Federal Equalization, Early Shocks, and Self-Reinforcing Regional Investment

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Abstract

This paper develops a model of federal equalization when early regional shocks create persistent differences in local investment capacity. Regions receive stochastic budget shocks and choose how much of any positive surplus to consume or invest in productive public infrastructure. Infrastructure investment updates a regional concentration parameter, generating a Dirichlet-Pólya urn reinforcement process in which early advantages raise the likelihood of future favorable outcomes. A central government can use equalization grants to offset unfavorable shock sequences, but the effectiveness of intervention depends on timing. Simulation results show that no equalization produces rising regional divergence, while continuous equalization sharply limits concentration. Early equalization is especially effective at reducing the cumulative concentration of favorable opportunities, even though late equalization can mechanically compress terminal inequality when it occurs near the end of the simulation horizon. Descriptive county-level applications using BEA (Bureau of Economic Analysis) and FEMA (Federal Emergency Management Agency) data provide empirical baselines for the model. BEA income and GDP data show mean reversion rather than simple positive growth persistence, while FEMA Public Assistance obligations are negatively associated with longer-horizon future growth in fixed-effects specifications. These patterns suggest that empirical tests of the reinforcement mechanism should distinguish growth persistence and disaster exposure from the specific investment and transfer channels that update regional productive capacity. The model provides a theoretical rationale for front-loaded federal equalization in federations exposed to asymmetric regional shocks.

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1 Introduction

Federal systems routinely confront regional shocks that differ in size, timing, and persistence. Some regions experience favorable shocks that expand their fiscal capacity, while others face adverse or stagnant conditions that limit their ability to invest in productive public infrastructure. These early differences may matter well beyond the period in which they occur. If a region uses temporary fiscal gains to finance infrastructure, improve public services, or attract mobile capital and labor, then an initially favorable shock can become self-reinforcing. Conversely, regions that fail to receive favorable shocks may fall behind not only because of the initial event, but because they are unable to make the investments that raise the probability of future growth. This paper develops a model of federal equalization in which early regional shocks generate persistent differences in local investment capacity. Regions receive stochastic budget shocks and choose how much of any positive surplus to consume or invest in productive public infrastructure. Infrastructure investment updates a regional concentration parameter, generating a Pólya urn-style reinforcement process in which early advantages increase the likelihood of future favorable outcomes. The model therefore captures a basic form of path dependence: regions that benefit early are better positioned to benefit again, while regions that miss early opportunities may face declining relative prospects even when their underlying characteristics are initially identical.

The central government faces a timing problem. Equalization grants can offset unfavorable shock sequences by supporting investment in lagging regions, but the effectiveness of those grants depends on when they are provided. Early transfers can alter the path of regional investment before productive advantages become entrenched. Late transfers, by contrast, may have weaker effects because accumulated infrastructure gaps have already shifted the distribution of future opportunities toward advantaged regions. In this setting, the marginal value of federal intervention is highest before regional concentration weights diverge substantially.

This model builds on public finance and insurance themes. It follows the federal insurance and equalization literature by treating central transfers as a mechanism for sharing risk across regions while also shaping local investment incentives. It also draws on models of fiscal competition and public input provision, where infrastructure investment affects a region's ability to attract productive factors and expand its future tax base. The paper connects these ideas through a reinforcement process based on the Dirichlet-multinomial distribution. This framework provides a tractable way to model how early fiscal advantages can compound over time and how central grants can alter the path of regional divergence. The main implication is straightforward: when regional shocks generate self-reinforcing investment advantages, federal equalization should be front-loaded. Small transfers in later periods may be insufficient once early winners have accumulated a larger share of productive infrastructure and future opportunity. A successful central government therefore does not simply insure regions against realized losses. It also preserves the capacity of

disadvantaged regions to invest before initial differences become durable regional inequality.

Our included empirical applications via BEA (1969 - 2024) and FEMA datasets do not directly estimate the Pólya urn update equation. Instead, they clarify the empirical challenge implied by our model. The BEA results show that realized county growth histories exhibit mean reversion rather than simple positive persistence, indicating that the model's concentration parameter should not be proxied by growth alone. The FEMA results move closer to the federal grant channel, but gross Public Assistance obligations combine recovery support with the severity of the disaster shocks that trigger federal assistance. These results motivate an empirical strategy focused on the investment and transfer components that update regional productive capacity.

2 Related Literature

This paper contributes to the public finance literature on federal insurance, fiscal competition, and the dynamic effects of regional shocks. The central question is how a federal government should design transfers when early shocks affect not only current regional welfare, but also the future distribution of productive investment. Existing work has studied regional risk-sharing, public input competition, and fiscal policy in settings with mobile factors. This paper connects those ideas through a dynamic reinforcement mechanism in which early fiscal advantages can compound over time.

A natural starting point is the literature on federal insurance and disaster relief. Goodspeed and Haughwout (2012) develop a model of optimal disaster insurance in a federation with two symmetric regional governments. In their framework, regional governments face uncertain second-period income and may invest in protective infrastructure before the state of nature is realized. A central government can provide transfers across regions to insure against adverse outcomes. Their key result is that the design of federal transfers affects regional investment incentives. When regions expect to be fully insured by the central government, they may underinvest in protective infrastructure. Optimal federal policy must therefore balance risk-sharing against moral hazard in regional investment decisions. The present paper follows Goodspeed and Haughwout (2012) in treating central transfers as both insurance instruments and incentive-shaping policy tools. However, the model developed here differs in two important ways. First, the setting is explicitly multi-period rather than two-period. This matters because regional investment decisions can accumulate over time. Further, shocks do not determine second-period income rather favorable shocks create fiscal surplus that may be invested in productive public infrastructure, thereby affecting the likelihood of future favorable outcomes. The central government's problem is therefore not only to insure against realized losses, but also to prevent unfavorable shock sequences from generating persistent regional divergence. Another related literature studies fiscal competition and public input provision. Bucovetsky (2005) analyzes public input competition, while Keen and Marchand (1997) show

how fiscal competition can distort the composition of public spending toward productive public inputs and away from local public goods. Public infrastructure can expand the local tax base by attracting productive factors, creating a link between current public spending and future fiscal capacity.

