Why Hasn't the Equity Premium Fallen? Risk-Sharing, Firm Dynamics and Growth*

Joel M. David† FRB Chicago

David Zeke‡ Federal Reserve Board

July 14, 2026

Abstract

Despite dramatic advances in financial development and the resulting diversification and risk-sharing opportunities, the equity premium has not trended downward through time. We document this fact using cross-country panel data for the United States and other developed economies over the last 150 years, and for the world's most important stock markets at each point in time over the last four centuries (the Netherlands, U.K. and U.S.). We develop a theory that rationalizes this fact, incorporating uncertainty and risk into a workhorse model of firm dynamics: when firms have heterogeneous risk profiles, increased diversification and risk-sharing reduce within-firm risk premia and simultaneously induce new entry by, and resource reallocation toward, high risk firms. We derive conditions under which these two effects offset and the aggregate equity premium, though fluctuating in the short-run in response to shocks, remains constant in the long-run. The aggregate equity premium is determined by the tail behavior of the distribution of firm-level risk premia, a novel notion of 'tail risk.' We empirically corroborate our findings using data on firm-level stock returns in the U.S. dating back to 1929. The theory has new implications for the contribution of financial development to economic growth via firm dynamics, e.g., entry, exit, and the reallocation of resources towards higher productivity, but riskier, technologies.

*We thank Ian Dew-Becker, Francois Gourio, Carolin Pflueger and various seminar and conference participants for helpful comments and suggestions, and Adalyn Schommer for excellent research assistance. The views expressed here are those of the authors and not necessarily those of the Federal Reserve Bank of Chicago, the Federal Reserve Board of Governors, or the Federal Reserve System.

†joel.david@chi.frb.org.

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Financial market development and the equity premium

Four centuries of financial market development

- Increased opportunities for diversification and risk-sharing

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Amsterdam, 1668



Job Berckheyde c. 1670

London, 1800s



Artist unknown

NYSE, today

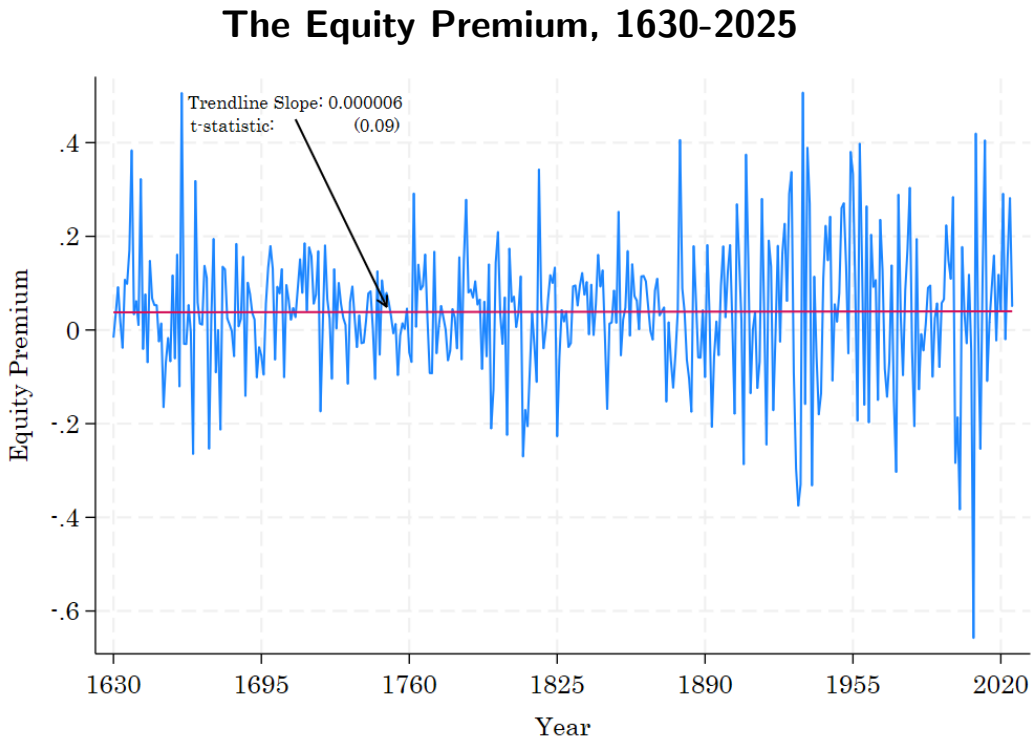


Julenne Schaer via NYC Tourism

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Four centuries of financial market development

- Increased opportunities for diversification and risk-sharing
- But, no trend in the equity premium...!



