Congested Waiting Lists and Organ Allocation

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Abstract

More than 25% of the kidneys that are recovered from deceased donors in the U.S. and are offered to patients on the national waiting list are not utilized. This paper shows that waiting list designs can suffer from a form of congestion that can lead to discarding valuable organs. Specifically, as organs have a limited "cold ischemia time" in which they can be offered before they expire, an organ may expire before being offered to a patient lower on the waiting list who would accept it. We develop a parsimonious framework to study equilibria of waiting lists with congestion, and show that congestion provides a strong enough externality to substantively affect welfare and wastage. The delegation of organ acceptance decisions to risk-averse doctors can worsen congestion and increase waiting times. We discuss how recent changes to waiting list designs affect congestion, and policies and market designs that can mitigate congestion.

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1 Introduction

In 2024, 3,571 patients died while waiting for a kidney transplant from a deceased donor in the United States, and 4,758 patients were removed from the waiting list due to being too sick to receive transplants (Health Resources and Services Administration, 2025). During the same year, 27% of kidneys recovered from deceased donors were not used—a rate that has been increasing in recent years (Mohan et al., 2023; Wood et al., 2025). This paper seeks to understand how practical features of the organ allocation system can lead to excess non-utilization of recovered organs, and investigates implications for market design.

The key issue we focus on is that organs are perishable and hence have a limited time to be offered before they expire. We show that this friction causes "congestion" in the sense that some patient on the waiting list would be willing to accept some organ, but the organ expires before being offered to that patient—leading to excess non-utilization. We argue that understanding the welfare effects of a market design requires accounting for its impact on congestion, and evaluate recent attempts to improve utilization rates.

We proceed in three steps. First, we provide descriptive evidence that suggests that organ expiry leads to congestion, and that the resulting under-utilization of organs is quantitatively significant. Second, we investigate the welfare implications of congestion by developing a tractable theoretical model of waiting lists with heterogeneous, expiring objects. We show that congestion leads to artificially lower endogenous demand for lower-quality organs—thereby generating severe inefficiencies and leading to policies sometimes having counterintuitive welfare implications. Third, we use our model to evaluate recent market design innovations and proposals. For example, we show that attempts to increase access to organ offers, such as in a 2021 reform in the US, can exacerbate congestion. On the other hand, the recent practices of accelerating placement of organs at risk of non-utilization via out-of-sequence offers may alleviate congestion and hence improve welfare.

In practice, when an organ becomes available, it is typically offered sequentially to patients in priority order. Once an offer is made, the patient can refuse the offer without penalty (i.e., without forgoing any priority). Patients near the top of the list may be inclined to refuse organs of reasonable quality, as such patients stand a chance of being offered a high-quality organ in the future. By contrast, patients further from

the top of the waiting list may accept lower quality organs, as they have less of a chance of surviving (and remaining healthy enough to undergo transplantation) until being offered a high-quality organ.

Refusals of potentially useful organs come at a cost because there is a limited "cold ischemia time" after recovery for which organs are viable for transplantation. We show that due to this limit on viability, waiting lists can enter into a counterintuitive state that we call *congestion*. Specifically, lower quality organs may expire before being offered to a patient who would be willing to accept them—despite there being such patients lower down the waiting list. Section 2 provides descriptive evidence suggesting that this form of congestion is prevalent and quantitatively significant in practice.

To investigate the welfare implications of congestion, we develop a parsimonious fluid-limit framework to study steady states of waiting lists with heterogeneous, expiring objects (Section 3). Patients and organs arrive as flows of constant rates. Patients stochastically depart unmatched (i.e., due to death or becoming too sick to undergo transplantation) at rates that can depend on their types (e.g., sicker or older patients may depart unmatched at a faster rate). To capture a key dimension of heterogeneity in organ allocation, we suppose that organs can be high-quality (safe) or low-quality (risky).[1] Patients are heterogeneous in how they value risky organs relative to safe organs. Last, to capture the limited cold ischemia time for which organs are viable, we assume that organs expire after being offered to γ mass of patients.[2]

A consequence of organs' limited viability is that organs—including risky organs—are never offered to patients who are not among the top γ on the waiting list. If enough patients strongly prefer safe organs, risky organs may never make it far enough down the waiting list to be offered to patients who would be willing to accept them. Hence, there may be congestion.

Congestion is relevant not only to patients who may be willing to accept risky organs. Indeed, congestion incentivizes such patients to wait for safe organs, thereby raising the waiting time for safe organs and hurting even patients who strongly prefer safe organs. When congestion occurs, all patients would strictly prefer a counterfactual with non-perishable organs, and hence no congestion. In this sense, when congestion arises, it hurts all patients.

[1]For interpretation, we focus on patients of a single blood type and abstract from other forms of incompatibility (e.g., tissue type incompatibility).

[2]We abstract from other forms of decay in organ quality with the elapse of cold ischemia time.

To illustrate the severity of the welfare loss caused by congestion, we consider a hypothetical policy that results in the procurement of more safe organs. Without congestion, this policy would obviously weakly benefit all patients and strictly benefit at least some. By contrast, we find that in the presence of congestion, such a policy can strictly hurt all patients. Indeed, an increase in the availability of safe organs results in more types waiting for safe organs. This can exacerbate congestion so severely that it raises waiting times for everyone.

Another feature of the organ allocation system can exacerbate congestion and harm patients. In practice, patients do not decide whether to refuse an organ offer themselves—rather, doctors make these decisions on behalf of patients. However, transplant centers are penalized for adverse post-transplant outcomes, but not deaths on the waiting list (Chan and Roth, 2024; Mildebrath et al., 2024)). As a result, doctors are incentivized to refuse risky organs even when patients may benefit from them.

This misalignment of incentives inflates the relative demand for safe organs. Without congestion, the change in demand increases the waiting time for safe organs but lowers the waiting time for risky organs. However, in the presence of congestion, the change in demand also worsens congestion, which we show increases wastage enough to raise the waiting time for risky organs as well. As a result, if the system would be congested under patient-optimal refusal decisions, the delegation of decisions to doctors harms all patients. Moreover, while some doctors may benefit from not using risky organs, congestion makes doctors worse off in the aggregate from making decisions on patients' behalves. Thus, impacts on congestion make the misalignment of incentives between patients and doctors particularly harmful.

Design choices can worsen congestion and potentially harm welfare. For example, we show that efforts to improve market thickness can worsen congestion and harm patients. This insight may shed light on some of the increase in organ discards. In March 2021, the Organ Procurement and Transplantation Network (OPTN) moved from offering a "local" priority to patients based on an Organ Procurement Organization (OPO) donor service service area to offer priority to patients within 250 nautical miles of the donor hospital. This change increased the number of patients that are given local priority. At the same time the discard rate of kidneys from deceased donors has increased from 21.2% in 2020 to 24.5% in 2021.

In response to the increasing rate of organ discards, organ procurement organizations have recently begun to *accelerate* organs by offering them to transplant

centers out of sequence. In 2023, more than 14% of transplanted kidneys from deceased donors were placed out of sequence (Liyanage et al., 2025). We argue why accelerated placement can help overcome congestion, and discuss the difficulties of alleviating congestion under sequential offers.

Methodologically, the possibility of congestion means that we cannot use standard approaches to analyze queuing systems. When patients' departure rates are heterogeneous, there may be multiple steady states that suffer from congestion, and we cannot solve for the steady states in closed form. Our analysis instead relies on monotone comparative statics methods (Milgrom and Roberts, 1994), which had not been previously used to study queues.

1.1 Related Literature

This paper contributes to the literature on dynamic allocation without transfers. A large body of work studies the design of waiting lists and priority mechanisms for allocating scarce objects as in organ allocation and public housing (Su and Zenios, 2004, 2005, 2006; Bloch and Cantala, 2017; Agarwal et al., 2021; Ata et al., 2021; Leshno, 2022; Arnosti and Shi, 2020; Waldinger, 2021; Murra-Anton and Thakral, 2024; Schummer, 2021). [3] We study a friction that is largely absent from this literature: implementing a priority order takes time, and an object may expire before the allocation process reaches an agent who would accept it. Moreover, our focus is not on design; instead we analyze the sequential priority mechanism used in the practice of organ allocation and study equilibrium consequences and comparative statics under this friction.

Several papers study and compare queueing systems with endogenous acceptance. Su and Zenios (2004) and Bloch and Cantala (2017) show how queueing discipline affects patients' incentives to reject lower-quality objects and, consequently, object utilization. They find that Last-Come-First-Serve and Random order increase utilization by reducing selectivity at the expense of welfare.[4] Closely related is Shi and Yin (2022), which shows that waiting-time priority can induce patients to reject acceptable organs while waiting for more desirable ones, and characterize mechanisms that

[3] See also (Baccara et al., 2020) for congestion and design of waiting lists in two-sided matching markets as well as Doval (2025) for Ashlagi and Roth (2021) for recent surveys on dynamic matching and organ allocation.

[4] See also Agarwal et al. (2021) for an empirical equilibrium analysis of alternative kidney waiting-list designs as well as Ata et al. (2021) for waitlist designs with endogenous choice.

eliminate the resulting waste. In these models, waste can arise because organs are rejected by all patients on the waiting list. The key difference in our setting is that an organ may expire before being offered to all patients on the list. Thus, an organ can be unused even though willing recipients remain on the waiting list and its equilibrium waiting time is positive. In a standard queue this cannot occur as a positive waiting time reflects excess demand. Congestion breaks this usual market-clearing interpretation of waiting times. Refusals in our model therefore create a congestion externality in addition to reflecting patients' endogenous demand. We study how this friction changes equilibrium acceptance, waiting times, and the welfare consequences of changes in the allocation environment.

Close to this paper on the offering-process friction is Castro et al. (2021), which study perishable ride requests offered sequentially to strategic drivers in an airport queue. They design direct and randomized FIFO mechanisms that allow unattractive requests to move through the queue more quickly, thereby reducing waste from repeated rejections. Our focus is complementary: rather than designing the queue to overcome this friction, we study its equilibrium consequences under the sequential priority mechanism used in organ allocation. Their model further assume homogeneous preferences and no departures. Heterogeneity in patients' willingness to wait for quality and in their rates of departure contribute to the congestion and acceptance behavior, and makes the analysis relevant for practical allocation policies.

Our analysis also complements literature studying why organ offers are refused. Zhang (2010), De Mel et al. (2021), and Doval et al. (2024) study informational forces generated by earlier acceptance and rejection decisions, while Chan and Roth (2024) and Mildebrath et al. (2024) study the incentives created by transplant-center regulation. These papers identify forces that affect acceptance decisions. We instead emphasize how refusals propagate through the market when the offering process itself is congested.

Finally, our fluid formulation also relates to the queueing literature (Kleinrock, 1974; Dai, 1995; Whitt, 2006; Talreja and Whitt, 2008); the possibility of multiple congested steady states leads us to use monotone comparative statics (Milgrom and Roberts, 1994) to characterize equilibrium responses.

2 Background

2.1 Overview

There are more than 808,000 people with end-stage renal disease in the United States (USRDS, 2024). Medicare provides near-universal coverage for this disease and has spent over $77 billion treating kidney failure in 2021 (USRDS, 2023). Transplantation is the preferred treatment: it not only improves health outcomes, but also saves Medicare more than $200,000 relative to dialysis costs over the life of a patient who receives a transplant (Wolfe et al., 1999; Held et al., 2016).

Kidneys from deceased donors is allocated through a waiting list, and there is a severe shortage of organs for transplantation. In 2024, 47,159 patients joined the waiting list, more than 66% of whom were already on dialysis. In the same year, there were 21,341 kidney transplants from deceased donors in the US, 3,571 patients died while waiting for a kidney transplant, and 4,758 patients were removed from the waiting list due to being too sick to be transplanted. Figure 1 shows the annual trends of the waiting list for kidneys from deceased donors.[5]

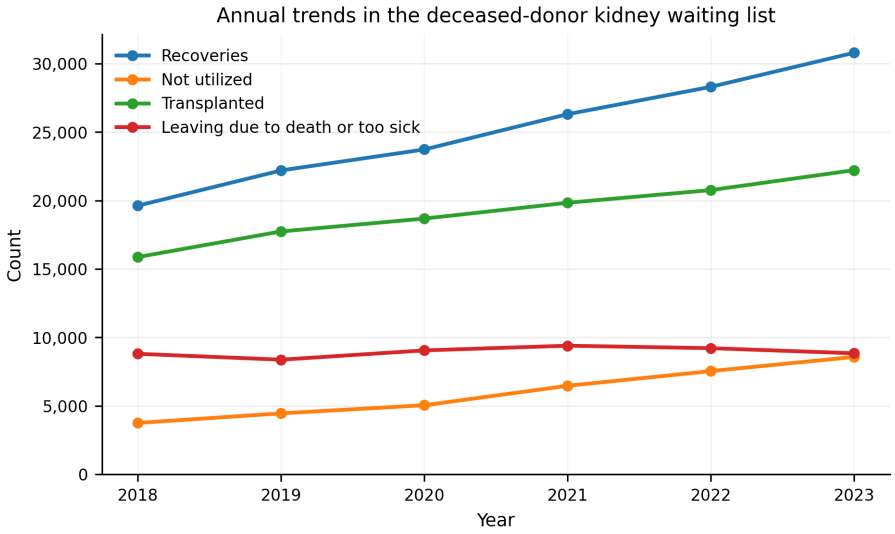


Figure 1: Annual trends in the deceased donor waiting list for cadaver kidneys.

[5]Data presented in this paper is from the United Network for Organ Sharing (UNOS), which includes the Standard Transplant Analysis and Research (STAR) files for de-identified information on all patients, as well as the Potential Transplant Recipient (PTR) files for de-identified organ offers.

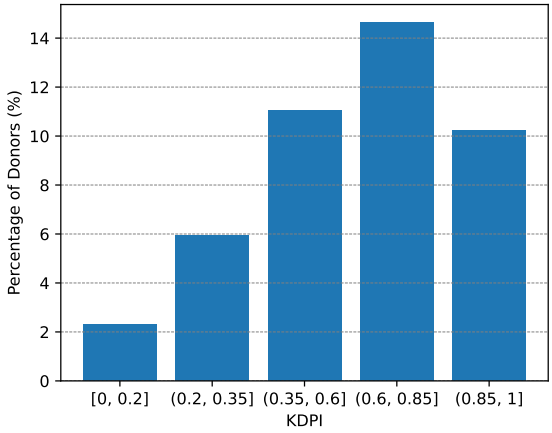


Figure 2: Percent of donors by (binned) KDPI for which two kidneys were recovered but only one transplanted.

2.2 Key Institutional Features and Stylized Facts

The economics of organ allocation are shaped by several institutional and empirical features that generate congestion in waiting lists.

