The Effect of Intangible Capital on Offshoring*

Jiyoung Lee†

University of Washington

This version: Aug, 2026

Abstract

This paper analyzes the impact of intangible capital on firms' offshoring decisions, aggregate productivity, and external competitiveness. I develop a two-country offshoring model with endogenous intangible investment that captures its unique scalability in a framework featuring heterogeneous firm entry and exit. The model successfully replicates the key post-GFC U.S. empirical regularities—the coexistence of aggregate productivity gains and real exchange rate appreciation. Following a positive productivity shock, the entry of domestic firms lowers average firm productivity and raises domestic prices, thereby amplifying the Harrod–Balassa–Samuelson effect. Furthermore, incorporating intangibles yields substantially larger aggregate productivity gains than non-intangible models by improving resource allocation. This research contributes by endogenously modeling the scalability of intangible capital—a crucial yet previously understudied factor in offshoring models.

Keywords: Intangible Capital, Offshoring, Aggregate Productivity, Real Exchange Rate

JEL Codes: E22, F12, F23, F41, O33

*I am deeply grateful to my advisors, Fabio Ghironi, Theo Eicher, and Andrei Zlate, for their support and guidance. I am also indebted to Gianluca Benigno, Paul Bergin, Lorenzo Caliendo, Jonathan Dingel, Aydan Dogan, Ye Li, Isabelle Mejean, Dmitry Mukhin, Felix Tintelnot, Michael Waugh, and Francesco Zanetti for their invaluable feedback. I would also like to thank the participants at the FIW Research Conference on International Economics (WU Vienna), 2026 SMU Trade Conference, the Boston College PhD Conference, the AEA CSWEP Mentorship Program, the HEC Paris Economics PhD Conference, and the University of Washington Macroeconomics Seminar for their helpful comments and suggestions. All errors are my own.

†Department of Economics, University of Washington, Savery Hall, Box 353330, Seattle, WA 98195, U.S.A. E-mail: jylee269@uw.edu.

1 Introduction

Since the Global Financial Crisis (GFC), the US economy has experienced a slight increase in aggregate productivity accompanied by a persistent appreciation of the real exchange rate (RER) and a weakening of external competitiveness (Figures 1 & 2). While the Harrod–Balassa–Samuelson (HBS) effect traditionally explains long-run RER appreciation through faster productivity growth in the tradable sector relative to the non-tradable sector, recent dynamics demand closer attention to structural changes within the US tradable sector. Central to these shifts is the sharp post-GFC increase in intangible investment. As recent studies highlight, the expansion of intangible capital, particularly among US firms participating in Global Value Chains (GVCs), drives endogenous productivity improvements (Karpowicz and Suphaphiphat, 2020) and can induce a persistent appreciation of the RER (Gornemann, Guerrón Quintana, and Saffie, 2025). Consequently, the surge in intangible investment serves as a key structural driver explaining both US endogenous growth and recent RER dynamics.

Crucially, this rise in intangibles extends beyond domestic borders. With US multinationals deepening their GVC integration, intangible investment is increasingly sourced globally alongside a rise in R&D offshoring (Baldwin and Evenett, 2015). Understanding this dynamic requires grasping a unique feature of intangible capital: its scalability. Unlike tangible assets fixed in location, offshored intangible capital—such as R&D—can be replicated and deployed across multiple foreign production sites at near-zero marginal cost (Haskel and Westlake, 2017; Hsieh and Rossi-Hansberg, 2023). This cross-border scalability provides a powerful incentive for firms to offshore intangibles to maximize global profits.

However, this cross-border relocation presents a tradeoff for US aggregate productivity. While offshoring can drive efficient resource reallocation by pushing the least efficient domestic firms out of the market (Melitz, 2003), shifting intangible capital and profits to foreign affiliates risks contracting domestic production and statistically dampening aggregate productivity gains (Guvenen, Mataloni, Rassier, and Ruhl, 2022). Despite its pivotal role, this international allocation of intangible capital remains under-explored as an endogenous driver of both productivity and RER dynamics. To fill this gap, this paper develops a two-country heterogeneous-firm model that endogenizes the scalability of intangible capital. Specifically, I analyze (1) what role intangible capital plays in firms' offshoring decisions, and (2) how it impacts aggregate productivity and external competitiveness in general equilibrium. Ultimately, this framework demonstrates how the post-GFC surge in intangible investment simultaneously explains both aggregate productivity growth and persistent RER appreciation.

This paper develops a two-country general equilibrium model that incorporates intangible capital investment as an endogenous decision for heterogeneous firms. Building on the framework of Ghironi and Melitz (2005, GM05), the model features endogenous firm entry,

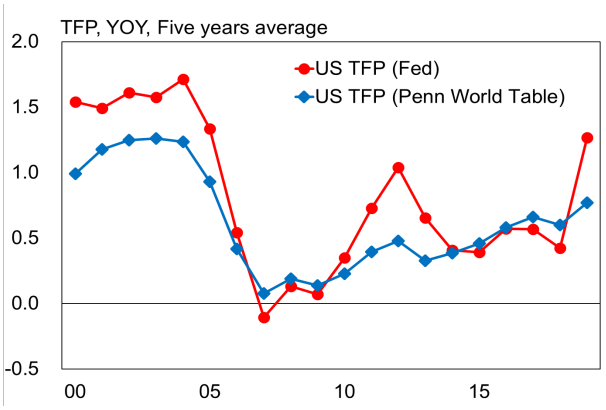


Figure 1: Total Factor Productivity in the U.S.
Source: San Francisco Fed, Penn World Table

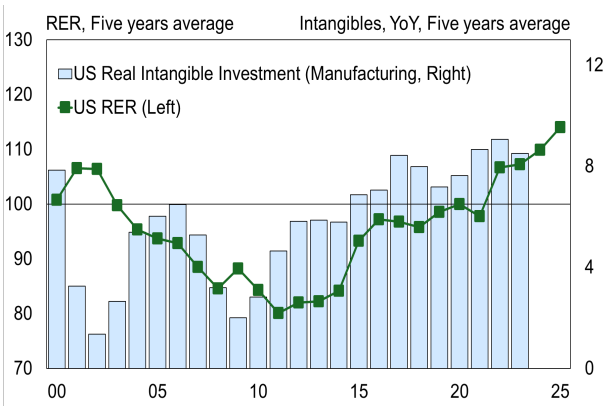


Figure 2: RER & Intangibles in the U.S.
Source: BIS, BEA

idiosyncratic productivity, and endogenous export decisions. Following Zlate (2016, Zlate16), I include an asymmetric offshoring framework where Home firms establish offshore plants in the Foreign economy to exploit wage differentials, reshipping output back to serve the domestic market.

The core innovation of the model is the incorporation of scalability of intangible capital. Adopting De Ridder's (2024, De Ridder24) framework, I model intangible investment as a fixed cost that reduces marginal costs. Unlike physical capital, intangibles exhibit unique scalability due to their low marginal replication costs (Haskel and Westlake, 2017; Hsieh and Rossi-Hansberg, 2023). This cost structure—characterized by recurring fixed expenses but negligible incremental costs—shifts the firms' cost structures.

After paying sunk entry costs, Home firms draw idiosyncratic productivity levels and choose their production mode on a period-by-period basis, subsequently optimizing price and quantity. Firms operate under three distinct modes: (1) domestic production without intangible investment, (2) domestic production with intangible investment, and (3) offshoring combined with intangible investment. Unlike Zlate (2016), offshoring firms in this paper must invest in intangibles—incurring additional fixed costs while benefiting from lower marginal costs.[1]

In a departure from Zlate (2016), domestic firms in this model are now differentiated by their intangible investment status. Home firms can serve the domestic market using domestic or foreign labor. Specifically, offshoring firms combine their idiosyncratic productivity with foreign labor, incurring offshoring fixed costs, intangible fixed costs, and iceberg trade costs each period; only those productive enough to cover these total costs choose to offshore. Similarly, domestic firms investing in intangible capital combine their productivity with

[1]This is consistent with empirical evidence showing that offshoring firms and multinationals exhibit greater R&D intensity and innovation rates than non-offshoring firms (Bøler, Moxnes, and Ulltveit-Moe, 2015; Dachs, Ebersberger, Kinkel, and Som, 2015; Fritsch and Gorg, 2015; Gumpert, Manova, Rujan, and Schnitzer, 2024).

domestic labor and incur intangible fixed costs. Only firms with productivity above the intangible investment cutoff invest in intangibles. A subset of firms exports, but only those with productivity above the export cutoff engage in exporting. For simplicity, while the intangible fixed cost is assumed to be uniform within each production method—domestic and offshoring—its payment reduces marginal costs heterogeneously across firms based on their idiosyncratic productivity.

The benchmark model incorporating intangible capital yields several key distinctions compared to the model without intangibles (Zlate16). First, the introduction of intangible capital predicts a stronger RER appreciation in response to positive productivity shocks, successfully reproducing post-GFC dynamics when calibrated with U.S. empirical TFP data. The mechanism is driven by an expansion in firm entry: A positive productivity shock boosts firm profits, triggering firm entry in the Home economy. This expansion increases Home household income and demand for Foreign goods, thereby inducing excess demand for Foreign labor and driving up Foreign wages. Consequently, the relative cost of Home labor falls (a short-run depreciation of the terms of labor), which further stimulates domestic entry. Crucially, this heightened entry allows lower-productivity firms to enter, leading to a decline in average domestic productivity, a rise in domestic prices, and ultimately, an amplification of RER appreciation.

In this context, the scalable nature of intangible capital—which lowers marginal costs—amplifies the increase in profits compared to the model without intangibles. This enhanced profitability further expands Home demand for foreign goods, exacerbating the rise in foreign wages and the depreciation of relative Home labor costs. As a result, domestic entry is significantly higher than in the model without intangibles, leading to a steeper rise in domestic prices and a stronger Harrod–Balassa–Samuelson (HBS) effect. This mechanism theoretically rationalizes the empirical pattern of RER appreciation accompanying productivity growth observed in the U.S. post-GFC (Figures 1 & 2), highlighting the importance of accounting for intangibles in understanding recent RER dynamics.

Second, using a Dynamic Olley-Pakes decomposition (Melitz and Polanec, 2015), I find that intangible investment amplifies aggregate productivity gains and enhances trade-induced resource allocation. In the short run, intangible investment expands Home income and raises foreign wages, which reduces the number of offshore firms while increasing their average productivity. This leads to larger short-run aggregate productivity gains compared to models without intangibles. In the long run, the surge in domestic labor demand raises wages, reducing the number of domestic firms. Driven by these rising wages, the sharp increase in fixed costs associated with intangibles further erodes the profits of domestic intangible firms, leading to a more pronounced decline in their numbers and a subsequent increase in average productivity. Thus, incorporating intangibles demonstrates that trade-induced productivity

improvements are substantially larger than previously suggested, reinforcing the resource allocation channel highlighted by Melitz (2003).

Third, the benchmark model better captures export expansion following a productivity shock, as intangible capital improves the price competitiveness of Home exporters by lowering marginal costs. This mechanism allows the model to better match the post-GFC procyclicality of U.S. export data, highlighting a key theoretical role for intangibles in driving export cyclicality.

Fourth, incorporating intangibles reduces the procyclicality of offshore value-added (VA), achieving a closer fit to post-GFC data compared to the model without intangibles. In the benchmark model, rising domestic income drives up foreign labor costs, which acts as a cost-push factor that dampens offshore VA even as Home income increases. This aligns precisely with the observed reduction in the procyclicality of U.S. offshore VA, confirming that the intangible-augmented framework better explains this stylized fact than Zlate16.

This research advances the offshoring literature by incorporating the scalable nature of intangible capital into firms' endogenous production decisions—a cost structure (shifting from marginal to fixed costs) that remains underexplored in offshoring models despite its relevance for productive firms (Haskel et al., 2017; Hsieh et al., 2023; De Ridder, 2024). Building on models of endogenous offshoring such as Zlate16, which did not fully account for this unique cost structure, this study endogenizes scalability within heterogeneous firms' offshoring choices. While the causal link from foreign outsourcing to increased R&D has been explored (Bernard et al., 2020; Bøler et al., 2015), the simultaneous determination of offshoring and intangible investment remains relatively under-researched. Unlike recent dynamic partial-equilibrium frameworks (Gumpert et al., 2025; Liu, 2024), this paper employs a general equilibrium model to comprehensively examine the economy-wide impact of intangible investment's scalability within the offshoring model.

To the best of my knowledge, this is the first paper to theoretically elucidate the empirical link between rising intangible investment, productivity growth, and RER appreciation within a framework of endogenous producer entry and exit. While Ghironi and Melitz (2005) established that endogenous producers' entry and exit decisions motivate RER appreciation—validating the HBS effect—and Zlate16 subsequently found that offshoring dampens RER appreciation by lowering consumer prices, this paper identifies a novel theoretical channel. I show that endogenously determined intangible capital stimulates domestic firm entry, which drives up domestic prices and amplifies RER appreciation. Thus, this study provides a theoretical mechanism that rationalizes the simultaneous increase in intangible investment, productivity, and RER appreciation, thereby explaining the strengthening of the HBS effect observed in the U.S. since the GFC.

Outline The remainder of the paper proceeds as follows: Section 2 presents the benchmark

two-country heterogeneous firm model with the inclusion of intangible capital under an offshoring setup. Section 3 details the calibration. Section 4 discusses the model dynamics and evaluates the model performance through empirical TFP calibration. Finally, Section 5 concludes. The Appendix includes analytical derivations and supplementary figures and tables. Furthermore, it outlines empirical results that back up the assumptions of the benchmark model and presents the findings of the event-study framework.

2 The Model

The model is composed of two economies, Home and Foreign, each with a representative household and a continuum of monopolistically competitive firms with heterogeneous labor productivity producing differentiated varieties. Home firms can produce domestically or offshore for their domestic market, while Foreign firms produce only domestically due to Home's higher labor costs. Additionally, a subset of firms from both countries produce domestically to export.

This section details the representative household's and firms' problems in the Home economy under financial autarky. For simplicity, labor is the sole production input. Appendix B provides full model summaries.

2.1 Household Problem

Households maximize their expected intertemporal utility, which depends on their consumption level (C_t). The expected intertemporal utility function is:

$$E_t \sum_{s=t}^{\infty} \beta^{s-t} \frac{C_s^{1-\gamma}}{1-\gamma} \tag{1}$$

The parameter $\beta \in (0,1)$ is a discount factor, and $\gamma > 0$ is the inverse of the intertemporal elasticity of substitution. The consumption basket (C_t) is defined over a continuum of goods Ω, where the elasticity of substitution across goods is constant, $\theta > 1$.

The household purchases the consumption basket C_t, including varieties produced domestically by Home firms without intangible investment ($\omega \in \Omega_t^{NI}$), with intangible investment ($\omega \in \Omega_t^{I}$), varieties produced offshore ($\omega \in \Omega_t^{V}$), and varieties produced by Foreign firms and imported by Home ($\omega \in \Omega_t^{F}$), all with the symmetric elasticity of substitution $\theta > 1$:

$$C_t = \left[\underbrace{\int_{z_{min}}^{z_{D,t}^I} y_{D,t}^{NI}(\omega)^{\frac{\theta-1}{\theta}} d\omega}_{\omega \in \Omega_t^{NI}} + \underbrace{\int_{z_{D,t}^I}^{z_{V,t}} y_{D,t}^{I}(\omega)^{\frac{\theta-1}{\theta}} d\omega}_{\omega \in \Omega_t^{I}} + \underbrace{\int_{z_{V,t}}^{\infty} y_{V,t}(\omega)^{\frac{\theta-1}{\theta}} d\omega}_{\omega \in \Omega_t^{V}} + \underbrace{\int_{z_{X,t}^*}^{\infty} y_{X,t}^{*}(\omega)^{\frac{\theta-1}{\theta}} d\omega}_{\omega \in \Omega_t^{F}} \right]^{\frac{\theta}{\theta-1}} \tag{2}$$

$[z_{min}, \infty)$ is the support interval for the idiosyncratic productivity of Home firms. Firms exceeding the endogenous threshold $z^I_{D,t}$ invest in intangible capital for the domestic market, while those above $z_{V,t}$ offshore production. The composition of the consumption basket changes over time since the number of firms is time-variant and firms re-optimize their intangible investment, offshoring and exporting strategies every period. $p_t(\omega)$ denotes the home currency price of a good $\omega \in \Omega_t$, and the consumption-based price index for the home economy is $P_t = (\int_{\omega\in\Omega} p_t(\omega)^{1-\theta} d\omega)^{1/(1-\theta)}$. The household's demand for each individual good ω is $c_t(\omega) = (p_t(\omega)/P_t)^{-\theta} C_t = \rho_t(\omega)^{-\theta} C_t$, where $\rho_t(\omega)$ is the real price of each variety and $\omega \in \Omega^{NI}_t \cup \Omega^I_t \cup \Omega^V_t \cup \Omega^F_t$.

2.2 Firm entry

In each period, an unbounded mass of potential entrants exists in both Home and Foreign. Following GM05, firm entry occurs each period in both countries and requires paying a sunk cost $f_{E,t}$, measured in effective labor units of the respective country. Upon entry, each firm draws its productivity level z from a common distribution $G(z)$ supported over the interval $[z_{min}, \infty)$, which remains fixed thereafter.

New entrants are forward-looking and calculate the expected stream of future profits $\tilde{d}_t$ in every period. Entrants at time t only start producing at time $t+1$, introducing a one-period time-to-build lag in the model. The exogenous exit shock occurs at the end of the time period with the probability δ. Prospective home entrants in period t compute their expected post-entry value $\tilde{v}_t$ given by the present discounted value of their expected stream of profit $\{\tilde{d}_s\}^{\infty}_{s=t+1}$:

$$\tilde{v}_t(z) = E_t \sum_{s=t+1}^{\infty} [\beta(1-\delta)]^{s-t} \left(\frac{C_s}{C_t}\right)^{-\gamma} \tilde{d}_s(z) \tag{3}$$

Thus, $N_{E,t}$ new firms are created every period t and $(1-\delta)N_{E,t}$ firms start producing at t+1.

The law of motion for the number of active firms is: $N_{t+1} = (1-\delta)(N_t + N_{E,t})$. The equation (3) also represents the average value of incumbent firms after production has occurred. Entry occurs until the average firm value is equalized with the entry cost, leading to the free entry condition $\tilde{v}_t = \frac{w_t}{Z_t} f_{E,t}$.

2.3 Firms' choice of markets and production strategies

Each period, the N_t active firms endogenously choose their production locations and whether to invest in intangible capital. The setup is as follows: (1) To serve the home market, domestic firms can choose three types of production methods: Producing domestically with intangible investment, producing domestically without intangible investment or producing in an offshore

plant with intangible investment. Intangible investment offers the advantage of reducing marginal costs of production but incurs intangible fixed costs each period. Similarly, offshoring provides the advantage of lower wage costs but entails offshoring fixed and trade costs per period. It is essential to note that all firms, regardless of their choices regarding intangible investment or offshore production, are exclusively focused on serving their domestic market and do not engage in foreign market activities. (2) A subset of firms in each economy exports to serve foreign markets. This study focuses on intangible-intensive manufacturing firms. Given the empirical prevalence of vertical foreign direct investment (FDI) in manufacturing, this paper abstracts from outsourcing driven by horizontal FDI aimed at capturing foreign market share. Consequently, I assume that firms serving foreign markets produce domestically and export, subject to a fixed cost, as in GM05.

2.3.1 Intangible investment decision

This paper models intangible capital investment as recurring fixed costs that reduce marginal production costs, reflecting the scalable nature of intangibles. Scalability refers to the minimal marginal costs of reproducing intangible assets, which alters firms' cost structures by shifting costs from marginal to fixed components (De Ridder24). For instance, incorporating intangible capital such as R&D or software, with negligible replication costs into production, raises fixed costs, such as recurring expenses for software licensing fees or R&D wages, while keeping marginal costs low. As a result, firms' production decisions increasingly hinge on fixed costs. Moreover, the impact of this investment depends on firms' heterogeneous productivity: more productive firms achieve larger marginal cost reductions from the same fixed investment (Veugelers et al., 2020)[2].

The following analysis will explore the cost dynamics of firms with and without intangible investments. If a firm with heterogeneous productivity z does not undertake intangible capital investment, its marginal cost is equivalent to wages measured in units of effective labor.

$$mc_t(z) = \frac{w_t}{Z_t z} \tag{4}$$

If a firm with heterogeneous productivity z invests in intangible capital, its marginal costs will decrease by a factor of ψ_t^{ξ}. ψ_t denotes the fraction by which marginal costs are reduced, and ξ represents the cost elasticity of intangibles.

$$mc_t(z) = \psi_t^{\xi} \frac{w_t}{Z_t z} \quad \text{where } \psi_t < 1 \tag{5}$$

However, firms must incur a fixed cost, measured in units of consumption, in order to

[2]The scalability—where marginal costs decline with output—can also be confirmed using Compustat Segment Historical data. Related analysis is provided in Appendix A, Figure 11.

engage in intangible capital investment.

$$\phi_t \frac{w_t}{Z_t}(\psi_t^{-\xi} - 1) \tag{6}$$

Here, ϕ_t represents the share of intangible fixed costs in total fixed costs. Countries with lower ϕ_t have more efficient firms, reflecting higher intangible capital efficiency, as they achieve greater marginal cost reductions for the same fixed costs.

Firms invest in intangible capital only if the resulting profit $d_{D,t}^I(z)$ meets or exceeds the profit without it, $d_{D,t}^{NI}(z)$ (i.e., $d_{D,t}^I(z) \geq d_{D,t}^{NI}(z)$). Since intangible investment entails fixed costs, only firms with sufficiently high productivity can justify the investment.

2.3.2 The timing of production and intangible investment decisions

After entry, firms draw their idiosyncratic productivity z from the common distribution $G(z)$. They then choose their production location and whether to invest in intangible capital based on their productivity. Without intangible investment, marginal costs are $\frac{w_t}{Z_t z}$; with investment, firms pay a fixed cost to reduce marginal costs to $\psi_t^{\xi} \frac{w_t}{Z_t z}$. Highly productive firms may both invest in intangibles and offshore production. Firms make these decisions simultaneously and then set prices and quantities to maximize profits. Figure 3 illustrates the timing of these decisions.

Since the decision to undertake intangible capital investment is made after the realization of productivity, firms cannot determine whether to make such investments prior to entering the market. Therefore, intangible capital investment decisions are made on a period-by-period basis according to the idiosyncratic productivity realized after entry. This setup can be considered a more realistic representation of the recurrent costs that firms must incur to maintain R&D, software, and information technology investments.

Since firms observe productivity only after entry, they cannot decide on intangible investment beforehand. Instead, they choose investment each period based on realized idiosyncratic productivity. This framework better captures the recurring costs firms incur to sustain R&D, software, and IT investments.

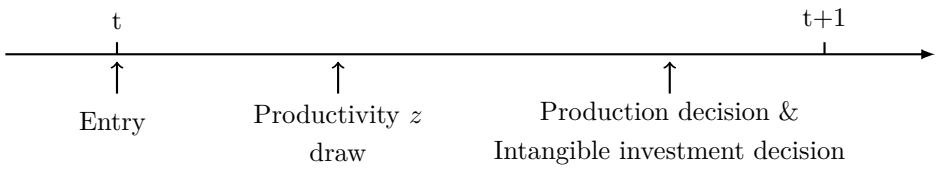


Figure 3: The timing of production and intangible investment decisions

2.3.3 Production decision

A home firm with idiosyncratic productivity z chooses between two production strategies each period to serve the home market: (a) Produce domestically, with output $y_{D,t}(z) = Z_t z l_t(z)$, as a function of aggregate productivity Z_t, the firm-specific labor productivity z, and domestic labor $l_t(z)$; or (b) Produce offshore, with output $y_{V,t}(z) = Z_t^* z l_t^*$, using foreign labor $l_t^*(z)$, subject to aggregate foreign productivity Z^*, while retaining its idiosyncratic productivity z abroad.

