Trade in Niches, Markups, and Firm Sorting in General Equilibrium*

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Abstract

This paper develops a general equilibrium model with heterogeneous firms that endogenously choose among various market segments/niches within a differentiated sector. I study the implications of both autarky and international trade between countries. In equilibrium, larger niches are featured by lower local prices. Positive assortative matching between niche size and firm productivity is driven by the demand side and implies that more productive firms sort into more competitive niches. It generates U–shaped equilibrium relation between markups and both niche and firm size. I prove that, in autarky, consumer welfare is increasing in niche size. Trade induces higher local competition and an unambiguous shift in the matching function that features tougher selection. I find that while consumers in middle-sized niches gain the most from opening the closed economy to trade, consumers in the largest niches may bear losses. The impact of small changes in trade costs is also highly uneven across consumers: niches previously unexposed to trade are those that lose from little trade liberalization. The sorting effect is sizable for most niches and contributes up to 59% of the total welfare gains for middle-sized niches. In contrast to costly trade patterns, gains from free trade are positive and monotonically decreasing in niche size. I also show that while Marshall's Second Law of Demand is a necessary and sufficient condition for positive sorting under many popular classes of preferences, it is actually neither necessary nor sufficient under the generalized Gorman-Pollak demand system.

Keywords: international trade, sorting, monopolistic competition, firm heterogeneity, consumer heterogeneity, variable markups, gains from trade, inequality.

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1 Introduction

The assumption of firm heterogeneity in differentiated product markets is the cornerstone ingredient of many studies in international trade and related fields since Melitz (2003). In contrast, the assumption of a representative consumer is still quite ubiquitous in modern trade theories. As a result, these models tend to be silent on how trade liberalization or other policies affect the welfare of different consumers or, to put it differently, on the distributional consequences of trade across different consumers. However, a change in a trade policy, while leading to large welfare gains for some groups of consumers, may also lead to relatively modest welfare gains, or even losses, for other consumers. One of the potential reasons is that different consumers tend to concentrate their spending on different products, even within quite narrowly defined product categories. For example, Neiman and Vavra (2019) document "The Rise of Niche Consumption" and make a succinct statement on these patterns:

> "*Over the last 15 years, the typical household [in the US] has increasingly concentrated its spending on a few preferred products. However, this is not driven by "superstar" products capturing larger market shares. Instead, households increasingly purchase different products from each other*".

Recent literature also points to the importance of the so-called firm "appeal" (quality or taste). For instance, Hottman, Redding, and Weinstein (2016) argue that it tends to explain at least 50% of the variance in the U.S. firm size. Related to this, Macedoni, Xu, and Feenstra (2020) find that heterogeneity across consumer segments accounts for 39% of the variation in product appeal.

As for the supply side, modern theories of international trade tend to impose highly tractable, yet very strict, assumption that all firms producing in a particular differentiated product sector directly compete with each other by serving the same consumers/niche, regardless of their productivity or export status. The fact that different firms tend to serve distinct niches is documented by Holmes and Stevens (2014), who estimate that within most industries in the US, more than 50% of the plants can be classified as specialty segment plants. As also noted by Macedoni, Xu, and Feenstra (2020), "segment targeting is a crucial driver of firm performance along with prices and product scope, and it further affects firm size heterogeneity".

In this paper, I develop a general equilibrium model allowing costly trade between countries. Firms are monopolistically competitive, heterogeneous in productivity as in Melitz (2003), and endogenously choose to operate in different consumer segments or, equivalently, market niches. I relax the CES assumption and exploit a subset of homothetic preferences that yield variable markups. I show that deviation from the CES is also crucial for endogenous decisions of heterogeneous firms on what market niches to target and sorting patterns.

Incorporating consumer niches into an otherwise standard Melitz-type model with variable

markups yields a number of additional insights. In particular, I prove that, in equilibrium, there will be an inverse relation between the value of the local price aggregator and niche size. As a result, each firm faces the trade-off between serving the more lucrative local market, on the one hand, and facing more benign local competition, reflected in the lower local price index, on the other hand. Given this trade-off, I prove that, in both autarky and costly trade scenarios, the condition on *Positive Assortative Matching* (PAM) between firm productivity and niche size is ensured by a simple condition in terms of primitives of the model – increasing the elasticity of the market share function with respect to a firm's price. It is shown that this condition is equivalent to Marshall's Second Law of Demand (MSLD). Positive assortative matching implies that more productive "high-markup" firms sort into more competitive niches that feature stronger downward pressure on markups. As a result, I show that under not very restrictive assumptions on the distribution of niche sizes and firm productivities, it may generate $\cup-$shaped relationship between markups and niche size as well as between markups and firm size. Hence, from an empirical point of view, the model is potentially able to capture a new stylized fact that has been recently unveiled by Díez, Fan, and Villegas-Sánchez (2021) and based on data from a number of advanced economies and emerging markets: the non-monotonic ($\cup-$shaped) relationship between markups and firm size.

I prove that in autarky, consumer welfare is strictly increasing in niche size under MSLD. I also show that niche-specific change in consumer welfare associated with a small shock to exogenous model parameters can be decomposed into the *sorting effect* and *competition effect*. Trade induces higher competition captured by lower local price indices for all niches. Moreover, selection into exporting creates a kink in the matching function at the export threshold niche and an unambiguous shift in the matching function: all niches, except for the largest one, are served by relatively more productive firms. In other words, firms relocate to "escape" competition. As for the relative impact on welfare, I find that consumers in medium-size niches generally tend to gain more than consumers in larger niches, which are directly exposed to trade, while consumers in some of the largest niches may actually bear losses from opening the closed economy to trade. The impact of small shocks on variable trade costs is also highly uneven across niches. In particular, while most niches tend to gain from small trade liberalization, niches previously unexposed to trade bear losses. The impact of the sorting effect is sizable for most niches and contributes up to 59% of the total welfare gains for middle-sized niches. I also find that, in contrast to costly trade patterns, gains from opening the closed economy to free trade are positive and monotonically decreasing in niche size.

Also, I compare the properties of the baseline model with its limiting case when the distribution of niche sizes degenerates and the economy reduces to a standard *spaceless* Melitz-type model with variable markups. I show that theoretical predictions of the *spatial* model imply more flexible patterns for markups and a number of other variables of interest. Moreover, firms charge higher markups than their counterparts in the limiting spaceless case of the model. In

addition to MSLD, I also emphasize the important role of a more restrictive assumption of an increasing elasticity of marginal revenue (MSLD′ assumption by Mayer, Melitz, and Ottaviano 2016) for reallocation patterns.

To shed more light on the role of the demand side for equilibrium outcomes, I also derive the model's properties under the Gorman-Pollak single-aggregator demand system that encompasses broad classes of both homothetic and non-homothetic preferences. In Appendix B, I analytically show that the model's property of strictly higher local competition measures in more populated markets/niches can be generalized to this demand system. In contrast, Marshall's Second Law of Demand appears to be neither necessary nor sufficient for positive assortative matching between firm productivity and niche size. I also highlight formal conditions on the demand side that provide particular sorting patterns.

Related Literature This paper is related to several strands of the literature. First of all, it complements theoretical trade models with monopolistic competition, variable markups, and heterogeneous firm-level productivity. This literature has emerged since Melitz (2003) to obviate salient limitations and empirical failings of the workhorse CES framework. For example, Melitz and Ottaviano (2008) replace the CES preferences with the linear demand system and develop an analytically solvable model with endogenous markups to examine the effects of different trade liberalization policies. Arkolakis, Costinot, and Rodríguez-Clare (2010) develop a multi-country trade model with a symmetric translog expenditure function. Arkolakis, Costinot, Donaldson, and Rodriguez-Clare (2019) and Fally (2019) recently provided extensions to more general demand systems that encompass broad classes of both homothetic and non-homothetic preferences. My paper complements this literature by introducing heterogeneous niches into the monopolistic competition trade model with heterogeneous firms and endogenous markups. To highlight the important role of heterogeneous niches, I also provide a comparative analysis of equilibrium outcomes and comparative statics between "spatial" and "spaceless" versions of the model.

This paper is also related to the recent literature on the role of "horizontal" consumer heterogeneity in monopolistic competition models. In particular, the model shares some common ingredients with Kokovin, Sharapudinov, Tarasov, and Ushchev (2020) (hereafter, KSTU 2020). For example, both papers feature PAM between heterogeneous firms and exogenously given niche sizes. Meanwhile, the papers also differ in several important aspects. First, while KSTU (2020) study the equilibrium properties of the closed economy and related comparative statics, the present paper also explores the implications of costly trade between countries. Second, I get rid of the quasi-linear setting and cope with a full-fledged general equilibrium setup. Third, both papers exploit different demand systems.[1] The homothetic demand structure that I use

[1] In particular, I use a subset of homothetic preferences, while KSTU (2020) exploit directly-additively separable preferences within the differentiated sector. To highlight formal conditions on the demand side that result in par-

in the baseline version of the model allows various sources of differences in niche sizes – such differences may be driven by either the total income of consumers within each niche, "horizontal" taste heterogeneity, or both.[2] Another related paper, albeit under a somewhat different setup, is Goryunov, Kokovin, and Tabuchi (2021). While providing a comprehensive analysis of a monopolistic competition model that features both spatial and price competition, they focus on the uniform distribution of niche sizes and do not consider costly trade between countries. In addition, they abstract from any sorting of firms based on their productivity.

To prove positive assortative matching between firm productivity and niche size, I use monotone comparative statics tools, which are reminiscent of those used in the assignment literature with heterogeneous firms and heterogeneous workers in monopolistic competition framework (see e.g. Sampson 2014, Costinot and Vogel 2015[3]) as well as the literature on sorting of heterogeneous firms across cities of different sizes (see, e.g., Gaubert 2018). In the meantime, the mechanism of PAM in my paper is intrinsically different. In particular, in Sampson (2014) and Costinot and Vogel (2015), the PAM between high-skill workers and high-technology firms is entirely driven by the supply-side characteristics (namely, the assumption that labor productivity is log-supermodular in firm technology and worker skill). In contrast to these studies, the PAM between firms and market niches in my paper is driven by the demand side. Specifically, it is closely linked with the behavior of an individual's price elasticity of demand and hinges on whether Marshall's Second Law of Demand holds or not.

The rest of the paper is organized as follows. In Section 2, I develop a closed economy setup and study its main properties. In Section 3, I extend the model to an open economy setup in the presence of costly trade between symmetric countries. Section 4 contrasts equilibrium and comparative statics properties of the model with its limiting case of degenerate distribution of niche sizes that represents the spaceless Melitz-type model with endogenous markups. Appendix A contains routine proofs. Appendix B provides a discussion of the model's properties under a more general demand system as well as some special cases. To check the robustness of the welfare implications obtained under the baseline model, I develop an alternative open economy setup in Appendix C.

ticular sorting patterns, I also examine the implications of the Gorman-Pollak single-aggregator demand system. It encompasses both homothetic and non-homothetic preferences with a single aggregator (see Appendix B).

[2]Aggregation properties of homothetic preferences are going to be particularly useful for extensions of the model to heterogeneous workers, as in Sampson (2014), or non-local consumption. Details and preliminary results are available upon request.

[3]While primarily focusing on perfectly competitive markets, Costinot and Vogel (2015) also consider an extension when good markets are monopolistically competitive.

2 Model

I start with a closed-economy version of the model. I introduce costly trade between countries in Section 3.

2.1 Set Up

Each individual in an economy is both a consumer and a worker. Labor is inelastically supplied and is the only factor of production. Within a country, workers are perfectly mobile across firms. Hereafter, the unit of labor is chosen as numéraire, and I normalize the unit labor wage to one.

Consumers derive utility from the consumption of varieties in the differentiated product sector.[4] Specifically, the sector is characterized by a set of "locations/niches", $\mathcal{X} \equiv [0, x_{\max}]$, where production and consumption take place. One can potentially treat $\mathcal{X}$ as either the set of consumer segments, niches, or types of consumers.[5] For the sake of unifying terminology, I will oftentimes treat $x \in \mathcal{X}$ as a particular market *niche*.

For the baseline version of the model, I assume that consumption is *local*, implying that each consumer buys only those varieties that are produced within her personal niche. Within each niche, there is a continuum of horizontally differentiated varieties, and consumers have symmetric homothetic preferences over them. I assume that the distribution of total market demand along the set of niches, $\mathcal{X}$, is exogenously given by $\ell(\cdot)$. Throughout the paper, I impose the following assumptions on function $\ell(\cdot)$:

Assumption 1. ***i)*** *$\ell(\cdot)$ is a C^2 function on $\mathcal{X}$;* ***ii)*** *$\ell'(\cdot) < 0$;* ***iii)*** *$\ell(x_{\max}) > 0$.*

This set of assumptions implies that a market becomes sparser as x rises. In what follows, I treat $\ell(\cdot)$ as *the distribution of niche sizes*. As will become apparent further on in this Section, the demand structure that I use possesses well-known aggregation properties of homothetic preferences within a particular niche. It implies that differences in niche sizes may be driven by either the total income of consumers within each niche, "horizontal" taste heterogeneity, or both. As a result, workers may be potentially endowed with different efficiency units of labor. However, to facilitate comparison with the model's outcomes under a more general demand system (see Appendix B), I assume that each worker is endowed with the same efficiency units of labor, E.[6] As a result, the total amount of efficiency units of labor in an economy can be denoted by $L \equiv E \int_{\mathcal{X}} \ell(x) \mathrm{d}x$.

[4] To highlight the implications of consumer heterogeneity, I consider a single-sector setup with a continuum of niches within the sector. It is straightforward to extend the model to the case of an arbitrary number of sectors. Moreover, as will be mentioned further, there are several potential interpretations of "a niche" in my model.

[5] It is possible to treat $\mathcal{X}$ as a "geographical" space. More details on the model's features under this setting are available upon request.

[6] In this case, $\ell(\cdot)$ controls for the mass of consumers (population density) at each niche x. Alternatively, E may be "location-specific" and one can allow for a correlation between niche index and a consumer's income. Note

Demand Recall that each consumer buys only those varieties that are produced at her personal niche. In what follows, Ω_x denotes the (endogenous) mass of varieties of type $x \in \mathcal{X}$ available on the market, while ω_x denotes a particular variety of type $x \in \mathcal{X}$.

A consumer located at $x \in \mathcal{X}$ (i.e., an x-type consumer) chooses an individual consumption profile of locally-produced varieties, $\mathbf{q}_x \equiv \{q(\omega_x) \mid \omega_x \in \Omega_x\}$ for a given profile of prices, $\mathbf{p}_x \equiv \{p(\omega_x) \mid \omega_x \in \Omega_x\}$, to maximize her homothetic utility function subject to the budget constraint:

$$\int_{\omega_x \in \Omega_x} p(\omega_x) q(\omega_x) \mathrm{d}\omega_x = E.$$

For the baseline setup, I exploit a subset of homothetic preferences. In particular, I assume that preferences are symmetric and described by (non-parametrically defined) homothetic demand systems with a single aggregator (henceforth, HSA). This class of demand systems implies the following form of an individual's demand for a variety $\omega_x \in \Omega_x$ of type $x \in \mathcal{X}$:[7]

$$q(\omega_x) = \frac{E}{p(\omega_x)} \Lambda \left(\frac{p(\omega_x)}{P(x)} \right), \tag{2.1}$$

where $P(x) \equiv P(\mathbf{p}_x)$ denotes *the (local) price aggregator* for varieties of niche/type $x \in \mathcal{X}$, while $\Lambda : \mathbb{R}_{++} \to \mathbb{R}_+$ represents *the "market share" function* that satisfies the standard set of assumptions: **i)** $\Lambda(\cdot)$ is a C^3 function on $\mathbb{R}_{++}$; **ii)** $\Lambda'(z) < 0 \; \forall \; z \in \mathbb{R}_{++}$; **iii)** $\lim_{z \to 0} \Lambda(z) = \infty$ and $\lim_{z \to \bar{z}} \Lambda(z) = 0$, where $\bar{z} = \inf \{z > 0 \mid \Lambda(z) = 0\} \leq \infty$ denotes the *choke price*, if any.

For each niche $x \in \mathcal{X}$, the (local) price aggregator $P(x)$ is linearly homogeneous in $\mathbf{p}_x$ and implicitly defined by

$$\int_{\omega_x \in \Omega_x} \Lambda \left(\frac{p(\omega_x)}{P(x)} \right) \mathrm{d}\omega_x = 1. \tag{2.2}$$

Notice that (2.2) implies that the budget constraint holds and "market shares" of all varieties sum up to one. The schedule of $P(x)$ is endogenous and represents the distribution of the inverse of a measure of *local competitive toughness* along the set of niches. Specifically, a higher value of $P(x)$ implies a lower level of local competition at $x \in \mathcal{X}$. Note also that $P(x)$ is proportional to the (local) ideal price index, $P^I(x)$, only for a special case CES preferences, $\Lambda(z) = z^{1-\sigma}$, where $\sigma > 1$ denotes the price elasticity of demand.[8]

also that Faber and Fally (2017) find that more productive firms sort into catering to the taste of relatively richer households. More details of the model's features under income heterogeneity are available upon request.

[7] General properties of this class of demand systems have been recently studied by Matsuyama and Ushchev (2017; 2020) and Fally (2018). In particular, Matsuyama and Ushchev (2017; 2020) also discuss "direct" and "alternative" formulations of HSA demand systems. In what follows, I maintain baseline restrictions they impose on the market share function for "direct" and "alternative" representations of HSA demand systems.

[8] According to Matsuyama and Ushchev (2017, 2020), a (common) price aggregator is proportional to the ideal price index only for the CES specification of HSA demand systems. In the context of my "spatial" model, this result holds true within every location/segment.

For quantitative analysis, I exploit a widely used (see e.g. Arkolakis, Costinot, and Rodríguez-Clare 2010, Feenstra 2003, Feenstra and Weinstein 2017) symmetric translog specification: for any $\zeta > 0$, $\Lambda(z) = \log z^{-\zeta}$ for $z \in (0, 1)$ with the choke price $\bar{z} = 1$. Translog specification is a special case of HSA demand as well as a flexible demand system developed by Arkolakis, Costinot, Donaldson, and Rodriguez-Clare (2019), and even more general, the Gorman-Pollak single aggregator demand system studied by Fally (2018).[9]

Throughout most parts of the paper, I will also impose an additional assumption that preferences feature Marshall's Second Law of Demand.

Assumption 2 (MSLD). *For any $x \in \mathcal{X}$, the price elasticity of demand falls with quantity consumed (Marshall's Second Law of Demand).*

Note that Marshall's Second Law of Demand is equivalent to the assumption that the elasticity of the market share function, $\mathcal{E}_\Lambda\left(p/P(x)\right) \equiv \frac{-\Lambda'(p/P(x))}{\Lambda(p/P(x))}\frac{p}{P(x)}$, is increasing with price.[10]

Market Structure and Technology Firms are monopolistically competitive and heterogeneous in their production efficiency, similar to that in Melitz (2003). In particular, each firm draws its marginal cost in units of labor, $c \in [c_{\min}, \infty)$ with $c_{\min} \geq 0$, from some exogenous distribution defined by a continuously differentiable p.d.f., $g(\cdot)$, with cumulative distribution defined by $G(\cdot)$. Production of a variety also requires fixed costs of F units of labor.[11]

I assume that each firm treats the (endogenous) schedule of the competitive toughness as given when it decides on its price and market (or, equivalently, segment/niche), $x \in \mathcal{X}$, to serve by further supplying a unique variety within this niche. A notable feature of the model is that the measure of competition is "location-specific" and captured by the local value of the price aggregator, $P(x)$. As it will become apparent from further discussion, each firm faces the trade-off between a larger local market and weaker local competition.

2.2 Firm Behavior and PAM

Note that consumers within a niche $x \in \mathcal{X}$ share the same symmetric preferences over all potential varieties $\omega \in \Omega_x$ of type x. Hence, if an active monopolistically competitive firm incurs the marginal cost c and produces a variety of type x, then its profit function can be represented as follows

$$\Pi(c, x) \equiv \max_p \left\{ (p - c)\ell(x)\frac{E}{p}\Lambda\left(\frac{p}{P(x)}\right) - F \right\}. \tag{2.3}$$

[9] In Appendix B, I study the robustness of the main results of the paper under a more general Gorman-Pollak single-aggregator demand system.

[10] One can readily verify that $\mathcal{E}'_\Lambda(z) > 0 \Leftrightarrow \mathcal{E}_\Lambda(z) + 1 > -z\Lambda''(z)/\Lambda'(z)$.

[11] Generally, it is assumed $F > 0$. However, details for the case of $F = 0$ (thereby implying that the selection of firms is driven by the existence of a choke price) are available upon request.

One can readily verify that maximization of a firm's profits w.r.t. its price implies:[12]

$$\frac{p}{c} = \frac{\mathcal{E}_\Lambda\left(p/P(x)\right)+1}{\mathcal{E}_\Lambda\left(p/P(x)\right)}, \tag{2.4}$$

where $\mathcal{E}_\Lambda\left(\frac{p}{P(x)}\right) \equiv \frac{-\Lambda'(p/P(x))}{\Lambda(p/P(x))}\frac{p}{P(x)}$ denotes the elasticity of the market share function, $\Lambda(\cdot)$.

It is straightforward to verify that MSLD assumption is a sufficient condition to ensure that (2.4) has a unique solution in terms of p for given $P(x)$ and c. Thus, solving (2.4) for p yields a firm's price as a function of its marginal cost and location/niche, $p(c,x)$.

As a result, the expression for a firm's profit can be re-written as follows

$$\Pi(c,x) = \left(p(c,x)-c\right)\ell(x)\frac{E}{p(c,x)}\Lambda\left(\frac{p(c,x)}{P(x)}\right) - F.$$

Exploiting the envelope theorem, the corresponding FOC w.r.t. x implies

$$-\frac{\ell(x)}{\ell'(x)}\frac{P'(x)}{P(x)} = \frac{1}{\mathcal{E}_\Lambda\left(p(c,x)/P(x)\right)}. \tag{2.5}$$

Solving equation (2.5) for x yields the optimal location/niche (or, equivalently, product type/attribute $x \in \mathcal{X}$) of a firm as a function of its marginal cost, $x(c)$.

Proposition 1 (Firm's Trade-off). *Consider any interval of niches $[\underline{x},\overline{x}] \subseteq \mathcal{X}$ that is served by firms in equilibrium. In the interior of this interval, larger local markets (niches) feature tougher competition, implying lower values of the local price aggregator, $P(x)$. Formally, $P(x)$ is strictly increasing on $(\underline{x},\overline{x})$.*

Proof. See Appendix A.1. □

Proposition 1 claims that larger niches are more competitive in a closed economy equilibrium. As shown in Appendix B, this property of the model is not restricted by the assumption of homotheticity and can also be proven for the Gorman-Pollak demand system that encompasses HSA demand systems as well as broad classes of non-homothetic demand systems. Now, note that combining (2.4) and (2.5) yields

$$\mu(c,x) \equiv \frac{p(c,x)}{c} = 1 - \frac{\ell(x)}{\ell'(x)}\frac{P'(x)}{P(x)}, \tag{2.6}$$

where $\mu(\cdot)$ is henceforth referred to as a firm's markup.

It is straightforward to see from Proposition 1 that each firm faces the trade-off between a larger local market and weaker local competition. Next, I formulate necessary and sufficient

[12] Note that (2.4) can be alternatively stated as $\frac{p}{c} = \frac{\mathcal{E}_D\left(p/P(x)\right)}{\mathcal{E}_D\left(p/P(x)\right)-1}$, where $\mathcal{E}_D\left(p/P(x)\right) \equiv \frac{-D'(p/P(x))}{D(p/P(x))}\frac{p}{P(x)}$ denotes the price elasticity of demand, taking $P(x)$ as given. It is straightforward to verify that $\mathcal{E}_D(\cdot) = 1 + \mathcal{E}_\Lambda(\cdot)$.

conditions for *Positive Assortative Matching* (PAM) between firm productivity and niche size, or, put differently, positive sorting of heterogeneous firms into market niches/segments.

Proposition 2. *The model equilibrium implies that:*

i) *more productive firms set lower prices,* $\mathrm{d}p/\mathrm{d}c > 0$*;*

ii) *there is PAM between firm productivity and niche size,* $\mathrm{d}x/\mathrm{d}c > 0$*, iff the elasticity of the market share function,* $\mathcal{E}_\Lambda\left(p/P(x)\right)$*, is increasing with price (or, equivalently, the MSLD assumption holds).*

Proof. See Appendix A.2. □

Proposition 2 implies that PAM is equivalent to assuming that $\mathcal{E}_\Lambda(\cdot)$ is increasing in price. One can readily verify that the CES preferences represent the "knife-edge" case, where $\mathcal{E}_\Lambda(\cdot) = \sigma - 1$ is constant and there can be multiple equilibria.[13] Hereafter, I will focus on properties of the model under MSLD assumption – an empirically relevant assumption on models with variable markups in international trade literature (see e.g. Mayer, Melitz, and Ottaviano 2016).

Taking into account PAM, the equation for a firm's markup (2.6) can be represented as a first-order differential equation for the local price aggregator, $P(x)$:

$$\frac{P'(x)}{P(x)} = -\frac{\ell'(x)}{\ell(x)}\frac{p\left(c(x),x\right) - c(x)}{c(x)}. \tag{2.7}$$

In further analysis, I will derive a differential equation for the matching function, $c(x)$, that, when combined with (2.6), forms a system of non-linear differential equations.

2.3 General Equilibrium

The entry and exit mechanism is similar to that exploited in static versions of Melitz-type models (see, e.g., Melitz and Redding 2014). In particular, it is assumed there is an unbounded pool of potential entrants that can enter the differentiated product sector by paying sunk entry cost of $F^e > 0$ units of labor. Once the entry cost is paid, a firm draws its marginal cost level and decides whether to produce or immediately exit the market.

The total mass of active firms, M, is, therefore, a fraction of entrants that survive, $M = G(c^*)\cdot M^e$, where $c^* = \sup\{c : \Pi(c,\cdot) > 0\}$ is the cutoff marginal cost level that makes a firm indifferent between starting to produce or immediately exiting the market; M^e – the mass of entrants, and $G(c^*)$ represents probability of successful entry. As a result, the conditional distribution of firm productivities is $g(c)/G(c^*)$ for $c \in [c_{\min}, c^*]$, and 0 otherwise.

[13] In Appendix B, I show that CES is not the only case with no clear sorting pattern under a more general Gorman-Pollak single aggregator demand system that encompasses HSA, directly-additively separable, and many other preferences. In addition, I show that MSLD is actually neither a necessary nor sufficient condition for PAM under this generalized demand system.

Note that the PAM implies $\mid \mathrm{d}c(x)/\mathrm{d}x \mid = \mathrm{d}c(x)/\mathrm{d}x \equiv c'(x)$. Hence, applying the standard "change of variables" theorem, the mass of firms that serve a particular location $x \in \mathcal{X}$ is given by $M(x) = M \cdot g\left(c(x)\right) c'(x)/G(c^*) = M^e \cdot g\left(c(x)\right) c'(x)$.

As a result, equation (2.2) that implicitly defines (the inverse of) a measure of *local competitive toughness*, $P(x)$, can be rewritten as $M(x) \cdot \Lambda\left(p\left(c(x), x\right)/P(x)\right) = 1$ that is equivalent to

$$M^e\, g\left(c(x)\right) c'(x)\, \Lambda\left(\frac{p\left(c(x), x\right)}{P(x)}\right) = 1. \tag{2.8}$$

Rearranging (2.8) yields a first-order differential equation for the matching function, $c(x)$. In equilibrium, (2.7) and (2.8) form the system of non-linear differential equations:

$$\begin{cases} \frac{P'(x)}{P(x)} = -\frac{\ell'(x)}{\ell(x)} \frac{p(c(x),x)-c(x)}{c(x)} \\ c'(x) = \frac{1}{M^e\, g(c(x))\Lambda(p(c(x),x)/P(x))}, \end{cases} \tag{2.9}$$

with two boundary conditions: $c(0) = c_{\min}$ and $c(x_{\max}) = c^*$.

Note that boundary conditions imply that the most populated niche, $x = 0$, is served by the most productive firms, $c(0) = c_{\min}$, where $c_{\min} \geq 0$ is exogenously given. The least populated niche, $x = x_{\max}$, is served by firms with a cutoff level of marginal cost, $c(x_{\max}) = c^*$, where c^* is endogenously determined by the *zero cutoff profit* (ZCP) condition:

$$\Pi\left(c(x_{\max}), x_{\max}\right) = 0. \tag{2.10}$$

As for the equilibrium mass of entrants, M^e, it is determined by the *free entry* (FE) condition that equates an entrant's ex-ante (expected) profits with the sunk entry cost, F^e:

$$\int_0^{x_{\max}} \Pi\left(c(x), x\right) g\left(c(x)\right) c'(x) \mathrm{d}x = F^e. \tag{2.11}$$

The following paragraph formulates the general equilibrium with PAM for the closed economy setup.

Definition. *A closed economy equilibrium with PAM is described by the set of endogenous variables* $\left\{ \left[\left(p(c,x)\right)_{c\in[c_{\min},c^*]}, P(x),\ c(x) \right]_{x\in[0,x_{\max}]}, c^*, M^e \right\}$ *such that:* ***i)*** *a firm's pricing formula, (2.4), defines* $p(c,x)$*;* ***ii-iii)*** *the schedule of the local price aggregators,* $P(x)$*, and the matching function,* $c(x)$*, satisfy the system of differential equations, (2.9);* ***iv)*** *the cutoff level of marginal cost,* c^**, satisfies the zero cutoff profit (ZCP) condition, (2.10);* ***v)*** *the mass of entrants,* M^e*, satisfies the free entry (FE) condition, (2.11).*

Equilibrium outcomes In addition to analytical results on the behavior of $c(x)$ and $P(x)$ (see Propositions 1 and 2), I show that imposing a few auxiliary assumptions yields unambiguous

predictions on the behavior of the model's main variables across the space of niches, $\mathcal{X}$. The following proposition summarizes some of these analytical findings.

Proposition 3. *Assume that MSLD holds. Then, in equilibrium:*

i) *prices, p, are increasing in x;*

ii) *total quantities, $q(\cdot)\ell(\cdot)$, revenues, $p(\cdot)q(\cdot)\ell(\cdot)$, and profits, $\Pi(\cdot)$, are decreasing in x.*[14]

If, in addition, $g(\cdot)$ is non-decreasing and $\ell(\cdot)$ is logarithmically concave, then: [15]

iii) *markups, $\mu(\cdot)$, and market shares of firms, $\Lambda(\cdot)$, are $\cup-$shaped in x;*

iv) *the distribution of the mass of firms, $M(\cdot)$, is $\cap-$shaped in x;*

Proof. See Appendix A.3. □

One can readily verify that Proposition 3 points out a number of distinctions of this model from standard ("spaceless") versions of Melitz (2003) with variable markups. For example, taking into account PAM between firm productivity and niche size, Proposition 3 allows for markups to be non-monotonic ($\cup-$shaped) not only in niche size but also in marginal cost. Recall that standard models with variable markups predict that markups are strictly decreasing in marginal cost under MSLD.[16] As for intuition for a $\cup-$shaped pattern, recall that in this model, firms endogenously sort into different niches/segments within the differentiated-product sector. More competitive niches feature lower values of the local price index, $P(x)$, which creates downward pressure on local markups. In the meantime, PAM implies that more productive "high-markup" firms endogenously choose to serve more lucrative niches, implying an upward pressure on local markups. Under additional assumptions on the distribution of firm productivities and niche sizes, the resulting interplay of these two forces tends to generate $\cup-$shaped markup in equilibrium.

More formally, one may readily verify from a firm's pricing rule (2.4) that if MSLD holds, then a firm's markup at niche x, $\mu(x)$, can be written as an increasing function of price aggregator over marginal cost, $P(x)/c(x)$. In "spaceless" Melitz-type models, the numerator is a scalar variable common for all firms in equilibrium. As a result, firms with lower marginal costs charge higher markups under MSLD. In contrast, this model's equilibrium implies that $P(\cdot)$ is a function that is increasing in x. Likewise, PAM implies that the equilibrium matching function, $c(\cdot)$, is increasing in x. Thus, the pattern of markups is determined solely by the interplay of matching function and competition schedule:

$$\mu'(x) \gtreqless 0 \iff \frac{P'(x)}{P(x)} \gtreqless \frac{c'(x)}{c(x)}.$$

[14] As for firms' total costs, $c(\cdot)q(\cdot)\ell(\cdot) + F$, it can be shown that the corresponding equilibrium pattern depends on whether an additional condition holds or not (see Appendix A.3 for details).

[15] For these results, I also impose mild auxiliary assumptions (see Appendix A.3).

[16] See Section 4 for more details.

Analyzing the behavior of U.S. multinational firms, Keller and Yeaple (2020) document a tendency of high-productivity (and high markup) firms to enter more competitive markets that imply relatively lower markups. As a result, they argue that "*...mark-up differences across locations taken at face value understate differences in the extent of competition across countries.*". A similar argument is potentially applied to implications of my model – markup differences between firms may not reflect true differences in firms' market power and understate differences in the extent of competition between any pair of "locations". Moreover, the model's theoretical prediction is in line with a new stylized fact that has been recently unveiled by Díez, Fan, and Villegas-Sánchez (2021) and based on data from a number of advanced economies and emerging markets: the non-monotonic ($\cup-$shaped) relationship between markups and firm size. Note that the model implies that firm size is decreasing in marginal cost. As a result, given the $\cup-$shaped relationship between a firm's markup and marginal cost, the model also predicts $\cup-$shaped pattern between markups and firm size.