Data for The Netherlands/U.K. 1630-1812, U.K. 1814-1870, U.S. 1872-2025

Sources: Golez and Koudijs (JFE, 2018), Schmelzing (BOE, 2022), Jordà et al. (QJE, 2019), authors' calculations

Financial market development and the equity premium

Four centuries of financial market development

- Increased opportunities for diversification and risk-sharing
- But, no trend in the equity premium...!

$\Rightarrow$ Why hasn't the equity premium fallen?

Implications for role of financial development in economic growth?

This paper

Data: Document trend-less equity premium across measures, time and space

Theory: incorporate risk/uncertainty into workhorse model of firm dynamics

- $\uparrow$ Diversification/risk-sharing $\rightarrow$ $\downarrow$ price of risk
  1. Fall in **within-firm** risk premia
  2. **Entry/reallocation** towards riskier, high productivity firms
  3. Financial development induces growth via **selection**

Main results: derive conditions for constant equity premium

- Equity premium determined by **tail** of firm-level risk premia
  1. Disciplines role of financial development in growth ($\sim$ 2.5 b.p./year)
  2. Reallocation is key; without it, equity premium would be falling

Empirical evidence using U.S. firm-level (not today)

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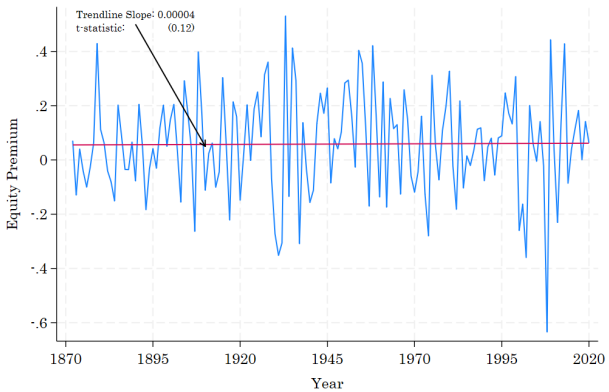
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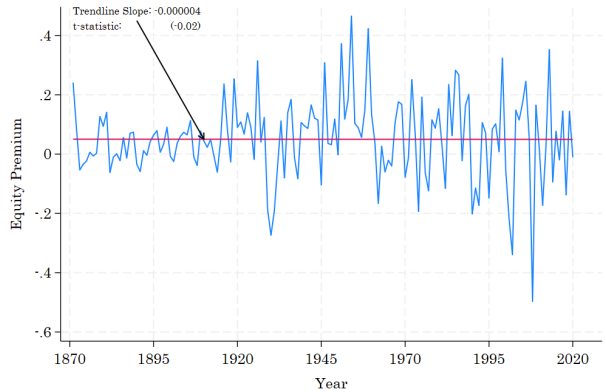
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The trend-less equity premium

