The Geoengineering Put: Ecological Debt, Termination Risk, and Asset Prices

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PRELIMINARY AND INCOMPLETE: RESULTS SUBJECT TO CHANGE

Abstract

Geoengineering, such as Solar Radiation Management, fundamentally inverts the standard economics of climate change. These interventions are technologically proven, inexpensive and fast-acting. This means they replace the classic free-rider problem of global mitigation with a free-driver problem Weitzman (2015), where a single vulnerable nation can unilaterally cool the planet. We develop a macro-finance framework to formalize the asset pricing implications of this technological regime. By capping realized temperatures without reducing the atmospheric carbon stock, geoengineering acts as a put option on climate damages funded by the accumulation of "Ecological Debt." This debt introduces a catastrophic, endogenous tail risk known as a termination shock. We show that the anticipation of geoengineering initially subsidizes fossil fuel incumbents by suppressing the carbon tax, but ultimately transforms them into toxic assets as Ecological Debt deepens. Geoengineering and Carbon Dioxide Removal switch from substitutes to strict complements once termination risk is present.

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1 Introduction

The economic framework to understand the implications of global warming typically revolves around the gargantuan costs to humanity of climate change (around $10 trillion per year)[1] and the immense cost required to address the problem (around $1 trillion per year)[2]. Further complicating the issue is the free-rider problem whereby sovereign actors face an incentive to under-invest in the transition to a low-carbon economy, as the benefits of one country's costly mitigation is globally shared. The potential deployment of climate geoengineering fundamentally alters this paradigm.

Geoengineering is a proven technology that can keep the earth's temperature stable even while carbon continues to accumulate in the atmosphere. It is cheap enough (about $10 to $20 billion per year) and technologically straightforward enough that a single country can unilaterally deploy the technology and dictate the global thermostat regardless of international consensus. Given the heterogeneous impact of global warming, countries facing relatively high costs of climate change may want to implement the technology, even in the face of international opposition. If implemented, geoengineering merely masks the symptoms of accumulating carbon. This introduces a risk of a termination shock, whereby the cessation of geoengineering would induce a rapid spike in global temperatures dictated by the high underlying carbon stock resulting in potentially massive economic costs. While interest in geoengineering is nascent, as carbon continues to accumulate it is likely that its deployment will become an appealing proposition for certain states leading to large economic and financial implications across the globe.

In this paper, we develop a tractable macro-finance framework to formalize the asset pricing implications of climate geoengineering in a general equilibrium setting. We define geoengineering broadly as any technological intervention that lowers realized global temperatures without altering the underlying atmospheric carbon stock. This primarily encompasses Radiative Forcing Modification (RFM) and albedo modification technologies. The most prominent example is Stratospheric Aerosol Injection (SAI), which involves spraying reflective particles such as sulfur dioxide into the stratosphere to reflect a portion of incoming sunlight back into space. Other forms include Marine Cloud Brightening (MCB), which seeds low-lying ocean clouds with sea salt to increase their reflectivity, and Cirrus Cloud Thinning (CCT), which reduces high-altitude ice clouds that trap terrestrial heat. The distinction between these technologies and Carbon Dioxide Removal is central to our analysis:

[1] For example, Bilal and Känzig (2026) estimates an impact of about 20% of GDP, around $20 trillion in today's dollars.

[2] For example, the IPCC estimates costs of about 1% to 4% of GDP, roughly $1 to $4 trillion `https://www.ipcc.ch/report/ar6/wg3/chapter/chapter-3/`

geoengineering masks the symptoms of carbon accumulation without treating the underlying cause, and it is precisely this gap that generates the novel risks we formalize.

The economic threat of unilateral deployment is highly credible because the physical efficacy and cost-effectiveness of these interventions are well established (Barrett, 2008; Moreno-Cruz and Keith, 2013). The physics of SAI, for instance, are demonstrated by natural volcanic activity. The 1991 eruption of Mount Pinatubo injected approximately 20 million tons of sulfur dioxide into the stratosphere, which observably cooled the planet by roughly 0.5 degrees Celsius over the following two years. Replicating this cooling effect artificially is strikingly cheap. Engineering estimates suggest a functional SAI program would cost between $10 billion and $20 billion per year (Smith and Wagner, 2018) — a fraction of a percent of the annual capital expenditure required for global decarbonization. Given this asymmetric cost structure, a sovereign nation facing existential heat damages possesses both the means and the rational incentive to act unilaterally.

We formalize the macroeconomic consequences of this free-driver regime by introducing two concepts that organize the analysis. The first is the "Geoengineering Put." By establishing a hard cap on global temperatures, unilateral deployment truncates the right tail of traditional climate damages, functioning as a put option on the climate. However, because these technologies only mask the warming effects of greenhouse gases without removing them from the atmosphere, exercising this put requires the continuous accumulation of what we call "Ecological Debt": the growing gap between the temperature the atmosphere would deliver absent geoengineering and the temperature the economy actually experiences. By emitting carbon and masking the associated warming with reflective aerosols, the global economy pulls growth forward today at the cost of accumulating a latent, catastrophic tail risk.

This tail risk is recognized in the geophysical literature as a termination shock (Parker and Irvine, 2018; Trisos et al., 2018). It stems from a fundamental mismatch in atmospheric physics: while carbon dioxide remains in the atmosphere for centuries, stratospheric aerosols precipitate out in one to two years. If geoengineering is suddenly halted, the climate system rapidly snaps back to the temperature dictated by the underlying carbon stock, generating warming velocities an order of magnitude faster than baseline climate change. The resulting rapid spike in global temperatures would cause unprecedented destruction of global capital and productivity. In our framework, a termination shock represents a sudden default on the world's Ecological Debt.

We assume that the termination hazard is not a fixed technological failure rate but increases endogenously with the intensity of deployment. The economic logic is straightforward: geoengineering generates severe regional side effects — disrupted monsoon pat-

terns, induced droughts — which primarily harm neighboring agricultural nations (Ricke et al., 2014). More aggressive deployment provokes stronger geopolitical opposition from affected regions, raising the probability of infrastructure disruption through economic sanctions, diplomatic pressure, or targeted interference. The termination shock is thus best understood as an endogenous, structural risk factor whose probability grows with the scale of intervention.

Our analysis is organized around three nested regimes that progressively activate the economic mechanisms at work. In the *baseline* (Regime 1), geoengineering does not exist and the economy faces standard climate risks. In *Regime 2*, a vulnerable nation deploys geoengineering with certainty of continuation, isolating the moral hazard channel. In the *full model* (Regime 3), geopolitical opposition introduces an endogenous termination hazard. A central insight of our framework is that the same physical model of geoengineering produces qualitatively opposite asset pricing implications depending on whether termination risk is present. This motivates our nested approach: by first deriving results without termination risk and then activating it, we isolate the mechanisms precisely and show that several conventional conclusions about geoengineering are reversed. We solve the planner's Hamilton-Jacobi-Bellman equation numerically, yielding the forward-looking social cost of carbon, equilibrium dynamics, and asset prices across all three regimes.

The model yields four main results.

First, the Geoengineering Put acts as a moral hazard device. By suppressing realized temperatures, it collapses the optimal carbon tax, inflates fossil fuel valuations, and stalls the green transition. In the limit where geoengineering perfectly masks all warming, the social cost of carbon falls to zero and climate risk disappears entirely from asset markets. This is the starkest formalization of the moral hazard concern — and the reason that evaluating geoengineering without considering termination risk is dangerously incomplete.

Second, the introduction of termination risk fundamentally alters the cross-section of expected returns. Fossil fuel firms carry a positive termination risk premium: they crash precisely when the representative investor's marginal utility is highest. We show that fossil fuel valuations are non-monotonic in the carbon stock. Geoengineering initially helps these firms by suppressing the carbon tax, but ultimately transforms them into toxic assets as Ecological Debt accumulates — a mechanism we call the "toxic put." Green firms and CDR firms, by contrast, carry negative termination risk premia, acting as endogenous disaster insurance.

Third, and most strikingly, the relationship between geoengineering and Carbon Dioxide Removal reverses sign across regimes. Without termination risk, geoengineering crowds out CDR: the suppressed carbon tax eliminates the revenue that funds removal operations, and

there is no tail-hedge motive for removal investment. With termination risk, they become strict complements: CDR capacity is uniquely valuable because it is the only technology that can safely deflate Ecological Debt without triggering a catastrophic termination shock. The economics are intuitive once stated: it is precisely the availability of cheap geoengineering that makes expensive CDR indispensable, because geoengineering creates the very tail risk that CDR is uniquely positioned to resolve. This substitutes-to-complements reversal is the central theoretical contribution of the paper, and its policy implication is immediate: any credible deployment of geoengineering should be accompanied by massive concurrent investment in CDR capacity.

Fourth, solving the full dynamic problem reveals substantial precautionary amplification that static or myopic analyses miss entirely. The forward-looking social cost of carbon exceeds a myopic benchmark by 88% at 600 ppm and 137% at 800 ppm, because the planner anticipates that the convexity of the termination damage function will compound along the equilibrium carbon path. Above approximately 612 ppm, the social cost of carbon under geoengineering exceeds the no-geoengineering baseline — meaning that carbon becomes more socially costly with intervention than without it. This finding has no analogue in the existing integrated assessment literature.

To bridge our theoretical model to observable market behavior, we propose a novel empirical strategy. Building on the climate news hedging methodology of Engle et al. (2020), we outline the construction of two high-frequency geoengineering news indices — one capturing shifts in the viability of deployment, the other tracking the endogenous termination hazard — and test their impact on the cross-section of sovereign CDS spreads and corporate equity returns.

This paper contributes to several strands of the literature. First, we build on the climate macro-finance literature surveyed by Giglio et al. (2021), extending the asset pricing framework of Acharya et al. (2025) and the biodiversity-sovereign risk analysis of Giglio et al. (2025) to the geoengineering setting. Our stochastic discount factor with endogenous jump risk connects to the rare disasters literature (Barro, 2006; Wachter, 2013; Gabaix, 2012), where we interpret the termination shock as a disaster whose probability is endogenously determined by geopolitical friction. The risk-neutral amplification of the termination hazard generates quantitatively significant risk premia even when the physical probability of termination is modest, paralleling the mechanism in Gabaix (2012).

Second, we contribute to the economics of geoengineering. Barrett (2008) first noted the "incredible economics" of solar geoengineering, and Weitzman (2015) formalized the free-driver externality. Moreno-Cruz and Keith (2013) and Heutel et al. (2018) study optimal geoengineering policy and its effect on the carbon price, while Belaia et al. (2024) consider

the joint optimization of mitigation, CDR, and solar geoengineering. Our contribution is to formalize the asset pricing consequences, showing how termination risk creates novel risk premia, and to solve for the forward-looking equilibrium, yielding quantitative magnitudes that connect to observable market data.

Third, we contribute to the integrated assessment literature. Standard IAMs (Nordhaus, 2017; Golosov et al., 2014; Barrage, 2020) derive the social cost of carbon under gradual warming. We show that geoengineering fundamentally alters the SCC by introducing a state-dependent regime switch between suppressed and tipping-augmented damages, connecting to the tipping points literature of Lemoine and Traeger (2014) and Cai and Lontzek (2019). Our finding that the SCC can be *higher* under geoengineering than without it is, to our knowledge, novel.

Fourth, our analysis of fossil fuel valuations and stranded asset risk complements the work of van der Ploeg and Rezai (2020), who study stranded assets under policy tipping, and Bolton and Kacperczyk (2021), who document the carbon premium in equity returns. The toxic put mechanism differs from standard stranded asset risk: it arises not from a deterministic policy transition but from the endogenous accumulation of latent catastrophic exposure during a period of apparent climate safety.

Finally, our empirical strategy builds on the hedging portfolio approach of Engle et al. (2020) and the climate uncertainty pricing framework of Barnett et al. (2020).

The remainder of the paper proceeds as follows. Section 2 develops the model. Section 3 presents the theoretical results under three nested regimes. Section 4 presents quantitative results from the solved model. Section 5 details our empirical strategy. Section 6 concludes.

2 Model

This section develops the model in five parts. We begin with the physical climate system (Section 2.1), then introduce geoengineering and the concept of Ecological Debt (Section 2.2). Section 2.3 specifies firm dividends as reduced-form functions of the carbon tax. Section 2.4 formulates and solves the planner's problem, which we present incrementally across three nested regimes. Section 2.5 develops the asset pricing framework.

2.1 Carbon Stock and Climate

The fundamental slow-moving state variable is the stock of atmospheric carbon, S_t, measured in parts per million (ppm). Its evolution is governed by aggregate emissions and Carbon

Dioxide Removal (CDR):

$$dS_t = \left[e(\tau_t) - R_t^{CDR}\right] dt + \sigma_S \, dZ_t \tag{1}$$

where $e(\tau_t)$ denotes aggregate emissions as a function of the prevailing carbon tax τ_t, R_t^{CDR} is the flow of physical carbon removal by CDR firms, $\sigma_S > 0$ parameterizes natural carbon cycle volatility, and Z_t is a standard Brownian motion. Emissions are strictly decreasing in the carbon tax: higher taxes induce substitution away from fossil fuels and toward cleaner alternatives.