The public input literature also clarifies why equalization policy cannot be evaluated only through contemporaneous redistribution. A transfer that supports infrastructure investment in a lagging region may have effects beyond the current period if it changes the region's future tax base or investment capacity. Conversely, a transfer that arrives after regional investment gaps have already widened may have limited long-run impact. The timing of equalization therefore becomes central. The model developed below formalizes this idea by allowing infrastructure investment to update a regional concentration parameter that determines future opportunity probabilities. The Mundell (1961) theory of optimum currency areas emphasizes the importance of labor mobility in adjustment to asymmetric regional shocks. In a currency union, regions cannot use independent monetary policy to respond to local shocks, so mobility and fiscal adjustment become central mechanisms of regional stabilization. Later work has extended this logic to settings in which mobility may undermine the effectiveness of policy. Baglioni et al. (2016), for example, examine how labor mobility can weaken fiscal stabilization in a currency union. If workers move toward regions experiencing positive shocks, the tax base of disadvantaged regions may shrink, reducing the effectiveness of local policy responses. This mechanism is closely related to the dynamic concern in the present paper. When early positive shocks attract mobile labor and capital, advantaged regions may experience a reinforcing expansion in fiscal capacity. Disadvantaged regions may face both weaker current revenues and reduced future investment capacity. Equalization policy in such a setting must account for the possibility that mobility amplifies early differences. Federal transfers may be most effective when they arrive before fiscal and productive factors have already reallocated toward initially favored regions.

The model also draws on the logic of reinforcement processes, especially the Pólya urn and Dirichlet-multinomial frameworks. In a standard Pólya urn process, an early draw increases the probability of drawing the same type in the future. This creates a simple form of path dependence: early outcomes influence later probabilities. The Dirichlet-multinomial distribution provides a natural generalization for multiple regions, where each region's concentration parameter represents its accumulated weight in the future opportunity distribution. In the present paper, we interpret the concentration parameter as accumulated productive advantage generated by prior infrastructure investment. Favorable shocks create fiscal resources, infrastructure investment updates the concentration parameter, and the updated concentration parameter affects the probability of future favorable opportunities. This reinforcement mechanism also relates to a broader literature on path dependence, increasing returns, and lock-in. David (1985) argues that historical sequences can shape economic outcomes when early choices alter the costs or benefits of later choices. Arthur

(1989) formalizes this logic in a model of competing technologies under increasing returns, where early random events can shift adoption dynamics and generate lock-in. Durlauf (1993) develops a related growth framework in which complementarities and incomplete markets can produce non-ergodic long-run outcomes. In the political economy space, Pierson (2000) emphasizes increasing returns as a social process through which early institutional choices become progressively harder to reverse, while Page (2006) clarifies different forms of path dependence and the conditions under which history affects long-run outcomes. The present model applies this logic to federal equalization and regional public investment. Early fiscal shocks matter because they change investment capacity, and investment capacity changes the probability distribution over future regional opportunities.

This reinforcement structure helps formalize a public finance intuition that is difficult to capture in a two-period model. When regions are initially symmetric, early shocks may appear temporary. But if early shocks affect investment, and investment affects future fiscal opportunities, then temporary shocks can have permanent consequences. A central government that waits until regional divergence is already large may need much larger transfers to restore balance than it would have needed earlier in the process. The model therefore provides a theoretical rationale for front-loaded equalization grants in federations exposed to asymmetric regional shocks.

3 A Motivating Example

This section presents two simple examples that illustrate the main mechanism of the model. The first example shows how an early favorable shock can generate a self-reinforcing regional advantage. The second example shows why the timing of federal equalization matters.

3.1 Example 1: Early Shocks and Reinforcement

Consider a federation with three initially identical regions, indexed by $k \in \{1, 2, 3\}$. At the beginning of the process, each region has the same concentration weight:

$$\alpha_0 = (1, 1, 1).$$

The probability that any region receives a favorable fiscal opportunity is therefore equal across regions:

$$p_{k0} = \frac{\alpha_{k0}}{\sum_{j=1}^{3} \alpha_{j0}} = \frac{1}{3}.$$

Suppose region 1 receives an early positive shock. The shock creates a temporary fiscal surplus, and region 1 invests the entire surplus in productive public infrastructure. Normalize the investment increment to one unit. Region 1's concentration weight then rises from 1 to 2, while the weights of

the other two regions remain unchanged:

$$\alpha_1 = (2, 1, 1).$$

The probability of receiving a future favorable opportunity is now:

$$p_{11} = \frac{2}{4} = \frac{1}{2},$$

while the probabilities for regions 2 and 3 are:

$$p_{21} = p_{31} = \frac{1}{4}.$$

A temporary shock has therefore changed the distribution of future opportunities. Region 1 is no longer merely lucky in the first period. By converting its fiscal surplus into productive infrastructure, it has increased its relative probability of receiving future favorable opportunities. The initial shock becomes self-reinforcing because investment changes the future concentration vector.

This example captures the central path-dependence mechanism of the paper. Even when regions are initially symmetric, early shocks can generate persistent differences if they affect investment capacity. The divergence does not require permanent differences in regional fundamentals. It can arise from the interaction between temporary shocks, local investment decisions, and a reinforcement process in the future opportunity distribution.

