Measuring Fiscal Risk: Evidence From a Century of Data*

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Abstract

How safe or risky is government debt? According to asset pricing theory, this depends on the covariance between the real holding period return on the public portfolio and the prevailing stochastic discount factor (SDF). Through the lens of a simple monetary-fiscal model with bond investors who price risk, we show that the sign of this covariance is a sufficient statistic for the underlying policy regime. We recover the SDF from the cross-section of returns, using a novel method which allows us to remain broadly agnostic about the underlying asset pricing model, and use this to evaluate the key covariance of interest using US and UK data from 1926-2024. We find that US government debt was i) risky in the pre-WWII period but became safe after the Treasury-Fed accord of 1951, ii) risky from 1965-1985, iii) safe from 1985-2020, and iv) seems to have become risky in the post-pandemic 2020-2024 period. We estimate that in 2024 the risk premium component was currently adding 100bps (annualised) to the average funding cost which appears in the US government budget constraint.

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1 Introduction

In common parlance, government debt in advanced economies is often referred to as a safe asset. As asset pricing theory tells us, this safety status is not an intrinsic quality — an asset is only safe if its real returns covary non-negatively with the stochastic discount factor (SDF). In other words, to be truly safe, the real holding period return (HPR) on government debt must be high in bad states of the world for the marginal investor. If this is indeed the case, bondholders should be willing to accept a zero or negative expected excess real return to hold the debt portfolio, since it provides a valuable hedge. Conversely, if this covariance is negative, government debt is risky and commands a risk premium to compensate bondholders as a result. The majority of work in macroeconomics employs linear models, and so disregards this risk premium term by construction. In this paper we measure the covariance between the SDF which prices the cross-section of returns and the real HPR on government debt, providing an explicit and theoretically consistent measure of the riskiness of government debt.

Through the lens of a simple model which builds upon Gómez-Cram et al. (2024) and Corhay et al. (2023), we show that the aforementioned covariance term is a sufficient statistic for identifying two distinct monetary-fiscal regimes. In the safe debt regime, deficits are backed in net present value terms by future surpluses. The real HPR on government debt is thus unresponsive to the shock, rendering debt a safe asset that has a beta of zero with the SDF. Monetary policy is active in this regime, meaning that inflation is kept under control and does not need to adjust to satisfy the government budget constraint. Bondholders are insured in this regime, as debt is kept riskless, but the government may not be able to provide as much insurance to taxpayers as it may wish to. In the risky debt regime, deficit shocks are accommodated via higher inflation rather than a rise in the net present value of future surpluses as there is active fiscal policy. This means that the HPR on the government debt portfolio falls in bad times, when the SDF is high, and thus means that there is a negative covariance and a risk premium on government debt. Taxpayers are provided with insurance in the form of countercyclical deficits, but bondholders cannot be insured simultaneously and debt carries a negative beta with the SDF.

A common reduced-form diagnostic of the riskiness of government debt is the bond–stock correlation: the correlation between the aggregate stock market return and the HPR on a long-term government bond, typically the 10-year Treasury in the US. This is a useful summary of whether long nominal bonds

hedge equity downturns, but it is not the object that determines the risk premium on government debt, which is the covariance between the SDF and the real return on the public debt portfolio. The two coincide only under two restrictive conditions: i) the CAPM holds, so that the market return spans the priced risks faced by the marginal bondholder;[1] and ii) a single long-term nominal bond adequately proxies the return on the public portfolio. Neither is innocuous. The well-documented failures of the CAPM make the market return an inadequate summary of the SDF, and since governments issue debt across the maturity spectrum, vary its composition over time, and issue inflation-indexed debt, a 10-year nominal return need not track the real HPR that enters the consolidated budget constraint. Our approach generalises the bond–stock correlation along both dimensions. We replace the market return with an SDF estimated from the cross-section of returns, and the single bond with the real return on the actual public portfolio. An additional benefit of our approa

Motivated by these restrictions, we directly measure the theoretically relevant covariance term. The HPR on US government debt is constructed by Hall et al. (2018), accounting for the rich and evolving maturity structure of US government debt, leaving the SDF as the only unknown object that is needed. We propose a novel parametric approach to recover this from the cross-section of returns which builds upon Di Tella et al. (2025). This method is of independent interest and i) does not assume that the zero-beta rate is directly observable or constant over time; ii) does not require the specification of a full structural asset pricing model. Crucially we do not use the return on government debt, or its constituents, to construct the SDF. The first of these is important since Di Tella et al. (2025) argue that the common assumption that the zero-beta rate coincides with the short-term Treasury bill rate is likely incorrect, as the true zero-beta rate is much more volatile. While we are not directly interested in the zero-beta rate, this indirectly affects the construction of the SDF since the zero-beta rate is the inverse of the conditional expectation of the SDF. The advantage of the second is that, rather than estimating a full structural model such as the consumption CAPM, the habits model of Cochrane and Campbell (1999), the rare disasters model of Barro (2006), or the intermediary-based model of He and Krishnamurthy (2013), we can instead model the SDF as being dependent on a number of asset pricing factors that have repeatedly been found to earn a risk premium in the cross-section of returns. We also propose a complementary method that is fully nonparametric and finds the SDF which satisfies the fundamental asset pricing equation for each asset ($1 = \mathbb{E}M_{t+1}R_{i,t+1}$ for asset i) and satisfies the

[1] With the implicit further condition that the stock market proxies the market portfolio (Roll, 1977).

Hansen and Jagannathan (1991) bound with equality.

Armed with this pair of necessary objects, we then examine the evolution of the key covariance term over time using a variety of methods for uncovering time-variation. We find substantial evidence of distinct, persistent debt regimes over the sample period of 1926-2024. Over the period of 1926-1965, we find that US government debt was risky pre-WWII and then became safe after the Treasury-Fed accord in 1951, although we do not definitively claim this as the cause of this change. We then find evidence of a distinct shift in the mid-1960s, which coincides with President Johnson's Great Society program that constituted a large, sustained and partially unfunded increase in government spending.[2] This regime lasts until the early 1980s which is also when Paul Volcker became chair of the Federal Reserve and is widely considered to have engineered a monetary regime change.[3] The period from the mid-1980s to 2020 is found to be a safe debt regime. This is consistent with previous work which has partially attributed the reduction in inflation volatility over this period to a structural shift in monetary policy (e.g. Clarida et al. (2000), Lubik and Schorfheide (2004), Beaudry et al. (2023)). Finally, we find evidence that the post-pandemic period may have featured a shift back towards the risky debt regime. The covariance between the SDF and the Treasury portfolio return is found to be more negative than at any previous point in the sample, indicative of a large implied risk premium on the US government debt portfolio. We emphasise that previous regimes identified lasted for multiple decades, and it is thus hard to draw firm conclusions given the limited amount of data in the post-pandemic period.

We also apply our method to the UK, building on the series for the HPR on government debt produced by Ellison and Scott (2020). We extend their series, and also limit it to the debt held by private investors, which in turn requires netting off the gilts held by the Bank of England as a result of its quantitative easing programme. We find that UK government debt has largely been safe over the last century, with three periods which form notable exceptions: i) the interwar period; ii) the 1975-1980 period; iii) the post-pandemic 2020-2024 period.

Finally, we conduct a number of further tests of the regimes via the lens of the model's testable predictions. We provide evidence that the regimes we identify are consistent with estimates of the inflation

[2] See Bianchi et al. (2023) who provide detailed evidence on the partially unfunded nature of the Great Society program.

[3] Bianchi and Ilut (2017) also identify this period as a fiscal regime, while Bianchi et al. (2023) find that the unfunded spending shocks of this period contributed substantially to the rise in inflation. Both of these papers rely on medium-scale, estimated DSGE models whereas ours is more parsimonious.

responsiveness parameter for both the US and UK, using the method of Bocola et al. (2024). We also find — consistent with the model — that the volatility of the real HPR is higher in the fiscal regimes vs the monetary regimes we identify.

Related Literature

Our paper contributes to three main literature areas. Primarily it contributes to the literature which 'puts the finance back in public finance' (Lustig, 2023), which, broadly speaking, describes a class of papers that bring an asset pricing perspective to the analysis of government debt dynamics and fiscal policy. Much of this in turn builds upon the fiscal theory of the price level (FTPL) literature, which has the intertemporal government budget constraint as its key object, and uses the expected path of discounted future surpluses to describe the joint dynamics of government debt and inflation. Seminal papers include Sargent and Wallace (1981), Leeper (1991), Sims (1994), Woodford (1995) and Cochrane (2005). Recent work has incorporated the SDF more explicitly in this analysis. Gómez-Cram et al. (2024) and Corhay et al. (2023), who form the basis of our model in Section 2, both develop models where risk is priced explicitly, and examine how this impacts the dynamics of government debt and inflation in an environment similar to Leeper (1991). Lustig et al. (2024) find empirical evidence that fiscal news produces an increase in the nominal term premium and expected inflation, consistent with our model. Jiang et al. (2024) examine the intertemporal government budget constraint empirically, using an asset pricing model to discount future surpluses and ultimately finding evidence that Treasuries are overpriced. Lustig et al. (2020) highlight a key tradeoff that a fiscal policymaker faces between insuring bondholders by keeping government debt riskless, or insuring taxpayers, which necessitates government debt being risky.

A closely related empirical literature studies the comovement of bond and equity returns directly. Campbell et al. (2017) document that the covariance between nominal bond and stock returns is far from constant and has in fact switched sign, with nominal Treasuries moving from carrying a positive covariance with equities to acting as a hedge around the turn of the millennium. A subsequent body of work attributes this time-variation to the prevailing macroeconomic shock structure and the stance of monetary policy. Campbell et al. (2020) and Song (2017) show, through the lens of New Keynesian models, that the sign of the bond-stock covariance turns on whether inflation is driven by supply or

demand disturbances and on how actively monetary policy leans against inflation, while Cieslak and Pang (2021) decompose the comovement empirically into growth, monetary, and risk premium shocks. Our paper relates closely to this literature but differs in two important respects. First, the bond-stock correlation is a sufficient statistic for the riskiness of government debt only under the joint assumption that the CAPM prices the cross-section of returns and that all government debt is nominal and of a single maturity; we instead measure the covariance of the holding-period return on the entire debt portfolio with an SDF recovered from the broad cross-section of returns. Second, and more importantly, by embedding this covariance in a model that permits both monetary and fiscal regimes, we are able to read its sign as informative about the underlying monetary-fiscal interaction, rather than about the shock structure alone.

Secondly, we contribute to the literature on the fiscal history of both the US and UK over the 20th and 21st centuries. Hall and Sargent (2011) analyse the dynamics of the US debt-to-GDP ratio in the post-WWII period through the lens of the intertemporal government budget, in turn constructing the one-period HPR on government debt that is a key object in this paper. Hall and Sargent (2022) extend this analysis, and argue that the Covid-19 pandemic should be classed alongside WWI and WWII in fiscal terms, since all three events have featured high deficit spending paid for by delivering low real returns to bondholders via subsequent interest rate hikes and high inflation. Nieuwerburgh et al. (2025) quantify the history of exorbitant privilege in the US, UK and the Netherlands. They find that the Netherlands and the UK lost the ability to sell government debt at a premium relative to their respective fiscal fundamentals after they suffered a large shock which meant bondholders suffered large losses. They warn that this may lie in store for the US given the fiscal response to the Covid-19 pandemic. Crucially none of these papers tries to quantify the riskiness of government debt as we do here. Our contribution is potentially complementary to the exorbitant privilege analysis, since part of this could reflect a safety premium of government debt. Indeed, Laarits and Acharya (2025) find that a substantial component of the convenience yield on US Treasuries can be explained by their negative covariance with stock returns (implying a positive covariance with the SDF if one assumes the CAPM holds). The negative covariance between the SDF and the Treasury portfolio we find from 2020 onwards suggests that the convenience yield of the US may be eroding for risk-based reasons.

Third and finally, we contribute to the literature on parametric and nonparametric estimation of the

SDF. Our benchmark method does not specify a structural, fully parametric asset pricing model, and instead relies on a number of factors which have been well-studied in the literature and have been shown to explain the cross-section of equity returns. Our approach builds upon Di Tella et al. (2025), who estimate the zero-beta rate using a set of factors, instruments and the cross-section of returns. Unlike us, they do not recover the SDF. Christensen (2017) develops a nonparametric framework to decompose the SDF into permanent and transitory components by solving a Perron–Frobenius eigenfunction problem. We also provide a very simple, nonparametric method of estimating the SDF. Ghosh et al. (2017) also nonparametrically estimate the SDF, finding that it has a strong relationship with the Fama-French factors that we use as the basis for our SDF.

The paper proceeds as follows. Section 2 presents a simple monetary-fiscal model which we use to motivate our subsequent analysis. Section 3 describes our empirical methodology and the data we use in practice. Section 4 presents the corresponding results for the US. Section 5 presents results for the UK, and describes a subset of the robustness tests we conducted, with the remainder relegated to the appendix. Section 6 provides additional analysis to further test the fiscal regimes we identify. Section 7 concludes.

2 Model

In this section, we develop a model which builds upon Corhay et al. (2023) and Gómez-Cram et al. (2024) to motivate our analysis. We initially maintain a high level of generality and then impose more structure to obtain some more concrete analytical results.

2.1 General Environment

We consider a discrete time economy with an infinite horizon. There is a government which runs an exogenous, stochastic real surplus of level s_t and issues nominal, zero-coupon bonds of varying maturities that are rolled over each period. Letting $B_{t-1}^{(j)}$ denote the face value of nominal bonds issued in $t-1$ which mature in $t-1+j$, $\tilde{Q}_t^{(j)}$ the price of a j-period nominal bond, and P_t the aggregate price level,

the period-by-period consolidated government budget constraint is given by:[4]

$$\sum_{j\geq 1} \tilde{Q}_t^{(j-1)} B_{t-1}^{(j)} = P_t s_t + \sum_{j\geq 1} \tilde{Q}_t^{(j)} B_t^{(j)}, \tag{1}$$

where $Q_t^{(0)} = 1$ as a normalisation. The real market value of total government debt is given by:

$$b_t = \frac{\sum_{j\geq 1} \tilde{Q}_t^{(j)} B_t^{(j)}}{P_t}. \tag{2}$$

The Treasury portfolio share of j-period bonds is given by:

$$\Omega_t^{(j)} = \frac{\tilde{Q}_t^{(j)} B_t^{(j)}}{\sum_{j\geq 1} \tilde{Q}_t^{(j)} B_t^{(j)}}. \tag{3}$$

We assume these are constant for the sake of simplicity, and so drop the time subscript from henceforth. P_t is the aggregate price level, and s_t is the level of real primary surpluses. As Corhay et al. (2023) demonstrate, the government budget constraint can alternatively be expressed compactly as:

$$\frac{\widetilde{R}_{g,t}}{\Pi_t} b_{t-1} = s_t + b_t, \tag{4}$$

where:

$$\widetilde{R}_{g,t} \equiv \sum_{j\geq 1} \Omega_{t-1}^{(j)} \widetilde{R}_t^{(j)}, \tag{5}$$

$$\widetilde{R}_t^{(j)} \equiv \frac{\tilde{Q}_t^{(j-1)}}{\tilde{Q}_{t-1}^{(j)}}, \tag{6}$$

$$\Pi_t \equiv \frac{P_t}{P_{t-1}}. \tag{7}$$

In words, $\widetilde{R}_{g,t}$ is the nominal holding period return on the Treasury portfolio , $\widetilde{R}_t^{(j)}$ is the nominal holding period return on a j-period bond, and Π_t is the (gross) inflation rate. By construction $\sum_{j\geq 1} \Omega^{(j)} = 1$.

We assume that there is the absence of arbitrage, and markets are complete, implying that there exists a real SDF, M_{t+1}, that is unique (Harrison and Kreps, 1979; Kreps, 1981b) and prices all traded securities. This implies that the bond prices and holding period returns are determined by the following

[4]The budget constraint consolidates the Treasury and central bank budget constraints since the central bank remits net earnings to the Treasury.

Euler equations:

$$\tilde{Q}_t^{(j)} = \mathbb{E}_t\left(M_{t+1}\frac{\tilde{Q}_{t+1}^{(j-1)}}{\Pi_{t+1}}\right), \tag{8}$$

$$1 = \mathbb{E}_t\left(M_{t+1}\frac{\widetilde{R}_{t+1}^{(j)}}{\Pi_{t+1}}\right). \tag{9}$$

Defining $R_{s,t} = (b_t + s_t)/b_{t-1}$ as the real return on surpluses, the government budget constraint can also be expressed as:

$$\frac{\widetilde{R}_{g,t}}{\Pi_t} = R_{s,t}. \tag{10}$$

This is the crucial identity we make use of, and tells us that analysing the real HPR on the Treasury portfolio is equivalent to analysing the return on an asset which replicates the stream of real surpluses as its payoffs. The Euler equations above imply that the following Euler equations are also satisfied:

$$1 = \mathbb{E}_t\left(M_{t+1}\frac{\widetilde{R}_{g,t+1}}{\Pi_{t+1}}\right), \tag{11}$$

$$1 = \mathbb{E}_t(M_{t+1}R_{s,t+1}). \tag{12}$$

Note that everything up to this point holds even in economies such as Brunnermeier et al. (2020) where there is a bubble in government debt. Imposing the appropriate transversality condition means that the government budget constraint can also be rearranged into the familiar FTPL equation by iterating it forward:

$$\frac{\widetilde{R}_{g,t}b_{t-1}}{\Pi_t} = \mathbb{E}_t\left[\sum_{j=0}^{\infty} M_{t,t+j}s_{t+j}\right],$$

where $M_{t,t+j} = \prod_{k=1}^{j} M_{t+k}$ is the j-period real pricing kernel, and $M_{t,t} = 1$.

The zero-beta rate, $R_{0,t}$, is determined by the following condition:

$$R_{0,t} = \frac{1}{\mathbb{E}_t M_{t+1}}. \tag{13}$$

By standard asset pricing algebra, and using (11), (12), and (13), the expected excess return — the risk

premium — on government debt is given by:

$$\mathbb{E}_t\left(\frac{\tilde{R}_{g,t+1}}{\Pi_{t+1}} - R_{0,t}\right) = -R_{0,t}\,\mathrm{Cov}_t\left(M_{t+1}, \frac{\tilde{R}_{g,t+1}}{\Pi_{t+1}}\right)$$
$$\approx -\,\mathrm{Cov}_t\left(M_{t+1}, \frac{\tilde{R}_{g,t+1}}{\Pi_{t+1}}\right). \tag{14}$$

The approximation holds exactly in continuous time, and holds accurately when the time period is short or $R_{0,t} \approx 1$. The right-hand side of (14) features the key covariance term that we are interested in measuring, since it is a sufficient statistic for measuring the riskiness of government debt. When this covariance is positive, it means that the Treasury portfolio pays off well in bad states of the world, making it a safe asset. When the covariance is negative, the Treasury portfolio pays off poorly in these bad states, making government debt risky and generating a risk premium on it.

2.2 Fiscal Consequences of Risky Debt

What are the fiscal consequences of having risky or safe debt? The evolution of the debt ratio, $d_t = b_t/Y_t$, where Y_t is real output, follows:

$$d_t = \frac{\widetilde{R}_{g,t}/\Pi_t}{Y_t/Y_{t-1}} d_{t-1} - \tilde{s}_t, \tag{15}$$

where $\tilde{s}_t = s_t/Y_t$. Note that this involves the *market* value rather than the *face* value of debt. Iterating this forward one period, taking expectations and dividing both sides by d_t yields:

$$\mathbb{E}_t\left[\frac{d_{t+1}}{d_t}\right] = \mathbb{E}_t\left[\frac{\widetilde{R}_{g,t+1}/\Pi_{t+1}}{Y_{t+1}/Y_t}\right] - \mathbb{E}_t\left[\frac{\tilde{s}_{t+1}}{d_t}\right] \tag{16}$$

This expression clearly illustrates that a higher risk premium on government debt, all else equal, causes the debt-to-GDP ratio to grow at a faster rate.

Suppose that the policy wants to stabilise the debt ratio (in expectation) at some fixed level, $d^\star$. Assuming that Y_{t+1}/Y_t is known at time t, the expected surplus ratio necessary to achieve this is given by:

$$\mathbb{E}_t\left[\tilde{s}^\star_{t+1}\right] = \frac{Y_t}{Y_{t+1}}\mathbb{E}_t\left(\frac{\widetilde{R}_{g,t+1}}{\Pi_{t+1}}\right) d_t - d^\star. \tag{17}$$

When debt is riskier (the covariance term in (14) is more negative), the above equation illustrates

that fiscal policy must run higher expected surpluses to achieve stability of the debt-ratio. This effect is strengthened when the starting debt ratio, d_t, is higher, growth is lower, or the target debt level is lower. The first two of these are relevant for many advanced economies currently, since they have experienced falling growth rates and rising debt-to-GDP ratios since the turn of the millennium. If debt has also become riskier, this suggests that the task of achieving debt stability has only grown more difficult. This is related to the $r - g$ logic of Blanchard (2019), who argues that when $r - g < 0$ the government can sustainably run deficits. His analysis abstracts from risk premia however. If the altered surplus process means that s_t is lower on average (higher deficits), but also involves running lower values of real surpluses in bad states of the world (recessions), by the risk premia effect this can raise the expected return on government debt. This in turn can increase $r-g$ and defeat the original purpose of the change in fiscal policy. We expand on this point via a simple version of the model in the appendix.

2.3 Adding More Structure

We now add more parametric structure to the model to produce some key analytical results.

We assume that the log SDF, $m_{t+1} = \log(M_{t+1})$, follows the process:

$$-m_{t+1} = \mu + \frac{\lambda^2}{2} + \lambda\varepsilon_{t+1}, \tag{18}$$

where $\mu > 0$ is the (constant) shadow log zero-beta rate ($r_{0,t}$), $\lambda < 0$ is the price of risk parameter and $\varepsilon_{t+1} \sim N(0,1)$. This can be thought of as capturing global bond arbitrageurs (e.g. hedge funds) who absorb the net supply of government debt, and are thus the marginal investor.

The government issues one- and two-period nominal, zero-coupon bonds, and so their period t budget constraint is:

$$B_{t-1}^{(1)} + \tilde{Q}_t^{(1)} B_{t-1}^{(2)} = P_t s_t + \tilde{Q}_t^{(1)} B_t^{(1)} + \tilde{Q}_t^{(2)} B_t^{(2)}. \tag{19}$$

We define $\Omega \equiv \Omega^{(2)} \equiv \frac{\tilde{Q}_{t-1}^{(2)} B_{t-1}^{(2)}}{\tilde{Q}_{t-1}^{(1)} B_{t-1}^{(1)} + \tilde{Q}_{t-1}^{(2)} B_{t-1}^{(2)}}$, which is the Treasury portfolio share of the long bond.

Monetary policy follows a Taylor rule:

$$i_t = i^\star + \rho_\pi \left(\pi_t - \pi^\star\right), \tag{20}$$

where $i_t = \widetilde{r}^{(1)}_{t+1}$ is the log nominal interest rate and $\pi_t = \log(\Pi_t)$ is log inflation. $i^\star$ is the (unconditional) mean of i_t, while $\pi^\star$ is the log inflation target rate. ρ_π is the inflation responsiveness parameter.

Fiscal policy follows a rule for real surpluses, where they respond to real debt levels in log deviation from their target level:

$$s_t = s^\star + \delta_b \left(\log\left(b_{t-1}\right) - \log\left(b^\star\right)\right) - x_t, \tag{21}$$

$$x_t = \rho x_{t-1} + \sigma \varepsilon_t. \tag{22}$$

$s^\star$ is the (unconditional) mean of real surpluses, $b^\star$ is the target level for real debt, and δ_b captures the responsiveness of real surpluses to real debt level deviations from target. x_t captures a deficit disturbance such as a positive government spending or negative tax shock. Because $\lambda < 0$, a deficit shock constitutes a high marginal utility state of the world such as a recession, war or pandemic.

2.3.1 Model Solution

We use the Campbell and Koo (1997) approximation of the log real return on surplus:

$$r_{s,t+1} \approx \kappa_0 + \kappa_1 \log\left(b_{t+1}\right) + \kappa_2 s_{t+1} - \log\left(b_t\right), \tag{23}$$

where:

$$\kappa_0 \equiv \log\left(b^\star + s^\star\right) - \frac{b^\star \log\left(b^\star\right) + s^\star}{b^\star + s^\star}, \qquad \kappa_1 \equiv \frac{b^\star}{b^\star + s^\star}, \qquad \kappa_2 \equiv \frac{1}{b^\star + s^\star}. \tag{24}$$

The second approximation we make use of, following Campbell and Viceira (2001), is for the log nominal Treasury portfolio return:

$$\begin{aligned} \widetilde{r}_{g,t+1} &= i_t + \log\left(1 + \Omega\left(\exp\left(\widetilde{r}^{(2)}_{t+1} - i_t\right) - 1\right)\right), \\ &\approx i_t + \Omega\left(\widetilde{r}^{(2)}_{t+1} - i_t\right) + \frac{1}{2}\Omega\left(1 - \Omega\right)\mathrm{Var}_t\left(\widetilde{r}^{(2)}_{t+1}\right), \end{aligned} \tag{25}$$

where $\widetilde{r}^{(2)}_{t+1} = \log(\widetilde{R}^{(2)}_{t+1})$. When $\Omega = 0$, the Treasury portfolio consists entirely of one-period nominal bonds, and so $\widetilde{r}_{g,t+1} = i_t$ in this special case.

2.3.2 Monetary and Fiscal Regimes

Definition 1: The fiscal regime is characterised by $\rho_\pi < 1$ and $\delta_b < s^\star$. The monetary regime is characterised by $\rho_\pi > 1$ and $\delta_b > s^\star$.

We do not consider any other parameter combinations for (ρ_π, δ_b) since these yield indeterminacy.

Fiscal regime. We guess that log real debt is affine in the deficit state: $\log(b_t) = A_0 + A_1 x_t$. Substituting this into the Euler equation for the log real return on surplus and matching coefficients (see Appendix A.1 for full details), the log real return on the surplus claim decomposes as:

$$r_{s,t+1} = \mathbb{E}_t r_{s,t+1} + \beta_s \varepsilon_{t+1}, \tag{26}$$

where:

$$\mathbb{E}_t r_{s,t+1} = \mu - \frac{\beta_s^2}{2} + \beta_s \lambda, \tag{27}$$

and:

$$\beta_s \equiv \sigma \frac{b^\star + s^\star - \delta_b}{(b^\star + s^\star)\left[b^\star(\rho - 1) + \delta_b - s^\star\right]}. \tag{28}$$

It holds that $\beta_s < 0$ in the active fiscal regime where $\delta_b < s^\star$. This means that the deficit shock erodes the realised return on the real surplus claim, making it a risky asset. The excess return on this asset is thus positive and given by:

$$\mathbb{E}_t r_{s,t+1} + \frac{1}{2}\operatorname{Var}_t r_{s,t+1} - r_{0,t} = \beta_s \lambda > 0. \tag{29}$$

Unexpected inflation in the fiscal regime is pinned down by the government budget identity. Expressing this in unexpected deviations:

$$\pi_{t+1} - \mathbb{E}_t \pi_{t+1} = \widetilde{r}_{g,t+1} - \mathbb{E}_t \widetilde{r}_{g,t+1} - (r_{s,t+1} - \mathbb{E}_t r_{s,t+1}). \tag{30}$$

The first term captures innovations in the nominal HPR on the Treasury portfolio; the second captures innovations in the real surplus return. Solving out the two-period bond return using the Taylor rule yields:

$$\pi_{t+1} - \mathbb{E}_t \pi_{t+1} = \underbrace{-\frac{\beta_s}{1 + \Omega \rho_\pi}}_{>0} \varepsilon_{t+1}. \tag{31}$$

This implies that unexpected inflation rises after a deficit shock in the fiscal regime. Bondholders cannot be insured fully in this regime since debt carries a negative beta with the SDF. The government can provide insurance to taxpayers, however, in the form of counteryclical deficits that are not offset in net present value terms by future surpluses.

