RECONCILING AFFIRMATIVE ACTION AND MERIT: EVIDENCE AND DESIGN FROM INDIA

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ABSTRACT: This paper analyzes India's affirmative action system through the lens of market design, identifying a fundamental conflict between the Supreme Court rulings of Indra Sawhney (1992) and Ashoka Kumar Thakur (2008). While the former established the "over-and-above" principle and "within-category fairness," the latter introduced a specific cutoff mandate and de-reservation recommendation. We establish that these legal requirements are jointly incompatible, as no assignment rule can satisfy all resulting axioms simultaneously. By exploring alternative relaxations, we identify a family of rules supporting soft reservations, proving that the Forward Transfer rule is merit-dominant. Finally, we propose a reinterpreted cutoff policy that resolves legal inconsistencies and eliminates endogenous feedback, offering a superior meritocratic rule ready for immediate institutional implementation.

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1. INTRODUCTION

Market design has informed and improved resource allocation and exchange in a range of consequential real-world settings. From the assignment of doctors to hospitals and children to daycare centers in Japan (Kamada and Kojima, 2015, 2024) to the assignment and exchange of teachers in France and Italy (Combe, Tercieux, and Terrier, 2022; Combe, Dur, Tercieux, Terrier, and Ünver, 2025; Cavallo and Dogan, 2024), carefully designed theoretical mechanisms have been shown to generate substantial welfare gains for market participants. We contribute to this literature by studying India's reservation system, whose extraordinary complexity has generated persistent confusion, legal debate, and policy ambiguity at the highest levels of Indian governance.

A central question in market design is how to allocate scarce resources when society values both meritocratic selection and diversity in the resulting allocation (Imamura (2025), Hafalir, Kojima, and Yenmez (2025), and Hafalir, Kojima, Yenmez, and Yokote (2026)). In many high-stakes environments—such as school choice, college admissions, and public-sector recruitment—allocative institutions are asked to reward performance while also expanding access for historically excluded groups.

These objectives are often treated as competing ethical ideals, but from a design perspective, they are also competing operational constraints: once diversity goals are encoded through quotas, reserves, or priority rules, they interact in nontrivial ways with merit-based rankings, eligibility thresholds, and the filling of available positions. The relevant economic problem is therefore not whether diversity should come at the expense of meritocracy in the abstract, but how institutional rules can reconcile these goals without generating avoidable inefficiency, arbitrariness, or legal incoherence.

This tension is especially sharp in India, which operates one of the world's most extensive affirmative action systems and where the legal framework simultaneously seeks to redress historical disadvantage and preserve standards of excellence. Our paper studies this tradeoff through the lens of market design and shows that, in this setting, the joint

pursuit of diversity and meritocracy imposes fundamental constraints on implementable allocation rules.

India is the world's most populous country, with a population exceeding 1.4 billion people, of whom approximately 800 million are of working age.[1] The scale of competition for positions subject to India's reservation system is staggering. Each year, tens of millions of candidates apply for central and state government employment: the Staff Selection Commission (SSC) and the Union Public Service Commission (UPSC) alone receive tens of millions of applications annually, and individual examinations—such as the Railway Recruitment Board (RRB) exams—regularly attract upward of 20–30 million applicants.[2] Similarly, admissions to centrally funded higher education institutions attract millions of applicants annually. Since reservations apply to a constitutionally mandated 59.5% of available seats at these institutions, India's affirmative action program directly determines the life outcomes of tens of millions of individuals annually. These figures underscore that the design of India's reservation system is not merely a theoretical concern, but one of the most consequential policy problems affecting human welfare at scale.

1.1. *Historical and Legal Institutional Details*

Reservation policies in India were initially implemented to ensure the representation of historically discriminated Scheduled Castes (SC) and Scheduled Tribes (ST) in public education and employment. Articles 15(4) and 16(4) of the Indian Constitution provided reservations for SCs and STs in educational institutions and government jobs. In 1954, the Ministry of Education suggested that 20% reservation for SCs and STs in educational institutions, with 15% for SCs and 5% for STs. The percentage of SC and ST reservations was increased to 15% and 7.5%, respectively, in central government jobs in 1982.

[1] See https://data.worldbank.org for details.

[2] See ssc.nic.in and upsc.gov.in/upsc-annual-reports for annual reports.

Reservations for Other Backward Classes (OBCs) were formally introduced in 1987, following the recommendations of the Mandal Commission, which suggested a 27% reservation for OBCs in government jobs. There was strong opposition from different segments of society against OBC reservations.[3] Shortly afterward, OBC reservations were challenged in court. In 1992, the Supreme Court of India (SCI), in the landmark case Indra Sawhney vs. Union of India (henceforth Indra Sawhney (1992)), upheld the 27% reservation to OBCs.[4] Reservations for OBCs still remain the most controversial in Indian society and have been challenged multiple times in the SCI after Indra Sawhney (1992).

Indra Sawhney (1992) refers to reservations for SC, ST, and OBC as **vertical** reservations.[5] The unreserved positions are classified as **open-category** and are accessible to all applicants, including those in vertical reserve categories. Applicants who do not identify with SC, ST, or OBC are classified in the **general category**. Indra Sawhney (1992) considers reservations for SC, ST, and OBC as **hard reserves** in the sense that the positions left from the SC, ST, and OBC categories cannot be provided to others.

Indra Sawhney (1992) established three fundamental principles..

(1) The **over-and-above** (OA) principle: Applicants with the highest scores must be admitted through the open-category.

(2) The **within-category fairness** (WCF) principle: If an applicant with a lower score obtains a position, then an applicant from the same category with a higher score must also obtain a position.

[3]Although this issue covers deep-rooted social and political dimensions, we maintain a neutral position, focusing solely on a theoretical analysis of reservations in India rather than offering personal opinions.

[4]The case can be accessed at https://indiankanoon.org/doc/1363234/. Last accessed on 04/02/2025. Sönmez and Yenmez (2022) analyzes Indra Sawhney (1992) from the matching theoretical perspective and formulates principles laid out in Indra Sawhney as formal mathematical axioms.

[5]There are also **horizontal reservations**—such as those for women or people with disabilities. These reservations intersect with vertical categories and introduce additional layers of complexity in the allocation process (Sönmez and Yenmez (2022)). To maintain analytical clarity and focus on the interaction between vertical reservation policies and endogenous eligibility constraints, we abstract from horizontal reservations in the present analysis. This simplification is without loss of generality for our main results: the model, axiomatic framework, and characterization results can be extended in a straightforward manner to incorporate horizontal reservations by refining the assignment rules.

(3) The **quota-filling subject to eligibility** (QFE) principle: In each category, all positions must be assigned as long as there are eligible applicants.

By admitting high-scoring applicants from vertical reserve categories into the open-category, the OA principle ensures that positions earmarked for reserved categories can provide an extra boost to applicants from these categories. The WCF principle is a fairness criterion that requires selecting candidates based on merit scores in each category. Finally, the QFE principle is a mild efficiency requirement.

When reserves are considered a **hard constraint**, the three principles above characterize the following simple assignment rule, which has been used in India for decades.

> *First, applicants are selected for open-category positions one at a time following the merit score ranking up to the capacity of the open-category. Then, applicants are selected for vertical reserve categories following the merit score ranking of applicants in each respective category up to their capacities.*

We refer to this assignment procedure as the "*Indra Sawhney (IS) assignment rule* ." In Theorem 0, we show that an assignment rule satisfies the OA, WCF, and QFE properties if and only if it is the Indra Sawhney assignment rule.[6] This characterization is the starting point of our axiomatic analysis.

The judicial challenges to reservations for OBCs persisted well beyond the Supreme Court's ruling in the Indra Sawhney case. A significant development occurred in 2006 when the constitutional validity of the **Central Educational Institutions (Reservation in Admission) Act** was challenged in the historic case Ashoka Kumar Thakur vs. Union of India (2008) (henceforth referred to as Ashoka Kumar Thakur (2008)). 27% reservation for OBCs in central educational institutions was challenged, once again, in this historic case. In its judgment, the SCI modified the eligibility criteria for OBC positions with the explicit aim of **maintaining the standard of excellence**. The Court specifically mandated that the minimum qualifying score for candidates in the OBC category must not fall by more than 10 points below that established for the open-category.

[6] Sönmez and Yenmez (2022) also provides a similar characterization in plain English without formalization and proof. Notably, they provided a verbal description of the OA principle that differs from our definition.

> *It is reasonable to balance reservation with other societal interests. To maintain standards of excellence, cutoff marks for OBCs should be set not more than 10 marks out of 100 below that of the general category.*[7]

In effect, the Court restricts the set of OBC applicants who are deemed eligible to obtain a position in the OBC category by making the OBC cutoff score endogenous, i.e., a function of the cutoff score of the open-category.

Recognizing the inefficiencies that could result from this elevated threshold, **one of the SCI Justices** at the time, **Dalveer Bhandari**, suggested the implementation of **flexible** (or "**soft**") reservations for the OBC category as a remedial measure in Ashoka Kumar Thakur (2008), as his personal opinion. His approach would allow the reallocation of unfilled OBC positions to candidates from other categories. A significant excerpt from the Ashoka Kumar Thakur (2008) judgment, in the section that articulates Justice Bhandari's personal opinion and suggestions for the future, articulates this position as follows.[8]

> *Only non-creamy layer OBCs can avail of reservations in college admissions, and once they graduate from college, they should no longer be eligible for post-graduate reservation. 27% is the upper limit for OBC reservations. The Government need not always provide the maximum limit. Reasonable cutoff marks should be set so that the standards of excellence are not significantly compromised. The unfilled seats should revert to the general category.*[9]

Justice Bhandari explains the connection of the OBC cutoff mandate and his recommendation of flexible reservation suggestion on page 235 of the same judgment as follows.

[7]There is widespread confusion in India about the type of position and the types of applicant. A cursory search on the Internet reveals that the term "general seats" is used to describe open-category positions. The same confusion also arises in SCI verdicts. In the above quote, by "general category," they mean the "open-category." Aygün and Turhan (2022) shows that there are significant unintended consequences if the remaining OBC positions are **only** provided to students in the general category. Consistent with our interpretation, vacant OBC positions have been assigned as an open-category in admissions to technical universities. See Baswana, Chakrabarti, Chandran, Kanoria, and Patange (2019) for further details.

[8]This excerpt can be found on page 165 of the judgment.

[9]The same confusion we pointed out in Footnote 4 applies here. The unfilled seats should revert to the "open-category" is what Justice Bhandari meant. See Aygün and Turhan (2022) for more on this point.

> *To this end, the Government shall set up a committee to look into the question of setting the OBC cut off at nor more than 10 marks below that of the general category. Under such a scheme, whenever the non-creamy layer OBCs fail to fill the 27% reservation, the remaining seats would revert to general category students.*[10]

However, no procedural framework has been provided in the judgment or any other legal document for simultaneously implementing the mandated cutoff-score differential between the open-category and OBC, as well as the recommended soft reservation policy for OBC positions. The soft reservation recommendation for the OBC category advanced by Justice Bhandari has been adopted in public university admissions through piecemeal administrative action, in the absence of a formally legislated implementation mechanism. The absence of explicit judicial directives on the implementation of soft reservations for OBCs presents an opportunity for innovative policy design.

Although OBC de-reservation was initially articulated as the recommendation of a Supreme Court justice, its implementation has since become prevalent throughout India. However, the absence of explicit guidelines for implementing OBC de-reservation has led to inconsistent practices across India. The ambiguity mainly concerns two critical aspects: (1) whether the reverted OBC positions should be exclusively allocated to candidates in the general category or made available to all applicants, including those in reserved categories; and (2) the appropriate sequencing to fill these reverted positions in the overall allocation process, specifically, whether they should be filled before or after processing the reserve categories. Both aspects may have significant distributional consequences. [11]

[10] By “general category" in the first sentence, Justice Bhandari means the “open-category.

[11] The admission process to engineering colleges in India was reformed in 2015 (Baswana, Chakrabarti, Chandran, Kanoria, and Patange (2019)). The design team identified OBC de-reservations as a primary challenge. The procedure was made clear that reverted OBC positions are relabeled as open-category. Policymakers required the open-category cutoff score to be higher than those of reserve categories. Given that open-category positions are allocated prior to OBC positions by law due to the OA principle, transferring vacant OBC slots to open-category while preserving the desired cutoff score ranking was particularly challenging. The design team solved the OBC de-reservation through a creative approach: re-executing the student-proposing deferred acceptance (DA) algorithm of Gale and Shapley (1962) by updating the capacities of open and OBC positions.

1.2. *Contributions and Overview of Results*

This paper contributes to the market design literature on reserve systems by identifying a previously unstudied design margin: the joint determination of eligibility cutoffs and de-reservation policy. Standard analyses of reserves have focused either on reserve sizes, precedence orders, or the hard-versus-soft nature of protected seats. By contrast, our framework shows that when eligibility for a reserved category is itself defined relative to the open-category cutoff, cutoff determination and seat reversion cannot be analyzed separately. In the Indian affirmative action system, the open-category cutoff endogenously interacts with the treatment of unfilled OBC seats, so that de-reserving those seats as open-category positions feeds back into who is eligible for OBC assignment in the first place. We formalize this interaction using **assignment rules** that record not only who is selected but also under which category they are assigned, prove an impossibility result for the joint implementation of the legally salient axioms, and then characterize the constrained-optimal resolutions on either side of the tradeoff. In particular, we identify the unique rule implementing the endogenous-cutoff mandate without de-reservation, characterize the class of soft-reserve rules when cutoff endogeneity is relaxed, show that the Forward Transfer rule merit-dominates every rule in that class, and finally establish that a fixed-cutoff reinterpretation strictly improves upon all of them. The broader theoretical lesson is that in reserve systems with soft seats, cutoff rules are not ancillary implementation details; they are part of the mechanism itself, and their interaction with transfer policy is a first-order determinant of merit, feasibility, and legal implementability.

A central theme running through both landmark judgments is the SCI's persistent preoccupation with **meritocratic standards**. In *Ashoka Kumar Thakur* (2008), the Court explicitly conditioned OBC eligibility on proximity to the open-category cutoff score, invoking the imperative to "maintain the standard of excellence" as the constitutional justification for restricting access to reserved seats. This judicial emphasis on merit provides the normative basis for our primary criterion for comparing assignment rules. Modifying the criterion introduced in Gale (1968), we say that a set of individuals *I merit-dominates* a set

J if there exists an injection from J into I such that every matched individual in I scores weakly higher than her counterpart in J, with at least one strict improvement. We extend this criterion to choice rules: a choice rule C *merit-dominates* C' if, for every applicant pool, C selects a set that merit-dominates the set selected by C', or selects a weakly larger set.

