Green Growth through Innovation and Production Networks*

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Abstract

This paper studies the impact of industrial policy on green technologies in a multi-sector economy characterized by input-output linkages and cross-sector knowledge spillovers. Using global patent data, we first document that, on average, green technologies generate substantial spillovers to other technologies and tend to arise in sectors that are central within the production network of the economy. On the theory side, we propose an endogenous growth model that incorporates directed technical change, a production network, an innovation network, and an energy sector with multiple energy sources and specific infrastructure. We establish that when the production and innovation networks are considered jointly, a well-designed industrial policy that fosters green innovation in highly central sectors can raise long-run growth. Quantitatively, we calibrate the model using Chinese data and assess how effective current industrial policies are, given the observed network structure. We then quantify the overall welfare gains of moving to a set of optimal policy with R&D subsidy, carbon tax, and energy-specific infrastructure investment implied by the theory.

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1 Introduction

The past decade has witnessed an expansion of green industrial policy across advanced and developing countries, including the *Inflation Reduction Act* of the United States, the REPowerEU and Net-Zero Industry Act packages of the European Union's, Japan's *Green Transformation* (GX) program, and analogous initiatives in China, Brazil, India, and other emerging economies. Despite the widening use of green industrial policies, governments face a fundamental tension between economic growth and green transition. Statically, environmental policy imposes an abatement cost, diverting resources from current consumption (Stern, 2007; Nordhaus, 2017). Dynamically, it redirects R&D from mature, high-productivity dirty technologies toward nascent clean alternatives, slowing aggregate TFP growth during the transition (Acemoglu et al., 2012, 2016). Standard frameworks therefore predict that the green transition and economic growth are , at least temporarily, in conflict.

However, in recent years, China stands out as one of the rare cases in which accelerated green transition appears to coincide with sustained aggregate growth. Over roughly a decade, China's renewable energy sector, such as solar photovoltaics (PV) and wind turbines, achieved significant cost reductions and widespread adoption, with an estimated contribution of over one third in its annual economic growth. Why has China's green transition been broader, faster, and more growth-enhancing than standard theory predicts? This paper argues that two structural features of the Chinese economy contributes to this green growth: the central position of green technologies in the production network, and the unusually strong cross-sector knowledge spillovers they generate.

Using global patent data and input-output (IO) tables, this paper documents these features of green technology innovation. First, green technologies generate significantly larger cross-sector knowledge spillovers than their non-green counterparts, with particularly pronounced effects across industries. A green innovation in one sector disproportionately raises R&D productivity in other sectors relative to a comparable non-green innovation. Second, green innovation is more concentrated in sectors that are highly central to China's production network, as measured by their weighted outdegrees in the input-output structure. Taken together, these facts imply that green innovations occupy a structurally strategic position in the Chinese economy, which amplifies the aggregate return to green industrial policy through both static production linkages and dynamic knowledge spillovers.

To formalize and quantify these mechanisms, we develop a tractable multi-sector general equilibrium framework with endogenous and directed innovation. The model integrates three features. First, production is structured as an input-output network, where cost and demand changes propagate through both direct and higher-order linkages. Second, within each production sector, firms choose between substitutable clean and dirty technologies that differ in productivity and emission intensity. An explicit energy sector supplies inputs to all production sectors, combining fossil fuel and renewable sources, each with resource-specific technology and infrastructure. Third, technology evolves through endogenous Schumpeterian innovation directed at either the clean or dirty technologies. Successful innovations replace incumbents and advance sector-level knowledge stocks. Crucially, innovation in one sector-technology node diffuses across the economy through a knowledge spillover network, raising R&D productivity in other nodes.

The model shows that the impact of green industrial policy depends not only on the size of policy support, but also on the network structure of the economy in which it operates. Policies that alter relative prices (e.g., a carbon tax) reallocate adoption towards the cheaper technology via a market-size effect, with the impact's strength depending on existing technology market shares. The aggregate impact of sector-specific policies thus depends on the interplay between the market-size effect, determined by technology substitutability and propagated through the production network, and the cross-technology spillovers, which in turn affect the speed and direction of future innovation.

Using this framework, the marginal impact of sector-specific policies can be characterized by sufficient statistics matrices that describe the production and innovation network structures. Specifically, an augmented IO matrix with four blocks summarizes the marginal policy impact on technology-specific market sizes, while the cross-technology spillover matrix with similar four blocks summarizes the marginal impact on innovation efficiency of each technology. Since these two channels are separable, the first-order policy impact can be characterized in closed form through a multiplication of these two matrices. When the knowledge spillovers from clean/renewable technologies to the dirty/fossil fuel technologies are substantial, targeted R&D subsidies alone would be insufficient to achieve the green transition. Any clean technology advance will benefit and allow some other dirty technology to catch up. Therefore, policies that reduce the market size of dirty technologies (e.g., a carbon tax) at the same time become essential to ensure a successful transition.

In our policy analysis, we consider a broad set of instruments: technology-specific R&D subsidies, sector-specific carbon taxes, and energy infrastructure investments. Our central policy finding is that the second-best industrial policy (i.e., correcting for network and environmental externalities but not monopoly rents) is characterized by a single sufficient statistic: the "green centrality". This measure combines three important channels: (i) a sector's production centrality in the IO network, (ii) its innovation centrality in the spillover network, and (iii) its within-sector relative productivity. This delivers closed-form targeting principles for second-best clean R&D subsidies that remain valid even when the carbon price is constrained. Under sufficient conditions of the two matrices, our model implies that a well-designed green industrial policy can amplify rather than dampen economic growth in the long run.

To quantify the impacts of green industrial policy in the green transition and economic growth in China, we calibrate the model using Chinese data from 2005 to 2024 as the transitional path from an initial steady state to a new one. With the calibrated model, we simulate the transitional path with observed China's major industrial policy in green sectors and examine the welfare and growth impacts of such policies. Moving from a historical baseline with a uniform 15% green R&D subsidy to the optimal portfolio (targeted R&D, second-best carbon price path, and clean infrastructure) raises aggregate TFP by 27.4% by 2055, increases the clean market share by 15.8 percentage points, and lifts welfare by 8.19%. The policy bundle also advances the emissions peak by roughly ten years and yields a steeper decline thereafter, lowering cumulative damages. Moreover, relying on a single policy instrument cannot deliver a green transition alongside robust economic growth. An R&D-only strategy speeds up technological frontier expansion but leaves the green transition lagging; a carbon tax boosts clean technology adoption but dampens productivity growth; and an infrastructure-only approach alleviates the constraints in renewable energy but delivers moderate gains. Comparative statics with respect to energy endowments yield different patterns: as renewable potential increases, the optimal levels of R&D subsidies decline, while the optimal renewable infrastructure investment increases. This provides insight into the sources of comparative advantage in the transition to green technologies.

Related Literature Our paper is closely related to and bridges several strands of recent literature. First, our paper connects two largely parallel strands of the network-macroeconomics literature. On the production side, we build on models showing how IO

linkages transmit and amplify sectoral shocks (Acemoglu et al., 2012, 2016b; Liu, 2019), and how network structure shapes macro sufficient statistics for distortions (Baqaee and Farhi, 2020), providing aggregation tools that map micro distortions and linkages into macro sufficient statistics. On the innovation side, we draw from empirical work and models of knowledge-spillover networks (Acemoglu et al., 2016a; Liu and Ma, 2021), which characterize optimal R&D allocations based on innovation centrality. Our contribution is to place both networks in one tractable environment with directed technology change, proving that the policy multiplier is the product of (production) Domar weight and (innovation) spillover centrality, scaled by the adoption elasticity. This yields implementable formulas for policy targeting and for transition dynamics that account for (i) immediate IO pass-through on prices and quantities, (ii) reallocation across clean and dirty lines within sectors, and (iii) the dynamic diffusion of knowledge across sectors and technologies. Furthermore, network structure provides a transparent mapping from empirical IO tables and estimated spillover matrices to policy weights.

Second, we build on the directed technical change literature that considers the substitution between clean and dirty technologies (Acemoglu et al., 2012, 2016)), and we connect it directly to the spillover-based account of innovation dynamics recently formalized in. Two recent works in this literature are closely related to our work. Donald (2025) builds a model of directed technology change in clean and dirty technologies with multiple sectors, emphasizing that the pattern of cross-technology spillovers shapes transitional dynamics and the effectiveness of incremental policy in avoiding lock-in. We contribute by integrating these insights with sectoral heterogeneity and IO propagation, and our results prove that sectoral interactions are of first-order importance. In this case, the market-size effect is no longer just an aggregate parameter, but it is endogenously determined by a sector's Domar weight and its centrality in the production network. Therefore, our "green centrality" sufficient statistic is a richer and more comprehensive guide for policy. Another recent work by Aghion et al. (2025) provides another rationale for targeted industrial policy that is grounded in the production structure. They model a vertical supply chain and show that strategic complementarities in adoption create a coordination failure which leads to multiple steady-states. Our model features endogenous innovation instead of technology adoption, and our imperfect substitutability between clean and dirty technology ensures that we have a unique equilibrium path. Furthermore, we study general input-output and spillover matrices, which can capture more complex, non-linear relationships than a vertical supply chain.

Third, our work is also related to the vast macroeconomic literature studying the impacts of climate externalities. This field is built on seminal frameworks like DICE (Nordhaus, 2017) and the policy calculus of the Stern Review (Stern, 2007). Subsequent work, such as Golosov et al. (2014), provided tractable formulations for optimal carbon-tax rules, typically within frictionless, single-sector general equilibrium models. Our primary departure is to move from a representative-agent, single-sector framework to a multi-sector economy structured by explicit production (IO) and innovation (spillover) networks. In such an environment, the welfare impact of a policy depends not just on the aggregate abatement costs but also on which sectors the policy targets. We believe this sectoral heterogeneity is central to the discussion of green growth, as the general-equilibrium multiplier of a given carbon price or subsidy is determined by sectoral Domar weights and innovation centrality.

Lastly, our paper is related to the emerging literature on industrial policy. Juhász et al. (2023) surveyed recent research and defined industrial policies as government policies that explicitly target the transformation of the structure of economic activity in pursuit of some public goal. While some studies challenge the effectiveness of industrial policies in generating systematic economic and social benefits (Buera et al., 2013), notable works suggest that well-designed and effectively implemented industrial policies can significantly contribute to economic growth in the context of network economies (Liu, 2019; Buera and Trachter, 2024), Marshallian externalities (Harrison and Rodriguez-Clare, 2010), and particularly in sectors that are strategically important or underdeveloped (Rodrik, 2009). Our framework contributes to this literature by delivering general implications and new rationales for industrial policy design in the green sectors. Besides, most of this work advocates industrial policies that take the form of import tariffs, taxes, or subsidies to sectoral production, and direct financial interventions. In addition to the carbon tax, our work complements this literature by focusing on R&D subsidies while also viewing infrastructure investment as a tool for industrial policy.

The remaining part of the paper is organized as follows. Section 2 presents our empirical findings and model-free evidence on the position of green technology innovations in networks. Section 3 outlines our baseline endogenous growth model of green transition in the innovation and production network. Section 4 shows the quantitative exercises that simulate the impact of a policy as well as the welfare improvement under the optimal industrial policy. Section 5 concludes the paper.

2 Motivational Facts

Using data from major economies, this section documents that green technologies and industries lie closer to the core of both networks than do their non-green counterparts. The evidence supplies a new theoretical rationale for the economic and social salience of green industrial policy.

2.1 Data

Our analysis uses a comprehensive patent dataset covering applications filed and published through December 2023 across the United States, China, the European Union (including the United Kingdom)[1], Japan, and South Korea. It includes each patent's publication number, patent family number, International Patent Classification (IPC) and Cooperative Patent Classification (CPC) codes, filing date, and citation publication numbers.[2]

The widely used IPC/CPC classification system does not directly map to either (1) green innovation categories or (2) industrial sector classifications. To address this, we first apply the OECD ENV-TECH classification system to label patents with two dummy variables: *RenewableEnergy* and *EmissionReduction*. If either dummy equals one, the patent is classified as a green innovation; otherwise, it is considered a non-green innovation.[3] The OECD ENV-TECH classification system has been widely adopted in the empirical economics literature to study the dynamics of green technology innovation (Kesidou and Wu, 2020; Cecere et al., 2014; Ghisetti and Quatraro, 2017). Next, we utilize the concordance tables provided by Dorner and Harhoff (2018) to assign patents to specific industries. The authors employ statistical methods to analyze the co-occurrence of industry codes and technology classifications using linked inventor-employment data from German patents and social security records, generating concordance tables based on fractional counts and relative frequencies. Combining these two classification schemes yields a patent-level indicator of whether an innovation contributes to renewable energy adoption or green-

[1] Our sample period largely predates the United Kingdom's withdrawal from the European Union. Accordingly, patents filed with or published by the UK Intellectual Property Office are included as part of the European patent system in our analysis.

[2] A common issue with patent data is that multiple patents may be filed for the same underlying invention to secure intellectual property protection. This practice mechanically inflates patent counts and citation measures, leading to biased inferences about innovative activity. To mitigate this, we group patents into families based on their family number and measure patents and their corresponding citations at the family level.

[3] We conduct a robustness check by using the World Intellectual Property Organization (WIPO) Green Inventory classification to categorize patents related to environmentally sound technologies.

Table 2.1: Descriptive Statistics of Yearly Patent Registration

	2009 # of Registration	2019 # of Registration	CAGR (2009–2019)
US:			
Total patents	261,897	278,691	0.62%
Non-green	240,595	251,979	0.46%
Renewable energy	9,889	10,797	0.88%
Emission reduction	14,710	19,530	2.88%
China:			
Total patents	84,556	999,960	28.02%
Non-green	68,944	854,070	28.62%
Renewable energy	4,498	32,697	21.94%
Emission reduction	11,114	113,193	26.12%
EU:			
Total patents	164,888	169.360	0.27%
Non-green	144,785	145,033	0.00%
Renewable energy	5,082	5,652	1.07%
Emission reduction	15,021	18,675	2.20%
Japan:			
Total patents	103,043	98,120	-0.49%
Non-green	83,433	76,537	-0.85%
Renewable energy	6,877	7,934	1.44%
Emission reduction	12,733	13,649	0.70%
South Korea:			
Total patents	92,246	126,818	3.23%
Non-green	80,596	110,150	3.17%
Renewable energy	3,090	4,601	4.06%
Emission reduction	8,560	12,067	3.49%

Notes: Patent registrations are measured at the patent family level. CAGR denotes the compound annual growth rate between 2009 and 2019. Counts may be subject to minor revisions due to publication lags for patents filed in earlier years.

house gas emissions mitigation within its assigned industry.

Table 2.1 reports descriptive statistics on yearly patent registrations by country. Green innovation continues to grow across all major economies. In developed economies such as the United States, the European Union, Japan, and South Korea, non-green patenting is largely stagnant over the period, while green innovation continues to expand, indicating a reallocation of innovative activity toward green technologies. Meanwhile, China exhibits rapid growth in both green and non-green patenting, with non-green patents expanding slightly faster than green ones, reflecting broad-based innovation catch-up from a low initial level. Nonetheless, green patenting in China grows at exceptionally high rates, suggesting that the expansion of green innovation occurs alongside, rather than at the expense of, overall technological development.

2.2 Knowledge Spillover

Existing knowledge plays a central role in the generation of new knowledge, serving both as a foundation for incremental improvements and as a source of ideas for novel recombinations. Such knowledge spillovers frequently occur across firm and industry boundaries, giving rise to innovation networks that shape the direction and pace of technological progress.

In empirical economic research, patent citations are commonly used as a proxy for knowledge spillovers: each instance in which a patent is cited by a subsequent patent is interpreted as one spillover event. Using our patent data, we construct citation linkages that allow us to measure the intensity of knowledge spillovers generated by green and non-green innovations across industries. These citation-based measures further enable the construction of innovation networks that characterize how knowledge diffuses through the production and innovation system.

Our analysis indicates that green innovations—encompassing both renewable energy and emission-reduction technologies—generate stronger knowledge spillovers than non-green innovations. Focusing on patents filed in China between 2014 and 2017,[4] we find

[4]Accordingly, we focus on patents filed between 2014 and 2017 and measure the number of forward citations they receive within five years. This choice reflects the fact that patents filed in more recent years—particularly from 2023 onward—might still require additional time to complete the examination and approval process, and their citation records remain incomplete. Including such patents would therefore lead to a systematic underestimation of citation-based spillovers.

that the average number of forward citations within five years of filing is 2.42 for non-green patents, compared to 2.84 for renewable energy patents and 2.57 for emission-reduction patents.

A similar pattern emerges across other major economies. In the European Union, the corresponding figures are 0.69 for non-green patents, 0.89 for renewable energy patents, and 0.92 for emission-reduction patents. In Japan, the averages are 0.74, 1.08, and 0.92, respectively. In South Korea, they are 1.18 for non-green patents, 1.61 for renewable energy patents, and 1.30 for emission-reduction patents.[5]

At the same time, knowledge spillovers vary substantially across industries. For instance, patents in telecommunications tend to generate far more citations than those in agriculture, regardless of whether the underlying innovation is green. This raises the concern that the higher average spillovers associated with green innovations may simply reflect their concentration in industries that are intrinsically more spillover-intensive.

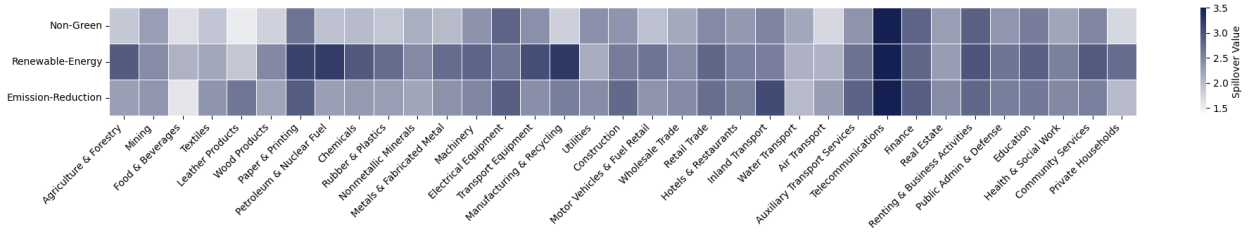


Figure 2.1: Industry-Level Knowledge Spillovers by Green Category (China)

Note: Each panel reports average total knowledge spillovers by industry and technology type, measured as the number of forward citations received within five years after patent registration. The sample includes patents filed between 2014 and 2017. Rows correspond to non-green, renewable energy, and emission-reduction patents; columns denote industries. Darker shading indicates higher spillover intensity.

To address this potential confounding factor, we compare green and non-green innovations within the same industry. Figure 2.1 presents the results for China, where the shading of each cell reflects the average spillover intensity for a given green category and industry. The figure shows that, within most industries, renewable energy and emission-reduction innovations generate more spillovers than non-green innovations. In Figures C.1,

[5] In the United States, the average number of forward citations within five years is 5.76 for non-green patents, compared to 5.03 for renewable energy patents and 5.81 for emission-reduction patents. However, after controlling for industry fixed effects, as discussed below, renewable energy innovations exhibit higher spillovers.

we further show that these results extend to other economies.