Welfare and Inequality in the Closed Economy Recall that the (local) ideal price index, $P^I(x)$, is not proportional to the local price aggregator, $P(x)$. Hence, to study welfare implications, one needs to use the niche-specific ideal price index, $P^I(x)$. Note that while the characterization of the behavior of $P(x)$ can be directly inferred from a firm's optimization program (see Proposition 1), a more involved proof and additional assumptions are required to detect the behavior of $P^I(x)$. Yet, the following proposition provides a succinct statement on the distribution of consumer welfare.

Proposition 4. *Assume that MSLD holds. Then, in equilibrium, consumer welfare, $W(x) = 1/P^I(x)$, is decreasing in x.*

Proof. See Appendix A.4. □

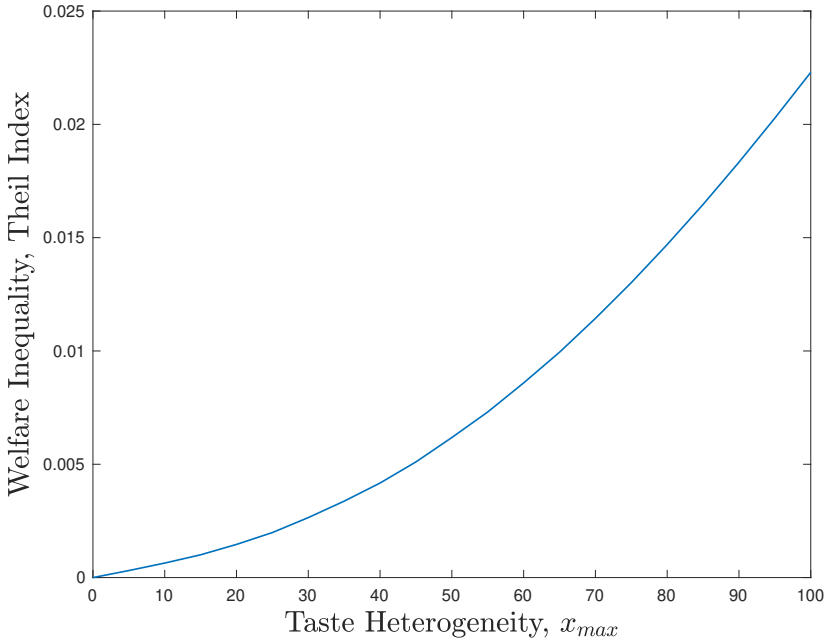
Proposition 4 explicitly states that under MSLD, consumers at sparsely settled locations/niches derive relatively lower welfare benefits. Introducing heterogeneity in niche sizes also allows an examination of welfare inequality across consumers from different niches, even under the closed economy scenario. For example, one can use the Theil index, T, as a measure of welfare inequality:

$$T = \int_{\mathcal{X}} \frac{W(x)}{\bar{W}} \cdot \log\left(\frac{W(x)}{\bar{W}}\right) \frac{\ell(x)}{L} \mathrm{d}x, \tag{2.12}$$

where $\bar{W} = \frac{1}{L}\int_{\mathcal{X}} W(x)\ell(x)\mathrm{d}x$ is the weighted mean of welfare across all niches/locations.

Figure 1 shows the evolution of the Theil index under different measures of taste heterogeneity while keeping the total size of the economy fixed.

Figure 1: Welfare Inequality and Taste Heterogeneity.



As one may expect, there is no welfare inequality under the assumption of consumer homogeneity. Figure 1 also demonstrates that the equilibrium welfare inequality is increasing in the level of taste heterogeneity that is captured by the length of the interval of product niches, $x_{\max}$. Note that a higher value of $x_{\max}$ can potentially represent a more developed industry/economy. Specifically, one may expect a higher level of taste heterogeneity in countries at later stages of economic development, where "worldly-wise" consumers tend to have specific preferences over varieties in differentiated product sectors. Note that this is generally in line with the findings of "The Rise of Niche Consumption" by Neiman and Vavra (2019) discussed in the Introduction. Relatedly, one may expect that firms in more technologically advanced industries/economies are increasingly able to cater to different consumer tastes/niches. Many industries also tend to be increasingly fragmented, with high customization of products and personalized marketing strategies.

3 Trade

In this Section, I introduce international trade into the model. To examine the interplay of firm and consumer heterogeneity in determining, among others, the distribution of the gains from trade, I extend the model from Section 2 to an open economy with costly trade between countries. To capture the essence of the underlying forces that affect the model's outcomes, I find it instructive to start with the case of trade between symmetric countries. However, the

setting potentially allows the consideration of countries that are different in terms of either distribution of consumer tastes/niches or total market size.[17]

Set Up

The world is comprised of $N+1 \geq 2$ countries. In the spirit of Melitz (2003), I abstract from endogenous relative wage movements by assuming that countries are symmetric. Hence, all countries have the same wage that is normalized to one. In further analysis, I consider an arbitrary "*home* (h)" country that trades with N "*foreign*" (f) countries indexed by $j = 1, .., N$.

The baseline structure of preferences is assumed to be the same as in the closed economy setup (see Section 2). However, a consumer located at $x \in \mathcal{X}$ now derives utility from the consumption of both domestic and foreign varieties. As a result, the (local) price aggregator in location/niche $x \in \mathcal{X}$ in "Home" is implicitly defined by

$$\int\limits_{\omega_x \in \Omega_x} \Lambda\left(\frac{p(\omega_x)}{P(x)}\right) \mathrm{d}\omega_x + \left[\sum_{j=1}^{N} \int\limits_{\omega_x^j \in \Omega_x^j} \Lambda\left(\frac{p(\omega_x^j)}{P(x)}\right) \mathrm{d}\omega_x^j\right] \cdot \mathbb{I}^{ex} = 1, \tag{3.1}$$

where $\mathbb{I}^{ex} \in \{0, 1\}$ is an indicator variable that equals one if a particular niche $x \in \mathcal{X}$ is served by foreign firms, and zero otherwise; $\omega_x \in \Omega_x$ denotes a niche x variety produced by a domestic firm; $\omega_x^j \in \Omega_x^j$ denotes a niche x variety produced by a foreign firm from country $j \in \{1, .., N\}$.

Following Melitz (2003), I assume that each active firm still incurs the fixed cost of production F, regardless of its export status. Exporting firms, in addition, bear a per unit iceberg transportation cost, $\tau \geq 1$. Moreover, in order to enter each foreign market, a firm incurs the fixed exporting ("market access") cost, $F^{ex} \geq 0$.[18] As a result, firms tend to charge different prices for domestic and foreign markets. However, each exporting firm is allowed to choose only one product type/niche, $x \in \mathcal{X}$, that will be common for both domestic and foreign markets.[19]

Firm Behavior and PAM

Note that the behavior of non-exporting firms is reminiscent of that discussed in the closed economy setup. So, here, I mainly focus on the behavior of exporting firms. Denote by p^h the price charged by a firm on the domestic market. Note also that, due to the assumption of symmetric countries, an exporting firm sets that same price, p^f, for all foreign markets. Moreover,

[17] Details are available upon request.

[18] Similar to the closed economy case, it is generally assumed that $F^{ex} > 0$. However, details for the case of $F^{ex} = 0$ and when the selection of firms is driven by the existence of a choke price are available upon request.

[19] This assumption can be relaxed (see Appendix C). It is mainly a matter of how we treat a product niche.

the symmetry assumption and zero domestic market access costs imply that no firm exports without serving the domestic market. Hence, the combined profit of an active firm that serves niche x and features marginal cost level c can be represented in the following form:

$$\Pi(c,x) = \Pi^h(c,x) + \max\left\{0, N \cdot \Pi^f(c,x)\right\}, \tag{3.2}$$

where

$$\Pi^h(c,x) \equiv \max_{p^h}\left\{\frac{p^h - c}{p^h}\ell(x)\, E\, \Lambda\left(\frac{p^h}{P(x)}\right) - F\right\}$$

$$\Pi^f(c,x) \equiv \max_{p^f}\left\{\frac{p^f - \tau c}{p^f}\ell(x)\, E\, \Lambda\left(\frac{p^f}{P(x)}\right) - F^{ex}\right\}.$$

One can readily verify that profit maximization w.r.t. p^h and p^f implies:

$$\frac{p^h}{c} = \frac{\mathcal{E}_\Lambda\left(p^h/P(x)\right)+1}{\mathcal{E}_\Lambda\left(p^h/P(x)\right)}, \qquad \frac{p^f}{\tau c} = \frac{\mathcal{E}_\Lambda\left(p^f/P(x)\right)+1}{\mathcal{E}_\Lambda\left(p^f/P(x)\right)}. \tag{3.3}$$

Similarly to the closed economy setup, MSLD is a sufficient condition to ensure that these two equations have unique solutions p^h and p^f, given $P(x)$ and c. Thus, solving (3.3) for p^h and p^f yields the relations between prices and location/niche for the given value of marginal cost: $p^h(c,x)$ and $p^f(c,x)$, respectively. It is straightforward to see from (3.3) that an exporting firm generally sets different factory gate prices for domestic and export markets, $p^h(c,x) \neq p^f(c,x)$. However, the next proposition states that Marshall's Second Law of Demand rules out any profitable arbitrage opportunities.

Proposition 5. *If the MSLD assumption holds, then* $p^h(c,x) < p^f(c,x) < \tau p^h(c,x)$ *for all* $x \in \mathcal{X}$.

Proof. See Appendix A.5. □

Note that a direct implication of Proposition 5 is that an exporting firm charges a relatively lower markup for a foreign market:

$$\frac{p^f(c,x)}{\tau c} < \frac{p^h(c,x)}{c}.$$

Now, exploiting the envelope theorem, the firm's profit maximization w.r.t. x implies:

$$-\frac{\ell(x)}{\ell'(x)}\frac{P'(x)}{P(x)} = \frac{\frac{p^h(c,x)-c}{p^h(c,x)}\Lambda\left(\frac{p^h(c,x)}{P(x)}\right) + N\frac{p^f(c,x)-\tau c}{p^f(c,x)}\Lambda\left(\frac{p^f(c,x)}{P(x)}\right)\mathbb{I}^{ex}}{\frac{p^h(c,x)-c}{p^h(c,x)}\Lambda\left(\frac{p^h(c,x)}{P(x)}\right)\mathcal{E}_\Lambda\left(\frac{p^h(c,x)}{P(x)}\right) + N\frac{p^f(c,x)-\tau c}{p^f(c,x)}\Lambda\left(\frac{p^f(c,x)}{P(x)}\right)\mathcal{E}_\Lambda\left(\frac{p^f(c,x)}{P(x)}\right)\mathbb{I}^{ex}}. \tag{3.4}$$

Using a similar approach as in Proposition 1, it can be shown that $P(x)$ is increasing in equilibrium. Moreover, one can verify that if a niche is not directly exposed to trade (i.e., $\mathbb{I}^{ex} =$

0), then (3.4) is simplified to a firm's FOC w.r.t. x for the closed economy setup, $-\frac{\ell(x)}{\ell'(x)}\frac{P'(x)}{P(x)} = 1/\mathcal{E}_\Lambda\left(\frac{p(c,x)}{P(x)}\right)$.

Note also that combining (3.3) with (3.4) yields the expression for the "weighted markup":

$$\frac{\Lambda\left(p^h(c,x)/P(x)\right) + N\cdot\Lambda\left(p^f(c,x)/P(x)\right)\cdot\mathbb{I}^{ex}}{\frac{c}{p^h(c,x)}\Lambda\left(p^h(c,x)/P(x)\right) + N\cdot\frac{\tau\cdot c}{p^f(c,x)}\Lambda\left(p^f(c,x)/P(x)\right)\cdot\mathbb{I}^{ex}} = 1 - \frac{\ell(x)}{\ell'(x)}\frac{P'(x)}{P(x)}. \tag{3.5}$$

If $\mathbb{I}^{ex} = 0$, then (3.5) simplifies to a firm's markup for the closed economy setup given by $\frac{p(c,x)}{c} = 1 - \frac{P'(x)}{P(x)}\frac{\ell(x)}{\ell'(x)}$.

Similarly to the closed economy case, each firm faces the trade-off between a larger local market and weaker local competition. The following proposition states that a necessary and sufficient condition for PAM in an open economy also relies on the MSLD assumption.

Proposition 6. *The open economy equilibrium implies that:*

i) *more productive firms set lower prices:* $\mathrm{d}p^h/\mathrm{d}c > 0$, $\mathrm{d}p^f/\mathrm{d}c > 0$;

ii) *there is PAM between firm productivity and niche size,* $\mathrm{d}x/\mathrm{d}c > 0$, *iff the elasticity of the market share function,* $\mathcal{E}_\Lambda(\cdot)$, *is increasing with price (MSLD).*

Proof. See Appendix A.6. □

Trade Equilibrium

Taking into account PAM, there exists a unique export threshold location/niche $\tilde{x} \in \mathcal{X}$ such that any niche $x \in [0, \tilde{x}]$ is served by both domestic and foreign firms, while any niche $x \in (\tilde{x}, x_{\max}]$ is served by domestic firms only. In what follows, the cutoff level of marginal cost for exporting firms is denoted by $\tilde{c}^* \equiv c(\tilde{x})$ and defined as $\tilde{c}^* = \sup\left\{c : c \le c^* \text{ and } \Pi^f(c,\cdot) > 0\right\}$, where, as previously, $c^* = \sup\{c : \Pi(c,\cdot) > 0\}$ denotes the cutoff level of marginal cost for successful entry. Both of these cutoffs are endogenous.

Hereafter, $\mathbb{I}_{\{x\le\tilde{x}\}}$ denotes the indicator function implying that $\mathbb{I}_{\{x\le\tilde{x}\}} = 1$ if $x \le \tilde{x}$. Now, taking into account PAM and applying a similar argument to that in Section 2.3, one can readily verify that (3.1) and (3.5) can be re-written as the system of (piecewise continuous) non-linear differential equations:

$$\begin{cases} \dfrac{P'(x)}{P(x)} = -\dfrac{\ell'(x)}{\ell(x)}\left(\dfrac{\Lambda\left(\frac{p^h(c(x),x)}{P(x)}\right)+N\cdot\Lambda\left(\frac{p^f(c(x),x)}{P(x)}\right)\mathbb{I}_{\{x\le\tilde{x}\}}}{\frac{c(x)}{p^h(c(x),x)}\Lambda\left(\frac{p^h(c(x),x)}{P(x)}\right)+N\cdot\frac{\tau\cdot c(x)}{p^f(c(x),x)}\Lambda\left(\frac{p^f(c(x),x)}{P(x)}\right)\mathbb{I}_{\{x\le\tilde{x}\}}} - 1\right) \\ c'(x) = \dfrac{1}{M^e\, g(c(x))\left(\Lambda\left(\frac{p^h(c(x),x)}{P(x)}\right)+N\cdot\Lambda\left(\frac{p^f(c(x),x)}{P(x)}\right)\mathbb{I}_{\{x\le\tilde{x}\}}\right)}, \end{cases} \tag{3.6}$$

with boundary conditions $c(0) = c_{\min}$ and $c(x_{\max}) = c^*$.

Note that both $P'(\cdot)$ and $c'(\cdot)$ are discontinuous at the export threshold niche, $\tilde{x}$. Yet, in equilibrium, both $P(\cdot)$ and $c(\cdot)$ are continuous with a kink at $x = \tilde{x}$. To ensure that $\lim\limits_{x \to \tilde{x}+0} P(x) = P(\tilde{x})$, the export threshold niche must satisfy the following condition:

$$\int_0^{\tilde{x}} P'(x)\mathrm{d}x + \int_{\tilde{x}}^{x_{\max}} P'(x)\mathrm{d}x \Leftrightarrow P(x_{\max}) - P(0). \tag{3.7}$$

The equilibrium mass of entrants, M^e, satisfies the open economy version of the *free entry* condition that equates an entrant's ex-ante (expected) profits with the sunk entry cost, F^e:

$$\int_0^{x_{\max}} \left(\Pi^h\left(c(x), x\right) + N \cdot \Pi^f\left(c(x), x\right) \mathbb{I}_{\{x \leq \tilde{x}\}}\right) g\left(c(x)\right) c'(x)\mathrm{d}x = F^e. \tag{3.8}$$

The following paragraph formulates an open economy equilibrium with PAM for the case of symmetric countries.

Definition. *An open economy equilibrium with PAM is described by the set of endogenous variables* $\left\{\left[\left(p^h(c,x)\right)_{c\in[c_{\min},c^*]}, \left(p^f(c,x)\right)_{c\in[c_{\min},\tilde{c}^*]}, P(x),\ c(x)\right]_{x\in[0,x_{\max}]}, c^*, \tilde{c}^*, \tilde{x}, M^e\right\}$ *such that:* ***i-ii)*** *a firm's pricing rules,* $p^h(c,x)$ *and* $p^f(c,x)$*, are given by (3.3);* ***iii-iv)*** *the schedule of the local price aggregators,* $P(x)$*, and the matching function,* $c(x)$*, satisfy the system of (piecewise continuous) differential equations given by (3.6);* ***v)*** the export threshold niche, $\tilde{x}$, is determined by *(3.7);* ***vi)*** *the cutoff level of marginal cost for production,* c^**, satisfies* $\Pi^h\left(c^*, x_{\max}(c^*)\right) = 0$*;* ***vii)*** *the cutoff level of marginal cost for exporting firms,* $\tilde{c}^*$*, satisfies* $\Pi^f\left(\tilde{c}^*, x(\tilde{c}^*)\right) = 0$*;* ***viii)*** *the mass of entrants,* M^e*, satisfies the free entry condition, (3.8).*

Free Trade

Note that the model's special case of costless trade (i.e., $\tau = 1$ and $F^{ex} = 0$) implies that any active firm exports and maximizes its profits by setting exporting prices at the same level as it sets for the domestic market, $p^h(c(x), x) = p^f(\tau c(x), x) = p(c(x), x)$. Moreover, any of the niches is served by both domestic and foreign firms, implying that $\tilde{x} = x_{\max}$.

In this case, the system of differential equations (3.6) is simplified to the following form:

$$\begin{cases} \frac{P'(x)}{P(x)} = -\frac{\ell'(x)}{\ell(x)} \frac{p(c(x),x) - c(x)}{c(x)} \\ c'(x) = \frac{1}{(N+1)M^e g(c(x)) \Lambda\left(\frac{p(c(x),x)}{P(x)}\right)} \end{cases}, \tag{3.9}$$

with two boundary conditions: $c(0) = c_{\min}$, $c(x_{\max}) = c^*$.

As previously, c^* and M^e are endogenous and pinned down by the corresponding ZCP and FE conditions:

$$\text{ZCP}: \quad \Pi\left(c(x_{\max}), x_{\max}\right) = 0,$$

$$\text{FE}: \quad \int_0^{x_{\max}} \Pi\left(c(x), x\right) g\left(c(x)\right) c'(x) \mathrm{d}x,$$

where $\Pi(c, x)$ denotes the combined profit of a c-type firm takes the following form:

$$\Pi(c, x) = \max_p \left\{ (N+1) \frac{p-c}{p} \ell(x) \, E \, \Lambda\left(\frac{p}{P(x)}\right) - F \right\}.$$

Notice that, apart from the presence of the term $N+1$, the system of differential equations and profit function under the free trade equilibrium resemble their counterparts for the closed economy setup. Thus, the free trade equilibrium must replicate the integrated equilibrium.

3.1 Quantitative Analysis

For quantitative analysis, I consider a world economy comprising two symmetric countries (as in quantitative exercises provided by Arkolakis, Costinot, Donaldson, and Rodriguez-Clare 2019 and Melitz and Redding 2015). Labor in one country is chosen as numéraire, which implies $w^h = w^f = 1$ under the symmetry assumption. For the demand side, I exploit specification of the market share function that corresponds to symmetric homothetic translog, $\Lambda\left(p(\omega_x)/P(x)\right) = \log\left(p(\omega_x)/P(x)\right)^{-\zeta}$, and set $\zeta = 0.19$ (Feenstra and Weinstein 2017).[20] The distribution of marginal costs is assumed to be Pareto distributed with the shape parameter $k = 3.61$ (Feenstra 2018). As has been mentioned, trade is subject to iceberg-type trade cost, τ, and the fixed exporting ("market access") cost, F^{ex}. I set $\tau = 1.7$, which is consistent with the estimates in Anderson and van Wincoop (2004) and in line with Arkolakis, Costinot, Donaldson, and Rodriguez-Clare (2019). The fixed export cost is calibrated as $F^{ex} = 0.0392$ to match the average fraction of U.S. manufacturing firms that export (0.18, as reported by Bernard, Jensen, Redding, and Schott 2007). The rest of the parameter values of the baseline parameterization can be found in Table 1.

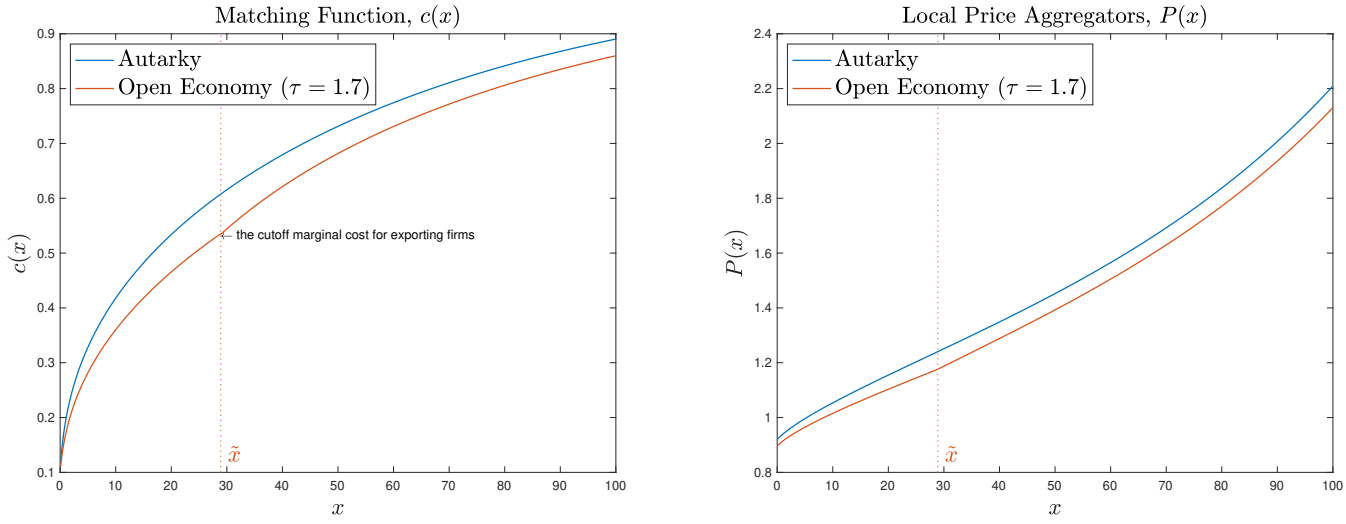
[20] See Appendix B.2 for detailed derivations with this specification.

Table 1: Parameterization

Functional form	Parameterization
$\Lambda\left(\frac{p(\omega_x)}{P(x)}\right) = \log\left(\frac{p(\omega_x)}{P(x)}\right)^{-\zeta}$	$\zeta = 0.19$ (Feenstra and Weinstein 2017)
$\ell(x) = \frac{\lambda \exp(-\lambda x)}{1-\exp(-\lambda x_{\max})} L$	$L = 1000$, $\lambda = 0.02$, $x_{\max} = 100$
$g(c) = \frac{k}{c_{\max}^k - c_{\min}^k} c^{k-1}$	$k = 3.61$ (Feenstra 2018), $c_{\min} = 0.1$, $c_{\max} = 1.0$
	$F^{ex} = 0.0392$ (18% of firms export, BJRS 2007)
	$F = 0.1$, $F^e = 0.1$, $\tau = 1.7$ (Anderson and van Wincoop 2004)

Open economy equilibrium under the benchmark parameterization results in the following equilibrium values: $c^* = 0.859$, $\tilde{c}^* = 0.534$, $M^e = 1,888$, and $\tilde{x} = 28.9$. The equilibrium *matching function*, $c(x)$, and schedule of the *(local) price aggregators*, $P(x)$, are depicted in Figure 2.

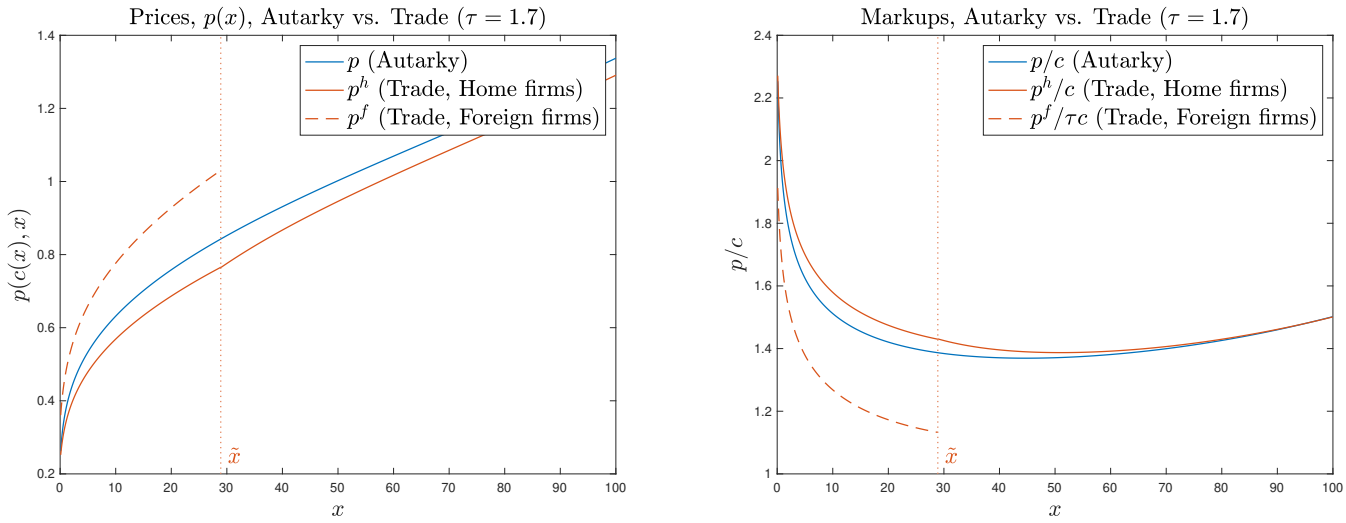
Figure 2: The matching function (left panel) and competition schedule (right panel).



One can see from Figure 2 that equilibrium patterns of $c(x)$ and $P(x)$ correspond to theoretical predictions of the model. Note also that, following trade liberalization, each niche is populated by more productive firms. The only exception is the largest niche, $x = 0$, that is served by firms with the lowest level of marginal cost, $c_{\min}$. Moreover, Figure 2 shows that each niche is now characterized by lower values of $P(x)$, implying higher local competition.

Equilibrium distributions of prices and markups are presented in Figure 3.

Figure 3: Firms' prices (left panel) and markups (right panel).

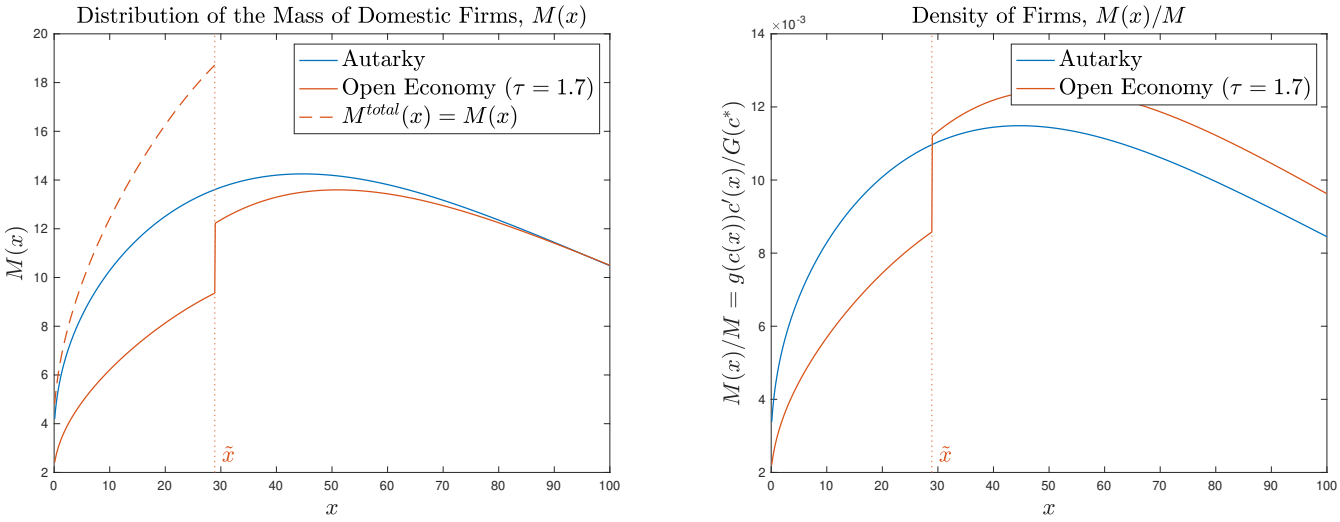


It is worth noting that, compared to the autarky case, prices for all domestic varieties have declined. The reason is that opening the closed economy to trade is associated with higher competition at each niche and relocation of firms toward less populated (see Figure 2), thereby lowering prices.

As for markups, at least three features of the corresponding patterns are worth emphasizing. First, as has been discussed earlier, markups are ∪–shaped across niches of different sizes (see also Proposition 3). Second, an exporting firm charges relatively higher markup for the domestic market, implying that $p^h/c > p^f/\tau c$. Recall that it is a direct corollary of Proposition 5. Third, after opening the closed economy to trade, niche-specific (domestic) markups have been increased for almost all niches. It might seem puzzling, taking into account predictions of standard (spaceless) trade models with variable markups under the MSLD assumption (see, e.g., Arkolakis, Costinot, Donaldson, and Rodriguez-Clare 2019). In particular, such models predict that an increase in competition lowers firm-level markups across the whole distribution. However, the present model implies that, following trade liberalization, all niches (except for the largest one) are populated by relatively more productive firms. It creates upward pressure on niche-specific markups. For most niches, this effect outweighs the (counteracting) impact of rising local competition captured by a decline in $P(\cdot)$.

Figure 4 shows distribution of the mass of domestic firms and density of domestic firms.

Figure 4: Distribution of the mass of domestic firms (left panel) and density of firms (right panel).



First, note that while the exogenous pdf of firm marginal cost $g(\cdot)$ is assumed to be non-decreasing as in most trade models, the distribution of the mass of (domestic) firms, $M(x) = M^e g\left(c(x)\right) c'(x)$, turns out to be non-monotonic in x. One can readily verify that $\frac{M'(x)}{M(x)} = \left[\mathcal{E}_g(\cdot)\frac{c'(x)}{c(x)} + \frac{c''(x)}{c'(x)}\right]$, where $\mathcal{E}_g(\cdot) \equiv cg'(\cdot)/g(\cdot)$. Thus, we have

$$M'(x) \gtreqless 0 \Leftrightarrow \mathcal{E}_g(\cdot) \gtreqless -\frac{c''(x)c(x)}{\left[c'(x)\right]^2}.$$

Recall from eq. (3.6) that $c'(x)$ is discontinuous at the export threshold niche, $\tilde{x} = 28.9$, implying that the mass of (domestic) firms, $M(x) = M^e g\left(c(x)\right) c'(x)$, is discontinuous at $\tilde{x}$ as well. So, there is an upward jump in the mass of domestic firms right after the export threshold niche. Note also that while the mass of domestic varieties decreased after opening the closed economy to trade, the total mass of varieties available at each trading niche increased. Figure 4 also shows that trade causes a shift in the density of (domestic) firms, $M(x)/M = g(c(x))c'(x)/G(c^*)$, towards less populated niches.[21]

3.2 Welfare Analysis

Recall that Proposition 4 states that consumer welfare is increasing in niche size or, equivalently, decreasing in x. In this Section, I study how welfare changes across consumers from different niches in response to opening the closed economy to trade, as well as the impact of small shocks to exogenous model parameters.

[21] One can readily verify that $\int_{\mathcal{X}} \frac{M(x)}{M}\mathrm{d}x = \int_{\mathcal{X}} \frac{g(c(x))c'(x)}{G(c^*)}\mathrm{d}x = 1$, so the areas under blue line and red lines from Figure 4 (right panel) are both equal to 1.

Small shocks in the closed economy Recall that nominal wage per unit of labor is normalized to one. Hence, change in niche-specific consumer welfare (real wage or income per unit of labor) can be written in the following way:

$$\mathrm{d}\log W(x) = -\mathrm{d}\log P^I(x),$$

In a closed economy equilibrium with PAM, the (niche-specific) ideal price index takes the following form: $\log P^I(x) = const + \log P(x) - M(x)\int_{\Upsilon(c(x)/P(x))}^{\bar{z}} \frac{\Lambda(\xi)}{\xi}\mathrm{d}\xi$, where $\bar{z}$ denotes a choke price, $\Upsilon(\cdot)$ is an increasing function implicitly defined from a firm's pricing formula (2.4) such that $p/P = \Upsilon(c/P)$.

The following proposition formally states that niche-specific change in consumer welfare associated with a small shock to exogenous model parameters can be decomposed into the *sorting* and *competition* effects.