Kidneys from deceased donors vary substantially in quality and expected graft survival. A widely used measure of donor quality is the Kidney Donor Profile Index (KDPI), where higher KDPI values correspond to lower expected graft longevity.[6] Even kidneys with relatively high KDPI values often provide substantial survival benefits relative to remaining on dialysis. While only around 40% of patients survive on dialysis for more than 5 years (USRDS, 2013), even organs in the lowest 15th percentiles of KDPI (i.e., the highest risk) have more than 60% five year graft survival. Moreover, many discarded kidneys have observable characteristics similar to kidneys that are ultimately transplanted (Mohan et al., 2018; Aubert et al., 2019). For example, in 2023 there were 1,547 deceased donors for whom both kidneys were recovered but only one was transplanted (Figure 2).

It is therefore plausible that many kidneys that are currently not utilized could in fact be valuable as grafts, leading to numerous excess deaths of patients on the waiting list.

Second, organs from deceased donors are offered to patients on a waiting list and offers can be refused by patients and doctors without penalty. The waiting lists for deceased donors are ordered according to priorities that include medical characteris-

[6]KDPI is a percentile version of Kidney Donor Risk Index (KDRI), which is based on a basic lab test prior to recovery.

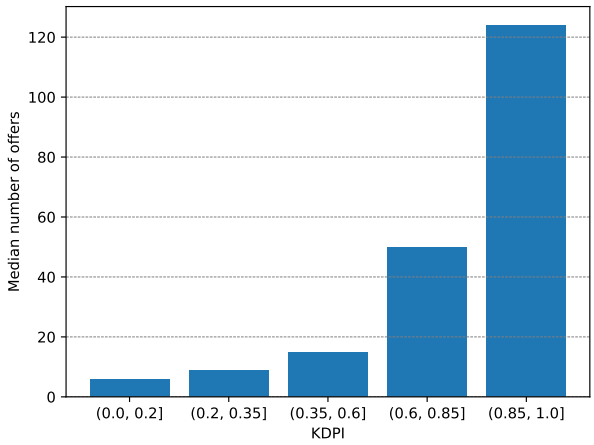


Figure 3: Median number of offers for transplanted organs by Kidney Donor Percentile Index (KDPI).

tics, demographics as well as waiting time. When an organ becomes available, it is offered to patients according to a priority order.[7] Once an offer is made, the patient or her doctor can refuse the offer without affecting her priority.[8]

Since patients at the "top" of the list may be inclined to refuse offers of organs with low quality, such organs may accumulate many refusals. Figure 3 shows the number of offers for organs that are eventually transplanted. This number increases substantially as the quality of the organ decreases. So the process of repeatedly declining organs affects which patients receive which organs, but not necessarily which organs are utilized.

Third, recovered organs have a limited time to be placed. A long cold ischemia time (the time on ice since recovery to being used) can have a substantial impact on the graft survival (Quiroga et al., 2006; Peters-Sengers et al., 2019; Debout et al., 2015). Recovered kidneys are medically viable for only around 24-36 hours after procurement and refuse in quality as CIT increases. Refusal reasons due to long cold ischemia time become very common already 6 hours after recovery (Figure 4).[9] So

[7]For details of the current organ allocation policies in the US see `https://optn.transplant.hrsa.gov/media/eavh5bf3/optn_policies.pdf`. For example, healthier patients are prioritized for highest quality organs. Increased priority is also given to patients with a perfect match or patients who are highly sensitized.

[8]In practice, they may need to do so because particular organs are not a good enough match for them (e.g., since the patient may refuse the organ due to existing antibodies, which is discovered through a crossmatch test.)

[9]See also Guan et al. (2025) for a detailed analysis of offer refusals.

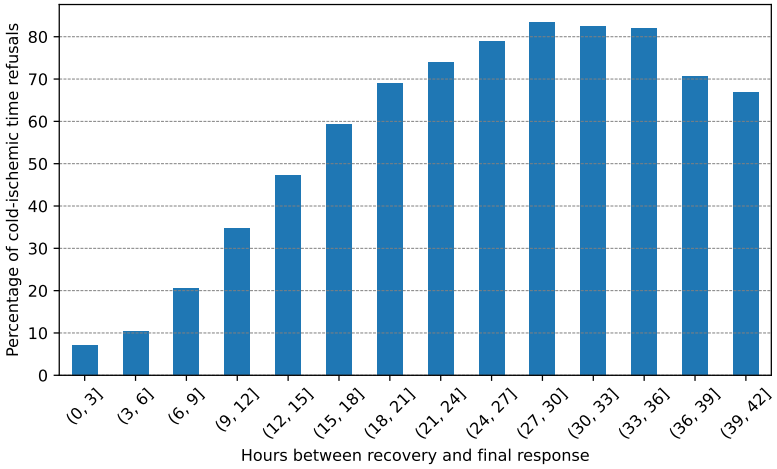


Figure 4: Percent of offer refusals that are due to long cold ischemia time, by (binned) number of hours after organ recovery.

lower quality organs, which require many offers to patients on the list, may therefore accrue excess cold ischemia time and *expire* (Barah et al., 2022; Guan et al., 2025) before reaching a patient who would be willing to accept the offer—despite there being such patients on the waiting list (Mehrotra et al., 2022).

Finally, in practice doctors often make decisions to refuse an offer on behalf of their patients. Guan et al. (2025) distinguish between a transplant center refusing an organ for a single patient or simultaneously refusing it for multiple patients and they document that the majority of the refusals are of the latter category.[10] At the same time survey data suggests that patients are more inclined to accept low quality organs than their doctors (Mehrotra et al., 2022; Sepulveda et al., 2023). Our analysis begins with patients' decisions, but we return in Section 7 to discuss how equilibrium outcomes change under moral hazard.

3 Model

This section develops our model of waiting lists with expiring objects. We take a continuous-time dynamic fluid approach, which makes the equilibrium analysis and comparative statics tractable by avoiding fluctuations due to stochastic arrivals and departures. Thus, we suppose a continuum of patients arrive on the waitlist at a flow of a constant rate $p > 0$. The analogue of position in a standard queue is that we

[10]In 2022-2023, more than 69% of refusals were multi-patient refusals.

refer to a patient who has waited for time t as being *position* t.

3.1 Organs

There are two types of organs, safe grafts (denoted by $s > 0$) and risky grafts (denoted by $r > 0$). A continuum of safe organs (resp. risky organs) arrives at a flow of constant rate sp (resp. rp). We assume that $s < 1$, capturing the shortage of safe organs.

Each arriving organ is offered sequentially to patients in descending order of waiting time. However, an organ expires after $\gamma > 0$ mass of patients refuse the organ. Here, γ can be interpreted as a measure of the lifetime of a graft (called "cold ischemia time" in the medical literature). Thus, each organ goes to a highest-ranking patient on the waitlist (at the time of the organ's arrival) who is willing to accept the organ, except that patients who have more than γ mass (of patients) ahead of them on the waitlist do not receive organs. We allow for the case of $\gamma = \infty$, which captures the case with organs that do not expire.

3.2 Patients' Preferences and Decisions

Patients are heterogeneous in their individual characteristics, as well as in the characteristics of transplant centers that manage their care. We summarize these characteristics in a type $\theta \in \Theta$, where Θ is a compact domain in a finite-dimensional Euclidean space $\mathbb{R}^k$. We assume that the distribution of characteristics of arriving patients is constant over time, and admits a (measurable) probability density function $f(\theta)$. A patient of type θ has Bernoulli utility

$$u_{\text{patient}} = \begin{cases} 1 & \text{if get safe organ} \\ \nu(\theta) & \text{if get risky organ} \\ 0 & \text{if do not get an organ} \end{cases},$$

where $\nu(\theta) \in [0, 1]$ is the willingness to accept a risky organ. Patients of type θ who remain unmatched depart at a constant rate $\delta(\theta) > 0$.[11] We assume that $\delta(\theta)$ is bounded away from 0; and that conditional on $\delta(\theta)$, the distribution of $\nu(\theta)$ is continuous.

Upon receiving an offer, a patient decides whether to accept it. We suppose that patients make these decisions to maximize their expected utility. In Section 7, we

[11] Here, $\nu : \Theta \to [0, 1]$ and $\delta : \Theta \to \mathbb{R}_{>0}$ are measurable functions.

introduce an extension in which decisions are delegated to hospitals.

3.3 Discussion of Assumptions

To isolate the impact of organ expiry due to excess cold ischemia time in a tractable framework, our model abstracts away some features of the kidney waiting list.

In practice, there are numerous different qualities of organs, parameterized by the KDPI. For tractability, we consider two types of organs to understand the impact of congestion on the utilization of lower-quality organs.

In practice, the waiting list involves priorities, which depend on medical and geographic characteristics of the organ and the patient, including waiting times. For example, in kidney allocation in the US, the healthy and young patients are prioritized for organs with the top 20% quality (measured by KDPI). However, a majority of organs from deceased donors are effectively allocated (within a local region) based on the amount of time on dialysis, which we interpret as waiting time in our framework. Indeed, for each organ, typically few patients are assigned a high priority (as few patients have a perfect match, or are very sensitized yet still compatible with the organ). For other patients compatible with the organ, priority is mainly determined by dialysis time.[12] Our analysis can be interpreted as applying within any of the majority of priority classes for which organs are effectively allocated in order of waiting times on dialysis.

Last, we impose functional form assumptions on the expiry of organs and departure of patients. In practice, organ quality can degrade continuously over time (Sampaio et al., 2018). For tractability, we assume that the quality of the organ remains fixed for γ offers after which it immediately expires. We also assume that patients depart unmatched at a constant, type-dependent rate, so each type's distribution of departure times (if they never receive organs) takes an exponential distribution. This assumption is also close to empirical observations and has been widely applied in the literature via COX regression models (Medin et al., 2000; Schaubel et al., 2009; Agarwal et al., 2021).

[12] See `https://optn.transplant.hrsa.gov/media/eavh5bf3/optn_policies.pdf` for organ allocation policies in the US.

4 Equilibria

In this section, we introduce steady states based on waiting times. Intuitively, due to the fluid-limit analysis, there will be two waiting times: a waiting time T_{safe} needed to obtain an offer of a safe organ, and a waiting time T_{risky} needed to obtain an offer of a risky organ. All patients accept safe organs if offered, and patients who wait at least T_{risky} continue to be offered risky organs at each moment in time (until they accept an offer or depart unmatched). In Appendix A, we formally define steady-state equilibria when patients can dynamically adapt their organ acceptance decision rules, and show that steady-state equilibria precisely correspond to the waiting-time steady states introduced in this section.

Suppose a patient of type θ is offered a risky organ at time t but knows that they will receive an offer of a safe organ at time $T_{\text{safe}} \geq t$. Maximizing expected utility taking into account the exogenous rate of unmatched departure, they will accept the risky organ if $T_{\text{safe}} - t > W(\theta)$, where

$$W(\theta) = \frac{-\log \nu(\theta)}{\delta(\theta)}, \tag{1}$$

and accept the safe organ if $T_{\text{safe}} - t < W(\theta)$. (If $T_{\text{safe}} - t = W(\theta)$, then the patient has multiple optimal strategies.)

We call the quantity $W(\theta)$ the *willingness-to-wait* for a patient with type θ. As the distribution of $\nu(\theta)$ is continuous conditional on $\delta(\theta)$, the distribution of W is continuous as well. Hence the organ assignment in steady-state equilibrium is determined by an agent's willingness-to-wait. Indeed,a patient of type θ refuses a risky organ at time T_{risky} and waits for a safe one if $W(\theta) > T_{\text{safe}} - T_{\text{risky}}$; such patients then continue to refuse any risky organs that are offered. Conversely, a patient of type θ accepts a risky organ at time T_{risky} if $W(\theta) < T_{\text{safe}} - T_{\text{risky}}$.

We can then define waiting-time steady states in terms of the distribution of the willingness-to-wait. Such steady states must satisfy four conditions. As safe organs are in short supply and all patients accept offers of safe organs, all safe organs must be allocated. Three further conditions constrain T_{risky}. First, the demand for risky organs cannot exceed the supply of risky organs. Second, risky organs expire if refused by γ mass of patients, which can increase T_{risky}. Third, risky organs are not intentionally withheld: one of the previous two conditions must hold with equality

unless everyone is offered a risky organ ($T_{\text{risky}} = 0$).

Definition 1 (Waiting-Time Steady state)**.** A *waiting-time steady state* consists of waiting times $0 \leq T_{\text{risky}} < T_{\text{safe}}$ that satisfy the following conditions:

1. All safe organs are allocated:

$$\text{SafeDemand}(T_{\text{risky}}, T_{\text{safe}}) := p \int_{\theta: W(\theta) > T_{\text{safe}} - T_{\text{risky}}} e^{-\delta(\theta) T_{\text{safe}}} f(\theta) d\theta = \ sp\,. \quad (2)$$

2. Demand for risky organs cannot exceed their supply:

$$\text{RiskyDemand}(T_{\text{risky}}, T_{\text{safe}}) := p \int_{\theta: W(\theta) < T_{\text{safe}} - T_{\text{risky}}} e^{-\delta(\theta) T_{\text{risky}}} f(\theta) d\theta \leq \ rp\,. \quad (3)$$

3. An organ expires after being refused by γ measure of agents:

$$\text{RefuseRisky}(T_{\text{risky}}, T_{\text{safe}}) := \int_{T_{\text{risky}}}^{T_{\text{safe}}} \underbrace{p \int_{\theta: W(\theta) > T_{\text{safe}} - T_{\text{risky}}} e^{-\delta(\theta) t} f(\theta) d\theta}_{\text{declining mass of patients at position } t} dt \leq \ \gamma\,. \quad (4)$$

4. Risky organs are not intentionally withheld:

$$\text{At least one of the inequalities (3),(4), and } T_{\text{risky}} \geq 0 \text{ holds with equality.} \quad (5)$$

In the case of $\gamma = \infty$, (4) cannot bind, and we obtain the standard case of a fluid limit queuing with non-expiring objects.

As patients make optimal decisions about organ acceptance given the waiting times, the expected utility of type θ in a waiting-time steady state $(T_{\text{safe}}, T_{\text{risky}})$ is

$$U_\theta(T_{\text{safe}}, T_{\text{risky}}) = \mathbb{E}[u_{\text{patient}}] = \max\left\{e^{-\delta(\theta) T_{\text{safe}}}, \nu(\theta) e^{-\delta(\theta) T_{\text{risky}}}\right\}.$$

4.1 Existence

We next establish the existence of a waiting-time steady state.

Proposition 1 (Existence)**.** *In each market, there exist waiting-time steady states.*

A proof with arbitrary numbers of types of organs can be given based on Kakutani's Fixed Point Theorem. We instead give a more direct argument that uses the two-type structure but helps to illustrate the logic behind equilibrium in our model.

Note that participants' decisions depend only on the difference $T_{\text{safe}} - T_{\text{risky}}$ in waiting times. A key consequence, that we use throughout our subsequent analysis, is that the flow balance for safe organs (2) can be expressed as a relationship between T_{safe} and the difference $T_{\text{safe}} - T_{\text{risky}}$ in waiting times. The larger the difference, the fewer people who wait for a safe organ, and hence the smaller the waiting time for safe organs. As $T_{\text{risky}} = T_{\text{safe}} - (T_{\text{safe}} - T_{\text{risky}})$, this also implies a relationship between T_{risky} and $T_{\text{safe}} - T_{\text{risky}}$.