3.2 Example 2: Timing of Equalization

The second example illustrates why the timing of federal equalization matters. Again consider three initially identical regions with:

$$\alpha_0 = (1, 1, 1).$$

Suppose region 1 receives an early positive shock and invests the resulting surplus. Without federal intervention, the concentration vector becomes:

$$\alpha_1 = (2, 1, 1),$$

and the future opportunity probabilities become:

$$p_1 = \left(\frac{1}{2}, \frac{1}{4}, \frac{1}{4}\right).$$

Now suppose the central government provides early equalization grants to regions 2 and 3 before the investment gap becomes entrenched. If each disadvantaged region receives one unit of

investment support, the concentration vector becomes:

$$\alpha_1^E = (2, 2, 2).$$

The future opportunity probabilities are restored to:

$$p_1^E = \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right).$$

In this case, early equalization prevents the initial shock from changing the long-run distribution of opportunities.

Contrast this with a delayed response. Suppose the central government does not intervene after the first shock. Region 1's higher concentration weight raises its probability of receiving another favorable opportunity. If region 1 receives and invests a second favorable shock, the concentration vector becomes:

$$\alpha_2 = (3, 1, 1).$$

The future opportunity probabilities are now:

$$p_2 = \left(\frac{3}{5}, \frac{1}{5}, \frac{1}{5}\right).$$

At this point, an equalization grant of one unit to each disadvantaged region produces:

$$\alpha_2^L = (3, 2, 2),$$

with future probabilities:

$$p_2^L = \left(\frac{3}{7}, \frac{2}{7}, \frac{2}{7}\right).$$

The late transfer improves the position of regions 2 and 3, but it does not restore symmetry. The same transfer that would have fully offset the early divergence is now insufficient because region 1's advantage has compounded.

The comparison highlights the timing problem facing the central government. Equalization grants are more powerful when they are provided before early advantages become embedded in the concentration vector. Once favorable shocks and infrastructure investment have compounded, larger transfers are required to produce the same equalizing effect. The central policy implication is that federal equalization should not merely respond to realized regional inequality. It should also prevent temporary shocks from becoming durable investment advantages.

The formal model generalizes these examples to K regions and T periods.

4 Model

We first develop a dynamic model of regional shocks, infrastructure investment, and federal equalization. The purpose of the model is to show how initially symmetric regions can diverge over time when early favorable shocks increase local investment capacity. The central government can offset this process through equalization grants, but the effectiveness of those grants depends on their timing.

4.1 Environment

Consider a federation with K regions indexed by $k \in \{1, \ldots, K\}$ and discrete time periods $t = 0, 1, \ldots, T$. Regions are initially symmetric. Each region begins with the same productive concentration weight:

$$\alpha_{k0} = 1 \quad \forall k. \tag{1}$$

The concentration parameter α_{kt} represents accumulated regional productive advantage. This advantage may reflect public infrastructure, fiscal capacity, public capital, administrative capacity, or other durable features that make a region more likely to receive favorable future economic opportunities. The vector of regional concentration weights is:

$$\boldsymbol{\alpha}_t = (\alpha_{1t}, \ldots, \alpha_{Kt}). \tag{2}$$

At each period, the probability that region k receives a favorable fiscal opportunity is determined by its relative concentration weight:

$$p_{kt} = \frac{\alpha_{kt}}{\sum_{j=1}^{K} \alpha_{jt}}. \tag{3}$$

This probability rule captures the reinforcement mechanism in the model. A region with a higher concentration weight is more likely to receive future favorable opportunities. Since concentration weights are updated by investment, early investment advantages can raise the likelihood of future favorable outcomes.

4.2 Regional Shocks and Fiscal Surplus

Each period, regions may receive favorable fiscal opportunities. Let D_{kt} be an indicator equal to one if region k receives a favorable opportunity in period t, and zero otherwise:

$$D_{kt} \in \{0, 1\}. \tag{4}$$

For the baseline model, assume that one favorable opportunity is realized in each period. The

probability that region k receives the opportunity is:

$$\Pr(D_{kt} = 1) = p_{kt}. \tag{5}$$

If region k receives a favorable opportunity, it receives fiscal surplus $s > 0$. If it does not receive the opportunity, it receives no surplus. Thus:

$$s_{kt} = sD_{kt}. \tag{6}$$

The surplus can be interpreted as an unexpectedly favorable budget shock, a temporary increase in fiscal capacity, a positive productivity realization, or a favorable inflow of mobile capital and labor. The important feature is that the shock creates resources that can either be consumed in the current period or invested in productive public infrastructure.

4.3 Regional Investment Choice

A region that receives fiscal surplus chooses how much to allocate to current consumption and how much to invest in productive public infrastructure. Let c_{kt} denote current public consumption and let w_{kt} denote infrastructure investment. The regional budget constraint is:

$$s_{kt} = c_{kt} + w_{kt}, \tag{7}$$

with:

$$c_{kt} \geq 0, \quad w_{kt} \geq 0. \tag{8}$$

For the baseline model, suppose each region invests a constant share $\rho \in [0, 1]$ of any fiscal surplus:

$$w_{kt} = \rho s_{kt}, \tag{9}$$

and consumes the remainder:

$$c_{kt} = (1 - \rho)s_{kt}. \tag{10}$$

The parameter ρ captures the investment intensity of regional governments. When $\rho = 0$, regions consume all favorable shocks and do not accumulate productive advantage. When $\rho = 1$, regions invest all favorable shocks, maximizing the reinforcement mechanism.

4.4 Updating the Concentration Parameter

Infrastructure investment raises the region's future concentration weight. The concentration parameter evolves according to:

$$\alpha_{k,t+1} = \alpha_{kt} + \delta w_{kt}, \tag{11}$$

where $\delta > 0$ measures the productivity of infrastructure investment in generating future regional advantage.

