Markups, Markdowns, and the Labor Share: A Two-Sector General Equilibrium Decomposition

Ricardo Gouveia-Mendes
Iscte – University Institute of Lisbon
ricardo.mendes@iscte-iul.pt

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Abstract. The U.S. labor share of income has declined persistently since 1980. I develop a two-sector general equilibrium model that nests investment-embodied technological progress (IETP), product-market markups, and labor-market wage markdowns, and use it to decompose their joint contribution to the post-1980 decline of the labor share. The calibration uses no income-share moments: capital-output elasticities are recovered from a within-industry panel regression, TFP growth rates from the long-run trends of the capital stock and the relative investment price, markups from an operating margin built on the Compustat sample of Anderson et al. (2025, ARW), and markdowns from marginal-product residuals. The labor share, output, the real wage, and the real rate of return on capital remain untargeted moments. Three findings follow. First, rising wage markdowns alone reproduce the labor share decline; the ARW level is by construction an upper bound on the model-consistent markup — its cost base excludes the return to capital that the model counts in marginal cost — so combining the markdown with the full ARW markup overshoots the data, and the joint fit is closest, among the markup levels examined, at roughly half that level. Second, IETP contributes essentially nothing to the labor share trend but accounts for nearly all of the rise in the relative investment price and for the level of real wages — without it, the real-wage index in 2024 reaches only about half its full-model level; distribution and growth are mechanically separable in this calibration. Third, the implied markdown path translates into a halving of the firm-level labor-supply elasticity, from about 6.4 to 3.5 within the aggregation scheme, the consumption-sector markdown carries roughly three times the leverage of the investment-sector markdown on the aggregate labor share. The substantive conclusion is that the intensification — not necessarily the structural expansion — of employer market power in the consumption sector is the dominant force behind the post-1980 U.S. labor share decline.

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1. Introduction

The labor share of income in the United States has declined persistently since the early 1980s (Grossman & Oberfield, 2022; Karabarbounis, 2024; Karabarbounis & Neiman, 2014). Accounting for this decline requires distinguishing among candidate mechanisms that are individually plausible but interact in ways a single-sector or single-channel framework cannot capture. Three forces stand out as simultaneously empirically disciplined and quantitatively relevant: investment-embodied technological progress (IETP), which has steadily reduced the relative cost of capital goods; product-market markup power, which shifts income from labor to firm owners; and labor-market wage[1] markdowns — the wedge between workers' marginal revenue product of labor and their compensation, arising from monopsonistic competition in local labor markets.[2] This paper builds a two-sector general equilibrium model that nests all three forces and uses it to decompose their joint contribution to the U.S. labor share decline over 1980–2024.

A two-sector framework is needed not because IETP drives the labor share decline directly — heterogeneous capital-output elasticities ($\alpha_C \neq \alpha_I$) do open a composition channel through which IETP can shift the labor share, even under Cobb-Douglas production technologies, but the small capital-intensity gap measured in the data makes that channel quantitatively modest — but because IETP shapes the prices and real wages against which any distributional account must be evaluated. The secular fall in the relative investment price has unambiguous effects on the relative composition of investment and consumption in national accounts, on the level of real wages workers earn, and on the long-run capital stock. Karabarbounis & Neiman (2014) showed that to attribute the labor share decline to capital-labor substitution alone, one needs an input-substitution elasticity above unity — at odds with the consensus micro estimates (Chirinko, 2008; Oberfield & Raval, 2021). This paper does not test that elasticity; it shows that once Cobb-Douglas is imposed at the production-function level, the work of accounting for the labor share trend must be done by market-power channels, and IETP separates the price and growth dimensions of the data from the distributional dimension that market-power forces can move.

The sources of wage markdowns vary across settings. Employer size and concentration (Berger et al., 2022; Yeh et al., 2022), product-market power (Arnold, 2025), heterogeneous worker preferences over jobs (Bhaskar et al., 2002; Manning, 2003), and search-and-matching frictions (Diamond, 1982; Mortensen & Pissarides, 1994; Pissarides, 2000) all generate firm-level labor-supply elasticities below infinity, allowing compensation to lie below the marginal revenue product of labor. I model this mechanism via monopsonistic competition: workers view jobs at different firms within a sector as imperfect substitutes, governed by a finite exogenous sector-specific labor-supply elasticity θ_j, implying a markdown $\nu_j =$

[1] Strictly speaking, the correct expression is *labor compensation*; I use *wages* for simplicity throughout.

[2] In a multi-sector model, the relevant comparison is between the wage and the marginal *revenue* product of labor ($\text{MRPL} = P_j \times \text{MPL}_j$), which converts the physical marginal product to a common numeraire. The distinction matters when sector output prices differ, as they do in a two-sector model with IETP.

$1/(1+\theta_j)$. This structure allows for substantial heterogeneity in markdowns across firms — consistent with the data (Yeh et al., 2022) — while generating a sector-level aggregation that connects micro-level wedges to macroeconomic dynamics. The aggregation is consistent with the evidence that markdowns are weakly correlated with firm size and that the relevant source of market power is workers' preferences over jobs rather than employer size per se (Yeh et al., 2022). Because the mechanism operates on the labor-supply side, it can persist even as local firm concentration declines (Rossi-Hansberg et al., 2021).

I build the model following Greenwood et al. (1997), with two production sectors (consumption and investment), IETP driven by Harrod-neutral productivity growing faster in the investment sector, and Greenwood-Hercowitz-Huffman (GHH) preferences (Greenwood et al., 1988) for endogenous labor supply. GHH preferences eliminate wealth effects on labor supply: hours depend only on the contemporaneous real wage, not on wealth. This property is consistent with micro evidence that households with different wealth levels exhibit similar labor-supply behavior (Attanasio et al., 2012; Browning et al., 1985), and it yields a tractable aggregate labor-supply schedule that depends only on the contemporaneous real wage — on which the sector-level monopsonistic structure then operates. The consumption sector operates under monopolistic competition in goods markets — generating a markup $\mu = 1/(\sigma_c - 1)$ over marginal cost, where σ_c is a finite exogenous elasticity of demand across consumption goods varieties implied by household preferences — and under monopsonistic competition in labor markets; the investment sector is competitive in product markets but faces the same monopsonistic friction in labor markets, with sector-specific markdowns ν_C and ν_I.

The calibration uses no income-share moments. Capital-output elasticities α_C and α_I come from a within-industry fixed-effects panel regression on Bureau of Economic Analysis (BEA) data; total factor productivity (TFP) growth rates from the long-run trends of the aggregate capital stock and the relative investment price; the markup path from an operating-profit markup built on the Compustat sample of Anderson et al. (2025, hereafter ARW) and the markdown path from a marginal-product residual identification — the sectoral gap between the marginal revenue product of labor and the wage, from BEA and Bureau of Labor Statistics (BLS) data. The labor share, output, the real wage, and the real rate of return on capital all remain untargeted moments that test the calibrated model.

Three findings follow. First, the markdown channel alone nearly reproduces the labor share decline. The symmetric markdown path, rising from about 13% to 22%, delivers a 1980–2024 decline of roughly 6 percentage points against 7 in the data, with the markup set to zero throughout. The ARW operating margin is, by construction, an upper bound on the model-consistent markup: its cost base omits the return to capital that the model counts in marginal cost, so combining the markdown with the full ARW markup overshoots the data. Among the three markup levels I examine — the full ARW path, one-half, and one-tenth — one-half gives the closest joint fit across the labor share, the relative investment price,

the capital stock, and output. Identifying the markup ceiling through general-equilibrium fit corroborates the methodological concerns about Compustat-based markup measures raised by Benkard et al. (2026) and Ren & Zhang (2025), and contrasts with the role assigned to product-market markups in De Loecker et al. (2020). A further refinement is that a constant wedge — whether markup or markdown — produces only a level shift in the labor share, not a transition path: because the capital-reallocation channel is negligible, the secular trend requires time-varying market power.

Second, IETP is quantitatively orthogonal to the labor share dimension but central to the price and real-wage dimensions. The composition channel opened by $\alpha_C \neq \alpha_I$ — through which IETP can shift m_ℓ even under Cobb-Douglas production — is active in principle, but the small capital-intensity gap in the data makes it negligible: removing the investment-sector TFP premium leaves the labor share decline essentially unchanged, while the same intervention collapses the rise in the relative investment price, and the real-wage index in 2024 reaches only about half its full-model level. The two leading narratives of the labor share decline — IETP through capital-labor substitution, and rising market power — therefore complement rather than compete in this calibration: market-power distortions govern distribution, while IETP governs the relative price and the level of real wages.

Third, the implied markdown path translates into a single structural reading. The firm-level labor-supply elasticity perceived by workers falls from about 6.4 in 1980 to about 3.5 in 2024 — workers' outside-option responsiveness has roughly halved. Within the aggregation scheme, the consumption-sector markdown carries roughly three times the leverage of the investment-sector markdown on the aggregate labor share, so the substantive answer to the paper's question is mechanically a consumption-sector phenomenon: rising employer market power in the production of consumption goods is the dominant force behind the post-1980 U.S. labor share decline.

This paper contributes to three literatures. First, to the literature on the labor share decline: by providing a general-equilibrium decomposition that nests IETP, markups, and markdowns simultaneously and identifies all parameters without using income shares, I show that declining labor-supply elasticities are the quantitatively dominant force, that the Compustat-implied markup series overstates the model-consistent markup by roughly a factor of two, and that IETP is essentially orthogonal to the income split (Autor et al., 2020; De Loecker et al., 2020; Grossman & Oberfield, 2022). Second, to the IETP literature: I show that the composition channel opened by heterogeneous α_j — through which IETP can in principle shift the labor share under Cobb-Douglas — is quantitatively small in the data, so IETP shapes the trajectory of the relative price and of real wages while leaving the labor share trend almost untouched, sharpening the role assigned to investment-specific technical change in a multi-channel account (Greenwood et al., 1997; Herrendorf et al., 2014; Karabarbounis & Neiman, 2014). Third, to the labor monopsony literature: by tracing the

general-equilibrium implications of micro-level markdowns for factor shares and aggregate dynamics over a multi-decade horizon, I document that the intensification — though not necessarily a structural expansion — of monopsony power is the quantitatively dominant channel for the labor share decline (Azar & Marinescu, 2024; Ren & Zhang, 2025; Yeh et al., 2022).

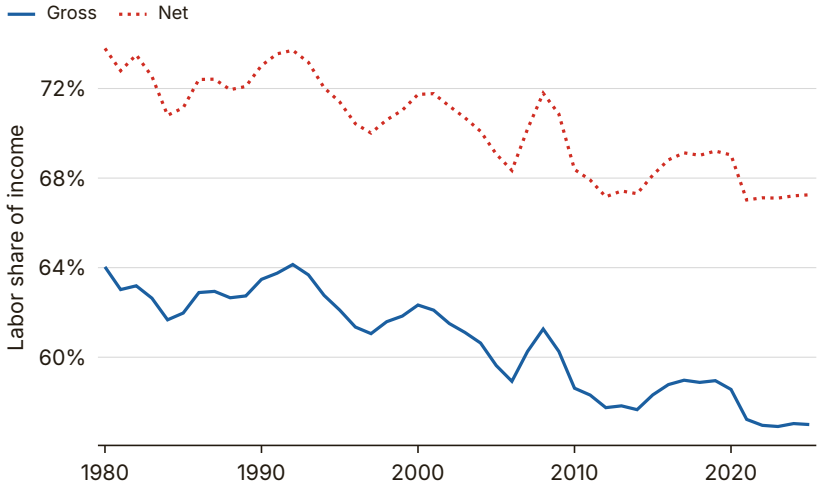
■ **Structure of the Paper.** Section 2 documents the decline in the labor share, presents the investment-price and market-power facts that motivate the three-channel model, and reviews the related literature. Section 3 introduces the two-sector general equilibrium model with IETP, endogenous labor supply, simultaneous goods-market and labor-market imperfections, and derives the implications for the labor share and for balanced growth. Section 4 calibrates the model to U.S. data and presents the quantitative decomposition results and robustness checks. Section 5 concludes.

2. Empirical Facts and Related Literature

2.1. The Labor Share: Measurement and U.S. Facts

Measuring the labor share is nontrivial. Two difficult choices matter for levels and trends: (i) how to treat proprietors' income, and (ii) whether to work with gross or net series. I focus on the United States and use U.S. Bureau of Economic Analysis (2025) National Income and Product Accounts (NIPA) data, treating proprietors' income as capital income. I report both gross and net labor shares (the latter subtracts depreciation from national income, i.e., divides labor compensation by net domestic product rather than GDP). While net shares are arguably more relevant for welfare accounting — because covering capital replacement costs constrains consumption possibilities — both measures point to a similar pattern. Figure 1 shows that both gross and net labor shares have declined since the early 1980s and makes the role of depreciation transparent.

Figure 1. Labor share of income in the U.S., 1980–2024.



Notes: Gross labor share: compensation of employees divided by GDP. Net: compensation divided by net domestic product (GDP minus depreciation). Proprietors' income is treated as capital income throughout. Source: BEA NIPA.

A key concern regarding labor share measurement is the 1999/2013 BEA capitalization of intellectual property products (IPP), which reclassified software, artistic originals, and R&D as capital rather than intermediate inputs. Koh et al. (2020) argue that the U.S. labor share decline disappears when reversing these changes or using milder assumptions. However, IPP shares key characteristics with other capital goods — identifiable ownership rights, tradable future benefits, multi-period service flows, and depreciation — and is central to recent technological progress. The BEA reclassification followed other major statistical agencies and aligns with standard firm accounting practices; treating IPP as intermediate consumption would understate productive capacity and overstate the labor share. This paper proceeds under the current BEA methodology, viewing the labor share decline as genuine.

The decline of the labor share is not confined to the United States. Karabarbounis & Neiman (2014) and Karabarbounis (2024) document a broad fall in labor shares across countries and industries, linking it to the concurrent decline in the relative price of investment goods. Their mechanism works through factor substitution: cheaper investment goods lower the rental rate of capital, inducing firms to shift production toward capital. Under standard production-function curvature, the resulting rise in the capital-labor ratio raises the marginal product of labor — and hence wages — while reducing that of capital, consistent with the lower equilibrium rental rate. Yet the labor share can still fall: if the elasticity of substitution between capital and labor, σ, exceeds one, capital expands so much that its income share rises despite the lower rental rate. Using cross-country variation in investment price and labor share trends, they estimate this elasticity to be approximately 1.25. However, micro-level evidence from firm and industry data consistently finds values below unity (Chirinko, 2008), though aggregate estimates are somewhat higher once real-

location effects are accounted for (Oberfield & Raval, 2021) — both supporting the Cobb-Douglas benchmark used in this paper and motivating the search for additional channels.

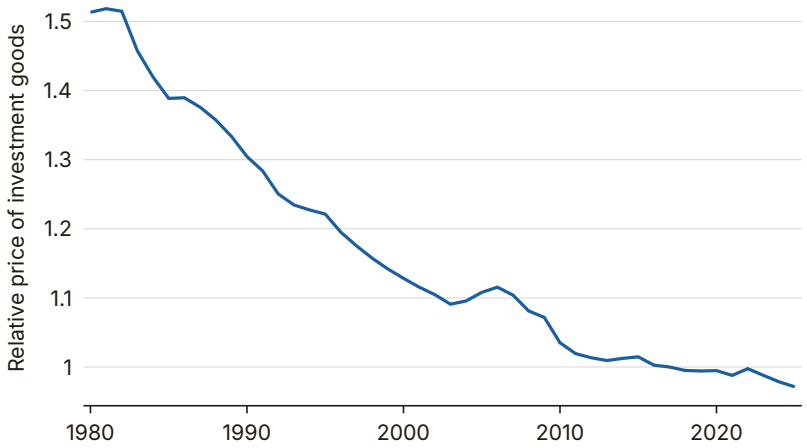
Any account of the labor share decline must be disciplined by the investment price facts, which motivates the two-sector structure adopted in this paper. I share the view of Karabarbounis (2024) that the decline has technological origins, but argue they alone are insufficient: they must be complemented by changes in market structure. On the technology side, investment-specific progress interacts with heterogeneous capital intensities across sectors (Basu et al., 2013; Valentinyi & Herrendorf, 2008) to generate a falling labor share through input sectoral reallocation under Cobb-Douglas production, without requiring $\sigma > 1$. On the market structure side, product-market markups and labor-market markdowns can compress the labor share independently of factor substitution. Together, these forces reconcile the aggregate evidence with the micro estimates of σ reviewed above.

The next subsection documents these investment facts — the fall in the relative prices of IPP and equipment and the shift in the composition of investment — that discipline the technological side of the model; the following one reviews the evidence on markups and markdowns.

2.2. Investment-Embodied Technological Progress

The investment-embodied technological progress (IETP) view — rooted in the classic Solow-Jorgenson debate and developed by Greenwood et al. (1997), with subsequent contributions by Hornstein (2004) and Herrendorf et al. (2014; 2019; 2021) — links growth to relatively faster productivity in the investment-goods sector. Under this view, the persistent decline in the relative price of investment goods over the last four decades (Figure 2) is the empirical signature of faster investment-sector productivity, which is not observed directly.

Figure 2. Relative price of investment goods in the U.S., 1980–2024.

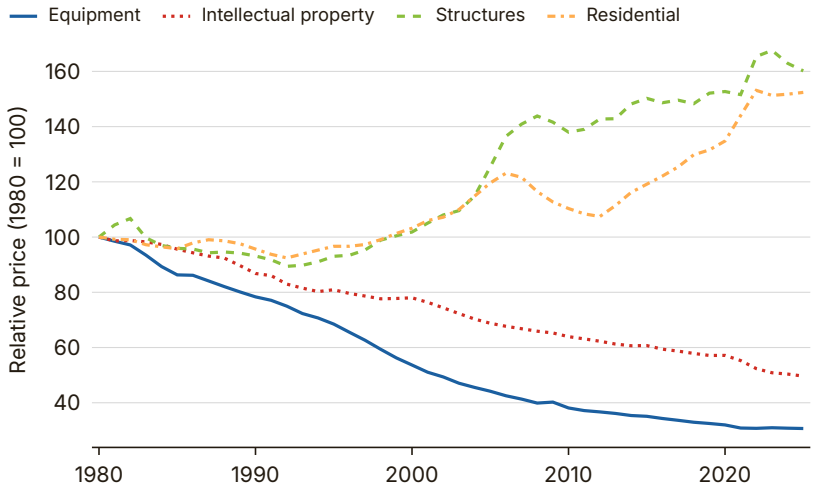


Notes: Ratio of the PCE chain-type price index to the GPDI chain-type price index (1980 = 1). A declining series indicates that investment goods are becoming cheaper relative to consumption goods. Source: BEA NIPA.

This aggregate decline reflects two distinct forces. First, a price effect: the relative prices of IPP and equipment fell sharply — driven by computers and semiconductors — while those of structures and residential investment rose (Figure 3). Second, a shift in the composition of investment: the investment basket tilted toward IPP (Figure 4), precisely the component whose relative price fell the most, so reweighting toward it lowers the aggregate relative price further still. These facts motivate the two-sector (consumption versus investment) structure with IETP used below. Following Greenwood et al. (1997), I use the falling relative price of investment goods to discipline IETP in the model. While this feature does not affect the labor share under Cobb-Douglas production with identical capital-output elasticities across sectors (Karabarbounis & Neiman, 2014), it interacts with capital reallocation across sectors when those elasticities differ.

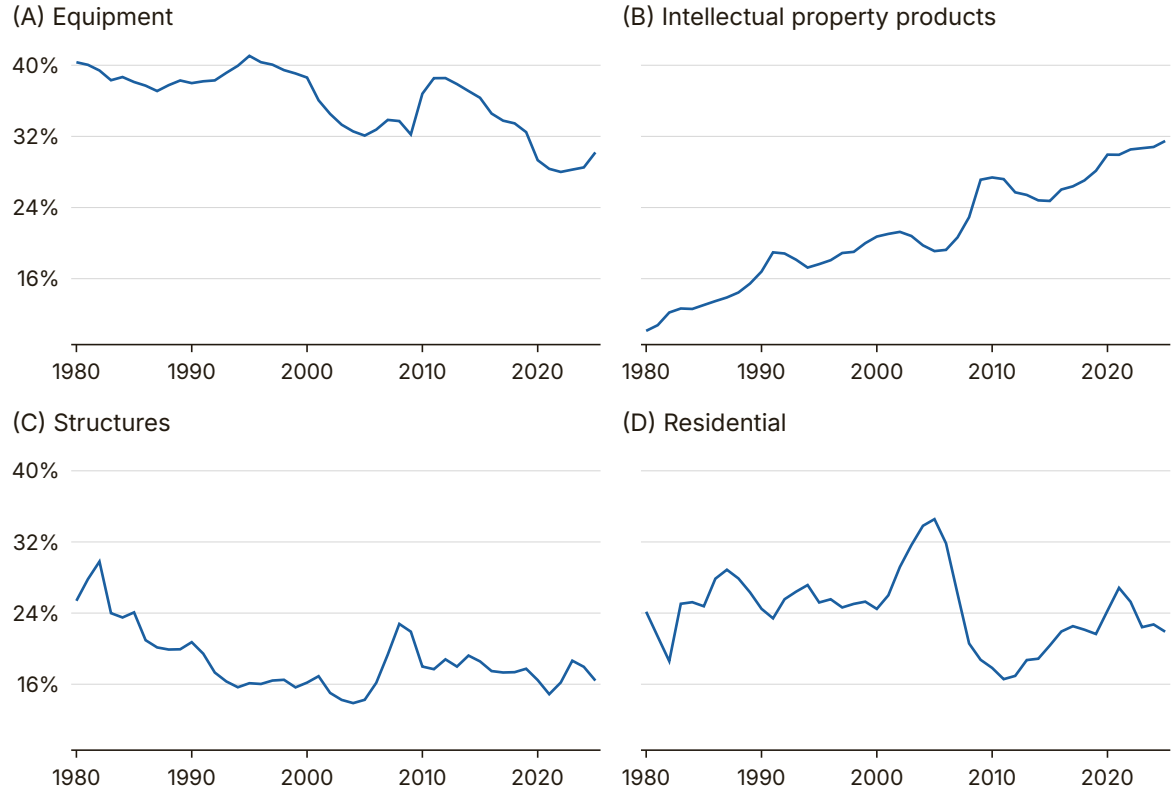
The quantitative role of IETP for the labor share depends critically on whether production functions differ across sectors. Under identical capital-output elasticities, sectoral reallocation has no first-order effect on the aggregate factor distribution: cheaper investment goods raise the capital stock without lowering the labor share (Karabarbounis & Neiman, 2014). Introducing heterogeneous capital-output elasticities across sectors breaks this neutrality — the channel developed in Gouveia-Mendes (2025). The mechanism is a composition effect: IETP lowers the relative price of investment goods (P_I falls), which raises the relative price of consumption ($q \equiv P_C/P_I$ rises) and thereby increases the nominal share of consumption-sector output in aggregate output. Since the consumption sector is more capital-intensive ($\alpha_C > \alpha_I$), a larger nominal weight on consumption raises the aggregate capital share — and lowers the labor share — even under Cobb-Douglas technology and without any market-power wedge. The mechanism is quantitatively present but modest: the small capital-intensity gap between sectors in the data implies the investment-price channel alone cannot account for the full magnitude of the observed decline. This paper therefore embeds that channel in a model with two further forces — product-market markups and labor-market markdowns — and decomposes their joint contribution to the decline; the market-power channels are reviewed next.

Figure 3. Relative prices of investment components in the U.S., 1980–2024.



Notes: Relative price of each investment component to personal consumption expenditure (1980 = 1). The relative prices of IPP and equipment fell sharply, driven by computers and semiconductors, while those of structures and residential rose. Source: BEA NIPA.

Figure 4. Composition of gross private domestic investment in the U.S., 1980–2024.



Notes: Share of each component in total gross private domestic investment (GPDI): equipment, intellectual property products (IPP), structures, and residential. IPP includes software, R&D, and artistic originals. Source: BEA NIPA.

2.3. Labor- and Goods-Market Power

■ **Wage Markdowns.** I define a wage markdown as $\nu \equiv 1 - w/\,\text{MRPL}$, the wedge between wages, w, and the marginal revenue product of labor, MRPL.[3] In canonical wage-posting models (Bhaskar et al., 2002; Burdett & Mortensen, 1998; Manning, 2003), a finite firm-level labor-supply elasticity θ implies $\text{MRPL}/w = 1 + 1/\theta$, or equivalently $\nu = 1/(1 + \theta)$: a monopsonist facing an inelastic labor supply curve compresses wages below marginal revenue product in proportion to its market power. In search-and-matching environments (Diamond, 1982; Mortensen & Pissarides, 1994; Pissarides, 2000), analogous wedges arise through bargaining, as firms capture part of the match surplus. Employer market power can originate from several sources: local employer concentration (Berger et al., 2022; Yeh et al., 2022), product-market power that reduces the derived demand for labor (Arnold, 2025), heterogeneous worker preferences over job characteristics that limit outside options (Bhaskar et al., 2002; Manning, 2003), and search frictions that reduce effective competition among potential employers (Marinescu & Rathelot, 2018). All of these sources share a common empirical signature: a gap between the marginal revenue product of labor and the wage.

