Endogenous Value Chain Formation

Organizational Capability, Coordination, and Productivity Diffusion

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August 2026

Abstract

This paper develops a theory of endogenous value chain formation and productivity diffusion in which coordination is supplied by anchor firms. Heterogeneous firms sort into anchor and partner roles according to organizational capability. Anchor firms raise partner productivity through costly relationship-specific coordination — including technology transfer, quality upgrading, training, and input financing — but appropriate only a share of the revenue increment their coordination generates in each relationship; appropriability erodes endogenously as anchor entry intensifies competition among coordinating firms. Incomplete appropriability distorts coordination effort, network breadth, and anchor entry at a unique equilibrium. We characterize the constrained-efficient allocation and derive three corrective instruments that decentralize it. A single diagnostic statistic separates insufficient-entry from excess-entry regimes, while the implementing entry instrument is determined separately under corrected conduct. The central mechanism links endogenous coordination to productivity diffusion and value creation. The scale of these gains depends on the capability distribution, which determines both the mass of coordinating firms and the productivity of their coordination relationships. Because corrective instruments operate through anchor firms, deterioration in the capability distribution that drives anchor density toward zero also drives aggregate policy responsiveness toward zero. Organizational capability therefore determines not only how much value coordination generates, but also whether corrective policy has any traction. In a numerical illustration, privately supplied coordination raises sectoral output by roughly 21 percent relative to the no-coordination benchmark. Corrective policy generates additional gains of roughly 2 percent in net social value and 22 percent in sectoral output relative to the decentralized equilibrium, with both the level and the shape of the capability distribution governing these gains.

JEL Classification: O14, O33, L14, L22, D24
Keywords: Endogenous value chain formation; productivity diffusion; organizational capability; anchor firms; coordination; production networks

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1. Introduction

Differences in aggregate productivity across countries far exceed differences in access to technology. Much of the productive knowledge that could raise output in low-income economies already exists—embodied in techniques, standards, and organizational practices used elsewhere—yet it remains undeployed. A long tradition in economics has argued that persistent differences in economic development arise, in part, from demand-side coordination failures that impede industrialization and structural transformation. Rosenstein-Rodan (1943) first articulated this idea, and Murphy, Shleifer, and Vishny (1989) formalized it as a demand-externality model in which increasing returns generate multiple equilibria, including persistent underdevelopment. Yet the canonical remedy—centralized coordination of investment across sectors—demands information, administrative capability, and credibility that governments in low-capacity environments rarely possess. This literature explains why firms may fail to invest simultaneously, but it does not explain how productive knowledge is transmitted across firms once production relationships form. Whether such coordination can instead emerge endogenously through firms on the supply side, and what determines when it does, remains an open question.

This paper develops a theory in which coordination is supplied by firms. High-capability firms — hereafter anchor firms[1] — find it privately profitable to organize production networks: they invest coordination effort in upstream and downstream partners, raising partner productivity through technology transfer, quality upgrading, training, input financing, and demand stabilization, and appropriate a share of the revenue increment each partnership generates. The model combines endogenous selection among heterogeneous firms with a Penrosian view of organizational capability (Penrose, 1959): firms differ in a single capability index, sort endogenously into anchor and partner roles according to a capability threshold, and each anchor's capability bounds the total coordination effort it can deploy across its network. Which firms coordinate, how intensely, and over how many partners are all equilibrium objects. Under stated regularity conditions the decentralized equilibrium exists and is unique: coordination here fails *partially, at a unique equilibrium*, rather than through multiplicity — so the policy problem is not equilibrium selection but the correction of identifiable, margin-specific distortions.

The mechanism is pervasive in practice. Across a wide range of global value chains, lead firms routinely provide training, technical assistance, quality upgrading, certification support, logistics, and other forms of supplier development to legally independent production partners. These investments are undertaken because they raise the quantity and quality of supply available to the coordinating firm. The same logic operates across agro-industrial value chains — including cotton, cocoa, coffee, and horticulture — as well as in manufacturing through supplier development programs and at the technological frontier through ecosystem management and standards

[1] The term *anchor firm* emphasizes the firm's role as the organizational center of a production network. The terminology builds on the "anchor tenant" of Agrawal and Cockburn (2003) and corresponds most closely to the "lead firm" of the global value chain literature (Gereffi, Humphrey, and Sturgeon, 2005); related notions include focal firms in supply-chain management and hub firms in network analysis. In this paper, the anchor is an organizing firm within a production network that deliberately invests in coordinating production partners. Productivity diffusion therefore arises through intentional coordination investments embedded in production relationships—investments whose value the anchor only partly appropriates—rather than as an incidental by-product of anchor activity operating through regional markets.

integration, as documented across diverse sectors and institutional environments (Gereffi, Humphrey, and Sturgeon, 2005; Otsuka, Nakano, and Takahashi, 2016; Brambilla and Porto, 2011). What varies across these environments is not the underlying mechanism but the capability of the firms involved, the cost and intensity of coordination, and the extent to which coordinating firms can appropriate the coordination-generated value they create.

The market solution is systematically partial. Because contracts are incomplete and partners retain outside options, an anchor appropriates only a fraction υ of the revenue increment its coordination generates in each partner relationship — and this fraction is endogenous: it erodes as anchor entry intensifies competition among coordinating firms, which improves partners' outside options and bids down the anchor's share of the value created. Incomplete appropriability generates three distinct distortions, one on each margin of the coordination decision: effort per relationship is underprovided, production networks are inefficiently narrow, and anchor entry itself is distorted — in either direction, since entry expands coordination but also dissipates the appropriability that sustains it. We characterize the constrained-efficient allocation and derive three corrective instruments that decentralize it: a Pigouvian effort subsidy, a relationship-level inclusion grant, and an entry instrument. A single diagnostic statistic classifies the direction of the entry distortion — the same model yields insufficient-entry and excess-entry regimes as opposite cases of one condition — while the implementing instrument may be a subsidy or a tax: the direction of the distortion and the sign of the instrument turn out to be distinct questions, a distinction Section 5 develops.

The framework's central positive result characterizes when coordination—and the policies built upon it—matter. Equilibrium coordination — and with it the output gain over the no-coordination benchmark — is governed by three fundamentals: the capability distribution, institutional quality operating through appropriability, and coordination costs, with the fixed cost of anchor status operating solely through the entry margin. When organizational capability becomes sufficiently scarce, the economy operates in a Capability-Constrained Coordination Regime: anchor density collapses, and with it the output gains generated by the corrective package. Aggregate policy responsiveness converges to zero as capability weakens — organizational capability determines not only how much an economy produces, but whether corrective policy has any traction at all. This failure mode is distinct in kind from the incentive failure in Murphy, Shleifer, and Vishny (1989): there, the right policy flips the economy to a good equilibrium that incentives alone could not reach; here, the instruments that correct the appropriability distortions lose their grip because the firms through which they operate do not exist. The result requires no equilibrium multiplicity; it is proved directly from the structure of the policy-transmission channel.

A numerical illustration demonstrates these mechanisms. Privately supplied coordination is quantitatively substantial even without corrective policy: in the baseline calibration, the decentralized equilibrium raises sectoral output by roughly 21 percent relative to the no-coordination benchmark. The corrective package generates additional gains, raising net social value by roughly 2 percent and sectoral output by approximately 22 percent relative to the decentralized equilibrium. These gains rise sharply with mean capability as anchor density and the productivity of coordination increase. Holding mean capability fixed, greater dispersion substantially reduces decentralized output, anchor density, and policy-induced gains. The

exercises therefore show that both the level and the shape of the capability distribution govern value-chain formation and the gains from corrective policy.

The paper relates to three major traditions. First, it complements the Big Push literature (Rosenstein-Rodan, 1943; Murphy, Shleifer, and Vishny, 1989), which emphasizes demand-side coordination failures across sectors, by focusing on supply-side coordination failures within production networks. By introducing firm heterogeneity, endogenous network formation, and incomplete appropriability, it also moves beyond the binary distinction between stagnation and industrialization toward intermediate cases in which coordination occurs in equilibrium but remains incomplete on the effort, inclusion, and entry margins. Second, it provides a microfoundation for Hirschman's (1958) linkage theory by modeling how anchor firms transform potential partner relationships into productivity-enhancing linkages through coordination effort and organizational capability. Third, it is conceptually complementary to endogenous growth theory: whereas models such as Romer (1990) emphasize the creation of new knowledge and its market-mediated transmission, the present framework emphasizes the organizational deployment of existing productive knowledge through production networks.

The paper also relates to the empirical literature on management and organizational capability (Bloom and Van Reenen, 2007; Bloom et al., 2013). That literature documents persistent productivity differences associated with management quality; the present framework gives organizational capability a general-equilibrium role by allowing it to determine firm selection into coordination, network formation, and the transmission of productivity gains across firms. Finally, the model speaks to the literature on value chains and supplier development (Gereffi, Humphrey, and Sturgeon, 2005) by explaining why lead firms invest in partner capabilities, why such investment remains underprovided, and how coordination varies with institutional and capability conditions.

More broadly, the model offers a reinterpretation of industrial coordination. Rather than requiring comprehensive state direction, coordination can emerge from decentralized production networks organized around high-capability firms, with corrective instruments operating on identifiable distortions in privately supplied coordination rather than on the allocation of investment itself. The model shifts attention from the absence of a coordinating state to the scarcity of firms with sufficient organizational capability as a constraint on structural transformation—and the same scarcity that limits output also limits what correction can achieve.

The remainder of the paper proceeds as follows. Section 2 describes the environment, in which heterogeneous firms occupy one of two roles — anchor or partner — separated by a capability threshold determined in equilibrium. Section 3 introduces the three frictions: coordination costs, the organizational capability constraint, and incomplete appropriability. Section 4 characterizes the decentralized equilibrium, derives the endogenous sorting of firms into anchor and partner roles, and identifies the three distortions. Section 5 derives the constrained-efficient allocation and the three corrective instruments that decentralize it. Section 6 maps organizational capability to aggregate output and characterizes the Capability-Constrained Coordination Regime. Section 7 illustrates the model's central mechanisms numerically. Proofs appear in Appendices A–D; computational methods, calibration, and robustness are documented in Appendix E. Section 8 concludes.

2. Economic Environment and Production

2.1 Environment

Consider a coordination sector, indexed by s[2], populated by a continuum of heterogeneous firms indexed by k. Each firm is endowed with organizational capability[3] denoted $M_k \geq 1$, drawn from a common distribution $F_s(M)$, where $M_k = 1$ represents the minimum economically relevant capability level. Organizational capability captures the firm's ability to coordinate, integrate, and deploy productive resources within and across organizational boundaries. It influences both firm-level productivity and the firm's capacity to establish, manage, and upgrade production relationships. Organizational capability is the scarce organizational resource that determines whether a firm can profitably operate as an anchor firm and how intensively it can engage in coordination activities.

In equilibrium, firms self-select into two endogenous roles. Firms with sufficiently high capability operate as anchor firms, indexed by $i \in I$, while lower-capability firms participate as partner firms, indexed by $j \in J$. The capability threshold required for anchor entry, M^*, is determined endogenously in equilibrium. Firms with $M_k \geq M^*$ operate as anchor firms, incur coordination costs, and form production networks, whereas firms with $M_k < M^*$ participate as partner firms. Firms differ only in organizational capability; anchor and partner roles emerge endogenously in equilibrium. Operating as an anchor firm requires an irreversible fixed coordination entry cost, $c_0 > 0$, representing the organizational investments necessary to establish and maintain coordination capacity, including management systems, partner identification, monitoring, and other network-specific infrastructure. In addition, anchor firms incur variable coordination costs, introduced in Section 3.1, that increase with the intensity and breadth of coordination activities.

Each partner firm is matched to at most one anchor firm, while each anchor may coordinate multiple partners, generating a one-to-many network structure.[4] The set of partners coordinated by anchor i is chosen optimally and denoted $N_i \subseteq J$.[5] Anchor firms compete to establish and maintain

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[2] The coordination sector is formally defined in Section 6 (Definition 3), where the framework is generalized to multiple sectors. For expositional simplicity, the sector subscript is suppressed on equilibrium objects throughout Sections 2–5.

[3] The notion of organizational capability builds on Penrose's (1959) view of the firm as a bundle of productive resources whose deployment is constrained by its organizational and managerial capacity. It is related to absorptive capacity (Cohen and Levinthal, 1990) and dynamic capabilities (Teece, Pisano, and Shuen, 1997). It is modeled here as a higher-order organizational asset governing the deployment and coordination of productive resources within and across firm boundaries. Management quality, as measured by Bloom and Van Reenen (2007) and Bloom, Eifert, Mahajan, McKenzie, and Roberts (2013), should therefore be viewed as one observable manifestation of organizational capability rather than as an equivalent concept.

[4] The assumption that each partner is affiliated with at most one anchor applies to coordination relationships rather than to arm's-length transactions. While suppliers may sell to multiple buyers, the transfer of knowledge, technical assistance, and production standards is typically concentrated within a dominant coordination relationship. The relevant network is therefore a coordination network, describing the flow of productive knowledge and support, rather than the full pattern of input-output transactions.

[5] Anchor density (I) is modeled as a continuous mass because it emerges from a continuum of heterogeneous firms. By contrast, partner relationships are discrete: an anchor either includes a given partner in its network, or it does not. Modeling (N_i) as discrete preserves the network structure of the coordination problem without affecting the core economics.

these relationships, and the intensity of competition is governed by the equilibrium density of active anchors, I. Variables associated with partner j coordinated by anchor i carry the double subscript ij, while single subscripts refer to anchor- or partner-specific variables when no ambiguity arises.

Anchor firms undertake costly relationship-specific coordination activities that raise partner productivity above baseline levels. These activities generate productivity gains for both partner firms and anchor firms through the total-factor-productivity channel. Organizational capability therefore determines not only whether firms become anchors but also the scale and intensity of coordination networks they can sustain.

Absent network interactions, firm k's baseline productivity is given by M_k^{α}, where $0 < \alpha < 1$. Organizational capability therefore constitutes the fundamental source of baseline productivity heterogeneity in the economy. It is modeled as a determinant of total factor productivity rather than as a direct production input, consistent with both the empirical management literature, which links managerial quality to persistent productivity differences across firms (Bloom and Van Reenen, 2007; Bloom, Eifert, Mahajan, McKenzie, and Roberts, 2013), and the capabilities-based view of the firm (Penrose, 1959; Teece, Pisano, and Shuen, 1997).

Let Y_k^0 denote the value added that firm k would produce under its baseline productivity M_k^{α}, in the absence of production-network coordination. Baseline output is therefore determined solely by baseline organizational capability and the firm's optimal use of primary factors under the benchmark technology. Throughout the paper, Y_k^0 serves as the reference level of production against which coordination gains are measured.

Production is modeled in value-added terms using only primary factors, so that aggregate output corresponds directly to GDP without double counting intermediate transactions. The baseline analysis abstracts from international trade in order to isolate the role of organizational capability and network coordination in shaping production structure and aggregate productivity.

All firms operate under perfect competition in both goods and factor markets and produce under constant returns to scale. Absent coordination activity, a firm of organizational capability M_k therefore earns zero operating profit, $\pi^0(M_k) = 0$. Consequently, any positive economic profit earned by an anchor firm arises exclusively from coordination activities. The share of the coordination-generated revenue increment that an anchor appropriates is developed in Section 3.3.

To isolate the supply-side determinants of value-chain formation, final demand is assumed to be exogenous and symmetric across sectors, and firms take output prices as given. Throughout, 'increasing' and 'decreasing' denote weak monotonicity; strict monotonicity is indicated explicitly.

2.2 Anchor Firm Production Technology

Anchor firms operate a standard Cobb–Douglas technology in primary factors, but all economically relevant heterogeneity enters through total factor productivity, which depends jointly on organizational capability and the productivity of network partners. Value added produced by anchor firm i is given by:

$$Y_i = A_i K_i^{\beta} L_i^{1-\beta}, \quad \beta \in (0,1) \qquad (1)$$[6]

where K_i denotes capital and L_i labor. Constant returns to scale are assumed in primary factors.

Total factor productivity A_i is endogenous and depends both on the organizational capability of the anchor firm and on the productivity of its network partners:

$$A_i = M_i^{\alpha} \prod_{j \in N_i} q_{ij}^{\phi}, \qquad 0 < \alpha < 1, \phi \in (0,1) \qquad (2)$$

where M_i[7] denotes the organizational capability of anchor firm i, q_{ij} denotes the productivity level of partner j within the network of anchor i, and ϕ measures the elasticity of anchor productivity with respect to partner j's productivity[8].

Equation (2) embeds two forms of complementarity. Productivity gains across partners are complementary: the productivity contribution of any relationship is larger when the other relationships in the network are more productive, so inefficiencies in any relationship reduce the productivity of the anchor firm. Organizational capability and partner productivity are also complementary: more capable anchors obtain larger productivity gains from more productive partners. This second complementarity underlies the assortative-matching result developed in Appendix C. Therefore, in the present framework, organizational capability determines the anchor firm's baseline productivity and further enhances it by improving partner performance through network coordination[9]. The restriction $0 < \alpha < 1$ implies diminishing marginal productivity of organizational capability.

The multiplicative structure of equation (2) is related to the weak-link production frameworks of Kremer (1993) and Jones (2011), in which complementarities across production tasks or intermediate inputs make aggregate production sensitive to the performance of individual stages. The connection to intermediate-input linkages is particularly direct for supplier relationships, including outgrower arrangements, where the productivity, quality, or quantity of upstream production affects the performance of the anchor firm and the production network as a whole. The

[6] The Cobb–Douglas specification is adopted for tractability and calibration. The paper's core results rely on constant returns to scale and competitive factor markets rather than on the Cobb–Douglas functional form itself. More generally, under standard CRS homothetic technologies, optimal revenue remains proportional to productivity, $pY^* = \Omega(p, r, w) \cdot A_i$, where $\Omega(\cdot)$ depends only on output and factor prices. Cobb–Douglas simply delivers a convenient closed-form expression for Ω, that is used in the quantitative illustration of Section 7.

[7] Organizational capability is treated as exogenous, reflecting the view that capability accumulation operates on a longer time scale than the coordination decisions modeled here. The analysis therefore takes the distribution of capabilities as given and examines the coordination and policy implications of capability scarcity.

[8] The aggregate effect of ϕ depends on the equilibrium network size N_i: under symmetric proportional improvements in partner productivity, the aggregate elasticity is approximately ϕN_i. Thus, relatively small values of ϕ remain economically meaningful in sectors with large supplier networks (e.g., agricultural outgrower schemes), whereas larger values may be appropriate for industries with smaller, more intensive supplier networks. Accordingly, ϕ should be calibrated jointly with the empirical interpretation of N_i.

[9] Partner productivity enters anchor productivity through the production network: coordination effort raises partner productivity, which in turn increases the productivity of the anchor's production network and hence the anchor's own productivity. The resulting returns to coordination effort are characterized in Section 4.2; see, in particular, equations (10) and (12).

present framework is broader: partners need not be suppliers, and q_{ij}—the productivity of partner j within the network of anchor firm i (equation (3))—may reflect the partner's role in supplying intermediate inputs, services, distribution, or other complementary activities performed by legally independent firms. Equation (2) therefore extends the same complementarity logic to an endogenously formed network of interfirm relationships.

2.3 Partner Firm Technologies

Partner firm $j \in N_i$ produces value added using the same constant-returns-to-scale technology as anchor firms, but with productivity endogenously enhanced through relationship-specific coordination investments undertaken by the anchor.

$$Y_{ij} = q_{ij} K_{ij}^{\beta} L_{ij}^{1-\beta}, \qquad \beta \in (0,1) \qquad (3)$$

where K_{ij} and L_{ij} denote capital and labor inputs, respectively, and q_{ij} denotes the productivity of partner j within the network of anchor firm i

Anchor firm i invests coordination effort $e_{ij} \geq 0$ in each partner $j \in N_i$. Coordination effort represents costly relationship-specific investments—including input upgrading, quality control, technology transfer, logistics support, market access facilitation, and demand guarantees—that raise partner productivity and, through production complementarities, increase the productivity of the anchor firm itself. Partner productivity is given by:

$$q_{ij}(e_{ij}) = q_{ij}^{0}\left(1+\theta_{ij} e_{ij}\right)^{\delta}, \qquad \theta_{ij} > 0, \delta \in (0,1) \qquad (4)$$

where $q_{ij}^{0} = M_j^{\alpha}$ denotes the baseline productivity of partner j, which depends positively on its organizational capability M_j. The parameter θ_{ij} captures the responsiveness of partner j to coordination effort—its absorptive capacity. Following Cohen and Levinthal (1990), the framework assumes that this capacity depends on the partner's pre-existing organizational capability M_j. It satisfies $\theta(1) = 0$ and $\theta'(M_j) > 0$: firms at the minimum organizational capability possess no absorptive capacity, while responsiveness to coordination increases monotonically with capability. The parameter $\delta \in (0,1)$ governs diminishing marginal returns to coordination. Initial coordination investments therefore generate large productivity gains, while additional effort yields progressively smaller marginal improvements.

The productivity-enhancing role of such coordination effort is consistent with evidence from multinational supply chains: Alfaro-Ureña, Manelici, and Vasquez (2022) document explicit buyer support through product specifications, process learning, audits, and improvement guidance, alongside persistent productivity gains among domestic suppliers.
The concavity of $q_{ij}(e_{ij})$ implies diminishing marginal returns to coordination effort at the partner level. Partner firms can in principle absorb arbitrarily large levels of coordination effort, but coordination effort is ultimately bounded by the coordination costs introduced in Section 3.1, operating through the anchor's problem analyzed in Section 4. Higher productivity increases the marginal productivity of primary factors. The partner's optimal factor demands and equilibrium output are characterized in Section 2.4.

Equation (2) specifies anchor TFP as depending on the level of partner productivity q_{ij} rather than solely on the coordination-induced gain, because production complementarities tie anchor performance to the overall productivity of its partner base. Partner capability M_j operates through two complementary channels — baseline productivity $q^0(M_j)$ and absorptive capacity $\theta(M_j)$— both increasing in M_j. Consequently, more capable partners are both intrinsically more productive and generate larger returns to coordination investments. This complementarity creates a natural incentive for positive assortative matching in production networks, with aggregate implications developed in CS2 and Section 6.3.

2.4. Partner Firm's Problem

Partner firms are competitive: they take the output price, p_{ij}, and the factor prices, r and w, as given and choose capital and labor subject to the technology in equation (3). Coordination with anchor firm i raises partner j's productivity to q_{ij}, and the anchor appropriates a share $\upsilon_{ij} \in (0,1)$ of the resulting coordination-generated revenue increment. The parameter υ_{ij} is taken as given at this stage; Section 3.3 characterizes the appropriability environment that determines it. The partner solves

$$\max_{K_{ij},\, L_{ij}} \Pi_{ij} = p_{ij}Y_{ij} - \upsilon_{ij}\, p_{ij}(Y_{ij} - Y_{ij}^0) - rK_{ij} - wL_{ij}, \qquad (5)$$

where Y_{ij}^0 denotes the partner's predetermined pre-coordination benchmark output, which serves as the contractual baseline; thus $Y_{ij} - Y_{ij}^0$ is the coordination-generated output increment on which the anchor's claim is written. The revenue accruing to the partner admits an equivalent decomposition:

$$p_{ij}Y_{ij} - \upsilon_{ij}\, p_{ij}(Y_{ij} - Y_{ij}^0) = (1 - \upsilon_{ij})\, p_{ij}Y_{ij} + \upsilon_{ij}\, p_{ij}Y_{ij}^0, \qquad (5a)$$

so that the arrangement is equivalent to a proportional claim by the anchor on the partner's output, with the benchmark output Y_{ij}^0 exempt from that claim. Contractual arrangements of this type — relationship-specific investment by a lead firm compensated through a claim on the resulting increment — are widely documented in global value chains (Macchiavello and Morjaria, 2015).

Because the technology exhibits constant returns to scale and the partner takes prices as given, the first-order conditions of problem (5) determine cost-minimizing factor proportions but not, by themselves, a unique scale of production; a finite production level requires a scale-determining condition, and the expressions that follow characterize factor payments and profit conditional on an equilibrium scale.[10]1 Because the contractual benchmark Y_{ij}^0 is fixed with respect to the partner's current factor choices, the conditions for an interior solution are:

[10] Under constant returns to scale and price taking, conditions (5b) determine only the cost-minimizing factor proportions. Production scale is determined elsewhere in the market equilibrium through demand for the relationship's output. The analytical characterization selects this scale by the normalization stated in Appendix A.1, which the numerical illustration in Section 7 maintains.

$$(1 - \upsilon_{ij})\, p_{ij}\, \frac{\partial Y_{ij}}{\partial K_{ij}} = r, \qquad (1 - \upsilon_{ij})\, p_{ij}\, \frac{\partial Y_{ij}}{\partial L_{ij}} = w. \qquad \text{(5b)}$$

The partner equates the marginal revenue products of capital and labor, net of the anchor's appropriation share, to the respective factor prices: at the margin, each unit of output enlarges the increment $Y_{ij} - Y_{ij}^0$ one for one, so the partner retains only the fraction $1 - \upsilon_{ij}$ of marginal revenue. Weighting the conditions in (5b) by K_{ij} and L_{ij} and invoking Euler's theorem for the linearly homogeneous technology yields

$$rK_{ij} + wL_{ij} = (1 - \upsilon_{ij})\, p_{ij}\, Y_{ij}. \qquad \text{(5c)}$$

Under competitive factor pricing, the scale-dependent component of the partner's retained revenue exactly equals its factor payments. Substituting (5c) into (5), and using the decomposition above, therefore leaves the exempt benchmark component as the partner's equilibrium profit:

$$\Pi_{ij}^{*} = \upsilon_{ij}\, p_{ij}\, Y_{ij}^0. \qquad \text{(5d)}$$

In the absence of coordination, the partner solves the analogous problem with $\upsilon_{ij} = 0$ at baseline productivity q_{ij}^0; constant returns and competitive pricing imply that factor payments exhaust revenue, so benchmark profit is zero. Since $\upsilon_{ij} > 0$, $p_{ij} > 0$, and $Y_{ij}^0 > 0$, equation (5d) implies that coordination raises the partner's equilibrium profit above the zero-profit benchmark; participation is therefore individually rational.

Equation (5d) clarifies the relationship between the contractual arrangement and equilibrium profits. The contract specifies claims over the coordination-generated revenue increment: the anchor's claim is $\upsilon_{ij}\, p_{ij}(Y_{ij} - Y_{ij}^0)$, and the partner's retained claim is $\left(1 - \upsilon_{ij}\right) p_{ij}(Y_{ij} - Y_{ij}^0)$. Equilibrium profits are determined only after all payment obligations — including the contractual transfer to the anchor and the competitive remuneration of production factors — have been satisfied. By (5c), the scale-dependent component of the partner's retained revenue is exhausted by factor payments, leaving the exempt benchmark component as residual profit. Consequently, contractual allocations describe the division of the value that coordination creates, whereas equilibrium profits describe the ultimate economic incidence of that value. This distinction plays a central role in the welfare accounting of Section 5 and organizes the presentation of the numerical results in Section 7.

3 Coordination Costs, Capability Constraints, and Incomplete Appropriability

This section introduces three frictions that jointly shape private incentives for production-network coordination. Coordination costs and organizational capability constraints limit the scale and intensity of anchor-firm investments, while incomplete appropriability creates a wedge between the private and social returns to coordination. Together, these forces determine equilibrium network formation and give rise to the coordination failures analyzed below.

3.1 Coordination Costs.

Conditional on paying the fixed coordination cost c_0, anchor firms incur variable coordination costs that increase with both effort intensity e_{ij} and network size N_i:

$$c_v(e_{ij}, N_i) = \frac{\lambda(N_i;\mathbf{z})}{2} \sum_{j \in N_i} e_{ij}^2, \quad \lambda'(N_i) > 0,\ \lambda''(N_i) \geq 0 \qquad (6)^{11}$$

where λ is a coordination congestion function capturing the increasing complexity of coordinating larger networks of partners and z indexes exogenous coordination-environment shifters — infrastructure, logistics, and organizational systems — with higher z denoting a more favorable environment and $\lambda_z(N_i; \boldsymbol{z}) < 0$ componentwise. The argument z is suppressed hereafter. The quadratic specification implies convex costs in coordination effort, while the dependence of λ on network size captures rising monitoring difficulties, managerial congestion, communication costs, and the declining quality of marginal partners as networks expand.

Coordination effort e_{ij} is measured in abstract organizational units capturing managerial attention, technical assistance, monitoring, training, and other coordination activities undertaken by the anchor firm. The congestion parameter $\lambda(N_i)$ converts these activities into value units and can therefore be interpreted as the shadow cost of organizational resources devoted to coordination. As the number of coordination relationships expands, these resources become increasingly stretched, causing the marginal cost of coordination effort to rise.

The specification generates an endogenous limit to network breadth: although additional partners may continue to generate positive expected returns, rising coordination costs eventually exhaust the gains from further network expansion. Together with the soft partner-side capacity constraint implied by diminishing returns to coordination, this ensures an interior optimum for both coordination effort and network size. As in Lucas (1978), organizational capability determines the scale of activity that a single organizing agent can profitably coordinate, while, as in Garicano (2000), communication, knowledge transmission, and related coordination activities consume scarce organizational resources, limiting organizational scale. Unlike both, however, the relevant organizational problem here concerns the coordination of legally independent firms within production networks rather than the allocation of productive inputs or knowledge within a single firm.

3.2 Capability Constraints

In addition to coordination congestion, anchor firms face an aggregate organizational capability constraint:

[11] The congestion technology is assumed common across anchor firms, so that differences in coordination outcomes arise solely from organizational capability M_i and endogenous network choices. More generally, the congestion parameter may depend on a vector of environmental factors z— including infrastructure quality, institutional effectiveness, and information technology — that affect the cost of managing partner relationships. The baseline analysis treats z as fixed in order to isolate the role of organizational capability. Improvements in these factors shift the congestion function downward, reducing coordination costs and supporting broader equilibrium production networks.

$$\sum_{j \in N_i} e_{ij} \leq M_i \qquad (7)$$

It is useful to distinguish between organizational capability and coordination effort. Organizational capability M_i is a firm-level organizational asset that captures the anchor's managerial, engineering, and coordination capacity. By contrast, e_{ij} is a relationship-specific choice variable representing the coordination services deployed toward partner j. Organizational capability determines what the anchor firm can do; coordination effort determines what it actually does. Constraint (7) therefore captures capability scarcity: the aggregate coordination effort deployed across the network cannot exceed the anchor firm's available organizational capability.

The treatment of organizational capability is closely related to Penrose's (1959) theory of firm growth. Penrose argued that the expansion of firms is ultimately constrained by the availability of managerial and organizational services rather than by physical resources or technology alone. The present framework extends this insight beyond the boundaries of the firm to production networks. Organizational capability determines not only a firm's own productivity but also its ability to coordinate and upgrade partners. The breadth-versus-depth tradeoff implied by constraint (7) can therefore be interpreted as a network analogue of the Penrose effect: the expansion of production networks is ultimately constrained by the organizational capability available to coordinate increasingly complex partner relationships.

The interpretation of constraint (7) as a scarce-capability constraint is related to Sutton (2012), who places firm capabilities at the center of industrial development. Sutton's formal object is revealed capability — the productivity and quality a firm displays in product-market competition — while its deeper organizational determinants, which he traces to Penrose (1959), remain outside his formal model. The present framework models that underlying layer directly: organizational capability M_i is a primitive that determines both the anchor firm's own productivity and its capacity to coordinate legally independent partner firms, with constraint (7) governing the allocation of that capacity across the partner network.

