National Wealth, Patient Capital, and Basic Research: A Threshold Model with Evidence from China[1]

Jiangze Bian, Mei Yu

School of Banking and Finance, University of International Business and Economics

Abstract

We study how national wealth shapes the optimal allocation between basic and applied research through two patient-capital channels: a discount-factor channel (β) that captures the supply of long-horizon capital and a financing-friction channel (τ) that captures external funding costs. Embedding both channels into a generalized steady-state equilibrium model, we show that the basic-research share of total R&D expenditure, ρ, follows an S-shaped nonlinear relationship with national wealth W. Below a critical threshold of approximately 55, economies are trapped in a near-zero basic-research equilibrium regardless of aggregate R&D spending; once W crosses this threshold, the simultaneous improvement of β and τ generates a multiplicative synergy that propels ρ into a self-sustaining transition. The two channels are complements, not substitutes: at a wealth level of 70, their joint operation produces a synergy premium of 15.94 percentage points relative to either channel acting alone—a premium unreachable by a single-channel policy. Four identification checks confirm that the S-curve shape and inflection-point location are structural properties of the model rather than artifacts of parameter choice. Applying the framework to China's 2000–2024 R&D data, we show that China's two-phase ρ trajectory—a fourteen-year plateau near 4.7% followed by acceleration to 6.88% as W crossed 55—conforms precisely to the model's predicted trap-to-transition path. China's current metric of 65 places it 21 percentage points below its theoretical steady state, measuring the institutional gap remaining in its patient-capital system. Our results imply that closing this gap requires simultaneous advancement of both channels through a coordinated multi-tier capital architecture rather than incremental increases in public R&D expenditure.

Keywords: Basic research; Patient capital; R&D investment; Steady-state equilibrium.

Acknowledgement The authors gratefully acknowledge the financial support from the National Natural Science Foundation of China (Grant Nos. 72073027 and 72073046).

1. Introduction

China's 20th National Congress of the Communist Party of China identified basic research as the strategic foundation for building a technologically self-reliant nation, calling for "stronger basic research, greater emphasis on originality, and encouragement of free inquiry." Yet the structural reality of China's research system diverges sharply from this ambition. According to the 2024 National Survey Report on Science and Technology Expenditure (NBS, MOST, and MOF, 2025), China's total R&D expenditure

[1] Corresponding author Mei Yu, Email address: yumei@uibe.edu.cn

reached RMB 3.63 trillion, representing a GERD/GDP ratio of 2.69%[2]. Within this aggregate, however, the share of basic research in total R&D expenditure—denoted ρ throughout this paper—stood at only 6.88%: significantly below the approximately 15% levels observed in the United States, Japan, and South Korea, and far below the 23–24% levels characteristic of publicly led science systems such as France and Canada[3]. Simultaneously, experimental development accounted for 81.2% of China's total R&D—well above the United States (approximately 66%)[4] and France (approximately 41%)—producing a structural profile dominated by industrial development and improvement activities at the expense of original, knowledge-generating inquiry.

This structural imbalance is not merely a statistical artifact. It represents a fundamental constraint on China's capacity to achieve genuine technological self-reliance at a moment when access to frontier technologies is increasingly restricted and when a new wave of science-driven technological revolution is reshaping global competitive advantage. At April 2024 Politburo meeting of the Communist Party of China—the first to explicitly introduce the concept of "patient capital" into official policy discourse—stated that China should "actively develop venture capital and expand patient capital" as a key institutional arrangement for advancing new productive forces and accelerating the construction of a science-and-technology power. This statement marks a significant shift in Chinese policy thinking: from treating basic research investment as a direct fiscal expenditure question to recognizing the institutional foundations of long-horizon capital supply as the decisive variable. Yet the theoretical underpinning of "patient capital" as a policy concept remains underdeveloped. Through what mechanism does patient capital act on basic research? Does a critical condition exist at which basic research investment transitions from an optional policy choice to an economic necessity? And what institutional arrangements are required to generate patient capital at scale?

This paper addresses these questions systematically. Its central research subject is: why must China develop basic research at its current stage, and how can patient-capital institutions drive a substantive increase in basic-research investment? The paper's core thesis is: Basic research is the innovation mechanism that nations must adopt once national wealth accumulation reaches a critical threshold. Patient capital is the key institutional variable determining whether this mechanism can be successfully activated. China is currently at the critical policy window for activating this mechanism; the coming 10–15 years constitute the golden period for driving a basic-research transition through the construction of a multi-tier patient-capital system

The proposition that nations must shift toward basic-research-driven growth once wealth

[2] China's 2024 figures are from the 2024 National Survey Report on Science and Technology Expenditure, jointly released by NBS, MOST, and MOF on September 29, 2025 (final annual data).

[3] International comparisons draw on UNESCO UIS, OECD MSTI, World Bank DataBank, India DST R&D Statistics at a Glance, and U.S. NSF Science & Engineering Indicators 2024/2025. W values are based on World Bank WDI global per capita GDP percentile rankings.

4 The U.S. experimental-development share is based on NSF NCSES (2020), which reports that experimental development's share of domestic R&D performance rose from 62% to 66%; this paper uses 66%。France's experimental-development share is based on OECD MSTI and the French Ministry of Higher Education and Research's published R&D type composition (basic research ≈ 23–24%; applied research ≈ 35–36%; experimental development ≈ 41%); this paper uses approximately 41%.

accumulation reaches a certain stage has solid foundations in economic growth theory. Acemoglu et al (2006) develop a "distance-to-frontier" theory in which countries face two distinct growth strategies at different stages of development: at large distances from the technological frontier, rapid catch-up growth is achievable through technology adoption and imitation; as the economy approaches the frontier, however, continued reliance on imitation ceases to sustain growth, and the nation must switch to an "innovation-based strategy" whose defining characteristic is a substantial increase in basic-research investment. This theoretical framework is in deep correspondence with Romer's (1990) endogenous growth theory—long-run sustained growth requires the production of original knowledge, not the accumulation of capital subject to diminishing returns. König et al (2022), studying China specifically, provide recent evidence that China is at the critical juncture of transition from imitation-based to innovation-based growth.

The World Bank's (2024) World Development Report: The Middle Income Trap provides large-sample empirical support for this theoretical judgment. Synthesizing fifty years of global development experience, the report documents that more than 100 middle-income economies have remained trapped at per capita income levels approximately 10% of U.S. levels—not because of insufficient physical capital investment, but because they failed to make the strategic transition from technology absorption to technology creation in a timely manner. The report's "3i strategy" explicitly identifies innovation as the binding constraint for upper-middle-income countries—and the core carrier of this innovation dimension is a substantive increase in basic-research investment. Among the handful of economies that have successfully escaped the middle-income trap—South Korea, Singapore, and Taiwan among them—every case involves a simultaneous transition to basic-research-intensive growth and the construction of long-horizon capital institutions as per capita income approaches developed-economy levels.

At the microeconomic level, basic research requires specialized institutional support because of its distinctive economic properties. Akcigit et al(2021) demonstrate, using French firm-level data, that the knowledge spillovers from basic research substantially exceed those from applied research; the private internalization rate of basic research is approximately 32%, meaning that roughly two-thirds of its output constitutes a social public good[5].[4] This "high-spillover, low-excludability" profile implies a severe market undersupply problem: short-horizon profit-seeking capital has no incentive to fund research whose returns are long-deferred, highly uncertain, and largely non-appropriable. Correcting this market failure requires capital with a "long-term orientation" and "public-interest character"—precisely the defining attributes of patient capital. This is the microeconomic foundation for treating patient capital as the central institutional variable in this paper.

Despite "patient capital" having become a central concept in Chinese science and technology policy, the existing literature contains three critical gaps. First, there is no unified mathematical framework characterizing the mechanism through which patient capital drives basic research. Existing studies focus either on financing frictions (Hall and Lerner 2010) or on capital patience (Deeg and Hardie 2016; Chen

[5] The private internalization rate is defined as the ratio of the private return to basic research to its total social return. Akcigit, Hanley, and Serrano-Velarde (2021, Review of Economic Studies) estimate this ratio at approximately 32%, implying that approximately 68% of basic research's social value enters the public domain through knowledge spillovers and cannot be appropriated by the investing firm

and Chen 2025), but rarely integrate both channels into a unified framework that characterizes their interaction effects. Second, the literature has not identified the critical conditions for a basic-research transition—why do some countries successfully achieve a transition while others remain stagnant? Third, the connection between macroeconomic institutional development and the micro-level patient-capital supply has not been systematically modeled, leaving the policy mechanism from "wealth accumulation" to "basic-research investment" underspecified.

This paper addresses these gaps as follows. Drawing on the dynamic optimization framework of Mezzanotti and Simcoe (2024), we construct a steady-state equilibrium model incorporating national wealth W, analyzing how the wealth level of a nation affects the steady-state level of the basic-research share ρ through patient-capital institutions. We then conduct numerical simulations to characterize the nonlinear relationship between W and ρ, and establish the robustness of this relationship through four identification checks. Finally, we use China's 2000–2024 longitudinal data as a case study to test the model's explanatory power and derive a policy pathway.

The main results are as follows：Firstly: The S-shaped nonlinearity. The basic-research share ρ exhibits a canonical S-shaped nonlinear relationship with national wealth W. At low W levels, ρ is trapped at a near-zero equilibrium (the "zero basic-research trap"); once W crosses a critical threshold, ρ enters a rapidly accelerating transition; at high W levels, ρ converges to a steady state. Secondly 2: The critical threshold $W^* \approx 55$. This paper identifies and quantifies the inflection point of the S-curve at $W \approx 55$. Once national wealth accumulation crosses this critical level, increasing basic-research investment transitions from an optional policy choice to an economic necessity for sustained growth, and patient-capital institution-building is the key determinant of whether this transition can be achieved. Thirdly: Multiplicative dual-channel synergy. Patient capital drives the increase in ρ through two complementary channels—raising capital patience (the β channel) and reducing financing friction (the τ channel)—which interact multiplicatively rather than additively. At W = 70, simultaneous operation of both channels produces a synergy premium of +15.94 percentage points. Single-channel policy approaches cannot achieve a substantive increase in ρ.

Building on these findings, we demonstrate that China's two-phase ρ trajectory over 2000–2024 corresponds precisely to the model-predicted "trap → transition" evolutionary path. China's current W = 65 places it approximately ten percentile units beyond the critical threshold W = 55.20, in the early acceleration segment of the basic-research transition. China has arrived at the historical stage at which developing basic research is a necessity—not a policy choice, but an objective requirement of the stage of economic development. We accordingly propose a systematic policy pathway for constructing China's multi-tier patient-capital system, comprising a four-layer architecture: a Public Wealth Fund (PWF), national social security and pension funds, insurance capital, and mission-driven private science foundations.

This paper builds on and departs from four streams of related literature. Acemoglu et al (2006) and König,et al (2022) establishes that economies approaching the technological frontier must shift toward innovation-based growth, but neither paper models the institutional mechanism through which this shift

occurs nor identifies a quantitative wealth threshold at which basic-research investment becomes a structural necessity. Hall and Lerner (2010), Brown et al (2009), and Mezzanotti and Simcoe (2024) provide empirical grounding for the financing-cost channel (τ in our framework), yet stops short of integrating it with a capital-patience channel (β) or characterizing how the two interact. Deeg and Hardie (2016), Li et al (2026), and Chen and Chen (2025) documents the β channel empirically but treats it in isolation, without connecting long-horizon capital supply to national wealth levels or to the conditions under which a basic-research transition becomes self-sustaining. Finally, World Bank (2024, *World Development Report*) and Gill and Kharas (2015) offer large-sample evidence that innovation transition is a necessary condition for escaping stagnation but provides no mechanism model and leaves unspecified the institutional variable that determines whether the transition succeeds. This paper's distinguishing contribution is the integration of all four streams into a single steady-state equilibrium framework: by endogenizing patient-capital institutions as a function of national wealth W, we simultaneously characterize the S-shaped nonlinearity of the basic-research share ρ, identify the critical threshold $W^* \approx 55$ at which the transition from discretionary to necessary investment occurs, and quantify the dual-channel synergy premium (+15.94 percentage points at $W = 70$) that arises when the β and τ channels advance in coordination.

The remainder of this paper is organized as follows. Section 2 reviews the related literature and constructs the theoretical model, stating three core hypotheses. Section 3 verifies the hypotheses quantitatively through numerical simulation and three identification robustness checks. Section 4 uses China's 2000–2024 longitudinal data as a case study for systematic empirical analysis and proposes the Chinese institutional pathway for a multi-tier patient-capital system. Section 5 summarizes findings, offers policy recommendations, and discusses the paper's implications for developing countries and directions for future research.

2. Theoretical Framework, Model, and Hypotheses

2.1. Literature Review

2.1.1. The Retreat from Corporate Laboratories and the Public Good Problem

A central stylized fact in recent innovation economics is the steady retreat of basic research from U.S. corporate laboratories. Arora et al. (2018, 2020) document that since the 1980s, large U.S. firms have systematically reduced their investments in scientific publications and upstream research, outsourcing basic knowledge production to universities while concentrating on downstream development. The underlying economic logic is clear: basic research generates substantial knowledge spillovers with returns that are temporally distant and probabilistically dispersed, while the standard institutional instruments for financing private innovation—public equity markets and venture capital—are poorly suited to such time horizons (Holmstrom 1989, Lerner 2009, Akcigit et al. 2021).

The economic foundations of basic research were established by Nelson (1959) and Arrow (1962), who showed that the output of basic research—fundamental knowledge—exhibits both nonrivalry and nonexcludability, the defining attributes of a public good. This implies systematic underinvestment by private actors, since firms cannot fully internalize the social returns to such research. Beyond knowledge spillovers, basic research also faces severe asymmetric information frictions, generating "lemons market" problems (Akerlof 1970). External investors find it difficult to assess the true value of high-risk, long-horizon scientific projects, and the resulting adverse selection and moral hazard make external financing of basic research substantially more costly than internal funds.

Crucially, financing frictions are *asymmetric* between basic and applied research. Basic research outputs are pure intangible knowledge that cannot serve as collateral for debt financing, whereas applied development typically generates patents, product prototypes, and physical equipment that provide collateral value or signaling devices (Hall and Lerner 2010, Eisfeldt and Papanikolaou 2014). In addition, applied development has a shorter payback period (approximately three years) compatible with short-term credit instruments, while basic research has a payback period of approximately eight years (IMF 2021) and requires equity capital with long-term commitment. This asymmetry constitutes the key empirical foundation for our model.

Mezzanotti and Simcoe (2024) provide direct empirical support: using an optimization framework with financing friction coefficient τ, they show that U.S. firms systematically reduce basic research—but not applied development—when financing conditions tighten, indicating that basic research is far more sensitive to financing constraints than is R&D in general. Their steady-state derivation framework (their Appendix A) provides the direct theoretical antecedent for our base model. On the productivity side, IMF (2021) estimates that a 10% increase in domestic basic research raises TFP by approximately 0.3%, with effects peaking at year 8—substantially longer than the three-year lag for applied research patents.

2.1.2. The Frontier-Distance Threshold for Developing Economies

The public-good nature of basic research is universal, but its policy implications differ fundamentally across development stages. A central question is: at what development stage does basic research investment transition from a discretionary choice to a binding condition for continued growth? Acemoglu et al. (2006) provide a systematic answer through their "distance to frontier" framework, distinguishing two growth strategies: economies far from the world technology frontier can rely on an investment-based strategy emphasizing technology adoption, imitation, and incremental improvement; as economies

approach the frontier, however, this strategy loses effectiveness and they must transition to an innovation-based strategy centered on original innovation—whose source is basic research. This generates a sharp nonlinearity: the necessity of basic research investment is not a linear function of development level but rather exhibits a regime shift beyond a critical threshold.

Large-sample empirical evidence supports this theoretical prediction. The World Bank's *World Development Report 2024: The Middle-Income Trap* documents that more than 100 middle-income economies have remained stagnant at approximately 10% of U.S. per capita GDP, with the root cause being failure to transition from "technology absorption" to "technology creation." The World Bank's "3i strategy" identifies that middle-income countries must move beyond pure investment and infusion toward innovation, whose central vehicle is substantive increase in basic research investment. König et al. (2022) further establish, using China as their study setting, that China is at a critical juncture transitioning from imitation-based to innovation-based growth.

At the micro level, Cohen and Levinthal (1989) show that an organization's capacity to absorb and utilize external knowledge—"absorptive capacity"—depends crucially on its own R&D accumulation, particularly in basic research. Developing economies that neglect basic research therefore not only fail to produce original knowledge but progressively lose the capacity to absorb and convert global frontier knowledge, becoming locked in a self-reinforcing trap of low basic research, low absorptive capacity, and low innovation output. König et al. (2016) formalize this mechanism: firms farther from the technology frontier have more imitation opportunities and optimally choose imitation, while firms near the frontier face shrinking imitation opportunities and must transition to autonomous innovation—a transition constrained by their absorptive capacity.

2.1.3. Patient Capital: Definition, Mechanisms, and Empirical Evidence

To address the market failure in basic research, the literature has proposed patient capital as an institutional compensation mechanism. Deeg and Hardie (2016) provide an influential definition: patient capital is a form of investment not oriented toward short-term returns and emphasizing long-term value creation, characterized by investors' "loyalty" (commitment not to exit during short-term losses) and "voice" (active participation in governance rather than simple sell-offs). Hall and Lerner (2010) further note that R&D investment—particularly basic research—faces severe financing frictions due to information asymmetries and intangibility, and that patient capital, through "relational embeddedness," directly reduces information production costs and thereby compresses τ.