Table 2.2: Poisson Regression Results for Citation Spillovers

	US	China	EU	Japan	South Korea
	(1)	(2)	(3)	(4)	(5)
Panel A: Spillover					
RenewableEnergy	0.083***	0.128***	0.252***	0.229***	0.316***
	(0.002)	(0.002)	(0.006)	(0.006)	(0.005)
EmissionReduction	0.141***	0.026***	0.128***	0.057***	0.018***
	(0.002)	(0.002)	(0.004)	(0.005)	(0.004)
Industry FE	✓	✓	✓	✓	✓
Observations	1,214,109	2,802,960	837,010	518,586	709,010
Pseudo R^2	0.05	0.04	0.02	0.02	0.01
Panel B: Cross-Industry Spillover					
RenewableEnergy	0.150***	0.193***	0.322***	0.282***	0.385***
	(0.003)	(0.002)	(0.007)	(0.006)	(0.005)
EmissionReduction	0.164***	0.055***	0.183***	0.095***	0.046***
	(0.002)	(0.002)	(0.005)	(0.005)	(0.005)
Industry FE	✓	✓	✓	✓	✓
Observations	1,214,109	2,802,960	837,010	518,586	709,010
Pseudo R^2	0.05	0.05	0.02	0.03	0.02

Notes: The dependent variable is the number of forward citations received by each patent within five years after registration. Panel A uses total forward citations as the dependent variable, while Panel B uses forward citations originating from patents in different industries. The sample includes patents registered between 2014 and 2017 across regions. Renewable Energy Patent and Emission Reduction Patent are indicator variables based on OECD ENV-TECH classifications. All regressions are estimated using Poisson models with industry fixed effects. Reported coefficients are log incidence-rate ratios; standard errors are in parentheses. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$.

To further quantify these differences while controlling for industry-level heterogeneity, we estimate Poisson regressions of citation counts on green innovation indicators with industry fixed effects. For each patent p, we estimate

$$\ln \mathbb{E}\left(\text{Spillover}_p\right) = \alpha + \beta_1 \cdot \text{RenewableEnergy}_p + \beta_2 \cdot \text{EmissionReduction}_p + \text{IndustryFE}_p, \tag{1}$$

where the dependent variable is the number of forward citations received within five

years after registration.

Table 2.2 reports the Poisson regression results. The coefficients indicate that, relative to non-green patents within the same industry, green innovations generate significantly stronger knowledge spillovers across different regions. In Panel A, both renewable energy and emission-reduction patents receive more forward citations than non-green patents after controlling for industry fixed effects. For example, in China, renewable energy patents are associated with substantially higher citation intensity than non-green patents, with a similar but slightly stronger effect for emission-reduction patents; comparable patterns hold in the European Union and other advanced economies.[6] Moreover, Panel B shows that these differences are even larger for cross-industry spillovers, suggesting that green innovations diffuse more broadly across sectors.

2.2.1 Why Green Innovations Generate Stronger Spillovers

The empirical evidence above shows that green innovations generate systematically stronger knowledge spillovers than non-green innovations, both within and across industries, even after controlling for industry fixed effects. These findings naturally raise the question of why green technologies diffuse more intensively than other types of innovations. This section provides an intuitive explanation, drawing on additional empirical evidence presented in Appendix A.

A useful lens for understanding these patterns is the theory of general-purpose technologies (GPTs) developed by Bresnahan and Trajtenberg (1995). In this framework, technologies generate unusually large spillovers when they are widely applicable across sectors and serve as enabling inputs for downstream innovations. Many green technologies fit this description. Clean-energy and environmental innovations—such as battery materials, power electronics, catalytic processes, and energy-efficiency technologies—are not confined to a single industry but are used as upstream inputs in a broad range of production and innovation activities. Consistent with this view, Appendix A shows that green patents exhibit significantly higher industry-entropy scores than non-green patents, indicating broader applicability across sectors. This wide applicability increases the likelihood that advances in green technologies are cited and recombined outside their origi-

[6] Because the Poisson coefficients are log incidence-rate ratios, they can be interpreted as percentage changes in expected citation counts. For instance, a coefficient of 0.128 implies a 13.7% increase in expected citations relative to non-green patents, computed as $e^{0.128} - 1$.

nating industries, leading to stronger cross-industry spillovers.

In addition, green innovations tend to draw on a more diverse set of technological knowledge. They frequently combine insights from materials science, chemistry, electrical engineering, and computer science, reflecting the complex engineering challenges involved in clean and low-emission technologies. Using CPC-based measures, Appendix A documents that green patents display greater technological breadth and larger maximum technological distance than non-green patents. This multidimensional knowledge base expands the set of potential recombination paths and citation channels, further amplifying spillover intensity.

Taken together, these mechanisms suggest that the stronger spillovers associated with green innovation are not incidental but arise from intrinsic features of the innovation process. By being both broadly applicable across industries and technologically diverse, green innovations resemble general-purpose technologies in their diffusion patterns, generating spillovers that extend well beyond their immediate technological domains. Detailed supporting evidence is provided in Appendix A.

2.3 Production Centrality

Among various industries analyzed, green innovations are particularly intense in certain sectors. These industries include (1) petroleum and fuel, (2) chemicals, (3) metal and fabricated metal, (4) machinery, (5) electrical equipment, (6) transport equipment, (7) utilities, (8) wholesale trade, and (9) renting and business activities, among others.[7]

Most of these industries share a common characteristic: high production network centrality. This implies that their outputs are extensively used as intermediate inputs across various industries, thereby playing a crucial role in shaping the economic structure and development. This can be quantified using various measures. For instance, we can utilize input-output tables to calculate the weighted outdegree, defined as

$$\text{Weighted Outdegree}_i = \sum_j w_{ij},$$

where w_{ij} denotes the share of intermediate inputs purchased by industry j that originates

[7]These industries either have the greatest number of green patents or have the highest proportion of green patents relative to all patents.

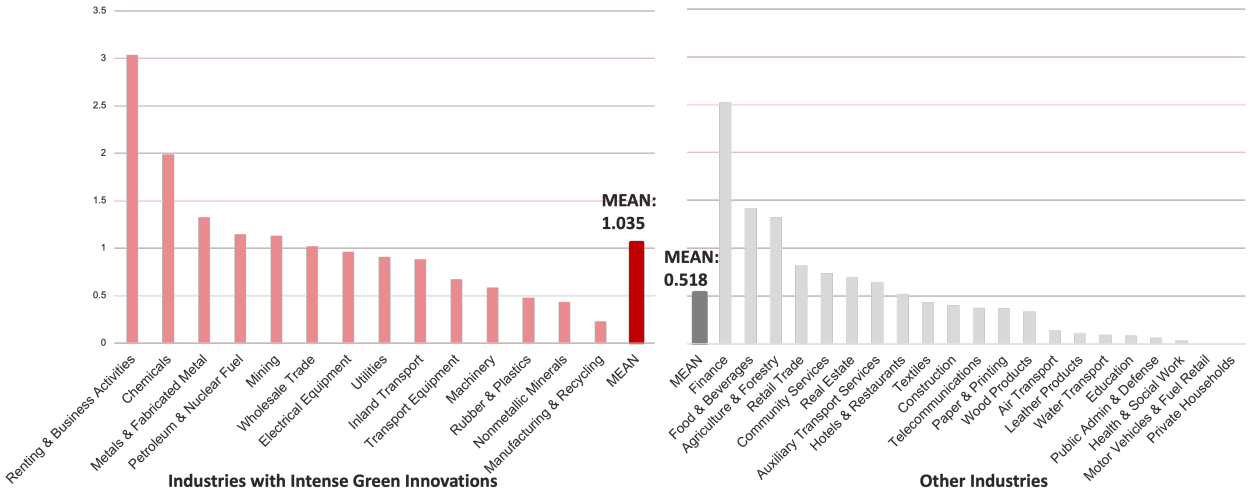


Figure 2.2: Production Network Centrality of Green and Non-Green Industries

from industry i, as measured using input–output tables.

We divide all industries into two groups: one comprising industries with intense green innovations and the other comprising the remaining industries. We conduct a simple t-test to compare the weighted outdegrees of the two groups, and the results indicate that industries with intensive green innovations have significantly higher weighted outdegrees than others, as illustrated by Figure 2.2.

The implication of finding that industries with intense green innovations have significantly higher weighted outdegrees is substantial. Innovations—particularly green innovations—in these central industries have the potential to propagate widely through the production network, influencing the technological advancement and environmental performance of a large portion of the economy.

Moreover, our previous observation that green innovations exhibit stronger spillovers than non-green innovations on the innovation network adds another layer of significance. Green innovations not only spread through the production network due to the centrality of their originating industries, but also enhance the innovation capabilities of other industries more effectively than non-green innovations. Fostering green innovations in industries with high centrality in the production network can lead to a multiplier effect. Specifically, it can accelerate the transition to a greener economy by enhancing both production efficiency and stimulating further innovations.

3 The Model

In the following section, we build a multi-sector general-equilibrium model that nests two types of network structure: an input–output (IO) production network and an innovation network across green and dirty technologies. The framework develops in Liu and Ma (2021) and extends to two new dimensions: (i) an explicit energy sector that endogenizes relative energy prices through its own directed technical change, and (ii) public infrastructure that lowers the delivered cost of renewable energy. We derive optimal policy rules, highlight novel comparative-advantage channels, and propose sufficient statistics that summarize the impact of industrial policies.

The model is in continuous time, indexed by t. We consider a multi-sector economy endowed with $\bar{L}$ amount of production labor, $\bar{H}$ amount of researchers, and ϑ_g level of renewable potential. There are two layers for the economy. At the outer layer, the output of each sector is combined to produce a final good for consumption as well as serve as the intermediate input used by other sectors. At the inner layer, each sector consists of a continuum of heterogeneous varieties, where firms produce using either clean and dirty technology with intermediate materials, energy, and labor.

3.1 Model Environment

3.1.1 Production and Market Structure

Consumption good sector. The final consumption good $\mathcal{Y}_t$ is a Cobb-Douglas aggregation in the energy sector e and $s \in \mathcal{N} = \{1, 2, ...N\}$ sectors:

$$\mathcal{Y}_t = \mathcal{E}_t \left(Y_{e,t}^{\gamma_e} \prod_{s \in \mathcal{N}} Y_{s,t}^{\gamma_s} \right), \qquad \gamma_e + \sum_{s \in \mathcal{N}} \gamma_s = 1, \tag{2}$$

where $\mathcal{E}_t$ is the damage from atmospheric carbon concentration (detailed below), γ_s denotes the expenditure share of sector s, and we let $\boldsymbol{\gamma}$ denote the column vector collecting all γ_s.

Production sector. Within each sector s, there is a measure of differentiated varieties $i \in [0, 1]$ produced by a monopolist using either clean $(j = c)$ technology, or dirty $(j = d)$ technology that emits pollution. The sectoral output Y_s is a CES aggregation over clean

and dirty outputs

$$Y_{s,t} = \left[(Y^c_{s,t})^{\frac{\varepsilon-1}{\varepsilon}} + (Y^d_{s,t})^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}}, \quad s \in \mathcal{N}, \tag{3}$$

where the elasticity of substitution between the clean and dirty technology satisfies $\varepsilon > 1$, and the technology-level output is aggregated as $\ln Y^j_{s,t} = \int_0^1 \ln y^j_{i,s,t}\, di$ for $j \in \{c, d\}$ [8]. The total output of sector s can be absorbed either by final-good demand or by intermediate use in downstream sectors. We therefore define total intermediate demand for sector-s output as

$$M_{s,t} \equiv \sum_{k \in \mathcal{N}} \sum_{j \in \{c,d\}} \int_0^1 m^j_{i,k,s,t}\, di, \tag{4}$$

that is, the sum of all input demands for sector-s varieties across downstream sectors, technologies, and product lines. The corresponding sectoral resource constraint is

$$C_{s,t} + M_{s,t} \leq Y_{s,t}, \qquad s \in \mathcal{N}. \tag{5}$$

The production function of variety i using the current leading technology is

$$y^j_{i,s,t} = B_s\, A^j_{i,s,t}\, \ell^{\alpha_s}_{i,s,t} e^{\beta_s}_{i,s,t} \prod_{k \in \mathcal{N}} m^{\sigma_{s,k}}_{i,s,k,t}, \qquad \alpha_s + \beta_s + \sum_{k \in \mathcal{N}} \sigma_{s,k} = 1 \tag{6}$$

where $A^j_{i,s,t}$ is the productivity of variety i, $\ell_{i,s,t}$ is the labor employed, $e_{i,s,t}$ is the energy input, $m_{i,s,k,t}$ is the intermediate-input used by sector s of sector k, and $B_s \equiv (\alpha_s^{\alpha_s} \beta_s^{\beta_s} \prod_{k \in \mathcal{N}} \sigma_{s,k}^{\sigma_{s,k}})^{-1}$ is a normalizing constant. Elasticities $\alpha_s, \beta_s, \sigma_{s,k}$ are the sector-specific input share for labor, energy, and intermediates in sector s, respectively. We define $A^j_{s,t}$ as a sector-specific knowledge stock of technology j: $\ln A^j_{s,t} \equiv \int_0^1 \ln a^j_{i,s,t}\, di$.

Each product variety is produced in a monopoly market by the producer with frontier technology, and it sets price in Bertrand competition with firms using the previous vintage of technology, which is 'one step' behind the current leader's productivity (innovation 'steps' are defined below).

Energy sector and infrastructure. The energy sector supplies energy input for all sectors with substitutable energy from two sources that match the universal clean/dirty classification: renewable generation (clean, $j = c$) and fossil-fuel generation (dirty, $j = d$). Physical quantities are still labelled by their source type g (renewable) and f (fossil) to

[8] This implies unit elasticity of substitution between different varieties. We can also micro-found firm dynamics with the fixed cost of operation. For each product line, there is a unit mass of potential entrants that can adopt a better technology and replace the incumbent with an i.i.d. shock of fixed cost $\kappa_s \sim \text{Gumbel}(\theta)$.

keep the production equations readable, but technology stocks, R&D effort, and all innovation objects carry the universal superscripts c and d. Each energy type is produced in a monopoly market with a fringe at $1/\lambda$ the incumbent's productivity. For renewable generation, the production function is $E_{g,t} = A^c_{e,t}\,\vartheta_g\,\ell_{g,t}$, where $A^c_{e,t}$ denotes the clean energy technology stock, ϑ_g is the economy's renewable potential, and $\ell_{g,t}$ is the labor input.[9] For fossil-fuel generation, the production function is $E_{f,t} = B_f A^d_{e,t}\,\ell^{\alpha_e}_{f,t} R^{1-\alpha_e}_{f,t}$, with $B_f = [(\alpha_e)^{\alpha_e}(1-\alpha_e)^{1-\alpha_e}]^{-1}$ a normalizing constant. $A^d_{e,t}$ denotes the dirty (fossil-fuel) energy technology stock, $\ell_{f,t}$ is the labor input for fossil generation, and $R_{f,t}$ denotes the fossil fuel input extracted at the constant exogenous price $\bar{p}_F$.

Transmission and distribution of electricity rely on grid services such as interconnection, balancing, storage, and demand management, which are particularly important for integrating intermittent renewables (Arkolakis and Walsh, 2023). We therefore introduce an energy-type-specific grid service sector that converts public infrastructure into effective grid efficiency.

For each electricity type $j \in \{g, f\}$, delivered electricity $X_{j,t}$ is produced by combining raw generation $E_{j,t}$ with a corresponding grid service input $G_{j,t}$:

$$X_{j,t} = \left[E_{j,t}^{\frac{\eta-1}{\eta}} + G_{j,t}^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}, \qquad \eta < 1, \tag{7}$$

where $\eta < 1$ implies that raw generation and grid services are complements. Grid services are produced competitively using the final good as the sole input:

$$G_{j,t} = \frac{x_{j,t}}{\kappa_j(K_{j,t})}, \qquad \kappa_j(K_{j,t}) > 0, \quad \kappa_j'(K_{j,t}) < 0, \tag{8}$$

where $x_{j,t}$ is the amount of final good allocated to grid services of type j and $\kappa_j(K_{j,t})$ is the unit cost of providing one unit of $G_{j,t}$, decreasing in the infrastructure stock $K_{j,t}$. The government invests in energy-specific infrastructure $K_{j,t}$, $j \in \{g, f\}$, which evolves according to

$$\dot{K}_{j,t} = I_{j,t} - \delta_K K_{j,t} \tag{9}$$

where the depreciation rate is $\delta_K > 0$. For tractability, to deliver $I_{j,t}$ amount of infrastruc-

[9] As defined by Arkolakis and Walsh (2023), the renewable potential is the sum of all renewable resources potential in the economy, including solar, wind, etc.

ture investment, we assume a convex total cost in units of the final good:

$$\mathcal{G}(I_{j,t}) \equiv \frac{c_K}{2} I_{j,t}^2, \qquad c_K > 0.$$

For $I_{j,t} > 2/c_K$, the convex cost satisfies $\mathcal{G}(I_{j,t}) > I_{j,t}$.
Finally, effective delivered electricity supplied to production sectors is a CES aggregate of the two delivered types:

$$Y_{e,t} = \left[X_{g,t}^{\frac{\varepsilon_e - 1}{\varepsilon_e}} + X_{f,t}^{\frac{\varepsilon_e - 1}{\varepsilon_e}} \right]^{\frac{\varepsilon_e}{\varepsilon_e - 1}}, \qquad \varepsilon_e > 1, \tag{10}$$

where ε_e governs substitution between renewable and fossil delivered electricity.

3.1.2 Innovation

Each sector-technology node (s, j) consists of a continuum of independent product lines of measure one. On every line, a unit mass of entrants chooses R&D effort taking prices, wages, subsidies, and the expected value of successful innovation as given. A successful innovation advances the line's frontier technology by a step size of $\lambda > 1$ and replaces the incumbent on that line. Because product lines are independent and there is a continuum of them, a law of large numbers applies: over a short interval Δt, the fraction of lines upgraded in node (s, j) is deterministically equal to $z_{s,t}^j \Delta t + o(\Delta t)$. Hence the sector-level technology frontier evolves according to

$$\frac{d \ln A_{s,t}^j}{dt} = z_{s,t}^j \ln \lambda. \tag{11}$$

This formulation makes clear why aggregate innovation outcomes are deterministic even though innovation at the product-line level is stochastic. It also captures the "building-on-the-shoulders-of-giants" externality: a successful innovator not only upgrades the current line, but also raises the knowledge stock on which future entrants build.

R&D technology. Innovation is stochastic and follows a Poisson process. The arrival rate of a successful innovation in the technology j of sector $s \in \mathcal{N} \cup \{e\}$ is determined by the R&D efforts while benefiting from knowledge spillovers from other sectors

$$z_{s,t}^j = \theta_s^j \Phi_{s,t}^j (h_{s,t}^j)^\phi \tag{12}$$

where θ_s^j is the parameter for innovation efficiency, $h_{s,t}^j$ is the R&D effort chosen by the representative entrant for node (s,j), $\phi < 1$ captures decreasing returns to R&D effort, and $\Phi_{s,t}^j$ is the endogenous term capturing knowledge spillovers, defined as the Cobb–Douglas aggregator of all sector-technology knowledge stocks *relative to the innovating node's own stock*,

$$\Phi_{s,t}^j \equiv \prod_{k\in\mathcal{N}\cup\{e\}} \prod_{j'\in\{c,d\}} \left(\frac{A_{k,t}^{j'}}{A_{s,t}^j}\right)^{\omega_{s,k}^{j,j'}}, \qquad \sum_{k\in\mathcal{N}\cup\{e\}} \sum_{j'\in\{c,d\}} \omega_{s,k}^{j,j'} = 1, \tag{13}$$

where $\omega_{s,k}^{j,j'}$ represents the elasticity of innovation in sector s using technology $j \in \{c,d\}$ with respect to the knowledge stock of sector k using technology $j' \in \{c,d\}$.[10] Equivalently, in log form $\ln \Phi_{s,t}^j = \sum_{k,j'} \omega_{s,k}^{j,j'} \ln A_{k,t}^{j'} - \ln A_{s,t}^j$. It is convenient to define the normalized spillover matrix by $\widetilde{\mathbf{\Omega}} \equiv \mathbf{\Omega} - \mathbf{I}$ that governs the technology dynamics. Besides, the R&D for technology j in sector s receives a proportional government subsidy at the rate $\xi_{s,t}^j$, which also affects the firm's innovation decisions. Let w_t^R denote the equilibrium wage for researchers, the cost for achieving the arrival rate $z_{s,t}^j$ is therefore

$$(1-\xi_{s,t}^j)\, w_t^R\, h_{s,t}^j = (1-\xi_{s,t}^j)\, w_t^R \left(\frac{z_{s,t}^j}{\theta_s^j \Phi_{s,t}^j}\right)^{1/\phi}.$$

3.1.3 Carbon Emissions and Welfare

Pollution originates from two distinct sources. The first is the emission of greenhouse gases (GHG) due to the reliance on fossil fuels for energy production. The second is pollution from sectors using dirty technology. The economy-wide pollution is generated according to

$$\Gamma_t \equiv \psi_f E_{ft} + \sum_{s\in\mathcal{N}} \psi_s Y_{s,t}^d \tag{14}$$

where ψ_f is the emission from fossil fuel energy, $Y_{s,t}^d$ is the output with dirty technology in sector s, and $\psi_s > 0$ is the per-unit emissions from producing with dirty technology. The damage function of the pollution is specified as the exponential damage function

[10] The normalization by $A_{s,t}^j$ ensures that the spillover term $\Phi_{s,t}^j$ is homogeneous of degree zero in knowledge stocks: spillover productivity depends only on *relative* knowledge levels, not the absolute scale, which admits a balanced growth path to exist under the law of motion implied by (12). Along the BGP, all knowledge stocks grow at a common rate g, the gross aggregator $\prod(A_{k,t}^{j'})^{\omega}$ and the own-stock denominator $A_{s,t}^j$ both grow at rate g, so $\Phi_{s,t}^j$ remains constant.

following Golosov et al. (2014) as

$$\mathcal{E}_t = \exp\left[\varphi\left(\int_0^{t-T_0}(1-e^{-\delta_p t'})\Gamma_{t-t'}\,dt' - \mathcal{S}_0\right)\right], \tag{15}$$

where φ is the parameter for semi-elasticity of atmospheric GHG concentrations on welfare, δ_p is the natural decay rate of atmospheric GHG concentrations, T_0 is the time at which emission started, and $\mathcal{S}_0$ is the pre-industrial atmospheric GHG concentrations.