Formally, let $\overline{\Delta T} = \operatorname{ess\,sup} W(\theta) \leq \infty$, which is the maximum amount of time anyone is willing to wait for a safe organ.

Lemma 1. *(a) There is a non-increasing, continuous function $h_{\text{safe}} : [0, \overline{\Delta T}) \to \mathbb{R}$ (dependent on the parameters s, $f(\theta)$, $\nu(\theta)$, and $\delta(\theta)$) with $h_{\text{safe}}(0) > 0$ and $\lim_{\Delta T \to \overline{\Delta T}^-} h_{\text{safe}}(\Delta T) < 0$ such that $(T_{\text{risky}}, T_{\text{safe}})$ satisfies (2) if and only if $T_{\text{safe}} = h_{\text{safe}}(T_{\text{safe}} - T_{\text{risky}})$. If $W(\theta)$ has full support on $[0, \overline{\Delta T})$, then h_{safe} is strictly decreasing.*

(b) Letting $h_{\text{risky}}(\Delta T) = h_{\text{safe}}(\Delta T) - \Delta T$, which is continuous and strictly decreasing on $[0, \overline{\Delta T})$, we have that $(T_{\text{risky}}, T_{\text{safe}})$ satisfies (2) if and only if $T_{\text{safe}} = h_{\text{safe}}(T_{\text{safe}} - T_{\text{risky}})$.

Proof. We write $\Delta T = T_{\text{safe}} - T_{\text{risky}}$, and consider the demand for safe organs fixing ΔT, which is given by $G(T_{\text{safe}}, \Delta T) = \text{SafeDemand}(T_{\text{safe}} - \Delta T, T_{\text{safe}})$. Note that for each $\Delta T \in [0, \overline{\Delta T})$, the quantity $G(T_{\text{safe}}, \Delta T)$ is strictly decreasing and continuous in T_{safe} and converges to ∞ as $T_{\text{safe}} \to -\infty$ and to $0 < sp$ as $T_{\text{safe}} \to \infty$. Hence, for each $\Delta T < \overline{\Delta T}$, there exists a unique value $T_{\text{safe}} = h_{\text{safe}}(\Delta T)$ such that $G(T_{\text{safe}}, \Delta T) = sp$—i.e., (2) holds.

As $G(T_{\text{safe}}, \Delta T)$ is non-increasing in ΔT and continuous in $(\Delta T, T_{\text{safe}})$, the function h is non-increasing and continuous. Since $s < 1$, we have $G(0, 0) = p > sp = G(h_{\text{safe}}(0), 0)$. As $G(T_{\text{safe}}, 0)$ is decreasing in T_{safe}, we must have $h_{\text{safe}}(0) > 0$. As the distribution of $W(\theta)$ is continuous, we have $\lim_{\Delta T \to (\overline{\Delta T})^-} G(0, \Delta T) = 0 < sp$. As $G(T_{\text{safe}}, \Delta T)$ is decreasing in T_{safe}, it follows that $h_{\text{safe}}(\Delta T) < 0$ for all ΔT sufficiently close to $\overline{\Delta T}$.

The second part of the lemma is immediate from the first as $T_{\text{risky}} = T_{\text{safe}} - \Delta T$. □

Conditioning on T_{safe} and ΔT clearing the safe organs (i.e. satisfying (2)), we can express for the risky demand and mass of agents that refuse risky organs as a function of ΔT:

$$\text{RiskyDemand}_h(\Delta T) := \text{RiskyDemand}(h_{\text{risky}}(\Delta T), h_{\text{safe}}(\Delta T))$$
$$\text{RefuseRisky}_h(\Delta T) := \text{RefuseRisky}(h_{\text{risky}}(\Delta T), h_{\text{safe}}(\Delta T))$$

The continuity of h_{risky} and h_{safe} and the absolute continuity of the distribution of W imply that $\text{RiskyDemand}_h(\Delta T)$ and $\text{RefuseRisky}_h(\Delta T)$ are continuous on $[0, \overline{\Delta T})$. To prove Proposition 1, we show the existence of a steady state value for ΔT using a simple continuity argument.

Proof of Proposition 1. Let $h_{\text{risky}}, h_{\text{safe}}$ be as in Lemma 1, and write $\Delta T = T_{\text{safe}} - T_{\text{risky}}$. Taking $T_{\text{risky}} = h_{\text{risky}}(\Delta T)$ and $T_{\text{safe}} = h_{\text{safe}}(\Delta T)$, based on the conditions for steady-state, let

$$S = \{\Delta T \in [0, \overline{\Delta T}) \mid \text{RiskyDemand}_h(\Delta T) \leq r, \text{RefuseRisky}_h(\Delta T) \leq \gamma, \text{ and } h_{\text{risky}}(\Delta T) \geq 0\}.$$

Note that $\text{RiskyDemand}_h(\Delta T) = 0 < r$ and $\text{RefuseRisky}_h(\Delta T) = 0 < \gamma$. Furthermore, since $h_{\text{safe}}(0) > 0$, we have $h_{\text{risky}}(0) > 0$. Thus, $0 \in S$, so in particular, S is non-empty.

As $\lim_{\Delta T \to (\overline{\Delta T})^-} h_{\text{safe}}(\Delta T)) < 0$ and $h_{\text{risky}}(\Delta T) \leq h_{\text{safe}}(\Delta T)$, the set S must be bounded away from $\overline{\Delta T}$. Since the functions $\text{RiskyDemand}_h(\Delta T)$, $\text{RefuseRisky}_h(\Delta T)$, and h_{risky} are continuous (due to Lemma 1), S is closed. Therefore, S has a maximum $(\Delta T^*) \in [0, \overline{\Delta T})$.

As $(\Delta T)^* + \kappa \notin S$ for all $\kappa > 0$, continuity also implies that one of the constraints in the definition of S must bind at $\Delta T = (\Delta T)^*$. By Lemma 1, $(h_{\text{risky}}((\Delta T)^*), h_{\text{safe}}((\Delta T)^*))$ gives a waiting time steady-state. □

5 Congestion

In standard queuing systems, a waiting time for a good is positive only if the good is in excess demand—in particular, only if all units of the good are allocated. By contrast, due to the perishability of organs, the system can enter a counterintuitive "congested"

state in which the waiting time for risky organs is positive but nevertheless some risky organs are wasted (as will also be illustrated in Example 2).

We formally define congestion as follows.

Definition 2. A waiting-time steady state $(T_{\text{safe}}, T_{\text{risky}})$ is *congested* if $T_{\text{risky}} > 0$ and a positive mass of risky organs are discarded—i.e., (3) holds with strict inequality.

The following example exhibits a congested waiting-time steady state.

Example 1 (Congested Steady State)**.** Patients arrive at a rate of $p = 1$, safe organs arrive at a rate of $sp = 0.2$ and risky organs arrive at a rate of $rp = 0.3$. All patients depart at a rate of $\delta = 0.2$. A patient utility $\nu(\theta)$ for a risky organ is drawn from a standard uniform distribution $F_\nu \sim \text{Unif}[0, 1]$. Organs expire after being offered to measure $\gamma = 0.5$ of patients.

We claim that the waiting times $T_{\text{risky}} \approx 3.993$ and $T_{\text{safe}} \approx 6.020$ comprise a waiting-time steady state. [13] Given these waiting times, types whose value for risky organs satisfy $\nu(\theta) < \frac{2}{3}$ have willingness-to-wait $W(\theta) > T_{\text{safe}} - T_{\text{risky}} \approx 2.027$. As a result, the mass of allocated safe organs at each moment is $\frac{2}{3} \cdot e^{-\delta T_{\text{safe}}} = \frac{2}{3} \cdot \frac{3}{10} = sp$. The demand for risky organs at each moment is $\text{RiskyDemand}(T_{\text{safe}}, T_{\text{risky}}) = \frac{1}{3} \cdot e^{-\delta T_{\text{risky}}} = \frac{1}{3} \cdot \frac{9}{20} = 0.15 < rp$. The mass of patients that refuse a risky organ is $\text{RefuseRisky}(T_{\text{risky}}, T_{\text{safe}}) = \int_{T_{\text{risky}}}^{T_{\text{safe}}} \frac{2}{3} e^{-\delta t} dt = \frac{2}{3} \cdot \frac{3}{4} = \gamma$. Hence, no organs are intentionally withheld.[14]

Note that despite the waiting time for risky organs being positive, half of the risky organs are not utilized. This is because half of the risky organs expire before being offered to someone who would accept them. That is, not enough patients with $\nu(\theta) > \frac{2}{3}$ survive to be allocated a risky organ, and T_{risky} does not fall to clear the market due to organ expiry.

If instead organs did not expire ($\gamma = \infty$), the waiting-time steady state would be given by $T_{\text{risky}} \approx 2.043$ and $T_{\text{safe}} \approx 5.045$, [15] and all organs would be utilized.

[13] Analytically, the waiting times are $T_{\text{risky}} = 5 \log \frac{20}{9}$ and $T_{\text{safe}} = 5 \log \frac{10}{3}$.

[14] In the sequel, we show the waiting-time steady state is unique due to the homogeneity of departure rates.

[15] Analytically, these waiting times are $T_{\text{risky}} = 5 \log \frac{40 - 10\sqrt{7}}{9}$ and $T_{\text{safe}} = 5 \log \frac{5\sqrt{7} - 5}{3}$. To see why this is a waiting-time steady state for $\gamma = \infty$, note that given these waiting times, types with $\nu(\theta) < \frac{\sqrt{7}-1}{3}$ have willingness-to-wait $W(\theta) > T_{\text{safe}} - T_{\text{risky}}$. As a result, at each moment, the mass of allocated safe organs is $\frac{\sqrt{7}-1}{3} \cdot e^{-\delta T_{\text{safe}}} = 0.2 = sp$, and the mass of allocated risky organs is $\left(1 - \frac{\sqrt{7}-1}{3}\right) \cdot e^{-\delta T_{\text{risky}}} = 0.3 = rp$.

Moreover, all patients would be better off, as the waiting times for both safe and risky kidneys would be lower.

To understand the impact of congestion, we first analyze the uncongested case and then compare other steady states to that case. If an uncongested steady state exists, it corresponds to a steady state of the limiting case of $\gamma = \infty$—a model we show has a unique steady state.

Proposition 2. *(a) For $\gamma = \infty$, there is a unique waiting-time steady state $(T^{\infty}_{\text{risky}}, T^{\infty}_{\text{safe}})$.*

(b) For any γ, a waiting-time steady state is uncongested if and only if it is $(T^{\infty}_{\text{risky}}, T^{\infty}_{\text{safe}})$.

It is not difficult to see that there exists $\gamma^\star < \infty$ (dependent on the other parameters of the model) such that $(T^{\infty}_{\text{risky}}, T^{\infty}_{\text{safe}})$ is a waiting-time steady state if and only if $\gamma \geq \gamma^\star$. Thus, congestion is inevitable in all steady states for γ sufficiently small—for larger γ, congestion may not arise, or may only arise if self-fulfilling.

To prove Proposition 2, we show that the difference in waiting times in any steady state for $\gamma = \infty$ is an upper bound on the difference in waiting times in any steady state for any γ. In the sequel, we use this property to study congested steady states.

Lemma 2. *Let $(T_{\text{risky}}, T_{\text{safe}})$ be a waiting-time steady state for any γ, and let $(T^{\infty}_{\text{risky}}, T^{\infty}_{\text{safe}})$ be a waiting-time steady state for $\gamma = \infty$. We have $T_{\text{safe}} - T_{\text{risky}} \leq T^{\infty}_{\text{safe}} - T^{\infty}_{\text{risky}}$.*

The proof of Lemma 2 uses the following lemma, which establishes that demand for risky organs increases with the difference in waiting times.

Lemma 3. *The function $\text{RiskyDemand}_h(\Delta T)$ is non-decreasing in ΔT, and strictly increasing in ΔT on the region of ΔT for which $\text{RiskyDemand}_h(\Delta T) > 0$.*

Proof. Let $h_{\text{risky}}, h_{\text{safe}}$ be as in Lemma 1. Note that $\text{RiskyDemand}(T_{\text{risky}}, T_{\text{risky}} + \Delta T)$ is non-increasing in T_{risky} and strictly decreasing in T_{risky} for $\Delta T \geq \operatorname{ess\,inf} W(\theta)$ (as fewer agents survive at higher waiting times). Moreover, $\text{RiskyDemand}(T_{\text{risky}}, T_{\text{risky}} + \Delta T)$ is non-decreasing in ΔT (as more types are willing to accept risky organs at higher ΔT). As h_{risky} is decreasing (by Lemma 1), it follows that $\text{RiskyDemand}_h(\Delta T)$ is non-decreasing in ΔT, and strictly increasing in ΔT for $\Delta T \geq \operatorname{ess\,inf} W(\theta)$. The lemma follows as $\text{RiskyDemand}_h(\Delta T) > 0$ for and only for $\Delta T > \operatorname{ess\,inf} W(\theta)$. □

Proof of Lemma 2. We divide into cases based on the value of $T^{\infty}_{\text{risky}}$.

- Case 1: $T^{\infty}_{\text{risky}} = 0$. Let h_{risky} be as in Lemma 1. By Lemma 1, we have $h_{\text{risky}}(T^{\infty}_{\text{safe}} - T^{\infty}_{\text{risky}}) = T^{\infty}_{\text{risky}} = 0$ and $h_{\text{risky}}(T_{\text{safe}} - T_{\text{risky}}) = T_{\text{risky}} \geq 0$. Since h_{risky} is decreasing (by Lemma 1), it follows that $T_{\text{safe}} - T_{\text{risky}} \leq T^{\infty}_{\text{safe}} - T^{\infty}_{\text{risky}}$.

- Case 2: $T^{\infty}_{\text{risky}} > 0$. By the definition of $(T^{\infty}_{\text{risky}}, T^{\infty}_{\text{safe}})$, (3) must hold with equality at those waiting times. By Lemma 1, we therefore have $\text{RiskyDemand}_h(T^{\infty}_{\text{safe}} - T^{\infty}_{\text{risky}}) = pr$. But $(T_{\text{risky}}, T_{\text{safe}})$ is a waiting-time steady state, (3) must hold at those waiting times. By Lemma 1, we therefore have $\text{RiskyDemand}_h(T_{\text{safe}} - T_{\text{risky}}) \leq pr$. By Lemma 3, it follows that $T_{\text{safe}} - T_{\text{risky}} \leq T^{\infty}_{\text{safe}} - T^{\infty}_{\text{risky}}$.

As the cases exhaust all possibilities, we have proven the lemma. □

Proposition 2 follows straightforwardly from Lemma 2 by using the lemma to rank differences in waiting times, which leads to rankings of waiting times by Lemma 1.