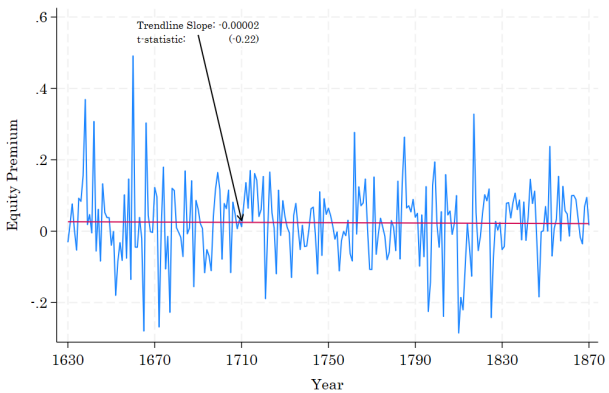
(a) U.S., 1872-2020



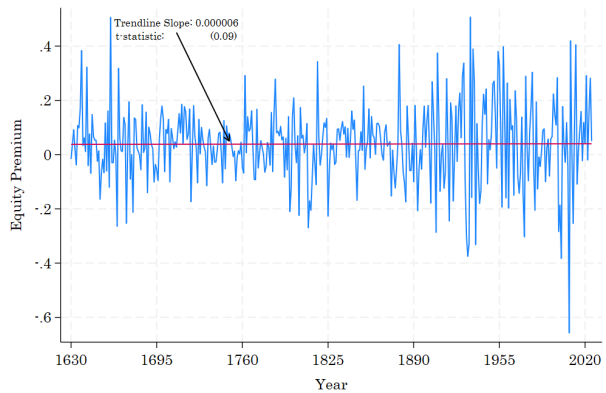
(b) 17 Advanced Economies, 1871-2020



(c) Netherlands/U.K. 1630-1870



(d) Netherlands/U.K. 1630-1870, U.S. 1872-2025



Data for The Netherlands/U.K. 1630-1812, U.K. 1814-1870, U.S. 1872-2020, Other economies 1871-2020

Sources: Golez and Koudijs (JFE, 2018), Schmelzing (BOE, 2022), Jordà et al. (QJE, 2019), authors' calculations

▶ Regressions ▶ Recent ▶ Lower frequency

Model

Main idea: introduce uncertainty/risk into workhorse firm dynamics model

Entrepreneurs operate firms, produce single good: $\hat{Y}_i = \hat{A}_i L_i^{\alpha}, \quad \alpha < 1$

- Productivity: expectation $A_i \equiv \mathbb{E}[\hat{A}_i]$, shock $\varepsilon_i = \frac{\hat{A}_i}{A_i} - 1$, $\text{var}(\varepsilon_i) = \sigma^2_{\varepsilon_i}$
- Firms choose L_i before shocks realized, pay fixed cost of operation

Value maximization: $\max_{L_i} \ \mathbb{E}[\Lambda_i \Pi_i], \quad \Pi_i = \hat{A}_i L_i^{\alpha} - W L_i - W f_c$

Labor and expected output:

$$L_i = \left(\frac{\alpha}{W} A_i (1 - \kappa_i)\right)^{\frac{1}{1-\alpha}}, \quad Y_i \equiv \mathbb{E}[\hat{Y}_i] = A_i^{\frac{1}{1-\alpha}} \left(\frac{\alpha}{W}(1 - \kappa_i)\right)^{\frac{\alpha}{1-\alpha}}$$

- $\kappa_i \equiv -\text{cov}\left(\frac{\Lambda_i}{\mathbb{E}[\Lambda_i]}, \varepsilon_i\right)$ is firm-specific risk premium in the labor choice
- $\uparrow \kappa_i \ \rightarrow \ \downarrow L_i$ and Y_i

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Valuations and returns

Firm valuations and stock market returns: $V(A_i, \kappa_i) = \mathbb{E}[\Lambda_i \Pi_i], \quad R_i = \frac{\Pi_i}{V_i}$

Firm values:

$$V(A_i, \kappa_i) = \mathbb{E}[\Lambda_i]\left((1-\alpha)(1-\kappa_i) Y_i - W f_c\right)$$

- Value increasing in A_i (through Y_i), decreasing in κ_i

Expected (excess) stock returns:

$$\mathbb{E}[R_i^e] \equiv \frac{\mathbb{E}[R_i]}{R_f} - 1 = \frac{\kappa_i Y_i}{(1-\alpha)(1-\kappa_i) Y_i - W f_c}$$

- Expected return increasing in κ_i, decreasing in A_i (increasing in $\frac{W f_c}{Y_i}$)

Volatility of returns:

$$\sigma_{r_i}^2 \equiv \text{var}(R_i^e) = \left(\frac{Y_i}{(1-\alpha)(1-\kappa_i) Y_i - W f_c}\right)^2 \sigma_{\varepsilon_i}^2 = \left(\frac{\mathbb{E}[R_i^e]}{\kappa_i}\right)^2 \sigma_{\varepsilon_i}^2$$