Theorem 1 establishes that the concurrent implementation of the OBC cutoff score mandate and the recommended OBC de-reservation is fundamentally incompatible with the principles established in *Indra Sawhney* (1992): no assignment rule simultaneously satisfies the OA principle, WCF, mQFE, and OBC de-reservation.

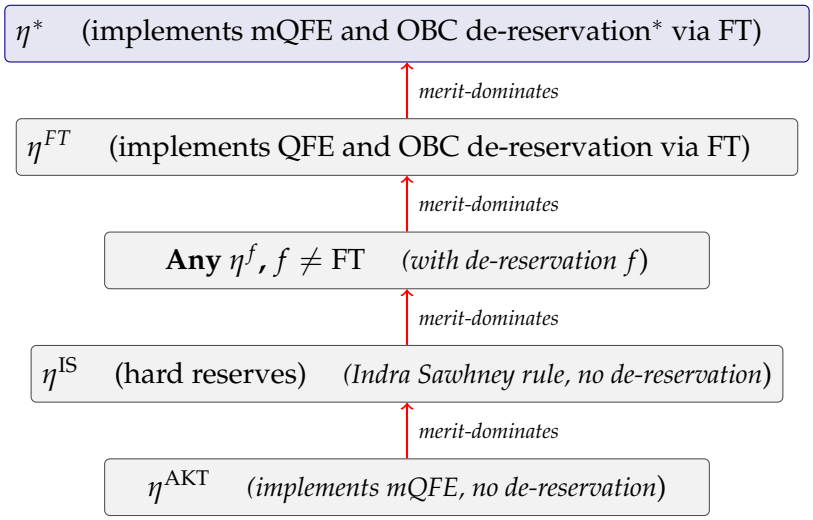
There is a natural way to implement the mandated mQFE principle in conjunction with the OA principle and WCF. We refer to this assignment rule the *Ashoka Kumar Thakur (AKT) assignment rule*, η^{AKT}. In Theorem 2, we show that η^{AKT} is the unique rule satisfying the OA principle, WCF, and mQFE. However, the choice rule C^{AKT} is merit-dominated by the choice rule C^{IS} induced by the assignment rule η^{IS} due to the fact that C^{AKT} may be severely wasteful. Thus, it is a worsening of the Indra Sawhney assignment rule η^{IS} because it can be vastly wasteful. Proposition 1 states that the choice rule induced by η^{AKT} is consistent and satisfies the substitutes property. However, this rule violates the size monotonicity.

We propose a family of assignment rules that satisfy the OA principle, WCF, and QFE established in the Indra Sawhney (1992), while accommodating OBC de-reservations. Assignment rules within this family differ primarily by the specific point in the precedence order at which unfilled positions from the OBC category are reallocated as open-category. Theorem 3 proves that among all assignment rules in this family, the choice rule induced by the Forward Transfer (FT) assignment rule η^{FT} merit-dominates choice rules induced by any other assignment rule in this family. Since η^{FT} consistently selects the most meritorious cohorts possible while respecting the core legal principles, it represents the optimal implementation if maintaining the standard of excellence is the objective. Moreover, Theorem 4 shows that every assignment rule in this family induce a choice rule that satisfies

the three critical conditions in the market design literature: the subsitutes property, consistency, and size monotonicity.

The impossibility result in Theorem 1 arises from the relabeling of de-reserved OBC positions as open-category positions. When OBC positions are reverted and reassigned as open-category, it alters the eligibility threshold for the OBC category and affects the set of applicants who qualify for OBC positions. An alternative interpretation of the cutoff requirement eliminates this source of endogeneity. Specifically, define the open-category cutoff solely with reference to the **original** open-category positions, i.e., as the score of the lowest-ranked applicant among the q_s^o original open-category slots. Under this formulation, reverted positions do not influence the open-category cutoff, thereby shutting down the endogenous feedback mechanism that gives rise to the incompatibility.

With this minor change, we design an assignment rule η^* that merit-dominates every assignment rule satisfying the OA principle, WCF, and mQFE under any de-reservation scheme (Theorem 6). Moreover, Theorem 5 shows that η^* merit-dominates the FT rule η^{FT}. By eliminating the endogenous feedback between de-reservation and cutoff determination, this reinterpretation resolves the incompatibility identified in Theorem 1 while achieving strictly greater merit-based selection than the FT rule. The resulting merit ranking of these rules is summarized as follows.



All assignment functions in the summary satisfy the OA principle and WCF. In each of them, a version of QFE is implemented. η^* implements rQFE, η^f, for any f including $f = FT$ and η^{IS} implement QFE. η^{AKT} implements mQFE.

1.3. *Empirical Motivation: Evidence on Unfilled OBC Positions*

Vacancies in the OBC category are not only theoretical but also empirically significant in both higher education and public employment in India. We document evidence on the prevalence of unfilled OBC positions *even when there is no cutoff score restriction*.

According to official data provided by the Government of India to the Rajya Sabha, Central Universities reported 1,543 vacant seats reserved for OBC as of February 2023.[12] This trend extends to other protected groups, with 878 vacancies for Scheduled Castes (SC) and 520 for Scheduled Tribes (ST); in aggregate, the total category-specific vacancies exceeded 6,000. These figures suggest that vacant reserved positions remain a quantitatively significant challenge, even within centralized high-demand allocation systems.

This phenomenon is further evidenced by the architectural design of engineering admissions. For example, the mechanism currently in use to match students with engineering colleges explicitly incorporates an iterative reallocation process, by which vacant OBC seats are transferred to the open category to minimize institutional waste.[13]

Evidence from public sector employment further supports this observation. A written response in the Lok Sabha reports that, across ten major ministries and departments, there were 40,562 OBC vacancies during the period 2012–2016, of which 13,535 remained unfilled as of January 2017.[14] Subsequent parliamentary committee reports indicate that

[12]The Rajya Sabha, officially known as the Council of States, is the upper house of the Parliament of India, representing India's states and union territories in the national legislative process.
See https://sansad.in/getFile/annex/259/AU3813.pdf?source=pqars for the report.

[13]See Baswana, Chakrabarti, Chandran, Kanoria, and Patange (2019).

[14]The Lok Sabha is the lower house of the Parliament of India and a central institution of Indian democracy. Also known as the "House of the People," it consists of directly elected representatives who exercise legislative authority and oversight over the executive branch of the Government of India. Please see https://sansad.in/getFile/loksabhaquestions/annex/171/AU4053.pdf for the report.

the stock of unfilled OBC positions remains large: as of January 2024, 22,286 OBC vacancies persisted in central government employment. These figures demonstrate that the underutilization of OBC positions is not episodic but persists over time.

These observations motivate our focus on the interaction between endogenous eligibility thresholds and the reallocation of unfilled OBC positions, which, we show, gives rise to fundamental incompatibilities in the design of assignment rules.

1.4. *Practical Implementability of the Proposed Assignment Rules*

An important advantage of the assignment rules η^* and η^{FT} are their practical implementability within existing allocation systems in India. These rules operate on primitives already available in current applications and do not require additional information. They can be incorporated into existing centralized platforms without changes to reporting requirements or institutional infrastructure. The mechanisms we propose are thus not only theoretically well-founded but also directly deployable.

From a computational perspective, all procedures can be implemented using simple sequential selection algorithms, with complexity comparable to the baseline procedures currently used in centralized admissions and public-sector recruitment.

1.5. *Structure of the Paper*

The paper proceeds as follows. Section 2 formulates the model and assignment rules. Section 3 translates the principles established in Indra Sawhney (1992) into formal mathematical axioms and characterizes the choice rule introduced in that judgment. Section 4 examines the Ashoka Kumar Thakur (2008) judgment, formalizing the mandated cutoff score requirement as an axiom and incorporating Justice Bhandari's recommendation on OBC de-reservation. Section 5 proposes new assignment rules as potential resolutions to the incompatibility between the two judgments. Section 6 introduces a minor reinterpretation of the cutoff requirement that simultaneously resolves the impossibility and achieves strict merit gains over all rules designed in Section 5. Section 7 discusses the related literature and Section 8 concludes. All proofs are relegated to the Appendix.

2. PRELIMINARIES

Consider an institution s with $\overline{q}_s$ positions. Let $R = \{SC, ST, OBC, EWS\}$ be the set of reserve categories. We denote by q_s^r the number of positions reserved for category $r \in R$ such that $\sum_{r \in R} q_s^r < \overline{q}_s$. We let $q_s^o = q_s - \sum_{r \in R} q_s^r$ be the number of open (unreserved) positions. The vector $\mathbf{q_s} = \left(q_s^o, q_s^{SC}, q_s^{ST}, q_s^{OBC}, q_s^{EWS}\right)$ denotes the initial allocation of positions in categories. Let I denote the set of all applicants.

The function $t : I \to \{g\} \bigcup R$ denotes the category membership of the applicants, where g denotes the "general category." $t(i) = g$ means that individual i does not belong to any vertical reserve category, or she did not report her category membership. In India, it is **optional** to report membership in the reserve categories. Members of the reserve categories who do not report their membership are considered general category applicants and are eligible only for open-category positions. Members of the reserve category $r \in R$ are eligible for both the open-category and reserved category r positions. We let $T = (t(i))_{i \in I}$ denote the profile of the applicants' reserve category membership. Let $\mathcal{T}$ denotes the set of all profiles of the applicants' reserve-category membership.

The function $\theta : I \to \mathbf{R}_+$ denotes the test scores of the applicants. $\theta(i) > \theta(j)$ means that applicant i has a higher test score than that of j. Applicants' merit scores induce a priority ranking for the institution s. We denote this linear order over $I \cup \{\emptyset\}$ by $\succ_s$.[15] We write $i \succ_s j$ to mean that the applicant i has higher priority (i.e., a higher merit score) than the applicant j in institution s. $i \succ_s \emptyset$ means that the applicant i is acceptable for institution s. Similarly, we write $\emptyset \succ_s i$ to say that applicant i is unacceptable for institution s.

The priority ranking for applicants of type $r \in R$, denoted by $\succ_s^r$, is obtained from $\succ_s$ in a straightforward way as follows:

(i) for $i, j \in A$ such that $t(i) = r$, $t(j) \neq r$, $i \succ_s \emptyset$, and $j \succ_s \emptyset$, we have $i \succ_s^r \emptyset \succ_s^r j$,[16]

(ii) for any other $i, j \in A$, $i \succ_s^r j$ if and only if $i \succ_s j$.

[15] In India, when two or more applicants have the same score, ties are broken with a given objective criterion.

[16] $\emptyset \succ_t^s j$ which means applicant j is unacceptable for category r at s.

The former property means that each category $r \in R$ is exclusive. That is, applicants who do not belong to category $r \in R$ are deemed unacceptable for positions in category-r. The latter states that the ranking $\succ_s$ is preserved among the applicants in category-r.

Given the set of applicants A and applicant $i \in A$, let $rank_A(i)$ be the rank of applicant i in the set A with respect to the merit ranking $\succ_s$. That is, $rank_A(i) = k$ if and only if $\mid \{j \in A \mid j \succ_s i\} \mid = k-1$. Let $A^r = \{i \in A \mid i \succ_s^r \emptyset\}$ be the set of applicants in A who are eligible for category $r \in R$.

2.1. *Institutions' Choice Rules*

Conventionally, institutions' selection from a given set of applicants under affirmative action constraints is modeled via *choice rules*. Choice rules, which select a subset from a given set of applicants, are insufficient for the problems we consider because they do not specify the position types under which applicants are selected.

Mathematically, for all $A \subseteq I$, a choice rule C is such that $C(A) \subseteq A$ and $\mid C(A) \mid \leq q$, where q denotes the number of positions. A choice rule $C : 2^A \longrightarrow 2^A$ satisfies the **substitutes property** if, for all $i, j \in I$ and $A \subseteq I \setminus \{i, j\}$,

$$j \notin C(A \cup \{j\}) \implies j \notin C(A \cup \{i, j\}).$$

A choice rule C is **consistent** if, for any $A \subseteq I$ and any $i \in I \setminus A$,

$$i \notin C(A \cup \{i\}) \text{ implies } C(A) = C(A \cup \{i\}).$$

A choice rule C satisfies **size monotonicity**[17] if, for all sets of individuals $A \subseteq I$ and all individuals $i \in I \setminus A$, we have

$$\mid C(A) \mid \leq \mid C(A \cup \{i\}) \mid .$$

[17]It is well-known that size monotonicity guaranties the strategy-proofness of the deferred acceptance mechanism when institutions' choice rules satisfy it along with substitutability.

2.2. *Institutions' Assignment Rules*

The choice rules tell only the set of admitted applicants from a given set. However, they do not specify the position types under which they are selected. In India, the type of positions to which applicants are assigned is made public. As such, we define an *assignment rule* that specifies the type of position to which individuals are assigned.

DEFINITION 1. *An **assignment rule** η is a function where, given a set of applicants A and a category membership profile T,*

$$\eta(A,T) = \{(i,\tau) | i \in A \text{ and } \tau \in \{o\} \cup R\} \text{ such that}$$

(i) for any $i \in A$, we have at most one $(i,\tau) \in \eta(A,T)$, where

$$\begin{cases} \tau = o & \text{if } t(i) = g, \\ \tau \in \{t(i), o\} & \text{if } t(i) \in R, \end{cases}$$

(ii) $|\eta_s(A,T)| \leq \overline{q}_s$, *and*

(iii) $|\{(i,\tau) \in \eta(A,T) : \tau = r\}| \leq q_s^r$, *for each $r \in R$.*

Condition (i) is a feasibility requirement that says that applicants' eligibility for different position types must be respected. Condition (ii) is the standard capacity constraint. Condition (iii) states that capacities of reserve categories must be respected.