Absent any climate intervention, the unmitigated global temperature anomaly is a strictly increasing, convex function of the carbon stock:

$$T_t^{un} = \Psi(S_t), \qquad \Psi' > 0, \quad \Psi'' \geq 0 \tag{2}$$

The convexity of Ψ reflects the logarithmic relationship between CO_2 concentration and radiative forcing, combined with positive climate feedbacks that amplify warming at higher concentrations.

Climate damages to aggregate productivity are captured by a tipping-augmented damage function:

$$\Omega(T) = \omega(T) + \zeta \cdot h(T - \bar{T}) \tag{3}$$

The first term, $\omega(T)$, represents standard climate damages (heat stress, agricultural yield losses, storm intensification) and is smooth, increasing, and convex. The second term introduces a critical temperature threshold $\bar{T}$ above which the climate system risks crossing irreversible tipping points — ice sheet disintegration, large-scale permafrost methane release, disruption of the Atlantic Meridional Overturning Circulation, and dieback of tropical forests. The function $h(\cdot)$ is smooth, increasing, and convex, with $h(x) \approx 0$ for $x \ll 0$ and $h'(x)$ large for $x > 0$. The parameter $\zeta > 0$ scales the severity of tipping-point damages. This specification is consistent with the formal treatment of stochastic tipping in Lemoine and Traeger (2014) and Cai and Lontzek (2019).

Assumption 1 (Damage Regularity)**.** Ω *is twice continuously differentiable with* $\Omega' > 0$ *and* $\Omega'' > 0$.

2.2 Geoengineering and Ecological Debt

We assume the existence of a climate-vulnerable sovereign actor (Region V) that deploys geoengineering at intensity $g_t \geq 0$. Geoengineering artificially lowers the realized global

temperature without altering the atmospheric carbon stock:

$$T_t = \Psi(S_t) - \theta\, g_t \tag{4}$$

where $\theta > 0$ captures the cooling efficiency of the intervention.

Assumption 2 (Geoengineering Policy)**.** *The equilibrium geoengineering intensity $g(S)$ is a given, exogenous policy function that is weakly increasing in the carbon stock: $g'(S) \geq 0$. A climate-vulnerable nation deploys solar radiation management to minimize local temperature damages. The optimal deployment intensity is increasing in the carbon stock because higher S implies higher unmitigated temperatures and greater marginal benefit of cooling. We take this policy function as given.*

The discrepancy between the unmitigated and realized temperatures constitutes the central risk object in our economy. We define the *Ecological Debt* as:

$$F(S) \equiv \theta\, g(S) \tag{5}$$

Ecological Debt measures the gap between what the atmosphere "wants" to deliver and what the economy actually experiences. When $g = 0$, $F = 0$: there is no masked warming. When $g > 0$, the global economy pulls growth forward by suppressing the temperature consequences of carbon accumulation. The Ecological Debt $F(S)$ is a hidden state variable that grows silently during deployment and is revealed catastrophically upon termination.

The risk of a termination shock stems from a fundamental mismatch in atmospheric physics: while carbon dioxide remains in the atmosphere for centuries, stratospheric aerosols precipitate out in one to two years. If geoengineering is suddenly halted, the climate system rapidly snaps back to the temperature dictated by the underlying carbon stock, generating a warming velocity an order of magnitude faster than baseline climate change.

Assumption 3 (Termination Hazard)**.** *The termination hazard rate is:*

$$\lambda(S) = \lambda_0 + \lambda_1\, g(S) \tag{6}$$

where $\lambda_0 \geq 0$ is a baseline hazard from exogenous infrastructure failure and $\lambda_1 > 0$ captures the endogenous geopolitical channel. The termination hazard increases with geoengineering intensity because more aggressive deployment provokes stronger geopolitical opposition from precipitation-affected nations (Region A), raising the probability of infrastructure disruption.

The permanent productivity destruction upon termination is:

$$\Gamma(F) = 1 - \exp\big(-\Gamma_0 F^{\beta}\big) \tag{7}$$

where $\Gamma_0 > 0$ scales the severity of termination damage and $\beta > 1$ controls the convexity. This functional form satisfies $\Gamma(0) = 0$ (no damage without Ecological Debt), $\Gamma' > 0$, $\Gamma'' > 0$ (damage is increasing and convex in the debt), and $\Gamma(F) \to 1$ as $F \to \infty$ (sufficiently large Ecological Debt is catastrophic). The convexity is economically important: it implies that the marginal danger of accumulating additional Ecological Debt accelerates as the stock grows, creating an endogenous tipping point even in the absence of a physical temperature threshold.

Aggregate productivity follows a jump-diffusion process:

$$\frac{dA_t}{A_t} = [\mu_A - \Omega(T_t)]\, dt + \sigma_A\, dW_t - \Gamma(F_t)\, dN_t \tag{8}$$

where μ_A is the expected baseline growth rate, $\sigma_A\, dW_t$ represents standard macroeconomic volatility, W_t is a Brownian motion independent of Z_t, and dN_t is a Poisson jump process with hazard rate $\lambda(S_t)$. If $dN_t = 1$, aggregate productivity is permanently destroyed by a fraction $\Gamma(F_t)$. Post-termination, geoengineering ceases ($g = 0$, $\lambda = 0$), and the economy reverts to a standard climate regime at the inherited carbon stock with diminished productivity.

2.3 Firms and Dividends

We model three types of firms: Brown (fossil fuel), Green (renewable energy), and CDR (Carbon Dioxide Removal). Rather than specifying the full industrial organization of energy markets, we work with reduced-form dividend functions that depend on the prevailing carbon tax.

Assumption 4 (Brown Dividends)**.** *Brown firm dividends $D^B(\tau)$ are strictly decreasing in the carbon tax τ, with $D^B(0) > 0$ and $D^B(\bar{\tau}_{shut}) = 0$ for some finite shutdown price $\bar{\tau}_{shut}$. Intuitively, a higher carbon tax erodes fossil fuel profitability by raising production costs and shifting energy demand toward cleaner substitutes.*

Assumption 5 (Green Dividends)**.** *Green firm dividends $D^G(\tau)$ are strictly increasing in the carbon tax τ, with $D^G(0) \geq 0$. A higher carbon tax stimulates demand for renewable energy and raises Green firm profitability.*

These reduced forms are consistent with competitive energy markets in which Brown and Green energy are substitutable inputs to aggregate production. Appendix A provides a

constant elasticity of substitution (CES) microfoundation that delivers Assumptions 4–5 as equilibrium outcomes.[3]

CDR firms operate physical carbon removal infrastructure (direct air capture plants, enhanced weathering installations) and earn revenue at the prevailing carbon price. The shadow value of one unit of installed CDR capacity is denoted $q(S)$ and reflects the present value of current and future carbon tax revenue from operating that capacity, net of depreciation.[4] Because CDR revenue depends directly on the carbon tax, $q(S)$ inherits the properties of the SCC: it is increasing in S under the baseline regime and responds to geoengineering through the same channels as the carbon tax.

Post-termination, all firms operate in an economy with permanently diminished productivity. Dividends for any firm i in the post-termination state are scaled by $(1-\Gamma(F))$ relative to what they would earn in a baseline economy at the same carbon stock, because aggregate output has been permanently reduced.

2.4 The Planner's Problem

A representative global investor has CRRA preferences with risk aversion $\gamma > 1$ and discount rate $\rho > 0$. Aggregate consumption equals output:

$$C_t = A_t \, \mathcal{E}(\tau_t) \tag{9}$$

where $\mathcal{E}(\tau)$ is the aggregate energy service flow, which depends on the carbon tax through the equilibrium composition of the energy mix.[5] A global social planner sets the Pigouvian carbon tax τ_t equal to the Social Cost of Carbon (SCC).

The planner takes Region V's geoengineering policy $g(S)$ as given when setting τ_t. This institutional asymmetry — coordinated carbon taxation combined with unilateral geoengineering — is a conservative modeling choice: decentralized national carbon taxes would be strictly lower than the global SCC, accelerating carbon accumulation and strengthening all of our results.

[3]The key properties required for our results are monotonicity (D^B decreasing, D^G increasing in τ) and the existence of a finite Brown shutdown price. Any reasonable model of the energy transition — including the CES specification, linear demand models, or capacity-constrained frameworks — delivers these properties.

[4]Formally, q is the Tobin's q of CDR capital. We abstract from the dynamic capacity accumulation problem in the main text and treat $q(S)$ as determined by the equilibrium carbon tax path. Appendix B provides the full investment problem with convex adjustment costs.

[5]The function $\mathcal{E}(\tau)$ is weakly decreasing in τ: a higher carbon tax contracts Brown energy output, and the expansion of Green output is insufficient to fully compensate. This property follows from the CES microfoundation in Appendix A under the empirically relevant case where Brown energy constitutes a non-trivial share of pre-policy energy production.

Separability

Under CRRA utility and multiplicative productivity, the value function separates as:

$$V(S, A) = \frac{A^{1-\gamma}}{1-\gamma}\, v(S) \tag{10}$$

where $v(S)$ is the carbon-stock-dependent component that solves the reduced HJB equation. This separation is the key simplification: the planner's problem reduces to a one-dimensional ODE in S.

The HJB Equation: Incremental Construction

We present the HJB equation incrementally across three nested regimes, each activating a distinct economic mechanism.

Regime 1: No Geoengineering. Set $g \equiv 0$, so $F = 0$, $\lambda = 0$, and $T = \Psi(S)$. This is the standard climate economy. The reduced HJB for $v(S)$ is:

$$\rho\, v = \max_{\tau} \left\{ \mathcal{E}(\tau)^{1-\gamma} \left[\mu_A - \Omega(\Psi(S))\right] v + \tfrac{1}{2}\sigma_S^2\, v'' + \left[e(\tau) - R^{CDR}\right] v' \right\} \tag{11}$$

where primes denote derivatives with respect to S. The planner chooses τ to balance the flow utility cost of taxing energy (through $\mathcal{E}(\tau)$) against the long-run benefit of slowing carbon accumulation (through $e(\tau)$). The first-order condition yields the Social Cost of Carbon:

$$\tau = SCC^{R1}(S) = -\frac{v'(S)}{v(S)} \cdot \frac{\mathcal{E}(\tau)^{1-\gamma}}{\partial[\mathcal{E}(\tau)^{1-\gamma}]/\partial\tau} \tag{12}$$

In this regime, all climate risk is priced through the flow damage channel: a higher carbon stock raises temperatures, which raises damages, which reduces the value function.

Regime 2: Geoengineering Without Termination Risk. Activate $g(S) > 0$ but keep $\lambda \equiv 0$. The realized temperature is now $T = \Psi(S) - \theta g(S) < \Psi(S)$, and the Ecological Debt $F(S) = \theta g(S) > 0$ accumulates but carries no consequence because termination cannot occur. The HJB becomes:

$$\rho\, v = \max_{\tau} \left\{ \mathcal{E}(\tau)^{1-\gamma} \left[\mu_A - \Omega(\Psi(S) - \theta g(S))\right] v + \tfrac{1}{2}\sigma_S^2\, v'' + \left[e(\tau) - R^{CDR}\right] v' \right\} \tag{13}$$

The only change from Regime 1 is that flow damages Ω are evaluated at the artificially low realized temperature $\Psi(S) - \theta g(S)$ rather than the unmitigated temperature $\Psi(S)$. This

suppression of realized damages is the moral hazard channel: geoengineering weakens the planner's incentive to tax.

Regime 3: The Full Model. Activate $\lambda(S) > 0$ and $\Gamma(F)\, dN_t$. Every mechanism is now live. The HJB acquires a jump term reflecting the expected value loss from termination:

$$\begin{aligned}\rho\, v = \max_{\tau} \Big\{ & \mathcal{E}(\tau)^{1-\gamma} \left[\mu_A - \Omega(\Psi(S) - \theta g(S))\right] v + \tfrac{1}{2}\sigma_S^2\, v'' + \left[e(\tau) - R^{CDR}\right] v' \\ & + \lambda(S) \left[(1 - \Gamma(F(S)))^{1-\gamma}\, v^{R1}(S) - v\right] \Big\}\end{aligned} \tag{14}$$

The new term $\lambda(S)[(1-\Gamma(F))^{1-\gamma} v^{R1}(S) - v]$ is the expected value change from a termination event. Upon termination, the economy jumps to Regime 1 at the same carbon stock S but with permanently diminished productivity: the continuation value is $(1-\Gamma(F))^{1-\gamma} v^{R1}(S)$, reflecting both the output destruction (through Γ) and the regime switch to the baseline climate economy (through v^{R1}). Because $\Gamma > 0$, this jump is value-destroying: $(1-\Gamma)^{1-\gamma} v^{R1} < v$ in the relevant range, so the planner faces a negative expected drift from the termination channel.

