When Open-Source and Proprietary Innovation Suppress Each Other

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Abstract

How do firm pricing and decentralized open-source software (OSS) development shape each other? I develop a dynamic quality-ladder model in which a proprietary firm chooses paid quality investment, user-developers contribute to OSS, and consumers choose between priced proprietary software and free OSS. When the proprietary product leads, equilibrium pricing caps a contributor's private gain from closing the quality gap, lowering the probability that OSS improves. Conversely, an OSS improvement that matches a newly created lead before pricing can remove the firm's current return and change its action from investing to waiting. The effects are linked: a tighter cap makes the lead more likely to survive but reduces OSS's ability to deter investment. Mutual suppression therefore occurs only at interior OSS-improvement probabilities. Outside parity, OSS affects firm investment through continuation values; in the no-catch-up region, the investment target coincides with the proprietary-only target. The model connects product-market pricing, decentralized technology development, and persistent proprietary advantage.

Keywords: open-source software; proprietary software; software competition; permissive licensing; public goods; quality ladders; pricing; industrial organization.

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I Introduction

How does a firm's technology investment respond when a competing technology is developed outside the firm, and how does the firm's market conduct feed back into that technology's development? This two-way feedback connects firm investment, technology adoption and diffusion, and the persistence of market structure. Open-source software (OSS) makes the connection especially sharp. A proprietary firm competes with a decentralized OSS project for users, but it may also incorporate compatible OSS improvements. Product-market pricing can therefore change user-developers' incentives to improve the competing technology, while the speed of the OSS response changes the return to a proprietary quality investment.

This paper develops a dynamic quality-ladder model of that feedback. Its central result is that open-source and proprietary innovation can suppress each other through asymmetric channels linked by a shared OSS-response probability. A proprietary lead discourages OSS contribution because equilibrium pricing places an upper bound on a user-developer's private gain from an OSS improvement that closes the quality gap. I call closing that gap *catch-up* and call the pricing-induced upper bound the *contributor-benefit cap*. Conversely, OSS can suppress proprietary lead initiation by matching a paid quality step before the firm monetizes it; this is *same-period OSS matching*. I use *mutual suppression* for two strict local comparisons at a common initial parity quality: the contributor-benefit cap lowers the equilibrium OSS-improvement probability, and same-period OSS matching changes the firm's action from investing to waiting relative to the payoff from carrying a newly initiated lead into current pricing.

Enterprise database competition makes the interaction concrete. PostgreSQL is available under a permissive license and depends on code authors, reviewers, testers, committers, release managers, and other contributors [PostgreSQL Global Development Group, 2026b,a]. Priced proprietary databases supply competing commercial technologies. A proprietary capability lead can change the private value of closing an OSS quality gap, while an OSS release that promptly matches a commercial improvement can remove the temporary advantage be-

fore it is monetized. The paper develops a theory benchmark, and this setting supplies its institutional illustration.

The model contains one proprietary firm, an OSS project, and heterogeneous user-developers. In each period the firm first decides whether to pay for a quality step. A fresh cohort of user-developers then plays a volunteer's dilemma in which one contribution is sufficient for an OSS step. Consumers choose last between priced proprietary software and free OSS. When OSS moves ahead, compatible reuse permits the firm to incorporate the open improvement before the next investment decision. The firm is forward-looking, while each contributor cohort solves the current volunteer problem. This timing separates the firm's investment decision, decentralized technology production, product-market pricing, and subsequent technology adoption.

Backward induction gives the proprietary-to-OSS channel. At parity, a contributor receives the full use value of an OSS improvement. Behind a proprietary product, the firm's equilibrium price limits that gain to the valuation of the marginal proprietary buyer. Holding OSS quality and the contributor population fixed, this transmission from the product-market valuation cutoff into a contributor's private benefit, which I call *contributor pass-through*, weakly lowers the OSS-improvement probability. *Complete OSS suppression* occurs where the contributor-benefit cap reduces the improvement probability to zero while contribution at parity remains feasible. The result derives the price-mediated effect from the product-market equilibrium.

The OSS-to-proprietary channel operates through the return to the firm's initial investment. At parity, an OSS step after proprietary investment matches the paid step. A match before pricing changes the investment payoff by the current operating return plus the continuation-value difference between a newly created one-step lead and the matched state. A match after pricing changes that continuation-value difference. When these differences are nonnegative, they are private losses from matching. The same OSS step after the firm waits instead advances the common quality level through compatible reuse. Same-period

matching can therefore reverse lead initiation, while a proprietary start partly protects itself by creating the lead that activates the contributor-benefit cap.

The shared no-improvement probability creates the paper's central tension. A cap that discourages OSS also raises the probability that a newly initiated proprietary lead carries beyond the contribution stage. The cost interval on which same-period matching reverses the firm's action has length equal to the residual OSS-response probability times the gross one-period return the firm would earn if that lead were certain to survive. It shrinks as the contributor-benefit cap raises the no-improvement probability and has zero length at the lead-regime feasibility cutoff. Mutual suppression is an interior outcome: OSS improvement remains feasible while same-period matching reverses the firm's action from investing to waiting.

Away from parity, OSS progress affects proprietary investment through a continuation-value cross-difference. Lower incremental continuation values discourage investment, while higher incremental continuation values encourage it. In the *exact no-catch-up region*, where no contributor type can profitably close the proprietary quality gap, the baseline OSS transition is action-independent and the firm's investment target is the proprietary-only target. The response is also zero whenever firm value separates additively into proprietary-quality and OSS-quality terms at all compared successor states.

The one-period and dynamic analyses are nested. Pricing and contribution are solved in the same way at every state, and setting $\beta = 0$ in the firm's Bellman comparison recovers the one-period investment thresholds. Positive β adds action-specific continuation values and extends the OSS-to-firm channel beyond parity. Dynamics then turn the local interaction into a race between two destination sets: parity states at which the firm no longer initiates a lead and strict-lead states from which contributors can no longer close the gap. Under bounded contributor support, every path eventually reaches one of three outcomes: same-period OSS matching before a lead carries over, catch-up after a lead carries over, or entry into the exact no-catch-up region. These paths distinguish temporary proprietary advantage,

delayed technology diffusion, and persistent proprietary dominance.

The comparative statics hold contributor composition fixed to isolate the contributor-benefit channel. Recruitment, experience, and sponsorship shift contributor composition and shape aggregate OSS activity. The welfare analysis follows how stronger OSS incentives change both the contributor transition and the firm's privately chosen investment path, thereby determining discounted user value and real costs.

The paper makes three contributions:

(i) it makes firm conduct and decentralized technology development jointly endogenous by deriving contributor pass-through from equilibrium product pricing and showing why mutual suppression is necessarily interior;

(ii) it gives a state-and-timing classification of proprietary investment responses, separating same-period matching, post-pricing matching, strict leads, and the no-catch-up region; and

(iii) it connects these local mechanisms to technology-diffusion paths, the persistence of proprietary advantage, paid proprietary investment, and welfare over time.

Sections III–V solve the model from the one-period mechanism to the classification of dynamic paths. Section VI develops comparative statics, welfare, and empirical interpretation. Proofs of all formal results are in the Appendix.

II Related Literature

Contributor incentives and OSS production. Lerner and Tirole [2002] explain OSS participation using own-use benefits, career concerns, reputation, and firm strategy. Johnson [2002] provides the closest foundation for the volunteer stage: OSS is a privately provided public good subject to free riding, duplicated effort, and possible underprovision. I retain the own-use benchmark in which one successful contribution is enough to produce the public

good, the standard *best-shot* technology, to isolate a different object. The proprietary product's equilibrium price changes the private return to the same contributor at the same quality state. The fixed-belief ordering is contributor by contributor, while the aggregate result resolves the symmetric volunteer equilibrium [Hirshleifer, 1983; Diekmann, 1985; Bergstrom et al., 1986].

Dynamic competition between OSS and proprietary software. Casadesus-Masanell and Ghemawat [2006] study forward-looking Linux–Windows competition driven by demand dynamics and network effects. Athey and Ellison [2014] model OSS quality together with a persistent community state and show that a commercial firm may price strategically to slow OSS growth. The paper's distinctive link is direct pass-through from the product-market cutoff to a user-developer's contribution payoff, combined with a state-and-timing classification of how OSS progress changes a paid proprietary step. The firm's action creates the lead that activates the contributor-benefit cap, so the two directions are jointly determined through the same state transition. This feedback yields the interior mutual-suppression result: a stronger cap both discourages OSS and raises the probability that a newly initiated proprietary lead survives the response that could reverse it. Fresh contributor cohorts make this link transparent by removing intertemporal learning and contributor composition feedback from the baseline mechanism.

Licensing, appropriation, and strategic contribution. August et al. [2020] show that licensing and strategic contribution can raise or lower effort and welfare depending on firm capabilities and the competitive environment. Related models endogenize open release, R&D, or platform investment [Economides and Katsamakas, 2006; August et al., 2013; Tesoriere and Balletta, 2017; Balletta and Tesoriere, 2020; Casadesus-Masanell and Llanes, 2015]. I instead hold the baseline reuse rule fixed and separate two reduced-form changes. Contributor pass-through changes private benefits; the rule that raises proprietary quality whenever OSS would otherwise move ahead changes state transitions.

Innovation races and lead persistence. The path analysis relates to preemption and step-by-step innovation [Gilbert and Newbery, 1982; Fudenberg and Tirole, 1985; Aghion et al., 2001; Denicolò and Zanchettin, 2012]. Here, a decentralized public-good project follows the proprietary firm, and the leader's market position changes the follower's success probability. The general result bounds the probability that catch-up occurs within any stated finite horizon, which is the paper's sense of *effective lead persistence*. With bounded contributor support, entry into the no-catch-up region creates exact irreversibility.

State-dependent innovation responses. The paper's central question is related to models in which competition can either discourage an incumbent by reducing appropriable returns or stimulate investment through an escape-competition or strategic-response effect [Aghion et al., 2001; Reisinger et al., 2014; Zhou and Choudhary, 2022]. The model maps the firm's response to OSS progress by state and within-period timing. At parity it depends on whether matching occurs before or after current pricing. At a strict lead it depends on a continuation-value cross-difference. The baseline investment target is exactly neutral to OSS quality in the no-catch-up region, while a perturbed-success cross-difference is zero when its successor states remain in that region. This state-and-timing map is the paper's connection to the broader ambiguous- competition literature.

Contributor composition and commercial participation. Commercial competition sorts OSS participation across contributor types. Nagaraj and Piezunka [2024] find that Google Maps entry discouraged new OpenStreetMap contributors but increased activity by established contributors. Haese and Peukert [2025] find that Microsoft's adoption of Chromium increased development activity, mainly through Microsoft and more modestly through external contributors. These findings motivate the paper's composition decomposition: the pass-through theorem holds within a fixed contributor distribution, while recruitment, experience, and sponsorship shape aggregate activity.

Welfare and quantitative analysis. The welfare analysis weighs OSS access and pooled development against the real cost of duplicated or misallocated decentralized effort [Johnson, 2002; Llanes and de Elejalde, 2013; Tesoriere and Balletta, 2017]. Nagle [2019] estimates productivity gains from OSS use for firms with complementary capabilities, and Hoffmann et al. [2024] documents the large value created by repeated reuse. These results motivate the social value of higher OSS quality and its strategic implications. Gortmaker [2026] estimates a substantially richer industry-equilibrium model in which firms choose software inputs and developer labor. The present paper develops a theory benchmark and uses numerical exercises to map its mechanisms and conditional orders.

III Model

Quality ladder. Time is discrete and indexed by $t = 0, 1, 2, \ldots$. At the start of a period, $q^I_{t,ini}$ denotes proprietary quality and $q^O_{t,ini}$ denotes open-source quality; the superscripts I and O refer to the proprietary incumbent and OSS, respectively. Both qualities lie on the nonnegative integer ladder,

$$q^I_{t,ini}, q^O_{t,ini} \in \mathbb{Z}_{\geq 0}.$$

A quality source is always identified by the superscript $k \in \{I, O\}$. For realized paths, the subscript lists calendar time first and the within-period stage second. Bare q^I and q^O are reserved for stationary function arguments and dummy state coordinates. A product at quality q^k, for $k \in \{I, O\}$, generates gross utility from product quality $\psi(q^k)$. Normalize $\psi(0) = 0$.