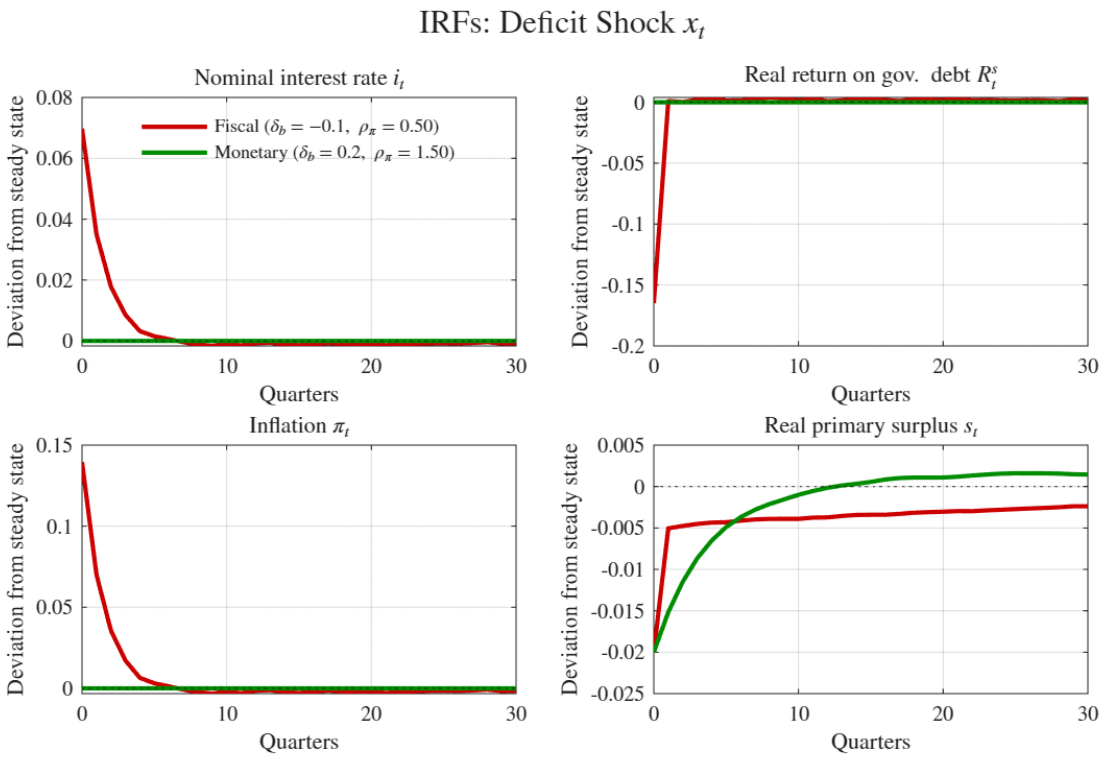
Monetary regime. Under active monetary policy with $\rho_\pi > 1$, the unique bounded solution features inflation constant at $\pi^\star$ each period. This in turn implies that the nominal interest rate and two-period bond price are also constant, so the Campbell-Viceira approximation (25) yields a constant nominal HPR on the Treasury portfolio. It follows from the government budget identity that:

$$r_{s,t+1} - \mathbb{E}_t r_{s,t+1} = \widetilde{r}_{g,t+1} - \mathbb{E}_t \widetilde{r}_{g,t+1} - (\pi_{t+1} - \mathbb{E}_t \pi_{t+1}) = 0. \tag{32}$$

Bondholders are thus fully insured in the monetary regime as government debt is kept riskless. Full derivations are provided in Appendix A.1.

Figure 1 presents the IRFs for several endogenous variables in the monetary regime (green lines) and the fiscal regime (red lines) following a deficit shock. In the monetary regime, monetary policy is sufficiently active such that the shock does not manifest itself via higher inflation. In turn, the shock does not result in a fall in the ex post real HPR on government debt, making this a safe asset in the monetary regime. This is due to the higher surpluses in the future which fully back the contemporary deficit. In the fiscal regime, the opposite is true. The government runs persistent deficits, and these are paid for by imposing real losses on bondholders in the form of elevated inflation. Accordingly, the real HPR decreases on impact of the shock. Since deficits occur in bad states of the world ($\lambda < 0$), government debt is a risky asset in the fiscal regime.

Figure 1: IRFs to Deficit Shock in Each Regime



Note: The graph plots the impulse response function to a deficit shock ε_t. The green lines depict the responses in the monetary regime, while the red lines depict the responses in the fiscal regime. We use the following quarterly calibration: $\mu = 0.005$, $\lambda = -0.25$, $s^\star = 0$, $b^\star = 1$, $\sigma = 0.02$, $\rho = 0.97$, $\Omega = 0.5$, $\pi^\star = 0$. In the monetary regime, we set $\delta_b = 0.2$, $\rho_\pi = 1.5$, while in the fiscal regime, we set $\delta_b = -0.1$, $\rho_\pi = 0.5$.

2.3.3 The Key Covariance of Interest

The following proposition is key to motivate our empirical analysis.[5]

Proposition 1. *In the fiscal regime, the covariance between the SDF and the real holding period return on the Treasury portfolio is negative, while this covariance is zero in the monetary regime:*

$$\textit{Fiscal regime}: \mathrm{Cov}_t(m_{t+1}, r_{g,t+1}) = -\beta_s \lambda < 0 \tag{33}$$

$$\textit{Monetary regime}: \mathrm{Cov}_t(m_{t+1}, r_{g,t+1}) = 0 \tag{34}$$

Proof. This follows from the above derivations. □

[5]Note that in the subsequent empirical work, we do not use the log SDF and log real HPR, and instead look at $\mathrm{Cov}_t(M_{t+1}, R_{g,t+1})$. In the model, this has the same sign as $\mathrm{Cov}_t(m_{t+1}, r_{g,t+1})$ from a property of the lognormal distribution.

This results in a risk premium on the Treasury portfolio equal to $\beta_s\lambda > 0$ in the fiscal regime and zero in the monetary regime. As such, the covariance term $\text{Cov}_t(m_{t+1}, r_{g,t+1})$ is crucial for distinguishing between risky and safe debt regimes, which correspond to periods of active fiscal and monetary policy respectively in the model.

2.3.4 Revisiting the Fiscal Consequences of Risky Debt

Section 2.2 treated output growth as known at t, which keeps the surplus-stabilisation expression exact. We now relax this and let real output growth load on the model's shock, which introduces an additional channel through which debt riskiness affects the debt ratio.[6] Suppose log growth $\Delta Y_{t+1} = \log(Y_{t+1}/Y_t)$ follows

$$\Delta Y_{t+1} = g - \phi\varepsilon_{t+1}, \qquad \phi > 0,$$

so that output falls in the bad state, consistent with governments running deficits during recessions. Evaluating the debt-ratio law of motion (16) under lognormality then yields

$$\mathbb{E}_t\left[\frac{d_{t+1}}{d_t}\right] = \exp\left(\overline{r}_g - g + \tfrac{1}{2}(\beta_s + \phi)^2\right) - \mathbb{E}_t\left[\frac{\tilde{s}_{t+1}}{d_t}\right], \tag{35}$$

where $\overline{r}_g = \mu - \frac{\beta_s^2}{2} + \beta_s\lambda$ is the expected log real HPR.

In the monetary regime $\beta_s = 0$, so the first term reduces to $\exp(\mu - g + \frac{1}{2}\phi^2)$ and the standard $r - g$ logic holds, with $r = \mu$ equal to the zero-beta rate. In the fiscal regime $\beta_s < 0$, and substituting for $\overline{r}_g$ the $\frac{1}{2}\beta_s^2$ terms cancel, leaving the first term as

$$\exp\Big(\mu - g + \tfrac{1}{2}\phi^2 + \beta_s(\lambda + \phi)\Big). \tag{36}$$

Relative to the monetary benchmark, the fiscal regime therefore scales the growth rate of the debt ratio by $\exp\big(\beta_s(\lambda + \phi)\big)$, which combines two channels.

The first is the risk-premium channel, $\exp(\beta_s\lambda)$. With $\lambda < 0$ and $\beta_s < 0$ we have $\beta_s\lambda > 0$, so the

[6] The two treatments are complementary: deterministic growth isolates the surplus-stabilisation arithmetic, while stochastic growth isolates the interaction between debt returns and the cycle that we study here.

expected return $\overline{r}_g$ is higher to compensate investors for holding risky debt, raising the growth rate of the debt ratio. This is the fiscal cost of risky debt emphasised in Section 2.2.

The second is a Jensen channel, $\exp(\beta_s\phi)$, which arises because the same shock drives both the debt return and growth. The relevant object is the growth-adjusted return $r_{g,t+1} - \Delta Y_{t+1}$, whose innovation loads $(\beta_s + \phi)\varepsilon_{t+1}$, so its conditional variance is $(\beta_s + \phi)^2$. When $\beta_s < 0$ and $\phi > 0$ the two loadings partially offset, making this variance *smaller* than $\beta_s^2 + \phi^2$, and the lower variance reduces $\mathbb{E}_t[d_{t+1}/d_t]$. In other words, because debt returns and growth move together, the growth-adjusted return is less volatile than its components, and this dampens expected debt-ratio growth.

The two channels work in opposite directions, so the net effect is destabilising precisely when $\beta_s(\lambda+\phi) > 0$. Since $\beta_s < 0$, this holds if and only if

$$\phi < |\lambda|, \tag{37}$$

that is, when the output loading is smaller than the absolute price of risk. Risky debt thus raises the growth rate of the debt ratio, reinforcing the difficulty of stabilisation discussed in Section 2.2, unless growth is sufficiently strongly co-driven with debt returns that the Jensen channel dominates the risk premium.

2.4 Model Extensions and Discussion

The model is stylised, but provides useful clarity by illustrating how the key covariance of interest can sharply identify different monetary-fiscal regimes. We consider a number of extensions in the appendix, including the robustness of the regime ordering to a priced monetary shock, the incorporation of convenience yields, inflation-linked debt, and a time-varying price of risk. We summarise these below.

Adding a monetary shock. A natural concern is whether the covariance remains informative about the regime once the SDF prices more than the deficit shock. We address this by adding a priced monetary policy shock to the Taylor rule, with a non-positive price of risk since a contractionary surprise raises the nominal rate and arrives in a high marginal utility state. We show in Appendix A that the regime ordering is not only preserved but strengthened: the covariance remains negative in the

fiscal regime and becomes non-negative in the monetary regime, so government debt is now safe rather than merely riskless there. The intuition is that the monetary shock is priced into the covariance only in the monetary regime. In the fiscal regime it has purely nominal effects, since the real HPR is pinned down by the present value of real surpluses and unexpected inflation absorbs the shock one-for-one, leaving the real return unaffected. In the monetary regime, a contractionary surprise disinflates and raises the real return on the long bond, so debt hedges the shock and the monetary channel reinforces rather than competes with the fiscal one.

Convenience yields. Several papers find that US government debt is overpriced, earning a convenience yield stemming from its liquidity services or pledgeability as collateral (Krishnamurthy and Vissing-Jorgensen, 2012; Greenwood et al., 2015; Nagel, 2016). Our equity-based SDF nonetheless prices the real HPR on US government debt in-sample, which we read as evidence against convenience yields playing a large role for the real HPR.[7] We nonetheless consider an extension in which the Euler equation becomes $1 = \mathbb{E}_t M_{t+1} R_{g,t+1} \exp(\psi_t)$, where the convenience term

$$\psi_t = \psi_0 - \psi_1[\log(b_t) - \log(b^\star)], \qquad \psi_0, \psi_1 \geq 0, \tag{38}$$

falls in the supply of debt, consistent with Krishnamurthy and Vissing-Jorgensen (2012) and a downward-sloping Treasury demand curve. In Appendix A we show that the key covariance continues to differentiate the regimes. Debt remains risky in the fiscal regime, though less so when ψ_1 is larger, since a more supply-sensitive convenience yield absorbs more of the deficit shock and leaves the surplus claim less exposed to it. In a quantitative calibration this reduction is modest.

Inflation-linked debt. We extend the model to allow the government to issue real as well as nominal debt. The US introduced this via Treasury Inflation-Protected Securities (TIPS) in 1997, and these now constitute around 7% of total marketable debt. The UK introduced index-linked gilts (linkers) in 1981, and these now make up around 25% of UK debt.

[7] One could conceive of a convenience yield that affects bond pricing but not the HPR, for example if it is multiplicative and constant across maturities and over time.

We assume that there is a constant share $\eta \in (0, 1]$ of nominal debt such that:

$$b_t = b_t^n + b_t^r \tag{39}$$

with $b_t^n = \eta b_t$, $b_t^r = (1 - \eta) b_t$, where b_t^n denotes the real market value of nominal debt and b_t^r denotes the real market value of inflation-linked debt. In Appendix A we show that the presence of real debt does not change the key covariance term in either the monetary or the fiscal regime. The riskiness of the public portfolio is thus independent of the composition between real and nominal debt. It does, however, influence the dynamics of inflation in the fiscal regime — a higher fraction of inflation-linked debt causes inflation to respond more to a deficit shock (Kawalec, 2025). With a smaller stock of nominal liabilities, a given fiscal shock requires a larger movement in the price level to reconcile the real value of debt with the present value of future primary surpluses.

Time-varying price of risk. Our benchmark specification assumes a constant price of risk λ. This is at odds with both the empirical asset pricing literature, which finds substantial time-variation in risk premia (Cochrane and Piazzesi, 2005; Cochrane, 2011), and with our own empirical approach, which uses instruments to capture conditional variation in the SDF. We therefore consider an extension where the price of risk varies with the state:

$$\lambda_t = \lambda_0 + \lambda_1 x_t, \tag{40}$$

with $\lambda_0 < 0$ and $\lambda_1 \leq 0$, so that bond investors demand a higher price of risk in bad states of the world. This is consistent with intermediary asset pricing models where effective risk aversion rises when balance sheets are stressed (He and Krishnamurthy, 2013). In Appendix A, we show that Proposition 1 continues to hold in terms of the sign of the covariance, so the regime partition of the model is preserved. In the fiscal regime, the price of risk now enters the solution for log real debt, and greater state-dependence in the price of risk makes β_s more negative, amplifying the inflation response to a deficit shock. Moreover, since

$$\mathrm{Cov}_t(m_{t+1}, r_{g,t+1}) = -\lambda_t \beta_s, \tag{41}$$

the conditional covariance becomes more negative when the deficit state deteriorates. This generates within-regime variation in debt risk. Finally, greater state-dependence in the price of risk tightens the parameter space over which the fiscal regime admits a bounded affine solution, providing a stylised

notion of fiscal fragility.

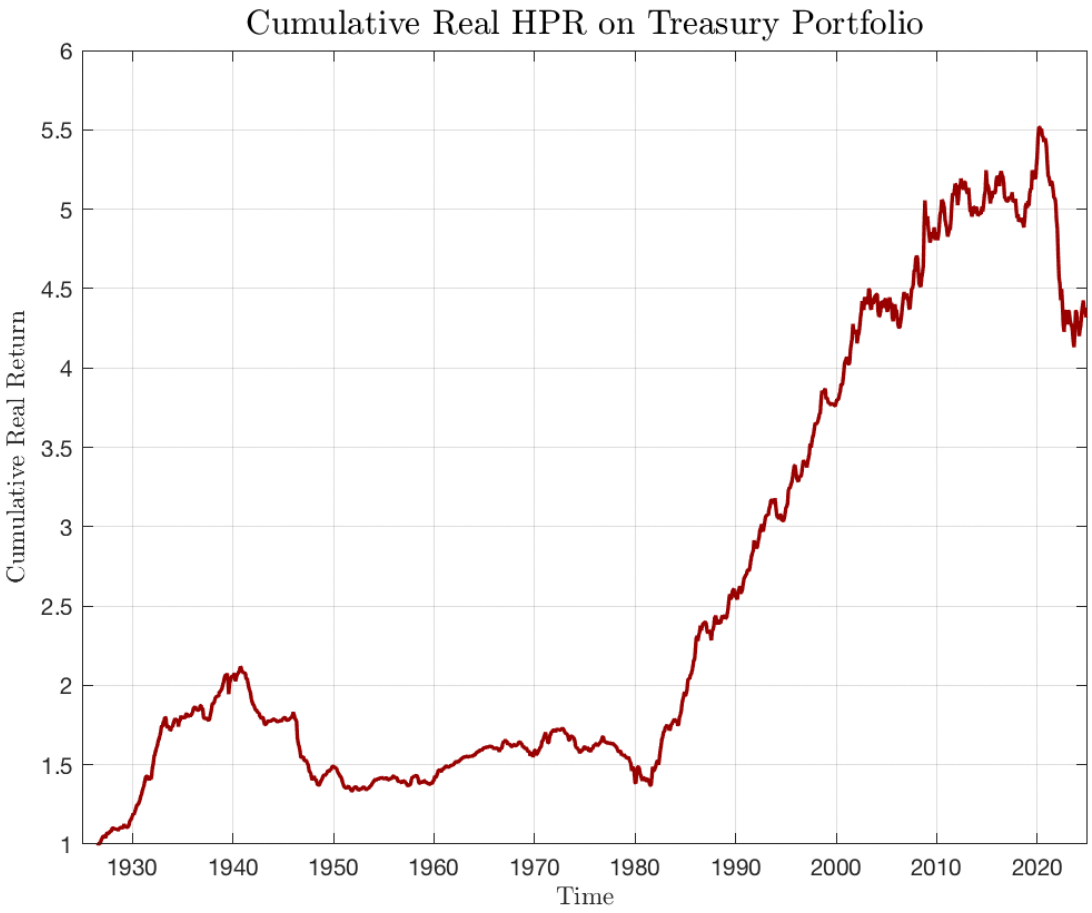
3 Empirical Methodology

Motivated by the model, we now attempt to measure the key covariance term empirically. For this, we need a measure of the real holding period return on government debt and a measure of the SDF. Starting with the first of these, we use the holding period return on government debt held by private investors constructed by Hall and Sargent (2011), extended to end in December 2024 by Hall and Sargent (2025), and posted on George Hall's website.[8] As Hall and Sargent (2025) describe, this measure, consistent with our model, nets out the holdings of government debt by the Federal Reserve, Government Agencies and Trust Funds. Crucially, the measure uses the market value of government debt obtained via the use of promised payment streams and bond price data. Finally, further adjustments are made to account for structural changes to the Federal Reserve's balance sheet from 2008 onwards. See Hall and Sargent (2025) for full details. To convert the nominal HPR into the real HPR, we use the rate of CPI inflation.

Figure 2 plots the cumulative real holding period return on the Treasury portfolio, $\widetilde{R}_{g,t}/\Pi_t$, over our full baseline sample period of July 1926 - December 2024. This illustrates four distinct periods of differing fortunes for bondholders. Firstly, bondholders actually did reasonably well over the period of 1926-1939 despite the 1929 Wall Street crash and subsequent Great Depression, in large part due to the deflation and flight to safety which occurred. Secondly, bondholders received negative real returns between 1939-1980. In the first part of this period, this was due to the high deficit spending and inflation which financed the US response to WWII, see Hall and Sargent (2011). In the second part of this period there was also high inflation which was met by high interest rates, both of which were deleterious for bondholders. Thirdly, bondholders received high real returns from 1980-2020. This bull market for fixed income stemmed from the combination of a secular decline in interest rates and the reduced rate of inflation which occurred in the Great Moderation and Great Financial Crisis periods. Fourth and finally, bondholders suffered large real losses from 2020-2024 as there were high fiscal deficits, interest rates rose and inflation accelerated significantly after the Covid-19 pandemic. Hall and Sargent (2025) liken this episode and the associated response to a third war accordingly.

[8] See https://sites.google.com/brandeis.edu/george-j-hall/research.

Figure 2: Cumulative Real HPR on the Treasury Portfolio, July 1926 - December 2024



Over the whole period, the average monthly real annualised HPR was 1.53%, with a standard deviation of 3.56% (again annualised). The monthly real one-month Treasury rate over this period was 0.29%, giving the real HPR a Sharpe ratio of 0.35. This is only slightly lower than the Sharpe ratio for the US stock market over the same period, giving an indication that the government debt portfolio is not lacking in volatility.

The second object we need is an estimate of the SDF. One approach would be to pick a structural asset pricing model, such as the consumption CAPM or the intermediary-based asset pricing model of He and Krishnamurthy (2013), and then use this to construct the SDF. However, we deliberately utilise a method of estimating the SDF which allows us to remain agnostic about the specific structural model, given the lack of consensus in the literature on this question. Instead, our approach is based on the assumption that there exists an SDF such that the following moment condition holds for a set of real

gross returns $i = 1, ..., N$:

$$1 = \mathbb{E}_t\left[M_{t+1}R_{i,t+1}\right] \qquad \forall i \in \mathcal{I}, \tag{42}$$

where $\mathcal{I}$ denotes a potentially exhaustive set assets. This is the fundamental asset pricing equation. The existence of such an SDF is guaranteed under no arbitrage (Harrison and Kreps, 1979; Kreps, 1981a). In our approach we do not assume that government bonds belong in the set $\mathcal{I}$, implying that we allow for convenience yields in these assets. For our fully nonparametric approach, this is the only equilibrium condition used. In our parametric approach, we place slightly more structure on the process which characterises the SDF.

3.1 Parametric Approach

Our primary approach to estimating the SDF is parametric, since we impose a degree of structure on its functional form.

We first make the assumption, following Di Tella et al. (2025), that the zero-beta rate is an affine function of a vector of de-meaned instruments, Z_t which is $L \times 1$:

$$R_{0,t} = \gamma_0 + \sum_{l=1}^{L} \gamma_l Z_{l,t}, \tag{43}$$

where the first element of Z_t is $R_{f,t} - 1$, meaning we nest the common case where the risk-free Treasury rate coincides with the zero-beta rate.[9] Importantly, we therefore allow for a time-varying zero-beta rate in contrast to many methods which assume it is constant or equal to a short-term Treasury yield, e.g. the well-known Fama and MacBeth (1973) procedure. While we are not directly interested in the zero-beta rate, as will become clear, it influences our estimate of the SDF, and so accounting for volatility remains important. Di Tella et al. (2025) provide strong evidence that the zero-beta rate i) does not coincide with the Treasury bill rate; ii) exhibits substantial volatility.

In our parametric approach, we assume that the SDF is described by:

$$M_{t+1} = R_{0,t}^{-1} + \sum_{j=1}^{K} \delta_j \left(F_{j,t+1} - \mathbb{E}_t F_{j,t+1}\right), \tag{44}$$

[9] This occurs when $\gamma_0 = 1, \gamma_1 = 1, \gamma_l = 0 \; \forall \; l \geq 2$.

where F_j is a specific factor j. We thus assume that the SDF is given by a linear factor model. This nests, or approximately nests, many of the leading asset pricing models. For example, the CAPM can be written as: $M_{t+1} = a + bR^M_{t+1}$, where R^M_{t+1} is the market return. The consumption CAPM and rare disasters model of Barro (2006) can be linearised as: $M_{t+1} \approx 1 - \tilde{\gamma}\Delta C_{t+1}$, where $\tilde{\gamma}$ is the coefficient of relative risk aversion, and $\Delta C_{t+1} = \frac{C_{t+1}-C_t}{C_t}$ is aggregate consumption growth.

Rather than tying ourselves to the mast of a specific structural asset pricing model, we instead assume in our benchmark specification that F_t consists of the three Fama and French (1993) factors: the market return, the size factor (SMB), and the value factor (HML). We use the excess return of the market over the zero-beta rate, $R_{M,t} - R_{0,t}$, as the first factor rather than the excess return over the 1-month Treasury bill. The logic underpinning this choice of factors is that they have repeatedly been demonstrated to have large explanatory power for the cross-section of returns, and can be interpreted as proxies for the underlying risk factors in structural models. While additional factors have also been found to explain the cross-section of returns — see the literature on the 'factor zoo' (Harvey et al., 2016; Feng et al., 2020) — we choose these three factors for two main reasons. The first is that they are the most well-studied in the empirical literature. The second, more practical in nature, is that data on these three factors goes back to 1926, which far precedes many other proposed factors.[10] We consider several robustness tests where we vary the set of factors in the appendix.

We next discuss our choice of instruments in the vector Z_t. As previously stipulated, the first is the one-month Treasury yield adjusted for CPI inflation. The choice of the remaining instruments largely follows Di Tella et al. (2025) with some considerations made for the availability of the data.[11] Our second instrument is the 12-month backwards-looking moving average of CPI inflation. This is motivated by the findings of Cohen et al. (2005), who document that the price of risk systematically differs between high- and low-inflation states of the world. The third instrument is the term spread, defined as the difference between the yield of a 10-year and 3-month Treasury. This has often been found to forecast recessions and we include it for that reason. Our fourth instrument is the inverse of the Shiller CAPE. A large literature documents that this inverse valuation ratio forecasts future excess returns, suggesting it

[10] For example, data for the profitability and investment factors additionally proposed by Fama and French (2015) only begins in 1963.

[11] The instruments that Di Tella et al. (2025) use (in addition to the 1-month Treasury yield) are as follows: i) the 12-month backwards-looking moving average of inflation, ii) the term spread, iii) the U3 unemployment rate, iv) the excess bond premium of Gilchrist and Zakrajšek (2012). These last two instruments are not available until 1948 and 1973 respectively, hence the reason we deviate from this instrument set.

is associated with the price or quantity of risk. Our fifth and final instrument is the bond spread, defined as the yield differential between BAA corporate bonds and Treasuries. Credit spreads have been found to forecast future economic activity and excess returns on risky assets (Gilchrist and Zakrajšek, 2012), making them relevant for capturing time variation in risk premia. We consider multiple robustness tests on the instrument set in the appendix, usually coming at the cost of truncating the sample period under consideration.

The last choice we need to make is the cross-section of returns used to recover the SDF. Again, we face a tradeoff between the richness of the portfolio set and data availability. Our benchmark cross-section consists of the 25 Fama–French portfolios formed on size and book-to-market, augmented with the 30 industry portfolios, which are available from 1926–2024. This follows Lewellen et al. (2010), who argue that the strong factor structure in the size-value Fama–French portfolios can overstate model performance. They recommend including a broader set of portfolios, such as those sorted by industry, to mitigate this bias. We also include the CRSP value-weighted market return and the one-month Treasury yield. While in principle we could include the entire maturity spectrum of zero-coupon government bond yields, doing so would implicitly assume that all maturities are priced solely by the SDF without any segmentation or preferred-habitat effects. While this would be consistent with our model, the empirical asset pricing literature suggests that bonds of different maturities may carry distinct convenience yields or liquidity premia. To avoid imposing this strong restriction, we include only the one-month Treasury yield, which serves as a proxy for the short-term risk-free rate and anchors the SDF estimation. We stress that this does not mean we assume that the one-month Treasury yield is the zero-beta rate. In the appendix, we consider a robustness test where we only use the cross-section of equities, as well as numerous other robustness tests where we vary the cross-section more substantively.

In Appendix B we describe the algorithm used to estimate the SDF fully. The method builds upon the process developed by Di Tella et al. (2025), jointly estimating both the zero-beta rate and the SDF together. Their original method only recovers the former.

3.2 Nonparametric Approach

Our secondary, nonparametric approach is much simpler and exclusively utilises the fundamental asset pricing equation (42) as the only moment condition. We search for an M_{t+1} such that the following sample analogue holds for all $i = 1, ..., N$:

$$\frac{1}{T}\sum_{t=0}^{T-1} M_{t+1}R_{i,t+1} = 1 \tag{45}$$

This yields an infinite set of M_{t+1} if $T > N$ since the system is underidentified. We thus need to impose an additional criterion in order to uniquely identify the SDF. We choose to search for the minimum variance SDF in the set of M_{t+1} which satisfy (45), corresponding to the Hansen-Jagannathan projection of the SDF onto the payoff space spanned by returns. In other words, it is the most parsimonious, valid SDF in the sense that it exactly achieves the Hansen-Jagannathan bound. Formally, we solve the linear-quadratic optimisation problem:

$$\min_{M_{t+1}>0} \text{Var}(M_{t+1}) \;\; s.t. \;\; 1 = \frac{1}{T}\sum_{t=0}^{T-1} M_{t+1}R_{i,t+1} \qquad \forall i = 1, ..., N \tag{46}$$

where we impose strict positivity of the SDF also. This approach is fully nonparametric since no functional form restrictions are placed on M_{t+1}. In our benchmark nonparametric approach, we do not incorporate conditioning information in the form of instruments for the purpose of maximum parsimony, but we examine this extension in the appendix. We refer to the estimated SDF obtained from this procedure as M_{t+1}^{NP} from here on.

3.3 Discussion of the Two Approaches

Each approach has distinct merits and limitations. The parametric method imposes structure on the SDF via a linear factor model, using well-established factors that explain the cross-section of returns. This structure improves interpretability and, under a correct specification, delivers efficiency by lowering the asymptotic variance of the estimator. Its main drawback is the risk of model misspecification; omitting relevant risk factors or instruments can introduce bias. Conversely, the nonparametric approach imposes no functional form on the SDF, making it highly flexible and robust to misspecification. It

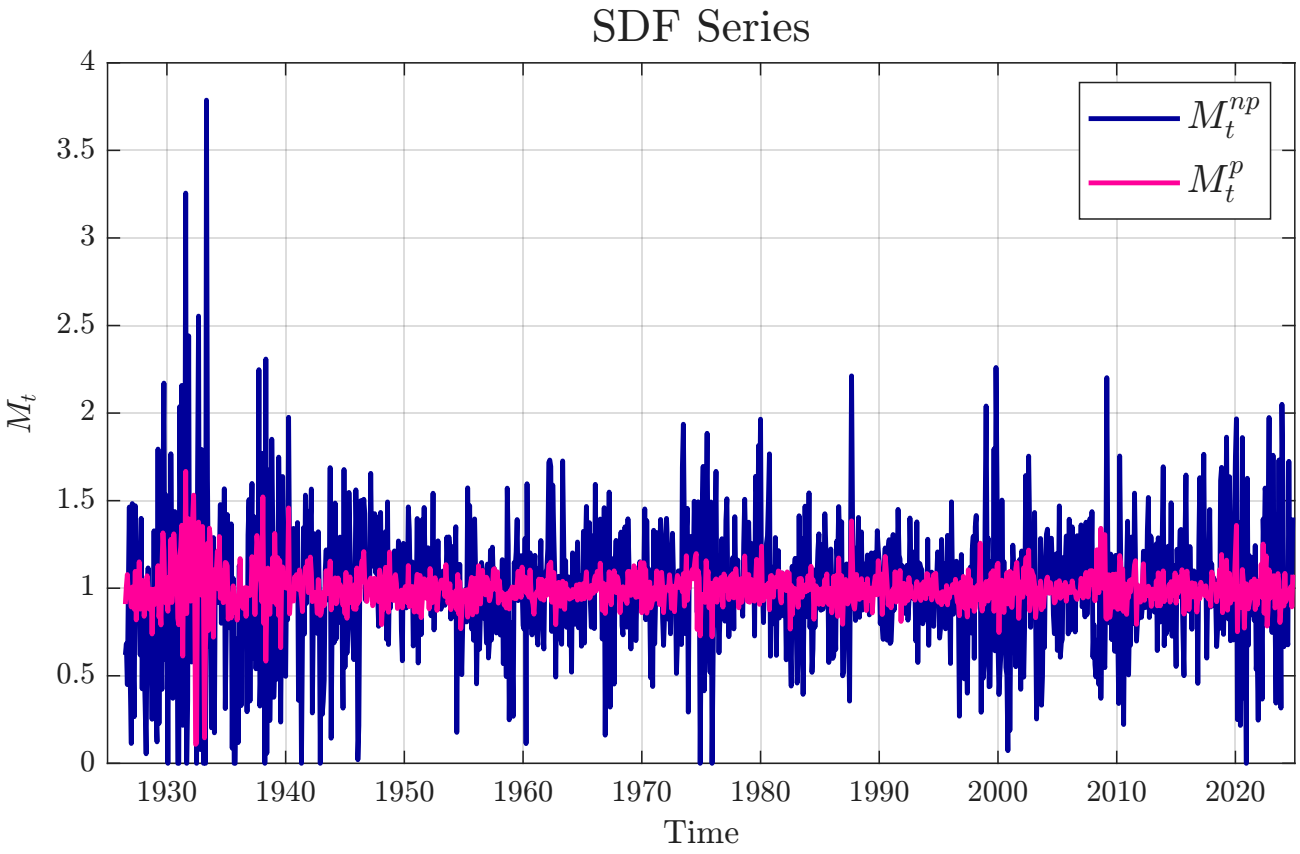
exactly prices the test assets, but sacrifices efficiency because the number of unknowns grows with the sample size. When $T > N$, the system is underidentified, and the solution, though variance-minimising, can be highly sample-specific and sensitive to noise. This can be thought of as a form of overfitting. In the appendix, we also consider an alternative semiparametric approach, which follows the nonparametric approach above, except that the SDF is approximated as a linear combination of basis functions of the factors and instruments. This sieve approximation greatly reduces the dimensionality of the problem but, as in the main parametric approach, requires the explicit choice of these variables.