The SCC Under the Full Model

The first-order condition of the Regime 3 HJB yields the full Social Cost of Carbon. The SCC decomposes into two channels:

$$SCC^{R3}(S) = \underbrace{SCC^{flow}(S)}_{\text{Flow Damage Channel}} + \underbrace{SCC^{term}(S)}_{\text{Termination Fragility Channel}} \tag{15}$$

The flow damage channel reflects the standard marginal cost of carbon through realized temperature increases, suppressed by geoengineering. The termination fragility channel reflects the marginal increase in termination severity from accumulating additional Ecological Debt: each unit of carbon raises $g(S)$ (by Assumption 2), which deepens $F(S)$ and steepens the damage $\Gamma(F)$ upon termination. The full expressions are derived from the value function:

$$SCC^{R3}(S) = -\frac{v'(S)}{v(S)} \cdot \frac{\mathcal{E}(\tau)^{1-\gamma}}{\partial[\mathcal{E}(\tau)^{1-\gamma}]/\partial\tau} \tag{16}$$

where now $v(S)$ solves the Regime 3 HJB (Equation 14), incorporating both flow damages and termination risk into the slope of the value function.

We solve the planner's HJB numerically via value function iteration on a grid over S (details in Section 4). This yields the true forward-looking SCC, which accounts for the entire

future trajectory of carbon accumulation, geoengineering deployment, and termination risk.

2.5 Asset Pricing

The Stochastic Discount Factor

Under CRRA utility, the stochastic discount factor (pricing kernel) evolves as:

$$\frac{d\Lambda_t}{\Lambda_t} = -r_t\,dt - \gamma\sigma_A\,dW_t - \gamma\eta_t\,dZ_t + (J_t^\Lambda - 1)dN_t - \lambda_t(J_t^\Lambda - 1)\,dt \tag{17}$$

where η_t is the market price of carbon cycle risk, reflecting the sensitivity of aggregate consumption to carbon stock shocks through the endogenous tax. The pricing kernel jump at termination is:

$$J_t^\Lambda = \left[(1 - \Gamma(F_t)) \cdot J_t^{\mathcal{E}}\right]^{-\gamma} \tag{18}$$

where $J_t^{\mathcal{E}} \equiv \mathcal{E}(\bar{\tau}_t)/\mathcal{E}(\tau_t)$ is the proportional change in the energy aggregate caused by the post-termination tax jump. Here $\bar{\tau}_t \equiv SCC^{R1}(S_t)$ is the carbon tax that prevails immediately after termination, when the economy reverts to Regime 1 at the inherited carbon stock. Both factors contribute to $J^\Lambda > 1$: $(1-\Gamma)^{-\gamma} > 1$ from direct productivity destruction, and $(J^{\mathcal{E}})^{-\gamma} > 1$ from the energy reallocation forced by the crisis-level tax.

Risk-Neutral Pricing

The change of measure to the risk-neutral probability $\mathbb{Q}$ transforms the physical termination hazard into the risk-neutral hazard rate:

$$\lambda_t^{\mathbb{Q}} = \lambda_t \cdot J_t^\Lambda = \lambda_t \cdot \left[(1 - \Gamma(F_t)) \cdot J_t^{\mathcal{E}}\right]^{-\gamma} \tag{19}$$

This is the standard Barro (2006) rare disaster mechanism applied to the termination shock. Because termination is a bad-state event (marginal utility is high when it occurs), the risk-neutral probability of termination exceeds the physical probability by the factor $J_t^\Lambda > 1$.

Valuation Equations

Pre-termination asset valuations are discounted at the effective rate $r + \lambda^{\mathbb{Q}}$ rather than $r + \lambda$, with the probability-weighted post-termination continuation value entering at the risk-neutral hazard rate. The pre-termination Brown firm valuation satisfies:

$$V^B(S) = \frac{D^B(\tau(S)) + \lambda^{\mathbb{Q}}(S) \cdot V^B_{post}(S)}{r + \lambda^{\mathbb{Q}}(S)} \tag{20}$$

where $V^B_{post}(S) = (1 - \Gamma(F(S))) \cdot D^B(\bar{\tau}(S))/r$ is the post-termination perpetuity value, reflecting both the productivity destruction and the tax jump. Analogous expressions hold for Green and CDR firms:

$$V^G(S) = \frac{D^G(\tau(S)) + \lambda^{\mathbb{Q}}(S) \cdot V^G_{post}(S)}{r + \lambda^{\mathbb{Q}}(S)} \tag{21}$$

$$q(S) = \frac{\tau(S) \cdot \phi + \lambda^{\mathbb{Q}}(S) \cdot q_{post}(S)}{r + \delta + \lambda^{\mathbb{Q}}(S)} \tag{22}$$

where ϕ is the removal flow per unit of installed CDR capacity and δ is the depreciation rate of CDR infrastructure.

Under Regimes 1 and 2, the termination terms vanish ($\lambda = 0$), and asset values reduce to standard perpetuities of their respective dividend flows: $V^B = D^B(\tau)/r$, $V^G = D^G(\tau)/r$, and $q = \tau\phi/(r + \delta)$.

Termination Risk Premia

The expected excess return of any firm i satisfies:

$$\mu^i - r = \underbrace{\gamma\sigma_A\,\sigma_{i,W} + \gamma\eta\,\sigma_{i,Z}}_{\text{Diffusion risk premia}} + \underbrace{TRP^i}_{\text{Termination risk premium}} \tag{23}$$

where the termination risk premium is:

$$TRP^i = -\lambda\,\mathbb{E}\big[(J^\Lambda - 1)(J^i - 1)\big] \tag{24}$$

and $J^i \equiv V^i_{post}/V^i_{pre}$ is the proportional jump in firm i's valuation at termination. The TRP is the compensation investors demand for bearing termination risk exposure. Its sign depends on J^i: firms that crash at termination ($J^i < 1$) covary positively with the pricing kernel jump ($J^\Lambda > 1$), generating a positive risk premium. Firms that appreciate at termination ($J^i > 1$) carry a negative TRP, acting as endogenous disaster insurance.

Three Nested Regimes

We analyze the economy under three nested parameter restrictions, each isolating a distinct economic mechanism:

	Regime 1 (Baseline)	Regime 2 (Geo, no term.)	Regime 3 (Full)
$g(S)$	$\equiv 0$	> 0 (Assumption 2)	> 0 (Assumption 2)
$\lambda(S)$	$\equiv 0$	$\equiv 0$	> 0 (Assumption 3)
$\Gamma(F)\, dN_t$	absent	absent	active

This completes the model. We now derive the theoretical results under each regime.

3 Theoretical Results Under Three Nested Regimes

3.1 Regime 1: Baseline Climate Risk

Set $g \equiv 0$, so $F = 0$, $\lambda = 0$, and $T = \Psi(S)$. The economy faces standard climate risks without geoengineering. The HJB reduces to Equation (11), and the SCC reflects only the flow damage channel.

Lemma 1 (Baseline Economy)**.**

(a) The SCC is strictly positive and strictly increasing in S: $\partial SCC^{R1}/\partial S > 0$. The equilibrium carbon tax rises as carbon accumulates, inducing a gradual transition from Brown to Green energy (Assumptions 4–5).

(b) Brown firm valuations $V^B = D^B(\tau)/r$ are strictly decreasing in S.

(c) Green firm valuations $V^G = D^G(\tau)/r$ are strictly increasing in S.

(d) The CDR shadow value $q^{R1}(S) = \tau(S)\phi/(r+\delta)$ is increasing in S.

Proof sketch. Part (a) follows from convexity of Ω and the monotonicity of Ψ: a higher carbon stock raises future temperatures, increasing the marginal damage $\Omega'(\Psi(S))$ at every future date, which steepens $v'(S)$ and raises the SCC. Parts (b)–(d) follow from the monotonicity of the carbon tax in S and Assumptions 4–5. □

3.2 Regime 2: The Geoengineering Put

Activate $g(S) > 0$ (Assumption 2) but keep $\lambda \equiv 0$. The geoengineering program operates with certainty of continuation, isolating the moral hazard channel from termination risk. The HJB is Equation (13): the only change from Regime 1 is that flow damages are evaluated at the artificially low realized temperature $\Psi(S) - \theta g(S)$.

Proposition 1 (The Geoengineering Put: Moral Hazard)**.**

(a) The SCC under geoengineering is strictly lower than the baseline: $SCC^{R2}(S) < SCC^{R1}(S)$ for all S where $g(S) > 0$. Two forces contribute: geoengineering lowers the realized temperature, reducing the marginal damage $\Omega'(T)$; and because $g'(S) > 0$, Region V's endogenous deployment response absorbs part of the marginal temperature impact of carbon.

(b) In the limit where geoengineering perfectly caps the temperature at $\bar{T}$ (that is, $g(S) \to (\Psi(S) - \bar{T})/\theta$ for $\Psi(S) > \bar{T}$), $SCC^{R2} \to 0$. The optimal carbon tax vanishes because geoengineering fully neutralizes the marginal damage of carbon.

(c) The lower carbon tax raises Brown firm valuations relative to the baseline: $V^{B,R2}(S) > V^{B,R1}(S)$. (By Assumption 4, lower τ raises D^B.)

(d) The lower carbon tax reduces Green firm valuations: $V^{G,R2}(S) < V^{G,R1}(S)$.

(e) In the perfect-capping limit, the carbon cycle risk premium vanishes for all firms. Carbon stock fluctuations become economically irrelevant because geoengineering fully insulates the economy from temperature changes.

Proof sketch. Compare the Regime 1 and Regime 2 HJBs (Equations 11–13). Because $\Omega(\Psi(S)-\theta g) < \Omega(\Psi(S))$, the effective growth rate is higher under Regime 2. The value function is less steeply declining in S (the planner is less concerned about carbon accumulation), yielding a lower SCC. Parts (c)–(d) follow from Assumptions 4–5. Part (b) follows because if $T = \bar{T}$ for all S, then $\Omega'(T) \cdot dT/dS = 0$, and $v'(S) \to 0$. Formal proof in Appendix C. □

Part (e) is a strong theoretical statement: if geoengineering works perfectly and permanently, climate risk disappears from asset markets entirely. This is the starkest formalization of the moral hazard concern — and the reason that evaluating geoengineering without considering termination risk is dangerously incomplete.

Corollary 1 (SRM–CDR Substitutability)**.** *Without termination risk, geoengineering crowds out CDR:*

(a) $q^{R2}(S) < q^{R1}(S)$ for all S: the suppressed carbon tax reduces CDR operating revenue.

(b) An increase in geoengineering effectiveness θ strictly reduces the CDR shadow value. Geoengineering and CDR are substitutes when termination risk is absent.

This is the "conventional wisdom" result: cheap geoengineering crowds out expensive CDR. The availability of a fast-acting temperature fix eliminates the urgency to invest in slow, capital-intensive carbon removal. We state it clearly to set up the reversal in Regime 3.

Corollary 2 (Sovereign Risk Redistribution)**.** *Geoengineering does not reduce aggregate sovereign risk; it redistributes it:*

(a) Region V's sovereign CDS spreads narrow: suppressed flow temperature damages lower default intensity.

(b) Region A's sovereign CDS spreads widen: precipitation side effects from geoengineering raise default intensity, and the widening is increasing in $g(S)$ and therefore in S.

3.3 Regime 3: Termination Risk and the Toxic Put

Activate $\lambda(S) > 0$ (Assumption 3) and $\Gamma(F)\, dN_t$. The HJB is now Equation (14), with the jump term capturing the expected value loss from termination. Every result in this subsection identifies a mechanism that is absent when $\lambda = 0$ and present when $\lambda > 0$.

Proposition 2 (The Social Cost of Carbon Under Termination Risk)**.**

(a) For λ sufficiently small, $SCC^{R2}(S) < SCC^{R3}(S) < SCC^{R1}(S)$. Termination risk partially restores the carbon tax relative to Regime 2, but the tax remains below the no-geoengineering level.

(b) In the perfect-capping limit, the flow damage channel vanishes and the SCC is sustained entirely by the termination fragility channel. Carbon is priced not because of the flow damages it causes (which are masked) but because of the latent catastrophe it deepens: each additional unit of carbon raises $g(S)$, which deepens $F(S)$ and steepens $\Gamma'(F)$.

(c) The SCC is increasing in S (more Ecological Debt raises Γ'), increasing in λ (higher hazard), and increasing in the convexity of Γ.

(d) If Γ is sufficiently convex, there exists $\hat{S}$ such that for $S > \hat{S}$, $SCC^{R3}(S) > SCC^{R1}(S)$. Above this crossing, the marginal unit of carbon is more socially costly under geoengineering than without it.

Proof sketch. Part (a): the Regime 3 HJB has an additional negative term $\lambda[(1-\Gamma)^{1-\gamma} v^{R1} - v]$ that steepens $v'(S)$ relative to Regime 2, raising the SCC. The SCC remains below Regime 1 because flow damages are still suppressed. Part (d): since Γ is convex, $\Gamma'(F)$ grows without bound as F increases, while the flow damage $\Omega'(\Psi(S))$ grows at the rate of the smooth damage function. The termination channel eventually dominates. Formal proof in Appendix C. □

Part (d) strengthens the "toxic" narrative: the geoengineering regime can make carbon *more dangerous* than a world without intervention. The crossing at $\hat{S}$ marks the point

where the latent catastrophe embedded in Ecological Debt outweighs the flow damages that geoengineering suppresses.