The empirical literature relies on two broad measurement approaches (Azar & Marinescu, 2024). The first estimates the firm-level labor-supply elasticity θ directly, using matched employer–employee data in structural wage-posting models (Berger et al., 2023; Yeh et al., 2022), vacancy and application elasticities from job-posting data (Marinescu & Rathelot, 2018), or platform field experiments (Caldwell & Oehlsen, 2022). The second recovers MRPL from production-function estimation and computes the markdown as the residual gap (De Loecker et al., 2020; Yeh et al., 2022). The two approaches agree on levels: markdowns are substantial. Using the production-function approach in U.S. manufacturing, Yeh et al. (2022) estimate a mean markdown of about 35%, implying workers receive roughly 65 cents per dollar of marginal revenue product. Economy-wide estimates for publicly traded firms place the markdown level at about 19% in 1977 (Ren & Zhang, 2025). Cross-sectional estimates across sectors span 15% to 50% (Azar & Marinescu, 2024).

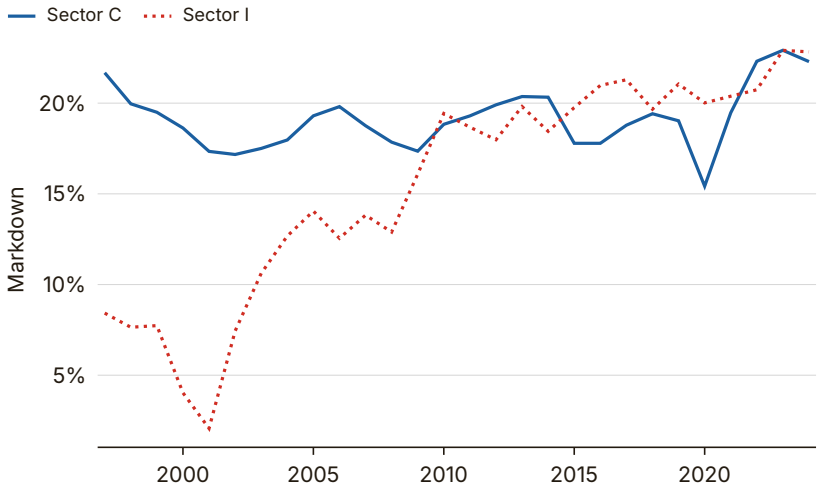
Time-series evidence on markdown trends is thin: few papers in the empirical literature document the secular evolution of markdowns. Yeh et al. (2022) document the evolution of markdowns in manufacturing at five-year intervals from 1977 to 2012. Their main translog specification yields a U-shaped pattern: the aggregate markdown index falls roughly 10% through the early 2000s, then rebounds to about 10% above its 1977 level by 2012. Under their alternative Cobb-Douglas specification — the functional form consistent with this paper's model — the trend is monotonically increasing throughout the period, providing direct micro evidence for a secular rise in labor-market power in manufacturing. Ren

[3] All wage measures throughout the paper are real, expressed in units of investment goods (the model's numeraire). In a two-sector model, what is marked down is the real marginal revenue product of labor, not the physical marginal product itself; the distinction matters when sector output prices differ.

& Zhang (2025) provide the most comprehensive time-series evidence to date, matching Compustat to confidential Census firm-level records (the Longitudinal Business Database) to observe wage bills and employment directly for publicly traded firms across all sectors from 1977 to 2019. They find that aggregate markdowns nearly double over the period — rising from 19% to 55% — driven primarily by a sharp decline in the gross labor cost share. Crucially, price markups are essentially flat over the same period in their data, isolating wage markdowns as the dominant driver of rising aggregate market power.

These time-series patterns are consistent with the view that the structural conditions for monopsony have not necessarily expanded, but that their exercise has intensified. Rising product-market power may have enabled firms to extract greater labor-market surplus: the link between markups and markdowns is documented empirically by Ren & Zhang (2025) and theoretically by Arnold (2025). Geographic concentration at the national level has risen even as local firm counts remained stable or increased (Rinz, 2020; Rossi-Hansberg et al., 2021), complicating concentration-based measures of monopsony and suggesting that frictions and preference heterogeneity, rather than employer count alone, are the operative mechanisms. This paper identifies markdowns from marginal-product conditions — which capture the combined effect of all sources of monopsony power — and traces their economy-wide evolution as an input to the general equilibrium decomposition. Figure 5 shows the sector-specific markdown paths — for the consumption sector C and the investment sector I[4] — recovered from the identification described in Section 4.

Figure 5. Identified markdown paths by sector, 1997–2024.



Notes: Wage markdown $\nu_j \equiv 1 - W/\mathrm{MRPL}_j$, where $\mathrm{MRPL}_j = (1-\alpha_j)P_jY_j/L_j$ is the revenue marginal product of labor under Cobb-Douglas technology. Series cover 1997–2024. Source: author's calculations from BEA NIPA sectoral data.

[4] Sector I comprises the 13 investment-good-producing NAICS industries: Construction (23); Information (51), which covers software, R&D, and artistic originals recorded as intellectual property investment in the national accounts; and 11 durable-goods manufacturing sub-industries (321, 327, 331, 332, 333, 334, 335, 3361MV, 3364OT, 337, 339). Manufacturing aggregates (33DG, 31G, 31ND) are excluded to avoid double-counting with the constituent sub-industries. Sector C consists of all remaining 60 private-sector industries.

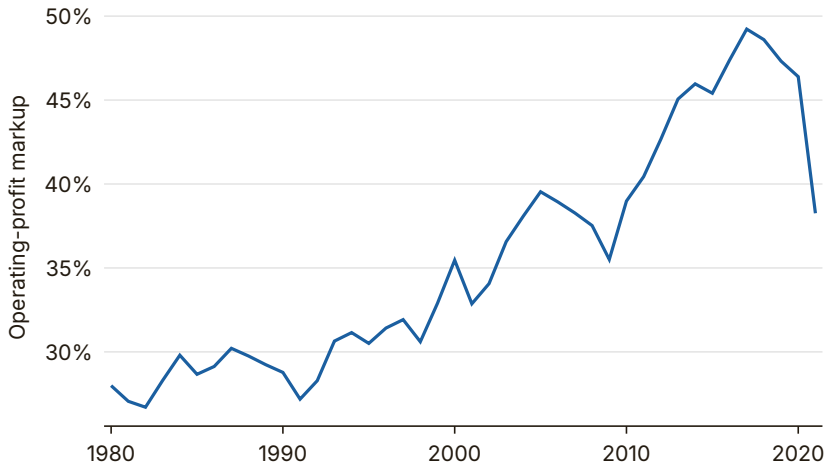
■ **Product-Market Markups.** I define a markup as $\mu \equiv p/\,\mathrm{MC} - 1$, the proportional gap between price and marginal cost. In a model with a single output and a single factor of production, markdowns and markups enter the labor share symmetrically: both drive a wedge between the output price and the factor payment, making them observationally indistinguishable from aggregate data alone. When more inputs and sectors are introduced, the connection persists but becomes more complex (Azar & Marinescu, 2024; Yeh et al., 2022): production-function-based markup estimates can absorb part of the labor-market wedge, and vice versa. This conflation likely contributes to the ongoing debate over whether markups have risen and by how much.

De Loecker et al. (2020) estimate that average U.S. markups roughly tripled between 1980 and 2016 — from 21% to 61% — with the increase occurring across all sectors and industries. Benkard et al. (2026) show, however, that this result is sensitive to unstated sample restrictions and is driven almost entirely by the *Finance and Insurance* sector; excluding it, markups rise only modestly from 30% to 35%, and within-firm markups have declined over time. In reply, De Loecker et al. (2025) attribute the mismatch to outliers in the *Pharmaceutical* sector (NAICS 3254); Benkard et al. (2026) show that dropping that sector entirely from the analysis leaves their muted-increase result unchanged. Using direct marginal-cost data from retail scanner prices, Anderson et al. (2025) document that markups are remarkably stable over time and mildly procyclical — a finding consistent with Ren & Zhang (2025) and with Benkard et al. (2026). Taken together, the most carefully constructed external markup series point to approximate stability in aggregate markups over 1980–2024: markups may contribute to the *level* of labor share compression, but they are a poor candidate for its *secular trend*.

The markup definition adopted here follows the standard monopolistic competition framework: the consumption sector faces a markup $\mu_C = 1/(\sigma_c - 1)$ over marginal cost arising from firms' pricing power over differentiated goods, while the investment sector is perfectly competitive ($\mu_I = 0$). The model's μ_C requires an external empirical time path, and constructing one over the full 1980–2024 sample at the firm level limits the choice to publicly available data: Compustat, covering publicly traded firms across all sectors. The direct marginal-cost evidence behind the stability finding is confined to retail scanner data, and the production-function markup estimates built on Compustat are, as the debate above shows, sensitive to sample and specification. Rather than import a contested point estimate, I use a transparent, reproducible margin: the cross-firm average operating-profit markup — operating income before depreciation over operating cost (cost of goods sold plus selling, general, and administrative expense) — computed on the firm-level sample assembled by ARW (hereafter the ARW path). This is not ARW's own markup measure: ARW estimate retail markups from scanner data and find them roughly stable, whereas the operating-profit markup over all Compustat firms rises over the sample, reproducing the trend documented by De Loecker et al. (2020). By construction it overstates the markup over the model's marginal cost on two counts (Section 4.2): its cost base omits the return to capital,

and operating profit already embeds the monopsony rents the model attributes separately to the markdown ν, so pairing the full margin with the markdown double-counts them. This is the conflation Ren & Zhang (2025) resolve directly: matching Compustat to confidential Census firm-level records (the Longitudinal Business Database) to observe wage bills and employment, they separate labor from the composite flexible input and recover a flat price markup, with the rising trend loaded onto wage markdowns. That decomposition requires restricted-access microdata; working with public data, I instead bound μ_C from above with the operating margin and identify ν separately from sectoral marginal products, reaching the same substantive conclusion — that markdowns, not markups, carry the trend. On that basis I use the ARW path as an upper bound rather than a measurement of μ_C: because the operating margin exceeds the model's markup at every date, feeding it in gives the markup channel its largest defensible role, in level and in trend alike. I scale it to 50% and 10% of its full value and let the general-equilibrium fit determine how much markup the data require (Figure 6); the stability evidence is then what that fit corroborates, not an assumption imposed on the input (Section 4).

Figure 6. Operating-profit markup from the ARW Compustat sample, 1980–2021.



Notes: Operating-profit markup $\mu \equiv \text{oibdp}/(\text{sale} - \text{oibdp})$, averaged across publicly traded U.S. firms (Compustat), winsorized at the 1st and 99th percentiles. Covers 1980–2021; the 2022–2024 extrapolation used in the simulation is excluded. Source: Anderson et al. (2025).

2.4. Three Channels, One Decomposition

Three empirical patterns emerge from the evidence reviewed in this section. First, the sustained decline in the relative price of investment goods is a robust, well-documented fact pointing to faster productivity growth in the investment sector; it provides external discipline for the IETP channel. Second, the most credible evidence — which separates price markups from labor-market rents — points to approximate stability in aggregate markups over 1980–2024 (Anderson et al., 2025; Benkard et al., 2026; Ren & Zhang, 2025): markups shape the level of labor share compression rather than its secular trend, and the rising operating margin I take from public data enters the calibration only as an upper

bound on the markup's contribution. Third, the time-series evidence on wage markdowns consistently points to a sustained upward trend — whether measured in manufacturing under a Cobb-Douglas specification (Yeh et al., 2022) or economy-wide for publicly traded firms (Ren & Zhang, 2025) — making markdowns the natural candidate for the secular transition path of the labor share.

These three patterns motivate a general equilibrium model that can (i) replicate the joint dynamics of the relative investment price, the capital stock, real wages, and the labor share simultaneously; (ii) hold all three channels active so that their interactions through factor markets are fully accounted for; and (iii) isolate the contribution of each force by shutting them down one at a time. The model developed in the next section does exactly this.

3. The Model

This paper builds on the framework presented in Gouveia-Mendes (2025), incorporating endogenous labor, wage markdowns, and goods-market markups. In the present work, the model structure follows Greenwood et al. (1997) in featuring two production sectors and investment-embodied technological progress. It also follows Berger et al. (2022) in incorporating labor-market monopsonistic competition, and introduces goods-market monopoly power in the consumption sector.

3.1. Set Up

■ **Households.** In this economy, there is a continuum of households $h \in [0,1]$ that make consumption, saving and firm-by-firm labor supply decisions by maximizing their lifetime utility. All households share the same intertemporal utility function, which takes the Greenwood-Hercowitz-Huffman (GHH) form (Greenwood et al., 1988):

$$U = \sum_{t=0}^{+\infty} \beta^t \frac{x_t^{1-\varphi}}{1-\varphi}, \quad x_t \equiv c_t - \Gamma_t \frac{\ell_t^{1+\eta}}{1+\eta}, \tag{1}$$

where the parameter $\beta \in (0,1)$ represents time discounting, $\varphi^{-1} \geq 0$ measures the elasticity of intertemporal substitution in consumption, $\eta^{-1} > 0$ is the elasticity of aggregate labor supply, and $\Gamma_t \equiv z_{It}^{\alpha_C} z_{Ct}^{1-\alpha_C}$ is a time-varying scaling factor whose functional form ensures the existence of a balanced growth path, as shown in Section 4.1. I choose these preferences for two reasons. First, they rule out wealth effects on labor supply, meaning that the labor supply decision depends only on the contemporaneous real wage, not on wealth. This is in line with micro evidence on labor supply decisions, according to which households with different wealth levels exhibit different consumption patterns but no significant differences in hours worked (Attanasio et al., 2012; Browning et al., 1985). Second, the quasi-homothetic structure of these preferences on consumption, c, is enough to satisfy the conditions

for Gorman (1961) aggregation (Correia, 1999), allowing the use of a representative agent framework.

Consumption c_t is a CES aggregate of sector-C firm-level varieties:

$$c_t \equiv \left(\int_0^1 c_{nt}^{\frac{\sigma_c - 1}{\sigma_c}} \, \mathrm{d}n \right)^{\frac{\sigma_c}{\sigma_c - 1}}, \tag{2}$$

where $\sigma_c > 1$ governs the degree of substitutability between consumption varieties. This disaggregation is the source of product-market power: with a finite σ_c, each variety is an imperfect substitute, so its producer faces a downward-sloping demand and prices above marginal cost, generating the consumption-sector markup $\mu_C = 1/(\sigma_c - 1)$ introduced below; as $\sigma_c \to \infty$ the varieties become perfect substitutes and the markup vanishes. The associated CES price index for the consumption aggregate is:

$$q_t \equiv \left(\int_0^1 q_{nt}^{1-\sigma_c} \, \mathrm{d}n \right)^{\frac{1}{1-\sigma_c}}, \tag{3}$$

where $q_{nt} \equiv P_{Cnt}/P_{It}$ is the relative price of variety n in units of investment goods.

The representative household members can work in either sector of the economy. Therefore, the *aggregate* labor disutility index is defined as a simple sum of the disutility index of labor supplied to each sector:

$$\ell_t \equiv \sum_j \ell_{jt}, \quad j \in \{C, I\}. \tag{4}$$

Workers view jobs at different firms within a sector as imperfect substitutes, with the degree of substitutability governed by a sector-specific parameter $\theta_j > 0$. Formally, the *sectoral* labor disutility index is defined as a CES aggregator of hours of labor supplied to each firm $n \in (0, 1)$ in sector j:

$$\ell_{jt} \equiv \left(\int_0^1 \ell_{jnt}^{\frac{1+\theta_j}{\theta_j}} \, \mathrm{d}n \right)^{\frac{\theta_j}{1+\theta_j}}, \tag{5}$$

where θ_j is a finite sector-specific labor supply elasticity and ℓ_{jnt} denotes the number of hours supplied by the household to firm n in sector j.[5]

[5] The index n labels firms within a sector and implies no ordering across sectors: each sector has its own unit mass of firms, so firm n in sector C bears no relation to firm n in sector I. Within sector C, firm n is the same object throughout — the producer of variety n and the employer of ℓ_{Cnt}.

Capital accumulation occurs through investment, i, with the capital stock k evolving according to:

$$k_{t+1} - k_t(1-\delta) = i_t, \quad \forall t \geq 0, \tag{6}$$

where the parameter $\delta \in (0,1)$ represents a constant rate of economic depreciation.

Household income derives from three sources: labor compensation, capital rental payments, and firm profits. Expressing all values in units of investment goods (the numeraire), the period-by-period budget constraint takes the form:

$$q_t c_t + k_{t+1} \leq (1 + r_t - \delta)k_t + w_t \ell_t + \pi_t, \quad \forall t \geq 0. \tag{7}$$

The notation $\pi_t \equiv \sum_j \Pi_{jt}/P_{It}$ represents real profits rebated from firms, and q_t is the aggregate relative price of consumption defined in Equation 3.

The solution of the household problem is trivial and is derived in Appendix A.1. However, it is worth noting some aggregation properties of the labor supply resulting from the solution and collected in the following lemma.

Lemma 1. *(Sectoral and Aggregate Labor Supplies).* Define the sectoral wage index as:

$$w_{jt} \equiv \left(\int_0^1 w_{jnt}^{1+\theta_j} \, \mathrm{d}n \right)^{\frac{1}{1+\theta_j}}. \tag{8}$$

Then, firm-level labor supplies can be aggregated into a sector-level labor supply given by:

$$w_{jt} = \Gamma_t q_t \ell_t^{\eta} = w_t, \tag{9}$$

which determines an equilibrium wage index identical across sectors. Therefore, the aggregate labor supply can be written as:

$$\ell_t = \left(\frac{w_t}{\Gamma_t q_t} \right)^{\frac{1}{\eta}} \tag{10}$$

Proof. See Appendix A.2. □

Lemma 1 states that the sectoral wage is pinned to the aggregate wage regardless of the sectoral allocation of labor, so that each sector faces a perfectly elastic labor supply at the prevailing aggregate wage. However, the aggregate labor supply has a positive finite elasticity of η^{-1}, allowing us to write the firm-level labor supply as:

$$\ell_{jnt} = \ell_{jt}\left(\frac{w_{jnt}}{w_t}\right)^{\theta_j}, \tag{11}$$

meaning that the labor supplied to a given firm n in sector j depends on how the firm wage deviates from the aggregate wage index w_t.

■ **Firms.** In this economy there are two distinct production sectors: sector C produces consumption goods, while sector I produces investment goods. Firms in sector $j \in \{C, I\}$ employ two inputs — capital k_j and labor ℓ_j — to generate output y_j. Production in each sector follows a Cobb-Douglas specification with constant returns to scale, although the capital-output elasticity α_j varies across sectors:

$$y_{jt} = F_j(k_{jt}, \ell_{jt}) = k_{jt}^{\alpha_j}\left(z_{jt}\ell_{jt}\right)^{1-\alpha_j}, \quad \alpha_j \in (0,1), \quad \alpha_C \neq \alpha_I. \tag{12}$$

The technology parameters, z_j, materialize Harrod-neutral (labor-augmenting) technical change, evolving with sector-specific growth rates:

$$z_{jt} = \left(1 + g_{z_j}\right)^t z_{j0}, \quad g_{z_j} \geq 0, \quad g_{z_I} > g_{z_C}, \quad z_{j0} \geq 1. \tag{13}$$

The condition $g_{z_I} > g_{z_C}$ captures the essence of investment-specific technological change as documented by Greenwood et al. (1997): production technologies are different across sectors, with technological progress being faster in the investment goods sector than in the consumption goods sector. When $g_{z_I} > g_{z_C}$, the relative price of investment goods, $q_t^{-1} \equiv P_{It}/P_{Ct}$, declines over time, making capital increasingly cheaper in terms of consumption goods. This embodiment mechanism implies that newer capital vintages incorporate superior technology, driving long-run economic growth through capital accumulation rather than solely through improvements in total factor productivity. Empirically, Greenwood et al. (1997) demonstrate that this investment-specific technological change accounts for a substantial portion of post-World War II U.S. growth. Under Cobb-Douglas production the output elasticity of labor is the constant $1 - \alpha_j$, invariant to the level and growth of technology; combined with estimating α_j directly from the production function (Section 4.2), this keeps factor-biased technology from being mistaken for a labor-market wedge, the identification concern raised by Grossman & Oberfield (2022).

Within each sector, a continuum of identical firms operates in output markets, taking the real capital rental rate r_t as given while utilizing the technology described in Equation 12. However, two forms of market power coexist in this economy. On the output side, firms in sector C produce differentiated consumption varieties and therefore possess monopoly power in goods markets, enabling them to charge a price above marginal cost. Sector I produces a homogeneous investment good and is perfectly competitive in the goods market. On the input side, firms in both sectors possess monopsony power in labor markets, enabling them to set wages below their marginal revenue product. The foundation of

this labor-market power lies in workers' heterogeneous preferences over jobs across firms. Such heterogeneity arises from non-wage job characteristics that impose no direct costs on firms — attributes like workplace location, skill specificity, search frictions, or occupational barriers[6]. Although labor itself is homogeneous, these subjective job-switching utility costs enable firms in each sector to engage in monopsonistic wage competition.

The firm-level first-order conditions (derived in Appendix A.4) reveal two wedges driving wages below the value of the marginal revenue product of labor. First, a markup $\mu_j \geq 0$ arising from goods-market monopoly power, defined as:

$$\mu_C \equiv \frac{1}{\sigma_c - 1} \geq 0, \quad \mu_I \equiv 0, \tag{14}$$

so that sector C charges a markup inversely related to the elasticity of demand σ_c, while sector I, being perfectly competitive, has no markup. Second, a markdown $\nu_j \in (0,1)$ arising from labor-market monopsony power, defined as:

$$\nu_j \equiv 1 - \frac{\theta_j}{1+\theta_j} = \frac{1}{1+\theta_j}, \tag{15}$$

which falls as labor supply becomes more elastic (higher θ_j) and equals zero only in the frictionless competitive limit ($\theta_j \to +\infty$). The wage paid by firm n in sector j satisfies:

$$w_{jnt} = \frac{1-\nu_j}{1+\mu_j} \cdot q_{nt}\left(\frac{k_{jnt}}{\ell_{jnt}}\right)^{\alpha_j} z_{jt}^{1-\alpha_j}(1-\alpha_j), \tag{16}$$

so that both wedges simultaneously compress the wage below the marginal revenue product of labor: a higher markup μ_j reduces the marginal revenue that firms capture from each worker, while a higher markdown ν_j allows firms to pay workers less than that reduced marginal revenue product.

Since technology is identical across firms within each sector and the labor supply elasticity is sector-specific, I can employ a representative firm framework for each sector, solving the profit maximization problem as described in Proposition 1.

Proposition 1. *(Representative Firm).* In each sector $j \in \{C, I\}$, firm n chooses its wage w_{jnt} and capital k_{jnt} to maximize profit

$$\max_{w_{jnt}, k_{jnt}} \quad \pi_{jnt} = q_{nt} y_{jnt} - w_{jnt}\ell_{jnt} - r_t k_{jnt}, \quad j \in \{C, I\},$$

[6] As Azar & Marinescu (2024) noted, the literature on firms' endogenous provision of job amenities usually assumes a perfectly competitive labor market, where wages equal marginal productivity. However, this is not the case under monopsony power. For simplicity, I will assume that the amenities are exogenous.

subject to the production technology in Equation 12, the downward-sloping demand for its variety, $q_{nt} = q_t(c_{nt}/c_t)^{-1/\sigma_c}$ (with $q_{nt} = q_t = 1$ in the perfectly competitive sector I), and the upward-sloping firm-level labor supply, $w_{jnt} = w_{jt}\big(\ell_{jnt}/\ell_{jt}\big)^{1/\theta_j}$. The firm takes as given the aggregate relative price of consumption q_t (defined in Equation 3) and aggregate consumption c_t, the sectoral wage and labor w_{jt} and ℓ_{jt}, and the real rental rate of capital $r_t \equiv R_t/P_{It}$.