Assumption 1 (Diminishing returns to quality)**.** $\psi : \mathbb{Z}_{\geq 0} \to \mathbb{R}_{\geq 0}$ is strictly increasing and strictly concave. For either product $k \in \{I, O\}$, the one-step gain $\psi(q^k + 1) - \psi(q^k)$ decreases strictly to zero as q^k rises.

The assumption says that another release always improves the product, but the incremental improvement becomes less valuable as the product matures. It is what eventually

stops both paid and volunteer development.

Users and demand. There is a mass $N > 0$ of users. User j values one unit of quality at v_j, so utility from a quality-q product before paying its price is $v_j\psi(q)$. Valuations have cumulative distribution function F_V on $[0, v_H]$, where v_H is the highest possible valuation. OSS is available at price zero; the proprietary firm chooses a nonnegative price p and has zero marginal serving cost. At final qualities, user j obtains

$$\max\{v_j\psi(q^O_{t,fin}), v_j\psi(q^I_{t,fin}) - p, 0\}.$$

The three entries are, respectively, utility from OSS, utility from the proprietary product net of price, and the outside option. The user chooses the largest entry. Thus the proprietary firm's operating profit equals price times demand. Zero marginal serving cost represents negligible software replication cost and keeps the real proprietary development cost in the one-step investment cost c^I.

Assumption 2 (Regular proprietary demand)**.** F_V is continuously differentiable and its survival function $1 - F_V$ is strictly log-concave.

Firm and contributors. The firm chooses $a_t \in \{0, 1\}$ at the beginning of period t. Action $a_t = 1$ purchases one proprietary quality step at real cost $c^I > 0$; action $a_t = 0$ means that the firm waits. The firm discounts next-period profit by $\beta \in (0, 1)$.

After the firm moves, a newly drawn cohort of $n \geq 2$ potential OSS contributors plays a volunteer's dilemma: OSS gains one quality step if at least one member contributes, so additional simultaneous contributions do not create additional quality. Contributor j is also a user with valuation v_j and incurs opportunity cost c_j if contributing. The pair (v_j, c_j) has joint distribution H, valuation marginal F_V, and cost marginal F_C. Value and cost may be correlated within a contributor, while draws are independent across contributors and across periods. The cohort is small relative to the user market, $n \ll N$. Using F_V as both the user-

demand distribution and the contributor valuation marginal is a baseline normalization: the pricing problem uses the former, whereas the contribution problem uses the joint distribution H of contributor value and cost. At a fixed contribution-stage quality, the proprietary lead maps each contributor's valuation into the capped private benefit defined below.

Each contributor cohort is short-lived in the strategic sense. A contributor evaluates the current-period private use gain from the OSS step net of the current opportunity cost and carries no continuation value into later periods. Consequently, the OSS stage is a newly solved static volunteer equilibrium at each contribution-stage state, while the proprietary firm is the model's forward-looking player.

Assumption 3 (Baseline exact-cutoff regularity)**.** The following conditions hold:

(i) valuations are bounded above by $v_H < \infty$ and contribution costs are bounded below by $c_L > 0$;

(ii) the joint type distribution H is absolutely continuous; and

(iii) H assigns positive probability to every sufficiently small neighborhood of the high-valuation, low-cost corner (v_H, c_L).

Bounded type support generates exact finite cutoffs and the resulting absorbing no-catch-up region. Absolute continuity and the positive lower bound on cost make the benefit–cost-ratio distributions introduced below atomless. Positive mass near (v_H, c_L) supports OSS improvement below the support cutoffs. At a fixed contribution-stage quality, a proprietary lead weakly lowers each contributor's benefit–cost ratio. With unbounded tails, the corresponding objects are quality levels at which OSS success falls below a stated positive tolerance.

Assumption 4 (Compatible contemporaneous reuse)**.** If an OSS contribution would leave OSS ahead after development, the baseline treats the firm as incorporating that quality improvement contemporaneously and at zero integration cost before pricing.

The timing below applies the compatible-reuse transition after defining the within-period quality variables.

III.1 Within-Period Timing

Given Assumption 4, each period proceeds as follows.

(1) *Initial state.* Three stage labels appear as the second component of the subscript: *ini* is the initial state, *mid* is the state after the proprietary firm moves, and *fin* is the state after OSS contribution and any permissive catch-up or reuse. The initial pair in period t is $(q^I_{t,ini}, q^O_{t,ini})$.

(2) *Proprietary action.* The firm chooses a_t, producing the intermediate contribution-stage pair

$$q^I_{t,mid} = q^I_{t,ini} + a_t, \qquad q^O_{t,mid} = q^O_{t,ini}.$$

Only the proprietary coordinate can change at this stage: $a_t = 1$ moves it one quality step, while $a_t = 0$ leaves the initial pair unchanged. OSS has not yet had an opportunity to respond.

(3) *OSS contribution.* Write $\mathbf{1}\{A\}$ for the indicator that equals one when statement A is true. The contribution regime is

$$r_t = \mathbf{1}\{q^I_{t,mid} > q^O_{t,mid}\}.$$

Thus $r_t = 0$ means parity at the contribution stage, analogous to the neck-and-neck state in step-by-step innovation models [Aghion et al., 2001], and $r_t = 1$ means that the proprietary product leads. Let $s_t \in \{0, 1\}$ indicate whether OSS improves. Before compatible reuse, OSS quality would be $q^O_{t,mid} + s_t$, while proprietary quality remains $q^I_{t,mid}$.

(4) *Compatible reuse and final qualities.* Assumption 4 determines the final pair:

$$q^O_{t,fin} = q^O_{t,mid} + s_t, \qquad q^I_{t,fin} = \max\{q^I_{t,mid}, q^O_{t,fin}\}.$$

The first equality records the OSS step. The maximum in the second leaves a strict proprietary lead unchanged when OSS remains behind and raises proprietary quality to the OSS level when the OSS step would otherwise move ahead. Reuse therefore changes the final proprietary coordinate only on that overtaking branch.

(5) *Pricing and consumption.* Given the final quality pair, the firm chooses its static profit-maximizing price. Users then choose between OSS, the proprietary product, and the outside option according to their final-quality utilities. The resulting price and consumption choices determine the firm's current operating profit.

At the start of the next period, the final pair becomes the initial pair:

$$q^I_{t+1,ini} = q^I_{t,fin}, \qquad q^O_{t+1,ini} = q^O_{t,fin}.$$

This identity is the intertemporal link: the next period's firm observes the current final qualities. Every pricing and continuation state consequently belongs to the state space

$$S := \{(q^I, q^O) \in \mathbb{Z}^2_{\geq 0} : q^I \geq q^O\}.$$

The compatible-reuse transition maps an OSS overtake into the parity diagonal before the next period begins.

Information and equilibrium. The firm observes the quality state and the type distribution H. Each contributor observes their own (v_j, c_j) and the common distribution; realized contributor types are private information. The mass-N demand curve is deterministic, whereas OSS improvement is stochastic.

The solution concept combines three stage-specific choices:

(i) the firm uses a stationary Markov investment policy on S;

(ii) contributors use a symmetric cutoff equilibrium, meaning that each volunteers when their private benefit relative to cost exceeds the same equilibrium threshold, conditional on OSS quality and regime $r \in \{0, 1\}$; and

(iii) the firm chooses the static profit-maximizing price after final qualities are realized.

The results below characterize the unique symmetric cutoff equilibrium. At threshold equality, the tie-breaking convention assigns purchase to the consumer, nonparticipation to the contributor, and investment to the firm. The last rule selects among payoff-equivalent stationary actions. The next section solves these choices backward in the one-period game. When a comparison changes contributor incentives, a *re-solved equilibrium comparison* means that both the volunteer fixed point and the firm's Bellman policy are solved again under the changed primitives.

IV One-Period Mutual Suppression

Within-period question. Within a period, how do the proprietary firm and OSS contributors respond to one another, and when does that interaction prevent either side from advancing? The answer is the current-period game embedded in every state of the dynamic model. Setting $\beta = 0$ also turns that game into the firm's terminal-period investment benchmark. The pricing and contribution solutions remain exact components of the dynamic model; only the terminal investment rule ceases to apply when $\beta > 0$.

Question 1: What are the agents' best responses within a period? The within-period game follows the timing introduced in Section III:

(1) the firm chooses whether to buy a proprietary quality step;

(2) contributors decide whether OSS advances;

(3) compatible reuse raises proprietary quality to the OSS quality if OSS moves ahead; and

(4) the firm prices after final qualities are known, and users then make their consumption choices.

Solving backward, final pricing determines what an OSS improvement is worth to contributors. Contributor incentives then determine the probability that a paid proprietary step survives. The firm anticipates both objects when making the initial investment decision.

Step 1: pricing at final qualities. Fix a final quality pair $(q^I_{t,fin}, q^O_{t,fin}) \in S$. At parity, the two products have the same quality and OSS is free, so the proprietary firm earns zero operating profit. Any price that generates zero demand is payoff-equivalent; the selected price is $p = 0$.

Now suppose that the proprietary product has a strict final-quality lead. Let x denote the valuation of the consumer who is indifferent between buying the proprietary product and using OSS. The price p and the final quality advantage determine that valuation cutoff:

$$x = \frac{p}{\psi(q^I_{t,fin}) - \psi(q^O_{t,fin})}.$$

A consumer buys the proprietary product exactly when $v_j \geq x$. Thus the mass of proprietary buyers is $N[1 - F_V(x)]$, and current revenue is

$$N[\psi(q^I_{t,fin}) - \psi(q^O_{t,fin})]x[1 - F_V(x)].$$

This is the standard indifferent-consumer construction for competition between a zero-priced OSS product and a proprietary product [Casadesus-Masanell and Ghemawat, 2006].

The bracketed quality difference is strictly positive at a proprietary lead and scales the normalized revenue expression. Maximizing the normalized expression therefore determines

the revenue-maximizing cutoff used below. This is the known-demand posted-pricing benchmark studied by Segal [2003]; zero marginal replication cost and the multiplicative quality advantage make the normalized cutoff independent of the final quality pair.

Final-stage pricing result. Under Assumption 2, let x^* denote the unique revenue-maximizing valuation cutoff and let e^* denote operating profit per unit of final quality advantage at that cutoff. They are

$$x^* := \arg\max_{x \in [0, v_H]} x[1 - F_V(x)], \qquad e^* := Nx^*[1 - F_V(x^*)].$$

At every strict proprietary lead, the optimal cutoff is x^*, and the corresponding price and operating profit are

$$p^*(q^I_{t,fin}, q^O_{t,fin}) = x^*[\psi(q^I_{t,fin}) - \psi(q^O_{t,fin})], \qquad \pi(q^I_{t,fin}, q^O_{t,fin}) = e^*[\psi(q^I_{t,fin}) - \psi(q^O_{t,fin})],$$

If the optimal cutoff is interior, its first-order condition is $1 - F_V(x^*) = x^* f_V(x^*)$, where f_V is the density of F_V. At parity, optimized profit is zero and the selected payoff-irrelevant price is $p^* = 0$.

Both x^* and e^* are therefore equilibrium objects derived from the consumer valuation distribution. The pricing solution supplies these two objects to the earlier stages:

- x^* is the marginal buyer's valuation and therefore the maximum price reduction that a high-valuation OSS contributor receives from an OSS step; and
- e^* converts the gross-utility difference $\psi(q^I_{t,fin}) - \psi(q^O_{t,fin})$ into proprietary operating profit and therefore determines the firm's return to an additional quality step.

Step 2: OSS contribution anticipating pricing. Move backward to the contribution stage. The OSS quality available when contributors decide is $q^O_{t,mid}$. The regime $r \in \{0, 1\}$ records the market position at that point: $r = 0$ is parity and $r = 1$ is a proprietary lead.