A representative household in the economy values aggregate consumption C_t:

$$U = \int_0^{\infty} e^{-\rho t}\ln C_t\,dt \tag{16}$$

Let $P_{C,t}$ denote the final-good price index. The household owns all factors and firms in the economy, and its nominal budget constraint is

$$P_{C,t}C_t = w_t\bar{L} + w_t^R\bar{H} + \sum_{s\in\mathcal{N}\cup\{e\}}\sum_{j\in\{c,d\}}\int_0^1 \pi_{i,s,t}^j\,di - T_t \tag{17}$$

where $\pi_{i,s,t}$ are the firm's profits of variety i in sector s and T_t is the lump-sum transfer/tax from the government. Since only relative prices matter, we take the final good as numeraire and set $P_{C,t} = 1$ in equilibrium; all real allocations in both the analytical and quantitative sections are therefore reported in final-good units.

3.2 Equilibrium

Price setting and firm profit. Since the sectoral output of each type of technology j is a Cobb-Douglas aggregation, the demand for the output of a monopolist firm producing variety i with technology j is given by $y_{i,s,t}^j = P_{s,t}^j Y_{s,t}^j / p_{i,s,t}^j$. Let w_t denote the wage, $p_{e,t}$ the unit price of delivered electricity, and $P_{k,t}$ the price index of the output of sector $k \in \mathcal{N}$. For a given output level $y_{s,t}^j$, the unit cost of producing one unit of a technology-j variety is

$$mc_{s,t}^j = \frac{w_t^{\alpha_s}\,p_{e,t}^{\beta_s}\prod_{k\in\mathcal{N}}P_{k,t}^{\sigma_{s,k}}}{A_{s,t}^j} \tag{18}$$

We assume *sector-specific* carbon taxes $\{\tau_{s,t}\}_{s\in\mathcal{N}}$ are levied as ad-valorem factors on the marginal cost of dirty production in each sector s. Specifically, producing with dirty technology in sector s generates ψ_s units of emissions per unit of production cost; the dirty producer's effective marginal cost is $(1+\tau_{s,t}\psi_s)\,mc_{s,t}^d$, incorporating both base production

costs and the emission compliance liability. Fossil fuel generation faces its own rate $\tau_{f,t}$, applying symmetrically. Each incumbent competes with an imitating fringe that can produce the previous vintage at $1/\lambda$ the current leading productivity, so the incumbent sets the highest price that keeps the fringe out (limit pricing). Equilibrium prices satisfy the constant markup rule:

$$p^c_{i,s,t} = \lambda\, mc^c_{s,t}, \qquad p^d_{i,s,t} = (1+\tau_{s,t}\psi_s)\,\lambda\, mc^d_{s,t} \tag{19}$$

The optimal factor demands are

$$\ell^j_{i,s,t} = \alpha_s \frac{mc^j_{s,t}}{w_t} y^j_{i,s,t}, \quad e^j_{i,s,t} = \beta_s \frac{mc^j_{s,t}}{p_{e,t}} y^j_{i,s,t}, \quad m^j_{i,s,k,t} = \sigma_{s,k} \frac{mc^j_{s,t}}{P_{k,t}} y^j_{i,s,t} \tag{20}$$

and per-variety operating profits are

$$\pi^c_{i,s,t} = \frac{\lambda-1}{\lambda} P^c_{s,t} Y^c_{s,t}, \qquad \pi^d_{i,s,t} = \frac{\lambda-1}{\lambda} P^d_{s,t} Y^d_{s,t} \tag{21}$$

Since all active varieties within sector s and technology j are symmetric, aggregate operating profits gives $\Pi^j_{s,t} = \int_0^1 \pi^j_{i,s,t}\, di = \pi^j_{i,s,t}$.

Aggregating across varieties within each technology and across technologies within a sector, let $P^j_{s,t}$ denote the technology-j price index in sector s, we define $\ln P^j_{s,t} \equiv \int_0^1 \ln p^j_{i,s,t}\, di$. Across clean and dirty technology outputs, the sector-level price index $P_{s,t}$ is defined as

$$P_{s,t} = \left[\left(P^c_{s,t}\right)^{1-\varepsilon} + \left(P^d_{s,t}\right)^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}}$$

The consumption-good price index is

$$\ln P_{C,t} = \gamma_e \ln P_{e,t} + \sum_{s\in\mathcal{N}} \gamma_s \ln P_{s,t} \;=\; \boldsymbol{\gamma}' \ln \boldsymbol{P}_t$$

where vector $\boldsymbol{P}_t$ collects $\{P_{e,t}, (P_{s,t})_{s\in\mathcal{N}}\}$. Under our numeraire choice, $P_{C,t} = 1$ for all t, so nominal and real final-good quantities coincide.

The fossil fuel resource price is an exogenous constant $\bar{p}_F$, calibrated to the observed average fossil electricity price. Renewable generation implies constant unit cost and limit

pricing,

$$mc_{E_g,t} = \frac{w_t}{A^c_{e,t}\vartheta_g}, \qquad p_{E_g,t} = \lambda\, mc_{E_g,t}$$

with labor demand $\ell_{g,t} = E_{g,t}/(A^c_{e,t}\vartheta_g)$. For fossil generation, cost minimization, and carbon tax yields unit cost

$$mc_{E_f,t} = \frac{w_t^{\alpha_e}\bar{p}_F^{\,1-\alpha_e}}{B_f A^d_{e,t}}, \qquad p_{E_f,t} = (1+\tau_{f,t}\psi_f)\,\lambda\, mc_{E_f,t} \tag{22}$$

The implied factor demands for fossil generation are $\ell_{f,t} = \alpha_e \frac{mc_{E_f,t}}{w_t} E_{f,t}$, $R_{f,t} = (1-\alpha_e)\frac{mc_{E_f,t}}{\bar{p}_F} E_{f,t}$. Given (8) and competitive production, the unit price of grid services is

$$p_{G_j,t} = \kappa_j(K_{j,t}), \qquad j \in \{g,f\}$$

Let $p_{E_j,t}$ denote the unit price of raw generation $E_{j,t}$. With the grid transmission technology and the CES aggregation (10), the demand for each delivered type $j \in \{g,f\}$ is

$$X_{j,t} = Y_{e,t}\left(\frac{p_{X_j,t}}{p_{e,t}}\right)^{-\varepsilon_e}, \qquad p_{X_j,t} = \left[p_{E_j,t}^{1-\eta} + p_{G_j,t}^{1-\eta}\right]^{\frac{1}{1-\eta}}. \tag{23}$$

The aggregate delivered energy price is given by $p_{e,t} = \left(p_{X_g,t}^{1-\varepsilon_e} + p_{X_f,t}^{1-\varepsilon_e}\right)^{\frac{1}{1-\varepsilon_e}}$.

Innovation intensities. Let $V^j_{s,t}$ denote the private value of becoming the incumbent in node (s,j). A representative entrant takes $\{V^j_{s,t}, w^R_t, \Phi^j_{s,t}, \xi^j_{s,t}\}$ as given and solves

$$\max_{h^j_{s,t}\geq 0}\; z^j_{s,t}V^j_{s,t} - (1-\xi^j_{s,t})w^R_t h^j_{s,t}.$$

The non-arbitrage condition implies that $V^j_{s,t}$ is proportional to the operating profit flow of the incumbent[11]. Given the first-order condition, the relative R&D intensity to clean and dirty technology in sector s is

$$\left(\frac{h^c_{s,t}}{h^d_{s,t}}\right)^{1-\phi} = \frac{1-\xi^d_{s,t}}{1-\xi^c_{s,t}}\,\frac{\theta^c_s\,\Phi^c_{s,t}\,\Pi^c_{s,t}}{\theta^d_s\,\Phi^d_{s,t}\,\Pi^d_{s,t}} \tag{24}$$

[11]For example, in a stationary environment with creative-destruction hazard $\bar{z}$, one can write $V^j_{s,t} = \Pi^j_{s,t}/(\rho+\bar{z})$. The proportionality is common across nodes for our allocation formulas, so the equilibrium R&D conditions can be written directly in terms of $\Pi^j_{s,t}$.

Therefore, the relative growth rate in clean technologies within a sector is determined by the R&D subsidy to clean research, the relative profits, and the knowledge spillover structure of the economy. In particular, the relative profit of clean and dirty technology is proportional to the relative market share of clean technology.

Between different sectors, the allocation of researchers to each sector in equilibrium is proportional to the expected profit of the sectors:

$$H_{s,t} = \frac{\sum_{j\in\{c,d\}} (\theta_s^j \, \Phi_{s,t}^j \, \Pi_{s,t}^j / (1 - \xi_{s,t}^j))^{\frac{1}{1-\phi}}}{\sum_{k\in\mathcal{N}\cup\{e\}} \sum_{j\in\{c,d\}} (\theta_k^j \, \Phi_{k,t}^j \, \Pi_{k,t}^j / (1 - \xi_{k,t}^j))^{\frac{1}{1-\phi}}} \bar{H} \tag{25}$$

where $H_{s,t} = \sum_j h_{s,t}^j$ is the total researchers conducting R&D in sector s. R&D labor market clearing implies that the wage for researchers is given by

$$w_t^R = \left[\frac{1}{\bar{H}} \sum_{s\in\mathcal{N}\cup\{e\}} \sum_{j\in\{c,d\}} \left(\frac{\phi \, \theta_s^j \, \Phi_{s,t}^j \, \Pi_{s,t}^j}{1 - \xi_{s,t}^j} \right)^{\frac{1}{1-\phi}} \right]^{1-\phi}. \tag{26}$$

Cost minimization by the final-good producer, together with Cobb–Douglas aggregation across sectors, implies

$$C_{s,t} = \gamma_s \, P_{s,t}^{-1} \, C_t, \qquad C_{e,t} = \gamma_e \, P_{e,t}^{-1} \, C_t. \tag{27}$$

Closing the model, the labor market clearing conditions are given by

$$\sum_{s\in\mathcal{N}\cup\{e\}} \sum_{j\in\{c,d\}} \ell_{s,t}^j = \bar{L}, \qquad \sum_{s\in\mathcal{N}\cup\{e\}} \sum_{j\in\{c,d\}} h_{s,t}^j = \bar{H}. \tag{28}$$

Goods markets clear in the final good, in each production sector $s \in \mathcal{N}$, and in delivered electricity:

$$C_t + \sum_{j\in\{g,f\}} \mathcal{G}(I_{j,t}) + \sum_{j\in\{g,f\}} x_{j,t} \;=\; \mathcal{Y}_t, \tag{29}$$

$$Y_{s,t} = C_{s,t} + M_{s,t}, \qquad s \in \mathcal{N}, \tag{30}$$

where $M_{s,t}$ is defined in (4). Delivered electricity clears according to

$$C_{e,t} + \sum_{s\in\mathcal{N}} E_{s,t} = Y_{e,t}, \tag{31}$$

where $E_{s,t}$ denotes the delivered electricity input used by sector s and $Y_{e,t}$ is the delivered electricity aggregate defined in (10).

The government implements R&D subsidies $\{\xi^j_{s,t}\}$, sector-specific carbon taxes $\{\tau_{s,t}\}_{s\in\mathcal{N}}$ on dirty production and $\tau_{f,t}$ on fossil fuel generation, and infrastructure investment $\{I_{g,t}, I_{f,t}\}$. Under the multiplicative tax, the carbon tax revenue collected from sector s is $\tau_{s,t}\psi_s mc^d_{s,t} Y^d_{s,t}$, and from fossil fuel generation is $\tau_{f,t}\psi_f mc_{E_f,t} E_{f,t}$. A balanced budget requires

$$\sum_{s\in\mathcal{N}\cup\{e\}}\sum_{j\in\{c,d\}} \xi^j_{s,t}\, w^R_t\, h^j_{s,t} + \sum_{j\in\{g,f\}} \mathcal{G}(I_{j,t}) = \sum_{s\in\mathcal{N}} \tau_{s,t}\,\psi_s\, mc^d_{s,t}\, Y^d_{s,t} + \tau_{f,t}\,\psi_f\, mc_{E_f,t}\, E_{f,t} + T_t. \tag{32}$$

Under the first-best sector-specific taxes $\tau^*_{s,t} = \mathrm{SCC}_t / mc^d_{s,t}$ and $\tau^*_{f,t} = \mathrm{SCC}_t / mc_{E_f,t}$ (derived below), the carbon revenue simplifies to $\mathrm{SCC}_t\,\Gamma_t$, where $\Gamma_t = \sum_{s\in\mathcal{N}} \psi_s Y^d_{s,t} + \psi_f E_{f,t}$ is total physical emissions.

Given these conditions, the competitive equilibrium is defined as follows:

Definition 1. *(Decentralized Equilibrium). Given initial technology stocks $\{A^j_{s,0}\}$, initial infrastructure stocks $(K_{g,0}, K_{f,0})$, exogenous fossil price $\bar{p}_F$, and policy sequences $\{\tau_{s,t}, \tau_{f,t}, I_{g,t}, I_{f,t}, \xi^j_{s,t}\}_{t\geq 0}$, a decentralized equilibrium is a collection of prices $\{p^j_{i,s,t}, P^j_{s,t}, P_{s,t}, P_{e,t}, P_{C,t}, w_t, w^R_t\}_{t\geq 0}$, allocations*

$$\{C_t, (C_{s,t})_{s\in\mathcal{N}}, C_{e,t}, E_{s,t}, M_{s,t}, Y_{s,t}, R_{f,t}, y^j_{i,s,t}, \ell_{i,s,t}, e_{i,s,t}, m_{i,s,k,t}, h^j_{s,t}\}_{t\geq 0},$$

and state paths $\{A^j_{s,t}, K_{g,t}, K_{f,t}\}_{t\geq 0}$ such that:
(1) the household chooses C_t subject to (17); final-good demand satisfies (27);
(2) within each sector, an active firm chooses price as (19) with profits (21);
(3) representative entrants choose innovation effort according to (24) and (25);
(4) factor and goods markets clear according to (28), (30), and (31);
(5) technology stock of each sector evolves as (11);
(6) emissions from fossil fuels and dirty technologies follow (14);
(7) budget constraints (17), (32) hold.

3.3 Sufficient Statistics and Planner's Allocation

To characterize the first-best allocations and the general equilibrium propagation of green industrial policies, we define two sufficient statistic matrices. These matrices summarize the complex interactions within the production network and the innovation network.

Input-output network matrix at technology level. First, we capture the relative importance of each sector-technology node (s, j) through the sectoral input-output structure and technology-level substitutability. Let $\mathbf{\Sigma} = [\sigma_{s,k}]_{(N+1)\times(N+1)}$ denote the matrix of input-output elasticities between sectors $s, k \in \mathcal{N} \cup \{e\}$. We define $\boldsymbol{L} \equiv (\boldsymbol{I} - \mathbf{\Sigma})^{-1}$ as the Leontief inverse of the production network (at sector level). As in Acemoglu et al. (2012), this matrix summarizes the first-order impact of a sector through the input-output network.

In the presence of substitutable clean and dirty production technologies within each sector, we must account for the substitution effects between technologies that affect the direction of innovations. To capture this, we let m_s^c and m_s^d denote the output shares of clean and dirty technology in sector s with $m_s^c + m_s^d = 1$, and let $\mathbf{M}^c \equiv \text{diag}(m_s^c)$, $\mathbf{M}^d \equiv \text{diag}(m_s^d)$ be diagonal matrices collecting the output shares. Then we define the within-sector substitution Jacobian

$$\mathbf{\Psi} \equiv \begin{bmatrix} \boldsymbol{I}_{N+1} + (\varepsilon - 1)\boldsymbol{M}^c & (1-\varepsilon)\boldsymbol{M}^d \\ (1-\varepsilon)\boldsymbol{M}^c & \boldsymbol{I}_{N+1} + (\varepsilon - 1)\boldsymbol{M}^d \end{bmatrix}. \tag{33}$$

Intuitively, $\mathbf{\Psi}$ maps technology-specific price changes into technology-specific demand changes through within-sector CES substitution with elasticity ε.

Next, we build a block-diagonal extension of the Leontief inverse to the technology space $\boldsymbol{I}_2 \otimes \boldsymbol{L}$ to define the augmented Leontief inverse of a technology-level input-output network matrix

$$\widetilde{\boldsymbol{L}} \equiv (\mathbf{\Psi} - \boldsymbol{I}_{2(N+1)})\,(\boldsymbol{I}_2 \otimes \boldsymbol{L}) = \begin{bmatrix} \Sigma^{cc} & \Sigma^{dc} \\ \Sigma^{cd} & \Sigma^{dd} \end{bmatrix} \tag{34}$$

This matrix compactly summarizes how technology-specific cost (or productivity) shocks propagate through production network amplification (via $\overline{\boldsymbol{L}}$) and the clean and dirty technology substitution (via $\mathbf{\Psi}$). The off-diagonal blocks Σ^{cd} and Σ^{dc} specifically capture the cross-technology propagation induced by substitution combined with network amplification[12].

Knowledge spillover network matrix. We denote the $2(N+1) \times 2(N+1)$ matrix $\mathbf{\Omega}$ as the innovation network of this economy, representing the cross-sector and cross-technology

[12] In this aspect, our paper also complements Donald (2025) that studies the substitution between technologies and innovation without the input-output network.

knowledge spillovers

$$\mathbf{\Omega} = \begin{bmatrix} \left(\omega_{s,k}^{cc}\right)_{s,k\in\mathcal{N}\cup\{e\}} & \left(\omega_{s,k}^{dc}\right)_{s,k\in\mathcal{N}\cup\{e\}} \\ \left(\omega_{s,k}^{cd}\right)_{s,k\in\mathcal{N}\cup\{e\}} & \left(\omega_{s,k}^{dd}\right)_{s,k\in\mathcal{N}\cup\{e\}} \end{bmatrix} = \begin{bmatrix} \Omega^{cc} & \Omega^{dc} \\ \Omega^{cd} & \Omega^{dd} \end{bmatrix}, \tag{35}$$

where each element $\omega_{s,k}^{j,j'}$ is defined in (13). Diagonal blocks $(\Omega^{cc}, \Omega^{dd})$ capture clean-to-clean and dirty-to-dirty technology spillovers, and off-diagonal blocks $(\Omega^{cd}, \Omega^{dc})$ capture the cross-technology spillovers. We refer to $\mathbf{\Omega}$ as the *gross* spillover matrix; its rows sum to one. The corresponding *net* spillover matrix that governs the growth dynamics is $\widetilde{\mathbf{\Omega}} \equiv \mathbf{\Omega} - \mathbf{I}_{2(N+1)}$, with entries $\tilde{\omega}_{s,k}^{j,j'} = \omega_{s,k}^{j,j'}$ off the diagonal and $\tilde{\omega}_{s,s}^{j,j} = \omega_{s,s}^{j,j} - 1$ on the diagonal.