Each assignment rule η induces an associated choice rule C^η, which specifies a selected subset of any given set of applicants. That is, for any given $A \subseteq I$, $C^\eta(A) \subseteq A$. Formally, given a set of applicants $A \subseteq I$, a category membership profile T, and an assignment function η_s, the **induced choice rule** C^η is defined as

$$C^\eta(A) = \{i \in A | \exists (i,\tau) \in \eta(A,T) \text{ for some } \tau \in R \cup \{o\}\}.$$

2.3. *Merit Comparison*

Following the tradition of *Gale* (1968) we use the following criterion to compare two sets of individuals based on merit.[18]

DEFINITION 2. *The set of individuals I* merit-dominates *the set of individuals J with $\mid I \mid \geq \mid J \mid$ at institution s if there exists an injection $g : J \to I$ such that*

(1) for all $j \in J$, $\theta(g(j), s) \geq \theta(j, s)$, and,

(2) there exists $j \in J$ such that $\theta(g(j), s) > \theta(j, s)$.

The merit-based comparison of two sets can be extended to compare choice rules.

DEFINITION 3. *A choice rule C* merit-dominates *another choice rule $C^{'}$ if, for all sets of individuals $A \subseteq I$, $C(A)$ merit-dominates $C^{'}(A)$ or $C^{'}(A) \subseteq C(A)$.*

3. INDRA SAWHNEY (1992) PRINCIPLES AS AXIOMS

Indra Sawhney (1992) defines SC, ST, and OBC reservations as **hard** constraints and formulates three desiderata on the selection procedure. The first requires that open-category positions be allocated to the highest-scoring applicants up to the open-category's capacity. In doing so, positions reserved for reserved categories can be more effectively assigned to individuals who would otherwise not qualify for them without the reservation policy.

DEFINITION 4. *An assignment rule η satisfies the **over-and-above (OA) principle** if, for all $(A, T) \in 2^I \times \mathcal{T}$, (i) $\mid \{j \mid (j, o) \in \eta(A, T)\} = min\{|A|, q_s^o\}$, and (ii) for all $i \in A$ such that $rank_A(i) \leq q_s^o$, $(i, o) \in \eta(A, T)$.*

Condition (i) states that if there are fewer than q_s^o applicants, they are all accepted through the open category. Moreover, it emphasizes the fact that reserves are considered as *hard* reserves in the sense that at most q_s^o open-category positions can be assigned. Condition (ii) states that top q_s^o applicants must be assigned open-category positions.

[18]Our merit domination criterion is stronger than the Gale domination introduced in Gale (1968), Gale domination induces a reflexive binary relation, while ours is irreflexive.

The second is a fairness condition that requires respect for merit scores of individuals in the same category. If an individual with a lower score is assigned a position, then an individual who belong to the same category with higher scores must also assign a position.

DEFINITION 5. *An assignment rule η satisfies **within-category fairness** (WCF) if, for any pair of individuals $i, j \in A$ such that $t_i = t_j = t$ and $i \succ_s j$,*

$$(j, \tau) \in \eta(A, T), \text{for some } \tau \in \{o, t\} \text{ implies } (i, \tau') \in \eta(A, T), \text{for some } \tau' \in \{o, t\}.$$

The last is a mild efficiency property.

DEFINITION 6. *An assignment rule η satisfies **quota-filling subject to eligibility** (QFE) if individual i with $t(i) = r$ is not assigned, then, for all $r \in R$,*

$$|i \in A : (i, r) \in \eta(A, T)| = q^r.$$

3.1. *Indra Sawhney Assignment Rule*

Given a set of applicants A and the profile of category membership $T = (t(i))_{i \in A}$, the assignment of applicants to categories is calculated as follows:

Stage 1. Only open-category positions are considered. For each applicant $i \in A$ such that $rank_A(i) \leq q_s^o$, add (i, o) to $\eta_s^o(A, T)$. Let $C_s^o(A)$ be the set of individuals chosen.

Stage 2. Among the remaining applicants $A^1 = A \setminus C_s^o(A)$, for each reserve category $r \in R$, applicants are chosen one at a time according to $\succ_s^r$ up to capacity q_s^r. Let us call the set of applicants chosen for the reserved category r by $C_s^r\left(A^1\right)$. For each individual i chosen in category r, add (i, r) to $\eta_s^r(A^1, T)$.

Then

$$\eta_s^{IS}(A, T) = \eta_s^o(A, T) \cup \bigcup_{r \in R} \eta_s^r\left(A^1, q_s^r\right).$$

The three conditions described above characterize the Indra Sawhney assignment rule that has been used in India for decades when the reserves in each vertical category are considered hard reserves.

THEOREM 0. *An assignment rule satisfies the OA principle, WCF, and QFE if and only if it is* η^{IS}.

Theorem 0 is the starting point of our axiomatic analysis.[19]

4. ASHOKA KUMAR THAKUR (2008) MANDATE AND RECOMMENDATION

The judgment in Ashoka Kumar Thakur (2008) brings about two crucial changes: (1) It changes the eligibility criteria for the OBC candidates, thereby modifying the QFE principle of Indra Sawhney (1992), and (2) it recommends OBC reservations to be implemented as soft reserves. Ashoka Kumar Thakur (2008) mandated a minimum cutoff score for the OBC category to **maintain the standard of excellence**. The SCI explicitly stated that there should be a reasonable difference in the OBC and open-category cutoff scores. Thus, the SCI **endogenizes** the OBC's cutoff as a function of the cutoff score of the open-category.

> *It is reasonable to balance reservation with other societal interests. To maintain standards of excellence, cutoff marks for OBCs should be set not more than 10 marks out of 100 below that of the general category.*

Thus, the directive on cutoff scores in Ashoka Kumar Thakur (2008) modifies the eligibility criteria for OBC candidates. We formulate this directive as a formal axiom below.

DEFINITION 7. *An assignment function* η *satisfies the modified quota-filling subject to eligibility (**mQFE**) if for any set of applicants* A *and* $T = (t(i))_{i \in A}$, *the following holds.*

(1) If an individual i *with* $t(i) = r \in R \setminus \{OBC\}$ *is not assigned, then the number of individuals who are matched to a category* r *position must be equal to* q_s^r.

[19] An indirect (in plain English) characterization of the same choice rule is given in Section 2.2 of Sönmez and Yenmez (2022), in which the OA principle is defined **differently** as follows: "*Next, VR protected positions must be allocated on an "over-and-above" basis; that is, positions that can be received without invoking the VR protections do not count against VR-protected positions*." Note that their definition does not require that the highest-scoring applicants be assigned to the open-category. The authors do not provide a formal proof for their characterization claim.

(2) If an individual i with $t(i) = OBC$ is not assigned and $\theta(i) \geq min\{\theta(j)|(j,o) \in \eta(A,T)\} - \lambda$, then the number of individuals assigned OBC positions must be q_s^{OBC}.

(3) If an individual i with $t(i) = OBC$ and $\theta(i) < min\{\theta(j)|(j,o) \in \eta(A,T)\} - \lambda$, then

$$(i, OBC) \notin \eta(A,T).$$

The mQFE principle may require that some OBC positions remain unfilled to maintain the standard of excellence, thereby making the eligibility requirement for the OBC category endogenous. Recognizing the potential allocative inefficiencies inherent in the elevated threshold for the OBC category, Justice Dalveer Bhandari advocated the implementation of soft reservations for the OBC category as a remedial institutional design. Under his proposal, vacant OBC positions would be reallocated to candidates from other categories as open-category positions, thereby mitigating efficiency losses. We mathematically formulate this recommendation as follows.

DEFINITION 8. *An assignment function η satisfies the **OBC de-reservation** if for any set of individuals A and $T = (t(i))_{i \in A}$,*

$$|\{(i,\tau) \in \eta(A,T) : \tau \in \{o, OBC\}\}| < q_s^o + q_s^{OBC} \implies |\eta(A,T)| = |A|.$$

Since OBC reservation is interpretted as *soft* reservation, the OA principle needs to a minor changed (replacing the equality in condition (i) with "$\geq$") as follows.

DEFINITION 9. *An assignment rule η satisfies the **over-and-above (OA) principle** if, for all $(A,T) \in 2^I \times \mathcal{T}$, (i) $|\{j|(j,o) \in \eta(A,T)\} \geq min\{|A|, q_s^o\}$, and (ii) for all $i \in A$ such that $rank_A(i) \leq q_s^o$, $(i,o) \in \eta(A,T)$.*

The OBC de-reservation policy requires reverting the vacancies in OBC category to open-category. When reverted open-category positions are filled, that is, more than q_s^o positions are allocated as open-category, the open-category cutoff score becomes lower. In turn, some candidates for the OBC who were ineligible due to their low scores may become eligible with the lower open-category cutoff score.

THEOREM 1. *No assignment rule satisfies the OA principle, WCF, OBC de-reservation, and mQFE.*

PROOF. Consider institution s with $\overline{q}_s = 4$ and $(q_s^o, q_s^{SC}, q_s^{ST}, q_s^{OBC}) = (1,1,1,1)$, and the set of individuals $I = \{i_1, i_2, i_3, i_4, i_5\}$ with the following scores and category membership:

Individual	*Category*	*Score*
i_1	g	100
i_2	g	90
i_3	OBC	89
i_4	SC	81
i_5	ST	80

The OA principle, WCF, and mQFE imply that $(i_1, o), (i_4, SC), (i_5, ST) \in \eta_s(I, T)$. Given the category memberships of i_2 and i_3, one of the following must hold.

(1) $\eta_s(I, T) = \{(i_1, o), (i_3, OBC), (i_4, SC), (i_5, ST)\}$.

(2) $\eta_s(I, T) = \{(i_1, o), (i_2, o), (i_4, SC), (i_5, ST)\}$.

(3) $\eta_s(I, T) = \{(i_1, o), (i_3, o), (i_4, SC), (i_5, ST)\}$.

(4) $\eta_s(I), T = \{(i_1, o), (i_4, SC), (i_5, ST)\}$.

The assignment function in (1) violates the mQFE because i_3 is assigned to an OBC position, even though her score is 11 points lower than the cutoff score of the open-category. The assignment function in (2) violates the mQFE because the open-category cutoff score is 90 (that is, the score of i_2) while i_3 was not chosen, although she is eligible for OBC. The assignment function in (3) violates WCF because i_2 has a higher score than i_3, assigned an open-category position. Finally, the assignment function in (4) violates the OBC de-reservation because, from the open-category and OBC combined, only i_1 is chosen: $|\eta_s(A, T)| = 3$ and $|A| = 5$. Thus, no assignment rule can satisfy them simultaneously.

■

5. RESOLVING THE INCOMPATIBILITY

The root cause of the incompatibility lies in the interaction between the endogenous cutoff score for the OBC category and OBC de-reservation in the presence of the OA principle and WCF. To resolve this incompatibility, we design assignment rules falling into two categories. In the first, we establish that there exists a unique rule satisfying the OA principle, WCF, and mQFE (Section 5.1). In the second, we identify a family of rules satisfying the OA principle, WCF, QFE, and OBC de-reservation (Section 5.2).

5.1. *A Simple Assignment Rule to Implement mQFE*

A simple modification of the Indra Sawhney assignment rule straightforwardly implements the mQFE principle in conjunction with the OA principle and WCF.

Given a set of applicants $A \subseteq I$, the profile of category membership $T = (t(i))_{i \in A}$, their test score $(\theta(i))_{i \in A}$, and the maximum cutoff score difference allowed between the open-category and OBC, λ, the assignment rule η^{AKT} is calculated as follows:

Stage 1. Only open-category positions are considered. For each applicant $i \in A$ such that $rank_A(i) \leq q_s^o$, add (i, o) to $\eta_s^o(A, T)$. Let $C_s^o(A)$ be the set of individuals chosen.

Stage 2. Among the remaining applicants $A^1 = A \setminus C_s^o(A)$, for reserve categories $r \in R \setminus \{OBC\}$, the applicants are chosen one at a time following $\succ_s^r$ to capacity q_s^r. Call the set of applicants chosen for the reserved category r by $C_s^r(A^1)$. For each individual i chosen in category $r \in R \setminus \{OBC\}$, add (i, r) to $\eta_s^r(A^1, T)$.

Stage 3. Among the remaining applicants, consider those in the OBC category. Let j^* be the lowest-scoring applicant in the open-category. For each individual i with $t(i) = OBC$ and $\theta(i) < \theta(j^*) - \lambda$, $(i, OBC) \notin \eta_s^{OBC}(A^1, T)$. Among the applicants in $\{i \in A^1 | t(i) = OBC \text{ and } \theta(i) \geq \theta(j^*) - \lambda\}$, select the highest scoring q_s^{OBC}. If there are fewer than q_s^{OBC} applicants, select all of them. For each chosen i, add (i, OBC) to $\eta_s^{OBC}(A^1, T)$.

Then $\eta^{AKT}(A, T) = \bigcup_{r \in R} \eta_s^r(A^1, q_s^r) \cup \eta_s^o(A, T)$.

The sole distinction between the assignment rule η^{AKT} and the Indra Sawhney assignment rule η^{IS} is that η^{AKT} restricts the eligibility of OBC seats to individuals whose score meets or exceeds the open category cutoff minus λ. Every individual assigned a position under η^{AKT} receives the same assignment under η^{IS}. However, the converse does not hold: some individuals assigned to OBC positions under η^{IS} may not receive an assignment under η^{AKT}. Thus, we have the following corollary.

COROLLARY 1. *C^{IS} merit dominates C^{AKT}.*

From the definitions of η^{IS} and η^{AKT} it follows that, for any $A \subseteq I$, $C^{AKT}(A) \subseteq C^{IS}(A)$. Consequently, η^{AKT} underperforms relative to η^{IS} because it frequently results in a significant number of unfilled OBC seats. This outcome underscores the critical necessity of the recommendation of Justice Bhandari on seat reservation.

Next, we characterize the assignment rule η^{AKT}.

THEOREM 2. *An assignment rule satisfies the OA principle, WCF, and mQFE if and only if it is η^{AKT}.*

Let C^{AKT} be the choice rule induced by the assignment rule η^{AKT}, which is a straightforward extension of the Indra Sawhney choice rule C^{IS} and satisfies a condition that has been a key in the market design literature.

PROPOSITION 1. *The choice rule C^{AKT} is consistent and satisfies the substitutes property.*

However, the choice rule C^{AKT} violates size monotonicity and can be very wasteful. The following example explains why.

EXAMPLE 1. *Consider an institution with six positions. One position is reserved for SC, one for ST, and two for OBC. The two remaining positions are open-category. Let $\lambda = 10$. Consider the set of individuals $I = \{i_1, i_2, i_3, i_4, i_5, i_6\}$ with the following scores and category membership:*

Individual	*Category*	*Score*
i_1	g	100
i_2	g	99
i_3	OBC	88
i_4	OBC	87
i_5	SC	81
i_6	ST	80

Let $B = \{i_3, i_4, i_5, i_6\}$. Note that $\eta^{AKT}(B, T) = \{(i_3, OBC), (i_4, OBC), (i_5, SC), (i_6, ST))\}$. The induced choice is $C^{AKT}(B) = \{i_3, i_4, i_5, i_6\}$. Thus, $|C^{AKT}(B)| = 4$.