Substituting the regional investment rule gives:

$$\alpha_{k,t+1} = \alpha_{kt} + \delta \rho s D_{kt}. \tag{12}$$

This law of motion captures the central mechanism of the model. A region that receives an early favorable opportunity can invest the surplus, raise its concentration weight, and increase its probability of receiving favorable opportunities in future periods. Thus, initially temporary shocks can create persistent differences across regions.

If no region invested its surplus, then the concentration vector would remain unchanged and favorable opportunities would remain equally likely across initially symmetric regions. Divergence arises because positive shocks create investable resources, and investment changes the future distribution of opportunity.

4.5 Federal Equalization

The central government can provide equalization grants to regions with relatively low concentration weights. Let $g_{kt} \geq 0$ denote the equalization grant received by region k in period t. Grants are assumed to support productive public investment rather than current consumption. The law of motion for regional concentration weights becomes:

$$\alpha_{k,t+1} = \alpha_{kt} + \delta w_{kt} + \gamma g_{kt}, \tag{13}$$

where $\gamma > 0$ measures the productivity of federal grant-supported investment.

The central government operates subject to a period-by-period budget constraint:

$$\sum_{k=1}^{K} g_{kt} \leq G_t, \tag{14}$$

where G_t is the total amount of federal equalization resources available in period t.

A simple equalization rule targets regions whose concentration weights fall below the cross-regional average. Define the average concentration weight as:

$$\bar{\alpha}_t = \frac{1}{K} \sum_{j=1}^{K} \alpha_{jt}. \tag{15}$$

Define the equalization gap for region k as:

$$h_{kt} = \max\{\bar{\alpha}_t - \alpha_{kt}, 0\}. \tag{16}$$

When the equalization program is active and at least one region has a positive gap, grants are allocated proportionally to the gaps:

$$g_{kt} = \begin{cases} G_t \dfrac{h_{kt}}{\sum_{j=1}^{K} h_{jt}} & \text{if } \sum_{j=1}^{K} h_{jt} > 0, \\ 0 & \text{if } \sum_{j=1}^{K} h_{jt} = 0. \end{cases} \tag{17}$$

This rule satisfies the federal budget constraint by construction. It does not require the central government to directly reassign probability weights. Instead, the government changes future opportunity probabilities indirectly by financing productive investment in lagging regions.

4.6 Timing of Federal Intervention

The model allows for different equalization regimes. Let E_t be an indicator equal to one if the central government operates an equalization program in period t, and zero otherwise. The effective grant rule can be written as:

$$g_{kt} = E_t \cdot \begin{cases} G_t \dfrac{h_{kt}}{\sum_{j=1}^{K} h_{jt}} & \text{if } \sum_{j=1}^{K} h_{jt} > 0, \\ 0 & \text{if } \sum_{j=1}^{K} h_{jt} = 0. \end{cases} \tag{18}$$

This notation permits direct comparison of early and late intervention. Under early equalization, $E_t = 1$ for low values of t and $E_t = 0$ later. Under late equalization, $E_t = 0$ initially and $E_t = 1$ only after regional concentration weights have already diverged.

The central prediction is that early equalization has a larger effect on the cumulative distribution of opportunities than an equal-sized late equalization program. Early grants affect the concentration vector before favorable shocks and infrastructure investment have compounded. Late grants arrive after some regions have already accumulated larger concentration weights, making the same grant less effective at preventing earlier opportunity concentration.

4.7 Regional Divergence

To evaluate regional divergence, define the cross-regional dispersion of concentration weights as:

$$V_t = \frac{1}{K} \sum_{k=1}^{K} (\alpha_{kt} - \bar{\alpha}_t)^2. \tag{19}$$

A higher value of V_t indicates greater inequality in accumulated productive advantage. The model predicts that, when $\rho > 0$ and $\delta > 0$, the absence of equalization leads to rising expected dispersion over time:

$$\mathbb{E}[V_{t+1}] > \mathbb{E}[V_t] \tag{20}$$

for sufficiently early periods in which initial shocks begin to accumulate.

Federal equalization reduces this dispersion by increasing the concentration weights of lagging regions. However, the magnitude of this effect depends on timing. An early grant to a lagging region affects not only its current concentration weight, but also its future probability of receiving favorable opportunities. A late grant affects the same law of motion, but it begins from a more unequal concentration vector and after more opportunities have already been allocated.

4.8 Implications

The model generates three main implications. First, favorable regional shocks can produce persistent divergence when regions invest part of their fiscal surplus in productive public infrastructure. Second, the strength of this divergence is increasing in the investment share ρ and the productivity of investment δ. Third, federal equalization is more effective at preventing cumulative opportunity concentration when it is provided early, before regional concentration weights have substantially diverged.

These implications provide the basis for the simulation exercise that follows. By comparing no equalization, early equalization, late equalization, and continuous equalization regimes, the model can evaluate how the timing and strength of federal grants affect long-run regional inequality.

5 Simulation Results

This section uses a simple simulation exercise to illustrate the model's main implications. The simulation is not intended to estimate structural parameters. Instead, it shows how the reinforcement mechanism generates regional divergence and how the timing of federal equalization affects the distribution of future opportunities.

5.1 Simulation Design

Consider a federation with $K = 50$ initially identical regions observed over $T = 60$ periods. Each region begins with the same concentration weight:

$$\alpha_{k0} = 1 \quad \forall k. \tag{21}$$

At the beginning of each period, the probability that region k receives a favorable fiscal opportunity is:

$$p_{kt} = \frac{\alpha_{kt}}{\sum_{j=1}^{K} \alpha_{jt}}. \tag{22}$$

One favorable opportunity is drawn in each period. If region k receives the opportunity, it

receives a normalized fiscal surplus equal to one:

$$s_{kt} = 1. \tag{23}$$

All other regions receive no surplus in that period. The region receiving the favorable opportunity invests share ρ of the surplus in productive public infrastructure:

$$w_{kt} = \rho s_{kt}. \tag{24}$$

For the baseline simulation, set $\rho = 1$, so that the full fiscal surplus is invested. The concentration parameter then evolves according to:

$$\alpha_{k,t+1} = \alpha_{kt} + \delta w_{kt} + \gamma g_{kt}, \tag{25}$$

where $\delta = 1$ and $\gamma = 1$. Thus, one unit of local investment or federal grant-supported investment raises the region's concentration weight by one unit.