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Factor structure of risk

Firms face both aggregate and idiosyncratic risk:

$$\varepsilon_i = \beta_i \varepsilon + \tilde{\varepsilon}_i \quad \Rightarrow \quad \sigma_{\varepsilon_i} = \sqrt{\beta_i{}^2 \sigma_\varepsilon^2 + \tilde{\sigma}_{\varepsilon_i}^2}$$

- Firm-specific exposure β_i to aggregate shock ε, with volatility σ_ε^2
- Firm-specific volatility $\tilde{\sigma}_{\varepsilon_i}^2$ of idiosyncratic shock $\tilde{\varepsilon}_i$

Simple model of frictional financial markets Details

- Entrepreneurs diversify via risk-free bond, risk aversion γ
- Risk-neutral mutual fund obtains funds from HHs, trades in firm shares
- Mutual fund retains fraction δ of firm returns, convex cost for shares (τ)

$\Rightarrow$ Linear factor structure of risk: $\kappa_i = \overline{\kappa} \Theta_i, \quad \overline{\kappa} = \sqrt{\frac{\tau \gamma}{\delta}}, \quad \Theta_i = \sigma_{\varepsilon_i}$

More generally: results hold for any linear factor structure of the form

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Entry, exit and equilibrium

Entrepreneurs pay entry cost f_e to draw values of Θ_i and A_i

- Mass of new entrants M_e
- For each A_i, firm will operate if $\kappa_i \leq \kappa^*(A_i)$ where $V(A_i, \kappa^*(A_i)) = 0$
- Free entry: $\mathbb{E}[V(A_i, \kappa_i)] = \mathbb{E}[\Lambda_e] W f_e$

Equilibrium:

- Mass of operating firms $M = P_e M_e$ where P_e is prob. of successful entry
- Labor market clearing: $L = L_p + L_f + L_e$

Exogenous distributions:

- Productivity is Pareto (shape ζ): $dG(A_i) = \zeta A_{min}^{\zeta} A_i^{-\zeta-1}, \quad A_i \in [A_{min}, \infty)$
- Risk is inverse Pareto (shape ω): $dF(\Theta_i) = \omega \Theta_{max}^{-\omega} \Theta_i^{\omega-1}, \quad \Theta_i \in [0, \Theta_{max}]$

  $\rightarrow$ κ_i also inverse Pareto: $dF(\kappa_i) = \omega \left(\frac{1}{\kappa \Theta_{max}}\right)^{\omega} \kappa_i^{\omega-1}$

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The trend-less equity premium

Aggregate equity premium: $\mathbb{E}[R^e] = \int \mathbb{E}[R_i^e] \frac{V_i}{\int V_i di} di$

Proposition: If $\overline{\kappa}$ is not too small ($\overline{\kappa} > \frac{1}{\Theta_{max}}$), then $\mathbb{E}[R^e] = \omega$!

- Equity premium constant; equals tail parameter of risk distribution
- Independent of all else, e.g., risk aversion, nature of shocks, etc.
- Does not change with financial market development ($\overline{\kappa}$)
- Result relies on distribution of Θ_i; holds for each level of A_i
- Reallocation is key: o/w equity premium falls with $\overline{\kappa}$

Intuition

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▸ Intuition

Diversification/risk-sharing and economic growth

Expected aggregate output and the wage:

$$Y = \overline{\kappa}^{-\frac{\omega}{\zeta}} \frac{\omega+\zeta}{\zeta} \left(\frac{\omega \mathcal{B}(\omega, 1+\zeta)}{\Theta_{max}^{\omega}} \right)^{\frac{1}{\zeta}} Y^*, \qquad W = \frac{\zeta}{\omega+\zeta} \frac{Y}{L}$$

- Elasticity of output to the price of risk: $\Delta \log Y = -\frac{\omega}{\zeta} \Delta \log \overline{\kappa}$

Decomposition:

$$Y = Const. \times \underbrace{M}_{\text{mass of firms}} \times \underbrace{A^*(0)}_{\text{min. productivity}} \times \underbrace{\mathcal{M}}_{\text{misallocation}} \times \underbrace{L_p^{\alpha}}_{\text{production labor}}$$

