Technology Standardization in a Network Industry with Overlapping Ownership*

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Abstract

This paper examines a differentiated duopoly with overlapping ownership where the network externality is present via cost channel. The firms choose technology simultaneously followed by quantities. This study found the existence unique or multiple equilibrium with standardization. If the degree of product differentiation and overlapping ownership is high, the industry is always standardized and characterized with socially desired equilibrium, whatever be the firms technology choice is. With overlapping ownership standardization equilibrium is found welfare-superior, which has serious policy implications for antitrust authorities.

Keywords: *Overlapping Ownership, Network Externality, Technology Standardization, Competition.*

JEL classification: *D43; L13; L14*

*The authors received no external funding or financial support for this research. The authors declare no financial relationships, competing interests, or other potential conflicts of interest relevant to this submission.

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1 Introduction

Overlapping ownership, whether it is common ownership, i.e., stakes in competing firms by institutional investors, or cross ownership, i.e., firms owning stakes in rivals, is an increasingly common feature in global economy.[1] [2] At the same time, with the prevalence of internet-related technologies, industries have shown strong presence of network externalities. Especially, industries like telecommunication, airlines, banking, retailing, etc. are featured with both network externality and overlapping ownership.

Following Rubinstein and Yaari (1983) and Rotemberg (1984), the markets with overlapping ownership are extensively studied in various contexts. In presence of overlapping ownership, firms (partially) internalize the effect of aggressive competition strategies on their rival's profit and as a result the competition is soften. This implies higher profits for the firms and reduction in consumer surplus as well as social welfare [O'Brien and Salop (1999), Vertager (2018), Azar(2020), Azar and Vives (2021)]. However, in several cases overlapping ownership helps to internalize externalities like technological spillovers and thus may be welfare-improving [López and Vives (2019), Sato and Matsumura (2020)]. Welfare improving overlapping ownership can be observed in case of (a) unionized labor market [Fanti (2014)], (b) knowledge transfers between the firms [Ghosh and Morita (2017)], (c) vertical quality differentiation [Wang and Wang (2023)] and (d) suitable tax/subsidy policy by the government [Cheng et al. (2024)]. But all these papers while considering overlapping ownership did not consider the presence of network externality.

Only a few papers have considered the implications of overlapping ownership in network industries. Pi and Zhang (2024) investigated the effects of common ownership in two-sided platform competition. When both sides singlehome, the consumer surplus of the side treated as a loss leader increases in presence of common ownership, but consumer surplus of the other side decreases. When one side multihome, consumer surplus of the singlehoming side decreases. But for both the cases, the social welfare remains same as the gain in platform profits balances the loss in consumer surplus. Buccella, Fanti and Gori (2025a) analyzed a network industry to determine the optimal cross-holdings. Buccella, Fanti and Gori (2025b) examined the role of compatibility in case of a network duopoly with passive unilateral cross-ownership and found that the profitability can fall and social welfare can rise for certain degree of product compatibility. However, all of these paper consider network externality as positive consumer externality and none of these papers examines the technology standardization process for a network industry with overlapping ownership.

Technology standardization is common in industries with positive network externality because by making its product compatible, a firm can manage to raise its profit serving greater market share by the stimulus of network effect. Starting from Katz and Saphiro (1985, 1986) and Ferrell and Saloner (1985, 1986), there are substantial researches on technology choice of the firms from various aspects [Katz and Saphiro (1994), Besen and Ferrell (1994), Economides (1996),

[1] for empirical studies in different industries see OECD (2017), Azar et al.(2018, 2022), He and Huang (2017), He, Huang and Zhao (2019), Gutiérrez and Philippon (2017), Boller and Morton (2020), Lewellen and Lowry (2021), Schmalz and Xie (2022), Newham et al. (2018), Gao et al.(2022), Banal-Estanol (2021), Adler and Mitkov (2023), Chen et al. (2024), Peng, Yin and Zhang (2023), Freeman (2025) among others.

[2] In this study, we have ignored the difference between common ownership and cross ownership as the objective function for the firms remain same. The terms 'common ownership', 'cross ownership' and 'overlapping ownership' are interchangeably used.

Matutes and Regibeau (1989), Regibeau and Rockett (1996), Kim (2002), Hahn and Kim (2012), Suleymanova and Wey (2012), Chen et al.(2009), Toshimitsu (2018), Buccella, Fanti and Gori (2023)]. Majority of these papers have considered network externality as demand side phenomenon. Only a few studies like Bloch (1995), Belleflamme (1998) have described network externality as supply side phenomenon acting through cost channel as the more firms implement any technology, the marginal cost of production using that technology decreases. Belleflamme (1998) considered a differentiated oligopoly with network externality acting via cost channel and analyzed the technology adoption decision for firms. The study found the existence of unique or multiple equilibria with technology standardization. However, there exists no such study that analyzes the technology adoption decisions by the firms for network industry featuring overlapping ownership.

Following Belleflamme (1998), we have considered a theoretical model with a differentiated duopoly featured by overlapping ownership where the network externality works via cost channel. The firms simultaneously decide what technology to adopt followed by quantity competition. We have found equilibrium technology standardization is always exists whatever be the degree of product substitutability or overlapping ownership. However, when the conflict of interest is high between the firms in terms of technology preference, the technology standardization does not happen. But if the overlapping ownership is high enough, technology standardization happens always whatever be the firm's preference of technology. Contrary to Belleflamme (1998), we found that the possibility of non-standardization rises with product substitutability when the overlapping ownership is high enough. The possibility of non-standardization always reduces as overlapping ownership rises. We have also found that with overlapping ownership, standardization equilibrium is welfare-maximizing. The socially optimal equilibrium always happens when the overlapping ownership is high enough given the product substitutability.

This paper contributes to the existing literature in several ways. Firstly, this is a novel attempt to investigate the technology standardization in a network industry featuring overlapping ownership. Secondly, Our study gives fresh insights on the interaction between product differentiation and overlapping ownership in presence of network externality working via cost channel. Thirdly, We found welfare-superior technology standardization with overlapping ownership, which has several policy implications for antitrust authorities. The antitrust authority should consider its policy concerning overlapping ownership in network industry observing the technology preferences of the firm keeping its welfare-improving effect in mind.

The rest of the paper is organized as follows. The section 2 provides the description of our model. In section 3, we solve the second stage of the game, the quantity competition while the choice of technology by the firms is discussed in section 4. Section 5 deals with the competitive statics exercises. In section 6 provides the welfare analysis and we concludes in section 7.

2 The Model

We consider a simple economy consisting of two sectors. One of the sector is a competitive sector producing a numeraire commodity (money). The other sector consists of a standard duopoly producing commodities that are not perfectly substitutes. We assume, the utility function of a representative consumer is given by

$$U(q) = m + a(q_1 + q_2) - \frac{1}{2}(q_1^2 + q_2^2 + 2\beta q_1 q_2) \tag{1}$$

where $m(>0)$ is the quantity of the numeraire good, $a > 0$ is the preference parameter. The quantity produced by firm $i(i = 1, 2)$ is given by q_i and $q = (q_1, q_2)$. The degree of product differentiation is given by $\beta \in [0, 1)$.

Under the utility function of the representative consumer specified by equation (1), if the price of the firm i's product is p_i, the (inverse) demand function for firm i is given by (Singh and Vives (1984))

$$p_i(q) = a - q_i - \beta q_j; i, j = 1, 2, i \neq j. \tag{2}$$

Hence, The consumer's surplus is given by

$$CS(q) = U(q) - p_1 q_1 - p_2 q_2 = \frac{q_1^2}{2} + \frac{q_2^2}{2} + \beta q_1 q_2. \tag{3}$$

The firms have two alternative choices of technology, namely, X and Y. They have different preferences for two available technologies. These preferences are represented by $\theta_i \in [0, 1]$ and are common knowledge. Without any loss of generality, we assume $0 \leq \theta_1 \leq \theta_2 \leq 1$. We assume the network effect works via cost channel a lá Belleflamme(1998). The size of the network depends on technology chosen by both firms. If $t_i \in \{X, Y\}$ denotes the technology chosen by firm i $(i = 1, 2)$ then for any $i, j = 1, 2$ and $i \neq j$ we have the size of the network resulting from the technology choices by the firms as

$$n(t_1, t_2) = \begin{cases} 2 & if \quad t_1 = t_2 \\ 1 & if \quad t_1 \neq t_2. \end{cases} \tag{4}$$

Further, the firms' choice of technology not only determine the network size, but the mode of production technology also. If $t_1 = X$ and $t_2 = X$, then we call the prevailing mode of the technologies by XX. Similarly, we define XY, YX and YY. [3] Therefore, the set of mode of technologies is $\Omega^M = \{XX, YX, XY, YY\}$. Given any mode of technology $t_1 t_2$ as well as the network size, $n(t_1, t_2)$, arises due to the mode of technology, and the preferences for technology $\theta = (\theta_1, \theta_2)$, the marginal cost of firm $i = 1, 2$, is

$$c_i^{t_1 t_2}(\theta) = \begin{cases} c - n(t_1, t_2)(1 - \theta_i) & \text{for } t_i = X \\ c - n(t_1, t_2)\theta_i & \text{for } t_i = Y. \end{cases} \tag{5}$$

where $c < a$. The vector of the marginal costs, $c^{t_1 t_2}(\theta) = (c_1^{t_1 t_2}(\theta_1), c_2^{t_1, t_2}(\theta_2))$ is given by Table 1.

$t_1 \backslash t_2$	X	Y
X	$c - 2 + 2\theta_1, c - 2 + 2\theta_2$	$c - 1 + \theta_1, c - \theta_2$
Y	$c - \theta_1, c - 1 + \theta_2$	$c - 2\theta_1, c - 2\theta_2$

Table 1: $c_i^{t_1 t_2}(\theta)$ for any $t_1 t_2 \in \Omega^M$

The profit function of the firm i is given by the following:

$$\Pi_i^{t_1 t_2}(q, \theta) = (p_i(q) - c_i^{t_1 t_2}(\theta_i))q_i; i, j = 1, 2; i \neq j. \tag{6}$$

[3] Therefore, given our definition, if $t_i = t_j$, firms are standardized in the same technology.

The social welfare is given by the sum of the consumer surplus and the total profits earned by the firms.

$$W^{t_i t_2}(q,\theta) = CS(q) + \Pi_1^{t_1 t_2}(q,\theta) + \Pi_2^{t_1 t_2}(q,\theta) \tag{7}$$

Moreover, the industry is featured by overlapping ownership between the firms. We assume symmetric overlapping ownership between the firms given by $\alpha \in [0, \frac{1}{2}]$. Thus, the objective function of firm i becomes

$$\begin{aligned} \bar{V}_i^{t_1 t_2}(q,\theta) &= (1-\alpha)\Pi_i^{t_1 t_2}(q,\theta) + \alpha\Pi_j^{t_1 t_2}(q,\theta) \\ &= (1-\alpha)[\Pi_i^{t_1 t_2}(q,\theta) + \frac{\alpha}{1-\alpha}\Pi_j^{t_1 t_2}(q,\theta)] \\ &= (1-\alpha)[\Pi_i^{t_1 t_2}(q,\theta) + \lambda\Pi_j^{t_1 t_2}(q,\theta)] \end{aligned} \tag{8}$$

where $\lambda = \frac{\alpha}{1-\alpha} \in [0,1]$. From the perspective of optimization, we can equivalently assume the objective function of the firm i is given as follows:

$$V_i^{t_1 t_2}(q,\theta) = \Pi_i^{t_1 t_2}(q,\theta) + \lambda\Pi_j^{t_1 t_2}(q,\theta); i,j = 1,2; i \neq j \tag{9}$$

Assumption 1. We assume the following parametric restrictions: $b := (a-c) > \frac{2\gamma}{2-\gamma} = b^S$ where $\gamma = \beta(1+\lambda)$.

We need this Assumption 1 to ensure that both firms produce a positive quantity under any mode of production technology. As we are interested in the standardization of production technology, it is essential to have both firms producing positive output. Therefore, this assumption is crucial for our analysis.