In a symmetric equilibrium, where all firms in a sector are identical, the input-demand conditions of this problem coincide with those of the reduced-form sector-specific representative-firm problem:

$$\max_{\{\ell_{jt},k_{jt}\}} \quad \pi_t = \frac{q_t}{1+\mu_j} k_{jt}^{\alpha_j}\big(z_{jt}\ell_{jt}\big)^{1-\alpha_j} - \frac{w_{jt}}{1-\nu_j}\ell_{jt} - r_t k_{jt}, \quad j \in \{C, I\}, \tag{17}$$

in which the firm hires $\{\ell_{jt}, k_{jt}\}$ taking as given q_t, the real wage $w_t \equiv W_t/P_{It}$, and the real rental rate $r_t \equiv R_t/P_{It}$, both measured in units of investment goods (the numeraire). The goods-market markup $\mu_j \geq 0$ (defined in Equation 14) and the labor-market markdown $\nu_j \in (0,1)$ (defined in Equation 15) enter as constant wedges generated, respectively, by the demand elasticity σ_c and the labor-supply elasticity θ_j.

Proof. See Appendix A.3. □

Note that in Equation 17, w_{jt} denotes the equilibrium wage received by workers; the effective labor cost to the firm, $w_{jt}/\big(1-\nu_j\big)$, equals the marginal revenue product of labor adjusted for the markdown. Similarly, the factor $1/\big(1+\mu_j\big)$ on revenues captures the markup in goods prices relative to the competitive benchmark. Together, the two wedges drive a gap between the value of what an additional worker produces and what they are actually paid.

The markdown ν_j depends on the sector labor supply elasticity θ_j — i.e., on within-sector labor mobility — and, in general, on the distribution of capital-labor ratios across firms through the share-weighted dispersion factor Ω_{jt} (derived in Appendix A.3). However, in a symmetric equilibrium where all firms within a sector are identical (equal size and equal capital-labor ratio), $\Omega_{jt} = 1$ and the markdown reduces to $\nu_j = 1/\big(1+\theta_j\big)$, making it a time-invariant structural parameter. In this symmetric case, $\theta_C \neq \theta_I$ directly implies $\nu_C \neq \nu_I$.

Yet, the economic interpretation of cross-sectoral differences in markdowns remains ambiguous given the sectoral aggregation scheme employed here. What would justify systematic markdown differentials between investment goods and consumption goods sectors? In fact, the local nature of labor markets — emphasized in the monopsony literature — suggests sector-agnostic markdowns. Moreover, in the current setup I assume atomistic firm structures in both sectors, implying no wage dispersion across firms. The economy's

aggregate labor market functions, then, as a duopsony[7], with two firms competing for the same worker pool and sectoral markdowns becoming exogenous to the representative firms. Even in this case, despite equal sectoral wages in equilibrium ($w_{Ct} = w_{It} = w_t$), the sectoral markdowns differ when $\theta_C \neq \theta_I$. Those differences are a direct consequence of the aggregation procedure, rather than reflecting a well-motivated theoretical stance. I return to this point when calibrating the model; indeed, as I show in the calibration, a similar markdown across sectors fits the data on the decline of the labor share better than asymmetric markdowns. The solution of the firms' problem in Equation 17 under a symmetric equilibrium is collected in Appendix A.4.

3.2. General Equilibrium

Market clearing conditions ensure that total supply equals total demand for each good and factor of production. In output markets, total production of each good equals total consumption or investment:

$$y_{Ct} = c_t, \quad \forall t \geq 0, \tag{18}$$

$$y_{It} = i_t, \quad \forall t \geq 0, \tag{19}$$

Similarly, input markets clear when the total quantity of each factor supplied by households matches the total quantity demanded by firms across both sectors:

$$\ell_{Ct} + \ell_{It} = \ell_t, \quad \forall t \geq 0, \tag{20}$$

$$k_{Ct} + k_{It} = k_t, \quad \forall t \geq 0. \tag{21}$$

With these market clearing conditions in hand, I can formally define equilibrium.

Definition 1. *(General Equilibrium).* In this economy, a general equilibrium is defined as a set of sequences of allocations $\{c_t, k_{t+1}\}_{t=0}^{+\infty}$, inputs $\{k_{jt}, \ell_{jt}\}_{t=0}^{+\infty}$, outputs $\{y_{jt}\}_{t=0}^{+\infty}$, and prices $\{w_t, r_t, q_t\}_{t=0}^{+\infty}$, such that, assuming that all firms are equal in the same sector and given the initial capital endowment $k_0 \geq 0$, the path for technologies $\{z_{jt}\}_{t=0}^{+\infty}$, and a vector of parameters $[\beta, \varphi, \eta, \delta, \alpha_j, \theta_j, \sigma_c]$, firms in each sector $j \in \{C, I\}$ maximize profits according to Appendix A.4, the representative household maximizes utility according to Appendix A.1, and markets clear according to Equation 18 to Equation 21.

While the complete system of optimality conditions characterizing general equilibrium comprises thirteen equations, judicious algebraic manipulation reduces it to a more tractable four-equation system. In each period t, four decisions are to be made: one forward-looking and three static. The first decision is the aggregate labor ℓ_t: the number of hours households will work given wages, wedges, and the current stock of capital, to balance the

[7] By analogy with duopoly, I use *duopsony* to denote a market with exactly two buyers of labor.

tradeoff between labor and the level of consumption they would like to attain. The second decision is the allocation of labor across sectors $s_{\ell t} \equiv \ell_{It}/\ell_t$: how much labor to allocate to the investment sector versus the consumption sector, given the prices of inputs and outputs, to produce the desired levels of consumption and investment goods in that period. The third decision is the allocation of capital across sectors $s_{kt} \equiv k_{It}/k_t$: how much capital to allocate to the investment sector versus the consumption sector, given the prices of inputs and outputs, to achieve the desired levels of consumption and investment goods in that period. And, finally, the forward-looking decision is the aggregate capital stock of the next period k_{t+1}: how much to invest in the current period, given the expected future path of wages, wedges, and technologies, to balance the tradeoff between current consumption and future consumption.

Proposition 2. *(Block-Recursive Solution).* Let the *transformed variables* be the sectoral capital and labor shares $s_{kt} \equiv k_{It}/k_t$ and $s_{\ell t} \equiv \ell_{It}/\ell_t$, and the scaled capital $\tilde{k}_t \equiv k_t/(z_{It}\ell_t)$, which render the otherwise trending system stationary. Then, the general equilibrium solution described by the conditions in Appendix A.1 and Appendix A.4, together with the market clearing conditions in Equation 18 to Equation 21, becomes block-recursive. This way, I can first solve the following system of four equations determining $\{s_{kt}, s_{\ell t}, \ell_t, \tilde{k}_{t+1}\}_{t=0}^{+\infty}$:

$$\ell_t = \left[(1-\alpha_C)\frac{1-\nu_C}{1+\mu}\left(\frac{1-s_{kt}}{1-s_{\ell t}}\tilde{k}_t\right)^{\alpha_C}\right]^{\frac{1}{\eta}} \tag{22}$$

$$s_{\ell t} = \left[\frac{\alpha_I}{1-\alpha_I}\frac{1-\alpha_C}{\alpha_C}\frac{1-\nu_C}{1-\nu_I}\frac{1-s_{kt}}{s_{kt}} + 1\right]^{-1} \tag{23}$$

$$\begin{aligned}
&\left[\frac{\left(\frac{1-s_{kt+1}}{1-s_{\ell t+1}}\tilde{k}_{t+1}\right)^{\alpha_C}(1-s_{\ell t+1}) - \frac{\ell_{t+1}^{\eta}}{1+\eta}}{\left(\frac{1-s_{kt}}{1-s_{\ell t}}\tilde{k}_t\right)^{\alpha_C}(1-s_{\ell t}) - \frac{\ell_t^{\eta}}{1+\eta}}\frac{\ell_{t+1}}{\ell_t}\right]^{\varphi} = \\
&= \left(\frac{s_{\ell t}}{s_{\ell t+1}}\frac{s_{kt+1}}{s_{kt}}\right)^{1-\alpha_I}\left(\frac{1-s_{kt}}{1-s_{\ell t}}\frac{1-s_{\ell t+1}}{1-s_{kt+1}}\right)^{1-\alpha_C}\left(\frac{\tilde{k}_t}{\tilde{k}_{t+1}}\right)^{\alpha_I-\alpha_C} \times \\
&\times \beta_\Gamma\left(\frac{1+g_{z_C}}{1+g_{z_I}}\right)^{1-\alpha_C}\left(1-\delta+\left(\frac{s_{\ell t+1}}{s_{kt+1}}\frac{1}{\tilde{k}_{t+1}}\right)^{1-\alpha_I}\alpha_I\right)
\end{aligned} \tag{24}$$

$$\tilde{k}_{t+1}(1+g_{z_I})\frac{\ell_{t+1}}{\ell_t} = (1-\delta)\tilde{k}_t + \left(\frac{s_{kt}}{s_{\ell t}}\tilde{k}_t\right)^{\alpha_I} s_{\ell t} \tag{25}$$

where $\mu \equiv \mu_C$ denotes the markup of sector C. Then, I determine the other variables from a trivial second block of equations.

Proof. See Appendix A.5. □

I must also formally delimit the economically feasible solution set for the first-block conditions of the equilibrium system in Proposition 2, which I do next.

Definition 2. *(Feasible Solution Set).* Solutions to the first-block equations of the equilibrium system in Proposition 2 must reside in the feasible set $\mathcal{S} \equiv \left\{\left(\tilde{k}_{t+1}, \ell_t, s_{kt}, s_{\ell t}\right) \in \mathbb{R}^4_{++} : s_{kt} \in (0,1), s_{\ell t} \in (0,1), \tilde{k}_{t+1} > 0, \ell_t > 0, \forall t \geq 0\right\}$.

Under the previous definition the following uniqueness result can be established, which will be useful later on.

Lemma 2. *(Uniqueness of $s_{\ell t}$).* Consider wedge values satisfying $\nu_C, \nu_I \in (0,1)$ and $\mu \geq 0$, for all $t \geq 0$. Then for any $(k_t, \ell_t, s_{kt}) \in \mathcal{S}$, Equation 23 yields a unique $s_{\ell t} \in \mathcal{S}$.

Proof. See Appendix A.8. □

■ **The First Best.** The solution to the planner's problem (derived in Appendix A.6) yields the same system of four equations as Proposition 2, evaluated at $\nu_C = \nu_I = 0$ and $\mu = 0$. Comparing the first-best allocation with the decentralized equilibrium reveals three channels through which markdowns and markups jointly distort the economy.

The first channel is the *intratemporal labor supply distortion*. In the decentralized equilibrium, the factor $(1 - \nu_C)/(1 + \mu)$ jointly depresses the effective return to labor. Because $\nu_C > 0$ and $\mu > 0$, the equilibrium features less aggregate labor than the efficient allocation for any given capital-labor ratio and sectoral shares: monopsony power in labor markets and monopoly power in goods markets both act as taxes on labor.

The second channel is the *sectoral misallocation of labor*. The decentralized equilibrium distorts the efficient allocation of inputs across sectors by the ratio $(1 - \nu_C)/(1 - \nu_I)$. When markdowns differ across sectors ($\nu_C \neq \nu_I$), the allocation of labor no longer equates sectoral marginal products, generating a misallocation wedge. Notably, the markup μ does not enter this ratio, because sector I is perfectly competitive in the goods market ($\mu_I = 0$) and the markup does not affect the relative incentives to allocate labor across sectors. When both markdowns are equal ($\nu_C = \nu_I$), this second channel is muted, but the level distortion from the labor supply channel persists.

The third channel is the *intertemporal distortion*. The capital accumulation equation (Equation 25) is the same in both the first best and the decentralized equilibrium — it is a purely technological relationship. However, the Euler equation (Equation 24) is affected indirectly: because markdowns and markups alter equilibrium employment, sectoral shares, and the relative price q_t, the intertemporal consumption-savings margin is evaluated at distorted quantities, resulting in a different capital accumulation path.

In summary, the decentralized equilibrium coincides with the first-best allocation if and only if $\nu_C = \nu_I = 0$ and $\mu = 0$, i.e., when both labor and goods markets are perfectly competitive. When wedges are positive but markdowns are symmetric ($\nu_C = \nu_I > 0$, $\mu > 0$), the sectoral misallocation channel shuts down but the intratemporal and intertemporal distortions remain. Only when all wedges are zero do all three channels vanish simultaneously.

■ **Impact of Markdowns and Markups on the Labor Share.** Because this model features two sectors with potentially distinct input-output elasticities, the aggregate shares of capital and labor in total income cannot be directly inferred from sectoral production parameters. Given that sectors produce output in different units using different technologies, measuring aggregate output requires weighting sectoral production by equilibrium relative prices. Once aggregate output is defined, I can proceed to calculate the labor share of income.

Corollary 1. *(Labor Share of Income).* Let y_t be the aggregate output, given by:

$$y_t \equiv y_{It} + q_t y_{Ct} = \left[\frac{\alpha_I}{\alpha_C}\frac{1-s_{kt}}{s_{kt}} + 1\right]\left(\frac{s_{kt}}{s_{\ell t}}\tilde{k}_t\right)^{\alpha_I} z_{It} s_{\ell t} \ell_t.$$

Then, it follows from Proposition 2 that the labor share of income $m_{\ell t} \equiv (w_t \ell_t)/y_t$ can be written as:

$$m_{\ell t} = \frac{\alpha_I(1-\alpha_C)}{\alpha_I + (\alpha_C - \alpha_I)s_{kt}}\frac{1-\nu_C}{1+\mu}(1-s_{kt}) + \frac{\alpha_C(1-\alpha_I)}{\alpha_I + (\alpha_C - \alpha_I)s_{kt}}\frac{1-\nu_I}{1+\mu}s_{kt}. \tag{26}$$

Proof. See Appendix A.7. □

Several notable properties emerge from Equation 26. First, the labor share depends on the capital allocation share s_{kt}, both markdowns ν_C and ν_I, the goods-market markup μ, and the structural parameters α_C and α_I — but not on productivity levels z_{It} and z_{Ct}. Note that $s_{\ell t}$ no longer appears in the formula, as it has been substituted out using Equation 23, which expresses $s_{\ell t}$ as a function of s_{kt}, the markdowns, and the structural parameters. Intuitively, $s_{\ell t}$ is determined by the same equilibrium forces as the wedges and s_{kt}, so the labor share is ultimately a function of the capital allocation share and the structural parameters alone.

Second, consider the case with no wedges ($\nu_C = \nu_I = 0$, $\mu = 0$). When additionally capital-output elasticities are uniform ($\alpha_C = \alpha_I = \alpha$), the factor allocation conditions imply $s_{kt} = s_{\ell t}$ and the labor share reduces to:

$$m_{\ell t} = 1 - \alpha, \quad \forall t \geq 0,$$

recovering the standard Cobb-Douglas result and a constant labor share over time. With differing sectoral elasticities ($\alpha_C \neq \alpha_I$) but still no wedges, the labor share becomes:

$$m_{\ell t} = \frac{\alpha_I(1-\alpha_C)}{\alpha_I + (\alpha_C - \alpha_I)s_{kt}}(1 - s_{kt}) + \frac{\alpha_C(1-\alpha_I)}{\alpha_I + (\alpha_C - \alpha_I)s_{kt}}s_{kt}. \tag{27}$$

Under these conditions, temporal variation in the labor share arises through two channels: changes in the capital allocation s_{kt}, or changes in the wedges ν_C, ν_I, and μ. Absent such reallocation or changes in wedges, the labor share remains stable. The direction of labor share movements due to reallocation depends on the relative magnitudes of sectoral capital-output elasticities.

Third, suppose markdowns are symmetric across sectors: $\nu_C = \nu_I \equiv \nu$. Then Equation 26 factorizes as:

$$m_{\ell t} = \frac{1-\nu}{1+\mu} m^0_{\ell t}, \tag{28}$$

where $m^0_{\ell t}$ denotes the labor share under no wedges (Equation 27). Under this restriction, markup and markdown enter the labor share only through their product $(1-\nu)/(1+\mu)$, so the two distortions are *observationally equivalent*: a markup μ has the same effect on the labor share as a uniform markdown $\nu = \mu/(1+\mu)$ (equivalently, $\theta = 1/\mu$). Since $\mu = 1/(\sigma_c - 1)$, the equivalence condition becomes $\theta = \sigma_c - 1$: the firm-level labor-supply elasticity equals the goods-demand elasticity minus one, assigning equal magnitude to each form of market power. This implies that the labor share alone cannot separately identify μ and ν when markdowns are symmetric; the paper identifies them separately — μ from the ARW path and ν from marginal-product identification (see Section 4.2). Conversely, when $\nu_C \neq \nu_I$, the markup μ scales the entire labor share uniformly while ν_C and ν_I shift the consumption- and investment-sector terms with weights $1 - s_{kt}$ and s_{kt} respectively.

Finally, the partial derivatives of the labor share with respect to each wedge, holding s_{kt} constant, are all unambiguously negative:

$$\left.\frac{\partial m_{\ell t}}{\partial \nu_C}\right|_{s_{kt}} = -\frac{\alpha_I(1-\alpha_C)(1-s_{kt})}{(\alpha_I + (\alpha_C - \alpha_I)s_{kt})(1+\mu)} < 0, \tag{29}$$

$$\left.\frac{\partial m_{\ell t}}{\partial \nu_I}\right|_{s_{kt}} = -\frac{\alpha_C(1-\alpha_I)s_{kt}}{(\alpha_I + (\alpha_C - \alpha_I)s_{kt})(1+\mu)} < 0, \tag{30}$$

$$\left.\frac{\partial m_{\ell t}}{\partial \mu}\right|_{s_{kt}} = -\frac{m_{\ell t}}{1+\mu} < 0. \tag{31}$$

All three wedges directly compress the labor share. The consumption-sector markdown ν_C operates through the labor allocated to sector C (weighted by $1 - s_{kt}$), while the investment-sector markdown ν_I operates through labor in sector I (weighted by s_{kt}). The markup μ scales the entire formula downward by a factor of $1/(1+\mu)$. Comparing the first two derivatives reveals that the relative impact of a change in ν_C versus ν_I depends on the capital allocation: a change in ν_C has a larger effect when $1 - s_{kt}$ is large (most capital in sector C), while a change in ν_I dominates when s_{kt} is large.

4. Calibration and Quantitative Results

In Section 2 I briefly mentioned the multiple channels through which the labor share can decline. By numerically solving the model described in Section 3, I am now able to quantify the importance of each particular mechanism. The numerical solution of the model requires parameter calibration — selecting values that align with the economy being studied. I target U.S. data spanning 1980–2024 to examine the labor share decline observed during this interval. The calibrated model then serves to investigate the labor share and some other macroeconomic variables. Before proceeding to calibration, I first specify the type of equilibrium solution sought.

4.1. Balanced Growth Path

Macroeconomic analysis of long-run dynamics typically centers on equilibrium paths capable of replicating the Kaldor (1961) facts.[8] Equilibrium paths consistent with these patterns are termed *balanced growth paths*.

Definition 3. *(Balanced Growth Path).* In the model framework presented above, a Balanced Growth Path (BGP) constitutes an equilibrium trajectory satisfying Proposition 2 where all allocations expand at constant (though potentially distinct) rates.

To investigate conditions under which such paths exist, I conjecture that a BGP requires stationary solutions for the first-block equations in the equilibrium system of Proposition 2. This implies examining: (i) existence and uniqueness of a stationary solution to the system, conditional on parameter values; (ii) convergence of the system's solution to that stationary state from an arbitrary initial capital stock k_0.[9]

[8] These stylized facts comprise six empirical patterns observed over extended time horizons: (i) roughly constant growth in output per worker; (ii) roughly constant growth in capital per worker; (iii) approximate constancy of the capital-output ratio; (iv) approximate constancy of the return on capital; (v) approximate constancy of labor and capital shares in national income; and (vi) substantial cross-country variation in output-per-worker growth rates, with no systematic relationship to output levels.

[9] The numerical exercises in Section 4.3 establish existence by deploying a shooting algorithm that selects s_{k0} conditional on k_0 to ensure convergence to the stationary solution. This paper version omits formal proofs of the uniqueness of such converging paths.

Proposition 3. *(Unique Steady State of the Transformed Variables).* A unique interior steady state $(\tilde{k}_*, \ell_*, s_{k*}, s_{\ell*}) \in \mathcal{S}$ for the first-block equations in Proposition 2 exists, given by:

$$\ell_* = \left[(1-\alpha_C)(1-\nu_C)\left(\frac{1-s_{k*}}{1-s_{\ell*}}\tilde{k}_*\right)^{\alpha_C}\right]^{\frac{1}{\eta}},$$

$$s_{\ell*} = \left[\frac{\alpha_I}{1-\alpha_I}\frac{1-\alpha_C}{\alpha_C}\frac{1-\nu_C}{1-\nu_I}\frac{1-s_{k*}}{s_{k*}} + 1\right]^{-1},$$

$$s_{k*} = \frac{s_{\ell*}}{\tilde{k}_*}\left[\frac{1+g_{z_I}}{\alpha_I\beta_\Gamma} - \frac{1-\delta}{\alpha_I}\right]^{\frac{1}{\alpha_I-1}},$$

$$\tilde{k}_* = \left[\frac{s_{\ell*}^{1-\alpha_I}s_{k*}^{\alpha_I}}{g_{z_I}+\delta}\right]^{\frac{1}{1-\alpha_I}}.$$

Proof. See Appendix A.9. □

Having established the steady state for transformed variables, characterizing the BGP in the sense of Definition 3 using the second-block equations in Appendix A.5 is immediate. The BGP growth rates are derived in Appendix A.10.

4.2. Calibration

Model variables are expressed in per-household units; all quantities are converted using household count data (U.S. Census Bureau, 2025). The calibration proceeds in two stages. First, I fix the structural technology parameters — the capital-output elasticities α_j and the TFP growth rates g_{z_j} — and the preference parameters, using results in the literature, data and estimation strategies that do not use income-share data. Second, I construct time-varying paths for the product-market markup μ_t and the wage markdown ν_t from external series and marginal-product identification, respectively. Table 2 collects the parameters common to all simulation exercises; initial conditions are presented in Table 3.

■ **Capital-Output Elasticities.** The most directly comparable estimates for capital-output elasticities, α_j, in the existing literature are those of Greenwood et al. (1997), Basu et al. (2013), and Valentinyi & Herrendorf (2008), who work with a consumption/investment sectoral split similar to the one employed here. These estimates, however, derive α_j from factor income shares under the competitive assumption — which makes them upper bounds on the true production-function elasticity. The standard approach sets $(1-\alpha_j)$ from the labor share of income. But under imperfect competition, the labor income share is compressed below its competitive value. Inverting this depressed share therefore

overestimates the true elasticity. Using such calibrations in a model where markups and markdowns are the primary object of interest would additionally conflate the technological parameter with the very distortions the model is designed to measure. I therefore estimate α_C and α_I directly from the production function using a fixed-effects panel regression on BEA industry-level data. For each industry i belonging to sector $j \in \{C, I\}$, the specification is:

$$\log\left(\frac{\text{VA}_{it}}{L_{it}}\right) = \alpha_j \log\left(\frac{K_{it}}{L_{it}}\right) + \gamma_i + \lambda_t + \varepsilon_{it}, \tag{32}$$

where γ_i are industry fixed effects, λ_t are year fixed effects, and standard errors are clustered at the industry level.

I estimate this specification as a single pooled regression across all 73 industries with sector-specific slopes, interacting $\log(K/L)$ with sector indicators and using the sector split defined in Section 2.3:

$$\log\left(\frac{\text{VA}_{it}}{L_{it}}\right) = \left(\alpha_C D_i^C + \alpha_I D_i^I\right) \log\left(\frac{K_{it}}{L_{it}}\right) + \gamma_i + \lambda_t + \varepsilon_{it}, \tag{33}$$

where $D_i^j = \mathbb{1}[s(i) = j]$ for $j \in \{C, I\}$ are sector indicator variables. Pooling shares the fixed-effect variation across both sectors; estimating each sector on its own instead leaves α_I unidentified.[10] The pooled specification draws on 2 044 observations and yields positive and significant estimates for both: $\hat{\alpha}_C = 0.282$ ($p = 0.003$) and $\hat{\alpha}_I = 0.264$ ($p = 0.019$). Table 1 reports the pooled estimates, which are used in all subsequent simulations.

This identification strategy is valid under imperfect competition. The Cobb-Douglas production function is a technological identity, not an optimality condition: markups and markdowns distort factor income shares but not factor elasticities. The coefficient on $\log(K/L)$ therefore recovers α_j from within-industry variation in the capital-labor ratio regardless of the prevailing degree of market power in either market. Figure 7 presents the Frisch–Waugh–Lovell (FWL) partial regression scatter for the pooled specification, obtained by residualizing both variables on industry and year fixed effects; the positive slopes are clearly visible for both sectors.

