceil \frac{N}{2} \right\rceil + 3$ or more agents have fully correlated preferences, there exists a majority-approved sequence of reassignments from any allocation to any other allocation, for $N \geq 6$.*

[9] I thank anonymous referee for their suggestion of this simpler path with $N - 1$ movements.
[10] Lagrange's theorem also tells us that the number of unique allocations spanned by any initial allocation through majority-approved paths of reassignments must be a factor of $N!$. Note, however, that the set of allocations under an arbitrary set of preferences and majority rule might not form a group.

Proof: With all N agents having fully correlated preferences over seats $1 \succ 2 \succ ... \succ N$, for N even, Figure 2 gives an path of allocations that is approved in each step by at least $N-2$ agents. For N even, if preferences of at most $\frac{N}{2}-3$ agents were arbitrarily changed, each step of this path is still approved by at least $N-2-(\frac{N}{2}-3)=\frac{N}{2}+1$, which is a majority. Similarly, for N odd, if preferences of at most $\frac{N+1}{2}-3$ agents were arbitrarily changed, each step of this path is still approved by at least $N-2-(\frac{N+1}{2}-3)=\frac{N+1}{2}$, which is a majority. □

More generally, as the correlation across agents' preferences increases, the set of allocations reachable through majority-approved sequences tends to expand.

4.2. **What does correlated preferences mean for organizations in practice?**

The preceding analysis—combining (i) the intuition that correlated preferences intensify competition, generate envy, and enable larger coalitions to mobilize to reform the institution, and (ii) the absence of institutional stability under plurality or majority rule and the unchecked agenda-setting power at extreme levels of preference correlation—raises a natural question: what are the implications for real-world organizations?

One implication is that institutional reform may be triggered by sudden increases in correlation across agents' preferences over seats. For example, if a political party experiences an electoral wave concentrated in a particular region, newly elected members may share similar committee preferences (e.g., prioritizing agriculture committee as top choice, followed by foreign affairs as second choice, etc.). Such a shift in the distribution of preferences can facilitate the formation of large coalitions advocating institutional reform. A prominent example is Newt Gingrich's 1994 "Republican Revolution," in which the influx of 52 new Republicans entering the US House of Representatives resulted in a significant overhauling of seniority-based systems/entitlements in the committee assignments: e.g., the introduction of term limits on committee and sub-committee chair positions led to a broad reshuffling of committee assignments.

Alternatively, changes in the relative power or jurisdiction of positions can also increase preference correlation. For instance, a financial crisis elevate the importance of Finance, Banking and Commerce committees thereby aligning legislators' preferences towards these positions.

At the same time, many organizations exhibit persistent correlation in members' preferences over positions. U.S. Senators consistently seek prestigious committees such as Appropriations and Rules, Indian civil servants tend to avoid underdeveloped or conflict-prone regions, and workers often prefer teams engaged in high-profile or innovative projects (Thakur, 2025, 2018).

Given this, a natural puzzle arises: why do institutions often remain stable despite such correlated preferences? The next section addresses this question by comparing across different voting rules. In particular, I show how organizations governed by (super-)majority rules can sustain institutional stability even in the presence of intermediate levels of preference correlation.

5. Existence of institutionally stable allocations: Popular Matching vs. Majority Stability

In which preference environments do institutionally stable allocations exist? How do different voting rules—corresponding to different notions of institutional stability—compare in their ability to sustain stability under varying levels of correlation across agents' preferences? Which mechanisms can attain such institutionally stable allocations when they exist and what are constraints on such mechanisms? How computationally difficult is it to determine whether a given allocation is institutionally stable for a particular set of preferences? I address these questions in this section.

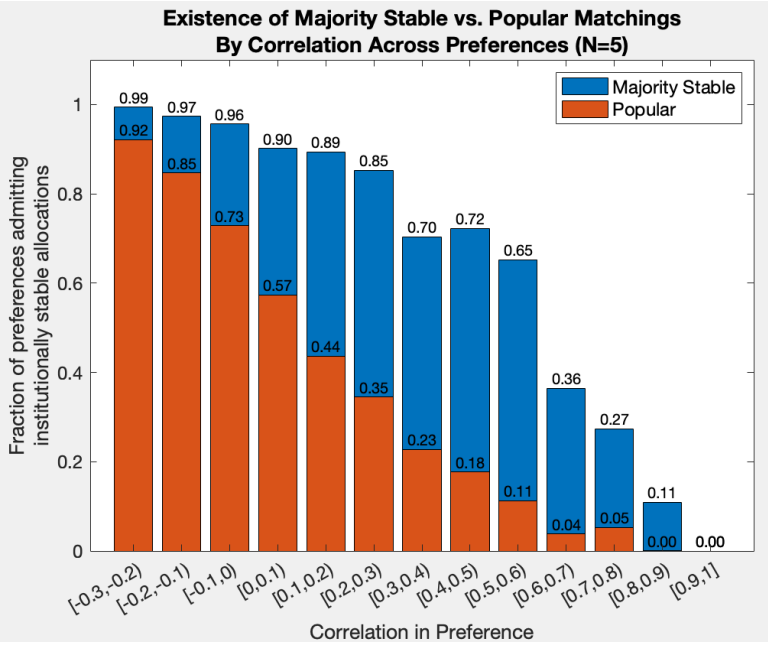
In economics, Pareto efficiency is often a central benchmark as it ensures that no surplus is left unrealized. In social allocation choice settings, Pareto efficient allocations always exist, regardless of the degree of correlation in preferences, and can be implemented using simple and widely used mechanisms such as Serial Dictatorship.[11]

[11]With strict preferences, Serial Dictatorship is Pareto efficient because the mechanism cannot make the nth most senior agent in the seniority ordering better off without making some $j \in \{1, ..., n-1\}$ seniorities worse off, otherwise agent j would have selected that alternative. Moreover, varying the seniority order allows Serial Dictatorship to span the entire set of Pareto efficient matchings. With weak preferences, however, tie-breaking can matter. For example, consider two agents $\{1, 2\}$ and two seats $\{A, B\}$ with $A \sim_1 B$ and $A \succ_2 B$, and suppose 1 is more senior to 2. Serial Dictatorship yields the allocation $(1-A, 2-B)$ which is pareto dominated by $(1-B, 2-A)$.

However, unanimity rule—under which every agent effectively holds veto power—can be overly permissive as a notion of institutional stability in practice. A single agent, or small group of agents, can block changes that would generate substantial gains for a large majority. As such, ruling out only unanimously preferred improvements sets a relatively low bar for institutional stability, especially in settings where institutional change typically produces both winners and losers.

To capture these dynamics, I next analyze environments in which endogenous coalitions seek to alter institutional arrangements in their favor, potentially at the expense of others. I examine this under plurality rule in Section 5.1 and under majority rule in Section 5.2.

Figure 3. Existence of Majority Stable vs. Popular Matchings By Correlation Across Preferences ($N = 5$).
Exhaustively checking whether there exists a majority stable allocation vs. popular matching for $N = 5$ for 100,000 simulated set of preferences of differing correlations for 5 agents over 5 seats, shows the striking robustness of majority stability existence—in comparison to popular matching existence—even when preferences across agents are considerably correlated. Each preference profile is generated independently by beginning with perfectly correlated rankings and then progressively reducing correlation through repeated random swaps of two positions in a randomly selected agent's ranking.



A central objective is to explain the striking robustness of majority-stable allocations, relative to popular matchings, in the presence of correlated preferences. The simulations in Figure

3 highlight this contrast. Even in the All-India Civil Service—where civil servants systematically rank under-developed and conflict-prone regions at the bottom of their preference lists and preference correlations are large enough to cause marked imbalances in allocations and outcomes—correlation coefficients lie in the range of 0.3 to 0.5 (Thakur, 2018). Even at these levels, approximately 70% of such preference profiles admit majority stable allocations. What explains this stark gap between this 70% existence rate for majority stability and the much lower (around 20%) existence rate of popular matching?[12]

5.1. **Plurality Rule and Popular Matching.**

Building on Abraham et al.'s (2007) characterization of popular matching, I show in this section that the existence of institutionally stable allocations under plurality rule—i.e., complete popular matchings—imposes extremely stringent conditions on both preferences and mechanisms.

Lemma 1. *(Abraham et al. (2007) Lemma 2.4)*
A matching M is popular if and only if

(1) *every seat that is someone's first choice (called a "f-post") is matched in M and*
(2) *each agent i is assigned to either their first choice (i.e., $f(i)$) or to their most preferred alternative that is not ranked first by any other agent (i.e., $s(i)$) called an "s-post"*

This characterization also allows for popular matchings that are not complete (i.e., some agents or seats are left unmatched) by introducing a "last resort option" $l(i)$ appended to the end of each agent i's preference ordering after all acceptable assignments as their least preferred option. This ensures that the s-post for any agent, $s(i)$, is non-empty, and assignment to $l(i)$ corresponds to remaining unmatched.

The key idea underlying Lemma 1 is as follows. If any agent i is assigned to a seat worse than their s-post, then either (i) i's s-post is empty– in which case assigning that seat to i would be favored by 1-to-0 vote—or (ii) i envies some agent j occupying j's s-post, while j in turn envies another agent k occupying their f-post (whereby a reassignment moving i to j's seat, j to k's

[12]The gap between existence of majority stable and popular matchings appears to widen with larger N, for reasons discussed below, although exhaustive simulations become computationally infeasible at larger scales.

seat, and k to i's seat would be favored by a 2-to-1 vote). Thus, proofing against 2-to-1 votes fully characterizes the set of popular matchings.