We can reinterpret Assumption 1 as $\gamma \leq \frac{2b}{2+b}$ as $b < 2$ when $\beta \in [0,1)$ and $\lambda \in [0,1)$. Further, after some algebraic manipulation, we can express the Assumption 1 as $\lambda \leq \frac{2b-(2+b)\beta}{(2+b)\beta} = \hat{\lambda}(\beta,b)$[4]. This means given any $b \in [0,\infty)$ we have

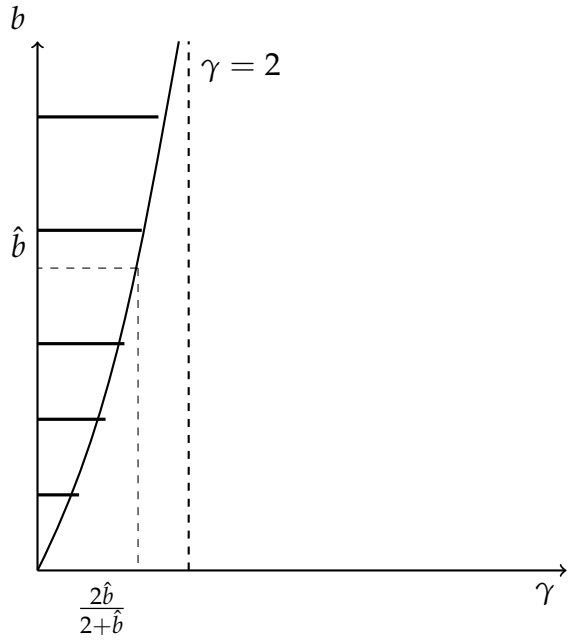
$$\lambda \begin{cases} \in [0,1] \forall \beta \in [0, \frac{b}{2+b}] \\ \in [0, \max[0, \hat{\lambda}(\beta,b)]] \forall \beta \in [\frac{b}{2+b}, 1) \end{cases}$$

The sequence of events is described in the following two-stage game.

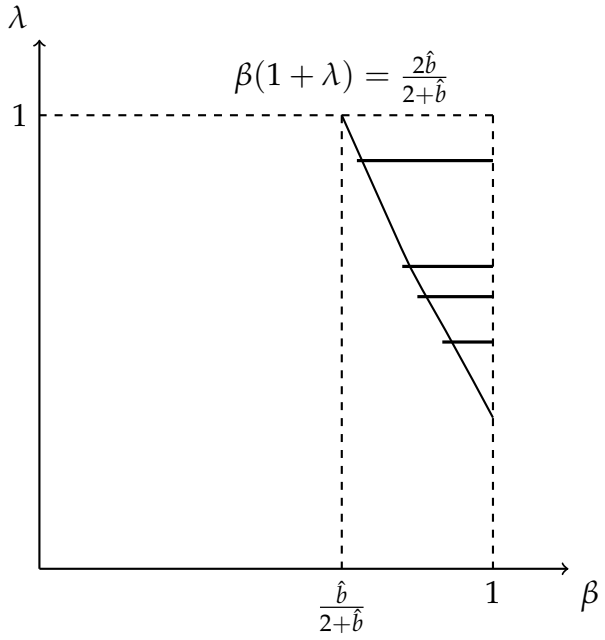
- **Stage-I:** Firms choose which of the available technologies is to be implemented. The firms choose production technology solving for the Nash equilibria for the game presented in Table 2.
- **Stage-II:** Given any mode of production technology, firms compete in Cournot fashion to choose quantity independently and simultaneously.

We apply the method of backward induction to solve the game.

[4] $\hat{\lambda}(\beta,b) \geq 0 \forall \beta \in [0,1)$ whenever $b \geq 2$.



(a) The restriction of b(shaded region)



(b) The restricted domain in $\beta - \lambda$ space for $\hat{b}$

Figure 1: Interpretation of Assumption 1.

$1\backslash 2$	X	Y
X	V_1^{XX}, V_2^{XX}	V_1^{XY}, V_2^{XY}
Y	V_1^{YX}, V_2^{YX}	V_1^{YY}, V_2^{YY}

Table 2: Incentives of technology choice under different cases

3 Equilibrium Analysis

3.1 Market Competition

At stage-II, the firms compete in quantities independently and simultaneously given the mode of production technology t_1t_2. Any firm, $i = 1, 2$, maximizes its objective function given by the equation (9) by choosing its own quantity q_i and taking its rival's quantity q_j fixed. If $q^{t_1t_2}(c^{t_1t_2}(\theta), \lambda, \beta) = (q_1^{t_1t_2}(c^{t_1t_2}(\theta), \lambda, \beta), q_2^{t_1t_2}(c^{t_1t_2}(\theta), \lambda, \beta))$ be the Stage-II optimum quantities, the following first order conditions are satisfied.

$$\frac{\partial V_1(q^{t_1t_2}(c^{t_1t_2}(\theta), \lambda, \beta), \theta)}{\partial q_1} = a - 2q_1^{t_1t_2}(c^{t_1t_2}(\theta), \lambda, \beta) - \beta(1+\lambda)q_2^{t_1t_2}(c^{t_1t_2}(\theta), \lambda, \beta) - c_1^{t_1t_2}(\theta_2) = 0 \tag{10}$$

$$\frac{\partial V_2(q^{t_1t_2}(c^{t_1t_2}(\theta), \lambda, \beta), \theta)}{\partial q_2} = a - 2q_2^{t_1t_2}(c^{t_1t_2}(\theta), \lambda, \beta) - \beta(1+\lambda)q_1^{t_1t_2}(c^{t_1t_2}(\theta), \lambda, \beta) - c_2^{t_1t_2}(\theta_2) = 0 \tag{11}$$

Solving equation(10) and equation(11), we get the optimal level of quantity produced by firm $i \in \{1,2\}$ and $i \neq j$ [5] as follows:

$$q_i^{t_1t_2} := q_i^{t_1t_2}(c^{t_1t_2}(\theta), \lambda, \beta) = \frac{2\left(a - c_i^{t_1t_2}(\theta_i)\right) - (1+\lambda)\beta\left(a - c_j^{t_1t_2}(\theta_j)\right)}{\Delta}. \tag{12}$$

where $\Delta = 4 - (1+\lambda)^2\beta^2$.

Therefore, the price charged, Stage-II Profit, and Stage-II objective function of the firm i are respectively as follows.

$$p_i^{t_1t_2} := P_i(q^{t_1t_2}) = c_i^{t_1t_2}(\theta) + q_i^{t_1t_2} + \beta\lambda q_j^{t_1t_2} \tag{13}$$

$$\Pi_i^{t_1t_2} := \Pi_i^{t_1t_2}(q^{t_1t_2}, \theta) = (q_i^{t_1t_2} + \beta\lambda q_j^{t_1t_2})q_i^{t_1t_2} \tag{14}$$

$$V_i^{t_1t_2} = V_i^{t_1t_2}(q^{t_1t_2}, \theta) = (q_i^{t_1t_2})^2 + \lambda(q_j^{t_1t_2})^2 + \beta\lambda(1+\lambda)q_i^{t_1t_2}q_j^{t_1t_2} \tag{15}$$

Putting the optimal value of the quantity produced in the equation(9) and the respective marginal costs of production $c_1^{t_1t_2}(\theta_1)$ and $c_2^{t_1t_2}(\theta_2)$ as shown in Table 1, we get the values of the objective functions for each case of technology choice.

Lemma 1. Given the Assumption 1, for all $\theta \in [0,1]^2$ such that $\theta_1 \leq \theta_2$, the Stage-II equilibrium quantities chosen by both the firms are positive for any mode of production technology $t_1t_2 \in \Omega^M$.

Given Lemma 1 combined with equation (13) to (15), one can conclude that if both the firms produce positive output, then all the relevant resulting outcomes are positive.

3.2 Choice of Technology

In the stage-I, the firms choose between technology X and Y comparing their incentives, $V_i^{t_1t_2}(\theta; \beta, \lambda)$. We define the following incentives of firms to choose technology X over technology Y: the incentive of firm 1 given that firm 2 chooses technology X,$V_1^X(\theta; \beta, \lambda)$, the incentive of firm 1 given that firm 2 chooses technology Y, $V_1^Y(\theta; \beta, \lambda)$, the incentive of firm 2 given that firm 1 chooses technology X, $V_2^X(\theta; \beta, \lambda)$ and the incentive of firm 2 given that firm 1 chooses technology Y, $V_2^Y(\theta; \beta, \lambda)$.

Definition 1. We define the following incentive of firm i to choose technology X over technology Y given that firm j chooses any technology T, $V_i^T(\theta, \beta, \lambda)$; where $T \in \{X, Y\}$.

1. $V_1^X(\theta; \beta, \lambda) = V_1^{XX}(q_1^{XX}(\theta), q_2^{XX}(\theta)) - V_1^{YX}(q_1^{YX}(\theta), q_2^{YX}(\theta))$
2. $V_1^Y(\theta; \beta, \lambda) = V_1^{XY}(q_1^{XY}(\theta), q_2^{XY}(\theta)) - V_1^{YY}(q_1^{YY}(\theta), q_2^{YY}(\theta))$
3. $V_2^X(\theta; \beta, \lambda) = V_2^{XX}(q_1^{XX}(\theta), q_2^{XX}(\theta)) - V_2^{XY}(q_1^{XY}(\theta), q_2^{XY}(\theta))$
4. $V_2^Y(\theta; \beta, \lambda) = V_2^{YX}(q_1^{YX}(\theta), q_2^{YX}(\theta)) - V_2^{YY}(q_1^{YY}(\theta), q_2^{YY}(\theta))$.

[5] We have checked for the second order condition and it is satisfied.

Now, given the incentives of firms defined in Definition 1, the firm i chooses technology X if $V_i^T(\theta;\beta,\lambda) > 0$, technology Y if $V_i^T(\theta;\beta,\lambda) < 0$ and remains indifferent between the technologies if $V_i^T(\theta;\beta,\lambda) = 0$. Technology standardization happens when both the firms choose same technology.

Lemma 2. Given Assumption 1, for all $\beta \in [0,1)$ and $\lambda \in [0,1)$ the following holds:

(a) $V_1^Y(\theta;\beta,\lambda,b) \geq V_2^Y(\theta;\beta,\lambda,b)$ for all $0 \geq \theta_1 \geq \theta_2 \geq 1$. Further, $V_1^Y(\theta;\beta,\lambda,b) = V_2^Y(\theta;\beta,\lambda,b)$ at $\theta_1 = \theta_2 = \theta^1$.[6]

(b) $V_1^X(\theta;\beta,\lambda,b) \geq V_2^X(\theta;\beta,\lambda,b)$ for all $0 \geq \theta_1 \geq \theta_2 \geq 1$. Further, $V_1^X(\theta;\beta,\lambda,b) = V_2^X(\theta;\beta,\lambda,b)$ at $\theta_1 = \theta_2 = \theta^2$.[7]

(c) $V_1^X(\theta;\beta,\lambda,b) \geq V_1^Y(\theta;\beta,\lambda,b)$ for all $0 \geq \theta_1 \geq \theta_2 \geq 1$.

Combining Lemma 2 (a) and (c), we have

$$V_1^X(\theta;\beta,\lambda,b) \geq V_1^Y(\theta;\beta,\lambda,b) \geq V_2^Y(\theta;\beta,\lambda,b).$$

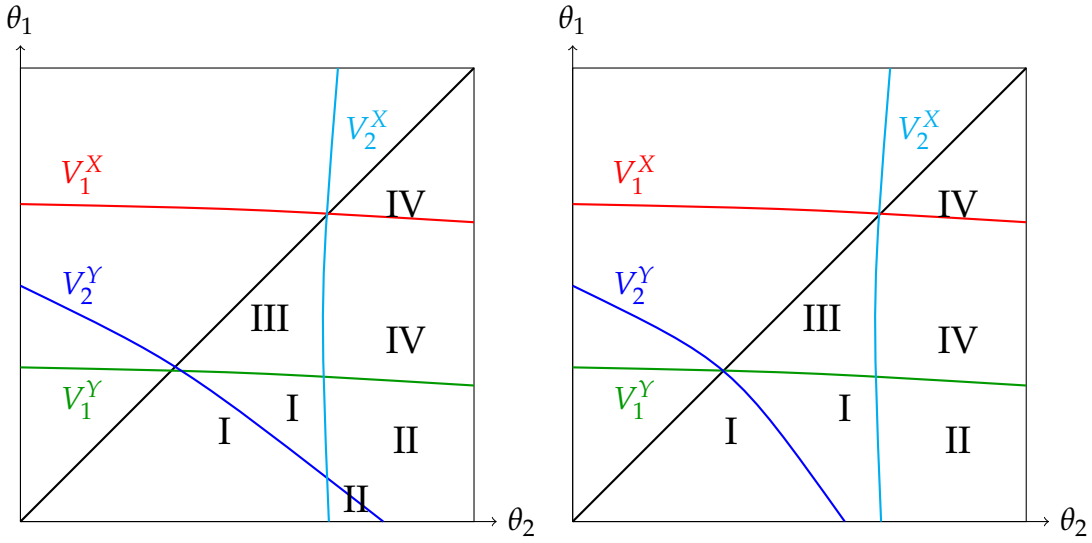
Therefore, we have the following possibilities.

