Innovation Potential, Firm Life Cycle, and Cross-Country Productivity Differences*

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March 1, 2026

Extended abstract: Technological innovation is pivotal for long-run productivity improvement, yet the design of innovation policies often overlooks heterogeneity in firms' innovation abilities. In this paper, I provide new facts on firms' innovation abilities across countries. In developed countries, firm type, innovation quality, and innovation frequency are mostly determined at birth. In developing countries, there are fewer innovative and radically innovative firms. Additionally, these firms have lower radical innovation frequency but higher incremental innovation frequency. As innovative firms age in these economies, they transition toward radical innovation. The cross-country differences are primarily observed in industries with high external financial dependence. To explain these facts and understand their macroeconomic implications, I build a semi-endogenous growth model with financial frictions and firm heterogeneity and demonstrate that collateral constraints combined with limited liability conditions impede innovative firm creation and distort firms' choice between radical and incremental innovation in developing countries. Radical innovation, which requires higher upfront costs, disproportionately burdens young firms in financially constrained environments. The model shows that financial frictions in developing countries suppress aggregate productivity primarily by reducing the creation of innovative firms and distorting innovation choices, highlighting the composition of firm types as a key driver of cross-country productivity gaps. I offer new policy implications for countries at different development stages: for developed countries, creating more innovative firms is more effective than subsidizing incumbent ones; for developing countries, both creating more innovative firms and subsidizing young, radically innovative firms are important. However, developing countries face greater challenges in accurately identifying firm types, as financial frictions distort firms' choices among different innovation types. Counterfactual analyses suggest that the efficiency loss from existing innovation policies is substantial. The feasibility of firm type-specific interventions is also discussed. By bridging macro-development and micro-innovation dynamics, this paper redefines how policymakers can leverage firm heterogeneity to unlock productivity gains across different development stages.

Keywords: innovation, ex-ante firm heterogeneity, financial frictions, TFP loss.

JEL classification: D24, E22, O16, O30, O40.

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*I am extremely grateful to Ufuk Akcigit, Francisco Buera, Yiming Cao, Gaurav Chiplunkar, Tuo Chen, Min Fang, Tilman Graff, Xing Guo, Zhen Huo, Chang Liu, Marta Morazzoni, Ezra Oberfield, Guillermo Ordonez, Tommaso Porzio, Lukasz Rachel, Thomas Sampson, Yongseok Shin, Vincent Sterk, Burak Uras, Lichen Zhang, Shenzhe Jiang, Yong Wang, Chunzan Wu, Anson Zhou, Lijun Zhu, and Xiaodong Zhu for their inspiring comments and suggestions. I thank seminar participants at BSE Summer Forum (Growth and Productivity Workshop), STEG Theme 1 Workshop, NEUDC Conference, China International Conference in Macroeconomics, Stone Centre Inaugural Affiliate Workshop, Peking University, National University of Singapore, Singapore Management University, and the University of Hong Kong for their helpful discussions. I also thank Zhengqi Tao, Guanxi Li, Boqi Zhang, and Tao Tang for their excellent research assistance. All errors are my own.

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1 Introduction

Technological innovation is pivotal for long-run productivity improvement in developed countries. While developing countries also rely on technology diffusion from developed nations for growth, a transition to innovation-driven growth becomes necessary for sustainability as they converge to the global technology frontier. Numerous policies have been implemented in both developed and developing countries to promote innovation within firms, particularly focusing on encouraging patent creation. For example, China has provided patent quantity-based subsidies irrespective of firm characteristics. Germany has launched a high-tech start-up fund aimed at small, young firms that are heavily involved in patenting activities. The U.S. has initiated the SBIR program to provide grants to small businesses for innovation. Subsidies for innovation in large (and old) incumbent firms are also common across many countries. Figure 1.1 provides a more comprehensive view of innovation policy instruments across countries, suggesting that most policies do not differentiate between firms of different ages or sizes.[1] However, as firms differ in their innovation capabilities and growth potential, they each have a different impact on aggregate productivity improvement. Consequently, the effect of innovation policies on growth depends on which types of firms are subsidized.

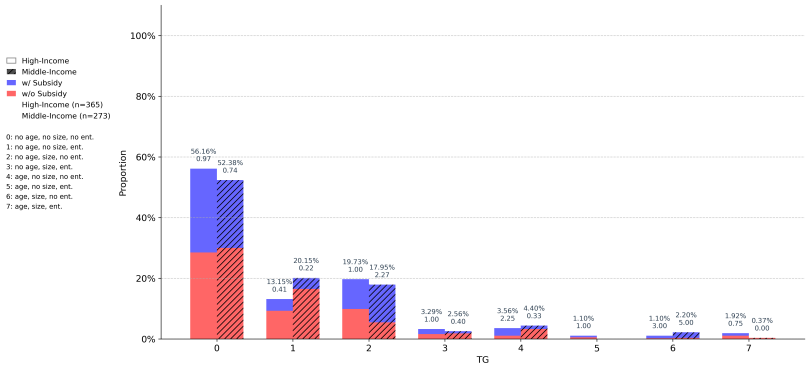
Given that a higher number of patents and citations are correlated with a firm's greater growth potential (cf. Acemoglu et al. (2022)), firms can be categorized into three types. A firm is considered *non-innovative* if it does not apply for patents; it is *incrementally innovative* if it applies for patents but receives few citations; and it is *radically innovative* if its patents attract a significant number of citations. The total number of citations in the economy in a given year, as demonstrated in equation (1.1), is the sum of citations from newly filed patents by all types of firms. These citations are generally positively correlated with aggregate productivity improvement (cf. Kelly et al. (2021)). For each firm i with age a that belongs to type k, its citations are products of two terms: citations per patent (innovation quality) and the number of patents it has (innovation frequency).

$$\underbrace{\text{Economy-wide citations}}_{\propto \text{ aggregate productivity}} = \sum_k \sum_{i^k(a)} \text{citations}_{i^k(a)} = \sum_k \sum_{i^k(a)} \Big(\underbrace{\frac{\text{citations}_{i^k(a)}}{\#\text{ patents}_{i^k(a)}}}_{\text{innovation quality}} \times \underbrace{\#\text{ patents}_{i^k(a)}}_{\text{innovation frequency}} \Big). \quad (1.1)$$

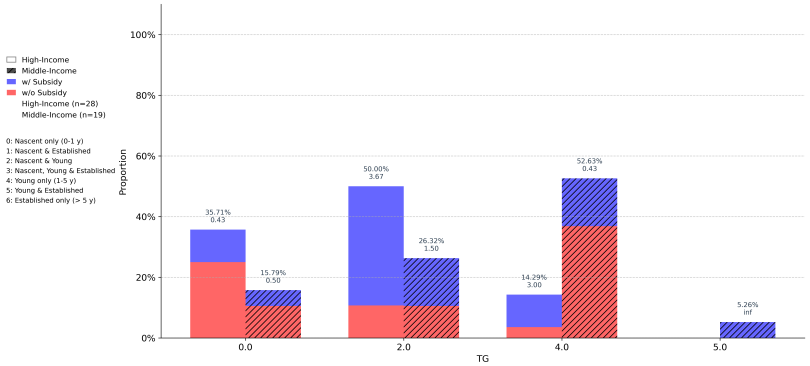
A firm's type, innovation quality, and innovation frequency may vary over its life cycle. Con-

[1] Figure 1.1 shows that policies which do not differentiate between firms of varying ages or sizes are more prevalent than size-dependent policies. These, in turn, are more common than policies targeting entrepreneurs or those dependent on firm age. Among age-dependent policies, young firms are most frequently subsidized. Among size-dependent policies, small firms are the ones most often subsidized.

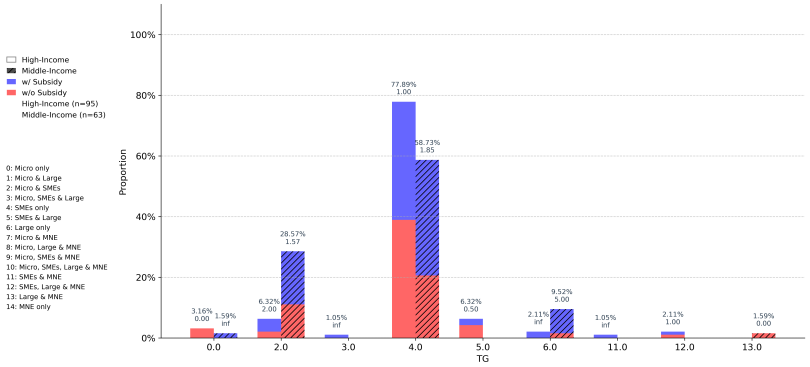
Figure 1.1: Science, Technology, and Innovation Policy (STIP): Various Targets



(a) All Targets



(b) If Target Age



(c) If Target Size

Notes: The X-axis in each panel denotes the groups of policies with different targets based on firm age, firm size, or entrepreneur characteristics. The Y-axis represents the fraction of policies targeting a specific group among all STIP policies targeting firms or entrepreneurs (Panel a), all STIP policies targeting firm age (Panel b), and all STIP policies targeting firm size (Panel c). In each group, one bar represents high-income countries, and the other shaded bar represents middle-income countries. The blue dots signify policies with subsidies, while the red dots indicate policies without subsidies. The ratio of subsidized policies to unsubsidized policies is also provided (below the fraction of policies targeting a specific group). The data is from OECD STIP Compass. The detailed data description is in Appendix B.1.

sequently, aggregate productivity improvement depends not only on the composition of firm types and their respective innovation quality and frequency but also on the life cycle dynamics of each type. If firms transition from being non-innovative to more innovative as they age, older firms will

drive aggregate growth, necessitating policies that support old incumbents. Conversely, if firms start as highly innovative but decline over time, younger firms will play a more critical role in aggregate growth, warranting policies that support new entrants.

To understand and increase aggregate productivity, this paper addresses the following questions: Do firm types, innovation quality, and frequency evolve over their life cycle, and if so, how? Are there differences across countries, and what could be the reasons? What are the policy implications? Are existing policies effective? These largely unexplored areas, possibly due to data limitations, are analyzed herein, contributing to empirical, theoretical, and policy discussions.

Empirically, I document that in developed (high-income) countries, firm type, innovation frequency, and innovation quality are mostly *invariant or decrease* over the life cycle, indicating that ex-ante factors are crucial and the highest innovation ability is determined at birth. In contrast, in developing (middle-income) countries, there is a lower number of innovative firms, specifically those that are radically innovative, along with a lower radical innovation frequency and a higher incremental innovation frequency. As innovative firms age, they become more radically innovative and exhibit higher radical innovation frequency, reducing the differences compared to high-income countries. These findings not only provide a novel picture of firms' life cycle dynamics in innovation but also reveal that cross-country differences are largely driven by a differing proportion of non-innovative firms. Previous studies often restrict the analysis to innovative firms, limiting the exploration of this extensive margin.

Furthermore, I demonstrate that cross-country differences in innovation are most pronounced in industries with high external financial dependence. This highlights the critical role of under-developed financial markets in middle-income countries—a well-documented phenomenon (e.g., Kremer et al. (2021))—in stifling innovation. These findings align with the nature of technological innovation, which often requires substantial, indivisible capital investments and involves lengthy, uncertain processes prone to failure (cf. Bassi et al. (2022), Ottonello and Winberry (2024)). As Tim Dettmers, a researcher at the Allen Institute for Artificial Intelligence, noted in a New York Times interview, "You have to invest a lot of money to try new things—and they often end in failure. This is why we don't see as much innovation: people are afraid that their massive investments will go to waste." I also provide additional evidence on leverage and the average product of capital (ARPK), emphasizing the important role of financial market development. To my knowledge, this paper uniquely shows how financial development could impact the innovations of different types of firms across countries, enhancing our understanding of how finance influences economic development and growth.

To explain the empirical findings and understand their macroeconomic implications, I build

a semi-endogenous growth model with financial frictions. My model focuses on the impact of financial frictions on the life cycle growth of firms involved in various types of innovation, as well as on the ex-ante factors that are crucial for firm growth. In my model, firm type is determined at birth subject to financial frictions. All firms produce homogeneous goods with decreasing returns to scale production technology and accumulate capital for production, while different firm types lead to different growth potential. Non-innovative firms have a fixed individual productivity level but rely on knowledge spillovers to expand the scale. Low-type innovative firms make incremental innovation investments to gradually increase productivity. High-type innovative firms make either incremental or radical innovation investments, with the latter generating dramatic increases in productivity at the expense of additional innovation costs. Firms can borrow and save with financial intermediaries. Financial frictions are represented by two financial constraints: a collateral constraint that limits the economy-wide credit, and a limited liability condition that imposes non-negative dividend constraints and hence restricts the amount of resources available for innovation investment.

A less developed financial market is modeled by a tighter collateral constraint, which not only limits capital accumulation, but also leads to a more binding limited liability condition that reduces firms' innovation investment. Furthermore, radical innovation investment is more negatively affected compared to incremental innovation investment, especially for young (and hence small) high-type innovative firms, because it is harder for these firms to overcome the additional innovation cost required by radical innovations. Lower innovation investment reduces innovation probability, slowing productivity improvement. Without financial frictions, high-type innovative firms are more productive, larger, and exhibit higher life cycle growth than low-type innovative firms, which in turn outperform non-innovative firms. However, in less developed financial markets, financial frictions reduce the aggregate TFP level through two channels: (1) reduced selection into innovative or radically innovative types, and (2) reduced innovation investment (and thus innovation probability), especially among young radically innovative firms.

Taking the model to the data, the results show that financial frictions reduce aggregate TFP level in middle-income countries mainly via the first channel— reduced selection into innovative or radically innovative types—as innovative firms can largely self-undo financial constraints as they age. While in high-income countries, which have more relaxed financial constraints, productivity gains obtained by further relaxing financial constraints are limited because ex-ante factors matter for firm types, innovation frequency, and innovation quality. By shutting down firm types or innovation types, the model is reduced to a model with life cycle growth of heterogeneous firms similar to Hsieh and Klenow (2014), among others. In this simplified version, relaxing constraints on incumbent

firms boosts life cycle growth for *all* incumbents. However, this counterfactual overestimates the TFP gains from relaxing constraints on incumbent firms by overvaluing the growth contributions of small, unproductive firms.

To mitigate the adverse effects of financial frictions on innovation, a natural policy response is to improve firms' access to credit. However, as the model shows, even when credit constraints are relaxed, firms may still choose to allocate additional resources to physical capital rather than to innovation. This limits the effectiveness of credit expansion as a stand-alone policy. A more targeted and practical approach is to subsidize innovation directly. Innovation subsidies both ease financial constraints and strengthen incentives to invest in innovative activities. The policy counterfactuals show that the impact of such subsidies on TFP and welfare depends critically on the severity of financial frictions and on how subsidies are targeted. In high-income countries, subsidizing entrants' investment in becoming innovative types is more effective in raising TFP and welfare than subsidizing the innovation expenditures of incumbent innovative firms.[2] This is because firm type and innovation capability are largely determined at birth, so influencing the type at entry has long-lasting effects. In middle-income countries, by contrast, both (i) subsidizing entrants' investment in becoming innovative types and (ii) supporting the innovation investment of young, radically innovative firms are crucial. These firms face more severe financial constraints, and without targeted support, they cannot fully exploit their growth potential.

These findings are important because most existing subsidy schemes do not differentiate between innovative and non-innovative firms, nor between radical and incremental innovation, as previously discussed. The counterfactual analyses indicate that this lack of targeting generates substantial efficiency losses. Moreover, these losses are larger in middle-income countries, where high-potential firms tend to be more financially constrained and require relatively large subsidies to grow—subsidies they are unlikely to receive under undifferentiated policies.

The natural implication for policymakers is to differentiate between firm types and innovation types when providing subsidies. But is this feasible in practice? In my empirical work, I identify innovation using granted patents, patent citations, and patent claims, measured by the patent application year rather than the grant year. This constitutes ex-post identification, since firm types are not known with certainty at the time of application. Lengthy grant lags and the risk of application failure further complicate early identification of firm types. I further provide empirical evidence, together with model-based mechanisms, showing that identifying firm types—especially early in the

[2] Supporting entrants' investment in becoming innovative types can be micro-founded by policies that encourage entrepreneurs to become innovative, including subsidies to human capital accumulation and related support measures.

life cycle—is particularly challenging in middle-income countries. In high-income countries, firm size (measured by sales, employment, or fixed assets) is a reasonably good proxy for firm type: both the data and the model suggest that radically innovative firms are typically larger than incrementally innovative firms, which in turn are larger than non-innovative firms. In middle-income countries, however, the data show that radically innovative firms are not necessarily larger and can even be smaller than incrementally innovative firms in their early stages. The model explains this pattern by showing that high-type firms in middle-income countries, despite potentially high productivity and a tendency to grow large, face tight financial constraints when young. As a result, they may initially choose incremental rather than radical innovation. Building on these insights, I propose several measures to reduce misidentification of firm types in middle-income countries and to improve the targeting of innovation policies.

Contribution to the Literature. First, I contribute new facts to the literature on heterogeneous innovation types and economic growth (see, e.g., Akcigit and Kerr (2018), Caggese (2019), Acemoglu et al. (2022), Cao et al. (2024)). Compared with previous studies that primarily examine the relationship between innovation and firm size in one specific country, I provide a novel cross-country empirical perspective on how firms' types, as well as the frequency and quality of their innovation, evolve with firm age. Furthermore, I uniquely document the extensive margin of innovation across-countries by considering the composition of non-innovative firms. This approach complements previous studies that restrict their analysis to innovative firms and proves to be vital for understanding aggregate productivity difference across countries. Additionally, I find that given the same innovation type, the cross-country difference in innovation is mainly driven by innovation frequency, rather than innovation quality.

Second, I contribute original model mechanisms to the literature that emphasizes the importance of taking into account heterogeneity in firms' innate growth potential for understanding firm dynamics and the resulting macroeconomic implications (see, e.g., Schoar (2010), de Mel et al. (2008), Luttmer (2011), Decker et al. (2014), Sterk et al. (2021), Akcigit et al. (2021), Peters and Zilibotti (2021), Haas et al. (2022)). I reveal the importance of a new channel—financial frictions in explaining the fewer (radically) innovative firms in middle-income countries, while having a limited role in affecting innovation in developed countries.[3] Moreover, I show the significance of financial frictions in explaining firms' radical and incremental innovation trade-off in developing countries, and its unique implications on innovation dynamics compared to other mechanisms.

[3] Akcigit et al. (2021) and Peters and Zilibotti (2021) attribute low productivity in developing countries to the shortage of dynamic firms, which may reflect the human capital of entrepreneurs and management practices.

Third, I contribute new insights to the literature on financial frictions, misallocation, innovation, and macro-development (see, e.g., Restuccia and Rogerson (2008, 2017), Hsieh and Klenow (2009, 2014), Arellano et al. (2012), Buera et al. (2011), Buera and Shin (2013), Midrigan and Xu (2014), Moll (2014), Gopinath et al. (2017), Ates and Saffie (2021), Vereshchagina (2023), Sui (2022), Kochen (2023), Ottonello and Winberry (2024), Ayerst et al. (2024)). Existing studies suggest that the cross-country disparity in TFP (and consequently GDP per capita) is partly driven by greater financial frictions in less developed countries. Recent advances underscore that endogenizing firm productivity distribution by modeling innovation investment results in larger TFP losses due to financial frictions. In this paper, I argue that overlooking heterogeneous innovation types and firm types, as well as their ex-ante differences in life cycle growth potential, could overestimate TFP losses due to financial frictions.[4] This insight is also relevant for studies examining other types of frictions. I also show how the interplay between ex-ante firm innovation potential and financial frictions redefines the implications of innovation policy for countries at different stages of development.

Fourth, I contribute new facts and mechanisms to the growing literature on declining entry rates, rising industrial concentration, rising R&D-to-GDP ratio, but productivity growth slowdown in OECD countries in recent decades (see, e.g., Akcigit and Ates (2021, 2023), Sui (2024), Liu et al. (2022), Olmstead-Rumsey (2022), Aghion et al. (2023), and Peters and Walsh (2022)). I empirically show that firm innovation quality decreases as firms age and that innovation type remains mostly invariant throughout a firm's life cycle. Theoretically, I show that larger (and older) firms face higher innovation investment costs. These findings suggest that any factors increasing the market share of large (older) firms or decreasing entry rates could contribute to the slowdown in productivity growth and the rise in the R&D-to-GDP ratio.

Layout. The rest of the paper is organized as follows. Section 2 provides the empirical facts. Section 3 presents the model. Section 4 conducts quantitative analysis. Section 5 concludes. The online Appendix contains additional details for the data and model.

[4]This paper is closely related to Sui (2022) and Ottonello and Winberry (2024), which emphasize that financial frictions affect the innovation and growth incentives of firms with heterogeneous sizes. Compared to these papers, I introduce life cycle growth of firms with ex-ante differences, as well as heterogeneous innovation types. Furthermore, I illustrate how these new elements in the model can reshape policy implications based on the extent of financial frictions.

2 Empirical Facts

In this section, I present new empirical facts that demonstrate how firm type, innovation frequency and innovation quality vary over firms' age using cross-country data. My focus is on high-income and middle-income countries in Europe, due to data limitations.[5]

2.1 Data

I use ORBIS Historical balance sheet data and patent data from ORBIS Intellectual Property, linked via firm ID. These datasets offer two main benefits: comparability across countries for cross-national analysis, and inclusion of both public and private firms for a nearly nationally representative dataset, thanks to standardized reporting even for smaller private firms.

I focus on 24 OECD countries with substantial data coverage, categorizing them into two groups based on GDP per capita. The *high-income* group comprises Italy (IT), France (FR), Germany (DE), Finland (FI), Great Britain (GB), Netherlands (NL), Austria (AT), Sweden (SE), Ireland (IE), Denmark (DK), Norway (NO), and Luxembourg (LU). The *middle-income* group includes Bulgaria (BG), Hungary (HU), Croatia (HR), Slovakia (SK), Latvia (LV), Estonia (EE), Slovenia (SI), Malta (MT), Portugal (PT), Greece (GR), Spain (ES), and Cyprus (CY). I use yearly data from 1999 to 2015.The details of data cleaning and representativeness are in Appendix B.

2.2 Sample Selection

To avoid double counting the same patents in multiple patent offices, I focus on *granted* patents filed in the U.S. patent office, as most significant patents relevant for productivity improvement are filed there (Cai et al. (2022)). However, using granted patents filed in European patent office generates similar results.

The data sample is limited to industries with patenting activities, acknowledging that firms in certain industries do not depend on patenting for growth. Industries are defined according to the two-digit NACE Rev.2 classification system.

[5]Therefore, innovative firms exist in middle-income countries. This sample could be different from poorer countries that rely solely on knowledge diffusion from global technology frontiers.

2.3 Variable Construction

Firm type. I construct annual patent data based on their *application year*. Subsequently, I categorize firms in each country into three types year by year, according to the newly granted patents they apply for: **non-innovative firms**, representing those without patents; **radically innovative firms**, representing those with at least one patent with citations above the median among all newly granted patents; and **incrementally innovative firms**, representing those with patents but are not radically innovative. For each firm, I construct two dummy variables $1_{\text{innovative},t}$ and $1_{\text{radical},\,t|1_{\text{innovative},\,t}=1}$ to represent a firm's type in year t:

$$1_{\text{innovative},t} = \begin{cases} 1, & \text{if the firm is innovative in year } t, \\ 0, & \text{if the firm is non-innovative in year } t, \end{cases} \tag{2.1}$$

$$1_{\text{radical},\,t|1_{\text{innovative},\,t}=1} = \begin{cases} 1, & \text{if the innovative firm is radically innovative in year } t, \\ 0, & \text{if the innovative firm is incrementally innovative in year } t. \end{cases} \tag{2.2}$$

Although firm types are identified year by year, firm type is fairly persistent, as shown in Table B.3. It is also important to note that a firm can have multiple new patents every year, but as long as one of its new patents has citations above the median, it is considered a radically innovative firm. The empirical facts remain robust to alternative definitions of radically innovative firms. On the other hand, Figure B.1 demonstrates that for radically innovative firms, the fraction of patents that are radical remains almost unchanged as the firms age. This also addresses concerns regarding the baseline measure of radically innovative firms.

To address the issue where older patents have more years to receive citations than newer patents and different technology classes could receive varying numbers of citations, I adopt the truncation correction weights developed by Hall et al. (2001). Consequently, all patents across countries over the years are pooled together and classified as either incremental or radical.

Innovation frequency, quality, and input. The innovation frequency of a radically or incrementally innovative firm is measured by the log of the number of all new patents of the firm for each year. The innovation quality of such firms is measured by the log of the average citations of all new patents of the firm for each year. Note that patents are essentially innovation output of the firm. Due to the lack of R&D expenditure data, I use intangible capital intensity (intangible fixed assets-to-tangible fixed assets ratio), which is closely related to R&D expenditure as discussed in Midrigan and Xu (2014) among others, as an approximation of innovation input of the firm.

2.4 Empirical Specification

To investigate how firm type, innovation frequency, innovation quality, and other variables of interest vary with a firm's age, I estimate the following non-parametric specification: for firm i in industry j, country c and year t,

$$y_{ijct} = \sum_{p \in \mathcal{P}} (\alpha_p + \alpha_p^{\text{MI}} 1_{\text{MI}}) D_{ijct}^p + \text{controls}_{ijct} + \kappa_c + \gamma_{jt} + \delta_i + \epsilon_{ijct}, \tag{2.3}$$

where y_{ijct} represents variables of interest, 1_{MI} is a dummy variable equal to 1 if firm i belongs to the middle-income country group and zero otherwise, D_{ijct}^p is a dichotomic variable equal to 1 if firm i is in age group p in year t. The set $\mathcal{P}$ includes 8 age groups: $\mathcal{P} \equiv \{(0,2], (2,4], (4,6], (6,8], (8,10], (10,15], (15,20], (20,\infty)\}$, i.e., age 0-2, age 3-4, and so on. This age group classification is based on the fact that most firms will not exist after twenty years in these countries.[6] Controls_{ijct} include firm size ($\log(\text{sales}_{ijct-1})$)[7] and firm exit propensity ($1_{\text{exit next period},ijct}$), κ_c denotes country fixed effects, γ_{jt} denotes industry-year fixed effects to absorb industry-specific patent creation or citation trends, and δ_i denotes firm fixed effects.[8] The standard errors are clustered at the firm level.