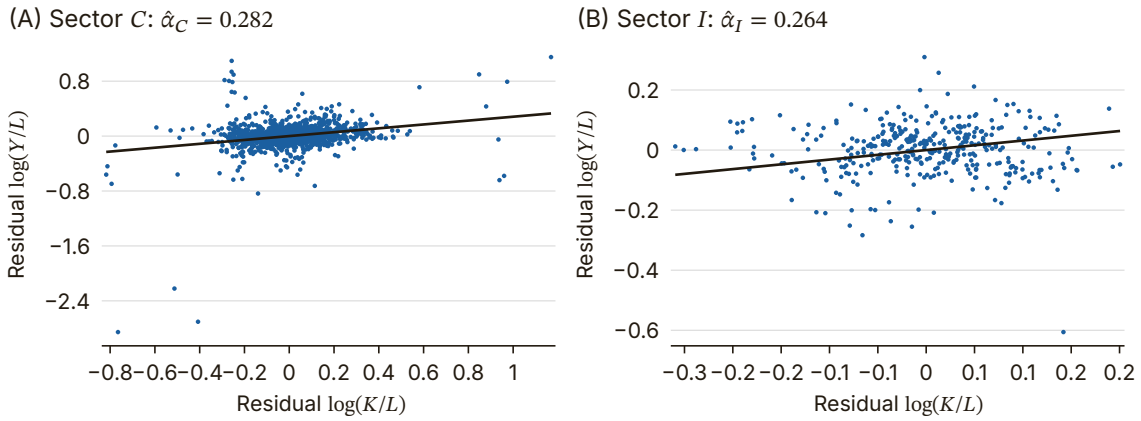
[10] Estimated sector by sector, the sector-C regression (60 industries, 1 680 observations) gives a positive, precisely estimated $\hat{\alpha}_C = 0.302$ ($p = 0.002$), but the sector-I regression (13 industries, 364 observations) gives a negative, statistically insignificant $\hat{\alpha}_I = -0.159$ ($p = 0.150$): with only 13 investment industries, within-variation in the capital-labor ratio is insufficient for separate identification once industry and year fixed effects are partialled out.

Table 1. Pooled panel estimates of capital-output elasticities.

Dependent variable: $\log(\mathrm{VA}/L)$	$\hat{\alpha}_C$	$\hat{\alpha}_I$
$\log(K/L)$	0.282^{***} (0.095)	0.264^{**} (0.113)
Industries	73	
Observations	2 044	
Years	28	
R^2	0.934	

Notes: Dependent variable: $\log(\mathrm{VA}_{it}/L_{it})$. Regressor: $\log(K_{it}/L_{it})$. The pooled specification includes industry and year fixed effects. Standard errors clustered by industry in parentheses. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

Figure 7. Frisch–Waugh–Lovell scatter: pooled capital-output elasticity regression.



Notes: Each axis shows residuals after partialling out industry and year fixed effects from $\log(\mathrm{VA}_{it}/L_{it})$ and $\log(K_{it}/L_{it})$, respectively. Each point is an industry-year observation. Fit lines correspond to the pooled sector-specific slope estimates from Table 1.

■ **TFP Growth Rates.** I identify TFP growth rates using two BGP conditions of the model applied to aggregate data. Both identifying moments — the long-run growth rate of the capital stock and the long-run growth rate of the relative price of investment goods — are invariant to the distortions, and neither relies on factor income shares. The labor share is therefore an untargeted moment: a check on the calibrated model, not a calibration input.

Along the model's BGP, the stationary scaled capital stock $\tilde{k}_*$ is constant by construction (see Proposition 3), so the aggregate capital stock k grows at the gross rate $\left(1 + g_{z_I}\right)$. I estimate g_{z_I} as the slope coefficient of an OLS regression of $\log k_t$ on a linear time trend over 1980–2024. This identification is distortion-robust: markups and markdowns shift the

steady-state level of the capital-labor ratio through $\tilde{k}_*$, but they do not affect the growth rate of k along the BGP, which is pinned entirely by g_{z_I}.

The relative price of consumption goods along the BGP satisfies:

$$1 + g_q = \left(\frac{1 + g_{z_I}}{1 + g_{z_C}} \right)^{1-\alpha_C}.$$

This relationship is likewise distortion-robust: at constant distortion levels the markup and markdown enter q as a multiplicative constant and cancel when computing growth rates, so g_q depends only on TFP growth rates.[11] Substituting the OLS estimate $\hat{g}_q$ from a log-linear time trend on q_t over 1980–2024 and inverting yields a closed-form estimate of g_{z_C}.

A final clarification is that neither moment requires the economy to be at its balanced growth path in 1980. Both conditions exploit only the long-run growth rates of k and q — the slopes of log-linear OLS trends estimated over 1980–2024 — and impose no restriction on the level of any variable in the base year. The economy may be off its BGP in 1980; the model accommodates this through transition dynamics, and the calibrated TFP growth rates discipline only the long-run direction of travel, not the starting point.

Together, these two moments discipline the model to reproduce the long-run trends in the capital stock and the relative price of investment goods. The model's fit to output, the labor share, and the returns on inputs then measures performance on untargeted moments, not a consequence of the calibration design.

■ **Depreciation and Preference Parameters.** The capital depreciation rate δ is set to the 1980–2024 BEA average. The household discount factor $\beta = 0.96$ follows the classic calibration of Prescott (1986). The elasticity of intertemporal substitution $\varphi^{-1} = 0.65$ is set to the midpoint of the stockholder estimates in Vissing-Jørgensen (2002), corresponding to a curvature parameter $\varphi = 1/0.65 \approx 1.54$. The inverse aggregate labor supply elasticity η is set to 1.5 — implying a Frisch elasticity $\eta^{-1} \approx 0.67$ — consistent with the macro evidence surveyed by Keane & Rogerson (2012).

Table 2 collects these parameters, which are common to all simulation scenarios.

[11] Time-varying distortions affect the level of q but not its long-run growth rate, since a slowly moving distortion path shifts q smoothly without altering its trend slope. The identification of g_{z_C} from g_q therefore remains valid under the gradual markdown and markup paths estimated here.

Table 2. Parameters common to all simulations.

Parameter	Value	Source/Target
α_C	0.2821	Pooled OLS panel estimation
α_I	0.2642	Pooled OLS panel estimation
g_{z_C}	1.23%	BGP-implied from OLS trend of q_t, 1980–2024 (U.S. Bureau of Economic Analysis, 2025)
g_{z_I}	2.68%	OLS trend of k_t, 1980–2024 (U.S. Bureau of Economic Analysis, 2025)
δ	5.48%	1980–2024 average (U.S. Bureau of Economic Analysis, 2025)
φ^{-1}	0.65	Midpoint of stockholder EIS estimates, Vissing-Jørgensen (2002)
β	0.96	Prescott (1986)
η	1.5	Keane & Rogerson (2012) (Frisch $= \eta^{-1} \approx 0.67$)

■ **Product-Market Markup.** I construct the markup path μ_t as the cross-firm average operating-profit markup in Compustat, computed on the firm-level sample assembled by Anderson et al. (2025) and covering all firms across sectors over 1980–2024 (the ARW path).[12] The series starts at roughly 28% in 1980, rises steadily through the sample, and reaches approximately 46% by 2020 before the OLS extrapolation brings it to around 41% by 2024 (Figure 8). The operating-profit markup over all Compustat firms rises over the sample, reproducing the Compustat-based trend of De Loecker et al. (2020) rather than the roughly stable retail markups ARW themselves estimate from scanner data. I treat its level and trend as an upper bound, following the critique of Compustat-based markup measures by Benkard et al. (2026).

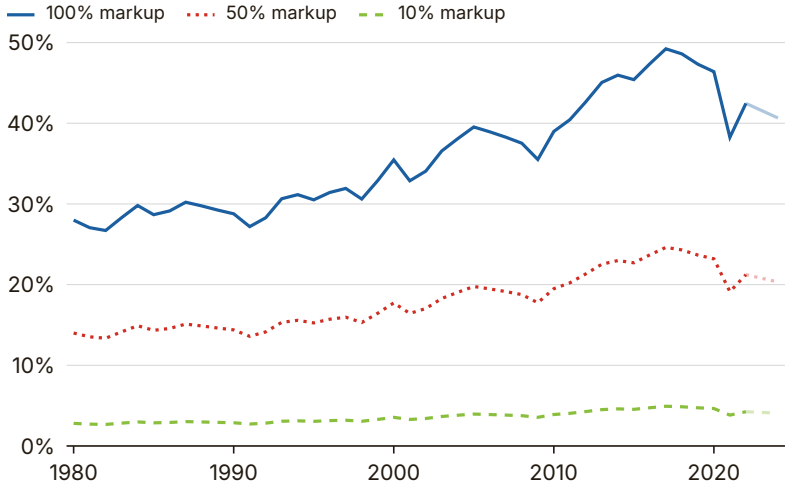
The ARW path, however, measures operating profit relative to operating cost — cost of goods sold plus selling, general, and administrative expense — a cost base that differs from the model's marginal cost. In the model's two-sector production structure there are no intermediate inputs: labor and capital are the only variable factors, so the model-consistent μ is the markup over factor costs. The ARW path is therefore an upper bound on the model-consistent markup, by construction rather than by fit, for two reasons. First, operating profit embeds the labor-market rents that the model attributes to the markdown: when the wage lies below the marginal revenue product of labor, the monopsony rent inflates operating profit, so assigning all of it to the product-market markup double-counts rents

[12]The ARW path is the average operating-profit markup — operating income before depreciation divided by operating cost (cost of goods sold plus selling, general, and administrative expense) — across all Compustat firms, winsorized at the 1st and 99th percentiles. Values for 2022–2024 are extrapolated via an OLS trend fitted to the 2015–2021 window.

the model identifies separately as ν. Second, operating cost omits the user cost of capital — the return on capital is part of operating profit, not of cost, and the series is measured before depreciation — so it understates the model's marginal cost and inflates the implied markup. Including selling and administrative expense in the cost base already removes much of the upward bias that affects markup series built on cost of goods sold alone (Traina, 2018); the two channels above are what remain, and both push the measured markup above the model-consistent object (Benkard et al., 2026).

I therefore examine robustness by scaling the ARW path by factors of 1/2 and 1/10, tracing the consequences of progressively larger overstatement of the model-consistent markup. Each scaled variant re-anchors the technology levels to the full-model initial steady state at the corresponding μ_0, so all sensitivity scenarios remain internally consistent at $t = 0$.

Figure 8. Product-market markup path and sensitivity variants (1980–2024).



Notes: The baseline series (100%) is the average operating-profit markup across all Compustat firms — operating income before depreciation divided by operating cost (COGS plus selling, general, and administrative expense) — winsorized at the 1st and 99th percentiles; values for 2022–2024 are extrapolated via an OLS trend fitted to the 2015–2021 window. The 50% and 10% variants scale the full ARW path by the indicated factor. Each variant anchors its own technology levels z_{I0} and z_{C0} to the full-model steady state at the corresponding μ_0, preserving internal consistency at $t = 0$. Source: Anderson et al. (2025), Compustat annual data (Jan 2022 file).

■ **Wage Markdowns.** Due to data constraints, the markdown path ν_t is constructed in two steps. For the 1997–2024 period, I use a marginal-product identification strategy. The wage markdown is defined as the gap between the marginal revenue product of labor and the wage, expressed as a fraction of the former:

$$\nu_{jt} \equiv 1 - \frac{W_t}{\text{MRPL}_{jt}},$$

where W_t is the aggregate nominal hourly wage and $\text{MRPL}_{jt} = (1-\alpha_j)P_{jt}Y_{jt}/L_{jt} = (1-\alpha_j)\text{VA}_{jt}/L_{jt}$ is the marginal revenue product of labor per hour — the physical marginal product $(1-\alpha_j)Y_{jt}/L_{jt}$ valued at sector j's output price P_{jt}, with $\text{VA}_{jt} \equiv P_{jt}Y_{jt}$ sector j's nominal value added (output price times real output) and L_{jt} hours worked. The $(1-\alpha_j)Y_{jt}/L_{jt}$ form of the marginal product follows from the Cobb-Douglas technology — the minimal structure standard in the literature for marginal-product identification, not a circular use of the model: the construction invokes only the definition of the markdown and the Cobb-Douglas functional form, no equilibrium condition of the model. Output Y_{jt} and hours L_{jt} come from BEA industry accounts and BLS data, and the elasticities α_j are the pooled-regression estimates of Equation 33, so the marginal product is built entirely from data and from parameters estimated outside the model. Both W_t and MRPL_{jt} are in currency per hour worked, so ν_{jt} is a pure fraction.[13]

The baseline simulation imposes a common markdown across sectors, $\nu_{Ct} = \nu_{It} = \nu_t^{\text{sym}}$, equal to the economy-wide markdown — the aggregate wage bill as a fraction of aggregate competitive labor income:

$$\nu_t^{\text{sym}} = 1 - \frac{W_t L_t}{(1-\alpha_C)\text{VA}_{Ct} + (1-\alpha_I)\text{VA}_{It}}, \quad L_t = L_{Ct} + L_{It}. \tag{34}$$

Because the two sectors have different labor elasticities, the denominator is the sum of the sectoral competitive labor incomes $(1-\alpha_j)\text{VA}_{jt}$, not a single elasticity times total value added. Equivalently, ν_t^{sym} is the value-added-weighted average of the two identified series ν_{Ct} and ν_{It}, with weights $(1-\alpha_j)\text{VA}_{jt}$ proportional to each sector's competitive labor income.

Sectoral value-added data that allow marginal-product identification are only available from 1997. For the earlier period, 1980–1996, I extend the markdown series backward using the Cobb-Douglas variant of the secular trend documented by Yeh et al. (2022, hereafter YMH). Under Cobb-Douglas production — consistent with this model's functional form — the YMH aggregate markdown rises monotonically from 1977, unlike the U-shaped path that emerges under their translog baseline. The quinquennial Census-year index[14] is interpolated to annual frequency and rescaled so that its 1997 value coincides with the OLS linear trend fitted to the identified series, yielding a smooth join at the overlap year, as shown in Figure 9.

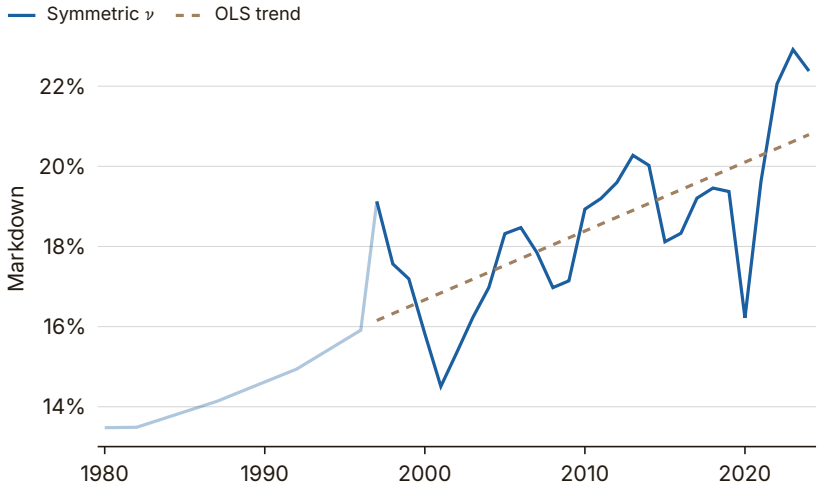
Applying the same marginal-product formula sector by sector reveals substantial heterogeneity: in 1997, $\nu_C \approx 22\%$ against $\nu_I \approx 8\%$, and both rise to around 22% by 2024, with sector I converging toward the consumption-sector level over the sample. The same

[13] Equivalently, $\nu_{jt} = 1 - (W_t/P_{jt})/\left[(1-\alpha_j)Y_{jt}/L_{jt}\right]$, the real wage in sector-j output units divided by the physical marginal product. The deflator P_{jt} cancels, so the markdown is identical whether the wage and the marginal product are both measured in nominal or both in real terms.

[14] Unfortunately, the markdown path is not publicly available, so I rely on approximations from visual inspection of their figures (1977≈1.000, 1982≈1.002, 1987≈1.050, 1992≈1.110, 1997≈1.200).

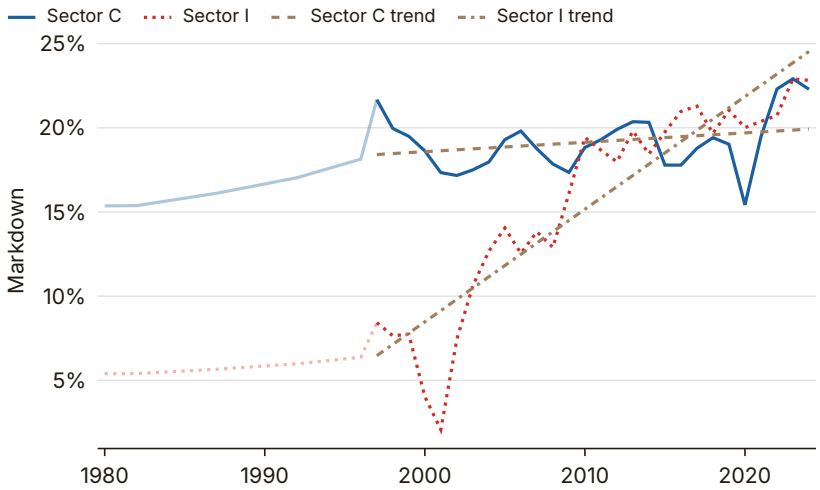
OLS-anchored YMH backfill is applied independently to each sector for 1980–1996, but with less smooth and convincing results (Figure 10). Although the baseline simulation uses only the symmetric aggregate markdown, the sectoral paths document that labor-market markdowns are substantially higher in the consumption sector throughout the sample.

Figure 9. Symmetric aggregate wage markdown path (1980–2024).



Notes: The symmetric aggregate ν^{sym}, a value-added-weighted average of ν_C and ν_I, rises from approximately 13% in 1980 to 22% by 2024. Solid line: 1997–2024, marginal-product identification using BEA and BLS data. Dashed line: OLS linear trend fitted to the identified series (1997–2024), used to anchor the backfill. Faded line: 1980–1996, Cobb-Douglas secular trend from Yeh et al. (2022, App. O.2.3), rescaled so its 1997 value matches the OLS trend at that year.

Figure 10. Sectoral wage markdown paths ν_C and ν_I (1980–2024).



Notes: Sector C starts near 15% in 1980 and sector I near 5% both converge to approximately 22% by 2024. Solid lines: 1997–2024, marginal-product identification applied independently to each sector. Dashed lines: OLS linear trend fitted to each sector's identified series (1997–2024), used to anchor the backfill. Faded lines: 1980–1996, same OLS-anchored (Yeh et al., 2022) Cobb-Douglas backfill applied sector by sector.

The identified path is broadly consistent with several independent strands of the microdata literature. Azar & Marinescu (2024) survey production-function-based structural estimates and find markdowns in the 15–50% range for the U.S.; the symmetric path ν^{sym} rising from 13% to 22% over the sample is well within this range. On the time trend, Ren & Zhang (2025) document — using broad U.S. Census data — that aggregate market power has risen since the early 2000s and is driven by wage markdowns rather than product markups, estimating roughly a doubling of markdowns between 1977 and 2019. Kirov & Traina (2023) corroborate this for U.S. manufacturing, finding that markdowns roughly doubled between 1972 and 2014 as marginal revenue products outpaced wages. However, not all evidence points in the same direction. Deb et al. (2022) find markdowns broadly stable between 1997 and 2016, attributing wage stagnation primarily to product markups rather than monopsony power, though this likely reflects differences in data coverage and time horizon.

■ **Initial Technology Levels.** The initial capital stock k_0 is set to the 1979 BEA value. Identifying the initial technology levels (z_{I0}, z_{C0}) requires care: if distortions were already present in 1980, anchoring to a competitive benchmark would be internally inconsistent with the model. I instead anchor both levels to the full-model initial steady state, setting $\mu_0 = 1/2\mu_{1980}^{\text{ARW}}$ — the best-case markup adopted as the main specification (Section 4.3) — and $\nu_0 = \nu_{1980}^{\text{sym}}$, solving for the stationary allocations $(\tilde{k}_*, \ell_*)$ from Proposition 3, and recovering $z_{I0} = k_0/(\tilde{k}_* \ell_*)$. The initial consumption-sector level z_{C0} is then backed out by inverting the distorted relative price at $t = 0$:

$$q_0 = (1 + \mu_0)\left(\frac{\alpha_I}{\alpha_C}\right)\left(\frac{z_{I0}}{z_{C0}}\right)^{1-\alpha_C} A_0,$$

where A_0 collects the steady-state factor allocation ratios. Every scenario entering the best-case decomposition — the competitive baseline, the markup-only and markdowns-only counterfactuals, and the no-IETP counterfactual — is anchored to this same steady state, so the counterfactual comparisons across them remain internally consistent. Each markup sensitivity variant instead recomputes z_{I0} and z_{C0} using its own scaled μ_0, preserving consistency within each variant.

Table 3. Initial conditions.

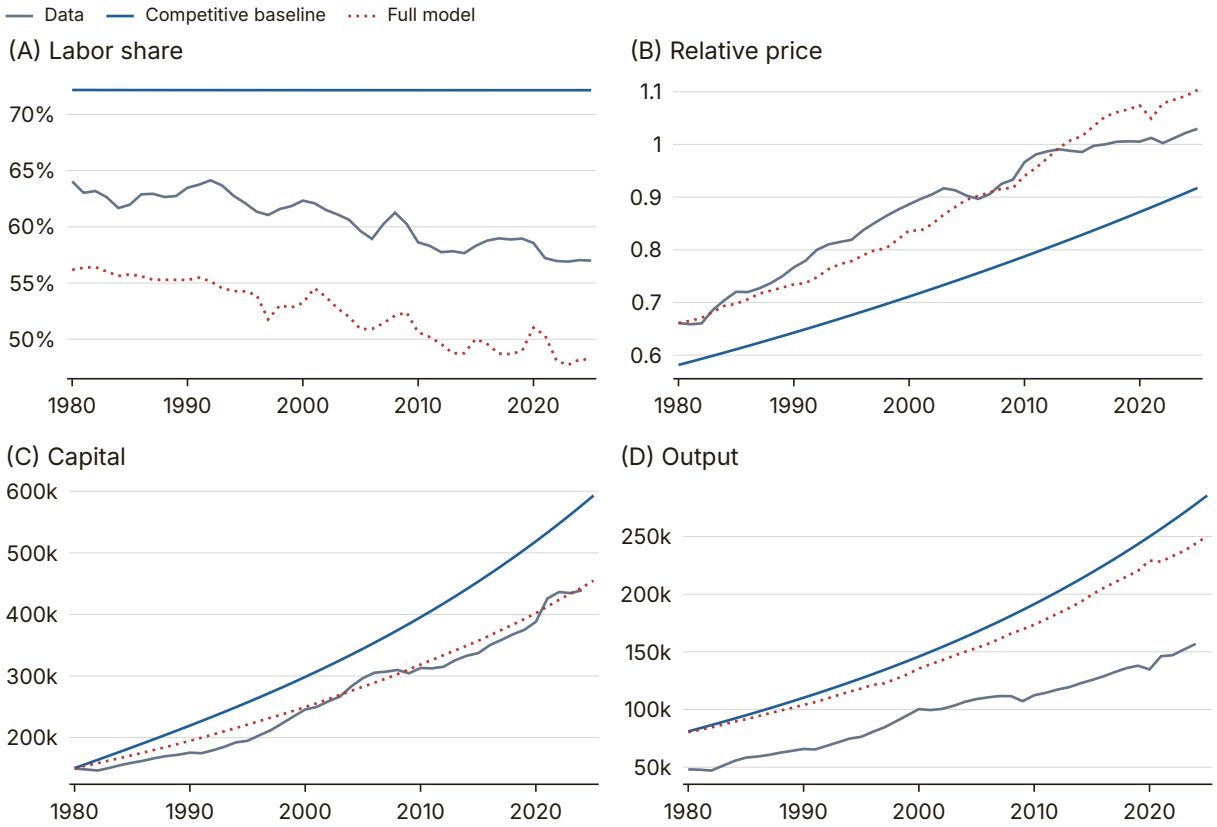
Variable	Value	Target
k_0	150 388.31	k_{1979} from U.S. Bureau of Economic Analysis (2025)
z_{I0}	68 756.89	Distortion-aware SS: $\mu_0 = 1/2\mu_{1980}^{\text{ARW}}$, $\nu_0 = \nu_{1980}^{\text{sym}}$
z_{C0}	143 440.95	Inverted distorted q formula at $t = 0$

With parameters and distortion paths calibrated to U.S. data over 1980–2024, I now examine the model's implications for the labor share and other macroeconomic aggregates.

