Pricing and Shock Propagation through Demand and Production Networks

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Abstract

A firm shock moves downstream through input costs, across final demand, and upstream through supplier revenues before joint repricing closes the loop. I develop a framework in which locally integrable demand permits substitution and complementarity. The pricing condition decomposes each markup wedge into final-use exposure, production cycles, and demand reallocation carried through the revenue network. Jointly differentiating all pricing conditions delivers the firm-level distribution of a productivity shock after endogenous repricing. Through downstream prices and upstream revenues, the production network transforms product-demand links into a directed strategic-pricing network. An exact spectral condition maps separately measured product links into admissible compensated-demand curvature. A completed 2022 LLM-assisted complementarity measure and a text-based substitution network supply inputs for two planned exercises: a reference-point markup decomposition and an equilibrium elasticity to a hypothetical common AI technology shock, with firm loadings constructed from AI Supply Chain relationships.

Keywords: demand networks, production networks, substitutes, complements, markups, shock propagation, artificial intelligence.

JEL: D21, D43, D57, D85, L13.

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1 Introduction

A productivity gain at one firm changes more than that firm's price. Its customers obtain a cheaper input, their costs change, and the resulting price vector reallocates final expenditure. Substitution moves spending toward alternatives; complementarity can move spending in the same direction across products used together. These final-expenditure changes return through the production network as supplier revenues. Firms then revise their markups, so the first price movement starts another round of adjustment. The incidence of the original shock depends on the interaction between demand and production networks.

This paper builds that interaction into a tractable equilibrium model. There are N differentiated goods and N firms. Every good can be consumed or used as an intermediate input, and each firm charges one price in both outlets. Final demand is represented by a translog expenditure system. Its compensated demand matrix admits positive off-diagonal entries for substitutes and negative entries for complements. Production is Cobb–Douglas with an arbitrary input–output matrix, and firms simultaneously choose markups while anticipating the conditional equilibrium generated by any unilateral change. This closes the path from downstream prices to demand reallocation, upstream revenues, and joint repricing.

The main results concern markup formation and shock incidence. Proposition 1 gives an exact decomposition of each equilibrium markup wedge. One term measures exposure to aggregate final use, a second captures production cycles, and a third carries substitution and complementarity through the upstream revenue network. Proposition 2 converts the same interaction into a directed strategic-repricing network and supplies the equilibrium bridge to shock incidence. Proposition 3 combines conditional propagation with joint repricing to derive firm-level price, final-demand, and revenue responses to productivity shocks.

To map measured product links into the model, Lemma 1 constructs the demand curvature used by the propositions. A complete share-weighted graph provides ordinary background substitution. Graph Laplacians add separately measured substitution and complementarity links while preserving Slutsky symmetry and adding-up. A generalized Rayleigh bound gives the exact complement curvature that the background and substitution layers can support. Reference shares and the admissible compensated-demand matrix then recover the local translog curvature.

The quantitative work starts from measured primitives. Hoberg–Phillips text-based product similarity supplies the substitution layer (Hoberg and Phillips, 2016). For complementarity, an LLM reads firms' 2022 10-K product descriptions and produces structured own-product catalogs. Transparent joint-use rules then form firm-pair links; embedding scores only rank candidates for additional confirmation. This pipeline has produced the 2022 complementarity matrices used in the current implementation. Two planned exercises use these measured objects. The first evaluates the markup decomposition over admissible parameter scenarios. The second uses directed AI Supply Chain records (Cen et al., 2025) to assign heterogeneous firm loadings on a hypothetical common AI technology shock and will compute the resulting equilibrium elasticities, including endogenous repricing.

The closest pricing comparison is Wu (2026), who derives a network Lerner condition under simultaneous markup choice in a production network. Here one-price production accounting

is combined with a measured final-demand system that includes substitutes and complements, so the joint structure determines both markup formation and shock incidence. The propagation mechanism builds on production-network analyses of shocks and distortions (Atalay et al., 2011; Acemoglu et al., 2012; Baqaee, 2018; Baqaee and Farhi, 2020).

The measurement bridge draws on classical integrability and flexible dual demand, which make symmetry, homogeneity, and negative semidefiniteness central local restrictions (Hurwicz and Uzawa, 1971; Diewert, 1974; Christensen et al., 1975; Moschini, 1999). The construction applies those restrictions to separately measured product links using weighted graph Laplacians (Zelazo and Bürger, 2014). Product-network models show how graph position shapes differentiated-product competition (Ushchev and Zenou, 2018; Pellegrino, 2025), while empirical work learns both signs of product relationships from text, baskets, and transactions (McAuley et al., 2015; Ruiz et al., 2020; Tian et al., 2021).

Section 2 develops demand, production, and the conditional equilibrium. Section 3 derives markup determination and strategic repricing. Section 4 studies productivity shocks. Section 5 describes the completed network measurement and the two quantitative exercises.

2 Signed Demand and the Conditional Production Equilibrium

2.1 Household and firms

There are $N \geq 3$ firms and N goods. Firm i produces good i, which can enter final use or intermediate production. Prices $p = (p_1, \ldots, p_N)'$ are common across the two outlets. All vectors are columns, a prime denotes transpose, and vector logarithms and divisions are elementwise. The vector e_i is the i-th coordinate vector and $\mathbf{1}$ is the N-vector of ones. The matrix I is the $N \times N$ identity, and $\operatorname{diag}(v)$ places a vector v on the diagonal of a square matrix. For a positive vector v, v^{-1} denotes elementwise reciprocals.