The model distinguishes between coordination congestion and capability scarcity. Whereas $\lambda(N_i)$ governs the marginal cost of coordination effort, constraint (7) limits the total coordination effort that can be deployed across the network. The former captures the increasing difficulty of managing larger supplier networks, while the latter reflects the finite organizational capability available to the anchor firm. These two constraints operate through distinct channels and jointly shape equilibrium network breadth and coordination effort.

3.3 Incomplete Appropriability.

Unlike benchmark production, where competitive markets eliminate economic profits under constant returns to scale (Section 2.4), coordination creates relationship-specific value whose contractual allocation depends on the appropriability environment. The appropriability parameter $\upsilon_{ij} \in (0,1)$ denotes the share of the coordination-generated revenue increment appropriated by the anchor firm. For tractability, this share is modeled as a reduced-form function of the appropriability environment:

$$v_{ij}(I) = v(I, i_q), \qquad \frac{\partial v(I, i_q)}{\partial I} < 0, \qquad (8)$$

where I denotes the density of active anchor firms. As anchor density increases, partner firms acquire stronger outside options, reducing $v_{ij}(I)$ for any individual anchor firm. Appropriability is assumed to depend only on anchor density and institutional quality, $v(I, i_q)$, and not on match-specific characteristics. Because institutional quality is treated as an exogenous parameter throughout the analysis, we suppress the argument i_q hereafter and write simply $v(I)$. As Section 2.4 shows, this contractual claim should be distinguished from equilibrium profits, which are determined only after competitive factor payments.

Institutional quality shifts the appropriability schedule: stronger contractual enforcement — through judicial quality, dispute resolution, and commercial law — strengthens the enforceability of the anchor's contractual claim over the coordination-generated revenue increment, thereby increasing the anchor's appropriated share and mitigating — but not eliminating — the wedge between private and social returns to coordination. Weak enforcement, limited legal recourse, or the scope for partner-side renegotiation lower appropriability and deepen the underprovision of coordination effort.

The function υ(I) is assumed continuously differentiable on its domain. This representation is related to the property-rights literature on buyer–supplier relationships (Antràs, 2003; Antràs and Helpmann, 2004): the analysis requires only that appropriability be decreasing in anchor density, $v'(I) < 0$, reflecting the strengthening of partner outside options as competition among anchors intensifies, and does not depend on a specific bargaining protocol. The treatment of the anchor's claim as partially enforceable within an ongoing commercial relationship is in the spirit of the relational-contracting literature (Baker, Gibbons, and Murphy, 2002). The precise shape of $v(I)$ is left unspecified at this stage. Importantly, equilibrium appropriability is an endogenous outcome rather than a policy instrument.

The coordination investments considered here are relationship-specific, linking the model to the literature on relationship-specific investment, appropriable quasi-rents, and the hold-up problem (Klein, Crawford, and Alchian, 1978; Grossman and Hart, 1986; Hart and Moore, 1990). Unlike that literature, however, the present framework takes firm boundaries as given and represents appropriability through a sharing rule over the coordination-generated revenue increment whose equilibrium value is determined endogenously rather than through ownership allocation.

Partner firms do not choose coordination effort, which is determined by anchor firms, but remain economically relevant through their participation in coordination relationships and through the appropriability environment governing the division of the coordination-generated revenue increment. Because $v(I) < 1$ in general, anchor firms internalize only part of the value generated through coordination. Higher appropriability strengthens the anchor's private return to coordination investment, mitigating the underprovision of coordination effort generated by incomplete appropriability. Since appropriability itself depends on anchor density, relationship-level coordination decisions and industry structure are jointly determined. Section 4 derives the resulting decentralized equilibrium and characterizes coordination effort, network breadth, and anchor entry.

4. Equilibrium

This section characterizes the decentralized equilibrium and identifies three distinct distortions—in coordination effort, network breadth, and industry structure—that arise from incomplete appropriability. We derive anchor firms' optimal coordination effort and network-formation decisions, and then characterize the endogenous entry and selection mechanism that determines equilibrium anchor density. Finally, we establish the model's comparative-static implications.

4.1 Anchor Firm's Problem.

Each anchor firm i chooses capital K_i, labor L_i, coordination effort $\{e_{ij}\}_{j\in N_i}$, and the size of its partner network N_i to maximize profits:

$$\max_{K_i, L_i, \{e_{ij}\}_{j\in N_i}, N_i} \Pi = P_i Y_i - wL_i - rK_i - \sum_{j\in N_i} \frac{\lambda(N_i)}{2} e_{ij}^2 + \sum_{j\in N_i} v(I)\ p_{ij}(Y_{ij} - Y_{ij}^0)\ -\ c_0 \qquad (9)$$

subject to the organizational capability constraint of equation 7.

Partner firms' optimization and the resulting transfer structure are characterized in Section 2.4, while the appropriability parameter v_{ij} is determined by the appropriability environment of Section 3.3.

In equation (9), coordination effort affects anchor profits through two channels: higher partner-productivity component of anchor TFP raises the anchor firm's own productivity and output, while coordination also generates incremental partner output from which the anchor appropriates the share $v(I)$. The anchor therefore chooses coordination effort, network size, and factor inputs subject to coordination congestion and the capability constraint, the two forces that jointly bound the scale and intensity of network coordination.

4.2 Optimal Coordination Effort

Let $\eta_i \geq 0$ denote the multiplier on the organizational capability constraint. Conditional on partner inclusion, optimal coordination effort (e_{ij}^*) equates the marginal benefit of coordination to its marginal cost:

$$\left[P_i \frac{\partial Y_i}{\partial A_i}\frac{\partial A_i}{\partial q_{ij}} + v(I)p_{ij}\frac{\partial Y_{ij}}{\partial q_{ij}}\right]\frac{\partial q_{ij}}{\partial e_{ij}} = \lambda(N_i)e_{ij} + \eta_i \qquad (10)^{12}$$

[12] The anchor-side term is written as a partial derivative by the envelope theorem: once the anchor firm's own capital and labor inputs are chosen optimally, the induced adjustments in those inputs make no first-order contribution to optimized profit. The partner-output term is evaluated under the scale normalization stated in Appendix A.1, whereby coordination-induced output gains are measured at a predetermined benchmark production bundle. This normalization isolates the productivity effect of coordination, is consistent with constant returns to scale, and abstracts from the separate partner factor-allocation margin that would otherwise arise from the anchor's claim on incremental revenue. The numerical illustration in Section 7 applies the same normalization.

The left-hand side is the marginal private benefit of effort: the TFP gain accruing to the anchor firm through higher partner-productivity component of anchor TFP plus the appropriated share of the marginal increment in partner revenue. The right-hand side is the marginal coordination cost plus the shadow cost of organizational capability when the constraint binds.

Equation (10) highlights two channels through which coordination effort creates value. First, improvements in partner productivity raise the productivity of the anchor's own production network through the mechanism described in equation (2), increasing the marginal return to coordination effort. Second, the anchor appropriates a share $\upsilon(I)$ of the revenue increment generated by improvements in partner productivity, providing an additional return to coordination effort

Distortion 1 (Incomplete Appropriability). Because $\upsilon(I) < 1$, anchor firms internalize only a fraction of the value generated through partner upgrading. Consequently, the marginal private benefit of coordination effort is strictly below the marginal social benefit, implying inefficiently low coordination effort in decentralized equilibrium.

4.3 Equilibrium Network Structure

Partner j is included in anchor firm i's network if and only if adding the relationship raises profits. The inclusion condition is:

$$P_i[Y_i(N_i+1) - Y_i(N_i)] + \upsilon(I)p_{ij}(Y_{ij} - Y_{ij}^0) - \frac{\lambda(N_i+1)}{2}e_{ij}^2 \;\geq\; \frac{\lambda(N_i+1) - \lambda(N_i)}{2}\sum_{j' \in N_i} e_{ij'}^2 \qquad (11)$$

The left-hand side is the net private gain generated by the new relationship: the impact of anchor-firm TFP gains plus appropriated partner surplus, net of the direct coordination cost. The right-hand side is the congestion cost imposed on existing relationships through the increase in λ. Partner j is included if and only if the net private gain exceeds this coordination cost.

Three forces jointly limit network expansion: diminishing returns to coordination effort ($\delta < 1$), rising coordination congestion through $\lambda'(N_i) > 0$, and the organizational capability constraint. Together they generate bounded coordination effort and a finite equilibrium network size.

Equations (10) and (11) are jointly determined: effort is optimal conditional on inclusion, while inclusion is optimal given the equilibrium effort profile. Anchor firms therefore select partners that generate the highest net private gain, expanding the network until the inclusion condition ceases to hold. This equilibrium structure formalizes the explicit coordination of legally independent suppliers emphasized in the global value chain literature (Gereffi, Humphrey, and Sturgeon, 2005), with coordination effort and network breadth determined here as endogenous firm choices.

The inclusion condition highlights that network expansion is limited not only by organizational capability but also by coordination congestion. As the number of relationships increases, managerial complexity, monitoring requirements, communication costs, and logistical frictions raise the marginal cost of coordination through $\lambda(N_i)$. Consequently, even anchors with substantial organizational capability may find it optimal to limit network breadth before reaching their capability ceiling, with congestion rather than capability becoming the binding constraint on

further network expansion. Capability heterogeneity determines who can become an anchor — with only sufficiently capable firms finding coordination profitable — while congestion determines how much of the network each anchor can efficiently coordinate. The capability constraint and the congestion mechanism therefore represent two distinct limits to coordination: the former reflects the scarcity of organizational capacity, while the latter reflects the rising marginal cost of managing increasingly complex production networks.

Distortion 2 (Network Under-Expansion). Because anchor firms appropriate only a fraction $v(I) < 1$ of the coordination-generated value, the private inclusion condition is stricter than the constrained-efficient one. Greater anchor density further reduces appropriability, shrinking the net private gain generated by new relationships and tightening the inclusion condition. Consequently, equilibrium production networks are inefficiently narrow: socially valuable partnerships may be excluded because anchor firms internalize only a fraction of the coordination-generated value while bearing the full congestion cost associated with network expansion.

Distortion 2 is distinct from Distortion 1 but compounds it. Because incomplete appropriability reduces equilibrium coordination effort relative to the constrained-efficient allocation, the net private gain generated by marginal relationships is also reduced, tightening the inclusion condition and amplifying network under-expansion. The two distortions therefore originate from the same primitive, $v(I) < 1$, but operate on different dimensions: Distortion 1 affects coordination effort conditional on inclusion, whereas Distortion 2 affects the inclusion decision itself. Correcting one distortion does not automatically eliminate the other.

4.4 Endogenous Entry and the Capability Threshold

This section treats the cutoff level of organizational capability M^* and the number of active anchor firms I as endogenous, both through a free entry condition combined with a selection mechanism based on firm heterogeneity in organizational capability.

The anchor firm's equilibrium profit $\Pi^*(M_i, I)$ depends on two state variables: organizational capability M_i and anchor density I. After substituting optimal factor demands — which cancel exactly under CRS, conditional on an equilibrium scale[13]— optimal coordination effort $e_{ij}^*(M_i, I)$ from equation (10), and optimal network size $N_i^*(M_i, I)$ from equation (11), equilibrium profit reduces to the difference between the total appropriated partner revenue increment and total coordination costs:

$$\Pi^*(M_i, I) = \sum_{j \in N_i^*(M_i, I)} v(I)\ p_{ij}(Y_{ij}(e_{ij}^*) - Y_{ij}^0) - \frac{\lambda(N_i^*(M_i, I))}{2} \sum_{j \in N_i^*(M_i, I)} e_{ij}^*\,(M_i, I)^2\ - c_0 \qquad (12)$$

Higher organizational capability M_i raises equilibrium profit by relaxing the capability constraint and expanding both network size and appropriated surplus. Conversely, higher anchor density I

[13] Anchors choose coordination effort and network composition taking output and factor prices as given, so the own-productivity channel operates at the margin in equations (10)–(11). This explains why equation (12) contains no own-output term (Appendix A.8.1, where the reduced entry problem is derived), even though partner productivity affects the marginal return to coordination effort through $A_i = M_i^{\alpha} \prod_{j \in N_i} q_{ij}^{\phi}$ in equation (10).

reduces equilibrium profit through two reinforcing channels: directly through the appropriability channel $v(I)$, which erodes coordination rents for incumbent anchors, and indirectly by reducing the marginal return to coordination effort, which dampens equilibrium effort $e_{ij}^*$and network size N_i^*.[14]

Firm i with organizational capability M_i chooses to enter as an anchor if and only if the equilibrium profit from operating as an anchor — that is, the total appropriated revenue increment from partner relationships net of the coordination costs of sustaining effort across the network — is non-negative. Formally: $\Pi^*(M_i, I) \geq 0$.

Since $\Pi^*(M_i, I)$ is strictly increasing in M_i, there exists a unique capability threshold M^* below which entry is unprofitable (Theorem A.8.1). Under free entry, M^* satisfies the zero-profit condition:

$$\Pi^*(M^*, I) = 0 \qquad (13)$$

Firms with $M_i \geq M^*$ operate as anchor firms, while firms with $M_i < M^*$remain ordinary producers. The equilibrium mass of anchor firms is:

$$I^* = 1 - F_s(M^*) \qquad (14)$$

Competition and organizational capability jointly determine equilibrium anchor mass. Equations (13) and (14) jointly determine $\{I^*, M^*\}$. Substituting equation (14) into equation (13) yields a fixed-point characterization of equilibrium anchor density. By the implicit function theorem applied to equation (12):

$$\frac{dM^*}{dI} = -\frac{\partial \Pi^* / \partial I}{\partial \Pi^* / \partial M}\Big|_{(M^*, I^*)} > 0 \qquad (15)$$

Since the numerator is negative and the denominator is positive, the capability threshold is increasing in anchor density. Greater entry intensifies competition among anchors and erodes appropriability, raising the organizational capability required to sustain profitable coordination. Competition therefore operates as an endogenous selection mechanism, increasing the threshold for anchor status and concentrating coordination activity among the most capable firms.

The resulting capability-threshold selection is related to the heterogeneous-firm selection mechanism in Melitz (2003). Here, however, firms sort into an organizational role within a production network rather than into export markets, and selected anchor firms subsequently choose their partner networks and coordination effort.

Distortion 3 (industry structure). The welfare implications of anchor entry are potentially ambiguous. On the one hand, entry expands the set of active production networks, increasing partner inclusion and facilitating the diffusion of coordination benefits. On the other hand, entry intensifies competition among anchors, reducing appropriability and weakening the incentives that

[14] The first-order condition (10) is derived in Appendix A.3 and the inclusion condition (11) in Appendix A.4. Formally, $\Pi^*(M, I)$ is weakly increasing in M (Proposition A.6.1), strictly so under Condition CS (Corollary A.6.1), and strictly decreasing in I (Lemma A.6.1).

sustain coordination investments. Consequently, insufficient entry may limit the formation and diffusion of productive networks, whereas excessive entry may dissipate the appropriated returns to coordination and weaken incentives for relationship-specific investment. Both failure modes originate from the same primitive—the dependence of $v(I)$ on the mass of active anchor firms (I)—but operate in opposite directions. This entry distortion is related to the literature on inefficient entry under fixed costs (Mankiw and Whinston, 1986), although the externality here operates through endogenous appropriability rather than business stealing. Section 5 characterizes the welfare implications of this tradeoff and derives the conditions under which decentralized equilibrium exhibits under-entry or over-entry relative to the constrained-efficient allocation.

Unlike Distortions 1 and 2, which operate at the relationship level, Distortion 3 operates through industry structure by altering equilibrium anchor density and the capability threshold for entry.

4.5 Comparative Statics and Economic Interpretation

The equilibrium characterized above yields four comparative-static predictions concerning organizational capability M_i, partner capability $M_j$operating through absorptive capacity θ_{ij}, the coordination environment z operating through the congestion schedule $\lambda(N_i; z)$, and institutional quality iq operating through the appropriability schedule $v(I, iq)$. The comparative statics in this subsection characterize the anchor's problem at given anchor density and prices; equilibrium adjustments in industry structure are considered subsequently. The implications of these comparative statics for industry equilibrium, sectoral output, and policy effectiveness are developed in Propositions 2 and 3.

Organizational Capability (CS1). Higher organizational capability expands the anchor's capacity to coordinate, weakly increasing anchor profitability, total coordination effort, and network breadth. When the capability constraint binds, additional capability is allocated across included relationships, raising coordination effort in each; when it is slack, the effort allocation is locally unchanged. Network breadth is weakly increasing, with strict expansion when an additional relationship crosses the inclusion threshold[15]. These firm-level effects provide the microfoundation for the role of the capability distribution in determining sectoral GDP in Proposition 2 and for the capability-constrained coordination regime characterized in Proposition 3.

Partner Capability and Absorptive Capacity (CS2). Higher partner capability, operating through absorptive capacity θ_{ij}, raises the return to coordination. For an individual partner, greater absorptive capacity raises the value generated by coordination and, when the capability constraint is slack, weakly increases coordination effort directed toward that relationship. When the capability constraint binds, total coordination effort remains fixed and effort is reallocated toward relationships with greater absorptive capacity. A common increase in absorptive capacity across the potential partner pool weakly expands network breadth under Condition NB, with strict

[15] Proposition A.6.1 establishes that anchor profitability is weakly increasing in organizational capability by feasible-set replication, and Corollary A.6.1 establishes strict monotonicity under Condition CS. Proposition B.1.1 characterizes the corresponding effort and network responses. Where the capability constraint binds, total coordination effort equals M_i and each included relationship's effort rises strictly; network breadth is weakly increasing under Condition NB.

expansion only when an additional relationship crosses the inclusion threshold[16]. Together with CS1, these results generate complementarity between anchor organizational capability and partner absorptive capacity. Under the additional conditions established in Appendix C, this complementarity gives rise to positive assortative matching: higher-capability anchors sort toward partners with greater absorptive capacity.

Coordination Environment (CS3). Improvements in the exogenous coordination environment z, which shift the congestion schedule $\lambda(N_i; z)$ downward, weakly raise anchor value and, when the capability constraint is slack, weakly increase coordination effort. When the capability constraint binds, lower congestion raises the value of coordination but may alter the allocation of effort across relationships rather than increasing effort in every relationship. Under Condition NB, improvements in the coordination environment weakly expand network breadth, with strict expansion only when an additional relationship crosses the inclusion threshold[17].

This result highlights the breadth–depth tradeoff at the core of the model. While broader networks create additional opportunities for coordination-generated productivity gains, they also generate organizational frictions: as the production network expands, congestion progressively increases coordination costs. Network expansion therefore eventually becomes self-limiting. Improvements in infrastructure, logistics, or management systems operate through z, shifting $\lambda(N_i; z)$ downward and lowering the cost of coordinating partner relationships. Such improvements do not correct incomplete appropriability; rather, they expand the coordination frontier within which coordination effort and network formation take place.

Institutional Quality and Appropriability (CS4). Improvements in institutional quality iq, which shift the appropriability schedule $v(I, iq)$ upward, strictly raise anchor value at a given anchor density by increasing the share of the coordination-generated revenue increment appropriated by the anchor. When the capability constraint is slack, coordination effort is weakly increasing in institutional quality; when the constraint binds, total coordination effort remains fixed and the resulting increase in appropriability may instead alter its allocation across relationships. Under Condition NB, improvements in institutional quality weakly expand network breadth, with strict expansion only when an additional relationship crosses the inclusion threshold[18]. Institutional quality therefore strengthens private incentives to coordinate through the appropriability channel rather than by reducing coordination costs.

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[16] Proposition B.2.1. The individual-partner and partner-pool experiments are distinct: an increase in one partner's absorptive capacity may reallocate coordination effort toward that relationship without expanding network breadth; weak expansion follows from a common increase across the potential partner pool under Condition NB. Under the maintained specification, the individual-effort response in the slack regime is strictly positive (Condition EM; Remark B.0.1). Matching frictions are maintained not to overturn gain-maximizing assignments; the complementarity and assortative-matching results are established in Appendix C.

[17] Proposition B.3.1, for schedule shifts satisfying the maintained slope condition of Appendix B.3. Under the maintained specification, Condition EM holds automatically wherever the joint-concavity criterion holds (Remark B.0.1).

[18] Proposition B.4.1. The corresponding equilibrium implications, combining this firm-level response with the endogenous adjustment of anchor density, are assembled in Appendix D.6.

5. Coordination Failures and Optimal Policy Design

Sections 2–4 characterized the decentralized equilibrium and identified three distortions arising from incomplete surplus appropriability: underprovision of coordination effort, under-expansion of production networks, and inefficient industry structure. This section characterizes the corresponding constrained-efficient allocation and derives the policy instruments that decentralize it.

The planner values the full surplus generated through production-network coordination — not merely the appropriated share — but cannot eliminate incomplete appropriability. Consequently, appropriability remains an endogenous equilibrium object jointly determined with anchor density, with $\upsilon(I^{**}) < 1$ even at the constrained-efficient allocation.

We first characterize the constrained-efficient levels of coordination effort and network inclusion. We then characterize the constrained-efficient industry structure and derive the policy package that decentralizes the constrained-efficient allocation.

5.1 The Planner's Objective

The planner does not directly choose coordination effort, network breadth, or anchor entry. Instead, it characterizes the constrained-efficient allocation and subsequently derives the policy instruments that decentralize it. Firms continue to choose (e_{ij}, N_i) by maximizing private profits subject to the organizational capability constraint (7), which the policy instruments cannot relax. The planner therefore characterizes efficient allocations directly and derives, in a second step, the instrument that implements each one — exactly the strategy followed throughout this section. The planner's problem is one of constrained efficiency in the sense of Greenwald and Stiglitz (1986): the underlying contractual and organizational frictions remain in place, and the planner chooses corrective instruments subject to those constraints rather than eliminating them.

Relative to private objectives, the planner values the full coordination-generated increase in partner revenue rather than only the share contractually appropriated by anchor firms. Social welfare consists of aggregate value-added, including the gains generated through anchor production networks and improvements in partner productivity, net of coordination resource costs. Because the contractual transfer $\upsilon(I)\, p_{ij}\left(Y_{ij} - Y_{ij}^{0}\right)$ is a payment between the anchor and the partner, it nets out of social welfare: the division of the coordination-generated revenue increment affects the distribution of value between firms but not the total value created (Section 2.4). Appropriability therefore enters the planner's problem only through behavior—the effort, inclusion, and entry decisions it induces—not through the welfare accounting itself.

This difference in valuation — anchors weight the coordination-generated revenue increment by the appropriated share υ(I), whereas the planner weights it fully —generates a wedge between private and social incentives along three dimensions—coordination effort, network inclusion, and industry structure—corresponding precisely to Distortions 1, 2, and 3 characterized in Section 4. The planner's problem is therefore to characterize the constrained-efficient coordination effort, network size, and anchor density jointly, recognizing that these outcomes are implemented through

decentralized firm optimization under the corrective instruments and the organizational capability constraint.

Definition 1 (Constrained-Efficient Allocation). The constrained-efficient allocation is the allocation of coordination effort, network relationships, and the set of anchor firms that maximizes social welfare subject to the model's technological and organizational constraints and to the requirement that coordination, network formation, and anchor entry remain privately undertaken. Incomplete appropriability remains part of the economic environment; in evaluating allocations, the planner values the full coordination-generated increase in output. Its components are denoted e_{ij}^{**}, N_i^{**}, and I^{**}, characterized along the effort, inclusion, and entry margins in Sections 5.2–5.4, respectively.

In the remainder of the paper, single stars on allocation objects denote the decentralized equilibrium, double stars denote the constrained-efficient allocation, and stars on instruments $(\sigma^*, \tau_{ij}^*, \xi^*)$ denote their optimal values.

5.2 Distortion 1: Constrained-Efficient Coordination Effort

Because anchor firms internalize only a fraction $\upsilon(I) < 1$ of the coordination-generated revenue increment, the private return to effort is strictly below the social return. This incomplete appropriability generates an externality gap equal to the uninternalized share of the marginal increment in partner revenue, causing coordination effort to be underprovided in the decentralized equilibrium.

The constrained-efficient effort condition internalizes the full marginal increment in partner revenue rather than only the fraction $\upsilon(I)$ appropriated by the anchor firm. Hence, conditional on a given industry structure, the constrained-efficient effort level exceeds the decentralized effort level whenever $\upsilon(I) < 1$.[19]

Comparing the social effort condition with the private first-order condition (equation 10) reveals that the two differ only in the valuation of the marginal increment in partner revenue. The planner internalizes the full marginal increment in partner revenue, whereas the anchor firm internalizes only the fraction $\upsilon(I)$. The resulting externality gap is exactly the uninternalized share of the marginal increment in partner revenue, $1 - \upsilon(I)$, identifying a Pigouvian subsidy proportional to coordination effort as the corrective instrument.

[19] Unlike a decentralized anchor firm, which internalizes only the appropriated fraction $\upsilon(I)$ of the partner's revenue increment, the planner values the full increase in partner revenue generated by coordination. Conditional on a given network N_i, the constrained-efficient effort allocation is characterized by $\max_{K_i, L_i, \{e_{ij}\}_{j \in N_i}} \Pi_S = P_i Y_i - wL_i - rK_i - \sum_{j \in N_i} \frac{\lambda(N_i)}{2} e_{ij}^2 + \sum_{j \in N_i} p_{ij} (Y_{ij} - Y_{ij}^0)$, subject to the same organizational capability constraint faced by decentralized firms, equation (7), and conditional on an equilibrium scale (Section 2.4).

A Pigouvian subsidy σ^*on coordination effort closes this gap by aligning private and social incentives. Setting the subsidized private first-order condition equal to the constrained-efficient condition yields:

$$\sigma^* = 1 - \upsilon(I^{**}) \qquad (16)$$

where I^{**}denotes the constrained-efficient anchor density, formally characterized in Section 5.4. The corrective subsidy depends only on the equilibrium degree of appropriability. As appropriability erodes, the gap between private and social incentives widens, and the corrective subsidy $\sigma^*$is correspondingly larger. Read at an arbitrary candidate density I, the same expression defines the schedule $\sigma^*(I) = 1 - \upsilon(I)$ that Lemma 1 employs.

Remark. Equation (16) holds regardless of whether the organizational capability constraint (7) binds; Appendix D.2 (Lemma D.2.1) shows that the KKT multiplier on (7) adjusts endogenously so that σ^* continues to decentralize the constrained-efficient effort level even when firms are capability-constrained.

Since $\upsilon(I^{**}) < 1$ even at the constrained-efficient allocation, the subsidy does not eliminate incomplete appropriability itself. Instead, it offsets the resulting incentive wedge, inducing anchors to choose the constrained-efficient effort level despite incomplete appropriability. The policy therefore induces the constrained-efficient effort choice; it does not restore the allocation that would obtain under full appropriability.

5.3 Distortion 2: Constrained-Efficient Network Inclusion

The planner includes partner *j* whenever the full social value generated by the relationship exceeds the marginal congestion cost imposed on the existing network, counting the entire incremental partner revenue rather than only the appropriated fraction.[20]

Comparing the social inclusion condition with the private inclusion condition (equation 11) reveals that the two differ only in the valuation of incremental partner revenue. The planner values the entire increase in partner output generated through coordination, whereas the anchor firm values only the appropriated fraction $\upsilon(I)$. Relative to the private inclusion condition, the planner's condition is strictly less demanding: some relationships that are socially valuable remain privately unprofitable because anchor firms do not internalize the full partner incremental revenue. Consequently, equilibrium production networks are inefficiently narrow.

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[20] Social Inclusion Condition**.** The planner includes partner j whenever $P_i[Y_i(N_i+1) - Y_i(N_i)] + p_{ij}Y_{ij}^0\left[\left(1+\theta_{ij}e_{ij}\right)^{\delta}-1\right] - \frac{\lambda(N_i+1)}{2}e_{ij}^2 \geq \frac{\lambda(N_i+1)-\lambda(N_i)}{2}\sum_{j'\in N_i} e_{ij'}^2$. The social condition differs from the private condition (11) only in the valuation of incremental partner revenue: the planner counts the full amount $p_{ij}Y_{ij}^0\left[(1+\theta_{ij}e_{ij})^{\delta}-1\right]$, whereas the private condition counts only the appropriated fraction, $\upsilon(I)p_{ij}Y_{ij}^0\left[(1+\theta_{ij}e_{ij})^{\delta}-1\right]$.

The resulting gap is therefore equal to the value of the uninternalized portion of the relationship's revenue increment, identifying an inclusion grant tied to that revenue increment generated by the relationship as the corrective instrument.

The corrective inclusion grant τ_{ij}^{*} compensates anchor firms for the uninternalized portion of the partner revenue increment, thereby aligning the private inclusion condition with the social one. The inclusion grant is the coordination-breadth counterpart to the effort subsidy in Section 5.2: whereas σ^{*}corrects underinvestment in the depth of existing relationships, τ_{ij}^{*} corrects the under-formation of new relationships.

$$\tau_{ij}^{*} = (1 - \upsilon(I^{**}))\, p_{ij}[\, Y_{ij}(e_{ij}^{**}) - Y_{ij}^{0}\,] \qquad (17)$$

Equation (17) is evaluated at the constrained-efficient anchor density I^{**} and the associated effort level e_{ij}^{**}, formally characterized in Section 5.4. The corrective grant is larger when appropriability is weaker and when coordination generates larger gains in partner output.

The grant is offered for every potential relationship of an active anchor and paid upon inclusion, with $e_{ij}^{**}(M_{i'}\, I)$ the constrained-efficient effort for that relationship at the corresponding candidate network size; evaluated at an arbitrary candidate density I, the same expression defines the schedule $\tau_{ij}^{*}(I)$ that Lemma 1 employs.

Equations (16) and (17) correct distinct margins—the effort and network breadth—of the same transfer schedule (Appendix D.2.5). Correcting the effort margin alone does not restore efficient network formation: even when the marginal incentive is fully aligned, incomplete appropriability continues to distort inclusion because the anchor's inclusion comparison depends on the level of appropriated relationship revenue increment and therefore continues to reflect the appropriability wedge $\upsilon(I) < 1$. Equation (17) supplies the corresponding level correction, compensating anchor firms for the portion of the partner revenue increment they fail to internalize and thereby decentralizing the constrained-efficient inclusion condition. Both margins must be corrected for constrained efficiency, and their joint calibration with the entry instrument is characterized in Section 5.5.

5.4 Distortion 3: Constrained-Efficient Industry Structure

Even after correcting the coordination effort and network-breadth distortions through the corrective effort subsidy σ^{*} and inclusion grant τ_{ij}^{*}, the decentralized equilibrium may still exhibit an inefficient industry structure because anchor entry affects appropriability itself. Distortion 3 differs fundamentally from Distortions 1 and 2. Whereas the first two distortions arise from incomplete internalization of relationship-level revenue increment, Distortion 3 arises because anchor entry simultaneously expands production networks and erodes appropriability. The planner therefore faces a three-way trade-off between the productivity gains from expanding production networks, the erosion of coordination incentives through reduced appropriability, and the resource costs of coordination associated with additional anchor entry.

The Constrained Planner's Objective. Because coordination effort, network formation, and anchor entry remain decentralized decisions, firms continue to maximize private profits subject to the organizational capability constraint. The planner therefore cannot directly choose the allocation. Instead, it characterizes the constrained-efficient industry structure — represented equivalently by the constrained-efficient anchor density I^{**} or capability threshold M^{**}— by evaluating, for each candidate threshold, the decentralized equilibrium induced by the corrective instruments σ^* and τ_{ij}^* already characterized in Sections 5.2 and 5.3. It then derives, in a second step, the entry instrument that implements this structure — the same two-stage strategy used throughout Section 5: characterize the efficient allocation first, then derive the instrument that decentralizes it.

