NONLINEAR PRICING WITH RESALE

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Abstract. We study nonlinear pricing by a monopolist when consumers can resell in a secondary market. In deterministic menus with no disclosure of primary-market choices before resale, resale changes screening through resale-adjusted virtual values. With convex production costs and linear utility, resale can raise profit by increasing the effective value of primary-market units. With constant marginal cost and concave utility, the seller prefers banning resale because a no-resale menu provides certainty-equivalent allocations at lower cost. Numerical exercises report profit, consumer-surplus, and total-surplus comparisons, including cases where consumers and total surplus favor resale while the monopolist does not.

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1. **Introduction**

Nonlinear pricing is one of the monopolist's central tools for screening privately informed consumers. Quantity discounts, two-part tariffs, subscription tiers, and capacity menus all allow firms to tailor allocations and payments to differences in willingness to pay. Yet the same feature that makes nonlinear pricing valuable also creates a natural arbitrage channel: once consumers receive different quantities at different effective unit prices, they may be able to trade with one another in a secondary market. Resale therefore affects not only the final allocation of goods, but also the incentive structure underlying the monopolist's primary-market screening problem.

Firms often try to prevent or limit this arbitrage channel. They may lease rather than sell, reduce durability, make the product non-transferable, integrate with

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downstream users, or create contractual and reputational barriers to resale. At the same time, resale cannot always be shut down. IBM and Xerox were able to lease their products only until the practice was interpreted as anticompetitive under U.S. antitrust law.[1] In 2012, the European Court of Justice ruled that customers who download software cannot be prevented from reselling it even if contractual provisions state otherwise.[2] In other markets, such as automobiles, the specialization required in distribution has limited manufacturers' ability to internalize downstream resale activity.[3]

This paper asks two related questions. First, if a monopolist can prevent resale, should he always do so? Second, if resale is unavoidable, how should the monopolist adjust his nonlinear price schedule? We answer these questions in a two-consumer durable-good model in which the monopolist offers a deterministic nonlinear menu in the primary market and consumers can subsequently trade with each other in a secondary market. Throughout the paper we focus on deterministic menus, no disclosure of primary-market choices before resale, and truthful behavior in the primary and resale stages. In the closed-form Case-I resale characterization, the resale continuation uses all-or-nothing unit-price offers. This framework isolates the interaction between nonlinear pricing, production incentives, and downstream reallocability.

The central insight is that resale generates two opposing forces. On one side, resale changes the monopolist's effective screening environment by endogenously modifying the virtual value associated with primary-market allocations. When production costs are convex, downstream reallocation weakens the need for sharply differentiated primary allocations and can lower expected production costs. On the other side, resale can expose consumers to uncertainty regarding final consumption. When consumers have concave utility from consumption, the monopolist may prefer to eliminate resale and provide consumers directly with certainty-equivalent allocations through the primary menu. Propositions 1 and 2 formalize these opposing forces at the two curvature endpoints. Resale therefore changes not only ex post efficiency, but also the structure of monopoly screening itself.

[1] See Wilson (1993).

[2] See *UsedSoft GmbH v. Oracle International Corp.*, Case C-128/11, Court of Justice of the European Union, 3 July 2012.

[3] See Scherer (1996).

We study two benchmark environments that isolate these forces. In the first, the monopolist has convex production costs and consumers have linear utility, as in the scarcity-driven allocation logic of Myerson (1981). This benchmark generalizes that scarcity logic by allowing endogenous production while preserving convex aggregate cost; as the cost function becomes increasingly convex, the allocation problem approaches the fixed-supply logic of Myerson (1981). In the second, the monopolist has constant marginal cost and consumers have concave utility, following the nonlinear-pricing tradition of Maskin and Riley (1984). The first benchmark is motivated by markets with convex production or capacity costs and economically meaningful downstream reallocation, such as capacity-constrained production environments. The second is motivated by digital goods and transferable access-right environments, where marginal reproduction costs are low and consumers value flexibility after uncertainty about future use is resolved. Used software licenses are a natural example of the second force, while textbook and other durable-information-good markets point toward the first.

The results show that resale need not be harmful to the monopolist. In the linear-utility case, the unit-price resale continuation generates a resale-adjusted virtual value that dominates the no-resale virtual value. Under a mild strictness condition, the monopolist earns strictly higher profit when resale is allowed. The sign of this comparison comes from the resale-adjusted virtual-value dominance: resale raises the effective willingness to pay for primary-market units and slows the growth of information rents. Convex production cost makes the benchmark well posed and gives the induced allocation flattening its economic bite, because the monopolist can economize on expected marginal production costs while relying less on sharply differentiated primary allocations. The same comparison rests on a separable resale-adjusted virtual-value representation, and the appendix records the properties under which other resale continuations generate the same representation. In the concave-utility case, the force goes in the opposite direction. A revealed-preference mimicking argument shows that the monopolist can ban resale and offer each type a certainty-equivalent allocation that delivers the same expected utility at weakly lower production cost. Resale then weakly lowers the monopolist's attainable profit in the class studied here.

The numerical exercises sharpen the welfare implications and answer how the nonlinear menu changes when resale becomes feasible. In the convex-cost environment, resale increases computed monopoly profit, consumer surplus, and total

surplus across all reported parameterizations, so the monopolist's incentive to facilitate resale is aligned with welfare. In the concave-utility environment, resale generates a conflict between monopoly incentives and welfare: the monopolist prefers banning resale, while consumers and total surplus favor allowing downstream transferability.

The contribution is to connect two literatures that are usually studied separately. Standard nonlinear-pricing models typically assume that resale is absent, limited, or controlled, while much of the auction-with-resale literature abstracts from production and nonlinear quantity choice. This paper allows both production and downstream trade, and shows that the profitability and welfare effects of resale depend critically on the interaction between cost curvature, utility curvature, and the resale protocol.

1.1. **Related Literature.** The paper sheds light on the complications created by resale markets for nonlinear pricing. The closest antecedent is Calzolari and Pavan (2006), who study revenue-maximizing mechanisms for a monopolist selling a single-unit durable good when buyers can resell. They distinguish resale to an outside third party from inter-bidder resale, and show that the optimal mechanism may require stochastic allocations and a richer disclosure policy than a simple announcement of the trading decision. Our model adopts a different informational environment–deterministic mechanisms with full non-disclosure–and moves from a single indivisible object to a divisible good and an explicit nonlinear price schedule. In this environment, our concave-utility case obtains a revenue-suppressing comparison that is close in spirit to their inter-bidder resale case, while our linear-utility case shows that resale need not be harmful when production costs are convex. Recent work by Flores (2024) is also closely related because it studies resale under monopoly two-part tariffs and shows that resale can benefit a price-discriminating monopolist by changing the composition of primary-market participants. The present paper differs by studying the primary nonlinear price schedule together with a resale continuation game among privately informed consumers.

A large resale literature instead studies auctions in the primary market. Zheng (2002) shows that, under regularity conditions, resale can help implement the Myerson allocation in a two-bidder market when the winner is committed to resell. Mylovanov and Tröger (2008) characterize the type-space conditions under which this logic extends beyond the uniform case. For ex-ante symmetric bidders, Hafalir and Krishna (2008) show that resale changes the revenue ranking of first- and

second-price auctions, while Lebrun (2012) shows that the second-price auction can implement Myerson's optimal allocation with resale under more general assumptions than Zheng (2002); Lebrun (2010) provides an equilibrium of the first-price auction with resale whose outcome is robust to bid disclosure. These papers clarify how resale affects auction formats when the seller has a single object and no production decision. Our contribution is to study how the possibility of resale changes a monopolist's nonlinear price schedule and production incentives.

Other auction-resale papers emphasize asymmetry, speculation, and bargaining power. For example, Hafalir and Krishna (2009), Garratt and Tröger (2006), Pagnozzi (2007), and Zhang and Wang (2013) show that resale can change primary-market incentives through bidder asymmetry, speculative entry, or resale bargaining. Our model instead studies a divisible good, an explicit nonlinear price menu, and the production-smoothing channel created by resale.

Information disclosure from the primary market to the resale market is another central theme. Calzolari and Pavan (2006) show that the monopolist may want to control the information revealed to secondary-market participants. Garratt et al. (2009) show that resale can facilitate collusion when auction histories generate public signals. Carroll and Segal (2019) study robustly optimal auctions when the seller does not know the resale opportunity. More recently, Dworczak (2020) characterizes optimal cutoff mechanisms for communication with aftermarkets. We hold this disclosure margin fixed: after the primary market, the menu is common knowledge, but individual primary-market choices are not revealed. This isolates the production and nonlinear-pricing channel of resale.

The paper is also connected to nonlinear pricing and durable-goods monopoly. Armstrong (2016) surveys the envelope and demand-profile methods used in nonlinear pricing, while Wilson (1993) provides the standard treatment of nonlinear price schedules under limited arbitrage. In durable-goods markets, Coase (1972) first emphasized how resale and durability can undermine monopoly power, and Bulow (1982) showed that durability can generate aggregate inefficiencies even when monopoly power is weakened. Loertscher and Muir (2022) study a continuum of consumers with single-unit demand and compare posted pricing with rationing when resale is possible. Hendel and Lizzeri (1999) interpret secondary-market trading as a source of adverse selection that raises firsthand-market prices, while Kumar (2002) and Schiraldi and Nava (2012) study related intertemporal resale channels. Relative to this literature, our model keeps the market static but

allows a monopolist to choose a nonlinear menu in the presence of strategic resale among consumers.

The remainder of the paper is organized as follows. Section 2 presents the main model and the two baseline cases. Sections 3 and 4 analyze the linear-utility/convex-cost case and the concave-utility/linear-cost case, respectively. Section 5 discusses the numerical illustrations, and Section 6 concludes. All proofs are collected in the appendix.

2. The Main Model

Consider a monopolist who can produce any amount, $q \geq 0$, of a single product at a total cost $c(q)$. The monopolist is profit oriented and risk neutral. If the monopolist produces q units in total and collects a total revenue of T then its profit is given by

$$\pi(q, T) = T - c(q). \tag{1}$$

The monopolist faces two consumers, labeled 1 and 2,[4] whose preferences are represented by

$$u(q_i, T_i; \theta_i) = \theta_i v(q_i) - T_i \tag{2}$$

where q_i and T_i denote the number of units purchased and the amount paid by consumer i, respectively, for $i = 1, 2$. Consumers can potentially differ in their taste towards the product. Consumer i's taste, θ_i, is privately known and is independently and identically distributed over the interval $[\underline{\theta}, \overline{\theta}] \subset \mathbb{R}_+$ according to the continuous cumulative distribution function F with associated density f. We suppose that F satisfies Myerson's (1981) regularity condition. That is, the virtual value function associated with F, namely

$$\psi(\theta) \equiv \theta - \frac{1 - F(\theta)}{f(\theta)},$$

is increasing.

The monopolist does not observe consumer tastes but is able to engage in indirect price discrimination by offering both consumers the same nonlinear price schedule (T, q) which is composed of pairs of payment and product amount, $(T(\theta), q(\theta))$, each of which is directed to consumers of taste θ. Each consumer chooses one

[4]The two-consumer environment keeps the resale subgame analytically tractable. With more than two consumers, the resale protocol must specify who can propose to whom, whether offers are sequential or simultaneous, and how competing resale opportunities interact. Extending the analysis to $N > 2$ consumers is therefore a distinct resale-protocol problem rather than a notational change.

menu option; in the direct-mechanism notation, choosing the option indexed by θ' is equivalent to reporting taste θ'.

After having purchased from the monopolist in the primary market, consumers can resell some or all units to each other, but to no one else, in the resale market. The monopolist does not participate in the resale market. Moreover, he does not reveal any information he obtains in the primary market regarding consumers' tastes, or the actions and the outcome he observes. Thus, in the resale market, consumers cannot update their prior beliefs regarding the other consumer's taste.

In the resale market, one of the two consumers is chosen randomly - with probability 0.5 - as the proposer. The other consumer observes the proposer's offer and is called the responder. The proposer offers the responder a menu (S, r) which is comprised of buy and sell offers of the form $(S(\theta), r(\theta))$, each of which is directed to the responder of taste θ. The product lasts for two periods and consumption takes place at the end of the second period in which resale takes place. Consumers are sequentially rational and they do not discount future payoffs.

Symmetry. The two consumers are ex-ante symmetric, the proposer role is assigned symmetrically, and the monopolist's mechanism treats the two consumers symmetrically. Throughout the paper, we therefore select symmetric resale continuations: consumers with the same type use the same proposer strategy and the same responder strategy. This is the continuation delivered constructively by the posted-price representation in Case I, and it is the continuation class used in the Case-II comparison.

Using the language of mechanism design, both (T, q) and (S, r) are direct mechanisms which ask players (here, consumers) to report their types (here, tastes) directly. We work with direct revelation mechanisms that are truthful in both markets.