Proposition 7. *Consider a closed economy described in Section 2. Then, the change in consumer welfare for a particular niche associated with a small shock to exogenous model parameters can be decomposed as:*

$$\frac{\mathrm{d}W(x)}{W(x)} = \underbrace{-(1-\vartheta(x))\frac{\mathrm{d}P(x)}{P(x)}}_{\text{competiton effect}(x)} \underbrace{-\vartheta(x)\frac{\mathrm{d}c(x)}{c(x)}}_{\text{sorting effect}(x)}, \tag{3.10}$$

where $\vartheta(x) \equiv \mathcal{E}_\Upsilon(\cdot) - M(x)\mathcal{E}_\Lambda(\cdot)\mathcal{E}_\Upsilon(\cdot)\int_{\Upsilon(c(x)/P(x))}^{\bar{z}} \frac{\Lambda(\xi)}{\xi}\mathrm{d}\xi$.

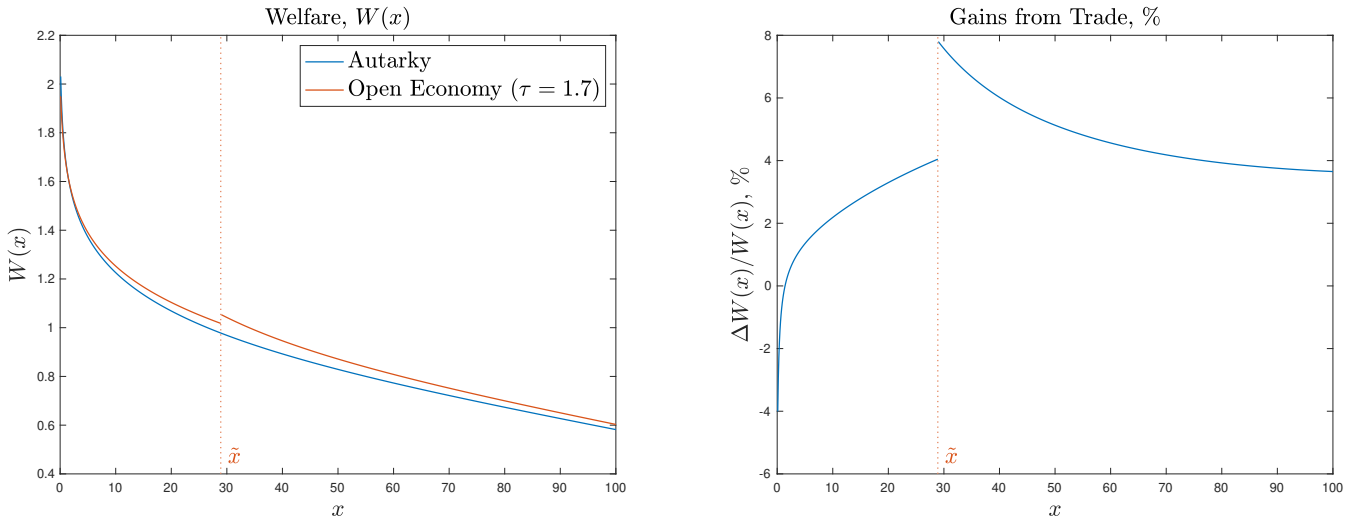
Proof. See Appendix A.7. □

Recall that the model's properties imply that the largest niche is always served by some positive mass of firms with the lowest (and exogenously given) value of marginal cost, $c(0) = c_{\min}$. Hence, the direct corollary of this observation and Proposition 7 is that the change in consumer welfare for the niche $x = 0$ associated with a small shock to exogenous model parameters does not contain "sorting" effect and can be computed as:

$$\frac{\mathrm{d}W(0)}{W(0)} = -(1-\vartheta(0))\frac{\mathrm{d}P(0)}{P(0)}.$$

Opening the closed economy to trade Now, I turn to the welfare analysis of an open economy. Figure 5 shows that the impact of opening the closed economy to (costly) trade is highly heterogeneous across consumers.

Figure 5: The impact of opening the closed economy to trade (autarky → costly trade, $\tau = 1.7$).



In particular, consumers in medium-size niches tend to gain relatively more than consumers in larger niches, which are directly exposed to trade, while consumers from some range of the largest niches actually lose from trade. These two findings are worth additional discussion.

As for the latter finding, consider some range of the largest niches $[0, \epsilon] \subset \mathcal{X}$, with $\epsilon \approx 0$, that bear losses from trade. Note that welfare in a particular niche depends positively on the mass of firms located at this niche and negatively on the prices of these firms: $W(x) \equiv W\left(\underset{-}{\mathbf{p}_x}, \underset{+}{M(x)}\right)$. Hence, for niches that lose from trade between two symmetric countries, we have,

$$W\left(p_x^h, p_x^f, M_x\right) < W\left(p_x^A, M_x^A\right),$$

where superscript A denotes the autarky.

Note that the mass of domestic varieties decreases, yet less than proportionally, so that the total mass of varieties is higher at those locations $M_x < M_x^A < 2 \cdot M_x = M_x^h + M_x^f$ (see Figure 4). In the meantime, while prices set by domestic firms decrease in the trade equilibrium, prices set by foreign firms are significantly higher due to high transportation costs, $p_x^h < p_x^A < p_x^f$. As a result, the negative impact on welfare from high prices set by foreign firms may outweigh the positive impact on welfare from an increase in the total number of varieties available and relatively lower prices set by domestic firms. Note that, in addition to the presence of the novel "sorting" effect that arises only in the case of heterogeneous niches, losses from trade are also driven by the market distortions generated by variable markups in standard monopolistic competition models.

A similar mechanism explains why consumers in medium-size niches tend to gain from trade relatively more than their counterparts in larger niches. Notice that, in a costly trade scenario, the welfare of consumers in the export threshold niche, $\tilde{x}$, is strictly less than consumer

welfare at neighbor niches that are not exposed to trade directly. For example, for a niche $x = \tilde{x} + \varepsilon$ we have:

$$W\left(p^h_{\tilde{x}}, p^f_{\tilde{x}}, M_{\tilde{x}}\right) < W\left(p_{\tilde{x}+\varepsilon}, M_{\tilde{x}+\varepsilon},\right)$$

For this comparison, recall that the mass of (domestic) firms, $M(x) = M^e g\left(c(x)\right) c'(x)$, is discontinuous at $\tilde{x}$, due to discontinuity jump of $c'(x)$ after this niche (see eq. (3.6)). So, there is an upward jump in the mass of domestic firms after the export threshold niche (see Figure 4). Yet, this jump is less than proportional so that the total mass of firms at $\tilde{x}$ is still higher than at a neighbor niche $\tilde{x} + \varepsilon$, which is not exposed to trade: $M_{\tilde{x}} < M_{\tilde{x}+\varepsilon} < 2 \cdot M_{\tilde{x}} = M^h_{\tilde{x}} + M^f_{\tilde{x}}$. In the meantime, while prices set by domestic firms at $\tilde{x}$ are slightly lower than those at $\tilde{x} + \varepsilon$, prices set by the positive mass of foreign firms (with the export cutoff level of marginal cost) at $\tilde{x}$ are much higher due to high transportation costs, $p^h_{\tilde{x}} < p_{\tilde{x}+\varepsilon} < p^f_{\tilde{x}}$. As a result, the negative impact on welfare from high prices set by foreign firms far outweighs the positive impact on welfare from an increase in the total number of varieties available and relatively lower prices of domestic varieties. It generates an upward jump in consumer welfare after the export threshold niche. Now, recall that consumer welfare in autarky is monotonically decreasing in niche size or, equivalently, increasing in x. So, it results in relatively higher welfare gains from trade for medium-size niches that are unexposed to trade in both equilibria.

To capture the broad picture of welfare gains patterns, Figure 6 shows the distributional impact of opening the closed economy to trade for different variable trade costs.

Figure 6: The impact of opening the closed economy to trade for different levels of τ.

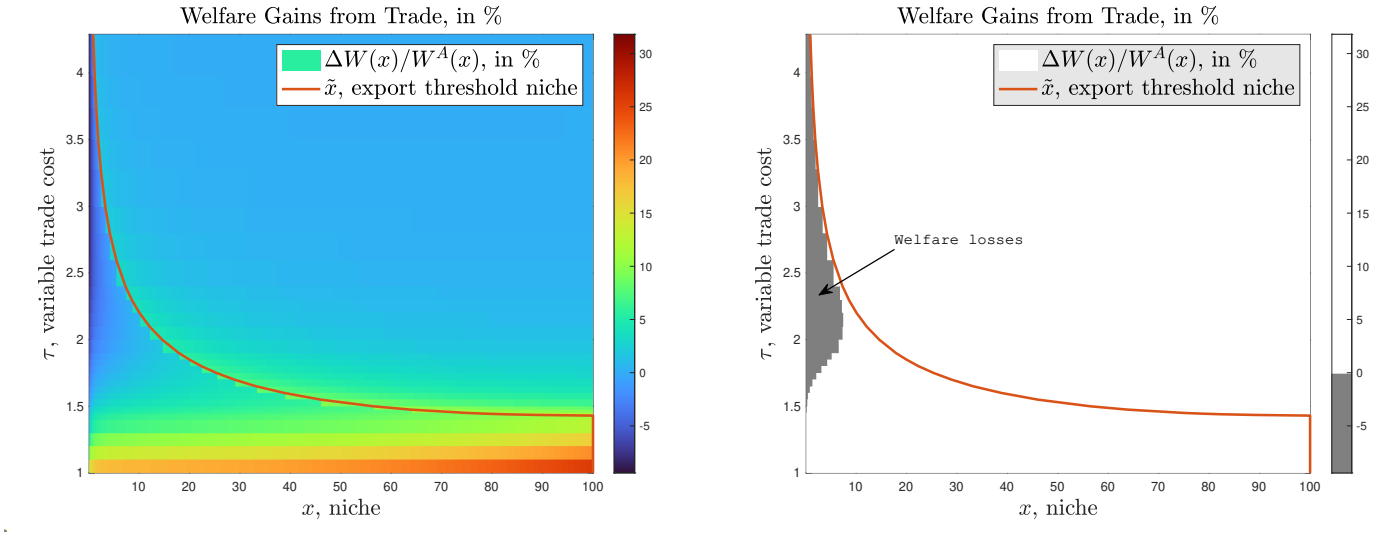


Figure 6 sheds light on several salient patterns. First, note that when variable trade costs are high enough, the range of niches that are directly exposed to trade, $[0, \tilde{x}]$, is very small, and the economy is reminiscent of the closed economy setup. As a result, the distribution of the gains from trade across niches is close to uniform and approaching 0% when τ is increasing.

Second, the largest niches that are directly exposed to trade tend to have negative welfare gains from opening the closed economy to trade. One can see from the right panel of Figure 6 that the range of niches that bear losses from trade is ∩–shaped in variable trade cost level τ. Moreover, middle-sized niches that are not directly exposed to trade and close to the export threshold niche tend to gain the most from opening the closed economy to trade. The mechanisms of these effects are the same as was previously discussed for the particular case of $\tau = 1.7$.

Third, note that when variable trade costs are low enough ($\tau = 1.43$ and lower), then $\tilde{x} = 100$, and the whole range of niches is directly exposed to trade, implying that they are served by both domestic and foreign firms. In this case, in addition to the relocation of relatively more productive firms, consumers in smaller niches now also gain from an increase in the total varieties available in these niches. As a result, while all consumers gain from free trade, gains are monotonically larger for smaller niches.

Small shocks to (variable) trade costs Now, consider the impact of a small change in variable trade costs from τ to $\tau + \mathrm{d}\tau$. The following proposition provides the decomposition of the niche-specific change in consumer welfare (real wage or income per unit of labor) associated with the corresponding small shock.

Proposition 8. *Consider an open economy described in Section 3. Then, the change in consumer welfare for a particular niche associated with a small shock in variable trade costs, τ, can be decomposed as:*

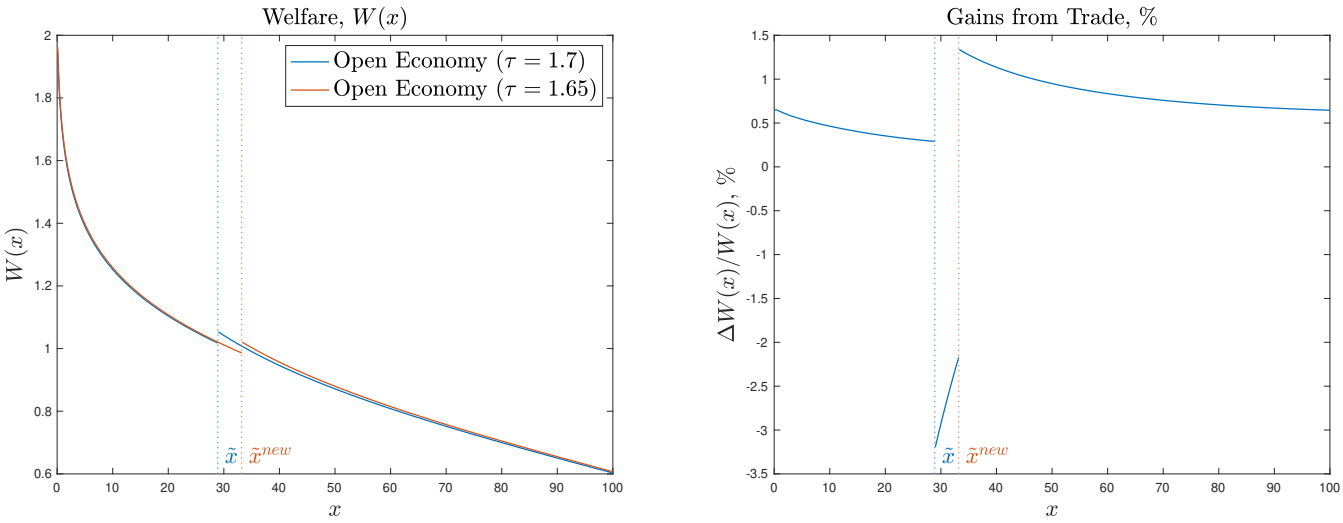
$$\frac{\mathrm{d}W(x)}{W(x)} = \underbrace{-\left(1-\phi(x)\right)\frac{\mathrm{d}P(x)}{P(x)}}_{\text{competiton effect}(x)} \underbrace{-\phi(x)\frac{\mathrm{d}c(x)}{c(x)}}_{\text{sorting effect}(x)} \underbrace{-\left[1-\Xi(x)M(x)\mathcal{E}_{\Lambda^f}\right]\frac{N\Lambda^f\mathcal{E}_{\Upsilon^f}}{\Lambda^h+N\Lambda^f}\mathbb{I}_{\{x\leq\tilde{x}\}}\frac{\mathrm{d}\tau}{\tau}}_{\text{direct effect of } \mathrm{d}\tau}, \tag{3.11}$$

where $\phi(x) \equiv \frac{\Lambda^h\mathcal{E}_{\Upsilon^h}+N\Lambda^f\mathcal{E}_{\Upsilon^f}\mathbb{I}_{\{x\leq\tilde{x}\}}}{\Lambda^h+N\Lambda^f\mathbb{I}_{\{x\leq\tilde{x}\}}} - \Xi(x)M(x)\frac{\Lambda^h\mathcal{E}_{\Lambda^h}\mathcal{E}_{\Upsilon^h}+N\Lambda^f\mathcal{E}_{\Lambda^f}\mathcal{E}_{\Upsilon^f}\mathbb{I}_{\{x\leq\tilde{x}\}}}{\Lambda^h+N\Lambda^f\mathbb{I}_{\{x\leq\tilde{x}\}}}$, $\Xi(x) \equiv \int_{\Upsilon\left(\frac{c(x)}{P(x)}\right)}^{\bar{z}}\frac{\Lambda(\xi)}{\xi}\mathrm{d}\xi + \mathbb{I}_{\{x\leq\tilde{x}\}}\cdot N\int_{\Upsilon\left(\frac{\tau c(x)}{P(x)}\right)}^{\bar{z}}\frac{\Lambda(\xi)}{\xi}\mathrm{d}\xi$.

Proof. See Appendix A.8. □

Figure 7 shows the impact of lowering variable trade costs from $\tau = 1.7$ to $\tau = 1.65$.

Figure 7: The impact of small changes in trade costs ($\tau = 1.7 \to \tau = 1.65$).



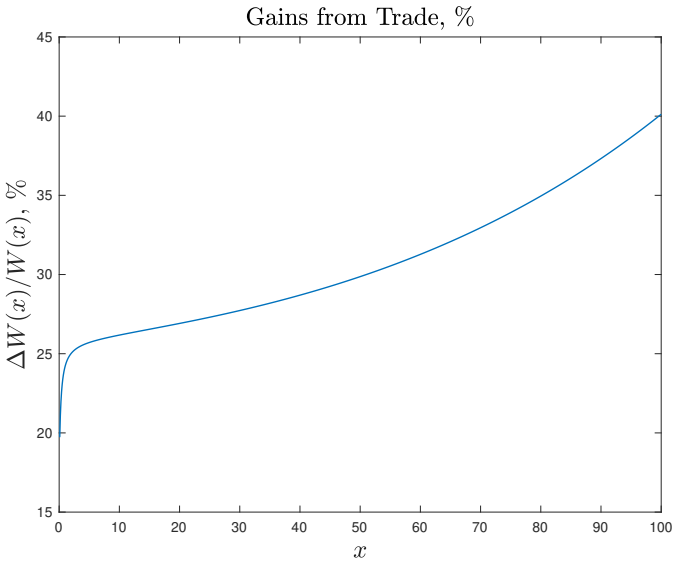
Notice that the impact of small changes in trade costs is also highly uneven and provides rich welfare implications. In particular, while most niches tend to gain from trade, niches previously unexposed to trade $(\tilde{x}, \tilde{x}^{new}]$, are those that lose from modest trade liberalization. To explain this pattern, first note that, as in the case of opening from autarky to trade, a small positive shock to variable trade costs also leads to a higher measure of competition for all niches (i.e., $P(x)$ is lower at each point in the new equilibrium) and relocation of firms to relatively less populated niches. Hence, prices of domestic varieties decrease for all niches. Moreover, due to the presence of foreign firms, the total mass of varieties available at niches previously unexposed to trade $(\tilde{x}, \tilde{x}^{new}]$, will increase in a new equilibrium. However, consumers in these niches still bear losses from a positive shock to variable costs. The reason is that, despite an increase in the total amount of varieties in these niches, the amount of domestic varieties has been significantly decreased. In the meantime, high variable trade costs imply that prices of foreign varieties are higher than those of domestic ones. As a result, the negative impact of high prices set by foreign firms far outweighs the positive impact of an increase in total number of varieties available and relatively lower prices of domestic varieties. So, the mechanism of losses from trade is similar to that discussed for the scenario of opening the closed economy to (costly) trade.

One can see from Figure 7 that consumers in all niches except $(\tilde{x}, \tilde{x}^{new}]$ gain from trade, despite a slight decrease in the total amount of varieties available at these niches. As for consumers from the range of niches exposed to trade in both equilibria, $[0, \tilde{x}]$, they gain from a decrease in prices for both domestic and foreign varieties. As for consumers from the range of niches unexposed to trade in both equilibria, $(\tilde{x}^{new}, x_{\max}]$, they gain from a decrease in prices of domestic varieties. Going back to the decomposition of welfare change provided in Proposition 8, it is worth emphasizing that the contribution of the sorting effect to the total gains is

sizable for most niches, especially the middle-sized and small ones. In particular, while the sorting effect contributes from 0% (for $x = 0$) to 59% of the total welfare gains at niches $[0, \tilde{x}]$, its contribution at niches $(\tilde{x}^{new}, x_{\max}]$ ranges from 30% to 49%.

Opening the closed economy to free trade Figure 8 provides the distribution of the gains from trade from opening the closed economy to the *costless* (*free*) trade scenario, where $\tau = 1$ and $F^{ex} = 0$.

Figure 8: Gains from free trade (autarky $\rightarrow$ free trade with $F^{ex} = 0, \tau = 1$).



Note that patterns of welfare gains in this scenario differ from the case of costly trade discussed earlier. In particular, gains from opening the closed economy to free trade are positive and monotonically decreasing in niche size or, equivalently, increasing in x. Note that all niches are now directly exposed to trade, implying that they are served by both domestic and foreign firms. Hence, consumers in small niches, in addition to the relocation of relatively more productive firms, now also gain from a significant increase in total varieties available in these niches. As a result, while all consumers gain from free trade, gains are monotonically decreasing in niche size. Note that this welfare gains pattern is similar to what we observe in Figure 6 for values of variable trade costs close to 1, which also implies that the whole range of niches is directly exposed to trade.

4 A Spaceless Model as a Limiting Case

One can readily verify that if the space of consumer niches, $\mathcal{X} \equiv [0, x_{\max}]$, becomes arbitrarily small (i.e., $x_{\max} \rightarrow 0$), whereas keeping the total amount of (efficiency units of) labor,

$L \equiv E \int_{\mathcal{X}} \ell(x)\mathrm{d}x$, fixed, then the economy converges to the "spaceless" monopolistic competition trade model alike Melitz (2003), yet with variable markups. In this Section, I provide both analytical and numerical comparisons between the baseline "spatial" model and the "spaceless" model as a limiting case with degenerated distribution of niche sizes.

4.1 Spatial vs. Spaceless Model. Closed Economy

Notice that in spaceless version of the baseline model an individual's demand for a variety $\omega \in \Omega$ takes the following form:

$$q(\omega) = \frac{E}{p(\omega)} \Lambda \left(\frac{p(\omega)}{P} \right), \tag{4.1}$$

where the price aggregator P is linearly homogeneous in $\mathbf{p}$ and is implicitly defined by

$$\int_{\omega \in \Omega} \Lambda \left(\frac{p(\omega)}{P} \right) \mathrm{d}\omega = 1. \tag{4.2}$$

One can notice that in this case, the (local) price aggregator, $P(x)$, is replaced with a single (common) price aggregator, P. It implies that all firms explicitly compete with each other within a single niche/market.

Hence, the corresponding profit function now takes the following form:

$$\Pi(c) \equiv \max_{p} \left\{ \frac{p-c}{p} \Lambda \left(\frac{p}{P} \right) LE - F \right\}.$$

One can readily verify that maximization of a firm's profit w.r.t. price implies,

$$\frac{p}{c} = \frac{\mathcal{E}_{\Lambda}\left(p/P\right) + 1}{\mathcal{E}_{\Lambda}\left(p/P\right)} \Leftrightarrow \frac{p}{c} = \frac{\mathcal{E}_{D}\left(p/P\right)}{\mathcal{E}_{D}\left(p/P\right) - 1}. \tag{4.3}$$

The Melitz-type zero cutoff profit (ZCP) condition yields,

$$\Pi(c^*) = 0. \tag{4.4}$$

Free entry (FE) condition now takes the following form:

$$\int_{c_{\min}}^{c^*} \Pi(c)\mathrm{d}G(c) = F^e. \tag{4.5}$$

Conditions (4.4) and (4.5) jointly determine the equilibrium cutoff marginal cost, c^*, and the price aggregator, P.

Note also that the corresponding condition that defines the (common) price aggregator (4.6)

can be re-written in the following way:

$$M^e \int_{c_{\min}}^{c^*} \Lambda\left(\frac{p(c)}{P}\right) \mathrm{d}G(c) = 1, \tag{4.6}$$

and the mass of entrants, M^e, is related with the mass of active firms, M, as $M^e \cdot G(c^*) = M$.

Definition. *An equilibrium in the "spaceless" version of the closed economy model under HSA demand systems (4.1) is the set of endogenous variables* $\left\{\{p(c)\}_{c\in[c_{\min},c^*]}, c^*, P, M^e\right\}$ *such that conditions (4.3)-(4.6) hold.*

The following proposition summarizes the behavior of the main variables of interest.

Proposition 9. *Consider a "spaceless" version of the model described above. There exists a unique equilibrium such that:*[22]

i) *a firm's price, p, is increasing in c;*

ii) *a firm's "market share", $\Lambda(\cdot) = pq/E$, is decreasing in c;*

iii) *a firm's production per capita, q, and total quantities sold, qL, are decreasing in c;*

iv) *a firm's total revenue, $p\,q\,L$, is decreasing in c;*

v) *a firm's total profits, $\Pi(\cdot)$, are decreasing in c;*

vi) *a firm's total costs, $c\,q\,L + F$, are increasing (decreasing) in c iff $\mathcal{E}_{\tilde{\Lambda}'}(\cdot) > (<) 1$;*

If, in addition, MSLD holds, then

vii) *a firm's markup, p/c, is decreasing in c;*

Proof. See Appendix A.9. □

Table 2 below provides a comparison of the equilibrium behavior of some main variables in a closed economy setup between "Spaceless" and "Spatial" models.

Table 2: Spatial vs. Spaceless model (the limiting case). Equilibrium outcomes

	Spatial (under MSLD)	Spaceless (under MSLD)[23]
p	increasing in c	increasing in c
p/c	$\cup-$shaped in c	decreasing in c
$\Lambda(\cdot)$	$\cup-$shaped in c	decreasing in c
q	$\cup-$shaped in c	decreasing in c
$\Pi(\cdot)$	decreasing in c	decreasing in c

[22] I also impose a mild additional assumption on a firm's market share function (see Appendix A for details).
[23] Note that MSLD is a sufficient, but not necessary, condition for most of these patterns to take place.

Table 2 points out several important differences between "spaceless" and "spatial" setups. First of all, notice that more productive firms tend to charge higher markups under MSLD in the "spaceless" model. In contrast, it has been proven that in the "spatial" setup with self-selection of firms to different market segments/niches/locations, firms' markups demonstrate $\cup$–shaped patterns. Similarly, more productive firms have higher "market shares", $\Lambda(\cdot)$, in the "spaceless" setup. The reason is that given the common measure of competition, more productive firms set lower prices. In contrast, as stated by Proposition 3, a firm's "market share" is $\cup$–shaped in x (as well as in marginal cost, c) under the baseline spatial setup.

Table 3 presents a quantitative comparison of the closed economy equilibrium outcomes between the baseline ("spatial") model (see Section 2) and its limiting ("spaceless") case under the baseline parameterization.

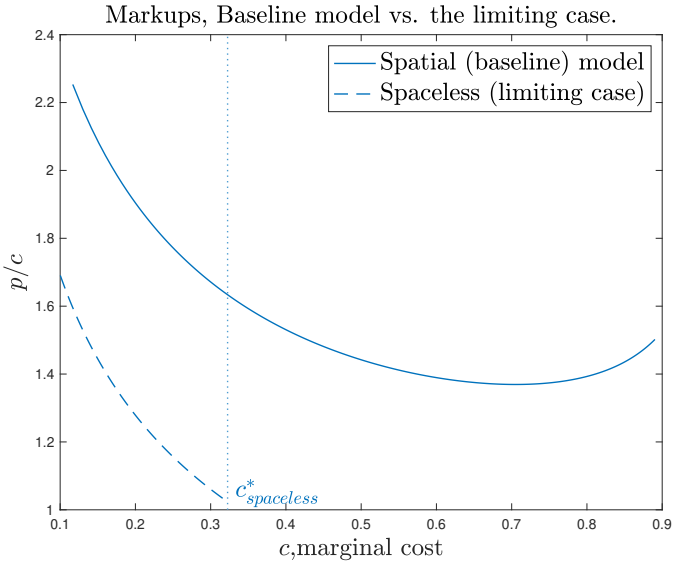
Table 3: Baseline model vs. the limiting case. Closed Economy

	Baseline model	Limiting case
c^*	0.89	0.32
M^e	$1,887$	$1,959$
M	$1,241$	32.96
P	$[0.92,\ 2.21]$	0.34
W	1.02 $[0.58, 2.03]$	3.54

There are several distinctions in outcomes between these two setups that are worth to be discussed. First of all, one can notice that the spaceless model implies a lower cutoff level of marginal cost c^* or, equivalently, tougher selection. This tougher selection results in a lower total mass of active firms, M, despite a relatively larger mass of entrants, M^e. Moreover, it also results in lower absolute values of the price aggregator, P, and a higher welfare measure than those observed for every niche in the baseline model. This result might seem puzzling at first sight. However, recall that in the limiting version of the model all mass of consumers is centered in one "point" rather than unevenly spread out throughout the space of niches. As a result, all firms have to explicitly compete with each other within the same market niche, eventually resulting in a higher measure of competition and consumer welfare.

In addition to the U-shaped equilibrium relation between markups and marginal costs, Figure 9 shows that firms tend to charge higher markups than their counterparts under the limiting (spaceless) case of the model. The feature that markups are larger in the presence of consumer heterogeneity is also in line with the findings of Macedoni, Xu, and Feenstra (2020), albeit for oligopolistic firms with different demand structure and setup.

Figure 9: Markups: the baseline model vs. the limiting case.



4.2 Spatial vs. Spaceless model. Comparative Statics

To discuss and compare comparative statics implications, it is helpful to use an alternative yet equivalent representation of single-aggregator homothetic demand systems that exploit an inverse demand function and quantity aggregator. Below, I provide a brief discussion of the corresponding setup.

In particular, the inverse demand function is represented as follows:[24]

$$p(\omega_x) = \tilde{\Lambda}\left(\frac{q(\omega_x)}{Q(x)}\right)\frac{E}{q(\omega_x)},$$

where $Q(x) \equiv Q(\mathbf{q}_x)$ denotes *the (local) quantity aggregator* for varieties of type $x \in \mathcal{X}$, while $\tilde{\Lambda} : \mathbb{R}_+ \to \mathbb{R}_+$ represents *the market share function* that basically satisfies the following set of assumptions: **i)** $\tilde{\Lambda}(\cdot)$ is a C^3 function on $\mathbb{R}_{++}$; **ii)** $\mathcal{E}_{\tilde{\Lambda}}(z) \equiv z\tilde{\Lambda}'(z)/\tilde{\Lambda}(z) \in (0,1) \;\; \forall\, z \in \mathbb{R}_{++}$; **iii)** $\tilde{\Lambda}(0) = 0$ and $\tilde{\Lambda}(\infty) = \infty$; **iv)** $\tilde{\Lambda}''(z) < 0\, \forall\, z \in \mathbb{R}_+$.[25]

For each location $x \in \mathcal{X}$, the (local) quantity aggregator $Q(x)$ is linearly homogeneous in $\mathbf{q}_x$ and implicitly defined by

$$\int\limits_{\omega_x \in \Omega_x} \tilde{\Lambda}\left(\frac{q(\omega_x)}{Q(x)}\right) \mathrm{d}\omega_x = 1. \tag{4.7}$$

[24]For expositional purposes, I discuss an alternative representation of the HSA demand systems only for the spatial version of the model. It is straightforward to provide a similar analysis for the spaceless version.

[25]Note that it is related to the previously introduced market share function as follows: $\tilde{\Lambda}\left(q/Q\right) = \Lambda\left(\frac{\tilde{\Lambda}(q/Q)}{q/Q}\right)$. Note also that MSLD ensures $\tilde{\Lambda}''(z) < 0$.

Note that a counterpart of a firm's profit function, $\Pi(c,x)$, introduced earlier, now takes the following form:

$$\Pi(c,x) \equiv \max_{q} \left\{ \left(\tilde{\Lambda}\left(\frac{q}{Q(x)} \right) \frac{E}{q} - c \right) q\, \ell(x) - F \right\}$$

One can readily verify that maximization of a firm's profit w.r.t. q implies:

$$\tilde{\Lambda}'\left(\frac{q}{Q(x)} \right) \frac{E}{Q(x)} = c. \tag{4.8}$$

Hence, a firm's relative quantity, $q/Q(x)$, can be expressed as $q/Q(x) = \tilde{\Lambda}'^{-1}\left(c\,Q(x)/E\right)$. Notice also that a firm's markup is now given by $p/c = \tilde{\Lambda}\left(q/Q(x)\right)\frac{E}{cq} = 1/\mathcal{E}_{\tilde{\Lambda}}\left(q/Q(x)\right)$.

One can also readily verify that combining (4.8) with a firm's FOC w.r.t. x yields[26]

$$\frac{Q'(x)}{Q(x)} = \frac{\ell'(x)}{\ell(x)} \frac{1 - \mathcal{E}_{\tilde{\Lambda}}\left(q(c,x)/Q(x)\right)}{\mathcal{E}_{\tilde{\Lambda}}\left(q(c,x)/Q(x)\right)}. \tag{4.9}$$

Recall that $\mathcal{E}_{\tilde{\Lambda}}(z) \in (0,1) \;\; \forall\, z \in \mathbb{R}_{++}$ and $\ell'(x) < 0$, by assumption. Hence, one can infer that $Q(\cdot)$ is strictly decreasing. Notice also that $\mathcal{E}_p(q) \equiv -qp'(q)/p(q) = 1 - \mathcal{E}_{\tilde{\Lambda}}\left(q/Q(x)\right)$. Hence, MSLD (i.e. $\mathcal{E}_p'(q) > 0$), is equivalent to assuming that $\mathcal{E}_{\tilde{\Lambda}}'(q) < 0$. One can readily verify that under this alternative representation of HSA demand systems, MSLD is equivalent to:[27]

$$\mathcal{E}_{\tilde{\Lambda}}\left(q/Q(x)\right) + \mathcal{E}_{\tilde{\Lambda}'}\left(q/Q(x)\right) > 1. \tag{4.10}$$

Now, let me formulate some propositions of comparative statics for a "spaceless" version of the model described above.[28]

Proposition 10. *Consider a "spaceless" version of the model. An increase in total market size leads to:*

i) *a higher measure of competition in a new equilibrium.*

If, in addition, MSLD holds, then an increase in total market size leads to:

ii) *a lower equilibrium cutoff marginal cost, c^*, regardless of the distribution of firm productivities.*

Proof. See Appendix A.10. □

Recall that the Translog specification of HSA demand systems satisfies MSLD. Hence, an increase in market size leads to tougher selection of firms. It can also be easily verified that c^* is

[26] Notice also that, in equilibrium, (4.9) and $c'(x) = 1/\left[M^e g(c(x))\tilde{\Lambda}\left(q(c(x),x)/Q(x)\right)\right]$ form a pair of DEs that is reminiscent of (2.9). Detailed intermediate derivations are available upon request.

