Reconciling Wage and Price Phillips Curves *

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Abstract

This paper highlights and rationalizes a steepening of the price Phillips curve beyond a linear wage Phillips curve during tight labor markets. This discrepancy conflicts with the conventional wage-driven Phillips curve view, and the standard model assumption of firms as wage takers. I resolve this puzzle with wage setting power for firms and transitory shocks under downward nominal wage rigidity. Adding wage setting power allows for wage-price trade-offs. Transitory shocks increase the cost of raising wages if wages fail to decline after the shock subsides. Consequently, firms adjust prices to reduce demand, resulting in subdued increases in employment and wages. I provide additional support for this channel using market concentration and firm survey data. I rationalize my findings in a New Keynesian model with firm wage setting power and downward nominal wage rigidity, leading to a three-region non-linear Phillips curve. My channel predicts almost painless disinflation after transitory demand and labor supply shocks, and challenges the notion that ever-tighter labor markets are unambiguously good for real wage growth.

Keywords: Phillips curve, Downward Nominal Wage Rigidity, User Cost of Labor, Profit Margins, Market Concentration, Monetary Policy.
JEL-Classification: E24, E31, E52, E6.

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1 Introduction

The Phillips curve connects tight labor markets to inflation and has been central to macroeconomics since its discovery by Phillips (1958). The Phillips curve is commonly explained by rising wage bargaining power for workers as labor markets tighten. As wages rise, the cost of labor increases, which in turn drives up prices.

This interpretation informs research and policy to this day.[1] Wage growth serves as the sole driver in most theoretical frameworks concerning the Phillips curve, and is frequently cited as the rationale in empirical analyses (Benigno and Eggertsson (2023),Bernanke and Blanchard (2023)). Consequently, tight labor markets are suggested to affect inflation by at most as much as wage growth.

However, recent results contradict this view. Evidence points to a non-linear price Phillips curve, with a flat region during normal and weak labor markets and a steep region during tight labor markets (Ball et al. (2022), Benigno and Eggertsson (2023), Gitti (2024), Hooper et al. (2020)). In contrast, the wage Phillips curve remains linear (Ball et al. (2022), Bernanke and Blanchard (2023)) even when considering different labor market measures (Heise et al. (2024)). Specifically, inflation rises faster than wage growth during periods of very tight labor markets.

This paper highlights and rationalizes this discrepancy. Specifically, I propose that tight labor markets have an effect on inflation, independent of prevailing wage growth. I call this the direct impact channel of the Phillips curve, in contrast to the usual channel which operates indirectly through wage growth. I test the existence of the direct impact channel by including wage growth controls into common price Phillips curve estimations. If the two Phillips curves have different functional forms, then tight labor markets should show a significant relation to price inflation, even after controlling for wage growth.

First, I evaluate the direct impact channel using U.S. aggregate data. The direct impact is robust to additional controls, excluding the pandemic, and on aggregate data back to the early 1960s.

Second, I address potential endogeneity issues from supply shocks and monetary and fiscal policies, in a cross-sectional analysis using Metropolitan Statistical Area (MSA) data. The direct impact is significant in both ordinary least squares and instrumental variable two-way fixed effects estimations. Further, I use a shift-share instrumental variable approach based on different MSA exposure to national employment trends to account for local supply shocks.

I present additional evidence supporting the direct impact on U.S. sectoral and cross-country data in the Appendix. For the sector analysis I construct NAICS two-digit CPI estimates from CPI component data to match existing sector level wage and labor market data. I find the direct impact across US service sectors, and to a lesser degree in goods sectors. Furthermore, I find the direct impact across 24 developed nations, confirming a similar direct

[1]For example Federal Reserve Governor Kugler stated in March 2024: "In a context of strong aggregate demand, firms will likely grant the wage increases to retain their workers — then pass through the resulting cost increase as higher prices".

impact for all but four of these countries, using a Classifier LASSO estimation.

The direct impact channel implies a decline in real wage growth and a rise in profit margins during especially tight labor markets. I test these implications in my data and find them to hold.

I explain the direct impact of labor market tightness on inflation with firms having both price and wage setting power, and facing downward rigid wages. Simultaneous wage and price-setting power allows for trade-offs between prices and wages, while downward rigid wages introduce additional cost to raising wages under transitory shocks. Under flexible wages, firms still raise wages and prices in proportion when faced with transitory demand shocks. However, wages stay elevated after the shock subsides if they are downward rigid. Consequently, marginal cost increases both now and in the future when firms raise wages today. Taking these future costs into account, firms price out demand which tempers increases in employment and wages.

I illustrate my idea in a two-period monopolist model with iso-elastic demand and labor supply schedules. Under the assumption of downward nominal wage rigidity, the monopolist takes higher future wages from today's wage decision into account when deciding on optimal prices and wages. As a result, temporary positive demand shocks or negative labor supply shocks create a gap between prices and wages on top of the usual steady-state markup.

The illustrative model provides two testable implications. First, it implies that market power and monopsony power are essential for the channel. I test this implication by interacting the Herfindahl-Hirschman Index as a measure of market concentration, with labor market tightness in my cross-country and cross-sector data. Specifically, lower concentration weakens the direct impact, while higher concentration increases the steepness of the price Phillips curve during times of tight labor markets.

Second, my mechanism implies that increases in expected future sales during tight labor markets reduce inflationary pressures. When expected demand rises, it lessens the expected effects of downward wage rigidity, which reduces the wedge between prices and wages. This prediction contrasts with those of the standard New Keynesian model. I test these differing predictions using survey data from the National Foundation of Independent Businesses. I find a negative interaction between expected future sales growth and labor market tightness, supportive of my channel.

Finally, I integrate the channel into a New Keynesian Search and Matching framework by introducing wages into firms' decision-making via wage posting. The interplay between firm wage-setting and labor market frictions generates monopsony power (Bloesch et al. (2024)), which I discipline with on-the-job search. I model general nominal frictions with Rotemberg adjustment costs and enforce downward nominal wage rigidity.

Firms under temporary demand shocks consider the increase in future marginal costs from raising wages today. This leads to a greater price and a smaller wage response during tight labor markets, creating an additional wedge between prices and wages.

I calibrate the model to micro-level labor market evidence and solve the dynamics using piece-wise linearization across its three distinct states: when downward nominal wage

rigidity (DNWR) binds, when wages can adjust freely, and when DNWR is expected to bind tomorrow. These three cases create a strictly convex Phillips curve in simulated data. I use lower adjustment costs for wages compared to prices to match evidence for a steeper wage than price Phillips curve during slack to moderately tight labor markets (Galí and Gambetti (2019)).

The model produces a steepening price Phillips curve during tight labor markets without relying on a similar steepening in the wage Phillips curve. This explains the observed drop in real wage growth and the increase in profit margins during particularly tight labor markets.

My channel bolsters the non-linear Phillips curve explanation for the low cost of disinflation observed after the post-pandemic inflation surge.[2] Since real wage growth is subdued or even negative in my framework, there is greater capacity to reduce inflation without rendering sections of the labor force too costly. As a result, increases in unemployment become smaller, leading to smaller reductions in output.

Additionally, my channel challenges the notion that tighter labor markets are always good for workers. The direct impact channel suggests that very tight labor markets can actually reduce real wage growth. Finally, the linear nature of the wage Phillips curve indicates the direct impact channel's key role in explaining Phillips curve non-linearity. This highlights the merit of further research on the channel.

1.1 Literature

First and foremost, this paper fills a gap in the empirical Phillips curve literature. The divergence I point out is apparent in the results of Ball et al. (2022), but remains unaddressed in their paper. I document and rationalize this divergence with a direct impact of labor market tightness on inflation. This explanation matters for other papers who point out a non-linear price Phillips curve, but explain it through a non-linearity in the wage Phillips curve (Benigno and Eggertsson (2023), Gitti (2024)). A discrepancy that has also been noted by Heise et al. (2024) and Dao et al. (2024) but left unaddressed.

Second, this paper offers a novel micro-founded explanation of Phillips curve non-linearity. Existing explanations predominantly attribute this non-linearity to a non-linear wage response (Benigno and Eggertsson (2024), Schmitt-Grohé and Uribe (2022), Gitti (2024)), which is inconsistent with my empirical findings. My approach offers an alternative explanation for the non-linear price Phillips curve that does not rely on the non-linearity of the wage Phillips curve. While the notion that downwardly rigid wages exert an influence on the Phillips curve is not novel (see Schmitt-Grohé and Uribe (2022), Gagnon and Collins (2019), Donayre and Panovska (2016), Gitti (2024)), prior literature has concentrated on how downward wage rigidity tends to flatten the Phillips curve during periods of slack labor markets. In contrast, I expand this discourse by establishing that downward nominal wage rigidity (DNWR) leads to a steepening of the Phillips curve in the context of tight labor markets. The resulting Phillips curve is three-fold. A flat part when DNWR is binding, a moderately steep part under dynamics unaffected by DNWR, and a steep part when DNWR

[2] This helps resolve the "Missing Unemployment Puzzle" mentioned by Schmitt-Grohé and Uribe (2022).

does not bind but is expected to bind in the future.

Third, this paper weighs in on the importance of the labor market during the post-pandemic inflation surge. While some studies (Bernanke and Blanchard (2023)) suggest a negligible impact, these assessments are primarily framed through the conventional wage growth channel. In contrast, other research (Ball et al. (2022), Benigno and Eggertsson (2023)) underscores the substantial influence of labor market tightness on inflation. My direct impact channel reconciles these divergent findings by demonstrating that Bernanke and Blanchard (2023) captures only the limited effect associated with the traditional Phillips curve channel, while overlooking the non-linear inflationary increases arising from the direct impact channel. For further details, refer to Osten (2025b).

Finally, this paper contributes to the emerging class of New Keynesian models that ascribe wage-setting power to firms, initiated by Sveen and Weinke (2009) and Thomas (2011). I incorporate downward nominal wage rigidity into its most recent branch based on wage posting (Bloesch and Larsen (2023), Bloesch et al. (2024), de la Barrera i Bardalet (2024)). This synthesis of monopsony power within the labor market and downward nominal wage rigidity culminates in the first New Keynesian model capable of yielding the user cost of labor (Kudlyak (2014)).

The rest of the paper is structured as follows. Section 2 provides the empirical evidence for the direct impact channel on inflation, real wage growth, and profit margins in US aggregate data. Section 3 addresses the known endogeneity issues of Phillips curve estimations using US regional data. Section 4 illustrates the mechanism of the direct impact channel in a simple framework. Section 5 provides the channel dynamics in a New Keynesian framework with monopsony power, and Section 6 concludes.

2 Direct Impact on Aggregate Data

I highlight puzzling empirical findings in the Phillips curve literature, where price Phillips curves are observed to steepen significantly during tight labor markets, while wage Phillips curves remain linear (Appendix A.1). In this section I document that this divergence during tight labor markets persists when the traditional wage channel of the Phillips curve is shut down with wage growth controls. Thus, I document a puzzle in the empirical Phillips curve literature.

I resolve the puzzle of diverging wage and price Phillips curves by introducing a direct effect from labor market tightness on inflation. I provide empirical evidence for this direct impact in four parts. First, I show that labor market tightness exerts significant influence on price inflation, even when wage growth is accounted for. Second, I explore the non-linearity of the direct impact to test if it contributes to Phillips curve non-linearity (Smith et al. (2023), Benigno and Eggertsson (2023)). Third, I estimate the direct impact on real wage growth. Fourth and finally, I investigate the origin of the direct impact by estimating it using profit margin data, which enables me to broadly control for marginal cost. This section provides the evidence for each step in turn.

2.1 Evidence on Inflation Data

I estimate a cubic Phillips curve as in Ball et al. (2022) while including controls for wage growth, energy inflation, and import prices.

$$\pi_t = \phi_1 \pi_t^w + \phi_2 \pi_{t-1}^w + \acute{\alpha}_1 (V/U)_t + \acute{\alpha}_2 (V/U)_t^2 + \acute{\alpha}_3 (V/U)_t^3 + \delta \pi_t^e + \rho X_t + c_\pi + \epsilon_t \quad (1)$$

where π is inflation, π^w wage growth (based on ECI wages and salaries), V/U labor market tightness (job openings to unemployment ratio), π^e inflation expectations (10-year SPF median forecast), ϵ_t is an error term, and X_t a vector of controls containing an eight quarter moving average of labor productivity growth, current and lagged energy inflation, as well as relative import price pressure (Yellen (2015)).[3]

Using SPF expectations is preferable to lags of inflation due to the regime shift from low inflation to high inflation. One also only gets the static impact of variables on inflation rather than the dynamic impact with auto-regressive lags.

Further, I impose the following homogeneity constraint

$$\phi_1 + \phi_2 + \delta = 1$$

I estimate my price equation on US aggregate data from 1985Q1 - 2023Q4. I test multiple inflation measures, and present results on CPI headline, traditional core CPI, median CPI and core services excluding shelter inflation in Table 1.

Models 1-4 provide estimates of the price equation on CPI headline, CPI excluding food and energy, median CPI, and core services excluding shelter inflation.[4] All four models exhibit a significant impact of labor market tightness on inflation as shown by the Wald test p-value in the third to last row ("No Impact"). The direct impact is also highly non-linear as shown by the Wald test in the second to last row ("Linear Impact"). The Null-Hypothesis here is that the higher order coefficients on labor market tightness are jointly zero. This direct impact on inflation holds even though past and current wage growth as well as inflation expectations, labor productivity, energy shocks, and import price shocks are controlled for.

Core services excluding shelter provide the strongest response of inflation to labor market tightness. This, fits with the generally stronger response I found on service sectors as opposed to manufacturing sectors (see Appendix D). Services are generally more labor intensive than manufacturing, leading to a greater role of downward nominal wage rigidity in these sectors. The stronger results on services than manufacturing also points to a lower role of pandemic supply chain issues in the estimation, as these mainly affected manufacturing sectors.

I base the number of wage growth lags on the Bayesian Information Criterion. I further explore the robustness of the direct impact to inclusions of additional wage lags in Appendix

[3] Yellen (2015) excludes petroleum products, natural gas, computers, and semiconductors from the import price index. I merely take out petroleum products as I control for energy inflation separately.

[4] I omit reporting the coefficients of the control variables. These can be found in Appendix A.3. For the PCE version of Table 1 see Appendix A.3.3, for the results on monthly data see Appendix A.3.4 and for density graphs see Appendix A.3.5.

Table 1: Direct Impact on Inflation

	CPI	XFE	Median	CSXS
(V/U)	4.69*	4.99*	4.11**	9.32***
	(2.65)	(2.60)	(1.74)	(2.82)
$(V/U)^2$	-5.04	-5.08	-5.11**	-11.84***
	(3.29)	(3.26)	(1.96)	(3.32)
$(V/U)^3$	2.11*	2.09*	2.20***	4.47***
	(1.13)	(1.12)	(0.64)	(1.12)
Expected Inflation	0.82***	1.02***	0.64***	1.17***
	(0.11)	(0.11)	(0.08)	(0.14)
Wage Growth	0.05	0.03	0.18***	-0.10
	(0.07)	(0.05)	(0.04)	(0.09)
Wage Growth Lag	0.10	-0.05	0.19***	-0.07
	(0.08)	(0.10)	(0.05)	(0.13)
Constant	-1.52**	-1.76***	-0.79	-1.51**
	(0.66)	(0.64)	(0.49)	(0.76)
R^2	0.87	0.69	0.84	0.60
N	156	156	156	155
No Impact (p-value)	0.0000	0.0000	0.0000	0.0000
Linear Impact (p-value)	0.0023	0.0035	0.0000	0.0000
Partial R^2	0.17	0.25	0.42	0.16
Controls	✓	✓	✓	✓

Newey-West standard errors in parentheses

* $p < .10$, ** $p < .05$, *** $p < .01$

Notes: "CPI" uses headline Consumer Price Index inflation as dependent variable. "XFE", "Median" and "CSXS" do the same with CPI excluding food and energy, median CPI inflation, and Core Services excluding Shelter CPI inflation, respectively. The "No Impact" Wald test has the hypothesis of all V/U coefficients equal to zero. The "Linear Impact" tests the higher order V/U coefficients being zero. Partial R^2 reports the variation explained by V/U and its higher order terms. The results are from estimating Equation 1 on US quarterly data from 1985Q1 - 2023Q4.

A.3.1. The coefficients are virtually unchanged by the inclusion of four lags of wage growth, while the Wald test strongly rejects both the non-existence and linearity of the direct impact. Additionally, I test the price equation on a plethora of other inflation measures, and all but

the Producer Price Index evince the same cubic pattern[5]. The failure of the PPI is unsurprising as it is almost exclusively based on manufacturing which is not labor-intensive.

Similarly, using different wage growth measures leads to the same significantly cubic direct impact of labor market tightness on inflation[6]. I further test robustness of my results in Table 1 to winsorizing labor market tightness and inflation measures to their respective 99th percentile. Winsorizing labor market tightness and inflation measures leaves the estimated coefficients essentially unchanged.

Finally, I constrain the estimation to the pre-pandemic period (1985Q1 - 2019Q4). Using purely pre-pandemic data turns the Wald tests on headline CPI inflation insignificant. However, core, median and core services less housing inflation still provide evidence for the cubic direct impact (see Appendix A.3.2). The noise in headline inflation complicates the identification of the direct impact. This is particularly true during the pre-pandemic estimation period (1985Q1 - 2019Q4), in which tight labor markets were rare.[7] The evidence on core inflation measures shows that the direct impact estimation is not singularly driven by the pandemic inflation surge.

In summary, Table 1 presents evidence for a direct impact and its contribution to the nonlinearity of the Phillips curve.[8] However, wage cost is still adding substantially to the Phillips curve, making the direct impact a complement to the traditional wage based Phillips curve channel. I quantify the contributions of both channels during the post-pandemic inflation surge in Appendix A.4.

I estimate a similar cubic wage Phillips curve (see Appendix A.6) and compare the estimated functional forms of these wage and price equations in Figure 1. The functions are plotted over the rough range of V/U in the data: $[0.15, 2.0]$.

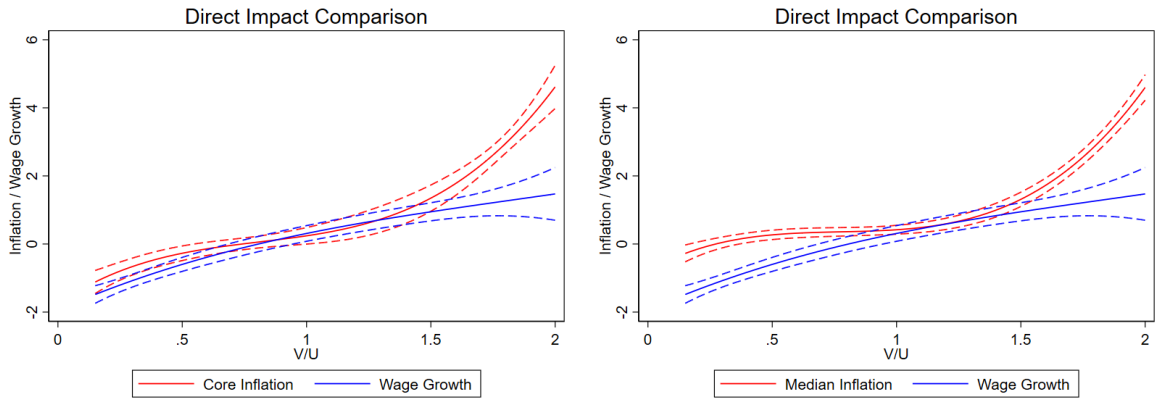
Figure 1 highlights the linear nature of the relation between labor market tightness and wage growth, once expectations and inflation are accounted for. Even though I include the higher order terms, the function is essentially linear over the range of observed labor market tightness. The direct impact on inflation and wage growth is similar for a majority of the observed V/U values, hinting at stable real wage gains under normal labor market conditions. However, from a V/U ratio of 1.25 onwards the price Phillips curve lifts off while the wage Phillips curve remains linear. These findings suggest an increasingly rapid decline in real

[5]CPI Trimmed Mean, CPI Services exc. Shelter, CPI Services exc. Energy Services, CPI Services exc. Energy Services and Shelter, PCE, PCEXFE, PCE Median, PCE Trimmed Mean, PPI, and the GDP Deflator. I point to the robustness of measuring inflation with core services excluding shelter due to the ongoing discussion on the role of shelter in the CPI (Adams et al. (2024)). The results are similar to those on median CPI inflation.

[6]Average Hourly Earnings, Average Hourly Compensation, Unit Labor Cost, and ECI Total Compensation. Some papers point to the importance of new hire wages and their greater pro-cyclicality, however Gertler et al. (2020) show that new hire wages are not more procyclical than existing employee wages once composition effects are controlled for.

[7]The scarcity of meaningful labor market tightness over the pre-pandemic period is conjectured to have caused the flatter Phillips curve estimates over the same period (Gagnon and Collins (2019)). The direct impact can explain this observation, as it is almost non-existent under average labor market tightness.

[8]In fact my price equation synthesizes the approaches of Ball et al. (2022) and Bernanke and Blanchard (2023) and the empirical evidence for the direct impact reconciles the divergent findings of high and low labor market importance for the post-pandemic inflation in these papers. See Osten (2025b) for details.



(a) CPIXFE

(b) Median CPI

Figure 1: Comparison of the direct impact on wage and price growth.
Subfigure a) shows the labor market's cubic impact on CPI inflation excluding food and energy against the cubic impact on ECI wage growth. Subfigure b) does the same for median CPI inflation.

wage growth. I explore this indication in the ensuing section.

There is evidence for a non-linear price Phillips curve. This non-linearity survives when controlling for wage growth, expected inflation, labor productivity, energy shocks and import price pressure. I present similar results on cross-country and US cross-sectoral data in Appendices D and C, respectively.

The impact of labor market tightness on wage growth on the other hand is linear. The non-linear reaction of inflation and the linear reaction of wage growth to labor market tightness imply that real wage growth also reacts non-linearly to labor market tightness. Specifically, it should decline rapidly during hot labor markets.

2.2 Direct Impact on Real Wage Growth

Real wage growth gives a direct comparison of the relative impact of labor market tightness on wages and prices. Therefore, I provide evidence for a direct impact on a real wage version of the previous equation in this section.

$$(\pi_t^w - \pi_t) = \bar{\alpha}_1(V/U)_t + \bar{\alpha}_2(V/U)_t^2 + \bar{\alpha}_3(V/U)_t^3 + \bar{\delta}\pi_t^e + \bar{\phi}(\pi_{t-1}^w - \pi_{t-1}) + \bar{\rho}X_t + c_\omega + \bar{\epsilon}_t \quad (2)$$

I measure real wage growth as the difference between annualized ECI and headline CPI growth rates. Table 2 presents the coefficient estimates of equation 2. I successively introduce the linear, quadratic and cubic labor market tightness terms in Models 1 to 3. I again omit the coefficients of energy inflation, and relative import prices in the table. The linear impact is entirely rejected by the Wald test, while the cubic functional form adds next to nothing in explanatory power. Thus, the data supports a quadratic functional form for real wage growth. Figure 2 shows the quadratic functional form on real wage growth.

Real wage growth is subdued during slack labor markets and rises with increasing tightness to form a maximum during moderately tight labor markets. After this point the direct

Table 2: Real Wage Growth

	Linear	Quadratic	Cubic
Expected Inflation	-0.48***	-0.58***	-0.58***
	(0.15)	(0.15)	(0.16)
Productivity Trend	0.30***	0.29***	0.29***
	(0.09)	(0.07)	(0.07)
L.Real Wage Growth	-0.15	-0.23***	-0.23***
	(0.09)	(0.08)	(0.08)
V/U	0.19	3.43***	3.92*
	(0.43)	(0.65)	(2.20)
$(V/U)^2$		-1.80***	-2.41
		(0.32)	(2.53)
$(V/U)^3$			0.21
			(0.82)
Constant	1.28*	0.47	0.38
	(0.72)	(0.62)	(0.72)
R^2	0.72	0.74	0.74
N	156	156	156
No Impact (p-value)	0.6669	0.0000	0.0000
Linear Impact (p-value)	-	0.0000	0.0000
Partial R^2 (V/U)	0.00	0.09	0.09
Controls	✓	✓	✓

Newey-West standard errors in parentheses

* $p < .10$, ** $p < .05$, *** $p < .01$

Notes: "Cubic" estimates Equation 2 while "Quadratic and "Linear" reduce the functional form of V/U to a quadratic and linear form, respectively. All three are estimated on US quarterly data from 1985Q1 - 2023Q4. The "No Impact" Wald test has the hypothesis of all V/U coefficients equal to zero. The "Linear Impact" tests the higher order V/U coefficients being zero. Partial R^2 reports the variation explained by V/U and its higher order terms.

impact leads to declining real wage growth, as increases in inflation surpass additional wage growth.[9]

[9]The negative effect on real wage growth reconciles Blanchflower et al. (2022) criticism of labor market tightness as the measure of labor market slack with its superior explanatory power (Domash and Summers (2022)).

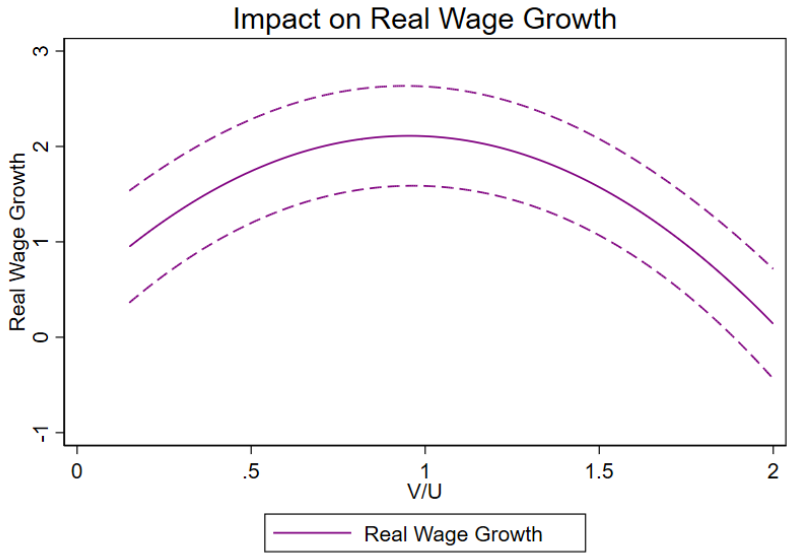


Figure 2: Functional Form on Real Wage Growth

The negative and non-linear impact of labor market tightness on real wages is robust to the use of winsorized and pre-pandemic (1985Q1-2019Q4) data. Finally, the results are robust to including additional auto-regressive lags in the real wage equation.

I test my price, wage and real wage equations' robustness to different measures of inflation, wage growth and inflation expectations. The non-linear direct impact on inflation, the limited linear impact on wage growth, and the negative non-linear impact on real wage growth are all robust to using a range of inflation, wage growth and expectations measures.[10]

2.3 Direct Impact on Profit Margins

There are two plausible explanations for my empirical findings: firs increases in marginal cost other than wages could drive inflation independently of wage growth, and second rising markups in response to tight labor markets could elevate prices beyond wage growth. To distinguish between these explanations, I estimate the response of profit margins to labor market tightness. A shock to marginal cost would reduce profit margins in the absence of

Blanchflower et al. (2022) argue that the concomitant fall in real wages and the rise of labor market tightness shows its unsuitability as measure of labor market slack. However, the direct impact reconciles the strong explanatory power of labor market tightness on inflation and the fall in real wage growth during especially tight labor markets. I thus contribute to the literature concerned with the question why labor market tightness outperformed traditional slack measures over the pandemic period (Furman and Powell (2021)).

[10] I check my results on headline CPI, CPIXFE, median CPI, CPI Trimmed Mean, CPI Services exc. Shelter, CPI Services exc. Energy Services, CPI Services exc. Energy Services and Shelter, PCE, PCEXFE, PCE Median, PCE Trimmed Mean, PPI, and the GDP Deflator. The only measure that fails is the PPI, which is unsurprising as it is almost exclusively based on non-labor intensive manufacturing. On the wage side my results are robust to using Average Hourly Earnings, Average Hourly Compensation, Unit Labor Cost, and ECI Total Compensation instead of the preferred measure of ECI wages and Salaries. Finally, the results are robust to using the Cleveland Fed's measure for 5, 10, 20, and 30 year ahead inflation expectations, instead of the median 10 year inflation forecast from the Survey of Professional Forecasters.

increased demand, while rising prices unrelated to marginal cost would result in increased profit margins. Thus, examining the relationship between profit margins and labor market tightness helps clarify the source of the direct impact.

The existing literature already documents the pro-cyclicality of profit margins and markups (Small (1997), Macallan et al. (2008), Nekarda and Ramey (2020)). Okun's law suggests that this pro-cyclicality translates into a positive correlation between profit margins and labor market tightness. However, this relationship has not been explored. Additionally, if optimal price setting drives the direct impact, the connection between profit margins and labor market tightness should exhibit some non-linearity, which is a novel aspect of the pro-cyclical profits and markup literature.

I use the Bureau of Economic Analysis' measure of profits per unit of real gross value added for non-financial corporations, adjusted for inventory values and capital cost. This measure, derived from NIPA Table 1.15, accounts for material input cost, employee compensation, depreciation, and taxes on production and imports (minus subsidies and transfers). According to Barkai and Benzell (2018), it is the best publicly available profit measure, improving upon aggregate profit margins from Palazzo (2023) by accounting for inventory value changes and miscellaneous cost, resulting in a raw aggregate operating profit margin before taxes.

I estimate the impact of labor market tightness on profit margins for the period 1985Q1-2023Q3 (2023Q4 data was not available during my latest data pull) in Table 3. The empirical model follows a similar structure as Equation 2 for real wages, examining the relationship between profit margins and linear, quadratic, and cubic labor market tightness. Additionally, I control for wage growth to exclude the traditional Phillips curve channel and include an eight-quarter moving average of labor productivity growth.

The linear estimation shows a significant and substantial impact of labor market tightness on profit margins, as indicated by the "No Impact" Wald test and Partial R^2. This impact increases further when accounting for a quadratic relationship. In contrast, the cubic estimation yields insignificant coefficients and adds minimal explanatory power, making the quadratic relationship preferable. The notable non-linear effect of labor market tightness on profit margins suggests that the direct impact is not merely a result of rising marginal cost alongside tight labor markets. This relationship remains robust even when winsorizing labor market tightness and limiting the estimation to the pre-pandemic period (1985Q1-2019Q4).

I show the quadratic functional form of profit margins in Figure 3.

The profit margin function increases with inflation during tight labor markets, indicating a common underlying cause for the rise in profit margins and price inflation. These findings suggest a profit-maximizing price-setting mechanism driving the direct impact, which I will explore in Section 4.1.[11]

There are well-known identification issues related to endogenous monetary and fiscal

[11] I test the robustness of my real wage growth and profit margin results using winsorized data and an alternative threshold model. The results remain largely unchanged. Similarly, my findings on inflation, wage growth, real wage growth, and profit margins are robust when using NFIB's percentage measure of small businesses reporting unfilled job openings in the current period instead of the V/U ratio.

Table 3: Profit Margins

	Linear	Quadratic	Cubic
Wage Growth	-1.59***	-1.37***	-1.24***
	(0.40)	(0.36)	(0.41)
Productivity Trend	-0.41	-0.35	-0.44
	(0.62)	(0.57)	(0.58)
V/U	7.66***	-3.96	-15.56
	(1.92)	(4.22)	(12.57)
$(V/U)^2$		6.12***	20.27
		(1.82)	(13.79)
$(V/U)^3$			-4.86
			(4.49)
Constant	10.93***	14.38***	16.79***
	(2.53)	(2.61)	(3.35)
R^2	0.38	0.47	0.48
N	155	155	155
No Impact (p-value)	0.0001	0.0000	0.0000
Linear Impact (p-value)	-	0.0010	0.0025
Partial R^2 (V/U)	0.31	0.42	0.43

Newey-West standard errors in parentheses

* $p < .10$, ** $p < .05$, *** $p < .01$

Notes: "Cubic" estimates the impact of a cubic functional form of V/U on aggregate profit margins, while "Quadratic and "Linear" reduce the functional form of V/U to a quadratic and linear form, respectively. All three are estimated on US quarterly data from 1985Q1 - 2023Q3. The "No Impact" Wald test has the hypothesis of all V/U coefficients equal to zero. The "Linear Impact" tests the higher order V/U coefficients being zero. Partial R^2 reports the variation explained by V/U and its higher order terms.

policy (Fitzgerald et al. (2020), McLeay and Tenreyro (2020)) and simultaneity bias from aggregate supply shocks when estimating the Phillips curve slope using aggregate data. I address these issues in the next section utilizing panel data on regional US data, as well as partially in cross-sector and cross-country data (Appendices D, C). While cross-country data allows for controls for global factors and time-invariant policy, US regional data enables me to address local supply shocks using an established instrumental variable approach.

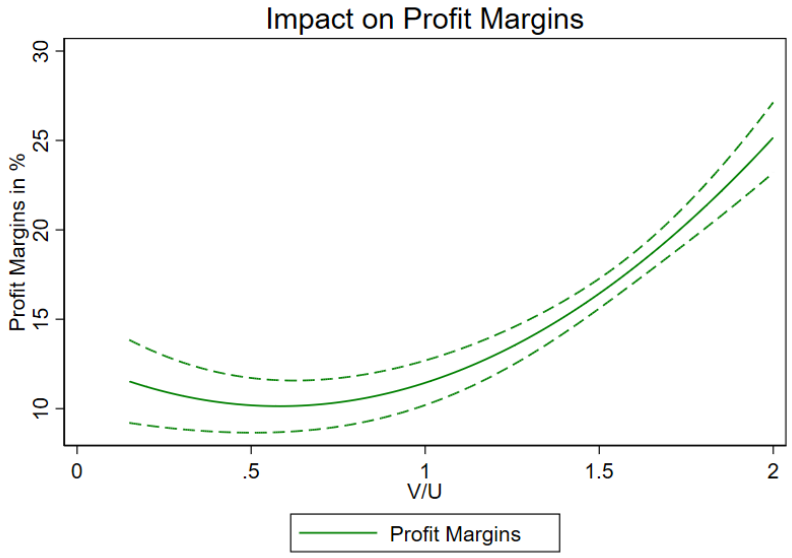


Figure 3: Functional Form on Profit Margins

3 Cross-Sectional Evidence from US Regional Data

Cross-sectional evidence from regional data provides several key advantages over aggregate data when estimating the Phillips curve. Primarily, it facilitates the inclusion of time fixed effects, which help to eliminate simultaneity bias that can stem from aggregate supply shocks, as well as endogeneity bias related to output-stabilizing monetary and fiscal policies and the effects of aggregate inflation expectations. Additionally, using regional data markedly increases the sample size and introduces greater variation in labor market conditions (Hooper et al. (2020)).

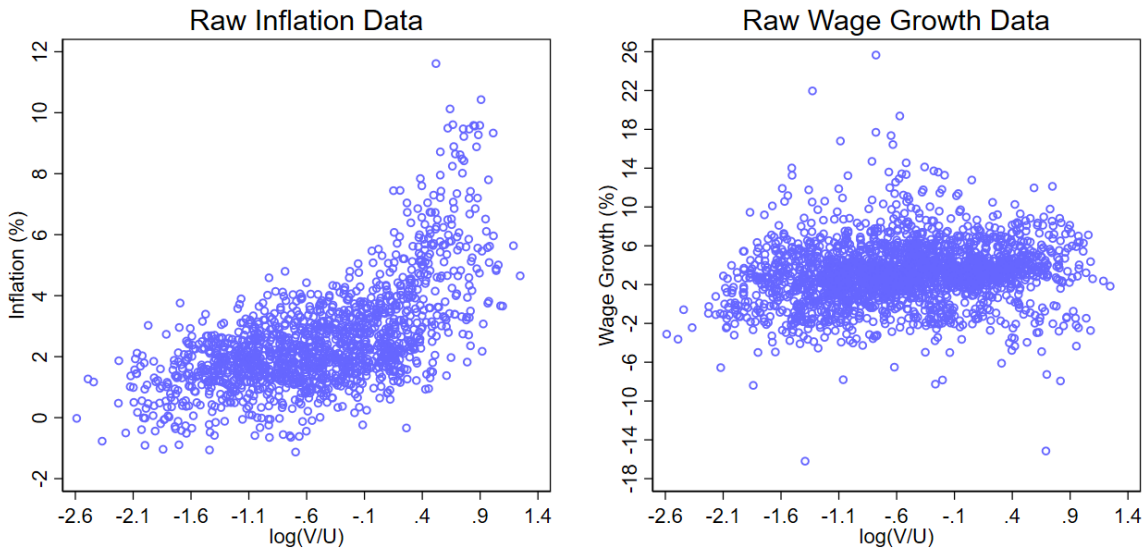
3.1 MSA Data

The Bureau of Labor Statistics provides data on average weekly wages, the Consumer Price Index (CPI), and unemployment rates for 21 major Metropolitan Statistical Areas (MSAs), while job vacancy data are only available at the state level. To address this limitation, I adopt the approach outlined by Gitti (2024) for imputing MSA-level vacancy data. Specifically, I calculate each MSA's share of state job vacancies based on its population relative to the total state population and aggregate these shares for MSAs that span state borders.

However, this imputation relies on the assumption of a homogeneous vacancy-to-employment ratio within each state, which may not accurately reflect the variability that exists across counties. This assumption introduces potential measurement error. To mitigate the impact of this error, I will employ an instrumental variables approach.

There is a mis-match in the data in terms of frequency. The unemployment and vacancy data is available at monthly frequency while the CPI data is available at a monthly frequency for only three major MSAs, while the remaining MSAs are sampled bi-monthly. Average weekly wages are collected quarterly for all MSAs. I aggregate the data to quarterly fre-

quency and show in Appendix B.3 that my results hold when using the linear interpolation to monthly data used in the literature (Beaudry et al. (2025), Gitti (2024)). Finally, I use year over year inflation and wage growth to get rid of seasonality in the data.



(a) Core Inflation

(b) Wage Growth

Figure 4: Raw Data Correlations

Subfigure a) scatters MSA core inflation against MSA $\log(V/U)$ on quarterly data. Subfigure b) does the same for wage growth.

I present the raw correlations between inflation and labor market tightness in the MSA data from 2001Q1 to 2024Q3 in Figure 4. Additionally, I display the correlations for MSA wage growth and labor market tightness. The raw MSA data suggests a non-linear relationship for the price Phillips curve. In contrast, wage growth, the expected conduit of labor market pressure onto prices, shows a flat Phillips curve in the raw data.

I present basic Ordinary Least Squares (OLS) estimations with step-wise MSA and time fixed effects for both inflation (π) and wage growth (π^w) in Table 4. Following the literature, I use a threshold of $V/U = 1$ to assess Phillips curve non-linearity.

Including controls for wage growth in the price Phillips curve yields results that are essentially the same as those from OLS estimation without these controls. Specifically, the impact of labor market tightness on inflation remains significant and non-linear, even when accounting for the traditional wage channel. Similar findings are also observed when using MSA data at the monthly frequency, as outlined in Appendix B.3.

3.2 Benefits of Regional Data

Figure 4 highlights the advantages of utilizing regional data, as it not only increases the overall number of observations but also enhances the variation in labor market tightness. In the MSA data 25% of quarterly labor markets are at or above $V/U = 1$, while only 17% of aggregate US data make that threshold. This provides a greater opportunity to observe

Table 4: OLS Results on Quarterly Data

	π	π	π^w	π^w
$\log(V/U)$	0.76***	0.62***	1.08**	0.42
	(0.246)	(0.122)	(0.471)	(0.307)
$I_{V/U>1} \times \log(V/U)$	4.39***	0.53**	-1.68	0.18
	(0.927)	(0.264)	(1.738)	(0.459)
Observations	1492	1492	2271	2271
MSA FEs	✓	✓	✓	✓
Year-Quarter FEs		✓		✓
MSA time trends		✓		✓

Driscoll-Kraay standard errors in parentheses

* (p<0.10), ** (p<0.05), *** (p<0.01)

Notes: π denotes headline inflation estimates, with log-labor market tightness $\log(V/U)$ and its interaction with tight labor markets as independent variables. MSA FEs and Year-Quarter FEs are the usual location and time fixed effects, and controls for MSA specific time trends.

inflation dynamics under tight labor markets.

Furthermore, regional data allows for the inclusion of time fixed effects, which help address various endogeneity issues, including those related to monetary and fiscal policy (Fitzgerald et al. (2014), McLeay and Tenreyro (2020)), aggregate inflation expectations (Hazell et al. (2022)), and confounding factors such as aggregate supply shocks.

This leaves regional supply shocks as potential confounding factors in the estimation of the Phillips curve. The literature addresses this issue by utilizing shift-share instruments, as proposed by Bartik (1991). Notably, Hazell et al. (2022), Gitti (2024), and Beaudry et al. (2025) all employ similar instruments in their analyses.

The shift-share approach leverages the variation in regional exposure to employment trends within the tradable goods or intermediate input sectors. A region that has a higher share of employment in a specific tradable sector will experience a tightening of its labor market relative to other regions in response to positive labor demand shocks in that sector.

For example, an aggregate increase in demand for processed grains will have a more pronounced impact on the Minneapolis-St. Paul area than on Pittsburgh, whereas an increase in demand for steel will have the opposite affect. Similarly, a rise in the price of oil will result in a greater increase in labor demand in Houston compared to other metropolitan areas.

The employment trend based shift-share instrumental variable is

$$\text{Shift Share}_{l,t} = \sum_i \bar{S}_{i,l} \times \Delta_{3YR} \log(E_{i,t})$$

where $\bar{S}_{i,l}$ is the average employment share of industry i in locality l, and $\Delta_{3YR} \log(E_{i,t})$ is the three year growth rate of national employment of industry i.

This requires average 2-digit NAICS industry employment shares for each Metropolitan Statistical Area (MSA), which I obtain from the Census Bureau, along with national employment data by industry sourced from the Current Population Survey.

The industries utilized in this instrument vary slightly among studies. Hazell et al. (2022) and Beaudry et al. (2025) primarily focus on the tradable goods sectors, while Gitti (2024) incorporates additional intermediate inputs such as wholesale trade, as well as the intermediate components of transportation, warehousing, utilities, and professional and business services.

For the shift-share instrumental variable to function properly, local supply shocks should not correlate with both the employment shifts $\Delta_{3YR} \log(E_{i,t})$ and the shares $S_{i,l}$ in the cross-section. In other words, while local supply shocks may influence an MSA's employment composition, they should not impact national employment trends in the affected industry. Finally, the instrumental variable approach helps address the classical measurement error present in regional labor market data.

3.3 Instrumental Variables Approach

I generalize the existing shift-share IV approach in its tradable goods exposure variation. I incorporate the relative price of tradable goods across MSAs and time ($\frac{PPI_t}{CPI_{l,t}}$) as an instrument, following Hazell et al. (2022) and Gitti (2024). Finally, I replace the interaction of the shift-share instrument with the tight labor market indicator used by both Beaudry et al. (2025) and Gitti (2024) with a quadratic term of the instrument. The indicator of tight local labor markets violates the exclusion restriction as it is affected by local supply shocks. The quadratic term of the shift-share instrument avoids this pitfall by relying exclusively on the exogenous variation provided by the shift-share IV.

I control for MSA and time fixed effects to account for stable regional labor market differences, as well as aggregate effects such as monetary policy or energy price shocks. Additionally, I control for MSA specific time trends to capture the differential impact of common shocks across MSAs. These additional fixed effects help mitigate the bias outlined by Borusyak et al. (2022).

Putting all of this together, the identification strategy is valid if, after conditioning on fixed effects, employment growth in tradables and their relative price are uncorrelated with local labor supply shocks that could otherwise confound the estimates of labor demand. Contemporaneous relative prices are affected by local supply shocks. Thus, I use two-year lagged relative prices and add one-year lagged relative prices as controls for recent supply shocks.

The instruments are quite strong, as indicated by both traditional and Cragg and Donald (1993) F-values (CD Wald F), as well as the Lagrange Multiplier Sargan statistic proposed by Stock and Wright (2000) (SW S stat.) in the first stage regression Table 5.

Table 5: First Stage Regressions

	$\log(V/U)$	$I_{V/U>1} \times \log(V/U)$
$IV_{i,t}$	5.46***	0.05
	(0.868)	(0.659)
$IV_{i,t}^2$	-4.76	14.61**
	(6.987)	(6.403)
L8.$log(PPI_t/CPI_{i,t})$	3.76***	1.92***
	(0.662)	(0.491)
L4.$log(PPI_t/CPI_{i,t})$	-2.20***	-1.44***
	(0.646)	(0.374)
Wage Growth	0.00	0.00
	(0.003)	(0.001)
CD Wald F	12.16	12.16
SW S stat.	29.32	29.32
Observations	1394	1394
MSA FEs	✓	✓
Year-Quarter FEs	✓	✓
MSA time trends	✓	✓

Driscoll-Kraay standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.010$

Notes: $\log(V/U)$ denotes log-labor market tightness and $I_{V/U>1} \times \log(V/U)$ its interaction with tight labor markets. $IV_{i,t}$ is the shift-share variable from different exposure to employment trends, and $log(PPI_t/CPI_{i,t})$ is the relative price level of tradable goods. CD Wald F reports the Cragg-Donaldson Wld F statistic and SW S stat. does the same for the Stock-Wright S statistic for weak identification. Both reject a weak identification issue. MSA FEs and Year-Quarter FEs are the usual location and time fixed effects, and controls for MSA specific time trends.

3.4 Direct Impact Estimation

In Table 6, I present the estimated direct impacts on MSA data from the instrumental variables approach. To this end, I include local wage growth as a control variable and account for the price level from four quarters ago to capture existing price pressures influencing the year-over-year inflation estimate. The first column presents estimates for headline CPI inflation, while the second column presents those on core CPI inflation.[12]

[12] A final difference to the approaches of Beaudry et al. (2025) and Gitti (2024) is that I do not control for the indicator of labor market tightness. Including this indicator results in a noticeable jump in the Phillips curve relationship, which is a peculiar choice. When included, the indicator is generally negative, although rarely significant, and it increases the interaction coefficient.

Table 6: MSA Quarterly IV Estimation

	CPI	Core
$\log(V/U)$	1.38	1.79*
	(0.871)	(0.951)
$I_{V/U>1} \times \log(V/U)$	4.65**	6.58**
	(2.294)	(2.715)
L4.$log(PPI_t/CPI_{i,t})$	15.34***	15.73***
	(2.243)	(2.448)
Wage Growth	0.05**	0.03*
	(0.018)	(0.018)
Observations	1394	1394
MSA FEs	✓	✓
Year-Quarter FEs	✓	✓
MSA time trends	✓	✓

Driscoll-Kraay standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.010$

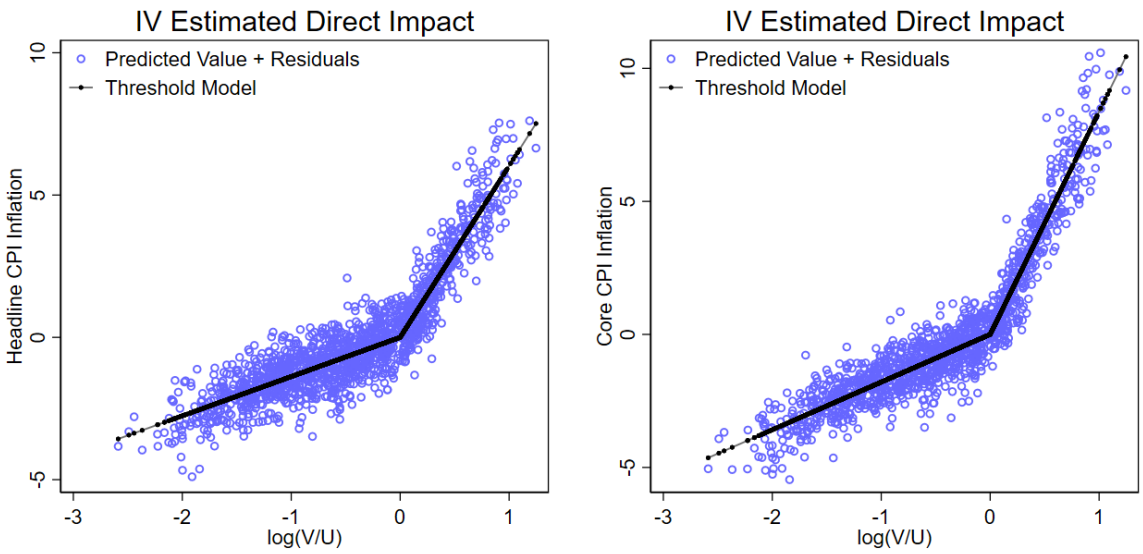
Notes: The CPI column uses headline CPI and the Core column core CPI as dependent variables. $\log(V/U)$ denotes log-labor market tightness and $I_{V/U>1} \times \log(V/U)$ its interaction with tight labor markets. $IV_{i,t}$ is the shift-share variable from different exposure to employment trends, and $log(PPI_t/CPI_{i,t})$ is the relative price level of tradable goods. MSA FEs and Year-Quarter FEs are the usual location and time fixed effects, and controls for MSA specific time trends.

I explore how the estimations change under different fixed effects in Appendix B.2 by successively introducing the three fixed effects discussed earlier. Additionally, I repeat the analysis from Table 34 using eight lags of wage growth in Appendix B.1. Finally, I replicate Table 1 of Gitti (2024) while including wage growth as a control variable. The results remain consistent across these analyses. I present the functional forms plotted against the residuals from the fixed effects and V/U, along with the IV-predicted labor market impact, in Figure 5.

3.5 Wage Phillips curve Estimation

Since most arguments for a non-linear Phillips curve are tied to wage growth (Benigno and Eggertsson (2024), Gitti (2024)), it is worthwhile to test this hypothesis using a similar estimation approach.

The employment trend-based shift-share instrument was designed to isolate local labor demand pressures, which are expected to translate into higher wage growth. However, only



(a) Direct Impact on Headline Inflation

(b) Direct Impact on Core Inflation

Figure 5: Direct impact on MSA Data

Subfigure a) shows the functional form for headline CPI inflation against IV predicted quarterly data plus inflation residuals from controls and fixed effects. Subfigure b) does the same for core CPI inflation.

after an additional step, the wage to price pass-through, does labor market pressure translate into inflation under the traditional understanding of the Phillips curve. Therefore, under the conventional Phillips curve framework employed by Hazell et al. (2022) and Gitti (2024), the instrument should be even better suited for estimating the wage Phillips curve than the price Phillips curve.

I test the wage Phillips curve by promoting wage growth from a control variable to the variable of interest. The results of the wage growth estimation are presented in Table 7, where $L4.log(WageLevel)$ denotes the four quarter lagged log level of local average weekly wages.

The IV-estimated wage Phillips curve provides no evidence of non-linearity, reinforcing the findings observed in aggregate data. I present the resulting estimated functional form in Figure 6.

I present the estimated wage and price Phillips curves together in Figure 7. The comparison shows that the wage Phillips curve (WPC) is steeper than the price Phillips curve (PPC) under slack to moderately tight labor markets. However, the PPC steepens in excess of the WPC under very tight labor markets leading to real wage growth declines.

Finally, I test the implication of a negative impact on real wage growth by subtracting year-over-year inflation from year-over-year wage growth to obtain an MSA measure of real wage growth. I do this for both headline and core inflation. The results of the fixed effects regressions are presented in Table 8, while controlling for workers' catch-up from lost real wage growth. To this end I include five lagged terms of real wage growth in annual steps

Table 7: Quarterly MSA Wage Phillips Curve

	π^W
$\log(V/U)$	3.00*
	(1.762)
$I_{V/U>1} \times \log(V/U)$	-0.11
	(6.001)
L4.$log(WageLevel)_{i,t}$	-0.17***
	(0.030)
Observations	1402
MSA FEs	✓
Year-Quarter FEs	✓
MSA time trends	✓

Driscoll-Kraay standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.010$

Notes: π^w denotes wage growth from QCEW average weekly wages as dependent variable, $\log(V/U)$ denotes log-labor market tightness, $I_{V/U>1} \times \log(V/U)$ its interaction with tight labor markets, and $log(WageLevel)$ is the level of average weekly wages. MSA FEs and Year-Quarter FEs are the usual location and time fixed effects, and controls for MSA specific time trends.

(l4,l8,l12,l16,l20).[13] The resulting impact is significant and negative.

