Growth, Volatility, Imperfect Implementation and Technological Mismatch*

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Abstract

This paper introduces a three-sector Schumpeterian-style growth model that simultaneously endogenizes balanced-growth-path growth and growth volatility. This model is developed on which the imperfect implementation of ideas generates endogenous growth volatility. Building on the standard distinction between invention, innovation, diffusion, and implementation, the paper studies how stochastic implementation success, technological mismatch, and queues of unimplemented ideas jointly determine both balanced-growth-path growth and growth volatility. While idea and technology creation sustains long-run growth, it also generates Schumpeterian waves that produce volatility. At the same time, by the conditional complementarity between the creation and implementation sectors of idea and technology, the higher balanced growth driven by greater research labor also generates an opposing force that reduces volatility through more effective idea adoption. Empirical evidence suggests a robust negative relationship between growth volatility and growth. The model further predicts that, along the transitional dynamics, a technological breakthrough initially induces a slowdown in growth, followed by transitional acceleration and a higher balanced-growth path.

Keywords: Schumpeterian growth; Growth volatility; Idea implementation; Technology adoption; Mismatch; Endogenous growth.

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1 Introduction

As a long-standing convention in economics, short-run volatility and long-run growth are often analyzed and modeled separately. Lucas (1987) argued that the potential welfare gains from reducing business-cycle fluctuations are small compared with the welfare gains from increasing long-run growth. This claim reflects his view that volatility is of second-order importance and implicitly suggests that growth and volatility can be treated as separable.

However, this view is challenged as the emerging empirical literature has begun to shed light on the relationship between growth and volatility. Arguably among the earliest empirical contributions, motivated by Black's (1979) hypothesis concerning a risk-return trade-off in aggregate technology, Kormendi and Meguire (1985) ran a cross-country growth regression on the standard deviation of the growth rate for 47 countries over the period 1950-1977, and found that growth was positively associated with volatility. In subsequent literature, Grier and Tullock (1989) reported preliminary evidence that supports a positive relationship between volatility and growth for 24 OECD countries and 89 rest-of-the-world (ROW) countries using similar methods as Kormendi and Meguire (1985). Nevertheless, when the population-weighted ROW group was further divided into Africa, Americas and Asia, the positive growth-volatility correlation vanished for Africa and the Americas, and even turns negative for Asia. Several empirical studies document a negative association between macroeconomic volatility and long-run growth, although the relationship appears heterogeneous across countries and levels of development (Ramey and Ramey, 1995; Fatás, 2002; Martin and Rogers, 2000; Siegler, 2005). Taken together, this literature provides strong evidence to support that growth and volatility are related, and the conflicting results suggest that the nature of this relationship might be more complex than a simple correlation.

In neoclassical and endogenous growth literature, models such as Solow (1956) and Romer (1990) focus on long-run dynamics and deliberately abstract from short-run business-cycle

volatility, generating smooth growth paths along balanced growth paths. Conversely, real business cycle models (e.g. Kydland and Prescott, 1982; King et al., 1988) successfully account for short-run business-cycle fluctuations, but attribute them to exogenous technological shock which moves around an exogenously given growth path, leaving growth-cycle interactions largely unexplored. The Schumpeterian growth framework by Aghion and Howitt (1992) endogenizes innovation through creative destruction, in which the economy's average growth rate and its variance are positively related to the amount of research labor. However, this implication is only partially consistent with the aforementioned empirical studies. In fact, early attempts to jointly model growth and fluctuations include Smithies (1957), who explains both through the dynamic interaction between aggregate demand and productive capacity. His framework shows that growth itself can generate cyclical fluctuations when demand and productive capacity adjust at different speeds. However, the model relies largely on behavioral postulates and lacks micro-foundations.