[27] The correctness of this condition can be double-checked by exploiting the Implicit Function Theorem (as in the proof of Proposition 2) and verifying that $\mathrm{d}x/\mathrm{d}c > 0 \Leftrightarrow \mathcal{E}_{\tilde{\Lambda}}\left(q/Q(x)\right) + \mathcal{E}_{\tilde{\Lambda}'}\left(q/Q(x)\right) > 1$. Detailed intermediate derivations are available upon request. As also noted in Mayer, Melitz, and Ottaviano (2016), MSLD holds iff the pass-through elasticity, $\mathcal{E}_p(\cdot)/\mathcal{E}_\phi(\cdot) = \left[1 - \mathcal{E}_{\tilde{\Lambda}}(\cdot)\right]/\mathcal{E}_{\tilde{\Lambda}'}(\cdot)$ is less than 1.

[28] More details and results on the comparative statics are available upon request.

independent of the market size only under the CES specification.[29] The following proposition characterizes the comparative statics behavior of other variables of interest.

Proposition 11. *Consider a "spaceless" version of the model described above. An increase in the total market size leads to:*

i) *lower "market shares", $\Lambda(\cdot) = pq/E$, for all firms among those survived.*

If, in addition, MSLD holds, then an increase in the total market size leads to:

ii) *lower prices, p, and markups, p/c, for all firms among those that survived;*

iii) *higher mass of entrants, M^e;*

iv) *exit of "high–c" firms; higher $\Pi(\cdot)$ for "low–c" firms; lower $\Pi(\cdot)$ for "middle–c" firms.*

Proof. See Appendix A.11. □

Table 4 provides a comparison of comparative statics for the main variables in a closed economy setup between "Spaceless" and "Spatial" models.

Table 4: Spatial vs. Spaceless model (the limiting case). Comparative statics (an increase in total market size, L, or proportional increase in size of all niches, $\ell(x)$).

	Spatial (under MSLD)[30]	Spaceless (under MSLD)[31]
c^*	$\searrow$	$\searrow$
P	$\searrow$ for low–c; $\nearrow$ for high–c $(*)$ ($\searrow$for all locations/niches, x)	$\searrow$
M^e	$\nearrow$	$\nearrow$
$\Lambda(\cdot)$	$\searrow$ for low–c; $\nearrow$ for high–c $(*)$	$\searrow$ for all firms survived
p	$\searrow$ for low–c; $\nearrow$ for high–c $(*)$	$\searrow$ for all firms survived
p/c	$\searrow$ for low–c; $\nearrow$ for high–c $(*)$	$\searrow$ for all firms survived
$\Pi(\cdot)$	$\nearrow$ for low–c; $\searrow$ for high–c	$\nearrow$ for low–c; $\searrow$ for high–c

Notice that, in addition to MSLD, Mayer, Melitz, and Ottaviano (2016) also introduce the so-called MSLD′ condition to explain some reallocation patterns in the context of multi-product firms and additive preferences. As will be shown below, this condition remains relevant under HSA demand systems and single-product firms. Mayer, Melitz, and Ottaviano (2016) define MSLD′ condition as the requirement that the elasticity of marginal revenue is increasing. Following their notation, let $\phi(q)$ denotes marginal revenue. One can readily verify that in the

[29] Details are available upon request. See also Zhelobodko, Kokovin, Parenti, and Thisse (2012) for the case of directly-additive preferences and heterogeneous firms.

[30] In this Table, $(*)$ indicates that a corresponding pattern for high–c firms holds only for not very large increase in market size. Specifically, if a (proportional) increase in market size is large enough, then high–c firms experience i) a decline in P (i.e. an increase in competition) for their niche in a new equilibrium; ii) a decline in their "market share", $\Lambda(\cdot)$; iii) a decline in their prices, p; iv) a decline in markups, p/c.

[31] Note that MSLD is sufficient, but not necessary condition for some comparative statics results. For example, as noted by Proposition 10 an increase in market size implies a higher measure of competition, regardless of whether MSLD holds or not. Additional details are available upon request.

context of HSA preferences, the elasticity of marginal revenue takes the following form:

$$\mathcal{E}_{\phi}(q) \equiv -\frac{\phi'(q)q}{\phi(q)} = \frac{-\tilde{\Lambda}''\left(q/Q\right)}{\tilde{\Lambda}'\left(q/Q\right)}\frac{q}{Q} \equiv \mathcal{E}_{\tilde{\Lambda}'}\left(q/Q\right).$$

One can readily verify that increasing elasticity of marginal revenue is equivalent to the following condition:

$$\mathcal{E}'_{\phi}(q) > 0 \Leftrightarrow \left[\frac{\tilde{\Lambda}'''\left(q/Q\right)}{-\tilde{\Lambda}''\left(q/Q\right)}\frac{q}{Q} - 1\right]\frac{\tilde{\Lambda}'\left(q/Q\right)}{-\tilde{\Lambda}''\left(q/Q\right)q/Q} < 1 \Leftrightarrow \frac{\tilde{\Lambda}'''\left(q/Q\right)}{-\tilde{\Lambda}''\left(q/Q\right)}\frac{q}{Q} - \mathcal{E}_{\tilde{\Lambda}'}\left(q/Q\right) < 1. \quad (4.11)$$

Given $\tilde{\Lambda}''(\cdot) < 0$, it is straightforward to verify that $\tilde{\Lambda}'''(\cdot) < 0$ and $\frac{\tilde{\Lambda}'''(q/Q)}{-\tilde{\Lambda}''(q/Q)}\frac{q}{Q} < 1$ are sufficient conditions for $\mathcal{E}'_{\phi}(q) > 0$. As shown by Mayer, Melitz, and Ottaviano (2016), under some mild assumptions, MSLD′ implies MSLD. They also show that if only MSLD holds, then the log marginal revenue curve would need not be globally concave, yet it would still have to be everywhere steeper than the log demand curve. The relevance of MSLD′ for reallocation patterns, even in a single-sector setup, can be demonstrated by the following proposition:[32]

Proposition 12. *MSLD′ is a sufficient condition for an increase in total market size to reallocate firms' "market shares" to more productive firms:* $\frac{\tilde{\Lambda}\left(\tilde{\Lambda}'^{-1}(c_1 Q)\right)}{\tilde{\Lambda}\left(\tilde{\Lambda}'^{-1}(c_2 Q)\right)}$ *increases whenever* $c_2 > c_1$. *The necessary condition is that* $\frac{\mathcal{E}_{\tilde{\Lambda}}(q)}{\mathcal{E}_{\tilde{\Lambda}'}(q)}$ *is decreasing.*[33]

Proof. See Appendix A.12. □

Notice that a number of results from the theoretical literature on distributional comparative statics can be readily applied to make the corresponding predictions. In particular, let $q \in \mathscr{Q} \subseteq \mathbb{R}$ and $c \in \mathscr{C} \subseteq \mathbb{R}$. Then, following Lemma 3 from Jensen (2018), a thrice differentiable "operational profit" function $\pi : \mathscr{Q}\text{x}\mathscr{C} \to \mathbb{R}$ exhibits quasi-concave differences if and only if:

$$2\pi''_{qq}(q,c)\pi''_{qc}(q,c)\pi'''_{qqc}(q,c) \geq \left[\pi''_{qq}(q,c)\right]^2\pi'''_{qcc}(q,c) + \left[\pi''_{qc}(q,c)\right]^2\pi'''_{qqq}(q,c). \quad (4.12)$$

As also noted by Jensen (2018), in the context of Melitz's model, the inequality above holds iff $R'''(q) \leq 0$ (that implies that the marginal revenue function is concave) and the inequality is reversed iff $R'''(q) \geq 0$ (that implies that the marginal revenue function is convex).

[32] This proposition provides just an example to demonstrate some resemblance with the findings of Mayer, Melitz, and Ottaviano (2016) for additive preferences and in the context of multi-product firms. More results on the role of MSLD′ and the condition $\frac{\mathrm{d}}{\mathrm{d}z}\left[\mathcal{E}_{\tilde{\Lambda}}(z)/\mathcal{E}_{\tilde{\Lambda}'}(z)\right] < 0$ are available upon request. Note also that conditions MSLD′ and decreasing $\mathcal{E}_{\tilde{\Lambda}}(\cdot)/\mathcal{E}_{\tilde{\Lambda}'}(\cdot)$ are also relevant for reallocation patterns in the baseline spatial model described in Section 2.

[33] Recall that $\mathcal{E}_{\tilde{\Lambda}'}(q)$ represents elasticity of marginal revenue. As for $\mathcal{E}_{\tilde{\Lambda}}(q)$, it can be easily verified that it represents the elasticity of revenue. In the terminology of Mayer, Melitz, and Ottaviano (2016), this condition is also equivalent to the requirement that $\left[1 - \mathcal{E}_p(q)\right]/\mathcal{E}_{\phi}(q)$ is decreasing.

One can readily verify that under HSA demand systems $\pi(q,c) = \tilde{\Lambda}\left(q/Q\right) E\, L - cq\, L$ with $R(q) \equiv \tilde{\Lambda}\left(q/Q\right) E\, L$. Hence, it is straightforward to show that $R'''(q) \leq 0 \Leftrightarrow \tilde{\Lambda}'''\left(q/Q\right) \leq 0$.

As a result, applying results from Jensen (2018), it can be concluded that if $\tilde{\Lambda}'''\left(q/Q\right) \leq 0$, then: *i*) "aggregate output decreases when firms become more diverse"; *ii*) "the distribution of the outputs of a more diverse set of firms will be second-order stochastically dominated by the distribution of outputs of a less diverse set of firms". Note that corresponding results are reversed under the "increasing returns to diversity" case, $\tilde{\Lambda}'''\left(q/Q\right) \geq 0$.[34]

4.3 Spatial vs. Spaceless Model. Open Economy

In this Subsection, I provide a similar analysis to that in Subsection 4.1 yet between open economy setups. To maintain relevance with Section 3, I abstract from endogenous relative wage movements by assuming that countries are symmetric. Hence, all countries have the same wage that is normalized to one.

Notice that the corresponding profit functions for the "home" market and a "foreign" market now take the following form:

$$\Pi^h(c) \equiv \max_{p^h}\left\{\frac{p^h - c}{p^h}\Lambda\left(\frac{p^h}{P}\right) E\, L - F\right\}, \quad \Pi^f(c) \equiv \max_{p^f}\left\{\frac{p^f - \tau c}{p^f}\Lambda\left(\frac{p^f}{P}\right) E\, L - F^{ex}\right\}.$$

Definition. *An equilibrium in the "spaceless" version of the open economy model with $N+1$ symmetric countries is the set of endogenous variables $\left\{\left\{p^h(c)\right\}_{c\in[c_{\min},c^*]}, \left\{p^f(c)\right\}_{c\in[c_{\min},\tilde{c}^*]}, c^*, \tilde{c}^*, P, M^e\right\}$ such that conditions (4.13)-(4.16) hold.*

$$\frac{p^h}{c} = \frac{\mathcal{E}_\Lambda\left(p^h/P\right)+1}{\mathcal{E}_\Lambda\left(p^h/P\right)}, \qquad \frac{p^f}{\tau c} = \frac{\mathcal{E}_\Lambda\left(p^f/P\right)+1}{\mathcal{E}_\Lambda\left(p^f/P\right)}. \tag{4.13}$$

$$\Pi^h(c^*) = 0, \quad \Pi^f(\tilde{c}^*) = 0. \tag{4.14}$$

$$\int_{c_{\min}}^{c^*} \Pi^h(c)\mathrm{d}G(c) + N\int_{c_{\min}}^{\tilde{c}^*} \Pi^f(c)\mathrm{d}G(c) = F^e. \tag{4.15}$$

$$M^e\left[\int_{c_{\min}}^{c^*} \Lambda\left(\frac{p^h(c)}{P}\right)\mathrm{d}G(c) + N\int_{c_{\min}}^{\tilde{c}^*} \Lambda\left(\frac{p^f(c)}{P}\right)\mathrm{d}G(c)\right] = 1. \tag{4.16}$$

Table 5 presents a quantitative comparison of (open economy) equilibrium outcomes of the baseline (spatial) model (see Section 3) vs. the limiting case (spaceless) model under the same

[34] In further versions of the draft, I will transfer the rest of my results on comparative statics for spatial vs. spaceless setups. I will also transfer the rest of my results on the role of the assumption of increasing elasticity of marginal revenue (MSLD') and decreasing $\mathcal{E}_{\tilde{\Lambda}}(\cdot)/\mathcal{E}_{\tilde{\Lambda}'}(\cdot)$.

parameterization as in Table 1.

Table 5: The baseline model vs. the limiting case. Open Economy ($\tau = 1.7$)

	Baseline model	Limiting case
c^*	0.86	0.32
$\tilde{c}^*$	0.53	0.19
M^e	$1,888$	$1,894$
P	$[0.90,\ 2.13]$	0.33
W	$[0.60,\ 1.95]$	3.79
$\frac{W-W^A}{W^A}$, %	3.70% $[-4.01\%, 7.80\%]$	7.09%

Table 5 demonstrates that, in an open economy, both the baseline model and its limiting case predict similar amounts of entrant firms. The reason is that in the limiting case of the model all firms directly compete with each other within the same market niche, thereby resulting in tougher selection. Note also that while the baseline model predicts relatively lower average welfare change from opening the closed economy to trade, this model allows studying distributional effects across different niches. In particular, it reveals that the impact of trade or a change in trade policy is highly heterogeneous across different consumers, ranging from -4.01% to 7.80%.[35]

[35]To be completed.

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Appendix

A Proofs

In Appendix A, I provide proofs of numerous propositions from the paper.

A.1 The Proof of Proposition 1

Proof. Recall that $\ell'(x) < 0$ for $\forall\ x \in \mathcal{X}$ by assumption. If $\Omega_x \neq \emptyset$, then equation (2.4) implies that $\mathcal{E}_D\left(p(c,x)/P(x)\right) > 1$. Hence, it directly follows from (2.5) that $P'(x) > 0$.[36] Note also that if $x' > x''$ for any $x', x'' \in \mathcal{X}$, then it is not possible to have $P(x') \leq P(x'')$ in equilibrium. Recall that a firm perceives the local price index $P(\cdot)$ as (the inverse of) a measure of local competition. Recall also that $\ell(\cdot)$ is decreasing by assumption. Hence, it is straightforward to see from (2.3) that $P(x') \leq P(x'')$ would imply that a firm located at x' can obtain strictly higher profit by reallocating to x''. □

A.2 The Proof of Proposition 2

Proof. One version of the proof is based on deriving conditions for (log-)supermodularity of a firm's variable profit function.[37] Specifically, let us define a firm's variable profit function as follows

$$\pi = (p-c)\ell(x)\,\frac{E}{p}\Lambda\left(p/P(x)\right).$$

Taking the logs yields

$$\log\pi = \log(p-c) + \log\ell(x) + \log\frac{E}{p} + \log\Lambda\left(p/P(x)\right).$$

To obtain conditions for $\mathrm{d}x/\mathrm{d}c > 0$, it is sufficient to derive conditions on (log-)supermodularity of a firm's variable profit function. In particular, it can be readily verified that:

$$\frac{\partial^2\log\pi}{\partial p\partial c} = \frac{1}{(p-c)^2} > 0,$$

$$\frac{\partial^2\log\pi}{\partial x\partial c} = 0,$$

$$\frac{\partial^2\log\pi}{\partial p\partial x} = \frac{P'(x)}{P(x)}\cdot\frac{\partial}{\partial p}\left\{\frac{-\Lambda'\left(p/P(x)\right)}{\Lambda\left(p/P(x)\right)}\frac{p}{P(x)}\right\}$$

[36] It is assumed that $P(x)$ is differentiable. A formal proof of it will be added in further versions of the draft.

[37] See also KSTU (2020) for proof of PAM in a closed economy setup under additive preferences in conjunction with a quasi-linear setup.

$$\frac{\partial^2 \log \pi}{\partial p \partial x} = \frac{P'(x)}{P(x)} \cdot \frac{\partial}{\partial p} \left\{ \mathcal{E}_\Lambda \left(p/P(x) \right) \right\} = \frac{P'(x)}{P(x)} \cdot \frac{\partial}{\partial p} \left\{ \mathcal{E}_D \left(p/P(x) \right) - 1 \right\}. \tag{A.1}$$

Now, recall that $P'(x) > 0$ in equilibrium (see Proposition 1). It follows from derivations above and standard theorems of monotone comparative statics that strict (log-)supermodularity of a firm's variable profit (demand) function in (p, x), $\frac{\partial^2 \log \pi}{\partial p \partial x} > 0$, implies positive assortative matching between firm productivity and niche size, $\mathrm{d}x/\mathrm{d}c > 0$. One can readily verify that $\frac{\partial^2 \log \pi}{\partial p \partial x} > 0 \Leftrightarrow \mathcal{E}'_\Lambda(\cdot) > 0$. Now recall that it is equivalent to the MSLD assumption implying that price elasticity of demand decreases with consumption. It is also equivalent to the condition that the elasticity of demand is increasing with price, $\mathcal{E}'_D(\cdot) > 0$. It is straightforward to see from (A.1) that it implies $\frac{\partial^2 \log \pi}{\partial p \partial x} > 0$.

Notice that one can alternatively use a more classical approach via the Implicit Function Theorem and study the first-order comparative statics effects of c on p and x at a solution (for expositional purposes, I omit specific notation that derivatives are evaluated at a solution that satisfies FOCs). In this case, we have

$$\begin{pmatrix} \frac{\mathrm{d}p}{\mathrm{d}c} \\ \frac{\mathrm{d}x}{\mathrm{d}c} \end{pmatrix} = - \begin{pmatrix} \pi''_{pp} & \pi''_{px} \\ \pi''_{xp} & \pi''_{xx} \end{pmatrix}^{-1} \begin{pmatrix} \pi''_{pc} \\ \pi''_{xc} \end{pmatrix},$$

which, in turn, implies

$$\frac{\mathrm{d}p}{\mathrm{d}c} = \frac{-\pi''_{pc}\pi''_{xx} + \pi''_{xc}\pi''_{px}}{\Delta}, \quad \frac{\mathrm{d}x}{\mathrm{d}c} = \frac{-\pi''_{pp}\pi''_{xc} + \pi''_{xp}\pi''_{pc}}{\Delta},$$

where $\Delta \equiv \pi''_{pp}\pi''_{xx} - \left[\pi''_{xp}\right]^2 > 0$.

Notice that a firm's FOC implies $\pi'_x = 0$, where

$$\pi'_x = \frac{p-c}{p} E \left[\ell'(x) \Lambda \left(\frac{p}{P(x)} \right) - \ell(x) \Lambda' \left(\frac{p}{P(x)} \right) \frac{p}{P(x)} \frac{P'(x)}{P(x)} \right]$$

$$= \frac{p-c}{p} E \ell(x) \Lambda \left(\frac{p}{P(x)} \right) \left[\frac{P(x)}{P'(x} \frac{\ell'(x)}{\ell(x)} + \mathcal{E}_\Lambda \left(\frac{p}{P(x)} \right) \right].$$

Taking into account $\pi'_x = 0$, one can readily verify that $\pi''_{xc} = 0$. As a result, expressions for $\mathrm{d}p/\mathrm{d}c$ and $\mathrm{d}x/\mathrm{d}c$ can be simplified as follows:

$$\frac{\mathrm{d}p}{\mathrm{d}c} = \frac{-\pi''_{pc}\pi''_{xx}}{\Delta}, \quad \frac{\mathrm{d}x}{\mathrm{d}c} = \frac{\pi''_{xp}\pi''_{pc}}{\Delta}.$$

Now, notice that $\pi'_p = 0$, where

$$\pi'_p = \ell(x) \, E \left[\frac{c}{p^2} \Lambda \left(\frac{p}{P(x)} \right) + \frac{p-c}{pP(x)} \Lambda' \left(\frac{p}{P(x)} \right) \right].$$

$$\pi''_{pc} = \ell(x)\, E\left[\frac{\Lambda\left(p/P(x)\right)}{p^2} - \frac{\Lambda'\left(p/P(x)\right)}{pP(x)}\right] = \ell(x)\Lambda\left(\frac{p}{P(x)}\right)\frac{E}{p^2}\left[1 - \frac{p}{P(x)}\frac{\Lambda'\left(p/P(x)\right)}{\Lambda\left(p/P(x)\right)}\right]$$

Recall that $\Lambda'\left(p/P(x)\right) < 0$ by assumption. Hence, $\pi''_{pc} > 0$. Combining it with $\pi''_{xx} < 0$ implies that $\mathrm{d}p/\mathrm{d}c > 0$. Now, notice that $\pi''_{pc} > 0$ implies $\operatorname{sign}\left\{\mathrm{d}x/\mathrm{d}c\right\} = \operatorname{sign}\left\{\pi''_{xp}\right\}$. Taking into account derivations above, one can readily verify that $\pi''_{xp} > 0$ iff elasticity of the market share function, $\mathcal{E}_{\Lambda}\left(p/P(x)\right)$, is increasing in price (i.e., the MSLD assumption holds). □

A.3 The Proof of Proposition 3

For this proof, I use an alternative yet equivalent representation of HSA demand systems that exploits an inverse demand function and replaces the price aggregator with the quantity aggregator (see more details in Section 4.2).

i) The result that firms' prices, p, are increasing in x is a corollary of Proposition . Recall that according to Proposition , firms with higher marginal cost, c, set higher prices and serve less "populated" niches, x, if, in addition, MSLD holds. Hence, firms' prices, p, are decreasing in niche size or, equivalently, increasing in x. To show it more formally, recall that

$$p(c(x), x) = \frac{c(x)}{\mathcal{E}_{\tilde{\Lambda}}\left(q(c(x), x)/Q(x)\right)} = \frac{c(x)}{\mathcal{E}_{\tilde{\Lambda}}\left(\tilde{\Lambda}'^{-1}\left(c(x)\, Q(x)/E\right)\right)}.$$

Hence, one can readily verify that

$$\frac{\mathrm{d}}{\mathrm{d}x}\left[p(c(x), x)\right] = \frac{c'(x)}{\mathcal{E}_{\tilde{\Lambda}}(\cdot)} - \frac{c(x)\mathcal{E}'_{\tilde{\Lambda}}(\cdot)}{\tilde{\Lambda}''(\cdot)\left[\mathcal{E}_{\tilde{\Lambda}}(\cdot)\right]^2}\frac{c(x)\, Q(x)}{E}\left[\frac{c'(x)}{c(x)} + \frac{Q'(x)}{Q(x)}\right].$$

Rearranging terms yields the following condition for $\frac{\mathrm{d}}{\mathrm{d}x}\left[p(c(x), x)\right] > 0$,

$$\frac{c'(x)}{c(x)}\left(1 - \frac{\mathcal{E}'_{\tilde{\Lambda}}(\cdot)}{\tilde{\Lambda}''(\cdot)\mathcal{E}_{\tilde{\Lambda}}(\cdot)}\frac{c(x)\, Q(x)}{E}\right) > \frac{\mathcal{E}'_{\tilde{\Lambda}}(\cdot)}{\tilde{\Lambda}''(\cdot)\mathcal{E}_{\tilde{\Lambda}}(\cdot)}\frac{c(x)\, Q(x)}{E}\frac{Q'(x)}{Q(x)}.$$

Recall that MSLD implies $\mathcal{E}'_{\tilde{\Lambda}}(\cdot) < 0$ and ensures that $\tilde{\Lambda}''(\cdot) < 0$. Moreover, $Q'(x) < 0$ and $c'(x) > 0$ (under MSLD). Hence, it is sufficient to show that

$$1 > \frac{\mathcal{E}'_{\tilde{\Lambda}}(\cdot)}{\tilde{\Lambda}''(\cdot)\mathcal{E}_{\tilde{\Lambda}}(\cdot)}\frac{c(x)\, Q(x)}{E} \Leftrightarrow 1 > \frac{\tilde{\Lambda}'(\cdot)\tilde{\Lambda}'(\cdot)}{\tilde{\Lambda}(\cdot)\tilde{\Lambda}''(\cdot)\mathcal{E}_{\tilde{\Lambda}}(\cdot)}\left[1 + \frac{q(c(x), x)}{Q(x)}\frac{\tilde{\Lambda}''(\cdot)}{\tilde{\Lambda}'(\cdot)} - \mathcal{E}_{\tilde{\Lambda}}(\cdot)\right]$$

$$\Leftrightarrow 1 > \frac{\tilde{\Lambda}'(\cdot)}{\frac{q(c(x), x)}{Q(x)}\tilde{\Lambda}''(\cdot)}\left[1 + \frac{q(c(x), x)}{Q(x)}\frac{\tilde{\Lambda}''(\cdot)}{\tilde{\Lambda}'(\cdot)} - \mathcal{E}_{\tilde{\Lambda}}(\cdot)\right] \Leftrightarrow \mathcal{E}_{\tilde{\Lambda}}\left(\frac{q(c(x), x)}{Q(x)}\right) < 1.$$

Recalling that $\mathcal{E}_{\tilde{\Lambda}}(z) \equiv z\tilde{\Lambda}'(z)/\tilde{\Lambda}(z) \in (0, 1)\ \forall z \in \mathbb{R}_{++}$ completes the proof.

ii) To show that firms' total quantities sold, $q(x)\ell(x)$, are decreasing in x, notice that

$$\frac{\mathrm{d}}{\mathrm{d}x}\left[q(x)\ell(x)\right] = \ell'(x)Q(x)\tilde{\Lambda}'^{-1}\left(c(x)Q(x)/E\right)$$

$$+\ell(x)\left[Q'(x)\tilde{\Lambda}'^{-1}\left(\frac{c(x)Q(x)}{E}\right) + \frac{Q(x)\tilde{\Lambda}'(\cdot)}{\tilde{\Lambda}''(\cdot)}\left[\frac{c'(x)}{c(x)} + \frac{Q'(x)}{Q(x)}\right]\right].$$

Hence, rearranging terms yields

$$\frac{\mathrm{d}}{\mathrm{d}x}\left[q(x)\ell(x)\right] < 0 \Leftrightarrow \frac{Q'(x)}{Q(x)}\left(\frac{\mathcal{E}_{\tilde{\Lambda}'}(\cdot)-1}{\mathcal{E}_{\tilde{\Lambda}'}(\cdot)}\right) - \frac{c'(x)/c(x)}{\mathcal{E}_{\tilde{\Lambda}'}(\cdot)} < -\frac{\ell'(x)}{\ell(x)}$$

$$\Leftrightarrow \frac{\ell'(x)}{\ell(x)}\frac{(1-\mathcal{E}_{\tilde{\Lambda}}(\cdot))}{\mathcal{E}_{\tilde{\Lambda}}(\cdot)}\frac{(\mathcal{E}_{\tilde{\Lambda}'}(\cdot)-1)}{\mathcal{E}_{\tilde{\Lambda}'}(\cdot)} - \frac{c'(x)/c(x)}{\mathcal{E}_{\tilde{\Lambda}'}(\cdot)} < -\frac{\ell'(x)}{\ell(x)}$$

$$\Leftrightarrow \frac{\ell'(x)}{\ell(x)}\left(\frac{\mathcal{E}_{\tilde{\Lambda}'}(\cdot)+\mathcal{E}_{\tilde{\Lambda}}(\cdot)-1}{\mathcal{E}_{\tilde{\Lambda}}(\cdot)\mathcal{E}_{\tilde{\Lambda}'}(\cdot)}\right) - \frac{c'(x)/c(x)}{\mathcal{E}_{\tilde{\Lambda}'}(\cdot)} < 0.$$

Recall from (4.10) that MSLD implies $\mathcal{E}_{\tilde{\Lambda}'}(\cdot)+\mathcal{E}_{\tilde{\Lambda}}(\cdot)-1 > 0$. Hence, recalling that $\ell'(x) < 0$ and $c'(x) > 0$, it can be directly concluded that firms' total quantities sold, $q(x)\ell(x)$, are decreasing in x.

To show that firms' total revenues, $p(x)q(x)\ell(x)$, are decreasing in x, notice that

$$\frac{\mathrm{d}}{\mathrm{d}x}\left[p(x)q(x)\ell(x)\right] = \tilde{\Lambda}\left(\frac{q(c(x),x)}{Q(x)}\right)E\ell(x) = \tilde{\Lambda}\left(\tilde{\Lambda}'^{-1}\left(\frac{c(x)\,Q(x)}{E}\right)\right)E\ell(x)$$

$$\Leftrightarrow \ell'(x)\tilde{\Lambda}\left(\tilde{\Lambda}'^{-1}\left(\frac{c(x)Q(x)}{E}\right)\right)E + \ell(x)c(x)Q(x)\frac{\tilde{\Lambda}'(\cdot)}{\tilde{\Lambda}''(\cdot)}\left(\frac{c'(x)}{c(x)}+\frac{Q'(x)}{Q(x)}\right).$$

Hence, one can readily verify that

$$\frac{\mathrm{d}}{\mathrm{d}x}\left[p(x)q(x)\ell(x)\right] < 0 \Leftrightarrow \ell(x)c(x)Q(x)\frac{\tilde{\Lambda}'(\cdot)}{\tilde{\Lambda}''(\cdot)}\left(\frac{c'(x)}{c(x)} + \frac{Q'(x)}{Q(x)}\right) < -\ell'(x)\tilde{\Lambda}\left(\tilde{\Lambda}'^{-1}\left(\frac{c(x)Q(x)}{E}\right)\right)E$$

$$\Leftrightarrow -\frac{\mathcal{E}_{\tilde{\Lambda}}(\cdot)}{\mathcal{E}_{\tilde{\Lambda}'}(\cdot)}\left(\frac{c'(x)}{c(x)}+\frac{Q'(x)}{Q(x)}\right) < -\frac{\ell'(x)}{\ell(x)}$$

$$\Leftrightarrow -\frac{\mathcal{E}_{\tilde{\Lambda}}\left(\frac{q(c(x),x)}{Q(x)}\right)}{\mathcal{E}_{\tilde{\Lambda}'}\left(\frac{q(c(x),x)}{Q(x)}\right)}\left(\frac{1}{c(x)M^e g(c(x))\tilde{\Lambda}\left(\frac{q(c(x),x)}{Q(x)}\right)} + \frac{\ell'(x)}{\ell(x)}\frac{1-\mathcal{E}_{\tilde{\Lambda}}(\cdot)}{\mathcal{E}_{\tilde{\Lambda}}(\cdot)}\right) < -\frac{\ell'(x)}{\ell(x)}$$

$$\Leftrightarrow -\frac{\mathcal{E}_{\tilde{\Lambda}}\left(\frac{q(c(x),x)}{Q(x)}\right)}{\mathcal{E}_{\tilde{\Lambda}'}\left(\frac{q(c(x),x)}{Q(x)}\right)}\frac{1}{c(x)M^e g(c(x))\tilde{\Lambda}\left(\frac{q(c(x),x)}{Q(x)}\right)} < -\frac{\ell'(x)}{\ell(x)}\left(1-\frac{1-\mathcal{E}_{\tilde{\Lambda}}\left(\frac{q(c(x),x)}{Q(x)}\right)}{\mathcal{E}_{\tilde{\Lambda}'}\left(\frac{q(c(x),x)}{Q(x)}\right)}\right).$$

Notice that MSLD implies $\mathcal{E}_{\tilde{\Lambda}'}\left(q/Q(x)\right)+\mathcal{E}_{\tilde{\Lambda}}\left(q/Q(x)\right)>1$. Hence, recalling that $\ell'(x)<0$, the RHS of the inequality above is always positive, while the LHS is, in contrast, negative. So, it can be concluded that firms' total revenues, $p(x)q(x)\ell(x)$, are decreasing in x.

To show that firms' total profits, $\Pi(x)$, are decreasing in x, notice that

$$\Pi(c(x),x)=\left[\tilde{\Lambda}\left(q(c(x),x)/Q(x)\right)E-c(x)q(c(x),x)\right]\ell(x)-F$$

$$=\left[\tilde{\Lambda}\left(\tilde{\Lambda}'^{-1}\left(c(x)\,Q(x)/E\right)\right)E-c(x)Q(x)\tilde{\Lambda}'^{-1}\left(c(x)\,Q(x)/E\right)\right]\ell(x)-F.$$

Hence,

$$\mathrm{d}\Pi(\cdot)/\mathrm{d}x=\ell(x)\left[\tilde{\Lambda}\left(\tilde{\Lambda}'^{-1}(\cdot)\right)E-c(x)Q(x)\tilde{\Lambda}'^{-1}(\cdot)\right]$$

$$+\ell'(x)\left[\frac{\tilde{\Lambda}'(\cdot)}{\tilde{\Lambda}''(\cdot)}\left(c'(\cdot)\,Q(\cdot)+c(\cdot)\,Q'(\cdot)\right)-\left(c'(\cdot)\,Q(\cdot)+c(\cdot)\,Q'(\cdot)\right)\tilde{\Lambda}'^{-1}(\cdot)-\frac{c(\cdot)Q(\cdot)}{\tilde{\Lambda}''(\cdot)}\frac{c'(\cdot)Q(\cdot)+c(\cdot)Q'(\cdot)}{E}\right].$$

Rearranging terms yields

$$\frac{\mathrm{d}\Pi(\cdot)}{\mathrm{d}x}=\ell'(x)\left[\tilde{\Lambda}\left(\tilde{\Lambda}'^{-1}(\cdot)\right)E-c(x)Q(x)\tilde{\Lambda}'^{-1}(\cdot)\right]$$

$$+\ell(x)\left(c'(x)\,Q(x)+c(x)\,Q'(\cdot)\right)\left[\frac{\tilde{\Lambda}'(\cdot)}{\tilde{\Lambda}''(\cdot)}-\tilde{\Lambda}'^{-1}(\cdot)-\frac{c(x)Q(x)/E}{\tilde{\Lambda}''(\cdot)}\right].$$

One can readily verify that

$$\frac{\mathrm{d}\Pi(\cdot)}{\mathrm{d}x}<0\Leftrightarrow\frac{\ell'(x)}{\ell(x)}\left[\frac{1}{\mathcal{E}_{\tilde{\Lambda}}(\cdot)}-1\right]<\frac{c'(x)}{c(x)}+\frac{Q'(x)}{Q(x)}$$

Now recall from a firm's FOCs that $1+\frac{Q'(x)}{Q(x)}\frac{\ell(x)}{\ell'(x)}=1/\mathcal{E}_{\tilde{\Lambda}}(\cdot)$. Hence,

$$\frac{\mathrm{d}\Pi(\cdot)}{\mathrm{d}x}<0\Leftrightarrow\frac{\ell'(x)}{\ell(x)}\frac{Q'(x)}{Q(x)}\frac{\ell(x)}{\ell'(x)}<\frac{c'(x)}{c(x)}+\frac{Q'(x)}{Q(x)}\Leftrightarrow 0<\frac{c'(x)}{c(x)}$$

Hence, it can be argued that firms' total profits, $\Pi(x)$, are decreasing in x.