Then, the monopolist's problem boils down to choosing the nonlinear price scheme (T, q) that maximizes his expected profit,

$$\int_{\underline{\theta}}^{\overline{\theta}} \int_{\underline{\theta}}^{\overline{\theta}} \left[T(\theta_1) + T(\theta_2) - c\left(q(\theta_1) + q(\theta_2)\right)\right] dF(\theta_1) dF(\theta_2), \tag{3}$$

given that consumers of each possible taste, θ, have an incentive to take the offer that the monopolist directed to them, namely $(T(\theta), q(\theta))$, and not some other offer, even after taking into account their expected payoff associated with the resale market (i.e., *incentive compatibility*); and that the monopolist's offers, $(T(\theta), q(\theta))$, yield

an expected payoff to consumers of taste, θ, that is at least as high as the payoff associated with their outside option, which is normalized to zero (i.e., *individual rationality*).[5]

Class of mechanisms considered. Throughout this paper we focus on deterministic menus, no disclosure of primary-market choices before resale, and truthful behavior in the primary and resale stages. Equivalently, we study deterministic, jointly truthful direct mechanisms with full non-disclosure: the monopolist commits to a deterministic menu (T, q) and discloses no information from the primary market into the resale stage. Two remarks are in order. First, deterministic production is without loss of generality in Case I, where c is strictly convex and any randomization in q is dominated by its mean via Jensen's inequality. In Case II, where c is linear, randomization in production is cost-neutral, but consumers' concave v implies they strictly prefer the certainty-equivalent allocation, so randomization does not help relax the relevant incentive constraints in our setting. Second, joint truthfulness is without loss of generality in Case I by the resale-stage truthfulness argument below and is the maintained direct-mechanism solution concept in Case II. For the closed-form Case-I resale characterization, we further focus on all-or-nothing unit-price resale offers; Lemma 2 optimizes within that resale protocol. This mechanism class complements the broader stochastic and disclosure-design environments studied by Calzolari and Pavan (2006).

In what follows below, we study the monopolist's nonlinear pricing problem for two representative cases, which we call baseline models. In the first case, we suppose that the monopolist's marginal cost function is increasing (i.e., $c(q)$ is convex) and that the consumers' marginal benefit is constant (i.e., $v(q)$ is linear). In the second case we consider a monopolist who produces at constant marginal cost (i.e., $c(q)$ is linear) and faces two consumers with diminishing marginal utilities from consumption (i.e., $v(q)$ is concave).

The two baseline cases isolate the two curvature forces. If marginal cost and marginal benefit are both constant, the monopolist's problem is degenerate: production is either zero or unbounded. If cost is convex and utility is concave, both resale channels operate at once; the two cases studied below are the endpoint environments that separate them cleanly.

[5]We treat individual menu choices and bilateral resale. Coalitional and account-splitting deviations, such as breaking a large order into smaller purchases across accounts or aggregating several small buyers through a coalition, require a richer contracting protocol.

3. **Case I: Convex Cost and Linear Utility**

In this section, we study the monopolist's nonlinear pricing problem in an environment that has two main features. First, we suppose that the monopolist's cost function is convex. More specifically, $c(q)$ in equation (1) satisfies $c(0) = 0$, $c'(q) > 0$, and $c''(q) > 0$ for all $q > 0$. Second, we assume that consumers have constant marginal utility equal to their tastes θ_i respectively. More specifically, $v(q)$ in equation (2) takes the form $v(q) = q$. All other features of our main model are maintained.

The section proceeds in three steps. We first recall the no-resale benchmark. We then characterize the resale continuation under linear utility and derive the resale-adjusted virtual value that enters the primary-market problem. Finally, we compare the with-resale and no-resale objectives at a fixed allocation, which isolates the revenue-side force behind Proposition 1; convex production costs then shape the optimal allocation response and make the resulting flattening valuable.

3.1. **Benchmark: Optimal Menu without Resale.**

Recall that the monopolist's expected profit is given by (3). The optimal nonlinear pricing schedule maximizes this expression over the domain of feasible schedules that induce consumers to report truthfully (i.e.,

$$u(\theta_i) \equiv \theta_i q(\theta_i) - T(\theta_i) \geq \theta_i q(\theta_i') - T(\theta_i') \equiv u(\theta_i, \theta_i'), \tag{4}$$

for all $\theta_i, \theta_i' \in [\underline{\theta}, \overline{\theta}]$ and $i = 1, 2$), and participate voluntarily (i.e.,

$$u(\theta_i) \geq 0,$$

for all $\theta_i \in [\underline{\theta}, \overline{\theta}]$ and $i = 1, 2$.)

Note that $u(\theta_i)$ is a maximum of a family of affine functions and is therefore convex. It is thus absolutely continuous and differentiable almost everywhere. A necessary condition for (4) to hold is

$$T'(\theta_i) - \theta_i q'(\theta_i) = 0$$

for all $\theta_i \in [\underline{\theta}, \overline{\theta}]$, which in turn implies that

$$u'(\theta_i) = q(\theta_i). \tag{5}$$

Since $u(\theta_i)$ is convex, (5) implies that q is a nondecreasing function. Integrating both sides of (5) we obtain

$$u(\theta_i) = u(\underline{\theta}) + \int_{\underline{\theta}}^{\theta_i} q(\tau)\, d\tau. \tag{6}$$

Hence, up to an additive constant, namely $u(\underline{\theta})$, the payoff of consumer i of taste θ_i in an incentive compatible pricing scheme (T, q) depends only on the allocation rule, q.

It is important to notice that incentive compatibility conditions (4) hold if and only if q is nondecreasing and (6) is satisfied. Also notice that, due to (6), individual rationality conditions are equivalent to $u(\underline{\theta}) \geq 0$ (or $T(\underline{\theta}) \leq \underline{\theta} q(\underline{\theta})$).

Since $u(\theta_i) = \theta_i q(\theta_i) - T(\theta_i)$ and $u(\underline{\theta}) = \underline{\theta} q(\underline{\theta}) - T(\underline{\theta})$ equality (6) can be written as

$$T(\theta_i) = T(\underline{\theta}) - \underline{\theta} q(\underline{\theta}) + \theta_i q(\theta_i) - \int_{\underline{\theta}}^{\theta_i} q(\tau)\, d\tau. \tag{7}$$

Substituting (7) in (3) we can rewrite the monopolist's problem as maximizing

$$\begin{aligned} &2\left[T(\underline{\theta}) - \underline{\theta} q(\underline{\theta})\right] + 2\int_{\underline{\theta}}^{\overline{\theta}} \left[\theta_1 q(\theta_1) - \int_{\underline{\theta}}^{\theta_1} q(\tau)\, d\tau\right] dF(\theta_1) \\ &- \int_{\underline{\theta}}^{\overline{\theta}} \int_{\underline{\theta}}^{\overline{\theta}} \left[c\left(q(\theta_1) + q(\theta_2)\right)\right] dF(\theta_1) dF(\theta_2) \end{aligned} \tag{8}$$

subject to the condition that $T(\underline{\theta})$ and q satisfies

(IC) q is nondecreasing and satisfies (6); and
(IR) $T(\underline{\theta}) \leq \underline{\theta} q(\underline{\theta})$.

Note that for any given q, (8) is maximized when $T(\underline{\theta}) = \underline{\theta} q(\underline{\theta})$. This holds because (8) is increasing in $T(\underline{\theta})$ and bounded from above by $\underline{\theta} q(\underline{\theta})$ due to **(IR)**. Thus the first term in (8) is zero. We can conclude that the optimal nonlinear pricing scheme leaves no surplus to the consumer of the lowest taste.

Next, interchanging the order of integration, we obtain

$$\int_{\underline{\theta}}^{\overline{\theta}} \int_{\underline{\theta}}^{\theta_1} q(\tau)\, d\tau dF(\theta_1) = \int_{\underline{\theta}}^{\overline{\theta}} \int_{\tau}^{\overline{\theta}} dF(\theta_1) q(\tau)\, d\tau = \int_{\underline{\theta}}^{\overline{\theta}} \left(\frac{1 - F(\tau)}{f(\tau)}\right) q(\tau)\, dF(\tau). \tag{9}$$

Then, using (9) we can write the monopolist's problem in (8) as

$$\begin{aligned} \max_{q(\cdot)} \Bigg[&2\int_{\underline{\theta}}^{\overline{\theta}} \psi(\theta) q(\theta) dF(\theta) \\ &- \int_{\underline{\theta}}^{\overline{\theta}} \int_{\underline{\theta}}^{\overline{\theta}} c\big(q(\theta_1) + q(\theta_2)\big) dF(\theta_1) dF(\theta_2) \Bigg]. \end{aligned} \tag{10}$$

Next, we assume that resale is possible and revisit the nonlinear pricing problem. Consumers are sequentially rational. Therefore the possibility of resale has implications on their behavior both in the resale market and the primary market. The

monopolist would take this into account when designing his pricing scheme. Thus, we work backwards and analyze the resale market before the primary market.

3.2. **Optimal Menu with Resale Market.** Recall that both consumers are equally likely to act as the proposer in the resale market and that the proposer is to present the responder a menu of buy and sell offers. Along the same lines as in the previous case we study consumer 1's decisions under both roles and note that consumer 2's problems are alike due to symmetry.

Consider consumer 1 of taste θ_1 who has reported her taste as θ_1' in the primary market and thereby has obtained $q(\theta_1')$ units at the lump sum price $T(\theta_1')$. Since consumer 1 does not obtain any new information regarding consumer 2's taste, θ_2, she would still believe that it is distributed over the interval $[\underline{\theta}, \overline{\theta}]$ according to the cumulative distribution function, F. Consumer 1 also believes that if consumer 2 is of taste θ_2 then she has purchased $q(\theta_2)$ units in the primary market and has paid $T(\theta_2)$. The latter follows because the monopolist's menu, (T, q), is incentive compatible.

When consumer 1 acts as the proposer she chooses a resale menu subject to the responder's incentive and participation constraints. We assume here that $v(q) = q$. Then the quasi-linear utility function becomes linear and both the proposer's and the responder's payoffs are additively separable across the two periods. For the analytical comparison below, Lemma 2 shows that the resale stage is represented by two all-or-nothing posted prices, $p_s(\theta_1)$ and $p_b(\theta_1)$, such that $p_s(\theta_1)$ maximizes

$$q(\theta_1')(1 - F(p_s))(p_s - \theta_1) \tag{11}$$

and $p_b(\theta_1)$ maximizes

$$\int_{\underline{\theta}}^{p_b} q(\theta_2)(\theta_1 - p_b)dF(\theta_2). \tag{12}$$

(11) represents the expected continuation payoff of the proposer when the responder accepts to buy all units that proposer has, namely $q(\theta_1')$, at unit price p_s, which happens whenever $\theta_2 > p_s$, an event that occurs with probability $(1 - F(p_s))$. Similarly, (12) represents the expected continuation payoff of the proposer when the responder accepts to sell all she has, namely $q(\theta_2)$, at unit price p_b, which happens whenever $\theta_2 < p_b$.

Lemma 1. (*Separability*) *In the case where the consumer's utility function is linear, the aggregated utility from the primary and secondary markets are additive separable. Moreover,*

any profitable resale trade is directionally separated: the proposer sells only to consumers with higher tastes and buys only from consumers with lower tastes.

Lemma 2. (*Posted-price resale representation*) *Suppose the sell-side virtual valuation is regular and, for the non-decreasing allocation q under consideration, the buy-side weighted virtual cost*

$$C_q(p) \equiv p + \frac{\int_{\underline{\theta}}^{p} q(t)dF(t)}{q(p)f(p)}$$

is non-decreasing on the set where $q(p) > 0$. Then, for this fixed q, a symmetric resale-stage equilibrium in all-or-nothing unit-price resale offers exists. After the directional separation in Lemma 1, this equilibrium is represented by the posted unit prices in (11) and (12). The induced price functions p_s and p_b are non-decreasing in the proposer type.

Resale-stage truthfulness. A consequence of Lemma 1 and Lemma 2 is that, given any monopolist menu (T, q), a consumer of type θ_1 who has misreported as θ_1' in the primary market still optimally reports her true type in the resale stage, irrespective of her role. As proposer, her equilibrium posted prices $p_s(\theta_1)$ and $p_b(\theta_1)$ are determined by (13) and (14), which depend only on her true type θ_1 and on F and q, not on θ_1'. As responder, the offered prices likewise depend only on the proposer's true type, and her best response (accept the sell offer iff $\theta_2 > p_s$, accept the buy offer iff $\theta_2 < p_b$) depends only on her own true type. Hence the joint incentive-compatibility constraint reduces to incentive compatibility in the primary market alone; together with monotonicity of q, the first-order condition $T'(\theta) = h(\theta)q'(\theta)$ used in Lemma 4 characterizes **(IC)**.

The responder's incentive conditions are satisfied since $p_s(\theta_1) > p_b(\theta_1)$. Note that, in line with Myerson and Satterthwaite (1983), final allocation in the resale market is ex-post inefficient.