The literature on general-purpose technologies (GPTs) provides a natural bridge between long-run growth and short-run fluctuations. Bresnahan and Trajtenberg (1995) characterize GPTs as technologies that are pervasive, improve over time, and generate complementarities with downstream innovation. In this sense, major technologies do not raise aggregate productivity merely because they are invented. Their productivity effects depend on whether firms, sectors, and institutions can re-organize around them. However, canonical endogenous-growth models such as Romer (1990) and Aghion and Howitt (1992) typically abstract from this implementation margin by treating newly created ideas or technologies as immediately productive once invented. This assumption is analytically convenient, but it does not capture the empirical pattern that major technologies often diffuse gradually and require complementary adjustment before their productivity effects are fully realized.

This view is closely related to David (1990), who argues that the productivity gains from the

electric dynamo were delayed because factories had to be reorganized before electricity could be fully exploited. Similarly, Nelson and Phelps (1966) emphasize that the implementation of new technologies depends on the ability of an economy to absorb and adopt them through complementary education and human capital. A related strand of the GPT literature emphasizes that adoption is gradual and can itself generate transitional dynamics. Helpman and Trajtenberg (1998) show that the gradual diffusion of a GPT across heterogeneous user sectors can generate two-phased cycles before sustained growth emerges. Jovanovic and Rousseau (2005) document related historical patterns for electricity and information technology, where the aggregate gains from major technologies appear only after a period of adjustment and diffusion. More generally, Hall and Khan (2003) argue that the contribution of a new technology to economic growth is realized only when the technology is widely diffused and used. Consistent with this view, Comin and Hobijn (2010) provide cross-country evidence that the adoption of major technologies often takes many years, and sometimes several decades.

This diffusion-based view also helps explain why major technological improvements may initially coincide with weak measured productivity. Greenwood and Yorukoglu (1997) interpret the post-1970s productivity slowdown as part of an information-technology-driven transformation that required learning and complementary adjustment. Hornstein and Krusell (1996) similarly show that improvements in technology can be associated with temporary productivity slowdowns. More recently, Basu and Fernald (2007) find that the productivity effects of information and communications technology are consistent with GPT mechanisms, where productivity gains in using sectors appear only with long lags. Brynjolfsson et al. (2021) formalize this idea as a productivity J-curve: early GPT adoption requires intangible complementary investment, which may depress measured productivity before later gains are realized. Firm-level evidence also supports this complementarity view, as Bresnahan and Greenstein (1996), Bresnahan et al. (2002), and Bloom et al. (2012) show that the pro-

ductivity effects of information technology depend on co-invention, workplace organization, management practices, and skilled labor.

Taken together, this literature shows that implementation, diffusion, and complementary adjustment are not peripheral to technological progress. Schumpeterian growth theory emphasises innovation and creative destruction; the diffusion and GPT literatures further show that new technologies often require organisational change, complementary investment, human capital, and time before their productivity effects are realised. Therefore, the relevant gap is not the mere absence of an idea–implementation distinction. The more specific gap is that implementation frictions are rarely used to derive a tractable growth-volatility mechanism.

This paper addresses that gap by treating implementation as stochastic, mismatch-dependent, and capacity-constrained. The model separates latent technological opportunities from realised productivity growth. Ideas may arrive, but they affect measured productivity only when they are successfully matched with compatible machines, institutions, or organisational structures. This distinction allows implementation frictions to become an endogenous source of volatility rather than a purely transitional delay. In this sense, the paper uses a well-established insight from Schumpeterian and diffusion-based theories of technological change to study a less-developed question: how imperfect implementation jointly shapes balanced-growth-path growth and the volatility of realised growth.

This paper develops a three-sector Schumpeterian-style framework in which imperfect implementation acts as a structural source of growth volatility. The contribution is not to claim that idea creation and implementation are conceptually distinct; that distinction is already central to Schumpeterian, diffusion-based, and general-purpose-technology views of technological change. Rather, the paper studies how this distinction affects the joint determination

of long-run growth and short-run growth volatility. In the model, newly generated ideas do not immediately raise the measured productivity frontier. They first enter a queue of unimplemented ideas, and their eventual contribution to productivity depends on a stochastic implementation process. The probability of successful implementation is governed by machine–idea compatibility and by labour allocated to implementation. Hence measured productivity growth reflects not only the arrival of new ideas, but also the economy's ability to convert latent ideas into realised productivity.