Let $\mathcal{U}$ be a degree-one homogeneous final-use aggregator. The representative household first chooses a final-use vector q^F for a given level of the consumption aggregate $\mathcal{C}$. The unit expenditure function $P(p)$ is defined by

$$P(p)\mathcal{C} = \min_{q^F \geq 0} \left\{ p' q^F : \mathcal{U}(q^F) \geq \mathcal{C} \right\}, \qquad q^F = \mathcal{C} \nabla_p P(p). \tag{1}$$

At the upper tier, $\nu > 0$ shifts willingness to consume and $\eta_C > 0$ is the price elasticity of aggregate final use. Direct utility is

$$V(\mathcal{C}; \nu, \eta_C) = \begin{cases} \nu \dfrac{\mathcal{C}^{1-1/\eta_C}}{1 - 1/\eta_C}, & \eta_C \neq 1, \\ \nu \log \mathcal{C}, & \eta_C = 1. \end{cases} \tag{2}$$

Let $w > 0$ denote the wage, let $\bar{L}$ be the labor endowment, and let Π denote aggregate firm profit. The household owns the firms and solves

$$\max_{\mathcal{C} \geq 0,\, 0 \leq L \leq \bar{L}} V(\mathcal{C}; \nu, \eta_C) - L \quad \text{subject to} \quad P(p)\mathcal{C} \leq wL + \Pi. \tag{3}$$

Here L is labor supplied. At an interior labor choice, utility (2) gives $\nu \mathcal{C}^{-1/\eta_C} = P(p)/w$.

Firm i has productivity $A_i > 0$, labor share $\beta_i = 1 - \sum_j \Omega_{ij} > 0$, and input cost shares $\Omega_{ij} \geq 0$, collected in $\Omega \in \mathbb{R}_+^{N \times N}$. Rows index buyers and columns index suppliers. Let l_i be firm i's labor and let x_{ij} be its use of good j. The cost of producing output y_i defines unit cost $\kappa_i(p, w, A_i)$:

$$\kappa_i(p, w, A_i) y_i = \min_{l_i, \{x_{ij}\} \geq 0} \left\{ w l_i + \sum_j p_j x_{ij} : A_i \left(\frac{l_i}{\beta_i} \right)^{\beta_i} \prod_{j : \Omega_{ij} > 0} \left(\frac{x_{ij}}{\Omega_{ij}} \right)^{\Omega_{ij}} \geq y_i \right\}. \tag{4}$$

Constant returns imply

$$\kappa_i(p, w, A_i) = A_i^{-1} w^{\beta_i} \prod_j p_j^{\Omega_{ij}}.$$

Firm i's gross markup is $\mu_i = p_i / \kappa_i \geq 1$, and $\mu = (\mu_1, \ldots, \mu_N)'$. Let $A = (A_1, \ldots, A_N)'$ collect productivities. Gross revenue and profit are

$$r_i = p_i y_i, \qquad \pi_i = \left(1 - \frac{1}{\mu_i} \right) r_i, \qquad \Pi = \sum_i \pi_i.$$

We set $w = 1$ as the nominal numeraire. The final-use price index $P(p)$ remains endogenous. The household first-order condition becomes

$$\mathcal{C} = \left(\frac{\nu}{P(p)} \right)^{\eta_C}, \qquad Y \equiv P(p) \mathcal{C} = \sum_i p_i q_i^F, \tag{5}$$

where Y is nominal final expenditure. Goods and labor markets clear when

$$y_i = q_i^F + \sum_j x_{ji}, \qquad L = \sum_i l_i.$$

The equilibrium has two layers. Conditional on a markup vector μ, the household problems (1) and (3), the cost problem (4), and market clearing determine the inner equilibrium. Firms then choose markups simultaneously. Specifically, μ_{-i} denotes the markups of all firms except i, and firm i solves

$$\max_{\mu_i \geq 1} \frac{\pi_i(\mu_i, \mu_{-i})}{P(p(\mu_i, \mu_{-i}))}. \tag{6}$$

A unilateral deviation holds A, Ω, ν, and μ_{-i} fixed and resolves the inner equilibrium. Maximization of own real profit in (6) is the model's conduct assumption.

2.2 A locally integrable demand system

For any price vector, let $s_i(p) = p_i q_i^F / Y = \partial \log P(p) / \partial \log p_i$ be good i's expenditure share. Let p^0 be a reference price vector, let $s^0 \gg 0$ be the associated share vector, and impose $\mathbf{1}' s^0 = 1$. The translog unit expenditure function has curvature $\Gamma \in \mathbb{R}^{N \times N}$:

$$\log P(p) = \log P(p^0) + (s^0)' \log(p / p^0) + \frac{1}{2} \log(p / p^0)' \Gamma \log(p / p^0), \tag{7}$$

where $\Gamma = \Gamma'$ and $\Gamma \mathbf{1} = 0$. Shephard's lemma gives

$$s(p) = s^0 + \Gamma \log(p / p^0), \qquad q_i^F = \frac{Y s_i(p)}{p_i}. \tag{8}$$

The expenditure-weighted compensated-demand matrix $K(p) \in \mathbb{R}^{N\times N}$ is

$$K(p) = \Gamma + s(p)s(p)' - \operatorname{diag}(s(p)). \tag{9}$$

If $h_i(p, \mathcal{C})$ is Hicksian demand for good i, then

$$\left.\frac{\partial \log h_i}{\partial \log p_j}\right|_{\mathcal{C}} = \frac{K_{ij}(p)}{s_i(p)}. \tag{10}$$

For $i \neq j$, $K_{ij} > 0$ denotes compensated substitution and $K_{ij} < 0$ denotes compensated complementarity. Symmetry, adding-up, and local concavity require $K(p) = K(p)'$, $K(p)\mathbf{1} = 0$, and $K(p) \preceq 0$, where $\preceq 0$ denotes negative semidefiniteness.

Let $\widehat{S}, \widehat{C} \in \mathbb{R}^{N\times N}$ be symmetric, nonnegative, zero-diagonal matrices of measured substitution and complementarity links. For any such matrix G, define its graph Laplacian as

$$\mathcal{L}(G) = \operatorname{diag}(G\mathbf{1}) - G. \tag{11}$$

The parameters θ_S and θ_C scale the two measured layers. The subspace of relative-price directions is $\mathbf{1}^\perp = \{x \in \mathbb{R}^N : x'\mathbf{1} = 0\}$.