[13]These lags are supported by both Bayesian and Akaike information criteria.

Table 8: Real Wage Growth Estimation on Quarterly MSA data

	RWG	RWG_{core}
$\log(V/U)$	5.08***	4.99***
	(1.439)	(1.749)
$I_{V/U>1} \times \log(V/U)$	-9.02**	-11.05**
	(4.015)	(4.561)
L4.$log(PPI_t/CPI_{i,t})$	-0.73	4.15
	(10.748)	(11.007)
Observations	1058	1058
Wage catch-up controls	✓	✓
MSA FEs	✓	✓
Year-Quarter FEs	✓	✓
MSA time trend	✓	✓

Driscoll-Kraay standard errors in parentheses

* (p<0.10), ** (p<0.05), *** (p<0.01)

Notes: RWG denotes real wage growth from differencing QCEW average weekly wage growth and CPI headline inflation as dependent variable, while RWG_{core} does the same for the difference between wage growth and core CPI inflation. $\log(V/U)$ denotes log-labor market tightness, $I_{V/U>1} \times \log(V/U)$ its interaction with tight labor markets, and $log(PPI_t/CPI_{i,t})$ is the relative price level of tradable goods. MSA FEs and Year-Quarter FEs are the usual location and time fixed effects, and controls for MSA specific time trends.

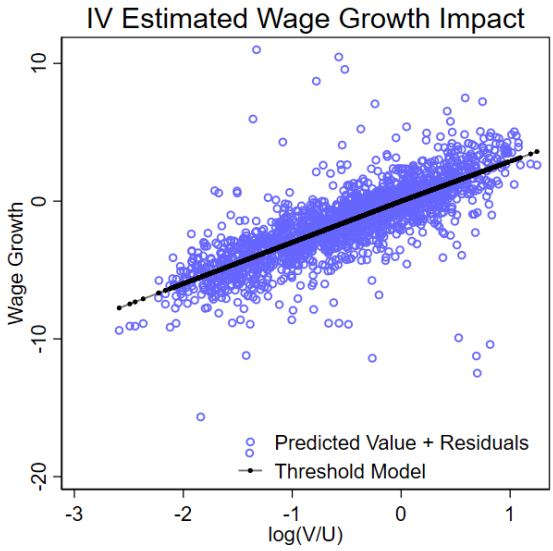
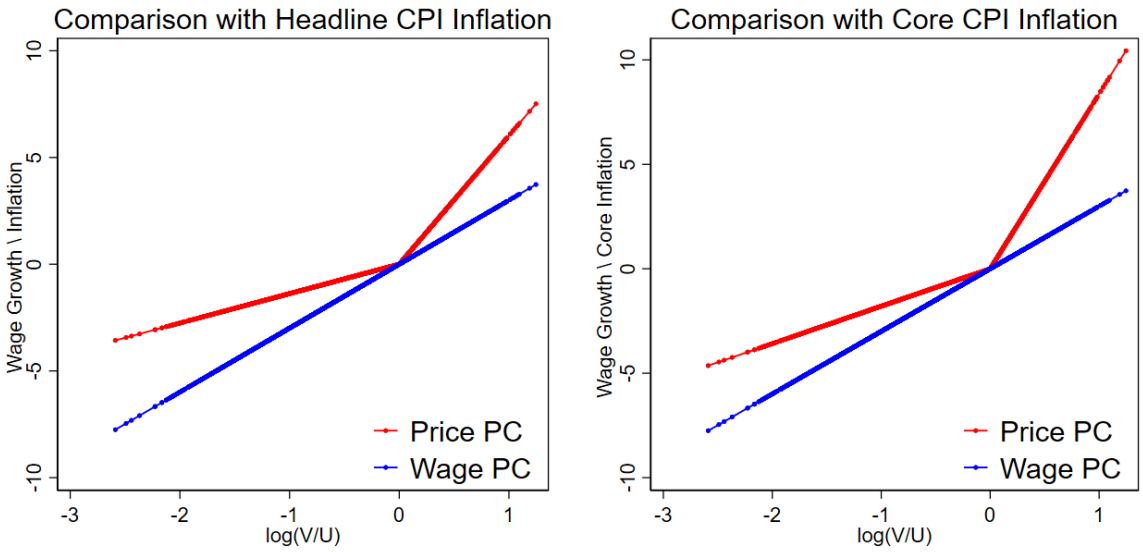


Figure 6: (Non-Linear) Wage Phillips Curve on quarterly MSA Average Weekly Wages



(a) PC Comparison on Headline

(b) PC Comparison on Core

Figure 7: Comparing Wage and Price Phillips curves on quarterly MSA Data

Subfigure a) shows the predicted wage and price Phillips curves using growth in average weekly wages and headline CPI inflation. Subfigure b) does the same using core CPI inflation.

4 The Direct Impact Channel

This section presents an explanation for the direct impact of tight labor markets on inflation. It concludes that the greater stickiness of wages than prices and associated optimal pricing behavior under transitory demand or labor supply shocks is the most likely driver of the direct impact channel.

I propose higher downward-stickiness for wages than for prices, as found by Spencer (1998), paired with firms choosing wages instead of taking wages as given. The last important ingredient are transitory upward wage pressure shocks. Transitory positive demand shock typically lead to production expansion alongside mild wage increases. During a tight labor market however, it is necessary to raise wages significantly to hire the workers needed to full-fill extra demand. If the demand shock is transitory, or at least perceived as such, then firms will resist raising wages which will stick around after the demand shock dissipates. It is thus more profitable to raise prices in the short run than raising wages and output. This mechanism gives rise to a natural non-linearity as prices do not rise during slack labor markets but are the main equalizer between demand and supply during a tight labor market.

A similar logic is at play for transitory negative labor supply shocks which were prevalent in the post-pandemic tightening cycle (Forbes et al. (2025)). With supply and demand in disequilibrium at long-run wages, the firm has an incentive to price out the demand overhang created by newly vacant positions instead of raising wages to bring supply and demand into equilibrium.

The channel ties the divergence of price and wage Phillips curves during tight labor markets to the observed rise in profit margins. Prices are raised with future marginal cost in mind leading to higher observed profits today. Similarly, the channel rationalizes the outperformance of V/U relative to the unemployment gap in the recent inflation episode (Blanchard et al. (2022)). If firms want to expand production but are unwilling to raise wages accordingly then more of their job postings will remain unfilled. Thus, rising V/U does not only signal a tighter labor market, but firms' resistance to increase wages to a level where their vacancies fill at a reasonable rate.

Thus, this paper extends the literature relating downward nominal wage rigidity to Phillips curve non-linearity from its focus on recessions (Schmitt-Grohé and Uribe (2022), Gagnon and Collins (2019), Donayre and Panovska (2016), Gitti (2024)) to how DNWR affects Phillips curve non-linearity during tight labor markets. The only other paper concerned with non-linearity during tight labor markets is Benigno and Ricci (2011). They argue that workers voluntarily reduce their wage demands during the boom to prevent getting fired during the bust. However, this leads to a flatter Phillips curve during tight labor markets, which is contrary to the evidence (Hooper et al. (2020) , Ball et al. (2022), Benigno and Eggertsson (2023), Gitti (2024)).

Empirical evidence for firms' resistance to wage increases due to DNWR comes from Elsby (2009). Elsby (2009) argues that firms will "compress" wages under sizeable DNWR to prevent getting stuck at the DNWR constraint. Elsby (2009) uses micro employment data from British establishment data and shows that firms resist wage increases when inflation is low, and they are more likely to get stuck at the DNWR constraint in future periods. Schaefer and Singleton (2022) update the evidence and Daly and Hobijn (2014) show that the wage compression is evident in a fitted model with DNWR during negative demand shocks. My proposed channel applies this logic to tight labor markets. When firms experience such a high level of demand that it cannot last, they also resist raising wages too far above the level they expect to be sufficient to fill their positions in the future.

Furthermore, the mechanism is in line with recent findings from Caldwell et al. (2024). The paper analyzes German firm survey data and finds that labor market conditions explain how firms negotiate wages with workers rather than the characteristics of the firm. I postulate that firms similarly change how they think about price increases depending on how tight their relevant labor market is.

Finally, the mechanism touches on the link between the Beveridge and the Phillips curve (Barnichon and Shapiro (2024), Michaillat and Saez (2024)). In particular, sectoral demand shifts have been linked to sectoral Beveridge Curve shifts (Elsby et al. (2010), Blanchard et al. (2022), Figura and Waller (2022)), which in turn are linked to aggregate Beveridge Curve shifts (Barnichon et al. (2012), Kindberg-Hanlon and Girard (2024), Kodua et al. (2025)). My proposed User-Cost of Labor channel offers a link between sectoral demand shift-induced Beveridge Curve shifts and Phillips curve non-linearity.

4.1 Illustrative Model

I illustrate the mechanism in a two period monopolist model. Downward nominal wage rigidity paired with labor and goods market power leads to price increases above and beyond wage increases under transitory positive demand or negative labor supply shocks. The goal of the model is to highlight the necessary ingredients of the channel before integrating it into a DSGE framework.

I begin by stating the assumptions on my simple two period economy. First, labor is the only factor of production

$$Y_t = L_t \text{ for } t \in \{1, 2\}$$

The monopolist firm faces a downward sloping demand schedule

$$D(P_t, x_t) = x_t P_t^{-\epsilon} \text{ where } \epsilon > 1 \text{ for } t \in \{1, 2\}$$

subject to a demand multiplier x_t, and an upward sloping labor supply constraint

$$W_t \geq (\eta_t x_t P_t^{-\epsilon})^{\frac{1}{\beta}}$$

subject to a labor supply shifter η_t. Where $\eta_t > 1$ leads to a negative labor supply shock, and $\eta_t < 1$ leads to a positive labor supply shock.

The monopolist maximizes profits

$$\max_{P_1,P_2} \Pi = \max_{P_1,P_2} P_1 D_1 - W_1 L_1 + P_2 D_2 - W_2 L_2 = \max_{P_1,P_2} x_1 P_1^{1-\epsilon} - x_1^{\frac{1+\beta}{\beta}} \eta_1^{\frac{1}{\beta}} P_1^{-\epsilon\frac{1+\beta}{\beta}} + x_2 P_2^{1-\epsilon} - x_2^{\frac{1+\beta}{\beta}} \eta_2^{\frac{1}{\beta}} P_2^{-\epsilon\frac{1+\beta}{\beta}}$$

under a downward nominal wage rigidity constraint

$$W_2 = \max\{W(L_2), W_1\}$$

Wages are chosen at the labor supply constraint in all but a small subset of the parameter space leading to a case where $W_2 > (\eta_2 x_2 P_2^{-\epsilon})^{\frac{1}{\beta}}$. I present the full Lagrangian approach with a discussion of all cases in Appendix G. The outcome is parallel to the main case presented in this section.

4.1.1 Flexible Wage Case

When DNWR is not binding the two periods are independent of each other and deliver the standard constant markup over marginal cost paired with a wage mark-down from monopsony power. This is the case in the no shock ($x_1 = x_2 = \eta_1 = \eta_2 = 1$) steady state for example.

The First Order Condition with respect to P_t yields optimal price P_t^*

$$P_t^* = \left[\frac{\epsilon}{\epsilon - 1}\frac{1+\beta}{\beta}(\eta_t x_t)^{1/\beta}\right]^{\frac{\beta}{\beta+\epsilon}}$$

The optimal price implies a wage of

$$W_t^* = (\eta_t L_t)^{1/\beta} = (\eta_t D(P_t, x_t))^{1/\beta} = (\eta_t x_t)^{1/\beta}(P_t^*)^{-\frac{\epsilon}{\beta}} = (\eta_t x_t)^{1/\beta}\left[\frac{\epsilon}{\epsilon - 1}\frac{1+\beta}{\beta}(\eta_t x_t)^{1/\beta}\right]^{\frac{-\epsilon}{\beta+\epsilon}}$$

Optimal price setting in the flexible wage case yields the constant markup over marginal cost typical for iso-elastic demand functions

$$\frac{P_t^*}{W_t^*} = \frac{P_t^*}{(\eta_t x_t)^{1/\beta}(P_t^*)^{-\frac{\epsilon}{\beta}}} = (P_t^*)^{\frac{\beta+\epsilon}{\beta}}(\eta_t x_t)^{-1/\beta} = \frac{\epsilon}{\epsilon - 1}\frac{1+\beta}{\beta}$$

Any shock to the demand shifter x_t or the labor supply shifter η_t does not change the markup. Wages and prices have the same growth rate with respect to demand and labor supply shocks. And inflation and wage growth increase in lockstep following a positive demand or negative labor supply shock. With the solution to the flexible wage case, I now move on to the case with absolute downward wage rigidity.

4.1.2 DNWR Binds

DNWR binds under a greater demand or labor supply shifter in period one than in period 2 ($x_1 > x_2$ or $\eta_1 > \eta_2$).[14] Under binding DWNR the monopolist's profit function becomes

$$\max_{P_1,P_2}\Pi = \max_{P_1,P_2} x_1 P_1^{1-\epsilon} - x_1^{\frac{1+\beta}{\beta}}\eta_1^{\frac{1}{\beta}}P_1^{-\epsilon\frac{1+\beta}{\beta}} + x_2 P_2^{1-\epsilon} - x_2 x_1^{\frac{1}{\beta}}\eta_1^{\frac{1}{\beta}}P_1^{-\frac{\epsilon}{\beta}}P_2^{-\epsilon}$$

P_2 is determined in terms of P_1 by the binding of DNWR and the labor supply constraint in period 2.[15]

$$\begin{aligned}(\eta_2 x_2 P_2^{-\epsilon})^{\frac{1}{\beta}} &= (\eta_1 x_1 P_1^{-\epsilon})^{\frac{1}{\beta}} \\ P_2 &= \left(\frac{\eta_1 x_1}{\eta_2 x_2}\right)^{-\frac{1}{\epsilon}} P_1\end{aligned}$$

which simplifies the profit function to

$$\max_{P_1}\Pi = \max_{P_1} x_1 P_1^{1-\epsilon}\left(1 + \left(\frac{\eta_1 x_1}{\eta_2 x_2}\right)^{-\frac{1}{\epsilon}}\left(\frac{\eta_1}{\eta_2}\right)\right) - x_1^{\frac{1+\beta}{\beta}}\eta_1^{\frac{1}{\beta}}P_1^{-\epsilon\frac{1+\beta}{\beta}}\left(1 + \left(\frac{\eta_1}{\eta_2}\right)\right)$$

[14] See Appendix G for a full discussion of when DNWR is binding.

[15] I discuss the subcase where period 2's labor supply constraint is not binding in Appendix G.

The First Order Condition of which yields

$$P_1^* = \left[\frac{\epsilon}{\epsilon - 1} \frac{1+\beta}{\beta} x_1^{\frac{1}{\beta}} \eta_1^{\frac{1}{\beta}} \left(1 + \frac{\eta_2}{\eta_1}\right) \frac{\left(\frac{\eta_1 x_1}{\eta_2 x_2}\right)^{\frac{1}{\epsilon}}}{1 + \left(\frac{\eta_1 x_1}{\eta_2 x_2}\right)^{\frac{1}{\epsilon}} \frac{\eta_2}{\eta_1}} \right]^{\frac{\beta}{\epsilon+\beta}}$$

Note that this reverts back to the optimal price under flexible wages when $x_1 = x_2$ and $\eta_1 = \eta_2$. I.e. when DNWR is not binding.

I obtain an expression for the direct impact channel by deriving the difference in the percent growth rates of prices and wages under DNWR.

$$\text{Direct Impact} := \frac{\partial \ln(P_1)}{\partial x_1}|_{DNWR} - \frac{\partial \ln(W_1)}{\partial x_1}|_{DNWR}$$

To derive this expression for the direct impact channel I first calculate the derivative of the optimal price under DNWR with respect to x_1. This yields

$$\frac{\partial \ln(P_1)}{\partial x_1}|_{DNWR} = \frac{\beta}{\beta + \epsilon} \left[\frac{1}{\beta x_1} + \frac{\frac{1}{\epsilon x_1}}{\left(1 + \left(\frac{x_1}{x_2}\frac{\eta_1}{\eta_2}\right)^{\frac{1}{\epsilon}} \frac{\eta_2}{\eta_1}\right)} \right]$$

To describe the direct impact we now need the expression for the percent increase in period one wages $\frac{\partial ln(W_1)}{\partial x_1}$. Using the expression of period one wages in terms of period one prices $W_1 = D_1^{\frac{1}{\beta}} = (x_1 \eta_1 P_1^{-\epsilon})^{\frac{1}{\beta}}$, its derivative with respect to x_1 simplifies to

$$\frac{\partial W_1}{\partial x_1}|_{DNWR} = \frac{1}{\beta} W_1 \left[\frac{1}{x_1} - \epsilon P_1^{-1} \frac{\partial P_1}{\partial x_1} \right]$$

$$\implies \frac{\partial W_1}{\partial x_1}|_{DNWR} \frac{1}{W_1} = \frac{\partial \ln(W_1)}{\partial x_1}|_{DNWR} = \frac{1}{\beta} \left[\frac{1}{x_1} - \epsilon \frac{\partial \ln(P_1)}{\partial x_1}|_{DNWR} \right]$$

Thus, the difference between inflation and the rate of wage growth is

$$\frac{\partial \ln(P_1)}{\partial x_1}|_{DNWR} - \frac{\partial \ln(W_1)}{\partial x_1}|_{DNWR} = \left(1 + \frac{\epsilon}{\beta}\right) \frac{\partial \ln(P_1)}{\partial x_1}|_{DNWR} - \frac{1}{\beta x_1}$$

under DNWR. This is the percent increase in the price level above and beyond the percent increase in the wage level, which describes the direct impact in the illustrative model.

Plugging in the expression for $\frac{\partial \ln(P_1)}{\partial x_1}|_{DNWR}$ simplifies to

$$\frac{\partial \ln(P_1)}{\partial x_1}|_{DNWR} - \frac{\partial \ln(W_1)}{\partial x_1}|_{DNWR} = \frac{\frac{1}{\epsilon x_1}}{\left(1 + \left(\frac{x_1}{x_2}\frac{\eta_1}{\eta_2}\right)^{\frac{1}{\epsilon}} \frac{\eta_2}{\eta_1}\right)} \geq 0$$

The direct impact goes to zero when market power goes to to zero ($\epsilon \to \infty$). Similarly, the direct impact goes to zero without monopsony power ($\beta \to \infty$). With $\beta \to \infty$ the wage

in both periods goes to one regardless of demand or labor supply shocks. Thus, DNWR does not bind and we revert back to the flexible wage solution (see Appendix G for details). Similarly, the channel does not exist if $\beta \to 0$ as labor supply becomes unresponsive to wages.

Parallel derivations lead to a similar direct impact under a negative labor supply shock $\eta_1 > 1$, which were sizeable during the pandemic period (Goda and Soltas (2023), Kindberg-Hanlon and Girard (2024)).

I compare the resulting wage and price Phillips curves in my illustrative model in Figure 8. To this end I make simple assumptions to introduce labor market tightness into the model. I assume that the firm has to hire every employee in every period again. Abstracting from search and matching frictions this gives $V = L$. Further, unemployment is described by $U = N - L$, where N is the size of the labor force. Thus, $\frac{V}{U} = \frac{L}{N-L}$. As $L = D(p)$ depends on demand and thus prices we get a smaller increase in employment under the channel than under flexible wages. Thus, the price Phillips curve steepens under the channel as both the price response to a given shock is greater and the labor market tightens less than under flexible wages. I show the derivations of V/U in terms of x in Appendix G.7.

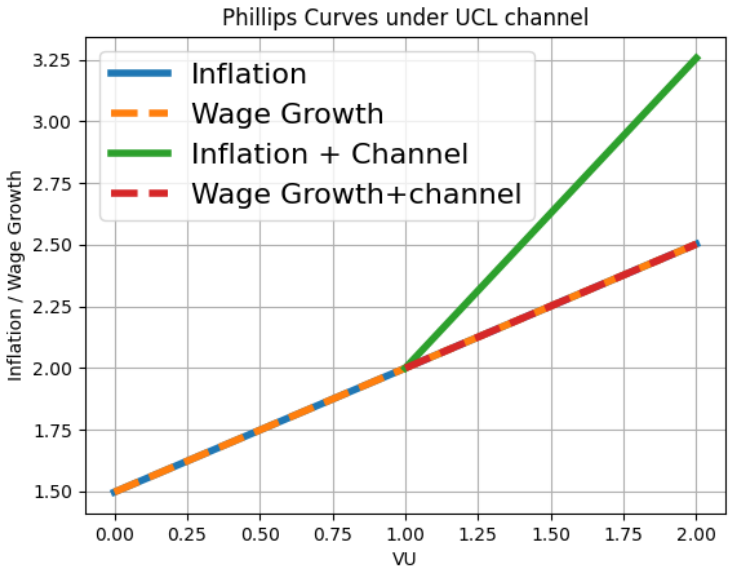


Figure 8: Comparing Wage and Price Phillips Curves in my Illustrative Model.

This simple model illustrates how a transitory wage pressure shock under DNWR leads to an additional wedge between wages and prices under tight labor markets. Furthermore, it explains the non-linearity of the direct impact. Positive demand shocks of a certain size will make the DNWR constraint bind in the future, while smaller or even negative shocks will not. Thus, the direct impact channel is only active for part of the spectrum of observed demand shocks, leading to the non-linearity in the data.

This on-off nature of the direct impact channel justifies the threshold approach to the non-linear Phillips curve taken in the literature (Smith et al. (2023), Benigno and Eggertsson (2023)). The observed smoothness of the direct impact under the cubic specification is

likely due to the varying levels of expected future wages, varying labor supply and demand elasticities across firms, and the prevalence of idiosyncratic shocks.

Furthermore, Table 3 indicates a substantial influence of labor market tightness on profit margins, which exhibit a similar non-linear pattern as inflation. My illustrative model can explain the concomitant rise in inflation and profit margins as prices rise beyond the usual markup over today's *measured* marginal cost. Therefore, another contribution of this paper is to substitute a more satisfying explanation to the concomitant rise in inflation and profit margins than the postulated increase in corporate greed purported under the popular "Greedflation" hypothesis (Vinod (2022)) or its close cousin "sellers' inflation" in which firms use shocks as signals for tacit collusion (Weber and Wasner (2023)).

Finally, the paper contributes another angle to the documented pro-cyclical nature of profit margins and markups (Macallan et al. (2008), Nekarda and Ramey (2020) Anderson et al. (2018), Qiu and Ríos-Rull (2022)). The pro-cyclicality is heretofore explained by the pro-cyclicality of productivity (Qiu and Ríos-Rull (2022)) and demand (Anderson et al. (2018)). Other colloquial reasons are pro-cyclical cost reduction due to the declining ratio of fixed cost to revenue, and financial leverage. The literature has explored neither these latter channels nor my proposed direct impact channel, driven by downward nominal wage rigidity. Therefore, offering wage-rigidity-driven price setting in response to labor market tightness as the driver of profit margin pro-cyclicality is another contribution of this paper.

4.2 Market Concentration

I am testing the role of market power on the direct impact in this section, as my proposed channel needs both market and monopsony power to determine prices and wages. This section finds that market concentration is indeed important for the direct impact channel.

To test the importance of market power I use the Herfindahl-Hirschman Index (HHI) of market concentration. The HHI is a measure of market power in both goods/services markets as well as the labor market. It thus functions as an indicator for the degree of both monopoly and monopsony power. I use my cross-sectional data on developed countries and US service sectors (Appendix D.1.1) to assess the importance of market concentration on the direct impact channel.

4.2.1 Cross-Country Evidence

I interact the HHI by country and year with labor market tightness. I re-estimate my cross-country panel regression (Equation 7), while adding interaction terms of labor market tightness with market concentration. Table 9 reports the estimation results.

$$\pi_{c,t} = \check{\alpha}_1(V/U)_{c,t} + \check{\alpha}_2(V/U)^2_{c,t} + \check{\alpha}_3(V/U)^3_{c,t} + \check{\beta}\pi^e_{c,t} + \check{\gamma}_1\pi^w_{c,t} + \check{\gamma}_2\pi^w_{c,t-1} + \check{\phi}\text{Prod}_{c,t} + \check{\delta}_c + \check{\delta}_t + \check{\epsilon}_{c,t}$$
$$+ \zeta_1(V/U)_{c,t} * \text{HHI}_{c,t} + \zeta_2(V/U)^2_{c,t} * \text{HHI}_{c,t} + \zeta_3(V/U)^3_{c,t} * \text{HHI}_{c,t} + \zeta_4\text{HHI}_{c,t}$$

Both Bayesian and Akaike Information Criteria prefer the inclusion of all three interaction terms. The reported p-values in the table are from the Wald tests on the joint insignificance

Table 9: Market Concentration

	Core	π
(V/U)	5.48	2.04
	(4.81)	(5.38)
$(V/U)^2$	-4.96	1.23
	(6.47)	(7.11)
$(V/U)^3$	0.54	-2.22
	(2.45)	(2.70)
$(V/U) \times$ HHI	0.37	34.30
	(52.58)	(60.66)
$(V/U)^2 \times$ HHI	-39.54	-102.87
	(70.44)	(78.68)
$(V/U)^3 \times$ HHI	32.06	59.32*
	(27.37)	(30.21)
HHI	-2.24	-8.89
	(10.13)	(12.08)
Constant	-1.01	-0.19
	(1.19)	(1.34)
R^2	0.66	0.69
N	1513	1513
No Impact p-value	0.0004	0.0025
Linear p-value	0.0001	0.0030
Controls	✓	✓

Two-way clustered standard errors in parentheses

* $p < .10$, ** $p < .05$, *** $p < .01$

Notes: $Core$ and π in the column headers denote core and headline inflation as dependent variables. V/U denotes labor market tightness which is included in cubic form and in interaction with the World Bank's Herfindahl Hirschman Index by country. "No Impact p-value" reports the p-value of the Wald test hypothesis that the interaction terms are jointly zero. "Linear p-value" does the same for testing the higher order interactions as jointly zero.

of the interaction terms instead of the labor market tightness coefficients. They indicate that market concentration leads to higher inflation during tight labor markets.

Furthermore, in Osten (2025a) I show that market concentration and international labor mobility flatten the wage Phillips curve. This is particularly true during tight labor markets. These results are in line with my proposed direct impact. Higher market power allows companies to resist upward wage pressure and price out the resulting demand overhang. This leads to lower wage growth and higher inflation during tight labor markets then under lower market power.

In Figure 9 I show the empirical effect of market concentration on core inflation. To this end I choose the mean HHI (0.11) plus minus one SD (0.09) of HHI in my international data to illustrate the change in the functional form between a country with elevated or low market concentration. I floor the low HHI at 0.05, as the minimal observed HHI in my data is 0.03. Note that the direct impact is gone when market concentration is very low, but lifts inflation during tight labor markets in countries with elevated market concentration.[16]

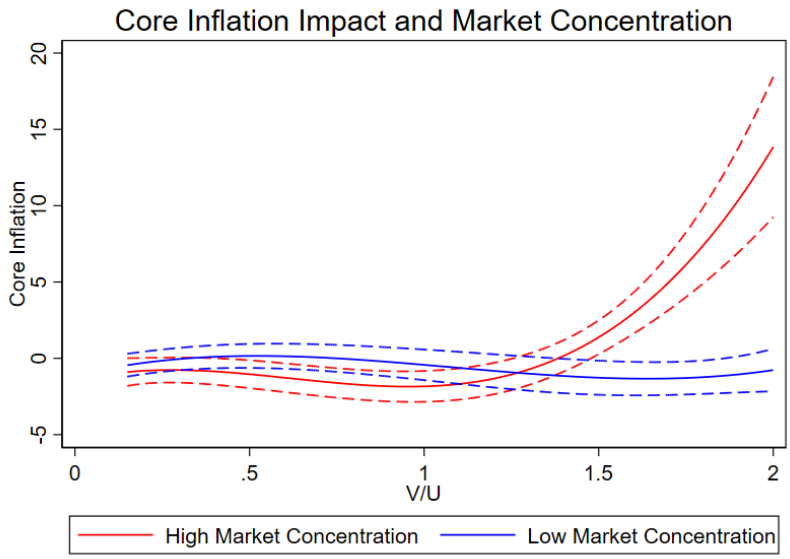


Figure 9: Direct Impact and Market Concentration

Note: Comparing the Direct Impact on Core Inflation under high (0.2) and low (0.05) market concentration as measured via the Herfindahl-Hirschman Index.

I find similar results when interacting non-linear labor market tightness with a sectoral HHI measure from the 2017 census on US service sectors (Appendix D.1.1).

4.3 NFIB Firm Expectations

Did firms encounter challenges in recruiting new employees despite modest wage growth and declining real wages during the post-pandemic recovery? The National Federation of Independent Business (NFIB) surveys small businesses, primarily with fewer than 40 em-

[16] High concentration countries are Canada (0.23-0.69), Portugal (0.08 - 0.51), Australia (0.07 - 0.49). Low concentration countries are Great Britain (0.045 - 0.074), France (0.041 - 0.088), Belgium (0.069 - 0.083), Bulgaria (0.047-0.067), Lithuania (0.041 - 0.088).

ployees, across all Census Regions. In 2019, the primary sectors represented by respondents included Retail (22.3%), Construction (18.2%), and Services (16.6%).

The NFIB poses the question: "Do you have any job openings that you are unable to fill right now?" During the post-pandemic inflationary period, the percentage of businesses answering "yes" sharply increased. The rise in difficulties to hire rises roughly in line with the overall V/U ratio. I show a comparison of the two measures and a robustness check based on NFIB reported hiring difficulties in Appendix G.8.

Furthermore, the NFIB offers data to test the second implication of my proposed mechanism, which is based on businesses' reluctance to raise offered wages for positions they characterize as "unfillable." I hypothesize that this reluctance is driven by expectations of declining demand. Decreasing demand would lessen the need for additional workers and could lock firms into uncompetitively high wages, which are difficult to adjust downward.

Unfortunately, the NFIB does not inquire about firms' expectations of future wages. However, it does ask, "Do you think that six months from now general business conditions will be better than they are now, about the same, or worse?"[17].

The NFIB constructs an indicator for expected business conditions by subtracting the percentage of firms responding "worse" from those responding "better." This indicator declined sharply in 2021 and continued to decrease, reaching its lowest point in June 2022, coinciding with the peak of inflation. While this measure may partially reflect the effects of inflation, as rising costs influence business outlooks, the decline in the indicator occurs a few months prior to the increase in inflation, suggesting that causality may run from business expectations to inflation.

Additionally, I utilize the NFIB measure on expected real sales growth over the next three months, derived from the survey question: "What do you expect to happen to the real volume (number of units) of goods and/or services that you will sell during the next 3 months?"[18]

The NFIB calculates the real sales expectations indicator by subtracting the percentage of respondents answering "lower" from those answering "higher." Outside of recessions, this indicator typically ranges between 15% and 20%. However, it steadily declined as the labor market tightened and inflation surged, reaching a low of nearly -30%. I show both measures of expected future demand in Appendix G.9.

While not conclusive, this suggests that firms anticipated a decline in sales volume even as the economy was booming and inflation was rising. The NFIB measures on expected business conditions in six months and expected real sales growth over the next three months enable me to test whether the proposed expected decline in future demand contributes to inflation when labor markets are tight. In contrast, the standard New Keynesian (NK) framework posits that expectations of higher future demand lead firms to raise prices more in tight labor market conditions than in slack labor markets.[19]

In contrast, my model predicts that firms increase their prices more when tight labor

[17] Question 5 at NFIB.com

[18] Question 8 at NFIB.com.

[19] See Appendix H for the derivation in the standard Galí (2015) framework.

markets coincide with an expected *decline* in future demand for their goods and services. This pricing strategy allows firms to avoid committing to high wage costs in anticipation of a future decrease in demand.[20]

To test between these two models I interact the business conditions and the real sales growth expectations with the NFIB's measure of hiring difficulty:

$$\pi_t = \alpha V/U_{NFIB,t} + \beta Z_t + \gamma V/U_{NFIB,t} * Z_t + \delta_1 \pi_t^w + \delta_2 \pi_{t-1}^w + \epsilon_t^{NFIB}$$

where Z_t is a place-holder for the two different expected demand measures of the NFIB. Under the traditional NK model γ should be positive. If my proposed channel holds, it should be negative instead.[21] Table 10 presents the results for expected business conditions in six month and for three month expected real sales growth on median CPI inflation, respectively. The results are similar for headline and traditional core inflation.

Both measures yield a highly significant and negative coefficient for the interaction term. These results remain robust when controlling for real GDP and sales growth, supporting my proposed channel over the traditional NK view. Additionally, this finding rules out a broad range of models, including the basic NK framework and the diminishing productivity approach of Bunn et al. (2024).

5 New Keynesian Model

In this section I introduce my channel into a New Keynesian framework. The model differs from the mainline New Keynesian framework with search and matching primarily through wage posting instead of wage bargaining, on-the-job-search, and downward nominal wage rigidity.

The standard New Keynesian framework is unable to accommodate my channel as its bargaining based wage setting leads to wages and prices growing in lock-step.[22] This is true even for the non-linear Phillips curve model of Benigno and Eggertsson (2023) or Gitti (2024). Their non-linearity is simply driven by a non-linearity in wages. Price setting still follows a constant markup over marginal cost behavior.

To incorporate my channel I need firms to decide both on prices and wages. Thus, I build on a family of New Keynesian models with simultaneous wage and price setting pioneered by Sveen and Weinke (2009), and Thomas (2011). My model extends the newest generation of these models, who use wage posting instead of wage bargaining to provide firms with monopsony power (Bloesch and Larsen (2023), de la Barrera i Bardalet (2024), Bloesch et al. (2024)).

However, monopsony power alone does not lead to an increasing profit share as labor markets tighten (Bloesch and Larsen (2023)). I introduce my channel into the model via

[20] Appendix G.6 derives the negative interaction term between tight labor markets and expected sales growth in my illustrative model.

[21] See derivation of negative interaction from the illustrative model in Appendix G.6.

[22] In Appendix H I show that the standard NK model fails to match my empirical findings as firms are price takers in the labor market.

Table 10: NFIB Business and Real Sales Expectations

	π	π
CantFill	5.03***	6.60***
	(0.93)	(0.88)
Improved Business Conditions	2.04***	
	(0.68)	
Real Sales Growth		3.97***
		(0.93)
CantFill × Improved BCs	-11.36***	
	(2.19)	
CantFill × Real Sales Growth		-22.85***
		(3.42)
Constant	-1.16***	-1.32***
	(0.22)	(0.21)
R^2	0.74	0.76
N	152	152
Controls	✓	✓

Newey-West standard errors in parentheses

* $p < .10$, ** $p < .05$, *** $p < .01$

Notes: π in the column header denotes core inflation as dependent variable. CantFill denotes the reported measure of hiring difficulties from the NFIB survey. Similarly, Improved Business Conditions and Real Sales Growth, are the measures of improved business condition and real sales growth measures from the NFIB firm survey data.

downward nominal wage rigidity. Downward nominal wage rigidity (DNWR) introduces a forward looking wage cost component into the optimal price setting behavior under transitory demand and labor supply shocks. The forward looking wage cost stems from the rise of wages beyond future expected wages. Due to DNWR these higher wages are then cemented into the future, leading to higher future marginal cost. The firm takes these additional cost into account when maximizing its profits and raises prices beyond the level optimal under flexible wages.

The model matches key micro evidence on labor market dynamics. First, it matches the well-identified evidence on the sensitivity of recruiting and quitting to changes in wages estimated in the monopsony literature (Hirsch et al. (2022), Azar et al. (2022), Datta (2023)). Second, it matches the close correlation of wage growth with job-to-job transitions (Faber-

man and Justiniano (2015), Moscarini and Postel-Vinay (2017), Karahan et al. (2022)). Third, it matches the low responsiveness of wages to flow benefits of unemployment (Jäger et al. (2020)). Fourth, it matches the prevalence of wage posting over wage bargaining in modern labor marketsHall and Krueger (2012). Fifth, the model matches the auto-correlation of a firm's past and current wage (Di Addario et al. (2023)). Finally, the model matches the growing evidence of substantial and almost absolute downward nominal wage rigidity (Grigsby et al. (2021), Lebow et al. (2003)).

5.1 Households

I begin the model with a standard household block, which consists of a representative household maximizing life time utility based on aggregate consumption C_{t+k}, subject to the discount factor β_t. Life time utility is described by

$$\mathcal{U}_t = E_t \left[\sum_{k=0}^{\infty} \beta_t^k u(C_{t+k}) \right]$$

which is maximized under the following intertemporal budget constraint

$$\int_0^1 p_{i,t} c_{i,t} \, di + Q_t B_t = \int_0^1 w_{i,t} n_{i,t} \, di + B_{t-1} + \Pi_t$$

where $p_{i,t}$, and $c_{i,t}$ are the price and the consumption of variety $i \in [0,1]$. Q_t is the price of a one period savings bond B_t. $w_{i,t}$ is the wage earned by workers $n_{i,t}$ producing variety i. Finally, Π_t are firms' profits distributed to households via dividends.

I impose standard constant elasticity of substitution (CES) preferences among consumption varieties, which gives the consumption bundle of the representative household

$$C_t = \left(\int_0^1 (c_{i,t})^{\frac{\epsilon_t - 1}{\epsilon_t}} \, di \right)^{\frac{\epsilon_t}{\epsilon_t - 1}}$$

The overall consumption expenditure is split among varieties to match overall

$$\text{Consumption Expenditure} = \int_0^1 p_{i,t} c_{i,t} \, di$$

Minimizing the cost of the optimal consumption bundle C_t yields household demand for a single variety

$$\left(\frac{p_{i,t}}{P_t} \right)^{-\epsilon_t} C_t$$

This expression delivers the relative consumption weights of each consumption variety. Incidentally, these weights can be used to derive the price index P_t of the aggregate consumption bundle C_t. The corresponding aggregate price index is

$$P_t = \left(\int_0^1 p_{i,t}^{1-\epsilon_t} \, di \right)^{\frac{1}{1-\epsilon_t}}$$

The final step in solving the household's optimal behavior is to determine the split of market resources between savings and consumption. Optimizing the households life time utility

function under the budget constraint, with respect to bond holdings B_t, yields the household's Euler Equation.

$$Q_t = \beta E_t \left[\frac{u'(C_{t+1})}{u'(C_t)} \frac{P_t}{P_{t+1}} \right]$$

This concludes the household side of the economy. I now move on to describe the labor market which will set the constraints under which firms choose the optimal wage to post on their vacancies and the amount of vacancies to post.

5.2 Labor Market

At the beginning of period t a fraction $\bar{\delta} \in (0, 1)$ of existing employees exogenously separate from their employer. Thus, at the beginning of period t, the total pool of unemployed workers consists of the unemployed at the end of the previous period and the newly separated workers.

$$U_t^{start} = U_{t-1} + \bar{\delta} N_{t-1}$$

In this expression U_t^{start} and U_t are the number of unemployed workers at the beginning and end of period t, respectively.[23] $\bar{\delta}$ is the exogenous separation rate, and N_t the number of employees in period t.

In the labor market, both employed and unemployed workers browse existing job postings V_t. The employed do so with a relative search intensity $s \leq 1$. The resulting effective amount of job seekers in the economy is

$$S_t = U_{t-1} + \bar{\delta} N_{t-1} + s(1 - \bar{\delta}) N_{t-1}$$

The amount of effective job seekers describes the amount of labor ready for hire in the labor market. Labor demand is described by the total amount of open jobs in the economy

$$V_t = \int_0^1 v_{i,t} \, di$$

Labor market tightness in the model is the ratio of job postings V_t to the effective job searchers S_t.[24]

Labor market tightness is a measure of relative labor demand to labor supply. If the ratio is large, then there is a lot of labor demand relative to available labor supply and firms will have a hard time filling a given vacancy. If the ratio is small, then there is ample labor supply and firms will find it relatively easy to fill a vacancy.

$$\theta_t = \frac{V_t}{U_{t-1} + \bar{\delta} N_{t-1} + s(1 - \bar{\delta}) N_{t-1}} =: \frac{V_t}{S_t}$$

[23] U_t^{start} evolves according to exogenous separation and the job finding rate of the unemployed: $U_t^{start} = U_{t-1} + \bar{\delta} N_{t-1} = U_{t-1}^{start} - p_{t-1}^U q(\theta_{t-1}) V_{t-1} + \bar{\delta} N_{t-1}$. Where V_t is the amount of vacancies, $\mathfrak{p}_t^U$ the likelihood of encountering an unemployed worker, and $q(\theta_t)$ the probability of filling a vacancy given labor market tightness θ_t.

[24] This indicates that the measure of labor market tightness from Abraham et al. (2020) based on effective job searchers would be a better measurement for my empirical work. However, they construct their measure only up to 2019. Furthermore their measure is closely correlated with the standard V/U measure. Thus, using their measure in future renditions is unlikely to change my results.

The likelihood that a current job seeker is matched to a current job posting is determined by how efficient the labor market works. I introduce labor market frictions via a standard search and matching process. Job seekers are matched to an available job vacancy with a standard Cobb-Douglas matching function

$$m(V_t, S_t) = \kappa_\theta S_t^\eta V_t^{(1-\eta)} \ , \ \eta \in (0,1)$$

The matching function determines the matching probability for a given job vacancy

$$q(\theta_t) = \frac{m(V_t, S_t)}{V_t} = \kappa_\theta \theta^{-\eta}$$

As $\eta > 0$, the tighter the labor market is (greater θ) the lower the probability of finding a worker to fill a given vacancy.

Once a job seeker is matched to a vacancy, the worker has to decide whether to accept the offered job or not. The likelihood of accepting the offered job potentially differs a lot between unemployed job seekers and those who are searching on-the-job. The probability that a matched worker is already employed is the ratio of effective on-the-job seekers relative to all job seekers.

$$\mathfrak{p}_t^E = \frac{s(1-\bar{\delta})N_{t-1}}{U_{t-1} + \bar{\delta}N_{t-1} + s(1-\bar{\delta})N_{t-1}}$$

Similarly, the probability that the matched worker is unemployed is the ratio of all unemployed job seekers to all effective job seekers.

$$\mathfrak{p}_t^U = \frac{U_{t-1} + \bar{\delta}N_{t-1}}{U_{t-1} + \bar{\delta}N_{t-1} + s(1-\bar{\delta})N_{t-1}}$$

For simplicity, I assume that there are no unemployment benefits, and an unemployed job seeker always accepts a job offer. I introduce an outside option following Appendix I.3 when estimating the model dynamics, but the assumption simplifies the mathematical expressions without changing my mechanism of interest.

Employed workers compare the offered salary with their current wages. The employed worker receives a taste shock $\psi \sim F_\psi$ for the offered job relative to its current position. Think of this taste shock as a preference for location, fringe benefits like health insurance, childcare etc, or simply lower work hours, free coffee or nicer colleagues. The main reason to introduce these shocks is the existence of a symmetric wage equilibrium which simplifies the derivations. The symmetric wage equilibrium exists if these shocks have sufficient variance as shown by Albrecht et al. (2018).

I specify F_ψ as log-normal distribution with mean μ_ψ and standard deviation σ_ψ. This taste shock is compared with the relative wage between the current and the offered job. If the taste shock is smaller than the ratio of the current wage to the offered wage, the worker's comparative preference for the new job is too small and the worker will decline the offer. Thus, offering a higher wage than the going wage rate increases the likelihood of poaching a worker from a competitor, while it lowers the likelihood of losing employees to a competitor.

On average, the worker accepts an offered job with wage $w_{i,t}$ relative to the going wage rate W_t with probability

$$1 - F_\psi\left(\frac{W_t}{w_{i,t}}\right)$$

The importance of taste shocks in the acceptance decision depends crucially on the variance of the taste shocks. If the variance goes to zero, a worker will always choose the higher paying job. On the other hand if the variance increases, the payoff from raising the offered wage diminishes as worker decisions for or against a job offering become increasingly random. The ex-ante likelihood of a matched worker accepting an offered job is one for unemployed workers, and $1 - F_\psi\left(\frac{W_t}{w_{i,t}}\right)$ for employed workers. Hence the overall acceptance probability is

$$\mathfrak{p}_t^U + \mathfrak{p}_t^E(1 - F_\psi\left(\frac{W_t}{w_{i,t}}\right)) = \mathfrak{p}_t^U + \mathfrak{p}_t^E - \mathfrak{p}_t^E F_\psi\left(\frac{W_t}{w_{i,t}}\right) = 1 - \mathfrak{p}_t^E F_\psi\left(\frac{W_t}{w_{i,t}}\right)$$

Combining the likelihood of matching a vacancy to a worker given labor market tightness θ_t, and the likelihood of being accepted by an unemployed or employed worker leads to the ex-ante acceptance probability of a posted vacancy depending on the wage posted with the vacancy $w_{i,t}$ relative to the going wage rate in the labor market W_t.

$$a_t\left(\frac{w_{i,t}}{W_t}\right) = q(\theta_t)\left(1 - \mathfrak{p}_t^E F_\psi\left(\frac{W_t}{w_{i,t}}\right)\right)$$

The ex-ante probability of filling a vacancy is the product of the probability of matching a vacancy to a worker in the current labor market described by tightness θ_t and the probability that a matched vacancy is accepted by the matched worker.

Job-to-job transition leads to an endogenous worker-firm separation additional to exogenous separation at the beginning of the period. This endogenous separation depends on the relative wage of the firm to the going wage rate, as it determines the likelihood of its employees to accept an offered job from a competitor. The endogenous separation rate is described by

$$\delta_t\left(\frac{w_{i,t}}{W_t}\right) = \frac{V_t q(\theta_t)\mathfrak{p}_t^E\left(1 - F_\psi\left(\frac{w_{i,t}}{W_t}\right)\right)}{(1-\bar{\delta})N_{t-1}}$$

Endogenous firm separation is the product of the likelihood of matching a vacancy to an existing employee $q(\theta_t)\mathfrak{p}_t^E$, the probability that this employee would accept the outside offer with wage W_t given the wage $w_{i,t}$ at its current employer firm i, $(1 - F_\psi\left(\frac{w_{i,t}}{W_t}\right))$, times the number of total vacancies. The numerator describes the number of endogenous separations, and is divided by the number of total employees still employed after exogenous separation $(1-\bar{\delta})N_{t-1}$.

On-the-job search disciplines firms' wage setting power. Firms with low wages constantly bleed employees to other firms and incur elevated vacancy cost. Thus, firms can only put their offered wage so low before they are losing more money on recruiting than they save on paid wages. Bloesch et al. (2024) show that this disciplining device leads to equilibrium wages way above the outside option, which is non-zero in their model. Their finding is in

line with the empirical evidence that unemployment is not the credible outside threat for employees, but other job offers (see Jäger et al. (2020)).

With the endogenous separation rate we can write out the law of motion for firm i's employment

$$n_{i,t} = \left[1 - \left(\bar{\delta} + (1-\bar{\delta})\delta_t\left(\frac{w_{i,t}}{W_t}\right)\right)\right] n_{i,t-1} + a_t\left(\frac{w_{i,t}}{W_t}\right) v_{i,t}$$

Employment $n_{i,t}$ for firm i, posting $v_{i,t}$ vacancies and setting its wage to $w_{i,t}$, equals its last period employment $n_{i,t-1}$ subject to exogenous $\bar{\delta}$ and endogenous δ_t separation plus the amount of newly hired employees, which is the product of the likelihood of filling a vacancy $a_t\left(\frac{w_{i,t}}{W_t}\right)$ times the amount of posted vacancies $v_{i,t}$.

The aggregate law of motion for employment is independent of the endogenous separation rate and only depends on the amount of matches with unemployed workers relative to the exogenous separation rate. The exogenous separation rate determines the flow of workers from employment to unemployment, while the level of vacancies times its matching rate with unemployed workers determines the flow from unemployment to employment.

$$N_t = (1-\bar{\delta})N_{t-1} + V_t q(\theta_t)\mathfrak{p}_t^U$$

I briefly describe the timing of the labor market to illustrate the employment law of motion. First, firms inherit their labor from last period $n_{i,t-1}$ subject to the exogenous separation rate $\bar{\delta}$. Second, firms decide on the optimal wage $w_{i,t}$ and job postings $v_{i,t}$ for this period to reach optimal employment $n_{i,t}$. Job vacancies are matched to employed and unemployed workers. If matched to an employed worker the firm poaches this worker from its current employer with probability $\delta_t\left(\frac{w_{i,t}}{W_t}\right)$, depending on the relative wage offered. This job-to-job transition leads to an endogenous separation on top of the exogenous separation. Unemployed workers accept the offered job with probability one, irrespective of the offered wage.[25] The overall vacancy filling rate $a_t\left(\frac{w_{i,t}}{W_t}\right)$ from employed and unemployed workers for firm i also depends on its relative wage. Finally, firm i realizes its employment $n_{i,t}$ and produces accordingly.

For simplicity I assume that labor is the only input to production, and all firms have the same production technology. The production function of variety $i \in [0,1]$ is

$$y_{i,t} = f(n_{i,t})$$

5.3 Firm's Problem

The firm producing variety i maximizes its expected present-discounted value of current and future profits. The firm's state variables are employment $n_{i,t}$, price $p_{i,t}$, wage $w_{i,t}$, and job postings $v_{i,t}$. The firm faces two major trade-offs in deciding the optimal level of each of these four decision variables.

[25] In my estimated model dynamics I use a version of the model with non-trivial unemployment benefits as laid out in Appendix I.3. Labor supply shocks enter the model through increases in the real wage replacement rate for unemployed workers.

First, increasing the price of its output is increasing profit per unit, but reducing demand due to households partial substitution by other varieties. Second, raising employment raises output and consequently profits due to a positive markup. But in order to increase employment the firm has to either increase vacancies, which cost κ_v to post, or raise its wage to shelter its employee base from competitors while making job offers to workers at other firms more attractive. However, raising the wage raises marginal cost.

These major trade-offs are incorporated in the firm's Bellman equation, and its constraints. The value optimized by the firm in the Bellman equation depends on current economic conditions such as total employment N_t, total production Y_t and expectations about the future. It further depends on the state the firm inherits from the previous period $x_{i,t-1} = (n_{i,t-1}, p_{i,t-1}, w_{i,t-1})$.

$$\begin{aligned} J(x_{i,t-1}) = \max_{n_{i,t},p_{i,t},w_{i,t},v_{i,t}} & \frac{p_{i,t}}{P_t} D_t\left(\frac{p_{i,t}}{P_t}\right) - \frac{w_{i,t}}{P_t} n_{i,t} \\ & - \kappa_v v_{i,t} - \frac{\kappa_p}{2}\left(\frac{p_{i,t}}{p_{i,t-1}} - 1\right)^2 Y_t - \frac{\kappa_w}{2}\left(\frac{w_{i,t}}{w_{i,t-1}} - 1\right)^2 N_t + E_t\left[\Lambda_{t,t+1} J_{t+1}(x_{i,t})\right] \end{aligned}$$

The firm's Bellman equation increases in the firm's current and expected future revenue, and decreases in its current and expected future wage, vacancy and price and wage adjustment costs. $\Lambda_{t,t+1}$ is the stochastic discount factor the firm is using to discount the future between period t and $t+1$.

I highlight the terms introduced by DNWR in red, to differentiate it from the nested flexible wage case in black. The optimization problem is constrained by downward nominal wage rigidity

$$w_{i,t} \geq w_{i,t-1}$$

as well as the law of motion for employment

$$n_{i,t} = \left[1 - \left(\bar{\delta} + (1-\bar{\delta})\delta_t\left(\frac{w_{i,t}}{W_t}\right)\right)\right] n_{i,t-1} + a_t\left(\frac{w_{i,t}}{W_t}\right) v_{i,t} =: \left[1 - \tilde{\delta}\left(\frac{w_{i,t}}{W_t}\right)\right] n_{i,t-1} + a_t\left(\frac{w_{i,t}}{W_t}\right) v_{i,t}$$

and the market clearing condition

$$D_t\left(\frac{p_{i,t}}{P_t}\right) = Y_{i,t} = f(n_{i,t})$$

The constraints are introduced into the optimization problem via Lagrange multipliers $\phi_{i,t}$, $\mu_{i,t}$ and $\lambda_{i,t}$ respectively. Profit maximization delivers four first order conditions. The first order condition with respect to the optimal job posting $v_{i,t}$ provides an expression for the value of an additional worker $\mu_{i,t}$, which is the Lagrange-multiplier of the employment law of motion.