4 Main US Results

We begin by describing the estimated SDF series we recover from each of the two approaches. A full set of parameter estimates for the parametric approach can be found in Table A1. The time series of these is plotted in Figure 3, and clearly illustrates the fact, highlighted previously, that the nonparametric approach produces a noisier series compared to the parametric procedure. Despite the significant differences in the two approaches, the two SDFs have a correlation coefficient of 0.34, and so are moderately correlated. Both pick up significant adverse macrofinance events, including the Wall Street crash of 1929, the outbreak of WWII in 1939, Black Monday in 1987, the Great Financial Crisis in 2008, and the Covid-19 pandemic in 2020. Table 1 presents a set of summary statistics for the real HPR and the pair of estimated SDFs. The plot of the estimated zero-beta rate from the parametric procedure can be found in the appendix.

The key covariance term in (14), evaluated unconditionally over the entire sample period, is negative for both SDFs, implying that, on average, US government debt has been a risky asset. Annualised, this risk premium component is estimated to add 12 bps to the real return on government debt when using the parametric SDF, and 32 bps when using the nonparametric SDF. As a separate test, we evaluate this covariance for the stock market return, which is an untargeted moment. We find that the parametric SDF produces an annualised equity premium of 6.81%, close to most empirical estimates of this. The nonparametric SDF yields an annualised equity premium of 8.21%, somewhat of an overestimate. We also consider the extent of mispricing of the HPR on US government debt, by testing the sample analogue of the Euler equation in (11), i.e. we test the null hypothesis that the average product of the SDF with the real HPR is equal to 1. We cannot reject this null for the parametric SDF ($p = 0.20$), or for the

Figure 3: Estimated SDFs



Note: The dark blue line plots the estimated SDF from the nonparametric approach, while the pink line plots the estimated SDF from the parametric approach.

nonparametric SDF ($p = 0.94$). This is effectively a test of how segmented the bond and equity markets are, given that we use the latter to recover the SDF.[12] If the two were segmented, for example due to a large convenience yield on US government bonds, we would not expect the SDF which prices stocks to also price bonds (and hence the HPR). This is not what we find, consistent with a small or negligible convenience yield on average over our sample period. This does not preclude the possibility that the convenience yield varies over time but is equal to zero on average.[13]

We now focus on evaluating how the key covariance term in (14) has evolved over time, with the idea of identifying distinct safe and risky debt regimes. To do so, we plot the rolling, backward-looking covariance between the SDF and the real HPR on government debt. We choose three years as the

[12]Rather than any market microstructure definition, we term the markets for asset i and asset j segmented if $1 = \mathbb{E}_t M^i_{t+1} R_{i,t+1}$ and $1 = \mathbb{E}_t M^j_{t+1} R_{j,t+1}$ but $M^i_{t+1} \neq M^j_{t+1}$.

[13]We include the one-month Treasury yield in our baseline cross-section of returns, but even when we omit this and focus purely on equities, we still find that we cannot reject the null that $1 = \mathbb{E}(M_{t+1} R_{g,t+1})$

Table 1: Summary Statistics

	Real HPR	SDF (Parametric)	SDF (Nonparametric)
Mean	0.13%	0.995	0.999
Standard Deviation	1.03%	0.111	0.392
Skewness	-0.02	-0.41	0.52
AR(1)	0.25	0.09	-0.01
Corr. w/ Parametric SDF	-0.09	1.00	0.34
Corr. w/ Nonparametric SDF	-0.07	0.34	1.00

Note: This table presents summary statistics for the real (non-annualised) HPR on US government debt, the parametric SDF, and the nonparametric SDF. AR(1) refers to the coefficient on the lag term in an AR(1) OLS regression. The sample period is monthly and spans July 1926 - December 2024.

window length for this, but also experiment with other values for this, as well as more data-driven procedures, in the appendix. This method also has the benefit of being nonparametric in nature, but in the appendix we also discuss various alternative parametric procedures.

Figure 4 plots this rolling covariance for the parametric SDF, while Figure 5 plots this for the nonparametric SDF. Both figures tell largely the same story qualitatively, identifying four clear fiscal regimes.

- **1926–1965: Debt was risky before the 1951 Treasury-Fed Accord and safe afterwards.** The evidence for this sub-period is less decisive than for subsequent regimes, reflecting both the volatility of the estimated covariance and the multiple structural changes that occurred. In the 1926–1951 period, the rolling covariance typically features negative point estimates, albeit with wide confidence intervals. This is consistent with the fiscal regime conditions being satisfied for much of this period. At this time, the Fed's monetary policy role was subordinated to Treasury debt management. Its task was to help the Treasury finance wartime and postwar debt cheaply by supporting the market for government securities and maintaining low — sometimes explicitly pegged — yields through open-market purchases, see Garbade (2020) for example. Jacobson et al. (2024) argue that the US departure from the gold standard in 1933 was itself consistent with a move towards a fiscal regime, since it enabled unbacked fiscal expansions. The period from around 1950–1965 is identified as a safe debt period despite its poor real returns. The Treasury-Fed Accord of 1951, which formally separated government debt management from monetary policy, is a plausible source of this regime shift, representing an institutional move towards $\rho_\pi > 1$. Fiscal policy was also consistent with the monetary regime: as Hall and Sargent (2022) outline, the

Korean War was financed with a balanced budget, driven in part by President Truman's own experience as a Liberty bondholder during WWI.

- **1965–1985: Debt was risky.** The covariance turns decisively negative in the early-to-mid 1960s and remains so until the early 1980s. In terms of our model, both conditions for the fiscal regime appear to have been satisfied. On the fiscal side, the high, persistent budget deficits associated with President Johnson's Great Society programmes and the Vietnam War are consistent with a low δ_b. As discussed by Bianchi et al. (2023), these were largely unfunded in the sense that they were not accompanied by expectations of future offsetting surpluses. On the monetary side, there was substantial political interference in the Fed, as documented by Drechsel (2024), which may have contributed to monetary policy being insufficiently responsive to inflation, consistent with $\rho_\pi < 1$. This began with Johnson's pressure on Fed Chair William McChesney Martin and continued throughout the Nixon administration.[14] Bianchi and Ilut (2017) also identify this period as an active fiscal regime using a medium-scale estimated DSGE model, lending support to our finding via a different methodology.

- **1985–2020: Debt was safe.** The covariance is approximately zero throughout this period, consistent with the monetary regime. On the monetary side, the Volcker disinflation marked a decisive shift towards active monetary policy ($\rho_\pi > 1$), which prior work has partially attributed to the reduction in macroeconomic volatility during the Great Moderation (e.g. Clarida et al. (2000), Lubik and Schorfheide (2004), Beaudry et al. (2023)). On the fiscal side, the deficit reduction of the late 1980s and the Clinton-era surpluses of the late 1990s are consistent with a high δ_b, with the surplus process responding actively to debt levels. The shift towards debt being riskless, interpreted through the lens of our model, supports this characterisation. Notably, debt remained safe through several large fiscal shocks during this period, including the deficits associated with the Bush tax cuts, the wars in Iraq and Afghanistan, and the fiscal response to the global financial crisis which approximately doubled the debt-to-GDP ratio. The persistence of the safe debt regime despite these shocks suggests that what mattered was not the level of debt per se, but rather the credibility of the monetary and fiscal policy frameworks, consistent with

[14] Fessenden (2016) argues that 1965 represents a watershed moment in relations between the President and the Fed. Two instances from LBJ's presidency stand out. First, in 1965 he invited Martin to his ranch and berated him over his decision to raise interest rates, even allegedly going so far as to physically assault him. This came in the wake of the 1964 tax cuts which Martin viewed as being a significant cause of inflationary pressure. Second, in 1967 Johnson and Martin came to an agreement whereby the Fed would lower interest rates and Johnson would, in return, accompany this with a tax increase. Martin would later admit to regretting this decision when he testified before Congress two years later.

the model's emphasis on the policy rules (ρ_π, δ_b) rather than the state variables.

- **2020–2024: Debt may have become risky once again.** The covariance turns sharply negative from 2020 onwards, with the parametric SDF producing the most negative point estimate over the entire sample period in 2024. The large fiscal transfers associated with the pandemic response (the CARES Act of 2020 and the American Rescue Plan of 2021) constitute precisely the type of deficit shock in a high marginal utility state that generates risky debt in our model. Crucially, and in contrast to the GFC episode, these transfers were not accompanied by comparable signals of future fiscal consolidation, consistent with a decline in δ_b. The subsequent inflation surge is consistent with the fiscal regime mechanism, where deficit shocks that are not backed by future surpluses manifest as higher inflation, eroding the real HPR on government debt. The quantitative magnitude is substantial: the point estimate implies that the risk premium component is currently adding around 100 bps to the annualised real HPR on government debt (95% CI: 57bps – 193bps). This represents a material increase in the government's funding cost which appears in the intertemporal budget constraint, with the fiscal consequences described in Section 2.2. We stress that previous regimes typically last multiple decades, and so it may be premature to draw firm conclusions given the lack of data in the post-pandemic period.

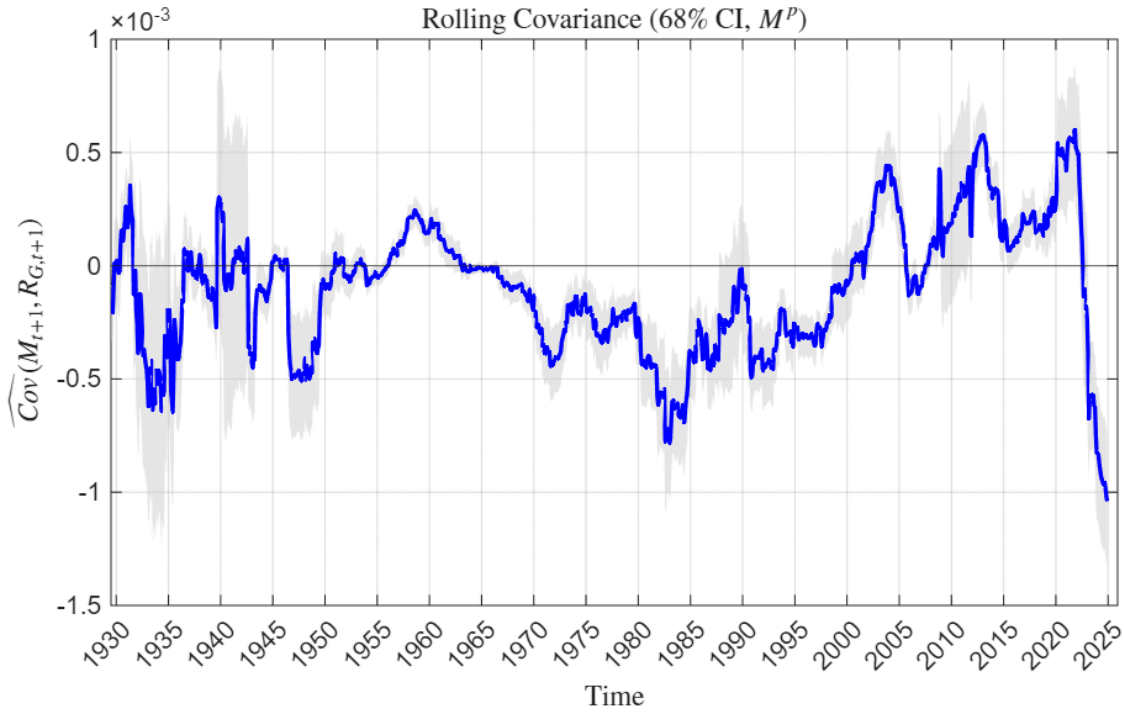
4.1 Robustness

Both the parametric and nonparametric procedures require us to make a set of meaningful choices, and as such, we consider a number of alternative choices to gauge the robustness of our results. For the parametric procedure, these include variations of:

1. *Cross-section of returns.* We look at richer cross-sections of returns used in the GMM procedure, which typically comes at the cost of a shorter sample length. We also consider broadening the cross-section beyond equities and including a set of government bond portfolios.

2. *Factors.* We consider the addition of other asset pricing factors in the SDF, for example the betting-against-beta factor of Frazzini and Pedersen (2014) and the additional two factors which comprise the five factor Fama and French (2015) model. We also include real consumption growth as a factor in the spirit of the CCAPM.

Figure 4: Rolling covariance between parametric SDF and real HPR on government debt.



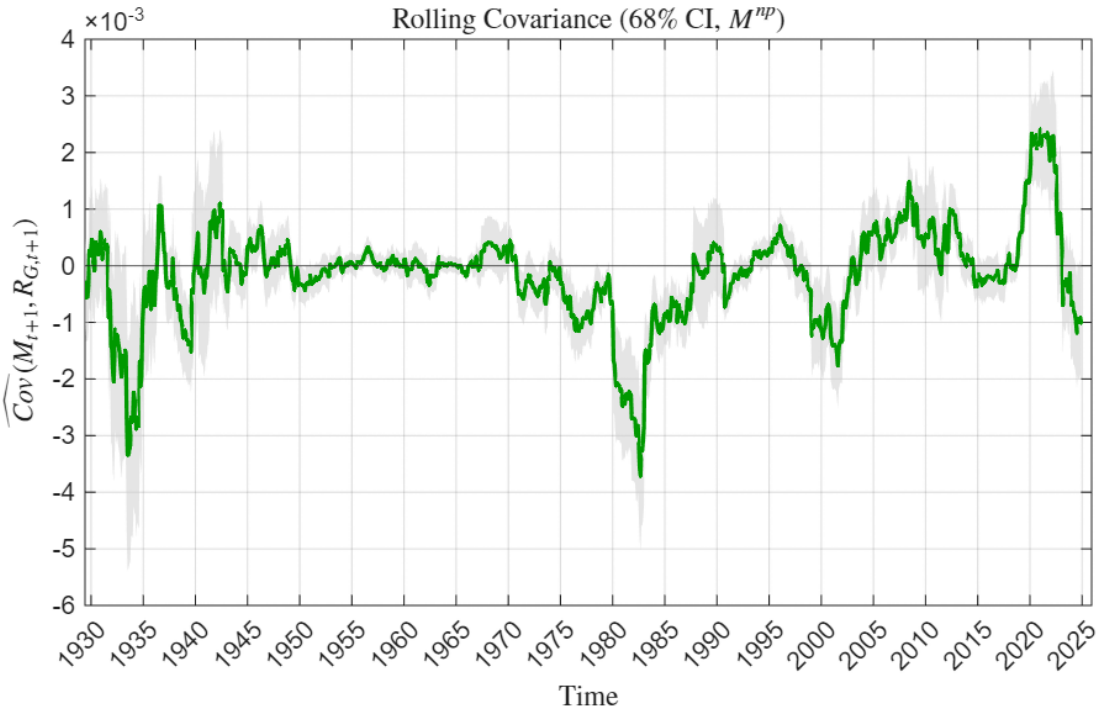
Note: This plots the three-year backward-looking covariance between the parametric SDF and the real HPR on government debt. The grey shaded area plots 68% confidence intervals.

3. *Instruments.* We consider varying the set of instruments by including additional variables which have been shown to forecast returns such as the cyclical consumption measure of Atanasov et al. (2020).

4. *Method.* We consider variations of parts of the parametric approach. For example, we look at using one-step rather than two-step GMM, and allowing for the risk prices (δ) to depend on the instruments.

For the nonparametric approach, we also consider the use of an alternative cross-section of returns, and the use of conditioning information. We also consider different windows for the rolling covariance, as well as alternative methods of detecting the safe and risky debt regimes. All of these robustness tests can be found in Appendix C. By and large, we find our key results to be robust to this myriad of variations.

Figure 5: Rolling covariance between nonparametric SDF and real HPR on government debt.



Note: This plots the three-year backward-looking covariance between the nonparametric SDF and the real HPR on government debt. The grey shaded area plots 68% confidence intervals.

5 Application to the UK

We also apply our approach to the UK, which, like the US, has rich, historical gilt-level data that allows us to compute the HPR on its government debt. To do so, we build on Ellison and Scott (2020) and develop a monthly series for the HPR on debt held by private investors, stripping out holdings of the Bank of England in its Asset Purchase Facility. Full details of our construction of this series can be found in Appendix F.[15]

To risk-adjust the real holding-period return on UK government debt, a measure of the sterling real SDF is again required. In principle, one could estimate this object from the cross-section of UK asset returns, following the same approach used for the US. In practice, this is difficult because data on the cross-section of UK returns is extremely limited, with no extant dataset spanning a sufficiently long sample. We therefore instead infer a sterling real pricing kernel from the previously estimated US real pricing kernel and the real exchange rate, following the international asset-pricing logic in Backus et al.

[15]We make this series available at: https://www.utsopm.com/data.

(2001).

Let $M_{t+1}^{US,n}$ denote a nominal dollar pricing kernel, let $M_{t+1}^{UK,n}$ denote a nominal sterling pricing kernel, and let $\mathcal{E}_t$ denote the nominal GBP/USD exchange rate, measured as pounds per dollar. No-arbitrage in currency markets implies that for any valid US nominal pricing kernel $M_{t+1}^{US,n}$, the object

$$M_{t+1}^{UK,n} = M_{t+1}^{US,n} \frac{\mathcal{E}_t}{\mathcal{E}_{t+1}} \tag{47}$$

is a valid nominal pricing kernel for assets traded in both markets. To express this relation in real terms, note that nominal and real pricing kernels are linked by

$$M_{t+1}^{i} = M_{t+1}^{i,n} \Pi_{t+1}^{i}, \qquad i \in \{US, UK\}, \tag{48}$$

where Π_{t+1}^{i} denotes gross CPI inflation in country i. It follows that the sterling real SDF satisfies

$$M_{t+1}^{UK} = M_{t+1}^{US} \frac{\mathcal{E}_t}{\mathcal{E}_{t+1}} \frac{\Pi_{t+1}^{UK}}{\Pi_{t+1}^{US}}. \tag{49}$$

The intuition is straightforward. The price of any payoff denominated in dollars must equal the price of the same payoff translated into sterling. Hence, once prices are expressed in a common currency, a valid US pricing kernel can be converted into a valid sterling pricing kernel via the rate of change of the exchange rate. In real terms, the relevant object is real exchange-rate growth. This provides a way to convert the US real SDF estimated previously into sterling real terms, and hence to risk-adjust UK government bond returns.

No-arbitrage guarantees that the constructed M_{t+1}^{UK} is a valid pricing kernel for internationally traded claims, but our application requires the additional assumption that it also prices UK government bonds. This amounts to assuming that gilts are not segmented from international capital markets—that is, gilts are spanned by, or at least priced consistently with, the set of assets accessible to international investors. This assumption may be violated by transaction costs, funding frictions, balance-sheet constraints, capital controls, and other forms of market segmentation. For example, Du et al. (2018) describe persistent deviations from covered interest parity in the post-GFC period, and historical episodes such as the Exchange Control Act 1947 imposed explicit barriers to cross-border capital flows. We therefore interpret

the mapping from the US pricing kernel into sterling real terms as a useful empirical approximation whose accuracy is likely to vary across historical subperiods, rather than as an exact frictionless identity. Regardless of whether or not no-aribtrage holds, the covariance we consider can be interpreted as measuring the risk of UK government debt to a US investor once real exchange rate risk is also accounted for.

We can directly test this assumption by evaluating whether the constructed UK SDF prices the real HPR on gilts, i.e. by testing the null hypothesis that

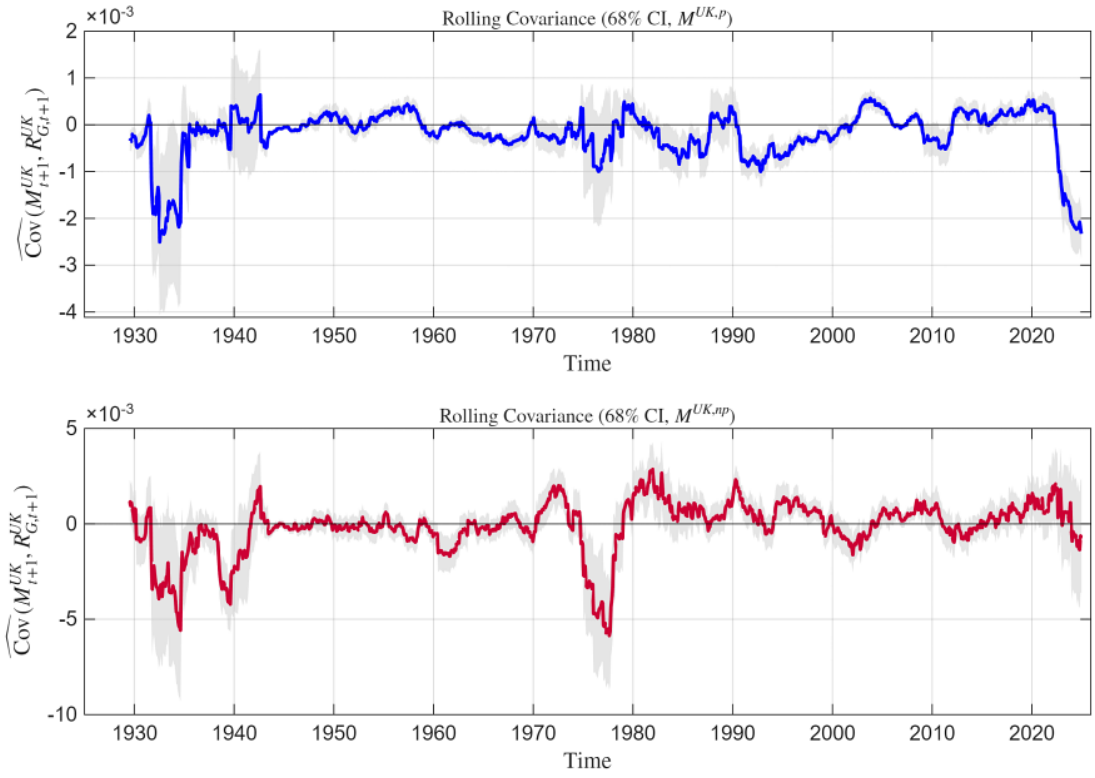
$$1 = \mathbb{E}\left[M_{t+1}^{UK} R_{g,t+1}^{UK}\right] \tag{50}$$

using the expression for M_{t+1}^{UK} in (49). Doing so yields a p-value of 0.24 and 0.64 when the parametric and nonparametric SDFs are used respectively for M_{t+1}^{US}, and so we cannot reject the null hypothesis in either case. It thus appears that the UK government debt portfolio is priced effectively by the UK SDF we obtain.

To examine how the riskiness of UK government debt has evolved over time, Figure 6 plots the rolling covariance between M_{t+1}^{UK} and $R_{g,t+1}^{UK}$, with either the parametric or nonparametric SDF used to construct the UK SDF via (49). Regardless of which SDF is used, the results imply that UK government debt has largely been safe over the past century, with three periods forming notable exceptions. We discuss each regime in turn.

- **1926–1939: Debt was risky.** Both conditions for the fiscal regime appear to have been satisfied during the interwar period. On the monetary side, the gold standard constrained ρ_π directly: the Bank of England targeted the exchange rate rather than domestic inflation. Its abandonment in 1931 did not restore active monetary policy; instead, the subsequent 'cheap money' policy kept rates low to support recovery, consistent with $\rho_\pi < 1$ continuing through the 1930s. Lennard et al. (2023) find an increase in inflation expectations during the first half of the 1930s, consistent with this passive monetary stance. On the fiscal side, the clearest evidence of a low δ_b comes from the 1932 War Loan conversion—effectively a forced restructuring of the government's largest single liability—which arguably demonstrated a revealed preference for imposing losses on bondholders

Figure 6: Rolling covariance between the SDF and the real HPR on UK government debt.



Note: The top panel plots the 36-month trailing covariance between the parametric SDF, adjusted as in (49), and the real HPR on UK government debt. The bottom panel repeats this for the nonparametric SDF.

rather than generating offsetting surpluses.[16] The 1934 default on war debts owed to the United States (Gill, 2023) further underscores this posture. The inherited wartime debt burden made conventional fiscal adjustment politically costly, as the 1931 government collapse over unemployment benefit spending cuts illustrated.

- **1945–c.1972: Debt was safe.** Despite emerging from WWII with a debt-to-GDP ratio of approximately 250%, both SDFs indicate safe debt. On the fiscal side, the Attlee and subsequent governments pursued meaningful consolidation: the Cripps austerity programme moved the primary balance into surplus, and the debt ratio fell steadily, reaching under 60% by the early 1970s, consistent with $\delta_b > s^\star$. On the monetary side, the Bretton Woods system provided an external nominal anchor, disciplining monetary policy via the exchange rate commitment and functioning analogously to a high ρ_π. The financial system was also subject to extensive repression—capital controls under the Exchange Control Act 1947, administered interest rates, and directed lending requirements—which insulated gilt holders from risk and likely compressed the covariance towards zero independently of the policy regime.

- **c.1973–1980: Debt was risky.** The mid-1970s represent the sharpest episode of risky debt in the UK sample, categorised as risky debt by both SDFs, although the nonparametric estimate is the more decisively negative of the two. Both conditions for the fiscal regime appear to have been satisfied. On the monetary side, the Heath government's Competition and Credit Control reforms in 1971 dismantled the direct lending controls that had insulated gilt holders, and the Barber boom of 1972–73 combined fiscal expansion with monetary accommodation. With the Bank of England lacking operational independence and interest-rate decisions effectively taken by the Chancellor, there was a direct mechanism for passive monetary policy ($\rho_\pi < 1$). On the fiscal side, the Social Contract between the Wilson and Callaghan governments and the trade unions from 1974 traded sustained public spending for wage restraint that repeatedly broke down, consistent with a weak surplus response to debt ($\delta_b < s^\star$). The result was the pattern our model predicts in the fiscal regime: inflation exceeding 25%, large deficits, and negative real returns to bondholders. The 1976 IMF programme forced a fiscal adjustment, but the rolling covariance remains negative through to approximately 1980, suggesting credibility was not restored until the

[16] While not in our sample, this attitude towards government debt management stands in contrast to what preceded it, where Victorian policymakers typically had a deep disinclination towards debt and deficits. This attitude is typified by a question posed by Edwin Montagu to Lloyd George in 1921: "Is it conceivable that England should prostitute itself to the level of France and budget for a deficit?". See Middleton (1996) and Cloyne et al. (2023).

Thatcher government's Medium-Term Financial Strategy. Our reading is consistent with Bush (2024) and Bordo et al. (2025), who find that fiscal policy deviated from its debt-stabilising norm during this period.

- **1980–2020: Debt was safe.** The shift back to safe debt reflects a joint move towards $\rho_\pi > 1$ and $\delta_b > s^\star$, progressively reinforced by institutional reforms: the Medium-Term Financial Strategy (1980), inflation targeting (1992), Bank of England independence (1997), and the fiscal rules framework culminating in the Office for Budget Responsibility (2010). Notably, debt remained safe through the global financial crisis despite the approximate doubling of the debt-to-GDP ratio. This is consistent with the model's emphasis on the policy rules (ρ_π, δ_b) rather than the state variables: the fiscal expansion of 2008–10 was accompanied by credible signals of future consolidation, and the monetary framework remained intact.