It follows directly from (11) that $p_s(\theta_1)$ solves

$$\theta_1 = p_s - \frac{1 - F(p_s)}{f(p_s)} \tag{13}$$

and from (12) that $p_b(\theta_1)$ solves

$$\theta_1 = p_b + \frac{1}{q(p_b)f(p_b)} \int_{\underline{\theta}}^{p_b} q(\theta_2)dF(\theta_2). \tag{14}$$

The right-hand side of (13) is the virtual value function and the right-hand side of (14) is the virtual cost function. Not surprisingly, the latter depends on the primary market allocation, $q(\cdot)$. By Lemma 2, both p_b and p_s are nondecreasing in θ_1.

Moreover, (13) implies $p_s(\theta) > \theta$ for all $\theta \in [\underline{\theta}, \overline{\theta})$ and $p_s(\overline{\theta}) = \overline{\theta}$; and (14) implies $p_b(\underline{\theta}) = \underline{\theta}$ and $p_b(\theta) \leq \theta$ for all $\theta \in (\underline{\theta}, \overline{\theta}]$.

Linearity of utility gives an analytical representation of consumers' resale-stage offers. Together with the separability lemma, this yields an analytical expression for the monopolist's expected revenue, analogous to (9).

To understand consumers' behavior in the primary market, start with the expected utility function when a consumer has purchased $q(\theta')$ in the primary market and acts as if she has taste θ_1'' in the resale market if she is the responder. Let

$$\begin{aligned} h(\theta_1, \theta_1'') \equiv \theta_1 &+ \frac{1}{2}\big(1 - F(p_s(\theta_1))\big)\big(p_s(\theta_1) - \theta_1\big) \\ &+ \frac{1}{2}\int_{\min\{p_b^{-1}(\theta_1''), \overline{\theta}\}}^{\overline{\theta}} \big(p_b(\theta_2) - \theta_1\big) dF(\theta_2) \end{aligned} \tag{15}$$

denote the expected benefit of consumer 1 which is associated with the consumption or the resale of $q(\theta_1')$ units she obtains in the primary market and

$$\begin{aligned} g(\theta_1, \theta_1'') \equiv \frac{1}{2}&\int_{\underline{\theta}}^{p_b(\theta_1)} q(\theta_2)\big(\theta_1 - p_b(\theta_1)\big) dF(\theta_2) \\ &+ \frac{1}{2}\int_{\underline{\theta}}^{\max\{p_s^{-1}(\theta_1''), \underline{\theta}\}} q(\theta_2)\big(\theta_1 - p_s(\theta_2)\big) dF(\theta_2) \end{aligned}$$

denote the expected additional surplus associated with the purchases she makes in the resale market.

Notation in the truth-telling equilibrium. For the rest of this section we evaluate the proposer's payoff at truthful reporting in both stages. By Lemma 1 and the discussion above, this means setting $\theta_1' = \theta_1$ in the primary market and $\theta_1'' = \theta_1$ in the resale market. Under truthful reporting, define

$$\theta_2^b \equiv p_b^{-1}(\theta_1),$$

the cutoff value of the *counterparty's* type above which a consumer of type θ_1 – when acting as responder – accepts the buy offer made by a proposer (since p_b is non-decreasing, this consumer agrees to sell at price $p_b(\theta_2)$ exactly when $\theta_2 > p_b^{-1}(\theta_1) = \theta_2^b$). The integral $\int_{\theta_2^b}^{\overline{\theta}} (p_b(\theta_2) - \theta_1)\, dF(\theta_2)$ in (16) collects the expected per-unit gains from such sales.[6] With this convention, $h(\theta_1) \equiv h(\theta_1, \theta_1)$ and (16) follows from the truth-telling specialization of the two-argument expression for $h(\cdot, \cdot)$.

[6]Equivalently, θ_2^b is the truth-telling specialization of the lower limit $\min\{p_b^{-1}(\theta_1''), \overline{\theta}\}$ in (15).

Under truthful resale-stage behavior, and for an arbitrary primary-market report θ_1', the expected payoff from the consumption or resale of the $q(\theta_1')$ units obtained in the primary market is summarized by the following per-unit term:

$$h(\theta_1) = \theta_1 + \frac{1}{2}\left(1 - F\left(p_s\left(\theta_1\right)\right)\right)\left(p_s\left(\theta_1\right) - \theta_1\right) + \frac{1}{2}\int_{\theta_2^b}^{\overline{\theta}} \left(p_b\left(\theta_2\right) - \theta_1\right) dF\left(\theta_2\right) \qquad (16)$$

Lemma 3. (*Case-I resale virtual values*) *For every non-decreasing primary allocation q considered in Case I, let h_q denote the function in (16) induced by q through p_b. Under the regularity conditions in Lemma 2, h_q is absolutely continuous and satisfies*

$$h_q(\theta) \geq \theta \quad \textit{and} \quad h_q'(\theta) = 1 - \frac{1}{2}\left(1 - F(p_s(\theta))\right) - \frac{1}{2}\left(1 - F(\theta_2^b(\theta))\right) \in [0,1]$$

for almost every θ, where $\theta_2^b(\theta) = p_b^{-1}(\theta)$.

Definition 1 (Resale-adjusted virtual valuation)**.** *For any feasible Case-I primary allocation q, let h_q be the resale-adjusted marginal benefit in (16) induced by q through the posted resale prices p_s and p_b. The associated resale-adjusted virtual valuation is*

$$\varphi_q(\theta) \equiv h_q(\theta) - \frac{1 - F(\theta)}{f(\theta)} h_q'(\theta).$$

When the allocation is clear from context, we write φ for φ_q.

Assumption 1 (Strict Case-I dominance)**.** *For the no-resale optimal allocation q^N, $\varphi_{q^N}(\theta) > \psi(\theta)$ on a set of positive measure on which q^N is positive, where φ_q is the resale-adjusted virtual valuation in Definition 1.*

This strictness condition is mild. By Lemma 3, $h_q'(\theta) < 1$ whenever either $p_s(\theta) < \overline{\theta}$ or $\theta_2^b(\theta) < \overline{\theta}$. Hence, under a continuous regular distribution, Assumption 1 is implied whenever the no-resale optimum serves a positive-measure set of interior types. The appendix records the fixed-point existence statement and the protocol-robust version of the Case-I comparison.

Lemma 4. *In Case I, fix a non-decreasing primary allocation q and the posted resale continuation in Lemma 2. For any incentive-compatible primary payment T implementing q, the expected revenue from a single consumer satisfies*

$$\int_{\underline{\theta}}^{\overline{\theta}} T(\theta) dF(\theta) = \int_{\underline{\theta}}^{\overline{\theta}} \varphi(\theta) q(\theta) dF(\theta) + g(\underline{\theta}) - u(\underline{\theta}),$$

where $u(\underline{\theta})$ is the lowest type's truthful rent. At a revenue-maximizing menu the lowest type's participation constraint binds, and under the posted resale continuation $g(\underline{\theta}) = 0$.

Hence the monopolist's expected revenue from a single consumer is

$$\int_{\underline{\theta}}^{\overline{\theta}} \left(h(\theta) - \frac{1-F(\theta)}{f(\theta)} h'(\theta) \right) q(\theta) dF(\theta) \equiv \int_{\underline{\theta}}^{\overline{\theta}} \varphi(\theta) q(\theta) dF(\theta) \tag{17}$$

Lemma 4 suggests that the monopolist's revenue in this case hinges only on consumers' willingness to buy in the primary market, which is a direct result of the separability of the two stages.

Dependence on the primary allocation. Although the resale-adjusted virtual valuation φ_q in Definition 1 is written in Lemma 4 as if exogenous, it is in fact a functional of q: while $p_s(\cdot)$ is determined by (13) and depends only on F, the buy-price $p_b(\cdot)$ is determined by (14), which involves q on both sides. Hence h_q, and therefore φ_q, depends on the underlying primary-market allocation. Lemma 3 is the key condition that keeps this dependence from overturning the comparison: for any feasible q, it implies that the with-resale objective evaluated at q weakly dominates the no-resale objective at the same q, with strict dominance under Assumption 1.

Next, since the two consumers are symmetric, the expected revenue of the monopolist can be simplified as

$$\int_{\underline{\theta}}^{\overline{\theta}} \int_{\underline{\theta}}^{\overline{\theta}} \left[T(\theta_1) + T(\theta_2) \right] dF(\theta_1) dF(\theta_2) = 2 \int_{\underline{\theta}}^{\overline{\theta}} T(\theta) dF(\theta).$$

The monopolist's problem in the primary market is then given by

$$\max_{q,T} \quad 2 \int_{\underline{\theta}}^{\overline{\theta}} T(\theta) dF(\theta) - \int_{\underline{\theta}}^{\overline{\theta}} \int_{\underline{\theta}}^{\overline{\theta}} c\left(q(\theta_1) + q(\theta_2) \right) dF(\theta_1) dF(\theta_2)$$

subject to

$$u(\theta) \geq u(\theta, \theta', \theta'')$$

and

$$u(\theta) \geq 0$$

for all $\theta, \theta', \theta'' \in [\underline{\theta}, \overline{\theta}]$ where $u(\theta, \theta', \theta'') = q(\theta') h(\theta, \theta'') + g(\theta, \theta'') - T(\theta')$ and $u(\theta) \equiv u(\theta, \theta, \theta)$.

These observations lead to the first main comparison.

Proposition 1. *Suppose the regularity conditions in Lemmas 2 and 3 hold. If the monopolist has a convex cost function, and the consumers have linear utility functions, then the maximal expected profit achievable with the all-or-nothing unit-price resale continuation in Lemma 2 is weakly higher than the maximal expected profit achievable in an environment without resale. Under Assumption 1, the comparison is strict.*

4. **Case II: Linear Cost and Concave Utility**

In this section, we suppose that the monopolist's cost function in equation (1) is linear and of the form $c(q) = cq$ where $c > 0$ is constant; and that consumers' marginal utility is decreasing. More specifically, $v(q)$ in equation (2) satisfies $v(0) = 0$, $v'(0) = \infty$, and $v'(q) > 0$, $v''(q) < 0$ for all $q > 0$. In this environment the resale stage is not separable in the way it is in Case I. The key comparison instead comes from a mimicking argument: any with-resale menu in the maintained mechanism class is weakly dominated, from the monopolist's perspective, by a no-resale certainty-equivalent menu.

The section again works backward. We first characterize the proposer and responder problems in the resale stage and record a sufficient condition under which the responder's resale-stage participation constraint is slack away from the lowest type. We then use the induced continuation values to write the primary-market problem. The profit comparison itself uses only the mimicking restrictions implied by joint truthfulness, together with concavity of v and constant marginal cost.

4.1. **Analysis of the Resale Market.**

In the resale market, each consumer has an equal chance of proposing a menu of buy and sell offers to the other. Focusing on consumer 1, we study the proposer's problem first and then move on to the responder's problem. Consumer 2's problems are analogous.

When we study both roles of consumer 1, we suppose that she is of taste θ_1 and has reported her taste as θ_1' in the primary market. Thus, she has already acquired $q(\theta_1')$ units from the monopolist and already paid $T(\theta_1')$ to him. Since no new information about consumer 2's taste is revealed, consumer 1 would still believe that consumer 2's taste, θ_2, is distributed over the interval $[\underline{\theta}, \overline{\theta}]$ according to the cumulative distribution function, F. Consumer 1 also believes that if consumer 2's taste is θ_2 then she has purchased $q(\theta_2)$ units in the primary market and has paid $T(\theta_2)$. The latter follows from the incentive compatibility of the menu offered by the monopolist.

4.1.1. *Proposer's Problem.*

When chosen to be the proposer, consumer 1 proposes consumer 2 a menu, (S, r), which is composed of buy and sell offers of the form $(S(\theta_2), r(\theta_2))$, each of which is directed to consumer 2 of taste θ_2. In a buy offer both $S(\theta_2)$ and $r(\theta_2)$ are positive, while in a sell offer both components of the offer are

negative.[7] Note that consumer 1 cannot buy more than $q(\theta_2)$ units from consumer 2 of taste θ_2 and cannot sell her more than $q(\theta_1')$ units. Thus, in any menu that consumer 1 could propose all offers must satisfy $r(\theta_2) \in [-q(\theta_1'), q(\theta_2)]$.

The proposed menu, (S, r), maximizes consumer 1's expected continuation payoff which - without loss of generality - ignores the payment she made to the monopolist. More formally, it solves

$$\max_{\substack{S(\theta_2),\\ -q(\theta_1')\leq r(\theta_2)\leq q(\theta_2)}} \int_{\underline{\theta}}^{\overline{\theta}} [\theta_1 v(q(\theta_1') + r(\theta_2)) - S(\theta_2)] dF(\theta_2)$$

subject to

$$(\mathbf{IC})\; Y(\theta_2) \geq Y(\theta_2, \theta_2') \tag{18}$$

$$(\mathbf{IR})\; Y(\theta_2) \geq \theta_2 v(q(\theta_2)) \tag{19}$$

for all $\theta_2, \theta_2' \in [\underline{\theta}, \overline{\theta}]$ where

$$Y(\theta_2, \theta_2') \equiv \theta_2 v(q(\theta_2) - r(\theta_2')) + S(\theta_2') \tag{20}$$

represents the continuation payoff of the responder of taste θ_2 if (after having reported to the monopolist that she is of taste θ_2) she reports to the proposer that she is of taste, θ_2'; and $Y(\theta_2) \equiv Y(\theta_2, \theta_2)$ represents her payoff when she reports truthfully also in the secondary market.