The simulation compares four policy regimes. The first regime has no federal equalization:

$$g_{kt} = 0 \quad \forall k, t. \tag{26}$$

The second regime provides early equalization grants. The central government operates an equalization program during periods $t = 1, \ldots, 5$ and provides a fixed grant budget B in each equalization period. The third regime provides late equalization grants during periods $t = 40, \ldots, 44$, using the same grant budget B and the same number of intervention periods. The fourth regime provides continuous equalization in every period.

When equalization is active, grants are allocated to regions with concentration weights below the cross-regional average. The gap for region k is:

$$h_{kt} = \max\{\bar{\alpha}_t - \alpha_{kt}, 0\}. \tag{27}$$

If the equalization program is active and at least one region is below the average, grants are allocated proportionally to these gaps:

$$g_{kt} = B \frac{h_{kt}}{\sum_{j=1}^{K} h_{jt}}. \tag{28}$$

If no region is below the average, then $g_{kt} = 0$ for all k. In the baseline simulation, the grant budget is set to $B = 5$ in each active equalization period. Each regime is evaluated over 1,000 simulation runs.

5.2 Outcomes

The simulation tracks two main outcomes. The first is dispersion in accumulated productive advantage, measured by the cross-regional variance of concentration weights:

$$V_t = \frac{1}{K}\sum_{k=1}^{K}(\alpha_{kt} - \bar{\alpha}_t)^2. \tag{29}$$

The second is concentration in realized favorable opportunities. Let N_{kT} denote the total number of favorable opportunities received by region k through period T. The simulation records the share of total opportunities received by the most favored region:

$$M_T = \max_k \frac{N_{kT}}{\sum_{j=1}^{K} N_{jT}}. \tag{30}$$

The first outcome measures inequality in productive capacity at the end of the simulation. The second outcome measures the concentration of realized opportunities over the full path of the simulation.

5.3 Results

Table 1 reports the simulation results by policy regime. The main outcomes are terminal dispersion in regional concentration weights, the Gini coefficient of terminal concentration weights, the maximum terminal concentration share, and the maximum cumulative share of favorable opportunities received by any one region.

Table 1: Simulation Results by Equalization Regime

Regime	Var(α_T)	Gini(α_T)	Max α_T Share	Max Opp. Share
No equalization	2.612	0.347	0.073	0.116
Early equalization	2.077	0.272	0.057	0.102
Late equalization	1.589	0.207	0.057	0.110
Continuous equalization	0.089	0.012	0.023	0.074

Notes: Averages across 1,000 runs with $K = 50$, $T = 60$, and grant budget $B = 5$. Early equalization is active in periods 1–5, late equalization in periods 40–44, and continuous equalization in all periods.

The no-equalization regime produces the highest level of terminal regional divergence. The variance of terminal concentration weights is 2.612, and the terminal Gini coefficient is 0.347. This reflects the reinforcement mechanism in the model. When a region receives a favorable opportunity, it invests the resulting surplus, raises its concentration weight, and becomes more likely to receive future favorable opportunities. Even though all regions begin symmetrically, early favorable draws generate persistent differences in accumulated productive advantage.

Early equalization reduces regional divergence relative to the no-equalization case. The variance of terminal concentration weights falls from 2.612 to 2.077, and the terminal Gini coefficient

falls from 0.347 to 0.272. The maximum terminal concentration share also falls from 0.073 to 0.057. These results show that early transfers can weaken the compounding process by raising the concentration weights of lagging regions before the opportunity distribution becomes highly unequal.

Late equalization produces an important comparison. Terminal dispersion is lower under late equalization than under early equalization: the variance of α_T falls to 1.589, and the terminal Gini coefficient falls to 0.207. This occurs because the late intervention takes place close to the end of the simulation horizon. As a result, late grants mechanically compress the final distribution of concentration weights. However, this does not mean that late equalization is more effective at preventing divergence over the full path of the model. The maximum cumulative opportunity share remains higher under late equalization, at 0.110, than under early equalization, where it is 0.102.

This distinction is central to the interpretation of the model. If the policy objective is to reduce measured inequality in concentration weights at a terminal date, late equalization can appear effective because it narrows gaps just before the final period. If the policy objective is to prevent the cumulative concentration of favorable opportunities, early equalization performs better. Early grants alter the opportunity path before reinforcement has fully unfolded. Late grants can reduce final observed gaps, but they cannot undo the earlier allocation of favorable opportunities.

Continuous equalization produces the lowest inequality across all measures. The variance of terminal concentration weights falls to 0.089, and the terminal Gini coefficient falls to 0.012. The maximum cumulative opportunity share also falls to 0.074. This result is unsurprising because continuous equalization repeatedly offsets concentration gaps throughout the simulation. The more informative comparison is therefore between early and late intervention, where the same basic grant mechanism produces different effects depending on when it is applied.

Figures 1 and 2 illustrate these dynamics. Figure 1 plots the cross-regional variance of concentration weights over time. In the absence of equalization, regional divergence rises steadily. Early equalization slows this increase, while late equalization generates a visible decline in dispersion when the policy activates near period 40. Continuous equalization keeps dispersion close to zero throughout the simulation.

Figure 2 plots the average maximum cumulative opportunity share. This outcome captures the extent to which favorable opportunities become concentrated among a small number of regions over the entire simulation path. The figure shows that late equalization has limited ability to reduce cumulative opportunity concentration, since many opportunities have already been allocated before the policy begins. Early equalization performs better on this outcome because it changes the future opportunity probabilities before early advantages fully compound.