- **2020–2024: Evidence of a shift towards risky debt.** The covariance turns negative from 2020 onwards, consistent with a move towards the risky debt regime. The large fiscal transfers of 2020–21 constitute a deficit shock arriving in a high marginal utility state, and the subsequent rise in inflation is consistent with the fiscal regime mechanism, where deficit shocks not fully backed by future surpluses contribute to higher inflation and erode the real HPR. The September 2022 gilt market episode is also relevant, though it operated through a distinct mechanism: an announced future fiscal loosening triggered a rapid repricing of gilts, illustrating the sensitivity of gilt pricing to expectations about the future surplus path. The subsequent fiscal correction partially stabilised the market, consistent with a partial restoration of perceived δ_b. Evidence on the Taylor rule presented in the next section suggests this regime may have been quite transitory.

In summary, we identify the following regimes for UK government debt: risky in the interwar period (1926–1939), safe in the post-war period (1945–c.1972), risky in the 1970s (c.1973–1980), safe from approximately 1980–2020, and a shift towards risky debt in the post-pandemic 2020-2024 period. We provide further supporting evidence in the subsequent section.

6 Additional Supporting Evidence of Monetary and Fiscal Regimes

In this section, we test additional predictions of the model in the regimes we identify previously as a way of providing independent validation of these.

6.1 Perceptions of the Monetary Policy Rule

Our model predicts that a fiscal regime should feature $\rho_\pi < 1$. To test this, we utilise the approach of Bocola et al. (2024), who develop a method to infer this from the joint response of nominal bond yields and inflation breakevens. The core logic is that a rise in expected inflation, as proxied by breakevens, should be proportional to the rise in expected interest rates, as proxied by nominal yields, according to the Taylor rule parameter ρ_π. Bocola et al. (2024) find considerable variation in this over time, and find a deviation of ρ_π below 1 in the 2020-2022 period. We extend their sample to cover 2000-2024, using daily data on 5-year nominal yields and 5-year inflation breakevens.[17] We also extend the analysis to the UK. A virtue of this is that the UK has issues inflation-linked gilts for a much longer time, meaning the sample used is 1985-2024 in this case. We use 5-year nominal gilts and 5-year breakvens also.

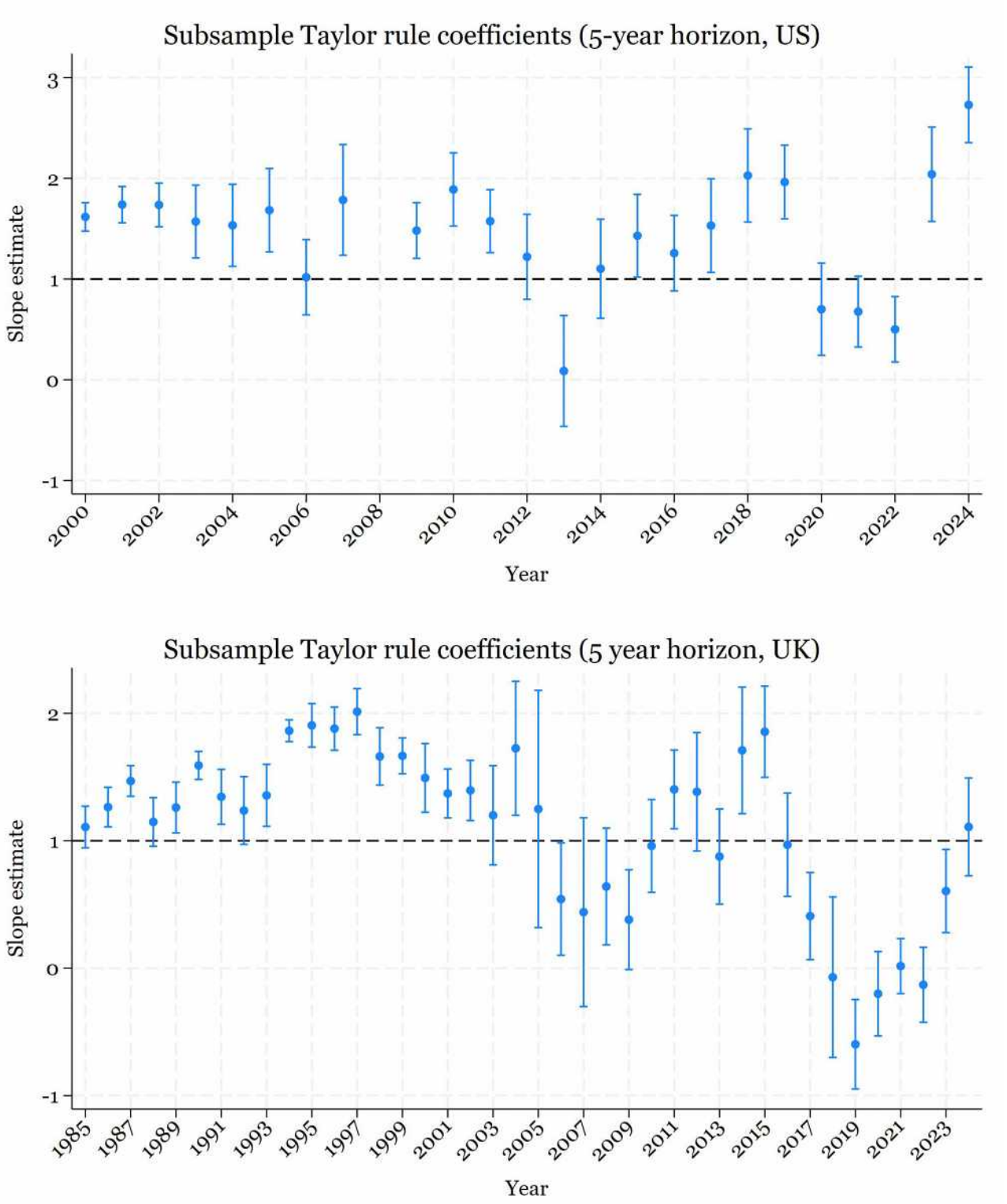
Figure 7 plots the results for both countries. For the US, the 2000-2020 period features $\rho_\pi > 1$, while the 2020-2022 period features $\rho_\pi < 1$, consistent with our previous results. The results suggests that the US may have returned to a monetary regime in 2023-2024. The UK estimates are also consistent with ours over the longer 40-year sample period. From 1985-2015, the estimate of ρ_π is firmly above 1 on a consistent basis. However, from 2016 onwards the parameter estimate dips below 1, before rising above again in 2024. While we find the 2020-2024 period to be consistent with this, this is not the case for 2016-2019. However, Bianchi et al. (2026) do find evidence of a rise in unfunded transfers in this period using a structural model.

[17] The method requires the pre-setting of the smoothing parameter ρ_i in the specification of the Taylor rule:

$$i_t = i^\star + \rho_i i_{t-1} + (1-\rho_i)\rho_\pi \pi_t$$

We follow Bocola et al. (2024) and set $\rho_i = 0.8$ for both the US and UK.

Figure 7: Results of Bocola et al. (2024) estimation of ρ_π for US and UK.



Note: This figure depicts estimates of ρ_π produced via the method of Bocola et al. (2024). The blue dots represent the poiunt estimates, and the bars represent the 95% confidence intervals.

6.2 Conditional Volatility of the Real HPR

In our model, the monetary regime features a constant real HPR on government debt, while this is positive in the fiscal regime. Although this should not be taken literally, we view a testable prediction of our framework as implying that the real HPR should exhibit higher volatility in the identified fiscal regimes vs the monetary regimes. Concretely, $\text{Var}_t(r_{g,t+1}) = \beta_s^2$ in the fiscal regime.To test this, we plot the 36-month trailing realised volatility of this variable for both the US and UK in Figure 8. Both countries exhibit higher and rising HPR volatility in the fiscal regimes we identify as compared to the monetary regimes, consistent with our model. In Appendix C, we also report estimates from a GARCH model — these look very similar.

6.3 Comovement with Fed Pressure Index of Drechsel (2024)

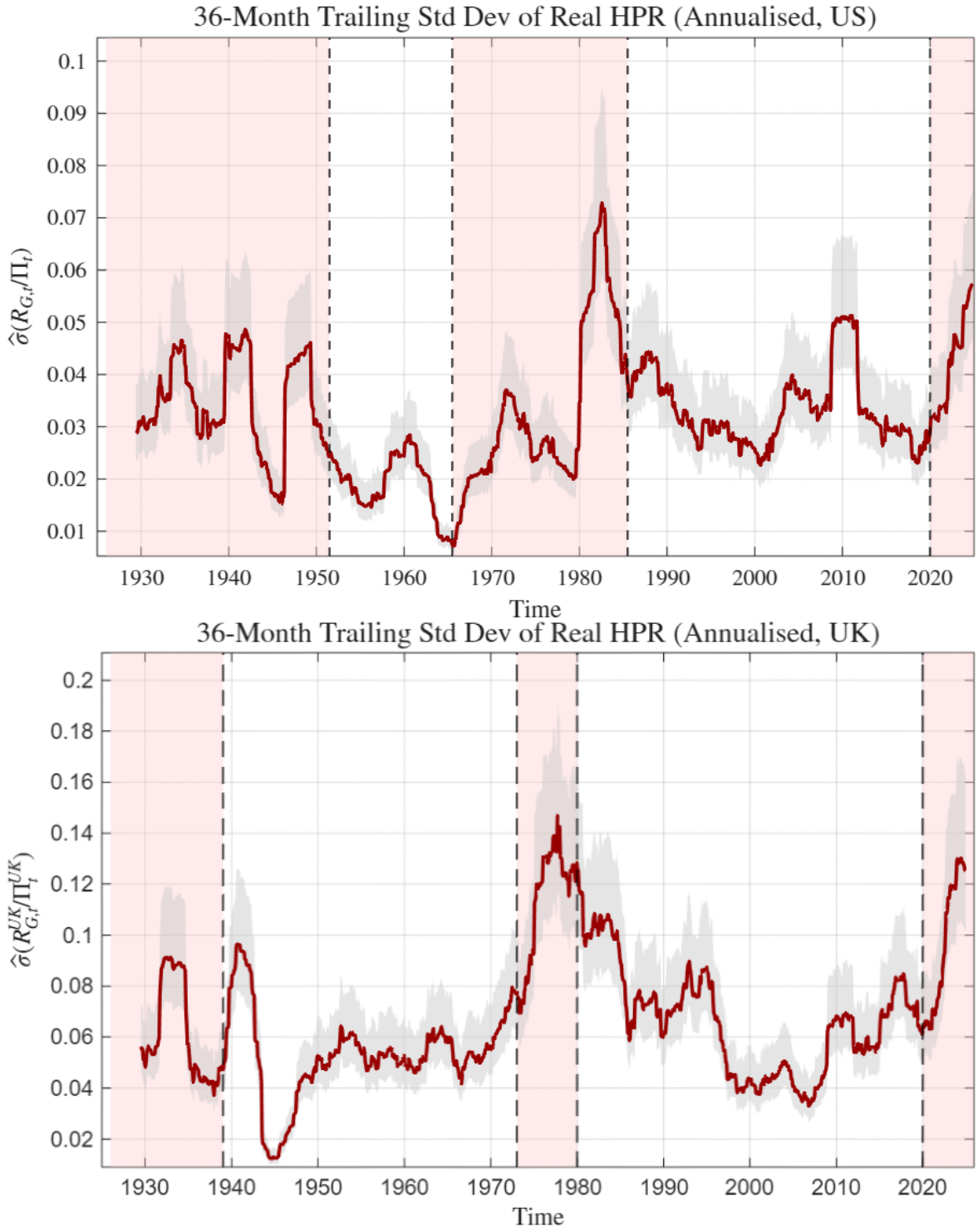
Drechsel (2024) uses detailed data from Presidential diaries to create an index of political pressure on the Fed. Through the lens of our model, this can be seen as an attempt by the fiscal authority to push ρ_π down by attempting to enact more accommodative monetary policy. In line with our theoretical framework, he finds that shocks to this Fed pressure variable generate inflation.

Figure 9 plots this (inverted) series against our trailing covariance measure obtained using the parametric SDF. The two co-move together reasonably closely, with Fed pressure receding after the Treasury-Fed accord, as we find US debt became safe, and then increasing again in the 1960s, as we find that US debt became risky. We view this as supportive evidence of our theoretical mechanism via which debt gains its status as a safe or risky asset via the interaction of monetary and fiscal policy.

6.4 Supply Shocks?

One alternative explanation for government debt becoming risky is a shift in prevalence of aggregate supply shocks relative to aggregate demand shocks. While these are not well-defined in our framework due to its frictionless nature, in a standard New Keynesian model supply shocks generate a negative correlation between the output gap and inflation. Insofar as the SDF is inversely proportional to the output gap (Campbell et al., 2020), this then means that government debt becomes a risky asset due

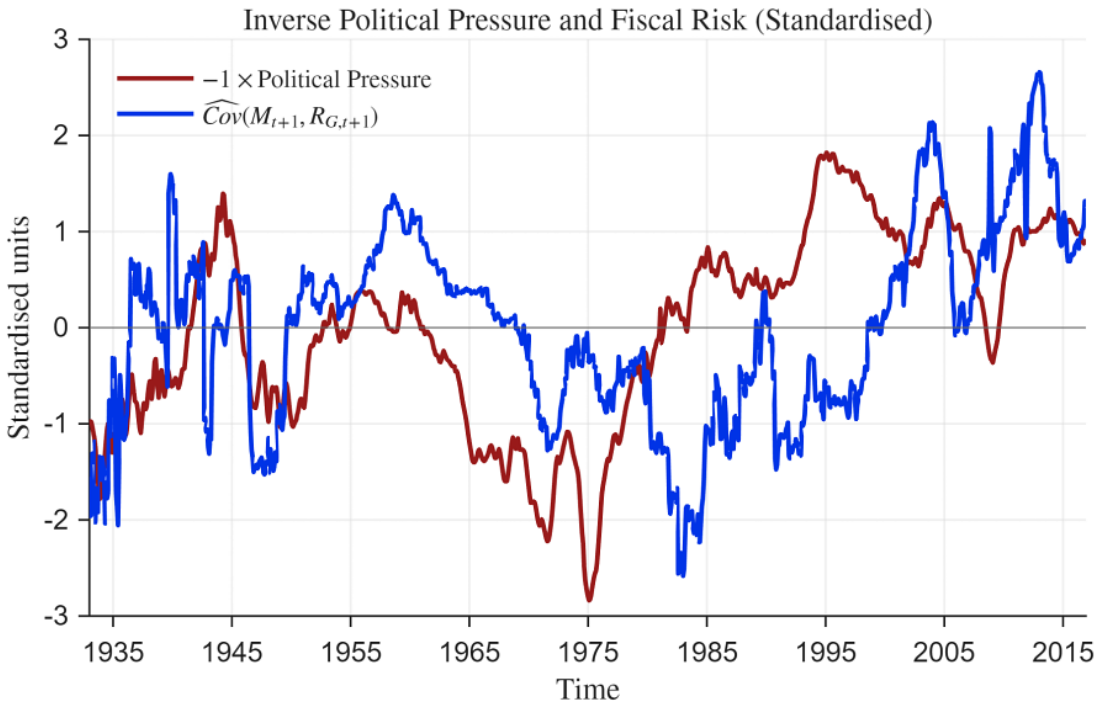
Figure 8: Real HPR Volatility for US and UK.



Note: The shaded red areas denote the risky debt regimes we identify respectively for the US and the UK. For the US these are: 1926-1945, 1965-1985, 2020-2024. For the UK these are: 1926-1939, 1973-1980, 2020-2024. The grey shaded areas represent 68% confidence intervals.

to its poor hedging properties.

Figure 9: Fed Political Pressure Index vs US Debt Risk

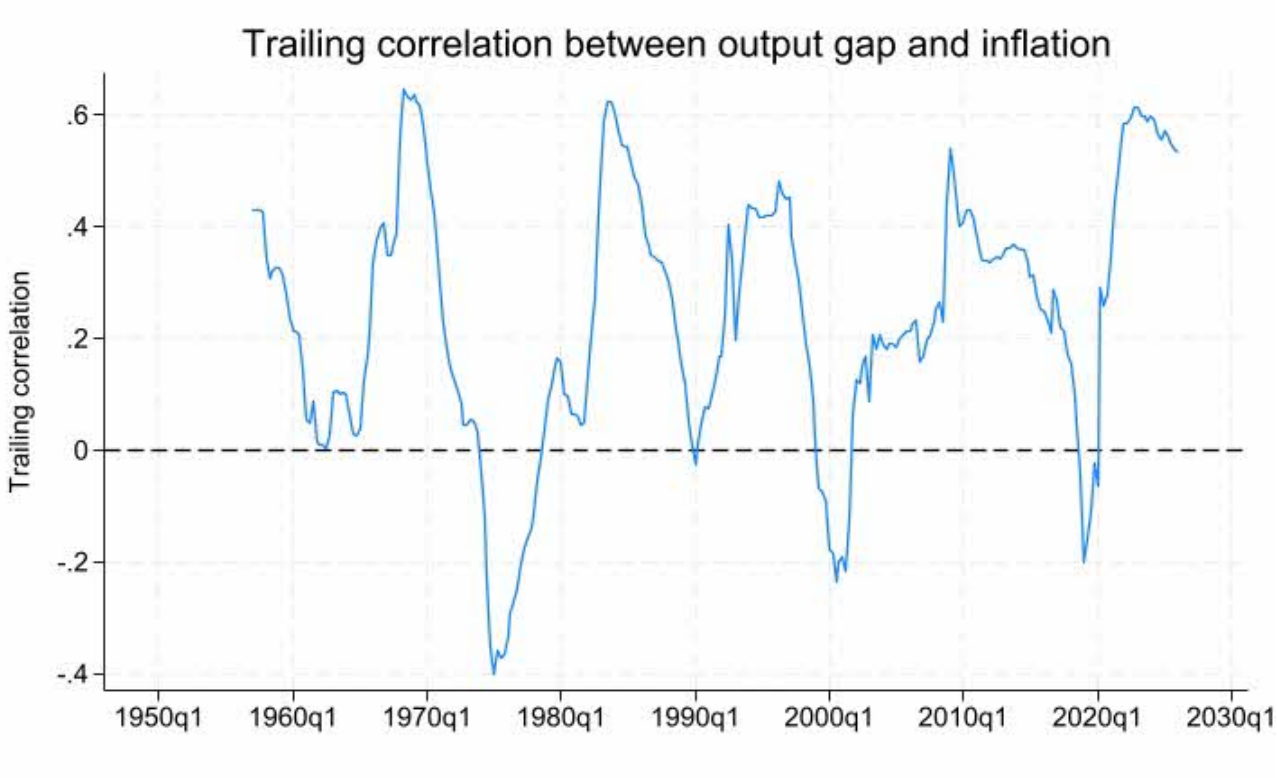


Note: The red line plots the Fed political pressure index from Drechsel (2024) multiplied by -1. The blue line plots our trailing covariance measure using our parametric SDF.

While we cannot definitively rule this out as a competing explanation, we make a number of arguments against this for the US. First, the timing of the covariance sign shift in the first regime, which occurs in the early-to-mid 1960s, does not align well with the traditional supply shock narrative. This instead typically points to the large oil shocks that occurred a decade later in the 1970s. TFP growth was robust in the 1960s also. We argue that a fiscal regime change engineered by President Johnson following his assumption of office in 1963 aligns much more naturally with the timing of the shift from safe to risky debt that we find. For the post-pandemic period, the debate very much remains active, but several papers have found that supply shocks play a less important role than demand shocks, towards which the large fiscal transfers that occurred in 2020 and 2021 may have contributed.[18] For example, Barro and Bianchi (2023) look cross-country and find that, consistent with the FTPL, countries which undertook more government expenditure, scaled by the duration-adjusted size of their public debt, experienced more inflation over the 2020-2023 period. This dovetails with our second point: the supply-shock and fiscal-regime stories are not mutually exclusive. In a fiscal regime, the central bank accommodates rather than offsets inflationary shocks – including cost-push shocks – so a given sequence of supply disturbances

[18] See for example Giannone and Primiceri (2025) and the references therein.

Figure 10: 3-year Trailing Correlation Between the US Output Gap and Inflation



will translate into larger, more persistent inflation and a more negative covariance between bond returns and the SDF. Third and finally, as illustrated in Figure 10, the output gap-inflation correlation did not turn negative until the mid-1970s and swiftly returned to positive territory before the beginning of the 1980s. Likewise, the correlation remained firmly positive in the post-pandemic period.

7 Conclusion

Most analysis of public debt sustainability in macroeconomics treats government debt as being riskless in linearised models. As a result the r in the $r - g$ debates is typically equal to the risk-free rate. In this paper, we deliberately deviate from this assumption and instead assume that the government has to sell its debt to bond investors who in turn risk-adjust the cash flows that this asset offers when formulating the price they are willing to pay. If bonds pay off poorly in bad states of the world, they will trade at a discount due to the risk premia they carry. This consequently increases the relevant average funding cost for the government, since it becomes more expensive to borrow.

We showed through the lens of a stylised model that monetary and fiscal policy interactions can jointly determine the risk properties of government debt. In the monetary regime, inflation is kept under control and deficit shocks do not lead to a negative realisation of the real HPR on government debt. The short-run deficits are backed by long-run future primary surpluses, and bondholders are insured from risk. Debt has a zero beta with the SDF in this regime, and is safe consequently. Conversely, in the fiscal regime, inflation is sensitive to deficit shocks and so the real HPR on government debt falls in this state of the world. Deficits are not backed by future surpluses, and taxpayers receive insurance via fiscal transfers. Because the shock carries a positive price of risk, the real HPR has a negative beta with the SDF and debt becomes risky. Thus, our model tells us that the covariance between the SDF and the HPR is informative about the underlying monetary-fiscal relationship.

With this in mind, we measure this covariance empirically for the US over the last century. To do so, we devise a novel method for recovering the SDF from the cross-section of returns which allows us to stay broadly agnostic about the true asset pricing model. We find that there have been a number of distinct regimes over the period. Debt was risky in the twenty years preceding WWII, but was then safe in the 15 years afterwards under the Truman and Eisenhower administrations. This flipped in the 1960s during the Johnson presidency, perhaps owing to his administration's Great Society program which featured unfunded spending increases. Debt then became safe once more at some point between 1985 and 2000, possibly due to a more robust Fed under Paul Volcker, and a shift towards fiscal prudence under the Clinton administration. This lasted until 2020, at which point we find that there may have been a shift back to the risky regime. Our baseline estimates imply that the risk premium component on US debt is adding around 100bps to the real HPR. Our UK results suggest that gilts have largely exhibited returns uncorrelated with the SDF over the last century, with the exception of the interwar period, the 1970s and the post-pandemic 2020-2024 period.

Future work could build upon this paper in several different ways. Firstly, we specified the SDF in a relatively reduced form way to capture the idea of global bond arbitraguers. This could be modified by modelling this more explicitly, for example explicitly incorporating preferred habitat investors and investors as in Vayanos and Vila (2021). Secondly, one could extend the analysis to other advanced economies such as Japan and EU countries. Thirdly, one could take more of a normative approach to examine the welfare implications of the two regimes we described.

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Measuring Fiscal Risk: Evidence From a Century of Data

Appendix

Utso Pal Mustafi Andrew Preston

A Additional Material for the Model

A.1 Model Solution: Full Derivations

A.1.1 Fiscal Regime

Substituting the surplus rule into the Campbell-Koo approximation (23):

$$\begin{aligned} r_{s,t+1} &= \kappa_0 + \kappa_2 s^\star - \kappa_2 \delta_b \log(b^\star) + \kappa_1 \log(b_{t+1}) + (\kappa_2 \delta_b - 1)\log(b_t) - \kappa_2 x_{t+1} \\ &= \bar{r}_s + \kappa_1 \log(b_{t+1}) + (\kappa_2 \delta_b - 1)\log(b_t) - \kappa_2 x_{t+1}, \end{aligned} \tag{51}$$

where $\bar{r}_s \equiv \kappa_0 + \kappa_2 (s^\star - \delta_b \log(b^\star))$. This illustrates that, all else equal, deficits serve to reduce the ex-post return on surpluses.

Using the Euler equation for the real log return on surplus:

$$\begin{aligned} 1 &= \mathbb{E}_t \exp(m_{t+1} + r_{s,t+1}) \\ &= \mathbb{E}_t \exp\left(-\mu - \frac{\lambda^2}{2} - \lambda\varepsilon_{t+1} + \bar{r}_s + \kappa_1 \log(b_{t+1}) + (\kappa_2\delta_b - 1)\log(b_t) - \kappa_2 x_{t+1}\right). \end{aligned} \tag{52}$$

The stability condition necessary to solve this equation forward for the real value of debt is $\delta_b < s^\star$. We guess that log real debt is affine in the state variable, x_t:

$$\log(b_t) = A_0 + A_1 x_t. \tag{53}$$

Substituting this into (52) yields:

$$1 = \mathbb{E}_t \exp\left(-\mu - \frac{\lambda^2}{2} - \lambda\varepsilon_{t+1} + \bar{r}_s + \kappa_1(A_0 + A_1 x_{t+1}) + (\kappa_2\delta_b - 1)(A_0 + A_1 x_t) - \kappa_2 x_{t+1}\right). \tag{54}$$

Using $x_{t+1} = \rho x_t + \sigma\varepsilon_{t+1}$ and collecting terms, the coefficient on ε_{t+1} inside the exponential is $\sigma(\kappa_1 A_1 - \kappa_2) - \lambda$, while the coefficient on x_t is $\kappa_1 A_1 \rho + (\kappa_2\delta_b - 1)A_1 - \kappa_2\rho$. Taking expectations using log-normality and matching the coefficient on x_t to zero:

$$0 = \kappa_1 A_1 \rho + (\kappa_2\delta_b - 1)A_1 - \kappa_2\rho, \tag{55}$$

which gives:

$$A_1 = \frac{\kappa_2 \rho}{\kappa_1 \rho + \kappa_2 \delta_b - 1} = \frac{\rho}{b^\star(\rho - 1) + \delta_b - s^\star}. \tag{56}$$

Matching the constant terms:

$$A_0 = \frac{\mu + \frac{\lambda^2}{2} - \bar{r}_s - \frac{1}{2}\left[\sigma\kappa_2 \frac{\kappa_2\delta_b - 1}{\kappa_1\rho + (\kappa_2\delta_b - 1)} + \lambda\right]^2}{\kappa_1 + \kappa_2\delta_b - 1}. \tag{57}$$

Substituting the affine solution for $\log(b_t)$ into (51) and separating into conditional mean and innovation:

$$r_{s,t+1} = \mathbb{E}_t r_{s,t+1} + \beta_s \varepsilon_{t+1}, \tag{58}$$

where:

$$\mathbb{E}_t r_{s,t+1} = \mu - \frac{\beta_s^2}{2} + \beta_s \lambda, \tag{59}$$

and:

$$\beta_s \equiv \sigma(\kappa_1 A_1 - \kappa_2) = \sigma \frac{b^\star + s^\star - \delta_b}{(b^\star + s^\star)\left[b^\star(\rho - 1) + \delta_b - s^\star\right]}. \tag{60}$$

Inflation solution. Taking expectations of the log government budget identity:

$$\mathbb{E}_t r_{s,t+1} = \mathbb{E}_t \tilde{r}_{g,t+1} - \mathbb{E}_t \pi_{t+1}. \tag{61}$$

Rearranging and expressing in unexpected deviations:

$$\begin{aligned} \pi_{t+1} - \mathbb{E}_t \pi_{t+1} &= \widetilde{r}_{g,t+1} - \mathbb{E}_t \widetilde{r}_{g,t+1} - (r_{s,t+1} - \mathbb{E}_t r_{s,t+1}) \\ &= \Omega(\widetilde{r}^{(2)}_{t+1} - \mathbb{E}_t \widetilde{r}^{(2)}_{t+1}) - \beta_s \varepsilon_{t+1}, \end{aligned} \tag{62}$$

where we have used the Campbell-Viceira approximation (25) and (58). To solve out the two-period bond return innovation, note that:

$$\begin{aligned} \widetilde{r}^{(2)}_{t+1} &= \tilde{q}^{(1)}_{t+1} - \tilde{q}^{(2)}_t \\ &= \tilde{q}^{(1)} - \rho_\pi(\pi_{t+1} - \pi^\star) - \tilde{q}^{(2)}_t, \\ \mathbb{E}_t \widetilde{r}^{(2)}_{t+1} &= \tilde{q}^{(1)} - \rho_\pi(\mathbb{E}_t \pi_{t+1} - \pi^\star) - \tilde{q}^{(2)}_t, \\ \widetilde{r}^{(2)}_{t+1} - \mathbb{E}_t \widetilde{r}^{(2)}_{t+1} &= -\rho_\pi(\pi_{t+1} - \mathbb{E}_t \pi_{t+1}), \end{aligned} \tag{63}$$

where $\tilde{q}^{(j)}_t = \log \tilde{Q}^{(j)}_t$. Substituting (63) into (62) and solving for the inflation innovation:

$$\pi_{t+1} - \mathbb{E}_t \pi_{t+1} = -\frac{\beta_s}{1 + \Omega\rho_\pi} \varepsilon_{t+1}. \tag{64}$$

A.1.2 Monetary Regime

Using the Euler equation for one-period nominal bonds:

$$\exp(-i_t) = \mathbb{E}_t \exp(m_{t+1} - \pi_{t+1}). \tag{65}$$

Substituting in the Taylor rule and the expression for the log real SDF:

$$\exp(-i^\star - \rho_\pi(\pi_t - \pi^\star)) = \mathbb{E}_t \exp\left(-\mu - \frac{\lambda^2}{2} - \lambda\varepsilon_{t+1} - \pi_{t+1}\right). \tag{66}$$

With $i^\star = \mu + \pi^\star$, under the unique bounded solution, inflation is constant and equal to $\pi^\star$ each period. The monetary rule then implies that $i_t = i^\star$ in every period.