Let $A = \{i_2, i_3, i_4, i_5, i_6\}$. $\eta^{AKT}(A, T) = \{(i_2, o), (i_5, SC), (i_6, ST))\}$. Note that $C^{AKT}(A) = \{i_2, i_5, i_6\}$. Thus, $|C^{AKT}(A)| = 3$. Since $B \subset A$, size monotonicity is violated.

Although C^{AKT} satisfies path independence, it can be wasteful. In example 1, two seats are wasted as $C^{AKT}(I, T) = \{i_1, i_2, i_5, i_6\}$. If the number of OBC candidates whose scores exceed the endogenous cutoff is low, the waste can be substantial.

5.2. *Designing for the Standard of Excellence*

In many applications in India—such as admissions to engineering and medical schools and faculty recruitment at public universities—numerous OBC positions remain unfilled each year.[20] Conditioning the OBC minimum cutoff score on that of the open category—a requirement motivated by the principle of "maintaining standards of excellence"—further aggravates the vacancies in the OBC category.[21] Given the impossibility result in Theorem 1, we reinterpret the principle of maintaining the standard of excellence as follows:

[20] Baswana, Chakrabarti, Chandran, Kanoria, and Patange (2019) documents a substantial number of vacancies in the OBC category for admissions to engineering colleges. A cursory examination of online resources reveals the widespread prevalence of unfilled OBC positions throughout India.

[21] A similar cutoff score restriction exists in admissions to high schools in China (Dur, Kesten, Wang, and Zhou (2024) and Hu and Zhang (2025)). Wuhan eliminated the 30-point cutoff score constraint for reserved seats in 2024. Hu and Zhang (2025) argues that China is trending toward completely eliminating this constraint.

Among all possible choice rules via different transfer policies, which rule consistently selects the most meritorious set of applicants?

We introduce assignment functions by removing the mQFE policy from consideration and focusing on the OBC de-reservation along with the three axioms from Indra Sawhney (1992). More precisely, we consider assignment rules that satisfy the OA principle, the WCF and QFE principles proposed by Indra Sawhney (1992), and the OBC de-reservation recommendation by Ashoka Kumar Thakur (2008). Throughout this section, we assume that any vacant position in the OBC category must be provided to others as an open-category position, whenever possible.

An institution always fills the open-category before the reserved categories to satisfy the OA principle. That is, the precedence ordering is

$$Open - c_1 - c_2 - c_3 - c_4, \tag{1}$$

where $c_1, c_2, c_3, c_4 \in R$ such that $c_i \neq c_j$ for $i \neq j$. There are $4! = 24$ such orderings. The order of the categories in R does not matter, as the reserve categories are assumed to be nonintersecting.[22] If there is no vacancy in the OBC category after applying η^{IS} according to one of the precedence orders above, then we are done.

However, the ordering does matter under the recommended OBC de-reservation policy. Because if there are vacancies in the OBC category, they must be allocated as open-category positions. The precedence order at which these additional open-category seats are filled has non-trivial distributional consequences. We expand the precedence order (1) above to locate new groups of open-category positions $\{T_1, T_2, T_3, T_4, T_5\}$, referred to as the **de-reserved open-categories** below.

[22]The implementation of the EWS reservation has raised significant legal and normative concerns regarding the integrity of the non-intersectionality principle governing the reserved categories. Specifically, critics argue that the introduction of the EWS quota—which operates across caste lines but excludes SC, ST, and OBC beneficiaries—risks blurring the constitutionally mandated separation between the socially and educationally backward classes and the economically disadvantaged among the forward castes, thereby undermining the structural independence of the reserved categories from one another. See Janhit Abhiyan v. Union of India (2022) for more details.

$$Open - T_1 - c_1 - T_2 - c_2 - T_3 - c_3 - T_4 - c_4 - T_5. \tag{2}$$

We call this precedence order *expanded precedence order*. Each de-reserved open-category T_i has an initial capacity of 0. When there are $\sum_{i=1}^{5} r_i$ many unfilled positions in the OBC category, the vector $(r_1, r_2, r_3, r_4, r_5)$ describes how these vacancies are distributed in the groups T_1, T_2, T_3, T_4, and T_5, where r_i is the number of positions allotted to the group T_i, for all $i = 1, ..., 5$.

Assignment Rule $\eta^{\mathbf{x}}$. Given the expanded precedence order (2), a set of individuals $A \subseteq I$, a category membership profile $T = (t(i))_{i \in A}$, and the capacities of the groups of positions $\mathbf{x} = (x_1, \ldots, x_{10}) \in \mathbb{Z}_+^{10}$, and scores of the applicants $(\theta(i))_{i \in A}$ the assignment rule $\eta^{\mathbf{x}}(A, T)$ induced by the capacity vector $\mathbf{x}$ is calculated as follows:

Step 1. Consider all applicants. Select the top-x_1 applicants according to the merit scores. If there are fewer than x_1 applicants, select all of them. Add (i, o) to $\eta^{\mathbf{x}}(A, T)$ for any applicant $i \in A$ selected. Let A^1 be the set of remaining applicants.

Step k.[$10 \geq k \geq 2$] Consider the relevant applicants[23] in A^{k-1}. The groups $\{T_i\}_{i=1}^{5}$ consider all applicants. Select top-x_k applicants according to merit scores. If there are fewer than x_k applicants, select all of them. For each applicant $i \in A^{k-1}$ selected in the groups $T_1, ..., T_5$, add (i, o) to $\eta^{\mathbf{x}}(A, T)$. Similarly, for each applicant i who is selected in category $r \in R$, add (i, r) to $\eta^{\mathbf{x}}(A, T)$. Let A^k be the set of remaining applicants.

Let $C^{\mathbf{x}}(A)$ be the induced choice function from $\eta^{\mathbf{x}}(A, T)$.

We first define a transfer scheme, where $\kappa \in \{0, ..., q_s^{OBC}\}$ denotes the number of vacant positions in OBC category.

DEFINITION 10. *A transfer scheme is a 5-dimensional vector function f such that*

[23]The relevant applicants for the de-reserved open categories T_1, T_2, T_3, T_4 and T_5 are **all** applicants in the set A^{k-1}. The relevant applicants for the SC (similarly for the ST / OBC / EWS) categories are the applicants who belong to SC (similarly to ST/OBC/EWS) in the set A^{k-1}.

(1) $f(\kappa) = (f_1(\kappa), f_2(\kappa), f_3(\kappa), f_4(\kappa), f_5(\kappa))$ *where* $f_i(\kappa) \geq 0$ *is the number of positions available in category* T_i *and* $\kappa = \sum_{j=1}^{5} f_j(\kappa)$*, for each* $\kappa \in \{0, \ldots, q_s^{OBC}\}$.

(2) $f_i(x+1) - f_i(x) \in \{0, 1\}$*, for all* $i \in \{1, 2, 3, 4, 5\}$ *and* $\sum_{i=1}^{5}[f_i(x+1) - f_i(x)] = 1$*, for all* $x \in \{0, \ldots, q_s^{OBC} - 1\}$.

In other words, given a number of unfilled positions κ in the OBC category, a transfer scheme $f(\kappa)$ allocates these positions among the groups of positions $T_1, ..., T_5$. The definition implies that each vacant position must be transferred to one of $T_1, ..., T_5$.

We denote by $\mathcal{F}$ the set of all transfer schemes. The assignment rule induced by the transfer scheme $f \in \mathcal{F}$, η^f is constructed as follows.

Assignment Rule Induced by Transfer Function f

Set the initial capacity of each group T_i to 0, $\forall i = 1, ..., 5$ and construct

$$\mathbf{x^1} = (q_s^o, 0, q_s^{c_1}, 0, q_s^{c_2}, 0, q_s^{c_3}, 0, q_s^{c_4}, 0).$$

Let $x_{r^*}^1 = q_s^{OBC}$ denote the initial capacity of the OBC category, where $r^* \in \{1, ..., 10\}$ denotes the position of the OBC seats in the precedence order.

Given a set of applicants A, a category membership profile $T = (t(i))_{i \in A}$, and the vector of initial position distribution $q_s = (q_s^o, q_s^{SC}, q_s^{ST}, q_s^{OBC}, q_s^{EWS})$, the assignment rule $\eta_s^f(A, T)$ selects the applicants in (possibly) multiple iterations as follows:

Iteration 1. Run $\eta_s^{\mathbf{x^1}}(A, T)$. Let κ^1 be the number of vacant positions in OBC. Update the number of positions by transferring κ^1 many positions to the de-reserved open categories via f. That is, $\mathbf{x^2} = (x_1^2, \ldots, x_{10}^2)$ is such that

$$x_{2j}^2 = f_j(\kappa^1), \text{ for all } j = 1, ..., 5,$$

$$x_{r^*}^2 = q_s^{OBC} - \kappa^1.$$

Note that the capacities of the Open, SC, ST, and EWS categories remain unchanged.

Iteration n (n≥2). Run $\eta_s^{\mathbf{x^n}}(A,T)$. Let κ^n be the number of vacant OBC positions. Update the number of positions by transferring κ^n many positions in the transferred slots. That is, $\mathbf{x^{n+1}} = (x_1^{n+1}, \ldots, x_{10}^{n+1})$ is such that

$$x_{2j}^{n+1} = x_{2j}^n + f_j(\kappa^n), \text{ for all } j = 1, ..., 5,$$

$$x_{r^*}^{n+1} = x_{r^*}^n - \kappa^n$$

Again, the capacities of the Open, SC, ST, and EWS categories remain unchanged.

The process ends when there are no vacant positions in the OBC category. Let N be the last iteration. Then

$$\eta_s^f(A,T) = \eta_s^{\mathbf{x^N}}(A,T).$$

Let $C_s^f(A)$ be the induced choice rule from $\eta_s^f(A,T)$.

We denote by $\mathbb{A} \equiv \left(\eta^f\right)_{f\in\mathcal{F}}$ the set of assignment rules generated by different transfer schemes. Let $\mathbb{C}$ be the set of choice rules induced by the assignment rules in $\mathbb{A}$.

Two notable assignment rules in $\mathbb{A}$ are the following.

Forward Transfer (FT) Assignment Rule. The FT assignment rule is such that $f(\kappa) = (0,0,0,0,\kappa)$ for all $\kappa \in \{0, \ldots, q_s^{OBC}\}$. We denote this assignment rule $\eta_s^{FT} \in \mathbb{A}$. Let $C_s^{FT} \in \mathbb{C}$ be the choice rule induced by η_s^{FT}, called the forward transfer choice rule.

Backward Transfer (BT) Assignment Rule. The BT assignment rule is such that $f(\kappa) = (\kappa,0,0,0,0)$ for all $\kappa \in \{0, \ldots, q_s^{OBC}\}$. We denote this assignment rule $\eta_s^{BT} \in \mathbb{A}$. Let $C_s^{BT} \in \mathbb{C}$ be the choice rule induced by η_s^{BT}, called the backward transfer choice rule.[24]

EXAMPLE 2. *Consider an institution s with $q_s = 7$. The initial distribution of positions over categories is $(o, SC, ST, OBC, EWS) = (3,1,1,2,0)$. Suppose that $I = \{i_1, ..., i_8\}$ with the following test scores and category memberships.*

[24]The backward transfer choice rule C^{BT} was introduced in Aygün and Turhan (2022).

Individual	*Category*	*Score*
i_1	g	100
i_2	g	99
i_3	g	98
i_4	SC	97
i_5	g	96
i_6	OBC	85
i_7	ST	84
i_8	SC	84

We first compute $\eta_s^{FT}(I,T)$. i_1, i_2, *and* i_3 *are selected for open-category positions.* i_4 *and* i_7 *are assigned to the reserved SC and ST positions, respectively.* i_6 *is chosen for a reserved OBC position. The other OBC position remains unfilled and is made an open-category. Among the unassigned individuals,* i_5 *has the highest score and is chosen for the open-category position. Thus,*

$$\eta_s^{FT}(I,T) = \{(i_1,o),(i_2,o),(i_3,o),(i_4,SC),(i_5,o),(i_6,OBC),(i_7,ST)\}.$$

We now compute $\eta_s^{BT}(I,T)$. *In the first iteration,* i_1, i_2, *and* i_3 *are selected in the open-category,* i_4 *is selected in the SC category,* i_7 *is selected in the ST category, and the only OBC individual* i_6 *is chosen in the OBC category. One reserved OBC seat remains vacant. Therefore, it becomes an open-category position by updating the capacity vector to* $q_s^2 = (4,1,1,1,0)$.

In the second iteration, i_1, i_2 , i_3, *and* i_4 *are selected in the open-category,* i_8 *is selected in the SC category,* i_7 *is selected in the ST category and* i_6 *is chosen in the OBC category. We end the procedure because there is no vacancy in the OBC category. Thus, we have*

$$\eta_s^{BT}(I,T) = \{(i_1,o),(i_2,o),(i_3,o),(i_4,o),(i_6,OBC),(i_7,ST),(i_8,SC)\}.$$

Note that $\eta_s^{FT}(A,T) \setminus \eta_s^{BT}(A,T) = \{(i_5,o)\}$ *and* $\eta_s^{BT}(A,T) \setminus \eta_s^{FT}(A,T) = \{(i_8,SC)\}$.

5.3. *Characterization of the FT Assignment Rule*

In Example 2, the set of individuals chosen induced by $\eta_s^{FT}(I,T)$ is a more meritorious set than the set of individuals chosen induced by $\eta_s^{BT}(I,T)$. We now generalize this observation and establish that the FT choice rule merit-dominates other choice rules induced by the assignment rules in the family $\mathbb{A}$.

THEOREM 3. *The choice rule induced by η^{FT} dominates any choice rule induced by any other assignment rule in $\mathbb{A}$.*

This result implies that if selecting the most meritorious set of applicants is the policy objective, then the FT assignment rules are the right choice with a forward transferring soft reserve policy.

