Does Automation Reverse the Macroeconomics of Aging?

A Two-Sector Model of Population, Production, and Effective Demand

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Abstract

Population aging is conventionally modeled as a labor-scarcity shock: in standard overlapping-generations economies, lower fertility raises the capital–labor ratio, depresses the natural rate of interest, and raises wages. We ask whether large-scale automation reverses this logic. We build a tractable two-sector economy in which a capital-goods (investment) sector absorbs its own output while a consumption-goods sector is bounded by household labor income, and in which automation capital substitutes for labor with elasticity σ. In a benchmark with fixed saving behavior, aging widens the consumption-demand gap if and only if $\sigma > 1$; the threshold is exact at the Cobb–Douglas knife-edge and coincides with the reversal of the labor-share response to capital deepening. When population is made endogenous—fertility responds to realized employment, and the labor force adjusts one generation later than capital—and automation accumulates in a profit-driven but demand-braked way, the balanced-growth path is locally unstable when $\sigma > 1$: the steady state is a repelling threshold, not a point of global collapse. The static and dynamic results are governed by the same parameter. The analysis is deliberately positive and conditional—on a demand-constrained closure and the CES specification—makes no policy recommendation, and identifies the empirical position of σ relative to one as the decisive forward-looking question.

Claims are positive and conditional, with no policy recommendation or normative judgment. Calibration values are illustrative and numerical thresholds are not estimates.

1 Introduction

Population aging is conventionally read as a labor-scarcity shock. In the textbook overlapping-generations economy, a fall in fertility lowers the growth of the labor force, raises the capital–labor ratio, and—through diminishing returns to capital—depresses the natural rate of interest while raising wages. This logic underpins much of the recent macroeconomics of demographics, in which aging is a leading candidate for lower real returns and secular stagnation in advanced economies. Auclert, Malmberg, Martenet, and Rognlie (2021), for example,

quantify how population aging raises wealth-to-GDP ratios and lowers returns to wealth, while Summers (2014) frames secular stagnation as a chronic difficulty of generating enough demand at the zero lower bound. In the benchmark aging story, fewer workers make labor scarce and capital abundant, and prices move accordingly.

This paper asks whether large-scale automation reverses that logic. The question is motivated by a simple observation about composition. When the capital that accumulates is increasingly automation capital—machines and algorithms that substitute for labor rather than complement it—the share of output accruing to labor need not be stable as capital deepens. If labor and capital are gross substitutes, capital deepening compresses the labor share; and since consumption demand is ultimately financed out of labor income, a compressed labor share can turn aging from a force that tightens labor markets into a force that slackens product markets. The same demographic change that the standard model reads as "labor scarce, wages up" may, under sufficient automation, read instead as "labor income eroded, demand short."

To make this precise we build a two-sector economy with a single, transparent margin of substitution. Output is produced from labor and (automation) capital through a constant-elasticity-of-substitution technology with elasticity σ, and it is used in two ways: as investment goods, which are absorbed within the production sector—the self-reinforcing accumulation of capital that needs no household buyer—and as consumption goods, whose realization requires household labor income. We do not assume any failure of the aggregate resource constraint; the identity $Y = C + I$ holds throughout. What does the work is the *composition* of demand: investment goods can be absorbed by firms, whereas consumption goods require purchasing power that flows, in the first instance, through wages. When that purchasing power falls short, the consumption-goods sector is demand-constrained. This demand-constrained closure is a deliberate modeling choice, not a market failure: we suppress the price, interest-rate, and fiscal adjustments that a fully flexible economy would use to clear the consumption-goods market, in order to isolate the composition channel in its sharpest form. The resulting gap is therefore conditional on that closure and on the CES specification; we return to this scope in Sections 4.6 and 7.

Two results follow. First, in a benchmark with fixed saving behavior and labor-income-constrained consumption demand, aging widens the economy's consumption-demand gap if and only if $\sigma > 1$ (Proposition 1). The threshold is exact in the benchmark—it sits precisely at the Cobb–Douglas knife-edge $\sigma = 1$—and it coincides with the point at which capital deepening switches from raising to lowering the labor share. Below unity, aging supports the labor-income base and the demand gap narrows; above unity, aging erodes that base and the gap widens. Second, when population is made endogenous—fertility responds positively to realized employment, with the labor force adjusting one generation later than capital—and automation accumulates in a profit-driven way that is nonetheless braked by weak demand, the balanced-growth path is locally unstable when $\sigma > 1$ (Proposition 3). At an interior balanced-growth steady state the linearized map has slope exceeding one, so the steady state is a repelling threshold: economies on its high-automation side move toward a low-utilization, low-fertility region. We are deliberately careful here. Local instability is not global collapse; sustained, unbounded growth of automation per worker is a separate, global property that requires an additional condition on the high-automation range and is not implied by the local result. The contribution is the threshold and its sign, not a prophecy.

A single mechanism drives both results. For the CES technology, the response of the labor share to capital deepening satisfies $\operatorname{sign}(\mathrm{d}s_L/\mathrm{d}k) = -\operatorname{sign}(\sigma - 1)$: the labor share rises with capital when factors are complements and falls when they are substitutes, with the sign flipping exactly at $\sigma = 1$. The static comparative-statics threshold of Proposition 1 and the dynamic stability threshold of Proposition 3 are therefore the same threshold viewed twice: the position of σ relative to one determines both the *sign* of the aging shock and whether the population–production feedback is self-correcting or self-reinforcing. This is the organizing idea of the paper.

Three features of the framework connect it to existing measurement and theory. As Karabarbounis and Neiman (2014) show, falling relative prices of investment goods can shift income away from labor when capital and labor are sufficiently substitutable; Acemoglu and Restrepo (2018, 2019) give the task-based version of the same displacement mechanism. The consumption-demand gap is, in the language of National Transfer Accounts (Lee and Mason 2011), the statement that the labor-income surplus financing the economy's lifecycle deficit erodes as the labor share falls. And the demand-side dynamics of r^* in the substitution region connect to the secular-stagnation literature (Summers 2014; Auclert et al. 2021), here arising from the composition of demand rather than from labor scarcity alone.

We emphasize what the paper does not claim. The exact location of the threshold at $\sigma = 1$ is a property of the benchmark closure, in which the saving rate, the consumption parameter, and the use of capital income are held fixed; richer closures move the threshold off unity, and the robust content is that the reversal occurs as σ crosses a value near one. The dynamic instability is local and conditional on the profit-driven component of automation being strong relative to the net demand brake. And throughout, the analysis is positive: we characterize the conditions under which aging and automation interact in one way rather than another, and identify the resulting open questions, without making policy recommendations or normative judgments.

The remainder proceeds as follows. Section 2 places the paper among four strands of literature. Section 3 sets out the two-sector circular-flow framework. Section 4 develops the benchmark model and proves Proposition 1. Section 5 introduces endogenous population and automation and proves Proposition 3. Section 6 studies, as positive comparative analysis, how alternative intergenerational transfer arrangements perform when the labor-income base erodes. Section 7 collects forward-looking research questions.