Corollary 3 (Post-Termination Regime Switch)**.** *If $dN_t = 1$ (termination occurs):*

(a) Geoengineering ceases ($g = 0$, $\lambda = 0$) and the economy reverts to Regime 1 at the inherited carbon stock S with permanently diminished productivity $A_{t+} = A_{t-}(1-\Gamma(F))$.

(b) The post-termination carbon tax $\bar{\tau}(S) \equiv SCC^{R1}(S)$ satisfies $\bar{\tau}(S) \gg \tau^{R3}(S)$ when $\Psi(S)$ is sufficiently above $\bar{T}$, because the economy now faces the full force of tipping-augmented damages at the unmitigated temperature.

(c) The tax jump forces Brown dividends toward zero and raises Green dividends (Assumptions 4–5). The energy aggregate contracts: $J^{\mathcal{E}} \equiv \mathcal{E}(\bar{\tau})/\mathcal{E}(\tau^{R3}) < 1$.

(d) The ratio $\bar{\tau}(S)/\tau^{R3}(S)$ is increasing in S: the longer geoengineering has operated, the more extreme the post-termination adjustment.

Proposition 3 (The Toxic Put: Termination Risk Premia and Brown Firm Non-Monotonicity)**.**
Part I: The Cross-Section of Termination Risk Premia. *For S such that $\tau^{R3}(S) < \bar{\tau}_{shut}$ (Brown firms earn positive pre-termination dividends):*

*(a) **Brown firms:** $J^B < 1$ (Brown valuations crash at termination because the post-termination tax crushes dividends). $TRP^B > 0$: Brown firms carry a strictly positive termination risk premium, increasing in F and therefore in S.*

*(b) **Green firms:** $J^G > 1$ (the tax surge shifts demand to renewables). $TRP^G < 0$: Green firms act as endogenous disaster insurance.*

*(c) **CDR firms:** $J^{CDR} = q_{post}/q_{pre} > 1$ (pre-installed removal capacity becomes scarce and valuable post-termination). $TRP^{CDR} < 0$. When CDR capacity is sufficiently scarce relative to the carbon overshoot at termination, $|TRP^{CDR}| > |TRP^G|$: CDR is the most potent hedge.*

Part II: The Toxic Put. *Brown firm valuations under Regime 3 are non-monotonic relative to the baseline:*

(d) At low S (near the onset of geoengineering), geoengineering helps Brown firms: the dividend boost from the suppressed carbon tax (Proposition 1c) dominates the small termination risk premium.

(e) There exists a threshold S^* such that for $S > S^*$, geoengineering destroys Brown firm value: $V^{B,R3}(S) < V^{B,R1}(S)$. The termination risk premium eventually dominates the dividend benefit, transforming geoengineering from a subsidy into a death sentence. Brown firms in this mature regime are toxic assets — high-yield securities whose excess returns compensate for catastrophic termination losses.

(f) The threshold S^* is decreasing in λ and in the convexity of Γ: higher termination risk and more severe damages accelerate the transition from "safe incumbent" to "toxic asset."

Proof sketch. Parts (a)–(c) follow from the valuation equations (Equations 20–22): the sign of each firm's valuation jump at termination determines the sign of its TRP through Equation (24). For (e): as S grows, TRP^B grows without bound because $\Gamma'(F)$ grows (convex Γ) and J^Λ grows (larger catastrophe amplified by risk aversion γ). The dividend benefit, by contrast, is bounded: $D^B(\tau)$ cannot exceed $D^B(0)$ regardless of how low the tax falls. Since TRP^B is unbounded from above and the dividend benefit is bounded, the crossing occurs at some finite S^*. Formal proof in Appendix C. □

The toxic put is the paper's most striking asset pricing result. It reveals a deep irony: geoengineering, which fossil fuel incumbents might initially celebrate as a reprieve from carbon taxation, ultimately transforms them into the most dangerous assets in the economy. The very technology that props up their near-term cash flows simultaneously creates the catastrophic tail risk that will destroy them. At $S > S^*$, investors holding Brown equity are effectively long a toxic put — they collect elevated dividends in exchange for bearing the risk of near-total value destruction upon termination.

Remark (TRP Inversion at Extreme S). For S sufficiently large, Proposition 2(d) implies that the precautionary tax τ^{R3} eventually exceeds $\bar{\tau}_{shut}$, at which point Brown pre-termination dividends fall to zero. In this extreme regime, $J^B > 1$: termination *rescues* Brown firms by reverting to the lower Regime 1 tax, and $TRP^B < 0$. Brown firm value becomes purely an option on governance failure. This inversion illustrates the logical endpoint of the ecological debt mechanism: at sufficiently high carbon concentrations, it is the rational policy response to the *threat* of catastrophe — not the catastrophe itself — that destroys fossil fuel firms.

Proposition 4 (Geoengineering–CDR Complementarity: The Substitutes-to-Complements Reversal)**.**

(a) At any S, the shadow value of CDR capacity is strictly higher in Regime 3 than in Regime 2: $q^{R3}(S) > q^{R2}(S)$. The increment reflects the probability-weighted option value of having installed removal capacity in the catastrophic termination state.

(b) The increment $q^{R3}(S) - q^{R2}(S)$ is increasing in S: larger Ecological Debt makes the post-termination state more severe (higher $\bar{\tau}$), raising the option value of pre-installed CDR capacity.

(c) Combined with Corollary 1(a) ($q^{R2} < q^{R1}$), the introduction of termination risk partially reverses the geoengineering–CDR interaction: from substitutes (Regime 2 vs. Regime 1) toward complements (Regime 3 vs. Regime 1) as S increases.

Proof sketch. From Equation (22), the Regime 3 CDR shadow value contains the term $\lambda^{\mathbb{Q}} q_{post}/(r + \delta + \lambda^{\mathbb{Q}})$, which is absent in Regime 2 (where $\lambda = 0$). Post-termination, CDR earns quasi-rents of $(1 - \Gamma(F))\bar{\tau}(S)\phi$ per unit of capacity, because the carbon price jumps to the crisis-level $\bar{\tau}$ and CDR is the only technology that can physically draw down the Ecological Debt. These quasi-rents are transitory (as CDR reduces S, $\bar{\tau}$ declines), but at high S they are very large. The risk-neutral measure amplifies the option value because $\lambda^{\mathbb{Q}} = \lambda \cdot J^{\Lambda} > \lambda$. Formal analysis in Appendix D. □

This substitutes-to-complements reversal is the central theoretical contribution of the paper. The economics are intuitive once stated: without termination risk, geoengineering eliminates the motive for carbon removal (why pay to remove carbon when you can cheaply mask its effects?). With termination risk, geoengineering *creates* the motive for carbon removal, because the Ecological Debt that geoengineering accumulates can only be safely deflated by physically removing carbon from the atmosphere. CDR is uniquely valuable because it is the only technology that can permanently resolve Ecological Debt without triggering a termination shock. Green energy, by contrast, can slow the accumulation of new debt but cannot retire the existing stock.

The policy implication is immediate: any credible deployment of geoengineering should be accompanied by massive concurrent investment in CDR capacity. The standard policy framework — which treats geoengineering and CDR as competing alternatives in the climate portfolio — is not merely incomplete but actively dangerous. It is precisely the availability of cheap geoengineering that makes expensive CDR capacity indispensable.

An additional mechanism strengthens this result. During the geoengineering era, CDR investment is suppressed by the substitutability channel (Corollary 1a). At termination, the economy suddenly requires massive carbon removal to draw down the Ecological Debt, but installed CDR capacity is far below the post-termination optimum. The shadow value of installed capacity during this transition exceeds the steady-state value by a scarcity premium. Crucially, this scarcity is itself a consequence of substitutability: the crowding-out during the geoengineering era creates the very capacity shortage that makes CDR invaluable at

termination.[6]

Proposition 5 (Sovereign CDS Term Structures Under Termination Risk)**.** *We formalize sovereign credit risk through a reduced-form default intensity* $\mu_{i,t} = \mu_i(A_{i,t}, L_{i,t})$*, with* $\partial\mu_i/\partial A_i < 0$ *and* $\partial\mu_i/\partial L_i > 0$*, where* $L_{V,t} = \Omega_V(T_t)$ *for temperature-vulnerable nations and* $L_{A,t} = \Omega_A(g_t)$ *for precipitation-vulnerable nations.*[7]

(a) Region V's CDS term structure steepens: short-maturity spreads narrow (geoengineering suppresses flow damages) while long-maturity spreads widen (the probability-weighted cost of a termination shock accumulates over the contract horizon). The expected default intensity over horizon $\mathcal{T}$ *is:*

$$\bar{\mu}_V(\mathcal{T}) \approx \mu_V^{pre} + \left(1 - \frac{1 - e^{-\lambda\mathcal{T}}}{\lambda\mathcal{T}}\right)(\mu_V^{post} - \mu_V^{pre}) \tag{25}$$

where the weight on μ_V^{post} *is strictly increasing in* $\mathcal{T}$*.*

(b) Region A's CDS spreads embed a termination risk premium on top of the flow precipitation widening from Corollary 2(b): sovereign defaults correlated with the termination state are priced at risk-neutral intensities exceeding physical intensities.

(c) At the moment of termination, the cross-sectional spike in CDS spreads for country i *is proportional to* $\Gamma_i(F)/A_{i,t}$*, the ratio of country-specific termination damages to fiscal capacity.*

Proof sketch. Part (a) follows from the mixture structure of default intensity over the contract horizon: at short maturities, the pre-termination intensity dominates; at long maturities, the probability of termination within the contract window is substantial, and the weight on the elevated post-termination intensity grows. Parts (b)–(c) follow from the risk-neutral pricing of jump risk. Formal proof in Appendix C. □

The steepening of Region V's CDS term structure is a novel prediction: standard climate risk models predict a flat or mildly upward-sloping sovereign CDS term structure, whereas our model generates pronounced steepening driven by the latent termination hazard. This prediction is directly testable using the empirical strategy developed in Section 5.

[6]The scarcity channel is specific to CDR. Brown firms face no analogous post-termination demand surge — there is no state of the world in which the economy desperately needs more fossil fuel capacity. This asymmetry works unambiguously in the direction of CDR complementarity.

[7]This reduced-form specification is a tractable stand-in for a structural sovereign debt model in the tradition of Eaton and Gersovitz (1981). A fully structural treatment would introduce an additional amplification channel: during the geoengineering regime, stabilized near-term output allows sovereigns to borrow at favorable rates, accumulating debt that magnifies the default spike at termination.

Summary of Cross-Regime Comparisons

Table 1 summarizes the directional predictions across regimes. We distinguish three ranges of S: low (near the onset of geoengineering), moderate (the toxic put region), and high (where the precautionary tax dominates). At moderate S, the standard cross-section obtains: Brown firms are toxic, CDR firms are complements. At high S, the precautionary Regime 3 tax can exceed the Regime 1 baseline (Proposition 2d), crushing Brown dividends so severely that termination *rescues* Brown firms — inverting the TRP signs (see the Remark following Proposition 3). The $\downarrow$ for Green firm valuations across all ranges reflects the bounded jump $J^G - 1$: unlike CDR, Green capacity cannot absorb the post-termination carbon drawdown requirement.

Table 1: Cross-Regime Comparative Statics

	τ	V^B	V^G	q^{CDR}	CDS (V)	CDS (A)
Regime 2 vs. 1	$\downarrow$	$\uparrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\uparrow$
Regime 3 vs. 2	$\uparrow$	$\downarrow$	$\uparrow$	$\uparrow$	steepens	$\uparrow$ (+TRP)
Regime 3 vs. 1 (low S)	$\downarrow$	$\uparrow$	$\downarrow$	ambig.	$\downarrow$	$\uparrow$
Regime 3 vs. 1 (mod. S)	$\downarrow$	$\downarrow$ (toxic)	$\downarrow$	$\uparrow$ (compl.)	steepens	$\uparrow$
Regime 3 vs. 1 (high S)	$\uparrow$	$\downarrow$ (prec. tax)	$\downarrow$	$\uparrow$	steepens	$\uparrow$

4 Quantitative Results

This section presents the quantitative output of the solved model. We solve the planner's HJB equation (Equations 11–14) via value function iteration on a grid of 600 points over $S \in [280, 1200]$ ppm. The converged value function yields the forward-looking SCC, equilibrium emissions, and all asset prices as functions of the carbon stock. All three regimes are computed from a single parameterization, varying only the geoengineering and termination risk channels. The carbon stock S ranges from current concentrations ($\sim$420 ppm) through high-emission scenarios. Firm valuations are computed by solving the asset pricing ODEs (Equations 20–22) on the same grid using a finite-difference method. The calibration parameters are reported in Table 2.

No circularity. The planner's HJB uses the physical hazard rate $\lambda(S)$, not the risk-neutral rate $\lambda^{\mathbb{Q}}$. Asset pricing is a downstream step: given the solved value function $v(S)$ and the resulting equilibrium carbon tax $\tau(S)$, firm valuations and risk premia are computed from the valuation ODEs. There is no feedback from asset prices to the planner's problem.