Define sectoral aggregate baseline output across all firms $k \in F$ as $Y_s^0 \equiv \sum_{k \in F} p_k Y_k^0$, the total value added the economy would generate absent any production-network coordination; since Y_k^0 is independent of coordination, so is Y_s^0. Define aggregate coordination gains generated by anchor firms and partner firms at candidate capability threshold M: $\Gamma_A(M) \equiv \sum_{M_i \geq M} p_i (Y_i - Y_i^0), \Gamma_P(M) \equiv \sum_{M_i \geq M} \sum_{j \in N_i^{**}} p_{ij} (Y_{ij}(e_{ij}^{**}) - Y_{ij}^0)$. Both measure the total increase in value added due to coordination, evaluated at the policy-induced decentralized equilibrium — at the corrected conduct $\left(e_{ij}^{**}(M), N_i^{**}(M)\right)$ prevailing once $\sigma^*$and τ_{ij}^* are applied (Lemma 1). For notational simplicity, this dependence is suppressed. Aggregate gross output is $Y_s(M) \equiv Y_s^0 + \Gamma_A(M) + \Gamma_P(M)$. Because coordination effort is a real resource cost, social welfare — as defined in Section 5.1 — nets this out. Define aggregate fixed coordination costs $C_0(M) \equiv c_0 \cdot I(M)$, aggregate variable coordination costs $C_v(M) = \sum_{M_i \geq M} \frac{\lambda(N_i^{**})}{2} \sum_{j \in N_i^{**}} (e_{ij}^{**})^2$, and total coordination costs $C(M) = C_0(M) + C_v(M)$. Total net social value is $W_s(M) = Y_s(M) - C(M)$. The function $W_s(M)$ is a planner-evaluation function: it evaluates each candidate industry structure under the corrected effort and network choices characterized in Sections 5.2–5.3. It should therefore be distinguished from realized welfare in the uncorrected decentralized equilibrium.

The planner values both the anchor productivity gains Γ_A and the full partner gains Γ_P, whereas the private entry decision is based on optimized private profit and internalizes only the appropriated share $v(I)$ of partner gains. The planner and the entrant therefore evaluate the value of an additional anchor differently, generating the entry distortion analyzed below. The entry instrument derived corrects the residual wedge remaining at the entry margin once $\sigma^*$and τ_{ij}^* are in place. Write $W_s(I) \equiv W_s(M(I))$, where $M(I) = F_s^{-1}(1 - I)$. The constrained planner characterizes the constrained-efficient anchor density as the solution to $I^{**} = \arg\max_I W_s(I)$. Since Y_s^0 is independent of the industry structure, this is equivalent to: $I^{**} = \arg\max_I [\Gamma_A(I) + \Gamma_P(I) - C(I)]$ — that is, choosing the industry structure that maximizes the net value generated by coordination; $M^{**} = F_s^{-1}(1 - I^{**})$. Realized welfare under the full corrective package is therefore $W_s^{**} = W_s(M^{**})$.

The Net Entry Distortion Indicator. Differentiating $W_s(I)$— since Y_s^0 is constant — and evaluating at the decentralized equilibrium I^*, define:

$$\Delta \equiv \frac{dW_s}{dI}\,|_{I^*} = \frac{d\Gamma_A}{dI}\,|_{I^*} + \frac{d\Gamma_P}{dI}\,|_{I^*} - \frac{dC}{dI}\,|_{I^*} \quad (18)^{21}$$

Δ measures the net marginal social value of anchor entry, inclusive of the resource cost of coordination. If $\Delta > 0$, more entry raises welfare; if $\Delta < 0$, less entry raises welfare; if $\Delta = 0$, the decentralized equilibrium is constrained efficient.

Proposition 1 (Characterization of the Constrained-Efficient Industry Structure). Under the maintained assumptions of the model (Section 2.1) and Assumption D.1 (Global Single-Peakedness), let $I^* = 1 - F_S(M^*)$ denote the decentralized equilibrium anchor density, M^* the decentralized equilibrium capability threshold, $W_S(\cdot)$ the planner's net social value objective, and $\Delta \equiv \frac{dW_S}{dI}\,|_{I^*}$ *the net marginal social value of anchor entry evaluated at the decentralized equilibrium. Then:*

(i) If $\Delta > 0$, then $I^{**} > I^*$: the decentralized equilibrium exhibits insufficient anchor entry.
(ii) If $\Delta < 0$, then $I^{**} < I^*$: the decentralized equilibrium exhibits excessive anchor entry.
(iii) If $\Delta = 0$, then $I^{**} = I^*$: the decentralized industry structure is constrained efficient.

Equivalently, $I^{**} > I^*$ corresponds to $M^{**} < M^*$, and $I^{**} < I^*$ corresponds to $M^{**} > M^*$.

Proof: ***Appendix D.4****.*

Economic Interpretation. The decomposition in equation (18) makes the underlying forces transparent. Network expansion increases partner gains by creating additional productive relationships, while appropriability erosion weakens coordination incentives and reduces coordination-generated value within incumbent networks; because anchor productivity is generated through the same relationships, both forces propagate to Γ_A. The coordination-cost term introduces a third force: additional entry commits real organizational resources — a fixed cost per entrant and induced variable coordination costs — that must be weighed against the productivity gains entry generates. Although appropriability does not appear explicitly after differentiation, it remains an equilibrium state variable because it determines the decentralized effort and network choices that jointly define Γ_A, Γ_P, and C.

Decentralization of the Inner Margins. The entry instrument is derived conditional on the effort subsidy and inclusion grant already being in place. The following result establishes that this conditioning is well defined.

Let $\left(e_{ij}^{**}, (M_i, I)\; N_i^{**}(M_i, I)\right)$ denote the constrained-efficient effort and network choices characterized in Sections 5.2 and 5.3, conditional on the candidate anchor density I.

[21] Conditional on the model primitives and parameter vector, I^*, $W_S(I)$, and hence Δ are uniquely determined. The sign of Δ therefore partitions the parameter space into insufficient-entry and excessive-entry regions, with $\Delta = 0$ defining the boundary between them.

Lemma 1 (Effort and Network Decentralization). Fix any candidate anchor density I, with corresponding capability threshold $M(I)$, and let $\sigma^*(I)$ and $\tau_{ij}^*(I)$ be given by equations (16) and (17), evaluated at I. Then, for every active anchor i with $M_i \geq M(I)$, the privately optimal effort and network choices under these instruments coincide with the constrained-efficient choices conditional on I: $e_{ij}^{\sigma^*,\tau^*}(M_i, I) = e_{ij}^{**}(M_i, I), \quad N_i^{\sigma^*,\tau^*}(M_i, I) = N_i^{**}(M_i, I)$. Here $e_{ij}^{\sigma^*,\tau^*}$ and $N_i^{\sigma^*,\tau^*}$ denote the anchor's privately optimal choices under the effort subsidy and inclusion grant.

Lemma 1 does not determine I^{**}; it establishes conditional decentralization of the effort and network margins at every candidate anchor density, thereby justifying the construction of $W_s(I)$ used below. The content of the lemma is that margin-by-margin construction of the effort subsidy and inclusion grant implies joint decentralization: despite the interaction between coordination effort and network breadth through the congestion schedule and the organizational capability constraint, the instrumented private solution coincides with the constrained-efficient allocation for every active anchor.

Proof: Appendix D.2.

Optimal Entry Instrument. Given the constrained-efficient threshold M^{**}, the entry instrument is obtained from the entry condition itself — the same logic by which σ^* is obtained from the effort first-order condition and τ_{ij}^* from the inclusion condition. Let $\Pi^{\sigma^*,\tau^*}(M_i, I)$ denote the profit of an anchor with capability M_i under the effort subsidy and inclusion grants, net of the fixed coordination cost c_0; by Lemma 1, it is evaluated at $\left(e_{ij}^{**}(M_i, I), N_i^{**}(M_i, I)\right)$. The explicit expression is given in Appendix D.3. The entry instrument is defined by the break-even condition of the marginal anchor:

$$\Pi^{\sigma^*,\tau^*}(M^{**}, I^{**}) + \xi^* = 0, \text{ equivalently } \xi^* = -\Pi^{\sigma^*,\tau^*}(M^{**}, I^{**}). \qquad (19)$$

When $\xi^* > 0$, the instrument compensates the marginal anchor for value its entry generates but does not capture even under σ^* and τ_{ij}^*; when $\xi^* < 0$, it discourages entry that remains privately profitable under the corrective instruments but is socially excessive.

Lemma 2 (Threshold Implementation). Under the policy vector $\left(\sigma^*, \tau_{ij}^*, \xi^*\right)$, an anchor enters if and only if $M_i \geq M^{**}$, and M^{**} is the unique decentralized capability threshold.

*Proof: **Appendix D.3**.*

As with σ^* and τ_{ij}^*, ξ^* is derived as the instrument that implements a directly-characterized efficient allocation; it is not itself the object of the planner's maximization. Each instrument is obtained from the equilibrium condition of the margin it corrects: σ^* from the effort first-order condition, τ_{ij}^* from the inclusion condition, and ξ^* from the entry condition. Because σ^* and τ_{ij}^* depend on the constrained-efficient allocation only through I^{**}, while ξ^*depends in addition on σ^* and τ_{ij}^* through the policy-inclusive profit in equation (19), the instruments are computed in the order $I^{**} \to (\sigma^*, \tau_{ij}^*) \to \xi^*$.

The entry instrument ξ^* is the industry-structure counterpart to σ^* and τ_{ij}^*. All three instruments correct distortions generated by incomplete appropriability — σ^*at the effort margin, τ_{ij}^* at the network breadth margin, and ξ^* at the industry structure margin. Their joint calibration is characterized in Section 5.5.

Concluding Remark. Distortion 3 differs fundamentally from Distortions 1 and 2 because it operates at the extensive margin of industry organization. Whereas the first two distortions govern the efficiency of coordination within production networks, Distortion 3 governs the equilibrium density of anchor firms through which those networks emerge. The planner therefore balances the productivity gains from expanding production networks, the economy-wide erosion of coordination incentives through appropriability, and the fixed and variable coordination costs associated with additional anchor entry. The sign of Δ is a sufficient statistic for whether the decentralized equilibrium exhibits excessive or insufficient anchor entry; the entry instrument that implements the constrained-efficient structure is determined by the break-even condition (19).

5.5 Decentralizing the Constrained Second-Best

The three corrective instruments derived in this section — the effort subsidy σ^*, the inclusion grant τ_{ij}^*, and the entry instrument ξ^* defined by the break-even condition (19)— jointly decentralize the constrained-efficient allocation, with one instrument correcting each distortion identified in Section 4.

The effort subsidy σ^* eliminates Distortion 1 by aligning private and social incentives along the coordination-depth margin — raising equilibrium coordination effort from the decentralized equilibrium level to the constrained-efficient level conditional on network structure. The inclusion grant τ_{ij}^* eliminates Distortion 2 by aligning private and social incentives along the coordination-breadth margin — expanding equilibrium network size from the privately optimal level to the constrained-efficient level conditional on anchor density. The entry instrument eliminates Distortion 3 by aligning decentralized anchor mass (I^*) with the constrained-efficient anchor density, moving the equilibrium from I^* to the constrained-efficient density I^{**} (equivalently, from $M^*$to M^{**}). Their joint effect on the equilibrium allocation is summarized in equation (20):

$$\left(\sigma^*, \tau_{ij}^*, \xi^*\right) \Rightarrow \left(I^{**}, M^{**}, \upsilon(I^{**}), e_{ij}^{**}, N_i^{**}\right) \qquad (20)$$

The three corrective instruments are interdependent and cannot be calibrated independently. The constrained-efficient anchor density I^{**}determines equilibrium appropriability, $\upsilon(I^{**})$, and hence the corrective effort subsidy σ^* and inclusion grant τ_{ij}^* in equations (16) and (17). The entry instrument ξ^* is then chosen to implement I^{**}, taking these conduct instruments into account. Consequently, implementation of the constrained-efficient allocation requires the instruments to be calibrated consistently with one another.

Policies that alter industry structure modify equilibrium anchor density and therefore equilibrium appropriability $\upsilon(I^{**})$, changing the coordination gains generated throughout production networks. Because the corrective effort subsidy and inclusion grant depend on these equilibrium coordination

incentives, all three instruments must be calibrated jointly. The constrained-efficient allocation therefore requires a coherent policy package rather than a collection of independent interventions.

Because baseline output, Y_s^0, is independent of anchor density, the three instruments jointly decentralize the constrained-efficient allocation that maximizes net social value, W_s (Section 5.4), or equivalently, aggregate coordination gains, $\Gamma_A + \Gamma_P - C$. Their joint calibration therefore characterizes the constrained-efficient policy package. As developed in Section 6, the effectiveness of this policy package depends on the economy possessing a sufficiently large mass of active anchor firms through which coordination gains can propagate.

The three corrective instruments therefore decentralize the constrained-efficient allocation conditional on the existing distribution of organizational capability. Section 6 relaxes this conditioning by examining how changes in the capability distribution alter the feasible set itself and, consequently, the effectiveness of the policy package.

6. Organizational Capability, Coordination, and Policy Effectiveness

Section 6 extends the framework to an economy comprising multiple coordination sectors. The sector index s, suppressed in Sections 2–5, now applies to all sectoral variables: the capability distribution $F_s(M)$, the equilibrium anchor density I_s^*, and sectoral output Y_s.

Definition 2 (Coordination Sector). A coordination sector is the maximal set of firms connected by potential coordination relationships and participating in the production of one or more related products. Firms' organizational capabilities are heterogeneous draws from the sector-specific distribution $F_s(M)$.

Within each coordination sector, firms are assumed to face a common production technology, coordination technology, and institutional environment.[22]. These commonalities are maintained for analytical tractability rather than as defining features of a coordination sector. Let $\mathcal{A}_s$ denote the set of active anchors in sector s, and $\mathcal{P}_s \equiv \bigcup_{i \in \mathcal{A}_s} N_i^*$ the set of firms participating as partners in their networks. Aggregate sectoral GDP can then be written as:

$$Y_s = \sum_{k \notin \mathcal{A}_s \cup \mathcal{P}_s} p_k \, Y_k^0 + \sum_{i \in \mathcal{A}_s} [\, P_i Y_i + \sum_{j \in N_i^*} p_{ij} \, Y_{ij}] \qquad (21)^{23}$$

The sectoral GDP (Y_s) therefore depends on the equilibrium mass of active anchors I_s^*, the capability composition of those anchors, the breadth of their partner networks $\{N_i^*\}$, and the coordination effort deployed across those networks $\{e_{ij}^*\}$. Coordination is itself a productive activity: it deploys the organizational capability and managerial resources of anchor firms to

[22] A potential coordination relationship is a technologically feasible relationship between two firms that may, but need not, be realized in equilibrium. Maximality makes a coordination sector a connected component of the network of potential coordination relationships: each firm belongs to exactly one coordination sector, and coordination sectors partition the economy of firms.

[23] Throughout Sections 6–7, Y_s denotes gross sectoral output (GDP), the object of the positive analysis. Net social welfare, $W_s = Y_s - C$, is introduced in Section 5.4 solely for the planner's normative problem.

generate additional value through production relationships. Gross sectoral output Y_s comprises baseline production together with the coordination-generated value created through production networks, while $W_s = Y_s - C$ (Section 5.4) measures the corresponding net social value. Organizational capability is the central state variable of the model. It determines which firms can operate as anchor firms, how many coordination relationships they can sustain, and how effectively they can upgrade their partners. Through these channels, organizational capability shapes both the structure of production networks and the productivity of the firms participating in them. This creates a natural hierarchy: Section 5 derives the corrective policy package that decentralizes the constrained-efficient allocation; Section 6 shows that the effectiveness of this package is itself limited by organizational capability—and that when capability becomes sufficiently scarce, active coordination networks disappear and the effectiveness of corrective policies approaches zero.

The central implication is that economies may exhibit both low output and weak policy responsiveness not because coordination incentives are insufficient, but because organizational capability is too scarce to sustain production networks at economically meaningful scale. Organizational capability therefore determines not only equilibrium sectoral output, but also the composition of active production networks and the extent to which coordination-oriented industrial policy can operate effectively—a result formalized in Proposition 3.

The section proceeds in three steps. Section 6.1 characterizes the capability-distribution channel through which organizational capability, jointly with the institutional environment, shapes equilibrium production networks and aggregate output. Section 6.2 characterizes the capability-constrained coordination regime. Section 6.3 draws the policy implications, distinguishing capability-building interventions from corrective interventions.

6.1 Development Determinants

Let $F_s(M; \nu_s)$ denote the sectoral distribution of organizational capability, where ν_s indexes the overall capability level of the sector. Larger values of ν_s shift the distribution toward higher levels of organizational capability, increasing both the prevalence and the capability of firms that can participate in production networks.

Combining the equilibrium effort and inclusion conditions (10)–(11) with the entry and selection conditions (12)–(14), subject to the capability constraint (7), determines the endogenous equilibrium variables I_s^*, N_i^*, and $\{e_{ij}^*\}$ as functions of the model's primitive determinants. Substituting these equilibrium choices into equation (21) yields the central development relationship at the sector level:

$$Y_s = \mathcal{Y}_s\left(F_s(M; \nu_s), v(\cdot), \lambda(\cdot), c_0\right) \qquad (22)$$

where the mapping $\mathcal{Y}_s$ embeds the production and quality technologies—equations (1), (2), and (4)—together with their parameters, which are held fixed throughout.

Under the maintained demand assumption (Section 2.1) and constant-returns technologies, and holding production parameters fixed, sectoral output admits the following representation.

Proposition 2 (Development Determinants). Sectoral GDP (Y_s) depends on four exogenous determinants: (i) the sectoral capability distribution F_s, which governs the selection and capability composition of anchor and partner firms; (ii) the appropriability function υ, which governs the anchor's appropriated share of the coordination-generated revenue increment; (iii) the congestion function λ, which governs firm-level coordination costs; and (iv) the fixed coordination cost c_0, incurred upon anchor entry. A first-order stochastic improvement in the capability distribution raises equilibrium anchor density and improves the capability composition of anchors and partners; an upward shift in the appropriability schedule raises equilibrium appropriability and anchor density; a pointwise upward shift in the congestion schedule reduces anchor density; and a higher fixed coordination cost reduces anchor density. The resulting effects on sectoral GDP operate through anchor density, network breadth, coordination effort, and the capability composition of production networks. Because changes in anchor density feed back into appropriability through $\upsilon_I < 0$, the gross-output effects of these determinants need not mechanically inherit the signs of their entry effects. Conditional on the maintained demand assumption and fixed production parameters, cross-sectoral differences in output arise solely through differences in F_s, υ, λ, and c_0.

Proof. Given the equilibrium mapping in equation (22), the stated equilibrium comparative statics follow from the effects of F_s, υ, λ, and c_0 on equilibrium coordination and entry established in Appendix D.6.

The first three determinants constitute the model's central development mechanisms. Organizational capability governs the selection of coordinating firms and the productivity of coordination relationships; appropriability governs the private return to coordination investment; and congestion governs the cost of supplying coordination. The fixed coordination cost c_0 plays a different role: rather than introducing an additional coordination mechanism, it enters through the zero-profit condition for anchor entry and shifts the equilibrium capability threshold. It is therefore an equilibrium determinant of anchor density without constituting a separate channel through which coordination raises productivity.

Discussion. Corrective instruments alter private incentives for coordination and entry, but they do not change the underlying capability distribution $F_s(M)$. When organizational capability becomes sufficiently scarce, the organizational channel through which these instruments operate contracts, limiting their aggregate effectiveness — a result formalized in Proposition 3.

The sectoral capability-distribution channel runs:

$$F_s(M) \longrightarrow \left\{ I_s^*, \mathbb{E}[M_i^\alpha \mid M_i \geq M_s^*], \mathbb{E}\left[M_j^\alpha \mid M_j < M_s^*\right] \right\} \longrightarrow Y_s \qquad (23)$$

The capability distribution influences aggregate output through two channels. First, together with the appropriability function $\upsilon(\cdot)$, the congestion schedule $\lambda(\cdot)$, and the fixed coordination cost c_0, it governs the endogenous capability threshold M_s^* and therefore the equilibrium density of active anchor firms I_s^*. Second, conditional on the resulting anchor density, F_s determines the capability composition of both anchors and partners, thereby shaping the productivity of coordination, as measured by the coordination-generated components of sectoral output in equation (21). The economic environment and the capability distribution therefore jointly determine the scale of

production-network coordination, while the capability distribution additionally determines network composition and the productivity of coordination within networks[24].

Appropriability and congestion operate through the private returns and costs of coordination. Higher appropriability raises the private return to coordination effort and partner inclusion, while greater congestion raises the cost of coordinating larger networks. Both also affect anchor entry and therefore interact with the endogenous appropriability response $\upsilon(I_s^*)$. The fixed coordination cost c_0, by contrast, operates through the entry condition and the equilibrium capability threshold. Thus, the four determinants affect sectoral output through the jointly determined intensive, network-breadth, entry, and composition margins characterized in Sections 4 and 5.

Aggregate economy-wide GDP follows by summation across sectors:

$$Y = \sum_s Y_s = \sum_s \mathcal{Y}_s\left(F_s(M;\nu_s), \upsilon(\cdot), \lambda(\cdot), c_0\right) = Y\left(\{F_s(M;\nu_s)\}_s, \upsilon(\cdot), \lambda(\cdot), c_0\right) \qquad (24)$$

Under the maintained demand assumption and fixed production parameters, and when $\upsilon(\cdot)$, $\lambda(\cdot)$, and c_0 are common across sectors, cross-sectoral heterogeneity in output originates from differences in the sectoral capability distributions $F_s(M;\nu_s)$. Economy-wide development is therefore shaped by the entire cross-sectoral architecture of organizational capability rather than by any single summary statistic.

6.2 The Capability-Constrained Coordination Regime

The development determinants characterized in Section 6.1 imply that sufficiently adverse capability distributions generate equilibria in which corrective policy instruments become increasingly ineffective. This section characterizes that possibility formally and traces its implications for development policy.

Most development models—including standard Big Push and coordination-failure frameworks—focus on incentive-based coordination failures: firms could coordinate but choose not to because incentives are distorted. The present model introduces a deeper constraint: firms cannot coordinate because the economy lacks a sufficient mass of capable coordinators. This distinction is fundamental. In the standard incentive-failure case, capable firms exist but find coordination unprofitable:

$$\Pi^*(M_i, I_s^*; \upsilon(\cdot)) < 0, \quad \forall I_s^* > 0 \qquad (25)$$

[24] Aggregating the firm-level capacity constraint $\sum_{j\in N_i^*} e_{ij}^* \le M_i$ across active anchors yields $\mathbb{E}\left[\sum_j e_{ij}^* \mid M_i \ge M_s^*\right] \le \mathbb{E}[M_i \mid M_i \ge M_s^*]$, confirming that average coordination effort per anchor is bounded by average anchor capability — itself a moment of the upper tail of $F_s(M)$.

Such failures can, in principle, be corrected through policies that improve appropriability or reduce coordination costs by operating on $v(\cdot)$ and $\lambda(\cdot)$. The capability-constrained coordination regime is different in kind.

Definition 3 (Capability-Constrained Coordination Regime). A sector operates in a capability-constrained coordination regime when its capability distribution $F_s(M)$ is sufficiently unfavorable that the equilibrium mass of active anchor firms $I_s^*$is small. In the limiting case considered here, deterioration of $F_s(M)$ drives $I_s^* \to 0$. Proposition 3 establishes that, in this limit, coordination-generated value and the responsiveness of the sector to coordination-oriented policy vanish.

Proposition 3 (Capability-Constrained Coordination Regime). Consider a sector operating in the capability-constrained coordination regime defined above.

(i) Organizational channel. Let $\mathcal{A}_s^* \equiv \{i: M_i \geq M_s^*\}$ denote the equilibrium set of active anchor firms. All output gains above the no-coordination benchmark are generated through active anchors: $Y_s^* - Y_s^0 = \sum_{i\in\mathcal{A}_s^*}(Y_i^* - Y_i^0) + \sum_{i\in\mathcal{A}_s^*}\sum_{j\in N_i^*}(Y_{ij}^* - Y_{ij}^0)$. Active anchor firms therefore constitute the organizational channel through which coordination raises sectoral output and through which the corrective instruments operate.

(ii) Capability-constrained limit. As equilibrium anchor density approaches zero, $I_s^* \to 0$, coordination-generated output vanishes and sectoral output converges to its no-coordination benchmark, $Y_s^* \to Y_s^0$. At the same time, the marginal effect of each corrective instrument on sectoral output converges to zero.[25] Thus, as the mass of active anchor firms vanishes, the organizational channel through which corrective policy operates disappears.
(iii) Capability environment. A first-order stochastic deterioration of the capability distribution reduces equilibrium anchor density I_s^* (Appendix D.6). Under the maintained log-normal specification, such a deterioration also reduces the coordination-relevant composition of the equilibrium partner pool, as measured by pool-average baseline productivity and absorptive capacity, notwithstanding the reclassification of marginal anchors into the pool as the entry threshold rises (Remark D.7.2). As the location parameter of the capability distribution declines without bound, equilibrium anchor density converges to zero (Remark D.7.1). Capability-poor environments therefore approach the limit characterized in part (ii): coordination-generated output and the marginal traction of the corrective instruments vanish.

Proof. See Appendices D.6 and D.7.

The policy-responsiveness result is local: it characterizes the marginal effects of the corrective instruments from the decentralized equilibrium and does not characterize the effects of large discrete interventions.

Discussion

[25] Appendix D.7 establishes the entry-instrument result for capability distributions satisfying a tail-regularity condition: as the mass of firms above the entry threshold vanishes, so does the density of firms at the threshold (Assumption (T)). The maintained log-normal specification satisfies this condition (Remark D.7.1). The effort and inclusion results do not require Assumption (T).

Part (ii) establishes the collapse of the coordination channel: as the mass of active anchor firms shrinks toward zero, coordination-generated output vanishes, sectoral output converges to the no-coordination benchmark, and the marginal effectiveness of all three corrective instruments converges to zero. The reason is the organizational channel identified in part (i): coordination operates through active anchor firms. Part (iii) connects this limiting result to the underlying capability environment. A deterioration of the capability distribution reduces equilibrium anchor density, and under the maintained log-normal specification sufficiently severe deterioration drives I_S^* toward zero. Organizational capability therefore determines whether the organizational channel through which coordination and corrective policy operate remains economically meaningful.

Even when equilibrium anchor density remains economically meaningful (because of favorable institutional environment), weak partner capability can limit the productivity of production-network coordination. In the present framework, partner capability determines both baseline productivity and absorptive capacity and therefore the value generated by coordination within production relationships. This mechanism is closely related to the role of "capabilities in the supply-base" in Gereffi, Humphrey, and Sturgeon (2005), who identify supplier competence as a central determinant of value-chain governance and note that suppliers unable to meet buyer requirements may face exclusion from the chain. A weak partner capability environment can consequently limit the productivity of coordination even when an economically meaningful mass of anchor firms remains active.

The analysis therefore identifies two dimensions of capability constraint on economic development. The first operates through the extensive margin: insufficient organizational capability limits the equilibrium mass of active anchor firms and therefore the scale of production-network coordination — Proposition 3 formalizes the limiting implication, with coordination-generated output and the marginal traction of corrective policy vanishing as anchor density does. The second operates through the intensive margin: even when the economic environment — appropriability $\upsilon(\cdot)$ and coordination costs $\lambda(\cdot)$— sustains an economically meaningful anchor density I_S^*, weak capability within the equilibrium partner pool reduces the returns to coordination effort and the value created within existing relationships. Production networks then form, but generate only limited coordination gains. Improvements in appropriability and in the coordination environment thus shape the emergence and scale of production networks, whereas organizational capability governs the productivity of coordination within them; the two classes of policy are complements rather than substitutes.

The distortions corrected in Section 5 arise from incomplete appropriability: capable firms exist but underinvest in coordination. The two dimensions of capability constraint identified above are different in kind. The binding constraint is not simply the incentive to coordinate but, in the extensive-margin case, the supply of firms capable of performing the coordinating function and, in the intensive-margin case, the supply of partners with sufficient organizational capability to translate coordination effort into productivity gains. Unlike traditional coordination-failure models such as Murphy, Shleifer, and Vishny (1989), where capable firms may fail to invest because demand is insufficient, these capability constraints arise from scarcity of organizational capability rather than insufficient demand. Demand expansion alone therefore need not resolve either constraint when the binding limitation lies on the supply side of organizational capability.

When the extensive capability constraint binds— so that scarcity of firms with sufficient organizational capability is a principal force limiting I_s^*—the corrective instruments of Section 5 generate only limited marginal gains, and policies that strengthen the organizational capability of potential anchor firms become increasingly important complements to the incentive and coordination conditions that also determine anchor entry. When the intensive capability constraint binds—I_s^* remains economically meaningful but capability is weak within the equilibrium partner pool—the corrective instruments remain theoretically valid but may generate limited quantitative gains, and policies that strengthen partner organizational capability become increasingly important complements. In both cases, the effectiveness of coordination policy depends on the organizational capability environment through which coordination gains are generated and propagated. Section 6.3 develops the resulting policy framework incorporating these two dimensions of capability constraints.

6.3 Policy Design under Coordination Constraints

The preceding sections established that coordination failures in production networks arise from three analytically distinct sources: insufficient organizational capability, coordination congestion, and incomplete surplus appropriability. These sources differ in their equilibrium manifestations and in the policy instruments required to address them. Table 1 summarizes the architecture before developing the policy implications.

Table 1. Barriers to Coordination and Policy Responses

Problem	Symptom	Policy
Capability constraints	(i) insufficient mass of active anchor firms, or (ii) low partner capability reducing the productivity of coordination	Capability building: managerial development, engineering capability, organizational upgrading.
Coordination costs	N_i^*, e_{ij}^* too low	Lower $\lambda(\cdot)$: infrastructure, logistics, digital platforms; reduce fixed coordination costs through measures that lower the cost of establishing coordination capacity.
Incomplete appropriability	e_{ij}^*, N_i^*, and I_s^* deviate from their constrained-efficient levels	σ^*, τ^*, and ξ^*

Incomplete appropriability affects three distinct margins of coordination — effort, network formation, and anchor entry — and therefore requires distinct corrective instruments for each margin.

The distinction between these barriers is important. Capability constraints and congestion are structural limits on the amount and productivity of coordination that can be sustained, whereas incomplete appropriability creates incentive distortions along the effort, network-formation, and anchor-entry margins — and these margins interact, so that correcting one distortion in isolation generates only partial gains. Incomplete appropriability therefore generally requires a combination of instruments targeting each margin simultaneously (σ^*, τ^*, and ξ^*), while capability-building

and congestion-reducing interventions address the underlying coordination environment itself rather than incentive distortions.

Capability constraints may operate through either the extensive or the intensive margin identified in the discussion related to Proposition 3. On the extensive margin, insufficient organizational capability limits the equilibrium mass of active anchor firms. However, because the equilibrium capability threshold is jointly determined by $F_s(\cdot)$, $v(\cdot)$, $\lambda(\cdot)$, and c_0 through equations (12)–(13), institutional reforms that strengthen appropriability or reduce coordination congestion also expand the equilibrium mass of active anchor firms by lowering the entry threshold, for a given capability distribution.

On the intensive margin, weak partner capability limits the productivity of coordination even when anchor density remains economically meaningful. Unlike the extensive margin, this constraint depends primarily on the capability composition of firms within the equilibrium partner pool and therefore responds mainly to capability-building policies rather than institutional reforms.

The relative importance of these barriers varies with both the sectoral capability distribution $F_s(M; \nu_s)$ and the institutional environment: when either dimension of capability constraint dominates, corrective instruments generate limited gains; once both constraints are sufficiently relaxed, the appropriability distortion becomes the primary constraint and the full instrument package becomes operative. This dependence gives rise to the two-phase framework developed below.