- M, $\mathcal{M}$, L_p constant, independent of $\overline{\kappa}$ $\rightarrow$ $Y \propto \frac{A^*(0)}{A^*} Y^*$

$\Rightarrow$ Diversification/risk-sharing affects output only via selection

▸ Intuition

Diversification/risk-sharing and economic growth

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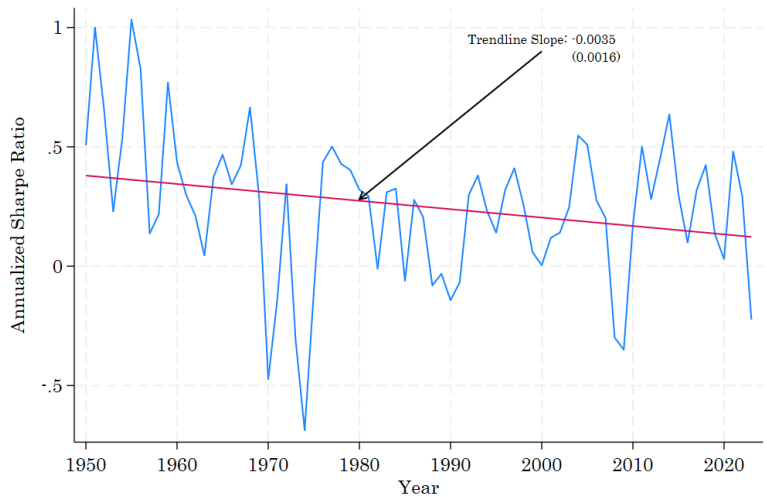
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Average U.S. Firm Sharpe Ratio, 1950-2023 ▸ Details



- $SR \downarrow$ from 0.38 to 0.12 $\rightarrow$ $\Delta \log Y = -\frac{\omega}{\zeta} \Delta \log SR \approx 1.7\%$
- Contribution to growth modest to date, $\approx$ 2.5 b.p./year
- Could be larger once $\overline{\kappa} < \frac{1}{\Theta_{max}}$; will know b/c equity premium will fall

Conclusion

Why hasn't the equity premium fallen with financial development?

- Entry of riskier firms keeps the equity premium constant
- Reallocation is key; without it, equity premium would fall

Implications for role of financial development in economic growth

- Diversification opportunities affect growth via selection
- Constant equity premium disciplines quantitative effects

To-do list

- Theory: within/reallocation decomposition, case when $\overline{\kappa}$ small
- Firm-level evidence of distribution of risk

The trend-less equity premium – regressions

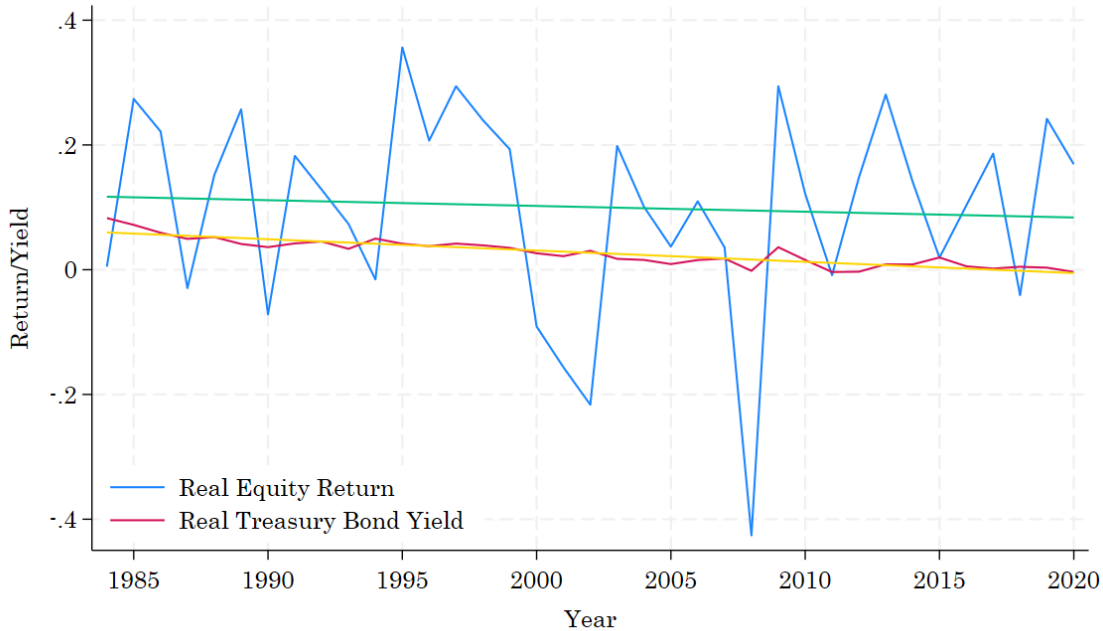
Estimate time-trend:

$$R_t^e = \alpha + \beta t + \varepsilon_t$$

	(1) U.S. 1872-2020	(2) U.S. 1872-2025	(3) Global 1871-2020	(4) Netherlands/U.K. 1630-1870	(5) Long Sample 1630-2025	(6) U.S. Bills 1872-2025	(7) U.S. Corp. 1872-2025
Time Trend	0.000042	0.00018	-0.0000044	-0.000022	0.0000056	0.00044	0.00023
	(0.12)	(0.52)	(-0.02)	(-0.22)	(0.09)	(1.35)	(0.71)
Observations	149	154	150	240	395	154	154

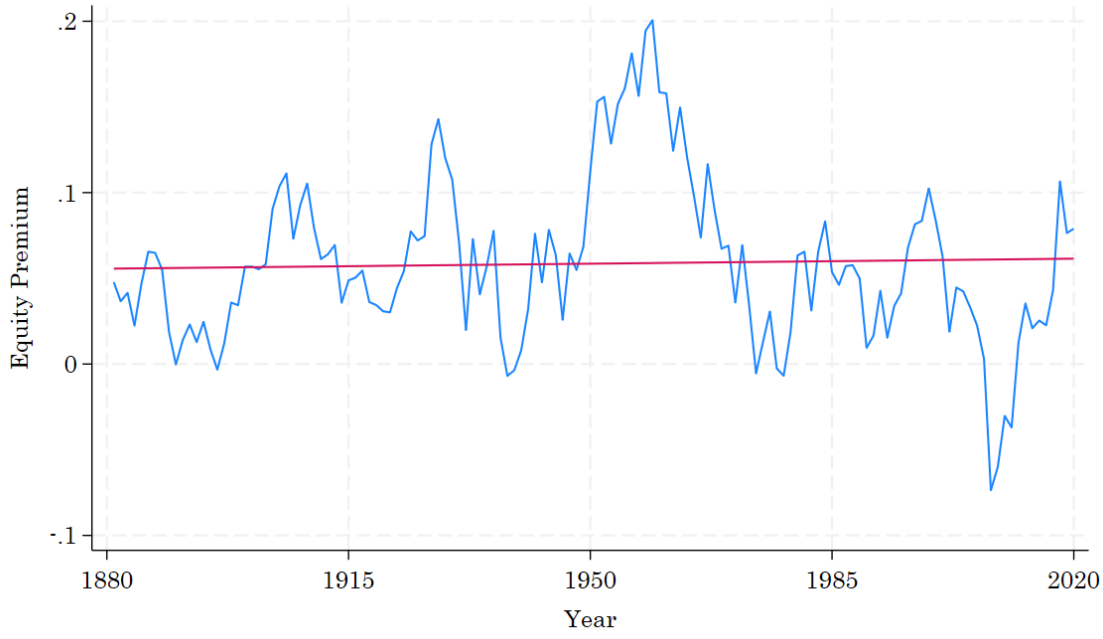
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U.S. real equity returns and Treasury bond yields, 1984-2020



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U.S. equity premium, decadal moving average