The simulation results support the main theoretical implication of the model. When favorable shocks create investable fiscal surplus, and investment raises the probability of future favorable opportunities, initially symmetric regions can diverge over time. Federal equalization can reduce

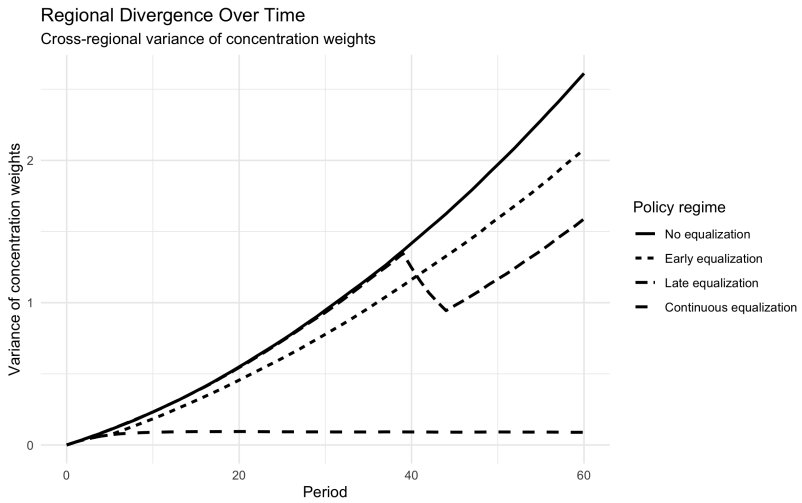


Figure 1: Regional Divergence Over Time

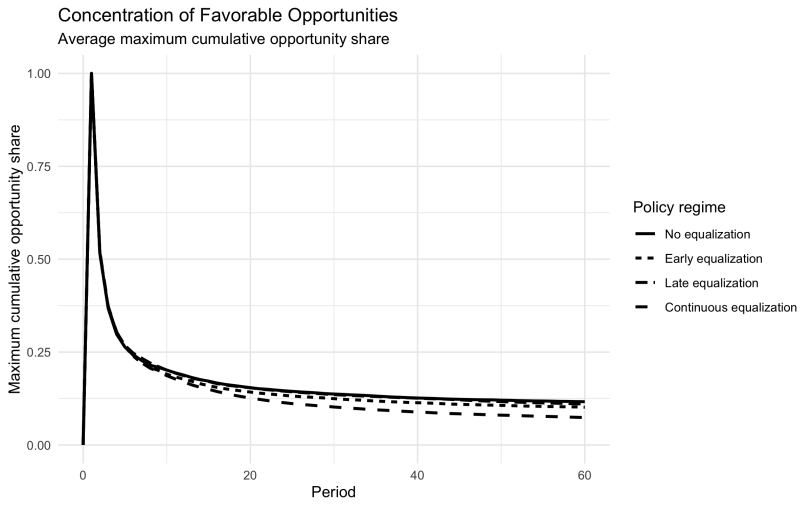


Figure 2: Concentration of Favorable Opportunities

this divergence, but its effects depend on timing and on the outcome being evaluated. Late transfers may reduce measured terminal inequality, but early transfers are more effective at preventing the cumulative concentration of opportunity. The results therefore support a preventive view of federal equalization policy: central governments should intervene before temporary regional shocks become durable investment advantages.

6 Empirical application: County-Level Reinforcement in BEA and FEMA Data

This section provides a first-pass empirical application of the reinforcement framework using county-level data from the Bureau of Economic Analysis. This application exercise asks whether an empirical analogue of the model's concentration parameter predicts future county economic outcomes. The analysis therefore serves as a descriptive test of whether prior favorable outcomes are associated with subsequent regional advantage. The county-year panel includes 3,149 county geographies observed from 1969 through 2024. The main outcomes are future per capita personal income growth and future real GDP growth. A favorable opportunity is defined as a county-year observation in which the county's growth rate exceeds the national county median in that year. Using this definition, an empirical concentration weight is constructed as:

$$\widehat{\alpha}_{ct} = 1 + \sum_{s<t} \mathbb{1}\{Growth_{cs} > MedianGrowth_s\}. \tag{31}$$

Thus, $\widehat{\alpha}_{ct}$ measures the accumulated number of prior years in which county c experienced above-median growth. The measure is lagged so that the concentration weight in year t is based only on prior outcomes.

The estimating equation is:

$$Y_{c,t+h} = \beta \log(\widehat{\alpha}_{ct}) + \mu_c + \lambda_t + \varepsilon_{ct}, \tag{32}$$

where $Y_{c,t+h}$ is future county growth over horizon h, μ_c denotes county fixed effects, and λ_t denotes year fixed effects. Standard errors are clustered by county. County fixed effects take into account time-invariant differences across places, while year fixed effects control for national shocks common to all counties.

Table 2 reports estimates for future per capita income growth and future real GDP growth. Across one-, three-, and five-year horizons, the coefficient on the lagged empirical concentration weight is negative and statistically significant. A higher value of $\log(\widehat{\alpha}_{ct})$ is associated with lower future per capita income growth and lower future real GDP growth. The estimates therefore do not support a simple positive persistence interpretation in which prior above-median growth

mechanically predicts stronger future growth.

