The Engineering of Consent
Strategic Cultural Transmission and the Boundaries of Social Control

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Abstract

This paper studies intergenerational cultural transmission when dynasties embedded in a social network strategically misrepresent their traits. A central planner operates two instruments—centralized schooling and mobility incentives—on a DeGroot network with strategic parental display. Under a quadratic specification of preferences, the equilibrium law of motion is governed by a modified transition matrix whose spectral properties yield four results. First, a Nash equilibrium in displayed traits exists and is unique, and traits converge to consensuses determined by eigenvector centrality. Second, long-run traits are necessarily convex combinations of initial traits, so the planner cannot engineer convergence to an arbitrary target—an impossibility result that holds whenever the transmission technology preserves the convex hull of displayed traits. Third, the planner's instruments decouple: schooling steers the consensus target within the convex hull, while mobility incentives govern only the speed of convergence. Fourth, under a budget constraint, optimal targeting concentrates incentives on dynasties whose cultural positions load most heavily on the convergence bottleneck—the second eigenvector of the transition matrix. Together, the impossibility theorem and the instrument separation characterize both the planner's constraint set and the best achievable policy within it. The framework is illustrated with an application to East German cultural dynamics.

JEL Classification: C72, D83, D85, Z13

1 Introduction

How do cultural traits persist when political institutions change dramatically? The answer this paper offers is stark: a regime that inherits a population with heterogeneous cultural beliefs cannot school or coerce those beliefs toward an arbitrary target. The achievable range of cultural outcomes is permanently bounded by the beliefs the regime inherited—no combination of propaganda, schooling, and mobility incentives can push the society beyond this inherited frontier. A large empirical literature documents that exposure to authoritarian regimes leaves lasting imprints on preferences,

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trust, and intergroup attitudes, often persisting decades after a regime collapses.[1] Yet the theoretical mechanisms through which regimes attempt to reshape culture—and the reasons they systematically fail to achieve full homogenization—remain underexplored. A key unresolved issue is how regimes combine informational influence with coercive enforcement. Although propaganda, censorship, and repression are well documented features of authoritarian rule, the economic logic governing their joint use—and whether persuasion and coercion reinforce or substitute for one another—remains poorly understood (Zhuravskaya, Guriev, and Markevich 2024). This paper develops a network-based theory of strategic cultural transmission that addresses these questions.

The framework builds on Buechel, Hellmann, and Pichler (2014), who introduce strategic cultural display on networks in a DeGroot–Bisin-Verdier setting: dynasties on a social network choose displayed traits that differ from their true preferences, and children's traits are formed as weighted averages of these displayed traits. The present paper extends this base model and adds a central planner who operates two policy instruments—a centralized schooling system that shifts socialization weights away from parental influence, and dynasty-specific mobility incentives that differentially reduce the intergenerational utility cost of cultural distance. Under a quadratic specification of preferences, the planner's instruments generate a modified transition matrix M that nests the standard DeGroot process as a special case and yields four results absent from the existing literature: an impossibility theorem, a spectral speed comparison, an instrument separation, and an optimal targeting result.

Specifically, a finite set of dynasties is embedded in a fixed social network, represented by a row-stochastic socialization matrix Ψ. Each generation, an adult with a true cultural trait chooses a *displayed* trait that is publicly observed. The child's trait is then formed as a weighted average of the parent's displayed trait and the displayed traits of other dynasties in the network, where the weights are determined by Ψ, the *network transmission matrix*. The parent faces a trade-off: displaying a trait far from one's true preference is costly (cognitive dissonance), but it allows the parent to offset the horizontal socialization influence of the network on the child (intergenerational utility).

[1] See, among others, Alesina and Fuchs-Schündeln (2007) on market preferences, Bursztyn and Cantoni (2014) on political attitudes, Voigtländer and Voth (2012) on the persistence of anti-Semitism, and Bondar and Fuchs-Schündeln (2023) for updated evidence that East-West convergence remains ongoing but decelerating thirty years after reunification.

Both components are captured by an additively separable utility function with single-peaked, strictly concave terms.

The analysis delivers three sets of results. The first set establishes the microeconomic foundations and convergence properties. The parental optimization problem has a unique solution: parents deviate from their true trait in the direction opposite to the network's influence, with the magnitude increasing in the cultural gap (Proposition 1). A Nash equilibrium in displayed traits exists under general preferences (via Brouwer's theorem); under the quadratic specification with symmetric positive definite Ψ, it is unique (Proposition 2). The quadratic specification yields linear best responses and a closed-form transition matrix $M = \Psi(I + \Pi\Psi)^{-1}(I + \Pi)$. Under a spectral condition on M, traits converge: within each essential communication class of the network, all dynasties reach a common consensus; dynasties in inessential classes converge to convex combinations of these consensuses (Propositions 4–5 and Theorem 1).

The second set—the paper's central contribution—characterizes the planner's constraint set and instrument structure. Long-run traits are convex combinations of initial traits weighted by eigenvector centrality, so complete cultural homogenization is impossible: the planner cannot engineer convergence to a target outside the convex hull of inherited traits (Theorem 1 and Proposition 7, part c). Moreover, convergence under strategic parental socialization is strictly slower than under the baseline DeGroot process, as the second-largest eigenvalue of M strictly exceeds that of Ψ (Proposition 6). Under uniform incentive pressure within essential classes (Assumption 2), the planner's instruments decouple: schooling effectiveness ψ determines the long-run consensus within the convex hull, while mobility incentives $\boldsymbol{\pi}$ govern only the speed of convergence (Proposition 7). Together, the impossibility theorem and the instrument separation constitute the paper's main contribution: they show that a planner inheriting a heterogeneous population faces both an inescapable constraint and a clean decomposition of the best achievable policy within it.

The third set derives the planner's optimal policy. When mobility incentives are dynasty-specific and budget-constrained, the optimal allocation targets dynasties whose cultural positions load most heavily on the convergence bottleneck—the second eigenvector of M (Proposition 8).

Taken together, these results render the impossibility constructive: the instrument separation characterizes not only the planner's constraint but the best the planner can do within it—schooling steers the achievable consensus, mobility incentives accelerate convergence toward it, and optimal

targeting identifies which dynasties to prioritize under a budget constraint. The framework is applied to three phases of East German cultural dynamics: selective emigration before the Wall (which narrowed the planner's feasible set), four decades of closed-network indoctrination that failed to achieve homogenization, and post-reunification network opening, where the model predicts the slow, asymmetric convergence to democratic norms documented by Mau (2024).

Related Literature

This paper contributes to three strands of the literature. The first is the theory of cultural transmission, pioneered by Boyd and Richerson (1985) and formalized in the economic framework of Bisin and Verdier (2001). In Bisin and Verdier (2001), parents with imperfect empathy invest in vertical socialization to counteract horizontal influence, generating long-run cultural heterogeneity through a mechanism of cultural substitution. Tabellini (2008) identifies conditions under which strategic complementarity in socialization—the opposite force—drives convergence to cultural homogeneity; the present model generates cultural substitution through the displayed-trait mechanism, but the impossibility result ultimately rests on the convex-hull constraint imposed by the network's essential-class structure. The most closely related paper is Buechel, Hellmann, and Pichler (2014), who embed strategic cultural display in a network-based DeGroot model with continuous traits and overlapping generations, and establish convergence of the resulting dynamics.[2] The present paper adopts this framework and adds a central planner with two instruments (schooling and mobility incentives). In the quadratic version of the model, this planner layer generates the modified transition matrix M and yields four results absent from Buechel, Hellmann, and Pichler (2014): an impossibility theorem, a spectral speed comparison, an instrument-separation result, and an optimal targeting result showing that budget-constrained mobility incentives should be concentrated on dynasties with the largest loadings on the second eigenvector of the transition matrix. The targeting result complements Almagro and Andrés-Cerezo (2020), who study a forward-looking cultural leader allocating fixed resources for nation-building in a setting where the zero-sum character of the conflict pushes dynamics toward homogeneous steady states; in the present framework, by contrast, the convex-hull constraint

[2] See also Buechel, Hellmann, and Klößner (2015) for a related network model of opinion formation with continuous traits, Spiro (2020) for a model of multigenerational cultural transmission, and Gradstein and Justman (2002) on how public education promotes cultural cohesion. Bisin and Verdier (2025) provide a comprehensive survey of the current literature.

binds and full homogenization remains impossible. Meng (2025) studies optimal network intervention when a planner targets social norms in a static setting; the present paper's targeting result operates within a dynamic intergenerational framework where the impossibility constraint binds and the planner must allocate across both a steering and an acceleration instrument. Panebianco (2014) studies intergenerational trait transmission on networks but without strategic display; Panebianco and Verdier (2017) extend Bisin-Verdier to random networks with paternalistic parents. Verdier and Zenou (2018) introduce a forward-looking cultural leader who engages in oblique socialization to influence assimilation dynamics, but the leader does not interact with a full network game in displayed traits. Prummer and Siedlarek (2017) study cultural leaders who preserve minority traits through community norms on networks; the present paper shares their focus on network-embedded cultural leadership but centres on a planner seeking homogenization rather than preservation. Della Lena and Panebianco (2021) add incomplete information to the Bisin-Verdier framework, showing how group-level misrepresentation affects cultural dynamics. In a complementary direction, Doepke and Zilibotti (2017) enrich the parental side of cultural transmission by endogenizing parenting styles in the development of an entrepreneurial culture during industrialization. In contrast, the present paper enriches the societal side by introducing networks and a planner.

The second strand is the DeGroot model of social learning and its extensions. DeMarzo, Vayanos, and Zwiebel (2003) study persuasion bias in a DeGroot framework and show that influence is determined by eigenvector centrality. Golub and Jackson (2010a) establish conditions for asymptotic learning, and Golub and Jackson (2012) show that the spectral gap of the socialization matrix governs the speed of convergence and best-response dynamics—the latter result is the direct benchmark for the speed comparison in this paper. The present paper departs from this literature by introducing *strategic* agents who choose displayed opinions that differ from their true beliefs, and by adding an intergenerational dimension with a central planner. The resulting modified transition matrix M has richer spectral properties than the standard DeGroot matrix Ψ, and the comparison of their eigenvalues yields the convergence-speed result. Here, Battigalli, Panebianco, and Pin (2023) study self-confirming equilibria when agents misperceive the network structure, showing that strategic misperceptions can sustain non-Nash outcomes—an issue related to the persistent deviation between displayed and true traits in the present framework. Relatedly, Amir et al. (2025) address the

robustness of DeGroot dynamics to stubborn agents, which connects to the planner's role as a persistent influence on network weights.[3]

The third strand concerns the political economy of regimes and the persistence of cultural traits under institutional change. The model speaks to the literature on communist legacies (Pop-Eleches and Tucker 2017; Darden and Grzymala-Busse 2006), the role of religious institutions as bulwarks against indoctrination (Wittenberg 2006; Grzymala-Busse 2015), and work examining how states attempt to shape beliefs and homogenize populations through propaganda, censorship, and other instruments of ideological influence (Zhuravskaya, Guriev, and Markevich 2024; Greif and Tadelis 2010). It also relates to research on nation-building policies, such as the use of mass education to foster cultural homogenization across both democratic and authoritarian regimes (Alesina, Giuliano, and Reich 2021). The impossibility result—showing that a planner cannot move cultural traits outside the convex hull of initial beliefs—provides a theoretical foundation for the empirical finding that regime change does not erase pre-existing cultural heterogeneity (Voigtländer and Voth 2015; Bursztyn and Cantoni 2014; Luttmer and Singhal 2011), and formalizes the identification challenge raised by Becker, Mergele, and Woessmann (2020), who show that pre-existing regional differences account for part of the observed East-West gap in Germany. On the empirical side, Bursztyn, Egorov, and Fiorin (2020) provide experimental evidence that individuals strategically conceal or reveal social attitudes depending on perceived norms, validating the displayed-trait mechanism at the core of the model. Cantoni, Chen, et al. (2017) establish a causal effect of school curricula on political beliefs in China, and Fuchs-Schündeln and Masella (2016) provide analogous evidence for the GDR, showing that additional years of socialist schooling causally shifted political preferences—both supporting the schooling instrument. Satyanath, Voigtländer, and Voth (2017) show that denser civic networks accelerated ideological diffusion in Weimar Germany, consistent with the role of network structure in the model's convergence results. In the same setting, Buggle, Deter, and Lange (2025) show that Protestant pastor networks—not geographic proximity—drove protest diffusion across GDR cities during the 1989 Peaceful Revolution, providing causal evidence that network centrality determines political influence.

[3] Other extensions of the DeGroot framework include Jadbabaie, Molavi, and Tahbaz-Salehi (2013) on repeated information arrival, Bohren (2016) on informational herding with misspecified beliefs, and Förster, Mauleon, and Vannetelbosch (2014) on information manipulation.

Outline

Section 2 presents the model and Section 3 states the results. Section 4 develops a historical application to the East German transition from fascism to state socialism. Section 5 concludes.

2 The Model

2.1 Setup

Consider an overlapping generations economy with a fixed set of n dynasties, $N = \{1, \ldots, n\}$, where each dynasty is represented by a single adult in each period.[4] At the start of each period $t \in \mathbb{N}$, each adult produces one offspring, ensuring a constant population. Assume that this society has recently undergone a regime change that resulted in a fundamental shift in its incentive structure and ideology, so that most dynasties were socialized under the old system and carry its cultural imprint into the new regime.

Each adult is characterized by a continuous cultural trait $\phi_i(t) \in \mathcal{I}$, where $\mathcal{I} \subseteq \mathbb{R}$ is a convex compact set containing all feasible trait intensities. The trait captures any persistent cultural characteristic—such as political ideology, risk attitudes, or social norms—that is transmitted intergenerationally. Its continuous nature represents different intensities or discount factors: if adherence to the new regime's ideology is represented by a discount factor, then $\mathcal{I} = [0, 1]$. Low values of $\phi_i(t)$ correspond to weaker adherence to the prevailing ideology (indicating orientation toward the old regime), while high values correspond to stronger adherence to the new one. Traits acquired during the formative years become a permanent part of a dynasty's personality by adulthood; studies in political socialization indicate that children are more susceptible to ideological influence than adults (Krosnick and Alwin 1989; Osborne, Sears, and Valentino 2011), and that the total number of years spent under a regime has a cumulative effect (Pop-Eleches and Tucker 2017). The model therefore focuses on how these traits are transmitted across generations rather than on within-lifetime learning.[5]

[4] The terms "dynasty" and "family" are used interchangeably throughout.

[5] The scalar trait is adopted for analytical tractability. The analysis extends to k-dimensional traits $\Phi \in \mathbb{R}^{n \times k}$ by applying the transmission technology dimension-by-dimension. If parental attachment β_i and mobility incentives π_i are dimension-specific, different cultural traits converge at different rates—consistent with the empirical observation that ethnocentric attitudes persist longer than economic preferences (Decker 2016; Alesina and Fuchs-Schündeln 2007; Elsayed et al. 2025).

Displayed versus true traits. In the standard DeGroot framework, agents report their beliefs truthfully (DeMarzo, Vayanos, and Zwiebel 2003). However, in many political environments—particularly authoritarian regimes—social advancement depends on overtly displayed and possibly dishonest allegiance to the ruling ideology.[6] To capture this, assume that true traits are unobservable to outsiders and that each adult selects a publicly *displayed* trait $\phi_i^d(t) \in \mathcal{I}$ that is observable by all other members of society.[7] Here, dishonesty can have various facets—strategic considerations about social mobility being one—but they share the commonality that different cultural characteristics are associated with different socioeconomic choices. An individual who openly rejects the majority opinion will have less access to influential positions; the displayed trait therefore represents an observable measure of loyalty that is crucial for socioeconomic advancement. The displayed trait may differ from the true trait: $\phi_i^d(t) \neq \phi_i(t)$. This discrepancy creates cognitive dissonance, which is costly and increasing in its magnitude (Festinger 1957; Bernheim 1994). Thus, any deviation from the true trait intensity reduces parental utility, with the cost increasing as the extent of cognitive dissonance increases. The distinction between true and displayed traits is the key departure from the standard DeGroot framework and the source of strategic behavior in the model.

Let $\Phi^d(t) := (\phi_1^d(t), \ldots, \phi_n^d(t))^\top \in \mathcal{I}^n$ denote the vector of all displayed traits. Since children can only observe the adults' publicly displayed actions, which coincide with their observable socioeconomic choices, the displayed trait becomes the relevant object for cultural transmission.

Trait formation. Children acquire their traits from the observable behavior of adults through two channels: *vertical* (parental) socialization and *horizontal* (network) socialization. While the former refers to the direct parental transmission of traits, the latter captures the influence of the social environment, i.e., the network of other dynasties with which the child interacts.[8] Let $\sigma_{ii} \in (0, 1]$ denote the weight of parental socialization, capturing the time and effort parents invest in their children's upbringing relative to societal influences. The child's trait is formed as:

$$\phi_i(t+1) = \sigma_{ii}\, \phi_i^d(t) + (1 - \sigma_{ii})\, \phi_{Ni}^d(t) \tag{1}$$

[6] See the preference falsification framework developed by Kuran (1995), and Solga (2006) for evidence from the case of East Germany.

[7] The assumption of strategic misrepresentation follows Buechel, Hellmann, and Pichler (2014).

[8] Reflecting the common proverb that "it takes a village to raise a child," highlighting the role of the broader social environment in shaping children's traits.

where $\phi^d_{Ni}(t)$ denotes the average displayed trait in dynasty i's social environment. The subscripts indicate that children are socialized by their individual social environment, resulting in different social networks influencing their trait formation and, thus, geographic variation in traits. Parental misrepresentation of traits therefore directly affects the formation of genuine cultural characteristics in the next generation: if parents strongly support the prevailing ideology, their children are more likely to adopt traits consistent with it; if parents are critical, their children may develop opposing traits.