2.5 Empirical Facts

I plot the regression coefficients of interest α_p or $\alpha_p + \alpha_p^{\text{MI}}$ for different variables of interest and summarize the following empirical facts.

Fact 1. In high-income countries, firm type is primarily determined at birth. Conversely, middle-income countries exhibit fewer (radically) innovative firms. Figure 2.1 demonstrates

[6] Haas et al. (2022) document that in the US Longitudinal Business Database, 71 percent of all firms were 20 years old or younger in 2019; the startup rate in that year was 8.2 percent; European startup rates are comparable. These numbers suggest that focusing on the dynamics of firms in their first twenty years of the life cycle is reasonable.

[7] I control for firm size to rule out potential size-related effects. Regressions conducted without firm size control reveal a more significant cross-country difference, suggesting that differences in size contribute to variations in innovation across countries. As detailed in the model, tighter collateral constraints in middle-income countries help to explain these results when firm size controls are implemented. The model also predicts that, on average, firms are smaller in middle-income countries due to financial frictions. This, in turn, amplifies the impact of financial frictions on innovation through the collateral constraint based on firm size. This is consistent with the empirical findings, which indicate a more significant cross-country difference when firm size control is not applied.

[8] I use ORBIS status identifiers to identify firms' exits. For the firms that ORBIS report to be in "Bankruptcy", "In liquidation", "Dissolved", or "Inactive", I define the exiting year as the last observation of the firm in the data.

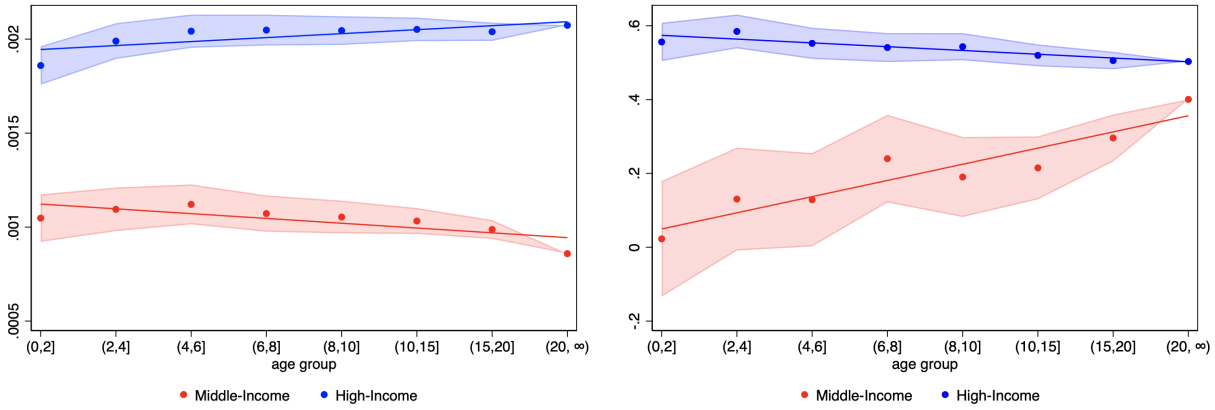
that in high-income countries, the probability of firms becoming innovative remains nearly constant, with a slight increase as they age. Meanwhile, the probability of becoming radically innovative experiences a slight decrease as they age. Conversely, in middle-income countries, although the probability of firms becoming innovative or radically innovative is significantly lower, the likelihood of becoming radically innovative increases as the firms mature. This suggests that these firms may overcome constraints or accumulate the necessary inputs and experience to engage in radical innovation. Of note, it is not surprising to see some firms being innovative or even radically innovative at birth, as they often start applying for patents upon entering the market. Some firms even have new ideas ready for patenting before their official inception. A prime example is Google's PageRank patent, the technology that powers their search engine. This patent was filed in 1998, the same year Google was founded, just two years after Larry Page and Sergey Brin initiated the company from their dorm rooms.

To my knowledge, this is the first paper to provide cross-country evidence on the extensive margin of innovation. This complements existing studies primarily focused on incumbent innovative firms, enhancing our comprehension of firms' innovation behaviors across different developmental stages. Figures B.2 and B.3 further demonstrate that using *public administrative* data combined with patent data reveals an increasing fraction of innovative firms as national income levels rise. This not only emphasizes the correlation between national income level and the fraction of innovative firms *within* both high- and low-income country groups, but also mitigates concerns regarding imperfect data coverage in ORBIS.

Fact 2. In high-income countries, innovation quality and innovation frequency are primarily determined at birth. In contrast, middle-income countries exhibit relatively low radical innovation frequency and high incremental innovation frequency. Figure 2.2 suggests a minor decline (around 10 percent) in firms' innovation quality over their life cycle, leading me to argue that a firm's highest innovation quality is established at birth. This pattern does not significantly vary across countries. Figure 2.3 displays similar cross-country patterns for incremental innovation frequency, but reveals significant differences for radical innovation frequency. High-income countries show a slight increase in firms' radical innovation frequency over time, while middle-income countries exhibit a more substantial rise, thereby reducing the cross-country disparity as firms age. Figure B.4 demonstrates that innovation input patterns closely resemble those of innovation frequency. Given the imperfect nature of the innovation input measure, it's unsurprising that its patterns don't entirely align with innovation frequency.

What accounts for these cross-country differences in innovation? To address this, I exploit industry variations and present fact 3.

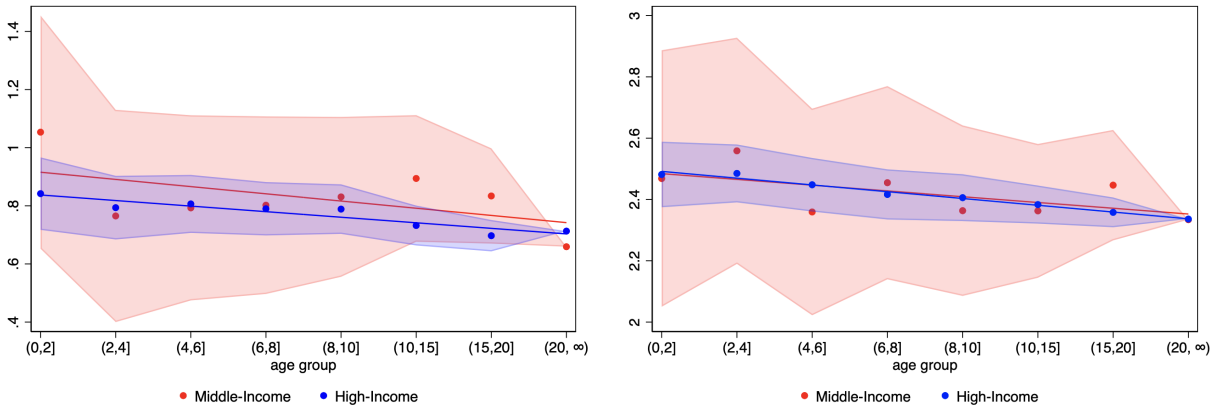
Figure 2.1: Firm Type over Age



(a) $1_{\text{innovative}}$ (b) $1_{\text{radical}|1_{\text{innovative}}=1}$

Notes: The Y-axis represents the dummy variables $1_{\text{innovative}}$ and $1_{\text{radically innovative}|1_{\text{innovative}}=1}$ for each firm, respectively. The X-axis in each panel denotes the age groups. The blue (red) dots signify the regression coefficients in the specification (2.3) for the high-income (middle-income) group. The lines represent the fitted line for the dots, and the shaded area plots the 90% confidence interval. The regression coefficients are scaled using the unconditional mean of the omitted group - age group $(20, \infty)$ in high-income countries. The regression coefficient for the age group $(20, \infty)$ in middle-income countries is omitted due to fixed effects. However, it is reintroduced using the unconditional difference compared to the age group $(15, 20]$ in middle-income countries.

Figure 2.2: Innovation Quality over Age

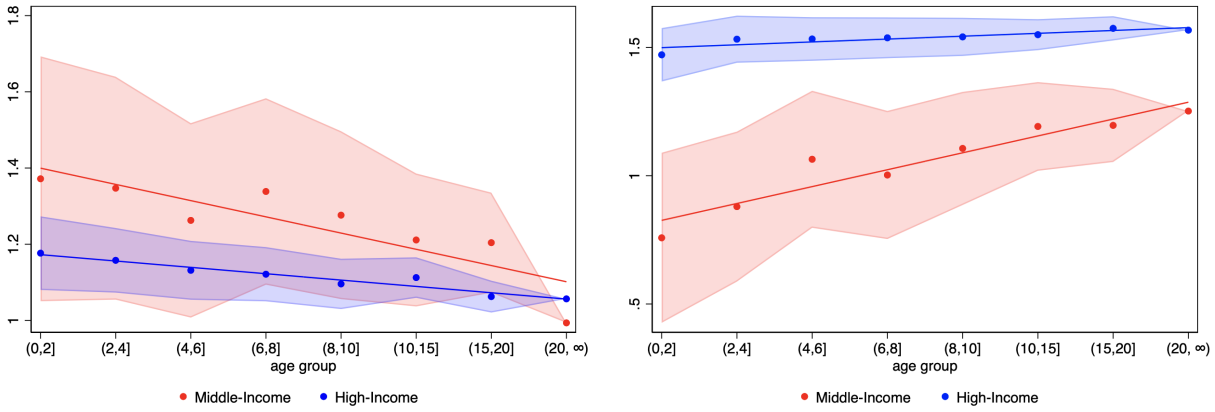


(a) incrementally innovative firms (b) radically innovative firms

Notes: The Y-axis represents the log(average citations of new patents) for incrementally innovative firms and radically innovative firms, respectively. All other descriptions are identical to those in Figure 2.1.

Fact 3. Most cross-country differences are in high external financial dependence industries. Technology, often embodied in large, indivisible capital inputs, requires access to external finance due to its long, risky innovation process (Holmstrom (1989), Bassi et al. (2022)). A well-functioning financial market is thus crucial for innovation, providing external finance and reducing financing costs. However, financial markets in middle-income countries are typically less developed than those in high-income countries (see, e.g., Kremer et al. (2021)). Figure B.6 shows that

Figure 2.3: Innovation Frequency over Age



(a) incrementally innovative firms (b) radically innovative firms

Notes: The Y-axis represents the log(number of new patents) for incrementally innovative firms and radically innovative firms, respectively. All other descriptions are identical to those in Figure 2.1.

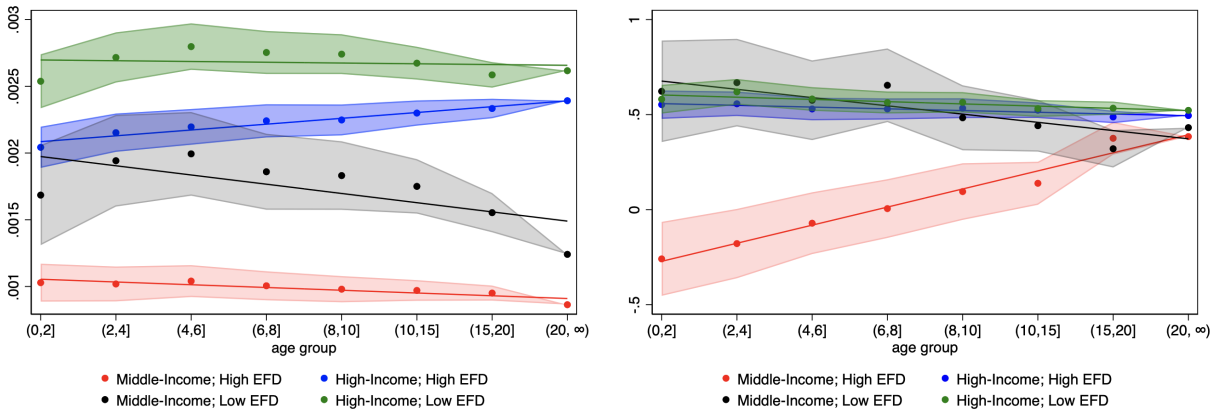
standard measures of financial development are positively correlated with national income levels across the countries in my sample.

To explore whether differences in external financing contribute to cross-country disparities in firm types and innovations, I divide industries into those with high and low external financial dependence (EFD) based on the median U.S. EFD level, as shown in Table B.4. The EFD data, derived from Rajan and Zingales (1998), uses U.S. Compustat data and defines EFD as (capital expenditures - cash flow from operations)/capital expenditures. I use U.S. data to mitigate endogeneity issues, given its highly developed financial markets and industry similarities across countries. My empirical specification, based on equation (2.3), is used for high EFD industries and low EFD industries separately. Note that the focus here is on cross-country comparisons, so running regressions for two types of industries separately is harmless.

Figures 2.4, 2.5, and B.5 show that high-income countries have similar patterns across high and low EFD industries. Middle-income countries, however, display different patterns, with the cross-country difference primarily evident in high EFD industries. There is a concern that factors other than external financial dependence (EFD), such as varying technology intensities across industries, may explain the observed cross-country differences. However, by classifying industries into high and low technology intensity categories, no distinct cross-country patterns emerge. On the other hand, by using firm-level leverage and average revenue product of capital (log(ARPK)), a common indicator of capital misallocation across diverse firms (see, e.g., Hsieh and Klenow (2009)), I demonstrate in Table B.5 that young innovative firms, especially those from middle-income countries, typically exhibit higher leverage but lower log(ARPK) than non-innovative firms. This implies that firms with fewer financial constraints have a higher probability

of becoming innovative. Interestingly, the discrepancies between innovative and non-innovative firms tend to diminish as the firms age, suggesting that firms manage to alleviate their financial constraints over their lifecycle. Additionally, no significant cross-country pattern is observed in the average product of labor (log(ARPL)). However, as a cautious remark, I do not dismiss any other possible explanations but consider these facts as suggestive evidence of the importance of external financial dependence.

Figure 2.4: Industry Variation: Firm Type over Age



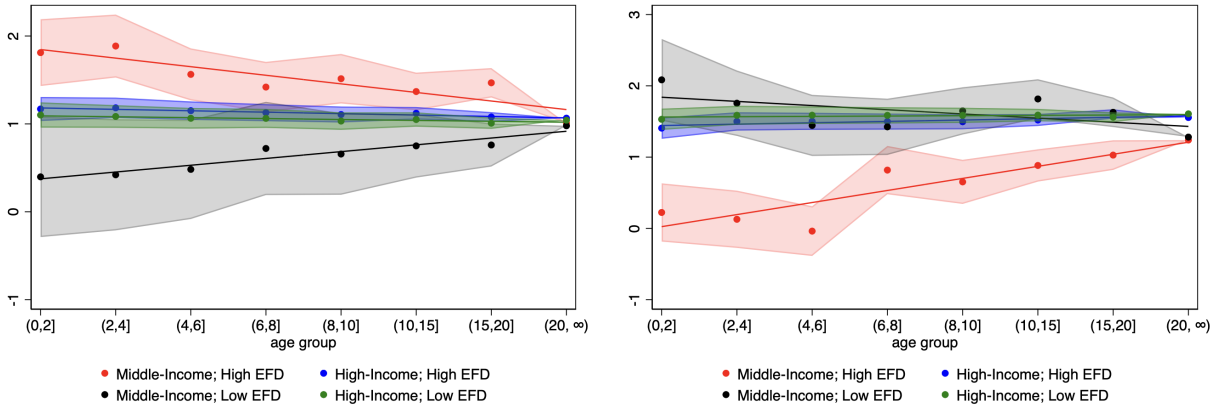
(a) $1_{\text{innovative}}$ (b) $1_{\text{radical}|1_{\text{innovative}}=1}$

Notes: The Y-axis represents the dummy variables $1_{\text{innovative}}$ and $1_{\text{radically innovative}|1_{\text{innovative}}=1}$ for each firm, respectively. The X-axis in each panel denotes the age groups. The dots signify the regression coefficients in a specification based on (2.3) for high external finance dependence (EFD) industries and low EFD industries separately. The blue, red, green, and black dots correspond to the high-income group with high EFD industries, middle-income group with high EFD industries, high-income group with low EFD industries, and middle-income group with low EFD industries, respectively. The lines represent the fitted lines for the dots, and the shaded area plots the 90% confidence interval. The regression coefficients are scaled using the unconditional mean of the omitted group - age group $(20, \infty)$ in high-income group. The regression coefficient for the age group $(20, \infty)$ in middle-income countries is omitted due to fixed effects; however, it is reintroduced using the unconditional difference compared to the age group (15, 20] in middle-income countries.

Fact 4. Different types of firms exhibit heterogeneous sizes at birth, and these size disparities amplify as firms age. Table 2.1 displays the connection between firm type and life cycle growth in sales, capital, and employment. Among *young* firms, innovative firms are larger than non-innovative firms in terms of sales, fixed assets, and the number of employees. Furthermore, across *all* firms, innovative firms are much larger than non-innovative firms, indicating that innovative firms experience more significant life cycle growth. In comparison to firms in middle-income countries, those in high-income countries display more substantial life cycle growth, as evident from the difference between column (1) and column (4), aligning with Hsieh and Klenow (2014).[9]

[9]Firms in high-income countries experience more substantial life cycle growth in sales and employment, but not in fixed assets. This is consistent with my model, which suggests that financial frictions are more severe

Figure 2.5: Industry Variation: Innovation Frequency over Age



(a) incrementally innovative firms (b) radically innovative firms

Notes: The Y-axis represents the log(number of new patents) for incrementally innovative firms and radically innovative firms, respectively. All other descriptions are identical to those in Figure 2.4.

Table 2.2 further underscores that in high-income countries, radically innovative firms are larger at birth and experience greater life cycle growth than incrementally innovative firms. Conversely, in middle-income countries, radically innovative firms are not significantly larger than incrementally innovative firms. As detailed in sections 3 and 4, this can be attributed to productive firms in middle-income countries choosing incremental innovation over radical innovation due to financial frictions.

2.6 Connection with Existing Empirical Facts

My observations on high-income and middle-income countries complement the findings in Ottonello and Winberry (2024), which document that U.S. public firms are more likely to engage in patenting activities as they age. Specifically, they show that compared to newly born firms, 30-year-old firms have a 10 percent higher probability of engaging in patenting activities. I document that in high-income European countries, among both public and private firms, the probability of engaging in patenting activities also has a slight increase as firms age. Specifically, firms older than twenty years have a 0.05 percent higher probability of engaging in patenting activities compared to newly born firms. In comparison to their paper, I contribute new facts on firm types and innovation types, as well as their variation over the firms' life cycle.

Additionally, my findings—that a firm's innovation frequency and innovation quality are mostly determined at birth—do not contradict previous empirical evidence that firm innovation intensity

in middle-income countries, leading firms to accumulate more capital to relax capital constraints over their life cycle.

Table 2.1: Innovative V.S. Non-Innovative: Life Cycle Difference

	(1)	(2)	(3)	(4)	(5)	(6)
	Among All Firms			**Among Young Firms**		
Panel A. log (sales)						
$1_{\text{innovative}}$	0.194***	0.040***	0.007**	0.140***	0.024***	0.005***
	(0.001)	(0.003)	(0.003)	(0.004)	(0.001)	(0.001)
$1_{\text{innovative}} \times 1_{\text{HI}}$	-0.036***	-0.001	-0.000	-0.038***	-0.004***	-0.002***
	(0.002)	(0.003)	(0.001)	(0.000)	(0.001)	(0.001)
Adjusted R^2	.18	.82	.89	.15	.7	.87
Panel B. log (fixed assets)						
$1_{\text{innovative}}$	3.701***	2.435***	0.117***	3.546***	1.759***	0.085**
	(0.022)	(0.068)	(0.010)	(0.064)	(0.019)	(0.036)
$1_{\text{innovative}} \times 1_{\text{HI}}$	-0.477***	-0.162**	-0.030	-0.436***	-0.153**	-0.023
	(0.024)	(0.082)	(0.033)	(0.071)	(0.075)	(0.042)
Adjusted R^2	.16	.46	.91	.14	.34	.92
Panel C. log (number of employees)						
$1_{\text{innovative}}$	0.655***	0.245***	0.012***	0.583***	0.093***	0.011***
	(0.005)	(0.017)	(0.003)	(0.018)	(0.004)	(0.002)
$1_{\text{innovative}} \times 1_{\text{HI}}$	-0.169***	-0.063***	0.006*	-0.235***	-0.081***	-0.002
	(0.020)	(0.019)	(0.003)	(0.006)	(0.004)	(0.015)
Adjusted R^2	.2	.62	.9	.17	.45	.89
Obs.	2.12e+07	1.42e+07	1.35e+07	5562965	2842715	2339914
Industry-Year FE	Y	Y	Y	Y	Y	Y
Country FE	Y	Y	Y	Y	Y	Y
Controls	N	Y	Y	N	Y	Y
Firm FE	N	N	Y	N	N	Y

Notes: This table presents the regression results under the following regression: for firm i in industry j, country c and year t, $y_{ijct} = x_{ijct} \times 1_{\text{HI}} + \text{controls}_{ijct} + \kappa_c + \gamma_{jt} + \delta_i + \epsilon_{ijct}$, where y_{ijct} are variables of interest, x_{ijct} is the firm type indicator, 1_{HI} is a country group dummy, with $1_{\text{HI}} = 1$ if the country is a high-income country, controls_{ijct} includes firm sales in the last period sales_{ijct-1} and exit indicator that is equal to 1 if the firm exits in the next period, κ_c denotes country fixed effect, γ_{jt} denotes industry-year fixed effect, and δ_i denotes firm fixed effect. The first three columns include all firms. The last three columns include firms with age less than five. * $p < 0.10$, ** $p < 0.05$,*** $p < 0.01$.

decreases with firm size (e.g., Akcigit and Kerr (2018) measure innovation intensity using patents per employee). Regarding innovation frequency, it represents a *flow* measure of patents rather than a *stock* measure. As firms age, they accumulate more patents. Given that firms grow in size as they age, my observation that firm innovation frequency is mostly invariant over the life cycle is consistent with prior evidence that the number of new patents relative to firm size decreases as firm size increases. Recall that I find that the highest innovation quality occurs at birth, with a slight decrease as firms age. My measure of innovation quality, which is based on the average citations per patent, could be decreasing due to "ideas are getting harder to find" (Bloom et al. (2020)), or due to firms' shift from external to internal innovation (Akcigit and Kerr (2018)). As the variation in innovation quality across countries and over firms' life cycles is limited, in the baseline model detailed below, I assume the firms' innovation quality over their life cycle is constant.

Table 2.2: Incrementally Innovative V.S. Radically Innovative: Life Cycle Difference

	(1)	(2)	(3)	(4)	(5)	(6)
	Among All Firms			**Among Young Firms**		
Panel A. log (sales)						
$1_{\text{radical}\|1_{\text{innovative}}=1}$	-0.004	-0.000	-0.000	-0.033*	-0.011	-0.010
	(0.004)	(0.002)	(0.002)	(0.019)	(0.010)	(0.016)
$1_{\text{radical}\|1_{\text{innovative}}=1} \times 1_{\text{HI}}$	0.035***	0.022*	0.013**	0.034***	0.015**	0.011*
	(0.006)	(0.012)	(0.006)	(0.004)	(0.006)	(0.006)
Adjusted R^2	.25	.89	.92	.28	.82	.93
Panel B. log (fixed assets)						
$1_{\text{radical}\|1_{\text{innovative}}=1}$	-0.106	-0.021	0.003	-0.614**	-0.177	0.006***
	(0.078)	(0.049)	(0.031)	(0.290)	(0.250)	(0.001)
$1_{\text{radical}\|1_{\text{innovative}}=1} \times 1_{\text{HI}}$	0.704***	0.586**	0.194***	0.658**	0.379*	0.057*
	(0.083)	(0.240)	(0.053)	(0.326)	(0.226)	(0.033)
Adjusted R^2	.26	.73	.94	.3	.65	.95
Panel C. log (number of employees)						
$1_{\text{radical}\|1_{\text{innovative}}=1}$	-0.036***	-0.021***	-0.003	-0.090***	-0.061***	-0.021
	(0.012)	(0.007)	(0.006)	(0.007)	(0.006)	(0.035)
$1_{\text{radical}\|1_{\text{innovative}}=1} \times 1_{\text{HI}}$	0.111***	0.091***	0.025*	0.109***	0.023***	0.021***
	(0.010)	(0.018)	(0.015)	(0.013)	(0.008)	(0.007)
Adjusted R^2	.25	.74	.91	.31	.7	.96
Obs.	65392	59129	50662	44469	4544	2848
Industry-Year FE	Y	Y	Y	Y	Y	Y
Country FE	Y	Y	Y	Y	Y	Y
Controls	N	Y	Y	N	Y	Y
Firm FE	N	N	Y	N	N	Y

Notes: The empirical specification and data sample are the same as Table 2.1. * $p < 0.10$, ** $p < 0.05$,*** $p < 0.01$.

Furthermore, considering the literature often finds that firm sizes in middle-income countries are smaller than those in high-income countries (Hsieh and Klenow (2014)), my Fact 4 not only captures this but also suggests that ex-ante firm differences (determined at birth) contribute to cross-country disparities in firm size.

2.7 Robustness

The empirical results above remain robust to: (i) **Alternative classification of firm types** (identifying firm type using patent data every 3 years instead of every year) to address the concern that patenting activity is infrequent; (ii) **Alternative classification of radical innovation** (calculating the citations of all new patents for each firm and using the median across firms rather than the median across patents, I classify the firm as either radically innovative or incrementally innovative); (iii) **Alternative definition of radical innovation** (using the median level of the length of patent claims over all years, rather than citations, we classify patents as either radical or incremental);[10]

[10] As discussed in Akcigit and Ates (2023), when a firm introduces a novel technology that makes a broad contribution to the field, the relevant patent is expected to have a relatively short claim, reflecting its broader

(iv) **Alternative fixed effect** (control for firm-five-year period fixed effects, instead of only using firm fixed effects, to address the concern that the observed pattern results from time trends rather than the heterogeneity of firms); and (v) **Alternative patent data** (patents filed in the European Patent Office) to address the concern that some firms do not rely on the U.S. market for growth and therefore do not have the incentive to apply for patents in the U.S. All robustness results are relegated to Appendix B.3.

3 Model

I introduce a semi-endogenous growth model that can rationalize the empirical findings and examine the macroeconomic implications of cross-country differences in firm types and innovations. I focus on a stationary equilibrium along the balanced growth path.