The second FOC, with respect to the optimal wage posted $w_{i,t}$ delivers an expression for the wage Phillips curve. Third, the FOC with respect to the optimal price $p_{i,t}$ delivers the price Phillips curve of the model. Finally, the FOC with respect to optimal employment $n_{i,t}$ delivers an expression for marginal cost $\lambda_{i,t}$, which is the Lagrange-multiplier of the market clearing condition.

I will take these in turn and provide the value of an additional worker $\mu_{i,t}$, marginal cost $\lambda_{i,t}$, and the wage and price Phillips curves in the following section. I close the model with a standard Taylor rule and the imposition of overall market clearing (see Appendix I.1). The Taylor rule also contains a monetary policy shock which provides the demand shock in the model. The steady state derivation is provided in Appendix I.2.

5.4 Optimization under Future DNWR

This section provides the solution to the optimization problem of the firm producing variety $i \in [0,1]$. Due to DNWR there are essentially three cases. First, the case where DNWR binds. Second, the case where DNWR is not binding nor expected to bind. And third, the case where DNWR does not bind in period t but is expected to bind in period $t+1$. This is the case under a sufficiently strong but transitive positive demand or negative labor supply shock.

The first order condition with respect to the number of posted vacancies $v_{i,t}$ is the same in all three cases and gives

$$\mu_{i,t} = \frac{\kappa_v}{a_t\left(\frac{w_{i,t}}{W_t}\right)}$$

5.4.1 Wage Phillips Curve Derivation

The first order condition with respect to $w_{i,t}$ gives

$$\begin{aligned} 0 = &- \frac{n_{i,t}}{P_t} - \kappa_w\left(\frac{w_{i,t}}{w_{i,t-1}} - 1\right)\frac{N_t}{w_{i,t-1}} + E_t\left[\Lambda_{t,t+1}\kappa_w\left(\frac{w_{i,t+1}}{w_{i,t}} - 1\right)\frac{w_{i,t+1}}{w_{i,t}^2}N_{t+1}\right] \\ &+ \mu_{i,t}\left[a_t'\left(\frac{w_{i,t}}{W_t}\right)\frac{v_{i,t}}{W_t} - \tilde{\delta}_t'\left(\frac{w_{i,t}}{W_t}\right)\frac{n_{i,t-1}}{W_t}\right] + \phi_{i,t} - E_t\left[\Lambda_{t,t+1}\phi_{i,t+1}\right] \end{aligned}$$

The red terms vanish if DNWR is neither binding nor expected to bind as the Lagrange multiplier ϕ_t and ϕ_{t+1} equal to zero. Reordering and using symmetry as well as multiplying by W_t/N_t gives

$$\begin{aligned} \phi_t = &\frac{W_t}{P_t} + \kappa_w\left(\Pi_t^w - 1\right)\Pi_t^w - E_t\left[\Lambda_{t,t+1}\kappa_w\left(\Pi_{t+1}^w - 1\right)\Pi_{t+1}^w\frac{N_{t+1}}{N_t}\right] \\ &-\mu_t\left[a_t'\left(1\right)\frac{V_t}{N_t} - \tilde{\delta}_t'\left(1\right)\frac{N_{t-1}}{N_t}\right] + E_t\left[\Lambda_{t,t+1}\phi_{i,t+1}\right] \end{aligned}$$

Plugging in the expressions for

$$\mu_t = \frac{\kappa_v}{a_t\left(1\right)}$$

and the outer derivatives

$$\begin{aligned} a_t'(1) &= q(\theta_t)\mathfrak{p}_t^E f_\psi(1) \\ \tilde{\delta}_t'(1) &= -\frac{V_t q(\theta_t)\mathfrak{p}_t^E}{N_{t-1}} f_\psi(1) \end{aligned}$$

leads to the expression for optimal flexible real wage $\left(\frac{\kappa_v}{a_t(1)} 2 \frac{V_t}{N_t} q(\theta_t) \mathfrak{p}_t^E f_\psi(1)\right) = \frac{W_t}{P_t}^*$, as I show in Appendix I.2, and thus

$$\phi_t = \frac{W_t}{P_t} - \frac{W_t}{P_t}^* + \kappa_w \left(\Pi_t^w - 1\right) \Pi_t^w - E_t \left[\Lambda_{t,t+1} \kappa_w \left(\Pi_{t+1}^w - 1\right) \Pi_{t+1}^w \frac{N_{t+1}}{N_t}\right] + E_t \left[\Lambda_{t,t+1} \phi_{i,t+1}\right]$$

Iterating forward until DNWR is not expected to bind in period $t+\tau$ anymore, which gives $E_t\left[\phi_{i,t+\tau}\right] = 0$ gives the expression

$$\begin{aligned}
\phi_t &= \sum_{k=0}^{\tau-1} E_t \left[\Lambda_{t,t+k} \left(\left(\frac{W_{t+k}}{P_{t+k}} - \frac{W_{t+k}}{P_{t+k}}^*\right) \frac{N_{t+k}}{N_t} + \kappa_w \left(\Pi_{t+k}^w - 1\right) \Pi_{t+k}^w \frac{N_{t+k}}{N_t} - \kappa_w \left(\Pi_{t+k+1}^w - 1\right) \Pi_{t+k+1}^w \frac{N_{t+k+1}}{N_t}\right)\right] \\
&= \sum_{k=0}^{\tau-1} E_t \left[\Lambda_{t,t+k} \left(\frac{W_{t+k}}{P_{t+k}} - \frac{W_{t+k}}{P_{t+k}}^*\right) \frac{N_{t+k}}{N_t}\right] + \kappa_w \left(\Pi_t^w - 1\right) \Pi_t^w - \kappa_w E_t \left[\left(\Pi_{t+\tau}^w - 1\right) \Pi_{t+\tau}^w \frac{N_{t+\tau}}{N_t}\right]
\end{aligned}$$

for the benefit of relaxing the DNWR constraint by one if DNWR is expected to bind from t to $t+\tau-1$. Where the second step follows from collapsing the alternating terms of expected wage growth.

DNWR not binding today gives $\phi_t = 0$, but expected binding tomorrow leaves $-E_t[\Lambda_{t,t+1}\phi_{t+1}]$. Thus, under no binding constraint today but expected binding constraint tomorrow until $t+\tau$ th wage Phillips curve is

$$\left(\Pi_t^w - 1\right) \Pi_t^w = \frac{1}{\kappa_w} \left(\frac{W_t}{P_t}^* - \frac{W_t}{P_t} + \sum_{k=1}^{\tau-1} E_t \left[\Lambda_{t,t+k} \left(\frac{W_{t+k}}{P_{t+k}}^* - \frac{W_{t+k}}{P_{t+k}}\right) \frac{N_{t+k}}{N_t}\right]\right) + E_t \left[\Lambda_{t,t+\tau} \left(\Pi_{t+\tau}^w - 1\right) \Pi_{t+\tau}^w \frac{N_{t+\tau}}{N_t}\right]$$

where $W_{t+k} = W_t$ due to DNWR binding. A first order approximation delivers the wage Phillips curve in its common form

$$\pi_t^w = E_t \left[\Lambda_{t,t+\tau} (\pi_{t+\tau}^w)\right] + \frac{1}{\kappa_w} \left(\hat{\omega}_t^* - \hat{w}_t + \hat{p}_t + \sum_{k=1}^{\tau-1} E_t \left[\Lambda_{t,t+k} (\hat{\omega}_{t+k}^* - \hat{w}_t + \hat{p}_{t+k})\right]\right)$$

where $\hat{\omega}_t^*$ is the deviation of flexible real wage from steady state flexible real wage. Thus, the expected binding of DNWR in future periods reduces the wage response in the model.

5.5 Price Phillips Curve

The first order condition with respect to the price of variety i is

$$\frac{D_t\left(\frac{p_{i,t}}{P_t}\right)}{P_t} + \frac{p_{i,t}}{P_t} D_t'\left(\frac{p_{i,t}}{P_t}\right) - \kappa_p \left(\frac{p_{i,t}}{p_{i,t-1}} - 1\right) \frac{Y_t}{p_{i,t-1}} + E_t \left[\Lambda_{t,t+1} \kappa_p \left(\frac{p_{i,t+1}}{p_{i,t}} - 1\right) \frac{p_{i,t+1}}{p_{i,t}^2} Y_{t+1}\right] - \lambda_t D_t'\left(\frac{p_{i,t}}{P_t}\right) \frac{1}{P_t} = 0$$

where $D_t'\left(\frac{p_{i,t}}{P_t}\right)$ is the derivative of $D_t\left(\frac{p_{i,t}}{P_t}\right)$, and λ_t is the Lagrange multiplier of the market clearing condition and thus represents marginal cost. Multiplying the FOC by P_t/Y_t and symmetry across firms yields

$$1 - \epsilon - \kappa_p (\Pi_t^p - 1)\Pi_t^p + E_t \left[\Lambda_{t,t+1} \kappa_p \left(\Pi_{t+1}^p - 1\right) \Pi_{t+1}^p \frac{Y_{t+1}}{Y_t}\right] + \epsilon \lambda_t = 0$$

where $\Pi_t^p = P_t/P_{t-1}$. Rearrange to derive the model's price Phillips curve

$$(\Pi_t^p - 1)\Pi_t^p = \frac{1}{\kappa_p}(1 - \epsilon + \epsilon\lambda_t) + E_t\left[\Lambda_{t,t+1}\left(\Pi_{t+1}^p - 1\right)\Pi_{t+1}^p \frac{Y_{t+1}}{Y_t}\right]$$

The price Phillips curve is determined by a markup over marginal cost λ_t, depending on the elasticity of demand ϵ, plus discounted inflation expectations.

The linear approximation once more provides the usual form of the price Phillips curve

$$\pi_t^p = \frac{1}{\kappa_p}(\hat{\mathcal{M}}_t + \hat{\lambda}_t) + E_t\left[\Lambda_{t,t+1}\pi_{t+1}^p\right]$$

However, the price Phillips curve is incomplete without the measure of marginal cost λ_t.

5.6 Marginal Cost

Marginal cost λ_t is obtained from the first order condition with respect to employment $n_{i,t}$.

$$0 = -\frac{w_{i,t}}{P_t} - \mu_{i,t} + E_t[\Lambda_{t,t+1}(1 - \tilde{\delta}_{t+1})\mu_{t+1}] + \lambda_{i,t}f'(n_{i,t})$$

Using symmetry across firms and re-arranging terms gives marginal cost in period t

$$\lambda_t = \frac{\frac{W_t}{P_t} + \mu_t - E_t[\Lambda_{t,t+1}(1 - \tilde{\delta}_{t+1})\mu_{t+1}]}{f'(N_t)}$$

A linear approximation of the marginal cost expression provides

$$\hat{\lambda}_t = \hat{w}_t - \hat{p}_t + \hat{\mu}_t - E_t[\Lambda_{t,t+1}(1 - \tilde{\delta}_{t+1})\hat{\mu}_{t+1}] - \hat{mpl}_t$$

where $\hat{mpl}_t$ is the deviation of the marginal product of labor from steady state.

Plugging the linear approximation of marginal cost into the linear approximation of the price Phillips curve provides the full linearized price Phillips curve of the model.

$$\pi_t^p = \frac{1}{\kappa_p}(\hat{\mathcal{M}}_t + \hat{w}_t - \hat{p}_t + \hat{\mu}_t - E_t[\Lambda_{t,t+1}(1 - \tilde{\delta}_{t+1})\hat{\mu}_{t+1}] - \hat{mpl}_t) + E_t\left[\Lambda_{t,t+1}\pi_{t+1}^p\right]$$

5.7 Direct Impact Channel in the Model

Above we have derived the dynamics of the model with and without DNWR. Expected DNWR introduces a reduction of today's wage response. However, so far it is not clear if or how the price response increases, as marginal cost and the price Phillips curve expressions are the same for binding or not binding DNWR in future periods.

To see the impact of DNWR on inflation let us start with the model's price Phillips curve again.

$$(\Pi_t^p - 1)\Pi_t^p = \frac{1}{\kappa_p}(1 - \epsilon_p + \epsilon_p\lambda_t) + E_t\left[\Lambda_{t,t+1}(\Pi_{t+1}^p - 1)\Pi_{t+1}^p \frac{Y_{t+1}}{Y_t}\right]$$

Iterating forward delivers

$$
\begin{aligned}
(\Pi_t^p - 1)\Pi_t^p &= \frac{1}{\kappa_p} E_t \left[\sum_{k=0}^{\infty} \Lambda_{t,t+k} (1 - \epsilon_p + \epsilon_p \lambda_{t+k}) \frac{Y_{t+k}}{Y_t} \right] \\
&= \underbrace{\frac{1}{\kappa_p} E_t \left[\sum_{k=0}^{\infty} \Lambda_{t,t+k} (1 - \epsilon_p) \frac{Y_{t+k}}{Y_t} \right]}_{=:\Theta_t} + \frac{\epsilon_p}{\kappa_p} E_t \left[\sum_{k=0}^{\infty} \Lambda_{t,t+k} \lambda_{t+k} \frac{Y_{t+k}}{Y_t} \right] \\
&= \Theta_t + \frac{\epsilon_p}{\kappa_p} E_t \left[\sum_{k=0}^{\infty} \Lambda_{t,t+k} \lambda_{t+k}|_{flex} \frac{Y_{t+k}}{Y_t} \right] + \frac{\epsilon_p}{\kappa_p} E_t \left[\sum_{k=0}^{\infty} \Lambda_{t,t+k} \left(\lambda_{t+k} - \lambda_{t+k}|_{flex} \right) \frac{Y_{t+k}}{Y_t} \right]
\end{aligned}
$$

where $\lambda_{t+k}|_{flex}$ is the marginal cost without the DNWR constraint in period $t+k$. Under small enough shocks such that $Y_{t+k} \approx Y_{t+k}|_{flex}$ we can rewrite this as

$$
(\Pi_t^p - 1)\Pi_t^p \approx (\Pi_t^p - 1)\Pi_t^p|_{flex} + \frac{\epsilon_p}{\kappa_p} E_t \left[\sum_{k=0}^{\infty} \Lambda_{t,t+k} \left(\lambda_{t+k} - \lambda_{t+k}|_{flex} \right) \frac{Y_{t+k}}{Y_t} \right]
$$

The difference between marginal cost under DNWR binding or not is mostly driven by the differences in real wages. Hiring cost are slightly larger if DNWR is not enforced, and wages can adjust downward. This dampens the difference between the two marginal costs λ_{t+k} and $\lambda_{t+k}|_{flex}$, but plays a minor role. Thus, inflation with DNWR expected to bind can be approximated by inflation under the same circumstances but without enforcing DNWR, plus a discounted difference between the real wages paid under DNWR and non-downward rigid wages.

$$
(\Pi_t^p - 1)\Pi_t^p \approx (\Pi_t^p - 1)\Pi_t^p|_{flex} + \frac{\epsilon_p}{\kappa_p} E_t \left[\sum_{k=0}^{\infty} \Lambda_{t,t+k} \left(\frac{\frac{W_{t+k}}{P_{t+k}} - \frac{W_{t+k}}{P_{t+k}}|_{flex}}{f'(N_{t+k})} \right) \frac{Y_{t+k}}{Y_t} \right]
$$

Finally, $Y_{t+k} = f(N_{t+k}) \approx f'(N_{t+k}) N_{t+k}$ giving

$$
(\Pi_t^p - 1)\Pi_t^p \approx (\Pi_t^p - 1)\Pi_t^p|_{flex} + \frac{\epsilon_p}{\kappa_p} E_t \left[\sum_{k=0}^{\infty} \Lambda_{t,t+k} \left(\frac{\frac{W_{t+k}}{P_{t+k}} - \frac{W_{t+k}}{P_{t+k}}|_{flex}}{f'(N_t)} \right) \frac{N_{t+k}}{N_t} \right]
$$

Thus, the additional term has the same structure as today's marginal cost. This translates the additional inflation into the higher marginal cost associated with the DNWR constraint, highlighting the connection to my illustrative model.

Furthermore, a first-order approximation of the model is equivalent to a model where the firm is allowed to pay newly hired workers a lower wage for one period, as I show in Appendix I.4. There, I further show how the model generates the user cost of labor of Kudlyak (2014).

Finally, it is worth noting that the direct impact steepening the price response enters the model through higher inflation expectations. This reconciles the findings of Beaudry et al. (2025) that the non-linear price Phillips curve is somewhat fragile to the inclusion of short-term inflation expectations.

5.8 Model Dynamics

I linearize the model around flexible wage steady state for the unconstrained dynamics in the model. I derive the dynamics of the constraint periods using the piece-wise linearization of Kulish et al. (2017) and Guerrieri and Iacoviello (2015).

5.8.1 Calibration

I calibrate the model to quarterly data using standard parameters such as a discount factor of $\beta = 0.995$, a labor share in production of $\alpha = 70\%$, normalizing production technology to one ($A = 1$), setting the elasticity of demand $\epsilon_p = 3$ and setting the Cobb-Douglas parameter of the matching function to 0.5, giving equal weight to vacancies and job seekers.

Table 11: Calibration

Parameter	Description	Value	Source/Target
β	Discount factor	0.995	Standard value in literature
α	Labor Share	0.7	Standard value in literature
η	Cobb-Douglas parameter of Matching Function	0.5	Standard value in literature
U	Unemployment Rate	5.8%	Average 2000-2019 level
$\bar{\delta}$	Exogenous Separation Rate	7.2%	LEHD data
EE/UE	Ratio of EE to UE flows	0.32	LEHD data
s	Search Intensity of Employed	0.23	Faberman et al. (2022)
HC/MC	Ratio of Hiring to Marginal Cost	1%	Blanchard and Galí (2010)
ϵ_p	Price Elasticity of Demand	3	Anderson et al. (1997)
NK PC slope	Slope of New Keynesian Phillips curve	0.075	Gagliardone et al. (2023)

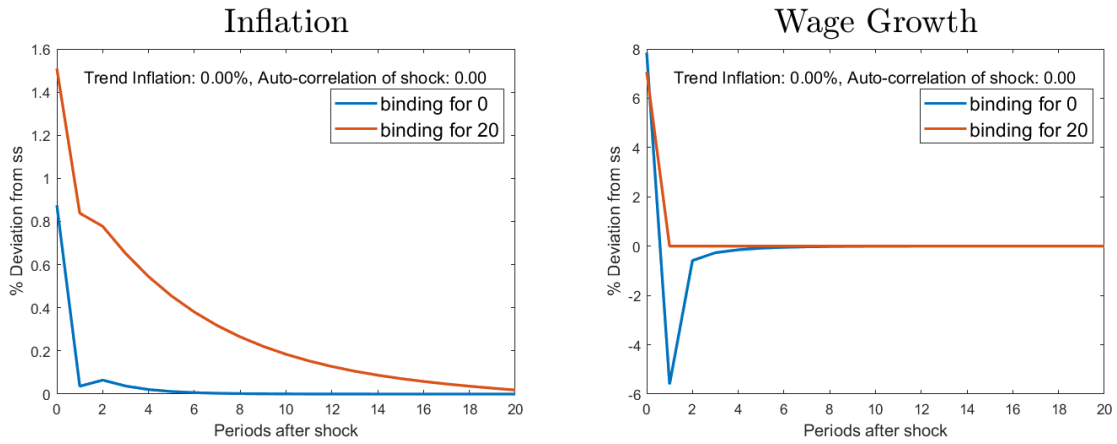
I calibrate the model to fit average 2000 to 2019 unemployment rate of 5.8% in steady state. Similarly, the ratio of employment-to-employment transitions relative to total quits is matched to its 0.32 average from Longitudinal Employer Household Dynamics data. LEHD also implicitly provides an exogenous separation rate of $\bar{\delta} = 7.2\%$. Relative search effort $s = 0.23$ of employees is taken from the Job Search Supplement of the Survey of Consumer Expectations (Faberman et al. (2022)).

I follow de la Barrera i Bardalet (2024) by inferring $1 - F_\psi = 0.31$ from the average portion of employed to unemployed workers among new hires, which roughly matches the 37% fraction of employment switchers who see declining wages (Sorkin (2018)). I calibrate the model to fit hiring costs to 1% of marginal cost matching the results from (Blanchard and Galí (2010), Gertler et al. (2008), Christiano et al. (2016)). This pins down the quits elasticity in the model. The quits elasticity and the portion of job switchers with wage declines allow me to calibrate the expected value and variance of the taste shock distribution. I assume that unemployed workers face the same distribution of taste shocks and have a non-zero outside option from unemployment insurance. I use the calibrated distribution and the EE/UE flow ratio to pin down the replacement rate of unemployment insurance at about 78%, which is in line with the rate suggested by Hall and Milgrom (2008).

Finally, I calibrate price rigidity for a slope of the New Keynesian Phillips curve of 0.075, in line with the estimates from Gagliardone et al. (2023). I choose nominal wage rigidity at an eighth of price rigidity to show-case how the user cost of labor flattens the wage response and steepens the price Phillips curve without relying on high nominal wage rigidity.

5.8.2 Dynamics

I show the IRFs following a transitory 5pp increase in nominal demand. I compare model dynamics with and without DNWR to highlight the contribution of the user cost of labor channel. As Figure 10 shows the user cost of labor channel increases the inflation response, and decreases the response of nominal and real wage growth. Similar dynamics follow a negative labor supply shock, see Appendix I.7.1.



(a) Inflation Response

(b) Wage Growth Response

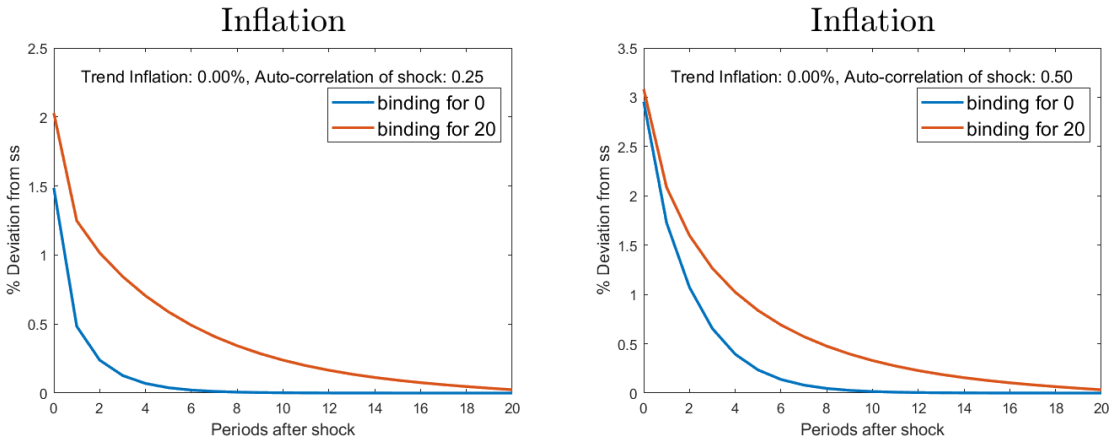
Figure 10: Impulse Response Functions to Demand Shock

Subfigure a) provides the IRF of inflation with respect to a transitory 5% increase in nominal demand. Subfigures b) does the same for nominal wage growth.

In Figure 11 I explore how the user cost of labor declines with rising auto-correlation of the shock. Note that a rising persistence of the shock raises the overall increase in demand and thus increases the inflation response. The relevant measure is the relative increase in the inflation response with and without DNWR. The relative size goes from roughly 50% under purely transitory shocks (Figure 10), over 40% under 25% auto-correlation, to roughly a third under 50% auto-correlation.

Increasing the persistence of the demand shock diminishes the channel as the discrepancy between today's and tomorrow's expected wages declines. The channel only plays a major role when shocks are perceived as transitory, as was widely the case under pandemic era stimulus and labor supply shocks colloquially termed the Great Resignation.

Higher trend inflation similarly diminishes the UCL channel as it takes larger shocks to raise current wages above expected future wages. Figure 12 shows how the channel decreases with rising trend inflation when keeping the shock size fixed. This highlights the diminished role of the channel under small shocks in an economy with trend inflation. Only



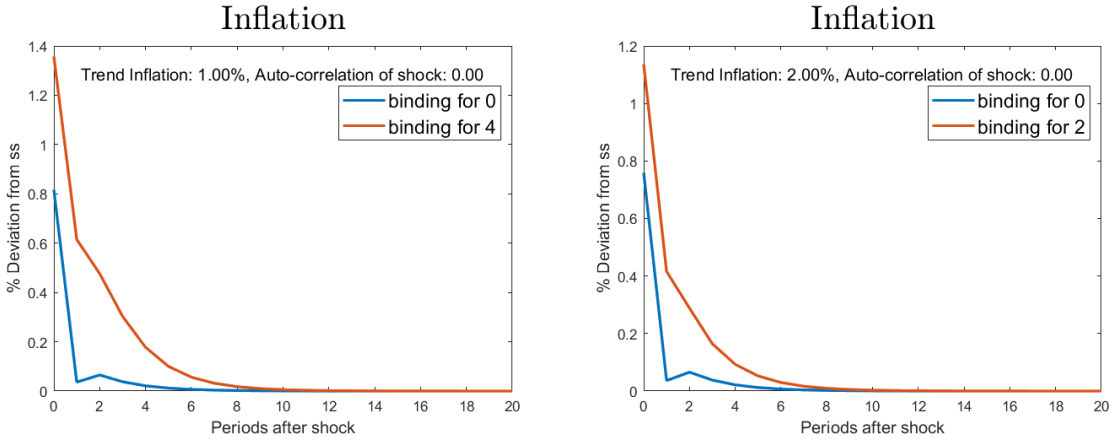
(a) 25% Auto-Correlation (b) 50% Auto-Correlation

Figure 11: Impulse Response Functions to Demand Shock

Subfigure a) provides the IRF of inflation with respect to a 5% increase in nominal demand and an auto-correlation of 25%. Subfigure b) does the same for demand shocks with auto-correlations of 50%.

under large shocks or tight enough labor markets from previous shocks does the channel play a role.

Trend inflation plays a similar role for negative shocks. Negative shocks have to be large enough to overcome trend inflation to turn desired wage growth negative and thus trigger the DNWR constraint. Trend inflation thus determines the size of the flexible wage region in which DNWR is neither binding nor expected to bind in the future.



(a) 1% Trend Inflation (b) 2% Trend Inflation

Figure 12: Impulse Response Functions to Demand Shock

Subfigure a) provides the IRF of inflation with respect to a transitory 5% increase in nominal demand under 1% trend inflation. Subfigure b) does the same under 2% trend inflation.

The general model already showcases the impact of the UCL channel, with inflation increasing, and nominal and real wage growth decreasing. Furthermore, the UCL channel tempers the employment and labor market tightness response of the shock. Thus, under greater inflation responses and smaller increases in labor market tightness the channel steepens the

Phillips curve significantly.

Figure 13 shows the steepening of the price Phillips curve in simulated data under two percent trend inflation. I mark the constrained, unconstrained and UCL periods in blue, gray, and red respectively for better distinction.

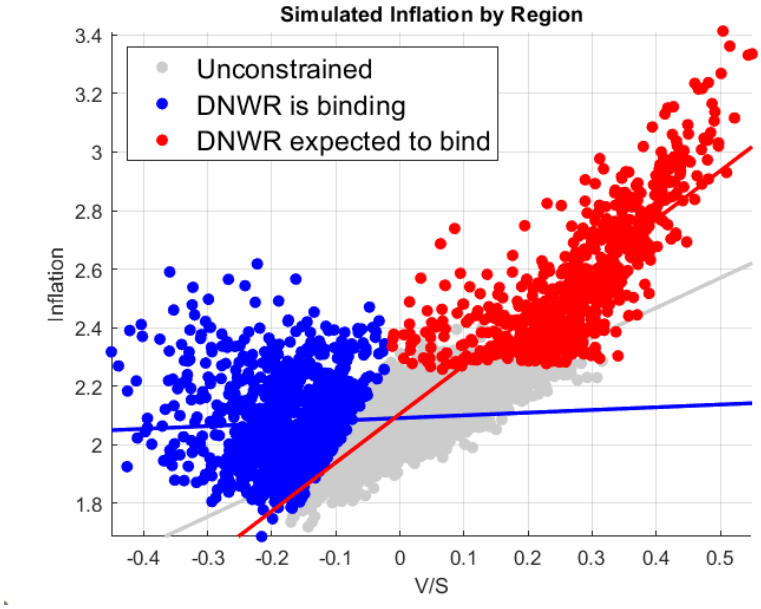


Figure 13: Simulated Phillips curve under 2% Trend Inflation

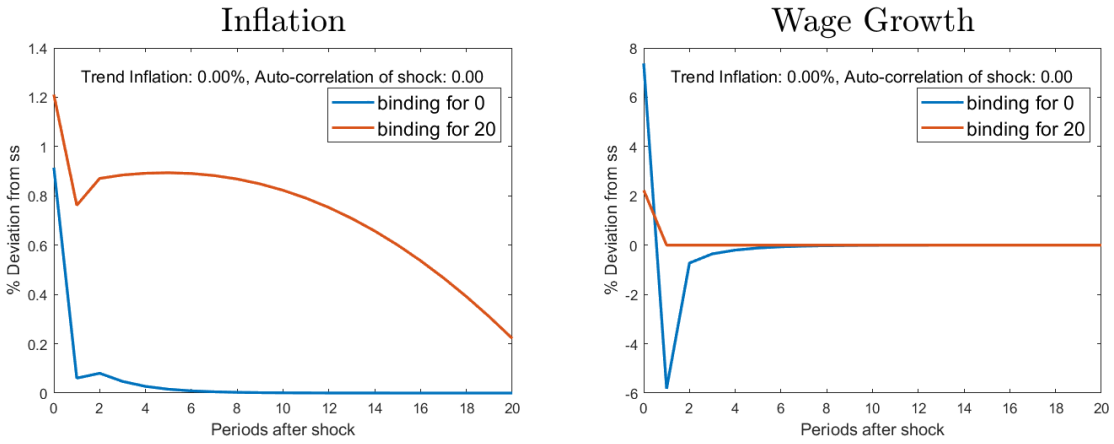
The UCL channel further scales with shock size. However, there is a lid on how far the UCL channel can go under purely aggregate shocks. Its inflation increase is limited to the wage growth response to the shock. The channel thus can lower the real wage growth response to zero, but not beyond. If real wages do not rise then there is no higher marginal cost in the future limiting any further increase in the channel.

This restraint on the channel stems from inflation targeting and the abstraction from idiosyncratic shocks. Inflation targeting mitigates the impact of raising wages today beyond their medium run level as the price level is allowed to catch up and bring real wages back to steady state level. This would not be the case under a price level target.

Idiosyncratic shocks have a similar impact to a price level targeting central bank however. Idiosyncratic demand shocks do not move the aggregate price level due to atomistic firms. Thus, under a mis-perception of aggregate demand shocks as idiosyncratic demand shocks as in Lucas (1972) firms will act as if nominal wage increases are real wage increases. This enhances their resistance to raising wages.

To illustrate this I show the dynamics under firms' mis-perception of demand shocks as fully idiosyncratic. As firms are atomistic they do not expect a change in the price level except for trend inflation. This leads firms to treat nominal wage increases beyond trend inflation as real wage increases. I first show these dynamics under no trend inflation in Figure 14, and then show how trend inflation again reduces the inflation impact in Figure 15.

Test

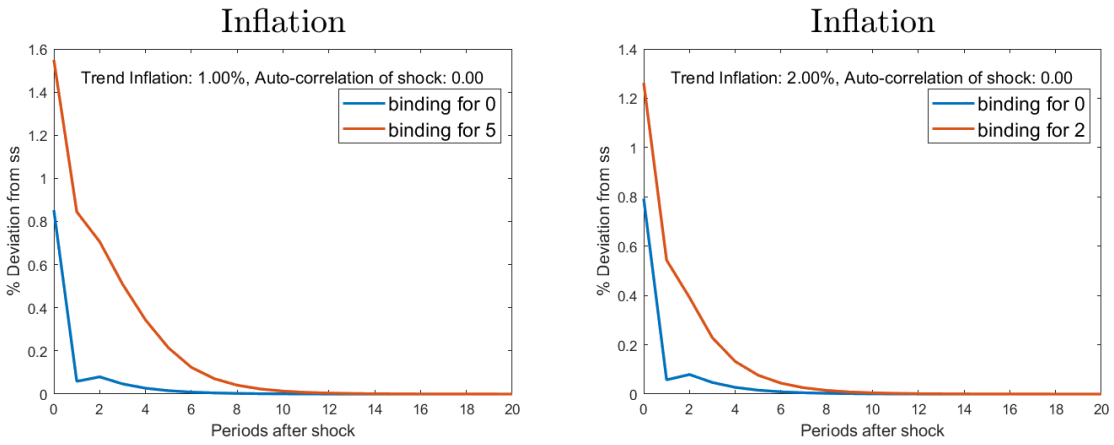


(a) Inflation Response (b) Wage Growth Response

Figure 14: Impulse Response Functions to Demand Shock.

Subfigure a) provides the IRF of inflation with respect to a transitory 5% increase in nominal demand under shocks perceived as idiosyncratic. Subfigure b) does the same for nominal wage growth.

Under informational frictions, or the prevalence of idiosyncratic shocks the channel can explain not just a decline in real wage growth but also a decline in real wages as Figure 14 shows.



(a) 1% Trend Inflation (b) 2% Trend Inflation

Figure 15: Impulse Response Functions to Demand Shock

Subfigure a) provides the IRF of inflation with respect to a transitory 5% increase in nominal demand perceived as idiosyncratic under 1% trend inflation. Subfigure b) does the same under 2% trend inflation.

Trend inflation tempers the negative impact on real wages as firms anticipate to return to steady state real wage levels over time even when they allow their real wage to rise today. However, the steepening of the price Phillips curve persists.

I show the divergence of the Phillips curves under the informational friction in Figure 16, by tracing out the two Phillips curves with demand shocks under 2% trend inflation. Here, I omit the link between periods to show the shape of the underlying Phillips curves. I also

reduce price rigidity to match the slope of the wage Phillips curve during unconstrained periods to illustrate the split happening during large demand shocks.

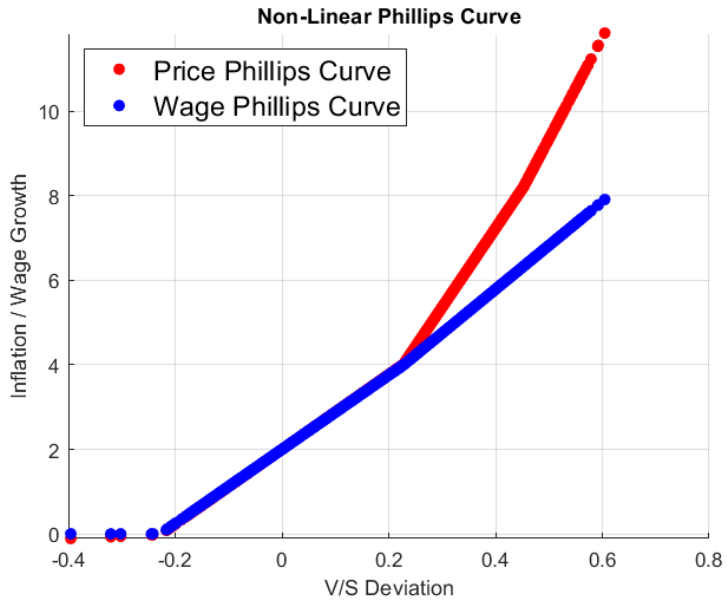


Figure 16: Phillips curve under 2% trend inflation and informational frictions

The model is able to match my empirical findings under tight labor markets when incorporating informational frictions and sufficiently low wage and price rigidities other than from downward nominal wage rigidity. The price Phillips curve is less steep than the wage Phillips curve under moderate shocks but steepens in excess of the wage Phillips curve under large shocks, while the wage Phillips curve remains approximately linear.

However, one discrepancy with my estimated Phillips curves remains. The wage Phillips curve is truly non-linear when DNWR is binding. Further extension of the model is thus needed to match wage growth dynamics during slack labor markets. A promising direction is to have a true dispersion of wages in the economy through heterogeneous firms along the productivity dimension.

Differing firm productivity would lead to a job ladder labor market structure where unemployed accept any job above their UI benefits, and move up the productivity ladder through on-the-job search. Thus, high productivity firms expand their payroll during booms and constrain their hiring during busts. Exogenous separation then leads to a prevalence of lower wages among new hires during slack labor markets, leading to measured wage declines. Implementing productivity heterogeneity among firms will be the next step in extending the model.

To summarize, the model introduces a user cost of labor channel into a New Keynesian Search and Matching framework with monopsony power. The combination of monopsony power and downward nominal wage rigidity introduces the possibility for an excess steepening of the price Phillips curve during tight labor markets. Furthermore, the user cost of labor is spread out over future periods instead of fully incurred in the current period. Conse-

quently measured profits in the current period increase with the User-Cost of Labor wedge between prices and wages. Thus, the model also explains the non-linear rise of profit margins with labor market tightness.

Potential extensions of the model include symmetric absolute rigidity of existing employee contracts, as suggested by Kudlyak and Miskanic (2024). This would extend the User-Cost of Labor to periods with negative demand shocks, albeit with the opposite sign. The upward wage rigidity of existing employees would result in greater hiring during transitory negative demand shocks, buffering the economy and aligning with firms' labor hoarding behavior early in a downturn.

Additionally, I plan to expound on the role of monopsony power in the channel, as firms cannot be pure price takers in the labor market for my framework to hold true. This necessity introduces monopsony power into the model implicitly. A natural next step is to investigate how the dynamics shift when a wage markdown is offset by a labor subsidy, or in a two-sided labor market where unions impose a wage markup to counterbalance the wage markdown from monopsony power.

6 Conclusion

Recent findings have hinted at a discrepancy between wage and price Phillips curves during tight labor markets. This paper confirms the discrepancy and provides evidence for a direct impact of labor market tightness on inflation as a second channel of the price Phillips curve.

In addition to looking at the Phillips curve the paper documents a non-linear rise in profit margins and a non-linear decline in real wage growth during especially tight labor markets.

I point out that these dynamics present a puzzle to the classic Phillips curve view, which posits that tight labor markets drive inflation primarily through wage growth. The dynamics are also hard to reconcile with the standard model assumption of firms as wage takers.

I resolve this puzzle with wage setting power for firms and transitory shocks under downward nominal wage rigidity. Adding wage setting power allows for wage-price tradeoffs. Transitory shocks increase the cost of raising wages if wages fail to decline after the shock subsides. Consequently, firms adjust prices to manage demand, resulting in subdued increases in employment and wages. As a result, transitory positive demand or negative labor supply shocks create a wedge between prices and wages on top of steady-state markups. Thus, this paper provides a novel micro-founded explanation for Phillips curve non-linearity.

The channel produces two testable implications that hold in the data. First, market power is necessary for the channel to operate. I find that this holds in country and sector level data using Herfindahl-Hirschman Indexes as measure of market concentration.

Second, my mechanism implies that rising expected future sales during tight labor markets reduce inflation. I test this implication using firm survey data from the National Foundation of Independent Businesses and find a negative interaction term between future sales growth expectations and labor market tightness, supporting my channel.

Finally, I integrate the channel into a New Keynesian model with search and matching by putting wages into firms' decision set via wage posting. Under downward nominal wage rigidity firms facing non-permanent demand shocks consider higher future marginal cost from increasing wages in their price setting. This increases the price and flattens the wage response during periods of tight labor markets, resulting in an additional wedge between prices and wages. The model thus rationalizes the divergence of wage and price Phillips curves, even to the point of declining real wages during tight labor markets when shocks are perceived as idiosyncratic.

My channel strengthens the non-linear Phillips curve explanation for the low cost of disinflation observed after the post-pandemic inflation surge. Since real wages hardly grow or may even fall in my framework, there is a greater ability to reduce inflation without making parts of the labor force too costly. As a result, increases in unemployment are smaller, leading to less loss in output.

Additionally, my channel challenges the notion that increasingly tight labor markets are unequivocally beneficial for workers. The direct impact channel suggests that overly tight labor markets can actually reduce real wage growth. Finally, the linear nature of the wage Phillips curve suggests that the direct impact is the primary driver of the Phillips curve non-linearity, highlighting the importance of further research on the channel.

Appendix:
Reconciling Wage and Price Phillips Curves

David Osten

November 5, 2025

A Additional Results on US Aggregate Data

A.1 Diverging Wage and Price Phillips curves - A Puzzle

This section presents the divergence of the wage and price Phillips curves found by the previous literature. In particular, Hooper et al. (2020) and Ball et al. (2022) find a greater steepening of the price than the wage Phillips curve during tight labor markets on cross-sectional and aggregate US data, respectively. I use the approach of Ball et al. (2022) on aggregate data to illustrate these findings of diverging wage and price Phillips curves. I simplify their equations to the following wage and price Phillips curves[26]:

$$\pi_t - \pi_t^e = \alpha_1(V/U)_t + \alpha_2(V/U)_t^2 + \alpha_3(V/U)_t^3 + \beta \text{Prod}_t + e_t \tag{3}$$

$$\pi_t^w - \pi_t^e = \acute{\alpha}_1(V/U)_t + \acute{\alpha}_2(V/U)_t^2 + \acute{\alpha}_3(V/U)_t^3 + \acute{\beta} \text{Prod}_t + \acute{e}_t \tag{4}$$

where π is median CPI inflation, π^e is the median 10 year inflation forecast from the Survey of Professional Forecasters, V/U is the ratio of job openings to unemployed, Prod is an eight quarter moving average of productivity growth and π^w is wage growth, based on ECI wages and salaries. I show estimation results of equations 3 and 4 on 1985Q1 to 2023Q4 US aggregate data in Table 12.

Note the discrepancy between the price and the wage Phillips curve, with inflation yielding a cubic relation, while wage growth exhibits merely a linear relation to labor market tightness. Figure 17 graphs the cubic labor market functions of Table 12.

The divergence of the wage and price Phillips curves under tight labor markets constitutes a puzzle under the conventional Phillips curve view of rising wages causing inflation. The divergence gives rise to two hypotheses. First, tight labor markets exert an influence on inflation above and beyond their influence on wage growth. Second, this direct impact is a key driver of Phillips curve non-linearity. I provide evidence on these hypothesis in the ensuing section.

[26] Ball et al. (2022) further include linear, quadratic and cubic "HeadShocks" which they define as the difference of headline inflation and median inflation. Similar results are found on other inflation measures.

Table 12: Discrepancy between Wage & Price Phillips Curves

	π	π^w
V/U	5.99^{***}	4.44^{*}
	(2.08)	(2.33)
$(V/U)^2$	-6.52^{***}	-2.47
	(2.46)	(2.74)
$(V/U)^3$	2.72^{***}	0.78
	(0.82)	(0.90)
Prod	-0.09^{*}	0.22^{***}
	(0.05)	(0.06)
Constant	-1.62^{***}	-2.20^{***}
	(0.49)	(0.54)
R^2	0.81	0.54
N	156	156

Newey-West standard errors in parentheses

* $p < .10$, ** $p < .05$, *** $p < .01$

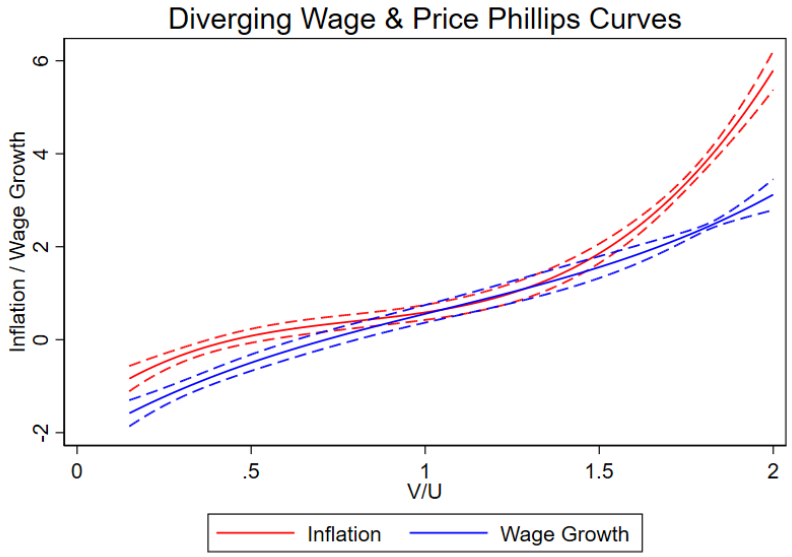


Figure 17: Comparing the cubic Wage and Price Phillips Curves

A.2 Robustness Checks

A.3 Full Flagship Table

This section reports the full extend of the coefficients in the flagship table testing for a direct impact of labor market tightness on inflation.

A.3.1 Robustness to Lagged Wage Growth

Table 14 below shows the robustness of the direct impact to the inclusion of further wage growth lags on CPI headline inflation. I again omit reporting the coefficients on the control variables.

Table 14 shows that the direct impact measured in Table 1 is not due to a negligence of further lags of wage growth. Including further wage lags leads to only miniscule changes in the estimated labor market tightness coefficients. Further the Wald tests reject the insignificance and linearity of the direct impact of labor market tightness on inflation in all specifications. Similar results are obtained when including further wage lags in the CPIXFE and median CPI estimations.

A.3.2 Pre-pandemic Results (1985Q1 - 2019Q4)

This section repeats the flagship table 1 on pre-pandemic data to show that the direct impact is not purely driven by the pandemic inflation surge. Although headline inflation now delivers insignificant results, the general shape of the cubic direct impact is still apparent. Furthermore, CPI excluding food and energy still delivers significant rejection of both non-existence and linearity of the labor market tightness coefficients, even though the single coefficients are insignificant. Finally, median CPI inflation provides both significant labor market tightness coefficients as well as significant Wald tests on both existence and non-linearity. I again omit reporting the coefficients on the control variables.

Thus, even though the direct impact is slightly less significant on pre-pandemic data, Table 15 shows that the direct impact is robust to excluding the inflation surge during the pandemic recovery phase. The slightly less significant results on the 1985-2019 data is likely due to the limited range of labor market tightness in that time frame. Previous contributions to the literature have already argued that this lack of tight labor markets leads to flatter Phillips curve estimations (Gagnon and Collins (2019)). Even during this period of relative labor market tranquility the median and traditional core inflation measures still provide evidence for both the existence and non-linearity of the direct impact.

A.3.3 PCE Estimation

This section repeats Table 1 on PCE data. Using PCE data instead of CPI data leads to insignificant coefficients on headline and traditional core inflation. However, the Wald tests still reject the non-existence or mere linearity of the direct impact. Furthermore, using median PCE inflation still leads to highly significant labor market tightness coefficients. I thus

Table 13: V/U Direct Impact

	CPI	XFE	Median	CSXS
(V/U)	4.69^{*}	4.99^{*}	4.11^{**}	9.32^{***}
	(2.65)	(2.60)	(1.74)	(2.82)
$(V/U)^2$	-5.04	-5.08	-5.11^{**}	-11.84^{***}
	(3.29)	(3.26)	(1.96)	(3.32)
$(V/U)^3$	2.11^{*}	2.09^{*}	2.20^{***}	4.47^{***}
	(1.13)	(1.12)	(0.64)	(1.12)
π^e	0.82^{***}	1.02^{***}	0.64^{***}	1.17^{***}
	(0.11)	(0.11)	(0.08)	(0.14)
ω	0.05	0.03	0.18^{***}	-0.10
	(0.07)	(0.05)	(0.04)	(0.09)
L.ω	0.10	-0.05	0.19^{***}	-0.07
	(0.08)	(0.10)	(0.05)	(0.13)
Prod	-0.08	-0.00	-0.17^{***}	0.01
	(0.07)	(0.08)	(0.05)	(0.09)
Energy	0.10^{***}	0.01	0.00	0.02^{*}
	(0.01)	(0.01)	(0.00)	(0.01)
L.Energy	0.00	0.00	-0.00	0.00
	(0.01)	(0.00)	(0.00)	(0.00)
Import	0.24	-0.16	-0.12	-0.51
	(0.38)	(0.23)	(0.14)	(0.35)
L.Import	0.19	0.15	0.16	0.45^{**}
	(0.31)	(0.27)	(0.15)	(0.23)
Constant	-1.52^{**}	-1.76^{***}	-0.79	-1.51^{**}
	(0.66)	(0.64)	(0.49)	(0.76)
R^2	0.87	0.69	0.84	0.60
N	156	156	156	155
No Impact (p-value)	0.0000	0.0000	0.0000	0.0000
Linear Impact (p-value)	0.0023	0.0035	0.0000	0.0000
Partial R^2	0.17	0.25	0.42	0.16

Newey-West standard errors in parentheses

* $p < .10$, ** $p < .05$, *** $p < .01$

Table 14: Robustness of Headline CPI Direct Impact to Further Wage Lags

	L0	L1	L2	L3	L4
V/U	3.83	3.35	3.16	3.19	3.20
	(2.52)	(2.37)	(2.44)	(2.42)	(2.35)
$(V/U)^2$	−3.93	−3.59	−3.42	−3.46	−3.48
	(3.19)	(3.02)	(3.07)	(3.05)	(2.98)
$(V/U)^3$	1.70	1.58	1.51	1.53	1.55
	(1.10)	(1.04)	(1.07)	(1.06)	(1.03)
π^e	0.93***	0.82***	0.78***	0.80***	0.84***
	(0.06)	(0.09)	(0.08)	(0.08)	(0.09)
π^w	0.07	0.06	0.05	0.06	0.06
	(0.06)	(0.06)	(0.06)	(0.06)	(0.06)
L.π^w		0.12	0.11	0.12	0.15*
		(0.07)	(0.07)	(0.08)	(0.09)
L2.π^w			0.07	0.07	0.10
			(0.07)	(0.07)	(0.08)
L3.π^w				−0.06	−0.05
				(0.06)	(0.05)
L4.π^w					−0.11
					(0.10)
Constant	−1.67***	−1.42**	−1.34**	−1.35**	−1.38**
	(0.60)	(0.57)	(0.60)	(0.59)	(0.57)
R^2	0.86	0.87	0.87	0.87	0.87
N	156	156	156	156	156
No Impact (p-value)	0.0000	0.0000	0.0000	0.0000	0.0000
Linear Impact (p-value)	0.0029	0.0050	0.0086	0.0043	0.0023
Partial R^2	0.25	0.17	0.14	0.15	0.15

Newey-West standard errors in parentheses

* $p < .10$, ** $p < .05$, *** $p < .01$

Table 15: Pre-Pandemic Direct Impact

	CPI	XFE	Median	CSXS
V/U	4.03	5.50*	9.18***	12.52**
	(3.81)	(2.79)	(2.20)	(5.21)
$(V/U)^2$	−4.12	−5.72	−12.86***	−17.95**
	(5.77)	(4.50)	(3.31)	(8.45)
$(V/U)^3$	1.38	1.99	5.72***	7.53*
	(2.64)	(2.12)	(1.50)	(4.00)
π^w	0.07	−0.01	0.18***	−0.06
	(0.06)	(0.04)	(0.04)	(0.07)
L.π^w	0.10	−0.06	0.15***	−0.01
	(0.07)	(0.05)	(0.05)	(0.08)
π^e	0.84***	1.06***	0.67***	1.07***
	(0.09)	(0.07)	(0.07)	(0.13)
Constant	−1.48*	−1.72***	−1.79***	−2.12**
	(0.85)	(0.53)	(0.45)	(1.01)
R^2	0.86	0.75	0.77	0.75
N	140	140	140	139
No Impact (p-value)	0.1301	0.0000	0.0000	0.0477
Linear Impact (p-value)	0.2710	0.0184	0.0008	0.0194
Partial R^2	0.05	0.11	0.13	0.07

Newey-West standard errors in parentheses

* $p < .10$, ** $p < .05$, *** $p < .01$

conclude that the direct impact is also robust to using PCE measures of inflation. Similar is also true for other sub-components of both CPI and PCE inflation, such as Trimmed Mean PCE and CPI, CPI core services, and CPI services excluding shelter, as well as for the GDP deflator. I again omit reporting the coefficients on the control variables.