Accordingly, a popular matching of some size always exists. However, in practice—particularly, in balanced markets or organizations when the number of people equal the number of seats—it is neither reasonable nor feasible to leave agents unmatched or seats vacant. I therefore restrict attention to the more practically relevant case of *complete popular matchings*, in which no agent or seat is left unassigned/vacant (i.e., no assignment to last resort $l(i)$ occurs). How robust, then, is the existence of complete popular matchings to correlation in agents' preferences?

Using Hall's Marriage Theorem,[13] I derive the following corollaries which illustrate the stringent restrictions that the characterization in Lemma 1 imposes on both the preference environment and matching mechanism for a complete popular matching to exist and be reached.

Corollary 1.1. *A necessary but not sufficient condition for the existence of a complete popular matching is that* $\bigcup_{i \in \mathbb{I}} \{f(i), s(i)\} = \mathbb{C}$*, where* $\mathbb{C}$ *is the set of distinct seats to be assigned.*

Example 2. *Agents* $\{1, 2, 3, 4, 5, 6\}$ *rank seats* $\{A, B, C, D, E, F\}$*. Observe that seat* F *is neither an f- nor an s-post (in bold) for any agent . Hence, no complete popular matching exists.*

1	*2*	*3*	*4*	*5*	*6*
A	***A***	***B***	***B***	***C***	***B***
C	***D***	*A*	*C*	***E***	*C*
D	*B*	*C*	***E***	*A*	***E***
F	*F*	***D***	*F*	*F*	*A*
B	*C*	*E*	*A*	*D*	*F*
E	*E*	*F*	*D*	*B*	*D*

[13]The graph theoretic formulation of Hall's Marriage Theorem (Hall, 1935) states that for any finite bipartite graph G with bipartite sets P and C ($G := (P + C, E)$). An P-saturating matching is a matching which covers every vertex in P. For a subset W of P, let $N_G(W)$ denote the set of all vertices in C adjacent to some element of W. Hall's Marriage Theorem states that there is an P-saturating matching if and only if for every subset W of P, $|W| \leq |N_G(W)|$. In other words, every subset W of P has sufficiently many adjacent vertices in C. Proofs of the corollaries rely on the graph G induced by f- and s-posts in Lemma 1 condition 2), along with Hall's Marriage Theorem.

Next, define preferences as having M blocks where $(B_1, B_2, \ldots, B_M)$ is the finest partition of the set of seats such that, for every agent, every seat in B_n is ranked below every seat in B_j whenever $j < n$, and above every seat in B_k whenever $k > n$.

Corollary 1.2. *No complete popular matching exists if preferences exhibit three or more blocks, as $f(i) \in B_1$, $s(i) \in \{B_1, B_2\}$ $\forall i \in \mathbb{I}$, while seats $B_3, B_4, \ldots$ are never s-posts for any agent.*

Example 3. *Consider agents $\{1, 2, 3, 4, 5, 6\}$ ranking seats $\{A, B, C, D, E, F\}$ with three blocks $B_1 = \{A, B\}$, $B_2 = \{C, D\}$, and $B_3 = \{E, F\}$. The f- and s-posts (in bold) only consist of seats in B_1 and B_2, while no seat in B_3 is an s-post. Hence no complete popular matching exists.*

1	*2*	*3*	*4*	*5*	*6*
A	***A***	***B***	***B***	***B***	***A***
B	*B*	*A*	*A*	*A*	*B*
C	***D***	***D***	***C***	***D***	***C***
D	*C*	*C*	*D*	*C*	*D*
F	*E*	*F*	*F*	*E*	*E*
E	*F*	*E*	*E*	*F*	*F*

Corollary 1.3. *If preferences exhibit two blocks, then for a complete popular matching to exist, every seat $c \in B_1$ must be the first choice of some agent and every seat $c \in B_2$ must be the most preferred seat within B_2 for at least one agent; otherwise, $\bigcup_{i \in \mathbb{I}} \{f(i), s(i)\} \neq \mathbb{C}$*

Example 4. *Consider agents $\{1, 2, 3, 4, 5\}$ ranking seats $\{A, B, C, D, E\}$ with two blocks $B_1 = \{A, B\}$ and $B_2 = \{C, D, E\}$. Seat E is not the top-ranked option within B_2, and hence neither an f- or s-post (in bold). Hence, no complete popular matching exists.*

1	*2*	*3*	*4*	*5*
A	***A***	***B***	***B***	***B***
B	*B*	*A*	*A*	*A*
C	***D***	***D***	***C***	***D***
E	*C*	*C*	*E*	*C*
D	*E*	*E*	*D*	*E*

These corollaries collectively impose strong restrictions on agents' preferences that are necessary for the existence of a complete popular matching, regardless of the mechanism used. I next turn to a specific matching mechanism—Serial Dictatorship—and show that complete popular matching imposes additional constraints on its design.

Corollary 1.4. *A complete popular matching is implemented by a Serial Dictatorship if and only if the seniority ordering places all agents assigned to their f-posts as more senior to those assigned to their s-posts.*

Example 5. *Consider agents* $\{1, 2, 3\}$ *ranking seats* $\{A, B, C\}$*. If the seniority order is* $1 > 2 > 3$*, the resulting matching* $(1 - A, 2 - B, 3 - C)$ *is not one of the two complete popular matchings—namely* $(1 - A, 2 - C, 3 - B)$ *and* $(1 - C, 2 - A, 3 - B)$*—because agent* 3*, who must be assigned to his f-post in any complete popular matching, is junior to both agents* 1 *and* 2*. Thus, only seniority orders in which agent* 3 *is more senior that at least one of agents* 1 *or* 2 *yield in a complete popular matching under Serial Dictatorship.*

1	*2*	*3*
A	***A***	***B***
B	*B*	*A*
C	***C***	***C***

Taken together, these four corollaries (also see simulations in Appendix A) establish that existence of complete popular matching is rare, particularly given the empirical prevalence of

correlated preferences. Such correlation may arise from shared agreement over the most desirable assignments (e.g., prestigious legislative committees (Thakur, 2025)), or from a common aversion to undesirable placements (e.g., distressed or underdeveloped regions in civil service assignments (Thakur, 2018)). Such shared preference structures naturally generate blocks, making complete popular matchings difficult to sustain.

Moreover, the requirement that seniority align perfectly with the partition of agents into those receiving f-posts and s-posts under Serial Dictatorship is itself highly restrictive and unlikely to hold in practice, e.g., in Republican committee assignment procedures (Thakur, 2025).

From an applied perspective, these results suggest that organizations relying on plurality rule for internal conflict resolution are likely to experience a lack of institutional stability, manifesting in frequent coalitional turnover and repeated institutional restructuring.

5.2. **Majority Rule and Majority Stability.**

In sharp contrast to complete popular matching, majority stability is strikingly robust to correlated preferences and, given the widespread use of (super-)majority rules in practice, may help explain the institutional stability observed in many real-world settings.

I illustrate the subtleties underlying the existence of majority stable allocations using Example 6.

Example 6. *Consider agents* $\{1,2,3,4,5\}$ *ranking seats* $\{A,B,C,D,E\}$.

(a) Preference correlation of $\rho = 0.84$

1	*2*	*3*	*4*	*5*
$\underline{A}$	A	A	A	A
B	$\underline{B}$	B	B	B
C	C	$\underline{C}$	$\underline{D}$	$\underline{E}$
D	D	D	C	C
E	E	E	E	D

(b) Preference correlation of $\rho = 0.82$

1	*2*	*3*	*4*	*5*
A	A	A	A	A
B	B	B	B	B
C	C	C	D	D
D	D	D	E	E
E	E	E	C	C

Focusing first on Example 6(a), where preferences are highly correlated ($\rho = 0.84$), with agents 1,2, and 3 having identical preferences and agents 4 and 5 also aligning at the top. As

a result, no complete popular matching exists, since the f- and s-posts cover only seats $\{A, B\}$ and preferences exhibit 3 blocks ($B_1 = \{A\}, B_2 = \{B\}, B_3 = \{C, D, E\}$). However, there exist six majority stable allocations—specifically, those assigning {A,B,C} to agents {1,2,3} and D and E to agents 4 and 5—including the underlined allocation as an example.

Why is the underlined allocation in Example 6(a) majority stable? All agents rank seats A and B as their top two choices, reflecting substantial correlation. Yet this very feature underpins majority stability. Four agents (2, 3, 4, and 5) envy agent 1's assignment to seat A, and three agents (3, 4, and 5) envy agent 2's assignment to seat B. However, a majority of three or more agents cannot be reassigned to just these two seats.

More generally, a *packing problem* arises: to overturn an allocation under majority rule, $\frac{N+1}{2}$ *distinct* agents must be reassigned to $\frac{N+1}{2}$ *distinct* seats that they prefer. Satisfying this condition requires the existence of at least one *chain of envy*: a structure where i is envious of j's seat, and j is envious of k's seat, and so on.[14]

The underlined majority stable allocation in Example 6(a) admits three chains of envy of length two: $5 \to 2 \to 1$ (i.e., 5 is envious of 2, who is envious of 1), $4 \to 2 \to 1$, and $3 \to 2 \to 1$. Nonetheless, majority stability holds because there is no additional envy to overcome the packing problem.

Thus, Example 6(a) illustrates how undermining majority stability requires (i) sufficient overall envy to move a majority and make its members strictly better off, (ii) the presence of chains of envy to overcome the packing problem, and (iii) envy in the "right" places. For instance, multiple chains originating from the same seats (here, A and B) are redunant and do not suffice to generate majority instability.