Let $B_{j,r}(q^O_{t,mid})$ denote contributor j's private current-period benefit from an OSS quality step in regime r. The pricing solution from Step 1 implies

$$B_{j,0}(q^O_{t,mid}) = v_j[\psi(q^O_{t,mid}+1) - \psi(q^O_{t,mid})],$$
$$B_{j,1}(q^O_{t,mid}) = \min\{v_j, x^*\}[\psi(q^O_{t,mid}+1) - \psi(q^O_{t,mid})].$$

At parity, the contributor receives the full value of the OSS quality gain. Behind a proprietary lead, a low-valuation contributor receives that gain directly, while a proprietary buyer gains through the induced price reduction. The valuation factor is therefore capped at x^* under a lead. This pricing-induced upper bound on contributor benefit is the mechanism used below.

Given the benefits $B_{j,r}(q^O_{t,mid})$, the contribution costs c_j, the contributor distribution H, and cohort size n, the symmetric volunteer's-dilemma equilibrium determines the probability that no contributor provides the OSS step. Let this cohort no-improvement probability be $\gamma_r(q^O_{t,mid})$ in regime r. Thus $\gamma_0(q^O_{t,mid})$ is the no-improvement probability at parity, and $\gamma_1(q^O_{t,mid})$ is the corresponding probability when the proprietary product leads. The Appendix adapts the cutoff-equilibrium argument of Johnson [2002] to the state-dependent benefits and costs in this model. It derives the symmetric fixed point, verifies the required regularity, and establishes uniqueness within the symmetric cutoff class. The main text uses only the resulting regime-specific no-improvement probabilities.

Proposition 1 (A proprietary lead lowers OSS improvement)**.** *Fix the contribution-stage quality pair* $(q^I_{t,mid}, q^O_{t,mid})$ *and the contributor type distribution. The capped and uncapped private benefits and the corresponding equilibrium no-improvement probabilities satisfy*

$$B_{j,1}(q^O_{t,mid}) \le B_{j,0}(q^O_{t,mid}), \qquad \gamma_1(q^O_{t,mid}) \ge \gamma_0(q^O_{t,mid}).$$

The first inequality is strict for $v_j > x^*$*. The second is strict whenever the contributor-benefit cap moves positive type mass across the regime-*0 *equilibrium contribution cutoff.*

Proof. See Appendix.

Economic intuition. A proprietary lead affects the private return to an OSS step through price. Once price adjusts, a high-valuation user-developer receives at most the marginal buyer's valuation x^* from closing part of the gap. The cap therefore shifts contributor types toward nonparticipation before free riding is re-solved at the cohort level. Re-solving the two equilibria changes contributor pivotality and yields the proposition's aggregate ordering $1 - \gamma_1(q^O_{t,mid}) \leq 1 - \gamma_0(q^O_{t,mid})$.

Figure 1 displays the resulting reduction in the OSS-improvement probability, $1-\gamma_r(q^O)$.

Contribution-feasibility cutoffs. The type-support bounds identify two OSS quality thresholds. The parity threshold is the first quality at which the maximum contributor benefit reaches the minimum contribution cost. The proprietary-lead threshold is the analogous point with the valuation factor capped at x^*. Formally,

Definition 1 (OSS contribution-feasibility cutoffs)**.**

$$Q^{\mathrm{OS}}_{\max,0} := \inf\left\{q^O \in \mathbb{Z}_{\geq 0} : \psi(q^O+1) - \psi(q^O) \leq \frac{c_L}{v_H}\right\},$$
$$Q^{\mathrm{OS}}_{\max,1} := \inf\left\{q^O \in \mathbb{Z}_{\geq 0} : \psi(q^O+1) - \psi(q^O) \leq \frac{c_L}{\min\{v_H, x^*\}}\right\}.$$

Here $Q^{\mathrm{OS}}_{\max,0}$ is the parity feasibility cutoff and $Q^{\mathrm{OS}}_{\max,1}$ is the proprietary-lead feasibility cutoff.

The threshold value is one because a contributor is pivotal with probability one at a candidate no-contribution outcome. At that point, contribution requires $B_{j,r}(q^O) > c_j$. If even the largest private benefit is no greater than the smallest contribution cost, no contribution is an equilibrium. Conversely, below the relevant cutoff, Assumption 3(iii) assigns positive probability to contributors whose private benefit exceeds their cost. The unique fixed point then has $\gamma_r < 1$.

Exhaustion of feasible contributor types. For either regime $r \in \{0, 1\}$, the feasibility cutoff partitions the contribution stage as follows: $q^O \geq Q^{\mathrm{OS}}_{\max,r}$ implies $\gamma_r(q^O) = 1$, while $q^O < Q^{\mathrm{OS}}_{\max,r}$ implies $\gamma_r(q^O) < 1$ under the local-full-support condition in Assumption 3(iii).

Ordering of the OSS cutoffs. The capped valuation factor, $\min\{v_H, x^*\}$, is bounded above by the uncapped factor v_H. The proprietary-lead cutoff is therefore weakly lower: $Q^{\mathrm{OS}}_{\max,1} \leq Q^{\mathrm{OS}}_{\max,0}$. The integer interval

$$Q^{\mathrm{OS}}_{\max,1} \leq q^O < Q^{\mathrm{OS}}_{\max,0}$$

is the *cap-induced OSS inactivity interval*. It contains qualities where parity supports OSS improvement and the proprietary lead generates lead-state OSS inactivity. The endpoint claims are proved in the Appendix.

Beyond bounded contributor types. With unbounded types, the corresponding tolerance frontiers order the quality levels beyond which OSS success remains below any stated positive tolerance: a proprietary lead reaches each such low-success tail weakly earlier. The corresponding effective-cutoff result is proved in the Appendix.

Step 3: the firm's initial investment choice. Return to the beginning of the period. The firm can buy one proprietary quality step at real cost c^I, denoted by action $a = 1$, or wait, denoted by $a = 0$. The one-period benchmark evaluates current payoff only.

To isolate the effect of OSS on this decision, compare the baseline with a lead-preserving benchmark in which a newly created proprietary lead at initial parity carries into current pricing. At common initial quality, $q^I_{t,ini} = q^O_{t,ini}$, a successful OSS step restores parity in the baseline but leaves the firm one step ahead in the counterfactual. The counterfactual changes neither the wait branch nor any other outcome. In the baseline, OSS matches the new step with probability $1 - \gamma_1(q^O_{t,mid})$.

One-period firm-response result. For any initial state $(q^I_{t,ini}, q^O_{t,ini}) \in S$, the firm compares the cost c^I with the current operating-profit gain from its quality step. At initial parity, that gain is multiplied by the probability $\gamma_1(q^O_{t,mid})$ that the new lead survives the

contributor stage. At an initial strict lead, one OSS step leaves the proprietary product ahead, so the current return to the incremental proprietary advantage takes its full value. The firm invests if and only if

$$c^I \leq \begin{cases} e^*\gamma_1(q^O_{t,mid})[\psi(q^I_{t,ini}+1)-\psi(q^I_{t,ini})], & q^I_{t,ini}=q^O_{t,ini}, \\ e^*[\psi(q^I_{t,ini}+1)-\psi(q^I_{t,ini})], & q^I_{t,ini}>q^O_{t,ini}. \end{cases}$$

At parity, proprietary investment suppression occurs when the return in the lead-preserving benchmark covers c^I and anticipated same-period OSS matching switches the baseline action to waiting, as formalized below.

The timing is essential. A pre-pricing OSS match changes the current proprietary return, while a post-pricing match changes the next state. The pre-pricing effect generates the loss displayed in the one-period firm-response result. The dynamic model retains a post-pricing continuation effect because the match still changes the next initial state.

At a strict lead, both actions leave the proprietary product ahead after one OSS step, so the current return to the proprietary increment takes its full value. At parity, OSS can match a new costly quality step before the firm monetizes the resulting advantage. Anticipating Step 2 therefore changes the Step 3 decision.

Question 2: When do the best responses generate mutual suppression? Proposition 1 gives the OSS side of the mechanism: a proprietary lead weakly reduces the probability that OSS improves. The one-period firm-response result gives the firm side: anticipated OSS matching can switch the firm from investing to waiting. At initial parity, both effects occur together precisely under the following conditions.

Proposition 2 (One-period mutual suppression at parity)**.** *Fix an initial parity state, so* $q^I_{t,ini}=q^O_{t,ini}$*. Mutual suppression requires two strict comparisons. First, the proprietary lead must lower the equilibrium probability of OSS improvement. Second, the firm's cost must be low enough for investment in the lead-preserving benchmark and high enough to induce*

waiting in the baseline. Equivalently,

$$\gamma_1(q^O_{t,mid}) > \gamma_0(q^O_{t,mid}), \qquad \gamma_1(q^O_{t,mid})e^*[\psi(q^I_{t,ini}+1)-\psi(q^I_{t,ini})] < c^I \leq e^*[\psi(q^I_{t,ini}+1)-\psi(q^I_{t,ini})].$$

Proof. See Appendix.

Interpretation and interiority. The second comparison defines a proprietary-cost interval. Its length is $[1-\gamma_1(q^O_{t,mid})]e^*[\psi(q^I_{t,ini}+1)-\psi(q^I_{t,ini})]$. Positive residual OSS-improvement probability gives the interval positive length. At complete OSS suppression, $\gamma_0(q^O_{t,mid}) < 1 = \gamma_1(q^O_{t,mid})$, the interval has zero length. In a fixed-state comparison that raises γ_1 while holding γ_0, e^*, and the quality state fixed, the contributor-discouragement gap weakly widens while the proprietary action-reversal interval weakly narrows. Thus mutual suppression is an interior outcome.

The neighboring one-period cases. The proposition separates four logically exhaustive cases:

(i) *interior mutual suppression*, when both conditions hold;

(ii) *contributor discouragement only*, when $\gamma_1 > \gamma_0$ but the firm's cost lies outside the action-reversal interval; complete OSS suppression belongs to this case;

(iii) *proprietary investment suppression only*, when same-period matching reverses the firm's action but the contributor-benefit cap does not strictly change aggregate OSS success; and

(iv) *neither strict suppression comparison*, when neither condition holds.

The second case can contain either investment under both firm comparisons or waiting under both. The third case shows that same-period OSS matching can matter even when the contributor-benefit cap is locally nonbinding.

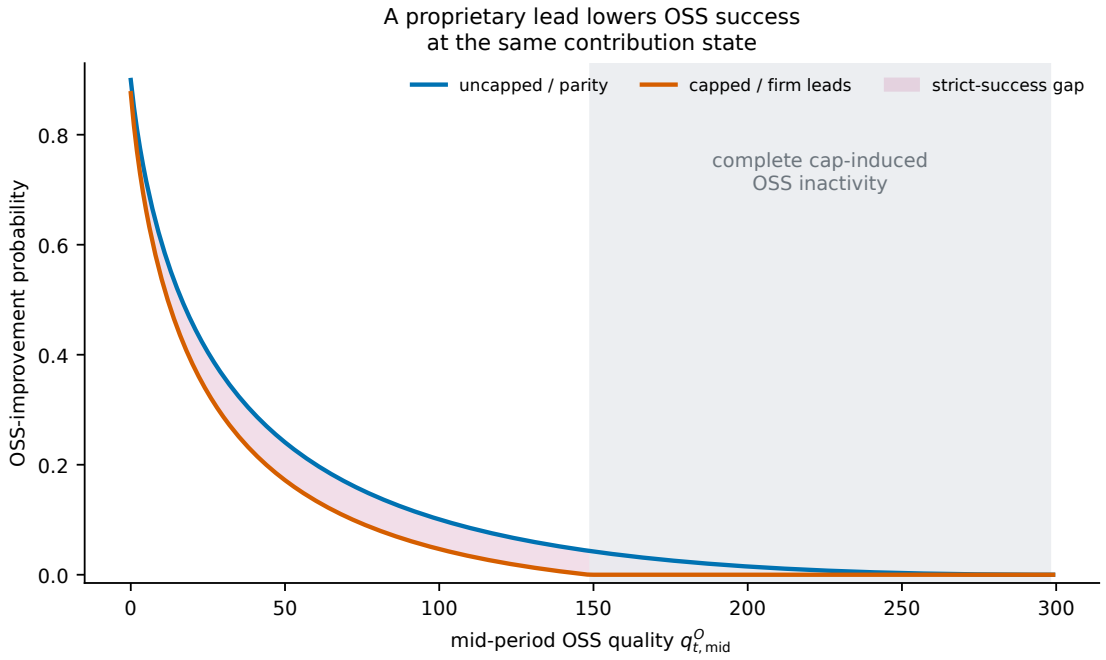


Figure 1: A proprietary lead lowers OSS improvement at the same contribution state. The solid blue curve is OSS-improvement probability at parity, $1 - \gamma_0(q^O_{t,mid})$; the solid orange curve is the corresponding probability when the proprietary product leads, $1 - \gamma_1(q^O_{t,mid})$. The pink region is the reduction caused by the pricing-induced upper bound on contributors' private benefit. The gray interval marks qualities where parity supports another OSS improvement and a proprietary lead creates lead-state OSS inactivity.