Table 2: Baseline Calibration

Parameter	Description	Value	Source
Climate			
T_{cs}	Climate sensitivity	3.0 °C/doubling	IPCC AR6
$\bar{T}$	Tipping threshold	1.5 °C	Paris Agreement; Armstrong McKay
ω_2	Quadratic damage coefficient	0.00236	DICE (Nordhaus, 2017)
ζ_s	Tipping damage scale	0.005	Calibrated to IPCC tipping estimat
σ_S	Carbon cycle volatility	2.0 ppm/yr$^{1/2}$	Standard
Geoengineering and Termination			
θ	Cooling efficiency	0.5 °C/unit	Robock et al. (2009)
λ_0	Base termination hazard	0.002/yr	Expert elicitation
λ_1	Hazard scaling with g	0.005/yr per unit	Calibrated
Γ_0	Termination damage scale	0.100	Burke et al. (2015); Trisos et al. (20
β	Termination damage convexity	2.5	Calibrated
Emissions and Abatement			
$\bar{e}$	Baseline emissions	3.0 ppm/yr	Current global emissions
ψ	Abatement cost parameter	120	$\bar{e}/\bar{\tau}_{shut}$; MAC $\approx$ \$320/tCO$_2$
Preferences and Pricing			
γ	Risk aversion	8	Barro (2006); Wachter (2013)
ρ	Time preference	0.02	Standard
r	Risk-free rate	0.04	Standard
δ	CDR depreciation	0.05	IEA
ϕ	CDR removal efficiency	0.5 ppm/yr per unit	Calibrated

4.1 Calibration

Table 2 reports the baseline parameter values. Climate sensitivity ($T_{cs} = 3.0$°C per doubling) and the tipping threshold ($\bar{T} = 1.5$°C) follow IPCC AR6 and the Paris Agreement, respectively. Quadratic flow damages ($\omega_2 = 0.0028$) are from DICE (Nordhaus, 2017). Geoengineering costs and cooling efficiency follow Smith and Wagner (2018). Risk aversion $\gamma = 8$ is within the range considered in the rare disaster literature (Barro, 2006; Wachter, 2013).

Two calibration choices merit discussion. First, we set the emissions cost parameter $\psi = \bar{e}/\bar{\tau}_{shut}$ so that full decarbonization and fossil fuel shutdown coincide, at a marginal abatement cost of \$320/tCO$_2$ consistent with estimates for hard-to-abate sectors. This coherence condition simplifies the mapping between the carbon tax and firm dividends. Second, the termination damage parameter $\Gamma_0 = 0.10$ implies output destruction of 3.8% at $S = 800$ ppm (where the Ecological Debt is $F \approx 0.69$°C of masked warming). This is conservative relative to the Burke et al. (2015) estimates of $\sim$1.5% GDP per °C (global average), considering that termination causes warming velocities 2–4 times faster than gradual climate change

(Trisos et al., 2018). The robustness analysis in Section 4.8 confirms that all qualitative results hold across $\Gamma_0 \in [0.05, 0.15]$.

4.2 Model Primitives

Figure 1 displays the calibrated geoengineering primitives that drive all subsequent results. Panel (a) plots the equilibrium deployment intensity $g(S)$, which activates at approximately 426 ppm — just above current atmospheric CO_2 concentrations — where the marginal benefit of cooling first exceeds the deployment cost. Above this threshold, $g(S)$ increases monotonically as the carbon stock grows (Assumption 2). Panel (b) shows the resulting Ecological Debt $F(S) = \theta g(S)$, which reaches 0.37°C of masked warming at 600 ppm and 0.69°C at 800 ppm. Panel (c) displays the termination hazard rate $\lambda(S) = \lambda_0 + \lambda_1 g(S)$, which rises from the baseline hazard of 0.2%/yr to 0.9%/yr at 800 ppm. The termination damage $\Gamma(F)$ (Equation 7) reaches 0.9% at 600 ppm, 2.1% at 700 ppm, and 3.8% at 800 ppm — modest individually, but amplified by risk aversion through the pricing kernel jump $J^{\Lambda} = [(1 - \Gamma) \cdot J^{\mathcal{E}}]^{-\gamma}$.

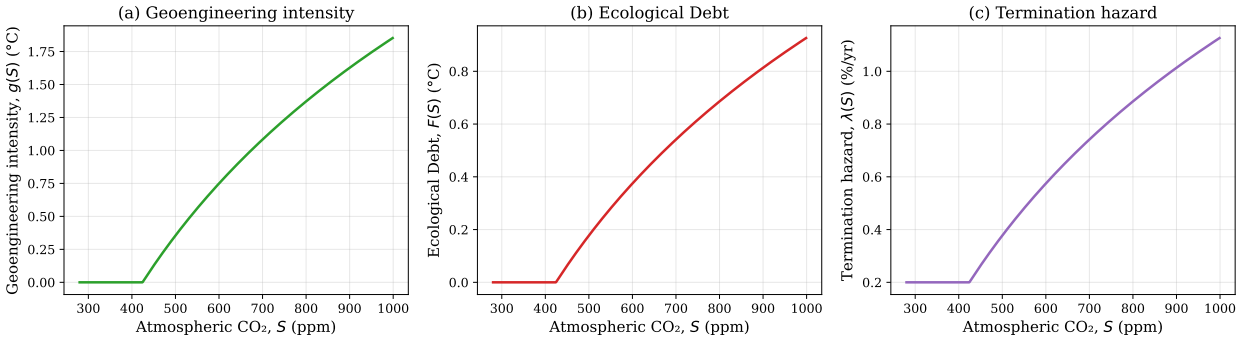


Figure 1: **Calibrated Model Primitives.** (a) Geoengineering intensity $g(S)$, activating at ∼426 ppm. (b) Ecological Debt $F(S) = \theta g(S)$. (c) Termination hazard rate $\lambda(S)$.

4.3 Equilibrium Carbon Tax

Figure 2 plots the Social Cost of Carbon under all three regimes. Panel (a) displays the three SCC curves. Regime 1 (no geoengineering) produces a smoothly increasing SCC: \$132/tCO$_2$ at current concentrations (420 ppm), rising to \$186/tCO$_2$ at 600 ppm and \$201/tCO$_2$ at 800 ppm (Lemma 1a). Regime 2 (geoengineering without termination risk) collapses the SCC dramatically to \$92/tCO$_2$ at 420 ppm and \$118/tCO$_2$ at 600 ppm — a reduction of approximately 30–37% relative to the baseline (Proposition 1a). This is the moral hazard

channel: the existence of a cheap climate backstop makes carbon appear less dangerous, weakening the policy response.

Regime 3 (full model) partially restores the SCC through the termination fragility channel: \$112/tCO$_2$ at 420 ppm, \$184/tCO$_2$ at 600 ppm, and \$262/tCO$_2$ at 800 ppm (Proposition 2a). At approximately $S \approx 612$ ppm, the Regime 3 SCC crosses above the Regime 1 baseline (Proposition 2d). Above this crossing, the marginal social cost of carbon is *higher* under geoengineering than without it, because each additional unit of carbon deepens a latent catastrophe rather than merely causing incremental flow damages.

Panel (b) isolates the precautionary amplification channel by comparing the forward-looking Regime 3 SCC to a myopic benchmark that includes the termination channel but ignores the dynamic path of carbon accumulation. The forward-looking SCC exceeds the myopic benchmark by 88% at $S = 600$ ppm, rising to 137% at $S = 800$ ppm. This gap reflects the planner's anticipation that termination damages will steepen dramatically along the equilibrium carbon path: the convexity of $\Gamma(F)$ means that future Ecological Debt is disproportionately more dangerous than current debt, and the forward-looking planner internalizes this acceleration.

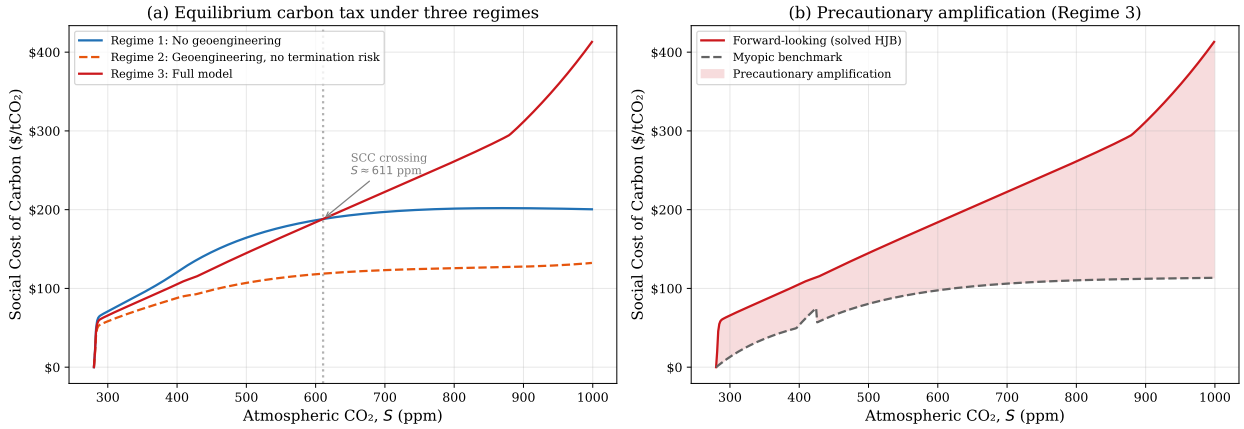


Figure 2: **Social Cost of Carbon.** (a) SCC under three regimes. The Regime 3 SCC crosses above Regime 1 at ∼612 ppm (Proposition 2d). (b) Precautionary amplification: the forward-looking Regime 3 SCC exceeds the myopic benchmark by 88% at 600 ppm, reflecting the planner's anticipation of accelerating termination damages.

4.4 Equilibrium Dynamics

Figure 3 presents the equilibrium carbon trajectories and Ecological Debt dynamics under Monte Carlo simulation (1,000 paths, starting at $S_0 = 420$ ppm).

Panel (a) shows that the three regimes produce strikingly different carbon paths. Regime 2 generates the highest carbon trajectory because the suppressed SCC keeps emissions elevated: equilibrium emissions under Regime 2 remain approximately 1.9 ppm/yr at 600 ppm, compared to 1.2 ppm/yr under Regimes 1 and 3. This is the moral hazard channel operating through the quantity margin: cheap geoengineering not only reduces the carbon tax but sustains high fossil fuel output. At 800 ppm, Regime 2 emissions are still 1.7 ppm/yr while Regimes 1 and 3 have driven emissions near zero.

Panel (b) tracks the Ecological Debt $F(t)$ under Regime 3. Under the median path, the toxic put threshold ($S^* \approx 585$ ppm) is reached around 2130 — approximately 80 years from today at current emission trends. This timeline is within a plausible institutional planning horizon, suggesting that the toxic put is not merely a theoretical curiosity but a risk that could materialize within the lifetimes of assets currently being financed.

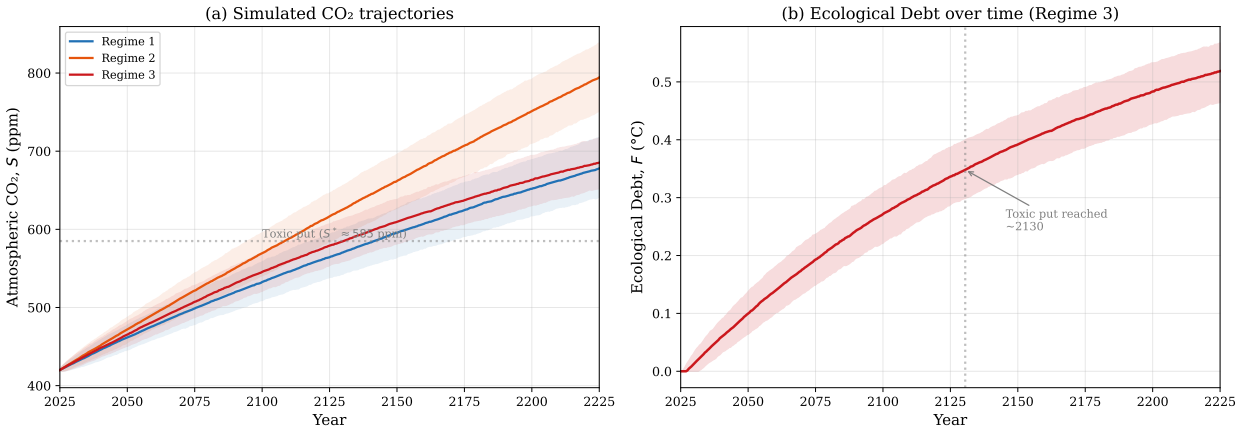


Figure 3: **Equilibrium Dynamics.** (a) Simulated CO_2 trajectories (median and 90% bands) under three regimes. Regime 2 produces the highest carbon path due to moral hazard. (b) Ecological Debt $F(t)$ under Regime 3, with the toxic put threshold ($S^* \approx 585$ ppm) reached around 2130.

4.5 Brown Firm Valuations and the Toxic Put

Figure 4 illustrates the toxic put mechanism (Proposition 3) through Brown firm valuations. The Regime 1 valuation declines monotonically in S as the rising carbon tax erodes fossil fuel dividends (Lemma 1b). Under Regime 2, Brown firms enjoy higher valuations at every S because the suppressed carbon tax preserves their dividend stream (Proposition 1c).