The comparison with Example 6(b), however, shows that these features are not fully captured by simple measures of correlation. Example 6(a) which admits majority stability, has a higher correlation ($\rho = 0.84$) than Example 6(b) ($\rho = 0.82$), where no majority stable allocations exist. In particular, the underlined majority stable allocation from Example 6(a) fails to be

[14]The smallest possible chain of envy needed to overturn majority stability has length two, with the remaining $\frac{N-3}{2}$ agents needing to envy $\frac{N-3}{2}$ distinct seats that remain.

majority stable in Example 6(b), since agent 5, now assigned seat E, also envies agent 4's seat D, thereby enabling a majority-sized coalition of three agents.

This comparison highlights the more intricate structure required for correlated preferences to undermine majority stability. Widespread envy directed at a small set of desirable seats is often insufficient; rather, instability requires enough envy, appropriately structured into chains, and located in configurations that overcome the packing problem. Simple scalar measures of correlation do not capture these combinatorial features. Simulation results in Appendix A further underscore this distinction.

Nevertheless, Figure 3 shows that, overall, higher correlation tends to reduce the likelihood of majority stability. At the same time, it highlights the striking robustness of majority stability, even when preferences are substantially correlated.

5.2.1. ***Validation: is a given allocation majority stable?***

Given a set of preferences, checking whether a candidate matching is majority stable and determining the size of the largest coalition that can benefit from an alternative matching is, in fact, computationally tractable. At first glance, one might expect this requires searching over all $N! - 1$ alternative allocations to check whether sufficiently large coalitions can strictly improve. However, I show that this problem can be solved efficiently using an integer linear program, which can in turn be relaxed to a linear program that admits integer solutions and can be solved in polynomial time.

The integer linear programming formulation operates on the envy graph in matrix form and identifies an alternative one-to-one matching that maximizes realized envy. Crucially, the requirement that the alternative allocation be a one-to-one matching imposes sufficient structure to ensure that the relaxed linear program yields integer solutions.

Let X denote the reassignment matrix, where $X_{ij} = \begin{cases} 1, \text{ if } i \text{ is reassigned to } j\text{'s seat} \\ 0, \text{ otherwise} \end{cases}$.

Note that $X_{ii} = 1$ indicates that agent i remains in their current seat.

Let A denote the envy matrix, where $A_{ij} = \begin{cases} 1, \text{ if } i \text{ is envious of } j\text{'s seat} \\ 0, \text{ otherwise} \end{cases}$.

That is, $A_{ij} = 1$ if agent i strictly prefers j's seat to their own.

The objective is to find a reassignment X that maximizes total realized envy, subject to the constraint that the resulting allocation is a one-to-one matching in a balanced market:

$$(1) \qquad \begin{aligned} \max_{X} \quad & \textstyle\sum_{diag} A'X \\ s.t. \quad & \textstyle\sum_i X_{ij} = 1 \quad \forall j \\ & \textstyle\sum_j X_{ij} = 1 \quad \forall i \end{aligned}$$

The first set of constraints ensures that each seat j is assigned to only one agent, while the second ensures that each agent is assigned exactly one seat. The objective function sums the diagonal entries of $A'X$, thereby counting the total realized envy without double counting.

This problem can be rewritten in vectorized form. Stack the columns of X and of A to obtain

$$\tilde{X} = \begin{bmatrix} X_{11} \\ ... \\ X_{N1} \\ X_{12} \\ ... \\ X_{N2} \\ ... \\ ... \\ X_{1N} \\ ... \\ X_{NN} \end{bmatrix} \text{ and } \tilde{A} = \begin{bmatrix} A_{11} \\ ... \\ A_{N1} \\ A_{12} \\ ... \\ A_{N2} \\ ... \\ ... \\ A_{1N} \\ ... \\ A_{NN} \end{bmatrix}.$$

Let b be a $2N$-by-1 vector of ones, and let C be the $2N$-by-N^2 constraint matrix,

$$
C = \left[\begin{array}{cccc|cccc|c|cccc}
\overbrace{1 & 1 & \dots & 1}^{\text{length N}} & \overbrace{0 & 0 & \dots & 0}^{\text{length N}} & \dots & \dots & \dots & \dots & \overbrace{0 & 0 & \dots & 0}^{\text{length N}} \\
0 & 0 & \dots & 0 & 1 & 1 & \dots & 1 & \dots & \dots & \dots & \dots & 0 & 0 & \dots & 0 \\
\dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\
\dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\
0 & 0 & \dots & 0 & 0 & 0 & \dots & 0 & \dots & \dots & \dots & \dots & 1 & 1 & \dots & 1 \\
\hline
1 & 0 & \dots & 0 & 1 & 0 & \dots & 0 & \dots & \dots & \dots & \dots & 1 & 0 & \dots & 0 \\
0 & 1 & \dots & 0 & 0 & 1 & \dots & 0 & \dots & \dots & \dots & \dots & 0 & 1 & \dots & 0 \\
\dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\
\dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\
\underbrace{0 & 0 & \dots & 1}_{\text{N-dim identity mtx}} & \underbrace{0 & 0 & \dots & 1}_{\text{N-dim identity mtx}} & \dots & \dots & \dots & \dots & \underbrace{0 & 0 & \dots & 1}_{\text{N-dim identity mtx}}
\end{array}\right]
$$

Hence, the integer linear program from Equation (1) can be equivalently expressed as

$$
\begin{aligned}
(2) \qquad & \max_{X} \tilde{A}'\tilde{X} \\
& s.t. \quad C\tilde{X} = b
\end{aligned}
$$

Partition C into submatrices on either side of the dashed line:

$$
C = \left[\begin{array}{c} \overline{C} \\ \hline \\ \underline{C} \end{array}\right]
$$

, where $\overline{C}x = b_{NxN^2}$ ensures that each seat is allotted to exactly one agent while $\underline{C}x = b_{NxN^2}$ ensures that each agent receives exactly one seat.

Importantly, $\tilde{A}$, C, and b have all integer entries and C is totally unimodular.[15] Hence, the relaxed linear program in Equation (2) will admit integer solutions, ensuring that it can be solved efficiently in polynomial time.

Solving this linear program gives us the maximum number of agents who can be made better off relative to the initial matching. If this number is strictly less than $\frac{N+1}{2}$, the initial matching is majority stable; otherwise, it is not.

5.2.2. ***Existence: is there a majority stable allocation?***

Determining whether a majority stable allocation exists for a given set of preferences is, in general, a difficult problem; it is an NP-complete problem, as shown by Aziz et al. (2023). In this section, I develop intuition for why this is the case.

First, I show that commonly used preference restrictions—weak single-peaked preferences and order restricted preferences—that are known to be well-behaved under majority rule and yield tractability in standard social choice settings, have no traction when applied to the preferences over matchings in social allocation choice. Second, I demonstrate that constructive approaches based on minimizing envy—either globally or greedily—are highly effective in identifying majority allocations when they exist, although they can, in rare cases, fail to detect them.

5.2.2.1. **Non-existence of Weakly Single-Peaked Preferences in Social Allocation Choice.**

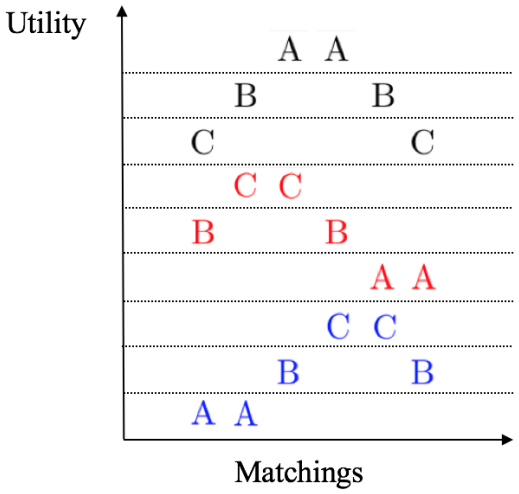
Under *weakly single-peaked preferences* in the social allocation choice setting, matchings are ordered along a left-right scale such that moving away from an agent's most preferred outcome corresponds to a weak decline in an agent's preference ranking (i.e., allowing indifference at the peak of preference order as well as to either side of the peak). I show that once preferences over allocations are restricted to depend solely on each agent's own assignment, this structure is too restrictive to sustain weak single-peakedness.

[15]Heller and Tompkins (1956) show that the coefficient matrix for a bipartite matching is totally unimodular. And $\tilde{A}$ is an unoriented incidence matrix of a bipartite graph. Equivalently, Hoffman and Gale (1956) characterize more general sufficiency conditions which $\tilde{A}$ satisfies.

Lemma 2. *There exists no set of weakly single-peaked preferences over matchings for markets of size $N > 3$.*

Proof: First restrict attention to any subset of agents $\{1, 2, 3\}$ with preferences over matchings, where $\{A, B, C\}$ denotes the matching 1-A, 2-B, and 3-C. Maintain the assumption that agents care only about their own assignments and that preferences are strict. Without loss of generality, let agent 1's preference be $A \succ_1 B \succ_1 C$. In a market of size three, it is straightforward to verify that–up to relabeling and reversal–there is a unique ordering of matchings consistent with weak single-peakedness: $\{C, B, A\}$, $\{B, C, A\}$, $\{A, C, B\}$, $\{A, B, C\}$, $\{B, A, C\}$, $\{C, A, B\}$ (see Figure 4).

Figure 4. Weakly single-peaked preferences for $N = 3$.
Restricting to a market of size 3 (any 3 seats and 3 agents), there is only one possible (ignoring relabeling of seats/agents and reverse order of preferences) ordering of preferences which satisfies weak single-peakedness. Observe the utility symmetry of 1's preference relative to 2 and 3, and polar asymmetry of 2 and 3's preferences.