Lemma 1 (Demand-system well-definedness)**.** *Suppose $s^0 \gg 0$, $\mathbf{1}'s^0 = 1$, $\theta_S > 0$, and $\theta_C \geq 0$. Define*

$$K^0 = -\left[\operatorname{diag}(s^0) - s^0(s^0)'\right] - \theta_S\mathcal{L}(\widehat{S}) + \theta_C\mathcal{L}(\widehat{C}). \tag{12}$$

Then:

(i) $K^0 = (K^0)'$, $K^0\mathbf{1} = 0$, *and, for* $i \neq j$,

$$K^0_{ij} = s^0_i s^0_j + \theta_S\widehat{S}_{ij} - \theta_C\widehat{C}_{ij}.$$

(ii) The matrix is demand-admissible at the reference point, $K^0 \preceq 0$, *if and only if*

$$0 \leq \theta_C \leq \inf_{\substack{x'\mathbf{1}=0 \\ x'\mathcal{L}(\widehat{C})x>0}} \frac{x'\left[\operatorname{diag}(s^0) - s^0(s^0)' + \theta_S\mathcal{L}(\widehat{S})\right]x}{x'\mathcal{L}(\widehat{C})x}, \tag{13}$$

where the infimum over an empty set is $+\infty$.

(iii) Given (s^0, K^0), *the unique translog curvature that matches the reference shares and compensated-demand matrix is*

$$\Gamma = K^0 - s^0(s^0)' + \operatorname{diag}(s^0). \tag{14}$$

Equations (7)–(9) *then satisfy* $s(p^0) = s^0$ *and* $K(p^0) = K^0$.

(iv) If the upper inequality in (13) *is strict,* K^0 *is negative definite on* $\mathbf{1}^\perp$. *Shares remain positive and* $P(p)$ *remains locally concave in a neighborhood of* p^0.

The lemma separates pairwise measurement from system-wide demand rationalizability. The off-diagonal equation gives measured product links their intended incremental signs around ordinary background substitution. The spectral condition asks whether background and measured substitution curvature jointly support the entire complementarity layer in every relative-price direction. Equation (14) then embeds the admissible object in a demand system without introducing another curvature parameter.

Hereafter, s and K without price arguments denote $s(p)$ and $K(p)$ at the current conditional equilibrium.

2.3 Conditional prices and revenues

Cost minimization in (4) gives

$$l_i = \frac{\beta_i r_i}{\mu_i}, \qquad p_j x_{ij} = \Omega_{ij} \frac{r_i}{\mu_i}.$$

Combining unit cost with $p_i = \mu_i \kappa_i$ yields the conditional price vector

$$\log p = (I - \Omega)^{-1}(\log \mu - \log A), \tag{15}$$

Thus $(I - \Omega)^{-1}$ carries costs and markups downstream into prices.

Let $r = (r_1, \ldots, r_N)'$ collect firm revenues. By (5), final users spend Ys_j on good j. Input purchases imply $r_j = Ys_j + \sum_i \Omega_{ij} r_i / \mu_i$, so

$$r = \left[I - \Omega' \operatorname{diag}(\mu^{-1}) \right]^{-1} Ys. \tag{16}$$

The inverse in (16) carries final expenditure upstream through suppliers. The operator $\rho(\cdot)$ below denotes spectral radius.

Assumption 1 (Regular inner equilibrium)**.** *At the equilibrium under study,*

$$\rho(\Omega) < 1, \qquad \rho\left(\Omega' \operatorname{diag}(\mu^{-1}) \right) < 1, \qquad \min_i \beta_i > 0, \qquad \min_i s_i(p) > 0, \qquad K(p) \preceq 0,$$

and the household labor choice is interior.

Lemma 1 establishes local demand regularity, while Assumption 1 makes downstream price and upstream revenue propagation well defined. The results below apply to a regular interior branch: Proposition 1 is an exact pricing condition at the current equilibrium, and Propositions 2 and 3 describe local changes along that branch. The analysis characterizes this regular branch locally; global existence and uniqueness are separate questions.

3 Markup Determination and Strategic Repricing

3.1 An exact markup-wedge decomposition

An interior solution to the conduct problem (6) satisfies

$$\frac{1}{\mu_i - 1} = -\frac{\partial \log(r_i / P)}{\partial \log \mu_i}. \tag{17}$$

The right side follows a unilateral markup change through conditional prices, final demand, and revenues. This gives the following structural equation.

Proposition 1 (Two-network decomposition of the markup wedge)**.** *Under Assumption 1, every interior markup in a simultaneous Nash equilibrium satisfies*

$$\begin{aligned} \frac{1}{\mu_i - 1} = {} & (1 + \eta_C) s' (I - \Omega)^{-1} e_i \\ & + \left\{ \left(\left[I - \Omega' \operatorname{diag}(\mu^{-1}) \right]^{-1} \right)_{ii} - 1 \right\} \\ & - \frac{e_i' \left[I - \Omega' \operatorname{diag}(\mu^{-1}) \right]^{-1} \left[\operatorname{diag}(s) + K \right] (I - \Omega)^{-1} e_i}{e_i' \left[I - \Omega' \operatorname{diag}(\mu^{-1}) \right]^{-1} s}. \end{aligned} \tag{18}$$

All objects on the right side are evaluated at the current conditional equilibrium: $p = p(\mu, A)$, $s = s(p)$, and $K = K(p)$.

The final ratio maps demand reallocation back into firm i's revenue. The matrix $\mathrm{diag}(s)$ revalues existing final quantities, while K reallocates compensated demand across substitutes and complements. The downstream price inverse appears to the right of this demand response and the upstream revenue inverse to its left. The term therefore records the interaction of the two networks.

The other terms complete the markup wedge. The first line measures exposure to aggregate final use. A unilateral increase in μ_i moves the price vector by $(I - \Omega)^{-1} e_i$, raising the exact price index and contracting aggregate final use. The second line captures production cycles: lower input spending per dollar of firm i's revenue partly returns through input-output loops.