2.2 The central planner

A central planner seeks to build a culturally homogeneous society aligned with the new regime's ideology, but faces a numerically overwhelming mass of dynasties whose traits were shaped by the old system. To effect change, the planner operates two policy instruments. The first is a *centralized schooling system* with an effectiveness parameter $\psi \in (0, 1]$, which interferes with parental socialization by reducing its success probability from σ_{ii} to $\psi_{ii} \coloneqq \sigma_{ii} - \psi \geq 0$. Higher ψ corresponds to more effective institutional indoctrination and a correspondingly lower probability of successful parental transmission. The remaining horizontal socialization success component is $1 - \psi_{ii} = 1 - (\sigma_{ii} - \psi) = 1 - \sigma_{ii} + \psi$, where σ_{ii} accounts for the parental success component and ψ captures the additional socialization accruing to collective institutions. The trait formation process under centralized schooling becomes:

$$\phi_i(t+1) = \psi_{ii}\, \phi^d_i(t) + (1 - \psi_{ii})\, \phi^d_{Ni}(t) \tag{2}$$

When $\psi = 0$, parental socialization is unimpeded. When $\psi = 1$, the school system fully determines the traits of children not socialized by their parents and negatively affects the likelihood of children with oppositional parents adopting oppositional traits. When $\psi_{ii} = 0$ (i.e., $\sigma_{ii} = \psi$), the centralized institution fully supplants parental influence and the child's trait is determined entirely by the social environment. Higher ψ therefore corresponds to a redistribution of socialization time in favor of collective institutions, during which children are continuously exposed to the regime's preferred values. The cost of implementing these interventions is abstracted from the analysis, and fiscal considerations are therefore set aside.[9]

[9] For analytical tractability, the cost of institutional interventions is not modeled explicitly, so that the planner chooses the intensity of institutional socialization directly. This reduced-form treatment follows the standard approach in

The social environment is formalized through a weighted, directed network. Let $\Sigma = (\sigma_{ij})$ be an $n \times n$ row-stochastic baseline socialization matrix, with σ_{ii} representing parental socialization and σ_{ij} $(j \neq i)$ capturing horizontal peer influence. The planner's schooling instrument $\psi \in [0, \min_i \sigma_{ii}]$ reduces each dynasty's parental weight and redistributes it proportionally among existing peer contacts, yielding the *network transmission matrix* $\Psi = (\psi_{ij})$:

$$\psi_{ii} = \sigma_{ii} - \psi, \qquad \psi_{ij} = \sigma_{ij} \frac{1 - \sigma_{ii} + \psi}{1 - \sigma_{ii}} \text{ for } j \neq i \qquad (\sigma_{ii} < 1) \tag{3}$$

When $\sigma_{ii} = 1$ (no peer exposure), the school cannot redistribute socialization weights and $\psi_{ii} = 1$.[10] The matrix Ψ is row-stochastic for all i. When $\psi = 0$, $\Psi = \Sigma$. The rescaling preserves two natural properties: row-stochasticity (socialization weights sum to one) and proportional redistribution (the weight displaced from parental influence is allocated to existing peer contacts in proportion to their baseline weights, rather than creating new network connections). Any rescaling satisfying these two properties yields the same qualitative results; the key requirement is that schooling reduces ψ_{ii} without altering the relative weights among peer contacts—consistent with the institutional reality that state schooling supplements the existing social environment rather than replacing it.[11] The entry ψ_{ij} captures the relative cognitive influence of dynasty j's displayed trait on dynasty i's child, reflecting exposure to role models through neighborhood interaction, school networks, or peer groups. The trait formation process for the entire society reduces to the matrix equation:

$$\Phi(t+1) = \Psi \, \Phi^d(t) \tag{4}$$

where Ψ describes a matrix of transition probabilities of a Markov chain with n states, from time t to $t+1$. The block structure of Ψ determines which subgroups of society will reach consensus in the long run, with dynasties within closed and strongly connected groups converging to a shared trait (DeMarzo, Vayanos, and Zwiebel 2003; Golub and Jackson 2010b).

models of cultural transmission, where institutional influences enter the transmission technology rather than arising from a separate budget-constrained optimization problem.

[10] Singleton essential classes satisfy $\sigma_{ii} = 1$ by closure. The school operates through peer exposure: with no network contacts, the displaced parental weight cannot be redirected to peers, so the dynasty is immune to schooling.

[11] In the GDR, the centralized schooling system operated through institutions embedded in children's existing social networks—youth organizations (FDJ), school classes, paramilitary training (GST)—rather than by isolating children from their network peers. The proportional redistribution in (3) reflects this institutional design.

2.3 Parental utility and optimization

Each adult $i \in N$ chooses a displayed trait $\phi_i^d(t) \in \mathcal{I}$ to maximize an additively separable utility function with two components: the cost of cognitive dissonance from misrepresenting one's true trait, and an intergenerational component reflecting the parent's preference over the child's adopted trait.

Assumption 1 (Parental utility function): *Let the utility function of an adult $i \in N$ at time $t \in \mathbb{N}$ be given by:*

$$u_i(\phi_i^d(t) \mid \phi_i(t)) + v_i(\phi_i(t+1) \mid \phi_i(t)) \tag{5}$$

where:

(a) $u_i(\cdot \mid \phi_i(t))$ is single-peaked at $\phi_i(t)$: strictly increasing for $\phi_i^d(t) < \phi_i(t)$ and strictly decreasing for $\phi_i^d(t) > \phi_i(t)$.

(b) $v_i(\cdot \mid \phi_i(t))$ is single-peaked at $\phi_i(t)$: strictly increasing for $\phi_i(t+1) < \phi_i(t)$ and strictly decreasing for $\phi_i(t+1) > \phi_i(t)$.

(c) $u_i(\cdot \mid \phi_i(t))$ and $v_i(\cdot \mid \phi_i(t))$ are continuous, twice continuously differentiable, and strictly concave.

The utility function differs from the standard DeGroot model, but its additive separability and the assumptions above make it analytically tractable and economically interpretable. Part a captures cognitive dissonance: utility decreases as the displayed trait diverges from the true trait, so that actions inconsistent with beliefs produce a utility loss. Part b captures imperfect empathy in the sense of Bisin and Verdier (2001): parents evaluate their child's future trait using their own preferences, and suffer a utility loss when the child's adopted trait differs from theirs. This admits two possible interpretations. The first is that children make independent socioeconomic choices and parents evaluate only these choices; in this view, children maximize their own utility share v_i. The second—more natural in the context of a planner striving for cultural homogeneity—is that parents suffer a utility loss if their child's adopted trait differs too much from their own, so that parents prefer their children to be exact copies of themselves. Part c ensures analytical tractability through standard regularity conditions. Finally, the choice of publicly displayed trait affects not only an adult's own child but also the children of all affiliated dynasties through the network.

2.4 The adult's decision

The parental optimization problem is:

$$\max_{\phi_i^d(t)\in\mathcal{I}} u_i(\phi_i^d(t) \mid \phi_i(t)) + v_i(\phi_i(t+1) \mid \phi_i(t)) \quad \text{s.t.} \quad \phi_i(t+1) = \psi_{ii}\,\phi_i^d(t) + (1-\psi_{ii})\,\phi_{Ni}^d(t) \tag{6}$$

The parent faces a trade-off between two components of utility loss. The first is the cognitive dissonance from deviating from one's true trait (via u_i). The second is the intergenerational cost of the child adopting traits that differ from the parent's own (via v_i). In economic terms, the first component captures the cost of forgoing social advancement associated with privileged access to scarce goods and networks, while the second reflects the family's socioeconomic gains from the child adopting traits aligned with the prevailing ideology. By strategically misrepresenting the displayed trait, parents can partially offset the horizontal socialization influence of the network on their children, at the cost of increased cognitive dissonance. The deviation from the true trait thus measures the parental socialization incentives—or disincentives—that parents are willing to accept for their intergenerational benefit. These correspond to the cultural substitution between vertical and horizontal socialization channels first identified by Bisin and Verdier (2001). The solution to this optimization problem yields the optimal displayed trait as a best response to both the displayed traits in the social environment $\phi_{Ni}^d(t)$ and the parent's true trait $\phi_i(t)$.

3 Results

3.1 Best responses and Nash equilibrium

Proposition 1 (Parental best responses): [12] *For any adult $i \in N$ and any trait $\phi_i(t) \in \mathcal{I}$, there exists a unique best-response displayed trait $\phi_i^{d*}(\phi_i(t), \phi_{Ni}^d(t))$ satisfying:*

(a) If $\psi_{ii} \notin (0,1)$, $\phi_{Ni}^d(t) = \phi_i(t)$, or $\phi_i(t)$ is on the boundary of $\mathcal{I}$, then $\phi_i^{d*} = \phi_i(t)$.

(b) If $\psi_{ii} \in (0,1)$, $\phi_{Ni}^d(t) > \phi_i(t)$, and $\phi_i(t)$ is in the interior of $\mathcal{I}$, then $\phi_i^{d*} < \phi_i(t)$.

(c) If $\psi_{ii} \in (0,1)$ and at least one of the best responses to $\phi_{Ni}^d(t)$ or $\tilde{\phi}_{Ni}^d(t)$ is interior, then $\phi_i^{d*}(\phi_i(t), \tilde{\phi}_{Ni}^d(t)) > \phi_i^{d*}(\phi_i(t), \phi_{Ni}^d(t))$ if and only if $\tilde{\phi}_{Ni}^d(t) < \phi_{Ni}^d(t)$.

[12] This result generalises Proposition 1 of Buechel, Hellmann, and Pichler (2014) to the planner-augmented transmission matrix Ψ.

Part a establishes that parents display their true trait when (i) the schooling system fully indoctrinates or is absent ($\psi_{ii} \notin (0,1)$), (ii) the network already matches their own trait, or (iii) they are at the boundary of the trait space. In such cases, children become exact copies of their parents. Part b captures the central mechanism of cultural substitution: when the network's displayed trait exceeds the parent's true trait, the parent *underrepresents* to counteract the network's influence on the child. The deviation is always in the direction *opposite* to the network's influence. For example, individuals hostile to the prevailing regime may counteract institutional indoctrination by privately transmitting oppositional values, so that each successive generation is less affected by the regime's message compared to those without such countervailing parental influence (Grzymala-Busse 2015; Wittenberg 2006). Comparable dynamics apply to adherents of previous regimes who use their parental influence to deter their children from adopting the new ideology. Part c establishes monotone comparative statics: the distance between publicly displayed and true traits increases with the cultural distance between the parent and the social environment.

Proposition 2 (Nash equilibrium): [13] *Under Assumption 1, for every $t \in \mathbb{N}$, there exists a Nash equilibrium in displayed traits $\Phi^{d*}(t) = (\phi_1^{d*}(t), \ldots, \phi_n^{d*}(t))^\top$. Under the quadratic specification of Section 3.3, if Ψ is symmetric positive definite, the equilibrium is unique.*

Two-dynasty example. To build intuition, consider two dynasties $N = \{1, 2\}$ with $\phi_1(t) < \phi_2(t)$. By Proposition 1, parents' displayed traits are more extreme than their true traits: $\phi_1^{d*}(t) \leq \phi_1(t) < \phi_2(t) \leq \phi_2^{d*}(t)$. The children's traits evolve according to $\phi_1(t+1) = \psi_{11}\phi_1^d(t) + (1-\psi_{11})\phi_2^d(t)$ and $\phi_2(t+1) = \psi_{22}\phi_2^d(t) + (1-\psi_{22})\phi_1^d(t)$. Each dynasty attempts to offset the influence of the other through investment in socialization. At the Nash equilibrium, the ordering $\phi_1(t+1) < \phi_2(t+1)$ is preserved if and only if $\psi_{11} + \psi_{22} \geq 1$, i.e., when children are primarily socialized by their parents. When $\psi_{ii} = 1$ (full parental self-weight, as when $\psi = 0$ and $\sigma_{ii} = 1$), children mirror their parents exactly. When $\psi_{11} + \psi_{22} < 1$—that is, when societal influence dominates parental socialization—the relative positions alternate each generation and the sign of the trait difference changes each period: $\text{sign}(\phi_2(t+1) - \phi_1(t+1)) = -\text{sign}(\phi_2(t) - \phi_1(t))$.

[13] Buechel, Hellmann, and Pichler (2014) establish existence (their Proposition 2) under the general assumptions of Assumption 1. Uniqueness under the quadratic specification and positive definiteness is proved in Appendix A via the linear first-order conditions.

3.2 Social network and steady states

The interaction structure of Ψ partitions society into communication classes, and heterogeneous traits may persist across these groups in the long run, even if convergence to a homogeneous trait is achieved within individual groups. The block structure of Ψ therefore determines which subgroups of society will reach consensus and which will remain heterogeneous. Following DeMarzo, Vayanos, and Zwiebel (2003) and Jackson (2008), two dynasties i and j *communicate* ($i \backsim j$) if there exist directed paths from i to j and from j to i in the network. This communication relation induces a partition of N into equivalence classes, each representing the communication behavior of a group of dynasties toward the rest of society.

Definition 1 (Communication classes): *Let $\mathcal{P}(\Psi) = \{L_1, \ldots, L_K, \mathcal{R}\}$ be a partition of N into $K \geq 1$ communication classes and a (possibly empty) residual set $\mathcal{R}$, such that:*

(a) Each class L_k is strongly connected*: for all $i, j \in L_k$, there exists an integer $m \geq 1$ such that $(\Psi^m)_{ij} > 0$.*

(b) Each class L_k is closed*: for all $i \in L_k$, $\psi_{ij} > 0$ implies $j \in L_k$.*

(c) Classes that are both strongly connected and closed are called essential.[14]

(d) $\mathcal{R} = N \setminus \bigcup_{k=1}^{K} L_k$ contains all inessential *dynasties.*

By suitable renumbering, Ψ admits the block structure:

$$\Psi = \begin{pmatrix} \Psi_{11} & \mathbf{0} & \cdots & \cdots & \mathbf{0} \\ \mathbf{0} & \ddots & \ddots & & \vdots \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ \mathbf{0} & \cdots & \mathbf{0} & \Psi_{KK} & \mathbf{0} \\ \Psi_{\mathcal{R}1} & \cdots & \cdots & \Psi_{\mathcal{R}K} & \Psi_{\mathcal{R}\mathcal{R}} \end{pmatrix} \tag{7}$$

where Ψ_{kk} is the submatrix corresponding to essential class L_k and $\Psi_{\mathcal{R}\mathcal{R}}$ corresponds to $\mathcal{R}$. Essential classes influence $\mathcal{R}$ (via $\Psi_{\mathcal{R}k}$) but are not influenced in return.

Proposition 3 (Steady states): [15] *Let $\Phi(t) \in \mathcal{I}^n$ be a steady state, i.e., $\Phi(t+1) = \Phi(t)$. Then:*

[14] The designation of $\mathcal{R}$ as the "rest of the world" (ROTW) follows Buechel, Hellmann, and Klößner (2015).

[15] Cf. Proposition 3 of Buechel, Hellmann, and Pichler (2014); the result extends directly to the planner-augmented matrix.

(a) Parents display their true traits: $\Phi^{d*}(t) = \Phi(t)$.

(b) Traits within each essential class coincide: $\phi_i(t) = \phi_j(t)$ *for all* $i, j \in L$, *for each* $L \in \mathcal{P}(\Psi)$.

(c) Traits in inessential classes are convex combinations of traits in the essential classes to which they are connected.

In a steady state, Part a implies that children inherit the same trait as their parents and that the centralized schooling system is ineffective at changing the distribution. If two dynasties are related and form an essential communication class, their children share the same trait in any steady state (Part b). The traits of inessential dynasties are determined by—but do not influence—those of the essential classes (Part c).

Dynasties in essential communication classes that interact regularly thus form a guiding political culture ("Leitkultur") (Golub and Sadler 2016). Their influence on $\mathcal{R}$ is unidirectional: essential classes shape the ROTW but are not shaped by it. Once these groups receive attention from dynasties in $\mathcal{R}$, they gain significant influence over other actors in the network. Even minor differences in the network weights can lead to significant deviations from the asymptotic predictions, particularly when inward-looking groups—initially ignored by the ROTW—start to have an increasing impact over time. This finding is consistent with existing research on political and academic persuasion (DeMarzo, Vayanos, and Zwiebel 2003).

3.3 Convergence

To establish the conditions under which traits converge to a stable long-run equilibrium, it is necessary to clearly separate the role of the social network from that of imperfect empathy. Following Section 4.2 of Buechel, Hellmann, and Pichler (2014), the utility components are specified as quadratic loss functions; the extension here is the addition of dynasty-specific mobility incentives π_i:

$$u_i(\phi_i^d(t) \mid \phi_i(t)) \coloneqq -\alpha_i\big(\phi_i^d(t) - \phi_i(t)\big)^2, \qquad v_i(\phi_i(t+1) \mid \phi_i(t)) \coloneqq -\beta_i\big(\phi_i(t+1) - \phi_i(t)\big)^2 \tag{8}$$

The parameter α_i represents the cost parents incur when their displayed trait deviates from their true trait, while β_i measures the strength of the parental socialization incentives. The planner's second instrument, *mobility incentives* $\pi_i \geq 0$, enters by reducing dynasty i's intergenerational utility weight from β_i to $(\beta_i - \pi_i)$. The dynasty-specific nature of π_i reflects the planner's ability to target incentives

differentially—for example, by offering greater career advancement opportunities to dynasties whose cultural transformation is most valuable.[16] In practice, the planner provides preferential access to resources and career advancement for those who display loyalty, thereby reducing the utility cost parents experience when their child's trait diverges from their own. Setting $\alpha_i = 1$ for normalization and allowing β_i to reflect the relative weight of parental imperfect empathy, the parental optimization problem becomes:

$$\min_{\phi_i^d(t)\in\mathcal{I}} \frac{1}{2}\big(\phi_i^d(t) - \phi_i(t)\big)^2 + \frac{1}{2}(\beta_i - \pi_i)\big(\phi_i(t+1) - \phi_i(t)\big)^2 \tag{9}$$

where $(\beta_i - \pi_i) > 0$ for all $i \in N$ ensures that socialization incentives remain positive. The assumption that $(\beta_i - \pi_i) > 0$ is critical: in the extreme scenario where $(\beta_i - \pi_i) = 0$, there are no socialization incentives, and the model reduces to a pure coordination game where each individual's displayed trait coincides with their true trait.

Under quadratic utility, the Nash equilibrium displayed traits are linear in the trait vector: $\Phi^{d*}(t) = (I + \Pi\Psi)^{-1}(I + \Pi)\,\Phi(t)$, where Π is a diagonal matrix with diagonal entries $(\beta_i - \pi_i)\psi_{ii}$.[17] The linearity of M is a direct consequence of the quadratic utility specification. The law of motion for traits becomes $\Phi(t+1) = M\,\Phi(t)$, where:

$$M \coloneqq \Psi(I + \Pi\Psi)^{-1}(I + \Pi) \qquad \Longrightarrow \qquad \Phi(t) = M^t\,\Phi(0) \tag{10}$$

The transformation from $\Phi(t)$ to $\Phi(t+1)$ via M is independent of the current trait vector, so the dynamics are fully described by the power sequence M^t. The matrix M generalizes the standard DeGroot transition matrix: when $\pi_i = \beta_i$ for all i (no socialization incentives), strategic incentives vanish and $M = \Psi$, reducing the model to the standard DeGroot process (DeGroot 1974). The comparison of the spectral properties of M and Ψ—in particular, how the planner's instruments alter the eigenvalue structure—is the basis for the convergence and speed results that follow. The convergence conditions depend on four components: the social network Ψ (comprising parental

[16] The concept draws on Henrich (1989)'s characterization of the GDR as a "custodial dictatorship" (*vormundschaftlicher Staat*), in which the state actively manipulated citizens' life opportunities to reshape intergenerational preferences.