Within a monopolistically competitive framework, a firm with idiosyncratic productivity z optimally chooses between three production modes, factoring in the effect of intangible investment on profits: domestic production with intangible investment, domestic production without intangible investment, and offshore production with intangible investment. Equations (7)–(9) present the profit functions for these modes.

$$\max_{\{\rho_{D,t}^I(z)\}} d_D^I(z) = \left(\rho_{D,t}^I(z) - \psi_{D,t}^{\xi} \frac{w_t}{Z_t z}\right) y_{D,t}(z) - \phi_{D,t} \frac{w_t}{Z_t} (\psi_{D,t}^{-\xi} - 1) \tag{7}$$

$$\max_{\{\rho_{D,t}^{NI}(z)\}} d_D^{NI}(z) = \left(\rho_{D,t}^{NI}(z) - \frac{w_t}{Z_t z}\right) y_{D,t}(z) \tag{8}$$

$$\max_{\{\rho_{V,t}(z)\}} d_V(z) = \left(\rho_{V,t}(z) - \tau Q_t \psi_{V,t}^{\xi} \frac{w_t^*}{Z_t^* z}\right) y_{V,t}(z) - f_V Q_t \frac{w_t^*}{Z_t^*} - Q_t \phi_{V,t} \frac{w_t^*}{Z_t^*} (\psi_{V,t}^{-\xi} - 1) \tag{9}$$

where $\rho_{D,t}^{NI}(z)$, $\rho_{D,t}^I(z)$, and $\rho_{V,t}(z)$ represent the relative prices for domestic production without intangible investment, domestic production with intangible investment, and offshore production with intangible investment, respectively. w_t and w_t^* denote real wages in the home and foreign markets, and Q_t represents the real exchange rate. Thus, the cost of producing a unit of output depends on whether production is domestic or offshore and whether intangible investment is undertaken. In other words, this decision is influenced not only by effective labor costs $\frac{w_t}{Z_t}$ and $\frac{w_t^* Q_t}{Z_t^*}$ across countries, but also by firm-specific labor productivity z and the marginal cost reduction fraction ψ_t.

Home firms investing in intangible capital incur fixed costs while gaining the ability to reduce marginal costs. For domestic producers, intangible investment lowers marginal costs by a factor of $\psi_{D,t}^{\xi}$; for offshore producers, by $\psi_{V,t}^{\xi}$. Following Gumpert, Manova, Rujan, and Schnitzer (2025), I assume $\psi_{D,t} > \psi_{V,t}$, reflecting higher fixed costs for intangible investment in an offshore plant. A lower $\psi_{V,t}$ indicates both higher fixed costs and greater marginal cost reduction efficiency for offshore firms. Fixed costs are independent of firm-specific productivity z. However, the fixed costs associated with intangible investment mean that only firms with relatively high idiosyncratic productivity z engage in such investment.

Moreover, home firms that offshore production incur a fixed cost of f_V units of foreign effective labor, covering the establishment and maintenance of the offshore facility. They also

face an iceberg trade cost $\tau > 1$ when transporting goods produced overseas back to the home country.

The demand for the variety of firm z, produced domestically or offshore considering intangible investment, is $y^I_{D,t}(z) = \rho^I_{D,t}(z)^{-\theta} C_t$, $y^{NI}_{D,t}(z) = \rho^{NI}_{D,t}(z)^{-\theta} C_t$ or $y_{V,t}(z) = \rho_{V,t}(z)^{-\theta}\ C_t$ respectively, where C_t is aggregate Home consumption. Profit maximization yields equilibrium prices: $\rho^I_{D,t}(z) = \frac{\theta}{\theta-1}\psi^{\xi}_{D,t}\frac{w_t}{Z_t z}$, $\rho^{NI}_{D,t}(z) = \frac{\theta}{\theta-1}\frac{w_t}{Z_t z}$ and $\rho_{V,t}(z) = \frac{\theta}{\theta-1}\tau Q_t \psi^{\xi}_{V,t}\ \frac{w^*_t}{Z^*_t z}$ for the three production methods. These prices show that firms with intangible capital investment price differently from non-intangible investing firms by a factor of ψ^{ξ}, allowing them to reduce prices proportionally to marginal cost reductions. The corresponding profits for each production cases are as follows: $d^I_{D,t}(z) = \frac{1}{\theta}\rho^I_{D,t}(z)^{1-\theta} C_t - \phi_{D,t}\frac{w_t}{Z_t}(\psi^{-\xi}_{D,t} - 1), d^{NI}_{D,t}(z) = \frac{1}{\theta}\rho^{NI}_{D,t}(z)^{1-\theta} C_t$ and $d_{V,t}(z) = \frac{1}{\theta}\rho_{V,t}(z)^{1-\theta} C_t - f_V Q_t \frac{w^*_t}{Z^*_t} - Q_t \phi_{V,t}\frac{w^*_t}{Z^*_t}(\psi^{-\xi}_{V,t} - 1)$.

Each period, domestic firms decide whether to invest in intangible capital. A firm with productivity z compares domestic profit with intangible investment $d^I_{D,t}(z)$ and without such investment $d^{NI}_{D,t}(z)$. The cutoff productivity $z^I_{D,t}$ on the support interval $[z_{min}, \infty)$ is where the two profits are equal; at this threshold, the firm is indifferent between investing and not investing.

$$z^I_{D,t} = \{z : d^I_{D,t}(z^I_{D,t}) = d^{NI}_{D,t}(z^I_{D,t})\} \tag{10}$$

Similarly, a firm with productivity z determines its production location each period by comparing profits from domestic production with intangible investment, $d^I_{D,t}(z)$, and from offshoring, $d_{V,t}(z)$. The productivity cutoff $z_{V,t}$ on the support interval $[z_{min}, \infty)$ denotes the level at which the firm is indifferent, earning the same profit from either location.

$$z_{V,t} = \{z : d^I_{D,t}(z_{V,t}) = d_{V,t}(z_{V,t})\} \tag{11}$$

The model implies that only relatively more productive domestic firms can invest in intangible capital, and even more productive firms among them can pursue offshore production with intangible investment. Since intangible investment entails fixed costs, only firms with productivity above the investment threshold ($z > z^I_{D,t}$) gain enough from intangible capital to offset these costs.[3]

Similarly, offshore production involves additional fixed and iceberg trade costs, such that only firms with idiosyncratic productivity above the offshoring cutoff ($z > z_{V,t}$) can generate sufficient gains to offset these costs. This implication is consistent with empirical evidences: multinational firms are more likely to invest in R&D (Castellani et al., 2015); and multinational firms are both more productive and earn higher R&D returns than domestic

[3]Intangible investing firms are more productive, a fact supported by the Compustat Historical Segment data showing high average productivity for firms in the highest intangible quantile. See empirical fact (iii) and Figure 13 in Appendix A for details. Based on empirical fact (ii), intangible investing firms exhibit higher net sales.

firms (Belderbos et al., 2014; Rahko, 2021). The Compustat Segment Historical data also backs up the finding that offshoring firms are more productive than non-offshoring firms, which is presented as stylized fact (iii) in Appendix A. Furthermore, the model's setup, where offshoring firms always undertake intangible investment and incur fixed costs, is motivated by stylized fact (iv) identified in Appendix A using Compustat Historical Segment data: offshoring firms are more likely to undertake intangible investments compared to domestic firms.

Additionally, the productivity cutoffs $z^I_{D,t}$ and $z_{V,t}$ respond to changes in relative effective labor costs across countries, thereby influencing the extensive margin over the business cycle. A decline in the relative cost of Home effective labor lowers domestic prices and raises Home firms' revenues, leading more Home firms to invest in intangibles at equilibrium. Similarly, a decline in the relative cost of Foreign effective labor increases the profitability of offshoring, resulting in a higher share of firms choosing to offshore.

Existence of the equilibrium productivity cutoff

The intangible investment cutoff $(z^I_{D,t})$ In equilibrium, the existence of intangible investment productivity cutoff $z^I_{D,t}$ requires lower price for domestically producing firms with intangible investment compared to the domestic firms without intangible investment. To clarify this, I rewrite the per-period profits from domestically producing firms with intangible investment and that of without intangible investment: $d^I_D(z^I_{D,t}) = \frac{1}{\theta}\left(\frac{\theta}{\theta-1}\psi^{\xi}_{D,t}\frac{w_t}{Z_t z^I_{D,t}}\right)^{1-\theta} C_t - \phi_{D,t}\frac{w_t}{Z_t}(\psi^{-\xi}_{D,t} - 1)$ and $d^{NI}_D(z^I_{D,t}) = \frac{1}{\theta}\left(\frac{\theta}{\theta-1}\frac{w_t}{Z_t z^I_{D,t}}\right)^{1-\theta} C_t$.

Figure 4 plots the three profit functions against the idiosyncratic productivity parameter $z^{\theta-1}$ over the support interval $[z_{min}, \infty)$. Domestic production without intangible investment incurs no fixed costs, resulting in a zero vertical intercept. Domestic production with intangible investment incurs intangible fixed costs, yielding a vertical intercept of $-\phi_{D,t}\frac{w_t}{Z_t}(\psi^{-\xi}_{D,t} - 1)$. Offshoring incurs both intangible and offshoring fixed costs, resulting in a vertical intercept of $-f_V\frac{w^*_t Q_t}{Z^*_t} - \phi_{V,t}\frac{w^*_t Q_t}{Z^*_t}(\psi^{-\xi}_{V,t} - 1)$.

The model predicts that the productivity cutoff $z^I_{D,t}$ exists in equilibrium only if the slope of the profit function for domestic production with intangible investment exceeds that of production without it: $slope\{d^I_{D,t}(z)\} > slope\{d^{NI}_{D,t}(z)\}$. When this condition holds, domestic production with intangible investment yields higher profits for firms with productivity above $z^I_{D,t}$ compared to domestic production without intangible investments. The slope inequality is equivalent to the condition $\psi^{\xi(1-\theta)}_{D,t} > 1$. Since $\psi_{D,t} < 1$, $\xi > 0$, and $\theta > 1$, this condition always holds. It implies that domestic firms investing in intangibles can charge lower prices than those without, ensuring a unique intangible investment cutoff. To avoid a corner solution, profits for domestic firms without intangible investment must exceed those with intangible

investment at z_{min} : $d^{NI}_{D,t}(z_{min}) > d^{I}_{D,t}(z_{min})$.

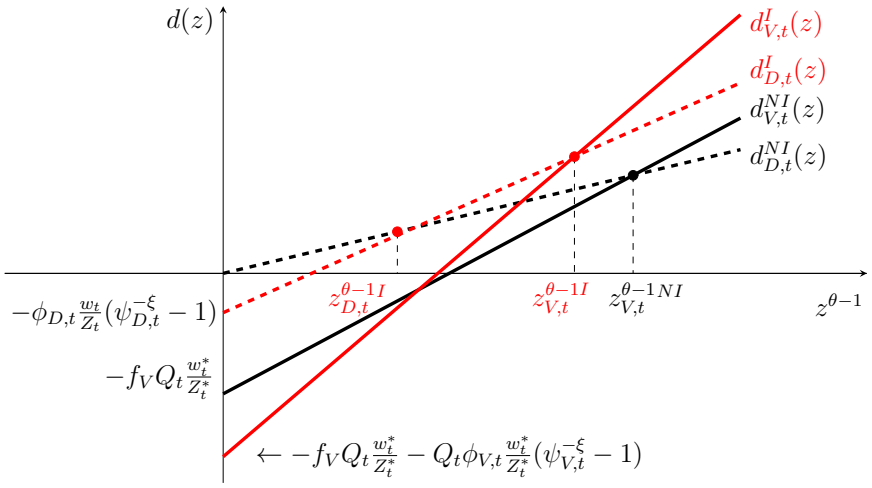


Figure 4: The productivity cutoff

The offshoring cutoff ($z_{V,t}$) In equilibrium, the existence of productivity cutoff $z_{V,t}$ requires a cross-country disparity in effective labor costs, ensuring that a subset of domestic firms find offshore production profitable. To illustrate this, I express per-period profits as: $d^{I}_{D,t}(z) = M_t\left(\psi^{\xi}_{D,t}\frac{w_t}{Z_t}\right)^{1-\theta} z^{\theta-1} - \phi_{D,t}\frac{w_t}{Z_t}(\psi^{-\xi}_{D,t}-1)$ and $d_{V,t}(z) = M_t\left(\tau\psi^{\xi}_{V,t}\frac{w^*_t Q_t}{Z^*_t}\right)^{1-\theta} z^{\theta-1}$ $-f_V\frac{w^*_t Q_t}{Z^*_t} - \phi_{V,t}\frac{w^*_t Q_t}{Z^*_t}(\psi^{-\xi}_{V,t}-1)$, where $M_t = \frac{1}{\theta}\left(\frac{\theta}{\theta-1}\right)^{1-\theta} C_t$ measures the size of Home market.

The model predicts that a productivity cutoff $z_{V,t}$ exists in equilibrium only if the slope of the offshoring profit function exceeds that of domestic production with intangible investment: $slope\{d_{V,t}(z)\} > slope\{d^{I}_{D,t}(z)\}$. This condition implies that offshoring is more profitable for firms with productivity above $z_{V,t}$.

The slope inequality is equivalent to the condition $\tau(\psi_{V,t}/\psi_{D,t})^{\xi} TOL_t < 1$, where $TOL_t = \frac{Q_t w^*/Z^*_t}{w_t/Z_t}$ denotes the relative cost of effective labor between the foreign and home country in the same currency. This condition requires the effective foreign wage, adjusted for relative marginal cost reductions, to be sufficiently low to offset fixed and iceberg trade costs ($\tau > 1$), incentivizing some domestic intangible investing firms to offshore. To avoid a corner solution, profits for domestic producers with intangible investment must exceed those of offshoring firms at z_{min} ($d^{I}_{D,t}(z_{min}) > d_{V,t}(z_{min})$). The model's calibration and the scale of macroeconomic shocks guarantee that this condition holds true in all periods.

2.3.4 Exporting

Firms in each economy can serve both domestic and foreign markets through exports, as in GM05. A Home firm with productivity z exports to the foreign market by using domestic

labor $l_{X,t}$ to produce $y_{X,t}(z) = Z_t z l_{X,t}(z)$. Foreign firms similarly export to Home. Profit maximization yields the equilibrium export price and profit function for a Home exporter with productivity z: $\rho_{X,t}(z) = \frac{\theta}{\theta-1}\tau^* Q_t^{-1}\frac{w_t}{Z_t z}$ and $d_{X,t}(z) = \frac{1}{\theta}\rho_{X,t}(z)^{1-\theta}C_t^* Q_t - f_X\frac{w_t}{Z_t}$, where C_t^* is foreign aggregate consumption. Exporting generates additional profits but requires fixed costs (f_X) in units of Home effective labor and iceberg trade costs τ^*. Only Home firms with $z \geq z_{X,t}$ can export profitably, where $z_{X,t} = inf\{z \mid d_{X,t}(z) > 0\}$.

2.4 Aggregation over heterogeneous firms

This section reformulates the offshoring model with four representative Home firms: (1) domestic production without intangible investment, (2) domestic production with intangible investment, (3) offshore production with intangible investment, and (4) domestic production for export. Offshoring occurs only from Home to Foreign. Accordingly, Foreign has three representative firms: domestic production without intangible investment, domestic production with intangible investment, and exporters to Home.

2.4.1 Average productivity, prices, and profits

Figure 5 shows average productivity for three representative Home firms: those producing domestically without intangible investment, those producing domestically with intangible investment, and those producing offshore. Specifically, Figure 5 plots the density of firm-specific labor productivity z over the support interval $[z_{min}, \infty)$. $N_{D,t}^{NI}$ firms with productivity $z < z_D^{NI}$ produce domestically without intangible investment, and their average productivity is $\tilde{z}_{D,t}^{NI}$. Additionally, $N_{D,t}^{I}$ firms with productivity $z_{D,t}^{I} < z < z_{V,t}$ have an average productivity of $\tilde{z}_{D,t}^{I}$. Finally, $N_{V,t}$ firms with productivity ($z > z_{V,t}$) have an average productivity of $\tilde{z}_{V,t}$. Given that firm-specific labor productivity levels z are randomly drawn from a common distribution $G(z)$ with density $g(z)$, the three average productivity levels are:

$$\tilde{z}_{D,t}^{NI} = \left[\frac{1}{G(z_{D,t}^{I})}\int_{z_{min}}^{z_{D,t}^{I}} z^{\theta-1}g(z)dz\right]^{\frac{1}{\theta-1}}, \quad \tilde{z}_{D,t}^{I} = \left[\frac{1}{G(z_{V,t}) - G(z_{D,t}^{I})}\int_{z_{D,t}^{I}}^{z_{V,t}} z^{\theta-1}g(z)dz\right]^{\frac{1}{\theta-1}},$$

$$\tilde{z}_{V,t} = \left[\frac{1}{1 - G(z_{V,t})}\int_{z_{V,t}}^{\infty} z^{\theta-1}g(z)dz\right]^{\frac{1}{\theta-1}} \tag{12}$$

Assuming firm-specific labor productivity z follows a Pareto distribution with density $g(z) = kz_{min}^k/z^{k+1}$ and cumulative distribution $G(z) = 1 - (z_{min}/z)^k$ over $[z_{min}, \infty)$, the average productivity levels can be expressed in terms of the cutoffs $z_{D,t}^{I}$ and $z_{V,t}$:

$$\tilde{z}_{D,t}^{NI} = \nu z_{min} z_{D,t}^{I}\left[\frac{z_{D,t}^{I\ k-(\theta-1)} - z_{min}^{k-(\theta-1)}}{z_{D,t}^{I\ k} - z_{min}^{k}}\right]^{\frac{1}{\theta-1}}, \quad \tilde{z}_{D,t}^{I} = \nu z_{D,t}^{I} z_{V,t}\left[\frac{z_{V,t}^{k-(\theta-1)} - z_{D,t}^{I\ k-(\theta-1)}}{z_{V,t}^{k} - z_{D,t}^{I\ k}}\right]^{\frac{1}{\theta-1}},$$

$$\tilde{z}_{V,t} = \nu z_{V,t} \tag{13}$$

where the cutoff is $z_{V,t} = z_{min}(N_t/N_{V,t})^{1/k}$, with parameters $\nu \equiv \left[\frac{k}{k-(\theta-1)}\right]^{\frac{1}{\theta-1}}$ and $k > \theta - 1$.

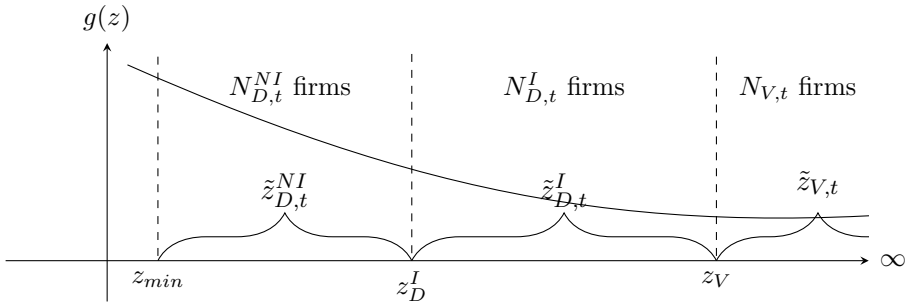


Figure 5: Average productivity of firms producing at Home

Foreign firms exclusively serve their domestic market through domestic production, with average productivity given by:

$$\tilde{z}_{D,t}^{NI*} = \nu z_{min}^{*} z_{D,t}^{I*} \left[\frac{z_{D,t}^{I*\ k-(\theta-1)} - z_{min}^{*k-(\theta-1)}}{z_{D,t}^{I*\ k} - z_{min}^{*k}}\right]^{\frac{1}{\theta-1}}, \quad \tilde{z}_{D,t}^{I*} = \nu z_{D,t}^{I*} \tag{14}$$

The average productivity levels of exporting firms are similar to GM05:

$$\tilde{z}_{X,t} = \nu z_{min}\left(\frac{N_t}{N_{X,t}}\right)^{1/k} \text{ and } \tilde{z}_{X,t}^{*} = \nu z_{min}^{*}\left(\frac{N_{D,t}^{*}}{N_{X,t}^{*}}\right)^{1/k} \tag{15}$$

Origin	Production	Market	Intangible	Average Prices	Average Profits
Home	Home	Home	Y	$\tilde{\rho}_{D,t}^{I} = \frac{\theta}{\theta-1}\psi_{D,t}^{\xi}\frac{w_t}{Z_t\tilde{z}_{D,t}^{I}}$	$\tilde{d}_{D,t}^{I} = \frac{1}{\theta}\tilde{\rho}_{D,t}^{I\ 1-\theta}C_t - \phi_{D,t}\frac{w_t}{Z_t}(\psi_{D,t}^{-\xi}-1)$
Home	Home	Home	N	$\tilde{\rho}_{D,t}^{NI} = \frac{\theta}{\theta-1}\frac{w_t}{Z_t\tilde{z}_{D,t}^{NI}}$	$\tilde{d}_{D,t}^{NI} = \frac{1}{\theta}\tilde{\rho}_{D,t}^{NI\ 1-\theta}C_t$
Home	Foreign	Home	Y	$\tilde{\rho}_{V,t} = \frac{\theta}{\theta-1}\tau Q_t\psi_{V,t}^{\xi}\frac{w_t^*}{Z_t^*\tilde{z}_{V,t}}$	$\tilde{d}_{V,t} = \frac{1}{\theta}\tilde{\rho}_{V,t}^{1-\theta}C_t - f_V Q_t\frac{w_t^*}{Z_t^*} - Q_t\phi_{V,t}\frac{w_t^*}{Z_t^*}(\psi_{V,t}^{-\xi}-1)$
Home	Home	Foreign	N	$\tilde{\rho}_{X,t} = \frac{\theta}{\theta-1}\tau^* Q_t^{-1}\frac{w_t}{Z_t\tilde{z}_{X,t}}$	$\tilde{d}_{X,t} = \frac{1}{\theta}\tilde{\rho}_{X,t}^{1-\theta}C_t^* Q_t - f_X\frac{w_t}{Z_t}$
Foreign	Foreign	Foreign	Y	$\tilde{\rho}_{D,t}^{*I} = \frac{\theta}{\theta-1}\psi_{D,t}^{*\xi*}\frac{w_t^*}{Z_t^*\tilde{z}_{D,t}^{*I}}$	$\tilde{d}_{D,t}^{*I} = \frac{1}{\theta}\tilde{\rho}_{D,t}^{*I\ 1-\theta}C_t^* - \phi_{D,t}^*\frac{w_t^*}{Z_t^*}(\psi_{D,t}^{*\ -\xi}-1)$
Foreign	Foreign	Foreign	N	$\tilde{\rho}_{D,t}^{*NI} = \frac{\theta}{\theta-1}\frac{w_t^*}{Z_t^*\tilde{z}_{D,t}^{*NI}}$	$\tilde{d}_{D,t}^{*NI} = \frac{1}{\theta}\tilde{\rho}_{D,t}^{*NI1-\theta}C_t^*$
Foreign	Foreign	Home	N	$\tilde{\rho}_{X,t}^{*} = \frac{\theta}{\theta-1}\tau Q_t\frac{w_t^*}{Z_t^*\tilde{z}_{X,t}^{*}}$	$\tilde{d}_{X,t}^{*} = \frac{1}{\theta}\tilde{\rho}_{X,t}^{*\ 1-\theta}C_t Q_t^{-1} - f_X^*\frac{w_t^*}{Z_t^*}$

Table 1: Average prices and profits

Based on the average productivity of firms engaged in domestic production (with and without intangible investment), offshoring, and exporting, Table 1 presents the average prices and profits for each production type. Therefore, the aggregate price indexes for Home and Foreign are: $1 = N_{D,t}^{NI}(\tilde{\rho}_{D,t}^{NI}(z))^{1-\theta} + N_{D,t}^{I}(\tilde{\rho}_{D,t}^{I}(z))^{1-\theta} + N_{V,t}(\tilde{\rho}_{V,t}(z))^{1-\theta} + N_{X,t}^{*}(\tilde{\rho}_{X,t}^{*}(z^*))^{1-\theta}$ and $1 = N_{D,t}^{*NI}(\tilde{\rho}_{D,t}^{*NI}(z^*))^{1-\theta} + N_{D,t}^{*I}(\tilde{\rho}_{D,t}^{*I}(z^*))^{1-\theta} + N_{X,t}(\tilde{\rho}_{X,t}(z))^{1-\theta}$. Similarly, the total profits of firms from Home and Foreign are $N_t\tilde{d}_t(z) = N_{D,t}^{NI}\tilde{d}_{D,t}^{NI}(z) + N_{D,t}^{I}\tilde{d}_{D,t}^{I}(z) + N_{V,t}\tilde{d}_{V,t}(z) + N_{X,t}\tilde{d}_{X,t}(z)$ and $N_t^*\tilde{d}_t^*(z^*) = N_{D,t}^{*NI}\tilde{d}_{D,t}^{*NI}(z^*) + N_{D,t}^{*I}\tilde{d}_{D,t}^{*I}(z^*) + N_{X,t}^{*}\tilde{d}_{X,t}^{*}(z^*)$.