Proof of Proposition 2. To prove Part (a), note that Proposition 1 provides existence. For uniqueness, let $(T_{\text{risky}}, T_{\text{safe}})$ and $(T^{\infty}_{\text{risky}}, T^{\infty}_{\text{safe}})$ be waiting-time steady states for $\gamma = \infty$. Lemma 2 implies that $T_{\text{safe}} - T_{\text{risky}} \leq T^{\infty}_{\text{safe}} - T^{\infty}_{\text{risky}}$. Exchanging the roles of the two steady states, Lemma 2 also implies that $T_{\text{safe}} - T_{\text{risky}} \geq T^{\infty}_{\text{safe}} - T^{\infty}_{\text{risky}}$. Hence, we have $T_{\text{safe}} - T_{\text{risky}} = T^{\infty}_{\text{safe}} - T^{\infty}_{\text{risky}}$. By Lemma 1, we then have that $T_{\text{safe}} = T^{\infty}_{\text{safe}}$, and hence $T_{\text{risky}} = T^{\infty}_{\text{risky}}$.

Now consider an arbitrary γ. If $(T^{\infty}_{\text{risky}}, T^{\infty}_{\text{safe}})$ is a waiting-time steady state, then as (3) or $T^{\infty}_{\text{risky}} = 0$ must hold, the steady state is uncongested. Conversely, consider any uncongested waiting-time steady state $(T_{\text{risky}}, T_{\text{safe}})$. By construction, $(T_{\text{risky}}, T_{\text{safe}})$ is also a waiting-time steady state for $\gamma = \infty$. By Part (a), we then have that $(T_{\text{risky}}, T_{\text{safe}}) = (T^{\infty}_{\text{risky}}, T^{\infty}_{\text{safe}})$. □

We can also show that the waiting times for the $\gamma = \infty$ case are lower than the waiting times in any congested steady state. This inequality holds for both the risky waiting times and the safe waiting times, and provides a formal sense in which congestion, if it arises, hurts everyone. Clearly, by leading to the discard of risky organs, congestion hurts those who would accept such organs. However, congestion also affects those who refuse risky organs: with a higher waiting time for risky organs, patients who are offered risky organs have a better chance of obtaining a safe organ, making congestion increase the demand for safe organs. Hence, congestion raises the waiting time for safe organs as well once patients' decisions are taken into account.

Proposition 3. *If* $(T_{\text{risky}}, T_{\text{safe}})$ *is a congested waiting time steady-state, then we have that* $T_{\text{risky}} > T^{\infty}_{\text{risky}}$ *and that* $T_{\text{safe}} \geq T^{\infty}_{\text{safe}}$*, with strict inequality if* $W(\theta)$ *has full support on* $[0, \overline{\Delta T})$.

Proof. By Lemma 2, we have $T_{\text{safe}} - T_{\text{risky}} \leq T^{\infty}_{\text{safe}} - T^{\infty}_{\text{risky}}$. If $T_{\text{safe}} - T_{\text{risky}} = T^{\infty}_{\text{safe}} - T^{\infty}_{\text{risky}}$, then it would follow from Lemma 1 that $(T_{\text{risky}}, T_{\text{safe}}) = (T^{\infty}_{\text{risky}}, T^{\infty}_{\text{safe}})$, which is not congested (by Proposition 2(b)). Hence, we must have $T_{\text{safe}} - T_{\text{risky}} < T^{\infty}_{\text{safe}} - T^{\infty}_{\text{risky}}$. The proposition then follows by Lemma 1. □

5.1 Multiplicity and Equilibrium Selection

The following example illustrates that the waiting-time steady state is not generally unique. In fact, we show that congestion can arise in one steady state but not another.

Example 2 (Multiple Steady States)**.** Patients are equally likely to be *healthy* (θ_h) and *unhealthy* (θ_u). Types' departure rates are $\delta(\theta_h) = 0.2$ and $\delta(\theta_u) = 0.4$—reflecting that healthy patients have longer expected lifetimes. Patients' utility from risky organs is $\nu(\theta_h) = \nu(\theta_u) = 0.1$.[16] Hence, types' willingness-to-wait for quality are $W(\theta_h) \approx 11.51$ and $W(\theta_u) \approx 5.76$.

Patients arrive at rate $p = 1$, Safe organs arrive at rate $sp = 0.15$, and risky organs arrive at rate $rp = 0.5$. Organs expire after refusal by mass $\gamma = 1.9$ of patients.

We identify two waiting time steady states—one uncongested and one congested:

- *An Uncongested Steady State.* The waiting times are $T_{\text{risky}} = 0$ and $T_{\text{safe}} \approx 6.02$.[17] As $W(\theta_u) < T_{\text{safe}} - T_{\text{risky}} < W(\theta_h)$, healthy patients refuse risky organs, while unhealthy patients accept risky organs. (2) is satisfied, and (3) holds with equality. Moreover, the left-hand-side of (4) is 1.75, which is smaller than γ; thus, the cold-ischemia-time limit does not bind.

- *A Congested Steady State.* The waiting times are $T_{\text{risky}} \approx 1.44$ and $T_{\text{safe}} \approx 7.10$.[18] Since $T_{\text{safe}} - T_{\text{risky}} < W(\theta_u) < W(\theta_h)$, both patient types refuse risky organs. (2) is satisfied, and (4) holds with equality, meaning that risky organs expire at their cold-ischemia-time limit. Since no patients accept risky organs, (3) also holds.

[16] While the distribution of $\nu(\theta)$ conditional on $\delta(\theta)$ is not continuous here, it can be made absolutely continuous by perturbation without affecting the conclusions of our example.

[17] Analytically, the waiting times are $T_{\text{risky}} = 0$ and $T_{\text{safe}} = 5 \log \frac{10}{3}$.

[18] Analytically, the waiting times are $T_{\text{risky}} = -5 \log \frac{\sqrt{232+10\sqrt{55}}-10}{10}$ and $T_{\text{safe}} = 5 \log \frac{5+\sqrt{55}}{3}$.

The above example of multiple steady states illustrates an interesting *self-fulfilling* congestion phenomenon. Starting from the uncongested steady state, if unhealthy patients switch to choosing safe organs, T_{safe} increases by only about 1.08 because they depart much faster than healthy patients. However, the shift creates congestion and raises T_{risky} by 1.44—which is enough to make risky organs unattractive to unhealthy patients and thereby sustain a congested steady state. The effect of congestion is stark enough that all risky organs are discarded, whereas all risky organs would be utilized in the uncongested steady state.[19]

More generally, while there is at most one uncongested waiting-time steady state (by Proposition 2(b)), there may be both congested and uncongested waiting-time steady states, as well as multiple congested steady states. Nevertheless, we show that all waiting-time steady states can be ranked in a total order. Indeed, waiting-time steady states can clearly be ordered by their difference in waiting times, which, by Lemma 1, ranks waiting times.

Proposition 4. *All waiting-time steady states are totally ordered.*

Proof. By Lemma 1, if $(T_{\text{risky}}, T_{\text{safe}})$ and $(T'_{\text{risky}}, T'_{\text{safe}})$ are waiting-time steady states, if $T_{\text{safe}} - T_{\text{risky}} \geq T'_{\text{safe}} - T'_{\text{risky}}$, then $T_{\text{risky}} \leq T'_{\text{risky}}$ and $T_{\text{safe}} \leq T'_{\text{safe}}$. □

Throughout the rest of the paper, we focus on waiting-time steady state with lowest waiting times, which exists by Proposition 4. This waiting-time steady state is Pareto-optimal among waiting-time steady states, as all types of patients prefer lower waiting times. This equilibrium selection rule minimizes the scope of the inefficiencies that we highlight. Propositions 2 and 3 tell us that this selection rule always selects the uncongested steady state if one exists.

5.2 Welfare and Congestion Effects of the Cold-Ischemia-Time Limit

We next show how changes in the cold-ischemia-time limit γ affect waiting times. Specifically, we show that starting from a congested lowest waiting-time steady state, raising γ lowers both waiting times—benefiting all patient types (and strictly so under a full support condition). Similarly, lowering γ raises both waiting times—hurting all

[19]The example relies on having heterogeneous departure rates. Indeed, as we show in Appendix B, when all patients have the same departure rate, there is a unique waiting-time steady state.

patient types (and strictly so under a full support condition). This comparative static formalizes a sense in which relaxing the cold-ischemia-time limit alleviates congestion, and tightening it worsens congestion.

Proposition 5. *If the lowest waiting-time steady state is congested, then raising (resp. lowering) γ strictly decreases (resp. increases) T_{risky} and weakly decreases (resp. increases) T_{safe} under the lowest waiting-time steady state selection. If furthermore the distribution of $W(\theta)$ has full support on $[0, \Delta T)$, then the decrease (resp. increase) in T_{safe} is strict.*

To prove Proposition 5, we first characterize in Lemma 4 the lowest waiting-time steady state in terms of the function h and the waiting-time steady state for $\gamma = \infty$.[20] We then complete the proof of Proposition 5 by using a monotone comparative statics argument based on Milgrom and Roberts (1994). (The details of our setting allow for a simple argument that does not rely on any results of Milgrom and Roberts (1994).)

Lemma 4. *The lowest waiting-time steady state $(T_{\text{risky}}, T_{\text{safe}})$ satisfies*

$$T_{\text{safe}} - T_{\text{risky}} = \max\left\{\Delta T \leq T^{\infty}_{\text{safe}} - T^{\infty}_{\text{risky}} \middle| \text{RefuseRisky}_h(\Delta T) \leq \gamma\right\}. \tag{6}$$

Proof. Denote the right-hand side of (6) by $(\Delta T)^*$, and write $\Delta T^{\infty} = T^{\infty}_{\text{safe}} - T^{\infty}_{\text{risky}}$.

If $(\Delta T)^* = \Delta T^{\infty}$, then (4) holds at $(T^{\infty}_{\text{risky}}, T^{\infty}_{\text{safe}})$ (due to Lemma 1), so $(T^{\infty}_{\text{risky}}, T^{\infty}_{\text{safe}})$ is a waiting-time steady state. By Propositions 2(b) and 3, it is the lowest one.

Hence, we can restrict attention to the case in which $(\Delta T)^* < \Delta T^{\infty}$. Letting $h_{\text{risky}}, h_{\text{safe}}$ be as in Lemma 1, we first show $(h_{\text{risky}}(\Delta T), h_{\text{safe}}(\Delta T))$ is a waiting-time steady state. It follows from Lemmata 1 and 3 that

$$pr \geq \text{RiskyDemand}_h(\Delta T^{\infty}) \geq \text{RiskyDemand}_h((\Delta T)^*)$$

—i.e., that (3) holds. By Lemma 1, we also have $h_{\text{risky}}((\Delta T)^*) > h_{\text{risky}}(\Delta T^{\infty}) \geq 0$. Last, the continuity of h (by Lemma 1) and the absolute continuity of the distribution $W(\theta)$) imply the continuity of $\text{RefuseRisky}_h(\Delta T)$, so (4) must hold with equality.

Letting $\Delta T = T_{\text{safe}} - T_{\text{risky}}$, we have that $(T_{\text{risky}}, T_{\text{safe}}) = (h_{\text{risky}}(\Delta T), h_{\text{safe}}(\Delta T))$ by Lemma 1. (4) then implies that $\text{RefuseRisky}_h(\Delta T) \geq \gamma$. Lemma 2 implies that

[20] A characterization of all waiting-time steady states is available along similar lines. Suppose that a pair of waiting times $(T_{\text{risky}}, T_{\text{safe}})$ satisfies (2). The proof of Lemma 4 shows that $(T_{\text{risky}}, T_{\text{safe}})$ is a waiting-time steady state if and only if we have $T_{\text{safe}} - T_{\text{risky}} \leq T^{\infty}_{\text{safe}} - T^{\infty}_{\text{risky}}$ and $\text{RefuseRisky}_h(T_{\text{safe}} - T_{\text{risky}}) \leq \gamma$, and one of these two inequalities holds with equality.

$\Delta T \leq \Delta T^{\infty}$, and hence we have that $\Delta T \leq (\Delta T)^{*}$. By Lemma 1, it follows that $(T_{\text{risky}}, T_{\text{safe}}) \geq (h_{\text{risky}}(\Delta T), h_{\text{safe}}(\Delta T))$.

Hence, we have $(T_{\text{risky}}, T_{\text{safe}}) = (h_{\text{risky}}(\Delta T), h_{\text{safe}}(\Delta T))$, so $T_{\text{safe}} - T_{\text{risky}} = (\Delta T)^{*}$. □

Proof of Proposition 5. Let the lowest waiting-time steady states be $(T_{\text{risky}}, T_{\text{safe}})$, which we assume to be congested. Let $\gamma' \neq \gamma$, and consider the new lowest waiting-time steady state $(T'_{\text{risky}}, T'_{\text{safe}})$. Let $\Delta T = T_{\text{safe}} - T_{\text{risky}}$ and let $\Delta T' = T'_{\text{safe}} - T'_{\text{risky}}$. As $(T_{\text{risky}}, T_{\text{safe}})$ is assumed congested, Lemma 2 implies that $\Delta T < T^{\infty}_{\text{safe}} - T^{\infty}_{\text{risky}}$.

First, suppose $\gamma' > \gamma$. By Lemma 4, we have $\text{RefuseRisky}_h(\Delta T) \leq \gamma < \gamma'$. By the continuity of $\text{RefuseRisky}_h(\Delta T)$ (which follows from Lemma 1 and the absolute continuity of the distribution $W(\theta)$) and Lemma 4, we then have that $\Delta T' > \Delta T$.[21] The conclusion of the proposition follows by Lemma 1.

Now suppose $\gamma' < \gamma$. By Lemma 4, we have $\text{RefuseRisky}_h(\Delta T') \leq \gamma' < \gamma$. Since $\Delta T < T^{\infty}_{\text{safe}} - T^{\infty}_{\text{risky}}$, Lemma 4 implies that $\text{RefuseRisky}_h(T^{\infty}_{\text{safe}} - T^{\infty}_{\text{risky}}) > \gamma$, and hence we have that $\Delta T' < T^{\infty}_{\text{safe}} - T^{\infty}_{\text{risky}}$. By the continuity of $\text{RefuseRisky}_h(\Delta T)$ and Lemma 4, we then have that $\Delta T' < \Delta T$. The conclusion of the proposition follows by Lemma 1. □

6 Constrained Inefficiency of Congested Waiting Lists

Intuitively, increasing the pool of safe organs should improve welfare. If not, then the equilibrium is constrained inefficient in that discarding safe organs might improve welfare even without changing the underlying queuing structure. In this section, we show that this form of cininefficiency can arise if the steady state is congested, but (as is intuitive) not if the steady state is uncongested.

We first show that without congestion or cold-ischemia-time limits, "artificially discarding" safe kidneys can never lower waiting times.

Proposition 6. *Lowering s strictly increases T^{∞}_{safe} and weakly increases $T^{\infty}_{\text{risky}}$.*

[21] Milgrom and Roberts (1994, Theorem 1) provides conditions to obtain a weak inequality that apply under weaker continuity conditions. We obtain a strict inequality using continuity and our functional forms.