1	2	3
A	C	C
B	B	B
C	A	A

Now, consider 4 agents $\{1, 2, 3, 4\}$. Agent 1 is indifferent across all matchings in which they receive their top choice, say D. For agents 2, 3, and 4 to satisfy weak single-peakedness over this subset of allocations, their ordering must follow the structure identified above: $\{D, C, B, A\}$, $\{D, B, C, A\}$, $\{D, A, C, B\}$, $\{D, A, B, C\}$, $\{D, B, A, C\}$, $\{D, C, A, B\}$. However, since agent 1 has strict preferences (e.g., $D \succ_1 C$), the set of allocations in which they receive C must be placed either entirely to one side or split

across both sides of the peak. In any such placement, there exists a matching in which agent 2 receives seat B in a way that violates weak single-peakedness for agent 2. □

5.2.2.2. **Non-existence of Order Restricted Preferences in Social Allocation Choice.**

Under *order-restricted preferences*, agents are ordered along a left-right scale such that for any pair of matchings, those who strictly prefer one matching lie on one side, those who strictly prefer the other lie on the opposite side, and any indifferent agents lie in between. I show that such structure is also incompatible in social allocation choice problems.

Lemma 3. *There exists no set of order-restricted preferences over matchings for markets of size* $N \geq 3$.

Proof: It suffices to consider any subset of three agents and three seats. Let A, B, C denote the matching (1-A, 2-B, 3-C), and maintain that agents care only about their own assignments and have strict preferences. Without loss of generality, suppose agent 1 has $A \succ_1 B \succ_1 C$.

Comparing the allocations A, B, C and A, C, B, agent 1 is indifferent, leaving four possible preference configurations for agents 2 and 3 over seats B and C.

- Case 1: if $B \succ_2 C$ and $B \prec_3 C$.

  Then, the order restriction is 2-1-3 (or reverse) given the comparison of the two allocations above. However, consider the comparison between $\{C, B, A\}$ and $\{B, C, A\}$. Preferences are $\succ_2$, $\prec_1$, and $\sim_3$, which violates the order restriction.

- Case 2: if $B \prec_2 C$ and $B \succ_3 C$.

  Then, the order restriction is 2-1-3 (or reverse) given the comparison of the two allocations above. However, consider the comparison between $\{C, A, B\}$ and $\{B, A, C\}$. Preferences are $\sim_2$, $\prec_1$, and $\succ_3$, which violates the order restriction.

- Case 3: if $B \succ_2 C$ and $B \succ_3 C$.

  Then, the order restriction is either 1-3-2 (or reverse) or 1-2-3 (or reverse) given the comparison of the two allocations above.

  - Case 3a: if order restriction is 1-3-2 (or reverse).

Consider the comparison between $\{C, A, B\}$ and $\{B, A, C\}$. Preferences are $\prec_1$, $\succ_3$, and $\sim_2$, which violates the order restriction.

- Case 3b: if order restriction is 1-2-3 (or reverse).

  Consider the comparison between $\{C, B, A\}$ and $\{B, C, A\}$. Preferences are $\prec_1$, $\succ_2$, and $\sim_3$, which violates the order restriction.

- Case 4: if $B \prec_2 C$ and $B \prec_3 C$.

  Then, the order restriction is either 1-3-2 (or reverse) or 1-2-3 (or reverse) given the comparison of the two allocations above.

  - Case 4a: if order restriction is 1-3-2 (or reverse).

    Consider the comparison between $\{C, B, A\}$ and $\{B, C, A\}$. Preferences are $\prec_1$, $\sim_3$, and $\prec_2$, which violates the order restriction.

  - Case 4b: if order restriction is 1-2-3 (or reverse).

    Consider the comparison between $\{C, A, B\}$ and $\{B, A, C\}$. Preferences are $\prec_1$, $\sim_2$, and $\prec_3$, which violates the order restriction.

□

5.2.2.3. **Constructive Approach to Finding a Majority Stable Allocation: Minimizing Envy.**

To assess the existence of majority stable allocations, I consider constructive approaches that aim to minimize realizable envy. Specifically, I use: (i) Algorithm 1, minimizing the average preference rank, and (ii) Algorithm 2, greedily minimizing assigned preference ranks.[16]

Lemma 4. *If preferences admit a majority stable allocation, there exists a Pareto efficient allocation that is also majority stable.*[17]

Proof: If a majority stable allocation is not Pareto efficient, applying Top Trading Cycles weakly reduces total envy: agents who improve experience less envy, while others are unaffected. The resulting allocation is both Pareto efficient and majority stable. □

[16]This matching can be computed by assigning weight N^r to a seat ranked rth and solving a maximum-weight matching problem via integer linear programming. Since $N^r > (N-1)N^{r-1}$, the solution lexicographically prioritizes higher-ranked assignments.

[17]Lemma 4 implies that that if there exists a majority stable allocation, then a majority stable allocation can be achieved by a strategyproof mechanism, e.g., a Serial Dictatorship. See Appendix B for more details.

Both algorithms return Pareto efficient allocations, which is useful in light of Lemma 4, and both minimize envy through different criteria. Moreover, the greedy algorithm identifies a complete popular matching whenever one exists.

Simulation results show that these algorithms are highly effective in practice. In simulations with $N = 7$, they fail to identify a majority stable allocation in only 1% of cases where such allocations exist. Nonetheless, they are not guaranteed to succeed. Example 7 illustrates a case in which both methods fail despite the existence of a majority stable allocation.

Example 7. *Consider agents* $\{1, 2, 3, 4, 5, 6, 7\}$ *ranking seats* $\{A, B, C, D, E, F, G\}$. *Although a majority stable matching exists (in bold), it is not found by either Algorithm 1) returning a minimum average preference matching or Algorithm 2) returning a greedy minimum preference matching (red or underlined).*

1	*2*	*3*	*4*	*5*	*6*	*7*
F	***G***	***D***	*G*	*F*	*D*	*G*
A	*F*	*A*	***A***	***B***	*A*	*F*
C	*A*	*E*	*D*	*C*	*E*	***E***
G	*C*	*F*	*C*	*D*	*F*	*B*
E	*D*	*C*	*B*	*G*	*B*	*D*
D	*E*	*G*	*E*	*A*	*G*	*C*
B	*B*	*B*	*F*	*E*	***C***	*A*

In Example 7, a majority stable matching exists $(1-F, 2-G, 3-D, 4-A, 5-B, 6-C, 7-E)$ however, neither the minimum average rank nor the greedy algorithm $(1-F, 2-C, 3-D, 4-A, 5-B, 6-E, 7-G$ and $1-F, 2-C, 3-A, 4-G, 5-B, 6-D, 7-E)$ identifies it. Both algorithms avoid assigning agent 6 to their least preferred seat C, as doing so worsens average rank and conflicts with the objective of prioritizing higher-ranked assignments. However, this assignment is necessary to prevent sufficient envy (over seats G, F) from accumulating to form a majority coalition.

5.3. **What are the implications of the voting rule for organizations in practice?**

The comparison between majority stability and complete popular matching suggests that, despite systematic correlation in preferences, organizations governed by majoritarian rules may exhibit substantial institutional stability. By contrast, organizations operating under plurality rule are generally prone to instability, even under modest correlation in preferences.

This contrast is especially stark in environments where agents share a common top choice. Such situations are empirically common—for instance, U.S. senators often place the highest value on being assigned to the Appropriations Committee as it controls the Senate's purse and carries tremendous power and prestige. In such cases, the existence of complete popular matching would require $N-1$ agents' second choices (their s-posts) to be distinct. By contrast, this might still leave ample scope for existence of majority stability since only one person can be reassigned to this top choice seat which everyone else is envious of—the crux of the packing problem.

6. Envy-minimizing mechanisms are more robust to institutional stability.

Which mechanisms are most robust to institutional stability? What features of their design account for differences in their ability to attain institutionally stable outcomes? While institutional stability is defined as a property of the allocation, the mechanisms that generate these allocations can also be evaluated along this dimension. I address these questions using simulations.

The simulation analysis in Appendix A compares Serial Dictatorship (under a fixed seniority order) with the Boston mechanism using the same ordering for tie-breaking, assuming truthful preference revelation. This comparison serves two purposes. First, it highlights mechanism features that promote institutional stability. Second, both mechanisms are used in practice by U.S. Senate Republicans and Democrats for committee assignments (Thakur, 2025). I calibrate my simulations to this particular application.

Two main findings emerge. First, the largest coalition that can be made strictly better off is typically larger under Serial Dictatorship than under the Boston mechanism. Second,

the Boston mechanism (under truthful reporting) is more robust to majority stability. Across 12,001 simulated preference profiles, both mechanisms yield majority stable outcomes in 5,154 cases and fail in 4,673 cases; in 2,169 cases, only the Boston mechanism is majority stable, whereas in just 5 cases only Serial Dictatorship is.

This difference is driven by the *favoring higher ranks property* of the Boston mechanism: any seat that agent i envies is assigned to some agent j who ranks that seat at least as highly as i does. This reduces envy relative to Serial Dictatorship. Algorithm 2 from Section 5.2.2.3 can be interpreted as globally optimizing this property of prioritizing higher-ranked assignments.