Denote the optimal solution by $(S(\theta_2|\theta_1, \theta_1'), r(\theta_2|\theta_1, \theta_1'))$ and its value function by $U^P(\theta, \theta')$ as the *expected continuation payoff of the proposer.*

(18) implies that $Y(\cdot)$ is a maximum of a family of functions given in (20). Then, using the envelope theorem, we can calculate how responder's expected continuation payoff changes with her taste:

$$Y'(\theta_2) = v(q(\theta_2) - r(\theta_2)) + \theta_2 v'(q(\theta_2) - r(\theta_2)) q'(\theta_2) \equiv k(\theta_2) \tag{21}$$

A marginal increase in taste θ_2 results in a twofold gain for the responder. The first term on the right-hand side of (21) represents the increase in the benefit received from the consumption of inframarginal $q(\theta_2) - r(\theta_2)$ units and the second term represents the marginal benefit due to infinitesimally higher consumption.

[7] In a buy offer, $(S(\theta_2), r(\theta_2))$, consumer 1 proposes to buy $r(\theta_2)$ units from consumer 2 at a payment of $S(\theta_2)$.

Integrating both sides of (21) yields

$$Y(\theta_2) = \int_{\underline{\theta}}^{\theta_2} k(\theta)d\theta + Y(\underline{\theta}). \tag{22}$$

Thus, for any given q proposed by the monopolist in the primary market, responder's expected continuation payoff is only determined by r up to an additive constant, namely $Y(\underline{\theta})$. Since $Y(\theta_2) \equiv Y(\theta_2, \theta_2)$, substituting the truthful continuation payoff into (22) and rearranging yields

$$S(\theta_2) = \int_{\underline{\theta}}^{\theta_2} k(\theta)d\theta - \theta_2 v(q(\theta_2) - r(\theta_2)) + Y(\underline{\theta}).$$

Then the expected payment that the proposer will make is

$$\begin{aligned}\int_{\underline{\theta}}^{\overline{\theta}} S(\theta_2)dF(\theta_2) &= \int_{\underline{\theta}}^{\overline{\theta}} \left[\int_{\underline{\theta}}^{\theta_2} k(\theta)d\theta - \theta_2 v(q(\theta_2) - r(\theta_2))\right] dF(\theta_2) + Y(\underline{\theta}) \\ &= \int_{\underline{\theta}}^{\overline{\theta}} \left[\frac{1 - F(\theta_2)}{f(\theta_2)} k(\theta_2) - \theta_2 v(q(\theta_2) - r(\theta_2))\right] dF(\theta_2) + Y(\underline{\theta})\end{aligned}$$

where the second equality follows from

$$\int_{\underline{\theta}}^{\overline{\theta}} \int_{\underline{\theta}}^{\theta_2} k(\theta)d\theta dF(\theta_2) = \int_{\underline{\theta}}^{\overline{\theta}} \int_{\theta}^{\overline{\theta}} dF(\theta_2)k(\theta)d\theta = \int_{\underline{\theta}}^{\overline{\theta}} \frac{1 - F(\theta_2)}{f(\theta_2)} k(\theta_2)dF(\theta_2).$$

Substituting the expected payment back, we can rewrite the proposer's problem as

$$\max_{r(\cdot), Y(\underline{\theta})} \int_{\underline{\theta}}^{\overline{\theta}} \left[\begin{array}{c} \theta_1 v(q(\theta_1') + r(\theta_2)) + \left(\theta_2 - \frac{1-F(\theta_2)}{f(\theta_2)}\right) v(q(\theta_2) - r(\theta_2)) \\ -\frac{1-F(\theta_2)}{f(\theta_2)} \theta_2 v'(q(\theta_2) - r(\theta_2))q'(\theta_2) \end{array}\right] dF(\theta_2) - Y(\underline{\theta}) \tag{23}$$

subject to the condition that $(r(\cdot), Y(\underline{\theta}))$ satisfies the following conditions

(IC) k is nondecreasing and satisfies (22); and
(IR) $\int_{\underline{\theta}}^{\theta_2} k(\theta)d\theta + Y(\underline{\theta}) - \theta_2 v(q(\theta_2)) \geq 0$ for all θ_2.[8]

Lemma 5. (*Sufficient condition for resale-stage IR slackness*) *Fix the compact class of absolutely continuous monotone primary allocations considered by the monopolist. For any q in this class and any solution r of the resale-stage proposer problem* (23)*, define*

$$D(\theta) \equiv Y(\theta) - \theta v(q(\theta)).$$

[8]Equivalently, this constraint requires $D(\theta_2) \equiv Y(\theta_2) - \theta_2 v(q(\theta_2)) \geq 0$. Lemma 5 gives the pointwise condition under which D is minimized at the lower endpoint. Without this condition, D can have interior minima and the primary-market envelope representation below would require additional kink terms.

If the induced functions satisfy

$$k(\theta) \geq \frac{d}{d\theta}\left[\theta v(q(\theta))\right] \tag{24}$$

for almost every θ, with strict inequality except possibly on a null set, then the proposer optimally chooses $Y(\underline{\theta})$ so that the responder's resale-stage **(IR)** *binds only at $\underline{\theta}$ and is slack almost surely.*

Checking the resale-stage participation condition. Using the definition of k in (21), condition (24) is equivalently the pointwise inequality

$$\begin{aligned}&\big[v(q(\theta)-r(\theta))-v(q(\theta))\big]\\&\quad+\theta q'(\theta)\big[v'(q(\theta)-r(\theta))-v'(q(\theta))\big]\geq 0\end{aligned}$$

almost everywhere. Thus Lemma 5 reduces the resale-stage participation issue to a check on the resale allocation induced by the primitives and the primary allocation. The profit comparison in Proposition 2 follows from mimicking and certainty equivalence; the IR-slack condition supports the kink-free envelope representation used in the numerical characterization of the with-resale menu. In numerical applications, the condition is verified on the computed resale allocation.

4.1.2. *Responder's Problem.* In the resale market, consumer 1 is equally likely to be the responder. In this case, she will be presented the menu, $(S(\cdot|\theta_2,\theta_2), r(\cdot|\theta_2,\theta_2))$. Although this menu carries information about consumer 2's taste, θ_2, it is no longer relevant for consumer 1. Being of taste θ_1 and having already reported θ_1' in the primary market, the responder either reports the taste

$$\arg\max_{\theta_1''}\rho(\theta_1,\theta_1',\theta_1''|\theta_2)$$

that maximizes her continuation payoff[9]

$$\rho(\theta_1,\theta_1',\theta_1''|\theta_2)\equiv\theta_1 v(q(\theta_1')-r(\theta_1''|\theta_2,\theta_2))+S(\theta_1''|\theta_2,\theta_2),$$

or opts out of the resale market and consumes $q(\theta_1')$. Opting out yields

$$\theta_1 v(q(\theta_1'))$$

[9]When acting as a responder, consumer 1 cannot feasibly report any taste θ_1'' for which $r(\theta_1''|\theta_2,\theta_2)) > q(\theta_1')$. This holds because she can sell at most what she purchased in the primary market.

as continuation payoff. Then, the *expected continuation payoff of the responder* (before being presented the menu $(S(\cdot|\theta_2,\theta_2), r(\cdot|\theta_2,\theta_2))$) is given by

$$U^R(\theta_1,\theta_1') \equiv \int_{\underline{\theta}}^{\overline{\theta}} \max\left\{\theta_1 v(q(\theta_1')), \max_{\theta_1''} \rho(\theta_1,\theta_1',\theta_1''|\theta_2)\right\} dF(\theta_2). \tag{25}$$

Notice that due to the incentive compatibility condition, $\arg\max_{\theta_1''} \rho(\theta_1,\theta_1,\theta_1''|\theta_2) = \theta_1$ must hold. Moreover, $\rho(\theta_1,\theta_1,\theta_1|\theta_2) \geq \theta_1 v(q(\theta_1))$ for all $\theta_1 \in [\underline{\theta},\overline{\theta}]$. The latter follows from the individual rationality condition.

4.2. **Analysis of the Primary Market.** Consumer i has equal chance of being proposer or responder. Then, we can write the overall expected payoff of consumer i who has taste θ_i but reports θ_i' in the primary market as

$$U(\theta_i,\theta_i') \equiv \frac{1}{2}U^P(\theta_i,\theta_i') + \frac{1}{2}U^R(\theta_i,\theta_i') - T(\theta_i').$$

Also, let $U(\theta) \equiv U(\theta,\theta)$ denote the overall expected payoff of consumer i when she reports her taste to the monopolist truthfully. Define $U^P(\theta) \equiv U^P(\theta,\theta)$ and $U^R(\theta) \equiv U^R(\theta,\theta)$.

Since consumers are identical from the perspective of the monopolist and the monopolist's cost function is linear, the nonlinear pricing problem described in (3) takes the following form

$$\max_{T,q} \quad 2\int_{\underline{\theta}}^{\overline{\theta}} [T(\theta) - cq(\theta)]\, dF(\theta) \tag{26}$$

subject to

$$U(\theta) \geq U(\theta,\theta') \tag{27}$$

and

$$U(\theta) \geq 0 \tag{28}$$

for all $\theta,\theta' \in [\underline{\theta},\overline{\theta}]$.

Incentive compatibility conditions in (27) imply that $U(\theta,\theta')$ is maximized at $\theta' = \theta$ for all $\theta \in [\underline{\theta},\overline{\theta}]$. Since $v'(0) = \infty$, this solution arises as an interior solution and satisfies the following first-order condition

$$\frac{1}{2}\int_{\underline{\theta}}^{\overline{\theta}} \theta q'(\theta) v'(q(\theta) + r(\tau \mid \theta)) dF(\tau) + \frac{1}{2}\int_{\underline{\theta}}^{\overline{\theta}} \theta q'(\theta) v'(q(\theta) - r(\theta \mid \tau)) dF(\tau) = T'(\theta) \tag{29}$$

where $r(\tau \mid \theta) \equiv r(\tau \mid \theta, \theta)$ and $S(\tau \mid \theta) \equiv S(\tau \mid \theta, \theta)$.[10]

The intuition for (29) is as follows. Any deviation from true taste θ results in a change in the primary market allocation $q(\theta)$. With equal chance, the consumer ends up being either the proposer or the responder. In each case, her expected gains from consumption is either the first term or the second term on the left-hand side, respectively. Note also that a deviation in the primary market has another implication on the secondary market. Following a misreport, if a consumer ends up being the proposer she will distort allocation function by r' but any change in benefit due to this distortion will be balanced out by a corresponding distortion in S. Since the terms associated with this second effect cancel out, they don't appear in (29). (See footnote 10) Moreover, misreporting the taste in the primary market does also change the outside option of the responder in the secondary market. For the envelope-based characterization, Lemma 5 gives a sufficient condition under which the individual rationality condition of the responder is not binding, so consumers cannot strategize on it. Finally, no information about the primary market is carried across to the secondary market; therefore, misreporting does not change the behavior of the other consumer in any direct way. In short, the left-hand side of (29) represents the expected net marginal benefit from reporting truthfully in the primary market and the right-hand side represents the marginal cost for this consumer.

Again, $U(\cdot)$ is a maximum of a family of functions, by envelope theorem, we obtain

$$U'(\theta) = \int_{\underline{\theta}}^{\overline{\theta}} \left[\frac{1}{2} v(q(\theta) + r(\tau \mid \theta)) + \frac{1}{2} v(q(\theta) - r(\theta \mid \tau)) \right] dF(\tau) \equiv \widetilde{v}(\theta). \tag{30}$$

Integrating both sides of (30) yields

$$U(\theta) = \int_{\underline{\theta}}^{\theta} \widetilde{v}(\tau) d\tau + U(\underline{\theta}).$$

Substituting $U(\theta)$ and rearranging we obtain

$$T(\theta) = \frac{1}{2} U^P(\theta) + \frac{1}{2} U^R(\theta) - \int_{\underline{\theta}}^{\theta} \widetilde{v}(\tau) d\tau - U(\underline{\theta})$$

The sum of the first two terms on the right-hand side represents the expected continuation payoff of consumer of taste θ when she reports truthfully in the primary

[10] Envelope theorem applies when differentiating the value function $U^P(\theta, \theta')$ with respect to θ'. Thus, r and S are treated as constant when differentiating this term with respect to θ'.

market. Evaluating U^P and U^R at $\theta' = \theta$ and rearranging yield

$$\frac{1}{2}U^P(\theta) + \frac{1}{2}U^R(\theta) = \theta\widetilde{v}(\theta) - l(\theta) \tag{31}$$

where $\widetilde{v}$ is defined in (30) and

$$l(\theta) \equiv \frac{1}{2}\int_{\underline{\theta}}^{\overline{\theta}} [S(\tau \mid \theta) - S(\theta|\tau)]dF(\tau).$$

$l(\theta)$ represents the payment that a consumer of taste θ expects to make in the secondary market when she reports truthfully in the primary market. Taking the expectation of $l(\theta)$ over $[\underline{\theta}, \overline{\theta}]$ and simplifying yields

$$\int_{\underline{\theta}}^{\overline{\theta}} l(\theta)\, dF(\theta) = 0.$$

Namely, the net payment that a consumer (ex ante) expects to make or receive in the secondary market is zero. This result is intuitive as the two consumers are identical and the game they play is symmetric.