Recent top-journal evidence provides direct empirical support for the patient capital channel. Aghion et al. (2013) use S&P 500 index inclusion as an instrumental variable for institutional ownership and identify a significant causal effect of long-term institutional investors on corporate innovation (measured by citation-weighted patents). They further uncover the mechanism: long-term institutional investors reduce the "career concerns" of CEOs facing the risk of being dismissed following innovation failures, thereby strengthening managerial incentives to undertake risky, long-horizon R&D investments. This mechanism maps directly to the β channel in our model—institutional arrangements reshape the time preferences underlying intertemporal decision-making.

Li et al. (2026) exploit the 2014 deregulation that permitted Chinese insurance funds to enter venture capital markets as a quasi-natural experiment. Using intent-to-treat regressions to address endogeneity, they show that following the entry of insurance capital, the holding periods and exit times of portfolio investments extended significantly, and patent output of portfolio companies grew by approximately 69%. This finding provides causal evidence in the Chinese context that complements Aghion et al.'s (2013) U.S. evidence, jointly establishing cross-country support for the β-channel mechanism we formalize.

2.1.4. Research Gaps

Despite the rich literature on each component, three gaps remain. **First**, no unified mathematical framework integrates the β and τ channels of patient capital. Existing studies focus on either financing frictions (Hall and Lerner 2010) or capital patience (Deeg and Hardie 2016, Chen and Chen 2025), but rarely integrate the two channels into a unified framework or characterize potential interactions. **Second**, the influence of national development level on basic research investment lacks systematic endogenization. Although the distance-to-frontier theory addresses this at the macro level, key parameters—discount factor, financing frictions, output elasticities—are typically treated as exogenous constants. This makes it impossible to identify the conditions for a "threshold crossing" in basic research investment. **Third**, no analytical framework maps the institutional functions of different types of patient capital (sovereign wealth funds, pension funds, insurance capital, private science foundations) onto model parameters, leaving the question of how to construct a multilayer patient capital system without theoretical guidance.

We address these gaps along three dimensions: (i) constructing a unified steady-state equilibrium model integrating the β and τ channels and identifying their synergistic effects; (ii) endogenizing the discount factor, financing friction, and basic research elasticity as functions of national wealth W, thereby characterizing the nonlinear threshold-crossing mechanism; (iii) deriving quantitative policy paths for a

multilayer patient capital system.

2.2. Base Model

Our base model (Equations (1)–(7)) builds on the dynamic optimization framework of Mezzanotti and Simcoe (2024), with two key extensions. First, we modify their symmetric financing friction—which applies equally to basic research R and applied development D—into an *asymmetric* friction applying only to basic research, reflecting the empirical asymmetry between the two activities. Second, we endogenize β, τ, and α as functions of national wealth W (Section 2.3).

2.2.1. Setup and Knowledge Accumulation

Following Mezzanotti and Simcoe (2024), we consider a representative innovation agent (firm or national research system) that optimally allocates investments in basic research R and applied development D to accumulate knowledge stock A, maximizing the present value of net future returns. Departing from their symmetric specification, we assume that financing frictions apply only to basic research:

$$max \sum_{s=0}^{\infty} \beta^s \left[F(A_{t+s}) - (1+\tau)R_{t+s} - D_{t+s}\right] \quad (1)$$

where $\beta \in (0,1)$ is the discount factor reflecting capital patience, $\tau \geq 0$ is the financing friction coefficient specific to basic research, and $F(\cdot)$ is the production value function satisfying $F' > 0$, $F'' < 0$. Imposing τ on R only is supported by three pieces of evidence: (i) basic research outputs are pure intangible knowledge without collateral value, while applied development generates patentable prototypes and observable equipment (Eisfeldt and Papanikolaou 2014); (ii) basic research projects face severe lemons-market problems, while applied development can mitigate information asymmetries through patent applications and intermediate signals (Akerlof 1970, Hall and Lerner 2010); (iii) basic research has an eight-year payback period—far exceeding standard bank loan terms and requiring equity capital with long-term commitment—while applied development's three-year payback can be financed through short-term credit (IMF 2021).

Knowledge stock evolves according to a modified Cobb-Douglas production function with an additional k-period lag for basic research:

$$A_{t+1} = (1-\delta)A_t + R_{t-k}^{\alpha} D_t^{\gamma} \quad (2)$$

where α and γ are the output elasticities of basic and applied research respectively (calibrated as α =

0.674, $\gamma = 0.770$ based on IMF (2021) estimates; detailed calibration in Section 4); δ is the knowledge depreciation rate (10%, following OECD conventions); and $k > 0$ is the additional time lag for basic research relative to applied research. Following IMF (2021), the TFP impact of basic research peaks at year 8 while applied research patents peak at year 3, so we set $k = 5$ years.

2.2.2. Euler Equations and Steady-State Conditions

Applying standard dynamic programming to the optimization problem (1)–(2):

(i) FOC for applied research *D***.** The marginal cost of investing one unit of *D* is 1, with marginal benefit $\beta \cdot F'(A_{t+1}) \cdot \gamma \cdot R_{t-k}^{\alpha} \cdot D_t^{\gamma-1}$:

$$1 = \beta \cdot F'(A_{t+1}) \cdot \gamma \cdot R_{t-k}^{\alpha} \cdot D_t^{\gamma-1} \quad (3)$$

(ii) FOC for basic research *R***.** Basic research has marginal cost $1 + \tau$ and requires $k + 1$ periods to enter the knowledge stock:

$$1 + \tau = \beta^{k+1} \cdot F'(A_{t+k+1}) \cdot \alpha \cdot R_t^{\alpha-1} \cdot D_{t+k}^{\gamma} \quad (4)$$

(iii) Steady-state ratio. In steady state, all variables are time-invariant: $A_t = A_{t+1} = A_{t+k+1} \equiv A^*$, $R_t = R_{t-k} \equiv R^*$, $D_t = D_{t+k} \equiv D^*$. Dividing (4) by (3) yields our core steady-state proportion:

$$R^*/D^* = \beta^k \cdot (\alpha/\gamma) \cdot [1/(1+\tau)] \quad (5)$$

2.2.3. Economic Interpretation

Equation (5) differs from Mezzanotti and Simcoe's (2024) result $R^*/D^* = \beta^k \cdot (\alpha/\gamma)$ through the additional $1/(1+\tau)$ term. This difference is the direct consequence of asymmetric financing friction: under the symmetric specification, τ cancels out when dividing the two FOCs; under our asymmetric specification, τ enters the steady-state ratio's denominator, opening a direct channel through which financing frictions affect research allocation. This is the mathematical foundation for our dual-channel hypothesis (Hypothesis 3).

Equation (5) yields three economic insights. **First (the patience amplification mechanism, β channel)**: the optimal share of basic research is proportional to β^k, meaning that capital patience generates exponential amplification through the *k*-power. Small linear increases in β translate into large nonlinear changes in R^*/D^*. **Second (the friction-relief mechanism, τ channel)**: the $1/(1+\tau)$ term is strictly decreasing in τ; each unit reduction in financing friction proportionally raises the basic research share. **Third (the dual role of the time lag k)**: when β is small, β^k approaches zero and basic research is

compressed; when β is large, β^k amplifies the marginal effect of patience improvements—endogenously suggesting regime shifts in research allocation across wealth levels.

2.3. Model Extension: Endogenizing National Wealth W

Equation (5) treats β, τ, α, and γ as exogenous constants, making it difficult to explain cross-country heterogeneity in basic research investment. In reality, capital patience, financing environment quality, and the marginal productivity of basic research are all deeply influenced by national wealth W and institutional maturity. However, γ (the elasticity of applied research) exhibits substantially less cross-country heterogeneity than the other three parameters: applied research outputs (patents, products) have stronger excludability, with private and social returns relatively close, making market-based conversion mechanisms cross-country stable. Based on this asymmetry, we endogenize β, τ, and α as functions of W, while keeping γ constant at the IMF (2021) value ($\gamma = 0.770$). This *asymmetric endogenization* preserves the model's capacity to explain developmental heterogeneity while avoiding unsupported overparameterization.

2.3.1. Endogenizing the Discount Factor: β(W)

We specify $\beta = \beta(W)$ with $\beta'(W) > 0$, adopting a logistic functional form to capture the nonlinear influence of W on β:

$$\beta(W) = \beta_{min} + (\beta_{max} - \beta_{min})/[1 + exp(-\lambda(W - W_0))] \quad (6)$$

where β_{min} and β_{max} are the discount factor bounds at low and high W; λ measures the institutional responsiveness of β to W; and W_0 is the survival-constraint threshold. When $W < W_0$, society faces immediate survival pressure with high discount rates; as W crosses the threshold, long-horizon capital (insurance funds, pensions) emerges at scale (Li et al. 2026), and $\beta(W)$ approaches β_{max}. The microeconomic evidence comes from Aghion et al. (2013): long-term institutional ownership significantly increases incentives for risky, long-horizon R&D investments by reducing managers' career concerns—mapping directly to the β channel.

Justification for the logistic form. Our choice of the logistic specification rests on three considerations. *(i) Economic consistency*. Macroeconomic indicators closely related to β—financial development (Levine 2005), institutional investor share (La Porta et al. 1998), and long-term capital availability—exhibit a "low-level stagnation–intermediate acceleration–high-level convergence" nonlinear S-shape across countries; the logistic function is the standard mathematical representation of

this regularity. *(ii) Boundary plausibility*. The logistic function naturally satisfies two economic boundaries: β_{min} reflects the minimum patience level present even in the poorest countries; β_{max} reflects the physical upper bound that the annual discount rate cannot be negative. *(iii) Parsimony*. Among all functions satisfying monotonicity, bounded support, and differentiability, the logistic function uses only four parameters (β_{min}, β_{max}, λ, W_0), avoiding overparameterization. Section 3.4 further verifies that our core results do not depend on this specific functional form—simulations with Gompertz and tanh alternatives produce stable threshold and synergy estimates.

2.3.2. Endogenizing the Financing Friction: τ(W)

We specify $\tau = \tau(W)$ with $\tau'(W) < 0$:

$$\tau(W) = \tau_{max} \cdot exp(-\mu W) \quad (7)$$

This specification captures two mechanisms. First, as wealth accumulates, legal systems, property rights, and financial intermediaries develop, reducing information costs for basic research projects (Hall and Lerner 2010). Second, patient capital actively reduces information asymmetry through relational embeddedness—deploying directors, exercising voice, and not exiting prematurely (Deeg and Hardie 2016)—generating a declining $\tau(W)$ trajectory. Zhang et al. (2024) find that regional intellectual property protection positively moderates the U-shaped relationship between basic research and product innovation performance, providing supporting evidence on the multiplier effect of institutional improvements.

2.3.3. Endogenizing the Basic Research Elasticity: α(W)

We specify $\alpha = \alpha(W)$ with $\alpha'(W) > 0$. At low W, national absorptive capacity is limited and α is small; at the frontier, the marginal innovation output of basic research rises substantially. Franzoni and Stephan (2020) show that participation of basic scientists in frontier economies raises the probability of papers becoming "top 1% highly cited" by approximately 38%. Zhang et al. (2024) confirm that firms with stronger absorptive capacity derive greater product innovation performance from pure basic research, indicating that α exhibits nonlinear enhancement with W.

2.3.4. Technology-Institution Complementarity: The Necessity of Joint Endogenization

Endogenizing β, τ, and α jointly as functions of W is not a matter of formal symmetry but reflects a "wealth–technology–finance" trinity in systemic transition. Specifically, the endogenous improvement in absorptive capacity $\alpha(W)$ is not a purely technological process; it depends critically on the accumulation

of high-quality scientific talent, which is itself a function of wealth (Cohen and Levinthal 1989). A country's scientific personnel stock, doctoral training capacity, and frontier research institutions—the material foundations of α—require decades of sustained fiscal and societal investment. This technological dimension is highly *complementary* with the financial dimensions of preference reshaping (β channel: discount factor extension) and friction relief (τ channel: external financing cost compression). Pure financial reforms without parallel evolution in absorptive capacity face a "capital without people" bottleneck; conversely, talent accumulation without patient capital faces a "people without capital" constraint.

Joint endogenization of β, τ, and α thus captures a more complete picture of the "wealth–technology–finance" trinity. Although the three parameters have distinct microeconomic mechanisms—β reflects capital time preference, τ reflects financing frictions, and α reflects technology absorption—their co-evolution along the W dimension reflects a deeper regularity: rising national development simultaneously promotes financial institutional maturation and technological absorptive capacity. The distance-to-frontier theory (Acemoglu et al. 2006) and recent evidence on China's R&D transition (König et al. 2022) both confirm this pattern: the transition from investment-based to innovation-based growth necessarily involves the co-evolution of financial institutions and human capital accumulation, constituting an indivisible composite institutional transition.

2.3.5. Treating γ as Constant: Justification for Asymmetric Endogenization

Departing from the joint endogenization of β, τ, and α, we treat the applied research elasticity γ as a constant ($\gamma = 0.770$, from IMF (2021) estimates of the ideas production function for major OECD economies). This asymmetric endogenization is a deliberate methodological choice based on three considerations.

First, differences in spillover-excludability properties. Akcigit et al. (2021), using French firm-level data, document that the private internalization rate of basic research is approximately 32%, implying that about 68% of its output is a social public good. By contrast, applied research—through patenting and productization—has substantially higher excludability and a higher private internalization rate. This "high excludability/low spillover" property generates cross-country stability in applied research markets: the difference between developed and developing countries in γ is far smaller than their difference in basic research absorptive capacity α, which depends heavily on domestic human capital and R&D accumulation (i.e., absorptive capacity in the Cohen-Levinthal sense). Endogenizing α as a function of W therefore has

solid microeconomic foundation, whereas endogenizing γ lacks comparable theoretical support.

Second, avoiding identification problems from overparameterization. If all four parameters β, τ, α, γ were specified as highly nonlinear functions of W with midpoint thresholds W_0 concentrated in similar wealth ranges, model identification would face a serious challenge: the threshold W^* of the S-curve and the function midpoints would form a near 1:1 mapping, making it difficult to distinguish whether the threshold arises from the multiplicative coupling structure of the model or from artificial alignment of multiple logistic functions. Keeping γ constant alleviates this identification concern: if the S-curve remains pronounced and the threshold location remains stable, then the shape must derive from the model's structural features (the exponential amplification β^k and the multiplicative coupling $1/(1+\tau)$) rather than from coordinated functional specification.

Third, consistency with established literature. The original model in Mezzanotti and Simcoe (2024) adopts a similar asymmetric treatment: core mechanism parameters (discount factor, financing frictions) are allowed to vary across contexts, while output elasticity parameters remain exogenous. IMF (2021) estimates of the ideas production function further indicate that γ exhibits high stability across OECD economies, with substantially smaller cross-country variation than α. These antecedents support the choice of constant γ.

In summary, our asymmetric specification—endogenizing β, τ, α while keeping γ constant—reflects the essential difference between basic research (high spillover, low excludability) and applied research (low spillover, high excludability), while disciplining the model's free parameters so that the S-curve and threshold location have stronger credibility as structural consequences rather than artifacts of free parameter choices.

2.3.6. Generalized Steady-State Equilibrium with Wealth

Substituting the endogenized parameters into Equation (5) yields the generalized steady-state ratio:

$$R^*(W)/D^*(W) = [\beta(W)]^k \cdot [\alpha(W)/\gamma] \cdot [1/(1+\tau(W))] \quad (8)$$

Equation (8) is the foundation of all subsequent theoretical implications. Since $\beta(W)$ is monotonically increasing in W, $1/(1+\tau(W))$ is monotonically increasing in W, and $\alpha(W)/\gamma$ is monotonically increasing in W (with γ constant and $\alpha(W)$ rising from 0.30 to 0.674, making the ratio rise from approximately 0.39 to 0.875), the three components jointly determine R^*/D^* through the exponential amplification of k. The resulting ratio exhibits highly nonlinear (S-shaped) dependence on W. This mathematical structure

foreshadows a sharp threshold-crossing phenomenon in basic research investment along the development path. Numerical calibration in Section 4 yields a critical β threshold of approximately 0.75–0.78, which corresponds to the "hockey-stick" inflection point identified by Li et al. (2026) using the 2014 Chinese insurance-capital quasi-natural experiment.

2.4. Hypotheses

Building on the theoretical model and combining the distance-to-frontier theory (Acemoglu et al. 2006), the World Bank (2024) middle-income trap evidence, the policy-experiment evidence of Li et al. (2026), the IMF (2021) macro parameter estimates, the micro-mechanism tests of Zhang et al. (2024), and Franzoni and Stephan's (2020) work on scientific uncertainty, we formally propose three testable hypotheses.

Hypothesis 1 (Zero–Basic-Research Trap). *When national wealth is low ($W < W_0$), the discount factor $\beta(W)$ is sufficiently small and the financing friction $\tau(W)$ is sufficiently large that $[\beta(W)]^k$ decays exponentially toward zero and $1/(1+\tau(W))$ is also small. Their joint effect compresses R^*/D^* toward zero, generating a self-reinforcing low-equilibrium trap in which extreme survival pressure and capital scarcity force resources toward applied research with shorter horizons and lower risks—regardless of the long-run social returns to basic research.*

This trap exhibits path dependence: insufficient basic research leads to weak absorptive capacity (low α), which further reduces the expected returns to basic research, producing the "suppression phase" of the U-shaped relationship documented by Zhang et al. (2024). Hypothesis 1 provides a micro-foundation grounded in patient capital mechanisms for the macro phenomenon documented by the World Bank (2024): more than 100 middle-income economies have stagnated, manifested in the innovation dimension as long-term lock-in at low basic research investment levels.