Equation (18) has a dual empirical role. At a model-consistent equilibrium it is an exact decomposition of the markup wedge. At a measured reference point, the difference between its two sides is a pricing residual that measures consistency with the maintained conduct and parameter choices. This distinction keeps accounting identities separate from the firm's optimality condition.

3.2 The strategic-repricing network

Define firm i's pricing condition by

$$\mathrm{FOC}_i(\log \mu, \log A) = \frac{1}{\mu_i - 1} + \frac{\partial \log(r_i / P)}{\partial \log \mu_i}. \tag{19}$$

After substituting the inner equilibrium, Proposition 1 is the system $\mathrm{FOC}_i = 0$, $i = 1, \ldots, N$. A change in another firm's markup moves downstream through prices, across demand, and upstream through revenue.

Proposition 2 (Strategic pricing relationships and joint repricing)**.** *Suppose Assumption 1 holds at an interior equilibrium. Assume the pricing conditions are continuously differentiable and satisfy the strict own second-order conditions*

$$\frac{\partial \mathrm{FOC}_i}{\partial \log \mu_i} < 0 \quad \textit{for every } i.$$

For $i \neq j$, define the local best-response slope

$$\mathsf{BR}_{ij} = -\frac{\partial \mathrm{FOC}_i / \partial \log \mu_j}{\partial \mathrm{FOC}_i / \partial \log \mu_i}, \qquad \mathsf{BR}_{ii} = 0. \tag{20}$$

The entries form $\mathsf{BR} \in \mathbb{R}^{N \times N}$. This matrix is generally directed. The full system $(\Omega, K, s, \mu, \eta_C)$ determines each local best-response slope and its sign.

For a productivity differential $\mathrm{d} \log A \in \mathbb{R}^N$, if $I - \mathsf{BR}$ is nonsingular, the equilibrium markup response is

$$\mathrm{d} \log \mu = (I - \mathsf{BR})^{-1} \Bigg\{ - \mathrm{diag}\left(\frac{\partial \mathrm{FOC}_1}{\partial \log \mu_1}, \ldots, \frac{\partial \mathrm{FOC}_N}{\partial \log \mu_N} \right)^{-1} \times \left[\frac{\partial \mathrm{FOC}_i}{\partial \log A_j} \right]_{i,j=1}^{N} \Bigg\} \mathrm{d} \log A. \tag{21}$$

If $\rho(\mathsf{BR}) < 1$, then

$$(I - \mathsf{BR})^{-1} = \sum_{m=0}^{\infty} \mathsf{BR}^m. \tag{22}$$

Each power collects another round of firms responding to previous markup changes.

The proposition closes the equilibrium feedback left open by conditional shock propagation. The demand matrix describes compensated expenditure reallocation between products. The strategic matrix describes how one firm's optimal markup responds after the entire conditional equilibrium has adjusted to another firm's markup. Production links connect these objects first through prices and again through revenue, and these two transformations also make the strategic relationship directed.

The following benchmark isolates the distinction analytically.

Corollary 1 (No-input benchmark)**.** *Suppose $\Omega = 0$ and retain the assumptions of Proposition 2. For every $i \neq j$,*

$$\mathsf{BR}_{ij} = \frac{(s_i s_j - K_{ij})\left[2\eta_C - \{s_i(\mu_i - 1)\}^{-1}\right]}{\dfrac{\mu_i}{(\mu_i - 1)^2} + \Gamma_{ii}\left[2\eta_C - \{s_i(\mu_i - 1)\}^{-1}\right]}, \tag{23}$$

where the denominator is positive under the strict own second-order condition. A Hicksian complement, $K_{ij} < 0$, is a strategic substitute for firm i when

$$s_i(\mu_i - 1) < \frac{1}{2\eta_C},$$

and a strategic complement when this inequality is reversed. For a strong Hicksian substitute, $K_{ij} > s_i s_j$, these strategic signs reverse. A weak Hicksian substitute, $0 < K_{ij} < s_i s_j$, has the same local strategic sign as a Hicksian complement. At the reference point,

$$s_i^0 s_j^0 - K_{ij}^0 = \theta_C \widehat{C}_{ij} - \theta_S \widehat{S}_{ij}.$$

The scalar threshold is available only after removing both production-network transformations. With intermediate inputs, a rival's markup changes a vector of downstream prices, and final expenditure returns through many suppliers. The general best-response slope remains computable from (18) and depends on the full two-network system.

4 Productivity Shocks and Equilibrium Elasticities

Consider a vector of firm productivity changes $\mathrm{d}\log A = (\mathrm{d}\log A_1, \ldots, \mathrm{d}\log A_N)'$. Preferences and Ω remain fixed, and all expressions are evaluated at a regular interior equilibrium. The first result holds markups fixed to separate conditional propagation from strategic repricing.

Lemma 2 (Conditional propagation at fixed markups)**.** *Holding every markup fixed, a productivity shock generates*

$$\mathrm{d}\log p\big|_{\mu} = -(I - \Omega)^{-1}\mathrm{d}\log A, \tag{24}$$

$$\mathrm{d}(Ys)\big|_{\mu} = -Y[\mathrm{diag}(s) + K - \eta_C ss'](I - \Omega)^{-1}\mathrm{d}\log A, \tag{25}$$

$$\mathrm{d}r\big|_{\mu} = -Y\left[I - \Omega'\,\mathrm{diag}(\mu^{-1})\right]^{-1}[\mathrm{diag}(s) + K - \eta_C ss'](I - \Omega)^{-1}\mathrm{d}\log A. \tag{26}$$

Equation (26) reads from right to left. Production links transmit productivity downstream into prices, demand reallocates final expenditure, and production links transmit expenditure upstream into supplier revenue. Equations (24)– (26) form the conditional part of the full equilibrium effect.

