Social Insurance During Lockdowns: Fiscal Policy with Systemic Occupational Risk

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Abstract

I characterize optimal catastrophe-state transfers in an economy where entrepreneurs face systemic income risk and the government is benevolent. Individuals choose between a risky entrepreneurial activity and a safe occupation. A rare disaster involving lockdown of small businesses, such as a war or a pandemic, reduces entrepreneurial returns and constrains fiscal capacity. A social planner sets taxes and transfers to balance efficiency, redistribution, and crisis insurance. Systemic entrepreneurial shocks generate large transfer needs that must be financed by workers not subject to the shock, creating the need for social insurance. I show that, under plausible conditions, fully or partially insuring systemic risk across the public and private sectors constitutes the optimal Ramsey allocation. Moreover, when social-insurance benefits depend on past reported income, the introduction of such insurance increases tax compliance in normal times, aligning private incentives with fiscal sustainability. I discuss why existing policy tools are insufficient to fully insure systemic occupational risks.

Keywords: systemic risk, public finance, social insurance.
JEL Codes: H10; H12; H21; H26; G01; L26.

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1 Introduction

When societies face catastrophe—whether a war, a pandemic, or a natural disaster—governments act swiftly to protect lives. Yet they rarely confront an equally urgent question: how to fairly distribute the economic burden of shutting down an economy. Government lockdowns have an asymmetric effect across the public and the private sector. Only small businesses are subject to government lockdowns. Recent history makes this asymmetry visible. During the war in Ukraine, as in many past conflicts, small firms closed, self-employment contracted sharply, and public-sector jobs became a safe harbor. In early 2022, a survey by the National Bank of Ukraine reported that 75 percent of small businesses had halted operations following the invasion (National Bank of Ukraine, 2022). Research by Audretsch et al. (2023) documents that after the 2014 Crimea invasion, the number of self-employed workers in Ukraine fell by approximately 675,000—a decline of roughly 20 percent relative to pre-conflict trends—and that small and medium-sized enterprises shrank by about 71,000 (14 percent), with most but not all of the losses recovered over the subsequent five years (Audretsch et al., 2023). Russia's experience was even more severe: five years into the conflict, it had lost more than 1.4 million small and medium-sized firms, a collapse of around 42 percent. These wartime reallocations reflect a recurrent historical pattern in which governments redirect resources to defense, constrain private enterprise, or shift entire regions into wartime production, with public employment—especially military employment—becoming the anchor. Okubo and Strobl (2017) provide evidence that large natural disasters sharply reduce small-firm survival and local entrepreneurial activity.

A similar occupational distortion was visible during the COVID-19 pandemic. Across high-income democracies, governments ordered business closures, restricted mobility, and froze entire sectors of the economy. Salaried employees, particularly in the public sector, continued receiving paychecks, often while working remotely. By contrast, shopkeepers, restaurateurs, hairdressers, and other self-employed or face-to-face service providers lost their revenues overnight. This was not the ordinary idiosyncratic risk of entrepreneurship—the risk that an individual business idea fails or demand turns out to be weak. It was a systemic risk: the risk that an entire occupation or sector, through no fault of its members, is ordered to close for the collective good.

The starting point is a simple observation. When a pandemic or a war shuts down economic activity, the wage earner in the public or formal sector continues to be paid, while the self-employed or small business owner is left exposed. Current safety nets, such as unemployment insurance or means-tested transfers, offer little protection to this latter group. These programs insure individuals against idiosyncratic risks—losing a job, suffering illness, or falling below the poverty line—but not against systemic occupational risks that strike entire groups at once.

The paper develops a model in which individuals choose between a safe occupation and an entrepreneurial one. Entrepreneurs face idiosyncratic success risk even in normal times, and on top of that a rare catastrophic state that wipes out their income entirely. Safe workers receive a stable wage but pay a tax that finances the catastrophe transfer. The government, observing the occupations but not individual abilities, chooses how much to insure the catastrophe state through a targeted transfer. The government wants to preserve a welfare enhancing allocation of talent, by encouraging people who create more wealth as entrepreneur to pick that occupation. This occupational-choice framework follows the tradition of models linking entrepreneurial selection to liquidity constraints and risk aversion (e.g., Lucas, 1978; Kihlstrom and Laffont, 1979; Evans and Jovanovic, 1989). This is an important gap in the social insurance theory.

The model delivers a simple prediction: there exists a cutoff ability level such that individuals above it become entrepreneurs and those below it choose safe employment. This cutoff depends on the generosity of catastrophe insurance and on the tax burden imposed on the safe sector. A higher catastrophe transfer encourages entrepreneurship by reducing systemic risk exposure, while higher taxes on the safe sector discourage it, improving the allocation of talent of the economy. The planner's problem, akin to a Ramsey social insurance problem, trades off the benefit of additional insurance and entrepreneurial entry against the normal-state fiscal cost of raising revenue from safe workers. Full catastrophe insurance—a transfer that perfectly smooths entrepreneurial consumption across states—is optimal when the catastrophe probability is high, when entrepreneurial spillovers are large, or when the fiscal base is broad and enforcement costs are low. Partial insurance arises when fiscal capacity or risk aversion of taxpayers is limited.

An important concern is that existing safety nets already insure the systemic entrepreneurial

risk of lockdowns during catastrophes. Recent systemic shocks, most visibly the COVID-19 pandemic, exposed a fundamental weakness in existing social insurance architectures: governments provided relief, but not systemic insurance. A growing empirical literature documents that emergency programs—while often large in scale—did little to preserve incentives for productive risk-taking. Europe's wage-subsidy systems, such as Germany's *Kurzarbeit* and the United Kingdom's furlough scheme, successfully maintained formal employer–employee attachments but did not protect those outside traditional payroll employment (Giupponi, Landais, and Lapeyre, 2022). In the United States, the Paycheck Protection Program deployed nearly $800 billion in forgivable loans to small businesses; it preserved jobs in the short run, but at a fiscal cost between $169,000 and $258,000 per job-year saved—three to five times median annual earnings (Autor et al., 2022). OECD (2021) reports provide similar evidence across dozens of countries: relief programs tended to exclude those most exposed to systemic occupational risk—self-employed workers, young firms, minority- and women-led businesses, and new entrants.

These empirical findings highlight the central conceptual point of this paper. Common policy tools such as universal stimulus payments, general transfers, or in-kind assistance do not insure occupational risk. By treating all individuals identically in the catastrophe state, universal transfers fail to alter the relative attractiveness of entrepreneurship versus safe employment. Means-tested or poverty-targeted benefits protect consumption floors but not the earnings volatility inherent in high-risk, high-return occupations; indeed, their phase-out rates may discourage post-shock recovery. In practice, as observed during COVID-19, these policies conveyed an implicit message: take a safe job, not a risky one; become a bureaucrat, not an entrepreneur. Over time, such distortions undermine dynamism and shift the economy away from innovation.

By contrast, this paper proposes and analyzes a targeted catastrophe-insurance policy that allocates transfers specifically to occupations whose incomes are most exposed to systemic shocks. The mechanism preserves ex ante incentives to undertake productive risk while providing ex post support precisely where the private market fails. My theoretical framework clarifies why this form of insurance cannot be replicated by existing policy instruments and quantifies the welfare gains of aligning public insurance with systemic occupational exposure.

An important extension of the framework introduces tax evasion, building on the classic

Allingham–Sandmo model, arguably the most important model in the theory of tax compliance. Entrepreneurs may underreport their income in normal times to evade taxes, facing a probability of audit and a penalty if caught. When catastrophe insurance depends on reported rather than true income, the incentive to report truthfully changes. In the absence of such linkage, an entrepreneur weighs the expected tax saving from evasion against the risk of audit and penalty. Once catastrophe benefits are based on reported income, however, underreporting also reduces expected catastrophe transfers. This acts as an additional enforcement mechanism, aligning the incentives of self-employed workers with the government's insurance objectives.

The analysis yields two main insights. First, catastrophe insurance and tax enforcement are complements. A more generous insurance scheme that is tied to reported income strengthens compliance even without increasing audit intensity. Second, truthful reporting can emerge as an equilibrium outcome if the marginal loss in catastrophe benefits from underreporting outweighs the expected tax saving in normal times. This condition is likely to hold when systemic insurance is substantial and the probability of catastrophic states is not negligible. In that sense, the design of catastrophe insurance can reduce the need for punitive enforcement: the promise of future protection disciplines behavior today.

Beyond taxation, the paper connects systemic risk in public finance with the long-run evolution of economic dynamism. If repeated crises leave entrepreneurs systematically uninsured while bureaucrats remain safe, the occupational distribution of talent shifts toward the public and corporate sectors. The result is a society less innovative but more risk-averse. In contrast, an economy that offers systemic insurance for risk-takers—analogous to how deposit insurance stabilizes the banking system—preserves incentives for entry and growth.

An important implication of my work is that catastrophe insurance for entrepreneurs need not take the form of ad hoc bailouts. The government support can be built as a formal social insurance program funded by small levies in normal times, possibly with eligibility linked to verified income reports and pre-registration. Such a program would provide automatic stabilizers in the face of future pandemics, wars, or climate shocks, distributing the burden of systemic events more evenly across society. The resulting policy problem parallels the macroprudential logic of Farhi and Werning (2016), in which optimal intervention requires state-contingent transfers that correct the externalities created by aggregate risk exposure.

The broader contribution of this paper is conceptual. It shows that the public finance logic of optimal insurance and taxation can be extended to systemic occupational risks, and that doing so reveals a new complementarity between social insurance and fiscal compliance. In the world of recurring crises, fair risk-sharing is not only a matter of equity but also of efficiency: it determines who dares to innovate, who prefers to hide, and who chooses safety over enterprise.

This paper, by asking what an efficient public response to such systemic entrepreneurial risk should look like, is related to three strands of literature. First, this paper connects to the large literature on occupational choice and entrepreneurship (Lucas, 1978; Kihlstrom and Laffont, 1979; Evans and Jovanovic, 1989), as well as the empirical work documenting its determinants and responses to shocks (Hurst and Lusardi, 2004; Hombert et al., 2020; Glaeser and Kerr, 2010; Fairlie, 2020). I contribute to this tradition by embedding the occupational-choice margin into a public-finance framework with aggregate risk and fiscal externalities, linking the microeconomics of entrepreneurship to the macro-public finance of systemic risk and fiscal policy.

The paper also contributes to the public-finance literature on optimal taxation and the allocation of fiscal resources (Ramsey, 1927, Atkinson and Stiglitz, 1976 and Saez, 2001). That literature studies how governments design tax instruments to raise revenue efficiently in the presence of behavioral distortions. I extend this framework by incorporating a systemic-risk component and an occupational-choice margin, which together determine the fiscal base and the social cost of public funds.[1]

Finally, this paper relates to the literature on systemic risk and government interventions. Barro (2006), Lorenzoni (2008), Farhi and Tirole (2012) and Dávila and Korinek (2018) study how private risk-taking creates aggregate vulnerabilities and how public policies can internalize or offset these externalities. Michaillat and Saez (2019) relate public policy with labor market inefficiencies and Goenka, Liu, and Nguyen (2024) model when is it optimal to introduce a lockdown. I contribute to this line of research by examining how rare disaster events—such as lockdowns or systemic disruptions to retail activity—affect entrepreneurial

[1]This formulation connects the theory of optimal fiscal policy with the economics of rare disasters, highlighting how the marginal cost of public funds and the efficiency–insurance trade-off depend on the structure of entrepreneurial risk in the economy.

entry and the design of optimal social insurance.[2]

The paper proceeds as follows. In Section 2, I present the model. In Section 3, I present the implications of the model for tax enforcement. In Section 4 I conclude.

2 Model

2.1 Environment

The economy is populated by a continuum of risk–averse individuals indexed by $i \in [0, 1]$. Each individual is endowed with one unit of labor and heterogeneous *entrepreneurial ability* $a \in \mathbb{R}_+$, distributed according to a continuous cumulative distribution function $F(a)$ with density $f(a)$. Individuals have expected utility defined over consumption, with common preferences represented by a strictly increasing and strictly concave utility function $u(\cdot)$ satisfying

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma}, \qquad \gamma > 0,$$

where γ denotes the coefficient of relative risk aversion.