1. If $V_2^Y(\theta;\beta,\lambda,b) \geq 0$, both $V_1^Y(\theta;\beta,\lambda,b) \geq 0$ and $V_1^X(\theta;\beta,\lambda,b) \geq 0$, therefore, it is dominant strategy for firm 1 to choose X. However, the firm 2's choice of technology will depend on $V_2^X(\theta;\beta,\lambda,b)$.
   (a) If $V_2^X(\theta;\beta,\lambda,b) \geq 0$ then firm 2 will choose X.
   (b) If $V_2^X(\theta;\beta,\lambda,b) < 0$ then firm 2 will choose Y.
2. If $V_1^X(\theta;\beta,\lambda,b) \geq V_1^Y(\theta;\beta,\lambda,b) \geq 0 \geq V_2^Y(\theta;\beta,\lambda,b)$, the similar outcomes are observed. The dominant strategy for firm 1 is to choose technology X. Moreover,the firm 2's choice of technology will depend on $V_2^X(\theta;\beta,\lambda,b)$.
   (a) If $V_2^X(\theta;\beta,\lambda,b) \geq 0$ then firm 2 will choose X.
   (b) If $V_2^X(\theta;\beta,\lambda,b) < 0$ then firm 2 will choose Y.
3. If $V_1^X(\theta;\beta,\lambda,b) \geq 0 \geq V_1^Y(\theta;\beta,\lambda,b) \geq V_2^Y(\theta;\beta,\lambda,b)$, then the best response of firm 1 is technology X (technology Y) if firm 2 chooses technology X (technology Y). Further, we have $V_2^Y(\theta;\beta,\lambda,b) \leq 0$, which implies firm 2 will always choose technology Y when firm 1 chooses Y. Hence, the mode of production technology YY is always an equilibrium outcome. Whether XX be a possible mode of production technology will be dependent on the sign of $V_2^X(\theta;\beta,\lambda,b)$. Here, we arrive with the following two possibilities:
   (a) When $V_2^X(\theta;\beta,\lambda,b) \geq 0$, we have multiple equilibria with different mode of production technology XX and YY.
   (b) When $V_2^X(\theta;\beta,\lambda,b) < 0$, the both firms will choose technology Y.

[6]The expression of θ^1 in the equation ().
[7]the expression of θ^2 in the equation ().

4. Finally, when $0 \geq V_1^X(\theta;\beta,\lambda,b) \geq V_1^Y(\theta;\beta,\lambda,b) \geq V_2^Y(\theta;\beta,\lambda,b)$, then by (b) from Lemma 2, we get $V_2^X(\theta;\beta,\lambda,b) \leq V_1^X(\theta;\beta,\lambda,b) \leq 0$. Therefore, both the firms' dominant strategy is technology Y and hence the prevalent mode of production technology is YY.

Thus, we have outcome-wise the following four cases.
Case I: Case I is a combination of possibility 1(a) and 2(a). If $V_1^Y(\theta;\beta,\lambda,b) \geq 0$ and $V_2^X(\theta;\beta,\lambda,b) \geq 0$, both the firms will choose technology X and the prevalent mode of production technology in the industry will be XX. The industry will be standardized to technology X.
Case II: Case II is a combination of possibility 1(b) and 2(b). If $V_1^Y(\theta;\beta,\lambda,b) \geq 0$ and $V_2^X(\theta;\beta,\lambda,b) < 0$, firm 1 will choose technology X and firm 2 will choose technology Y. As a result, the prevalent mode of production technology in the industry will be XY. Here, the industry will not be standardized to any single technology.
Case III: Possibility 3(a) is Case III. If $V_1^Y(\theta;\beta,\lambda,b) \leq 0$ and $V_2^X(\theta;\beta,\lambda,b) \geq 0$, then both the firms best response will be to choose technology X when the other firm chooses technology X and technology Y when the other firm chooses technology Y. Hence, multiple equilibria with different mode of production technology (XX or YY) arises. Here, two equilibria with two different technology standardization (X or Y) is possible.
Case IV: Combination of possibility 3(b) and 4 gives Case IV. If $V_1^X(\theta;\beta,\lambda,b) \leq 0$ and $V_1^X(\theta;\beta,\lambda,b) \leq 0$, then both the firms will choose technology Y and the prevalent mode of production technology in the industry will be YY. Here, the industry will be standardized to technology Y.



(a) Possibility 1 (b) Possibility 2

Figure 2: Partition of the $\theta_2 - \theta_1$ plane into regions with different signs of the incentives. The blue line indicates $V_2^Y(\theta;\beta,\lambda,b)$, the red line indicates $V_1^X(\theta;\beta,\lambda,b)$, the green line indicates $V_1^Y(\theta;\beta,\lambda,b)$ and the cyan line indicates $V_2^X(\theta;\beta,\lambda,b)$. Representative figure based on authors' calculation.

Observing the above discussion involving signs of the incentives, we can have the following remark.

Remark 1. *The firms' choices of technology and technology standardization will depend only on the signs of* $V_1^Y(\theta;\beta,\lambda,b)$ *and* $V_2^X(\theta;\beta,\lambda,b)$.

Equating $V_1^Y(\beta,\lambda,b)=0$, we solve for $\theta_1=\theta_1(\theta_2)$ to get

$$\tilde{\theta_1}(\theta_2;\beta,\lambda,b)=\frac{\Gamma_1(\theta_2;\beta,\lambda)}{6(\lambda(\lambda+1)^2\beta^2-4)} \tag{16}$$

where $\Gamma_1(\theta_2;\beta,\lambda,b)=(5\lambda^4\beta^3+15\lambda^3\beta^3+15\lambda^2\beta^3+5\lambda\beta^3-20\lambda\beta-20\beta)\theta_2+3\lambda^4\beta^3b+9\lambda^3\beta^3b-6\beta^2b\lambda^3+9\lambda^2\beta^3b-12\beta^2b\lambda^2+3\lambda\beta^3b-2\beta^2\lambda^3-6\beta^2b\lambda-4\beta^2\lambda^2-12\beta b\lambda-2\beta^2\lambda-12\beta b+24b+8-(9((\lambda+1)^4\lambda(b+(5\theta_2)/3)^2\beta^4-4(\lambda+1)^3(b+(4\theta_2)/3)\lambda(b+2/3)\beta^3+4(\lambda+1)^2((b^2+(-(4\theta_2)/3+4/3)b-2\theta_2^2+4/9)\lambda-(b+(4\theta_2)/3)^2)\beta^2+16(\lambda+1)(b+(4\theta_2)/3)(b+2/3)\beta+(32\theta_2(b+(3\theta_2)/2)\lambda)/3-16(b+2/3)^2)(-4+\lambda(\lambda+1)^2\beta^2))^{\frac{1}{2}}$. Similarly, equating $V_2^X(\beta,\lambda)=0$, we solve for $\theta_2=\theta_2(\theta_1)$ to get

$$\tilde{\theta_2}(\theta_1;\beta,\lambda,b)=\frac{\Gamma_2(\theta_1;\beta,\lambda,b)}{6(\lambda(\lambda+1)^2\beta^2-4)} \tag{17}$$

where $\Gamma_2(\theta_1;\beta,\lambda)=(5\lambda^4\beta^3+15\lambda^3\beta^3+15\lambda^2\beta^3+5\lambda\beta^3-20\lambda\beta-20\beta)\theta_1-3\lambda^4\beta^3b-9\lambda^3\beta^3b-5\lambda^4\beta^3+6\beta^2b\lambda^3-9\lambda^2\beta^3b-15\lambda^3\beta^3+12\beta^2b\lambda^2-3\lambda\beta^3b+8\beta^2\lambda^3-15\lambda^2\beta^3+6\beta^2b\lambda+16\beta^2\lambda^2-5\lambda\beta^3+12\beta b\lambda+8\beta^2\lambda+12\beta b+20\lambda\beta-24b+20\beta-32+(9((\lambda+1)^4\lambda(b-(5\theta_1)/3+5/3)^2\beta^4-4(\lambda+1)^3(b-(4\theta_1)/3+4/3)\lambda(b+2/3)\beta^3+4(\lambda+1)^2((b^2+4/3b\theta_1-2\theta_1^2+4\theta_1-14/9)\lambda-(b-(4\theta_1)/3+4/3)^2)\beta^2+16(\lambda+1)(b-(4\theta_1)/3+4/3)(b+2/3)\beta-32(\theta_1-1)(-(3\theta_1)/2+b+3/2)\lambda/3-16(b+2/3)^2)(-4+\lambda(\lambda+1)^2\beta^2))^{\frac{1}{2}}$.

Now, depending upon $\tilde{\theta_1}(\theta_2,\beta,\lambda)$ and $\tilde{\theta_2}(\theta_1,\beta,\lambda)$ given in equations (16) and (17), the best response function of the firms are given in the equations (18) and (19).

$$BR_1(\theta;\beta,\lambda,b)=\begin{cases} X & \text{if} \quad \theta_1<\tilde{\theta_1}(\theta_2;\beta,\lambda,b) & & \\ X & \text{if} \quad \theta_1\geq\tilde{\theta_1}(\theta_2;\beta,\lambda,b) & \text{and} & BR_2(\theta;\beta,\lambda,b)=X \\ Y & \text{if} \quad \theta_1\geq\tilde{\theta_1}(\theta_2;\beta,\lambda,b) & \text{and} & BR_2(\theta;\beta,\lambda,b)=Y \end{cases} \tag{18}$$

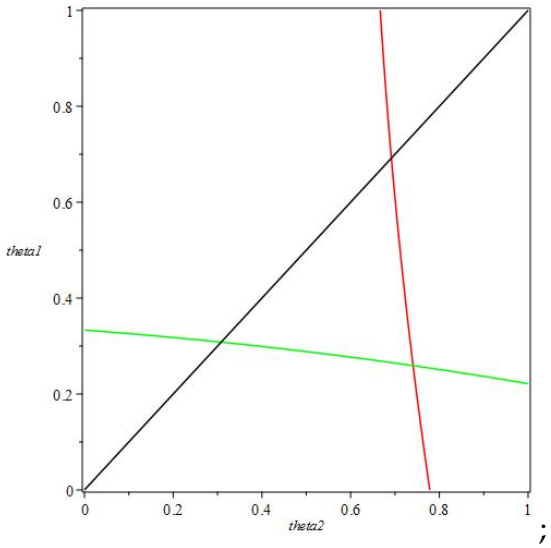
$$BR_1(\theta;\beta,\lambda,b)=\begin{cases} Y & \text{if} \quad \theta_2>\tilde{\theta_2}(\theta_1;\beta,\lambda,b) & & \\ X & \text{if} \quad \theta_2\leq\tilde{\theta_2}(\theta_1;\beta,\lambda,b) & \text{and} & BR_1(\theta;\beta,\lambda,b)=X \\ Y & \text{if} \quad \theta_1\leq\tilde{\theta_2}(\theta_1;\beta,\lambda,b) & \text{and} & BR_1(\theta;\beta,\lambda,b)=Y \end{cases} \tag{19}$$

We found that Nash Equilibrium with standardization always exists for all possible values of the degree of product differentiation, β and the degree of overlapping ownership, λ in the domain. It can be unique or multiple depending on the values of the firms' preferences of technology, θ_1 and θ_2. Also, there exists some values of firms' technology preference (θ_1,θ_2), where the firms choose different technology and standardization does not happen. If the conflict of interest regarding technology choice between the firms is sufficiently low, the technology standardization happens and if the conflict of interest regarding technology choice is high, the technology standardization is not possible. Given any degree of product differentiation, the non-standardization can happen if the degree of overlapping ownership is low. But, if the overlapping ownership is high enough, the technology standardization always happens given any degree of product differentiation. We summarize the results in the following proposition.