The preceding fixed-state comparisons supply the dynamic model's within-period transition kernel and terminal-period start–wait comparison. At every dynamic state, the pricing objects x^* and e^* and the contribution schedules $\gamma_r(q^O_{t,mid})$ are evaluated exactly as above. The dynamic firm evaluates the action-specific continuation states generated when the period-final pair becomes the next period's initial pair. Section V makes this nesting explicit.

V Dynamic Mutual Suppression and Lead Persistence

The one-period mutual-suppression proposition identifies when both comparisons hold at the same parity quality and why that coexistence is interior. Dynamics preserve the same contributor stage but replace the gross one-period return conditional on lead survival with the flow-plus-continuation value of a lead created from initial parity. They classify paths into same-period OSS matching, catch-up after a lead carries over, and effective lead persistence, defined by a stated finite-horizon catch-up bound.

What is inherited and what dynamics adds.

(i) Final-stage pricing and the contributor equilibrium are inherited unchanged. At each current state, the same x^*, e^*, and state-dependent schedule $\gamma_r(q^O_{t,mid})$ determine the within-period transition. Setting $\beta = 0$ in the firm's start–wait comparison recovers the one-period investment rule.

(ii) When $\beta > 0$, the final quality pair becomes the next initial pair. A post-pricing match can therefore remove continuation value, and an OSS step at a strict lead can change the marginal continuation value of proprietary quality.

(iii) The Bellman comparison combines fixed-state terms with action-specific successor values to determine the re-solved dynamic policy and complete paths.

The section first embeds the one-period start–wait comparison in the Bellman problem, then uses the resulting policy to construct two long-run destinations: a parity region in which the firm initiates no further lead and a strict-lead no-catch-up region under bounded contributor support. Their hitting order classifies equilibrium paths. The analysis separates three proprietary-side consequences of OSS: *proprietary investment suppression*, when OSS reverses a lead-initiation decision; *paid-step reduction*, when OSS lowers the expected discounted number of proprietary quality steps; and a reduction in the firm's discounted operating profit when OSS improves, even if the firm's investment policy is unchanged.

V.1 Bellman Extension of the One-Period Game

Given the contributor transition probabilities, the firm solves a discounted Markov decision process [Puterman, 1994]. At each initial state and firm action, the contributors' static equilibrium first determines $\gamma_{r_t}(q^O_{t,mid})$, which enters the firm's transition kernel. Independently redrawn contributor cohorts make this a single-agent decision problem with an equilibrium-induced transition kernel.

Let $V(q^I_{t,ini}, q^O_{t,ini})$ be the firm's start-of-period value on S. For a proposed action a_t, its transition has three steps:

(1) *Firm action and intermediate state.* Proprietary quality rises by a_t, while OSS quality is unchanged:

$$q^I_{t,mid} = q^I_{t,ini} + a_t, \qquad q^O_{t,mid} = q^O_{t,ini}.$$

(2) *OSS response.* The intermediate state determines the contribution regime

$$r_t = \mathbf{1}\{q^I_{t,mid} > q^O_{t,mid}\}.$$

Conditional on that regime, an OSS step ($s_t = 1$) occurs with probability $1 - \gamma_{r_t}(q^O_{t,mid})$.

(3) *Final and next-period state.* The OSS step raises OSS quality, and compatible reuse

determines final proprietary quality:

$$q^O_{t,fin} = q^O_{t,mid} + s_t, \qquad q^I_{t,fin} = \max\{q^I_{t,mid}, q^O_{t,fin}\}.$$

The final pair becomes next period's initial pair:

$$q^I_{t+1,ini} = q^I_{t,fin}, \qquad q^O_{t+1,ini} = q^O_{t,fin}.$$

Action-contingent payoffs. For $a \in \{0, 1\}$, let V_a be the firm's expected current operating profit plus continuation value after choosing action a:

$$V_a(q^I_{t,ini}, q^O_{t,ini}) := \mathbb{E}\big[\pi(q^I_{t,fin}, q^O_{t,fin}) + \beta V(q^I_{t+1,ini}, q^O_{t+1,ini}) \mid q^I_{t,ini}, q^O_{t,ini}, a_t = a\big] .$$

This expectation averages over the current OSS outcome generated after action a. It includes current optimized operating profit and next-period firm value; the real cost c^I enters only when the firm starts.

Bellman equation and start–wait decision. After charging c^I only to the start branch, the Bellman equation is

$$V(q^I_{t,ini}, q^O_{t,ini}) = \max\big\{V_1(q^I_{t,ini}, q^O_{t,ini}) - c^I, V_0(q^I_{t,ini}, q^O_{t,ini})\big\} .$$

The start–wait payoff difference before the paid-step cost is

$$\mathcal{M}(q^I_{t,ini}, q^O_{t,ini}) := V_1(q^I_{t,ini}, q^O_{t,ini}) - V_0(q^I_{t,ini}, q^O_{t,ini}).$$

Equivalently,

$$V(q^I_{t,ini}, q^O_{t,ini}) = V_0(q^I_{t,ini}, q^O_{t,ini}) + \max\big\{\mathcal{M}(q^I_{t,ini}, q^O_{t,ini}) - c^I, 0\big\} .$$

Thus the firm starts exactly when

$$\mathcal{M}(q^I_{t,ini}, q^O_{t,ini}) \geq c^I;$$

otherwise, it waits.

The one-period comparison as a special case. Every expectation in the following identity conditions on the current initial quality pair. Subtracting the two action values gives the exact bridge to the one-period problem:

$$\begin{aligned}\mathcal{M}(q^I_{t,ini}, q^O_{t,ini}) = & \underbrace{\Big\{\mathbb{E}[\pi(q^I_{t,fin}, q^O_{t,fin}) \mid a_t = 1] - \mathbb{E}[\pi(q^I_{t,fin}, q^O_{t,fin}) \mid a_t = 0]\Big\}}_{\substack{\text{difference in current expected operating profit} \\ \text{after the action-specific OSS response}}} \\ & + \beta \underbrace{\Big\{\mathbb{E}[V(q^I_{t+1,ini}, q^O_{t+1,ini}) \mid a_t = 1] - \mathbb{E}[V(q^I_{t+1,ini}, q^O_{t+1,ini}) \mid a_t = 0]\Big\}}_{\substack{\text{difference in the value of next period's initial state} \\ \text{after the current OSS outcome and compatible reuse}}}. \end{aligned} \tag{1}$$

The first bracket compares the current operating profit generated by starting with that generated by waiting. It already accounts for the fact that the two actions can induce different OSS-improvement probabilities. The second bracket compares the firm values of the two next-period initial states reached after the current OSS outcome and compatible reuse. It is the continuation effect of starting today. When $\beta = 0$, this second term disappears. The remaining current-profit comparison is exactly the one underlying the one-period firm-response result in Section IV; charging c^I then gives that section's start-or-wait decision rule.

Existence and uniqueness on the full ladder, together with the Bellman-residual and boundary certificates for the finite computation, are proved in the Appendix. In the main text, the dynamic program determines whether the start–wait payoff difference exceeds the paid-step cost at each initial quality pair.

V.2 Proprietary Investment Responses Across Initial States

The Bellman rule in the preceding subsection asks whether the start–wait payoff difference $\mathcal{M}$ covers the paid-step cost c^I. Whether OSS can change that comparison depends on the initial quality pair. OSS affects the firm through both the probability of an improvement and the final and continuation states reached after that improvement.

What an OSS step changes across states. Begin at parity, with $q^I_{t,ini} = q^O_{t,ini} = q$. Conditional on OSS improvement, the final pair is the same whether the firm starts a lead or waits:

$$(a_t, s_t) \in \{(0,1),(1,1)\} \quad \Longrightarrow \quad (q^I_{t,fin}, q^O_{t,fin}) = (q+1, q+1). \tag{2}$$

If the firm starts, OSS has matched the paid increment before pricing. If the firm waits, compatible reuse incorporates the OSS increment into proprietary quality. At a one-step proprietary lead, an OSS improvement restores parity if the firm waits but preserves a one-step lead if it invests. At a larger lead, one OSS improvement narrows the gap but leaves the firm strictly ahead under either action.

This state map explains why OSS contribution depends only on the binary lead regime (Section IV), whereas the firm's start–wait comparison depends on the full initial pair $(q^I_{t,ini}, q^O_{t,ini})$. The complete branch table is derived in the Appendix. This leader–follower state language treats OSS as a decentralized project following the proprietary firm [Aghion et al., 2001].

Proprietary-only reference point. If OSS quality is fixed, the firm invests if and only if $q^I < Q^{I,M}_{\max}$, where

$$Q^{I,M}_{\max} := \inf\left\{ q^I \in \mathbb{Z}_{\geq 0} : \frac{e^*[\psi(q^I+1) - \psi(q^I)]}{1-\beta} < c^I \right\}.$$

The expression inside the braces compares the discounted value of a permanently retained one-step proprietary quality increment with its real cost. The infimum selects the stopping quality on the paid-only path. It is the reference point for the live OSS comparisons below; compatible reuse can later raise both products through an OSS improvement.

Three state cases. Equation (1) adds successor-state values to the one-period firm-response result. The effect of those values is most transparent when the state space is divided into three cases:

(i) an equal-quality initial state, where a start creates a lead that OSS can match before pricing;

(ii) a strict proprietary lead in the no-catch-up region, where the investment comparison coincides with the proprietary-only reference point; and

(iii) a strict proprietary lead below that boundary, where an OSS step still changes the relative state and continuation values.

V.2.1 Equal-Quality Initial States

At an equal-quality start, the two firm actions induce different contribution regimes. Starting creates a mid-period proprietary lead and uses γ_1, whereas waiting preserves parity and uses γ_0. If the firm starts and OSS improves, OSS matches the paid step before pricing. If the firm waits and OSS improves, compatible reuse raises proprietary quality through the OSS increment. The first branch can reduce the return to starting; the second can increase the value of waiting.

Definition 2 (Same-start lead payoff difference)**.** At an equal-quality start $q^I_{t,ini} = q^O_{t,ini}$, a proprietary start creates a one-unit lead before pricing. If OSS improves, it matches that newly created step in the same period and pricing occurs at parity. Define the payoff

difference between those two branches by

$$V_1^{\text{same}}(q_{t,ini}^I, q_{t,ini}^O) := e^*[\psi(q_{t,ini}^I + 1) - \psi(q_{t,ini}^I)] + \beta[V(q_{t,ini}^I + 1, q_{t,ini}^O) - V(q_{t,ini}^I + 1, q_{t,ini}^O + 1)], \quad \text{where } q_{t,ini}^I = q_{t,ini}^O.$$

The new-lead branch earns current operating rent and continues from a one-step lead, whereas the matched branch continues from parity. Their difference is V_1^{same}, the parity-specific signed payoff difference between these branches. Its superscript records the equal-quality start, and its subscript records the one-unit proprietary lead created at that start. Its first term is the current operating-rent difference; its bracket is the continuation-value difference.

The difference between the lead-preserving and baseline start branches is

$$[1 - \gamma_1(q_{t,mid}^O)]V_1^{\text{same}}(q_{t,ini}^I, q_{t,ini}^O).$$

At $\beta = 0$, this product is nonnegative and, whenever $1 - \gamma_1(q_{t,mid}^O) > 0$, positive. It then reduces to the one-period expected loss from same-period OSS matching in the one-period firm-response result. With continuation, it is nonnegative only when $V_1^{\text{same}} \geq 0$; without that condition it remains an exact signed transition adjustment.