Proof. Consider $s > s'$, and let $(T^{\infty}_{\text{risky}}, T^{\infty}_{\text{safe}})$ and $(T^{\infty,\prime}_{\text{risky}}, T^{\infty,\prime}_{\text{safe}})$ denote the corresponding waiting-time steady states for $\gamma = \infty$. Let $\Delta T = T^{\infty}_{\text{safe}} - T^{\infty}_{\text{risky}}$ and let $\Delta T' = T^{\infty,\prime}_{\text{safe}} - T^{\infty,\prime}_{\text{risky}}$.

We do casework based on the relative values of ΔT and $\Delta T'$ to complete the proof.

- Case 1: $\Delta T' \leq \Delta T$. Due to (2), as $s' < s$, we must then have $T^{\infty,\prime}_{\text{safe}} > T^{\infty}_{\text{safe}}$. It follows that $T^{\infty,\prime}_{\text{risky}} = T^{\infty,\prime}_{\text{safe}} - \Delta T' > T^{\infty}_{\text{safe}} - \Delta T = T^{\infty}_{\text{risky}}$.
- Case 2: $\Delta T' > \Delta T$. Due to (3) and (5), and since the arrival rate of risky organs is unchanged, we must have $T^{\infty,\prime}_{\text{risky}} \geq T^{\infty}_{\text{risky}}$. It follows that $T^{\infty,\prime}_{\text{safe}} = T^{\infty,\prime}_{\text{risky}} + \Delta T' > T^{\infty}_{\text{risky}} + \Delta T = T^{\infty}_{\text{safe}}$.

The cases exhaust all possibilities, which completes the proof. □

The following example shows that when the state is congested, discarding safe organs may result in a Pareto improvement by decreasing both steady state waiting times. (Equivalently, the arrival of more safe organs can harm all patients by increasing both steady state waiting times.) This can happen even when there is a unique waiting-time steady state.

Example 3 (Pareto improvement when artificially discarding safe organs under congestion). Patients arrive at a rate of $p = 1$, safe organs arrive at a rate of $sp = 0.05$ and risky organs arrive at a rate of $rp = 0.8$. All patients depart at a rate of $\delta = 0.2$. A patient's utility $\nu(\theta)$ for a risky organ is drawn from the distribution $F_\nu \sim \text{Beta}(2, 1)$, whose cdf is $F_\nu(x) = x^2$. Organs expire after being offered to mass $\gamma = 0.25$ of patients.

We claim that $T_{\text{risky}} \approx 4.581$ and $T_{\text{safe}} \approx 8.047$ comprise a waiting-time steady state.[22] Given these waiting times, types whose values for risky organs satisfy $\nu(\theta) < \frac{1}{2}$ have willingness-to-wait $W(\theta) > T_{\text{safe}} - T_{\text{risky}} \approx 3.466$—such types comprise $\frac{1}{4}$ fraction of types according to the distribution F_ν. Hence, the mass of safe organs allocated at each moment is $\frac{1}{4} \cdot e^{-\delta T_{\text{safe}}} = \frac{1}{4} \cdot \frac{1}{5} = sp$. The demand for risky organs at each moment is $\frac{3}{4} \cdot e^{-\delta T_{\text{risky}}} = \frac{3}{4} \cdot \frac{2}{5} = 0.3 < rp$. The mass of patients that refuse a risky organ is $\text{RefuseRisky}(T_{\text{risky}}, T_{\text{safe}}) = \int_{T_{\text{risky}}}^{T_{\text{safe}}} \frac{1}{4} e^{-\delta t} dt = 0.25$. Hence, no organs are intentionally withheld.

Now suppose that the arriving mass of safe organs falls to $s'p = 0.02$. We claim that $T_{\text{risky}} \approx 0.769$ and $T_{\text{safe}} \approx 7.032$ comprise a waiting-time steady state.[23]

[22] Analytically, the waiting times are $T_{\text{risky}} = 5 \log \frac{5}{2}$ and $T_{\text{safe}} = 5 \log 5$.

[23] Analytically, the waiting times are $T_{\text{risky}} = 5 \log \frac{400}{343}$ and $T_{\text{safe}} = 5 \log \frac{200}{49}$.

Given these waiting times, types whose values for risky organs satisfy $\nu(\theta) < \frac{2}{7}$ have willingness-to-wait $W(\theta) > T_{\text{safe}} - T_{\text{risky}} \approx 6.264$—such types comprise $\frac{4}{49}$ fraction of types according to the distribution F_{ν}.. As a result, the mass of allocated safe organs at each moment is $\frac{4}{49} \cdot e^{-\delta T_{\text{safe}}} = \frac{4}{49} \cdot \frac{49}{200} = s'p$. The demand for risky organs at each moment is $\text{RiskyDemand}(T_{\text{risky}}, T_{\text{safe}}) = \frac{45}{49} \cdot e^{-\delta T_{\text{risky}}} = \frac{45}{49} \cdot \frac{343}{400} < 0.79 < rp$. The mass of patients that refuse a risky organ is $\text{RefuseRisky}(T_{\text{risky}}, T_{\text{safe}}) = \int_{T_{\text{risky}}}^{T_{\text{safe}}} \frac{4}{49} e^{-\delta t} dt = 0.25$. Hence, no organs are intentionally withheld.

As we demonstrate in Appendix B, the homogeneity of departure rates across patients ensures that the waiting-time steady state is unique. Therefore, lowering the safe arrival rate from s to s' lower the (unique) steady state waiting time for both risky and safe organs under congestion.

For intuition, when the system is congested, an increase in safe organs results in more types waiting for safe organs. This worsens congestion, and the example shows it can exacerbate congestion so severely as to increase waiting times for both safe and risky organs.

While our example is stylized, it illustrates the severity of the congestion externality. The externality can be so severe that a benevolent social planner who is forced to use a queuing system might in principle prefer to destroy safe organs. This result shows that the queuing system is constrained inefficient under congestion, in that the inefficiency could be mitigated by taking a feasible action without changing the underlying structure of the queuing system (or priorities). In the remainder of the paper, we show how the congestion externality is relevant to other features of the market and to studying the welfare effects of alternative waiting list policies.

7 Moral Hazard and the Congestion Externality

In practice, acceptance and refusal decisions are delegated to the patient's hospital, which may have less of an incentive to transplant risky organs. To allow for the possibility of moral hazard in this delegation problem, we suppose that acceptance

and refusal of offered organs is instead decided based on the (Bernoulli) utility function

$$u_{\text{decision}} = \begin{cases} 1 & \text{if get safe organ} \\ \widehat{\nu}(\theta) & \text{if get risky organ} \\ 0 & \text{if do not get an organ} \end{cases}.$$

where $\widehat{\nu}(\theta) \in [0,1]$ is the effective willingness to accept a risky organ. Throughout, we assume that conditional on $\delta(\theta)$, the distribution of $\widehat{\nu}(\theta)$ is continuous.

A similar definition to (1) shows that the transplant center's willingness-to-wait for quality for a patient of type θ is

$$\widehat{W}(\theta) = \frac{-\log \widehat{\nu}(\theta)}{\delta(\theta)}.$$

As transplant centers are penalized for adverse post-transplant outcomes but not deaths on the waiting list (Mehrotra et al., 2022; Sepulveda et al., 2023; Chan and Roth, 2024; Mildebrath et al., 2024), we assume that hospitals are overall less willing to accept risky organs than patients would be themselves: i.e., that $\widehat{W}$ first-order stochastically dominates $W(\theta)$. We then show that if the steady state with patient decisions is congested, decisions being made by hospitals weakly hurts all patients. If there exists $\varepsilon > 0$ such that $\widehat{W}$ first-order stochastically dominates $W(\theta) + \varepsilon$, then delegation strictly hurts all patients. Intuitively, hospitals' risk aversion worsens congestion enough to cause substantial harm.

We introduce an additional assumption on each of ν and $\widehat{\nu}$ under which we can demonstrate the comparative static.

Definition 3. We say that W satisfies *the healthy-more-willing-to-wait-condition* if for all $\theta, \theta' \in \Theta$, if $\delta(\theta) < \delta(\theta')$, then $W(\theta) \geq W(\theta')$.

Consider for example a market with older patients, as well as younger and healthier patients who have a lower departure rate δ than the older patients. The healthy-more-willing-to-wait condition would then require that the younger and healthier patients all have a weakly higher willingness-to-wait than all the older patients. We impose this condition on both patients' and hospitals' preferences.

Proposition 7. *Suppose that $\widehat{W}(\theta)$ first-order stochastically dominates $W(\theta)$, and*

that both W and $\widehat{W}$ satisfy the healthy-more-willing-to-wait condition. Suppose furthermore that $W(\theta)$ has full support on $[0, \overline{\Delta T})$.

(a) Moving from patient to hospital decisions weakly increases T_{safe} under the lowest waiting-time steady state selection.

(b) If the lowest waiting-time steady state is congested under patient decisions, then moving from patient to hospital decisions weakly increases T_{risky} under the lowest waiting-time steady state selection and the lowest waiting-time steady state must also be congested under hospital decisions.

If there exists $\varepsilon > 0$ such that $\widehat{W}(\theta)$ first-order stochastically dominates $W(\theta) + \varepsilon$, then both increases in waiting times are strict.

The proof is in Appendix C and also uses monotone comparative statics ideas.

By contrast, without congestion or a cold-ischemia-time limit, delegation to hospitals would lower the waiting time for risky organs by reducing demand for them. Thus, it is through the congestion externality that moral hazard raises the waiting time for risky organs and exacerbates the inefficiencies of the waiting list design.

8 Market Design Implications

Our framework and findings can help in understanding how innovations in the market design can affect congestion, which is relevant to welfare.

8.1 Market Thickness

In March 2021, the Organ Procurement and Transplantation Network (OPTN) increased the regions in which patients are given a (local) geographic priority.[24] In the same year, the discard rate increased by 3%. Motivated by this, we are interested in comparative statics with respect to the market thickness p.

Mathematically, this is equivalent to changing γ in the opposite direction, so Proposition 5 provides comparative statics. For simplicity, we focus on the practically relevant case of increasing market thickness.

[24] Instead of a "small" fixed service area patients within 250NM from the donor hospital are given priority.

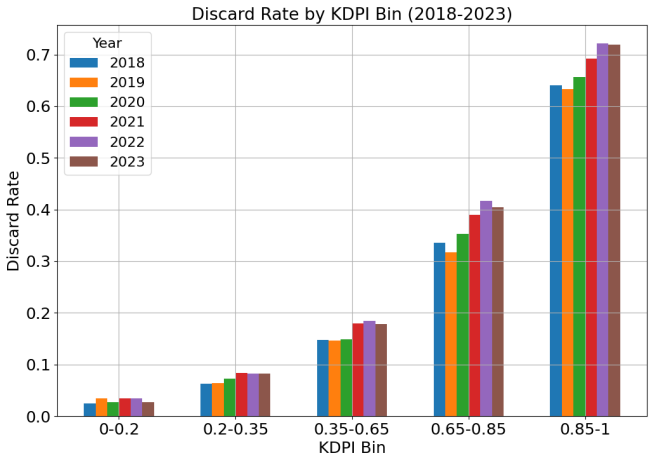


Figure 5: Annual trends of discard rate by KDPI

Proposition 8. *If the lowest waiting-time steady state is congested, then raising p strictly increases T_{risky} and weakly increases T_{safe} under the lowest waiting-time steady state selection. If furthermore the distribution of $W(\theta)$ has full support on $[0, \Delta T)$, then the increase in T_{safe} is strict.*

Proof. The steady state conditions (2) and (3) are homogeneous of degree 1 in p on both sides. As the left-hand side of (4) is homogeneous of degree 1 in p and the right-hand side is γ, changing p to p' is equivalent to changing γ to $\gamma' = p\gamma/p'$. The result then follows from Proposition 5. □

Consistent with Proposition 8, following the modification of local prioritization in March 2021, the discard rate increased especially for low-quality organs (i.e., organs with high KDPI) as depicted in Figure 5.

Note that market thickness may still benefit patients when preferences exhibit substantial horizontal heterogeneity, for example due to compatibility differences (Agarwal et al., 2021). Our fluid-limit formulation abstracts from these forms of heterogeneity to highlight the role of congestion on waiting lists.

8.2 Offering Processes

In the U.S., a deceased donor kidney is offered sequentially to patients based on a fixed priority order. While high quality kidneys are accepted quickly, kidneys with marginal quality are refused by many patients (see Figure 3). Guan et al. (2025) find that refusals due to long cold ischemia time increase substantially 4 hours after

recovery, and excess cold ischemia time becomes the main refusal reason 10 hours after recovery (see also Figure 4).[25] The large non-utilization rate has led Organ Procurement Organizations to expedite the offering process and in 2023, more than 20% of accepted organs were placed out-of-sequence (Liyanage et al., 2025).

It is straightforward to see that in partial equilibrium (i.e., not taking into account effects on patients' decisions and equilibrium waiting times), expediting raises welfare by allocating risky organs to patients who would otherwise not receive an organ. However, expediting many organs will also have general equilibrium effects due to patients' decisions and equilibrium waiting times. Our model allows us to formally study these broader impacts.

Consider the following modification for the first-come-first-serve offering policy, denoted by Π: Safe organs are offered first-come-first-serve, and risky organs are offered in sequence to patients who have not yet refused a risky organ.[26] Observe that this policy achieves the same waiting times for any γ for safe and risky organs. Since in a fluid limit, there is no uncertainty about when a patient will receive an offer of safe or risky kidney, their decisions will be based solely on (steady-state) waiting time. As a result, the following result is immediate.

Proposition 9. *The offering policy* Π *achieves the same welfare (for any* γ*) as the the first-come-first-serve policy achieves for* $\gamma = \infty$.

Evidence from other markets supports this intuition.[27] In the U.K., organs refused more than three times are offered simultaneously to all transplant centers and the organ is given to the highest priority accepting patient (Callaghan et al., 2017). In the U.S., organ procurement organizations (OPOs) have increasingly adopted ad-hoc expedited placement practices by making open offers to transplant centers (Jadlowiec et al., 2023) and in 2023 more than 14% of recovered kidneys have been expedited. These practices, however, have raised equity concerns (such as "list-skipping") prompting the Health Resources and Services Administration (HRSA) to direct the OPTN to develop a standardized policy for expedited placements.

A complementary approach to reducing congestion is to elicit more refined preferences from physicians and patients, thereby avoiding unnecessary offers. Although

[25] Guan et al. (2025) also find that 4 hours after recovery, an organ has been offered to patients from only 11 transplant centers (out of 220).

[26] This is similar to a buffer queue as studied in Leshno (2022).

[27] Castro et al. (2021) show that expediting unattractive options increases utilization in ride-sharing platforms.

current systems allow transplant centers to filter certain types of offers, refusal rates remain high (Guan et al., 2025).