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Financial markets

Entrepreneurs hold shares in their own firm or trade them for risk-free bond

- Fraction of shares held by entrepreneur: $P_i = \frac{1}{\gamma}\frac{\mathbb{E}\left[R_i^e\right]}{\sigma_{r_i}^2} = \frac{1}{\gamma}\frac{\kappa_i^2}{\mathbb{E}[R_i^e]\sigma_{\varepsilon_i}^2}$

Mutual fund also trades in firm shares, holds fraction F_i

- Obtains funds from HHs, keeps fraction δ of excess return, cost $\frac{\tau}{1-F_i}$
- No arbitrage condition: $\delta\mathbb{E}\left[R_i^e\right] = \frac{\tau}{1-F_i}$

Market clearing: $P_i + F_i = 1$

$$\Rightarrow \quad \kappa_i = \overline{\kappa}\Theta_i, \quad \text{where} \quad \overline{\kappa} = \sqrt{\frac{\tau\gamma}{\delta}} \quad \text{and} \quad \Theta_i = \sigma_{\varepsilon_i} = \sqrt{\beta_i^2\sigma_\varepsilon^2 + \tilde{\sigma}_{\varepsilon_i}^2}$$

HHs (net worth W_h) supply funds to mutual fund or hold risk-free bond

- Supply all funds demanded by mutual fund if $\frac{1-\delta}{\gamma}\frac{\mathbb{E}\left[R_p^e\right]}{\sigma_{r_p}^2}W_h \geq \int F_i V_i di$
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Financial development and the equity premium

With financial development, τ and hence $\overline{\kappa}$ fall; for a given A_i:

- Expected returns fall for incumbent firms
- Riskier, higher expected return firms enter and produce

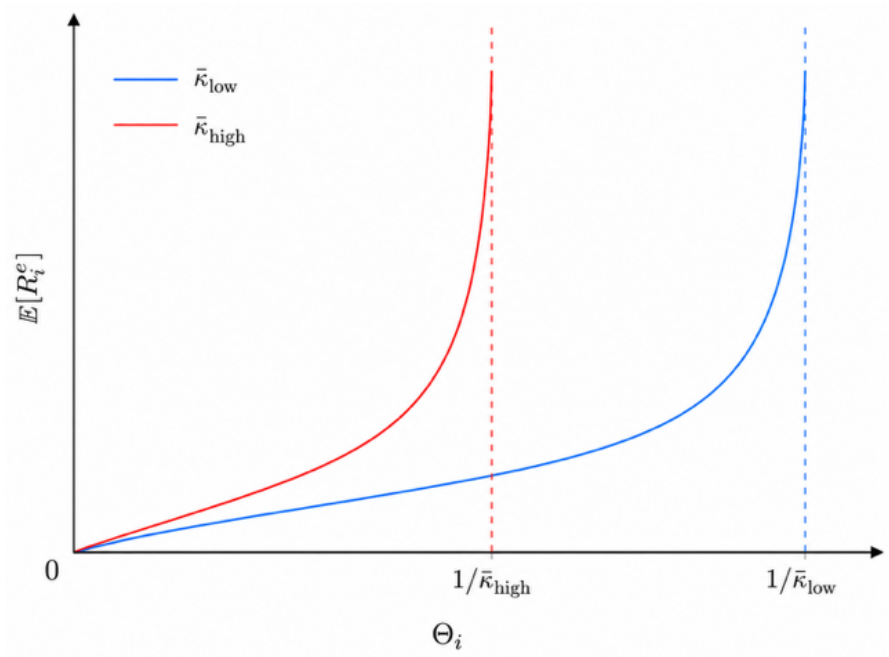
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Financial development and selection

Threshold condition:

$$V\left(A^*\left(\overline{\kappa}\Theta_i\right), \overline{\kappa}\Theta_i\right) = 0 \quad \Rightarrow \quad A^*\left(\overline{\kappa}\Theta_i\right) = \frac{W f_c^{1-\alpha}}{\alpha^\alpha \left(1-\alpha\right)^{1-\alpha}} \frac{1}{1-\overline{\kappa}\Theta_i}$$

- $\downarrow \overline{\kappa} \rightarrow \uparrow W \rightarrow \uparrow A^*(0)$ and $\downarrow \overline{\kappa}\Theta_i$