Proposition 3 (Same-period OSS matching can reverse proprietary investment)**.** *Fix the Bellman value V and an initial parity state $q_{t,ini}^I = q_{t,ini}^O$. The lead-preserving comparison holds a new proprietary step in place when OSS improves and changes the gain from starting by*

$$[1 - \gamma_1(q_{t,mid}^O)]V_1^{\text{same}}(q_{t,ini}^I, q_{t,ini}^O).$$

If $V_1^{\text{same}} \geq 0$, the lead-preserving comparison weakly raises this gain, or equivalently same-period matching before pricing weakly lowers it. Matching changes the firm's action from

investment in the lead-preserving comparison to waiting in the baseline exactly when

$$\mathcal{M}(q^I_{t,ini}, q^O_{t,ini}) < c^I \leq \mathcal{M}(q^I_{t,ini}, q^O_{t,ini}) + [1 - \gamma_1(q^O_{t,mid})]V_1^{\text{same}}(q^I_{t,ini}, q^O_{t,ini}). \tag{3}$$

Proof. See Appendix.

The left inequality says that the baseline gain does not cover c^I; the right says that preventing same-period matching adds enough value to cover it. The result therefore identifies a change in the firm's action, not merely a change in its investment value. Timing also matters. A match after current pricing but before the next initial state changes only continuation value. A match before pricing additionally removes current operating rent from the new proprietary quality step. Either difference is a loss from matching only when it is nonnegative.

How the contributor-benefit cap makes a new proprietary lead less likely to be matched. At initial parity, the gross gain from initiating a proprietary lead can be written as

$$\begin{aligned}\mathcal{M}(q^I_{t,ini}, q^O_{t,ini}) = {} & \gamma_0(q^O_{t,mid})V_1^{\text{same}}(q^I_{t,ini}, q^O_{t,ini}) \\ & + \big[\gamma_1(q^O_{t,mid}) - \gamma_0(q^O_{t,mid})\big]V_1^{\text{same}}(q^I_{t,ini}, q^O_{t,ini}) \\ & + \beta\gamma_0(q^O_{t,mid})\big[V(q^I_{t,ini}+1, q^O_{t,ini}+1) - V(q^I_{t,ini}, q^O_{t,ini})\big].\end{aligned} \tag{4}$$

This identity is evaluated at $q^I_{t,ini} = q^O_{t,ini}$. The first line weights the same-start lead payoff difference by the probability that OSS does not match a new proprietary lead. The second puts the additional no-match probability created by the contributor-benefit cap, $\gamma_1 - \gamma_0 \geq 0$, on the same signed payoff difference. The third is the remaining continuation adjustment between moving one quality level up the parity diagonal and remaining at the current parity state. If $V_1^{\text{same}} \geq 0$, the second line is weakly nonnegative. By creating a mid-period proprietary lead, a start activates the same contributor-benefit cap that gives $\gamma_1 \geq \gamma_0$, thereby making the attempted lead less likely to be matched before pricing. The identity is derived in the Appendix.

Within the start branch, the risk from an uncapped parity response separates into the cap-induced increase from γ_0 to γ_1 and the residual probability of OSS matching. When $V_1^{\text{same}} \geq 0$, raising γ_1 shifts probability weight one for one from matching to no match. At $\gamma_1 = 1$, the residual term vanishes. This is the dynamic counterpart of the one-period interiority result, although an action reversal still requires the full dynamic start–wait gain to cross c^I.

This feedback connects the two directions without making them symmetric. Contributor discouragement acts at the contribution stage by lowering the probability that OSS responds; it becomes complete OSS suppression only when that probability falls to zero. Proprietary investment suppression is the fixed-parity action reversal created when a response changes the final and continuation states. The former makes an attempted proprietary lead less likely to be matched before pricing; the latter can match it or increase the value of waiting.

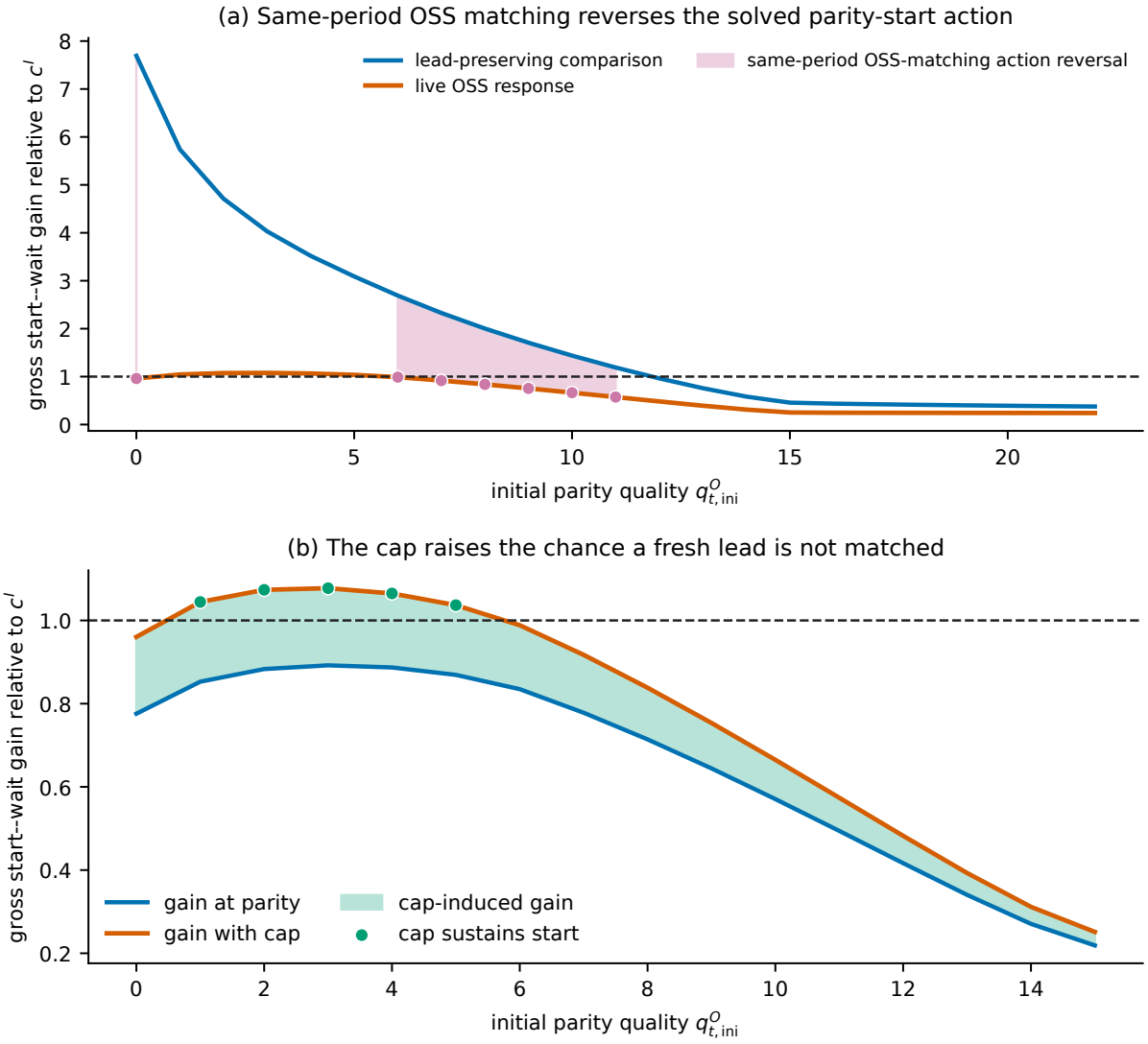


Figure 2: The dynamic bridge at the high-cost benchmark $c^I = 100{,}000$, where c^I is the real cost of one paid proprietary quality step. The horizontal coordinate is common initial parity quality $q^I_{t,ini} = q^O_{t,ini}$. Panel (a) compares the solved gross parity-start gain—the invest value minus the wait value before subtracting c^I—with the gain in the lead-preserving comparison, which holds a fresh proprietary step in place when OSS improves while holding the Bellman value function V fixed; shaded states are action reversals. Panel (b) isolates $(\gamma_1 - \gamma_0)V_1^{\text{same}}$, where γ_0 and γ_1 are the parity and proprietary-lead OSS no-improvement probabilities and V_1^{same} is the signed payoff difference, at an equal-quality start, between a newly created one-step lead and the matched state before pricing. Markers identify starts sustained by the term $(\gamma_1 - \gamma_0)V_1^{\text{same}}$, the added expected value generated by the cap-induced increase in the probability of no same-period match when $V_1^{\text{same}} \geq 0$. State locations are calibration-specific; the displayed branch-value identities are exact.

The long-run parity policy. Equation (2) also moves the next initial pair one step up the parity diagonal. A higher common quality lowers the value of another quality increment, raises the probability that a new lead survives, and changes continuation values. The policy-defined upper tail accommodates these competing forces and the resulting nonmonotone start incentives.

Definition 3 (Upper parity investment cutoff and start set)**.** Under the stationary policy a^*, define

$$Q^I_{\mathrm{NS}} := \inf\{q : a^*(q', q') = 0 \text{ for every } q' \geq q\}, \qquad \mathcal{S}_{\mathrm{start}} := \{q : a^*(q, q) = 1\}.$$

If $\mathcal{S}_{\mathrm{start}}$ is finite, $Q^I_{\mathrm{NS}} = 1 + \sup \mathcal{S}_{\mathrm{start}}$; if it is empty, the cutoff is zero.

Unlike the contribution-feasibility cutoffs, these objects depend on n and the continuation value. The start set need not be contiguous. Under the bounded-support part of Assumption 3, OSS improvement becomes infeasible by $Q^{\mathrm{OS}}_{\max,0}$. Diminishing quality increments and $c^I > 0$ stop proprietary-only investment by $Q^{I,M}_{\max}$. Together these conditions make the start set finite and imply the explicit bound

$$Q^I_{\mathrm{NS}} \leq \max\{Q^{\mathrm{OS}}_{\max,0}, Q^{I,M}_{\max}\}.$$

Upper no-further-investment region at parity. The set

$$\mathcal{R}_{\mathrm{NS}} := \{(q, q) \in S : q \geq Q^I_{\mathrm{NS}}\}$$

is invariant under a^*. Once it is reached, the firm never initiates another proprietary lead; the only possible motion is OSS-driven movement up the parity diagonal, which stops no later than $Q^{\mathrm{OS}}_{\max,0}$.

V.2.2 The No-Catch-Up Proprietary-Lead Region

Under Assumption 3, define the exact strict-lead no-catch-up region

$$\mathcal{R}_{\mathrm{NC}} := \{(q^I_{t,ini}, q^O_{t,ini}) \in S : q^I_{t,ini} > q^O_{t,ini},\ q^I_{t,ini} \geq Q^{\mathrm{OS}}_{\max,1} + 1\}.$$

Once the firm enters this region, bounded contributor support prevents OSS from completing the final step needed to restore parity. The region is therefore invariant.

Investment is independent of OSS quality in this region. On $\mathcal{R}_{\mathrm{NC}}$, the investment rule coincides with the proprietary-only reference point:

$$a^*(q^I_{t,ini}, q^O_{t,ini}) = \mathbf{1}\{q^I_{t,ini} < Q^{I,M}_{\max}\}.$$

After entry, the remaining paid proprietary target is $\max\{q^I_{t,ini}, Q^{I,M}_{\max}\}$, and the proprietary lead is persistent. Within $\mathcal{R}_{\mathrm{NC}}$, the common OSS transition enters the value of both actions while the investment comparison follows the proprietary-only reference point. See Appendix for proof.

V.2.3 Catch-Up-Feasible Proprietary Leads

What happens after a proprietary start survives same-period matching, but before the lead becomes impossible for OSS to close? The relevant initial states satisfy

$$q^O_{t,ini} < q^I_{t,ini} \leq Q^{\mathrm{OS}}_{\max,1}.$$

We call this catch-up-feasible transition region the *strict-lead corridor*. It connects the parity and no-catch-up boundary regimes. Within the corridor, a paid firm step adds one to proprietary quality and an OSS improvement adds one to OSS quality. Consequently, along

every such episode,

$$q^I_{t,ini} - q^O_{t,ini} = q^I_{0,ini} - q^O_{0,ini} + \sum_{k=0}^{t-1} a_k - \sum_{k=0}^{t-1} s_k. \tag{5}$$

Here subscript 0 denotes the first initial state of the strict-lead episode. $a_k = 1$ records a paid proprietary step and $s_k = 1$ records an OSS improvement in period k. Each paid step adds one unit that OSS must recover, and each OSS improvement removes one unit from the lead. Equation (5) tracks this lead accounting through the corridor episode.