2 Related literature

The paper sits at the intersection of four literatures and contributes by joining elements that are usually studied apart.

Aging, the natural rate, and secular stagnation. A large body of work links demographic change to lower real returns and weak demand. Auclert et al. (2021) use a quantitative life-cycle framework to show how population aging raises wealth-to-GDP ratios, lowers returns to wealth, and widens global imbalances. Aksoy, Basso, Smith, and Grasl (2019) estimate that aging and low fertility reduce output growth, investment, and real interest rates in OECD economies. Summers (2014) revived the secular-stagnation hypothesis as a

chronic shortfall of demand relative to full-employment saving. In these accounts the demographic force operates primarily through saving and investment quantities. We retain the falling-return implication but locate an additional channel in the *composition* of demand under automation, and we show that the sign of aging's effect on the demand gap is governed by the substitution elasticity.

Automation, the labor share, and task models. Karabarbounis and Neiman (2014) document a broad global decline in the labor share and argue that a falling relative price of investment goods induced firms to substitute capital for labor. Acemoglu and Restrepo (2018) show in a task-based growth model that automation can reduce employment and the labor share, while Acemoglu and Restrepo (2019) emphasize the displacement effect of automation and the countervailing reinstatement effect of new labor-intensive tasks. Our CES specification is the reduced-form counterpart of that mechanism: $\operatorname{sign}(\mathrm{d}s_L/\mathrm{d}k) = -\operatorname{sign}(\sigma - 1)$. We take this mechanism as given and ask what it implies, when combined with demand-constrained consumption and endogenous fertility, for the macroeconomics of aging. The task-based microfoundation is a natural extension and is left to the discussion.

National Transfer Accounts and intergenerational transfers. Lee and Mason (2011) and the National Transfer Accounts project measure, by age, the gap between consumption and labor income—the lifecycle deficit—and the public and private transfers that finance it. This framework gives our consumption-demand gap an operational meaning: a widening gap is an erosion of the working-age labor-income surplus that funds the consumption of dependents and retirees. Section 6 uses this language to compare, on positive grounds, a labor-income-funded pay-as-you-go arrangement with a transfer indexed to capital or automation income when the labor-income base shrinks.

Endogenous fertility and demographic–economic feedback. A long tradition models fertility as responsive to economic conditions. Myrskylä, Kohler, and Billari (2009), for example, show that the relationship between development and fertility can change at high levels of development, while Lutz, Skirbekk, and Testa (2006) formulate a low-fertility-trap mechanism in which demographic, sociological, and economic feedbacks reinforce sub-replacement fertility. Our dynamic extension contributes a specific feedback: when consumption is demand-constrained, low realized employment depresses fertility, which—one generation later—raises automation per worker and, under substitution, further depresses employment. The novelty is not endogenous fertility per se but its coupling to a demand-constrained two-sector structure, which makes the substitution elasticity the pivot between a self-correcting and a self-reinforcing demography.

Contribution. Relative to these strands, the paper's contribution is to show that a single parameter—the elasticity of substitution between automation capital and labor—simultaneously governs (i) the sign of aging's static effect on the consumption-demand gap and (ii) the local stability of the joint population–production dynamics, and that the two thresholds coincide. This unifies the "aging lowers r^*" and "automation lowers the labor share" observations

within one demand-constrained framework and identifies the empirical position of σ relative to unity as the decisive forward-looking question.

3 The two-sector circular-flow framework

Figure 1 fixes ideas before the algebra. The economy is a circular flow around a financial market, organized into four quadrants: the enterprise system (the *production module*, I), the family–household system (the *population module*, III), and the two markets that link them—the commodity market (II) and the labor market (IV). Real flows (output Y, final consumption goods C, labor N) and the matching money flows run in opposite directions around the loop.

Two annotations carry the paper's content. Along the II–IV diagonal, the commodity and labor markets are mutually dependent: labor income earned in IV finances consumption demand in II, and the consumption-goods market is bounded by that income, $C \leq w \cdot N$. Along the I–III diagonal, the production and population modules are *asymmetric*: the structure of labor and output the enterprise sector demands, D_f, need not match the age–skill structure the household sector supplies, D_h, and the mismatch $M = \mathrm{dist}(D_f, D_h)$ widens when the two sides adjust at different speeds. Crucially, investment goods recirculate within the production sector—the self-loop on the enterprise box—whereas consumption goods must reach households. The sections that follow formalize these two annotations: the demand bound $C \leq w \cdot N$ becomes the demand-constrained consumption sector, and the mismatch becomes the dynamic object whose stability depends on σ.

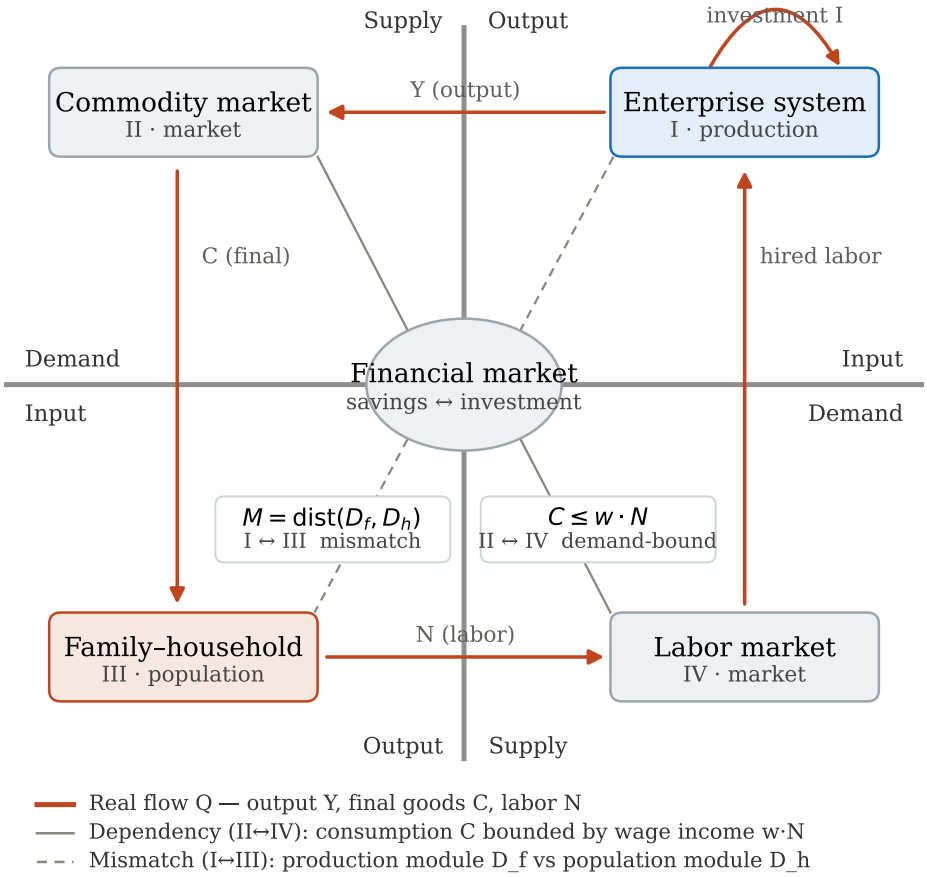


Figure 1: The two-sector circular flow. Enterprise (I, production module) and family (III, population module) sit on the dashed mismatch diagonal; the commodity (II) and labor (IV) markets sit on the dependency diagonal, where consumption is bounded by labor income, $C \le w \cdot N$. Investment goods recirculate within the firm sector (the self-loop); final goods flow to households. Bold axes carry the input/output and supply/demand structure of each quadrant.