The central mechanism is a two-force relationship between implementation capacity, growth, and volatility. Greater research effort raises the flow of new ideas and therefore supports higher balanced-growth-path growth, but it also increases the scale of implementation events and can generate Schumpeterian waves of realised productivity growth. At the same time, stronger implementation capacity raises the probability that queued ideas are successfully adopted, reducing residual implementation risk and stabilising growth. The model therefore generates a structural link between growth and volatility: the relationship may be positive, negative, or non-monotonic depending on whether the idea-expansion effect or the stabilising implementation effect dominates. The empirical evidence presented below is consistent with the stabilising-implementation channel, as residual growth volatility is negatively associated with average growth in the baseline specifications.

2 The Basic Model

2.1 Production and Idea Accumulation

Time starts at 0 and is assumed to be discrete, so that $t \in \mathbb{N}$. In each period, new ideas are generated. Some are successfully implemented through matching with compatible machines, while others remain unmatched and wait in the queue. Let M_t denote the stock of ideas in the representative economy at period t. For tractability, these ideas are summarized by a

representative type $z_t \in \zeta = (0, 1]$, which captures their technical requirements. Similarly, the implementation side is summarized by a representative machine type $x_t \in \chi = (0, 1]$, which captures the technical capability of the installed machine stock.

The terms "ideas" and "machines" are interpreted broadly. Ideas represent technologies in a general sense, while machines represent the institutional or organizational structures that enable technology to be implemented in practice. The present model adopts the simple language of ideas and machines as a reduced-form way of examining how productive knowledge can be translated into realized output. In this sense, output productivity is rooted in ideas, whereas machines affect output only through implementation.

The representative household supplies labor ($\bar{L}$) inelastically. The labor is allocated across three production departments: final-good department ($L_{Y,t}$), which is used for producing consumption goods; research department ($L_{z,t}$), which is employed to generate ideas; and engineering department ($L_{x,t}$), which is hired to implement the ideas generated by the research department.

$$\bar{L} = L_{Y,t} + L_{z,t} + L_{x,t} = 1 \tag{1}$$

Final goods are produced using a technology that is linear in labor, depending on the compatibility between ideas and machines

$$Y_t = A_t q_t L_{Y,t} \tag{2}$$

where A_t denotes the measured productivity, and q_t is an index of the compatibility between ideas and machines. Its exact definition and functional specification are introduced later.

Following Aghion and Howitt (1992), the arrival of new ideas is assumed to follow a Poisson process

$$N_t^{Arrival} \sim \text{Poisson}(\lambda_t), \quad \lambda_t \geq 0$$

$$\lambda_t = \phi z^{\theta} L_{z,t}^{\nu} A_t^{\bar{\chi}} \tag{3}$$

where ϕ denotes the baseline productivity of idea creation; L_z denotes the labor devoted to the creation of ideas, whose contribution is subject to diminishing marginal returns, i.e. $\nu \in (0,1)$; the contribution of the idea type is also subject to diminishing marginal returns, i.e., $\theta \in (0,1)$; and $A_t^{\bar{\chi}}$ measures the "spread-of-knowledge effect". In particular, $\bar{\chi} > 0$ implies a standing-on-the-shoulders-of-giants effect, $\bar{\chi} < 0$ implies a fishing-out effect, and $\bar{\chi} = 0$ implies that the two forces balance each other.

Idea implementation takes place independently of previous trials, following a binomial (Bernoulli) process. Each idea waiting in the queue is implemented with success rate $s_t \in [0,1]$, of which whose functional specification is introduced below. Conditional on the stock of ideas M_t, the number of successfully implemented ideas in period t is

$$N_t^{impl} | M_t \sim \text{Binomial}(M_t, s_t)$$

After the inflows and outflows of ideas are defined, the expected law of motion of the idea queue can be derived naturally

$$\mathbb{E}_t[M_{t+1}] = (1 - s_t)M_t + \lambda_t \tag{4}$$

2.2 Machine Adaptation, Mismatch, and Compatibility

The type of machines is not randomly drawn. Engineers design machines based on the average types of machines and ideas. The representative machine type follows the partial adjustment rule below

$$x_{t+1} = (1-\rho)x_t + \rho z_t, \quad \rho \in (0,1) \tag{5}$$

where ρ measures the speed of machine adaptation. A larger ρ implies faster adjustment of machines toward the current type of ideas. Conversely, a smaller ρ implies stronger technological inertia.