The position within this region determines what one more action or OSS improvement can accomplish:

(i) At a *parity-adjacent state*, $q^I_{t,ini} = q^O_{t,ini} + 1 < Q^{\mathrm{OS}}_{\max,1}$, waiting followed by an OSS improvement restores parity, whereas investing followed by an OSS improvement preserves a one-unit lead.

(ii) In the *strict interior*, $q^O_{t,ini} + 1 < q^I_{t,ini} < Q^{\mathrm{OS}}_{\max,1}$, one more OSS improvement narrows the lead and one paid step moves the firm toward the no-catch-up boundary.

(iii) At a *no-catch-up-adjacent state*, $q^I_{t,ini} = Q^{\mathrm{OS}}_{\max,1}$ and $q^O_{t,ini} < q^I_{t,ini} - 1$, investing carries the process into $\mathcal{R}_{\mathrm{NC}}$ next period; waiting leaves OSS an opportunity to narrow the lead.

(iv) At the *junction*, $q^I_{t,ini} = Q^{\mathrm{OS}}_{\max,1}$ and $q^O_{t,ini} = q^I_{t,ini} - 1$, waiting until the next OSS improvement restores parity, whereas investing enters $\mathcal{R}_{\mathrm{NC}}$ immediately.

Two cutoffs locate the corridor. $Q^{\mathrm{OS}}_{\max,1}$ is the highest proprietary quality from which OSS can still catch up under the contributor-benefit cap. $Q^{I,M}_{\max}$ is the stopping quality for paid growth on the proprietary-only path. The solved policy $a^*(q^I_{t,ini}, q^O_{t,ini})$ selects transitions using the current lead, distance to no-catch-up, and continuation values.

The corridor identity yields three exact benchmarks. First, if the firm always waits, OSS

reaches parity almost surely, with expected duration

$$\sum_{j=q^O_{0,ini}}^{q^I_{0,ini}-1} \frac{1}{1-\gamma_1(j)}.$$

$1-\gamma_1(j)$ is the OSS-improvement probability at OSS quality j while the proprietary product leads. Each reciprocal is the expected waiting time for the next OSS improvement at that quality. Second, if the firm invests at every reached corridor state, the process reaches the no-catch-up boundary after exactly $Q^{\mathrm{OS}}_{\max,1}+1-q^I_{0,ini}$ periods. Third, if the solved policy has a proprietary-quality ceiling strictly below that boundary and waits at every corridor state at or above the ceiling, only finitely many additional paid steps can occur and OSS returns to parity almost surely. The Appendix proves these statements and gives the recursions for exit probabilities, expected duration, and discounted arrivals.

These benchmarks separate three effects of OSS activity on the firm. OSS can reduce the expected number of paid proprietary steps before corridor exit, shorten the expected time over which a proprietary lead earns operating rent, and change the firm's current invest–wait decision through successor-state values. The last effect is state dependent. Holding the solved Bellman value function V fixed, the Appendix's cross-difference maps lower continuation values of another proprietary quality step into investment discouragement and higher continuation values into investment encouragement. The stationary policy can therefore vary nonmonotonically with lead size.

Return to parity completes the current strict-lead episode. At parity qualities below Q^I_{NS}, the firm may initiate another lead. Section V.3 therefore composes repeated parity starts and corridor episodes to determine which invariant region is reached.

Summary of the three cases. The equal-quality and no-catch-up cases define the two boundary regimes, while catch-up-feasible proprietary leads connect them:

(i) At an equal-quality start, OSS can match a new lead or raise the value of waiting,

potentially reversing investment.

(ii) In $\mathcal{R}_{\text{NC}}$, OSS changes firm value through the common transition, while the investment decision follows the proprietary-only rule.

(iii) Between those cases, proprietary investment adds to the lead and moves the firm toward no-catch-up, while OSS improvement subtracts from the lead and moves the process toward parity.

V.3 Lead Persistence and the Classification of Dynamic Paths

What are the long-run destinations? Section V.2 identified two invariant regions. At an equal-quality state in $\mathcal{R}_{\text{NS}}$, the firm never initiates another lead. In a strict proprietary lead in $\mathcal{R}_{\text{NC}}$, OSS can no longer catch up and the firm's investment rule is the proprietary-only rule. The states between them are transient: firm investment moves the process toward $\mathcal{R}_{\text{NC}}$, while OSS improvement moves it toward parity and potentially $\mathcal{R}_{\text{NS}}$.

Let the process start at equal quality (q_0, q_0). Define the first entry date into either invariant region by

$$\tau := \inf\left\{t \geq 0 : (q^I_{t,ini}, q^O_{t,ini}) \in \mathcal{R}_{\text{NS}} \cup \mathcal{R}_{\text{NC}}\right\}. \tag{6}$$

At this date, entry into $\mathcal{R}_{\text{NS}}$ means that the process has reached parity and the firm will never initiate another proprietary lead. Entry into $\mathcal{R}_{\text{NC}}$ means instead that a proprietary lead has become impossible for OSS to close. The probability that the equal-quality no-further-investment region is reached first is

$$\Pr_{(q_0,q_0)}\left(\tau < \infty, \quad (q^I_{\tau,ini}, q^O_{\tau,ini}) \in \mathcal{R}_{\text{NS}}\right). \tag{7}$$

We refer to this in prose as the dominance-prevention probability. It measures whether OSS prevents a permanent no-catch-up proprietary lead, not merely whether OSS remains active for another period.

Theorem 1 (Exhaustive dynamic path classification)**.** *Maintain the model assumptions, including all three parts of Assumption 3, and suppose that* $1 - \gamma_0(q) > 0$ *at every parity state* $q < Q^I_{\mathrm{NS}}$ *at which the firm waits. From any parity state* (q_0, q_0)*,* $\tau < \infty$ *almost surely, and exactly one of the following outcomes occurs up to probability-zero histories:*

(i) the state enters $\mathcal{R}_{\mathrm{NS}}$ *at* τ*, and* $q^I_{t,ini} = q^O_{t,ini}$ *for every* $t < \tau$*, so no proprietary lead carries into the next period;*

(ii) the state enters $\mathcal{R}_{\mathrm{NS}}$ *at* τ*, but* $q^I_{t,ini} > q^O_{t,ini}$ *for at least one* $t < \tau$*, so a lead carries over before OSS later restores parity; or*

(iii) the state enters $\mathcal{R}_{\mathrm{NC}}$ *at* τ*, after which the firm follows the proprietary-only benchmark and the proprietary lead persists.*

Proof. See Appendix.

The theorem establishes finite arrival at an economically distinct destination. Diminishing quality gains eventually close the firm's lead-initiation region on the parity diagonal or OSS catch-up from behind. The first boundary reached classifies the path as same-period matching, post-lead catch-up, or persistent proprietary lead.

How likely is each route, and how long does it take? The three routes in Theorem 1 have the following probabilities:

(i) *No carried-over proprietary lead.* If the remaining start set is empty, this route occurs immediately. Otherwise OSS must match every remaining proprietary start in the same period. Cross-period independence gives

$$\Pr_{(q_0,q_0)}(\text{route (i)}) = \prod_{q \in \mathcal{S}_{\mathrm{start}},\ q \geq q_0} [1 - \gamma_1(q)]. \tag{8}$$

Here $\mathcal{S}_{\mathrm{start}}$ is the set of equal-quality states at which the firm initiates a lead. When this set is empty, the product is one.

(ii) *A temporary carried-over lead followed by OSS catch-up.* This route reaches $\mathcal{R}_{\text{NS}}$ first, but excludes route (i). Its probability is

$$\Pr_{(q_0,q_0)}\left((q^I_{\tau,ini}, q^O_{\tau,ini}) \in \mathcal{R}_{\text{NS}}\right) - \Pr_{(q_0,q_0)}(\text{route (i)}).$$

(iii) *Irreversible proprietary dominance.* This route reaches the strict-lead no-catch-up region first. Its probability is

$$1 - \Pr_{(q_0,q_0)}\left((q^I_{\tau,ini}, q^O_{\tau,ini}) \in \mathcal{R}_{\text{NS}}\right).$$

Eventual arrival and economically timely arrival are different. Conditional on reaching $\mathcal{R}_{\text{NS}}$ with positive probability,

$$\mathbb{E}\left[\beta^\tau \mathbf{1}\{(q^I_{\tau,ini}, q^O_{\tau,ini}) \in \mathcal{R}_{\text{NS}}\}\right] = \Pr_{(q_0,q_0)}\left((q^I_{\tau,ini}, q^O_{\tau,ini}) \in \mathcal{R}_{\text{NS}}\right) \mathbb{E}[\beta^\tau \mid \mathcal{R}_{\text{NS}} \text{ is reached}].$$

The first factor records whether permanent dominance is prevented; the second discounts prevention that arrives only after a long catch-up episode. The Appendix derives this first-entry expression and an exact product formula for an always-wait strict-lead corridor.

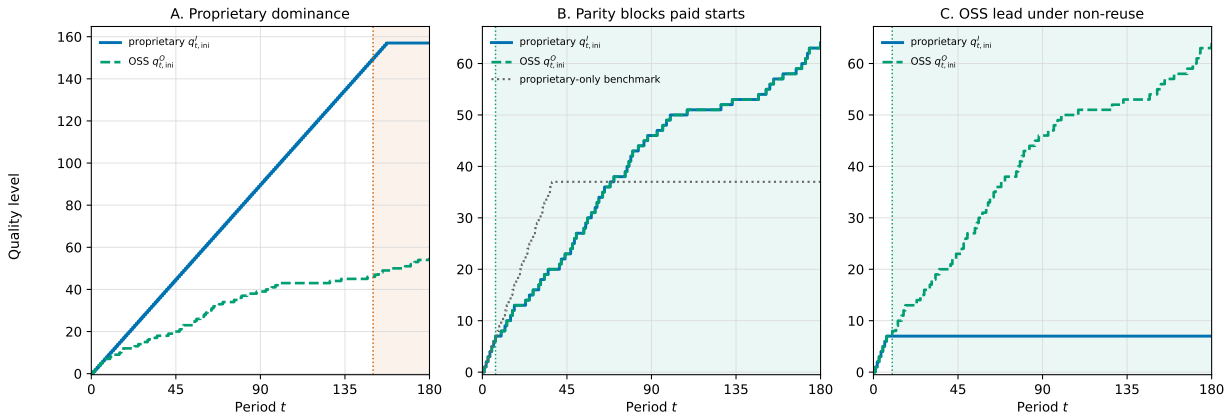


Figure 3: **Three realized quality paths.** The horizontal axis is period t; the solid blue path is initial proprietary quality $q^I_{t,ini}$ and the dashed green path is initial OSS quality $q^O_{t,ini}$. Panel A is a baseline compatible-reuse realization at $c^I = 24{,}000$ in which the proprietary firm enters the no-catch-up region and ends permanently ahead. Panel B is a baseline realization at $c^I = 100{,}000$ in which every attempted proprietary start is matched by OSS within the same period; the state remains at parity and eventually reaches qualities at which the firm initiates no further paid lead. The dotted gray line is the proprietary-only quality path at the same cost, making the reduction in paid investment visible. Panel C depicts the permanent non-reuse extension at $c^I = 100{,}000$, where OSS attains and retains $q^O_{t,ini} > q^I_{t,ini}$. Each panel displays one simulated realization from its solved stationary policy. In each panel, the dotted vertical line and lightly shaded area mark, respectively, the first arrival at the stated path configuration and the periods after that arrival. All panels use $n = 20$ and $\beta = 0.9$.

Economic consequences for the firm. The path classification measures whether OSS prevents permanent no-catch-up dominance. The action accounting below measures the paid-investment response. OSS can prevent dominance after the firm completes its paid investment path or arrive early enough to delay or eliminate paid steps.