4 The benchmark two-sector economy

4.1 Technology

Time is discrete and indexed by generations t. A single final good is produced from effective capital services K_t—which include automation (AI) capital—and labor L_t, through a constant-elasticity-of-substitution (CES) technology. In per-worker terms, with $k_t \equiv K_t/L_t$,

$$y(k) \;=\; \big[\,\alpha\, k^{\rho} + (1-\alpha)\,\big]^{1/\rho}, \qquad \rho \;=\; \frac{\sigma-1}{\sigma}, \tag{1}$$

where $\sigma > 0$ is the elasticity of substitution between capital (automation) and labor, $\alpha \in (0,1)$, and $\sigma = 1$ ($\rho \to 0$) is the Cobb–Douglas limit $y = k^{\alpha}$. Factor prices equal marginal products:

$$w(k) = (1-\alpha)\, y(k)^{\,1-\rho}, \qquad R(k) = \alpha\,\big(y(k)/k\big)^{\,1-\rho}. \tag{2}$$

The labor share is

$$s_L(k) \;=\; \frac{w(k)\,L}{Y} \;=\; \frac{w(k)}{y(k)} \;=\; (1-\alpha)\, y(k)^{-\rho} \;=\; \frac{1-\alpha}{\alpha k^{\rho} + (1-\alpha)}, \tag{3}$$

and the capital share is $s_K(k) = 1 - s_L(k)$.

4.2 Two uses of output, and the composition of demand

Output is used in two ways: as investment goods I, absorbed within the production sector (automation capital that is reinvested—the "self-loop"), and as consumption goods C, which must be purchased out of household labor income. We do *not* assume any failure of the aggregate resource constraint. The accounting identity

$$Y \;=\; C + I \tag{4}$$

holds throughout. What matters is the *composition*: investment goods can be absorbed by firms, whereas consumption goods require household purchasing power. We summarize the consumption side by

$$C^{d} \;=\; \varphi\, w\, L, \qquad \varphi \in (0,1], \tag{5}$$

so that consumption demand is bounded by labor income, with φ folding in the consumption of dependants and retirees financed out of labor income and transfers. Capital income is reinvested rather than consumed.

Assumption 1 (Benchmark closure)**.** *Saving behavior is fixed: a constant share $s \in (0,1)$ of output finances the capital stock, so in a steady demographic state with per-generation population growth n and depreciation δ the capital–output ratio satisfies $k/y(k) = s/(n+\delta)$. Consumption demand is given by* (5) *with constant φ.*

4.3 The consumption-demand gap

At full employment, investment absorbs a share of output and the residual is the supply of consumption goods. Writing the full-employment investment requirement per worker as $(n+\delta)k$ and using Assumption 1 ($(n+\delta)k = s\,y$), the per-worker excess supply of consumption goods—the *consumption-demand gap*—is

$$\frac{g}{Y} \;=\; \frac{y - \varphi w - (n+\delta)k}{y} \;=\; (1-s) - \varphi\, s_L(k). \tag{6}$$

A positive gap means that, at full employment, consumption-goods supply exceeds labor-income-financed demand; the consumption-goods sector is then demand-constrained.

4.4 The engine: capital deepening and the labor share

Lemma 1 (Labor-share response)**.** *For the CES technology* (1)*,*

$$\frac{\mathrm{d}s_L}{\mathrm{d}k} = -\,\frac{(1-\alpha)\,\alpha\,\rho\,k^{\rho-1}}{\big(\alpha k^{\rho} + 1 - \alpha\big)^2}, \qquad \textit{hence} \qquad \operatorname{sign}\!\left(\frac{\mathrm{d}s_L}{\mathrm{d}k}\right) = -\operatorname{sign}(\sigma - 1). \tag{7}$$

Proof. Differentiate the last expression in (3). With $D(k) \equiv \alpha k^{\rho} + (1-\alpha) > 0$, $s_L = (1-\alpha)D^{-1}$ and $s_L' = -(1-\alpha)D^{-2}D'$ where $D' = \alpha\rho k^{\rho-1}$, giving (7). Since $(1-\alpha)$, α, $k^{\rho-1}$ and D^2 are positive, the sign is that of $-\rho = -(\sigma-1)/\sigma$, i.e. $-\operatorname{sign}(\sigma-1)$. □

Thus capital deepening raises the labor share when capital and labor are gross complements ($\sigma < 1$), lowers it when they are gross substitutes ($\sigma > 1$), and leaves it unchanged at $\sigma = 1$.

4.5 Aging and the reversal threshold

We model aging as a fall in n. Under Assumption 1, lower n raises the steady-state capital–labor ratio. Combining this with (6) and Lemma 1 yields the first main result.

Proposition 1 (Reversal threshold)**.** *In the benchmark two-sector CES economy (Assumption 1) with labor-income-constrained consumption demand* (5)*, aging—a fall in n—widens the consumption-demand gap g/Y if and only if $\sigma > 1$. Equivalently, the gap is unaffected by aging exactly at $\sigma = 1$, narrows for $\sigma < 1$, and widens for $\sigma > 1$. The reversal threshold in the benchmark model is therefore exactly $\sigma^* = 1$.*

Proof. From (6), $g/Y = (1-s) - \varphi s_L(k)$ with $(1-s)$ constant under Assumption 1, so

$$\frac{\mathrm{d}(g/Y)}{\mathrm{d}n} \;=\; -\varphi\,\frac{\mathrm{d}s_L}{\mathrm{d}k}\cdot\frac{\mathrm{d}k^*}{\mathrm{d}n}.$$

Aging is $\mathrm{d}n < 0$, so the change in the gap from aging has sign $-\operatorname{sign}\big(\mathrm{d}(g/Y)/\mathrm{d}n\big) = \operatorname{sign}\big(\frac{\mathrm{d}s_L}{\mathrm{d}k}\cdot\frac{\mathrm{d}k^*}{\mathrm{d}n}\big)$. Monotonicity of the capital-deepening response follows from Assumption 1: $k/y(k) = s/(n+\delta)$ with $\mathrm{d}\log(k/y)/\mathrm{d}\log k = 1 - \mathrm{d}\log y/\mathrm{d}\log k = 1 - s_K(k) = s_L(k) > 0$, so k/y is strictly increasing in k; since the right-hand side $s/(n+\delta)$ is decreasing in n, lower n raises

k^*, i.e. $\mathrm{d}k^*/\mathrm{d}n < 0$. With this and Lemma 1 ($\mathrm{sign}(\frac{\mathrm{d}s_L}{\mathrm{d}k}) = -\mathrm{sign}(\sigma - 1)$), the displayed sign equals $\mathrm{sign}(\sigma - 1)$. Hence aging widens the gap iff $\sigma > 1$, leaves it unchanged at $\sigma = 1$, and narrows it for $\sigma < 1$. □

Remark (Scope of the threshold). The *exact* location $\sigma^* = 1$ follows from the gap (6) being an affine function of the labor share, so that its response to aging reverses sign exactly where the labor-share response does (Lemma 1). What preserves or moves the threshold is therefore whether a given closure keeps the gap affine in s_L. Consuming a constant fraction of capital income does *not* move it (Section 4.6); allowing the investment-demand channel to vary with demography, the saving rate to respond to the age structure, or the propensity to consume capital income to depend on the capital share shifts the threshold off unity. The robust qualitative content is that the response of the gap to aging reverses sign as σ crosses a threshold near unity, coinciding with the reversal of the labor-share response in Lemma 1.