Proposition 3 (Firm-level distribution of productivity-shock effects)**.** *Let* $\mathrm{d}\log\mu$ *be the joint markup response in Proposition 2. The local equilibrium responses of prices, final demand, and revenue are*

$$\mathrm{d}\log p = (I-\Omega)^{-1}(\mathrm{d}\log\mu - \mathrm{d}\log A), \tag{27}$$

$$\mathrm{d}\log q^F = \left[-\eta_C \mathbf{1}s' + \mathrm{diag}(s)^{-1}K\right]\mathrm{d}\log p, \tag{28}$$

$$\mathrm{d}r = \left[I - \Omega'\,\mathrm{diag}(\mu^{-1})\right]^{-1}\Big\{Y[\mathrm{diag}(s) + K - \eta_C ss']\mathrm{d}\log p - \Omega'\,\mathrm{diag}(\mu^{-1})\,\mathrm{diag}(r)\mathrm{d}\log\mu\Big\}. \tag{29}$$

For any differentiable equilibrium outcome X*, its response has the exact first-order decomposition*

$$\mathrm{d}X = \left.\frac{\partial X}{\partial \log A}\right|_{\mu}\mathrm{d}\log A + \left.\frac{\partial X}{\partial \log \mu}\right|_{A}\mathrm{d}\log\mu. \tag{30}$$

The first term is conditional propagation and the second is the effect of endogenous repricing, including all strategic rounds in (22)*.*

The proposition delivers a firm-level incidence map. Demand curvature affects final demand and revenue directly through K, and it affects every outcome indirectly through firms' pricing conditions. The production matrix weights both channels. As a result, the same productivity shock can generate different price, demand, and revenue responses for firms with similar direct exposure.

Let μ^0, r^0, and Y^0 denote reference markups, revenues, and final expenditure. Local log responses and revenue changes normalized by Y^0 are determined by

$$s^0, \quad K^0, \quad \Omega, \quad \mu^0, \quad r^0/Y^0, \quad \eta_C, \quad \text{and} \quad \mathrm{d}\log A.$$

Revenue changes in levels additionally require Y^0. Equation (14) recovers Γ from (s^0, K^0), and differentiating (18) supplies the joint markup response. Appendix C gives the corresponding price-index, aggregate-consumption, output, profit, labor, and welfare formulas.

5 Measured Networks and Two Quantitative Directions

The theory requires reference shares, a production matrix, two demand layers, markups, and a productivity-shock vector. Completed measurement supplies the network and accounting inputs. Quantitative magnitudes require a model-consistent pricing baseline solved across the predeclared demand-curvature scenarios.

5.1 Mapping product text into demand curvature

The 2022 implementation begins with Compustat firms, BEA Input–Output accounts, and Hoberg–Phillips TNIC links (S&P Global Market Intelligence, 2026; U.S. Bureau of Economic

Analysis, 2026; Hoberg and Phillips, 2026). BEA broad final use is allocated across firms within output commodities using positive sales. This gives s^0. BEA use shares give Ω, with rows retaining the buyer-industry technology and columns assigned across suppliers in the corresponding output commodity. TNIC cosine similarity supplies the raw substitution layer.

The completed 2022 catalog covers 4,437 firms and contains 54,951 extracted product rows. On this roster, the deterministic core contains 10,548 firm pairs and the local-balanced matrix contains 33,085. The joint model restricts all inputs to a common connected roster of 3,769 firms.

The complementarity layer is built in three stages. First, an LLM reads each firm's 2022 10-K Item 1 and Item 7 and extracts a structured catalog of the firm's own final-use products and services. The fields record product group, economic role, use episode, customer type, compatibility, and application domain. The LLM converts unstructured disclosure into product objects, while the next stage classifies pairwise mechanisms.

Second, deterministic rules form candidate product pairs when their roles, domains, or compatibility fields describe a concrete joint-use episode. The rules retain mechanisms such as device–content, application–platform, payment–merchant, and diagnostic–treatment pairings. They exclude ordinary same-market similarity, vertical raw inputs, generic cloud dependence, and vague business relationships. Accepted product pairs are aggregated to firm pairs, symmetrized, and assigned zero diagonal. A strict rule-based core provides the lower-bound specification, while a transparent local-balanced matrix adds rule-supported candidates.

Third, product-level embeddings rank possible omissions. Embedding similarity is a search device: an embedding-discovered pair enters the complementarity matrix only after a second mechanism check by a strict LLM prompt or a human audit. Pair admission therefore remains tied to an explicit joint-use mechanism while machine learning searches a large candidate space.

On the common roster, each raw substitution and complementarity score is multiplied by $s_i^0 s_j^0$ before becoming $\widehat{S}_{ij}$ and $\widehat{C}_{ij}$. By (10), this scaling makes the incremental Hicksian cross-elasticity proportional to the destination firm's share instead of the inverse of the origin firm's share.

The spectral frontier provides the theoretical discipline for complement curvature. A provisional micro cross-elasticity serves as a calibration reference for θ_S; the first exercise reports one-half, one, and twice that value. Conditional on each substitution scale, θ_C is evaluated at 0, 25, 50, 75, and 90 percent of the exact frontier in (13). Every scenario must pass symmetry, row-sum, share-positivity, generalized-eigenvalue, and negative-semidefiniteness checks.

5.2 Direction I: markup decomposition

The first exercise targets the decomposition in Proposition 1 at the 2022 reference point. For each admissible demand scenario, it will compute the final-use, production-cycle, and demand-reallocation terms using the same firm roster, production matrix, expenditure shares, and markup measure. The pricing residual will record how closely the measured point satisfies the simultaneous markup conditions. The exercise will then solve the full pricing system for a model-consistent baseline on a regular interior branch.

The central comparison changes the demand layer while holding the accounting objects fixed. It starts from background substitution and production links, adds measured substitution, and

then adds complementarity along the admissible frontier. The output will report the distribution of each exact term, its sales-weighted mean, firm and sector rankings, and changes in those rankings across demand scenarios. This design uses the decomposition as a theoretical measurement equation and keeps fit diagnostics visible.

5.3 Direction II: an AI technology-shock elasticity

The completed AI Supply Chain (AISC) mapping links dated actor-relationship records to Compustat using audited aliases and exact normalized names. Each strict input relationship is directed from an AI provider or input supplier to the public firm that uses the input. Partnerships, investment links, rivalry statements, internal relationships, and vague records remain separate audit categories.