2.4.2 Indifference conditions for offshoring and exporting

At the productivity cutoff, the two production strategies yield equivalent average productivity levels, establishing a linkage between the average profits of firms at distinct productivity cutoffs. First, at the intangible investment cutoff $z^I_{D,t}$, a connection emerges between the average profits of domestic firms with and without intangible investment. Second, at the offshoring cutoff $z_{V,t}$, a link appears between the average productivity of offshore firms and those producing domestically with intangible investment.

$$\begin{aligned}\tilde{d}^I_{D,t} = &\frac{k}{k-(\theta-1)}\left(\frac{z^I_{D,t}z_{V,t}}{\tilde{z}^{NI}_{D,t}}\right)^{\theta-1}\left[\frac{z^{k-\theta+1}_{V,t}-z^{I\ k-\theta+1}_{D,t}}{z^k_{V,t}-z^{I\ k}_{D,t}}\right]\tilde{d}^{NI}_{D,t}\\ &+\left\{\frac{k}{k-(\theta-1)}z^{\theta-1}_{V,t}\left[\frac{z^{k-\theta+1}_{V,t}-z^{I\ k-\theta+1}_{D,t}}{z^k_{V,t}-z^{I\ k}_{D,t}}\right]-1\right\}\phi_{D,t}\frac{w_t}{Z_t}(\psi^{-\xi}_{D,t}-1)\end{aligned} \tag{16}$$

$$\begin{aligned}\tilde{d}_{V,t} = &\frac{k}{k-(\theta-1)}\left(\frac{z_{V,t}}{\tilde{z}^I_{D,t}}\right)^{\theta-1}\left\{\tilde{d}^I_{D,t}+\phi_{D,t}\frac{w_t}{Z_t}(\psi^{-\xi}_{D,t}-1)\right\}-\frac{k}{k-(\theta-1)}\phi_{D,t}\frac{w_t}{Z_t}(\psi^{-\xi}_{D,t}-1)\\ &+\frac{\theta-1}{k-(\theta-1)}\left\{f_V Q_t\frac{w^*_t}{Z^*_t}+\phi_{V,t}Q_t\frac{w^*_t}{Z^*_t}(\psi^{-\xi}_V-1)\right\}\end{aligned} \tag{17}$$

Additionally, since firms at the productivity cutoff $z_{X,t}$ earn zero profits from exporting, the average export profits are derived as shown in GM05:

$$\tilde{d}_{X,t}=\frac{\theta-1}{k-(\theta-1)}f_X\frac{w_t}{Z_t},\quad \tilde{d}^*_{X,t}=\frac{\theta-1}{k-(\theta-1)}f^*_X\frac{w^*_t}{Z^*_t} \tag{18}$$

The derivations of equations (16)-(18) are provided in Appendix D.

2.5 Household Budget Constraint and Intertemporal Choices

The representative Home household entering period t holds two types of assets: domestic risk-free bond holdings B_t, and mutual fund share holdings x_t. Each period, the household begins with share holdings x_t in a mutual fund of N_t firms with average market value $\tilde{v}_t$. It also holds Home risk-free real bonds, B_t, denominated in units of the Home consumption basket. The household receives dividends equal to the average firm profit $\tilde{d}_t$ in proportion with the stock of firms N_t, the real wage w_t for $L \equiv 1$ supplied inelastically, and real rates of return r_t from the Home bonds. The household allocates these resources between purchases of bonds and shares to be carried into next period and consumption. Each period, the household purchases two types of assets, risk-free bonds B_{t+1} and mutual fund shares x_{t+1}. The mutual fund comprises N_t incumbent Home firms (producing domestically or offshore) and $N_{E,t}$ new

entrants in period t. The average market value of each firm is $\tilde{v}_t$, representing the net present value of expected future profits.

The period budget constraint (in units of consumption) is:

$$(1+r_t)B_t + (\tilde{v}_t + \tilde{d}_t)N_t x_t + w_t L \geq B_{t+1} + \tilde{v}_t(N_t + N_{E,t})x_{t+1} + C_t \tag{19}$$

The household maximizes its expected intertemporal utility subject to equation (19). The Euler equations for bonds is:

$$C_t^{-\gamma} = \beta(1+r_{t+1})E_t(C_{t+1})^{-\gamma} \tag{20}$$

The Euler equations for stocks is as follows, with the rate of firm exit δ described in Section 2.2:

$$\tilde{v}_t = \beta(1-\delta)E_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}(\tilde{v}_{t+1} + \tilde{d}_{t+1})\right] \tag{21}$$

With the consumption basket C_t set as the numeraire, the Home price index is normalized such that $1 = \left(\int_{\Omega_t} \rho_t(\omega)^{1-\theta} d\omega\right)^{\frac{1}{1-\theta}}$.

2.6 Aggregation over heterogeneous firms

In a financial autarky, household income from labor, stocks, and bonds is used for consumption and investment in new firms within each country:

$$C_t + N_{E,t}\tilde{v}_t + B_{t+1} = w_t L + N_t \tilde{d}_t + (1+r_t)B_t \tag{22}$$

$$C_t^* + N_{E,t}^*\tilde{v}_t^* + B_{t+1}^* = w_t^* L^* + N_{D,t}^*\tilde{d}_t^* + (1+r_t^*)B_t^* \tag{23}$$

The current account in the Home is given by:

$$\begin{aligned} CA_t = \underbrace{N_{X,t}(\tilde{\rho}_{X,t}(z))^{1-\theta}C_t^* Q_t}_{\text{(a) Exports}} + \underbrace{N_{V,t}\tilde{d}_{V,t}(z)}_{\text{(b) Repatriated offshore profits}} \\ - \underbrace{N_{V,t}(\tilde{\rho}_{V,t}(z))^{1-\theta}C_t}_{\text{(c) Value added offshore in Foreign}} - \underbrace{N_{X,t}^*(\tilde{\rho}_{X,t}^*(z^*))^{1-\theta}C_t}_{\text{(d) Imports from Foreign}} \end{aligned} \tag{24}$$

Under financial autarky, the balanced current account condition ($CA_t = 0$) requires that the sum of (a) export value added and (b) repatriated offshore profits equals the sum of (c) the value added by offshoring affiliates imported into Home and (d) imports from Foreign firms.

As a result, the benchmark model of financial autarky for the Home economy consists of 19 equations and 19 endogenous variables: $N_t, N_{D,t}^I, N_{D,t}^{NI}, N_{V,t}, N_{X,t}, N_{E,t}, \tilde{d}_t, \tilde{d}_{D,t}^I, \tilde{d}_{D,t}^{NI}, \tilde{d}_{V,t}, \tilde{d}_{X,t}, \tilde{z}_{D,t}^I, \tilde{z}_{D,t}^{NI}, \tilde{z}_{V,t}, \tilde{z}_{X,t}, \tilde{v}_t, r_t, w_t, C_t$.

Since Foreign firms do not produce in Home, the Foreign economy reduces to 16 equations and variables, with no counterparts for $N_{V,t}, \tilde{d}_{V,t}, \tilde{z}_{V,t}$. The domestic productivity of a representative Foreign firm $\tilde{z}^*_{D,t}$ is constant over time.

3 Calibration

The model calibrates $\beta = 0.99$ to match a 4% annual interest rate, and the relative risk aversion coefficient is $\gamma = 2$. Consistent with GM05, the intra-temporal elasticity of substitution (θ), the firm exit probability (δ), and the iceberg trade cost (τ) are set to 3.8, 0.025, and 1.2, respectively. Additionally, the Pareto distribution parameter (k) is calibrated to 3.8.

Following Zlate16, the fixed costs of offshoring (f_V) and exporting (f_X and f^*_X) are set to 0.095, 0.040, and 0.025, respectively. These parameter values ensure that the steady-state model replicates the observed significance of offshoring in the Foreign economy, as well as the role of exports in both the Foreign and Home economies. The lower bound of productivity support is normalized to $z_{min} = z^*_{min} = 1$.

According to Zlate16, the sunk entry cost is set to be higher in Foreign than in Home ($f^*_E = 4f_E$ and $f_E = 1$) to reflect the greater regulatory burden of starting a business abroad, thereby introducing an asymmetry in the effective labor costs between the two countries in steady state. Consequently, the steady-state output, the number of firms, the labor demand, and the effective wage are relatively lower in Foreign. This calibration is consistent with the 1.5 times higher cost of business start-up procedures in China compared to the United States in purchasing power parity terms in 2015 (World Bank Database, 2015).

The calibrated values for k, τ, f_V, f_X, f^*_X, along with asymmetric sunk entry costs, result in a steady-state value for the terms of labor that is less than one ($TOL = \frac{Qw^*/Z^*}{w/Z} = 0.76$). This indicates that the steady-state cost of effective labor in Foreign is 76% of that in Home, providing an incentive for some Home firms to offshore production.

Following De Ridder24's estimates of intangible cost share (ϕ), I set $\phi = 0.215$ (Home) and $\phi^* = 0.279$ (Foreign), reflecting higher US intangible investment efficiency. I assume US R&D efficiency (ϕ) is lower than trading partners (ϕ^*), using France's (ϕ^*) as a proxy for Foreign. The intangible cost elasticities, ξ and ξ^*, are set to 2, following De Ridder24. Setting these to 0 nests the model in Zlate16's framework without intangible investment.

The reduction fraction of marginal costs for Home and Foreign firms with intangible investment is set at 0.96. This indicates that these firms can reduce their marginal costs by $1 - \psi^{\xi}_{D,t} = 1 - 0.96^2 = 0.078$, or 7.8%. Estimated using Compustat Segment data, the event study indicates a 7% decline in marginal cost at the timing of the event when U.S. manufacturing firms increased their intangible investments (see Appendix A, Table 5, column (2) for details). For simplicity of comparison, I assume that foreign firms experience the same rate of marginal cost reduction from intangible investments as domestic firms.

Parameter	Description	Value	Source
β	Discount rate	0.99	GM05
γ	The coefficient of relative risk aversion	2	GM05
θ	The intra-temporal elasticity of substitution	3.8	GM05
δ	The probability of firm exit	0.025	GM05
k	The Pareto distribution parameter	3.8	
τ	The iceberg trade cost	1.2	Zlate16
f_E	The sunk entry cost, Home	1	Zlate16
f_E^*	The sunk entry cost, Foreign	4	Zlate16
f_V	The fixed costs of offshoring	0.095	Zlate16
f_X	The exporting fixed cost, Home	0.040	Zlate16
f_X^*	The exporting fixed cost, Foreign	0.025	Zlate16
ξ, ξ^*	Intangibles cost elasticity	2	De Ridder24
ϕ	Share of intangible costs in fixed costs, Home	0.215	De Ridder24
ϕ^*	Share of intangible costs in fixed costs, Foreign	0.279	De Ridder24
ψ_D	The reduction fraction of marginal costs, Home	0.96	Compustat Segment
ψ_V	The reduction fraction of marginal costs for offshoring, Home	0.93	Compustat Segment
ψ_D^*	The reduction fraction of marginal costs, Foreign	0.96	Compustat Segment

Table 2: Calibration parameters and steady-state targets

The reduction fraction for Home firms engaged in offshoring is set at 0.93, implying a marginal cost reduction of $1 - \psi_{V,t}^{\xi} = 1 - 0.93^2 = 0.135$, or 13.5%. Following Gumpert et al. (2025), this paper assumes $\psi_{D,t} > \psi_{V,t}$; therefore offshore intangible investment requires higher fixed costs than domestic intangible investment. Estimated using Compustat Segment data, the event study indicates a 13.6% decline in marginal cost at the timing of the event when U.S. offshoring manufacturing firms increased their intangible investments (see Appendix A, Table 5, column (6) for details).

Therefore, the most productive offshoring firms can achieve an average marginal cost reduction of 13.5% through intangible investment, while domestic Home firms can reduce costs by 7.8%, and Foreign domestic firms by 7.8%. All parameterizations are summarized in Table 2.

4 Results

4.1 Impulse responses

To illustrate the mechanisms and aggregate effects of intangible investment and offshoring, I log-linearize the model around the steady state and compute impulse responses to a permanent 1% productivity shock in Home. Aggregate productivity follows $logZ_{t+1} = \rho\ logZ_t + u_t$, with persistence $\rho = 0.98$.

4.1.1 Benchmark model

Figures 6 show impulse responses, with solid lines representing the benchmark model and dashed lines the model without intangible investment (Zlate16). The shock makes Home

more attractive to firms, increasing firm entry and boosting Home income. Higher income allows Home consumers to increase demand for both Home and Foreign goods. The rise in demand for Foreign varieties raises Foreign labor demand and wages, causing a short-run appreciation of Foreign wages and a depreciation of the terms of labor ($TOL_t = \frac{Qw_t^*/Z_t^*}{w_t/Z_t}$). TOL measures the cost of effective labor in the Foreign country relative to that in the Home country. In the long run, however, TOL appreciates. As aggregate productivity remains above its steady-state level for several periods after the shock, firm entry gradually rises, increasing the total number of firms (N). The resulting increase in labor demand pushes up the cost of effective labor in Home relative to Foreign, driving a gradual appreciation (decline) of TOL over time.

The benchmark model generates a greater increase in the average productivity of offshoring firms ($\tilde{z}_V$) following a positive productivity shock than the model without intangible investment. The short-run TOL depreciation raises offshoring costs, limiting firm entry into offshore production ($N_V \downarrow$). As a result, only more productive firms choose to offshore, increasing the average productivity of offshoring firms.

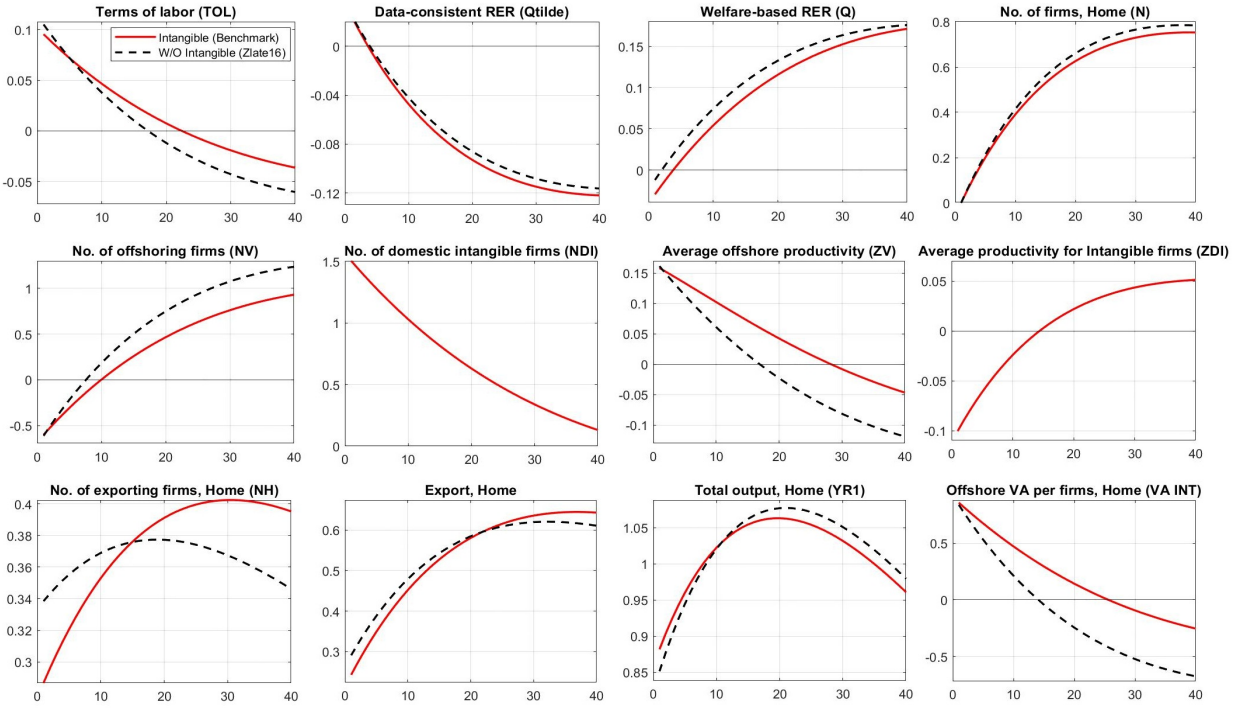


Figure 6: Comparing Impulse Responses: Benchmark Model and Model without Intangibles

The introduction of intangible capital leads to a larger increase in Home firms' profits, which further raises Home income. This prolongs the excess demand for Foreign labor, extending the period of TOL depreciation. Thus, the subsequent appreciation of TOL is also dampened over time compared to Zlate16. As a result, the prolonged depreciation restrains

the expansion of offshoring firms, leading to a more pronounced increase in average offshore productivity.

Intangible capital amplifies RER appreciation under a positive productivity shock, strengthening the HBS effect for two reasons. First, intangible capital induces short-run TOL depreciation, increasing Home firm entry, which lowers average productivity and raises domestic prices, further amplifying RER appreciation. Second, the increase in domestic entry leads to higher consumption of Home-produced varieties, shifting expenditure from Foreign-produced varieties to Home-produced varieties, further strengthening RER appreciation (A detailed explanation of how expenditure switching induces RER appreciation can be found in section 4.2.). This mechanism helps explain the empirical pattern in Figure 1 & 2: the post-GFC increase in U.S. intangible investment, combined with productivity growth, may have driven the observed RER appreciation.

The introduction of intangible capital boosts Home exports by reducing firms' marginal costs, enhancing price competitiveness during a Home productivity shock. This allows Home firms to offer lower prices than Foreign firms, increasing Home exports.

	Benchmark	Zlate16	Difference
Offshoring cutoff, Home (z_V)	3.094	3.283	-0.189
Welfare-consistent RER (Q)	1.278	1.303	-0.025
Data-consistent RER ($\widetilde{Q}$)	0.670	0.725	-0.055
Share of the number of firms ($\frac{N}{N^*}$)	10.386	7.847	2.539

Table 3: Steady State Results: Benchmark vs. Zlate16

Compared to the steady state in the model without intangible investment (Zlate16), the steady state of our model offers a foundation for assessing how intangible investment influences firms' long-term offshoring decisions. As shown in Figure 4, incorporating intangible capital lowers the offshoring productivity cutoff ($z_V \downarrow$), which in turn increases the number of offshore firms. This outcome stems from the mechanism in which intangible investment raises fixed costs but reduces marginal costs, thereby boosting offshore profits and encouraging more firms to choose offshoring. As a result, the productivity cutoff for offshoring falls relative to the model without intangible capital—consistent with our calibration results in Table 3 and the findings in Figure 4.

Furthermore, the introduction of intangible capital leads to a dampened depreciation of the welfare-consistent RER. Welfare-consistent RER measures differences in a consumer's welfare derived from spending a given normal amount in each market ($Q_t \equiv \epsilon P_t^*/P_t$), capturing pure variety effects. A value above 1 indicates depreciation, meaning consumers in Home enjoy greater welfare due to access to more varieties. In Zlate16, the steady-state RER was 1.303—above 1—due to asymmetries in wages and sunk entry costs between Home

and Foreign. A favorable productivity shock that enhances the attractiveness of the Home economy stimulates Home labor demand, driving up real wages and causing Home prices to rise relative to Foreign, which results in RER appreciation. When intangible capital is introduced, it further boosts firm profitability and induces an even greater increase in Home labor demand, leading to sharper wage hikes. Consequently, in the long run, the heightened wage pressure serves to moderate firm entry, which in turn limits the expansion of varieties. As a result, the depreciation of the welfare-consistent RER is dampened to 1.278, due to the relatively smaller variety effect.

In addition, incorporating intangible investment amplifies the appreciation of the data-consistent RER in the steady state. The data-consistent RER differs from the welfare-consistent RER in that it captures real price data and purges out pure variety effects. As shown in Table 3, the RER decreases from 0.725 to 0.670 with intangible investment. Although the RER remains below 1—indicating continued appreciation—the extent of appreciation is clearly strengthened. This effect arises for several reasons.

First, relative to the Zlate16 model, intangible investment enhances the profits and income of Home firms, which stimulates demand for Foreign-produced varieties and induces a short-run depreciation of TOL. This dynamic facilitates an increase in the number of domestic firms, which subsequently reduces their average productivity. This decline in average productivity, in turn, drives up the prices of domestically produced varieties, leading to an appreciation of the RER.

Second, the depreciation of TOL induced by intangible investment stimulates domestic firm entry, thereby expanding the production of domestic varieties. This process entails an expenditure switching effect, as Home consumers substitute imported varieties with those produced domestically. This transition constitutes a channel for RER appreciation, as previously lower-priced imports are replaced by relatively more expensive domestic goods.

4.2 Real Exchange Rate Dynamics

The welfare consistent RER ($Q_t \equiv \epsilon P_t^*/P_t$) measures differences in a consumer's welfare derived from spending a given normal amount in each market, capturing pure variety effects. When the number of Home varieties increases more than that of Foreign, the welfare consistent RER (Q_t) depreciates and exceeds 1. In contrast, the data-consistent RER captures real price data, but it purges out the pure variety effect. As in GM05, the data-consistent RER ($\widetilde{Q}_t$) is related to the welfare-consistent RER (Q_t) by the equation:

$$\widetilde{Q}_t = \left(\frac{N_t^*}{N_t}\right)^{\frac{1}{\theta-1}} Q_t \tag{25}$$

where $N_t = N_{D,t}^{NI} + N_{D,t}^{I} + N_V + N_X^*$ denotes the total number of varieties available to

Home consumers, and $N_t^* = N_{D,t}^{NI*} + N_{D,t}^{I*} + N_X$ represents the total number of products available to Foreign consumers.