3.1 Model Setup

Consider an economy populated by a large number of infinitely lived firms $i = 1, ..., N$ that produce homogeneous goods. The productivity level varies across these firms. Time is discrete and denoted by t. Firms, owned by representative households, make decisions on production, innovation, and financing to maximize the present value of dividends. Firms have three types $s \in \{n, h, l\}$, where n represents non-innovative firm, h represents high-type innovative firm, and l represents low-type innovative firm. Firms determine their types upon entry. At the beginning of each period, incumbent firms exit at an exogenous rate γ, with an exit value of zero. New entrants join the market with some productivity level, ensuring the total mass of firms remains constant over time. Next, given the current productivity level, incumbent firms and entrants make capital input decision, net worth accumulation decision, and innovation decision. These decisions will reshape firms' net worth level and productivity level in the next period.

Households. Representative households choose consumption and asset levels to maximize utility:

$$\max_{\{C_t, A_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \phi^t \frac{C_t^{1-\sigma}}{1-\sigma}, \tag{3.1}$$

subject to the per-period budget constraint $C_t + A_{t+1} = (1 + \tilde{r}_t)A_t$, where ϕ represents households'

scope. Conversely, if a patent makes a marginal contribution to an already crowded area, the claims are likely to be much longer and narrower in scope. Thus, the length of the claim can serve as an indicator of the incremental nature of a patent.

discount factor, σ governs the elasticity of intertemporal substitution, C_t denotes consumption of goods produced by firms (with the price of goods normalized to 1), $A_t \equiv \int_f V_t df$ represents assets (the value of firms), and $\tilde{r}_t \equiv \frac{1+r_t}{1-\gamma} - 1$ denotes the risk-free interest rate r_t, adjusted for the firm exit rate γ such that lenders break even. The risk-free interest rate r_t is determined by the usual Euler equation derived from the utility maximization problem:

$$\frac{1}{1+\tilde{r}_t} = \phi \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma}. \tag{3.2}$$

Firm Production. Firms produce homogeneous goods using a decreasing returns to scale production technology, à la Hopenhayn (1992):

$$y_{it}(s) = (Q_t z_{it}(s))^{1-\alpha} k_{it}(s)^{\alpha}, \tag{3.3}$$

where α denotes the output elasticity with respect to inputs ($0 < \alpha < 1$), $z_{it}(s)$ represents the individual productivity of firm i that belongs to type s ($s \in \{n, h, l\}$) in period t, and

$$Q_t \equiv Q_t^F \Big(\sum_s \int z_{it}(s) di \Big)^{\iota} \tag{3.4}$$

defines an aggregate productivity index of the economy. Here, ι indicates that knowledge spillovers from other domestic firms can benefit a firm's own production. Q_t^F represents the world frontier technology level, which grows at an exogenous constant rate g^F, so that $Q_{t+1}^F = (1+g^F) Q_t^F$. Therefore, Q_t^F captures the knowledge spillovers from the world technology frontier and $z_{it}(s)$ can be interpreted as the firm-specific efficiency relative to the frontier. By investing in innovation and increasing $z_{it}(s)$, the firm improves its relative productivity $z_{it}(s)$ and thus reduces its gap to the world frontier.

$k_{it}(s)$ and $y_{it}(s)$ represent a firm's physical capital stock and output level, respectively. Firms own the capital stock and augment it through capital investment with capital depreciation rate δ. I adopt the convenient assumption made in Midrigan and Xu (2014), Moll (2014), Buera and Moll (2015), and Sui (2022), among others, that productivity in period $t+1$ is known at the end of t, before firms make decisions for period $t+1$. This makes the capital decision static.

Incumbent Firm Type Transition. Non-innovative firms ($s = n$) do not engage in innovation but may choose to pay a fixed cost $G^{\text{inno}}(Q_t, z_t, s)$ in order to become innovative in the future:

$$G^{\text{inno}}(Q_t, z_t, s) = F Q_t z_t,$$

where $F > 0$ and $Q_t z_t$ ensures the existence of a balanced growth path. If they choose not to pay this fixed cost, they remain non-innovative and draw a productivity level z, which is determined at birth. If they do pay the fixed cost, then with some probability ε^n they become innovative firms ($s \in \{h, l\}$) in the next period.

Innovative firms can be either low-type ($s = l$) or high-type ($s = h$). Low-type innovative firms cannot transition to high-type innovative firms over their life cycle. While it would be possible to introduce a fixed cost for transitioning from low-type to high-type, the additional economic insights from doing so would be limited. The distinction between low-type and high-type firms is intended to capture cross-country variation in innovation frequencies across firms, as detailed below.

Incumbent Firm Innovation. Innovative firms invest in innovation throughout their life cycle to increase their productivity. The low-type innovation firm ($s = l$) chooses incremental innovation probability x to maximize its value. Note that the innovation probability captures the *innovation frequency* of the firm. The high-type innovation firm ($s = h$) chooses either incremental innovation probability x or radical innovation probability q to maximizes its value. The successful radical innovation leads to higher productivity increase of the firm, captured by the innovation step size (*innovation quality*):

$$\log z_{t+1} = \begin{cases} \log z_t + J^{\text{inc}}, & \text{with prob.} = x, \\ \log z_t - J^{\text{inc}}, & \text{with prob.} = \varepsilon, \\ \log z_t, & \text{elsewhere,} \end{cases} \tag{3.5}$$

where J^{inc} denotes the incremental innovation step size with $J^{\text{inc}} > 0$, and

$$\log z_{t+1} = \begin{cases} \log z_t + J^{\text{rad}}, & \text{with prob.} = q, \\ \log z_t - J^{\text{inc}}, & \text{with prob.} = \varepsilon, \\ \log z_t, & \text{elsewhere,} \end{cases} \tag{3.6}$$

where J^{rad} denotes the radical innovation step size with $J^{\text{rad}} > J^{\text{inc}}$, implying that radically innovative firms have larger life cycle growth than incrementally innovative firms.[11] With probability ε, firms suffer a technological obsolescence shock that reduces $\log(z)$ by J^{inc}. This shock represents the depreciation of firm-specific know-how. Technologies and organizational practices lose value over time as demand, regulations, and complementary inputs change, or as key employees leave. In addition, competitors' innovations can render a firm's existing products or processes less useful, a standard "creative destruction" mechanism in Schumpeterian models. Both forces can be parsimoniously captured by the technological obsolescence shock.

Innovation costs are measured in units of goods produced by firms. To achieve an incremental

[11] For simplicity and without loss of generality, the innovation step size is the same across countries and constant throughout a firm's life cycle. Alternatively, I could make the step size decrease with a firm's productivity level to capture the decrease in innovation quality over its life cycle. This would better match the data and strengthen the macroeconomic implications of financial frictions and policy implications.

innovation probability x, a firm must invest

$$G^{\text{inc}}(Q_t, z_t, s, x) = \psi_s Q_t z_t x^{\eta},$$

where $\psi_s > 0$ captures the institutional or social costs of innovations for $s \in \{h, l\}$, and $\eta > 1$ indicates that incremental innovation costs are increasing and convex in innovation probability x. This suggests that firms must pay higher costs to increase innovation probability. Consequently, the innovation probability effectively captures the firm's innovation incentive. The innovation cost is scaled by the aggregate productivity index of the economy to ensure the existence of a balanced growth path, and the functional form implies that more productive firms face higher innovation costs.

To achieve a radical innovation probability q, a firm must invest

$$G^{\text{rad}}(Q_t, z_t, s, q) = \kappa \psi_s Q_t z_t q^{\eta} + F^{\text{rad}} Q_t z_t,$$

where $\psi_h > \psi_l$ highlights the higher innovation efficiency of high-type firms compared to low-type firms, leading to a higher innovation frequency for high-type firms. This difference in innovation efficiency explains the previously mentioned empirical observation that radically innovative firms have higher innovation frequencies than incrementally innovative firms in high-income countries. Additionally, middle-income countries exhibit higher incremental innovation frequencies than high-income countries. This is because high-type firms in middle-income countries tend to favor incremental innovation over radical innovation due to financial frictions, as detailed later.[12]

The terms $F^{\text{rad}} > 0$ and $\kappa > 1$ represent the additional fixed cost and variable cost of radical innovation, respectively.[13] Compared to incremental innovation, radical innovations often necessitate new infrastructure or factory construction. For instance, Tesla's decision to produce electric cars required the company to invest in building charging stations, a significant fixed and variable cost. Moreover, radical innovation can render much of a firm's existing investment in technical skills, knowledge, designs, production techniques, plants, and equipment obsolete. Given that innovation investments are irreversible (see, e.g., Utterback (1996)), the existing investment superseded by radical innovation becomes the fixed cost of radical innovation.[14]

[12] Among firms of the same type, their growth rate is independent of firm size in the absence of financial constraints, supporting Gibrat's Law. However, financial frictions disrupt this pattern by creating systematic differences in how firms of varying sizes access capital. Smaller firms, often facing tighter borrowing constraints, exhibit slower growth compared to larger firms. This is consistent with a broad set of empirical evidence.

[13] The fixed cost of incremental innovation in this model is zero. This is because the key model mechanism to be highlighted is the trade-off between incremental and radical innovation, and the extra cost of radical innovation is sufficient to highlight this trade-off.

[14] This dynamic is exemplified by Kodak's situation. Despite inventing the digital camera in 1975 - a radical

External Financing and Financial Frictions. Firm i of type s in period t can save and borrow using a one-period bond $b_{it}(s)$ at an interest rate level $\tilde{r}_t$, determined by households and adjusted for the firm exit rate. The amount a firm can borrow is subject to a collateral constraint, which is common in macro-finance literature (see, e.g., Kiyotaki and Moore (1997)):

$$\theta k_{it}(s) \geq b_{it}(s), 0 < \theta < 1, \tag{3.7}$$

where θ governs the tightness of the collateral constraint. This constraint can be derived from an environment in which firms lack the commitment to repay debts, and lenders can seize a fraction θ of the firms' capital in the event of default.Firms also face a limited liability condition, imposing a non-negative constraint on the dividend:

$$div_{it}(s) \geq 0, \tag{3.8}$$

where

$$\begin{aligned} div_{it}(s) \equiv (Q_t z_{it}^{1-\alpha} k_{it}(s)^{\alpha} + (1-\delta)k_{it}(s) - (1+\tilde{r}_t)b_{it}(s) - k_{it+1}(s) - G^{\text{inno}}(Q_t, z_{it}, s) \cdot 1_{\text{inno},it}(s) \\ - G^{\text{inc}}(Q_t, z_{it}, s, x) \cdot (1 - 1_{\text{rad},it}(s)) - G^{\text{rad}}(Q_t, z_{it}, s, q) \cdot 1_{\text{rad},it}(s) + b_{it+1}(s), \end{aligned} \tag{3.9}$$

$1_{\text{inno},it}(s)$ a dummy variable that equals 1 if a non-innovative firm chooses to pay the transition cost, and 0 otherwise. Similarly, $1_{\text{rad},it}(s)$ is a dummy variable that equals 1 if a high-type innovative firm chooses to undertake radical innovation, and 0 otherwise.

Note that innovation investment is essentially internally financed due to the limited liability condition, as innovations have low collateral value. This is consistent with the evidence discussed in Caggese (2019), among others.

Financial frictions are modeled by both the collateral constraint and the limited liability condition. The collateral constraint restricts available external financing, affecting firms' capital accumulation, while the limited liability condition impacts firms' resource allocation among capital accumulation, borrowing, and innovation investment.

Entrants (Entrepreneurs). Incumbent firms face an exogenous exit rate γ, with an exit value of zero. New entrants join the market, ensuring the total mass of firms remains constant over time. For simplicity without loss of generality, assume that the total mass of firms in the economy is one. New entrants start with an initial productivity level, with a mean among entrants equal to $\zeta^n \bar{z}_t$,

innovation at the time - the company failed to fully embrace and develop this technology due to fears it would cannibalize their existing film business.

where

$$\bar{z}_t \equiv \sum_s \int_a \int_z z_t(s)\Gamma_t(z,a,s)dzda$$

represents the average productivity of the firm and ζ^n denotes a productivity shifter of entrants relative to incumbent firms. They also have an initial net capital stock level with a mean among entrants equal to k^0 and a zero bond level with a mean among entrants equal to b^0; both variables grow along the balanced growth path.

Entrants also produce goods, and the production technology is the same as that of the incumbents. Entrants with productivity level $\zeta^n z_t$ invest

$$G^e(Q_t, \zeta^n z_t, e) \equiv \varphi Q_t \zeta^n z_t e^\nu$$

to become innovative, where $\varphi > 0, \nu > 1$, and are subject to financial frictions.[15] They have a probability e to become innovative firms ($s \in \{h, l\}$), a probability determined by the entrants themselves. If they fail to become innovative, they are categorized as non-innovative firms ($s = n$). Upon becoming an innovative firm, an entrant has an exogenous probability ξ to be a high-type innovative firm ($s = h$) with a mean productivity level $\zeta^h \bar{z}$, and a probability $1 - \xi$ to be a low-type innovative firm ($s = l$) with a mean productivity level $\zeta^l \bar{z}$. It is assumed that $\zeta^n < \zeta^l < \zeta^h < 1$ to reflect the empirical observation in high-income countries where the entrants are smaller than the average incumbent firms of the same type. Further, high-type innovative firms have a larger initial size than low-type innovative firms, which in turn are larger than non-innovative firms. The firm type is heterogeneous at birth, reflecting empirical observations (fact 1).

3.2 Recursive Formulation and Decision Rules

Define the net worth level of firm i of type k in period t as $a_{it}(s) \equiv k_{it}(s) - b_{it}(s) \geq 0$.Define the firm discount factor $\beta_t \equiv 1/(1 + \tilde{r}_t)$. The timing assumption on capital implies that firm profit is solely a function of net worth, not of capital or debt. Using primes to denote next-period variables and omitting the notation i, which is uniquely mapped to productivity z, net worth a, and firm type s, we can rewrite the firm's problem in recursive form as follows.

Non-Innovative Firms. For non-innovative firms ($s = n$), they choose net worth in the next period a' and capital input in the current period k to maximize their current and future dividends

[15] Entrants' investment in different firm types can be explained by the accumulation of entrepreneurs' human capital or their investment in production techniques to establish the firm.

subject to limited liability condition and collateral constraint:

$$
\begin{aligned}
V(Q,z,a,s) &= \max_{1_{\text{inno}}}\{V^{\text{non}}(Q,z,a,s), V^{\text{inno}}(Q,z,a,s)\}, \text{where} \\
V^{\text{non}}(Q,z,a,s) &= \max_{a_1'} div(Q,z,a,s) + \beta(1-\gamma)V(Q',z,a_1',s), \\
V^{\text{inno}}(Q,z,a,s) &= \max_{a_2'} div(Q,z,a,s) + \beta(1-\gamma)[(1-\varepsilon^n)V(Q',z,a_2',s) \\
&\quad +\varepsilon^n((1-\xi)V(Q',z,a_2',l) + \xi V(Q',z,a_2',h))], \\
\text{s.t.} \quad div(Q,z,a,s) &= \pi(Q,z,a,s) + (1+\tilde{r})a - a_1' \cdot (1-1_{\text{inno}}) - a_2' \cdot 1_{\text{inno}} - FQz \cdot 1_{\text{inno}}, \\
\text{where} \quad \pi(Q,z,a,s) &= \max_k (Qz)^{1-\alpha}k^{\alpha} - (\delta+\tilde{r})k \\
\text{s.t.} \quad a &\geq (1-\theta)k.
\end{aligned}
\tag{3.10}
$$

Non-innovative firms decide to pay fixed cost for changing their types according to

$$
1_{\text{inno}} = \begin{cases} 1, & \text{if } V^{\text{inno}}(Q,z,a,s) \geq V^{\text{non}}(Q,z,a,s), \\ 0, & \text{elsewhere.} \end{cases}
\tag{3.11}
$$

The implied capital input decision, as shown in equation (3.12), demonstrates that the Lagrange multiplier associated with the collateral constraint, $\chi(Q,z,a,s) \geq 0$, reduces capital input. In other words, borrowing constraints limit capital formation. To relax borrowing constraints, firms have to accumulate net worth.

$$
k = Qz\left(\frac{\alpha}{\delta+\tilde{r}+(1-\theta)\chi(Q,z,a,s)}\right)^{\frac{1}{1-\alpha}}.
\tag{3.12}
$$

The implied net worth decision, as shown in equation (3.13), clearly illustrates that the Lagrange multiplier associated with the limited liability condition, $\mu(Q,z,a,s) \geq 0$, is binding (greater than 0) if the collateral constraint is expected to be binding in the future. The intuition behind this is simple: a future binding collateral constraint increases the shadow cost of funds and reduces the firm's dividends, which makes the limited liability condition more likely to be binding.

$$
1+\mu(Q,z,a,s) = \beta(1-\gamma)\mathbb{E}[\chi(Q',z',a',s) + (1+\tilde{r}')(1+\mu(Q',z',a',s))].
\tag{3.13}
$$

Low-Type Innovative Firms. For low-type innovative firms ($s = l$), they not only choose the net worth in the next period a' and capital input k in the current period, but also choose incremental innovation probability x to maximize their current and future dividends subject to limited liability

condition and collateral constraint:

$$
\begin{aligned}
V(Q,z,a,s) = \max_{a',x\in[0,1]} & \; div(Q,z,a,s) + \beta(1-\gamma)[(1-x-\varepsilon)V(Q',z,a',s) \\
& + xV(Q',z\cdot\exp(J^{\text{inc}}),a',s) + \varepsilon V(Q',z/\exp(J^{\text{inc}}),a',s)] \\
\text{s.t.} \quad & div(Q,z,a,s) = \pi(Q,z,a,s) + (1+\tilde{r})a - a' - G^{\text{inc}}(Q,z,s,x) \geq 0, \\
\text{where} \quad & \pi(Q,z,a,s) = \max_{k}(Qz)^{1-\alpha}k^{\alpha} - (\delta+\tilde{r})k \\
& \text{s.t.} \quad a \geq (1-\theta)k.
\end{aligned}
\tag{3.14}
$$

The implied capital input decision and net worth decision are the same as those of non-innovative firms. The incremental innovation decision x, as shown in equation (3.15), indicates that a binding limited liability condition ($\mu(Q,z,a,s) > 0$) reduces a firm's innovation probability x. This is because fewer resources are available for innovation investment, and lower innovation investment leads to lower innovation probability. Furthermore, a more binding limited liability condition (represented by a larger $\mu(Q,z,a,s)$) reduces the firm's innovation probability x by a greater extent, all other things being equal.

$$
x = \left(\frac{\beta(1-\gamma)[V(Q',z\cdot\exp(J^{\text{inc}}),a',s) - V(Q',z,a',s)]}{[1+\mu(Q,z,a,s)]\psi_s Qz\eta}\right)^{\frac{1}{\eta-1}}. \tag{3.15}
$$

High-Type Innovative Firms. For high-type innovative firms ($s = h$), they also choose between incremental or radical innovation types and determine the innovation probability for their chosen type:

$$
\begin{aligned}
V(Q,z,a,s) &= \max_{1_{\text{rad}}}\{V^{\text{inc}}(Q,z,a,s), V^{\text{rad}}(Q,z,a,s)\}, \text{where} \\
V^{\text{inc}}(Q,z,a,s) &= \max_{a_1',x} div(Q,z,a,s) + \beta(1-\gamma)[(1-x-\varepsilon)V(Q',z,a_1',s) \\
&\quad + xV(Q',z\cdot\exp(J^{\text{inc}}),a_1',s) + \varepsilon V(Q',z/\exp(J^{\text{inc}}),a_1',s)], \\
V^{\text{rad}}(Q,z,a,s) &= \max_{a_2',q} div(Q,z,a,s) + \beta(1-\gamma)[(1-q-\varepsilon)V(Q',z,a_2',s) \\
&\quad + qV(Q',z\cdot\exp(J^{\text{inc}}),a_2',s) + \varepsilon V(Q',z/\exp(J^{\text{inc}}),a_2',s)], \\
\text{s.t.} \quad & div(Q,z,a,s) = \pi(Q,z,a,s) + (1+\tilde{r})a - (a_1' + G^{\text{inc}}(Q,z,s,x))\cdot(1-1_{\text{rad}}) - (a_2' + G^{\text{rad}}(Q,z,s,q))\cdot 1_{\text{rad}}, \\
& \text{where} \quad \pi(Q,z,a,s) = \max_{k}(Qz)^{1-\alpha}k^{\alpha} - (\delta+\tilde{r})k \\
& \text{s.t.} \quad a \geq (1-\theta)k.
\end{aligned}
\tag{3.16}
$$

The implied capital input decision and net worth decision are the same as those of non-innovative firms. The incremental and radical innovation decision $1_{\text{rad}}, x, q$, are summarized in

equations (3.17), (3.18) and (3.19).

$$1_{\text{rad}} = \begin{cases} 1, & \text{if } V^{\text{rad}}(Q,z,a,s) \geq V^{\text{inc}}(Q,z,a,s), \\ 0, & \text{elsewhere.} \end{cases} \tag{3.17}$$

$$x = \left(\frac{\beta(1-\gamma)[V(Q', z\cdot\exp(J^{\text{inc}}), a'^{\text{inc}}, s) - V(Q', z, a'^{\text{inc}}, s)]}{[1+\mu^{\text{inc}}(Q,z,a,s)]\psi_s Q z \eta} \right)^{\frac{1}{\eta-1}}. \tag{3.18}$$

$$q = \left(\frac{\beta(1-\gamma)[V(Q', z\cdot\exp(J^{\text{inc}}), a'^{\text{rad}}, s) - V(Q', z, a'^{\text{rad}}, s)]}{[1+\mu^{\text{rad}}(Q,z,a,s)]\kappa\psi_s Q z \eta} \right)^{\frac{1}{\eta-1}}. \tag{3.19}$$

Entrants. The entrants choose the probability of becoming innovative firms e, capital input level k^e, and next-period net worth level a^e, in order to maximize their current dividends and future dividends, which depend on their realized types:

$$\begin{aligned} V^{\text{entrant}} &= \max_{e,a^e} \{ div^e + e[\xi V(Q,\zeta^h z, a^e, h) + (1-\xi)V(Q,\zeta^l z, a^e, l)] + (1-e)V(Q,\zeta^n z, a^e, n)\} \\ &\text{s.t.} \quad div^e \equiv \pi^e(Q,\zeta^n z, a^0) + (1+\tilde{r})a^0 - a^e - \varphi Q \zeta^n z e^\nu \geq 0, \\ &\text{where} \quad \pi^e(Q,\zeta^n z, a^0) = \max_{k^e} (Q\zeta^n z)^{1-\alpha}(k^e)^\alpha - (\delta+\tilde{r})k^e \\ &\text{s.t.} \quad a^0 \geq (1-\theta)k^e. \end{aligned} \tag{3.20}$$

The decision rules for k^e, a^e, and e are summarized by equations (3.21), (3.22), and (3.23).

$$k^e = \left(\frac{\alpha(Q\zeta^n z)^{1-\alpha}}{\delta + \tilde{r} + (1-\theta)\chi(Q,\zeta^n z, a^0, s)} \right)^{\frac{1}{1-\alpha}}, \tag{3.21}$$

$$1 + \mu(Q,\zeta^n z, a^0, s) = \beta(1-\gamma)\mathbb{E}[\chi(Q,z',a^e,s) + (1+\tilde{r}')(1+\mu(Q,z',a^e,s))], \tag{3.22}$$

$$e = \left(\frac{\xi V(Q,\zeta^h z, a^e, h) + (1-\xi)V(Q,\zeta^l z, a^e, l) - V(Q,\zeta^n z, a^e, n)}{[1+\mu(Q,\zeta^n z, a^0, s)]\varphi Q \zeta^n z \nu} \right)^{\frac{1}{\nu-1}}. \tag{3.23}$$

3.3 Aggregation and Equilibrium

Equilibrium Definition. A *balanced growth path equilibrium* of the economy consists of price $\{r_t\}^{t\in[0,\infty)]}$ and an allocation $\{y_{i,t}, k_{i,t}, a_{i,t+1}, x_{i,t}, q_{i,t}, 1_{\text{rad}i,t}, 1_{\text{inno}i,t}, k^e_{i,t}, a^e_{i,t}, e_{i,t}, Y_t, C_t, A_t\}^{t\in[0,\infty)]}_{i\in[0,1]}$ such that for all firm i and period t,

(i) representative households choose C_t and A_t to solve their utility maximization problem;

(ii) incumbent firms choose $y_{i,t}, k_{i,t}, a_{i,t+1}, x_{i,t}, q_{i,t}, 1_{\text{rad}i,t}, 1_{\text{inno}i,t}$ to solve their production, borrowing, and innovation decision;

(iii) entrants choose $k^e_{i,t}, a^e_{i,t}, e_{i,t}$ to solve their production, borrowing, and type selection decision;

(iv) the asset market clears, pinning down r_t via the household's Euler equation;

(v) the goods market clears:

$$C_t \equiv Y_t - I_t - R_t^{inc} - R_t^{rad} - R_t^{inno} - R_t^{entrant}, \tag{3.24}$$

where Y_t denotes the aggregate output, I_t is the aggregate capital investment, R_t^{inc}, R_t^{rad}, R_t^{inno} and $R_t^{entrant}$ denote the aggregate investments in incremental innovation, radical innovation, non-innovative firms' type transition, and entrants' firm type, respectively;[16]

and (vi) the distribution of firms over detrended productivity and net worth is stationary along the balanced growth path and consistent with firms' decision rules and the evolution of aggregate variables.

Aggregate Productivity and Output. Define an aggregate productivity Z_t as

$$Z_t \equiv \sum_s \int_z \int_a Q_t z_t(s) \Gamma_t(z,a,s) dadz = Q_t^F \left(\sum_s \int_z \int_a z_t(s) \Gamma_t(z,a,s) dadz \right)^{1+\iota}, \tag{3.25}$$

where $\Gamma_t(z,a,s)$ denotes the distribution of firms over states z, a and s.