[17] The invertibility of $(I + \Pi\Psi)$ is guaranteed when Ψ is symmetric positive definite. To see this, note that $(I + \Pi\Psi)$ and $(I + \Pi^{1/2}\Psi\Pi^{1/2})$ share the same eigenvalues. Since Ψ is symmetric and $\Pi \geq 0$, $\Pi^{1/2}\Psi\Pi^{1/2}$ is positive semidefinite, so $\Pi\Psi$ has nonnegative real eigenvalues and all eigenvalues of $(I + \Pi\Psi)$ are nonzero.

socialization success σ_{ii} minus schooling effectiveness ψ), the imperfect-empathic preferences β_i, and the planner's mobility incentives π_i.

Assumption 2 (Uniform net incentive pressure): *For each essential class $L_k \in \mathcal{P}(\Psi)$, the net incentive parameter $(\beta_i - \pi_i)\psi_{ii}$ is constant across all $i \in L_k$.*

Assumption 2 requires the planner to apply uniform incentive pressure within each essential class. Since essential classes are tightly connected groups whose members cannot be distinguished by their network position alone—they form a single strongly connected component—this is a natural informational constraint: the planner targets groups, not individuals within groups. The assumption is automatically satisfied in the baseline model of Buechel, Hellmann, and Pichler (2014), where $\pi_i = 0$ for all i, provided $\beta_i\psi_{ii}$ is constant within each essential class. It is also satisfied whenever each essential class is a singleton.

Remark 1 (Relation to Buechel, Hellmann, and Pichler (2014)): *When $\psi = 0$ and $\boldsymbol{\pi} = \mathbf{0}$, the planner is absent and $M = \Psi$, recovering the baseline framework. Assumption 2 is automatically satisfied in this case, since $(\beta_i - \pi_i)\psi_{ii} = \beta_i\sigma_{ii}$ and homogeneity within essential classes is a standard regularity condition. The planner layer generates four results with no analogue in the baseline model: (i) the impossibility theorem (Theorem 1 combined with Proposition 7c), which requires a* target *to be impossible—without a planner there is no target; (ii) the spectral speed comparison (Proposition 6), which requires comparing M and Ψ—objects that coincide when $\boldsymbol{\pi} = \mathbf{0}$; (iii) the instrument separation (Proposition 7), which arises only when two distinct policy instruments interact; and (iv) the optimal targeting result (Proposition 8), which requires the planner's budget constraint. All four results depend on the spectral properties of M relative to Ψ.*

Proposition 4 (Convergence I): *Under the parental optimization problem* (9)[18]*:*

(a) If Ψ is symmetric positive definite, then for any $(\beta_i - \pi_i) \in \mathbb{R}^n_+$, the sequence $\{M^t\,\Phi(0)\}_{t\to\infty}$ converges for arbitrary $\Phi(0)$.

(b) If Ψ has an eigenvalue λ satisfying $\Re(\lambda) < |\lambda|^2$, then there exists $(\beta_i - \pi_i) \in \mathbb{R}^n_+$ such that $\rho(M) > 1$, and the sequence diverges.

[18] Generalises Proposition 4 of Buechel, Hellmann, and Pichler (2014), replacing the socialization-incentive parameter β_i with the net parameter $(\beta_i - \pi_i)$.

Corollary 1: *Convergence for arbitrary* $(\beta_i - \pi_i) \in \mathbb{R}^n_+$ *is achieved if and only if* Ψ *is symmetric positive semidefinite.*

Part a establishes the sufficient condition for convergence: when parental socialization weights dominate horizontal influence (positive definiteness of Ψ), all eigenvalues of M lie in $(0, 1]$ and traits converge regardless of initial conditions. This holds whenever the diagonal entries of Ψ—the parental socialization success shares—exceed the off-diagonal entries. Part b shows that when the network allows eigenvalues with large imaginary parts relative to their real parts, excessive parental investment in socialization can generate oscillatory, divergent dynamics. This scenario is plausible in a pervasive authoritarian regime where parents have less influence on their children than the social environment, leading to overshooting of cultural traits across generations.[19]

The Corollary establishes the sharp characterization: convergence obtains if and only if Ψ is symmetric positive semidefinite. Convergence fails when imperfect-empathic preferences are too strong and the eigenvalue condition is violated—that is, when the intergenerational utility is significantly prioritized, causing traits to oscillate rather than settle. Importantly, convergence within essential classes does not guarantee convergence in $\mathcal{R}$, since eigenvalues of $M_{\mathcal{R}\mathcal{R}}$ may exceed 1. The following proposition establishes conditions under which the entire society converges:

Proposition 5 (Convergence II): [20] *Let* Ψ *be irreducible with strictly positive diagonal elements. Then there exists a neighborhood* $\mathcal{N}(\mathbf{0} \mid \Psi) \subset \mathbb{R}^n_+$ *such that for any* $(\beta_i - \pi_i) \in \mathcal{N}(\mathbf{0} \mid \Psi) \cup \{\mathbf{0}\}$*, the sequence* $\{M^t \, \Phi(0)\}_{t\to\infty}$ *converges for arbitrary* $\Phi(0)$*.*

Proposition 5 establishes that convergence depends not only on initial conditions but also on the network structure. The irreducibility of Ψ ensures that information and opinions can spread across the entire society, while the strictly positive diagonal elements guarantee that each dynasty retains non-zero self-influence, preventing the consensus state from being dominated by any single dynasty. If the diagonals of Ψ are strictly positive, then Ψ has a simple Perron–Frobenius eigenvalue of 1 with all other eigenvalues in $(0, 1)$. Since eigenvalues are continuous functions of matrix entries, Ψ can be perturbed to ensure that M^t also has spectral radius 1 with all other eigenvalues in $(0, 1)$, implying convergence.

[19] If $\phi_i(t+1) = \Psi^t \phi_i(t)$, true traits $\phi_i(t)$ converge if and only if Ψ^t converges. Additionally, displayed traits $\phi^d_i(t)$ converge if and only if true traits converge. If both converge, the limits coincide.

[20] Generalises Proposition 5 of Buechel, Hellmann, and Pichler (2014) to the planner-augmented setting.

Taken together, Propositions 4 and 5 provide conditions for convergence and consensus while highlighting the importance of the social network structure as a strongly connected graph. The planner achieves convergence through sufficiently large mobility incentives—which reduce parental utility loss from having a child that is culturally distant—and an effective schooling system that tips the balance of socialization toward collective influence. The consensus is determined by the linear combinations of initial traits weighted by the network centrality of the respective dynasties (Golub and Sadler 2016).

3.4 Long-run traits

Upon establishing the convergence conditions, the asymptotic behavior of traits can be fully characterized. Specifically, the long-run trait profile depends on each dynasty's initial trait, its position in the social network Ψ, and the planner's ability to alleviate the limitations of imperfect parental empathy. Under the convergence conditions of Propositions 4–5:

Theorem 1 (Asymptotic traits): *Let Ψ and M be organized as in (7). Under Assumption 2, for each essential class $L_k \in \mathcal{P}(\Psi)$, let $v_{|L_k}$ be the left eigenvector of Ψ_{kk} corresponding to its Perron–Frobenius eigenvalue, normalized so that $\sum_{i \in L_k} v_i = 1$. If M^t converges to $M^\infty := \lim_{t\to\infty} M^t$, then:*

(a) Within each essential class, all dynasties converge to a common consensus:

$$c_k := \phi_i(\infty) = \phi_j(\infty) = v'_{|L_k}\, \Phi(0)_{|L_k} \quad \text{for all } i, j \in L_k \tag{11}$$

(b) The long-run traits of dynasties in $\mathcal{R}$ are given by:

$$\Phi(\infty)_{|\mathcal{R}} = \Gamma\, \ell \tag{12}$$

where $\ell = (c_1, \ldots, c_K)'$ and $\Gamma := (I - \Psi_{\mathcal{R}\mathcal{R}})^{-1}(\Psi_{\mathcal{R}1}\mathbb{1}_{|L_1}, \ldots, \Psi_{\mathcal{R}K}\mathbb{1}_{|L_K})$ is row-stochastic.

Theorem 1 establishes that the long-run dynamics of true traits converge to $\Phi(\infty) = M^\infty\, \Phi(0)$, analogously to Theorem 10 in DeMarzo, Vayanos, and Zwiebel (2003). Each essential communication class L_k, being a tightly connected and self-contained group, eventually reaches a consensus c_k. The consensus is a weighted average of the initial traits of the class members, with weights given

by eigenvector centrality—the left eigenvector $v_{|L_k}$ of Ψ_{kk} corresponding to the Perron eigenvalue, normalized to sum to 1. The standardized eigenvector $v'_{|L_k}$ thus captures the effect of each dynasty's initial trait on the consensus of its essential class. Since Ψ is row-stochastic and each essential class L_k is closed, the submatrix Ψ_{kk} is itself row-stochastic with Perron eigenvalue 1. The normalized left eigenvector $v_{|L_k}$ is therefore a probability vector, so c_k is a convex combination of the initial traits in L_k—the source of the impossibility result.

For dynasties in $\mathcal{R}$, the long-run trait is governed by the weighted mean of the essential-class consensuses $c_1, \ldots, c_K$, with weights determined by the network structure but *not* by the level of parental imperfect empathy. Crucially, the initial traits of dynasties in $\mathcal{R}$ have no effect on the long-run trait profile: they ultimately obtain a weighted average of the traits displayed by dynasties in the essential classes and are therefore asymptotically irrelevant. When there is more than one essential communication class, dynasties in $\mathcal{R}$ may have different weighted averages of the essential-class consensuses and thus need not reach consensus among themselves—heterogeneity in the ROTW persists in the long run.

Remark 2 (Robustness of the convex-hull constraint): *The row-stochasticity of* Ψ *ensures that each generation's true traits are a weighted average of the previous generation's* displayed *traits:* $\Phi(t{+}1) \in \mathrm{conv}(\Phi^d(t))$. *This property holds regardless of the utility specification. Under the quadratic specification, the Nash equilibrium displayed traits are linear in true traits, so the composite one-period map* $\Phi(t) \mapsto \Phi(t{+}1) = M\Phi(t)$ *is itself row-stochastic and nonnegative, yielding the nested inclusion* $\mathrm{conv}(\Phi(t{+}1)) \subseteq \mathrm{conv}(\Phi(t))$ *and hence the long-run convex-hull constraint of Theorem 1. Under more general utility functions, best responses are typically nonlinear and displayed traits may be more extreme than true traits (Proposition 1b), so the composite map* $\Phi(t) \mapsto \Phi(t{+}1)$ *need not be affine or preserve the convex hull of initial traits; extending the impossibility result to the nonlinear setting would require additional structure on the best-response map. The spectral speed comparison (Proposition 6) and optimal targeting result (Proposition 8) likewise rely on the linear best-response structure of the quadratic specification.*

Remark 3 (Counterexample: additive-bias transmission): *The convex-hull constraint is not vacuous: it depends on the weighted-average structure of trait formation and fails under alternative transmission technologies. Replace* (4) *with an additive-bias technology in which the planner directly*

injects its preferred trait value:

$$\phi_i(t+1) = (1-\delta)\sum_{j\in N}\psi_{ij}\,\phi_j^d(t) + \delta\,\phi^* \tag{13}$$

where $\delta \in (0,1)$ *governs the intensity of the planner's ideological injection and* ϕ^* *is the target trait. This remains a convex combination, but the input set now includes* ϕ^**, which may lie outside* $\mathrm{conv}(\Phi(0))$*. Under this technology, the impossibility result breaks: long-run traits converge to a value incorporating* ϕ^**, and the planner can steer society beyond the inherited trait frontier. The distinction is that the planner in* (13) *acts as an independent cultural source—effectively, a new essential communication class with its own trait—rather than re-weighting existing transmission channels as in the baseline model. Theorem 1 therefore provides a sharp boundary: the impossibility is a property of the weighted-average (DeGroot) transmission technology, not of strategic interaction per se.*

3.5 Speed of convergence

Traits exhibit persistence over time due to network connections, albeit with diminishing differences between individuals as relative positions become more important. The speed of convergence varies depending on the trait being analyzed—childhood political socialization necessitates more generational turnover to converge than other attitudes. To disentangle the factors that influence convergence speed, the spectral properties of the transition matrices are compared. The convergence rate is determined by the second-largest eigenvalue in modulus.[21] Order eigenvalues by decreasing magnitude:

$$1 = |\lambda_1(\Psi)| \geq |\lambda_2(\Psi)| \geq \cdots \geq |\lambda_n(\Psi)|, \qquad 1 = |\lambda_1(M)| \geq |\lambda_2(M)| \geq \cdots \geq |\lambda_n(M)|.$$

Then:

Proposition 6 (Speed of convergence): *Let* Ψ *be irreducible, symmetric positive definite, and suppose* $(\beta_i - \pi_i)\psi_{ii} > 0$ *for all* $i \in N$*. Then*

$$|\lambda_k(M)| > |\lambda_k(\Psi)| \quad \textit{for all } 2 \leq k \leq n.$$

[21] Since Ψ is row-stochastic for all $\psi \in [0, \min_i \sigma_{ii}]$, the Perron–Frobenius eigenvalue is 1 and the second-largest eigenvalue governs the convergence rate.

In particular, $|\lambda_2(M)| > |\lambda_2(\Psi)|$*, so* M^t *converges strictly slower than* Ψ^t*. Moreover, as* $(\beta_i - \pi_i) \to 0$ *for all* i*,* $M \to \Psi$*, and the convergence rate approaches the baseline rate of* Ψ^t*.*

Proposition 6 establishes that strategic parental socialization is inherently self-limiting. When parents have no socialization incentives ($\beta_i = 0$), all dynasties publicly express their true traits, producing a society of truthful "brokers." However, when $\beta_i > 0$, dynasties invest in preserving their true convictions, and the second-largest eigenvalue of M strictly exceeds that of Ψ, slowing convergence. The planner can accelerate convergence by increasing mobility incentives π_i, which reduces dynasty i's effective parental weight $(\beta_i - \pi_i)$. Greater incentives for social mobility thus reduce imperfect-empathic preferences, preventing overshooting in traits and leading to faster convergence.[22]

However, the planner cannot alter the *target* of convergence: long-run traits remain within the convex hull of initial traits, weighted by eigenvector centrality (Theorem 1). The planner faces a fundamental dilemma: state intervention can expedite convergence to a new cultural equilibrium, but it simultaneously disrupts cultural change by diminishing the success of parental socialization, which can cause overshooting unless the diagonal elements of Ψ are strictly positive. The transition matrix M^t implies that long-run traits are a convex combination of initial traits, which means that an "engineering of consent" toward a cultural target outside this convex hull is impossible. Even with centralized indoctrination and increased social mobility incentives, a planner who inherits a population with heterogeneous priors cannot drive the society to an arbitrary cultural target—the limits of cultural engineering are determined by the initial distribution of traits and the network structure, not by the intensity of intervention.

3.6 Optimal policy

The preceding results establish the dynamical system and its convergence properties. This subsection asks the normative question: given a target trait $\phi^* \in \mathcal{I}$ that the planner seeks to impose on society,

[22] Propositions 4 and 6 assume Ψ symmetric positive definite. The symmetry assumption is sufficient but stronger than necessary: the results extend to any Ψ whose symmetric part $(\Psi + \Psi')/2$ is positive definite, which holds whenever diagonal entries exceed the sum of off-diagonal entries in each row. The numerical example in Section 4 has a reducible, asymmetric Ψ whose full symmetric part is not positive definite; the sufficient conditions hold within each irreducible essential class (where the submatrices are symmetric positive definite), consistent with the class-by-class qualification in Section 4.5.

what is the optimal choice of instruments? Let the planner's objective be:

$$\min_{(\psi,\boldsymbol{\pi})} \sum_{i\in N} \omega_i \big(\phi_i(\infty) - \phi^*\big)^2 \quad \text{s.t.} \quad \psi \in [0, \min_i \sigma_{ii}],\ 0 \le \pi_i < \beta_i \text{ for all } i \in N,\ \sum_{i\in N} \pi_i \le B \tag{14}$$

where $\omega_i \ge 0$ are dynasty-specific importance weights and the constraints ensure that the socialization matrix Ψ remains well-defined and that parental socialization incentives remain positive; B denotes the planner's total budget for mobility incentives. The key insight is that the planner's two instruments play fundamentally different roles.

Proposition 7 (Instrument separation): *Under the convergence conditions of Propositions 4–5 and Assumption 2:*

(a) The long-run consensus $c_k(\psi)$ within each essential class L_k depends on schooling effectiveness ψ but is independent of mobility incentives $\boldsymbol{\pi}$.

(b) The convergence rate, governed by $\lambda_2(M(\psi,\boldsymbol{\pi}))$, depends on both instruments.

(c) For any admissible $(\psi,\boldsymbol{\pi})$, $c_k(\psi) \in \operatorname{conv}(\Phi(0)_{|L_k})$.

Part a follows directly from Theorem 1: the consensus $c_k = v'_{|L_k}\Phi(0)_{|L_k}$ is determined by the left unit eigenvector $v_{|L_k}$ of Ψ_{kk}, and Ψ depends on ψ (through both diagonal and off-diagonal entries; see (3)) but not on $\boldsymbol{\pi}$. Part b is immediate from the definition $M = \Psi(I + \Pi\Psi)^{-1}(I + \Pi)$, where both Ψ and Π enter. Part c holds because $v_{|L_k}$ is a probability vector: its entries are nonnegative and sum to one, so c_k is a convex combination of the initial traits in L_k.