Phase 1 (Catalytic). Phase 1 is characterized by the presence of one or both dimensions of capability constraints identified above. Either the equilibrium mass of active anchor firms is too small to sustain meaningful coordination — the extensive margin — or anchor density is economically meaningful but the partner pool is too weak to generate substantial coordination gains — the intensive margin. In either case, Proposition 3 implies that sectoral output is only weakly responsive to interventions operating through σ^*, τ^*, and ξ^*, because the channel through which these instruments operate is itself underdeveloped. In this regime, the primary policy objective is to relax the binding capability constraint by expanding organizational capability, improving the institutional environment, or both, depending on whether the extensive or intensive margin is binding.

A defining feature of the framework is that potential anchors and potential partners are drawn from a common capability distribution. The capability distribution F_s plays a central role in both margins: together with the institutional environment it governs the extensive margin of anchor formation, while it directly governs the intensive margin of coordination productivity within existing networks. By raising organizational capability, capability-building policies increase both the number of firms able to operate as anchors and the absorptive capacity of potential partner firms. They therefore relax both the extensive margin of network formation and the intensive margin of coordination productivity. Development depends on the shape of the entire capability distribution, because that distribution governs both the equilibrium mass of active anchor firms and the productivity of coordination within existing production networks. Even when anchor density is sufficient, high coordination congestion may continue to limit network breadth —

implying that capability-enhancing and congestion-reducing interventions can be complementary even within Phase 1.

Phase 2 (Corrective). Once both dimensions of capability constraint are sufficiently relaxed, the corrective instruments derived in Section 5 — the effort subsidy σ^*, the inclusion grant τ^*, and the entry instrument ξ^*— become quantitatively effective. Improvements that reduce coordination congestion — better logistics, information systems, quality-certification mechanisms, and managerial practices — complement both σ^* and τ^* by increasing the amount of coordination that can be sustained for a given level of organizational capability, without directly correcting the appropriability distortion. Even within Phase 2, capability-building and corrective interventions remain complementary, because stronger partner capability increases the quantitative effectiveness of the effort and inclusion instruments by raising the productivity of coordination relationships that those instruments are designed to incentivize.

The resulting framework implies not only a sequencing of policies but also a division of roles. Institutional reforms determine whether productive production networks emerge and sustain, whereas capability-building policies determine how much value those networks generate once they exist. When capability constraints bind, policies that relax those constraints — through capability building, institutional reform, or both — typically generate larger gains than policies that operate solely by improving coordination incentives. Once both dimensions of capability constraint are sufficiently relaxed, the binding constraints shift toward appropriability and coordination costs, making the corrective instruments increasingly effective.

The capability-based sequencing principle developed here is related to the recent industrial-policy literature, which has increasingly shifted attention from whether governments should intervene to how industrial policies are designed and implemented (Juhász, Lane, and Rodrik, 2024). The present framework derives this sequencing principle from the equilibrium interaction between organizational capability, coordination, and surplus appropriability, characterizing how the effects of different policy instruments vary across capability environments.

At the aggregate level, the framework applies sector by sector. Since $\upsilon(\cdot)$ and $\lambda(\cdot)$ are common across sectors while the capability distributions differ, some sectors may remain capability-constrained — on either the extensive or the intensive margin — while others benefit primarily from corrective interventions. Aggregate development strategy should therefore differentiate across sectors according to the capability constraint that binds in each sector, rather than relying on uniform policy instruments economy-wide. The relevant diagnostic is not the mean of the capability distribution alone, but the entire distribution of organizational capability, which determines the selection, capability composition, and coordination capacity of anchor and partner firms.

7. Numerical Illustration of the Theory

This section illustrates the model's central mechanisms numerically. Figure 1 traces how institutional quality and organizational capability shape the decentralized equilibrium, as characterized in Proposition 2. Figure 2 illustrates the capability-constrained mechanism of Proposition 3: as the capability environment improves, a larger mass of active anchor firms,

drawing on a stronger partner pool, sustains production-network coordination, raising both coordination-generated output and the gains from corrective policy (Proposition 3(i)–(iii)). Figure 3 shows that these gains also depend on the shape of the capability distribution independently of its mean: greater dispersion at a constant mean weakens the coordination-relevant composition of the partner pool and, through entry, reduces anchor density, extending the composition mechanism that Proposition 3(iii) establishes for first-order shifts. A common logic runs through the three exercises: the economic environment and the capability distribution determine whether coordination emerges and at what scale, while the capability composition of participating firms determines how productive that coordination becomes.

The objective is not quantitative calibration. The exercises clarify the economic forces established analytically in Sections 4–6 rather than match any particular economy, and none of the numerical results is new relative to the theory. The distributional experiments also have a direct policy interpretation: because capability-building policy operates on the capability distribution F_s itself (Section 6.3), the shifts in location and shape considered below are the numerical analogue of capability-building interventions, while the corrective package $(\sigma^*, \tau_{ij}^*, \xi^*)$ is evaluated at each resulting equilibrium.

The numerical illustration maintains the scale normalization introduced in Appendix A.1. Accordingly, the effects of coordination are evaluated at benchmark production bundles. This isolates the direct productivity channel developed in the theoretical model — coordination raises partner productivity from q_{ij}^0 to $q_{ij}(e)$, and the resulting output and value gains are evaluated at pre-coordination factor allocations — while abstracting from endogenous factor reallocation. Section 2.4 characterizes the associated equilibrium profit incidence under flexible factors, conditional on the equilibrium scale of production.

The numerical results are organized using the distinction established in Section 2.4: the contract allocates claims over the coordination-generated revenue increment, whereas equilibrium profits are residuals determined after all payment obligations — the contractual transfer to the anchor and the competitive remuneration of production factors — have been satisfied. Accordingly, every quantity reported below is assigned to one of four layers, each answering a distinct question: (i) *coordination-generated value* — how much value coordination creates, measured by $Y_{ij} - Y_{ij}^0$ and $p_{ij}(Y_{ij} - Y_{ij}^0)$; (ii) *contractual allocation* — how the contract divides that value between the anchor's claim, $\upsilon(I^*)p_{ij}(Y_{ij} - Y_{ij}^0)$, and the partner's retained claim, $(1 - \upsilon(I^*))p_{ij}(Y_{ij} - Y_{ij}^0)$; (iii) *equilibrium incidence* — who ultimately earns what after all payments are made, as represented by $\Pi^*$and Π_{ij}^* and (iv) *welfare* — whether coordination raises social surplus once transfers are netted out and real resource costs are deducted, measured by W_s, with Γ_A and Γ_P recording the anchor- and partner-side contributions to total welfare. The four layers mirror the sequence of the model: coordination creates value, contracts divide it, markets determine its incidence, and the planner evaluates the result. Each table and figure explicitly identifies the layer to which its reported quantities belong.

Organizational capability follows a shifted log-normal distribution, $M = 1 + X$ with $X \sim \text{Lognormal}(\nu_s, \sigma_s^2)$, where $\nu_s$indexes the location of the sectoral capability distribution and σ_s governs its dispersion; higher ν_s shifts F_s upward in the sense of first-order stochastic dominance.

Table 2 reports the baseline calibration, held fixed except where a figure explicitly varies it. The calibration is intended to illustrate the model's mechanisms rather than reproduce any particular economy. The baseline calibration yields an equilibrium network size of $N^* = 31$ partner firms per anchor. As noted in Footnote 8, the empirical interpretation of network size depends on the unit of observation, ranging from numerous individual producers to much smaller numbers of cooperatives or larger production partners. The baseline composition of welfare gains, $\Gamma_P/\Gamma_A = 7.7$, keeps both coordination channels quantitatively operative, while the resulting net social gain — approximately 2 percent of sectoral welfare — is an equilibrium outcome rather than a calibration target.

Table 2. Baseline calibration

Parameter	Symbol	Baseline
Capability location	ν_s	2.00
Capability dispersion	σ_s	0.80
Institutional appropriability (baseline)	υ_0	0.99
Appropriability erosion curvature	κ	2.00
Capability elasticity	α	0.40
Coordination elasticity	δ	0.35
Partner-productivity elasticity	ϕ	0.007
Baseline congestion	λ_0	1.00
Congestion slope	λ_1	0.003
Fixed coordination cost	c_0	3.34
Absorptive-capacity ceiling	$\bar{\theta}$	2.00
Absorptive-capacity curvature	ρ	0.193

Note: Absorptive capacity is heterogeneous across partners, $\theta(M_j) = \bar{\theta}\left(1-e^{-\rho(M_j-1)}\right)$.

7.1 Baseline Equilibrium: A Four-Layer Decomposition

Before turning to comparative statics, this subsection decomposes the baseline equilibrium into the four analytical layers introduced above. The subsequent figures then trace how this equilibrium responds to changes in institutional quality and the capability distribution.

Methodological note. The first, second, and fourth layers are evaluated numerically at benchmark production bundles. Equilibrium-incidence objects are reported only where they follow directly from the analytical results of Section 2.4.

Table 3 makes two central theoretical distinctions concrete. First, equilibrium appropriability is jointly determined: competition among anchors erodes the institutional appropriability ceiling $\upsilon_0 = 0.99$ to an equilibrium value of $\upsilon(I^*) = 0.868$, so the contractual division of coordination-generated value reflects both the institutional environment and the equilibrium market structure rather than institutions alone. Second, contractual claims and equilibrium profits are distinct objects: the partner's equilibrium profit — the exempt benchmark component of Section 2.4 — is

roughly sixty-five times larger than its retained contractual claim, because the scale-dependent component of retained revenue accrues to competitive factor payments rather than to profit. The welfare rows then show that contractual transfers vanish from the social evaluation: welfare depends only on the coordination-generated value created and the real coordination resources employed. This numerical decomposition mirrors the planner's objective in Section 5, where contractual transfers cancel from welfare because they redistribute existing value rather than create additional value. The comparative statics of anchor density reinforce the first distinction: because $\upsilon'(I) < 0$, per-relationship partner profit declines as anchor density rises while the retained contractual claim rises, yet aggregate partner gains roughly double under the corrective package (Γ_P: 0.508 to 1.049) through the formation of additional relationships. Table 3 therefore provides the numerical counterpart to the analytical distinction between value creation, contractual allocation, equilibrium incidence, and welfare developed in Sections 2–5.

Table 3. The Four Layers at the Baseline Equilibrium

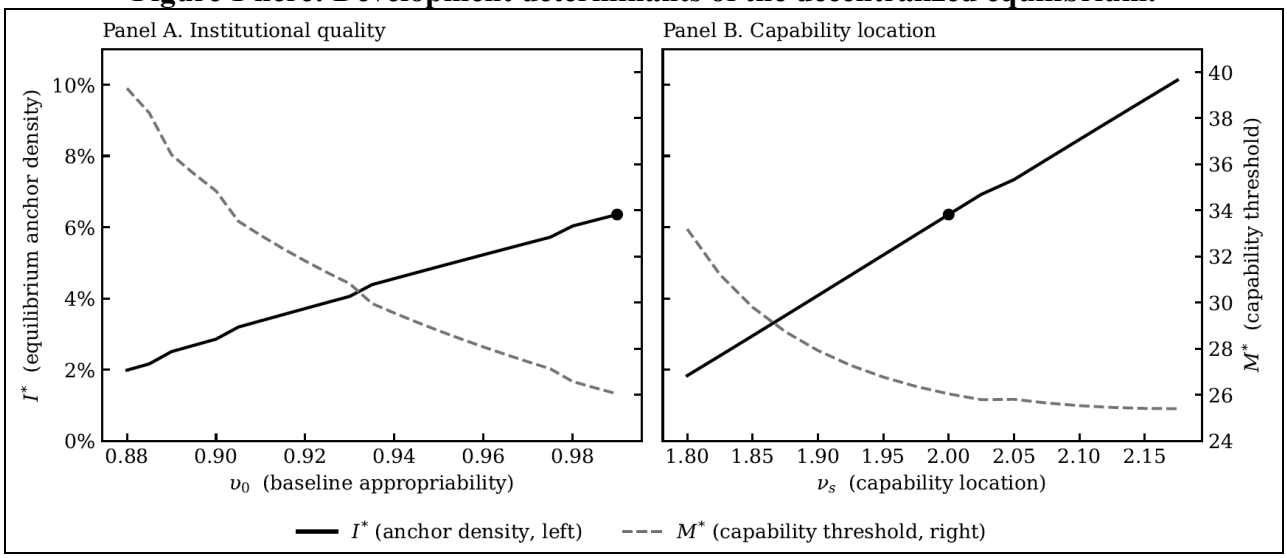
	Object	Expression	Per relationship	Aggregate
Layer 1: Coordination-generated value	Partner-side coordination-generated value	$q(e^*) - q^0$	0.232	$\Gamma_P = 0.508$
	Anchor-side coordination-generated value	$P_i(Y_i - Y_i^0)$	0.841 (per anchor)	$\Gamma_A = 0.066$
Layer 2: Contractual allocation	Equilibrium appropriability	$\upsilon(I^*) = \upsilon_0(1 - I^*)^\kappa$	0.868 (ceiling $\upsilon_0 = 0.99$)	—
	Anchor contractual claim	$\upsilon(I^*)\,(q(e^*) - q^0)$	0.202 (6.252 per anchor)	0.441
	Partner retained claim	$(1 - \upsilon(I^*))\,(q(e^*) - q^0)$	0.031 (0.949 per anchor)	0.067
Layer 3: Equilibrium incidence (analytically identified objects)	Partner profit	$\Pi_{ij}^* = \upsilon(I^*)\,q^0$	2.038	—
Layer 4: Welfare	Contractual transfers	net to zero in W_s	—	0
	Net social value	$W_s = Y_s - C_v - C_0$	—	2.595
	Net coordination gain	$W_s - Y_s^0$	—	0.127
	Composition of welfare gains	Γ_P/Γ_A	—	7.74

Notes: q denotes pool-average partner productivity at the threshold anchor; subscripts suppressed. Output units ($P_i = p_{ij} = r = w = 1$); per-relationship values are evaluated at the threshold anchor ($M^* = 26.04$, $N^* = 31$). The anchor's contractual claim in Layer 2 also equals its transfer receipts evaluated at benchmark bundles; equilibrium-incidence objects beyond Π_{ij}^* are not separately computed (see the methodological note).

7.2 Development Determinants of the Decentralized Equilibrium

Having established the baseline equilibrium decomposition in Table 3, we now examine how the equilibrium responds to changes in institutional quality and organizational capability. Figure 1 illustrates how the decentralized equilibrium responds to the development determinants identified in Proposition 2. The equilibrium pair (I^*, M^*) is jointly determined by the institutional environment and the capability distribution through the entry condition $\Pi^*(M^*, I^*) = 0$ and the identity $I^* = 1 - F_s(M^*)$; the two panels therefore perturb the two classes of primitives separately.

Figure 1 here: Development determinants of the decentralized equilibrium.



Notes: This figure reports the equilibrium determination of anchor density and the capability threshold, the equilibrium objects that underline the coordination-generated value, contractual allocation, equilibrium incidence, and welfare decomposition presented in Table 3. Each panel plots equilibrium anchor density, I^*(solid line, left axis), and the equilibrium capability threshold, M^* (dashed line, right axis). Panel A varies baseline appropriability, υ_0; Panel B varies the location parameter of the sectoral capability distribution, ν_s. All other parameters are held at their baseline values reported in Table 2. The marker identifies the baseline calibration.

Although both panels move the same equilibrium objects, they operate through fundamentally different mechanisms. Institutions change the incentives of a given population of firms: Panel A varies baseline appropriability υ_0, and because anchors retain a larger share of the coordination-generated value they generate, the profitability of anchor entry rises, the zero-profit threshold M^* falls, and anchor density I^* increases. The capability set is unchanged; what changes is whether coordination pays. Organizational capability, by contrast, changes the set of firms capable of coordinating production: Panel B shifts the location of F_s, a first-order stochastic dominance improvement under which more firms possess the capability required to operate profitably as anchors. An increase in $\nu_s$shifts the distribution of organizational capability to the right. This improves the capability composition of the potential partner pool. Since partner capability M_j determines both baseline productivity $q_{ij}^0(M_j)$ and absorptive capacity $\theta(M_j)$, the expected return

to coordination rises. Anchoring therefore becomes profitable for firms with lower own capability, so the zero-profit threshold M^*declines[26] and equilibrium anchor density I^* increases.

This distinction — *institutions govern the incentive to coordinate, capability governs the capacity to coordinate* — is the conceptual content of Proposition 2's separation of the determinants, and it organizes the policy analysis that follows: corrective instruments act on incentives, capability-building policy acts on the distribution. No policy appears in Figure 1; the panels characterize the decentralized equilibrium that the remaining exercises take as their starting point.

The decline in M^* does not continue indefinitely. Extending the simulations beyond the reported range shows that the threshold eventually reaches a minimum and then rises gradually, as further improvements in the capability composition of the partner pool become progressively smaller while continued entry erodes the coordination surplus appropriated by each anchor.

7.3 Capability and the Returns to Corrective Policy

Figure 2 examines how the returns to the optimal corrective package vary with organizational capability. Mean capability, $E[M]$, is increased through the location parameter while dispersion is held at its baseline value. Panel A reports the net social value from the full corrective package, $\Delta W_S/W_S^*$,[27] and Panel B the corresponding output gain, $\Delta Y_S/Y_S^*$. Both panels also report the decentralized and constrained-efficient anchor densities, $I^*$and I^{**}, on the right axis.

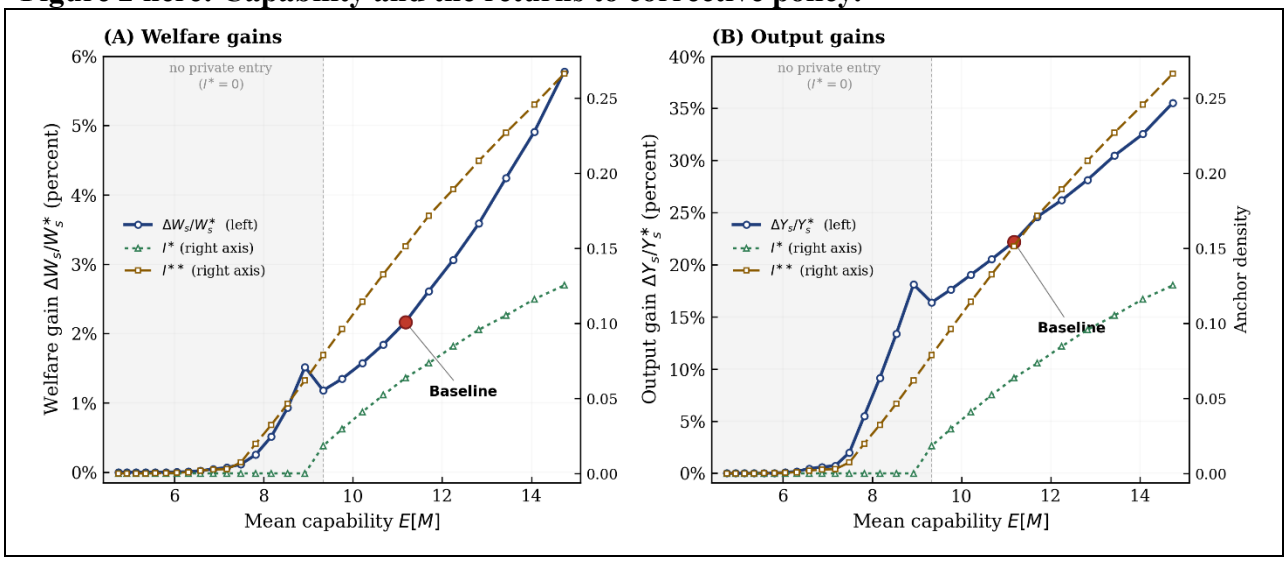
At low levels of mean capability — the shaded region — no firm finds anchor entry privately profitable and $I^* = 0$. Consistent with the capability-constrained coordination regime characterized in Proposition 3(ii), the corrective instruments are ineffective through the decentralized margin in this region: instruments that operate through coordination relationships have nothing to operate on when no such relationships exist. The constrained-efficient allocation nonetheless features positive coordination ($I^{**} > 0$) before private entry begins, because incomplete appropriability makes coordination socially valuable before it becomes privately profitable; measured gains therefore appear to the left of the entry point, and in that range the entire gain reflects coordination that only the corrected economy sustains. The measured gain peaks and briefly dips at the onset of private entry: once the market begins coordinating on its own, the decentralized benchmark W_S^* rises above the no-coordination baseline Y_S^0, narrowing the gap the corrective package can close, before the widening distortion again dominates.

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[26] Formally, the decline in M^* reflects the dominance of the direct partner-pool effect over the appropriability loss generated by additional anchor entry. The two forces converge near the upper edge of the reported range, where the threshold reaches an interior minimum before rising gradually.

[27] Here $W_S^*$and Y_S^* denote welfare and output at the decentralized equilibrium of Section 4, and W_S^{**}, Y_S^{**} their counterparts under the full corrective package $(\sigma^*, \tau_{ij}^*, \xi^*)$, i.e., at the constrained-efficient allocation. Where the decentralized equilibrium involves no entry ($I^* = 0$), $W_S^* = Y_S^* = Y_S^0$.

Figure 2 here: Capability and the returns to corrective policy.



Notes: This figure reports the comparative-static responses of coordination-generated value and welfare outcomes (Layers 1 and 4 of Table 3). Panel A plots the net social value from the full corrective package, $\Delta W_s/W_s^* = [W_s^{**} - W_s^*]/W_s^*$ (left axis); Panel B plots the corresponding output gain, $\Delta Y_s/Y_s^*$ (left axis). Both panels plot the decentralized and constrained-efficient anchor densities, $I^*$and I^{**}(right axis). Mean organizational capability, $E[M]$, is varied through the location parameter ν_S; all other parameters are held at their baseline values reported in Table 2. The shaded region indicates the range with no private anchor entry ($I^* = 0$). The marker indicates the equilibrium under the baseline calibration. At the lowest capability levels ($E[M] \leq 6.6$), the planner's network size at the threshold reaches the computational limit $N_{\max} = 400$; the associated gains in this region are below 0.02 percent, so the plotted series are unaffected at the resolution of the figure. Network size is integer-valued; the small steps in the output-gain series reflect discrete changes in partner inclusion as capability rises.

As capability rises past the activation threshold, private entry begins and the economy's response becomes sharply nonlinear. Anchor density increases rapidly, coordination relationships multiply, and the welfare and output gains from the corrective package accelerate, reflecting the expansion of the organizational channel identified in Proposition 3(i). Capability improvements generate disproportionately large policy returns near and above this threshold because they expand the scale of coordination and increase the productivity of each coordination relationship. The baseline economy, marked in both panels, sits well inside the coordination-active region with a net social value of approximately 2 percent. The difference between Panels A and B reflects coordination costs: Panel B reports the gross increase in coordination output, whereas Panel A reports the corresponding increase in net social value.

Panel B also makes the entry distortion of Proposition 1 visible: the gap between I^{**} and I^* widens as capability rises, indicating insufficient anchor entry throughout the plotted range.[28] Taken

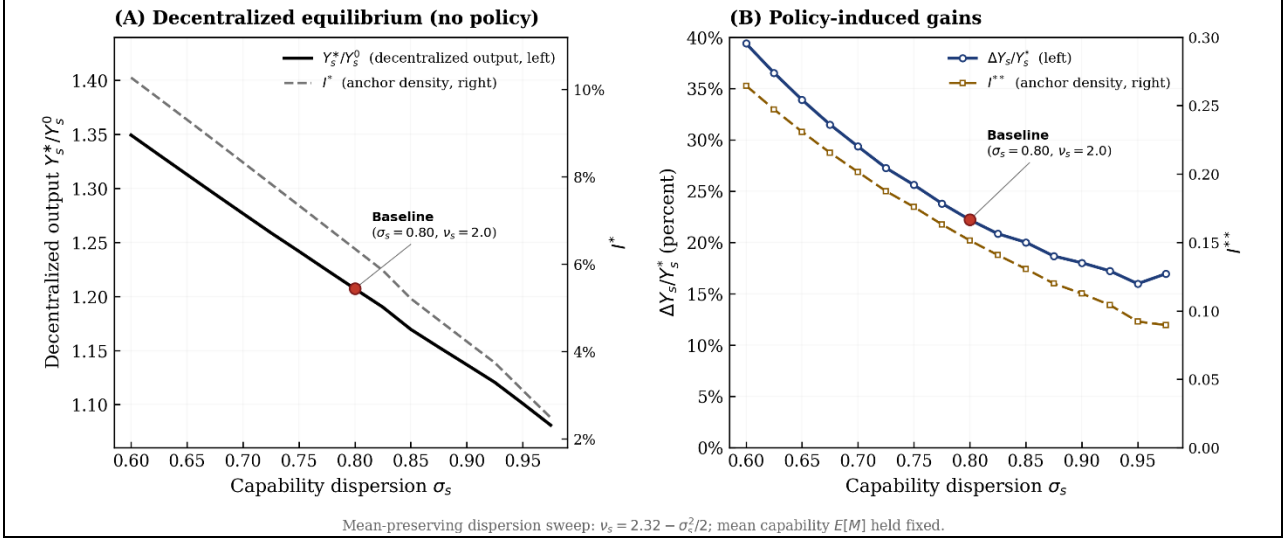
[28] The vertical distance between the two density curves, $I^{**} - I^*$, is the graphical counterpart of the entry diagnostic Δ defined in Proposition 1: a positive gap indicates that the decentralized equilibrium generates too little anchor entry. The diagnostic satisfies $\Delta > 0$ at every point of the sweeps in Figures 1–3 at which private entry is positive. The entry instrument that holds the marginal anchor to its break-even condition, $\xi^* = -\Pi^{(\sigma^*,\tau^*)}(M^{**}, I^{**})$, is strictly negative throughout, except in a narrow neighborhood of the boundary of coordination viability, where the effort and inclusion instruments are not yet generous enough to make the corrected marginal anchor profitable and the residual instrument is a small subsidy. Away from that neighborhood, unrestricted entry under the transfers would overshoot the

together, the two panels establish the section's second implication: the returns to a given corrective package increase with the capability distribution, because a stronger distribution supports a denser network of coordination relationships through which the instruments operate. The same corrective package, applied to economies that differ only in organizational capability, generates markedly different welfare and output gains.

7.4 Capability Composition and the Returns to Policy

Figure 3 illustrates the compositional constraint of Proposition 3(iii): the capability composition of the partner pool governs the surplus that coordination relationships generate and, through the entry margin, the scale of coordination the economy sustains — and, through both, the gains from corrective policy. Dispersion is the experiment, not the result. The exercise holds mean capability $E[M]$ fixed while varying dispersion over $\sigma_s \in [0.60, 0.975]$, adjusting the location parameter along $\nu_s = 2.32 - \sigma_s^2/2$ so that every distribution in the family has the same mean; any variation across the sweep is therefore attributable to the shape of the capability distribution alone. The experiment thus differs from Panel B of Figure 1, which shifts the *location* of the distribution: here the *shape* changes at a fixed mean. This comparative static is absent from models in which policy effectiveness depends only on average productivity or technology levels. We first show how the change in shape affects the decentralized equilibrium (Panel A), and then ask how it alters the gains from optimal coordination policy (Panel B).

Figure 3: Capability dispersion and policy effectiveness.



Notes: This figure reports the comparative-static responses of coordination-generated value and welfare outcomes to changes in the capability distribution (Layers 1 and 4 of Table 3). Panel A plots decentralized output relative to the no-coordination benchmark, Y_s^*/Y_s^0 (left axis), and decentralized anchor density, I^* (right axis). Panel B plots the policy-induced output gain, $\Delta Y_s/Y_s^*$ (left axis), and the constrained-efficient anchor density, I^{**}(right axis). Dispersion in organizational capability, σ_s, is varied along the mean-preserving locus $\nu_s = 2.32 - \sigma_s^2/2$, holding mean organizational capability $E[M]$ fixed; all other parameters are held at their baseline values reported in Table 2. The marker indicates the equilibrium under the baseline calibration.

constrained optimum, and the entry instrument operates as a tax. The sign of ξ^* is not established analytically (Appendix D.5); these computations determine it across the grid.

Panel A shows that greater dispersion at constant mean weakens the decentralized equilibrium: anchor density falls substantially across the sweep, and decentralized output relative to the no-coordination benchmark declines with it. Panel B shows that the gains from the corrective package decline in parallel: the same instruments, applied to distributions with identical means, generate progressively smaller gains as the composition of the economy deteriorates.

The mechanism operates through the composition of the partner pool, amplified by the equilibrium entry response. Holding mean capability fixed, greater dispersion leaves a small group of firms with exceptionally high capability while shifting most firms toward lower capability. The typical partner therefore shifts toward lower capability; because partners' organizational capability determines both baseline productivity and absorptive capacity, the surplus generated within each coordination relationship falls. The decline in relationship surplus then operates on the entry margin: the marginal anchor earns only a small residual over the fixed coordination cost, so the capability required to break even rises sharply and anchor density contracts. It is this contraction in the scale of coordination, rather than the decline in per-relationship productivity, that accounts for most of the fall in coordination output. The result is an equilibrium phenomenon, not a mechanical one: holding the initial threshold fixed, greater dispersion would place *more* mass above it, so the collapse arises entirely from the equilibrium adjustment of entry. Notably, the surviving anchors are substantially more capable than before, yet coordination nevertheless contracts, because the economy can profitably sustain far fewer coordinating firms and, with them, far fewer coordination relationships — the effect is not a reallocation toward intrinsically more productive anchors but a contraction in aggregate coordination (Appendix E.3).

The implication is important. Two economies with identical average organizational capability can nevertheless exhibit very different equilibrium coordination and very different returns to the same corrective policy package, because their capability distributions differ.

Development therefore depends not only on the existence of highly capable firms but on the quality of the broader ecosystem of potential partners through which coordination gains are transmitted; a small number of highly capable firms cannot compensate for a weak partner base when production requires coordination across many firms. The relevant diagnostic is the entire capability distribution — through the composition of the coordinated segment it induces — rather than any single summary statistic such as its mean. One clarification completes the exercise: the sweep in Figure 3 lets anchor density adjust endogenously, and the entry margin amplifies the compositional effect rather than offsetting it.

7.5 Synthesis

Taken together, the numerical exercises deliver the paper's central message: the institutional and coordination environment shapes the emergence and scale of production networks, while organizational capability shapes both selection into coordination and the productivity of coordination within the networks that form. Institutions determine whether coordination is privately sustainable — appropriability governs the return to entering as an anchor, and the entry condition determines how many firms enter. The capability distribution determines how productive coordination becomes: it determines the equilibrium composition of production networks, and partner capability governs the surplus generated by each coordination relationship.

Because corrective policies operate through the organizational channel of active anchor firms, their returns are jointly determined by these two forces: the same package produces large gains where institutions and capability sustain dense, productive networks, and small gains where they do not.

Capability-building and corrective coordination policy are therefore complements rather than substitutes, with a natural sequencing. Capability-building improves organizational capability throughout the distribution; in equilibrium, this both expands the set of firms able to operate as anchors and — in the simulations reported here — much of the improvement operates through more capable partners. Corrective policy, by contrast, improves the efficiency of the coordination relationships that exist. Because the second operates through the first, the returns to correction are small in low-capability environments precisely because the relationships it would improve are scarce, and rise as capability accumulates. The entry-side results point in the same direction: away from the boundary of coordination viability, the entry instrument satisfies $\xi^* < 0$, so the constrained optimum pairs effort subsidies and inclusion grants with a tax on anchor entry, reallocating resources within the coordinated segment rather than expanding the number of anchors (footnote 28). Capability determines not only the structure of value chains but also the returns to the policies designed to improve them.