The aggregate output Y_t can then be written as

$$Y_t = Q_t \sum_s \int_z \int_a z_t(s) \left(\frac{\alpha}{\delta + \tilde{r}_t + (1-\theta)\chi(Q,z,a,s)} \right)^{\frac{\alpha}{1-\alpha}} \Gamma_t(z,a,s) dadz = \hat{Y}_t \bar{Y}_t, \tag{3.26}$$

where

$$\hat{Y}_t \equiv Z_t \left(\frac{\alpha}{\delta + \tilde{r}_t} \right)^{\frac{\alpha}{1-\alpha}}, \tag{3.27}$$

$$\bar{Y}_t \equiv 1 + \frac{\sum_s \int_z \int_a z_t(s) \Gamma_t(z,a,s) [\left(\frac{\delta + \tilde{r}_t}{\delta + \tilde{r}_t + (1-\theta)\chi(Q,z,a,s)} \right)^{\frac{\alpha}{1-\alpha}} - 1] 1_{\chi(Q,z,a,s)>0} dadz}{\sum_s \int_z \int_a z_t(s) \Gamma_t(z,a,s) dadz} \leq 1. \tag{3.28}$$

$\hat{Y}_t$ reflects the level of aggregate output that depends on aggregate productivity, and $\bar{Y}_t$ reflects the effect of financial frictions that contributes to aggregate output loss.

Aggregate Growth Rates. The growth rate of aggregate productivity Z_t, therefore, is

$$g_t \equiv \frac{Z_{t+1}}{Z_t} - 1 = \left(\frac{Q_{t+1}^F \sum_s \int_z \int_a z_{t+1}(s) \Gamma_{t+1}(z,a,s) dadz}{Q_t^F \sum_s \int_z \int_a z_t(s) \Gamma_t(z,a,s) dadz} \right)^{1+\iota} - 1 = (1 + \tilde{g}_t)^{1+\iota} - 1, \tag{3.29}$$

where $\tilde{g}_t$ denotes the average growth rate of firms:

$$\tilde{g}_t \equiv \frac{Q_{t+1}^F \sum_s \int_z \int_a z_{t+1}(s) \Gamma_{t+1}(z,a,s) dadz}{Q_t^F \sum_s \int_z \int_a z_t(s) \Gamma_t(z,a,s) dadz} - 1. \tag{3.30}$$

[16] The definition of dividend and household's budget constraint imply that aggregate consumption is financed out of dividends and changes in financial assets; the net worth accumulation decision of firms does not add another real use of goods beyond capital investment and innovation investment.

It is easy to show that along the balanced growth path, aggregate productivity, aggregate output, and aggregate consumption grow at a time-invariant rate g_t, where $g_t = g, \forall t$ and average firm productivity grows at a time-invariant rate with $\tilde{g}_t = \tilde{g}$. The state variables in the detrended economy become $\{\tilde{z}, \tilde{a}, s\}$, and the detrended innovation step size for incremental innovation and radical innovation is $\tilde{J}^{\text{inc}} \equiv \log(\frac{\exp(J^{\text{inc}})}{1+\tilde{g}})$ and $\tilde{J}^{\text{rad}} \equiv \log(\frac{\exp(J^{\text{inc}})}{1+\tilde{g}})$, respectively.

Semi-Endogenous Growth. As cross-country differences in productivity are mostly in levels rather than growth rates (Hsieh and Klenow (2009), Klenow and Rodriguez-Clare (2004)), I consider a special case in which $\tilde{g}_t = g^F$. In this case, cross-country differences in productivity levels arise from differences in the stationary distribution of z (i.e., in firm-level efficiency relative to the world frontier), not from differences in the growth rate, which is common across countries. The condition $\tilde{g}_t = g^F$ is satisfied when the probability of technology obsolescence is sufficiently high (i.e., when ε is large enough). The world frontier growth rate g^F is exogenously given, consistent with the idea of semi-endogenous growth model.

Welfare. Welfare in the balanced growth path equilibrium is measured by time zero utility, which depends on the associated initial level of consumption C_0, household discount factor ϕ, and growth rate of aggregate consumption g_C:

$$W = \frac{C_0^{1-\sigma}}{1-\sigma} \frac{1}{1-\phi(1+g_C)^{1-\sigma}}, \tag{3.31}$$

where $g_C = g$.[17]

The aggregate consumption $C_t \equiv Y_t - I_t - R_t^{inc} - R_t^{rad} - R_t^{entrant}$, where Y_t denotes the aggregate output, I_t is the aggregate capital investment, R_t^{inc}, R_t^{rad}, and $R_t^{entrant}$ denote the aggregate investments in incremental innovation, radical innovation, and entrants' firm type, respectively. Higher capital or innovation investment can increase output and thus consumption; however, it may also reduce consumption because output does not rise proportionally and fewer resources are available for consumption. Therefore, in maximizing welfare, there is a trade-off between consumption and investment in capital or innovation.

3.4 Illustrating the Mechanism

In the model, incumbent firms make net worth accumulation decision a' (and capital investment decision k), innovation type decisions $1_{\text{rad}}, 1_{\text{inno}}$, and innovation probability decisions x, q. Among

[17] By substituting $C_t = C_0(1+g_C)^t$ into the utility function and summing over time, we directly obtain W.

these three decisions, firms face two key trade-offs. The first is between net worth accumulation (capital investment) and innovation investment. The second, exclusive to high-type innovative firms, is between radical and incremental innovation.[18] In this section, I illustrate the mechanisms at play in a detrended stationary equilibrium for scenarios with and without financial frictions.

For expositional simplicity, I will discuss the decisions of high-type firms below, while noting that the analysis for low-type firms, non-innovative firms, and entrants is given symmetrically.

3.4.1 If No Financial Frictions

In the absence of financial frictions, i.e., when the Lagrange multipliers associated with the collateral constraint and limited liability condition are both zero, a firm's net worth level has no impact on its capital investment decision and innovation decision.

Net worth accumulation. When firms are financially unconstrained, they are indifferent to external financing (by issuing bonds) or internal financing (through positive dividends), or any combination of the two. To address this indeterminacy, I assume firms accumulate the maximum amount of debt (or have the minimum amount of savings) that still leaves them financially unconstrained, i.e., the minimum savings policy in Khan and Thomas (2013) and Ottonello and Winberry (2024), among others. This policy implies that dividends are zero when firms accumulate the maximum amount of debt and strictly positive otherwise.

Innovation probability. In the absence of financial frictions, a firm's productivity level does not impact its innovation probability. This is due to the model's construction, which ensures that the innovation cost and benefit are scaled equivalently.[19] Moreover, firms face no trade-off between capital investment and innovation investment, as they can freely optimize their decisions.

Innovation type. In the absence of financial frictions, high-type innovation firms face a trade-off between incremental and radical innovation. As illustrated in Figure 3.1, without financial frictions, a firm prefers radical innovation to incremental innovation only if its productivity is above a certain cutoff level. Otherwise, it prefers incremental innovation. This result is generated by three key factors. First, the model features an exponential growth. Second, there is a fixed radical innovation cost, F^{rad}, that is the same for all firms. Third, a firm's profit increases with its productivity

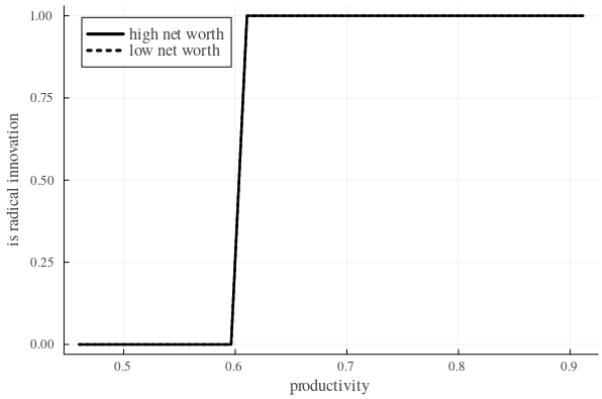
[18] I omit the discussion of non-innovative firms' trade-off between becoming innovative or remaining non-innovative, as this is not the key mechanism of the paper. The estimated model indicates that very few firms choose to become innovative, which is consistent with the data.

[19] Firms' optimal profits are expressed by $\pi(\tilde{z}) = \tilde{z}[\left(\frac{\alpha}{\delta+\tilde{r}}\right)^{\frac{\alpha}{1-\alpha}} - (\tilde{r}+\delta)\left(\frac{\alpha}{\delta+\tilde{r}}\right)^{\frac{1}{1-\alpha}}]$, which is proportional to the productivity level $\tilde{z}$. It is straightforward that firms' innovation costs (net of fixed innovation costs) are also proportional to the productivity level $\tilde{z}$.

level, $\tilde{z}$. These three factors ensure that the additional profits from radical innovation compared to incremental innovation increase with firms' productivity. Firms are willing to choose radical innovation only when the additional profits from radical innovation are larger than the additional cost of radical innovation. Productive firms are more likely to choose radical innovation than unproductive firms as they are more able to cover the fixed radical innovation cost.

Indeed, assuming firms are myopic, such that they only care about current and next-period production profit, the statements above can be theoretically proven as detailed in Appendix A.3. Intuitively speaking, when firms are myopic, the extra profit from radical innovation is proportional to $\tilde{z}(\exp(\tilde{J}^{\text{rad}}) - \exp(\tilde{J}^{\text{inc}}))$, which is larger than F^{rad} if $\tilde{z}$ is large enough.

Figure 3.1: Incremental and Radical Innovation Trade-off



Notes: This figure illustrates the innovation type selection for high-type innovative firms without financial frictions, i.e., with Lagrange multipliers $\mu = 0$ and $\chi = 0$. The y-axis indicates whether the innovation is radical (1_{rad} =1) or incremental (1_{rad} =0), while the x-axis displays the firm's productivity level. The solid line represents high net worth firms, and the dashed line corresponds to low net worth firms.

3.4.2 Effects of Financial Frictions

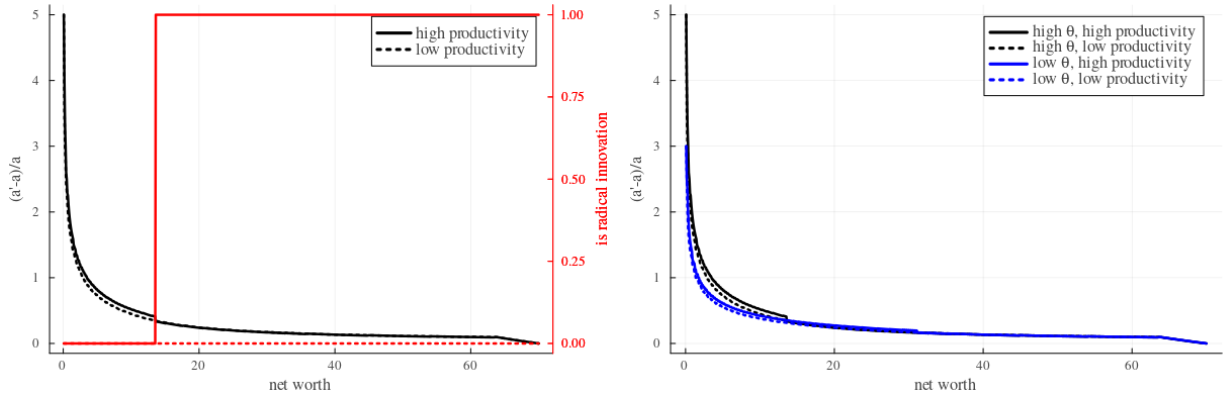
Who are more negatively affected? To understand the effects of financial frictions, it is crucial to first determine which firms are more collaterally constrained in the model economy. Figure A.1 illustrates that low net worth or high productivity firms are more likely to face binding collateral constraints. Low net worth firms lack sufficient collateral, making them more susceptible to these constraints. High productivity firms, aiming to expand their production scale and borrow more, are also more likely to face binding collateral constraints. This is demonstrated theoretically (i.e., $\frac{\partial \chi(\tilde{z},\tilde{a},s)}{\partial \tilde{a}} \leq 0$, $\frac{\partial \chi(\tilde{z},\tilde{a},s)}{\partial \tilde{z}} \geq 0$) in Appendix A.3.

Financial frictions incentivize net worth accumulation and reduce capital investment. The left panel in Figure 3.2 shows that all other things being equal, firms with lower net worth or higher productivity level have a faster growth rate of net worth accumulation, as they are more financially

constrained and are thus more inclined to relax borrowing constraints to accumulate net worth and expand their production scale, leveraging their current productivity level. The y-axis on the right-hand side in this left panel further underscores that, given the same productivity level, higher net worth firms accumulate net worth at a slower pace, especially when they choose radical innovation over incremental innovation, as radical innovation is more costly and hence fewer resources are allocated for net worth accumulation.

The left panel in Figure 3.3 illustrates that, all other things being equal, firms with lower net worth or higher productivity level have a lower capital input, as they are more financially constrained. As firms accumulate net wealth and become financially unconstrained, their capital input becomes independent of the net worth level.

Figure 3.2: Net Worth Accumulation with Financial Frictions

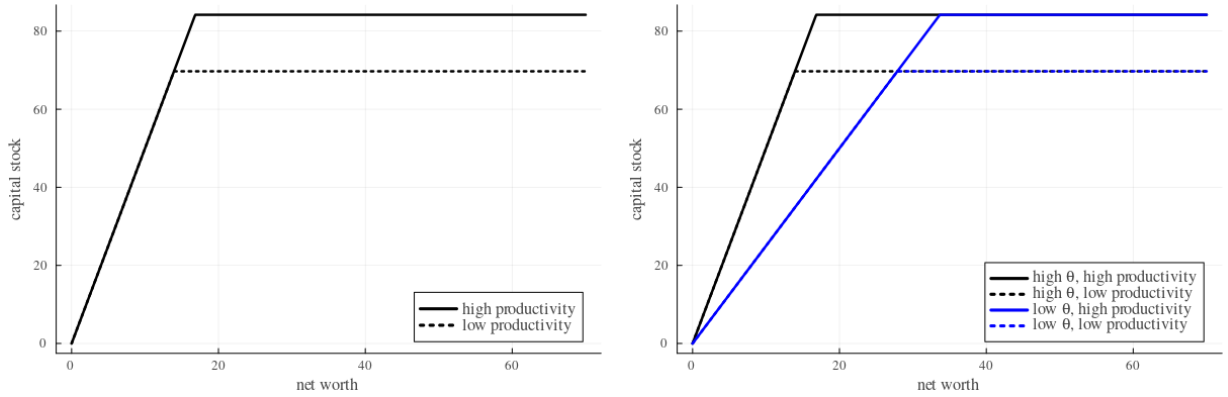


(a) high θ (b) low θ

Notes: This figure presents the net worth accumulation decision for high-type innovative firms in the presence of financial frictions. The y-axis indicates the growth rate of net worth accumulation $\frac{a'-a}{a}$, while the x-axis displays the firm's net worth level. The solid line represents high productivity firms, while the dotted line corresponds to low productivity firms. The left panel denotes a low degree of collateral constraint tightness (high θ). The right panel, building on the left panel, adds two additional lines, signifying a high degree of collateral constraint tightness (low θ). The left panel has an additional y-axis on the right-hand side, indicating whether the innovation is radical ($1_{\text{rad}} = 1$) or incremental ($1_{\text{rad}} = 0$).

Financial frictions reduce innovation probability. The left panel in Figures 3.4 and 3.5 shows that low net worth or high productivity firms exhibit lower incremental and radical innovation rates due to more binding financial constraints, consistent with innovation decisions (3.15), (3.18), and (3.19). Given that innovation investments are proportional to innovation rates, and considering the firms' net worth accumulation decisions previously illustrated, these results highlight that firms prefer to reallocate more resources to net worth (and hence capital) accumulation instead of innovation when they are more financially constrained. As firms accumulate enough net worth and become financially unconstrained, their innovation rates become independent of the net worth level.

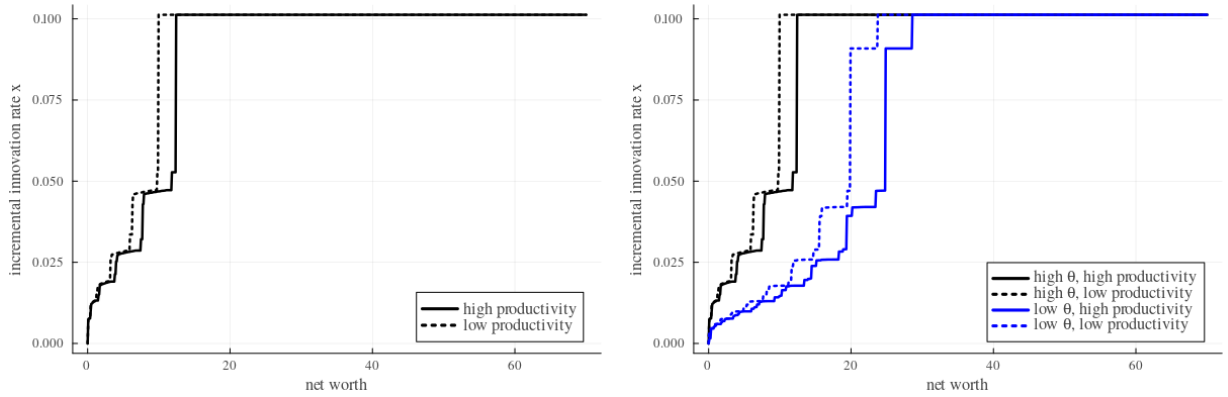
Figure 3.3: Capital Investment with Financial Frictions



(a) high θ (b) low θ

Notes: This figure presents the capital investment decision for high-type innovative firms in the presence of financial frictions. The y-axis indicates the capital input k, while the x-axis displays the firm's net worth level. The solid line represents high productivity firms, while the dotted line corresponds to low productivity firms. The left panel denotes a low degree of collateral constraint tightness (high θ). The right panel adds two additional lines, signifying a high degree of collateral constraint tightness (low θ).

Figure 3.4: Incremental Innovation Probability with Financial Frictions

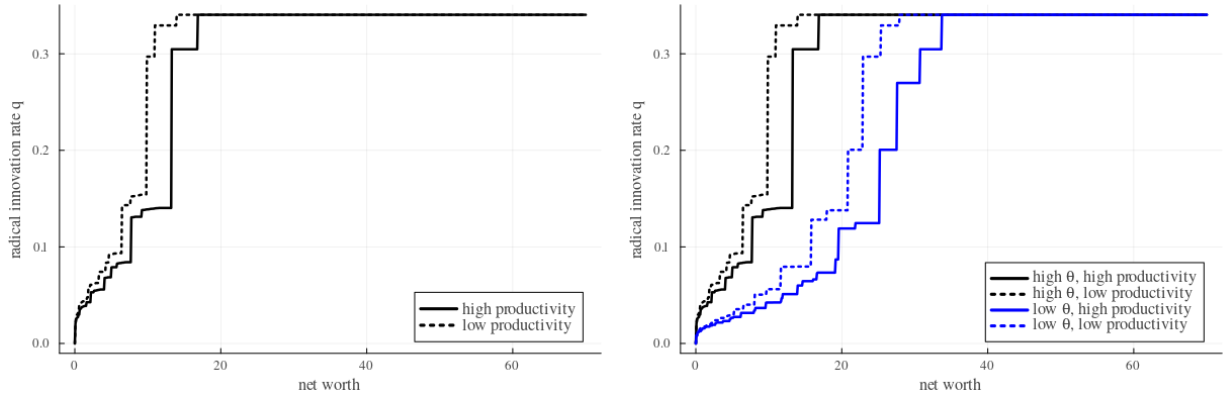


(a) high θ (b) low θ

Notes: This figure presents the incremental innovation decision for high-type innovative firms in the presence of financial frictions. The y-axis indicates the incremental innovation probability x, while the x-axis displays the firm's net worth level. The solid line represents high productivity firms, while the dotted line corresponds to low productivity firms. The left panel denotes a low degree of collateral constraint tightness (high θ). The right panel adds two additional lines, signifying a high degree of collateral constraint tightness (low θ).

Financial frictions reduce radical innovation. The left panel in Figure 3.6 conveys two key points. First, in the presence of financial frictions, firms with higher net worth are more inclined to choose radical innovation over incremental innovation, given the same productivity level. This is because firms with higher net worth are less financially constrained, and hence, they are more willing to decelerate their net worth accumulation and allocate more resources to costly radical innovation. Second, depending on the net worth level, firms with either very low or very high productivity may choose incremental innovation, while firms with a medium productivity level typically opt for radical

Figure 3.5: Radical Innovation Probability with Financial Frictions



(a) high θ (b) low θ

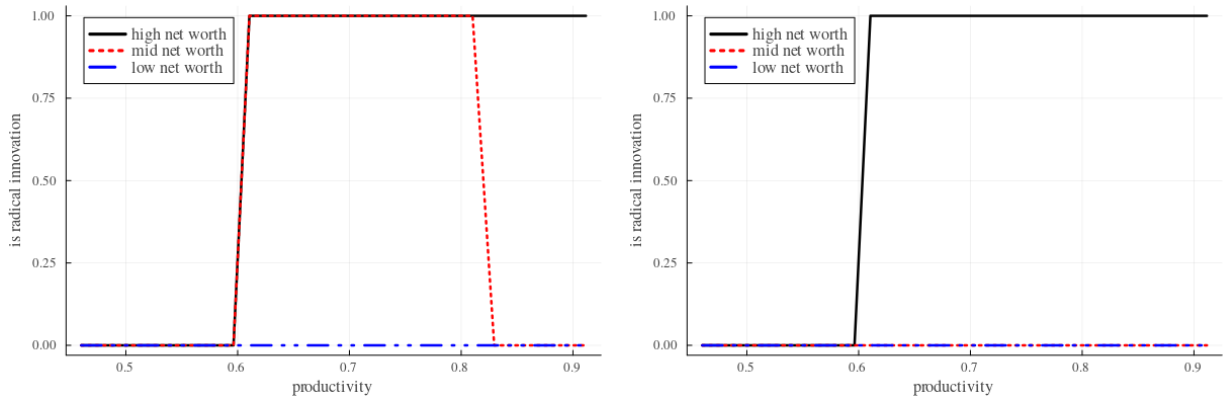
Notes: This figure presents the radical innovation decision for high-type innovative firms in the presence of financial frictions. The y-axis indicates the radical innovation probability x, while the x-axis displays the firm's net worth level. The solid line represents high productivity firms, while the dotted line corresponds to low productivity firms. The left panel denotes a low degree of collateral constraint tightness (high θ). The right panel adds two additional lines, signifying a high degree of collateral constraint tightness (low θ).

innovation. Firms with very low productivity choose incremental innovation because the additional profits from radical innovation cannot compensate for the extra costs, as discussed in the scenario without financial frictions. Very high productivity firms opt for incremental innovation because they are financially constrained and prefer to accumulate net worth to expand their production scale, capitalizing on their current productivity level. From their perspective, pursuing radical innovation would be excessively costly. On the other hand, firms with a medium productivity level have a weaker incentive to accumulate net worth and are willing to invest in the more costly radical innovation. A theoretical proof for these results is provided in Appendix A.3. As detailed later, this heterogeneous trade-off among firms accounts for the empirical observation that in middle-income countries, which typically have more financial frictions, radically innovative firms may not be larger than those that innovate incrementally. This is largely due to highly productive firms, which have the potential for radical innovation, opting instead for incremental innovation to expand their production scale in these countries.

It is worth noting that this empirical observation in middle-income countries can be exclusively attributed to financial frictions, as opposed to other potential frictions present in the model. For instance, some existing empirical evidence suggests that middle-income countries may face significantly higher costs for radical innovation than high-income countries. In the absence of financial frictions, this would indicate fewer radically innovative firms in middle-income countries due to these higher costs. However, this would also imply that radically innovative firms in these countries are more productive and larger than those implementing incremental innovation, a conclusion that is inconsistent with the data. This underscores the merits of incorporating financial frictions when

explaining cross-country differences in firm innovation and growth.

Figure 3.6: Incremental and Radical Innovation Trade-off with Financial Frictions



(a) high θ (b) low θ

Notes: This figure presents the innovation type selection for high-type innovative firms in the presence of financial frictions. The y-axis indicates whether the innovation is radical (1_{rad} =1) or incremental (1_{rad} =0), while the x-axis displays the firm's productivity level. The solid black line represents high-net-worth firms that are financially unconstrained. The dotted red line corresponds to mid-net-worth firms, which have a lower net worth than high-net-worth firms and are financially constrained. The dash-dot blue line represents low-net-worth firms, which have a lower net worth than mid-net-worth firms and are more financially constrained. The left panel denotes a low degree of collateral constraint tightness (high θ), while the right panel signifies a high degree of collateral constraint tightness (low θ).

Tightness of collateral constraint θ affects net worth accumulation, capital investment, innovation probability, and innovation type. In the model, it's not only the presence of financial frictions that matters, but also their severity. More severe financial frictions are represented by tighter collateral constraints (lower θ). Theoretically, it can be shown that tighter collateral constraints make firms more likely to face binding collateral constraints (i.e., $\frac{\partial \bar{a}}{\partial \theta} \leq 0$, $\frac{\partial a^*}{\partial \theta} \leq 0$), as proved in Appendix A.3. Tighter collateral constraints lead to slower net worth accumulation, less capital investment, lower innovation rates, and the lower likelihood of choosing radical innovation over incremental innovation, as illustrated in the right panel of Figures 3.2, 3.3, 3.4, 3.5, and 3.6.

Financial frictions affect not only individual firms' decisions, but also the distribution of firms as a whole. Specifically, the reduced innovation rate resulting from more severe financial frictions slows firms' productivity growth. As a result, firms are less likely to increase their productivity to a certain level throughout their life cycle, making it more difficult for them to overcome the fixed costs of radical innovation and reducing their likelihood for such innovation. This leads to less radical innovation overall, contributing to lower aggregate productivity.

Financial frictions similarly influence entrants' selection of firm types, with a lower probability of becoming innovative firms when facing more severe financial frictions.

3.4.3 Market Inefficiencies and Role of Policy Intervention

The equilibrium is not socially efficient for several reasons. First, firms do not internalize the positive externalities from their innovations. Second, some firms are financially constrained, leading to sub-optimal levels of capital accumulation and innovation. Third, firms vary in their innovation capabilities, and certain firms may invest excessively in innovation relative to others. For example, excessively high incumbent innovation could crowd out the formation of new innovative firms by increasing the equilibrium growth rate, which in turn raises the capital cost (interest rate) and adversely affects the formation of new innovative firms as well as the production of incumbents. This third reason typically emerges in a general equilibrium setting; in a partial equilibrium setting, enhancing the innovation of one type of firm without affecting others could contribute to higher aggregate productivity.

To this end, I decompose aggregate productivity and its growth to highlight the mechanisms behind market inefficiencies, and discuss how feasible policies in the decentralized equilibrium might be employed to mitigate these inefficiencies.[20] Furthermore, I explore how policies could vary in markets with different magnitudes of financial frictions.