Next, we define the sector-technology node's innovation centrality following Liu and Ma (2021), $\boldsymbol{\chi} \equiv [\chi_s^j]_{j\in\{c,d\},s\in\mathcal{N}\cup\{e\}}$, which is the dominant left-eigenvector of the innovation network $\mathbf{\Omega}$ with eigenvalue 1 that satisfies $\boldsymbol{\chi}' = \boldsymbol{\chi}'\mathbf{\Omega}$ and $\sum_s \sum_j \chi_s^j = 1$[13]. Taking into account the network effects, the extent of impact of a sector's technology on the economy's innovation efficiency is captured by this $2(N+1)$ vector, and the higher this centrality is, the more fundamental the technology is to technology innovation.
Taken together, $\widetilde{\boldsymbol{L}}$ determines the static gain from innovations through market size, while $\mathbf{\Omega}$ determines the dynamic gain of innovations by governing future research productivity.

3.3.1 First Best Allocations

Based on the sufficient statistic matrices, we can then characterize the first-best allocation in the economy.

Planner's problem. The planner chooses paths for allocations to maximize discounted welfare

$$\max_{\{\cdot\}_{t\geq 0}} \int_0^\infty e^{-\rho t} \ln C_t \, dt \tag{36}$$

subject to production functions, feasibility constraints (2) to (3), law-of-motion constraints for technology and infrastructure, GHG damages (11) to (15), and resource constraints

$$C_t + \sum_{j\in\{g,f\}} \mathcal{G}\left(I_{j,t}\right) + \sum_{j\in\{g,f\}} x_{j,t} \leq \mathcal{Y}_t, \qquad C_{s,t} + M_{s,t} \leq Y_{s,t} \; \forall s \in \mathcal{N}, \qquad C_{e,t} + \sum_{s\in\mathcal{N}} E_{s,t} \leq Y_{e,t},$$

[13] if the spectral radius of $\mathbf{\Omega}$ is larger than one, the innovation centrality is given by $\lambda_{\max}\boldsymbol{\chi}' = \boldsymbol{\chi}'\mathbf{\Omega}$ and $\sum_s \sum_j \chi_s^j = 1$ where $\lambda_{\max}$ is the Perron–Frobenius eigenvalue.

with $M_{s,t}$ defined in (4), labor and research labor

$$\sum_{s\in\mathcal{N}\cup\{e\}}\sum_{j\in\{c,d\}}\ell^j_{s,t}\leq\bar{L},\qquad\sum_{s\in\mathcal{N}\cup\{e\}}\sum_{j\in\{c,d\}}h^j_{s,t}\leq\bar{H}$$

Between-sector production factor allocations. Based on the input-output structure of the economy and the market structure of the sectors, the aggregate output elasticity can be expressed as

$$\boldsymbol{\mu}'=\boldsymbol{\gamma}'(\mathbf{I}-\boldsymbol{\Sigma})^{-1},\tag{37}$$

where $\boldsymbol{\Sigma}$ is the input-output cost share matrix collecting all the input elasticity $\sigma_{s,k}$ of sector k's output in sectors $s\in\mathcal{N}$, and $(\boldsymbol{I}-\boldsymbol{\Sigma})^{-1}$ is the Leontief inverse that summarizes the direct and subsequent rounds of amplification from a change in intermediate input sectors: $\boldsymbol{I}+\boldsymbol{\Sigma}+(\boldsymbol{\Sigma})^2+(\boldsymbol{\Sigma})^3...$ As in Acemoglu et al. (2012), this reflects how strongly the demand for sector s output is amplified down the production network, and is equivalent to the sectoral Domar weights by Hulten's theorem.

The planner allocates the production labor and energy inputs between sectors according to:

$$L_s\equiv\ell^c_{s,t}+\ell^d_{s,t}=\frac{\alpha_s\,\mu_s}{\sum_k\alpha_k\,\mu_k}\bar{L},\qquad E_s\equiv e^c_{s,t}+e^d_{s,t}=\frac{\beta_s\,\mu_s}{\sum_k\beta_k\,\mu_k}Y_e$$

This result shows that the planner allocates production inputs across sectors in proportion to the sector's factor intensity and its sectoral Domar weight. Hence, even if two sectors have the same technology and the same pollution intensity, the planner assigns more resources to the sector that is more central in the production network.

Within-sector production factor allocations. Let $\mathrm{SSC_t}$ denote the planner's shadow value of one unit of emissions, which is the social cost of carbon (SCC) in consumption units (derived below). The associated first-order condition implies that the planner's factor allocation across technologies satisfies

$$\frac{\ell^c_{s,t}}{\ell^d_{s,t}}=\frac{e^c_{s,t}}{e^d_{s,t}}=\left(\frac{mc^d_{s,t}+\psi_s\mathrm{SSC_t}}{mc^c_{s,t}}\right)^{\varepsilon-1}\tag{38}$$

where $mc^j_{s,t}$ is the marginal cost of production determined by the productivity in each sector. It follows that the planner internalizes GHG damages from the marginal change in the input allocation to clean technologies. This planner allocation is decentralized by sector-specific carbon taxes $\tau^*_{s,t}=\mathrm{SCC}_t/mc^d_{s,t}$, which make the private effective dirty cost

$(1+\tau^*_{s,t}\psi_s)mc^d_{s,t} = mc^d_{s,t} + \psi_s \,\mathrm{SCC}_t$ coincide with the planner's shadow cost.

We can further define the augmented weight for each technology based on the augmented Leontief inverse as $\widetilde{\boldsymbol{\mu}}' \equiv \widetilde{\boldsymbol{\gamma}}'\widetilde{\boldsymbol{L}}$, where

$$\widetilde{\boldsymbol{\gamma}} = \begin{bmatrix} \mathbf{M}^c \\ \mathbf{M}^d \end{bmatrix} \boldsymbol{\gamma},$$

and $\mathbf{M}^c, \mathbf{M}^d$ are the matrices for sectoral technology market shares as defined above. Intuitively, the technology-level welfare weights depend on the sector's Domar weights μ_s, and within-sector substitution governed by $\boldsymbol{\Psi}$.

R&D factor allocations across technologies. Beyond the static Domar weights $\widetilde{\boldsymbol{\mu}}$, the dynamic network spillovers introduce a discounted innovation-network resolvent that accumulates all future rounds of spillover amplification. Solving the planner's Hamiltonian shows that the discount factor inside this resolvent equals the ratio of the BGP growth rate to the household discount rate. The following proposition characterizes the optimal allocation of researchers. In Appendix B, we also provide the growth-maximizing R&D resource allocation.

Proposition 1. *At a BGP with innovation rates* $\mathbf{Z}^* = \mathrm{diag}(z_s^{j*})$*, define the welfare co-state vector*

$$\boldsymbol{\nu}^* = \left(\rho\,\mathbf{I} - \ln\lambda \cdot \widetilde{\boldsymbol{\Omega}}^\top \mathbf{Z}^*\right)^{-1} \widetilde{\boldsymbol{\mu}}.$$

The welfare-maximizing R&D resource allocation is time-invariant at the BGP and given by

$$\boldsymbol{\chi}^{R\prime} = \frac{\boldsymbol{\nu}^{*\prime}\mathbf{Z}^*}{\boldsymbol{\nu}^{*\prime}\mathbf{Z}^*\,\mathbf{1}}.$$

In a symmetric BGP where $z_s^{j*} = z^*$ *for all* (s, j)*:*

$$\boldsymbol{\chi}^{R\prime} = \frac{\widetilde{\boldsymbol{\mu}}'\left(\mathbf{I} - \dfrac{z^*\ln\lambda}{\rho}\widetilde{\boldsymbol{\Omega}}\right)^{-1}}{\widetilde{\boldsymbol{\mu}}'\left(\mathbf{I} - \dfrac{z^*\ln\lambda}{\rho}\widetilde{\boldsymbol{\Omega}}\right)^{-1}\mathbf{1}},$$

Proof. Refer to Appendix B. □

3.4 Network Externalities and Optimal Industrial Policy

In this section, we discuss why market outcomes diverge from the social optimum allocations, and under what structural conditions a given policy magnitude is sufficient to redirect the economy toward the green steady state. Then we characterize the optimal industrial policy to achieve the second-best results.

Network externality in production. A sector that is a critical supplier to many other productive sectors (i.e., has high network centrality) imposes a larger positive externality when it raises its productivity. A successful innovation raises final consumption via the downstream production network, but the innovator captures only own-sector profit Π_s^j. The social surplus is proportional to the Domar weight of the sector, depending on the input-output network $\mathbf{\Sigma}$. As a result, sectors with greater production network centrality experience an insufficient R&D effort compared to the optimal allocation, resulting in fewer innovations in such sectors.

Knowledge-spillover externality. A successful innovation earns the current monopoly profit. Research labs choose h_s^j by equating the private marginal return to the researcher's wage, while treating the knowledge spillovers as exogenous. Since a higher h_s^j accelerates all lines (k, j') for which $\omega_{s,k}^{j,j'} > 0$, a lab ignores the spillover of its innovation to downstream sectors and to other firms through the spillover matrix. It results in an undersupply of research effort proportional to the columns of the innovation network $\mathbf{\Omega}$.

Environmental externality. Neither fossil energy producers nor dirty-technology firms internalized the disutility caused by their emissions, thus the dirty output and fossil energy are over-consumed in the decentralized equilibrium. For the former, competitive extractors set the energy price equal to marginal extraction cost, while the social cost of fossil fuel energy is adjusted based on resource constraints and the negative externality under the social planner's allocation. For the latter, firms' dirty production generates emissions that create a flow of negative externality $\mathcal{E}_t$ in every time period, which is not taken into account by the producers using dirty technology. If the initial knowledge stocks of the clean technologies are lower than the dirty technologies, the high market share of the dirty technology in all sectors results in too few innovation efforts directed to the clean technologies[14].

[14]This aspect is discussed extensively in the directed technology change literature, e.g. Acemoglu et al. (2012), Acemoglu et al. (2016)

Infrastructure as a public good. The final source of inefficiency arises from the public good nature of renewable energy infrastructure (e.g., smart grids, charging stations, and storage facilities). In our framework, the specific infrastructure stock lowers the effective price of the renewable energy bundle. However, private firms fail to internalize the positive spillover effects on production sectors that rely on energy inputs. Consequently, the decentralized equilibrium results in underinvestment in infrastructure. This creates a bottleneck: even if renewable technology productivity rises through innovation, the lack of complementary infrastructure keeps the delivered price of renewable energy high.

The potential market failures in such a decentralized equilibrium growth path result in too little R&D in input-output-central and knowledge-spillover-central technologies, too much innovation with dirty technology and fossil fuel extraction, as well as a static misallocation of labor and intermediates caused by monopoly pricing.

3.4.1 Optimal Policies

This section characterizes a set of policy instruments that decentralize the planner's allocation in an economy with an IO production network, endogenous innovation with knowledge spillovers, and emissions externalities generated by dirty production. In particular, we suppose the social planner does not correct the monopoly rents to preserve innovation incentives. She decides sector-specific carbon taxes $\{\tau_{s,t}\}_{s\in\mathcal{N}}$ and $\tau_{f,t}$ applied as ad-valorem factors on dirty production costs (so that the effective dirty marginal cost in sector s is $(1+\tau_{s,t}\psi_s)mc^d_{s,t}$, internalizing emission compliance costs), a R&D subsidy $\xi^j_{s,t}$ that reduces the effective research wage from $(1-\xi^j_{s,t})w^R_t$, and an investment in renewable infrastructure $I_{g,t}$[15]. Proofs of the optimal policies can be found in Appendix B.

Carbon tax. Define the social cost of carbon (SCC) as the planner's shadow value of a marginal increase in emissions at time t:

$$\text{SCC}_t \;\equiv\; C_t \int_t^\infty e^{-\rho(t'-t)}\,\frac{\partial \mathcal{E}_{t'}}{\partial \Gamma_t}\,d\,t'. \tag{39}$$

Under the multiplicative carbon tax, the effective emission price per physical tonne of CO_2 in sector s is $\tau_{s,t}\cdot mc^d_{s,t}$ (the ad-valorem rate times the marginal cost, which is the tax paid per unit of physical emission ψ_s). The Pigouvian condition requires this effective

[15]In interest of studying the green industry policies, the study currently focus on the optimal renewable infrastructure investment policy, while leave the optimal fossil fuel infrastructure investment policy for further discussion.

emission price to equal the SCC in every sector, yielding the *sector-specific* first-best carbon tax:

$$\tau^*_{s,t} \;=\; \frac{\text{SCC}_t}{mc^d_{s,t}}, \quad s \in \mathcal{N}; \qquad \tau^*_{f,t} \;=\; \frac{\text{SCC}_t}{mc_{E_f,t}}. \tag{40}$$

Sectors with lower marginal cost (higher dirty productivity $A^d_{s,t}$) receive a *higher* ad-valorem rate, ensuring the effective emission price $\tau^*_{s,t}\, mc^d_{s,t} = \text{SCC}_t$ is uniform across all sectors. As dirty technology improves over time ($mc^d_{s,t}$ falls), the optimal tax rate $\tau^*_{s,t}$ rises endogenously to maintain the Pigouvian condition. Under (40), total carbon revenue equals $\text{SCC}_t\, \Gamma_t$ (see government budget in Section **??**), recovering the same revenue as a uniform per-unit emission tax of SCC_t per tonne.

R&D subsidy. A subsidy for research that reduces the effective wage paid by each research lab for technology j within sector s could shift away innovation from over-invested technologies in the market allocations. To coincide with the planner's allocation, the innovation subsidy is set according to $\xi^j_{s,t} = 1 - (\Pi^j_{s,t}/\mathcal{V}^j_{s,t})$, and

$$\rho\, \mathcal{V}^j_{s,t} = \ln\lambda \cdot \widetilde{\mu}^j_{s,t} \;+ \sum_{k\in\mathcal{N}\cup\{e\}} \sum_{j'\in\{c,d\}} z^{j'}_{k,t}\, \omega^{j',j}_{k,s}\, \mathcal{V}^{j'}_{k,t} + \frac{\partial \mathcal{V}^j_{s,t}}{\partial t} \;+\; \nabla_{\mathcal{A}} \mathcal{V}^j_{s,t} \cdot \dot{\mathcal{A}}_t. \tag{41}$$

where $\mathcal{V}^j_{s,t}$ denotes the social value of innovation in technology j of sector s, and $\mathcal{A}_t$ collects the log of sectoral productivity in a $2(N+1)$ vector

$$\mathcal{A}_t \equiv (\ln A^c_{1,t}, \ldots, \ln A^c_{N+1,t}, \ln A^d_{1,t}, \ldots, \ln A^d_{N+1,t})'.$$

In the expression, the first term represents the benefit to production from having more advanced technology. This effect is captured by the augmented technology-level weight $\widetilde{\mu}^j_{s,t}$ defined above. Then, the second term represents the knowledge spillover benefits generated in the following periods. The $\omega^{j,j'}_{s,k}$ terms correspond to compensating the R&D misallocation due to knowledge spillovers. In a stationary equilibrium, the above expression can be written in vector form as

$$\boldsymbol{V} = \left(\rho\, \boldsymbol{I} - \boldsymbol{Z}\, \widetilde{\boldsymbol{\Omega}}^\top\right)^{-1} \ln\lambda \cdot \widetilde{\boldsymbol{\mu}}, \qquad \boldsymbol{\xi} = \mathbf{1} - \text{diag}(\boldsymbol{V})^{-1}\boldsymbol{\Pi}, \tag{42}$$

where $\boldsymbol{\Pi} \equiv [\Pi^j_s]_{s\in\mathcal{N}\cup\{e\}, j\in\{c,d\}}$ collects all steady state profits in vector, and the $2(N+1)\times 2(N+1)$ diagonal matrix $\mathbf{Z} \equiv \text{diag}(z^j_s), s \in \mathcal{N}\cup\{e\}, j \in \{c,d\}$ collects all steady state innovation intensities in each sector and technology.

Renewable infrastructure investment. The optimal infrastructure capital under the first-best policy follows the F.O.C. such that the marginal cost of infrastructure investment equals the marginal social benefit from reduced input cost in energy. The optimal infrastructure investment path is characterized by a system of differential equations:

$$\dot{I}^{\text{FB}}_{g,t} = (\rho + \delta_K + g_{Ct})\, I^{\text{FB}}_{g,t} - \frac{C_t}{c_K} \frac{\partial \ln C_t}{\partial K_{g,t}}. \tag{43}$$

where $g_{Ct} = \dot{C}_t / C_t$, together with the law of motion of infrastructure stock (9), and the transversality condition $\lim_{t\to\infty} e^{-\rho t} \nu_{g,t} K_{g,t} = 0$, with $\nu_{g,t}$ as the co-state w.r.t. renewable infrastructure investment. Since $K_{g,t}$ affects the aggregate economy only through the energy bundle price, we can write the marginal benefit term as

$$MB_t = \frac{\partial \ln C_t}{\partial K_{g,t}} = -\left(\sum_{s\in\mathcal{N}} \gamma_s \beta_s + \gamma_e\right) \frac{\partial \ln p_{e,t}}{\partial K_{g,t}}. \tag{44}$$

Hence, for any $t \geq 0$, the optimal infrastructure investment is

$$I^{\text{FB}}_{g,t} = e^{\left(\int_0^t \rho + \delta_K + g_{C\tau}\, d\tau\right)} \left[I_{g,0} - \int_0^t \exp\left(- \int_0^{t'} \rho + \delta_K + g_{C\tau}\, d\tau \right) \frac{C_t}{c_K} MB'_t\, dt' \right], \tag{45}$$

$I^{\text{FB}}_{g,0}$ is pinned down by the transversality condition.

3.5 Impact of Industrial Policy on Green Growth

With the equilibrium conditions, we can characterize how the economy's welfare and growth rate change in response to a change in industrial policy. We first focus on the balanced-growth steady state outcomes in response to the changes in carbon taxes, R&D subsidies, and renewable infrastructure investment.

Proposition 2. *An interior balanced-growth path (BGP) exists of the economy, which is a decentralized equilibrium such that there exist constant growth and time-invariant shares with: (1) all decentralized equilibrium conditions hold for all $t \geq 0$;*
(2) aggregate consumption, wages, and gross outputs grow at a constant rate g;
(3) all relative technology levels are constants; growth in sectoral technology stock is constant and follows $\frac{\dot{A}^c_{s,t}}{A^c_{s,t}} = \frac{\dot{A}^d_{s,t}}{A^d_{s,t}} = g_s$;
(4) allocations of R&D resource, production labor, and output share of clean technology are constants.

To examine the short- and long-run effects of a policy change on aggregate growth and

the green transition, we leverage the sufficient statistics derived in Section 3.3, which incorporate the network structure and the degree of substitutability between clean and dirty technologies within a sector. We can derive the following proposition to characterize the steady state impacts of a policy reform.