The Regime 3 valuation reveals the toxic put. At low carbon concentrations, the Regime 3 curve lies above Regime 1: the dividend benefit from the suppressed tax dominates the

small termination risk premium (Proposition 3d). At $S^* \approx 585$ ppm, the Regime 3 and Regime 1 curves cross: geoengineering transitions from asset to liability for fossil fuel firms, precisely because the Ecological Debt at this concentration ($F \approx 0.35$°C) generates a termination shock severe enough to destroy more value than the suppressed tax creates. Above S^*, Brown firms under geoengineering are *worse off* than in a world without intervention (Proposition 3e). The geoengineering put has transformed them into toxic assets.

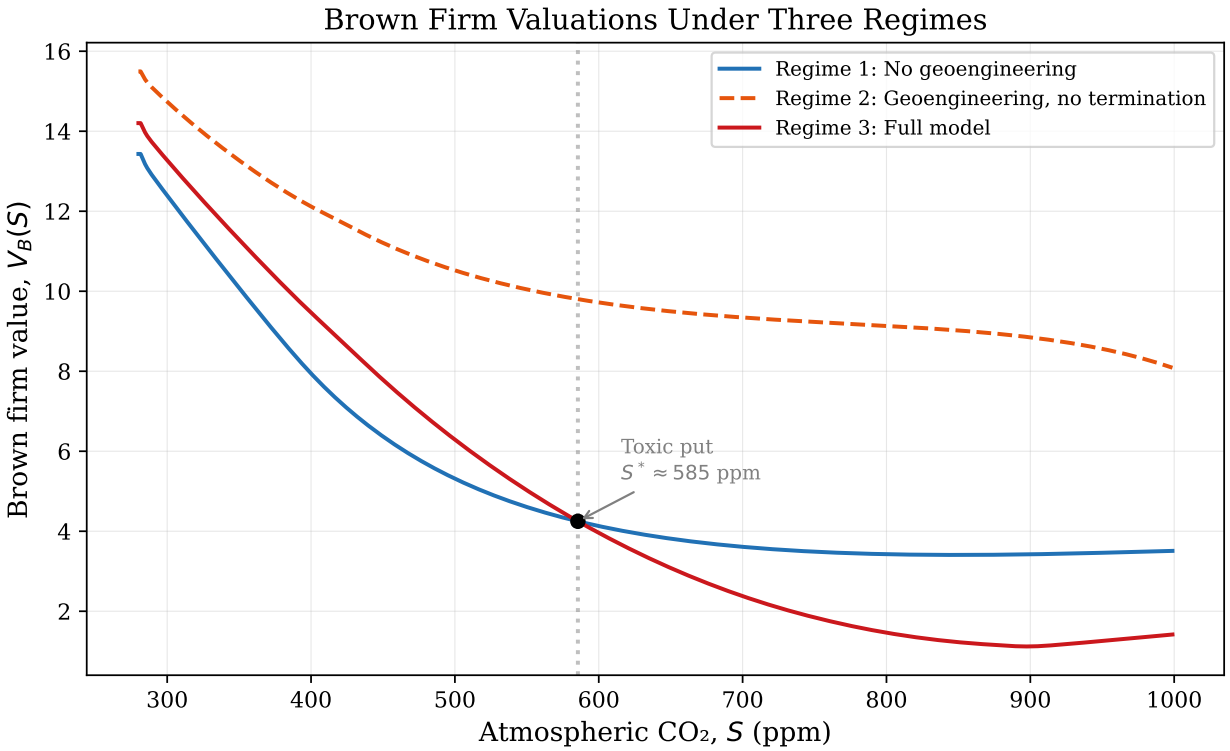


Figure 4: **Brown Firm Valuations and the Toxic Put.** Regime 3 valuations cross below Regime 1 at $S^* \approx 585$ ppm (Proposition 3e). Below the crossing, geoengineering helps Brown firms; above it, termination risk transforms them into toxic assets.

4.6 CDR Shadow Values and the Complementarity Reversal

Figure 5 displays the shadow value of CDR capacity q under all three regimes. The Regime 1 CDR shadow value rises with S as the increasing carbon tax sustains CDR operating revenue (Lemma 1d). Under Regime 2, CDR valuations collapse: the suppressed carbon tax eliminates CDR revenue, and without termination risk there is no tail-hedge motive (Corollary 1a).

Regime 3 reverses this conclusion. The forward-looking planner internalizes the growing

tail risk of termination and sets a precautionary carbon tax that capitalizes the catastrophic option value of CDR into current quasi-rents. At $S = 600$ ppm, the Regime 3 CDR shadow value fully recovers to the Regime 1 level ($q^{R3} = q^{R1} = 0.082$). At $S = 800$ ppm, the Regime 3 shadow value *exceeds* the Regime 1 value by 26% ($q^{R3} = 0.110$ vs. $q^{R1} = 0.087$), reflecting the full complementarity crossing: geoengineering makes CDR *more* valuable than in a world without intervention (Proposition 4). The shaded region in the figure highlights the complementarity premium — the increment by which Regime 3 exceeds Regime 2.

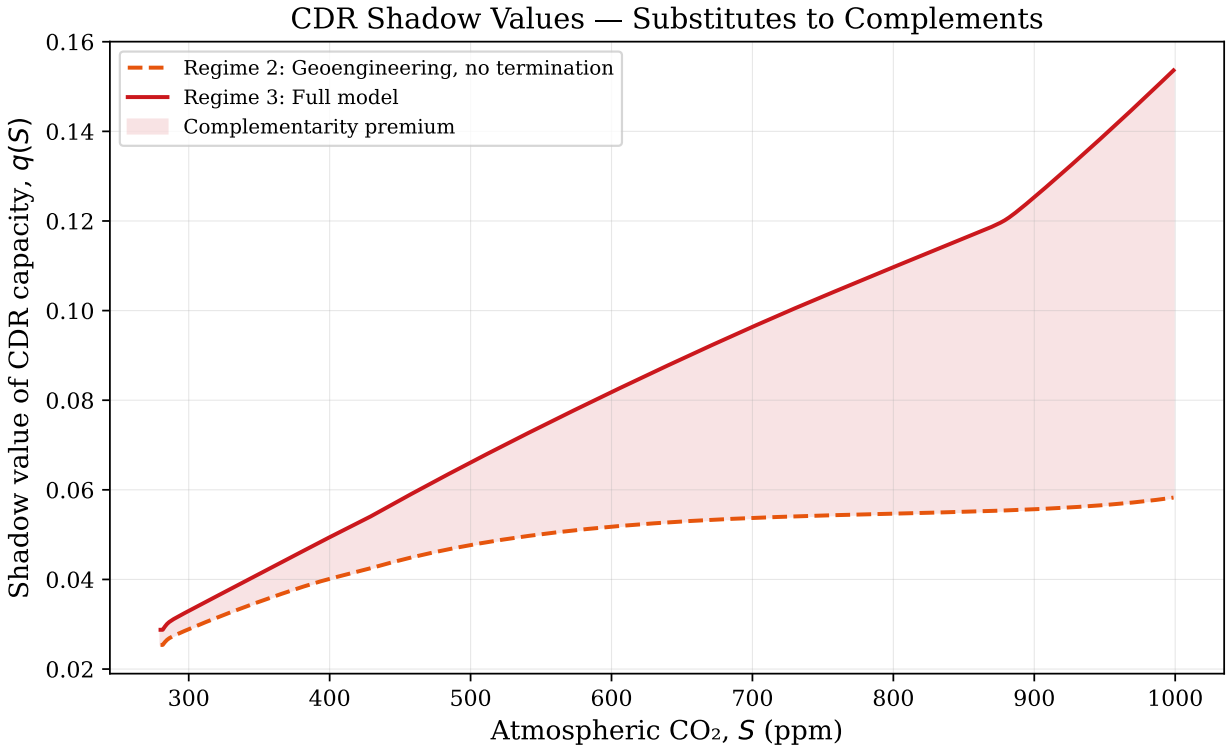


Figure 5: **CDR Shadow Values: Substitutes to Complements.** Regime 2 collapses CDR valuations (Corollary 1). Regime 3 fully reverses the crowding-out effect at high S, with the CDR shadow value exceeding the no-geoengineering baseline by 26% at 800 ppm (Proposition 4).

4.7 Termination Risk Premia and the Cross-Section Inversion

Figure 6 presents the cross-section of termination risk premia.

Panel (a) shows the TRP for all three firm types. At moderate carbon concentrations (below ∼589 ppm), the cross-section matches the theoretical predictions of Proposition 3(a–c): Brown firms carry a positive TRP (compensation for termination losses), while Green

and CDR firms carry negative TRPs (they act as disaster insurance). At 700 ppm, the Brown TRP reaches 14 basis points and CDR reaches -14 basis points. At 800 ppm, the Brown TRP is 73 basis points — a substantial premium driven by the risk-neutral hazard rate $\lambda^{\mathbb{Q}} = 1.47\%/\text{yr}$ (compared to the physical rate $\lambda = 0.89\%/\text{yr}$), amplified by risk aversion through $J^{\Lambda} = 1.65$.

Above approximately 589 ppm, the cross-section *inverts*: the Brown TRP turns negative and the CDR TRP turns positive. This inversion, anticipated in the Remark following Proposition 3, occurs because the precautionary Regime 3 carbon tax becomes so aggressive at high S that it crushes Brown dividends more severely than termination would. Termination — the very event that the climate system cannot survive — becomes the scenario under which fossil fuel firms *recover* value, because the post-termination Regime 1 tax is lower than the precautionary Regime 3 tax ($J^B > 1$ above 589 ppm). This two-phase structure — first the standard toxic put (moderate S), then the precautionary inversion (high S) — reveals that geoengineering destroys Brown firm value through two distinct channels that operate at different carbon concentrations.

Panel (b) displays the Brown jump ratio $J^B = V^B_{post}/V^B_{pre}$. Below 589 ppm, $J^B < 1$: termination destroys Brown value. Above 589 ppm, $J^B > 1$: termination rescues Brown firms. J^B rises sharply at high S, reaching 2.3 at 800 ppm, because the growing gap between the precautionary tax τ^{R3} and the post-termination tax $\bar{\tau}$ generates an increasingly large value recovery upon termination.

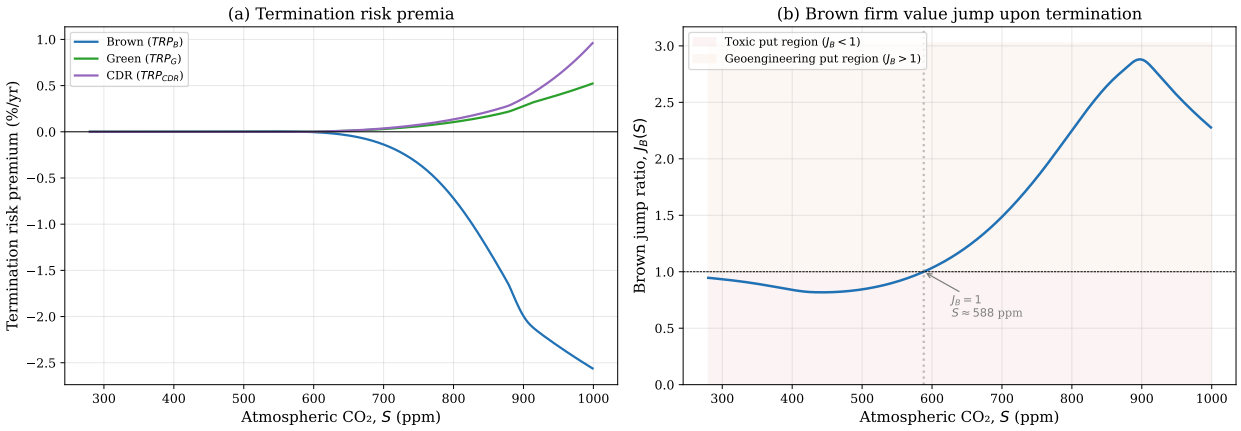


Figure 6: **Termination Risk Premia.** (a) TRP cross-section for Brown, Green, and CDR firms. The standard cross-section (Brown positive, Green/CDR negative) inverts above ∼589 ppm as the precautionary tax dominates termination risk. (b) Brown jump ratio J^B: below unity at moderate S (termination destroys value), above unity at high S (termination rescues Brown firms from the precautionary tax).

4.8 Robustness

Figure 7 examines the sensitivity of the key results to the three most important parameters: the termination hazard scaling λ_1, risk aversion γ, and the termination damage intensity Γ_0.

Panels (a)–(b) vary $\lambda_1 \in \{0.002, 0.005, 0.010, 0.020\}$. Higher λ_1 amplifies the SCC restoration and shifts the toxic put crossing leftward (Proposition 3f): at $\lambda_1 = 0.020$, the toxic put threshold falls to approximately 520 ppm, well within plausible near-term concentrations. All qualitative results — the signs, monotonicity, and existence of crossings — are preserved across the full range.