Let z_i^{AISC} denote firm i's measured exposure loading from this mapping. The baseline uses a strict input-user indicator; a normalized and winsorized count of strict relationships provides a continuous robustness measure. The second exercise maps this loading to the productivity primitive in Section 4. Let T_{AI} be a hypothetical common AI technology index and let λ_A be the productivity elasticity for a unit-exposure firm. The scenario maps the common shock into firm productivity according to

$$\mathrm{d}\log A_i = \lambda_A z_i^{\text{AISC}} \mathrm{d}\log T_{\text{AI}}. \tag{31}$$

Thus $\lambda_A z_i^{\text{AISC}}$ is firm i's productivity elasticity with respect to the hypothetical technology index. AISC supplies the heterogeneous loading, while transparent scenarios supply λ_A.

Substituting (31) into Proposition 3 gives the equilibrium elasticity of each price, markup, final-demand quantity, and revenue to T_{AI}. The same calculation gives aggregate consumption, labor, profit, and welfare. Reporting both terms in (30) separates propagation at fixed markups from the additional equilibrium effects of endogenous repricing.

Conclusion

Demand and production networks jointly determine how firms price and how shocks are distributed. Production links transmit cost changes downstream and final expenditure upstream. Substitution and complementarity govern the reallocation between those two passages. Joint markup choice then feeds the resulting revenue changes back into prices.

The model turns this mechanism into three usable objects: an admissible map from measured product relationships to compensated-demand curvature, an exact markup-wedge decomposition, and equilibrium shock elasticities that include strategic repricing. The completed text pipeline supplies a firm-level complementarity matrix, and the AISC mapping supplies heterogeneous firm loadings for a hypothetical AI technology shock. The composition of downstream price propagation, demand reallocation, upstream revenue propagation, and joint repricing governs both markup wedges and the firm-level incidence of productivity shocks.

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A Demand admissibility

A.1 Proof of Lemma 1

From (11), any symmetric nonnegative matrix G satisfies

$$x'\mathcal{L}(G)x = \frac{1}{2}\sum_{i,j} G_{ij}(x_i - x_j)^2 \geq 0. \tag{32}$$

Thus $\mathcal{L}(G)$ is symmetric and positive semidefinite, and $\mathcal{L}(G)\mathbf{1} = 0$. The matrix $\mathrm{diag}(s^0) - s^0(s^0)'$ is also a Laplacian: its off-diagonal edge weights are $s_i^0 s_j^0$. These observations establish symmetry and adding-up in part (i). For $i \neq j$, taking the off-diagonal entry of (12) gives

$$K_{ij}^0 = s_i^0 s_j^0 + \theta_S \widehat{S}_{ij} - \theta_C \widehat{C}_{ij}.$$

For any x,

$$x'K^0 x = -x'\left[\mathrm{diag}(s^0) - s^0(s^0)' + \theta_S \mathcal{L}(\widehat{S}) - \theta_C \mathcal{L}(\widehat{C})\right] x.$$

All three terms in brackets vanish in the common-price direction. On $\mathbf{1}^\perp$, $K^0 \preceq 0$ holds if and only if

$$\theta_C x'\mathcal{L}(\widehat{C})x \leq x'\left[\mathrm{diag}(s^0) - s^0(s^0)' + \theta_S \mathcal{L}(\widehat{S})\right] x$$

for every x. Dividing in directions with a positive denominator and taking the infimum gives (13). Directions with $x'\mathcal{L}(\widehat{C})x = 0$ satisfy the inequality because the remaining matrix is positive semidefinite. This proves part (ii).

Define Γ by (14). It is symmetric and

$$\Gamma\mathbf{1} = K^0\mathbf{1} - s^0\{(s^0)'\mathbf{1}\} + s^0 = 0.$$

At $p = p^0$, equation (8) gives $s(p^0) = s^0$. Substitution into (9) gives

$$K(p^0) = \Gamma + s^0(s^0)' - \mathrm{diag}(s^0) = K^0.$$

The same equality shows that any matching Γ must equal (14), proving uniqueness.

Finally, the Hessian of the unit expenditure function obeys

$$\frac{p_i p_j}{P(p)} \frac{\partial^2 P(p)}{\partial p_i \partial p_j} = K_{ij}(p).$$

Because $s^0 \gg 0$, the background share graph is connected and its Laplacian is positive definite on $\mathbf{1}^\perp$. A strict inequality in (13) therefore makes K^0 negative definite on this subspace. Continuity preserves negative definiteness on the relative-price subspace and positivity of shares in a neighborhood of p^0. This proves part (iv). □

B Conditional equilibrium and the markup condition

B.1 Cost minimization and accounting

The first-order conditions of (4) allocate the share β_i of variable cost to labor and the share Ω_{ij} to input j. Because total cost is r_i/μ_i,

$$w l_i = \frac{\beta_i r_i}{\mu_i}, \qquad p_j x_{ij} = \frac{\Omega_{ij} r_i}{\mu_i}.$$

Substituting these demands into the production constraint gives the unit cost reported below (4). Taking logs of $p_i = \mu_i \kappa_i$ after setting $w = 1$ gives

$$\log p = \log \mu - \log A + \Omega \log p,$$

which yields (15). Goods clearing and the input demand above give

$$r_j = Y s_j + \sum_i \Omega_{ij} \frac{r_i}{\mu_i},$$

which yields (16). Constant returns and market clearing also imply the household budget identity $P\mathcal{C} = L + \Pi$.