Proposition 3. *Consider a policy reform in sectoral relative R&D subsidy* $\mathbf{\Xi} \equiv [\Xi_s^j]$ *and through a* technology-level unit-cost wedge $\mathbf{R} \equiv [R_s^j]$. *Let* $\Delta\mathcal{A} \in \mathbb{R}^{2(N+1)}$ *denote the change in log technology levels across all sector-technology nodes after the reform. Define the amplification operators*

$$\underbrace{\mathbf{M}_\Xi \equiv \left(\widetilde{\mathbf{\Omega}} - \frac{\phi}{1-\phi}\widetilde{\boldsymbol{L}}\right)^{-1}}_{\textit{innovation amplification}}, \qquad \underbrace{\mathbf{M}_R \equiv \left(\widetilde{\mathbf{\Omega}} - \frac{\phi}{1-\phi}\widetilde{\boldsymbol{L}}\right)^{-1}\widetilde{\boldsymbol{L}}}_{\textit{production \& substitution amplification}}. \tag{46}$$

Steady state technology change for all sectors is characterized by

$$\Delta\mathcal{A} = \frac{\phi}{1-\phi}\left[\mathbf{M}_\Xi \Delta \ln \mathbf{\Xi} - \mathbf{M}_R \Delta \ln \mathbf{R}\right].$$

The amplification operators $\mathbf{M}_\Xi$ and $\mathbf{M}_R$ depend jointly on both network structures: the production network $\widetilde{\boldsymbol{L}}$ and the innovation network $\widetilde{\mathbf{\Omega}}$, and the steady-state formula gives the long-run response of technology levels to policy.

Technology Lock-in via the Innovation Network. The second mechanism operates through the row-stochastic structure of $\widetilde{\mathbf{\Omega}}$. Since each row sums to one, the spillover productivity for technology j in sector s is a geometric average of existing knowledge levels:

$$\phi_{s,t}^j = \prod_{k,j'} \left(A_{k,t}^{j'}\right)^{\omega_{sk}^{jj'}}.$$

When dirty technology currently leads, $\Lambda_s \equiv A_s^d / A_s^c > 1$, the innovation network translates the technology gap into a spillover productivity advantage for dirty R&D, compounding the profit advantage from market size. Define the *aggregate dirty R&D advantage*:

$$\Gamma_s \equiv \frac{\Pi_s^d}{\Pi_s^c} \cdot \frac{\phi_s^d}{\phi_s^c} = \underbrace{\frac{m_s^d}{m_s^c}}_{\text{market-size}} \cdot \underbrace{\frac{1}{1+\tau_s\psi_s}}_{\text{carbon-tax discount}} \cdot \underbrace{\Lambda_s^{\bar{\delta}_s^{cc}+\bar{\delta}_s^{dd}-1}}_{\text{spillover amplification}}, \tag{47}$$

where $\bar{\delta}_s^{cc} = \sum_k \omega_{sk}^{cc}$ and $\bar{\delta}_s^{dd} = \sum_k \omega_{sk}^{dd}$ are the own-type citation shares—the fraction of knowledge spillovers that clean (resp. dirty) sector s draws from other clean (resp. dirty) sources.

Proposition 4 (Technology Lock-in)**.** *Suppose the innovation network satisfies:*

[(a)]*Own-type dominance*: $\bar{\delta}_s^{cc} + \bar{\delta}_s^{dd} > 1$ for all s (equivalently, $\bar{\delta}_s^{cd} > \bar{\delta}_s^{dc}$: clean-to-dirty spillovers exceed dirty-to-clean spillovers); and *Dirty leads*: $\Lambda_s > 1$ for all s.

Then: (i) *dirty R&D has strictly higher spillover productivity,* $\phi_s^d > \phi_s^c$*;* (ii) *the ratio* $\phi_s^d/\phi_s^c = \Lambda_s^{\bar{\delta}_s^{cc}+\bar{\delta}_s^{dd}-1}$ *is strictly increasing in* Λ_s*; and* (iii) *the market devotes the overwhelming majority of researchers to dirty technology when* $\Gamma_s > 1$*.*
The critical clean R&D subsidy *required to redirect innovation is:*

$$\xi_s^*(\Lambda_s) \;=\; 1 - \Gamma_s^{-(1-\phi)}, \tag{48}$$

with $\xi_s^* \in (0,1)$ *whenever* $\Gamma_s > 1$*. Any policy* $\xi_s^c < \xi_s^*$ *leaves dirty technology growing faster, raising* Λ_s *over time, which strictly increases* $\xi_s^*(\Lambda_s)$*. Proofs are in Appendix B.*

The lock-in is self-reinforcing: insufficient policy today makes intervention tomorrow strictly more costly. Differentiating (48) yields $\partial\xi_s^*/\partial\Lambda_s > 0$ whenever own-type dominance holds, so the critical threshold rises with each period of delay. As Λ_s grows, $\xi_s^*(\Lambda_s) \to 1$: there exists an approximate threshold $\Lambda_s^{\max}(\tau_s)$ beyond which the required subsidy becomes fiscally infeasible, at which point a sufficiently large sector-specific carbon tax τ_s to reduce Γ_s below unity is also required. This provides a precise dynamic rationale for early, pre-emptive intervention: the social cost of delay is not linear but accelerating in the technology gap.

Corollary 1 (Policy Complementarity)**.** *The sector-specific carbon tax* τ_s *and the clean R&D subsidy* ξ_s^c *are* policy complements *in overcoming technology lock-in:*

[(i)]**Threshold reduction.** ξ_s^* is strictly decreasing in τ_s:

$$\frac{\partial \xi_s^*}{\partial \tau_s} \;=\; (1-\phi)\,\Gamma_s^{-(2-\phi)}\,\frac{\partial \Gamma_s}{\partial \tau_s} \;<\; 0. \tag{49}$$

Every unit increase in τ_s strictly relaxes the minimum subsidy required to redirect innovation toward clean technology. **Strict joint necessity under severe lock-in.** Define $\bar{\tau}_s$ as the minimum sector-specific carbon tax sufficient to reduce Γ_s below unity (i.e., $\tau_s\psi_s > m_s^d\Lambda_s^{\bar{\delta}_s^{cc}+\bar{\delta}_s^{dd}-1}/m_s^c - 1$). When Λ_s is large enough that $\xi_s^*(\tau_s) \to 1$ under any fiscally feasible $\tau_s < \bar{\tau}_s$, neither instrument is individually sufficient: no feasible subsidy $\xi_s^c \in [0,1)$ alone achieves the transition, and a sector-specific carbon tax alone cannot redirect R&D without also raising clean research returns. A joint policy (τ_s, ξ_s^c) with $\tau_s \geq \bar{\tau}_s$ and $\xi_s^c > \xi_s^*(\tau_s)$ is both necessary and sufficient. **Dynamic complementarity.** Even when current policy is insufficient ($\xi_s^c < \xi_s^*$), the

sector-specific carbon tax slows lock-in accumulation. Since $\dot{\Lambda}_s = \Lambda_s(g_s^d - g_s^c)$ and $\partial(g_s^d - g_s^c)/\partial\tau_s < 0$ (a higher τ_s compresses dirty profits in sector s and thereby dirty R&D growth), the rate at which the critical threshold $\xi_s^*(\Lambda_s)$ rises over time is strictly lower under a positive τ_s. Formally,

$$\left.\frac{d\xi_s^*}{dt}\right|_{\tau_s>0} = \left.\frac{\partial\xi_s^*}{\partial\Lambda_s}\cdot\dot{\Lambda}_s\right|_{\tau_s>0} < \left.\frac{d\xi_s^*}{dt}\right|_{\tau_s=0},$$

so the joint policy buys time: the cost of delay is strictly lower when a sector-specific carbon tax is in place.

The complementarity has a transparent structural origin in the multiplicative decomposition of Γ_s in (47). The three factors attack lock-in from distinct angles. The sector-specific carbon tax enters the *carbon-tax discount* factor $1/(1+\tau_s\psi_s)$ directly: a higher τ_s shrinks this factor, reducing Γ_s and lowering ξ_s^*. Through market equilibrium, the higher dirty price $(1+\tau_s\psi_s)\lambda mc_s^d$ also shifts market share toward clean ($m_s^d/m_s^c \downarrow$, further compressing Γ_s through the market-size factor). The R&D subsidy, independently, reduces the *private cost of clean research* by subsidizing the effective research wage. Because Γ_s is multiplicative in these factors, reducing any one of them lowers the threshold that the others must overcome, and combining both instruments requires strictly less of each than either alone. This superadditive structure means that the marginal return to the subsidy—measured in additional clean researchers attracted—is strictly increasing in the sector-specific carbon tax, and vice versa. The optimal policy in (42) internalizes this interaction: the social value $\mathcal{V}_s^j$ rises when the sector-specific tax compresses Π_s^d, which endogenously raises the optimal subsidy level.

The structure of Γ_s in (47) reveals why both networks matter jointly: the market-size component m_s^d/m_s^c is itself amplified by the IO network through Φ_s, so sectors that are highly central in the production network also face a more severe lock-in in the innovation network. Mechanism 1 and Mechanism 2 are therefore not independent—IO centrality inflates Π_s^d, enlarging Γ_s and raising ξ_s^* in the same sectors that attract excessive market R&D.

4 Quantitative Analysis

In this section, we apply the theoretical framework to quantify the impact of green industrial policy on welfare and growth. First, we construct a baseline consistent with observed production and innovation structure, and the scale of clean technology adoption. Second,

we use the calibrated model to quantify the welfare gains and transitional paths. Specifically, we calibrate our model to the Chinese economy, which we model as a system of non-energy production sectors (based on the CIP input-output classification) plus an upstream energy sector. The model is parametrized using a combination of direct empirical estimation for the network structures (the sufficient statistics matrices) and take from literature and estimates from macroeconomics data for the rest of the structural parameters. This analysis allows us to compute the welfare loss from constrained policy environments and measure the consumption equivalent variation of moving from the current regime to the optimal, network-targeted policy. Throughout the quantitative analysis, all nominal variables are reported in final-good units under the same normalization as in the theory, namely $P_{C,t} = 1$.

4.1 Parameterization

We start with the construction of the two network objects derived from the model's sufficient statistics: the input-output (IO) matrix ($\mathbf{\Sigma}$) and the cross-technology spillover matrix ($\mathbf{\Omega}$). The IO structure is constructed using the IO tables from the CIP Database, which provides the domestic inter-sectoral flows necessary to calculate the input elasticities and the production centrality of each sector. The second and more challenging step is the empirical measurement of the innovation network, $\mathbf{\Omega}$. Following the methodology of Acemoglu et al. (2016a) and Liu and Ma (2021), we construct this knowledge spillover matrix using patent citation data from Google Patents, classifying patents into clean and dirty technology based on OECD's green patent inventory, and assign the patents to each of the $N + 1 = 37$ sectors by the industry of application firms. Specifically, we map the technological classes (CPC codes) to their corresponding IO sectors, and then estimate the strength of the cross-technology spillovers (i.e., the frequency of a technology $j \in \{c, d\}$ patent in sector s citing any patent in technology $j' \in \{c, d\}$ of sector k) to construct the spillover matrix.

The calibrated $2(N + 1) \times 2(N + 1)$ knowledge spillover matrix $\mathbf{\Omega}$ from Chinese patent citations by sector and technology is shown in Figure 4.1. Rows index receiving sectors s and columns index source sectors k; the first $N + 1$ columns/rows are clean and renewable energy technologies, the last $N + 1$ are dirty and fossil fuel technologies. Each row is normalized to sum to one, so $\omega_{s,k}^{jj'}$ is the share of citations in (s, j) that goes to (k, j'). The heat maps for 2015–2018 and 2018–2021 show a stable spillover pattern. The two diagonal blocks for clean-to-clean and dirty-to-dirty knowledge diffusion exhibit rel-

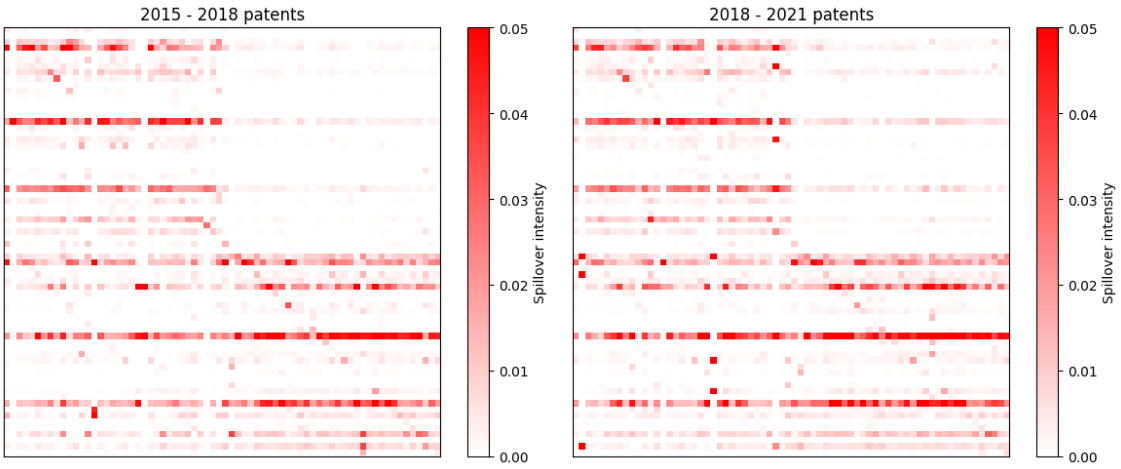


Figure 4.1: Constructed Innovation Network Elasticities (2015-2021)

atively strong spillovers; own-diffusion within a sector (the main diagonal within each block) is not dominant, suggesting the importance of considering the cross-technology knowledge spillovers rather than pure within-sector learning. The southwest block for clean-to-dirty diffusion is noticeably stronger than the northeast block of dirty-to-clean diffusion, revealing that clean technology spills over strongly into dirty sectors, while the reverse channel is weak. This is also consistent with our empirical findings in Section 2.

Second, we introduce the calibration of the model's remaining structural parameters. Four sets of parameters determine the equilibrium: aggregate and time-invariant variables, sector- and technology-specific parameters, and industrial policy parameters. We set the discount rate ρ to 0.02, consistent with a standard annual real interest rate of 2%. We take the discount and calibrate the elasticity of substitution between clean and dirty technologies based on the estimation by Papageorgiou et al. (2017) with $\varepsilon = 2.86$, implying relatively high substitutability between clean and dirty varieties in consumption. For the energy sector, the renewable potential and fossil fuel stock are taken from the IRENA Database, which provides the aggregate potential of countries with respect to solar, wind, and other renewable energy sources. We set the elasticity of substitution between renewable and fossil electricity generation to $\varepsilon_e = 1.45$ following Hassler et al. (2021). Following the estimation in Arkolakis and Walsh (2023), we set the elasticity of substitution between energy generation and grid infrastructure to $\eta = 0.15$, which implies a strong complementarity. On the innovation side, we adopt the standard Schumpeterian growth parameters from Acemoglu et al. (2018), setting the innovation step size $\lambda = 1.07$ and the R&D cost elasticity $\phi = 0.5$.

To capture the distinct technological characteristics of different industries, we calibrate sector-specific parameters using microdata. The labor shares, energy shares, and intermediate input elasticities are calculated using cost shares from the growth accounting and input-output tables of the CIP database. We estimate the spillover elasticities $\omega_{s,k}^{j,j'}$ using patent citation data from Google Patents to measure how much the innovation in one sector relies on prior knowledge from others. Emission intensities ψ_s are calibrated to match the sectoral CO_2 emissions data from the CEADs database. And we estimate the innovation efficiencies in the model internally to match the sets of moments of the observed clean R&D investment revenue shares in China.

Next, we introduce the calibration of policy-related parameters. It would be ideal to identify and quantify China's actual green industrial subsidies for each sector, however, such subsidies take various forms, including direct subsidies reported by firms, as well as indirect subsidies in the forms of preferential credit policy, government sponsored venture capital investments, etc. Unfortunately, there is a lack of comprehensive time series data that encompasses all these different types of subsidies and supporting policies in the green technologies. As a result, we calibrate a common carbon tax rate $\bar{\tau}$ (uniform across sectors) as the observed policy benchmark for China, acknowledging that this is a second-best approximation relative to the first-best sector-specific rates $\tau_s^* = \text{SCC}/mc_s^d$ derived in Section 3.4.1. In particular, the baseline common rate $\bar{\tau}$ is set based on the punishment for violating environmental regulations. The R&D subsidy to the green technologies is set to 15% as estimated from the green technology R&D tax rebate policy in China. For energy-specific infrastructure investment policy, we parameterize the function related to infrastructure as $\kappa_j(K_{j,t}) \equiv \bar{\kappa}_j K_{j,t}^{-1}$. We normalize the initial stock of fossil-fuel infrastructure, and using data from the Global Energy Monitor database on the relative installed capacity, we set the initial clean infrastructure stock $K_{g,0} = 0.18$, reflecting the nascent state of the renewable infrastructure relative to the established fossil-fuel capacity. Along the transitional path, we set the path of fossil fuel infrastructure stock to match the observed fossil fuel electricity capacity. The parameters $\bar{\kappa}_g, \bar{\kappa}_f$ are estimated to match the observed price wedges in electricity markets for renewable energy and fossil fuels. The infrastructure depreciation rate is set to $\delta_K = 0.03$, which is taken from the calibration in Gibson and Rioja (2020).

Finally, the preference for environmental quality is captured by the parameters of the environmental damage function ($\mathcal{E}_t$), which are set according to the estimation in Golosov et al. (2014), with a carbon decay rate of $\delta_p = 0.14$ and a damage semi-elasticity of

$\varphi = 5.3 \times 10^{-5}$. The initial pre-industrial atmospheric GHG concentration is taken from the estimation by IPCC. We validate this parameterization by matching the estimated Social Cost of Carbon (SCC) from integrated assessment models (IAMs) under a baseline scenario.

The parameter values used in the quantitative model are summarized in Table 4.1 and 4.2.

Table 4.1: Summary of Model Parameter Values

Parameter	Description	Value	Source
Panel A: Aggregate and time-invariant parameters			
ρ	Discount rate	0.02	Real interest rate
ε	Clean/Dirty EoS	2.86	Papageorgiou et al. (2017)
λ	Innovation Step Size	1.07	Acemoglu et. al. (2018)
ϕ	R&D Cost Elasticity	0.5	Acemoglu et. al. (2018)
ϑ_g	Renewable potential	1.79	IRENA (2024)
$\bar{p}_F$	Fossil fuel resource price	1.35	Average fossil electricity price
η	Energy-Infra EoS	0.15	Arkolakis & Walsh (2023)
ε_e	Renewable and fossil EoS	1.45	Hassler et al. (2021)
φ	Damage semi-elasticity	5.3×10^{-5}	Golosov et al. (2014)
δ_p	Carbon-cycle persistence parameter	0.14	Golosov et al. (2014)
$\mathcal{S}_0$	Initial GHG stock	581	IPCC (2017)
Panel B: Sector- and technology-specific parameters			
γ_s, γ_e	Final-good expenditure shares	Table 4.2	CIP database
α_s	Labor share	Table 4.2	CIP database
β_s	Energy share	Table 4.2	CEADs database
$\sigma_{s,k}$	Input elasticity	Table link	CIP database
$\omega_{s,k}^{j,-j}$	knowledge spillover elasticity	Table link	Google patents
$A_{s,0}^d$	Initial dirty technology level	Table 4.2	Google patents
$N_{s,0}^c$	Initial clean technology gaps	Table 4.2	Google patents
ψ_s	Emission intensity	Table 4.2	CEADs database
θ_s^j	Innovation efficiency	Table 4.2	Sectoral clean R&D share
Panel C: Policy-related parameters			
$\bar{\tau}$	Common (baseline) carbon tax	0.03	Environmental regulation counts
ξ_s^c	Sectoral clean R&D subsidy	0.15	Take from R&D policy
c_K	Cost of Infrastructure	2.18	Estimation from public spending
δ_K	Depreciation of Infrastructure	0.03	Gibson & Rioja (2020)
$\bar{\kappa}_g$	renewable grid parameter	2.43	Estimation from electricity price
$\bar{\kappa}_f$	fossil grid parameter	1.35	Estimation from electricity price
K_{f0}	Initial Dirty Infra Stock	1	Normalization
K_{g0}	Initial Clean Infra Stock	0.18	Global Energy Monitor

Note: This table shows the calibrated parameter values for the quantitative analysis. The input-output table and knowledge spillover matrix are provided in external links.