THEOREM 4. *The choice rule induced by every assignment rule in the family $\mathbb{A}$ is consistent and satisfies the substitutes property and size monotonicity.*

Theorem 4 establishes the admissibility of any assignment rule in the family $\mathbb{A}$ for institutions in centralized markets.[25]

6. A BETTER SOLUTION

The impossibility result in Theorem 1 originates from relabeling de-reserved OBC positions as open-category. When OBC positions are reverted and reallocated as open-category positions, the cutoff score for the open-category is endogenously altered. This, in turn, modifies the eligibility cutoff score for the OBC category and consequently affects which applicants qualify for OBC positions.

An alternative interpretation of the cutoff score requirement bypasses this endogeneity: Define the open-category cutoff exclusively with reference to the **original** open-category positions—that is, set the eligibility threshold as the score of the lowest-ranked applicant

[25]The results established by Hatfield and Milgrom (2005) ensure that the deferred acceptance mechanism in conjunction with any assignment rule in the family $\mathbb{A}$ is stable and strategy-proof.

among the q_s^o original open-category positions. Under this specification, reverted positions are precluded from influencing the open-category cutoff, thereby constraining the endogenous feedback mechanism that generates the incompatibility.

A natural implementation of this idea is to assign de-reserved OBC positions a distinct label “d", reserving the label “o" exclusively for original open-category positions, and to determine the OBC eligibility cutoff solely by reference to the lowest-scoring applicant admitted through an “o" position.

Formulating this idea requires a minor extension of the model. Let D be the new position category representing de-reserved OBC positions. We say that an individual is assigned a position in category D under an assignment rule η, if $(i, d) \in \eta$. With this change, the assignment rule definition is updated as follows.

DEFINITION 11. *An **assignment rule** η is a function in which, given a set of applicants A and a category membership profile T,*

$$\eta(A, T) = \{(i, \tau) | i \in A \text{ and } \tau \in \{o, d\} \cup R\} \text{ such that}$$

(i) for any $i \in A$, we have at most one $(i, \tau) \in \eta(A, T)$, where

$$\begin{cases} \tau \in \{o, d\} & \text{if } t(i) = g, \\ \tau \in \{t(i), o, d\} & \text{if } t(i) \in R, \end{cases}$$

(ii) $|\eta_s(A, T)| \leq \overline{q}_s$, *and*

(iii) $|\{(i, \tau) \in \eta(A, T) : \tau = r\}| \leq q_s^r$, *for each $r \in R$.*

6.1. *Assignment Rule η^**

Given a set of applicants A, the profile $T = (t(i))_{i \in A}$, their test score $(\theta(i))_{i \in A}$, and the maximum cutoff score difference allowed between the open-category and OBC, λ, the assignment of applicants to categories is calculated as follows:

Stage 1. Only open-category positions are considered. For each applicant $i \in A$ such that $rank_A(i) \leq q_s^o$, add (i, o) to $\eta_s^o(A, T)$. Let $C_s^o(A)$ be the set of individuals chosen.

Stage 2. Among the remaining applicants $A^1 = A \setminus C_s^o(A)$, for reserve categories $r \in R \setminus \{OBC\}$, the applicants are chosen one at a time according to $\succ_s^r$ to capacity q_s^r. Call the set of applicants chosen for the reserved category r by $C_s^r\left(A^1\right)$. For each individual i chosen in category r, add (i, r) to $\eta_s^r(A^1, T)$.

Stage 3. Among the remaining applicants, consider those in the OBC category. Each individual i with $t(i) = OBC$ and $\theta(i) \geq \theta(j^*) - \lambda$ where $rank_A(j^*) = q_s^o$, select the highest scoring applicants q_s^{OBC}. If the number of such applicants is less than q_s^{OBC}, then choose all of them. Let λ be the number of vacant OBC positions.

Stage 4. Consider the remaining applicants A''. Among them, select the highest-scoring λ applicants. If the number of remaining applicants is less than λ, select all of them. For each individual i chosen in this stage, add (i, d) to $\eta_s^D(A'', T)$ where d denotes a de-reserved OBC position. Then,

$$\eta^*(A, T) = \eta_s^o(A, T) \cup \bigcup_{r \in R} \eta_s^r\left(A^1, q_s^r\right) \cup \eta_s^D(A'', T).$$

The assignment rule η^* runs η^{AKT} in its first three stages. Unlike η^{AKT}, it utilizes all vacant OBC positions in its final stage and fills them based on merit scores as category-D positions.

Let C^* be the choice rule induced by the assignment rule η^*.

EXAMPLE 3. *Consider an institution s with $q_s = 7$. Let be distribution of positions over categories be $(q_s^o, q_s^{SC}, q_s^{ST}, q_s^{OBC}, q_s^{EWSE} = (3, 1, 1, 2, 0)$. Let $\lambda = 10$. Suppose that $I = \{i_1, ..., i_8\}$ with the following test scores and category memberships.*

Individual	*Category*	*Score*
i_1	g	100
i_2	g	99
i_3	SC	97
i_4	g	96
i_5	g	90
i_6	OBC	85
i_7	ST	84
i_8	SC	84

We have $\eta^{FT}(I,T) = \{(i_1,o),(i_2,o),(i_3,o),(i_8,SC),(i_7,ST),(i_6,OBC),(i_4,o)\}$ with the induced choice $C^{FT}(I) = \{i_1,i_2,i_3,i_4,i_6,i_7,i_8\}$.

Moreover, $\eta^*(I,T) = \{(i_1,o),(i_2,o),(i_3,o),(i_8,SC),(i_7,ST),(i_4,o),(i_5,o)\}$. The induced choice is $C^*(I) = \{i_1,i_2,i_3,i_4,i_5,i_7,i_8\}$.

Note that η^* selects a better set of candidates than η^{FT}. Our next result shows that this is not a coincidence, but it holds in general.

Moreover, in the example provided in the proof of Theorem 1, the assignment rule η^* assigns i_1 to the open-category position and i_4 and i_5 to the SC and ST positions, respectively. The open-category cutoff is 100, which raises the OBC cutoff to 90, rendering i_3 ineligible. The vacant OBC position is consequently reallocated as a category-D position to i_2. As in the preceding example, η^* merit-dominates η^{FT}, which would have selected i_3 over i_2.

THEOREM 5. *The induced choice of η^* merit-dominates that of η^{FT}.*

Theorem 5 implies that the assignment rule η^* merit-dominates every assignment rule in the family $\mathbb{A}$ developed in Section 5.2.

The assignment rule η^* can be considered as a compromise between implementing mQFE from Ashoka Kumar Thakur (2008) and QFE from Indra Sawhney (1992) with OBC de-reservation. Revising the cutoff-score-difference policy and implementing mQFE with

OBC de-reservation yields the most meritorious set of assignments considered in this paper. Thus, we propose η^* as a viable solution that upholds the standard of excellence while remaining consistent with the SCI mandates to the extent possible.

6.2. *Characterization*

Next, we characterize the choice rule induced by η^*. To do so, we first restate our axioms with the relabeling de-reserved OBC positions as category-D positions.

DEFINITION 12. *An assignment rule η satisfies the over-and-above* (OA^*) principle if, for all $A \subseteq I$ and $T \in \mathcal{T}$, (i) $| \{j | (j,o) \in \eta(A,T)\} = min \{|A|, q_s^o\}$, and (ii) for all $i, j \in I$ such that $(i,o) \in \eta(A,T)$ and $(j,\tau) \in \eta(A,T)$ where $\tau \in \{d\} \cup R$, we have $i \succ_s j$.*

The OA* property states that at most q_s^o positions can be assigned as original open-category. If there are fewer than q_s^o applicants, all are assigned original open-category positions. Otherwise, applicants who are assigned original open-category positions must have higher scores than applicants who are assigned to reserved category positions and de-reserved positions.[26]

DEFINITION 13. *An assignment rule η satisfies within-category fairness* (WCF^*) if, for any pair of individuals $i, j \in A$ such that $t_i = t_j \in \{g\} \cup R$ and $i \succ_s j$,*

$$(j,\tau) \in \eta(A,T), \text{for some } \tau \in \{o,d\} \cup R \text{ implies } (i,\tau') \in \eta(A,T), \text{for some } \tau' \in \{o,d\} \cup R.$$

The mQFE property remains unchanged. Finally, the OBC de-reservation property is slightly changed to reflect that unfilled OBC seats are allocated as category-D positions.

DEFINITION 14. *An assignment function η satisfies the **OBC de-reservation*** if for any set of individuals A and $T = (t(i))_{i \in A}$,*

$$|\{(i,\tau) \in \eta(A,T) : \tau \in \{d, OBC\}\}| < q_s^{OBC} \implies |\eta(A,T)| = |A|.$$

[26]Condition (ii) requires the top q_s^o applicants to be assigned through original open-category positions, consistent with the preceding OA conditions. To accommodate the newly introduced category-D and to underscore that original open positions must be allocated to the highest-priority candidates, we have reworded condition (ii) accordingly.

THEOREM 6. *The choice rule induced by the assignment rule η^* merit dominates any other choice rule induced by an assignment rule that satisfies the OA principle*, WCF*, mQFE, and OBC de-reservation*.*

OBC de-reservation admits multiple implementations under the reinterpretation of the cutoff requirement as in Section 5.2. Let $\mathbb{A}^*$ denote the family of assignment rules satisfying the four axioms in Theorem 6. Theorem 6 then establishes that η^* is optimal with respect to merit within $\mathbb{A}^*$. Taken together, Theorems 5 and 6 characterize η^* as the best rule in the family $\mathbb{A} \cup \mathbb{A}^*$.

PROPOSITION 2. *The choice rule induced by η^* is consistent and satisfies the substitutes property and size monotonicity.*

Proposition 2 establishes that η^* is admissible as an assignment rule for institutions in a centralized market under the deferred acceptance algorithm. By the results of Hatfield and Milgrom (2005), the deferred acceptance mechanism coupled with η^* is both stable and strategy-proof in such markets.

7. RELATED LITERATURE

Reconciling equity and meritocracy is one of the central challenges of market design. The growing theoretical literature has studied how the structure of affirmative action determines its distributional consequences. Kojima (2012) showed that strengthening the majority quotas—an instrument introduced in the context of school choice by Abdulkadiroglu and Sönmez (2003) to promote diversity—can paradoxically harm all members of the intended beneficiary group under any stable mechanism. Hafalir, Yenmez, and Yildirim (2013) demonstrated that replacing majority quotas with soft minority reserves resolves this perversity: the deferred acceptance mechanism with minority reserves Pareto dominates its majority-quota counterpart, improving diversity while limiting meritocratic costs. Ehlers, Hafalir, Yenmez, and Yildirim (2014) extended this analysis to systems with multiple protected categories, characterizing the admissible assignment

rules under both hard and soft bounds. Beyond reserve size, Dur, Kominers, Pathak, and Sönmez (2018) established that the precedence order in which seat types are filled has distributional consequences comparable in magnitude to adjusting reserve quotas. The present paper identifies a dimension of the diversity–meritocracy tradeoff that neither the general theory nor the India-specific literature has yet confronted: the joint design of eligibility thresholds and de-reservation policies, which together determine both who qualifies for reserved positions and what happens to those that go unfilled.

The two papers most related to ours are Sönmez and Yenmez (2022) and Aygün and Turhan (2022). Both articles take an axiomatic approach, formulating policy goals as formal axioms. Sönmez and Yenmez (2022) studies the joint implementation of vertical and horizontal reservations. Aygün and Turhan (2022) examines the concurrent implementation of vertical reservations and OBC de-reservations, explicitly focusing on the admission procedures for technical universities in India. Sönmez and Yenmez (2022) takes OBC reservations as hard reserves and proposes a choice rule to deal with overlapping horizontal reservations as their main contribution. Aygün and Turhan (2022) introduced the *backward transfers* (BT) assignment rule that ensures higher cutoff scores for the open-category than those for the reserve categories.

Echenique and Yenmez (2015) is the first paper to present Indian affirmative action as an example of controlled school choice. Aygün and Turhan (2017) explores the specific challenges faced in admissions to the Indian Institutes of Technology (IIT). Aygün and Turhan (2020) and Aygün and Turhan (2026) offer a comprehensive treatment of vertical reservations and de-reservations by introducing a family of *dynamic reserves* and *generalized sequential* choice rules, respectively.

A growing literature examines resource allocation mechanisms in India by analyzing specific government legislation. Evren and Khanna (2021) study the Central Educational Institutions Act (2019), which requires reservations for teaching positions in India's central educational institutions for beneficiaries of affirmative action policies. Mishra, Sarkar,

Sen, Sethuraman, and Yadav (2025) analyzes the Right to Free and Compulsory Education Act (2009), which establishes student-teacher ratios for state schools and advocates teacher redeployment from schools with surpluses to those with deficits. Our paper provides the first theoretical study of the joint implementation of vertical reservations, endogenous cutoff restrictions, and de-reservation policies in the Indian context.

Dur, Kesten, Wang, and Zhou (2024) analyzes China's ZBS high school admission policy, where the problem is a design flaw in how bonus points ("privileges") are allocated to students from disadvantaged middle schools. Both their paper and ours formalize real-world diversity policies as axiomatic constraints, design choice rules that satisfy those constraints, and prove optimality results. Both also identify a central tension between competing policy goals — fairness/merit versus representation — and propose novel rules to resolve it. The most fundamental difference is the institutional setting. We analyze India's reservation system, where the core problem is the legal incompatibility between two Supreme Court rulings. The nature of the affirmative action instrument also differs structurally. India's system does not use bonus points.

This paper contributes to the economic theory literature on resource allocation with diversity considerations. Other prominent papers include Doğan and Yildiz (2023), Hafalir, Kojima, and Yenmez (2025), Doğan, Imamura, and Yenmez (2025), Evren (2023), Celebi (2023), Imamura (2025), and Celebi and Flynn (2025), among many others.

8. Conclusion

Beyond its applied contributions, this paper advances the theoretical literature on reserve systems in three dimensions. First, we introduce endogenous eligibility cutoffs as a design primitive. Previous work on reserve systems treats eligibility for reserved positions as an exogenous characteristic of applicants. We show that once the eligibility threshold for a reserve category is tied to the allocation outcome itself, the resulting feedback generates significant incompatibilities.

Second, we characterize the optimal soft reserve policy within a rich family of de-reservation schemes. The merit-dominance of the FT rule over this entire family provides a sharp and constructive criterion for optimal policy design under soft reserves.