Next, we price the two-period nominal bond using its Euler equation:

$$\exp\left(\tilde{q}_t^{(2)}\right) = \mathbb{E}_t \exp\left(m_{t+1} - \pi_{t+1} + \tilde{q}_{t+1}^{(1)}\right). \tag{67}$$

Using that $\tilde{q}_{t+1}^{(1)} = -i_{t+1} = -i^\star = -\mu - \pi^\star$ and $\pi_{t+1} = \pi^\star$:

$$\exp\left(\tilde{q}_t^{(2)}\right) = \mathbb{E}_t \exp\left(-\mu - \frac{\lambda^2}{2} - \lambda\varepsilon_{t+1} - \pi^\star - \mu - \pi^\star\right), \tag{68}$$

which implies that:

$$\tilde{q}_t^{(2)} = -2i^\star, \tag{69}$$

and the price of the two-period nominal bond is constant. From the Campbell-Viceira approximation (25), $\widetilde{r}_{g,t+1}$ is therefore constant every period. The unexpected real log return on the surplus claim is then, using the government budget identity:

$$r_{s,t+1} - \mathbb{E}_t r_{s,t+1} = \widetilde{r}_{g,t+1} - \mathbb{E}_t \widetilde{r}_{g,t+1} - (\pi_{t+1} - \mathbb{E}_t \pi_{t+1}) = 0. \tag{70}$$

A.2 Adding a Monetary Shock

We extend the benchmark of Section 2.3 by adding a priced monetary policy shock, leaving the fiscal block unchanged. Let $\boldsymbol{\varepsilon}_{t+1} = (\varepsilon_{t+1}, \nu_{t+1})'$ collect two mutually independent standard normal innovations: the deficit shock ε and a monetary policy shock ν, where $\nu_{t+1} > 0$ denotes a contractionary surprise. The log SDF is

$$-m_{t+1} = \mu + \tfrac{1}{2}\boldsymbol{\lambda}'\boldsymbol{\lambda} + \boldsymbol{\lambda}'\boldsymbol{\varepsilon}_{t+1}, \qquad \boldsymbol{\lambda} = (\lambda_\varepsilon, \lambda_\nu)', \tag{71}$$

with $\lambda_\varepsilon \equiv \lambda < 0$ the baseline price of deficit risk, and the monetary rule gains a policy shock,

$$i_t = i^\star + \rho_\pi(\pi_t - \pi^\star) + \omega_t, \qquad \omega_t = \rho_\omega \omega_{t-1} + \sigma_\omega \nu_t. \tag{72}$$

We impose the natural sign restriction $\lambda_\nu \leq 0$, since a contractionary surprise raises the nominal rate and is a high marginal utility state.

Proposition 2. *With the two-shock structure above and $\lambda_\nu \leq 0$, the conditional covariance is additive over the orthogonal shocks, $\mathrm{Cov}_t(m_{t+1}, r_{g,t+1}) = -\boldsymbol{\lambda}'\boldsymbol{\beta}^{(j)}$, with regime-specific loadings $\boldsymbol{\beta}^{(F)} = (\beta_s, 0)$ and $\boldsymbol{\beta}^{(M)} = (0, \beta_\nu^{(M)})$, where*

$$\beta_s = -\frac{\sigma\,(b^\star + s^\star - \delta_b)}{(b^\star + s^\star)\,D} < 0, \qquad \beta_\nu^{(M)} = \frac{\sigma_\omega(1 + \Omega\rho_\omega)}{\rho_\pi - \rho_\omega} > 0, \tag{73}$$

with $D \equiv b^\star(1-\rho) + (s^\star - \delta_b) > 0$. The covariance in each regime is therefore

$$\mathrm{Cov}_t^{(F)} = -\lambda_\varepsilon \beta_s < 0, \qquad \mathrm{Cov}_t^{(M)} = -\lambda_\nu \beta_\nu^{(M)} \geq 0, \tag{74}$$

so government debt is risky in the fiscal regime and safe in the monetary regime.

Proof. The one-period nominal bond satisfies $e^{-i_t} = \mathbb{E}_t e^{m_{t+1} - \pi_{t+1}}$. Writing the regime-specific inflation innovation as $\pi_{t+1} - \mathbb{E}_t \pi_{t+1} = \boldsymbol{b}_\pi' \boldsymbol{\varepsilon}_{t+1}$ and applying lognormality,

$$i_t = \mu + \mathbb{E}_t \pi_{t+1} - \boldsymbol{\lambda}'\boldsymbol{b}_\pi - \tfrac{1}{2}\|\boldsymbol{b}_\pi\|^2. \tag{75}$$

The risk-adjustment term is constant and is absorbed into the steady-state nominal rate, so in deviations $i_t - i^\star = \mathbb{E}_t \pi_{t+1} - \pi^\star$. Equating to the policy rule,

$$\mathbb{E}_t \hat{\pi}_{t+1} = \rho_\pi \hat{\pi}_t + \omega_t, \qquad \hat{\pi}_t \equiv \pi_t - \pi^\star. \tag{76}$$

Solving (76) forward, guess $\hat{\pi}_t = a_\omega \omega_t$. Matching gives $a_\omega(\rho_\omega - \rho_\pi) = 1$, so

$$a_\omega = -\frac{1}{\rho_\pi - \rho_\omega} < 0, \qquad \pi_{t+1} - \mathbb{E}_t \pi_{t+1} = a_\omega \sigma_\omega \nu_{t+1}. \tag{77}$$

The nominal rate is $i_{t+1} = i^\star + (\rho_\pi a_\omega + 1)\omega_{t+1}$, and since $\rho_\pi a_\omega + 1 = a_\omega \rho_\omega$,

$$i_{t+1} - \mathbb{E}_t i_{t+1} = a_\omega \rho_\omega \sigma_\omega \nu_{t+1}. \tag{78}$$

The two-period nominal bond return is $\tilde{r}_{t+1}^{(2)} = \tilde{q}_{t+1}^{(1)} - \tilde{q}_t^{(2)} = -i_{t+1} - \tilde{q}_t^{(2)}$, with innovation $-(i_{t+1} - \mathbb{E}_t i_{t+1})$. Applying the Campbell-Viceira approximation (25), the nominal portfolio return innovation is Ω times this, and the real return innovation subtracts the inflation surprise:

$$r_{g,t+1} - \mathbb{E}_t r_{g,t+1} = -a_\omega \sigma_\omega (1 + \Omega\rho_\omega)\, \nu_{t+1}. \tag{79}$$

Hence $\beta_\varepsilon^{(M)} = 0$ and $\beta_\nu^{(M)} = -a_\omega\sigma_\omega(1+\Omega\rho_\omega) = \sigma_\omega(1+\Omega\rho_\omega)/(\rho_\pi - \rho_\omega) > 0$.

Equation (76) is now explosive forward, so inflation is pinned down by the budget identity. Guess that log real debt is affine in the full state,

$$\log b_t = A_0 + A_1 x_t + A_3 \omega_t. \tag{80}$$

Substituting into the Campbell-Koo approximation (23) gives the real return on the surplus claim,

$$r_{s,t+1} = \bar{r}_s + \kappa_1 \log b_{t+1} + (\kappa_2\delta_b - 1)\log b_t - \kappa_2 x_{t+1}, \tag{81}$$

whose shock loadings are $\beta_s = \sigma(\kappa_1 A_1 - \kappa_2)$ and $\beta_\nu^{(F)} = \kappa_1 A_3 \sigma_\omega$. Imposing the Euler equation $1 = \mathbb{E}_t e^{m_{t+1}+r_{s,t+1}}$ and using lognormality,

$$\mathbb{E}_t r_{s,t+1} = \mu + \boldsymbol{\lambda}'\boldsymbol{\beta}_s - \tfrac{1}{2}\|\boldsymbol{\beta}_s\|^2, \qquad \boldsymbol{\beta}_s \equiv (\beta_s, \beta_\nu^{(F)})'. \tag{82}$$

Matching the coefficients on x_t and ω_t between this expression and the conditional mean of $r_{s,t+1}$ delivers

$$A_1 = \frac{\kappa_2\rho}{\kappa_1\rho + \kappa_2\delta_b - 1}, \qquad A_3 = 0. \tag{83}$$

The first reproduces the baseline solution (56), so β_s is unchanged; substituting $\kappa_1 = b^\star/(b^\star + s^\star)$ and $\kappa_2 = 1/(b^\star + s^\star)$ writes (28) as $\beta_s = -\sigma(b^\star + s^\star - \delta_b)/[(b^\star + s^\star)D] < 0$, with $D \equiv b^\star(1-\rho) + (s^\star - \delta_b) > 0$. The second gives $\beta_\nu^{(F)} = 0$: the surplus claim's payoff does not depend on the monetary shock, so the real holding period return cannot load on it.

The inflation innovation follows from the budget identity $r_{g,t+1} = \tilde{r}_{g,t+1} - \pi_{t+1} = r_{s,t+1}$. Using $\tilde{r}_{g,t+1} - \mathbb{E}_t\tilde{r}_{g,t+1} = -\Omega(i_{t+1} - \mathbb{E}_t i_{t+1})$ with $i_{t+1} - \mathbb{E}_t i_{t+1} = \rho_\pi(\pi_{t+1} - \mathbb{E}_t\pi_{t+1}) + \sigma_\omega\nu_{t+1}$,

$$\pi_{t+1} - \mathbb{E}_t\pi_{t+1} = -\frac{1}{1+\Omega\rho_\pi}\Big[\beta_s\,\varepsilon_{t+1} + \Omega\sigma_\omega\,\nu_{t+1}\Big]. \tag{84}$$

The ε-loading reproduces (31). Substituting back into $r_{g,t+1} = \tilde{r}_{g,t+1} - \pi_{t+1}$ confirms the real-return loadings $\boldsymbol{\beta}^{(F)} = (\beta_s, 0)$; in particular the ν contributions to $\tilde{r}_{g,t+1}$ and π_{t+1} cancel exactly, so the monetary shock is neutral for the real holding period return in the fiscal regime.

Since $m_{t+1} - \mathbb{E}_t m_{t+1} = -\boldsymbol{\lambda}'\boldsymbol{\varepsilon}_{t+1}$ and the shocks are orthonormal, $\mathrm{Cov}_t(m_{t+1}, r_{g,t+1}) = -\boldsymbol{\lambda}'\boldsymbol{\beta}^{(j)}$, giving $\mathrm{Cov}_t^{(F)} = -\lambda_\varepsilon\beta_s < 0$ and $\mathrm{Cov}_t^{(M)} = -\lambda_\nu\beta_\nu^{(M)} \geq 0$. □

A.3 Convenience Yields

In the first of the two cases we consider for convenience yields in the model, we make two simplifying assumptions which aid tractability. Firstly, we assume that the one-period nominal bond earns no

convenience yield. Secondly, we assume that the government finances itself entirely using the long bond ($\Omega = 1$). We assume that the following Euler equation holds for the two-period bond:

$$Q_t^{(2)} = \mathbb{E}_t M_{t+1} \exp(\psi_t) \left(\frac{Q_{t+1}^{(1)}}{\Pi_{t+1}} \right) \tag{85}$$

Rearranging this, and using the fact that $R_{t+1}^{(2)} = R_{g,t+1}$ from the second assumption above, we have that:

$$1 = \mathbb{E}_t M_{t+1} \exp(\psi_t) R_{g,t+1} \tag{86}$$

Here, ψ_t captures the convenience component which the marginal investor obtains from holding government debt. We assume it follows the process:

$$\psi_t = \psi_0 - \psi_1 [\log(b_t) - \log(b^\star)] \tag{87}$$

with $\psi_0 \geq 0$, $\psi_1 \geq 0$. This implies that the convenience yield is reduced by an increase in the supply of debt, consistent with a downward sloping Treasury demand curve.

Proposition 3. *In the fiscal regime, the covariance between the SDF and the real holding period return on the Treasury portfolio is negative, while this covariance is zero in the monetary regime:*

$$\textit{Fiscal regime} : \mathrm{Cov}_t(m_{t+1}, r_{g,t+1}) = -\tilde{\beta}_s \lambda < 0 \tag{88}$$

$$\textit{Monetary regime} : \mathrm{Cov}_t(m_{t+1}, r_{g,t+1}) = 0 \tag{89}$$

Proof. We first analyse the fiscal regime. Once again, we guess that log real debt is affine in x_t:

$$\log(b_t) = \tilde{A}_0 + \tilde{A}_1 x_t \tag{90}$$

Plugging this into the Euler equation for the log real return on surplus, and making use of the Campbell-Shiller approximation yields the following solution after matching coefficients:

$$\tilde{A}_0 = \frac{(b^\star + s^\star) \left[\mu + \frac{\lambda^2}{2} - \psi_0 - \psi_1 \log b^\star - \bar{r}_s - \frac{1}{2} \left(\frac{\sigma}{b^\star + s^\star} \left(b^\star \tilde{A}_1 - 1 \right) - \lambda \right)^2 \right]}{\delta_b - s^\star - \psi_1 (b^\star + s^\star)} \tag{91}$$

$$\tilde{A}_1 = \frac{\rho}{b^\star \rho + \delta_b - (b^\star + s^\star)(1 + \psi_1)} < 0 \tag{92}$$

$\tilde{A}_1$ is increasing in ψ_1, since a larger ψ_1 means that the convenience yield is more sensitive to debt, and hence a smaller response of $\log b_t$ to the deficit shock is needed to satisfy the Euler equation.

Once again substituting the affine solution for $\log(b_t)$ into the approximation for $r_{s,t+1}$ to yield:

$$r_{s,t+1} = \mathbb{E}_t r_{s,t+1} + \tilde{\beta}_s \varepsilon_{t+1} \tag{93}$$

where:

$$\mathbb{E}_t r_{s,t+1} = \mu + \frac{\lambda^2}{2} - \frac{1}{2}\left[\beta_s - \lambda\right]^2 - \psi_t \tag{94}$$

$$\tilde{\beta}_s = \sigma \frac{(b^\star + s^\star)(1+\psi_1) - \delta_b}{(b^\star + s^\star)\left[\, b^\star \rho + \delta_b - (b^\star + s^\star)(1+\psi_1)\,\right]} < 0 \tag{95}$$

The key covariance (in log terms) in the fiscal regime is then given by:

$$\mathrm{Cov}_t(m_{t+1}, r_{g,t+1}) = -\tilde{\beta}_s \lambda < 0 \tag{96}$$

It holds that $\tilde{\beta}_s \geq \beta_s$ and so the presence of a time-varying convenience yield makes debt less risky in the fiscal regime. Intuitively, the convenience yield can absorb some of the shock, meaning that the surplus claim's payoff becomes less responsive to the shock which reduces its risk. It holds that the beta of the surplus claim converges to a negative limit as $\psi_1 \to \infty$:

$$\lim_{\psi_1 \to \infty} \tilde{\beta}_s = -\frac{\sigma}{b^\star + s^\star} \tag{97}$$

This eliminates the debt valuation component of risk since $\tilde{A}_1 \to 0$, but cannot eliminate the contemporaneous deficit component.

We now move on to the monetary regime. The solution is unchanged in this regime, and debt remains riskless.

□

We also consider a more general case with $\Omega \in [0, 1]$ and where there is convenience yield on both the short and long bond. The above solution continues to hold in the fiscal regime, but the monetary regime lacks an analytical solution. We thus consider numerical results for this case. We calibrate the convenience yield parameters following Krishnamurthy and Vissing-Jorgensen (2012). They estimate an average convenience yield of 73bps at an annual frequency, so we set $\psi_0 = 0.0073/4 = 0.0018$. They also estimate that an increase in the debt-to-GDP ratio of roughly 20 percentage points reduces the convenience yield by 44bps. This implies a value of $\psi_1 = 0.0224/4 = 0.0056$, again adjusting to the quarterly frequency.[19] We keep all other parameters the same as in Figure 1.

We do not present the IRFs in each regime, since they look identical to those in Figure 1. The convenience yield is not sufficiently responsive to cause any quantitative impact. The size of the risk premium on government debt (over the zero-beta rate) is reduced slightly by around 10% in the fiscal regime compared to the benchmark results, in accordance with the analytical results above.

[19] Here we are implicitly interpreting the units of b as being scaled by GDP.

A.4 Inflation-linked Debt

The government issues one- and two-period nominal zero-coupon bonds, with nominal face values $B_t^{N,(1)}$ and $B_t^{N,(2)}$, and a two-period real (inflation-linked) bond with face value $B_t^{R,(2)}$. Let $\tilde{Q}_t^{N,(j)}$ denote the period-t nominal price of a j-period nominal bond and $\tilde{Q}_t^{R,(2)}$ the nominal price of the two-period real bond. The period-t nominal government budget constraint is then

$$B_{t-1}^{N,(1)} + \tilde{Q}_t^{N,(1)} B_{t-1}^{N,(2)} + \tilde{Q}_t^{R,(2)} B_{t-1}^{R,(2)} = P_t s_t + \tilde{Q}_t^{N,(1)} B_t^{N,(1)} + \tilde{Q}_t^{N,(2)} B_t^{N,(2)} + \tilde{Q}_t^{R,(2)} B_t^{R,(2)}. \tag{98}$$

Let the total nominal market value of government debt at $t-1$ be

$$V_{t-1} \equiv \tilde{Q}_{t-1}^{N,(1)} B_{t-1}^{N,(1)} + \tilde{Q}_{t-1}^{N,(2)} B_{t-1}^{N,(2)} + \tilde{Q}_{t-1}^{R,(2)} B_{t-1}^{R,(2)}. \tag{99}$$

We define the nominal share of debt as

$$\eta \equiv \frac{\tilde{Q}_{t-1}^{N,(1)} B_{t-1}^{N,(1)} + \tilde{Q}_{t-1}^{N,(2)} B_{t-1}^{N,(2)}}{V_{t-1}}, \tag{100}$$

and, within the nominal portfolio, the share of the long nominal bond as

$$\Omega \equiv \frac{\tilde{Q}_{t-1}^{N,(2)} B_{t-1}^{N,(2)}}{\tilde{Q}_{t-1}^{N,(1)} B_{t-1}^{N,(1)} + \tilde{Q}_{t-1}^{N,(2)} B_{t-1}^{N,(2)}}. \tag{101}$$

Thus η controls the split between nominal and inflation-linked debt, while Ω controls the composition of nominal debt between short and long bonds. These are constant over time.

The nominal holding period return on the inflation-linked bond is given by:

$$\tilde{R}_{g,t+1}^{r} = \frac{\tilde{Q}_{t+1}^{R,(1)}}{\tilde{Q}_t^{R,(2)}}. \tag{102}$$

Evaluating the prices using the fundamental asset pricing condition, it holds that in log terms this is given by:

$$r_{g,t+1}^{r} = \mu, \tag{103}$$

and thus the log real HPR on the inflation-linked part of the public portfolio is constant and equal to the log zero-beta rate.

In the monetary regime, the introduction of inflation-linked debt does not change the equilibrium solution. We thus analyse the fiscal regime. The log real return on surplus remains equal to the log real HPR on the public portfolio:

$$\begin{aligned} r_{s,t+1} &= r_{g,t+1} \\ &= \eta r_{g,t+1}^{n} + (1-\eta) r_{g,t+1}^{r}, \end{aligned} \tag{104}$$

where $r^n_{g,t+1}$ is the log real HPR on the nominal component of the public portfolio. From the Euler equation for the log real return on surplus, it continues to hold that $r_{s,t+1} = \mathbb{E}_t r_{s,t+1} + \beta_s \varepsilon_{t+1}$, and therefore it also holds that $\text{Cov}_t(m_{t+1}, r_{g,t+1}) = -\lambda\beta_s$. The riskiness of govt. debt is unaffected by the composition of the public portfolio between nominal and inflation-linked bonds.

It must also hold that:

$$r_{g,t+1} - \mathbb{E}_t r_{g,t+1} = \eta\big(r^n_{g,t+1} - \mathbb{E}_t r^n_{g,t+1}\big). \tag{105}$$

From the derivations in the baseline, we have that:

$$r^n_{g,t+1} - \mathbb{E}_t r^n_{g,t+1} = -\left(1 + \rho_\pi \Omega\right)\left(\pi_{t+1} - \mathbb{E}_t \pi_{t+1}\right), \tag{106}$$

which implies that:

$$r_{g,t+1} - \mathbb{E}_t r_{g,t+1} = -\eta\left(1 + \rho_\pi \Omega\right)\left(\pi_{t+1} - \mathbb{E}_t \pi_{t+1}\right). \tag{107}$$

Rearranging yields the inflation deviation solution:

$$\pi_{t+1} - \mathbb{E}_t \pi_{t+1} = -\frac{\beta_s}{\eta(1 + \rho_\pi \Omega)} \varepsilon_{t+1}. \tag{108}$$

This implies that when more debt is inflation-linked (lower η), inflation responds more to deficit shocks.

A.5 Time-varying Price of Risk

We replace the constant price of risk in (18) with a state-dependent specification:

$$-m_{t+1} = \mu + \frac{\lambda_t^2}{2} + \lambda_t \varepsilon_{t+1}, \qquad \lambda_t = \lambda_0 + \lambda_1 x_t, \tag{109}$$

with $\lambda_0 < 0$ and $\lambda_1 \leq 0$. The interpretation is that bond arbitraguers demand a higher price of risk when the deficit state deteriorates, consistent with intermediary asset pricing frictions in which effective risk aversion rises in bad states (He and Krishnamurthy, 2013). The rest of the model is unchanged.

Using log-normality:

$$R_{0,t}^{-1} = \mathbb{E}_t M_{t+1} = \exp\left(-\mu - \frac{\lambda_t^2}{2} + \frac{\lambda_t^2}{2}\right) = \exp(-\mu), \tag{110}$$

so $r_{0,t} = \mu$ remains constant. The time-varying price of risk enters only second-moment terms that cancel in the zero-beta rate.

Fiscal regime. The Euler equation for the log real return on surplus is

$$1 = \mathbb{E}_t \exp\left(-\mu - \frac{\lambda_t^2}{2} - \lambda_t \varepsilon_{t+1} + r_{s,t+1}\right). \tag{111}$$

We guess that log real debt is affine in the deficit state:

$$\log(b_t) = A_0 + A_1 x_t. \tag{112}$$

Substituting into the Campbell-Koo approximation for $r_{s,t+1}$:

$$r_{s,t+1} = \bar{r}_s + \kappa_1(A_0 + A_1 x_{t+1}) + (\kappa_2\delta_b - 1)(A_0 + A_1 x_t) - \kappa_2 x_{t+1}, \tag{113}$$

and using $x_{t+1} = \rho x_t + \sigma\varepsilon_{t+1}$:

$$r_{s,t+1} = \text{const} + \left[\kappa_1 A_1\rho + (\kappa_2\delta_b - 1)A_1 - \kappa_2\rho\right] x_t + \sigma(\kappa_1 A_1 - \kappa_2)\varepsilon_{t+1}. \tag{114}$$

Define $\beta_s \equiv \sigma(\kappa_1 A_1 - \kappa_2)$, consistent with the baseline notation. Since m_{t+1} and $r_{s,t+1}$ are conditionally jointly normal under the affine guess, log-normality applied to (111) yields:

$$0 = -\mu - \frac{\lambda_t^2}{2} + \mathbb{E}_t r_{s,t+1} + \frac{(\beta_s - \lambda_t)^2}{2}. \tag{115}$$

Expanding $(\beta_s - \lambda_t)^2 = \beta_s^2 - 2\beta_s\lambda_t + \lambda_t^2$, the λ_t^2 terms cancel exactly, yielding the simplified Euler equation:

$$0 = -\mu + \mathbb{E}_t r_{s,t+1} + \frac{\beta_s^2}{2} - \beta_s\lambda_t. \tag{116}$$

This holds exactly under the affine guess, without further approximation. Matching coefficients on x_t:

$$0 = \kappa_1 A_1\rho + (\kappa_2\delta_b - 1)A_1 - \kappa_2\rho - \lambda_1\beta_s. \tag{117}$$

Substituting $\beta_s = \sigma(\kappa_1 A_1 - \kappa_2)$ and solving:

$$A_1(\lambda_1) = \frac{\kappa_2(\rho - \lambda_1\sigma)}{\kappa_1\rho + \kappa_2\delta_b - 1 - \lambda_1\sigma\kappa_1} = \frac{\rho - \lambda_1\sigma}{b^\star(\rho - 1 - \lambda_1\sigma) + \delta_b - s^\star}. \tag{118}$$

The shock loading admits the closed-form expression:

$$\beta_s(\lambda_1) = \frac{\sigma(b^\star + s^\star - \delta_b)}{(b^\star + s^\star)\left[b^\star(\rho - 1 - \lambda_1\sigma) + \delta_b - s^\star\right]}. \tag{119}$$

When $\lambda_1 = 0$, this collapses to the baseline expression in (34). Crucially, λ_0 does not appear in β_s: the level of the price of risk affects only A_0 and the unconditional risk premium, while the state-dependence parameter λ_1 governs the entire feedback into equilibrium dynamics.

The denominator of A_1 must be negative for $\beta_s < 0$ in the fiscal regime, i.e.

$$b^\star(\rho - 1 - \lambda_1\sigma) + \delta_b - s^\star < 0, \qquad \text{equivalently} \qquad \lambda_1 > \lambda_1^\star \equiv \frac{b^\star(\rho - 1) + \delta_b - s^\star}{\sigma b^\star}. \tag{120}$$

Since $\lambda_1^\star < 0$ in the fiscal regime, this provides a critical lower bound on λ_1: as $\lambda_1 \to \lambda_1^\star$ from above, $|\beta_s|$ diverges. Greater state-dependence in the price of risk pushes the system toward an explosive boundary,

providing a stylised notion of fiscal fragility.

The following proposition formalises the comparison with the baseline.

Proposition 2. *In the fiscal regime, suppose the determinacy condition (120) holds. Then* $\beta_s(\lambda_1) < \beta_s^{\text{base}} < 0$ *for all* $\lambda_1 < 0$*, and* $\partial|\beta_s|/\partial|\lambda_1| > 0$*.*

Proof. Differentiating (119):

$$\frac{\partial \beta_s}{\partial \lambda_1} = \frac{\sigma^2 b^\star (b^\star + s^\star - \delta_b)}{(b^\star + s^\star)\left[b^\star(\rho - 1 - \lambda_1 \sigma) + \delta_b - s^\star\right]^2} > 0, \tag{121}$$

since the numerator is strictly positive in the fiscal regime ($\delta_b < b^\star + s^\star$) and the denominator is squared. Hence β_s strictly increases in λ_1. Since $\beta_s(0) = \beta_s^{\text{base}} < 0$ and $\lambda_1 < 0$ implies $\beta_s(\lambda_1) < \beta_s(0)$, we have $|\beta_s(\lambda_1)| > |\beta_s^{\text{base}}|$. □

The amplification operates entirely through the determinacy denominator:

$$\frac{\beta_s(\lambda_1)}{\beta_s^{\text{base}}} = \frac{b^\star(\rho - 1) + \delta_b - s^\star}{b^\star(\rho - 1 - \lambda_1 \sigma) + \delta_b - s^\star} > 1. \tag{122}$$

A more state-dependent price of risk brings the system closer to the explosive boundary identified in (120), amplifying the response of debt and surpluses to any given shock.

The government budget identity continues to yield:

$$\pi_{t+1} - \mathbb{E}_t \pi_{t+1} = -\frac{\beta_s}{1 + \Omega \rho_\pi} \varepsilon_{t+1}, \tag{123}$$

but β_s is now given by (119). With $\lambda_1 < 0$ and $|\beta_s|$ amplified by Proposition 2, the inflation response to a deficit shock is larger than in the baseline fiscal regime. The amplification grows without bound as λ_1 approaches the fragility threshold $\lambda_1^\star$.

The conditional covariance between the SDF and the real HPR on government debt is:

$$\text{Cov}_t(m_{t+1}, r_{g,t+1}) = -\lambda_t \beta_s = -(\lambda_0 + \lambda_1 x_t)\beta_s. \tag{124}$$

Both the unconditional level of the covariance and its within-regime conditional variation depend on the price of risk parameters. The level reflects the amplified $|\beta_s|$ from Proposition 2 multiplied by the average price of risk λ_0. The conditional variation comes from $\lambda_1 x_t$: in the fiscal regime with $\lambda_1 < 0$, a deterioration in the fiscal state ($x_t > 0$) makes λ_t more negative, which makes the covariance more negative, raising the conditional risk premium on government debt. This provides a mechanism for the within-regime variation in the rolling covariance that we document empirically, and in particular

rationalises the finding that the post-pandemic covariance reading is more negative than any previous fiscal-regime episode.

Monetary regime. In the monetary regime, active monetary policy ($\rho_\pi > 1$) pins down inflation independently of the surplus process. The unexpected real return on the surplus claim is zero, so $\beta_s = 0$ and:

$$\mathrm{Cov}_t(m_{t+1}, r_{g,t+1}) = 0, \tag{125}$$

regardless of λ_t. The monetary regime is inherently certainty-equivalent: the policy rule pins down real bond returns exactly, and so the price of risk plays no role in either determinacy or dynamics. Proposition 1 therefore continues to hold in terms of the sign of the covariance, and the regime partition of the model is preserved.