Time is one period, but uncertainty has two layers: (i) an aggregate, *systemic* state of nature, and (ii) idiosyncratic shocks to entrepreneurial projects. The aggregate state $S \in \{N, C\}$ can be either *Normal* (N) or *Catastrophe* (C), with probabilities

$$\Pr(S = N) = 1 - \pi, \qquad \Pr(S = C) = \pi, \qquad \pi \in (0, 1).$$

The catastrophe state can be interpreted as a pandemic, war, or natural disaster that shuts down face–to–face activities and prevents small businesses from operating. The parameter π denotes the probability of a systemic downturn. This abstraction parallels the treatment of aggregate shocks in models of collective moral hazard (Farhi and Tirole, 2012), where private agents do not internalize the fiscal externalities of aggregate bailouts or systemic support.

The realization of S is publicly observable.

2.2 Occupational choice

Each individual chooses between two occupations:

[2]In contrast to models where systemic risk arises from financial fragility or maturity mismatch, here aggregate shocks originate from non-financial crises that suppress small-scale entrepreneurial income and force governments to reallocate fiscal resources across sectors.

1. **Safe employment.** The safe sector provides a fixed wage w_S in both aggregate states. This occupation is fully insured against systemic risk—public–sector or salaried jobs continue to pay regardless of S.

2. **Entrepreneurship.** The entrepreneur invests her time and ability a into running a project. In the normal state $S = N$, the project succeeds with probability $p(a)$ and yields revenue $R(a)$; with probability $1 - p(a)$ it fails and yields 0. In the catastrophe state $S = C$, all entrepreneurial activity ceases and output is zero regardless of a. The functions $p(a)$ and $R(a)$ are continuously differentiable and strictly increasing in a:

$$p'(a) > 0, \quad R'(a) > 0.$$

Hence, higher–ability individuals have both a higher probability of success and a higher return conditional on success, which provides a natural sorting mechanism: only sufficiently able individuals choose entrepreneurship in equilibrium. This selection mechanism follows the Roy model of self-selection based on comparative advantage (Roy, 1951; Heckman and Honoré, 1990) and its applications to entrepreneurship (Lucas, 1978; Kihlstrom and Laffont, 1979; Evans and Jovanovic, 1989).

Let a^* denote the ability cutoff at which an individual is indifferent between occupations; those with $a \geq a^*$ become entrepreneurs, and those with $a < a^*$ choose safe employment. The mass of entrepreneurs is $m = 1 - F(a^*)$.

2.3 Government policy and the instrument θ

The government has access to state–contingent taxes and transfers and acts as a social insurer. Its goal is to provide partial insurance against systemic risk while maintaining incentives for occupational choice.

I focus on a simple and transparent policy instrument: a *catastrophe–state, entrepreneur–targeted transfer* $\theta \geq 0$, financed by a proportional tax on safe–sector income in both states. Formally, policy is described by two states of the world. In both states $S = N$ and $S = C$, safe workers pay a tax τ_N on their wage income, while entrepreneurs pay no taxes.[3] In the

[3]This asymmetry simplifies notation; in the calibration or extension, τ_N can be levied on both groups with possibly different bases.

catastrophe state $S = C$, entrepreneurs receive a transfer θ while safe workers receive none:

$$b_C^E = \theta, \qquad b_C^S = 0.$$

The government budget must balance in expectation:

$$\tau_N B_S = \pi\, \theta\, m, \tag{1}$$

where B_S denotes the total tax base of safe workers (identical in both states) and m is the mass of entrepreneurs.

Equation (1) captures the essence of the public–finance problem: resources collected from the systemically protected sector are used to provide insurance to the systemically exposed sector in catastrophe. The design of θ thus determines both the extent of risk sharing and the degree of occupational reallocation.

In the baseline environment, taxation is modeled as falling entirely on safe–sector income. This choice has two motivations. First, it captures the practical difficulty of taxing income which is not third-party reported (Pissarides and Weber, 1989), especially for small or self–employed firms whose revenues are volatile, irregular, and often only imperfectly observable. Second, restricting taxation to the safe sector yields a simple balanced–budget identity and makes the fiscal incidence of catastrophe insurance transparent without altering the qualitative mechanisms of the model.[4]

2.4 Timing and information

The sequence of events is as follows:

[4]In the baseline formulation, taxation is levied exclusively on safe–sector income. This assumption is adopted for expositional clarity and does not drive the welfare results. The relevant object in the Ramsey problem is the *effective tax base* available to finance catastrophe insurance, not whether entrepreneurs contribute directly to revenue. In most advanced economies, including the United States, the overwhelming majority of taxable labor income arises from salaried employment: approximately 90% of workers receive third–party–reported W–2 earnings, whereas only about 10% are self–employed (Bureau of Labor Statistics, 2016). Consequently, the tax base is empirically concentrated in the safe sector even when entrepreneurs are taxed. Allowing entrepreneurs to pay taxes would simply augment B_S by their reported income and would leave the structure of the optimality conditions unchanged; only the magnitude of the fiscal pass–through from insurance benefit to tax rate would be affected. The assumption is relaxed in Section 3, where entrepreneurs may misreport taxable income. In that environment, the tax base becomes endogenous through the reporting decision r^*, and entrepreneurs contribute to financing catastrophe insurance through their declared income. The baseline specification should therefore be interpreted as a tractable benchmark that is fully consistent with the microfoundations developed in the extended model.

1. The government sets (τ_N, θ) subject to the budget constraint (1).

2. Each individual observes her ability a and chooses an occupation.

3. The aggregate state S and, if applicable, the idiosyncratic entrepreneurial success shock are realized.

4. Consumption and utilities are realized according to the policy in place.

Individuals take (τ_N, θ) as given when deciding whether to become entrepreneurs. Hence, equilibrium occupational choice and the ability cutoff a^* depend endogenously on policy. The fact that a rare-disaster risk changes the risk-profile of a choice is broadly consistent with the empirical calibrations of Gourio (2012).

The transfer θ should be interpreted broadly as a state–contingent social–insurance payment triggered by systemic shocks and conditioned on entrepreneurial status. It can take many real–world forms: for example, automatic grants to registered self–employed workers during pandemics, catastrophe–state business–interruption insurance, or pre–funded "entrepreneurial FDIC" programs financed by small levies in both states.

The crucial feature of θ is that it is *state–contingent and occupation–specific*. This distinguishes it from universal disaster relief, which gives identical transfers to all individuals in $S = C$ and therefore does not affect relative incentives between occupations, and from means–tested transfers, which insure only extreme poverty rather than systemic business risk. By calibrating θ optimally, the government can correct the asymmetric exposure of entrepreneurs to systemic shocks while maintaining incentives for entry.

2.5 Private choice and the cutoff

This subsection lays out (i) safe employment, (ii) entrepreneurship, (iii) the utility difference that pins down occupational choice, and (iv) the definition of the cutoff. Each step states the object first and then gives the intuition in words.

Policies are a tax on safe workers, $\tau_N \geq 0$, applied in both states, and a catastrophe–state transfer to entrepreneurs, $\theta \geq 0$. Let

$$x \equiv \tau_N$$

denote the (state-invariant) tax bite that appears in utilities.

A safe worker earns a fixed wage w_S that does not vary with the aggregate state. She pays the tax in both states, so consumption is identical across states and expected utility collapses to a single term:

$$U_S \;=\; u\big(w_S - x\big).$$

Safe jobs are insulated from systemic shocks (professors, civil servants, salaried corporate workers). Because income is guaranteed in both states, the systemic state does not enter U_S separately.

An entrepreneur with ability a faces idiosyncratic success risk in the normal state and complete shutdown in the catastrophe state. Expected utility is

$$U_E(a) \;=\; (1-\pi)\Big[p(a)\,u\big(R(a)\big) + \big(1-p(a)\big)\,u(0)\Big] \;+\; \pi\,u(\theta),$$

with $p'(a) \geq 0$, $R'(a) > 0$, and u strictly increasing and strictly concave. Ability raises both the chance and the payoff of success in normal times; in catastrophes, market income is wiped out but the policy θ can partially insure entrepreneurs.[5]

Define the normal–state entrepreneurship premium

$$G(a;x) \;\equiv\; p(a)\,u\big(R(a)\big) + \big(1-p(a)\big)\,u(0) \;-\; u\big(w_S - x\big).$$

The difference between entrepreneurship and safe work is

$$\begin{aligned}\Delta U(a;\theta,x) &\equiv U_E(a) - U_S \\ &=\; (1-\pi)\,G(a;x) \;+\; \pi\Big[u(\theta) - u\big(w_S - x\big)\Big]. \end{aligned} \tag{2}$$

The first term compares risky entrepreneurial utility in normal times with the certain safe wage net of the tax bite. The second term tilts the catastrophe state toward entrepreneurs: the safe worker keeps $w_S - x$ even in catastrophes, so only a *differential* transfer to entrepreneurs, $u(\theta) - u(w_S - x)$, changes relative incentives.

There exists a unique threshold ability $a^* = a^*(\theta, x)$ such that

$$\Delta U(a^*;\theta,x) = 0, \qquad m(\theta,x) \equiv 1 - F(a^*) \text{ is the mass of entrepreneurs.} \tag{3}$$

[5] Cagetti and De Nardi (2006) highlight how wealth, risk, and borrowing constraints shape entrepreneurial entry, reinforcing the role of heterogeneity in the occupational-choice margin studied in this paper.

Individuals with $a \geq a^*$ become entrepreneurs; those with $a < a^*$ choose safe employment. Because higher ability raises the normal–state premium $G(a; x)$, the utility difference crosses zero once: a single cutoff organizes occupational sorting.

There are two states of the world to consider when reading ΔU. In the normal state: $G(a; x)$ increases with a. A larger x lowers $u(w_S - x)$ and therefore raises G. In the catastrophe state: the safe worker consumes $w_S - x$; the entrepreneur receives θ. The relative tilt is $u(\theta) - u(w_S - x)$. Increasing θ shifts ΔU up in catastrophe only; increasing x shifts G up by lowering the safe benchmark in normal times, but must be financed (addressed in the Ramsey section).

Lemma 2.1 (Existence and single crossing)**.** *Assume u is strictly increasing and strictly concave, $p'(a) \geq 0$, $R'(a) > 0$, and $R(a)$ is positive in a neighborhood of the cutoff. Then for any (θ, x) there exists a unique $a^*(\theta, x)$ solving $\Delta U(a^*; \theta, x) = 0$. Moreover,*

$$\partial_a \Delta U(a; \theta, x) \;=\; (1-\pi)\, \partial_a G(a; x) \;=\; (1-\pi)\Big\{ p'(a)\big[u(R(a)) - u(0)\big] + p(a)\, u'(R(a))\, R'(a) \Big\} \;>\; 0$$

in a neighborhood of a^*.

Proof. Recall

$$\Delta U(a; \theta, x) = (1 - \pi)\, G(a; x) + \pi\, [u(\theta) - u(w_S - x)],$$

$$G(a; x) = p(a)\, u(R(a)) + \big(1 - p(a)\big)\, u(0) - u(w_S - x).$$

Assume throughout:

(A1) $u : \mathbb{R}_+ \to \mathbb{R}$ is C^2, strictly increasing and strictly concave; in particular $u'(c) > 0$ for $c > 0$.

(A2) $p, R : \mathbb{R}_+ \to \mathbb{R}_+$ are C^1 with $p'(a) \geq 0$, $R'(a) > 0$ for all a, and $0 \leq p(a) \leq 1$.

(A3) There exists an interval $\mathcal{N}$ containing the (a priori unknown) cutoff such that $R(a) > 0$ for all $a \in \mathcal{N}$.

(A4) There exist $a_L < a_H$ such that

$$G(a_L; x) < 0 \quad \text{and} \quad G(a_H; x) > 0.$$

A sufficient condition is: $p(0) = 0$ and $R(0) = 0$ so that $G(0; x) = u(0) - u(w_S - x) < 0$, and either $\lim_{a\to\infty} p(a) = 1$ with $\lim_{a\to\infty} R(a) = +\infty$ (so $u(R(a)) > u(w_S - x)$ eventually) or, more generally, there exists a_H with $p(a_H)u(R(a_H)) > u(w_S - x)$.