Remark 4 (Role of uniform incentive pressure): *Without Assumption 2, the consensus c_k depends on both instruments: the left eigenvector determining consensus weights is that of M_{kk}, not Ψ_{kk}, and M_{kk} depends on $\boldsymbol{\pi}$ through Π. The assumption ensures that the left eigenvectors of M_{kk} and Ψ_{kk} coincide within each essential class, restoring the separation. To see why, note that for $v'_{|L_k}$ to be a left eigenvector of both Ψ_{kk} and M_{kk}, one requires $v'_{|L_k}\Pi_{kk}(I - \Psi_{kk}) = 0$. Since the left null space of $(I - \Psi_{kk})$ is spanned by $v'_{|L_k}$, this holds if and only if $v'_{|L_k}\Pi_{kk} = \alpha_k\, v'_{|L_k}$ for some scalar α_k. Because Π_{kk} is diagonal with entries $(\beta_i - \pi_i)\psi_{ii}$, this requires precisely that $(\beta_i - \pi_i)\psi_{ii}$ be constant within L_k.*

Corollary 2 (Decomposition of the planner's problem): *The planner's problem* (14) *decomposes into two independent subproblems:*

(a) Target subproblem*: choose schooling effectiveness ψ^* to minimize the distance between the achievable consensus and the target:*

$$\psi^* \in \arg \min_{\psi \in [0,\, \min_i \sigma_{ii}]} \sum_{k=1}^{K} \sum_{i \in L_k} \omega_i \big(c_k(\psi) - \phi^*\big)^2 \tag{15}$$

(b) Speed subproblem*: conditional on ψ^*, choose the allocation of mobility incentives $\boldsymbol{\pi}^*$ to maximize the convergence rate:*

$$\boldsymbol{\pi}^* \in \arg \min_{\boldsymbol{\pi} \in \mathcal{A}(B)} \lambda_2(M(\psi^*, \boldsymbol{\pi})) \tag{16}$$

where $\mathcal{A}(B) \coloneqq \{\boldsymbol{\pi} \in \mathbb{R}^n_+ : \pi_i < \beta_i \text{ for all } i,\ \sum_i \pi_i \leq B\}$ is the set of admissible allocations under budget B.

The decomposition reveals a sharp division of labor between the planner's instruments. Schooling is the *steering* instrument: by varying ψ, the planner shifts the eigenvector centrality weights $v_{|L_k}$ and thereby redirects the long-run consensus within the convex hull of initial traits. Mobility incentives are the *accelerator*: by increasing π_i, the planner reduces dynasty i's effective parental weight $(\beta_i - \pi_i)$ and shrinks the second-largest eigenvalue of M, speeding convergence to whatever target the schooling instrument has selected. Crucially, the accelerator cannot substitute for the steering instrument: no amount of mobility incentives can alter the direction of convergence.

The heterogeneity of the instrument $\boldsymbol{\pi}$ introduces a targeting dimension: rather than uniformly reducing all parental weights, the planner must allocate a fixed budget of mobility incentives across dynasties. The optimal allocation depends on each dynasty's contribution to the convergence bottleneck. The following proposition characterizes the marginal effect of targeting a given dynasty and the resulting optimal allocation rule.

Proposition 8 (Optimal targeting): *Let Ψ be symmetric positive definite, and suppose the second-largest eigenvalue $\lambda_2(M) \in (0,1)$ of M is simple. Let x and y denote the right and left eigenvectors of M corresponding to λ_2, normalized so that $y'x = 1$. Define $d_i \coloneqq 1 + (\beta_i - \pi_i)\psi_{ii}$.*

(a) The marginal effect of increasing π_i[23] *on the convergence rate is:*

$$\frac{\partial \lambda_2}{\partial \pi_i} = -\frac{\psi_{ii}\,\lambda_2\,(1-\lambda_2)}{d_i}\,x_i\,y_i \tag{17}$$

where d_i summarizes dynasty i's effective parental persistence after accounting for mobility incentives. Under symmetric Ψ, $x_i\,y_i \geq 0$ for all i, so increasing π_i weakly accelerates convergence.

(b) At any interior stationary point of the budget-constrained speed subproblem (16) *(i.e., $\pi_i^* \in (0, \beta_i)$ for all i), the optimal allocation satisfies the equimarginal condition:*

$$\frac{\psi_{ii}\,\lambda_2\,(1-\lambda_2)}{d_i}\,x_i\,y_i = \mu \quad \textit{for all } i \in N \tag{18}$$

where $\mu > 0$ is the shadow price of the budget constraint, which binds at any optimum since $\partial\lambda_2/\partial\pi_i \leq 0$.

(c) At any stationary point with strictly positive shadow value $\mu > 0$, dynasties with $x_i\,y_i = 0$ have zero marginal speed gain and receive no incentives: $\pi_i^* = 0$.

Proposition 8 provides a complete characterization of the optimal allocation of mobility incentives. The gradient formula (17) decomposes the marginal speed gain from targeting dynasty i into three factors: (i) the parental socialization weight ψ_{ii}, reflecting the dynasty's integration into the network; (ii) the spectral gap $(1-\lambda_2)$, capturing the distance between the system and the convergence frontier; and (iii) the product $x_i\,y_i$, which measures dynasty i's loading on the second eigenvector of M—the cultural mode governing the slowest-converging component of trait heterogeneity. Dynasties with large loadings on this bottleneck mode have the highest marginal returns to targeting: they are the ones whose cultural resistance most constrains the rate of societal convergence.

Part b establishes the equimarginal principle: at a stationary point, the marginal speed gain per unit of incentive is equalized across all targeted dynasties. This is the standard efficiency

[23] The expression follows from standard eigenvalue perturbation results for simple eigenvalues. Differentiating the second eigenvalue of M with respect to π_i using $\partial\lambda_2/\partial\theta = y'(\partial M/\partial\theta)x$ for normalized eigenvectors $y'x = 1$, and applying the representation of M^{-1} from Proposition 4, yields eq. (17). The equimarginal condition in Part (b) then follows from the Karush–Kuhn–Tucker first-order conditions of the budget-constrained speed subproblem. See Appendix A for details.

condition from optimal resource allocation, applied to the spectral domain—the planner allocates incentives until the marginal spectral return is equal across all dynasties receiving positive incentives. Part c provides the complementary slackness condition: dynasties whose cultural positions do not contribute to the convergence bottleneck receive no incentives.

The targeting result has a direct network interpretation. The product $x_i\, y_i$ is large when dynasty i occupies a structurally important position in the second eigenvector of M. In a network with clear community structure, the second eigenvector identifies the principal cultural divide—the partition of society into groups with the most divergent long-run traits. Dynasties at the interface of these groups, or those with the highest loading on the bottleneck mode, are precisely those whose cultural transformation yields the greatest acceleration of societal convergence. When Ψ is doubly stochastic, M is similar to a symmetric matrix with $x = y$, so $x_i\, y_i = v_i^2$ where v is the eigenvector corresponding to λ_2—the algebraic connectivity vector, or Fiedler vector (Fiedler 1973). The optimal allocation then ranks dynasties by their squared Fiedler-vector loadings, connecting the targeting problem to spectral graph partitioning and algebraic connectivity.

The targeting result has a complementary relationship with the nation-building model of Almagro and Andrés-Cerezo (2020), where a forward-looking state allocates a fixed resource to promote a national identity trait. In their framework, the zero-sum character of the allocation pushes cultural dynamics toward homogeneous steady states. In the present model, by contrast, the impossibility result (Theorem 1) ensures that the convex hull of initial traits binds even under optimal targeting: the planner can accelerate convergence but cannot escape the structural constraint imposed by essential communication classes with heterogeneous priors.

When the planner's target ϕ^* lies outside the convex hull $\mathrm{conv}(\Phi(0)_{|L_k})$, no admissible policy $(\psi, \boldsymbol{\pi})$ achieves it. The planner's constrained optimum is the point on the boundary of the convex hull closest to ϕ^*, attained by the schooling intensity ψ^* that tilts eigenvector centrality as far as possible toward the most favorable initial traits. This renders the impossibility result constructive: rather than merely asserting that full cultural engineering is infeasible, the separation characterizes the *best the planner can do* and identifies the binding constraint as the initial distribution of traits, not the intensity of intervention.

4 Application: The East German Double Dictatorship

This section applies the framework to the transition from fascism to state socialism in the Soviet Occupation Zone (SBZ) and subsequent German Democratic Republic (GDR), 1945–1989. The case is well suited to the model for three reasons: (i) it involves a complete reversal of the official ideology imposed on a population whose prior traits were shaped by the preceding regime; (ii) the new regime deployed both centralized schooling and mobility incentives as explicit instruments of cultural transformation; and (iii) the persistence of ethnocentrism and right-wing attitudes despite four decades of anti-fascist doctrine provides direct evidence for the impossibility result of Theorem 1.

4.1 Historical setting

After World War II, the SBZ—later the GDR—was placed under Soviet control and led by a small communist elite composed primarily of former Nazi prisoners and returning exiles (Bursztyn and Cantoni 2014, p. 27). This elite confronted a population in which former NSDAP members and sympathizers constituted a substantial proportion (Neubacher 1994, pp. 18–19). By 1953, approximately 25% of the ruling Socialist Unity Party (SED) members were former NSDAP members or functionaries of affiliated organizations, with concentrations as high as 46% in some districts (Hafeneger and Buddrus 1994, p. 92). In July 1950, the SED declared the "roots of fascism" eradicated[24] and discontinued critical public discourse on the Nazi past (Münkler 2002).

The regime operated two principal instruments of cultural transformation, corresponding to the planner's policy variables in the model. First, a *centralized schooling system* was established to indoctrinate successive generations. Anti-fascism and Marxism–Leninism formed the core of the official curriculum, reinforced through compulsory education, youth organizations (FDJ), paramilitary training, and national memorial visits (Schubarth, Pschierer, and Schmidt 1991; Giordano 1987). Compulsory military education was introduced in 1978 for the 9th and 10th grades, and the paramilitary Society for Sport and Technology (GST) maintained over 500,000 members (Waibel

[24] The SED's definition of fascism followed the 1935 resolution of the VII World Congress of the Communist International, authored by Georgi Dimitrov, Wilhelm Pieck, and Palmiro Togliatti, which characterized fascism as "the openly terrorist dictatorship of the most reactionary, chauvinist, imperialist elements of finance capital" (Pieck, Dimitrov, and Togliatti 1957, p. 87). This narrow Marxist-Leninist interpretation attributed fascism exclusively to class antagonisms, enabling the SED to claim that the nationalization of industry had eliminated its material basis.

1996, pp. 174–176).[25] This corresponds to the parameter ψ in the model, which shifts socialization weights away from parental influence.

Second, a system of *mobility incentives* tied career advancement to political loyalty. Party membership functioned as a filtering mechanism for access to higher education and the upper service class. By 1989, male party members with university degrees constituted 63% of the upper service class, compared to 31% for nonmembers (Eyal, Szelenyi, and Townsley 1997; Solga 1994). Access to higher education was tightly rationed and restricted to roughly 10% of each cohort by 1970, with admission decisions strongly conditioned on political reliability and family background (Solga 2006; Solga 1995). In practice, this system disproportionately favored the children of politically loyal families and regime elites, creating strong incentives for dynasties to align publicly with socialist ideology in order to secure educational and career opportunities for the next generation. This corresponds to the dynasty-specific parameter π_i, which reduces dynasty i's intergenerational utility cost ($\beta_i - \pi_i$) and thereby weakens parental opposition to cultural convergence. The heterogeneity of π_i is historically grounded: party membership requirements and career advancement opportunities varied across occupational groups, with political screening concentrated among those aspiring to professional and managerial positions (Solga 2006).

Importantly, these incentives operated across the entire population and were not limited to individuals with a Nazi past. Even among cohorts socialized entirely under socialism, access to education and career advancement depended on signals of political conformity, making ideological display a rational strategy for dynasties seeking upward mobility. Despite these extensive interventions, substantial heterogeneity in ethnocentric and xenophobic attitudes persisted. Historian Harry Waibel documented approximately 8,600 cases of racist or anti-Semitic incidents in the GDR (Waibel 2014, p. 19). Right-wing youth subcultures emerged in the early 1980s and radicalized throughout the decade, culminating in neo-Nazi attacks such as the 1987 assault on the Zion Church in Berlin (Stöss 2010, p. 108). Crucially, these movements were not confined to socially marginalized individuals but included working-class and middle-class youth embedded in the socialist social order (Bergmann and Erb 1994). East Germans born after 1986 consistently express more far-right views (23.7%) than their West German peers (13.7%) (Decker 2016, p. 39).

[25] Fuchs-Schündeln and Masella (2016) exploit the natural experiment of reunification to show that additional years of exposure to the socialist curriculum causally shifted political preferences leftward, providing direct empirical support for the schooling mechanism modeled by ψ.

4.2 Preserved mentalities and the failure of anti-fascism

A central premise of the model is that initial traits $\Phi(0)$ are inherited from the preceding regime and cannot be reset by fiat. The GDR illustrates this vividly. Although the SED declared fascism eradicated by 1950, the leadership and population retained persistent mentalities and personal continuities from the Nazi era (Leonhard 1990, pp. 411). Many former Nazi officials held positions in science, media, administration, and the military that were indispensable to the new state. The SBZ leadership—itself composed of former prisoners and exiles—had to forge informal accommodations with these individuals (Neubacher 1994, p. 24). The premature termination of public reckoning with the Nazi past, combined with the Marxist-Leninist reinterpretation of fascism as a purely capitalist phenomenon, meant that the underlying attitudes were never directly confronted (Münkler 2002).

The consequences of these preserved mentalities manifested in three domains. First, persistent *xenophobia* toward foreign contract workers. As of 1989, approximately 90,500 contract workers from Vietnam, Mozambique, Cuba, and other states lived in the GDR under restrictive conditions: maximum stays of 4–5 years, gender-segregated dormitories with nightly inspections, prohibition of family reunification, and deportation in case of pregnancy (Waibel 2014, p. 13). The regime actively prevented integration while simultaneously promoting an official ideology of international solidarity (Siegler 1991, pp. 138–150; Müggenburg 1996, p. 18). In the model's terms, the gap between displayed traits (ϕ_i^d, official solidarity) and true traits (ϕ_i, social hostility) widened precisely as Proposition 1 part b predicts.

Second, persistent *anti-Semitism* in the absence of a significant Jewish population.[26] The Ministry of the Interior reported 595 incidents of anti-Semitic agitation and 2,977 neo-fascist incidents to the SED Politburo in 1960 alone (Timm 1997, p. 422). The GDR's anti-Zionist foreign policy—aligned with Moscow and directed against Israel—provided institutional cover for latent anti-Semitism (Poliakov 1992, pp. 15; Waibel 2014). The remaining Jewish communities were classified as political rather than religious organizations, restricting their ability to criticize state policy (Waibel 2014, pp. 82).

[26] Before 1933, approximately 85,000 people of Jewish origin lived in what would become the SBZ/GDR. The Holocaust reduced this number to a few hundred survivors and returnees (Waibel 2014, pp. 82). A further wave of emigration followed the anti-Semitic purges accompanying the Slánský trial in Prague in early 1953, when the SED leadership—on Soviet instruction—registered all Jewish party members in a special card index and transferred their property to national ownership, prompting more than 400 East German Jews to flee to the West (Groehler and Kessler 1995; Timm 1997).

Third, the emergence of organized *right-wing youth subcultures* from the early 1980s onward. The skinhead scene radicalized throughout the decade, drawing recruits not only from marginalized youth but also from working-class and middle-class individuals embedded in the socialist social order (Bergmann and Erb 1994). Many held stable employment and were recognized in their work collectives (Niederländer 1987, p. 39; Brück 1992, p. 44). Crucially, these were not remnants of old Nazi networks but a new generation born decades after the war and socialized entirely under the GDR's anti-fascist system (Neubacher 1994, p. 17)—in the model's terms, members of inessential communication classes whose traits nonetheless incorporated fascist-era elements through the intergenerational network transmission mechanism. The 1987 attack on the Zion Church in Berlin, in which thirty skinheads assaulted concertgoers with anti-Semitic slogans (Menhorn 2001, pp. 157), demonstrated both the persistence of fascist-era traits and the state's inability to suppress them—despite an omnipresent surveillance apparatus.

In the model's framework, these phenomena are not anomalies but predictions. Think of an essential communication class as a self-sufficient cultural enclave: its members take cues only from within, never from outsiders, so their traits are not diluted by external contact but reproduced with full fidelity in each generation—the community's prior values do not asymptotically approach zero but persist indefinitely. When the initial trait vector $\Phi(0)$ contains essential communication classes with fascist-era priors, the impossibility result (Theorem 1) guarantees that these traits propagate indefinitely through the network under any admissible policy—only an exogenous restructuring of the network itself can break the essential-class structure.[27][28]

[27] A natural question is why the state cannot act as an independent *creator* of new traits—injecting values outside the range of existing beliefs through propaganda or schooling. The answer is structural: trait formation is a weighted average of displayed traits ($\Phi(t+1) = \Psi\Phi^d(t)$), so the output is always constrained to lie within the convex hull of the inputs. Adjusting the weights Ψ—what the planner's instruments do—cannot move the output outside this hull. Introducing genuinely new trait values would require adding a new essential communication class to the network, one that itself embodies those values. Post-1990, this is precisely what happens: West German institutions enter the network as a dominant essential class, restructuring the network by breaking the closure of previously impervious groups and thereby initiating convergence. The GDR's Marxist-Leninist ideal was itself imported from an external essential class (Moscow); the model accommodates this as an expansion of the Soviet zone's original trait distribution, but the convex-hull constraint then applied to that expanded distribution.

[28] The GDR's premature termination of denazification is the historical source of this lock-in. The regime declared fascism eradicated by 1950, but the informal accommodations with former Nazi elites described above meant that fascist-era priors were absorbed into $\Phi(0)$ without confrontation. These priors establish that the initial trait vector $\Phi(0)$ encodes cultural constraints that no single regime could have engineered away (Becker and Pascali 2019). By contrast, the Federal Republic began the process of *Vergangenheitsbewältigung* relatively late but continued it with increasing intensity into the present (Neubacher 1994, p. 24). In the model's terms, this ongoing process corresponds not to the planner's instruments $(\psi, \boldsymbol{\pi})$ operating on a fixed network, but to a gradual restructuring of the network itself—breaking the closure of fascist-era essential classes over successive generations. Section 4.7 shows the analogous mechanism at work when reunification breaks dynasty 3's essential-class status.