Panels (c)–(d) vary $\gamma \in \{3, 5, 8, 12\}$. Higher risk aversion amplifies the pricing kernel jump J^Λ (Equation 18), which magnifies the risk-neutral hazard rate and therefore all termination risk premia. The CDR complementarity ratio q^{R3}/q^{R2} increases sharply with γ, confirming that the substitutes-to-complements reversal is driven by the SDF weighting of the catastrophic termination state.

Panels (e)–(f) vary $\Gamma_0 \in \{0.05, 0.075, 0.10, 0.15\}$ and reveal a striking asymmetry: the SCC is nearly invariant to Γ_0 (panel f), while the Brown jump ratio J^B is highly sensitive (panel e). The intuition is that Γ_0 enters the planner's HJB only through the expected termination cost, weighted by the small hazard rate λ (0.2–0.9%/yr). This makes Γ_0 second-order for the planner relative to the direct climate damage and abatement cost channels.

Asset prices, by contrast, reflect the *conditional* jump: investors price what happens *if* termination occurs, not just its probability-weighted expectation. The social cost of carbon is first-order determined by the hazard rate λ and risk aversion γ, not the damage severity Γ_0. This asymmetry implies that uncertainty about termination damage severity is a *pricing* problem, not a *policy* problem — the optimal tax is robust even when the magnitude of termination damages remains deeply uncertain.

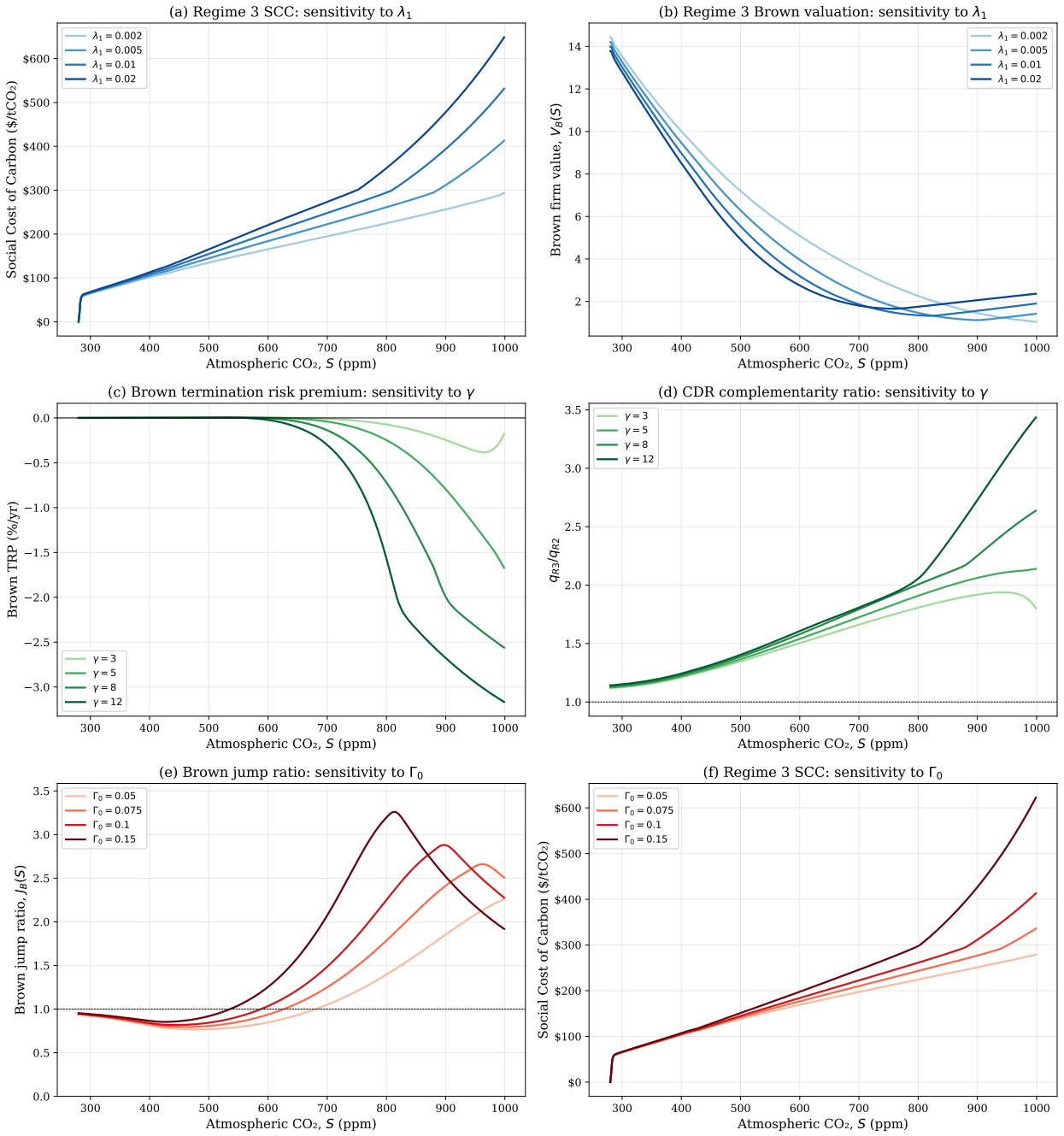


Figure 7: **Robustness.** (a)–(b): Sensitivity to λ_1. (c)–(d): Sensitivity to γ. (e)–(f): Sensitivity to Γ_0. The SCC is robust to Γ_0 uncertainty while J^B is highly sensitive, implying that termination damage uncertainty is a pricing problem, not a policy problem.

5 Empirical Strategy

To test the theoretical predictions of our model, we need to map the unobservable state variables, namely the probability of geoengineering deployment and the latent termination fragility, to observable, high-frequency data. We propose an empirical strategy that constructs text-based Geoengineering News Indices and evaluates their impact on forward-looking asset prices across sovereign and corporate markets.

5.1 Constructing the Geoengineering News Indices

Because geoengineering technology is rapidly evolving but not yet globally deployed, we rely on high-frequency news coverage to proxy for shifts in market expectations. Following the climate news hedging methodology of Engle et al. (2020), we utilize large language models to construct specialized news indices from major newspaper archives.

We construct two distinct weekly indices. The first index, SRM_News_t, captures shifts in the viability and likelihood of albedo modification and solar radiation management. It spikes during technological breakthroughs, cost reductions, or political signaling from vulnerable nations indicating a willingness to deploy geoengineering. A positive shock to this index implies that the market assigns a higher probability to the exercise of the Geoengineering Put.

The second index, $Term_News_t$, captures news related to the endogenous termination hazard. It tracks reports on the severe regional side effects of geoengineering, international legal disputes, threats of military retaliation, and scientific warnings about masked warming. A positive shock to this index indicates an increase in the perceived probability of a catastrophic termination jump.

To ensure clean identification, we orthogonalize both series against a baseline general climate risk news index and extract the $AR(1)$ innovations of the residuals, yielding our final orthogonalized shock measures: $\widetilde{SRM_News_t}$ and $\widetilde{Term_News_t}$.

5.2 Sovereign CDS Panel

Our model predicts that unilateral geoengineering deployment lowers flow damages for temperature-vulnerable countries (Region V) but imposes involuntary variance shocks on

precipitation-vulnerable agricultural countries (Region A). We test this by estimating:

$$\begin{aligned}\Delta CDS_{i,t} = \alpha_i + \delta_t &+ \beta_1 \widetilde{SRM_News}_t \times TempVuln_i \\ &+ \beta_2 \widetilde{SRM_News}_t \times PrecipVuln_i \\ &+ \beta_3 \widetilde{Term_News}_t \times Fragility_{i,t} + \epsilon_{i,t}\end{aligned} \tag{26}$$

where α_i and δ_t are country and time fixed effects. We identify Region V using $TempVuln_i$, measured by the country-level temperature-to-GDP sensitivities established by Burke et al. (2015), and Region A using $PrecipVuln_i$, measured by the Notre Dame Global Adaptation Initiative (ND-GAIN) indices for agricultural and water-stress vulnerability.

Predictions: $\beta_1 < 0$ (Corollary 2a and Proposition 5a), $\beta_2 > 0$ (Corollary 2b), $\beta_3 > 0$ (Proposition 5c).

5.3 Corporate Cross-Section

We test the corporate asset pricing predictions by examining market-hedged weekly stock returns:

$$R_{j,t} = \alpha_j + \beta_1 \widetilde{SRM_News}_t \times FirmType_j + \beta_2 \widetilde{Term_News}_t \times FirmType_j + \gamma X_{j,t} + \epsilon_{j,t} \tag{27}$$

For Brown firms, we predict $\beta_1 > 0$ (moral hazard, Proposition 1c) and $\beta_2 \ll 0$ (toxic put, Proposition 3). For Green firms, we predict $\beta_1 < 0$ (suppressed urgency) and $\beta_2 > 0$ (disaster insurance, Proposition 3b). For CDR-exposed firms, we predict $\beta_1 > 0$ and $\beta_2 > 0$ (complementarity, Proposition 4).

We proxy CDR exposure using the carbon capture and storage (CCS) patent intensity of publicly traded industrial firms, and where available, the installed or contracted CDR capacity of firms with operational carbon removal projects. We acknowledge that CCS (capturing emissions at the point of combustion) is technologically distinct from CDR (removing legacy carbon from the ambient atmosphere). We rely on CCS patent intensity as a proxy because the underlying chemical engineering and fluid transport expertise is highly correlated with the capacity to pivot to direct air capture operations if the post-termination shadow price of carbon spikes.

5.4 Discussion of Statistical Power

Geoengineering news is thinner than general climate news. A null result would be informative: if markets do not yet price geoengineering risk, this is consistent with the "sleepwalking"

narrative in the model, where the economy accumulates Ecological Debt past the tipping threshold without experiencing consequences until the intervention fails.

6 Conclusion

The potential deployment of climate geoengineering fundamentally disrupts traditional climate economics. By providing a cheap, fast-acting mechanism to artificially cool the planet, geoengineering replaces the global free-rider problem with a perilous free-driver problem. In this paper, we develop a macro-finance framework to characterize the asset pricing implications of this technological regime and solve for the forward-looking equilibrium.

Our analysis, organized around three nested regimes, reveals three main results. First, geoengineering without termination risk acts as a pure moral hazard device: it collapses the social cost of carbon (from \$132 to \$92/tCO$_2$ at current concentrations), inflates fossil fuel valuations, stalls the green transition, and crowds out CDR investment. Second, the introduction of endogenous termination risk transforms the cross-section of asset prices. Fossil fuel firms carry a positive termination risk premium that eventually dominates their near-term dividend benefit, transforming them into toxic assets at $S^* \approx 585$ ppm — a threshold reached around 2130 under the equilibrium carbon trajectory. Third, and most importantly, geoengineering and CDR switch from substitutes to strict complements: the forward-looking planner internalizes the growing tail risk and sets a precautionary carbon tax that makes CDR capacity *more* valuable under geoengineering than without it.

A new quantitative finding emerges from the solved model: substantial precautionary amplification. The forward-looking SCC exceeds its myopic counterpart by 88% at 600 ppm and 137% at 800 ppm. This gap reflects the planner's anticipation that the convexity of the termination damage function will compound along the equilibrium carbon path. Static or myopic analyses of geoengineering policy miss this channel entirely, and as a result understate the optimal carbon tax by roughly a factor of two at high concentrations.

Our findings carry immediate implications for climate policy and financial regulation. The standard framework for evaluating geoengineering — which treats it as a cost-effective substitute for mitigation — is not merely incomplete but actively dangerous. It is precisely the availability of cheap geoengineering that makes expensive CDR capacity indispensable, because geoengineering creates the very tail risk that CDR is uniquely positioned to resolve. Policymakers should treat concurrent CDR investment not as an alternative to geoengineering but as its necessary complement.

For financial markets, the toxic put mechanism implies that fossil fuel assets may be systematically mispriced if markets treat geoengineering as a reprieve from carbon taxation

without internalizing the latent termination exposure. The robustness analysis offers a partial reassurance: the optimal carbon tax is robust to deep uncertainty about the severity of termination damages (Γ_0). However, fossil fuel valuations are not — they are highly sensitive to Γ_0 through the conditional jump ratio. This asymmetry suggests that the welfare cost of uncertainty is small (the planner's policy is robust) but the financial cost may be large (asset prices are fragile).

As geoengineering deployment approaches — and the physical and geopolitical conditions for unilateral action become increasingly plausible — the predictions of this model become directly testable. The cross-sectional predictions for equities and sovereign CDS spreads can be evaluated using the news-based identification strategy outlined in Section 5. We leave the empirical implementation to future work.

A CES Energy Market Microfoundation

This appendix provides the competitive energy market structure that delivers Assumptions 4–5 as equilibrium outcomes.

Total energy is an aggregate of fossil fuel (Brown) and renewable (Green) energy, combined via a constant elasticity of substitution technology:

$$\mathcal{E} = \left[\alpha \left(Y^B\right)^{\frac{\sigma-1}{\sigma}} + (1-\alpha)\left(Y^G\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} \tag{28}$$

where $\alpha \in (0,1)$ is the Brown energy share parameter and $\sigma > 1$ is the elasticity of substitution. Brown and Green energy are produced by competitive firms with upward-sloping supply curves: $C^B(Y^B) = m_B Y^B + \frac{\psi_B}{2}(Y^B)^2$ and $C^G(Y^G) = m_G Y^G + \frac{\psi_G}{2}(Y^G)^2$, with $m_G > m_B$ (absent policy, fossil fuel energy is cheaper). A carbon tax τ per unit of emissions raises the effective Brown cost by $\tau\kappa_B$ per unit of output.