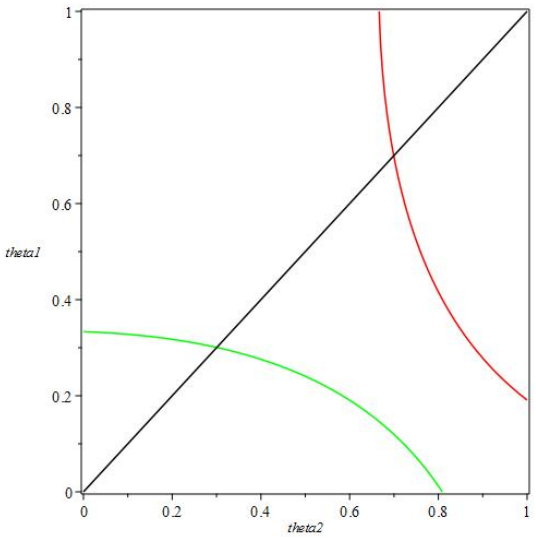
Proposition 1. *For a differentiated duopoly with cross ownership and network externality working via cost channel, the following results hold:*

(i) For all β and λ in the domain, there exists always an equilibrium with standardization. The standardization can be unique or multiple depending on (θ_1, θ_2).

(ii) For all $\beta \in [0,1)$ and $\lambda \in [0, \bar{\lambda})$, there exists some (θ_1, θ_2) for which standardization does not happen. But, for $\beta \in [0,1)$ if $\lambda \in [\bar{\lambda}, 1)$, standardization happens for all (θ_1, θ_2) in the domain.



(a) $\beta = 0.5, \lambda = 0.5$



(b) $\beta = 0.9, \lambda = 0.9$

Figure 3: The red line indicates $V_1^Y(\theta; \beta, \lambda) = 0$ and the green line indicates $V_2^X(\theta; \beta, \lambda) = 0$. In the left panel (a), all the cases A, B, C and D appears. The technology is standardized to technology X in case A and to technology Y in case C. In case C, technology standardization happens to either technology X or technology Y. In case D, the technology will not be standardized. In the right panel (b), the case D does not appear and the technology always gets standardized.

4 Comparative Statics

In this section, we have performed a comparative static exercise to assess the effect of change in the degree of product differentiation at a given level of the degree of overlapping ownership and the effect of change in the degree of overlapping ownership at a given level of product differentiation.

4.1 The Change in the Degree of Product Differentiation given the Degree of Overlapping Ownership

In our model, if we fix the degree of overlapping ownership at low level, we found that Schedule-A rotates leftwards and Schedule-B rotates upwards as the degree of product differentiation in-

creases. As a result, the area with multiple equilibrium decreases, and the area with no standardization increases. But when the degree of cross ownership is given at a level high enough, the story got reversed. the area with multiple equilibrium increases, and the area with no standardization decreases and eventually vanishes. We summarize this findings in the following proposition.

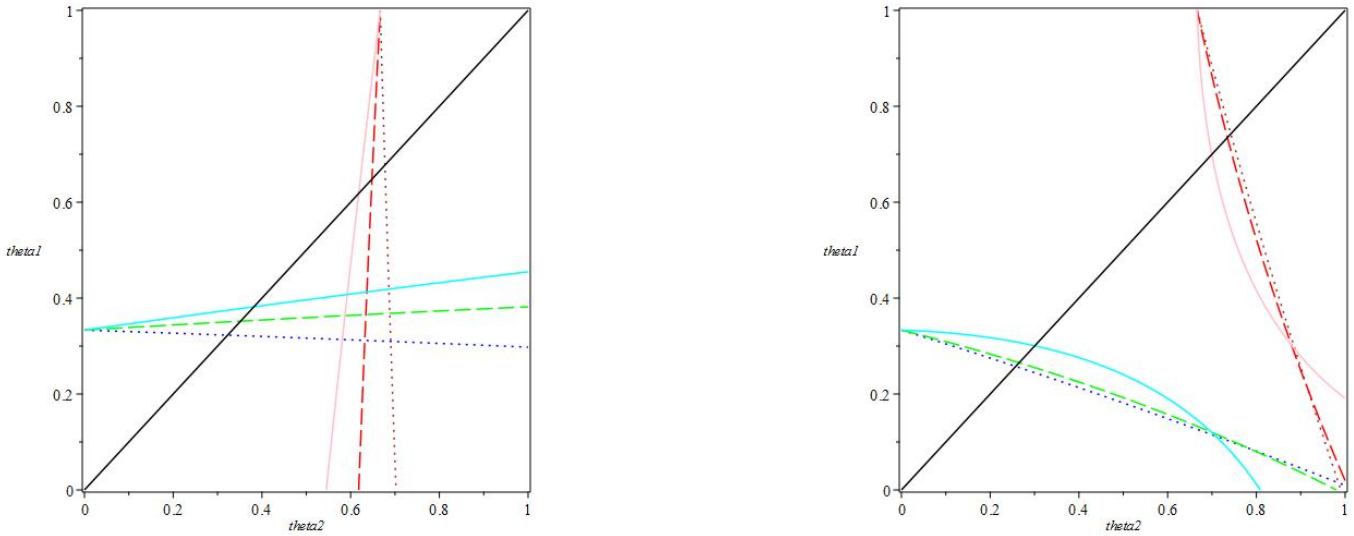
Proposition 2. *For a differentiated duopoly with cross ownership and network externality working via cost channel, the following results hold:*

(i) If λ is given at low level ($\lambda \in [0, \lambda_L]$, scope of non-standardization increases as β increases.

(ii) When $\lambda \in (\lambda_L, \bar{\lambda})$, scope of non-standardization decreases as β increases.

(iii) When $\lambda \in [\bar{\lambda}, 1)$, the non-standardization area vanishes and technology standardization always happens irrespective of the technology preferences of the firms.

In Figure 4a, the cases with different levels of product differentiation is shown when the degree of cross ownership is given at $\lambda = 0.1$ and in Figure 4b, the cases with different levels of product differentiation is shown when the degree of cross ownership is given at $\lambda = 0.9$. In Figure 4a, the area with multiple equilibrium, scope of multiple eqilibria decreases and the area with no standardization, Area with non-standardization increases as the degree of product differentiation, β increases. In Figure 4b, the area with multiple equilibria increases and the area with no standardization decreases as the degree of product differentiation, β increases.



(a) low $\lambda(= 0.1)$ (b) high $\lambda(= 0.9)$

Figure 4: Cases with different levels of β given high and low level of λ

(a) The dotted lines represent $\beta = 0.1$, the dashed lines represent $\beta = 0.5$ and the firm lines represent $\beta = 0.9$. In Panel (a), Area with multiple equilibria decreases and area with non-standardization increases with the increase in β and in Panel (b),area with multiple equlibria increases and area with non-standardization decreases with the increase in β.

4.2 The Change in the Degree of Overlapping Ownership given the Degree of Product Differentiation

When the degree of product differentiation is at a given level, the possibility of standardization increases and the possibility of non-standardization reduces as the degree of overlapping ownership rises. The area with multiple equilibrium expands while the area with no standardization shrinks as overlapping ownership rises.

Proposition 3. *For a differentiated duopoly with overlapping ownership and network externality working via cost channel, at a given level of β, scope of non-standardization decreases as λ rises for $\lambda \in [0, \bar{\lambda})$. Eventually, when $\lambda \in [\bar{\lambda}, 1)$ the non-standardization area vanishes as technology standardization always happens irrespective of firms' preference for technology.*

In Figure 6a we have depicted cases with different values of β when the degree of overlapping ownership is given at $\lambda = 0.1$. As the product differentiation increases, area with non-standardization shrinks.

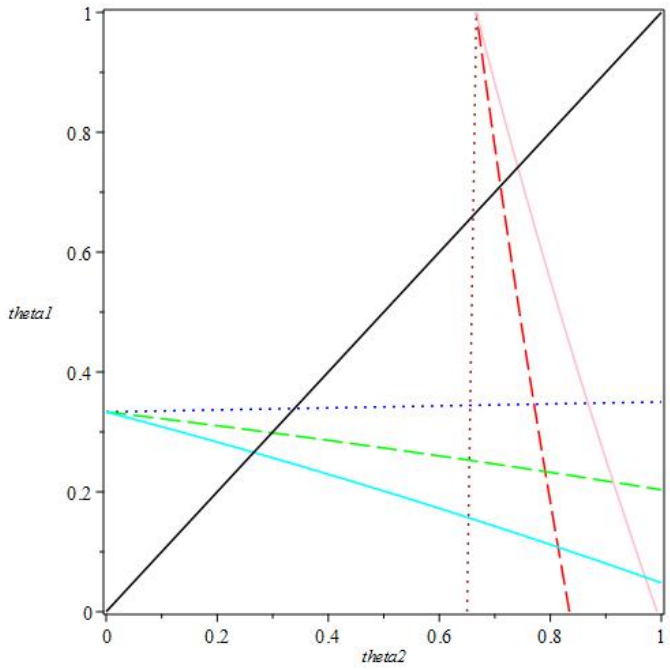


Figure 6: Case with different levels of λ at given $\beta = 0.1$

(a) The dotted lines represent $\lambda = 0.1$, the dashed lines represent $\lambda = 0.5$ and the firm lines represent $\lambda = 0.9$. Non-standardization area decreases with the increase in λ.

5 Welfare Analysis

Whether we start from a viewpoint of a first-best regulator dictating particular technology choice as well as marginal cost pricing in the product market or a second-best criterion, where the regulator imposes a technology for each firm and the firms compete in the product market, we end

with similar conclusion. That is, social welfare is maximized when the industry is standardized to technology X if the mean of the technology preference by the firms is less than 0.5 and to technology Y if the mean of the technology preference by the firms is greater than 0.5. In our model setup, it means that firms production decisions are deviated only in the non-standardization area (case II) in $\theta_1\theta_2$ plane where standardization does not happen. [8]. When the degree of product differentiation and the degree of overlapping ownership is high, then the non-standardization area vanishes and standardization occurs for all possible technology preferences of the firms in the domain. Hence, there firms always in socially desirable equilibrium.

When the degree of overlapping ownership is given, the scope of non-standardization increases with product differentiation when the overlapping ownership is low and the scope of non-standardization decreases with product differentiation when the overlapping ownership is high. Accordingly, the possibility of divergence from socially desirable equilibrium increases as product differentiation rises when the overlapping ownership is given at low level and the possibility of divergence from socially desirable equilibrium decreases as product differentiation reduces when the overlapping ownership is given at high enough level. Eventually, when the product differentiation is high enough, the socially desirable equilibrium always occurs. When the degree of product differentiation is at a given level, the possibility of divergence from socially desirable equilibrium increases as overlapping ownership increases. And when the overlapping ownership is high enough, the industry is always characterized by socially desirable equilibrium. We have summarized these findings in the following proposition:

Proposition 4. *For a differentiated duopoly with overlapping ownership and network externality working via cost channel, the following results hold:*

(i) Standardization of technology X is socially desired if $\theta_1 + \theta_2 < 1$. Conversely, Standardization of technology Y is desired if $\theta_1 + \theta_2 > 1$.

(ii) For $\beta \in [0, 1)$ and $\lambda \in [0, \bar{\lambda})$, there exists some (θ_1, θ_2) for which the firms' decisions are not socially optimal. But, when $\lambda \in [\bar{\lambda}, 1)$, firms' decisions are socially optimal for all (θ_1, θ_2) in the domain.