Table 4.2: Sector- and technology-specific parameters

Sector	γ_s	α_s	β_s	θ_s^j	ψ_s	$A_{s,0}^d$	$N_{s,0}^c$
1	0.081	0.512	0.008	0.074	0.031	0.421	-21
2	0.001	0.186	0.056	0.302	0.128	4.233	-17
3	0.131	0.074	0.008	0.166	0.034	14.785	-17
4	0.033	0.095	0.015	0.105	0.022	2.089	-19
5	0.014	0.106	0.006	0.113	0.01	1.403	-12
6	0.004	0.098	0.018	0.134	0.025	1.828	-17
7	0.003	0.098	0.018	0.148	0.072	2.677	-20
8	0.007	0.043	0.024	0.141	0.142	7.608	-22
9	0.017	0.06	0.046	0.139	0.118	4.375	1
10	0.002	0.079	0.019	0.113	0.024	2.001	-20
11	0.001	0.102	0.052	0.161	0.539	2.159	-22
12	0.002	0.061	0.04	0.129	0.234	2.778	-23
13	0.002	0.096	0.017	0.126	0.021	3.668	-20
14	0.027	0.072	0.011	0.121	0.011	2.835	-19
15	0.021	0.085	0.009	0.114	0.025	3.481	-24
16	0.006	0.089	0.017	0.293	0.027	1.765	-19
17	0.013	0.077	0.282	0.197	0.271	7.055	-15
18	0.002	0.145	0.013	0.108	0.01	1.179	-13
19	0.016	0.185	0.011	0.471	0.023	0.848	-17
20	0.015	0.188	0.013	0.467	0.021	0.847	-18
21	0.016	0.181	0.012	0.470	0.025	0.842	-20
22	0.046	0.193	0.015	0.208	0.009	0.895	-22
23	0.007	0.157	0.016	0.246	0.178	1.733	-23
24	0.027	0.167	0.016	0.247	0.111	1.733	-20
25	0.019	0.154	0.015	0.242	0.199	1.733	-23
26	0.008	0.158	0.013	0.247	0.049	1.733	-17
27	0.02	0.156	0.011	0.358	0.020	10.526	-20
28	0.037	0.186	0.005	0.443	0.006	11.413	-21
29	0.069	0.094	0.005	0.682	0.008	29.422	-21
30	0.035	0.175	0.007	0.19	0.005	3.321	-20
31	0.125	0.502	0.009	0.076	0.002	2.125	-22
32	0.085	0.553	0.013	0.113	0.004	1.329	-15
33	0.079	0.311	0.009	0.091	0.005	1.803	-16
34	0.031	0.246	0.014	0.221	0.002	1.804	-21
35	0.037	0.286	0.015	0.101	0.002	1.094	-22
36	0.029	0.252	0.014	0.104	0.004	1.039	-22
Energy	0.012	0.073 (α_e)	–	0.251	2.272	5.053	-16

Note: This table shows the external calibrated parameter values. Sector names corresponding to the code can be found in this link to CIP Database.
Data Source: CIP database, Google Patents, CNRDS, National S&T Yearbook 2005-2020.

4.2 Transitional Path with Industrial Policies

4.2.1 Decomposition of Network Effects

In this section, we conduct quantitative exercises to show the importance of considering network effects in examining the industrial policy impacts. Our theoretical results emphasized the role of both knowledge spillover and market size effects from input-output linkages in shaping the growth and welfare gains of the economy from industrial policies. We quantify how the two networks amplify green growth by running two counterfactual paths that keep all primitives, policies, and initial conditions fixed to the baseline: (i) No IO network scenario: we shut down market-size propagation by setting the Leontief inverse to identity; (ii) No spillover scenario: we shut down cross-sector knowledge diffusion by setting $(\boldsymbol{I} - \boldsymbol{\Omega})$ to identity matrix, so growth comes only from own R&D. We compute each full transitional path and compare to baseline.

Figure 4.2 shows the log differences of aggregate TFP and the aggregate clean market share. First, market-size propagation through the IO network is a major growth multiplier. Removing it lowers cumulative TFP growth substantially by the terminal date, and delays the acceleration of clean adoption—the maroon path stays below baseline for most of the transition. Second, innovation spillovers raise the long-run clean technology share. Shutting the knowledge spillover channel down reduces the 2055 clean market share by roughly 28% relative to baseline (orange path). The differences are modest early on but widen nonlinearly as clean sectors scale up. This shows the amplification implied by the production network and innovation network compounding over time, indicating that propagation primarily governs the level and slope of aggregate TFP, while the innovation network primarily governs the speed of the green transition. Intuitively, the loss of cross-technology knowledge spillovers removes the mechanism that enables the less productive technologies to "catch up" by building on advancements in other technologies. Moreover, the results underscore that without IO linkages, the immediate market-size effect of any policy or initial clean cost advantage is localized and cannot propagate demand or cost shocks downstream. This lack of propagation dampens the profitability of clean R&D across the economy, demonstrating that the market-size channel, as amplified by the production network structure, is a first-order determinant of the transition speed and policy effectiveness.

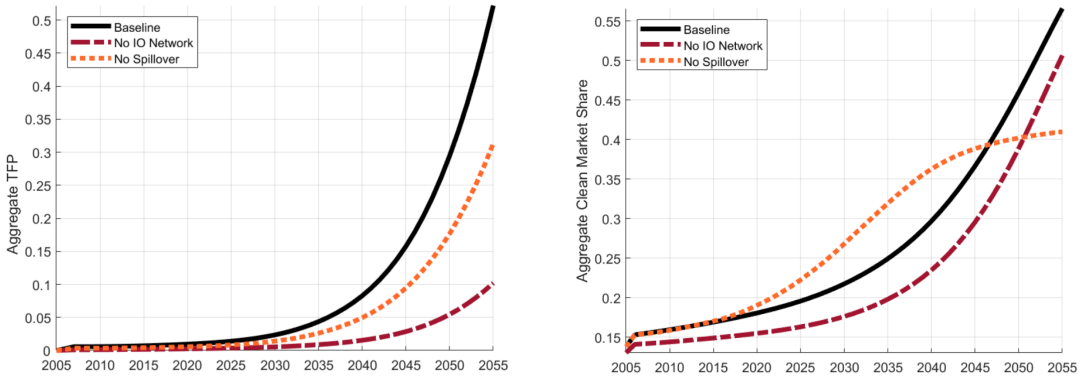


Figure 4.2: Aggregate Technology Level and Green Technology Share Path

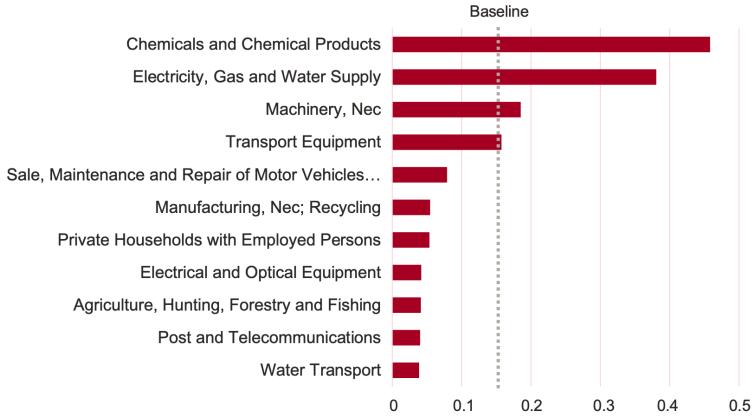


Figure 4.3: Optimal Clean R&D Subsidy Across Sectors

4.2.2 Optimal Industrial Policy and Welfare Impacts

We compute the vector of optimal ad-valorem R&D subsidies $\zeta_s^{j,\text{FB}}$ by solving the planner problem as in Section 3.4.1. The ranking is proportional to each technology's network centrality implied by the product of the augmented technology-level Domar weight and the innovation network's Leontief inverse matrix. Figure 4.3 displays the implied optimal R&D subsidy rates for clean technologies in different sectors.

This implies that for the green technologies, the R&D subsidy intensity is highly heterogeneous. Technologies with high innovation centrality receive the highest subsidies, such as Chemicals, Energy Supply, Machinery, and Transport Equipment. Taken together, optimal R&D subsidy is more intensive in green central technologies, while still allocating smaller but positive subsidies to non-green technologies that generate large spillovers along production chains.

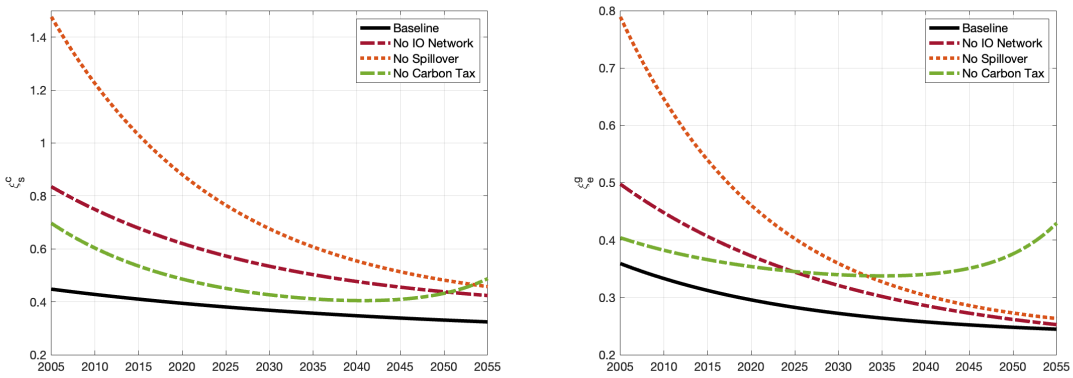


Figure 4.4: Optimal R&D Subsidy Path in Chemicals & Renewable Energy

Figure ?? further shows the path for optimal green industrial policy in two sepcific sectors. On the left panel, we present the optimal R&D subsidy for the clean technology in chemicals and chemical product sector, and optimal R&D subsidy for renewable energy is presented in the right panel. It shows that the optimal R&D subsidy declines over time in both Chemicals and Renewable Energy in the baseline (solid black), consistent with the idea that early-stage innovation has the largest dynamic payoff because it compounds through future knowledge accumulation and propagates through the production network as downstream sectors adopt cheaper inputs. The counterfactuals clarify the mechanisms. Shutting down knowledge spillovers (orange dotted lines) requires a much larger initial subsidy, especially in chemicals, because the planner must replace the missing intertemporal amplification that would otherwise raise future innovation productivity across nodes; as spillover-driven amplification fades, the needed subsidy falls and eventually converges toward the baseline. Removing the IO network (red dashed lines) also raises the subsidy path relative to baseline in both sectors, reflecting that without upstream–downstream propagation, each innovation has a smaller contemporaneous market-size and welfare footprint, so stronger direct incentives are needed to implement the same innovation direction. Finally, without a carbon tax (green dashed lines), the subsidy path becomes higher and more persistent. This is most visibly for renewable energy sector, where the subsidy even turns upward late in the horizon, because innovation policy must partially substitute for the missing adoption incentive. Without tax on emissions, clean technologies face weaker market expansion through substitution away from fossil energy, so the planner leans more on R&D support to keep the economy on a transition to clean technology.

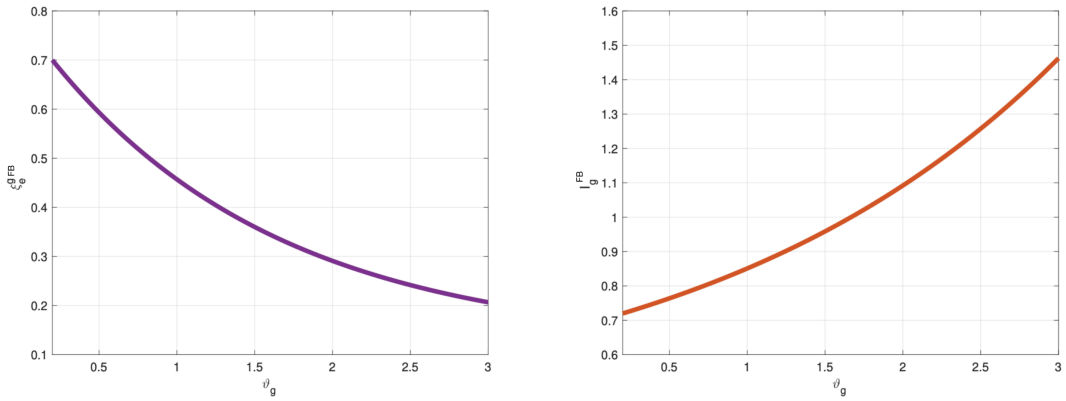


Figure 4.5: Optimal R&D Subsidy and Infrastructure w.r.t. Renewable Potentials

We next perform a set of comparative statics experiments on the renewable potential ϑ_g that governs the marginal cost of renewable electricity generation, and re-solve for optimal policies. As shown in Figure 4.5, two patterns emerge. First, the average optimal clean-energy R&D subsidy in renewable energy $\zeta_e^{c,\mathrm{FB}}$ declines in ϑ_r. Higher renewable potential raises the private return to clean power (through a lower unit cost of X_g), shrinking the social–private wedge corrected by the subsidy; the innovation network still creates positive spillovers, but the marginal external benefit for R&D subsidies falls as comparative advantage shifts toward renewable resources. Second, interestingly, the opposite pattern holds for infrastructure investment due to the strong complementarity of infrastructure and energy generation, which is also consistent with our theory. Since the marginal benefit from infrastructure, as stated in (43) increases with the renewable potential, it implies a higher steady state investment in renewable infrastructures. Intuitively, holding the initial technology level unchanged, when renewable resources are more abundant, the renewable energy generation is cheaper in the initial period, so improving K_g reduces the electricity price on a larger base and with stronger leverage, hence the optimal infrastructure investment rises with the renewable potentials.

Based on the model-implied optimal policies, we compare the growth and welfare impact of the historical policy in the baseline scenario, which is a 15% uniform R&D subsidy on green technologies plus a carbon price calibrated to the environmental regulations over time, with the optimal policy bundle. Under the optimal policy bundle, the R&D profile is front-loaded in central green nodes but declines over time, as private incentives strengthen and marginal spillovers to the rest of the economy saturate. The welfare gain comes predominantly from higher consumption via TFP growth, with the remainder from reduced climate damages. Figure 4.6 shows the impact of moving to the optimal policy.

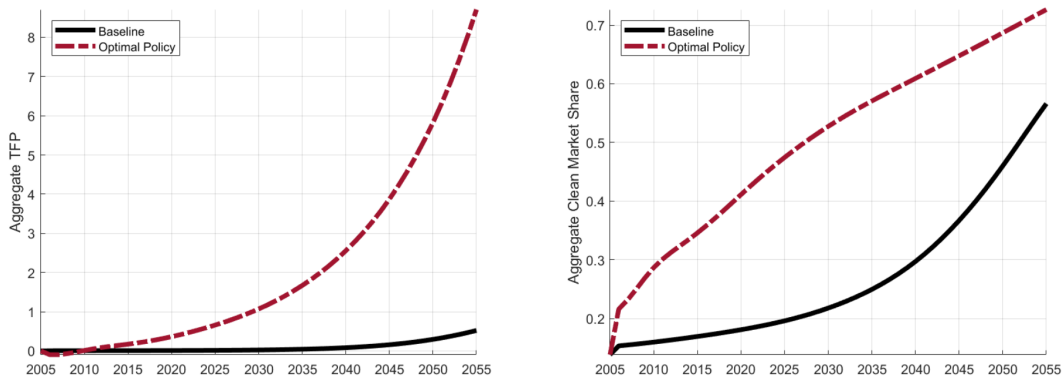


Figure 4.6: Gains from Optimal Green Industrial Policy

The red dashed line shows the impact on aggregate TFP and clean technology output share under the optimal policy, and the black solid line shows the baseline results.

As predicted in the model, the optimal bundle delivers substantial dynamic gains: the economy's aggregate TFP level becomes significantly higher over the 50-year horizon, the clean market share grows rapidly from the initial stage and increases by 15.8% by 2055, and total consumption-equivalent welfare rises by 8.19% relative to the baseline. With calibrated input-output linkages, knowledge spillovers, and substitution patterns, the optimal policy allows reallocation of resources to central sectors in the networks, raises the private payoff of using a clean production technology through amplified demand, and allows clean technologies to gain a lead over their dirty counterparts as the economy transitions to a steady-state. Consequently, clean adoption starts earlier, and scales faster, and aggregate productivity accelerates with increased innovation efficiencies.

Figure 4.7 shows that the optimal policy bundle advances the emissions peak by roughly a decade and lowers emissions at every horizon relative to the baseline. The mechanism operates on both margins: (i) targeted R&D into innovation-central clean nodes accelerates the productivity growth in the clean technologies, and (ii) the carbon price and infrastructure investments ensure the clean technologies have broad adoption and increased market sizes through the IO network, yielding a faster decline in total GHG emissions. Because the damage kernel is convex in cumulative emissions, this earlier and sharper peak implies lower climate damage and sizable welfare gains. Overall, the results highlight that targeted policy is essential compared with a uniform policy in the baseline scenario.

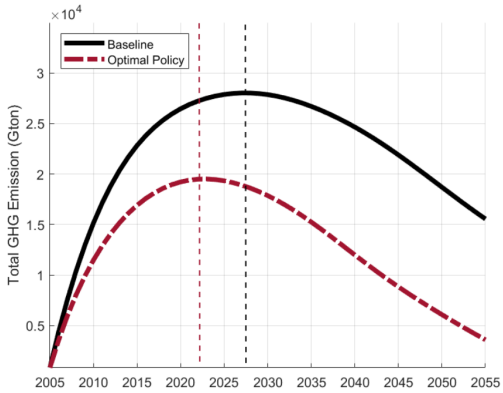


Figure 4.7: Carbon Emission Path under Baseline and Optimal Policy

To unpack the relative importance of each type of policy, we implement each policy instrument in isolation at its model-implied optimum level to show the distinct wedges they correct and discuss the broader implications. Figure 4.8 shows the results for the aggregate TFP path and clean technology shares. The black solid lines indicate baseline results, and the dashed lines represent the path under every single policy. As shown in the figures, targeted R&D subsidy reallocates research toward innovation and production network central sector-technology nodes, which raises the aggregate TFP substantially through the spillover network, and delivers the highest aggregate TFP growth. However, absent a production-side cost adjustment, the initial market size favors the dirty technologies and is reinforced through input-output linkages in the production network; thus, clean adoption plateaus and the long-run transition remain low. On the other hand, imposing only an optimal carbon price changes relative prices in all dirty technologies and yields the largest increase in the aggregate clean share, yet it results in weaker TFP growth due to the efficiency loss from imposing heavy taxes on more productive sectors. An optimal infrastructure investment lowers the effective cost of delivered energy bundle and stimulates steady diffusion, but generates smaller productivity gains than R&D and slower adoption than carbon pricing. Taken together, these counterfactual exercises demonstrate that the R&D, carbon-pricing, and renewable infrastructure investments are complementary. Intuitively, the R&D subsidy offsets the relative efficiency loss of carbon taxes by amplifying innovation along the production network, while the carbon tax and renewable infrastructure investments further incentivize the transition to clean technologies by raising the relative market size of clean technologies. The limits of unilateral policy along both productivity and transition margins show the importance of implementing the full policy bundle.