To derive the data-consistent RER, I decompose welfare-based price indexes P_t and P_t^* into components representing (i) product variety and (ii) average prices as $P_t = (N_{D,t}^I + N_{D,t}^{NI} + N_{V,t} + N_{X,t}^*)^{\frac{1}{\theta-1}} \widetilde{P}_t$ for the Home country and $P_t^* = (N_{D,t}^{*I} + N_{D,t}^{*NI} + N_{X,t})^{\frac{1}{\theta-1}} \widetilde{P}_t^*$ for the Foreign country. The resulting expression for the data-consistent RER is:

$$\widetilde{Q}_{CPI,t}^{1-\theta} = \left(\frac{N_{D,t}^I + N_{D,t}^{NI} + N_{V,t} + N_{X,t}^*}{N_{D,t}^{*I} + N_{D,t}^{*NI} + N_{X,t}}\right) \frac{N_{D,t}^{*NI}\left(\frac{TOL_t}{\widetilde{z}_{D,t}^{*NI}}\right)^{1-\theta} + N_{D,t}^{*I}\left(\psi_{D,t}^{*\xi*}\frac{TOL_t}{\widetilde{z}_{D,t}^{*I}}\right)^{1-\theta} + N_{X,t}\left(\frac{\tau_t^*}{\widetilde{z}_{X,t}}\right)^{1-\theta}}{N_{D,t}^{NI}\left(\frac{1}{\widetilde{z}_{D,t}^{NI}}\right)^{1-\theta} + N_{D,t}^{I}\left(\frac{\psi_{D,t}^{\xi}}{\widetilde{z}_{D,t}^{I}}\right)^{1-\theta} + N_{V,t}\left(\frac{\psi_{V,t}^{\xi}\tau TOL_t}{\widetilde{z}_{D,t}^{I}}\right)^{1-\theta} + N_{X,t}^*\left(\frac{\tau TOL_t}{\widetilde{z}_{X,t}^*}\right)^{1-\theta}} \tag{26}$$

where the terms of labor $TOL_t = \frac{Qw_t^*/Z_t^*}{w_t/Z_t}$ measures the cost of effective labor in the Foreign country relative to that in the Home country; the iceberg trade costs τ_t and τ_t^* affect the imports of Home and Foreign, respectively. I log-linearize the data-consistent RER in equation (26) to derive economic implications. The derivation process is provided in the appendix F, and the final equation is as follows.

Analytical results The log-linearized version of (26) is:

$$\begin{aligned}
\widetilde{\mathsf{Q}}_t = {} & (s_D - s_V + s_D^* - 1)\mathsf{TOL}_t && \text{(C1)}\\
& + s_V(\tilde{\mathsf{z}}_{V,t} - \mathsf{t}_t) + (s_D - s_D^I - s_V)\tilde{\mathsf{z}}_{D,t}^{NI} + s_D^I\tilde{\mathsf{z}}_{D,t}^I && \text{(C2,C3,C4)}\\
& + (1 - s_D)(\tilde{\mathsf{z}}_{X,t}^* - \mathsf{t}_t) - (1 - s_D^*)(\tilde{\mathsf{z}}_{X,t} - \mathsf{t}_t^*) - (s_D^* - s_D^{*I})\tilde{\mathsf{z}}_{D,t}^{*NI} - s_D^{*I}\tilde{\mathsf{z}}_{D,t}^{*I} && \text{(C5)}\\
& - \ s_V\xi\Psi_{V,t} - s_D^I\xi\Psi_{D,t} + s_D^{*I}\xi^*\Psi_{D,t}^* && \text{(C6)}\\
& + \frac{1}{\theta-1}\left(s_V - \frac{N_V}{N_D + N_V + N_X^*}\right)\left(\mathsf{N}_{V,t} - \mathsf{N}_{X,t}^*\right) && \text{(C7)}\\
& + \frac{1}{\theta-1}\left[\begin{array}{l}\left(\frac{N_D^{*NI}}{N_D^{*NI} + N_D^{*I} + N_X} - (s_D^* - s_D^{*I})\right)\left(\mathsf{N}_{D,t}^{*NI} - \mathsf{N}_{X,t}\right)\\ \quad - \left(\frac{N_D^{NI}}{N_D^{NI} + N_D^I + N_V + N_X^*} - (s_D - s_D^I - s_V)\right)\left(\mathsf{N}_{D,t}^{NI} - \mathsf{N}_{X,t}^*\right)\end{array}\right] && \text{(C8)}\\
& + \frac{1}{\theta-1}\left[\begin{array}{l}\left(\frac{N_D^{*I}}{N_D^{*NI} + N_D^{*I} + N_X} - s_D^{*I}\right)\left(\mathsf{N}_{D,t}^{*I} - \mathsf{N}_{X,t}\right)\\ \quad - \left(\frac{N_D^I}{N_D^{NI} + N_D^I + N_V + N_X^*} - s_D^I\right)\left(\mathsf{N}_{D,t}^I - \mathsf{N}_{X,t}^*\right)\end{array}\right] && \text{(C9)}
\end{aligned}$$

Variables denoted in sans-serif font represent percent deviations from their respective steady states. Parameter s_D represents the steady-state share of Home's spending on varieties produced by Home firms, both domestically and offshore. Within s_D, s_D^I is the share spent on varieties produced by the domestic intangible firms, and s_V on varieties produced by offshoring firms. The remainder, s_D^{NI}, is denoted $s_D - s_D^I - s_V$, representing spending on

varieties produced by domestic firms without intangible investment. Similarly, s_D^* is the steady-state share of Foreign's spending on varieties produced by Foreign firms domestically, with s_D^{*I} and $(s_D^* - s_D^{*I})$ denoting the shares for intangible and non-intangible firms, respectively. The average productivities of Foreign intangible and non-intangible firms, $\tilde{z}_D^{*I}$ and $\tilde{z}_D^{*NI}$, remain constant over time.

The calibration ensures that: (a) $(s_D - s_V + s_D^*) > 1$, as domestically produced varieties represent more than half of the consumption expenditure in each country; (b) $\left(\frac{N_D^I}{N_D^{NI}+N_D^I+N_V+N_X^*} - s_D^I\right) > 0$ and $\left(\frac{N_D^{*I}}{N_D^{*NI}+N_D^{*I}+N_X} - s_D^{*I}\right) > 0$, indicating that less productive intangible-investing firms have smaller market shares than their share in the total number of varieties in both countries; (c) $\left(\frac{N_D^{NI}}{N_D^{NI}+N_D^I+N_V+N_X^*} - (s_D - s_D^I - s_V)\right) > 0$ and $\left(\frac{N_D^{*NI}}{N_D^{*NI}+N_D^{*I}+N_X} - (s_D^* - s_D^{*I})\right) > 0$, meaning that less productive non-intangible firms also have smaller market shares than their share in the total number of varieties in both countries; and (d) $\left(s_V - \frac{N_V}{N_D+N_V+N_X^*}\right) > 0$, showing that more productive offshoring firms hold larger market shares than their share in total varieties. As in Zlate16, the model captures that more productive offshoring firms dominate market shares compared to less productive domestic firms, consistent with the empirical findings of Kurz (2006).

The log-linearized equation (26) shows nine channels (C1-C9) through intangible capital that affect the data-consistent RER: (1) price of non-traded varieties (via TOL_t); (2) offshore prices (via the average offshore productivity $\tilde{\mathsf{z}}_{V,t}$, and the magnitude of trade costs t_t); (3) price of domestic non-intangible varieties (through average non-intangible firm productivity $\tilde{\mathsf{z}}_{D,t}^{NI}$); (4) price of domestic intangible varieties (via the average productivity of intangible firms $\tilde{\mathsf{z}}_{D,t}^{I}$); (5) relative import prices (via the average productivity of Foreign exporters $(\tilde{\mathsf{z}}_{X,t}^* - \mathsf{t}_t)$ and that of Home exporters $(\tilde{\mathsf{z}}_{X,t} - \mathsf{t}_t^*)$; (6) marginal cost reductions $(-\, s_V\xi\Psi_{V,t} - s_D^I\xi\Psi_{D,t} + s_D^{*I}\xi^*\Psi_{D,t}^*)$; (7) relative difference in varieties between offshore varieties and imported varieties $(\mathsf{N}_{V,t} - \mathsf{N}_{X,t}^*)$; (8) relative difference in varieties between domestic non-intangible varieties and imported varieties $(\mathsf{N}_{D,t}^{NI} - \mathsf{N}_{X,t}^*)$; and (9) relative difference in varieties between domestic intangible varieties and imported varieties $(\mathsf{N}_{D,t}^{I} - \mathsf{N}_{X,t}^*)$.

Impulse responses Intangible investment amplifies data-consistent RER appreciation following a positive Home aggregate productivity shock. This is because it boosts domestic firm entry, lowering average productivity and raising domestic prices, leading to RER appreciation. The greater variety of Home goods further shifts consumer spending from Foreign to domestic, reinforcing this appreciation. Thus, intangibles strengthen the HBS effect, explaining the post-GFC RER appreciation alongside increased US intangible investment. This operates through channels C3, C4, C8, and C9. Figure 6 illustrates this pronounced RER appreciation, and we explore the contributing channels, specifically the additional impact of intangible investment compared to Zlate (2010).

(C1) Price changes of non-traded varieties Without intangibles, a Home aggregate productivity increase encourages firm entry and lead to an appreciation of the terms of labor in the long run (i.e., TOL_t decreases). This results in a relative increase in the average price of non-traded varieties in Home, causing an appreciation of the data-consistent RER (i.e., $\widetilde{Q}_t^{CPI}$ decreases). Intangible investment can dampen this RER appreciation through two channels: (a) It leads to a dampened appreciation of TOL compared to Zlate (2010) (i.e., causes $\mathsf{TOL_t}$ to decrease by a smaller amount) because offshoring firms' intangible investments more significantly transfer wage pressures abroad. (b) Simultaneously, intangibles increase the impact of TOL on RER by lowering the spending share on offshore varieties and raising it on domestic varieties. This is seen in the coefficient of TOL_t in channel C1 ($s_D - s_V + s_D^* - 1$), which increases as the offshore share (s_V) falls and the domestic share (s_D) rises. Without intangible capital, TOL appreciation decreases the domestic spending share (s_D). However, intangible capital dampens the TOL appreciation in the long run, preventing a sufficient decrease in the relative cost of hiring foreign labor for offshoring firms and mitigating upward pressure on domestic labor costs. This limits the expansion of offshoring firms, reducing the offshore spending share (s_V), and conversely, increases the domestic spending share (s_D). Consequently, intangible capital increases the coefficient ($s_D - s_V + s_D^* - 1$), further contributing to the dampened TOL appreciation.

(C2) Price changes of offshore varieties This channel mirrors Zlate (2010). Rising Foreign wages reduce the number of offshoring firms but increase their average productivity. Intangible investment amplifies this average productivity increase ($\tilde{z}_{V,t}$), leading to a larger price drop in offshore varieties and dampening RER appreciation. However, long-run TOL appreciation increases offshoring profitability, growing the number of offshoring firms and decreasing their average productivity $\tilde{z}_{V,t}$, raising their average price over time and contributing to RER appreciation.

(C3 and C4) Price changes of domestically produced goods (without and with intangible capital) Intangible capital incorporation increases Home firms' profits and income, depreciating TOL. This initially raises the number of both non-intangible (N_D^{NI}) and intangible (N_D^I) firms, decreases their respective average productivities ($\tilde{z}_{D,t}^{NI}$ and $\tilde{z}_{D,t}^I$), and increases their average prices, leading to RER appreciation. However, as TOL appreciates over time, the increase in the number of both types of domestic firms slows, causing their average productivities to rise and average prices to fall, thus dampening the RER appreciation effect.

(C5) Relative import price changes Without intangible investment, offshoring increases Foreign labor demand, putting upward pressure on Foreign wages and weakening the export competitiveness of Foreign firms. As a result, the average productivity of surviving Foreign exporters ($\tilde{z}_{X,t}^*$) rises relative to that of their Home counterparts ($\tilde{z}_{X,t}$), lowering the average price of imports in Home relative to Foreign and dampening RER appreciation.

With intangible investment, this effect is amplified. Offshoring firms that invest in intangibles reduce marginal costs, further undermining the price competitiveness of Foreign firms. As a result, the average productivity of surviving Foreign exporters ($\tilde{z}^*_{X,t}$) rises further relative to their Home counterparts ($\tilde{z}_{X,t}$), lowering the average price of imports in Home and further dampening RER appreciation.

(C6) Changes in the fraction of marginal cost reduction The incorporation of intangible investment reduces marginal costs, leading to lower production prices. When the price of Home's offshore-produced varieties falls by $\Psi_{V,t}$, it causes RER depreciation. Similarly, a price decline in domestically produced varieties by $\Psi_{D,t}$ due to intangible investment also contributes to RER depreciation. In contrast, a decline in the price of Foreign's domestically produced varieties by $\Psi^*_{D,t}$ leads to RER appreciation.

As discussed in (C1), intangible investment raises the spending share of domestically produced varieties with intangible investment in total spending (s^I_D) and reduces the share on offshore varieties (s_V). As a result, the depreciation effect on the RER caused by offshore varieties is diminished, while that from domestic intangible varieties becomes more pronounced.

(C7) Expenditure switching from imports towards offshore varieties Similar to the channel discussed in Zlate (2010), offshoring reduces Foreign export competitiveness. Since offshoring requires intangible investment, Foreign export competitiveness decreases even further. Consequently, Home consumers shift their expenditure from less competitive Foreign varieties to relatively cheaper offshore varieties, more significantly dampening RER appreciation.

(C8 and C9) Expenditure switching from imports towards domestic varieties (without and with intangible capital) When there is a positive aggregate productivity shock, the incorporation of intangibles generates higher profits for Home firms, resulting in a rise in Home income. Consequently, Home consumers spend more on Foreign-produced varieties, which increases Foreign labor demand and Foreign wages. This short-run TOL depreciation causes more domestic firms—both those investing in intangibles and those not—to enter the Home market. This firm entry in the Home country increases the number of domestic varieties ($\mathsf{N}^I_{D,t}$ and $\mathsf{N}^{NI}_{D,t}$) compared to the number of imported foreign varieties ($\mathsf{N}^*_{X,t}$). Consequently, Home consumers shift their expenditure away from imported varieties toward domestically produced varieties, despite their relatively higher prices. This shift leads to an RER appreciation. However, as TOL appreciates in the long run, the number of domestically producing firms decreases. This reduction leads to a rise in average productivity and a decrease in average prices, easing the appreciation pressure on RER.

4.3 Model Performance: Empirical TFP Calibration

To assess the benchmark model's ability to explain actual U.S. data, I evaluate its

performance using quarterly U.S. TFP data from the San Francisco Fed. The Foreign TFP series was constructed by linearly interpolating annual China TFP data from the Penn World Table 10.01 into quarterly observations.

To evaluate the relative performance of the models under more realistic conditions, I depart from the uniform parameter settings introduced in Section 4.1. Specifically, our benchmark model maintains a Pareto parameter of $k = 3.8$, whereas the Zlate (2016) model is assigned its original value of $k = 4.2$ to ensure its results remain consistent with the existing literature. This lower k for the benchmark reflects the intensified firm-level heterogeneity and market concentration characteristic of the modern intangible-driven economy. Mathematically, a decrease in k signifies a greater market share for top-tier firms, aligning with UNCTAD data showing that Top 100 MNE sales nearly doubled between 2015 and 2023 (from $7,612 billion to $14,182 billion). This structural shift is supported by findings that increasing intangibles drive market power among leading U.S. public firms (Crouzet and Eberly, 2019) and that production concentration in the U.S. has risen steadily throughout the last century (Kwon, Ma, and Zimmermann, 2024).

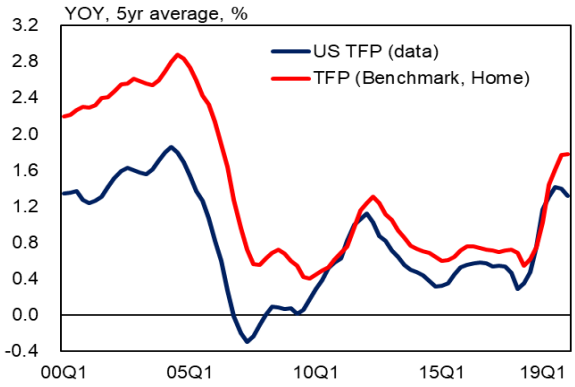


Figure 7: Productivity Growth in the U.S.
Source: Federal Reserve Bank of San Francisco

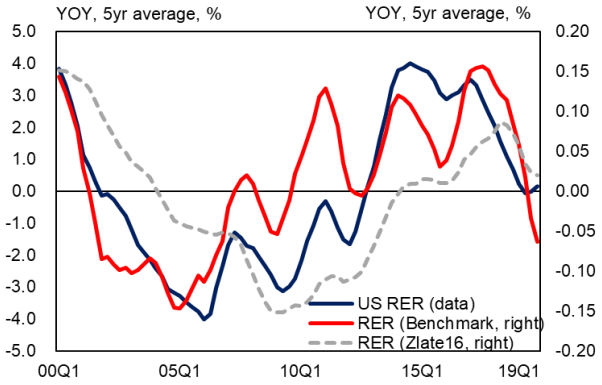


Figure 8: Real Exchange Rate Changes in the U.S.
Source: BIS

Simulations using real TFP data demonstrate that the model effectively captures post-GFC TFP improvements and RER appreciation in the U.S. Figure 7 illustrates the productivity shock dynamics generated by the model when injected with real TFP shocks. The model accurately replicates the historical trend of U.S. TFP declining after 2000 and subsequently increasing following the GFC.

Furthermore, RER dynamics generated using actual U.S. TFP shocks closely match the observed U.S. RER trajectory (Figure 8). The benchmark model, which incorporates intangible capital, proves highly effective at capturing the pattern of RER depreciation in the 2000s followed by appreciation after the GFC. In particular, the benchmark model significantly outperforms the Zlate16 model—which excludes intangibles—in tracking these

depreciation and appreciation trends. Specifically, Zlate16 overestimates the pre-crisis RER depreciation and underestimates the appreciation effect in the post-crisis period. In contrast, the benchmark model accurately replicates the pre-crisis RER depreciation and captures the subsequent appreciation with far greater precision than the Zlate16 framework. Furthermore, the benchmark model more precisely identifies the turning points in the empirical RER data compared to Zlate16.

These results suggest that the rise in firm profits and the consequent increase in domestic firm entry driven by intangible investment are pivotal in explaining the post-GFC RER appreciation and the Harrod–Balassa–Samuelson (HBS) effect. Thus, incorporating intangibles into an offshoring framework is essential for explaining productivity enhancements in the tradable sector and the associated real exchange rate appreciation.

4.4 The Business Cycle Properties

The aggregate productivies Z_t and Z_t^* from the steady state follow the bivariate process:

$$\begin{bmatrix} logZ_t \\ logZ_t^* \end{bmatrix} = \begin{bmatrix} \rho_Z & \rho_{ZZ^*} \\ \rho_{Z^*Z} & \rho_{Z^*} \end{bmatrix} \begin{bmatrix} logZ_{t-1} \\ logZ_{t-1}^* \end{bmatrix} + \begin{bmatrix} \varepsilon_t \\ \varepsilon_t^* \end{bmatrix}$$

with the persistence parameters ρ_Z and $\rho_Z^* < 1$, spillovers ρ_{ZZ^*} and $\rho_{Z^*Z} \geq 1$, and normally-distributed, zero-mean technology shocks ε_t and ε_t^*.

To compare the effects of introducing intangibles, I derived the moments generated by the benchmark model using the parameters of Zlate16. Home exhibits a more persistent productivity process than Foreign ($\rho_Z = 0.996 > \rho_{Z^*} = 0.951$). Spillovers from Foreign to Home are negligible ($\rho_{ZZ^*} = 0.003$), whereas Home-to-Foreign spillovers remain positive ($\rho_{Z^*Z} = 0.049$). To ensure comparability, the shock variance is set equal for both countries using the average standard deviation across the two economies (0.0095^2), and the covariance is fixed at 0.2422×10^{-4}.

The model generates volatilities for output, consumption, and firm entry that are close to the US data, although the volatilities of offshore VA and exports are generally less than in the data. The introduction of intangibles to Zlate16 increased the standard deviation of output, offshore VA, and exports, allowing for volatility closer to real data.

The correlation between U.S. real GDP and offshore value-added, exports, consumption, and firm entry was found to be procyclical. The introduction of intangibles reduces the procyclicality of offshore value-added while increasing that of exports, yielding a closer fit to post-GFC data than Zlate16. The inclusion of intangibles raises domestic income, which boosts demand for foreign-produced varieties and, in turn, increases foreign labor demand and wages, raising offshore costs. This mechanism dampens the rise in offshore value-added relative to the model without intangibles, consistent with the observed post-GFC decline in

business cycle procyclicality. Conversely, by enhancing the price competitiveness of exporters, intangibles expand exports and amplify their procyclicality, aligning with U.S. export patterns after the GFC.

		Data		Model		
		2000–23	2010–23	Benchmark	Zlate16	GM05
% Standard deviation						
Output [(1)]		1.41	1.39	1.18	1.14	1.08
Standard deviation relative to GDP						
OffshoreVA [(2)]	BEA	4.20	5.06	1.08	0.99	–
	Census	4.13	4.30			
Export		3.27	3.09	1.07	1.02	0.95
Consumption		1.06	1.26	0.76	0.82	0.81
New entrants [(3)]	Census	2.28	2.01	2.83	2.62	2.91
Correlation with GDP						
OffshoreVA [(4)]	BEA	0.82	0.58	0.73	0.84	–
	Census	0.58	0.59			
Export		0.87	0.84	0.84	0.82	0.83
Consumption		0.92	0.95	1.00	0.99	0.99
New entrants [(5)]	Census	0.71	0.29	0.99	1.02	0.95
Cross-country correlation						
(Y, EX^*) [(6)]	US, G20	0.55	0.50	0.82	0.84	0.87
	US, G10	0.66	0.65			
(Y, Y^*)	US, G20 [(7)]	0.56	0.67	0.32	0.35	0.22
	US, G10	0.77	0.81			
(C, C^*)	US, G20	0.37	0.54	0.84	0.40	0.80
	US, G10	0.58	0.78			
(Y, N_V)	No. of Foreign Affiliates [(8)]	0.48	0.36	0.59	0.97	–
$(C/C^*, \widetilde{Q})$	US, G20	0.01	-0.01	0.51	0.63	0.37
	US, G10	-0.12	-0.15			

Table 4: International Business Cycles with Intangible Investment

Source: BEA, BIS, BLS, Census Bureau, IMF

Note: (1) The real GDP is used. (2) BEA data: Manufacturing imports from affiliates, 2000–2022; Census data: Imports from related trade in manufacturing industries, 2005–2023. All offshore VA data are deflated using the BLS import price index excluding petroleum. (3) Center for Economic Studies, Business Dynamics Statistics: Firm age 0 –5 Years. (4) BEA data: Manufacturing imports from affiliates, 2000–2022; Census data: Imports from related trade in manufacturing industries, 2005–2023. All offshore VA data are deflated using the BLS import price index excluding petroleum. (5) Center for Economic Studies, Business Dynamics Statistics: Firm age 0 –5 Years. (6) Average of correlations between individual countries' HP-filtered cycles and the U.S. cycle. Deviations from the HP-filtered series of U.S. imports by country are averaged. Nominal imports are used due to the lack of real import data by country. (7) Average of correlations between individual countries' HP-filtered cycles and the U.S. cycle. (8) Number of affiliates with assets, sales, or net income (+/-) greater than $25 million, AMNE data, year: 2000–2020.