As for firms' total cost, notice that

$$\frac{\mathrm{d}}{\mathrm{d}x}\left[c(x)q(x)\ell(x)+F\right]=\frac{\mathrm{d}}{\mathrm{d}x}\left[c(x)Q(x)\tilde{\Lambda}'^{-1}\left(\frac{c(x)Q(x)}{E}\right)\ell(x)\right]=$$

$$= \left[\frac{c'(x)}{c(x)} + \frac{Q'(x)}{Q(x)}\right] c(x)Q(x)\tilde{\Lambda}'^{-1}\left(\frac{c(x)Q(x)}{E}\right)\ell(x) +$$

$$+c(x)Q(x)\left[\tilde{\Lambda}'^{-1}\left(\frac{c(x)Q(x)}{E}\right)\ell'(x) + \ell(x)\frac{\tilde{\Lambda}'(\cdot)}{\tilde{\Lambda}''(\cdot)}\left[\frac{c'(x)}{c(x)} + \frac{Q'(x)}{Q(x)}\right]\right]$$

$$= \left(\frac{c'(x)}{c(x)} + \frac{Q'(x)}{Q(x)}\right) c(x)Q(x)\tilde{\Lambda}'^{-1}\left(\frac{c(x)Q(x)}{E}\right)\ell(x)\left[1 - \frac{1}{\mathcal{E}_{\tilde{\Lambda}'}(\cdot)}\right] + c(x)Q(x)\tilde{\Lambda}'^{-1}\left(\frac{c(x)Q(x)}{E}\right)\ell'(x).$$

Now, notice that

$$\frac{\mathrm{d}}{\mathrm{d}x}\left[c(x)q(x)\ell(x) + F\right] \gtreqless 0 \Leftrightarrow \left(\frac{c'(x)}{c(x)} + \frac{Q'(x)}{Q(x)}\right)\left[1 - \frac{1}{\mathcal{E}_{\tilde{\Lambda}'}(\cdot)}\right] \gtreqless -\frac{\ell'(x)}{\ell(x)}$$

$$\Leftrightarrow \frac{c'(x)}{c(x)}\left[1 - \frac{1}{\mathcal{E}_{\tilde{\Lambda}'}(\cdot)}\right] + \frac{\ell'(x)}{\ell(x)}\left[\frac{1 - \mathcal{E}_{\tilde{\Lambda}}(\cdot)}{\mathcal{E}_{\tilde{\Lambda}}(\cdot)}\right]\left[\frac{\mathcal{E}_{\tilde{\Lambda}'}(\cdot) - 1}{\mathcal{E}_{\tilde{\Lambda}'}(\cdot)}\right] \gtreqless -\frac{\ell'(x)}{\ell(x)}$$

$$\Leftrightarrow \frac{c'(x)}{c(x)}\left[1 - \frac{1}{\mathcal{E}_{\tilde{\Lambda}'}(\cdot)}\right] + \frac{\ell'(x)}{\ell(x)}\left[\frac{\mathcal{E}_{\tilde{\Lambda}'}(\cdot) - 1 - \mathcal{E}_{\tilde{\Lambda}}(\cdot)\mathcal{E}_{\tilde{\Lambda}'}(\cdot) + \mathcal{E}_{\tilde{\Lambda}}(\cdot)}{\mathcal{E}_{\tilde{\Lambda}}(\cdot)\mathcal{E}_{\tilde{\Lambda}'}(\cdot)}\right] \gtreqless -\frac{\ell'(x)}{\ell(x)}$$

$$\Leftrightarrow \frac{c'(x)}{c(x)}\left[1 - \frac{1}{\mathcal{E}_{\tilde{\Lambda}'}(\cdot)}\right] \gtreqless -\frac{\ell'(x)}{\ell(x)}\left[\frac{\mathcal{E}_{\tilde{\Lambda}'}(\cdot) + \mathcal{E}_{\tilde{\Lambda}}(\cdot) - 1}{\mathcal{E}_{\tilde{\Lambda}}(\cdot)\mathcal{E}_{\tilde{\Lambda}'}(\cdot)}\right].$$

Recall that MSLD implies $\mathcal{E}_{\tilde{\Lambda}'}(\cdot) + \mathcal{E}_{\tilde{\Lambda}}(\cdot) > 1$. Recalling that $\ell'(x) < 0$, it can be concluded that the RHS of the inequality above is positive. As for the LHS, recall that $c'(x) > 0$. However, the sign of $[1 - 1/\mathcal{E}_{\tilde{\Lambda}'}(\cdot)]$ is generally ambiguous under the baseline assumptions on $\Lambda(\cdot)$.

Before proving the remaining parts of the proposition, it is instructive to prove an auxiliary lemma that describes the behavior of $c(x)Q(x)$.

Lemma 1. *Assume that MSLD holds, the distribution of marginal cost, $g(\cdot)$, is non-decreasing, and the distribution of niche sizes, $\ell(\cdot)$, is logarithmically concave. Then, imposing mild additional assumptions (see below) ensures that $c(x)Q(x)$ is $\cap-$shaped in x in a model's equilibrium.*

Proof. Recall that $Q'(x) < 0$ and that MSLD implies $c'(x) > 0$. Note also that

$$\frac{\mathrm{d}}{\mathrm{d}x}\left[\log\left(c(x)Q(x)\right)\right] = \frac{c'(x)}{c(x)} + \frac{Q'(x)}{Q(x)} = \frac{1}{c(x)M^e g(c(x))\tilde{\Lambda}(\cdot)} + \frac{\ell'(x)}{\ell(x)}\frac{1 - \mathcal{E}_{\tilde{\Lambda}}(\cdot)}{\mathcal{E}_{\tilde{\Lambda}}(\cdot)}.$$

Recall that $c_{\min}$ and $\ell(\cdot)$ are exogenously given. Hence, one can readily verify that if $c(0) = c_{\min}$ is sufficiently close to 0, whereas $\ell'(x) > -\infty$, then $c'(0)/c(0) + Q'(0)/Q(0) > 0$. Similarly, if $\ell(x_{\max})$ is sufficiently close to 0, while $\ell'(x_{\max})$ is large enough, then $c'(x_{\max})/c(x_{\max}) + Q'(x_{\max})/Q(x_{\max}) < 0$.

If $c'(0)/c(0) + Q'(0)/Q(0) > 0$ and $c'(x_{\max})/c(x_{\max}) + Q'(x_{\max})/Q(x_{\max}) < 0$, then there exists at least one $x = \mathring{x}$ such that $c'(\mathring{x})/c(\mathring{x}) + Q'(\mathring{x})/Q(\mathring{x}) = 0$. By showing that for any $\mathring{x}$,

$(c'(x)/c(x)+Q'(x)/Q(x))'_{x=\mathring{x}} = (c'(x)/c(x))'_{x=\mathring{x}} + (Q'(x)/Q(x))'_{x=\mathring{x}} < 0$, it can be ensured that there exists only one such point. Now, notice that

$$\left(\frac{Q'(x)}{Q(x)}\right)'_x = \frac{\ell''(x)\ell(x)-[\ell'(x)]^2}{[\ell(x)]^2}\left[\frac{1}{\mathcal{E}_{\tilde{\Lambda}}\left(\frac{q(c(x),x)}{Q(x)}\right)}-1\right] - \frac{\ell'(x)}{\ell(x)}\frac{\mathcal{E}'_{\tilde{\Lambda}}\left(\frac{q(c(x),x)}{Q(x)}\right)\frac{c(x)\,Q(x)}{E}\left[\frac{c'(x)}{c(x)}+\frac{Q'(x)}{Q(x)}\right]}{\left[\mathcal{E}_{\tilde{\Lambda}}\left(\frac{q(c(x),x)}{Q(x)}\right)\right]^2\tilde{\Lambda}''\left(\frac{q(c(x),x)}{Q(x)}\right)}.$$

Recall also that $c'(x)/c(x)+Q'(x)/Q(x)=0$ for $x=\mathring{x}$. Hence, assuming that $\ell(\cdot)$ is logarithmically concave, we have $\ell''(\mathring{x})\ell(\mathring{x}) \leq [\ell'(\mathring{x})]^2\ \forall x\in\mathcal{X}$. Hence,

$$\left(\frac{Q'(x)}{Q(x)}\right)'_{x=\mathring{x}} = \frac{\ell''(\mathring{x})\ell(\mathring{x})-[\ell'(\mathring{x})]^2}{[\ell(\mathring{x})]^2}\left[1/\mathcal{E}_{\tilde{\Lambda}}\left(\frac{q(c(\mathring{x}),\mathring{x})}{Q(\mathring{x})}\right)-1\right] < 0.$$

Next, notice that

$$\left(\frac{c'(x)}{c(x)}\right)'_x = \frac{[c'(x)g(\cdot)+c(x)g'(c(x))c'(x)]\,\tilde{\Lambda}(\cdot)+c(x)g(\cdot)\frac{\tilde{\Lambda}'(\cdot)}{\tilde{\Lambda}''(\cdot)}\frac{c(x)\,Q(x)}{E}\left[\frac{c'(x)}{c(x)}+\frac{Q'(x)}{Q(x)}\right]}{-M^e\left[c(x)g(c(x))\tilde{\Lambda}(\cdot)\right]^2}.$$

Again, recall that $c'(x)/c(x)+Q'(x)/Q(x)=0$ for $x=\mathring{x}$. Hence,

$$\left(\frac{c'(x)}{c(x)}\right)'_{x=\mathring{x}} = \frac{[c'(\mathring{x})g(c(\mathring{x}))+c(\mathring{x})g'(c(\mathring{x}))c'(\mathring{x})]\,\tilde{\Lambda}\left(q(c(\mathring{x}),\mathring{x})/Q(\mathring{x})\right)}{-M^e\left[c(\mathring{x})g(c(\mathring{x}))\tilde{\Lambda}\left(q(c(\mathring{x}),\mathring{x})/Q(\mathring{x})\right)\right]^2}$$

$$= \frac{[c'(\mathring{x})/c(\mathring{x})+c'(\mathring{x})g'(c(\mathring{x}))/g(c(\mathring{x}))]}{-M^e c(\mathring{x})g(c(\mathring{x}))\tilde{\Lambda}\left(q(c(\mathring{x}),\mathring{x})/Q(\mathring{x})\right)} = \frac{(1+\mathcal{E}_g(c(\mathring{x})))c'(\mathring{x})/c(\mathring{x})}{-M^e c(\mathring{x})g(c(\mathring{x}))\tilde{\Lambda}\left(q(c(\mathring{x}),\mathring{x})/Q(\mathring{x})\right)}.$$

Assuming that $g(\cdot)$ is non-decreasing yields $\mathcal{E}_g(c)\equiv cg'(c)/g(c)\geq 0$. Hence, $(c'(x)/c(x))'_{x=\mathring{x}} < 0$. Combining it with $(Q'(x)/Q(x))'_{x=\mathring{x}}\leq 0$ yields $(c'(x)/c(x)+Q'(x)/Q(x))'_{x=\mathring{x}}<0$. So, it can be argued that there exists a unique $\mathring{x}$ such that $c'(\mathring{x})/c(\mathring{x})+Q'(\mathring{x})/Q(\mathring{x})=0$. As a result, $c(x)Q(x)$ is $\cap-$shaped in x. This concludes the proof. □

Now, I provide proof of the remaining parts of Proposition 3.

iii) To show that firms' markups, p/c, are $\cup-$shaped in x, recall that

$$\frac{p(c(x),x)}{c(x)} = 1/\mathcal{E}_{\tilde{\Lambda}}\left(q(c(x),x)/Q(x)\right) = 1/\mathcal{E}_{\tilde{\Lambda}}\left(\tilde{\Lambda}'^{-1}\left(c(x)\,Q(x)/E\right)\right).$$

One can readily verify that

$$\frac{\mathrm{d}}{\mathrm{d}x}\left[\frac{p(c(x),x)}{c(x)}\right] = \frac{\mathcal{E}'_{\tilde{\Lambda}}(\cdot)}{-\tilde{\Lambda}''(\cdot)\left[\mathcal{E}_{\tilde{\Lambda}}(\cdot)\right]^2}\frac{c(x)\,Q(x)}{E}\left[\frac{c'(x)}{c(x)}+\frac{Q'(x)}{Q(x)}\right].$$

Recall that MSLD implies $\mathcal{E}'_{\tilde{\Lambda}}(\cdot) < 0$ and ensures $\tilde{\Lambda}''(\cdot) < 0$. Hence

$$\text{sign}\left\{\frac{\mathrm{d}}{\mathrm{d}x}\left[\frac{p(c(x),x)}{c(x)}\right]\right\} = -\text{sign}\left\{\frac{c'(x)}{c(x)} + \frac{Q'(x)}{Q(x)}\right\}.$$

Recall that $\text{sign}\left\{\frac{\mathrm{d}}{\mathrm{d}x}\left[\frac{p(c(x),x)}{c(x)}\right]\right\} = -\text{sign}\left\{\frac{c'(x)}{c(x)} + \frac{Q'(x)}{Q(x)}\right\}$. So, applying results of Lemma 1 yields that firms' markups, p/c, are $\cup-$shaped in x.

A similar strategy is used to show that fraction of consumers' spending on each variety, $p(x)q(x)/E$ (and, equivalently, a firm's "market share", $\Lambda\left(q(x)/Q(x)\right) = p(x)q(x)/E$), is $\cup-$shaped in x. In particular, notice that

$$\frac{\mathrm{d}}{\mathrm{d}x}\left[\frac{p(x)q(x)}{E}\right] = \frac{\mathrm{d}}{\mathrm{d}x}\left[\tilde{\Lambda}\left(\tilde{\Lambda}'^{-1}\left(\frac{c(x)\,Q(x)}{E}\right)\right)\right] = \frac{\tilde{\Lambda}'(\cdot)}{\tilde{\Lambda}''(\cdot)}\frac{c(x)Q(x)}{E}\left[\frac{c'(x)}{c(x)} + \frac{Q'(x)}{Q(x)}\right].$$

Recall that $\tilde{\Lambda}''(\cdot) < 0$. As a result, $\text{sign}\left\{\frac{\mathrm{d}}{\mathrm{d}x}\left[\frac{p(x)q(x)}{E}\right]\right\} = -\text{sign}\left\{\frac{c'(x)}{c(x)} + \frac{Q'(x)}{Q(x)}\right\}$. Hence, going through the same steps as in part **(ii)** of this proposition, it can be inferred that $\Lambda\left(q(x)/Q(x)\right) = p(x)q(x)/E$ is $\cup-$shaped in x.

iv) It is straightforward to see that the distribution of the mass firms, $M(x)$, is $\cap-$shaped in x from the fact that $M(x) = 1/\Lambda\left(q(x)/Q(x)\right)$. Recall that it is already proven that $\Lambda\left(q(x)/Q(x)\right) = p(x)q(x)/E$ is $\cup-$shaped in x. So, it can be directly inferred that the distribution of the mass firms, $M(x)$, is $\cap-$shaped in x.

A.4 The Proof of Proposition 4

Proof. Note that properties of homothetic preferences imply $E = P^I(x) \cdot U(x)$, where $U(x)$ is equivalent to the local welfare, $W(x)$. Hence, taking into account the general relationship between $Q(x)$ and $U(x)$,[38] we get

$$\log\left(\frac{W(x)}{Q(x)}\right) = const + M(x)\int_0^{q(c(x),x)/Q(x)}\frac{\tilde{\Lambda}(\xi)}{\xi}\mathrm{d}\xi = const + M(x)\int_0^{\tilde{\Lambda}'^{-1}\left(\frac{c(x)\,Q(x)}{E}\right)}\frac{\tilde{\Lambda}(\xi)}{\xi}\mathrm{d}\xi.$$

Differentiating both sides w.r.t. x yields,

$$\frac{W'(x)}{W(x)} = \frac{Q'(x)}{Q(x)} + M'(x)\int_0^{\tilde{\Lambda}'^{-1}\left(\frac{c(x)Q(x)}{E}\right)}\frac{\tilde{\Lambda}(\xi)}{\xi}\mathrm{d}\xi + M(x)\frac{\tilde{\Lambda}\left(q(c(x),x)/Q(x)\right)}{q(c(x),x)/Q(x)}\frac{\mathrm{d}}{\mathrm{d}x}\left[\tilde{\Lambda}'^{-1}\left(\frac{c(x)\,Q(x)}{E}\right)\right]$$

[38] See Matsuyama and Ushchev (2017, 2020) for more details on the relationship between the ideal price index and (common) price aggregator under the HSA demand systems.

$$\frac{W'(x)}{W(x)} = \frac{Q'(x)}{Q(x)} + \underbrace{\frac{M'(x)}{=\frac{[\tilde{\Lambda}'(\cdot)]^2\left(\frac{c'(x)}{c(x)}+\frac{Q'(x)}{Q(x)}\right)}{-[\tilde{\Lambda}(\cdot)]^2\tilde{\Lambda}''(\cdot)}}} \int_0^{\tilde{\Lambda}'^{-1}\left(\frac{c(x)Q(x)}{E}\right)} \frac{\tilde{\Lambda}(\xi)}{\xi}\mathrm{d}\xi + \underbrace{M(x)}_{=1/\tilde{\Lambda}(\cdot)} \frac{\tilde{\Lambda}(\cdot)\tilde{\Lambda}'(\cdot)\left[\frac{c'(x)}{c(x)}+\frac{Q'(x)}{Q(x)}\right]}{\frac{q(c(x),x)}{Q(x)}\tilde{\Lambda}''(\cdot)}$$

$$\frac{W'(x)}{W(x)} = \frac{Q'(x)}{Q(x)} + \frac{\mathcal{E}_{\tilde{\Lambda}}(\cdot)/\tilde{\Lambda}(\cdot)}{\mathcal{E}_{\tilde{\Lambda}'}(\cdot)}\left[\frac{c'(x)}{c(x)}+\frac{Q'(x)}{Q(x)}\right]\int_0^{\tilde{\Lambda}'^{-1}\left(\frac{c(x)Q(x)}{E}\right)}\frac{\tilde{\Lambda}(\xi)}{\xi}\mathrm{d}\xi - \frac{1}{\mathcal{E}_{\tilde{\Lambda}'}(\cdot)}\left[\frac{c'(x)}{c(x)}+\frac{Q'(x)}{Q(x)}\right].$$

Rearranging the corresponding terms yields,

$$\frac{W'(x)}{W(x)} = \frac{\mathcal{E}_{\tilde{\Lambda}'}(\cdot) - 1 + \frac{\mathcal{E}_{\tilde{\Lambda}}(\cdot)}{\tilde{\Lambda}(\cdot)}\int_0^{\tilde{\Lambda}'^{-1}\left(\frac{c(x)Q(x)}{E}\right)}\frac{\tilde{\Lambda}(\xi)}{\xi}\mathrm{d}\xi}{\mathcal{E}_{\tilde{\Lambda}'}(\cdot)}\frac{Q'(x)}{Q(x)} + \frac{\frac{\mathcal{E}_{\tilde{\Lambda}}(\cdot)}{\tilde{\Lambda}(\cdot)}\int_0^{\tilde{\Lambda}'^{-1}\left(\frac{c(x)Q(x)}{E}\right)}\frac{\tilde{\Lambda}(\xi)}{\xi}\mathrm{d}\xi - 1}{\mathcal{E}_{\tilde{\Lambda}'}(\cdot)}\frac{c'(x)}{c(x)}. \quad \text{(A.2)}$$

Now, recall that $\mathcal{E}_{\tilde{\Lambda}}(z) \equiv z\tilde{\Lambda}'(z)/\tilde{\Lambda}(z)$ is decreasing under MSLD and $\tilde{\Lambda}(0) = 0$. Hence,

$$\int_0^z \frac{\tilde{\Lambda}(\xi)}{\xi}\mathrm{d}\xi = \int_0^z \frac{\tilde{\Lambda}'(\xi)}{\mathcal{E}_{\tilde{\Lambda}}(\xi)}\mathrm{d}\xi < \int_0^z \frac{\tilde{\Lambda}'(\xi)}{\mathcal{E}_{\tilde{\Lambda}}(z)}\mathrm{d}\xi = \frac{\tilde{\Lambda}(z)}{\mathcal{E}_{\tilde{\Lambda}}(z)}.$$

One can readily verify from (A.2) that $\int_0^z \frac{\tilde{\Lambda}(\xi)}{\xi}\mathrm{d}\xi < \frac{\tilde{\Lambda}(z)}{\mathcal{E}_{\tilde{\Lambda}}(z)}$ implies that the coefficient in front of $\frac{c'(x)}{c(x)}$ is negative.

The next step is to verify the sign of the first term of (A.2). What determines its sign is $\mathcal{E}_{\tilde{\Lambda}'}(\cdot) - 1 + \frac{\mathcal{E}_{\tilde{\Lambda}}(\cdot)}{\tilde{\Lambda}(\cdot)}\int_0^{\tilde{\Lambda}'^{-1}\left(\frac{c(x)Q(x)}{E}\right)}\frac{\tilde{\Lambda}(\xi)}{\xi}\mathrm{d}\xi$. Recall that MSLD implies $\mathcal{E}_{\tilde{\Lambda}'}(\cdot) + \mathcal{E}_{\tilde{\Lambda}}(\cdot) > 1$. Hence, it is crucial to show that $\frac{\int_0^{\frac{q(c(x),x)}{Q(x)}}\frac{\tilde{\Lambda}(\xi)}{\xi}}{\tilde{\Lambda}(q(c(x),x)/Q(x))}\mathrm{d}\xi > 1$ for any $\frac{q(c(x),x)}{Q(x)} \geq 0$. Adding some confusion to the previous notation, recall that $\tilde{\Lambda}'(0) = \bar{z}$ and $\lim_{z\to 0}\tilde{\Lambda}(z)/z = \bar{z}$, where $\bar{z}$ denotes the choke price. It also implies that $\lim_{z\to 0}\mathcal{E}_{\tilde{\Lambda}}(z) \equiv \lim_{z\to 0} z\tilde{\Lambda}'(z)/\tilde{\Lambda}(z) = 1$. Now, exploiting L'Hospital's Rule yields:

$$\lim_{z\to 0}\frac{\int_0^z \tilde{\Lambda}(\xi)/\xi}{\tilde{\Lambda}(z)} = \lim_{z\to 0}\frac{\tilde{\Lambda}(z)/z}{\tilde{\Lambda}'(z)} = \lim_{z\to 0} 1/\mathcal{E}_{\tilde{\Lambda}}(z) = 1.$$

The next step is to show that $\frac{\mathrm{d}}{\mathrm{d}z}\left[\frac{1}{\tilde{\Lambda}(z)}\int_0^z \frac{\tilde{\Lambda}(\xi)}{\xi}\right] > 0$.

$$\frac{\mathrm{d}}{\mathrm{d}z}\left[\frac{1}{\tilde{\Lambda}(z)}\int_0^z \frac{\tilde{\Lambda}(\xi)}{\xi}\right] = \frac{1}{z} - \frac{\tilde{\Lambda}'(z)}{\left[\tilde{\Lambda}(z)\right]^2}\int_0^z \frac{\tilde{\Lambda}(\xi)}{\xi} = \frac{1}{z}\left[1 - \frac{\mathcal{E}_{\tilde{\Lambda}}(z)}{\tilde{\Lambda}(z)}\int_0^z \frac{\tilde{\Lambda}(\xi)}{\xi}\right].$$

Recall that it has already been shown that $\int_0^z \frac{\tilde{\Lambda}(\xi)}{\xi} < \frac{\tilde{\Lambda}(z)}{\mathcal{E}_{\tilde{\Lambda}}(z)}$. Hence, $\frac{\mathrm{d}}{\mathrm{d}z}\left[\frac{1}{\tilde{\Lambda}(z)}\int_0^z \frac{\tilde{\Lambda}(\xi)}{\xi}\right] > 0$ that, in turn, implies $\mathcal{E}_{\tilde{\Lambda}'}(\cdot) - 1 + \frac{\mathcal{E}_{\tilde{\Lambda}}(\cdot)}{\tilde{\Lambda}(\cdot)} \int_0^{\tilde{\Lambda}'^{-1}\left(\frac{c(x)Q(x)}{E}\right)} \frac{\tilde{\Lambda}(\xi)}{\xi} \mathrm{d}\xi > 0$ (recall that MSLD implies $\mathcal{E}_{\tilde{\Lambda}'}(\cdot) + \mathcal{E}_{\tilde{\Lambda}}(\cdot) > 1$). Thus, in equilibrium, $W'(x)/W(x) < 0$. □

A.5 The Proof of Proposition 5

Proof. It is straightforward to see that (3.3) implies $p_x^h < p_x^f$. Moreover, the ratio of markups is as follows:

$$\left(\frac{p^h(c,x) - c}{p^h(c,x)}\right) / \left(\frac{p^f(c,x) - \tau c}{p^f(c,x)}\right) = \frac{\mathcal{E}_D\left(p^f(c,x)/P(x)\right)}{\mathcal{E}_D\left(p^h(c,x)/P(x)\right)}. \tag{A.3}$$

Recall that the price elasticity of demand , $\mathcal{E}_D\left(p/P(x)\right) = 1 + \mathcal{E}_\Lambda\left(p/P(x)\right)$, is increasing by assumption (MSLD). Hence, if $p^h(c,x) < p^f(c,x)$, then (A.3) implies that

$$\frac{p^h(c,x) - c}{p^h(c,x)} > \frac{p^f(c,x) - \tau c}{p^f(c,x)} \Leftrightarrow \frac{p^h(c,x)}{p^f(c,x)} > \frac{1}{\tau} \Leftrightarrow p^f(c,x) < \tau p^h(c,x).$$

□

A.6 The Proof of Proposition 6

Proof. (Sketch) Without loss of generality, I normalize N to 1 throughout the proof. Now, let the profit function of an exporting firm be given by:

$$\pi = (p^h - c)\ell(x)\frac{E}{p^h}\Lambda\left(\frac{p^h}{P(x)}\right) - F + N \cdot \left[(p^f - \tau c)\ell(x)\frac{E}{p^f}\Lambda\left(\frac{p^f}{P(x)}\right) - F^{ex}\right]$$

$$= \left[\left(\frac{p^h - c}{p^h}\right)\Lambda\left(\frac{p^h}{P(x)}\right) + N \cdot \left(\frac{p^f - \tau c}{p^f}\right)\Lambda\left(\frac{p^f}{P(x)}\right)\right] E\,\ell(x) - F - NF^{ex}.$$

The proof can be based on theorems for monotone comparative statics. In particular, if $\pi\left(p^h, p^f, x; c\right)$ has increasing differences in $\left((p^h, p^f, x), c\right)$ and is supermodular in $\left(p^h, p^f, x\right)$, then: i) $\frac{\partial p^h(c)}{\partial c} \geq 0$; ii)$\frac{\partial p^f(c)}{\partial c} \geq 0$; iii) $\frac{\partial x(c)}{\partial c} \geq 0$. Note that if $\pi\left(p^h, p^f, x; c\right)$ is twice continuously differentiable, then $\left(p^h, p^f, x, c\right)$ has increasing differences in $\left((p^h, p^f, x), c\right)$ iff the following conditions hold: i) $\frac{\partial^2 \pi}{\partial p^h \partial c} \geq 0$; ii) $\frac{\partial^2 \pi}{\partial p^f \partial c} \geq 0$; iii) $\frac{\partial^2 \pi}{\partial x \partial c} \geq 0$. In addition, $\pi\left(p^h, p^f, x, c\right)$ is supermodular in $\left(p^h, p^f, x\right)$ iff: i) $\frac{\partial^2 \pi}{\partial p^h \partial p^f} \geq 0$; ii) $\frac{\partial^2 \pi}{\partial p^h \partial x} \geq 0$; iii) $\frac{\partial^2 \pi}{\partial p^f \partial x} \geq 0$.

First of all, recall that

$$\frac{\partial \pi}{\partial p^h} = E\ell(x)\left[\frac{c}{(p^h)^2}\Lambda\left(\frac{p^h}{P(x)}\right) + \frac{p^h - c}{p^h P(x)}\Lambda'\left(\frac{p^h}{P(x)}\right)\right],$$

$$\frac{\partial \pi}{\partial p^f} = E\ell(x)\left[\frac{\tau c}{(p^f)^2}\Lambda\left(\frac{p^f}{P(x)}\right) + \frac{p^f - \tau c}{p^f P(x)}\Lambda'\left(\frac{p^f}{P(x)}\right)\right].$$

Hence, one can easily verify that

$$\frac{\partial^2 \pi}{\partial p^h \partial c} = E\ell(x)\left[\frac{\Lambda\left(p^h/P(x)\right)}{(p^h)^2} - \frac{\Lambda'\left(p^h/P(x)\right)}{p^h P(x)}\right] = \frac{E\ell(x)\Lambda\left(p^h/P(x)\right)}{(p^h)^2}\left[1 + \mathcal{E}_\Lambda\left(\frac{p^h}{P(x)}\right)\right] > 0,$$

$$\frac{\partial^2 \pi}{\partial p^f \partial c} = \tau E\ell(x)\left[\frac{\Lambda\left(p^f/P(x)\right)}{(p^f)^2} - \frac{\Lambda'\left(p^f/P(x)\right)}{p^f P(x)}\right] = \tau\frac{E\ell(x)\Lambda\left(p^f/P(x)\right)}{(p^f)^2}\left[1 + \mathcal{E}_\Lambda\left(\frac{p^f}{P(x)}\right)\right] > 0.$$

The next step is to show that $\frac{\partial^2 \pi}{\partial x \partial c} \geq 0$. First, recall that

$$\frac{\partial \pi}{\partial x} = E\ell'(x)\left[\frac{p^h - c}{p^h}\Lambda\left(\frac{p^h}{P(x)}\right) + \frac{p^f - \tau c}{p^f}\Lambda\left(\frac{p^f}{P(x)}\right)\right]$$

$$-E\ell(x)\frac{P'(x)}{P(x)}\left[\frac{p^h - c}{P(x)}\Lambda'\left(\frac{p^h}{P(x)}\right) + \frac{p^f - \tau c}{P(x)}\Lambda'\left(\frac{p^f}{P(x)}\right)\right].$$

Recall also that $\partial \pi / \partial x = 0$ under FOCs. Now, note that

$$\frac{\partial^2 \pi}{\partial x \partial c} = E\ell'(x)\left(-\frac{\Lambda\left(\frac{p^h}{P(x)}\right)}{p^h} - \tau\frac{\Lambda\left(\frac{p^f}{P(x)}\right)}{p^f}\right) - E\ell(x)\frac{P'(x)}{P(x)}\left(-\frac{\Lambda'\left(\frac{p^h}{P(x)}\right)}{P(x)} - \tau\frac{\Lambda'\left(\frac{p^f}{P(x)}\right)}{P(x)}\right).$$

Recalling that $P'(x) > 0$ yields,

$$\frac{\partial^2 \pi}{\partial x \partial c} \geq 0 \Leftrightarrow -\frac{\ell'(x)}{\ell(x)}\frac{P(x)}{P'(x)}\left(\frac{\Lambda\left(p^h/P(x)\right)/p^h + \tau\Lambda\left(p^f/P(x)\right)/p^f}{-\Lambda'\left(p^h/P(x)\right)/P\left(x\right) - \tau\Lambda'\left(p^f/P(x)\right)/P\left(x\right)}\right) \geq 1.$$

Taking into account FOCs, one can readily verify that

$$\frac{\partial^2 \pi}{\partial x \partial c} \geq 0 \Leftrightarrow \frac{\Lambda\left(\frac{p^h}{P(x)}\right) + \Lambda\left(\frac{p^f}{P(x)}\right)}{\frac{c}{p^h}\Lambda\left(\frac{p^h}{P(x)}\right) + \frac{\tau \cdot c}{p^f}\Lambda\left(\frac{p^f}{P(x)}\right)} - 1 \leq \frac{\frac{\Lambda\left(\frac{p^h}{P(x)}\right)}{p^h} + \tau\frac{\Lambda\left(\frac{p^f}{P(x)}\right)}{p^f}}{\Lambda\left(\frac{p^h}{P(x)}\right)\frac{c/p^h}{p^h - c} + \Lambda\left(\frac{p^f}{P(x)}\right)\frac{\tau c/p^f}{p^f - c}}.$$

Tedious derivations result in:

$$\frac{\partial^2 \pi}{\partial x \partial c} \geq 0 \Leftrightarrow \left[\Lambda\left(\frac{p^f}{P(x)}\right)\right]^2 \frac{p^h \tau / p^f}{p^f - c}\left[1 - \tau\right] \leq$$

$$\Lambda\left(\frac{p^f}{P(x)}\right)\frac{\tau}{p^f-c}\Lambda\left(\frac{p^h}{P(x)}\right)\left[1-p^h/p^f\right]+\frac{\Lambda\left(\frac{p^h}{P(x)}\right)}{p^h-c}\Lambda\left(\frac{p^f}{P(x)}\right)\left[\frac{p^h\tau}{p^f}-1\right].$$

Recall that $\tau \geq 1$, $p^h/p^f < 1$, and $p^h > p^f/\tau$ by MSLD (see Proposition 5). Hence, it is straightforward to see that the inequality above holds.