Third, perhaps most importantly, by introducing the category of de-reserved positions as a distinct label, we demonstrate that a minor reinterpretation of the cutoff requirement—fixing the eligibility threshold with respect to the original open-category positions rather than the endogenously expanded pool—simultaneously resolves the impossibility and achieves strict merit gains over all previously considered designs. This reinterpretation establishes that the source of the incompatibility is not an irreducible tension between meritocracy and diversity, but a modeling choice about what the cutoff measures, a distinction with direct axiomatic and welfare consequences.

REFERENCES

ABDULKADIROGLU, A., AND T. SÖNMEZ (2003): "School choice: A mechanism design approach," *American Economic Review*, 93(3), 729–747.

AYGÜN, O., AND B. TURHAN (2017): "Large-scale affirmative action in school choice: Admissions to IITs in India," *American Economic Review*, 107(5), 210–213.

——— (2020): "Dynamic reserves in matching markets," *Journal of Economic Theory*, 188, 105069.

——— (2022): "How to de-reserve reserves: Admissions to technical colleges in India," *Management Science*, 69(10), 5695–6415.

——— (2026): "Matching with Generalized Sequential Choice Rules," *Working Paper*.

BASWANA, S., P. P. CHAKRABARTI, S. CHANDRAN, Y. KANORIA, AND U. PATANGE (2019): "Centralized admissions for engineering colleges in India," *INFORMS Journal on Applied Analytics*, 45(5), 338–354.

CAVALLO, M., AND B. DOGAN (2024): "When geography shapes preferences: Redesigning teacher assignment in italy," in *Proceedings of the 25th ACM Conference on Economics and Computation*, pp. 419–419.

CELEBI, O. (2023): "Diversity preferences, affirmative action and choice rules," *Working Paper*.

CELEBI, O., AND J. FLYNN (2025): "Adaptive priority mechanisms," *Working Paper*.

COMBE, J., U. DUR, O. TERCIEUX, C. TERRIER, AND M. U. ÜNVER (2025): "Market design for distributional objectives in (re) assignment: An application to improve the distribution of teachers in schools," *Working Paper*.

COMBE, J., O. TERCIEUX, AND C. TERRIER (2022): "The design of teacher assignment: Theory and evidence," *The Review of Economic Studies*, 89(6), 3154–3222.

DOĞAN, B., K. IMAMURA, AND M. B. YENMEZ (2025): "Market Design with Deferred Acceptance: A Recipe for Policymaking," *Journal of Economic Theory*.

DOĞAN, B., AND K. YILDIZ (2023): "Choice with affirmative action," *Management Science*, 69, 2284–2296.

DUR, U., O. KESTEN, T. WANG, AND C. ZHOU (2024): "Who gets the bonus? Affirmative Action Reforms in High School Admissions in China," *Working Paper*.

DUR, U., S. D. KOMINERS, P. A. PATHAK, AND T. SÖNMEZ (2018): "Reserve design: Unintended consequences and the demise of Boston's walk zones," *Journal of Political Economy*, 126(6), 2457–2479.

ECHENIQUE, F., AND M. B. YENMEZ (2015): "How to control controlled school choice," *American Economic Review*, 105(8), 2679–94.

EHLERS, L., I. E. HAFALIR, M. B. YENMEZ, AND M. A. YILDIRIM (2014): "School choice with controlled choice constraints: Hard bounds versus soft bounds," *Journal of Economic Theory*, 153, 648–683.

EVREN, H., AND M. KHANNA (2021): "Affirmative Action's Cumulative Fractional Assignments," *arXiv preprint arXiv:2111.11963*.

EVREN, S. (2023): "Reserve Matching with Thresholds," *arXiv preprint arXiv:2309.13766*.

GALE, D. (1968): "Optimal assignments in an ordered set: an application of matroid theory," *Journal of Combinatorial Theory*, 4(2), 176–180.

GALE, D., AND L. S. SHAPLEY (1962): "College admissions and the stability of marriage," *The American Mathematical Monthly*, 69(1), 9–15.

HAFALIR, I. E., F. KOJIMA, AND M. B. YENMEZ (2025): "Efficient Market Design with Distributional Objectives," *Journal of Economic Theory*.

HAFALIR, I. E., F. KOJIMA, M. B. YENMEZ, AND K. YOKOTE (2026): "Market Design with Distributional Objectives," *Mathematics of Operation Research*.

HAFALIR, I. E., M. B. YENMEZ, AND M. A. YILDIRIM (2013): "Effective affirmative action in school choice," *Theoretical Economics*, 8(2), 325–363.

HATFIELD, J. W., AND P. R. MILGROM (2005): "Matching with contracts," *American Economic Review*, 95(4), 913–935.

HU, X., AND J. ZHANG (2025): "Verifiable affirmative action in Chinese high school admissions," *arXiv preprint arXiv:2504.04689*.

IMAMURA, K. (2025): "Meritocracy versus diversity," *Journal of Economic Theory*, p. 106047.

KAMADA, Y., AND F. KOJIMA (2015): "Efficient matching under distributional constraints: Theory and applications," *American Economic Review*, 105(1), 67–99.

——— (2024): "Fair matching under constraints: Theory and applications," *Review of Economic Studies*, 91(2), 1162–1199.

KOJIMA, F. (2012): "School choice: Impossibilities for affirmative action," *Games and Economic Behavior*, 75(2), 685–693.

MISHRA, D., S. SARKAR, A. SEN, J. SETHURAMAN, AND S. YADAV (2025): "Teacher transfers: equalizing deficits across schools," *arXiv preprint arXiv:2510.18708*.

SÖNMEZ, T., AND M. B. YENMEZ (2022): "Affirmative action in India via vertical, horizontal, and overlapping reservations," *Econometrica*, 90(3), 1143–1176.

9. ONLINE APPENDIX

PROOF OF THEOREM 0. ($\Longleftarrow$) By the definition of η_s^{IS}, the first q_s^o highest-scoring applicants are assigned open-category positions. If there are fewer than q_s^o applicants, all are assigned open-category positions. Thus, the OA principle is satisfied.

Since each category fills its positions following the merit score rankings, the WFC property is trivially satisfied. The only way for an individual i to be unable to be assigned is that the positions i is eligible for are filled with individuals with higher merit scores. Thus, the QFE principle is also satisfied.

($\Longrightarrow$) Let η_s be an assignment rule that satisfies the three axioms. By the OA principle, we have $\{(i,o) : (i,o) \in \eta_s(A,T)\} = \{(i,o) : (i,o) \in \eta_s^{IS}(A,T)\}$. If $\mid A \mid \leq q^o$, then we are done. Let $\mid A \mid > q^o$. The QFE requires that if an individual i with $t(i) \in R$ is rejected, then $\mid C^{t(i)} \mid = q_s^{t(i)}$. The QFE and WCF jointly imply that C^v chooses the top q_s^v individuals of the set $(A \setminus C^o(A)) \bigcap A^v$, for all $v \in R$. Thus, $\eta(A,T) = \eta^{IS}(A,T)$. ■

PROOF OF THEOREM 2. *($\Longleftarrow$)* By construction, η^{AKT} satisfies the OA principle: in Stage 1 it assigns the q_s^o highest-scoring applicants in A to the open-category positions, or all applicants if $|A| < q_s^o$.

η^{AKT} also satisfies the WCF. Indeed, in Stage 1 the open-category positions are assigned in decreasing order of score. In Stage 2, for each $r \in R \setminus \{OBC\}$, the category-$r$ positions are assigned based on merit scores among the remaining applicants of type r. In Stage 3, the OBC slots are assigned based on merit scores among the remaining OBC applicants whose scores satisfy the cutoff requirement. Hence no lower-scoring applicant of a given type is assigned unless every higher-scoring applicant of the same type is also assigned.

Finally, η^{AKT} satisfies mQFE by definition of Stages 2 and 3. For each $r \in R \setminus \{OBC\}$, all category-$r$ positions are filled whenever there are enough remaining applicants of type r, and otherwise all such applicants are selected. For the OBC category, Stage 3 assigns the highest-scoring remaining OBC applicants whose scores are at least the open-category cutoff minus λ, up to capacity q_s^{OBC}; thus, if an eligible OBC applicant remains

unmatched, then all OBC positions must already be filled, and no ineligible OBC applicant is assigned.

(⇒) Let η be any assignment rule satisfying the OA principle, WCF, and mQFE. We show that $\eta = \eta^{AKT}$. Fix a set of applicants A and a category profile $T = (t(i))_{i \in A}$. We will prove that $\eta(A, T) = \eta^{AKT}(A, T)$.

By the OA principle, $\{\, i \in A : (i, o) \in \eta(A, T) \,\} = \{\, i \in A : \text{rank}_A(i) \leq q_s^o \,\}$. This is exactly the set assigned to the open-category in Stage 1 of η^{AKT}. Hence, the open-category assignments under η and η^{AKT} coincide.

Let $A^1 = A \setminus \{\, i \in A : (i, o) \in \eta(A, T) \,\}$. Since the open-category assignments under η and η^{AKT} are identical, the set A^1 is also exactly the set of applicants remaining after Stage 1 of η^{AKT}. Fix $r \in R \setminus \{OBC\}$, and let $A_r^1 = \{\, i \in A^1 : t(i) = r \,\}$ be the set of remaining applicants of type r after seats in the open-category are filled.

We claim that under η, the applicants assigned to category r are exactly the highest-scoring $\min\{|A_r^1|, q_s^r\}$ applicants in A_r^1.

First, suppose that $|A_r^1| \leq q_s^r$. Then every applicant in A_r^1 must be assigned to category-r. Otherwise, if some $i \in A_r^1$ were not assigned, then since $t(i) = r \neq OBC$, part (1) of mQFE would require that all q_s^r category-r seats be filled. But, there are fewer than q_s^r applicants of type r remaining, which is impossible because category-r positions can only be assigned to applicants of type r. Hence, all applicants in A_r^1 must be assigned.

Now suppose that $|A_r^1| > q_s^r$. Then exactly q_s^r applicants of type r must be assigned, again by part (1) of mQFE. Moreover, WCF implies that these q_s^r assignees must be the highest-scoring q_s^r applicants in A_r^1: if a lower-scoring applicant $j \in A_r^1$ were assigned while a higher-scoring applicant $i \in A_r^1$ were not assigned, then WCF would be violated. Thus, for every $r \in R \setminus \{OBC\}$, the category-$r$ assignments under η coincide with those in Stage 2 of η^{AKT}.

Let $A^1_{OBC} = \{\, i \in A^1 : t(i) = OBC \,\}$ be the set of OBC applicants remaining after the open-category seats are filled. Let $\underline{\theta}^o(A) = \min\{\theta(j) : (j,o) \in \eta(A,T)\}$ be the open-category cutoff score. Since open-category assignments are already determined by OA, this is exactly the same cutoff score used in Stage 3 of η^{AKT}.

Let $E = \{\, i \in A^1_{OBC} : \theta(i) \geq \underline{\theta}^o(A) - \lambda \,\}$ be the set of remaining eligible OBC applicants. We now show that under η, the applicants assigned to the OBC category are exactly the highest-scoring $\min\{|E|, q_s^{OBC}\}$ applicants in E.

Take any $i \in A^1_{OBC} \setminus E$. Then $\theta(i) < \underline{\theta}^o(A) - \lambda$. By part (3) of mQFE, such an applicant cannot be assigned to an OBC slot. Hence every OBC assignee under η must belong to E. Thus, no ineligible OBC applicant can be assigned.

If $|E| \leq q_s^{OBC}$, then every applicant in E must be assigned. Suppose, to the contrary, that some $i \in E$ is not assigned. Since i is eligible and unassigned, part (2) of mQFE implies that all q_s^{OBC} OBC seats must be filled. But by part (i), only applicants in E can be assigned to OBC seats, and $|E| \leq q_s^{OBC}$. Hence, it is impossible to fill all q_s^{OBC} OBC seats while leaving some applicants in E unmatched. Contradiction. Therefore, every applicant in E must be assigned.

If $|E| > q_s^{OBC}$, then exactly the highest-scoring q_s^{OBC} applicants in E must be assigned. Because there are more than q_s^{OBC} eligible OBC applicants, capacity implies that at most q_s^{OBC} of them can be assigned. On the other hand, if some eligible OBC applicants were left unmatched, part (2) of mQFE requires that all q_s^{OBC} OBC seats be filled. Thus, exactly q_s^{OBC} applicants must be assigned to OBC seats.

Finally, WCF implies that these assignees must be the highest-scoring q_s^{OBC} applicants in E: if a lower-scoring eligible OBC applicant $j \in E$ were assigned while a higher-scoring eligible OBC applicant $i \in E$ were not assigned, then $t(i) = t(j) = OBC$ and $i \succ_s j$, contradicting WCF.

We can conclude that the OBC assignments under η coincide exactly with the OBC assignments in Stage 3 of η^{AKT}. Therefore, $\eta(A,T) = \eta^{AKT}(A,T)$. Since A and T were arbitrary, it follows that $\eta = \eta^{AKT}$. ■

PROOF OF PROPOSITION 1. We first show that C_s^{AKT} satisfies the substitutes condition. Consider $i, j \in I$ and $A \subset I \setminus \{i, j\}$ such that $i \notin C_s^{AKT}(A \cup \{i\}, q_s)$. We need to show that $i \notin C_s^{AKT}(A \cup \{i, j\}, q_s)$.

Since $i \notin C_s^{AKT}(A \cup \{i\}, q_s)$, $\nexists t \in \mathcal{C}$ such that $(i, t) \in \eta_s^{AKT}(A \cup \{i\})$. Let $\mathbf{t}(i) = GC$. Then, it must be the case that $rank_{A\cup\{i\}}(i) > q_s^o$. When an individual j joins, we still have $rank_{A\cup\{i,j\}}(i) > q_s^o$. Thus, we have $i \notin C_s^{AKT}(A \cup \{i, j\}, q_s)$.

Now, let $\mathbf{t}(i) \in \{SC, ST, EWS\}$. Without loss of generality, suppose that $\mathbf{t}(i) = SC$. $i \notin C_s^{AKT}(A \cup \{i\}, q_s)$ implies that (1) $rank_{A\cup\{i\}}(i) > q_s^o$ so that i could not obtain an open-category position and (2) $rank_{A^1_{SC}\cup\{i\}}(i) > q_s^{SC}$, where $A^1_{SC} \cup \{i\}$ is the set of remaining unassigned SC applicants after the open-category positions were filled. When j is added to the set $A \cup \{i\}$, both (1) and (2) still hold. Thus, $i \notin C_s^{AKT}(A \cup \{i, j\}, q_s)$.