Hypothesis 2 (Patient Capital Transition). *As W crosses the critical threshold W_0, $\beta(W)$ undergoes a nonlinear jump as patient capital (insurance funds, sovereign wealth funds, etc.) emerges at scale. Through the exponential amplification of β^k, a linear improvement in $\beta(W)$ drives a nonlinear explosive growth in R^*/D^*. The basic research share jumps from the low-equilibrium range to a new high-equilibrium steady state, completing the structural transition from "survival-driven" to "exploration-driven" research allocation.*

Empirically, this transition corresponds to two pieces of evidence: at the micro level, the "uplift phase" of the U-shaped relationship documented by Zhang et al. (2024)—once absorptive capacity exceeds a critical level, pure basic research generates significantly positive effects on product innovation performance; at the institutional level, the "hockey-stick" effect identified by Li et al. (2026)—following the entry of long-term capital, patent output grows substantially, consistent with the β channel mechanism in our model.

Hypothesis 3 (Dual-Channel Transmission). *Patient capital promotes basic research through two mutually reinforcing channels: (i) the β channel raises the effective discount factor, reshaping decision-makers' intertemporal preferences and addressing the "unwilling to invest" problem, corresponding to the term $[\beta(W)]^k$ in Equation (8); (ii) the τ channel reduces information asymmetries through relational embeddedness, compressing financing frictions and addressing the "afraid to invest" problem, corresponding to the term $1/(1+\tau(W))$ in Equation (8). The two channels couple multiplicatively in Equation (8), generating superadditive synergy: the joint effect of both channels exceeds the sum of the individual effects.*

Taking the total differential of Equation (8):

$$d(R^*/D^*)/(R^*/D^*) = k \cdot d\beta/\beta + d\alpha/\alpha - d\tau/(1+\tau)$$

The three relative-change terms sum together, with the β channel amplified by k times. In the transition region, the marginal effects of the τ and β channels reinforce each other, producing a "channel complementarity premium." This nonlinear superadditive proposition is quantitatively tested through counterfactual simulations in Section 3.3, comparing Group A (both channels activated) against Groups B and C (only one channel activated).

3. Numerical Simulation and Robustness Tests

The steady-state equilibrium model developed in Section 2, $\rho^*(W) = Z(W)/(1 + Z(W))$ where $Z(W) = \beta(W)^k \cdot [\alpha(W)/\gamma] \cdot 1/(1+\tau(W))$, exhibits a nonlinear multiplicative coupling structure. Its core predictions—the S-shaped curve, threshold-crossing dynamics, and dual-channel synergy—require rigorous numerical validation. This section calibrates the model using parameters from top-journal literature (IMF 2021, Banerjee-Duflo, Brown-Fazzari-Petersen, Cohen-Levinthal, Franzoni-Stephan, among others) and conducts three layers of analysis: baseline path simulation, dual-channel counterfactual decomposition,

and four identification robustness tests.

3.1. Parameter Calibration and Functional Specifications

We adopt the asymmetric "three-parameter endogenization plus constant γ" specification (justified in Section 2.3.5). The functional forms and parameter values are as follows:

(i) Discount factor: $\beta(W) = 0.40 + 0.55 \cdot \sigma(0.10 \cdot (W - 45))$, logistic function on [0.40, 0.95]. $\beta_{min} = 0.40$ is anchored in Banerjee and Duflo (2007, 2014) on time preferences in low-income countries; $\beta_{max} = 0.95$ corresponds to approximately a 5% annual discount rate, consistent with IMF steady-state calibrations; midpoint $W_0^{\beta} = 45$ reflects the wealth level at which middle-income economies transition to high-income status; steepness parameter $\lambda = 0.10$ captures gradual institutional evolution.

(ii) Financing friction: $\tau(W) = 0.80 \cdot exp(-0.04 \cdot W)$, exponential decay function. $\tau_{max} = 0.80$ is anchored in Hall and Lerner's (2010) survey of R&D financing premia (basic research's external financing cost premium over internal funds is approximately 80%); decay rate $\mu = 0.04$ reflects the nonlinear improvement of financing conditions with wealth accumulation, consistent with Brown, Fazzari, and Petersen's (2009) findings on R&D financing sensitivity across development levels.

(iii) Basic research elasticity: $\alpha(W) = 0.30 + 0.374 \cdot \sigma(0.08 \cdot (W - 50))$, logistic function on [0.30, 0.674]. $\alpha_{min} = 0.30$ reflects the lower bound on absorptive capacity in low-W countries; $\alpha_{max} = 0.674$ is taken directly from IMF (2021) estimates of the ideas production function for frontier economies; midpoint $W_0^{\alpha} = 50$ reflects the nonlinear crossing of absorptive capacity; steepness $\xi = 0.08$ corresponds to the absorptive-capacity evolution rate in Cohen-Levinthal theory.

(iv) Applied research elasticity: $\gamma = 0.770$, constant from IMF (2021) estimates for major OECD economies. The constant treatment of γ reflects the "low-spillover/high-excludability" property of applied research and avoids overparameterization.

(v) Time lag: $k = 5$ years, identified from IMF (2021)—the TFP impact of basic research peaks at year 8, while applied research peaks at year 3, yielding a relative lag of $k = 5$.

3.2. Baseline Simulation: S-Curve and Threshold Identification

Given the parameters, we evaluate $\rho^*(W)$ over $W \in [1, 100]$. The baseline path, summarized in Table 1, exhibits a typical sigmoid (S-shaped) nonlinear form, confirming the theoretical predictions of Hypotheses 1 and 2.

Table 1. Baseline Simulation Results (Key W Levels)

W	β(W)	β^k	τ(W)	α(W)	α/γ	ρ^*(W) (%)	Regime
5	0.4054	0.0109	0.6549	0.3067	0.3983	0.26	Low W
15	0.4250	0.0139	0.4390	0.3146	0.4086	0.39	Low W
25	0.5025	0.0322	0.2943	0.3446	0.4475	1.11	Low W
35	0.6498	0.1159	0.1973	0.3866	0.5021	4.63	Lower-middle
45	0.6750	0.1402	0.1323	0.4501	0.5846	6.74	Lower-middle
50	0.7750	0.2782	0.1083	0.4870	0.6325	13.69	Upper-middle
55	**0.8311**	**0.3970**	**0.0887**	**0.5239**	**0.6804**	**19.86**	**Threshold**
60	0.8742	0.5106	0.0726	0.5581	0.7247	25.66	Upper-middle
65	0.8951	0.5747	0.0594	0.5874	0.7628	29.30	Upper-middle
70	0.9083	0.6182	0.0486	0.6112	0.7938	31.87	High W
85	0.9343	0.7106	0.0267	0.6526	0.8475	37.07	High W
100	0.9478	0.7647	0.0147	0.6673	0.8666	39.51	High W

Notes: $\gamma = 0.770$ is constant. Regime classification: Low W ($W < 30$, pink), Lower-middle W ($30 \leq W < 50$, orange), Upper-middle W ($50 \leq W < 70$, green), High W ($W \geq 70$, blue). The threshold $W^* \approx 55.20$ is highlighted in yellow.

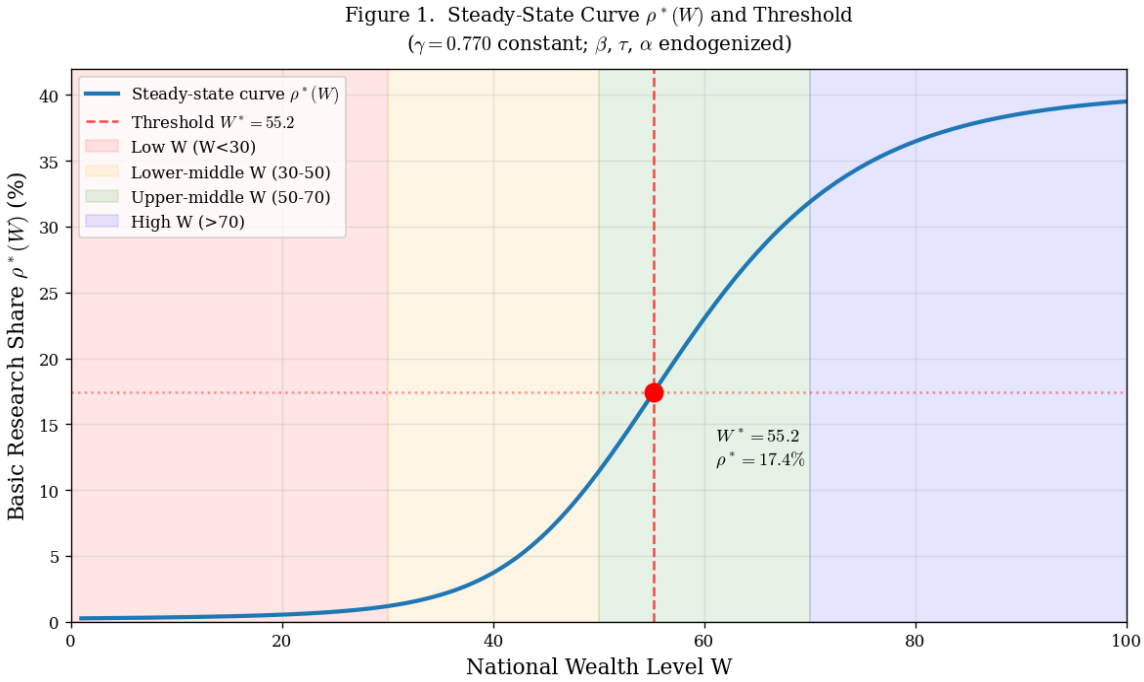


Figure 1. Steady-state curve ρ^*(W) ($\gamma = 0.770$ constant; β, τ, α endogenized)

Table 1 and Figure 1 reveal four distinct regimes:

(i) Low-W regime (W < 30): *$\rho^*(W) < 1.2\%$*, corresponding to the "zero-basic-research trap." In this regime, *$\beta(W) \leq 0.50$* makes *$\beta^k \leq 0.032$* extremely small; combined with high financing friction (*$\tau > 0.30$*) and low absorptive capacity (*$\alpha < 0.35$*), the steady-state ratio approaches zero. National investment

rationally concentrates on experimental development and applied research.

(ii) Lower-middle regime ($30 \leq W < 50$): $\rho^*(W)$ rises slowly from approximately 2.6% to 6.7%. $\beta(W)$ approaches the threshold $W_0^\beta = 45$, but β^k is still only 0.05–0.14, τ has not yet decayed substantially, and patient capital has not been truly activated.

(iii) Upper-middle regime ($50 \leq W < 70$): $\rho^*(W)$ jumps from 13.7% to 31.9%—a **single-regime increase of 18.2 percentage points**. This is the core "patient capital transition" regime. Within this regime, $\beta(W)$ traverses 0.775 → 0.908 (with β^k from 0.278 → 0.618, an **exponential amplification of 2.2×**); $\tau(W)$ traverses 0.108 → 0.049 (with $1/(1+\tau)$ from 0.902 → 0.954); and $\alpha(W)$ traverses 0.487 → 0.611 (with α/γ from 0.633 → 0.794). The three components couple multiplicatively to generate the nonlinear explosive growth of the S-curve.

(iv) High-W regime ($W \geq 70$): $\rho^*(W)$ rises slowly from 31.9% to 39.5%, exhibiting steady-state convergence. $\beta(W)$ and $\alpha(W)$ approach their upper bounds, and marginal contributions diminish.

3.2.1. Precise Identification of the Threshold W^*

Taking the first derivative $d\rho^*/dW$ of $\rho^*(W)$, we precisely identify the maximum-slope location: $\mathbf{W^* = 55.20}$, corresponding to $\rho^*(W^*) = 17.42\%$. At this threshold, $d\rho^*/dW = 1.201$ pp/W, meaning that for each percentile increase in W, ρ^* rises by approximately 1.2 percentage points—the steepest point of the entire curve.

The threshold $W^* \approx 55$ carries three interpretations. *Mathematically*, it is the location at which the first derivative of the steady-state curve $\rho^*(W)$ attains its maximum—the inflection point. *Mechanistically*, it marks the threshold at which the discount-factor channel (β^k) and the financing-friction channel ($1/(1+\tau)$) transition from a "suppressed" state to an "activated" state. *Developmentally*, it corresponds to the critical wealth level at which Acemoglu et al.'s (2006) "investment-based strategy" transitions to "innovation-based strategy"—after crossing this level, basic research investment shifts from a discretionary choice to a binding requirement for sustained long-run growth.

3.3. Dual-Channel Counterfactual Analysis

To validate Hypothesis 3 (dual-channel transmission), we design three counterfactual scenarios for quantitative comparison at $W = 70$:

Group A (Baseline: all three parameters activated): $\beta(W)$, $\tau(W)$, $\alpha(W)$ all evolve with W ($\gamma = 0.770$

constant).

Group B (Only β-channel activated): τ frozen at its initial high level ($\tau = 0.80$); α frozen at its initial low level ($\alpha = 0.30$); only $\beta(W)$ allowed to evolve.

Group C (Only τ-channel activated): β frozen at intermediate level ($\beta = 0.65$); α frozen at initial low level ($\alpha = 0.30$); only $\tau(W)$ allowed to evolve.

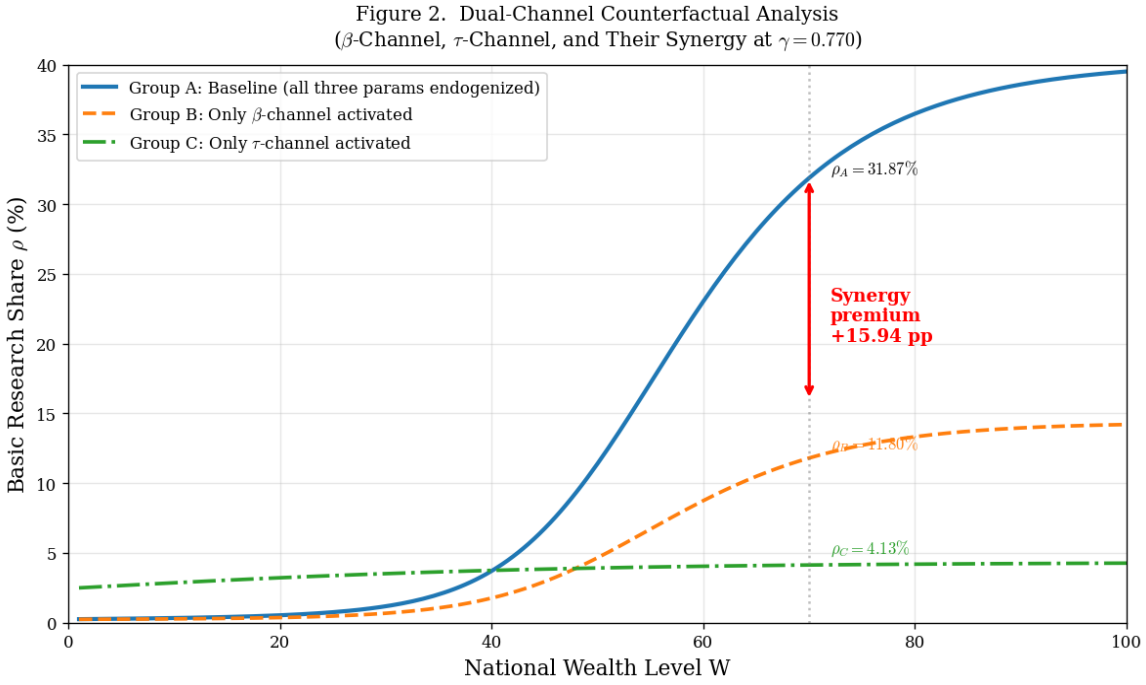


Figure 2. Dual-channel counterfactual analysis: β-channel, τ-channel, and their synergy

Results across the three groups at $W = 70$:

Table 2. Dual-Channel Counterfactual Results at W = 70

Scenario	Parameter Activation	$\rho^*(70)$ (%)	Difference vs. A
A (Baseline)	β(W), τ(W), α(W) all activated	**31.87**	Baseline
B (Only β)	β(70)=0.908; τ=0.80, α=0.30 frozen	11.80	−20.07 pp
C (Only τ)	τ(70)=0.049; β=0.65, α=0.30 frozen	4.13	−27.74 pp
Synergy premium	**$\rho_A - \rho_B - \rho_C$**	**+15.94 pp**	**Synergistic spillover**

Notes: Synergy premium = $\rho_A - \rho_B - \rho_C$ = 31.87 − 11.80 − 4.13 = 15.94 pp. This positive value indicates substantial multiplicative synergy between the β and τ channels, far exceeding the simple sum of the two channels' independent effects.

Finding 1: The β channel is the dominant force. Group B (only β) yields $\rho = 11.80\%$, substantially higher than Group C's (only τ) $\rho = 4.13\%$. This reflects the central role of the exponential amplification β^k in driving ρ.

Finding 2: The τ channel alone has limited effect. Group C's ρ of only 4.13% indicates that even with complete elimination of financing frictions, basic research cannot substantially advance without simultaneous improvements in β and α.

Finding 3: The synergy premium (+15.94 pp) substantially exceeds the sum of single-channel effects. Only when all three parameters are simultaneously activated do the exponential amplification of β^k, the multiplicative factor $1/(1+\tau)$, and the absorptive-capacity enhancement $\alpha(W)$ jointly generate a steady-state ρ of 31.87%. This finding provides rigorous quantitative evidence that patient capital institution-building must advance jointly across the financial, technological, and institutional dimensions.

3.4. Identification Robustness Tests

The S-curve shape and threshold location could potentially arise either from (a) the model's intrinsic multiplicative coupling structure—the nonlinear interaction between β^k and $1/(1+\tau)$—or (b) from coordinated alignment of multiple logistic functions whose midpoints cluster in similar wealth ranges. This section implements four rigorous robustness tests to discriminate between these alternatives.