Remark (Wages versus realized labor income). The full-employment marginal-product wage $w(k) = (1-\alpha)y(k)^{1-\rho}$ rises with k for all σ, since $\mathrm{d}w/\mathrm{d}k = (1-\alpha)(1-\rho)y^{-\rho}\,\mathrm{d}y/\mathrm{d}k > 0$. This does not contradict a deterioration of household purchasing power. Realized household labor income is $u\,w\,L$, where $u \in (0, 1]$ is the utilization rate of the demand-constrained consumption-goods sector (defined in Section 5). For $\sigma > 1$, aging compresses s_L and widens the gap, lowering u; the full-employment marginal-product wage w may rise while $u\,w\,L$ falls.

Illustration. Figure 2 shows the benchmark threshold. Panel A plots the labor share against the capital–labor ratio for $\sigma < 1$, $\sigma = 1$, and $\sigma > 1$, illustrating Lemma 1. Panel B plots the consumption-demand gap g/Y against σ for a young and an aged economy; the two curves cross exactly at $\sigma = 1$, with aging widening the gap only for $\sigma > 1$. (Calibration is illustrative.)

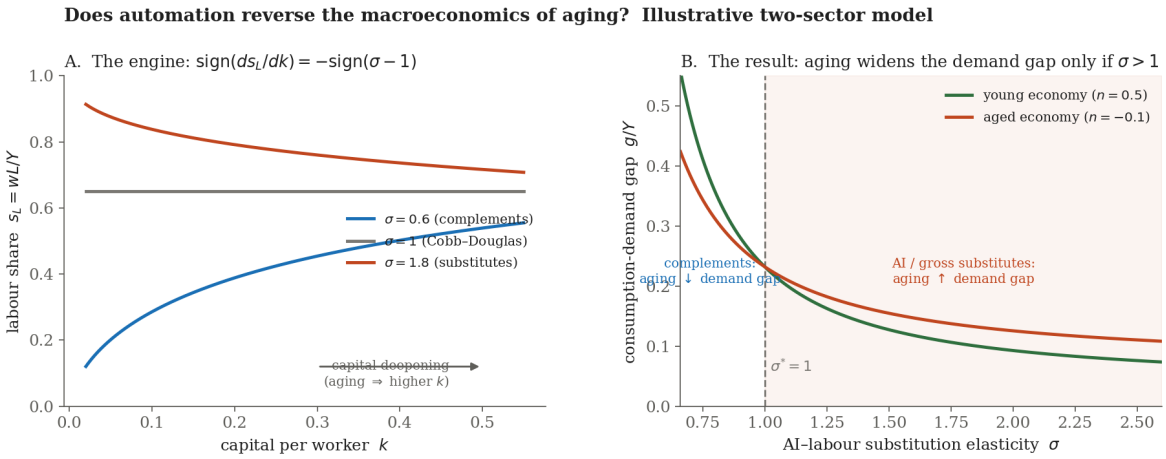


Figure 2: Benchmark reversal threshold. **A**: $\mathrm{sign}(\mathrm{d}s_L/\mathrm{d}k) = -\mathrm{sign}(\sigma - 1)$. **B**: the consumption-demand gap g/Y for a young ($n = 0.5$) versus aged ($n = -0.1$) economy; the curves cross at $\sigma^* = 1$ and aging widens the gap only for $\sigma > 1$. Illustrative calibration; values are not estimates.

4.6 Consuming capital income and the location of the threshold

The exact location $\sigma^* = 1$ might appear to be an artifact of the benchmark assumption that capital income is entirely reinvested. It is natural to ask how the threshold moves once part of capital income is consumed. We show that the most direct relaxation leaves the threshold exactly at unity, and identify precisely what does move it.

Suppose a constant fraction $\chi \in [0, 1)$ of capital income is consumed rather than reinvested, so that consumption demand per worker becomes $C^d/y = \varphi\, s_L + \chi\, s_K$. The consumption-demand gap (6) generalizes to

$$\frac{g}{Y} = (1 - s) - \varphi\, s_L - \chi\, s_K = (1 - s - \chi) - (\varphi - \chi)\, s_L(k), \tag{8}$$

using $s_K = 1 - s_L$. This is still an *affine* function of the labor share: the intercept shifts to $(1 - s - \chi)$ and the slope scales to $(\varphi - \chi)$, but the only k-dependence runs through $s_L(k)$. Since $\mathrm{d}(g/Y)/\mathrm{d}n = -(\varphi - \chi)\, s_L'(k)\, \mathrm{d}k^*/\mathrm{d}n$ and $s_L'(k)$ changes sign exactly at $\sigma = 1$ (Lemma 1), the aging effect reverses sign at the same point. We record this.

Proposition 2 (Threshold invariance under constant capital-income consumption)**.** *If a constant fraction $\chi < \varphi$ of capital income is consumed, the reversal threshold is unchanged: $\sigma^* = 1$. Consuming capital income at a constant rate rescales the magnitude of the aging effect by $(\varphi - \chi)/\varphi$ but does not move the threshold. Only in the extreme $\chi > \varphi$—capital income consumed at a higher rate than labor income—does the sign of the aging effect flip globally.*

Proof. Immediate from (8): with $(1 - s - \chi)$ constant, $\mathrm{d}(g/Y)/\mathrm{d}n$ has the sign of $(\varphi - \chi)\, s_L'(k)\, \mathrm{d}k^*/\mathrm{d}n$. For $\chi < \varphi$ the factor $(\varphi - \chi) > 0$, so the sign is that of the benchmark, reversing only where $s_L'(k) = 0$, i.e. at $\sigma = 1$. For $\chi > \varphi$ the factor is negative and every sign is reversed. □

The threshold is thus not a fragile coincidence: it is tied to the labor-share reversal of Lemma 1 and survives the natural relaxation of the reinvestment assumption. What moves the threshold is not consuming capital income *per se*, but doing so in a way that makes the gap depend on the capital stock beyond the affine channel—for instance, a propensity to consume capital income that itself rises with the capital share, $\chi(k) = \chi_0 + \chi_1 s_K(k)$. The gap (8) then carries a term $-\chi_1 s_K(k)^2$, quadratic in s_L, and an additional switching point appears below unity. Figure 3 illustrates: with χ constant the aging effect crosses zero at $\sigma = 1$ regardless of its level, whereas with a sufficiently capital-share-sensitive $\chi(k)$ a second threshold $\sigma^* < 1$ emerges, enlarging the region in which aging widens the gap. The direction is intuitive—consuming relatively more capital income precisely when the capital share is high partly offsets the labor-share channel—but it requires the propensity itself to respond to factor shares, not merely to be positive. The calibration is illustrative.