9 Conclusion

This paper shows that the perishability of organs—driven by the undesirability of organs with long cold ischemia times—can generate congestion on deceased-donor waiting lists and thereby lead to substantial inefficiencies and externalities. When the waiting list is congested, many risky organs are not utilized because they fail to reach patients who are willing to accept them quickly enough.

Congestion is also exacerbated by moral hazard in acceptance decisions. One way to mitigate these inefficiencies is to involve patients more directly in refusal decisions (Mohan and Husain, 2022). Patients who recently joined the waiting list may be more willing to accept lower-quality organs (Mehrotra et al., 2022; Sepulveda et al., 2023). This opens the door to mechanisms such as dedicated lists for risky organs—e.g., requiring explicit prospective consent from the transplant center to receive offers with KDPI above 85%. Another option is to relax the disincentives faced by physicians and transplant centers when accepting higher-risk organs.

Our findings have broader implications for organ allocation policy. When the waitlist is congested, thickening the market by expanding service regions may exacerbate congestion. A natural approach to alleviate congestion and increase utilization of low quality organs is to expedite their offering process.

A Formal Definition of Steady State

We formally define a stationary first-come-first-served (FCFS) fluid game. Its equilibrium outcomes are exactly the waiting-time steady states in Definition 1. Neither waiting time is a primitive of the game. Instead, both arise as the youngest waiting ages reached by the endogenous organ-offer processes.

Let $\mathbb{R}_+ = [0, +\infty)$ denote the set of waiting ages and let $\mu(d\theta) = f(\theta)d\theta$ denote the type distribution. Choose $\underline{\delta} > 0$ such that $\delta(\theta) \geq \underline{\delta}$ for μ-almost every θ. For pointwise completeness, extend (1) by setting $W(\theta) = +\infty$ when $\nu(\theta) = 0$. Conditional continuity of $\nu(\theta)$ given $\delta(\theta)$ implies $\mu\{\theta : \nu(\theta) = 0\} = 0$, so this convention does not change any aggregate object in the main text.

A.1 The stationary FCFS game

For $x \in \{s, r\}$, a *residual-offer schedule* b_x records the flow of type-x organs that reaches waiting age t after all patients with waiting ages greater than t have been considered, but before patients of age t are considered. An admissible residual-offer schedule is a pair $b = (b_s, b_r)$ such that

- $b_s : \mathbb{R}_+ \to [0, sp]$ and $b_r : \mathbb{R}_+ \to [0, rp]$ are nondecreasing and right-continuous;
- $\lim_{t\to+\infty} b_s(t) = sp,\ \lim_{t\to+\infty} b_r(t) = rp.$

For each $x \in \{s, r\}$, define the corresponding *offer frontier* by

$$h_x(b) := \inf\{t \geq 0 : b_x(t) > 0\}. \tag{A.1}$$

The limiting-flow condition implies that the set in (A.1) is nonempty and hence that $h_x(b) < +\infty$. We call the residual-offer schedule b *admissible* if both offer frontiers are attained:

$$b_x\big(h_x(b)\big) > 0 \qquad \text{for each } x \in \{s, r\}. \tag{A.2}$$

By monotonicity, an admissible schedule satisfies

$$\{t \geq 0 : b_x(t) > 0\} = [h_x(b), +\infty).$$

Thus $h_x(b)$ is the actual first waiting age reached by the type-x offer process. The frontier-attainment condition is a continuous-time well-posedness requirement: it

rules out schedules under which offers occur arbitrarily close above the frontier but no offer occurs at the frontier itself.

A stationary pure strategy for type θ is a Borel map

$$\sigma_\theta : \mathbb{R}_+ \to \{0, 1\},$$

where one denotes acceptance of a risky offer; safe offers are always accepted. Fix an admissible schedule b. We call σ_θ *admissible given* b if, for every $c \in [h_r(b), h_s(b))$, the set

$$\{t \in [c, h_s(b)) : \sigma_\theta(t) = 1\}$$

is empty or has a least element. Hence, after any scheduled risky offer, the strategy has a well-defined first future risky offer to accept whenever it prescribes acceptance before the safe frontier.

For $c \geq h_r(b)$, let

$$\tau_{\theta,c}(\sigma_\theta; b) := \begin{cases} \min\{t \in [c, h_s(b)) : \sigma_\theta(t) = 1\}, & \text{if this set is nonempty,} \\ h_s(b), & \text{otherwise.} \end{cases} \tag{A.3}$$

For $c \in [h_r(b), h_s(b))$, the continuation payoff, conditional on survival to age c, is

$$v_{\theta,c}(\sigma_\theta; b) := \begin{cases} \nu(\theta) e^{-\delta(\theta)(\tau_{\theta,c}(\sigma_\theta;b)-c)}, & \tau_{\theta,c}(\sigma_\theta; b) < h_s(b), \\ e^{-\delta(\theta)(h_s(b)-c)}, & \tau_{\theta,c}(\sigma_\theta; b) = h_s(b). \end{cases} \tag{A.4}$$

A rule is *sequentially optimal given* b if

$$v_{\theta,c}(\sigma_\theta; b) = \sup_{\widetilde{\sigma}_\theta} v_{\theta,c}(\widetilde{\sigma}_\theta; b) \qquad \text{for every } c \in [h_r(b), h_s(b)), \tag{A.5}$$

where the supremum is over admissible continuation rules.

A profile $\sigma = (\sigma_\theta)_{\theta \in \Theta}$ is admissible for b if $(\theta, t) \mapsto \sigma_\theta(t)$ is jointly measurable, and $\theta \mapsto \tau_{\theta,h_r(b)}(\sigma_\theta; b)$ is measurable. Abbreviate this last stopping age by τ_θ.

A.2 Stationary accounting and equilibrium

Given an admissible pair (b, σ), define the risky- and safe-allocation measures on $\mathbb{R}_+ \times \Theta$ by

$$M_{r,\sigma,b}(C) := p \int_\Theta \mathbf{1}\{\tau_\theta < h_s(b)\}\mathbf{1}\{(\tau_\theta, \theta) \in C\} e^{-\delta(\theta)\tau_\theta}\, \mu(d\theta), \tag{A.6}$$
$$M_{s,\sigma,b}(C) := p \int_\Theta \mathbf{1}\{\tau_\theta = h_s(b)\}\mathbf{1}\{(h_s(b), \theta) \in C\} e^{-\delta(\theta)h_s(b)}\, \mu(d\theta).$$

The total refusal mass accumulated while a risky organ moves from the safe frontier to the risky frontier is

$$\mathcal{R}_{\sigma,b} := \int_{[h_r(b), h_s(b))} p \int_\Theta e^{-\delta(\theta)t} \mathbf{1}\{\tau_\theta > t\}\, \mu(d\theta)\, dt. \tag{A.7}$$

The interval is empty, and the integral is zero, when $h_r(b) \geq h_s(b)$.

Definition 4. A *steady-state FCFS equilibrium* is a tuple $\mathcal{E} = (b, \sigma, e_r)$, where b is an admissible residual-offer schedule, σ is an admissible profile for b, and $e_r \geq 0$ is the flow of risky organs discarded at the risky offer frontier, such that:

(i) σ_θ is sequentially optimal given b for μ-almost every θ;

(ii) safe-organ flow is conserved and no safe organ is discarded:

$$M_{s,\sigma,b}(\mathbb{R}_+ \times \Theta) = sp, \tag{A.8}$$
$$b_s(t) = sp - M_{s,\sigma,b}((t, +\infty) \times \Theta), \qquad t \geq 0;$$

(iii) risky-organ flow is conserved:

$$M_{r,\sigma,b}(\mathbb{R}_+ \times \Theta) + e_r = rp, \tag{A.9}$$
$$b_r(t) = rp - M_{r,\sigma,b}((t, +\infty) \times \Theta) - e_r \mathbf{1}\{t < h_r(b)\}, \qquad t \geq 0;$$

(iv) the refusal limit is respected:

$$\mathcal{R}_{\sigma,b} \leq \gamma; \tag{A.10}$$

(v) risky organs are not intentionally withheld. If $e_r > 0$, then

$$h_r(b) = 0 \quad \text{or} \quad \mathcal{R}_{\sigma,b} = \gamma. \tag{A.11}$$

The open upper tails in the flow identities mean that $b_x(t)$ is measured immediately before allocations or discard at age t. In particular, the term $e_r\mathbf{1}\{t < h_r(b)\}$ records discard at the risky frontier. The weak inequality in (A.10) adopts the same boundary convention as (4): an organ that has accumulated exactly γ units of refusal mass may still be allocated or discarded at its current age.

A.3 Endogenous frontiers and optimal strategies

Lemma 5 (Endogenous offer frontiers)**.** *Let $\mathcal{E} = (b, \sigma, e_r)$ be a stationary FCFS equilibrium and set*

$$T_{\text{risky}} := h_r(b), \qquad T_{\text{safe}} := h_s(b). \tag{A.12}$$

Then

$$0 \le T_{\text{risky}} < T_{\text{safe}} \le T_{\max} := \underline{\delta}^{-1} \log(s^{-1}).$$

Proof. The limiting flows make both frontiers finite, and admissibility makes them attained. Suppose that $T_{\text{risky}} \ge T_{\text{safe}}$. Then no risky offer is available before the safe frontier, so $\tau_\theta = T_{\text{safe}}$ for every type and $M_{r,\sigma,b} = 0$. Safe clearing gives

$$s = \int_\Theta e^{-\delta(\theta)T_{\text{safe}}}\, \mu(d\theta),$$

so $T_{\text{safe}} > 0$. Risky-organ balance then gives $e_r = rp > 0$. Because $T_{\text{risky}} \ge T_{\text{safe}}$, the interval in (A.7) is empty and $\mathcal{R}_{\sigma,b} = 0$. Since $T_{\text{risky}} \ge T_{\text{safe}} > 0$ and $\gamma > 0$, this contradicts (A.11). Hence $T_{\text{risky}} < T_{\text{safe}}$.

Every safe allocation occurs at T_{safe}. Therefore,

$$sp = M_{s,\sigma,b}(\mathbb{R}_+ \times \Theta) \le pe^{-\underline{\delta} T_{\text{safe}}},$$

which yields the upper bound. □

Write $\Delta T := T_{\text{safe}} - T_{\text{risky}}$ whenever the frontier pair is ordered.

Lemma 6 (Optimal stationary strategies)**.** *Fix an admissible schedule b with $T_{\text{risky}} := h_r(b) < h_s(b) =: T_{\text{safe}}$. For every type θ and every $c \in [T_{\text{risky}}, T_{\text{safe}})$,*

$$\sup_{\widetilde{\sigma}_\theta} v_{\theta,c}(\widetilde{\sigma}_\theta; b) = \max\{\nu(\theta), e^{-\delta(\theta)(T_{\text{safe}}-c)}\}. \tag{A.13}$$

An admissible rule is sequentially optimal if and only if, for every $c \in [T_{\text{risky}}, T_{\text{safe}})$,

$$\sigma_\theta(c) = \begin{cases} 1, & W(\theta) < T_{\text{safe}} - c, \\ 0, & W(\theta) > T_{\text{safe}} - c, \end{cases} \tag{A.14}$$

with either action allowed at equality. Consequently, except at indifference,

$$\tau_\theta = \begin{cases} T_{\text{risky}}, & W(\theta) < \Delta T, \\ T_{\text{safe}}, & W(\theta) > \Delta T. \end{cases} \tag{A.15}$$

Moreover,

$$\mu\{\theta : W(\theta) = c\} = 0 \qquad \textit{for every finite } c. \tag{A.16}$$

Proof. Conditional on survival to age c, accepting the current risky offer gives $\nu(\theta)$. Accepting a later risky offer at $u > c$ gives $\nu(\theta)e^{-\delta(\theta)(u-c)} \leq \nu(\theta)$, while rejecting all remaining risky offers and waiting for safe gives $e^{-\delta(\theta)(T_{\text{safe}}-c)}$. This proves (A.13).

The strict comparisons in (A.14) are therefore necessary. They are also sufficient. If accepting is strictly better, the rule accepts immediately. If waiting for safe is strictly better at age c, then it remains strictly better than risky acceptance at every later age, so the rule rejects all remaining risky offers. At equality, either immediate acceptance or waiting for safe is optimal. Applying the rule at $c = T_{\text{risky}}$ gives (A.15).

For finite c, conditional on $\delta(\theta)$, the event $\{W(\theta) = c\}$ is $\{\nu(\theta) = e^{-c\delta(\theta)}\}$. Conditional continuity assigns zero probability to this singleton; integrating over $\delta(\theta)$ proves (A.16). □

A.4 Correspondence with waiting-time steady states

Proposition 10 (Equilibrium-frontier correspondence)**.** *Under the maintained assumptions, a pair* $(T_{\text{risky}}, T_{\text{safe}})$ *is a waiting-time steady state in the sense of Definition 1 if and only if there exists a stationary FCFS equilibrium with* $h_r(b) = T_{\text{risky}}$ *and* $h_s(b) = T_{\text{safe}}$*. Every such equilibrium satisfies*

$$b_s(t) = sp\,\mathbf{1}\{t \geq T_{\text{safe}}\}, \qquad b_r(t) = rp\,\mathbf{1}\{t \geq T_{\text{risky}}\}. \tag{A.17}$$

Up to a μ*-null set, types with* $W(\theta) < T_{\text{safe}} - T_{\text{risky}}$ *receive a risky organ at* T_{risky}*, and types with* $W(\theta) > T_{\text{safe}} - T_{\text{risky}}$ *receive a safe organ at* T_{safe}*.*

Proof. Let (b, σ, e_r) be a stationary FCFS equilibrium and define $(T_{\text{risky}}, T_{\text{safe}})$ by (A.12). Lemma 6 and (A.16) imply, up to a null set, that $\tau_\theta = T_{\text{risky}}$ when $W(\theta) < \Delta T$ and $\tau_\theta = T_{\text{safe}}$ when $W(\theta) > \Delta T$. Hence

$$
\begin{aligned}
M_{s,\sigma,b}(\mathbb{R}_+ \times \Theta) &= p \int_{\{W > \Delta T\}} e^{-\delta(\theta) T_{\text{safe}}}\, \mu(d\theta) = \text{SafeDemand}(T_{\text{risky}}, T_{\text{safe}}), \\
M_{r,\sigma,b}(\mathbb{R}_+ \times \Theta) &= p \int_{\{W < \Delta T\}} e^{-\delta(\theta) T_{\text{risky}}}\, \mu(d\theta) = \text{RiskyDemand}(T_{\text{risky}}, T_{\text{safe}}), \\
\mathcal{R}_{\sigma,b} &= \int_{T_{\text{risky}}}^{T_{\text{safe}}} p \int_{\{W > \Delta T\}} e^{-\delta(\theta) t}\, \mu(d\theta)\, dt = \text{RefuseRisky}(T_{\text{risky}}, T_{\text{safe}}).
\end{aligned}
$$

Thus safe clearing, risky feasibility, and expiry feasibility are exactly (2), (3), and (4). If $e_r = 0$, risky demand equals rp. If $e_r > 0$, (A.11) implies that either $T_{\text{risky}} = 0$ or the expiry constraint binds. Therefore (5) also holds.