Now, consider $\mathbf{t}(i) = OBC$. $i \notin C_s^{AKT}(A \cup \{i\}, q_s)$ implies $rank_{A\cup\{i\}}(i) > q_s^o$ and $rank_{A^1_{OBC}\cup\{i\}}(i) > q_s^{OBC}$, where $A^1_{OBC} \cup \{i\}$ is the set of unassigned remaining OBC applicants after the open-category positions were filled, or $\theta(i) < min\{\theta(j)|(j, o) \in \eta_s(A)\} - \lambda$. Including j to $A \cup \{i\}$ does not affect either condition. Therefore, $i \notin C_s^{AKT}(A \cup \{i, j\}, q_s)$.

Next, we show that C_s^{AKT} satisfies consistency. Suppose $i \notin C_s^{AKT}(A \cup \{i\}, q_s)$. We must have $rank_{A\cup\{i\}}(i) > q_s^o$. By the definition of C_s^{AKT}, the same set of individuals is assigned to open-category positions when i is removed from the set of alternatives.

If $\mathbf{t}(i) \in \{SC, ST\}$, then the positions in category $\mathbf{t}(i)$ must be filled with individuals with higher scores. Thus, removing i from the set of alternatives does not affect who is chosen in the SC and ST categories.

Finally, if $\mathbf{t}(i) = OBC$, then $i \notin C_s^{AKT}(A \cup \{i\}, q_s)$ implies that $\theta(i) < min\{\theta(j)|(j, o) \in \eta_s(A)\} - \lambda$ or $rank_{A^1_{OBC}\cup\{i\}}(i) > q_s^{OBC}$, where $A^1_{OBC} \cup \{i\}$ is the set of remaining unassigned OBC applicants after the open-category positions were filled. In both cases, removing i from the set of individuals available does not change the set of individuals chosen for the OBC positions.

Thus, we conclude $C_s^{AKT}(A \cup \{i\}, q_s) = C_s^{AKT}(A, q_s)$. ■

PROOF OF THEOREM 3. Fix (A, T). Let $C^{FT}(A)$ and $C^{f}(A)$ denote the sets selected under η^{FT} and η^{f}, respectively, for some $f \in F$.

Let $C^{o}(A) = \{i \in A : \text{rank}_A(i) \leq q_s^o\}$ be the set of applicants assigned to the original open-category positions. By the OA principle, $C^{o}(A)$ is identical under all rules in $\mathbb{A}$.

Let $A^1 = A \setminus C^{o}(A)$. Fix $r \in R$. Under η^{FT}, no de-reserved positions are allocated before any reserve category is processed. Hence, $C_r^{FT}(A)$ is the top q_s^r elements of A_r^1, where $A_r^1 = \{i \in A^1 : t(i) = r\}$, with truncation if $|A_r^1| < q_s^r$.

Under η^{f}, some de-reserved open positions may be allocated prior to category r. Let $D_r^f(A) \subseteq A_r^1$ denote the set of type-r applicants selected into earlier de-reserved positions. Then, $C_r^f(A)$ is the top q_s^r elements of $A_r^1 \setminus D_r^f(A)$.

Since $A_r^1 \setminus D_r^f(A) \subseteq A_r^1$, there exists an injection $g_r : C_r^f(A) \to C_r^{FT}(A)$ such that

$$\theta(g_r(i)) \geq \theta(i) \quad \text{for all } i \in C_r^f(A).$$

Because the reserve categories are disjoint, the maps $\{g_r\}_{r \in R}$ can be combined into an injection

$$g_R : \bigcup_{r \in R} C_r^f(A) \to \bigcup_{r \in R} C_r^{FT}(A)$$

satisfying $\theta(g_R(i)) \geq \theta(i)$ for all i.

Now let us define

$$U^f(A) = C^f(A) \setminus \Big(C^o(A) \cup \bigcup_{r \in R} C_r^f(A) \Big),$$

$$U^{FT}(A) = C^{FT}(A) \setminus \Big(C^o(A) \cup g_R\big(\bigcup_{r \in R} C_r^f(A) \big) \Big).$$

Then $|U^f(A)| = |U^{FT}(A)|$.

Order the elements of both sets in decreasing merit:

$$i_1^f \succ_s \cdots \succ_s i_m^f, \qquad i_1^{FT} \succ_s \cdots \succ_s i_m^{FT}.$$

Under η^{FT}, all de-reserved positions are allocated only after all reserve categories are processed; hence $U^{FT}(A)$ consists of the highest-scoring remaining applicants. Under any

η^f, some of these positions may be allocated earlier, which can only remove high-scoring applicants from later reserve-category consideration and cannot improve the final merit composition. Therefore, $\theta(i_\ell^{FT}) \geq \theta(i_\ell^f)$ for all $\ell = 1, \ldots, m$.

Define $g_U(i_\ell^f) = i_\ell^{FT}$. Then $g_U : U^f(A) \to U^{FT}(A)$ is injective and satisfies

$$\theta(g_U(i)) \geq \theta(i) \quad \text{for all } i \in U^f(A).$$

We can define $g : C^f(A) \to C^{FT}(A)$ by

$$g(i) = \begin{cases} i & \text{if } i \in C^o(A), \\ g_R(i) & \text{if } i \in \bigcup_{r \in R} C_r^f(A), \\ g_U(i) & \text{if } i \in U^f(A). \end{cases}$$

Then g is injective and satisfies $\theta(g(i)) \geq \theta(i)$ for all $i \in C^f(A)$.

If $C^{FT}(A) \neq C^f(A)$, then at least one inequality is strict. Hence $C^{FT}(A)$ strictly merit-dominates $C^f(A)$ whenever the two sets differ. Therefore, for all $A \subseteq I$, $C^{FT}(A)$ merit-dominates $C^f(A)$ or $C^f(A) \subseteq C^{FT}(A)$. ■

PROOF OF THEOREM 4. Fix a transfer scheme $f \in F$, and let C^f denote the choice rule induced by the assignment rule η^f. For any fixed capacity vector $x = (x_1, \ldots, x_{10}) \in \mathbb{Z}_+^{10}$, let C^x denote the choice rule induced by the one-shot assignment rule η^x defined on the expanded precedence order $\text{Open} - T_1 - c_1 - T_2 - c_2 - T_3 - c_3 - T_4 - c_4 - T_5$.

We proceed in two steps.

Step 1: In this step, we show that the rule C^x satisfies consistence, the substitute property, and size monotonicity. Fix x. The rule C^x is a sequential priority rule with fixed capacities. At each block, the highest-priority remaining applicants are selected subject to the block's capacity constraint.

Consistency. Let $A \subseteq I$ and $i \in I \setminus A$. Suppose $i \notin C^x(A \cup \{i\})$. Then i is never selected at any block. Removing i does not affect the set of applicants available at any stage of the procedure. Therefore, $C^x(A \cup \{i\}) = C^x(A)$. Hence, C^x is consistent.

Substitutes property. Let $i, j \in I$ and $A \subseteq I \setminus \{i, j\}$. Suppose $j \notin C^x(A \cup \{j\})$.

Then at the stage where j could potentially be selected, either the relevant capacity is already filled with higher-priority applicants or j is ineligible. Adding i weakly increases competition and does not increase capacity. Therefore, $j \notin C^x(A \cup \{i, j\})$. Hence, C^x satisfies the substitutes property.

Size monotonicity. Let $i \in I \setminus A$. In each group of positions, the number of selected applicants is the minimum of the capacity of that group and the number of relevant applicants. Adding i weakly increases the number of relevant applicants at each stage, and the capacities remain fixed. Therefore, $|C^x(A)| \leq |C^x(A \cup \{i\})|$.

Step 2: In this step, we prove the same properties for C^f.

The rule η^f is implemented via a sequence of capacity vectors $x^1, x^2, \ldots, x^{N(A)}$, for any $A \subseteq I$, where $x^{N(A)}$ is the terminal capacity vector. By construction, $C^f(A) = C^{x^{N(A)}}(A)$.

Observe that enlarging the applicant set weakly reduces the number of vacant OBC positions at each iteration. Therefore, the total number of transferred positions to the de-reserved open categories is weakly smaller when the applicant set expands.

Consistency. Let $i \notin C^f(A \cup \{i\})$. Then i does not affect the number of vacant OBC positions at any iteration. Hence, $x^{N(A \cup \{i\})} = x^{N(A)}$. Therefore, $C^f(A \cup \{i\}) = C^{x^{N(A)}}(A \cup \{i\})$. Since $C^{x^{N(A)}}$ is consistent, $C^f(A \cup \{i\}) = C^{x^{N(A)}}(A) = C^f(A)$.

Substitutes property. Let $i, j \in I$ and $A \subseteq I \setminus \{i, j\}$. Suppose $j \notin C^f(A \cup \{j\})$.

Adding i weakly reduces the number of transferred open-category positions and increases competition. Therefore, j cannot become selected: $j \notin C^f(A \cup \{i, j\})$.

Size monotonicity. Let $i \in I \setminus A$. If $i \notin C^f(A \cup \{i\})$,then $C^f(A \cup \{i\}) = C^f(A)$ by consistency. If $i \in C^f(A \cup \{i\})$, then the number of filled positions weakly increases. Therefore, $|C^f(A)| \leq |C^f(A \cup \{i\})|$. ■

PROOF OF THEOREM 5. Fix a set of applicants A and a category profile T. We compare the sets of selected applicants under η^* and η^{FT}.

Let $C^o(A) = \{i \in A : \text{rank}_A(i) \leq q_s^o\}$ be the set of applicants assigned to the original open-category seats. By construction, both η^* and η^{FT} assign exactly the applicants

in $C^o(A)$ to the original open-category positions. Hence, the two rules have the same original open-category cutoff, call it $\underline{\theta}^o(A) = \min\{\theta(i) : i \in C^o(A)\}$.

Let $A^1 = A \setminus C^o(A)$ be the set of applicants remaining after the original open-category positions are filled. For each $r \in \{SC, ST, EWS\}$, both η^* and η^{FT} assign the category-r positions to the highest-scoring applicants in $A^1_r = \{i \in A^1 : t(i) = r\}$ up to q^r_s. Thus, the sets of applicants assigned to SC, ST, and EWS seats are identical under the two rules.

Thus, the only possible difference between η^* and η^{FT} arises in the treatment of OBC positions and of the seats generated by de-reservation. Define the set of OBC applicants who are eligible under the fixed cutoff interpretation by

$$E^*(A) = \Big\{ i \in A^1 : t(i) = OBC \text{ and } \theta(i) \geq \underline{\theta}^o(A) - \lambda \Big\}.$$

Let $m^* = \min\{q^{OBC}_s, |E^*(A)|\}$. By the definition of η^*, the rule assigns the m^* highest-scoring applicants in $E^*(A)$ to OBC positions.

Let m^{FT} denote the number of applicants assigned to OBC positions under η^{FT}.

CLAIM 1. $m^* \geq m^{FT}$.

PROOF. Under η^* the OBC cutoff is determined solely by the original open-category positions, and therefore it is unaffected by any later de-reservation of vacant OBC seats. Hence, every applicant in $E^*(A)$ is eligible for an OBC position throughout the procedure, and η^* fills all OBC positions that can be filled subject to this fixed eligibility requirement.

By contrast, under η^{FT}, vacant OBC positions are converted into open-category positions through forward transfer. This transformation does not create new OBC positions; it only changes how vacant OBC positions are reallocated. Therefore, the number of seats filled as OBC positions under η^{FT} cannot exceed the number of seats filled as OBC positions under η^*. This proves the claim. ■

Let $\widetilde{A} = C^o_s(A) \cup C^{SC}_s(A) \cup C^{ST}_s(A) \cup C^{EWS}_s(A)$, where $C^{SC}_s(A), C^{ST}_s(A), C^{EWS}_s(A)$ denote the sets of applicants selected for these categories. As argued above, these sets are identical under η^* and η^{FT}.

Since both rules fill all seats whenever possible and both have the same total capacity, the number of applicants selected outside $\widetilde{A}$ is the same under the two rules. Denote this number by $x = \overline{q}_s - |\widetilde{A}|$.

Under η^*, exactly m^* of these x applicants are selected for OBC slots, and the remaining $x - m^*$ applicants are chosen from the set of applicants not yet selected, in decreasing order of merit, regardless of category.

Under η^{FT}, exactly m^{FT} of these x applicants are selected for OBC slots, and the remaining $x - m^{FT}$ applicants are selected through forward transfer.

We now construct an injection $g : C^{FT}(A) \to C^*(A)$ to show merit-domination. For every applicant $i \in \widetilde{A}$, define $g(i) = i$.

List the applicants assigned to OBC positions under η^{FT} in decreasing order of score: $i_1^{FT} \succ_s i_2^{FT} \succ_s \cdots \succ_s i_{m^{FT}}^{FT}$.

Similarly, list the applicants assigned to OBC positions under η^* in decreasing order of score: $i_1^* \succ_s i_2^* \succ_s \cdots \succ_s i_{m^*}^*$.

Since $m^* \geq m^{FT}$, for each $\ell = 1, \ldots, m^{FT}$ define $g(i_\ell^{FT}) = i_\ell^*$. Because η^* selects the highest-scoring eligible OBC applicants under the fixed cutoff, we have

$$\theta(i_\ell^*) \geq \theta(i_\ell^{FT}) \qquad \text{for each } \ell = 1, \ldots, m^{FT}.$$

After removing the applicants in $\widetilde{A}$ and the matched OBC assignees above, let us define

$$R^{FT}(A) = C^{FT}(A) \setminus \left(\widetilde{A} \cup \{i_1^{FT}, \ldots, i_{m^{FT}}^{FT}\}\right),$$

$$R^*(A) = C^*(A) \setminus \left(\widetilde{A} \cup \{i_1^*, \ldots, i_{m^{FT}}^*\}\right).$$

Then $|R^{FT}(A)| = x - m^{FT} = |R^*(A)|$.

Order the elements of these two sets in decreasing order of merit:

$$j_1^{FT} \succ_s j_2^{FT} \succ_s \cdots \succ_s j_{x-m^{FT}}^{FT},$$

$$j_1^* \succ_s j_2^* \succ_s \cdots \succ_s j_{x-m^{FT}}^*.$$

By construction, η^* fills all remaining vacancies by taking the highest-scoring remaining applicants, regardless of category. Hence, position by position,

$$\theta(j_\ell^*) \geq \theta(j_\ell^{FT}) \qquad \text{for each } \ell = 1, \ldots, x - m^{FT}.$$

Define $g(j_\ell^{FT}) = j_\ell^*$ for each $l = 1, \ldots, x - m^{FT}$. The map g is injective by construction. Moreover, for every $i \in C^{FT}(A)$, $\theta(g(i)) \geq \theta(i)$.