Substituting $T(\theta)$ into (31), we can calculate the expected payment that a consumer makes to the monopolist as

$$\begin{aligned}
\int_{\underline{\theta}}^{\overline{\theta}} T(\theta)dF(\theta) &= \int_{\underline{\theta}}^{\overline{\theta}} \theta\widetilde{v}(\theta)dF(\theta) - \int_{\underline{\theta}}^{\overline{\theta}} l(\theta)dF(\theta) - \int_{\underline{\theta}}^{\overline{\theta}}\int_{\underline{\theta}}^{\theta} \widetilde{v}(\tau)d\tau dF(\theta) - U(\underline{\theta}) && (32)\\
&= \int_{\underline{\theta}}^{\overline{\theta}} \theta\widetilde{v}(\theta)dF(\theta) - \int_{\underline{\theta}}^{\overline{\theta}}\int_{\underline{\theta}}^{\theta} \widetilde{v}(\tau)d\tau dF(\theta) - U(\underline{\theta}) && (33)\\
&= \int_{\underline{\theta}}^{\overline{\theta}} \theta\widetilde{v}(\theta)dF(\theta) - \int_{\underline{\theta}}^{\overline{\theta}} (1 - F(\theta))\widetilde{v}(\theta)d\theta - U(\underline{\theta}) && (34)\\
&= \int_{\underline{\theta}}^{\overline{\theta}} \left(\theta - \frac{1 - F(\theta)}{f(\theta)}\right)\widetilde{v}(\theta)dF(\theta) - U(\underline{\theta}) && (35)
\end{aligned}$$

where the second equality follows from symmetry and the third equality follows from changing the order of integration in the second term of (33) and simplifying it.

Now we can rewrite the monopolist's objective as[11]

$$\max_{q(\cdot),U(\underline{\theta})} \; 2\left(\int_{\underline{\theta}}^{\overline{\theta}} \left[\left(\theta - \frac{1 - F(\theta)}{f(\theta)}\right)\widetilde{v}(\theta) - cq(\theta)\right] dF(\theta) - U(\underline{\theta})\right). \tag{36}$$

[11]Note that $\widetilde{v}(\theta)$ is a function of q and r and that r solves (23).

We find that when the producer's cost is linear and consumers' utility is concave in q, resale is weakly detrimental to the monopolist within the maintained mechanism class. The envelope representation above is useful for characterizing and computing the with-resale menu. The profit comparison below uses only the mimicking restrictions implied by joint truthfulness, together with concavity of v and constant marginal cost.

Lemma 6. (*Mimicking implementability*) *For every jointly truthful with-resale menu considered in Proposition 2, let $\widetilde{v}(\theta)$ be type θ's expected value of final consumption and let $\widetilde{T}(\theta)$ be type θ's expected total payment, including primary- and resale-market transfers. These induced functions satisfy*

$$\theta\widetilde{v}(\theta) - \widetilde{T}(\theta) \geq \theta\widetilde{v}(\theta') - \widetilde{T}(\theta') \quad \textit{for all } \theta, \theta',$$

and

$$\theta\widetilde{v}(\theta) - \widetilde{T}(\theta) \geq 0 \quad \textit{for all } \theta.$$

Equivalently, a consumer of type θ cannot gain by selecting the primary-market option intended for type θ' and then following, in both resale roles, the continuation behavior prescribed for type θ'.

Proposition 2. *Suppose the monopolist has constant marginal cost and consumers have concave utility. For every deterministic, jointly truthful with-resale menu with full nondisclosure in the maintained mechanism class, there exists a no-resale menu that is incentive compatible, individually rational, and yields weakly higher expected profit. Consequently, the supremum expected profit achievable with resale cannot exceed the supremum expected profit achievable without resale.*

Curvature endpoints. Within the deterministic mechanism class with full nondisclosure studied in this paper, Propositions 1 and 2 show opposite profit effects at the two curvature endpoints: convex production cost with linear utility makes resale strictly profit enhancing, while constant marginal cost with concave utility makes resale weakly profit reducing. This endpoint summary organizes the two forces rather than giving an if-and-only-if boundary for all pairs of cost and utility curvatures. When both forces are present, resale creates a production-smoothing channel that can raise the monopolist's profit and a certainty-equivalent channel that can lower it. The numerical exercises below therefore report parameter-grid robustness checks within each endpoint case and nested checks for the concave-utility environment.

5. **Numerical Exploration**

The numerical exercises in this section combine baseline examples with parameter-grid robustness checks. The baseline examples use the canonical parameterizations that illustrate the two forces in the analytical results. The full grid results and numerical-implementation details are reported in the Online Appendix. In Case I, the with-resale computations use the resale-adjusted virtual-value representation. In Case II, we report no-resale optima, a diagnostic for the envelope-based resale-stage participation condition, and nested with-resale computations that solve the proposer's resale allocation inside the primary-market calculation.

Exercise	specs	profit change (%)	CS change (%)	TS change (%)
Case I baseline	1	$+6.73$	$+23.55$	$+11.38$
Case I robustness checks	16	$+5.67$ to $+12.43$	$+9.21$ to $+39.71$	$+8.79$ to $+19.01$
Case II baseline	1	-5.43	$+16.15$	$+1.82$
Case II nested checks	9	-10.30 to -0.89	$+6.39$ to $+41.91$	$+0.52$ to $+5.42$

TABLE 1. Summary of the numerical welfare comparisons. Changes are measured relative to the corresponding no-resale benchmark. "CS" denotes ex-ante consumer surplus and "TS" denotes total surplus. Baseline rows use the baseline welfare computations in Table 2; the Online Appendix parameter-grid tables use coarser grids for the broader numerical checks, so overlapping cells can differ slightly.

5.1. **Case I: linear utility function and convex cost function:** $c(q) = q^2, v(q) = q, F(\theta) = \theta$**.** We first consider a specific example where $c(q) = q^2$ and $F(\theta) = \theta$ for $\theta \in [0, 1]$. For this example, we *numerically* solve the monopolist's nonlinear pricing problem, first assuming that the resale market is absent and then that it is present.

Suppose first that resale is not possible. Then the monopolist's problem in (10) can be written as

$$\max_{q(\cdot)} \left[2 \int_0^1 (2\theta - 1)\, q(\theta)\, d\theta - \int_0^1 \int_0^1 \left(q(\theta_1) + q(\theta_2) \right)^2 d\theta_1 d\theta_2 \right].$$

Rearranging the second term and simplifying the entire expression, we obtain

$$\max_{q(\cdot)} 2 \int_0^1 \left(2\theta - 1 - q(\theta) - \int_0^1 q(\tau)\, d\tau \right) q(\theta)\, d\theta.$$

Using the deterministic projected-gradient computation used for the parameter-grid checks reported in the Online Appendix, the optimal no-resale profit is 0.0621.

Next suppose that resale is possible. Then the monopolist maximizes the resale-adjusted objective characterized in Lemma 4 by offering the nonlinear pricing scheme shown in Figure 1 and obtains profit of 0.0663.

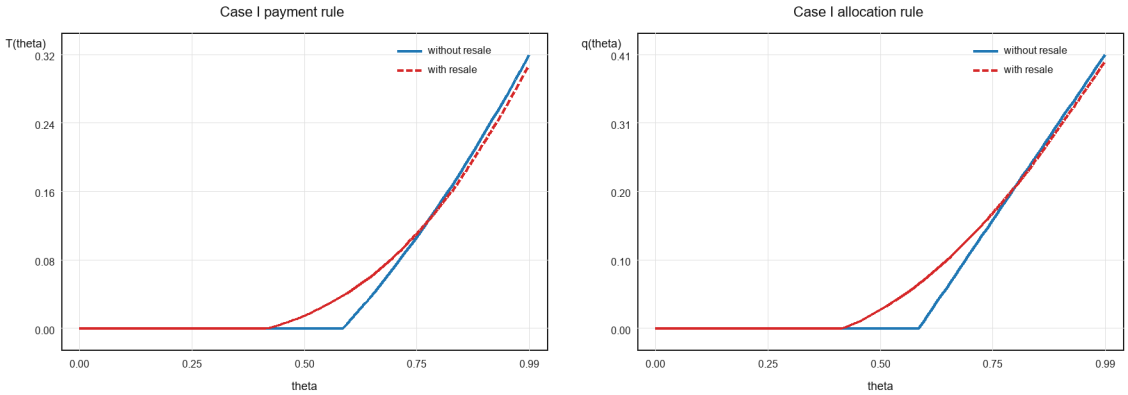


FIGURE 1. Optimal payment and allocation rules with and without resale.

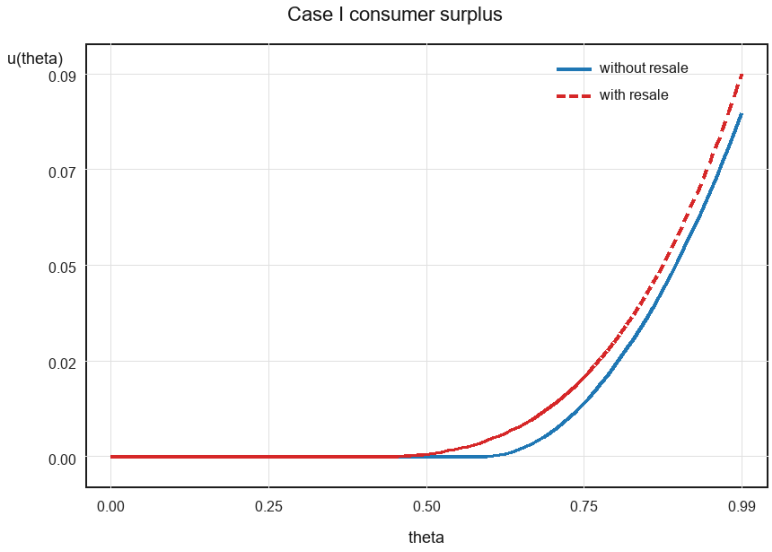


FIGURE 2. Consumer surplus with and without resale

Figure 1 illustrates the impact of resale on the monopolist's nonlinear price schedule. In the right panel, when resale is possible, the monopolist serves some types that are excluded without resale, while still excluding the lowest types. Relative to the no-resale menu, he also offers smaller quantities to the highest types. Resale increases computed monopoly profit by approximately 6.7%. The numerical example illustrates the mechanism behind this change:

As shown above, resale gives consumers an additional benefit from the option to sell to higher-taste consumers in the resale market (i.e., $h(\theta) \geq \theta$ for all $\theta \in [\underline{\theta}, \overline{\theta}]$, with strict inequality on the relevant interior types). This additional marginal benefit decreases with the consumer's taste for the good (i.e., $h'(\theta) \leq 1$ for all $\theta \in [\underline{\theta}, \overline{\theta}]$). A consumer can also gain positive expected surplus from buying in the resale market when purchase opportunities are active (namely, $g(\theta) > 0$ on the served interior region where resale purchases can occur).

The right panel of Figure 1 shows that the allocation function q is flatter when resale is possible. Lemma 3 identifies the force behind the profit comparison: $h(\theta) \geq \theta$ and $h'(\theta) \leq 1$, so the resale-adjusted virtual value weakly dominates the no-resale virtual value, with strict dominance on the served interior types covered by Assumption 1. Convex production costs then make the flatter allocation economically valuable by lowering the monopolist's expected marginal production cost. Thus, the monopolist sells more on average in equilibrium when resale is available.

With the introduction of the resale market, the seller does not merely redistribute a fixed amount of production across consumers. Instead, he sells less to the highest types, redirects part of that output toward lower types, and expands total production.

5.2. **Case I parameter-grid robustness checks .** To check whether the qualitative conclusion is sensitive to the chosen parameterization, we re-solve the Case-I problem on a grid of cost curvatures $\alpha \in \{1.5, 2.0, 2.5, 3.0\}$ (so $c(q) = q^{\alpha}$) and four type distributions: uniform on $[0, 1]$, $\mathrm{Beta}(2, 2)$, $\mathrm{Beta}(2, 5)$, and $\mathrm{Beta}(5, 2)$. The first is the symmetric uniform benchmark, the second is symmetric and bell-shaped, the third concentrates mass on low types, and the fourth concentrates mass on high types. For each (F, α) pair we compute the no-resale solution and a with-resale fixed-point solution on a 60-point grid. The numerical routine uses projected gradient ascent on the cumulative monotonicity cone, with PAV projection at each step; in the with-resale case it iterates between computing the resale-adjusted virtual value induced by the current allocation and re-solving the primary-market problem. The Online Appendix reports the full table and accompanying figure.