Table 1: Mapping GDR institutions to model parameters

GDR institution	Parameter	Model mechanism	Empirical implication
Anti-fascist curriculum, FDJ, GST, military education	ψ	Shifts socialization weights from parental to collective influence; steers consensus target within conv($\Phi(0)$) (Prop. 7)	More schooling exposure → stronger left-leaning preferences (Fuchs-Schündeln and Masella 2016)
Party-gated education and career advancement	π_i	Reduces intergenerational cost ($\beta_i - \pi_i$); accelerates convergence without changing target (Prop. 7)	63% upper service class = party members with degrees; education rationed to ∼10% (Solga 2006)
SED Politburo (Wandlitz) / Nazi-era holdouts	Essential classes L_1, L_2	Anchor vertices of convex hull; impossibility of full homogenization (Thm. 1)	Persistent fascist-era traits despite anti-fascist doctrine (Waibel 2014)
MfS surveillance; social pressure	$\phi_i^d \neq \phi_i$	Increases displayed conformity without changing true traits; widens display-trait wedge (Prop. 1b)	Public compliance coexists with private dissent (Wittenberg 2006; Lichter, Löffler, and Siegloch 2021)
Religious institutions (Catholic / Protestant churches)	Counter-socialization (β_i high)	Parents transmit oppositional values privately; stronger when cultural distance is larger	Religious adherence → resilience to indoctrination (Darden and Grzymala-Busse 2006; Grzymala-Busse 2015)
Selective emigration (1945–1961)	Network exit; conv($\Phi(0)$) shrinks	Culturally distant dynasties leave; narrower feasible set for planner (Prop. 7c)	2.6M+ refugees, disproportionately educated (Central Intelligence Agency, Office of Current Intelligence 1961)

4.3 Mapping the model to the GDR

The historical setting maps onto the model parameters as follows. Table 1 provides a compact overview; the detailed discussion follows.

- **Essential communication classes.** The communist elite—the Politburo and senior SED functionaries, physically isolated in the Wandlitz compound outside Berlin (Schroeder 2013, pp. 276–277)—constitutes an essential communication class in the sense of Definition 1.[29] Their trait intensity is high (strong commitment to the new socialist ideology). However, an unreformed adherent of the old fascist culture who maintains influence through the network—

[29] The aloofness of the Wandlitz elite, whether by design or indifference, ensured that this group could influence the rest of society without being influenced in return—precisely the one-directional influence that characterizes essential communication classes.

such as a former Nazi official in an indispensable administrative position[30]—can constitute a second essential communication class with opposing traits, provided that this individual's connections are closed and strongly connected.

- **Inessential classes (ROTW).** The mass of former NSDAP members and sympathizers, absorbed into the SED and state apparatus, constitutes the inessential communication classes. Their initial traits reflect the prior regime, and Proposition 3 part c predicts that their long-run traits will be convex combinations of the essential-class consensuses—incorporating elements of both the new socialist and the old fascist ideologies.

- **Schooling parameter ψ.** The centralized schooling system—compulsory anti-fascist education, FDJ youth organizations, paramilitary training (GST), and national memorial visits—corresponds to the indoctrination effectiveness ψ. A value of $\psi = 0.2$ in the numerical example reflects imperfect indoctrination: the school system shifts socialization weights toward collective influence but does not eliminate parental transmission.[31]

- **Mobility incentives π_i.** The party membership requirement for career advancement corresponds to dynasty-specific π_i. The GDR allocated mobility incentives heterogeneously: access to higher education, foreign travel, and professional advancement was distributed based on political reliability assessments, which varied across individuals and occupational groups (Solga 2006; Solga 1995). By tying these opportunities to political loyalty, the regime reduced the intergenerational utility cost $(\beta_i - \pi_i)$ of cultural distance. This weakened parental opposition to convergence but, as the instrument separation result (Proposition 7) establishes, could not alter the direction of convergence. By Proposition 8, the regime's optimal strategy would concentrate incentives on dynasties with the largest loadings on the convergence bottleneck—which need not coincide with essential communication classes but may instead sit in the inessential part of the network.

[30] The regime's propaganda apparatus itself reflected these contradictions: Albert Norden, a Jewish émigré whose father perished in a concentration camp, oversaw anti-Zionist campaigns that provided institutional cover for latent anti-Semitism (Wolffsohn 1995, p. 202).

[31] The calibration is anchored by Fuchs-Schündeln and Masella (2016), who estimate that each additional year of socialist schooling in the GDR shifted political preferences leftward by approximately 1 percentage point on a 10-point scale. Over a formative period of roughly 10 years of compulsory education, this cumulative effect implies a schooling-induced shift of approximately 10% of the trait range—consistent with $\psi = 0.2$ reducing the parental weight by one-fifth and redistributing it to collective influence.

- **Counter-socialization.** Religious institutions served as organized counter-socialization, consistent with Proposition 1 part b. Parents with religious backgrounds displayed exaggerated loyalty to the regime while privately transmitting oppositional values, increasing the gap between displayed and true traits (Wittenberg 2006; Grzymala-Busse 2015). The model predicts that such counter-socialization is stronger precisely when the cultural distance between a dynasty and its network environment is larger—a prediction consistent with the empirical finding that religious adherence increased resilience to communist indoctrination (Darden and Grzymala-Busse 2006). Indeed, Buggle, Deter, and Lange (2025) provide causal evidence that Protestant pastor networks facilitated protest diffusion during the 1989 revolution, with network centrality—not geographic proximity—determining influence across cities, confirming that the Church's counter-socialization operated through the network structure the model formalizes. Two institutional features sharpen this mechanism.[32]

[32] The Catholic Church, though a small minority in the predominantly Protestant GDR, maintained its organizational independence through the Vatican—a foreign sovereign entity whose authority over episcopal appointments, canon law, and doctrinal matters the regime could not subsume. In the model's terms, the Catholic Church constituted a communication class with an external link to an essential class (Rome) outside the GDR's network, making it structurally more resistant to co-optation than domestically organized institutions (Grzymala-Busse 2015). The Protestant churches, by contrast, were the primary organized space for civil society but were heavily infiltrated by the Ministry for State Security (MfS), which placed informal collaborators (*Inoffizielle Mitarbeiter*) among clergy and lay members (Knabe 1999; Lichter, Löffler, and Siegloch 2021). In the model's framework, this infiltration partially penetrated the Protestant churches' communication class, weakening its closure and reducing the effectiveness of counter-socialization from within. More broadly, the Stasi's *Zersetzung* (*decomposition*) strategy—which targeted informal dissident networks by spreading disinformation and sowing suspicion between members—constitutes a form of network restructuring in the model's terms: it operated by reducing the σ_{ij} weights between connected dissidents, fragmenting their essential communication class, rather than acting through the standard policy levers ψ and $\boldsymbol{\pi}$ (Hager and Krakowski 2022).

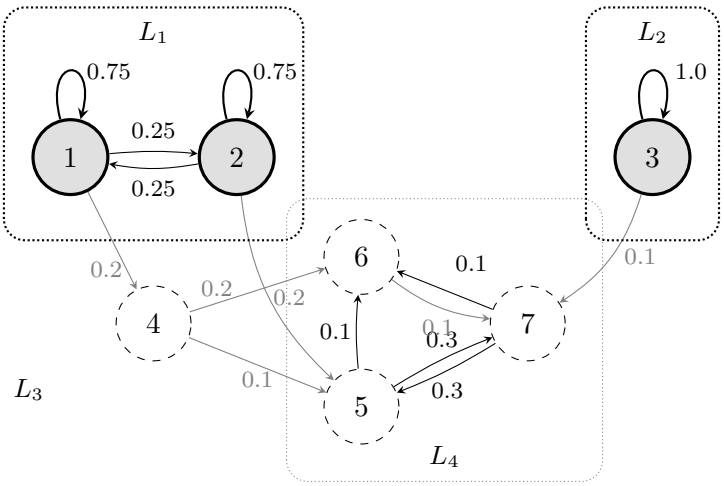


Figure 1: Network structure of the baseline socialization matrix Σ. Solid nodes denote essential communication classes; dashed nodes denote inessential dynasties (ROTW). Black arrows represent within-class socialization; gray arrows represent cross-class influence. Edge labels show the corresponding entries σ_{ij}. Self-loops (baseline parental weights σ_{ii}) are shown for essential classes only. The essential classes L_1 and L_2 influence inessential dynasties but receive no influence in return.

4.4 Numerical example

Consider a society of seven dynasties, $N = \{1, \ldots, 7\}$, with imperfect centralized indoctrination ($\psi = 0.2$). The baseline socialization matrix is:

$$\Sigma = \begin{pmatrix} 0.75 & 0.25 & 0 & 0 & 0 & 0 & 0 \\ 0.25 & 0.75 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0.2 & 0 & 0 & 0.8 & 0 & 0 & 0 \\ 0 & 0.2 & 0 & 0.1 & 0.4 & 0 & 0.3 \\ 0 & 0 & 0 & 0.2 & 0.1 & 0.6 & 0.1 \\ 0 & 0 & 0.1 & 0 & 0.3 & 0.1 & 0.5 \end{pmatrix} \tag{19}$$

Applying the rescaling formula (3) with $\psi = 0.2$ yields the row-stochastic transmission matrix Ψ. Since $\sigma_{33} = 1$, dynasty 3 is immune to schooling ($\psi_{33} = 1$); for the remaining dynasties, the diagonal is reduced by ψ and the displaced weight is redistributed proportionally to existing peer contacts.

The network structure is depicted in Figure 1. The matrix decomposes into four communication classes: $L_1 = \{1, 2\}$, $L_2 = \{3\}$, $L_3 = \{4\}$, and $L_4 = \{5, 6, 7\}$. Only L_1 and L_2 are essential—they

represent the dominant political cultures. Classes L_3 and L_4 are inessential and constitute the ROTW.

Initial trait intensities are $\Phi(0) = (90, 100, 10, 105, 15, 80, 5)^\top$, where higher values indicate stronger support for the new socialist regime. Dynasties 1 and 2 (the communist elite, L_1) hold strong pro-regime traits. Dynasty 3 (L_2) represents an unreformed adherent of the old fascist ideology ($\phi_3 = 10$), whose trait remains constant as a singleton essential class with $\sigma_{33} = 1$. As such, dynasty 3 is immune to the schooling instrument ($\psi_{33} = 1$): the school system cannot penetrate a closed community with no peer contacts—a deeply entrenched ideological commitment impervious to indoctrination.

The model's predictions align with the historical record in three respects:

1. *Convergence within the elite.* Dynasties 1 and 2, forming the essential communication class L_1 with $\sigma_{11} + \sigma_{22} > 1$, converge smoothly to a shared consensus trait. This reflects the ideological consolidation within the SED leadership.

2. *Cultural contamination in the ROTW.* Dynasties in L_3 and L_4, including those with initial pro-regime traits (dynasty 6, $\phi_6 = 80$), converge to convex combinations of the essential-class consensuses. Since L_2 represents the old fascist culture, the long-run traits of ROTW dynasties incorporate elements of both ideologies. This corresponds to the erosion of purist communist values into a national-chauvinist synthesis—the model's explanation for the failure of state-socialist anti-fascism.

3. *Impossibility of full homogenization.* Because long-run traits are convex combinations of initial traits weighted by eigenvector centrality (Theorem 1), the planner cannot drive the population to a uniform anti-fascist consensus as long as any essential communication class retains fascist-era traits. Dynasty 3's persistent influence on dynasty 7 through the network ensures that pre-existing heterogeneity is transmitted indefinitely—even though dynasty 3 is not directly influenced by any other dynasty. This result is independent of the content of the ideology and arises purely from the network structure.

4.5 Convergence speed and the planner's dilemma

Within each irreducible essential class, Proposition 6 establishes that convergence under strategic parental socialization is strictly slower than under the baseline DeGroot process. Although the full network in the numerical example is reducible, the mechanism operates within each component: parental counter-socialization retards consensus formation within L_1, while the singleton class L_2 is immune by construction. In the GDR context, this implies that the very instruments deployed by the regime to accelerate cultural transformation—schooling and mobility incentives—simultaneously provoked parental counter-socialization that retarded convergence. Parents with pre-communist priors had incentives to adjust their *displayed* traits ϕ_i^d toward the regime's preferred value—without any change in their true traits ϕ_i—displaying exaggerated loyalty while privately transmitting oppositional values, a phenomenon well documented in the GDR context (Wittenberg 2006; Grzymala-Busse 2015).

The planner thus faces a fundamental dilemma.[33] Increasing π_i (dynasty-specific mobility incentives) reduces the intergenerational utility cost $(\beta_i - \pi_i)$ and accelerates convergence by shrinking the gap between displayed and true traits. However, it cannot alter the *target* of convergence, which remains within the convex hull of initial traits. Increasing ψ (schooling effectiveness) shifts socialization weights toward collective influence but simultaneously triggers stronger parental countermeasures from dynasties whose true traits diverge from the regime's preferred outcome (Proposition 1, part b).

The optimal targeting result (Proposition 8), derived for irreducible networks, provides a qualitative lens on the GDR's incentive allocation. The regime concentrated career advancement opportunities among the intelligentsia and administrative classes rather than the industrial working class (Solga 2006). The model's prescription—targeting dynasties with the largest loadings on the convergence bottleneck—suggests that the regime's focus was rational if the principal cultural divide ran between the educated urban elite and the broader population, but suboptimal if the key bottleneck lay among rural or working-class families with stronger fascist-era traits.[34]

[33] A structurally analogous tension arises in Bisin and Verdier (2005), where welfare state policies weaken parental incentives to transmit work ethic, initially fostering cultural homogenization but ultimately undermining the very preferences that sustain political support for redistribution.

[34] The evidence supports this interpretation. Voigtländer and Voth (2012) document that NSDAP support was concentrated in rural Protestant communities, and Cantoni, Hagemeister, and Westcott (2020) show that municipalities with higher 1933 Nazi vote shares exhibit significantly stronger support for the AfD after the party's nationalist turn in 2015—a correlation absent when the AfD ran on a moderate fiscal-conservative platform, suggesting dormant ideological persistence that the model predicts for untargeted essential classes.

This trade-off provides a theoretical account of a well-documented empirical pattern: despite four decades of intensive indoctrination, the GDR never achieved the cultural homogenization it proclaimed. The persistence of xenophobic attitudes, the emergence of right-wing youth subcultures from within the socialist order, and the consistently higher levels of ethnocentrism among post-reunification East Germans (Decker 2016; Zick, Krause, and Küpper 2015) are all consistent with the model's prediction that complete cultural engineering is impossible when the initial population contains heterogeneous essential communication classes.

4.6 Post-reunification persistence

The model's predictions extend beyond the GDR's collapse. Since long-run traits are convex combinations of initial traits weighted by eigenvector centrality, the cultural heterogeneity accumulated over four decades of imperfect homogenization does not disappear with regime change—it is transmitted to subsequent generations through parental socialization. This is consistent with a broad empirical literature documenting persistent East-West differences in market skepticism (Alesina and Fuchs-Schündeln 2007), trust (Jacob and Tyrell 2010; Lichter, Löffler, and Siegloch 2021), consumption patterns (Friehe and Mechtel 2014), financial attitudes (Laudenbach, Malmendier, and Niessen-Ruenzi 2020), and gender norms (Campa and Serafinelli 2019). These persistent differences are often attributed to communist institutions, though Becker, Mergele, and Woessmann (2020) caution that pre-existing regional differences account for part of the observed East-West gap—an identification challenge that the model addresses directly, since initial traits $\Phi(0)$ encode pre-existing heterogeneity and the impossibility result guarantees its propagation regardless of the intervening regime. Crucially, these differences persist even among cohorts born after reunification, which the model explains through the intergenerational transmission channel: parents socialized under the GDR transmit their (partially converged) traits to children who have no direct exposure to the old regime (Kollmorgen 2005, p. 162)—a pattern confirmed by Avdeenko and Siedler (2016), who document strong intergenerational correlations in far-right party preferences.

Moreover, the model clarifies why post-communist cultural persistence is not uniform across traits. The qualitative mechanism underlying Proposition 6—that strategic parental resistance widens the gap between M and Ψ—suggests that traits for which parental imperfect-empathic preferences β_i are large—such as deeply held ethnocentric attitudes—converge more slowly than traits for which

parents are more indifferent. This is consistent with the empirical observation that childhood political socialization requires more generational turnover to dissipate than other attitudes, and that xenophobic attitudes in East Germany have proven particularly resistant to change (Decker 2016; Zick, Krause, and Küpper 2015). The heterogeneity extends to a gender dimension: authoritarian indoctrination persists almost exclusively among East German men, while women adapted more readily to post-reunification norms (Elsayed et al. 2025)—a concrete instance of differential β_i across dynasties generating the heterogeneous convergence rates the model predicts.

4.7 Emigration, network opening, and democratic convergence

The model's parameters can be varied across three historical phases to capture the full arc of East German cultural dynamics: the open network before the Wall (1945–1961), the closed network under the GDR (1961–1989), and the network expansion after reunification (1990–present). The instrument separation result (Proposition 7) provides the analytical framework for understanding each transition. Figure 2 traces the resulting dynamics, including counterfactual paths without each structural break.

Phase 1: Selective emigration (1945–1961). Before the construction of the Berlin Wall, more than 2.6 million registered refugees fled East Germany for the West, with a disproportionate share of professionals, engineers, and intellectuals (Central Intelligence Agency, Office of Current Intelligence 1961)[35]—plausibly the dynasties with strong prior convictions (high β_i) whose cultural transformation would have been most costly for the planner to achieve. In the model's terms, their departure constitutes a selective exit from the network: dynasties whose cultural distance $|\phi_i(0) - \phi^*|$ from the regime's target was largest had the strongest incentive to leave. This selective exit shrinks the convex hull of initial traits in the remaining population, $\text{conv}(\Phi(0)_{|N \setminus N_{\text{exit}}}) \subset \text{conv}(\Phi(0))$, which by Proposition 7 part c narrows the planner's feasible target set. Paradoxically, emigration *helped* the planner: with fewer culturally extreme dynasties, the post-Wall population was more amenable to the schooling and mobility instruments. The Wall, by closing the exit option, locked in a population

[35] A CIA memorandum dated 10 August 1961—three days before the Wall went up—lists 3.5 million including expellees since the end of World War II, and documents the severity of the brain drain: "From 1954 through 1960, the refugees included 4,334 doctors and dentists, 15,536 engineers and technicians, 738 professors, 15,885 other teachers and more than 11,700 college graduates" complemented by a significant number of industrial managers (Central Intelligence Agency, Office of Current Intelligence 1961, p. 2). The exodus extended to firms: some 18,000 companies relocated from the Soviet occupation zone to the West between 1945 and 1950 alone (Donges and Streb 2025, citing Held (1957) on p. 9).

with a narrower—but still heterogeneous—distribution of initial traits, on which the planner operated for the next three decades.