It can be inferred that $\left(p^h, p^f, x, c\right)$ has increasing differences in $\left((p^h, p^f, x), c\right)$. The last step is to show that π is supermodular in $\left(p^h, p^f, x\right)$.

Recall that

$$\frac{\partial \pi}{\partial p^h}=\frac{E\ell(x)}{(p^h)^2}\left[c\,\Lambda\left(\frac{p^h}{P(x)}\right)+(p^h-c)\frac{p^h}{P(x)}\Lambda'\left(\frac{p^h}{P(x)}\right)\right].$$

Hence, one can readily verify that

$$\frac{\partial^2 \pi}{\partial p^h \partial x}=\frac{E\ell'(x)}{(p^h)^2}\left[c\,\Lambda(\cdot)+(p^h-c)\frac{p^h}{P(x)}\Lambda'(\cdot)\right]$$

$$+\frac{E\ell(x)}{(p^h)^2}\left[-c\frac{p^h\Lambda'(\cdot)}{P(x)}\frac{P'(x)}{P(x)}+(p^h-c)p^h\left(-\frac{p^h\Lambda''(\cdot)}{P^2(x)}\frac{P'(x)}{P(x)}-\frac{\Lambda'(\cdot)}{P(x)}\frac{P'(x)}{P(x)}\right)\right].$$

Taking into account that $\partial\pi/\partial p^h = 0$ under the FOCs yields

$$\frac{\partial^2 \pi}{\partial p^h \partial x}\geq 0 \Leftrightarrow -c\frac{p^h\Lambda'(\cdot)}{P(x)}\frac{P'(x)}{P(x)}+(p^h-c)p^h\left(-\frac{p^h\Lambda''(\cdot)}{P^2(x)}\frac{P'(x)}{P(x)}-\frac{\Lambda'(\cdot)}{P(x)}\frac{P'(x)}{P(x)}\right)\geq 0$$

$$\Leftrightarrow -c\frac{p^h\Lambda'(\cdot)}{P(x)}+(p^h-c)p^h\left(-\frac{p^h\Lambda''(\cdot)}{P^2(x)}-\frac{\Lambda'(\cdot)}{P(x)}\right)\geq 0$$

$$\Leftrightarrow \frac{p^h-c}{c}\left(-\frac{p^h\Lambda''(\cdot)}{P(x)\Lambda'(\cdot)}-1\right)\leq 1 \Leftrightarrow -\frac{p^h\Lambda''(\cdot)}{P(x)\Lambda'(\cdot)}\leq \mathcal{E}_\Lambda(p^h/P(x))+1.$$

Recall that MSLD implies $\mathcal{E}'_\Lambda(z)>0 \Leftrightarrow \mathcal{E}_\Lambda(z)+1 > -z\Lambda''(z)/\Lambda'(z)$. Hence, $\frac{\partial^2\pi}{\partial p^h\partial x}>0$. Similar argument applies to verify that $\frac{\partial^2\pi}{\partial p^f\partial x}>0$. □

A.7 The Proof of Proposition 7

Let $\Psi(x)\equiv\int_{\Upsilon(c(x)/P(x))}^{\bar{z}}\frac{\Lambda(\xi)}{\xi}\mathrm{d}\xi$. Then, we have

$$\frac{\mathrm{d}W(x)}{W(x)}=-\frac{\mathrm{d}P^I(x)}{P^I(x)}=-\frac{\mathrm{d}P(x)}{P(x)}+\Psi(x)\mathrm{d}M(x)+M(x)\mathrm{d}\Psi(x).$$

Now, one can readily verify that $\mathrm{d}\Psi(x)$ can be decomposed as follows:

$$\mathrm{d}\Psi(x)=\Lambda(\cdot)\mathcal{E}_\Upsilon(\cdot)\left[\frac{\mathrm{d}P(x)}{P(x)}-\frac{\mathrm{d}c(x)}{c(x)}\right],$$

where $\mathcal{E}_\Upsilon(z)\equiv\frac{z\Upsilon'(z)}{\Upsilon(\cdot)}$.

Now, recall that $M(x) = M^e g\left(c(x)\right) c'(x)$. One can readily verify that routine derivations result in the following decomposition:

$$\frac{\mathrm{d}M(x)}{M(x)} = \mathcal{E}_{\Lambda}(\cdot)\mathcal{E}_{\Upsilon}(\cdot)\left[\frac{\mathrm{d}c(x)}{c(x)} - \frac{\mathrm{d}P(x)}{P(x)}\right].$$

Now, substituting these two auxiliary expressions into the original one yields,

$$\frac{\mathrm{d}W(x)}{W(x)} = -\left(1-\vartheta(x)\right)\frac{\mathrm{d}P(x)}{P(x)} - \vartheta(x)\frac{\mathrm{d}c(x)}{c(x)},$$

where $\vartheta(x) \equiv \mathcal{E}_{\Upsilon}(\cdot) - \Phi(x)M(x)\mathcal{E}_{\Lambda}(\cdot)\mathcal{E}_{\Upsilon}(\cdot)$.

A.8 The Proof of Proposition 8

Note that (niche-specific) ideal price index takes the following form:

$$\log P^I(x) = const + \log P(x) - M(x)\left[\int_{\frac{p^h}{P(x)}}^{\bar{z}} \frac{\Lambda(\xi)}{\xi}\mathrm{d}\xi + \mathbb{I}_{\{x\leq\tilde{x}\}}\cdot N\int_{\frac{p^f}{P(x)}}^{\bar{z}} \frac{\Lambda(\xi)}{\xi}\mathrm{d}\xi\right]$$

Let $\Xi(x) \equiv \int_{\Upsilon\left(\frac{c(x)}{P(x)}\right)}^{\bar{z}} \frac{\Lambda(\xi)}{\xi}\mathrm{d}\xi + \mathbb{I}_{\{x\leq\tilde{x}\}}\cdot\int_{\Upsilon\left(\frac{\tau c(x)}{P(x)}\right)}^{\bar{z}} \frac{\Lambda(\xi)}{\xi}\mathrm{d}\xi$. Then, we have

$$\frac{\mathrm{d}W(x)}{W(x)} = -\frac{\mathrm{d}P^I(x)}{P^I(x)} = -\frac{\mathrm{d}P(x)}{P(x)} + \Xi(x)\mathrm{d}M(x) + M(x)\mathrm{d}\Xi(x).$$

Now, routine derivations imply the following decomposition for $\mathrm{d}\Xi(x)$:

$$\mathrm{d}\Xi(x) = \left(\Lambda^h(\cdot)\mathcal{E}_{\Upsilon^h}(\cdot) + N\Lambda^f(\cdot)\mathcal{E}_{\Upsilon^f}(\cdot)\right)\left[\frac{\mathrm{d}P(x)}{P(x)} - \frac{\mathrm{d}c(x)}{c(x)}\right] - N\Lambda^f(\cdot)\mathcal{E}_{\Upsilon^f}(\cdot)\frac{\mathrm{d}\tau}{\tau},$$

where $\mathcal{E}_{\Upsilon^h}\left(\frac{c(x)}{P(x)}\right) \equiv \frac{c(x)}{P(x)}\frac{\Upsilon'(c(x)/P(x))}{\Upsilon(c(x)/P(x))}$, $\mathcal{E}_{\Upsilon^f}\left(\frac{c(x)}{P(x)}\right) \equiv \frac{\tau c(x)}{P(x)}\frac{\Upsilon'(\tau c(x)/P(x))}{\Upsilon(\tau c(x)/P(x))}$,$\Lambda^h(\cdot) \equiv \Lambda(p^h/P(x))$, $\Lambda^f(\cdot) \equiv \Lambda(p^f/P(x))$.

Hence, one can readily verify that

$$M(x)\mathrm{d}\Xi(x) = \left(\frac{\Lambda^h(\cdot)\mathcal{E}_{\Upsilon^h}(\cdot) + N\Lambda^f(\cdot)\mathcal{E}_{\Upsilon^f}(\cdot)}{\Lambda^h(\cdot) + N\Lambda^f(\cdot)}\right)\left[\frac{\mathrm{d}P(x)}{P(x)} - \frac{\mathrm{d}c(x)}{c(x)}\right] - \frac{N\Lambda^f(\cdot)\mathcal{E}_{\Upsilon^f}(\cdot)}{\Lambda^h(\cdot) + N\Lambda^f(\cdot)}\frac{\mathrm{d}\tau}{\tau}.$$

Now, recall that $M(x) = M^e g\left(c(x)\right) c'(x)$. One can readily verify that routine derivations result in the following decomposition:

$$\frac{\mathrm{d}M(x)}{M(x)} = \left(\frac{\Lambda^h\mathcal{E}_{\Lambda^h}\mathcal{E}_{\Upsilon^h} + N\Lambda^f\mathcal{E}_{\Lambda^f}\mathcal{E}_{\Upsilon^f}}{\Lambda^h + N\Lambda^f}\right)\left[\frac{\mathrm{d}c(x)}{c(x)} - \frac{\mathrm{d}P(x)}{P(x)}\right] + \frac{N\Lambda^f\mathcal{E}_{\Lambda^f}\mathcal{E}_{\Upsilon^f}}{\Lambda^h + N\Lambda^f}\frac{\mathrm{d}\tau}{\tau},$$

where $\mathcal{E}_{\Lambda^h} \equiv \frac{-\Lambda'(p^h/P(x))}{\Lambda(p^h/P(x))}\frac{p^h}{P(x)}$, $\mathcal{E}_{\Lambda^f} \equiv \frac{-\Lambda'(p^f/P(x))}{\Lambda(p^f/P(x))}\frac{p^f}{P(x)}$.

Now, substituting these two auxiliary expressions into the original one yields,

$$\frac{\mathrm{d}W(x)}{W(x)} = -\frac{\mathrm{d}P(x)}{P(x)}\left(1-\phi(x)\right) - \frac{\mathrm{d}c(x)}{c(x)}\phi(x) - \frac{\mathrm{d}\tau}{\tau}\left[1-\Xi(x)M(x)\mathcal{E}_{\Lambda f}\right]\frac{N\Lambda^f\mathcal{E}_{\Upsilon f}}{\Lambda^h+N\Lambda^f}\mathbb{I}_{\{x\leq\tilde{x}\}},$$

where $\phi(x) \equiv \frac{\Lambda^h\mathcal{E}_{\Upsilon h}+N\Lambda^f\mathcal{E}_{\Upsilon f}\mathbb{I}_{\{x\leq\tilde{x}\}}}{\Lambda^h+N\Lambda^f\mathbb{I}_{\{x\leq\tilde{x}\}}} - \Xi(x)M(x)\frac{\Lambda^h\mathcal{E}_{\Lambda h}\mathcal{E}_{\Upsilon h}+N\Lambda^f\mathcal{E}_{\Lambda f}\mathcal{E}_{\Upsilon f}\mathbb{I}_{\{x\leq\tilde{x}\}}}{\Lambda^h+N\Lambda^f\mathbb{I}_{\{x\leq\tilde{x}\}}}$.

A.9 The Proof of Proposition 9

To prove this Proposition, I again impose the assumption of $\tilde{\Lambda}''(\cdot) < 0$ that implies decreasing marginal revenue (or concavity of a firm's profit w.r.t. q). I also exploit an alternative yet equivalent representation of HSA demand systems via an inverse demand function (instead of a direct demand function) and the quantity aggregator (instead of the price aggregator).

First of all, let me provide a discussion of the existence and uniqueness of equilibria. Note that taking into account a firm's FOC w.r.t. q, the *zero cutoff profit* (ZCP) and the *free entry* (FE) conditions take the following form:

$$\left[\tilde{\Lambda}\left(\tilde{\Lambda}'^{-1}\left(\frac{c^*Q}{E}\right)\right) - \frac{c^*Q}{E}\tilde{\Lambda}'^{-1}\left(\frac{c^*Q}{E}\right)\right]EL - F = 0.$$

$$\int\limits_{c_{\min}}^{c^*}\left\{\left[\tilde{\Lambda}\left(\tilde{\Lambda}'^{-1}\left(\frac{cQ}{E}\right)\right) - \frac{cQ}{E}\tilde{\Lambda}'^{-1}\left(\frac{cQ}{E}\right)\right]EL - F\right\}\mathrm{d}G(c) = F^e.$$

These two conditions jointly determine equilibrium values of Q and c^*. One can readily verify that a firm's profit can be represented in the following way

$$\Pi(cQ) = \left[1-\mathcal{E}_{\tilde{\Lambda}}\left(\tilde{\Lambda}'^{-1}\left(cQ/E\right)\right)\right]\tilde{\Lambda}\left(\tilde{\Lambda}'^{-1}\left(cQ/E\right)\right)EL - F.$$

As will be shown further in the proof, if $\tilde{\Lambda}''(\cdot) < 0$, then a firm's profit is decreasing in cQ. Now, it is straightforward to see that if, for example, Q increases, then the ZCP condition requires c^* to decrease. Hence, the ZCP curve is downward sloping in (c^*, Q) – plane. Similarly, one can readily verify that if Q increases, then the FE condition requires c^* to increase as well. Hence, the FE curve is upward sloping in (c^*, Q) – plane. So, if ZCP and FE curves intersect, then they intersect at only one point at most.

As for existence, one can use a similar argument to that provided in Aghion, Bergeaud, Lequien, and Melitz (2018) for the case of additively separable preferences. In particular, it is sufficient to show that the ZCP curve lies above the FE curve for sufficiently low values of c^*, and that the ZCP curve lies above the FE curve for sufficiently high values of c^*. So, one can readily verify that if c^* becomes close to $c_{\min}$, then Q becomes arbitrarily close to 0 in order to ensure that the FE condition continues to hold. At the same time, if c^* becomes close to $c_{\min}$,

the ZCP condition implies that Q is still bounded away from 0. Now, note that the ZCP must be below the curve $Q = \bar{z}E/c^*$, where $\bar{z}$ denotes the choke price ($\tilde{\Lambda}'(0) = \bar{z}$). One can readily verify that for sufficiently high $\bar{z}$, the curve $Q = \bar{z}E/c^*$ must cross the FE curve at sufficiently high c^*. It is trivial to infer that, at this point, the FE curve lies above the ZCP curve.

Proof. **i**) To show that firms' prices, p, are decreasing in c, notice that the indirect demand function and a firm's FOC jointly imply:

$$p = c/\mathcal{E}_{\tilde{\Lambda}}\left(\tilde{\Lambda}'^{-1}\left(cQ/E\right)\right).$$

Note also that

$$\frac{\mathrm{d}p}{\mathrm{d}c} = \left[\mathcal{E}_{\tilde{\Lambda}}(\cdot) - \frac{cQ}{E}\mathcal{E}'_{\tilde{\Lambda}}/\tilde{\Lambda}''(\cdot)\right] / \left[\mathcal{E}_{\tilde{\Lambda}}(\cdot)\right]^2$$

$$\frac{\mathrm{d}p}{\mathrm{d}c} > 0 \Leftrightarrow 1 > \frac{\mathcal{E}'_{\tilde{\Lambda}}(\cdot)}{\mathcal{E}_{\tilde{\Lambda}}(\cdot)}\frac{\tilde{\Lambda}'(\cdot)}{\tilde{\Lambda}''(\cdot)}$$

Recall that $\mathcal{E}'_{\tilde{\Lambda}}\left(\frac{q}{Q}\right) = \left[\left[\tilde{\Lambda}''\left(\frac{q}{Q}\right)\frac{q}{Q} + \tilde{\Lambda}'\left(\frac{q}{Q}\right)\right]\tilde{\Lambda}\left(\frac{q}{Q}\right) - \tilde{\Lambda}'\left(\frac{q}{Q}\right)\frac{q}{Q}\tilde{\Lambda}'\left(\frac{q}{Q}\right)\right] / \left[\tilde{\Lambda}\left(q/Q\right)\right]^2$. So, the inequality above takes the following form:

$$\frac{\mathrm{d}p}{\mathrm{d}c} > 0 \Leftrightarrow 1 > \frac{\left\{\left[\tilde{\Lambda}''\left(\frac{q}{Q}\right)\frac{q}{Q} + \tilde{\Lambda}'\left(\frac{q}{Q}\right)\right]\tilde{\Lambda}\left(\frac{q}{Q}\right) - \tilde{\Lambda}'\left(\frac{q}{Q}\right)\frac{q}{Q}\tilde{\Lambda}'\left(\frac{q}{Q}\right)\right\}\tilde{\Lambda}'\left(\frac{q}{Q}\right)}{\left[\tilde{\Lambda}\left(q/Q\right)\right]^2\mathcal{E}_{\tilde{\Lambda}}(\cdot)\tilde{\Lambda}''\left(q/Q\right)}$$

$$\Leftrightarrow 1 > \frac{\tilde{\Lambda}'\left(\frac{q}{Q}\right)\left\{\left[\frac{\tilde{\Lambda}''(q/Q)}{\tilde{\Lambda}'(q/Q)}\frac{q}{Q} + 1\right] - \mathcal{E}_{\tilde{\Lambda}}(\cdot)\right\}\tilde{\Lambda}'\left(\frac{q}{Q}\right)}{\tilde{\Lambda}\left(q/Q\right)\mathcal{E}_{\tilde{\Lambda}}(\cdot)\tilde{\Lambda}''\left(q/Q\right)} \Leftrightarrow 1 > \frac{\mathcal{E}_{\tilde{\Lambda}'}(\cdot) + \mathcal{E}_{\tilde{\Lambda}}(\cdot) - 1}{\mathcal{E}_{\tilde{\Lambda}'}(\cdot)}.$$

Recall that $\tilde{\Lambda}''(\cdot) < 0$ (thereby implying that $\mathcal{E}_{\tilde{\Lambda}'}(\cdot) > 0$). Hence, the inequality above is always satisfied because $1 > \mathcal{E}_{\tilde{\Lambda}}(\cdot)$. Notice that this condition is assumed to be always satisfied by basic assumptions on $\tilde{\Lambda}(\cdot)$ and implies that an inverse demand function has a negative slope (see also the proof of Proposition 3 for more details).

ii) To show that a firm's "market share", $\tilde{\Lambda}(\cdot)$, is decreasing in c, notice that

$$\tilde{\Lambda}\left(q(c,Q)/Q\right) = \tilde{\Lambda}\left(\tilde{\Lambda}'^{-1}\left(cQ/E\right)\right)$$

$$\frac{\mathrm{d}}{\mathrm{d}c}\left[\tilde{\Lambda}(\cdot)\right] = \frac{\tilde{\Lambda}'(\cdot)}{\tilde{\Lambda}''(\cdot)}\frac{Q}{E}.$$

Recall that $\tilde{\Lambda}''(\cdot) < 0$. Hence, $\frac{\mathrm{d}}{\mathrm{d}c}\left[\tilde{\Lambda}(\cdot)\right] < 0$.

iii) To show that firms' quantities sold per capita, q, are decreasing in c, notice that[39]

$$q = Q\tilde{\Lambda}'^{-1}\left(cQ/E\right).$$

$$\frac{\mathrm{d}q}{\mathrm{d}c} = \frac{Q}{\tilde{\Lambda}''(\cdot)}\frac{Q}{E}.$$

One can readily verify that if $\tilde{\Lambda}''(\cdot) < 0$, then $\mathrm{d}q/\mathrm{d}c < 0$.

iv) To show that firms' total revenues, $p\,q\,L$, are decreasing in c, notice that

$$R \equiv p(q)qL = \tilde{\Lambda}\left(q/Q\right) = \tilde{\Lambda}\left(\tilde{\Lambda}'^{-1}\left(cQ/E\right)\right)EL.$$

$$\mathrm{d}R/\mathrm{d}c = QL\tilde{\Lambda}'(\cdot)/\tilde{\Lambda}''(\cdot).$$

One can readily verify that if $\tilde{\Lambda}''(\cdot) < 0$, then $\mathrm{d}R/\mathrm{d}c < 0$.

v) To show that firms' total profits, $\Pi(\cdot)$, are decreasing in c, notice that a firm's profit can be re-written as follows[40]

$$\Pi(c) = \max_q \left\{\left[\tilde{\Lambda}\left(q/Q\right)E/q - c\right]qL - F\right\} = \max_q \left\{\left[\tilde{\Lambda}\left(q/Q\right)E - cq\right]L - F\right\}$$

$$= \left[\tilde{\Lambda}\left(\frac{q(c)}{Q}\right) - \tilde{\Lambda}'\left(\frac{q(c)}{Q}\right)\frac{q(c)}{Q}\right]EL - F = \left[1 - \frac{q(c)}{Q}\tilde{\Lambda}'\left(\frac{q(c)}{Q}\right)/\tilde{\Lambda}\left(\frac{q(c)}{Q}\right)\right]\tilde{\Lambda}\left(\frac{q(c)}{Q}\right)EL - F$$

$$= \left[1 - \mathcal{E}_{\tilde{\Lambda}}\left(\frac{q(c)}{Q}\right)\right]\tilde{\Lambda}\left(\frac{q(c)}{Q}\right)EL - F = EL\,\tilde{\Lambda}\left(\frac{q(c)}{Q}\right)/\Upsilon\left(\frac{q(c)}{Q}\right) - F.$$

Recall that it has just been shown that $q(c)$ is decreasing in c. Now, notice that

$$\left[\frac{\tilde{\Lambda}(z)}{\Upsilon(z)}\right]'_z = \frac{\tilde{\Lambda}'(z)\Upsilon(z) - \tilde{\Lambda}(z)\Upsilon'(z)}{\Upsilon^2(z)}$$

$$\left[\frac{\tilde{\Lambda}(z)}{\Upsilon(z)}\right]'_z > 0 \Leftrightarrow \frac{z\tilde{\Lambda}'(z)}{\tilde{\Lambda}(z)} > \frac{z\Upsilon'(z)}{\Upsilon(z)} \Leftrightarrow \mathcal{E}_{\tilde{\Lambda}}(z) > \frac{-\mathcal{E}_{\tilde{\Lambda}}(z)\mathcal{E}_{\tilde{\Lambda}'}(z) + \mathcal{E}_{\tilde{\Lambda}}(z) - \left[\mathcal{E}_{\tilde{\Lambda}}(z)\right]^2}{\left[1 - \mathcal{E}_{\tilde{\Lambda}}(z)\right]}$$

$$\Leftrightarrow 1 > \frac{1 - \mathcal{E}_{\tilde{\Lambda}}(z) - \mathcal{E}_{\tilde{\Lambda}'}(z)}{1 - \mathcal{E}_{\tilde{\Lambda}}(z)} \Leftrightarrow 0 > -\mathcal{E}_{\tilde{\Lambda}'}(z).$$

[39] Note also that product-level performance measures can alternatively be written in terms of the elasticities of demand and marginal revenue (in the spirit of Mayer, Melitz, and Ottaviano 2016). For example, one can readily verify that the elasticity of q w.r.t. marginal cost can be alternatively written as 1 over elasticity of marginal revenue, $\mathcal{E}_{q,c} = -1/\mathcal{E}_{\phi}$, where $\mathcal{E}_{\phi}(q) \equiv -qR''(q)/R'(q) = -\left(q/Q\right)\tilde{\Lambda}''\left(q/Q\right)/\tilde{\Lambda}'\left(q/Q\right)$.

[40] Alternatively, $\Pi(c) = \left[\tilde{\Lambda}\left(\tilde{\Lambda}'^{-1}\left(cQ/E\right)\right) - \left(cQ/E\right)\tilde{\Lambda}'^{-1}\left(cQ/E\right)\right]EL - F$.

Now, recall that $\mathcal{E}_{\tilde{\Lambda}'}(q/Q) \equiv \frac{-\tilde{\Lambda}''(q/Q)}{\tilde{\Lambda}'(q/Q)}\frac{q}{Q}$. Hence, the inequality above holds iff $\tilde{\Lambda}''(q/Q) < 0$.[41] Recall that it has just been shown that $q(c)$ is decreasing in c. Hence, it can be concluded that a firm's profit, $\Pi(c)$, is decreasing in marginal cost.

vi) To show that firms' total costs, $TC \equiv c\,q\,L + F$, are increasing (decreasing) in c iff $\mathcal{E}_{\tilde{\Lambda}'}(\cdot) > 1$ ($\mathcal{E}_{\tilde{\Lambda}'}(\cdot) < 1$), notice that

$$c\,q\,L + F = cQ\tilde{\Lambda}'^{-1}(cQ/E)\,L + F.$$

$$\frac{\mathrm{d}}{\mathrm{d}c}[TC] = \frac{cQ}{\tilde{\Lambda}''(\cdot)}\frac{QL}{E} + \tilde{\Lambda}'^{-1}\left(\frac{cQ}{E}\right)QL.$$

$$\frac{\mathrm{d}}{\mathrm{d}c}[TC] \gtreqless 0 \Leftrightarrow 1 \gtreqless -\frac{\tilde{\Lambda}'(\cdot)}{(q/Q)\,\tilde{\Lambda}''(\cdot)} \Leftrightarrow \mathcal{E}_{\tilde{\Lambda}'}(q/Q) \gtreqless 1.$$

Hence, it can be concluded that firms' total costs, $TC \equiv c\,q\,L + F$, are increasing (decreasing) in c iff $\mathcal{E}_{\tilde{\Lambda}'}(\cdot) > (<)1$.[42]

vii) To show that if, in addition, MSLD holds, then firms' markups, p/c, are decreasing in c, notice that the indirect demand function and a firm's FOC jointly imply:

$$p/c = 1/\mathcal{E}_{\tilde{\Lambda}}(q(c,Q)/Q) = 1/\mathcal{E}_{\tilde{\Lambda}}\left(\tilde{\Lambda}'^{-1}(cQ/E)\right).$$

Recall that MSLD implies $\mathcal{E}'_{\tilde{\Lambda}}(\cdot) < 0 \Leftrightarrow \mathcal{E}_{\tilde{\Lambda}'}(\cdot) + \mathcal{E}_{\tilde{\Lambda}}(\cdot) > 1$ and also ensures that $\tilde{\Lambda}'^{-1}(\cdot)$ is decreasing. Hence, it is straightforward to see that a firm's markup, p/c, is decreasing in marginal cost under MSLD. □

A.10 The Proof of Proposition 10

To show that an increase in total market size leads to a higher measure of competition in a new equilibrium, I will exploit an alternative yet equivalent representation of HSA demand systems via an inverse demand function (instead of a direct demand function) and the quantity aggregator (instead of the price aggregator).[43] It is straightforward to see that under this formulation, a higher level of competition is captured by a higher value of a quantity aggregator, Q. Note that the zero-cutoff-profit (ZCP) and the free entry (FE) conditions can be represented as the

[41] Recall also that $\tilde{\Lambda}''(\cdot) < 0$ implies that a firm's profit is strictly concave in q and that marginal revenue function is decreasing in q. It is straightforward to see that it is a less restrictive condition than MSLD.

[42] One can also readily verify that $\mathcal{E}_{\tilde{\Lambda}'}(q/Q) \gtreqless 1 \Longleftrightarrow R'(q)q \lesseqgtr 0$.

[43] It is straightforward to show that an increase in L also leads to a lower value of P if we represent HSA demand systems via direct demand function and price aggregator. For brevity, this alternative proof is omitted. Details are available upon request.

system of two equations with two variables, Q and c^*:

$$\Phi_1(Q, c^*; L) \equiv \left[\tilde{\Lambda}\left(\tilde{\Lambda}'^{-1}\left(\frac{c^*Q}{E}\right)\right) - \frac{c^*Q}{E}\tilde{\Lambda}'^{-1}\left(\frac{c^*Q}{E}\right)\right] EL - F = 0, \tag{A.4}$$

$$\Phi_2(Q, c^*; L) \equiv \int_{c_{\min}}^{c^*} \left\{\left[\tilde{\Lambda}\left(\tilde{\Lambda}'^{-1}\left(\frac{cQ}{E}\right)\right) - \frac{cQ}{E}\tilde{\Lambda}'^{-1}\left(\frac{cQ}{E}\right)\right] EL - F\right\} \mathrm{d}G(c) - F^e = 0. \tag{A.5}$$

Exploiting the Implicit Function Theorem yields,

$$\begin{pmatrix} \mathrm{d}Q/\mathrm{d}L \\ \mathrm{d}c^*/\mathrm{d}L \end{pmatrix} = -\begin{pmatrix} \Phi'_{1,Q} & \Phi'_{1,c^*} \\ \Phi'_{2,Q} & \Phi'_{2,c^*} \end{pmatrix}^{-1} \begin{pmatrix} \Phi'_{1,L} \\ \Phi'_{2,L} \end{pmatrix},$$

which, in turn, implies

$$\frac{\mathrm{d}Q}{\mathrm{d}L} = \frac{-\Phi'_{1,L}\Phi'_{2,c^*} + \Phi'_{2,L}\Phi'_{1,c^*}}{\Delta}, \qquad \frac{\mathrm{d}c^*}{\mathrm{d}L} = \frac{-\Phi'_{1,Q}\Phi'_{2,L} + \Phi'_{2,Q}\Phi'_{1,L}}{\Delta},$$

where $\Delta \equiv \Phi'_{1,Q}\Phi'_{2,c^*} - \Phi'_{1,c^*}\Phi'_{2,Q}$.

One can readily verify that $\Phi'_{2,c^*} = 0$. Hence, $\Delta = -\Phi'_{1,c^*}\Phi'_{2,Q}$. As a result,

$$\mathrm{d}Q/\mathrm{d}L = \Phi'_{2,L}\Phi'_{1,c^*}/\Delta = \Phi'_{2,L}\Phi'_{1,c^*}/\left[-\Phi'_{1,c^*}\Phi'_{2,Q}\right] = -\Phi'_{2,L}/\Phi'_{2,Q}.$$

As for the sign of Δ, notice that:

$$\Phi'_{1,c^*} = \left[\frac{\tilde{\Lambda}'(\cdot)}{\tilde{\Lambda}''(\cdot)}\frac{Q}{E} - \frac{Q}{E}\left(\tilde{\Lambda}'^{-1}\left(\frac{c^*Q}{E}\right) + \frac{c^*Q/E}{\tilde{\Lambda}''(\cdot)}\right)\right] EL$$

$$\Phi'_{1,c^*} < 0 \Leftrightarrow \frac{\tilde{\Lambda}'(\cdot)}{\tilde{\Lambda}''(\cdot)} < \left(\tilde{\Lambda}'^{-1}\left(\frac{c^*Q}{E}\right) + \frac{\tilde{\Lambda}'(\cdot)}{\tilde{\Lambda}''(\cdot)}\right) \Leftrightarrow 0 < \tilde{\Lambda}'^{-1}\left(\frac{c^*Q}{E}\right)$$

As a result, it can be concluded that $\Phi'_{1,c^*} < 0$. Now, notice that

$$\Phi'_{2,Q} = \int_{c_{\min}}^{c^*} \left[\frac{\tilde{\Lambda}'\left(q/Q\right)}{\tilde{\Lambda}''\left(q/Q\right)}\frac{c}{E} - \frac{c}{E}\left(\tilde{\Lambda}'^{-1}\left(\frac{cQ}{E}\right) + \frac{Qc/E}{\tilde{\Lambda}''\left(q/Q\right)}\right)\right] ELg(c)\mathrm{d}c$$

$$\Leftrightarrow \frac{\tilde{\Lambda}'\left(q/Q\right)}{\tilde{\Lambda}''\left(q/Q\right)}\frac{c}{E} - \frac{c}{E}\left(\tilde{\Lambda}'^{-1}\left(\frac{cQ}{E}\right) + \frac{\tilde{\Lambda}'\left(q/Q\right)}{\tilde{\Lambda}''\left(q/Q\right)}\right) < 0 \Leftrightarrow -\frac{c}{E}\tilde{\Lambda}'^{-1}\left(\frac{cQ}{E}\right) < 0.$$

Hence, it can be readily concluded that $\Phi'_{2,Q} < 0$ and $\Delta = -\Phi'_{1,c^*}\Phi'_{2,Q} < 0$.

Now, note that

$$\Phi'_{2,L} = \int_{c_{\min}}^{c^*} \left\{ \left[\tilde{\Lambda}\left(\tilde{\Lambda}'^{-1}\left(\frac{cQ}{E} \right) \right) - \frac{cQ}{E} \tilde{\Lambda}'^{-1}\left(\frac{cQ}{E} \right) \right] E \right\} \mathrm{d}G(c) > 0.$$

Hence, it can be concluded that

$$\mathrm{d}Q/\mathrm{d}L = \Phi'_{2,L}\Phi'_{1,c^*}/\Delta = \Phi'_{2,L}\Phi'_{1,c^*}/\left[-\Phi'_{1,c^*}\Phi'_{2,Q} \right] = -\Phi'_{2,L}/\Phi'_{2,Q} > 0.$$

Note that there is a more straightforward way to show that $\mathrm{d}Q/\mathrm{d}L > 0$, regardless of whether MSLD holds or not. In particular, recall that after an increase in L, cutoff marginal cost, c^*, and common quantity aggregator, Q, change such that conditions (A.4) and (A.5) continue to hold. Note that a firm's profit is decreasing in $c \cdot Q$. Hence, the two equations above directly imply that it is impossible that equilibrium Q decreases in response to an increase in L. Specifically, consider two cases:

1) $\Delta c^* \geq 0$, $\Delta Q \leq 0$. One can readily verify that, in this case, the FE condition, (A.5), does not hold anymore.