4.3. Simulation Results

The calibration in Section 4.2 deliberately uses no income-share data on either side: the capital-output elasticities α_j come from a within-industry panel regression, the TFP growth rates from the long-run trends of the capital stock and the relative price, the markup path from an external accounting series, and the markdown path from marginal-product residuals. Beyond these trends, no aggregate trajectory is fit: the labor share and output enter the calibration nowhere, and only the long-run slopes — not the year-to-year dynamics — of the capital stock and the relative price discipline the two TFP growth rates. The joint behavior of the four aggregates in Figure 11 — the labor share, the relative price, capital, and output — is therefore a test of the model on moments the calibration did not target.

Figure 11. Joint dynamics: full model vs. data (1980–2024).



Notes: Each panel plots BEA data (solid gray) against the competitive baseline and the full model (best case: ARW markup scaled to one-half, symmetric markdowns). Output and capital are per household. Sources: BEA NIPA, BEA Fixed Assets.

Figure 11 reports the test result: the full model — the best case, pairing the ARW markup scaled to one-half with symmetric markdowns — tracks the data on all four aggregates from 1980 to 2024. The competitive baseline misses both the level and the trend of the labor share and also the level of the relative price; the full model matches the relative price in

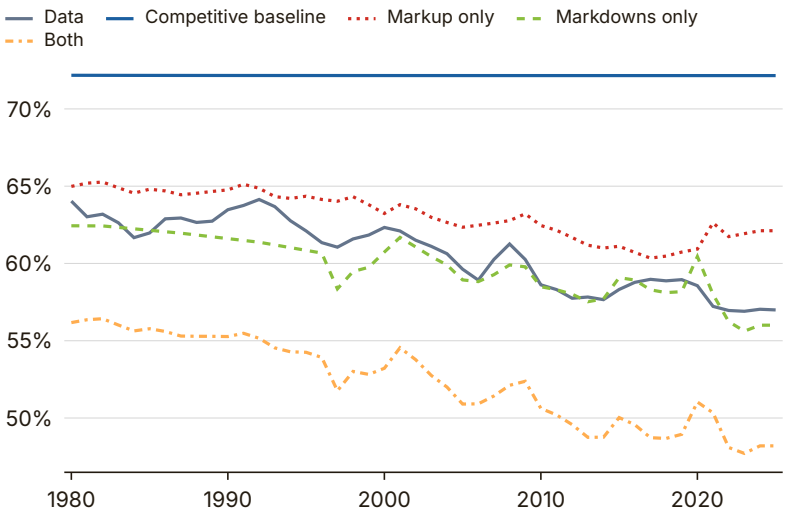
both level and trend and reproduces the dynamics of the labor share — though not its level — while capturing both the level and the trend of capital but only the trend of output.

The scenarios reported below decompose this joint fit into its constituent channels. The full model pairs the symmetric markdown path ν^{sym} with the ARW markup scaled to one-half, the best-case level identified below. Three counterfactuals, all sharing its $t = 0$ technology anchor, switch off one channel at a time: the competitive baseline ($\mu = 0$, $\nu^{\text{sym}} = 0$), the markup-only path ($\nu^{\text{sym}} = 0$), and the markdowns-only path ($\mu = 0$). A fourth retains the full distortions but sets $g_{z_C} = g_{z_I}$, isolating the investment-specific TFP contribution. Table 4 reports 1980–2024 changes in the labor share, the relative price, and the real wage across these scenarios; Table 5 reports normalized RMSE fit statistics across the four headline aggregates. The robustness exercises that follow restore the face-value and one-tenth ARW markup, and replace the symmetric markdown with the sector-specific identified paths $\nu_C \neq \nu_I$.

The remainder of this section organizes the findings around four claims: (i) wage markdowns alone reproduce the labor share decline, with the consumption-sector markdown carrying most of the leverage; (ii) the ARW markup at face value overstates the model-consistent level — the best-fitting calibration scales it down by half; (iii) the investment-specific TFP wedge is essentially orthogonal to the distribution channel — it shapes the relative price and the level of real wages but not the labor share trajectory; (iv) the implied path of the firm-level labor-supply elasticity falls from about 6.4 to about 3.5, a structural reading of the markdown rise as a halving of workers' outside-option responsiveness.

■ **Markdowns as the Primary Lever.** Figure 12 decomposes the labor share into the markup-only, markdown-only, and full-model paths. Two facts dominate the picture.

Figure 12. Labor share: data and simulations.



Notes: Markdowns alone nearly reproduce the data; the markup alone accounts for less. Combining the two in the full model (half-scaled markup) tracks the data with a slight overshoot. Sources: BEA NIPA, BEA Fixed Assets, BLS.

First, the markdowns-only counterfactual nearly reproduces the data. Feeding the symmetric markdown path ν^{sym} into the model — with $\mu = 0$ throughout — yields a 1980–2024 decline of 6.4 pp against 7.0 pp in the data (Table 4). The markup-only counterfactual delivers a smaller decline of 2.9 pp. Since neither distortion path was calibrated against the labor share, this contrast is informative: markdowns account for the bulk of the decline, while the markup channel explains substantially less.

Second, the half-scaled markup adds to the markdown channel. Combining the symmetric markdown with the ARW markup scaled to one-half — the full model — delivers a decline of 8.0 pp, modestly past the 7.0 pp in the data, while the implied 1980 level of 56% sits below the observed 64% — the model reproduces the trajectory of the decline but not its level. Holding the markup at one-half rather than its face value is what keeps the model close to the data: below I show that the unscaled ARW markup overshoots the labor share decline and that one-half gives, among the scales examined, the best joint fit to the trajectory of m_ℓ, q, k, and y — the overstatement flagged in Section 4.2.

The symmetric markdown specification masks an asymmetry that is structural in the model. Comparing Equation 29 and Equation 30, conditional on s_k, a unit change in ν_C produces the same effect on the aggregate labor share as a change in ν_I scaled by:

$$\chi_t = \frac{\alpha_I}{1-\alpha_I}\frac{1-\alpha_C}{\alpha_C}\frac{1-s_{kt}}{s_{kt}}.$$

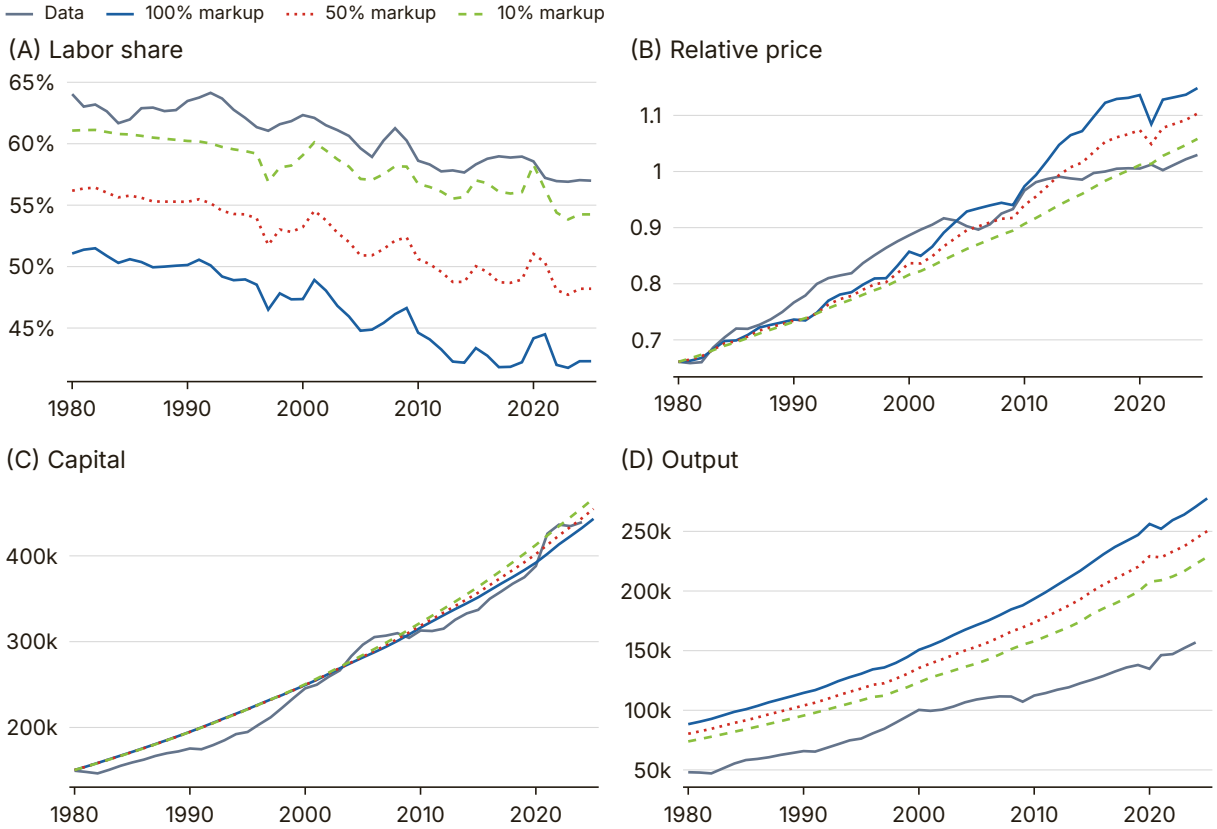
Evaluated at the 1980 baseline ($s_k = 0.210$, $\alpha_C = 0.282$, $\alpha_I = 0.264$), $\chi_0 = 3.44$: one percentage point of ν_C moves the aggregate labor share as much as roughly 3.44 percentage points of ν_I. Two features generate this leverage gap. Sector C absorbs the majority of the capital stock ($s_k < 1 - s_k$) and is therefore the larger sector in production; a markdown applied there compresses a larger share of the aggregate wage bill than the same markdown in sector I. On top of that, ν_C enters the labor-supply decision directly (Equation 22), whereas ν_I reaches it only indirectly through input allocation. The substantive implication is that the labor share decline is, mechanically, a consumption-sector phenomenon — even an economy-wide markdown path acts on the aggregate primarily through its incidence in the consumption sector. This χ_t asymmetry also explains why the asymmetric robustness exercise below produces a labor share path close to the symmetric baseline despite the sizeable sectoral gap between the identified ν_C and ν_I paths.

The model-implied markdown path translates into a single structural parameter: the firm-level labor-supply elasticity $\theta_t \equiv (1-\nu_t)/\nu_t$. The symmetric markdown rising from 13% in 1980 to 22% in 2024 implies θ falling from approximately 6.4 to 3.5 — workers' outside-option responsiveness has roughly halved over the sample. As discussed in Section 4.2, this range sits inside the cross-sectional band documented by the production-function literature surveyed by Azar & Marinescu (2024), and the implied roughly two-fold rise in ν^{sym} is consistent with the doubling of public-firm markdowns documented by Ren & Zhang

(2025) and the comparable doubling of the manufacturing labor wedge documented by Kirov & Traina (2023). The structural reading is also the substantive answer to the paper's research question: the rising exercise of employer market power — concretely, a decline in the labor-supply elasticity perceived by firms — is the dominant force behind the post-1980 U.S. labor share decline.

■ **The Model-Consistent Markup.** The one-half markup scaling used in the full model is not assumed, but rather selected by joint fit among the markup levels examined. As argued in Section 4.2, the ARW operating margin is an upper bound on the model-consistent markup by construction — its denominator omits the cost of capital, which enters marginal cost in this model. To trace the consequences of this overstatement quantitatively, I re-run the model at the face-value ARW markup and at one-tenth, alongside the one-half best case, re-anchoring the initial technology levels to each variant's steady state at $t = 0$.

Figure 13. Markup sensitivity under symmetric markdowns.



Notes: Main aggregates under the full model with symmetric markdowns and three markup scaling variants: 100%, 50%, and 10% of the ARW path. Each variant re-anchors technology levels to the full-model steady state at its own μ_0. Sources: BEA NIPA, BEA Fixed Assets, BLS.

Figure 13 shows the four headline aggregates under each scaling. The qualitative pattern is robust to the markup level: every variant produces a declining labor share, a rising relative price of investment goods, and slower aggregate growth than the competitive baseline. Quantitatively, the level of the labor share shifts mechanically with the markup, while the

trajectory of output and capital improves materially as the markup shrinks — the 100% ARW scenario somewhat overpredicts the growth slowdown.

Table 5 formalizes the trade-off. The joint normalized RMSE — a simple average of the four variable-level NRMSE, computed on 1980-indexed series[15] — is minimized at one-half of ARW (joint NRMSE = 0.095), below both the face-value ARW (0.104) and the one-tenth variant (0.100). The improvement at 50% comes almost entirely from the labor share fit (NRMSE drops from 0.047 to 0.024): halving the markup leaves room for the markdown channel to account for the labor share decline without overshooting, while the markup retains a non-trivial role in the relative-price dynamics. At 10% the markup contribution to Δm_ℓ collapses and the joint fit deteriorates, though it remains below the 100% ARW level.

The 50% ARW result is best read as a direct GE-based corroboration of the methodological concerns raised by Benkard et al. (2026) and Ren & Zhang (2025): the Compustat operating margin overstates the model-consistent markup by roughly a factor of two. Identified through the joint trajectory of m_ℓ, q, k, and y rather than through micro re-estimation, this conclusion is independent of the markup-measurement debate and supports it from the demand side.

■ **Distribution vs. Growth: The IETP Decomposition.** The third channel of Section 2 is investment-specific technological change, captured here by the wedge $g_{z_I} > g_{z_C}$. The no-IETP counterfactual sets $g_{z_C} = g_{z_I} = g_{z_C}^{ss} \approx 1.23\%$ while retaining the full distortion paths at the best-case markup, isolating the contribution of the IETP wedge to each variable.

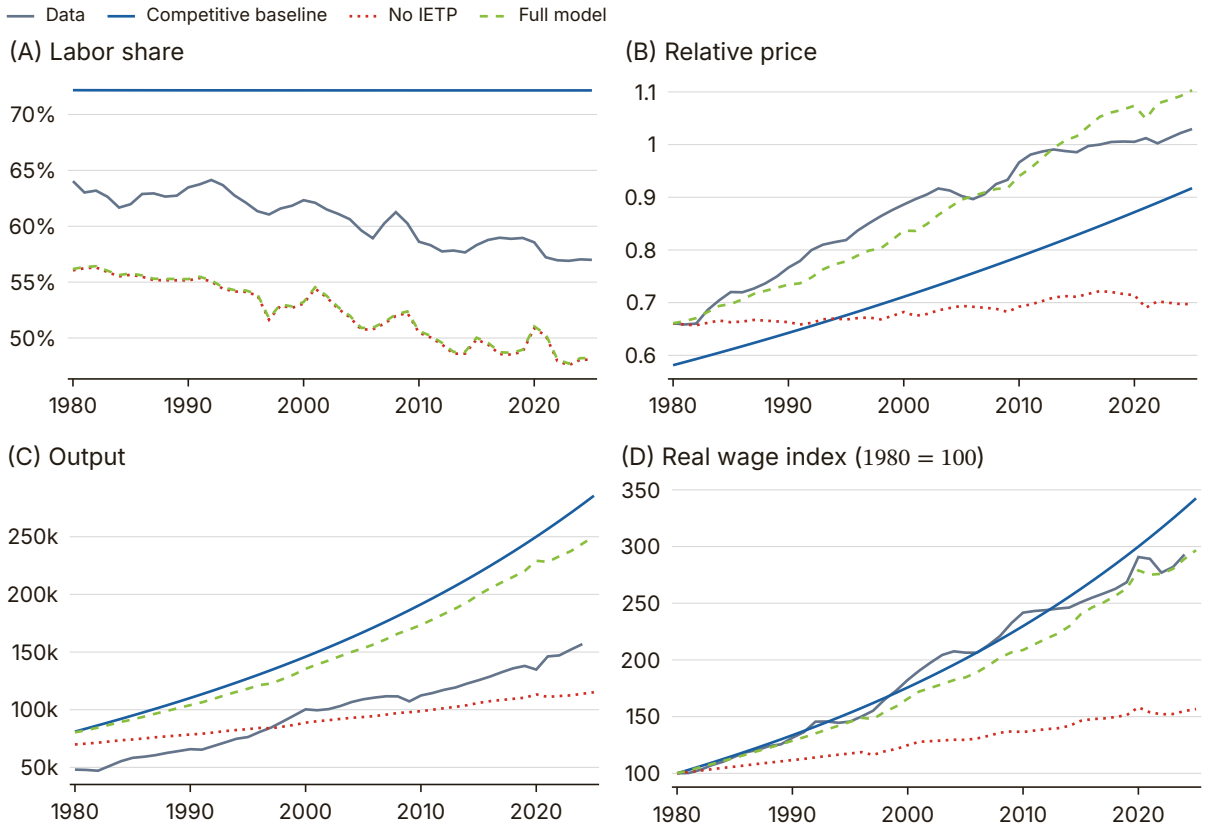
The four panels of Figure 14 deliver a clean orthogonal decomposition between distribution and growth.

Labor share. The full model and the no-IETP counterfactual are virtually indistinguishable: a 1980–2024 decline of 8.0 pp with IETP, 8.0 pp without. The IETP contribution to the labor share trend is +0.02 pp — economically zero. Heterogeneous capital-output elasticities ($\alpha_C \neq \alpha_I$) do, in principle, open a composition channel through which IETP can move m_ℓ: faster TFP growth in sector I raises q, which tilts nominal output toward the capital-intensive consumption sector and compresses the labor share even without market-power wedges (Gouveia-Mendes, 2025). In the calibration, however, the measured $\alpha_C - \alpha_I$ gap is too narrow for this channel to leave a quantitatively visible mark; only the wedge between marginal product and wage moves m_ℓ in a significant way.

Relative price. The two scenarios diverge sharply on q. Without IETP, the model produces $\Delta q = 0.04$ over 1980–2024, far short of the $\Delta q = 0.36$ in the data. With IETP, the model delivers $\Delta q = 0.43$, bracketing the data from above. IETP accounts for 0.40 of the 0.43 rise in q predicted by the full model — essentially all of it.

[15] All series are indexed to their 1980 value before computing differences, so the metric captures how well each scenario tracks the normalized trajectory of each variable, not whether it matches the level.

Figure 14. IETP isolation: competitive baseline, no-IETP counterfactual, and full model.



Notes: The no-IETP counterfactual sets $g_{z_C} = g_{z_I} = g_{z_C}^{ss}$ ($\approx 1.23\%$), eliminating the investment-sector TFP premium while preserving the best-case (half-scaled) ARW markup and symmetric markdown paths. Sources: BEA NIPA, BLS.

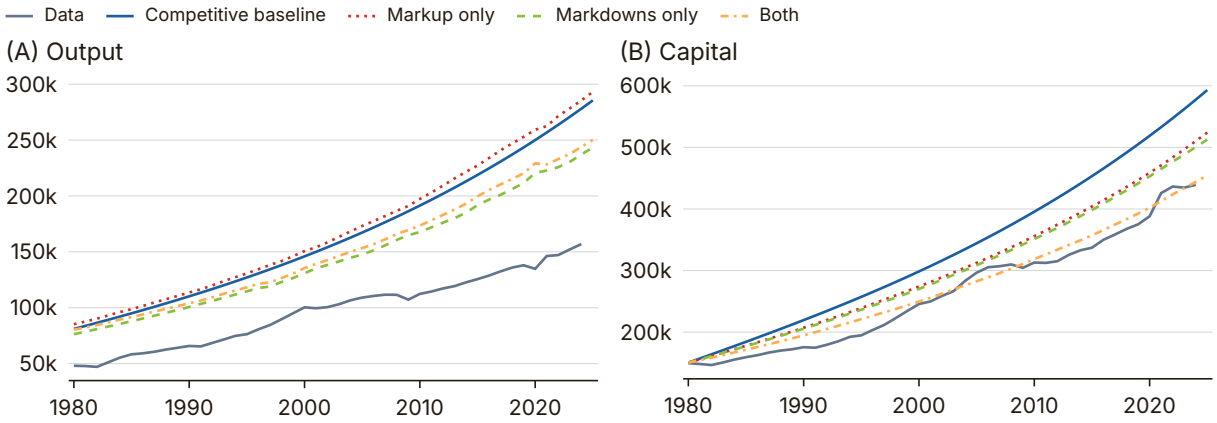
Real wage. Without IETP the wage index reaches only 155 by 2024 (base 1980 = 100), against 289 in the full model — about half the full-model level. IETP therefore accounts for the bulk of the gap between the markdown-suppressed wage trajectory and the level observed in the data: most of post-1980 real-wage growth originates in cheaper investment goods (the model numeraire) rather than in compensation paid for labor's marginal product.

Output. Output per household follows the same pattern: the no-IETP trajectory grows visibly more slowly than the full model, since shutting down the investment-sector TFP premium removes the dominant source of long-run growth in the calibration. Like the relative price and the real wage, the long-run level of output is essentially an IETP phenomenon and is largely unaffected by the distortion paths.

The decomposition has a clear implication for the broader literature: explanations of the labor share decline that lean on IETP — through capital–labor substitution at $\sigma > 1$, as in the Karabarbounis–Neiman tradition — operate in this calibration on the price trend and on the level of real wages, not on the income split. The two stories therefore complement rather than compete: market-power distortions govern distribution, while IETP governs the relative price and the secular real-wage gain.

■ **Other Macroeconomic Aggregates.** I now sweep output, capital, the factor returns, and the relative price across the decomposition scenarios; together they confirm the behavior anticipated by the model's analytical structure. The first-best comparison in Section 3 isolates three distortions: an *intratemporal labor supply distortion*, a *sectoral misallocation of labor*, and an *intertemporal distortion* of capital accumulation. Under the symmetric markdown used throughout this section the sectoral-misallocation channel is shut down — the markdown enters the sectoral labor allocation (Equation 23) only through the ratio $(1-\nu_C)/(1-\nu_I)$, which equals one when $\nu_C = \nu_I$ — so the wedges move the levels of the aggregates, not their sectoral composition, and the sectoral shares stay virtually unchanged in the simulations. Reallocation enters only with sector-specific markdowns, the subject of the robustness exercise that follows.

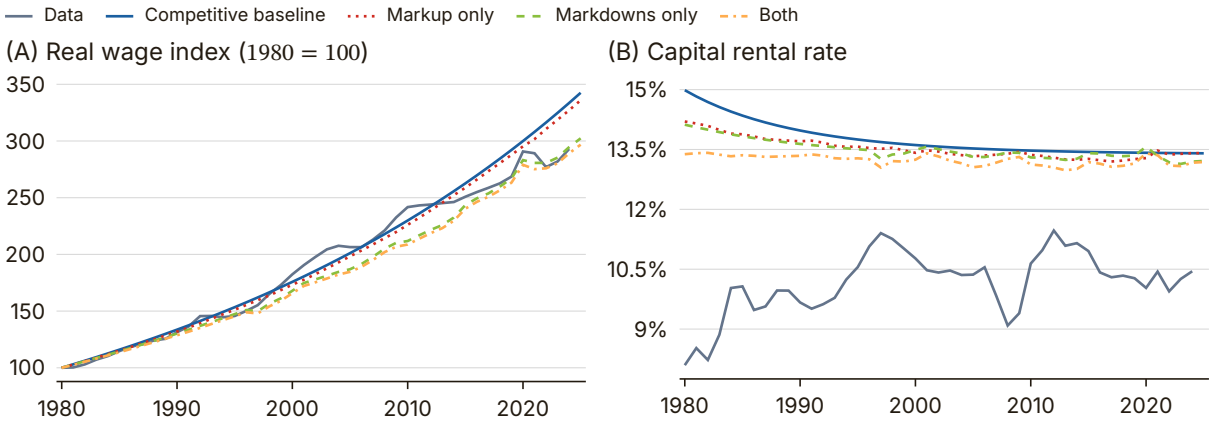
Figure 15. Output and capital.



Notes: Output per household (left) and capital per household (right), plotted against BEA data. Sources: BEA NIPA, BEA Fixed Assets.

Output and capital (Figure 15). All distorted scenarios grow more slowly than the competitive baseline in both output and capital. The markup-only output path nonetheless sits above the baseline in level, because aggregate output is valued at the markup-inflated relative price q. Capital growth matches the data closely under the full specification. Output growth is overpredicted at the 100% ARW level — a discrepancy that shrinks to a near-fit at 50% ARW (Figure 13). Mechanically, the slowdown is a level effect, not sectoral misallocation (muted when $\nu_C = \nu_I$, Figure 18): the intratemporal wedge $(1-\nu_C)/(1+\mu)$ in Equation 22 lowers aggregate labor, and the lower labor together with the distorted Euler condition (Equation 24) lowers the capital stock the economy sustains.