Assumptions (A1)–(A3) are standard. Assumption (A4) guarantees a sign change of G and hence of ΔU (since the catastrophe term is constant in a), which will ensure existence of a root.

Because u, p, and R are C^1, G is C^1 in a, and by the chain rule

$$\begin{aligned}\partial_a G(a; x) &= p'(a)\, u\big(R(a)\big) + p(a)\, u'\big(R(a)\big)\, R'(a) - 0 - p'(a)\, u(0) \\ &= p'(a)\, \Big[u\big(R(a)\big) - u(0)\Big] + p(a)\, u'\big(R(a)\big)\, R'(a).\end{aligned}$$

On the neighborhood $\mathcal{N}$, Assumption (A3) gives $R(a) > 0$, whence by (A1) I have $u(R(a)) > u(0)$ and $u'(R(a)) > 0$. Together with $p'(a) \geq 0$, $p(a) \geq 0$, and $R'(a) > 0$ from (A2), each summand is nonnegative; at least one is strictly positive for all $a \in \mathcal{N}$ (indeed $R'(a) > 0$ and either $p(a) > 0$ or $p'(a) > 0$ in any economically relevant region), so

$$\partial_a G(a; x) > 0 \quad \text{for all } a \in \mathcal{N}.$$

Since $\Delta U(a; \theta, x) = (1 - \pi)G(a; x) + \text{constant in } a$ with $1 - \pi > 0$, I obtain

$$\partial_a \Delta U(a; \theta, x) = (1 - \pi)\, \partial_a G(a; x) > 0 \quad \text{for all } a \in \mathcal{N}.$$

Hence $\Delta U(\cdot\,; \theta, x)$ is *strictly increasing* in a on $\mathcal{N}$.

By (A4), choose $a_L < a_H$ with $G(a_L; x) < 0 < G(a_H; x)$. Then

$$\Delta U(a_L; \theta, x) = (1 - \pi)\, G(a_L; x) + \pi\, [u(\theta) - u(w_S - x)],$$

$$\Delta U(a_H; \theta, x) = (1 - \pi)\, G(a_H; x) + \pi\, [u(\theta) - u(w_S - x)].$$

Because the catastrophe term $\pi[u(\theta) - u(w_S - x)]$ is the same at a_L and a_H, ΔU inherits the sign change from G:

$$\Delta U(a_L; \theta, x) < \Delta U(a_H; \theta, x).$$

Moreover, $\Delta U(\cdot\,; \theta, x)$ is continuous in a. Therefore, by the Intermediate Value Theorem, there exists at least one $a^* \in [a_L, a_H] \cap \mathcal{N}$ such that

$$\Delta U(a^*; \theta, x) = 0.$$

From Step 2, $\partial_a \Delta U(a;\theta,x) > 0$ for all $a \in \mathcal{N}$, so $\Delta U(\cdot\,;\theta,x)$ is strictly increasing on $\mathcal{N}$. A strictly increasing continuous function has at most one root. Hence the solution a^* to $\Delta U(a^*;\theta,x) = 0$ is unique within $\mathcal{N}$; since any root must lie in $\mathcal{N}$ by construction, uniqueness follows globally.

I have shown (i) $\partial_a \Delta U(a;\theta,x) = (1-\pi)\partial_a G(a;x) > 0$ in a neighborhood of the cutoff and (ii) the existence and uniqueness of $a^*(\theta,x)$ solving $\Delta U(a^*;\theta,x) = 0$. This establishes the lemma. □

The derivative decomposes into two strictly positive forces at the margin: (i) a higher success chance multiplies a positive utility gap between success and failure; (ii) a higher success payoff, weighted by the success probability, raises utility given concavity. Hence the utility difference is strictly increasing in ability, crosses zero exactly once, and the cutoff is well defined.

2.5.1 Comparative statics of the cutoff

I next characterize how a^* moves with the policy instruments.

Proposition 2.2 (Cutoff comparative statics)**.** *For any (θ,x) with interior cutoff a^*,*

$$\frac{\partial a^*}{\partial \theta} = -\frac{\pi\, u'(\theta)}{(1-\pi)\,\partial_a G(a^*;x)} \;<\; 0, \tag{4}$$

$$\frac{\partial a^*}{\partial x} = -\frac{\partial_x G(a^*;x)}{\partial_a G(a^*;x)}, \qquad \textit{where} \quad \partial_x G(a;x) \;=\; u'\big(w_S - x\big) - p(a)\,u'\big(R(a)\big). \tag{5}$$

Proof sketch. /The full proof is in the Appendix. Here a sketch follows. Apply the Implicit Function Theorem to $\Delta U(a^*;\theta,x) = 0$. I have $\partial_\theta \Delta U = \pi\, u'(\theta)$, $\partial_x \Delta U = (1-\pi)\,\partial_x G(a^*;x)$, and $\partial_a \Delta U = (1-\pi)\,\partial_a G(a^*;x) > 0$ by Lemma 2.1. Equations (4) and (5) follow immediately. □

A higher catastrophe–state transfer θ raises entrepreneurial utility only in $S = C$, leaving the safe job unchanged; therefore the indifference curve shifts up uniformly in a, and the cutoff falls. This is the precise sense in which *targeted* catastrophe insurance (occupation–specific, state–contingent) encourages entry. The proposition is broadly consistent with the empirical finding of Gottlieb, Townsend, and Xu (2022), who find that the introduction of women's safety nets in Canada increases female entrepreneurship.

Raising the tax on the safe sector (x) reduces $u(w_S - x)$ and thus raises $G(a; x)$; whether the cutoff rises or falls depends on the relative marginal utilities $u'(w_S - x)$ and $p(a)u'(R(a))$ at the margin. If at the cutoff the safe job is *less* marginally valuable than the risky return weighted by success probability (i.e., $p(a^*)u'(R(a^*)) > u'(w_S - x)$), then $\partial_x G(a^*; x) < 0$ and so $\partial a^*/\partial x > 0$: a higher safe–sector tax *raises* the cutoff, discouraging entry. This captures the financing trade–off that will enter the Ramsey problem.

It is important to connect the social insurance I propose for systemic entrepreneurial risk and existing social safety nets. Deryugina (2017) shows that natural disasters trigger large increases in general social safety-net spending, implying that households in developed countries receive substantial ex post insurance. These programs, however, are not designed to insure the occupational-income component of systemic entrepreneurial risk, nor do they target the sector-specific losses that motivate the fiscal externalities in this paper. The mechanism studied here concerns the ex ante design of state-contingent transfers that preserve entrepreneurial entry rather than broad post-disaster assistance. Consistent with this distinction, research on small-business disasters indicates that traditional safety-net programs provide limited protection to small firms, whose revenues and survival rates are highly sensitive to systemic shocks (Marshall, 2025, Basker and Miranda, 2018). A similar conclusion can be obtained by looking at the empirical results of Chetty et al. (2020): they document that small businesses were severely affected by the COVID-19 pandemic, and that the existing safety nets were insufficient to fully insure consumption.

I can consider existing program type by type. The first program to consider is unemployed insurance, which requires to be unemployed and actively looking for job and it is legally less relevant for the margin discussed in this paper. The second program I consider is universal relief. If in $S = C$ the government pays a per–capita amount b_C to *all* individuals, then $u(b_C)$ enters both U_E and U_S and cancels out of ΔU. Formally, setting $\theta = b_C$ for entrepreneurs and adding the same b_C for safe workers leaves (2) unchanged, hence a^* is unaffected (holding financing constant). Universal relief smooths consumption but does not change relative incentives across occupations. Programs that provide a minimum floor $\underline{b}$ (food stamps, housing vouchers) similarly do not differentially insure systemic entrepreneurial risk: they activate at low individual income rather than in the aggregate catastrophe state, and they apply across occupations. In our notation this corresponds to replacing $u(0)$ by $u(\underline{b})$ in both

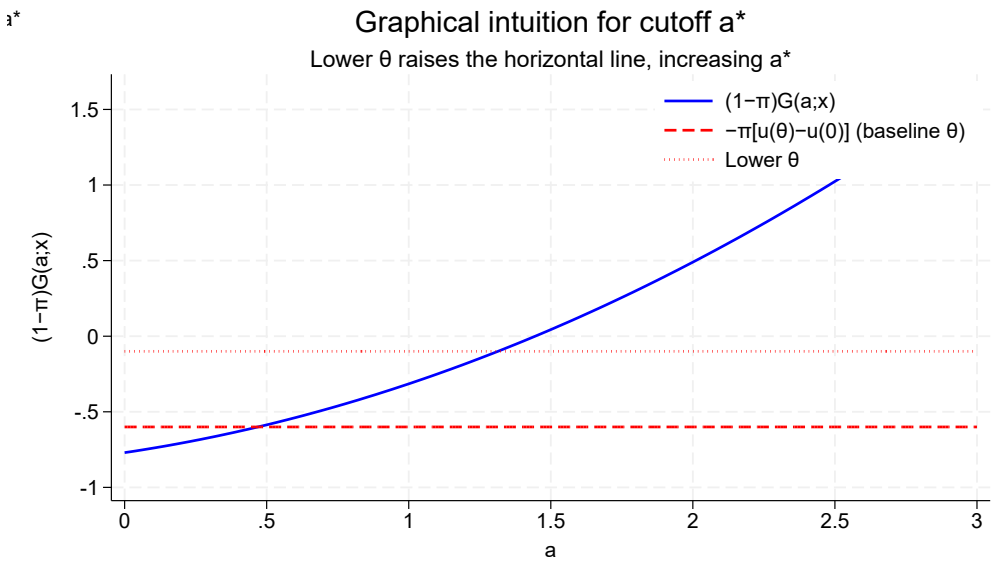


Figure 1: Graphical intuition for the cutoff a^*.

U_E and U_S, leaving ΔU and thus a^* unchanged.

2.5.2 Entry and its elasticity

Let $m(\theta, x) = 1 - F(a^*)$ denote the mass of entrepreneurs. Differentiating,

$$\frac{\partial m}{\partial \theta} \;=\; f(a^*)\,\frac{\pi\, u'(\theta)}{(1-\pi)\,\partial_a G(a^*;x)} \;>\; 0, \qquad \frac{\partial m}{\partial x} \;=\; f(a^*)\,\frac{\partial_x G(a^*;x)}{\partial_a G(a^*;x)}.$$

Define the (local) entry elasticity with respect to θ:

$$\eta_{m,\theta} \;\equiv\; \frac{\partial m}{\partial \theta}\cdot\frac{\theta}{m} \;=\; \frac{f(a^*)}{m}\,\frac{\pi\, u'(\theta)}{(1-\pi)\,\partial_a G(a^*;x)}\,\theta.$$

This elasticity will be the sufficient statistic that prices the extensive–margin benefit of insurance in the Ramsey condition below.

Figure 1 illustrates the comparative statics underlying equation (3). The blue curve plots the increasing function $(1-\pi)G(a;x)$, which represents the expected utility differential between entrepreneurship and the safe occupation as a function of entrepreneurial ability a. The horizontal red dashed line corresponds to the right-hand side of (3), $-\pi[u(\theta) - u(w_S - x)]$, for a baseline value of the safe-activity payoff θ. The intersection of the two curves identifies the equilibrium cutoff a^*, above which individuals choose entrepreneurship and below which they prefer the safe occupation.

The red dotted line corresponds to a lower value of θ, which raises the horizontal intercept because the utility gap $u(\theta) - u(w_S - x)$ shrinks. When the safe alternative becomes less attractive, the opportunity cost of entrepreneurship declines, and a larger set of individuals—those with lower ability a—are willing to enter. Consequently, the cutoff shifts from the baseline a^* to a higher $a^{*'}$. Conversely, an increase in θ would lower the red line and shift the intersection to the left, reducing entrepreneurial entry.

The parametrization underlying the figure is purely illustrative but economically meaningful. I set $\pi = 0.3$ to represent a 30% probability of failure in entrepreneurial activity and normalize utility so that $u(0) = 0.5$ and $u(\theta) = 2.5$ in the baseline. The function $G(a; x) = 0.4a - 1.1 + 0.25a^2$ produces an increasing and convex expected-return profile, consistent with the idea that higher-ability entrepreneurs face both larger potential rewards and greater variability. The baseline and alternative horizontal lines are placed at $-\pi[u(\theta) - u(w_S - x)]$ and $-\pi[u(\theta') - u(w_S - x)]$, respectively, where the latter corresponds to a smaller θ' (a less generous safe-sector income). These numerical choices are chosen solely to generate a clear visual separation between the two intersections; they preserve the qualitative comparative statics implied by the theory while keeping the scale interpretable.