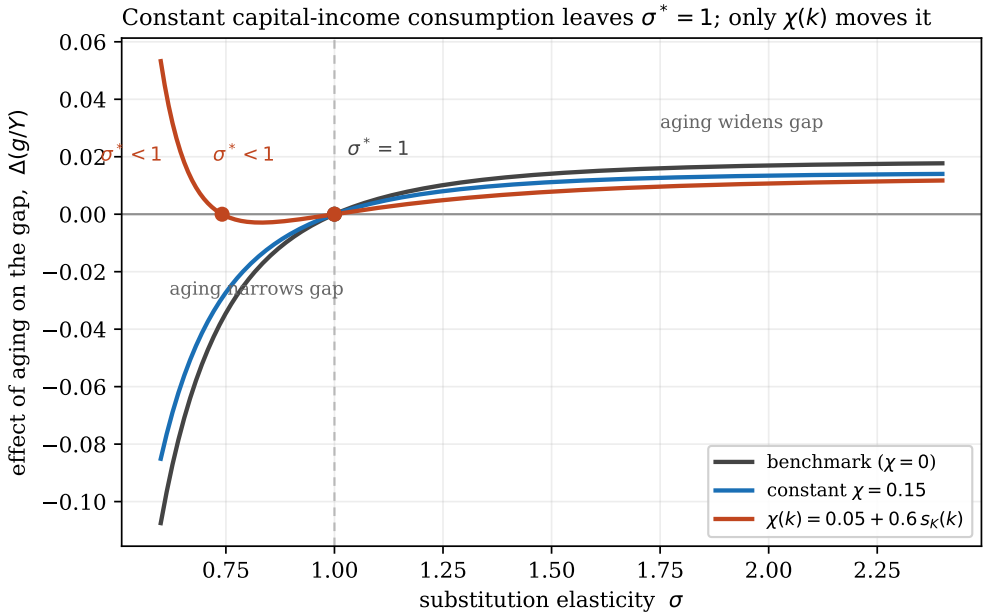


Figure 3: Effect of aging on the consumption-demand gap, $\Delta(g/Y)$, as a function of σ. The benchmark ($\chi = 0$) and a constant consumption rate out of capital income ($\chi = 0.15$) both cross zero at $\sigma^* = 1$: constant χ rescales the magnitude but not the threshold (Proposition 2). Only when the consumption rate rises with the capital share, $\chi(k) = \chi_0 + \chi_1 s_K(k)$, does an additional threshold $\sigma^* < 1$ appear. Illustrative calibration; values are not estimates.

5 Dynamic extension: endogenous population and automation

In this section a denotes the automation component of effective capital per worker; it plays the role that k plays in Section 4, so the sign results of Lemma 1 apply with a replacing k. Population and automation are now both endogenous.

5.1 Demand-constrained utilization

When the gap is positive the consumption-goods sector is rationed by demand. Treating investment as autonomous (anchored on the predetermined capital stock) and consumption as induced with marginal propensity φs_L out of realized output, the Keynesian fixed point gives realized output $y_{\text{real}} = i/(1 - \varphi s_L)$, capped at full employment $y(k)$. The utilization rate is

$$u(k) \;=\; \min\left\{ 1,\; \frac{s}{1 - \varphi\, s_L(k)} \right\}. \tag{9}$$

For $\sigma > 1$, higher k lowers s_L, raises $1 - \varphi s_L$, and lowers u: automation deepening tightens the demand constraint.

5.2 Endogenous life-force and productive force

Population (life-force) responds with a one-generation lag. Fertility responds positively to realized employment, proxied by utilization:

$$n(k) \;=\; n_{\min} + (n_{\max} - n_{\min})\,\Lambda\!\left(\frac{u(k) - u_{\text{ref}}}{u_{\text{scale}}}\right), \qquad \Lambda(x) = \frac{1}{1+e^{-x}}, \tag{10}$$

with $n_{\min} < n_{\max}$. The labor force evolves as $L_{t+1} = (1 + n_t)L_t$, so fertility chosen this generation affects labor supply only next generation: the population adjusts with a one-generation lag, whereas capital adjusts within the generation. This timing asymmetry is the formal content of "productive force runs ahead, life-force lags."

Automation (productive force): the weak, demand-braked rule. Rather than postulating fully autonomous automation growth, we let the per-generation growth of automation per worker be *partly autonomous, profit-driven, and braked by weak demand*:

$$\gamma(k) \;=\; \gamma_0 + \gamma_1\, s_K(k) - \gamma_2\,\big(1 - u(k)\big), \qquad \gamma_0, \gamma_1, \gamma_2 \geq 0, \tag{11}$$

where $s_K = 1 - s_L$ is the capital (profit) share and $(1 - u)$ is the demand shortfall. With a_t denoting automation capital per worker, the one-generation map is

$$a_{t+1} \;=\; F(a_t) \;\equiv\; \frac{1 + \gamma(a_t)}{1 + n(a_t)}\, a_t. \tag{12}$$

A balanced-growth steady state a^* satisfies $a_{t+1} = a_t$, i.e.

$$\gamma(a^*) \;=\; n(a^*), \tag{13}$$

so the growth of the productive force equals that of the life-force. Crucially, under (11) the steady state is *endogenous*: it is the solution of (13), not an externally imposed growth rate.

5.3 Local stability and the role of σ

Linearizing (12) at a steady state and using (13),

$$F'(a^*) \;=\; 1 \;+\; \frac{a^*\left[\gamma'(a^*) - n'(a^*)\right]}{1 + n(a^*)}. \tag{14}$$

The steady state is locally stable iff $|F'(a^*)| < 1$, which requires both $\gamma'(a^*) < n'(a^*)$ (upper bound) and a not-too-strong feedback (lower bound $F' > -1$, ruling out over-correcting oscillations). The sign of the decisive term $\gamma' - n'$ is governed by σ through Lemma 1: for $\sigma > 1$, deeper automation lowers s_L, which raises s_K and hence the profit-driven term $\gamma_1 s_K$, and lowers u, which lowers n via (10). The demand brake works in the opposite, stabilizing direction: a lower u raises $(1 - u)$ and so makes $-\gamma_2(1 - u)$ *more* negative, lowering γ. Whether $\gamma' - n' > 0$ (instability) therefore turns on whether the profit-driven response dominates the net demand brake; the proof below makes the condition exact. For $\sigma < 1$ the comparative statics reverse.