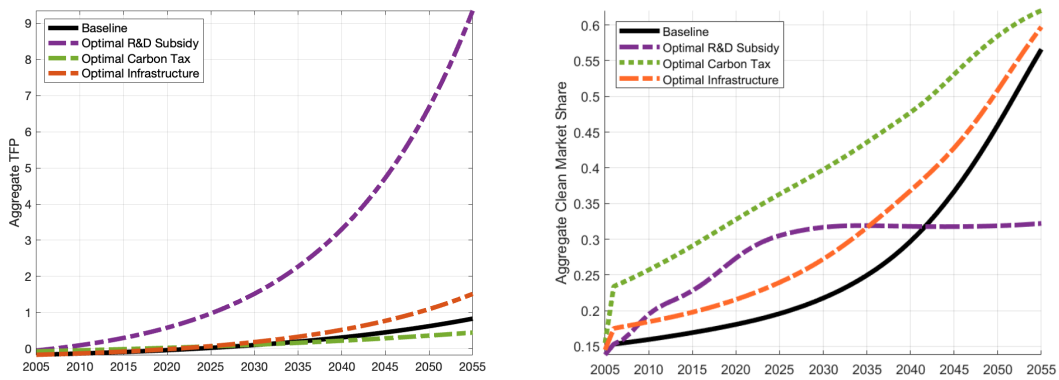


Figure 4.8: Impacts from Optimal Policy Instruments in Isolation

5 Conclusion

This paper develops a multi-sector growth model for that embeds both production linkages and an innovation spillover network featuring clean and dirty technologies. The theory delivers two core results. First, the social return to R&D is proportional to a technology's network marginal benefit—its IO centrality times its position in the innovation network. Second, policy instruments are complements: R&D subsidies correct knowledge externalities that are amplified through supply chains, while carbon pricing and infrastructure direct the innovation to green technologies.

We calibrate the model to China from 2005 to 2020, estimating the innovation matrix from patent citation shares and the IO structure from national accounts of input-output tables. The citation data reveal a stable block pattern, with strong clean-to-dirty and weak dirty-to-clean technology spillovers, and limited own-diffusion. Decomposition exercises show that removing market-size propagation from the IO network eliminates roughly 20% of cumulative TFP growth, while shutting down innovation spillovers reduces the long-run clean share by about 28%. These facts rationalize why policies that internalize both networks deliver sizable effects.

Policy counterfactual exercises confirm the mechanism. Moving from a historical baseline with a uniform 15% green R&D subsidy to the optimal portfolio (targeted R&D, optimal carbon price path, and clean infrastructure) raises aggregate TFP substantially by 2055, increases the clean market share by 15.8%, and lifts total welfare by 8.19%. The bundle also advances the emissions peak by roughly ten years and yields a steeper decline thereafter, lowering cumulative damages. Single-instrument runs clarify the role of each tool: R&D-

only policy accelerates frontier growth but stalls the green transition; carbon tax pushes adoption but slows productivity growth; infrastructure-only eases bottlenecks with intermediate gains. Comparative statics with respect to energy endowments yield clear policy rules: as renewable energy potential increases, the optimal level of R&D subsidies falls, while the optimal investment in renewable infrastructure rises, providing intuition for the sources of comparative advantage in the green transition.

The framework has clear policy messages and can be extended to richer scenarios. First, cross-technology knowledge spillovers and input-output linkages imply a systematic R&D underinvestment in green technologies in the decentralized equilibrium, and policy should target the most central sector-technology nodes. Second, an optimal policy should combine a carbon tax with targeted R&D support that reflects technology-level welfare weights. Third, infrastructure investment is a key complement to renewable energy and green innovation. One of the central mechanisms in the paper is that renewable infrastructure is not a substitute for green industrial policies; instead, it complements the industrial policies. Renewable potential (solar radiation, wind resources, etc.) does not automatically translate into cheap delivered clean electricity. The infrastructure that turns raw generation into reliable electricity for firms and households lowers the unit cost of delivered clean electricity and strengthens the downstream productivity gains that propagate through the input–output network. This implies that policymakers should prioritize such investments that relax the binding delivery constraint. Finally, our evidence and quantitative results indicate that network-targeted policy portfolios can reconcile rapid decarbonization with faster growth.

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A Why Green Innovations Generate Stronger Spillovers

Section 2.2 documents that green innovations generate larger knowledge spillovers than non-green ones. Even within the same industry, green patents receive significantly more forward citations than non-green patents. This appendix provides supporting evidence on two mechanisms that can account for this regularity: (i) green technologies are more pervasive across industries, consistent with the general-purpose technology (GPT) perspective of Bresnahan and Trajtenberg (1995); and (ii) green technologies draw on more diverse and technologically distant knowledge bases, which expands the set of potential recombination channels and citation pathways. The goal is not to establish a single definitive mechanism, but to document systematic differences in technological characteristics that align closely with the spillover patterns in the main text.

A.1 Cross-Industry Applicability and Pervasiveness

A central feature of GPTs is pervasiveness: the ability of a technology to support innovation and productivity improvements across a wide range of downstream sectors (Bresnahan and Trajtenberg, 1995). To assess whether green technologies exhibit this characteristic, we quantify the cross-industry applicability of each patent using an industry-probability vector $p_i = (p_{i1}, \ldots, p_{iK})$, constructed from IPC-to-industry mappings. Intuitively, p_{ik} captures the relevance of patent i for industry k among K sectors. As a summary measure, we compute an entropy index,

$$H_i \;=\; -\sum_{k=1}^{K} p_{ik} \ln p_{ik}, \tag{50}$$

where higher values indicate that a patent is relevant for a broader set of industries. This measure operationalizes pervasiveness in a way that is closely related to the "span of use" notion in the GPT literature.

Table A.1 compares the distribution of H_i between green and non-green patents. Green patents have higher entropy on average (1.762 vs. 1.641), and the median difference is similar, indicating that the gap reflects a broad distributional shift rather than being driven by a small set of outliers. A two-sample Welch's t-test strongly rejects equality of means. Together, these patterns imply that green inventions are systematically more cross-sector applicable, consistent with the view that many clean-energy and environmental innovations function as enabling inputs for downstream applications in multiple industries.

Table A.1: Industry-Entropy of Green vs. Non-Green Patents

	Mean	Median	Std. Dev.	Observations
Non-Green	1.641	1.666	0.380	7,193,903
Green	1.762	1.750	0.366	1,471,228
t-statistic			363.4	
p-value			< 0.001	

A.2 Multidisciplinarity and Technological Recombination

A second mechanism is that green innovations may be more multidisciplinary, combining knowledge from multiple technological domains. Such diversity can increase spillovers by expanding the number of potential follow-on directions and the set of innovators who can build on the invention. We measure multidisciplinarity using the CPC system and construct two patent-family-level indicators.

First, CPC breadth is defined as the number of distinct CPC subclasses (CPC4) assigned to the patent family. A larger value indicates that the invention spans more technological fields. Second, CPC maximum distance captures the extent to which the invention combines technologically distant knowledge. Specifically, for each patent family, we compute the maximum pairwise distance between any two CPC subclasses associated with the family. The distance metric equals 0 for codes in the same subclass, 1 for codes in different subclasses within the same class, 2 for codes in different classes within the same CPC section, and 3 for codes in different CPC sections.

Table A.2 reports summary statistics for green and non-green patents. Green patents exhibit substantially greater breadth and distance than non-green patents. On average, green patents are assigned 2.577 distinct CPC4 subclasses, compared with 1.582 among non-green patents, and their average maximum CPC distance is 2.316, compared with 0.825 among non-green patents. Welch's t-tests reject equality of means with very large test statistics for both measures. These results indicate that green innovations systematically integrate knowledge from more diverse and more technologically distant domains, which is consistent with higher spillover intensity through broader recombination opportunities.

Table A.2: Multidisciplinarity of Green vs. Non-Green Patents

	CPC Breadth			CPC Max Distance		
	Mean	Median	Std. Dev.	Mean	Median	Std. Dev.
Non-Green	1.582	1.0	0.948	0.825	0.0	1.189
Green	2.577	2.0	1.388	2.316	3.0	1.218
Observations (Non-Green)		2,200,717			2,200,717	
Observations (Green)		315,306			315,306	
t-statistic		389.4			644.4	
p-value		< 0.001			< 0.001	

The evidence in this appendix supports two complementary interpretations of why green innovations generate stronger citation spillovers. First, green patents exhibit higher cross-industry applicability, consistent with the pervasiveness dimension emphasized in the GPT literature. Second, green patents are more multidisciplinary, combining knowledge from a wider and more technologically distant set of fields, which increases the scope for follow-on recombination and diffusion. These technological characteristics provide a natural explanation for why, in the main text, green patents receive more citations even when comparing patents within the same industry and controlling for industry fixed effects.

B Model Details and Proofs

B.1 Proof for First Best Allocation

Consider the planner's static allocation problem at time t conditional on (A_t, K_t) and on aggregate feasibility. By Cobb–Douglas aggregation at the variety level, aggregating within sector yields a sectoral unit-cost function that is homogeneous of degree one in $(w, p_e, \{P_k\})$ in the decentralized problem; in the planner problem, the analogous object is the shadow-cost of producing $Y_{s,t}$ using labor, energy, and intermediates, with marginal contribution to $\ln C_t$ weighted by μ_s from (37).
Because sector-s gross output uses labor share α_s and energy share β_s, the planner equalizes weighted marginal products:

$$\mu_s \cdot \frac{\partial \ln Y_{s,t}}{\partial L_s} = \mu_k \cdot \frac{\partial \ln Y_{k,t}}{\partial L_k}, \qquad \mu_s \cdot \frac{\partial \ln Y_{s,t}}{\partial E_s} = \mu_k \cdot \frac{\partial \ln Y_{k,t}}{\partial E_k}.$$

Under Cobb–Douglas, $\partial \ln Y_{s,t}/\partial L_s = \alpha_s / L_s$ and $\partial \ln Y_{s,t}/\partial E_s = \beta_s / E_s$. Solving the two proportionality systems subject to $\sum_s L_s = \bar{L}$ and $\sum_s E_s = Y_{e,t}$ yields the between-sector allocation.
For sector s and time t, the sectoral aggregator is CES across $Y^c_{s,t}$ and $Y^d_{s,t}$ with elasticity ε:

$$Y_{s,t} = \left[(Y^c_{s,t})^{\frac{\varepsilon-1}{\varepsilon}} + (Y^d_{s,t})^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}}.$$

In the planner problem, producing one unit of dirty output carries an extra marginal social cost $\psi_s \, \mathrm{SSC}_t$ in addition to the resource marginal cost $mc^d_{s,t}$. Hence the planner chooses $(Y^c_{s,t}, Y^d_{s,t})$ to minimize social cost for a given $Y_{s,t}$:

$$\min_{Y^c_{s,t}, Y^d_{s,t}} \; mc^c_{s,t} Y^c_{s,t} + \left(mc^d_{s,t} + \psi_s \, \mathrm{SSC}_t\right) Y^d_{s,t} \quad \text{s.t.} \quad Y_{s,t} = \left[\left(Y^c_{s,t}\right)^{\frac{\varepsilon-1}{\varepsilon}} + \left(Y^d_{s,t}\right)^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}}.$$

Standard CES expenditure-share conditions imply

$$\frac{Y^c_{s,t}}{Y^d_{s,t}} = \left(\frac{mc^d_{s,t} + \psi_s \, \mathrm{SSC}_t}{mc^c_{s,t}} \right)^{\varepsilon}.$$

since the social marginal cost of dirty production needs to account for carbon emissions. Because both technologies share the same Cobb–Douglas factor shares $(\alpha_s, \beta_s, \{\sigma_{s,k}\})$ in (6), rewriting with (20)

$$\frac{\ell^c_{s,t}}{\ell^d_{s,t}} = \frac{\alpha_s \, mc^c_{s,t} \, Y^c_{s,t} / w_t}{\alpha_s \left(mc^d_{s,t} + \psi_s \, \mathrm{SSC}_t\right) Y^d_{s,t} / w_t}, \qquad \frac{e^c_{s,t}}{e^d_{s,t}} = \frac{\beta_s \, mc^c_{s,t} \, Y^c_{s,t} / p_{e,t}}{\beta_s \left(mc^d_{s,t} + \psi_s \, \mathrm{SSC}_t\right) Y^d_{s,t} / p_{e,t}},$$

and this delivers (38).

B.2 Proof for Proposition 1

Index technology–sector nodes by $i \in \{1, \ldots, J\}$ where $J = 2(N+1)$ collects (s, j) with $s \in \mathcal{N} \cup \{e\}$ and $j \in \{c, d\}$. Let $\mathcal{A}_{i,t} \equiv \ln A_{i,t}$ and $\mathcal{A}_t \equiv (\mathcal{A}_{1,t}, \ldots, \mathcal{A}_{J,t})'$.

Innovation block objective. By the chain rule the flow benefit of raising $\mathcal{A}_{i,t}$ is $\partial \ln C_t / \partial \ln A_{i,t} = \widetilde{\mu}_i$. Treating $\widetilde{\boldsymbol{\mu}}$ as constant (evaluated at BGP shares), the planner's innovation block is

$$\max_{\{h_{i,t} \geq 0\}} \int_0^\infty e^{-\rho t} \widetilde{\boldsymbol{\mu}}' \mathcal{A}_t \, dt \tag{51}$$

subject to the law of motion

$$\dot{\mathcal{A}}_{i,t} = z_{i,t} \ln \lambda, \quad z_{i,t} = \theta_i \, \Phi_{i,t} \, h^{\phi}_{i,t}, \quad \Phi_{i,t} = \exp\big((\boldsymbol{\Omega} \mathcal{A}_t)_i\big), \tag{52}$$

and the research constraint $\sum_i h_{i,t} = \bar{H}$.

Hamiltonian and first-order conditions. Let $\boldsymbol{\nu}_t$ be the current-value co-state vector on $\mathcal{A}_t$ and ζ_t the multiplier on $\sum_i h_{i,t} = \bar{H}$. The current-value Hamiltonian is

$$\mathcal{H} = \widetilde{\boldsymbol{\mu}}' \mathcal{A} + \ln \lambda \sum_i \nu_i \, \theta_i \, \Phi_{i,t} \, h^{\phi}_i - \zeta \left(\sum_i h_i - \bar{H} \right).$$

The first-order condition for each interior node i is

$$\frac{\partial \mathcal{H}}{\partial h_i} = \phi \ln \lambda \, \nu_i \, \theta_i \, \Phi_{i,t} \, h_i^{\phi-1} - \zeta = 0. \tag{53}$$

Multiplying both sides by h_i/ϕ and using $z_i = \theta_i \Phi_i h_i^{\phi}$:

$$\ln \lambda \, \nu_i \, z_i = \frac{\zeta}{\phi} h_i \qquad \Longrightarrow \qquad h_i = \frac{\phi \ln \lambda}{\zeta} \nu_i \, z_i. \tag{54}$$

Hence, for any $\phi \in (0,1)$, the optimal research effort at node i is proportional to the *product* of the co-state and the BGP innovation rate:

$$\chi_i^R \equiv \frac{h_i}{\bar{H}} = \frac{\nu_i \, z_i}{\sum_m \nu_m \, z_m}. \tag{55}$$

Co-state dynamics. Since $\partial \ln \Phi_k / \partial \mathcal{A}_i = \Omega_{ki}$ (element (k,i) of $\mathbf{\Omega}$) we have $\partial z_k / \partial \mathcal{A}_i = z_k \Omega_{ki}$. Differentiating $\mathcal{H}$ with respect to $\mathcal{A}_i$:

$$\frac{\partial \mathcal{H}}{\partial \mathcal{A}_i} = \widetilde{\mu}_i + \ln \lambda \sum_k \nu_k \, z_k \, \Omega_{ki} = \widetilde{\mu}_i + \ln \lambda \left[\mathbf{\Omega}^\top (\mathbf{Z}_t \boldsymbol{\nu}_t) \right]_i, \tag{56}$$

where $\mathbf{Z}_t = \operatorname{diag}(z_{i,t})$. The co-state evolves as

$$\dot{\boldsymbol{\nu}}_t = \rho \boldsymbol{\nu}_t - \widetilde{\boldsymbol{\mu}} - \ln \lambda \, \mathbf{\Omega}^\top \mathbf{Z}_t \boldsymbol{\nu}_t = \left(\rho \, \mathbf{I} - \ln \lambda \, \mathbf{\Omega}^\top \mathbf{Z}_t \right) \boldsymbol{\nu}_t - \widetilde{\boldsymbol{\mu}}. \tag{57}$$

BGP co-state. At a BGP the innovation rates stabilise at $\mathbf{Z}^* = \operatorname{diag}(z_i^*)$. Setting $\dot{\boldsymbol{\nu}} = \mathbf{0}$ in (57):

$$\boldsymbol{\nu}^* = \left(\rho \, \mathbf{I} - \ln \lambda \, \mathbf{\Omega}^\top \mathbf{Z}^* \right)^{-1} \widetilde{\boldsymbol{\mu}}. \tag{58}$$

The matrix $\rho \mathbf{I} - \ln \lambda \, \mathbf{\Omega}^\top \mathbf{Z}^*$ is an M-matrix (hence invertible with non-negative inverse) whenever $\rho > \ln \lambda \cdot \rho(\mathbf{Z}^* \mathbf{\Omega})$, where $\rho(\cdot)$ denotes spectral radius; this holds at any stable BGP with $\rho > g^*$. Substituting (58) into (55) establishes the general formula in Proposition 1.

Symmetric BGP. When $z_i^* = z^*$ for all i so that $\mathbf{Z}^* = z^* \mathbf{I}$, equation (58) simplifies to

$$\boldsymbol{\nu}^* = \frac{1}{\rho} \left(\mathbf{I} - \frac{g^*}{\rho} \mathbf{\Omega}^\top \right)^{-1} \widetilde{\boldsymbol{\mu}}, \qquad g^* = z^* \ln \lambda.$$

Because z^* cancels in the ratio (55), the optimal allocation reduces to

$$\chi^{R\prime} = \frac{\boldsymbol{\nu}^{*\prime}}{\mathbf{1}'\boldsymbol{\nu}^*} = \frac{\widetilde{\boldsymbol{\mu}}'\left(\mathbf{I} - \frac{g^*}{\rho}\boldsymbol{\Omega}\right)^{-1}}{\widetilde{\boldsymbol{\mu}}'\left(\mathbf{I} - \frac{g^*}{\rho}\boldsymbol{\Omega}\right)^{-1}\mathbf{1}}. \tag{59}$$

The resolvent $\left(\mathbf{I} - \frac{g^*}{\rho}\boldsymbol{\Omega}\right)^{-1} = \sum_{k=0}^{\infty}(g^*/\rho)^k\boldsymbol{\Omega}^k$ accumulates all higher-order spillover paths through the innovation network, with each additional round discounted by g^*/ρ (BGP growth rate relative to the discount rate). Applying the transversality condition $\lim_{t\to\infty} e^{-\rho t}\boldsymbol{\nu}_t'\mathcal{A}_t = 0$ (satisfied when $\rho > g^*$) confirms the stable solution. □

The growth-maximizing allocation of R&D resource is given by the following proposition.

Proposition 5. *The R&D allocation that maximizes the economy's BGP growth rate is the product of two weights summarizing the network structures. The share of research labor allocated to sector s technology j is*

$$h_{s,t}^{j,\text{Growth}} = \frac{\widetilde{\mu}_{s,t}^{j}\,\chi_s^j}{\sum_{k\in\mathcal{N}\cup\{\lceil\}}\sum_{j'\in\{c,d\}}\widetilde{\mu}_{k,t}^{j'}\,\chi_k^{j'}}\bar{H}.$$

B.3 Proof for Optimal Industrial Policy

Carbon Tax In the planner problem, a marginal increase of GHG concentration $d\Gamma_t$ changes welfare by

$$-\int_t^\infty e^{-\rho(t'-t)}\frac{\partial\mathcal{E}_{t'}}{\partial\Gamma_t}\,dt'\cdot d\Gamma_t.$$

Decentralization under the multiplicative tax requires that the private emission price in each sector equals the social cost. The tax paid by sector s per unit of dirty output is $\tau_{s,t}\psi_s mc_{s,t}^d$, while the physical emission per unit of output is ψ_s. Hence the private emission price (tax per tonne) is $\tau_{s,t}\cdot mc_{s,t}^d$. Setting this equal to SCC_t yields the sector-specific first-best tax:

$$\tau_{s,t}^* = \frac{\text{SCC}_t}{mc_{s,t}^d}, \quad s\in\mathcal{N}; \qquad \tau_{f,t}^* = \frac{\text{SCC}_t}{mc_{E_f,t}}.$$

Under these taxes the planner's within-sector factor ratio (38) is replicated, since $(1+\tau_{s,t}^*\psi_s)mc_{s,t}^d = mc_{s,t}^d + \psi_s\,\text{SCC}_t$, exactly matching the planner's shadow cost of dirty production.