A.6 Inflation Expectations in Each Regime

In the monetary regime, it always holds that:

$$\mathbb{E}_t \pi_{t+j} = \pi^\star \quad \forall t, j$$

We now show that inflation expectations are sensitive to deficit shocks in the fiscal regime. Define:

$$b_\pi \equiv -\frac{\beta_s}{1 + \Omega\rho_\pi},$$

so that in the fiscal regime inflation satisfies:

$$\pi_{t+1} = E_t[\pi_{t+1}] + b_\pi \varepsilon_{t+1}.$$

Using the one-period nominal bond Euler equation:

$$e^{-i_t} = E_t\left[e^{m_{t+1} - \pi_{t+1}}\right],$$

together with:

$$m_{t+1} = -\mu - \frac{\lambda^2}{2} - \lambda\varepsilon_{t+1},$$

we obtain:

$$e^{-i_t} = E_t\left[\exp\left(-\mu - \frac{\lambda^2}{2} - E_t[\pi_{t+1}] - (\lambda + b_\pi)\varepsilon_{t+1}\right)\right].$$

By lognormality:

$$E_t\left[e^{-(\lambda + b_\pi)\varepsilon_{t+1}}\right] = \exp\left(\frac{(\lambda + b_\pi)^2}{2}\right),$$

so:

$$i_t = \mu + E_t[\pi_{t+1}] - \lambda b_\pi - \frac{b_\pi^2}{2}.$$

Combining this with the Taylor rule:

$$i_t = i^\star + \rho_\pi(\pi_t - \pi^\star), \qquad i^\star = \mu + \pi^\star,$$

gives:

$$E_t[\pi_{t+1}] = \rho_\pi \pi_t + (1-\rho_\pi)\pi^\star + \lambda b_\pi + \frac{b_\pi^2}{2}.$$

Hence inflation follows the AR(1) process:

$$\pi_{t+1} = c_\pi + \rho_\pi \pi_t + b_\pi \varepsilon_{t+1},$$

where:

$$c_\pi \equiv (1-\rho_\pi)\pi^\star + \lambda b_\pi + \frac{b_\pi^2}{2}.$$

Iterating forward yields:

$$E_t[\pi_{t+j}] = \rho_\pi^j \pi_t + \sum_{h=0}^{j-1} \rho_\pi^h c_\pi = \rho_\pi^j \pi_t + \frac{1-\rho_\pi^j}{1-\rho_\pi} c_\pi.$$

Substituting for c_π:

$$E_t[\pi_{t+j}] = \rho_\pi^j \pi_t + \frac{1-\rho_\pi^j}{1-\rho_\pi}\left[(1-\rho_\pi)\pi^\star + \lambda b_\pi + \frac{b_\pi^2}{2}\right].$$

Equivalently:

$$E_t[\pi_{t+j}] = \bar{\pi}_F + \rho_\pi^j(\pi_t - \bar{\pi}_F),$$

where:

$$\bar{\pi}_F = \pi^\star + \frac{\lambda b_\pi + \frac{1}{2} b_\pi^2}{1-\rho_\pi}.$$

Finally, substituting:

$$b_\pi = -\frac{\beta_s}{1+\Omega\rho_\pi},$$

gives:

$$E_t[\pi_{t+j}] = \bar{\pi}_F + \rho_\pi^j(\pi_t - \bar{\pi}_F),$$

with:

$$\bar{\pi}_F = \pi^\star + \frac{-\lambda \frac{\beta_s}{1+\Omega\rho_\pi} + \frac{1}{2}\left(\frac{\beta_s}{1+\Omega\rho_\pi}\right)^2}{1-\rho_\pi}.$$

Therefore:

$$\frac{\partial E_t(\pi_{t+j})}{\partial \epsilon_t} = \rho_\pi^j \frac{\partial \pi_t}{\partial \epsilon_t} = \rho_\pi^j b_\pi = \rho_\pi^j \left(-\frac{\beta_s}{1+\Omega\rho_\pi}\right) > 0.$$

In cumulative expected inflation terms for horizon k:

$$\frac{\partial}{\partial \epsilon_t} E_t\left(\sum_{h=1}^{j} \pi_{t+h}\right) = -\frac{\beta_s}{1+\Omega\rho_\pi} \cdot \frac{\rho_\pi(1-\rho_\pi^j)}{1-\rho_\pi} > 0.$$

B Algorithm For Parametric SDF Estimation

We now describe our procedure to implement the parametric procedure. Throughout, we assume that the factors have a constant conditional mean, denoted by $\overline{F}_j$. An infeasible procedure would involve using GMM with the following moment conditions:

$$\mathbb{E}\left\{\left(\left[R_{0,t}^{-1} + \sum_{j=1}^{K} \delta_j \left(F_{j,t+1} - \overline{F}_j\right)\right] R_{i,t+1} - 1\right) \tilde{Z}_t\right\}, \tag{126}$$

where $\tilde{Z}_t = [1, Z_t']'$. This yields $N \times (L+1)$ moment conditions in total. This is infeasible since it requires a measure of $R_{0,t}$. We instead outline a feasible two-step GMM approach which jointly estimates both γ and δ:

1. **Specify the parameter vector**

$$\theta \;=\; \big(\gamma;\, \delta_1, \ldots, \delta_K\big) \in \mathbb{R}^{(L+1+K)\times 1}.$$

   Choose starting values $\theta^{(0)}$.

2. **GMM Step 1 (initial weighting, $W = I$):**

   (a) *Compute the zero–beta rate and SDF for a given θ:*

$$R_{0,t} \;=\; \gamma_0 + \sum_{l=1}^{L} \gamma_l Z_{l,t}, \qquad M_{t+1} \;=\; \big(R_{0,t}\big)^{-1} + \sum_{j=1}^{K} \delta_j \big(F_{j,t+1} - \bar{F}_j\big). \tag{127}$$

   (b) *Estimate betas and residuals for each test asset $i = 1, \ldots, N$:*

$$R_{i,t+1} - R_{0,t} \;=\; \alpha_i + \beta_i' F_{t+1} + \varepsilon_{i,t+1}.$$

Stack $\beta = \big(\beta_1, \ldots, \beta_N\big)'$ and collect residuals.

(c) *Estimate the residual covariance $\widehat{\Sigma}_R$ via the Ledoit and Wolf (2017) method of analytical shrinkage.*

(d) *Compute minimum–variance, factor–neutral weights*

$$\Omega \;=\; \big[\mathbf{1}_N \;\; \beta\big], \qquad w \;=\; \widehat{\Sigma}_R^{-1}\Omega\Big(\Omega'\widehat{\Sigma}_R^{-1}\Omega\Big)^{-1}\begin{bmatrix}1\\ 0_K\end{bmatrix}.$$

(e) *Form the moments at each t:*

$$\begin{aligned}
g_{0,t}(\theta) &= \big(M_{t+1}R_{0,t} - 1\big)\,\tilde{Z}_t, && \text{(Zero–beta pricing)}\\
g_{1,t}(\theta) &= \big(w'R_{t+1} - R_{0,t}\big)\,\tilde{Z}_t, && \text{(Replicating portfolio pricing)}\\
g_{2,i,t}(\theta) &= \big(M_{t+1}R_{i,t+1} - 1\big)\,\tilde{Z}_t, \quad i = 1, \ldots, N, && \text{(Asset pricing, stacked).}
\end{aligned}$$

(f) *Initial objective ($W = I$):*

$$\bar{g}_T(\theta) \;=\; \frac{1}{T}\sum_{t=1}^{T} g_t(\theta), \qquad J_T^{(1)}(\theta) \;=\; \bar{g}_T(\theta)'\bar{g}_T(\theta).$$

(g) *Preliminary estimate:*

$$\widehat{\theta}^{(1)} \;=\; \arg\min_{\theta}\; J_T^{(1)}(\theta).$$

3. **GMM Step 2 (optimal weighting, $W = \widehat{S}^{-1}$):**

(a) *Estimate the long–run covariance of moments at $\widehat{\theta}^{(1)}$ (HAC/Newey–West):*

$$\widehat{S} \;=\; \Gamma_0 + \sum_{\ell=1}^{L_b} w_\ell\big(\Gamma_\ell + \Gamma_\ell'\big), \qquad \Gamma_\ell \;=\; \frac{1}{T}\sum_{t=\ell+1}^{T}\big(g_t(\widehat{\theta}^{(1)}) - \bar{g}_T(\widehat{\theta}^{(1)})\big)\big(g_{t-\ell}(\widehat{\theta}^{(1)}) - \bar{g}_T(\widehat{\theta}^{(1)})\big)'.$$

Choose a bandwidth L_b and kernel weights w_ℓ.

(b) *Set the optimal weight*

$$W^\star \;=\; \widehat{S}^{-1}.$$

(c) *Define the optimal–weight objective and re–estimate:*

$$J_T^{(2)}(\theta) \;=\; \bar{g}_T(\theta)'W^\star\,\bar{g}_T(\theta), \qquad \widehat{\theta}^{(2)} \;=\; \arg\min_{\theta}\; J_T^{(2)}(\theta).$$

We iterate this procedure until the parameter estimates converge, setting $L_b = 6$ for the Newey-West bandwidth.

We give intuition behind each of the three types of moment included in the GMM objective. The first, the zero-beta pricing condition, enforces that the zero-beta portfolio is correctly priced by the

SDF, tying these two objects together in a theoretically consistent manner. The second, the replicating portfolio pricing condition, enforces that the expected return on a minimum-variance, factor-neutral portfolio (constructed using weights, $w(\theta)$) is equal to the zero-beta rate. This is essentially a no-arbitrage condition which demands internal consistency within the model. The third set of moments are the standard asset pricing moments, which enforce that the fundamental asset pricing equation holds for each of the test assets. Each moment is multiplied by instruments Z_t to approximate conditional pricing restrictions and exploit additional information for identification. In this procedure, the time-series regressions are placed outside of the GMM moment conditions, which is equivalent to including the relevant OLS moments, but weighting them very heavily. Henceforth, we denote M_{t+1}^P as the estimated SDF obtained from this parametric approach.

C Additional Material for the Main Results

C.1 Parameter Estimates for the Parametric Approach

The table below presents estimates of the γ and δ parameters from the benchmark parametric approach with two-step GMM.

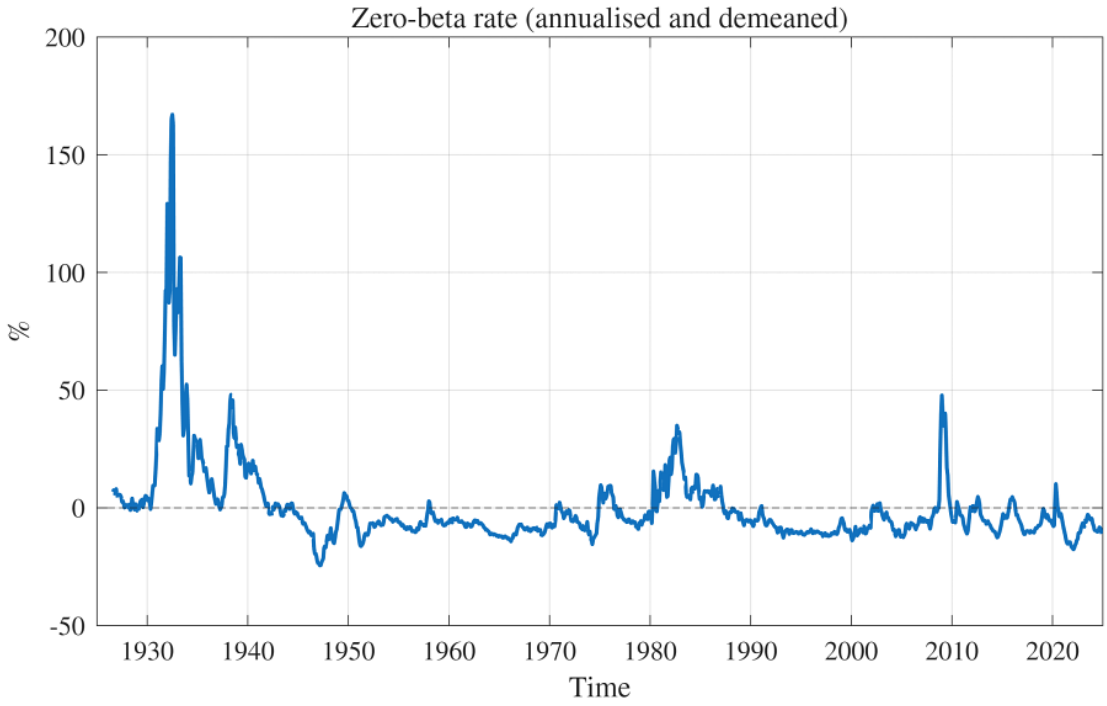
Table A1: Parameter estimates for parametric approach.

Parameter	Estimate	Std. Error	t-stat
γ_0	1.01	0.00	2278.21
γ_{treas}	0.61	0.28	2.19
γ_{infl}	-1.24	0.17	-7.42
γ_{ts}	-0.04	0.03	-1.76
γ_{cape}	0.09	0.02	3.86
γ_{bs}	1.32	0.06	22.12
δ_{mkt}	-1.99	0.14	-14.01
δ_{hml}	-0.85	0.24	-3.53
δ_{smb}	0.50	0.34	1.47

Note: The instruments, in order, are a constant, the 1-month Treasury Bill yield, the 12-month moving average of CPI inflation, the term spread, the inverse Shiller CAPE, and the bond spread. See text for full details. The factors are the market excess return over the zero-beta rate, the HML factor and the SMB factor.

C.2 Plot of Estimated Zero-beta Rate

Figure A1: Estimated Zero-beta rate



Note: This plots the demeaned and annualised zero-beta rate estimated as a by-product of the parametric approach to estimating the SDF.

D Robustness Tests

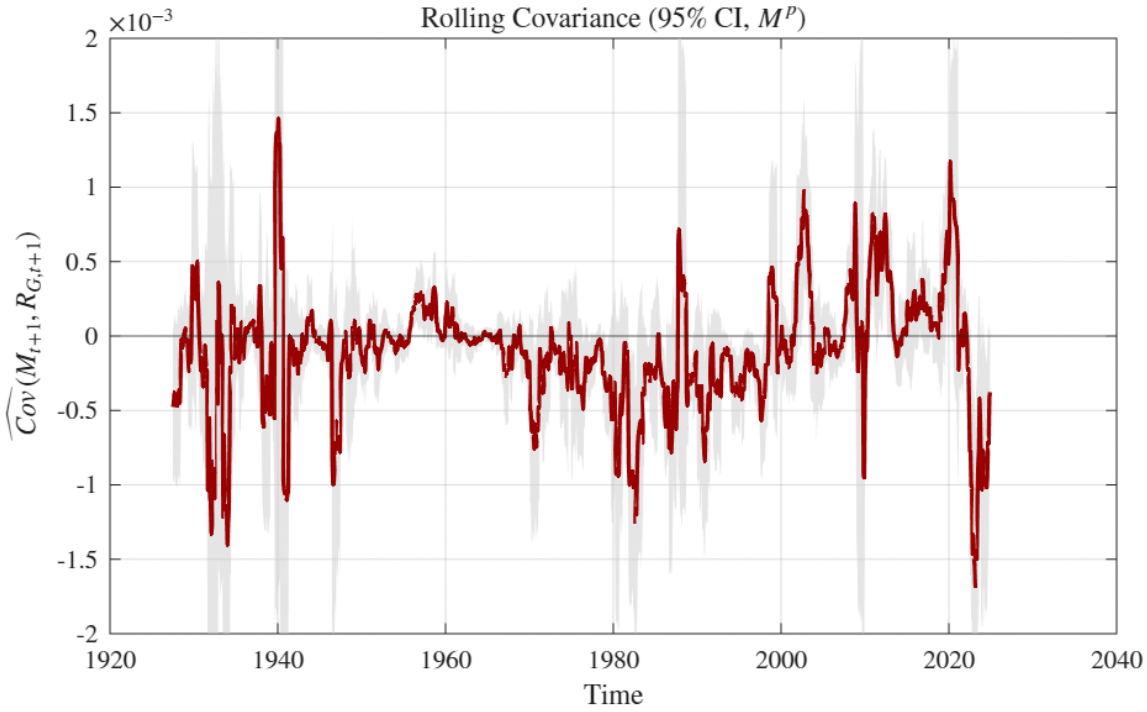
D.1 Alternative Methods for Computing Time-varying Covariance

We consider alternative ways of examining time-variation in the covariance between the SDF and the real HPR on government debt. We look at alternative window lengths in the rolling covariance estimation, a more data-driven procedure which makes use of cross-validation, and a GARCH model.

D.1.1 Alternative Window Lengths

We consider a shorter window of length one year, as well as a longer window of length six years, for the backward-looking rolling covariance. The figures below plots these.

Figure A2: Shorter window for backwards-looking covariance between parametric SDF and real HPR on government debt.



Note: This plots the backwards-looking covariance between the parametric SDF and the real HPR on government debt, where the window length is set to one year. The grey shaded area plots 95% confidence intervals.

D.1.2 Alternative Data-driven Procedure for Conditional Covariance

Instead of an equal-weighted, backwards-looking approach to computing the covariance term, we consider a more data-driven approach.

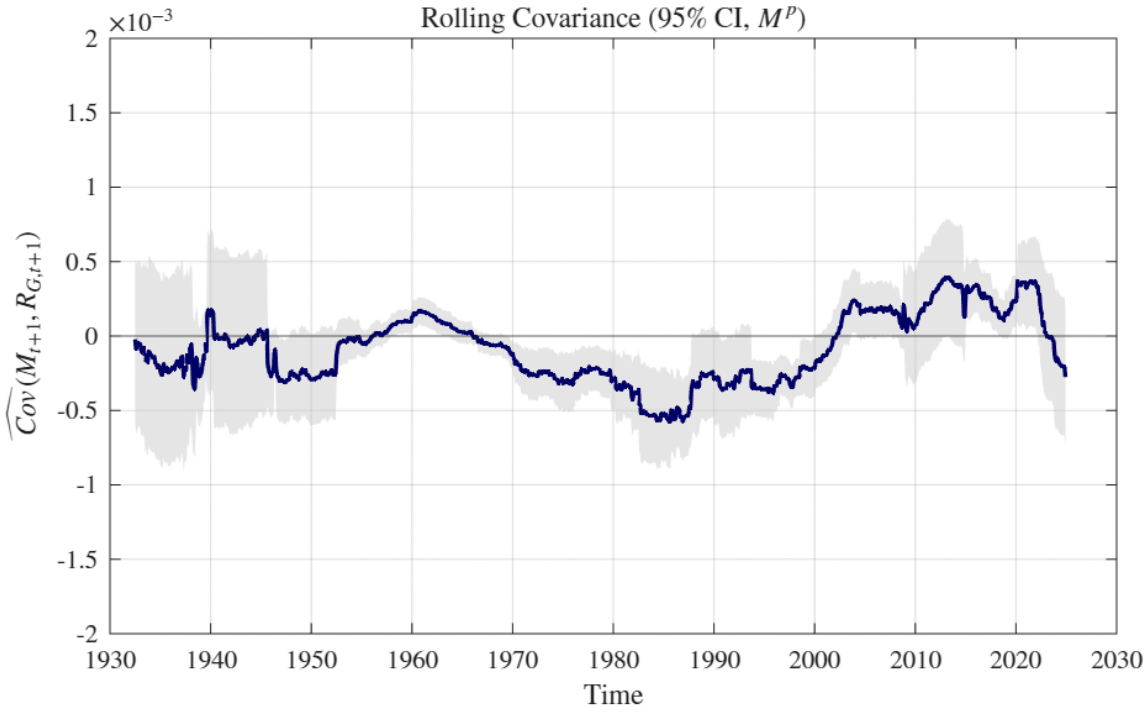
Let $R_t^g \equiv \widetilde{R}_t^g / \Pi_t$ denote the real *gross* holding–period return on government debt and let M_t be our SDF proxy. We seek an estimate of the conditional covariance

$$\sigma_{MR,t} \equiv \mathrm{Cov}_t(M_t, R_t^g),$$

updated at the monthly frequency. We employ a bivariate exponentially–weighted moving average (EWMA) for $(M_t, R_t^g)'$, with the smoothing parameter $\lambda \in (0,1)$ chosen by out–of–sample quasi–likelihood (QLIKE) cross–validation.

Define $w_t \equiv \begin{pmatrix} M_t \\ R_t^g \end{pmatrix}$, a time–varying mean $\mu_t \in \mathbb{R}^2$, demeaned observation $\tilde{w}_t \equiv w_t - \mu_t$, and a conditional

Figure A3: Longer window for backwards-looking covariance between parametric SDF and real HPR on government debt.



Note: This plots the backwards-looking covariance between the parametric SDF and the real HPR on government debt, where the window length is set to six years. The grey shaded area plots 95% confidence intervals.

covariance matrix $\Sigma_t \in \mathbb{R}^{2\times 2}$. For a given λ we update

$$\mu_t = \lambda\,\mu_{t-1} + (1-\lambda)\,w_t, \tag{128}$$

$$\Sigma_t = \lambda\,\Sigma_{t-1} + (1-\lambda)\,\tilde{w}_t\tilde{w}_t^{\top}. \tag{129}$$

We initialise (μ_0, Σ_0) with sample moments from a short burn–in. Our empirical object is the $(1,2)$ entry of Σ_t,

$$\widehat{\sigma}_{MR,t} \;\equiv\; e_1^{'}\Sigma_t e_2,$$

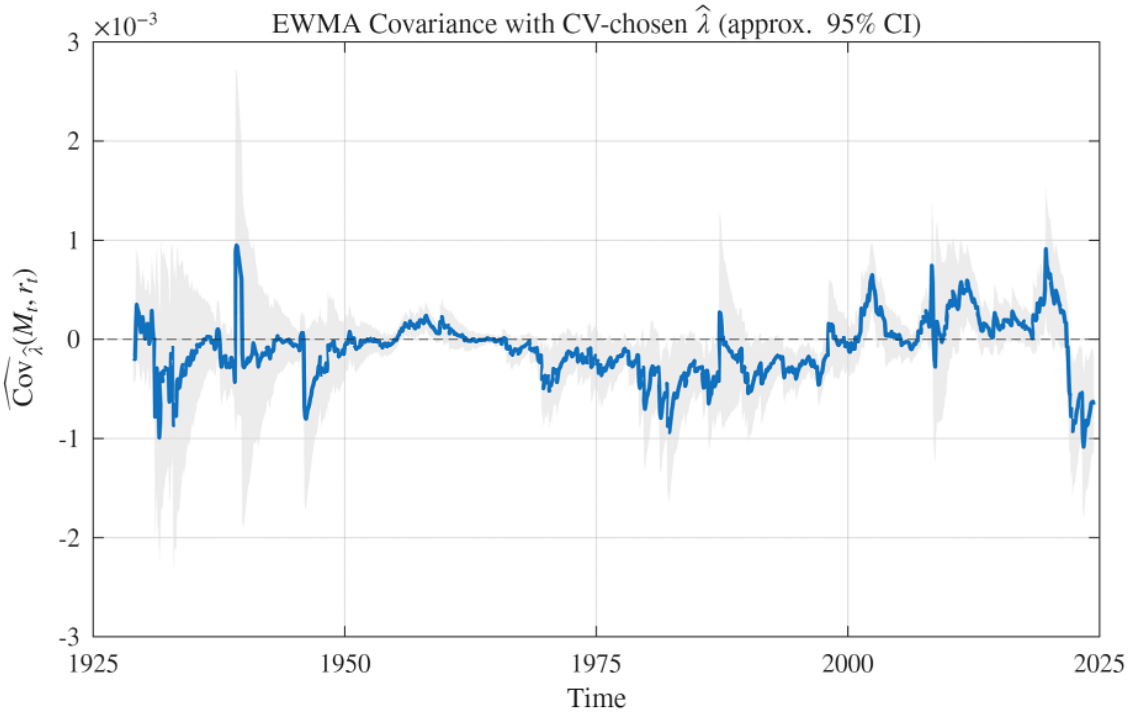
where e_j is the j-th unit vector. Rather than fixing λ ad hoc, we set it by minimising the average one–step–ahead QLIKE score on a hold–out comprising the last 15% of the sample. Let $\bar{w}_{t-1}$ be the expanding–sample mean up to $t-1$ and $d_t \equiv w_t - \bar{w}_{t-1}$ the corresponding de–meaned observation. Given information through $t-1$, the EWMA covariance forecast is

$$H_t(\lambda) \;=\; \lambda\,H_{t-1}(\lambda) + (1-\lambda)\,d_{t-1}d_{t-1}^{'}, \quad H_b(\lambda) = \mathrm{Cov}\big(d_1,\ldots,d_b\big), \tag{130}$$

and the QLIKE loss for realised d_t and forecast H_t is

$$\ell_t(\lambda) \;=\; \tfrac{1}{2}\Big(\mathrm{logdet}\,H_t(\lambda) + d_t^{'}H_t(\lambda)^{-1}d_t\Big)\,. \tag{131}$$

Figure A4: Exponentially-weighted covariance between parametric SDF and real HPR on government debt.



Note: This plots the exponentially-weighted covariance between the parametric SDF and the real HPR on government debt, where the smoothing parameter is chosen by out-of-sample cross-validation. See text for full details. The grey shaded area plots 95% confidence intervals computed using the delta method.

We choose

$$\widehat{\lambda} \in \arg\min_{\lambda \in (0,1)} \frac{1}{T_{\text{val}}} \sum_{t \in \mathcal{T}_{\text{val}}} \ell_t(\lambda), \tag{132}$$

where $\mathcal{T}_{\text{val}}$ indexes the validation window. With $\widehat{\lambda}$ in hand, we run (128)–(129) on the full sample and obtain $\{\widehat{\sigma}_{Mr,t}\}_t$. Figure A4 plots this over time for the parametric SDF. This looks similar to the benchmark results in Figure 4.

D.1.3 GARCH Model

To capture time-varying second moments, we employ the Dynamic Conditional Correlation (DCC) GARCH model of Engle (2002). Let $\mathbf{y}_t = (M_t, R_{g,t})'$ denote the vector of our variables of interest at time t. Conditional on past information $\mathcal{F}_{t-1}$, we assume:

$$\mathbf{y}_t \mid \mathcal{F}_{t-1} \sim \mathcal{N}(\mathbf{0}, \mathbf{H}_t),$$

where $\mathbf{H}_t$ is the conditional covariance matrix. The DCC specification decomposes $\mathbf{H}_t$ as:

$$\mathbf{H}_t = \mathbf{D}_t \mathbf{R}_t \mathbf{D}_t,$$

with $\mathbf{D}_t = \text{diag}(\sigma_{M,t}, \sigma_{R,t})$ containing conditional standard deviations from univariate GARCH(1,1) models:

$$\sigma^2_{i,t} = \omega_i + \alpha_i \varepsilon^2_{i,t-1} + \beta_i \sigma^2_{i,t-1}, \quad i \in \{M, R\}.$$

The dynamic correlation matrix $\mathbf{R}_t$ is obtained by standardising residuals $z_t = \mathbf{D}_t^{-1}\varepsilon_t$ and updating:

$$\mathbf{Q}_t = (1 - \alpha - \beta)\bar{\mathbf{Q}} + \alpha z_{t-1} z'_{t-1} + \beta \mathbf{Q}_{t-1},$$

where $\bar{\mathbf{Q}}$ is the unconditional correlation of z_t. Finally, $\mathbf{R}_t$ is the correlation matrix implied by $\mathbf{Q}_t$:

$$\mathbf{R}_t = \text{diag}(\mathbf{Q}_t)^{-1/2} \mathbf{Q}_t \text{diag}(\mathbf{Q}_t)^{-1/2}.$$

This structure allows for time-varying covariances while maintaining parsimony and positive definiteness. Figure A5 plots the covariance estimated via this method.

D.2 Alternative Cross-Sections

We consider varying the set of test assets used in estimation, for both the parametric and non-parametric procedure.