Phase 2: Closed network (1961–1989). This is the regime analyzed in the preceding subsections. The planner operates $(\psi, \boldsymbol{\pi})$ on the reduced population. By the instrument separation result, schooling ψ steers the consensus target within the narrowed convex hull, while mobility incentives $\boldsymbol{\pi}$ accelerate convergence. Despite four decades of intensive intervention, the impossibility result binds: the convex hull still contains essential communication classes with fascist-era traits (dynasty 3 in the numerical example), ensuring that complete homogenization remains unattainable.

Phase 3: Network expansion and democratic convergence (1990–present). Reunification fundamentally alters the model's parameters in three ways. First, the planner's instruments are removed: $\psi \to 0$ (no centralized indoctrination) and $\pi_i \to 0$ for all i (no politically conditioned mobility). Second, the network Ψ expands from $n_E \times n_E$ to $(n_E + n_W) \times (n_E + n_W)$, as East German dynasties are integrated into a larger social network that includes West German dynasties with democratic and market-oriented traits. Third, West German institutions—media, political parties, universities, the legal system—become the dominant essential communication classes in the expanded network, while East German dynasties enter these networks as *inessential* classes with no reciprocal influence on the Western core.[36] Crucially, dynasty 3—the fascist holdout that remained impervious to indoctrination throughout the GDR era as a singleton essential class ($\sigma_{33} = 1$)—acquires peer connections to Western institutions in the expanded network ($\sigma_{33} = 0.85$) and loses its essential-class status. What four decades of centralized schooling could not achieve, the structural expansion of the network initiates: dynasty 3 begins converging toward democratic norms for the first time (Figure 2, blue trajectory after $t = 20$). Yet without state instruments to accelerate the process, convergence is glacially slow—after ten generations, dynasty 3 remains far below the West German consensus, consistent with the persistent far-right attitudes documented among post-reunification East German cohorts (Decker 2016). This sluggishness reflects a further endogenous mechanism: the Treuhand's mass privatisation triggered economic dislocations that intensified parental resistance to Western norms (Kellermann 2024), while the exclusion of East German managers from post-privatisation

[36] See also Bindler et al. (2026) for evidence from the post-reunification replacement of academic staff in East Germany, analyzed as a labor-demand shock affecting institutional composition.

network leadership concentrated socialization weights toward Western elites (Hager, Hilbig, and Vlandas 2025)—raising effective β_i precisely when the network opening should have accelerated convergence.

The model's predictions for this phase are sharp. By Proposition 3, the long-run traits of inessential classes are convex combinations of the essential-class consensuses. East German dynasties therefore converge toward West German cultural norms—but the process is one-directional: West German essential classes shape the East without being shaped in return. This asymmetry is precisely the "unequal union" documented by Mau (2024), who argues that reunification proceeded not as a merger of equals but as an institutional absorption that sidelined East German civic preferences.

The speed of democratic convergence is governed by the spectral gap of M with $\boldsymbol{\pi} = \mathbf{0}$ and $\psi = 0$. Without state acceleration, convergence relies entirely on passive network diffusion, which is slow when parental attachment β_i is high. This formalizes Mau's observation that reunification proceeded on "autopilot" without active democratic socialization (Mau 2024). The Alesina and Fuchs-Schündeln (2007) prediction of 20–40 years for convergence of market preferences is consistent with moderate β_i. Three decades on, this trajectory has been confirmed but shown to be decelerating (Bondar and Fuchs-Schündeln 2023)—precisely the asymptotic slowdown that the model's M^t dynamics predict. However, for traits with high parental attachment—ethnocentric attitudes, institutional distrust—convergence takes much longer, consistent with the persistent electoral divergence that East-West gaps in democratic attitudes have not closed 35 years after reunification. For gender norms, the direction of influence even reverses: Boelmann, Raute, and Schönberg (2025) show that East German egalitarian norms persist intergenerationally while West German traditional norms are displaced upon exposure, implying that the essential-class structure may differ across cultural dimensions.

A second emigration wave compounds the problem. After 1990, young educated East Germans—disproportionately women—moved West, plausibly those with lower β_i (weaker attachment to GDR-era norms) and higher openness to democratic traits. This selective exit depletes the East's most integration-prone dynasties and raises the effective β_i of the remaining population, creating communities where convergence to democratic norms is structurally impeded. Mau (2024) documents the resulting gender imbalances and their correlation with anti-democratic attitudes. In the model's framework, the post-1990 emigration represents a second shrinking of the convex hull—but this time

working *against* democratic convergence, as the dynasties most likely to facilitate it are the ones who leave.

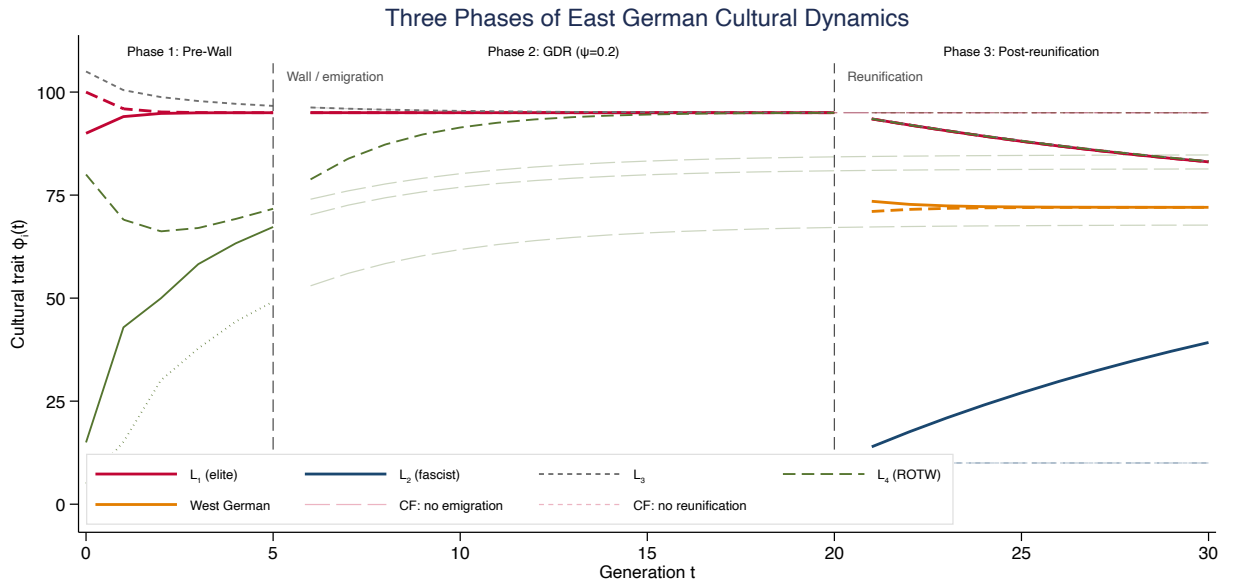


Figure 2: Cultural dynamics across three historical phases. Solid lines trace trait trajectories under the actual sequence of structural breaks: selective emigration at $t = 5$ and reunification at $t = 20$ (vertical dashed lines). Faded lines show counterfactual paths: *CF-A* continues the pre-emigration 7-dynasty network past $t = 5$; *CF-B* continues the closed GDR network past $t = 20$. Emigration at $t = 5$ removes two dynasties from the network, altering subsequent dynamics; the convex-hull contraction of initial traits (Proposition 7, part c) motivates the theoretical interpretation in Section 4.7. At reunification, the removal of state instruments ($\psi \to 0$) and the addition of West German nodes alter both the long-run target and the network's essential-class structure, breaking dynasty 3's immunity to external influence. Colours denote communication classes: red = L_1 (communist elite), blue = L_2 (fascist holdout), gray = L_3 (inessential), green = L_4 (rest of the world), orange = West German nodes. Parameters: $\beta = 2$, $\psi = 0.2$ (Phases 1–2), $\psi = 0$ (Phase 3), $\pi_i = 0$ throughout.

4.8 Empirical implications

The model generates several implications that connect its structural parameters to observable regional variation, suggesting directions for future empirical work.

Heterogeneous convergence across cultural dimensions. The model predicts that convergence speed depends on parental attachment β_i, which may vary both across dynasties and across cultural dimensions. Existing evidence supports this heterogeneity. For political and economic preferences, East Germans converge toward Western norms, albeit at a decelerating rate (Bondar and Fuchs-Schündeln 2023). For gender norms, however, the direction reverses: Boelmann, Raute, and Schönberg

(2025) show that East German egalitarian norms persist intergenerationally while West German traditional norms are displaced upon exposure—implying that β_i for gender egalitarianism is higher among East German dynasties than among their Western counterparts. For ethnocentric attitudes, persistence is itself gendered: authoritarian indoctrination persists almost exclusively among East German men (Elsayed et al. 2025). These patterns are consistent with the model's core mechanism—β_i governs convergence speed—but suggest that a multidimensional extension (Section 5) is needed to capture the full structure of post-reunification cultural dynamics.

Religious adherence and rate of cultural change. Religious families invested more heavily in counter-socialization during the GDR era, transmitting oppositional values with greater intensity (Darden and Grzymala-Busse 2006; Wittenberg 2006)—consistent with high β_i. However, these same families were already *closer* to West German democratic norms before reunification, precisely because their high β_i preserved pre-communist civic values. The model therefore predicts not that religious districts are farther from Western norms after 1990, but that they exhibit a slower *rate* of cultural change in any direction: high β_i dynasties resist assimilation to both communist and post-communist cultural environments. This prediction is testable using the German Allbus or European Social Survey by regressing the rate of attitudinal change—controlling for initial cultural distance—on pre-reunification religious adherence at the district level.

Pre-Wall emigration and post-reunification convergence. Phase 1's selective emigration (1945–1961) narrowed the convex hull of initial traits by removing dynasties whose starting values were far from the regime's target. Bauernschuster et al. (2012) provide evidence on the composition of these emigrants: pre-Wall movers were disproportionately educated, white-collar, and self-reliant in their preferences, suggesting that emigration selected on *trait values* (cultural distance from the regime) rather than on β_i directly. The model predicts that districts with higher pre-1961 out-migration rates should have narrower post-reunification trait distributions. If, additionally, emigrants also had high β_i—a joint hypothesis, since strong convictions and high intergenerational transmission need not coincide—the remaining population would converge faster toward democratic norms after 1990. Since district-level refugee flows are documented in administrative records (Central Intelligence Agency, Office of Current Intelligence 1961), these predictions can be tested by regressing post-

1990 attitudinal convergence on pre-Wall emigration intensity, controlling for pre-existing regional characteristics (Becker, Mergele, and Woessmann 2020).

Targeting mismatch and differential persistence. Proposition 8 establishes that optimal incentive allocation concentrates on dynasties at the structural fault lines of the network—those with the largest loadings on the second eigenvector of M. The GDR concentrated career incentives on the intelligentsia and administrative classes (Solga 2006), which may or may not have corresponded to this structural bottleneck. The model predicts that, conditional on initial traits, occupational groups receiving higher mobility incentives π_i during the GDR era should exhibit faster post-reunification convergence than under-targeted groups. An empirical challenge is that the post-1990 economic shocks—mass unemployment from the Treuhand's privatisation (Kellermann 2024) and the replacement of East German elites (Hager, Hilbig, and Vlandas 2025)—hit the previously targeted groups hardest, confounding the effect of past π_i with the effect of post-reunification disruption. Disentangling these channels would require variation in targeting intensity that is orthogonal to post-1990 exposure, a task left for future research.

5 Conclusion

This paper has studied intergenerational cultural transmission when dynasties embedded in a social network strategically misrepresent their traits. The framework nests the standard DeGroot model as a special case and introduces two instruments—centralized schooling and dynasty-specific mobility incentives—through which a planner can attempt to reshape cultural traits. Under a quadratic specification of preferences, the equilibrium law of motion is governed by a modified transition matrix whose spectral properties yield closed-form results.

The analysis delivers four results. First, long-run traits are convex combinations of initial traits weighted by eigenvector centrality, so a planner who inherits a heterogeneous population cannot engineer convergence to an arbitrary cultural target—an impossibility result that holds whenever the transmission technology preserves the convex hull of displayed traits. Second, the planner's instruments decouple: schooling steers the achievable consensus within the convex hull, while mobility incentives govern only the speed of convergence. Third, the very interventions designed to accelerate

convergence provoke parental counter-socialization that slows the process. Fourth, under a budget constraint, optimal targeting concentrates incentives on dynasties whose loading on the second eigenvector of M is largest—the structural fault lines of the network. Together, the impossibility theorem and the instrument separation characterize both the planner's constraint set and the best achievable policy within it. The application to the East German transition illustrates both the mechanisms and a non-obvious symmetry: selective emigration shrinks the convex hull of initial traits, paradoxically *helping* the regime before the Wall but—through the identical mechanism—impeding democratic convergence after 1990 (Mau 2024), when the dynasties most likely to facilitate integration are the ones who leave.

The quadratic specification is essential to these results: it delivers linear best responses and a closed-form transition matrix, but restricts the class of equilibria. Whether the impossibility and separation results extend to more general preference specifications—or whether richer strategic interactions can break the convex-hull constraint—is an open question that determines the scope of the policy prescriptions derived here.

Three extensions merit investigation. First, the network structure is exogenous, and this is the source of the convex-hull constraint: since next-generation traits are weighted averages of displayed traits ($\Phi(t{+}1) = \Psi\Phi^d(t)$), adjusting the weights—what the planner's instruments do—cannot move outcomes outside the hull; only introducing a new essential communication class that embodies the target values can break the constraint, as reunification did when West German institutions entered the network. Endogenizing link formation would allow the analysis of sorting and segregation as responses to the planner's interventions. If dynasties can sever ties with culturally distant neighbours (homophily), essential communication classes would form endogenously—potentially strengthening the impossibility result, since cultural enclaves emerge as equilibrium outcomes rather than exogenous assumptions. Conversely, if the planner can directly restructure the network—as the Stasi's *Zersetzung* strategy did by fragmenting dissident networks (Hager and Krakowski 2022)—the impossibility result breaks, but the analysis shifts from optimising $(\psi, \boldsymbol{\pi})$ on a fixed network to optimising Σ itself, a combinatorial problem of a different character. Second, the model assumes a single cultural dimension; a multidimensional extension could capture the empirical observation that different traits converge at different rates and even in different directions—Elsayed et al. (2025) show that authoritarian indoctrination persists almost exclusively among East German men, while Boelmann, Raute, and

Schönberg (2025) find that East German egalitarian gender norms displace Western traditional norms upon exposure, implying that the essential-class structure itself may vary across cultural dimensions. Third, embedding the planner's problem within a joint model of cultural and institutional change—along the lines of Bisin and Verdier (2017)—would capture how the distribution of cultural traits feeds back into institutional design; the post-reunification experience, in which the Treuhand's mass privatisation endogenously raised parental resistance to Western norms (Kellermann 2024), illustrates precisely the two-way feedback that this extension would formalize.

A Proofs

Proof of Proposition 1

Consider a scenario with initial traits $\phi_i(t)$, socialization incentives σ_{ii}, the effectiveness of indoctrination ψ, and traits within the rest of society $\phi^d_{Ni}(t)$. Define $f\colon \mathbb{R} \to \mathbb{R}$ as $f(x) = v_i((\sigma_{ii} - \psi)x + (1 - \sigma_{ii} + \psi)\phi^d_{Ni}(t))$, which evaluates the intergenerational utility of a parent at each displayed trait. With $\psi_{ii} = (\sigma_{ii} - \psi) \geq 0$, this becomes $f(x) = v_i(\psi_{ii}x + (1 - \psi_{ii})\phi^d_{Ni}(t))$.

Assuming that both the intergenerational and own utility components are strictly concave, the maximization problem of an adult can be rewritten as $\max_{x\in\mathcal{I}} u_i(x) + f(x)$, where $\mathcal{I}$ is the set of feasible traits. The second derivative of $f(x)$ is $f''(x) = v''(\psi_{ii}x + (1 - \psi_{ii})\phi^d_{Ni}(t))\psi^2_{ii}$. As $u_i''(x) + f''(x) < 0$ for all $x \in \mathcal{I}$, the maximization delivers a unique solution, x^*, since a continuous and strictly concave function over a compact set is maximized at a unique point.

Proof of Part a. Consider the following cases:

(a) When $\psi_{ii} = 0$, the function $f(x)$ becomes a constant for all values of x, and thus the maximization problem simplifies to $\arg\max_{x\in\mathcal{I}} u_i(x)$. As $u_i(x)$ is strictly concave and continuous over the compact set $\mathcal{I}$, there exists a unique solution $x^* = \phi_i(t)$.

(b) When $\psi_{ii} = 1$, the function $f(x)$ becomes $v_i(x)$, and maximizing $u_i(x) + f(x)$ is equivalent to maximizing $v_i(x) + u_i(x)$. As both $v_i(x)$ and $u_i(x)$ are strictly concave and continuous over $\mathcal{I}$, the maximization problem has a unique solution $x^* = \phi_i(t)$.

(c) When $\phi^d_{Ni}(t) = \phi_i(t)$, the next generation's traits will be the same as the current generation's traits, and thus the optimal solution for an individual is to maintain their current traits, i.e., $x^* = \phi_i(t)$ maximizes both $u_i(x)$ and $f(x)$.

(d) When $\phi_i(t) = \min \mathcal{I}$, the function $u_i(x)$ is decreasing in x and $f(x)$ is non-increasing in x. Hence, the unique maximizer of $u_i(x)$ is $\phi_i(t)$, while $f(x)$ does not affect the choice of x^*. The same argument applies when $\phi_i(t) = \max \mathcal{I}$.

In summary, the maximum of the parent's optimization problem, $\max_{x \in \mathcal{I}} u_i(x) + f(x)$, depends on the value of the parameters ψ_{ii}, $\phi^d_{Ni}(t)$, and $\phi_i(t)$, as well as the properties of the functions $u_i(x)$ and $f(x)$.