The aggregate productivity Z_t can be decomposed as follows:

$$Z_t = Q_t^F \Big(\sum_s \int_z \int_a z_t(s)\Gamma_t^i(z,a,s)1_{\chi(Q,z,a,s)=0}dadz + \sum_s \int_z \int_a z_t(s)\Gamma_t^i(z,a,s)1_{\chi(Q,z,a,s)>0}dadz + \int_z \int_a z_t\Gamma_t^e(z,a)1_{\chi(Q,z,a)=0}dadz + \int_z \int_a z_t\Gamma_t^e(z,a)1_{\chi(Q,z,a)>0}dadz\Big)^{1+\iota}, \tag{3.32}$$

where $\Gamma_t^i(z,a,s)$ denotes the distribution of incumbent firms that do not exit in period t, $\Gamma_t^e(z,a)$ denotes the distribution of entrants that enter the economy in period t, and $1_{\chi(Q,z,a,s)=0}$ is an indicator equal to 1 if the financial constraint is not binding (i.e., $\chi(Q,z,a,s) = 0$). Firms' innovation probability, financial frictions, and the composition of firm types are all directly related to aggregate productivity as they all affect the distribution of firms.

The aggregate productivity growth rate depends on the average productivity growth rate among firms, $\tilde{g}_t$, which can be decomposed as

$$1 + \tilde{g}_t = (1 + \tilde{g}_t^n)s_t^n + (1 + \tilde{g}_t^h)s_t^h + (1 + \tilde{g}_t^l)s_t^l + (1 + \tilde{g}_t^e)s_t^e, \tag{3.33}$$

[20] Although it is useful to characterize a social planner's problem to understand the implications of these market inefficiencies, in practice, achieving the social planner's allocation is challenging. Therefore, I focus on feasible policies in the decentralized equilibrium that could be employed to mitigate these market inefficiencies.

where $s_t^m \equiv \frac{\int_z \int_a z_t(m)\Gamma_t^i(z,a,m)dadz}{\sum_s \int_z \int_a z_t(s)\Gamma_t(z,a,s)dadz}$, $s_t^e \equiv \frac{\int_z \int_a z_t(e)\Gamma_t^e(z,a)dadz}{\sum_s \int_z \int_a z_t(s)\Gamma_t(z,a,s)dadz}$, $\tilde{g}_t^m \equiv \frac{Q_{t+1}^F \int_z \int_a z_{t+1}(m)\Gamma_{t+1}^i(z,a,m)dadz}{Q_t^F \int_z \int_a z_t(m)\Gamma_t^i(z,a,m)dadz} - 1$ for $m \in \{n, h, l\}$, and $\tilde{g}_t^e \equiv \frac{Q_{t+1}^F \int_z \int_a z_{t+1}\Gamma_{t+1}^e(z,a)dadz}{Q_t^F \int_z \int_a z_t\Gamma_t^e(z,a)dadz} - 1$.

The feasible policies I consider include: (i) production profit tax reduction; (ii) investment subsidies for entrants (entrepreneurs) for becoming innovative types; and (iii) innovation subsidies for type-specific incumbents. The key distinction between profit tax reduction and innovation subsidy is that the former directly impacts both firm net worth accumulation and innovation investment, while the latter specifically targets innovation investment.

These policies may not perfectly differentiate firm types due to practical limitations, reflecting real-world phenomena. For example, the UK's Research and Development Tax Relief Scheme provides cash incentives for firms to accelerate innovation. It applies to both entrants and incumbents but requires firms to demonstrate how their work resolved scientific or technological uncertainty. Only successful innovations qualify, meaning this policy primarily encourages firms confident in their "innovative-type" status. In contrast, Canada's Scientific Research and Experimental Development (SR&ED) tax incentives reward efforts in innovation, without requiring the outcomes to be successful. Eligible work must aim for scientific or technological advancement through systematic investigation or experimentation. This approach encourages innovation across all firms, including those that may not initially recognize their innovative potential.

Since market inefficiencies and the scope for policy intervention depend heavily on firm distribution and the relative strength of underlying mechanisms, a quantitative analysis is essential to evaluate policy implications effectively.

3.5 Model Extensions

In Appendix A.5, I discuss how the key model mechanisms are robust to various model extensions.

4 Quantitative Analysis

In this section, I perform a balanced growth path analysis for high-income and middle-income countries separately to provide insights into macroeconomic implications and policy lessons.

4.1 Parameterization

The model economy is calibrated to annual data from high-income countries (1999-2015). To highlight the effect of financial frictions, I re-estimate the financial friction parameters in middle-income countries while keeping all other parameters the same as those in high-income countries. As a robustness check, I re-estimate all parameters in middle-income countries. I categorize parameters as either externally or internally estimated, as shown in Table 4.1.

4.1.1 Externally Estimated Parameters

The output elasticity relative to the inputs of the firm, denoted as α, is defined to align with the estimate provided in Cooper and Haltiwanger (2006). The exogenous exit rate of the firm, denoted as γ, is extracted from Eurostat, considering my dataset encompasses European countries. The probability of innovative firms transitioning to become a high-type innovative firm is set to correspond with my empirical fact concerning the likelihood of firms becoming radically innovative, conditional upon innovating. The innovation cost function for incumbents is quadratic with a parameter η equal to 2, a prevalent estimate within empirical innovation literature (cf. Acemoglu et al. (2018)). The investment of entrants in types is also set as quadratic with a parameter ν set to 2, for simplicity and without any loss of generality. Common estimates in the literature are utilized to assign values for the elasticity of intertemporal substitution (EIS), the household discount factor, and the capital depreciation rate.

4.1.2 Internally Estimated Parameters

The remaining parameters are calibrated to align with three distinct sets of data moments: firm innovation-related moments, industry distribution, and financial friction-related moments, as detailed in Table 4.1. Although these parameters are jointly determined using the simulated method of moments technique, certain parameters exhibit a close correlation with specific moments.

The innovation cost scale closely aligns with the R&D to GDP ratio, based on data from the World Bank. The cost scale and step size associated with incremental and radical innovation are adjusted to match the sales share and growth rate of each firm type. The model suggests that a larger innovation step size can contribute to a higher sales growth rate and a larger firm sales share. The differential innovation cost scales for high-type and low-type firms in implementing incremental innovation, represented by ψ_h/ψ_l, allow for flexibility and lead to varying innovation probabilities for these firms. Higher cost scales predict lower innovation probabilities, according to the firms'

innovation decision rule. As a result, ψ_h/ψ_l effectively captures the relative innovation efficiency of high-type firms compared to low-type firms. Estimation results suggest that high-type firms exhibit greater incremental innovation efficiency and probability than low-type firms. This aligns with empirical observations that radically innovative firms generate more patents than incrementally innovative firms in high-income countries. Moreover, it reflects the empirical observation that middle-income countries have a higher incremental innovation probability than high-income countries, as high-type firms in middle-income countries opt for incremental innovation over radical innovation due to financial frictions. The data indicates a relatively faster capital growth compared to sales growth in middle-income countries. This aligns with the model's suggestion that due to capital constraints, firms in these countries are inclined to accumulate capital.

Regarding industry distribution-related moments, knowledge spillovers significantly influence the dispersion of firms' sales distribution. Greater knowledge spillovers enhance the productivity and thus the sales of less productive firms, thereby reducing sales dispersion. The model also specifies that entrants' investment in types positively correlates with the proportion of innovative firms in the economy. Entrants' productivity is lower than that of the average incumbent firms, reflecting the reality that firms grow over their life cycle. The relative net worth level of entrants to incumbent firms is also considered to account for the financing needs of entrants. The parameters F and ε^n are estimated to match the differences in size and life cycle growth between innovative firms and non-innovative firms. The estimation results suggest that the non-innovative firms have a relatively high fixed cost of transitioning to innovative firms and a low successful rate ε^n. This is consistent with the empirical observation that only a few non-innovative firms become innovative as they age.

The financial friction parameter, specifically the collateral constraint θ, is calibrated to match the leverage level, in accordance with Buera et al. (2011), Midrigan and Xu (2014), Sui (2022), and Ottonello and Winberry (2024), among others.

4.1.3 Model Fit, Mechanism Validation, and Unique Role of Financial Frictions

Table 4.2 presents both the targeted moments in the data and their corresponding values in the model, illustrating the model's accurate representation of the data. This is evident in the 'HI' and 'MI2' columns. The model moments in the 'MI' column, however, do not quantitatively align well with the data moments in the middle-income countries, since the parameters are not re-estimated to match these data moments, as previously discussed. Nevertheless, the model moments in the 'MI' column qualitatively match the data moments, indicating that financial frictions contribute to explaining the cross-country differences in innovation and industry distribution.

Table 4.1: Parameterization

Parameter	Notation	Value			Identification
		HI	MI	MI2	
External Parameterization					
Output elas w.r.t. inputs	α	0.59			Cooper and Haltiwanger (2006)
Exit rate	γ	0.09			Eurostat
High-type prob.	ξ	0.55			Fact 1 (Prob. of radical innovation if innovate)
Innovation cost curvature	η	2			Common estimates
Entrant cost curvature	ν	2			Assigned
EIS	$1/\sigma$	1.5			Common estimates
Household discount factor	ϕ	0.97			Common estimates
Capital depreciation rate	δ	0.08			Common estimates
World frontier growth rate	g^F	0.01			Penn World Table
Internal Parameterization					
Panel A. Innovation					
Innovation cost scale	ψ_l	4.52	4.63		R&D/GDP, %
Inc innovation step size	J^{inc}	0.015	0.015		1-year capital growth - 1-year sales growth, %
Rad innovation cost scale	F^{rad}	2.81	2.85		Rad innovative firms' sales share among innovative firms
	κ	1.07	1.08		Innovative firms' sales share among all firms
Rad innovation step size	J^{rad}	0.03	0.03		Rad innovative firms' sales growth rel to non-innovative, %
High-type efficiency	ψ_h/ψ_l	0.94	0.93		Rad innovative firms' sales growth rel to inc innovative, %
Panel B. Industry Distribution					
Knowledge spillovers	ι	0.55	0.60		Std(log(sales)) among all firms
Entrants' investment in types	φ	1.12	1.14		Fraction of innovative firms
Entrants' productivity	ζ^n	0.03	0.02		Rel sales of non-innovative firms with age $\leq$ 4 vs age $\geq$ 20
	ζ^l	0.28	0.25		Rel sales of inc innovative firms with age $\leq$ 4 vs age $\geq$ 20
	ζ^h	0.39	0.36		Rel sales of rac innovative firms with age $\leq$ 4 vs age $\geq$ 20
Entrants' net worth	$\tilde{a}^0$	0.01	0.01		Rel net worth of firms with age $\leq$ 4 vs age $\geq$ 20
Type transition cost	F	0.76	0.78		Innovative firms' log(sales) rel to non-innovative (among age $\leq$ 4)
Type transition prob.	ε^n	0.1	0.09		Innovative firms' log(sales) rel to non-innovative (among all)
Technology obsolescence	ε	0.25	0.25		$\tilde{g} = g^F$
Panel C. Financial Frictions					
Collateral constraint	θ	0.9	0.4	0.42	Mean leverage ((total debt - cash)/total assets)

Notes: HI denotes high-income countries, MI denotes middle-income countries under benchmark parameterization (only financial friction parameter is re-estimated), and MI2 denotes middle-income countries under alternative parameterization (all internal parameters are re-estimated). For the internal parameterization, all parameters are estimated jointly. The data moments are computed from high-income countries unless specified. The parameters in middle-income countries are the same as those in high-income countries unless specified.

Table 4.2 also displays non-targeted moments which serve to further validate the model. The first set of moments presents firms' capital accumulation and financing patterns over their life cycle. Specifically, the data reveals that young innovative firms, particularly those from middle-income countries, typically have higher leverage but lower log(ARPK) than non-innovative firms. This suggests that firms with fewer financial constraints are more likely to become innovative.[21] Conversely, the discrepancies between innovative and non-innovative firms diminish as firms age,

[21] log(ARPK), a common indicator of capital misallocation across heterogeneous firms (see, e.g., Hsieh and Klenow (2009)), is computed as the firm's log of the output to fixed asset ratio relative to the industry mean in its country, i.e., $\log(\frac{Y_{ijct}}{K_{ijct}})$ - $\log(\overline{\frac{Y_{ijct}}{K_{ijct}}})$.

suggesting that firms alleviate their financial constraints over their life cycle.

The second set of moments shows the size of different types of firms and their persistent life cycle growth differences. Specifically, in high-income countries, young radically innovative firms are larger than young incrementally innovative firms. These differences increase over the firms' life cycle, suggesting that radically innovative firms experience faster life cycle growth than incrementally innovative firms. In middle-income countries, radically innovative firms are not larger than incrementally innovative firms. My model accounts for this phenomenon, suggesting that high-type firms in middle-income countries, despite potentially having high productivity levels and large sizes, face financial constraints in their early stages, leading them to favor incremental over radical innovation.

The 'MI' column shows that these non-targeted moments in the model are qualitatively consistent with the data counterparts, suggesting that financial frictions can explain these data patterns. However, there are concerns that other parameter differences across countries could also explain the empirical facts I document, thereby weakening the argument for financial frictions. For instance, middle-income countries could have higher innovation costs, particularly for radical innovation, higher costs for entrant investment in types, or lower entrant productivity and net worth compared to high-income countries. However, all of these alternatives predict some counterfactuals. Without financial frictions, a higher innovation cost cannot explain firms' life cycle change of radical innovation probability in middle-income countries; a higher radical innovation cost cannot explain why there are fewer (incrementally) innovative firms in middle-income countries; a higher cost of entrant investment in types cannot explain the incumbents' innovation difference across countries; a lower entrant productivity or net worth cannot explain the innovation probability difference across countries. Instead, I argue that these alternatives could reduce firms' life cycle growth or size, thereby strengthening the effects of financial frictions on innovation, as detailed later.

4.2 Effects of Financial Frictions on Cross-Country TFP Difference

4.2.1 Aggregate Implications of Financial Frictions

When comparing TFP between high-income and middle-income countries, financial frictions significantly reduce the TFP level in middle-income countries. Table 4.3 illustrates this: under the benchmark parameterization (Panel A), where only the financial friction parameter varies across countries, financial frictions lower the TFP level by 9.01 percent in middle-income countries relative to high-income countries. Under an alternative parameterization (Panel B), where all parameters differ across countries, the reductions are more pronounced—13.12 percent. The results in Panel

Table 4.2: Targeted and Non-Targeted Moments

Identification	Data		Model		
	HI	MI	HI	MI	MI2
Targeted Moments					
Panel A. Innovation					
R&D/GDP, %	2.03	0.73	1.95	1.19	0.73
1-year capital growth − 1-year sales growth, %	-3.10	2.30	-2.53	2.16	2.27
Rad innovative firms' sales share among innovative firms	0.76	0.55	0.84	0.64	0.55
Innovative firms' sales share among all firms	0.10	0.01	0.09	0.05	0.02
Rad innovative firms' 1-year sales growth rate rel to non-innovative, %	7.8	7.4	6.9	6.1	7.2
Rad innovative firms' 1-year sales growth rate rel to inc innovative, %	5.2	0.9	5.0	2.2	1.1
Panel B. Industry Distribution					
Std(log(sales)) among all firms	3.38	2.39	3.34	2.87	2.41
Fraction of innovative firms, %	0.20	0.09	0.20	0.14	0.09
Rel log(mean) in sales of non-innovative firms with age ≤ 4 vs age ≥ 20	1.21	0.83	1.14	0.91	0.76
Rel log(mean) in sales of inc innovative firms with age ≤ 4 vs age ≥ 20	1.29	1.34	1.28	1.16	1.31
Rel log(mean) in sales of rac innovative firms with age ≤ 4 vs age ≥ 20	1.46	1.43	1.54	1.39	1.40
Rel log(mean) in net worth of firms with age ≤ 4 vs age ≥ 20	1.20	1.22	1.19	1.25	1.23
Innovative firms' log(sales) rel to non-innovative (among age ≤ 4)	0.10	0.14	0.11	0.13	0.14
Innovative firms' log(sales) rel to non-innovative (among all)	0.16	0.19	0.16	0.20	0.19
Panel C. Financial Frictions					
Mean leverage among all firms ((total debt - cash)/total assets)	0.35	0.15	0.35	0.15	0.15
Non-Targeted Moments					
Panel A. Financing Patterns Over Life Cycle					
Innovative firms' leverage rel to non-innovative (among age ≤ 4)	0.03	0.10	0.02	0.06	0.09
Innovative firms' leverage rel to non-innovative (among all)	-0.03	0.06	-0.03	0.04	0.06
Innovative firms' log(ARPK) rel to non-innovative (among age ≤ 4)	-0.96	-1.30	-0.42	-0.92	-1.06
Innovative firms' log(ARPK) rel to non-innovative (among all)	-0.15	-0.85	-0.12	-0.46	-0.74
Panel B. Firm Size and Growth Over Life Cycle					
Rad innovative firms' log(sales) rel to inc innovative (among age ≤ 4)	0.001	-0.03	0.002	-0.01	-0.03
Rad innovative firms' log(sales) rel to inc innovative (among all)	0.03	-0.004	0.04	-0.000	-0.002

Notes: This table presents both the targeted and non-targeted moments for high-income (HI) and middle-income countries, based on the data and the model. Among the targeted moments for middle-income countries, all moments in the 'MI' column are non-targeted, except for those related to financial frictions; while all moments in the 'MI2' column are targeted. The non-targeted data moments are from Table B.5, controlling for the industry-year fixed effect and country fixed effect.

A represent the lower bounds of financial friction effects, while Panel B highlights that these effects are amplified when additional factors are considered. For example, firms in middle-income countries tend to be smaller and exhibit lower innovation efficiency (due to any reasons), and hence rely more heavily on external finance for innovation, further exacerbating the impact of financial frictions.

It is worth noting that the cross-country TFP level comparisons are based on detrended TFP, implicitly assuming that firms in all countries operate within a common detrended productivity space. These comparisons can be equivalently interpreted as measuring TFP level differences relative to the world technology frontier. The observed TFP gaps arise from differences in the distribution of firms: middle-income countries tend to have a more left-skewed productivity distribution, which results in lower aggregate productivity.[22]

[22]Relatedly, Midrigan and Xu (2014) develop a model with an exogenous productivity process and one-time

4.2.2 Decomposing the Effects of Financial Frictions

Decomposing the factors contributing to TFP differences across countries, the results shown in Panel A of Table 4.3 suggest that the composition and growth of innovative firms, particularly young radically innovative firms, play significant roles. Specifically, aligning the proportion of innovative and radically innovative firms in middle-income countries with those in high-income countries substantially reduces the TFP level difference. If only the fraction of radically innovative firms is matched, while keeping incrementally innovative firms different, the TFP difference still decreases, albeit to a lesser extent. These findings emphasize the importance of firm composition in determining TFP levels.

Furthermore, if the growth rates of innovative firms in middle-income countries were identical to those in high-income countries, TFP differences would be reduced, although the impact is less pronounced. Similarly, matching the growth rates of young radically innovative firms in middle-income countries with those in high-income countries, while maintaining different growth rates for other types of innovative firms, also decreases the TFP difference with a similar magnitude to the previous scenario. This highlights the substantial contribution of young innovative firms, among all innovative firms, to the overall TFP level.

4.2.3 Role of Firm Types and Innovation Types

Heterogeneous firm types and innovation types have been under-explored in previous studies focusing on financial frictions and aggregate TFP loss. In this section, I conduct counterfactuals to emphasize that these factors are crucial for understanding the aggregate implications of financial frictions and their micro-level decomposition. The results are summarized in Panels C and D of Table 4.3 and are explained below.

Without heterogeneous firm types and innovation types: Suppose all firms can innovate incrementally to increase their productivity. A recalibrated model suggests that it overestimates the growth contribution of small, unproductive firms (young firms are typically small and unproductive in the model), leading to a larger effect of financial frictions on TFP loss compared to the baseline. Moreover, it struggles to match the right tail of the firm size distribution, i.e., the dispersion of sales among all firms is smaller than that in the data.

technology adoption, finding that financial frictions account for at most 10 percent TFP loss (shown in their Table 4). Gopinath et al. (2017) develop a model with an exogenous productivity process and size-dependent financial frictions, estimating that financial frictions account for around 4 percent TFP loss. In my model, TFP level differences are larger because financial frictions distort firms' innovation investments, reshaping the productivity distribution of firms.

Table 4.3: Compare Middle-Income and High-Income Countries, %

	Difference in TFP level
Panel A. Aggregate Effects of Financial Frictions	9.01
Decomposition (if ... the same as in high-income)	
Fraction of innovative and radically innovative firms	3.03
Fraction of radically innovative firms	4.26
Innovative firms' growth rate	6.79
Radically innovative firms' growth rate	7.05
Young (age ≤ 4) innovative firms' growth rate	7.28
Young (age ≤ 4) radically innovative firms' growth rate	7.44
Panel B. Alternative Parameterization	13.12
Panel C. No Heterogeneous Firm Types and Innovation Types	11.13
Panel D. Exogenous Productivity Process	4.32

Notes: Panel A displays the percentage difference in TFP level between high-income (HI) and middle-income (MI) countries, where only the financial friction parameter varies across countries. The TFP level difference is computed as $(\tilde{Z}^{\text{HI}} - \tilde{Z}^{\text{MI}})/\tilde{Z}^{\text{MI}}$. It additionally breaks down the factors that could contribute to the TFP discrepancy between the two groups of countries, by setting the moment of interest in middle-income countries to be the same as that in high-income countries, while keeping all other moments unchanged as per the benchmark case. Panel B presents the percentage difference in TFP level between high-income and middle-income countries, where all parameters for the latter are re-estimated to match their respective data counterparts. Panel C eliminates heterogeneous firm types and innovation types in the baseline model, assuming that all firms innovate incrementally. Panel D eliminates heterogeneous firm types and innovation in the baseline model, assuming that all firms' productivity follows an AR(1) process. Panels C and D display the results of recalibrated models.

Additionally, I consider there are only non-innovative firms and innovative firms that do incremental innovation in the economy. Similar to a model where all firms innovate incrementally, a recalibration for this economy overestimates the growth contribution of small, unproductive firms and struggles to match the right tail of the firm size distribution.

Without heterogeneous firm types and innovation: The model reduces to an exogenous firm productivity model with financial frictions. The negative effects of financial frictions on aggregate TFP occur through the standard static misallocation channel and are much smaller than in the baseline model or any counterfactual models listed above. This highlights that the primary negative effects of financial frictions on aggregate TFP stem from dynamic firm innovation rather than the standard static misallocation channel, consistent with Sui (2022).

4.3 Policy Implications

To mitigate the adverse effects of financial frictions on productivity, a direct policy solution is to enhance firms' access to credit. However, even with greater credit availability, firms may not allocate it toward innovation investment due to the trade-off between physical capital accumulation and innovation, as demonstrated in this paper. A more common approach in the presence of financial frictions is to provide innovation subsidies to address firms' financial constraints and incentivize

innovation. In practice, policymakers across countries implement various forms of subsidies—with or without differentiating firm types or innovation types—to encourage innovation investment. In this section, I conduct policy counterfactuals to show that the effectiveness of these policies on TFP levels varies significantly, depending on the extent of financial frictions.

Policy instruments. The policies I consider include: (i) Investment subsidies for entrants (entrepreneurs) to become innovative types, τ^e, where entrants pay

$$G^e(Q, \zeta^n z, e) \equiv (1 - \tau^e)\varphi Q \zeta^n z e^{\nu};$$

(ii) Innovation subsidies for type-specific incumbents, τ^i, where low-type incumbents pay

$$G^{\text{inc}}(Q, z, s, x) = (1 - \tau^i)\psi_s Q z x^{\eta}$$

and high-type incumbents pay

$$G^{\text{rad}}(Q, z, s, q) = (1 - \tau^i)\kappa\psi_s Q z q^{\eta} + F^{\text{rad}} Q z;$$

and (iii) production profit tax rebate, τ^p, where firms' production profit becomes

$$\pi(Q, z, a, s) = (1 + \tau^p)[(Qz)^{1-\alpha} k^{\alpha} - (\delta + \tilde{r})k].$$

The production profit tax rebate captures non-innovation targeted interventions, as the tax rebate can be used for any investment, including capital investment.

Across all policies, total government expenditure T is financed by representative households:

$$C = Y - I - R^{inno} - R^{inc} - R^{rad} - R^{entrant} - T.$$

For simplicity without loss of generality, I assume that all targeted firms face the same policies. For example, innovation subsidies for low-type incumbents are represented by the same τ^i for all low-type incumbents.

To ensure comparability across policy counterfactuals, I standardize the total government expenditure in all scenarios. Specifically, I set the total government expenditure in the detrended equilibrium to 0.3 percent of the aggregate output level, where this aggregate output level corresponds to the baseline parameterization for high-income countries. The number 0.3 percent is derived from the average percentage of Gross Domestic Expenditure on R&D (GERD) performed by the government sector as a share of GDP, based on data from the Main Science and Technology Indicators (MSTI) database.[23]

Policy implications depend on the magnitude of financial frictions. Table 4.4 presents the

[23] This average is calculated across all economies in the database from 1999 to 2015. The calculation involves multiplying the percentage of GERD performed by the government sector by the total GERD as a share of GDP, yielding an average of 0.3 percent.

results of policy counterfactuals. For high-income countries, creating more innovative firms through entrant subsidies (Panel A) outperforms subsidizing existing incumbents (Panel B) in improving TFP level and welfare. In contrast, for middle-income countries, both strategies—subsidizing entrants to create more innovative firms and subsidizing incumbent innovative firms—are equally important for increasing TFP. Among policies targeting incumbent firms, subsidies for high-type firms are particularly effective in enhancing TFP level and welfare. These findings align with empirical evidence and model mechanisms, which suggest that firms' innovation frequency and quality are largely determined at birth. In middle-income countries, innovative and radically innovative firms face greater financial constraints, making innovation subsidies essential to alleviate these barriers. Creating more innovative firms not only increases aggregate innovation frequency but also enhances knowledge spillovers, thereby promoting growth for other firms.

Panel C of Table 4.4 demonstrates that production profit tax rebates are less effective than innovation subsidies in increasing TFP level and welfare. This is because tax rebates promote both capital and innovation investment, while subsidies directly target innovation investment, which is more critical for TFP improvements.