Table 16: PCE Direct Impact

	PCE	XFE	Median
V/U	1.08	1.70	4.40^{***}
	(2.05)	(2.11)	(1.40)
$(V/U)^2$	-1.74	-2.18	-5.48^{***}
	(2.56)	(2.76)	(1.64)
$(V/U)^3$	1.05	1.19	2.29^{***}
	(0.90)	(0.98)	(0.54)
π^w	0.15^{**}	0.15^{**}	0.15^{***}
	(0.07)	(0.07)	(0.05)
L.π^w	0.12^{**}	-0.00	0.11^{**}
	(0.05)	(0.06)	(0.04)
π^e	0.73^{***}	0.85^{***}	0.75^{***}
	(0.08)	(0.09)	(0.08)
Constant	-0.94^{*}	-1.09^{**}	-1.23^{***}
	(0.51)	(0.49)	(0.37)
R^2	0.85	0.68	0.80
N	156	156	156
No Impact (p-value)	0.0000	0.0000	0.0000
Linear Impact (p-value)	0.0005	0.0010	0.0000
Partial R^2	0.21	0.21	0.41

Newey-West standard errors in parentheses

* $p < .10$, ** $p < .05$, *** $p < .01$

A.3.4 Monthly CPI Estimation

This section provides evidence for the robustness of the direct impact to using monthly instead of quarterly data. Due to the relative noise in monthly data the significance of the coefficients is subdued for all three measures of inflation. The noise further obscures the non-linearity of the direct impact on headline inflation. However, the Wald test rejects non-existence of the direct impact even on headline inflation. Furthermore, both traditional core and median CPI inflation yield significant rejections of both non-existence as well as mere lin-

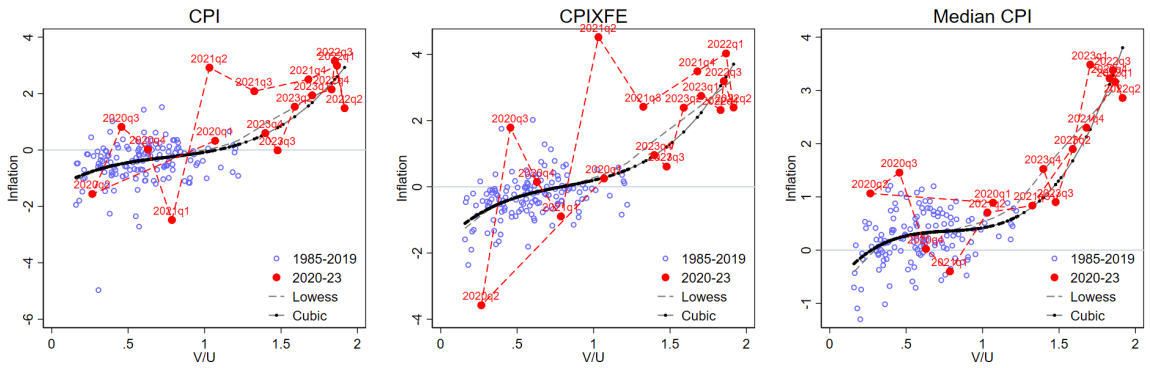
earity of the direct impact, and median inflation still gives significant labor market tightness coefficients. Thus, the direct impact is robust to measuring inflation on a monthly instead of a quarterly basis. Controls for labor productivity and relative import prices are omitted in the monthly estimation due to data availability issues on the monthly frequency.

Table 17: V/U Direct Impact on MoM Data

	CPI	CPIXFE	Median	CSXS
V/U	1.95	3.72	4.67**	7.77***
	(3.28)	(2.55)	(2.13)	(2.81)
$(V/U)^2$	−0.76	−3.41	−4.70*	−10.11***
	(3.89)	(3.21)	(2.42)	(3.36)
$(V/U)^3$	0.41	1.42	1.99**	3.82***
	(1.28)	(1.09)	(0.79)	(1.13)
π^w	0.08*	0.07	0.04	−0.02
	(0.05)	(0.04)	(0.03)	(0.06)
L.π^w	−0.03	0.01	0.05*	0.03
	(0.04)	(0.04)	(0.03)	(0.07)
π^e	0.91***	0.93***	0.90***	0.995***
	(0.06)	(0.05)	(0.04)	(0.09)
Energy	0.07***	−0.00	0.00***	0.00
	(0.01)	(0.00)	(0.00)	(0.00)
L.Energy	0.01**	0.01***	−0.00	0.01
	(0.00)	(0.00)	(0.00)	(0.00)
Constant	−1.76**	−1.50**	−1.55***	−1.20*
	(0.86)	(0.59)	(0.55)	(0.67)
R^2	0.76	0.44	0.64	0.32
N	468	468	468	467
No Impact (p-value)	0.0000	0.0000	0.0000	0.0000
Linear Impact (p-value)	0.5558	0.0428	0.0000	0.0004
Partial R^2	0.16	0.26	0.52	0.08

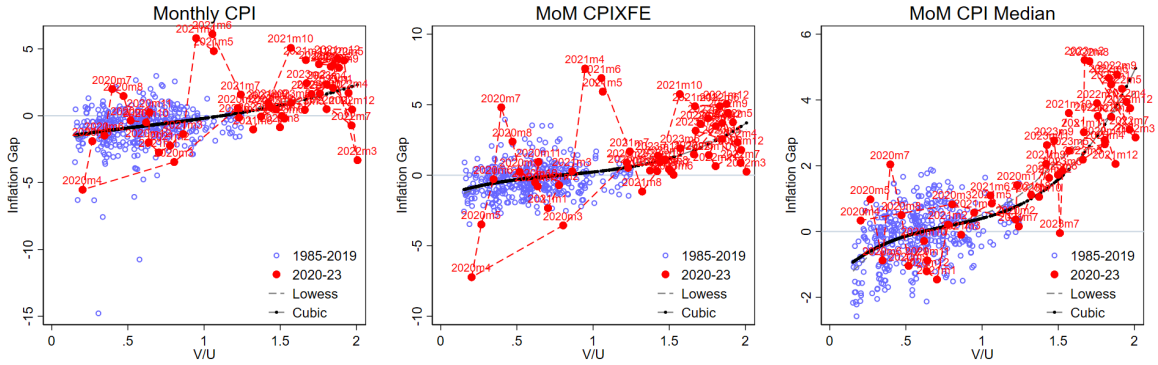
Newey-West standard errors in parentheses

* $p < .10$, ** $p < .05$, *** $p < .01$



(a) CPI (b) CPIXFE (c) Median CPI

Figure 18: Density graphs on annualized quarterly inflation.
Subfigure a) shows the cubic functional form of the annualized quarterly CPI inflation.
Subfigure b), and c) do the same for CPIXFE, median CPI inflation, respectively.



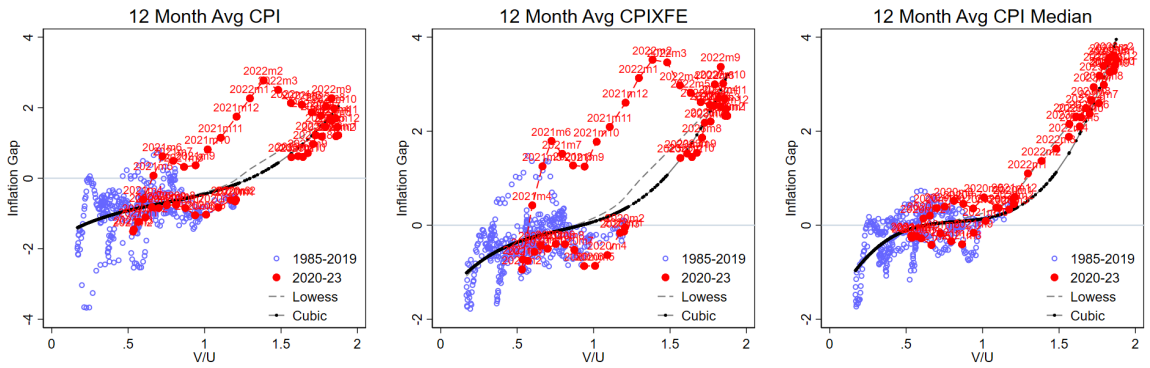
(a) CPI (b) CPIXFE (c) Median CPI

Figure 19: Density graphs on annualized month over month inflation.
Subfigure a) shows the cubic functional form of the annualized monthly CPI inflation.
Subfigure b), and c) do the same for CPIXFE, and median CPI inflation, respectively.

A.3.5 Density Graphs

This section provides data density graphs. I cleaned the data from the influence of the control variables, taking the homogeneity into account, and plot this against the fitted Phillips curve. First on the quarterly then on the monthly data.

The annualized month-over-month inflation is rather noisy and thus almost drowns out the functional form. Using 12 month moving averages of monthly inflation data gives a clearer picture. In this last figure I plot 12 month inflation averages against 12 month averages of V/U.



(a) CPI (b) CPIXFE (c) Median CPI

Figure 20: Density graphs on 12 month moving average of annualized month over month inflation.
Subfigure a) shows the cubic functional form of the 12 month average annualized monthly CPI inflation. Subfigure b), and c) do the same for CPIXFE, and median CPI inflation, respectively.

A.4 Quantification of the Pandemic Inflation Surge

This section quantifies the extent to which the direct impact contributed to the post-pandemic inflation surge, how much wage pressure contributed, and how the direct impact does on explaining the fall in real wages and the rise in profit margins. I leverage the findings on US aggregate data in Section 2.

In particular I use the estimated coefficients on labor market tightness from Equation 1 to quantify the direct impact. Recall that these coefficients were obtained under controls for past and current wage growth. I use these to compare average 2019 levels ($V/U = 1.19$) with peak labor market tightness in 2022Q2 ($V/U = 1.9$) and use the estimated functional forms to calculate how much of the inflation increase is explained by the direct impact. For example, I use the functional form of the headline CPI based model from Table 1: $\pi = 3.35 * (V/U) - 3.59*(V/U)^2 + 1.58*(V/U)^3$ to calculate the explained rise in inflation. I plug the relevant V/U numbers for 2019 and 2022Q2 into the functional form and calculate the difference between the predicted inflation levels. The difference (2.68 percentage points (pp)) is the estimated contribution of the direct impact to the rise in headline inflation between average 2019 levels and peak labor market tightness in 2022Q2.

I present the results of this and similar exercises for core and median inflation in Table 18. The first two rows provide the average 2019 and 2022Q2 peak inflation rates from the data. The difference between the two is provided in row three. Row four finally presents the estimated increase in inflation due to the direct impact.

In row five I provide the estimated total impact from the direct and traditional wage channel. I calculate the wage channel by computing the impact of labor market tightness on wage growth. The linear wage estimation on ECI wages and salaries gave a coefficient of 1.73. Thus, the increase of V/U from 2019 levels to 2022Q2 is estimated to have increased

Table 18: Direct Impact Quantification

	CPIXFE	**Median**	**CPI**
Avg 2019	2.33%	2.89%	2.05%
2022Q2	5.87%	6.98%	9.66%
Δ	3.54pp	4.09pp	7.61pp
Direct Impact	3.12pp	3.1pp	2.68pp
Total Effect	3.18pp	3.55pp	2.86pp

wage growth by about $1.73 \times 0.7 = 1.21$. Multiplying this by the respective sum of coefficients on wage growth in Table 1 I estimate an inflation impact from the traditional channel of 0.06, 0.45, and 0.18, for core, median and headline CPI inflation respectively.[27] Adding these estimates to the direct impact from row four, the last row provides an estimate of the total impact of labor market tightness between the two periods. If a pass-through of one for one from wage growth to inflation is forced instead of the estimated coefficients, the total effect increases to 4.27pp, 4.31pp, and 3.89pp on core, median and headline CPI inflation, respectively.

The direct impact accounts for almost all of the rise in core inflation (3.12pp out of 3.54pp), and a good portion of the rise in median inflation (3.1pp out of 4.09pp). The fit is less good for headline CPI inflation (2.68pp out of 7.61pp). Including the impact from wage growth (see "Total Effect") further increases the estimated impact of labor market tightness on inflation, although only by a comparatively small amount. Wage pressures added to median inflation, but seemed to have added little to traditional core inflation.

The quantification highlights the importance of allowing for the direct impact when assessing the labor market impact on inflation. The channel reconciles the discrepancy between the small impact found by Bernanke and Blanchard (2023) and the sizeable impact found by Ball et al. (2022). Bernanke and Blanchard (2023) only measure the impact stemming from wage growth, while Ball et al. (2022) allow for both the traditional wage and the direct impact on inflation. For more details see Osten (2025b).

Using estimation results from Bernanke and Blanchard (2023) yields a total impact of 0.93pp on headline inflation for the period. B&B estimate a sum of V/U coefficients on wage growth of 0.7, and a sum of wage growth lag coefficients of 0.46. Thus, the dynamic impact of labor market tightness on wage growth in their model is $0.7/(1 - 0.46) \approx 1.3$. Given the rise in V/U of roughly 0.7 this yields an estimated increase in wage growth of 0.91pp between average 2019 and 2022Q2. I use B&B's price equation to translate this into an estimated impact of 0.93pp on headline CPI inflation. This comes from multiplying the wage growth impact of 0.91pp with the dynamic coefficient on the sum of wage growth coefficients $(0.67/(1 - 0.34) = 1.02)$. Thus, the estimated impact from B&B is less than a third of the total impact on headline inflation estimated under the model that allows for a direct impact of labor market tightness on inflation.

[27] I drop the negative coefficient estimated on lagged wage growth for core inflation.

I repeat the comparison of average 2019 and 2022Q2 peak levels for wage growth, real wage growth and profit margins in Table 19. The direct effect on nominal wage growth explains less than half of the actual increase (1.23pp out of 2.95pp). The rest of the increase in wage growth is likely due to workers' attempt to catch-up with inflation. Multiplying the sum of inflation coefficients (0.2) with the increase in headline inflation over this period (7.61pp) does indeed roughly make up for the gap between the observed increase in wage growth and the increase due to tighter labor markets ($0.2 \times 7.61 = 1.52$pp; which is 88% of the 1.72pp gap).

Similarly, the drop in real wage growth is under-predicted (-1.52pp out of -5.07pp; a gap of -3.55pp), likely a result of the substantial shocks in food and energy prices. The unexplained variation in headline inflation (7.61pp -2.86pp $= 4.75$pp) does indeed more than explain the additional fall in real wage growth. Finally, profit margins are over-predicted by the direct impact (10pp compared to the observed 7pp). Higher input cost from headline shocks are again more than enough to explain the lower than predicted profit margins.

Table 19: Direct Impact Quantification Part 2

	Wage	**Real Wage**	**Profit**
Avg 2019	2.97%	0.91%	13%
2022Q2	5.92%	-4.16%	20%
Δ	2.95pp	-5.07pp	7pp
Predicted Δ	1.23pp	-1.52pp	10pp

A.5 Historical Perspective: 1960 - 2023

I re-do the analysis from Benigno and Eggertsson (2023), but control for the wage growth they claim is driving the non-linear rise in inflation. Benigno and Eggertsson (2023) perform this analysis for the sub-periods 1960-1969, 1970-1987, 1987-2008, and 2008-2022. I simplify their approach by focusing on the periods 1960-1969, 1970-1985, and 1985-2023. I estimate the following equation:

$$\pi_t = \alpha log(V/U) + \beta log(V/U)\mathbb{1}_{V/U>1} + \gamma X_t + e_t \tag{5}$$

where X_t is a vector of controls including current and lagged wage growth, energy inflation and labor productivity.

The direct impact is significant across all periods, except for the 1970s, where the major oil price shocks caused a decline in labor market tightness alongside rising inflation. Furthermore, I test the wage growth hypothesis of Benigno and Eggertsson (2023), which is at best linear across all sub-periods. I also examine real wage growth (ω) and profit margins (PMs). Real wage growth falls non-linearly with labor market tightness in all sub-periods except for the 1970s. As for Profit Margins, they fail to exhibit the non-linear rise in the pre-1985 periods. This failure is likely due to the oil price shocks in the 1970s. However, I am still unsure why PMs fail to rise in the 1960s, despite the hot labor markets.

Finally, my 1985-2023 results hold when estimating them over the entire period 1960-2023. This is true whether using a threshold model (Benigno and Eggertsson (2023)) or my quadratic and cubic specifications from previous sections. However, the threshold estimation of real wage growth is only significant at the 10% level. Therefore, the quadratic specification is preferred.

In Table 20 I present the results of equation 5 for the 1960Q1 - 1969Q4 period. The steepening of the price Phillips curve remains even when controlling for current and lagged wage growth, labor productivity and energy inflation.

Table 21 shows that in 1960Q1 - 1969Q4 real wage growth and even nominal wage growth were suffering during the especially tight labor market. The results on profit margins are insignificant.

In Table 22 I present the results of equation 5 for the 1970Q1 - 1984Q4 period. During the 70s there is no detectable steepening of the price Phillips curve.

Table 23 shows that in 1970Q1 - 1984Q4 real and nominal wage growth were not suffering during tight labor markets. However, profit margins were suffering during the 70s as high energy cost and wage indexing hit profits.

In Table 24 I present the results of equation 5 for the 1985Q1 - 2023Q4 period. The table presents a robustness check to the threshold model of the results presented before.

Table 25 presents a robustness check of my 1985Q1 - 2023Q4 results to using a threshold model. Wages are still failing non-linearity, while real wage growth falls and profit margins soar when labor markets get very tight.

In Table 26 I present the results of equation 5 for the entire period: 1960Q1 - 2023Q4. Due to the immense energy shocks in the 70s I control for raw CPI energy inflation instead of the

Table 20: V/U Direct Impact 1960-1969

	CPI	XFE	Median
log(V/U)	0.54	-0.03	0.09
	(0.36)	(0.53)	(1.01)
$V/U \geq 1$	3.68**	6.50***	7.58**
	(1.67)	(2.04)	(3.65)
π^w	-0.01	0.11	0.11*
	(0.11)	(0.07)	(0.06)
L.π^w	0.09	-0.10	0.07
	(0.13)	(0.07)	(0.09)
π^e	0.78***	0.82***	0.99***
	(0.10)	(0.08)	(0.12)
Energy	-0.05	-0.11**	0.03
	(0.08)	(0.05)	(0.06)
L.Energy	-0.02	-0.03	0.03
	(0.04)	(0.05)	(0.03)
Prod	-0.24*	-0.07	-0.28
	(0.14)	(0.08)	(0.17)
Constant	1.15**	0.42	-0.24
	(0.52)	(0.55)	(1.25)
R^2	0.64	0.75	0.76
N	40	40	40

Newey-West standard errors in parentheses

* $p < .10$, ** $p < .05$, *** $p < .01$

difference between energy inflation and headline inflation. Due to likely regime changes I also shorten the length of the Newey-West SE autocorrelation to 4 quarters.

My previous findings remain mostly the same, with median inflation's non-linearity only significant at the 10% level, but core and headline inflation still providing significant evidence for the non-linearity of the direct impact of V/U on inflation.

Table 27 presents the wage, real wage and profit margin results on the entire 1960-2023 period. Wage growth remains linear at best, while real wage growth suffers under very tight labor markets alongside rising profit margins.

Table 21: V/U Wage and Profit Impact 1960-1969

	W	ω	PMs
log(V/U)	0.69	0.06	0.04
	(0.47)	(0.70)	(0.05)
$V/U \geq 1$	-5.66*	-5.97**	-0.16
	(3.13)	(2.68)	(0.29)
π	0.17		
	(0.31)		
π	0.54**		
	(0.26)		
π^e	0.29	0.73	
	(0.49)	(0.71)	
Prod	0.19	0.25	0.04**
	(0.23)	(0.22)	(0.02)
Energy	-0.23**	-0.24**	-0.03*
	(0.09)	(0.09)	(0.02)
L.Energy	-0.08	-0.05	-0.01
	(0.09)	(0.11)	(0.01)
L4.ω		-0.21***	
		(0.07)	
W			0.01
			(0.01)
L.W			-0.02
			(0.02)
Constant	2.04**	1.33	0.01
	(0.85)	(1.56)	(0.10)
R^2	0.27	0.28	0.41
N	40	39	40

Newey-West standard errors in parentheses

* $p < .10$, ** $p < .05$, *** $p < .01$

Table 22: V/U Direct Impact 1970-1984

	CPI	XFE	Median
log(V/U)	3.88***	2.61*	3.54***
	(0.94)	(1.37)	(1.09)
$V/U \geq 1$	-19.57	-18.86	-19.13
	(13.23)	(15.68)	(13.74)
π^w	0.13	0.09	0.10
	(0.12)	(0.13)	(0.14)
L.π^w	0.10	0.06	0.10
	(0.13)	(0.16)	(0.14)
π^e	0.76***	0.77***	0.70***
	(0.22)	(0.26)	(0.24)
Energy	0.11***	0.02	0.05***
	(0.02)	(0.01)	(0.02)
L.Energy	-0.02	0.01	-0.03
	(0.03)	(0.03)	(0.03)
Prod	-0.59***	-0.82***	-0.87***
	(0.17)	(0.30)	(0.24)
Constant	4.52***	4.16***	4.60***
	(1.02)	(1.33)	(1.04)
R^2	0.74	0.56	0.62
N	60	60	60
Nixon Controls	✓	✓	✓

Newey-West standard errors in parentheses

* $p < .10$, ** $p < .05$, *** $p < .01$

A.6 Evidence on Wage Growth

This section provides evidence against the non-linearity of the underlying wage Phillips curve relation, once inflation and expected inflation are controlled for. It concludes that the direct impact is the sole driver of the Phillips curve non-linearity found by in the literature (Benigno and Eggertsson (2023), Gitti (2024)). I estimate a wage equation similar to my price equation, by simply switching the role of inflation and wage growth as dependent and independent variables. This is a slight divergence from Bernanke and Blanchard (2023) as they only use controls for past inflation. However, leaving out contemporary inflation has

Table 23: V/U Wage and Profit Impact 1970-1984

	W	ω	PMs
log(V/U)	0.70	-2.71***	-0.10
	(0.79)	(0.59)	(0.12)
$V/U \geq 1$	-7.16	-23.41	-5.68**
	(13.31)	(18.17)	(2.36)
π	0.09		
	(0.17)		
L.π	0.15*		
	(0.09)		
π^e	0.72***	-0.54***	
	(0.15)	(0.20)	
Energy	-0.03	-0.11***	-0.00
	(0.03)	(0.02)	(0.00)
L.Energy	0.01	0.04	-0.01*
	(0.03)	(0.04)	(0.00)
Prod	0.25	0.38*	0.00
	(0.29)	(0.22)	(0.03)
L4.ω		-0.01	
		(0.10)	
π^w			0.02
			(0.02)
L.π^w			-0.01
			(0.02)
Constant	0.81	0.45	0.05
	(0.94)	(1.52)	(0.20)
R^2	0.34	0.62	0.17
N	60	60	60
Nixon Controls	✓	✓	✓

Newey-West standard errors in parentheses

* $p < .10$, ** $p < .05$, *** $p < .01$

Table 24: V/U Direct Impact

	CPI	XFE	Median
log(V/U)	0.53*	0.65**	0.25
	(0.27)	(0.25)	(0.17)
$V/U \geq 1$	3.96***	3.95***	3.75***
	(0.74)	(0.64)	(0.55)
π^w	0.05	0.02	0.16***
	(0.07)	(0.06)	(0.05)
L.π^w	0.10	-0.05	0.19***
	(0.08)	(0.07)	(0.06)
π^e	0.83***	1.03***	0.65***
	(0.11)	(0.09)	(0.08)
Energy	0.10***	0.01	0.00
	(0.01)	(0.01)	(0.00)
L.Energy	0.00	0.00	-0.00
	(0.01)	(0.00)	(0.00)
Prod	-0.10	-0.02	-0.19***
	(0.09)	(0.09)	(0.05)
RIMP	0.27	-0.14	-0.10
	(0.36)	(0.22)	(0.14)
L.RIMP	0.17	0.13	0.15
	(0.27)	(0.25)	(0.15)
Constant	0.19	0.18	0.42***
	(0.26)	(0.20)	(0.16)
R^2	0.83	0.50	0.79
N	154	154	154

Newey-West standard errors in parentheses

* $p < .10$, ** $p < .05$, *** $p < .01$

no influence on the results.

The wage equation:

$$\pi_t^w = \tilde{\phi}_1 \pi_t + \tilde{\phi}_2 \pi_{t-1} + \tilde{\alpha}_1 (V/U)_t + \tilde{\alpha}_2 (V/U)_t^2 + \tilde{\alpha}_3 (V/U)_t^3 + \tilde{\delta} \pi_t^e + \tilde{\rho} X_t + c_{\pi^w} + \tilde{\epsilon}_t \quad (6)$$

where π^w is wage growth, π inflation, V/U labor market tightness, π^e 10-year ahead expected

Table 25: V/U Wage and Profit Impact 1985 - 2023

	W	ω	PMs
log(V/U)	1.11***	0.59***	-0.19*
	(0.18)	(0.21)	(0.11)
$V/U \geq 1$	1.31*	-3.03***	1.02**
	(0.70)	(0.53)	(0.41)
π	0.01		
	(0.05)		
L.π	0.14		
	(0.09)		
Energy	0.01	-0.10***	0.01*
	(0.01)	(0.01)	(0.01)
L.Energy	-0.01	-0.01	0.00
	(0.01)	(0.01)	(0.01)
Prod	0.22***	0.21***	0.09
	(0.05)	(0.07)	(0.06)
π^e	0.42***	-0.64***	
	(0.09)	(0.12)	
L.ω		-0.01	
		(0.02)	
π^w			-0.03
			(0.07)
L.π^w			-0.07
			(0.05)
Constant	0.38*	2.34***	0.05
	(0.21)	(0.39)	(0.14)
R^2	0.50	0.70	0.12
N	154	154	154

Newey-West standard errors in parentheses

* $p < .10$, ** $p < .05$, *** $p < .01$

inflation, $\tilde{\epsilon}_t$ is an error term, and X_t is the same vector of controls containing an eight quarter moving average of labor productivity growth, current and lagged energy inflation, as well

Table 26: V/U Direct Impact 1960-2023

	CPI	XFE	Median
log(V/U)	1.06***	0.86***	1.05***
	(0.28)	(0.28)	(0.29)
$V/U \geq 1$	2.12**	2.39**	1.93*
	(0.93)	(1.15)	(1.08)
π^w	0.07***	0.04	0.07**
	(0.03)	(0.04)	(0.03)
L.π^w	0.06**	0.06**	0.04
	(0.03)	(0.03)	(0.03)
π^e	0.87***	0.90***	0.89***
	(0.05)	(0.08)	(0.12)
Energy	0.10***	0.02***	0.02***
	(0.01)	(0.01)	(0.01)
L.Energy	0.01	0.01*	0.01
	(0.01)	(0.01)	(0.01)
Prod	-0.16*	-0.26**	-0.43***
	(0.10)	(0.12)	(0.11)
Constant	0.55*	1.04***	1.44***
	(0.30)	(0.36)	(0.35)
R^2	0.76	0.46	0.51
N	254	254	254
Nixon Controls	✓	✓	✓

Newey-West standard errors in parentheses

* $p < .10$, ** $p < .05$, *** $p < .01$

as relative import price pressures as in Yellen (2015). The wage equation obeys the following homogeneity constraint:

$$\tilde{\phi}_1 + \tilde{\phi}_2 + \tilde{\delta} = 1$$

Table 28 explores the possibility of a non-linear wage Phillips curve as found by part of the literature (Nalewaik (2016)). I estimate the wage Phillips curve on US aggregate data from 1985Q1-2023Q4 for a one-to-one comparison with the non-linearity of the price equation. I measure wage growth using the Employment Cost Index (ECI) for wages and salaries in Table 28.

Model 1 estimates the cubic form of the wage equation while model 2 estimates a linear

Table 27: V/U Wage and Profit Impact 1960-2023

	W	ω	PMs
log(V/U)	0.59	-0.28	-0.34***
	(0.46)	(0.51)	(0.08)
$V/U \geq 1$	-1.45	-3.33*	0.84***
	(1.43)	(1.69)	(0.32)
π	0.13		
	(0.10)		
L.π	0.34*		
	(0.18)		
π^e	0.42***	-0.41***	
	(0.13)	(0.10)	
Prod	0.73***	0.62***	0.07**
	(0.22)	(0.23)	(0.03)
Energy	-0.04*	-0.13***	0.01*
	(0.03)	(0.03)	(0.01)
L.Energy	-0.01	0.01	0.00
	(0.01)	(0.01)	(0.00)
L4.ω		-0.02	
		(0.06)	
W			-0.03**
			(0.01)
L.W			0.05
			(0.03)
Constant	-0.13	0.83	-0.42**
	(0.62)	(0.67)	(0.19)
R^2	0.14	0.36	0.19
N	254	253	254
Nixon Controls	✓	✓	✓

Newey-West standard errors in parentheses

* $p < .10$, ** $p < .05$, *** $p < .01$

Table 28: Impact on Wage Growth

	Cubic	Linear
V/U	3.03^{*}	1.73^{***}
	(1.83)	(0.26)
$(V/U)^2$	-0.99	
	(2.10)	
$(V/U)^3$	0.13	
	(0.73)	
π	0.06	0.03
	(0.06)	(0.06)
L.π	0.21^{*}	0.17^{*}
	(0.11)	(0.09)
π^e	0.74^{***}	0.80^{***}
	(0.11)	(0.09)
Prod	0.24^{***}	0.24^{***}
	(0.05)	(0.06)
Constant	-1.84^{***}	-1.48^{***}
	(0.45)	(0.26)
R^2	0.53	0.52
N	156	156
No Impact (p-value)	0.0000	0.0000
Linear Impact (p-value)	0.1551	$-$
Partial R^2	0.24	0.22
Controls	✓	✓

Newey-West standard errors in parentheses

* $p < .10$, ** $p < .05$, *** $p < .01$

Notes: "Cubic" estimates Equation 6 while "Linear" reduces the functional form of V/U to a linear form. Both are estimated on US quarterly data from 1985Q1 - 2023Q4. The "No Impact" Wald test has the hypothesis of all V/U coefficients equal to zero. The "Linear Impact" tests the higher order V/U coefficients being zero. Partial R^2 reports the variation explained by V/U and its higher order terms.

version of the same. I omit reporting the coefficients on the control variables besides labor productivity. The estimates yield no evidence for non-linearity. The higher order terms are close to zero and the Wald Test ("Linear Impact") fails to reject their joint insignificance. Hence the non-linearity found by Ball et al. (2022), Benigno and Eggertsson (2023) and others seems entirely driven by the direct impact.[28]

However, labor market tightness exerts a significant influence on wage growth, as the Wald test on the "No Impact"-Hypothesis shows. The direct impact channel and the conventional Phillips curve channel are thus complementary rather than mutually exclusive. However, the impact of labor market tightness on wage growth is linear in nature and fails to explain the steep rise of inflation during hot labor markets.

The results are robust to alternative wage measures[29]. All measures fail to give significant non-linearity, except for Average Hourly Earnings (AHE), which features a significant but negative third order labor market tightness coefficient. The negative non-linearity is likely due to composition effects, which cause the AHE infamously to spike upwards during sizeable labor market downturns. Furthermore, the results of Table 28 are robust to limiting the estimation period to pre-pandemic and winsorizing the data. Finally, I exclude the contemporaneous inflation control to match the lag structure of Bernanke and Blanchard (2023). The coefficients and the verdict against non-linearity are essentially unchanged by these robustness checks.

B Additional MSA Results

B.1 Additional Quarterly MSA Results

This appendix provides a robustness check using twelve lags as well as current wage growth as controls in the IV estimated price Phillips curve.

B.2 Successive Fixed Effects

In this appendix I present results of step-wise introduction of fixed effects into the IV estimations on headline (Table 30) and core inflation (Table 31)

[28] Some contributions to the literature (Nalewaik (2016), Smith et al. (2023)) find a non-linear wage Phillips curve. These papers do not control for price inflation and expected inflation however. Thus, they are likely picking up the non-linearity of the price Phillips curve, which affects wage growth due to a desire to preserve real wages. To test this I exclude the controls for inflation and inflation expectations. The wage Phillips curve is significantly non-linear when excluding these controls. This explains non-linear wage Phillips curve estimations in the literature. However, once these controls are included, the wage Phillips curve is clearly linear in nature. Hence, the direct impact is the source of the non-linear Phillips curve as found by Ball et al. (2022) and others.

[29] Average Hourly Earnings, Average Hourly Compensation, Unit Labor Cost, and ECI Total Compensation.

Table 29: Direct Impact on MSA Data

	(1) Core	(2) Headline
$\log(\theta)$	2.25**	2.04**
	(0.975)	(0.865)
$I_{\theta_{i,t}>1} \times \log(\theta)$	5.78**	4.23**
	(2.439)	(2.036)
L4.log(PPI/CPI)	15.91***	15.62***
	(2.717)	(2.464)
Wage Controls L0-L4	✓	✓
Observations	1334	1334
MSA FEs	✓	✓
Year-Quarter FEs	✓	✓
MSA × Year-Quarter FEs	✓	✓

Driscoll-Kraay standard errors in parentheses

* (p<0.10), ** (p<0.05), *** (p<0.01)

Table 30: Step-by-step FEs on Headline CPI

	(1) π	(2) π	(3) π
$\log(\theta)$	0.17	1.86**	1.38
	(0.388)	(0.812)	(0.871)
$I_{\theta_{i,t}>1} \times \log(\theta)$	10.00***	9.54**	4.65**
	(1.638)	(4.190)	(2.294)
L4.log(PPI/CPI)	-5.98***	3.18	15.34***
	(1.627)	(2.885)	(2.243)
π^w	0.04	0.07***	0.05**
	(0.031)	(0.018)	(0.018)
Observations	1394	1394	1394
MSA FEs	✓	✓	✓
Year-Quarter FEs		✓	✓
MSA × Year-Quarter FEs			✓

Driscoll-Kraay standard errors in parentheses

* (p<0.10), ** (p<0.05), *** (p<0.01)

Table 31: Step-by-step FEs on Core CPI

	(1)	(2)	(3)
	π	π	π
$\log(\theta)$	0.61***	2.42**	1.79*
	(0.118)	(1.010)	(0.951)
$I_{\theta_{i,t}>1} \times \log(\theta)$	5.45***	13.40**	6.58**
	(0.636)	(5.604)	(2.715)
L4.log(PPI/CPI)	3.27***	3.74	15.73***
	(0.777)	(3.032)	(2.448)
π^w	0.02	0.05**	0.03*
	(0.015)	(0.022)	(0.018)
Observations	1394	1394	1394
MSA FEs	✓	✓	✓
Year-Quarter FEs		✓	✓
MSA $\times$ Year-Quarter FEs			✓

Driscoll-Kraay standard errors in parentheses

* (p<0.10), ** (p<0.05), *** (p<0.01)

B.3 Monthly MSA Results

I present the monthly MSA estimation results in this appendix.

The Bureau of Labor Statistics provides data on average weekly wages, the Consumer Price Index (CPI), and unemployment rates for 21 major Metropolitan Statistical Areas (MSAs), while job vacancy data are only available at the state level. To address this limitation, I adopt the approach outlined by Gitti (2024) for imputing MSA-level vacancy data. Specifically, I calculate each MSA's share of state job vacancies based on its population relative to the total state population and aggregate these shares for MSAs that span state borders.

However, this imputation relies on the assumption of a homogeneous vacancy-to-employment ratio within each state, which may not accurately reflect the variability that exists across counties. This assumption introduces potential measurement error. To mitigate the impact of this error, I will employ an instrumental variables approach.

CPI data is available at a monthly frequency only for three major MSAs, while the remaining MSAs are sampled bi-monthly. Average weekly wages are collected quarterly for all MSAs. To optimize the use of monthly labor market data, I interpolate the missing monthly values for both the CPI and average weekly wages.

The classical measurement error in inflation introduced by the interpolation should not affect my estimates of the price Phillips curve. Similarly, for average weekly wages and the wage Phillips curve, the interpolation should not have a significant impact. However, the measurement error in weekly wages may attenuate my direct impact estimation, as it represents a classical measurement error in the control variable. To address this attenuation bias, I demonstrate in Appendix **??** that the results remain robust when using quarterly MSA data.

Finally, I use year over year inflation and wage growth to reduce measurement error and get rid of seasonality in the data.

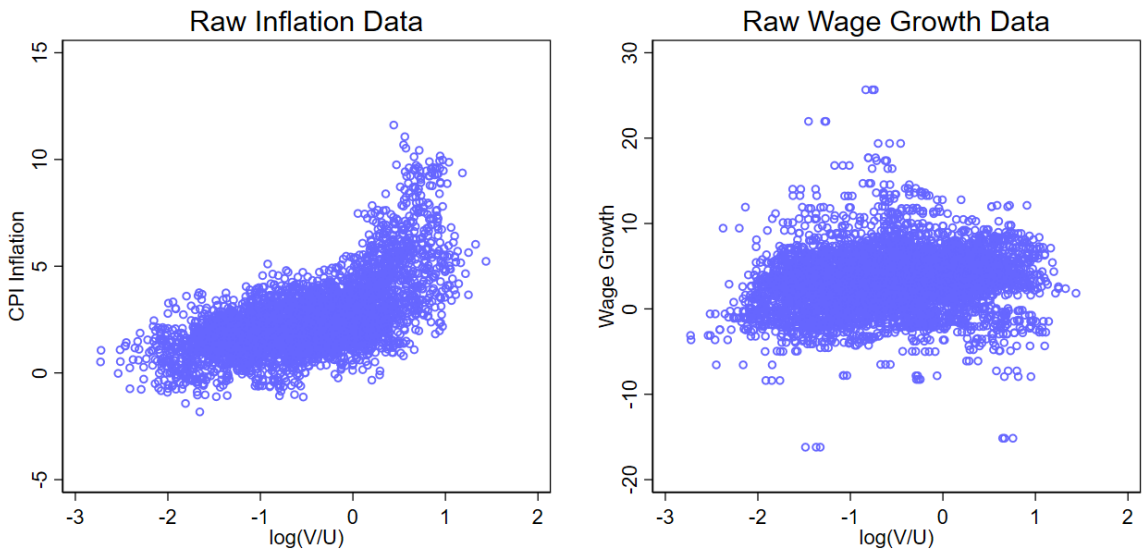
I present the raw correlations between inflation and labor market tightness in the MSA data from January 2001 to August 2024 in Figure 21. Additionally, I display the correlations for MSA wage growth and plot the fitted cubic relationship for comparison with my aggregate results.

The non-linear relation between inflation and labor market tightness is readily apparent in raw MSA data. In contrast, wage growth, the expected conduit of labor market pressure onto prices, shows a flat Phillips curve in the raw data.

B.4 Ordinary Least Squares Estimation

I present basic Ordinary Least Squares (OLS) estimations with step-wise MSA and time fixed effects for both inflation (π) and wage growth (π^w) in Table 32, where $\theta = V/U$. Following the literature, I use a threshold of $V/U = 1$ to assess Phillips curve non-linearity. The raw data indicates that the price Phillips curve is non-linear, while the wage Phillips curve appears to be essentially flat.

Including controls for wage growth in the price Phillips curve yields results that are essentially the same as those from OLS estimation without these controls. Specifically, the



(a) CPI Core Inflation

Figure 21: Raw Data Correlations

Subfigure one scatters MSA core inflation against MSA $\log(V/U)$. Subfigure two does the same for wage growth.

impact of labor market tightness on inflation remains significant and non-linear, even when accounting for the traditional wage channel. Similar findings are also observed when using MSA data at the quarterly frequency, as outlined in Appendix **??**.

B.5 Benefits of Regional Data

Figure 21 highlights the advantages of utilizing regional data, as it not only increases the overall number of observations but also enhances the variation in labor market tightness. In the MSA data 24% of monthly labor markets are at or above $V/U = 1$, while only 17% of aggregate US data make that threshold. This provides a greater opportunity to observe inflation dynamics under tight labor markets.

Furthermore, regional data allows for the inclusion of time fixed effects, which help address various endogeneity issues, including those related to monetary and fiscal policy (Fitzgerald et al. (2014), McLeay and Tenreyro (2020)), aggregate inflation expectations (Hazell et al. (2022)), and confounding factors such as aggregate supply shocks.

This leaves regional supply shocks as potential confounding factors in the estimation of the Phillips curve. The literature addresses this issue by utilizing shift-share instruments, as proposed by Bartik (1991). Notably, Hazell et al. (2022), Gitti (2024), and Beaudry et al. (2025) all employ similar instruments in their analyses.

The shift-share approach leverages the variation in regional exposure to employment trends within the tradable goods or intermediate input sectors. A region that has a higher share of employment in a specific tradable sector will experience a tightening of its labor market relative to other regions in response to positive labor demand shocks in that sector.

Table 32: OLS Direct Impact Estimation

	π	π	π^w	π^w
$\log(\theta)$	0.73***	0.55***	1.05***	0.38
	(0.076)	(0.119)	(0.124)	(0.230)
$I_{\theta_{i,t}>1} \times \log(\theta)$	4.28***	0.48*	-1.61***	0.18
	(0.321)	(0.256)	(0.483)	(0.462)
π^w	0.06***	0.05***		
	(0.012)	(0.016)		
Cons	2.37***	2.69***	3.82***	3.29***
	(0.081)	(0.096)	(0.110)	(0.151)
Observations	4398	4398	6788	6788
MSA FEs	✓	✓	✓	✓
Year-Month FEs		✓		✓
MSA × Year-Month FEs		✓		✓

MSA-Year-Quarter clustered standard errors in parentheses

* (p<0.10), ** (p<0.05), *** (p<0.01)

For example, an aggregate increase in demand for processed grains will have a more pronounced impact on the Minneapolis-St. Paul area than on Pittsburgh, whereas an increase in demand for steel will have the opposite affect. Similarly, a rise in the price of oil will result in a greater increase in labor demand in Houston compared to other metropolitan areas.

The employment trend based shift-share instrumental variable is

$$\text{Shift Share}_{l,t} = \sum_i \bar{S}_{i,l} \times \Delta_{3YR} \log(E_{i,t})$$

where $\bar{S}_{i,l}$ is the average employment share of industry i in locality l, and $\Delta_{3YR} \log(E_{i,t})$ is the three year growth rate of national employment of industry i.

This requires average 2-digit NAICS industry employment shares for each Metropolitan Statistical Area (MSA), which I obtain from the Census Bureau, along with national employment data by industry sourced from the Current Population Survey.

The industries utilized in this instrument vary slightly among studies. Hazell et al. (2022) and Beaudry et al. (2025) primarily focus on the tradable goods sectors, while Gitti (2024) incorporates additional intermediate inputs such as wholesale trade, as well as the intermediate components of transportation, warehousing, utilities, and professional and business services.

For the shift-share instrumental variable to function properly, local supply shocks should not correlate with both the employment shifts $\Delta_{3YR} \log(E_{i,t})$ and the shares $S_{i,l}$ in the cross-section. In other words, while local supply shocks may influence an MSA's employment composition, they should not impact national employment trends in the affected industry. Finally, the instrumental variable approach helps address the classical measurement error present in regional labor market data.

B.6 Instrumental Variables Approach

I generalize the existing shift-share IV approach in its tradable goods exposure variation. I incorporate the relative price of tradable goods across MSAs and time ($\frac{PPI_t}{CPI_{l,t}}$) as an instrument, following Hazell et al. (2022) and Gitti (2024). Finally, I replace the interaction of the shift-share instrument with the tight labor market indicator used by both Beaudry et al. (2025) and Gitti (2024) with a quadratic term of the instrument. The indicator of tight local labor markets violates the exclusion restriction as it is affected by local supply shocks. The quadratic term of the shift-share instrument avoids this pitfall by relying exclusively on the exogenous variation provided by the shift-share IV.

Table 33: First Stage Regression

	$\log(\theta)$	$I_{\theta>1} \times \log(\theta)$
$IV_{l,t}$	5.05***	-0.06
	(1.076)	(0.550)
$IV_{l,t}^2$	-8.76	12.60**
	(9.724)	(6.172)
$L24.log(PPI/CPI)$	3.73***	1.82***
	(0.645)	(0.360)
$L12.log(PPI/CPI)$	-2.11***	-1.33***
	(0.657)	(0.345)
π^w	0.00	0.00
	(0.002)	(0.001)
CD Wald F	30.42	30.42
SW S stat.	24.52	24.52
Observations	4146	4146
MSA FEs	✓	✓
Year-Month FEs	✓	✓
MSA × Year-Month FEs	✓	✓

MSA-Quarter clustered standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.010$

I control for MSA and time fixed effects to account for stable regional labor market differences, as well as aggregate effects such as monetary policy or energy price shocks. Additionally, I incorporate the interaction of MSA and time fixed effects to capture the differential impact of common shocks across MSAs. These additional fixed effects help mitigate the bias outlined by Borusyak et al. (2022).

Putting all of this together, the identification strategy is valid if, after conditioning on fixed effects, employment growth in tradables and their relative price are uncorrelated with local labor supply shocks that could otherwise confound the estimates of labor demand. Contemporaneous relative prices are affected by local supply shocks. Thus, I use two-year lagged relative prices and add one-year lagged relative prices as controls for recent supply

shocks.

The instruments are quite strong, as indicated by both traditional and Cragg and Donald (1993) F-values (CD Wald F), as well as the Lagrange Multiplier Sargan statistic proposed by Stock and Wright (2000) (SW S stat.).

B.7 Direct Impact Estimation

In Table 34, I present the estimated direct impacts on MSA data from the instrumental variables approach. To this end, I include local wage growth as a control variable and account for the price level from twelve months ago to capture existing price pressures influencing the year-over-year inflation estimate. The first column presents estimates for headline CPI inflation, while the second column presents those on core CPI inflation.[30]

Table 34: Direct Impact on MSA Data

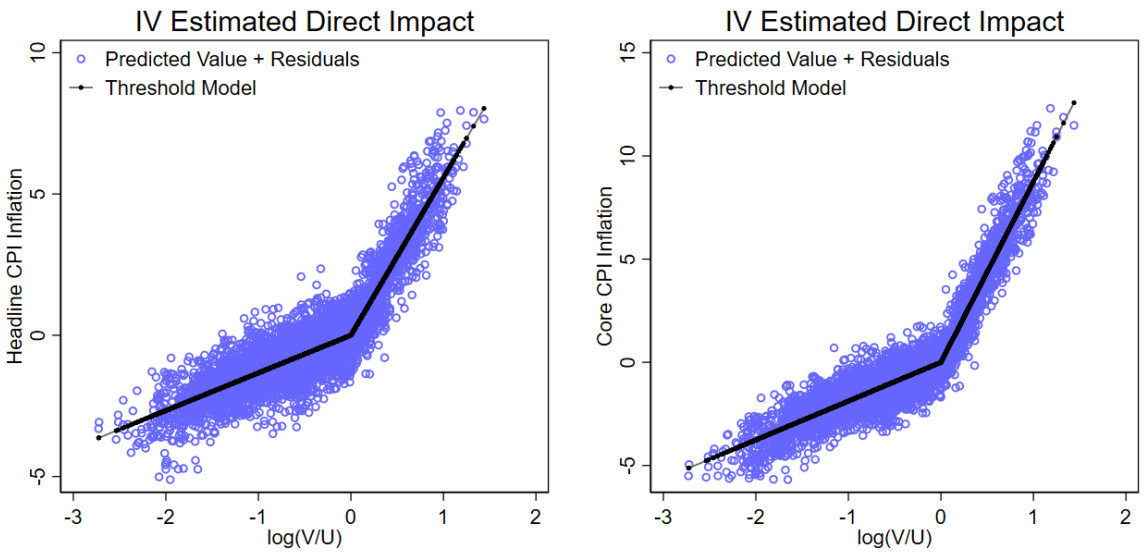
	CPI	Core
$\log(\theta)$	1.33**	1.88**
	(0.637)	(0.800)
$I_{\theta>1} \times \log(\theta)$	4.25**	6.86***
	(1.664)	(2.119)
$L12.log(PPI/CPI)$	15.81***	15.29***
	(1.798)	(2.386)
π^w	0.04***	0.03
	(0.017)	(0.019)
Observations	4146	4146
MSA FEs	✓	✓
Year-Month FEs	✓	✓
MSA × Year-Month FEs	✓	✓

MSA-Year-Quarter clustered standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.010$

I explore how the estimations change under different fixed effects in Appendix B.9 by successively introducing the three fixed effects discussed earlier. Additionally, I repeat the analysis from Table 34 using twelve lags of wage growth in Appendix B.10. Finally, I replicate Table 1 of Gitti (2024) while including wage growth as a control variable. The results remain consistent across these analyses. I present the functional forms plotted against the residuals from the fixed effects and V/U, along with the IV-predicted labor market impact, in Figure 22.

[30] A final difference to the approaches of Beaudry et al. (2025) and Gitti (2024) is that I do not control for the indicator of labor market tightness. Including this indicator results in a noticeable jump in the Phillips curve relationship, which is a peculiar choice. When included, the indicator is generally negative, although rarely significant, and it increases the interaction coefficient.



(a) Direct Impact on Headline Inflation

(b) Direct Impact on Core Inflation

Figure 22: Direct impact on MSA Data

Subfigure a) shows the functional form for headline CPI inflation against IV predicted data plus inflation residuals from controls and fixed effects. Subfigure b) does the same for core CPI inflation.

B.8 Wage Phillips curve Estimation

Since most arguments for a non-linear Phillips curve are tied to wage growth (Benigno and Eggertsson (2024), Gitti (2024)), it is worthwhile to test this hypothesis using a similar estimation approach.

The employment trend-based shift-share instrument was designed to isolate local labor demand pressures, which are expected to translate into higher wage growth. However, only after an additional step, the wage to price pass-through, does labor market pressure translate into inflation under the traditional understanding of the Phillips curve. Therefore, under the conventional Phillips curve framework employed by Hazell et al. (2022) and Gitti (2024), the instrument should be even better suited for estimating the wage Phillips curve than the price Phillips curve.

I test the wage Phillips curve by promoting wage growth from a control variable to the variable of interest. The results of the wage growth estimation are presented in Table 35, where $L12.log(AvgWW)$ denotes the log level of local average weekly wages.

The IV-estimated wage Phillips curve provides no evidence of non-linearity, reinforcing the findings observed in aggregate data. I present the resulting estimated functional form in Figure 23.

I present the estimated wage and price Phillips curves together in Figure 24. The comparison shows that the wage Phillips curve (WPC) is steeper than the price Phillips curve (PPC) under slack to moderately tight labor markets. However, the PPC steepens in excess of the WPC under very tight labor markets leading to real wage growth declines.

Finally, I test the implication of a negative impact on real wage growth by subtracting

Table 35: MSA Wage Phillips Curve

	π^W
$\log(\theta)$	2.84*
	(1.670)
$I_{\theta>1} \times \log(\theta)$	0.12
	(5.987)
$L12.log(AvgWW)$	-0.16***
	(0.029)
Observations	4177
MSA FEs	✓
Year-Month FEs	✓
MSA × Year-Month FEs	✓

MSA-Year-Quarter clustered standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.010$

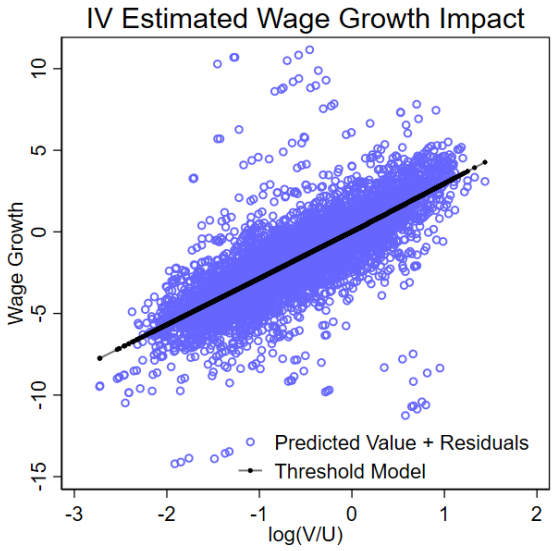
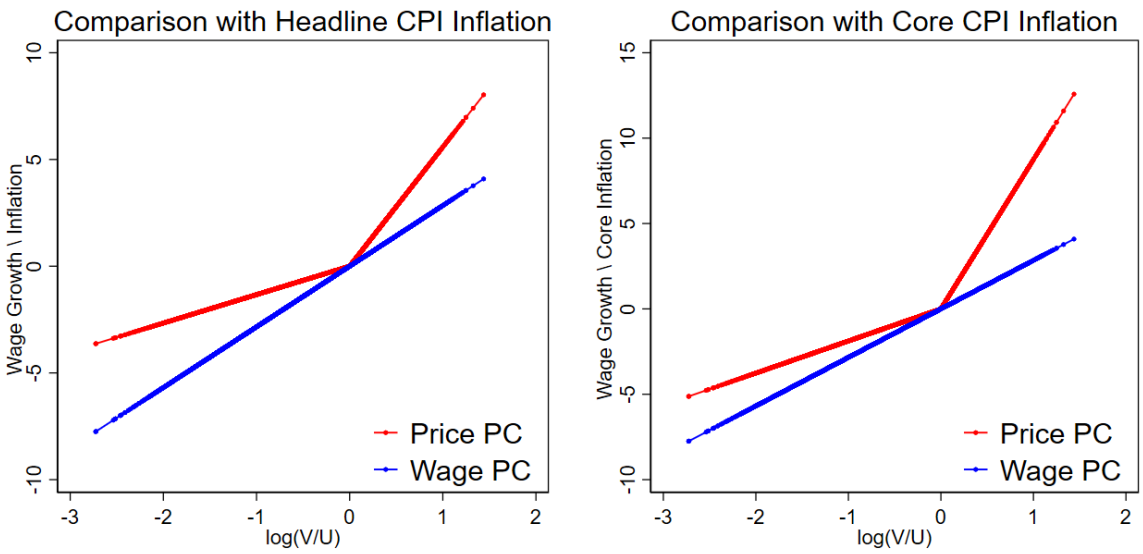


Figure 23: (Non-Linear) Wage Phillips Curve on MSA Avgerage Weekly Wages

year-over-year inflation from year-over-year wage growth to obtain an MSA measure of real wage growth. The results of the fixed effects regressions are presented in Table 36, while controlling for workers' catch-up bargaining from lost real wage growth twelve months ago. Under year-month fixed effects, the impact of labor market tightness is insignificant. However, under year-quarter fixed effects, which are preferred by Cerrato and Gitti (2022), the impact is fully significant and negative.