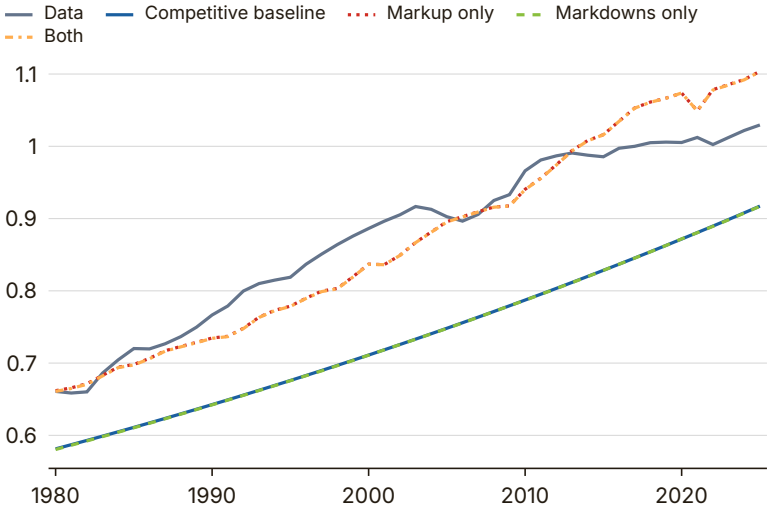
Figure 16. Factor returns.



Notes: Real wage index (left, 1980 = 100) and real return on capital (right), plotted against BEA and BLS data. Sources: BEA NIPA, BEA Fixed Assets, BLS.

Wages and returns (Figure 16). Real wage growth is dampened in every distorted scenario — the immediate manifestation of the intratemporal wedge $(1-\nu_C)/(1+\mu)$ that drives the labor share decline (Equation 28). The model overpredicts the level of the real return on capital across all scenarios — an artifact of the frictionless Euler equation, which pins r_{ss} to $\left(1+g_{z_I}\right)/\beta_\Gamma - 1 + \delta$ and leaves no room for the risk premia, taxes, and other wedges that compress observed returns — but it reproduces the broadly flat trend.

Figure 17. Relative price of consumption goods.



Notes: Relative price of consumption goods (q), plotted against BEA data. Symmetric markdowns leave the q path close to the baseline because input reallocation across sectors is minimal; sector-specific markdowns produce diverging paths through their differential effect on sectoral output supplies. Source: BEA NIPA.

Relative price (Figure 17). With symmetric markdowns, q tracks the data closely — by construction, since g_{z_C} is calibrated from the long-run trend in q. The IETP channel pins down

the long-run slope; the markup adds a level shift through the consumption-sector price. Sector-specific markdowns alter this fit by inducing visible cross-sectoral reallocation.

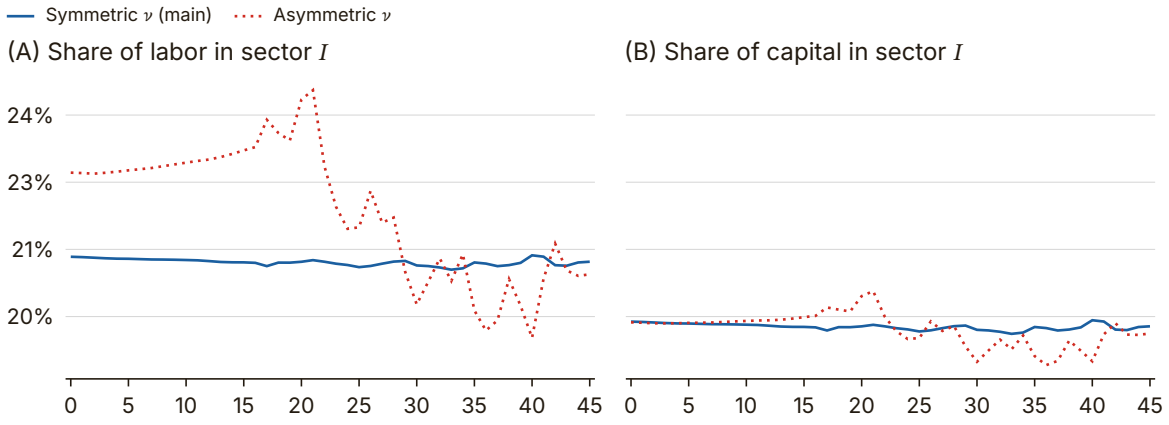
Table 4. Channel decomposition: 1980–2024 changes across scenarios.

Scenario	Channels active	Δm_ℓ	m_ℓ (1980)	Δq	w (2024)
Data (BLS/BEA)	—	−7.0 pp	64.0%	+0.36	—
Competitive	IETP	0.0 pp	72.2%	+0.33	334
Markup only	IETP + markup	−2.9 pp	65.0%	+0.43	327
Markdowns only	IETP + markdown	−6.4 pp	62.4%	+0.33	295
Full model	**IETP + markup + markdown**	**−8.0 pp**	**56.2%**	**+0.43**	**289**
No IETP	Markup + markdown	−8.0 pp	56.1%	+0.04	155

Notes: Δm_ℓ is the change in labor share, 1980–2024. m_ℓ (1980) is the initial labor share level. Δq is the change in relative price of consumption goods, 1980–2024. w (2024) is the real wage index (1980 = 100) in 2024. IETP contribution to Δm_ℓ: full model minus no-IETP counterfactual =−8.0 - (−8.0) = +0.02 pp. IETP contribution to Δq: 0.43 - 0.04 = 0.40.

■ **Robustness: Symmetric vs. Asymmetric Markdowns.** The baseline simulation uses the value-added-weighted symmetric markdown ν^{sym}. The asymmetric specification relaxes this restriction, feeding the sector-specific marginal-product-identified paths (Figure 10) — $\nu_C \neq \nu_I$, with $\nu_C > \nu_I$ throughout most of the sample — into the full model alongside the half-scaled markup. The asymmetric specification carries strictly more information than the aggregate, which may improve the joint fit.

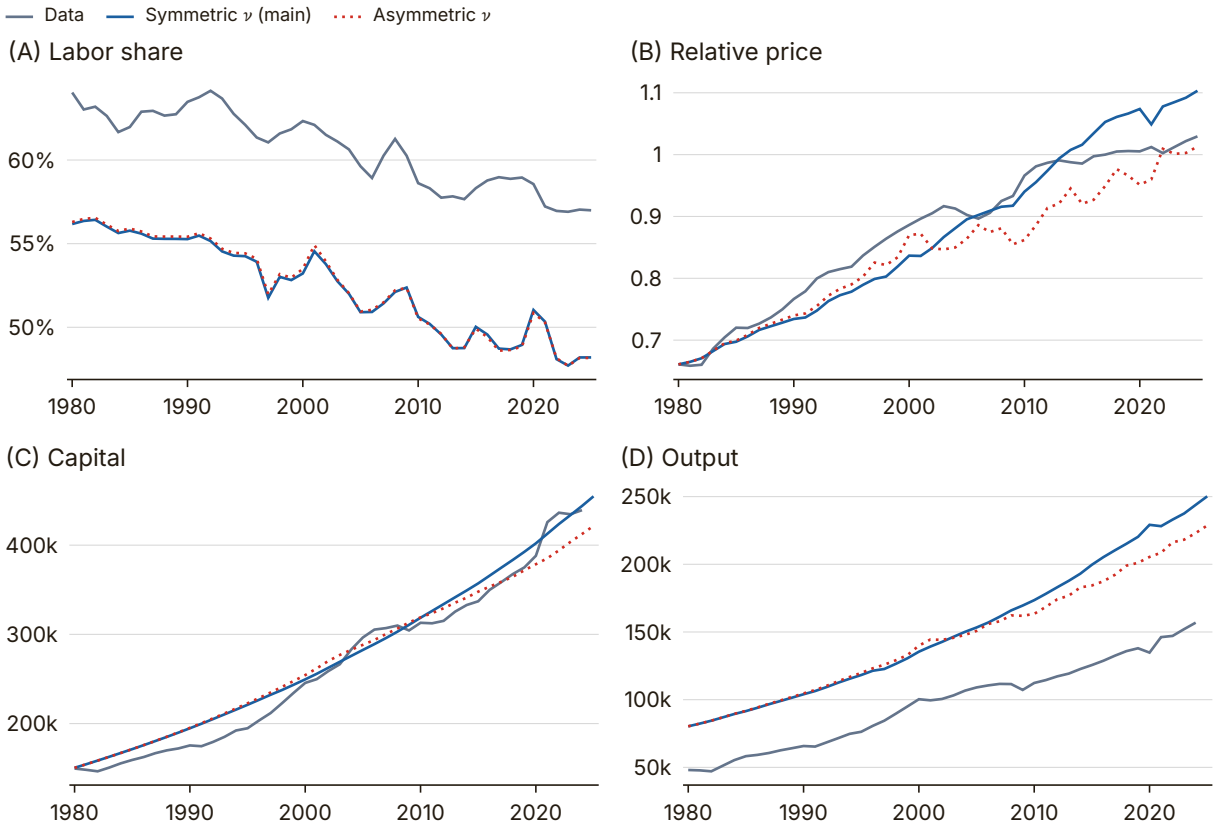
Figure 18. Input allocation to the investment sector: symmetric versus asymmetric markdowns.



Notes: Share of aggregate labor (left) and capital (right) allocated to sector I under the symmetric ($\nu_C = \nu_I$) and asymmetric ($\nu_C \neq \nu_I$) specifications of the full model. The symmetric specification produces virtually no cross-sectoral reallocation; the asymmetric specification, where $\nu_C > \nu_I$ throughout the sample, shifts labor toward sector I over much of the sample, with a markedly smaller effect on capital. Sources: BEA NIPA, BEA Fixed Assets, BLS.

The mechanical effect of moving to the asymmetric specification is visible in Figure 18. Under the symmetric specification, both the share of labor and the share of capital allocated to sector I move very little: with $\nu_C = \nu_I$ the markdown ratio $(1-\nu_C)/(1-\nu_I)$ in the sectoral labor allocation (Equation 23) equals one, shutting down the *sectoral misallocation of labor* channel, so there is no compositional incentive to reallocate inputs. Under the asymmetric specification, $\nu_C > \nu_I$ drives that ratio below one and raises $s_{\ell t}$: the heavier markdown in sector C lifts its effective labor cost $w_t/(1-\nu_C)$ relative to sector I's, pulling labor toward sector I over much of the sample (Equation 23); capital, pinned by the intertemporal margin (Equation 24), reallocates far less, so the shift operates mainly through labor. This labor reallocation makes consumption goods relatively scarcer, tilting the relative-price path. This is precisely the channel anticipated by the χ_t asymmetry derived above: the consumption-sector markdown is the dominant lever, and once it is moved independently of ν_I the model produces visible cross-sectoral input reallocation.

Figure 19. Main aggregates: symmetric versus asymmetric markdowns.



Notes: Labor share, relative price, capital, and output under the full model with symmetric ν and with asymmetric $\nu_C \neq \nu_I$, compared with the competitive baseline. Sources: BEA NIPA, BEA Fixed Assets, BLS.

The extra information does not improve the joint fit at the best-case markup. Figure 19 shows the four headline aggregates under both specifications: the two paths are nearly identical on the labor share, close on the relative price, and differ most on output and capital, though both stay comparable to the data. Table 5 confirms it: at the best-case markup, sym-

metric markdowns deliver a joint NRMSE of 0.095 versus 0.119 for asymmetric — driven mainly by output (0.198 vs 0.267) and capital (0.096 vs 0.116), with the labor share (0.024 vs 0.025) and the relative price (0.061 vs 0.068) close. At the face-value markup the asymmetric specification fits the relative price markedly better (0.043 vs 0.092 for symmetric), but that advantage disappears once the markup is scaled to its best-case level.

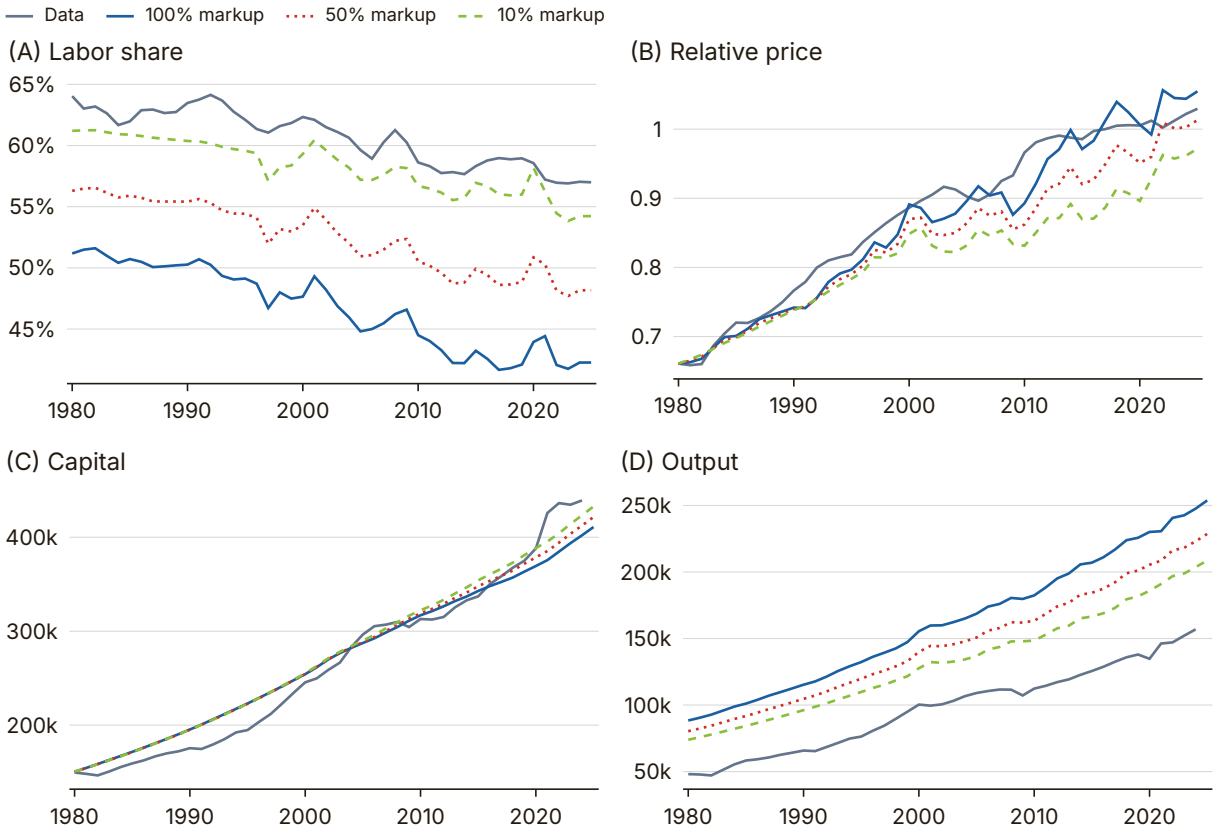
The ordering is robust to the markup level. Figure 20 repeats the markup-scaling exercise under the asymmetric specification, and Table 5 confirms that symmetric also wins the joint fit at the face-value markup (0.104 vs 0.116) and at one-tenth (0.100 vs 0.129). The robustness of the symmetric specification has a structural reading consistent with the previous analysis: the value-added-weighted aggregate captures the dominant ν_C lever cleanly, while the additional information in the sector-specific identification introduces cross-sectoral reallocation that the data do not seem to require.

Table 5. NRMSE comparison: symmetric versus asymmetric markdowns across markup levels.

Markup	**Case**	NRMSE(y)	NRMSE(m_ℓ)	NRMSE(q)	NRMSE(k)	**Joint**
100% ARW	3 (sym)	0.180	0.047	0.092	0.096	**0.104**
	4 (asym)	0.243	0.049	0.043	0.131	0.116
50% ARW	5 (sym)	0.198	0.024	0.061	0.096	**0.095**
	7 (asym)	0.267	0.025	0.068	0.116	0.119
10% ARW	6 (sym)	0.215	0.015	0.065	0.107	**0.100**
	8 (asym)	0.288	0.014	0.108	0.106	0.129

Notes: Normalized RMSE computed on indexed series (1980 = 1) over 1980–2024. Joint NRMSE is the simple average of the four variable-level NRMSE. Bold entries indicate the winner within each markup-level pair. All scenarios use the full ARW markup path scaled by the indicated factor and, for symmetric cases, $\nu_C = \nu_I = \nu^{\text{sym}}$; for asymmetric cases, the sector-specific marginal-product-identified paths.

Figure 20. Markup sensitivity under asymmetric markdowns.



Notes: As Figure 13, but using asymmetric sector-specific markdowns ($\nu_C \neq \nu_I$). Sources: BEA NIPA, BEA Fixed Assets, BLS.

■ **Summary.** Three numerical takeaways. First, the symmetric markdown path identified from marginal-product residuals accounts for 6.4 pp of the 7.0 pp labor share decline in isolation, while the half-scaled ARW markup contributes 2.9 pp on its own; at the face-value markup the combined model overshoots the data, as expected given that the ARW level is an upper bound on the model-consistent markup. Second, the joint NRMSE across the four headline aggregates is minimised at a markup scaling of roughly 1/2 of the ARW operating margin, a general equilibrium-based corroboration of the methodological critiques of Compustat-based markup estimates. Third, although heterogeneous α_j opens a composition channel through which IETP can affect m_ℓ, the measured capital-intensity gap is too narrow for it to register quantitatively: the investment-specific TFP wedge contributes essentially nothing to the labor share decline while accounting for nearly all of the rise in the relative price of investment and roughly half of post-1980 real-wage growth — distribution and growth dimensions are quantitatively separable in this calibration.

The substantive answer to the paper's research question follows from the structural reading of the markdown path: a fall in the firm-level labor-supply elasticity from about 6.4 to about 3.5 — workers' outside-option responsiveness has roughly halved — is the dominant force behind the post-1980 U.S. labor share decline. Product-market markups contribute, but at a level closer to half the ARW operating margin than the headline value,

and the investment-specific TFP wedge is essentially neutral for the income split, however essential it remains for explaining the relative price and the secular rise in real wages.

5. Conclusions

This paper has examined the joint role of investment-embodied technological progress (IETP), product-market markups, and labor-market wage markdowns in the post-1980 U.S. labor share decline. I built a two-sector general equilibrium model with Cobb-Douglas production, Greenwood-Hercowitz-Huffman preferences for endogenous labor supply, monopolistic competition in the consumption sector, and monopsonistic competition in both sectors' labor markets. The calibration uses no income-share moments: capital-output elasticities come from a within-industry panel regression, TFP growth rates from the long-run trends of the capital stock and the relative investment price, the markup path from an operating-profit markup in ARW's Compustat sample, and the markdown path from marginal-product residuals. The labor share, real wage, output, and capital stock remain untargeted moments that test the calibrated model.

Three substantive findings emerge. First, the markdown channel alone nearly reproduces the labor share decline. The marginal-product-identified symmetric markdown path, fed into the model with the markup set to zero, generates a 1980–2024 decline of roughly 6 percentage points against 7 in the data; combining the markdown with the ARW markup overshoots the data. The ARW level is an upper bound on the model-consistent markup by construction, and the overshoot at face value bears this out: among the markup levels examined, scaling the ARW path to roughly half gives the closest joint fit across the four headline aggregates. This corroborates the methodological concerns about Compustat-based markup measures raised by Benkard et al. (2026) and Ren & Zhang (2025) through a general-equilibrium criterion rather than micro re-estimation. By contrast, a constant wedge — whether markup or markdown — produces only a level shift in the labor share, confirming that, with the capital-reallocation channel negligible, the secular trend requires time-varying market power.

Second, IETP is quantitatively orthogonal to the labor share dimension. Heterogeneous capital-output elasticities ($\alpha_C \neq \alpha_I$) do open a composition channel through which IETP can shift m_ℓ — faster TFP growth in sector I raises q, tilts nominal output toward the capital-intensive consumption sector, and compresses the labor share even without market-power wedges (Gouveia-Mendes, 2025) — but the measured gap is too narrow to make the channel quantitatively visible. Removing the investment-sector TFP premium leaves the labor share decline essentially unchanged, while the same intervention collapses the rise in the relative investment price, and the real-wage index in 2024 reaches only about half its full-model level. Market-power distortions govern distribution; IETP governs the relative price and the level of real wages. The two dominant narratives of the labor share decline — IETP through capital-labor substitution and rising market power — therefore complement

rather than compete in this calibration: IETP shapes the level of welfare and the price trend, while monopsony power shapes the income split.

Third, the model-implied markdown path translates into a single structural parameter — the firm-level labor-supply elasticity perceived by workers — falling from about 6.4 in 1980 to about 3.5 in 2024. Workers' outside-option responsiveness has roughly halved. Mechanically, this is the consumption sector at work: the leverage of the consumption-sector markdown on the aggregate labor share is roughly three times that of the investment-sector markdown, so even an economy-wide markdown path acts on the aggregate primarily through its incidence in the consumption sector. This finding also alleviates the difficulty of motivating systematic markdown differences between the consumption and investment sectors under the aggregation scheme used here — what matters is the leverage of the consumption-sector markdown, not the sectoral asymmetry of the identified paths.

Why might labor-supply elasticities be declining? A plausible interpretation is that the exercise of monopsony power has intensified by drawing on latent structural potential, rather than through expansion of that potential. This view is consistent with three patterns: (i) a documented link between labor-market markdowns and product-market power (Ren & Zhang, 2025; Yeh et al., 2022); (ii) rising national concentration alongside declining local firm concentration (Rinz, 2020; Rossi-Hansberg et al., 2021); and (iii) growing markup dispersion since the 1980s, with a small set of firms sustaining high markups while many others lost market share without increasing their markups (Benkard et al., 2026; De Loecker et al., 2020; 2025). Together, these facts suggest that firms have increasingly been relying on labor-market power to protect margins.

Two broader implications follow. First, the exercise of labor-market power — arising from frictions that do not raise firms' costs — can shape both aggregate dynamics and the distribution of factor income, even when labor supply is endogenous and responds to changing wages. Second, because the data-consistent scenario is an economy-wide markdown whose aggregate incidence runs mainly through the consumption sector, the results point to cross-sector forces and suggest that policies reducing mobility frictions (housing and commuting constraints, licensing barriers, non-compete enforcement), increasing wage transparency, and incorporating labor-market effects into antitrust analysis may be warranted.

Several avenues for future research remain. On the welfare side, three channels are relevant but unexplored here. First, the deadweight loss from labor misallocation due to wedges, which requires modeling the extensive margin of labor supply and firm-level heterogeneity in equilibrium. Second, the sectoral misallocation of inputs, likely limited when the data-consistent case involves balanced economy-wide distortions. And third, distributional effects operating through relative prices on households' lifetime utility. A welfare analysis incorporating household wealth heterogeneity is left for future work. In addition, endogenizing the decline in labor-supply elasticities — linking it to product-

market outcomes, the evolution of labor-market institutions, or changes in the firm-size distribution — would help distinguish among competing explanations for the intensification of monopsony power documented here.