Across all sixteen parameterizations the computed with-resale profit exceeds the computed no-resale profit, consistent with Proposition 1. Two patterns emerge from the parameter-grid results. First, the gain from allowing resale increases with cost convexity α. The sign of the comparison comes from the resale-adjusted

virtual-value dominance, while the size of the gain reflects how valuable the induced allocation flattening is when production costs are more curved. Second, the gain is largest when the type distribution puts more mass on high types ($\mathrm{Beta}(5,2)$), where without resale the monopolist would otherwise serve only the upper tail, and resale induces him to spread production toward middle types as well.

5.3. **Case II: Concave utility function and linear cost function:** $c(q) = q, v(q) = \sqrt{q}, F(\theta) = \theta$**.** We next consider the case where $c(q) = q, v(q) = \sqrt{q}, F(\theta) = \theta$. The results are shown in Figure 3. The computed with-resale profit is 0.0788, compared with the no-resale optimum of approximately 0.0833 (the analytic value is $1/12$). The numerical comparison thus verifies Proposition 2 in this baseline.

Equilibrium check and sensitivity of the IR-slack condition. Proposition 2 uses the mimicking argument in Lemma 6. Lemma 5 plays a computational role: it supports the envelope-based characterization of the with-resale menu. The nested computations reported in the Online Appendix solve the proposer's resale allocation inside the primary-market calculation; in all nine reported specifications, the computed resale-stage participation slack is nonnegative on the grid and the with-resale profit lies below the no-resale benchmark. The Online Appendix also reports a sensitivity exercise using non-equilibrium smoothing candidates to illustrate the content of the IR-slack condition.

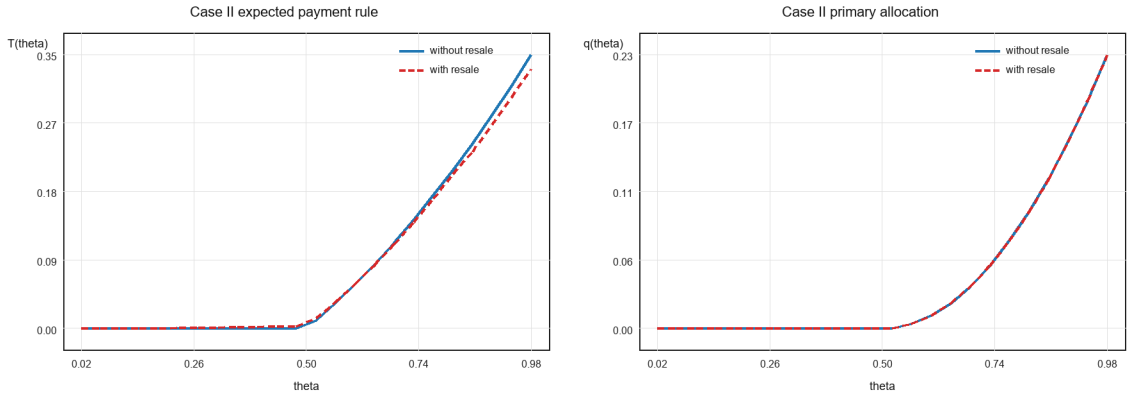


FIGURE 3. Optimal payment and allocation rules with and without resale.

Figure 4 shows that consumers on the lower end of the valuation distribution can gain from resale even though monopoly profit falls.

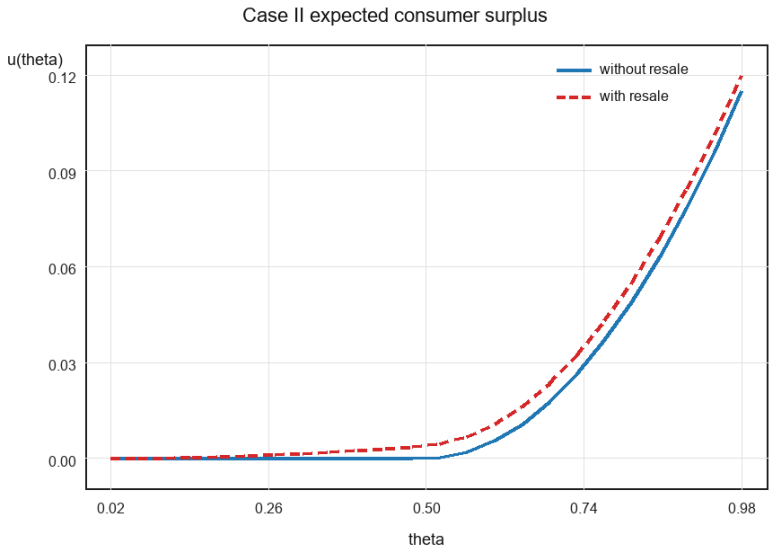


FIGURE 4. Expected consumer surplus with and without resale

5.4. **Case II parameter-grid robustness checks .** For Case II we compute the no-resale optimum across the same four distributions as in Section 5.2 and three utility curvatures $\beta \in \{0.3, 0.5, 0.7\}$ (so $v(q) = q^\beta$). The no-resale objective is concave in q for every $\beta \in (0, 1)$ and is solved by the same projected-gradient method as Case I. The Online Appendix reports the resulting no-resale profits.

By Proposition 2, the with-resale profit π^R is bounded above by π^{NR} throughout the maintained mechanism class. The Online Appendix also reports the sensitivity exercise for the envelope-based resale computation and the stronger 3×3 grid of nested with-resale computations. In those nested specifications the computed resale-stage participation slack is nonnegative and the with-resale profit lies below the no-resale benchmark. Finer grids, additional type distributions, and more utility curvatures would further sharpen this numerical comparison.

5.5. **Welfare accounting .** The preceding comparisons focus on monopoly profit. To see whether the monopolist's preference over resale is aligned with consumer and social surplus, we also compute ex-ante consumer surplus and total surplus. Since primary- and secondary-market transfers cancel in the aggregate, total surplus equals monopoly profit plus ex-ante consumer surplus. Table 2 reports the two baseline specifications. In Case I, resale raises profit, consumer surplus, and total surplus: the monopolist serves a larger set of types and the secondary market

reallocates units toward higher-use consumers. In Case II, the nested resale computation lowers monopoly profit but raises consumer surplus and total surplus. The resale continuation need not be ex-post efficient, but in the computed examples it can still raise total surplus because secondary trade partially offsets monopoly under-provision. In this environment, banning resale benefits the monopolist by extracting certainty-equivalent surplus, but consumers prefer the flexibility and reallocation generated by resale.

Rent computation in Case I with resale. In the no-resale case, type θ's rent is the usual envelope term $\int_{\underline{\theta}}^{\theta} q(\tau)d\tau$. In Case I with resale, Lemma 4 gives the truthful payoff representation $u(\theta) = g_q(\theta)+h_q(\theta)q(\theta)-T(\theta)$, where g_q is the expected surplus from resale purchases and h_q is the resale-adjusted marginal benefit of primary-market units. Using $T'(\theta) = h_q(\theta)q'(\theta)$ and normalizing the lowest type's primary-market rent to zero gives

$$u(\theta) = g_q(\theta) + \int_{\underline{\theta}}^{\theta} h_q'(\tau)q(\tau)d\tau,$$

which is the formula used in the consumer-surplus computation.

Case	Environment	Profit	CS	TS	served share
I	no resale	0.06209	0.02369	0.08578	0.410
I	resale	0.06627	0.02927	0.09554	0.580
II	no resale	0.08332	0.04192	0.12520	0.500
II	resale	0.07879	0.04869	0.12748	0.500

TABLE 2. Baseline welfare accounting. "CS" is ex-ante consumer surplus across the two consumers and "TS" is total surplus. The Case-II with-resale row uses the nested resale-continuation solver based on (23) and (30).

The Case-I welfare pattern is robust across the 16 parameterizations reported in the Online Appendix. In every specification, resale raises computed monopoly profit, consumer surplus, and total surplus. Proposition 1 establishes the profit comparison; the consumer- and total-surplus comparisons are numerical welfare patterns. Consumer-surplus gains range from 9.21% to 39.71%, and total-surplus gains range from 8.79% to 19.01%. Thus, in the convex-cost/linear-utility case, the monopolist's incentive to facilitate resale is aligned with both consumer and total surplus in the computed examples.

For Case II, the nested with-resale checks in the Online Appendix cover three type distributions and three utility curvatures. These computations solve the proposer's resale allocation inside the primary-market calculation and verify that the resale-stage participation slack is nonnegative on the computed grid. In every reported entry, resale lowers computed monopoly profit while raising consumer surplus and total surplus. The concave-utility environment therefore gives a policy-relevant conflict over resale rights: the monopolist would like to restrict resale, but consumers and total surplus favor allowing it. On the parameter region covered by these nested checks, a legal regime that prevents the monopolist from enforcing contractual resale bans yields strictly higher computed consumer surplus and total surplus than a regime that lets the monopolist contract around the secondary market.

These welfare patterns close the loop with the examples in the introduction. The Case-I gains correspond to textbook-style resale, where secondary trade reallocates durable copies and makes broader primary-market production attractive. The Case-II conflict corresponds to used-software-style resale, where downstream transferability insures consumers against future-use risk even though the seller would prefer to restrict it.

6. **Conclusion**

This paper studies how resale changes nonlinear pricing when a monopolist sells a durable divisible good to privately informed consumers. Within the deterministic framework with full non-disclosure introduced in the model, resale affects the monopolist through two channels: it changes consumers' willingness to choose primary-market quantities, and it changes the final allocation of goods after primary-market trade.

The two baseline cases show that the effect of resale is not one-sided. With linear consumer utility, the posted unit-price resale continuation raises the resale-adjusted virtual value relative to the no-resale benchmark. Under the stated regularity and strictness conditions, allowing resale is therefore profitable for the monopolist; convex production costs make the resulting allocation flattening a natural and economically meaningful response. With constant marginal cost and concave consumer utility, the certainty-equivalent argument goes in the opposite direction: for every with-resale menu in the maintained class, the monopolist can ban resale

and replicate consumers' expected utility with weakly lower production, so resale cannot raise profit.

The numerical exercises illustrate how these forces appear in the computed allocation and payment rules. In the linear-utility case, resale raises the effective virtual value of primary-market units and flattens the allocation schedule; with convex production costs, this lets the monopolist economize on expected marginal production costs. Section 5 reports a 16-specification parameter-grid robustness check for this case, and the computed gains are positive for profit, consumer surplus, and total surplus throughout. In the concave-utility case, resale exposes consumers to variation in final consumption, and banning resale lets the monopolist extract the associated certainty-equivalent surplus. The Case-II no-resale parameter-grid checks, sufficient-condition check, and nested resale computations clarify both the benchmark implied by the theorem and the welfare conflict: computed resale lowers monopoly profit but raises consumer surplus and total surplus in all nine nested checks. This provides a direct economic rationale for legal protections of resale rights, such as first-sale-style limits on contractual resale bans, when resale improves consumer and total surplus even though the seller would prefer to restrict it.

The framework also points to natural extensions. Stochastic disclosure mechanisms, richer resale protocols, and alternative equilibrium selections would add further strategic margins. Higher-resolution nested computations–using finer grids, additional type distributions, and more utility curvatures–would further sharpen the numerical evidence. Taken together, the analytical and numerical results show that the profitability and welfare effects of resale depend critically on cost curvature, utility curvature, and the resale protocol.

Appendix A. Proofs

Proof of Lemma 1. We prove the two claims in turn.

Step 1: Additive separability across stages. Under the linear specification $v(q) = q$, consumer i's payoff from the primary stage and the resale stage equals

$$\theta_i\big[q(\theta_i') + r(\theta_2)\big] - T(\theta_i') - S(\theta_2) = \big[\theta_i q(\theta_i') - T(\theta_i')\big] + \big[\theta_i r(\theta_2) - S(\theta_2)\big],$$

which is additively separable in the two stages.

Step 2: Separability of the proposer's menu. Recall the sign convention $r(\theta_2) \in [-q(\theta_1'), q(\theta_2)]$: $r(\theta_2) < 0$ means the proposer transfers $|r(\theta_2)|$ units to the responder in exchange for $|S(\theta_2)|$ (a sell offer); $r(\theta_2) > 0$ means the proposer receives $r(\theta_2)$ units against payment $S(\theta_2)$ (a buy offer).

Suppose, for contradiction, that the optimal menu prescribes $r(\theta_2) < 0$ for some $\theta_2 < \theta_1$ (selling to a lower-type responder). The responder's individual rationality requires

$$\theta_2\big[q(\theta_2) - r(\theta_2)\big] + S(\theta_2) \geq \theta_2 q(\theta_2),$$

which simplifies to $S(\theta_2) \geq \theta_2 r(\theta_2)$ (note that both sides are non-positive since $r(\theta_2) < 0$). The proposer's continuation payoff on this offer is

$$\theta_1 r(\theta_2) - S(\theta_2) \leq \theta_1 r(\theta_2) - \theta_2 r(\theta_2) = (\theta_1 - \theta_2)\, r(\theta_2) < 0,$$

where the final inequality uses $\theta_1 > \theta_2$ and $r(\theta_2) < 0$. Setting $r(\theta_2) = S(\theta_2) = 0$ for that type yields a strictly higher payoff. Hence the optimal menu never sells to lower-type responders. By a symmetric argument applied to $r(\theta_2) > 0$ with $\theta_2 > \theta_1$, the optimal menu never buys from higher-type responders.