D.2.1 Larger Cross-Section of Equities

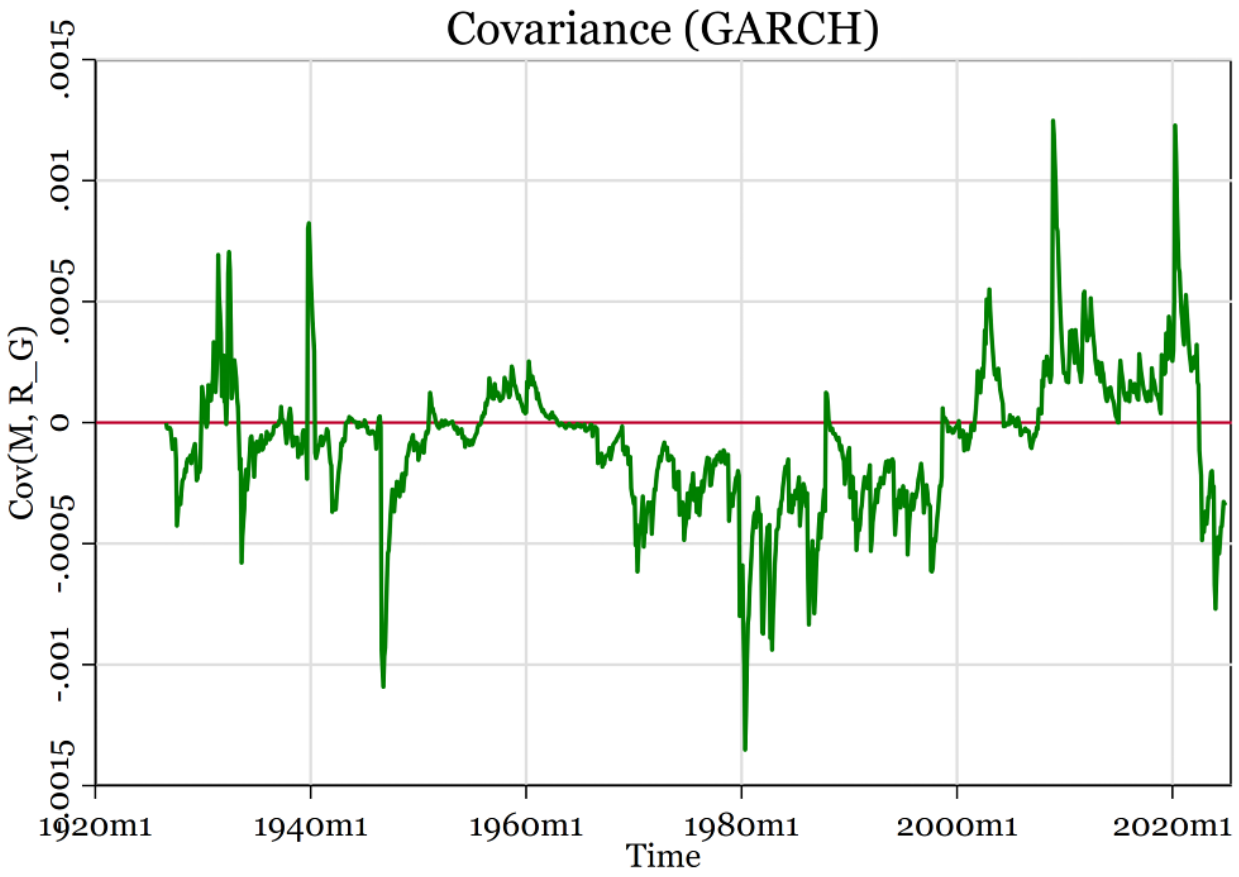
We add 25 portfolios sorted by size and momentum, as well as 6 portfolios sorted by size and dividend yield, to our original set of test assets. This gives us 88 portfolios in total. The sample is slightly different at January 1928 - December 2024 due to the availability of this portfolio data.

Figure A6 and Figure A7 plot the rolling covariances for these respective SDF series. Both imply that debt did not turn risky until the late 1960s, rather than the earlier part of this decade. They both pick up the move from a safe to a risky regime which occurs post-2020.

D.3 Alternative Factors

We look at perturbations of the set of factors which are used in the parametric approach to estimating the SDF.

Figure A5: Covariance from GARCH model between parametric SDF and real HPR on government debt.

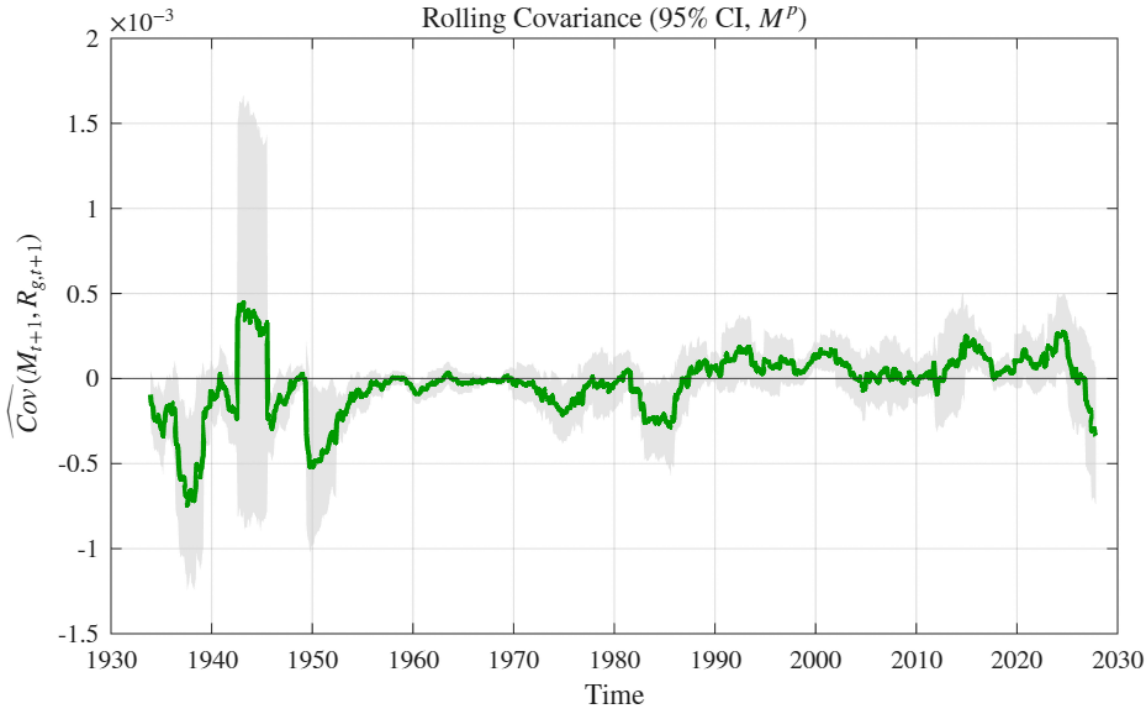


Note: This plots the covariance obtained from a GARCH model (Engle, 2002) between the parametric SDF and the real HPR on government debt. See text for full details.

D.3.1 Betting Against Beta Factor of Frazzini and Pedersen (2014)

Frazzini and Pedersen (2014) find that the security markets line is flat, implying that a portfolio which goes long low-beta assets and short high-beta assets earns a risk premium. We include their betting against beta (BAB) factor alongside the three Fama-French factors when using our parametric approach to estimate the SDF. Figure A8 depicts the rolling covariance with this estimated SDF. Table A2 presents parameter estimates. The correlation between the SDF with the BAB factor added and the benchmark SDF is 0.74, while it has a correlation of 0.33 with the nonparametric SDF.

Figure A6: Rolling covariance between parametric SDF and real HPR on government debt, larger cross-section of equities.



Note: This plots the three-year backward-looking covariance between the parametric SDF and the real HPR on government debt, with the larger cross-section of equities used during estimation of the SDF. The grey shaded area plots 95% confidence intervals.

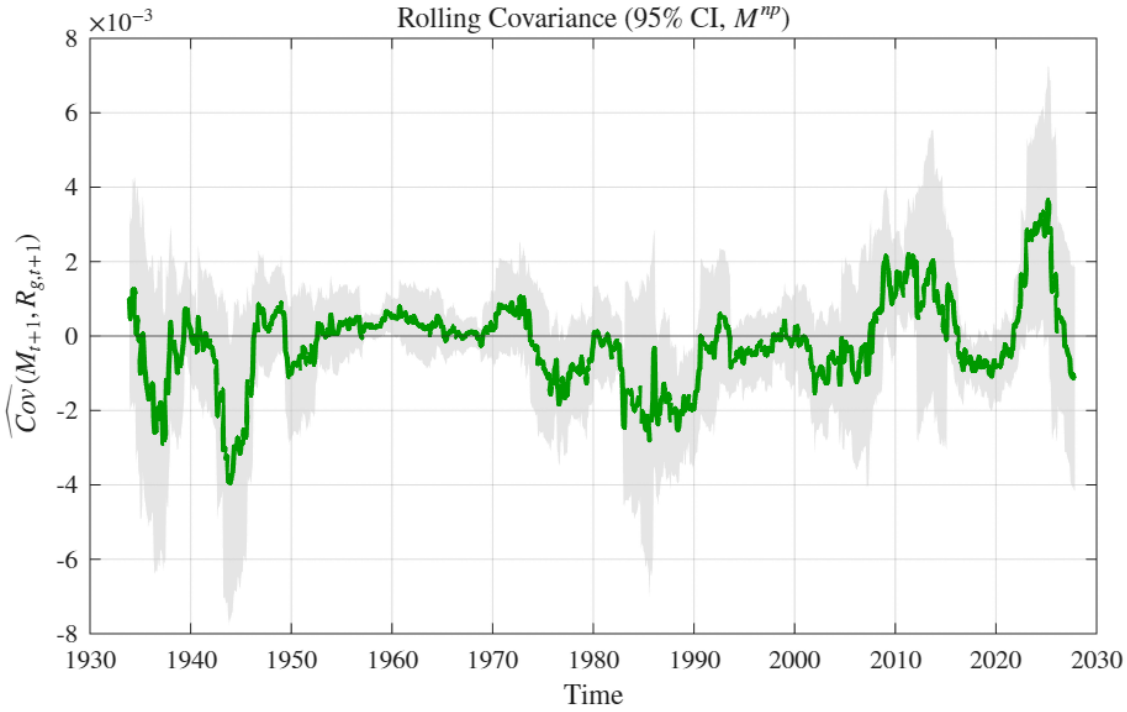
D.3.2 Fama and French (2015) Five Factor Model

We consider using the five factor model of Fama and French (2015), which adds the profitability factor (RMW) and the investment factor (CMA) to the Fama and French (1993) three factor model. A considerable drawback of doing this is that the five factors are only available from August 1963 onwards, which both reduces statistical power in estimation and also precludes us from looking at the riskiness of government debt in the period prior to this. Figure A9 presents the rolling covariance, while Table A3 presents the parameter estimates. The covariance once again implies that debt was risky in the 1965-1985 period, but the confidence intervals are wider. The covariance does not change significantly at the end of this period and does not turn positive until the late 2000s. As in the main results, the covariance turns negative in the post-pandemic period. The SDF obtained from this approach has a correlation of 0.06 with the baseline parametric SDF and a correlation of -0.04 with the nonparametric SDF.

D.4 Adding Consumption Growth as a Factor

We also consider a factor model with the three Fama-French factors as well as real consumption growth (nondurables and services) added in the spirit of the CCAPM. This requires moving to the quar-

Figure A7: Rolling covariance between nonparametric SDF and real HPR on government debt, larger cross-section of equities.



Note: This plots the three-year backward-looking covariance between the nonparametric SDF and the real HPR on government debt, with the larger cross-section of equities used during estimation of the SDF. The grey shaded area plots 95% confidence intervals.

terly frequency given that consumption is not available at a monthly rate. The sample is reduced to 1947Q2:2024Q4 also. Both of these significantly reduce the length of the sample, inhibiting the degree of statistical power. Figure A10 plots the rolling covariance, while Table A4 plots the parameter estimates. Both are qualitatively similar to the baseline results. The consumption growth factor is not found to be statistically significant.

D.5 Alternative Instruments

D.5.1 Adding the Atanasov et al. (2020) Cyclical Consumption Variable as an Instrument

Alongside our regular set of instruments, we also experiment with including the Atanasov et al. (2020) cyclical consumption variable. This captures deviations of consumption from its trend, in line with the spirit of the habits model of Cochrane and Campbell (1999). This requires moving to the quarterly frequency, and the sample ends up being 1953Q4:2024Q4. Figure A11 depicts the rolling covariance, while Table A5 presents the parameter estimates.

Table A2: Parameter estimates for parametric approach with BAB factor added.

Parameter	Estimate	Std. Error	t-stat
γ_0	1.01	0.00	1952.48
γ_{treas}	1.53	0.52	2.95
γ_{infl}	-1.43	0.22	-6.62
γ_{ts}	-0.04	0.05	0.77
γ_{cape}	0.13	0.03	5.42
γ_{bs}	0.67	0.09	7.26
δ_{mkt}	-2.51	0.29	-8.56
δ_{hml}	-1.04	0.32	-3.28
δ_{smb}	0.32	0.44	0.72
δ_{bab}	-3.61	0.40	-8.84

Note: The instruments, in order, are a constant, the 1-month Treasury Bill yield, the 12-month moving average of CPI inflation, the term spread, the inverse Shiller CAPE, and the bond spread. See text for full details. The factors are the market excess return over the zero-beta rate, the HML factor, the SMB factor and the BAB factor of Frazzini and Pedersen (2014).

D.6 Alternative SDF Estimation Methods

D.6.1 One-Step GMM

Rather than using two-step GMM in the parametric approach, we consider using one-step GMM with an identity weight matrix. Figure A12 depicts the rolling covariance, while Table A6 presents parameter estimates. The parameter estimates are broadly similar, albeit with higher standard errors. The rolling covariance plot looks extremely similar to the baseline.

D.6.2 Time-Varying Risk Prices

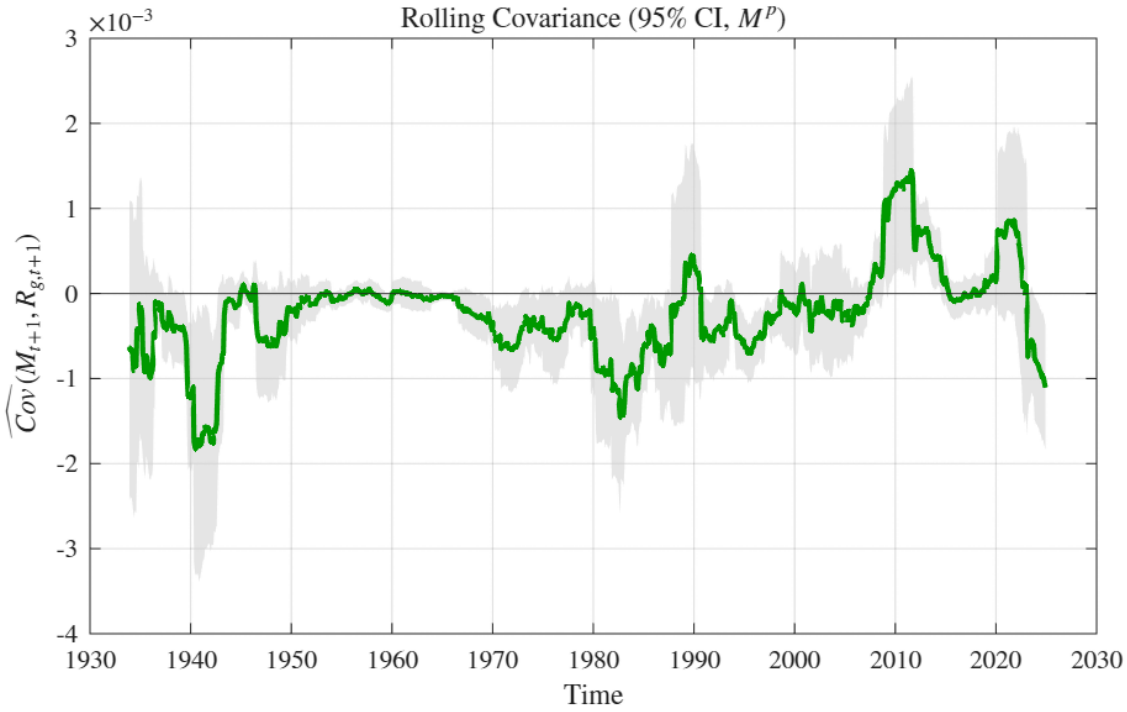
In our benchmark parametric specification, we assumed that the risk prices associated with the factors, δ_j, were constant, such that all time-variation in the SDF was driven by the conditional zero-beta rate. As a robustness exercise, we extend the model to allow the risk prices to vary conditionally on the same set of instruments, Z_t, used to model the zero-beta rate.

We specify the time-varying risk price for factor j, denoted $\delta_{j,t}$, as an affine function of the instrument vector:

$$\delta_{j,t} = \delta_{j,0} + \boldsymbol{\delta}'_{j,Z} Z_t \tag{133}$$

where $\delta_{j,0}$ represents the unconditional mean risk price and $\boldsymbol{\delta}_{j,Z}$ is an $L \times 1$ vector of sensitivity parameters capturing the response of the risk price to the instruments.

Figure A8: Covariance between parametric SDF and real HPR on government debt, BAB factor added.



Note: This plots the backwards-looking covariance between the parametric SDF and the real HPR on government debt, where the BAB factor of Frazzini and Pedersen (2014) is added. See text for full details. The grey shaded area plots 95% confidence intervals.

Substituting this into our linear factor model, the extended SDF takes the form:

$$M_{t+1} = R_{0,t}^{-1} + \sum_{j=1}^{K} \left(\delta_{j,0} + \boldsymbol{\delta}'_{j,Z} Z_t\right) \left(F_{j,t+1} - \bar{F}_j\right) \tag{134}$$

where $R_{0,t} = \gamma_0 + \boldsymbol{\gamma}' Z_t$.

This specification significantly expands the parameter space. The estimation proceeds via the same two-step GMM procedure outlined in Section 3.1. Identification of the additional sensitivity parameters, $\boldsymbol{\delta}_{j,Z}$, is achieved through the interaction of the instruments and the factor innovations within the moment conditions:

$$\mathbb{E}_t \left[\left(M_{t+1}(\theta) R_{i,t+1} - 1\right) \otimes \tilde{Z}_t \right] = 0 \tag{135}$$

where $\tilde{Z}_t = [1, Z_t']'$. The cross-products of the instruments in Z_t with the factor surprises pin down the time-variation in the factor loadings.

The parameter estimates are omitted for brevity. Figure A13 plots the moving covariance, which looks very similar to the baseline estimates.

Table A3: Parameter estimates for parametric approach, five Fama-French factors.

Parameter	Estimate	Std. Error	t-stat
γ_0	1.00	0.00	5806.51
γ_{treas}	0.39	0.15	2.54
γ_{infl}	-0.26	0.15	-1.79
γ_{ts}	0.06	0.02	2.53
γ_{cape}	-0.02	0.02	-0.87
γ_{bs}	0.52	0.06	8.73
δ_{mkt}	-4.09	1.39	-2.95
δ_{hml}	-6.78	3.02	-2.25
δ_{smb}	-1.64	1.84	-0.89
δ_{rmw}	-5.84	2.76	-2.12
δ_{cma}	-2.33	4.91	-0.48

Note: The instruments, in order, are a constant, the 1-month Treasury Bill yield, the 12-month moving average of CPI inflation, the term spread, the inverse Shiller CAPE, and the bond spread. See text for full details. The factors are the market excess return over the zero-beta rate, the HML factor, the SMB factor, the RMW factor, and the CMA factor.

D.6.3 Optimal-IV Sieve Estimation of a Conditional SDF

This section outlines a semiparametric estimator of a conditional stochastic discount factor (SDF) based on the optimal-instrument approach of Nagel and Singleton (2011). The method is useful when the SDF is conditionally affine in a set of factors, but the factor prices of risk are allowed to vary flexibly with observables.

Let $R_{i,t+1}$ denote gross real returns on the N test assets, $f_{t+1} \in \mathbb{R}^{K_f}$ the (demeaned) factor vector observed at $t+1$, and $Z_t \in \mathbb{R}^L$ a vector of conditioning instruments observed at t with $Z_{1,t} \equiv 1$ and all non-constant components demeaned. We consider a *conditionally affine* SDF of the form

$$m_{t+1} = \phi_{0,t} + \phi_{f,t}' f_{t+1}, \qquad \phi_{0,t} = a_0 + a_1' Z_t, \qquad \phi_{f,t} = b_0 + B_1 Z_t, \tag{136}$$

where $a_0 \in \mathbb{R}$, $a_1 \in \mathbb{R}^{L-1}$, $b_0 \in \mathbb{R}^{K_f}$, and $B_1 \in \mathbb{R}^{K_f \times (L-1)}$ are unknown parameters. Stacking terms yields a *completely affine* representation,

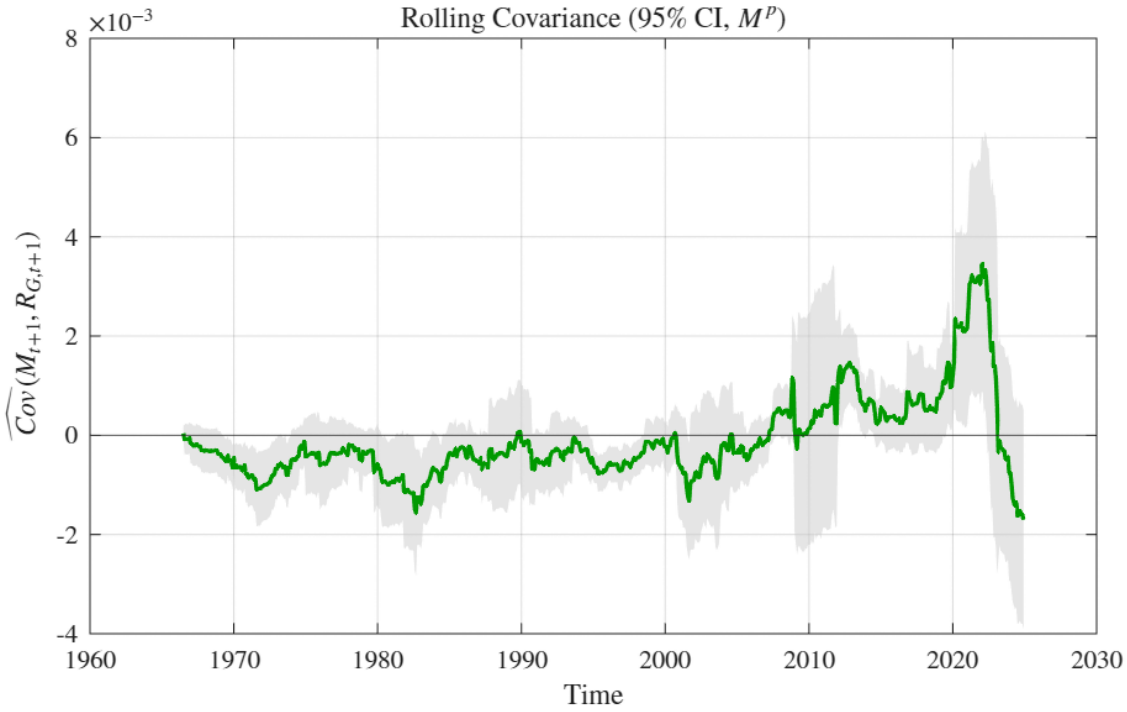
$$m_{t+1} = \theta' f_{t+1}^{\#}, \qquad f_{t+1}^{\#} \equiv \left[1,\ Z_{t,2:L}',\ f_{t+1}',\ (f_{t+1} \otimes Z_{t,2:L})'\right]', \tag{137}$$

for a parameter vector θ collecting $\{a_0, a_1, b_0, \mathrm{vec}(B_1)\}$ and pseudo-factors $f_{t+1}^{\#}$ comprising instruments, factors and their interactions (excluding interactions with the constant).

The conditional pricing restrictions are

$$\mathbb{E}[m_{t+1} R_{i,t+1} - 1 \mid Z_t] = 0, \qquad i = 1, \ldots, N, \tag{138}$$

Figure A9: Rolling covariance between parametric SDF and real HPR on government debt, five factor Fama-French model.



Note: This plots the three-year backward-looking covariance between the parametric SDF and the real HPR on government debt when the five factor Fama-French model is used. The grey shaded area plots 95% confidence intervals.

which we convert to unconditional moments using an instrument matrix A_t:

$$\mathbb{E}[A_t\, h_{t+1}(\theta)] = 0, \qquad h_{t+1}(\theta) \equiv m_{t+1}(\theta) R_{t+1} - \mathbf{1}, \tag{139}$$

where $R_{t+1} = (R_{1,t+1}, \ldots, R_{N,t+1})'$.

Among all admissible A_t, Nagel and Singleton (2011) show that the asymptotically efficient choice is

$$A_t^* = \Psi_{\theta,t}' \, \Sigma_t^{-1}, \qquad \Psi_{\theta,t} \equiv \mathbb{E}\left[\frac{\partial h_{t+1}(\theta)}{\partial \theta'} \middle| Z_t\right], \qquad \Sigma_t \equiv \mathbb{E}\left[h_{t+1}(\theta) h_{t+1}(\theta)' \mid Z_t\right]. \tag{140}$$

In a completely affine model $m_{t+1} = \theta' f_{t+1}^{\#}$, the Jacobian does not depend on θ and simplifies to

$$\Psi_{\theta,t}(i,k) = \mathbb{E}\left[R_{i,t+1}\, f_{k,t+1}^{\#} \mid Z_t\right]. \tag{141}$$

We estimate each element of (141) by projecting $R_{i,t+1} f_{k,t+1}^{\#}$ on a polynomial sieve in Z_t,

$$\mathbb{E}\left[R_{i,t+1} f_{k,t+1}^{\#} \mid Z_t\right] \approx \pi_{ik}' \, x(Z_t), \tag{142}$$

Table A4: Parameter estimates for parametric approach with consumption growth factor.

Parameter	Estimate	Std. Error	t-stat
γ_0	1.00	0.00	429.90
γ_{treas}	-1.31	0.67	-1.97
γ_{infl}	-10.75	3.12	-3.44
γ_{ts}	0.75	0.17	4.29
γ_{cape}	0.11	0.26	0.43
γ_{bs}	4.39	0.76	5.76
δ_{mkt}	-2.64	0.42	-6.32
δ_{hml}	-4.38	1.39	-3.16
δ_{smb}	2.98	1.35	2.21
δ_c	-1.01	5.72	-0.18

Note: The instruments, in order, are a constant, the 1-month Treasury Bill yield, the 12-month moving average of CPI inflation, the term spread, the inverse Shiller CAPE, and the bond spread. See text for full details. The factors are the market excess return over the zero-beta rate, the HML factor, the SMB factor, and real consumption growth.

Table A5: Parameter estimates for parametric approach, cyclical consumption of Atanasov et al. (2020) added as an instrument.

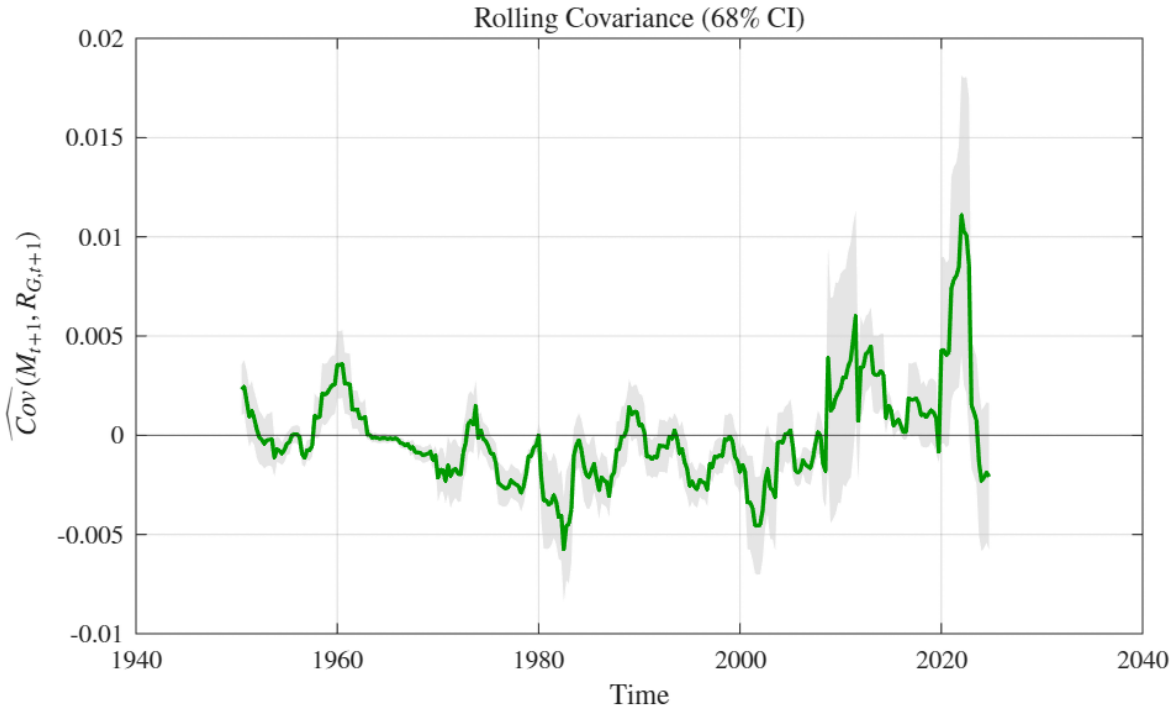
Parameter	Estimate	Std. Error	t-stat
γ_0	1.00	0.01	163.66
γ_{treas}	2.19	3.38	0.65
γ_{infl}	-1.97	8.67	-0.23
γ_{ts}	0.67	0.69	0.98
γ_{cape}	-1.10	1.21	-0.91
γ_{bs}	6.41	5.28	1.21
γ_{cc}	-0.51	0.25	-2.02
δ_{mkt}	-1.47	0.65	-2.26
δ_{hml}	-7.04	2.91	-2.42
δ_{smb}	-0.87	2.35	-0.37

Note: The instruments, in order, are a constant, the quarterly risk-free rate, the 12-month moving average of CPI inflation, the term spread, the inverse Shiller CAPE, the bond spread, and the cyclical consumption measure of Atanasov et al. (2020). See text for full details. The factors are the market excess return over the zero-beta rate, the HML factor, and the SMB factor.

where $x(Z_t)$ contains a constant, linear and quadratic terms in the demeaned instruments. We follow Nagel and Singleton (2011) in using a low-dimensional sieve to avoid overfitting and preserve finite-sample stability.

Given $\widehat{\Psi}_{\theta,t}$ and an estimate $\widehat{\Sigma}$ of Σ_t (we use the unconditional covariance of h_{t+1} as in Nagel and Singleton (2011), who find little evidence of strong state dependence), we form A_t^* from (140). Substituting A_t^* into (139) yields K optimal moment equations. Because $h_{t+1}(\theta)$ is linear in θ, the resulting estimator can be computed by iterating a sequence of linear systems until convergence.