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A. Appendix

A.1. Solution of the Household Problem

Let $\Gamma_* \equiv \Gamma_{t+1}/\Gamma_t = \left(1+g_{z_I}\right)^{\alpha_C}\left(1+g_{z_C}\right)^{1-\alpha_C}$, $\beta_\Gamma \equiv \beta\Gamma_*^{1-\varphi}$, $c_t \equiv c_t/\Gamma_t$, $k_t \equiv k_t/\Gamma_t$, $w_t \equiv w_t/\Gamma_t$, and $w_{jnt} \equiv w_{jnt}/\Gamma_t$. Then I can rewrite the household problem as:

$$\max_{\{c_{nt},k_{t+1},\ell_{jnt}\}_{t=0}^{+\infty}} \quad \mathcal{U} \equiv \frac{U}{\Gamma_0^{1-\varphi}} = \sum_{t=0}^{+\infty}\beta_\Gamma^t \frac{\left(c_t - \frac{\ell_t^{1+\eta}}{1+\eta}\right)^{1-\varphi}}{1-\varphi}$$

$$\text{s.t.} \int_0^1 q_{nt}c_{nt}\,\mathrm{d}n + \Gamma_* k_{t+1} \leq (1+r_t-\delta)k_t + w_t\ell_t + \frac{\pi_t}{\Gamma_t}$$

Let λ_t be the Lagrange multiplier associated with the budget constraint. The first-order condition with respect to c_{nt} is:

$$\lambda_t q_{nt} = \beta_\Gamma^t\left(c_t - \frac{\ell_t^{1+\eta}}{1+\eta}\right)^{-\varphi}\left(\frac{c_{nt}}{c_t}\right)^{-\frac{1}{\sigma_c}}.$$

Using the definition of q_t in Equation 3 and integrating over n pins down the Lagrange multiplier:

$$\lambda_t = \beta_\Gamma^t \frac{\left(c_t - \frac{\ell_t^{1+\eta}}{1+\eta}\right)^{-\varphi}}{q_t}, \tag{A.1}$$

and yields the CES demand for variety n:

$$c_{nt} = c_t\left(\frac{q_t}{q_{nt}}\right)^{\sigma_c}. \tag{A.2}$$

Substituting Equation (A.2) into the budget constraint and using Equation 3 shows that $\int_0^1 q_{nt}c_{nt}\,\mathrm{d}n = q_t c_t$, so the budget constraint simplifies to:

$$q_t c_t + \Gamma_* k_{t+1} \leq (1+r_t-\delta)k_t + w_t\ell_t + \frac{\pi_t}{\Gamma_t}.$$

The first-order condition with respect to labor ℓ_{jnt} is:

$$\lambda_t w_{jnt} = \beta_\Gamma^t\left(c_t - \frac{\ell_t^{1+\eta}}{1+\eta}\right)^{-\varphi}\ell_t^\eta\left(\frac{\ell_{jnt}}{\ell_{jt}}\right)^{\frac{1}{\theta_j}}. \tag{A.3}$$

The first-order condition with respect to $\not{k}_{t+1}$ is:

$$\frac{\lambda_t}{\lambda_{t+1}} = \frac{1 + r_{t+1} - \delta}{\Gamma_*}. \tag{A.4}$$

Finally, the first order condition with respect to λ_t is:

$$q_t c_t + \Gamma_* \not{k}_{t+1} = (1 + r_t - \delta)\not{k}_t + w_t \ell_t + \frac{\pi_t}{\Gamma_t}. \tag{A.5}$$

Using Equation (A.1) and Equation (A.4), I obtain the Euler equation:

$$\left[\frac{c_t - \ell_t^{1+\eta}/(1+\eta)}{c_{t+1} - \ell_{t+1}^{1+\eta}/(1+\eta)}\right]^{-\varphi} = \beta_\Gamma \frac{q_t}{q_{t+1}} \frac{1 + r_{t+1} - \delta}{\Gamma_*}. \tag{A.6}$$

Using Equation (A.1) and Equation (A.3), I obtain:

$$w_{jnt} = q_t \ell_t^\eta \left(\frac{\ell_{jnt}}{\ell_{jt}}\right)^{\frac{1}{\theta_j}}. \tag{A.7}$$

Rewriting Equation (A.5), Equation (A.6), and Equation (A.7) in non-efficient terms:

$$q_t c_t + k_{t+1} = (1 + r_t - \delta)k_t + w_t \ell_t + \pi_t, \tag{A.8}$$

$$\left[\frac{c_t/\Gamma_t - \ell_t^{1+\eta}/(1+\eta)}{c_{t+1}/\Gamma_{t+1} - \ell_{t+1}^{1+\eta}/(1+\eta)}\right]^{-\varphi} = \beta_\Gamma \frac{q_t}{q_{t+1}} \frac{1 + r_{t+1} - \delta}{\Gamma_*}, \tag{A.9}$$

$$w_{jnt} = q_t \Gamma_t \ell_t^\eta \left(\frac{\ell_{jnt}}{\ell_{jt}}\right)^{\frac{1}{\theta_j}}. \tag{A.10}$$

A.2. Proof of Lemma 1

Inverting Equation (A.10), I obtain the firm-level labor supply:

$$\ell_{jnt} = \ell_{jt} \left(\frac{w_{jnt}}{\Gamma_t q_t \ell_t^\eta}\right)^{\theta_j}. \tag{A.11}$$

Define the sectoral wage index:

$$w_{jt} \equiv \left(\int_0^1 w_{jnt}^{1+\theta_j} \, \mathrm{d}n\right)^{\frac{1}{1+\theta_j}}. \tag{A.12}$$

Substituting the firm-level labor supply into the definition of the CES labor aggregator in Equation 5:

$$\begin{aligned}\ell_{jt} &= \left[\int_0^1 \left[\ell_{jt}\left(\frac{w_{jnt}}{\Gamma_t q_t \ell_t^{\eta}}\right)^{\theta_j}\right]^{\frac{1+\theta_j}{\theta_j}} \mathrm{d}n\right]^{\frac{\theta_j}{1+\theta_j}} \\ &= \ell_{jt}\left(\Gamma_t q_t \ell_t^{\eta}\right)^{-\theta_j}\left(\int_0^1 w_{jnt}^{1+\theta_j}\,\mathrm{d}n\right)^{\frac{\theta_j}{1+\theta_j}} \\ &= \ell_{jt}\left(\frac{w_{jt}}{\Gamma_t q_t \ell_t^{\eta}}\right)^{\theta_j}.\end{aligned}$$

This simplifies to:

$$w_{jt} = \Gamma_t q_t \ell_t^{\eta} = w_t, \quad \forall j \in \{C, I\}. \tag{A.13}$$

Invert to obtain aggregate labor supply:

$$\ell_t = \left(\frac{w_t}{\Gamma_t q_t}\right)^{\frac{1}{\eta}} \tag{A.14}$$

Using Equation (A.13), I can rewrite firm-level labor supply and its inverse as:

$$w_{jnt} = w_t\left(\frac{\ell_{jnt}}{\ell_{jt}}\right)^{\frac{1}{\theta_j}} \tag{A.15}$$

$$\ell_{jnt} = \ell_{jt}\left(\frac{w_{jnt}}{w_t}\right)^{\theta_j} \tag{A.16}$$

A.3. Proof of Proposition 1

In each period $t \geq 0$, a firm $n \in [0,1]$ in sector j chooses capital, k_{jnt}, and wage, w_{jnt}, internalizing its labor supply $\ell_{jnt} = f(w_{jnt})$, to maximize its profit function. Sector C firms produce differentiated varieties and face a downward-sloping output demand (see Equation 2), while sector I firms produce a homogeneous good in a perfectly competitive market. The firm n in sector j solves:

$$\max_{w_{jnt}, k_{jnt}} \pi_{jnt} = q_{nt} y_{jnt} - w_{jnt}\ell_{jnt} - r_t k_{jnt}$$

$$\text{s.t. } y_{jnt} = k_{jnt}^{\alpha_j}\left(z_{jt}\ell_{jnt}\right)^{1-\alpha_j}$$

where $q_{nt} \equiv P_{jnt}/P_{It}$ is the relative price of the variety produced by firm n (for sector C) or the homogeneous good (for sector I, with $q_{nt} = q_t = 1$).

■ Deriving Markups.

For sector C firms, the inverse demand from Equation 2 gives $q_{nt} = q_t c_t^{\frac{1}{\sigma_c}} c_{nt}^{-\frac{1}{\sigma_c}}$. The marginal revenue factor satisfies:

$$1 + \frac{\partial q_{nt}}{\partial y_{Cnt}} \frac{y_{Cnt}}{q_{nt}} = 1 - \frac{1}{\sigma_c} = \frac{\sigma_c - 1}{\sigma_c} \equiv \frac{1}{1 + \mu_C}, \tag{A.17}$$

where the **markup** in sector C is defined as $\mu_C \equiv 1/(\sigma_c - 1) \geq 0$. For sector I, $q_{nt} = 1$ and there is no markup: $\mu_I = 0$, so $1/(1 + \mu_I) = 1$.

■ Deriving Markdowns.

The first-order condition of the firm's problem with respect to w_{jnt} is:

$$\frac{1}{1 + \mu_j} q_{nt} k_{jnt}^{\alpha_j} z_{jt}^{1-\alpha_j} (1 - \alpha_j) \ell_{jnt}^{-\alpha_j} \frac{\partial \ell_{jnt}}{\partial w_{jnt}} = \ell_{jnt} + w_{jnt} \frac{\partial \ell_{jnt}}{\partial w_{jnt}}. \tag{A.18}$$

Defining the labor-supply elasticity as $\theta_j \equiv \left(\partial \ell_{jnt}/\partial w_{jnt}\right)\left(w_{jnt}/\ell_{jnt}\right)$, the first-order condition simplifies to:

$$\frac{1}{1 + \mu_j} q_{nt} \left(\frac{k_{jnt}}{\ell_{jnt}}\right)^{\alpha_j} z_{jt}^{1-\alpha_j} (1 - \alpha_j) = w_{jnt} \frac{1 + \theta_j}{\theta_j}.$$

Defining the **markdown** in sector j as:

$$\nu_j \equiv 1 - \frac{\theta_j}{1 + \theta_j} = \frac{1}{1 + \theta_j} \in (0, 1), \tag{A.19}$$

the wage of firm n in sector j satisfies:

$$w_{jnt} = \frac{1 - \nu_j}{1 + \mu_j} q_{nt} \left(\frac{k_{jnt}}{\ell_{jnt}}\right)^{\alpha_j} z_{jt}^{1-\alpha_j} (1 - \alpha_j). \tag{A.20}$$

The first-order condition with respect to k_{jnt} is:

$$\frac{1}{1 + \mu_j} q_{nt} \left(\frac{k_{jnt}}{\ell_{jnt}}\right)^{\alpha_j - 1} z_{jt}^{1-\alpha_j} \alpha_j = r_t. \tag{A.21}$$

■ **Labor Demand Aggregation.**

Define the firm input shares in sector j as $s_{\ell njt} \equiv \ell_{jnt}/\ell_{jt}$, and $s_{knjt} \equiv k_{jnt}/k_{jt}$. From the household's problem (Equation (A.15)), the labor supply to firm n is:

$$w_{jnt} = w_{jt}\left(s_{\ell njt}\right)^{\frac{1}{\theta_j}}. \tag{A.22}$$

Substituting into Equation (A.18) and rearranging, the firm's wage FOC becomes:

$$\frac{q_{nt}}{1+\mu_j}\left(\frac{k_{jt}}{\ell_{jt}}\right)^{\alpha_j} z_{jt}^{1-\alpha_j}\left(1-\alpha_j\right) = w_{jt}\frac{1+\theta_j}{\theta_j}\left(s_{\ell njt}\right)^{\frac{1}{\theta_j}}\left(\frac{s_{\ell njt}}{s_{knjt}}\right)^{\alpha_j}. \tag{A.23}$$

Integrating over $n \in [0,1]$:

$$\frac{\overline{q}_t}{1+\mu_j}\left(\frac{k_{jt}}{\ell_{jt}}\right)^{\alpha_j} z_{jt}^{1-\alpha_j}\left(1-\alpha_j\right) = w_{jt}\frac{1+\theta_j}{\theta_j}\int_0^1 \left(s_{\ell njt}\right)^{\frac{1}{\theta_j}+\alpha_j}\left(s_{knjt}\right)^{-\alpha_j}\mathrm{d}n, \tag{A.24}$$

where $\overline{q}_t \equiv \int_0^1 q_{nt}\,\mathrm{d}n$ is the average relative price in the sector.

Define the **share-weighted wage dispersion factor**:

$$\Omega_{jt} \equiv \int_0^1 \left(s_{\ell njt}\right)^{\frac{1}{\theta_j}+\alpha_j}\left(s_{knjt}\right)^{-\alpha_j}\mathrm{d}n. \tag{A.25}$$

Under symmetric equilibrium, all firms in sector j are identical, so $s_{\ell njt} = s_{knjt} = 1$ for all $n \in [0,1]$, which implies $\Omega_{jt} = 1$ and $\overline{q}_t = q_t$. The sector markdown reduces to its firm-level counterpart: $\nu_{jt} = \nu_j = 1/\left(1+\theta_j\right)$.

The sector labor demand then simplifies to:

$$\frac{q_t}{1+\mu_j}\left(\frac{k_{jt}}{\ell_{jt}}\right)^{\alpha_j} z_{jt}^{1-\alpha_j}\left(1-\alpha_j\right) = \frac{1+\theta_j}{\theta_j}w_{jt}, \tag{A.26}$$

which can be rewritten as:

$$q_t\frac{1-\nu_j}{1+\mu_j}\left(\frac{k_{jt}}{\ell_{jt}}\right)^{\alpha_j} z_{jt}^{1-\alpha_j}\left(1-\alpha_j\right) = w_{jt}. \tag{A.27}$$

■ **Capital Demand Aggregation.**

The firm's capital demand FOC can be written using shares as:

$$\frac{q_{nt}}{1+\mu_j}\left(\frac{s_{\ell njt}\ell_{jt}}{s_{knjt}k_{jt}}\right)^{1-\alpha_j} z_{jt}^{1-\alpha_j}\alpha_j = r_t. \tag{A.28}$$

Under symmetric equilibrium, this reduces to:

$$\frac{q_t}{1+\mu_j}\left(\frac{\ell_{jt}}{k_{jt}}\right)^{1-\alpha_j} z_{jt}^{1-\alpha_j}\alpha_j = r_t. \tag{A.29}$$

These symmetric-equilibrium input demand conditions are precisely those implied by the representative-firm problem in Equation 17, which concludes the proof.

A.4. Solution of the Representative Firm's Problem in a Symmetric Equilibrium

In each period $t \geq 0$, the representative firm in sector $j \in \{C, I\}$ solves:

$$\begin{aligned} \max_{k_{jt},\ell_{jt}} \quad & \pi_{jt} = \frac{q_t}{1+\mu_j} y_{jt} - \frac{w_t}{1-\nu_j}\ell_{jt} - r_t k_{jt} \\ \text{s.t.} \quad & y_{jt} = \left(k_{jt}\right)^{\alpha_j}\left(z_{jt}\ell_{jt}\right)^{1-\alpha_j} \end{aligned}$$

given the aggregate relative price of consumption $q_t \equiv P_{Ct}/P_{It}$, the real returns to inputs $w_t \equiv W_t/P_{It}$ and $r_t \equiv R_t/P_{It}$, the technological levels z_{jt}, markup μ_j (with $\mu_C = 1/(\sigma_c - 1)$, $\mu_I = 0$), and markdown $\nu_j = 1/\left(1+\theta_j\right)$.

Then, the first-order conditions of this problem are:

$$\frac{q_t}{1+\mu}\alpha_C\left(\frac{z_{Ct}\ell_{Ct}}{k_{Ct}}\right)^{1-\alpha_C} = r_t \tag{A.30}$$

$$\frac{q_t}{1+\mu}(1-\alpha_C)z_{Ct}\left(\frac{k_{Ct}}{z_{Ct}\ell_{Ct}}\right)^{\alpha_C}(1-\nu_C) = w_t \tag{A.31}$$

$$\alpha_I\left(\frac{z_{It}\ell_{It}}{k_{It}}\right)^{1-\alpha_I} = r_t \tag{A.32}$$

$$(1-\alpha_I)z_{It}\left(\frac{k_{It}}{z_{It}\ell_{It}}\right)^{\alpha_I}(1-\nu_I) = w_t \tag{A.33}$$

$$y_{Ct} = k_{Ct}^{\alpha_C}(z_{Ct}\ell_{Ct})^{1-\alpha_C} \tag{A.34}$$

$$y_{It} = k_{It}^{\alpha_I}(z_{It}\ell_{It})^{1-\alpha_I} \tag{A.35}$$

From Equation (A.30) and Equation (A.32), I also conclude that:

$$\frac{q_t}{1+\mu} = \frac{\alpha_I}{\alpha_C}\left(\frac{z_{It}\ell_{It}}{k_{It}}\right)^{1-\alpha_I}\left(\frac{k_{Ct}}{z_{Ct}\ell_{Ct}}\right)^{1-\alpha_C}, \tag{A.36}$$

and from equating Equation (A.31) and Equation (A.33) (using Equation (A.36) to eliminate q_t):

$$\frac{k_{Ct}}{\ell_{Ct}} = \frac{\alpha_C}{1-\alpha_C}\frac{1-\alpha_I}{\alpha_I}\frac{1-\nu_I}{1-\nu_C}\frac{k_{It}}{\ell_{It}}. \tag{A.37}$$

A.5. Proof of Proposition 2

Define $\tilde{c}_t \equiv c_t/\Gamma_t - \ell_t^{1+\eta}/(1+\eta)$. Then, an equilibrium is defined by the following conditions:

$$\frac{q_t}{1+\mu}\Gamma_t\left(\frac{1-s_{kt}}{1-s_{\ell t}}\tilde{k}_t\right)^{\alpha_C}(1-\alpha_C) = \frac{w_t}{1-\nu_C} \tag{A.38}$$

$$\left(\frac{s_{kt}}{s_{\ell t}}\tilde{k}_t\right)^{\alpha_I} z_{It}(1-\alpha_I) = \frac{w_t}{1-\nu_I} \tag{A.39}$$

$$\frac{q_t}{1+\mu}\left(\frac{1-s_{\ell t}}{1-s_{kt}}\frac{1}{\tilde{k}_t}\right)^{1-\alpha_C}\left(\frac{z_{Ct}}{z_{It}}\right)^{1-\alpha_C}\alpha_C = r_t \tag{A.40}$$

$$\left(\frac{s_{\ell t}}{s_{kt}}\frac{1}{\tilde{k}_t}\right)^{1-\alpha_I}\alpha_I = r_t \tag{A.41}$$

$$\left[\frac{\tilde{c}_{t+1}}{\tilde{c}_t}\right]^{\varphi} = \beta_\Gamma\frac{q_t}{q_{t+1}}\frac{1+r_{t+1}-\delta}{\Gamma_*} \tag{A.42}$$

$$q_t\frac{c_t}{\Gamma_t} + \frac{\Gamma_{t+1}}{\Gamma_t}\mathcal{k}_{t+1} = (1-\delta)\mathcal{k}_t + q_t\frac{y_{Ct}}{\Gamma_t} + \frac{y_{It}}{\Gamma_t} \tag{A.43}$$

$$w_t = \Gamma_t q_t \ell_t^{\eta} \tag{A.44}$$

$$y_{Ct} = c_t \tag{A.45}$$

$$y_{It} = i_t \tag{A.46}$$

$$y_{Ct} = ((1-s_{kt})k_t)^{\alpha_C}(z_{Ct}(1-s_{\ell t})\ell_t)^{1-\alpha_C} \tag{A.47}$$

$$y_{It} = (s_{kt}k_t)^{\alpha_I}(z_{It}s_{\ell t}\ell_t)^{1-\alpha_I} \tag{A.48}$$

Using Equation (A.41) to solve for r_t, Equation (A.44) to solve for w_t, Equation (A.46) to solve for i_t, Equation (A.47) to solve for y_{Ct}, and Equation (A.48) to solve for y_{It}, and using the definition of $\tilde{c}_t$, I obtain:

$$\left(\frac{1-s_{kt}}{1-s_{\ell t}}\tilde{k}_t\right)^{\alpha_C}(1-\alpha_C) = (1+\mu)\frac{\ell_t^{\eta}}{1-\nu_C} \tag{A.49}$$

$$\left(\frac{s_{kt}}{s_{\ell t}}\tilde{k}_t\right)^{\alpha_I} z_{It}(1-\alpha_I) = \Gamma_t q_t \frac{\ell_t^{\eta}}{1-\nu_I} \tag{A.50}$$

$$\frac{q_t}{1+\mu} = \frac{\alpha_I}{\alpha_C}\left(\frac{z_{It}}{z_{Ct}}\right)^{1-\alpha_C}\left(\frac{s_{\ell t}}{s_{kt}}\right)^{1-\alpha_I}\left(\frac{1-s_{kt}}{1-s_{\ell t}}\right)^{1-\alpha_C}\tilde{k}_t^{\alpha_I-\alpha_C} \tag{A.51}$$

$$\left[\frac{\tilde{c}_{t+1}}{\tilde{c}_t}\right]^{\varphi} = \beta_\Gamma \frac{q_t}{q_{t+1}} \frac{1-\delta+\left(\frac{s_{\ell t+1}}{s_{kt+1}}\frac{1}{\tilde{k}_{t+1}}\right)^{1-\alpha_I}\alpha_I}{\Gamma_*} \tag{A.52}$$

$$\tilde{k}_{t+1}(1+g_{z_I})\frac{\ell_{t+1}}{\ell_t} = (1-\delta)\tilde{k}_t + \left(\frac{s_{kt}}{s_{\ell t}}\tilde{k}_t\right)^{\alpha_I} s_{\ell t} \tag{A.53}$$

$$c_t = \left(\frac{1-s_{kt}}{1-s_{\ell t}}\tilde{k}_t\right)^{\alpha_C}\Gamma_t(1-s_{\ell t})\ell_t \tag{A.54}$$

From Equation (A.49), I can determine ℓ_t:

$$\ell_t = \left[(1-\alpha_C)\frac{1-\nu_C}{1+\mu}\left(\frac{1-s_{kt}}{1-s_{\ell t}}\tilde{k}_t\right)^{\alpha_C}\right]^{\frac{1}{\eta}} \tag{A.55}$$

where I used $w_t = \Gamma_t q_t \ell_t^{\eta}$ from Equation (A.44) and $\mu \equiv \mu_C$.

Dividing Equation (A.50) by Equation (A.49), I obtain:

$$\left(\frac{s_{kt}}{s_{\ell t}}\tilde{k}_t\right)^{\alpha_I}\left(\frac{1-s_{\ell t}}{1-s_{kt}}\frac{1}{\tilde{k}_t}\right)^{\alpha_C} z_{It}\frac{1-\alpha_I}{1-\alpha_C} = \Gamma_t q_t \frac{1-\nu_C}{(1+\mu)(1-\nu_I)}$$

Substituting now $q_t/(1+\mu)$ from Equation (A.51), I can determine $s_{\ell t}$:

$$s_{\ell t} = \left[\frac{\alpha_I}{1-\alpha_I}\frac{1-\alpha_C}{\alpha_C}\frac{1-\nu_C}{1-\nu_I}\frac{1-s_{kt}}{s_{kt}} + 1\right]^{-1} \tag{A.56}$$

From Equation (A.51), the growth rate of the relative price is given by:

$$\frac{q_t}{q_{t+1}} = \left(\frac{s_{\ell t}}{s_{\ell t+1}}\frac{s_{kt+1}}{s_{kt}}\right)^{1-\alpha_I}\left(\frac{1-s_{kt}}{1-s_{\ell t}}\frac{1-s_{\ell t+1}}{1-s_{kt+1}}\right)^{1-\alpha_C}\left(\frac{1+g_{z_C}}{1+g_{z_I}}\right)^{1-\alpha_C}\left(\frac{\tilde{k}_t}{\tilde{k}_{t+1}}\right)^{\alpha_I-\alpha_C}$$

Substituting the previous result into Equation (A.52):

$$\left[\frac{\tilde{c}_{t+1}}{\tilde{c}_t}\right]^{\varphi} = \left(\frac{s_{\ell t}}{s_{\ell t+1}}\frac{s_{kt+1}}{s_{kt}}\right)^{1-\alpha_I}\left(\frac{1-s_{kt}}{1-s_{\ell t}}\frac{1-s_{\ell t+1}}{1-s_{kt+1}}\right)^{1-\alpha_C}\left(\frac{\tilde{k}_t}{\tilde{k}_{t+1}}\right)^{\alpha_I-\alpha_C} \times$$
$$\times\,\beta_\Gamma\left(\frac{1+g_{z_C}}{1+g_{z_I}}\right)^{1-\alpha_C}\left(1-\delta+\left(\frac{s_{\ell t+1}}{s_{kt+1}}\frac{1}{\tilde{k}_{t+1}}\right)^{1-\alpha_I}\alpha_I\right)$$

Plugging in the definition of $\tilde{c}_t$, I can determine s_{kt}:

$$\left[\frac{\left(\frac{1-s_{kt+1}}{1-s_{\ell t+1}}\tilde{k}_{t+1}\right)^{\alpha_C}(1-s_{\ell t+1}) - \Gamma_{t+1}\frac{\ell_{t+1}^{\eta}}{1+\eta}}{\left(\frac{1-s_{kt}}{1-s_{\ell t}}\tilde{k}_t\right)^{\alpha_C}(1-s_{\ell t}) - \Gamma_t\frac{\ell_t^{\eta}}{1+\eta}}\frac{\Gamma_t}{\Gamma_{t+1}}\frac{\ell_{t+1}}{\ell_t}\right]^{\varphi} =$$
$$= \left(\frac{s_{\ell t}}{s_{\ell t+1}}\frac{s_{kt+1}}{s_{kt}}\right)^{1-\alpha_I}\left(\frac{1-s_{kt}}{1-s_{\ell t}}\frac{1-s_{\ell t+1}}{1-s_{kt+1}}\right)^{1-\alpha_C}\left(\frac{\tilde{k}_t}{\tilde{k}_{t+1}}\right)^{\alpha_I-\alpha_C} \times \tag{A.57}$$
$$\times\,\beta_\Gamma\left(\frac{1+g_{z_C}}{1+g_{z_I}}\right)^{1-\alpha_C}\left(1-\delta+\left(\frac{s_{\ell t+1}}{s_{kt+1}}\frac{1}{\tilde{k}_{t+1}}\right)^{1-\alpha_I}\alpha_I\right)$$

Finally, Equation (A.53) determines $\tilde{k}_{t+1}$:

$$\tilde{k}_{t+1}(1+g_{z_I})\frac{\ell_{t+1}}{\ell_t} = (1-\delta)\tilde{k}_t + \left(\frac{s_{kt}}{s_{\ell t}}\tilde{k}_t\right)^{\alpha_I}s_{\ell t} \tag{A.58}$$

The other variables can be determined from the following second block of equations:

$$k_t = z_{It}\ell_t\tilde{k}_t, \tag{A.59}$$

$$k_{It} = s_{kt}k_t, \tag{A.60}$$

$$k_{Ct} = (1-s_{kt})k_t, \tag{A.61}$$

$$\ell_{It} = s_{\ell t}\ell_t, \tag{A.62}$$

$$\ell_{Ct} = (1-s_{\ell t})\ell_t, \tag{A.63}$$

$$y_{Ct} = \left(\frac{1-s_{kt}}{1-s_{\ell t}}\tilde{k}_t\right)^{\alpha_C}\left(\frac{z_{It}}{z_{Ct}}\right)^{\alpha_C}z_{Ct}(1-s_{\ell t})\ell_t, \tag{A.64}$$

$$y_{It} = \left(\frac{s_{kt}}{s_{\ell t}}\tilde{k}_t\right)^{\alpha_I} z_{It} s_{\ell t} \ell_t, \tag{A.65}$$

$$c_t = y_{Ct}, \tag{A.66}$$

$$i_t = y_{It}, \tag{A.67}$$

$$r_t = \alpha_I \left(\frac{s_{kt}}{s_{\ell t}}\tilde{k}_t\right)^{\alpha_I - 1}, \tag{A.68}$$

$$w_t = (1-\nu_I)(1-\alpha_I) z_{It} \left(\frac{s_{kt}}{s_{\ell t}}\tilde{k}_t\right)^{\alpha_I}, \tag{A.69}$$

$$q_t = (1+\mu)\frac{\alpha_I}{\alpha_C}\left(\frac{z_{It}}{z_{Ct}}\right)^{1-\alpha_C} \tilde{k}_t^{\alpha_I - \alpha_C} \left(\frac{s_{\ell t}}{s_{kt}}\right)^{1-\alpha_I} \left(\frac{1-s_{kt}}{1-s_{\ell t}}\right)^{1-\alpha_C}. \tag{A.70}$$

A.6. First Best

In each period $t \geq 0$, a benevolent planner solves:

$$\begin{aligned} \max_{\{c_t, k_{t+1}, \ell_t, s_{kt}, s_{\ell t}\}_{t=0}^{+\infty}} \quad & \mathcal{U} = \sum_{t=0}^{+\infty} \beta_\Gamma^t \frac{\left(c_t - \frac{\ell_t^{1+\eta}}{1+\eta}\right)^{1-\varphi}}{1-\varphi} \\ \text{s.t. } & c_t \leq y_{Ct} \Gamma_t^{-1} \\ & k_{t+1}\Gamma_* \leq k_t(1-\delta) + y_{It}\Gamma_t^{-1} \end{aligned}$$

Recall that $y_{Ct} \equiv ((1-s_{kt})k_t)^{\alpha_C}(z_{Ct}(1-s_{\ell t})\ell_t)^{1-\alpha_C}$ and $y_{It} \equiv (s_{kt}k_t)^{\alpha_I}(z_{It}s_{\ell t}\ell_t)^{1-\alpha_I}$ denote the sectoral outputs, and let λ_{at} and λ_{bt} be the multipliers on the consumption and capital-accumulation constraints, respectively.