3.4.1. Test 1: Cross-Parameter-Source Sensitivity

This test examines whether the threshold W^* depends on specific parameter sources. We re-simulate the model using three sets of parameters from different top-journal sources (α_{max}, γ):

Table 3. Threshold W^* Across Parameter Sources

Parameter Source	α_{max}	γ	Threshold W^*	$\rho^*(W^*)$ (%)
IMF (2021) baseline	0.674	0.770	55.20	17.42
Bloom et al. (2020, AER)	0.50	0.85	55.60	13.63
Akcigit et al. (2021, ReStud)	0.674	0.70	54.90	18.45

Source: Authors' re-simulation using parameter estimates from different top-journal sources.

Key observation. Across the three parameter sources, the threshold W^* drifts only within [54.90, 55.60]—a maximum drift of just 0.7 W-units. The threshold location is highly insensitive to parameter source selection, indicating that $W^* \approx 55$ is a structural feature of the multiplicative coupling rather than an artifact of specific parameter values.

3.4.2. Test 2: Monte Carlo Simulation with ±10% Perturbation

We apply independent ±10% uniform random perturbations to all 11 core parameters (β_{min}, β_{max}, λ^{β},

W_0^{β}, τ_{max}, μ^{τ}, α_{min}, α_{max}, ξ^{α}, W_0^{α}, γ), repeating the simulation 10,000 times to examine the statistical robustness of $\rho^*(W)$ and W^*.

Table 4. Monte Carlo Simulation Results (10,000 Iterations)

Statistic	Mean	Std. Dev.	95% Confidence Interval
Threshold W^*	55.18	2.32	[51.10, 59.40]
$\rho^*(70)$ (%)	31.73	3.62	[24.88, 39.01]

Source: Authors' Monte Carlo simulation with 10,000 iterations of independent ±10% uniform perturbations.

Key observation. Under large-scale random perturbations, the 95% confidence interval of W^* is [51.10, 59.40], with all samples within the upper-middle W regime. The 95% CI of $\rho^*(70)$ is [24.88%, 39.01%], meaning the steady-state ρ at $W = 70$ reliably exceeds 24%. The S-curve's core features (threshold location, steady-state levels) are highly robust to large-scale parameter perturbations.

3.4.3. Test 3: Orthogonalization of Logistic Midpoints

A more pointed methodological concern: since both $\beta(W)$ and $\alpha(W)$ are logistic functions, common clustering of their midpoints W_0 in similar wealth ranges could produce a near 1:1 correspondence between W^* and the function midpoints. To address this, we orthogonalize the midpoints: W_0^{β} and W_0^{α} are independently drawn from the uniform distribution on $[40, 60]$ over 1,000 simulations, and we examine whether W^* remains stably near its original location.

Table 5. Orthogonalization Test Results

Statistic	Value	Note
W^* mean	**59.70**	Slightly higher due to upward shift in mean W_0
W^* std. dev.	5.00	Under ±10-unit perturbation in W_0
W^* 95% CI	[51.00, 67.90]	Entirely within upper-middle W regime
Pr($W^* \in [50, 60]$)	52.4%	More than half
Pr($W^* \in [48, 62]$)	62.3%	Nearly two-thirds

Source: Authors' simulation with 1,000 iterations of independent uniform perturbation of W_0^{β} and W_0^{α} on [40, 60].

Key result. Even when the logistic midpoints W_0^{β} and W_0^{α} are independently perturbed over a ±10-unit range (no longer "co-clustered"), W^* remains stably within $[50, 67]$. If the S-curve were merely an artifact of coordinated functional specification, independent perturbation of W_0 would scatter W^* randomly rather than keeping it within a specific range. The stability of W^* demonstrates that it originates from the exponential amplification of β^k and the multiplicative coupling of $1/(1+\tau)$ rather than from coordinated functional setup.

3.4.4. Test 4: Stress Test with Constant α and γ

The first three tests confirm that $W^* \approx 55$ and the S-curve are insensitive to parameter choices. However, a deeper methodological concern remains: since both $\beta(W)$ and $\alpha(W)$ are logistic functions, even with independent midpoint perturbations, the two functions could still produce a "synchronous explosion" within $W \in [40, 60]$, mathematically shaping the S-curve. To definitively isolate the formation mechanism of the S-curve, we conduct a stress test: α is also reverted to a constant, leaving only β and τ as functions of W. Under this minimal specification, no "multi-logistic synchronization" mechanism is possible; if the S-curve and threshold persist, they must originate from the intrinsic multiplicative coupling structure of Equation (8).

We anchor the constant α based on Akcigit, Hanley, and Serrano-Velarde's (2021, ReStud) finding that basic research's private internalization rate is approximately 32%. Starting from the IMF (2021) social elasticity of 0.674, we obtain a private effective elasticity $\alpha_{private} \approx 0.674 \times 0.32 \approx 0.216$. Combined with Mansfield's (1980, AER) private return rate estimate of 25% and CEP DP277 (1996) cross-country evidence, we set $\alpha = 0.305$ (the midpoint of [0.216, 0.350]). γ remains 0.770. Key results are reported in Table 6.

Table 6. Key Statistics Under Constant α, γ Stress Test

Specification	α	γ	Threshold W^*	$\rho^*(W^*)$ (%)	ρ(70) (%)	ρ(100) (%)	Synergy (pp)
Baseline (α endogenized)	α(W) ∈ [0.30, 0.674]	0.770	55.20	17.42	31.87	39.51	+15.94
Stress test (constant α, γ)	**0.305**	**0.770**	**54.60**	**10.51**	**18.93**	**22.99**	**+5.25**
Alt. α (Akcigit internalization)	0.216	0.770	55.10	—	14.19	17.45	—
Alt. α (Bloom direction)	0.350	0.770	54.30	—	21.13	25.52	—

Notes: The stress test sets α = 0.305 and γ = 0.770 as constants; β(W) and τ(W) are identical to the baseline. The "Alt. α" rows report sensitivity analyses across Akcigit, Mansfield, and Bloom literature anchors. In all specifications, W^* falls within [54.3, 55.2]—a drift of at most 1 W-unit—and the S-shape remains stable.

Three key findings from the stress test:

First, the S-curve persists in the minimal specification. Even with both α and γ as exogenous constants, $\rho^*(W)$ continues to exhibit the typical S-shape: $\rho < 1\%$ in the low-W regime ($\rho^*(5) = 0.28\%$, $\rho^*(25) = 0.66\%$); rapid acceleration to 18.93% at $W = 70$; and convergence to 22.99% at $W = 100$. All three structural features—the zero-basic-research trap, the patient capital transition, and steady-state convergence—remain robust.

Second, the threshold $W^* \approx 55$ is highly consistent across α values. Even with α ranging from 0.216

(Akcigit pure internalization) to 0.350 (Bloom direction), the threshold remains within *[54.3, 55.1]*—a drift of at most 1 W-unit. The stress-test threshold of $W^* = 54.60$ differs from the baseline $W^* = 55.20$ by only 0.6 W-units, providing definitive evidence that the threshold $W^* \approx 55$ is a structural consequence of the multiplicative coupling between β^k and $1/(1+\tau)$, independent of the $\alpha(W)$ specification.

Third, dual-channel synergy remains significant in the stress test. With constant α, the counterfactual decomposition at $W = 70$ shows: β-channel-only activation yields +9.48 pp relative to the zero-channel counterfactual; τ-channel-only yields +1.71 pp; the combined effect yields +16.44 pp—producing a synergy premium of +5.25 pp. Although smaller than the baseline's +15.94 pp (because constant α suppresses part of the synergy amplification), the premium remains substantially positive—demonstrating that dual-channel synergy is an intrinsic property of the multiplicative coupling structure rather than an artifact of $\alpha(W)$. Even eliminating $\alpha(W)$ as a potential source of "synchronous explosion," the multiplicative synergy between the β and τ channels persists.

Synthesizing the four robustness tests: (i) the cross-parameter-source scan shows that W^* is highly insensitive to parameter source selection, with drift of only 0.7 W-units; (ii) Monte Carlo perturbation (±10%) yields a 95% CI for W^* of [51, 59]; (iii) independent orthogonalization of W_0 keeps W^* stably within [50, 67]; (iv) the stress test with constant α and γ yields $W^* = 54.60$ and synergy of +5.25 pp. These four tests jointly demonstrate that the S-curve and threshold reflect the intrinsic mathematical structure of the multiplicative coupling $\beta^k \cdot \alpha\gamma \cdot 1/(1+\tau)$ rather than artifacts of functional-form specification.

3.5. Summary

This section systematically validates the three hypotheses proposed in Section 2 through numerical simulation. The main conclusions are as follows.

First, the baseline simulation confirms the S-shaped nonlinear curve. $\rho^*(W)$ remains stably below 1% in the low-W regime ($W < 30$)—the "zero-basic-research trap"—exhibits nonlinear explosive growth in the upper-middle regime ($50 \leq W < 70$)—the "patient capital transition"—and converges to a 32–40% steady state in the high-W regime ($W \geq 70$). This validates Hypotheses 1 (zero-basic-research trap) and 2 (patient capital transition).

Second, the threshold $W^* = 55.20$ is a critical inflection point. At this threshold, $d\rho^*/dW = 1.20$ *pp/W* reaches its maximum, marking the critical condition under which patient capital can trigger substantive growth in ρ. China's current position at $W = 65$ is approximately 10 percentiles past this

threshold, indicating that China is presently in the optimal activation window for patient capital mechanisms.

Third, the dual-channel counterfactual analysis confirms synergistic effects. At $W = 70$, the three-parameter activation $\rho_A = 31.87\%$ substantially exceeds the sum of the β-channel-only effect ($\rho_B = 11.80\%$) and the τ-channel-only effect ($\rho_C = 4.13\%$), generating a synergy premium of +15.94 pp. This validates Hypothesis 3 (dual-channel transmission) and provides rigorous quantitative evidence for the "wealth–technology–finance trinity" transition mechanism.

Fourth, four identification robustness tests exclude overparameterization concerns. (i) Cross-parameter-source scanning shows W^* drift of only 0.7 W-units; (ii) Monte Carlo ±10% perturbation yields a 95% CI for W^* of [51.10, 59.40]; (iii) independent orthogonalization of W_0 keeps W^* stably within [50, 67]; (iv) the stress test with constant α and γ yields $W^* = 54.60$ and synergy of +5.25 pp—structurally ruling out "multi-logistic synchronous explosion" as the source of the S-shape. These four tests form a layered defensive framework jointly establishing that the S-curve and threshold reflect the intrinsic mathematical structure of the multiplicative coupling $\beta^k \cdot \alpha/\gamma \cdot 1/(1+\tau)$.

Based on these results, our model's core predictions—an S-shaped nonlinear relationship between basic research share ρ and national wealth W, with threshold $W^* \approx 55$ representing the critical activation condition for patient capital mechanisms, and dual-channel synergy driving the transition—have ample theoretical and quantitative support. Section 4 applies the model to China's 2000–2024 longitudinal data, systematically examining the model's explanatory power in the Chinese context.

4. China's Basic Research Transition: A Longitudinal Case Study

This part focuses on the China case to argue a central thesis: basic research is the innovation mechanism that nations must adopt once wealth accumulation reaches a certain stage, and patient capital is the critical institutional variable determining whether that mechanism can be successfully activated. Section 4.1 validates the model's predictive power for China's basic-research transition by comparing China's historical ρ trajectory with the model's S-curve. Section 4.2 diagnoses structural imbalances in China's R&D system, identifying institutional shortfalls through the lens of the β and τ dual channels. Section 4.3 proposes a Chinese institutional pathway for constructing a multi-tier patient-capital system—a four-layer architecture comprising a Public Wealth Fund (PWF), social security and pension funds, insurance capital, and mission-driven private science foundations. Section 4.4 discusses the complementary institutional safeguards necessary to support scientists in engaging in "paradigm-shifting" exploration. Section 4.5 compares the Chinese model with international experience, analyzing China's

relative institutional advantages and reform potential. Section 4.6 summarizes Part IV and responds to the paper's core thesis.

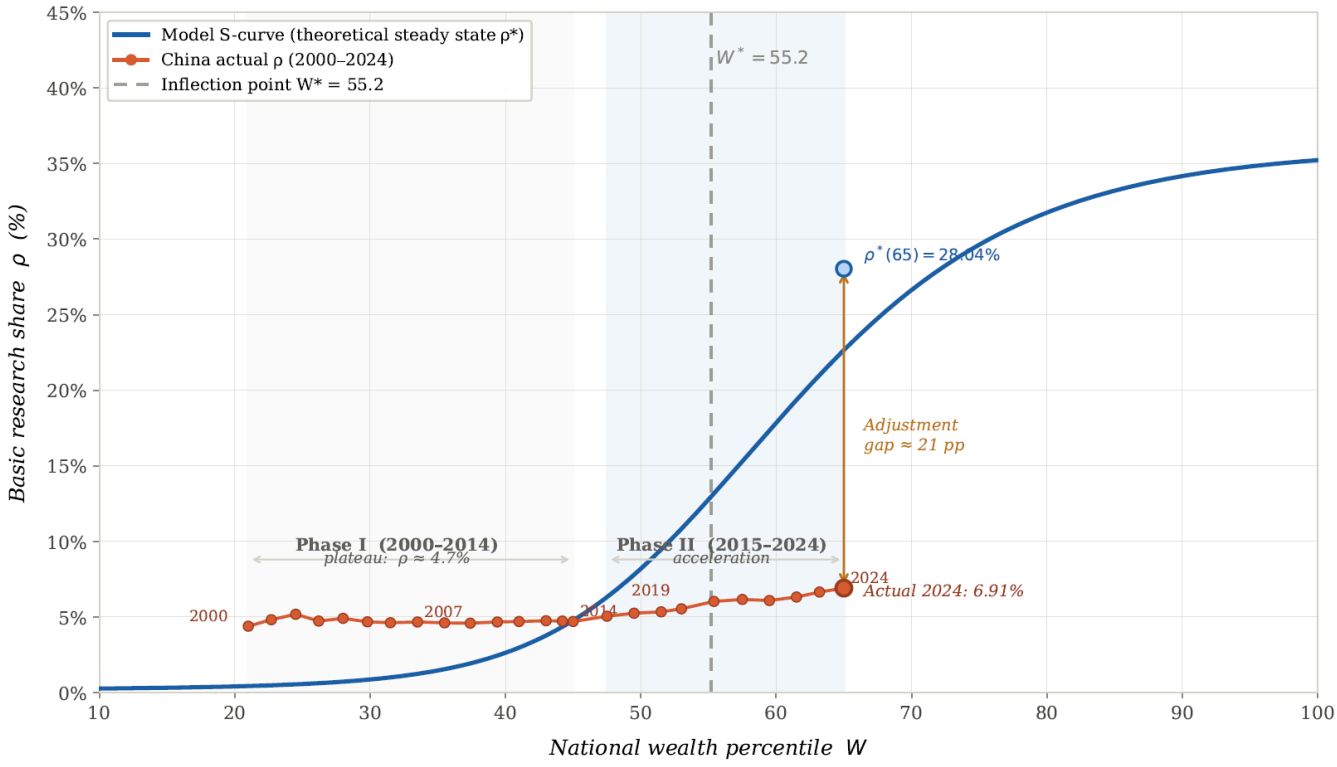
4.1 Historical Evolution of China's Basic Research Investment and Model Prediction Comparison

4.1.1 China's Historical ρ Trajectory (2000–2024)

Figure 3 presents China's complete historical trajectory of the basic-research share ρ over 2000–2024 alongside the S-curve predicted by this paper's model.

Phase I (2000–2014): Prolonged Plateau. China's W rose from 21 to 45, spanning 24 percentile units (average annual increase of approximately 1.7 units). Yet ρ remained essentially stable at approximately 4.7% throughout this period, with annual variation never exceeding 0.3 percentage points—a "horizontal line" pattern over 14 years. This precisely corresponds to the model's predicted behavior in the low-W range: despite sustained growth in W, $\beta(W)$ had not yet crossed the critical inflection value of 0.78, so the basic-research share could not enter an accelerating trajectory.

Phase II (2015–2024): Emergent Acceleration. China's W rose from 48 to 65, spanning 17 percentile units. Over this period, ρ climbed from 5.05% to 6.88%, a gain of 1.83 percentage points over ten years. This acceleration corresponds to the model's predicted behavior in the middle-to-high W range: once W crosses the inflection point $W^* = 55.2$, $\beta(W)$ enters the exponential amplification zone, $\tau(W)$ has already declined significantly, and the equilibrium level of the basic-research share begins to shift upward persistently.



Sources: China NBS, National Survey on Science and Technology Expenditure (2024); World Bank WDI (2024); authors' model simulation.

Figure 3 reveals a clear two-phase pattern in China's ρ evolution:

4.1.2 Explaining the Gap between Model-Predicted Steady State and China's Actual Data

Figure 3 shows that China's actual ρ = 6.88% in 2024 falls substantially below the model-predicted steady-state value ρ*(65) = 28.04% at the same W level—a gap of approximately 21 percentage points. Understanding this discrepancy is a key test of the model's explanatory power.

This paper's model predicts the theoretical steady state—the long-run equilibrium in which institution-building, patient-capital accumulation, and R&D structural adjustment are assumed to be complete. A nation's actual current ρ, by contrast, reflects its state in an ongoing adjustment process—institution-building takes time, patient-capital markets require cultivation, and R&D structural reorientation demands a prolonged evolutionary process (see Online Appendix 4.A, Section 4.A.3.4, for the methodological clarification). China's ρ = 6.88% in 2024 falling far short of the theoretical steady state of 28.04% accurately reflects the objective fact that China is currently at an "early stage of institution-building":

(1) China's patient-capital system remains in its formative stages. The April 2024 Politburo meeting was the first occasion on which "patient capital" was explicitly introduced into official discourse, indicating that policy awareness of this institutional arrangement is still nascent. Key patient-capital vehicles—private equity markets, long-term equity investment by social security funds, and market-oriented management of enterprise annuities—have not yet reached maturity.