In competitive equilibrium, the CES first-order conditions yield:

$$\frac{Y^B}{Y^G} = \left(\frac{\alpha}{1-\alpha}\right)^{\sigma}\left(\frac{p^G}{p^B}\right)^{\sigma} \tag{29}$$

where $p^B = m_B + \psi_B Y^B + \tau\kappa_B$ and $p^G = m_G + \psi_G Y^G$. Because $\sigma > 1$, an increase in τ raises p^B relative to p^G, shifting the equilibrium toward Green energy. The following properties hold:

(a) $Y^B(\tau)$ is strictly decreasing and $Y^G(\tau)$ is strictly increasing in τ.

(b) Brown dividends $D^B(\tau) = \frac{\psi_B}{2}(Y^B(\tau))^2$ are strictly decreasing in τ (Assumption 4).

(c) Green dividends $D^G(\tau) = \frac{\psi_G}{2}(Y^G(\tau))^2$ are strictly increasing in τ (Assumption 5).

(d) For α sufficiently large (Brown energy constitutes a non-trivial share), $\mathcal{E}(\tau)$ is weakly decreasing in τ.

Part (d) ensures that a higher carbon tax reduces aggregate energy output and therefore consumption, which is required for the carbon cycle risk premium and the energy reallocation ratio $J^{\mathcal{E}} < 1$ at termination. This is the empirically relevant case: the contraction of Brown output dominates the expansion of Green output in the CES aggregate. □

B CDR Capacity Dynamics

This appendix provides the full dynamic CDR investment problem underlying the shadow value $q(S)$ used in the main text.

CDR firms operate physical infrastructure (direct air capture plants, enhanced weathering installations) that requires time and capital to build. Installed CDR capacity K_t^{CDR} evolves according to:

$$\dot{K}_t^{CDR} = I_t - \delta K_t^{CDR} \tag{30}$$

where $I_t \geq 0$ is gross investment and $\delta > 0$ is the depreciation rate. Physical carbon removal is bounded by installed capacity:

$$R_t^{CDR} \leq \phi \, K_t^{CDR} \tag{31}$$

where $\phi > 0$ converts installed capacity into a maximum annual removal flow.

Investment faces convex adjustment costs:

$$C^{CDR}(I_t) = p_I I_t + \frac{\xi}{2} I_t^2 \tag{32}$$

CDR firms earn revenue at the prevailing carbon price. The dividend flow is:

$$D_t^{CDR} = \tau_t \, R_t^{CDR} - C^{CDR}(I_t) \tag{33}$$

The market value of the CDR firm is $V_t^{CDR} = q_t K_t^{CDR}$, where q_t is Tobin's q. The optimal investment policy satisfies $q_t = p_I + \xi I_t$, giving $I_t = (q_t - p_I)/\xi$ when $q_t > p_I$ and $I_t = 0$ otherwise.

In the main text, we abstract from the dynamic capacity accumulation and treat $q(S)$ as determined by the equilibrium carbon tax path (Equations 22). This is equivalent to the full dynamic problem when the capacity constraint (31) does not bind and K^{CDR} adjusts smoothly. The key economic insight from the full model is that the convex adjustment costs create a premium for *pre-installed* capacity: when CDR is suddenly needed (e.g., post-termination), the cost of rapidly scaling capacity exceeds the steady-state cost, generating scarcity rents for existing installations. This scarcity channel, noted in the discussion following Proposition 4, strengthens the complementarity result.

C Formal Proofs

C.1 Proof of Lemma 1

Parts (a)–(d) follow from the Regime 1 HJB (Equation 11). Because Ω is increasing and convex (Assumption 1) and Ψ is increasing, $v'(S) < 0$ and $v''(S) < 0$: the value function is decreasing and concave in S. The SCC $= -v'/v \cdot$ [elasticity term] is therefore positive

and increasing. Firm valuations follow from $V^i = D^i(\tau)/r$ and the monotonicity of $\tau(S)$ combined with Assumptions 4–5. □

C.2 Proof of Proposition 1

Compare Equations (11) and (13). Because $\Omega(\Psi(S) - \theta g) < \Omega(\Psi(S))$ for $g > 0$, the effective growth rate under Regime 2 exceeds Regime 1. By a comparison argument on the value functions, $|v'_{R2}(S)| < |v'_{R1}(S)|$ at each S, yielding $SCC^{R2} < SCC^{R1}$ (part a). For part (b), if $g \to (\Psi(S) - \bar{T})/\theta$, then $T \to \bar{T}$ and $\partial T/\partial S \to 0$, so $v'_{R2} \to 0$ and $SCC^{R2} \to 0$. Parts (c)–(e) follow from Assumptions 4–5 and the monotonicity of the carbon tax. □

C.3 Proof of Proposition 2

The Regime 3 HJB (Equation 14) adds the jump term $\lambda[(1-\Gamma)^{1-\gamma}v^{R1} - v]$ to the Regime 2 HJB. Because $(1-\Gamma)^{1-\gamma}v^{R1} < v$ (termination is value-destroying), this term is negative. By a comparison argument, $v_{R3}(S) < v_{R2}(S)$ and $|v'_{R3}(S)| > |v'_{R2}(S)|$, so $SCC^{R3} > SCC^{R2}$ (part a). For part (d), note that $\Gamma'(F)$ grows without bound as F increases (because $\Gamma(F) = 1 - \exp(-\Gamma_0 F^\beta)$ with $\beta > 1$), while $\Omega'(\Psi(S))$ grows at the rate of the smooth damage function. The jump term in v'_{R3} therefore eventually dominates, yielding $|v'_{R3}| > |v'_{R1}|$ and hence $SCC^{R3} > SCC^{R1}$ above some $\hat{S}$. □

C.4 Proof of Proposition 3

Part I (TRP signs). From Equation (24), $TRP^i = -\lambda(J^\Lambda - 1)(J^i - 1)$. At termination, the post-termination tax $\bar{\tau} = SCC^{R1}(S) > \tau^{R3}(S)$ for moderate S (part a of Proposition 2). By Assumption 4, $D^B(\bar{\tau}) < D^B(\tau^{R3})$, so $V^B_{post} < V^B_{pre}$ and $J^B < 1$. Combined with $J^\Lambda > 1$ (Equation 18), $TRP^B > 0$. The arguments for Green ($J^G > 1$ by Assumption 5) and CDR ($J^{CDR} > 1$ by the post-termination quasi-rent analysis in Appendix B) are analogous.

Part II (Non-monotonicity). For part (d), at low S, $F(S)$ is small, $\Gamma(F) \approx 0$, $\lambda^{\mathbb{Q}} \approx \lambda$, and TRP^B is small. The dividend benefit $D^B(\tau^{R3}) - D^B(\tau^{R1}) > 0$ (from $\tau^{R3} < \tau^{R1}$) dominates, so $V^{B,R3} > V^{B,R1}$. For part (e), as S grows, TRP^B grows without bound (because $\Gamma'(F)$ grows and J^Λ grows), while the dividend benefit is bounded above by $D^B(0) - D^B(\tau^{R1})$. Single crossing at some finite S^*. Part (f) follows because higher λ or steeper Γ accelerate the growth of TRP^B. □

C.5 Proof of Proposition 4

From Equation (22), $q^{R3} = [\tau\phi + \lambda^{\mathbb{Q}} q_{post}]/(r + \delta + \lambda^{\mathbb{Q}})$. Under Regime 2 ($\lambda = 0$), $q^{R2} = \tau^{R2}\phi/(r + \delta)$. The difference $q^{R3} - q^{R2}$ includes the term $\lambda^{\mathbb{Q}} q_{post}/(r + \delta + \lambda^{\mathbb{Q}})$, which is strictly positive when $\lambda > 0$ (part a). This term is increasing in S because q_{post} is increasing in $\bar{\tau}(S)$ (part b). Combined with $q^{R2} < q^{R1}$ (Corollary 1), the reversal from substitutes toward complements obtains (part c). □

C.6 Proof of Proposition 5

Part (a): over contract horizon $\mathcal{T}$, the expected default intensity is a mixture of pre- and post-termination intensities, weighted by the probability of termination within $\mathcal{T}$. The mixture weight on μ^{post} is $1 - (1 - e^{-\lambda\mathcal{T}})/(\lambda\mathcal{T})$, which is increasing in $\mathcal{T}$, yielding the steepening. Part (b): the change of measure to $\mathbb{Q}$ replaces λ with $\lambda^{\mathbb{Q}} > \lambda$, amplifying the termination-linked default component. Part (c): at the moment of termination, output drops by $\Gamma_i(F)$ for country i, and the default intensity spike is proportional to the ratio of this loss to fiscal capacity. □

C.7 Derivation of the SCC Decomposition

The planner's value function $V(S, A)$ satisfies the HJB equation. The SCC is $-V_S/V_C$. The planner takes $g(S)$ as given (Assumption 2). The marginal effect of S on welfare operates through two channels:

1. *Flow damages:* $dT/dS = \Psi'(S) - \theta g'(S)$, so the marginal flow damage is $\Omega'(T) \cdot [\Psi'(S) - \theta g'(S)]$.

2. *Termination severity:* $dF/dS = \theta g'(S)$, so the marginal increase in termination damage is $\Gamma'(F) \cdot \theta g'(S)$.

Integrating over all future dates with the stochastic discount factor yields Equation (15). □

C.8 Derivation of the Stochastic Discount Factor

The representative investor has CRRA utility over $C_t = A_t \mathcal{E}_t(\tau_t)$. Marginal utility is proportional to $C_t^{-\gamma}$. By Ito's lemma for jump-diffusion processes, accounting for the two Brownian risk factors (dW_t, dZ_t) and the Poisson jump (dN_t), we obtain Equation (17). The pricing kernel jump follows from:

$$\frac{C_{t+}}{C_{t-}} = (1 - \Gamma(F_t)) \cdot J_t^{\mathcal{E}} \tag{34}$$

and $J_t^\Lambda = (C_{t+}/C_{t-})^{-\gamma}$. □

C.9 Derivation of the Expected Return Equation

The no-arbitrage condition $\mathbb{E}_t[d(\Lambda_t V_t^i)] + \Lambda_t D_t^i dt = 0$, expanded via Ito's lemma for jump-diffusions:

$$d(\Lambda_t V_t^i) = V_{t-}^i d\Lambda_t + \Lambda_{t-} dV_t^i + d\langle \Lambda^c, V^{i,c} \rangle_t + (\Delta\Lambda_t)(\Delta V_t^i) \tag{35}$$

Dividing by $\Lambda_{t-} V_{t-}^i$, taking conditional expectations, and substituting $\mathbb{E}_t[d\Lambda_t/\Lambda_t] = -r_t\, dt$ yields Equation (23). □

D Post-Termination Dynamics

After termination ($dN_t = 1$), geoengineering ceases ($g = 0$, $\lambda = 0$) and the economy reverts to Regime 1 with inherited carbon stock S_t and damaged productivity $A_{t+} = A_{t-}(1 - \Gamma(F_t))$. Dividends and quasi-rents are scaled by $(1 - \Gamma(F_t))$ relative to an undamaged Regime 1 economy. CDR is the primary mechanism for recovery; the carbon stock evolves as:

$$dS = -\phi K^{CDR}\, dt + \sigma_S\, dZ_t \tag{36}$$

As S falls, $\bar{\tau}(S)$ falls, and CDR quasi-rents decay. The post-termination shadow value is:

$$q_{t+} = (1 - \Gamma(F_t)) \cdot \mathbb{E}_{t+} \int_t^\infty e^{-(r+\delta)(s-t)} \bar{\tau}_s\, \phi\, ds \tag{37}$$

bounded above by $(1-\Gamma)\bar{\tau}_t\phi/(r+\delta)$ and below by $(1-\Gamma)\tau_t^{R1}\phi/(r+\delta)$. The CDR valuation jump satisfies:

$$1 < J_t^{CDR} = \frac{q_{t+}}{q_{t-}} \leq \frac{r + \delta + \lambda^{\mathbb{Q}}}{\lambda^{\mathbb{Q}}} \tag{38}$$

confirming Proposition 3(c).

E Ocean Acidification Extension

The baseline model assumes that all climate damages operate through the temperature channel. In reality, ocean acidification $\Omega^{acid}(S_t)$ accumulates with the carbon stock regardless of geoengineering, providing a floor on the SCC even in the tipping limit of Regime 2:

$$SCC^{R2} \geq \mathbb{E}_t \int_t^\infty e^{-\rho(s-t)} \frac{U'(C_s)}{U'(C_t)} (\Omega^{acid})'(S_s)\, ds > 0 \tag{39}$$

The optimal carbon tax cannot literally fall to zero because the acidification externality persists. This strengthens all Regime 3 results: the higher baseline SCC means the Regime 2 collapse is less extreme, but the Regime 3 termination channel still dominates at high S.

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