B.2 Proof of Proposition 1

A unilateral change in firm i's log markup changes prices according to

$$\frac{\partial \log p}{\partial \log \mu_i} = (I - \Omega)^{-1} e_i. \tag{33}$$

Equation (33) implies that the price index, aggregate consumption, and nominal final expenditure satisfy

$$\begin{aligned}
\frac{\partial \log P}{\partial \log \mu_i} &= s'(I - \Omega)^{-1} e_i, \\
\frac{\partial \log \mathcal{C}}{\partial \log \mu_i} &= -\eta_C s'(I - \Omega)^{-1} e_i, \\
\frac{\partial \log Y}{\partial \log \mu_i} &= (1 - \eta_C) s'(I - \Omega)^{-1} e_i.
\end{aligned}$$

Differentiating (16) with respect to firm i's log markup gives

$$\frac{\partial r}{\partial \log \mu_i} = \left[I - \Omega' \operatorname{diag}(\mu^{-1})\right]^{-1} \left\{ Y[\Gamma + (1-\eta_C)ss'](I-\Omega)^{-1} e_i - \frac{r_i}{\mu_i}\Omega' e_i \right\}. \tag{34}$$

Use

$$\Gamma = K - ss' + \operatorname{diag}(s), \qquad e_i' \left[I - \Omega' \operatorname{diag}(\mu^{-1})\right]^{-1} s = \frac{r_i}{Y},$$

and the cycle identity

$$\frac{1}{\mu_i} e_i' \left[I - \Omega' \operatorname{diag}(\mu^{-1})\right]^{-1} \Omega' e_i = \left(\left[I - \Omega' \operatorname{diag}(\mu^{-1})\right]^{-1} \right)_{ii} - 1.$$

Substituting these relations into the i-th component of (34), dividing by r_i, and subtracting the price-index elasticity gives $\partial \log(r_i/P)/\partial \log \mu_i$. Taking its negative gives the right side of (17). Rearrangement yields (18). □

C Strategic pricing and productivity responses

C.1 Proof of Proposition 2

For any firm j, a unilateral markup change produces

$$\frac{\partial \log p}{\partial \log \mu_j} = (I-\Omega)^{-1} e_j, \tag{35}$$

$$\frac{\partial s}{\partial \log \mu_j} = \Gamma (I-\Omega)^{-1} e_j, \tag{36}$$

$$\frac{\partial \log Y}{\partial \log \mu_j} = (1-\eta_C) s' (I-\Omega)^{-1} e_j, \tag{37}$$

$$\frac{\partial r}{\partial \log \mu_j} = \left[I - \Omega' \operatorname{diag}(\mu^{-1})\right]^{-1} \left\{ Y[\operatorname{diag}(s) + K - \eta_C ss'](I-\Omega)^{-1} e_j - \Omega' \operatorname{diag}(\mu^{-1}) \operatorname{diag}(r) e_j \right\}. \tag{38}$$

Equations (35)–(38), together with the derivative of (18), make every cross derivative in (20) explicit. They also show why the cross derivative depends on both production transformations and the full demand matrix.

For $i \neq j$, the implicit-function theorem applied to $\mathrm{FOC}_i = 0$ gives (20). By its definition, the full pricing Jacobian factors as

$$\left[\frac{\partial \mathrm{FOC}_i}{\partial \log \mu_j}\right]_{i,j=1}^{N} = \operatorname{diag}\left(\frac{\partial \mathrm{FOC}_1}{\partial \log \mu_1}, \ldots, \frac{\partial \mathrm{FOC}_N}{\partial \log \mu_N}\right) (I - \mathsf{BR}).$$

Totally differentiating all pricing conditions with respect to productivity and substituting this factorization gives (21). The Neumann series proves (22) when $\rho(\mathsf{BR}) < 1$. □

C.2 Proof of Corollary 1

Set $\Omega = 0$. Then $r_i = Ys_i$, and equation (19) becomes

$$\mathrm{FOC}_i = \frac{1}{\mu_i - 1} - \eta_C s_i + \frac{\Gamma_{ii}}{s_i}.$$

For $j \neq i$,

$$\frac{\partial \,\mathrm{FOC}_i}{\partial \log \mu_j} = -\left(\eta_C + \frac{\Gamma_{ii}}{s_i^2}\right)\Gamma_{ij},$$

while

$$\frac{\partial \,\mathrm{FOC}_i}{\partial \log \mu_i} = -\frac{\mu_i}{(\mu_i - 1)^2} - \left(\eta_C + \frac{\Gamma_{ii}}{s_i^2}\right)\Gamma_{ii}.$$

The pricing condition and (9) imply

$$\eta_C + \frac{\Gamma_{ii}}{s_i^2} = 2\eta_C - \frac{1}{s_i(\mu_i - 1)}, \qquad -\Gamma_{ij} = s_i s_j - K_{ij}.$$

Substitution into (20) yields (23). The denominator is minus the own derivative and is positive under the strict second-order condition. The sign cases follow directly. At p^0, the final equality follows from the off-diagonal entry of (12). □

C.3 Proofs of Lemma 2 and Proposition 3

For fixed preferences and fixed Ω, the conditional equilibrium obeys

$$\mathrm{d}\log p = (I - \Omega)^{-1}(\mathrm{d}\log \mu - \mathrm{d}\log A), \tag{39}$$

$$\mathrm{d}s = \Gamma(I - \Omega)^{-1}(\mathrm{d}\log \mu - \mathrm{d}\log A), \tag{40}$$

$$\mathrm{d}\log Y = (1 - \eta_C)s'(I - \Omega)^{-1}(\mathrm{d}\log \mu - \mathrm{d}\log A). \tag{41}$$

Differentiating the revenue equation gives

$$\begin{aligned}\mathrm{d}r = \left[I - \Omega' \,\mathrm{diag}(\mu^{-1})\right]^{-1} \Big\{ & Y[\mathrm{diag}(s) + K - \eta_C s s'](I - \Omega)^{-1}(\mathrm{d}\log \mu - \mathrm{d}\log A) \\ & - \Omega' \,\mathrm{diag}(\mu^{-1})\,\mathrm{diag}(r)\,\mathrm{d}\log \mu \Big\}.\end{aligned} \tag{42}$$