2) $\Delta c^* < 0$, $\Delta Q \leq 0$. Note that the operational profit of a firm with a cutoff level of marginal cost will increase in this case. Hence, it is straightforward to see that the ZCP condition (A.4) does not hold anymore.

As a result, one can readily verify that Q must increase in response to an increase in L.

Now, I turn to the second part of the proposition. Recall that $\Delta < 0$. Hence, one can readily verify that

$$\mathrm{d}c^*/\mathrm{d}L < 0 \Leftrightarrow \Phi'_{2,Q}\Phi'_{1,L} > \Phi'_{1,Q}\Phi'_{2,L}.$$

We have already found that $\Phi'_{2,Q} < 0$, $\Phi'_{2,L} > 0$. Now, one can readily verify that $\Phi'_{1,L} > 0$:

$$\Phi'_{1,L} = \left[\tilde{\Lambda}\left(\tilde{\Lambda}'^{-1}\left(\frac{c^*Q}{E} \right) \right) - \frac{c^*Q}{E} \tilde{\Lambda}'^{-1}\left(\frac{c^*Q}{E} \right) \right] E > 0.$$

One can also readily verify that $\Phi'_{1,Q} < 0$ from:

$$\Phi'_{1,Q} = \left[\frac{\tilde{\Lambda}'(\cdot)}{\tilde{\Lambda}''(\cdot)} \frac{c^*}{E} - \frac{c^*}{E} \left(\tilde{\Lambda}'^{-1}\left(\frac{c^*Q}{E} \right) + \frac{Qc^*/E}{\tilde{\Lambda}''(\cdot)} \right) \right] EL.$$

Notice that $\Phi'_{2,Q}/\Phi'_{1,Q} < \Phi'_{2,L}/\Phi'_{1,L}$ is equivalent to

$$\frac{\int_{c_{\min}}^{c^*}\left[\frac{\tilde{\Lambda}'\left(\frac{q_c}{Q}\right)\frac{c}{E}}{\tilde{\Lambda}''\left(\frac{q_c}{Q}\right)}-\frac{c}{E}\left(\tilde{\Lambda}'^{-1}\left(\frac{cQ}{E}\right)+\frac{Qc/E}{\tilde{\Lambda}''\left(\frac{q_c}{Q}\right)}\right)\right]\mathrm{d}G(c)}{\left[\frac{\tilde{\Lambda}'(\cdot)}{\tilde{\Lambda}''(\cdot)}\frac{c^*}{E}-\frac{c^*}{E}\left(\tilde{\Lambda}'^{-1}\left(\frac{c^*Q}{E}\right)+\frac{Qc^*/E}{\tilde{\Lambda}''(\cdot)}\right)\right]}<\frac{\int_{c_{\min}}^{c^*}\left[\tilde{\Lambda}\left(\tilde{\Lambda}'^{-1}\left(\frac{cQ}{E}\right)\right)-\frac{cQ}{E}\tilde{\Lambda}'^{-1}\left(\frac{cQ}{E}\right)\right]\mathrm{d}G(c)}{\left[\tilde{\Lambda}\left(\tilde{\Lambda}'^{-1}\left(\frac{c^*Q}{E}\right)\right)-\frac{c^*Q}{E}\tilde{\Lambda}'^{-1}\left(\frac{c^*Q}{E}\right)\right]}$$

$$\Leftrightarrow\frac{\int_{c_{\min}}^{c^*}\left[\frac{\tilde{\Lambda}'(q_c/Q)}{\tilde{\Lambda}''(q_c/Q)}-\left(\frac{q_c}{Q}+\frac{\tilde{\Lambda}'(q_c/Q)}{\tilde{\Lambda}''(q_c/Q)}\right)\right]c\mathrm{d}G(c)}{\left[\frac{\tilde{\Lambda}'(q_{c^*}/Q)}{\tilde{\Lambda}''(q_{c^*}/Q)}-\left(\frac{q_{c^*}}{Q}+\frac{\tilde{\Lambda}'(q_{c^*}/Q)}{\tilde{\Lambda}''(q_{c^*}/Q)}\right)\right]c^*}<\frac{\int_{c_{\min}}^{c^*}\left\{\left[\tilde{\Lambda}\left(\frac{q_c}{Q}\right)-\tilde{\Lambda}'\left(\frac{q_c}{Q}\right)\frac{q_c}{Q}\right]\right\}\mathrm{d}G(c)}{\left[\tilde{\Lambda}\left(\frac{q_{c^*}}{Q}\right)-\tilde{\Lambda}'\left(\frac{q_{c^*}}{Q}\right)\frac{q_{c^*}}{Q}\right]}$$

$$\Leftrightarrow\int_{c_{\min}}^{c^*}\frac{q_c}{q_{c^*}}\frac{c}{c^*}\mathrm{d}G(c)<\int_{c_{\min}}^{c^*}\frac{\tilde{\Lambda}\left(q_c/Q\right)}{\tilde{\Lambda}\left(q_{c^*}/Q\right)}\frac{1-\mathcal{E}_{\tilde{\Lambda}}\left(q_c/Q\right)}{1-\mathcal{E}_{\tilde{\Lambda}}\left(q_{c^*}/Q\right)}\mathrm{d}G(c)$$

$$\Leftrightarrow\int_{c_{\min}}^{c^*}\frac{q_c}{q_{c^*}}\frac{c}{c^*}\mathrm{d}G(c)<\int_{c_{\min}}^{c^*}\frac{q_c}{q_{c^*}}\frac{p_c}{p_{c^*}}\frac{1-\mathcal{E}_{\tilde{\Lambda}}\left(q_c/Q\right)}{1-\mathcal{E}_{\tilde{\Lambda}}\left(q_{c^*}/Q\right)}\mathrm{d}G(c)$$

$$\Leftrightarrow\int_{c_{\min}}^{c^*}\frac{q_c}{q_{c^*}}\frac{c}{c^*}\mathrm{d}G(c)<\int_{c_{\min}}^{c^*}\frac{q_c}{q_{c^*}}\frac{c}{c^*}\frac{1/\mathcal{E}_{\tilde{\Lambda}}\left(q_c/Q\right)-1}{1/\mathcal{E}_{\tilde{\Lambda}}\left(q_{c^*}/Q\right)-1}\mathrm{d}G(c)$$

$$\Leftrightarrow\int_{c_{\min}}^{c^*}\frac{q_c}{q_{c^*}}\frac{c}{c^*}\left[1/\mathcal{E}_{\tilde{\Lambda}}\left(q_{c^*}/Q\right)-1\right]\mathrm{d}G(c)<\int_{c_{\min}}^{c^*}\frac{q_c}{q_{c^*}}\frac{c}{c^*}\left[1/\mathcal{E}_{\tilde{\Lambda}}\left(q_c/Q\right)-1\right]\mathrm{d}G(c).$$

Recall that $\mathcal{E}_{\tilde{\Lambda}}(\cdot)>0$ by basic assumptions. Recall also that MSLD implies $\mathcal{E}'_{\tilde{\Lambda}}(\cdot)<0$. Hence, $\mathcal{E}_{\tilde{\Lambda}}\left(q_{c^*}/Q\right)>\mathcal{E}_{\tilde{\Lambda}}\left(q_c/Q\right)$ for all $c\in[c_{\min},c^*)$. Hence, it is straightforward to see that the inequality above holds under MSLD. Hence, $\mathrm{d}c^*/\mathrm{d}L<0$ under MSLD. So, MSLD is what ensures that an increase in market size will result in a lower c^* in a new equilibrium.

A.11 The Proof of Proposition 11[44]

i) Recall that $pq/E=\tilde{\Lambda}\left(q/Q\right)$ and that a firm's FOC implies: $q/Q=\tilde{\Lambda}'^{-1}\left(cQ/E\right)$. Hence, $pq/E=\tilde{\Lambda}\left(\tilde{\Lambda}'^{-1}\left(cQ/E\right)\right)$. Then,

$$\frac{\mathrm{d}}{\mathrm{d}Q}\left[\tilde{\Lambda}\left(\tilde{\Lambda}'^{-1}\left(cQ/E\right)\right)\right]=\frac{\tilde{\Lambda}'\left(q/Q\right)}{\tilde{\Lambda}''\left(q/Q\right)}\frac{cQ}{E}<0.$$

Recall that $\uparrow L\longrightarrow\uparrow Q$ and $\tilde{\Lambda}''\left(q/Q\right)<0$ (the corresponding discussion on that was provided earlier). So, one can readily verify that $pq/E=\tilde{\Lambda}\left(q/Q\right)$ will decrease after $\uparrow L$ for all

[44]Note also that product-level performance measures can alternatively be written in terms of the elasticities of demand and marginal revenue (in the spirit of Mayer, Melitz, and Ottaviano 2016). More details are available upon request.

firms among those that survived. (Similarly, it can be readily verified that $\frac{\mathrm{d}}{\mathrm{d}Q}\left[\frac{q}{Q}\right]=\frac{c/E}{\tilde{\Lambda}''(\cdot)}<0$).

ii) Recall that $p/c=1/\mathcal{E}_{\tilde{\Lambda}}\left(q/Q\right)$ and that MSLD implies that $\mathcal{E}'_{\tilde{\Lambda}}(\cdot)<0$. Recall also that in a new equilibrium with a larger market size (i.e., after $\uparrow L$), q/Q will decrease among survived firms. Hence, one can readily verify from the formula above that firms' markups, p/c, and prices, p, will decrease for all c's among surviving firms. It is straightforward to see that MSLD is crucial for these patterns to take place.

iii) Notice that $M^e=1/\int_{c_{\min}}^{c^*}\tilde{\Lambda}\left(q/Q\right)g(c)\mathrm{d}c$. It has been already shown that an increase in L leads to lower $\tilde{\Lambda}(\cdot)$ for all firms among those that survived. Recall also that c^* decreases as well under MSLD. Hence, it is straightforward to see that M^e must increase in response to an increase in L.

iv) Recall that

$$\Pi=\left[\tilde{\Lambda}\left(\tilde{\Lambda}'^{-1}\left(cQ/E\right)\right)-\left(cQ/E\right)\tilde{\Lambda}'^{-1}\left(cQ/E\right)\right]EL-F$$

$$=\left[1-\mathcal{E}_{\tilde{\Lambda}}\left(q/Q\right)\right]\tilde{\Lambda}\left(q/Q\right)EL-F=\left[1-\mathcal{E}_{\tilde{\Lambda}}(z)\right]\tilde{\Lambda}(z)\,EL-F.$$

Notice that $\Pi'(z)=\tilde{\Lambda}'(z)\left[1-\mathcal{E}_{\tilde{\Lambda}}(z)\right]-\tilde{\Lambda}(z)\mathcal{E}'_{\tilde{\Lambda}}(z)$. Hence,

$$\Pi'(z)>0\Leftrightarrow\tilde{\Lambda}'(z)\left[1-\mathcal{E}_{\tilde{\Lambda}}(z)\right]>\tilde{\Lambda}(z)\mathcal{E}'_{\tilde{\Lambda}}(z)\Leftrightarrow\left[1-\mathcal{E}_{\tilde{\Lambda}}(z)\right]\tilde{\Lambda}'(z)/\tilde{\Lambda}\left(z\right)>\mathcal{E}'_{\tilde{\Lambda}}(z)$$

Note also that MSLD (i.e., $\mathcal{E}'_{\tilde{\Lambda}}(\cdot)<0$) is sufficient to ensure that $\Pi'(z)>0$. Recall also that it has just been shown that an increase in market size L leads to a lower marginal cost cutoff level, c^*, under MSLD. So, the ZCP condition and monotonicity of a firm's profit in marginal cost implies that at least some range of the least productive firms (among those that survived) earn relatively lower profits after an increase in total market size. Now, taking into account that a firm's profit is monotonic in c, it is straightforward to see that, to ensure that the FE condition holds in a new equilibrium, some range of low–c (i.e., the most efficient) firms should earn relatively higher profits after an increase in total market size. Hence, it can be concluded that an increase in total market size leads to relatively higher profits only for some range of low–c firms.

A.12 The Proof of Proposition 12

Recall that $pq/E=\tilde{\Lambda}\left(q/Q\right)=\tilde{\Lambda}\left(\tilde{\Lambda}'^{-1}\left(cQ/E\right)\right)$. Let $c_2>c_1$. Then, it is straightforward to verify that:

$$\frac{\tilde{\Lambda}\left(\tilde{\Lambda}'^{-1}\left(c_1Q/E\right)\right)}{\tilde{\Lambda}\left(\tilde{\Lambda}'^{-1}\left(c_2Q/E\right)\right)}>1.$$

To examine the impact of an increase in total market size on reallocation patterns, recall that Proposition 10 argues that an increase in L leads to an increase in Q under MSLD. Now, notice that

$$\frac{\mathrm{d}}{\mathrm{d}Q}\left[\frac{\tilde{\Lambda}\left(\tilde{\Lambda}'^{-1}\left(c_1Q/E\right)\right)}{\tilde{\Lambda}\left(\tilde{\Lambda}'^{-1}\left(c_2Q/E\right)\right)}\right]=\frac{\frac{\tilde{\Lambda}'(q_1/Q)}{\tilde{\Lambda}''(q_1/Q)}\frac{c_1}{E}\tilde{\Lambda}\left(q_2/Q\right)-\frac{\tilde{\Lambda}'(q_2/Q)}{\tilde{\Lambda}''(q_2/Q)}\frac{c_2}{E}\tilde{\Lambda}\left(q_1/Q\right)}{\left[\tilde{\Lambda}\left(q_2/Q\right)\right]^2}$$

$$\frac{\mathrm{d}}{\mathrm{d}Q}\left[\frac{\tilde{\Lambda}\left(\tilde{\Lambda}'^{-1}\left(c_1Q/E\right)\right)}{\tilde{\Lambda}\left(\tilde{\Lambda}'^{-1}\left(c_2Q/E\right)\right)}\right]>0\Leftrightarrow\frac{\tilde{\Lambda}'\left(q_1/Q\right)}{\tilde{\Lambda}''\left(q_1/Q\right)}\frac{c_1}{E}\tilde{\Lambda}\left(q_2/Q\right)>\frac{\tilde{\Lambda}'\left(q_2/Q\right)}{\tilde{\Lambda}''\left(q_2/Q\right)}\frac{c_2}{E}\tilde{\Lambda}\left(q_1/Q\right)$$

$$\Leftrightarrow\frac{\tilde{\Lambda}'\left(q_1/Q\right)}{\tilde{\Lambda}''\left(q_1/Q\right)}\frac{\tilde{\Lambda}'\left(q_1/Q\right)}{\tilde{\Lambda}\left(q_1/Q\right)}>\frac{\tilde{\Lambda}'\left(q_2/Q\right)}{\tilde{\Lambda}''\left(q_2/Q\right)}\frac{\tilde{\Lambda}'\left(q_2/Q\right)}{\tilde{\Lambda}\left(q_2/Q\right)}\Leftrightarrow\frac{\mathcal{E}_{\tilde{\Lambda}}\left(q_1/Q\right)}{\mathcal{E}_{\tilde{\Lambda}'}\left(q_1/Q\right)}<\frac{\mathcal{E}_{\tilde{\Lambda}}\left(q_2/Q\right)}{\mathcal{E}_{\tilde{\Lambda}'}\left(q_2/Q\right)},$$

where, as previously, $\mathcal{E}_{\tilde{\Lambda}}(z)\equiv z\tilde{\Lambda}'(z)/\tilde{\Lambda}(z)$, $\mathcal{E}_{\tilde{\Lambda}'}(z)\equiv -z\tilde{\Lambda}''(z)/\tilde{\Lambda}'(z)$.

$$\frac{\mathrm{d}}{\mathrm{d}z}\left[\frac{\mathcal{E}_{\tilde{\Lambda}}(z)}{\mathcal{E}_{\tilde{\Lambda}'}(z)}\right]=\frac{\mathcal{E}'_{\tilde{\Lambda}}(z)\mathcal{E}_{\tilde{\Lambda}'}(z)-\mathcal{E}_{\tilde{\Lambda}}(z)\mathcal{E}'_{\tilde{\Lambda}'}(z)}{\left[\mathcal{E}_{\tilde{\Lambda}'}(z)\right]^2}.$$

Now, notice that

$$\frac{\mathrm{d}}{\mathrm{d}z}\left[\mathcal{E}_{\tilde{\Lambda}}(z)/\mathcal{E}_{\tilde{\Lambda}'}(z)\right]<0\Leftrightarrow\mathcal{E}'_{\tilde{\Lambda}}(z)\mathcal{E}_{\tilde{\Lambda}'}(z)<\mathcal{E}_{\tilde{\Lambda}}(z)\mathcal{E}'_{\tilde{\Lambda}'}(z).$$

Before proceeding with the proof, notice that

$$\mathcal{E}'_{\tilde{\Lambda}}(z)=\frac{\mathrm{d}}{\mathrm{d}z}\left[z\tilde{\Lambda}'(z)/\tilde{\Lambda}(z)\right]=\left(z\tilde{\Lambda}''(z)+\tilde{\Lambda}'(z)\right)/\tilde{\Lambda}(z)-z\left[\tilde{\Lambda}'(z)/\tilde{\Lambda}(z)\right]^2=$$

$$=\left(\frac{z\tilde{\Lambda}''(z)}{\tilde{\Lambda}'(z)}+1\right)\frac{\tilde{\Lambda}'(z)}{\tilde{\Lambda}(z)}-z\left[\frac{\tilde{\Lambda}'(z)}{\tilde{\Lambda}(z)}\right]^2=\frac{\tilde{\Lambda}'(z)}{\tilde{\Lambda}(z)}\left(1-\mathcal{E}_{\tilde{\Lambda}'}(z)-\mathcal{E}_{\tilde{\Lambda}}(z)\right).$$

$$\mathcal{E}'_{\tilde{\Lambda}'}(z)=\frac{\mathrm{d}}{\mathrm{d}z}\left[-z\tilde{\Lambda}''(z)/\tilde{\Lambda}'(z)\right]=-\frac{\left(z\tilde{\Lambda}'''(z)+\tilde{\Lambda}''(z)\right)\tilde{\Lambda}'(z)-z\left[\tilde{\Lambda}''(z)\right]^2}{\left[\tilde{\Lambda}'(z)\right]^2}=$$

$$=z\left[\frac{\tilde{\Lambda}''(z)}{\tilde{\Lambda}'(z)}\right]^2-\frac{z\tilde{\Lambda}'''(z)+\tilde{\Lambda}''(z)}{\tilde{\Lambda}'(z)}=-\frac{\tilde{\Lambda}''(z)}{\tilde{\Lambda}'(z)}\left[\mathcal{E}_{\tilde{\Lambda}'}(z)+\frac{z\tilde{\Lambda}'''(z)}{\tilde{\Lambda}''(z)}+1\right].$$

Hence,

$$\frac{\mathrm{d}}{\mathrm{d}z}\left[\mathcal{E}_{\tilde{\Lambda}}(z)/\mathcal{E}_{\tilde{\Lambda}'}(z)\right]<0\Leftrightarrow\mathcal{E}'_{\tilde{\Lambda}}(z)\mathcal{E}_{\tilde{\Lambda}'}(z)<\mathcal{E}'_{\tilde{\Lambda}'}(z)\mathcal{E}_{\tilde{\Lambda}}(z)$$

$$\Leftrightarrow\mathcal{E}_{\tilde{\Lambda}'}(z)\frac{\tilde{\Lambda}'(z)}{\tilde{\Lambda}(z)}\left[1-\mathcal{E}_{\tilde{\Lambda}'}(z)-\mathcal{E}_{\tilde{\Lambda}}(z)\right]<\mathcal{E}_{\tilde{\Lambda}}(z)\left(-\frac{\tilde{\Lambda}''(z)}{\tilde{\Lambda}'(z)}\right)\left[\mathcal{E}_{\tilde{\Lambda}'}(z)+\frac{z\tilde{\Lambda}'''(z)}{\tilde{\Lambda}''(z)}+1\right]$$

$$\Leftrightarrow \mathcal{E}_{\tilde{\Lambda}'}(z)\mathcal{E}_{\tilde{\Lambda}}(z)\left[1-\mathcal{E}_{\tilde{\Lambda}'}(z)-\mathcal{E}_{\tilde{\Lambda}}(z)\right] < \mathcal{E}_{\tilde{\Lambda}}(z)\mathcal{E}_{\tilde{\Lambda}'}(z)\left[\mathcal{E}_{\tilde{\Lambda}'}(z)+z\tilde{\Lambda}'''(z)/\tilde{\Lambda}''(z)+1\right]$$

$$\Leftrightarrow 1-\mathcal{E}_{\tilde{\Lambda}'}(z)-\mathcal{E}_{\tilde{\Lambda}}(z) < \mathcal{E}_{\tilde{\Lambda}'}(z)+z\tilde{\Lambda}'''(z)/\tilde{\Lambda}''(z)+1.$$

Recall that MSLD implies $1-\mathcal{E}_{\tilde{\Lambda}'}(z)-\mathcal{E}_{\tilde{\Lambda}}(z)<0$. Hence, if $\mathcal{E}_{\tilde{\Lambda}'}(z)+z\tilde{\Lambda}'''(z)/\tilde{\Lambda}''(z)+1>0$, then the inequality above holds. Notice also that it is equivalent to the so-called MSLD′ condition from Mayer, Melitz, and Ottaviano (2016) about increasing the elasticity of marginal revenue. More specifically,

$$\mathcal{E}'_{\phi}(q)>0 \Leftrightarrow \mathcal{E}_{\tilde{\Lambda}'}(z)+z\tilde{\Lambda}'''(z)/\tilde{\Lambda}''(z)+1>0,$$

where $\mathcal{E}_{\phi}(q)\equiv -\frac{\phi'(q)q}{\phi(q)}=\frac{-\tilde{\Lambda}''(q/Q)}{\tilde{\Lambda}'(q/Q)}\frac{q}{Q}\equiv\mathcal{E}_{\tilde{\Lambda}'}(q/Q)$.[45]

B The Gorman-Pollak Single-Aggregator Demand System

In this Section, I briefly outline the model's properties under what is often called the generalized Gorman-Pollak demand system (Fally 2018) or the Gorman-Pollak single-aggregator demand system. This flexible demand system encompasses HSA demand systems as well as *directly-additive* (*directly-additively separable*) preferences (see, e.g., Krugman 1979) and *indirectly-additive* (*indirectly-additively separable*) preferences (as in, e.g., Bertoletti, Etro, and Simonovska 2018) as special cases.

In the description of this demand system and its main properties, I closely follow Fally (2018). The main feature of this demand system is that an individual's demand for a variety depends on income, a firm's price for this variety, and a single aggregator that is itself a scalar function of normalized prices of all varieties. In the context of my model, it implies that an individual's demand takes the following form:

$$q(\omega_x)=\frac{\mathcal{D}\left(\mathcal{P}\left(\Theta(x)\right)p(\omega_x)/E\right)}{\mathcal{Q}\left(\Theta(x)\right)},$$

where $\mathcal{D}$, $\mathcal{P}$, and $\mathcal{Q}$ are differentiable functions; $\Theta(x)\equiv\Theta(\mathbf{p}_x/E)$ denotes the local market aggregator that is a scalar function of normalized prices of all varieties within a niche $x\in\mathcal{X}$.

Following Fally (2018), who provides conditions that ensure that such a demand system is integrable, I impose the following regularity assumptions:

i) $\mathcal{E}_{\mathcal{P}}\mathcal{E}_{\mathcal{D}}<\mathcal{E}_{\mathcal{Q}}$ for all $\Theta(x)$ and $\mathbf{p}_x/E$, where $\mathcal{E}_{\mathcal{P}}\equiv\frac{\partial\log\mathcal{P}}{\partial\log\Theta}$, $\mathcal{E}_{\mathcal{Q}}\equiv\frac{\partial\log\mathcal{Q}}{\partial\log\Theta}$, $\mathcal{E}_{\mathcal{D}}\equiv\frac{\partial\log\mathcal{D}}{\partial\log p}$.[46]

[45] Recall that $\mathcal{E}_p(q)\equiv -qp'(q)/p(q)=1-\mathcal{E}_{\tilde{\Lambda}}(q/Q)$. Hence, one can readily verify that the condition $\frac{\mathrm{d}}{\mathrm{d}z}\left[\mathcal{E}_{\tilde{\Lambda}}(z)/\mathcal{E}_{\tilde{\Lambda}'}(z)\right]<0$ is a counterpart of the condition from Mayer, Melitz, and Ottaviano (2016) that $\left[1-\mathcal{E}_p\left(q_i\right)\right]/\mathcal{E}_{\phi}\left(q_i\right)$ is decreasing . They also show that this condition is ensured by MSLD′ and, in turn, implies MSLD. As has been already discussed, they also show that MSLD′ implies MSLD under some mild conditions. Details in the context of HSA preferences are available upon request.

[46] As noted by Fally (2018), one can instead assume that $\mathcal{E}_{\mathcal{P}}\mathcal{E}_{\mathcal{D}}-\mathcal{E}_{\mathcal{Q}}$ has the same sign. Yet, the restriction

ii) for any set of normalized local prices, $\mathbf{p}_x/E$, the budget constraint admits a solution in Θ:

$$\int\limits_{\omega_x \in \Omega_x} \mathcal{D}\left(\mathcal{P}\left(\Theta(x)\right)\frac{p(\omega_x)}{E}\right)\frac{p(\omega_x)}{\mathcal{Q}\left(\Theta(x)\right)}\mathrm{d}\omega_x = E.$$

Note that directly-additively separable preferences correspond to the special case when the quantity shifter, $\mathcal{Q}(\cdot)$, is constant that implies $\mathcal{E}_{\mathcal{Q}} = 0$ and $\mathcal{E}_{\mathcal{P}} > 0$. In contrast, indirectly-additively separable preferences correspond to the special case when the price shifter, $\mathcal{P}(\cdot)$, is constant that implies $\mathcal{E}_{\mathcal{P}} = 0$ and $\mathcal{E}_{\mathcal{Q}} > 0$. As for HSA demand systems, they correspond to the special case when $\mathcal{P}\left(\Theta(\cdot)\right) = 1/\mathcal{Q}\left(\Theta(\cdot)\right) = \Theta(\cdot)$.

A counterpart of a firm's profit function (2.3) takes the following form:

$$\Pi(c,x) = \max_{p}\left\{(p-c)\frac{\ell(x)}{\mathcal{Q}\left(\Theta(x)\right)}\mathcal{D}\left(\mathcal{P}\left(\Theta(x)\right)\frac{p}{E}\right) - F\right\}.$$

One can readily verify that optimization w.r.t. p yields the standard pricing rule:

$$\frac{p-c}{p} = -1/\mathcal{E}_{\mathcal{D}},$$

where $\mathcal{E}_{\mathcal{D}} < -1$.

The equation above yields a firm's price as a function of its marginal cost and location/niche, $p(c,x)$. Now, exploiting the envelope theorem, the corresponding FOC w.r.t. x implies

$$\frac{\Theta'(x)}{\Theta(x)}\left(\mathcal{E}_{\mathcal{Q}} - \mathcal{E}_{\mathcal{P}}\mathcal{E}_{\mathcal{D}}\right) = \frac{\ell'(x)}{\ell(x)}.$$

Recalling that $\ell(x)$ is decreasing and $\mathcal{E}_{\mathcal{P}}\mathcal{E}_{\mathcal{D}} < \mathcal{E}_{\mathcal{Q}}$ yields that $\Theta(x)$ is decreasing. One can readily verify that $\mathcal{E}_{\mathcal{P}}\mathcal{E}_{\mathcal{D}} < \mathcal{E}_{\mathcal{Q}}$ also implies that an individual's demand is decreasing in $\Theta(\cdot)$. So, a higher $\Theta(\cdot)$ indicates tougher local competition. Hence, Proposition 1 can be readily generalized to the case of the Gorman-Pollak demand system.

Proposition 13. *Consider any interval of niches $[\underline{x},\overline{x}] \subseteq \mathcal{X}$ that is served by firms in equilibrium. In the interior of this interval, larger local markets (niches) feature tougher competition, implying higher values of the local market aggregator, $\Theta(x)$. Formally, $\Theta(x)$ is strictly decreasing on $(\underline{x},\overline{x})$.*

The formal proof mostly resembles the corresponding proof of Proposition 1. The proposition above implies that larger niches/markets feature strictly higher competition levels.

Now, denote a firm's variable profit function as follows,

$$\pi = (p-c)\frac{\ell(x)}{\mathcal{Q}\left(\Theta(x)\right)}\mathcal{D}\left(\mathcal{P}\left(\Theta(x)\right)\frac{p}{E}\right).$$

$\mathcal{E}_{\mathcal{P}}\mathcal{E}_{\mathcal{D}} < \mathcal{E}_{\mathcal{Q}}$ is without loss of generality.

We can apply the same analysis as in Section A.2. In particular, we can use an approach based on the Implicit Function Theorem and study the first-order comparative statics effects of c on p and x at a solution. Taking into account that $\pi'_p = 0$ and $\pi'_x = 0$ by FOCs, one can readily verify that $\pi''_{xc} = 0$. As a result, we obtain (for notational brevity, I omit specific notation that derivatives are evaluated at a solution that satisfies FOCs):

$$\frac{\mathrm{d}p}{\mathrm{d}c} = \frac{-\pi''_{pc}\pi''_{xx}}{\Delta}, \qquad \frac{\mathrm{d}x}{\mathrm{d}c} = \frac{\pi''_{xp}\pi''_{pc}}{\Delta},$$

where $\Delta \equiv \pi''_{pp}\pi''_{xx} - \left[\pi''_{xp}\right]^2 > 0$.

Recall that $\mathcal{D}'(\cdot) < 0$ yields $\pi''_{pc} > 0$. Hence, given that $\pi''_{xx} < 0$, one can infer that $\mathrm{d}p/\mathrm{d}c > 0$. As for the sorting pattern, it is straightforward to verify that $\operatorname{sign}\{\mathrm{d}x/\mathrm{d}c\} = \operatorname{sign}\left\{\pi''_{xp}\right\}$. Again, taking into account FOCs, notice that π''_{xp} can be represented as follows:

$$\pi''_{xp} = \frac{\ell(x)\mathcal{D}'(\cdot)\mathcal{P}'(\cdot)\Theta'(x)\frac{p}{E}}{\mathcal{Q}\left(\Theta(x)\right)}\left[1 - \left(\frac{\mathcal{D}''(\cdot)}{\mathcal{D}'(\cdot)}\frac{\mathcal{P}\left(\Theta(x)\right)p}{E} + 1\right)/\mathcal{E}_{\mathcal{D}}\right].$$

Recall that $\mathcal{D}'(\cdot) < 0$ and $\Theta'(\cdot) < 0$. Hence,

$$\operatorname{sign}\left\{\frac{\mathrm{d}x}{\mathrm{d}c}\right\} = \operatorname{sign}\left\{\mathcal{P}'(\cdot)\left[1 - \left(\frac{\mathcal{D}''(\cdot)}{\mathcal{D}'(\cdot)}\frac{\mathcal{P}\left(\Theta(x)\right)p}{E} + 1\right)/\mathcal{E}_{\mathcal{D}}\right]\right\}. \tag{B.1}$$

One can readily verify that condition $1 - \left(\frac{\mathcal{D}''(\cdot)}{\mathcal{D}'(\cdot)}\frac{\mathcal{P}(\Theta(x))p}{E} + 1\right)/\mathcal{E}_{\mathcal{D}} > 0$ is equivalent to Marshall's Second Law of Demand. Specifically, notice that the absolute value of the price elasticity of demand is given by

$$-\frac{\partial \log q(\omega_x)}{\partial \log p(\omega_x)} = -\frac{\mathcal{D}'(\cdot)}{\mathcal{D}(\cdot)}\frac{\mathcal{P}\left(\Theta(x)\right)p(\omega_x)}{E} = -\mathcal{E}_{\mathcal{D}}.$$

Now, notice that Marshall's Second Law of Demand requires that $-\mathcal{E}_{\mathcal{D}}$ is increasing in its argument/price. Note also that

$$-\mathcal{E}'_{\mathcal{D}}(p) > 0 \Leftrightarrow \frac{-p\mathcal{D}''(p)}{\mathcal{D}'(p)} - 1 + \mathcal{E}_{\mathcal{D}}(p) < 0 \Leftrightarrow 1 - \left(p\frac{\mathcal{D}''(p)}{\mathcal{D}'(p)} + 1\right)/\mathcal{E}_{\mathcal{D}}(p) > 0.$$

Hence, it follows from (B.1) that even if Marshall's Second Law of Demand holds, one still needs $\mathcal{P}'(\cdot) > 0$ to ensure positive assortative matching between firm productivity and niche size. A similar condition can be obtained under an alternative approach via log-supermodularity of variable profit function (see Section A.2 for more details):[47]

[47] One can also readily verify that $\frac{\partial^2 \log \pi}{\partial x \partial c} = 0$, $\frac{\partial^2 \log \pi}{\partial c \partial p} = \frac{1}{(p-c)^2} > 0$.

$$\frac{\partial^2 \log \pi}{\partial x \partial p} = -\frac{\Theta'(x)}{\Theta(x)} \mathcal{E}_{\mathcal{P}} \cdot \left\{ \frac{\partial}{\partial p} \left[-\mathcal{E}_{\mathcal{D}} \right] \right\}. \tag{B.2}$$

Thus, under this flexible demand system, Marshall's Second Law of Demand is neither necessary nor sufficient condition for positive assortative matching between firm productivity and niche size.