(a) PC Comparison on Headline

(b) PC Comparison on Core

Figure 24: Comparing Wage and Price Phillips curves on MSA Data

Subfigure a) shows the predicted wage and price Phillips curves using growth in average weekly wages and headline CPI inflation. Subfigure b) does the same using core CPI inflation.

Table 36: MSA Real Wage Impact

	Real π^w	Real π^w
$\log(\theta)$	2.87	0.62
	(1.786)	(0.733)
$I_{\theta_{i,t}>1} \times \log(\theta)$	-2.38	-2.65**
	(4.628)	(1.347)
L12.Real π^w	-0.19***	-0.18***
	(0.049)	(0.045)
$L12.log(PPI/CPI)$	-9.85*	-2.69
	(5.410)	(3.656)
Observations	4146	4146
MSA FEs	✓	✓
Year-Month FEs	✓	
MSA × Year-Month FEs	✓	
Year-Quarter FEs		✓
MSA × Year-Quarter FEs		✓

MSA-Year-Quarter clustered standard errors in parentheses

* (p<0.10), ** (p<0.05), *** (p<0.01)

B.9 IV Results with step-wise FEs introduction

Table 37 presents the IV estimation of the direct impact on headline inflation under different FEs. I introduce MSA, time and MSA time interaction FEs successively. Table 38 does the same on core CPI inflation.

Table 37: Step-by-step FEs on Headline CPI

	CPI	CPI	CPI
$\log(\theta)$	0.16	1.83***	1.33**
	(0.135)	(0.605)	(0.637)
$I_{\theta>1} \times \log(\theta)$	9.87***	8.86***	4.25**
	(0.734)	(3.369)	(1.664)
$L12.log(PPI/CPI)$	-6.00***	2.86	15.81***
	(0.740)	(2.421)	(1.798)
π^w	0.04***	0.06***	0.04***
	(0.014)	(0.020)	(0.017)
Observations	4146	4146	4146
MSA FEs	✓	✓	✓
Year-Month FEs		✓	✓
MSA × Year-Month FEs			✓

MSA-Year-Quarter clustered standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.010$

Table 38: Step-by-step FEs on Core CPI

	Core	Core	Core
$\log(\theta)$	0.60***	2.49***	1.88**
	(0.082)	(0.844)	(0.800)
$I_{\theta>1} \times \log(\theta)$	5.40***	13.81***	6.86***
	(0.514)	(4.612)	(2.119)
$L12.log(PPI/CPI)$	3.25***	2.89	15.29***
	(0.535)	(3.430)	(2.386)
π^w	0.02**	0.05*	0.03
	(0.010)	(0.026)	(0.019)
Observations	4146	4146	4146
MSA FEs	✓	✓	✓
Year-Month FEs		✓	✓
MSA × Year-Month FEs			✓

MSA-Year-Quarter clustered standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.010$

B.10 Robustness to further lags of Wage Growth

In this subsection I test the robustness of the direct impact estimation to the inclusion of further wage lags. The results are presented in Table 39, and are essentially the same as in the table of the body of the paper. I omit lag two to eleven to reign in table size. I am happy to provide the full table on request.

Table 39: Direct Impact on MSA Data

	Core	CPI
$\log(\theta)$	2.18**	1.65**
	(0.906)	(0.735)
$I_{\theta>1} \times \log(\theta)$	6.85***	4.32**
	(2.156)	(1.703)
$L12.log(PPI/CPI)$	14.80***	15.24***
	(2.515)	(1.922)
π^w	0.01	0.03*
	(0.018)	(0.016)
$L.\pi^w$	0.01	0.01
	(0.008)	(0.007)
⋮	⋮	⋮
$L12.\pi^w$	-0.03	-0.02
	(0.022)	(0.019)
Observations	4146	4146
MSA FEs	✓	✓
Year-Month FEs	✓	✓
MSA × Year-Quarter FEs	✓	✓

MSA-Year-Quarter clustered standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.010$

B.11 Gitti Table 1 Replication

Table 40: Gitti Table 1 Replication with Wage Controls

	IV	OLS	OLS
$\log(\theta)$	1.53**	0.78***	0.62***
	(0.666)	(0.082)	(0.108)
$I_{\theta>1} \times \log(\theta)$	3.70**	4.89***	0.51**
	(1.529)	(0.352)	(0.235)
$I_{\theta>1}$	-0.61***	-0.40***	-0.05
	(0.178)	(0.142)	(0.074)
$L12.log(PPI/CPI)$	15.75***	-2.77***	8.58***
	(1.791)	(0.517)	(1.138)
π^w	0.04***	0.04***	0.05***
	(0.016)	(0.012)	(0.014)
Constant		1.93***	4.46***
		(0.141)	(0.237)
Observations	4146	4398	4398
MSA FEs	✓	✓	✓
Year-Quarter FEs	✓		✓
MSA $\times$ Year-Quarter FEs	✓		✓

MSA-Year-Quarter clustered standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.010$

C Cross-Country Evidence

This section estimates the direct impact on cross-country panel data from 1985Q1 to 2024Q1. I use job vacancy, unemployment, employment cost, labor productivity, inflation expectations, market concentration, headline and core inflation for 24 developed countries.[31] I find evidence for the existence of the direct impact on both inflation and profit margin data, as well as support for the role of market concentration in the panel data. Furthermore, this paper is the first to test and provide evidence for a non-linear V/U based Phillips curve on a large panel of countries.

I estimate the following equation on headline and core inflation to test for the direct impact in the panel

$$\pi_{c,t} = \check{\alpha}_1(V/U)_{c,t} + \check{\alpha}_2(V/U)^2_{c,t} + \check{\alpha}_3(V/U)^3_{c,t} + \check{\beta}\pi^e_{c,t} + \check{\gamma}_1\pi^w_{c,t} + \check{\gamma}_2\pi^w_{c,t-1} + \check{\phi}\text{Prod}_{c,t} + \check{\delta}_c + \check{\delta}_t + \check{\epsilon}_{c,t} \quad (7)$$

where π is either headline consumer price or core (less food and energy (in some cases also less alcohol and tobacco)) inflation, V/U is the ratio of job openings to unemployed, π^e is the 5 year forward inflation prediction from the IMF's World Economic Outlook, π^w is nominal employment cost growth, Prod is an eight quarter moving average of labor productivity growth, and $\check{\epsilon}_{c,t}$ is the error term. I thus estimate a two-way fixed effects version of equation 1, with the exception of using a shorter inflation expectations horizon (5 vs 10 years), and omitting controls for energy inflation and relative import price pressures.

I test for the rejection of the following homogeneity constraint. I do this to test if common shifts in inflation expectations and time fixed effects are present, which may violate the homogeneity constraint.

$$1 = \check{\beta} + \check{\gamma}_1 + \check{\gamma}_2$$

The homogeneity constraint is not rejected by the Wald test.[32] Thus, I impose the homogeneity constraint on the TWFEs estimation. The labor market tightness coefficients are almost unchanged by the constraint.

The two-way fixed effects allow for a cleaner identification of the channel as time fixed effects filter out aggregate variables such as energy inflation, cost of global imports, global demand and supply shocks. Furthermore, the country fixed effects control for most social and policy differences across countries. Finally, clustered standard errors at time and country level control for serial correlation of the error term.

I present the estimation results in Table 41. The panel data provides evidence for the existence of the direct impact outside the US. The joint insignificance of the labor market tightness coefficients is rejected by the Wald test for both headline and core inflation. The joint insignificance of the quadratic and cubic terms is further rejected for core inflation. The same hypothesis is only rejected at the 10% confidence level for headline inflation.

[31] Australia, Canada, Belgium, Germany, Estonia, Ireland, Spain, France, Cyprus, Latvia, Lithuania, Luxembourg, Malta, Netherlands, Austria, Poland, Portugal, Slovenia, Slovakia, Finland, Great Britain, Switzerland, Greece, and Japan

[32] The p-value is 0.46 for headline, and 0.53 for core inflation.

Table 41: Cross-Country Direct Impact Estimation

	π	Core
(V/U)	7.17**	7.37**
	(3.30)	(2.82)
$(V/U)^2$	-12.24**	-11.82***
	(4.65)	(3.98)
$(V/U)^3$	5.11**	4.82***
	(1.93)	(1.63)
π^w	0.01	0.02*
	(0.01)	(0.01)
L.π^w	0.01	0.01
	(0.01)	(0.01)
5Y π^e	0.98***	0.97***
	(0.02)	(0.02)
Prod	0.07	0.03
	(0.08)	(0.04)
R^2	0.69	0.67
N	1412	1412
No Impact p-value	0.0226	0.0423
Linear p-value	0.0593	0.0208

Two-way clustered standard errors in parentheses

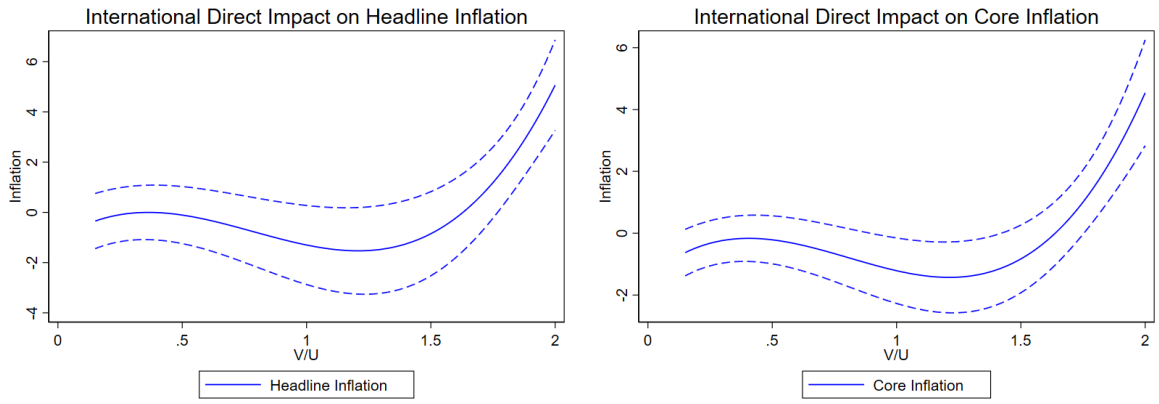
* $p < .10$, ** $p < .05$, *** $p < .01$

The direct impact clearly shows up on the cross-section of developed countries, without even including the US data. These results further bolster the growing body of evidence on the non-linearity of the Phillips curve around the world (Dao et al. (2024), Smith et al. (2024), Ho and Iyke (2019)). This paper is the first to test a non-linear Phillips curve specification based on the job openings to unemployed ratio on a large group of countries.[33] Furthermore, the evidence for the direct impact and its non-linearity indicates that as in US aggregate data the direct impact is a key driver of the Phillips curve's non-linearity.

I provide the functional form of the direct impact channel as estimated on headline and core inflation of the cross-country panel in Figure 25. I graph both functional forms on the range of labor market tightness observed in US data [0.15,2.0] for comparison with my previous results. Both functional forms are essentially flat for slack and normal labor market tightness, and soar when labor markets get really tight. Thus, the functional forms from my

[33] Dao et al. (2024) test it only for US and Canadian data.

cross-country panel follow a similar shape as the one estimated on US aggregate data, even though the US was excluded from the exercise.



(a) Headline

(b) Core

Figure 25: Comparison of the international direct impact on headline (a) and core inflation (b).

To test if the majority of countries in the panel experiences a similar direct impact as the US I now include the US data and run a linear Classifier-Lasso model to test for differences in the direct impact of labor market tightness on core inflation. The linear Classifier-Lasso model is allowed to both determine the number of distinct groups as well as assigning each single country to a fitting group. The model should provide a single group as the best fit if all countries experience a similar direct impact. Countries with different direct impacts should be sorted into separate groups. Thus, if the US data behaves significantly different than the data from other countries it should end up in its own sub-group.

Classifier-Lasso Linear Model	**Number of obs**	=	1,608			
Post-estimation with 2 groups	**Number of units**	=	25			
Fixed effect estimation with Group 1	**R-squared**	=	0.3190			
Absorbing: country	**Adj R-squared**	=	0.2887			
No. of obs	=	212	**Within R-sq.**	=	0.2758	
No. of units	=	4	**Root MSE**	=	2.1862	
Core	**Coefficient**	**Std. err.**	**z**	**P>|z|**	**[95% conf. interval]**	
V/U	65.85566	88.42338	0.74	0.456	[-107.451, 239.1623]	
$(V/U)^2$	-553.6134	1404.459	-0.39	0.693	[-3306.303, 2199.077]	
$(V/U)^3$	2162.271	5844.606	0.37	0.711	[-9292.946, 13617.49]	
π^w	0.1009044	0.0356007	2.83	0.005	[0.0311282, 0.1706805]	
L1.π^w	0.0139388	0.0270538	0.52	0.606	[-0.0390857, 0.0669633]	
5Y π^e	0.0090785	0.9808463	0.01	0.993	[-1.913345, 1.931502]	
Constant	-0.5967207	1.631794	-0.37	0.715	[-3.794978, 2.601537]	
Fixed effect estimation with Group 2	**R-squared**	=	0.3880			
Absorbing: country	**Adj R-squared**	=	0.3764			
No. of obs	=	1,396	**Within R-sq.**	=	0.2775	
No. of units	=	21	**Root MSE**	=	1.8364	
Core	**Coefficient**	**Std. err.**	**z**	**P>|z|**	**[95% conf. interval]**	
V/U	12.32452	3.909859	3.15	0.002	[4.661337, 19.9877]	
$(V/U)^2$	-11.34077	4.657104	-2.44	0.015	[-20.46853, -2.213018]	
$(V/U)^3$	3.720736	1.690654	2.20	0.028	[0.4071145, 7.034357]	
π^w	0.0326818	0.0195904	1.67	0.095	[-0.0057147, 0.0710783]	
L1.π^w	0.0365847	0.0185398	1.97	0.048	[0.0002474, 0.072922]	
5Y π^e	0.7687929	0.3582984	2.15	0.032	[0.0665409, 1.471045]	
Constant	-2.619952	1.053437	-2.49	0.013	[-4.68465, -0.5552529]	

The model determines two groups when estimating it on core inflation, with the majority of countries in group 2. Group 2 includes the US indicating again that the direct impact is not a phenomenon unique to the US. Group 1 consists of Spain, Portugal, Greece, and Slovakia. Their difference from the rest is likely due to the economic downturn intensity

Table 42: Country by Automated Group

Group 1	**Group 2**
Spain	Australia
Portugal	Canada
Greece	Japan
Slovakia	Finland
	Slovenia
	Cyprus
	Switzerland
	Poland
	United Kingdom
	Malta
	Estonia
	Latvia
	Lithuania
	Belgium
	Germany
	Ireland
	France
	Luxembourg
	Netherlands
	Austria
	United States

they experienced during the Euro Crisis. My data on the four countries only starts in 2008 at the earliest, and their labor market tightness levels ranges from 0.008 to 0.178 across the time frame. This is small compared with the average labor market tightness of 0.39 across all the countries in my automated grouping exercise.[34] Using headline inflation, this group also includes France, Estonia, Latvia, and Finland.

The direct impact is significant on all but 4 countries. These countries (see Group 1) had only poor labor markets over the estimation period. Thus, it is not surprising that they are not providing evidence for the direct impact which is only measurable during tight labor markets.

I also find that the direct impact is significant for the vast majority of single country regressions in my panel and follows a similar functional form as in US data (see Appendix E). However, due to international demand, supply, and commodity shocks, as well as policy differences between countries, the two-way fixed effects estimation presented provides cleaner results. Furthermore, I test the importance of market concentration for the direct impact. I

[34]Spain's V/U range (2010Q1 - 2024Q1): 0.009 - 0.054; Slovakia's range (2008Q1 - 2024Q1): 0.032 - 0.159; Portugal's range (2010Q1 - 2024Q1): 0.012 - 0.178; Greece's range (2014Q1 - 2024Q1): 0.008 - 0.118.

do this by interacting the Herfindahl-Hirschman Index with the labor market tightness measure by country. I find evidence that market concentration does play an important role in the direct impact channel. I present these results in Section 4.2.1.

C.1 Unit Profits

Finally, I estimate the profit margin to labor market tightness relation on country panel data. There is already evidence that "extraordinary unit profits" (Philip Lane (2023)) have played a crucial role in the post-pandemic Euro Area inflation surge (Hansen et al. (2023)). Hansen et al. (2023) further find that unit profits in the service sector were a major channel in the inflation surge and that they rose with the growth in real service consumption. I estimate Hansen's measure of unit profits on cubic labor market tightness in this sub-section.[35]

I use Equation 7 as basis for my empirical approach and simply switch out the dependent variable with profit margins. However, the profit margin measure is not available on all the countries in my dataset. I thus now include the US into the estimation of the direct impact on profit margins.[36] Table 43 presents the results of this estimation.

Table 43: International Direct Impact on Profit Margins

	PMs
(V/U)	2.09
	(3.03)
$(V/U)^2$	-4.41
	(4.10)
$(V/U)^3$	3.14*
	(1.48)
Constant	3.55*
	(1.74)
R^2	0.95
N	943
No Impact p-value	0.0000
Linear p-value	0.0000
Controls	✓

Two-way clustered standard errors in parentheses.

* $p < .10$, ** $p < .05$, *** $p < .01$

[35] Hansen et al. (2023) use value added per unit minus unit labor cost as measure of unit profits. They thus use a similar but less clean measure than the US measure I use in my aggregate results.

[36] Countries with profit margin data: Australia, Canada, Belgium, Germany, Estonia, Ireland, Spain, France, Netherlands, Austria, Poland, Portugal, Finland, the UK, Greece, and the USA. I am missing profit margin data for Switzerland, Japan, Luxembourg, Slovenia, Cypress, Malta, Slovakia, Latvia, and Lithuania.

The Wald Test supports a direct impact of labor market tightness on profit margins, as well as its non-linear rise. The results on profit margin data further support an optimal price setting mechanism behind the direct impact. I provide the estimated functional form graphed on the US labor market tightness data range in Figure 26. The functional form is again similar to the estimated functional form on US aggregate profit margins.

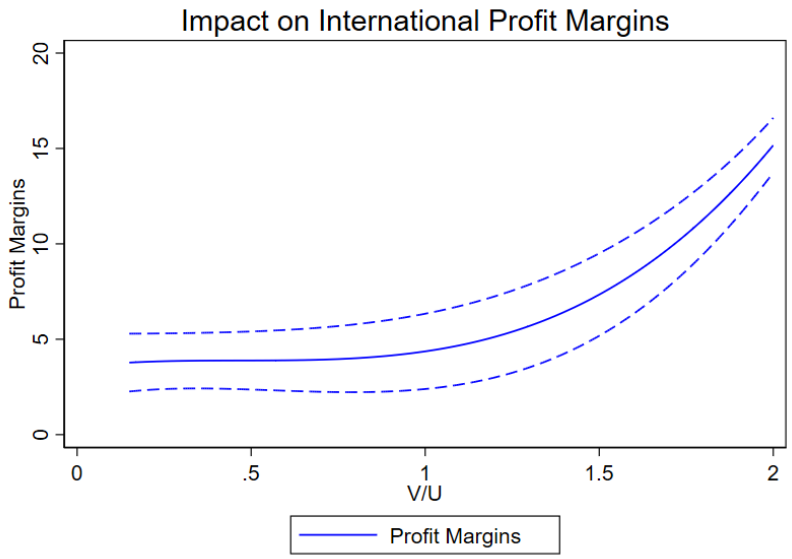


Figure 26: International Direct Impact on Profit Margins.

D Evidence from US Sectors

In this section I estimate the direct impact on a cross-section of US service sectors, which allows me to exclude non-labor intensive manufacturing sectors and use time fixed effects. Time fixed effects control for endogenous monetary policy, potential endogeneity from fiscal policy and inflation expectations as argued by McLeay and Tenreyro (2020) and Hazell et al. (2022). I estimate my channel across the sectors of Construction, Transportation, Warehousing and Utilities, Finance and Insurance, Real estate and Leasing, Professional Business Services, Arts, Entertainment and Recreation, Accommodation and Food Services, as well as Other services.

Inflation is measured using sectoral CPI, while wage growth is based on sectoral ECI wages and salaries. Job openings stem from the JOLTS survey and unemployment from the Current Population Survey. CPS' unemployment measure assigns the sector of last employment to an unemployed respondent. It is thus prone to mis-measurement from cross-sector labor mobility. In taking them at face value I make the implicit assumption of no labor mobility across sectors. I will later discuss on how to alleviate this assumption using a Shift-Share IV approach.

The sectoral CPI data stems from seasonal adjusted quarterly CPI component data from 1996 to early 2024. The component inflation is aggregated up to match the sectors at which

V, U and ECI data are available. I use annual Consumption Expenditure spending weights of the pertaining CPI series to create weighted average inflation rates for the two-digit NAICS sectors. For example, I aggregate Food Away From Home, Alcoholic Beverages Away From Home and Lodging Away From Home to get a CPI measure for the Accommodation and Food Services Sector.

The ECI is reported on the same NAICS level as V and U, except for "Transportation and Warehousing" and "Utilities". These are reported separately in the ECI data. I use average 2000-2024 employment levels to aggregate the two ECI series together, where Transportation and Warehousing takes a weight of about 90%.

Another benefit of using cross-sectional data is the greater prevalence of tight labor markets compared to aggregate data. Hooper et al. (2020) noted this for MSA level labor market data. I find that this observation holds true in the cross-section of US service sectors as well. Labor market tightness ranges from 0.026 to 3.78 in quarterly sectoral data, with a mean of 0.79. 17.6% of the sectoral labor markets lie on or above the 1.25 level of extremely tight labor markets. Thus, tight labor markets are much more prevalent in sectoral data than in the aggregate US economy, where V/U values on or above 1.25 only constitute about 8.5% of labor market data points.

D.1 Empirical Results

I provide estimation results for all sectors for which V/U data is available in Appendix F. The single sector estimation provides evidence for the direct impact on most sectors.[37] This includes the Durable and Non-durable goods sectors, as well as the Education, Health Services, and Information sectors. The only sector that is missing is the Mining and Logging sector due to missing wage data. Education and Health Services are excluded from the cross-section due to large government involvement in these sectors.[38] Finally, Information is excluded as it is not labor intensive.[39] Thus, it is unlikely to exhibit the direct impact.

[37] First, core services sectors "Accommodation & Food", "Arts, Entertainment, & Recreation", "Health Care", "Other", "Business", "Finance & Insurance", and "Real Estate & Leisure" all show a non-linear rise in inflation beyond wage growth during tight labor markets. Second, sectors that blend services and goods like "Transportation, Warehousing & Utilities" and "Construction" also show a significant rise in inflation beyond wage growth when labor markets are tight. The "Education" and "Information" sectors are not showing much evidence for a direct impact. Finally, even "Durables" and "Non-durables" show a non-linear rise in inflation beyond wage growth under tight labor markets.

[38] "In 2023, the public sector accounted for 14 percent of annual average employment (21.8 million jobs). Of this 14 percent, two-thirds were mostly concentrated in two sectors: education and healthcare." from Thompson (2024).

[39] NAICS 51 "Information" consists of Publishing, Telecommunications, Software, Data Processing and Hosting, and Broadcasting ordered by relative revenue in 2022 from large to small. None of these sectors faces a noticeable labor constraint on producing a marginal unit. This is also reflected in its relative size of total wages paid to non-supervisory workers to total revenue which is about 10%. Total revenue in 2022 was $2.2 Trillion, while I estimate the total wage bill for the peak employment year of 2022 (Employment = 3.06 Million) by multiplying average hourly earnings for non-supervisory employees (just under $38) by average weekly hours for non-supervisory employees (36.5) times 52 weeks, to get an average income of about $72108. Multiplied by 3.06 million workers gives a wage bill of $220.65 Billion. Data Sources: FRED and BLS.

To test the indication of the single-sector analysis I estimate the service sectors in a two-way fixed effects panel regression with two-way clustered standard errors. I estimate the following empirical model

$$\text{CPI}_{i,t} = \hat{\alpha}_1(V/U)_{i,t} + \hat{\alpha}_2(V/U)^2_{i,t} + \hat{\alpha}_3(V/U)^3_{i,t} + \hat{\gamma}_1\pi^w_{i,t} + \hat{\gamma}_2\pi^w_{i,t-1} + \hat{\phi}\text{Prod}_{i,t} + \hat{\delta}_i + \hat{\delta}_t + \hat{\epsilon}_{i,t} \quad (8)$$

I estimate this on all service sectors, barring those dominated by government price setting (Education, and Health), and the low-marginal cost information sector. The estimated sectors are Construction, Finance and Insurance, Real Estate and Leasing, Business Services, Transportation, Warehousing and Utilities, Arts, Entertainment, and Recreation, Accommodation and Food, as well as Other Services. I exclude all four quarters of 2020 from the estimation. The initial estimation is of a quadratic nature (Appendix F.1). The cubic nature of the direct impact is partially masked by inflation volatility in the business services sector which has thin coverage in CPI data. Excluding the business services sector clears the noise and leads to significant cubic coefficients. These are further strengthened when omitting the Transportation and Warehousing sector which exhibits a strong inflation cyclicality outside common seasonality.

The resulting estimates are presented in Table 44. I further estimate a threshold model specification, with the threshold splitting the range of sectoral V/U values in half. The Wald tests reject the null hypothesis of a linear direct impact for both models. The results are again robust to the inclusion of further lags of wage growth.

For visualization I plot the cubic functional form of model 1 in Figure 27. The sectoral direct impact follows the aggregate impact closely. The direct impact is non-existent under slack to mildly tight labor markets, while it soars during especially tight labor markets.

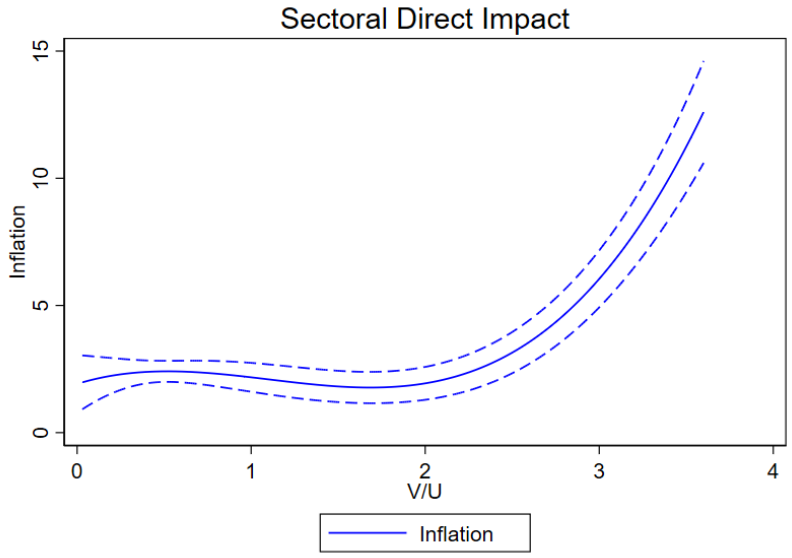


Figure 27: Sectoral Cubic Model

Note: Estimated cubic model of the direct impact on sectoral data. V/U ranges from 0.03 to 3.6 in the sectors used in Table 44.

The sectoral results imply a delayed increase in the direct impact, as the functional form

Table 44: Sectoral Direct Impact

	Cubic	Threshold
(V/U)	2.12	-1.01
	(2.90)	(0.57)
$(V/U)^2$	-2.67	
	(1.73)	
$(V/U)^3$	0.81**	
	(0.31)	
tight=1 $\times$ (V/U)		4.93***
		(0.97)
tight=1		-9.22***
		(1.84)
Constant	1.92	2.90***
	(1.14)	(0.67)
R^2	0.59	0.58
N	436	436
No Impact (p-value)	0.0004	0.0103
Linear Impact (p-value)	0.0002	0.0038
Controls	✓	✓

Two-way clustered standard errors in parentheses

* $p < .10$, ** $p < .05$, *** $p < .01$

is still flat at the aggregate labor market peak of $V/U = 2$. This is likely due to the fact that my panel regression provides estimates for an isolated tight labor market shock to one sector while all other sectors are unaffected. Thus, if labor mobility across sectors is non-negligible the affected sector can poach employees from less tight sectors, and delay the effects of a tight labor market on pricing until it is really tight. Whereas when all sectors are tight due to a general demand or labor supply shock, that reservoir of workers in less tight sectors is not existent.

I repeat the TWFE panel regression on wage growth and sectoral wage-price differentials to proxy for real wage growth (see Appendix F.2). All labor market coefficients fail to reach significance in the two non-linear wage growth models. Even a linear specification fails significance, while the Wald tests fail to reject the no impact hypothesis. I conjecture that labor mobility across sectors may be behind this discrepancy with aggregate data. Wage price differentials are only significant at a 10% level, but point to a decline of real wages during especially tight labor markets.

By using time fixed effects I control for aggregate supply shocks, the endogeneity of

monetary and fiscal policy (Fitzgerald et al. (2020), McLeay and Tenreyro (2020)) as well as long-run inflation expectations informed by the monetary policy regime (Hazell et al. (2022)). However, the problem of sectoral supply shocks remains. One way to control for disaggregate supply shocks is to use shift-share innovations in employment to proxy for labor demand shocks to US regions (Bartik (1991), Gitti (2024)).

Furthermore, there is potential measurement error in the calculation of sectoral unemployment. The number of unemployed by sector come from the Current Population Survey. Sectoral affiliation of an unemployed respondent are assigned as the sector the unemployed worked in last. However, workers often switch between sectors when transitioning from unemployment back to employment. Thus, the sectoral unemployment measure is imperfect and likely introduces noise into the measurement of sectoral V/U.

A Shift-Share instrumental variable (SSIV) can reduce the measurement error in an independent variable and deliver an identification independent of sector-specific supply shocks. Thus, SSIV offers an improved estimation of the underlying relation between sectoral inflation and labor market tightness.

I design the SSIV based on state level economic activity shocks, by using the deviation of annual state employment growth from national employment growth. That is if state A has an economic boom relative to the rest of the country this will be reflected in faster employment growth than the rest of the nation. I then use the employment share that the shocked state has in the sector to translate these regional economic activity shocks into sectoral labor market variation. This translates the regional activity shocks into sectoral labor demand shocks via the variation in expenditure baskets across states.

The bundle of services purchased varies quite a bit between states (e.g. New Yorkers spend on average 1.3% of their income on public transport, whereas Texans only spend 0.5%, Californians spend 1.9% on Arts and Entertainment services, while New Jerseyans only spend 0.7%. Similarly, Lodging away form home is bigger in California (1.8%) than in Texas (0.9%), while both Californians and Texans spend big for outside meals (5 & 4.9% respectively), while this is more muted in New Jersey and Florida (3.3 & 3.4%). Utility services take up more of household income in less densely populated states (CA: 5.8%, TX: 5.6%; NJ:4.1%, NY:4.3%). Similarly for Health Care Services: CA:1.4% vs NJ: 0.9%, and Personal Care Services: CA 1.3% vs TX or FL:0.9%.[40]

Because of this variation in the consumption bundles across states I can use a shift share instrument to estimate V/U shocks in the service sectors. An economic shock to a state with a high share of sector i in spending will increase sector i's overall demand relative to other sectors. Thus, the shift-share instrument is cleaned of unemployment measurement errors and sectoral supply shocks.

I multiply the exposure share with the regional economic shock and finally sum up over all fifty states plus the District of Columbia (DC). This gives:

[40]Source: BLS Research Tables for CA, FL, NJ, NY, TX: (State CE Tables). I used the all consumer expenditure weights from the most recent available research tables (2021-2022); except for NJ (2019-2020).

$$SSIV_{i,t} = \sum_{s=1}^{51} \text{Empl. Share}_{i,s,t}[g_{s,t} - g_{National,t}]$$

where Empl. Share$_{i,s,t}$ is the employment share that industry i has in state s at time t, and $g_{s,t} - g_{National,t}$ is the difference between annual employment growth rates of the state and the nation as a whole. I use this measure and non-linear versions of it to instrument for $V/U, (V/U)^2, (V/U)^3$.

The first stage regression results are borderline (see F-values in Table 45). Furthermore, the instruments are weak according to the Cragg-Donald-Stock-Yogo test. I will use a method robust to weak identification, like Lewis and Mertens (2022), in the next rendition. The second stage TWFEs results are presented in Table 45. Model 1 reports the estimation of the cubic model, Model 2 reports the results of the quadratic model, and Model 3 presents the threshold model.

The coefficients and Wald test results all support a positive non-linearity in the direct impact as in the non-IV panel estimation. To summarize, the direct impact is alive and well in sectoral data. Sectoral inflation increases non-linearly above and beyond wage growth in response to tightening sectoral labor markets, causing wage-price differentials to suffer during especially tight labor markets. This excess steepening is stronger for concentrated sectors, hinting at an optimal price setting mechanism behind the impact. Furthermore, taking out aggregate effects obliterates the traditional wage channel. The latter finding indicates that other cross-sectional Phillips curve estimations may be driven by the direct impact channel rather than the traditional wage channel.

D.1.1 US service Sector Evidence

Finally, I test the implied importance for market concentration from my cross-country results on the sectoral cross-section. I use 2017 Census firm survey data to construct sectoral Herfindahl-Hirschman Indexes (HHIs). The Census data provides brackets of firm size by revenue in the industry as well as total revenue in the sector. From these I impute the average revenue share per firm in each bracket and applying the appropriate weights I then construct the HHI for each sector. I interact these HHIs with labor market tightness to test if concentration increases or dampens the direct impact during tight labor markets. The results are presented in Table 46.

The interaction models are preferred over the non-interaction models by Bayesian and Akaike Information Criteria. The cubic interaction model provides a negative highest order coefficient that is nonetheless dominated by the positive quadratic interaction term. Reducing the functional form to quadratic delivers a positive highest order interaction coefficient. I show in Figure 28 that the resulting functional forms under maximal HHI in the estimated data are almost identical for the cubic and quadratic estimations. The Wald test rejects the hypothesis that the interaction terms are jointly zero or only linear in both specifications. The positive quadratic interaction term hints at a strengthening of the direct impact in concentrated industries under tight labor markets. The negative linear interaction term on the other

Table 45: Sectoral Phillips curves

	Cubic	Quad	Thresh
V/U	1.46	-4.81***	-2.89*
	(4.36)	(1.56)	(1.54)
$(V/U)^2$	-3.35	1.81***	
	(3.41)	(0.53)	
$(V/U)^3$	1.17		
	(0.76)		
π^w	0.26***	0.32***	0.18***
	(0.07)	(0.06)	(0.04)
L.π^w	0.25***	0.30***	0.12***
	(0.07)	(0.06)	(0.04)
Prod	0.06	0.04	
	(0.05)	(0.05)	
V/U_tight			1.60**
			(0.73)
R^2	0.24	0.18	0.04
N	436	436	436
No Impact (p-value)	0.0021	0.0033	0.0854
Linear Impact (p-value)	0.0007	0.0007	-
F-value V/U	9.11	9.11	5.08
F-value $(V/U)^2$ / $V/U\times$ tight	9.1	9.1	5.08
F-value $(V/U)^3$	8.08	-	-

Two-way clustered standard errors in parentheses

* $p < .10$, ** $p < .05$, *** $p < .01$

hand hints at lower inflation rates for concentrated industries under slack and normal labor markets. This is likely due to already elevated prices and margins in concentrated sectors. This finding fits with the thin existing literature on Phillips curves and market concentration (Firat (2024)).

I highlight the role of market concentration in the direct impact channel by comparing the quadratic functional form with minimal market concentration (HHI = 0.03), mean market concentration (HHI = 0.2), and maximal market concentration (HHI = 0.56) as observed in the estimated services sectors. The resulting functional forms are presented in Figure 29. The figure highlights the importance of market concentration for the direct impact channel. Under minimal market concentration the impact is at most flat, and even shows a slight

Table 46: HHI Direct Impact Interaction

	Cubic	Quadratic
V/U	7.23**	0.35
	(2.34)	(1.68)
$(V/U)^2$	-6.54**	-0.33
	(2.09)	(0.56)
$(V/U)^3$	1.57**	
	(0.57)	
$V/U \times$ HHI	-15.54**	-7.68*
	(4.69)	(3.63)
$(V/U)^2 \times$ HHI	12.66**	3.48**
	(3.43)	(1.17)
$(V/U)^3 \times$ HHI	-2.54**	
	(0.92)	
Constant	1.05	2.67**
	(0.95)	(0.71)
R^2	0.61	0.59
N	436	436
No Interaction (p-value)	0.0424	0.0490
Linear Interaction (p-value)	0.0290	0.0308
Controls	✓	✓

Two-way clustered standard errors in parentheses

* $p < .10$, ** $p < .05$, *** $p < .01$

indication for a decline under tight labor markets. Under normal market concentration the direct impact steepens slightly during tight labor markets, and soars under high market concentration.

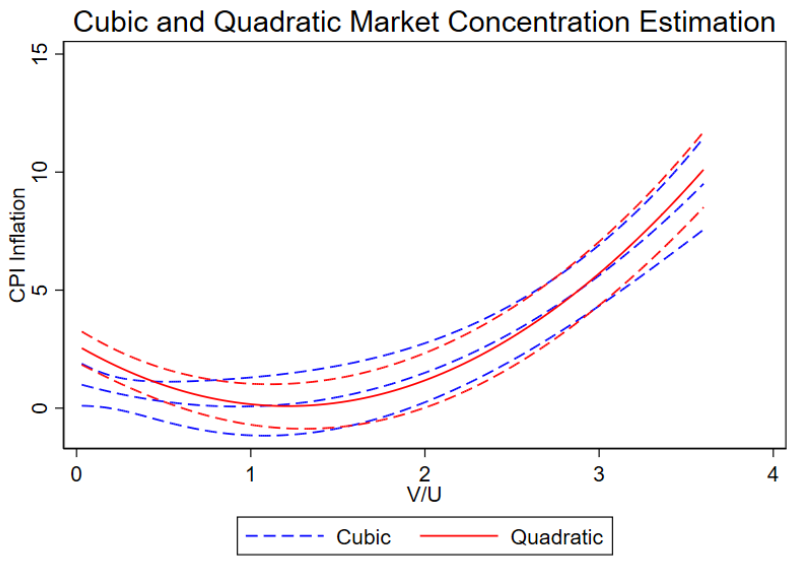


Figure 28: Sectoral Cubic Model

Note: Comparison of the cubic and quadratic market concentration interaction models. The comparison is done under the assumption of maximal level of market concentration; HHI = 0.56 to better visualize the difference in the impact of the interaction terms.

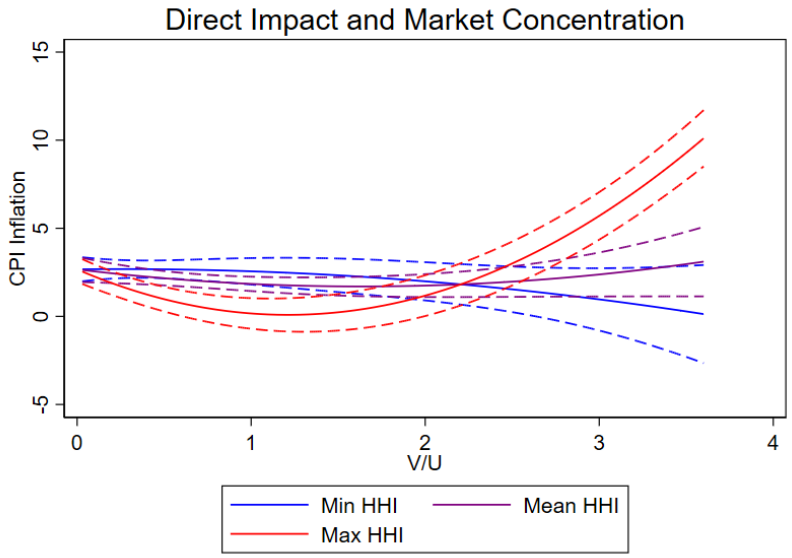


Figure 29: Sectoral Cubic Model

Note: Comparison of direct impact functional forms under different levels of market concentration, with minimal market concentration (HHI = 0.03; Construction), mean market concentration (HHI = 0.2; close to Arts, Entertainment, and Recreation), and maximal market concentration (HHI = 0.56; Finance & Insurance) as observed in the estimated services sectors.

E Single Country Regression Results

This Appendix tests the direct impact in single country regressions.[41] I estimate the following equation per country to test for the direct impact in the respective country:

$$\text{Core}_{c,t} = \check{\alpha}_1(V/U)_{c,t} + \check{\alpha}_2(V/U)^2_{c,t} + \check{\alpha}_3(V/U)^3_{c,t} + \check{\beta}\pi^e_{c,t} + \check{\gamma}_1\pi^w_{c,t} + \check{\gamma}_2\pi^w_{c,t-1} + \check{\phi}\text{Prod}_{c,t} + \check{c} + \check{\epsilon}_{c,t} \quad (9)$$

where Core is inflation of the consumer price index less food and energy (in some cases also less alcohol and tobacco), V/U is the ratio of job openings to unemployed, π^e is the 5 year forward inflation prediction from the IMF's World Economic Outlook, π^w is nominal wage growth, and Prod is an eight quarter moving average of labor productivity growth. I thus estimate the same equation as on US aggregate data before, with the exception of using a shorter inflation expectations horizon (5 vs 10 years), and omitting controls for energy inflation and relative import price pressures.

I graph the functional form of the V/U coefficients and the constant together with error bands by country, against the V/U range of the respective country. I compile the 24 country graphs in Figure 30. Most countries yield a non-linear upward sloping functional form of the direct impact. These results are similar to those on US data. I include a US based version of the graph at the end for better comparison.

Most countries evince a non-linear impact of labor market tightness on core inflation, providing evidence for the direct impact as an international phenomenon. The results of the per country estimation indicate that the direct impact is not restricted to the US. However, global demand and supply shocks, commodity price shocks, as well as correlation of monetary policy between advanced economies can confound the estimation of Phillips curve channels. Thus, a two-way fixed effects panel estimation as presented in the body of the paper allows for a cleaner identification of the channel. Furthermore, the country fixed effects control for most social and policy differences across countries. Finally, clustering standard errors at the country level controls for serial correlation of the error terms.

F Single Sector Regressions

For the single-sector results I estimate the following equation per sector

$$\text{CPI}_{i,t} = \grave{\alpha}_1(V/U)_{i,t} + \grave{\alpha}_2(V/U)^2_{i,t} + \grave{\alpha}_3(V/U)^3_{i,t} + \grave{\beta}\pi^e_t + \grave{\gamma}_1\pi^w_{i,t} + \grave{\gamma}_2\pi^w_{i,t-1} + \grave{c}_i + \grave{\epsilon}_{i,t}$$

The estimated functional form of each sector is presented in 32.

The single sector estimation provides evidence for the direct impact on most sectors. First, the core services sectors "Accommodation & Food", "Arts, Entertainment, & Recreation", "Health Care", "Other", "Business", "Finance & Insurance", and "Real Estate & Leisure"

[41] Australia, Canada, Belgium, Germany, Estonia, Ireland, Spain, France, Cyprus, Latvia, Lithuania, Luxembourg, Malta, Netherlands, Austria, Poland, Portugal, Slovenia, Slovakia, Finland, Great Britain, Switzerland, Greece, and Japan



Figure 30: Direct Impact Estimation by Country

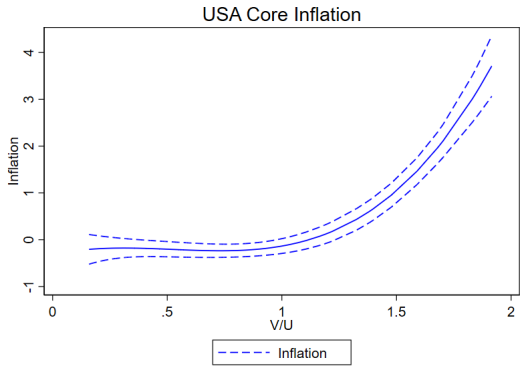


Figure 31: US Direct Impact Functional Form

all show a non-linear rise in inflation beyond wage growth during tight labor markets. Second, sectors that blend services and goods like "Transportation, Warehousing & Utilities" and "Construction" also show a significant rise in inflation beyond wage growth when labor markets are tight. The "Education" and "Information" sectors are not showing much evidence for a direct impact. Finally, even "Durables" and "Non-durables" show a non-linear rise in inflation beyond wage growth under tight labor markets.

F.1 Full Service Sector Regression Table

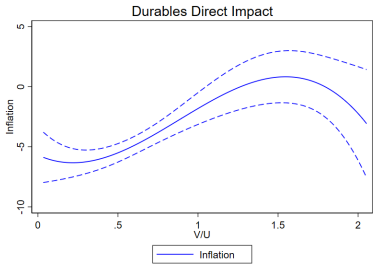
I present the quadratic and threshold estimations of the service sector cross-section including the noisy Business Services sector and the cyclical Transportation and Warehousing sector.

F.2 Sectoral Wage and Real Wage Estimations

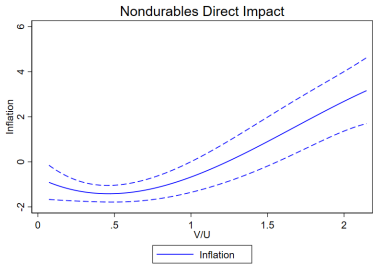
I repeat my aggregate wage and real wage growth estimation on sectoral data. Table 48 presents wage growth estimates in the cross-section with two-way fixed effects and two-way clustered standard errors. All labor market coefficients fail to reach significance in the two non-linear models 1 (cubic) and 2 (threshold). Even a linear specification fails significance, while the Wald tests fail to reject the no impact hypothesis.

This is puzzling as the aggregate data gave a significant, albeit linear, impact of labor market tightness on wage growth. The results are significant when I drop time fixed effects. This indicates that wage growth is a much more aggregate phenomenon than price changes. I am not aware of any paper exploring the correlation of wage growth across sectors, but the question of why wage growth moves in accord across sectors would be an interesting one for future research. I suspect that labor mobility is greater across sectors than previously assumed (Alonso-Carrera and Raurich (2018)).

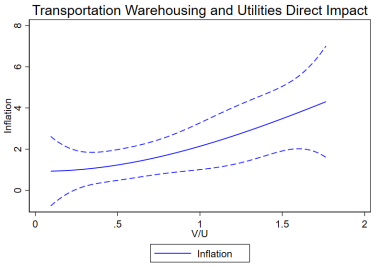
Further, I estimate wage-price differentials on sector panel data. I subtract sector-specific CPI inflation from the sector's ECI wage growth and estimate the impact of labor market tightness on this differential. The results are presented in Models 3 and 4. The non-linear terms fail significance at the 5% level in Model 3 but indicate the same direction as on aggregate data. Furthermore, the Wald test rejects a purely linear impact. In model 4 the highest



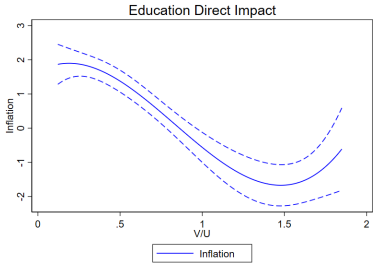
Durable Goods



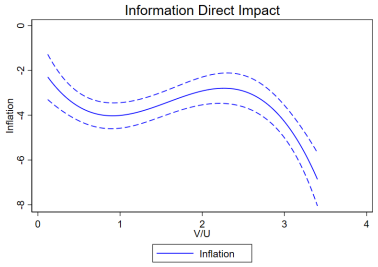
Non-Durable Goods



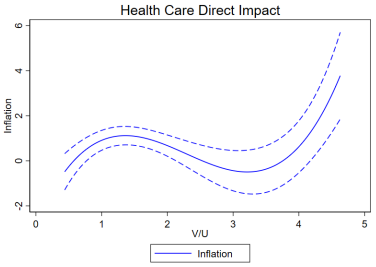
Transportation, Warehousing & Utilities



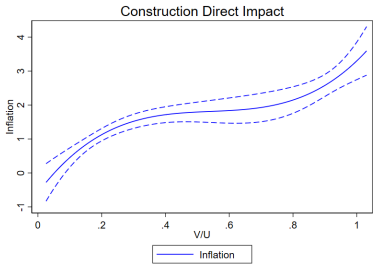
Education



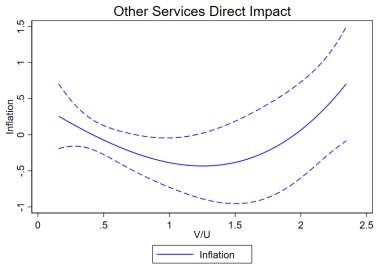
Information



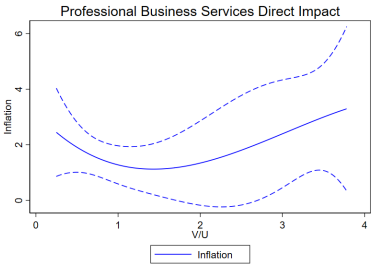
Medical Services



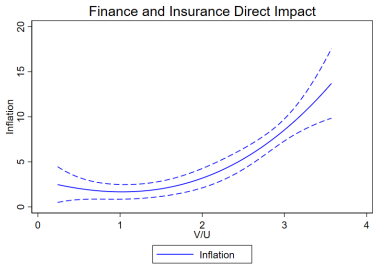
Construction



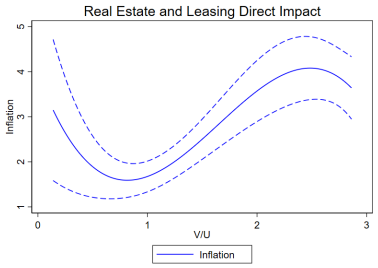
Other Services



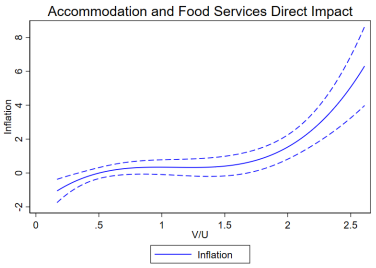
Business Services



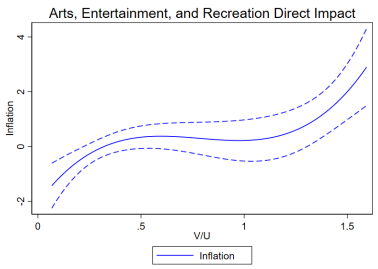
Finance & Insurance



Real Estate & Leasing



Accommodation & Food



Arts, Entertainment & Recreation

Figure 32: Direct Impact by Sector

Table 47: Sectoral Phillips curves

	Cubic	Threshold
(V/U)	-3.83***	-2.04**
	(0.98)	(0.85)
$(V/U)^2$	0.86**	
	(0.31)	
tight=1 $\times$ (V/U)		2.46*
		(1.23)
tight=1		-3.76
		(2.91)
Constant	5.27***	4.59***
	(0.93)	(1.00)
R^2	0.41	0.41
N	599	599
No Impact (p-value)	0.0177	0.0116
Linear Impact (p-value)	0.0287	0.0861
Controls	✓	✓

Two-way clustered standard errors in parentheses

* $p < .10$, ** $p < .05$, *** $p < .01$

order coefficient is significantly negative at the ten percent level. Using a cubic term instead of a quadratic term in model 4 leads to a fully significantly negative coefficient. Finally, the coefficient of a purely linear estimation is negative, albeit insignificantly so. Thus, the results are broadly in line with those on aggregate US data.