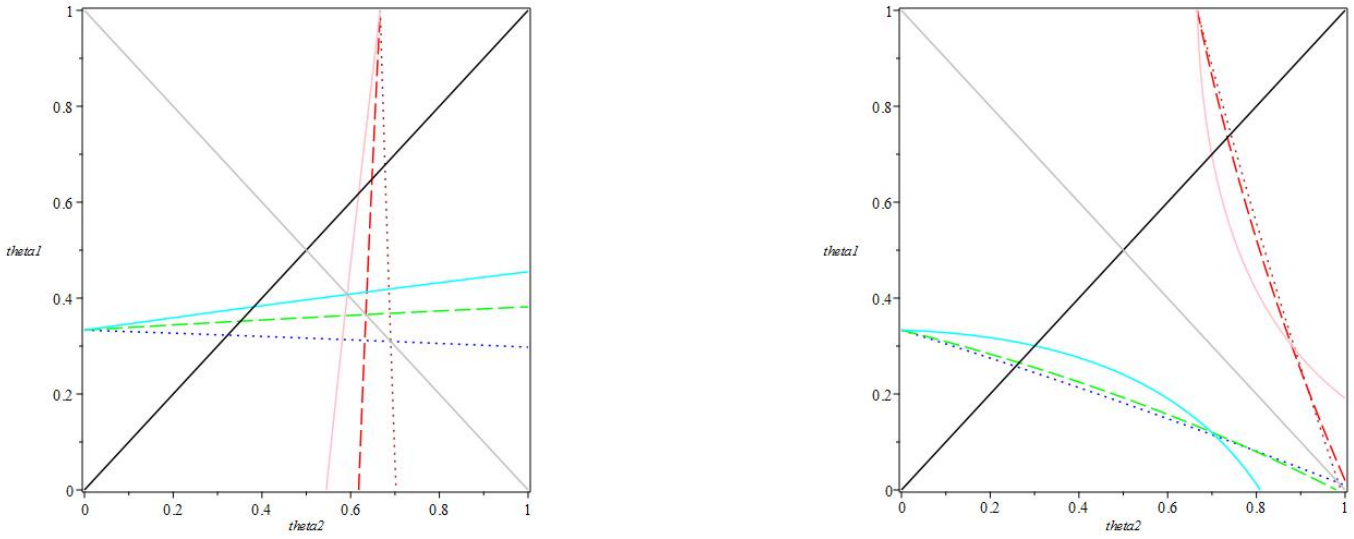
(iii) At a given level of λ, the possibility of divergence from socially desirable equilibrium increases as β rises, when $\lambda \in [0, \lambda_L]$. But, the possibility of divergence from socially desirable equilibrium decreases as β rises, when $\lambda \in (\lambda_L, \bar{\lambda})$.

(iv) At a given level of β, the possibility of divergence from socially desirable equilibrium decreases as λ increases when $\lambda \in [0, \bar{\lambda})$.

In Figures 8a and 9a, the gray line represents $\theta_1 + \theta_2 = 1$. In the left of the gray line, standardization to technology X is socially desired and in the right, standardization to technology Y is socially desired. In Figure 8a, we have depicted cases with different levels of product differentiation for a given level of overlapping ownership. In Panel (a), the degree of overlapping ownership is given at $\lambda = 0.1$ and in Panel (b), the degree of overlapping ownership is given at $\lambda = 0.9$. We can see the non-standardization area is increasing with product differentiation in Panel (a) and

[8] We assume that in the Area with possibility of multiple equilibria, where standardization is possible to both technology X and Y, Standardization to technology X happens when the mean of the technology preferences by the firms is less than 0.5, i.e., $\theta_1 + \theta_2 < 1$ and standardization to technology Y happens when the mean of the technology preferences by the firms is greater than 0.5, i.e., $\theta_1 + \theta_2 > 1$

Area Ω_4 is decreasing with product differentiation in Panel (b). Note that with in panel (b), the non-standardization area vanishes for high values of product differentiation ($\beta = 0.5 \& \beta = 0.9$). In Figure 9a, we have depicted cases with different levels of overlapping ownership when the degree of product differentiation is given at $\beta = 0.5$. Area with non-standardization shrinks as overlapping ownership rises and it vanishes when $\lambda = 0.9$.



(a) low $\lambda(= 0.1)$ (b) high $\lambda(= 0.9)$

Figure 7: Cases with different levels of β given high and low level of λ

(a) The dotted lines represent $\beta = 0.1$, the dashed lines represent $\beta = 0.5$ and the firm lines represent $\beta = 0.9$. The gray line represents $\theta_1 + \theta_2 = 1$. In the left of the gray line, standardization to technology X is socially desired and in the right, standardization to technology Y is socially desired.

In this section, we have found welfare-improving effect of overlapping ownership contrary to the common wisdom. At a given level of product differentiation, with overlapping ownership a firm internalizes the rival's profit and as a result, soften the competition. This competition-softening effect makes it easier to come to an standard agreement between the firms. However, greater the product is differentiated, the standardization is likely to be harder for the firms. Hence, the possibility of reaching a technology standard reduces as the product differentiation rises when the degree of overlapping ownership is given at a low level. But when the overlapping ownership is given at a high level, the shrinking effect of product differentiation is dominated by the expansive effect of overlapping ownership. As a result, Area Ω_4 and subsequently the possibility to deviate from the socially desirable equilibrium is reduced when the overlapping ownership is given at a high level. When both the effect is high enough, then the possibility to deviate from socially desired equilibrium is vanished and the industry is always featured with socially desirable equilibrium whatever be the firms' preference of technology be.

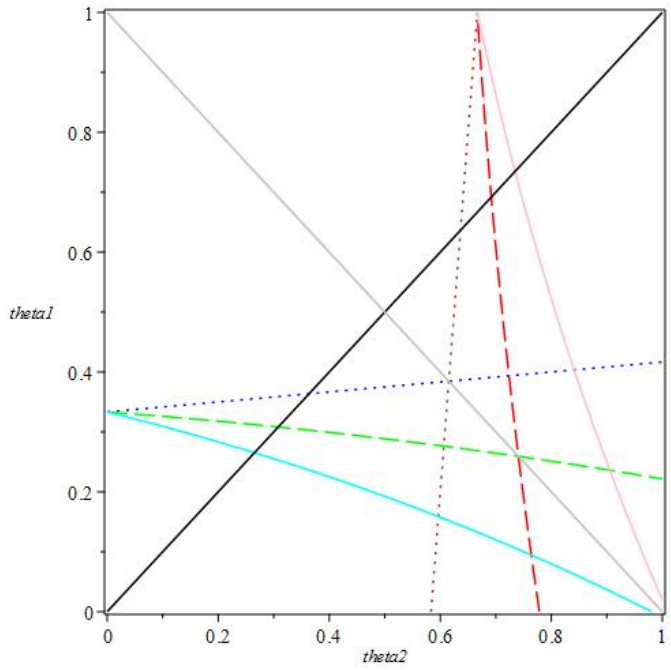


Figure 9: Cases with different λ when β is fixed at $\beta = 0.5$

(a) The dotted lines represent $\lambda = 0.1$, the dashed lines represent $\lambda = 0.5$ and the firm lines represent $\lambda = 0.9$. The gray line represents $\theta_1 + \theta_2 = 1$. In the left of the gray line, standardization to technology X is socially desired and in the right, standardization to technology Y is socially desired.

6 Conclusion

In this paper, we have considered the effect of overlapping ownership on a network industry. We have constructed a model of differentiated duopoly with overlapping ownership where the network externality works via cost channel á la Belleflamme (1998) where the firms choose between technologies followed by quantity competition. We have found equilibrium technology standardization is always exists whatever be the degree of product substitutability or overlapping ownership. However, when the conflict of interest is high between the firms in terms of technology preference, the technology standardization does not happen. But if the overlapping ownership is high enough, technology standardization happens always whatever be the firm's preference of technology. Contrary to Belleflamme (1998), we found that the possibility of non-standardization rises with product substitutability when the overlapping ownership is high enough. The possibility of non-standardization always reduces as overlapping ownership rises. We found that with overlapping ownership, standardization equilibrium will be welfare-superior. The socially optimal equilibrium always happens when the overlapping ownership is high enough given the product substitutability.

This paper is a novel attempt to investigate the technology standardization in a network industry featuring overlapping ownership. Our study gives fresh insights on the interaction between product differentiation and overlapping ownership in presence of network externality working via cost channel. We found welfare-improving technology standardization with overlapping ownership, which has several policy implications for antitrust authorities. The antitrust

authority should consider its policy concerning overlapping ownership in network industry observing the technology preferences of the firm keeping its welfare-improving effect in mind.

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7 Appendix

7.1 Proof of Lemma 1

Substituting $c_1^{XX}(\theta)$ and $c_2^{XX}(\theta)$ from table 1 in equation (12) for $i = 1$ and $t_1 t_2 = XX$, we get the Stage-II quantity of the firm 1 as follows.

$$q_1^{XX} = \frac{2b - 4\theta_1 + 4 - \beta(1+\lambda)(b + 2 - 2\theta_2)}{\Delta} \tag{20}$$

where $b > 0$. Given $\Delta > 0$, $q_1^{XX} \geq 0$ if and only if $2b - 4\theta_1 + 4 - \beta(1+\lambda)(b + 2 - 2\theta_2) \geq 0$ which can be reduced to

$$[2 - \beta(1+\lambda)]b + 2[2(1-\theta_1) - \beta(1+\lambda)(1-\theta_2)] \geq 0. \tag{21}$$

The inequality (25) always holds for any $\Gamma > 0$ because

(a) $\forall \beta \in [0,1) and \lambda \in [0,1), \beta(1+\lambda) \in [0,2) \implies 2 - \beta(1+\lambda) > 0 \implies [2 - \beta(1+\lambda)]\Gamma > 0$

(b) Given $\theta_1 \leq \theta_2 \implies 1 - \theta_1 \geq 1 - \theta_2 \geq 0$ as $\theta_2 \in [0,1]$. Further from (a), $\beta(1+\lambda) \in [0,2)$, hence, $2(1-\theta_1) - \beta(1+\lambda)(1-\theta_2) \geq 0$.

Thus, both the terms in the inequality (25) are non-negative. Hence q_1^{XX} is always non-negative for every $b > 0$. Using a similar kind of argument, one can establish that q_2^{YY} is always non-negative for every $b > 0$.

Substituting $c_1^{XY}(\theta)$ and $c_2^{XY}(\theta)$ from table 1 in equation (12) for $i = 1$ and $t_1t_2 = XY$, we get the Stage-II quantity of the firm 1 as follows.

$$q_1^{XY} = \frac{2b - 2\theta_1 + 2 - \beta(1+\lambda)(b+\theta_2)}{\Delta}. \tag{22}$$

Given $\Delta > 0$, $q_1^{XY} \geq 0$ if and only if $2[b + (1-\theta_1)] - \beta(1+\lambda)[b+\theta_2] \geq 0$ which can be reduced to

$$[2 - \beta(1+\lambda)]b + 2 - K_1^{XY} \geq 0 \tag{23}$$

where $K_1^{XY} = 2\theta_1 + \beta(1+\lambda)\theta_2$. Note that,

$$\max_{\substack{\theta_1,\theta_2 \in [0,1] \\ \theta_1 \leq \theta_2}} K_1^{XY} = 2.1 + \beta(1+\lambda).1 = 2 + \beta(1+\lambda) = \overline{K}_1^{XY}.$$

Therefore, to show $q_1^{XY} \geq 0$ it is sufficient to have $[2 - \beta(1+\lambda)]b + 2 - \overline{K}_1^{XY} = [2 - \beta(1+\lambda)]b - \beta(1+\lambda) \geq 0$, solving we have $b \geq \frac{\beta(1+\lambda)}{2-\beta(1+\lambda)} = b^N$. Therefore, given any $\lambda, \beta \in [0,1)$, if $b \geq b^N$ then $q_1^{XY} \geq 0$. Using a similar kind of argument, one can establish that $q_2^{YX} \geq 0$ for all $b \geq b^N$.

Substituting $c_1^{YX}(\theta)$ and $c_2^{YX}(\theta)$ from table 1 in equation (12) for $i = 1$ and $t_1t_2 = YX$, we get the Stage-II quantity of the firm 1 as follows.

$$q_1^{YX} = \frac{2b + 2\theta_1 - \beta(1+\lambda)(b+1-\theta_2)}{\Delta}. \tag{24}$$

Given $\Delta > 0$, $q_1^{YX} \geq 0$ if and only if $2[b + \theta_1] - \beta(1+\lambda)[b+1-\theta_2] \geq 0$ which can be reduced to

$$[2 - \beta(1+\lambda)]b - \beta(1+\lambda) + K_1^{YX} \geq 0 \tag{25}$$

where $K_1^{YX} = 2\theta_1 + \beta(1+\lambda)\theta_2$. Note that,

$$\min_{\substack{\theta_1,\theta_2 \in [0,1] \\ \theta_1 \leq \theta_2}} K_1^{YX} = 0 = \underline{K}_1^{YX}.$$

Therefore, to show $q_1^{YX} \geq 0$ it is sufficient to have $[2 - \beta(1+\lambda)]b - \beta(1+\lambda) + \underline{K}_1^{XY} = [2 - \beta(1+\lambda)]b - \beta(1+\lambda) \geq 0$, solving we have $b \geq \frac{\beta(1+\lambda)}{2-\beta(1+\lambda)} = b^N$. Therefore, given any $\lambda, \beta \in [0,1)$, if $b \geq b^N$ then $q_1^{YX} \geq 0$. Using a similar kind of argument, one can establish that $q_2^{XY} \geq 0$ for all $b \geq b^N$.