Table 2: Empirical Concentration Weights and Future County Growth

	Per Capita Income Growth			Real GDP Growth		
	1-year	3-year	5-year	1-year	3-year	5-year
$\log(\widehat{\alpha}_{ct})$	-2.942^{***}	-9.544^{***}	-16.13^{***}	-4.480^{***}	-13.98^{***}	-22.08^{***}
	(0.109)	(0.272)	(0.411)	(0.197)	(0.673)	(1.164)
County fixed effects	Yes	Yes	Yes	Yes	Yes	Yes
Year fixed effects	Yes	Yes	Yes	Yes	Yes	Yes
Clustered SEs	County	County	County	County	County	County
Observations	170,619	164,409	158,199	71,173	64,981	58,789
R^2	0.201	0.408	0.546	0.086	0.195	0.293
Within R^2	0.004	0.017	0.029	0.008	0.018	0.017

Notes: The table reports descriptive fixed-effects regressions using county-year BEA data. The explanatory variable is the log of a lagged empirical concentration weight, $\log(\widehat{\alpha}_{ct})$, constructed from the accumulated number of prior years in which a county experienced above-median growth. All models include county and year fixed effects. Standard errors are clustered by county and reported in parentheses. $^{***}p < 0.001$, $^{**}p < 0.01$, $^{*}p < 0.05$.

Figure 3 provides a descriptive view. Counties are sorted into deciles based on their lagged empirical concentration weight, and the figure plots average future three-year per capita income growth by decile. The relationship is not increasing. If anything, the highest prior-advantage deciles display lower subsequent growth. This pattern is consistent with the fixed-effects estimates and suggests mean reversion in broad county income growth rather than simple positive reinforcement.

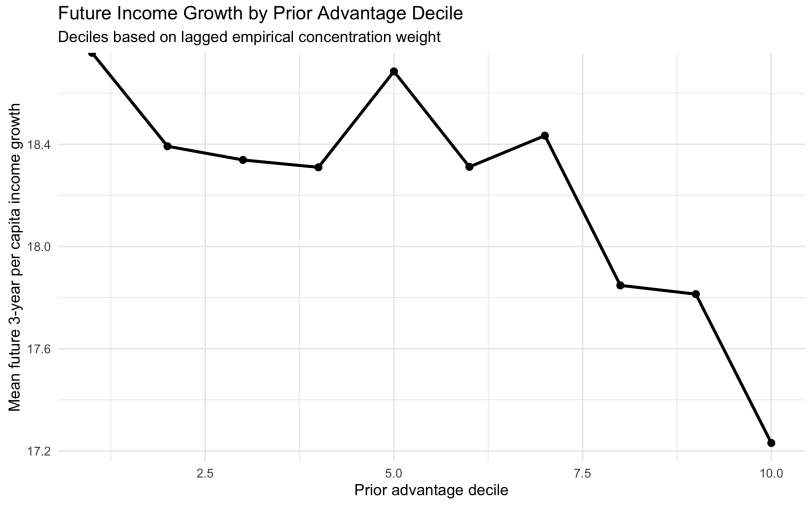


Figure 3: Future Income Growth by Prior Advantage Decile

The empirical exercise lends itself a caveat as the reinforcement mechanism developed in the theoretical model should not be equated with persistence in county income or GDP growth. In the BEA data, prior above-median growth is negatively associated with future growth once county and year fixed effects are included. This may reflect mean reversion, convergence dynamics, or the fact

that the empirical concentration weight used here is based on realized growth rather than direct measures of public investment, infrastructure, or federal grants.

The model's mechanism is not simply that fast-growing counties keep growing faster. The mechanism is that favorable shocks generate investable fiscal resources, and those resources raise future productive capacity when they are converted into public investment. Future empirical work should therefore incorporate direct measures of public investment and federal transfers, such as FEMA Public Assistance grants, Census government finance capital outlays, or other infrastructure-related spending. The BEA-only exercise is best interpreted as a descriptive baseline: broad county growth histories alone do not provide evidence of positive reinforcement, so the next empirical step is to test whether reinforcement appears specifically through investment and transfer channels.

6.1 FEMA public assistance and county-level growth

The BEA application above shows that broad county growth histories do not, by themselves, provide evidence of positive reinforcement. This section takes a step closer to the Pólya urn mechanism developed in the model by introducing a direct measure of federal disaster-related transfers. In the theoretical model, a region's concentration weight evolves according to:

$$\alpha_{k,t+1} = \alpha_{kt} + \delta w_{kt} + \gamma g_{kt}, \tag{33}$$

where local investment and federal grants increase the region's future opportunity weight. In the empirical application, FEMA Public Assistance obligations provide a potential observable counterpart to the federal grant term, g_{kt}. These obligations are therefore interpreted as a proxy for federally supported recovery and infrastructure investment that may help rebuild or preserve a county's productive capacity after an adverse shock. FEMA obligations are not randomly assigned, and they respond to disaster damage. A positive or negative association between FEMA obligations and future growth should therefore not be interpreted as a causal effect of federal transfers. Instead, the exercise provides a descriptive bridge between the paper's theoretical mechanism and future empirical work. In the Pólya urn framework, federal transfers can be interpreted as additions to the concentration weights of affected regions. However, in our data, FEMA obligations are observed precisely when counties have experienced damaging shocks. The same variable therefore captures both a potential positive update to the county's productive capacity and the adverse event that made the transfer necessary.

To connect the data to the model, we define an empirical FEMA-based update term:

$$\widehat{g}_{ct} = \log(1 + FEMApc_{ct}), \tag{34}$$

where $FEMApc_{ct}$ denotes allocated FEMA Public Assistance obligations per capita in county c

and year t. The empirical analogue of the model's concentration update can then be written as:

$$\widehat{\alpha}_{c,t+1} = \widehat{\alpha}_{ct} + \widehat{\delta} I_{ct} + \widehat{\gamma}\widehat{g}_{ct}. \tag{35}$$

Because direct county-year measures of public investment are not yet included, the exercise below focuses on the reduced-form relationship between lagged FEMA exposure and future county growth. This does not fully estimate the urn update equation and posits whether one observable component of the federal grant channel is associated with subsequent county outcomes.