Proposition 3 (Local instability under substitution)**.** *Consider the dynamic extension with utilization* (9)*, fertility* (10) *(n increasing in u), and the weak automation rule* (11)*, and let* a^* *be an interior balanced-growth steady state with* $u(a^*) < 1$*. If the profit-driven component of automation accumulation is sufficiently strong relative to the net demand brake—precisely,* $\gamma_1 > B(a^*)\,[\gamma_2 - \eta(a^*)]$ *with* B, η *as defined in the proof—then* a^* *is locally unstable when* $\sigma > 1$*:* $F'(a^*) > 1$*, so* a^* *is a repelling threshold. Under the benchmark calibration* a^* *is locally stable when* $\sigma < 1$*. In the numerical simulations, trajectories starting on the high-automation side of* a^* *move toward a low-utilization, low-fertility region; sustained, unbounded growth of automation per worker is a global property requiring the additional condition* $\gamma(a) > n(a)$ *over the relevant high-a range, which is not implied by local instability alone.*

Proof. Assume the steady state is interior and locally differentiable, with $u(a^*) < 1$ (the demand-constrained region). Define

$$B(a) \;\equiv\; \frac{s\,\varphi}{\big(1-\varphi\, s_L(a)\big)^2} \;>\; 0, \qquad \eta(a) \;\equiv\; \frac{n_{\max}-n_{\min}}{u_{\text{scale}}}\,\Lambda\big(z(a)\big)\big(1-\Lambda(z(a))\big) \;>\; 0,$$

where $z(a) = (u(a) - u_{\text{ref}})/u_{\text{scale}}$. Differentiating (9) in the constrained region and (10),

$$u'(a) = B(a)\, s_L'(a), \qquad n'(a) = \eta(a)\, u'(a) = \eta(a)\, B(a)\, s_L'(a).$$

Since $s_K = 1 - s_L$, the weak rule (11) gives

$$\gamma'(a) = -\gamma_1\, s_L'(a) + \gamma_2\, u'(a) = \big[-\gamma_1 + \gamma_2 B(a)\big] s_L'(a).$$

Therefore

$$\gamma'(a) - n'(a) = \big[-\gamma_1 + (\gamma_2 - \eta(a))\, B(a)\big]\, s_L'(a).$$

For $\sigma > 1$, Lemma 1 gives $s_L'(a) < 0$, so $\gamma'(a^*) - n'(a^*) > 0$ if and only if

$$\gamma_1 \;>\; B(a^*)\,\big[\gamma_2 - \eta(a^*)\big].$$

This holds automatically when $\eta(a^*) \geq \gamma_2$, and otherwise requires the profit-driven response γ_1 to dominate the net demand brake. Under this condition, $F'(a^*) > 1$ by (14), so a^* is locally unstable. Economically, the condition says that the response of automation investment to a rising capital share is strong enough to offset the direct investment brake from weak demand, net of the fertility response generated by employment. For $\sigma < 1$, $s_L'(a) > 0$ and, under the same sufficient condition, $\gamma'(a^*) - n'(a^*) < 0$, hence $F'(a^*) < 1$; the lower bound $F'(a^*) > -1$ is not implied analytically and is verified under the benchmark calibration. □

Remark (What "sufficiently strong" does and does not mean). The instability condition involves the profit-driven coefficient γ_1 and the net brake $\gamma_2 - \eta(a^*)$, *not* the autonomous level γ_0: γ_0 does not enter $\gamma'(a)$ and so does not affect local stability, though it does affect the existence and location of a^*. The correct qualifier is therefore "sufficiently profit-driven relative to the net demand brake," not "sufficiently autonomous."

Remark (Numerical robustness). Two computational checks support Proposition 3. (i) Over the grid $u_{\text{scale}} \in \{0.03, 0.05, 0.10, 0.20\}$, $\gamma \in \{0.10, 0.20, 0.25, 0.35\}$, $\varphi \in \{0.60, 0.72, 0.80\}$ with an exogenous-growth specification, every cell admitting a balanced path is unstable

for $\sigma = 1.8$ and stable for $\sigma = 0.6$, with no $F' < -1$ case arising; the calibration moves the tipping location a^* but not the sign of $F'(a^*) - 1$. (ii) Under the weak rule (11) with $\gamma_1 \in \{0.2, 0.3, 0.5\}$ and demand brake $\gamma_2 \in \{0.1, 0.3, 0.5, 0.8\}$, all cells remain unstable for $\sigma = 1.8$ ($F'(a^*) \approx 1.3$–1.4) and stable for $\sigma = 0.6$ ($F' \approx 0.4$): the demand brake postpones the tipping point but does not remove it within the range examined. A sufficiently strong brake (or $\gamma_0 < 0$) can in principle restore stability; this bound is not claimed to be uniform. Figure 5 reports the $F'(a^*)$ surface.

Remark (Existence caveat). For $\sigma < 1$ with low consumption pass-through φ and strong complementarity, a positive balanced-growth path may fail to exist (the CES capital–output ratio falls below its lower bound). The $\sigma < 1$ stability statement applies where such a path exists; the admissible range of (φ, σ) is reported in Appendix A.

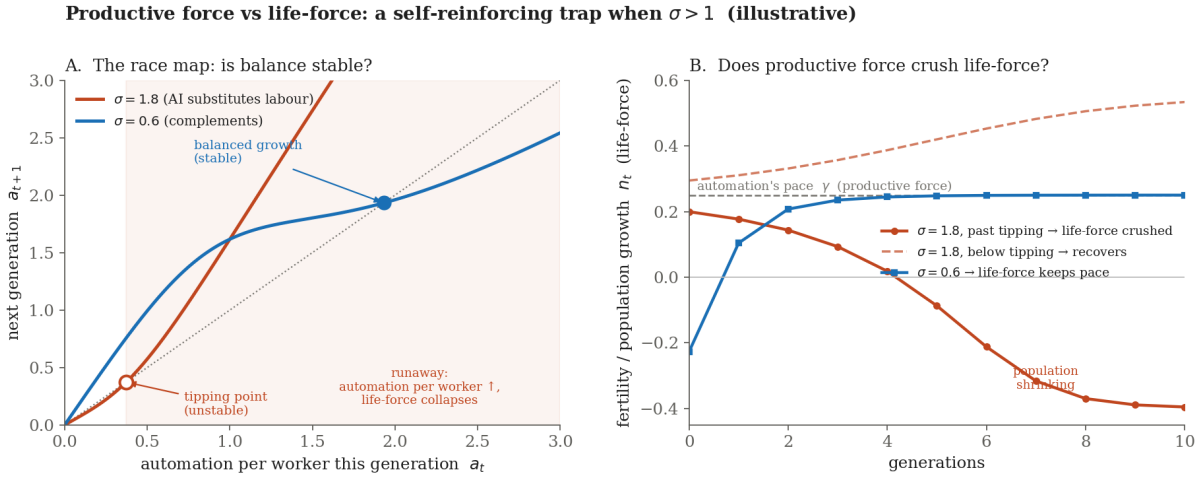


Figure 4: Productive force versus life-force. **A**: the race map a_{t+1} versus a_t; for $\sigma = 0.6$ the balanced point is stable, for $\sigma = 1.8$ it is an unstable (repelling) threshold. **B**: fertility trajectories—under $\sigma = 1.8$ past the threshold, realized population growth falls toward its lower bound; under $\sigma = 0.6$ it keeps pace with automation. Illustrative calibration.