R&D Subsidy In the planner problem, the value of successful innovation is the *social* value of innovation, denoted as $\mathcal{V}_{s,t}^j$.

$$\mathcal{V}_{s,t}^j \equiv \frac{\partial}{\partial\ln A_{s,t}^j}\left[\int_t^\infty e^{-\rho(u-t)}\Big(\ln C_u - \mathcal{E}_u\Big)\,du\right].$$

Let v_t be the multiplier on the research labor constraint. Then the planner's FOC is

$$\phi\, \theta_s^j \Phi_{s,t}^j \, (h_{s,t}^j)^{\phi-1} \, \mathcal{V}_{s,t}^j = v_t. \tag{60}$$

As derived in our augmented weight, the marginal increase in $\ln A_{s,t}^j$ raises contemporaneous $\ln C_t$ by $\widetilde{\mu}_{s,t}^j$, where $\widetilde{\mu}_{s,t}^j$ is the element of $\widetilde{\boldsymbol{\mu}}' \equiv \boldsymbol{\gamma}' \boldsymbol{\Sigma}_{\text{aug}}$. Since a one-step innovation changes $\ln A_{s,t}^j$ by $\ln \lambda$, the instantaneous social gain from having an innovation is: $\ln \lambda \cdot \widetilde{\mu}_{s,t}^j$.
The knowledge spillovers enter R&D productivity via (13). Since R&D production function for sector k technology j' is $z_{k,t}^{j'} = \theta_k^{j'} \Phi_{k,t}^{j'} (h_{k,t}^{j'})^{\phi}$, we can derive the impact on the innovation intensity from the knowledge stock of technology j in sector s

$$\frac{\partial z_{k,t}^{j'}}{\partial \ln A_{s,t}^j} = z_{k,t}^{j'} \, \omega_{s,k}^{j',j}, \quad j, j' \in \{c, d\},\, k, s \in \mathcal{N} \cup \{e\}.$$

The value associated with technology j in sector s, therefore, satisfies the HJB equation

$$\rho \, \mathcal{V}_{s,t}^j = \ln \lambda \cdot \widetilde{\mu}_{s,t}^j \; + \sum_{k \in \mathcal{N} \cup \{e\}} \sum_{j' \in \{c,d\}} z_{k,t}^{j'} \, \omega_{k,s}^{j',j} \, \mathcal{V}_{k,t}^{j'} + \frac{\partial \mathcal{V}_{s,t}^j}{\partial t} \; + \; \nabla_{\mathcal{A}} \mathcal{V}_{s,t}^j \cdot \dot{\mathcal{A}}_t.$$

where $\mathcal{A}_t$ is the vector collecting all $\ln A_{s,t}^j$. To align the private innovation incentives with the social optimal one, the R&D subsidy satisfies

$$1 - \xi_{s,t}^j = \frac{\Pi_{s,t}^j}{\mathcal{V}_{s,t}^j} \qquad \Longrightarrow \qquad \xi_{s,t}^j = 1 - \frac{\Pi_{s,t}^j}{\mathcal{V}_{s,t}^j}. \tag{61}$$

In a stationary environment, the above equation that determines the optimal R&D subsidy becomes

$$\rho \, \mathcal{V}_s^j = \ln \lambda \cdot \widetilde{\mu}_{s,t}^j \; + \sum_{k \in \mathcal{N} \cup \{e\}} \sum_{j' \in \{c,d\}} z_{k,t}^{j'} \, \omega_{k,s}^{j',j} \, \mathcal{V}_{k,t}^{j'}. \tag{62}$$

we can write the profits in vector form as $\boldsymbol{\Pi} \equiv [\Pi_s^j]_{s \in \mathcal{N} \cup \{e\}, j \in \{c,d\}}$ and define the $2(N+1) \times 2(N+1)$ diagonal matrix $\mathbf{Z} \equiv \text{diag}(z_s^j)$, we have

$$\rho \, \mathbf{V} = \ln \lambda \cdot \widetilde{\boldsymbol{\mu}} + \mathbf{Z} \, \boldsymbol{\Omega}^\top \mathbf{V},$$

$$\mathbf{V} = \left(\rho \, \mathbf{I} - \mathbf{Z} \, \boldsymbol{\Omega}^\top \right)^{-1} \ln \lambda \cdot \widetilde{\boldsymbol{\mu}}.$$

Infrastructure Investment The government chooses the investment flow $I_{g,t}$, which affects only the renewable side through the infrastructure stock in the delivery technology

$X_{g,t} = X_g(E_{g,t}, \kappa_g(K_{g,t}))$. The unit cost of effective delivered electricity $Y_{e,t}$ is

$$p_{e,t}(K_{g,t}) \;=\; \left[p_{X_g,t}(K_{g,t})^{\,1-\varepsilon_e} + p_{X_f,t}(K_{f,t})^{\,1-\varepsilon_e}\right]^{\frac{1}{1-\varepsilon_e}}.$$

Differentiating we can get the expression with cost shares $s_{g,t}$ and $s_{f,t}$.

$$\frac{\partial \ln p_{e,t}}{\partial K_{g,t}} = s_{g,t}\frac{\partial \ln p_{X_g,t}}{\partial K_{g,t}}. \tag{63}$$

Let $\nu_{g,t}$ denote the current-value co-state associated with $K_{g,t}$, the current-value Hamiltonian is

$$\mathcal{H}_t = \ln\Big(\mathcal{Y}_t - \sum_{j\in\{g,f\}} \mathcal{G}(I_{j,t})\Big) - \mathcal{E}_t + \sum_{j\in\{g,f\}} \nu_{j,t}\,(I_{j,t} - \delta_K K_{j,t}). \tag{64}$$

Differentiating (64) with respect to $I_{g,t}$ yields

$$\frac{\partial \mathcal{H}_t}{\partial I_{g,t}} = \frac{1}{C_t}\Big(-\mathcal{G}'(I_{g,t})\Big) + \nu_{g,t} = 0 \quad\Longrightarrow\quad \nu_{g,t} = \frac{c_K I_{g,t}}{C_t}. \tag{65}$$

given the sector-specific carbon taxes, we can further derive

$$\frac{\partial \mathcal{H}_t}{\partial K_{g,t}} = \frac{1}{C_t}\frac{\partial C_t}{\partial K_{g,t}} - (\mathrm{SCC}_t - \bar{\tau}_t^{\mathrm{eff}})\frac{\partial \Gamma_t}{\partial K_{g,t}} - \delta_K \nu_{g,t},$$

where $\bar{\tau}_t^{\mathrm{eff}} \equiv \sum_s \tau_{s,t} mc_{s,t}^d \psi_s Y_{s,t}^d/\Gamma_t$ is the emission-weighted average effective emission price. Since under the first best $\tau_{s,t}^* = \mathrm{SCC}_t/mc_{s,t}^d$ for all s, we have $\bar{\tau}_t^{\mathrm{eff}} = \mathrm{SCC}_t$, and we can substitute this back into the co-state equation, which yields

$$\dot{\nu}_{g,t} = (\rho + \delta_K)\nu_{g,t} - \frac{1}{C_t}\frac{\partial C_t}{\partial K_{g,t}} \tag{66}$$

Differentiate (65) with respect to time:

$$\dot{\nu}_{g,t} = \frac{d}{dt}\Big(\frac{c_K I_{g,t}}{C_t}\Big) = \frac{c_K}{C_t}\dot{I}_{g,t} - \frac{c_K I_{g,t}}{C_t^2}\dot{C}_t = \frac{c_K}{C_t}\Big(\dot{I}_{g,t} - I_{g,t}\frac{\dot{C}_t}{C_t}\Big).$$

Now substitute (65) and (B.3) into (66):

$$\frac{c_K}{C_t}\Big(\dot{I}_{g,t} - I_{g,t}\frac{\dot{C}_t}{C_t}\Big) = (\rho + \delta_K)\frac{c_K I_{g,t}}{C_t} - \frac{1}{C_t}\frac{\partial C_t}{\partial K_{g,t}}$$

Multiply both sides by C_t/c_K and rearrange:

$$\dot{I}_{g,t}^{\mathrm{FB}} = \Big(\rho + \delta_K + \frac{\dot{C}_t}{C_t}\Big) I_{g,t}^{\mathrm{FB}} - \frac{C_t}{c_K}\frac{\partial \ln C_t}{\partial K_{g,t}}. \tag{67}$$

Since $K_{g,t}$ affects the economy only through the delivered-electricity unit cost $p_{e,t}$, we can write

$$\frac{\partial \ln C_t}{\partial K_{g,t}} = \frac{\partial \ln C_t}{\partial \ln p_{e,t}} \cdot \frac{\partial \ln p_{e,t}}{\partial K_{g,t}}. \tag{68}$$

Under Cobb–Douglas aggregation across sectors and electricity input elasticities β_s, the price elasticity is

$$\frac{\partial \ln C_t}{\partial \ln p_{e,t}} = -\left(\gamma_e + \sum_{s\in\mathcal{N}} \gamma_s \beta_s\right), \tag{69}$$

hence

$$MB_t \equiv \frac{\partial \ln C_t}{\partial K_{g,t}} = -\left(\gamma_e + \sum_{s\in\mathcal{N}} \gamma_s \beta_s\right) \frac{\partial \ln p_{e,t}}{\partial K_{g,t}}. \tag{70}$$

Substituting (70) into (67) delivers the first-best infrastructure investment rule. Define

$$a_t \equiv \rho + \delta_K + \frac{\dot{C}_t}{C_t}, \qquad b_t \equiv \frac{1}{c_K}\frac{\partial C_t}{\partial K_{g,t}} = \frac{C_t}{c_K} MB_t.$$

Then (67) can be written as $\dot{I}^{\mathrm{FB}}_{g,t} = a_t I^{\mathrm{FB}}_{g,t} - b_t$, whose solution is

$$I^{\mathrm{FB}}_{g,t} = \exp\left(\int_0^t a_\tau \, d\tau\right)\left[I^{\mathrm{FB}}_{g,0} - \int_0^t \exp\left(-\int_0^{t'} a_\tau \, d\tau\right) b_{t'} \, dt'\right]. \tag{71}$$

together with the transversality condition $\lim_{t\to\infty} e^{-\rho t}\, \nu_{g,t}\, K_{g,t} = 0$ determines the level of $I^{\mathrm{FB}}_{g,0}$.

Second best infrastructure investment. Suppose carbon pricing is constrained so that the emission-weighted average effective emission price $\bar{\tau}^{\mathrm{eff}}_t \equiv \sum_s \tau_{s,t} mc^d_{s,t} \psi_s Y^d_{s,t}/\Gamma_t \leq \mathrm{SCC}_t$ (i.e., taxes are uniformly below the first-best sector-specific rates). Define the residual externality wedge $\Delta_t \equiv \mathrm{SCC}_t - \bar{\tau}^{\mathrm{eff}}_t \geq 0$. Infrastructure affects firms only through the unit cost of effective electricity $p_{e,t}(K_{g,t})$ induced by the delivery technology.

The marginal welfare benefit of K_g becomes $\frac{\partial C_t}{\partial K_{g,t}} - \Delta_t \frac{\partial \mathcal{E}_{g,t}}{\partial K_{g,t}}$. Substituting into (66) yields

$$\dot{\nu}_{g,t} = (\rho + \delta_K)\nu_{g,t} - \left[\frac{\partial \ln C_t}{\partial K_{g,t}} - \Delta_t \frac{\partial \mathcal{E}_t}{\partial K_{g,t}}\right]. \tag{72}$$

The second-best law of motion for infrastructure investment $I^{\mathrm{SB}}_{g,t}$ is defined by

$$\dot{I}^{\mathrm{SB}}_{g,t} = (\rho + \delta_K + g_t)\, I^{\mathrm{SB}}_{g,t} - \frac{1}{c_K}\left[\frac{\partial \ln C_t}{\partial K_{g,t}} - \Delta_t \frac{\partial \mathcal{E}_t}{\partial K_{g,t}}\right], \tag{73}$$

with Equation (44) together with $\dot{K}_{g,t} = I_{g,t} - \delta_K K_{g,t}$ define the optimal second-best infrastructure investment policy. Uncorrected markups $\lambda > 1$ and the IO structure enter

through $\partial C_t / \partial \ln p_{e,t}$ and $\partial \Gamma_t / \partial \ln p_{e,t}$, which govern how a change in the electricity price transmits to gross output, intermediate demand, and fossil use in the equilibrium.

B.4 Proof for Proposition 2

For the growth-maximizing R&D allocations, we use the augmented Domar weight for each technology as stated above. By the chain rule, the growth rate can be written as

$$g_{C,t} \equiv \frac{d \ln C_t}{dt} = \sum_s \sum_j \frac{\partial \ln C_t}{\partial \ln A^j_{s,t}} \cdot \frac{d \ln A^j_{s,t}}{dt} = \ln \lambda \sum_s \sum_j \widetilde{\mu}^j_{s,t} z^j_{s,t}. \tag{74}$$

Since $\ln \lambda > 0$ is a constant, maximizing $g_{C,t}$ is equivalent to maximizing

$$\sum_s \sum_j \widetilde{\mu}^j_{s,t} \theta^j_s \Phi^j_{s,t} (h^j_{s,t})^\phi \quad \text{s.t.} \quad \sum_s \sum_j h^j_{s,t} = \bar{H}, \;\; h^j_{s,t} \geq 0.$$

Form the Lagrangian

$$\mathcal{L} = \sum_s \sum_j \widetilde{\mu}^j_{s,t} \theta^j_s \Phi^j_{s,t} (h^j_{s,t})^\phi + \lambda_t \Big(\bar{H} - \sum_s \sum_j h^j_{s,t} \Big).$$

The first-order condition for an interior node (s, j) is

$$\phi \, \widetilde{\mu}^j_{s,t} \theta^j_s \Phi^j_{s,t} (h^j_{s,t})^{\phi - 1} = \lambda_t.$$

Solving for $h^j_{s,t}$ yields

$$h^j_{s,t} = \left(\frac{\phi \, \widetilde{\mu}^j_{s,t} \theta^j_s \Phi^j_{s,t}}{\lambda_t} \right)^{\frac{1}{1-\phi}}.$$

Imposing the research resource constraint pins down λ_t and gives the share form

$$h^j_{s,t} = \frac{\left(\widetilde{\mu}^j_{s,t} \theta^j_s \Phi^j_{s,t} \right)^{\frac{1}{1-\phi}}}{\sum_k \sum_{j'} \left(\widetilde{\mu}^{j'}_{k,t} \theta^{j'}_k \Phi^{j'}_{k,t} \right)^{\frac{1}{1-\phi}}} \bar{H},$$

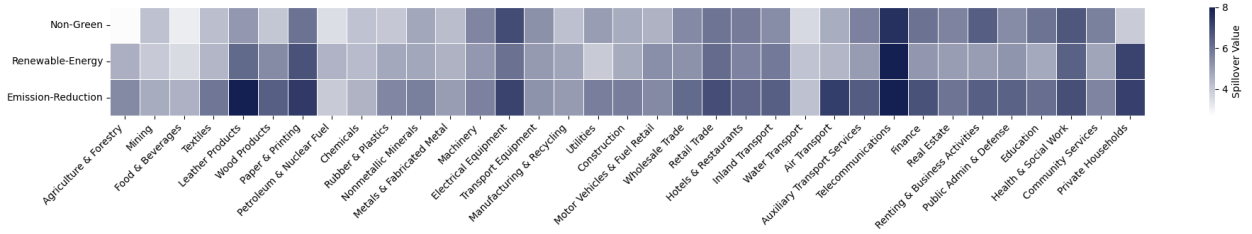
where the constant ϕ cancels between numerator and denominator. Finally, within any given sector s, dividing the two FOCs gives

$$\left(\frac{h^c_{s,t}}{h^d_{s,t}} \right)^{1-\phi} = \frac{\widetilde{\mu}^c_{s,t} \theta^c_s \Phi^c_{s,t}}{\widetilde{\mu}^d_{s,t} \theta^d_s \Phi^d_{s,t}},$$

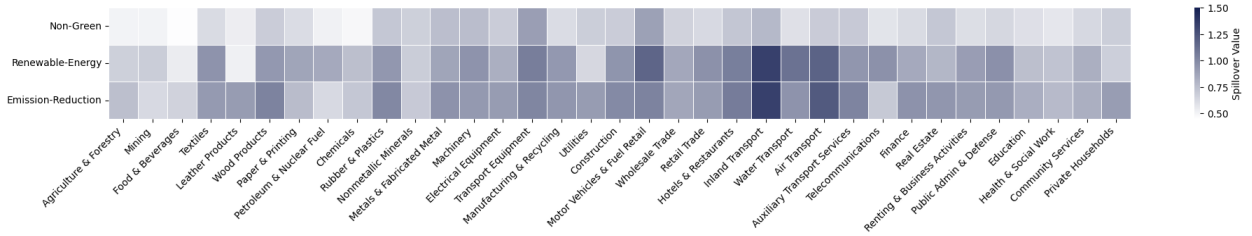
which can be written in the form of Proposition 2.

C Other Figures

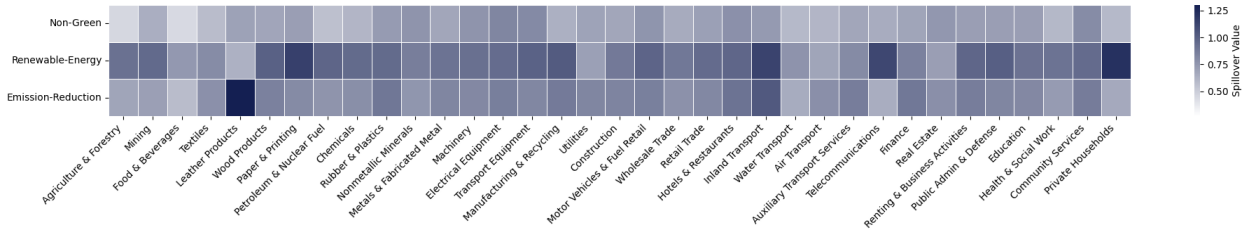
Figure C.1: Industry-Level Knowledge Spillovers by Green Category and Country: Total Spillovers



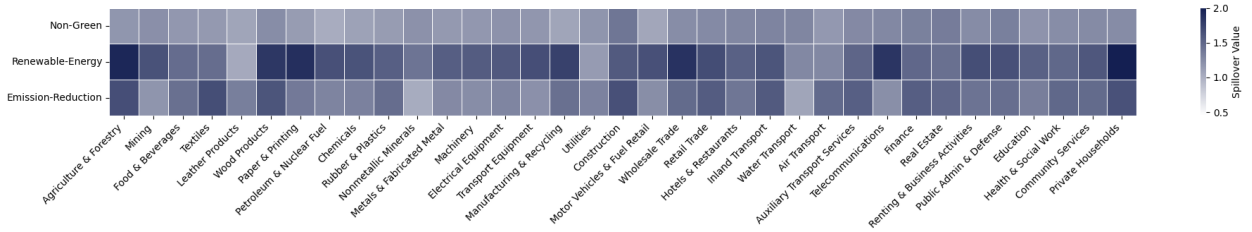
(a) United States



(b) European Union



(c) Japan



(d) South Korea

Notes: Each panel reports average total knowledge spillovers by industry and technology type, measured as the number of forward citations received within five years after patent registration. The sample includes patents filed between 2014 and 2017. Rows correspond to non-green, renewable energy, and emission-reduction patents; columns denote industries. Darker shading indicates higher spillover intensity.