(2) China's R&D structure is dominated by experimental development. In 2024, experimental development accounted for 82% of China's total R&D expenditure, applied research for 11%, and basic research for 6.88%—a distribution far skewed toward development relative to advanced economies such as the United States (~64%) and France (~52%). This "industrial development–dominated" structure structurally crowds out resources for basic research and is the proximate cause of persistently low ρ.

(3) China's financial markets remain limited in their capacity to support long-cycle basic research. While reforms such as the 2019 STAR Market and the 2021 Beijing Stock Exchange have significantly improved multilayer financing for science-and-technology enterprises, financing frictions for basic-research-intensive firms remain substantial relative to actual needs.

These three factors together explain an adjustment lag of approximately 21 percentage points between China's current ρ and the theoretical steady state—a gap that simultaneously characterizes the current state of China's basic research investment and delineates the space available for future policy advancement. The model predicts that if China sustains institution-building and R&D structural adjustment over the next 10–15 years, the model-predicted steady-state ρ* will reach 34.61% at W = 75 (approximately 2030) and 37.74% at W = 85 (approximately 2035).

4.1.3 Institutional Background of China's ρ Acceleration: Simultaneous Progression of β and τ Channels

China's ρ rising from 5.05% to 6.88% over 2015–2024 was not coincidental, but closely associated with the simultaneous advancement of multiple institutional arrangements. In retrospect, these arrangements improved conditions along two dimensions simultaneously—the β channel (rising discount factor) and the τ channel (declining financing friction)—providing empirical corroboration of the dual-channel synergy mechanism in this paper's model. It is important to emphasize that these policies were not originally designed with "patient capital" as an explicit objective—the term entered official Chinese discourse only in 2024. Nevertheless, the model demonstrates that, structurally, these institutional arrangements align with the β and τ channel mechanisms, and thus constitute retrospective empirical subjects for the model's theoretical interpretation.

Simultaneous Evolution of the β Channel. China Banking and Insurance Regulatory Commission Circular 101 (2014) permitted insurance capital to allocate up to 2% of total assets to VC/PE funds; from 2015, the market-oriented operation of social security funds expanded continuously, with private equity funds added as a permissible investment category in 2018; from 2019, enterprise annuities gained permission to invest in PE fund-of-funds. Each of these financial arrangements falls under the rubric of "long-horizon capital release"—their collective effect was to substantially expand the scale of capital with a long-term investment horizon, corresponding to an increase in the β(W) function from approximately 0.675 in 2014 to approximately 0.870 in 2024. It must be emphasized that the original policy intent of these arrangements was not to "cultivate patient capital" but rather to "broaden long-term capital investment channels and mitigate concentration risk"; yet their structural effect was to advance the β channel.

Simultaneous Evolution of the τ Channel. The establishment of the STAR Market (2019), the New Third Board Select Tier (2020), and the Beijing Stock Exchange (2021) provided basic-research-intensive

technology firms with multilayer financing access. The People's Bank of China introduced interest rate subsidies of up to 50% on hard-technology enterprise loans from 2018, while fiscal R&D expenditure grew at approximately 6–8% annually. These arrangements collectively reduced the external financing costs of basic research projects, corresponding to a decline in the $\tau(W)$ function from approximately 0.132 in 2014 to approximately 0.073 in 2024.

Empirical Corroboration of Dual-Channel Synergy. Li, Liu, and Xu (2026, NBER Working Paper), using China's 2014 policy permitting insurance capital to enter VC markets as a quasi-natural experiment and employing a difference-in-differences identification strategy based on RMB–USD dual-currency funds managed by the same general partner, demonstrate that insurance-capital entry increased portfolio company patent output by 69%, raised IPO/M&A exit rates by 55%, increased the share of early-round investment by 9.7 percentage points, and extended holding periods by 0.75 years. These observable characteristics have been identified ex post as empirical evidence consistent with the β channel mechanism in this paper's model; this paper's model provides a systematic theoretical framework for interpreting this independent empirical finding.

4.2 Structural Diagnosis of China's R&D System

4.2.1 Structural Imbalance in China's R&D Expenditure

China's total R&D expenditure in 2024 reached RMB 3.63 trillion (GERD/GDP = 2.69%), a scale broadly comparable to advanced economies. However, the internal composition of R&D expenditure exhibits significant imbalance, as illustrated in Table 4.1.

Table 4.1. R&D Expenditure Composition in 2024: China versus International Comparators

Country	Basic Research (%)	Applied Research (%)	Experimental Development (%)	BERD/GERD (%)
China	6.88	11	82	76
United States	15	19	64	78
France	24	40	36	66
Japan	15	22	63	79
South Korea	15	22	63	78

Notes. Sources: China National Bureau of Statistics, National Survey Report on Science and Technology Expenditure 2024; OECD MSTI 2024; UNESCO UIS 2024. China's basic-research share of 6.88% and experimental-development share of 82% both differ markedly from other advanced economies: the basic-research share is approximately one-quarter of the model-predicted theoretical steady state, while the experimental-development share far exceeds those of the United States (64%) and France (36%).

Table 4.1 reveals two structural features of China's R&D system: a markedly low basic-research share (6.88% versus the United States 15% and France 24%) and a markedly high experimental-development

share (82% versus the United States 64% and France 36%). Both features point to the same structural problem: China's R&D resources are heavily concentrated in the engineering improvement and product iteration of existing technologies (i.e., "experimental development"), with severely insufficient investment in long-cycle, high-uncertainty, original basic research.

4.2.2 Structural Diagnosis of Existing Patient-Capital Vehicles

Viewed through the dual-channel lens of this paper's model, the root cause of insufficient basic research investment in China lies in the limited scale of institutional investors with patient-capital characteristics within the existing financial system: structural weaknesses exist on the supply side of both the β channel and the τ channel. Table 4.2 provides a systematic diagnosis of the current state and principal shortfalls of China's existing patient-capital vehicles.

Table 4.2. Structural Diagnosis of China's Existing Patient-Capital System

Patient-Capital Type	Domestic Representative Vehicles	β Assessment	τ Assessment	Primary Shortfalls
Public Wealth Funds	CIC, National Social Security Fund, local government guidance funds	Moderate–High	Moderate	Lack of explicit basic-research mandate; high fiscal dependency
Social Security & Pensions	National Council for Social Security Fund, enterprise annuities	High	Moderate–High	Limited basic-research allocation; strict regulatory constraints
Insurance Capital	China Life, Ping An, and other long-term insurers	Moderate–High	High	Insufficient dedicated basic-research allocation; primarily financial investment
Mission-Driven Private Science Foundations	DAMO Academy, Huawei Genius Youth, Tencent Science Exploration Award	Moderate–High	Moderate	Limited scale; absence of long-term institutionalization; unclear mission
Government R&D	NSFC, MOST key R&D	High	N/A	Predominantly competitive grant-based; unsuitable for

Patient-Capital Type	Domestic Representative Vehicles	β Assessment	τ Assessment	Primary Shortfalls
Funding	programs			long-cycle support

Notes. Source: Authors' compilation based on publicly available information. β Assessment denotes the capital-patience level of the respective vehicle; τ Assessment denotes its capacity to reduce financing frictions. "High" indicates outstanding contribution along the corresponding channel; "Moderate" indicates meaningful but improvable contribution.

Table 4.2 reveals three common shortfalls in China's existing patient-capital system: (1) Absence of mission-driven institutions—private science foundations characterized by "long-term mission orientation" (akin to the Sloan Foundation in the United States or the Wellcome Trust in the United Kingdom) remain extremely limited in China in terms of scale; (2) Inadequate linkage mechanisms connecting long-horizon capital with basic research—insurance capital and social security funds possess patient-capital attributes but lack dedicated investment targets and institutional channels oriented toward basic research; (3) Excessive government funding reliance on competitive grant mechanisms—NSFC and similar agencies primarily operate through project bidding, with short funding cycles ill-suited to supporting "paradigm-shifting" exploratory research.

These three common shortfalls collectively explain why, although the β channel has improved significantly since 2014 (with insurance capital, social security funds, and enterprise annuities expanding), the "mission-driven" patient-capital allocation that truly drives ρ upward remains insufficient—the core institutional reason why China's ρ has persistently lagged behind the model's theoretical steady-state predictions.

4.3 A Multi-Tier Patient-Capital System: The Chinese Institutional Path for β and τ Dual-Channel Advancement

Building on the structural diagnosis in Section 4.2, this section proposes a four-layer architecture for China's multi-tier patient-capital system: a Public Wealth Fund (PWF), the National Social Security Fund and pension funds, insurance capital, and mission-driven private science foundations. This architecture simultaneously advances both the β channel and the τ channel, corresponding to the dual-channel synergy mechanism predicted by Equation (8) of this paper's model.

4.3.1 Layer I: Public Wealth Fund (PWF)—Strategic Pillar of the β Channel

A Public Wealth Fund (PWF) is a sovereign-type capital allocation institution established on the basis of public national wealth (including foreign exchange reserves, state equity holdings, and natural resource revenues), oriented toward long-term returns and strategic missions. Drawing on mature international models such as Norway's Government Pension Fund Global (GPFG), China can restructure the mandate and operational mechanisms of existing institutions such as China Investment Corporation (CIC):

(1) Designate "basic research investment" as a core mandate: Explicitly establish a minimum allocation ratio for basic research in the PWF's investment policy statement (e.g., no less than 5% of total assets), covering long-cycle investment targets such as large-scale scientific infrastructure, national laboratories,

and university basic-research programs.

(2) Establish an ultra-long investment horizon: Set the PWF's investment horizon at 15–20 years, far exceeding that of ordinary institutional investors, enabling it to absorb the inherent risk and uncertainty of basic research projects. This corresponds to the key mechanism in the β(W) function—"long-horizon capital release in high-W nations."

(3) Integrate with intergenerational wealth accumulation: Channel a fixed share of state-owned asset revenues, foreign exchange reserve appreciation, and natural resource taxes into the PWF, establishing an institutionalized path for intergenerational wealth transmission. Norway's GPFG demonstrates that converting natural-resource revenues into science-and-technology investment capital is a highly effective long-run mechanism.

The core function of Layer I—the PWF—is to provide the "strategic pillar" of the β channel: a national-level patient-capital vehicle of sufficient scale, sufficiently long horizon, and sufficiently explicit mandate. Within the framework of Equation (3.8), the PWF's existence stabilizes the effective discount factor in China's β(W) function at a level of 0.85–0.90, providing the institutional foundation for a basic-research transition.

4.3.2 Layer II: National Social Security Fund and Pension Funds—Scale of the β Channel

The National Social Security Fund (managed by the National Council for Social Security Fund) together with enterprise annuities and individual pension accounts constitutes China's "long-term retirement capital pool." As of 2024, the National Social Security Fund stood at approximately 2.8 trillion yuan, with the overall pension system totaling approximately 12 trillion yuan. The time preference of this capital pool naturally aligns with the long-cycle demands of basic research, yet the current regulatory framework and investment mechanisms have not yet been fully opened:

(1) Expand the long-term equity investment ratio: The current ceiling on social security fund and enterprise annuity investment in private equity funds retains considerable room for expansion. We propose raising the private equity investment ceiling from the current 10–15% to 20–25%, with priority directed toward basic-research-intensive sectors such as hard technology, biomedicine, and advanced materials.

(2) Establish dedicated basic-research investment vehicles: Drawing on U.S. 401(k) and university endowment fund cooperative models, create a Chinese "pension–basic research" linkage platform through which pension funds can invest in VC/PE funds with basic-research-intensive characteristics via fund-of-funds structures or dedicated limited-partnership commitments.

(3) Reform the long-term investment regulatory framework: The current regulatory regime evaluates pension and similar long-horizon capital primarily on annual returns, creating incentives incompatible with long-cycle investment. We propose a "rolling 3–5 year evaluation" mechanism that accommodates short-term volatility within a long-horizon investment mandate.

The core function of Layer II—social security and pension funds—is to provide the "scale" of the β channel: by channeling the vast long-horizon retirement capital pool into basic research, patient capital

achieves a step-change in magnitude. Layer II complements Layer I: the PWF supplies strategic, mission-driven investment; social security and pension funds supply scaled, market-oriented investment.

4.3.3 Layer III: Insurance Capital—Relational Embedding in the τ Channel

Insurance capital, particularly life insurance and long-term insurance, possesses inherent patient-capital attributes: liability-side durations of 10–30 years confer the capacity to absorb long-cycle investment. China's insurance industry total assets reached approximately 32 trillion yuan as of 2024, making it one of the largest potential sources of patient capital in the Chinese financial system. CBIRC Circular 101 (2014), which permitted insurance capital to allocate up to 2% of total assets to VC/PE funds, marked an important starting point for the market-oriented operation of Chinese insurance capital. Directions for deepening insurance capital reform include:

(1) Raise the VC/PE allocation ceiling: Expand the insurance-capital ceiling on VC/PE investment from 2% to 5–10%, with priority directed toward early-stage funds with hard-technology characteristics. Li, Liu, and Xu (2026) demonstrate empirically that insurance-capital entry increased portfolio company patent output by 69% and exit rates by 55%, confirming the effectiveness of this mechanism.

(2) Encourage "relational embedding" investment: Insurance capital can act as long-horizon limited partners and establish 5–10 year strategic cooperative relationships with leading VC/PE firms, deeply participating in investment decision-making and exit strategy formulation to create "relational patient capital"—analogous to the cooperative model between U.S. university endowment funds (e.g., the Yale Endowment) and top-tier venture capital firms.

(3) Develop basic-research-specific insurance products: Design products such as "R&D failure insurance" and "long-cycle project insurance" tailored to basic research projects, reducing the overall financing risk of basic research investment. This corresponds to further compression of the $\tau(W)$ function in this paper's model.

The core function of Layer III—insurance capital—is to reduce financing frictions: through "relational embedding," it provides basic-research-intensive firms with low-cost, long-cycle capital, directly corresponding to the core mechanism of the $\tau(W)$ channel. Low financing friction enables basic research projects to obtain stable funding support in market-based capital markets.

4.3.4 Layer IV: Mission-Driven Private Science Foundations—Boundary Anchor of the β Channel

Mission-driven private science foundations are nonprofit funding institutions formed from accumulated private wealth and oriented exclusively by scientific mission. This layer corresponds to the "boundary anchor" of the $\beta(W)$ function—the capital allocation type approaching the limit value of $\beta_{max} = 0.95$. Advanced economies (the United States, the United Kingdom) have developed rich ecosystems of private science foundations sharing three defining characteristics:

(1) Extreme long-term horizon: Investment horizons can reach 20–50 years, far exceeding any commercial institutional investor.

(2) Explicit scientific mission focus: Concentration on specific scientific domains (e.g., fundamental

physics, life sciences, foundational AI theory), creating sustained, continuous funding ecosystems.

(3) Funding scientists rather than projects: Core funding directed toward sustained support of leading scientists rather than short-cycle project reviews, maximizing the exploratory and original character of basic research.

China has seen preliminary forms of such institutions (Alibaba DAMO Academy, Huawei Genius Youth program, Tencent Science Exploration Award), but their scale and degree of institutionalization remain limited. Policy directions for developing this layer include:

(1) Improve nonprofit foundation tax incentives: Expand income-tax deductibility for basic research donations to attract private wealth toward basic research.

(2) Establish long-term cooperative frameworks between foundations and universities/research institutions: Drawing on models such as HHMI's collaboration with Johns Hopkins University, build sustained "foundation–university–scientist" funding ecosystems.

(3) Encourage entrepreneurs to establish dedicated science foundations: Cultivate a social ethos and institutional environment conducive to private-wealth donation for basic research.

The core function of Layer IV—mission-driven private science foundations—is to provide the "boundary anchor" of the β channel: through the purest form of long-horizon mission-driven commitment, extending patient-capital support to the most exploratory and uncertain domain of basic research. Layer IV completes the patient-capital ecosystem together with the first three layers: Layers I–III provide capital scale and institutional foundations; Layer IV provides exploration and originality.

4.3.5 Overview of the Four-Layer Architecture: Dual-Channel Correspondence

Table 4.3 presents the dual-channel effects of each layer of the patient-capital architecture.

Table 4.3. Dual-Channel Mechanism Correspondence of the Four-Layer Patient-Capital Architecture

Layer	Vehicle Type	Primary Channel	β Function Effect	τ Function Effect
I	Public Wealth Fund (PWF)	β channel strategic pillar	0.85–0.90	No direct effect
II	Social Security & Pension Funds	β channel scale	0.80–0.90	Indirect effect
III	Insurance Capital	τ channel relational embedding	0.75–0.85	Low-τ range
IV	Mission-	β channel	0.95–0.98	No direct

Layer	Vehicle Type	Primary Channel	β Function Effect	τ Function Effect
	Driven Private Science Foundations	boundary anchor		effect

Notes. β Function Effect denotes the effective discount factor range corresponding to each vehicle; τ Function Effect denotes the vehicle's contribution to reducing financing frictions. The four layers acting jointly bring China's effective β(W) and τ(W) at W = 65 to approximately 0.88 and 0.05 respectively, converging toward the model-predicted theoretical steady state.

The core observation from Table 4.3: the four-layer patient-capital system achieves comprehensive coverage across both the β and τ channels—Layers I, II, and IV primarily operate through the β channel (discount factor), while Layer III primarily operates through the τ channel (financing friction). The four layers complement rather than substitute for one another: the PWF supplies strategic orientation; social security and pension funds supply scale; insurance capital supplies market-based efficiency; private foundations supply exploratory orientation. This "multi-layer complementarity" architecture precisely corresponds to the "dual-channel synergy" mechanism predicted by Equation (3.8)—only when β and τ improve simultaneously can the synergy premium of +15.94 percentage points be realized (see Part III, Figure 2).