Nonetheless, neither mechanism strictly dominates the other. As Example 8 illustrates, there exist environments in which one mechanism yields a majority stable allocation while the other does not (even when agents are assumed to naively report their true preferences in the Boston mechanism). In Example 8(a), the Boston mechanism with truthful preference and same seniority ordering for tie-breaking resulting in $(1-A, 2-C, 3-D, 4-E, 5-B)$ is majority stable, while the matching from Serial Dictatorship $(1-A, 2-B, 3-D, 4-E, 5-C)$ is not as alternative matching $(1-C, 2-E, 3-A, 4-B, 5-D)$ is preferred by a 3:2 vote. On the other hand, in Example 8(b), the Serial Dictatorship resulting in $(1-A, 2-B, 3-E, 4-C, 5-D)$ is majority stable, while Boston mechanism with truthful preference and same seniority ordering for tie-breaking resulting in $(1-A, 2-C, 3-B, 4-E, 5-D)$ is not majority stable as alternative matching $(1-E, 2-A, 3-D, 4-C, 5-B)$ is preferred by a 3:2 vote.

Example 8. *Consider agents* $\{1, 2, 3, 4, 5\}$ *ranking seats* $\{A, B, C, D, E\}$ *given a seniority ordering* $1 \succ 2 \succ 3 \succ 4 \succ 5$.

(a) Boston mechanism is majority stable, but Serial Dictatorship is not.

1	*2*	*3*	*4*	*5*
A	A	A	A	B
B	B	B	B	A
C	C	D	E	D
D	D	C	D	E
E	E	E	C	C

(b) Serial Dictatorship is majority stable, but Boston mechanism is not.

1	*2*	*3*	*4*	*5*
A	A	B	A	A
B	B	A	B	B
C	C	E	C	D
D	D	D	D	C
E	E	C	E	E

7. Inter-dependent Preferences.

Thus far, I have maintained Assumption 4 that agent i's preferences over allocations depend only on his own seat assignment, and not on others' assignments (what I refer to as "*individual preferences*"). In this section I relax this assumption and consider "*selfish but inter-dependent preferences*:" agent i prioritizes his own assignment lexicographically, but may hold *any* preferences over allocations that leave his assignment unchanged. This is a broad class of preferences, encompassing caring for allies' welfare, deriving utility from the misfortune of rivals, caring about one's coalition of partners, caring about one's coalition in an ordered way (caring about the well-being of agent j the most, followed by k,...), and even very idiosyncratic preferences like, as long as I get A, I want j to get C and k to get D; but if I get B, I want j to get E and k to get F.[18]

Lemma 5. *If an allocation is a majority stable under individual preferences, there may exist selfish but inter-dependent preferences under which it ceases to be a majority stable.*

Proof: by Example 9.

Example 9. *Consider agents* $\{1,2,3,4,5\}$ *ranking seats* $\{A,B,C,D,E\}$.

1	*2*	*3*	*4*	*5*
A	A	A	D	E
B	B	B	A	A
C	C	C	B	B
D	D	D	C	C
E	E	E	E	D

With individual preferences, assigning agents 1, 2, and 3 to seats A, B, and C in any permutation and matching $4-D$ and $5-E$, yields six different majority stable allocations.

However, suppose 4 forms an alliance with 1 and 5 allies with 2. Then no majority stable allocation exists. Not assigning 4 or 5 seats D and E leads to majority instability.

[18]The only restriction on selfish but inter-dependent preferences is that agent i never votes against his own assignment; for instance, pure altruism that sacrifices i's assignment to benefit another falls outside of this class.

Furthermore, assigning $1-A$ is overturned by $\{2,3,5\}$; assigning $2-A$ is overturned by $\{1,3,4\}$; and assigning $3-A$ is overturned by $\{1,2,4,5\}$. □

Lemma 6. *If an allocation is not a complete popular matching under individual preferences, there may exist selfish but inter-dependent preferences under which it becomes a complete popular matching.*

Proof: In Example 9, there is no complete popular matching exists under individual preferences. However, if 4 allies with 1, then, $(1-A, 2-B, 3-C, 4-D, 5-E)$ becomes a complete popular matching. Under individual preferences, agents 2 and 3 would both gain at the expense of 1, but with inter-dependent preferences and 1 valuing 4's support, the vote over such an alternative is tied at 2:2. □

Lemma 7. *If an allocation is a complete popular matching under individual preferences, there may exist selfish but inter-dependent preferences under which it ceases to be a complete popular matching.*

Proof: In Example 5, $(1-A, 2-C, 3-B)$ and $(1-C, 2-A, 3-B)$ are both complete popular matchings under individual preferences. However, if 3 allies with 1, then, $(1-C, 2-A, 3-B)$ ceases to be a complete popular matching as it is defeated by a 2:1 vote by agents 1 and 3 in favor of $(1-A, 2-C, 3-B)$, the only complete popular matching under these self but inter-dependent preferences. □

Theorem 2. *(i) The set of majority stable allocations under selfish but inter-dependent preferences is a subset of that under individual preferences. (ii) In contrast, the set of complete popular matchings with selfish but inter-dependent preferences may be either a subset or a superset of that under individual preferences.*

Proof: (i) If an allocation is not majority stable under individual preferences, then by definition there exists a majority that strictly prefers an alternative based on their own assignments. Inter-dependent preferences cannot reverse these strict improvements,

so such an allocation cannot be made majority stable. Combined with Lemma 5, this establishes the claim. (ii) This follows directly from Lemmas 6 and 7. □

Theorem 2 implies that introducing selfish but inter-dependent preferences can only (weakly) expand the scope for majority instability, whereas no analogous monotonicity holds for complete popular matchings.[19]

Taken together, these results point to a central political economy implication for legislative committee assignments, our running application. When legislators' preferences are primarily selfish, majority rule can sustain remarkably stable assignment outcomes even in the presence of substantial agreement over top committees. The packing problem limits the ability of dissatisfied legislators to assemble winning coalitions, thereby insulating existing allocations from frequent renegotiation.

However, once preferences become selfish but inter-dependent—reflecting alliances, rivalries, or coordinated bloc behavior—the same environment can unravel. Coalitions can be mobilized not only around individual gains but also around the reallocation of benefits within allied groups, expanding the scope for majority-supported deviations. Institutional stability then becomes contingent not just on the distribution of preferences over committees, but on the underlying coalition structure of the legislature.

8. **Conclusions.**

I have developed a formal framework for endogenous institutional stability–the question of when an organization's internal allocations of members to positions, and the assignment procedures that produce them, are robust to coalitional challenges from within. By integrating matching theory and social choice, the analysis identifies preference correlation as the central source of potential institutional instability, while the underlying voting rule and the structure of chains of envy determine whether instability materializes.

[19]Under selfish but inter-dependent preferences I have assumed preferences over one's own assignment are independent of others' assignments. Under fully inter-dependent preferences—where an agent's ranking over their own assignment depends on others' assignments (e.g., if agent 2 is assigned C, agent 1 prefers A to B, but if agent 3 is assigned C agent 1 prefers B to A)—the sets of both majority stable allocations and complete popular matchings may either expand or contract.

The central result is that majority stability is strikingly robust to preference correlation, whereas complete popular matching—the plurality rule analog—is not. Moreover, majority stable allocations can be implemented by widely used mechanisms such as Serial Dictatorship and the Boston mechanism.[20] The key to this robustness lies in the packing problem which protects existing allocations even when envy is widespread because envy must be arranged in chain structures to improve majority coalitions.

This helps explain a puzzle in comparative institutional analysis: in many organizations exhibiting correlation in members' preferences over positions—e.g., U.S. Senators consistently ranking prestige committees at the top of their preference lists (Thakur, 2025), Indian civil servants consistently ranking underdeveloped, distressed states at the bottom of their preference lists (Thakur, 2018), or workers preferring high-profile teams within firms (Cowgill and Koning, 2018)—existing assignments and institutional procedures persist without internal reform.

The framework has broad applicability. While U.S. congressional committee assignments provide a canonical case, analogous allocation problems arise in cabinet and shadow-cabinet portfolios, parliamentary groups in the European Parliament and national legislatures, civil service postings, and intra-firm role assignment. In systems where formal committees are weak—such as Westminster-type parliaments—the relevant allocation decisions often occur within party caucuses, cabinets, or parliamentary groups. The notion of majority stability is most naturally suited to settings in which such assignments are shaped, formally or informally, by collective agreement or remain subject to coalition support or challenge.

Finally, the analysis also highlights when institutional reform and reorganization are likely. Shocks that increase preference alignment—such as electoral landslides from a certain region or constituency, or crises that elevate the value of specific positions (e.g., Finance or Commerce committees) with new jurisdiction, power, and credit-claiming opportunities—can tip an organization from majority stability to instability by aligning preferences and creating chains of envy. Testing such comparative statics empirically is a productive direction for future work.

[20]See Appendix B for a discussion of the properties of mechanism and voting rules that attain institutional stability.

9. Declaration of generative AI and AI-assisted technologies in the manuscript preparation process.

During the preparation of this work the author used ChatGPT-5.3 and Claude Sonnet 4.6 for grammar, spelling, language proofing, and copyediting to help refine exposition and improve clarity and organization. After using this tool/service, the author reviewed and edited the content as needed and takes full responsibility for the content of the published article.

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ONLINE APPENDIX for Endogenous Institutional Stability

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Online Appendix A. **Simulations: Majority Stability vs. Popular Matching.**

A.1. **Calibration to the U.S. Senate.**

To situate and illustrate the results of this paper within the context of a practical application, I base my simulations on the application of assigning US Senators to legislative committees (Thakur, 2025). Institutional stability of legislative committee assignment mechanisms is undermined by i) increased correlation across agents' preferences, which increases competition over seats and leads to increased likelihood of envy, ii) increased impatience (e.g., close elections or retirement which imply a short time horizon or large discounting of utility from committee assignments in the future), and iii) a large freshman class since freshmen have lowest priority and no existing tenant assignment with property rights.