Figure A10: Covariance between parametric SDF and real HPR on government debt, real cons. growth added as factor.



Note: This plots the three-year backward-looking covariance between the parametric SDF (where real cons. growth is added as a factor) and the real HPR on government debt. The grey shaded area plots 95% confidence intervals.

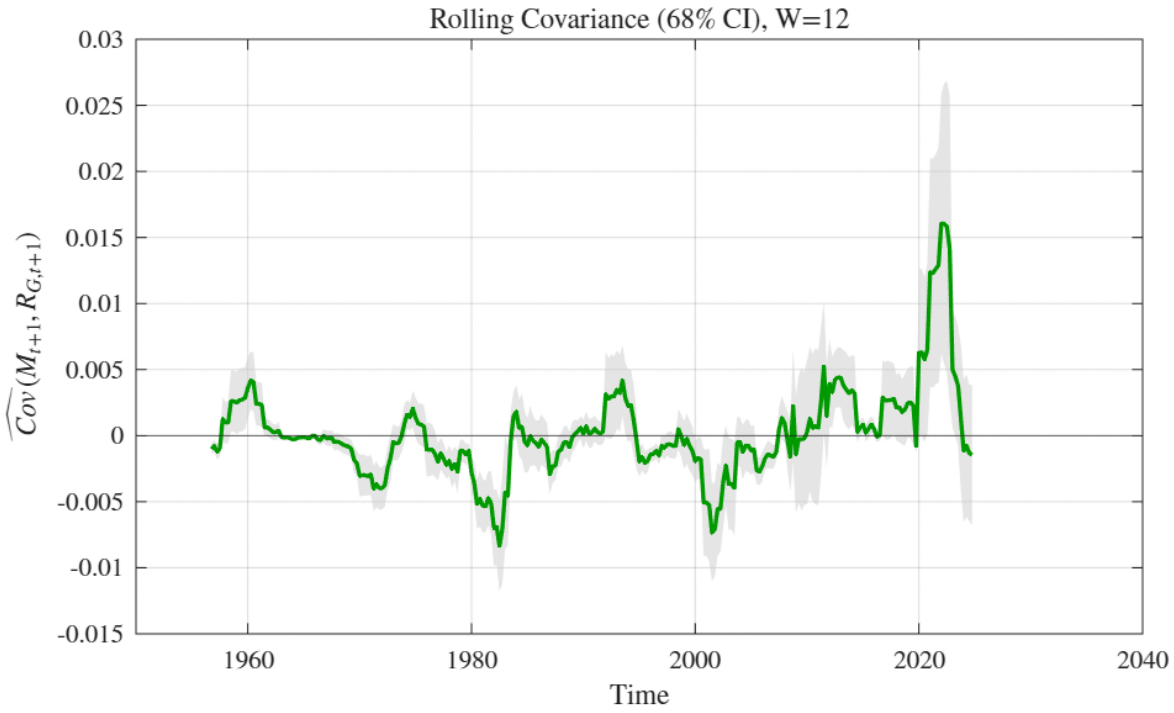
This approach has two practical benefits. First, it delivers *asymptotically efficient* estimation within the conditional affine class by weighting Euler errors with instruments that are tailored to the model and the conditional second moments of pricing errors. This reduces sampling noise in the recovered SDF, especially when the interaction basis $f^{\#}_{t+1}$ is high-dimensional. Second, the method provides *locally most powerful* specification tests against nearby alternatives: the optimal instruments focus estimation on the directions in return space that are most informative about conditional misspecification. In our setting, this is attractive because the key objects of interest (e.g. the covariance of the SDF with government-bond returns) are sensitive to weakly identified time-variation in risk prices.

Figure A14 plots the moving covariance. This yields less precise estimates, but largely detects the same regimes. It does not yet detect a risky debt regime post-2020.

D.7 GARCH Model for Real HPR

To further examine how $\sigma_t(r_{g,t+1})$ changes over time, and how this compares with our identified regimes, we also estimate a GARCH(1,1) model for $r_{g,t+1}$. The estimates of $\sigma_t(r_{g,t+1})$ which come from this are plotted below in Figure A15. These look very similar to those obtained from the trailing realised volatility.

Figure A11: Covariance between parametric SDF and real HPR on government debt, cyclical consumption added as instrument.



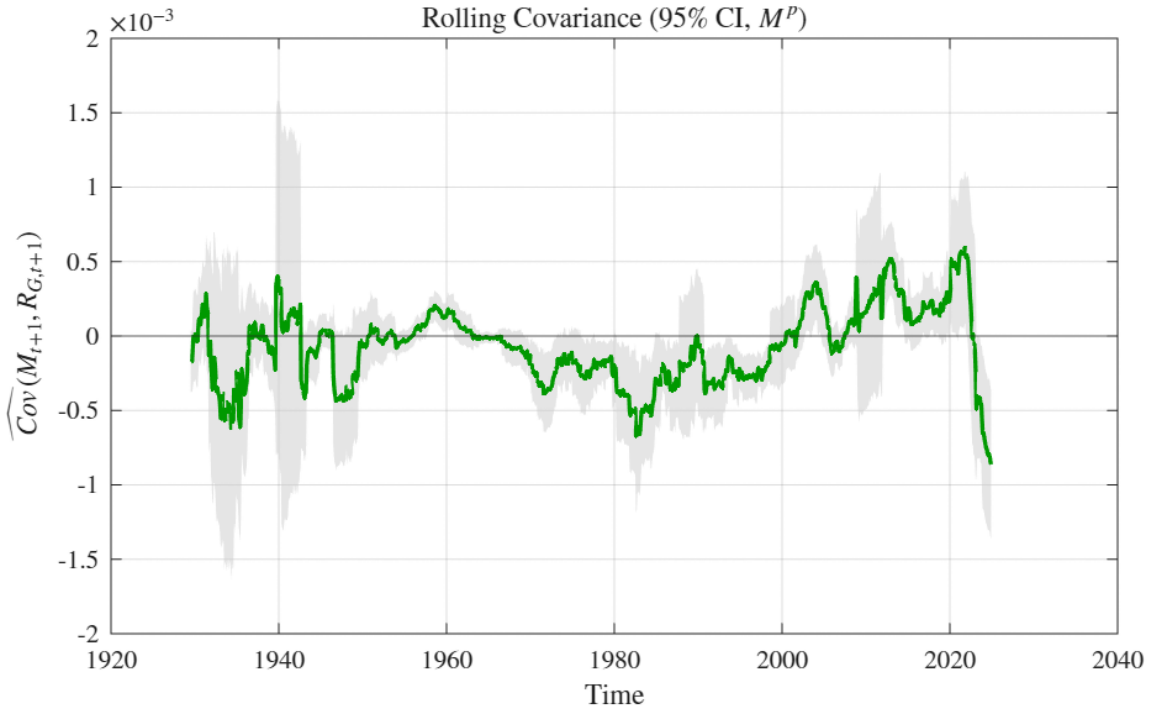
Note: This plots the three-year backward-looking covariance between the parametric SDF (where cyclical consumption is added as an instrument) and the real HPR on government debt. The grey shaded area plots 95% confidence intervals.

Table A6: Parameter estimates for parametric approach (one-step GMM).

Parameter	Estimate	Std. Error	t-stat
γ_0	1.00	0.00	419.36
γ_{treas}	-0.03	137.85	-0.00
γ_{infl}	-0.75	35.28	-0.02
γ_{ts}	-0.12	11.65	-0.01
γ_{cape}	0.24	2.46	0.10
γ_{bs}	1.32	16.38	0.08
δ_{mkt}	-1.65	0.60	-2.75
δ_{hml}	-1.02	1.03	-0.98
δ_{smb}	0.29	1.01	0.29

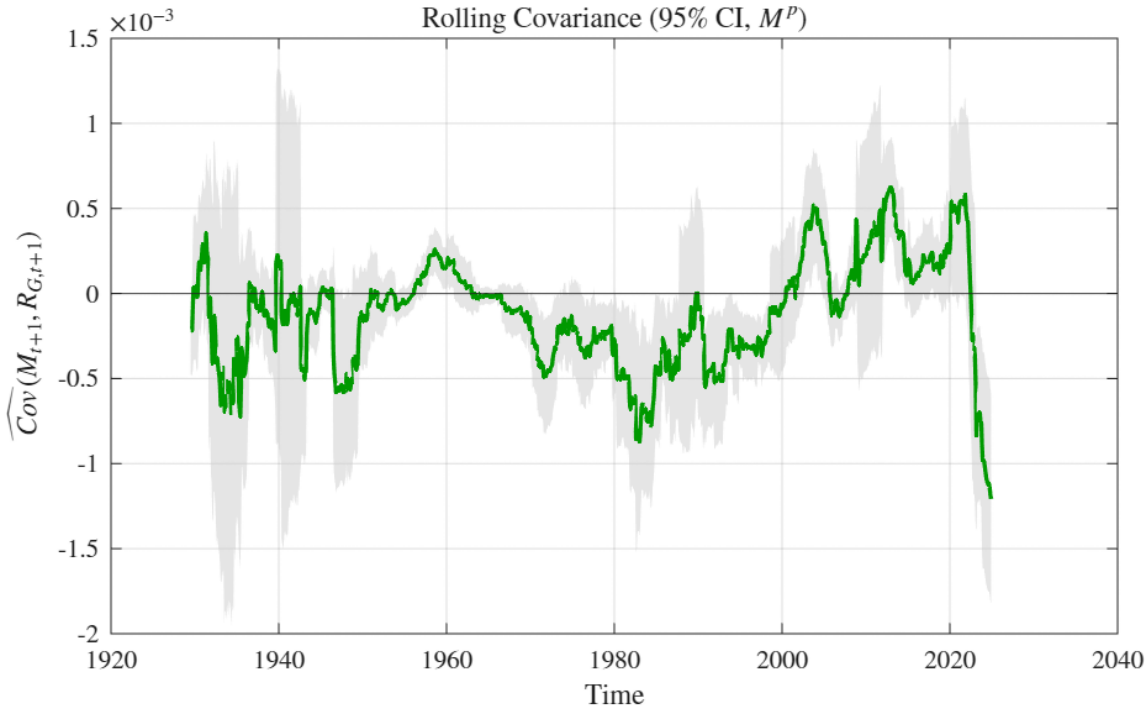
Note: The instruments, in order, are a constant, the 1-month Treasury Bill yield, the 12-month moving average of CPI inflation, the term spread, the inverse Shiller CAPE, and the bond spread. See text for full details. The factors are the market excess return over the zero-beta rate, the HML factor and the SMB factor.

Figure A12: Covariance between parametric SDF (one-step GMM) and real HPR on government debt.



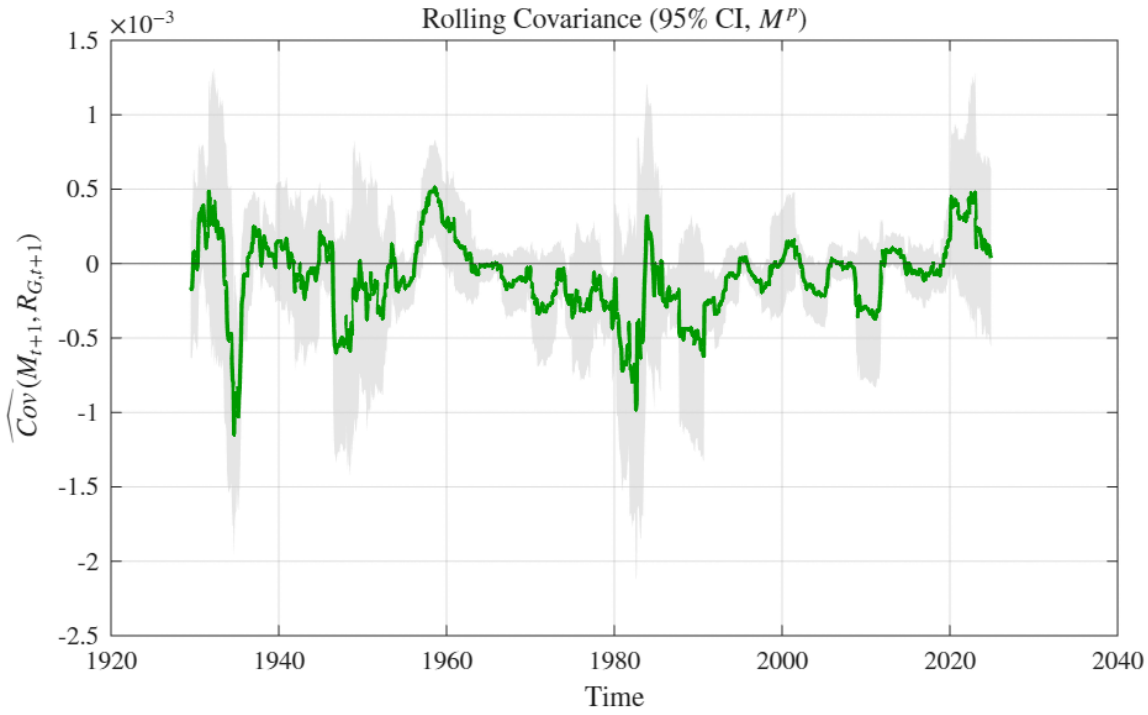
Note: This plots the three-year backward-looking covariance between the parametric SDF (where one-step GMM is used during estimation) and the real HPR on government debt. The grey shaded area plots 95% confidence intervals.

Figure A13: Covariance between parametric SDF (with time-varying risk prices) and real HPR on government debt.



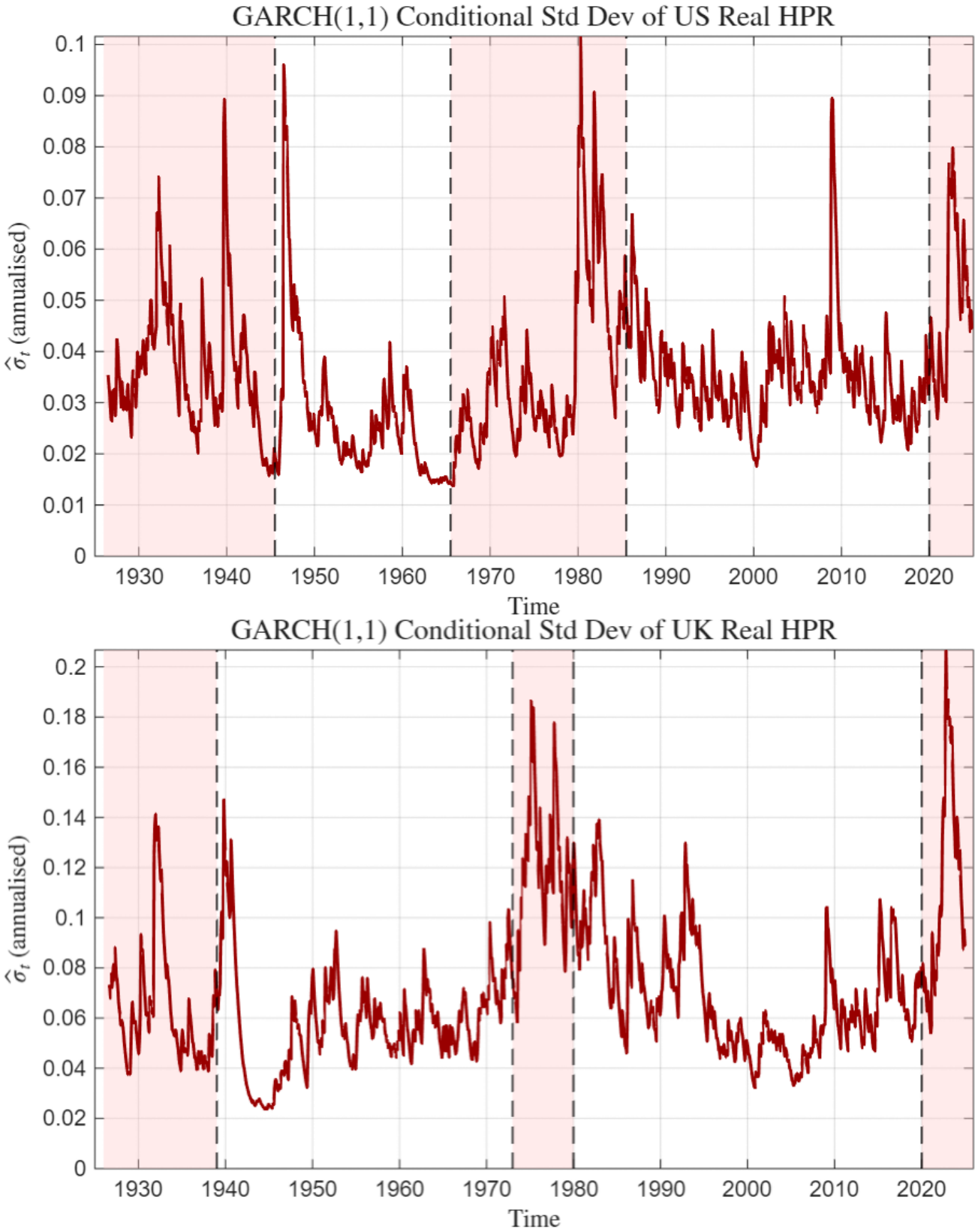
Note: This plots the three-year backward-looking covariance between the parametric SDF (where the risk prices are time-varying) and the real HPR on government debt. The grey shaded area plots 95% confidence intervals.

Figure A14: Covariance between parametric SDF (estimated using the Nagel and Singleton (2011) method) and real HPR on government debt.



Note: This plots the three-year backward-looking covariance between the parametric SDF (estimated using the Nagel and Singleton (2011) method) and the real HPR on government debt. The grey shaded area plots 95% confidence intervals.

Figure A15: GARCH(1,1) estimates of real HPR volatility for US and UK.



Note: The shaded red areas denote the risky debt regimes we identify respectively for the US and the UK. For the US these are: 1926-1945, 1965-1985, 2020-2024. For the UK these are: 1926-1939, 1973-1980, 2020-2024.

E Estimating the Fiscal Rule Parameter

In this section we provide auxiliary evidence on the fiscal rule parameter that underlies the regime interpretation in the main text. The exercise is not meant to replace the asset-pricing regime classification. Rather, it asks whether the regimes identified using the covariance between the SDF and the real holding-period return on government debt are also associated with different systematic responses of primary surpluses to debt.

Constructing the surplus from the return identity The main text shows that the consolidated government budget constraint can be written as

$$\frac{\widetilde{R}_{g,t}}{\Pi_t} b_{t-1} = s_t + b_t, \tag{143}$$

where b_t is the real market value of government debt, s_t is the real primary surplus, $\widetilde{R}_{g,t}$ is the nominal holding-period return on the Treasury portfolio, and Π_t is gross inflation. Equivalently,

$$R_{s,t} = \frac{b_t + s_t}{b_{t-1}} = \frac{\widetilde{R}_{g,t}}{\Pi_t}. \tag{144}$$

Since we observe the real holding-period return on government debt and the real market value of debt, we construct the empirical counterpart of the surplus by rearranging (143):

$$s_t^{ID} = \frac{\widetilde{R}_{g,t}}{\Pi_t} b_{t-1} - b_t. \tag{145}$$

The superscript ID emphasises that this is the surplus implied by the debt return identity. It is not an official NIPA or Treasury measure of the primary surplus. This distinction is useful because the object in (145) is exactly the surplus counterpart of the real holding-period return used in the main asset-pricing exercise.

For the empirical implementation, we scale real surpluses and real debt by real output. Let

$$s_t^Y \equiv \frac{s_t^{ID}}{Y_t}, \qquad b_t^Y \equiv \frac{b_t}{Y_t}, \tag{146}$$

where Y_t denotes real output. These scaled variables are used only for the empirical fiscal-rule estimation. The underlying debt-return identity remains (143). This construction is analogous to the standard procedure of recovering aggregate equity dividends from a price index and a total-return index. Just as the equity dividend satisfies $D_{t+1} = R_{t+1}P_t - P_{t+1}$, the surplus payoff on government debt satisfies $s_t = (\widetilde{R}_{g,t}/\Pi_t)b_{t-1} - b_t$.

Empirical fiscal rule In regime $j \in \{M, F\}$, we write the empirical fiscal rule as

$$s_t^Y = s_j^{Y,\star} + \delta_j \left(\log b_{t-1}^Y - \log b_j^{Y,\star} \right) - x_t + \zeta_t, \tag{147}$$

where M denotes the monetary regime and F denotes the fiscal regime. The parameter δ_j governs the systematic response of the identity-implied surplus-to-output ratio to lagged debt-to-output in regime j. Since the empirical rule is written in scaled variables, δ_j is the empirical counterpart of the model parameter δ_b, rather than a numerically identical object.

The term x_t is the exogenous deficit-news component, while ζ_t captures measurement error and residual variation in the identity-implied surplus not explained by the parsimonious fiscal rule. We assume that the component of x_t induced by an identified fiscal news shock satisfies

$$\frac{\partial x_{t+h}}{\partial \eta_t} = a_j \rho_j^h, \qquad h = 0, 1, \ldots, H, \tag{148}$$

where η_t is an identified fiscal news shock and a_j maps the shock into the direct deficit component in regime j. A positive value of η_t denotes news that raises expected deficits. Hence the direct effect of the shock lowers the surplus, while the systematic component of fiscal policy responds to the debt that subsequently accumulates.

Identified fiscal shocks For the US, we use the high-frequency fiscal shocks of Fritsch and Mazlish (2026). Their shocks measure changes in expected future deficits that are attributed to fiscal news, rather than to the endogenous response of fiscal expectations to macroeconomic conditions. The high-frequency nature of the shocks is useful because our regime classification is monthly and is based on asset prices.

We aggregate the daily fiscal shocks to the monthly frequency:

$$\eta_t = \sum_{\tau \in \mathcal{D}(t)} \Delta PDV_\tau^{fiscal}, \tag{149}$$

where $\mathcal{D}(t)$ is the set of trading days in month t, and ΔPDV_τ^{fiscal} is the change in the present discounted value of expected deficits attributed to fiscal news on day τ. Positive values of η_t correspond to expansionary deficit news. When available, we also control for the component of the forecast update classified as endogenous to macroeconomic conditions, aggregated to the same monthly frequency.

Local projection impulse responses For each horizon $h = 0, 1, \ldots, H$, we estimate regime-specific impulse responses of the identity-implied surplus-to-output ratio and log debt-to-output ratio using

local projections:

$$y_{t+h} - y_{t-1} = \alpha_y^h + \beta_{y,M}^h G_{t-1}\eta_t + \beta_{y,F}^h(1 - G_{t-1})\eta_t + \boldsymbol{\lambda}_y^{h\prime}\mathbf{X}_{t-1} + u_{y,t+h}^h, \tag{150}$$

where y_{t+h} is, in separate regressions, s_{t+h}^Y and $\log b_{t+h}^Y$. Using $y_{t+h} - y_{t-1}$ rather than $y_{t+h} - y_t$ ensures that the impact response at $h = 0$ is well defined. The vector $\mathbf{X}_{t-1}$ includes lags of s_t^Y, $\log b_t^Y$, output growth, inflation, and the short rate. In specifications using the full Fritsch–Mazlish decomposition, $\mathbf{X}_{t-1}$ is augmented with the monthly aggregation of forecast updates classified as endogenous fiscal news.

The regime variable G_{t-1} is predetermined. In the baseline specification, $G_{t-1} = 1$ in months classified as monetary-regime observations by the rolling covariance measure in the main text, and $G_{t-1} = 0$ otherwise. We use the lagged regime indicator to avoid mechanically conditioning on information realised contemporaneously with the fiscal shock. As a robustness check, we also estimate smooth-transition specifications in which G_{t-1} is a logistic function of the lagged rolling covariance.

The estimated coefficients

$$\left\{\hat{\beta}_{s,j}^h, \hat{\beta}_{b,j}^h\right\}_{h=0}^H$$

give the regime-specific responses of s_t^Y and $\log b_t^Y$ to a fiscal news shock.

Recovering the fiscal feedback parameter Equation (147) implies that the surplus response at horizon h can be decomposed into a systematic response to lagged debt and a direct deficit-news component. Since the debt stock relevant for the fiscal rule at horizon h is the debt stock inherited from $h - 1$, the impulse responses satisfy

$$\beta_{s,j}^h = \delta_j \beta_{b,j}^{h-1} - a_j \rho_j^h, \qquad h = 0, 1, \ldots, H, \tag{151}$$

with the convention that $\beta_{b,j}^{-1} = 0$. This convention captures the fact that the debt stock inherited at the time of the shock is predetermined.

The coefficient a_j is not imposed to equal the impact response of the surplus. This is important because the Fritsch–Mazlish shock is a shock to the expected present value of deficits, not necessarily a one-period innovation to the contemporaneous primary deficit. Estimating a_j allows the data to determine how much of the present-value deficit news loads onto the near-term identity-implied surplus response.

Minimum distance estimation For each regime j, define

$$m_{j,h}(\theta_j) = \hat{\beta}_{s,j}^h - \delta_j \hat{\beta}_{b,j}^{h-1} + a_j \rho_j^h, \qquad h = 0, 1, \ldots, H, \tag{152}$$

where

$$\theta_j = (\delta_j, a_j, \rho_j)'.$$

The minimum distance estimator solves

$$\hat{\theta}_j = \arg\min_{\theta_j} \mathbf{m}_j(\theta_j)' \hat{\mathbf{W}}_j^{-1} \mathbf{m}_j(\theta_j), \tag{153}$$

where

$$\mathbf{m}_j(\theta_j) = (m_{j,0}(\theta_j), m_{j,1}(\theta_j), \ldots, m_{j,H}(\theta_j))'.$$

The weighting matrix $\hat{\mathbf{W}}_j$ is the joint variance–covariance matrix of the estimated surplus and debt impulse responses, computed from the same bootstrap draws used to estimate the local projections. We also report equally weighted estimates as a robustness check.

Standard errors are obtained by block bootstrapping the full procedure. In each bootstrap draw, we re-estimate the local projections, re-compute the regime-specific impulse responses, and re-estimate θ_j. When the regime indicator is based on the rolling covariance measure, the baseline bootstrap treats the historical regime classification as fixed; a more conservative robustness check re-computes the rolling covariance and regime indicator in each bootstrap sample.

Interpretation The object of interest is the regime-specific fiscal feedback coefficient δ_j. The model predicts a stronger surplus response to debt in the monetary regime than in the fiscal regime. Hence the main empirical test is whether

$$\hat{\delta}_M > \hat{\delta}_F. \tag{154}$$

A stronger result would be that the estimated feedback coefficient crosses the model-implied stability threshold across regimes. Since the empirical rule is written using s_t^Y and $\log b_t^Y$, we interpret this threshold comparison cautiously and emphasise the cross-regime ordering of the estimates.

The identifying assumption is that the Fritsch–Mazlish fiscal shock is orthogonal to the systematic fiscal response to lagged debt, conditional on the controls and the lagged regime indicator. This is stronger than the standard fiscal-shock assumption because we use the shocks to learn about the fiscal rule itself, rather than only the dynamic effects of fiscal news. The exercise should therefore be read as an over-identifying validation of the regime interpretation: if the covariance-based fiscal regimes are meaningful, then fiscal-regime observations should display a weaker systematic surplus response to debt than monetary-regime observations.

F Holding period returns series for the UK

In this section, we describe how we construct holding-period returns for UK government debt. Our aim is to obtain the nominal holding-period return on debt held by private agents, that is, net of Bank of England (BoE) holdings. This is the UK analogue of the privately-held US federal debt return series constructed by Hall et al. (2018). Our starting point is the bond-level dataset of Ellison and Scott (2020), which provides monthly prices and quantities outstanding at the individual-gilt level. We use the Ellison–Scott files until their sample period ends in 2017 and, for the period beyond it, the updated series published by ESCoE[20], which we convert to the same format.

To net out gilts held by the BoE, we draw on the bond-level QE and QT data published by the Bank of England. Bonds in these files are identified by ISIN, whereas the price and quantity files identify them by Sequence number. We reconcile the two using the mapping in the ESCoE's most recent 'Details' file. After making the BoE series compatible with the ESCoE data, we compute, for each gilt, the monthly flow of QE purchases and QT sales.

The Ellison - Scott series reports the stock of debt outstanding each month, so to net out BoE holdings on a consistent basis we must convert the BoE flow data into monthly stocks. We therefore cumulate the QE and QT series. Cumulative QE purchases of a gilt give the BoE's running holding, which we subtract from the amount outstanding, while cumulative QT sales return that debt to private hands and are added back. We take each gilt's redemption date from the ESCoE quantity files. Once a gilt is redeemed it is no longer outstanding, and the Ellison–Scott quantities net it out automatically. We therefore set its net BoE holding to zero from that date, as there is nothing left to net out. This yields, for every month, the amount of each gilt outstanding net of BoE holdings.

Whereas Hall et al. (2018) obtain monthly bond-level returns from CRSP, no comparable source exists for UK gilts. Thus, we compute them directly as $R_t^i = (P_t^i + C_t^i)/P_{t-1}^i - 1$. Following Hall et al. (2018), we aggregate to a monthly series by value-weighting individual gilt returns. The total holding-period return weights each gilt by its face value outstanding. The private-sector holding-period return weights instead by face value outstanding net of BoE holdings. We use the latter for our external validity analysis in the main text.

[20] Economic Statistics Centre of Excellence.