Setting $\mathrm{d}\log \mu = 0$ proves Lemma 2. Substituting the equilibrium markup response (21) into (39) and (42) gives (27) and (29). From $q^F = \mathcal{C}\nabla_p P(p)$,

$$\mathrm{d}\log \mathcal{C} = -\eta_C s' \mathrm{d}\log p, \qquad \mathrm{d}\log q^F = \left[-\eta_C \mathbf{1}s' + \mathrm{diag}(s)^{-1}K\right]\mathrm{d}\log p,$$

which proves (28). The chain rule yields (30). □

For completeness, the price index, aggregate final use, physical output, and real profit respond according to

$$\mathrm{d}\log P = s'\mathrm{d}\log p, \tag{43}$$

$$\mathrm{d}\log \mathcal{C} = -\eta_C s'\mathrm{d}\log p, \tag{44}$$

$$\mathrm{d}\log y_i = \frac{\mathrm{d}r_i}{r_i} - \mathrm{d}\log p_i, \tag{45}$$

$$\mathrm{d}\log(\pi_i/P) = \frac{\mathrm{d}r_i}{r_i} + \frac{1}{\mu_i - 1}\mathrm{d}\log \mu_i - \mathrm{d}\log P. \tag{46}$$

Aggregate labor, profit, and welfare satisfy

$$\mathrm{d}L = \sum_i \frac{\beta_i}{\mu_i}\mathrm{d}r_i - \sum_i \frac{\beta_i r_i}{\mu_i}\mathrm{d}\log\mu_i, \tag{47}$$

$$\mathrm{d}\Pi = \mathbf{1}'[I - \mathrm{diag}(\mu^{-1})]\mathrm{d}r + r'\,\mathrm{diag}(\mu^{-1})\mathrm{d}\log\mu, \tag{48}$$

$$\mathrm{d}\{V(\mathcal{C};\nu,\eta_C) - L\} = \mathrm{d}\Pi - Y\mathrm{d}\log P. \tag{49}$$

The upstream revenue inverse changes with markups according to

$$\begin{aligned}\mathrm{d}\left\{\left[I - \Omega'\,\mathrm{diag}(\mu^{-1})\right]^{-1}\right\} = &-\left[I - \Omega'\,\mathrm{diag}(\mu^{-1})\right]^{-1}\Omega'\,\mathrm{diag}(\mu^{-1}) \\ &\times \mathrm{diag}(\mathrm{d}\log\mu)\left[I - \Omega'\,\mathrm{diag}(\mu^{-1})\right]^{-1}.\end{aligned} \tag{50}$$

Together with

$$\mathrm{d}[\mathrm{diag}(s) + K] = (\mathrm{d}s)s' + s(\mathrm{d}s)',$$

equation (50) makes the derivative matrices in (21) computable from the local sufficient statistics.

D Quantitative construction and audit protocol

D.1 Reference objects

The empirical construction uses one identifier spine for final-use shares, the two demand networks, the production matrix, markups, and AISC exposure. The 2022 product-catalog roster contains 4,437 firms. Positive sales, a BEA buyer-industry mapping, broad final use, and membership in the largest positive-TNIC component produce the 3,769-firm common network used by the current model implementation.

Within each represented BEA output commodity, broad final use is allocated by firm sales. Retail firms retain their retail buyer technology while their output is assigned to the retail-margin commodity. BEA use shares determine the buyer's total intermediate intensity; missing supplier commodities are reallocated across represented commodities without changing that intensity. Within a supplier commodity, sales shares allocate purchases across firms. This construction preserves the distinction between a firm's buyer technology and its output commodity.

Both raw product networks are symmetric and have zero diagonal. Multiplying each raw pair score by $s_i^0 s_j^0$ gives the matrices used in (12). The numerical audit verifies symmetry, row sums of each Laplacian, and the identity $K^0\mathbf{1} = 0$. It computes the frontier in (13) as a generalized eigenvalue on $\mathbf{1}^\perp$, checks its residual, and verifies negative semidefiniteness of K^0. Each scenario is also checked for positive expenditure shares at the reference point and along any reported local perturbation.

D.2 LLM complementarity pipeline

The reproducible pipeline separates extraction, economic classification, and search. The script `03_code/stage6_own_product_catalog.py` extracts own-product rows from Item 1 and Item 7. Deterministic scripts create mechanism layers and the core and local-balanced firm-pair matrices.

Product embeddings rank candidates outside these matrices; a separate economic-pair judge file records evidence for any proposed promotion. The build retains source text, product examples, rule names, and audit status for every accepted pair.

The completed full-roster outputs contain 10,548 core pairs and 33,085 local-balanced pairs. The broad candidate universe and embedding queue are kept as audit objects. A pair from those objects enters the measured complementarity layer only after mechanism confirmation. Before a quantitative run, all variants are restricted to the common 3,769-firm identifier spine and the matrix checks above are repeated.

D.3 AISC mapping and current status

The AISC build applies local date filters to relationship records and uses a no-look-ahead public-firm roster. High-confidence actor matches rely on manual aliases or exact normalized company names; fuzzy matches remain in an audit queue. Relation text is classified before actor-type fallbacks determine direction. The strict exposure measure retains relationships in which an AI provider or input supplier serves a downstream public user.

Table 1 distinguishes completed measurement from the two planned equilibrium exercises.

Table 1: Current implementation status

Object	Completed work	Next output
Final-use complementarity	2022 LLM product catalog; deterministic mechanism layers; core and local-balanced firm-pair matrices; embedding audit queue	Human or strict-LLM validation rates by mechanism and rank tranche
Substitution and production	TNIC firm links; BEA final-use allocation; buyer-technology input shares; common 3,769-firm identifier spine	Scenario-level regularity dashboard and sensitivity to network scaling
Markup decomposition	Exact pricing equation and local derivative code verified on small systems	Model-consistent large-network baseline and decomposition distributions
AISC exposure mapping	Dated AISC actor crosswalk; strict directed input-user layer; binary and count loadings	Equilibrium elasticities over transparent values of λ_A

For every reported baseline, the numerical solver must satisfy the pricing conditions, goods-market clearing, the household budget, both spectral-radius conditions in Assumption 1, the own second-order conditions, and the demand-curvature checks. Analytic pricing and productivity derivatives are compared with central differences on the same branch before applying them to the large network.