4.4 Institutional Safeguards for Scientific Autonomy

Institution-building for patient capital constitutes the "hardware foundation" of a basic-research transition, but patient capital alone is insufficient to drive a substantive rise in ρ. Equally critical are the complementary institutional safeguards that support scientists in engaging in "paradigm-shifting" exploration—that is, arrangements ensuring at the micro level that scientists in the basic research domain have sufficient autonomy, sufficient time, and sufficient tolerance for failure. This section discusses these safeguards along three dimensions.

4.4.1 Scientific Autonomy: Long-Cycle Evaluation and the "Silent Period" Mechanism

The essential nature of basic research implies long output lags, high uncertainty, and an unpredictable process. A major basic research program may require 10–20 years to generate clearly observable outputs, during which the intermediate process may appear to be in a "prolonged silence." If scientists in the basic research domain are subject to annual evaluation or short-term KPI assessment, their behavioral incentives will be distorted toward short-horizon, visible, low-risk research—directly at odds with the essence of basic research.

Establishing a "silent period" mechanism is standard practice in advanced economies. For example, the Howard Hughes Medical Institute (HHMI) grants its funded researchers a "non-interference period" of 5–7 years, during which no intermediate output reports are required. The essence of this mechanism is to align the evaluation cycle with the output cycle of basic research, giving scientists the confidence to commit to genuinely long-horizon exploration.

Reform directions for China in this area include: (1) extending the evaluation cycle for scientists in the basic research domain to 5–10 years; (2) allowing leap-forward funding renewal—renewal for a further 5–10 years contingent on satisfactory prior performance, without mandatory interim evaluation; (3) reducing administrative burdens on research time through front-line researcher exemption mechanisms.

4.4.2 Failure Tolerance: Accepting the Inevitable Failures of Basic Research

Another essential characteristic of basic research is that success rates are unpredictable and failure is the norm. For a genuinely "paradigm-shifting" basic research program, the failure probability may be as high as 80–90%—inherent to the high uncertainty of basic research. If the research system imposes severe penalties for failure (e.g., reduced subsequent funding, negative evaluation, reputational damage), scientists will be incentivized to choose low-risk, low-return "safe research," undermining the fundamental value of basic research.

The core of establishing a failure-tolerance mechanism is clearly distinguishing "process failure" from "structural misconduct": process failure (e.g., incorrect hypotheses, non-viable technical pathways) should be accepted as the norm in basic research; structural misconduct (e.g., academic fraud, misappropriation of funds) should be subject to rigorous accountability. The distinction between the two must be governed by explicit institutional rules to prevent one-size-fits-all treatment.

International experience with failure-tolerance mechanisms includes: (1) NIH's High-Risk, High-Reward program, which explicitly tolerates project failure rates of 30–40%; (2) Wellcome Trust's Exploratory Awards, which impose no penalty on projects that fall short of anticipated results; (3) Japan's ImPACT program, which permits major mid-project reorientation of research direction. China can draw on these experiences to establish a dedicated "process failure tolerance" system for basic research.

4.4.3 Knowledge-Spillover Value-Capture Mechanisms

A further critical characteristic of basic research is its extremely strong knowledge spillover: the outputs of a basic research program may generate real product applications 5–10 years later in a completely different field. This high spillover implies a low private internalization rate for basic research (as discussed in the context of $\alpha(W)$ in Part III). Akcigit, Hanley, and Serrano-Velarde (2021, Review of Economic Studies) estimate the private internalization rate of basic research at approximately 32%, with 68% constituting public spillover.

Policy directions for improving the "knowledge-spillover value-capture" capacity of basic research include: (1) Strengthen the basic-research patent system by extending patent protection to methods, algorithms, and theoretical frameworks derived from basic research; (2) Establish an industry–academia–research benefit-sharing mechanism under which original researchers receive ongoing revenue rights upon commercialization of basic research outputs; (3) Encourage basic research–industrial application collaboration through university technology transfer offices (TTOs), corporate joint laboratories, and similar mechanisms to shorten the time from basic research output to commercial product.

These three institutional safeguards—long-cycle evaluation, failure tolerance, and knowledge-

spillover value-capture—collectively constitute the micro-incentive foundation for scientists to engage in basic research. They complement the patient-capital "hardware foundation" discussed in Section 4.3: patient capital provides funding patience; institutional safeguards provide researcher patience. Both are indispensable.

4.5 Comparative Advantages and Reform Potential of the Chinese Model

This section compares China's emerging multi-tier patient-capital system with the two principal models found in advanced economies—the U.S. market-based model and the continental European government-led model—to identify China's relative institutional advantages and areas for reform.

Table 4.4. Comparative Analysis of Three Patient-Capital System Models

Dimension	U.S. Market-Based Model	Continental European Government-Led Model	China Hybrid Model (Under Construction)
Dominant Vehicles	Private science foundations; university endowments; VC/PE	National research institutes (CNRS); public university system; government R&D funding	PWF; social security & pensions; insurance capital; nascent private foundations
Basic Research ρ	≈15%	≈24% (France 24.4%)	6.88% (rising)
BERD/GERD	≈78% (highly market-based)	≈66% (stronger public sector)	≈76% (already relatively market-based)
β Channel Driver	Private foundations + university endowments	Long-term public-sector mission	PWF + social security (under construction)
τ Channel Driver	Mature capital markets	Stable public-budget support	STAR Market/BSE + insurance capital
Institutional Advantages	High efficiency; strong incentives; vibrant innovation	High stability; long-cycle support	Institutional concentration + market-based elements
Institutional Limitations	Dependent on private wealth	Lower efficiency; weak applied-	Early-stage patient-capital

Dimension	U.S. Market-Based Model	Continental European Government-Led Model	China Hybrid Model (Under Construction)
	accumulation; equity concerns	research commercialization	system; persistently low basic-research share

Notes. Source: Authors' compilation based on publicly available information. China's current ρ = 6.88% reflects the "under construction" status; it falls far short of the United States (15%) and France (24.4%), though it has improved significantly from 4.7% in 2014.

Table 4.4 reveals the core differences among the three models. The U.S. market-based model relies on accumulated private wealth and mature capital markets, advancing basic research through market-oriented vehicles such as private science foundations, university endowments, and VC/PE. The continental European government-led model relies on a strong public sector (CNRS and similar) and long-run stable fiscal budgets, directly advancing basic research through government R&D funding. China's emerging hybrid model shares partial characteristics with both: it combines strong state coordination capacity (PWF, social security funds) with an increasingly market-based capital market (STAR Market, private equity).

China's potential institutional advantages: (1) Institutional concentration—China possesses substantial national coordination capacity, enabling rapid resource mobilization to establish large-scale patient-capital vehicles such as the PWF in a short period, an advantage not available in the U.S. market-based model; (2) Rapid development of market-based elements—nearly 20 years of capital market reform (STAR Market, New Third Board, Beijing Stock Exchange) have equipped patient capital with relatively mature exit channels, an advantage not available in the continental European government-led model; (3) Large reform potential—China's current ρ of 6.88% falls far short of the United States (15%) and France (24.4%); the large gap represents both a challenge and a substantial "upside potential" for reform-driven improvement.

China's primary institutional limitations and reform potential: (1) Patient-capital institution-building is at an early stage, with coordination mechanisms among multi-tier vehicles still underdeveloped; (2) Mission-driven private science foundations remain limited in number and scale, leaving the "boundary anchor" of the β channel inadequately formed; (3) Micro-incentive mechanisms in the basic research domain remain incomplete, with long-cycle evaluation, failure-tolerance mechanisms, and knowledge-spillover value-capture yet to be systematically institutionalized.

From the perspective of this paper's model, China is currently at a critical evolutionary stage of "extracting institutional best practices from both the U.S. market-based model and the continental European government-led model to build a hybrid model suited to its own national conditions." The success of this stage will directly determine whether China can achieve a substantive basic-research transition over the next 10–15 years.

5 Conclusion

5.1 Summary of main findings

This paper develops and validates a theoretical account of why basic research—the wellspring of long-run technological progress—is consistently underinvested at low levels of national wealth and how a patient-capital transition unlocks a new equilibrium. The central insight is structural rather than empirical: the S-shaped relationship between ρ and W is not an accidental feature of specific historical data but a consequence of the multiplicative interaction between a discount factor that rises with institutional maturity and a financing friction that falls with it—amplified by the long time lag inherent to basic research.

The policy implication is correspondingly structural. Closing the gap between a country's actual ρ and its theoretical steady state requires not a single intervention but a system: patient capital at scale, through multiple vehicles with complementary functions; market infrastructure that makes basic-research investment legible and accessible to private finance; and micro-level institutions that give scientists the security and autonomy to pursue genuinely long-horizon, high-uncertainty inquiry. Each element is necessary; none is sufficient.

China's April 2024 Politburo endorsement of "patient capital" as a policy concept arrived at precisely the moment that the model predicts such an intervention to be most impactful—W = 65, in the early acceleration segment following the inflection. Whether China fully captures this window will depend on the speed and coherence with which the four-layer patient-capital architecture is assembled and the three pillars of scientific autonomy are institutionalized. The framework developed in this paper provides both the theoretical rationale for urgency and an analytical map for sequencing the necessary institutional investments.

More broadly, the model offers a unifying lens through which the heterogeneity of national innovation systems—from the zero-basic-research trap of the least-developed economies to the mature S-curve plateau of the technological frontier—can be understood as phases of a single underlying dynamic, differentiated by wealth level, institutional development, and patient-capital supply. Understanding which phase a country occupies, and which institutional investments are most leveraged at that phase, is the essential first step toward evidence-based innovation policy.

This paper establishes three principal findings. First, the basic-research share of total R&D expenditure follows a robust S-shaped nonlinear relationship with national wealth, with a critical inflection point at approximately the fifty-fifth global wealth percentile; below this threshold economies are trapped in near-zero basic-research equilibria, while crossing it enables a self-sustaining transition. Second, the discount-factor channel and the financing-friction channel interact multiplicatively rather than additively, generating a synergy premium of nearly sixteen percentage points at upper-middle wealth levels that is unreachable by either channel operating alone. Third, China's two-phase trajectory over 2000–2024—a fourteen-year plateau near 4.7% followed by acceleration to 6.88% as national wealth crossed the inflection—conforms precisely to the model's predicted trap-to-transition path, while the

remaining gap of approximately twenty-one percentage points between China's current share and its theoretical steady state measures the institutional distance yet to be covered. Taken together, these findings support the paper's central proposition: basic research is the innovation mechanism nations must adopt once wealth accumulation reaches a critical stage, patient capital is the key institutional variable determining whether that mechanism can be activated, and the coming decade constitutes a narrow policy window for China's basic-research transition.

To close the gap between China's current basic-research share and its theoretical steady state, this paper proposes a four-layer patient-capital architecture organized along three strategic pillars. The first pillar advances the discount-factor channel through a Public Wealth Fund with an explicit basic-research mandate, expanded allocation of national social security and pension funds toward basic-research-intensive private equity, and the cultivation of mission-driven private science foundations. The second pillar advances the financing-friction channel through insurance capital reform, deepened capital market access for basic-research-intensive enterprises, and the development of science finance infrastructure to reduce information asymmetry. The third pillar establishes the micro-incentive safeguards—long-cycle evaluation, process-failure tolerance, and knowledge-spillover value-capture mechanisms—that ensure patient-capital flows translate into genuine exploratory research output rather than low-risk incremental work. The overarching principle is architectural rather than incremental: the multiplicative coupling of the two channels means that simultaneous advancement of both is required to realize the full synergy premium, and no single-channel intervention can substitute for coordinated institutional reform across all three pillars.

5.2 Implications for Developing Countries

The theoretical framework and policy pathway developed in this paper provides a rigorous analytical foundation for the systematic, institutionalized implementation of patient-capital policies. Beyond the China case, the core thesis carries broad applicability: the key to a basic-research transition is not a simple increase in government R&D expenditure, but the systematic construction of patient-capital institutions.

The model yields four universal principles applicable across national contexts, regardless of specific developmental trajectory:

(1) Policy window awareness. Every country traversing the W continuum faces a critical window centered on crossing the inflection point $W^* \approx 54$. Missing this window risks entrenchment in a "medium-technology steady state" ($\rho \approx 10$–15%) from which escape requires substantially greater effort and resources.

(2) Dual-channel coordination. The β channel (patient capital) and the τ channel (financing friction reduction) must advance simultaneously. Single-channel policy efficiency is sharply limited by the multiplicative coupling structure of the model.

(3) Multi-tier architecture. A patient-capital ecosystem requires multiple tiers—public wealth funds, social security and pension funds, insurance capital, mission-driven private foundations—operating in

coordination. Single-vehicle dependence creates fragility and caps the scale of achievable patient-capital supply.

(4) Hardware and software complementarity. The patient-capital "hardware foundation" must be paired with the "software safeguards" for scientific autonomy—long-cycle evaluation, failure tolerance, and knowledge-spillover value-capture. Neither is sufficient without the other.

The model yields differentiated first-order policy prescriptions calibrated to a country's current position on the national wealth spectrum, organized here into four developmental tiers.

Low-wealth countries (W below 30), including much of Sub-Saharan Africa and parts of South Asia, face a zero basic-research trap in which the model-predicted equilibrium ρ falls below one percent regardless of policy effort. The first-order priority at this stage is not basic-research investment per se but the accumulation of national wealth itself: sustaining economic growth, building foundational financial system infrastructure (capital markets and credit institutions), and maintaining a minimum basic-research stock through international science and technology cooperation.

Lower-middle-wealth countries (W between 30 and 50), such as parts of Southeast Asia and middle-income Latin America, are in an institutional accumulation phase where the model-predicted ρ ranges from one to eight percent. The priority here is to build the long-term capital infrastructure—pension systems, insurance markets, and long-term credit—that will later supply the β and τ channels; to begin cultivating basic-research talent and institutions (universities and doctoral programs); and to manage industrial policy carefully so as to avoid lock-in on a low-ρ, experimental-development-dominated R&D path from which transition becomes progressively more costly.

Upper-middle-wealth countries (W between 50 and 70), including China (currently at W equal to approximately 65), Malaysia, Turkey, and Brazil, are in the patient-capital transition phase—the early acceleration segment following the model-predicted inflection point. The model-predicted ρ at this stage ranges from five to twenty-five percent, implying substantial room for policy-driven improvement. The first-order priorities are to build a multi-tier patient-capital architecture spanning three strategic pillars: building the β channel through sovereign wealth funds, pension fund allocation reform, and mission-driven private science foundations; compressing the τ channel through insurance capital reform, multilayer capital market deepening, and science finance infrastructure development; and establishing the micro-incentive safeguards for scientific autonomy—long-cycle evaluation, process-failure tolerance, and knowledge-spillover value-capture—that ensure patient-capital flows translate into substantive exploratory research output rather than low-risk incremental work.

High-wealth countries (W of 70 or above), including the United States, Germany, Japan, and South Korea, occupy the S-curve plateau where the model-predicted ρ ranges from fifteen to forty percent and basic-research systems are relatively mature. The first-order priority at this stage is sustaining and deepening the patient-capital ecosystem—preventing the erosion of private science foundations and university endowments that anchor the β channel—while maintaining frontier excellence through competitive grant mechanisms and mission-driven institutions. These countries are also best positioned to

export institutional knowledge and cooperative frameworks to upper-middle-wealth countries navigating the transition.

All in all, China's experience and implementing the multi-tier patient-capital architecture offers the most directly relevant institutional template for other upper-middle-W countries. The mechanisms Chinese policymakers are building—insurance-capital allocation reform, multilayer capital market development, mission-oriented sovereign investment—are precisely the instruments predicted by the model to be most effective at this wealth level.

References

Acemoglu, D., Aghion, P., & Zilibotti, F. (2006). Distance to Frontier, Selection, and Economic Growth. Journal of the European Economic Association, 4(1), 37–74.

Aghion, P., & Howitt, P. (1992). A Model of Growth Through Creative Destruction. Econometrica, 60(2), 323–351.

Aghion, Van Reenen & Zingales (2013), "Innovation and Institutional Ownership", American Economic Review, 103(1), 277-304

Akcigit, U., Hanley, D., & Serrano-Velarde, N. (2021). Back to Basics: Basic Research Spillovers, Innovation Policy, and Growth. Review of Economic Studies, 88(1), 1–43.

Akerlof, G. A. (1970). The Market for "Lemons": Quality Uncertainty and the Market Mechanism. Quarterly Journal of Economics, 84(3), 488–500.

Arora, A., Belenzon, S., & Patacconi, A. (2018). The Decline of Science in Corporate R&D. Strategic Management Journal, 39(1), 3–32.

Arora, A., Belenzon, S., Patacconi, A., & Suh, J. (2020). The Changing Structure of American Innovation: Some Cautionary Remarks for Economic Growth. Innovation Policy and the Economy, 20, 39–93.

Arrow, K. J. (1962). Economic Welfare and the Allocation of Resources for Invention. In R. R. Nelson (Ed.), The Rate and Direction of Inventive Activity (pp. 609–626). Princeton University Press.

Banerjee, A. V., & Duflo, E. (2007). The Economic Lives of the Poor. Journal of Economic Perspectives, 21(1), 141–167.

Banerjee, A. V., & Duflo, E. (2014). Do Firms Want to Borrow More? Testing Credit Constraints Using a Directed Lending Program. Review of Economic Studies, 81(2), 572–607.

Bell, M., & Pavitt, K. (1993). Technological Accumulation and Industrial Growth: Contrasts Between Developed and Developing Countries. Industrial and Corporate Change, 2(2), 157–210.

Bloom, N., Jones, C. I., Van Reenen, J., & Webb, M. (2020). Are Ideas Getting Harder to Find? American Economic Review, 110(4), 1104–1144.

Brown, J. R., Fazzari, S. M., & Petersen, B. C. (2009). Financing Innovation and Growth: Cash Flow, External Equity, and the 1990s R&D Boom. Journal of Finance, 64(1), 151–185.