8. Conclusion

This paper develops a theory of endogenous value chain formation and productivity diffusion in which coordination is supplied by firms. Heterogeneous firms sort into anchor and partner roles according to organizational capability. Anchors form production networks, invest coordination effort in their partners, and appropriate a share of the revenue increment their coordination generates in each partner relationship. Three equilibrium objects — the intensity of coordination, the breadth of production networks, and the mass of coordinating firms — emerge jointly from private decisions at a unique equilibrium. Incomplete appropriability creates distortions along all three margins, although the entry distortion is fundamentally different because anchor entry both expands production networks and endogenously erodes appropriability. We characterize the constrained-efficient allocation and derive three corrective instruments that decentralize it.

The direction of the entry distortion is classified by a single diagnostic statistic separating insufficient-entry from excess-entry regimes. The sign of the implementing instrument is determined separately by the exact break-even condition under corrected conduct. The corrective instruments raise the anchor's effective appropriated share; the sign of the entry instrument therefore depends on how this share effect interacts with the change in the capability of the marginal entrant induced by the shift in the entry threshold and the resulting change in the composition of the partner pool. When decentralized entry is excessive, all three effects reinforce one another, and the implementing entry instrument is a tax throughout the calibrated region. When decentralized entry is insufficient, by contrast, the share effect is opposed by the other two, leaving the sign of the entry instrument theoretically ambiguous so that it must be determined quantitatively. Insufficient entry therefore does not necessarily imply an entry subsidy: the direction of the entry distortion and the sign of the instrument required to decentralize the constrained-efficient allocation are distinct economic questions.

The deeper result concerns the limits of correction. The corrective instruments operate through coordinating firms, and the existence of those firms is itself an equilibrium outcome governed by the capability distribution and the coordination environment. As deterioration in the capability distribution drives equilibrium anchor density toward zero, the organizational channel through which the corrective instruments operate disappears and aggregate policy responsiveness converges to zero. Even when anchor density remains economically meaningful, weak capability within the partner pool can limit the productivity of coordination and reduce the quantitative gains from correction. Organizational capability therefore determines not only the structure and productivity of production networks, but also the extent to which distortions within those networks can be effectively corrected.

The analysis therefore separates two forces that are often conflated. The institutional and coordination environment shapes the incentives and costs governing the emergence and scale of production networks: appropriability and coordination congestion affect the entry threshold and therefore the equilibrium density of active anchors. Organizational capability shapes both which firms can perform the coordinating function and how productive coordination is within the networks that form. The two forces are complementary. Improvements in the coordination environment can expand the scale of network formation, but their aggregate returns depend on the capability base through which coordination operates.

The framework also suggests a broader developmental question concerning the changing form of coordination. In low-capability environments, firm-led coordination may primarily compensate for weak supplier capabilities and missing organizational infrastructure. In more advanced production systems, coordination may increasingly concern technological alignment, standards integration, and the management of complex complementarities across specialized firms. The present model holds the form of coordination fixed while varying the capability and institutional environment in which it occurs. Endogenizing the form of coordination along the development path is a natural extension.

Several further extensions merit attention. The model is static: organizational capability is a primitive, whereas in practice it accumulates — potentially through coordination itself, as partner firms learn from anchor investment. A dynamic version in which today's coordination shifts tomorrow's capability distribution could characterize transitions into and out of capability-constrained coordination regimes. The multi-sector aggregation developed in Section 6 treats coordination sectors as distinct components of the economy. Allowing firms to occupy different network roles across sectors, or allowing coordination relationships to transmit capability and productivity across sectoral boundaries, would link the cross-sectoral architecture of organizational capability more directly to aggregate development. Institutional structure is also held exogenous. Intermediary organizations such as producer cooperatives may alter both the cost and the appropriability of coordination by aggregating fragmented suppliers and changing bargaining conditions within production networks.

Finally, the framework predicts that coordination effort, production-network breadth, the productivity gains from coordination, and the effectiveness of corrective instruments should vary systematically with organizational capability, institutional quality, and the degree of competition among coordinating firms. These predictions provide a natural bridge from the theory to firm-level

evidence on management practices, supplier-development programs, technology transfer, quality upgrading, and other relationship-specific investments observed in buyer–supplier and outgrower networks. Testing these predictions offers a promising agenda for future empirical research on the organizational foundations of productivity diffusion and value chain formation.

Production networks organized around capable firms are neither a substitute for markets nor a rival to institutions. Rather, they are an organizational form that emerges within markets, enabling the deployment of productive knowledge through relationship-specific coordination that arm's-length exchange alone cannot sustain. Understanding when this organizational form emerges, why privately supplied coordination remains partial, and what governs its reach is therefore central to understanding why some economies convert existing productive knowledge into output more effectively than others.

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APPENDIX A. FOUNDATIONS OF THE ANCHOR PROBLEM AND EQUILIBRIUM

A.1 Setup, Notation, and the Benchmark-Output Mapping

Fix an anchor i with capability M_i, partner set N_i(we write N_i both for the set and, where no confusion arises, for its cardinality), and effort vector $e_i = (e_{ij})_{j \in N_i}$; $e_{i,-j}$denotes the vector excluding partner j. Throughout this appendix the anchor's factor choices (K_i, L_i) and network size $N_i$are held fixed, and the output price P_i is parametric.

Scale normalization. For the analytical characterization of the anchor firm's coordination problem, the coordination-generated partner-output increment $Y_{ij} - Y_{ij}^0$ is evaluated at a predetermined benchmark production bundle. Under this scale normalization, equation (4) implies

$$Y_{ij}(e_{ij}) = Y_{ij}^0(1 + \theta_{ij} e_{ij})^{\delta}, \quad \text{(A.1)}$$

where Y_{ij}^0 denotes pre-coordination output, since total factor productivity scales output one-for-one at a fixed factor bundle. The resulting coordination-generated revenue increment entering equation (9) is therefore:

$$p_{ij}[Y_{ij}(e_{ij}) - Y_{ij}^0].$$

The scale normalization applies only to the evaluation of the coordination-generated output increment entering the anchor's coordination problem; it does not alter the partner firm's factor-demand problem analyzed in Section 2.4.

The normalization is consistent with optimal factor conduct. Coordination raises total factor productivity but leaves the cost-minimizing capital-labor ratio unchanged because productivity scales the marginal products of both factors proportionally. Under constant returns to scale, however, the firm's first-order conditions determine the optimal input mix but not production scale (Section 2.4), so the benchmark production bundle selects one admissible production scale. This normalization isolates the productivity effect of coordination, abstracts from the separate partner factor-allocation margin that would otherwise arise from the anchor's claim on incremental revenue, and permits the coordination-generated output increment to enter the welfare accounting of Section 5 without requiring a separate treatment of factor reallocation.

Auxiliary notation. Define

$$a \equiv \phi\delta \in (0,1); \qquad x_{ij} \equiv \frac{\theta_{ij}}{1 + \theta_{ij} e_{ij}} > 0;$$

$$C \equiv P_i K_i^{\beta} L_i^{1-\beta} M_i^{\alpha} \prod_{k \in N_i} (q_{ik}^0)^{\phi} > 0,$$

the collection of all terms in anchor revenue that are independent of the effort vector; and

$$G(e_i) \equiv P_i Y_i = C \prod_{k \in N_i} (1 + \theta_{ik} e_{ik})^a, \qquad (A.2)$$

anchor revenue as a function of effort, obtained by substituting (4) into (2) into (1). Finally, define the own-curvature coefficient of the appropriated revenue increment,

$$b_{ij} \equiv \upsilon(I)\, p_{ij}\, Y_{ij}^0\, \delta(1-\delta)\, \theta_{ij}^2\, (1 + \theta_{ij} e_{ij})^{\delta - 2} > 0. \qquad (A.3)$$

With this notation, the effort-dependent part of the anchor objective Π in equation (9) is

$$\Pi(e_i) = G(e_i) + \upsilon(I) \sum_{j \in N_i} p_{ij}\, Y_{ij}^0 [(1 + \theta_{ij} e_{ij})^{\delta} - 1] - \frac{\lambda(N_i)}{2} \sum_{j \in N_i} e_{ij}^2 + \text{const.} \qquad (A.4)$$

The feasible set is $\mathcal{E}_i \equiv \{e_i \geq 0 : \sum_{j \in N_i} e_{ij} \leq M_i\}$, which is convex and compact.

Matrix notation. For a symmetric matrix A, $A \succ 0 (A \prec 0)$ denotes positive (negative) definiteness; $x^\top$ denotes the transpose, $xx^\top$ the outer product, and $\mathrm{diag}(d_1, \ldots, d_n)$ the diagonal matrix with entries d_j.

A.2 Concavity of the Anchor Objective in Coordination Effort

Lemma A.2.1 (Strict concavity of the revenue component). For each $j \in N_i$, anchor revenue $G(e_i)$ is strictly concave in e_{ij}, holding $e_{i,-j}$ fixed, with

$$\frac{\partial^2 G}{\partial e_{ij}^2} = a(a-1)\, x_{ij}^2\, G(e_i) < 0.$$

Proof. Isolating the dependence of (A.2) on e_{ij}, write $G = C_{-j}(1 + \theta_{ij} e_{ij})^a$ with $C_{-j} \equiv C \prod_{k \neq j} (1 + \theta_{ik} e_{ik})^a > 0$ constant in e_{ij}. Differentiating twice,

$$\frac{\partial^2 G}{\partial e_{ij}^2} = C_{-j}\, a(a-1)\, \theta_{ij}^2 (1 + \theta_{ij} e_{ij})^{a-2} = a(a-1)\, x_{ij}^2\, G < 0,$$

since $a = \phi\delta \in (0,1)$ by $\phi, \delta \in (0,1)$, hence $a(a-1) < 0$.

Remark A.2.1. Under the Cobb–Douglas specification of equations (1)–(2), strict concavity of the revenue channel is a derived property, not an assumption: the composite $e_{ij} \to q_{ij} \to A_i \to Y_i$ is a power function with exponent $\phi\delta < 1$. Economically, the marginal revenue return to coordination effort is diminishing, which rules out increasing returns through the productivity channel; absent this property, an anchor would optimally concentrate all effort on a single partner, and the one-to-many network structure central to the model could not arise. For non-Cobb–Douglas generalizations, the statement of Lemma A.2.1 may be maintained as an assumption in its place.

Lemma A.2.2 (Coordinate-wise strict concavity). Holding fixed (K_i, L_i, N_i) and $e_{i,-j}$, the anchor objective Π in equation (9) is strictly concave in e_{ij} for all $e_{ij} \geq 0$, with

$$\frac{\partial^2 \Pi}{\partial e_{ij}^2} = \frac{\theta_{ij}^2}{\left(1 + \theta_{ij} e_{ij}\right)^2} [\phi\delta(\phi\delta - 1)\, G(e_i) + \delta(\delta - 1)\, \upsilon(I)\, p_{ij}\, Y_{ij}(e_{ij})] - \lambda(N_i) < 0. \qquad (A.5)$$

Proof. Three components of (A.4) depend on e_{ij}. (a) Revenue channel: by Lemma A.2.1, $\partial^2 G / \partial e_{ij}^2 = a(a-1)x_{ij}^2 G < 0$; using (A.1) and the definition of x_{ij}, this equals the first bracketed term of (A.5) times $\theta_{ij}^2(1 + \theta_{ij} e_{ij})^{-2}$. (b) Appropriated revenue increment: by the mapping (A.1), the appropriated term for partner j is $\upsilon(I) p_{ij} Y_{ij}^0 [(1 + \theta_{ij} e_{ij})^{\delta} - 1]$, whose second derivative is $-b_{ij} < 0$by $\delta \in (0,1)$; equivalently, the second bracketed term of (A.5) times $\theta_{ij}^2(1 + \theta_{ij} e_{ij})^{-2}$. (c) Coordination cost: the term $-\frac{\lambda(N_i)}{2} e_{ij}^2$contributes the constant $-\lambda(N_i) < 0$. Summing (a)–(c) yields (A.5); each bracketed term is strictly negative and $-\lambda(N_i) < 0$, so $\partial^2 \Pi / \partial e_{ij}^2 < 0$ for all $e_{ij} \geq 0$.

Remark A.2.2 (conditional best response). Combined with the boundary behavior established in Lemma A.3.1 below, strict concavity guarantees a unique solution $e_{ij}^*(e_{i,-j})$ to the first-order condition (10) — a unique conditional best response for partner j, given effort directed at other partners. Uniqueness of the full effort vector is a strictly stronger property and is treated in Proposition A.5.1.

Remark A.2.3 (constrained optimum). Since the capability constraint (7) is linear in the effort vector, the feasible set $\mathcal{E}_i$ is convex, and coordinate-wise strict concavity delivers a unique conditional best response on $\mathcal{E}_i$ whether or not (7) binds.

A.3 Optimal Coordination Effort

A.3.1 The problem and existence. The anchor maximizes (A.4) over $e_i \in \mathcal{E}_i$, conditional on (K_i, L_i, N_i); the fixed cost c_0 and factor payments are constants at this stage and are omitted. Since $\mathcal{E}_i$ is compact and Π is continuous, a maximizer exists (Weierstrass); no concavity is required for existence.

A.3.2 Concavity and sufficiency. The objective is strictly concave in each e_{ij} (Lemma A.2.2), and jointly strictly concave on $\mathcal{E}_i$ under the criterion of Proposition A.5.1 — automatically whenever $N_i\phi\delta \leq 1$. Under joint concavity on the convex set $\mathcal{E}_i$, the Karush–Kuhn–Tucker conditions below are both necessary and sufficient for the global optimum, and the maximizing effort vector is unique.

A.3.3 KKT Lagrangian and conditions. Let $\eta_i \geq 0$ denote the multiplier on the capability constraint (7). The Lagrangian is

$$\mathcal{L} = P_i Y_i + \sum_{j \in N_i} \frac{\lambda(N_i)}{2} e_{ij}^2 + \sum_{j \in N_i} \upsilon(I)\ p_{ij}(Y_{ij} - Y_{ij}^0) + \eta_i(M_i - \sum_{j \in N_i} e_{ij}),$$

with conditions, for each $j \in N_i$: $\partial \mathcal{L} / \partial e_{ij} = 0$; $\eta_i \geq 0$; $M_i - \sum_j e_{ij} \geq 0$; and $\eta_i(M_i - \sum_j e_{ij}) = 0$ (complementary slackness).

By equation (A.2) and the mapping (A.1), this maximand coincides with (A.4) up to the constants omitted per A.3.1; the Lagrangian is stated in unsubstituted form so that the four channels in A.3.4 can be differentiated term by term.

A.3.4 Expanding the first-order condition. Differentiating $\mathcal{L}$ with respect to e_{ij} yields four terms.

Term 1 — anchor productivity channel. By the chain rule and equations (1), (2), (4),

$$\frac{\partial}{\partial e_{ij}}(P_i Y_i) = P_i \frac{\partial Y_i}{\partial A_i} \frac{\partial A_i}{\partial q_{ij}} \frac{\partial q_{ij}}{\partial e_{ij}}, \qquad \frac{\partial Y_i}{\partial A_i} = \frac{Y_i}{A_i}, \qquad \frac{\partial A_i}{\partial q_{ij}} = \phi \frac{A_i}{q_{ij}}, \qquad \frac{\partial q_{ij}}{\partial e_{ij}} = q_{ij}^0 \theta_{ij} \delta (1 + \theta_{ij} e_{ij})^{\delta - 1}.$$

Term 2 — appropriated revenue increment channel. Evaluating partner output at the benchmark production bundle by the mapping (A.1),

$$\frac{\partial}{\partial e_{ij}}[\upsilon(I) p_{ij}(Y_{ij} - Y_{ij}^0)] = \upsilon(I)\ p_{ij}\ Y_{ij}^0\ \theta_{ij} \delta (1 + \theta_{ij} e_{ij})^{\delta - 1}.$$

Term 3 — coordination cost. $\frac{\partial}{\partial e_{ij}}(-\frac{\lambda(N_i)}{2} e_{ij}^2) = -\lambda(N_i)\ e_{ij}$.

Term 4 — capability constraint. $\frac{\partial}{\partial e_{ij}}(-\eta_i \sum_j e_{ij}) = -\eta_i$.

A.3.5 Collecting terms. Setting $\partial \mathcal{L} / \partial e_{ij} = 0$:

$$P_i \frac{\partial Y_i}{\partial A_i} \frac{\partial A_i}{\partial q_{ij}}\ q_{ij}^0 \theta_{ij} \delta (1 + \theta_{ij} e_{ij})^{\delta - 1} + \upsilon(I)\ p_{ij}\ Y_{ij}^0\ \theta_{ij} \delta (1 + \theta_{ij} e_{ij})^{\delta - 1} = \lambda(N_i)\ e_{ij} + \eta_i.$$

This is equation (10), characterizing optimal coordination effort conditional on partner inclusion. In the notation of Section A.1, it reads compactly as

$$a\ x_{ij}\ G(e_i) + \upsilon(I)\ p_{ij}\ Y_{ij}^0\ \delta \theta_{ij} (1 + \theta_{ij} e_{ij})^{\delta - 1} = \lambda(N_i)\ e_{ij} + \eta_i, \qquad (A.6)$$

the form differentiated in Section A.5.

Remark A.3.1 (congestion bookkeeping). Because partner $j \in N_i$, the network containing j has size N_i and the marginal coordination cost is $\lambda(N_i)e_{ij}$, consistent with equations (6) and (9). The

parameter $\lambda(N_i + 1)$ appears only in the inclusion comparison of Section A.4, where network size itself is varied.

Lemma A.3.1 (Boundary behavior and interiority). For each $j \in N_i$, holding $e_{i,-j}$ fixed: (i) $\partial\Pi/\partial e_{ij}\,|_{e_{ij}=0} = a\,\theta_{ij}\,G(e_{i,-j}, 0) + \upsilon(I)\,p_{ij}\,Y_{ij}^0\,\delta\,\theta_{ij} > 0$; (ii) $\partial\Pi/\partial e_{ij} < 0$ for all e_{ij} sufficiently large. Consequently the conditional best response satisfies $e_{ij}^* > 0$, and absent a binding capability constraint it is the unique interior root of the first-order condition (10).

Proof. (i) At $e_{ij} = 0$: the marginal revenue term is $a\,\theta_{ij}\,G(e_{i,-j}, 0) > 0$; the marginal appropriated increment is $\upsilon(I)p_{ij}Y_{ij}^0\delta\theta_{ij} > 0$; the marginal coordination cost is $\lambda(N_i) \cdot 0 = 0$. The sum is strictly positive, so $e_{ij} = 0$ is never optimal. (ii) The marginal benefit is decreasing in e_{ij} (both terms have exponents below one), while the marginal cost $\lambda(N_i)e_{ij}$ increases without bound; hence $\partial\Pi/\partial e_{ij} < 0$ for e_{ij} large. By continuity and Lemma A.2.2, the first-order condition has exactly one root, which is the conditional best response.

A.3.6 Complementary slackness and two regimes. Case 1 (unconstrained, $\sum_j e_{ij} < M_i$): $\eta_i = 0$ and equation (10) reduces to marginal benefit equal to $\lambda(N_i)e_{ij}$; effort on each relationship is bounded only by convex costs, relationships are optimized independently, and the breadth-versus-depth tradeoff is inoperative. Case 2 (constrained, $\sum_j e_{ij} = M_i$): $\eta_i > 0$, the shadow cost of organizational capability enters (10) directly, effort falls below the unconstrained level, and allocation across partners activates the breadth-versus-depth tradeoff of Section 3.2.

A.4 Optimal Partner Inclusion

A.4.1 Setup. Partner j is included in anchor i's network if and only if the incremental profit generated by the relationship is non-negative: $\Pi(N_i + 1) - \Pi(N_i) \geq 0$. Using the objective (9), this difference expands as

$$\Delta\Pi = [P_iY_i(N_i + 1) - P_iY_i(N_i)] + \upsilon(I)p_{ij}(Y_{ij} - Y_{ij}^0) - \frac{\lambda(N_i + 1)}{2}e_{ij}^2 - \frac{\lambda(N_i + 1) - \lambda(N_i)}{2}\sum_{j' \in N_i} e_{ij'}^2.$$

A.4.2 The four terms. Term 1 — anchor TFP gain: $P_iY_i(N_i + 1) - P_iY_i(N_i) = P_i[A_i(N_i + 1) - A_i(N_i)]K_i^{\beta}L_i^{1-\beta}$ with $A_i(N_i + 1) - A_i(N_i) = M_i^{\alpha}(\prod_{j' \in N_i \cup \{j\}} q_{ij'}^{\phi} - \prod_{j' \in N_i} q_{ij'}^{\phi}) > 0$ since $q_{ij} > 0$ and $\phi > 0$. Term 2 — appropriated revenue increment: $\upsilon(I)p_{ij}(Y_{ij} - Y_{ij}^0) = \upsilon(I)p_{ij}Y_{ij}^0[(1 + \theta_{ij}e_{ij})^{\delta} - 1] > 0$ by (A.1). Term 3 — direct coordination cost: $\frac{\lambda(N_i+1)}{2}e_{ij}^2$, evaluated at the expanded network since network size is discrete and the comparison varies it. Term 4 —

congestion imposed on existing relationships: $\frac{\lambda(N_i+1)-\lambda(N_i)}{2}\sum_{j'\in N_i} e_{ij'}^2 > 0$ by $\lambda'(N_i) > 0$— the negative externality that expansion imposes on the existing network.

A.4.3 Collecting terms. Rearranging $\Delta\Pi \geq 0$, partner jis included if and only if

$$P_i[Y_i(N_i+1) - Y_i(N_i)] + \upsilon(I)\, p_{ij}\, Y_{ij}^0[(1+\theta_{ij}e_{ij})^{\delta} - 1] - \frac{\lambda(N_i+1)}{2} e_{ij}^2 \geq \frac{\lambda(N_i+1)-\lambda(N_i)}{2}\sum_{j'\in N_i} e_{ij'}^2 .$$

This is equation (11): the left-hand side is the net coordination gain from the new relationship — anchor TFP gain plus the appropriated revenue increment, net of direct coordination cost — and the right-hand side is the congestion cost imposed on existing relationships.

A.4.4 Properties. Observation 1. Because anchors appropriate only the fraction $\upsilon(I) < 1$ of the coordination-generated revenue increment, the private inclusion condition values new relationships below the constrained-efficient condition; this wedge is the source of Distortion 2. Observation 2. Since $\upsilon'(I) < 0$, greater anchor density reduces the appropriated increment from new relationships without affecting the congestion cost of expansion; the inclusion condition tightens. Observation 3. Equations (10) and (11) are jointly determined: e_{ij}^* enters the inclusion comparison through both the increment and the direct cost, so network size and coordination effort are jointly determined.

A.5 Joint Concavity and Uniqueness of the Anchor's Optimum

Proposition A.5.1 (Joint concavity criterion). Let $H(e_i) \equiv [\partial^2\Pi/\,\partial e_{ij}\,\partial e_{ik}]_{j,k\in N_i}$ denote the Hessian of the anchor objective in the effort vector, holding (K_i, L_i, N_i) fixed, and let $x = (x_{ij})_{j\in N_i}$. Then:

(i) The Hessian has the rank-one structure

$$H = a^2 G\, xx^{\top} - D, D \equiv \mathrm{diag}(a\, G\, x_{ij}^2 + b_{ij} + \lambda(N_i)) \succ 0. \qquad (A.7)$$

(ii) $H(e_i) \prec 0$ if and only if

$$\Phi(e_i) \equiv \sum_{j\in N_i} \frac{a^2\, G\, x_{ij}^2}{a\, G\, x_{ij}^2 + b_{ij} + \lambda(N_i)} < 1. \qquad (A.8)$$

(iii) A sufficient condition, independent of the effort vector and of all equilibrium objects, is $N_i\, \phi\delta \leq 1$.

(iv) If (A.8) holds for every $e_i \in \mathcal{E}_i$, then Π is jointly strictly concave on $\mathcal{E}_i$; the Karush–Kuhn–Tucker conditions of Section A.3 are then sufficient as well as necessary, and the maximizing effort vector is unique.

Proof. (i) From (A.2), $\partial G/\partial e_{ij} = a\,x_{ij}G$ and $\partial^2 G/\partial e_{ij}\,\partial e_{ik} = a^2 x_{ij}x_{ik}G$ for $k \neq j$, while $\partial^2 G/\partial e_{ij}^2 = a(a-1)x_{ij}^2 G = a^2 x_{ij}^2 G - a\,x_{ij}^2 G$. The increment and cost terms are additively separable across partners, contributing $-b_{ij}$ and $-\lambda(N_i)$ to the diagonal only. Collecting the common $a^2 x_{ij}x_{ik}G$terms — including on the diagonal — yields the rank-one term $a^2 G\,xx^\top$; the residual diagonal entries are $-(aGx_{ij}^2 + b_{ij} + \lambda(N_i))$, each strictly negative. (ii) Since $D \succ 0$, $H \prec 0 \iff D^{-1/2}HD^{-1/2} \prec 0 \iff a^2 G(D^{-1/2}x)(D^{-1/2}x)^\top \prec I$. A rank-one matrix $vv^\top$satisfies $vv^\top \prec I$iff $\| v \|^2 < 1$; here $\| v \|^2 = a^2 G\,x^\top D^{-1}x = \Phi(e_i)$. (iii) Each summand in (A.8) is strictly below $a^2 Gx_{ij}^2/(aGx_{ij}^2) = a$, so $\Phi(e_i) < N_i a = N_i \phi\delta$ for every $e_i \geq 0$. (iv) A twice-differentiable function with negative definite Hessian at every point of a convex set is strictly concave on that set; strict concavity on $\mathcal{E}_i$ with linear constraints implies any KKT point is the unique global maximizer.

Remark A.5.1 (complementarity and the scope of Lemma A.2.2). The off-diagonal entries of H are $a^2 x_{ij}x_{ik}G > 0$: coordination efforts directed at different partners are complements through the multiplicative aggregator (2). Coordinate-wise concavity therefore does not by itself imply joint concavity; criterion (A.8) is precisely the condition under which own curvature and congestion dominate the aggregate complementarity.

Remark A.5.2 (pointwise versus feasible-set verification). Criterion (A.8) evaluated at a single point establishes negative definiteness at that point only — a local second-order condition. Global KKT sufficiency and uniqueness of the effort vector require the feasible-set version in part (iv). Condition (iii) is valuable precisely because it is effort-independent: when it holds, the feasible-set version follows with no further verification.

Remark A.5.3 (computational domain). Under the Section 7 calibration ($\phi = 0.007$, $\delta = 0.35$), $\phi\delta = 0.00245$, so condition (iii) holds for every network size $N_i \leq 408$. Since the numerical implementation never evaluates networks larger than its search cap of 400 (Appendix E.1.4), the anchor's effort problem is jointly strictly concave throughout the entire computational domain, and the solved effort vector is unique for every network size evaluated. For network sizes exceeding $1/(\phi\delta)$, joint concavity is governed by the exact criterion (A.8) rather than by (iii).

A.6 Profit Monotonicity in Capability and Anchor Density

Proposition A.6.1 (Weak monotonicity in capability). $\Pi^*(M_i, I)$ is weakly increasing in M_i: for $M_i' > M_i$, $\Pi^*(M_i', I) \geq \Pi^*(M_i, I)$.

Proof. The capability constraint $\sum_{j \in N_i} e_{ij} \leq M_i$ defines a feasible set strictly expanding in M_i, so any allocation feasible at M_i remains feasible at M_i'; the anchor can replicate it, and optimization can only improve on replication. Where differentiable, $\partial\Pi^*/\partial M_i \geq 0$.

Condition CS (Capability Scarcity). For every active anchor, either $\eta_i > 0$(the multiplier on constraint (7)), or there exists at least one excluded supplier whose inclusion would generate strictly positive net coordination gain if additional capability became available.

Corollary A.6.1 (Strict monotonicity in capability). Under Condition CS, $\Pi^*(M_i, I)$is strictly increasing in $M_i$at the threshold: $\Pi^*(M_i', I) > \Pi^*(M_i, I)$for $M_i' > M_i$; where differentiable, $\Pi_M^*(M^*, I) > 0$.

Proof. Under Condition CS, additional capability either relaxes a binding constraint — raising effort along existing relationships with strictly positive marginal value η_i— or permits the profitable inclusion of at least one additional supplier. In either case the intensive or extensive margin yields a strict profit gain on top of the replication argument of Proposition A.6.1.

Lemma A.6.1 (Monotonicity in anchor density). $\Pi^*(M, I)$ is strictly decreasing in I: for $I' > I$, $\Pi^*(M, I') < \Pi^*(M, I)$; where differentiable, $\Pi_I^*(M, I) < 0$.

Proof. Since $\upsilon'(I) < 0$, $\upsilon(I') < \upsilon(I)$. Let $\left(e_{I'}^*, N_{I'}^*\right)$ denote the optimal allocation at density I'. Because υenters profit positively, $\Pi(M, I'; e_{I'}^*, N_{I'}^*) < \Pi(M, I; e_{I'}^*, N_{I'}^*)$; and since (e_I^*, N_I^*) is optimal at I, $\Pi(M, I; e_{I'}, N_{I'}) \leq \Pi^*(M, I)$. Combining, $\Pi^*(M, I') < \Pi^*(M, I)$.

A.7 The Capability Threshold and Anchor Density

Corollary A.7.1 (Sign of equation (15)). Under Condition CS, and at points of differentiability of Π^*, applying the Implicit Function Theorem to the zero-profit condition $\Pi^*(M^*, I) = 0$ yields $\Pi_M^* \frac{dM^*}{dI} + \Pi_I^* = 0$, so that

$$\frac{dM^*}{dI} = -\frac{\Pi_I^*(M^*, I)}{\Pi_M^*(M^*, I)} > 0,$$

where the numerator is strictly negative by Lemma A.6.1 and the denominator strictly positive by Corollary A.6.1. Greater anchor density therefore raises the capability threshold required for profitable anchor entry.

A.8 Existence and Uniqueness of the Decentralized Equilibrium —

A.8.1 Definitions and regularity. Throughout this section, $\Pi^*(M, I)$denotes the value of the reduced anchor problem conditional on active operation: after optimal factor choice, constant returns and competitive pricing imply $P_i Y_i - wL_i - rK_i = 0$ conditional on an equilibrium scale (Euler's theorem; Section 2.4, footnote 12), so that, consistent with equation (12),

$$\Pi^*(M, I) = \max_{\substack{N_i \geq 1 \\ e_i \in \mathcal{E}_i(M)}} \left\{ \sum_{j \in N_i} \upsilon(I)\, p_{ij}(Y_{ij}(e_{ij}) - Y_{ij}^0) - \frac{\lambda(N_i)}{2} \sum_{j \in N_i} e_{ij}^2 - c_0 \right\}.$$

The maximization is over the compact set $\mathcal{E}_i(M)$ and, by (R4) below, a finite network range; factor choices have been eliminated. By Lemma A.3.1, active operation entails $e_{ij}^* > 0$ for every included partner, so the appropriated revenue increment is strictly positive at the optimum. The following conditions are in force:

(R1) $F_s$is continuously differentiable with strictly positive density on its support $[1, \bar{M})$, $\bar{M} \leq \infty$.

(R2) $\upsilon(\cdot)$is continuous and strictly decreasing on $[0,1)$with $\upsilon(I) \to 0$as $I \to 1^-$(satisfied by $\upsilon(I) = \upsilon_0(1-I)^\kappa$).

(R3) $c_0 > 0$.

(R4) (Finite networks.) For every compact capability set $\mathcal{M}$, there exists a finite $\bar{N}_{\mathcal{M}}$such that the incremental profit from adding a relationship is strictly negative for all $M \in \mathcal{M}$, $I \in [0,1)$, and $N \geq \bar{N}_{\mathcal{M}}$.