Proof of Part b. Assume that $\phi^d_{Ni}(t) > \phi_i(t)$ and that $\phi_i(t)$ is interior, i.e., $\phi_i(t) \in \mathring{\mathcal{I}}$; the goal is to show $\phi_i^{d^*}(t)(\phi_i(t), \phi^d_{Ni}(t)) < \phi_i(t)$. Using the above definition of f, note that for $\sigma_{ii} \in (0,1)$, $f(x)$ is strictly decreasing for all $x \geq \phi_i(t)$. Since $u_i(x)$ is also non-increasing for $x > \phi_i(t)$, it follows that $x > \phi_i(t)$ cannot be optimal. At $x = \phi_i(t)$, $u_i'(\phi_i(t)) = 0$ and $f'(\phi_i(t)) < 0$ (since f is strictly decreasing at $\phi_i(t)$ when $\phi^d_{Ni}(t) > \phi_i(t)$), so $x = \phi_i(t) - \epsilon$ is an improvement for small enough $\epsilon > 0$. Moreover, for small enough ϵ, this is interior by the assumption that $\phi_i(t)$ is interior. The argument is analogous for $\phi^d_{Ni}(t) < \phi_i(t)$.

Proof of Part c. Let $\psi_{ii} \in (0,1)$ and assume $\tilde{\phi}^d_{Ni}(t) < \phi^d_{Ni}(t)$. It suffices to show that $\phi_i^{d^*}(\phi_i(t), \tilde{\phi}^d_{Ni}(t)) > \phi_i^{d^*}$, where $\phi_i^{d^*}$ is the optimal displayed trait. Consider three cases:

(a) Assume $\tilde{\phi}^d_{Ni}(t) \leq \phi_i(t) \leq \phi^d_{Ni}$, where at least one of the two inequalities is strict. Using Proposition 1 parts a and b, $\phi_i^{d^*}(\phi_i(t), \tilde{\phi}^d_{Ni}(t)) \geq \phi_i(t) \geq \phi_i^{d^*}(\phi_i(t), \phi^d_{Ni}(t))$, with strict inequalities preserved.

(b) Assume $\tilde{\phi}^d_{Ni}(t) < \phi_i(t) \leq \phi^d_{Ni}$. Let x^* be the best response to $\phi^d_{Ni}(t)$ as the solution to $\max_{x \in \mathcal{I}} u_i(x) + f(x)$. Now, instead of $\phi^d_{Ni}(t)$, consider $\tilde{\phi}^d_{Ni}(t)$ and define $\tilde{f}\colon \mathbb{R} \to \mathbb{R}$ such that $\tilde{f}(x) = f\left(x - \left[\frac{1-\psi_{ii}}{\psi_{ii}}(\phi_{Ni}(t) - \tilde{\phi}_{Ni}(t))\right]\right)$ is a right-shifted version of f. Since f is strictly concave for $\psi_{ii} \in (0,1)$, its derivative f' is strictly decreasing, so $\tilde{f}'(x) > f'(x)$ for any x. Since x^* is an interior solution, it satisfies $f'(x^*) = -u_i'(x^*)$, which implies $\tilde{f}'(x^*) > -u_i'(x^*)$. Hence, $u_i'(x) + \tilde{f}'(x) > 0$ at $x = x^*$, so $u_i(x) + \tilde{f}(x)$ is increasing at x^*, yielding $\tilde{x}^* > x^*$ whenever x^* is interior. If instead x^* is on the boundary while $\tilde{x}^*$ is interior (which is

the remaining case under the hypothesis that at least one best response is interior), then $x^* \in \{\min \mathcal{I}, \max \mathcal{I}\}$. If $x^* = \min \mathcal{I}$, then $\tilde{x}^* > \min \mathcal{I} = x^*$ since $\tilde{x}^*$ is interior. If $x^* = \max \mathcal{I}$, then the boundary first-order condition requires $u_i'(\max \mathcal{I}) + f'(\max \mathcal{I}) \geq 0$ (the objective is non-decreasing at the right endpoint). Since $\tilde{f}'(x) > f'(x)$ for all x (established above), $u_i'(\max \mathcal{I}) + \tilde{f}'(\max \mathcal{I}) > u_i'(\max \mathcal{I}) + f'(\max \mathcal{I}) \geq 0$, so $u_i + \tilde{f}$ is also non-decreasing at $\max \mathcal{I}$, giving $\tilde{x}^* = \max \mathcal{I}$. This contradicts the assumption that $\tilde{x}^*$ is interior, so this subcase cannot arise.

(c) Assume $\tilde{\phi}^d_{Ni}(t) \leq \phi_i(t) < \phi^d_{Ni}$, which is analogous to case (ii).

Furthermore, if $\phi^d_{Ni}(t) < \tilde{\phi}^d_{Ni}(t)$, then exchanging the values of $\phi^d_{Ni}(t)$ and $\tilde{\phi}^d_{Ni}(t)$ yields $\tilde{x}^* < x^*$. If $\phi^d_{Ni}(t) = \tilde{\phi}^d_{Ni}(t)$, the unique solution to both maximization problems coincides: $\tilde{x}^* = x^*$. □

Proof of Proposition 2

Existence under Assumption 1. Fix a period t and the opponents' displayed traits $\Phi^d_{-i}(t)$. Player i solves

$$\max_{x \in \mathcal{I}} \; u_i(x \mid \phi_i(t)) + v_i\Big(\psi_{ii} x + (1 - \psi_{ii}) \phi^d_{Ni}(t) \mid \phi_i(t)\Big).$$

By Assumption 1, this objective is continuous and strictly concave in x on the compact convex set $\mathcal{I}$, so the best response $BR_i(\Phi^d_{-i}(t))$ exists and is unique for every profile of opponents' displayed traits. By Berge's maximum theorem, BR_i is continuous. Hence the aggregate best-response map

$$BR(\Phi^d(t)) \coloneqq \big(BR_1(\Phi^d_{-1}(t)), \ldots, BR_n(\Phi^d_{-n}(t))\big)$$

is a continuous self-map of the compact convex set $\mathcal{I}^n$. By Brouwer's fixed-point theorem, there exists $\Phi^{d*}(t) \in \mathcal{I}^n$ such that $BR(\Phi^{d*}(t)) = \Phi^{d*}(t)$. This fixed point is a Nash equilibrium.

Uniqueness under quadratic utility and positive definite Ψ. Under the quadratic utility specification, the first-order condition of each parent's optimization problem is linear in $\phi^d_i(t)$. Stacking across all n dynasties, the system of first-order conditions can be written as:

$$(I + \Pi\Psi)\, \Phi^{d*}(t) = (I + \Pi)\, \Phi(t) \tag{20}$$

where $\Pi = \text{diag}((\beta_i - \pi_i)\psi_{ii})$ collects the net incentive parameters. It remains to show that $(I + \Pi\Psi)$ is invertible when Ψ is symmetric positive definite, so that (20) has a unique solution.

Consider the similarity transformation $\Pi^{1/2}\Psi\Pi^{1/2}$, where $\Pi^{1/2}$ denotes the diagonal matrix with entries $\sqrt{(\beta_i - \pi_i)\psi_{ii}} \geq 0$. Since Ψ is symmetric positive definite and $\Pi^{1/2}$ is a nonnegative diagonal matrix, the congruence $\Pi^{1/2}\Psi\Pi^{1/2}$ is positive semidefinite (positive definite whenever all $(\beta_i - \pi_i) > 0$). Therefore the eigenvalues of $\Pi\Psi$, which coincide with those of $\Pi^{1/2}\Psi\Pi^{1/2}$, are real and nonnegative. It follows that every eigenvalue of $(I + \Pi\Psi)$ is at least 1, so $(I + \Pi\Psi)$ is invertible.

The unique Nash equilibrium is therefore:

$$\Phi^{d*}(t) = (I + \Pi\Psi)^{-1}(I + \Pi)\,\Phi(t) \tag{21}$$

which yields the transition matrix $M = \Psi(I + \Pi\Psi)^{-1}(I + \Pi)$ as stated. □

Proof of Proposition 3

Proposition 1 directly implies that parents choose their best-reply displayed trait in any steady state. In a steady state, $\phi_i(t+1) = \phi_i(t)$ for all i, so the intergenerational utility component is maximised at zero loss. The first-order condition reduces to $u_i'(\phi_i^d) = 0$, which yields $\phi_i^{d*} = \phi_i$: in steady state, displayed traits coincide with true traits. The trait formation process $\Phi(t+1) = \Psi\Phi^d(t)$ then simplifies to $\Phi = \Psi\Phi$. Given the definition of steady states, the set of steady states equals $\{\Phi \in \mathcal{I}^n \mid \Psi\Phi = \Phi\}$. Therefore, if the traits of individuals within an essential communication class are identical, then $\Psi_L\Phi_L = \Phi_L$, where Ψ_L is the restriction of Ψ to an essential communication class L, and Φ_L is the vector of adopted traits restricted to that set.

To prove that steady-state traits cannot differ within an essential communication class, proceed by contradiction. Let L be an essential communication class with $|L| \geq 2$ and suppose there exist $i, j \in L$ with $\phi_i \neq \phi_j$. Let $\tilde{\phi}_L \coloneqq \max\{\phi_i \mid i \in L\}$ be the maximal trait in class L. Since L is a communication class, there exists $i \in \{l \in L\colon \phi_l = \tilde{\phi}_L\}$ and $j \in \{l \in L\colon \phi_l \neq \tilde{\phi}_L\}$, such that $\psi_{ij} > 0$.

Furthermore, due to the maximality of $\tilde{\phi}_L$ and the fact that L is essential, $\psi_{ik} = 0$ for all $k \in N$ with $\phi_k > \tilde{\phi}_L$. Therefore, if $e_i'\Psi\Phi_L \neq \phi_i$, where e_i denotes the i-th unit vector, then this cannot be a steady state. This completes the proof that steady-state traits cannot differ within an essential communication class.

Now assume that there exists an inessential communication class $L' \in \mathcal{P}(\Psi)$ with connections to other dynasties $J \coloneqq \{j \in N \mid i \to j,\, i \in L'\}$ such that the set of traits $\Phi_{L'}$ is not included in the convex hull of $\{\phi_j \mid j \in J\}$. Without loss of generality, assume $\tilde{\phi}_{L'} \coloneqq \max\{\phi_i \mid i \in L'\} > \max\{\phi_j \mid j \in J\}$.

Since L' is an inessential communication class and all connections to outside dynasties are with traits strictly less than $\tilde{\phi}_{L'}$, there exists a player k with $\phi_k = \tilde{\phi}_{L'}$ and a j with $\phi_j < \tilde{\phi}_{L'}$ such that $\psi_{kj} > 0$. By the maximality of $\tilde{\phi}_{L'}$ and all other connections being to dynasties with traits strictly less than $\tilde{\phi}_{L'}$, it follows that $e_k' \Psi \Phi_{L'} \neq \phi_k$. Therefore, this configuration cannot be a steady state. Hence, all traits of dynasties in inessential communication classes $L' \in \mathcal{P}(\Psi)$ are convex combinations of the traits of the essential communication classes $L \in \mathcal{P}(\Psi)$ such that $J \cap L \neq \emptyset$. □

Proof of Proposition 4

The proof uses the Gershgorin Disc Theorem, which states that for any square substochastic matrix A, $\rho(A) \leq 1$; see Horn and Johnson (2012, p. 388). If A is stochastic, then $\rho(A) = 1$ since A has an eigenvector of ones, $\mathbb{1} \coloneqq (1, \ldots, 1)^\top : \Psi \mathbb{1} = \mathbb{1}$. The following lemma is therefore sufficient:

Lemma 1 (Convergence): *Let A be a square matrix with complex and real entries. Then the sequence $\{A^t\}_{t \to \infty}$ converges if and only if:*

(a) If λ is an eigenvalue of A, then either $\lambda = 1$ or λ lies in the open unit circle, as per the Gershgorin Disc Theorem.

(b) If 1 is an eigenvalue of A, then its algebraic multiplicity equals its geometric multiplicity; see Horn and Johnson (2012, p. 181).

Denote the set of eigenvalues of A by $\Lambda(A)$, and let $\lambda(A) \in \Lambda(A)$. For a complex number z, denote by $\Re(z)$ the real part and by $\mathrm{Im}(z)$ the imaginary part.

The first condition of the lemma is verified for symmetric positive definite matrices Ψ. Recall that:

$$M \coloneqq \Psi(I + \Pi\Psi)^{-1}(I + \Pi) = (\Pi + \Psi^{-1})^{-1}(I + \Pi) \tag{22}$$

This is well-defined when Ψ is positive definite. If so, M is invertible and $M^{-1} = (I + \Pi)^{-1}(\Pi + \Psi^{-1})$. Let $\tilde{\Pi}$ be the diagonal matrix defined by $\tilde{\pi}_{ii} = \frac{1}{1 + (\sigma_{ii} - \psi)(\beta_i - \pi_i)}$ for all $i \in \{1, \ldots, n\}$. Then $\tilde{\pi}_{ii} \in (0, 1)$

for all i, and $\tilde{\Pi}\Pi = I - \tilde{\Pi}$. It follows that:

$$M^{-1} = \tilde{\Pi}(\Pi + \Psi^{-1}) = I - \tilde{\Pi} + \tilde{\Pi}\Psi^{-1} = I + \tilde{\Pi}(\Psi^{-1} - I) \tag{23}$$

Since Ψ is symmetric positive definite, so is Ψ^{-1}, and $(\Psi^{-1} - I)$ is positive semidefinite (with eigenvalue 0 corresponding to the Perron eigenvector of Ψ). Hence, the eigenvalues of these matrices are real and nonnegative. Consider the matrices $\tilde{\Pi}(\Psi^{-1}-I) = \tilde{\Pi}^{1/2}[\tilde{\Pi}^{1/2}(\Psi^{-1}-I)]$ and $\tilde{\Pi}^{1/2}(\Psi^{-1}-I)\tilde{\Pi}^{1/2}$. These matrices share the same eigenvalues, since for any $n \times n$ matrices A and B, the eigenvalues of AB coincide with those of BA. Here, $\tilde{\Pi}^{1/2}$ denotes the diagonal matrix with entries $(\tilde{\Pi}^{1/2})_{ii} = \sqrt{\tilde{\pi}_{ii}}$. The matrix $\tilde{\Pi}^{1/2}(\Psi^{-1} - I)\tilde{\Pi}^{1/2}$ is symmetric and positive semidefinite, implying that it has only real and nonnegative eigenvalues. Consequently, both M and M^{-1} have real and positive eigenvalues (the zero eigenvalue of $(\Psi^{-1} - I)$ maps to zero under the congruence, contributing the unit eigenvalue of M).

Since Ψ is row-stochastic, $|\lambda(\Psi)| \leq 1$, which implies $\lambda(\Psi^{-1}) \geq 1$. Subtracting I from Ψ^{-1} reduces the eigenvalues by 1, so $\lambda(\Psi^{-1} - I) \geq 0$. From the preceding result, $\lambda(\tilde{\Pi}(\Psi^{-1} - I)) \geq 0$, which implies $\lambda(I + \tilde{\Pi}(\Psi^{-1} - I)) \geq 1$. Consequently, $\lambda(M^{-1}) \geq 1$, and hence all eigenvalues of M are real and lie within $(0, 1]$. Furthermore, because M has row sum 1, at least one eigenvalue equals 1. This follows from the definition of row-stochasticity and the Gershgorin Disc Theorem, as proved in Horn and Johnson (2012, p. 391), which implies $\rho(M) \leq 1$ for any square substochastic matrix M. If M is stochastic, then $\rho(M) = 1$, as M has the eigenvector $\mathbb{1} \coloneqq (1, \ldots, 1)'$: $M\mathbb{1} = \mathbb{1}$. The number of eigenvalues equal to 1 and their associated linearly independent eigenvectors equals the number of essential communication classes, satisfying condition (ii) of the lemma.

By the Perron–Frobenius theorem, the spectral radius $\rho(M) \geq 0$ of a nonnegative matrix M is an eigenvalue of M for which a real nonnegative eigenvector exists. If M is irreducible, then $\rho(M)$ is a simple eigenvalue and is positive. The theorem also applies to the transposed matrix M', so M has a left nonnegative eigenvector w' such that $w'M = \rho(M)w'$. In addition to $\rho(M)$, a nonnegative matrix may have other eigenvalues λ with $|\lambda| = \rho(M)$. If λ is an eigenvalue of a nonnegative matrix M with $|\lambda| = \rho(M)$, then the algebraic and geometric multiplicities of λ coincide (all Jordan blocks corresponding to λ are trivial). Therefore, M^t converges, i.e., $M^\infty \coloneqq \lim_{t\to\infty} M^t$ exists, and since 1

is an eigenvalue of M, $M^\infty \neq \mathbf{0}$. It is straightforward to verify that $\Phi(\infty) \coloneqq M^\infty \Phi(0)$ is a steady state, as $M\Phi(\infty) = MM^\infty\Phi(0) = M^\infty\Phi(0) = \Phi(\infty)$.

Extension to the positive semidefinite case. The preceding argument uses Ψ^{-1} and therefore requires positive definiteness. To treat symmetric positive semidefinite Ψ, define, for $\varepsilon \in (0,1)$,

$$\Psi_\varepsilon \coloneqq (1-\varepsilon)\Psi + \varepsilon I.$$

Because Ψ is symmetric, nonnegative, and row-stochastic, so is Ψ_ε. Since I commutes with Ψ, the eigenvalues of Ψ_ε are

$$(1-\varepsilon)\lambda_i(\Psi) + \varepsilon, \qquad i = 1, \ldots, n.$$

As Ψ is symmetric positive semidefinite, each $\lambda_i(\Psi) \in [0,1]$, so every eigenvalue of Ψ_ε lies in $[\varepsilon, 1]$. Hence Ψ_ε is symmetric positive definite for every $\varepsilon > 0$. Let

$$M_\varepsilon \coloneqq \Psi_\varepsilon(I + \Pi\Psi_\varepsilon)^{-1}(I + \Pi).$$

By Part a, all eigenvalues of M_ε lie in $(0,1]$. Moreover, $M_\varepsilon \to M$ as $\varepsilon \to 0$, so by continuity of eigenvalues, every eigenvalue of M lies in $[0,1]$. Finally, M is row-stochastic, hence $\|M^t\|_\infty = 1$ for all $t \in \mathbb{N}$. Therefore eigenvalue 1 cannot have a nontrivial Jordan block, since such a block would make $\|M^t\|_\infty$ grow polynomially in t. Lemma 1 then implies that M^t converges.