To make the simulations more tractable and aligned with the focus on this paper, I make a few simplifications. First, instead of a many-to-many matching (where each committee is assigned multiple agents and each agent is assigned multiple committees), I consider a many-to-one matching where each committee is assigned many agents but each agent is assigned only one committee. Second, I assume maximal impatience, so that all agents care about the static one-shot matching. Lastly, I do not consider existing tenants, i.e., incumbents who have existing committee assignments from previous Congress. Instead, I assume that there is a strict order of seniority, fixed across all senators.

Table 4. Final list of vacancies by committee for simulation.
The first column is the type of committee which is a commonly used classification in the political science literature on the committee system. The second column has the name of the committee and the third column has the number of seats on the committee.

Committee Type	Committee Name	# Seats for Simulation
Prestige	Appropriations	6
Prestige	Budget	4
Prestige	Rules	2
Constituency	Agriculture	3
Constituency	Small Business	4
Constituency	Veterans Affairs	2
Constituency	Armed	3
Constituency	Energy	4
Policy	Banking	3
Policy	Commerce	3
Policy	Environment	3
Policy	Finance	4
Policy	Foreign Relations	3
Policy	Govt Affairs	3
Policy	Judiciary	3
Policy	Labor	3
	TOTAL	**53**

I calibrate the vacancies to the 97th Congress (1981-1983), which had the largest proportion of 16/53 freshmen amongst the Republican Senators from the 83rd-113th Congresses. The 97th

Congress had 292 seats across the 16 Standing committees. First, I assume that Republican seats resemble the 53% of seats in each of these committees (in practice, similar to this assumption, seats on a committee for each party resemble the party ratio in the Senate). Next, since the average number of seats held by a senator was around 3, to simplify the matching to be many-to-one (i.e., consider each politician being assigned to only one committee), I divide the seats in each committee by 3. Finally, I round to obtain the total number of vacancies in each of the 16 standing committees. The calibration to the 97th Congress with many-to-one matching gives 53 vacancies across 16 standing committees for the 53 Senators shown in Table 4.

Table 5. Generation of Correlated Preferences for Simulations.
For ease of exposition, varying degrees of preference correlation were generated by considering 13 different preference block structures across the 16 committees. Fully random preferences generated preferences in the correlation range of [-.0139, .0404], then fixing a block of 1 such that all agents agreed on the top choice and then randomly generating the remaining 15 committees in the next block generated preferences in the correlation range [.1625,.2109], etc. Thus, each row of this table shows the block structure used to generate simulated preferences using randomization within blocks in column 1, the minimum and maximum correlation ρ this produced in my simulations in columns 2 and 3, and the number of simulated preferences in column 4. The different preference block schemes are grouped and numbered such that they correspond to close clusters on the simulation figures on the horizontal axis.

	Preference Generation	**Min Correl**	**Max Correl**	**# simulations**
1)	16 (*Fully Random*)	-.0147	.0247	1000
2)	1;15	.1633	.2035	1000
3)	2;14	.3189	.3510	1000
4)	3;13	.4496	.4783	1000
5)	4:12	.5578	.5787	1000
	12:4	.5576	.5815	1000
6)	8;8	.7493	.7592	1000
7)	2;2;2;2;8	.8683	.8757	1000
	8;2;2;2;2	.8682	.8788	1000
8)	4;4;4;4	.9402	.9440	1000
9)	2;2;2;2;2;2;2;2	.9880	.9888	1000
	1;1;1;1;2;2;2;2;2;2	.9910	.9917	1000
10)	1;1;1;1;1;1;1;1;1;1;1;1;1;1;1;1: (*Identical Pref*)	1	1	1

For ease of exposition, as described in Table 5, I generate many different sets of preferences with varying degrees of correlation across the 16 committees. Moreover, Thakur (2025) finds that block structured preferences are a good approximation to senators' preferences in practice given the different types of legislative committees: "i) powerful Policy and Prestige committees like Finance and Appropriations have large common values and relatively small idiosyncratic differences, ii) Constituency committees such as Small Business and Armed Services depend on particulars of the politician's constituency characteristics and hence have lower common value, and iii) Policy committees such as Environment and Labor have relatively large idiosyncratic values, depending on the politician's own views and beliefs and that of their constituency,

which dominate the common value component" while it is agreed upon that Administrative committees like Post Office are of little value to any politician.

In these simulations, I compare the Republican mechanism of Serial Dictatorship in order of seniority with the Democrat mechanism of Boston mechanism (assuming truthful preferences and tie-breaking based on seniority).

A.2. **Fragility of Popular Matching existence.**

Common mechanisms such as Serial Dictatorship in order of seniority and Boston mechanism (with the assumption of truthful revelation) generally fail to guarantee a complete popular matching (see Figure 5).

To better understand why, I consider the necessary but not sufficient condition existence of a complete popular matching, that the f- and s-posts span the 16 committees from Corollary 1.1 (Figure 6). This illustrates how even with a slight correlation (e.g., $\rho \approx 0.2$), the necessary condition begins to be violated. As preferences become increasingly correlated, fewer and fewer of the 16 committees constitute f- and s-posts. Of course, when preferences are perfectly correlated with $\rho = 1$ and everyone shares the same preference, only the first and second choice committees are f- and s-posts.

A.3. **Existence of Majority Stability.**

The matchings produced by Serial Dictatorship and Boston mechanisms are majority stable for preference environments with low and intermediate correlations across agents, however, for highly correlated preferences, as expected the matchings are not majority stable (Figures 7 left and 8 left).

The relationship between correlation across agents' preferences and the maximal sized coalition trying to overturn the matching is positive on the whole but does not hold point-by-point (Figures 7 right and 8 right).

A.4. **Comparing Majority stability under Boston Mechanism vs. Serial Dictatorship.**

For most preference environments, the maximal sized coalition that can be made better off is weakly larger under Serial Dictatorship than under the Boston mechanism (see Figure 9 left).

Moreover, the Boston mechanism under truthful revelation is more robust with respect to the existence of majority stability (see Figure 9 right). Out of 12,001 simulated preference environments with varying levels of correlation, there were i) 5154 instances where both were majority stable, ii) 4673 instances where both were not majority stable, iii) 2169 instances where Boston mechanism is majority stable but Serial Dictatorship is not, and iv) only 5 instances where Serial Dictatorship was majority stable but Boston mechanism was not. The differential robustness with regards to majority stability between the two mechanisms highlights the Boston mechanism's envy-minimizing advantage, which comes from its favoring higher ranks property.

A.5. **Summary of Simulation Takeaways.**

Thus, the key takeaways from the simulations are that i) popular matchings fail to exist with even slight correlation across preferences, ii) Majority stable matchings often exist even with a significant amount of correlation across agents' preferences, iii) Serial Dictatorship and Boston mechanism very rarely produce popular matchings but they do produce majority stable allocations much more frequently, iv) as preferences become more and more correlated across agents, maximal coalition size wanting to overturn is larger, and v) Boston mechanism assuming truthful revelation is more robust to majority stability than Serial Dictatorship.

Figure 5. Existence of Popular Matching under Serial Dictatorship and Boston Mechanism.
Under Serial Dictatorship (*Left*) and Boston mechanism assuming truthful revelation and tie-breaking in order of seniority (*Right*), these figures plot whether the matching is popular or not. Observe that the matchings produced by these mechanisms are rarely (in fact never in these simulations!) complete popular matchings.

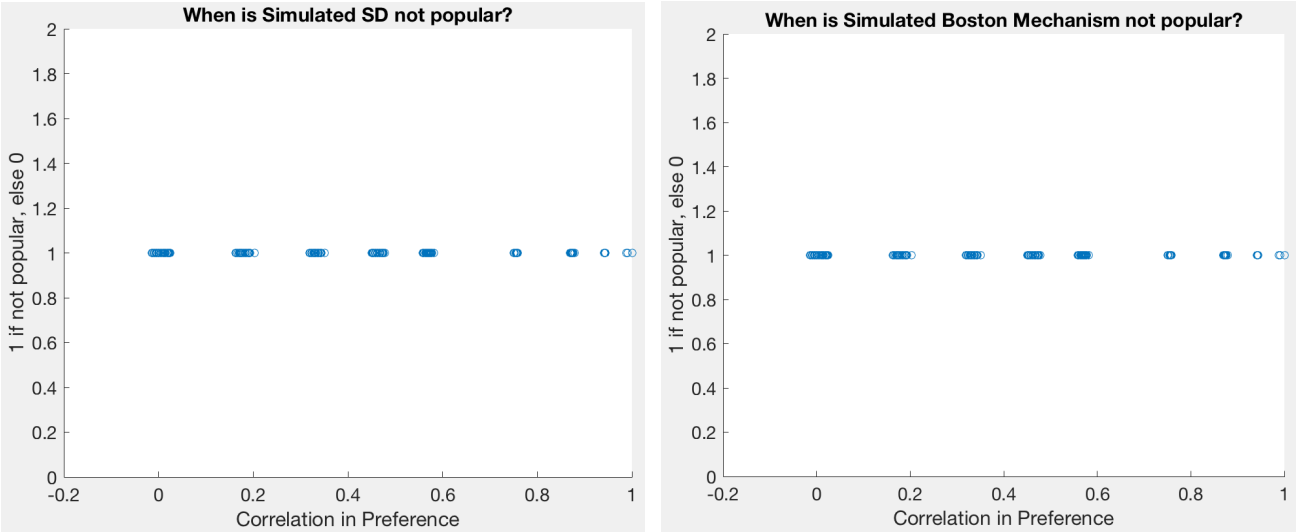


Figure 6. Necessary but not sufficient condition for complete Popular Matching across different correlations in preferences.
For each simulated set of preferences with a given correlation on the horizontal axis, I plot the number of committees (out of 16 total) which are f- and s-posts on the vertical axis. For there to exist a popular matching, it is necessary but not sufficient that all 16 committees be spanned by the f- and s-posts.