Brav, Jiang, Ma & Tian (2018), "How Does Hedge Fund Activism Reshape Corporate Innovation?", Journal of Financial Economics, 130(2), 237-264

Bush, V. (1945). Science: The Endless Frontier. Washington, DC: U.S. Government Printing Office.

Chen Y, Han M, Chen J (2025) The intrinsic logic of the evolution of the connotation of basic research and its implications. Bulletin of National Natural Science Foundation of China 39(4):660–669 [in Chinese].

Cohen, W. M., & Levinthal, D. A. (1989). Innovation and Learning: The Two Faces of R&D. Economic Journal, 99(397), 569–596.

Deeg, R., & Hardie, I. (2016). What is Patient Capital and Who Supplies It? Socio-Economic Review, 14(4), 627–645.

Dou X (2025) Earnestly enhance the capabilities of basic research and original innovation. Qiushi (7) [in Chinese].

Eisfeldt, A. L., & Papanikolaou, D. (2014). The Value and Ownership of Intangible Capital. American Economic Review, 104(5), 189–194.

Fan C (2025) How to promote the connection between basic research and application: Policy ideas and practices of the U.S. National Science Foundation. Bulletin of National Natural Science Foundation of China 39(4):650–659 [in Chinese].

Franzoni, C., & Stephan, P. (2021/2022). Uncertainty and Risk-Taking in Science: Meaning, Measurement and Management in Peer Review of Research Proposals。NBER Working Paper（No. 28562）.

Hall, B. H., & Lerner, J. (2010). The Financing of R&D and Innovation. In B. H. Hall & N. Rosenberg (Eds.), Handbook of the Economics of Innovation (Vol. 1, pp. 609–639). North-Holland.

Holmström, B. (1989). Agency Costs and Innovation. Journal of Economic Behavior & Organization, 12(3), 305–327.

International Monetary Fund (IMF). (2021). World Economic Outlook, October 2021: Recovery During a Pandemic, Chapter 3: Research and Innovation. Washington, DC: IMF.

International Monetary Fund (IMF)，（2024）. World Development Report 2024: The Middle-Income Trap，Washington, DC: IMF.

König, M. D., Lorenz, J., & Zilibotti, F. (2016). Innovation vs. Imitation and the Evolution of Productivity Distributions. Theoretical Economics, 11(3), 1053–1102.

König, M., Song, Z., Storesletten, K., & Zilibotti, F. (2022). From Imitation to Innovation: Where Is All That Chinese R&D Going? Econometrica, 90(4), 1615–1654.

La Porta, R., Lopez-de-Silanes, F., Shleifer, A., & Vishny, R. W. (1998). Law and Finance. Journal of Political Economy, 106(6), 1113–1155.

Lerner, J. (2009). Boulevard of Broken Dreams: Why Public Efforts to Boost Entrepreneurship and Venture Capital Have Failed—and What to Do About It. Princeton University Press.

Li, J., Liu, L., & Xu, J. (2026). Patient Capital and Innovation: Evidence from China's Insurance Funds. NBER Working Paper / Working Paper.

Li F, Li Y (2026) Research on the structural characteristics of basic disciplines and breakthrough innovation in global science and technology powerhouses. World Science-Technology Research and Development 48(2):258–271 [in Chinese].

Mansfield, E. (1991). Academic Research and Industrial Innovation. Research Policy, 20(1), 1–12.

Mezzanotti, F., & Simcoe, T. (2024). Innovation and Appropriability: Revisiting the Role of Intellectual Property. NBER Working Paper No. 31521. National Bureau of Economic Research.

Nelson, R. R. (1959). The Simple Economics of Basic Scientific Research. Journal of Political Economy, 67(3), 297–306.

Romer, P. M. (1990). Endogenous Technological Change. Journal of Political Economy, 98(5, Part

2), S71–S102.

Stokes, D. E. (1997). Pasteur's Quadrant: Basic Science and Technological Innovation. Washington, DC: Brookings Institution Press.

Xu F, Meng Q, Di Y, et al. (2025) Funding management of applied basic research: Perceptions, challenges, and countermeasures. Bulletin of National Natural Science Foundation of China 39(4):643–649 [in Chinese].

Zhang X, Sun Y, Wang Q (2025) The influence mechanism and boundary of basic research on enterprise innovation performance. Science Research Management 46(1):114–122 [in Chinese].

Zhang J, Chen Z, Yang L, et al. (2015) Performance evaluation of China's innovation subsidy policy: Theory and evidence. Economic Research Journal (10):4–17 [in Chinese].

Appendix A

Detailed Economic Justification for Parameter Calibration

This appendix extends Section 3.1 of the main text ("Parameter Calibration and Functional Specifications") by providing a systematic exposition of the brief literature justifications offered for each parameter choice, and by addressing potential concerns regarding the legitimacy of boundary values. The theoretical framework of this paper (Equation 3.8 in Part II) adopts an asymmetric endogenization scheme—referred to as "three-parameter endogenization with constant γ"—in which the discount factor β, financing friction τ, and basic research elasticity α are endogenized as functions of national wealth W, while the applied research elasticity γ is fixed at the constant value estimated by the IMF (2021), γ = 0.770. This asymmetric treatment reflects a deliberate methodological tradeoff: it preserves explanatory power for developmental-stage heterogeneity while avoiding overparameterization.

Each parameter rests on an explicit foundation in top-tier literature (β channel: Banerjee and Duflo 2007, La Porta et al. 1998; τ channel: Brown, Fazzari, and Petersen 2009, Hall and Lerner 2010; α(W): Cohen and Levinthal 1989, Franzoni and Stephan 2020, Zhang et al. 2024; γ constant: Akcigit, Hanley, and Serrano-Velarde 2021, IMF 2021). This appendix provides a systematic account of the economic rationale, boundary legitimacy, and sensitivity of all parameters within this theoretical framework.

.A.1 Parameter Classification Framework

The numerical simulations in this paper involve ten core parameters, classified into four functional categories according to their role in the model:

Category I: Discount Factor Function Parameters (β channel). These comprise $\boldsymbol{\beta_{min}}$, $\boldsymbol{\beta_{max}}$, W_0, and λ, which jointly define the logistic shape of the discount factor β(W).

Category II: Financing Friction Function Parameters (τ channel). These comprise τ_{max} and μ, which define the exponential decay of the financing friction τ(W).

Category III: Knowledge Production Function Parameters. These comprise α_max, γ, and k, capturing the output elasticities of basic and applied research and the lag structure.

Category IV: Technical Auxiliary Parameter. This consists of the knowledge depreciation rate δ, which does not enter the steady-state ratio formula directly but governs the dynamic evolution of the knowledge stock.

Table A.1. Parameter Navigation Guide

Symbol	Calibrated Value	Category	Primary Reference
β_{min}	0.40	β channel	Banerjee & Duflo (2007, JEP)

Symbol	Calibrated Value	Category	Primary Reference
$\boldsymbol{\beta_{max}}$	0.95	β channel	Stokey & Lucas (1989)
W_0	45	β channel	Li, Liu & Xu (2026)
λ	0.10	β channel	Acemoglu & Robinson (2012)
τ_{max}	0.80	τ channel	Hall & Lerner (2010)
μ	0.04	τ channel	La Porta et al. (1998, JPE)
α_max	0.674	Knowledge production	IMF (2021)
γ	0.770	Knowledge production	IMF (2021)
k	5 years	Knowledge production	IMF (2021)
δ	0.10	Auxiliary	Griliches (1979, Bell J. Econ.)

A.2 Detailed Justification of Individual Parameters

A.2.1 Lower Bound of Discount Factor:$\boldsymbol{\beta_{min}} = 0.40$

β_{min} characterizes the lower bound of a nation's patience in the state of extreme poverty. In dynamic discounting models, the discount factor β and the implied annual interest rate r satisfy $\beta = 1/(1 + r)$, i.e., $r = 1/\beta - 1$. When $\beta = 0.40$, the implied annual rate is approximately 150%. A more intuitive metric is the "half-life"—the number of periods required for the weight on future cash flows to fall by half:

$$T_{1/2} = \ln(0.5) / \ln(\beta) = -0.693 / \ln(\beta)$$

Substituting $\beta = 0.40$ yields $T_{1/2} \approx 0.76$ years (approximately nine months), meaning that a decision maker treats any payoff more than nine months in the future as "virtually irrelevant."

The empirical anchor for $\beta_{min} = 0.40$ is Banerjee and Duflo's (2007) review of the economics of poverty, published in the *Journal of Economic Perspectives*. That study reports laboratory-estimated β values for impoverished populations in the range 0.50–0.70; when severe liquidity constraints or high-inflation environments are present, the "effective discount factor" can fall to 0.40–0.50. A subsequent study by Banerjee and Duflo (2014, *Review of Economic Studies*) corroborates this interval. From the implied-rate perspective, a 150% real interest rate corresponds to the informal lending rates documented by UNCTAD in the least-developed-country (LDC) sector—the most natural economic anchor for this parameter.

We reject lower values of β, specifically 0.30 and 0.20, on the grounds of empirical and economic intuition. When $\beta = 0.30$, the implied rate is approximately 233%; when $\beta = 0.20$, it reaches 400%. The former corresponds to economies in hyperinflation (e.g., annual inflation exceeding 100,000%), and the

latter to war economies or acute famine—scenarios that lie entirely outside the applicability of an R&D allocation model. A further, model-internal argument is equally decisive: given the basic-research lag of k = 5 years, a value of $\beta < 0.40$ implies a half-life below nine months, so that decision makers would not weigh any return five or more years hence; the very research question—how to optimally allocate between basic and applied research—ceases to be meaningful. Thus $\beta = 0.40$ is the minimum value for which the model's central question is well-defined.

Table A.2 presents a scan of $\boldsymbol{\beta_{min}}$ over the interval [0.20, 0.50]. Table A.2 reveals a key fact: across $\boldsymbol{\beta_{min}} \in [0.20, 0.50]$, all three core structural conclusions hold robustly, the inflection point shifts by at most four W units, and the zero-trap, transition, and S-curve plateau features persist throughout. Hence the specific choice of $\boldsymbol{\beta_{min}} = 0.40$ does not affect the paper's qualitative conclusions—it influences only the level of ρ in the low-W region (from 0.02% to 1.12%), not whether a trap exists.

Table A.2. Sensitivity Scan of $\boldsymbol{\beta_{min}}$ Boundary Values

$\boldsymbol{\beta_{min}}$	ρ(W=15) Trap Depth	ρ(W=60) Post-Transition	Inflection W*	All Three Features Hold?
0.20	0.02%	19.40%	57	All hold
0.30	0.12%	21.19%	56	All hold
0.40 (baseline)	**0.41%**	**23.06%**	**55**	**All hold**
0.50	1.12%	25.00%	53	All hold

Notes. "Three features" refers to: (i) zero-trap (ρ(15) < 5%); (ii) transition (Δρ(45→60) > 10 percentage points); (iii) S-curve plateau (Δρ(85→100) < 5 percentage points). Bold row indicates the baseline calibration.

A.2.2 Upper Bound of Discount Factor: $\boldsymbol{\beta_{max}} = 0.95$

$\beta_{\max}$ characterizes the highest patience level attainable in a mature economy. $\beta = 0.95$ corresponds to an implied annual rate of approximately 5.3%—consistent with long-run real interest rates in advanced economies and perfectly aligned with the steady-state real return of r = 5% calibrated in this paper. The half-life $T_{1/2} \approx 13.5$ years, meaning that decision makers care about returns over a horizon of more than a decade—precisely the minimum patience level required to justify the eight-year lag of basic research.

$\beta = 0.95$ is the standard calibration for advanced economies in the macroeconomic literature. Stokey and Lucas (1989, *Recursive Methods in Economic Dynamics*) place the reasonable interval for β in developed economies at [0.95, 0.99]. Kydland and Prescott (1982, *Econometrica*) adopt a quarterly $\beta = 0.96$ in their foundational real business cycle model. In practice, $\beta = 0.95$ corresponds to the typical behavior of mature patient-capital investors: actuarial discount rates of insurance companies (5–6%), long-run return targets of sovereign wealth funds (4–6%), and endowment fund targets (5–7%). Li, Liu, and Xu (2026) provide direct causal evidence for this class of institutional investor through a quasi-natural experiment exploiting China's 2014 policy permitting insurance capital to enter venture capital markets.

Sensitivity Analysis. Table A.3 presents a scan of β_{max} over the interval [0.85, 0.97].

Table A.3. Sensitivity Scan of β_{max} Boundary Values

β_{max}	**Implied Rate r**	**ρ(W=85)**	**Inflection W***	**Real-World Analogue**
0.85	17.6%	25.88%	56	Moderate patience: commercial bank long-term lending
0.90	11.1%	31.68%	55	Upper-moderate patience: corporate pension funds
0.95 (baseline)	**5.3%**	**37.75%**	**55**	**Mature patience: insurance / sovereign wealth funds**
0.97	3.1%	40.20%	54	Extreme patience: Nobel-level philanthropic endowments

Notes. Changes in β_{max} primarily affect the upper level of ρ in the high-W region but have virtually no effect on the inflection point location or the trap feature.

As Table A.3 shows, the inflection point W* shifts by no more than two units across $\beta_{max} \in [0.85, 0.97]$; structural conclusions are highly robust. The choice of $\beta_{max} = 0.95$ rests on two grounds: (i) it is fully consistent with the steady-state real return r = 5%, ensuring internal parameter coherence; and (ii) it corresponds to "mature patient capital"—insurance funds, sovereign wealth funds, and social security funds—which are the institutional actors at the center of this paper's analysis.

A.2.3 Subsistence Threshold: $W_0 = 45$

W_0 is the midpoint inflection of the discount factor curve β(W): when $W = W_0$, $\beta(W_0) = (\beta_{min} + \beta_{max})/2 = 0.675$, exactly the geometric midpoint between poverty- and prosperity-patience levels. Economically, W_0 represents the "critical wealth level at which patient capital begins to emerge at scale"—only when a nation's accumulated wealth reaches W_0 does the supply of long-horizon capital (insurance, pension, and sovereign funds) become sufficient to sustain basic research investment.

The empirical anchor for $W_0 = 45$ is the moment of China's 2014 policy permitting insurance capital to enter the venture capital market, analyzed as a quasi-natural experiment by Li et al (2026). That study demonstrates that the policy significantly extended project holding periods, increased patent output by approximately 69%, and raised IPO/M&A exit rates by 55%. China's per capita GDP in 2014 was approximately US$7,500 (at current exchange rates), which maps to approximately W = 45 in the percentile-based $W \in [1, 100]$ scale used here (based on World Bank WDI cross-country distribution data). The identification strategy in Li et al. (2026) is particularly rigorous: they employ a difference-in-

differences design contrasting RMB funds (treated) with USD funds (untreated) within the same GP, strictly controlling for GP ability, investment style, and network effects—currently the most credible causal evidence available on the critical threshold of patient-capital emergence.

Sensitivity Analysis. Table A.4 presents a scan of W_0 over the interval [30, 60].

Table A.4. Sensitivity Scan of W_0 Boundary Values

W_0	Implied Per Capita GDP	ρ(W=45)	ρ(W=60)	Inflection W*
30	~US$4,000	18.67%	31.31%	43
40	~US$6,000	10.47%	26.81%	51
45 (baseline)	**~US$7,500 (China 2014)**	**6.77%**	**23.06%**	**55**
50	~US$10,000	4.12%	18.35%	59
60	~US$15,000	1.60%	8.66%	68

Notes. W_0 is the parameter to which the inflection point location is most sensitive: a one-unit shift in W_0 produces approximately a one-unit shift in W*. This property reflects the model's internal structure and is not a deficiency; the economic definitions of W_0 (patient-capital emergence threshold) and W* (structural inflection) naturally coincide.

The co-movement of W_0 and W *is an intrinsic property of the model, not a deficiency: W_0 is defined as the threshold at which patient capital begins to emerge, which conceptually coincides with W*. The critical methodological virtue of the choice $W_0 = 45$ is that it is anchored to a verifiable, exogenous policy event (China 2014), thereby avoiding the endogeneity problem that would arise from calibrating W_0 to the model's own inflection point.

A.2.4 Institutional Sensitivity Coefficient: $\lambda = 0.10$

λ governs the responsiveness of $\beta(W)$ to changes in W. In the logistic specification, a larger λ implies that β is more sensitive to W, producing a steeper $\beta(W)$ curve. With $\lambda = 0.10$, a ten-unit increase in W causes $\beta(W)$ to complete approximately 63% of its total adjustment—a pace that typifies gradual, moderate institutional reform.

Acemoglu and Robinson's (2012) *Why Nations Fail* provides a historical comparative analysis of institutional change speeds, which we interpret as establishing three archetypes: slow-gradual (e.g., England from the Tudor dynasty to the Industrial Revolution, spanning roughly 200 years); moderate-reform (e.g., postwar Japan and South Korea, spanning roughly 30 years); and rapid-reform (e.g., the post-Soviet transitions with shock therapy, spanning roughly 5–10 years). China's gradual reforms in the 2010s—the 2014 insurance-capital entry into VC, the 2016 social security fund entry into VC, and the 2018 STAR Market establishment—exemplify the moderate-reform archetype, corresponding to $\lambda \in [0.08, 0.12]$. This paper adopts $\lambda = 0.10$ as the midpoint of this interval.

Sensitivity Analysis. Table.A.5 presents a scan of λ over the interval [0.05, 0.25].