F.3 Dual Impact of Labor Mobility and Market Concentration on Wages

This appendix estimates the impact of international labor mobility and market concentration on the wage Phillips curve. I add interactions of labor market tightness with market power and similar interactions with the percent of Foreign Born Population to equation 7. The goal of this exercise is to see if the level of international labor mobility and labor productivity can explain the linearity of the wage channel in the US. I thus allow myself to now include the US in the estimation as well. The estimation results are presented in Table 49. I omit reporting the coefficients of control variables such as inflation, expected inflation and labor productivity to simplify the table.

The US has an average FBP of 12.57% and an average market concentration of 0.066 over the estimation period. Thus, the relevant labor market tightness coefficients for the US are given by $\pi^w = 12.31V/U - 16.87(V/U)^2 + 4.95(V/U)^3$. These are all within one standard

Table 48: Sectoral Wage Impact

	W_Cubic	W_Linear	ω_C	ω_Q
(V/U)	1.20	-0.25	-0.89	4.95*
	(4.25)	(0.47)	(5.89)	(2.40)
$(V/U)^2$	-0.21		2.21	-1.64*
	(2.43)		(3.44)	(0.79)
$(V/U)^3$	-0.07		-0.78	
	(0.43)		(0.60)	
Prod	0.04	0.05	0.03	0.04
	(0.08)	(0.09)	(0.08)	(0.08)
CPI	0.28*	0.25		
	(0.13)	(0.13)		
L.CPI	-0.04	-0.05		
	(0.16)	(0.16)		
L.ω			-0.12	-0.11
			(0.10)	(0.11)
Constant	1.59	2.51***	-0.36	-2.38*
	(1.73)	(0.41)	(2.21)	(1.16)
R^2	0.43	0.42	0.40	0.39
N	442	442	436	436
No Impact (p-value)	0.6074	0.6201	0.0052	0.2063
Linear Impact (p-value)	0.4367	-	0.0027	0.0916

Two-way clustered standard errors in parentheses

* $p < .10$, ** $p < .05$, *** $p < .01$

error of the basic V/U function $(22.65, 40.04, 18.98)$ from zero. The function is also rather flat on the US range of labor market tightness data (0.15,2.0), and declines rather than rises for tighter labor markets in the range. I provide a graphical comparison to the function estimated on aggregate US data in the body of the paper. The cross-sectional results indicate that international labor mobility and US levels of market concentration can explain the missing non-linearity in the US wage Phillips curve once inflation expectations are controlled for.

Table 49: FBP and HHI Impact on Wage Growth

	Wage Growth
(V/U)	69.65***
	(22.65)
$(V/U)^2$	-109.52**
	(40.04)
$(V/U)^3$	48.94**
	(18.98)
$(V/U) \times$ HHI	-630.74**
	(296.76)
$(V/U)^2 \times$ HHI	1038.25*
	(504.97)
$(V/U)^3 \times$ HHI	-511.52*
	(248.83)
HHI	83.79*
	(41.79)
$(V/U) \times$ FBP	-1.25
	(0.73)
$(V/U)^2 \times$ FBP	1.92
	(1.27)
$(V/U)^3 \times$ FBP	-0.81
	(0.64)
FBP	-0.17
	(0.42)
Constant	-7.50
	(6.46)
R^2	0.23
N	1183

Two-way clustered standard errors in parentheses

* $p < .10$, ** $p < .05$, *** $p < .01$

G Illustrative Model Parts

This appendix provides the full details of my illustrative model and its different cases. I begin by stating the assumptions of my simple two period economy. First, labor is the only factor of production

$$Y_t = L_t \text{ for } t \in \{1, 2\}$$

The monopolist faces a downward sloping demand schedule, with a demand multiplier x_t

$$D(P_t, x_t) = x_t P_t^{-\epsilon} \text{ where } \epsilon > 1 \text{ for } t \in \{1, 2\}$$

and chooses optimal prices and wages to maximize its profits

$$\max_{P_1,P_2,W_1,W_2} \Pi = \max_{P_1,P_2,W_1,W_2} P_1 D_1 - W_1 L_1 + P_2 D_2 - W_2 L_2 = \max_{P_1,P_2,W_1,W_2} x_1 P_1^{1-\epsilon} - W_1 x_1 P_1^{-\epsilon} + x_2 P_2^{1-\epsilon} - W_2 x_2 P_2^{-\epsilon}$$

under a downward nominal wage rigidity constraint

$$W_2 \geq W_1$$

and constraints to labor supply in both periods

$$W_1 \geq (\eta_1 x_1 P_1^{-\epsilon})^{\frac{1}{\beta}}$$

$$W_2 \geq (\eta_2 x_2 P_2^{-\epsilon})^{\frac{1}{\beta}}$$

where η_t is a labor supply shifter with $\eta_t > 1$ leading to a negative labor supply shock, and $\eta_t < 1$ leading to a positive labor supply shock.

I use a Lagrangian approach with multipliers ϕ_1 through ϕ_3

$$\max_{P_1,P_2,W_1,W_2} x_1 P_1^{1-\epsilon} - W_1 x_1 P_1^{-\epsilon} + x_2 P_2^{1-\epsilon} - W_2 x_2 P_2^{-\epsilon} + \phi_1(W_2 - W_1) + \phi_2(W_1 - (x_1 \eta_1 P_1^{-\epsilon})^{\frac{1}{\beta}}) + \phi_3(W_2 - (x_2 \eta_2 P_2^{-\epsilon})^{\frac{1}{\beta}})$$

The Lagrangian has the following Karush-Kuhn-Tucker conditions, which include the above constraints and the first order conditions.

$$\begin{aligned}
\phi_1, \phi_2, \phi_3 &\geq 0 \\
\phi_1(W_2 - W_1) &= 0 \\
\phi_2(W_1 - (\eta_1 x_1 P_1^{-\epsilon})^{\frac{1}{\beta}}) &= 0 \\
\phi_3(W_2 - (\eta_2 x_2 P_2^{-\epsilon})^{\frac{1}{\beta}}) &= 0 \\
W_2 &\geq W_1 \\
W_1 &\geq (\eta_1 x_1 P_1^{-\epsilon})^{\frac{1}{\beta}} \\
W_2 &\geq (\eta_2 x_2 P_2^{-\epsilon})^{\frac{1}{\beta}} \\
(1-\epsilon) P_1^{-\epsilon} x_1 + \epsilon P_1^{-(\epsilon+1)} x_1 W_1 + \phi_2 \frac{\epsilon}{\beta} P_1^{-\frac{\epsilon+\beta}{\beta}} x_1^{\frac{1}{\beta}} \eta_1^{\frac{1}{\beta}} &= 0 \\
(1-\epsilon) P_2^{-\epsilon} x_2 + \epsilon P_2^{-(\epsilon+1)} x_2 W_2 + \phi_3 \frac{\epsilon}{\beta} P_2^{-\frac{\epsilon+\beta}{\beta}} x_2^{\frac{1}{\beta}} \eta_2^{\frac{1}{\beta}} &= 0 \\
-x_1 P_1^{-\epsilon} - \phi_1 + \phi_2 &= 0 \\
-x_2 P_2^{-\epsilon} + \phi_1 + \phi_3 &= 0
\end{aligned}$$

These conditions imply that $\phi_1 = 0$ when $W_2 > W_1$, $\phi_2 = 0$ when $W_1 > (x_1\eta_1 P_1^{-\epsilon})^{\frac{1}{\beta}}$, and $\phi_3 = 0$ when $W_2 > (x_2\eta_2 P_2^{-\epsilon})^{\frac{1}{\beta}}$.

There are eight different combinations of the three constraints binding or not, which I discuss below. Due to the KKT conditions, whenever a constraint is not binding the pertaining Lagrange multiplier must be zero. Thus, written in tuples, the case where no constraint is binding can be written as $(\phi_1, \phi_2, \phi_3) = (0, 0, 0)$. Remember that ϕ_1 is associated with the DNWR constraint, ϕ_2 with the labor supply schedule in period one, and ϕ_3 with the labor supply schedule in period two. The eight cases are thus

$$
\begin{aligned}
&1.(0,0,0) \rightarrow \text{no constraint binds} \\
&2.(0,>0,0) \rightarrow W_1 = (x_1\eta_1 P_1^{-\epsilon})^{\frac{1}{\beta}} \\
&3.(0,0,>0) \rightarrow W_2 = (x_2\eta_2 P_2^{-\epsilon})^{\frac{1}{\beta}} \\
&4.(0,>0,>0) \rightarrow W_1 = (x_1\eta_1 P_1^{-\epsilon})^{\frac{1}{\beta}} \;\&\; W_2 = (x_2\eta_2 P_2^{-\epsilon})^{\frac{1}{\beta}} \\
&5.(>0,0,0) \rightarrow W_2 = W_1 \\
&6.(>0,>0,0) \rightarrow W_2 = W_1 \;\&\; W_1 = (x_1\eta_1 P_1^{-\epsilon})^{\frac{1}{\beta}} \\
&7.(>0,0,>0) \rightarrow W_2 = W_1 \;\&\; W_2 = (x_2\eta_2 P_2^{-\epsilon})^{\frac{1}{\beta}} \\
&8.(>0,>0,>0) \rightarrow W_2 = W_1 \;\&\; W_1 = (x_1\eta_1 P_1^{-\epsilon})^{\frac{1}{\beta}} \;\&\; W_2 = (x_2\eta_2 P_2^{-\epsilon})^{\frac{1}{\beta}}
\end{aligned}
$$

There are some trivial cases. For example when DNWR and one of the other constraints are not binding. In this case $\phi_1 = 0$ and either $\phi_2 = 0$ or $\phi_3 = 0$, or both. Using the FOCs with respect to wages W_1 and W_2

$$
\begin{aligned}
-x_1 P_1^{-\epsilon} - \phi_1 + \phi_2 = 0 \\
-x_2 P_2^{-\epsilon} + \phi_1 + \phi_3 = 0
\end{aligned}
$$

gives

$$
\begin{aligned}
x_t P_t^{-\epsilon} = 0 \\
\implies P_t = \infty
\end{aligned}
$$

for $t = 1$ when both ϕ_1 and ϕ_2 equal to zero, and for $t = 2$ if both ϕ_1 and ϕ_3 equal to zero. This makes cases 1-3 redundant as demand and thus profits go to zero. Similar reasoning rules out any case where $W_1 > (x_1 P_1^{-\epsilon})^{\frac{1}{\beta}}$. This leaves cases 4, 6, and 8.

G.1 Flexible Wage Case

Case 4, where DNWR is not binding ($\phi_1 = 0$) but the labor supply constraints are both binding ($W_t = (x_t P_t^{-\epsilon})^{\frac{1}{\beta}}$) yields the optimal behavior under flexible wages presented in Section 4.1.

$$
\begin{aligned}
\phi_2 = x_1 P_1^{-\epsilon} - \phi_1 = x_1 P_1^{-\epsilon} \\
\phi_3 = x_2 P_2^{-\epsilon} + \phi_1 = x_2 P_2^{-\epsilon}
\end{aligned}
$$

Plugging these into the FOCs from optimal price setting gives

$$\begin{aligned}
&(1-\epsilon)P_t^{-\epsilon}x_t + \epsilon P_t^{-(\epsilon+1)}x_t W_t + x_t P_t^{-\epsilon}\frac{\epsilon}{\beta}P_t^{-\frac{\epsilon+\beta}{\beta}}x_t^{\frac{1}{\beta}}\eta_t^{\frac{1}{\beta}} = 0 \\
\iff &(\epsilon-1)P_t^{-\epsilon}x_t = \epsilon P_t^{-(\epsilon+1)}x_t\left((x_t\eta_t P_t^{-\epsilon})^{\frac{1}{\beta}}\right) + \frac{\epsilon}{\beta}P_t^{-\frac{\epsilon+\beta}{\beta}-\epsilon}x_t^{\frac{1+\beta}{\beta}}\eta_t^{\frac{1}{\beta}} \\
\iff &\frac{\epsilon-1}{\epsilon}P_t = x_t^{\frac{1}{\beta}}\eta_t^{\frac{1}{\beta}}P_t^{-\frac{\epsilon}{\beta}} + \frac{1}{\beta}P_t^{-\frac{\epsilon}{\beta}}x_t^{\frac{1}{\beta}}\eta_t^{\frac{1}{\beta}} \\
\iff &P_t^{\frac{\epsilon+\beta}{\beta}} = \frac{\epsilon}{\epsilon-1}\frac{1+\beta}{\beta}x_t^{\frac{1}{\beta}}\eta_t^{\frac{1}{\beta}} \\
\iff &P_t^* = \left[\frac{\epsilon}{\epsilon-1}\frac{1+\beta}{\beta}x_t^{\frac{1}{\beta}}\eta_t^{\frac{1}{\beta}}\right]^{\frac{\beta}{\epsilon+\beta}}
\end{aligned}$$

The optimal price implies an optimal wage of

$$W_t^* = (\eta_t L_t)^{1/\beta} = (\eta_t D(P_t, x_t))^{1/\beta} = (\eta_t x_t)^{1/\beta}(P_t^*)^{-\frac{\epsilon}{\beta}} = (\eta_t x_t)^{1/\beta}\left[\frac{\epsilon}{\epsilon-1}\frac{1+\beta}{\beta}(\eta_t x_t)^{1/\beta}\right]^{\frac{-\epsilon}{\beta+\epsilon}}$$

Optimal price and wage setting yield the constant markup over marginal cost typical for iso-elastic demand functions

$$\frac{P_t^*}{W_t^*} = \frac{P_t^*}{(\eta_t x_t)^{1/\beta}(P_t^*)^{-\frac{\epsilon}{\beta}}} = (P_t^*)^{\frac{\beta+\epsilon}{\beta}}(\eta_t x_t)^{-1/\beta} = \frac{\epsilon}{\epsilon-1}\frac{1+\beta}{\beta}$$

Any shock to the demand shifter x_t or the labor supply shifter η_t leads to the same markup, and thus wages and prices grow at the same rate following a demand or labor supply shock.

G.2 All Constraints Binding

Case 8, where all three constraints are binding, is also straight forward. A short reminder of the Lagrangian's FOCs

$$(1-\epsilon)P_1^{-\epsilon}x_1 + \epsilon P_1^{-(\epsilon+1)}x_1 W_1 + \phi_2\frac{\epsilon}{\beta}P_1^{-\frac{\epsilon+\beta}{\beta}}x_1^{\frac{1}{\beta}}\eta_1^{\frac{1}{\beta}} = 0 \tag{10}$$

$$(1-\epsilon)P_2^{-\epsilon}x_2 + \epsilon P_2^{-(\epsilon+1)}x_2 W_2 + \phi_3\frac{\epsilon}{\beta}P_2^{-\frac{\epsilon+\beta}{\beta}}x_2^{\frac{1}{\beta}}\eta_2^{\frac{1}{\beta}} = 0 \tag{11}$$

$$-x_1 P_1^{-\epsilon} - \phi_1 + \phi_2 = 0 \tag{12}$$

$$-x_2 P_2^{-\epsilon} + \phi_1 + \phi_3 = 0 \tag{13}$$

Reordering equations 12 & 13 to get expressions for ϕ_1 and then setting these equal gives an expression of ϕ_2 in terms of ϕ_3.

$$\begin{aligned}
-x_1 P_1^{-\epsilon} + \phi_2 &= x_2 P_2^{-\epsilon} - \phi_3 \\
\phi_2 &= x_1 P_1^{-\epsilon} + x_2 P_2^{-\epsilon} - \phi_3
\end{aligned}$$

Plugging this into equation 10 gives

$$(1-\epsilon)P_1^{-\epsilon}x_1 + \epsilon P_1^{-(\epsilon+1)}x_1 W_1 + \left[x_1 P_1^{-\epsilon} + x_2 P_2^{-\epsilon} - \phi_3\right]\frac{\epsilon}{\beta}P_1^{-\frac{\epsilon+\beta}{\beta}}x_1^{\frac{1}{\beta}}\eta_1^{\frac{1}{\beta}} = 0 \tag{14}$$

$$\frac{\epsilon-1}{\epsilon}P_1^{-\epsilon}x_1 = P_1^{-(\epsilon+1)}x_1 W_1 + \frac{1}{\beta}P_1^{-\frac{\epsilon+\beta}{\beta}}x_1^{\frac{1}{\beta}}\eta_1^{\frac{1}{\beta}}\left[x_1 P_1^{-\epsilon} + x_2 P_2^{-\epsilon} - \phi_3\right] \tag{15}$$

$$\frac{\epsilon-1}{\epsilon}P_1 = W_1 + \frac{1}{\beta}P_1^{\epsilon-\frac{\epsilon}{\beta}}x_1^{\frac{1}{\beta}}\eta_1^{\frac{1}{\beta}}\left[P_1^{-\epsilon} + \frac{x_2}{x_1}P_2^{-\epsilon} - \frac{\phi_3}{x_1}\right] \tag{16}$$

As all three constraints are binding ($\phi_1, \phi_2, \phi_3 > 0$, $W_2 = W_1$, $W_1 = (x_1\eta_1 P_1^{-\epsilon})^{\frac{1}{\beta}}$, and $W_2 = (x_2\eta_2 P_2^{-\epsilon})^{\frac{1}{\beta}}$) this leads to

$$(x_1\eta_1 P_1^{-\epsilon})^{\frac{1}{\beta}} = (x_2\eta_2 P_2^{-\epsilon})^{\frac{1}{\beta}}$$

In the case of finite β this is equivalent to

$$\begin{aligned} & x_1\eta_1 P_1^{-\epsilon} = x_2\eta_2 P_2^{-\epsilon} \\ \iff & P_2 = \left(\frac{x_1\eta_1}{x_2\eta_2}\right)^{-\frac{1}{\epsilon}} P_1 \end{aligned}$$

Plugging this expression for P_2, $W_1 = (x_1\eta_1 P_1^{-\epsilon})^{\frac{1}{\beta}}$, and $W_1 = W_2$ into 16 yields

$$\begin{aligned} \frac{\epsilon-1}{\epsilon}P_1 &= (x_1\eta_1 P_1^{-\epsilon})^{\frac{1}{\beta}} + \frac{1}{\beta}P_1^{\epsilon-\frac{\epsilon}{\beta}}x_1^{\frac{1}{\beta}}\eta_1^{\frac{1}{\beta}}\left[P_1^{-\epsilon} + \frac{x_2}{x_1}\left(\frac{x_1\eta_1}{x_2\eta_2}\right)P_1^{-\epsilon} - \frac{\phi_3}{x_1}\right] \\ &= x_1^{\frac{1}{\beta}}\eta_1^{\frac{1}{\beta}}P_1^{-\frac{\epsilon}{\beta}} + \frac{1}{\beta}P_1^{\epsilon-\frac{\epsilon}{\beta}}x_1^{\frac{1}{\beta}}\eta_1^{\frac{1}{\beta}}\left[\left(1+\frac{\eta_1}{\eta_2}\right)P_1^{-\epsilon} - \frac{\phi_3}{x_1}\right] \\ &= P_1^{-\frac{\epsilon}{\beta}}x_1^{\frac{1}{\beta}}\eta_1^{\frac{1}{\beta}}\left[1 + \frac{1+\frac{\eta_1}{\eta_2}}{\beta}\right] - \frac{1}{\beta}P_1^{\epsilon-\frac{\epsilon}{\beta}}x_1^{\frac{1}{\beta}}\eta_1^{\frac{1}{\beta}}\frac{\phi_3}{x_1} \\ \iff \frac{\epsilon-1}{\epsilon}P_1^{\frac{\epsilon+\beta}{\beta}}x_1^{-\frac{1}{\beta}}\eta_1^{-\frac{1}{\beta}} &= \frac{\beta+1+\frac{\eta_1}{\eta_2}}{\beta} - \frac{1}{\beta}P_1^{\epsilon}\frac{\phi_3}{x_1} \\ \implies \phi_3 &= (\beta+1+\frac{\eta_1}{\eta_2})P_1^{-\epsilon}x_1 - \beta x_1^{\frac{\beta-1}{\beta}}\eta_1^{-\frac{1}{\beta}}\frac{\epsilon-1}{\epsilon}P_1^{\frac{\epsilon+\beta}{\beta}-\epsilon} \end{aligned}$$

Plugging the expression for ϕ_3, as well as for W_1, W_2, and P_2 into equation 11 finally yields

$$(1-\epsilon)P_1^{-\epsilon}x_1\frac{\eta_1}{\eta_2}+\epsilon\left(\left(\frac{x_1}{x_2}\frac{\eta_1}{\eta_2}\right)^{-\frac{1}{\epsilon}}P_1\right)^{-(\epsilon+1)}x_2(x_1\eta_1P_1^{-\epsilon})^{\frac{1}{\beta}}$$

$$+\left[\left(1+\beta+\frac{\eta_1}{\eta_2}\right)P_1^{-\epsilon}x_1-\beta x_1^{\frac{\beta-1}{\beta}}\eta_1^{-\frac{1}{\beta}}\frac{\epsilon-1}{\epsilon}P_1^{\frac{\epsilon+\beta}{\beta}-\epsilon}\right]\frac{\epsilon}{\beta}\left(\left(\frac{x_1}{x_2}\frac{\eta_1}{\eta_2}\right)^{-\frac{1}{\epsilon}}P_1\right)^{-\frac{\epsilon+\beta}{\beta}}(x_2\eta_2)^{\frac{1}{\beta}}=0$$

$$\iff(\epsilon-1)P_1^{-\epsilon}x_1\frac{\eta_1}{\eta_2}=\epsilon\left(\frac{x_1}{x_2}\frac{\eta_1}{\eta_2}\right)^{\frac{\epsilon+1}{\epsilon}}P_1^{-(\epsilon+1)}x_2x_1^{\frac{1}{\beta}}\eta_1^{\frac{1}{\beta}}P_1^{-\frac{\epsilon}{\beta}}$$

$$+P_1^{-\epsilon}x_1\left[1+\beta+\frac{\eta_1}{\eta_2}-\beta x_1^{-\frac{1}{\beta}}\eta_1^{-\frac{1}{\beta}}\frac{\epsilon-1}{\epsilon}P_1^{\frac{\epsilon+\beta}{\beta}}\right]\frac{\epsilon}{\beta}\left(\frac{x_1}{x_2}\frac{\eta_1}{\eta_2}\right)^{\frac{\epsilon+\beta}{\beta\epsilon}}P_1^{-\frac{\epsilon+\beta}{\beta}}(x_2\eta_2)^{\frac{1}{\beta}}$$

$$\iff\frac{\epsilon-1}{\epsilon}=\left(\frac{x_1}{x_2}\frac{\eta_1}{\eta_2}\right)^{\frac{1}{\epsilon}}x_1^{\frac{1}{\beta}}\eta_1^{\frac{1}{\beta}}P_1^{-\frac{\epsilon+\beta}{\beta}}+\left[1+\beta+\frac{\eta_1}{\eta_2}-\beta x_1^{-\frac{1}{\beta}}\eta_1^{-\frac{1}{\beta}}\frac{\epsilon-1}{\epsilon}P_1^{\frac{\epsilon+\beta}{\beta}}\right]\frac{1}{\beta}\left(\frac{x_1}{x_2}\frac{\eta_1}{\eta_2}\right)^{\frac{1}{\epsilon}}\frac{\eta_2}{\eta_1}P_1^{-\frac{\epsilon+\beta}{\beta}}x_1^{\frac{1}{\beta}}\eta_1^{\frac{1}{\beta}}$$

$$\iff P_1^{\frac{\epsilon+\beta}{\beta}}=\frac{\epsilon}{\epsilon-1}\left(\frac{x_1}{x_2}\frac{\eta_1}{\eta_2}\right)^{\frac{1}{\epsilon}}x_1^{\frac{1}{\beta}}\eta_1^{\frac{1}{\beta}}+\frac{\epsilon}{\epsilon-1}\frac{1+\beta+\frac{\eta_1}{\eta_2}}{\beta}\left(\frac{x_1}{x_2}\frac{\eta_1}{\eta_2}\right)^{\frac{1}{\epsilon}}x_1^{\frac{1}{\beta}}\eta_1^{\frac{1}{\beta}}\frac{\eta_2}{\eta_1}-P_1^{\frac{\epsilon+\beta}{\beta}}\left(\frac{x_1}{x_2}\frac{\eta_1}{\eta_2}\right)^{\frac{1}{\epsilon}}\frac{\eta_2}{\eta_1}$$

$$\iff P_1^{\frac{\epsilon+\beta}{\beta}}\left[1+\left(\frac{x_1}{x_2}\frac{\eta_1}{\eta_2}\right)^{\frac{1}{\epsilon}}\frac{\eta_2}{\eta_1}\right]=\frac{\epsilon}{\epsilon-1}\left(\frac{x_1}{x_2}\frac{\eta_1}{\eta_2}\right)^{\frac{1}{\epsilon}}x_1^{\frac{1}{\beta}}\eta_1^{\frac{1}{\beta}}\left[1+\frac{1+\beta+\frac{\eta_1}{\eta_2}}{\beta}\frac{\eta_2}{\eta_1}\right]$$

$$\iff P_1^{\frac{\epsilon+\beta}{\beta}}\left[1+\left(\frac{x_1}{x_2}\frac{\eta_1}{\eta_2}\right)^{\frac{1}{\epsilon}}\frac{\eta_2}{\eta_1}\right]=\frac{\epsilon}{\epsilon-1}\left(\frac{x_1}{x_2}\frac{\eta_1}{\eta_2}\right)^{\frac{1}{\epsilon}}x_1^{\frac{1}{\beta}}\eta_1^{\frac{1}{\beta}}\left[\frac{1+\beta}{\beta}\left(1+\frac{\eta_1}{\eta_2}\right)\frac{\eta_2}{\eta_1}\right]$$

If $x_1 = x_2$ and $\eta_1 = \eta_2$ this yields the same solution for optimal P_1 as under flexible wage optimization

$$P_1=\left[\frac{2\frac{\epsilon}{\epsilon-1}\frac{1+\beta}{\beta}}{1+1}x_1^{\frac{1}{\beta}}\eta_1^{\frac{1}{\beta}}\right]^{\frac{\beta}{\beta+\epsilon}}=\left[\frac{\epsilon}{\epsilon-1}\frac{1+\beta}{\beta}x_1^{\frac{1}{\beta}}\eta_1^{\frac{1}{\beta}}\right]^{\frac{\beta}{\beta+\epsilon}}$$

In general the optimal period one price is

$$P_1^*=\left[\left(1+\frac{\eta_1}{\eta_2}\right)\frac{\epsilon}{\epsilon-1}\frac{1+\beta}{\beta}x_1^{\frac{1}{\beta}}\eta_1^{\frac{1}{\beta}}\frac{\left(\frac{x_1}{x_2}\frac{\eta_1}{\eta_2}\right)^{\frac{1}{\epsilon}}\frac{\eta_2}{\eta_1}}{1+\left(\frac{x_1}{x_2}\frac{\eta_1}{\eta_2}\right)^{\frac{1}{\epsilon}}\frac{\eta_2}{\eta_1}}\right]^{\frac{\beta}{\beta+\epsilon}}$$

which I derived previously in Section 4.1. I obtain an expression for the direct impact channel by deriving the difference in the percent growth rates of prices and wages under DNWR.

$$\text{Direct Impact} := \frac{\partial \ln(P_1)}{\partial x_1}|_{DNWR} - \frac{\partial \ln(W_1)}{\partial x_1}|_{DNWR}$$

To derive this expression for the direct impact channel I first calculate the derivative of the optimal price under DNWR with respect to x_1.

$$\frac{\partial P_1}{\partial x_1}|_{DNWR} = \frac{\beta}{\beta+\epsilon}\left[\left(1+\frac{\eta_2}{\eta_1}\right)\frac{\epsilon}{\epsilon-1}\frac{1+\beta}{\beta}x_1^{\frac{1}{\beta}}\eta_1^{\frac{1}{\beta}}\frac{\left(\frac{x_1}{x_2}\frac{\eta_1}{\eta_2}\right)^{\frac{1}{\epsilon}}}{1+\left(\frac{x_1}{x_2}\frac{\eta_1}{\eta_2}\right)^{\frac{1}{\epsilon}}\frac{\eta_2}{\eta_1}}\right]^{-\frac{\epsilon}{\beta+\epsilon}}\left(1+\frac{\eta_2}{\eta_1}\right)\frac{\epsilon}{\epsilon-1}\frac{1+\beta}{\beta}\eta_1^{\frac{1}{\beta}}\times$$

$$\left[\frac{1}{\beta}x_1^{\frac{1-\beta}{\beta}}\frac{\left(\frac{x_1}{x_2}\frac{\eta_1}{\eta_2}\right)^{\frac{1}{\epsilon}}}{1+\left(\frac{x_1}{x_2}\frac{\eta_1}{\eta_2}\right)^{\frac{1}{\epsilon}}\frac{\eta_2}{\eta_1}}+x_1^{\frac{1}{\beta}}\frac{\frac{1}{\epsilon}\left(\frac{x_1}{x_2}\frac{\eta_1}{\eta_2}\right)^{\frac{1}{\epsilon}}x_1^{-1}\left(1+\left(\frac{x_1}{x_2}\frac{\eta_1}{\eta_2}\right)^{\frac{1}{\epsilon}}\frac{\eta_2}{\eta_1}\right)-\left(\frac{x_1}{x_2}\frac{\eta_1}{\eta_2}\right)^{\frac{1}{\epsilon}}\frac{1}{\epsilon}\left(\frac{x_1}{x_2}\frac{\eta_1}{\eta_2}\right)^{\frac{1}{\epsilon}}x_1^{-1}}{\left(1+\left(\frac{x_1}{x_2}\frac{\eta_1}{\eta_2}\right)^{\frac{1}{\epsilon}}\frac{\eta_2}{\eta_1}\right)^2}\right]$$

$$=\frac{\beta}{\beta+\epsilon}\left[\left(1+\frac{\eta_2}{\eta_1}\right)\frac{\epsilon}{\epsilon-1}\frac{1+\beta}{\beta}x_1^{\frac{1}{\beta}}\eta_1^{\frac{1}{\beta}}\frac{\left(\frac{x_1}{x_2}\frac{\eta_1}{\eta_2}\right)^{\frac{1}{\epsilon}}}{1+\left(\frac{x_1}{x_2}\frac{\eta_1}{\eta_2}\right)^{\frac{1}{\epsilon}}\frac{\eta_2}{\eta_1}}\right]^{-\frac{\epsilon}{\beta+\epsilon}}\left(1+\frac{\eta_2}{\eta_1}\right)\frac{\epsilon}{\epsilon-1}\frac{1+\beta}{\beta}\eta_1^{\frac{1}{\beta}}x_1^{\frac{1-\beta}{\beta}}\frac{\left(\frac{x_1}{x_2}\frac{\eta_1}{\eta_2}\right)^{\frac{1}{\epsilon}}}{1+\left(\frac{x_1}{x_2}\frac{\eta_1}{\eta_2}\right)^{\frac{1}{\epsilon}}\frac{\eta_2}{\eta_1}}$$

$$\left[\frac{1}{\beta}+\frac{\frac{1}{\epsilon}}{\left(1+\left(\frac{x_1}{x_2}\frac{\eta_1}{\eta_2}\right)^{\frac{1}{\epsilon}}\frac{\eta_2}{\eta_1}\right)}\right]$$

$$=\frac{\beta}{\beta+\epsilon}\left[\left(1+\frac{\eta_2}{\eta_1}\right)\frac{\epsilon}{\epsilon-1}\frac{1+\beta}{\beta}x_1^{\frac{1}{\beta}}\eta_1^{\frac{1}{\beta}}\frac{\left(\frac{x_1}{x_2}\frac{\eta_1}{\eta_2}\right)^{\frac{1}{\epsilon}}}{1+\left(\frac{x_1}{x_2}\frac{\eta_1}{\eta_2}\right)^{\frac{1}{\epsilon}}\frac{\eta_2}{\eta_1}}\right]^{\frac{\beta}{\beta+\epsilon}}\left[\frac{1}{\beta x_1}+\frac{\frac{1}{\epsilon x_1}}{\left(1+\left(\frac{x_1}{x_2}\frac{\eta_1}{\eta_2}\right)^{\frac{1}{\epsilon}}\frac{\eta_2}{\eta_1}\right)}\right]$$

$$=\frac{\beta}{\beta+\epsilon}P_1^*\left[\frac{1}{\beta x_1}+\frac{\frac{1}{\epsilon x_1}}{\left(1+\left(\frac{x_1}{x_2}\frac{\eta_1}{\eta_2}\right)^{\frac{1}{\epsilon}}\frac{\eta_2}{\eta_1}\right)}\right]$$

Bringing P_1 to the other side yields an expression for the relative increase $\frac{\partial P_1}{\partial x_1}|_{DNWR}\frac{1}{P_1} = \frac{\partial \ln(P_1)}{\partial x_1}|_{DNWR}$.

$$\frac{\partial \ln(P_1)}{\partial x_1}|_{DNWR} = \frac{\beta}{\beta+\epsilon}\left[\frac{1}{\beta x_1}+\frac{\frac{1}{\epsilon x_1}}{\left(1+\left(\frac{x_1}{x_2}\frac{\eta_1}{\eta_2}\right)^{\frac{1}{\epsilon}}\frac{\eta_2}{\eta_1}\right)}\right]$$

To describe the direct impact we now need the expression for the percent increase in period one wages $\frac{\partial ln(W_1)}{\partial x_1}$. Using the expression of period one wages in terms of period one prices $W_1 = D_1^{\frac{1}{\beta}} = (x_1\eta_1 P_1^{-\epsilon})^{\frac{1}{\beta}}$, its derivative with respect to x_1 is

$$\begin{aligned}\frac{\partial W_1}{\partial x_1}|_{DNWR} =& \frac{1}{\beta}\left(x_1\eta_1(P_1)^{-\epsilon}\right)^{\frac{1-\beta}{\beta}}\left[\eta_1 P_1^{-\epsilon}-\epsilon x_1\eta_1 P_1^{-(\epsilon+1)}\frac{\partial P_1}{\partial x_1}\right]\\ =&\frac{1}{\beta}W_1^{1-\beta}\left[W_1^{\beta}\frac{1}{x_1}-\epsilon W_1^{\beta}P_1^{-1}\frac{\partial P_1}{\partial x_1}\right]\\ =&\frac{1}{\beta}W_1\left[\frac{1}{x_1}-\epsilon P_1^{-1}\frac{\partial P_1}{\partial x_1}\right]\end{aligned}$$

$$\implies \frac{\partial W_1}{\partial x_1}|_{DNWR}\frac{1}{W_1} = \frac{\partial \ln(W_1)}{\partial x_1}|_{DNWR} = \frac{1}{\beta}\left[\frac{1}{x_1}-\epsilon\frac{\partial \ln(P_1)}{\partial x_1}|_{DNWR}\right]$$

Thus, the difference between inflation and the growth rate of wages is

$$\frac{\partial \ln(P_1)}{\partial x_1}|_{DNWR} - \frac{\partial \ln(W_1)}{\partial x_1}|_{DNWR} = \left(1 + \frac{\epsilon}{\beta}\right) \frac{\partial \ln(P_1)}{\partial x_1}|_{DNWR} - \frac{1}{\beta x_1}$$

under DNWR. This is the percent increase in the price level above and beyond the percent increase in the wage level, which describes the direct impact in the illustrative model.

Plugging in the expression for $\frac{\partial \ln(P_1)}{\partial x_1}|_{DNWR}$ yields:

$$\begin{aligned}
\frac{\partial \ln(P_1)}{\partial x_1}|_{DNWR} - \frac{\partial \ln(W_1)}{\partial x_1}|_{DNWR} &= \frac{\beta + \epsilon}{\beta} \frac{\partial \ln(P_1)}{\partial x_1} - \frac{1}{\beta x_1} \\
&= \frac{1}{\beta x_1} + \frac{\frac{1}{\epsilon x_1}}{\left(1 + \left(\frac{x_1}{x_2}\frac{\eta_1}{\eta_2}\right)^{\frac{1}{\epsilon}} \frac{\eta_2}{\eta_1}\right)} - \frac{1}{\beta x_1} \\
&= \frac{\frac{1}{\epsilon x_1}}{\left(1 + \left(\frac{x_1}{x_2}\frac{\eta_1}{\eta_2}\right)^{\frac{1}{\epsilon}} \frac{\eta_2}{\eta_1}\right)} \geq 0
\end{aligned}$$

The direct impact goes to zero when market power goes to to zero ($\epsilon \to \infty$). Similarly, the direct impact goes to zero as $\beta \to \infty$, as DNWR becomes non-binding and we revert back to the flexible wage solution (see Appendix G for details). Parallel derivations lead to a similar direct impact under a negative labor supply shock $\eta_1 > 1$, which were sizeable during the pandemic period (Goda and Soltas (2023), Kindberg-Hanlon and Girard (2024)).

G.3 DNWR leads to non-binding Labor Supply Constraint in Period 2

Case 6, where $W_2 = W_1$ and $W_1 = (x_1\eta_1 P_1^{-\epsilon})^{\frac{1}{\beta}}$, but $W_2 > (x_2\eta_2 P_2^{-\epsilon})^{\frac{1}{\beta}}$ is a tougher nut to crack. Setting $\phi_3 = 0$ changes the FOCs to

$$\begin{aligned}
(1-\epsilon)P_1^{-\epsilon}x_1 + \epsilon P_1^{-(\epsilon+1)}x_1 W_1 + \phi_2 \frac{\epsilon}{\beta} P_1^{-\frac{\epsilon+\beta}{\beta}} x_1^{\frac{1}{\beta}} \eta_1^{\frac{1}{\beta}} &= 0 \\
(1-\epsilon)P_2^{-\epsilon}x_2 + \epsilon P_2^{-(\epsilon+1)}x_2 W_2 &= 0 \\
-x_1 P_1^{-\epsilon} - \phi_1 + \phi_2 &= 0 \\
-x_2 P_2^{-\epsilon} + \phi_1 &= 0
\end{aligned}$$

This yields

$$\begin{aligned}
\phi_1 &= x_2 P_2^{-\epsilon} \\
\phi_2 &= x_1 P_1^{-\epsilon} + x_2 P_2^{-\epsilon} \\
P_2 &= \frac{\epsilon}{\epsilon - 1} W_2
\end{aligned}$$

Putting these and the binding constraints into the FOC with respect to P_1 gives

$$(1-\epsilon)P_1^{-\epsilon}x_1 + \epsilon P_1^{-(\epsilon+1)}x_1 W_1 + \phi_2 \frac{\epsilon}{\beta} P_1^{-\frac{\epsilon+\beta}{\beta}} x_1^{\frac{1}{\beta}} \eta_1^{\frac{1}{\beta}} = 0$$

$$\frac{\epsilon-1}{\epsilon}P_1^{-\epsilon}x_1 = x_1^{\frac{1+\beta}{\beta}} \eta_1^{\frac{1}{\beta}} P_1^{-(\epsilon+1)-\frac{\epsilon}{\beta}} + \frac{1}{\beta} P_1^{-\frac{\epsilon+\beta}{\beta}} x_1^{\frac{1}{\beta}} \eta_1^{\frac{1}{\beta}} \left[x_1 P_1^{-\epsilon} + x_2 P_2^{-\epsilon}\right]$$

$$\frac{\epsilon-1}{\epsilon}P_1 = x_1^{\frac{1}{\beta}} \eta_1^{\frac{1}{\beta}} P_1^{-\frac{\epsilon}{\beta}} + \frac{1}{\beta} P_1^{\epsilon-\frac{\epsilon}{\beta}} x_1^{\frac{1}{\beta}} \eta_1^{\frac{1}{\beta}} \left[P_1^{-\epsilon} + \frac{x_2}{x_1} P_2^{-\epsilon}\right]$$

$$\frac{\epsilon-1}{\epsilon}P_1^{\frac{\epsilon+\beta}{\beta}} = x_1^{\frac{1}{\beta}} \eta_1^{\frac{1}{\beta}} \left(1 + \frac{1}{\beta}\left[1 + P_1^{\epsilon} \frac{x_2}{x_1}\left(\frac{\epsilon}{\epsilon-1}\right)^{-\epsilon} W_2^{-\epsilon}\right]\right)$$

$$\frac{\epsilon-1}{\epsilon}P_1^{\frac{\epsilon+\beta}{\beta}} = x_1^{\frac{1}{\beta}} \eta_1^{\frac{1}{\beta}} \left(1 + \frac{1}{\beta}\left[1 + P_1^{\epsilon} \frac{x_2}{x_1}\left(\frac{\epsilon}{\epsilon-1}\right)^{-\epsilon} \left((x_1\eta_1 P_1^{-\epsilon})^{\frac{1}{\beta}}\right)^{-\epsilon}\right]\right)$$

$$\frac{\epsilon-1}{\epsilon}P_1^{\frac{\epsilon+\beta}{\beta}} = x_1^{\frac{1}{\beta}} \eta_1^{\frac{1}{\beta}} \left(1 + \frac{1}{\beta}\left[1 + P_1^{\epsilon} \frac{x_2}{x_1}\left(\frac{\epsilon}{\epsilon-1}\right)^{-\epsilon} x_1^{-\frac{\epsilon}{\beta}} \eta_1^{-\frac{\epsilon}{\beta}} P_1^{\frac{\epsilon^2}{\beta}}\right]\right)$$

$$\frac{\epsilon-1}{\epsilon}P_1^{\frac{\epsilon+\beta}{\beta}} = x_1^{\frac{1}{\beta}} \eta_1^{\frac{1}{\beta}} \left(\frac{1+\beta}{\beta} + \frac{1}{\beta}\frac{x_2}{x_1}\left(\frac{\epsilon}{\epsilon-1}\right)^{-\epsilon} x_1^{-\frac{\epsilon}{\beta}} \eta_1^{-\frac{\epsilon}{\beta}} P_1^{\epsilon\frac{\epsilon+\beta}{\beta}}\right)$$

which is an implicit function for optimal price setting in Case 6. Due to the implicit nature of optimal Case 6 behavior I need to simulate it numerically for different parameter values (ϵ, β, x_1), simplifying the simulation grid by setting $x_2 = 1$, $\eta_1 = \eta_2 = 1$. Thus, I simulate period one demand shocks from a no shock steady state in which all multipliers are equal to one. However, before jumping into the numeric simulation it is worth deriving the conditions under which the three different cases hold.

G.4 When is which Case optimal?

The three non-degenerate cases hold under different conditions. For example in the flexible wage case (0,>0,>0), where $W_1 = (x_1\eta_1 P_1^{-\epsilon})^{\frac{1}{\beta}}$ & $W_2 = (x_2\eta_2 P_2^{-\epsilon})^{\frac{1}{\beta}}$, the KKT conditions still specify that $W_2 \geq W_1$. Otherwise, the DNWR constraint is binding and we go to either case 8: (>0,>0,>0), where all three conditions bind, or case 6: (>0,>0,0), where DNWR binds and lifts period two wages above the level needed to satisfy the labor supply constraint $W_2 \geq (x_2\eta_2 P_2^{-\epsilon})^{\frac{1}{\beta}}$.

So when is DNWR binding? This can be derived by looking at the flexible wage case, where optimal price setting is characterized by

$$P_t = \left[\frac{\epsilon}{\epsilon-1}\frac{1+\beta}{\beta} x_t^{\frac{1}{\beta}} \eta_t^{\frac{1}{\beta}}\right]^{\frac{\beta}{\epsilon+\beta}}$$

For DNWR to be non-binding, we need $W_2 \geq W_1$. Given that both labor supply con-

straints are binding in the flexible wage case this is equivalent to

$$\begin{aligned}
(x_2\eta_2 P_2^{-\epsilon})^{\frac{1}{\beta}} &\geq (x_1\eta_1 P_1^{-\epsilon})^{\frac{1}{\beta}} \\
\iff x_2\eta_2 P_2^{-\epsilon} &\geq x_1\eta_1 P_1^{-\epsilon} \\
\iff x_2\eta_2 \left[\frac{\epsilon}{\epsilon-1}\frac{1+\beta}{\beta}x_2^{\frac{1}{\beta}}\eta_2^{\frac{1}{\beta}}\right]^{-\frac{\epsilon\beta}{\epsilon+\beta}} &\geq x_1\eta_1 \left[\frac{\epsilon}{\epsilon-1}\frac{1+\beta}{\beta}x_1^{\frac{1}{\beta}}\eta_1^{\frac{1}{\beta}}\right]^{-\frac{\epsilon\beta}{\epsilon+\beta}} \\
\iff (x_2\eta_2)^{\frac{\beta}{\epsilon+\beta}} &\geq (x_1\eta_1)^{\frac{\beta}{\epsilon+\beta}} \\
\iff x_2\eta_2 &\geq x_1\eta_1
\end{aligned}$$

The first step of the derivation makes the assumption that $\beta < \infty$. The equality holds trivially if $\beta \to \infty$. Thus, if the labor supply elasticity goes to zero DNWR is never binding, no matter what relative size the shocks in period one and two are. Outside zero labor supply elasticity DNWR is binding if $x_1 > x_2$, in absence of labor supply shocks ($\eta_1 = \eta_2 = 1$). Similarly, if $\eta_1 > \eta_2$, in absence of demand shocks, or when the combined wage pressure from demand and labor supply shocks is greater in period one than in period two.

The question of which non-degenerate DNWR case (6 or 8) we would enter is trickier. This depends on if the labor supply constraint is binding under optimal price setting or not. We have to start from Case 6, where the labor supply constraint is not binding, and check when optimal pricing leads to a binding constraint in Case 6. Remember that Case 6 gave us an implicit function for optimal period one price P_1.

$$\frac{\epsilon-1}{\epsilon}P_1^{\frac{\epsilon+\beta}{\beta}} = (x_1\eta_1)^{\frac{1}{\beta}}\left(\frac{1+\beta}{\beta} + \frac{1}{\beta}\frac{x_2}{x_1}\left(\frac{\epsilon}{\epsilon-1}\right)^{-\epsilon}(x_1\eta_1)^{-\frac{\epsilon}{\beta}}P_1^{\epsilon\frac{\epsilon+\beta}{\beta}}\right)$$

It also gave us optimal P_2

$$P_2 = \frac{\epsilon}{\epsilon-1}W_2$$

and of course period one wage W_1 by constraint.

$$W_1 = (x_1\eta_1 P_1^{-\epsilon})^{\frac{1}{\beta}}$$

Among the KKT conditions is the specification for the non-binding of period two's labor supply constraint.

$$W_2 \geq (x_2\eta_2 P_2^{-\epsilon})^{\frac{1}{\beta}}$$

Given the binding of DNWR in Case 6 ($W_2 = W_1$), we can bring these together to gain a

condition on P_1.

$$(x_1\eta_1)^{\frac{1}{\beta}}P_1^{-\frac{\epsilon}{\beta}} = W_1 = W_2 \geq (x_2\eta_2 P_2^{-\epsilon})^{\frac{1}{\beta}} = (x_2\eta_2)^{\frac{1}{\beta}}\left(\frac{\epsilon}{\epsilon-1}W_2\right)^{-\frac{\epsilon}{\beta}} = (x_2\eta_2)^{\frac{1}{\beta}}\left(\frac{\epsilon}{\epsilon-1}\right)^{-\frac{\epsilon}{\beta}}(x_1\eta_1)^{-\frac{\epsilon}{\beta^2}}P_1^{\frac{\epsilon^2}{\beta^2}}$$

$$\iff \left(\frac{x_1\eta_1}{x_2\eta_2}\right)^{\frac{1}{\beta}}P_1^{-\frac{\epsilon}{\beta}\left(1+\frac{\epsilon}{\beta}\right)} \geq \left(\frac{\epsilon}{\epsilon-1}\right)^{-\frac{\epsilon}{\beta}}(x_1\eta_1)^{-\frac{\epsilon}{\beta^2}}$$

$$\iff P_1^{\frac{\epsilon}{\beta}\left(\frac{\epsilon+\beta}{\beta}\right)} \leq \left(\frac{x_1\eta_1}{x_2\eta_2}\right)^{\frac{1}{\beta}}\left(\frac{\epsilon}{\epsilon-1}\right)^{\frac{\epsilon}{\beta}}(x_1\eta_1)^{\frac{\epsilon}{\beta^2}}$$

$$\iff P_1 \leq \left[\frac{x_1\eta_1}{x_2\eta_2}\left(\frac{\epsilon}{\epsilon-1}\right)^{\epsilon}(x_1\eta_1)^{\frac{\epsilon}{\beta}}\right]^{\frac{\beta}{\epsilon(\epsilon+\beta)}}$$

$$\iff P_1 \leq \left[\left(\frac{x_1\eta_1}{x_2\eta_2}\right)^{\frac{1}{\epsilon}}\frac{\epsilon}{\epsilon-1}(x_1\eta_1)^{\frac{1}{\beta}}\right]^{\frac{\beta}{\epsilon+\beta}}$$

Hence, for period two's labor supply constraint to remain non-binding in Case 6, the optimal period one price cannot be larger than a level that depends on the relative size of period one and two demand and labor supply shocks, as well as parameter values ϵ and β. If the optimal period one price in Case 6 were to be higher than this constraint, we would leave Case 6 and enter Case 8 where the constraint is binding. Thus, we need to check under what parameter values this constraint is congruent with the implicit function describing optimal pricing behavior in Case 6.

G.5 Numerical Simulation of Case 6

I now proceed in presenting the numerical results for Case 6. I simplify the grid by setting $x_2 = 1$, $\eta_1 = \eta_2 = 1$. I.e. I look at period one demand shocks from a no shock steady state in which all multipliers are equal to one. We have two criteria for sorting out (ϵ, β, x_1)-parametrization cases. First, cases in which the optimal prize violates period two's labor supply constraint. Second, cases in which period two's labor supply constraint remains non-binding, but deliver profits below those of Case 8 under the same parametrization.

To explore the different sub-cases of Case 6 I construct a "sieve". I start with a (β, ϵ, x_1) grid of parameter values in $[0.1, 10] \times [1.01, 11] \times [1, 2]$. I approximate optimal Case 6 profits for each of these grid points. I then check if period two's labor supply constraint is violated by checking if the profit maximizing price is greater than the maximum period one price derived above. I flag those that breach the maximal price level. I then take all non-flagged parameter combinations and calculate optimal Case 8 profits under these parameters. If Case 8 profits are larger than the optimal Case 6 profits I flag those grid points as well.

The sieve funneled the 124950 ($49 \times 50 \times 51$) grid points as follows. 121164 violated period two's labor supply constraint and thus lead to Case 8. Of the 3786 that obtained a maximum without violating the labor supply constraint 1388 delivered strictly greater profits in Case 6 than under Case 8 with the same parameter values.[42] Similar results are obtained under a

[42] 127 cases were clearly dominated by Case 8, and 2271 deviated by less than 0.1% from Case 8 profits.

negative labor supply shock.[43]

Do the optimal Case 6 parameter combinations yield a positive direct impact? I approximate the percentage change in the price level minus the percentage change in the wage level, as the direct impact measure. I initiate this approximation at the flexible wage steady state ($x_1 = x_2 = \eta_1 = \eta_2 = 1$) for each parameter set (ϵ, β, x_1).

$$DIC = \frac{P_1^* - P_1^{flex}}{P_1^{flex}} - \frac{W_1(P_1^*) - W_1^{flex}}{W_1^{flex}}$$

The resulting approximations of the direct impact channel (DIC) are all sizable and positive as expected. This is also true for negative period one labor supply shocks $\eta_1 > 1$.

G.6 Interaction of Future Demand and Labor Market Tightness

To wrap up I derive the impact of a period 2 demand multiplier x_2 increase on optimal price setting under DNWR. An increase in future demand is moving towards relaxing the DNWR constraint and thus decreasing the positive wedge between prices and wages stemming from the direct impact channel. This approximates the interaction term of future expected real sales growth with the "hard to hire" measure from the NFIB in the illustrative model. A tight labor market in which it is hard to hire is approximated in the model with $x_1 > x_2 = 1$, or $\eta_1 > \eta_2 = 1$ in the model. Thus, DNWR is binding in period 2 and we are in either Case 8 or 6.