Examining cross-country correlations, foreign exports are procyclical with home output, but their cyclicality weakens with the introduction of intangibles—consistent with the post-GFC decline ($corr(Y, EX^*) \downarrow$). The introduction of intangibles reduces the cross-country correlation of output while increasing the cross-country correlation of consumption. Therefore, compared to Zlate16, intangibles exacerbate the consumption–output anomaly (BKK puzzle,

$corr(Y, Y^*) \ll corr(C, C^*)$). Intangibles lower output–extensive margin correlation, aligning more closely with real data ($corr(Y, N_V) \downarrow$). While intangibles slightly reduce the relative consumption–RER correlation ($corr(C/C^*, \widetilde{Q}) \downarrow$), the correlation remains strongly positive, failing to resolve the consumption–RER anomaly (Backus–Smith puzzle).

4.5 A Dynamic Decomposition of Aggregate Productivity

I apply the dynamic Olley-Pakes decomposition (Melitz and Polanec, 2015) to analyze how intangible investment affects aggregate productivity dynamics. Aggregate productivity change from period t to t+1 is decomposed into: (1) changes in average productivity among survivors, (2) changes in the covariance between market share and productivity, (3) the market share-weighted productivity contribution of entrants relative to survivors, and (4) that of exiters. In this paper's model, components (1) and (2) are further disaggregated by non-intangible investing firms, intangible investing firms, and offshoring firms. The decomposition is formalized in equation (27).

$$
\begin{aligned}
\Delta\, z_{t+1} &= \tilde{z}_{S,t+1} - \tilde{z}_{S,t} + \Delta\, cov_{S,t+1} + s_{E,t+1}(\Phi_{E,t+1} - \Phi_{S,t+1}) - s_{EX,t}(\Phi_{EX,t} - \Phi_{S,t}) \\
&= \underbrace{\Delta\, \tilde{z}^{NI}_{D,t+1} + \Delta\, \tilde{z}^{I}_{D,t+1} + \Delta\, \tilde{z}_{V,t+1}}_{\substack{\text{Average Productivity Changes} \\ \text{(Non-intangible / Intangible / Offshore firms)}}} \\
&\quad + \underbrace{\Delta\, cov_{NI,t+1} + \Delta\, cov_{I,t+1} + \Delta\, cov_{V,t+1}}_{\substack{\text{Changes in the Covariance Between Market Share and Productivity} \\ \text{(Non-intangible / Intangible / Offshore firms)}}} \\
&\quad + \underbrace{s_{E,t+1}\left\{\tilde{z}_{E,t+1} - \frac{s^{NI}_{D,t+1}\tilde{z}^{NI}_{D,t+1} + s^{I}_{D,t+1}\tilde{z}^{I}_{D,t+1} + s_{V,t+1}\tilde{z}_{V,t+1}}{s^{NI}_{D,t+1} + s^{I}_{D,t+1} + s_{V,t+1}}\right\}}_{\text{Market Share–Weighted Productivity Contribution of Entrants Relative to Survivors}} \\
&\quad + \underbrace{s_{EX,t}\left\{\tilde{z}_{EX,t} - \frac{s^{NI}_{D,t}\tilde{z}^{NI}_{D,t} + s^{I}_{D,t}\tilde{z}^{I}_{D,t} + s_{V,t}\tilde{z}_{V,t}}{s^{NI}_{D,t} + s^{I}_{D,t} + s_{V,t}}\right\}}_{\text{Market Share–Weighted Productivity Contribution of Exiters Relative to Survivors}}
\end{aligned}
\tag{27}
$$

where S: surviving firms (non-intangible/ intangible/ offshore firms), E: entrant firms, EX: exit firms, $s_{E,t}$: aggregate market share of entrant firms, $s_{EX,t}$: aggregate market share of exit firms, $\Phi_{E,t}$: aggregate productivity of entrant firms, $\Phi_{S,t}$: aggregate productivity of surviving firms

Figure 9 shows the aggregate productivity decomposition of the benchmark model with intangibles in response to a permanent aggregate productivity shock. The introduction of intangibles increases Home income in the short run, raising foreign labor demand and wages, which slows the growth in the number of offshore firms. This drives an increase in average offshore productivity due to short-run TOL depreciation, contributing to productivity gains until approximately period 25, as shown in Figure 9. Meanwhile, both the adoption of

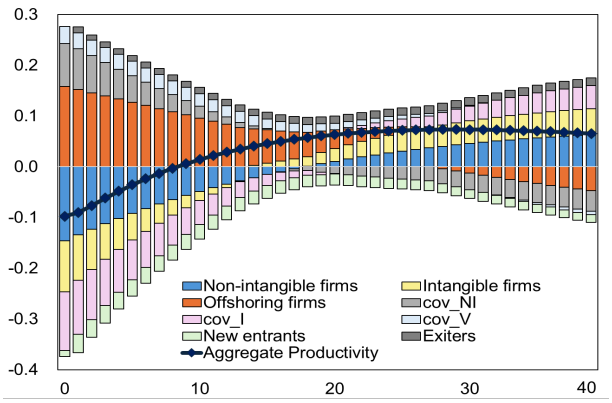


Figure 9: Aggregate Productivity Decomposition

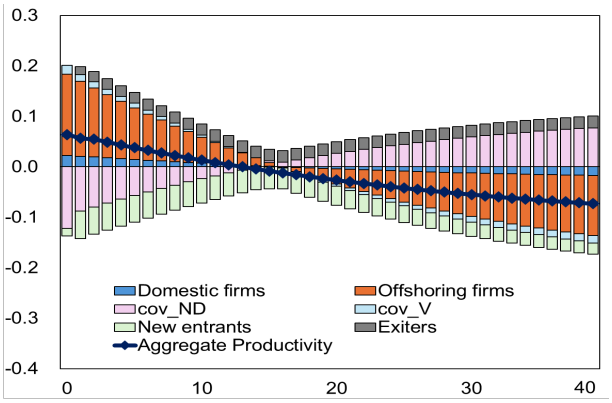


Figure 10: Aggregate Productivity Decomposition (Zlate16)

intangibles and the productivity shock significantly increase the number of domestic intangible firms, suggesting that domestic productivity gains may decrease in the short run. Over time, as domestic firm entry increases significantly, the TOL appreciates, burdening domestic production while offering cost-reduction benefits for offshore production. Consequently, the slowing growth rate of domestic firms contributes to productivity gains, whereas the slowing growth rate of offshore firms serves to reduce them. Especially, in the long-run, the sharp increase in fixed costs associated with intangibles further erodes the profits of domestic intangible firms, leading to a more pronounced decline in their numbers and a subsequent increase in average productivity. In short, the incorporation of intangibles drives trade-induced productivity improvements by fostering offshore productivity gains in the short run and inducing additional gains from domestic intangible firms in the long run.

The introduction of intangible investment enhanced resource allocation by enabling more productive domestic firms to expand their market share more effectively through price reductions as their productivity improved. The covariance terms reveal the relationship between changes in market share and productivity across firm production groups. Domestic intangible firms contributed to productivity gains through covariance as they increased their market share alongside productivity improvements over time. In contrast, domestic non-intangible firms reduced productivity gains via covariance because they failed to expand their market share despite gains in average productivity. Offshore firms expanded their market share while driving short-run productivity gains.

Figure 10 presents the results for Zlate16, which excludes intangibles. Identical Pareto distribution parameters are employed to compare the impact of incorporating intangible investment. When a productivity shock occurs, the increase in Home income leads to a depreciation of the TOL, which in turn causes a short-term decrease in the number of offshore firms and an increase in their average productivity. This results in productivity gains from offshore firms until period 16, as confirmed in Figure 10. Meanwhile, although domestic

firm productivity improves slightly in the short run due to the shock, the productivity gain diminishes over time as the number of domestic firms increases following TOL depreciation. Over time, as the TOL appreciates, the number of offshore firms increases, causing their productivity gain to turn negative after period 16. Finally, the covariance term (COV_{ND}) shows that as the productivity of domestic firms decreases over time, their market share also declines significantly.

Furthermore, in the long run, TOL appreciation raises the fixed costs associated with intangible investment, leading to a significant decline in the profits and the number of domestic intangible firms. Consequently, this reduction in firm numbers generates aggregate productivity gains even in the long run, a key factor that differentiates this from the model without intangibles. Incorporating intangibles into an offshoring model induces aggregate productivity gains, stemming from the different sensitivity of domestic intangible firms to changes in the TOL. Thus, the benchmark model reaffirms Melitz's (2003) argument—the productivity-enhancing role of trade-within the context of intangible offshoring.

5 Conclusion

This paper develops a general equilibrium offshoring model where firms endogenously choose both offshoring and intangible investment. By integrating the scalable nature of intangible capital—modeled as a fixed cost that lowers marginal costs—we examine how intangibles influence firms' offshoring decisions, aggregate productivity, and external competitiveness.

The model successfully reproduces the U.S. RER appreciation following the GFC and provides a theoretical mechanism for the intensified Harrod–Balassa–Samuelson (HBS) effect. We show that following a positive productivity shock, intangible capital boosts profits, inducing the entry of domestic firms. This lowers average domestic productivity and raises domestic prices, thereby amplifying RER appreciation.

Using the Dynamic Olley-Pakes decomposition, I find that the introduction of intangible investment amplifies aggregate productivity gains and enhances the efficiency of trade-induced resource allocation. In the short run, the introduction of intangibles increases Home income more significantly, leading to a depreciation of the TOL and expanding the productivity contribution of offshore firms. In the long run, the surge in domestic labor demand drives up wages, reducing the number of domestic firms. The high fixed costs associated with intangibles further erode the profits of domestic intangible firms, leading to a more pronounced decline in their numbers and a subsequent increase in average productivity. Consequently, explicitly incorporating intangible investment demonstrates that trade-induced productivity improvements are substantially larger than previously thought, reinforcing the resource allocation channel highlighted by Melitz (2003).

Furthermore, the benchmark model captures the expansion of exports following a productivity

shock more effectively than a model without intangibles, closely matching the increased export procyclicality observed in U.S. data. By reducing marginal costs, intangibles enhance the price competitiveness of exporters, leading to export growth. This consistency with empirical data suggests that intangibles are a crucial factor in explaining the procyclicality of exports.

On the other hand, the introduction of intangibles raise the foreign wages associated with production in Foreign plants, thereby reducing the value added (VA) and the extensive margin of offshore firms. Since intangible investment leads to a simultaneous increase in domestic production and a decrease in offshore VA, a pattern that parallels the declining procyclicality of offshore VA in U.S. data. These results underscore the necessity of incorporating intangibles to accurately evaluate the decisions of offshore firms.

In conclusion, this paper highlights the central role of intangibles in determining aggregate productivity and the RER. By bridging the gap between endogenous intangible investment and open economy macroeconomics, the findings offer significant implications for policy. As the global economy increasingly shifts toward intangible assets, our framework suggests that traditional trade models may underestimate the speed and magnitude of productivity gains from trade, while misdiagnosing the drivers of exchange rate fluctuations.

References

Baldwin, E. Richard, and Simon J. Evenett. 2015. "Value Creation and Trade in 21st Century Manufacturing." *Journal of Regional Science* 55(1): 31–50.

Bartel, Ann, Casey Ichniowski, and Kathryn Shaw. 2007. "How Does Information Technology Affect Productivity? Plant-Level Comparisons of Product Innovation, Process Improvement, and Worker Skills." *Quarterly Journal of Economics* 122(4): 1721–58.

Bergin, Paul R., Robert C. Feenstra, and Gordon H. Hanson. 2009. "Offshoring and Volatility: Evidence from Mexico's Maquiladora Industry." *The American Economic Review*, 99 (4), 1664–1671.

Bernard, Andrew B., Teresa C. Fort, Valerie Smeets, and Frederic Warzynski. 2020. "Heterogeneous Globalization: Offshoring and Reorganization." *NBER Working Paper* No. 26854.

Burstein, Ariel, Christopher Kurz, and Linda Tesar. 2008. "Trade, Production Sharing, and the International Transmission of Business Cycles." *Journal of Monetary Economics* 55(4): 775–95.

Bøler, Esther Ann, Andreas Moxnes, and Karen Helene Ulltveit-Moe. 2015. "R&D, International Sourcing, and the Joint Impact on Firm Performance." *American Economic Review* 105(12): 3704–39.

Coe, David T., and Elhanan Helpman. 1993. "International R&D Spillovers." *NBER Working Paper* No. 4444.

Corrado, Carol A., Charles R. Hulten, and Daniel E. Sichel. 2006. "Intangible Capital and Economic Growth." *Review of Income and Wealth* 55: 661–85.

Corsetti, Giancarlo, Luca Dedola, and Sylvain Leduc. 2008. "High Exchange-Rate Volatility and Low Pass-Through." *Journal of Monetary Economics* 55(6): 1113–28.

Crouzet, Nicolas, Janice C. Eberly, Andrea L. Eisfeldt, and Dimitris Papanikolaou. 2022. "The Economics of Intangible Capital." *Journal of Economic Perspectives* 36(3): 29–52.

De Ridder, Maarten. 2024. "Market Power and Innovation in the Intangible Economy." *American Economic Review* 114(1): 199–251.

Ghironi, Fabio, and Marc J. Melitz. 2005. "International Trade and Macroeconomic Dynamics with Heterogeneous Firms." *Quarterly Journal of Economics* 120(3): 865–915.

Gornemann, Nils, Pablo A. Guerrón Quintana, and Felipe Saffie. 2025. "Real Exchange Rates and Endogenous Productivity." *American Economic Journal: Macroeconomics* 17(4): 204–261.

Griliches, Zvi. 1998. "R&D and Productivity Growth at the Industry Level: Is There Still a Relationship?" *NBER Working Paper* No. 213–240.

Grossman, Gene M., Elhanan Helpman, and Alejandro Sabal. 2024. "Optimal Resilience in Multitier Supply Chains." *Quarterly Journal of Economics* 139(4): 2377–2425.

Grossman, Gene M., and Esteban Rossi-Hansberg. 2008. "Trading Tasks: A Simple Theory of Offshoring." *American Economic Review* 98(5): 1978–97.

Gumpert, Anna, Kalina Manova, Cristina Rujan, and Monika Schnitzer. 2025. "Multinational Firms and Global Innovation." *CEPR Discussion Paper* No. 20045.

Guvenen, Fatih, Raymond J. Mataloni Jr., Dylan G. Rassier, and Kim J. Ruhl. 2022. "Offshore Profit Shifting and Aggregate Measurement: Balance of Payments, Foreign Investment, Productivity, and the Labor Share." *American Economic Review* 112(6): 1848–84.

Haskel, Jonathan, and Stian Westlake. 2017. Capitalism Without Capital: The Rise of the Intangible Economy. *Princeton University Press.*

Helpman, Elhanan, Marc J. Melitz, and Stephen R. Yeaple. 2004. "Export Versus FDI with Heterogeneous Firms." *American Economic Review* 94(1): 300–16.

Hsieh, Chang-Tai, and Esteban Rossi-Hansberg. 2023. "The Industrial Revolution in Services." *Journal of Political Economy: Macroeconomics.*

Karpowicz, Izabela, and Nujin Suphaphiphat. 2020. "Productivity Growth and Value Chains in Four European Countries." *IMF Working Paper* 018.

Jona-Lasinio, Cecilia, Stefano Manzocchi, and Valentina Meliciani. 2019. "Knowledge Based Capital and Value Creation in Global Supply Chains." *Technological Forecasting and Social Change* 148(C).

Liu, Jin. 2024. "Multinational Production and Innovation in Tandem." *Census Working Paper* CES-24-64.

Méjean, Isabelle. 2008. "Can Firms' Location Decisions Counteract the Balassa–Samuelson Effect?" *Journal of International Economics* 76(2): 139–54.

Melitz, Marc J. 2003. "The Impact of Trade on Intra-Industry Reallocations and Aggregate Industry Productivity." *Econometrica* 71(6): 1695–1725.

Melitz, Marc J., and Sašo Polanec. 2015. "Dynamic Olley-Pakes Productivity Decomposition with Entry and Exit." *RAND Journal of Economics* 46(2): 362–75.

Rabanal, Pau, and Juan F. Rubio-Ramirez. 2015. "Can International Macroeconomic Models Explain Low-Frequency Movements of Real Exchange Rates?" *Journal of International Economics* 96(1): 199–211.

Zlate, Andrei. 2016. "Offshore Production and Business Cycle Dynamics with Heterogeneous Firms." *Journal of International Economics* 100: 34–49.

Appendix A. Empirical Facts and Event Study

This study utilizes Compustat Segment data to establish stylized facts regarding the impact of intangible capital investment on offshoring decisions and aggregate productivity. These empirical findings provide the rationale for the assumptions adopted in the benchmark model presented in the main text.

The Compustat Segment data covers 28,900 North American firms, representing 67% of the total database. This dataset is instrumental for analyzing the operating structures and geographical segmentation of firms expanding into foreign markets. It is uniquely suited for this study as it not only facilitates the identification of offshore plant locations but also contains the micro-level data—specifically R&D and Selling, General, & Administrative (SG&A) expenses—necessary to construct aggregate measures of intangible investment.

Firms are identified using the "GEOTP" variable within the operating segment type classification. Because this variable details the geographic locations of operations, it allows for a clear distinction between the domestic plants and foreign affiliates of multinational corporations. Following Corrado et al. (2006) and Eisfeldt and Papanikolaou (2013), intangible investment is defined as the sum of R&D expenses and organizational capital investment, the latter calculated as 30% of SG&A expenses. For the purpose of this analysis, total factor productivity (TFP) is approximated as sales per employee, while marginal cost is proxied by the ratio of cost of goods sold (COGS) to sales, assuming COGS represents variable costs. Manufacturing industries are defined by NAICS codes 311–339. Based on this data, I document the following stylized facts:

(i) The scalability of intangible investment (Figure 11): Manufacturing firms demonstrate scalability, characterized by a negative correlation between intangible investment and marginal costs. This finding validates the assumption in the Section 3 benchmark model that intangible investment drives reductions in marginal cost.

(ii) Revenue growth with intangible investment (Figure 12): There is a positive relationship between intangible investment and average sales. As shown in Figure 12, average sales rise distinctively as manufacturing firms progress to higher R&D quantiles. This corroborates the model's implication that intangible capital accumulation contributes to increased firm profitability.

(iii) Productivity sorting into intangible investment and offshoring (Figure 13): More productive firms make higher intangible investments and engage more in offshoring. The benchmark model relies on the assumption that only productive firms can overcome the fixed costs of intangible investment and offshoring. Figure 13 validates this assumption empirically: average productivity clearly increases alongside intangible investment quantiles. Additionally, offshore firms display higher average productivity than their domestic counterparts. These

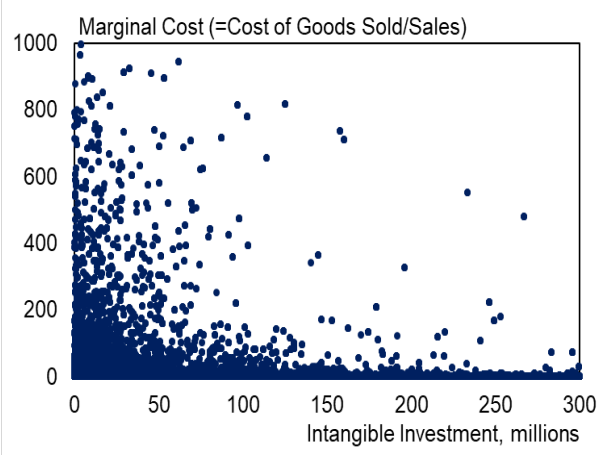


Figure 11: Scalability of Intangible Investment
Source: Compustat Segment

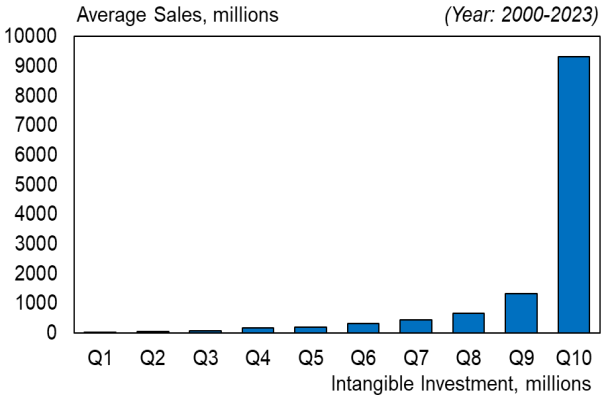


Figure 12: Quantiles of Intangible Investment and Average Sales (Manufacturing)
Source: Compustat Segment

findings provide empirical evidence that high-productivity firms are significantly more likely to undertake both offshoring and intangible investment.

(iv) Offshore firms tend to have higher intangible investment and higher sales compared to non-offshore firms (Figure 14): Figure 14 presents a scatter plot of intangible investment against sales, distinguishing between offshore and domestic entities. The data reveals a significant positive correlation between sales and intangible investment for offshore firms relative to non-offshore firms. This pattern is consistent with the benchmark model, in which offshore firms achieve higher profits through intangible investment and therefore face stronger incentives to invest in intangibles. Based on this empirical fact, I assume—purely for simplicity in the benchmark—that offshore firms necessarily undertake intangible investment.

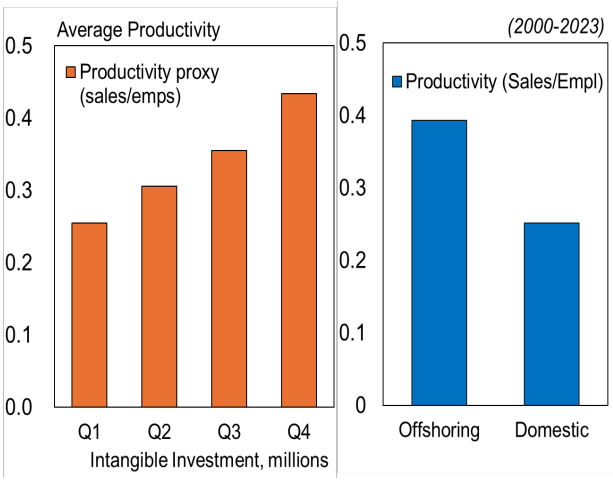


Figure 13: Intangibles Quantile & Productivity, Offshore vs. Domestic Productivity
Source: Compustat Segment

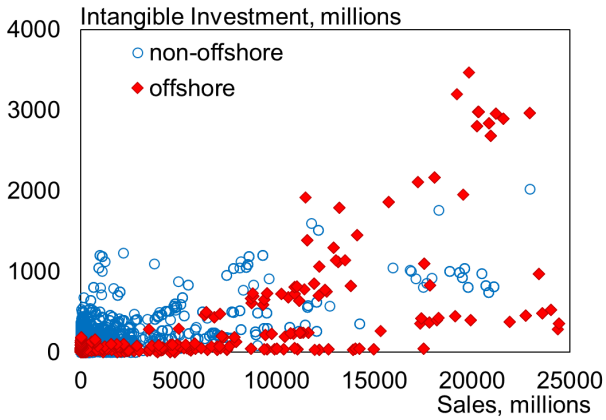


Figure 14: Offshore vs. Non-offshore Intangibles
Source: Compustat Segment
Note: Exclude data for non-offshore firms whose segments are classified as market, division, or product and service.