Dominance prevention and paid steps. Let $K^{I,M}$ be the number of paid steps in the proprietary-only benchmark from (q_0, q_0). Define the benchmark discounted paid-step count, the corresponding hybrid-equilibrium count, and their difference by

$$A^{I,M} := \sum_{k=0}^{K^{I,M}-1} \beta^k, \qquad A^I := \mathbb{E}_{(q_0,q_0)}\left[\sum_{t\geq 0} \beta^t a_t\right], \qquad D^A := A^{I,M} - A^I.$$

The superscript A denotes *action accounting*: these objects count discounted paid proprietary investment actions. A step taken in period t contributes β^t, so $A^{I,M}$ is the discounted number of paid steps in the proprietary-only benchmark, while A^I is the expected discounted number under OSS competition. Their difference D^A records the change in discounted paid steps and, after multiplication by c^I, gives the investment-cost term in the firm-value identity below.

When $A^{I,M} > 0$, the normalized quantity $D^A/A^{I,M}$ answers a specific question: what fraction of the benchmark's discounted paid-step exposure is removed by OSS competition? A value of zero records equal discounted paid-step counts, and a value of one records avoidance of every benchmark paid step. Intermediate values combine later timing and omitted steps.

The probability of reaching $\mathcal{R}_{\mathrm{NS}}$ first records the market structure path, while D^A aggregates paid-step timing and omission. A path that reaches $\mathcal{R}_{\mathrm{NS}}$ after the paid-only path is complete prevents irreversible dominance and preserves the benchmark paid-step count. Earlier arrival contributes delayed or omitted benchmark steps, together with probabilities, discounting, and any offsetting additional steps.

Firm value. $D^A = 0$ isolates a firm-value channel through which OSS converts a permanent proprietary gap into a temporary one. Define the baseline discounted operating-gap exposure and its signed difference from the proprietary-only benchmark by using the superscript hyb for the coexistence economy, in which both OSS and the proprietary firm are active, and the superscript M for the proprietary-only benchmark:

$$G^{\mathrm{hyb}} := \mathbb{E}_{(q_0,q_0)}\left[\sum_{t\geq 0}\beta^t\left[\psi(q^I_{t,fin}) - \psi(q^O_{t,fin})\right]_+\right],$$
$$D^G := G^M - G^{\mathrm{hyb}},$$

Here G^M evaluates the same final-state operating-gap sum in the proprietary-only benchmark. Let $V^{\text{hyb}} := V(q_0, q_0)$ and let V^M be the corresponding proprietary-only net firm value. Since operating profit is e^* times the final-state gap in each period,

$$V^M - V^{\text{hyb}} = e^* D^G - c^I D^A. \tag{9}$$

This signed accounting identity decomposes the firm-value difference into operating-rent exposure generated by quality gaps and real investment-cost changes generated by paid steps. See the Appendix for the derivation.

Which routes are feasible? The support thresholds distinguish three environments:

$$\begin{aligned} Q^{I,M}_{\max} \le Q^{\text{OS}}_{\max,1} &\Rightarrow \text{catch-up from behind is feasible given contributor types,} \\ Q^{\text{OS}}_{\max,1} < Q^{I,M}_{\max} \le Q^{\text{OS}}_{\max,0} &\Rightarrow \text{a carried-over lead places OSS in the inactivity interval,} \\ Q^{I,M}_{\max} > Q^{\text{OS}}_{\max,0} &\Rightarrow \text{the firm moves beyond both OSS contribution cutoffs.} \end{aligned}$$

Same-period OSS matching reaches $\mathcal{R}_{\text{NS}}$ before a strict-lead path carries over. The middle environment isolates lead-state OSS inactivity created by the contributor-benefit cap. The final environment captures ordinary exhaustion, where both OSS support cutoffs lie below the proprietary-only stopping quality.

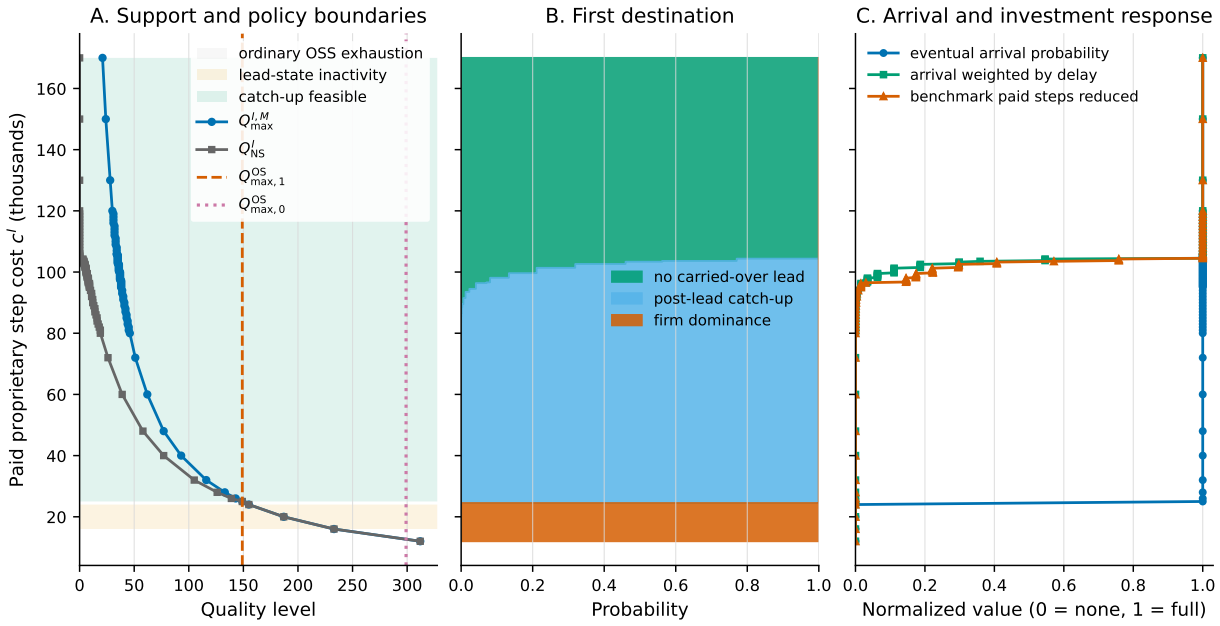


Figure 4: **From support to path outcomes and proprietary investment.** Each point re-solves the stationary equilibrium while varying the real cost c^I of one paid proprietary quality step; the shared vertical axis reports that cost in thousands. Panel A's horizontal axis plots the proprietary-only investment stopping quality $Q^{I,M}_{\max}$, the upper parity no-further-investment boundary Q^{I}_{NS}, and the last OSS qualities at which another improvement is feasible under a proprietary lead, $Q^{\mathrm{OS}}_{\max,1}$, and at equal quality, $Q^{\mathrm{OS}}_{\max,0}$. The background distinguishes ordinary OSS exhaustion, lead-state OSS inactivity attributable to the lower lead-regime cutoff, and contributor-type support for catch-up. Panel B partitions unit probability into paths with same-period matching or no start, post-lead catch-up, and irreversible proprietary dominance. Panel C deliberately places three bounded but distinct objects on the same horizontal scale. The blue line is the probability of eventually reaching the equal-quality no-further-investment region first. The green line is the same arrival event weighted by β^τ, so delay lowers it even when eventual arrival is certain. The orange line is $D^A/A^{I,M}$, the fraction of benchmark discounted paid-step exposure removed by OSS competition. The benchmark has contributor cohort size $n = 20$ and $\beta = 0.9$. The cost grid has 1,000-dollar spacing on [80,000, 120,000] and 250-dollar spacing on [95,000, 106,000]. Lines join adjacent exact discrete-state evaluations for readability.

Figure 4 makes the order of the mechanism visible. The support crossing changes which destination is attainable. Within the catch-up-feasible region, the policy boundary Q^{I}_{NS} determines how many equal-quality states still induce a proprietary start. Entry can remain economically remote even when eventual arrival is certain, and paid investment can change discontinuously when the Bellman comparison switches action at one of those states.

Thresholds and realized paths. The support cutoffs determine which branches are feasible; the policy objects Q^I_{NS} and $\mathcal{S}_{\mathrm{start}}$, together with the destination reached at τ and its entry date, determine which branch is realized and when OSS changes paid investment. Thus support cutoffs, policy thresholds, and the stochastic first-entry time separately identify the feasible branch, the chosen branch, and the timing of that branch.

From path classification to welfare. The same first-entry date also organizes the welfare comparison in Section VI. Before τ, welfare accumulates along the transient path through user surplus, paid proprietary costs, and contribution costs. At τ, the continuation value is evaluated in whichever invariant region was reached. Support therefore determines feasible destinations, transition probabilities determine how strongly distant destinations are discounted, and the re-solved firm policy determines the proprietary quality and cost path. Section VI.2 applies this decomposition to welfare over time in the coexistence economy.

VI Comparative Statics and Welfare

The model separates feasibility, speed, and policy. Contributor type support determines whether another OSS step is feasible; the volunteer equilibrium determines its probability; and the firm's Bellman problem determines whether that transition changes paid proprietary investment. Section V.3 then combines them through the first entry date τ: feasibility determines which invariant destination can be reached, speed determines the discounted weight placed on that destination, and policy determines the quality and real cost path before entry. Welfare is evaluated only after these objects have been distinguished.

VI.1 Comparative Statics

Under the uniform benchmark $F_V = U[0, v_H]$, $x^* = v_H/2$ and $e^* = Nv_H/4$. The proprietary-only stopping quality depends on the value and cost of paid investment, whereas the OSS

cutoffs depend on the highest contributor benefit and lowest contribution cost. These are direct frontier comparisons. They identify which primitive enters each closed-form object. The hybrid Bellman problem then determines the equilibrium path, firm value, and welfare.

Comparative-static summary. Under Assumptions 1–3, the closed-form objects e^*, $Q^{\mathrm{OS}}_{\max,0}$, $Q^{\mathrm{OS}}_{\max,1}$, and $Q^{I,M}_{\max}$ move in the directions reported in Table I. A plus sign records a weak increase, a minus sign records a weak decrease, and zero records invariance to that primitive in the table. Because the quality cutoffs are integer-valued, their responses are weak and stepwise.

Table I: Direct comparative statics. The columns vary proprietary investment cost c^I, discount factor β, market size N, contributor cohort size n, the lower bound c_L of contribution cost, and the upper bound v_H of user and contributor valuations. The rows report the optimized proprietary operating-profit coefficient e^*, which converts a proprietary quality advantage into operating profit, the largest OSS quality at which another step is feasible at parity $Q^{\mathrm{OS}}_{\max,0}$, the corresponding cutoff under a proprietary lead $Q^{\mathrm{OS}}_{\max,1}$, and the proprietary-only stopping quality $Q^{I,M}_{\max}$.

Outcome	c^I	β	N	n	c_L	v_H
e^*	0	0	+	0	0	+
$Q^{\mathrm{OS}}_{\max,0}$	0	0	0	0	−	+
$Q^{\mathrm{OS}}_{\max,1}$	0	0	0	0	−	+
$Q^{I,M}_{\max}$	−	+	+	0	0	+

Economic interpretation. The signs in Table I describe four economically different forces:

(i) *The cost of paid proprietary innovation.* A higher c^I raises the return required to justify one more proprietary quality step, so $Q^{I,M}_{\max}$ falls. Its direct effect operates through the paid-development path, making it more likely that OSS catches up before the firm reaches a quality at which catch-up is infeasible.

(ii) *Market size and patience.* A larger user market N scales the operating return to a proprietary quality advantage and therefore raises both e^* and $Q^{I,M}_{\max}$. A higher

discount factor β raises the value of the future operating-profit stream generated by a paid step and also extends the proprietary-only path. The table records their direct effects on the proprietary path. In the hybrid equilibrium, β also raises the value of waiting for OSS and the value of future catch-up, so the re-solved firm policy combines direct and continuation-value effects.

(iii) *Contributor costs.* A lower minimum contribution cost c_L makes positive contribution feasible at higher OSS qualities. Both OSS support cutoffs therefore rise. The lead-state cutoff remains weakly lower because a proprietary lead replaces the contributor's valuation v_j by the capped benefit $\min\{v_j, x^*\}$. Lower contributor costs can consequently convert a permanent proprietary lead into a catch-up path, although the additional participation may also increase real contribution costs in the welfare calculation.