■ **First-Order Conditions.** The first-order condition with respect to c_t is:

$$\lambda_{at} = \beta_\Gamma^t \left(c_t - \frac{\ell_t^{1+\eta}}{1+\eta}\right)^{-\varphi} \tag{A.71}$$

The first-order condition with respect to ℓ_t is:

$$\beta_\Gamma^t \left(c_t - \frac{\ell_t^{1+\eta}}{1+\eta}\right)^{-\varphi} \ell_t^\eta = \lambda_{at}(1-\alpha_C)\frac{y_{Ct}}{\ell_t \Gamma_t} + \lambda_{bt}(1-\alpha_I)\frac{y_{It}}{\ell_t \Gamma_t} \tag{A.72}$$

The first-order condition with respect to k_{t+1} is:

$$\lambda_{bt}\Gamma_* = \lambda_{at+1}\alpha_C \frac{y_{C,t+1}}{k_{t+1}\Gamma_{t+1}} + \lambda_{bt+1}\left(1 - \delta + \alpha_I \frac{y_{I,t+1}}{k_{t+1}\Gamma_{t+1}}\right) \tag{A.73}$$

The first-order condition with respect to s_{kt} is:

$$\frac{\lambda_{bt}}{\lambda_{at}} = \frac{\alpha_C}{\alpha_I}\frac{s_{kt}}{1 - s_{kt}}\frac{y_{Ct}}{y_{It}} \tag{A.74}$$

The first-order condition with respect to $s_{\ell t}$ is:

$$\frac{\lambda_{bt}}{\lambda_{at}} = \frac{1 - \alpha_C}{1 - \alpha_I}\frac{s_{\ell t}}{1 - s_{\ell t}}\frac{y_{Ct}}{y_{It}} \tag{A.75}$$

The first-order condition with respect to λ_{at} is:

$$c_t = y_{Ct}\Gamma_t^{-1} \tag{A.76}$$

The first-order condition with respect to λ_{bt} is:

$$k_{t+1}\Gamma_* = k_t(1 - \delta) + y_{It}\Gamma_t^{-1} \tag{A.77}$$

■ **Efficient Allocation of Inputs Condition.** Combining Equation (A.74) and Equation (A.75), I obtain the efficient allocation condition:

$$s_{\ell t} = \left[\frac{\alpha_I}{1 - \alpha_I}\frac{1 - \alpha_C}{\alpha_C}\frac{1 - s_{kt}}{s_{kt}} + 1\right]^{-1} \tag{A.78}$$

■ **Euler Equation.** Combining Equation (A.73) and Equation (A.74):

$$\lambda_{bt}\Gamma_* = \lambda_{bt+1}\left[1 - \delta + \alpha_I \frac{y_{It+1}}{s_{kt+1}k_{t+1}\Gamma_{t+1}}\right] \tag{A.79}$$

Using Equation (A.74) to substitute λ_{bt} and λ_{bt+1} and then Equation (A.71) to substitute λ_{at}:

$$\beta_\Gamma^t\left(c_t - \frac{\ell_t^{1+\eta}}{1+\eta}\right)^{-\varphi}\frac{\alpha_C}{\alpha_I}\frac{s_{kt}}{1 - s_{kt}}\frac{y_{Ct}}{y_{It}}\Gamma_* =$$

$$= \beta_\Gamma^{t+1}\left(c_{t+1} - \frac{\ell_{t+1}^{1+\eta}}{1+\eta}\right)^{-\varphi}\frac{\alpha_C}{\alpha_I}\frac{s_{kt+1}}{1 - s_{kt+1}}\frac{y_{Ct+1}}{y_{It+1}}\left[1 - \delta + \alpha_I \frac{y_{It+1}}{s_{kt+1}k_{t+1}\Gamma_{t+1}}\right]$$

Rearranging:

$$\left[\frac{\mathcal{C}_{t+1}-\frac{\ell_{t+1}^{1+\eta}}{1+\eta}}{\mathcal{C}_t-\frac{\ell_t^{1+\eta}}{1+\eta}}\right]^{\varphi}=\frac{1}{\beta_\Gamma}\frac{s_{k,t+1}}{1-s_{k,t+1}}\frac{y_{C,t+1}}{y_{I,t+1}}\frac{1-s_{kt}}{s_{kt}}\frac{y_{It}}{y_{Ct}}\Gamma_*\left[1-\delta+\alpha_I\frac{y_{It+1}}{s_{kt+1}k_{t+1}\Gamma_{t+1}}\right]^{-1} \quad \text{(A.80)}$$

Notice that, in the first best, the planner's shadow relative price of consumption goods is $q_t^* \equiv \lambda_{at}/\lambda_{bt}$ (λ_{at} values consumption goods and λ_{bt} values investment goods) and the shadow rental rate of capital is $r_t^* \equiv \alpha_I y_{It}/(s_{kt}k_t\Gamma_t)$. Both correspond to their decentralized equilibrium counterparts in Proposition 2 with $\nu_j = 0$ and $\mu = 0$.

Using $\lambda_{bt} = \lambda_{at}/q_t$ in Equation (A.79):

$$\frac{\lambda_{at}}{q_t}\Gamma_* = \frac{\lambda_{at+1}}{q_{t+1}}[1-\delta+r_{t+1}].$$

Substituting $\lambda_{at} = \beta_\Gamma^t\left(c_t/\Gamma_t - \ell_t^{1+\eta}/(1+\eta)\right)^{-\varphi}$ and dividing:

$$\left[\frac{c_{t+1}/\Gamma_{t+1}-\ell_{t+1}^{1+\eta}/(1+\eta)}{c_t/\Gamma_t-\ell_t^{1+\eta}/(1+\eta)}\right]^{\varphi}=\beta_\Gamma\frac{q_t}{q_{t+1}}\frac{1-\delta+r_{t+1}}{\Gamma_*},$$

which is the Euler equation of the decentralized economy (see Appendix A.1) evaluated at the first-best allocation.

■ **Labor Condition.** From Equation (A.72), substitute λ_{at} using Equation (A.71). Then, use Equation (A.75), which implies $\lambda_{bt}(1-\alpha_I)y_{It} = \lambda_{at}(1-\alpha_C)y_{Ct}s_{\ell t}/(1-s_{\ell t})$:

$$\ell_t = \left[(1-\alpha_C)\left(\frac{1-s_{kt}}{1-s_{\ell t}}\tilde{k}_t\right)^{\alpha_C}\right]^{\frac{1}{\eta}} \quad \text{(A.81)}$$

where $\tilde{k}_t \equiv k_t/(z_{It}\ell_t)$ and the last step uses $y_{Ct}/((1-s_{\ell t})\ell_t\Gamma_t) = \left((1-s_{kt})/(1-s_{\ell t})\tilde{k}_t\right)^{\alpha_C}$. This is Equation 22 with $\nu_C = 0$ and $\mu = 0$.

■ **Capital Condition.** From the binding capital-accumulation constraint (Equation (A.77)), using $k_t = z_{It}\ell_t\tilde{k}_t$ and $y_{It} = \left(s_{kt}/s_{\ell t}\tilde{k}_t\right)^{\alpha_I} z_{It}s_{\ell t}\ell_t$:

$$\tilde{k}_{t+1}\left(1+g_{z_I}\right)\frac{\ell_{t+1}}{\ell_t} = (1-\delta)\tilde{k}_t + \left(\frac{s_{kt}}{s_{\ell t}}\tilde{k}_t\right)^{\alpha_I} s_{\ell t}, \quad \text{(A.82)}$$

which is identical to Equation 25.

■ **Condition for the Share of Capital in Sector I.** Using the definition of $\tilde{k}_t$ and q_t^*, Equation (A.80) can be rewritten as Equation 24, which determines s_{kt} given the other variables.

A.7. Proof of Corollary 1

Aggregate output is given by:

$$y_t \equiv q_t y_{Ct} + y_{It} = \left[1 + (1+\mu)\frac{\alpha_I}{\alpha_C}\frac{1-s_{kt}}{s_{kt}}\right]\left(\frac{s_{kt}}{s_{\ell t}}\tilde{k}_t\right)^{\alpha_I} z_{It} s_{\ell t} \ell_t \tag{A.83}$$

where I used Equation (A.51) to substitute $q_t = (1+\mu)\alpha_I/\alpha_C \cdot (z_{It}/z_{Ct})^{1-\alpha_C} \cdot (s_{\ell t}/s_{kt})^{1-\alpha_I} \cdot ((1-s_{kt})/(1-s_{\ell t}))^{1-\alpha_C} \cdot \tilde{k}_t^{\alpha_I-\alpha_C}$ in the first term and then simplified using Equation (A.48).

Labor income is given by:

$$M_{\ell t} \equiv w_t \ell_t = \Gamma_t(1+\mu)\frac{\alpha_I}{\alpha_C}\left(\frac{z_{It}}{z_{Ct}}\right)^{1-\alpha_C}\left(\frac{s_{\ell t}}{s_{kt}}\right)^{1-\alpha_I}\left(\frac{1-s_{kt}}{1-s_{\ell t}}\right)^{1-\alpha_C}\tilde{k}_t^{\alpha_I-\alpha_C}\ell_t^{1+\eta} \tag{A.84}$$

where I substituted $w_t = \Gamma_t q_t \ell_t^{\eta}$ and then q_t from Equation (A.51).

Then, the labor share of income is given by:

$$m_{\ell t} \equiv \frac{M_{\ell t}}{y_t} = (1+\mu)\frac{\alpha_I}{\alpha_C}\frac{\Gamma_t}{z_{It}s_{kt}}\left(\frac{1-s_{kt}}{1-s_{\ell t}}\right)^{1-\alpha_C}\tilde{k}_t^{-\alpha_C}\ell_t^{\eta}\frac{\alpha_C s_{kt}}{(1+\mu)\alpha_I(1-s_{kt})+\alpha_C s_{kt}}$$

Substituting ℓ_t from Equation (A.55):

$$m_{\ell t} = \frac{1}{z_{It}}\left(\frac{z_{It}}{z_{Ct}}\right)^{1-\alpha_C}\left(\frac{1-s_{kt}}{1-s_{\ell t}}\right)^{1-\alpha_C}\frac{\tilde{k}_t^{-\alpha_C}\alpha_I\Gamma_t\ell_t^{\eta}}{(1+\mu)\alpha_I\frac{1-s_{kt}}{s_{kt}}+\alpha_C}$$

Substituting ℓ_t from Equation (A.55) and simplifying (using $\ell_t^{\eta} = (1-\alpha_C)(1-\nu_C)/(1+\mu)\cdot\left((1-s_{kt})/(1-s_{\ell t})\tilde{k}_t\right)^{\alpha_C}$):

$$m_{\ell t} = \frac{1}{z_{It}}\left(\frac{z_{It}}{z_{Ct}}\right)^{1-\alpha_C}\frac{1-s_{kt}}{1-s_{\ell t}}\frac{1-\nu_C}{1+\mu}\frac{\alpha_I\Gamma_t(1-\alpha_C)}{(1+\mu)\alpha_I\frac{1-s_{kt}}{s_{kt}}+\alpha_C}$$

Since $\Gamma_t \equiv z_{It}^{\alpha_C} z_{Ct}^{1-\alpha_C}$, it follows that $\frac{\Gamma_t}{z_{It}}\cdot\left(\frac{z_{It}}{z_{Ct}}\right)^{1-\alpha_C} = 1$. Rearranging:

$$m_{\ell t} = \frac{1-\nu_C}{1+\mu}\frac{1-s_{kt}}{1-s_{\ell t}}\frac{\alpha_I(1-\alpha_C)\alpha_C s_{kt}}{\alpha_C((1+\mu)\alpha_I(1-s_{kt})+\alpha_C s_{kt})}$$

Substituting $1 - s_{\ell t}$ from Equation (A.56) gives:

$$\begin{aligned} 1 - s_{\ell t} &= 1 - \left[\frac{\alpha_I}{1-\alpha_I} \frac{1-\alpha_C}{\alpha_C} \frac{1-\nu_C}{1-\nu_I} \frac{1-s_{kt}}{s_{kt}} + 1 \right]^{-1} \\ &= \frac{\frac{\alpha_I}{1-\alpha_I} \frac{1-\alpha_C}{\alpha_C} \frac{1-\nu_C}{1-\nu_I} \frac{1-s_{kt}}{s_{kt}}}{\frac{\alpha_I}{1-\alpha_I} \frac{1-\alpha_C}{\alpha_C} \frac{1-\nu_C}{1-\nu_I} \frac{1-s_{kt}}{s_{kt}} + 1} \\ &= \frac{1-\nu_C}{1-\nu_I} \frac{\alpha_I}{1-\alpha_I} \frac{1-\alpha_C}{\alpha_C} \frac{1-s_{kt}}{s_{kt}} s_{\ell t} \end{aligned}$$

where the last step uses the fact that the denominator equals $1/s_{\ell t}$ by the definition of $s_{\ell t}$ in Equation (A.56). So the labor share simplifies to:

$$m_{\ell t} = \frac{\alpha_C(1-\alpha_I)}{\alpha_I + (\alpha_C - \alpha_I)s_{kt}} \frac{1-\nu_I}{1+\mu} \frac{s_{kt}}{s_{\ell t}}.$$

Finally, substituting $s_{kt}/s_{\ell t} = A(1-s_{kt}) + s_{kt}$ where $A \equiv \alpha_I/(1-\alpha_I) \cdot (1-\alpha_C)/\alpha_C \cdot (1-\nu_C)/(1-\nu_I)$ from Equation (A.56):

$$m_{\ell t} = \frac{\alpha_I(1-\alpha_C)}{\alpha_I + (\alpha_C - \alpha_I)s_{kt}} \frac{1-\nu_C}{1+\mu}(1-s_{kt}) + \frac{\alpha_C(1-\alpha_I)}{\alpha_I + (\alpha_C - \alpha_I)s_{kt}} \frac{1-\nu_I}{1+\mu} s_{kt}. \quad \text{(A.85)}$$

A.8. Proof of Lemma 2

Write Equation (A.56) as:

$$s_{\ell t} = f(s_{kt}) \equiv \left[A \frac{1-s_{kt}}{s_{kt}} + 1 \right]^{-1}, \quad A \equiv \frac{\alpha_I}{1-\alpha_I} \frac{1-\alpha_C}{\alpha_C} \frac{1-\nu_C}{1-\nu_I}.$$

For $s_{kt} \in (0,1)$:

- The term $(1-s_{kt})/s_{kt} \in (0, +\infty)$
- Since $A > 0$ for $\nu_C, \nu_I \in (0,1)$, the bracketed expression is greater than 1
- Therefore, $s_{\ell t} \in (0,1)$

Now, define $g(s_{kt}) \equiv A(1-s_{kt})/s_{kt} + 1$. Then $f(s_{kt}) = g(s_{kt})^{-1}$. The derivative of g with respect to s_{kt} is:

$$\frac{\mathrm{d}g}{\mathrm{d}s_{kt}} = A \frac{\mathrm{d}}{\mathrm{d}s_{kt}} \left[\frac{1-s_{kt}}{s_{kt}} \right] = A \frac{\mathrm{d}}{\mathrm{d}s_{kt}} \left[\frac{1}{s_{kt}} - 1 \right] = -A \frac{1}{s_{kt}^2} < 0.$$

Since g is strictly decreasing, by the chain rule:

$$\frac{\mathrm{d}f}{\mathrm{d}s_{kt}} = \frac{\mathrm{d}}{\mathrm{d}s_{kt}}\left[\frac{1}{g(s_{kt})}\right] = -\frac{1}{g(s_{kt})^2}\frac{\mathrm{d}g}{\mathrm{d}s_{kt}} = -\frac{1}{g(s_{kt})^2}\left(-A\frac{1}{s_{kt}^2}\right) = \frac{A}{s_{kt}^2 g(s_{kt})^2} > 0.$$

Since $A > 0$, $s_{kt} > 0$, and $g(s_{kt}) > 0$ for all $s_{kt} \in (0,1)$, it follows that $\mathrm{d}f/\mathrm{d}s_{kt} > 0$. Therefore, f is strictly increasing in s_{kt}, establishing a one-to-one mapping.

Note that $\lim_{s_{kt}\to 0^+} f(s_{kt}) = 0$ and $\lim_{s_{kt}\to 1^-} f(s_{kt}) = 1$. Thus, $f : (0,1) \to (0,1)$ is a bijection.

A.9. Proof of Proposition 3

Assume that $\nu_C, \nu_I \in (0,1)$ and $\mu \geq 0$. Assume also that there exists a steady state of the transformed model characterized by: $s_{kt+1} = s_{kt} = s_{k*}$, $s_{\ell t+1} = s_{\ell t} = s_{\ell *}$, $\ell_{t+1} = \ell_t = \ell_*$, and $\tilde{k}_{t+1} = \tilde{k}_t = \tilde{k}_*$.

From Equation (A.55), there is a single value ℓ_* that satisfies the solution system for constant s_{k*}, $s_{\ell *}$, and $\tilde{k}_*$, given by:

$$\ell_* = \left[(1-\alpha_C)\frac{1-\nu_C}{1+\mu}\left(\frac{1-s_{k*}}{1-s_{\ell *}}\tilde{k}_*\right)^{\alpha_C}\right]^{\frac{1}{\eta}}. \tag{A.86}$$

From Equation (A.56), there is a single value $s_{\ell *}$ that satisfies the solution system for constant s_{k*}, given by:

$$s_{\ell *} = \left[\frac{\alpha_I}{1-\alpha_I}\frac{1-\alpha_C}{\alpha_C}\frac{1-\nu_C}{1-\nu_I}\frac{1-s_{k*}}{s_{k*}} + 1\right]^{-1}. \tag{A.87}$$

From Equation (A.58), there is a single value $\tilde{k}_*$ that satisfies the solution system for constant s_{k*}, $s_{\ell *}$, and ℓ_*, given by:

$$\tilde{k}_* = \left[\frac{s_{\ell *}^{1-\alpha_I} s_{k*}^{\alpha_I}}{g_{z_I} + \delta}\right]^{\frac{1}{1-\alpha_I}}. \tag{A.88}$$

From Equation (A.57), there is a single value s_{k*} that satisfies the solution system for constant $s_{\ell *}$, ℓ_*, and $\tilde{k}_*$, given by:

$$s_{k*} = \frac{s_{\ell *}}{\tilde{k}_*}\left[\frac{1+g_{z_I}}{\alpha_I \beta_\Gamma} - \frac{1-\delta}{\alpha_I}\right]^{\frac{1}{\alpha_I - 1}}, \tag{A.89}$$

which confirms the initial assumption.

A.10. BGP Growth Rates

$$g_\ell \equiv \frac{\ell_{t+1}}{\ell_t} - 1 = \frac{\ell_*}{\ell_*} - 1 = 0 = g_{\ell_C} = g_{\ell_I} \tag{A.90}$$

$$g_k \equiv \frac{k_{t+1}}{k_t} - 1 = \frac{\tilde{k}_{t+1}}{\tilde{k}_t}\frac{z_{It+1}}{z_{It}}\frac{\ell_{t+1}}{\ell_t} = \left(1 + g_{z_I}\right) - 1 = g_{k_C} = g_{k_I} \tag{A.91}$$

$$\begin{aligned} g_{y_C} \equiv \frac{y_{Ct+1}}{y_{Ct}} - 1 &= \frac{((1-s_{kt+1})k_{t+1})^{\alpha_C}(z_{Ct+1}(1-s_{\ell t+1})\ell_{t+1})^{1-\alpha_C}}{((1-s_{kt})k_t)^{\alpha_C}(z_{Ct}(1-s_{\ell t})\ell_t)^{1-\alpha_C}} - 1 \\ &= \Gamma_* - 1 = g_c \end{aligned} \tag{A.92}$$

$$\begin{aligned} g_{y_I} \equiv \frac{y_{It+1}}{y_{It}} - 1 &= \frac{(s_{kt+1}k_{t+1})^{\alpha_I}(z_{It+1}s_{\ell t+1}\ell_{t+1})^{1-\alpha_I}}{(s_{kt}k_t)^{\alpha_I}(z_{It}s_{\ell t}\ell_t)^{1-\alpha_I}} - 1 \\ &= \left(1 + g_{z_I}\right) - 1 \end{aligned} \tag{A.93}$$

$$\begin{aligned} g_q &\equiv \frac{q_{t+1}}{q_t} - 1 \\ &= \left(\frac{s_{\ell t+1}}{s_{\ell t}}\frac{s_{kt}}{s_{kt+1}}\right)^{1-\alpha_I}\left(\frac{1-s_{kt+1}}{1-s_{\ell t+1}}\frac{1-s_{\ell t}}{1-s_{kt}}\right)^{1-\alpha_C}\left(\frac{1+g_{z_I}}{1+g_{z_C}}\right)^{1-\alpha_C}\left(\frac{\tilde{k}_{t+1}}{\tilde{k}_t}\right)^{\alpha_I-\alpha_C} - 1 \\ &= \left(\frac{1+g_{z_I}}{1+g_{z_C}}\right)^{1-\alpha_C} - 1 \end{aligned} \tag{A.94}$$

$$g_w \equiv \frac{w_{t+1}}{w_t} - 1 = \frac{\Gamma_{t+1}q_{t+1}\ell_{t+1}}{\Gamma_t q_t \ell_t} - 1 = g_{z_I} \tag{A.95}$$

$$g_r \equiv \frac{r_{t+1}}{r_t} - 1 = \left(\frac{s_{\ell t+1}}{s_{kt+1}}\frac{1}{\tilde{k}_{t+1}}\right)^{1-\alpha_I}\left(\frac{s_{kt}}{s_{\ell t}}\tilde{k}_t\right)^{1-\alpha_I} - 1 = 0 \tag{A.96}$$