Remark on (R4). Condition (R4) is economically consistent with the maintained specification. Partner selection proceeds assortatively (consistent with the matching pattern of Appendix C), so lower-ranked partners have lower absorptive capacity and baseline productivity; since $\theta(M_j) = \bar{\theta}(1 - e^{-\rho(M_j - 1)})$ vanishes at the lower support, the gain from the marginal relationship declines as the pool is worked down, while inclusion continues to impose the strictly positive congestion externality $\frac{\lambda(N+1)-\lambda(N)}{2}\sum_{j'} e_{ij'}^2$ on inframarginal relationships. Rising congestion alone does not suffice, since effort per relationship may shrink as networks expand; it is the interaction of congestion with pool composition that bounds equilibrium networks. Appendix E.1 verifies that the network bound is slack over the reported parameter domain.

Definition A.8.1 (Interior equilibrium). A pair (M^*, I^*) with $I^* \in (0,1)$satisfying the zero-profit condition (13), $\Pi^*(M^*, I^*) = 0$, and the consistency condition (14), $I^* = 1 - F_s(M^*)$.

Definition A.8.2 (No-coordination equilibrium). The aggregate corner configuration $I^* = 0$such that no firm can earn strictly positive profit by entering as an anchor at ceiling appropriability: $\Pi^*(M, 0) \leq 0$ for every Min the support of F_s. Every firm produces at baseline and $Y_s^* = Y_s^0$. (The qualifier "aggregate" reflects that a measure-zero set of highest-capability firms may be indifferent without affecting $I^* = 0$.)

A.8.2 Scalar reduction. By (R1), $F_s$is strictly increasing, so (14) inverts: define the marginal-anchor capability $M(I) \equiv F_s^{-1}(1 - I)$, continuous and strictly decreasing on $(0, 1)$ with $M(I) \to \bar{M}$as $I \to 0^+$, and the marginal-anchor profit function

$$\psi(I) \equiv \Pi^*(M(I), I). \qquad \text{(A.9)}$$

A pair (M^*, I^*) with $I^* \in (0,1)$is an interior equilibrium if and only if $\psi(I^*) = 0$ and $M^* = M(I^*)$. The economy is coordination-viable if $\limsup_{I \to 0^+} \psi(I) > 0$.

Theorem A.8.1 (Existence and uniqueness of the decentralized equilibrium). Under (R1)–(R4):

(i) If the economy is coordination-viable, there exists a unique interior equilibrium (M^*, I^*), with $I^* \in (0,1)$and $M^* = M(I^*)$.

(ii) If the economy is not coordination-viable, no equilibrium with positive anchor density exists, and the unique aggregate equilibrium is the no-coordination equilibrium of Definition A.8.2.

Proof. The argument proceeds in five steps.

Step 1 (Local continuity of ψ). Fix $I_0 \in (0,1)$ and a compact neighborhood $K \subset (0,1)$. On K, $M(I)$ranges over a compact set $\mathcal{M}_K$, so by (R4) the network search is confined to $\{1, \dots, \bar{N}_{\mathcal{M}_K}\}$. For each such N, the value function $V_N(M, I)$of the reduced problem at fixed network size is continuous in $(M, I)$by the theorem of the maximum — the objective is continuous, and the constraint set $\mathcal{E}_i(M)$is compact-valued and continuous in M— with continuity in Ithrough $\upsilon(I)$by (R2). $\Pi^* = \max_N V_N$ is a maximum of finitely many continuous functions, hence continuous; composing with the continuous $M(I)$, $\psi$is continuous on K, hence on $(0, 1)$.

Step 2 (Strict monotonicity of ψ). For $I_2 > I_1$ in $(0, 1)$:

$$\psi(I_2) = \Pi^*(M(I_2), I_2) \leq \Pi^*(M(I_1), I_2) < \Pi^*(M(I_1), I_1) = \psi(I_1).$$

The first inequality applies Proposition A.6.1 (weak monotonicity in capability) with $M(I_2) < M(I_1)$. The second applies Lemma A.6.1, whose strictness is valid here because $\Pi^*$is conditional on active operation: by Lemma A.3.1 every included partner receives $e_{ij}^* > 0$, so $\upsilon(I)$ multiplies a strictly positive appropriated increment at the optimum, and a fall in υstrictly reduces value. Hence $\psi$is strictly decreasing on $(0, 1)$; only weak capability-monotonicity is used, and Condition CS is not invoked. Since $\psi$is strictly decreasing, its one-sided endpoint limits exist in the extended reals; hereafter we write $\lim_{I\to 0^+} \psi(I)$ and $\lim_{I\to 1^-} \psi(I)$, and coordination viability is equivalently $\lim_{I\to 0^+} \psi(I) > 0$.

Step 3 (Boundary behavior as $I \to 1^-$). By the reduced problem of A.8.1, profit equals appropriated revenue increments net of nonnegative coordination costs and c_0; hence

$$\Pi^*(M, I) \leq \upsilon(I)\, B(M) - c_0,$$

where $B(M) < \infty$is the finite upper bound on the aggregate revenue increment over the endogenous network implied by (R4) and the effort bound of constraint (7). As $I \to 1^-$, $M(I) \to 1$so $B(M(I))$is bounded, and $\upsilon(I) \to 0$by (R2); hence $\lim_{I\to 1^-} \psi(I) \leq -c_0 < 0$by (R3).

Step 4 (Case (i): existence and uniqueness of the interior equilibrium). Under viability, $\psi$is continuous (Step 1) and strictly decreasing (Step 2) on $(0, 1)$, with $\lim_{I\to 0^+} \psi(I) > 0$ and $\lim_{I\to 1^-} \psi(I) < 0$ (Step 3). By the intermediate value theorem a root I^*exists; by strict monotonicity it is unique. $M^* = M(I^*)$is then unique by strict monotonicity of $F_s$under (R1).

Step 5 (Case (ii): the corner is the unique equilibrium). Suppose $\lim_{I\to 0^+} \psi(I) \leq 0$. First, no interior equilibrium exists: by strict monotonicity, $\psi(I) < 0$ for every $I \in (0,1)$, so at any positive density the marginal anchor earns strictly negative profit and (13) cannot hold. Second, the corner

satisfies Definition A.8.2. Fix any Min the support of F_s. Since $M(I) \to \bar{M}$ as $I \to 0^+$, there exists $\bar{I} > 0$ such that $M(I) > M$ for all $I \in (0, \bar{I})$; by Proposition A.6.1,

$$\Pi^*(M, I) \leq \Pi^*(M(I), I) = \psi(I) \text{ for all } I \in (0, \bar{I}).$$

Taking $I \to 0^+$and using continuity of $\Pi^*(M,\cdot)$at $I = 0$— which holds by the argument of Step 1 with Mfixed, since $\upsilon$is continuous at 0by (R2) —

$$\Pi^*(M, 0) = \lim_{I \to 0^+} \Pi^*(M, I) \leq \lim_{I \to 0^+} \psi(I) \leq 0.$$

Since Mwas arbitrary, no firm in the support can profit from entry at ceiling appropriability; the no-coordination configuration is an equilibrium, and by the first part it is the only one.

Remark A.8.1 (dichotomy). Every economy satisfies exactly one of the two cases: viability or its negation partitions the parameter space, and the theorem assigns a unique equilibrium to each. Case (ii) is precisely the incentive-failure configuration of equation (25) in Section 6.2, realized along the capability sweep as the shaded region of Figure 2 where $I^* = 0$.

Remark A.8.2 (the economics of uniqueness). Uniqueness reflects two reinforcing negative-feedback channels. An increase in anchor density lowers the capability of the marginal entrant through $M(I) = F_s^{-1}(1 - I)$ and reduces appropriability through $\upsilon'(I) < 0$. Marginal-anchor profit therefore declines strictly with entry, and the zero-profit condition is crossed at most once. Equation (15) is the local expression of this feedback.

Remark A.8.3 (role of c_0). (R3) enters only in Step 3. When $c_0 = 0$, strict monotonicity is unaffected, so uniqueness is preserved: at most one interior root exists. Existence under viability additionally requires $\psi(\bar{I}) < 0$at some $\bar{I} \in (0,1)$, a condition that must be verified from the remaining primitives rather than following from them. Thus, at $c_0 = 0$, uniqueness is preserved while existence requires an additional interior negativity condition.

Remark A.8.4 (what the theorem licenses downstream). Theorem A.8.1 establishes that I^*, $W_s(I)$, and Δ are uniquely determined by the model's primitives: Δ is a well-defined diagnostic evaluated at the decentralized equilibrium, the comparative statics of Appendix B are statements about functions rather than correspondences, and every counterfactual in Section 7 is a comparison of two well-defined objects. Proposition 1 and the implementation results of Appendix D presuppose exactly this.

APPENDIX B — COMPARATIVE STATICS

Scope

Throughout this appendix, comparative statics are firm-level. Anchor density I— and therefore the appropriability level $\upsilon(I)$, except in Section B.4 where appropriability is itself the experiment — is held fixed, as are prices. Equilibrium adjustments in Ioperate through Theorem A.8.1 and equation (15) and are treated in the body.

All results condition on active operation. Existence and uniqueness of the optimal effort allocation for a fixed network are supplied by Appendix A. Because $N_i$is integer-valued, network comparative statics are stated as weak monotonicity rather than derivatives. Strict changes in network breadth occur only when a parameter variation crosses an inclusion threshold.

For a fixed network size N, let $V_N(x)$ denote the optimized value of the anchor problem when the parameter under consideration is x, after optimizing over the effort vector. Define the optimized inclusion differential

$$D_N(x) \equiv V_{N+1}(x) - V_N(x).$$

The optimal network is $N_i^*(x) \in \arg\max_{N\geq 1} V_N(x)$.

The following condition is used only for network comparative statics.

Condition NB (Ordered network margin). For a parameter xthat raises coordination returns, $D_N(x)$is weakly increasing in xfor every N. For a parameter xthat raises coordination costs, $D_N(x)$is weakly decreasing in xfor every N.

Condition NB states that the parameter shifts the optimized gain from an additional relationship in the same direction at every candidate network size. It is stronger than a pointwise shift of equation (11), because both $V_N$and V_{N+1}incorporate the reoptimization of all inframarginal efforts. Equivalently, $V_N(x)$has increasing differences in $(N, x)$when xraises coordination returns and decreasing differences when xraises coordination costs. Under the uniqueness result of Appendix A — or, at ties, selection of the largest maximizing network — the optimal network inherits the corresponding weak monotonicity. Without a tie convention, the correspondence of maximizers is monotone, but an arbitrarily chosen member of the correspondence need not be; because Appendix A supplies uniqueness except at isolated inclusion thresholds, this is a selection convention rather than an additional assumption.

For individual-effort comparative statics, let $H_i$denote the Hessian of the fixed-network effort problem evaluated at the optimum, and write $A_i \equiv -H_i$. Appendix A establishes that $H_i$is negative definite under the maintained concavity condition. The following condition provides the sign of the vector response when the parameter enters the objective directly.

Condition EM (Monotone effort response). For the parameter experiment considered, the vector $A_i^{-1}g_x$is componentwise weakly positive when xraises marginal coordination returns and componentwise weakly negative when xraises marginal coordination costs, where $g_x$is the vector of direct parameter derivatives of the effort first-order conditions.

Condition EM is automatically satisfied in the one-relationship case. With multiple relationships, it rules out cross-relationship effects strong enough to overturn the direct marginal-return effect.

Remark B.0.1 (Condition EM holds under the maintained specification). Under the joint-concavity criterion of Proposition A.5.1, $A_i = D - a^2 G\, xx^\top$ is symmetric positive definite with non-positive off-diagonal entries — a Stieltjes matrix — so A_i^{-1} is entrywise non-negative; it is entrywise strictly positive when A_i is irreducible, as it is for an active network with strictly positive cross-relationship complementarities. Hence $A_i^{-1} g_x$ is componentwise positive for any non-negative, nonzero g_x, and Condition EM holds automatically wherever criterion (A.8) holds — in particular, whenever $N_i \phi \delta \leq 1$, and therefore throughout the computational domain of Appendix E. Economically, coordination efforts are complements (Remark A.5.1), so a positive direct shock to the marginal return of any relationship weakly raises effort on all relationships. Thus, wherever the joint-concavity criterion holds, under the maintained Hessian structure, Condition EM is automatically satisfied. Condition NB is therefore the only additional condition required for the network comparative statics.

B.1 Organizational Capability

Proposition B.1.1 (Organizational capability). Holding I fixed:

(i) the optimized anchor value $\Pi^*(M_i, I)$ is weakly increasing in M_i, and strictly increasing under Condition CS;

(ii) for a fixed network N_i, total coordination effort $E_i^*(M_i) \equiv \sum_{j \in N_i} e_{ij}^*(M_i)$ is weakly increasing in M_i; where the capability constraint binds locally, $E_i^*(M_i) = M_i$;

(iii) for a fixed network N_i, with the network locally unchanged: if the capability constraint binds, every included relationship's effort rises strictly with M_i— specifically,

$$\frac{\partial e_i^*}{\partial M_i} = \frac{A_i^{-1}\mathbf{1}}{\mathbf{1}^\top A_i^{-1}\mathbf{1}} > 0 \text{ componentwise;}$$

if the constraint is slack, a marginal increase in M_i leaves the effort allocation unchanged;

(iv) under Condition NB, $N_i^*(M_i)$ is weakly increasing in M_i; the increase is strict whenever the capability change crosses an inclusion threshold.

Proof. For $M_i' > M_i$, the capability constraint $\sum_{j \in N_i} e_{ij} \leq M_i$ implies $\mathcal{E}_i(M_i) \subseteq \mathcal{E}_i(M_i')$. Every effort allocation feasible at M_i is therefore feasible at M_i'; replicating the old allocation gives $\Pi^*(M_i', I) \geq \Pi^*(M_i, I)$. Strictness under Condition CS is established by Proposition A.6.1 and Corollary A.6.1.

For a fixed network, if the capability constraint binds in a neighborhood of M_i, complementary slackness gives $\sum_{j \in N_i} e_{ij}^*(M_i) = M_i$; total coordination effort rises one-for-one with capability locally. If the constraint is slack, additional capability need not change total effort.

For part (iii), fix the network locally. Capability enters the reduced problem only through the constraint, so the direct parameter derivative of each first-order condition is zero. In the binding regime, differentiating the first-order system gives $H_i\, de_i^* - \mathbf{1}\, d\eta_i = 0$, and differentiating the constraint gives $\mathbf{1}^\top de_i^* = dM_i$. Since $H_i = -A_i$, the first equation yields $de_i^* = -A_i^{-1}\mathbf{1}\, d\eta_i$; substituting into the constraint, $d\eta_i = -dM_i/(\mathbf{1}^\top A_i^{-1}\mathbf{1})$, and therefore

$$\frac{\partial e_i^*}{\partial M_i} = \frac{A_i^{-1}\mathbf{1}}{\mathbf{1}^\top A_i^{-1}\mathbf{1}},$$

which is componentwise strictly positive by Remark B.0.1 and has components summing to one: a marginal unit of capability is allocated across all included relationships in strictly positive shares determined by the complementarity structure. The associated multiplier response is $\partial \eta_i / \partial M_i = -1/(\mathbf{1}^\top A_i^{-1}\mathbf{1}) < 0$, consistent with the shadow-value logic of Remark B.1.1. In the slack regime, $\eta_i = 0$ locally and M_i appears nowhere in the first-order system, so the effort allocation is locally invariant.

Finally, under Condition NB, the optimized inclusion differential $D_N(M_i) = V_{N+1}(M_i) - V_N(M_i)$ is weakly increasing in capability. By the monotone-choice reasoning stated with Condition NB, the optimal integer network is weakly increasing in M_i, with a strict increase only when some $D_N(M_i)$crosses zero.

Remark B.1.1 (Envelope identity). In the reduced anchor problem of Appendix A.8, where $M_i$enters through the capability constraint, the envelope theorem gives, wherever the value function is differentiable, $\partial \Pi^* / \partial M_i = \eta_i$. Thus the marginal private value of organizational capability equals the shadow value of the capability constraint. If an unreduced formulation retains an additional direct M_i-dependence in the objective, the envelope derivative also includes that direct term.

B.2 Partner Absorptive Capacity

Consider first an increase in the absorptive capacity of a particular partner j, holding the characteristics of all other partners fixed.

Proposition B.2.1 (Partner absorptive capacity). Holding (I, M_i) fixed — parts (i)–(iii) for a fixed network N_i, part (iv) for the endogenous network choice:

(i) the optimized value is weakly increasing in θ_{ij}, and strictly increasing when relationship ijreceives positive effort;

(ii) if the capability constraint is slack, then under Condition EM, $\partial e_{ij}^* / \partial \theta_{ij} > 0$;

(iii) if the capability constraint binds, higher θ_{ij} raises the relative return to effort on relationship ij, but total effort remains fixed at M_i; consequently, effort may be reallocated from other relationships toward j;

(iv) a rise in a single partner's θ_{ij} does not by itself imply an expansion of N_i^*. For a common upward shift in absorptive capacity across the potential partner pool, N_i^* is weakly increasing under Condition NB, with strict expansion only when an inclusion threshold is crossed.

Proof. For any fixed positive effort, $q_{ij}(e_{ij}) = q_{ij}^0(1 + \theta_{ij}e_{ij})^\delta$ is increasing in θ_{ij}, and $\partial q_{ij} / \partial e_{ij} = q_{ij}^0 \delta \theta_{ij}(1 + \theta_{ij}e_{ij})^{\delta-1}$ has a positive direct response to θ_{ij}over the maintained parameter domain. Thus an increase in θ_{ij}raises the return generated by any given positive effort on relationship ij. Replicating the old allocation yields $\Pi^*(\theta_{ij}') \geq \Pi^*(\theta_{ij})$for $\theta_{ij}' > \theta_{ij}$, with strict inequality when the old optimum assigns positive effort to j.

When the capability constraint is slack, the multiplier remains locally zero; differentiating the full effort system and applying Condition EM gives a positive own-effort response.

When the capability constraint binds, $\sum_{k\in N_i} e_{ik}^{*} = M_i$, so $\sum_{k\in N_i} \partial\, e_{ik}^{*} / \partial\theta_{ij} = 0$. A strict increase in e_{ij}^{*}must therefore be accompanied by a weak decrease in effort assigned to at least one other relationship. The comparative static is an effort-reallocation result, not an increase in every component.

For network breadth, improving a single existing partner may increase concentration rather than breadth. Under a common upward shift of the partner pool and Condition NB, however, $D_N(\theta') \geq D_N(\theta)$ for every N, which implies weakly broader optimal networks.

Remark B.2.1. The complementarity between anchor capability and partner absorptive capacity, and the positive assortative matching pattern used to order the partner pool, are established in Appendix C.

B.3 Coordination Congestion

Let $\lambda(N;\gamma)$denote a congestion schedule indexed by a scalar parameter γ. An upward shift satisfies $\partial\lambda(N;\gamma)/\partial\gamma > 0$ for every N. For network comparative statics, the maintained experiment also satisfies

$$\frac{\partial}{\partial\gamma}[\lambda(N+1;\gamma) - \lambda(N;\gamma)] \geq 0.$$

Under the quadratic specification $\lambda(N) = \lambda_0 + \lambda_1 N^2$, an increase in λ_0 raises the level of direct effort costs, while an increase in $\lambda_1$raises both the level and the discrete slope of congestion.

Proposition B.3.1 (Coordination congestion). Holding (I, M_i)fixed — parts (i)–(iii) for a fixed network N_i, part (iv) for the endogenous network choice:

(i) the optimized anchor value is weakly decreasing under a pointwise upward shift of the congestion schedule;

(ii) if the capability constraint is slack, individual efforts are weakly decreasing under Condition EM;

(iii) if the capability constraint binds, a higher congestion schedule weakly lowers the value of coordination but may also induce effort reallocation or network contraction; a strict decline in every retained relationship's effort is not implied without additional restrictions;

(iv) under Condition NB, $N_i^{*}$is weakly decreasing in the congestion parameter; the decline is strict when the shift crosses an exclusion threshold.

Proof. Let $\gamma' > \gamma$. For every feasible allocation,

$$-\frac{\lambda(N_i;\gamma')}{2}\sum_{j\in N_i} e_{ij}^2 \leq -\frac{\lambda(N_i;\gamma)}{2}\sum_{j\in N_i} e_{ij}^2.$$

Evaluating the new objective at the old optimum and then maximizing gives $\Pi^{*}(\gamma') \leq \Pi^{*}(\gamma)$.

For a fixed network and a slack capability constraint, the direct parameter derivative of each effort first-order condition is negative; Condition EM therefore implies a componentwise weak decline in effort.

When the capability constraint binds, congestion may induce the anchor to reduce network breadth and concentrate the remaining capability among fewer relationships. The value effect remains negative, but the sign of every retained effort need not be negative.

For network breadth, Condition NB implies that $D_N(\gamma) = V_{N+1}(\gamma) - V_N(\gamma)$ is weakly decreasing in congestion. Therefore the maximizing integer network is weakly decreasing in γ, with strict contraction only when an optimized inclusion differential crosses zero.

B.4 Appropriability

This comparative static appears as CS4 in Section 4.5 and complements the results reported there. It also supplies the firm-level response to the appropriability level required by Proposition 2(ii); its equilibrium implications are assembled in Appendix D.6.

The experiment is a parallel upward shift in the appropriability schedule: a higher level υat a given anchor density I. In equilibrium, this corresponds to an increase in the institutional ceiling υ_0, as in Figure 1, Panel A.

Proposition B.4.1 (Appropriability). Holding (I, M_i)fixed — parts (ii)–(iii) for a fixed network N_i:

(i) the optimized anchor value, allowing for the endogenous network choice, is strictly increasing in υ; wherever differentiable, its derivative is given by the envelope formula below;

(ii) if the capability constraint is slack, individual relationship efforts are weakly increasing under Condition EM, and strictly increasing wherever the corresponding direct response is strictly positive;

(iii) if the capability constraint binds, total effort remains equal to M_i, so higher appropriability changes the allocation of effort across relationships rather than increasing every effort;

(iv) under Condition NB, $N_i^*$is weakly increasing in υ, with strict expansion only when an inclusion threshold is crossed.

Proof. The reduced anchor objective contains the appropriated revenue increment $\upsilon \sum_{j \in N_i} p_{ij} [Y_{ij}(e_{ij}) - Y_{ij}^0]$. Where $\Pi^*$is differentiable — equivalently, away from a network-switching threshold — the envelope theorem gives:

$$\frac{\partial \Pi^*}{\partial \upsilon} = \sum_{j \in N_i^*} p_{ij} [Y_{ij}(e_{ij}^*) - Y_{ij}^0] > 0,$$

strictly positive because, under active operation, every included relationship receives strictly positive effort by Lemma A.3.1 and therefore generates a strictly positive revenue increment. At a network-switching threshold, N_i^*can jump and Π^*may have a kink; the same replication argument as in B.2 — the old allocation remains feasible and its appropriated increment is strictly more valuable at higher υ— establishes strict monotonicity of Π^*in υglobally, although the ordinary derivative need not exist exactly at the switch.

For a fixed network and a slack capability constraint, higher υraises the direct marginal return to coordination effort; Condition EM then implies a componentwise weak increase in the optimal effort vector.

If the capability constraint binds, $\sum_{j \in N_i} e_{ij}^{*} = M_i$, and therefore $\sum_{j \in N_i} \partial\, e_{ij}^{*} / \partial \upsilon = 0$. It is consequently impossible for every included effort to rise strictly. Higher appropriability instead reallocates capability toward relationships with larger appropriated returns and may alter network breadth.

For the discrete network choice, Condition NB implies that the optimized incremental value $D_N(\upsilon) = V_{N+1}(\upsilon) - V_N(\upsilon)$ is weakly increasing in appropriability. Thus $N_i^{*}(\upsilon)$ is weakly increasing, with a strict increase only when $D_N(\upsilon)$ crosses zero.

Remark B.4.1. The direct firm-level effect required in Appendix D.6 is therefore unambiguous: $\Pi^{*}$is strictly increasing in υ, with the envelope formula giving its derivative wherever differentiable. The effects on the composition and breadth of coordination depend on whether the capability constraint binds and, for network breadth, on Condition NB. Appendix D.6 combines this direct profitability effect — as a monotonicity statement, not a derivative assumed to exist at every υ— with the equilibrium response of anchor density.

APPENDIX C — MATCHING

Scope. This appendix establishes the results invoked by CS2 and the maintained frictionless-assignment assumption stated with CS2 in Section 4.5: the ranking of partners by absorptive capacity, and the complementarity between organizational capability and absorptive capacity that generates positive assortative matching. Throughout, (I, M_i) are held fixed and the network is fixed at $N_i$with strictly positive efforts (Lemma A.3.1). Define the net coordination gain from including partner j,

$$S_{ij} \equiv P_i[Y_i(N_i + 1) - Y_i(N_i)] + \upsilon(I)\, p_{ij}\, Y_{ij}^0[(1 + \theta_{ij}e_{ij})^{\delta} - 1] - \frac{\lambda(N_i + 1)}{2} e_{ij}^2$$

the gain gross of the congestion externality imposed on inframarginal relationships. The externality — the right-hand side of equation (11) — is common to every candidate for a given network slot and therefore does not affect the ranking of candidates; it governs network breadth, which is treated through the optimized differential D_N of Appendix B.

C.1 Partner Selection and Absorptive Capacity

Proposition C.1.1 (Ranking by absorptive capacity). Holding the effort profile fixed at any strictly positive level, and holding baseline productivity fixed across candidate partners so as to isolate the absorptive-capacity channel, $S_{ij}$is strictly increasing in θ_{ij}. Where the capability constraint is slack, the same holds for the optimized gain $S_{ij}(e_{ij}^*)$. The gain-maximizing ordering of candidate partners is therefore in decreasing order of θ_{ij}; the marginal partner is the one with the lowest θ_{ij} for which the inclusion condition (11) is satisfied. In the maintained specification, both baseline productivity q_{ij}^0 and absorptive capacity θ_{ij}increase with partner capability M_j; since S_{ij} is increasing in q_{ij}^0 as well — both revenue terms scale with baseline output while the cost term does not — the net coordination gain at a fixed positive effort profile is strictly increasing in $M_j$through both channels. By the slot-comparison logic of Remark C.1.1, this fixed-profile ranking is the comparison relevant for partner selection.

Proof. At a fixed strictly positive effort e_{ij}, differentiating $S_{ij}$with respect to θ_{ij}:

$$\frac{\partial S_{ij}}{\partial \theta_{ij}} = P_i \frac{\partial[Y_i(N_i + 1) - Y_i(N_i)]}{\partial \theta_{ij}} + \upsilon(I)\, p_{ij}\, Y_{ij}^0\, \delta(1 + \theta_{ij}e_{ij})^{\delta - 1} e_{ij}.$$

The first term is strictly positive: higher θ_{ij} raises $q_{ij}(e_{ij}) = q_{ij}^0(1 + \theta_{ij}e_{ij})^{\delta}$ at any positive effort, which raises anchor TFP through equation (2) and hence the TFP gain from inclusion. The second term is strictly positive since $\upsilon(I), p_{ij}, Y_{ij}^0, \delta, e_{ij} > 0$. Hence $\partial S_{ij} / \partial \theta_{ij} > 0$ at any fixed positive effort profile. Where the capability constraint is slack, and holding the remaining effort profile fixed, $e_{ij}^*$is an interior maximizer; by the envelope theorem, the indirect effect through the optimal adjustment of e_{ij}^*vanishes, so the optimized gain inherits the same strict monotonicity.

Remark C.1.1 (binding regime). When the capability constraint binds, the envelope step does not apply, and the optimized gain may respond to θ_{ij} through effort reallocation across relationships (Proposition B.2.1(iii)). The fixed-profile ranking of Proposition C.1.1 is unaffected: at any common effort level, a higher-θ partner generates a strictly larger net coordination gain, which is the comparison relevant for filling a given network slot.

C.2 Complementarity and Assortative Matching

Proposition C.2.1 (Supermodularity). Holding the effort profile fixed at any strictly positive level, the net coordination gain is supermodular in (M_i, θ_{ij}):

$$\frac{\partial^2 S_{ij}}{\partial M_i \, \partial \theta_{ij}} > 0.$$

Proof. At fixed efforts, only the TFP gain depends on M_i. From equation (2), $A_i = M_i^{\alpha} \prod_{j' \in J_i} q_{ij'}^{\phi}$, so

$$\frac{\partial^2 A_i}{\partial M_i \, \partial q_{ij}} = \frac{\partial}{\partial M_i}\left[\phi \frac{A_i}{q_{ij}}\right] = \alpha\phi \, M_i^{\alpha-1} \frac{\prod_{j'} q_{ij'}^{\phi}}{q_{ij}} > 0,$$

since $\alpha, \phi, M_i, q_{ij} > 0$. Because $q_{ij}(e_{ij}) = q_{ij}^0(1 + \theta_{ij} e_{ij})^{\delta}$ is strictly increasing in θ_{ij} at any fixed positive effort, the chain rule gives $\partial^2 A_i / \partial M_i \, \partial \theta_{ij} > 0$, and hence

$$\frac{\partial^2 [Y_i(N_i + 1) - Y_i(N_i)]}{\partial M_i \, \partial \theta_{ij}} > 0.$$

The remaining terms of $S_{ij}$are affine — indeed constant — in M_i at fixed efforts, so the displayed cross-partial signs the full expression.

Remark C.2.1 (endogenous-effort reinforcement). Allowing efforts to adjust reinforces the complementarity: in the binding regime, higher capability strictly raises effort on every included relationship (Proposition B.1.1(iii)), and more intensive coordination generates larger returns when directed at high-θ_{ij} partners. The fixed-profile supermodularity of Proposition C.2.1 is the primitive result; the endogenous-effort channel operates in the same direction.

Remark C.2.2 (assortative matching). Supermodularity of the net coordination gain in (M_i, θ_{ij}) implies that high-capability anchors value high-absorptive-capacity partners disproportionately more than low-capability anchors do. Under the maintained assumption stated with CS2 in Section 4.5 that matching frictions do not overturn gain-maximizing assignments, this complementarity generates positive assortative matching in the sense of Becker (1973): high-capability anchors sort toward high-θ_{ij} partners. This is the matching pattern invoked in CS2, in Appendix B's ordering of the partner pool, and in the Remark on condition (R4) of Appendix A.8.

Economic interpretation. Proposition C.2.1 identifies the primitive source of complementarity: under the multiplicative TFP structure, organizational capability amplifies the productivity return to partner absorptive capacity. Proposition B.1.1 shows that endogenous effort adjustments reinforce this complementarity in the binding regime. Together these channels provide the economic foundation for the positive assortative matching result of Remark C.2.2.

APPENDIX D — PLANNER AND DECENTRALIZATION

Scope. This appendix supports Sections 5 and 6. Section D.1 characterizes the planner's problem and establishes existence of an interior constrained optimum; D.2 and D.3 establish the decentralization results (Lemmas 1 and 2); D.4 proves Proposition 1; D.5 characterizes the entry-policy regimes; D.6 and D.7 prove Propositions 2 and 3. Throughout, the foundations of Appendix A — existence, uniqueness, joint concavity, profit monotonicity, and Theorem A.8.1 — and the comparative statics of Appendix B are used without restatement.

D.1 Planner Setup and Existence of the Interior Optimum

D.1.1 Planner's objective. Section 5.4 establishes that total net social value under the decentralized equilibrium at candidate capability threshold M is

$$W_s(M) = Y_s^0 + \Gamma_A(M) + \Gamma_P(M) - C(M), \qquad (D.1)$$

where $Y_s^0 \equiv \sum_k p_k Y_k^0$ is aggregate baseline output, independent of M;

$$\Gamma_A(M) \equiv \sum_{M_i \geq M} p_i (Y_i - Y_i^0), \Gamma_P(M) \equiv \sum_{M_i \geq M} \sum_{j \in N_i^*} p_{ij} (Y_{ij}(e_{ij}^*) - Y_{ij}^0)$$

are aggregate coordination-generated gains to anchors and partners; and $C(M) \equiv \sum_{M_i \geq M} \frac{\lambda(N_i^*)}{2} \sum_{j \in N_i^*} (e_{ij}^*)^2$ is aggregate coordination cost. All three are evaluated at the policy-induced decentralized equilibrium $\left(e_{ij}^{**}(M), N_i^{**}(M)\right)$— inclusive of $\sigma^*$and τ_{ij}^*— with this dependence suppressed notationally. Since Y_s^0 is independent of M, maximizing $W_s$is equivalent to maximizing $\Gamma_A + \Gamma_P - C$. Writing $W_s(I) \equiv W_s(M(I))$ with $M(I) = F_s^{-1}(1 - I)$, the planner solves $I^{**} = \arg\max_{I \in [0,1]} W_s(I)$; the feasible set follows from $I = 1 - F_s(M)$ and the normalization $M_{min} = 1$.