Substituting $c_1^{YY}(\theta)$ and $c_2^{YY}(\theta)$ from table 1 in equation (12) for $i = 1$ and $t_1t_2 = YY$, we get the Stage-II quantity of the firm 1 as follows.

$$q_1^{YX} = \frac{2b - 4\theta_1 - \beta(1+\lambda)(b+2\theta_2)}{\Delta}. \tag{26}$$

Given $\Delta > 0$, $q_1^{YY} \geq 0$ if and only if $2[b + 2\theta_1] - \beta(1+\lambda)[b+2\theta_2] \geq 0$ which can be reduced to

$$[2 - \beta(1+\lambda)]b + K_1^{YY} \geq 0 \tag{27}$$

where $K_1^{YY} = 4\theta_1 - 2\beta(1+\lambda)\theta_2$. Note that,

$$\min_{\substack{\theta_1,\theta_2 \in [0,1] \\ \theta_1 \leq \theta_2}} K_1^{YY} = -2\beta(1+\lambda) = \underline{K}_1^{YY}.$$

Therefore, to show $q_1^{YY} \geq 0$ it is sufficient to have $[2 - \beta(1+\lambda)]b + \underline{K}_1^{YY} = [2 - \beta(1+\lambda)]b - 2\beta(1+\lambda) \geq 0$, solving we have $b \geq \frac{2\beta(1+\lambda)}{2-\beta(1+\lambda)} = b^S$. Therefore, given any $\lambda, \beta \in [0,1)$, if $b \geq b^S$ then $q_1^{YY} \geq 0$. Using a similar kind of argument, one can establish that $q_2^{XX} \geq 0$ for all $b \geq b^S$.

Finally since $\max\{b^S, b^N\} = b^S$ therefore for all $i \in \{1,2\}$ and $t_1t_2 \in \Omega^M$ we have $q_i^{t_1t_2} \geq 0$ for all $b \geq b^S$. Hence, the result.

7.2 Proof of Lemma 2

Using Definition 1 and replacing $q_i^{t_1 t_2}$ from equation (12), we have found the expressions (28) to (31).

$$V_1^X(\theta;\beta,\lambda) = \frac{1}{\Delta^2}[(A_1\theta_1 + A_2\theta_2 + A_3)b + A_4\theta_1^2 + A_5\theta_2^2 + A_6\theta_1\theta_2 + A_7\theta_1 + A_8\theta_2 + A_9] \tag{28}$$

where $A_1(\beta,\lambda) = (-3(\beta(\lambda+1)-2)(\lambda^3\beta^2+2\lambda^2\beta^2+\lambda\beta^2-4)$, $A_2(\beta,\lambda) = -((\beta(\lambda+1)-2)(\lambda^3\beta^2+2\lambda^2\beta^2-2\lambda^2\beta+\lambda\beta^2-4\lambda+2\beta))$, $A_3(\beta,\lambda) = (\lambda\beta+\beta-2)(3\lambda^3\beta^2+6\lambda^2\beta^2-2\lambda^2\beta+3\lambda\beta^2-4\lambda+2\beta-8)$, $A_4(\beta,\lambda) = 3(4-\beta^2\lambda(\lambda+1)^2)$, $A_5(\beta,\lambda) = (3(\lambda+1)^2(1-2\lambda)\beta^2+12\lambda)$, $A_6(\beta,\lambda) = 5(\lambda(\lambda+1)^3\beta^3-4\beta(\lambda+1))$, $A_7(\beta,\lambda) = (-5\lambda(\lambda+1)^3\beta^3+8\beta^2\lambda(\lambda+1)^2+20(\lambda+1)\beta-32)$, $A_8(\beta,\lambda) = (-4\lambda(\lambda+1)^3\beta^3+6(\lambda+1)^2(2\lambda-1)\beta^2+16((\lambda+1)\beta-24\lambda)$, $A_9(\beta,\lambda) = 4\lambda(\lambda+1)^3\beta^3+(\lambda+1)^2(3-10\lambda)\beta^2-16(\lambda+1)\beta+12\lambda+16$.

$$V_1^Y(\theta;\beta,\lambda) = \frac{1}{\Delta^2}[(B_1\theta_1 + B_2\theta_2 + B_3)b + B_4\theta_1^2 + B_5\theta_2^2 + B_6\theta_1\theta_2 + B_7\theta_1 + B_8\theta_2 + B_9] \tag{29}$$

where $B_1(\beta,\lambda) = -3(\beta(1+\lambda)-2)(\lambda^3\beta^2+2\lambda^2\beta^2+\lambda\beta^2-4)$, $B_2(\beta,\lambda) = -(\beta(1+\lambda)-2)(\lambda^3\beta^2+2\lambda^2\beta^2-2\lambda^2\beta+\lambda\beta^2-4\lambda+2\beta)$, $B_3(\beta,\lambda) = (\beta(1+\lambda)-2)(\lambda^3\beta^2+2\lambda^2\beta^2+\lambda\beta^2-4)$, $B_4(\beta,\lambda) = (3(\lambda+1)^2\lambda\beta^2-12)$, $B_5(\beta,\lambda) = (3((\lambda+1)^2(2\lambda-1)\beta^2-4\lambda))$, $B_6(\beta,\lambda) = 5(-\lambda(\lambda+1)^3\beta^3+4\beta(\lambda+1))$, $B_7(\beta,\lambda) = 2((\lambda+1)^2\lambda\beta^2-4)$, $B_8(\beta,\lambda) = (\lambda(\lambda+1)^3\beta^3-4\beta(\lambda+1))$, $B_9(\beta,\lambda) = 4-(\lambda+1)^2\lambda\beta^2$.

$$V_2^X(\theta;\beta,\lambda) = \frac{1}{\Delta^2}[(H_1\theta_1 + H_2\theta_2 + H_3)b + H_4\theta_1^2 + H_5\theta_2^2 + H_6\theta_1\theta_2 + H_7\theta_1 + H_8\theta_2 + H_9] \tag{30}$$

where $H_1(\beta,\lambda) = -(\lambda\beta+\beta-2)(\lambda^3\beta^2+2\lambda^2\beta^2-2\lambda^2\beta+\lambda\beta^2-4\lambda+2\beta)$,$H_2(\beta,\lambda) = -3(l\lambda\beta+\beta-2)(\lambda^3\beta^2+2\lambda^2\beta^2+\lambda\beta^2-4)$,$H_3(\beta,\lambda) = (\lambda\beta+\beta-2)(3\lambda^3\beta^2+6\lambda^2\beta^2-2\lambda^2\beta+3\lambda\beta^2-4\lambda+2\beta-8)$,$H_4(\beta,\lambda) = 2(1-2\lambda)(\lambda+1)^2\beta^2+12\lambda)$,$H_5(\beta,\lambda) = (12-3(\lambda+1)^2\lambda\beta^2)$,$H_6(\beta,\lambda) = 5(\lambda(\lambda+1)^3\beta^3+4(\lambda+1)\beta)$,$H_7(\beta,\lambda) = (-2\beta(\lambda+1)(2\lambda^3\beta^2+4\lambda^2\beta^2-6\lambda^2\beta+2\lambda\beta^2-3\lambda\beta+3\beta-8)-24\lambda)$,$H_8(\beta,\lambda) = -(5\beta(1+\lambda)-8)(\lambda^3\beta^2+2\lambda^2\beta^2+\lambda\beta^2-4))$,$H_9(\beta,\lambda) = 4\lambda(\lambda+1)^3\beta^3+(3-10\lambda)(\lambda+1)^2\beta^2-16(\lambda+1)\beta+12\lambda+16$.

$$V_2^Y(\theta;\beta,\lambda) = \frac{1}{\Delta^2}[(I_1\theta_1 + I_2\theta_2 + I_3)b + I_4\theta_1^2 + I_5\theta_2^2 + I_6\theta_1\theta_2 + I_7\theta_1 + I_8\theta_2 + I_9] \tag{31}$$

where $I_1(\beta,\lambda) = -(\lambda\beta+\beta-2)(\lambda^3\beta^2+2\lambda^2\beta^2+2\lambda^2\beta+\lambda\beta^2+4\lambda-2\beta-8)$, $I_2(\beta,\lambda) = -(\lambda\beta+\beta-2)(3\lambda^3\beta^2+6\lambda^2\beta^2-4\lambda^2\beta+3\lambda\beta^2-8\lambda+4\beta-4)$, $I_3(\beta,\lambda) = (\beta(1+\lambda)-2)(\lambda^3\beta^2+2\lambda^2\beta^2+\lambda\beta^2-4)$, $I_4(\beta,\lambda) = ((2\lambda+1)(\lambda+1)^2\beta^2+4\lambda-16)$, $I_5(\beta,\lambda) = ((7\lambda-4)(\lambda+1)^2\beta^2-16\lambda+4)$, $I_6(\beta,\lambda) = (-\lambda(\lambda+1)^3\beta^3+4(\lambda+1)\beta)$, $I_7(\beta,\lambda) = (\beta(\lambda+1)(\lambda^3\beta^2+2\lambda^2\beta^2+\lambda\beta^2-4))$, $I_8(\beta,\lambda) = (2(\lambda+1)^2\lambda\beta^2-8)$, $I_9(\beta,\lambda) = -(\lambda+1)^2\lambda\beta^2+4$.

At first, we shall show $V_1^Y(\theta;\beta,\lambda) - V_2^Y(\theta;\beta,\lambda) \geq 0$ for all $0 \leq \theta_1 \leq \theta_2 \leq 1$. From equations (29) and (31), we get

$$V_1^Y(\theta;\beta,\lambda) - V_2^Y(\theta;\beta,\lambda) = (\theta_2-\theta_1)[D_1(\beta,\lambda)b + D_2(\beta,\lambda) + (\theta_1+\theta_2)D_3(\beta,\lambda)] \tag{32}$$

where $D_1(\beta,\lambda) = 2(\lambda+1)(\lambda\beta+1)(\beta(1+\lambda)-2)^2$, $D_2(\beta,\lambda) = (-4+\lambda\beta^2(1+\lambda)^2)(\beta(1+\lambda)-2)$ and $D_3(\beta,\lambda) = (\lambda-1)(\lambda\beta+\beta+2)(\lambda\beta+\beta-2)$. Therefore, given $\theta_2 \geq \theta_1$, $V_1^Y(\theta;\beta,\lambda) - V_2^Y(\theta;\beta,\lambda) \geq 0$ when $(D_1(\beta,\lambda)b + D_2(\beta,\lambda) + (\theta_1+\theta_2)D_3(\beta,\lambda) \geq 0$. For all $\beta \in [0,1)$ and $\lambda \in [0,1)$, $D_1(\beta,\lambda) > 0, D_2(\beta,\lambda) > 0$ and $D_3(\beta,\lambda) > 0$ which implies $(D_1(\beta,\lambda)b + D_2(\beta,\lambda) + (\theta_1+\theta_2)D_3(\beta,\lambda) > 0$. Thus, we can conclude $V_1^Y(\theta;\beta,\lambda) - V_2^Y(\theta;\beta,\lambda) \geq 0$ for all $0 \leq \theta_1 \leq \theta_2 \leq 1$ and for all $\beta \in [0,1)$ and $\lambda \in [0,1)$. Further, equality holds when $\theta_1 = \theta_2 = \theta^1$ where $\theta^1(\beta,\lambda,b) = \frac{D_{14}(\beta,\lambda,b)}{(2(12+5\lambda(\lambda+1)^2\beta^3+(-9\lambda^2-6\lambda+3)\beta^2-20\beta)(\lambda+1))}$ where $D_{14}(\beta,\lambda,b) = (((-4b+1)\lambda^4+(-12b+3)\lambda^3+(-12b+3)\lambda^2+(-4b+1)\lambda)\beta^3+((10b+2)\lambda^3+(18b+4)\lambda^2+(6b+2)\lambda-2b)\beta^2+((16b-4)\lambda+16b-4)\beta-8-24b-8+((-2b(\lambda\beta+\beta-2)(2\lambda^3\beta^2+4\lambda^2\beta^2-\lambda^2\beta+2\lambda\beta^2-2\lambda+\beta-6)+(\lambda\beta+\beta+2)(\lambda^3\beta^2+2\lambda^2\beta^2+\lambda\beta^2-4))^2+4(\lambda+1)(5\lambda^3\beta^3+10\lambda^2\beta^3-9\lambda^2\beta^2+5\lambda\beta^3-6\lambda\beta^2+3\beta^2-20\beta+12)(\lambda^3\beta^2+2\lambda^2\beta^2+\lambda\beta^2-4)(b\lambda\beta+b\beta-2b-1))^{\frac{1}{2}})$.