(iv) *The valuation bound.* A higher v_H strengthens both sides of the market. It raises proprietary operating returns through e^*, but it also raises the largest contributor benefit at parity and, because $x^* = v_H/2$, the largest capped benefit under a proprietary lead. Thus all three quality frontiers expand. The net effect on mutual suppression is intrinsically relative: it depends on whether the proprietary stopping quality or the OSS support cutoffs cross first.

Cohort size changes probabilities, timing, and policy. The four direct objects in Table I are invariant to n, while the OSS success probability $1 - \gamma_r(q^O; n)$ changes with cohort size. A larger cohort changes the chance that at least one contributor acts and intensifies free riding, generating potentially nonmonotone probability effects. At an equal-quality state, these probabilities operate on both sides of the firm's start–wait comparison. If the firm starts, $1-\gamma_1(q^O; n)$ is the probability that OSS matches the new proprietary step within the same period. If the firm waits, $1-\gamma_0(q^O; n)$ is the probability that OSS advances the common frontier.

Because those two probabilities enter the Bellman comparison, re-solving the firm's problem after changing n can change the parity start set $\mathcal{S}_{\text{start}}$ and its upper no-further-investment boundary Q^I_{NS}. Cohort size therefore affects route probabilities both directly, through the transition probabilities along a fixed policy, and indirectly, through the firm's policy response. It also changes first-entry times and hence the discounted amount of proprietary investment delayed or omitted. Thus the zero in Table I records support invariance; the Bellman analysis tracks the associated probability, policy, and outcome changes.

From frontier movements to dynamic outcomes. The ordering of the cutoffs determines the economic environment. A change that lowers $Q^{I,M}_{\max}$ or raises $Q^{\text{OS}}_{\max,1}$ can move the economy into the catch-up-feasible environment, where OSS can close a carried-over proprietary lead. If $Q^{\text{OS}}_{\max,1} < Q^{I,M}_{\max} \leq Q^{\text{OS}}_{\max,0}$, the proprietary benefit cap is what makes OSS inactive after a lead. If $Q^{I,M}_{\max} > Q^{\text{OS}}_{\max,0}$, failure of catch-up instead reflects ordinary exhaustion of contributor support. Within any one of these environments, the contribution probabilities and the firm's state-dependent policy still determine which path is realized and how quickly. The table therefore predicts boundary movements, while the dynamic analysis supplies path probabilities, timing, and investment responses. The cohort-size case also provides the bridge to recursive welfare. Changing n alters the distribution of next period's state, the expected real cost of contribution, and potentially the firm's re-solved policy. The next subsection therefore follows welfare over time and later separates the direct contributor-side change from the welfare effect of the induced firm-policy response.

VI.2 Welfare Over Time

Welfare is discounted user value net of real proprietary investment and contribution costs. Prices and profits are transfers. As in the firm-value accounting above, the superscript hyb denotes the coexistence economy in which both OSS and the proprietary firm are active, and the superscript eq denotes welfare evaluated along the privately chosen firm policy and

contributor equilibrium. Let v denote a generic valuation draw from F_V. At a final state $(q^I, q^O) \in S$, gross flow surplus is

$$\mathcal{U}(q^I, q^O) = N\mathbb{E}[v]\psi(q^O) + N\mathbb{E}[v\mathbf{1}\{v \geq x^*\}][\psi(q^I) - \psi(q^O)]_+. \tag{10}$$

The first term gives every user the zero-price OSS quality as a common baseline. The second adds the quality advantage consumed by users whose valuation exceeds the proprietary pricing cutoff; the positive part selects the proprietary quality advantage at a lead. Prices and profits offset as transfers, leaving $\mathcal{U}$ as user value. Expected real contribution cost in regime r is

$$C_r^O(q^O) := n\mathbb{E}\left[c_j \mathbf{1}\left\{\frac{B_{j,r}(q^O)}{c_j} > \gamma_r(q^O)^{(1-n)/n}\right\}\right]. \tag{11}$$

The indicator selects contributor types whose benefit–cost ratio exceeds the equilibrium volunteering threshold. Multiplication by c_j, expectation over types, and then multiplication by the cohort size n give total expected real cost, including simultaneous contributions that are redundant under the best-shot technology.

Denote the firm's private equilibrium policy in the coexistence economy by a^{hyb}. Its contribution-stage regime is

$$r^{\text{hyb}}(q^I, q^O) = \mathbf{1}\{q^I + a^{\text{hyb}}(q^I, q^O) > q^O\}.$$

The corresponding welfare value over time is

$$\begin{aligned} W^{\text{hyb,eq}}(q^I, q^O) = & -c^I a^{\text{hyb}}(q^I, q^O) - C^O_{r^{\text{hyb}}(q^I, q^O)}(q^O) \\ & + \mathbb{E}\left[\mathcal{U}(q^I_{t,fin}, q^O_{t,fin}) + \beta W^{\text{hyb,eq}}(q^I_{t,fin}, q^O_{t,fin})\right]. \end{aligned} \tag{12}$$

The first line subtracts the current paid proprietary step and expected volunteer cost. The expectation then adds user surplus at the realized final pair and welfare from that same pair when it becomes next period's initial state. The recursion evaluates discounted surplus

along the firm's private policy and the contributor equilibrium.

Welfare decomposition for stronger OSS incentives. For any two contributor environments, the change in $W^{\text{hyb,eq}}$ is exactly the sum of:

(i) the mechanical change in OSS success and contribution cost while the original firm policy is held fixed; and

(ii) the welfare effect of the firm's private policy response in the new contributor environment.

An exact add-and-subtract decomposition, given in the Appendix, separates the change in contributor behavior under the old firm policy from the effect of re-solving the firm policy in the new contributor environment. Each term is governed by its induced surplus-and-cost balance. Holding the firm's policy fixed, a higher probability of OSS progress raises gross user surplus as OSS quality rises and changes expected real contribution cost. The mechanical term is positive when the induced user-surplus gain exceeds that additional cost. The policy-response term is positive when saved real investment cost exceeds the associated loss of user surplus from proprietary quality. The reverse inequalities generate the corresponding welfare losses.

The main-text welfare exercise compares equilibrium welfare across contributor environments within the coexistence economy.

VI.3 Empirical Counterparts

An empirical implementation would measure $(q^I_{t,ini}, q^O_{t,ini})$ immediately before the proprietary decision, the intermediate pair after that decision but before the OSS response, and the final pair after the response and any observed adoption. A proprietary lead is therefore a relative-quality event. Same-period OSS matching means that OSS matches the proprietary step within the same period and before current pricing; post-lead catch-up restores parity only in a later period.

The first prediction compares OSS progress behind a lead with progress at parity, conditional on OSS quality and the contributor pool. The second compares proprietary lead initiation across parity states with different predicted probabilities of same-period OSS matching. An empirical design combines these comparisons with demand, project composition, governance, and release-timing measures.

Composition is especially important. Let H^{new} and H^{core} be the type distributions of newly recruited and established contributors. If ω is the established-contributor share, the contributor distribution is $(1-\omega)H^{\text{new}} + \omega H^{\text{core}}$. Write $\gamma_r(q^O;\omega)$ for the unique symmetric fixed point induced by this mixture in regime r. For a parity composition ω_0 and a lead-state composition ω_1, the difference in OSS-improvement probabilities is the sum of a population-composition effect and the opposing fixed-population effect of the contributor-benefit cap. The exact decomposition is given in the Appendix. The composition effect holds H^{new} and H^{core} fixed. Recruitment that changes ω operates through it. Sponsorship that instead changes costs or either component distribution is a separate contributor-environment shift to which the same add-and-subtract logic applies. The fixed-population cap effect is nonnegative by Proposition 1. Aggregate activity combines the fixed-population effect with the composition response and can rise after proprietary entry when the composition response dominates. This decomposition organizes evidence that commercial participation changes both contribution incentives and who participates [Nagaraj and Piezunka, 2024; Haese and Peukert, 2025]. Identification uses variation in appropriation or contributor support together with predecision quality and contributor-composition measures.

The broader evidence provides institutional context for the mechanism. User-developed modifications support private use value [von Hippel, 2005]; OSS is a productive input and has large replacement value across firms [Nagle, 2019; Hoffmann et al., 2024]; and the Netscape–Mozilla episode illustrates strategic interaction but bundles pricing, licensing, and governance changes [Raymond, 2001].

Institutional application: enterprise databases. Enterprise database software provides institutional grounding for the model. PostgreSQL is available under a permissive license that allows use, modification, and distribution without a fee [PostgreSQL Global Development Group, 2026b]. Its project recognizes code authors, reviewers, testers, committers, release managers, infrastructure maintainers, and support providers as contributors [PostgreSQL Global Development Group, 2026a]. These features map into the model's user-developer benefit, integrable contribution cost, and compatible-reuse transition. Priced proprietary database products provide the competing commercial technology.

The state variables are measured as comparable release capabilities. A proprietary step is a newly commercialized capability, performance improvement, or reliability gain; an OSS match is an integrated release that closes the same measured gap. The timing distinction places the match before the current commercial return is realized or in a later period. The model then yields four testable implications:

(i) conditional on OSS quality and contributor composition, OSS matching should be less likely when the proprietary product already leads than at parity;

(ii) among parity states, proprietary lead initiation should be less likely when same-period OSS matching is more likely, with a larger effect for matches that arrive before rather than after current monetization;

(iii) behind a strict lead, the proprietary response can change sign with the state, while OSS progress should have little effect on the investment target once catch-up is infeasible; and

(iv) contributor recruitment, corporate sponsorship, and the share of experienced reviewers must be measured separately because composition can reverse the aggregate activity comparison.

These implications motivate an event-level panel of comparable releases with predecision quality, response timing, contributor composition, and proprietary investment outcomes.

VII Conclusion

This paper asks when open-source and proprietary innovation suppress each other. At a fixed contribution-stage state, product-market pricing caps a user-developer's private gain from OSS catch-up and can strictly lower the aggregate improvement probability. At initial parity, same-period OSS matching can remove its current operating return and change its continuation value relative to the newly created one-step lead. A post-pricing match changes the continuation value. Under the stated nonnegative payoff-difference condition, either effect is a private loss from matching and the pre-pricing effect can reverse the firm's investment action. Mutual suppression occurs when both strict comparisons hold at the same common quality.

The shared no-improvement probability links the two directions. A tighter upper bound on contributor benefits raises γ_1, discouraging OSS and protecting the newly initiated lead. The proprietary-cost interval on which same-period matching reverses investment shrinks with γ_1 and has zero length at the lead-regime feasibility cutoff. The model maps an interior mutual-suppression region, a proprietary-to-OSS suppression region, a proprietary investment suppression region, and states with neither strict effect. At a strict lead, the sign follows the continuation-value cross-difference; in the no-catch-up region, the investment target coincides with the proprietary-only target.

The one-period game identifies the local mechanisms and supplies the exact intraperiod transition kernel used at every dynamic state. The dynamic model evaluates the action-specific successor states with the Bellman equation. With bounded contributor types, equilibrium paths reach the upper no-further-investment region at parity or the exact no-catch-up region behind a proprietary lead. With unbounded support, fixed-population probability orders and a binomial tail bound characterize finite-horizon catch-up. Dominance prevention and paid investment are separately measured: late parity restoration preserves the completed paid path, while early arrival delays or omits paid steps.

Recursive welfare combines the contributor transition with the firm's private policy re-

sponse. Stronger OSS incentives raise the probability of OSS progress and can induce a retreat in proprietary quality, producing nonmonotone total welfare. A reimbursement for contributions made while the proprietary product leads provides an implementable counterpart to contributor pass-through; its financing and induced firm-policy response determine its welfare effect. Delayed or costly proprietary adoption changes the wait-and-success branch.

Three empirical distinctions follow. First, OSS response is measured relative to both products' predecision qualities and within a prespecified release interval. Second, contributor composition and sponsorship are measured alongside the fixed-population suppression channel. Third, observed catch-up is decomposed into the contributor-benefit cap and ordinary exhaustion using a relaxation of the pricing-induced upper bound on contributor benefits.

OSS constrains proprietary dominance by matching a paid step within the same period or by restoring parity during subsequent catch-up. The central tension is that proprietary pricing changes the probability of the OSS response that, in turn, determines whether the firm's lead survives.

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