D.1.2 Derivation of Δ. The problem is stated in $M$and differentiated in Ivia $I = 1 - F_s(M)$; since $F_s$is continuously differentiable with density f_s(condition (R1) of Appendix A.8), $dM/dI = -1/f_s(M)$, a strictly monotone differentiable reparametrization. Differentiating (D.1), $dW_s/dI = d\Gamma_A/dI + d\Gamma_P/dI - dC/dI$.

Regularity of the equilibrium mapping. $F_s(\cdot)$, $\upsilon(\cdot)$, $\theta(\cdot)$, and the production functions are continuously differentiable. Equilibrium effort $e_{ij}^*(I)$ is continuously differentiable within each regime: under a slack capability constraint, by the implicit function theorem applied to equation 12. $N_i^*(I)$ solves an integer-valued problem (equation (11)) and is piecewise constant in I, with discrete jumps at inclusion thresholds. The derivation is therefore local: on any interval over which the regime and $N_i^*(I)$ are constant, Γ_A, Γ_P, and $C$are continuously differentiable in Ithrough $e_{ij}^*(I)$ alone; at the finitely many switching values, $W_s(I)$ is continuous but only piecewise continuously differentiable. $\Delta$is evaluated at I^*, which — except on a measure-zero set of parameter values — lies in the interior of one such region.

Economic decomposition. Both Γ_A and Γ_P respond to anchor density through two opposing forces. The network expansion effect: each additional anchor establishes new coordination relationships, contributing positively to dW_s/dI. The appropriability erosion effect: additional entry lowers $\upsilon(I)$, weakening coordination incentives — the value of coordination is strictly increasing in the appropriability level (Proposition B.4.1(i)), with effort and inclusion responding on the margins characterized in Appendix B — contributing negatively. Because anchor productivity is generated through coordination relationships, both forces propagate to Γ_A. The coordination-cost effect: within a region of constant N^*, dC/dI operates through the same equilibrium-effort channel and contributes negatively, independent of the erosion effect's sign.

No double counting. Γ_A, Γ_P, and Care distinct functions of the same equilibrium allocation: the first two measure the gross output that equilibrium coordination generates; Cmeasures the real organizational resources it consumes. Consistency of the evaluation point — not cancellation between terms — is what makes the decomposition additive without overlap.

Define the net entry distortion indicator:

$$\Delta \equiv \frac{dW_s}{dI}\Big|_{I^*} = \frac{d\Gamma_A}{dI}\Big|_{I^*} + \frac{d\Gamma_P}{dI}\Big|_{I^*} - \frac{dC}{dI}\Big|_{I^*}$$

the marginal net social value of anchor entry evaluated at the decentralized equilibrium I^*, inclusive of the resource cost of coordination. This is equation (18) of the body.

D.1.3 Existence of an interior constrained optimum.

Proposition D.1.1 (Existence of an Interior Optimum). Under the maintained assumptions, $W_s(I)$attains a maximum on $[0, 1]$, and there exists an interior maximizer $I^{**} \in (0,1)$.

Proof. Part 1 (Existence). $W_s(I)$ is continuous on $[0, 1]$(D.1.2); by the Extreme Value Theorem on the compact interval, a maximizer exists. Part 2 (Interiority). We show $W_s(I^*) > W_s(0) = W_s(1)$. Boundary values. At $I = 1$: the threshold is $M^* = 1$, so the partner set $\{M_j < M^*\}$ is empty; $N_i^* = 0$and $e_{ij}^* = 0$for all i, giving $\Gamma_P(1) = C(1) = 0$; with no partners, $q_{ij} = q_{ij}^0$, so $A_i = A_i^0$ and $\Gamma_A(1) = 0$; hence $W_s(1) = Y_s^0$. At $I = 0$: no firm operates as an anchor, so $\Gamma_A(0) = \Gamma_P(0) = C(0) = 0$ and $W_s(0) = Y_s^0$. Strict dominance of I^*. By Theorem A.8.1, the coordination-viable economy has a unique interior equilibrium $I^* \in (0,1)$ at which active anchors weakly prefer entry and every included relationship generates strictly positive coordination-generated value (Lemma A.3.1). The planner internalizes the full increment $\Gamma_P$rather than the appropriated fraction $\upsilon(I)\Gamma_P$, and Cis evaluated identically in both problems, so $W_s(I^*) > Y_s^0$ strictly whenever $\upsilon(I^*) < 1$, which holds by the incomplete-appropriability assumption of Section 2.1. Hence $W_s(I^*) > Y_s^0 = W_s(0) = W_s(1)$, and the maximizer of Part 1 is interior.

Remark D.1.1 (role of $c_0$and of $\theta(1) = 0$). Condition (R3) of Appendix A.8 supplies $c_0 > 0$; the graceful degradation of this section's boundary logic at $c_0 = 0$ parallels Remark A.8.3. The proof does not use the absorptive-capacity property $\theta(1) = 0$: the argument relies on the endogenous definition of partners and the normalization $M_{min} = 1$, which imply an empty partner set at $I = 1$.

The property $\theta(1) = 0$ retains its economic interpretation but is not a mathematical ingredient here.

Assumption D.1 (Global Single-Peakedness). $W_s(I)$ is continuous on $[0, 1]$, piecewise continuously differentiable on $(0, 1)$— consistent with the piecewise-constant nature of $N^*(I)$established in D.1.2 — and possesses a unique interior maximizer I^{**}. Equivalently, dW_s/dI changes sign exactly once on $(0, 1)$, from positive to negative.

This assumption is not implied by the maintained primitives of Sections 2–4, nor is it in tension with them; it is a genuinely independent restriction on the equilibrium mapping from anchor density to net social value. It is strictly weaker than global concavity: it signs neither d^2W_s/dI^2nor requires differentiability at the finitely many switching points. Under the power-form appropriability of Section 7, the numerical illustrations confirm exactly one interior peak across the calibrated range — precisely what the assumption states, and no more.

Remark D.1.2 (second-order condition). Under Assumption D.1, the single sign change of dW_s/dI directly identifies I^{**}as the unique global maximizer — W_s is increasing on $(0, I^{**})$ and decreasing on $(I^{**}, 1)$ by definition — so no separate second-order verification is required. Section 7 verifies the single sign-change property numerically across the calibrated range.

D.2 Lemma 1 — Inner-Margin Decentralization

D.2.1 The planner's effort problem. The planner maximizes aggregate net social value — internalizing both anchor-firm gains and partner-level productivity improvements, per the welfare criterion of Section 5.1. For given network size and anchor density, the planner's effort problem differs from the private problem in one respect: the planner values the full coordination-generated increment in partner revenue rather than the appropriated fraction $\upsilon(I)$:

$$\max_{\{e_{ij}\},\, K_i,\, L_i} \; P_i Y_i - wL_i - rK_i + \sum_{j \in N_i} p_{ij}\,(Y_{ij} - Y_{ij}^0) - \frac{\lambda(N_i)}{2} \sum_{j \in N_i} e_{ij}^2 \;\text{s.t.} \sum_{j \in N_i} e_{ij} \le M_i.$$

Conditional on optimal factor demands, the indirect effects through K_i^*, L_i^*vanish by the envelope theorem, and the problem reduces to the effort choice with the full increment $\sum_j p_{ij}\,(Y_{ij} - Y_{ij}^0)$ replacing the appropriated share, the capability constraint (7) maintained.

D.2.2 Concavity and sufficiency. The planner's reduced objective values the coordination-generated revenue increment at p_{ij}rather than $\upsilon(I)p_{ij}$; its own-curvature coefficients therefore exceed those of the private objective, and the joint-concavity criterion of Proposition A.5.1 applies a fortiori — in particular, the sufficient condition $N_i\phi\delta \le 1$ covers the planner's problem independently of υ. By Proposition A.5.1(iv), the KKT conditions below are necessary and sufficient, and the maximizing effort vector is unique. Coordinate-wise strict concavity and interiority are Lemmas A.2.2 and A.3.1.

D.2.3 KKT system and the social effort condition. Let $\eta_i^{**} \geq 0$denote the multiplier on (7) at the social optimum. The Lagrangian is

$$\mathcal{L}^S = P_i Y_i + \sum_{j \in N_i} p_{ij} (Y_{ij} - Y_{ij}^0) - \frac{\lambda(N_i)}{2} \sum_{j \in N_i} e_{ij}^2 + \eta_i^{**}(M_i - \sum_{j \in N_i} e_{ij})$$

with stationarity, dual feasibility, primal feasibility, and complementary slackness for each $j \in N_i$. Differentiating yields the conditionally efficient effort condition:

$$P_i \frac{\partial Y_i}{\partial A_i} \frac{\partial A_i}{\partial q_{ij}} q_{ij}^0 \theta_{ij} \delta \left(1 + \theta_{ij} e_{ij}^{**}\right)^{\delta - 1} + p_{ij} Y_{ij}^0 \theta_{ij} \delta \left(1 + \theta_{ij} e_{ij}^{**}\right)^{\delta - 1} = \lambda(N_i) e_{ij}^{**} + . \eta_i^{**}$$

Because partner $j \in N_i$, the marginal coordination cost is $\lambda(N_i)e_{ij}^{**}$, consistent with Remark A.3.1; the parameter $\lambda(N_i + 1)$ appears only in the inclusion comparison of D.2.5.

D.2.4 The externality gap and derivation of σ^*. The private effort condition (10) carries $\upsilon(I)p_{ij}$ where (D.2) carries p_{ij}. Defining the social and private marginal benefits SMB and PMB(I) — identical anchor-TFP channels, full versus appropriated revenue-increment channels — the gap is

$$\text{SMB} - \text{PMB(I)} = (1 - \upsilon(I))\, p_{ij} Y_{ij}^0 \theta_{ij} \delta (1 + \theta_{ij} e_{ij})^{\delta - 1} > 0,$$

Since $\upsilon(I) < 1$by equation (8), the private marginal benefit lies strictly below the social marginal benefit at every effort level and every anchor density, generating underinvestment in coordination effort (Distortion 1). A Pigouvian subsidy paid on the uninternalized portion of the marginal revenue increment yields the subsidized private marginal benefit $\text{PMB}^\sigma = \text{TFP channel} + [\upsilon(I) + \sigma] p_{ij} Y_{ij}^0 \theta_{ij} \delta \left(1 + \theta_{ij} e_{ij}\right)^{\delta - 1}$. Setting $\text{PMB}^\sigma = \text{SMB}$ at I^{**}, and canceling the common terms, requires $\upsilon(I^{**}) + \sigma^* = 1$.

$$\sigma^* = 1 - \upsilon(I^{**}), \qquad (16)$$

strictly positive since $\upsilon(I) < 1$, it is fully determined by equilibrium appropriability at the constrained optimum and increases with the severity of the coordination externality. Complementary slackness generates the two regimes of Section A.3.6 at the social optimum as well, and by Proposition A.5.1(iv), together with Lemma A.3.1, $e_{ij} \in (0, \infty)$ is unique. The same cancellation at an arbitrary candidate anchor density defines the effort-subsidy schedule $1 - \upsilon(I)$, of which equation (16) is the value at the constrained-efficient anchor density. Lemma 1 below is stated for this schedule.

Lemma D.2.1 (Multiplier Invariance). Suppose constraint (7) binds in both the subsidized private problem and the planner's problem. Then $\sigma^* = 1 - \upsilon(I^{**})$ equates the private and social multipliers, $\eta_i(\sigma^*) = \eta_i^S$, so the subsidized private choice coincides with the planner's constrained-efficient effort whether or not (7) binds.

Proof. When the constraint binds, effort is pinned by feasibility, and each multiplier is the residual value satisfying its own KKT system at the constrained allocation. Evaluating the two stationarity conditions at the common allocation and subtracting:

$$\eta_i^S - \eta_i(\sigma) = [1 - (\upsilon(I) + \sigma)]\, p_{ij} \frac{\partial Y_{ij}}{\partial q_{ij}} \frac{\partial q_{ij}}{\partial e_{ij}}.$$

Setting $\sigma = \sigma^*$ makes the bracket vanish, so the multipliers coincide and the KKT systems are identical. The argument uses only that the bracket vanishes, so the conclusion holds at every candidate density under the subsidy schedule $1 - \upsilon(I)$ — the form in which Lemma 1 invokes it.

D.2.5 The social inclusion condition and derivation of τ_{ij}^*. The planner includes partner jwhenever the full welfare difference $\Delta\mathcal{W} = \mathcal{W}(N_i + 1) - \mathcal{W}(N_i) \geq 0$, evaluated at constrained-efficient effort for each candidate network size. The planner evaluates the full social increment; conditional on optimal factor demands — whose indirect effects vanish by the envelope theorem where the capability constraint is slack, with the binding case covered by Lemma D.2.1 — this yields the social inclusion condition:

$$P_i[Y_i(N_i + 1) - Y_i(N_i)] + p_{ij} Y_{ij}^0 [(1 + \theta_{ij} e_{ij}^{**})^\delta - 1] - \frac{\lambda(N_i + 1)}{2} e_{ij}^{**2} \geq \frac{\lambda(N_i + 1) - \lambda(N_i)}{2} \sum_{j' \in N_i} e_{ij'}^{**2}. \qquad (D.3)$$

Comparing (D.3) with the private condition (11) at a common effort level eand generic density I, the inclusion gap is

$$SIS(e) - PIS(I, e) = (1 - \upsilon(I))\, p_{ij} Y_{ij}^0 [(1 + \theta_{ij} e)^\delta - 1] > 0:$$

for any given effort level, the private inclusion comparison understates the social one, so some socially valuable relationships remain privately unprofitable — Distortion 2, operating on the formation margin where Distortion 1 operates on the investment margin, which is why the two distortions require separate corrective instruments (Section 5.3). A grant paid conditional on inclusion — affecting the extensive margin without directly altering the effort choice — closes the gap when $PIS^\tau = SIS$ at $\left(I^{**}, e_{ij}^{**}\right)$. The inclusion gap is defined for every potential relationship at every candidate network size, so the grant is a schedule over candidate relationships at each candidate density: at density I it equals $1 - \upsilon(I)$ times the uninternalized revenue increment, evaluated at the conditionally efficient effort of that candidate. Equation (17) is its value at the constrained-efficient density:

$$\tau_{ij}^* = \left(1 - \upsilon(I^{**})\right) p_{ij} Y_{ij}^0 \left[\left(1 + \theta_{ij} e_{ij}^{**}\right)^\delta - 1\right]$$

strictly positive, compensating the anchor for the uninternalized share of the full revenue increment the new relationship generates. Complementary slackness again yields the two regimes at the social optimum.

Lemma 1 (Inner-Margin Decentralization). Fix an arbitrary interior candidate anchor density I, and set the instruments at that candidate: the effort subsidy $1 - \upsilon(I)$ of D.2.4 and, for every potential relationship, the inclusion grant of D.2.5 evaluated at I. Then for every anchor active at that candidate — that is, with capability at least $M(I)$ — the solution to the anchor's instrumented private problem coincides with the constrained-efficient pair $\left(e_{ij}^{**}(M_i), N_i^{**}(M_i)\right)$ characterized in Sections 5.2–5.3, conditional on I. Equations (16) and (17) are the values of these schedules at I^{**}, for which the conclusion holds in particular.

Proof. By Proposition A.5.1(iv), the instrumented private problem and the planner's inner problem each admit a unique solution on $\mathcal{E}_i \times \{1, \dots, \bar{N}\}$; the planner's objective values the increment at p_{ij} rather than at the appropriated fraction $\upsilon(I)$ of it, so its curvature is larger and the criterion covers both problems independently of υ (D.2.2). At the candidate density I, the effective appropriated share under the subsidy $1 - \upsilon(I)$ equals one, so the instrumented private objective and the planner's inner objective differ only by the inclusion grants and terms independent of (e_i, N_i); by the construction of D.2.5, the grant equals the wedge between the private and social inclusion comparisons at every potential relationship and every candidate network size, so the two objectives induce identical optimality conditions on both the continuous effort margin and the discrete inclusion margin. Two problems with identical optimality conditions and unique solutions have identical solutions; the private optimum is therefore $\left(e_{ij}^{**}(M_i), N_i^{**}(M_i)\right)$. Lemma D.2.1, whose argument turns only on the vanishing of the wedge bracket, extends the conclusion to anchors for which (7) binds. No step of the argument uses the outer optimality of I^{**}: it holds candidate by candidate, which is what the planner-evaluation function $W_s(I)$ of Section 5.4 requires.

Remark D.2.2 (uniform instrument). A single scalar subsidy σ^*decentralizes the effort margin for every active anchor — including those with binding capability constraints — because the wedge it corrects, $1 - \upsilon(I^{**})$, depends only on equilibrium appropriability and not on anchor characteristics; this is the theoretical payoff of the reduction of match-specific appropriability to $\upsilon(I)$in equation (8). The mechanism is that of Proposition B.4.1: the subsidy raises the anchor's effective appropriated share, and effort and inclusion respond on exactly the margins characterized there.

Remark D.2.3 (single transfer schedule). The two instruments target distinct margins — σ^* the investment in existing relationships, τ_{ij}^* the formation of new ones — and correcting Distortion 1 alone does not eliminate Distortion 2: even after σ^*restores constrained-efficient effort, the inclusion gap $(1 - \upsilon(I^{**}))\, p_{ij} Y_{ij}^0 [(1 + \theta_{ij} e_{ij}^{**})^{\delta} - 1] > 0$ remains, so the grant is a separate and necessary instrument. Equations (16) and (17) are, respectively, the intensity and breadth margins of a single transfer schedule on the uninternalized revenue increment.

D.3 Lemma 2 — Threshold Implementation and the Entry Instrument

D.3.0 Derivation of the entry instrument. The entry instrument implements I^{**}— characterized in D.1 — by shifting private entry incentives until the decentralized threshold reaches M^{**}: the same characterize-then-implement logic used for $\sigma^*$and τ_{ij}^*. The instrument operates on the zero-profit condition pinning the equilibrium threshold. Under the mean-field convention adopted for the numerical illustrations (Section 7), individual partner heterogeneity at the margin is replaced

by the average gain accruing to partners per active anchor at the constrained optimum, $\gamma^{**} \equiv \Gamma_P(M^{**})/I^{**}$; paralleling the Pigouvian logic of the inner margins, the mean-field expression for the instrument applies the wedge $1 - \upsilon(I^{**})$ to this average increment. The exact definition, which governs throughout, is the break-even condition (D.4) below: the transfer that makes the marginal anchor M^{**} exactly indifferent under the corrected inner margins. The sign of the instrument is not determined by the sign of Δ— the corrected inner margins raise anchor profitability, so implementing I^{**}may require an entry tax even when $\Delta > 0$; the sign question is treated in Remark D.3.1 and Section D.5.

D.3.1 The policy-inclusive profit function. Let $\Pi^{\sigma^*,\tau^*}(M, I)$ denote the value of the reduced anchor problem (Section A.8.1) under the instruments of D.2: the effort subsidy $\sigma^*$of equation (16) and the inclusion grants τ_{ij}^* of equation (17), both being the schedules of D.2 evaluated at the constrained-efficient density $I^{**}$and held fixed. Under the instruments, the anchor's effective share of the coordination-generated revenue increment is $\upsilon(I) + 1 - \upsilon(I^{**})$; it equals one at $I = I^{**}$ and inherits the strict monotonicity of $\upsilon(\cdot)$. By Lemma 1 (D.2), for every $M \geq M^{**}$ the solution of the instrumented problem at $I = I^{**}$ is the constrained-efficient pair $\left(e_{ij}^{**}(M), N_i^{**}(M)\right)$. The entry instrument is defined by the exact break-even condition

$$\xi^* \equiv - \Pi^{\sigma^*,\tau^*}(M^{**}, I^{**}), \qquad (D.4)$$

and a firm of capability Menters if and only if its policy-inclusive payoff $V(M) \equiv \Pi^{\sigma^*,\tau^*}(M, I^{**}) + \xi^*$is non-negative.

Lemma D.3.1 (Policy-inclusive profit monotonicity). (i) $\Pi^{\sigma^*,\tau^*}(M, I)$ is weakly increasing in M; strictly increasing under Condition CS. (ii) $\Pi^{\sigma^*,\tau^*}(M, I)$ is strictly decreasing in I, conditional on active operation.

Proof. (i) The instruments alter the objective but not the constraint set: the capability constraint (7) is unchanged, so the feasible-set replication argument of Proposition A.6.1 applies verbatim to the instrumented objective, giving weak monotonicity. Under Condition CS, the strictness argument of Corollary A.6.1 also applies: additional capability either relaxes a binding constraint with strictly positive shadow value or permits a strictly profitable inclusion — profitable a fortiori under the instruments, which raise both the effort and inclusion margins' returns (Proposition B.4.1's mechanism at the corrected share). (ii) The effective share $\upsilon(I) + 1 - \upsilon(I^{**})$ is strictly decreasing in Iby (R2), τ_{ij}^* and ξ^* are fixed transfers, and by Lemma A.3.1 — whose interiority argument applies to the instrumented objective, whose marginal benefit at zero effort is weakly larger — every included partner receives strictly positive effort, so the share multiplies a strictly positive increment; the two-allocation comparison of Lemma A.6.1 then applies verbatim.

Lemma 2 (Threshold Implementation). Fix the instruments $\left(\sigma^*, \tau_{ij}^*\right)$of equations (16)–(17) and the entry instrument $\xi^*$of equation (19). Then: (i) a firm enters as an anchor if and only if $M_i \geq M^{**}$; (ii) $M^{**}$is the unique zero-payoff threshold; (iii) the resulting anchor density is I^{**}, and the policy-augmented economy has no other equilibrium density.

Proof. The argument proceeds in four steps.

Step 1 (Break-even at the threshold). By equation (D.4),

$$V(M^{**}) = \Pi^{\sigma^*,\tau^*}(M^{**}, I^{**}) + \xi^* = 0.$$

Thus, the marginal anchor is exactly indifferent between entering and not entering.

Step 2 (Entry above the threshold, exclusion below). A firm that does not enter operates as a baseline producer or partner. Its outside option is therefore its baseline payoff, normalized to zero, and no instrument applies to non-entrants. Under Condition CS, Lemma D.3.1(i) implies that $V(M)$ is strictly increasing in M. Hence, for any $M > M^{**}$,

$$V(M) > V(M^{**}) = 0,$$

so entry is strictly optimal. At $M = M^{**}$, the firm is indifferent; we adopt the standard convention that the measure-zero marginal firm enters. For any $M < M^{**}$,

$$V(M) < V(M^{**}) = 0,$$

so non-entry is strictly optimal. This establishes part (i).

Step 3 (Uniqueness of the threshold). By Step 2, $V(M)$ crosses zero exactly once, at M^{**}: it is strictly negative below M^{**}, equal to zero at M^{**}, and strictly positive above it. Any alternative threshold $\widetilde{M} \neq M^{**}$would therefore violate entry optimality for a positive-measure set of firms. If $\widetilde{M} < M^{**}$, firms with

$$M \in (\widetilde{M}, M^{**})$$

would enter despite having strictly negative payoff. If $\widetilde{M} > M^{**}$, firms with

$$M \in (M^{**}, \widetilde{M})$$

would remain out despite having strictly positive payoff. Therefore $M^{**}$is the unique decentralized capability threshold. This establishes part (ii).

Step 4 (Implementation and uniqueness of the density). Given the entry rule of Step 2, the entering mass is $1 - F_S(M^{**}) = I^{**}$ by the definition of M^{**} in D.1 — the candidate density reproduces itself, so $I^{**}$is an equilibrium of the policy-augmented economy. For uniqueness, define the policy-inclusive marginal-anchor payoff $\psi^{\sigma^*,\tau^*}(I) \equiv \Pi^{\sigma^*,\tau^*}(M(I), I) + \xi^*$ with $M(I) = F_S^{-1}(1 - I)$ as in Section A.8.2. The single-crossing argument of Theorem A.8.1 applies unchanged: ψ^{σ^*,τ^*} is continuous by the argument of Step 1 of Theorem A.8.1 (the instruments are fixed transfers and a share rescaling, preserving continuity), and strictly decreasing by the two reinforcing channels — $M(I)$ decreasing with Lemma D.3.1(i), and Lemma D.3.1(ii) — so it has at most one root; since $\psi^{\sigma^*,\tau^*}(I^{**}) = V(M^{**}) = 0$, that root is I^{**}. This establishes (iii).

Closing paragraph (equation (20)). Combining the two lemmas delivers the implementation claim of Section 5.5. At the density I^{**}, Lemma 1 establishes that every active anchor's inner margins coincide with the constrained-efficient pair $\left(e_{ij}^{**}(M_i), N_i^{**}(M_i)\right)$; Lemma 2 establishes that the entry margin selects exactly the constrained-efficient anchor set $\{i: M_i \geq M^{**}\}$, that the resulting density is I^{**}, and that no other density is self-consistent under the instruments. Equilibrium appropriability is then $\upsilon^{**} = \upsilon(I^{**})$ by construction, and the effective share under σ^*equals one, so the private optimality conditions coincide with the planner's at every margin. The instrument package $\left(\sigma^*, \tau_{ij}^*, \xi^*\right)$ therefore decentralizes the constrained-efficient allocation $\left(I^{**}, M^{**}, \upsilon^{**}, \{e_{ij}^{**}\}, \{N_i^{**}\}\right)$, which is equation (20).

Remark D.3.1 (sign and interpretation of ξ^*). Definition (D.4) fixes the magnitude of the entry instrument but not its sign, which depends on whether the corrected inner margins raise or lower marginal-anchor profitability relative to the entry externality: $\xi^* < 0$ (an entry tax) when $\Delta > 0$ and excess entry must be curbed. The regime characterization is given in D.5.[29]

Remark D.3.2 (role of c_0). Unlike the planner's existence argument (D.1), where $c_0 > 0$ supplies the boundary condition for an interior I^{**}, no step of the present proof uses c_0: the threshold's uniqueness rides entirely on Condition CS through Lemma D.3.1(i). The lemma therefore degrades gracefully at $c_0 = 0$ in the same sense as Theorem A.8.1.

Remark D.3.3 (no multiplicity introduced). Step 4 shows the corrected economy inherits the uniqueness structure of the decentralized one: the instruments rescale the appropriated share and add fixed transfers, leaving both negative-feedback channels of Remark A.8.2 intact. Correcting the three distortions does not open the door to coordination-failure multiplicity — the policy analysis compares two unique equilibria, not equilibrium sets.

D.4 Proposition 1

Proposition 1. Under the maintained assumptions, Assumption D.1, and Proposition D.1.1, let Δ denote the net entry distortion indicator of D.1.2, evaluated at the decentralized equilibrium (M^*, I^*) of Theorem A.8.1. Then: $\Delta > 0 \Rightarrow I^{**} > I^*$; $\Delta < 0 \Rightarrow I^{**} < I^*$; $\Delta = 0 \Rightarrow I^{**} = I^*$; and in each case $I^{**} \gtrless I^* \Leftrightarrow M^{**} \lesseqgtr M^*$.

Proof. If $\Delta > 0$: $dW_s/dI\,|_{I^*} > 0$, so the planner's objective is locally increasing at I^*; under Assumption D.1, W_s is increasing on $(0, I^{**})$ and decreasing thereafter, so a point of increase lies left of the peak: $I^{**} > I^*$. If $\Delta < 0$: symmetrically, $I^{**} < I^*$. If $\Delta = 0$: I^*satisfies the planner's first-order condition (interior by Theorem A.8.1), and by uniqueness of the peak, $I^{**} = I^*$. The threshold equivalences follow from strict monotonicity of $I = 1 - F_s(M)$ under (R1).

Remark D.4.1. Δ diagnoses the direction of the entry distortion — whether the constrained-efficient allocation features more or less entry than the decentralized equilibrium. It does not by itself determine the sign of the implementing transfer ξ^*, which is governed by the break-even

[29] Under the baseline calibration, $\Delta > 0$and $\xi^* < 0$across all reported parameter configurations; see Appendix E.

condition (D.4) under the corrected inner margins; see Remark D.3.1 and Section D.5. (The proof above cites D.1 and Lemma 1 only; it nowhere uses Lemma 2.)

D.5 Entry-Policy Regimes under Endogenous Appropriability

D.5.1 Setup. At the decentralized equilibrium, the sign-relevant part of Δ takes the form (equation (18) of the body)

$$\Delta = \underbrace{(1-\upsilon(I^*))\,\bar{S}}_{\text{uninternalized diffusion benefit}} + \underbrace{\upsilon'(I^*)\,I^*\,\bar{S}}_{\text{economy-wide appropriability-erosion effect}} \quad [30]$$

where $\bar{S} > 0$ is the average coordination-generated revenue increment per anchor, so $\text{sign}(\Delta) = \text{sign}[(1-\upsilon(I^*)) + \upsilon'(I^*)I^*]$.

Proposition D.5.1 (Regime Characterization). Under Assumption D.1: $\Delta > 0 \Leftrightarrow 1-\upsilon(I^*) > -\upsilon'(I^*)I^* \Rightarrow I^{**} > I^*$(Regime A: insufficient entry); $\Delta < 0 \Leftrightarrow 1-\upsilon(I^*) < -\upsilon'(I^*)I^* \Rightarrow I^{**} < I^*$(Regime B: excess entry); $\Delta = 0 \Rightarrow I^{**} = I^*$(Regime C: no entry distortion).

Proof. By Proposition 1, the sign of Δ determines the position of I^*relative to the planner's optimum: $\Delta > 0$ places the decentralized equilibrium on the increasing branch of W_s, so $I^{**} > I^*$; $\Delta < 0$ places it on the decreasing branch, so $I^{**} < I^*$; $\Delta = 0$ places it at the peak. The decomposition of D.5.1 identifies which force generates each case: in Regime A, the uninternalized diffusion benefit $(1-\upsilon(I^*))\bar{S}$ dominates the erosion effect $-\upsilon'(I^*)I^*\bar{S}$— the value that additional anchors create for partners exceeds the coordination-incentive damage their entry inflicts economy-wide, so the decentralized equilibrium features too little entry. In Regime B the erosion effect dominates: the marginal entrant's uninternalized contribution is smaller than the appropriability damage it imposes on all inframarginal anchors, so entry is excessive. Regime C is the knife-edge at which the two forces exactly offset.

Remark D.5.1 (regimes versus instruments). The regime labels classify the distortion at the uncorrected equilibrium, not the sign of the implementing transfer: under the full package, $\xi^*$is determined by (D.4), and the corrected inner margins can require an entry tax even in Regime A — as they do under the baseline calibration ($\Delta > 0$, $\xi^* < 0$; Appendix E). The sign of $\xi^*$is not established analytically here; it is reported for the calibrated region in Appendix E.