Panel C also reveals that production profit tax rebates are less effective in middle-income countries compared to high-income countries, as higher financial frictions slow down firms' innovation and incentivize firms' capital accumulation. These results underscore the model mechanism that financial frictions distort firms' trade-off between capital accumulation and innovation investment. They also imply that in economies with significant financial frictions, policymakers may prefer innovation subsidies over profit tax rebates to improve TFP.

Efficiency loss from incorrectly identifying firm types is non-negligible. Panel B of Table 4.4 reveals that if policymakers fail to accurately identify between innovative firms and non-innovative firms, the policy effect on TFP and welfare is less positive, especially in middle-income countries. Similarly, if policymakers fail to distinguish between high-type and low-type innovative firms, the policy effect on TFP and welfare is less positive, especially in middle-income countries. Middle-income countries experience higher efficiency losses from misidentification, as firms with high growth potential are often more financially constrained and require substantial subsidies to grow, which they may not receive under such policies.

The quantitative results suggest that the efficiency loss from existing innovation policies is non-negligible, as most do not differentiate between types of firms, and only a limited number of policies support entrepreneurs and the creation of innovative firms (cf. Figure 1.1).

Table 4.4: Increase in TFP Level and Welfare Compared to Benchmark,%

High-income		Middle-income	
Z	W	Z	W
Panel A. Relaxing barriers to become innovative firms			
Subsidy to entrants' investment in innovation types			
1.863	1.692	1.722	1.454
Panel B. Innovation subsidy			
If cannot identify firm types correctly			
To all incumbents (innovative and non-innovative)			
0.726	0.865	0.897	1.269
To innovative incumbents (high-type and low-type)			
1.193	1.025	1.647	1.396
If can identify firm types correctly			
To high-type innovative incumbents			
1.467	1.253	1.658	1.512
To young high-type innovative incumbents (age ≤ 4)			
1.658	1.497	1.824	1.716
Panel C. Production profit tax rebate			
To all firms (entrants and incumbents)			
0.318	0.057	0.166	0.028

Notes: I conduct counterfactuals given the parameterization specified in Table 4.1, where the sole variation across countries is the tightness of the collateral constraint. The results present the percentage change in variables of interest relative to the baseline economy without policy interventions. All exercises maintain total government expenditure at 0.3 percent of aggregate output. In Panel B, I model scenarios where policies fail to accurately identify firm types by assuming that non-innovative firms also receive subsidies. This scenario reflects a situation where non-innovative firms invest in innovation (with costs the same as those of low-type innovative firms) every period but never succeed in increasing productivity, thus illustrating the challenge of accurately distinguishing firm types in a simplified manner. The Z column records detrended TFP level. W denotes welfare.

4.3.1 How to Identify Firm Types in Practice?

The policy analysis above demonstrates that inefficiencies arise when policies fail to accurately identify firm types. However, identifying firm types, especially in their early life cycle stages, remains challenging in practice. In my approach, I use granted patents, patent citations, and patent claims for identification, assessing innovation based on the patent application year rather than the grant year. This method amounts to an ex-post identification, as firm types cannot be determined with certainty at the time of patent application. Given the lengthy patent granting process and the possibility of application failures, identifying firm types early in their life cycle remains complex. This raises the question: Are there other observable measures available during a firm's early stages that could improve firm type identification?

I find that in high-income countries, firm size, as measured by sales, number of employees, or fixed assets, is a useful determinant of firm types. As illustrated in Table 2.2, early life cycle

data indicates that radically innovative firms are typically larger than incrementally innovative firms, which in turn are larger than non-innovative firms. However, in middle-income countries, radically innovative firms may not be significantly larger, and can even be smaller than incrementally innovative firms, particularly during the early stages. My model accounts for this phenomenon by suggesting that high-type firms in middle-income countries, despite potentially having high productivity levels and large sizes, face financial constraints during their early stages, leading them to opt for incremental over radical innovation.

This discrepancy in middle-income countries presents a challenge for policymakers trying to differentiate between incrementally and radically innovative firms based on size alone. Are there other observable measures? One potential solution is to examine the role of inventors, who are crucial to patent creation. As detailed in Appendix B.4, using inventor data from PatentsView, which provides inventor identities from the U.S. patent office and can be integrated with my data, I found that metrics such as the experience or number of inventors, and the experience or number of productive inventors, do not offer clear insights. However, it appears that radically innovative firms typically have more creative inventors, and these creative inventors tend to be more experienced. Thus, when size measurement presents challenges, inferring firm type based on the creativity and experience of inventors could serve as a valuable alternative approach. It is important to note that the presence of more experienced and creative inventors in radically innovative firms does not necessarily mean these inventors cause the firms to be radically innovative. This pattern could result from a sorting mechanism: firms that are radically innovative (for various reasons) tend to attract experienced, creative inventors. However, from a policymaker's perspective, as long as firm types can be accurately identified based on inventor characteristics, the underlying causal relationships are secondary. The study of sorting between firms and inventors is beyond the scope of this paper and is left for future research.

5 Conclusion

In this paper, I uniquely document that in high-income countries, firm type, innovation quality, and innovation frequency are largely determined at birth. In contrast, middle-income countries have fewer radically innovative firms, lower radical innovation frequency, but higher incremental innovation frequency. Furthermore, the cross-country difference is primarily observed in industries with high external financial dependence. I construct a rich semi-endogenous growth model incorporating financial frictions, firm heterogeneity and innovation heterogeneity to explain these facts. The model demonstrates that collateral constraints, combined with limited liability conditions, hinder

innovative firm creation and distort firms' choice between radical and incremental innovation in middle-income countries, offering new policy implications for countries at different developmental stages.

My study underscores the importance of considering heterogeneity in firms' innate growth potential to understand firm dynamics and the resulting macroeconomic implications. By utilizing balance sheet data and patent data, I provide an initial set of evidence and a comprehensive model linking a firm's ex-ante characteristics to its life cycle growth in innovation. This paves the way for numerous research questions for future investigation on the key ex-ante factors determining firm growth and the resulting policy implications in countries at different stages of development.

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Online Appendix for "Innovation Potential, Firm Life Cycle, and Cross-Country Productivity Differences"

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A Model Appendix

A.1 Detrended Equilibrium Conditions

To detrend the model, apply the following formulas: $\tilde{k} \equiv \frac{k}{Z_t}; \tilde{y} \equiv \frac{y}{Z_t}; \tilde{a} \equiv \frac{a}{Z_t}; \tilde{z} \equiv \frac{z}{\sum_s \int_z \int_a z(s)\Gamma(z,a,s)dadz} = \frac{z}{\bar{z}}$. Note that $\frac{Z}{Q} = \bar{z}$. For non-innovative firms ($s = n$), the detrending process proceeds as follows:

$$\begin{aligned}
&\frac{V(Q,z,a,s)}{Z'}\frac{Z'}{Z} = \max_{a'} \frac{div(Q,z,a,s)}{Z'}\frac{Z'}{Z} + \beta(1-\gamma)\mathbb{E}[\frac{V(Q',z',a',s)}{Z'}\frac{Z'}{Z}],\\
\text{s.t.}\quad &\frac{div(Q,z,a,s)}{Z'}\frac{Z'}{Z} = \frac{\pi(Q,z,a,s)}{Z'}\frac{Z'}{Z} + (1+\tilde{r})\frac{a}{Z'}\frac{Z'}{Z} - \frac{a'}{Z'}\frac{Z'}{Z} - \frac{FQz}{Z'}\cdot 1_{\text{inno}}\cdot\frac{Z'}{Z} \geq 0\\
&\text{where}\quad \frac{\pi(Q,z,a,s)}{Z'}\frac{Z'}{Z} = \max_{k} \frac{(Qz)^{1-\alpha}k^{\alpha}}{Z'}\frac{Z'}{Z} - (\delta+\tilde{r})\frac{k}{Z'}\frac{Z'}{Z},\\
&\qquad\qquad \text{s.t.}\quad \frac{a}{Z'}\frac{Z'}{Z} \geq (1-\theta)\frac{k}{Z'}\frac{Z'}{Z},
\end{aligned} \tag{A.1}$$

and a similar process is applied for other firms.

The successful incremental innovation in a detrended economy generates a productivity improvement as follows:

$$\log(\tilde{z}_{t+1}) = \log(\tilde{z}_t) + \log(\frac{\exp(J^{\text{inc}})}{1+\tilde{g}_t}), \tag{A.2}$$

where $\tilde{g}_t$ denotes the growth rate of $\bar{z}_t$ and the growth rate of aggregate productivity Z_t, g_t, satisfies that $1 + g_t = (1+\tilde{g}_t)^{1+\iota}$, as demonstrated in the main text. Note that equation (3.5) can be rewritten as

$$\frac{z_{t+1}}{\bar{z}_{t+1}}\frac{\bar{z}_{t+1}}{\bar{z}_t} = \frac{z_t}{\bar{z}_t}\exp(J^{\text{inc}}), \tag{A.3}$$

which directly implies equation (A.2). A similar derivation applies to radical innovation.

The detrended equilibrium conditions are summarized as follows:

Non-Innovative Firms ($s = n$) :

$$\begin{aligned}
&\tilde{V}(\tilde{z},\tilde{a},s) = \max_{\tilde{a}',1_{\text{inno}}} div(\tilde{z},\tilde{a},s) + \beta(1-\gamma)(1+g)\mathbb{E}[\tilde{V}(\tilde{z}',\tilde{a}',s)],\\
\text{s.t.}\quad &div(\tilde{z},\tilde{a},s) = \pi(\tilde{z},\tilde{a},s) + (1+\tilde{r})\tilde{a} - \tilde{a}'(1+g) - F\cdot 1_{\text{inno}} \geq 0\\
&\text{where}\quad \pi(\tilde{z},\tilde{a},s) = \max_{\tilde{k}} \tilde{z}^{1-\alpha}\tilde{k}^{\alpha} - (\delta+\tilde{r})\tilde{k},\\
&\qquad\qquad \text{s.t.}\quad \tilde{a} \geq (1-\theta)\tilde{k}.
\end{aligned} \tag{A.4}$$

$$1_{\text{inno}} = \begin{cases} 1, & \text{if } \tilde{V}^{\text{inno}}(\tilde{z}, \tilde{a}, s) \geq \tilde{V}^{\text{non}}(\tilde{z}, \tilde{a}, s), \\ 0, & \text{elsewhere.} \end{cases} \tag{A.5}$$

$$[\tilde{k}]: \quad \tilde{k} = \left(\frac{\alpha \tilde{z}^{1-\alpha}}{\delta + \tilde{r} + (1-\theta)\chi(\tilde{z}, \tilde{a}, s)} \right)^{\frac{1}{1-\alpha}} \tag{A.6}$$

$$[\tilde{a}']: \quad 1 + \mu(\tilde{z}, \tilde{a}, s) = \beta(1-\gamma)\mathbb{E}[\chi(\tilde{z}', \tilde{a}', s) + (1+\tilde{r}')(1+\mu(\tilde{z}', \tilde{a}', s))] \tag{A.7}$$

Low-Type Innovative Firms ($s = l$):

$$[x]: \quad x = \left(\frac{\beta(1-\gamma)(1+g)[\tilde{V}(\tilde{z} \cdot \frac{\exp(J^{\text{inc}})}{1+\tilde{g}}, \tilde{a}', s) - \tilde{V}(\tilde{z}, \tilde{a}', s)]}{[1+\mu(\tilde{z}, \tilde{a}, s)]\psi_s \tilde{z} \eta} \right)^{\frac{1}{\eta-1}} \tag{A.8}$$

High-Type Innovative Firms ($s = h$):

$$1_{\text{rad}} = \begin{cases} 1, & \text{if } \tilde{V}^{\text{rad}}(\tilde{z}, \tilde{a}, s) \geq \tilde{V}^{\text{inc}}(\tilde{z}, \tilde{a}, s), \\ 0, & \text{elsewhere.} \end{cases} \tag{A.9}$$

$$[a^{\text{rad}'}]: \quad 1 + \mu^{\text{rad}}(\tilde{z}, \tilde{a}, s) = \beta(1-\gamma)\mathbb{E}[\chi(\tilde{z}'^{\text{rad}}, \tilde{a}'^{\text{rad}}, s) + (1+\tilde{r}')(1+\mu(\tilde{z}'^{\text{rad}}, \tilde{a}'^{\text{rad}}, s))] \tag{A.10}$$

$$[q]: \quad q = \left(\frac{\beta(1-\gamma)(1+g)[\tilde{V}(\tilde{z} \cdot \frac{\exp(J^{\text{inc}})}{1+\tilde{g}}, \tilde{a}'^{\text{rad}}, s) - \tilde{V}(\tilde{z}, \tilde{a}'^{\text{rad}}, s)]}{[1+\mu^{\text{rad}}(\tilde{z}, \tilde{a}, s)]\kappa\psi_s \tilde{z} \eta} \right)^{\frac{1}{\eta-1}} \tag{A.11}$$

$$[a^{\text{inc}'}]: \quad 1 + \mu^{\text{inc}}(\tilde{z}, \tilde{a}, s) = \beta(1-\gamma)\mathbb{E}[\chi(\tilde{z}'^{\text{inc}}, \tilde{a}'^{\text{inc}}, s) + (1+\tilde{r}')(1+\mu(\tilde{z}'^{\text{inc}}, \tilde{a}'^{\text{inc}}, s))] \tag{A.12}$$

$$[x]: \quad x = \left(\frac{\beta(1-\gamma)(1+g)[\tilde{V}(\tilde{z} \cdot \frac{\exp(J^{\text{inc}})}{1+\tilde{g}}, \tilde{a}'^{\text{inc}}, s) - \tilde{V}(\tilde{z}, \tilde{a}'^{\text{inc}}, s)]}{[1+\mu^{\text{inc}}(\tilde{z}, \tilde{a}, s)]\psi_s \tilde{z} \eta} \right)^{\frac{1}{\eta-1}} \tag{A.13}$$

Entrants:

$$[\tilde{k}^e] := \left(\frac{\alpha(\zeta^n \tilde{z})^{1-\alpha}}{\delta + \tilde{r} + (1-\theta)\chi(\zeta^n \tilde{z}, \tilde{a}^0)} \right)^{\frac{1}{1-\alpha}} \tag{A.14}$$

$$[\tilde{a}^e]: \quad 1+\mu(\zeta^n\tilde{z},\tilde{a}^0)=\beta(1-\gamma)\mathbb{E}[\chi(\tilde{z}',\tilde{a}^e,s)+(1+\tilde{r}')(1+\mu(\tilde{z}',\tilde{a}^e,s))] \tag{A.15}$$

$$[e]: e=\left(\frac{\xi\tilde{V}(\zeta^h\tilde{z},\tilde{a}^e,h)+(1-\xi)\tilde{V}(\zeta^l\tilde{z},\tilde{a}^e,l)-\tilde{V}(\zeta^n\tilde{z},\tilde{a}^e)}{[1+\mu(\zeta^n\tilde{z},\tilde{a}^0)]\varphi\zeta^n\tilde{z}\nu}\right)^{\frac{1}{\nu-1}} \tag{A.16}$$

Additional Optimality Conditions (detrended equilibrium):

$$\chi(\tilde{z},\tilde{a},s)\begin{cases}=0, & \text{if } \tilde{a}+(\theta-1)\tilde{k}>0,\\ >0, & \text{if } \tilde{a}+(\theta-1)\tilde{k}=0.\end{cases} \tag{A.17}$$

$$\mu(\tilde{z},\tilde{a},s)\begin{cases}=0, & \text{if } \tilde{div}>0,\\ >0, & \text{if } \tilde{div}=0.\end{cases} \tag{A.18}$$

$$R(\tilde{z},\tilde{a},s)=\begin{cases}\tilde{r}, & \text{if } \tilde{a}+(\theta-1)\tilde{k}>0,\\ \tilde{r}+(1-\theta)\chi(\tilde{z},\tilde{a},s), & \text{if } \tilde{a}+(\theta-1)\tilde{k}=0.\end{cases} \tag{A.19}$$

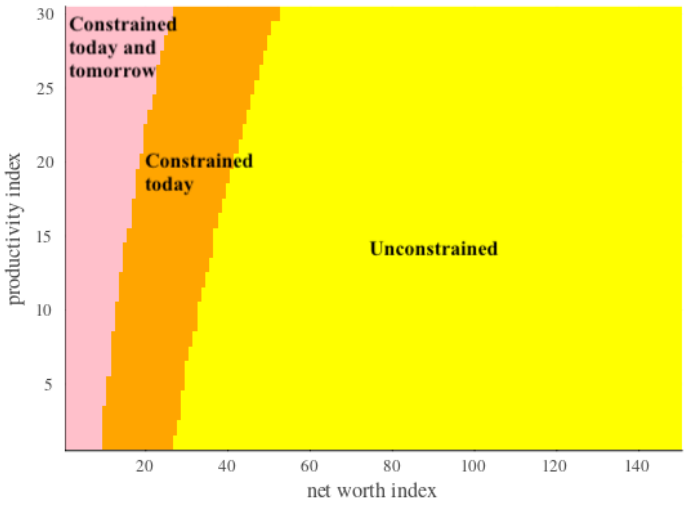
$$\tilde{k}(\tilde{z},\tilde{a},s)=\tilde{z}\left(\frac{\alpha}{\delta+R(\tilde{z},\tilde{a},s)}\right)^{\frac{1}{1-\alpha}}=\begin{cases}\tilde{z}\left(\frac{\alpha}{\delta+\tilde{r}}\right)^{\frac{1}{1-\alpha}}, & \text{if } \tilde{a}+(\theta-1)\tilde{k}>0\\ \frac{\tilde{a}}{1-\theta}, & \text{if } \tilde{a}+(\theta-1)\tilde{k}=0.\end{cases} \tag{A.20}$$

$$\begin{aligned}\pi(\tilde{z},\tilde{a},s)&=(\tilde{z})^{1-\alpha}\tilde{k}^{\alpha}-(\delta+\tilde{r})\tilde{k}=\tilde{z}[\left(\frac{\alpha}{\delta+R(\tilde{z},\tilde{a},s)}\right)^{\frac{\alpha}{1-\alpha}}-(\tilde{r}+\delta)\left(\frac{\alpha}{\delta+R(\tilde{z},\tilde{a},s)}\right)^{\frac{1}{1-\alpha}}]\\ &=\begin{cases}\tilde{z}[\left(\frac{\alpha}{\delta+\tilde{r}}\right)^{\frac{\alpha}{1-\alpha}}-(\tilde{r}+\delta)\left(\frac{\alpha}{\delta+\tilde{r}}\right)^{\frac{1}{1-\alpha}}], & \text{if } \tilde{a}+(\theta-1)\tilde{k}>0,\\ (\tilde{z})^{1-\alpha}(\frac{\tilde{a}}{1-\theta})^{\alpha}-(\delta+\tilde{r})\frac{\tilde{a}}{1-\theta}, & \text{if } \tilde{a}+(\theta-1)\tilde{k}=0.\end{cases}\end{aligned} \tag{A.21}$$

A.2 Additional Figures for Model Mechanisms

Using the net worth decision rule in equation (3.13) from the main text, I categorize the state space into three sections based on firms' collateral constraints, as shown in Figure A.1. Unconstrained firms have Lagrange multipliers $\mu(\tilde{z},\tilde{a},s)$ and $\chi(\tilde{z},\tilde{a},s)$ equal to 0. Firms constrained today exhibit $\chi(\tilde{z},\tilde{a},s)>0$. Firms constrained both today and tomorrow have $\chi(\tilde{z},\tilde{a},s)>0$ and $\mu(\tilde{z},\tilde{a},s)>0$.

Figure A.1: Partition of State Space



A.3 Proofs of Model Mechanisms

Proof for "without financial frictions, firms do radical innovation if their productivity is above a certain level":

Assume the following: (i) firms are not financially constrained and the minimum savings rule applies; (ii) firms are myopic, meaning they only consider next period's profit when making innovation decisions; and (iii) $\kappa = 1$, indicating there are no additional variable costs for radical innovation compared to incremental innovation. The optimal production profits of firms in a detrended stationary equilibrium are given by $\tilde{z}[\left(\frac{\alpha}{\delta+\tilde{r}}\right)^{\frac{\alpha}{1-\alpha}} - (\tilde{r}+\delta)\left(\frac{\alpha}{\delta+\tilde{r}}\right)^{\frac{1}{1-\alpha}}]$. High-type firms choose to undertake radical innovation in the next period if the following inequality holds: $\tilde{z}\frac{\exp(J^{\text{inc}})}{1+\tilde{g}}[\left(\frac{\alpha}{\delta+\tilde{r}}\right)^{\frac{\alpha}{1-\alpha}} - (\tilde{r}+\delta)\left(\frac{\alpha}{\delta+\tilde{r}}\right)^{\frac{1}{1-\alpha}}] - \tilde{z}\frac{\exp(J^{\text{inc}})}{1+\tilde{g}}[\left(\frac{\alpha}{\delta+\tilde{r}}\right)^{\frac{\alpha}{1-\alpha}} - (\tilde{r}+\delta)\left(\frac{\alpha}{\delta+\tilde{r}}\right)^{\frac{1}{1-\alpha}}] > F^{\text{rad}}$, given that the same probabilities of incremental innovation and radical innovation generate the same innovation cost.

Proofs for $\frac{\partial\chi(\tilde{z},\tilde{a},s)}{\partial\tilde{z}} \geq 0$, $\frac{\partial\chi(\tilde{z},\tilde{a},s)}{\partial\tilde{a}} \leq 0$, $\frac{\partial\bar{a}}{\partial\tilde{z}} \geq 0$ and $\frac{\partial\bar{a}}{\partial\theta} \leq 0$:

The expression for $R(\tilde{z},\tilde{a},s)$ and FOC for $\tilde{k}$ lead to

$$\chi(\tilde{z},\tilde{a},s) = \begin{cases} 0, & \text{if } \tilde{a}+(\theta-1)\tilde{k} > 0, \\ \frac{\alpha(\tilde{z})^{1-\alpha}(\frac{\tilde{a}}{1-\theta})^{\alpha-1}-\delta-\tilde{r}}{1-\theta}, & \text{if } \tilde{a}+(\theta-1)\tilde{k} = 0. \end{cases} \tag{A.22}$$

It's easy to see that there exists some cutoff $\bar{a}$ such that $\bar{a} = (1-\theta)\tilde{k}$ and for any $\tilde{a} > \bar{a}$, $\tilde{a} > (1-\theta)\tilde{k}$.

Furthermore,

$$\bar{a} = (1-\theta)\tilde{k} = (1-\theta)\tilde{z}\left(\frac{\alpha}{\delta+\tilde{r}}\right)^{\frac{1}{1-\alpha}}. \tag{A.23}$$

It's clear that $\bar{a}$ is independent of $\tilde{a}$. Therefore, rewrite $\chi(\tilde{z},\tilde{a},s)$ as

$$\chi(\tilde{z},\tilde{a},s) = \begin{cases} 0, & \text{if } \tilde{a} > \bar{a}, \\ \frac{\alpha(\tilde{z})^{1-\alpha}(\frac{\tilde{a}}{1-\theta})^{\alpha-1}-\delta-\tilde{r}}{1-\theta}, & \text{if } \tilde{a} \le \bar{a}. \end{cases} \tag{A.24}$$

It's direct that $\frac{\partial\bar{a}}{\partial\tilde{z}} \ge 0$ and $\frac{\partial\bar{a}}{\partial\theta} \le 0$, which indicates that more productive firms are more likely to face binding collateral constraint and less binding collateral constraint leads to lower level of cutoff of net worth. On the other hand, it's direct that $\frac{\partial\chi(\tilde{z},\tilde{a},s)}{\partial\tilde{z}} \ge 0$ and $\frac{\partial\chi(\tilde{z},\tilde{a},s)}{\partial\tilde{a}} \le 0$.

Proofs for $\frac{\partial\mu(\tilde{z},\tilde{a},s)}{\partial\tilde{a}} \le 0$, $\frac{\partial a^*}{\partial\tilde{z}}$, and $\frac{\partial a^*}{\partial\theta} \le 0$:

Given $\tilde{z}, s, \tilde{a}'$, there exists some cutoff a^* s.t. $div(a^*) = 0$, and for any $\tilde{a}' > a^*$, $div(\tilde{a}') > 0$. Furthermore, a^* depends on $\bar{a}$. The FOC for x (or q) leads to

$$\mu(\tilde{z},\tilde{a},s) = \begin{cases} 0, & \text{if } \tilde{a} > a^*, \\ \frac{\beta(1-\gamma)(1+g)\mathbb{E}[\tilde{V}(\tilde{z}',\tilde{a}',s|\omega=1)-\tilde{V}(\tilde{z}',\tilde{a}',s|\omega=0)]}{\eta x^{\eta-1}(\kappa)\psi_s\tilde{z}} - 1, & \text{if } \tilde{a} \le a^*, \end{cases} \tag{A.25}$$

where $\omega = 1$ means the firm innovates successfully. Using $\tilde{div}(\tilde{a}) = 0$ for $\tilde{a} \le a^*$, $\mu(\tilde{z},\tilde{a},s)$ can be rewritten as

$$\mu(\tilde{z},\tilde{a},s) = \begin{cases} 0, & \text{if } \tilde{a} > a^*, \\ \frac{x\beta(1-\gamma)(1+g)\mathbb{E}[\tilde{V}(\tilde{z}',\tilde{a}',s|\omega=1)-\tilde{V}(\tilde{z}',\tilde{a}',s|\omega=0)]}{\eta[\pi(\tilde{z},\tilde{a},s)+(1+\tilde{r})\tilde{a}-\tilde{a}'(1+g)]} - 1, & \text{if } \tilde{a} \le a^*. \end{cases} \tag{A.26}$$

Furthermore,

$$a^* = \frac{\tilde{a}'_{un}(1+g) + G^{\text{inc}}(\tilde{z},x_{un},s)\cdot 1_{s\neq n} + G^{\text{rad}}(\tilde{z},q_{un},s)\cdot 1_{s=h} - \pi(\tilde{z},a^*,s)}{1+\tilde{r}}, \tag{A.27}$$

where $\tilde{a}'_{un}$, g, x_{un}, and q_{un} are solutions to the unbinding limited liability condition case. It's easy to see that a^* is decreasing in $\pi(\tilde{z},a^*,s)$. Given $\pi(\tilde{z},\tilde{a},s)$ is decreasing in $\chi(\tilde{z},\tilde{a},s)$, we have

$$a^* = \begin{cases} a_1^*, & \text{if } \tilde{a} > \bar{a} \quad (\chi(\tilde{z},\tilde{a},s) = 0), \\ a_2^* > a_1^*, & \text{if } \tilde{a} \le \bar{a} \quad (\chi(\tilde{z},\tilde{a},s) > 0). \end{cases} \tag{A.28}$$

A direct implication is that $\frac{\partial a^*}{\partial \theta} \leq 0$ given $\frac{\partial \bar{a}}{\partial \theta} \leq 0$ and $\frac{\partial \chi(\tilde{z},\tilde{a},s)}{\partial \tilde{a}} \leq 0$.