Secondly, we shall show that $V_1^X(\theta;\beta,\lambda) - V_2^X(\theta;\beta,\lambda) \geq 0$ for all $0 \leq \theta_1 \leq \theta_2 \leq 1$. From equations (28) and (30), we get

$$V_1^X(\theta;\beta,\lambda) - V_2^X(\theta;\beta,\lambda) = (\theta_1-\theta_2)[D_4(\beta,\lambda)b + D_5(\beta,\lambda) + (\theta_1+\theta_2)D_6(\beta,\lambda)] \tag{33}$$

where $D_4(\beta,\lambda) = -2(\beta(1+\lambda)-2)(\lambda\beta^2(1+\lambda)^2+(\lambda^2-1)\beta+2\lambda-6)$, $D_5(\beta,\lambda) = -(\lambda\beta^2(1+\lambda)^2+(6\lambda^2-6)\beta+12\lambda-16)(\beta(1+\lambda)-2)$ and $D_6(\beta,\lambda) = 3(\lambda-1)(\lambda\beta+\beta+2)(\lambda\beta+\beta-2)$. Therefore, given $\theta_2 \geq \theta_1$, $V_1^X(\theta;\beta,\lambda) - V_2^X(\theta;\beta,\lambda) \geq 0$ when $D_4(\beta,\lambda)b + D_5(\beta,\lambda) + (\theta_1+\theta_2)D_6(\beta,\lambda) \leq 0$. Now, if θ_1 and θ_2 takes their maximum value

1, and we consider the minimum value of $b = b^S$, we get $D_4(\beta,\lambda)b^S + D_5(\beta,\lambda) + 2D_6(\beta,\lambda) = -8 + 3\lambda(1+\lambda)^3\beta^3 + (6\lambda^3 + 8\lambda^2 - 2\lambda - 4)\beta^2 + (8\lambda^2 - 12\lambda - 20)\beta < 0 \ \forall\beta \in [0,1), \forall\lambda \in [0,1)$. Therefore, Thus, we can conclude $V_1^X(\theta;\beta,\lambda) - V_2^X(\theta;\beta,\lambda) \geq 0$ for all $0 \leq \theta_1 \leq \theta_2 \leq 1$ and for all $\beta \in [0,1)$ and $\lambda \in [0,1)$. Further, equality holds when $\theta_1 = \theta_2 = \theta^2$ where $\theta^2(\beta,\lambda,b) = \frac{D_{15}(\beta,\lambda,b)}{(2(12+5\lambda(\lambda+1)^2\beta^3+(-9\lambda^2-6\lambda+3)\beta^2-20\beta)(\lambda+1))}$ where $D_{15}(\beta,\lambda,b) = (-2+\beta(\lambda+1)(16(\beta^2\lambda^3 + (2\beta^2 - 1/2\beta)\lambda^2 + (\beta^2-1)\lambda + \beta/2 - 3)^2b^2 + 12(\lambda^3\beta^2 + 2\lambda^2\beta^2 + \lambda\beta^2 - 4)(\beta^2\lambda^3 + (2\beta^2 - 2\beta)\lambda^2 + (\beta^2 - 4/3\beta - 8/3)\lambda + (2\beta)/3 - 4)b + (\lambda^3\beta^2 + 2\lambda^2\beta^2 + \lambda\beta^2 - 4)(-16 + \beta^2\lambda^3 + (2\beta^2 - 12\beta)\lambda^2 + (\beta^2 - 12\beta - 12)\lambda))^{\frac{1}{2}} + 4(\lambda+1)^3\lambda(b + 9/4)\beta^3 - 10(\lambda+1)^2((b+2)\lambda - b/5 - 3/5)\beta^2 - 16(\lambda+1)(b+9/4)\beta + (8b+24)\lambda + 24b + 32)$.

Lastly,we shall show that $V_1^X(\theta;\beta,\lambda) - V_1^Y(\theta;\beta,\lambda) > 0$ for all $0 \leq \theta_1 \leq \theta_2 \leq 1$. From equations (28) and (29), we get

$$V_1^X(\theta;\beta,\lambda) - V_1^Y(\theta;\beta,\lambda) = \begin{bmatrix} D_7(\beta,\lambda)b + D_8(\beta,\lambda)\theta_1^2 + D_9(\beta,\lambda)\theta_2^2 + 10D_{10}\theta_1\theta_2 \\ +D_{11}\theta_1 + D_{12}(\beta,\lambda)\theta_2 + D_{13}(\beta,\lambda) \end{bmatrix} \tag{34}$$

where $D_7(\beta,\lambda) = 2(1+\lambda)(\lambda\beta+1)(\beta(1+\lambda)-2)^2$, $D_8(\beta,\lambda) = 4 - \lambda\beta^2(1+\lambda)^2$, $D_9(\beta,\lambda) = (1+\lambda)^2(1-2\lambda)\beta^2 + 4\lambda$, $D_{10}(\beta,\lambda) = \lambda(1+\lambda)^3\beta^3 - 4\beta(1+\lambda)$, $D_{11}(\beta,\lambda) = -(-4+\lambda\beta^2(1+\lambda)^2)(-6+5\beta(1+\lambda))$, $D_{12}(\beta,\lambda) = -5\lambda(1+\lambda)^3\beta^3 + (12\lambda^3 + 18\lambda^2 - 6)\beta^2 + (20\lambda+20)\beta - 24\lambda$, $D_{13}(\beta,\lambda) = (1+\lambda)(12 + 4\lambda(1+\lambda)^2\beta^3 + (-9\lambda^2 - 6\lambda + 3)\beta^2 - 16\beta)$. By rearranging terms, we get $D_{11}(\beta,\lambda) = -D_8(\beta,\lambda) - 5D_{10}(\beta,\lambda)$ and $D_{12}(\beta,\lambda) = -D_9(\beta,\lambda) - 5D_{10}(\beta,\lambda)$. Therefore, the whole expression can be rewritten as

$$V_1^X(\theta;\beta,\lambda) - V_1^Y(\theta;\beta,\lambda) = \begin{bmatrix} D_8(\beta,\lambda)\theta_1(\theta_1-1) + D_9(\beta,\lambda)\theta_2(\theta_2-1) - 5D_{10}(\beta,\lambda)(\theta_1+\theta_2-2\theta_1\theta_2) \\ +[D_{13}(\beta,\lambda) + D_7(\beta,\lambda)b] \end{bmatrix} \tag{35}$$

Now, this expression in equation (35) is minimized

$$\begin{cases} (\theta_1,\theta_2) = (\frac{1}{2},\frac{1}{2}) & \text{when} \quad H(\beta,\lambda) > 0 \\ (\theta_1,\theta_2) = (0,0) & \text{when} \quad H(\beta,\lambda) \leq 0 \end{cases}$$

where the second-order condition $H(\beta,\lambda) = (-12\lambda\beta^2(1+\lambda)^2 + 48)(-6(4\lambda-2)(1+\lambda)^2\beta^2 + 48\lambda) - (10\lambda(1+\lambda)^3\beta^3 - 40\beta(1+\lambda))^2$. When $\theta_1 = \frac{1}{2}$ and $\theta_2 = \frac{1}{2}$, the minimum value of $V_1^X(\theta;\beta,\lambda) - V_1^Y(\theta;\beta,\lambda)$ becomes $D_7(\beta,\lambda)b + \frac{3}{2}(D_8(\beta,\lambda) + D_9(\beta,\lambda)) - \frac{5}{2}D_{10}(\beta,\lambda) + D_{13}(\beta,\lambda)$, Further, assuming the minimum value of $b = b^S = \frac{2\beta(1+\lambda)}{2-\beta(1+\lambda)}$ and further simplifying which we get $\min[V_1^X(\theta;\beta,\lambda) - V_1^Y(\theta;\beta,\lambda)] = (1+\lambda)(\beta\lambda+1)(2-\beta(1+\lambda))(6+5\beta(1+\lambda)) > 0 \forall\beta \in [0,1)\forall\lambda \in (0,1]$. Now, for the other case, if we consider $\theta_1 = 0$ and $\theta_2 = 0$, then the minimum value of $V_1^X(\theta;\beta,\lambda) - V_1^Y(\theta;\beta,\lambda)$ becomes $(-5\lambda^3 - 11\lambda^2 - 7\lambda - 1)\beta^2 + (8\lambda^2 - 8)\beta + 12\lambda + 12 > 0 \forall\beta \in [0,1)\forall\lambda \in (0,1]$. Thus, we can conclude $V_1^X(\theta;\beta,\lambda) - V_1^Y(\theta;\beta,\lambda) > 0$ for all $0 \leq \theta_1 \leq \theta_2 \leq 1$ and for all $\beta \in [0,1)$ and $\lambda \in [0,1)$. This implies if $V_1^Y(\theta;\beta,\lambda) > 0$, then $V_1^X(\theta;\beta,\lambda) > 0$. Hence, Proved.

7.3 Proof of Proposition 1

Comparing the expressions of $V_1^Y(\theta;\beta,\lambda)$ and $V_2^X(\theta;\beta,\lambda)$ in equations (29) and (30), we observe that

$$V_1^Y(\theta;\beta,\lambda)|_{\theta_1=(1-\theta^*),\theta_2=\theta^*} = -V_2^X(\theta;\beta,\lambda)|_{\theta_1=(1-\theta^*),\theta_2=\theta^*} \tag{36}$$

. That is, $V_1^Y(\theta;\beta,\lambda) = -V_2^X(\theta;\beta,\lambda)$ only when $\theta_1(\beta,\lambda) = (1-\theta^*(\beta,\lambda)), \theta_2(\beta,\lambda) = \theta^*(\beta,\lambda)$. Now, to get the solution of $V_1^Y(\theta;\beta,\lambda) = 0 = V_2^X(\theta;\beta,\lambda)$, we used equation (36) to have $V_1^Y(\theta^*;\beta,\lambda) = V_2^X(\theta^*;\beta,\lambda) = 0$, which is expressed in the following equation:

$$\frac{1}{\Delta^2}\left[\ \Psi_2(\beta,\lambda)\theta^{*2} + \Psi_1(\beta,\lambda,b)\theta^* + \Psi_0(\theta,\lambda,b)\ \right] = 0 \tag{37}$$

where $\Psi_2(\beta,\lambda) = (5\lambda^3\beta^3 + 10\lambda^2\beta^3 + 9\lambda^2\beta^2 + 5\lambda\beta^3 + 6\lambda\beta^2 - 3\beta^2 - 20\beta - 12)(\lambda+1)$, $\Psi_1(\beta,\lambda,b) = 2(\lambda\beta+\beta-2)(\lambda^3\beta^2 + 2\lambda^2\beta^2 + \lambda^2\beta + \lambda\beta^2 + 2\lambda - \beta - 6)b - 4(\lambda\beta+\beta+2)(\lambda^3\beta^2 + 2\lambda^2\beta^2 + \lambda\beta^2 - 4)$and $\Psi_0(\beta,\lambda,b) = -2(\lambda^3\beta^2 + 2\lambda^2\beta^2 + \lambda\beta^2 - 4)((\lambda\beta+\beta-2)b - 2)$. Solving $\theta^*(\beta,\lambda,b)$ from equation (37), we get

$$\theta^*(\beta,\lambda,b) = \frac{\begin{bmatrix} (-\lambda(\lambda+1)^3(b-2)\beta^3 + (\lambda+1)^2((b+4)\lambda+b)\beta^2 + 4(\lambda+1)(b-2)\beta \\ +4b\lambda - 12b - 16 + \sqrt{\Phi(\beta,\lambda,b)} \end{bmatrix}}{5(\lambda+1)(-12/5 + (\lambda+1)^2\lambda\beta^3 + (9/5\lambda^2 + 6/5\lambda - 3/5)\beta^2 - 4\beta)} \tag{38}$$

where $\Phi(\beta,\lambda) = ((\beta^2\lambda^3+(2\beta^2+\beta)\lambda^2+(\beta^2+2)\lambda-\beta-6)^2b^2+6(\beta^2\lambda^3+2\beta^2\lambda^2+\beta^2\lambda-4)(\beta^2\lambda^3+(2\beta^2+3\beta)\lambda^2+(\beta^2+8/3\beta+10/3)\lambda-\beta/3-2)b+4(-1+\beta^2\lambda^3+(2\beta^2+3\beta)\lambda^2+(\beta^2+3\beta+3)\lambda)(\beta^2\lambda^3+2\beta^2\lambda^2+\beta^2\lambda-4))(\beta\lambda+\beta-2)^2)$. Thus, $\theta_1^*(\beta,\lambda,b) = 1-\theta^*(\beta,\lambda,b)$ and $\theta_2^*(\beta,\lambda,b) = \theta^*(\beta,\lambda,b)$. We can show for all $\beta \in [0,1)$ and $\lambda \in [0,1)$, $\theta^*(\beta,\lambda) > \frac{1}{2}$. This always ensure the existence of cases (A), (B) and (C). Therefore, there always exist some (θ_1,θ_2) for which standardization happens. Multiple standardization exists in case (B): (θ_1,θ_2) in that region satisfies $V_1^Y(\theta;\beta,\lambda) < 0$ and $V_2^X(\theta;\beta,\lambda) > 0$.