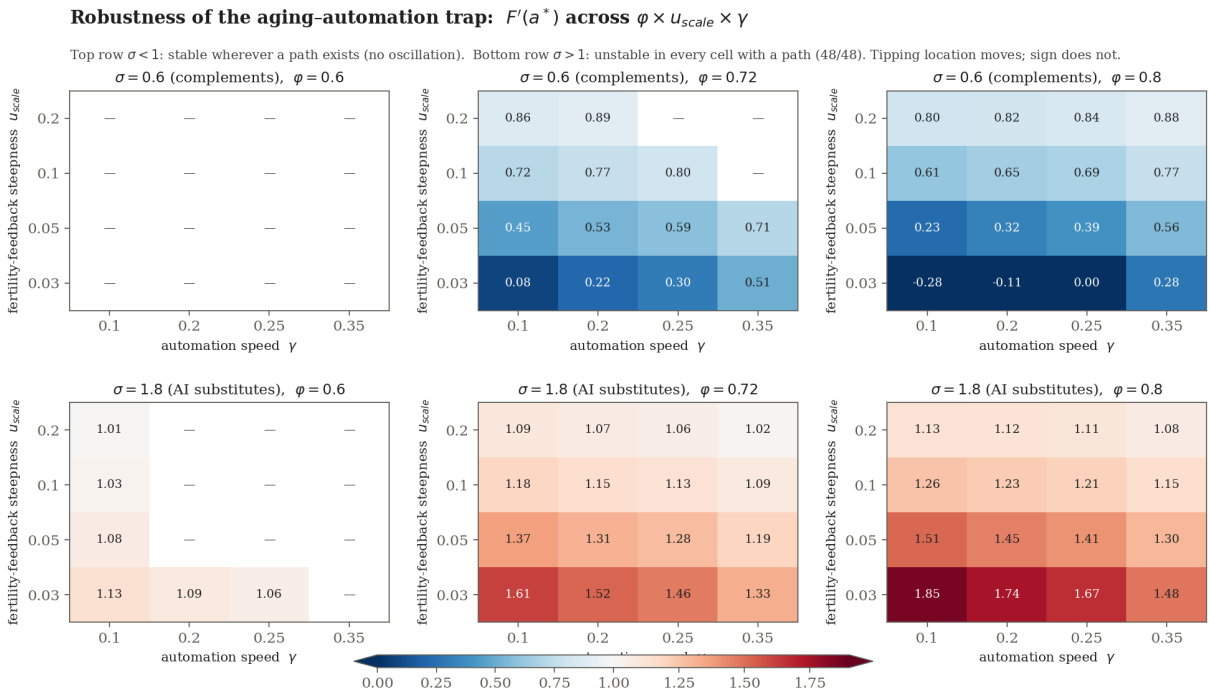


Figure 5: Robustness of $F'(a^*)$ over $\varphi \times u_{\text{scale}} \times \gamma$. Top row ($\sigma < 1$): stable wherever a balanced path exists, with no oscillatory case. Bottom row ($\sigma > 1$): unstable in every cell with a path. The calibration moves the tipping location, not the sign of $F'(a^*) - 1$.

6 Transfer arrangements when the labor-income base erodes

The model isolates a financing problem that sharpens under substitution. Consumption demand is financed out of labor income, $C^d = \varphi w L$, while the consumption of dependants and retirees is part of what that income must cover—the lifecycle deficit, in the language of National Transfer Accounts. When $\sigma > 1$, aging compresses the labor share (Lemma 1), so the working-age labor-income surplus that funds the deficit shrinks exactly as the dependency burden rises. This section asks, on positive grounds only, how two stylized transfer arrangements behave in that environment. We do not evaluate them against a welfare criterion or recommend either; we report their model-implied effects and leave the comparison to the reader.

6.1 Two tax bases

Both arrangements raise revenue T per worker and rebate it to non-working cohorts, leaving the production side unchanged. They differ only in the tax base.

(A) Labor-income (pay-as-you-go) transfer. $T_A = \tau_A \, w = \tau_A \, s_L(k) \, y$. Because the base is labor income, T_A inherits the dynamics of the labor share.

(B) Capital/automation-income transfer. $T_B = \tau_B\, Rk = \tau_B\, s_K(k)\, y$, whose base is the capital share. This is the formal counterpart of a transfer indexed to automation rents; it is treated here purely as one modeled option among others, not as a proposal.

6.2 Positive comparison

The contrast is governed by the same elasticity that drives the rest of the paper.

Base dynamics under aging. Aging raises k. By Lemma 1, for $\sigma > 1$ the labor share $s_L(k)$ falls and the capital share $s_K(k)$ rises. Holding rates fixed, the base of arrangement (A) contracts in the very region where the gap widens, while the base of arrangement (B) expands. For $\sigma < 1$ the comparison reverses. The two arrangements are thus not ranked independently of σ.

Effect on the consumption-demand gap. Rebating revenue to non-working cohorts who consume it adds to consumption demand. Writing realized consumption demand inclusive of transfers as $\varphi wL + T$, the gap becomes $g/Y = (1-s) - \varphi s_L - (T/y)$. The gap-closing term is $\tau_A s_L$ under (A) and $\tau_B s_K$ under (B). For $\sigma > 1$ and aging, the (A) term shrinks with s_L while the (B) term grows with s_K: at fixed rates, the labor-income transfer does less to offset a widening gap precisely when the gap is widening, whereas the capital-base transfer does more. The sign is reversed for $\sigma < 1$.

Effect on the dynamic margin. In the dynamic extension, transfers enter through realized household income and hence fertility (10). A transfer that sustains non-working consumption supports utilization u and fertility; this raises n' at the steady state and so lowers $F'(a^*)$ toward the stable region. Whether either arrangement can move $F'(a^*)$ below one—restoring local stability when $\sigma > 1$—depends on the transfer rate and on the same condition that appears in Proposition 3. We report this as a comparative-statics direction, not as a stabilization result: the model does not deliver, and we do not assert, that any particular rate restores stability.

6.3 What the comparison does and does not establish

The positive content is a single conditional statement: *the relative behavior of the two transfer bases under aging is governed by* $\operatorname{sign}(\sigma - 1)$—the labor-income base is the one that mechanically does not contract when factors are complements, and the capital/automation base is the one that mechanically does not contract when they are substitutes, where "does not contract" refers only to the movement of the base as the gap widens. This is a statement about model mechanics, not about desirability. A normative ranking would require a welfare criterion, incidence and behavioral-response assumptions, and considerations—political economy, administration, intergenerational fairness—that the model does not contain. We deliberately stop at the positive comparison.

7 Forward-looking research questions

The framework is a starting point rather than a settled account. We close by identifying questions that the analysis brings into focus.