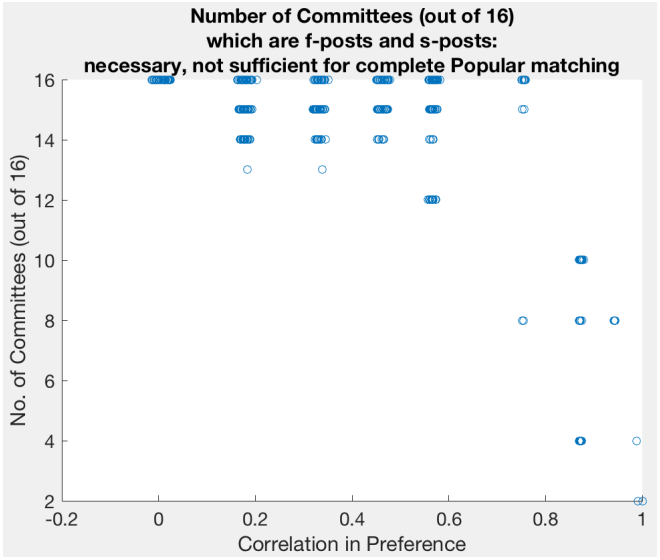


Figure 7. Majority Stability Under Serial Dictatorship.
Simulating the Serial Dictatorship in order of seniority across various preference environments with varying levels of correlation across agents' preferences, *Left* shows the fraction of times a majority overturns the matching produced by the Serial Dictatorship, while *Right* shows the maximal sized coalition that can improve from each Serial Dictatorship matching.

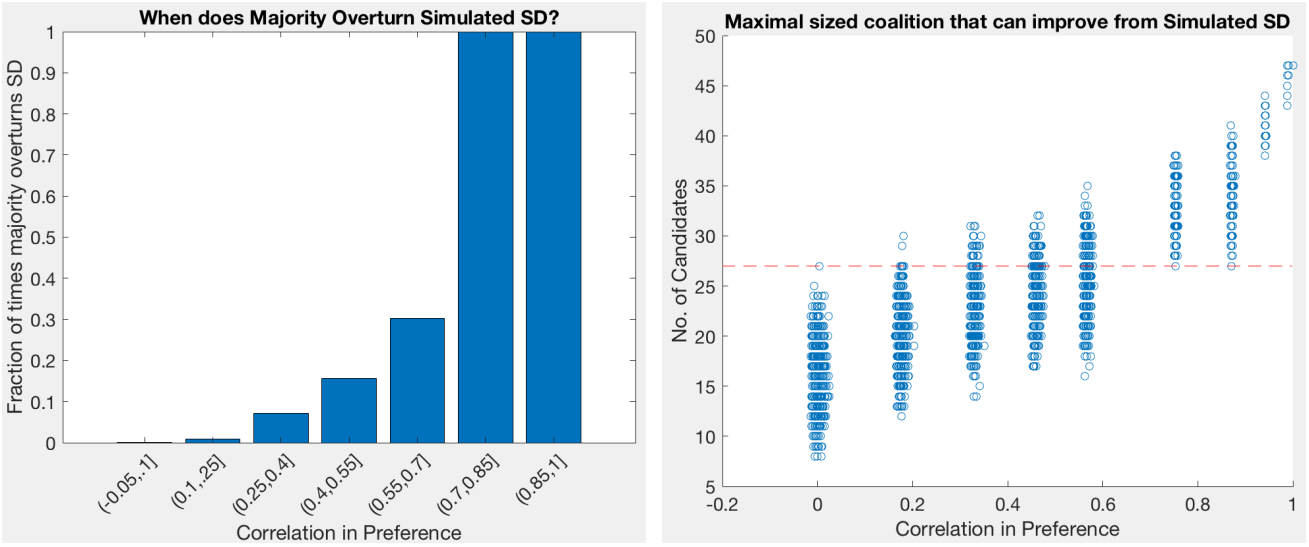


Figure 8. Majority Stability Under Boston Mechanism.
Simulating the Boston mechanism (assuming truthful preference revelation and tie-breaks in order of seniority) across various preference environments with varying levels of correlation across agents' preferences, *Left* shows the fraction of times a majority overturns the matching produced by the Boston mechanism, while *Right* shows the maximal sized coalition that can improve from each Boston mechanism matching.

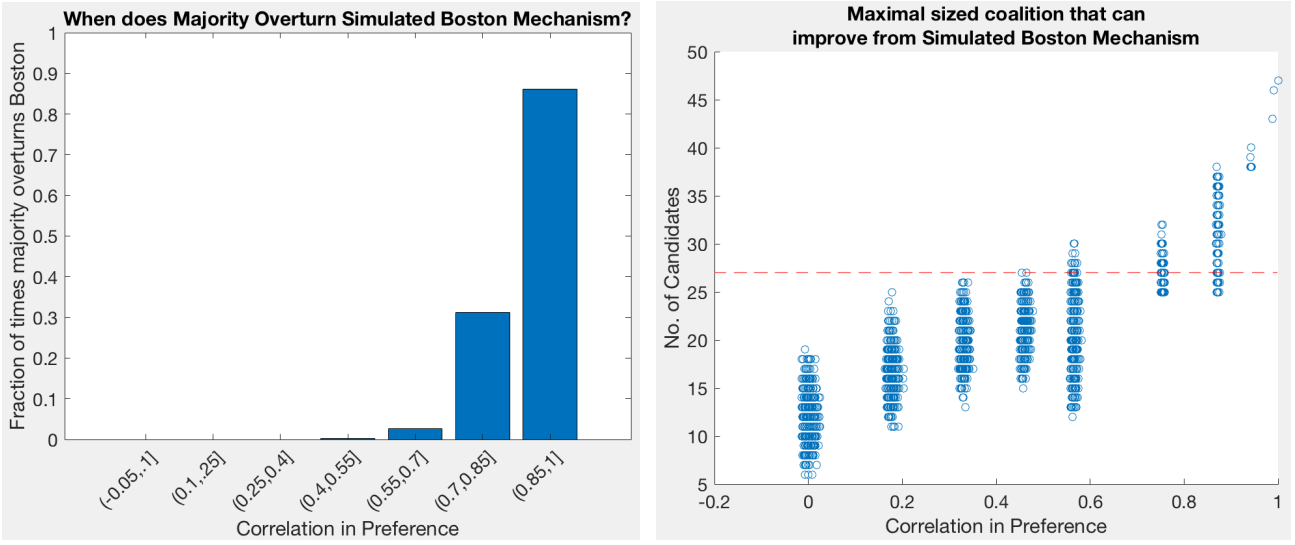
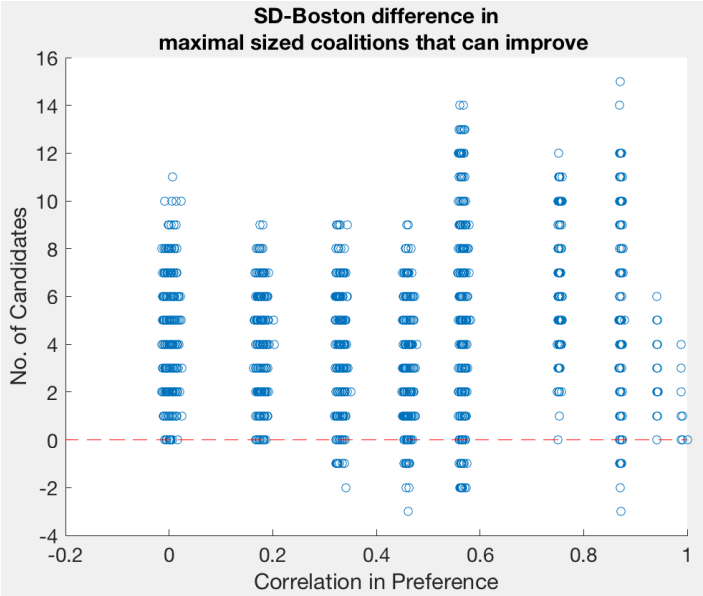
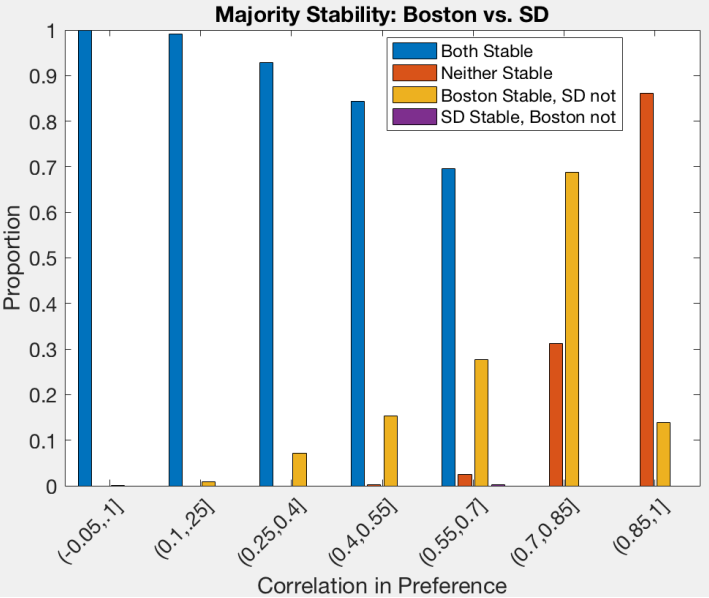


Figure 9. Comparing Majority Stability Under Serial Dictatorship v.s. Boston Mechanism.
Simulating Serial Dictatorship in order of seniority and Boston mechanism (assuming truthful preference revelation and tie-breaks in order of seniority) across varying levels of correlation across agents' preferences, these figures compare the performance of both mechanisms with regards to majority stability. *Left*: shows the difference between the sizes of the maximal sized coalitions which can be strictly improved under an alternative matching, across Serial Dictatorship and Boston mechanism. *Right*: shows that when matchings are majority stable for both mechanisms (blue), under neither mechanism (red), under the Boston mechanism but not Serial Dictatorship (orange), and under Serial Dictatorship but not Boston mechanism (purple).