Moreover, whenever $\theta^*(\beta,\lambda) \leq 1$, the case (D), i.e., region with non-standardization appears. Though we can not solve for $\bar{\lambda}$ such that$\theta^*(\beta,\lambda) = 1$, we simulate values of $\bar{\lambda}$ for different values of β and b (Table 3).

$b\backslash\beta$	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
b=2	0.714	0.689	0.664	0.636	0.605	0.571	0.535	-	-	-
b = 4	0.818	0.801	0.784	0.766	0.746	0.723	0.697	0.662	0.629	-
b = 8	0.895	0.885	0.874	0.864	0.852	0.840	0.825	0.806	0.779	0.743
b = 16	0.943	0.937	0.932	0.926	0.920	0.913	0.906	0.897	0.885	0.861
b = 32	0.970	0.967	0.964	0.961	0.958	0.955	0.951	0.948	0.942	0.932

Table 3: $\bar{\lambda}$ for different β and b (Approximated after 3 digits of decimal).

7.4 Proof of Proposition 2

$$\frac{\partial\theta^*(\beta,\lambda,b)}{\partial\beta} = \frac{\mu_1^\beta(\beta,\lambda,b)}{\mu_2^\beta(\beta,\lambda,b)} \tag{39}$$

where $\mu_1^\beta(\beta,\lambda,b) = (((2b^2+69b+46)\lambda+2b^2-13b-12)(\lambda+1)^6l^2\beta^6+(\lambda+1)^5\lambda((14b^2+251b+174)\lambda^2+(-12b^2-126b-78)\lambda-2b^2+3b)\beta^5+((b+6)(52b+34)\lambda^2+(-240b^2-768b-420)\lambda-4b^2+6b)(\lambda+1)^4\lambda\beta^4+((120b^2+108b+48)\lambda^3+(376b^2-1676b-1104)\lambda^2+(254b^2+532b+288)\lambda-8b^2+12b)(\lambda+1)^3\beta^3+(\lambda+1)^2((144b^2+168b+144)\lambda^3+(-784b^2-2664b-1680)\lambda^2+(1552b^2+3144b+11)\lambda-144b^2-8b+48)\beta^2+((192b^2+480b+216)\lambda^3+(-992b^2-1968b-1056)\lambda^2+(1664b^2+4416b+2496)\lambda-864b^2-880b-192)(\lambda+1)\beta+(-64b^2+96b)\lambda^2+(768b^2+2816b+1728)\lambda-1728b^2-2400b-832)(\beta(1+\lambda)-2)-(((14b+2)\lambda+2b+6)(\lambda+1)^4\lambda\beta^4-8(\lambda+1)^2(3(b+3)\lambda^2+(-20(b+1)\lambda+b+3)\beta^2-96(\lambda+1)(\lambda-1)^2(b+1)\beta+(32b+96)\lambda-288b-224)((\lambda\beta+\beta-2)^2((\beta^2\lambda^3+(2\beta^2+\beta)\lambda^2+(\beta^2+2)\lambda-\beta-6)^2b^2+(6\beta^2\lambda^3+6(2\beta^2+3\beta)\lambda^2+(6\beta^2+16\beta+20)\lambda-2\beta-12)(\lambda^3\beta^2+2\lambda^2\beta^2+\lambda\beta^2-4)b+4(\lambda^3\beta^2+2\lambda^2\beta^2+\lambda^2-4)(\beta^2\lambda^3-1+(2\beta^2+3\beta)\lambda^2+(\beta^2+3\beta+3)\lambda))^{\frac{1}{2}}$ and $\mu_2^\beta(\beta,\lambda,b) = ((\lambda\beta+\beta-2)^2((\beta^2\lambda^3+(2\beta^2+\beta)\lambda^2+(\beta^2+2)\lambda-\beta-6)^2b^2+(6\beta^2\lambda^3+6(2\beta^2+3\beta)\lambda^2+(6\beta^2+16\beta+20)\lambda-2\beta-12)(\lambda^3\beta^2+2\lambda^2\beta^2+\lambda\beta^2-4)b+4(\lambda^3\beta^2+2\lambda^2\beta^2+\lambda\beta^2-4)(\beta^2\lambda^3-1+(2\beta^2+3\beta)\lambda^2+(\beta^2+3\beta+3)\lambda)))^{\frac{1}{2}}(-12+5(\lambda+1)^2\lambda\beta^3+(9\lambda^2+6\lambda-3)\beta^2-20\beta)^2(\lambda+1)$. Now, $\mu_2^\beta(\beta,\lambda,b) > 0$ for all $\beta \in [0,1)$ and $\lambda \in [0,1)$. The sign of $\frac{\partial\theta^*(\beta,\lambda,b)}{\partial\beta}$ will depend on the sign of $\mu_1^\beta(\beta,\lambda,b)$. By simulating values of β and b, we get the value of λ_L where $\frac{\partial\theta^*(\beta,\lambda,b)}{\partial\beta} \leq 0$ if $\lambda \in [0,\lambda_L]$ and $\frac{\partial\theta^*(\beta,\lambda,b)}{\partial\beta} > 0$ if $\lambda \in (\lambda_L,\bar{\lambda})$, given in Table 4. From this we can say, for $\lambda \in [0,\lambda_l]$, area with standardization falls and area with non-standardization rises, and for $\lambda \in (\lambda_L,\bar{\lambda})$, area with standardization rises and area with non-standardization falls.

7.5 Proof of Proposition 3

$$\frac{\partial\theta^*(\beta,\lambda,b)}{\partial\lambda} = \frac{\mu_1^l(\beta,\lambda,b)}{\mu_2^l(\beta,\lambda,b)} \tag{40}$$

where $\mu_1^l(\beta,\lambda,b) = (((14b^2+69b+46)\lambda^2+(4b^2-26b-24)\lambda+28b^2-26b-12)\lambda(\lambda+1)^6\beta^7+((251b+14b^2+174)\lambda^3+(2b^2-175b-120)\lambda^2+(10b^2-43b-42)\lambda-2b^2+6b)(\lambda+1)^5\beta^6+((b+6)(52b+34)*\lambda^3+(-240b^2-768b-402)\lambda^2+(-28b^2+34b+48)\lambda-12b^2+28b+60)(\lambda+1)^4\beta^5+((120b^2+108b+48)\lambda^3+(-232b^2-148b-$

$b\backslash\beta$	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
b=2	0.583	0.570	0.555	0.541	0.526	0.510	0.494	-	-	-
b = 4	0.708	0.699	0.690	0.679	0.668	0.656	0.641	0.625	0.608	-
b = 8	0.816	0.812	0.807	0.802	0.796	0.788	0.779	0.767	0.751	0.732
b = 16	0.894	0.893	0.891	0.889	0.886	0.883	0.877	0.872	0.862	0.846
b = 32	0.943	0.942	0.942	0.941	0.940	0.939	0.938	0.935	0.931	0.922

Table 4: λ_L for different β and b (Approximated after 3 digits of decimal).

$924)\lambda^2 + (120b^2 + 828b + 600)\lambda - 64b^2 + 112b + 132)(\lambda+1)^3\beta^4 + ((144b^2 + 168b + 144)\lambda^3 + (-880b^2 - 2552b - 1632)\lambda^2 + (1648b^2 + 3544b + 1776)\lambda + 48b^2 + 280b + 192)(\lambda+1)^2\beta^3 + ((192b + 288)\lambda^3 + (-960b - 1248)\lambda^2 + (3360b + 2016)\lambda - 320b - 480)(b+1)(\lambda+1)\beta^2 + (b+1)((192b + 288)\lambda^2 + ((640b + 960)\lambda - 2112b - 1888)\beta + (b+1)((384b + 576)\lambda - 1152b - 960))(\beta(1+\lambda) - 2) - (((14b + 2)\lambda^2 + ((4b + 12)\lambda + 2b + 6)(\lambda+1)^4\beta^5 - 12(\lambda+1)^4(b+1)\beta^4 - (24(b+3)\lambda^2 + (-(176b - 208)\lambda - 8b - 24)(\lambda+1)^2\beta^3 - 96\lambda(\lambda+1)(\lambda-3)(b+1)\beta^2 - 320(b+1)\beta - 192b - 192)((\lambda\beta + \beta - 2)^2(\beta^2\lambda^3 + (2\beta^2+\beta)\lambda^2 + (\beta^2+2)\lambda - \beta - 6)^2b^2 + (6\beta^2\lambda^3 + 6(2\beta^2+3\beta)\lambda^2 + (6\beta^2 + 16\beta + 20)\lambda - 2\beta - 12)(\lambda^3\beta^2 + 2\lambda^2\beta^2 + \lambda\beta^2 - 4)b + 4(\lambda^3\beta^2 + 2\lambda^2\beta^2 + \lambda\beta^2 - 4)(\beta^2\lambda^3 - 1 + (2\beta^2 + 3\beta)\lambda^2 + (\beta^2 + 3\beta + 3)\lambda))^{\frac{1}{2}}$ and $\mu_2^l(\beta, \lambda, b) = ((\beta\lambda + \beta - 2)^2((\beta^2\lambda^3 + (2\beta^2+\beta)\lambda^2 + (\beta^2+2)\lambda - \beta - 6)^2b^2 + 6(\beta^2\lambda^3 + (2\beta^2+3\beta)\lambda^2 + (\beta^2 + 8/3\beta + 10/3)\lambda - \beta/3 - 2)(\beta^2\lambda^3 + 2\beta^2\lambda^2 + \beta^2\lambda - 4)b + 4(\beta^2\lambda^3 + 2\beta^2\lambda^2 + \beta^2\lambda - 4)(\beta^2\lambda^3 - 1 + (2\beta^2 + 3\beta)\lambda^2 + (\beta^2 + 3\beta + 3)\lambda)))^{\frac{1}{2}}(-12 + 5(\lambda + 1)^2\lambda\beta^3 + (9\lambda^2 + 6\lambda - 3)\beta^2 - 20\beta)^2(\lambda+1)^2$. We found $\frac{\partial\theta^*(\beta,\lambda,b)}{\partial\lambda} > 0$ for all $\beta \in [0,1)$ and $\lambda \in [0, \bar{\lambda}]$.

7.6 Proof of Proposition 4

We have compared $W^{XX}(\theta_1, \theta_2; \beta, \lambda)$, $W^{XY}(\theta_1, \theta_2; \beta, \lambda)$ and $W^{YY}(\theta_1, \theta_2; \beta, \lambda)$ and found that

$$W^{XX}(\theta_1, \theta_2; \beta, \lambda) - W^{XY}(\theta_1, \theta_2; \beta, \lambda) > 0 \qquad for \qquad \theta_1 + \theta_2 < 1$$

and

$$W^{YY}(\theta_1, \theta_2; \beta, \lambda) - W^{XY}(\theta_1, \theta_2; \beta, \lambda) > 0 \qquad for \qquad \theta_1 + \theta_2 > 1$$

.

Now, we assume that in the multiple equilibrium area the firms will choose to standardize in socially optimal technology, as both type of technology standardization is possible in that area. Thus, the possibility of divergence from socially desirable equilibrium essentially corresponds to the non-standardization area.. Thus, rest of the proof comes from the earlier propositions. Q.E.D.