1. **The empirical position of σ.** Everything turns on whether the effective elasticity of substitution between automation capital and labor exceeds one, and it is worth stating plainly that most existing estimates place the aggregate capital–labor elasticity *below* unity. Two considerations qualify how that evidence bears on the mechanism here. First, the relevant object is not the economy-wide elasticity between all capital and all labor but the elasticity along the specific margin at which automation capital substitutes for the tasks labor performs; task-based analyses suggest this margin can be considerably more elastic than the aggregate, and that its effective value rises as the set of automatable tasks expands. Second, the aggregate elasticity is itself a composite that can drift toward and through unity as the composition of the capital stock shifts toward automation. The paper is therefore explicitly conditional: for $\sigma < 1$ its mechanism runs in reverse and aging *narrows* the gap, so the substitution case is a possibility whose empirical relevance is an open question, not a maintained assumption. What is the relevant σ for the automation-versus-labor margin, how is it best identified, and is it moving through one over time?

2. **Locating the threshold.** In the benchmark the static reversal is exact at $\sigma = 1$, but richer closures move it. How far, and in which direction, does the threshold shift under empirically disciplined closures?

3. **From local instability to global dynamics.** Proposition 3 establishes local instability for $\sigma > 1$ but not global runaway. What investment behavior—in particular, what dependence of automation accumulation on expected demand and returns—determines whether the repelling threshold leads to a bounded low-utilization region or to sustained divergence?

4. **The strength of the demand brake.** The weak automation rule shows that a demand-responsive brake postpones but, within the range examined, does not remove the threshold. Is there an empirically plausible strength of the demand brake at which stability is restored, and what would generate it microeconomically?

5. **Measuring the gap with transfer accounts.** The consumption-demand gap maps onto the erosion of the labor-income surplus in National Transfer Accounts. Can the gap, and its response to automation, be measured directly using NTA-style age profiles?

6. **Care labor and the limits of substitution.** Some labor—notably care of the old and young—resists automation. How does a low-substitutability sector interact with a high-σ production sector, and does its presence bound the trap or merely relocate it?

7. **Migration as a partial offset.** Migration adjusts the labor force without the one-generation lag that constrains fertility. To what extent can it relax the timing asymmetry between fast-adjusting capital and slow-adjusting population, and under what conditions does it move a substitution economy back across the threshold?

These questions are positive and empirical. The paper's contribution is to show that a single elasticity organizes them—fixing both the sign of aging's effect on effective demand and the stability of the population–production feedback—and thereby to suggest where measurement and further modeling would be most decisive.

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A Robustness of the dynamic threshold

This appendix reports the admissible parameter range for the dynamic extension and the linearized slope $F'(a^*)$ across the calibration grid, complementing Remark 5.3 and the existence caveat in Section 5.

Existence of a balanced-growth path. For $\sigma < 1$ the CES capital–output ratio is bounded below by $\alpha^{-1/\rho}$, so a positive balanced-growth path with the benchmark saving rule exists only when the consumption pass-through φ is large enough that the implied k/y stays above this bound; with low φ and strong complementarity no interior path exists. For $\sigma \geq 1$ no such lower bound binds. Across the grid below, an interior balanced path exists in 29 of the 48 cells for $\sigma = 0.6$ and in 38 of the 48 cells for $\sigma = 1.8$; the stability statements concern only the cells in which a path exists.

Linearized slope across the grid. Table 1 reports the range of $F'(a^*)$ over the grid $u_{\text{scale}} \in \{0.03, 0.05, 0.10, 0.20\}$, $\gamma \in \{0.10, 0.20, 0.25, 0.35\}$, $\varphi \in \{0.60, 0.72, 0.80\}$ (exogenous-growth specification). For $\sigma < 1$ every cell with a path is locally stable and none is oscillatory ($F' > -1$ throughout); for $\sigma > 1$ every cell with a path is locally unstable.

	cells with path	$\min F'(a^*)$	$\max F'(a^*)$	all in $(-1, 1)$	all > 1
$\sigma = 0.6$ (complements)	29/48	-0.281	0.893	yes	—
$\sigma = 1.8$ (substitutes)	38/48	1.013	1.854	—	yes

Table 1: Range of the linearized map slope $F'(a^*)$ over the calibration grid. The sign of $F'(a^*) - 1$ is invariant across the grid; the calibration moves the tipping location, not the verdict. No $F' < -1$ (oscillatory) case arises for $\sigma < 1$.

Weak-rule grid. Under the demand-braked rule (11), sweeping $\gamma_1 \in \{0.2, 0.3, 0.5\}$ and $\gamma_2 \in \{0.1, 0.3, 0.5, 0.8\}$ leaves the verdict unchanged: all cells are locally unstable for $\sigma = 1.8$ ($F'(a^*) \in [1.28, 1.39]$) and locally stable for $\sigma = 0.6$ ($F'(a^*) \in [0.41, 0.58]$). The demand brake postpones the tipping location but does not, within this range, remove it.

References

[1] Acemoglu, Daron, and Pascual Restrepo. 2018. "The Race between Man and Machine: Implications of Technology for Growth, Factor Shares, and Employment." *American Economic Review* 108 (6): 1488–1542. https://doi.org/10.1257/aer.20160696.

[2] Acemoglu, Daron, and Pascual Restrepo. 2019. "Automation and New Tasks: How Technology Displaces and Reinstates Labor." *Journal of Economic Perspectives* 33 (2): 3–30. https://doi.org/10.1257/jep.33.2.3.

[3] Aksoy, Yunus, Henrique S. Basso, Ron P. Smith, and Tobias Grasl. 2019. "Demographic Structure and Macroeconomic Trends." *American Economic Journal: Macroeconomics* 11 (1): 193–222. https://doi.org/10.1257/mac.20170114.

[4] Auclert, Adrien, Hannes Malmberg, Frédéric Martenet, and Matthew Rognlie. 2021. "Demographics, Wealth, and Global Imbalances in the Twenty-First Century." NBER Working Paper No. 29161. https://doi.org/10.3386/w29161.

[5] Karabarbounis, Loukas, and Brent Neiman. 2014. "The Global Decline of the Labor Share." *Quarterly Journal of Economics* 129 (1): 61–103. https://doi.org/10.1093/qje/qjt032.

[6] Lee, Ronald, and Andrew Mason, eds. 2011. *Population Aging and the Generational Economy: A Global Perspective*. Cheltenham, UK: Edward Elgar; Ottawa: IDRC.

[7] Lutz, Wolfgang, Vegard Skirbekk, and Maria Rita Testa. 2006. "The Low-Fertility-Trap Hypothesis: Forces That May Lead to Further Postponement and Fewer Births in Europe." *Vienna Yearbook of Population Research* 2006: 167–192.

[8] Myrskylä, Mikko, Hans-Peter Kohler, and Francesco C. Billari. 2009. "Advances in Development Reverse Fertility Declines." *Nature* 460: 741–743. https://doi.org/10.1038/nature08230.

[9] Summers, Lawrence H. 2014. "U.S. Economic Prospects: Secular Stagnation, Hysteresis, and the Zero Lower Bound." *Business Economics* 49 (2): 65–73. https://doi.org/10.1057/be.2014.13.