Online Appendix B. A Discussion on Properties of Voting Rules & Mechanisms Yielding Institutional Stability.

In this appendix, I first analyze the properties of some mechanisms that can yield majority stability and then analyze what is the structure on voting rules that maintain certain normatively appealing properties of collective choice in social allocation choice problems.

First, I show that if a preference environment admits majority stable allocations, there exists a strategyproof mechanism that delivers some majority stable allocation. Here I build on Lemma 4:

Corollary 4.1. *If there exists a majority stable allocation, there exists a Serial Dictatorship order which achieves a majority stable allocation.*

> **Proof**: Serial Dictatorship spans all Pareto efficient allocations. Hence it can also implement a Pareto efficient allocation that is also majority stable (from Lemma 4).

Corollary 4.2. *If there exists a majority stable allocation, there always exists a strategyproof mechanism to attain it.*

> **Proof**: Serial Dictatorship with a given seniority order is strategyproof.[21]

Corollary 4.4. *If there exists a majority stable allocation, a seniority order of Serial Dictatorship which attains a majority stable allocation has all those getting their first preference seat being senior to those getting their second preference seat, ... (with any seniority order amongst agents being assigned seats of equal preference ranks.).*

> **Proof**: If not, there exists a pareto improving cycle and this contradicts the Pareto efficient majority stable allocation.

Next, in the spirit of Arrow's Impossibility Theorem from social choice problems, I discipline the collective choice of the social allocation choice problems to adhere to certain

[21]Serial Dictatorship is also group-strategyproof. It is important to distinguish between institutional stability which is a property of an allocation in a given preference environment and group-strategyproofness which is a property of the mechanism across all preference environments.

properties. It is important to realize that results from social choice often cannot be imported directly to social allocation choice because under Assumption 4, arbitrary individual preferences over matchings are not feasible.

Theorem 3. *Any preference aggregation rule f that is acyclic and weakly Paretian is collegial.*[22]

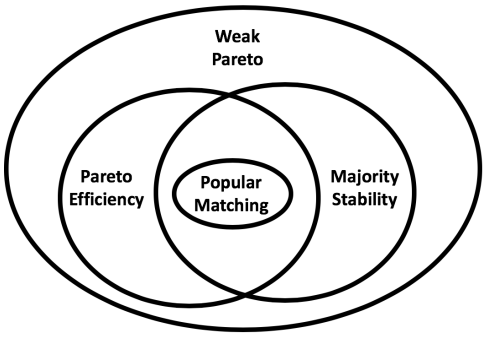
Proof[23]: Since f is weakly Paretian, there exists a decisive set and hence $\mathbb{L}(f) \neq \emptyset$. Now suppose by contradiction that f is non-collegial. Then for any $i \in \mathbb{I}$, there exists some $L \in \mathbb{L}(f)$ such that $i \notin L$. Since $\mathbb{L}(f)$ is monotonic (Lemma 2.2 from Austen-Smith and Banks (1999)), for every $i \in \mathbb{I}$, $\mathbb{I} \backslash \{i\} \in \mathbb{L}(f)$. Now consider the preference profile where all N agents have the same rank order preference over seats: $A > B > ....$ Take the set of matchings where i gets his top choice. The size of this set is $(N-1)!$ since, in this set of matchings, the remaining $N-1$ agents can be allotted the $N-1$ remaining seats. For each such permutation, consider all the matchings which are translations over agents of these permutations (e.g., for $N = 3$, a given permutation 1-A,2-B,3-C leads to a set of translation matchings: $M_1 = (1-A, 2-B, 3-C)$, $M_2 = (2-A, 3-B, 1-C)$, and $M_3 = (3-A, 1-B, 2-C)$). Each i has a strict preference over each of these translation matchings. Label these matchings as M_i where i gets everyone's top choice seat, A. Now, order each i's individual preference: i prefers matching where he gets A, over that where he gets B, etc. For all $j = 2, ..., N$, all agents except j prefer M_{j-1} to M_j, but since $\mathbb{I}\{j\} \in \mathbb{L}(f)$, $M_{j-1} \succ M_j$. Also, all agents except 1 prefer M_N to M_1, hence $M_N \succ M_1$. Therefore we have a cycle on $\{M_1, ..., M_N\}$: $M_N \succ M_1 \succ M_2 \succ ... \succ M_N$, which contradicts acyclicity.

[22]I follow Austen-Smith and Banks (1999) definitions: (1) A preference aggregation rule f is weakly Paretian if, for every profile and for any matchings M_1 and M_2, and all $i \in \mathbb{I}$, $x \succ_i y$ implies $x \succ_f y$. (2) A binary relation $\succeq$ is acyclic on the set of all matchings M if for all $\{M_1, M_2, ..., M_m\} \in M$, $M_1 \succ M_2$, $M_2 \succ M_3$,..., $M_{m-1} \succ M_m$ implies $M_1 \succeq M_m$. (3) A set is decisive for M_1 against M_2 if for every preference profile, $M_1 \succ_i M_2$ for all $i \in \mathbb{I}$ implies $M_1 \succ M_2$. (4) For any preference aggregation rule f, let $\mathbb{L}(f)$ denote the set of decisive coalitions associate with f. (5) A preference aggregation rule f is collegial if and only if $K(\mathbb{L}(f)) = \cap_{L \in \mathbb{L}(f)} L$ is non-empty. The set $K(\mathbb{L}(f))$ is called the collegium.

[23]This proof closely follows Theorem 2.4 from Austen-Smith and Banks (1999), but does not follow immediately since social allocation choice problems with Assumption 4 restrict the domain of preferences.

Online Appendix C. **Set inclusions of matchings across Institutional Stability notions.**

This appendix provides some examples and proofs of the set inclusion from Figure 1 replicated below:



- **Remark.** *All popular matchings are Pareto efficient.*

  Proof: if allocation M is not Pareto efficient, then there exists an alternative allocation M' such that $|M' \succ_i M| > 0$ and $|M \succ_i M'| = 0$. Thus, $|M' \succ_i M| > |M \succ_i M'| = 0$, thus M is not a popular matching.

- **Remark.** *All popular matchings are majority stable.*

  Proof: if allocation M is not majority stable, then there exists an alternative allocation M' such that $|M' \succ_i M| \geq \frac{N+1}{2}$. Thus, $|M' \succ_i M| < \frac{N+1}{2}$. Hence $|M' \succ_i M| > |M \succ_i M'| = 0$, thus M is not a popular matching.

- **Remark.** *There can exist a Pareto efficient matching that is not popular and not majority stable.*

  Proof: Consider agents $\{1, 2, 3\}$ ranking seats $\{A, B, C\}$. Consider 1-A, 2-B, 3-C.

1	2	3
A	A	B
B	B	A
C	C	C

Since Serial Dictatorship with seniority $1 > 2 > 3$ implements it, it is Pareto efficient. However, 1-C, 2-A, 3-B is strictly preferred by 3 and 2, hence it is not popular or majority stable.

- **Remark.** *There can exist a Pareto efficient matching that is majority stable, but not popular.*

  Proof: Consider agents $\{1, 2, 3, 4, 5\}$ ranking seats $\{A, B, C, D, E\}$. Consider 1-A, 2-B, 3-C, 4-D, 5-E.

1	2	3	4	5
A	A	A	A	A
B	B	B	B	B
C	C	C	D	E
D	D	D	C	D
E	E	E	E	C

  Since Serial Dictatorship with seniority $1 > 2 > 3 > 4 > 5$ implements it, it is Pareto efficient. Since envy is only over seats A and B, it is majority stable due to the packing problem. Since there are 3 blocks, there is no complete popular matching.

- **Remark.** *There can exist a majority stable matching that is not Pareto efficient nor popular.*

  Proof: Consider agents $\{1, 2, 3, 4, 5\}$ ranking seats $\{A, B, C, D, E\}$. Consider 1-B, 2-A, 3-C, 4-D, 5-E.

1	2	3	4	5
A	B	C	D	E
B	A	A	A	A
C	C	B	B	B
D	D	D	C	C
E	E	E	E	D

  The only envy is between agents 1 and 2 being envious of each other's assignments so it is majority stable, but fails Pareto efficiency and popular matching due to 1 and 2 strictly preferring $1 - A, 2 - B, 3 - C, 4 - D, 5 - E$.

- **Remark.** *There can exist a matching that is majority stable, popular and Pareto efficient.*

  Proof: Consider agents $\{1, 2, 3, 4, 5\}$ ranking seats $\{A, B, C, D, E\}$ given seniority $1 \succ 2 \succ 3 \succ 4 \succ 5$. Consider 1-A, 2-B, 3-C, 4-D, 5-E.

1	2	3	4	5
A	B	C	D	E
B	A	A	A	A
C	C	B	B	B
D	D	D	C	C
E	E	E	E	D