(v) Positive sales–intangible correlation in high-tech sectors (Figure 15): In manufacturing, intangible investments exhibit a stronger positive correlation with net sales growth within high-tech industries.[4] This observation confirms that among firms with significant intangible assets, technology-intensive firms achieve superior sales growth rates. Consequently, this suggests that the benchmark model is particularly effective for analyzing the dynamics of high-technology manufacturing firms.

(vi) Positive correlation between exports and intangible investment (Figure 16): For manufacturing firms, a rise in intangible investment is associated with increased export activity. In the benchmark model, both exporting and intangible investment entail fixed costs, meaning only firms capable of covering these costs can engage in them. In other words, firms with higher productivity are more likely to export and invest in intangibles. Figure 16 demonstrates this fact, showing that highly productive firms tend to increase both their exports and intangible investments.

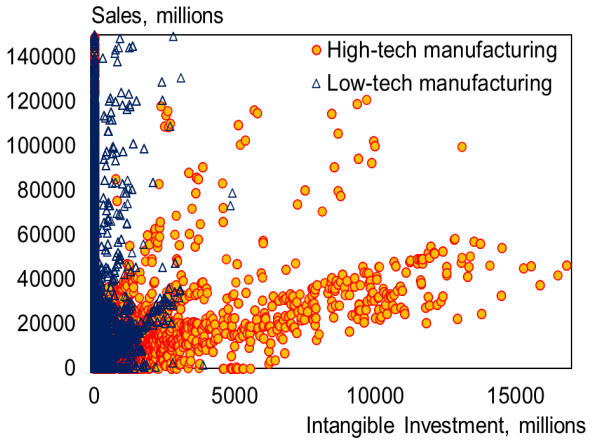


Figure 15: High- and Low-Tech Sales & Intangibles
Source: Compustat Segment

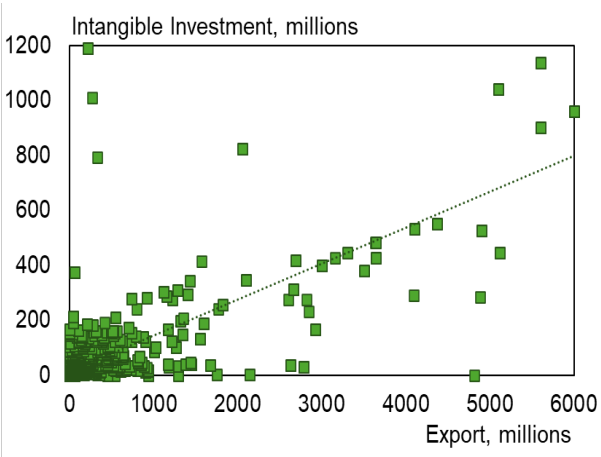


Figure 16: Export and Intangible Investment
Source: Compustat Segment

I executed an Event study using Equation (28) and Compustat Segment data to investigate the impact of intangible capital on sales, exports, offshore value-added, and productivity. To investigate how the introduction of intangible investment affects the decision-making of offshoring firms, I defined the event as a firm's R&D-to-sales ratio exceeding 10% compared to the previous year ($D_{i,t+j} = 1$ if firm i's R&D-to-sales ratio $\geq 10\%$ at relative year τ). To account for highly productive firms that already conduct intangible investment, I chose not to define the event as a transition from zero R&D investment to positive R&D. Relative year $j = 0$ corresponds to the event year when the R&D-to-sales ratio first exceeds 10%, and

[4]High-tech manufacturing includes industries classified under NAICS codes 325 (Chemicals), 334 (Computer and Electronic Products), 336 (Transportation Equipment), 333 (Machinery), 335 (Electrical Equipment), and 339 (Miscellaneous Manufacturing).

$j = -1$ is used as the omitted reference period. The unit of the event window is years.

$$y_{i,t} = \sum_{j=-5}^{7} \gamma_j D_{i,t+j} + \delta_i + \delta_t + \beta X_{i,t} + \epsilon_{i,t}, \tag{28}$$

$y_{i,t}$ represents sales, offshore value-added (VA), marginal cost, exports, and productivity, and γ_j captures the effect at relative year j. δ_i and δ_t capture firm and year fixed effects, respectively, and $X_{i,t}$ represents control variables.

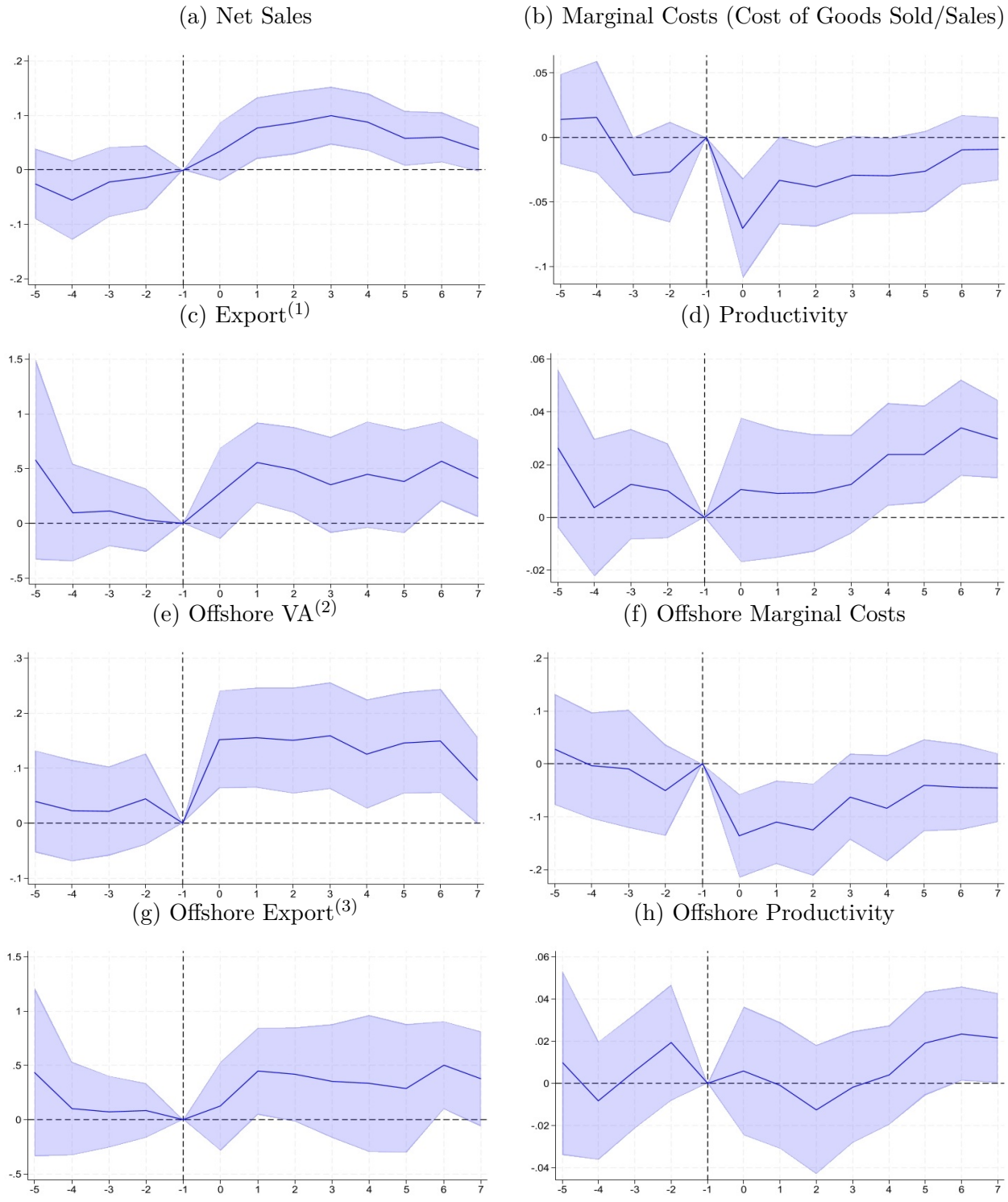
The event study was conducted on manufacturing firms and, specifically, on offshore firms within that group. First, the Intangible investment of manufacturing firms was found to significantly increase net sales, export, and productivity at the 95% confidence level, relative to the control group, which experienced a comparatively smaller increase in R&D investment. The investment also significantly decreased marginal cost, thereby demonstrating a scalable nature. Specifically, as shown in Figure 17, the increase in intangible investment led to a significant rise in manufacturing firms' net sales starting from one year post-event (Figure 17 (a)), and a significant immediate decrease in marginal cost for two years following the event (Figure 17 (b)). Furthermore, export significantly increased for up to two years post-event following the rise in intangible investment (Figure 17 (c)), and productivity significantly increased starting from the fourth year post-event (Figure 17 (d)).

For offshore firms, the increase in intangible investment significantly increased offshore VA (Value Added) and also significantly reduced marginal cost. Offshore VA showed a significant increase over seven years post-event (Figure 17 (e)), and marginal cost significantly decreased for two years post-event, demonstrating scalability (Figure 17 (f)). Conversely, the degree of significant increase in offshore firms' exports and productivity was weaker compared to the overall manufacturing firms. Nevertheless, export significantly increased in the first year post-event following the increase in offshore firms' intangible investment (Figure 17 (g)), and offshore productivity also showed a significant increase starting from the sixth year (Figure 17 (h)).

The detailed table of the Event study estimation results can be found in Appendix A.

Based on these results, we confirm, using micro-level evidence, that an increase in intangible investment significantly boosts the sales, export, and productivity of both manufacturing and offshore firms, while significantly lowering their marginal cost. This finding implies that the assumptions in the benchmark model—that firms with intangible investment experience increased profit and lower marginal cost—are supported by empirical evidence. Furthermore, the model's foundation, which posits that firms with intangible investment expand their exports and have higher productivity, is also corroborated by this event study.

Figure 17: Manufacturing Firms & Offshore Firms



Source: Compustat Segment

Note: Sample period is 2000–2023. For the dependent variables—net sales, offshore VA, export, marginal costs, and productivity measures—the inverse hyperbolic sine (IHS) transformation ($\text{asinh}(x) = \ln(x + \sqrt{x^2 + 1})$) was applied. (1) Missing values for exports are removed. (2) Offshoring VA is measured using sales of vertical offshore plants. (3) Missing values for exports are removed.

Table 5: Event Study Results

	Manufacturing Firms							
Dependent Variables	Log(Sales)	Log(Marg. Costs)	Log(Export)	Log(Productivity)	Log(Offsh. VA)	Log(Offsh. MC)	Log(Offsh. Export)	Log(Offsh. Prod.)
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
4 years before	-0.025	0.014	0.583	0.026	0.040	0.028	0.439	0.010
	(0.033)	(0.018)	(0.464)	(0.015)	(0.047)	(0.053)	(0.394)	(0.022)
3 years before	-0.055	0.016	0.099	0.004	0.023	-0.003	0.104	-0.008
	(0.037)	(0.022)	(0.227)	(0.013)	(0.047)	(0.051)	(0.218)	(0.014)
2 years before	-0.022	-0.029	0.112	0.013	0.022	-0.009	0.074	0.006
	(0.033)	(0.015)	(0.162)	(0.011)	(0.041)	(0.057)	(0.168)	(0.014)
1 year before	-0.014	-0.027	0.029	0.010	0.044	-0.050	0.086	0.019
	(0.030)	(0.020)	(0.146)	(0.009)	(0.042)	(0.044)	(0.127)	(0.014)
Event	0.034	-0.070***	0.276	0.010	0.152***	-0.136***	0.124	0.006
	(0.027)	(0.020)	(0.212)	(0.014)	(0.045)	(0.040)	(0.208)	(0.016)
1 year after	0.077***	-0.033*	0.556***	0.009	0.155***	-0.111***	0.447**	-0.001
	(0.029)	(0.017)	(0.187)	(0.012)	(0.046)	(0.040)	(0.203)	(0.015)
2 years after	0.087***	-0.038**	0.489**	0.009	0.150***	-0.125***	0.417*	-0.012
	(0.029)	(0.016)	(0.199)	(0.011)	(0.049)	(0.045)	(0.220)	(0.016)
3 years after	0.100***	-0.029*	0.353	0.013	0.159***	-0.062	0.356	-0.002
	(0.027)	(0.015)	(0.223)	(0.010)	(0.049)	(0.041)	(0.266)	(0.014)
4 years after	0.088***	-0.030**	0.446*	0.024**	0.126**	-0.084	0.334	0.004
	(0.027)	(0.015)	(0.247)	(0.010)	(0.051)	(0.051)	(0.320)	(0.012)
5 years after	0.058**	-0.026*	0.385	0.024**	0.146***	-0.041	0.291	0.019
	(0.026)	(0.016)	(0.239)	(0.009)	(0.047)	(0.044)	(0.301)	(0.013)
6 years after	0.060**	-0.010	0.566***	0.034***	0.150***	-0.044	0.503**	0.024**
	(0.023)	(0.014)	(0.186)	(0.009)	(0.048)	(0.041)	(0.206)	(0.011)
7 years after	0.038*	-0.009	0.411**	0.030***	0.078*	-0.045	0.376*	0.022**
	(0.021)	(0.012)	(0.179)	(0.008)	(0.040)	(0.033)	(0.222)	(0.011)
Pre-trend	No	Yes	Yes	Yes	No	No	Yes	No
Firm lin. time trends	Yes	No	No	Yes	No	No	No	Yes
FE (year)	No	No	Yes	No	Yes	Yes	Yes	No
FE (firm)	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Mean dep. Var.	4.6886	0.8716	3.0913	0.3459	5.0892	0.9215	3.0651	0.3766
# Cluster (firm)	4,307	4,421	413	4,525	1,404	1,403	360	1,573
N	95,996	100,763	5,172	127,255	21,823	21,617	4,449	30,247
Prob > F	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
R square	0.9696	0.8508	0.9457	0.6242	0.9599	0.7361	0.9499	0.8301

Notes: Robust standard errors in parentheses, clustered at the firm level. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$. Dependent variables (export, offshore VA, productivity) are inverse hyperbolic sine transformed. Event study coefficients are relative to year -1 (normalized to 0). All regressions include firm fixed effects.

Appendix B. Model Summary (Financial Autarky)

Average profit (Home)	$\tilde{d}^I_{D,t}(z) = \frac{1}{\theta}\tilde{\rho}^I_{D,t}(z)^{1-\theta}C_t - \phi_{D,t}\frac{w_t}{Z_t}(\psi^{-\xi}_{D,t} - 1)$
	$\tilde{d}^{NI}_{D,t}(z) = \frac{1}{\theta}\tilde{\rho}^{NI}_{D,t}(z)^{1-\theta}C_t$
	$\tilde{d}_{V,t}(z) = \frac{1}{\theta}\tilde{\rho}_{V,t}(z)^{1-\theta}C_t - f_V Q_t\frac{w^*_t}{Z^*_t} - Q_t\phi_{V,t}\frac{w^*_t}{Z^*_t}(\psi^{-\xi}_{V,t} - 1)$
	$\tilde{d}_{X,t}(z) = \frac{1}{\theta}\tilde{\rho}_{X,t}(z)^{1-\theta}C^*_t Q_t - f_X\frac{w_t}{Z_t}$
Average profit (Foreign)	$\tilde{d}^{I*}_{D,t}(z^*) = \frac{1}{\theta}\tilde{\rho}^{*I}_{D,t}(z^*)^{1-\theta}C^*_t - \phi^*_{D,t}\frac{w^*_t}{Z^*_t}(\psi^{*-\xi^*}_{D,t} - 1)$
	$\tilde{d}^{NI*}_{D,t}(z^*) = \frac{1}{\theta}\tilde{\rho}^{*NI}_{D,t}(z^*)^{1-\theta}C^*_t$
	$\tilde{d}^*_{X,t}(z^*) = \frac{1}{\theta}\tilde{\rho}^*_{X,t}(z^*)^{1-\theta}C_t Q_t^{-1} - f^*_X\frac{w^*_t}{Z^*_t}$
Average price (Home)	$\tilde{\rho}^I_{D,t}(z) = \frac{\theta}{\theta-1}\psi^{\xi}_{D,t}\frac{w_t}{Z_t\tilde{z}^I_{D,t}}$
	$\tilde{\rho}^{NI}_{D,t}(z) = \frac{\theta}{\theta-1}\frac{w_t}{Z_t\tilde{z}^{NI}_{D,t}}$
	$\tilde{\rho}_{V,t}(z) = \frac{\theta}{\theta-1}\tau Q_t\psi^{\xi}_{V,t}\frac{w^*_t}{Z^*_t\tilde{z}_{V,t}}$
	$\tilde{\rho}_{X,t}(z) = \frac{\theta}{\theta-1}\tau^* Q_t^{-1}\frac{w_t}{Z_t\tilde{z}_{X,t}}$
Average price (Foreign)	$\tilde{\rho}^{I*}_{D,t}(z^*) = \frac{\theta}{\theta-1}\psi^{*\xi^*}_{D,t}\frac{w^*_t}{Z^*_t\tilde{z}^{I*}_{D,t}}$
	$\tilde{\rho}^{NI*}_{D,t}(z^*) = \frac{\theta}{\theta-1}\frac{w^*_t}{Z^*_t\tilde{z}^{NI*}_{D,t}}$
	$\tilde{\rho}^*_{X,t}(z^*) = \frac{\theta}{\theta-1}\tau Q_t\frac{w^*_t}{Z^*_t\tilde{z}^*_{X,t}}$
Free entry	$\tilde{v}_t = \frac{w_t}{Z_t}f_{E,t}$
	$\tilde{v}^*_t = \frac{w^*_t}{Z^*_t}f^*_{E,t}$
Total number of firms	$N_t = N^I_{D,t} + N^{NI}_{D,t} + N_{V,t}$
	$N^*_t = N^{I*}_{D,t} + N^{NI*}_{D,t}$
Law of motion, firms	$N_{t+1} = (1-\delta)(N_t + N_{E,t})$
	$N^*_{t+1} = (1-\delta)(N^*_t + N^*_{E,t})$
Aggregate accounting	$C_t + N_{E,t}\tilde{v}_t(z) = w_t + N_t\tilde{d}_t(z)$
	$C^*_t + N^*_{E,t}\tilde{v}^*_t(z^*) = w^*_t + N^*_{D,t}\tilde{d}^*_t(z^*)$
Consumption price index	$1 = N^I_{D,t}(\tilde{\rho}^I_{D,t}(z))^{1-\theta} + N^{NI}_{D,t}(\tilde{\rho}^{NI}_{D,t}(z))^{1-\theta} + N_{V,t}(\tilde{\rho}_{V,t}(z))^{1-\theta} + N^*_{X,t}(\tilde{\rho}^*_{X,t}(z))^{1-\theta}$
	$1 = N^{I*}_{D,t}(\tilde{\rho}^{I*}_{D,t}(z^*))^{1-\theta} + N^{NI*}_{D,t}(\tilde{\rho}^{NI*}_{D,t}(z^*))^{1-\theta} + N_{X,t}(\tilde{\rho}_{X,t}(z^*))^{1-\theta}$
Total profits	$N_t\tilde{d}_t(z) = N^I_{D,t}\tilde{d}^I_{D,t}(z) + N^{NI}_{D,t}\tilde{d}^{NI}_{D,t}(z) + N_{V,t}\tilde{d}_{V,t}(z) + N_{X,t}\tilde{d}_{X,t}(z)$
	$N^*_t\tilde{d}_t(z^*) = N^{I*}_{D,t}\tilde{d}^{I*}_{D,t}(z^*) + N^{NI*}_{D,t}\tilde{d}^{NI*}_{D,t}(z^*) + N^*_{X,t}\tilde{d}^*_{X,t}(z^*)$
Euler equations (Bonds)	$C_t^{-\gamma} = \beta(1+r_{t+1})E_t(C_{t+1})^{-\gamma}$
	$C_t^{*-\gamma} = \beta(1+r^*_{t+1})E_t(C^*_{t+1})^{-\gamma}$

Euler equations (Shares)	$\tilde{v}_t = \beta(1-\delta)E_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}(\tilde{v}_{t+1}+\tilde{d}_{t+1})\right]$ $\tilde{v_t}^* = \beta^*(1-\delta^*)E_t\left[\left(\frac{C^*_{t+1}}{C^*_t}\right)^{-\gamma}(\tilde{v}^*_{t+1}+\tilde{d}^*_{t+1})\right]$
Average productivity	$\tilde{z}^I_{D,t} = \nu z_{min} z_{V,t}\left[\frac{z_{V,t}^{k-(\theta-1)}-z_{min}^{k-(\theta-1)}}{z_{V,t}^k - z_{min}^k}\right]^{\frac{1}{\theta-1}}$ $\tilde{z}^{NI}_{D,t} = \nu z_{min} z_{V,t}\left[\frac{z_{V,t}^{k-(\theta-1)}-z_{min}^{k-(\theta-1)}}{z_{V,t}^k - z_{min}^k}\right]^{\frac{1}{\theta-1}}$ $\tilde{z}_{V,t} = \nu z_{min}\left(\frac{N_t}{N_{V,t}}\right)^{1/k}$ $\tilde{z}_{X,t} = \nu z_{min}\left(\frac{N_t}{N_{X,t}}\right)^{1/k}$ $\tilde{z}^{I*}_{D,t} = \nu z^*_{min}\left(\frac{N^*_t}{N^{I*}_{D,t}}\right)^{1/k}$ $\tilde{z}^*_{X,t} = \nu z^*_{min}\left(\frac{N^*_t}{N^*_{X,t}}\right)^{1/k}$
Balanced trade	$N_{X,t}(\tilde{\rho}_{X,t}(z))^{1-\theta}C^*_t Q_t + N_{V,t}\tilde{d}_{V,t}(z) = N_{V,t}(\tilde{\rho}_{V,t}(z))^{1-\theta}C_t + N^*_{X,t}(\tilde{\rho}^*_{X,t}(z^*))^{1-\theta}C_t$

Appendix C. Average Productivity

C.1 The Average Productivity of Offshoring Firms

The average productivity of Home firms that produce offshore is obtained by integrating over the range of the support interval above the offshoring productivity cutoff:

$$\tilde{z}_{V,t} = \left[\frac{1}{1-G(z_{V,t})}\int_{z_{V,t}}^{\infty} z^{\theta-1}g(z)dz\right]^{\frac{1}{\theta-1}} = \left[\left(\frac{z_{V,t}}{z_{min}}\right)^{k}\int_{z_{V,t}}^{\infty} z^{\theta-1}\frac{kz_{min}^{k}}{z^{k+1}}dz\right]^{\frac{1}{\theta-1}}$$

$$= \left[\left(\frac{z_{V,t}}{z_{min}}\right)^{k}\frac{kz_{min}^{k}}{k-(\theta-1)}z_{V,t}^{\theta-1-k}\right]^{\frac{1}{\theta-1}} = \nu z_{V,t}, \text{ where } \nu \equiv \left[\frac{k}{k-(\theta-1)}\right]^{\frac{1}{\theta-1}}$$