All safe allocations occur at T_{safe}, and all risky allocations and discard occur at T_{risky}. Substitution into the two flow identities yields (A.17). Hence $(T_{\text{risky}}, T_{\text{safe}})$ is a waiting-time steady state.

Conversely, let $(T_{\text{risky}}, T_{\text{safe}})$ be a waiting-time steady state and set $\Delta T = T_{\text{safe}} - T_{\text{risky}}$. Define b by (A.17) and, for every θ, set

$$
\sigma^*_\theta(t) := \mathbf{1}\{T_{\text{risky}} \le t < T_{\text{safe}},\ W(\theta) < T_{\text{safe}} - t\}. \tag{A.18}
$$

This profile is admissible and jointly measurable. It satisfies (A.14), so it is sequentially optimal. Up to the null indifference set, its stopping age is T_{risky} on $\{W < \Delta T\}$ and T_{safe} on $\{W > \Delta T\}$. Set

$$
e^*_r := rp - \text{RiskyDemand}(T_{\text{risky}}, T_{\text{safe}}) \ge 0. \tag{A.19}
$$

The induced allocation measures are

$$
\begin{aligned}
M_{r,\sigma^*,b}(dt, d\theta) &= \mathfrak{d}_{T_{\text{risky}}}(dt)\, p e^{-\delta(\theta) T_{\text{risky}}} \mathbf{1}_{\{W < \Delta T\}}(\theta)\, \mu(d\theta), \\
M_{s,\sigma^*,b}(dt, d\theta) &= \mathfrak{d}_{T_{\text{safe}}}(dt)\, p e^{-\delta(\theta) T_{\text{safe}}} \mathbf{1}_{\{W > \Delta T\}}(\theta)\, \mu(d\theta),
\end{aligned}
$$

where $\mathfrak{d}_t$ denotes unit point mass at t. The safe and risky flow equations follow from (2) and (3), while

$$
\mathcal{R}_{\sigma^*,b} = \text{RefuseRisky}(T_{\text{risky}}, T_{\text{safe}}) \le \gamma.
$$

If $e_r^* > 0$, risky feasibility is strict, so (5) implies that either $T_{\text{risky}} = 0$ or the expiry constraint binds. Thus the no-withholding condition also holds, and (b, σ^*, e_r^*) is a stationary FCFS equilibrium. □

B Uniqueness with Homogeneous Departure Rates

When the departure rate of patients is homogeneous, we can establish the uniqueness of waiting-time steady state.

Proposition 11. *If $\delta(\theta) \equiv \delta$, then the waiting-time steady state is unique.*

The key to the proof of Proposition 11 is the following lemma.

Lemma 7. *If $\delta(\theta) \equiv \delta$, then* $\text{RefuseRisky}_h(\Delta T)$ *is strictly increasing on* $[0, \overline{\Delta T})$.

Proof. When $\delta(\theta) \equiv \delta$, we have that

$$\begin{aligned}\text{RefuseRisky}_h(\Delta T) &= \int_{h_{\text{risky}}(\Delta T)}^{h_{\text{safe}}(\Delta T)} p e^{-\delta t} \Pr{}_\nu[W(\theta) > \Delta T] dt \\ &= \frac{p}{\delta}(e^{-\delta h_{\text{risky}}(\Delta T)} - e^{-\delta h_{\text{safe}}(\Delta T)}) \Pr{}_\nu[W(\theta) > \Delta T] \\ &= \big(e^{\delta(h_{\text{safe}}(\Delta T) - h_{\text{risky}}(\Delta T))} - 1\big) \frac{p}{\delta} e^{-\delta h_{\text{safe}}(\Delta T)} \Pr{}_\nu[W(\theta) > \Delta T].\end{aligned}$$

Since $\text{SafeDemand}(T_{\text{risky}}, T_{\text{safe}}) = p e^{-\delta T_{\text{safe}}} \Pr{}_\nu[W(\theta) > T_{\text{safe}} - T_{\text{risky}}]$ when $\delta(\theta) \equiv \delta$, by the definitions of $h_{\text{safe}}(\Delta T)$ and $h_{\text{risky}}(\Delta T)$, it follows that

$$\text{RefuseRisky}_h(\Delta T) = (e^{\delta \Delta T} - 1)\frac{s}{\delta},$$

which is strictly increasing in ΔT. □

Proof of Proposition 11. Suppose for the sake of deriving a contradiction that there are two distinct waiting-time steady states $(T_{\text{risky}}, T_{\text{safe}})$ and $(T'_{\text{risky}}, T'_{\text{safe}})$. Let $\Delta T = T_{\text{safe}} - T_{\text{risky}}$ and let $\Delta T' = T'_{\text{safe}} - T'_{\text{risky}}$. By Lemma 1, we have $(T_{\text{risky}}, T_{\text{safe}}) = (h_{\text{risky}}(\Delta T), h_{\text{safe}}(\Delta T))$ and $(T'_{\text{risky}}, T'_{\text{safe}}) = (h_{\text{risky}}(\Delta T'), h_{\text{safe}}(\Delta T'))$. In particular, we have $\Delta T \neq \Delta T'$. Without loss of generality, assume that $\Delta T' < \Delta T$.

By the definition of steady state, we must have that $\text{RiskyDemand}_h(\Delta T) \leq rp$, that $\text{RefuseRisky}_h(\Delta T) \leq \gamma$, and that $h_{\text{risky}}(\Delta T) \geq 0$. By Lemmata 3 and 7, we then have that $\text{RiskyDemand}_h(\Delta T') < rp$ and that $\text{RefuseRisky}_h(\Delta T') < \gamma$. By Lemma 1,

we have that $h_{\text{risky}}(\Delta T') > 0$. This contradicts the hypothesis that $(T'_{\text{risky}}, T'_{\text{safe}}) = (h_{\text{risky}}(\Delta T'), h_{\text{safe}}(\Delta T'))$ is a waiting-time steady state (specifically, (5)). □

C Proof of Proposition 7

Let F_W and $F_{\widehat{W}}$ denote the cdfs of $W(\theta)$ and $\widehat{W}(\theta)$, respectively. For $\alpha \in [0, 1)$, define their lower quantile functions by

$$q_-(\alpha) = \inf\{\Delta T \geq 0 \mid F_W(\Delta T) \geq \alpha\},$$
$$\widehat{q}_-(\alpha) = \inf\{\Delta T \geq 0 \mid F_{\widehat{W}}(\Delta T) \geq \alpha\}.$$

We first record the quantile implication of first-order stochastic dominance.

Lemma 8. *The random variable $\widehat{W}(\theta)$ first-order stochastically dominates $W(\theta)$ if and only if*

$$\widehat{q}_-(\alpha) \geq q_-(\alpha) \qquad \text{for every } \alpha \in (0, 1).$$

Moreover, if there exists $\varepsilon > 0$ such that $\widehat{W}(\theta)$ first-order stochastically dominates $W(\theta) + \varepsilon$, then

$$\widehat{q}_-(\alpha) \geq q_-(\alpha) + \varepsilon \qquad \text{for every } \alpha \in (0, 1).$$

Proof. Suppose first that $F_{\widehat{W}}(x) \leq F_W(x)$ for all x. Fix $\alpha \in (0, 1)$ and $x < q_-(\alpha)$. Then $F_W(x) < \alpha$, so $F_{\widehat{W}}(x) < \alpha$ as well. Hence $x < \widehat{q}_-(\alpha)$. Taking the supremum over all $x < q_-(\alpha)$ gives $\widehat{q}_-(\alpha) \geq q_-(\alpha)$.

Conversely, suppose that $\widehat{q}_-(\alpha) \geq q_-(\alpha)$ for all $\alpha \in (0, 1)$. Fix $x \geq 0$ and $\alpha < F_{\widehat{W}}(x)$. By the definition of the lower quantile, $\widehat{q}_-(\alpha) \leq x$. Hence $q_-(\alpha) \leq x$, which implies $F_W(x) \geq \alpha$. Letting $\alpha \uparrow F_{\widehat{W}}(x)$ gives $F_W(x) \geq F_{\widehat{W}}(x)$. Thus $\widehat{W}(\theta)$ first-order stochastically dominates $W(\theta)$.

For the final claim, apply the first part to $\widehat{W}(\theta)$ and $W(\theta) + \varepsilon$, and use the fact that the lower quantile function for $W(\theta) + \varepsilon$ equals $q_-(\alpha) + \varepsilon$. □

To prove Proposition 7, we apply monotone comparative statics ideas, but based on a parameterization that is invariant across the change in decision-makers. Rather than considering the difference in waiting times, we consider the fractions α of patients who would accept a risky organ if offered at time T_{risky}. Let F_δ denote the cdf of the

departure rate $\delta(\theta)$. For $t \geq 0$ and $\alpha \in [0,1)$, define

$$G(t,\alpha) := p\int_0^\infty e^{-xt}\, d\min\{F_\delta(x), 1-\alpha\}, \tag{C.1}$$

$$D(t,\alpha) := p\int_0^\infty e^{-xt}\, d\Big(F_\delta(x) - \min\{F_\delta(x), 1-\alpha\}\Big). \tag{C.2}$$

Thus, $G(t,\alpha)$ is the mass at position t among the lowest-departure-rate fraction $1-\alpha$, while $D(t,\alpha)$ is the mass at position t among the highest-departure-rate fraction α. G and D are continuous in (t,α). Moreover, G is strictly decreasing in both t and α, while D is strictly increasing in α and, whenever $\alpha > 0$, strictly decreasing in t.

Claim 1. *Suppose that W satisfies the healthy-more-willing-to-wait condition. Fix $c \geq 0$ with $F_W(c) < 1$, and let $F_W(c) = \alpha$. Then, for every $t \geq 0$, we have*

$$p\int_{\{\theta: W(\theta) > c\}} e^{-\delta(\theta)t} f(\theta) d\theta = G(t,\alpha), p\int_{\{\theta: W(\theta) < c\}} e^{-\delta(\theta)t} f(\theta) d\theta \qquad = D(t,\alpha),$$

where $G(t,\alpha)$ and $D(t,\alpha)$ are as defined in (C.1) *and* (C.2)*, respectively.*

Proof. Because it is assumed that $\nu(\theta)$ is continuous conditioned on $\delta(\theta)$, W is atomless. Therefore, the two sets in the statement of the claim have masses $1-\alpha$ and α. Moreover, the set $\{W(\theta) > c\}$ is a lower set in departure rates: to see this, if $W(\theta') > c$ and $\delta(\theta) < \delta(\theta')$, then the healthy-more-willing-to-wait condition gives $W(\theta) \geq W(\theta') > c$. Hence $\{W(\theta) > c\}$ and $\{\delta(\theta) \leq x\}$ are nested up to null sets: otherwise one could choose $\theta \in \{W(\theta) > c, \delta(\theta) > x\}$ and $\theta' \in \{W(\theta) < c, \delta(\theta) \leq x\}$, contradicting the preceding lower-set property. Consequently, for every x,

$$\Pr\big(W(\theta) > c,\ \delta(\theta) \leq x\big) = \min\{F_\delta(x), 1-\alpha\}\,.$$

Thus the departure-rate subdistribution on $\{W(\theta) > c\}$ is $\min\{F_\delta, 1-\alpha\}$, while that on its complement $\{W(\theta) < c\}$ is $F_\delta - \min\{F_\delta, 1-\alpha\}$. Integrating pe^{-xt} against these two subdistributions proves the claim. □

The following lemma provides an analogue of Lemma 1.

Lemma 9. *(a) There is a strictly decreasing, continuous function $\tilde{h}_{\text{safe}} : [0, 1-s] \to \mathbb{R}_{\geq 0}$ independent of $\nu(\theta)$ (but dependent on s, $f(\theta)$, and $\delta(\theta)$) with $\tilde{h}_{\text{safe}}(1-s) = 0$ such if W satisfies the healthy-more-willing-to-wait condition, then a pair*

$(T_{\text{risky}}, T_{\text{safe}})$ *of waiting times satisfies* (2) *if and only if* $T_{\text{safe}} = \tilde{h}_{\text{safe}}(F_W(T_{\text{safe}} - T_{\text{risky}}))$.

(b) Suppose that $(T_{\text{risky}}, T_{\text{safe}})$ *satisfies* (2). *Writing* $\alpha = F_W(T_{\text{safe}} - T_{\text{risky}})$, *we have*

$$T_{\text{risky}} \leq \tilde{h}_{\text{safe}}(\alpha) - q_-(\alpha).$$

with equality if W *has full support on* $[0, \overline{\Delta T})$.

Proof. Note that $G(T_{\text{safe}}, \alpha)$ represents the demand for safe organs if patients with the lowest $1-\alpha$ fraction of departure rates wait for safe organs. For every $\alpha \in [0, 1-s]$, the function $G(t, \alpha)$ is continuous and strictly decreasing in $t \geq 0$, with

$$G(0, \alpha) = p(1-\alpha) \geq sp \quad \text{and} \quad \lim_{t\to\infty} G(t, \alpha) = 0.$$

Hence there is a unique $T_{\text{safe}} = \tilde{h}_{\text{safe}}(\alpha) \geq 0$ such that $G(T_{\text{safe}}, \alpha) = sp$. Moreover, $\tilde{h}_{\text{safe}}(1-s) = 0$. Since $G(t, \alpha)$ is strictly decreasing in α and continuous in (t, α), the function $\tilde{h}_{\text{safe}}$ is strictly decreasing and continuous.

To prove Part (a), suppose that W satisfies the healthy-more-willing-to-wait condition, and consider a pair $(T_{\text{risky}}, T_{\text{safe}})$ of waiting times. Let $\alpha = F_W(T_{\text{safe}} - T_{\text{risky}})$. Because the distribution of W is atomless, Claim 1 gives that the demand for safe organ under waiting times $(T_{\text{safe}}, T_{\text{risky}})$ is $G(T_{\text{safe}}, \alpha)$. If (2) holds, then $T_{\text{safe}} > 0$ for a pair of waiting times, and hence

$$p(1-\alpha) = G(0, \alpha) > G(T_{\text{safe}}, \alpha) = sp.$$

Thus $\alpha < 1 - s$, and uniqueness of the root gives $T_{\text{safe}} = \tilde{h}_{\text{safe}}(\alpha)$. Conversely, if $\alpha < 1-s$ and $T_{\text{safe}} = \tilde{h}_{\text{safe}}(\alpha)$, then the fact that demand for safe is $G(T_{\text{safe}}, \alpha)$ gives (2).