This graphical device provides a transparent visualization of the planner's trade-off: policies that increase the attractiveness of the safe sector (higher θ) reduce the measure of active entrepreneurs, whereas conditions that make the safe option less rewarding or raise expected entrepreneurial returns expand entry. The same logic extends to shifts in x, which pivot the $G(\cdot; x)$ curve through changes in the safe-sector utility $u(w_S - x)$, as described analytically in (5).

The next subsection formulates the planner's Ramsey problem, uses the government budget constraint

$$\tau_N B_S = \pi \, \theta \, m(\theta, x)$$

to eliminate τ_N, and characterizes the optimal θ via a first–order condition that balances (i) catastrophe–state insurance gains and (ii) extensive–margin reallocation gains against (iii) financing costs.

2.6 Ramsey problem: design of catastrophe insurance

I now study the optimal policy for systemic entrepreneurial risk assuming that safe workers earn the same income in both states. The planner chooses a catastrophe–state, entrepreneur–targeted transfer $\theta \geq 0$, financed by a tax τ_N on safe workers applied in both states, in order to maximize expected welfare while respecting the government budget and the private occupational choice margin summarized by the cutoff $a^*(\theta, x)$, where $x \equiv \tau_N$.

Safe workers consume $w_S - \tau_N$ in both states. Entrepreneurs consume $R(a)$ in the normal state with probability $p(a)$ and 0 otherwise, and receive θ in the catastrophe state. The mass of entrepreneurs is $m(\theta, x) = 1 - F(a^*(\theta, x))$.

Expected welfare is

$$W(\theta, \tau_N) = (1-\pi)\Big[(1-m)\,u(w_S - \tau_N) + \int_{a^*}^{\infty} \big(p(a)u(R(a)) + (1-p(a))u(0)\big) f(a)\,da\Big] + \pi\Big[(1-m)\,u(w_S - \tau_N) + m\,u(\theta)\Big] + (1-\pi)\int_{a^*}^{\infty} p(a)\,\Delta(a)\,f(a)\,da - \lambda\,\tau_N B_S. \tag{6}$$

The first two brackets aggregate private utilities across states. The third term represents a social externality from successful entrepreneurs (e.g. aggregate innovation or fiscal spillovers). Formally, each successful entrepreneur with ability a generates a social benefit beyond private profits that increases linearly with ability above the entry cutoff:

$$\Delta(a) = \delta\,(a - a^*)_+, \qquad (x)_+ \equiv \max\{x, 0\}, \quad \delta > 0.$$

Hence, only entrepreneurs who enter ($a \geq a^*$) generate external spillovers, and higher–ability entrepreneurs yield proportionally larger benefits. This reduced–form representation captures the idea that the private return to entrepreneurship understates its social value due to agglomeration economies, knowledge diffusion, and local demand spillovers. Empirical evidence shows that high–productivity firms and skilled workers raise the productivity and wages of nearby firms and employees (Moretti, 2004; Glaeser and Kerr, 2010; Chatterji, Glaeser, and Kerr, 2014). I model these effects parsimoniously, in the spirit of a Pigouvian correction for positive externalities, rather than microfounding the full agglomeration process. The planner internalizes the positive externality associated with successful entrepreneurship, represented by $\Delta(a)$, which captures the gap between the social and private return. This

formulation parallels the treatment of pecuniary externalities in economies with financial frictions (Dávila and Korinek, 2018), where individual actions generate aggregate effects through price or credit channels.

The last term in (6) captures the welfare cost of raising revenue, parameterized by $\lambda \geq 0$. Here λ represents the *marginal cost of public funds* (MCPF): each unit of revenue raised through distortionary taxation entails an additional welfare loss of λ beyond the direct utility cost already reflected in $u(w_S - \tau_N)$. When $\lambda = 0$, taxation is lump–sum and costless; for $\lambda > 0$, the planner internalizes the deadweight loss from behavioral responses such as reduced labor supply, avoidance, or evasion. This reduced–form approach follows the standard practice in applied Ramsey models, incorporating fiscal inefficiencies through a constant MCPF rather than explicitly modeling individual labor–supply distortions (Feldstein, 1999 and Ballard, Fullerton, and Shoven, 1985). Conceptually, this term is analogous to the shadow price of revenue in the classic Ramsey rule (Ramsey, 1927; Atkinson and Stiglitz, 1976; Saez, 2001).[6]

Revenues are collected from safe workers in both states and spent on catastrophe insurance in the catastrophe state:

$$\tau_N B_S \;=\; \pi\, m(\theta, x)\, \theta, \qquad x = \tau_N.$$

The left-hand side is revenue (since safe workers pay in both states), while the right-hand side is expected expenditure on insurance payments to entrepreneurs in catastrophic times.

Define the Lagrangian

$$\mathcal{L}(\theta, \tau_N, \mu) = W(\theta, \tau_N) + \mu \Big[\tau_N B_S - \pi\, m(\theta, x)\, \theta\Big],$$

where μ is the multiplier on the balanced-budget constraint.

Differentiating W with respect to θ, taking into account that a^* and τ_N adjust endogenously, yields

$$\frac{dW}{d\theta} = \pi\, m\, u'(\theta) - (1-\pi)\, f(a^*)\, p(a^*)\, \Delta(a^*)\, \frac{\partial a^*}{\partial \theta} - \Big[\lambda B_S + (1-m)\, u'(w_S - \tau_N)\Big]\, \frac{d\tau_N}{d\theta}. \quad (7)$$

[6]For tractability, the model abstracts from explicit household labor–supply decisions and instead captures the efficiency cost of taxation in reduced form. I include only the additional cost $\lambda \tau_N B_S$ in the planner's objective, since the direct utility loss from taxes already appears inside $u(w_S - \tau_N)$. This ensures no double counting while preserving the comparative statics of the marginal cost of funds. The formulation is equivalent to a standard Ramsey problem with a Lagrange multiplier λ on the government budget constraint (e.g. Ballard, Fullerton, and Shoven, 1985).

From the indifference condition $\Delta U(a^*;\theta,x)=0$,

$$\frac{\partial a^*}{\partial \theta} = -\frac{\pi\, u'(\theta)}{(1-\pi)\,\partial_a G(a^*;x)} \;<\; 0.$$

Increasing θ makes entrepreneurship more attractive in catastrophes, inducing lower-ability individuals to enter (reducing a^*).

Assumption 2.3 (Nondegenerate entrepreneurship at zero insurance)**.** *At $(\theta,\tau_N)=(0,0)$ there exists $\bar{a}$ such that*

$$(1-\pi)\Big[p(\bar{a})u(R(\bar{a}))+(1-p(\bar{a}))u(0)-u(w_S)\Big] \;\geq\; \pi\big[u(w_S)-u(0)\big],$$

so that a positive measure of individuals optimally chooses entrepreneurship and $m(0,0)>0$.

Assumption 2.3 rules out the degenerate case in which no one would ever become an entrepreneur even without catastrophe insurance. It is satisfied, for example, if sufficiently high-ability types have very high success probability and/or payoff in normal times.

Remark 2.4 (Strict insurance motive at zero insurance)**.** *Under Assumption 2.3, at $(\theta,\tau_N)=(0,0)$ a marginal increase in θ strictly increases expected utility for the set of existing entrepreneurs, because they receive zero consumption in the catastrophe state and do not finance the transfer in either state. In particular, holding τ_N fixed,*

$$\left.\frac{\partial W}{\partial \theta}\right|_{(\theta,\tau_N)=(0,0)} = \pi\, m(0,0)\, u'(0) \;>\; 0.$$

The planner will not choose $\theta=0$ unless the marginal cost of funds at $\theta=0$ is arbitrarily large; in what follows I focus on interior solutions with $\theta>0$.

Differentiating the budget constraint $\tau_N B_S = \pi m \theta$ with respect to θ gives

$$B_S\,\frac{d\tau_N}{d\theta} = \pi\Big[m+\theta\frac{\partial m}{\partial \theta}+\theta\frac{\partial m}{\partial x}\frac{dx}{d\theta}\Big], \qquad x=\tau_N.$$

This expression is unchanged algebraically except for the removal of the factor $(1-\pi)$, since safe workers pay the tax in both states.

At an interior optimum $\theta^\star>0$, the derivative (7) must vanish, implying

$$\boxed{\pi\, m\, u'(\theta) \;+\; (1-\pi)\, f(a^*)\, p(a^*)\, \Delta(a^*)\left|\tfrac{\partial a^*}{\partial \theta}\right| \;=\; \Big[\lambda B_S + (1-m)u'(w_S-\tau_N)\Big]\,\frac{d\tau_N}{d\theta}.} \qquad (8)$$

Relative to the baseline specification, the right-hand side now represents the utility cost of higher taxation on safe workers (who experience lower consumption in both states) plus any additional efficiency cost λ associated with public financing.

The first term on the left-hand side represents the marginal insurance value of raising θ for existing entrepreneurs in the catastrophe state. The second term captures the entry externality from newly induced entrepreneurs in the normal state, weighted by the probability that they succeed and the externality value $\Delta(a^*)$. The right-hand side gives the social cost of financing this transfer: every marginal increase in θ requires higher taxation on safe workers whose after-tax income is state-invariant, thus depressing their utility in both states. Consequently, with truly safe income, the marginal cost of redistribution rises and the optimal level of catastrophe insurance $\theta^\star$ is (weakly) lower than in the baseline model where safe income vanished in catastrophes.

I now characterize when the optimal policy entails *full* catastrophe insurance for entrepreneurs versus *partial* insurance, derive the balanced–budget optimality condition in sufficient–statistics form, and state comparative statics with respect to primitives. Throughout, (θ, τ_N) are chosen to maximize the planner's objective (6) subject to the feasibility constraint $\tau_N B_S = \pi m(\theta, x)\theta$ and the behavioral condition $\Delta U(a^*; \theta, x) = 0$, with $x = \tau_N$ and $m(\theta, x) = 1 - F(a^*)$.

2.6.1 Full vs. partial catastrophe insurance

I define *full catastrophe insurance for entrepreneurs* as the policy that equalizes marginal utility between entrepreneurs and safe workers across states. Since safe workers now consume $w_S - \tau_N$ in both states, the benchmark marginal utility is $u'(w_S - \tau_N)$. Full insurance therefore corresponds to

$$u'(\theta^{\text{full}}) \simeq u'(w_S - \tau_N),$$

that is, the entrepreneur's consumption in the catastrophe state delivers the same marginal utility as the safe worker's secure income.

Proposition 2.5 (Full vs. partial insurance)**.** *Let θ^{full} be the full–insurance candidate defined above, and let (a^*, m, τ_N) denote the associated equilibrium objects. Suppose Assumption 2.3 holds and the marginal cost of funds is finite at $\theta = 0$. Then the planner's problem admits an*

interior solution $\theta^\star > 0$. *Evaluate the Ramsey first–order condition* (8) *at* $\theta = \theta^{\text{full}}$. *Then:*

$$\textit{If} \quad \underbrace{\pi\, m\, u'(\theta^{\text{full}})}_{\textit{catastrophe insurance gain}} + \underbrace{(1-\pi)\, f(a^*)\, p(a^*)\, \Delta(a^*) \left|\tfrac{\partial a^*}{\partial \theta}\right|_{\theta^{\text{full}}}}_{\textit{entry externality gain}} \geq \underbrace{\tfrac{d\tau_N}{d\theta}\Big|_{\theta^{\text{full}}} \left[\lambda B_S + (1-m)\, u'(w_S - \tau_N)\right]}_{\textit{financing cost with safe income}}, \tag{9}$$

then full insurance is (weakly) optimal: $\theta^\star \geq \theta^{\text{full}}$.