C.2 The Average Productivity of Domestic Firms

The average productivity of Home firms that produce domestically with intangible investment is obtained by integrating over the range of the support interval above the intangible productivity cutoff and below the offshoring productivity cutoff:

$$\tilde{z}_{D,t}^{I} = \left[\frac{1}{G(z_{V,t})-G(z_{D,t}^{I})}\int_{z_{D,t}^{I}}^{z_{V,t}} z^{\theta-1}g(z)dz\right]^{\frac{1}{\theta-1}} = \left[\frac{(z_{D,t}^{I}z_{V,t})^{k}}{z_{min}^{k}(z_{V,t}^{k}-z_{D,t}^{I\,k})}\int_{z_{D,t}^{I}}^{z_{V,t}} z^{\theta-1}\frac{kz_{min}^{k}}{z^{k+1}}dz\right]^{\frac{1}{\theta-1}}$$

$$= \left[\frac{(z_{D,t}^{I}z_{V,t})^{k}}{z_{V,t}^{k}-z_{D,t}^{I\,k}}\frac{k}{k-(\theta-1)}(z_{D,t}^{I\,\theta-1-k}-z_{V,t}^{\theta-1-k})\right]^{\frac{1}{\theta-1}} = \nu\left[\frac{(z_{D,t}^{I}z_{V,t})^{k}}{z_{V,t}^{k}-z_{D,t}^{I\,k}}\left(\frac{1}{z_{D,t}^{I\,k-(\theta-1)}}-\frac{1}{z_{V,t}^{k-(\theta-1)}}\right)\right]^{\frac{1}{\theta-1}}$$

$$= \nu z_{D,t}^{I}z_{V,t}\left[\frac{z_{V,t}^{k-(\theta-1)}-z_{D,t}^{I\,k-(\theta-1)}}{z_{V,t}^{k}-z_{D,t}^{I\,k}}\right]^{\frac{1}{\theta-1}}$$

The average productivity of Home firms that produce domestically without intangible investment is obtained by integrating over the range of the support interval above the minimum level of the productivity and below the intangible productivity cutoff:

$$\tilde{z}_{D,t}^{NI} = \left[\frac{1}{G(z_{D,t}^{I})}\int_{z_{min}}^{z_{D,t}^{I}} z^{\theta-1}g(z)dz\right]^{\frac{1}{\theta-1}} = \left[\frac{z_{D,t}^{Ik}}{z_{D,t}^{I\,k}-z_{min}^{k}}\int_{z_{min}}^{z_{D,t}^{I}} z^{\theta-1}\frac{kz_{min}^{k}}{z^{k+1}}dz\right]^{\frac{1}{\theta-1}}$$

$$= \left[\frac{z_{D,t}^{Ik}}{z_{D,t}^{I\,k}-z_{min}^{k}}\frac{kz_{min}^{k}}{k-(\theta-1)}(z_{min}^{\theta-1-k}-z_{D,t}^{I\theta-1-k})\right]^{\frac{1}{\theta-1}} = \nu\left[\frac{(z_{min}z_{D,t}^{I})^{k}}{z_{D,t}^{I\,k}-z_{min}^{k}}\left(\frac{1}{z_{min}^{k-(\theta-1)}}-\frac{1}{z_{D,t}^{I\,k-(\theta-1)}}\right)\right]^{\frac{1}{\theta-1}}$$

$$= \nu z_{min}z_{D,t}^{I}\left[\frac{z_{D,t}^{I\,k-(\theta-1)}-z_{min}^{k-(\theta-1)}}{z_{D,t}^{I\,k}-z_{min}^{k}}\right]^{\frac{1}{\theta-1}}$$

Appendix D. Average Profits from Domestic Production with and without Intangible Investment, and from Offshore Production

D.1 The relationship between the average profit of offshoring firms and the average profit of domestic intangible firms

The average profit of Home firms that produce domestically with intangible investment is:

$$\begin{aligned}
\tilde{d}^I_{D,t} &= d^I_{D,t}(\tilde{z}^I_{D,t}) = \frac{1}{\theta}\left[\frac{\theta}{\theta-1}\psi^{\xi}_{D,t}\frac{w_t}{Z_t\tilde{z}^I_{D,t}}\right]^{1-\theta} C_t - \phi_{D.t}\frac{w_t}{Z_t}(\psi^{-\xi}_{D,t}-1) \\
&= \frac{1}{\theta}\left[\frac{\theta}{\theta-1}\psi^{\xi}_{D,t}\frac{w_t}{Z_t}\right]^{1-\theta} C_t\tilde{z}^{I\theta-1}_{D,t} - \phi_{D.t}\frac{w_t}{Z_t}(\psi^{-\xi}_{D,t}-1) \\
&= \frac{1}{\theta}\left[\frac{\theta}{\theta-1}\psi^{\xi}_{D,t}\frac{w_t}{Z_t}\right]^{1-\theta} C_t(\nu z^I_{D,t}z_{V,t})^{\theta-1}\left[\frac{z^{k-(\theta-1)}_{V,t}-z^{Ik-(\theta-1)}_{D,t}}{z^k_{V,t}-z^{Ik}_{D,t}}\right] - \phi_{D,t}\frac{w_t}{Z_t}(\psi^{-\xi}_{D,t}-1) \\
&= \frac{1}{\theta}\left[\frac{\theta}{\theta-1}\psi^{\xi}_{D,t}\frac{w_t}{Z_tz_{V,t}}\right]^{1-\theta} C_t(\nu z^I_{D,t})^{\theta-1}\left[\frac{z^{k-(\theta-1)}_{V,t}-z^{Ik-(\theta-1)}_{D,t}}{z^k_{V,t}-z^{Ik}_{D,t}}\right] - \phi_{D,t}\frac{w_t}{Z_t}(\psi^{-\xi}_{D,t}-1) \\
&= d^I_{D,t}(z_{V,t})(\nu z^I_{D,t})^{\theta-1}\left[\frac{z^{k-(\theta-1)}_{V,t}-z^{Ik-(\theta-1)}_{D,t}}{z^k_{V,t}-z^{Ik}_{D,t}}\right] - \phi_{D,t}\frac{w_t}{Z_t}(\psi^{-\xi}_{D,t}-1)
\end{aligned}$$

The average profit of the Foreign firms producing offshore is:

$$\begin{aligned}
\tilde{d}_{V,t} &= d_{V,t}(\tilde{z}_{V,t}) = \frac{1}{\theta}\left[\frac{\theta}{\theta-1}\psi^{\xi}_{V,t}\tau Q_t\frac{w^*_t}{Z^*_t\tilde{z}_{V,t}}\right]^{1-\theta} C_t - f_V Q_t\frac{w^*_t}{Z^*_t} - \phi_{V,t}Q_t\frac{w^*_t}{Z^*_t}(\psi^{-\xi}_{V,t}-1) \\
&= \frac{1}{\theta}\left[\frac{\theta}{\theta-1}\psi^{\xi}_{V,t}\tau Q_t\frac{w^*_t}{Z^*_t}\right]^{1-\theta} C_t\tilde{z}^{\theta-1}_{V,t} - f_V Q_t\frac{w^*_t}{Z^*_t} - \phi_{V,t}Q_t\frac{w^*_t}{Z^*_t}(\psi^{-\xi}_{V,t}-1) \\
&= \left\{\frac{1}{\theta}\left[\frac{\theta}{\theta-1}\psi^{\xi}_{V,t}\tau Q_t\frac{w^*_t}{Z^*_tz_{V,t}}\right]^{1-\theta} C_t - f_V Q_t\frac{w^*_t}{Z^*_t} - \phi_{V,t}Q_t\frac{w^*_t}{Z^*_t}(\psi^{-\xi}_{V,t}-1)\right\}\nu^{\theta-1} \\
&\quad + (\nu^{\theta-1}-1)\left\{f_V Q_t\frac{w^*_t}{Z^*_t} + \phi_{V,t}Q_t\frac{w^*_t}{Z^*_t}(\psi^{-\xi}_{V,t}-1)\right\} \\
&= \frac{k}{k-(\theta-1)}d_{V,t}(z_{V,t}) + \frac{\theta-1}{k-(\theta-1)}\left\{f_V Q_t\frac{w^*_t}{Z^*_t} + \phi_{V,t}Q_t\frac{w^*_t}{Z^*_t}(\psi^{-\xi}_{V,t}-1)\right\}
\end{aligned}$$

The Home firm with productivity equal to the cutoff $z_{V,t}$ is indifferent between producing domestically or offshore. I use the equality of profits at the productivity cutoff, $d_{D,t}(z_{V,t}) = d_{V,t}(z_{V,t})$:

$$\begin{aligned}
\tilde{d}_{V,t} &= \frac{k}{k-(\theta-1)} d_{V,t}(z_{V,t}) + \frac{\theta-1}{k-(\theta-1)} \left\{ f_V Q_t \frac{w_t^*}{Z_t^*} + \phi_{V,t} Q_t \frac{w_t^*}{Z_t^*} (\psi_{V,t}^{-\xi} - 1) \right\} \\
&= {z_{D,t}^I}^{1-\theta} \left[\frac{z_{V,t}^{k-(\theta-1)} - {z_{D,t}^I}^{k-(\theta-1)}}{z_{V,t}^k - {z_{D,t}^I}^k} \right]^{-1} \left\{ \tilde{d}_{D,t}^I(z_{V,t}) + \left(1 - (\nu z_D^I)^{\theta-1} \left[\frac{z_{V,t}^{k-(\theta-1)} - {z_{D,t}^I}^{k-(\theta-1)}}{z_{V,t}^k - {z_{D,t}^I}^k} \right] \right) \right. \\
&\quad \left. \phi_{D,t} \frac{w_t}{Z_t} (\psi_{D,t}^{-\xi} - 1) \right\} + \frac{\theta-1}{k-(\theta-1)} \left\{ f_V Q_t \frac{w_t^*}{Z_t^*} + \phi_{V,t} Q_t \frac{w_t^*}{Z_t^*} (\psi_{V,t}^{-\xi} - 1) \right\} \\
&= \left(\frac{z_{V,t}}{\tilde{z}_{D,t}^I} \right)^{\theta-1} \left\{ \tilde{d}_{D,t}^I(z_{V,t}) + \phi_{D,t} \frac{w_t}{Z_t} (\psi_{D,t}^{-\xi} - 1) \right\} - \frac{k}{k-(\theta-1)} \phi_{D,t} \frac{w_t}{Z_t} (\psi_{D,t}^{-\xi} - 1) \\
&\quad + \frac{\theta-1}{k-(\theta-1)} \left\{ f_V Q_t \frac{w_t^*}{Z_t^*} + \phi_{V,t} Q_t \frac{w_t^*}{Z_t^*} (\psi_{V,t}^{-\xi} - 1) \right\}
\end{aligned}$$

The calculation process utilizes the domestic intangible firms' average productivity equation derived in Appendix C.

$$\tilde{z}_{D,t}^I = \nu z_{D,t}^I z_{V,t} \left[\frac{z_{V,t}^{k-(\theta-1)} - {z_{D,t}^I}^{k-(\theta-1)}}{z_{V,t}^k - {z_{D,t}^I}^k} \right]^{\frac{1}{\theta-1}}$$

$$\Rightarrow {z_{D,t}^I}^{1-\theta} \left[\frac{z_{V,t}^{k-(\theta-1)} - {z_{D,t}^I}^{k-(\theta-1)}}{z_{V,t}^k - {z_{D,t}^I}^k} \right]^{-1} = \nu^{\theta-1} \left(\frac{z_{V,t}}{\tilde{z}_{D,t}^I} \right)^{\theta-1} = \frac{k}{k-(\theta-1)} \left(\frac{z_{V,t}}{\tilde{z}_{D,t}^I} \right)^{\theta-1}$$

D.2 The relationship between the average profit of domestic intangible firms and the average profit of domestic non-intangible firms

The average profit of Home firms that produce domestically without intangible investment is:

$$\begin{aligned}
\tilde{d}_{D,t}^{NI} &= d_{D,t}^{NI}(\tilde{z}_{D,t}^{NI}) = \frac{1}{\theta}\left[\frac{\theta}{\theta-1}\frac{w_t}{Z_t\tilde{z}_{D,t}^{NI}}\right]^{1-\theta} C_t = \frac{1}{\theta}\left[\frac{\theta}{\theta-1}\frac{w_t}{Z_t}\right]^{1-\theta} C_t\tilde{z}_{D,t}^{NI\theta-1} \\
&= \frac{1}{\theta}\left[\frac{\theta}{\theta-1}\frac{w_t}{Z_t}\right]^{1-\theta} C_t(\nu z_{min} z_{D,t}^{I})^{\theta-1}\left[\frac{z_{D,t}^{Ik-(\theta-1)} - z_{min}^{k-(\theta-1)}}{z_{D,t}^{Ik} - z_{min}^{k}}\right] \\
&= \frac{1}{\theta}\left[\frac{\theta}{\theta-1}\frac{w_t}{Z_t z_{D,t}^{I}}\right]^{1-\theta} C_t(\nu z_{min})^{\theta-1}\left[\frac{z_{D,t}^{Ik-(\theta-1)} - z_{min}^{k-(\theta-1)}}{z_{D,t}^{Ik} - z_{min}^{k}}\right] \\
&= d_{D,t}^{NI}(z_{D,t}^{I})(\nu z_{min})^{\theta-1}\left[\frac{z_{D,t}^{Ik-(\theta-1)} - z_{min}^{k-(\theta-1)}}{z_{D,t}^{Ik} - z_{min}^{k}}\right]
\end{aligned}$$

The average profit of Home firms that produce domestically with intangible investment is:

$$\begin{aligned}
\tilde{d}_{D,t}^{I} &= d_{D,t}^{I}(\tilde{z}_{D,t}^{I}) = \frac{1}{\theta}\left[\frac{\theta}{\theta-1}\psi_{D,t}^{\xi}\frac{w_t}{Z_t\tilde{z}_{D,t}^{I}}\right]^{1-\theta} C_t - \phi_{D.t}\frac{w_t}{Z_t}(\psi_{D,t}^{-\xi} - 1) \\
&= \frac{1}{\theta}\left[\frac{\theta}{\theta-1}\psi_{D,t}^{\xi}\frac{w_t}{Z_t}\right]^{1-\theta} C_t\tilde{z}_{D,t}^{I\theta-1} - \phi_{D.t}\frac{w_t}{Z_t}(\psi_{D,t}^{-\xi} - 1) \\
&= \frac{1}{\theta}\left[\frac{\theta}{\theta-1}\psi_{D,t}^{\xi}\frac{w_t}{Z_t}\right]^{1-\theta} C_t(\nu z_{D,t}^{I} z_{V,t})^{\theta-1}\left[\frac{z_{V,t}^{k-(\theta-1)} - z_{D,t}^{Ik-(\theta-1)}}{z_{V,t}^{k} - z_{D,t}^{Ik}}\right] - \phi_{D,t}\frac{w_t}{Z_t}(\psi_{D,t}^{-\xi} - 1) \\
&= \frac{1}{\theta}\left[\frac{\theta}{\theta-1}\psi_{D,t}^{\xi}\frac{w_t}{Z_t z_{D,t}^{I}}\right]^{1-\theta} C_t(\nu z_{V,t})^{\theta-1}\left[\frac{z_{V,t}^{k-(\theta-1)} - z_{D,t}^{Ik-(\theta-1)}}{z_{V,t}^{k} - z_{D,t}^{Ik}}\right] - \phi_{D,t}\frac{w_t}{Z_t}(\psi_{D,t}^{-\xi} - 1) \\
&= (\nu z_{V,t})^{\theta-1}\left[\frac{z_{V,t}^{k-(\theta-1)} - z_{D,t}^{Ik-(\theta-1)}}{z_{V,t}^{k} - z_{D,t}^{Ik}}\right]\left\{\frac{1}{\theta}\left[\frac{\theta}{\theta-1}\psi_{D,t}^{\xi}\frac{w_t}{Z_t z_{D,t}^{I}}\right]^{1-\theta} C_t - \phi_{D.t}\frac{w_t}{Z_t}(\psi_{D,t}^{-\xi} - 1)\right\} \\
&\quad + \left\{(\nu z_{V,t})^{\theta-1}\left[\frac{z_{V,t}^{k-(\theta-1)} - z_{D,t}^{Ik-(\theta-1)}}{z_{V,t}^{k} - z_{D,t}^{Ik}}\right] - 1\right\}\phi_{D,t}\frac{w_t}{Z_t}(\psi_{D,t}^{-\xi} - 1) \\
&= (\nu z_{V,t})^{\theta-1}\left[\frac{z_{V,t}^{k-(\theta-1)} - z_{D,t}^{Ik-(\theta-1)}}{z_{V,t}^{k} - z_{D,t}^{Ik}}\right] d_{D,t}^{I}(z_{D,t}^{I}) \\
&\quad + \left\{(\nu z_{V,t})^{\theta-1}\left[\frac{z_{V,t}^{k-(\theta-1)} - z_{D,t}^{Ik-(\theta-1)}}{z_{V,t}^{k} - z_{D,t}^{Ik}}\right] - 1\right\}\phi_{D,t}\frac{w_t}{Z_t}(\psi_{D,t}^{-\xi} - 1)
\end{aligned}$$

The Home firm with productivity equal to the cutoff $z^I_{D,t}$ is indifferent between producing domestically with intangible investment or without it. I use the equality of profits at the productivity cutoff, $d^{NI}_{D,t}(z^I_{D,t}) = d^I_{D,t}(z^I_{D,t})$:

$$
\begin{aligned}
\tilde{d}^I_{D,t} &= (\nu z_{V,t})^{\theta-1}\left[\frac{z_{V,t}^{k-(\theta-1)} - z_{D,t}^{Ik-(\theta-1)}}{z^k_{V,t} - z^{Ik}_{D,t}}\right] d^I_{D,t}(z^I_{D,t}) \\
&\quad + \left\{(\nu z_{V,t})^{\theta-1}\left[\frac{z_{V,t}^{k-(\theta-1)} - z_{D,t}^{Ik-(\theta-1)}}{z^k_{V,t} - z^{Ik}_{D,t}}\right] - 1\right\}\phi_{D,t}\frac{w_t}{Z_t}(\psi_{D,t}^{-\xi} - 1) \\
&= (\nu z_{V,t})^{\theta-1}\left[\frac{z_{V,t}^{k-(\theta-1)} - z_{D,t}^{Ik-(\theta-1)}}{z^k_{V,t} - z^{Ik}_{D,t}}\right]\left\{(\nu z_{min})^{1-\theta}\left[\frac{z_{D,t}^{I\ k-(\theta-1)} - z_{min}^{k-(\theta-1)}}{z^{I\ k}_{D,t} - z^k_{min}}\right]^{-1}\tilde{d}^{NI}_{D,t}(z^I_{D,t})\right\} \\
&\quad + \left\{(\nu z_{V,t})^{\theta-1}\left[\frac{z_{V,t}^{k-(\theta-1)} - z_{D,t}^{Ik-(\theta-1)}}{z^k_{V,t} - z^{Ik}_{D,t}}\right] - 1\right\}\phi_{D,t}\frac{w_t}{Z_t}(\psi_{D,t}^{-\xi} - 1) \\
&= (z_{V,t})^{\theta-1}\left[\frac{z_{V,t}^{k-(\theta-1)} - z_{D,t}^{Ik-(\theta-1)}}{z^k_{V,t} - z^{Ik}_{D,t}}\right](z_{min})^{1-\theta}\left[\frac{z_{D,t}^{I\ k-(\theta-1)} - z_{min}^{k-(\theta-1)}}{z^{I\ k}_{D,t} - z^k_{min}}\right]^{-1}\tilde{d}^{NI}_{D,t}(z^I_{D,t}) \\
&\quad + \left\{(\nu z_{V,t})^{\theta-1}\left[\frac{z_{V,t}^{k-(\theta-1)} - z_{D,t}^{Ik-(\theta-1)}}{z^k_{V,t} - z^{Ik}_{D,t}}\right] - 1\right\}\phi_{D,t}\frac{w_t}{Z_t}(\psi_{D,t}^{-\xi} - 1) \\
&= \frac{k}{k-(\theta-1)}\left(\frac{z^I_{D,t}z_{V,t}}{\tilde{z}^{NI}_{D,t}}\right)^{\theta-1}\left[\frac{z_{V,t}^{k-(\theta-1)} - z_{D,t}^{Ik-(\theta-1)}}{z^k_{V,t} - z^{Ik}_{D,t}}\right]\tilde{d}^{NI}_{D,t}(z^I_{D,t}) \\
&\quad + \left\{\frac{k}{k-(\theta-1)}z_{V,t}^{\theta-1}\left[\frac{z_{V,t}^{k-(\theta-1)} - z_{D,t}^{Ik-(\theta-1)}}{z^k_{V,t} - z^{Ik}_{D,t}}\right] - 1\right\}\phi_{D,t}\frac{w_t}{Z_t}(\psi_{D,t}^{-\xi} - 1)
\end{aligned}
$$

The calculation process utilizes the domestic non-intangible firms' average productivity equation derived in Appendix C.

$$
\tilde{z}^{NI}_{D,t} = \nu z_{min} z^I_{D,t}\left[\frac{z_{D,t}^{I\ k-(\theta-1)} - z_{min}^{k-(\theta-1)}}{z^{I\ k}_{D,t} - z^k_{min}}\right]^{\frac{1}{\theta-1}}
$$

$$
\Rightarrow z_{min}^{1-\theta}\left[\frac{z_{D,t}^{I\ k-(\theta-1)} - z_{min}^{k-(\theta-1)}}{z^{I\ k}_{D,t} - z^k_{min}}\right]^{-1} = \nu^{\theta-1}\left(\frac{z^I_{D,t}}{\tilde{z}^{NI}_{D,t}}\right)^{\theta-1} = \frac{k}{k-(\theta-1)}\left(\frac{z^I_{D,t}}{\tilde{z}^{NI}_{D,t}}\right)^{\theta-1}
$$

Appendix E. Asymmetric Steady State

In this appendix, I derive the steady-state solution for the offshoring model incorporating intangible capital, accounting for cross-country asymmetry in the cost of effective labor (i.e., the terms of labor, $TOL < 1$).

To ascertain the unique steady-state solution, I obtain a numerical solution by resolving a system of 17 non-linear equations with 17 endogenous variables. The specific equations, numbered (29) through (45), are detailed herein. The variables in question represent the steady-state magnitudes of: z_V (the offshoring productivity cutoff in Home); z_H (the exporting productivity cutoff in Home); TOL (the terms of labor); $\frac{C}{C^*Q}$ (the real consumption ratio in units of the same consumption basket); Q (the real exchange rate); $\frac{d_D^I}{w}$, $\frac{d_D^{NI}}{w}$, $\frac{d_V}{w}$, $\frac{d_X}{w}$ (Home's real firm profits, specifically for domestic production with and without intangible investment, offshore production, and export market production, all divided by the real wage in Home); $\frac{d_D^{I*}}{w^*}$, $\frac{d_D^{NI*}}{w^*}$ (Foreign's real firm profits from domestic production, with and without intangible investment, divided by the real wage in Foreign); z_X^* (the exporting productivity cutoff in Foreign); ρ_X (the average exporting price in Home), ρ_X^*(the average exporting price in Foreign), $\frac{N}{N^*}$(the ratio of the number of firms between Home and Foreign). Once these 17 variables are numerically determined, I utilize them to derive the steady-state values of all remaining variables. For the purpose of calculating the steady-state solution, the following price and profit formulations are instrumental.