Proposition 14. *Consider the Gorman-Pollak single-aggregator demand system. The model's equilibrium implies PAM between firm productivity and niche size under the following mix of conditions on* $\mathcal{E}_{\mathcal{P}}$, $\mathcal{E}_{\mathcal{Q}}$, *and* $\mathcal{E}_{\mathcal{D}}$ *(given that regularity assumptions hold):*

i) $-\mathcal{E}'_{\mathcal{D}}(p) > 0$ *(MSLD),* $\mathcal{E}_{\mathcal{Q}} = 0$, $\mathcal{E}_{\mathcal{P}} > 0$. *This case corresponds to directly-additively separable preferences (Krugman 1979);*

ii) $-\mathcal{E}'_{\mathcal{D}}(p) > 0$ *(MSLD),* $\mathcal{E}_{\mathcal{Q}} > 0$, $\mathcal{E}_{\mathcal{P}} > 0$;

iii) $-\mathcal{E}'_{\mathcal{D}}(p) > 0$ *(MSLD),* $\mathcal{E}_{\mathcal{Q}} < 0$, $\mathcal{E}_{\mathcal{P}} > 0$;

iv) $-\mathcal{E}'_{\mathcal{D}}(p) > 0$ *(MSLD),* $\mathcal{E}_{\mathcal{Q}} < 0$, $\mathcal{E}_{\mathcal{P}} > 0$;

v) $-\mathcal{E}'_{\mathcal{D}}(p) < 0$, $\mathcal{E}_{\mathcal{Q}} > 0$, $\mathcal{E}_{\mathcal{P}} < 0$.

Using (B.1) or (B.2), it is straightforward to formulate the corresponding proposition for the opposite case of negative assortative matching (NAM) between firm productivity and niche size.

Now, recall that *indirectly-additively separable* preferences (Bertoletti, Etro, and Simonovska 2018) imply $\mathcal{E}_{\mathcal{P}} = 0$ and $\mathcal{E}_{\mathcal{Q}} > 0$. Hence, one can easily verify from (B.1) or (B.2) that $\mathrm{d}x/\mathrm{d}c = 0$, irrespective of whether Marshall's Second Law of Demand holds or not. As a result, similar to the case of CES, there can potentially be multiple equilibria. It follows from the analysis above that such patterns may arise if either $\mathcal{E}'_{\mathcal{D}}(p) = 0$ or $\mathcal{E}_{\mathcal{P}} = 0$.[48]

B.1 Directly-Additively Separable Preferences ($\mathcal{E}_{\mathcal{Q}} = 0$, $\mathcal{E}_{\mathcal{P}} > 0$)

Here, I briefly discuss the system of differential equations (2.9) under directly-additive preferences (Krugman 1979) within the differentiated product sector.

Note that under directly-additive structure of preferences, we lose homotheticity, except for the CES specification. The measure of *local competitive toughness* is now captured by the corresponding (niche-specific) Lagrange multiplier, $\lambda(x)$, derived from a consumer's maximization problem. As a result, it can be shown that the system of differential equations that describes the differentiated-product market equilibrium is the following:

$$\begin{cases} \lambda'(x) = \lambda(x) \frac{\ell'(x)}{\ell(x)} \frac{p(c(x),x) - c(x)}{p(c(x),x)} \\ c'(x) = \frac{\lambda(x) \cdot E}{M^e \cdot g(c(x)) \cdot u'(q(c(x),x)) \cdot q(c(x),x)}, \end{cases}$$

[48] More details on the model's properties under the Gorman-Pollak demand system and corresponding proofs are available upon request. For brevity, I do not include them in this version of the draft.

with two boundary conditions: $c(0) = c_{\min}$ and $c(x_{\max}) = c^*$.

One can readily verify that under a quadratic utility specification, $u(q(\omega_x)) = aq(\omega_x) - \frac{b}{2}\left(q(\omega_x)\right)^2$, the system above takes the following form:

$$\begin{cases} \lambda'(x) = \lambda(x)\frac{\ell'(x)}{\ell(x)}\frac{a-\lambda(x)\cdot c(x)}{a+\lambda(x)\cdot c(x)} \\ c'(x) = \frac{4b\cdot\lambda(x)\cdot E}{M^e\cdot g(c(x))[a^2-(\lambda(x)\cdot c(x))^2]}. \end{cases}$$

Note that this system of differential equations does not coincide with the corresponding system obtained in KSTU (2020), where preferences for the differentiated product sector are directly-additively separable, albeit embedded within a quasilinear upper-tier utility. In my model, I develop a full-fledged general equilibrium setup. As in the baseline case of homothetic preferences, all niches are served by (positive mass of) firms in my setup. It provides additional evidence that differences between the two models go beyond the use of different classes of preferences and the extension to the costly trade scenario.

B.2 A Special Case: Translog

For quantitative analysis, I exploit the market share function, $\Lambda(\cdot)$, that corresponds to the symmetric homothetic translog: $\Lambda\left(p(\omega_x)/P(x)\right) = \log\left(p(\omega_x)/P(x)\right)^{-\zeta}$.

Hence, an individual's demand function for a variety $\omega_x \in \Omega_x$ is given by:

$$q(\omega_x) = \log\left(p(\omega_x)/P(x)\right)^{-\zeta}\frac{E}{p(\omega_x)}.$$

Note that in the case of symmetric translog expenditure function, the local price aggregator, $P(x)$, is generally defined as follows:

$$\log P(x) = \frac{1}{\zeta\cdot\mid\Omega_x\mid} + \frac{1}{\mid\Omega_x\mid}\int_{\omega'_x\in\Omega_x}\log p(\omega'_x)\mathrm{d}\omega'_x.$$

A firm's pricing formula (2.4) implies that $p(c,x) = c\left(1 - \log\left(p(c,x)/P(x)\right)\right)$.

Note that for the translog case we have $\mathcal{E}_\Lambda\left(p/P(x)\right) = -1/\log\left(p/P(x)\right)$. One can readily verify that $\mathcal{E}'_\Lambda(\cdot) > 0$, implying that MSLD holds. So, the model's equilibrium implies PAM between firm productivity and niche size.

Recall that (see Section 2.3) PAM implies that the mass of firms, $M(x)$, that serve a particular location $x \in \mathcal{X}$ is given by $M(x) = M\cdot g\left(c(x)\right)c'(x)/G(c^*) = M^e\cdot g\left(c(x)\right)c'(x)$.[49]

The corresponding system of differential equations, (2.9), takes the following form:

[49] Hence, in equilibrium, the local price aggregator is related with $M(x)$ as $P(x) = p\left(c(x),x\right)\cdot\exp\left(\frac{1}{\zeta M(x)}\right)$.

$$\begin{cases} P'(x) = \frac{\ell'(x)}{\ell(x)} P(x) \log\left(p(c(x), x)/P(x)\right) \\ c'(x) = \left\{ M^e \cdot g\left(c(x)\right) \log\left(p(c(x), x)/P(x)\right)^{-\zeta} \right\}^{-1}, \end{cases}$$

with boundary conditions $c(0) = c_{\min}$ and $c(x_{\max}) = c^*$.

Note also that the zero cutoff profit and the free entry conditions are as follows:

$$\Pi\left(c(x_{\max}), x_{\max}\right) = \frac{\zeta \, \log^2\left(p\left(c(x_{\max}), x_{\max}\right)/P(x_{\max})\right)}{1 - \log\left(p\left(c(x_{\max}), x_{\max}\right)/P(x_{\max})\right)} E\,\ell(x_{\max}) - F = 0.$$

$$\int_{\mathcal{X}} \left[\frac{\zeta \, \log^2\left(p\left(c(x), x\right)/P(x)\right)}{1 - \log\left(p\left(c(x), x\right)/P(x)\right)} E\,\ell(x) - F \right] g\left(c(x)\right) c'(x) \mathrm{d}x = F^e.$$

C An Alternative Trade Setup

According to the baseline trade setup described in Section 3, an exporting firm is assumed to choose a niche that is common for both domestic and foreign markets. In this section, I consider an alternative setup when a firm is potentially allowed to choose a domestic niche, x^h, and a foreign niche, x^f, such that $x^h \neq x^f$. Other baseline assumptions are left unchanged. In particular, I consider trade between $N+1$ symmetric countries with identical distribution of niche sizes, $\ell(\cdot)$, over the exogenous interval of niches, $\mathcal{X} = [0, x_{\max}]$. Hence, in equilibrium, an exporting firm tends to choose the same foreign niche, x^f, for any of N foreign markets.

After paying the sunk entry cost F^e a firm draws its marginal cost c from the exogenous distribution with p.d.f. $g(\cdot)$ over the support $[c_{\min}, c_{\max}]$. Production of each variety also involves fixed production cost $F \geq 0$. In addition, international trade is subject to fixed investment ("market access") costs $F^{ex} \geq 0$ per each foreign market as well as iceberg variable cost $\tau \geq 1$.

The total (combined) profits of an active firm with marginal cost c are:

$$\Pi(c, x^h, x^f) \equiv \Pi^h(c, x^h) + \max\left\{0, N \cdot \Pi^f(c, x^f)\right\},$$

where "domestic", h, and "foreign", f, profits are as follows:[50]

$$\Pi^h(c, x^h) \equiv \max_{p^h} \left\{ (p^h - c)\ell(x^h) \frac{E}{p^h} \Lambda\left(\frac{p^h}{P(x^h)}\right) - F \right\},$$

$$\Pi^f(c, x^f) \equiv \max_{p^f} \left\{ (p^f - \tau c)\ell(x^f) \frac{E}{p^f} \Lambda\left(\frac{p^f}{P(x^f)}\right) - F^{ex} \right\}.$$

[50] By incorporating the fixed production cost into $\Pi^h(c, x^h)$, I anticipate an equilibrium where no firm exports without serving the domestic market first.

Note that maximization of profits w.r.t. p^h and p^f implies:

$$\frac{p^h}{c} = \frac{\mathcal{E}_\Lambda\left(p^h/P(x^h)\right)+1}{\mathcal{E}_\Lambda\left(p^h/P(x^h)\right)}, \qquad \frac{p^f}{\tau c} = \frac{\mathcal{E}_\Lambda\left(p^f/P(x^f)\right)+1}{\mathcal{E}_\Lambda\left(p^f/P(x^f)\right)}, \tag{C.1}$$

where $\mathcal{E}_\Lambda(z) \equiv \frac{-z\Lambda'(z)}{\Lambda(z)}$ is the price elasticity of the market share function, $\Lambda(\cdot)$.

Thus, solving these equations for p^h and p^f yields a firm's prices as functions of a firm's marginal cost and niches served: $p^h = p^h(c, x^h)$, $p^f = p^f(c, x^f)$. Note that the symmetry assumption implies that the (endogenous) distribution of local price aggregators, $P(\cdot)$, over the interval of niches $\mathcal{X} = [0, x_{\max}]$ is identical for each country. Hence, one can readily verify from (C.1) that equilibrium "exporting" price of a firm with marginal cost c that serves an arbitrary niche x^f is equal to the domestic price of a firm with marginal cost τc for this specific niche: $p^f(c, x^f) = p^h(\tau c, x^f)$.[51]

Now, exploiting the envelope theorem, it can be shown that the firm's FOCs w.r.t. x^h and x^f imply:

$$-\frac{\ell(x^h)}{\ell'(x^h)}\frac{P'(x^h)}{P(x^h)} = \frac{1}{\mathcal{E}_\Lambda\left(p^h(c,x^h)/P(x^h)\right)}, \tag{C.2}$$

$$-\frac{\ell(x^f)}{\ell'(x^f)}\frac{P'(x^f)}{P(x^f)} = \frac{1}{\mathcal{E}_\Lambda\left(p^f(c,x^f)/P(x^f)\right)}, \tag{C.3}$$

where $\mathcal{E}_\Lambda\left(\frac{p^f(c,x^f)}{P(x^f)}\right) = \mathcal{E}_\Lambda\left(\frac{p^h(\tau c,x^f)}{P(x^f)}\right)$ (by recalling that $p^f(c, x^f) = p^h(\tau c, x^f)$).

These two conditions implicitly define optimal niches of a firm as functions of its marginal cost level: $x^h = x^h(c)$, $x^f = x^f(c)$. Hereafter, I consider only equilibria with positive assortative matching (PAM) between firm productivity and niche size.[52] It is straightforward from (C.2)-(C.3) that the optimal *exporting* niche of a firm with marginal cost c is equivalent to the optimal *domestic* niche of a firm with marginal cost τc:

$$x^f(c) = x^h(\tau \cdot c). \tag{C.4}$$

Now, let $c^h = c^h(x)$ denotes the *domestic matching function* (i.e., the marginal cost of a firm as a function of a niche served at the domestic market), where $c^h(\cdot) \equiv \left(x^h\right)^{-1}(\cdot)$ is the corresponding inverse function. Analogously, $c^f = c^f(x)$ denotes the *foreign matching function*, where $c^f(\cdot) \equiv \left(x^f\right)^{-1}(\cdot)$. Hence, we obtain the following relation between the matching functions:

$$c^f(x) = \frac{c^h(x)}{\tau}. \tag{C.5}$$

[51] It can also be shown that "*relative prices*" of a firm, $p^h/P(x^h)$ and $p^f/P(x^f)$, are increasing functions of the firm's "*relative inefficiency*", c/P: $p^h/P(x^h) = \Upsilon\left(c/P(x^h)\right)$ and $p^f/P(x^f) = \Upsilon\left(\tau c/P(x^f)\right)$.

[52] Note that, given the demand system described earlier, Marshall's Second Law of Demand is the necessary and sufficient condition for PAM. Details are available upon request.

It is straightforward from (C.5) that derivatives of the matching functions w.r.t. x are related as follows

$$c^{f\prime}(x) = \frac{c^{h\prime}(x)}{\tau}. \tag{C.6}$$

C.1 Trade Equilibrium

Combining FOCs of a firm (i.e., conditions (C.1)-(C.3)) yields equilibrium expressions for markups:

$$\frac{p^h(c^h(x),x)}{c^h(x)} = 1 - \frac{\ell(x)}{\ell'(x)}\frac{P'(x)}{P(x)}, \tag{C.7}$$

$$\frac{p^f(c^f(x),x)}{\tau c^f(x)} = 1 - \frac{\ell(x)}{\ell'(x)}\frac{P'(x)}{P(x)}. \tag{C.8}$$

Proposition 15. *Consider an arbitrary trading niche x. In equilibrium, domestic and foreign prices are equal at this niche x: $p^f(c^f(x),x) = p^h(c^h(x),x)$.*

Proof. It has been already shown that $p^f(c,x^f) = p^h(\tau c, x^f)$ and $c^f(x) = c^h(x)/\tau$. Hence, $p^f(c^f(x),x) = p^h(\tau c^f(x),x) = p^h(\tau c^h(x)/\tau, x) = p^h(c^h(x),x)$. □

Proposition 15 states that, despite different productivities between domestic and foreign firms that populate a specific trading niche, foreign firms set the same price as their domestic rivals. The mechanism is that the difference between domestic and foreign matching functions offsets the price impact of transportation cost, τ. Also, it is straightforward to show that, within a trading niche, markups charged by domestic firms are equal to markups charged by foreign firms.

Proposition 16. *Consider an arbitrary trading niche x. In equilibrium, domestic and foreign markups are equal at this niche x.*

Proof. Recall that $p^f(c,x^f) = p^h(\tau c, x^f)$ and $c^f(x) = c^h(x)/\tau$. Hence, we obtain $\frac{p^f(c^f(x),x)}{\tau c^f(x)} = \frac{p^h(\tau c^f(x),x)}{\tau c^h(x)/\tau} = \frac{p^h(c^h(x),x)}{c^h(x)}$. □

Proposition 2 implies that (C.7) and (C.8) replicate each other. As a result, they both form a single differential equation of the following form:

$$\frac{P'(x)}{P(x)} = -\frac{\ell'(x)}{\ell(x)}\frac{p^h(c^h(x),x) - c^h(x)}{c^h(x)}. \tag{C.9}$$

There is also another differential equation that comes from a consumer's budget constraint. It is straightforward to infer that, as long as $\tau > 1$, (costly) trade equilibrium implies that some range of the largest niches is left unserved by foreign firms. Formally, there exists an

export threshold niche, $\tilde{x}$, such that any $x \in [0, \tilde{x})$ is served by domestic firms only, while any $x \in [\tilde{x}, x_{\max}]$ is served by both domestic and foreign firms.[53] Note that $\tilde{x} = x^f(c_{\min}) = x^h(\tau \cdot c_{\min})$ and can be implicitly determined by $c^h(\tilde{x})/\tau = c_{\min}$. Hereafter, $\mathbb{I}_{\{x>\tilde{x}\}}$ is an indicator variable that equals one if $x \geq \tilde{x}$, and zero otherwise.

The budget constraint implies that for each x we have:

$$M^h(x)\,\Lambda\left(\frac{p^h(c^h(x),x)}{P(x)}\right) + N \cdot M^f(x)\,\Lambda\left(\frac{p^f(c^f(x),x)}{P(x)}\right)\cdot \mathbb{I}_{\{x\geq\tilde{x}\}} = 1, \tag{C.10}$$

where $M^h(x) = M^e g(c^h(x))c^{h\prime}(x)$ and $N \cdot M^f(x) = M^e g(c^f(x))c^{f\prime}(x)$ denotes the mass of active domestic and foreign varieties, respectively.[54]

Recall that $c^f(x) = c^h(x)/\tau$, $c^{f\prime}(x) = c^{h\prime}(x)/\tau$ and $p^f(c^f(x),x) = p^h(c^h(x),x)$. Hence, the budget constraint (C.10) can be rewritten as follows:

$$M^e g(c^h(x))c^{h\prime}(x)\Lambda\left(\frac{p^h(c^h(x),x)}{P(x)}\right) + \mathbb{I}_{\{x\geq\tilde{x}\}}\; N\cdot M^e g\left(\frac{c^h(x)}{\tau}\right)\frac{c^{h\prime}(x)}{\tau}\Lambda\left(\frac{p^h(c^h(x),x)}{P(x)}\right) = 1. \tag{C.11}$$

As a result, (C.9) and (C.11) form an equilibrium system of differential equations:

$$\begin{cases} P'(x) = -P(x)\frac{\ell'(x)}{\ell(x)}\frac{p(c^h(x),x)-c^h(x)}{c^h(x)}, \\ c^{h\prime}(x) = \dfrac{1}{M^e\Lambda\left(\frac{p^h(c^h(x),x)}{P(x)}\right)g\left(c^h(x)\right)\left[1+\mathbb{I}_{\{x\geq\tilde{x}\}}\; N\,\frac{g(c^h(x)/\tau)}{g(c^h(x))\tau}\right]} \end{cases}, \tag{C.12}$$

with two boundary conditions: $c^h(0) = c_{\min}$, $c^h(x_{\max}) = c^*$.

Similarly to the baseline setup, the endogenous mass of entrants per country (M^e) and the cutoff marginal cost level for production successful entry (c^*) are pinned down by the *zero cutoff profit* (ZCP) and *free entry* (FE) conditions:

$$\text{ZCP}: \quad \Pi^h(c^h(x_{\max}),(x_{\max})) = 0. \tag{C.13}$$

$$\text{FE}: \quad \int_0^{x_{\max}} \left[\Pi^h\left(c^h(x),x^h\right) + \mathbb{I}_{\{x\geq\tilde{x}\}}\; N\,\Pi^f\left(c^f(x),x\right)\right] g(c^h(x))c^{h\prime}(x)\mathrm{d}x = F^e, \tag{C.14}$$

where F^e denotes the sunk entry cost.

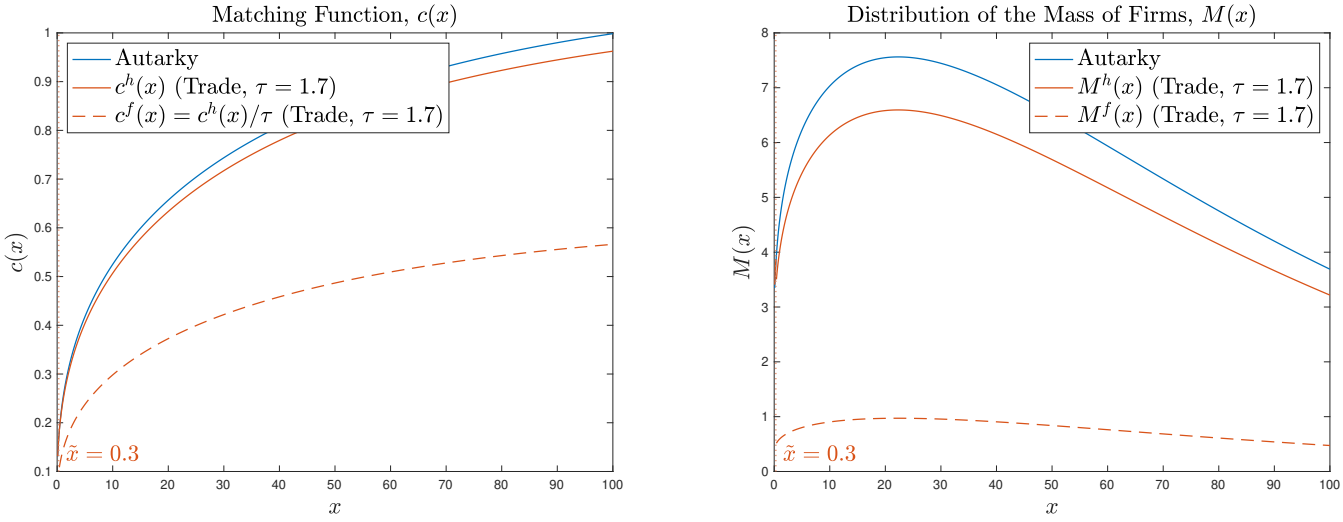
Recall that costly trade equilibrium implies that some (endogenous) range of the largest

[53] As will be discussed later, there can be equilibria where some range of the smallest niches is also left unpopulated by foreign firms. Such patterns take place if $F^{ex} > F$.

[54] One can readily verify that $\int_{\mathcal{X}} M^h(x)\mathrm{d}x = M\int_{\mathcal{X}}\frac{g(c^h(x))c^{h\prime}(x)}{G(c^*)}\mathrm{d}x = M = M\int_{c_{\min}}^{c^*}\frac{g(c)\mathrm{d}c}{G(c^*)}$, where $M = G(c^*)M^e$ is the (endogenous) total mass of active domestic firms.

niches, $[0, \tilde{x})$, is left unserved by foreign firms. To ensure that any $x \in [\tilde{x}, x_{\max}]$ is populated by both home and foreign firms in equilibria, I impose the assumption that fixed "market access" costs of entering a foreign market are equal to fixed production costs: $F^{ex} = F$. In this case, the cutoff marginal cost level for exporting firms is equal to $c^{**} \equiv c^f(x_{\max}) = c^*/\tau$. If $F^{ex} > F$, then only some range of middle-sized niches are populated by both home and foreign firms, while the largest and the smallest niches are served solely by domestic firms.[55] For quantitative exercise, I impose the assumption that $F^{ex} = F = 0.1$. It is the only difference from the parameterization of the baseline model. The figures below demonstrate equilibrium outcomes of the main parameters of interest.

Figure 10: The impact of opening the closed economy on the matching function and distribution of the mass of firms.



Note that the export threshold niche is close to zero, $\tilde{x} = 0.3$, implying that only the very large niches such that $x \in [0, \tilde{x})$ are not engaged in trade. So, it might be difficult to catch from Figure 10, that $c^f(x)$ and $M^f(x)$ are defined solely on the interval $[\tilde{x}, x_{\max}]$.

[55] In this case, the cutoff marginal cost level for exporting firms is determined from the condition of zero exporting profit. Details are available upon request.

Figure 11: The impact of opening the closed economy on prices and markups.

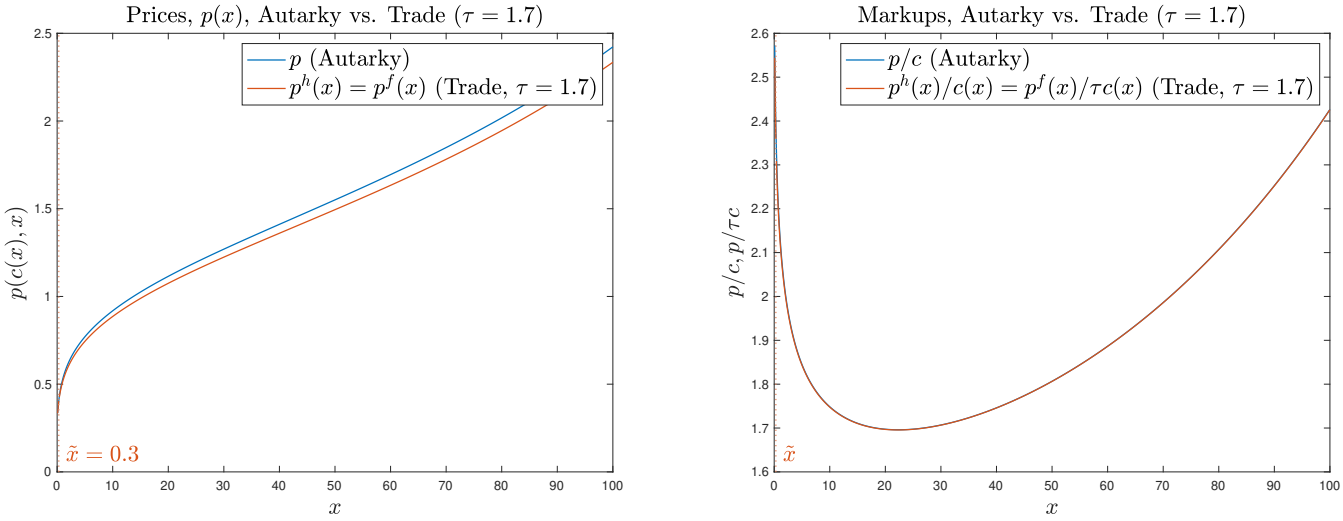
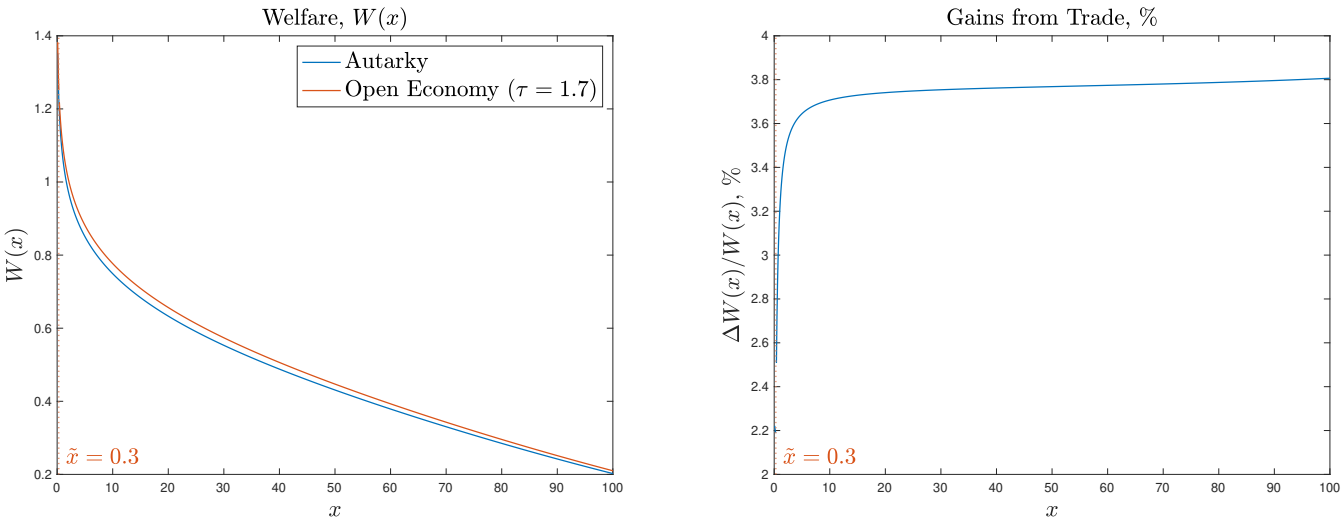


Figure 11 expectedly demonstrates that prices go down in trade equilibrium. However, niche-specific markups are mostly unaffected by opening the closed economy to trade. The reason is that, in addition to the decline in prices, the matching function shifts inward as well, thereby offsetting the impact of lower prices.

Figure 12: The impact of opening the closed economy to trade (autarky $\rightarrow$ costly trade, $\tau = 1.7$).

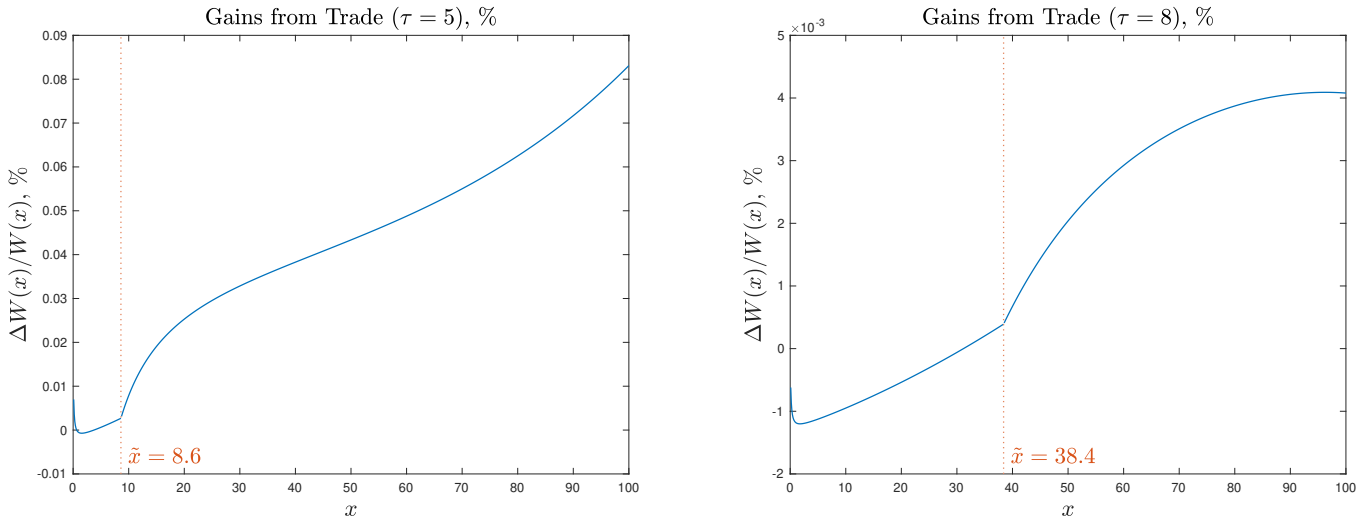


Recall that the baseline setup (where an exporting firm is assumed to choose a niche that is common for both domestic and foreign markets) predicts that the very large niches bear losses from opening the closed economy to trade (under $\tau = 1.7$). As for the setup when firms are allowed to choose domestic and foreign niches that may differ from each other, Figure 12 shows that all niches gain from trade (under $\tau = 1.7$). As for the intuition, recall that niche-specific welfare depends negatively on firms' prices while depending positively on the mass

of varieties consumed within a particular niche. Note that while the total amount of varieties consumed ($M^h(x) + M^f(x)$) across niches remains mostly unchanged compared to the Autarky scenario, prices have fallen. As a result, all niches gain from opening the closed economy to trade.

However, note that this alternative setup does not intrinsically rule out losses from trade. For example, the figures below show the impact of opening the closed economy to trade under $\tau = 5$ and $\tau = 8$.

Figure 13: The impact of opening the closed economy to trade (autarky $\rightarrow$ costly trade, $\tau = 5$ vs. $\tau = 8$).



One can note that if the iceberg transportation costs are large enough ($\tau \approx 5$), then consumers in some niches start to lose from opening the closed economy to trade. Let's consider the case of $\tau = 8$.A large fraction of niches that are not engaged in trade bear slight losses from trade. The reason is that firms' prices have increased in these niches, and such adverse effect on welfare outweighs the positive one that comes from the increase in the mass of varieties consumed in these niches. The corresponding increase in prices is, in turn, driven by the sophisticated shift of the matching function compared to the Autarky case – while preserving PAM, it has shifted up for large niches and has shifted down for smaller ones. It implies that large niches are populated by relatively less productive firms compared to the Autarky scenario.

C.2 Free Trade

One can readily verify that the case of free trade ($\tau = 1$, $F^{ex} = 0$) between $N + 1$ symmetric countries implies that $c^h(x) = c^f(x) = c(x)$ all niches are covered by both domestic and foreign firms. Moreover, regardless of whether domestic and foreign niches are allowed to be different

for a particular active firm – free trade equilibrium outcomes are identical for either of these setups. In particular, note that the system of differential equations is identical to that obtained under the free trade of the baseline open economy setup (see 3.9):

$$\begin{cases} \frac{P'(x)}{P(x)} = -\frac{\ell'(x)}{\ell(x)} \frac{p(c(x),x)-c(x)}{c(x)} \\ c'(x) = \frac{1}{(N+1)M^e g(c(x))\Lambda\left(\frac{p(c(x),x)}{P(x)}\right)} \end{cases},$$

with two boundary conditions: $c(0) = c_{\min}$, $c(x_{\max}) = c^*$.