If condition (9) *is strict in the opposite direction, then the optimum is partial:* $0 < \theta^\star < \theta^{\text{full}}$.[7]

Proof sketch. The full proof is in the Appendix. Here a sketch follows. At θ^{full}, evaluate $dW/d\theta$ using (8). If the left–hand side weakly exceeds the right–hand side, then $dW/d\theta\big|_{\theta^{\text{full}}} \geq 0$, implying that any $\theta \leq \theta^{\text{full}}$ is weakly dominated and full insurance is (weakly) optimal. If the inequality reverses, $dW/d\theta < 0$ at θ^{full}, so the maximizer lies strictly below θ^{full}. Concavity follows from standard second–order conditions (strict concavity of u, single crossing, and a broad base ensuring small fiscal feedback). □

Full insurance is favored by (i) a high catastrophe probability π, (ii) many entrepreneurs m, (iii) high marginal utility of catastrophe consumption $u'(\theta^{\text{full}})$ (strong risk aversion or low θ^{full}), and (iv) large social spillovers $p(a^*)\Delta(a^*)$ and responsive entry $\left|\partial a^*/\partial\theta\right|$. It is opposed by (v) limited fiscal capacity (large λ or small B_S) and (vi) high marginal utility for safe workers $u'(w_S - \tau_N)$, whose income is secure in both states. Relative to the baseline model without safe income, the financing term on the right–hand side is larger, implying that full insurance is harder to attain and the optimal $\theta^\star$ is (weakly) smaller.

2.6.2 Balanced–budget sufficient–statistics rule

Proposition 2.6. *Suppose the government levies a tax* τ_N *on a broad base* B_S *and, in the disaster state (probability* π*), pays an insurance transfer* θ *only to entrepreneurs, whose*

[7] The fact that assumption 2.3 holds and that the marginal cost of funds is finite at $\theta = 0$ rules out zero-insurance as optimal. Please note this conclusion holds even if entrepreneurs finance their own catastrophe insurance, with mild assumptions (concavity). If entrepreneurs were to finance catastrophe insurance entirely from their own normal-state income, the usual concavity-insurance logic would rule out $\theta = 0$ given risk aversion: a small, actuarially fair increase in θ raises expected utility by concavity of $u(\cdot)$. In the baseline formulation of this model, entrepreneurs do not finance the transfer directly, so the planner's insurance motive is even stronger. Therefore, the assumption that entrepreneurs do not finance their own catastrophe insurance in this part of the model is not driving this conclusion.

measure is $m(\theta) = \int_{a^*(\theta)}^{\infty} f(a)\, da$. *The government budget is*

$$\tau_N B_S \;=\; \pi\, m(\theta)\, \theta.$$

Let the entrepreneurship cutoff $a^*(\theta)$ *be defined implicitly by*

$$(1-\pi)\, G(a^*; x) \;+\; \pi\, [u(\theta) - u(w_S - x)] \;=\; 0, \tag{10}$$

and let $m(\theta) = 1 - F(a^*(\theta))$ *with density* $f(\cdot)$. *Define the elasticity*

$$\eta_{m,\theta}(\theta) \;\equiv\; \frac{\theta}{m(\theta)}\, \frac{\partial m(\theta)}{\partial \theta}.$$

If the tax base B_S *is broad so that the feedback term* $(\partial m/\partial x)\,(dx/d\theta)$ *can be neglected, then the optimal* $\theta^\star$ *approximately satisfies*[8]

$$\boxed{\frac{\pi\, m(\theta^\star)\, u'(\theta^\star)}{1-\pi} \;+\; \frac{m(\theta^\star)}{B_S}\, p(a^\star)\, \Delta(a^\star)\, \frac{\eta_{m,\theta}(\theta^\star)}{\theta^\star / B_S} \;\approx\; \Big[\lambda B_S + (1 - m(\theta^\star))\, u'(w_S - \tau_N)\Big]\, \frac{d\tau_N}{d\theta}\bigg|_{\theta^\star}.} \tag{11}$$

Moreover, differentiating the balanced-budget constraint

$$\tau_N B_S = \pi\, m(\theta)\, \theta$$

yields the financing pass–through

$$\frac{d\tau_N}{d\theta}\bigg|_{\theta^\star} \;\approx\; \frac{\pi}{B_S}\Big[m(\theta^\star) \;+\; \theta^\star\, \frac{\partial m}{\partial \theta}(\theta^\star)\Big] \;=\; \frac{\pi}{B_S}\, m(\theta^\star)\Big[1 + \eta_{m,\theta}(\theta^\star)\Big]. \tag{12}$$

[8] A "broad" tax base means that the mass and taxable income of safe–sector workers is large relative to the number of entrepreneurs. Equivalently, financing a marginal increase in θ requires only a very small increase in the tax τ_N, so the effect of taxation on the occupational cutoff is negligible. Hence the feedback term $(\partial m/\partial x)\,(dx/d\theta)$ can be ignored.

To see this formally, write $x = \tau_N$ and differentiate the budget constraint

$$\tau_N B_S = \pi\, m(\theta)\theta.$$

Total differentiation gives

$$B_S\, \frac{d\tau_N}{d\theta} = \pi\Big[m(\theta) + \theta \frac{\partial m}{\partial \theta} + \theta \frac{\partial m}{\partial x}\frac{dx}{d\theta}\Big], \qquad \frac{dx}{d\theta} = \frac{d\tau_N}{d\theta}.$$

If the entrepreneurial mass $m(\theta)$ and its derivatives are bounded on the relevant policy domain, the bracketed expression on the right–hand side is also bounded. Thus $\left|d\tau_N/d\theta\right|$ must be no larger than a constant times $1/B_S$. When B_S is large (a "broad" base), this implies that both $d\tau_N/d\theta$ and $dx/d\theta$ are very small, so the induced change in the entry margin—captured by $\theta(\partial m/\partial x)(dx/d\theta)$—is negligible in first order. Intuitively, with many safe–sector taxpayers, the tax rate barely moves when θ changes, and therefore fiscal adjustments do not meaningfully affect occupational choice.

Proof sketch. The full proof is in the Appendix. Here a sketch follows. Start from the FOC (8). Use $\partial a^*/\partial\theta$ from Lemma 2.1, define $\eta_{m,\theta}$, and neglect the small feedback term $\partial m/\partial x \cdot dx/d\theta$ (broad base). This gives (11). Differentiating the budget constraint $\tau_N B_S = \pi\, m(\theta)\theta$ and substituting $dm = (\partial m/\partial\theta)\, d\theta$ yields (12). □

The left-hand side of (11) summarizes the benefits of raising θ: the first term measures direct insurance to existing entrepreneurs, while the second captures the social value of new entrants induced by greater insurance. The right-hand side summarizes the financing cost: the utility loss of safe workers who pay higher taxes in both states and the additional efficiency cost λ associated with public funds. The rule highlights the key trade-off: social insurance for systemic risk is desirable when the insurance and entry gains outweigh the combined fiscal and efficiency costs.

Reading (11)–(12)**.** The left side combines (i) the *inframarginal* insurance value for existing entrepreneurs (first term) and (ii) the *extensive–margin* surplus generated by additional entrepreneurial entry, scaled by its elasticity and the fiscal base (second term). The right side represents the *total financing cost* of raising θ, now felt in both states because safe workers always consume their taxed income. This cost increases with the marginal cost of public funds λ and with the safe workers' marginal utility $u'(w_S - \tau_N)$, making systemic insurance more fiscally expensive when fiscal capacity is limited.

Corollary 2.7 (Comparative statics in systemic risk)**.** *Suppose u is CRRA and the tax base B_S is broad. Then, under the regularity conditions used above,*

$$\frac{d\theta^\star}{d\pi} > 0, \qquad \left.\frac{dm}{d\pi}\right|_{\theta=0} < 0, \qquad \left.\frac{dm}{d\pi}\right|_{\theta=\theta^\star} \textit{is weakly larger (less negative).}$$

Proof Sketch. A higher π raises the welfare weight on catastrophe utilities, increasing the marginal benefit of insurance and therefore $d\theta^\star/d\pi > 0$. Without targeted insurance ($\theta = 0$), a higher catastrophe probability lowers expected entrepreneurial income and increases the required tax, pushing a^* upward (so $dm/d\pi < 0$). At the optimum, the increase in $\theta^\star$ partially offsets this negative effect. □

Corollary 2.8 (Fiscal capacity and optimal insurance)**.** *Holding other primitives fixed and under a broad base,*

$$\frac{d\theta^\star}{d\lambda} < 0, \qquad \frac{d\theta^\star}{dB_S} > 0.$$

Proof Sketch. From (8)–(12), a higher λ raises the marginal cost of public funds, increasing the right–hand side of the first–order condition and requiring a lower $\theta^\star$ for equality. A larger fiscal base B_S spreads the financing burden over more taxpayers, lowering the cost of raising each unit of revenue and permitting a higher $\theta^\star$. □

Corollary 2.9 (Spillovers and entry responsiveness)**.** *Let $\Delta(a)$ increase (pointwise) or let the entry elasticity $\eta_{m,\theta}$ increase locally around the optimum. Then*

$$\frac{\partial \theta^\star}{\partial \Delta(a^*)} > 0, \qquad \frac{\partial \theta^\star}{\partial \eta_{m,\theta}} > 0.$$

Sketch. Both $\Delta(a^*)$ and $\eta_{m,\theta}$ appear multiplicatively on the left side of (11), scaling the extensive–margin benefit of higher insurance. An increase in either variable therefore requires a higher $\theta^\star$ to restore equality with the (unchanged) right–hand side. □

The optimal catastrophe insurance $\theta^\star$ rises with the salience of systemic risk (π), entrepreneurial spillovers $\Delta(a^*)$, and entry responsiveness $\eta_{m,\theta}$, and falls with the marginal cost of funds λ. It also increases with fiscal capacity B_S. Because safe workers now retain income in both states, the effective financing burden is larger, implying a slightly smaller $\theta^\star$ relative to the baseline model without safe income. Nonetheless, the qualitative comparative statics and the logic of targeted systemic insurance remain unchanged.

3 Extension: Tax evasion, Allingham–Sandmo, and report–based catastrophe insurance

This section augments the baseline entrepreneurship environment with *normal–state tax evasion* in the spirit of Allingham and Sandmo (1972). I show that when catastrophe–state benefits are *report–based*—that is, depend on reported normal–state income rather than true income—the planner's catastrophe insurance θ becomes a complement to tax enforcement: it raises optimal reporting and can eliminate evasion under a simple condition. By contrast, *universal* catastrophe aid (independent of reports) does not affect reporting incentives. Throughout this section I focus on evasion by entrepreneurs and, for simplicity, abstract from tax evasion by safe workers. This assumption is without loss for the mechanisms of interest. In practice, the overwhelming majority of taxable labor income for safe workers takes the

form of third–party–reported W–2 earnings, for which evasion is negligible. By contrast, entrepreneurial income is self–reported and is empirically the category in which evasion is concentrated (Pissarides and Weber, 1989).[9]

Consider an entrepreneur in the normal state $S = N$ with *true* income $y \geq 0$ (e.g., $y = R(a)$ upon success and $y = 0$ upon failure).[10] The entrepreneur chooses a *report* $r \in [0, y]$ to the tax authority.

Let $T(r)$ be the tax liability as a function of the report, with $T'(r) > 0$ and $T''(r) \geq 0$. With probability $1 - q$ there is no audit and tax due is $T(r)$; with probability $q \in (0, 1]$ there is an audit, in which case the authority reconstructs the true base y, assesses tax $T(y)$, and imposes a penalty $\phi(y-r)$ on the unreported base, where $\phi(0) = 0$, $\phi'(z) > 0$, and $\phi''(z) \geq 0$. Normal–state consumption is therefore

$$c_N(r) = y - T(r) \quad \text{(no audit)}, \qquad \tilde{c}_N(r) = y - T(y) - \phi(y - r) \quad \text{(audit)}.$$

Let the catastrophe–state benefit be *report–based*: $b_C(r)$, with $b_C'(r) \geq 0$ and $b_C''(r) \leq 0$. This nests the universal–aid case as $b_C(r) \equiv \bar{b}$ (constant), and the baseline θ–policy as $b_C(r) \equiv \theta$ for eligible entrepreneurs, independent of r. Catastrophe consumption is $c_C(r) = b_C(r)$ for entrepreneurs, while safe workers consume their stable income $w_S - \tau_N$ in both states.