To construct a county-year measure of FEMA exposure, Public Assistance obligations are aggregated by disaster and obligation year. FEMA Disaster Declarations Summaries are then used to identify counties designated under each disaster declaration.Our FEMA application uses a county-year panel constructed from two OpenFEMA sources. The first source is FEMA Public Assistance Grant Award Activities, which reports Public Assistance obligation activity by disaster number, state, incident type, applicant, project title, obligation date, and federal share obligated. The second source is FEMA Disaster Declarations Summaries, which identifies declared disasters and the counties designated under each declaration. Public Assistance obligations are first aggregated to the disaster-year level using the obligation date. Disaster-year obligations are then linked to the counties designated under the corresponding disaster declaration. The resulting measure, $FEMA_{ct}$, captures allocated FEMA Public Assistance obligations received by county c in year t. The merged FEMA-BEA panel includes 176,776 county-year observations, of which 43,143 have positive allocated FEMA Public Assistance exposure. Conditional on receiving FEMA Public Assistance, mean per capita exposure is approximately 71.70.

The estimating equation is:

$$Y_{c,t+h} = \beta \log(1 + FEMApc_{c,t-1}) + \mu_c + \lambda_t + \varepsilon_{ct}, \tag{36}$$

where $Y_{c,t+h}$ is future county growth over horizon h, $FEMApc_{c,t-1}$ is lagged allocated FEMA Public Assistance obligations per capita, μ_c denotes county fixed effects, and λ_t denotes year fixed effects. Standard errors are clustered by county. The coefficient β should be interpreted as a descriptive association between one lagged component of the urn-update process and future county growth, conditional on county and year fixed effects.

Table 3 reports the results. The estimates are generally negative. For future per capita income growth, the one-year coefficient is small and statistically insignificant. The three-year coefficient is negative and statistically significant at conventional levels, and the five-year coefficient is larger in magnitude and statistically significant. For real GDP growth, the one- and three-year coefficients are negative but not statistically significant, while the five-year coefficient is negative and statistically significant.

Table 3: FEMA Public Assistance and Future County Growth

	Per Capita Income Growth			Real GDP Growth		
	1-year	3-year	5-year	1-year	3-year	5-year
$\log(1 + FEMApc_{c,t-1})$	-0.0143	-0.0796^{*}	-0.1678^{***}	-0.0122	-0.1243	-0.5226^{***}
	(0.0122)	(0.0313)	(0.0432)	(0.0304)	(0.0897)	(0.1194)
County fixed effects	Yes	Yes	Yes	Yes	Yes	Yes
Year fixed effects	Yes	Yes	Yes	Yes	Yes	Yes
Clustered SEs	County	County	County	County	County	County
Observations	158,565	155,102	150,483	62,251	58,806	54,205
R^2	0.203	0.410	0.540	0.081	0.170	0.269
Within R^2	0.000005	0.000063	0.000170	0.000004	0.000100	0.000710

Notes: The table reports descriptive fixed-effects regressions using the merged FEMA-BEA county-year panel. The explanatory variable is lagged allocated FEMA Public Assistance obligations per capita, transformed as $\log(1 + FEMApc_{c,t-1})$. All models include county and year fixed effects. Standard errors are clustered by county and reported in parentheses. FEMA Public Assistance obligations are allocated from disaster-year obligations to declared counties, so the estimates should not be interpreted as causal effects of federal transfers. $^{***}p < 0.001$, $^{**}p < 0.01$, $^{*}p < 0.05$.

The negative coefficients are best understood as evidence that FEMA obligations combine two opposing components: federal recovery investment and the adverse disaster shock that triggered the obligation. In the theoretical model, federal grants enter positively into the regional concentration update. In the FEMA data, however, federal obligations are bundled with the adverse shocks that trigger them. Counties receiving larger FEMA obligations have likely experienced more severe disaster damage, and this damage may depress subsequent income or output even after recovery spending is provided. The negative coefficients therefore do not imply that FEMA assistance reduces growth. Rather, they indicate that a simple fixed-effects regression does not separately identify the positive urn-update effect of federal grants from the negative shock that generated the transfer. The model predicts that federal transfers should matter when they increase future productive capacity, not when they are observed following a damaging event. Future applications should therefore separate disaster severity from grant-supported investment. In the language of the model, the empirical challenge is to isolate the component of g_{ct} that updates α_{ct} from the adverse shock that lowers current and future outcomes.

7 Conclusion

This paper develops a dynamic model of federal equalization when early regional shocks create self-reinforcing investment advantages. Regions begin symmetrically, but favorable fiscal opportunities allow some regions to invest in productive public infrastructure. These investments increase future concentration weights, increasing the probability that advantaged regions will receive additional favorable opportunities. Temporary shocks can therefore generate persistent regional divergence

through a simple reinforcement process.

The simulation results illustrate the central implication of the model. In the absence of equalization, regional concentration weights diverge over time, and favorable opportunities become more concentrated among a smaller number of regions. Federal equalization reduces this divergence, but its effects depend on timing. Late equalization can reduce the measured terminal inequality when it occurs close to the end of the simulation horizon, but it cannot undo the earlier concentration of favorable opportunities. Early equalization is more effective in changing the opportunity path before the initial advantages fully compound. Continuous equalization produces the lowest inequality for all simulated outcomes. Descriptive county-level applications using BEA and FEMA data provide empirical baselines for interpreting the simulation. BEA income and GDP data show mean reversion rather than simple positive growth persistence, while FEMA Public Assistance obligations are negatively associated with longer-horizon future growth in fixed-effects specifications. These patterns do not contradict the simulation. They show that the reinforcement mechanism must be tested through the specific investment and transfer channels that update regional productive capacity, rather than gross disaster-related obligations.

An empirical extension would test whether early public investment following fiscal shocks predicts further regional income, employment, inequality or growth. Such evidence would help assess whether the reinforcement mechanism developed here is visible in actual regional economic data. And in particular disaster scenarios.

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