Once cross-direction trades are excluded, the proposer's expected payoff decomposes additively into a sell-side problem (over $\theta_2 > \theta_1$) and a buy-side problem (over $\theta_2 < \theta_1$). □

Proof of Lemma 2. After Lemma 1, the sell side and the buy side are two independent one-agent screening problems with linear per-unit payoffs.

On the sell side, the proposer has $q(\theta_1')$ units and faces a responder with value θ_2 per unit. The standard one-agent monopoly argument (Myerson, 1981) allocates the whole stock whenever the responder's virtual valuation exceeds the proposer's own value θ_1, and allocates nothing otherwise. Under regularity this allocation is monotone and is implemented by a posted unit price $p_s(\theta_1)$ satisfying

$$\theta_1 = p_s - \frac{1 - F(p_s)}{f(p_s)}.$$

Equivalently, $p_s(\theta_1)$ maximizes (11).

On the buy side, the proposer procures the responder's endowment $q(\theta_2)$ from a responder whose opportunity cost is θ_2 per unit. Within the all-or-nothing unit-price protocol, a buy offer is summarized by a unit price p: every responder with $\theta_2 \leq p$ sells her entire endowment and every responder with $\theta_2 > p$ refuses. The proposer's expected buy-side payoff is therefore

$$B(p;\theta_1) = \int_{\underline{\theta}}^{p} q(t)(\theta_1 - p)dF(t).$$

At an interior optimum,

$$\frac{\partial B(p;\theta_1)}{\partial p} = q(p)f(p)(\theta_1 - p) - \int_{\underline{\theta}}^{p} q(t)dF(t) = 0,$$

or equivalently

$$\theta_1 = p_b + \frac{\int_{\underline{\theta}}^{p_b} q(t)dF(t)}{q(p_b)f(p_b)}.$$

Thus the proposer buys from exactly those responder types whose weighted virtual cost C_q is below θ_1. Under the assumed monotonicity of C_q, this cutoff is globally optimal within the all-or-nothing unit-price protocol and is equivalently the maximizer of (12).

Finally, $p_s(\theta_1)$ is non-decreasing because it is the inverse of the regular sell-side virtual valuation. Similarly, $p_b(\theta_1)$ is non-decreasing because it is the inverse of the non-decreasing weighted virtual cost C_q. □

Proof of Lemma 3. The inequality $h_q(\theta) \geq \theta$ follows directly from (16). The sell-side resale term is non-negative because $p_s(\theta) \geq \theta$, and the responder-side sale term is non-negative because $p_b(t) \geq \theta$ for every $t \geq \theta_2^b(\theta)$.

It remains to compute the derivative. Write

$$M_s(\theta) = \frac{1}{2}\left(1 - F(p_s(\theta))\right)\left(p_s(\theta) - \theta\right)$$

for the sell-side option value. Since $p_s(\theta)$ maximizes the expression in (11), the envelope theorem gives

$$M_s'(\theta) = -\frac{1}{2}\left(1 - F(p_s(\theta))\right)$$

at every differentiability point. Next write

$$M_b(\theta) = \frac{1}{2}\int_{\theta_2^b(\theta)}^{\overline{\theta}} \left(p_b(t) - \theta\right) dF(t).$$

By the Leibniz rule, the derivative of the lower limit drops out because $p_b(\theta_2^b(\theta)) = \theta$. Hence

$$M_b'(\theta) = -\frac{1}{2}\left(1 - F(\theta_2^b(\theta))\right).$$

Since $h_q(\theta) = \theta + M_s(\theta) + M_b(\theta)$, we obtain

$$h_q'(\theta) = 1 - \frac{1}{2}\left(1 - F(p_s(\theta))\right) - \frac{1}{2}\left(1 - F(\theta_2^b(\theta))\right).$$

Both probability terms lie in $[0,1]$, so $h_q'(\theta) \in [0,1]$ almost everywhere. Absolute continuity follows from the absolute continuity of these envelope value functions under the maintained regularity conditions. □

Proof of Lemma 4. The expected payoff of a consumer who has taste θ and announces her taste as θ' in the primary market can be written as

$$u\left(\theta, \theta', \theta''\right) = q\left(\theta'\right) h\left(\theta, \theta''\right) + g\left(\theta, \theta''\right) - T\left(\theta'\right).$$

Applying the primary-market IC constraint at truthful reports implies the following first-order conditions for all $\theta \in [\underline{\theta}, \overline{\theta}]$:

$$T'\left(\theta\right) = h\left(\theta\right) q'\left(\theta\right)$$

Integrating both sides and then integration by parts yields

$$\begin{aligned} T\left(\theta\right) &= \int_{\underline{\theta}}^{\theta} h\left(\tau\right) q'\left(\tau\right) d\tau + T\left(\underline{\theta}\right) \\ &= h\left(\theta\right) q\left(\theta\right) - h\left(\underline{\theta}\right) q\left(\underline{\theta}\right) - \int_{\underline{\theta}}^{\theta} h'\left(\tau\right) q\left(\tau\right) d\tau + T\left(\underline{\theta}\right) \\ &= h\left(\theta\right) q\left(\theta\right) - \int_{\underline{\theta}}^{\theta} h'\left(\tau\right) q\left(\tau\right) d\tau + g\left(\underline{\theta}\right) - u\left(\underline{\theta}\right). \end{aligned}$$

It is optimal for the monopolist to leave no surplus to the consumer with the lowest taste. Thus, we have $u(\underline{\theta}) = 0$. Moreover, $g(\underline{\theta}) = 0$. That is, the consumer with the lowest taste cannot gain any surplus by making purchases in the resale market. This holds because $p_b(\underline{\theta}) = \underline{\theta}$, so she cannot make a buy offer that will please the responder, and because no sell offer is accepted by the lowest type since $p_s(\theta_2) > \underline{\theta}$ for all $\theta_2 < \overline{\theta}$. Therefore, the last two terms vanish. As a result, we can compute the expected payment that a consumer makes to the monopolist as

$$\int_{\underline{\theta}}^{\overline{\theta}} T(\theta) dF(\theta) = \int_{\underline{\theta}}^{\overline{\theta}} h(\theta) q(\theta) dF(\theta) - \int_{\underline{\theta}}^{\overline{\theta}} \int_{\underline{\theta}}^{\theta} h'(\tau) q(\tau) d\tau dF(\theta).$$

Next, we take the second term on the right-hand side and apply integration by parts to obtain

$$\begin{aligned} -\int_{\underline{\theta}}^{\overline{\theta}} \int_{\underline{\theta}}^{\theta} h'(\tau)q(\tau)d\tau dF(\theta) &= (1-F(\theta)) \int_{\underline{\theta}}^{\theta} h'(\tau)q(\tau)d\tau \Big|_{\theta=\underline{\theta}}^{\theta=\overline{\theta}} \\ &\quad - \int_{\underline{\theta}}^{\overline{\theta}} h'(\theta)q(\theta)(1-F(\theta))d\theta \\ &= -\int_{\underline{\theta}}^{\overline{\theta}} h'(\theta)q(\theta)\frac{1-F(\theta)}{f(\theta)}dF(\theta) \end{aligned}$$

Substituting back, we obtain

$$\int_{\underline{\theta}}^{\overline{\theta}} T(\theta)dF(\theta) = \int_{\underline{\theta}}^{\overline{\theta}} \left(h(\theta) - \frac{1-F(\theta)}{f(\theta)}h'(\theta) \right) q(\theta)dF(\theta) \equiv \int_{\underline{\theta}}^{\overline{\theta}} \varphi(\theta)q(\theta)dF(\theta)$$

□

Protocol robustness and incentive compatibility in Case I. The proof of Proposition 1 uses the posted-price resale continuation only to verify three properties. First, truthful expected utility is separable in the primary allocation and payment:

$$u(\theta, \theta') = h_q(\theta)q(\theta') + g_q(\theta) - T(\theta').$$

Second, the induced resale-adjusted marginal benefit satisfies $h_q(\theta) \geq \theta$ and $h_q'(\theta) \in [0,1]$ almost everywhere. The lower bound makes the primary menu single-crossing; the upper bound delivers the virtual-value dominance used in the profit comparison. Third, the no-resale and with-resale comparisons use the same monotone primary allocation domain, so holding q fixed also holds the production-cost term fixed. Any resale continuation with full non-disclosure satisfying these three properties gives the same weak profit comparison, with the revenue formula understood at a revenue-maximizing payment schedule where the lowest type's rent is normalized to zero.

A primitive way to generate the first two properties is a linear per-unit resale tariff for each consumer's own units, with the per-unit resale price independent of that consumer's primary-market report and primary holding except through the number of units available for trade. If the unit resale price faced by an individual is P and is taken as exogenous to her report, then the continuation value of an additional primary-market unit is $h_q(\theta) = E[\max\{\theta, P\}]$, so $h_q(\theta) \geq \theta$ and $h_q'(\theta) = \Pr\{P < \theta\} \in [0,1]$ wherever differentiable; purchases from others enter the additive term $g_q(\theta)$. Lemma 2 is one transparent two-consumer protocol that verifies the properties. A competitive price-taking secondary market provides

the corresponding large-market analogue: an individual treats the market-clearing unit price as parametric, so the same per-unit-value logic applies.

The first-order condition $T'(\theta) = h(\theta)q'(\theta)$ used in the proof of Lemma 4 is necessary for incentive compatibility but not, on its own, sufficient. Under the linear-utility specification the consumer's expected payoff from reporting θ' in the primary market and behaving truthfully thereafter takes the form $u(\theta, \theta') = h(\theta)q(\theta') + g(\theta) - T(\theta')$. Since Lemma 3 makes h non-decreasing in θ, u has the Spence-Mirrlees single-crossing property in (θ, θ'). Combined with q non-decreasing–which is verified at the optimum–single-crossing yields global incentive compatibility by standard arguments (see, e.g., Armstrong, 2016). Thus $T'(\theta) = h(\theta)q'(\theta)$ together with monotonicity of q characterizes the incentive-compatible menus.

Proof of Proposition 1. Let q be any non-decreasing primary allocation that is feasible in the no-resale problem. Under Lemma 2, the resale continuation is summarized by the posted prices p_s and p_b, and Lemma 4 gives the single-consumer revenue term

$$\int_{\underline{\theta}}^{\overline{\theta}} \varphi_q(\theta)q(\theta)dF(\theta), \quad \varphi_q(\theta) \equiv h_q(\theta) - \frac{1 - F(\theta)}{f(\theta)}h'_q(\theta).$$

The corresponding no-resale virtual value is $\psi(\theta) = \theta - \frac{1-F(\theta)}{f(\theta)}$. By Lemma 3,

$$\varphi_q(\theta) - \psi(\theta) = \big[h_q(\theta) - \theta\big] + \frac{1 - F(\theta)}{f(\theta)}\big[1 - h'_q(\theta)\big] \geq 0$$

for almost every θ. Therefore, holding the same allocation q fixed, the expected revenue term with resale is weakly larger than the expected revenue term without resale, while the production-cost term is identical:

$$\begin{aligned} &2\int_{\underline{\theta}}^{\overline{\theta}} \varphi_q(\theta)q(\theta)dF(\theta) - \int_{\underline{\theta}}^{\overline{\theta}}\int_{\underline{\theta}}^{\overline{\theta}} c(q(\theta_1) + q(\theta_2))\, dF(\theta_1)dF(\theta_2) \\ \geq\ &2\int_{\underline{\theta}}^{\overline{\theta}} \psi(\theta)q(\theta)dF(\theta) - \int_{\underline{\theta}}^{\overline{\theta}}\int_{\underline{\theta}}^{\overline{\theta}} c(q(\theta_1) + q(\theta_2))\, dF(\theta_1)dF(\theta_2). \end{aligned}$$

Both comparisons are over the common non-decreasing allocation domain: in the no-resale problem this is the standard one-dimensional incentive-compatibility requirement, and in the resale problem it is sufficient by the single-crossing argument in the proof appendix. Taking the supremum over this common domain gives that optimal profit with resale is weakly higher than optimal profit without resale. If q^N is a no-resale maximizer and Assumption 1 holds, evaluating the resale objective at q^N gives a strictly higher value than the no-resale optimum. Hence the result follows. □

Auxiliary Case-I notes.