An entrepreneur's expected utility given true income y and report r is

$$U(r; y) = (1 - \pi)\Big[(1 - q)\, u\big(c_N(r)\big) \;+\; q\, u\big(\tilde{c}_N(r)\big)\Big] \;+\; \pi\, u\big(b_C(r)\big). \tag{13}$$

3.1 Allingham–Sandmo first–order condition with report–based insurance

Lemma 3.1 (FOC for optimal reporting)**.** *If the optimal report is interior, $r^* \in (0, y)$, it satisfies*

$$(1-\pi)\Big[-(1-q)\, T'(r^*)\, u'\big(c_N(r^*)\big) \;+\; q\, \phi'(y-r^*)\, u'\big(\tilde{c}_N(r^*)\big)\Big] \;+\; \pi\, b_C'(r^*)\, u'\big(b_C(r^*)\big) \;=\; 0. \tag{14}$$

[9] Allowing safe workers to misreport would add a symmetric, linear term to the government budget constraint without affecting the cutoff condition for entrepreneurial entry or the structure of the Ramsey problem. What matters for the results below is that (i) the safe tax base is stable across aggregate states, and (ii) evasion responds to the incentives created by the catastrophe–state transfer, which only applies to entrepreneurs. Thus the simplified treatment used here is a transparent and without–loss representation of the general environment.

[10] For clarity, I analyze the reporting decision holding y fixed; the occupational choice analysis from the baseline continues to apply on top of the reporting margin.

Proof. Differentiate (13) with respect to r, using $c_N(r) = y - T(r)$ and $\tilde{c}_N(r) = y - T(y) - \phi(y - r)$. We have

$$c'_N(r) = -T'(r), \qquad \tilde{c}'_N(r) = \phi'(y - r),$$

since y and $T(y)$ do not depend on r.

Thus

$$\frac{\partial U(r;y)}{\partial r} = (1-\pi)\Big[(1-q)\,u'\big(c_N(r)\big)\,c'_N(r) \;+\; q\,u'\big(\tilde{c}_N(r)\big)\,\tilde{c}'_N(r)\Big] \;+\; \pi\,b'_C(r)\,u'\big(b_C(r)\big),$$

that is,

$$\frac{\partial U(r;y)}{\partial r} = (1-\pi)\Big[(1-q)\,u'\big(c_N(r)\big)\,(-T'(r)) \;+\; q\,u'\big(\tilde{c}_N(r)\big)\,\phi'(y-r)\Big] \;+\; \pi\,b'_C(r)\,u'\big(b_C(r)\big).$$

The first–order condition for an interior optimum r^* is $\partial U(r^*;y)/\partial r = 0$, which yields (14). Concavity of u and convexity of T and ϕ ensure a unique optimum. □

Equation (14) extends the standard Allingham–Sandmo condition with an *insurance term* $\pi b'_C(r)\,u'(b_C(r))$. The bracketed term is the usual evasion trade–off: the marginal tax saving from under–reporting (proportional to $T'(r)$) versus the expected marginal penalty when audited (proportional to $\phi'(y-r)$). The last term says that higher reported income raises catastrophe benefits, increasing expected utility in $S = C$. If $b'_C(r) \equiv 0$ (universal aid), the last term vanishes and I recover the classic Allingham–Sandmo condition.

3.2 Insurance–enforcement complementarity and truthful reporting

Proposition 3.2 (Insurance–enforcement complementarity)**.** *Suppose $b_C(r) = \theta\,\tilde{b}(r)$ with $\theta \geq 0$ and $\tilde{b}'(r) > 0$. Under the assumptions above, the optimal report is increasing in the generosity of report–based insurance:*

$$\frac{\partial r^*}{\partial \theta} \;>\; 0.$$

Proof Sketch. The full proof is in the Appendix. Here a sketch follows. Differentiate (14) with respect to θ at $r^*(\theta)$ and apply the Implicit Function Theorem. The derivative of the FOC with respect to θ is $\pi\,\tilde{b}'(r^*)\,u'\big(b_C(r^*)\big) > 0$. Since $U''(r^*) < 0$ by concavity, it follows that $dr^*/d\theta > 0$. □

Catastrophe insurance that depends on reported income rewards truthful behavior: a higher θ strengthens the incentive to report, easing enforcement without raising audits or penalties. Because safe workers' consumption is stable across states, the marginal social value of increased reporting stems entirely from entrepreneurs' state–contingent utility.

Proposition 3.3 (Truthful reporting under (approximate) full insurance)**.** *Suppose the planner chooses transfers so that entrepreneurs are (approximately) fully insured across the normal and catastrophe states, in the sense that their marginal utilities are comparable at $r = y$:*

$$u'\big(b_C(y)\big) \;\simeq\; u'\big(w_S - \tau_N\big).$$

Assume additionally that the marginal penalty at zero evasion is zero, $\phi'(0) = 0$, so that a marginal deviation from truthful reporting affects normal–state consumption only through $T'(y)$.

If at $r = y$ the following condition holds,

$$\boxed{\pi\, b_C'(y)\, u'\big(b_C(y)\big) \;\geq\; (1-\pi)(1-q)\, T'(y)\, u'\big(c_N(y)\big),} \tag{15}$$

then truthful reporting $r^* = y$ is optimal for the entrepreneur. If the inequality is strict, $r^* = y$ is the unique maximizer of $U(r; y)$.

The proof is in the Appendix.

At the truth $r = y$, shaving a dollar saves tax in the *no–audit* branch but reduces catastrophe benefits by $b_C'(y)$. Under near–full insurance, marginal utilities across states are approximately equalized, so (15) directly compares tax saved versus insurance lost. When insurance lost dominates, truthful reporting is optimal. This mechanism is absent under universal aid ($b_C'(r) = 0$), where evasion persists unless enforcement is strong.[11]

3.3 Integration with the Ramsey problem

In the full Ramsey problem, the planner chooses both the level θ and, if administratively feasible, the mapping $r \mapsto b_C(r)$, subject to the government budget and private margins

[11] A related empirical mechanism appears in Casaburi and Troiano (2016), who show that improvements in tax enforcement not only raise compliance but also expand local public–good provision, reinforcing the idea that enforcement and expenditure policies are jointly determined and that the fiscal effects of reporting incentives extend beyond private consumption to the provision of public goods.

(entry a^* and reporting r^*). The Ramsey first–order condition (8) gains an additional term:

$$+\,\pi\,\mathbb{E}\left[b_C'(r^*)\,u'(b_C(r^*))\,\frac{\partial r^*}{\partial \theta}\right],$$

representing the *enforcement dividend* from improved reporting. This term raises the marginal benefit of catastrophe insurance but only partly offsets the higher financing cost borne by safe workers in both states. Hence, report–based insurance remains a complement to enforcement, though the overall $\theta^\star$ may be slightly smaller in equilibrium than in the baseline model without safe income.

Report–based catastrophe insurance continues to perform double duty: it *insures* systemic entrepreneurial risk (improving entry) and *disciplines* evasion by rewarding truthful reporting. This complementarity persists, but the fiscal cost of full insurance is higher because taxes reduce safe workers' welfare in both states and, if $\lambda > 0$, entail an additional efficiency cost of raising public funds.

4 Conclusion

This paper has developed a framework to analyze how public finance should respond to systemic risk. When economies face catastrophes such as wars, pandemics, or climate-related disasters, the risk is not idiosyncratic but collective: entire groups of workers and firms lose their income at once. Yet current welfare systems and tax structures are designed for the opposite case, insuring individual misfortune rather than correlated occupational shocks. The result, visible during both wartime and the COVID-19 pandemic, is that bureaucrats and salaried employees remain protected, while entrepreneurs and the self-employed bear a disproportionate share of the loss. Over time, this asymmetry discourages risk-taking and reallocates talent away from entrepreneurship.

The model presented here provides a theoretical foundation for addressing this imbalance. Individuals choose between a safe occupation and an entrepreneurial one. Entrepreneurs face both idiosyncratic risk and an additional, low-probability systemic shock that eliminates income entirely. The government can provide targeted catastrophe insurance to entrepreneurs, financed by taxation of safe workers. The planner's problem is to determine how much insurance to provide, balancing the welfare gain from insurance and from additional entrepreneurial entry against the fiscal cost of raising revenue. The model shows that the opti-

mal insurance policy is state-contingent and occupation-specific. Full insurance—equalizing consumption across states for entrepreneurs—is efficient when systemic risk is large, fiscal capacity is broad, and entrepreneurial spillovers are significant. Otherwise, the planner provides partial insurance, limited by fiscal distortions and the marginal utility of safe workers.

The framework also clarifies why universal transfers or in-kind benefits, though politically popular, do not substitute for targeted systemic insurance. Universal payments smooth consumption but leave occupational incentives unchanged, while means-tested or in-kind programs protect against poverty rather than risk-taking. In contrast, a well-designed catastrophe insurance system directly compensates those occupations most exposed to systemic shocks, maintaining ex ante incentives for entrepreneurship and innovation.

The paper further extends the analysis to tax compliance. Incorporating an Allingham–Sandmo type evasion decision, it shows that when catastrophe insurance depends on reported income, truthful reporting becomes more attractive. The expected loss of insurance benefits in the catastrophic state acts as an implicit penalty for evasion. Consequently, catastrophe insurance and tax enforcement are complements: generous, report-based insurance not only insures systemic risk but also raises compliance, reducing the need for audits and punitive penalties. This insight links fiscal capacity and social insurance in a new way, suggesting that systems designed to share risks fairly can also strengthen the integrity of the tax base.

Extending my framework to incorporate *politically motivated social planners* is an exciting direction for new research. In reality, governments do not maximize utilitarian welfare but respond to political incentives, lobbying, and electoral pressures. Politicians may over-insure groups that are politically powerful or geographically concentrated, while under-insuring others with less representation. Extending the model to a setting where policy choices are made by self-interested or vote-seeking governments would shed light on why catastrophe assistance often favors incumbents or formal sectors. It would also connect the theory of optimal systemic insurance to the broader literature on political economy and distributive politics.

A second direction for future research concerns *intertemporal moral hazard and repeated catastrophes*. In practice, societies face a sequence of shocks—wars, pandemics, climate events—that occur unpredictably but not independently. If individuals anticipate govern-

ment bailouts, they may take excessive systemic exposure or underinsure privately. Future work could endogenize these expectations and study optimal policy over time, allowing the planner to commit or not commit to future bailouts. This extension would merge the present framework with the dynamic moral hazard models of fiscal policy and banking crises, providing a richer view of how reputation and credibility shape systemic insurance.

Finally, I abstracted from *heterogeneous fiscal capacity and international coordination*. Systemic shocks are rarely confined to national borders, and the ability to finance catastrophe insurance varies across countries. Extending the analysis to an international environment where richer states can borrow cheaply while poorer states face tight fiscal constraints would illuminate the global distributional consequences of systemic risk. Such an extension would require embedding the model in an open-economy framework with cross-border externalities and possibly a supranational insurance authority. This could help explain the uneven recovery from COVID-19 and the asymmetric fiscal responses to climate disasters.