To prove Part (b), the definition of the lower quantile gives

$$q_-(\alpha) \leq T_{\text{safe}} - T_{\text{risky}}.$$

Combining this inequality with $T_{\text{safe}} = \tilde{h}_{\text{safe}}(\alpha)$ from Part (a) proves the inequality. Full support of W makes F_W strictly increasing on $[0, \overline{\Delta T})$. Thus

$$T_{\text{safe}} - T_{\text{risky}} = q_-(\alpha),$$

and equality holds. □

Proof of Proposition 7. Let $(\widehat{T}_{\text{risky}}, \widehat{T}_{\text{safe}})$ be the lowest waiting-time steady state under hospital decisions. Let $\widehat{\alpha}^* = F_{\widehat{W}}(\widehat{T}_{\text{safe}} - \widehat{T}_{\text{risky}})$. Let $(T_{\text{risky}}, T_{\text{safe}})$ be the lowest waiting-time steady state under patient decisions, and let $\alpha^* = F_W(T_{\text{safe}} - T_{\text{risky}})$. Safe-organ flow balance and Lemma 9 (applied to both patient and hospital decisions) imply

$$T_{\text{safe}} = \tilde{h}_{\text{safe}}(\alpha^*), \qquad T_{\text{risky}} = \tilde{h}_{\text{safe}}(\alpha^*) - q_-(\alpha^*), \tag{C.3}$$

$$\widehat{T}_{\text{safe}} = \tilde{h}_{\text{safe}}(\widehat{\alpha}^*), \qquad \widehat{T}_{\text{risky}} \leq \tilde{h}_{\text{safe}}(\widehat{\alpha}^*) - \widehat{q}_-(\widehat{\alpha}^*). \tag{C.4}$$

To prove the proposition, we first show that

$$\alpha^* \geq \widehat{\alpha}^*, \qquad \alpha^* > \widehat{\alpha}^* \quad \text{under strict dominance.} \tag{C.5}$$

To show this claim, we divide into cases based on whether $\widehat{\alpha}^* = 0$.

- Case 1: $\widehat{\alpha}^* = 0$. In this case, the weak inequality is immediate. Note that (5) requires that $\text{RiskyDemand}(T_{\text{risky}}, T_{\text{safe}}) = rp > 0$ or $\text{RefuseRisky}(T_{\text{risky}}, T_{\text{safe}}) = \gamma > 0$ or $T_{\text{risky}} = 0 < T_{\text{safe}}$. In any of these cases, we must have $T_{\text{risky}} < T_{\text{safe}}$, and hence $\alpha^* > 0$ as W has full support on $[0, \overline{\Delta T})$.
- Case 2: $\widehat{\alpha}^* > 0$. By Lemma 9, we have $\widehat{T}_{\text{safe}} = \tilde{h}_{\text{safe}}(\widehat{\alpha}^*) > 0$, and hence $\widehat{\alpha}^* < 1 - s$. Define functions

$$\begin{aligned} \tilde{h}_{\text{risky}}(\alpha) &= \tilde{h}_{\text{safe}}(\alpha) - q_-(\alpha) \\ \text{RiskyDemand}_{\tilde{h}}(\alpha) &= \text{RiskyDemand}(\tilde{h}_{\text{risky}}(\alpha), \tilde{h}_{\text{safe}}(\alpha)) \\ \text{RefuseRisk}_{\tilde{h}}(\alpha) &= \text{RefuseRisky}(\tilde{h}_{\text{risky}}(\alpha), \tilde{h}_{\text{safe}}(\alpha)). \end{aligned}$$

  Lemma 8, (C.4), and the definition of $\tilde{h}_{\text{risky}}$ give

$$\tilde{h}_{\text{risky}}(\widehat{\alpha}^*) = \tilde{h}_{\text{safe}}(\widehat{\alpha}^*) - q_-(\widehat{\alpha}^*) \geq \tilde{h}_{\text{safe}}(\widehat{\alpha}^*) - \widehat{q}_-(\widehat{\alpha}^*) \geq \widehat{T}_{\text{risky}}.$$

It follows that

$$\begin{aligned}\text{RiskyDemand}_{\tilde{h}}(\widehat{\alpha}^*) &= p\int_{\theta:W(\theta)<q_-(\widehat{\alpha}^*)} e^{-\delta(\theta)\tilde{h}_{\text{risky}}(\widehat{\alpha}^*)}f(\theta)d\theta \\ &= p\int_{\theta:\widehat{W}(\theta)<\widehat{T}_{\text{safe}}-\widehat{T}_{\text{risky}}} e^{-\delta(\theta)\tilde{h}_{\text{risky}}(\widehat{\alpha}^*)}f(\theta)d\theta \\ &\le p\int_{\theta:\widehat{W}(\theta)<\widehat{T}_{\text{safe}}-\widehat{T}_{\text{risky}}} e^{-\delta(\theta)\widehat{T}_{\text{risky}}}f(\theta)d\theta \le rp.\end{aligned}$$

Here the second equality follows from Claim 1: both integration regions select the highest-departure-rate fraction $\widehat{\alpha}^*$. The first inequality uses $\tilde{h}_{\text{risky}}(\widehat{\alpha}^*) \ge \widehat{T}_{\text{risky}}$, and the last inequality holds due to the hospital risky-flow constraint (3). Similarly, we have that

$$\begin{aligned}\text{RefuseRisk}_{\tilde{h}}(\widehat{\alpha}^*) &= \int_{\tilde{h}_{\text{risky}}(\widehat{\alpha}^*)}^{\tilde{h}_{\text{safe}}(\widehat{\alpha}^*)} p\int_{\theta:W(\theta)>q_-(\widehat{\alpha}^*)} e^{-\delta(\theta)t}f(\theta)d\theta dt \\ &= \int_{\tilde{h}_{\text{risky}}(\widehat{\alpha}^*)}^{\widehat{T}_{\text{safe}}} p\int_{\theta:\widehat{W}(\theta)>\widehat{T}_{\text{safe}}-\widehat{T}_{\text{risky}}} e^{-\delta(\theta)t}f(\theta)d\theta dt \\ &\le \int_{\widehat{T}_{\text{risky}}}^{\widehat{T}_{\text{safe}}} p\int_{\theta:\widehat{W}(\theta)>\widehat{T}_{\text{safe}}-\widehat{T}_{\text{risky}}} e^{-\delta(\theta)t}f(\theta)d\theta dt \le \gamma,\end{aligned}$$

Here the equality again follows from Claim 1, now for the lowest-departure-rate fraction $1-\widehat{\alpha}^*$; the first inequality uses $\tilde{h}_{\text{risky}}(\widehat{\alpha}^*) \ge \widehat{T}_{\text{risky}}$, and the last is the hospital expiry constraint (4).

Based on the steady-state inequalities, let

$$\begin{aligned}I &:= \{\alpha \in [\widehat{\alpha}^*, 1-s] : \tilde{h}_{\text{risky}}(\alpha) \ge 0\}, \\ S &:= \{\alpha \in I : \text{RiskyDemand}_{\tilde{h}}(\alpha) \le rp \text{ and } \text{RefuseRisk}_{\tilde{h}}(\alpha) \le \gamma\}.\end{aligned}$$

We have shown above that $\widehat{\alpha}^* \in S$, so S is non-empty. Full support and atomlessness of W make $q_-(\alpha)$ continuous on $(0,1)$. For $\alpha \in I$, Claim 1 gives the useful identities

$$\text{RiskyDemand}_{\tilde{h}}(\alpha) = D(\tilde{h}_{\text{risky}}(\alpha), \alpha), \tag{C.6}$$

$$\text{RefuseRisk}_{\tilde{h}}(\alpha) = \int_{\tilde{h}_{\text{risky}}(\alpha)}^{\tilde{h}_{\text{safe}}(\alpha)} G(t,\alpha)\,dt. \tag{C.7}$$

Lemma 9 therefore implies that $\tilde{h}_{\text{risky}}$ is continuous on $[\widehat{\alpha}^*, 1-s]$ and that $\text{RiskyDemand}_{\tilde{h}}$ and $\text{RefuseRisk}_{\tilde{h}}$ are continuous on I. Thus I and S are closed. Moreover, $1-s>0$, so full support gives $q_-(1-s)>0$; since $\tilde{h}_{\text{safe}}(1-s)=0$, we have $\tilde{h}_{\text{risky}}(1-s)<0$ and therefore $1-s \notin I$.

Because S is a nonempty closed subset of the compact interval $[\widehat{\alpha}^*, 1-s]$, it has a maximum α^{**}; the preceding paragraph gives

$$\widehat{\alpha}^* \leq \alpha^{**} < 1-s.$$

At least one of the risky-flow, expiry, and nonnegativity inequalities binds at α^{**}. Otherwise, continuity and $\alpha^{**} < 1-s$ would provide $\eta > 0$ such that $\alpha^{**}+\eta \in S$, contradicting maximality. Consequently, Since full support and atomlessness give $F_W(q_-(\alpha^{**})) = \alpha^{**}$, the pair $(\tilde{h}_{\text{risky}}(\alpha^{**}), \tilde{h}_{\text{safe}}(\alpha^{**}))$ satisfies safe-organ balance by Lemma 9. It satisfies the risky-flow and expiry inequalities by membership in S, and it satisfies the no-withholding condition (5) by the binding just established. Since $\alpha^{**}>0$, full support gives $q_-(\alpha^{**})>0$, so these waiting times also satisfy $0 \leq \tilde{h}_{\text{risky}}(\alpha^{**}) < \tilde{h}_{\text{safe}}(\alpha^{**})$. Thus they form a waiting-time steady state under patient decisions. But since $(T_{\text{risky}}, T_{\text{safe}})$ was the lowest steady state under patient decisions, we then have that $T_{\text{safe}} \leq \tilde{h}_{\text{safe}}(\alpha^{**})$. Since $T_{\text{safe}} = \tilde{h}_{\text{safe}}(\alpha^*)$ and $\tilde{h}_{\text{safe}}$ is strictly decreasing, it follows that $\alpha^* \geq \alpha^{**} \geq \widehat{\alpha}^*$, proving the weak inequality in (C.5).

Now suppose that there exists $\varepsilon > 0$ such that $\widehat{W}$ first-order stochastically dominates $W+\varepsilon$. When $\widehat{\alpha}^* > 0$, Lemma 8 and (C.4) imply

$$\tilde{h}_{\text{risky}}(\widehat{\alpha}^*) = \tilde{h}_{\text{safe}}(\widehat{\alpha}^*) - q_-(\widehat{\alpha}^*) \geq \widehat{T}_{\text{risky}} + \varepsilon. \tag{C.8}$$

Consequently, $\widehat{\alpha}^* > 0$ gives

$$\text{RiskyDemand}_{\tilde{h}}(\widehat{\alpha}^*) = D(\tilde{h}_{\text{risky}}(\widehat{\alpha}^*), \widehat{\alpha}^*) < D(\widehat{T}_{\text{risky}}, \widehat{\alpha}^*) \leq rp.$$

Likewise, $G(t, \widehat{\alpha}^*) > 0$ and (C.8) give

$$\text{RefuseRisk}_{\tilde{h}}(\widehat{\alpha}^*) = \int_{\tilde{h}_{\text{risky}}(\widehat{\alpha}^*)}^{\widehat{T}_{\text{safe}}} G(t, \widehat{\alpha}^*)\, dt < \int_{\widehat{T}_{\text{risky}}}^{\widehat{T}_{\text{safe}}} G(t, \widehat{\alpha}^*)\, dt \leq \gamma,$$

and (C.8) also gives $\tilde{h}_{\text{risky}}(\widehat{\alpha}^*) > 0$. Thus all three feasibility inequalities are strict at $\widehat{\alpha}^*$. Continuity provides some $\alpha > \widehat{\alpha}^*$ in S, so $\alpha^{**} > \widehat{\alpha}^*$ and hence $\alpha^* > \widehat{\alpha}^*$.

Since $\tilde{h}_{\text{safe}}$ is strictly decreasing, (C.5) and the safe-time identities in (C.3)–(C.4) imply

$$T_{\text{safe}} \leq \widehat{T}_{\text{safe}},$$

with strict inequality under the strict-dominance hypothesis. This proves Part (a).

Suppose now that the patient-decision lowest steady state is congested. Then $T_{\text{risky}} > 0$ and $\text{RiskyDemand}(T_{\text{risky}}, T_{\text{safe}}) < rp$, so the no-withholding condition (5) forces the patient expiry constraint to bind. Part (a) and (C.5) give

$$\alpha^* \geq \widehat{\alpha}^* \quad \text{and} \quad T_{\text{safe}} \leq \widehat{T}_{\text{safe}},$$

with both inequalities strict under the strict-dominance hypothesis. By Claim 1,

$$\gamma = \text{RefuseRisky}(T_{\text{risky}}, T_{\text{safe}}) = \int_{T_{\text{risky}}}^{T_{\text{safe}}} G(t, \alpha^*)\, dt. \tag{C.9}$$

Because $G(t, \alpha)$ is decreasing in α and positive, it follows that

$$\gamma \leq \int_{T_{\text{risky}}}^{T_{\text{safe}}} G(t, \widehat{\alpha}^*)\, dt \leq \int_{T_{\text{risky}}}^{\widehat{T}_{\text{safe}}} G(t, \widehat{\alpha}^*)\, dt. \tag{C.10}$$

Under strict dominance, both inequalities in (C.10) are strict. On the other hand, the hospital expiry constraint is

$$\int_{\widehat{T}_{\text{risky}}}^{\widehat{T}_{\text{safe}}} G(t, \widehat{\alpha}^*)\, dt \leq \gamma. \tag{C.11}$$

For fixed $\widehat{T}_{\text{safe}}$, the map $a \mapsto \int_a^{\widehat{T}_{\text{safe}}} G(t, \widehat{\alpha}^*)\, dt$ is strictly decreasing because $G(t, \widehat{\alpha}^*) > 0$. Comparing (C.10) and (C.11) therefore yields

$$\widehat{T}_{\text{risky}} \geq T_{\text{risky}}.$$

Under the strict-dominance hypothesis, (C.10) is strict. Hence

$$\int_{\widehat{T}_{\text{risky}}}^{\widehat{T}_{\text{safe}}} G(t, \widehat{\alpha}^*)\, dt \leq \gamma < \int_{T_{\text{risky}}}^{\widehat{T}_{\text{safe}}} G(t, \widehat{\alpha}^*)\, dt.$$

Because the integral is strictly decreasing in its lower limit, this implies $\widehat{T}_{\text{risky}} > T_{\text{risky}}$.

It remains to verify the congestion conclusion in Part (b). Claim 1 gives

$$\begin{aligned}\text{RiskyDemand}(\widehat{T}_{\text{risky}}, \widehat{T}_{\text{safe}}) &= D(\widehat{T}_{\text{risky}}, \widehat{\alpha}^*) \\ &\leq D(T_{\text{risky}}, \alpha^*) = \text{RiskyDemand}(T_{\text{risky}}, T_{\text{safe}}) < rp.\end{aligned}$$

Moreover, $\widehat{T}_{\text{risky}} \geq T_{\text{risky}} > 0$. Thus, at the hospital steady state, both the risky-flow constraint and the nonnegativity constraint are slack. The no-withholding condition (5) therefore forces the hospital expiry constraint to bind. Hence the hospital steady state has a positive risky waiting time and discards a positive mass of risky organs, so it is congested. This completes the proof of Part (b), including the strict risky-waiting-time comparison under strict dominance. □

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