How is optimal price setting under future DNWR changed by an increase in future demand x_2? I start with the analytical solution to Case 8 and will discuss the numeric solutions to case 6 afterwards. For simplicity I focus on demand shocks, setting labor supply shocks to zero ($\eta_1 = \eta_2 = 1$). In Case 8 the optimal period one price is then

$$P_1^* = \left[2\frac{\epsilon}{\epsilon - 1}\frac{1+\beta}{\beta} x_1^{\frac{1}{\beta}} \frac{\left(\frac{x_1}{x_2}\right)^{\frac{1}{\epsilon}}}{1 + \left(\frac{x_1}{x_2}\right)^{\frac{1}{\epsilon}}} \right]^{\frac{\beta}{\beta+\epsilon}}$$

The interaction term is described by

$$\frac{\partial \ln(P_1)}{\partial x_2}|_{DNWR} - \frac{\partial \ln(W_1)}{\partial x_2}|_{DNWR}$$

To obtain an expression for the interaction term I first calculate the derivative of the optimal

[43] 122317 grid points lead to a violations of the labor supply constraint and thus to Case 8. Another 529 lead to indistinguishable profits from Case 8, and 2104 lead to optimal Case 6 solutions.

price under DNWR with respect to x_2.

$$\frac{\partial P_1^*}{\partial x_2} = \frac{\beta}{\beta+\epsilon}\left[2\frac{\epsilon}{\epsilon-1}\frac{1+\beta}{\beta}x_1^{\frac{1}{\beta}}\frac{\left(\frac{x_1}{x_2}\right)^{\frac{1}{\epsilon}}}{1+\left(\frac{x_1}{x_2}\right)^{\frac{1}{\epsilon}}}\right]^{-\frac{\epsilon}{\beta+\epsilon}} 2\frac{\epsilon}{\epsilon-1}\frac{1+\beta}{\beta}x_1^{\frac{1}{\beta}}\times$$
$$\left[\frac{-\frac{1}{\epsilon}x_2^{-1}\left(\frac{x_1}{x_2}\right)^{\frac{1}{\epsilon}}\left(1+\left(\frac{x_1}{x_2}\right)^{\frac{1}{\epsilon}}\right)+\frac{1}{\epsilon}x_2^{-1}\left(\frac{x_1}{x_2}\right)^{\frac{1}{\epsilon}}\left(\frac{x_1}{x_2}\right)^{\frac{1}{\epsilon}}}{\left(1+\left(\frac{x_1}{x_2}\right)^{\frac{1}{\epsilon}}\right)^2}\right]$$
$$= \frac{\beta}{\beta+\epsilon}\left[2\frac{\epsilon}{\epsilon-1}\frac{1+\beta}{\beta}x_1^{\frac{1}{\beta}}\frac{\left(\frac{x_1}{x_2}\right)^{\frac{1}{\epsilon}}}{1+\left(\frac{x_1}{x_2}\right)^{\frac{1}{\epsilon}}}\right]^{-\frac{\epsilon}{\beta+\epsilon}} 2\frac{\epsilon}{\epsilon-1}\frac{1+\beta}{\beta}x_1^{\frac{1}{\beta}}\left(-\frac{1}{\epsilon x_2}\right)\frac{\left(\frac{x_1}{x_2}\right)^{\frac{1}{\epsilon}}}{\left(1+\left(\frac{x_1}{x_2}\right)^{\frac{1}{\epsilon}}\right)^2}$$
$$= \frac{\beta}{\beta+\epsilon}\left[2\frac{\epsilon}{\epsilon-1}\frac{1+\beta}{\beta}x_1^{\frac{1}{\beta}}\frac{\left(\frac{x_1}{x_2}\right)^{\frac{1}{\epsilon}}}{1+\left(\frac{x_1}{x_2}\right)^{\frac{1}{\epsilon}}}\right]^{\frac{\beta}{\beta+\epsilon}}\left(-\frac{1}{\epsilon x_2}\right)\frac{1}{1+\left(\frac{x_1}{x_2}\right)^{\frac{1}{\epsilon}}}$$
$$= -\frac{\beta}{\beta+\epsilon}\frac{1}{\epsilon x_2}P_1^*\frac{1}{1+\left(\frac{x_1}{x_2}\right)^{\frac{1}{\epsilon}}} < 0$$

or in relative terms

$$\frac{\partial \ln(P_1^*)}{\partial x_2} = -\frac{\beta}{\beta+\epsilon}\frac{1}{\epsilon x_2}\frac{1}{1+\left(\frac{x_1}{x_2}\right)^{\frac{1}{\epsilon}}} < 0$$

which is negative as $\epsilon, \beta, x_1, x_2 > 0$.

To describe the interaction term we now need the expression for the percent increase in period one wages $\frac{\partial ln(W_1)}{\partial x_2}$. Using the expression of period one wages in terms of period one prices $W_1 = W_2 = (x_2 P_2^{-\epsilon})^{\frac{1}{\beta}}$, its derivative with respect to x_2 is

$$\begin{aligned}\frac{\partial W_1}{\partial x_2}|_{DNWR} =& \frac{1}{\beta}\left(x_2(P_2)^{-\epsilon}\right)^{\frac{1-\beta}{\beta}}\left[P_2^{-\epsilon} - \epsilon x_2 P_2^{-(\epsilon+1)}\frac{\partial P_2}{\partial x_2}\right]\\ =& \frac{1}{\beta}W_1^{1-\beta}\left[W_1^{\beta}\frac{1}{x_2} - \epsilon W_1^{\beta}P_2^{-1}\frac{\partial P_2}{\partial x_2}\right]\\ =& \frac{1}{\beta}W_1\left[\frac{1}{x_2} - \epsilon P_2^{-1}\frac{\partial P_2}{\partial x_2}\right]\end{aligned}$$

Using the fact that $P_2 = \left(\frac{x_1}{x_2}\right)^{-\frac{1}{\epsilon}} P_1$ in Case 8, we can derive

$$\frac{\partial P_2}{\partial x_2} = \frac{1}{\epsilon x_2}\left(\frac{x_1}{x_2}\right)^{-\frac{1}{\epsilon}} P_1 + \left(\frac{x_1}{x_2}\right)^{-\frac{1}{\epsilon}}\frac{\partial P_1}{\partial x_2}$$

$$
\begin{aligned}
\implies \frac{\partial W_1}{\partial x_2}|_{DNWR}\frac{1}{W_1} &= \frac{\partial \ln(W_1)}{\partial x_2}|_{DNWR} \\
&= \frac{1}{\beta}\left[\frac{1}{x_2} - \epsilon\left(\frac{x_1}{x_2}\right)^{\frac{1}{\epsilon}} P_1^{-1}\left(\frac{1}{\epsilon x_2}\left(\frac{x_1}{x_2}\right)^{-\frac{1}{\epsilon}} P_1 + \left(\frac{x_1}{x_2}\right)^{-\frac{1}{\epsilon}}\frac{\partial P_1}{\partial x_2}\right)\right] \\
&= \frac{1}{\beta}\left[-\epsilon P_1^{-1}\frac{\partial P_1}{\partial x_2}\right] \\
&= -\frac{\epsilon}{\beta}\frac{\partial \ln(P_1)}{\partial x_2}
\end{aligned}
$$

Thus, the difference between inflation and the growth rate of wages is

$$
\frac{\partial \ln(P_1)}{\partial x_2}|_{DNWR} - \frac{\partial \ln(W_1)}{\partial x_2}|_{DNWR} = \left(1 + \frac{\epsilon}{\beta}\right)\frac{\partial \ln(P_1)}{\partial x_2}|_{DNWR}
$$

under DNWR. This is the percent change in the price level above and beyond the percent increase in the wage level caused by an increase in expected future demand x_2. It measures how the direct impact caused by DNWR binding in period 2 is changing when future demand increases and moves the firm's situation towards non-binding DNWR.

Plugging in the expression for $\frac{\partial \ln(P_1)}{\partial x_2}|_{DNWR}$ yields

$$
\begin{aligned}
\frac{\partial \ln(P_1)}{\partial x_2}|_{DNWR} - \frac{\partial \ln(W_1)}{\partial x_2}|_{DNWR} &= \frac{\beta + \epsilon}{\beta}\frac{\partial \ln(P_1)}{\partial x_2} \\
&= -\frac{\frac{1}{\epsilon x_1}}{\left(1 + \left(\frac{x_1}{x_2}\right)^{\frac{1}{\epsilon}}\right)} \leq 0
\end{aligned}
$$

The direct impact decreases with rising period 2 demand as it moves towards non-binding DNWR. This negative impact from expected future demand under tight labor markets goes to zero when market power goes to to zero (i.e. $\epsilon \to \infty$), as the direct impact itself vanishes under zero market power. The same is true for $\beta \to \infty$, as DNWR becomes non-binding under perfectly rigid wages. Parallel derivations also lead to a falling direct impact under a negative labor supply shock $\eta_2 > 1$.

Thus, the interaction term of tight labor markets, leading to DNWR in future periods, with expected future sales growth should be negative in the NFIB firm survey data. This plays out exactly in the data (see 4.3).

In the side case where period two's labor supply constraint is not binding (Case 6) the interaction term is actually positive. This is due the odd nature of this case (high $\frac{x_1}{x_2} > 1.5$, high market power $\epsilon < 1.8$, and high elasticity of labor supply $\beta > 4$), which leads to a disregard of period 2 in face of very high profits from period one. This, leads to a diminishing of the direct impact channel as $\frac{x_1}{x_2}$ grows beyond the ratio of 1.5. In this region an increase in x_2 shifts focus back to period 2 and thus increases the direct impact channel. This leads to higher inflation under increased real growth expectations.

This is a weird finding when pairing it with reality. Most estimations of discount factors of the future are in the $[0.9, 0.99]$ range, which puts a weight of 10 to 100 on future periods

relative to today's quarter. In my illustrative model they have equal weight. Shifting more weight onto period 2, as the collective measure of the future, would increase the necessary $\frac{x_1}{x_2}$ ratio to reach the state where current profits are so large relative to future profits that future periods lose weight in the optimal price setting decision.

I run this experiment by putting 0.1 and 0.9 weights on period one and two profits respectively, and re-running the numeric estimation. Under this specification there are no feasible solutions in my initial parameter set ($(\beta, \epsilon, x_1) \in [0.1, 10] \times [1.01, 11] \times [1, 2]$) where the labor supply constraint in period two remains non-binding. Only when the range of demand multipliers is extended to $x_1 \in [1, 10]$ do feasible solutions show up again for $x_1 > 2.4$.

G.7 Translate Illustrative Model into V/U terms

I begin by simple assumptions, including that of no labor supply shocks ($\eta = 1$), and parameter values of $\epsilon = 2$, $\beta = 1$. First, I assume that the firm has to hire every employee again each period, but that the labor market is frictionless. Thus, the number of vacancies posted equals the number of workers employed each period.

$$V = L$$

I assume a maximum amount of workers N, which allows me to describe unemployment

$$U = N - L = N - V$$

Therefore

$$V/U = \frac{L}{N - L} = \frac{\eta_t x_t P_t^{-\epsilon}}{N - \eta_t x_t P_t^{-\epsilon}}$$

Labor market tightness depends on aggregate demand, labor supply shocks, and the price level. The latter is different between the flexible wage case and the case where DNWR is binding in period two. In the flexible wage case

$$P_1 = \left(\frac{\epsilon}{\epsilon - 1} \frac{1 + \beta}{\beta} (\eta_1 x_1)^{1/\beta} \right)^{\frac{\beta}{\epsilon + \beta}} = 4^{1/3} x_1^{1/3}$$

which implies

$$W_1 = (\eta_1 x_1)^{1/\beta} \left(\frac{\epsilon}{\epsilon - 1} \frac{1 + \beta}{\beta} (\eta_1 x_1)^{1/\beta} \right)^{\frac{\beta}{\epsilon + \beta}} = 4^{1/3} x_1^{1/3}$$

$$(V/U)_1 = \frac{\eta_1 x_1 \left(\frac{\epsilon}{\epsilon - 1} \frac{1+\beta}{\beta} (\eta_1 x_1)^{1/\beta} \right)^{-\frac{\epsilon\beta}{\epsilon+\beta}}}{N - \eta_1 x_1 \left(\frac{\epsilon}{\epsilon - 1} \frac{1+\beta}{\beta} (\eta_1 x_1)^{1/\beta} \right)^{-\frac{\epsilon\beta}{\epsilon+\beta}}} = \frac{x_1^{\frac{1}{3}} 4^{-\frac{2}{3}}}{N - x_1^{\frac{1}{3}} 4^{-\frac{2}{3}}}$$

I linearize around the steady state by first calculating the derivative of $P_1, W_1, V/U_1$ with respect to x_1.

$$\frac{\partial P_1}{\partial x_1} = \frac{1}{3} 4^{\frac{1}{3}}$$

$$\frac{\partial W_1}{\partial x_1} = \frac{1}{3} 4^{-\frac{2}{3}}$$

$$\frac{\partial (V/U)_1}{\partial x_1} = \frac{\frac{1}{3}x_1^{-\frac{2}{3}}4^{-\frac{2}{3}}(N - x_1^{\frac{1}{3}}4^{-\frac{2}{3}}) + x_1^{\frac{1}{3}}4^{-\frac{2}{3}}\frac{1}{3}x_1^{-\frac{2}{3}}4^{-\frac{2}{3}}}{(N - x_1^{\frac{1}{3}}4^{-\frac{2}{3}})^2} = \frac{N\frac{1}{3}x_1^{-\frac{2}{3}}4^{-\frac{2}{3}}}{(N - x_1^{\frac{1}{3}}4^{-\frac{2}{3}})^2} = \frac{N\frac{1}{3}4^{-\frac{2}{3}}}{(N - 4^{-\frac{2}{3}})^2}$$

Thus, under flexible wages the linearized impact of a demand shock is

$$P_1 \approx 4^{\frac{1}{3}} + \frac{1}{3}4^{\frac{1}{3}}\Delta x_1$$

$$W_1 \approx 4^{-\frac{2}{3}} + \frac{1}{3}4^{-\frac{2}{3}}\Delta x_1$$

$$(V/U)_1 \approx= \frac{4^{-\frac{2}{3}}}{N - 4^{-\frac{2}{3}}} + \frac{N\frac{1}{3}4^{-\frac{2}{3}}}{(N - 4^{-\frac{2}{3}})^2}\Delta x_1$$

I normalize steady state $V/U = 1$, which gives $N \approx 0.8$. In growth rates:

$$\frac{\partial \ln(P_1)}{\partial x_1}\Delta x_1 = \frac{1}{\beta + \epsilon}\Delta x_1 = \frac{1}{3}\Delta x_1$$

$$\frac{\partial \ln(W_1)}{\partial x_1}\Delta x_1 = \frac{1}{\beta + \epsilon}\Delta x_1 = \frac{1}{3}\Delta x_1$$

Translating this into V/U terms by using the relation $\Delta(V/U)_1 \approx \frac{N\frac{1}{3}4^{-\frac{2}{3}}}{(N-4^{-\frac{2}{3}})^2}\Delta x_1$ we get

$$\frac{\partial \ln(W_1)}{\partial x_1}\Delta x_1 = \frac{\partial \ln(P_1)}{\partial x_1}\Delta x_1 \approx \frac{1}{3}\frac{34^{\frac{2}{3}}(N - 4^{-\frac{2}{3}})^2}{N}\Delta(V/U)_1 = \frac{4^{\frac{2}{3}}(N - 4^{-\frac{2}{3}})^2}{N}\Delta(V/U)_1$$

Under binding DNWR in period two optimal price is set to

$$P_1 = \left(\frac{\epsilon}{\epsilon - 1}\frac{1+\beta}{\beta}(\eta_1 x_1)^{1/\beta}2\frac{\left(\frac{x_1}{x_2}\right)^{\frac{1}{\epsilon}}}{1 + \left(\frac{x_1}{x_2}\right)^{\frac{1}{\epsilon}}}\right)^{\frac{\beta}{\epsilon+\beta}} = x_1^{\frac{1}{2}}8^{\frac{1}{3}}\left(\frac{1}{1 + x_1^{\frac{1}{2}}}\right)^{\frac{1}{3}}$$

leading to

$$W_1 = (\eta_1 x_1)^{1/\beta}\left(\frac{\epsilon}{\epsilon - 1}\frac{1+\beta}{\beta}(\eta_1 x_1)^{1/\beta}2\frac{\left(\frac{x_1}{x_2}\right)^{\frac{1}{\epsilon}}}{1 + \left(\frac{x_1}{x_2}\right)^{\frac{1}{\epsilon}}}\right)^{-\frac{\epsilon\beta}{\epsilon+\beta}} = 8^{-\frac{2}{3}}\left(1 + x_1^{\frac{1}{2}}\right)^{\frac{2}{3}}$$

$$(V/U)_1 = \frac{x_1\left(\frac{\epsilon}{\epsilon-1}\frac{1+\beta}{\beta}(\eta_1 x_1)^{1/\beta}2\frac{\left(\frac{x_1}{x_2}\right)^{\frac{1}{\epsilon}}}{1+\left(\frac{x_1}{x_2}\right)^{\frac{1}{\epsilon}}}\right)^{-\frac{\epsilon\beta}{\epsilon+\beta}}}{N - x_1\left(\frac{\epsilon}{\epsilon-1}\frac{1+\beta}{\beta}(\eta_1 x_1)^{1/\beta}2\frac{\left(\frac{x_1}{x_2}\right)^{\frac{1}{\epsilon}}}{1+\left(\frac{x_1}{x_2}\right)^{\frac{1}{\epsilon}}}\right)^{-\frac{\epsilon\beta}{\epsilon+\beta}}} = \frac{8^{-\frac{2}{3}}\left(1 + x_1^{\frac{1}{2}}\right)^{\frac{2}{3}}}{N - 8^{-\frac{2}{3}}\left(1 + x_1^{\frac{1}{2}}\right)^{\frac{2}{3}}}$$

Linearizing around steady state

$$\frac{\partial P_1}{\partial x_1} = 2^{-\frac{1}{3}}\frac{5}{6} \implies P_1 \approx 4^{\frac{1}{3}} + 2^{-\frac{1}{3}}\frac{5}{6}\Delta x_1$$

$$\frac{\partial W_1}{\partial x_1} = \frac{1}{6}4^{-\frac{2}{3}} \implies W_1 \approx 4^{-\frac{2}{3}} + \frac{1}{6}4^{-\frac{2}{3}}\Delta x_1$$

$$\frac{\partial (V/U)_1}{\partial x_1} = \frac{8^{-2/3}\frac{2}{3}(1+x_1^{1/2})^{-\frac{1}{3}}\frac{1}{2}x_1^{-\frac{1}{2}}\left[N - 8^{-\frac{2}{3}}\left(1+x_1^{\frac{1}{2}}\right)^{\frac{2}{3}}\right] + 8^{-2/3}\frac{2}{3}(1+x_1^{1/2})^{-\frac{1}{3}}\frac{1}{2}x_1^{-\frac{1}{2}}8^{-\frac{2}{3}}\left(1+x_1^{\frac{1}{2}}\right)^{\frac{2}{3}}}{\left[N - 8^{-\frac{2}{3}}\left(1+x_1^{\frac{1}{2}}\right)^{\frac{2}{3}}\right]^2}$$

$$= \frac{\frac{N}{12}2^{-\frac{1}{3}}}{\left[N - 4^{-\frac{2}{3}}\right]^2} \implies (V/U)_1 \approx \frac{4^{-\frac{2}{3}}}{N - 4^{-\frac{2}{3}}} + \frac{\frac{N}{12}2^{-\frac{1}{3}}}{\left[N - 4^{-\frac{2}{3}}\right]^2}\Delta x_1$$

In growth rates

$$\frac{\partial \ln(P_1)}{\partial x_1}\Delta x_1 = \frac{5}{12}\Delta x_1$$

$$\frac{\partial \ln(W_1)}{\partial x_1}\Delta x_1 = \frac{1}{6}\Delta x_1$$

Translating this into V/U terms by using the relation $\Delta(V/U)_1 \approx \frac{\frac{N}{12}2^{-\frac{1}{3}}}{\left[N-4^{-\frac{2}{3}}\right]^2}\Delta x_1$ we get

$$\frac{\partial \ln(P_1)}{\partial x_1}\Delta x_1 \approx \frac{5}{12}\frac{12\left[N - 4^{-\frac{2}{3}}\right]^2 2^{\frac{1}{3}}}{N}\Delta(V/U)_1 = 5\frac{\left[N - 4^{-\frac{2}{3}}\right]^2 2^{\frac{1}{3}}}{N}\Delta(V/U)_1$$

$$\frac{\partial \ln(W_1)}{\partial x_1}\Delta x_1 \approx \frac{1}{6}\frac{12\left[N - 4^{-\frac{2}{3}}\right]^2 2^{\frac{1}{3}}}{N}\Delta(V/U)_1 = 2\frac{\left[N - 4^{-\frac{2}{3}}\right]^2 2^{\frac{1}{3}}}{N}\Delta(V/U)_1$$

G.8 NFIB Robustness Check

As noted by Kudlyak and Miskanic (2024), this rise occurred before the increase in the commonly used vacancy-to-unemployment (V/U) ratio. I plot both series for comparison in Figure 33.

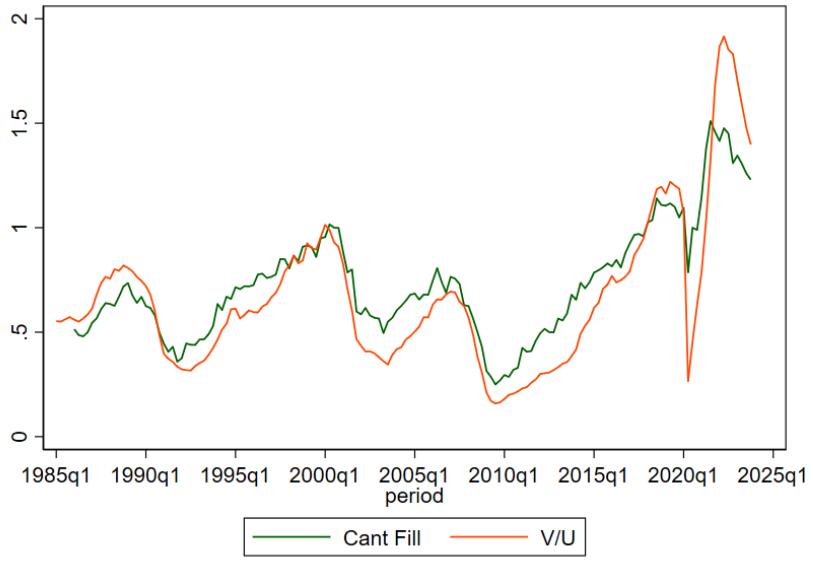


Figure 33: NFIB & BLS Labor Market Tightness

Note: Comparing BLS' V/U measure with the NFIB fraction of firms reporting unfillable positions, scaled by a factor of three to roughly match the range in the V/U data.

I use this measure as a robustness check for the V/U ratio, as it aligns with my focus on the challenges of expanding production through hiring. I find that the NFIB metric for unfillable positions effectively predicts higher inflation, even alongside subdued wage growth (see Table 50). This indicates that firms' difficulties in filling positions, and thereby meeting demand, may be a crucial driver of the V/U-inflation relationship, beyond the role of wage growth.

Table 50: NFIB Cannot Fill Vacancy Impact

	Median	XFE	CPI	ECI
CantFill	31.46***	41.14***	53.90***	12.03
	(9.06)	(12.80)	(15.12)	(13.57)
CantFill^2	-125.50***	-176.18***	-221.90***	-14.32
	(33.98)	(48.00)	(56.68)	(52.01)
CantFill^3	166.36***	247.73***	294.38***	13.65
	(39.63)	(55.98)	(66.10)	(62.06)
Expected Inflation	0.60***	1.00***	0.83***	0.82***
	(0.08)	(0.12)	(0.14)	(0.10)
Constant	-2.16***	-3.25***	-4.22***	-2.40**
	(0.80)	(1.13)	(1.33)	(1.14)
R^2	0.81	0.71	0.86	0.62
N	152	152	152	152
Controls	✓	✓	✓	✓
No Impact p-value	0.0000	0.0000	0.0000	0.0000
Linear p-value	0.0000	0.0000	0.0000	0.8634

Two-way clustered standard errors in parentheses

* $p < .10$, ** $p < .05$, *** $p < .01$

Models 1–4 in Table 50 provide estimates of the cubic NFIB "Cannot Fill Vacancy" measure on median, core, headline, and ECI wages and salaries inflation. The results remain robust when restricted to the pre-pandemic period. The impact on inflation reflects a pattern similar to that observed with the V/U ratio, while its effect on wage growth (Model 4) is again linear. This suggests that firms' reported hiring difficulties are closely aligned with labor market tightness, as emphasized by Kudlyak and Miskanic (2024). Additionally, this may explain why inflation exhibits a strong correlation with labor market tightness, even when controlling for wage growth.

G.9 NFIB Expected Demand

Figure 34 shows the aggregated index of future business conditions from NFIB survey data. Figure 35 does the same for expected real sales growth.



Figure 34: NFIB Business Condition Expectations

Note: Fraction of firms expecting business conditions to improve minus fraction of firms expecting conditions to worsen. Source: http://www.nfib-sbet.org/indicators/

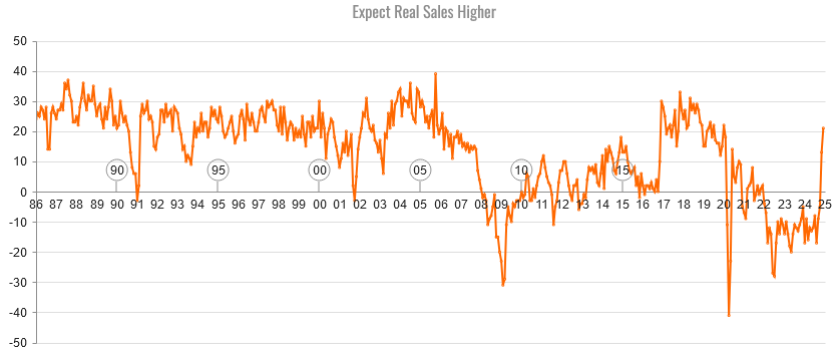


Figure 35: NFIB Real Sales Growth Expectations

Note: Fraction of firms expecting real sales volume to increase minus fraction of firms expecting real sales volume to decrease. Source: http://www.nfib-sbet.org/indicators/

H Gali Chapter 6 + DNWR

This appendix presents a standard New Keynesian framework with wage and price rigidity (Gali (2015) chapter 6, which I will follow closely), while adding downward nominal wage rigidity on top of Calvo wage rigidity. After the presentation of the model I will highlight its limitations in explaining the discrepancy between wage and price Phillips curves in the non-linear fashion found in the data. I will also derive the positive impact of an anticipated increase in demand on inflation under tight labor markets, to show that the model cannot explain my findings on NFIB survey data.

Firms

Firm's production function

$$Y_t(i) = A_t N_t(i)^{1-\alpha} \tag{17}$$

where A_t is the technology level. However, to get wage rigidity into the model Gali creates varieties of labor

$$N_t(i) = \left(\int_0^1 N_t(i,j)^{\frac{\epsilon_w - 1}{\epsilon_w}} dj \right)^{\frac{\epsilon_w}{\epsilon_w - 1}} \tag{18}$$

where ϵ_w is the elasticity of substitution among labor types. CES aggregation leads to optimal labor demand for each variety

$$N_t(i,j) = \left(\frac{W_t(j)}{W_t} \right)^{-\epsilon_w} N_t(i) \tag{19}$$

where $W_t(j)$ is the nominal wage of type j labor and W_t is the aggregate wage index

$$W_t = \left(\int_0^1 W_t(j)^{1-\epsilon_w} dj \right)^{1/(1-\epsilon_w)} \tag{20}$$

A fraction θ of firms cannot reset prices each period (Calvo pricing). Firms that can reset prices choose P_t^* to maximize expected profits

$$\max_{P_t^*} E_t \sum_{k=0}^{\infty} \theta^k E_t \left[Q_{t,t+k} \left(P_t^* Y_{t+k|t} - \Psi_{t+k|t}(Y_{t+k|t}) \right) \right] \tag{21}$$

subject to demand constraints

$$Y_{t+k|t} = \left(\frac{P_t^*}{P_{t+k}} \right)^{-\epsilon} C_{t+k} \tag{22}$$

where $Q_{t,t+k} = \beta^k (C_{t+k}/C_t)^{-\sigma} (P_t/P_{t+k})$ is the stochastic discount factor, and $\Psi_t(.)$ is the firms cost function.

The maximization problem leads to the following first order condition

$$E_t \sum_{k=0}^{\infty} \theta^k E_t \left[Q_{t,t+k} \left(P_t^* - \mathcal{M} \psi_{t+k|t} \right) Y_{t+k|t} \right] \tag{23}$$

where $\psi_{t+k|t} = \Psi'_{t+k}(Y_{t+k|t})$ is marginal cost and $\mathcal{M} = \frac{\epsilon}{\epsilon-1}$ is the optimal markup. A log-linear approximation of the FOC, divided by P_{t-1}, around the zero inflation steady state gives a condition for $\log(P_t^*) = p_t^*$.

$$p_t^* - p_{t-1} = (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k E_t\left[\hat{mc}_{t+k|t} + p_{t+k}\right]$$

where $\hat{mc}_{t+k|t} = mc_{t+k|t} - mc$ is the log-deviation of marginal cost from its steady state $mc = -\log(\mathcal{M}) =: -\mu$.

Given that $(1-\theta)$ of firms reset their price to P_t^*, while the other θ fraction cannot change their price, the aggregate price ratio is $\Pi_t^{1-\epsilon} = \left(\frac{P_t}{P_{t-1}}\right)^{1-\epsilon} = \theta + (1-\theta)\left(\frac{P_t^*}{P_{t-1}}\right)^{1-\epsilon}$. Its log-linearized deviation from steady state is $\pi = (1-\theta)(p_t^* - p_{t-1})$, which gives the inflation equation of the model.

Households

Households maximize utility over an infinite horizon:

$$E_0\sum_{t=0}^{\infty}\beta^t U(C_t(j), N_t(j)) \tag{24}$$

where

$$C_t(j) = \left(\int_0^1 C_t(i,j)^{\frac{\epsilon_p-1}{\epsilon_p}}di\right)^{\frac{\epsilon_p}{\epsilon_p-1}}$$

is the consumption index for the household specializing in labor supply $j \in [0,1]$, N_t is labor supply, and β the discount factor.

Each household has wage setting power, but only a $1-\theta_w$ fraction of households can reoptimize their wage in a given period. A reoptimizing household will maximize

$$E_t\{\sum_{k=0}^{\infty}\beta^k U(C_{t+k|t}, N_{t+k|t})\} = E_t\{\sum_{k=0}^{\infty}(\beta\theta_w)^k U(C_{t+k|t}, N_{t+k|t})\} + E_t\{\sum_{k=1}^{\infty}\beta^k(1-\theta_w^k)U(\tilde{C}_{t+k|t}, \tilde{N}_{t+k|t})\}$$

where $C_{t+k|t}$ and $N_{t+k|t}$ respectively denote the consumption and labor supply in period $t+k$ of a household that last reset its wage in period t. This optimization is subject to a downward nominal wage rigidity constraint

$$W_t^* \geq W_{t-1}$$

as well as the sequence of labor demand schedules

$$N_{t+k|t} = (\frac{W_t^*}{W_{t+k}})^{-\epsilon_W}N_{t+k}$$

$$\tilde{N}_{t+k|t} = (\frac{W_{t+k}^*}{W_{t+k}})^{-\epsilon_W}N_{t+k}$$

and budget constraints for the household.

$$P_{t+k}C_{t+k|t} + E_{t+k}\{Q_{t+k,t+k+1}D_{t+k+1|t}\} \leq D_{t+k|t} + W_t^*N_{t+k|t} - T_{t+k}$$

$$P_{t+k}\tilde{C}_{t+k|t} + E_{t+k}\{Q_{t+k,t+k+1}D_{t+k+1|t}\} \leq D_{t+k|t} + W^*_{t+k}\tilde{N}_{t+k|t} - T_{t+k}$$

for $k = 0, 1, 2, ...$ where $N_{t+k} \equiv \int_0^1 N_{t+k}(i)di$ denotes aggregate employment in period $t+k$, $D_{t+k|t}$ is the market value in period $t+k$ of the portfolio of securities held at the beginning of that period by households that last reoptimized their wage in period t, while $E_{t+k}\{Q_{t+k,t+k+1}D_{t+k+1|t}\}$ is the corresponding market value as of period $t+k$ of the portfolio purchased in that period, which yields a random payoff $D_{t+k+1|t}$. P_{t+k} is the aggregate price level, Q_{t+k} the price of a one-period bond, and T_{t+k} a lump-sum transfer. Finally, ϵ_w is the constant elasticity of substitution among labor varieties.

The first-order condition with respect to W^*_t is given by

$$\sum_{k=0}^{\infty}(\beta\theta_w)^k E_t\{N_{t+k|t}U_c(C_{t+k|t}, N_{t+k|t})\frac{W^*_t}{P_{t+k}} + \mathcal{M}_w U_n(C_{t+k|t}, N_{t+k|t})\} + \Phi^{DNWR}_t - \sum_{k=1}^{\infty}\beta^k(1-\theta^k_w)E_t\{\Phi^{DNWR}_{t+k}\} = 0$$

where $\mathcal{M}_w \equiv \frac{\epsilon_w}{\epsilon_w - 1}$ is the optimal wage markup, and Φ^{DNWR}_t is the Lagrangian multiplier of the DNWR constraint. $\Phi^{DNWR}_t = 0$ if DNWR is not binding in period t. Similarly, $E_t[\Phi^{DNWR}_{t+k}] = 0$ if DNWR is not expected to bind at W^*_t in future periods.

Log-linearizing around that steady state and simplifying the expression yields an approximated optimal wage setting rule

$$w^*_t = \mu^w + (1-\beta\theta_w)\sum_{k=0}^{\infty}(\beta\theta_w)^k E_t\{mrs_{t+k|t} + p_{t+k}\} + (1-\beta\theta_w)|\phi^{DNWR}_t| - (1-\beta\theta_w)\sum_{k=1}^{\infty}\beta^k(1-\theta^k_w)E_t\{|\phi^{DNWR}_{t+k}|\}$$

where $\mu^w \equiv log\mathcal{M}_w$. A note on log-linearizing Lagrangian multiplier: As the Lagrange multiplier is zero in steady state, small log-linear deviations from steady state $\phi_{t+k} < 0$. To get the direction of the impact on wage setting I draw the negative sign out and use absolute values. This can be seen when DNWR binds in period t and is not expected to bind in any later period. This happens if the unconstrained part of the wage equation is negative, and thus without a constraint $w_t < 0$. As the constraint binds $w_t = 0$ by force, which sets $\mu^w + (1-\beta\theta_w)\sum_{k=0}^{\infty}(\beta\theta_w)^k E_t\{mrs_{t+k|t} + p_{t+k}\} = (1-\beta\theta_w)\phi^{DNWR}_t$. The LHS is negative, as the unconstrained optimal w_t equals the LHS. Thus, $\phi^{DNWR}_t < 0$. This does not violate the positivity constraint on $\Phi \geq 0$, as we only look at small shocks to steady state: $\Phi = 0 + \epsilon_\Phi = 0 + e^\phi$. To get a small $\epsilon_\Phi = e^\phi$, $\phi < 0$.

Thus, the DNWR constraint prevents downward deviations from steady state, when in steady state. It also lowers wage increases when DNWR is expected to bind in the future. I.e. workers lower their markup today when expecting a negative shock to their ideal wage; a point made by Benigno and Ricci (2011). The negative shock to their ideal wage could be from a shock to the elasticity of substitution between labor varieties, a negative shock to expected inflation, or their marginal rate of substitution between leisure and consumption.

Following derivation steps of Gali (2015) chapter 6, the wage equation can be further simplified by putting in the expression for the economy's average marginal rate of substitution $mrs_{t+k} = \sigma c_{t+k} + \varphi n_{t+k}$ and writing

$$mrs_{t+k|t} = mrs_{t+k} + \varphi(n_{t+k|t} - n_{t+k}) = mrs_{t+k} - \epsilon_w\varphi(w^*_t - W_{t+k})$$

This gives the following wage equation

$$w_t^* = \beta\theta_w E_t[w_{t+1}^*] + (1-\beta\theta_w)(w_t - (1+\epsilon_w\varphi)^{-1}\tilde{\mu}_t^w) + \frac{(1-\beta\theta_w)}{(1+\epsilon_w\varphi)}\left(|\phi_t^{DNWR}| - \sum_{k=1}^{\infty}\beta^k(1-\theta_w^k)E_t\{|\phi_{t+k}^{DNWR}|\}\right)$$

The aggregate wage index evolves as follows

$$W_t = \left[\theta_w W_{t-1}^{1-\epsilon_w} + (1-\theta_w)(W_t^*)^{1-\epsilon_w}\right]^{\frac{1}{1-\epsilon_w}}$$

which in log-linear approximation is

$$w_t = \theta_w w_{t-1} + (1-\theta_w)w_t^*$$

Plugging in the wage equation and defining $\pi_t^w = w_t - w_{t-1}$ yields the basic wage inflation equation

$$\pi_t^w = \beta E_t\left[\pi_{t+1}^w\right] - \lambda_w\hat{\mu}_t^w + \lambda_w\left(|\phi_t^{DNWR}| - \sum_{k=1}^{\infty}\beta^k(1-\theta_w^k)E_t\{|\phi_{t+k}^{DNWR}|\}\right)$$

where $\lambda_w = \frac{(1-\theta_w)(1-\beta\theta_w)}{\theta_w(1+\epsilon_w\varphi)}$.

Further, defining the real wage gap $\tilde{\omega}_t = \omega_t - \omega_t^n$, where $\omega_t^n = \log(1-\alpha) + (y_t^n - n_t^n) - \mu^p$ is the natural real wage as in Gali (2015) chapter 6. This enables us to write the log-linear wage Phillips Curve

$$\pi_t^w = \beta E_t\left[\pi_{t+1}^w\right] + \kappa_w\tilde{y}_t - \lambda_w\tilde{\omega}_t + \lambda_w\left(|\phi_t^{DNWR}| - \sum_{k=1}^{\infty}\beta^k(1-\theta_w^k)E_t\{|\phi_{t+k}^{DNWR}|\}\right)$$

where $\kappa_w = \lambda_w(\sigma + \frac{\varphi}{1-\alpha}) = \frac{(1-\theta_w)(1-\beta\theta_w)}{\theta_w(1+\epsilon_w\varphi)}(\sigma + \frac{\varphi}{1-\alpha})$. It also finally allows us to write out the price Phillips curve of the model. Through deriving an expression for marginal cost's log deviation from steady state and linking it to the log deviation of output and the real wage from steady state (output and real wage gaps) we can derive the New Keynesian Phillips curve (for the derivations see Gali Chp 6).

$$\pi_t^p = \beta E_t\{\pi_{t+1}\} + \kappa_p\tilde{y}_t + \lambda_p\tilde{\omega}_t$$

where

$$\kappa_p = \lambda_p\frac{\alpha}{1-\alpha} = \frac{(1-\theta_p)(1-\beta\theta_p)}{\theta_p}\frac{1-\alpha}{1-\alpha+\alpha\epsilon_p}\frac{\alpha}{1-\alpha} = \frac{(1-\theta_p)(1-\beta\theta_p)}{\theta_p}\frac{\alpha}{1-\alpha+\alpha\epsilon_p}$$

On the consumption side households $j \in [0,1]$ split their consumption according to the optimal consumption index

$$C_t(j) = \left(\int_0^1 C_t(i,j)^{\frac{\epsilon_p-1}{\epsilon_p}}di\right)^{\frac{\epsilon_p}{\epsilon_p-1}}$$

which implies optimal consumption weights according to their relative prices

$$C_t(i,j) = \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon} C_t(j)$$

where $P_t = \left[\int_0^1 P_t(i)^{1-\epsilon} di\right]^{\frac{1}{1-\epsilon}}$ is the aggregate price index. Following this optimal choice of relative consumption among varieties gives

$$\int_0^1 P_t(i)C_t(i)di = P_t C_t$$

Optimal consumption/savings is described by the Euler equation

$$Q_t = \beta E_t \left[\frac{U_{C,t+1}}{U_{C,t}} \frac{P_t}{P_{t+1}}\right] \tag{25}$$

I follow Gali in specifying the utility function as

$$U(C_t, N_t) = \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\phi}}{1+\phi}$$

where σ is the degree of risk aversion, and ϕ is the Frisch elasticity of labor supply. With this assumption the log-linearized Euler equation is

$$c_t = E_t[c_{t+1}] - \frac{1}{\sigma}(i_t - E_t[\pi_{t+1}^p] - \rho)$$

where $i_t = -\log(Q_t)$, and ρ is the time preference factor.

The other key equation governing the dynamics of this model is the dynamic IS equation

$$\tilde{y}_t = E_t[\tilde{y}_{t+1}] - \frac{1}{\sigma}(i_t - E_t[\pi_{t+1}^p] - r_t^n), \tag{26}$$

where $r_t^n = \rho + \sigma E_t[\Delta y_t^n] = \rho + \sigma\psi_{ya}^n E_t[\Delta a_{t+1}] + (1-\rho_z)z_t$ is the natural rate of interest , ρ is the time preference rate and $y_t^n = \psi_{ya}^n a_t + \upsilon_y^n$ is the level of natural output. Where $\psi_{ya}^n = \frac{1+\varphi}{\sigma(1-\alpha)+\varphi+\alpha}$ and $\upsilon_y^n = -\frac{(1-\alpha)(\mu-\log(1-\alpha))}{\sigma(1-\alpha)+\varphi+\alpha}$. The TFP shock $a_{t+1} = \rho_a a_t + \epsilon_a$ and the time preference shock $z_{t+1} = \rho_z z_t + \epsilon_z$ follow AR(1) processes.

Monetary policy is modeled using a Taylor-type rule

$$i_t = \rho + \phi_\pi \pi_t + \phi_y y_t + \phi_w \pi_t^w + \nu_t \tag{27}$$

where ϕ_π, ϕ_y, and ϕ_w are policy weights on inflation, the output gap, and wage inflation, respectively. Finally, $\nu_t = \rho_\nu \nu_{t-1} + \epsilon_\nu$ is an AR(1) monetary policy shock.

Monetary policy is allowed to affect real variables through an adhoc money demand function

$$m_t - p_t = y_t - \eta i_t$$

paired with a money growth equation

$$\Delta m_t = \Delta y_t - \eta \Delta i_t + \pi_t^p$$

Parameter	Symbol	Value
Inverse of intertemporal elasticity of substitution	σ	1
Inverse of the Frisch elasticity of labor supply	φ	5
Coefficient on inflation in the Taylor rule	ϕ_π	1.5
Coefficient on output gap in the Taylor rule	ϕ_y	0.125
Coefficient on wage inflation in the Taylor rule	ϕ_{π^w}	0
Persistence of the monetary policy shock	ρ_ν	0.5
Persistence of the preference shock	ρ_z	0.5
Persistence of the technology shock	ρ_a	0.9
Discount factor	β	0.99
Semi-elasticity of money demand	η	3.77
Capital share	α	1/4
Elasticity of substitution between differentiated goods	ϵ_p	9
Elasticity of substitution between labor varieties	ϵ_w	4.5
Calvo price parameter	θ_p	3/4
Calvo wage parameter	θ_w	3/4

Table 51: Calibration of Parameters

Table 52: Essential Empirical Findings

Nr	**Finding**
1	Linear wage Phillips curve (WPC)
2	Non-Linear price Phillips curve (PPC)
3	WPC slope $>$ PPC slope during normal/slack labor markets
4	PPC slope $>$ WPC slope during especially tight labor markets
5	Expected higher growth during tight labor markets lowers inflation

H.1 Parametrization

H.2 Findings to explain

Before discussing the extend to which a standard NK framework with DNWR can explain my empirical findings, I boil my findings down to a list of essentials.

The standard NK framework can hit multiple of these findings depending on its parametrization. For example a standard NK framework with both wage and price rigidity creates linear wage and price Phillips curves, where the relative slope of these depends on the relative size of the rigidities. For example an economy with small price rigidity ($\theta_p = 10^{-6}$) and high wage rigidity ($\theta_w = 3/4$) creates a steeper PPC than WPC. However, the PPC of this model is steeper during both slack and tight labor markets, which contradicts finding three. Furthermore, there is no non-linearity in this Phillips curve.

Analytically, this can be seen by looking at the Phillips curve coefficients

$$\kappa_p = \frac{(1-\theta_p)(1-\beta\theta_p)}{\theta_p}\frac{\alpha}{1-\alpha+\alpha\epsilon_p}$$
$$\kappa_w = \frac{(1-\theta_w)(1-\beta\theta_w)}{\theta_w(1+\epsilon_w\varphi)}(\sigma+\frac{\varphi}{1-\alpha})$$

Both are decreasing in their respective rigidity, θ_p and θ_w. Thus, the model can produce steeper WPCs, steeper PPCs, or equally sloped curves. I highlight this by plotting the IRFs from a monetary policy shock for three different rigidity specifications (see Figure 36).

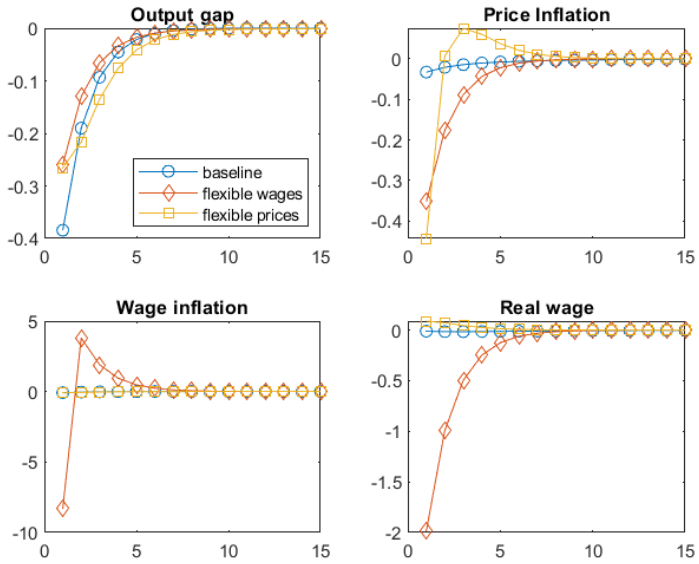


Figure 36: Gali Chp. 6 IRFs to Monetary Policy Shock

Note: The baseline model has significant rigidity ($\theta = 3/4$) in both wages and prices. The flexible wage model sets $\theta_w = 10^{-6}$, and the flexible price model sets $\theta_p = 10^{-6}$, while keeping the other rigidity at three quarters.

Under the baseline model with the same high rigidity for prices and wages the output loss is naturally largest. In this specification prices, wages and real wages do not move much. In contrast, under flexible wages but rigid prices, the output gap is slightly smaller, and wages adjust dramatically, while prices follow wages down partially, leading to a large decline in real wages. Flexible prices and rigid wages on the other hand lead to a slight increase in real wages.

This exemplifies that the PPC is steeper than the WPC if prices are flexible and wages are rigid, and vice versa if wages are flexible but prices are rigid. Crucially though, both PCs are symmetric around natural output. So no matter which curve is steeper during times of slack is also steeper during times of high demand and tight labor markets.

I simulate 1000 periods of monetary policy shocks under flexible prices and rigid wages and regress inflation, wage growth and real wages on the output gap, which proxies for labor market tightness in this simple model. The correlations are given in Table 53, where R^2 describes the fit for each separate regression. Due to linearity the Phillips curve slopes are symmetric around $y_t^{gap} = 0$, and there is no non-linearity in either the PPC or the WPC.

Dependent Variable	Slope	p-value	R-squared
π_p	0.6876	0.0000	0.3310
π_w	0.2991	0.0000	0.9850
ω	-0.3333	0.0000	1.0000

Table 53: Phillips curve Slopes of Gali Chapter 6.

Finally, I test for the existence of a direct impact of labor markets on inflation, when wage growth is controlled for. The results are presented in Table 54. Surprisingly, there is a significant, but negative impact on inflation when controlling for wage growth. This is not due to other shocks, but likely a negative endogeneity bias from monetary policy. I test this by varying the Taylor coefficients for inflation and the output gap. A larger Taylor coefficient on inflation leads to a smaller negative direct impact coefficient, as the range of inflation is compressed. Decreasing the output gap coefficient on the other hand does reduce the bias as expected. However, neither shows enough aptitude to explain the negative coefficient on the output gap completely. I leave the exploration of this to future research.

Variable	Coefficient	p-value
Intercept	-0.0000	0.7839
Y_Gap	-7.2278	0.0000
Pi_W	26.4681	0.0000

Table 54: Estimated Direct Impact on Gali Chapter 6.

Non-linearity of the PCs is achieved with DNWR if normal wage rigidity is not too large. If wage rigidity is small enough the slope of the WPC is steep during tight labor markets, and flat during slack labor markets. The PPC mimics this as firms seek to keep their optimal markup over marginal cost. However, then the non-linearity is as much in the WPC as in the PPC, which violates point one of my empirical findings.

The model tells us that the WPC is clearly non-linear, as its slope is zero when DNWR binds, and positive when it is not. As firms target their optimal markup over marginal cost, the PPC inherits this non-linearity from the underlying WPC. If the PCs are non-linear depends on the size of general wage rigidity θ_w. To demonstrate this I set $\theta_w = 0.95$, while keeping prices flexible ($\theta_p = 10^{-6}$) and estimate a threshold model of the price and wage Phillips curves. I present the findings in Table 55. Note that the coefficient on positive output gaps is insignificant on both price and wage inflation.

However, if wage rigidity is small enough the non-linearity becomes detectable in the simulated data. To this end I parametrize the model with flexible prices ($\theta_p = 10^{-6}$) and standard wage rigidity $\theta_w = 3/4$. I simulate the model's response to monetary policy shocks across 1000 periods and estimate a threshold model around $y^{gap} = 0$. The results are presented in Table 56.

Note that the WPC is significantly non-linear while the PPC is not. If wage rigidity is reduced further, then the PPC picks up the non-linearity as well (see Table 57). The important

	π^P		π^W	
Variable	Coeff.	p-value	Coeff.	p-value
Intercept	0.0078	0.7116	0.0000	0.7116
y^{gap}	0.4270	0.0000	0.0111	0.0000
$y^{gap} > 0$	-0.1037	0.2841	-0.0002	0.2841

Table 55: Undetectable Non-Linearity under high wage rigidity.

	π^P		π^W	
Variable	Coeff.	p-value	Coeff.	p-value
Intercept	-0.0057	0.7722	-0.0167	0.0000
y^{gap}	0.5410	0.0000	0.1133	0.0000
$y^{gap} > 0$	0.1026	0.3012	0.2170	0.0000

Table 56: Threshold Regression Results for $\theta_w = 3/4$

fact here is that the WPC is more likely to be estimated as non-linear than the PPC, as it is the source of the non-linearity. This contradicts my empirical findings.

	π^P		π^W	
Variable	Coeff.	p-value	Coeff.	p-value
Intercept	0.0604	0.0001	-0.0218	0.0000
y^{gap}	0.6573	0.0000	0.1167	0.0000
$y^{gap} > 0$	6.3813	0.0000	6.9508	0.0000

Table 57: Threshold Regression Results for $\theta_w = 1/4$

Finally, I estimate the direct impact in the DNWR augmented model with $\theta_p = 10^{-6}$, and $\theta_w = 3/4$. Table 58 presents the findings. The estimated direct impact is positive and small for negative output gaps, and negative and large for positive output gaps. This is in stark contrast with my empirical findings.

Variable	Coefficient	p-value
Intercept	0.1215	0.0000
y^{gap}	0.3310	0.0000
$y^{gap} > 0$	-13.0592	0.0000
π^W	2.7969	0.0000

Table 58: Direct Impact Estimation for

Now that I have documented the inaptitude of the baseline macro model in accounting for the first four of my five core empirical findings I move on to point five: "Expected higher growth during especially tight labor markets leads to lower inflation".

The difficulty in assessing the joint impact of tight labor markets and future expected

sales growth in the standard NK model is that they both affect the same variable: $y_t^{gap} =: \tilde{y}_t$. The only influence in which higher expected demand affects inflation today is through the IS equation

$$\tilde{y}_t = E_t[\tilde{y}_{t+1}] - \frac{1}{\sigma}(i_t - E_t[\pi_{t+1}] - r_t^n)$$

To evoke the joint impact of tight labor markets and expected future demand I look at two specific shocks. First a time preference shock increasing current demand at the expense of future demand. Second, a separate shock to expected future demand $E_t[\tilde{y}_t]$.