Using the expression for optimal profit, a^*, and $\bar{a}$, we have $a_1^* = \frac{\tilde{a}'_{un}(1+g)+G^{\text{inc}}(\tilde{z},x_{un},s)\cdot 1_{s\neq n}+G^{\text{rad}}(\tilde{z},q_{un},s)\cdot 1_{s=h}+\frac{\bar{a}}{1-\theta}(\tilde{r}+\delta)(1-\frac{1}{\alpha})}{1+\tilde{r}}$. It's direct that $\frac{\partial a_1^*}{\partial \bar{a}} > 0$ and $\frac{\partial a_1^*}{\partial \tilde{z}} \geq 0$, which suggests that lower cutoff for collateral constraint generates that limited liability condition is more likely to be binding; more productive firms are less likely to face a binding limited liability condition when they are not collaterally constrained ($\chi(\tilde{z},\tilde{a},s) = 0$) because they have higher production profits and hence more resources available for innovation investment and net worth accumulation.

Using the expression for optimal profit and functional form for innovation cost, we have $a_2^* = \frac{\tilde{a}'_{un}(1+g)+G^{\text{inc}}(\tilde{z},x_{un},s)\cdot 1_{s\neq n}+G^{\text{rad}}(\tilde{z},q_{un},s)\cdot 1_{s=h}-(\tilde{z})^{1-\alpha}(\frac{a_2^*}{1-\theta})^{\alpha}+(\delta+\tilde{r})\frac{a_2^*}{1-\theta}}{1+\tilde{r}}$. It's easy to show that $\frac{\partial a_2^*}{\partial \tilde{z}} < 0$ if $1-\delta-\theta(1+\tilde{r}) > 0$, i.e., if θ is low enough. This suggests that when the collateral constraint is tight enough, more productive firms subject to this constraint are more likely to face a binding limited liability condition. This is due to their higher shadow cost of external financing, which reduces the resources available for innovation investment and net worth accumulation. However, if $1-\delta-\theta(1+\tilde{r}) < 0$, i.e., if θ is high enough, $\frac{\partial a_2^*}{\partial \tilde{z}}$ has an ambiguous sign because the shadow cost of external financing for net worth accumulation and innovation investment could be not very high.

On the other hand, using $\frac{\partial \pi(\tilde{z},\tilde{a},s)}{\partial \tilde{a}} = \chi(\tilde{z},\tilde{a},s)$ and equation (A.26), it's direct that $\frac{\partial \mu(\tilde{z},\tilde{a},s)}{\partial \tilde{a}} \leq 0$. It also shows that the multiplier for limited liability condition is affected by the collateral constraint via optimal profit of the firm.

Proof for "in the presence of financial frictions, high-productivity firms do incremental innovation while low-productivity firms do radical innovation, all else equal":

Assume the following: (i) firms are myopic, meaning they only consider next period's profit when making innovation decisions and (ii) $\kappa = 1$, indicating there are no additional variable costs for radical innovation compared to incremental innovation. Theoretical proofs for a special case are provided, and other cases can be shown numerically.

Consider that there is one high-type firm with $\chi(\tilde{z},\tilde{a},3) = \mu(\tilde{z},\tilde{a},3) = 0$. Based on the proof for $\frac{\partial \chi}{\partial \tilde{z}}$, suppose $\chi(\tilde{z}\frac{\exp(J^{\text{inc}})}{1+\tilde{g}},\tilde{a},3) > 0$, $\chi(\tilde{z}\frac{\exp(J^{\text{inc}})}{1+\tilde{g}},\tilde{a},3) = 0$, $\mu(\tilde{z}\frac{\exp(J^{\text{inc}})}{1+\tilde{g}},\tilde{a},3) = 0$, and $\mu(\tilde{z}\frac{\exp(J^{\text{inc}})}{1+\tilde{g}},\tilde{a},3) = 0$. The same variable costs for incremental innovation and radical innovation (and hence, the same incremental innovation probability and radical innovation probability) lead to that firms only compare the extra production profits from two types of successful innovation and the fixed radical innovation cost F^{rad}. Specifically, the firm compares zero with $\Delta\pi^{\text{rad}} - \Delta\pi^{\text{inc}} - F^{\text{rad}} = \tilde{z}^{1-\alpha}(\frac{\exp(J^{\text{inc}})}{1+\tilde{g}})^{1-\alpha}(\frac{\tilde{a}'^{\text{rad}}}{1-\theta})^{\alpha} - (\delta+\tilde{r})\frac{\tilde{a}'^{\text{rad}}}{1-\theta} - \tilde{z}(\frac{\exp(J^{\text{inc}})}{1+\tilde{g}})[(\frac{\alpha}{\delta+\tilde{r}})^{\frac{\alpha}{1-\alpha}} - (\tilde{r}+\delta)(\frac{\alpha}{\delta+\tilde{r}})^{\frac{1}{1-\alpha}}] - F^{\text{rad}}$. It is easy to show that $\frac{\partial(\Delta\pi^{\text{rad}}-\Delta\pi^{\text{inc}}-F^{\text{rad}})}{\partial \tilde{z}} = (1-\alpha)\tilde{z}^{-\alpha}(\frac{\exp(J^{\text{inc}})}{1+\tilde{g}})^{1-\alpha}(\frac{\tilde{a}'^{\text{rad}}}{1-\theta})^{\alpha} - (\frac{\exp(J^{\text{inc}})}{1+\tilde{g}})[(\frac{\alpha}{\delta+\tilde{r}})^{\frac{\alpha}{1-\alpha}} - (\tilde{r}+\delta)(\frac{\alpha}{\delta+\tilde{r}})^{\frac{1}{1-\alpha}}$

< 0 if $\tilde{a}'^{\text{rad}}$ is low enough and other parameters have values that fit the data. Therefore, there exists some level of F^{rad} that firms with high $\tilde{z}$ but low net worth accumulation have $\Delta\pi^{\text{rad}} - \Delta\pi^{\text{inc}} - F^{\text{rad}} < 0$ and choose incremental innovation, while firms with low $\tilde{z}$ have $\Delta\pi^{\text{rad}} - \Delta\pi^{\text{inc}} - F^{\text{rad}} > 0$ and choose radical innovation, all else equal. The key is the binding collateral constraint due to radical innovation reduces firms' expected profits from choosing radical innovation compared to incremental innovation.

Suppose $\chi(\tilde{z}\frac{\exp(J^{\text{inc}})}{1+\tilde{g}}, \tilde{a}, 3) > 0, \chi(\tilde{z}\frac{\exp(J^{\text{inc}})}{1+\tilde{g}}, \tilde{a}, 3) = 0, \mu(\tilde{z}\frac{\exp(J^{\text{inc}})}{1+\tilde{g}}, \tilde{a}, 3) > 0$, and $\mu(\tilde{z}\frac{\exp(J^{\text{inc}})}{1+\tilde{g}}, \tilde{a}, 3) = 0$. Given the previous proof that $\frac{\partial \tilde{a}_2^*}{\partial \tilde{z}} < 0$ if θ is low enough, the mechanism mentioned above will be strengthened, as the binding limited liability condition due to choosing radical innovation further reduces firms' expected profits from choosing radical innovation compared to incremental innovation.

A.4 Computational Algorithm

The algorithm for solving the balanced growth path equilibrium involves four steps:

Step 1. Guess the aggregate productivity growth rate, denoted as g^1.

Step 2. Given g^1, generate the state space for detrended productivity $\tilde{z}$ and net worth $\tilde{a}$. Compute the production, innovation, and type selection decisions for incumbent and entrant firms. Specifically, compute $\tilde{r}$ using Households' Euler equation (3.2); compute $\bar{a}(\tilde{z})$, the net worth cutoff for binding collateral constraint, using equation (A.23); compute $\tilde{k}$ and $\tilde{y}$ given $\tilde{r}$ and $\bar{a}(\tilde{z})$; compute $a^*(\tilde{z}, s)$, the net worth cutoff for binding limited liability condition, using equation (A.27). Given cutoffs $\bar{a}(\tilde{z})$ and $a^*(\tilde{z}, s)$, partition the whole state space into three parts: unconstrained, constrained today, and constrained today and tomorrow as shown in Figure A.1. Derive $\chi(\tilde{z}, \tilde{a}, s)$ using equation (A.24). Derive $\mu(\tilde{z}, \tilde{a}, s)$ using equation (3.13). First compute firm decisions for the unconstrained part, then use these decisions to determine the firms' decisions for the other two parts.

Step 3. Compute the stationary distribution of firms over the detrended state space based on their innovation decision rules and net worth accumulation rules, and then determine the implied aggregate productivity growth rate g^2 based on these decision rules.

Step 4. Check if $|g^1 - g^2| < 1e-05$. If not, update g^1 and repeat from step 2 until convergence.

A.5 Model Extensions and Mechanism Robustness

In this section, I discuss how the key model mechanisms are robust to various model extensions. More details are available upon request.

Endogenous entry and exit. In the current setting, the total mass of firms is constant. Does the introduction of an endogenous number of firms, characterized by a free entry condition, alter the model mechanisms? In the baseline model, while the total mass of firms remains constant, entrants' incentive to become innovative firms is endogenous and influenced by the equilibrium aggregate productivity growth rate. Suppose this growth rate increases due to faster innovation by incumbents (driven by any reason). On one hand, the expected value of becoming an innovative incumbent rises for entrants. On the other hand, external borrowing becomes more expensive due to a higher equilibrium interest rate, increasing the costs of becoming innovative incumbents. The free entry condition captures similar mechanisms.[24]

Equity financing and collateralizable innovation. In the baseline model with debt financing, the collateral constraint implies that firms must accumulate capital stock, as opposed to focusing on innovation, to ease the borrowing constraint. However, equity financing is also a prevalent practice, especially when considering that high-tech firms in developed countries often raise venture capital. In the model extension, I incorporate an earnings-based constraint, which is similar to incorporating equity issuance because firms' expected future earnings (influenced by present or future innovation) can be used as collateral, as discussed in Ottonello and Winberry (2024). Specifically, $\theta \mathbb{E}(\pi_{it+1}(s)) \geq b_{it}(s)$, and greater financial frictions are modeled by a smaller θ.[25] Compared to the baseline model, this setup could enhance the return on innovation, as successful innovation increases future earnings. However, whether this matters is a quantitative question rather than a model mechanism alteration.

Technology adoption-innovation tradeoff. In the baseline model, firms can only choose whether to innovate to increase productivity. However, in middle-income countries, technology adoption is often preferred over innovation (König et al. (2022)). Does this preference alter the model mechanisms? As long as productivity gains from technology adoption are assumed to be smaller than those from innovation, the baseline model's mechanisms remain intact.[26] Specifi-

[24] Under free entry, more entrants are incentivized by the higher expected value of becoming innovative incumbents, but the equilibrium interest rate also rises, offsetting this incentive.

[25] Substantial evidence indicates that equity financing is less developed in developing countries compared to developed ones.

[26] This assumption reflects the idea that productivity gains from technology adoption plateau once the technology is fully integrated, whereas ongoing innovation sustains productivity growth.

cally, suppose low-type innovative firms can choose between technology adoption and incremental innovation, both of which incur investment costs. Incremental innovation, however, requires an additional fixed cost (e.g., for setting up production techniques) but yields higher productivity gains per unit of investment compared to technology adoption. Financial frictions incentivize more firms to opt for technology adoption, reducing the number of innovative firms in middle-income countries and lowering aggregate productivity. Similarly, high-type innovative firms choose among technology adoption, incremental innovation, and radical innovation, with financial frictions again skewing choices toward technology adoption.

Endogenizing firms' highest innovation quality is determined at birth. Empirical evidence suggests a slight decline in firms' innovation quality over their lifecycle in both high-income and middle-income countries, with firms in high-income countries becoming slightly less likely to pursue radical innovation as they age. Given that these lifecycle variations are small, the baseline model assumes that firms' innovation quality (step size) remains constant over time. Theoretically, the model can be extended to explain these patterns as follows: When a firm becomes radically or incrementally innovative, it develops a new technology. Over its lifecycle, the firm can choose to either improve its existing technology (subject to diminishing returns, where the α^{th} improvement yields a productivity increase of J^{α}, $J > 1$) or develop a brand-new technology that replaces the existing one. While the new technology initially has lower productivity and requires additional fixed cost, it offers the potential for higher future gains. As firms age, their existing technology becomes more productive, making them more likely to focus on incremental improvements rather than pursuing disruptive innovations. This leads to smaller innovation steps and a greater tendency toward incremental innovation. Such a model extension strengthens the policy implications, particularly for high-income countries.

B Data Appendix

B.1 Policy Data from OECD STIP Compass

Data Description. STIP Compass is a collaborative initiative between the European Commission (EC) and the OECD. It aims to centralize both qualitative and quantitative data on national science, technology, and innovation policy (STIP) trends in one location. The primary data source for STIP Compass is the EC-OECD STIP Survey, which gathers countries' responses. Countries are primarily required to list and describe their policy initiatives aimed at addressing specific challenges, such as the innovation deficit in small and medium-sized enterprises. These initiatives are cataloged

using a standardized template that employs taxonomies of policy instruments and target groups, facilitating comparisons both within and between countries.[27]

The data includes more than ten thousand policies dating back to 1900 on science, technology, and innovation, covering 64 countries. Each policy is annotated with extensive information, including the responsible organization(s), target group(s), a short description, an instrument description, detailed characteristics related to the instrument description, and additional relevant attributes such as the policy's acronym, background, instrument name, etc.

Sample Construction. I focus on firm-targeted policies implemented between 1995 and 2015 in high-income and middle-income countries. The high-income group includes Italy, France, Germany, Finland, the United Kingdom, the Netherlands, Austria, Sweden, Ireland, Denmark, Norway, Luxembourg, and the United States. The middle-income group includes Bulgaria, Hungary, Croatia, the Slovak Republic, Latvia, Estonia, Slovenia, Malta, Portugal, Greece, Spain, Cyprus, and China.

I keep policies that target firms, private investors, or entrepreneurs. Specifically, I include policies that involve TG25, TG26, TG27, TG28, TG29, TG30, TG31, TG32, TG33, TG17, and TG18, as described in Table B.1.

Next, I mark each policy with two dummy variables to indicate whether it addresses firm age and firm size according to the target. Specifically, if TG25, TG26, TG27, and TG28 all equal 0, or if TG25 equals 1 and TG26, TG27, TG28 equal 0, the policy does not concern firm age; otherwise, it does. Similarly, if TG29, TG30, TG31, TG32, and TG33 all equal 0, or if TG29 equals 1 and TG30, TG31, TG32, TG33 equal 0, the policy does not concern firm size; otherwise, it does. Based on whether the policy concerns firm age, firm size, and whether it targets entrepreneurs (equals 0 or 1), I assign the policy a comprehensive target label.

Furthermore, if the policy concerns firm age, I assign a dummy label of 'age detail' based on the values of TG26, TG27, and TG28. Similarly, if the policy concerns firm size, I assign a dummy label of 'size detail' based on the values of TG30, TG31, TG32, and TG33.

Additionally, based on the instrument type information of each policy, I classify the policy as either a subsidy or a non-subsidy policy and label it with a dummy indicating the subsidy type. The detailed classification is presented in Table B.2.

[27] Despite efforts by the EC and OECD to standardize reporting, variations still exist in how countries report their policies. While some countries provide comprehensive and detailed information, others do not. This variability should be considered when conducting cross-country comparisons. This paper utilizes this dataset to offer suggestive evidence on innovation policies among countries, while the conclusions of the paper are not based on this dataset.

Table B.1: Science, Technology, and Innovation Policy (STIP): Target Groups

Code	Description
TG17	Private investors (e.g., enture capitalists and angel investors)
TG18	Entrepreneurs (individuals who start and manage their own businesses, at least in its earliest stages)
TG25	Firms of any age
TG26	Nascent firms (0 to less than 1 year old)
TG27	Young firms (1 to 5 years old)
TG28	Established firms (more than 5 years old)
TG29	Firms of any size
TG30	Micro-enterprises (with fewer than ten employees)
TG31	Small and medium-sized enterprises (SMEs) (with fewer than 250 employees)
TG32	Large firms (with more than 250 employees)
TG33	Multinational enterprises (MNEs) (operate in multiple countries worldwide)

Notes: This table presents data from OECD STIP Compass.

Table B.2: Science, Technology, and Innovation Policy (STIP): Instrument Types

Instrument	Type
Information services and access to datasets	Non-subsidy
Centres of excellence grants	Subsidy
Strategies, agendas and plans	Non-subsidy
Dedicated support to research and technical infrastructures	Non-subsidy
Grants for business R&D and innovation	Subsidy
Science and technology regulation and soft law	Non-subsidy
Creation or reform of governance structure or public body	Non-subsidy
Tax relief for individuals supporting R&D and innovation	Subsidy
Networking and collaborative platforms	Non-subsidy
Equity financing	Subsidy
Technology extension and business advisory services	Non-subsidy
Innovation vouchers	Subsidy
Science and innovation challenges, prizes and awards	Subsidy
Project grants for public research	Subsidy
Tax or social contributions relief for firms investing in R&D and innovation	Subsidy
Public awareness campaigns and other outreach activities	Non-subsidy
Standards and certification for technology development and adoption	Non-subsidy
Horizontal STI coordination bodies	Non-subsidy
Procurement programmes for R&D and innovation	Non-subsidy
Policy intelligence (e.g. evaluations, benchmarking and forecasts)	Non-subsidy
Loans and credits for innovation in firms	Subsidy
Intellectual property regulation and incentives	Non-subsidy
Labour mobility regulation and incentives	Non-subsidy
Debt guarantees and risk sharing schemes	Subsidy
Fellowships and postgraduate loans and scholarships	Subsidy
Institutional funding for public research	Subsidy
Regulatory oversight and ethical advice bodies	Non-subsidy
Formal consultation of stakeholders or experts	Non-subsidy

Notes: Types are determined by the author.

B.2 Additional Figures and Tables for Main Empirical Facts

B.2.1 Firm Type Identification and Persistence

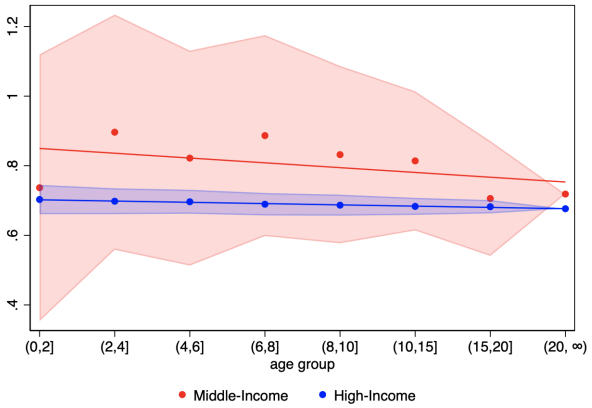
Table B.3 demonstrates that despite the annual identification of firm type, there is a significant persistence in the firm's type. Figure B.1 shows that the fraction of new patents that are radical for radically innovative firms is almost invariant over their life cycle.

Table B.3: Transition of Innovation Types

Identification	Data	
	High-Income	Middle-Income
Prob. $(1_{\text{innovative},t'} = 1, \forall t' \in [t+1, ..., t+5] \mid 1_{\text{innovative},t} = 1)$	0.87	0.79
Prob. $(1_{\text{innovative},t'} = 1, \forall t' \in [t+1, ..., t+5] \mid 1_{\text{innovative},t} = 0)$	0.01	0.00
Prob. $(1_{\text{rad innovate},t'} = 1, \forall t' \in [t+1, ..., t+5] \mid 1_{\text{rad innovate},t} = 1)$	0.85	0.73

Notes: Prob. $(1_{\text{innovative},t'} = 1, \forall t' \in [t+1, ..., t+5] | 1_{\text{innovative},t} = 1)$ represents the probability that a firm, innovative in year t, remains innovative at least once in the next five years. Prob. $(1_{\text{innovative},t'} = 1, \forall t' \in [t+1, ..., t+5] | 1_{\text{innovative},t} = 0)$ signifies the probability that a firm, non-innovative in year t, becomes innovative at least once in the next five years. Prob. $(1_{\text{rad innovate},t'} = 1, \forall t' \in [t+1, ..., t+5] | 1_{\text{rad innovate},t} = 1)$ illustrates the probability that a radically innovative firm in year t continues to be radically innovative at least once over the next five years. To avoid truncation issues, only data from years prior to 2010 are utilized to compute these statistics.

Figure B.1: Radically innovative firms: fraction of new patents that are radical



Notes: The Y-axis represents the fraction of new patents that are radical for radically innovative firms. All other descriptions are identical to those in Figure 2.1.

B.2.2 Raw Data for Fact 1 and Data Representativeness

Figure B.2 and B.3 illustrate an increasing fraction of innovative firms with rising national income levels. Innovative firms are defined by two criteria: those applying for at least one new patent annually and those applying for at least one new patent throughout the data sample, to account for potential infrequent patenting outcomes. The identification data for these firms comes from granted patents filed in the U.S. and European patent offices, merged with ORBIS Intellectual Property via firm ID. Consequently, the data sample only retains firms with patents merged with ORBIS. The fraction of innovative firms is calculated as the total number of innovative firms over the total number of firms at the county-industry-year level. The total number of firms is derived from three data series: (i) the total number of active firms from Eurostat Structural business statistics (2005-2022), (ii) the number of active employer firms from OECD SDBS Business Demography Indicators (post-2008), and (iii) the number of firms with at least one employee from ORBIS.

The study primarily focuses on European countries to ensure comparability of the firm-level data sample, but also includes the U.S. due to its innovation and global economic influence. However, ORBIS only covers public firms, in contrast to both public and private firms in Europe.[28]

B.2.3 Innovation Input and Life Cycle Dynamics

Figures (B.4) and B.5 demonstrate that innovation input patterns closely resemble those of innovation frequency.

B.2.4 Financial Development and Income Level in Europe

Figure B.6 shows that the income level is positively correlated with standard financial development indexes among the 24 European countries examined in this paper.[29] This positive correlation is consistent with the financial frictions story in the paper.

B.2.5 External Financial Dependence Index, Leverage, ARPK, and ARPL

Table B.4 lists high and low external financial dependence (EFD) industries. Table B.5 shows the financing pattern, ARPK, and ARPL of firms.

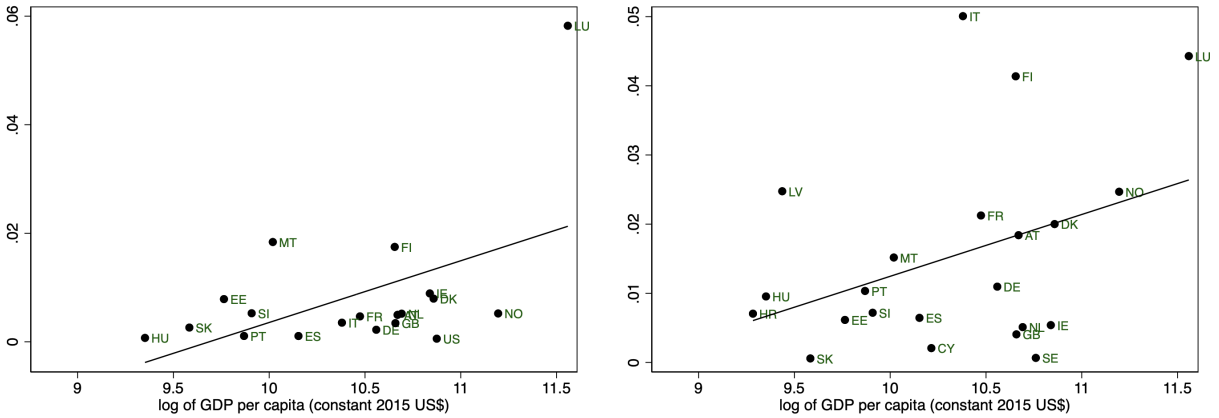
B.3 Robustness Checks for Main Empirical Results

Coming soon.

[28] U.S. data is not available in Eurostat Structural business statistics.

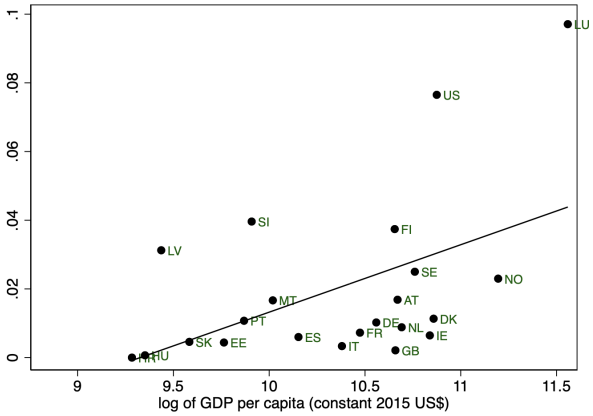
[29] In certain countries such as France, credit scores are not utilized, resulting in zero private credit bureau coverage.