Part b: Divergence. Consider uniform incentives $(\beta_i - \pi_i)\psi_{ii} = c > 0$ for all i, so that $\tilde{\Pi} = (1/(1+c))\, I$. Then:

$$M = \Psi(I + c\Psi)^{-1}(I + cI) = (1+c)\,\Psi(I + c\Psi)^{-1} \tag{24}$$

Since Ψ and $(I + c\Psi)^{-1}$ commute (they are polynomials in Ψ), the eigenvalues of M are:

$$\lambda_k(M) = \frac{(1+c)\,\lambda_k(\Psi)}{c + \lambda_k(\Psi)} \tag{25}$$

For an eigenvalue $\lambda_k(\Psi) = \alpha + i\beta$ satisfying $\Re(\lambda_k) < |\lambda_k|^2$ (i.e., $\alpha < \alpha^2 + \beta^2$, writing $r = |\lambda_k|^2$), compute:

$$|\lambda_k(M)|^2 = \frac{(1+c)^2\, r}{|c + \lambda_k|^2} = \frac{(1+c)^2\, r}{c^2 + 2c\alpha + r} \tag{26}$$

This exceeds 1 when $(1+c)^2 r > c^2 + 2c\alpha + r$, i.e., $c(2r + cr - 2\alpha) > 0$. For sufficiently small $c > 0$, the dominant term is $2c(r - \alpha) > 0$, which holds since $r = |\lambda_k|^2 > \Re(\lambda_k) = \alpha$ by hypothesis. More precisely, $|\lambda_k(M)|^2 > 1$ for all $c \in (0,\, 2(r-\alpha)/(1-r))$ when $r < 1$. Hence $\rho(M) > 1$ and the dynamics diverge. □

Proof of Theorem 1

Part 1(a): Consensus within essential classes. Within each essential class L_k, the submatrix M_{kk} is row-stochastic and nonnegative. Since L_k is a communication class, M_{kk} is irreducible. Moreover, the diagonal entries of M_{kk} are strictly positive (inherited from the strictly positive diagonals of Ψ_{kk} and the structure of M), so M_{kk} is primitive (irreducible and aperiodic).

By the Perron–Frobenius theorem for primitive row-stochastic matrices (Horn and Johnson 2012, Theorem 8.5.1), M_{kk}^t converges to $\mathbb{1}\, w'_{|L_k}$, where $w_{|L_k}$ is the unique left eigenvector of M_{kk} for eigenvalue 1, normalized so that $\sum_{i \in L_k} w_i = 1$.

Under Assumption 2, $(\beta_i - \pi_i)\psi_{ii} = \alpha_k$ is constant across all $i \in L_k$, so $\Pi_{kk} = \alpha_k I$ within each essential class. Let $v_{|L_k}$ denote the left eigenvector of Ψ_{kk} for eigenvalue 1, normalized to sum to one. Then:

$$v'_{|L_k} M_{kk} = v'_{|L_k} \Psi_{kk}(I + \alpha_k \Psi_{kk})^{-1}(I + \alpha_k I) = v'_{|L_k}(I + \alpha_k \Psi_{kk})^{-1}(1 + \alpha_k) = v'_{|L_k} \tag{27}$$

where the second equality uses $v'_{|L_k}\Psi_{kk} = v'_{|L_k}$, and the third uses the scalar factorization: for any eigenvector v' of Ψ_{kk} with eigenvalue 1, $(I + \alpha_k\Psi_{kk})^{-1}(1+\alpha_k)v = (1+\alpha_k)/(1+\alpha_k) \cdot v = v$. Hence $w_{|L_k} = v_{|L_k}$, and the long-run consensus within L_k is:

$$c_k = v'_{|L_k}\, \Phi(0)_{|L_k} \tag{28}$$

Part 1(b): Long-run traits of inessential dynasties. Since M^t converges, the limit $\Phi(\infty) = M^\infty \Phi(0)$ exists and satisfies $M\Phi(\infty) = \Phi(\infty)$. By Proposition 3, in any steady state the displayed

traits coincide with true traits: $\Phi^{d*}(\infty) = \Phi(\infty)$. The trait formation process therefore gives $\Phi(\infty) = \Psi\,\Phi^{d*}(\infty) = \Psi\,\Phi(\infty)$.

Partitioning into essential classes $L_1, \ldots, L_K$ and the inessential set $\mathcal{R}$, the identity $\Phi(\infty) = \Psi\,\Phi(\infty)$ restricted to $\mathcal{R}$ reads:

$$\Phi(\infty)_{|\mathcal{R}} = \Psi_{\mathcal{R}\mathcal{R}}\,\Phi(\infty)_{|\mathcal{R}} + \sum_{k=1}^{K} \Psi_{\mathcal{R}k}\,\Phi(\infty)_{|L_k} \tag{29}$$

From Part (a), $\Phi(\infty)_{|L_k} = c_k\,\mathbb{1}_{|L_k}$. Hence:

$$(I - \Psi_{\mathcal{R}\mathcal{R}})\,\Phi(\infty)_{|\mathcal{R}} = \sum_{k=1}^{K} \Psi_{\mathcal{R}k}\,\mathbb{1}_{|L_k}\,c_k \tag{30}$$

Since $\Psi_{\mathcal{R}\mathcal{R}}$ is a substochastic matrix (its row sums are strictly less than one, as some weight goes to essential classes), $\rho(\Psi_{\mathcal{R}\mathcal{R}}) < 1$ by the Gershgorin Disc Theorem. Hence $(I - \Psi_{\mathcal{R}\mathcal{R}})$ is invertible with Neumann series $(I - \Psi_{\mathcal{R}\mathcal{R}})^{-1} = \sum_{t=0}^{\infty} \Psi_{\mathcal{R}\mathcal{R}}^t$, which is nonnegative. Therefore:

$$\Phi(\infty)_{|\mathcal{R}} = \Gamma\,\ell \tag{31}$$

where $\Gamma \coloneqq (I - \Psi_{\mathcal{R}\mathcal{R}})^{-1}(\Psi_{\mathcal{R}1}\mathbb{1}_{|L_1}, \ldots, \Psi_{\mathcal{R}K}\mathbb{1}_{|L_K})$ and $\ell = (c_1, \ldots, c_K)'$. To verify that Γ is row-stochastic, note that Ψ being row-stochastic implies $\Psi_{\mathcal{R}\mathcal{R}}\,\mathbb{1} + \sum_k \Psi_{\mathcal{R}k}\,\mathbb{1}_{|L_k} = \mathbb{1}$, so $(I - \Psi_{\mathcal{R}\mathcal{R}})\,\mathbb{1} = \sum_k \Psi_{\mathcal{R}k}\,\mathbb{1}_{|L_k}$. Pre-multiplying both sides by $(I - \Psi_{\mathcal{R}\mathcal{R}})^{-1}$ gives $\mathbb{1} = \Gamma\,\mathbb{1}$, confirming row-stochasticity. □

Proof of Proposition 5

When $(\beta_i - \pi_i) = 0$ for all i, $\Pi = 0$ and $M = \Psi$. Since Ψ is irreducible with strictly positive diagonal entries, it is primitive (irreducible and aperiodic). By the Perron–Frobenius theorem for primitive row-stochastic matrices, eigenvalue 1 is simple and all other eigenvalues satisfy $|\lambda| < 1$. In particular, M^t converges when $\Pi = 0$.

The map $(\beta_1 - \pi_1, \ldots, \beta_n - \pi_n) \mapsto M = \Psi(I + \Pi\Psi)^{-1}(I + \Pi)$ is continuous in a neighborhood of the origin, since $(I + \Pi\Psi)$ is invertible at $\Pi = 0$ (where it equals I) and invertibility is an open condition. Since eigenvalues are continuous functions of matrix entries (Horn and Johnson 2012,

Appendix D), and the eigenvalues of M at $\Pi = 0$ satisfy the convergence conditions of Lemma 1 (simple eigenvalue 1, all others strictly inside the open unit disk $\{z \in \mathbb{C} : |z| < 1\}$), there exists a neighborhood $\mathcal{N}(\mathbf{0} \mid \Psi) \subset \mathbb{R}^n_+$ such that for all $(\beta_i - \pi_i) \in \mathcal{N}(\mathbf{0} \mid \Psi)$, M retains the same spectral properties and M^t converges. □

Proof of Proposition 6

The convergence of Ψ^t follows from Ψ being symmetric positive definite and row-stochastic. Since Ψ is also irreducible, the Perron eigenvalue 1 is simple, hence

$$1 = \lambda_1(\Psi) > \lambda_2(\Psi) \geq \cdots \geq \lambda_n(\Psi) > 0.$$

The convergence of M^t was established in Proposition 4. The objective is now to show that M^t converges strictly slower than Ψ^t.

Since $(\beta_i - \pi_i)\psi_{ii} > 0$ for all i,

$$0 < \tilde{\pi}_{ii} = \frac{1}{1 + (\beta_i - \pi_i)\psi_{ii}} < 1.$$

Hence every eigenvalue of $\tilde{\Pi}$ lies strictly in $(0, 1)$.

Now $\Psi^{-1} - I$ is symmetric positive semidefinite. For every $k \geq 2$, the simplicity of the Perron root implies $\lambda_k(\Psi) < 1$, so

$$\lambda_k(\Psi^{-1} - I) = \lambda_k(\Psi)^{-1} - 1 > 0.$$

Applying Ostrowski's theorem (Horn and Johnson 2012, p. 413) to $\tilde{\Pi}^{1/2}(\Psi^{-1} - I)\tilde{\Pi}^{1/2}$ yields

$$\lambda_k\Big(\tilde{\Pi}^{1/2}(\Psi^{-1} - I)\tilde{\Pi}^{1/2}\Big) = \theta_k \, \lambda_k(\Psi^{-1} - I)$$

with $0 < \theta_k < 1$ for all $k \geq 2$. Therefore,

$$\lambda_k(M^{-1}) = 1 + \lambda_k\Big(\tilde{\Pi}(\Psi^{-1} - I)\Big) < 1 + \lambda_k(\Psi^{-1} - I) = \lambda_k(\Psi^{-1}),$$

and hence

$$|\lambda_k(M)| > |\lambda_k(\Psi)| \qquad \text{for all } 2 \leq k \leq n.$$

This proves the claim. □

Proof of Proposition 7

Part a. The long-run consensus within essential class L_k is determined by M_{kk}^{∞}, which converges to $\mathbb{1}\, w'_{|L_k}$ where $w_{|L_k}$ is the left eigenvector of M_{kk} for eigenvalue 1. By Assumption 2, $\Pi_{kk} = \alpha_k I$ for some $\alpha_k > 0$ within each essential class (since $(\beta_i - \pi_i)\psi_{ii}$ is constant). Then for $v_{|L_k}$, the left unit eigenvector of Ψ_{kk}: $v'_{|L_k}\Pi_{kk} = \alpha_k\, v'_{|L_k}$, so $v'_{|L_k}M_{kk} = v'_{|L_k}\Psi_{kk}(I + \Pi_{kk}\Psi_{kk})^{-1}(I + \Pi_{kk}) = v'_{|L_k}(I+\alpha_k\Psi_{kk})^{-1}(I+\alpha_k I) = v'_{|L_k}$, where the last step uses $v'_{|L_k}\Psi_{kk} = v'_{|L_k}$ and the scalar factorization under uniform Π_{kk}. Hence $w_{|L_k} = v_{|L_k}$: the left eigenvectors of M_{kk} and Ψ_{kk} coincide. The consensus is therefore $c_k = v'_{|L_k}\,\Phi(0)_{|L_k}$, where $v_{|L_k}$ is a function of Ψ_{kk} alone. The matrix Ψ has diagonal entries $\psi_{ii} = \sigma_{ii} - \psi$ and off-diagonal entries determined by the socialization structure, so Ψ depends on ψ but not on $\boldsymbol{\pi}$. Since $v_{|L_k}$ is a function of Ψ_{kk} alone, c_k depends on ψ but is independent of $\boldsymbol{\pi}$.

Part b. By definition, $M = \Psi(I + \Pi\Psi)^{-1}(I + \Pi)$, where Π is a diagonal matrix with diagonal entries $(\beta_i - \pi_i)\psi_{ii}$. Both Ψ (through ψ) and Π (through $\boldsymbol{\pi}$) enter the expression for M, so the eigenvalues of M—and in particular $\lambda_2(M)$—depend on both instruments.

Part c. By the Perron–Frobenius theorem applied to the irreducible, row-stochastic submatrix Ψ_{kk}, the left unit eigenvector $v_{|L_k}$ has strictly positive entries summing to one. Therefore $c_k = \sum_{i\in L_k} v_i\,\phi_i(0)$ is a convex combination of the initial traits $\{\phi_i(0)\}_{i\in L_k}$, and hence $c_k \in \mathrm{conv}(\Phi(0)_{|L_k})$. □

Proof of Proposition 8

The proof proceeds in three steps: computing the gradient of $\lambda_2(M)$ with respect to $\boldsymbol{\pi}$ via eigenvalue perturbation theory, establishing the sign under symmetric Ψ, and applying the Karush–Kuhn–Tucker conditions to the constrained optimization problem.

Step 1: Eigenvalue perturbation. Recall from the proof of Proposition 4 that $M^{-1} = I + \tilde{\Pi}(\Psi^{-1} - I)$, where $\tilde{\Pi}$ is the diagonal matrix with entries $\tilde{\pi}_{ii} = 1/d_i$, $d_i \coloneqq 1 + (\beta_i - \pi_i)\psi_{ii}$. The eigenvalues of M^{-1} are $\mu_k = 1/\lambda_k(M)$, with the same eigenvectors. Let λ_2 be a simple eigenvalue of M with right eigenvector x and left eigenvector y, normalized so that $y'x = 1$. By the classical

perturbation formula for simple eigenvalues (Horn and Johnson 2012, Theorem 6.3.12):

$$\frac{\partial(1/\lambda_2)}{\partial \pi_i} = y' \, \frac{\partial M^{-1}}{\partial \pi_i} \, x \tag{32}$$

Since $\tilde{\pi}_{ii} = 1/d_i$ and $\partial d_i / \partial \pi_i = -\psi_{ii}$, one obtains $\partial \tilde{\pi}_{ii} / \partial \pi_i = \psi_{ii}/d_i^2$. Hence $\partial M^{-1}/\partial \pi_i = (\psi_{ii}/d_i^2)\, e_i e_i' (\Psi^{-1} - I)$, where e_i is the i-th standard basis vector. Substituting:

$$\frac{\partial(1/\lambda_2)}{\partial \pi_i} = \frac{\psi_{ii}}{d_i^2} \, y_i \left[(\Psi^{-1} - I)\, x \right]_i \tag{33}$$

From $M^{-1}x = (1/\lambda_2)\, x$, the identity $\tilde{\Pi}(\Psi^{-1} - I)\, x = (1/\lambda_2 - 1)\, x$ holds. Pre-multiplying by $\tilde{\Pi}^{-1}$ and extracting the i-th component: $[(\Psi^{-1} - I)\, x]_i = d_i \, (1/\lambda_2 - 1)\, x_i$. Therefore:

$$\frac{\partial(1/\lambda_2)}{\partial \pi_i} = \frac{\psi_{ii}}{d_i} \left(\frac{1}{\lambda_2} - 1 \right) x_i \, y_i \tag{34}$$

Applying the chain rule, $\partial \lambda_2 / \partial \pi_i = -\lambda_2^2 \, \partial(1/\lambda_2)/\partial \pi_i$, yields:

$$\frac{\partial \lambda_2}{\partial \pi_i} = -\frac{\psi_{ii} \, \lambda_2 \, (1 - \lambda_2)}{d_i} \, x_i \, y_i \tag{35}$$

establishing Part a.

Step 2: Sign under symmetric Ψ. When Ψ is symmetric positive definite, M^{-1} is similar to the symmetric matrix $\tilde{\Pi}^{1/2} \, M^{-1} \, \tilde{\Pi}^{-1/2} = I + \tilde{\Pi}^{1/2} (\Psi^{-1} - I) \tilde{\Pi}^{1/2}$ via the similarity transformation $Q = \tilde{\Pi}^{-1/2}$. If z is an eigenvector of the symmetric matrix corresponding to $1/\lambda_2$, then $x = \tilde{\Pi}^{-1/2} z$ is the right eigenvector and $y = \tilde{\Pi}^{1/2} z$ is the left eigenvector of M^{-1} (and hence of M). Therefore $x_i \, y_i = (\tilde{\pi}_{ii})^{-1/2} \, z_i \cdot (\tilde{\pi}_{ii})^{1/2} \, z_i = z_i^2 \geq 0$, with equality if and only if $z_i = 0$. Since $\lambda_2 \in (0, 1)$ and $(1 - \lambda_2) > 0$, it follows that $\partial \lambda_2 / \partial \pi_i \leq 0$.

Step 3: KKT conditions. The Lagrangian of the speed subproblem (16) is:

$$\mathcal{L}(\boldsymbol{\pi}, \mu, \boldsymbol{\nu}, \boldsymbol{\eta}) = \lambda_2(M(\boldsymbol{\pi})) + \mu \Big(\sum_{i \in N} \pi_i - B \Big) - \sum_{i \in N} \nu_i \, \pi_i + \sum_{i \in N} \eta_i \, (\pi_i - \beta_i) \tag{36}$$

where $\mu \geq 0$ is the multiplier on the budget constraint, $\nu_i \geq 0$ on the non-negativity constraint, and $\eta_i \geq 0$ on the upper bound $\pi_i < \beta_i$. The first-order conditions are:

$$\frac{\partial \lambda_2}{\partial \pi_i} + \mu - \nu_i + \eta_i = 0 \quad \text{for all } i \in N \tag{37}$$

with complementary slackness: $\nu_i \, \pi_i = 0$ and $\eta_i \, (\beta_i - \pi_i) = 0$.

At an interior optimum ($\pi_i^* \in (0, \beta_i)$ for all i), $\nu_i = \eta_i = 0$, so $-\psi_{ii} \, \lambda_2 \, (1 - \lambda_2) \, x_i \, y_i / d_i = -\mu$ for all $i \in N$, establishing Part b.

For Part c, consider a stationary point with $\mu > 0$. If $x_i \, y_i = 0$, then $\partial \lambda_2 / \partial \pi_i = 0$, so the first-order condition becomes

$$\mu - \nu_i + \eta_i = 0.$$

Since $\mu > 0$ and $\eta_i \geq 0$, it follows that

$$\nu_i = \mu + \eta_i > 0.$$

By complementary slackness, $\nu_i \, \pi_i^* = 0$, hence $\pi_i^* = 0$. □

Declaration of generative AI and AI-assisted technologies in the manuscript preparation process

During the preparation of this manuscript, the author used ChatGPT to improve the clarity and presentation of the text prior to submission. After using these tools, the author reviewed and edited the content as needed and takes full responsibility for the content of the published article.

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