Using average profit and price in appendix B, this appendix computes the steady-state solution. Introducing $v = \frac{\beta(1-\delta)}{1-\beta(1-\delta)}d$, $N_E = \frac{\delta}{1-\delta}N$, and $v = f_E w$ into the Home firms' total profits, we can derive the first equation of the system.

$$\frac{1-\beta(1-\delta)}{\beta(1-\delta)}f_E = \frac{N_D^I}{N}\frac{\widetilde{d}_D^I}{w} + \frac{N_D^{NI}}{N}\frac{\widetilde{d}_D^{NI}}{w} + \frac{N_V}{N}\frac{\widetilde{d}_V}{w} + \frac{N_X}{N}\frac{\widetilde{d}_X}{w} \tag{29}$$

where $\frac{N_X}{N} = \left(\frac{1}{z_X}\right)^k$, $\frac{N_D^I}{N} = \left(\frac{1}{z_D^I}\right)^k - \left(\frac{1}{z_V}\right)^k$, $\frac{N_D^{NI}}{N} = 1 - \left(\frac{1}{z_D^I}\right)^k$, $\frac{N_V}{N} = \left(\frac{1}{z_V}\right)^k$.

Then, the profit formulas in the Home country are represented as follows:

$$\frac{\widetilde{d}_D^I}{w} = \frac{k}{k-(\theta-1)}f_X\frac{C}{C^*Q}Q^{1-\theta}\tau^{*(\theta-1)}\psi_D^{\xi(1-\theta)}\left(\frac{z_D^I z_V}{z_X}\right)^{\theta-1}\left[\frac{z_V^{k-(\theta-1)} - z_D^{I\ k-(\theta-1)}}{z_V^k - z_D^{I\ k}}\right] - \phi_D(\psi_D^{-\xi}-1) \tag{30}$$

$$\frac{\widetilde{d}_D^{NI}}{w} = \frac{k}{k-(\theta-1)}f_X\frac{C}{C^*Q}Q^{1-\theta}\tau^{*(\theta-1)}\left(\frac{z_{min} z_D^I}{z_X}\right)^{\theta-1}\left[\frac{z_D^{I\ k-(\theta-1)} - z_{min}^{k-(\theta-1)}}{z_D^{I\ k} - z_{min}^k}\right] \tag{31}$$

$$\frac{\widetilde{d}_V}{w} = \frac{k}{k-(\theta-1)}f_X\frac{C}{C^*Q}Q^{1-\theta}\left(\frac{\tau^*}{\tau}\right)^{(\theta-1)}\psi_V^{\xi(1-\theta)}TOL^{1-\theta}\left(\frac{z_V}{z_X}\right)^{\theta-1} - f_V TOL - \phi_V TOL(\psi_V^{-\xi}-1) \tag{32}$$

$$\frac{\widetilde{d}_X}{w} = \frac{\theta-1}{k-(\theta-1)} f_X \tag{33}$$

$$\begin{aligned}\frac{\widetilde{d}_V}{w} =& {z_D^I}^{1-\theta}\left[\frac{z_V^{k-(\theta-1)} - {z_D^I}^{k-(\theta-1)}}{z_V^k - {z_D^I}^k}\right]^{-1}\left\{\frac{\widetilde{d}_D^I}{w} + \phi_D(\psi_D^{-\xi}-1)\right\} \\ &- \frac{k}{k-(\theta-1)}\phi_D(\psi_D^{-\xi}-1) + \frac{\theta-1}{k-(\theta-1)} TOL\left\{f_V + \phi_V(\psi_V^{-\xi}-1)\right\}\end{aligned} \tag{34}$$

$$\begin{aligned}\frac{\widetilde{d}_D^I}{w} =& \left(\frac{z_V}{z_{min}}\right)^{\theta-1}\left[\frac{z_V^{k-(\theta-1)} - {z_D^I}^{k-(\theta-1)}}{z_V^k - {z_D^I}^k}\right]\left[\frac{{z_D^I}^{k-(\theta-1)} - z_{min}^{k-(\theta-1)}}{{z_D^I}^k - z_{min}^k}\right]^{-1}\frac{\widetilde{d}_D^{NI}}{w} \\ &+ \left\{\frac{k}{k-(\theta-1)} z_V^{\theta-1}\left(\frac{z_V^{k-(\theta-1)} - {z_D^I}^{k-(\theta-1)}}{z_V^k - {z_D^I}^k}\right) - 1\right\}\phi_D(\psi_D^{-\xi}-1)\end{aligned} \tag{35}$$

Then, the profit formulas in the Foreign country are represented as follows:

$$\frac{\widetilde{d}_D^{*\,I}}{w^*} = \frac{k}{k-(\theta-1)} f_X^* \frac{C^*Q}{C}(\tau^*Q)^{(\theta-1)}\psi_D^{*\xi^*(1-\theta)}\left(\frac{z_D^{I*}}{z_X^*}\right)^{\theta-1} - \phi_D^*(\psi_D^{*-\xi^*}-1) \tag{36}$$

$$\frac{\widetilde{d}_D^{*\,NI}}{w^*} = \frac{k}{k-(\theta-1)} f_X^* \frac{C^*Q}{C}(\tau^*Q)^{(\theta-1)}\left(\frac{z_{min}^* z_D^{I*}}{z_X^*}\right)^{\theta-1}\left[\frac{{z_D^{I*}}^{k-(\theta-1)} - z_{min}^{*k-(\theta-1)}}{{z_D^{I*}}^k - z_{min}^{*k}}\right] \tag{37}$$

$$\frac{\widetilde{d}_D^{*\,I}}{w^*} = {z_{min}^*}^{1-\theta}\left[\frac{{z_D^{I*}}^{k-(\theta-1)} - z_{min}^{*k-(\theta-1)}}{{z_D^{I*}}^k - z_{min}^{*k}}\right]^{-1}\frac{\widetilde{d}_D^{NI*}}{w^*} - \frac{\theta-1}{k-(\theta-1)}\phi_D^*(\psi_D^{*-\xi^*}-1) \tag{38}$$

The total profit expression in the Foreign country is represented as follows:

$$\begin{aligned}\frac{1-\beta^*(1-\delta^*)}{\beta^*(1-\delta^*)} f_E^* =& \frac{k}{k-(\theta-1)} f_X^* \frac{C^*Q}{C} Q^{\theta-1}\tau^{\theta-1}\left(\frac{z_{min}^* z_D^{I*}}{z_X^*}\right)^{\theta-1}\left[\frac{{z_D^{I*}}^{k-(\theta-1)} - z_{min}^{*k-(\theta-1)}}{{z_D^{I*}}^k - z_{min}^{*k}}\right](1 - z_D^{I*-k}) \\ &+ \left\{\frac{k}{k-(\theta-1)} f_X^* \frac{C^*Q}{C} Q^{\theta-1}\tau^{\theta-1}\left(\frac{z_D^{I*}}{z_X^*}\right)^{\theta-1}\psi_D^{*\xi^*(1-\theta)} - \phi_D^*(\psi_D^{*\xi^*}-1)\right\} z_D^{I*-k} \\ &+ \frac{\theta-1}{k-(\theta-1)} f_X^* z_X^{*-k}\end{aligned} \tag{39}$$

The consumption ratio in units of the Home consumption basket is:

$$\frac{C}{C^*Q} = \frac{f_X^*}{f_X} TOL^\theta Q^{\theta-1}\left(\frac{\tau^* z_X}{\tau z_X^*}\right)^{\theta-1} \tag{40}$$

Under the condition of a balanced current account, it follows that:

$$\begin{aligned} & z_V^{-k} TOL\left[\frac{k(\theta-1)}{k-(\theta-1)}\left(\frac{z_V}{z_X^*}\right)^{\theta-1}\psi_V^{\xi(1-\theta)} f_X^* - \left(f_V + \phi_V(\psi_V^{-\xi}-1)\right)\right] \\ & = \frac{k\theta}{k-(\theta-1)}\left\{\frac{1}{z_X^{*k}}\left(\frac{N}{N^*}\right)^{-1} f_X^* TOL - \frac{1}{z_X^k} f_X\right\} \end{aligned} \tag{41}$$

The real exchange rate in the steady state is as follows:

$$Q^{1-\theta} = \left(\frac{N}{N^*}\right)^{-1}\frac{\Lambda}{\Omega} \tag{42}$$

$$\begin{aligned} \Lambda =& (1-z_D^{I*-k})TOL^{1-\theta}(z_{min}^* z_D^{I*})^{\theta-1}\left[\frac{z_D^{I*\ k-(\theta-1)} - z_{min}^{*k-(\theta-1)}}{z_D^{I*\ k} - z_{min}^{*k}}\right] + z_D^{I*-k}\psi_D^{*\xi^*(1-\theta)}\left(\frac{TOL}{z_D^{I*}}\right)^{1-\theta} \\ & + z_X^{-k}\frac{N}{N^*}\left(\frac{\tau^*}{z_X}\right)^{1-\theta} \\ \Omega =& (1-z_D^{I\ -k})(z_{min} z_D^I)^{\theta-1}\left[\frac{z_D^{I\ k-(\theta-1)} - z_{min}^{k-(\theta-1)}}{z_D^{I\ k} - z_{min}^k}\right] \\ & + (z_D^{I\ -k} - z_V^{-k})\psi_D^{\xi(1-\theta)}(z_D^I z_V)^{\theta-1}\left[\frac{z_V^{k-(\theta-1)} - z_D^{I\ k-(\theta-1)}}{z_V^k - z_D^{I\ k}}\right] + z_V^{-k}\psi_V^{\xi(1-\theta)}\left(\frac{\tau TOL}{z_V}\right)^{1-\theta} \\ & + z_X^{*-k}\left(\frac{N}{N^*}\right)^{-1}\left(\frac{\tau TOL}{z_X^*}\right)^{1-\theta} \end{aligned}$$

The equations for the export price are as follows:

$$\frac{\theta k}{k-(\theta-1)} f_X \frac{\tilde{\rho}_X^{\theta-1}}{TOL} = 1 + \frac{1-\beta^*}{\beta^*(1-\delta^*)} f_E^* \frac{\tilde{\rho}_X^{\theta-1}}{\Xi} \tag{43}$$

$$\frac{\theta k}{k-(\theta-1)} f_X^* TOL\,\tilde{\rho}_X^{*\theta-1} = 1 + \frac{1-\beta}{\beta(1-\delta)} f_E \frac{\tilde{\rho}_X^{*\theta-1}}{\Gamma} \tag{44}$$

$$\frac{N}{N^*}\left(\frac{\tilde{\rho}_X}{\tilde{\rho}_X^*}\right)^{\theta-1} = \frac{\Xi}{\Gamma} \tag{45}$$

where

$$\begin{aligned}\Xi =&(1-z_D^{*I\ -k})\left(\frac{\tau^*}{TOL}\right)^{\theta-1}\left(\frac{z^*_{min}z_D^{I*}}{z_X}\right)^{\theta-1}\left[\frac{z_D^{I*\ k-(\theta-1)}-z_{min}^{*k-(\theta-1)}}{z_D^{I*\ k}-z_{min}^{*k}}\right]\\&+z_D^{*I\ -k}\left(\frac{\tau^*}{TOL}\right)^{\theta-1}\psi_D^{*-\xi^*(\theta-1)}\left(\frac{z_D^{I*}}{z_X}\right)^{\theta-1}+z_X^{-k}\frac{N}{N^*}\end{aligned}$$

$$\begin{aligned}\Gamma =&(1-z_D^{I\ -k})(\tau TOL)^{\theta-1}\left(\frac{z_{min}z_D^I}{z_X^*}\right)^{\theta-1}\left[\frac{z_D^{I\ k-(\theta-1)}-z_{min}^{k-(\theta-1)}}{z_D^{I\ k}-z_{min}^{k}}\right]\\&+(z_D^{I\ -k}-z_V^{-k})(\tau TOL)^{\theta-1}\psi_D^{-\xi(\theta-1)}\left(\frac{z_D^Iz_V}{z_X^*}\right)^{\theta-1}\left[\frac{z_V^{k-(\theta-1)}-z_D^{I\ k-(\theta-1)}}{z_V^{k}-z_D^{I\ k}}\right]\\&+z_V^{-k}\psi_V^{-\xi(\theta-1)}\left(\frac{z_V}{z_X^*}\right)^{\theta-1}+z_X^{*-k}\left(\frac{N}{N^*}\right)^{-1}\end{aligned}$$

Appendix F. The Real Exchange Rate Log-linearization

The data-consistent real exchange rate is $\widetilde{Q}_{CPI,t}^{1-\theta} = \frac{N_t{}^*}{N_t}\left(\frac{P_t^*\epsilon_t}{P_t}\right)^{1-\theta} = \frac{N_t^*}{N_t}Q_t^{1-\theta}$, where $N_t \equiv N_{D,t}^{NI} + N_{D,t}^{I} + N_{V,t} + N_{X,t}^*$ and $N_t{}^* \equiv N_{D,t}^{NI*} + N_{D,t}^{I*} + N_{X,t}$ represents the total number of available varieties for consumers in both Home and Foreign countries.

The real exchange rate is derived by using the equilibrium prices for each type of production in the respective countries.

$$\begin{aligned}
\widetilde{Q}_{CPI,t}^{1-\theta} &= \frac{N_t}{N_t^*}\frac{N_{D,t}^{NI*}(\tilde{\rho}_{D,t}^{NI*}P_t^*\epsilon_t)^{1-\theta} + N_{D,t}^{I*}(\tilde{\rho}_{D,t}^{I*}P_t^*\epsilon_t)^{1-\theta} + N_{X,t}(\tilde{\rho}_{X,t}P_t^*\epsilon_t)^{1-\theta}}{N_{D,t}^{NI}(\tilde{\rho}_{D,t}^{NI}P_t)^{1-\theta} + N_{D,t}^{I}(\tilde{\rho}_{D,t}^{I}P_t)^{1-\theta} + N_{V,t}(\tilde{\rho}_{V,t}P_t)^{1-\theta} + N_{X,t}^*(\tilde{\rho}_{X,t}^*P_t)^{1-\theta}} \\
&= \frac{N_t}{N_t^*}\frac{N_{D,t}^{NI*}\left(\frac{w_t^*P_t^*\epsilon_t}{Z_t^*\tilde{z}_{D,t}^{NI*}}\right)^{1-\theta} + N_{D,t}^{I*}\left(\frac{\psi_{D,t}^{*\xi^*}w_t^*P_t^*\epsilon_t}{Z_t^*\tilde{z}_{D,t}^{I*}}\right)^{1-\theta} + N_{X,t}\left(\frac{\tau_t^*Q^{-1}w_tP_t^*}{Z_t\tilde{z}_{X,t}}\right)^{1-\theta}}{N_{D,t}^{NI}\left(\frac{w_tP_t}{Z_t\tilde{z}_{D,t}^{NI}}\right)^{1-\theta} + N_{D,t}^{I}\left(\frac{\psi_{D,t}^{\xi}w_tP_t}{Z_t\tilde{z}_{D,t}^{I}}\right)^{1-\theta} + N_{V,t}\left(\frac{\tau_t\psi_{V,t}^{\xi}Q_tw_t^*P_t}{Z_t^*\tilde{z}_{V,t}}\right)^{1-\theta} + N_{X,t}^*\left(\frac{\tau_tw_t^*P_tQ_t}{Z_t^*\tilde{z}_{X,t}^*}\right)^{1-\theta}} \\
&= \frac{\frac{N_{D,t}^{NI*}}{N_t^*}\left(\frac{TOL_t}{\widetilde{z}_{D,t}^{NI*}}\right)^{1-\theta} + \frac{N_{D,t}^{I*}}{N_t^*}\left(\frac{\psi_{D,t}^{*\xi^*}TOL_t}{\widetilde{z}_{D,t}^{I*}}\right)^{1-\theta} + \frac{N_{X,t}}{N_t^*}\left(\frac{\tau_t^*}{\widetilde{z}_{X,t}}\right)^{1-\theta}}{\frac{N_{D,t}^{NI}}{N_t}\left(\frac{1}{\widetilde{z}_{D,t}^{NI}}\right)^{1-\theta} + \frac{N_{D,t}^{I}}{N_t}\left(\frac{\psi_{D,t}^{\xi}}{\widetilde{z}_{D,t}^{I}}\right)^{1-\theta} + \frac{N_{V,t}}{N_t}\left(\frac{\psi_{V,t}^{\xi}\tau_tTOL_t}{\widetilde{z}_{V,t}}\right)^{1-\theta} + \frac{N_{X,t}^*}{N_t}\left(\frac{\tau_tTOL_t}{\widetilde{z}_{X,t}^*}\right)^{1-\theta}}
\end{aligned}$$

In the steady state, I assume $f_E = f_E^*, \tau = \tau^*$. I adhere to the notation used in Zlate (2010), $s_D \equiv N_D^{NI}\left(\frac{w}{Z\tilde{z}_D^{NI}}\right)^{1-\theta} + N_D^{I}\left(\psi_{D,t}^{\xi}\frac{w}{Z\tilde{z}_D^{I}}\right)^{1-\theta} + N_V\left(\tau TOL\psi_V^{\xi}\frac{w}{Z\tilde{z}_V}\right)^{1-\theta}$. Expression s_D represents the steady state share of Home spending on varieties produced by Home firms, both domestically and offshore. Expression $s_V \equiv N_V\left[\left(\frac{\tau TOL\psi_V^{\xi}}{\psi_D^{\xi}}\right)^{1-\alpha}\frac{w}{Zz_V}\right]^{1-\theta}$ denotes the steady-state share of Home spending on varieties produced by the offshoring Home firms only. (Note that $s_D - s_V > 0$) Expression s_D^* denotes the steady state share of Home spending on varieties produced by Home intangible firms. Expression $s_D^* \equiv N_D^*\left(Q\frac{\psi_D^{*\,\xi^*}}{\psi_D^{\xi}}\frac{w^*}{Z\tilde{z}_D}\right)^{1-\theta}$ represents the steady-state share of foreign spending on varieties produced by Foreign domestic firms. Expression $s_D^{I*} \equiv N_D^*\left(Q\frac{\psi_D^{*\,\xi^*}}{\psi_D^{\xi}}\frac{w^*}{Z\tilde{z}_D}\right)^{1-\theta}$ represents the steady-state share of foreign spending on varieties produced by Foreign intangible firms. The average productivity of foreign firms producing domestically, denoted by $\tilde{z}_D^*$, remains constant over time.

The log-linearized version of the data-consistent real exchange rate ($\widetilde{\mathsf{Q}}_t^{1-\theta}$) can be written as:

$$\widetilde{\mathsf{Q}}_t = \frac{\overline{NUM}dDEN_t - \overline{DEN}dNUM_t}{(\theta-1)\overline{NUMDEN}} = \frac{1}{\theta-1}\left[\frac{dDEN_t}{\overline{DEN}} - \frac{dNUM_t}{\overline{NUM}}\right]$$

Using above information, I derived the log-linearized version of the data-consistent real exchange rate ($\widetilde{\mathsf{Q}}_t^{1-\theta}$):

$$
\begin{aligned}
\widetilde{\mathsf{Q}}_t = \frac{1}{\theta-1}\Bigg[& (s_D - s_D^* - s_V)\Big\{\widetilde{\mathsf{N}}_{D,t}^{NI} - \widetilde{\mathsf{N}}_t - (1-\theta)\tilde{\mathsf{z}}_{D,t}^{NI}\Big\} \\
& + s_D^I\Big\{\widetilde{\mathsf{N}}_{D,t}^{I} - \widetilde{\mathsf{N}}_t + (1-\theta)(\xi\Psi_{D.t} - \tilde{\mathsf{z}}_{D,t}^{I})\Big\} \\
& + s_V\Big\{\widetilde{\mathsf{N}}_{V,t} - \widetilde{\mathsf{N}}_t + (1-\theta)(\mathsf{t}_t + \mathsf{TOL}_t - \tilde{\mathsf{z}}_{V,t} - \xi\Psi_{V,t})\Big\} \\
& + (1-s_D)\Big\{\widetilde{\mathsf{N}}_{X,t}^{*} - \widetilde{\mathsf{N}}_t + (1-\theta)(\mathsf{t}_t + \mathsf{TOL}_t - \tilde{\mathsf{z}}_{X,t}^{*})\Big\} \\
& - (s_D^* - s_D^{I*})\Big\{\widetilde{\mathsf{N}}_{D,t}^{NI*} - \widetilde{\mathsf{N}}_t^{*} + (1-\theta)(\mathsf{TOL}_t - \tilde{\mathsf{z}}_{D,t}^{NI*})\Big\} \\
& - s_D^{I*}\Big\{\widetilde{\mathsf{N}}_{D,t}^{I*} - \widetilde{\mathsf{N}}_t^{*} + (1-\theta)(\xi^*\Psi_{D.t}^{*} + \mathsf{TOL}_t - \tilde{\mathsf{z}}_{D,t}^{I*})\Big\} \\
& - (1-s_D^*)\Big\{\widetilde{\mathsf{N}}_{X,t} - \widetilde{\mathsf{N}}_t^{*} + (1-\theta)(\mathsf{t}_t^{*} - \tilde{\mathsf{z}}_{X,t})\Big\}\Bigg]
\end{aligned}
$$

$$
\begin{aligned}
= {} & (s_D - s_V + s_D^* - 1)\mathsf{TOL}_t && \text{(C1)} \\
& + s_V(\tilde{\mathsf{z}}_{V,t} - \mathsf{t}_t) + (s_D - s_D^I - s_V)\tilde{\mathsf{z}}_{D,t}^{NI} + s_D^I\tilde{\mathsf{z}}_{D,t}^{I} && \text{(C2–C4)} \\
& + (1-s_D)(\tilde{\mathsf{z}}_{X,t}^{*} - \mathsf{t}_t) - (1-s_D^*)(\tilde{\mathsf{z}}_{X,t} - \mathsf{t}_t^*) - (s_D^* - s_D^{*I})\tilde{\mathsf{z}}_{D,t}^{*NI} - s_D^{*I}\tilde{\mathsf{z}}_{D,t}^{*I} && \text{(C5)} \\
& - s_V\xi\Psi_{V,t} - s_D^I\xi\Psi_{D,t} + s_D^{*I}\xi^*\Psi_{D,t}^{*} && \text{(C6)} \\
& + \frac{1}{\theta-1}\left(s_V - \frac{N_V}{N_D + N_V + N_X^*}\right)(\mathsf{N}_{V,t} - \mathsf{N}_{X,t}^{*}) && \text{(C7)} \\
& + \frac{1}{\theta-1}\left[\begin{array}{l}\left(\dfrac{N_D^{*NI}}{N_D^{*NI} + N_D^{*I} + N_X} - (s_D^* - s_D^{*I})\right)(\mathsf{N}_{D,t}^{*NI} - \mathsf{N}_{X,t}) \\ \quad - \left(\dfrac{N_D^{NI}}{N_D^{NI} + N_D^{I} + N_V + N_X^*} - (s_D - s_D^I - s_V)\right)(\mathsf{N}_{D,t}^{NI} - \mathsf{N}_{X,t}^{*})\end{array}\right] && \text{(C8)} \\
& + \frac{1}{\theta-1}\left[\begin{array}{l}\left(\dfrac{N_D^{*I}}{N_D^{*NI} + N_D^{*I} + N_X} - s_D^{*I}\right)(\mathsf{N}_{D,t}^{*I} - \mathsf{N}_{X,t}) \\ \quad - \left(\dfrac{N_D^{I}}{N_D^{NI} + N_D^{I} + N_V + N_X^*} - s_D^{I}\right)(\mathsf{N}_{D,t}^{I} - \mathsf{N}_{X,t}^{*})\end{array}\right] && \text{(C9)}
\end{aligned}
$$