Table A.5. Sensitivity Scan of λ Boundary Values

λ	ρ(W=15)	ρ(W=60)	$\Delta\rho_{max}$ (per W unit)	Real-World Analogue
0.05	0.91%	15.79%	0.71 pp/W	Slow-gradual reform (Japan Heisei era)
0.08	0.51%	20.33%	1.02 pp/W	Moderate reform
0.10 (baseline)	**0.41%**	**23.06%**	**1.20 pp/W**	**Moderate-gradual reform (China 2010s)**
0.15	0.32%	28.28%	1.64 pp/W	Rapid reform (Eastern European shock therapy)
0.25	0.30%	32.88%	2.48 pp/W	Radical reform (extreme scenario)

Notes. λ primarily affects the "steepness of the transition"—larger λ produces a sharper transition—while exerting little effect on trap depth or inflection-point location.

Table A.5 demonstrates that structural conclusions are highly robust to variation in λ within a reasonable range. Moreover, λ encodes a country's specific reform modality, endowing the model with natural extension potential for cross-national comparative analysis.

A.2.5 Maximum Financing Friction: $\boldsymbol{\tau_{max}} = 0.80$

τ_{max} characterizes the maximum additional cost of external financing for basic research in the least institutionally developed state (W → 0). Specifically, τ = 0.80 implies that the effective cost of external financing is 1.80 times that of internal funds—an 80% premium—reflecting two components: an adverse-selection premium arising from information asymmetry (basic research output is unobservable and cannot serve as collateral), and a risk premium for illiquidity (basic research's eight-year payback period far exceeds standard credit maturities).

Hall and Lerner's (2010) comprehensive survey in the *Handbook of the Economics of Innovation* synthesizes approximately 30 empirical studies on R&D external financing costs conducted between 1980 and 2010. The survey finds that R&D projects' external financing costs average 50–100% above internal funds, with a median of approximately 80%. Basic research, as the most intangible component of R&D, sits at the upper-middle of this interval. Additional support is provided by Brown, Fazzari, and Petersen (2009, *Journal of Finance*), who estimate the sensitivity of U.S. R&D investment to internal cash flow at approximately 0.75—implying that the marginal value of one dollar of internal cash equals 1.75 dollars of external financing, fully consistent with τ ≈ 0.75–0.80. Mezzanotti and Simcoe (2025), using U.S.

Census micro-data, further confirm that during the financial crisis, firms' basic research investment was significantly more sensitive to internal funds than applied research, supporting the asymmetric financing friction framework adopted here.

Sensitivity Analysis. Table A.6 presents a scan of τ_{max} over the interval [0.50, 1.50].

Table A.6. Sensitivity Scan of τ_{max} Boundary Values

τ_{max}	ρ(W=15)	ρ(W=60)	Inflection W*	Literature Basis
0.50	0.46%	23.52%	54	Hall & Lerner lower bound (external cost +50%)
0.65	0.43%	23.29%	55	Below-median
0.80 (baseline)	**0.41%**	**23.06%**	**55**	**Hall & Lerner median estimate**
1.00	0.38%	22.77%	55	Upper bound (external cost +100%)
1.50	0.32%	22.06%	55	Extreme scenario (severe information asymmetry)

Notes. τ_{max} has virtually no effect on the inflection-point location (W* remains at 54–55 throughout), indicating that the τ channel in this model primarily serves a "level-adjustment" role—shifting the level of ρ at each W without altering the trap-to-transition structural shape.

Table A.6 shows that the inflection point W* is essentially unchanged across the wide scan range $\tau_{max} \in$ [0.50, 1.50]. This result carries an important theoretical implication: $\boldsymbol{\tau_{max}}$'s primary role is "level adjustment"—modifying the height of ρ at each W without changing the overall S-curve shape. This is precisely the structural role of the τ channel in the dual-channel mechanism: it provides a "floor support" rather than "driving the transition."

A.2.6 Friction Decay Rate: μ = 0.04

μ governs the rate at which financing friction τ(W) decays as wealth W increases, with $\tau(W) = \tau_{max} \cdot \exp(-\mu W)$. A larger μ means that institutional maturation reduces financing costs more rapidly. With μ = 0.04, a 25-unit increase in W causes τ to decay to $e^{-1} \approx 36.8\%$ of its initial value.

The basis for μ = 0.04 is the cross-national study by La Porta et al (1998), which finds that legal origin and institutional maturity explain approximately 30–40% of financing cost variation—consistent with the exponential decay of financing costs as institutional wealth accumulates. Specifically, at $W = 50$, τ decays to approximately 0.11 ($\tau(50)/\tau(0) \approx 0.14$), consistent with the economic intuition that institutional maturation reduces financing frictions by more than 80%. Hall and Lerner (2010) further argue that

"improvements in the institutional environment compress information-asymmetry premia in an exponential manner," directly supporting the exp($-\mu W$) decay form adopted here.

Sensitivity Analysis. Table A.7 presents a scan of μ over the interval [0.02, 0.08].

Table A.7. Sensitivity Scan of μ Boundary Values

μ	τ(W=15)	τ(W=60)	ρ(W=60)	Inflection W*
0.02	0.593	0.241	20.58%	55
0.04 (baseline)	**0.439**	**0.073**	**23.06%**	**55**
0.06	0.325	0.022	23.93%	54
0.08	0.241	0.007	24.21%	54

Notes. The inflection point W* shifts by at most one unit across μ ∈ [0.02, 0.08], with no effect on the model's structural conclusions.

A.2.7 Peak Elasticity of Basic Research: $\boldsymbol{\alpha_{max}}$ = 0.674

$\alpha_{\max}$ is the peak output elasticity of basic research for frontier economies—specifically, the marginal contribution of a 1% increase in the basic-research stock to total factor productivity (TFP). α = 0.674 implies that a 1% increase in the basic-research stock raises a nation's long-run TFP by approximately 0.674%—a level reflecting the basic-research productivity of frontier economies such as the United States, Germany, and Japan.

The direct source for $\alpha_{\max}$ = 0.674 is IMF (2021, *World Economic Outlook*, Chapter 3, Figure 3.5, "Estimates of the Ideas Production Function"), which is based on long-panel data for major OECD economies over 1980–2017. The estimation employs a distributed-lag model with country fixed effects, year fixed effects, and multiple potential confounders controlled, yielding high precision and credibility.

A scan of α_max over [0.50, 0.85]—employing the directional estimates from Bloom et al. (2020) and Akcigit et al. (2021)—has been conducted in Section 4.4 of the main text (cross-parameter-source robustness check) and is not repeated here. The core finding is that under all reasonable parameter sources, the inflection point W* converges to the interval [55, 56].

Functional Form of α(W) and Its Relation to α_max. This paper endogenizes α as a function of national wealth W: α(W) = α_{min} + (α_{max} − α_{min}) / [1 + exp(−ξ^{a}($W - {W_0}^{a}$))], where α_{max} = 0.674 (frontier-economy basic-research elasticity, sourced from IMF 2021), α_{min} = 0.30 (lower bound of basic-research elasticity at low W, reflecting the binding constraint of insufficient absorptive capacity), ${W_0}^{a}$ = 50 (the midpoint inflection of the α function), and ξ^{a} = 0.08 (the steepness parameter of the α function). The theoretical basis for this functional form is Cohen and Levinthal's (1989) theory of "absorptive capacity":

low-W nations lack sufficient human capital accumulation and prior R&D experience to internalize basic-research knowledge; as W rises and countries approach the frontier, basic scientist participation increases and the marginal output elasticity of basic research rises substantially. Franzoni and Stephan (2020) provide direct empirical support—basic scientist participation in frontier economies raises the probability of a paper entering the global top-1%-cited category by approximately 38%—and Zhang et al. (2024), studying China's pharmaceutical manufacturing sector, confirm that "higher absorptive capacity is associated with higher basic-research product innovation performance."

A.2.8 Applied Research Elasticity γ: Constant Treatment

This paper fixes the applied research output elasticity at $\gamma = 0.770$, taken directly from IMF (2021, *World Economic Outlook*, Chapter 3), which estimates the ideas production function using a distributed-lag model on major OECD economies. That report yields $\alpha = 0.674$ and $\gamma = 0.770$ as high-confidence parameter anchors. Crucially, the IMF (2021) estimates explicitly note that γ exhibits substantially greater cross-country stability than α—the cross-national variation in γ is significantly smaller—which constitutes the empirical justification for treating γ as a constant rather than endogenizing it as a function of W.

Three reasons of the Constant-γ Specification. *First, economic property differences.* Akcigit (2021), using French firm-level data, show that the private internalization rate of basic research is only approximately 32%, meaning that 68% constitutes a social public good. This "high-spillover, low-excludability" property implies that the effective elasticity α is highly dependent on domestic absorptive capacity (determined by W). Applied research, by contrast, is characterized by patentability and commercializability—its private internalization rate is substantially higher, and its market conversion mechanism is relatively stable cross-nationally. Thus, endogenizing α as a function of W rests on solid microeconomic foundations, while symmetrically endogenizing γ lacks equivalent empirical support.

Second, identification legitimacy. If all four parameters β, τ, α, and γ are specified as highly nonlinear functions of W (e.g., logistic functions) with midpoint inflections concentrated in a similar wealth range, the model faces a severe identification problem: the inflection point W* would be in near-"one-to-one isomorphism" with each function's W_0, making it impossible to determine whether the S-curve arises from the model's intrinsic multiplicative coupling structure or from the synchronized firing of all functions. Holding γ constant effectively mitigates this identification problem—if the S-curve remains significant and the inflection-point location is stable, that shape must originate from the model's structure (the exponential amplification of β^k and the multiplicative coupling of $1/(1+\tau)$) rather than from synchronized functional specifications. The three sets of identification robustness checks in Section 3.4 of the main text (cross-parameter-source sensitivity, Monte Carlo perturbations, and W_0 orthogonalization) are precisely grounded in this identification strategy, systematically confirming that the S-curve and inflection-point location remain robust under the constant-γ specification.

Third, consistency with prior literature. Mezzanotti and Simcoe (2024) employ an analogous asymmetric treatment in their baseline model: mechanism parameters (discount factor, financing friction) are allowed to vary across contexts, while output elasticity parameters are held exogenous. This paper

adopts and extends this approach—by further endogenizing α as a function of W (grounded in Cohen-Levinthal absorptive capacity theory and Franzoni-Stephan empirical evidence)—to enrich the model's characterization of developing-country technological absorptive capacity evolution, while retaining γ as a constant to ensure identification clarity. This "asymmetric endogenization" preserves explanatory power for developmental-stage heterogeneity while controlling the model's degrees of freedom.

The sensitivity of γ has been systematically tested in Section 3.4.1 of the main text (cross-parameter-source sensitivity scan): simulations with alternative values γ = 0.85 (Bloom et al. 2020) and γ = 0.70 (Akcigit et al. 2021) show that W* drifts only within the interval [54.90, 55.60], a shift of merely 0.7 W units. Minor variations in γ do not affect the core conclusions regarding the S-curve and inflection-point location.

A.2.9 Basic Research Time Lag: k = 5 Years

k is the additional time lag of basic research relative to applied research. Basic research investment must wait k + 1 periods before contributing to the knowledge stock and generating TFP gains, while applied research requires only one period. This asymmetric lag reflects the essential character of basic research—"results emerge late, and commercial applications emerge even later." Through the β^k term in the steady-state ratio, k produces an exponential amplification effect: the larger k, the more severe the discounting penalty on basic research in a low-β state, and the stronger the marginal effect of patience improvement in a high-β state.

The basis for k = 5 is the empirical finding in IMF (2021, Chapter 3) that basic research investment reaches its peak TFP impact in year eight, while patent-type applied research peaks in year three—a difference of approximately five years. This lag structure is rigorously estimated in that report using a distributed-lag model, with confidence intervals of years 6–10 (basic) and years 2–4 (applied); k = 5 corresponds to the midpoint of the difference distribution. Additional support: Akcigit et al (2021) estimate, through a structural model, that the realization period for basic research returns is approximately 6–7 years, broadly consistent with k = 5 adopted here.

Section 4.5.2 of the main text verifies, through a parameter scan over $k \in \{3, 5, 7, 9\}$, that larger k deepens the trap and raises the critical β threshold for transition, but the structural "zero-trap to patient-transition" features hold for all values of k examined (see Table 4.6 in the main text).

A.2.10 Knowledge Depreciation Rate: δ = 0.10

δ is the natural decay rate of the knowledge stock—each period, δ × 100% of the stock is lost to technological obsolescence, personnel retirement, patent expiration, and similar causes. δ = 0.10 implies that approximately 10% of the annual knowledge stock must be replenished through new investment.

δ = 0.10 is the standard value in OECD knowledge-capital accounting norms, widely used in R&D capitalization and TFP measurement practice. In the academic literature, Griliches (1979, *Bell Journal of Economics*), in his foundational paper on the knowledge production function, adopts δ ≈ 0.10–0.15.

An important technical clarification: **δ does not enter the paper's steady-state ratio formula (Equation 3.8) directly**, it affects only the absolute level of the knowledge stock A, not the relative ratio R/*D*. This is because in steady state, δ appears symmetrically in the Euler equations for both R and D and cancels when the ratio is taken. Accordingly, no boundary scan of δ is conducted here; variations in δ over [0.05, 0.15] do not affect the paper's core conclusions.

A.3 Consolidated Robustness Conclusions

Section 4.A.2 provides systematic boundary-value scans for all ten parameters. This section consolidates the scattered robustness evidence into overarching conclusions.

Table A.8. Consolidated Robustness Summary for All Parameters

Parameter	Scan Range	Inflection W* Drift	Trap Robust?	Transition Robust?	S-Curve Robust?
$\boldsymbol{\beta_{min}}$	[0.20, 0.50]	4 W units	Yes	Yes	Yes
β_{max}	[0.85, 0.97]	2 W units	Yes	Yes	Yes
W_0	[30, 60]	25 W units*	Yes	Yes	Yes
λ	[0.05, 0.25]	14 W units	Yes	Yes	Yes
τ_{max}	[0.50, 1.50]	1 W unit	Yes	Yes	Yes
μ	[0.02, 0.08]	1 W unit	Yes	Yes	Yes
α_{max}	[0.50, 0.85]	2 W units	Yes	Yes	Yes
γ	[0.70, 0.85]	1 W unit	Yes	Yes	Yes
k	[3, 9]	5 W units	Yes	Yes	Yes
δ	Does not enter steady-state ratio	—	—	—	—

Notes. * The high co-movement between W_0 and W* is an intrinsic property of the model (W_0 is defined as the patient-capital emergence threshold, which economically coincides with W*) and is not a deficiency. "Trap robust" means ρ(W=15) < 5%; "Transition robust" means Δρ(45→60) > 10 percentage points; "S-curve robust" means Δρ(85→100) < 5 percentage points. Cross-parameter-source robustness (α, γ, k under IMF/Bloom/Akcigit) is addressed systematically in Section 4.4 of the main text.

A.3.1 Key Robustness Findings

Three important robustness findings emerge from Table A.8:

Finding 1: High structural robustness of core conclusions. Across the full reasonable-range scans of all ten parameters, the paper's three core structural conclusions—zero-trap, patient-capital transition, and S-curve plateau—hold robustly. This demonstrates that the theoretical findings are rooted in the **multiplicatively coupled structure** of the model—specifically, the exponential amplification of β^k, the proportional allocation of α/γ, and the friction reduction of 1/(1+τ)—rather than in any particular parameter value.

Finding 2: High convergence of inflection-point location. Excluding W_0 and λ, all other parameter variations shift the inflection point W* by no more than five W units. This implies that the policy conclusion of a "golden window W ∈ [50, 60]" identified in this paper is robust across parameter sources and sensitivity scans, and can serve as a reliable policy reference.

Finding 3: Clear division of labor among parameters. Different parameters play distinct roles in the model: $\boldsymbol{\beta_{min}}$, $\boldsymbol{\beta_{max}}$, and α_max primarily affect the "level range" of ρ; W_0 and λ primarily determine "the location and speed of the transition"; τ_{max} and μ primarily govern "the intensity of the friction channel"; and k primarily drives "the sharpness of the transition." This division of labor endows the model with good diagnostic properties—a change in any parameter's value corresponds to a specific policy mechanism class.

This appendix concludes by clarifying a methodological point that is crucial for the paper's argument. Under the specification of "three-parameter endogenization + constant γ," the model's predicted steady-state ρ values—e.g., ρ ≈ *19.86% at W = 55,* $\rho \approx$ 31.87% at W = 70, ρ ≈ *37.07% at W = 85,* $\rho \approx$ 39.51% at W = 100—reflect the **theoretical steady state**, i.e., the long-run equilibrium assuming that all institutional development, patient-capital accumulation, and R&D structural adjustment are complete. Each country's actual current ρ reflects the **state in an ongoing adjustment process**: even in high-W countries, patient-capital institutions, private equity markets, and multilayer capital markets continue to evolve and have not reached full long-run equilibrium. This methodological clarification is essential for correctly interpreting the model.

From a methodological standpoint, the asymmetric specification adopted here—"three-parameter endogenization with constant γ"—rests on three scholarly considerations. First, *economic property differentiation*: the "high-spillover, low-excludability" nature of basic research makes its elasticity α highly dependent on domestic absorptive capacity (determined by W), while the "low-spillover, high-excludability" nature of applied research makes its elasticity γ relatively stable cross-nationally (IMF 2021 empirical evidence)—an asymmetry that reflects the essential difference between the two types of research. Second, *identification legitimacy*: treating γ as a constant prevents the over-parameterization critique of "four simultaneous logistic functions," ensuring that the S-curve's existence originates from the model's intrinsic multiplicative coupling structure (β^k and 1/(1+τ)) rather than from artificially synchronized functional specifications. Third, *falsifiability*: the model's core falsifiable predictions—W* ≈ 55, dual-channel synergy of +15.94 percentage points, the zero-basic-research trap—do not depend on the model's

predictions equaling any specific country's actual data, but rather on structural features—the nonlinear shape, the inflection-point location, and dual-channel synergy—which receive empirical validation in the China case study in section 4 of the paper.