$\Rightarrow$ Selection: less risky low prod. firms exit, riskier high prod. firms enter

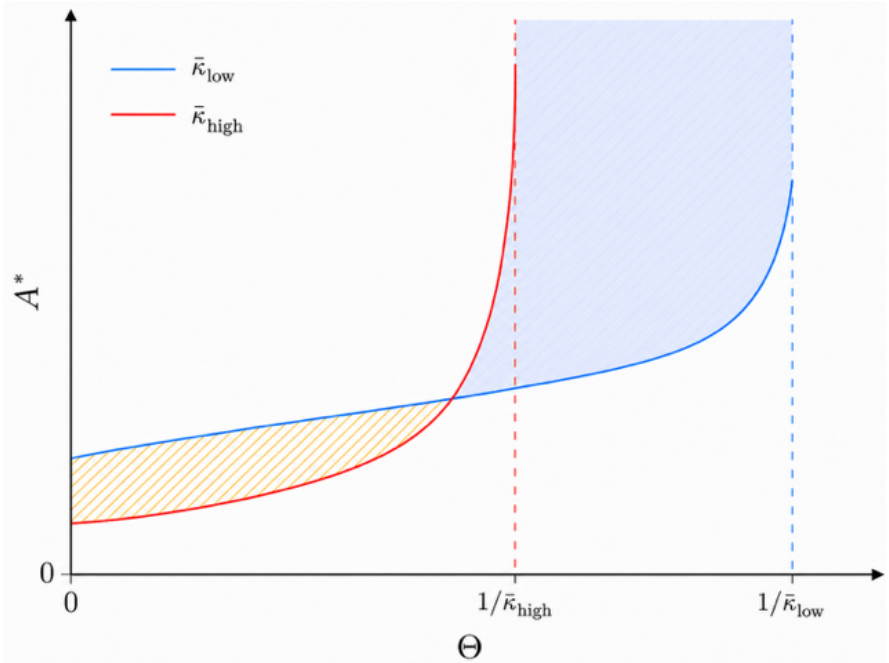
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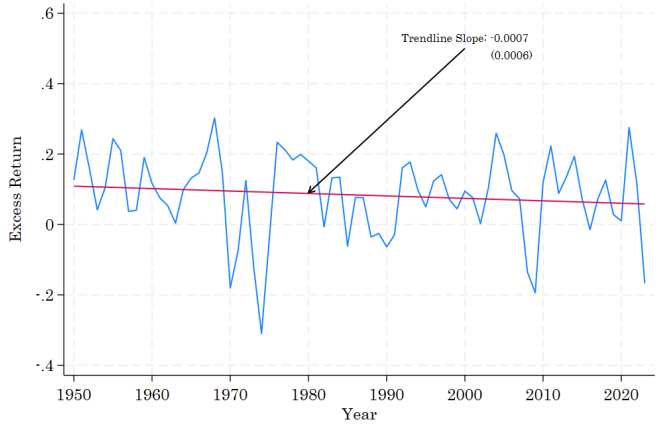
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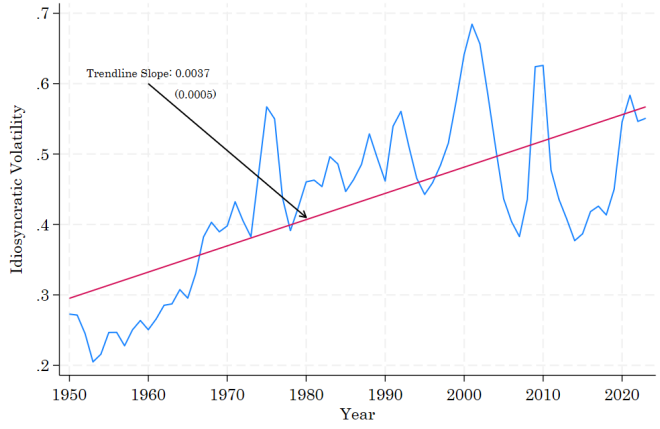
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Components of firm-level Sharpe ratio





- Average firm return virtually flat; large increase in average firm volatility

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