Figure B.2: Fraction of innovative firms (identify every year)

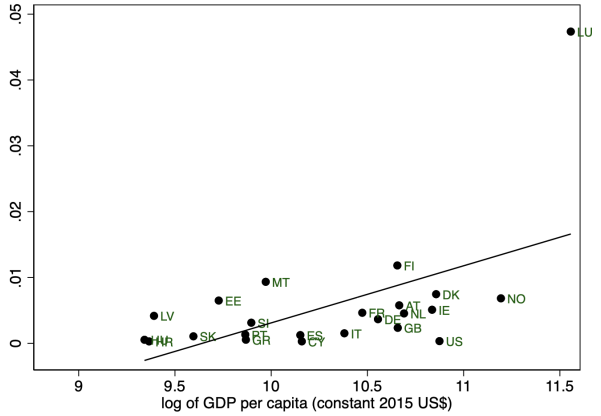


(a) USPTO: by number of active employer firms

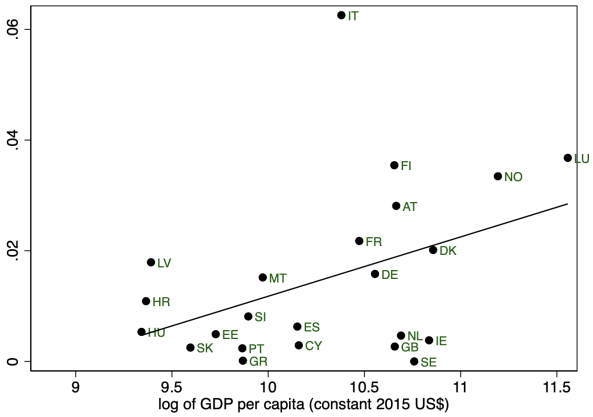
(b) USPTO: by number of active firms



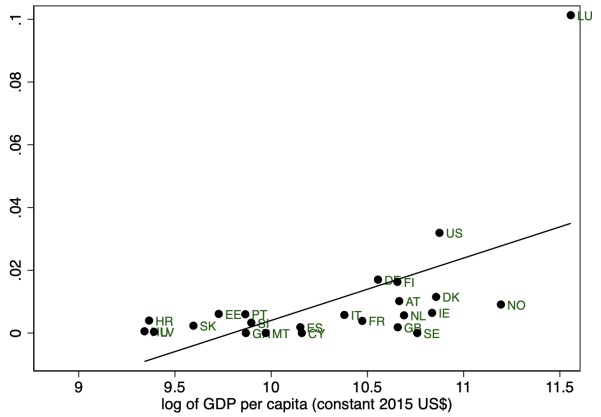
(c) USPTO: by number of firms in ORBIS



(d) EPO: by number of active employer firms



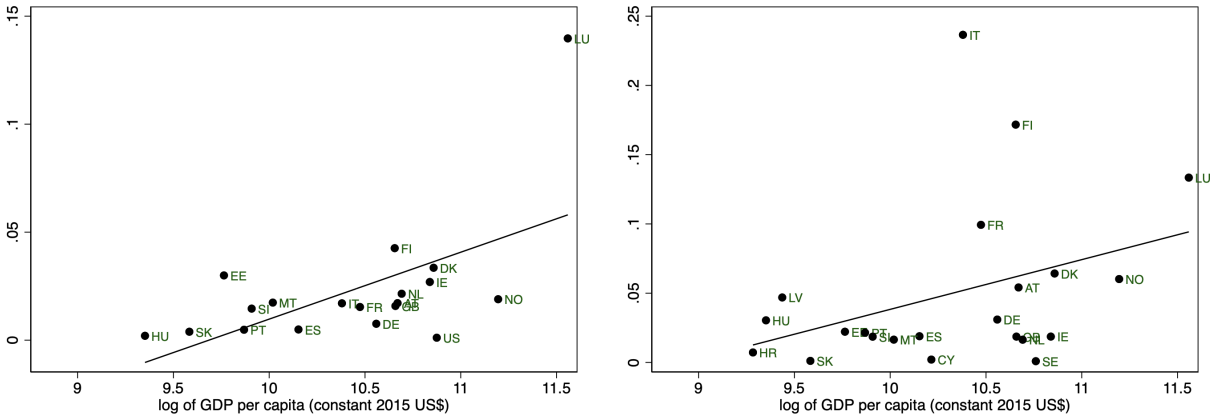
(e) EPO: by number of active firms



(f) EPO: by number of firms in ORBIS

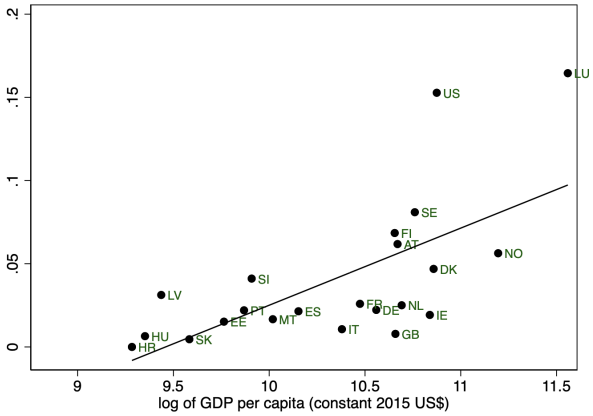
Notes: The X-axis depicts the GDP per capita of 24 European countries and the U.S., expressed as a natural logarithm of constant 2015 US dollars. The Y-axis displays three measures of the fraction of innovative firms, defined as those applying for at least one new patent annually, utilizing patent data from the U.S. and European Patent Offices (USPTO and EPO). Each country is represented by its 2-digit ISO country code. The solid line is the fitted line of the scatter plot. These figures represent the average values from 1999 to 2015. More details are described in section B.2.2.

Figure B.3: Fraction of innovative firms (identify from whole sample)

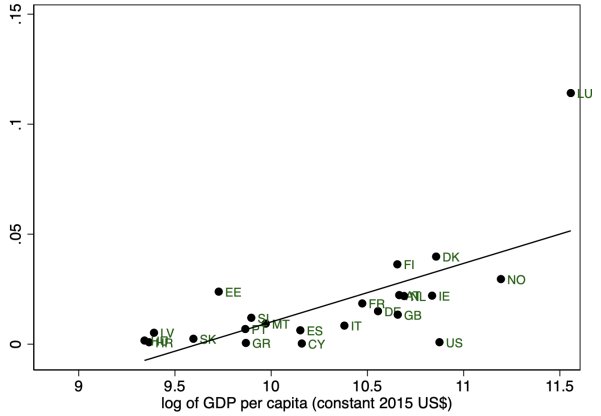


(a) USPTO: by number of active employer firms

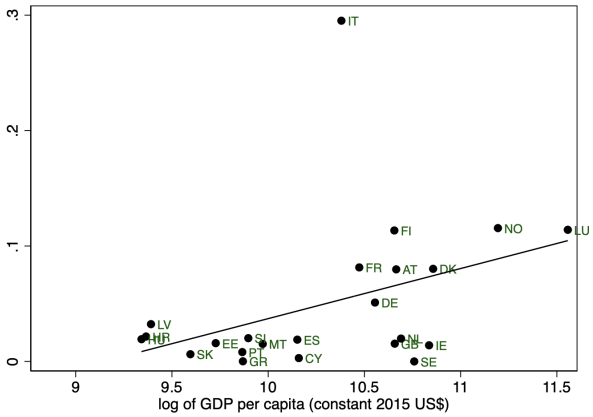
(b) USPTO: by number of active firms



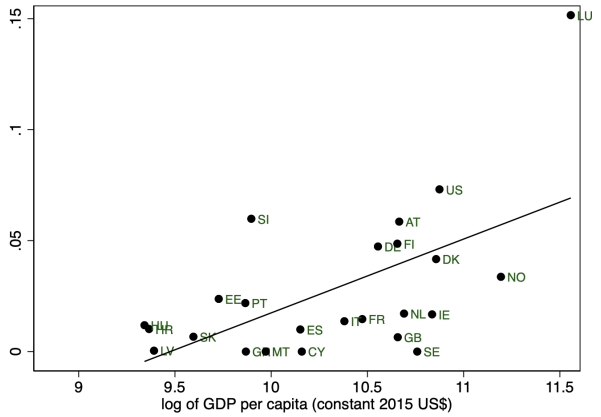
(c) USPTO: by number of firms in ORBIS



(d) EPO: by number of active employer firms



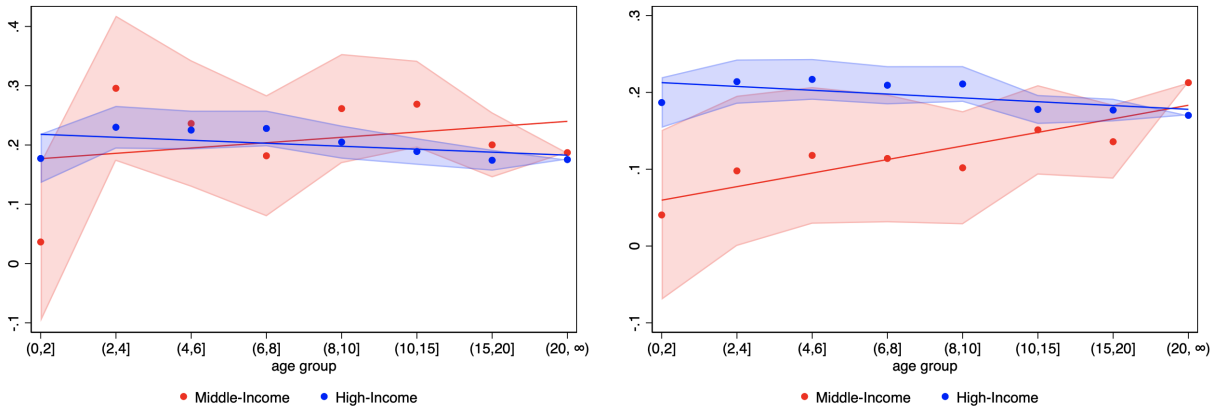
(e) EPO: by number of active firms



(f) EPO: by number of firms in ORBIS

Notes: The X-axis depicts the GDP per capita of 24 European countries and the U.S., expressed as a natural logarithm of constant 2015 US dollars. The Y-axis displays three measures of the fraction of innovative firms, defined as those applying for at least one new patent throughout the data sample, utilizing patent data from the U.S. and European Patent Offices (USPTO and EPO). Each country is represented by its 2-digit ISO country code. The solid line is the fitted line of the scatter plot. These figures represent the average values from 1999 to 2015. More details are described in section B.2.2.

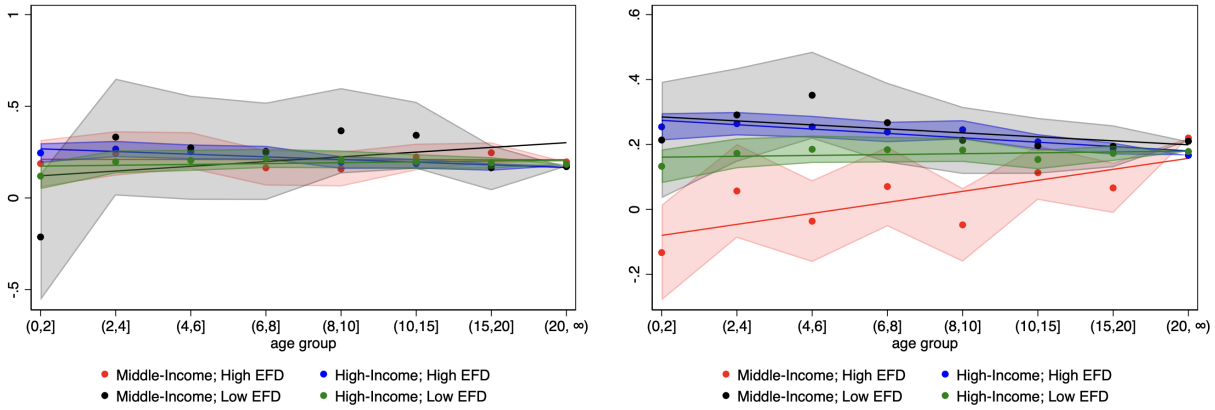
Figure B.4: Innovation Input over Age



(a) incrementally innovative firms (b) radically innovative firms

Notes: The Y-axis represents the intangible fixed assets-to tangible fixed assets ratio for incrementally innovative firms and radically innovative firms, respectively. All other descriptions are identical to those in Figure 2.1.

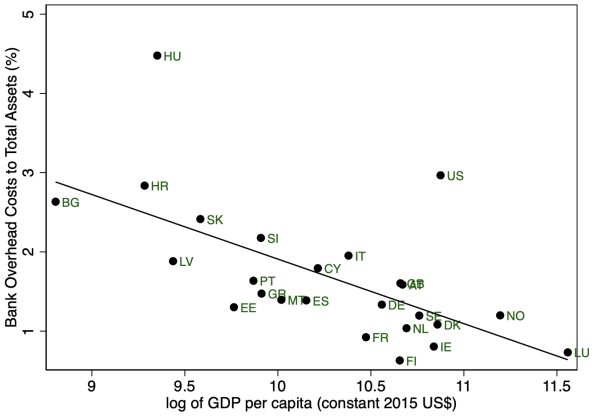
Figure B.5: Industry Variation: Innovation Input over Age



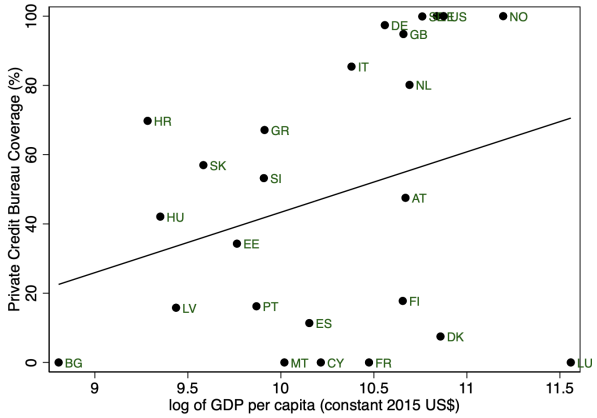
(a) incrementally innovative firms (b) radically innovative firms

Notes: The Y-axis represents the intangible fixed assets-to tangible fixed assets ratio for incrementally innovative firms and radically innovative firms, respectively. All other descriptions are identical to those in Figure 2.4.

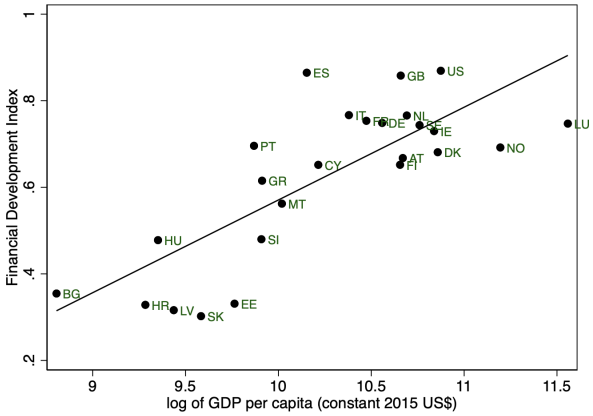
Figure B.6: Financial Development and Income Level



(a) Bank Overhead Costs to Total Assets, %



(b) Private Credit Bureau Coverage, %



(c) Financial Development Index

Notes: The X-axis depicts the GDP per capita of 24 European countries and the U.S., expressed as a natural logarithm of constant 2015 US dollars. The Y-axis measures three financial development indicators: bank overhead costs to total assets, private credit bureau coverage (both from the World Bank), and the IMF's financial development index. Each country is represented by its 2-digit ISO country code. The solid line is the fitted line of the scatter plot. These figures represent the average values from 1999 to 2015.

Table B.4: Industry Code (NACE Rev. 2)

Panel A. High external financial dependence industries	
H50	Water transport
F	Construction
H51	Air transport
B	Mining and quarrying
I55	Accommodation
L68	Real estate activities
G47	Retail trade, except of motor vehicles and motorcycles
C20_C21	Manufacture of chemicals and chemical products; basic pharmaceutical products and pharmaceutical preparations
H49	Land transport and transport via pipelines
I56	Food and beverage service activities
D	Electricity, gas, steam and air conditioning supply
E	Water supply; sewerage, waste management and remediation activities
C26_C27	Manufacture of computer, electronic and optical products; manufacture of electrical equipment
G46	Wholesale trade, except of motor vehicles and motorcycles
G45	Wholesale and retail trade and repair of motor vehicles and motorcycles
J59	Motion picture, video and television programme production, sound recording and music publishing activities
C19	Manufacture of coke and refined petroleum products
C33	Repair and installation of machinery and equipment
H52	Warehousing and support activities for transportation
C16	Manufacture of articles of straw and plaiting materials; wood and of products of wood and cork, except furniture
C22	Manufacture of rubber and plastic products
C28	Manufacture of machinery and equipment n.e.c.
C29_C30	Manufacture of motor vehicles, trailers, semi-trailers and of other transport equipment
C17_C18	Manufacture of paper and paper products; printing and reproduction of recorded media
Panel B. Low external financial dependence industries	
J62	Computer programming, consultancy and related activities
M69	Legal and accounting activities
M70	Activities of head offices; management consultancy activities
M71	Architectural and engineering activities; technical testing and analysis
M72	Scientific research and development
M74	Other professional, scientific and technical activities
J58	Publishing activities
J60	Programming and broadcasting activities
J61	Telecommunications
M73	Advertising and market research
C24_C25	Manufacture of basic metals and fabricated metal products, except machinery and equipment
J63	Information service activities
N77	Rental and leasing activities
N78	Employment activities
N79	Travel agency, tour operator and other reservation service and related activities
N80	Security and investigation activities
N81	Services to buildings and landscape activities
N82	Office administrative, office support and other business support activities
C23	Manufacture of other non-metallic mineral products
C31_C32	Manufacture of furniture; other manufacturing
C13_C14	Manufacture of textiles and wearing apparel
C10-C12	Manufacture of food products; beverages and tobacco products
C15	Manufacture of leather and related products
K65	Insurance, reinsurance and pension funding, except compulsory social security

Notes: This table presents the industry codes based on the two-digit NACE Rev.2 classification system, separating high and low external financial dependence (EFD) industries. The EFD data, obtained from Rajan and Zingales (1998), is derived from U.S. Compustat data. Industries are classified as high or low EFD based on the median level of EFD.

Table B.5: Financing Patterns and Innovation Probability

	(1)	(2)	(3)	(4)	(5)	(6)
	Among All Firms			**Among Young Firms**		
Panel A. leverage						
$1_{\text{innovative}}$	0.060***	0.007*	0.005*	0.092***	0.042***	0.008**
	(0.004)	(0.004)	(0.003)	(0.012)	(0.014)	(0.003)
$1_{\text{innovative}} \times 1_{\text{HI}}$	-0.086***	-0.067***	-0.005*	-0.066***	-0.059***	0.007
	(0.004)	(0.004)	(0.003)	(0.014)	(0.016)	(0.013)
Adjusted R^2	.087	.093	.7	.087	.087	.76
Panel B. leverage 2						
$1_{\text{innovative}}$	0.041***	0.041***	0.002***	0.050***	0.045***	0.002***
	(0.003)	(0.003)	(0.001)	(0.009)	(0.011)	(0.001)
$1_{\text{innovative}} \times 1_{\text{HI}}$	-0.079***	-0.072***	-0.003	-0.041***	-0.048***	0.008
	(0.003)	(0.003)	(0.002)	(0.010)	(0.012)	(0.010)
Adjusted R^2	.091	.086	.66	.11	.099	.73
Panel C. log(ARPK)						
$1_{\text{innovative}}$	-0.854***	-0.451***	-0.045***	-1.296***	-1.126***	-0.090**
	(0.016)	(0.017)	(0.013)	(0.048)	(0.056)	(0.041)
$1_{\text{innovative}} \times 1_{\text{HI}}$	0.702***	0.613***	0.039	0.335***	0.221***	0.013
	(0.053)	(0.063)	(0.048)	(0.018)	(0.018)	(0.015)
Adjusted R^2	.24	.26	.74	.22	.23	.79
Panel D. log(ARPL)						
$1_{\text{innovative}}$	-0.520***	-0.001	0.001	-0.602***	-0.017	0.011
	(0.010)	(0.013)	(0.008)	(0.033)	(0.027)	(0.032)
$1_{\text{innovative}} \times 1_{\text{HI}}$	0.545***	-0.019	0.002	0.508***	0.004	0.004
	(0.011)	(0.014)	(0.008)	(0.037)	(0.032)	(0.037)
Adjusted R^2	.36	.41	.82	.34	.36	.83
Obs.	2.12e+07	1.42e+07	1.35e+07	5562965	2842715	2339914
Industry-Year FE	Y	Y	Y	Y	Y	Y
Country FE	Y	Y	Y	Y	Y	Y
Controls	N	Y	Y	N	Y	Y
Firm FE	N	N	Y	N	N	Y

Notes: The empirical specification and data sample are the same as Table 2.1. Leverage is defined as (total debt - cash)/total assets. Leverage 2 is defined as the total debt to total assets ratio. log(ARPK) is the firm's log of output to fixed asset ratio relative to industry mean in its country, i.e., $\log(\frac{Y_{ijct}}{K_{ijct}})$ - $\log(\overline{\frac{Y_{ijct}}{K_{ijct}}})$. log(ARPL) is the firm's log of output to costs of employees ratio relative to industry mean in its country. * $p < 0.10$, ** $p < 0.05$,*** $p < 0.01$.

B.4 Additional Data and Facts for Identifying Firm Types in Practice

Data Source and Variable Definition. PatentsView provides unique inventor IDs, generated by a disambiguation algorithm, which can be merged with patents filed in the U.S. Patent Office. I incorporate this inventor information from PatentsView into my baseline data sample, as described in the main text.

Inventor Experience: At the inventor-application year level, experience is calculated based on the number of years since the inventor first appeared in the PatentsView dataset.[30] At the firm-application year level, experience is calculated as the average experience of inventors employed by the firm.

Inventor Productivity: At the inventor-application year level, productivity is measured by the number of new patents filed by the inventor in that year. I rank the productivity of all inventor-year observations from highest to lowest. An inventor is considered productive in a particular year if their productivity is in the top five percent, indicating that productivity can vary over time.

Inventor Creativity: At the inventor-application year level, creativity is assessed by the number of citations received by the inventor's new patents. I rank the creativity of all inventor-year observations from highest to lowest. An inventor is considered creative in a particular year if their creativity is in the top five percent, similarly indicating that creativity can vary over time. Citations are adjusted to ensure comparability across years, as detailed in the main text.

Firm-Level Metrics: The average number of (productive or creative) inventors at the firm-application year level is calculated as the average number of such inventors per new patent filed by the firm each year. Similarly, the average experience of inventors at this level is computed as the average experience of inventors per new patent filed by the firm each year. If a firm does not have productive or creative inventors, the corresponding firm-level measure is set to zero.

Empirical Facts. Table B.6 shows that metrics such as the experience or number of inventors, and the experience or number of productive inventors, do not offer clear insights and are sensitive to empirical specifications. However, radically innovative firms typically have more creative inventors who tend to be more experienced, a robust finding in both high-income (HI) and middle-income (MI) countries.

It is worth noting that the creativity and productivity of inventors can be used in lagged terms in empirical specifications, given that the identification of radically innovative firm types in a year can be positively correlated with the creativity and productivity of inventors. I find that results in

[30] Since my baseline data sample starts from 1999 while PatentsView data begins in 1976, the truncation issue for identifying inventor experience is not problematic.

these alternative empirical specifications are similar, but estimates become less precise due to a smaller number of observations.

Table B.6: Inventors and Radically Innovative Firms

	(1)	(2)	(3)	(4)	(5)	(6)
	Among All Firms			**Among Young Firms**		
Panel A. Average number of inventors						
$1_{\text{rad innovative}}$	-0.060***	-0.068***	0.027	0.045	0.042	0.048
	(0.013)	(0.014)	(0.016)	(0.046)	(0.058)	(0.124)
$1_{\text{rad innovative}} \times 1_{\text{HI}}$	0.119***	0.123***	0.042**	-0.013	-0.025	-0.034
	(0.014)	(0.015)	(0.017)	(0.049)	(0.062)	(0.131)
Adjusted R^2	.16	.17	.51	.1	.11	.52
Panel B. Average number of productive inventors						
$1_{\text{rad innovative}}$	0.032***	0.030***	0.007*	0.033***	0.030**	-0.004
	(0.004)	(0.004)	(0.004)	(0.011)	(0.014)	(0.029)
$1_{\text{rad innovative}} \times 1_{\text{HI}}$	0.042***	0.045***	0.006	0.042***	0.047***	-0.001
	(0.004)	(0.004)	(0.004)	(0.012)	(0.015)	(0.031)
Adjusted R^2	.18	.19	.66	.14	.14	.73
Panel C. Average number of creative inventors						
$1_{\text{rad innovative}}$	0.129***	0.125***	0.094***	0.151***	0.129***	0.111**
	(0.004)	(0.005)	(0.006)	(0.016)	(0.019)	(0.050)
$1_{\text{rad innovative}} \times 1_{\text{HI}}$	0.072***	0.076***	0.040***	0.073***	0.103***	0.030
	(0.005)	(0.005)	(0.006)	(0.017)	(0.021)	(0.052)
Adjusted R^2	.26	.26	.49	.24	.25	.56
Panel D. Average experience of inventors						
$1_{\text{rad innovative}}$	0.151***	0.131***	-0.048	0.195**	0.216*	-0.376*
	(0.029)	(0.031)	(0.032)	(0.088)	(0.111)	(0.222)
$1_{\text{rad innovative}} \times 1_{\text{HI}}$	-0.064**	-0.040	0.024	-0.110	-0.082	0.333
	(0.030)	(0.032)	(0.033)	(0.093)	(0.119)	(0.234)
Adjusted R^2	.14	.14	.6	.099	.1	.68
Panel E. Average experience of productive inventors						
$1_{\text{rad innovative}}$	0.361***	0.338***	0.098**	0.425***	0.453***	-0.118
	(0.038)	(0.041)	(0.043)	(0.106)	(0.138)	(0.266)
$1_{\text{rad innovative}} \times 1_{\text{HI}}$	0.050	0.085**	-0.042	-0.051	-0.025	0.087
	(0.039)	(0.043)	(0.045)	(0.113)	(0.147)	(0.280)
Adjusted R^2	.13	.13	.59	.076	.079	.73
Panel F. Average experience of creative inventors						
$1_{\text{rad innovative}}$	0.492***	0.493***	0.408***	0.470***	0.462***	0.192***
	(0.023)	(0.025)	(0.032)	(0.071)	(0.093)	(0.068)
$1_{\text{rad innovative}} \times 1_{\text{HI}}$	0.163***	0.175***	0.014	0.131*	0.192*	0.023
	(0.024)	(0.026)	(0.033)	(0.075)	(0.099)	(0.262)
Adjusted R^2	.17	.17	.43	.12	.14	.53
Obs.	63427	54729	47607	5127	3256	1671
Industry-Year FE, Country FE	Y	Y	Y	Y	Y	Y
Controls	N	Y	Y	N	Y	Y
Firm FE	N	N	Y	N	N	Y

Notes: The empirical specification and data sample are the same as Table 2.1. * $p < 0.10$, ** $p < 0.05$,*** $p < 0.01$.