In the linearized model the interaction of these two shocks is naturally zero. To assess the cross-derivative of the two shocks on $\tilde{y}_t$ I go back to the Euler equation which underlies the IS equation together with the market clearing condition. Enforcing the market clearing condition in the Euler equation and rearranging gives

$$\begin{aligned} Q_t &= \beta E_t \left[\frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}} \frac{P_t}{P_{t+1}} \right] = \beta E_t \left[\frac{Y_{t+1}^{-\sigma}}{Y_t^{-\sigma}} \frac{P_t}{P_{t+1}} \right] \\ \iff Y_t &= \left(\frac{\beta}{Q_t} E_t[Y_{t+1}^{-\sigma}] E_t \left[\frac{P_t}{P_{t+1}} \right] \right)^{-\frac{1}{\sigma}} \end{aligned}$$

To assess the interaction term we need derive the joint impact of two shocks on Y_t, and thus the output gap. A negative shock on β increases today's consumption and thus leads to tight labor markets. An increase in $E_t[Y_{t+1}]$ gives us the impact of higher expected future demand. For simplicity I transform this shock into a negative shock on $E_t[Y_{t+1}^{-\sigma}]$. Thus, we seek the cross-derivative of Y_t with respect to negative shocks to β and $E_t[Y_{t+1}^{-\sigma}]$. This is the same as the cross derivative of Y_t with respect to positive shocks to β and $E_t[Y_{t+1}^{-\sigma}]$.

I first calculate the derivative of Y_t with respect to β.

$$\frac{\partial Y_t}{\partial \beta} = -\frac{1}{\sigma} \left[\frac{\beta}{Q_t} E_t[Y_{t+1}^{-\sigma}] E_t \left[\frac{P_t}{P_{t+1}} \right] \right]^{-\frac{1+\sigma}{\sigma}} \frac{E_t[Y_{t+1}^{-\sigma}] E_t \left[\frac{P_t}{P_{t+1}} \right]}{Q_t} < 0$$

Taking the derivative of this with respect to $E_t[Y_{t+1}^{-\sigma}]$ gives

$$\begin{aligned}
\frac{\partial^2 Y_t}{\partial\beta\partial E_t[Y_{t+1}^{-\sigma}]} &= \frac{1+\sigma}{\sigma^2}\left[\frac{\beta}{Q_t}E_t[Y_{t+1}^{-\sigma}]E_t\left[\frac{P_t}{P_{t+1}}\right]\right]^{-\frac{1+2\sigma}{\sigma}}\frac{\beta}{Q_t}E_t\left[\frac{P_t}{P_{t+1}}\right]\frac{E_t[Y_{t+1}^{-\sigma}]E_t\left[\frac{P_t}{P_{t+1}}\right]}{Q_t} \\
&\quad - \frac{1}{\sigma}\left[\frac{\beta}{Q_t}E_t[Y_{t+1}^{-\sigma}]E_t\left[\frac{P_t}{P_{t+1}}\right]\right]^{-\frac{1+\sigma}{\sigma}}\frac{E_t\left[\frac{P_t}{P_{t+1}}\right]}{Q_t} \\
&= \frac{E_t\left[\frac{P_t}{P_{t+1}}\right]}{\sigma Q_t}\left[\frac{\beta}{Q_t}E_t[Y_{t+1}^{-\sigma}]E_t\left[\frac{P_t}{P_{t+1}}\right]\right]^{-\frac{1+\sigma}{\sigma}}\times \\
&\quad \left[\frac{1+\sigma}{\sigma}\left[\frac{\beta}{Q_t}E_t[Y_{t+1}^{-\sigma}]E_t\left[\frac{P_t}{P_{t+1}}\right]\right]^{-1}\frac{\beta E_t[Y_{t+1}^{-\sigma}]E_t\left[\frac{P_t}{P_{t+1}}\right]}{Q_t} - 1\right] \\
&= \frac{E_t\left[\frac{P_t}{P_{t+1}}\right]}{\sigma Q_t}\left[\frac{\beta}{Q_t}E_t[Y_{t+1}^{-\sigma}]E_t\left[\frac{P_t}{P_{t+1}}\right]\right]^{-\frac{1+\sigma}{\sigma}}\left[\frac{1+\sigma}{\sigma} - 1\right] \\
&= \frac{E_t\left[\frac{P_t}{P_{t+1}}\right]}{\sigma Q_t}\left[\frac{\beta}{Q_t}E_t[Y_{t+1}^{-\sigma}]E_t\left[\frac{P_t}{P_{t+1}}\right]\right]^{-\frac{1+\sigma}{\sigma}}\frac{1}{\sigma} > 0
\end{aligned}$$

Our two shocks, an increase in current demand, and an increase in expected future demand, raise today's output, the output gap and thus inflation. Taken together they lead to more inflation than just the sum of their parts. Thus, the standard NK framework predicts a positive interaction term between tight labor markets (positive output gap) and higher expectations about future demand. This is the opposite of what I find on NFIB firm survey data.

I NK Model Parts

I.1 Closing of the Model

To close the model monetary policy is described by a Taylor Rule

$$i_t = e^{m_t}(\pi_t^p)^{\phi_\pi} \tag{28}$$

where i_t is the interest paid on the savings bond B_t, π_t^p is aggregate price inflation, e^{m_t} is the monetary policy shock multiplier ($m_t = \bar{m} + \rho_m(m_{t-1} - \bar{m}) + \eta_t^m$), and ϕ_π is the Taylor coefficient. In words, the price of the bond falls with inflation, which raises the bond's rate of return. The Taylor coefficient determines how much the policy rate reacts to inflation.

Finally, aggregate market clearing is imposed.

$$Y_t = C_t + \kappa_v V_t + \frac{\kappa_p}{2}\left(\frac{p_{i,t}}{p_{i,t-1}} - 1\right)^2 Y_t + \frac{\kappa_w}{2}\left(\frac{w_{i,t}}{w_{i,t-1}} - 1\right)^2 N_t \tag{29}$$

The vacancy, price, and wage adjustment costs are sunk into the sea.

I.2 Steady State Derivation

To derive the zero inflation steady state I start with the FOC with respect to $p_{i,t}$. In steady state where $P_t = P_{t-1}$ it simplifies to

$$1 = \mathcal{M}_p \frac{\frac{W_t}{P_t} + \frac{\kappa_v}{a_t(1)} - E_t\left[\Lambda_{t,t+1}(1 - \tilde{\delta}_{t+1}(1))\frac{\kappa_v}{a_{t+1}(1)}\right]}{f'(N_t)}$$

where $\mathcal{M}_p = \frac{\epsilon}{\epsilon - 1}$ is the steady state markup. In equilibrium demand is equal across periods. Thus, the stochastic discount factor $\Lambda_{t,t+1} = \beta\frac{U'(C_{t+1})}{U'(C_t)}$ equals the discount factor β. Further substituting $a_t(1) = a_{t+1}(1) =: a$ as well as $\tilde{\delta}_{t+1}(1) =: \tilde{\delta}$, and generally dropping t subscripts gives

$$1 = \mathcal{M}_p \frac{\frac{W}{P} + \frac{\kappa_v}{a} - \beta(1 - \tilde{\delta})\frac{\kappa_v}{a}}{Af'(N)} = \mathcal{M}_p \frac{\frac{W}{P} + \frac{\kappa_v}{a}(1 - \beta(1 - \tilde{\delta}))}{Af'(N)}$$

I use the derived wage Phillips curve, where DNWR is neither binding nor expected to bind, to derive the wage setting behavior in equilibrium

$$\left(\Pi_t^w - 1\right)\Pi_t^w = \frac{1}{\kappa_w}\left(-\frac{W_t}{P_t} + \left(\frac{\kappa_v}{a_t(1)} 2\frac{V_t}{N_t} q(\theta_t) \mathfrak{p}_t^E f_\psi(1)\right)\right) + E_t\left[\Lambda_{t,t+1}\left(\Pi_{t+1}^w - 1\right)\Pi_{t+1}^w \frac{N_{t+1}}{N_t}\right]$$

The ratio $\Pi_t^w = \frac{W_t}{W_{t-1}} = 1$ in equilibrium for all t. This gives the optimal real wage prevailing under flexible wages, which equals the recruitment and retaining cost of an employee.

$$\frac{W_t}{P_t} = \frac{\kappa_v}{a_t(1)} 2\frac{V_t}{N_t} q(\theta_t) \mathfrak{p}_t^E f_\psi(1)$$

This makes intuitive sense. If the real wage was higher than recruitment cost for a new employee, firms would increase vacancies instead of raising wages in order to increase their

employee base. Only when the two are equal are firms indifferent in raising wages or vacancies when looking to hire more workers.

Finally, I use the fact that $\epsilon_a = \epsilon_{\tilde{\delta}} = \frac{q(\theta)p^E f_\psi(1)}{a(1)}$ and $\frac{V}{N} = \frac{\tilde{\delta}}{a}$ in steady state to derive

$$\frac{W_t}{P_t} = \frac{\kappa_v}{a(1)} 2\tilde{\delta}\epsilon_{\tilde{\delta}}$$

I combine the price and wage setting behavior to derive the following real wage in steady state

$$\frac{W_t}{P_t} = \frac{1}{\mathcal{M}_p} \frac{2\tilde{\delta}\epsilon_{\tilde{\delta}}}{2\tilde{\delta}\epsilon_{\tilde{\delta}} + 1 - \beta(1-\tilde{\delta})} Af'(N)$$

The real wage is the marginal product of labor less the price markup stemming from pricing power in the product market, and subject to a wage mark-down stemming from monopsony power. The pricing power of firms is due to the imperfect substitution of consumption between consumption varieties. Monopsony power stems from the firms' power in setting wages and vacancies in a labor market with search and matching frictions.

I.3 Unemployment Benefits

In this subsection I lay out the necessary changes to my model to incorporate non-trivial UI benefits. I introduce the benefits via a replacement rate $0 < b < 1$. Unemployed workers receive b times the market real wage as their outside option. Thus, they weigh the replacement rate against the wage and taste shock associated with an offered position.

An unemployed worker accepts an offered position with wage $w_{i,t}$ if the taste-remuneration combination of the position is not below UI benefits

$$\text{Acceptance Probability of Unemployed} = 1 - F_\psi\left(b\frac{W_t}{w_{i,t}}\right)$$

For an offered position at the market rate the taste shock only has to clear the hurdle of the outside option b. If the offered wage is larger than the going market wage rate then an increasingly smaller taste shock is sufficient to make the worker accept the position.

This alters the probability of filling a vacancy to

$$a_t\left(\frac{w_{i,t}}{W_t}\right) = q(\theta_t)\left(1 - p_t^E F_\psi\left(\frac{W_t}{w_{i,t}}\right) - p_t^U F_\psi\left(b\frac{W_t}{w_{i,t}}\right)\right)$$

and changes the optimal real wage obtained under flexible wages to

$$\frac{{W_t}^*}{P_t} = \frac{\kappa_v}{a_t(1)}\frac{V_t}{N_t} q(\theta_t)\left(2p_t^E f_\psi(1) + bp_t^U f_\psi(b)\right)$$

Finally, I introduce a shock to the outside option to model labor supply shocks. I add $\epsilon_{b,t} = \rho_b\epsilon_{b,t-1} + \epsilon_{ls,t}$ to the replacement rate. Thus, a positive $\epsilon_{ls,t}$ constitutes a negative labor supply shock as it raises the outside option of unemployed workers.

I.4 Model with Wage Discrimination for New Hires

This sub-appendix provides the derivation of the model with wage discrimination against new hires.

In this version the firm producing variety i maximizes its expected present-discounted value of current and future profits. The firm's decision variables are employment $n_{i,t}$, price $p_{i,t}$, job postings $v_{i,t}$, and the wage for new hires with which they are posted $w_{i,t}^{new}$. The firm hires new workers with $w_{i,t}^{new}$ while existing workers get the overall wage rate $w_{i,t}$ determined by $w_{i,t} = \max\{w_{i,t-1}, w_{i,t}^{new}\}$. Thus, existing workers earn at least as much as new hires, but potentially more if optimal wages decline below past wages. New hires go on to earn the firm's going wage rate from period $t+1$ onwards.

Firms face two major trade-offs. First, increasing the price of its output increases profits per unit, but reduces demand due to households partial substitution towards other varieties. Second, raising employment raises output and consequently profits due to a positive markup. But in order to increase employment the firm has to either increase vacancies, which cost κ_v to post, or raise its wage to attract and retain more workers.

These trade-offs are incorporated in the firm's Bellman equation, and its constraints. The firm's value optimized in the Bellman equation depends on current economic conditions such as total employment N_t, total production Y_t and expectations about the future. It further depends on the state the firm inherits from the previous period $x_{i,t-1} = (n_{i,t-1}, p_{i,t-1}, w_{i,t-1})$.

$$\begin{aligned} J_t(x_{i,t-1}) = \max_{n_{i,t}, p_{i,t}, w_{i,t}^{new}, v_{i,t}} & \frac{p_{i,t}}{P_t} D_t\left(\frac{p_{i,t}}{P_t}\right) - \kappa_v v_{i,t} - \frac{\kappa_p}{2}\left(\frac{p_{i,t}}{p_{i,t-1}} - 1\right)^2 Y_t \\ & - \frac{w_{i,t}}{P_t}\left(1 - \tilde{\delta}\left(\frac{w_{i,t}}{W_t}\right)\right) n_{i,t-1} - \frac{w_{i,t}^{new}}{P_t}\left(n_{i,t} - \left(1 - \tilde{\delta}\left(\frac{w_{i,t}}{W_t}\right)\right) n_{i,t-1}\right) \\ & - \frac{\kappa_w}{2}\left(\frac{w_{i,t}}{w_{i,t-1}} - 1\right)^2 N_t + E_t\left[\Lambda_{t,t+1} J_{t+1}(x_{i,t})\right] \end{aligned}$$

The firm's Bellman equation increases in the firm's current and expected future revenue, and decreases in its current and expected future wage, vacancy and Rotemberg adjustment costs. $\Lambda_{t,t+1}$ is the firm's stochastic discount factor between period t and $t+1$.

The optimization is constrained by absolute downward nominal wage rigidity for existing workers

$$w_{i,t} = \max\{w_{i,t-1}, w_{i,t}^{new}\}$$

the employment law of motion

$$\begin{aligned} n_{i,t} &= \left[1 - \left(\bar{\delta} + (1-\bar{\delta})\delta_t\left(\frac{w_{i,t}}{W_t}\right)\right)\right] n_{i,t-1} + a_t\left(\frac{w_{i,t}}{W_t}\right) v_{i,t} \\ &=: \left[1 - \tilde{\delta}\left(\frac{w_{i,t}}{W_t}\right)\right] n_{i,t-1} + a_t\left(\frac{w_{i,t}}{W_t}\right) v_{i,t} \end{aligned}$$

where I simplify the expression by defining the overall separation function $\tilde{\delta}$. The Bellman equation is further constrained by market clearing

$$D_t\left(\frac{p_{i,t}}{P_t}\right) = Y_{i,t} = f(n_{i,t})$$

which equalizes the optimal household demand for firm i's variety with firm i's output $Y_{i,t}$ described by its production function and labor input.

Profit maximization delivers four first order conditions. The first order condition with respect to the optimal job posting $v_{i,t}$ provides an expression for the value of an additional worker $\mu_{i,t}$, which is the Lagrange-multiplier of the employment law of motion. The second FOC, with respect to the optimal wage posted $w_{i,t}^{new}$ to new workers delivers an expression for the wage Phillips curve. Third, the FOC with respect to optimal employment $n_{i,t}$ delivers an expression for marginal cost $\lambda_{i,t}$, which is the Lagrange-multiplier of the market clearing condition. Finally, the FOC with respect to the optimal price $p_{i,t}$ delivers the price Phillips curve of the model.

I will take these in turn and provide the value of an additional worker $\mu_{i,t}$, marginal cost $\lambda_{i,t}$, and the wage and price Phillips curves in the following section. I close the model with a standard Taylor rule and the imposition of overall market clearing (see Appendix I.1). The Taylor rule also contains a monetary policy shock which provides the demand shock in the model.

I.5 Optimization under Future DNWR

This section provides the solution to the optimization problem of the firm producing variety $i \in [0, 1]$. Due to DNWR there are essentially three cases. First, the case where DNWR binds. Second, the case where DNWR is not binding nor expected to bind. And third, the case where DNWR does not bind in period t but is expected to bind in period $t + 1$. This is the case under a sufficiently strong but transitive positive demand or negative labor supply shock.

The employment law of motion and demand constraints are introduced into the optimization problem via Lagrange multipliers $\mu_{i,t}$ and $\lambda_{i,t}$ respectively, while the downward nominal wage rigidity constraint is solved case by case. The first order condition with respect to the number of posted vacancies $v_{i,t}$ is the same in all three cases and gives

$$\mu_{i,t} = \frac{\kappa_v}{a_t\left(\frac{w_{i,t}}{W_t}\right)}$$

I.5.1 Wage Phillips curve Derivation

I start with the simplest case: downward nominal wage rigidity is binding today. Thus, $w_{i,t} = w_{i,t-1}$ and is independent of the choice of $w_{i,t}^{new}$. The first order condition with respect to $w_{i,t}^{new}$ gives

$$0 = -\frac{1}{P_t}\left(n_{i,t} - \left(1 - \tilde{\delta}\left(\frac{w_{i,t}}{W_t}\right)\right) n_{i,t-1}\right)$$

It follows that $n_{i,t} = \left(1 - \tilde{\delta}\left(\frac{w_{i,t}}{W_t}\right)\right) n_{i,t-1}$ which implies zero hires $v_{i,t} = 0$ when downward nominal wage rigidity binds.

The second case is where downward nominal wage rigidity is not binding, nor expected to bind tomorrow. Thus, $w_{i,t} = w_{i,t}^{new}$ and $E_t[w_{i,t+1}] = E_t[w_{i,t+1}^{new}]$. As the wage of existing and

new hired workers is the same in this case, the wage cost collapses to $\frac{w_{i,t}^{new}}{P_t} n_{i,t}$ in the Bellman equation.

The FOC with respect to $w_{i,t}^{new}$ is

$$\begin{aligned} 0 = &- \kappa_w \left(\frac{w_{i,t}}{w_{i,t-1}} - 1 \right) \frac{1}{w_{i,t-1}} N_t - \frac{n_{i,t}}{P_t} + \mu_{i,t} \left[a_t' \left(\frac{w_{i,t}}{W_t} \right) \frac{v_{i,t}}{W_t} - \tilde{\delta}_t' \left(\frac{w_{i,t}}{W_t} \right) \frac{n_{i,t-1}}{W_t} \right] \\ &+ E_t \left[\Lambda_{t,t+1} \kappa_w \left(\frac{w_{i,t+1}}{w_{i,t}} - 1 \right) \frac{1}{w_{i,t}} N_{t+1} \right] \end{aligned}$$

By symmetry across firms, multiplying the equation by $\frac{W_t}{N_t}$, and defining $\Pi_t^w = \frac{W_t}{W_{t-1}}$

$$\begin{aligned} \left(\Pi_t^w - 1 \right) \Pi_t^w = &\frac{1}{\kappa_w} \left(\mu_t \left[a_t' \left(1 \right) \frac{V_t}{N_t} - \tilde{\delta}_t' \left(1 \right) \frac{N_{t-1}}{N_t} \right] - \frac{W_t}{P_t} \right) \\ &+ E_t \left[\Lambda_{t,t+1} \left(\Pi_{t+1}^w - 1 \right) \Pi_{t+1}^w \frac{N_{t+1}}{N_t} \right] \end{aligned}$$

Plugging in the expressions for

$$\mu_t = \frac{\kappa_v}{a_t \left(1 \right)}$$

and the outer derivatives

$$\begin{aligned} a_t'(1) &= q(\theta_t) \mathfrak{p}_t^E f_\psi(1) \\ \tilde{\delta}_t'(1) &= - \frac{V_t q(\theta_t) \mathfrak{p}_t^E}{N_{t-1}} f_\psi(1) \end{aligned}$$

provides the wage Phillips curve in terms of fundamentals

$$\left(\Pi_t^w - 1 \right) \Pi_t^w = \frac{1}{\kappa_w} \left(\frac{\kappa_v}{a_t \left(1 \right)} 2 \frac{V_t}{N_t} q(\theta_t) \mathfrak{p}_t^E f_\psi(1) - \frac{W_t}{P_t} \right) + E_t \left[\Lambda_{t,t+1} \left(\Pi_{t+1}^w - 1 \right) \Pi_{t+1}^w \frac{N_{t+1}}{N_t} \right]$$

I show in Appendix I.2 that $\frac{\kappa_v}{a_t(1)} 2 \frac{V_t}{N_t} q(\theta_t) \mathfrak{p}_t^E f_\psi(1)$ is the steady state real wage, which prevails in absence of wage rigidity ${\frac{W_t}{P_t}}^*$.

$$\left(\Pi_t^w - 1 \right) \Pi_t^w = \frac{1}{\kappa_w} \left({\frac{W_t}{P_t}}^* - \frac{W_t}{P_t} \right) + E_t \left[\Lambda_{t,t+1} \left(\Pi_{t+1}^w - 1 \right) \Pi_{t+1}^w \frac{N_{t+1}}{N_t} \right]$$

In the third case which contains my channel downward nominal wage rigidity is not binding, but expected to bind tomorrow. Thus, $w_{i,t} = w_{i,t}^{new}$ and $E_t[w_{i,t+1}] = E_t[w_{i,t}]$. As the wage of existing and new hired workers is the same in period t, the results nest those of case two. I mark the innovation of expected downward nominal wage rigidity in red.

The FOC with respect to $w_{i,t}^{new}$ is

$$\begin{aligned} 0 = &- \kappa_w \left(\frac{w_{i,t}}{w_{i,t-1}} - 1 \right) \frac{1}{w_{i,t-1}} N_t - \frac{n_{i,t}}{P_t} + \mu_{i,t} \left[a_t' \left(\frac{w_{i,t}}{W_t} \right) \frac{v_{i,t}}{W_t} - \tilde{\delta}_t' \left(\frac{w_{i,t}}{W_t} \right) \frac{n_{i,t-1}}{W_t} \right] \\ &+ E_t \left[\Lambda_{t,t+2} \kappa_w \left(\frac{w_{i,t+2}}{w_{i,t+1}} - 1 \right) \frac{1}{w_{i,t+1}} N_{t+2} \right] \\ &+ E_t \left[\Lambda_{t,t+1} \left(- \frac{n_{i,t}}{P_{t+1}} \left(1 - \tilde{\delta}_{t+1} \left(\frac{w_{i,t}}{W_{t+1}} \right) \right) + \frac{n_{i,t}}{P_{t+1}} \frac{w_{i,t}}{W_{t+1}} \tilde{\delta}_{t+1}' \left(\frac{w_{i,t}}{W_{t+1}} \right) - \frac{n_{i,t}}{P_{t+1}} \frac{w_{i,t+1}^{new}}{W_{t+1}} \tilde{\delta}_{t+1}' \left(\frac{w_{i,t}}{W_{t+1}} \right) \right) \right] \\ &+ E_t \left[\Lambda_{t,t+1} \mu_{i,t+1} \left[a_{t+1}' \left(\frac{w_{i,t}}{W_{t+1}} \right) \frac{v_{i,t+1}}{W_{t+1}} - \tilde{\delta}_{t+1}' \left(\frac{w_{i,t}}{W_{t+1}} \right) \frac{n_{i,t}}{W_{t+1}} \right] \right] \end{aligned}$$

as $E_t\left[\frac{w_{i,t+1}}{w_{i,t}}-1\right]=0$ due to expected downward nominal wage rigidity in period $t+1$.

By symmetry across firms and multiplying the equation by $\frac{W_t}{N_t}$

$$\begin{aligned}(\Pi_t^w-1)\,\Pi_t^w=&\frac{1}{\kappa_w}\left(\mu_t\left[a_t'(1)\,\frac{V_t}{N_t}-\tilde{\delta}_t'(1)\,\frac{N_{t-1}}{N_t}\right]-\frac{W_t}{P_t}\right)\\&+E_t\left[\Lambda_{t,t+2}\left(\Pi_{t+2}^w-1\right)\Pi_{t+2}^w\frac{N_{t+2}}{N_t}\right]\\&+E_t\left[\Lambda_{t,t+1}\left(-\frac{W_t}{P_{t+1}}\left(1-\tilde{\delta}_{t+1}(1)\right)+\frac{W_t}{P_{t+1}}\tilde{\delta}_{t+1}'(1)-\frac{W_t}{P_{t+1}}\frac{w_{i,t+1}^{new}}{W_{t+1}}\tilde{\delta}_{t+1}'(1)\right)\right]\\&+E_t\left[\Lambda_{t,t+1}\mu_{t+1}\left[a_{t+1}'(1)\,\frac{V_{t+1}}{N_t}-\tilde{\delta}_{t+1}'(1)\right]\right]\end{aligned}$$

Due to no hiring under DNWR in $t+1$, $N_{t+1}=(1-\bar{\delta})N_t$. Using this, the expressions for

$$\mu_t=\frac{\kappa_v}{a_t(1)}$$

and the outer derivatives

$$a_t'(1)=q(\theta_t)\mathfrak{p}_t^E f_\psi(1)$$

$$\tilde{\delta}_{t+1}'(1)=-\frac{V_{t+1}q(\theta_{t+1})\mathfrak{p}_{t+1}^E}{N_t}f_\psi(1)$$

gives

$$\begin{aligned}(\Pi_t^w-1)\,\Pi_t^w=&\frac{1}{\kappa_w}\left(\frac{\kappa_v}{a_t(1)}2\frac{V_t}{N_t}q(\theta_t)\mathfrak{p}_t^E f_\psi(1)-\frac{W_t}{P_t}\right)\\&+E_t\left[\Lambda_{t,t+2}\left(\Pi_{t+2}^w-1\right)\Pi_{t+2}^w\frac{N_{t+2}}{N_t}\right]\\&+E_t\left[\Lambda_{t,t+1}\left(-\frac{W_t}{P_{t+1}}\left(1-\tilde{\delta}_{t+1}(1)\right)+\frac{W_t}{P_{t+1}}\tilde{\delta}_{t+1}'(1)-\frac{W_t}{P_{t+1}}\frac{w_{i,t+1}^{new}}{W_{t+1}}\tilde{\delta}_{t+1}'(1)\right)\right]\\&+E_t\left[\Lambda_{t,t+1}(1-\bar{\delta})\frac{\kappa_v}{a_{t+1}(1)}2\frac{V_{t+1}}{N_{t+1}}q(\theta_{t+1})\mathfrak{p}_{t+1}^E f_\psi(1)\right]\end{aligned}$$

with the real wage in absence of wage rigidities $\frac{W_t}{P_t}^*=\frac{\kappa_v}{a_t(1)}2\frac{V_t}{N_t}q(\theta_t)\mathfrak{p}_t^E f_\psi(1)$ this becomes

$$\begin{aligned}(\Pi_t^w-1)\,\Pi_t^w=&\frac{1}{\kappa_w}\left(\frac{W_t}{P_t}^*-\frac{W_t}{P_t}\right)+E_t\left[\Lambda_{t,t+2}\left(\Pi_{t+2}^w-1\right)\Pi_{t+2}^w\frac{N_{t+2}}{N_t}\right]\\&+E_t\left[\Lambda_{t,t+1}\left(-\frac{W_t}{P_{t+1}}\left(1-\tilde{\delta}_{t+1}(1)\right)+\frac{W_t}{P_{t+1}}\tilde{\delta}_{t+1}'(1)-\frac{W_t}{P_{t+1}}\frac{w_{i,t+1}^{new}}{W_{t+1}}\tilde{\delta}_{t+1}'(1)\right)\right]\\&+E_t\left[\Lambda_{t,t+1}(1-\bar{\delta})\frac{W_{t+1}}{P_{t+1}}^*\right]\end{aligned}$$

Furthermore, with $V_{t+1}=0$ we get $\left(1-\tilde{\delta}_{t+1}(1)\right)=(1-\bar{\delta})$ and

$$\tilde{\delta}_{t+1}'(1)=-\frac{V_{t+1}q(\theta_{t+1})\mathfrak{p}_{t+1}^E}{N_t}f_\psi(1)\frac{1}{W_{t+1}}=-\frac{V_{t+1}^{1-\eta}S_{t+1}^\eta\mathfrak{p}_{t+1}^E}{N_t}f_\psi(1)\frac{1}{W_{t+1}}=0$$

as $\eta<1$. This simplifies the wage Phillips curve to

$$(\Pi_t^w - 1)\,\Pi_t^w = \frac{1}{\kappa_w}\left(\frac{W_t}{P_t}^* - \frac{W_t}{P_t}\right) + E_t\left[\Lambda_{t,t+2}\left(\Pi_{t+2}^w - 1\right)\Pi_{t+2}^w \frac{N_{t+2}}{N_t}\right]$$
$$+ E_t\left[\Lambda_{t,t+1}\left(1-\bar{\delta}\right)\left(\frac{W_{t+1}}{P_{t+1}}^* - \frac{W_t}{P_{t+1}}\right)\right]$$

which delivers the same flattening of the wage response as in the baseline model without wage discrimination.

A first order approximation delivers the wage Phillips curve in its common form

$$\pi_t^w = E_t\left[\Lambda_{t,t+1}\pi_{t+1}^w\right] + \frac{1}{\kappa_w}\left(\hat{\omega^*}_t - \hat{w}_t + \hat{p}_t + E_t\left[\Lambda_{t,t+1}(1-\bar{\delta})(\hat{\omega^*}_{t+1} - \hat{w}_t + \hat{p}_{t+1})\right]\right)$$

where $\hat{\omega^*}_t$ is the deviation of flexible real wage from steady state flexible real wage.

I.5.2 Price Phillips Curve

The first order condition with respect to the price of variety i is the same in all three cases. Together with symmetry across firms it gives the model's price Phillips curve

$$(\Pi_t^p - 1)\Pi_t^p = \frac{1}{\kappa_p}(1 - \epsilon + \epsilon\lambda_t) + E_t\left[\Lambda_{t,t+1}\kappa_p\left(\Pi_{t+1}^p - 1\right)\Pi_{t+1}^p \frac{Y_{t+1}}{Y_t}\right]$$

The price Phillips curve is determined by a markup depending on the elasticity of demand ϵ over marginal cost λ_t, plus a discounted term for expected inflation.

The linear approximation once more provides the usual form of the price Phillips curve

$$\pi_t^p = \frac{1}{\kappa_p}(\hat{\mathcal{M}}_t + \hat{\lambda}_t) + E_t\left[\Lambda_{t,t+1}\pi_{t+1}^p\right]$$

However, the price Phillips curve is incomplete without the measure of marginal cost λ_t.

I.5.3 Marginal Cost

Marginal cost λ_t is obtained from the first order condition with respect to employment $n_{i,t}$. The first order condition with respect to $n_{i,t}$ is

$$0 = -\frac{w_{i,t}}{P_t} - \mu_{i,t} + \lambda_{i,t}f'(n_{i,t}) + E_t\left[\Lambda_{t,t+1}(1-\tilde{\delta}_{t+1})\mu_{i,t+1}\right]$$
$$- E_t\left[\Lambda_{t,t+1}\left(1-\tilde{\delta}_{t+1}\right)\left(\frac{w_{i,t+1}}{P_{t+1}} - \frac{w_{i,t+1}^{new}}{P_{t+1}}\right)\right]$$

It makes a difference for marginal cost if firms expect the downward wage rigidity to bind in period $t+1$ or not. If thy do not expect it to bind $E_t\left[w_{i,t+1}\right] = E_t\left[w_{i,t+1}^{new}\right]$ the last term drops out. However, if they anticipate downward wage rigidity to get wages of existing workers to get stuck above optimal wages then the last term adds to current marginal cost as firms anticipate higher marginal cost from their wage decision today.

Furthermore, if DNWR is expected to bind $w^{new}_{i,t+1}$ does not affect $w_{i,t+1}$, and can be set without Rotemberg adjustment cost. Thus, $\frac{w^{new}_{i,t+1}}{P_{t+1}} = \frac{W^*_{t+1}}{P_{t+1}}$, the optimal real wage under flexible wages. Reordering and using symmetry across firms yields the marginal cost.

$$\lambda_t = \frac{\frac{W_t}{P_t} + \mu_t + E_t\left[\Lambda_{t,t+1}\left(\left(1-\bar{\delta}\right)\left(\frac{W_{t+1}}{P_{t+1}} - \frac{W^*_{t+1}}{P_{t+1}}\right) - \left((1-\tilde{\delta}_{t+1})\mu_{t+1}\right)\right)\right]}{f'(n_{i,t})}$$

Linear approximation of the marginal cost expression provides

$$\begin{aligned}\hat{\lambda}_t =& \hat{w}_t - \hat{p}_t - \hat{mpl}_t + \hat{\mu}_t - E_t\left[\Lambda_{t,t+1}(1-\bar{\delta})\hat{\mu}_{t+1}\right] \\ &+ E_t\left[\Lambda_{t,t+1}(1-\bar{\delta})(\hat{w}_t - \hat{p}_{t+1} - \hat{\omega}^*_{t+1})\right]\end{aligned}$$

where I denote the deviation of the marginal product of labor by $\hat{mpl}_t$.

Plugging the linear approximation of marginal cost into the linear approximation of the price Phillips curve provides the ultimate price Phillips curve of the model in its usually reported form.

$$\begin{aligned}\pi^p_t = \frac{1}{\kappa_p}(\hat{\mathcal{M}}_t + \hat{w}_t - \hat{p}_t - \hat{mpl}_t + \hat{\mu}_t - E_t\left[\Lambda_{t,t+1}(1-\bar{\delta})\hat{\mu}_{t+1}\right]) + E_t\left[\Lambda_{t,t+1}\pi^p_{t+1}\right] \\ + E_t\left[\Lambda_{t,t+1}(1-\bar{\delta})(\hat{w}_t - \hat{p}_{t+1} - \hat{\omega}^*_{t+1})\right]\end{aligned}$$

I.5.4 Relation to Baseline Model

Thus, the baseline model and the model with wage discrimination for new hires yield equivalent inflation and wage growth dynamics. We can use a similar derivation as in the body of the paper to see the latter.

Starting point is the baseline model's price Phillips curve again.

$$(\Pi^p_t - 1)\Pi^p_t = \frac{1}{\kappa_p}(1 - \epsilon_p + \epsilon_p\lambda_t) + E_t\left[\Lambda_{t,t+1}(\Pi^p_{t+1} - 1)\Pi^p_{t+1}\frac{Y_{t+1}}{Y_t}\right]$$

Iterating forward delivers

$$\begin{aligned}(\Pi^p_t - 1)\Pi^p_t &= \frac{1}{\kappa_p}E_t\left[\sum_{k=0}^{\infty}\Lambda_{t,t+k}(1-\epsilon_p+\epsilon_p\lambda_{t+k})\frac{Y_{t+k}}{Y_t}\right] \\ &= \underbrace{\frac{1}{\kappa_p}E_t\left[\sum_{k=0}^{\infty}\Lambda_{t,t+k}(1-\epsilon_p)\frac{Y_{t+k}}{Y_t}\right]}_{=:\Theta_t} + \frac{\epsilon_p}{\kappa_p}E_t\left[\sum_{k=0}^{\infty}\Lambda_{t,t+k}\lambda_{t+k}\frac{Y_{t+k}}{Y_t}\right] \\ &= \Theta_t + \frac{\epsilon_p}{\kappa_p}E_t\left[\sum_{k=0}^{\infty}\Lambda_{t,t+k}\lambda^*_{t+k}\frac{Y_{t+k}}{Y_t}\right] + \frac{\epsilon_p}{\kappa_p}E_t\left[\sum_{k=0}^{\infty}\Lambda_{t,t+k}\left(\lambda_{t+k} - \lambda^*_{t+k}\right)\frac{Y_{t+k}}{Y_t}\right]\end{aligned}$$

where λ^*_{t+k} is the marginal cost prevailing without rigidities in period $t+k$. Under small enough shocks such that $Y_{t+k} \approx Y_{t+k}|_{flex}$ we can rewrite this as

$$(\Pi^p_t - 1)\Pi^p_t \approx [(\Pi^p_t - 1)\Pi^p_t]^* + \frac{\epsilon_p}{\kappa_p}E_t\left[\sum_{k=0}^{\infty}\Lambda_{t,t+k}\left(\lambda_{t+k} - \lambda^*_{t+k}\right)\frac{Y_{t+k}}{Y_t}\right]$$

The difference between marginal cost under DNWR binding and flexible wages is mostly driven by the differences in real wages, as hiring cost are only about one percent of total marginal cost in the data and calibration. Hiring cost are slightly larger if DNWR is not enforced, and wages can adjust downward. This abates the difference between the two marginal costs λ_{t+k} and λ^*_{t+k}, but plays a minor role. Thus, inflation with DNWR expected to bind can be approximated by inflation under the same circumstances but without enforcing DNWR, plus a discounted difference between the real wages paid under DNWR and flexible wages.

$$(\Pi^p_t - 1)\Pi^p_t \approx [(\Pi^p_t - 1)\Pi^p_t]^* + \frac{\epsilon_p}{\kappa_p} E_t \left[\sum_{k=0}^{\infty} \Lambda_{t,t+k} \left(\frac{\frac{W_{t+k}}{P_{t+k}} - \frac{W_{t+k}}{P_{t+k}}^*}{f'(N_{t+k})} \right) \frac{Y_{t+k}}{Y_t} \right]$$

The linear approximation of $Y_{t+k} = f(N_{t+k}) \approx f'(N_{t+k})N_{t+k}$ gives

$$(\Pi^p_t - 1)\Pi^p_t \approx [(\Pi^p_t - 1)\Pi^p_t]^* + \frac{\epsilon_p}{\kappa_p} E_t \left[\sum_{k=0}^{\infty} \Lambda_{t,t+k} \left(\frac{\frac{W_{t+k}}{P_{t+k}} - \frac{W_{t+k}}{P_{t+k}}^*}{f'(N_t)} \right) \frac{N_{t+k}}{N_t} \right]$$

Finally, optimal no hiring under wage discrimination for new hires implies optimal no hiring in the baseline model as well. Then $N_{t+k} = (1 - \bar{\delta})^k N_t$ giving

$$(\Pi^p_t - 1)\Pi^p_t \approx [(\Pi^p_t - 1)\Pi^p_t]^* + \frac{\epsilon_p}{\kappa_p} E_t \left[\sum_{k=0}^{\infty} \Lambda_{t,t+k} (1 - \bar{\delta})^k \left(\frac{\frac{W_{t+k}}{P_{t+k}} - \frac{W_{t+k}}{P_{t+k}}^*}{f'(N_t)} \right) \right]$$

which is exactly the added up extra marginal cost derived under the wage discrimination version of the model.

I.6 Kudlyak's User Cost of Labor in the Model

This appendix shows how the model is related to the user cost of labor as defined by Kudlyak (2014).

The forward looking wage cost introduced by DNWR in my model are similar to the user cost of labor (UCL) as defined by Kudlyak (2014).[44] The UCL is defined as the present discounted value of the difference in wages for a worker hired today instead of tomorrow. If wages are upward flexible but downward rigid the UCL is only at play during boom phases of the economy, when expected future wages necessary to hire an additional worker are lower than today's wage rate. This increases the cost of hiring an additional worker beyond the traditional wage and vacancy cost.

The true wage cost of a worker in Kudlyak (2014) is

$$w_{i,t} + E_t \left[\sum_{k=1}^{\infty} (\beta(1 - \bar{\delta}))^k (w_{i,t} - w_{i,t+k}) \right]$$

[44] Another connected paper is Sbordone (2005) who contemplate the role of expected future marginal cost on inflation dynamics in a general sense.

β is the discount rate and $\bar{\delta}$ is the separation rate for employees, which together deliver the effective discount factor for the expected future wage differentials. An additional piece to the total user cost of labor from Kudlyak (2014) is the difference between hiring costs today and tomorrow. This brings the total cost of a worker to

$$w_{i,t} + E_t\left[\sum_{k=1}^{\infty}(\beta(1-\bar{\delta}))^k(w_{i,t}-w_{i,t+k})\right] + \frac{\kappa_v}{a_t\left(\frac{w_{i,t}}{W_t}\right)} - \beta(1-\bar{\delta})E_t\left[\frac{\kappa_v}{a_{t+1}\left(\frac{w_{i,t+1}}{W_{t+1}}\right)}\right]$$

where $a_t\left(\frac{w_{i,t}}{W_t}\right)$ is the probability of turning a vacancy into an employee and κ_v is the cost of a vacancy.[45]

To derive the User Cost of Labor I start from the model with wage discrimination for new hires. Taking into account that optimal hiring under DNWR is zero we get $n_{i,t+k} = (1-\bar{\delta})^k n_{i,t}$.

Thus, the FOC under future DWNR is

$$0 = -\frac{w_{i,t}}{P_t} - \mu_{i,t} + \lambda_{i,t}f'(n_{i,t}) + \sum_{k=1}^{\tau-1} E_t\left[\Lambda_{t,t+k}(1-\bar{\delta})^k\left(\lambda_{i,t+k}f'(n_{i,t+k}) - \frac{w_{i,t}}{P_{t+k}}\right)\right]$$

I plug in the expression for marginal cost under DNWR derived for marginal cost under binding DNWR, which yields

$$\begin{aligned}0 = &-\frac{w_{i,t}}{P_t} - \mu_{i,t} + \lambda_{i,t}f'(n_{i,t}) + \sum_{k=1}^{\tau-1} E_t\left[\Lambda_{t,t+k}(1-\bar{\delta})^k\left(\frac{w_{i,t+k}}{P_{t+k}} - \frac{w_{i,t}}{P_{t+k}}\right)\right]\\ &+ E_t[\Lambda_{t,t+1}(1-\bar{\delta})\mu_{i,t+1}]\\ &+ \sum_{k=2}^{\tau-1} E_t\left[\Lambda_{t,t+k}(1-\bar{\delta})^k\mu_{i,t+k}\frac{\tilde{\delta}_{i,t+k}-\bar{\delta}}{1-\bar{\delta}}\right]\\ &- E_t\left[\Lambda_{t,t+\tau}(1-\bar{\delta})^{\tau-1}(1-\tilde{\delta}_{i,t+\tau})\mu_{i,t+\tau}\right]\end{aligned}$$

By symmetry across firms, and reordering we get marginal cost under future DNWR

Under future DWNR and my simplifying assumption, that workers with wages above market rates do not look for jobs, and symmetry marginal cost is

$$\begin{aligned}\lambda_t = &\frac{\frac{W_t}{P_t} + \mu_t + \sum_{k=1}^{\tau-1} E_t\left[\Lambda_{t,t+k}(1-\bar{\delta})^k\left(\frac{W_t}{P_{t+k}} - \frac{W_{t+k}}{P_{t+k}}\right)\right] - E_t[\Lambda_{t,t+1}(1-\bar{\delta})\mu_{t+1}]}{f'(n_t)}\\ &- \frac{\sum_{k=2}^{\tau-1} E_t\left[\Lambda_{t,t+k}(1-\bar{\delta})^k\mu_{t+k}\frac{\tilde{\delta}_{t+k}-\bar{\delta}}{1-\bar{\delta}}\right] - E_t\left[\Lambda_{t,t+\tau}(1-\bar{\delta})^{\tau-1}(1-\tilde{\delta}_{t+\tau})\mu_{t+\tau}\right]}{f'(n_t)}\end{aligned}$$

This differs from Kudlyak's user cost of labor in the assumption that DNWR is only binding for $\tau-1$ periods, instead of forever, and in the form that Kudlyak does not have endogenous separation on top of exogenous separation. Thus, under her assumptions $\tilde{\delta}_{t+k} = \bar{\delta}$, and

[45] Evidence for the pro-cyclicality of the User Cost of labor is provided by Kudlyak (2014) & Bils et al. (2023).

$\tau \to \infty$. Under these assumptions the second fraction equals to zero, and we are left with the familiar user cost of labor as stated by Kudlyak (2014).

Linear approximation of the marginal cost expression provides

$$\begin{aligned}\hat{\lambda}_t =& \hat{w}_t - \hat{p}_t - \hat{A}_t + \sum_{k=1}^{\tau-1} E_t\left[\Lambda_{t,t+k}(1-\bar{\delta})^k(\hat{w}_t - \hat{w}_{t+k} - \hat{p}_{t+k})\right] \\ &+ \hat{\mu}_t - E_t\left[\Lambda_{t,t+1}(1-\bar{\delta})\hat{\mu}_{t+1}\right] - \sum_{k=2}^{\tau-1} E_t\left[\Lambda_{t,t+k}(1-\bar{\delta})^k\hat{\mu}_{t+k}\frac{\tilde{\delta}_{t+k}-\bar{\delta}}{1-\bar{\delta}}\right] \\ &+ E_t\left[\Lambda_{t,t+\tau}(1-\bar{\delta})^\tau\frac{(1-\tilde{\delta}_{t+\tau})}{1-\bar{\delta}}\hat{\mu}_{t+\tau}\right]\end{aligned}$$

Plugging the linear approximation of marginal cost into the linear approximation of the price Phillips curve provides the ultimate price Phillips curve of the model in its usually reported form.

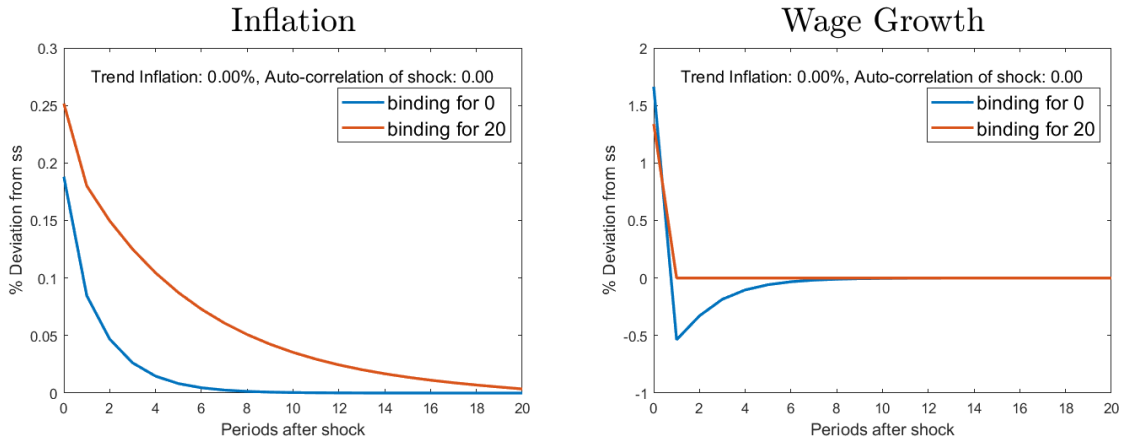
$$\begin{aligned}\pi_t^p =& \frac{\epsilon}{\kappa_p}\left(\hat{w}_t - \hat{p}_t - \hat{A}_t + \sum_{k=1}^{\tau-1} E_t\left[\Lambda_{t,t+k}(1-\bar{\delta})^k(\hat{w}_t - \hat{w}_{t+k} - \hat{p}_{t+k})\right]\right) \\ &+ \frac{\epsilon}{\kappa_p}\left(\hat{\mu}_t - E_t\left[\Lambda_{t,t+1}(1-\bar{\delta})\hat{\mu}_{t+1}\right] - \sum_{k=2}^{\tau-1} E_t\left[\Lambda_{t,t+k}(1-\bar{\delta})^k\hat{\mu}_{t+k}\frac{\tilde{\delta}_{t+k}-\bar{\delta}}{1-\bar{\delta}}\right]\right) \\ &+ \frac{\epsilon}{\kappa_p}\left(E_t\left[\Lambda_{t,t+\tau}(1-\bar{\delta})^\tau\frac{(1-\tilde{\delta}_{t+\tau})}{1-\bar{\delta}}\hat{\mu}_{t+\tau}\right]\right) + E_t\left[\Lambda_{t,t+1}\pi_{t+1}^p\right]\end{aligned}$$

The steady state derivation is provided in Appendix I.2. To the best of my knowledge this is the first New Keynesian model incorporating Kudlyak's user cost of labor.

I.7 Additional Dynamics

I.7.1 Labor Supply Shocks

This appendix presents the IRFs following a negative labor supply shock in the NK monopsony model, see Figure 37.



(a) Inflation Response

(b) Wage Growth

Figure 37: Impulse Response Functions to negative Labor Supply Shock

Subfigure a) provides the IRF of inflation with respect to a transitory 20% increase in the replacement rate. Subfigures b) and c) do the same for nominal and real wage growth, respectively.

J Downward Bias from the Inclusion of Short-Term Expectations

It is well documented (e.g. Weber et al. (2022)) that inflation causes inflation expectations to rise. This is particularly true for short-term expectations, while long-term expectations stay more anchored. Furthermore, short-term inflation expectations are strongly driven by slack variables, such as labor market tightness. Hence, the structural equation of expected inflation is described as follows:

$$\pi^e_{ST} = \gamma\pi + \phi(V/U) + \psi\pi^e_{LT} + \epsilon_1$$

If the relation of interest is between labor market tightness (or any other slack variable) and inflation, while the latter is also influenced by long-term inflation expectations:

$$\pi = \alpha(V/U) + \beta\pi^e_{LT} + \epsilon_2$$

then the inclusion of short-term instead of long-term expectations leads to an endogeneity bias in $\hat{\alpha}$, for two reasons. The first is mentioned but not further explained by Gagnon and Collins (2019). It is an endogeneity bias due to the impact of inflation on short-term inflation expectations. The second bias stems from the fact that labor market tightness also drives π^e. This leads to a second endogeneity bias, as I will show below. If one naively estimates the structural inflation equation, while using short-term instead of long-term expectations:

$$\pi = \hat{\alpha}(V/U) + \hat{\beta}\pi^e_{ST} + \hat{\epsilon}$$

then $\hat{\alpha}$ is biased due to the endogeneity of π^e_{ST} to both π and V/U. I.e. plugging in the structural equation of π^e_{ST} gives:

$$\pi = \hat{\alpha}(V/U) + \hat{\beta}[\gamma\pi + \phi(V/U) + \psi\pi^e_{LT} + \epsilon_1] + \hat{\epsilon}$$

or

$$(1 - \hat{\beta}\gamma)\pi = \hat{\alpha}(V/U) + \hat{\beta}\phi(V/U) + \hat{\beta}\psi\pi^e_{LT} + \hat{\beta}\epsilon_1 + \hat{\epsilon}$$

Inserting the true relation between inflation, labor market tightness and expected inflation on the lhs yields

$$(1 - \hat{\beta}\gamma)[\alpha(V/U) + \beta\pi^e + \epsilon_2] = \hat{\alpha}(V/U) + \hat{\beta}\phi(V/U) + \hat{\beta}\psi\pi^e_{LT} + \hat{\beta}\epsilon_1 + \hat{\epsilon}$$

Hence the estimated labor market tightness coefficient $\hat{\alpha}$ has the following form:

$$\hat{\alpha} = (1 - \hat{\beta}\gamma)\alpha - \hat{\beta}\phi$$

I.e. the estimated impact is biased downwards by two endogeneity biases. The first one, stemming from the endogeneity of inflation expectations to actual inflation, yields a downward bias in proportion to the correlation between inflation and inflation expectations. The second endogeneity bias, derived from the impact of labor market tightness on expectations, gives a downward bias that is one to one with the impact of labor market tightness on inflation expectations, and its impact on inflation.

As short-term expectations are much more responsive to movements in inflation than long-term expectations during inflation surges, their endogeneity bias on the labor market coefficient is more sizeable. Further, the impact of slack variables, such as V/U, is much greater on short-term expectations than on long-term ones. Hence, it is advisable to use long-term expectations.

Furlanetto and Lepetit (2024) make a similar point emphasizing the importance of the auto-correlation of the forcing variable (V/U here). If the variable has a high auto-correlation then a shock to it will influence inflation expectations into the near future. If the desired outcome is to measure the total impact of a shock to the forcing variable then controlling for short-term inflation expectations is ill-advised, as these expectations will control for all the future inflationary pressure of the forcing variable. This explains why Beaudry et al. (2024) or Del Negro et al. (2020) find only a very flat Phillips curve, as they control for one year inflation expectations.

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