Lemma 7. (*Existence in the Case-I mechanism class*) *Let $\mathcal{Q}$ be a nonempty, convex, compact set of non-decreasing primary allocations, bounded uniformly by some $\overline{q} < \infty$, and suppose the regularity conditions in Lemmas 2 and 3 hold for every $q \in \mathcal{Q}$. If $q \mapsto h_q$ is continuous and, for each continuation h_q, the monopolist's maximization over $\mathcal{Q}$ has a unique solution in $\mathcal{Q}$, then a symmetric two-stage equilibrium exists in Case I within this mechanism class.*

Lemma 7 combines the constructive resale-subgame equilibrium in Lemma 2 with a fixed point for the primary allocation. For example, a closed convex equicontinuous subset of non-decreasing functions $q : [\underline{\theta}, \overline{\theta}] \to [0, \overline{q}]$ satisfies the compactness and convexity requirements by Arzela–Ascoli. The lemma establishes existence for the symmetric deterministic class with full non-disclosure used in the Case-I comparison.

Proof of Lemma 7. For each $q \in \mathcal{Q}$, Lemma 2 gives a symmetric posted-price resale continuation and Lemma 3 gives the associated resale-adjusted willingness-to-pay function h_q. Holding this continuation fixed, define the monopolist's best-response problem over $\mathcal{Q}$ by

$$\max_{\widehat{q} \in \mathcal{Q}} \left[2 \int_{\underline{\theta}}^{\overline{\theta}} \varphi_q(\theta) \widehat{q}(\theta) dF(\theta) - \int_{\underline{\theta}}^{\overline{\theta}} \int_{\underline{\theta}}^{\overline{\theta}} c(\widehat{q}(\theta_1) + \widehat{q}(\theta_2)) dF(\theta_1) dF(\theta_2) \right],$$

where $\varphi_q(\theta) = h_q(\theta) - \frac{1-F(\theta)}{f(\theta)} h_q'(\theta)$. The objective is continuous in $(q, \widehat{q})$ by the assumed continuity of $q \mapsto h_q$ and the continuity of c. Since $\mathcal{Q}$ is compact, the maximum exists. By the assumed uniqueness, the best response is a single-valued map $\Gamma : \mathcal{Q} \to \mathcal{Q}$; Berge's maximum theorem implies that Γ is continuous. Schauder's fixed-point theorem then gives $q^* \in \mathcal{Q}$ such that $q^* = \Gamma(q^*)$. Together with the resale continuation induced by q^*, this fixed point is a symmetric two-stage equilibrium within the deterministic full non-disclosure mechanism class. □

Proof of Lemma 5. For a fixed q and resale allocation r, the responder's resale-stage surplus relative to opting out is

$$D(\theta) = Y(\theta) - \theta v(q(\theta)).$$

Using (21), its derivative is

$$D'(\theta) = k(\theta) - \frac{d}{d\theta} \left[\theta v(q(\theta)) \right]$$

almost everywhere. Condition (24) therefore implies that D is non-decreasing, and the strict part of the condition implies that it is strictly increasing except possibly on a null set. Since $Y(\underline{\theta})$ enters the proposer's objective in (23) with coefficient -1, the proposer chooses the smallest value of $Y(\underline{\theta})$ consistent with **(IR)**. Because D is minimized at $\underline{\theta}$, all resale-stage participation constraints are satisfied if and only if $D(\underline{\theta}) \geq 0$. If $D(\underline{\theta}) > 0$, the proposer can reduce $Y(\underline{\theta})$ by the same amount, keep every participation constraint satisfied, and strictly increase her objective. Hence optimality requires $D(\underline{\theta}) = 0$. Then $D(\theta) > 0$ for almost every $\theta > \underline{\theta}$, so the responder's resale-stage participation constraint binds only at the lower endpoint and is slack almost surely. □

Proof of Lemma 6. Fix a jointly truthful with-resale menu and two types θ and θ'. Let $D(\theta, \theta')$ be the highest expected payoff type θ can obtain after selecting the primary-market option intended for type θ' and then choosing any feasible continuation behavior in the resale stage. One feasible continuation is the mimicking continuation in which the consumer follows, in both resale roles, the continuation behavior prescribed for type θ'. This continuation is feasible because after choosing the primary option intended for θ', the consumer holds the same primary quantity as a truthful type θ'. Full non-disclosure implies that the opponent's continuation behavior and beliefs do not condition on the consumer's true type or primary report, so the mimicking continuation reproduces the final-consumption distribution and total payment of a truthful type θ'. Therefore, by the definitions of $\widetilde{v}$ and $\widetilde{T}$, this particular continuation gives payoff

$$\theta\widetilde{v}(\theta') - \widetilde{T}(\theta').$$

Therefore,

$$D(\theta, \theta') \geq \theta\widetilde{v}(\theta') - \widetilde{T}(\theta').$$

Joint truthfulness of the with-resale menu implies that truthful reporting in the primary market and truthful continuation in resale weakly dominate every such deviation:

$$\theta\widetilde{v}(\theta) - \widetilde{T}(\theta) \geq D(\theta, \theta').$$

Combining the two inequalities yields the mimicking incentive inequality stated in the lemma. The participation inequality stated in the lemma is the primary-market participation constraint evaluated at the truthful continuation, again using the definitions of $\widetilde{v}$ and $\widetilde{T}$. □

Proof of Proposition 2. Fix an arbitrary deterministic, jointly truthful with-resale menu $M = (T, q)$ with full non-disclosure and its truthful resale continuation. Let $\widetilde{v}(\theta)$ denote the expected value $E[v(\text{final consumption}) \mid \theta]$, let $\widetilde{q}(\theta)$ denote expected final consumption, and let $\widetilde{T}(\theta)$ denote expected total payment, including the primary payment and the net payment made in the resale market.

Because resale only reallocates units and all resale transfers net to zero across the two consumers, ex-ante symmetry gives

$$\int_{\underline{\theta}}^{\overline{\theta}} \widetilde{q}(\theta) dF(\theta) = \int_{\underline{\theta}}^{\overline{\theta}} q(\theta) dF(\theta), \qquad \int_{\underline{\theta}}^{\overline{\theta}} \widetilde{T}(\theta) dF(\theta) = \int_{\underline{\theta}}^{\overline{\theta}} T(\theta) dF(\theta). \tag{37}$$

Lemma 6 implies the mimicking incentive and participation inequalities

$$\theta\widetilde{v}(\theta) - \widetilde{T}(\theta) \geq \theta\widetilde{v}(\theta') - \widetilde{T}(\theta') \quad \text{for all } \theta, \theta', \tag{38}$$

$$\theta\widetilde{v}(\theta) - \widetilde{T}(\theta) \geq 0 \quad \text{for all } \theta. \tag{39}$$

Construct a no-resale certainty-equivalent allocation $\widehat{q}$ by

$$v(\widehat{q}(\theta)) = \widetilde{v}(\theta). \tag{40}$$

Since v is increasing, $\widehat{q}$ is well defined on the range of realized final consumption. Since v is concave, Jensen's inequality gives

$$v(\widetilde{q}(\theta)) \geq \widetilde{v}(\theta) = v(\widehat{q}(\theta)), \tag{41}$$

and hence $\widehat{q}(\theta) \leq \widetilde{q}(\theta)$ for every θ. Combining this pointwise inequality with (37) yields

$$\int_{\underline{\theta}}^{\overline{\theta}} \widehat{q}(\theta) dF(\theta) \leq \int_{\underline{\theta}}^{\overline{\theta}} q(\theta) dF(\theta). \tag{42}$$

Now consider the no-resale menu $(\widetilde{T}, \widehat{q})$. Using (40), inequalities (38) and (39) become

$$\theta v(\widehat{q}(\theta)) - \widetilde{T}(\theta) \geq \theta v(\widehat{q}(\theta')) - \widetilde{T}(\theta') \quad \text{for all } \theta, \theta', \tag{43}$$

$$\theta v(\widehat{q}(\theta)) - \widetilde{T}(\theta) \geq 0 \quad \text{for all } \theta. \tag{44}$$

Thus $(\widetilde{T}, \widehat{q})$ is incentive compatible and individually rational when resale is not possible. No separate monotonicity argument is needed: global incentive compatibility is the implementability condition for the direct no-resale menu, and monotonicity follows from standard single-crossing arguments.

Finally, constant marginal cost and (37)-(42) imply that this no-resale menu gives weakly higher expected profit than the original with-resale menu:

$$\int_{\underline{\theta}}^{\overline{\theta}} \big[\widetilde{T}(\theta) - c\widehat{q}(\theta)\big] dF(\theta) \geq \int_{\underline{\theta}}^{\overline{\theta}} \big[T(\theta) - cq(\theta)\big] dF(\theta).$$

This is the per-consumer profit comparison; multiplying both sides by two gives the monopolist's total expected profit comparison. The construction applies to every with-resale menu M in the maintained mechanism class. Therefore the supremum of no-resale profits is weakly at least the supremum of with-resale profits. □

Case-II existence note. A parallel in-class existence result for Case II follows under standard fixed-point hypotheses: compactness and convexity of the admissible monotone allocation class, nonempty compact upper-hemicontinuous resale-stage solution correspondences for (23), continuity of the induced primary-market payoff, and a nonempty convex-valued upper-hemicontinuous primary best-response correspondence. Under these conditions a Kakutani–Fan–Glicksberg argument gives a symmetric equilibrium in the same deterministic class with full nondisclosure. Proposition 2 is a dominance result for every jointly truthful with-resale menu in the maintained class, so the profit ranking is independent of selecting a particular optimal with-resale menu.

References

Armstrong, Mark, "Nonlinear Pricing," *Annual Review of Economics*, 2016, *8* (1), 583–614.

Bulow, Jeremy I., "Durable-Goods Monopolists," *Journal of Political Economy*, 1982, *90* (2), 314–332.

Calzolari, Giacomo and Alessandro Pavan, "Monopoly with resale," *The RAND Journal of Economics*, 2006, *37* (2), 362–375.

Carroll, Gabriel and Ilya Segal, "Robustly Optimal Auctions with Unknown Resale Opportunities," *The Review of Economic Studies*, 2019, *86* (4), 1527–1555.

Coase, R. H., "Durability and Monopoly," *The Journal of Law and Economics*, 1972, *15* (1), 143–149.

Dworczak, Piotr, "Mechanism Design With Aftermarkets: Cutoff Mechanisms," *Econometrica*, 2020, *88* (6), 2629–2661.

Flores, Daniel, "A two-part tariff monopolist with resale," *Economics Letters*, 2024, *243*, 111909.

Garratt, Rodney J. and Thomas Tröger, "Speculation in Standard Auctions with Resale," *Econometrica*, 2006, *74* (3), 753–769.

_____, _____, and Charles Z. Zheng, "Collusion via Resale," *Econometrica*, 2009, *77* (4), 1095–1136.

Hafalir, Isa and Vijay Krishna, "Asymmetric Auctions with Resale," *American Economic Review*, 2008, *98* (1), 87–112.

_____ **and** _____, "Revenue and Efficiency Effects of Resale in First-Price Auctions," *Journal of Mathematical Economics*, 2009, *45* (9-10), 589–602.

Hendel, Igal and Alessandro Lizzeri, "Adverse Selection in Durable Goods Markets," *American Economic Review*, 1999, *89* (5), 1097–1115.

Kumar, Praveen, "Price and quality discrimination in durable goods monopoly with resale trading," *International Journal of Industrial Organization*, 2002, *20* (9), 1313–1339.

Lebrun, Bernard, "First-price auctions with resale and with outcomes robust to bid disclosure," *The RAND Journal of Economics*, 2010, *41* (1), 165–178.

_____, "Optimality and the English and second-price auctions with resale," *Games and Economic Behavior*, 2012, *75* (2), 731–751.

Loertscher, Simon and Ellen V. Muir, "Monopoly Pricing, Optimal Randomization, and Resale," *Journal of Political Economy*, 2022, *130* (3), 566–635.

Maskin, Eric and John Riley, "Monopoly with Incomplete Information," *The RAND Journal of Economics*, 1984, *15* (2), 171–196.

Myerson, Roger B., "Optimal Auction Design," *Mathematics of Operations Research*, 1981, *6* (1), 58–73.

_____ **and Mark A. Satterthwaite**, "Efficient mechanisms for bilateral trading," *Journal of Economic Theory*, 1983, *29* (2), 265–281.

Mylovanov, Tymofiy and Thomas Tröger, "Optimal auction with resale—a characterization of the conditions," *Economic Theory*, 2008, *40* (3), 509–528.

Pagnozzi, Marco, "Bidding to lose? Auctions with resale," *The RAND Journal of Economics*, 2007, *38* (4), 1090–1112.

Scherer, Frederic M, *Industry structure, strategy, and public policy*, Prentice Hall, 1996.

Schiraldi, Pasquale and Francesco Nava, "Resale and Collusion in A Dynamic Market for Semidurable Goods," *The Journal of Industrial Economics*, 2012, *60* (2), 274–298.

Wilson, Robert B., *Nonlinear Pricing*, Oxford University Press, 1993.

Zhang, Jun and Ruqu Wang, "Optimal mechanism design with resale via bargaining," *Journal of Economic Theory*, 2013, *148* (5), 2096–2123.

Zheng, Charles Zhoucheng, "Optimal Auction with Resale," *Econometrica*, 2002, *70* (6), 2197–2224.