D.5.2 Institutional dependence. The functional form of $\upsilon(I)$ is institutionally determined, and three channels govern the regime: contracting institutions — stronger enforcement raises $\upsilon(I^*)$, shrinking the uninternalized component, with an ambiguous effect on the erosion rate; information quality — better partner-side information raises $-\upsilon'(I^*)$ through credible outside options, with an

[30] Relationship level. Under the scale normalization (A.1), the increment on relationship ij at equilibrium conduct is $s_{ij} = p_{ij}(Y_{ij}(e^*_{ij}) - Y^0_{ij}) = p_{ij}\,Y^0_{ij}[(1+\theta_{ij}\,e^*_{ij})^{\delta} - 1]$. Anchor level. Summing over the anchor's equilibrium network, with conduct evaluated at the decentralized equilibrium given I^*(so at the appropriability-eroded effort, under $\upsilon(I^*)$): $S(M) = \sum_{j\in N^*(M)} p_{ij}\;Y^0_{ij}[(1+\theta_{ij}\,e^*_{ij}(M;I^*))^{\delta} - 1] = N^*(M)\mathbb{E}[\,p\,Y^0((1+\theta e^*)^{\delta} - 1)]$; and $\bar{S} \equiv \frac{1}{I^*}\int_{M^*}^{\infty} S\,(M)\;dF_s(M)$.

ambiguous effect on the level; and competitive structure and development — higher I^*amplifies the erosion term $-\upsilon'(I^*)I^*$, so the same sector may transition between regimes as capability accumulates. The policy implication is that optimal entry policy is sector-specific and development-contingent: two sectors in one economy may require opposite interventions, a sector may switch regimes as it develops, and the regime is in principle estimable from sector-level measures of $\upsilon(I^*)$ and $-\upsilon'(I^*)$.

Corollary D.5.1 (Parametric regime condition). Under $\upsilon(I) = \upsilon_0(1-I)^\kappa$, Regime A obtains if and only if

$$1 > \upsilon_0(1-I^*)^{\kappa-1}[1+(\kappa-1)I^*], \qquad (D.5)$$

with the reverse inequality for Regime B and equality for Regime C. The regime depends jointly on $(\upsilon_0, \kappa, I^*)$— baseline appropriability (the contracting environment), the erosion elasticity (competitive structure and partner mobility), and equilibrium density (organizational development) — and no single parameter determines it in isolation; the boundary (D.5) is estimable from sector-level information.

D.6 Proof of Proposition 2

Sectoral output satisfies the decomposition of equation (21): $Y_s = Y_s^0 + \Gamma_A + \Gamma_P$— baseline output plus coordination-generated value flowing through anchors and partners; transfers between firms net out (Section 5.1). By Theorem A.8.1, the equilibrium (M^*, I^*) is unique, so each comparative static below is a statement about a function of primitives.

(i) FOSD shifts of F_s. A first-order improvement of F_s raises $M(I) = F_s^{-1}(1-I)$ pointwise, hence raises $\psi(I) = \Pi^*(M(I), I)$ pointwise by Proposition A.6.1; the unique crossing of Theorem A.8.1 moves right, so I_s^* rises. This density conclusion is exact: it uses only the pointwise ranking of $\psi(I)$ and the single-crossing property, and places no restriction on the induced adjustment of conduct. The consequences for coordination-generated value, by contrast, are two-sided and are not signed by the ranking of F_s alone. The direct channel raises it: the anchor set strictly expands, each entrant contributing non-negative net gains by the inclusion logic of (11), and coordination value per anchor is increasing in capability (Proposition B.1.1(i)–(iii)). The equilibrium channel works against it: because appropriability erodes in anchor density, the induced entry lowers $\upsilon(I_s^*)$, so every continuing anchor operates at a strictly smaller appropriated share and reduces coordination effort and, weakly, network breadth (Proposition B.4.1(i)–(ii), applied to a downward shift of the appropriability level). Composition is two-sided for the same reason: capability rises at every rank of F_s, while the entry response draws the active set deeper into the distribution, so that the movement of the threshold $M_s^* = M(I_s^*)$ — and with it the capability composition of the anchor set and of the partner pool — is not signed by the improvement alone. Proposition 2 accordingly states the determinant and the direction of the density response, and does not assert an unconditional sign for Y_s; the net movement in the calibrated region is reported in Section 7 and Appendix E (Figure 1).

(ii) Appropriability. Lemma D.6.1 (Equilibrium appropriability rises under an upward shift of the appropriability schedule). An upward shift of the appropriability schedule — a strictly higher

appropriability level at every anchor density, realized in the model by an increase in the institutional ceiling υ_0 — strictly raises equilibrium appropriability $\upsilon(I_s^*)$.

Proof. An upward shift of the schedule, realized by higher υ_0, raises $\psi(I)$ pointwise (Proposition B.4.1(i) applied at the marginal anchor), so I_s^* rises and $M_s^* = M(I_s^*)$ falls. At the new equilibrium the marginal anchor is strictly less capable yet still earns zero profit; since Π^* is weakly increasing in capability (Proposition A.6.1) and strictly increasing in the appropriability level (Proposition B.4.1(i)), zero profit at strictly lower capability requires strictly higher equilibrium appropriability under the shifted schedule: $\upsilon(I_s^{*\prime}) > \upsilon(I_s^*)$.

The entry response therefore partially but never fully offsets the upward shift: selection guarantees the net appropriability gain. Output then rises through three channels: every inframarginal anchor faces strictly higher $\upsilon(I_s^*)$, raising effort on every relationship in the slack regime (Condition EM, automatic by Remark B.0.1) and weakly expanding networks (Condition NB); the anchor set strictly expands, adding non-negative coordination gains; and in the binding regime, reallocation shifts capability toward relationships with larger revenue increments — which, because the appropriated share is common across relationships, are also the relationships with larger coordination-generated value. Remark D.6.1. In the binding regime the reallocation argument signs the direction of compositional change but not a strict aggregate increase at fixed total effort; the strict aggregate result rides on Lemma D.6.1 together with the extensive and network margins, and is verified quantitatively across the full parameter sweep in Appendix E (Figure 1, Panel A).

(iii) Congestion. An upward shift of $\lambda(\cdot)$ lowers $\psi(I)$ pointwise (Proposition B.3.1(i) at the marginal anchor), so I_s^* falls. At fixed $(I, \upsilon(I))$ the direct effects on conduct are unambiguous: coordination effort weakly falls in the slack regime and networks weakly contract (Proposition B.3.1(ii), (iv)). In equilibrium these effects are partially offset. The contraction of the anchor set raises $\upsilon(I_s^*)$, which raises the appropriated share on every continuing relationship and pushes effort and network breadth back up (Proposition B.4.1(i)–(ii)). The direct and feedback channels therefore act in opposite directions on inframarginal conduct, and the net effect on Y_s is not signed at this level of generality. Proposition 2 claims congestion as a determinant and the direction of the density response, not an unconditional output sign.

(iv) Fixed coordination cost. Higher c_0 lowers $\psi(I)$ pointwise by exactly the increment in c_0, so I_s^* falls. Because c_0 enters no optimality condition governing conduct (its demoted role, Section 5.4), it has no direct effect on coordination effort or network breadth at given $(I, \upsilon(I))$. It does not follow that the entire effect operates through the extensive margin: the induced contraction of the anchor set raises $\upsilon(I_s^*)$, and inframarginal conduct responds to that change in the appropriated share (Proposition B.4.1(i)–(ii)). The extensive-margin loss and this intensive-margin gain act in opposite directions, so the net effect on Y_s cannot be signed without additional restrictions; as in (i) and (iii), Proposition 2 asserts the determinant and the direction of the density response only.

D.7 Proof of Proposition 3

D.7.1 Context and proof strategy. Proposition 3's three claims — the decomposition identity, vanishing policy responsiveness in the capability-poor limit, and output convergence — share one transmission mechanism: equilibrium anchor density $I_s^*(\mu_F) = 1 - F(M_s^*; \mu_F)$. Policy

responsiveness is $\mathcal{R}(\mu_F) \equiv \| \partial Y_s^* / \partial(\sigma, \tau, \xi) \|_{(\sigma,\tau,\xi)=0} \|$. The capability distribution operates through two structurally distinct margins: the stock margin I_s^*— the mass of active anchors — governing σand τ, which operate through existing networks; and the flow margin $f(M_s^*; \mu_F)$— the density of firms at the entry threshold — governing ξ, which activates firms near it. The σand τresults require only the stock margin; the ξresult additionally requires:

Assumption (T) (Tail Regularity). $I_s^*(\mu_F) \to 0 \Rightarrow f(M_s^*; \mu_F) \to 0$: as the mass of firms above the entry threshold vanishes, so does the density of firms at it.

Economically, (T) rules out distributions whose negligible upper mass is concentrated in a spike just above M_s^*— a thin upper tail is thin throughout the relevant range, the natural property of unimodal distributions whose support does not pile up at the threshold.

Remark D.7.1 (verification for the log-normal). Under the log-normal specification with location μ_F: $1 - F(M_s^*; \mu_F) = 1 - \Phi((\ln M_s^* - \mu_F)/\sigma_s)$ and $f(M_s^*; \mu_F) = \phi((\ln M_s^* - \mu_F)/\sigma_s)/(M_s^* \sigma_s)$; as $\mu_F \to -\infty$ the standardized argument diverges to $+\infty$, so the tail mass and the density at the threshold vanish together. Assumption (T) is therefore satisfied by the Section 7 specification.

Proof architecture: D.7.3 decomposition identity and uniform bound; D.7.4 boundedness lemma; D.7.5–D.7.7 vanishing responsiveness of σ, τ(stock margin, no (T)), and ξ (flow margin, requires (T)); D.7.8 assembly; D.7.9 output convergence; D.7.10 upper-tail dominance.

Remark D.7.2 (Partner-pool composition under capability deterioration). Under the maintained log-normal specification, a deterioration of the capability distribution — represented by a decline in the location parameter $\nu_s$at a given σ_s— deteriorates the coordination-relevant composition of the equilibrium partner pool. Along the computed capability grid used in Section 7 (Figure 2; $\nu_s \in [1.80, 2.30]$, the region with active anchor entry), pool-average baseline productivity $\mathbb{E}[M_j^{\alpha} \mid M_j < M_s^*]$ and pool-average absorptive capacity $\mathbb{E}[\theta(M_j) \mid M_j < M_s^*]$ both decline monotonically as $\nu_s$falls, notwithstanding the rise in the equilibrium entry threshold M_s^*, which reclassifies marginal anchors into the partner pool. Below $\nu_s = 1.80$, private anchor entry ceases, the partner pool is no longer defined relative to an active anchor set, and the sector converges to its no-coordination benchmark. As with Remark D.7.1, this result is established for the maintained specification over the computed parameter region rather than as a distribution-free result: first-order dominance of a parent distribution does not in general order the moments of its truncation below an endogenous threshold.

D.7.3 Decomposition identity and uniform bound. Claim (Proposition 3(i)).

$$Y_s^* - Y_s^0 = \sum_{i \geq M_s^*} \sum_{j \in N_i^*} [Y_{ij}^*(e_{ij}^*) - Y_{ij}^0] + \sum_{i \geq M_s^*} [Y_i^* - Y_i^0],$$

and $0 \leq Y_s^* - Y_s^0 \leq I_s^* \cdot S^{max}$for all $I_s^* \in [0,1]$, with $S^{max} < \infty$ uniform in I_s^*.

Proof. Summing output across active anchors and their networks and subtracting the benchmark: firms below M_s^* contribute zero to the coordination-generated value; at $I_s^* = 0$both sums are empty

and $Y_s^* = Y_s^0$. For the bound, each active anchor's contribution is bounded by $S^{max} \equiv \sup_{I^* \in [0,1]} \mathbb{E}_{i,j}^{A}[p_{ij} Y_{ij}^0((1 + \theta_{ij} e_{ij}^*(I^*))^{\delta} - 1)] < \infty$— finite because $\upsilon(I^*) \to \upsilon(0) \in (0,1)$ by continuity ((R2)) bounds equilibrium effort, and networks are finite by (R4). Summing over the anchor mass gives the bound. The bound holds for all values of the policy vector, not merely at the origin — the foundation for every derivative bound below.

D.7.4 Boundedness. Lemma D.7.1 (Boundedness). The per-match output responses satisfy $|\partial Y_{ij}^*/\partial\sigma|_{\sigma=0}| \leq K_\sigma < \infty$ and $|\partial Y_{ij}^*/\partial\tau|_{\tau=0}| \leq K_\tau < \infty$, uniformly over active matches and $I_s^* \in [0,1]$; no Assumption (T) required.

Proof. From the effort condition (10), $\partial e_{ij}^*/\partial\sigma|_{\sigma=0} = -\Pi_{e\sigma}/\Pi_{ee}$, with $\Pi_{ee} < 0$(Lemma A.2.2) and $\Pi_{e\sigma} = p_{ij} Y_{ij}^0 \theta_{ij} \delta(1 + \theta_{ij} e_{ij}^*)^{\delta-1} > 0$; equilibrium effort is bounded on the feasible set, so all terms are finite and $|\partial e_{ij}^*/\partial\sigma| \leq C_e < \infty$, whence $|\partial Y_{ij}^*/\partial\sigma| \leq \tilde{\Omega}\, q_{ij}^0\, \delta\theta_{ij}\, C_e \equiv K_\sigma$. For τ: $|\partial Y_{ij}^*/\partial\tau| \leq |\partial Y_{ij}^*/\partial N_i^*| \cdot |\partial N_i^*/\partial\tau| \leq \bar{Y} \cdot B \equiv K_\tau$, with $\bar{Y} < \infty$ the maximum partner output and $B < \infty$ the uniformly finite number of additional viable partnerships per anchor, by (R4).

D.7.5 Vanishing responsiveness: σ (no Assumption (T)). The effort subsidy operates only through active anchor–partner relationships: $\partial Y_s^*/\partial\sigma|_{\sigma=0} = \sum_{i \geq M_s^*} \sum_{j \in N_i^*} \partial Y_{ij}^*/\partial\sigma$, the outer sum over mass I_s^*, each network bounded by $\bar{N} < \infty$ by (R4). By Lemma D.7.1, $0 \leq |\partial Y_s^*/\partial\sigma| \leq I_s^* \bar{N} K_\sigma \to 0$ as $I_s^* \to 0$; the squeeze theorem completes the claim. Interpretation: however generous, σ can only intensify coordination within existing networks; when $I_s^* \approx 0$, there are almost none.

D.7.6 Vanishing responsiveness: τ (no Assumption (T)). Identical structure: $0 \leq |\partial Y_s^*/\partial\tau| \leq I_s^* \bar{N} K_\tau \to 0$. However generous, τcan only broaden the networks of active anchors; when $I_s^* \approx 0$, there are almost no anchors whose networks can be broadened.

D.7.7 Vanishing responsiveness: ξ (requires Assumption (T)). Step 1 (entry-density response vanishes). $I_s^* = 1 - F(M_s^*; \mu_F)$, so $\partial I_s^*/\partial\xi = -f(M_s^*; \mu_F)\ \partial M_s^*/\partial\xi$. From the zero-profit condition $\Pi(M_s^*, I_s^*, \xi) = 0$, the implicit function theorem gives $\partial M_s^*/\partial\xi = -\Pi_\xi/\Pi_M$; the denominator is bounded away from zero at the threshold by Corollary A.6.1, and $\Pi_\xi = 1$since the transfer enters additively, so $|\partial M_s^*/\partial\xi| \leq C < \infty$ and $|\partial I_s^*/\partial\xi| \leq f(M_s^*; \mu_F) \cdot C \to 0$ under (T). Step 2 (chain rule). Under the mean-field convention of Section 7 — partner characteristics independent of anchor capability conditional on activity — Y_s^* depends on ξonly through $I_s^*(\xi)$, not through the conditional capability distribution as the threshold shifts. The convention is what licenses this step: because conditional partner characteristics are treated as independent of anchor identity, a marginal movement of the threshold changes the number of active networks but not the average value of one. From the D.7.3 bound, $0 \leq \partial Y_s^*/\partial I_s^* \leq S^{max} < \infty$; the per-anchor contribution need not vanish — the vanishing comes entirely from the flow margin: $|\partial Y_s^*/\partial\xi| \leq S^{max} \cdot f(M_s^*; \mu_F) \cdot C \to 0$. Role of (T): without it, a density spike just above the threshold would keep $|\partial I_s^*/\partial\xi|$ positive even as $I_s^* \to 0$, and the entry instrument would retain local traction; (T) rules this out, and the log-normal satisfies it by Remark D.7.1.

D.7.8 Assembly. Under Assumption (T), all three components of the policy gradient vanish as $I_s^* \to 0$, so $\mathcal{R}(\mu_F) \to 0$under any equivalent norm on $\mathbb{R}^3$; without (T), the result holds unconditionally for the (σ, τ) gradient on $\mathbb{R}^2$. Equilibrium anchor density is thus the common transmission mechanism linking organizational capability, industrial performance, and policy effectiveness; the capability-constrained regime exhibits severe underdevelopment of the coordination margin when both the stock and flow margins are thin — jointly implied by (T) whenever $I_s^* \to 0$.

D.7.9 Output convergence (no Assumption (T)). From D.7.3, $0 \le Y_s^* - Y_s^0 \le I_s^* \cdot S^{max} \to 0$ as $I_s^* \to 0$; the squeeze theorem gives $Y_s^* \to Y_s^0$.

D.7.10 Upper-tail dominance. Corollary D.7.1 (Upper-Tail Dominance). This corollary formalizes, at corollary level, the cross-sector comparison discussed in Section 6.2 and illustrated by the capability sweep of Figure 2; it is not among the numbered results of the body, and the responsiveness ranking is accordingly established through the bounds of D.7.5–D.7.7 rather than as an exact monotonicity theorem.

Statement. Let capability distributions Aand Bsatisfy $1 - F_A(M) \ge 1 - F_B(M)$for all M(upper-tail dominance), strictly at the equilibrium thresholds. Then $I_{s,A}^* > I_{s,B}^*$ and $Y_{s,A}^* > Y_{s,B}^*$; and under Assumption (T), sectors further from the capability-poor limit exhibit strictly greater policy responsiveness on every margin bounded by the stock and flow margins.

Proof sketch. Dominance raises M(I)pointwise, hence $\psi(I)$ pointwise (Proposition A.6.1), so the unique crossing of Theorem A.8.1 rises: $I_{s,A}^* > I_{s,B}^*$— the argument of D.6(i). The output ranking follows from the decomposition of D.7.3 with the extensive and compositional margins as in D.6(i). The responsiveness ranking follows from the bounds of D.7.5–D.7.7 evaluated at the respective equilibria.

What is not claimed. The sign of $\partial\mathcal{R}/\partial\mu_F$ is not claimed to equal the sign of Δ: Δ governs the direction of optimal entry policy (D.5), while responsiveness is governed by the stock and flow margins. The results complement rather than compete.

APPENDIX E — NUMERICAL METHODS

E.1 Solution Algorithm

E.1.1 Environment and replication protocol. The model is implemented in Python (NumPy; Matplotlib for figures) in two modules: `model_final.py` (primitives, firm problem, welfare integrals) and `solvers.py` (equilibrium search, planner optimization, Δ, ξ^*). The validation script `validate_baseline.py` reproduces all canonical baseline quantities to reported precision and reports any failure, directing that no sweep be run; it is the required first step of every session and the entry point of the replication package. Sweep scripts (`sweep_fig1.py`, `sweep_nu.py`, `sweep_fig3.py`) write per-point checkpoints; figure renders read only from the checkpointed grid logs, which are included in the package as the authoritative outputs. All reported digits in this appendix were regenerated from this code.

E.1.2 Mean-field approximation and symmetric effort. For numerical tractability, partner heterogeneity within an anchor's network is replaced by pool averages: at candidate threshold M, all matched partners carry the average baseline productivity $\tilde{q}^0(M)$ and average absorptive capacity $\bar{\theta}(M)$ of the pool $\{M_j < M\}$. Coordination effort is accordingly symmetric within a network, reducing the firm problem to two dimensions, (N_i, e_i). Baseline aggregates and pool averages are computed by quadrature over the log-normal capability distribution; welfare tails use 80-node Gauss–Legendre quadrature on the standardized domain $[z^*, 6]$.

E.1.3 The seventeen-step protocol. *Layer 1 (economy, once per candidate threshold):* parameters; pool averages $\tilde{q}^0(M)$, $\bar{\theta}(M)$. *Layer 2a (firm problem, unregulated):* the effort condition (10) solved by bisection on [10^{-14}, 15] with a fixed count of 60 iterations (final bracket width of order 10^{-17}, far below any reported precision — no tolerance-based stop), after which the capability cap $e \leq M_i/N_i$ is imposed; network size by greedy inclusion — partners added while the inclusion condition (11) holds, with $c_0$excluded from (11) and subtracted once in profit, per equation (12). *Layer 3a (equilibrium):* the threshold M^*located by scanning 48 geometrically spaced candidate thresholds on [2, 150] for the first negative-to-positive crossing of $\Pi^*(M) - c_0$ at which both bracket endpoints satisfy the moderate-network validity condition ($N < 200$, excluding the runaway region of E.1.4), followed by bisection for 90 iterations or until the bracket width falls below 10^{-11}; $I^* = 1 - F_s(M^*)$; a seven-point firm-level diagnostic profile confirms interior behavior. *Layer 2b (policy-induced):* the constrained-efficient pair (e^{**}, N^{**}) from the same firm problem with effective share $\upsilon_{\text{eff}} = 1$, implementing $\sigma^*$and τ_{ij}^*(Lemma 1). Planner: $W_s(M) = Y_s^0 + \Gamma_A + \Gamma_P - C_v - c_0 I(M)$ evaluated under $\upsilon_{\text{eff}} = 1$; grid scan (90–120 points), then golden-section refinement to 10^{-8}, with the planner boundary flag recorded per point — clear (interior maximizer) at every reported configuration. Entry margin: $\Delta$by the M-to-I chain rule (Step 14); $\xi^*$by the exact break-even definition (D.4) — the policy-inclusive profit at (M^{**}, I^{**})computed at full effective share and net of c_0, consistent with Lemma 1's effective-share-one characterization; the Pigouvian object $(1 - \upsilon(I^{**}))\gamma^{**}$ is reported separately for interpretation only (D.3.0) and is not the implementation formula. *Verification:* Step 16 confirms the global sign-crossing $\Pi^{**}(0.8M^{**}) < 0 = \Pi^{**}(M^{**}) < \Pi^{**}(1.2M^{**})$; Step 17 reports the sequencing decomposition explaining the coexistence of $\Delta > 0$ with $\xi^* < 0$ (Remarks D.3.1 and D.5.1).

E.1.4 Network ceilings and condition (R4). Two ceilings operate, with distinct statuses. At every *equilibrium determination* — the threshold search, the planner's threshold, and the entry instrument — the network search is capped at the numerical search cap of 400, and the cap is slack at every reported configuration: across the baseline, all six calibration candidates, and every grid point of Figures 1–3, the largest decentralized threshold network is $N^* = 51$ (Candidate 1; $N^* = 35$across the figure grids) and the largest constrained-efficient threshold network is $N^{**} = 48$(29 across the figure grids). In the *welfare integrals*, which aggregate over the full capability distribution, networks are capped at the economic truncation of 80, and this cap binds for far-tail firms: under the mean-field approximation, pool composition is held fixed at threshold averages, which suppresses exactly the pool-exhaustion force that grounds condition (R4) in the full model — the marginal partner's quality never declines as networks expand, so the inclusion differential remains positive indefinitely for sufficiently capable firms (at the baseline, the uncapped network diverges for $M_i \gtrsim 48$). The ceiling $N_{\max} = 80$ therefore *implements* (R4)'s finite-network property within the approximation, as a documented truncation whose level lies well above every equilibrium network. One further ceiling event is disclosed: at the lowest capability levels of the Figure 2 grid ($\nu_s \leq 1.40$, equivalently $E[M] \leq 6.6$), the planner-side threshold network reaches the 400 cap and the entry-instrument computation reports non-convergence at the bound; the affected constrained-efficient quantities are reported as truncated, consistent with the figure notes, and the associated welfare gains are below figure resolution. On the same grid, private entry fails entirely for $\nu_s \leq 1.75$— the no-coordination equilibrium of Definition A.8.2 realized along the capability sweep, the shaded region of Figure 2. Uniform joint concavity is preserved throughout: every effort problem evaluated anywhere in the computation — up to and including the network search cap of 400 — satisfies $N \leq 400 < 1/(\varphi\delta) = 408$ at the baseline $\varphi\delta = 0.00245$, so the effort problem is jointly strictly concave and the solved effort vector unique at every point, by Proposition A.5.1(iii).

E.1.5 Cross-validation. All baseline quantities were reproduced by an independent reimplementation; the Figure 3 series reported in E.3 was additionally regenerated end-to-end from the canonical modules and found digit-identical to the archived grid log. Figures were re-rendered from the checkpointed series after each protocol revision.

E.2 Baseline and Alternative Calibrations

E.2.1 Fixed parameters and functional forms. $\alpha = 0.40$, $\delta = 0.35$, $\nu_s = 2.00$, $\sigma_s = 0.80$, $\upsilon_0 = 0.99$, $\kappa = 2.0$, $\lambda_0 = 1.0$, $\bar{\theta} = 2.0$, $\rho = 0.193$; absorptive capacity $\theta(M_j) = \bar{\theta}(1 - e^{-\rho(M_j - 1)})$, vanishing at the lower support and bounded by the ceiling $\bar{\theta}$; congestion $\lambda(N) = \lambda_0 + \lambda_1 N^2$; appropriability $\upsilon(I) = \upsilon_0(1 - I)^\kappa$; capability log-normal, $M = 1 + e^{\nu_s + \sigma_s z}$. Factor allocations do not appear: the implementation solves the reduced problem of Section A.8.1, in which constant returns and competitive pricing eliminate factor choices by Euler's theorem.

E.2.2 Calibration discipline. The calibration is illustrative rather than moment-matched, disciplined by three targets: the zero-profit condition holding at a threshold near $M^* \approx 26$ (an anchor at roughly the 94th capability percentile); a partner-to-anchor gains ratio Γ_P/Γ_A inside the empirically documented 3–10 range for outgrower and anchor-supplier arrangements; and equilibrium networks in the several-dozen range characteristic of documented anchor schemes.

The three free parameters (ϕ, λ_1, c_0) were selected by evaluating six candidate vectors against all seventeen protocol steps; every candidate passed every step.

E.2.3 The candidate set. All values below are computed from the canonical code of E.1.1 and supersede interim session records. Candidate 2 is the baseline.

C	ϕ	λ_1	c_0	M^*	I^*	N^*	e^*	Γ_P/Γ_A	M^{**}	I^{**}	N^{**}	Δ	ξ^*	**Welfare gain**	**Output gain**
1	0.006	0.001	5.85	26.01	0.0638	51	0.234	9.47	18.31	0.144	48	+2.04	−0.813	3.18%	31.1%
2	**0.007**	**0.003**	**3.34**	**26.04**	**0.0635**	**31**	**0.220**	**7.74**	**17.85**	**0.151**	**29**	**+1.30**	**−0.425**	**2.17%**	**22.2%**
3	0.008	0.005	2.49	26.00	0.0638	27	0.188	5.23	16.80	0.171	23	+1.25	−0.314	1.74%	21.2%
4	0.008	0.010	1.76	25.94	0.0642	19	0.189	4.65	16.74	0.172	17	+0.93	−0.197	0.58%	15.2%
5	0.006	0.003	3.39	26.02	0.0637	29	0.239	10.11	18.49	0.141	28	+1.13	−0.471	1.76%	19.6%
6	0.007	0.005	2.58	25.96	0.0640	24	0.220	7.84	17.78	0.153	22	+0.93	−0.342	1.58%	17.9%

Notes: Γ_P/Γ_A is the aggregation-layer ratio; welfare gain is the full-package gain $(W^{**} - W^*)/W^*$; output gain is $(Y_S^{**} - Y_S^*)/Y_S^*$. Every candidate hits the zero-profit target by construction ($M^* \in [25.9, 26.0]$), equilibrium density and appropriability are essentially invariant across candidates ($I^* \approx 0.064$, $\upsilon(I^*) \approx 0.868$), networks span 19–51, and the gains ratio spans 4.7–10.1, bracketing the target band. The robustness content: $\Delta > 0$ and $\xi^* < 0$ in every candidate — the sequencing result of Section 5.4, Regime A entry distortion implemented by an entry tax under the corrected inner margins, is a property of the calibrated region, not of the baseline point. Candidate 2 was selected for the most balanced gains ratio and the largest welfare gain among candidates with networks in the target range.

E.2.4 Baseline equilibrium. Decentralized: $M^* = 26.041$, $I^* = 0.06355$, $N^* = 31$, $e^* = 0.21996$, $\upsilon(I^*) = 0.868$. Constrained-efficient: $M^{**} = 17.848$, $I^{**} = 0.1514$, $\upsilon(I^{**}) = 0.713$, $N^{**} = 29$, $e^{**} = 0.2511$. Diagnostics: $\Delta = +1.30 > 0$ (Regime A); $\xi^* = -0.4248$ (entry tax); package welfare gain 2.167%; output gain 22.20%. The package gain decomposes sequentially (conduct-first) into 0.069 percentage points from corrected conduct at the private threshold and 2.098 percentage points from the entry margin — an accounting decomposition whose dominance of the entry component is the quantitative counterpart of the sequencing architecture of D.3 and D.5. All digits reproduced by `validate_baseline.py`.

E.3 The Dispersion Decomposition (Figure 3)

E.3.1 Design. Figure 3 sweeps capability dispersion σ_s over $[0.600, 0.975]$ in steps of 0.025, holding mean capability fixed by the mean-preserving adjustment $\nu_s = 2.32 - \sigma_s^2/2$, so that $E[M]$ is constant across the grid and the experiment isolates dispersion.

E.3.2 The decomposition. Write coordination-generated value as $\Gamma \equiv \Gamma_A + \Gamma_P = I^* \cdot \bar{\gamma}$, where $\bar{\gamma} \equiv \Gamma/I^*$ is value per active anchor. Between grid points, the change decomposes exactly as

$$\Delta\Gamma = \underbrace{\Delta I^* \cdot \bar{\gamma}_0}_{\text{extensive margin}} + \underbrace{I_0^* \cdot \Delta\bar{\gamma}}_{\text{per-anchor intensity}} + \underbrace{\Delta I^* \cdot \Delta\bar{\gamma}}_{\text{interaction}}.$$

E.3.3 Result. From the baseline ($\sigma_s = 0.80$) to maximum dispersion ($\sigma_s = 0.975$), Γ falls from 0.511 to 0.194. The extensive margin accounts for 98.1 percent of the decline (−0.311 of −0.317); per-anchor intensity contributes −0.016 and the interaction +0.010. Over the full grid ($0.60 \rightarrow 0.975$), the extensive share is 97.0 percent. The composition of surviving coordination is nearly invariant: per-anchor value moves only from 8.05 to 7.80, network size from 31 to 34, and effort per relationship from 0.220 to 0.198, while equilibrium anchor density collapses from 0.0636 to 0.0249 — a 61 percent contraction — and the threshold rises from 26.0 to 43.8. Greater dispersion at constant mean capability therefore reduces coordination-generated value not by reallocating coordination toward intrinsically more productive anchors — per-anchor coordination value is approximately constant along the sweep — but by contracting the mass of firms for which anchor operation is viable: the selection margin, not the intensity margin. This is the decomposition referenced in Section 7.4. Two ancillary facts from the same grid: $\Delta > 0$ at every point (Regime A throughout), and ξ^*shrinks monotonically in magnitude from −0.895 to below 0.003 in absolute value at the grid boundary, where its sign is not numerically resolvable on the planner's flat plateau (the threshold network jumps from 29 to 30) — the entry instrument losing traction as the economy approaches the thin-anchor limit, the quantitative counterpart of Proposition 3's vanishing-responsiveness result (D.7.7). The plotted gain series are piecewise smooth; visible jumps, including the dip at $\sigma_s = 0.95$ (gain 15.98 percent), coincide with integer transitions of the marginal anchor's network and are not solver artifacts; no smoothing is applied (Protocol V8.1, Section 4).