It remains to show that, when $C^{FT}(A) \neq C^*(A)$, at least one of these inequalities is strict.

If $m^* > m^{FT}$, then η^* fills strictly more genuine OBC positions than η^{FT}. Since both rules ultimately select the same total number of applicants, η^* must compensate by selecting a strictly more meritorious residual applicant at some stage. Hence, there exists $i \in C^{FT}(A)$ such that $\theta(g(i)) > \theta(i)$.

If $m^* = m^{FT}$ but $C^{FT}(A) \neq C^*(A)$, then the difference must come from the residual selections. Since η^* chooses the highest-scoring remaining applicants while η^{FT} does not necessarily do so, there exists some ℓ such that $\theta(j_\ell^*) > \theta(j_\ell^{FT})$, and again there exists $i \in C^{FT}(A)$ such that $\theta(g(i)) > \theta(i)$.

Hence, for every set of applicants A, $C^{FT}(A) \subseteq C^*(A)$ or $C^*(A)$ merit-dominates $C^{FT}(A)$. Therefore, C^* merit-dominates C^{FT}. ■

PROOF OF THEOREM 6. Fix an arbitrary set of applicants A and a category profile T. Let C' be the choice rule induced by an arbitrary assignment rule η' satisfying OA*, WCF*, mQFE, and OBC de-reservation*.

By OA*, every admissible assignment rule must assign exactly the top $\min\{|A|, q_s^o\}$ applicants to the original open-category positions labeled o. Therefore, the set $C^o(A) = \{i \in A : \text{rank}_A(i) \leq q_s^o\}$ is selected in both C^* and C'. Hence, the applicants selected for the original open-category positions are the same under C^* and C'.

We now show that after removing individuals who are assigned open-category positions, the assignments in SC, ST, EWS, and OBC positions are also the same under

C^* and C'. Let $A^1 = A \setminus C^o(A)$. Consider first a category $r \in \{SC, ST, EWS\}$. Let $A^1_r = \{i \in A^1 : t(i) = r\}$.

CLAIM 2. *Any assignment rule satisfying WCF* and mQFE must assign the category-r seats to the highest-scoring* $\min\{q^r_s, |A^1_r|\}$ *applicants in* A^1_r.

PROOF OF CLAIM 2. WCF* implies that if a lower-scoring applicant of category r is assigned, then every higher-scoring applicant of category r must also be assigned. Thus, the selected applicants of type r must form an upper contour set of A^1_r with respect to merit. Moreover, mQFE requires that if some applicant of category r remains unassigned, then all q^r_s seats in category r must already be filled. Thus, the rule must assign exactly the highest-scoring $\min\{q^r_s, |A^1_r|\}$ applicants in A^1_r. Hence, the SC, ST, and EWS assignments are identical in C^* and C'.

Now consider the OBC category. Under mQFE, the eligible OBC applicants are those whose score is at least the original open-category cutoff minus λ. Since OA* fixes the set of individuals who are assigned open-category positions, the open-category cutoff is the same under C^* and C'. Let $E_{OBC}(A) = \left\{i \in A^1 : t(i) = OBC \text{ and } \theta(i) \geq \theta(j^*) - \lambda\right\}$, where j^* is the lowest-scoring applicant among the q^o_s open-category winners.

By WCF*, among eligible OBC applicants, any admissible rule must choose higher-scoring applicants before lower-scoring ones. By mQFE, it must fill all q^{OBC}_s OBC seats whenever there are at least q^{OBC}_s eligible OBC applicants, and otherwise it must assign all eligible OBC applicants and leave the remaining OBC seats vacant. Therefore, both C^* and C' choose exactly the same set of applicants for the genuine OBC seats, namely the highest-scoring $m = \min\{q^{OBC}_s, |E_{OBC}(A)|\}$ members of $E_{OBC}(A)$.

■

Thus, C^* and C' coincide in the set of individuals assigned open-category, SC, ST, EWS, and OBC positions. Let $\widetilde{A}$ denote this common set of applicants. Thus, the only possible difference concerns the de-reserved positions labeled d.

The number of vacant OBC seats is $\kappa = q_s^{OBC} - m$. These are precisely the seats that can be labeled as category d. Let $R(A) = A \setminus \widetilde{A}$ be the set of remaining applicants after all assignments forced by OA*, WCF*, and mQFE have been made in reserved categories.

By construction of η^*, the rule fills the d-positions by choosing the highest-scoring $\min\{\kappa, |R(A)|\}$ applicants from $R(A)$, regardless of category.

Now consider the arbitrary admissible rule η'. Since it also satisfies OBC de-reservation*, if there are vacant OBC seats and enough remaining applicants, it must fill those vacancies by assigning applicants to d-positions. More precisely:

- if $|R(A)| \geq \kappa$, then C' must select exactly κ applicants from $R(A)$ for the d-positions;
- if $|R(A)| < \kappa$, then OBC de-reservation* forces the rule to choose all remaining applicants, so $C'(A) = A = C^*(A)$.

The only nontrivial case is $|R(A)| \geq \kappa$. In that case, both rules select exactly κ additional applicants from $R(A)$, but C^* selects the top κ applicants by score, whereas C' selects some κ-element subset of $R(A)$.

Define $D^*(A) = C^*(A) \setminus \widetilde{A}$ and $D'(A) = C'(A) \setminus \widetilde{A}$. Then $|D^*(A)| = |D'(A)| = \kappa$, and $D^*(A)$ is the set of the κ highest-scoring applicants in $R(A)$.

Order the elements of $D^*(A)$ and $D'(A)$ from highest to lowest score:

$$i_1^* \succ_s i_2^* \succ_s \cdots \succ_s i_\kappa^*, \qquad i_1' \succ_s i_2' \succ_s \cdots \succ_s i_\kappa'.$$

Since $D^*(A)$ consists of the top κ applicants of $R(A)$, for every $\ell = 1, \ldots, \kappa$ we have $\theta(i_\ell^*) \geq \theta(i_\ell')$. We now define an injection $g : C'(A) \to C^*(A)$ as follows:

- for each applicant $i \in \widetilde{A}$, let $g(i) = i$;
- for each $\ell = 1, \ldots, \kappa$, let $g(i_\ell') = i_\ell^*$.

This map is well-defined and injective. Moreover, every applicant selected by $C'(A)$ is mapped to an applicant selected by $C^*(A)$ with weakly higher score.

If $D'(A) \neq D^*(A)$, then at least one of the inequalities $\theta(i_\ell^*) \geq \theta(i_\ell')$ is strict. Thus, $C^*(A)$ merit-dominates $C'(A)$. If $D'(A) = D^*(A)$, then $C'(A) = C^*(A)$. Hence, for

every set of applicants A, $C^*(A)$ merit dominates $C'(A)$ or $C^*(A) = C'(A)$. Since A was arbitrary, C^* merit-dominates C'.

Because C' was an arbitrary choice rule induced by an assignment rule satisfying OA*, WCF*, mQFE, and OBC de-reservation*, it follows that C^* merit-dominates every such choice rule. ■

PROOF OF PROPOSITION 2. Fix a set of applicants $A \subseteq I$. Let $C^o(A) = \{i \in A : \text{rank}_A(i) \leq q_s^o\}$ be the set of applicants assigned to the original open-category positions under η^*, and let

$$\theta^o(A) = \min\{\theta(i) : i \in C^o(A)\}$$

denote the original open-category cutoff score. Let $A^1 = A \setminus C^o(A)$ be the set of applicants remaining after the original open-category positions are filled. For each $r \in \{SC, ST, EWS\}$, let $A_r^1 = \{i \in A^1 : t(i) = r\}$, and let $C^r(A)$ be the set of the highest-scoring applicants in A_r^1, chosen up to capacity q_s^r.

Next, define the set of OBC applicants who are eligible under the fixed-cutoff interpretation by $E^*(A) = \{i \in A^1 : t(i) = OBC \text{ and } \theta(i) \geq \theta^o(A) - \lambda\}$. Let $m^*(A) = \min\{q_s^{OBC}, |E^*(A)|\}$, and let $C^{OBC*}(A)$ be the set of the $m^*(A)$ highest-scoring applicants in $E^*(A)$.

Finally, let $R(A) = A \setminus \left(C^o(A) \cup C^{SC}(A) \cup C^{ST}(A) \cup C^{EWS}(A) \cup C^{OBC*}(A)\right)$ be the set of applicants remaining after the original open-category, the SC, ST, EWS, and the OBC assignments are made. Let $\kappa(A) = q_s^{OBC} - m^*(A)$ be the number of vacant OBC positions. Under η^*, these $\kappa(A)$ positions are relabeled as category-D positions and are assigned to the highest-scoring $\min\{\kappa(A), |R(A)|\}$ applicants in $R(A)$. Denote this set by $C^D(A)$. Hence,

$$C^*(A) = C^o(A) \cup C^{SC}(A) \cup C^{ST}(A) \cup C^{EWS}(A) \cup C^{OBC*}(A) \cup C^D(A).$$

We now prove the three claims.

Consistency. Let $A \subseteq I$ and let $i \in I \setminus A$ be such that $i \notin C^*(A \cup \{i\})$. Since $i \notin C^*(A \cup \{i\})$, applicant i is not selected in any of the six stages of the construction of η^*.

First, $i \notin C^o(A \cup \{i\})$. Since removing an unchosen applicant cannot affect the set of top q_s^o applicants, we have $C^o(A \cup \{i\}) = C^o(A)$. In particular, $\theta^o(A \cup \{i\}) = \theta^o(A)$, and hence $A^1(A \cup \{i\}) \setminus \{i\} = A^1(A)$.

Second, for each $r \in \{SC, ST, EWS\}$, applicant i is not selected in category r. Since the sets of applicants considered for these categories are unchanged except for the presence of the unchosen applicant i, the selected sets remain unchanged: $C^r(A \cup \{i\}) = C^r(A)$ for each $r \in \{SC, ST, EWS\}$.

Third, because $\theta^o(A \cup \{i\}) = \theta^o(A)$ and because no earlier selected set changes, the eligible OBC set satisfies $E^*(A \cup \{i\}) \setminus \{i\} = E^*(A)$.

Since $i \notin C^{OBC*}(A \cup \{i\})$, removing i does not affect the set of the top $m^*(A \cup \{i\})$ applicants in $E^*(A \cup \{i\})$. Thus, $C^{OBC*}(A \cup \{i\}) = C^{OBC*}(A)$. Consequently, $m^*(A \cup \{i\}) = m^*(A)$ and $\kappa(A \cup \{i\}) = \kappa(A)$.

Finally, since all earlier selected sets are unchanged, the remaining set for the D-positions satisfies $R(A \cup \{i\}) \setminus \{i\} = R(A)$. Because $i \notin C^D(A \cup \{i\})$ and the number of d-positions is unchanged, removing i does not alter the set of highest-scoring applicants chosen for the d-positions. Thus, $C^D(A \cup \{i\}) = C^D(A)$.

Combining the equalities above yields $C^*(A \cup \{i\}) = C^*(A)$.

Substitutes property. Let $A \subseteq I \setminus \{i, j\}$ and suppose $j \notin C^*(A \cup \{j\})$. Consider the stages of the construction of η^*.

At the original open-category stage, the rule selects the top q_s^o applicants by merit. Since $j \notin C^*(A \cup \{j\})$, applicant j is not selected at this stage from $A \cup \{j\}$. Adding one more applicant i cannot improve the rank of j. Hence, $j \notin C^o(A \cup \{i, j\})$.

At the SC, ST, and EWS stages, the capacities are fixed, and the rule selects the remaining eligible applicants with the highest scores in each category. Adding i can only weakly increase competition and cannot create a new seat in any reserve category. Thus, if j is not selected in one of these categories from $A \cup \{j\}$, then j is not selected there from $A \cup \{i, j\}$.

In the OBC stage, the relevant cutoff is determined solely by the original seats in the open-category. Adding i cannot reduce the original open-category cutoff. Indeed, when

one additional applicant is added, the score of the q_s^o-th ranked applicant weakly increases. Thus, $\theta^o(A \cup \{i,j\}) \geq \theta^o(A \cup \{j\})$, which implies that the OBC eligibility threshold weakly increases: $\theta^o(A \cup \{i,j\}) - \lambda \geq \theta^o(A \cup \{j\}) - \lambda$. Therefore, adding i cannot make j newly eligible for a genuine OBC position if j was previously ineligible and if j was already eligible, adding i can only weakly increase competition for the fixed number q_s^{OBC} of OBC seats. Hence, $j \notin C^{OBC*}(A \cup \{i,j\})$.

Finally, at the d-stage, the rule chooses the highest-scoring remaining applicants for the vacant OBC positions relabeled as d-positions. Adding i cannot increase the number of applicants selected before the d-stage by less than zero, nor can it improve the relative rank of j among the remaining applicants. Thus, if j is not among the highest-scoring applicants chosen for the d-positions from $A \cup \{j\}$, then j cannot become chosen after adding one more competitor i. Hence, $j \notin C^D(A \cup \{i,j\})$. Thus, $j \notin C^*(A \cup \{i,j\})$, and so C^* satisfies the substitutes property.

Size monotonicity. Let $A \subseteq I$ and $i \in I \setminus A$. If $|A| < \overline{q}_s$, then $|C^*(A \cup \{i\})| = 1 + |C^*(A)|$ because under η^*, after the original open-category, SC, ST, EWS, OBC assignments are made, every remaining vacant OBC seat is filled as a d-position whenever there is a remaining applicant. That is, η^* fills all seats whenever possible.

Suppose $|A| \geq \overline{q}_s$. Then $|C^*(A)| = \overline{q}_s$. If $\theta(i) < \min_{j \in C^*(A)} \theta(j)$, then i is rejected and $|C^*(A \cup \{i\})| = |C^*(A)|$ by consistency. Suppose now that there exists some $j \in C^*(A)$ such that $\theta(i) > \theta(j)$. Then $i \in C^*(A \cup \{i\})$. Since $C^*(A \cup \{i\})| = \overline{q}_s$, i replaces another individual, call it j'. The individual j' may replace someone else in a category considered later in the process but total number of chosen individuals will remain unchanged at $\overline{q}_s$.

■