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Appendix

Proof of Proposition 2.2

Proof. Under the assumption that safe workers consume $w_S - \tau_N$ in both states, the planner's objective is

$$W(\theta, \tau_N) = (1-\pi)\int_{a\geq a^*}\Big[p(a)\,u(R(a)) + (1-p(a))\,u(0)\Big]\,dF(a) + (1-\pi)(1-m)\,u(w_S-\tau_N) + \pi\Big[(1-m)\,u(w_S-\tau_N) + m\,u(\theta)\Big] + (1-\pi)\int_{a\geq a^*} p(a)\,\Delta(a)\,dF(a) - \lambda\,(1-\pi)\,\tau_N B_S. \tag{16}$$

Here $m = 1-F(a^*)$, and $a^*(\theta, x)$ satisfies $\Delta U(a^*; \theta, x) = 0$ with $x = (1-\pi)\tau_N$. For shorthand, define the normal–state entrepreneurial utility

$$I_N(a) \equiv p(a)\,u(R(a)) + (1-p(a))\,u(0).$$

I treat $\theta \mapsto a^*(\theta, x(\theta))$ and $\theta \mapsto \tau_N(\theta)$ as endogenous and use

$$dm = -f(a^*)\,da^*, \qquad \frac{\partial a^*}{\partial \theta} = -\frac{\pi\,u'(\theta)}{(1-\pi)\,\partial_a G(a^*; x)} < 0.$$

(i) Catastrophe term.

$$\frac{d}{d\theta}\big[\pi((1-m)\,u(w_S-\tau_N) + m\,u(\theta))\big] = \pi\Big[-(\partial m/\partial\theta)\,u(w_S-\tau_N) + (\partial m/\partial\theta)\,u(\theta) + m\,u'(\theta)\Big] = \pi\Big[m\,u'(\theta) + \big(u(\theta) - u(w_S-\tau_N)\big)\frac{\partial m}{\partial\theta}\Big].$$

(ii) Normal-state private utilities. Consider

$$(1-\pi)\Big[(1-m)u(w_S-\tau_N) + \int_{a^*}^{\infty} I_N(a)\,f(a)\,da\Big],$$

where $I_N(a) \equiv p(a)u(R(a)) + (1-p(a))u(0)$. Differentiating with respect to θ yields

$$\frac{d}{d\theta}\Big[(1-\pi)(1-m)u(w_S-\tau_N)\Big] = (1-\pi)\Big[-\frac{\partial m}{\partial\theta}u(w_S-\tau_N) - (1-m)u'(w_S-\tau_N)\frac{d\tau_N}{d\theta}\Big], \tag{17}$$

$$\frac{d}{d\theta}\Big[(1-\pi)\int_{a^*}^{\infty} I_N(a)\,f(a)\,da\Big] = -(1-\pi)f(a^*)I_N(a^*)\frac{\partial a^*}{\partial\theta}. \tag{18}$$

Using $\partial m/\partial\theta = -f(a^*)\,\partial a^*/\partial\theta$ and collecting terms,

$$\text{(ii)} = (1-\pi)\Big\{ f(a^*)\frac{\partial a^*}{\partial\theta}\big[u(w_S-\tau_N) - I_N(a^*)\big] - (1-m)u'(w_S-\tau_N)\frac{d\tau_N}{d\theta}\Big\}. \tag{19}$$

Note that

$$G(a;x) \equiv I_N(a) - u(w_S - x), \quad \text{so} \quad u(w_S-\tau_N) - I_N(a^*) = -G(a^*;x).$$

At the cutoff a^* I also have the indifference condition

$$\Delta U(a^*;\theta,x) = (1-\pi)G(a^*;x) + \pi\big[u(\theta) - u(w_S-\tau_N)\big] = 0.$$

Hence

$$u(\theta) - u(w_S-\tau_N) = -\frac{1-\pi}{\pi}G(a^*;x).$$

Now combine the terms in (i) and (19) that involve $\partial a^*/\partial\theta$ and $\partial m/\partial\theta$. The cross term from the catastrophe component is

$$\pi\big[u(\theta) - u(w_S-\tau_N)\big]\frac{\partial m}{\partial\theta} = -\pi f(a^*)\big[u(\theta) - u(w_S-\tau_N)\big]\frac{\partial a^*}{\partial\theta},$$

and the corresponding term from (ii) is

$$(1-\pi)f(a^*)\frac{\partial a^*}{\partial\theta}\big[u(w_S-\tau_N) - I_N(a^*)\big] = -(1-\pi)f(a^*)G(a^*;x)\frac{\partial a^*}{\partial\theta}.$$

Adding these two pieces gives

$$\begin{aligned}
&f(a^*)\frac{\partial a^*}{\partial\theta}\Big[(1-\pi)\big(u(w_S-\tau_N) - I_N(a^*)\big) - \pi\big(u(\theta) - u(w_S-\tau_N)\big)\Big] \\
&= -f(a^*)\frac{\partial a^*}{\partial\theta}\Big[(1-\pi)G(a^*;x) + \pi\big(u(\theta) - u(w_S-\tau_N)\big)\Big] \\
&= -f(a^*)\frac{\partial a^*}{\partial\theta}\,\Delta U(a^*;\theta,x) = 0,
\end{aligned}$$

where the last equality follows from the cutoff condition $\Delta U(a^*;\theta,x) = 0$. Thus these cross terms cancel exactly.

Collecting the remaining terms (the inframarginal insurance gain, the entry externality, and the financing cost), I obtain

$$\frac{dW}{d\theta} = \pi m u'(\theta) - (1-\pi)f(a^*)p(a^*)\Delta(a^*)\frac{\partial a^*}{\partial\theta} - \Big[\lambda(1-\pi)B_S + (1-\pi)(1-m)u'(w_S-\tau_N)\Big]\frac{d\tau_N}{d\theta},$$

Setting $dW/d\theta = 0$ at the interior optimum gives

$$\boxed{\pi\, m\, u'(\theta) + (1-\pi)\, f(a^*)\, p(a^*)\, \Delta(a^*) \left|\tfrac{\partial a^*}{\partial \theta}\right| = \Big[\lambda(1-\pi)B_S + (1-\pi)(1-m)\, u'(w_S - \tau_N)\Big] \frac{d\tau_N}{d\theta}.} \tag{20}$$

$$\frac{\partial a^*}{\partial \theta} = -\frac{\pi u'(\theta)}{(1-\pi)\partial_a G(a^*; x)} < 0, \qquad \tau_N B_S = \pi\, \theta\, m(\theta, x),$$

so differentiating the budget constraint yields

$$\frac{d\tau_N}{d\theta} = \frac{\pi}{B_S}\left[m + \theta \frac{\partial m}{\partial \theta} + \theta \frac{\partial m}{\partial x}\frac{dx}{d\theta}\right], \qquad \frac{dx}{d\theta} = \frac{d\tau_N}{d\theta}.$$

Substituting these relations into (20) gives the stated first-order condition.

The first term represents the inframarginal insurance value for existing entrepreneurs in the catastrophe state. The second term captures the social surplus created by new marginal entrants in the normal state. The right-hand side represents the marginal cost of financing the insurance—now borne in both states—comprising (i) the extra efficiency cost of raising revenue, $\lambda(1-\pi)B_S$, and (ii) the private utility loss of safe workers, $(1-\pi)(1-m)u'(w_S-\tau_N)$. Because safe income is constant across states, taxation reduces their utility in both, making systemic insurance more fiscally expensive and shifting the optimal $\theta^\star$ downward relative to the baseline. □

Proof of Proposition 2.5

Proof. Let $W(\theta)$ denote the planner's welfare in (16) after substituting $\tau_N(\theta)$ from the government budget constraint and the induced cutoff $a^*(\theta, x(\theta))$. W is continuously differentiable around θ^{full} by the regularity assumptions on u, p, R and by the implicit-function theorem for a^*.

From the past Proposition, the total derivative of welfare with respect to θ can be written as

$$\frac{dW}{d\theta} = \pi\, m\, u'(\theta) - (1-\pi)\, f(a^*)\, p(a^*)\, \Delta(a^*)\, \frac{\partial a^*}{\partial \theta} - \Big[\lambda(1-\pi)B_S + (1-\pi)(1-m)\, u'(w_S - \tau_N)\Big] \frac{d\tau_N}{d\theta}. \tag{21}$$

Because $\partial a^*/\partial\theta < 0$ by the previous Lemma this is equivalent to:

$$\frac{dW}{d\theta} = \underbrace{\pi\, m\, u'(\theta)}_{\text{catastrophe insurance gain}} + \underbrace{(1-\pi)\, f(a^*)\, p(a^*)\, \Delta(a^*) \left|\tfrac{\partial a^*}{\partial\theta}\right|}_{\text{entry externality gain}} - \underbrace{\frac{d\tau_N}{d\theta}\Big[\lambda(1-\pi)B_S \;+\; (1-\pi)(1-m)\, u'(w_S-\tau_N)\Big]}_{\text{financing cost (both states)}}. \tag{22}$$

By definition, θ^{full} equalizes marginal utility between entrepreneurs in the catastrophe state and safe workers who consume $w_S - \tau_N$ in both states. Thus $u'(\theta^{\text{full}}) \simeq u'(w_S - \tau_N)$. It is therefore a feasible interior point. Evaluating (22) at $\theta = \theta^{\text{full}}$ yields

$$\frac{dW}{d\theta}\Big|_{\theta^{\text{full}}} = \pi\, m\, u'(\theta^{\text{full}}) + (1-\pi)\, f(a^*)\, p(a^*)\, \Delta(a^*) \left|\tfrac{\partial a^*}{\partial\theta}\right|_{\theta^{\text{full}}} - \tfrac{d\tau_N}{d\theta}\Big|_{\theta^{\text{full}}} \Big[\lambda(1-\pi)B_S + (1-\pi)(1-m)\, u'(w_S-\tau_N)\Big]. \tag{23}$$

To evaluate the financing term, use the budget constraint

$$\tau_N B_S = \pi\, \theta\, m(\theta, x), \qquad x = (1-\pi)\tau_N,$$

and differentiate it with respect to θ:

$$\frac{d\tau_N}{d\theta} = \frac{\pi}{B_S}\Big[m + \theta\frac{\partial m}{\partial\theta} + \theta\frac{\partial m}{\partial x}\frac{dx}{d\theta}\Big], \qquad \frac{dx}{d\theta} = \frac{d\tau_N}{d\theta}.$$

There are two possibilities.

Case A (marginal benefit dominates): If

$$\pi\, m\, u'(\theta^{\text{full}}) + (1-\pi)\, f(a^*)\, p(a^*)\, \Delta(a^*) \left|\tfrac{\partial a^*}{\partial\theta}\right|_{\theta^{\text{full}}} \geq \tfrac{d\tau_N}{d\theta}\Big|_{\theta^{\text{full}}} \big[\lambda(1-\pi)B_S + (1-\pi)(1-m)\, u'(w_S-\tau_N)\big],$$

then (23) implies $\frac{dW}{d\theta}\big|_{\theta^{\text{full}}} \geq 0$. Since W is differentiable, increasing θ locally cannot decrease welfare; hence the maximizer must satisfy $\theta^\star \geq \theta^{\text{full}}$. If the inequality is strict, $\frac{dW}{d\theta} > 0$ at θ^{full}, implying $\theta^\star > \theta^{\text{full}}$.

Case B (marginal cost dominates): If the inequality reverses, the derivative in (23) is negative, so a marginal reduction in θ increases welfare. Thus, any interior maximizer must satisfy $\theta^\star < \theta^{\text{full}}$.

The argument uses standard regularity: $W(\theta)$ is continuously differentiable, the induced mappings $\tau_N(\theta)$ and $a^*(\theta, x(\theta))$ are continuous, and θ^{full} is interior. Hence, the sign of $\frac{dW}{d\theta}$ at a feasible point determines whether the local optimum lies above or below that point.

At the full–insurance benchmark, the planner weighs the marginal benefit of increasing coverage—both the direct insurance to current entrepreneurs and the entry externality—against the marginal cost of financing this coverage. Because safe workers pay taxes in both states, the financing term is heavier, and the efficiency cost λ further increases the shadow price of public funds. If the benefit side dominates, $\theta^\star \geq \theta^{\text{full}}$ (full or overinsurance); if the cost side dominates, $\theta^\star < \theta^{\text{full}}$ (partial insurance). The qualitative structure of the trade–off is unchanged, but the fiscal constraint now bites more strongly. □

Proof of Proposition 2.6

Proof. I proceed in three steps.

By definition, the entrepreneurship cutoff $a^*(\theta)$ satisfies:

$$(1-\pi)\,G(a^*;x) + \pi\,[u(\theta) - u(0)] = 0.$$

Totally differentiating with respect to θ gives

$$(1-\pi)\,G_a(a^*;x)\,\frac{da^*}{d\theta} + \pi\,u'(\theta) = 0,$$

so that

$$\frac{da^*}{d\theta} = -\frac{\pi\,u'(\theta)}{(1-\pi)\,G_a(a^*;x)}.$$

Since $m(\theta) = \int_{a^*(\theta)}^{\infty} f(a)\,da$, it follows that

$$\frac{dm}{d\theta} = -f(a^*)\,\frac{da^*}{d\theta} = f(a^*)\,\frac{\pi\,u'(\theta)}{(1-\pi)\,G_a(a^*;x)} \;>\; 0.$$

Define the elasticity

$$\eta_{m,\theta}(\theta) = \frac{\theta}{m(\theta)}\frac{dm}{d\theta} = \frac{\theta}{m(\theta)}\,f(a^*)\,\frac{\pi\,u'(\theta)}{(1-\pi)\,G_a(a^*;x)}. \tag{24}$$

The balanced–budget condition implies

$$\tau_N B_S = \pi\,m(\theta)\,\theta.$$

Totally differentiating yields

$$B_S\,\frac{d\tau_N}{d\theta} = \pi\Big[m(\theta) + \theta\frac{dm}{d\theta}\Big],$$

and therefore

$$\frac{d\tau_N}{d\theta} = \frac{\pi}{B_S}\Big[m(\theta) + \theta\frac{dm}{d\theta}\Big] = \frac{\pi}{B_S}\, m(\theta)\Big[1 + \eta_{m,\theta}(\theta)\Big], \tag{25}$$

which gives (12).

Let $W(\theta, \tau_N, a^*)$ denote expected welfare as in (16):

$$\begin{aligned} W = {} & (1-\pi)\Big[(1-m)\, u(w_S - \tau_N) + \cdots\Big] \\ & + \pi\Big[(1-m)\, u(w_S - \tau_N) + m\, u(\theta)\Big] \\ & + (1-\pi)\int_{a^*}^{\infty} p(a)\, \Delta(a)\, f(a)\, da \\ & - \lambda(1-\pi)\, \tau_N B_S. \end{aligned} \tag{26}$$

where the omitted term collects the normal–state entrepreneurial utility, which depends on a^* but not directly on θ.

Differentiating W with respect to θ, taking into account $\tau_N = \tau_N(\theta)$ and $a^* = a^*(\theta)$, yields three contributions:

(i) *Direct insurance effect:*

$$\pi\, m(\theta)\, u'(\theta).$$

(ii) *Entry externality effect:*

$$\frac{d}{d\theta}\Big[(1-\pi)\int_{a^*}^{\infty} p(a)\, \Delta(a)\, f(a)\, da\Big] = -(1-\pi)\, p(a^*)\, \Delta(a^*)\, f(a^*)\, \frac{da^*}{d\theta}.$$

Using (4) and (24),

$$(1-\pi)\, p(a^*)\, \Delta(a^*)\, \frac{dm}{d\theta} = (1-\pi)\, p(a^*)\, \Delta(a^*)\, m(\theta)\, \frac{\eta_{m,\theta}(\theta)}{\theta}. \tag{27}$$

(iii) *Financing cost:* A higher θ requires raising τ_N, which reduces welfare through both safe workers' utility and the fiscal inefficiency cost λ. The total marginal cost is therefore

$$-\Big[\lambda(1-\pi)B_S + (1-\pi)(1-m(\theta))\, u'(w_S - \tau_N)\Big]\frac{d\tau_N}{d\theta}. \tag{28}$$

Combining (i)–(iii) and setting $dW/d\theta = 0$ gives

$$\pi\, m(\theta)\, u'(\theta) + (1-\pi)\, p(a^*)\, \Delta(a^*)\, m(\theta)\, \frac{\eta_{m,\theta}(\theta)}{\theta} = \Big[\lambda(1-\pi)B_S + (1-\pi)(1-m(\theta))\, u'(w_S - \tau_N)\Big]\frac{d\tau_N}{d\theta}.$$

Dividing both sides by $(1-\pi)$ yields

$$\frac{\pi\, m(\theta)\, u'(\theta)}{1-\pi} + \frac{m(\theta)}{B_S}\, p(a^*)\, \Delta(a^*)\, \frac{\eta_{m,\theta}(\theta)}{\theta/B_S} = \Big[\lambda B_S + (1-m(\theta))\, u'(w_S - \tau_N)\Big] \frac{d\tau_N}{d\theta},$$

which is the sufficient–statistics condition (11). Combining with (12) completes the proof. □

Proof of Proposition 3.2

Proof. Suppose that catastrophe–state benefits are report–based and increasing in a policy parameter θ, so that $b_C(r) = b_C(r;\theta)$ with

$$\frac{\partial}{\partial r} b_C(r;\theta) \;=\; b_C'(r;\theta) \;\geq 0, \qquad \frac{\partial}{\partial \theta} b_C(r;\theta) \;\geq 0,$$

and assume that the catastrophe term in the first–order condition,

$$\pi\, b_C'(r;\theta)\, u'\big(b_C(r;\theta)\big),$$

is strictly increasing in θ for all relevant (r,θ).[12]

Let $\Psi(r,\theta)$ denote the left–hand side of the reporting first–order condition in Lemma 3.1:

$$\Psi(r,\theta) \equiv (1-\pi)\Big[-(1-q)\, T'(r)\, u'\big(c_N(r)\big) \;+\; q\, \phi'(y-r)\, u'\big(\tilde{c}_N(r)\big)\Big] \;+\; \pi\, b_C'(r;\theta)\, u'\big(b_C(r;\theta)\big),$$

where $c_N(r) = y - T(r)$ and $\tilde{c}_N(r) = y - T(y) - \phi(y-r)$. For each θ, the optimal report $r^*(\theta)$ solves

$$\Psi\big(r^*(\theta), \theta\big) = 0.$$

By the implicit–function theorem,

$$\frac{\partial r^*}{\partial \theta} = -\,\frac{\Psi_\theta\big(r^*(\theta),\theta\big)}{\Psi_r\big(r^*(\theta),\theta\big)}.$$

I now sign numerator and denominator.

Numerator. By construction, only the catastrophe–state term depends on θ, so

$$\Psi_\theta(r,\theta) = \pi\, \frac{\partial}{\partial \theta}\Big[b_C'(r;\theta)\, u'\big(b_C(r;\theta)\big)\Big] \;>\; 0$$

by the maintained assumption that this term is strictly increasing in θ.

[12] A sufficient condition is, for example, $b_C(r;\theta) = \theta\, \tilde{b}(r)$ with $\tilde{b}(r) \geq 0$, $\tilde{b}'(r) > 0$, and $u'' < 0$ small enough in the relevant region so that $\partial_\theta\big[b_C'(r;\theta) u'(b_C(r;\theta))\big] > 0$.

Denominator. Differentiate $\Psi(r,\theta)$ with respect to r, using

$$c_N'(r) = -T'(r), \qquad \tilde{c}_N'(r) = \phi'(y-r),$$

and the chain rule.

For the no–audit term:

$$\frac{\partial}{\partial r}\Big[-(1-q)\,T'(r)\,u'\big(c_N(r)\big)\Big] = -(1-q)\Big[T''(r)\,u'\big(c_N(r)\big) \;+\; T'(r)\,u''\big(c_N(r)\big)\,c_N'(r)\Big].$$

Substituting $c_N'(r) = -T'(r)$ gives

$$\frac{\partial}{\partial r}\Big[-(1-q)\,T'(r)\,u'\big(c_N(r)\big)\Big] = -(1-q)\Big[T''(r)\,u'\big(c_N(r)\big) \;-\; T'(r)^2\,u''\big(c_N(r)\big)\Big],$$

which is negative since $T'(r) \geq 0$, $T''(r) \geq 0$, $u' > 0$, and $u'' < 0$.

For the audit term:

$$\frac{\partial}{\partial r}\Big[q\,\phi'(y-r)\,u'\big(\tilde{c}_N(r)\big)\Big] = q\Big[-\phi''(y-r)\,u'\big(\tilde{c}_N(r)\big) \;+\; \phi'(y-r)\,u''\big(\tilde{c}_N(r)\big)\,\tilde{c}_N'(r)\Big],$$

and substituting $\tilde{c}_N'(r) = \phi'(y-r)$ yields

$$\frac{\partial}{\partial r}\Big[q\,\phi'(y-r)\,u'\big(\tilde{c}_N(r)\big)\Big] = q\Big[-\phi''(y-r)\,u'\big(\tilde{c}_N(r)\big) \;+\; \phi'(y-r)^2\,u''\big(\tilde{c}_N(r)\big)\Big],$$

which is also negative because $\phi''(z) \geq 0$, $u' > 0$, and $u'' < 0$.

Finally, the catastrophe–state term yields

$$\frac{\partial}{\partial r}\Big[b_C'(r;\theta)\,u'\big(b_C(r;\theta)\big)\Big] = b_C''(r;\theta)\,u'\big(b_C(r;\theta)\big) \;+\; b_C'(r;\theta)^2\,u''\big(b_C(r;\theta)\big) \;<\; 0,$$

by $b_C'(r;\theta) \geq 0$, $b_C''(r;\theta) \leq 0$, $u' > 0$, and $u'' < 0$.

Combining all three pieces gives $\Psi_r(r,\theta) < 0$.

Since $\Psi_\theta > 0$ and $\Psi_r < 0$, the implicit–function formula implies

$$\frac{\partial r^*}{\partial \theta} = -\,\frac{\Psi_\theta}{\Psi_r} \;>\; 0.$$

Thus a more generous catastrophe–state, report–based transfer increases the optimal reported amount, establishing insurance–enforcement complementarity. □

Truthful reporting under full insurance - Proposition 3.3

Proof. Recall

$$U(r;y) = (1-\pi)\Big[(1-q)\,u\big(c_N(r)\big) + q\,u\big(\tilde{c}_N(r)\big)\Big] + \pi\,u\big(b_C(r)\big),$$

with

$$c_N(r) = y - T(r), \qquad \tilde{c}_N(r) = y - T(y) - \phi(y-r), \qquad c_C(r) = b_C(r).$$

Differentiating $U(r;y)$ with respect to r and using $c_N'(r) = -T'(r)$, $\tilde{c}_N'(r) = \phi'(y-r)$, and $c_C'(r) = b_C'(r)$ yields

$$U'(r) = (1-\pi)\Big[(1-q)\,u'\big(c_N(r)\big)\,(-T'(r)) + q\,u'\big(\tilde{c}_N(r)\big)\,\phi'(y-r)\Big] + \pi\,u'\big(b_C(r)\big)\,b_C'(r).$$

At $r = y$ I have $\phi(0) = 0$, hence

$$c_N(y) = y - T(y), \qquad \tilde{c}_N(y) = y - T(y) - \phi(0) = y - T(y) = c_N(y),$$

so $u'\big(\tilde{c}_N(y)\big) = u'\big(c_N(y)\big)$. Evaluating $U'(r)$ at $r = y$ gives

$$U'(y) = (1-\pi)\Big[-(1-q)\,T'(y) + q\,\phi'(0)\Big]\,u'\big(c_N(y)\big) + \pi\,b_C'(y)\,u'\big(b_C(y)\big). \tag{29}$$

Truthful reporting $r^* = y$ is optimal if $U'(y) \geq 0$ and $U(r)$ is concave in r on $[0,y]$. Concavity follows from u concave, T weakly convex, ϕ weakly convex, and b_C weakly concave, exactly as in Lemma 3.1.

Condition $U'(y) \geq 0$ is therefore equivalent to

$$\pi\,b_C'(y)\,u'\big(b_C(y)\big) \;\geq\; (1-\pi)\Big[(1-q)\,T'(y) - q\,\phi'(0)\Big]\,u'\big(c_N(y)\big). \tag{30}$$

Under the additional, standard assumption that the marginal penalty at zero evasion is zero, $\phi'(0) = 0$ (for example, when $\phi(z)$ is strictly convex with $\phi'(0) = 0$), inequality (30) simplifies to

$$\pi\,b_C'(y)\,u'\big(b_C(y)\big) \;\geq\; (1-\pi)(1-q)\,T'(y)\,u'\big(c_N(y)\big), \tag{31}$$

which is the stated condition for truthful reporting.

Intuitively, at $r = y$ a marginal reduction in the report saves the entrepreneur $(1-\pi)(1-q)T'(y)$ in expected tax in the no–audit branch, but reduces catastrophe–state benefits by $b_C'(y)$. Under approximate full insurance, $u'\big(b_C(y)\big) \simeq u'\big(c_N(y)\big)$ so (31) compares directly:

- the marginal utility gain from lower taxes in the normal state, versus
- the marginal utility loss from reduced catastrophe insurance.

If the insurance–loss term dominates, $U'(y) \geq 0$ and, by concavity, $r^* = y$ is optimal. If the inequality is strict, $U'(y) > 0$ and strict concavity implies that $r^* = y$ is the unique maximizer. □