Define the mismatch gap as

$$\omega_t \equiv z_t - x_t \tag{6}$$

The mismatch gap captures the distance between the technical requirement of ideas and the technical capability of machines. Following the definitions of the mismatch gap and the partial engineering adjustment rule, the mismatch law of motion is derived

$$\omega_{t+1} = (1-\rho)\omega_t + \Delta z_{t+1}, \quad \rho \in (0,1) \tag{7}$$

where $\Delta z_{t+1} \equiv z_{t+1} - z_t$. The mismatch law of motion is a stable first-order autoregressive transition equation.

Proposition 1 *A positive innovation shock raises mismatch on impact, and under the partial machine adjustment rule, the mismatch gap decays gradually over time in the absence of further shocks.*

When the type of ideas shifts upward suddenly, existing machines become less aligned with the frontier, which generates a transient mismatch gap. Since machines only adjust partially

each period, this mismatch does not disappear immediately but instead declines gradually over time. This transitional mismatch is the source of temporary implementation difficulty in this model.

The compatibility between ideas and machines is defined as

$$q_t \equiv q(\omega_t) = e^{-\kappa|\omega_t|^{\psi}} \tag{8}$$

This reduced-form functional specification implies that compatibility declines as the mismatch gap widens. κ governs the overall mismatch severity and ψ controls how sharply compatibility decreases with mismatch.

Embedded in the binomial process of idea implementation, the implementation probability, which is the per-idea success rate of implementation mentioned earlier, is specified as

$$s_t = 1 - e^{-\eta q_t L_{x,t}} \tag{9}$$

The success rate is strictly decreasing with mismatch due to lower compatibility and is strictly increasing with implementation labor.

2.3 Conditional Growth and Volatility

Definition 1 *A balanced growth path (BGP) is a path along which M_t, s_t, q_t, $L_{z,t}$, and $L_{x,t}$ are constant (denoted by the superscript *) and the expected growth rate of productivity is constant.*

Theorem 1 (Conditional growth and volatility) *Suppose productivity evolves according to*

$$A_{t+1} = A_t e^{\gamma N_t^{impl}} \tag{10}$$

Then conditional expected productivity growth is

$$E_t[\Delta \ln A_{t+1} \mid M_t] = \gamma s_t M_t \tag{11}$$

and conditional productivity-growth variance is

$$\mathrm{Var}_t(\Delta \ln A_{t+1} \mid M_t) = \gamma^2 s_t (1 - s_t) M_t \tag{12}$$

Equivalently,

$$\mathrm{Var}_t(\Delta \ln A_{t+1} \mid M_t) = \gamma (1 - s_t) E_t[\Delta \ln A_{t+1} \mid M_t] \tag{13}$$

Holding the implementation success rate fixed, higher expected growth is associated with higher conditional growth volatility. However, holding expected growth fixed, a higher implementation success rate lowers conditional volatility through the term $1 - s_t$. This is the first indication that implementation can simultaneously support growth and stabilize fluctuations.

Corollary 2 (Balanced-growth-path growth and volatility) *On a balanced growth path with a stationary idea queue,*

$$M^* = \frac{\lambda^*}{s^*} \tag{14}$$

If $\bar{\chi} = 0$, then balanced-growth-path expected productivity growth is

$$g_A^{BGP} = \gamma \phi (z^*)^\theta (L_z^*)^\nu \tag{15}$$

and balanced-growth-path productivity-growth variance is

$$\mathrm{Var}^* = \gamma^2 (1 - s^*) \phi (z^*)^\theta (L_z^*)^\nu \tag{16}$$

Once the idea queue is stationary, the long-run growth rate is governed by the arrival rate of new ideas rather than by the stock of queued ideas itself. Research labor L_z^* raises balanced-growth-path growth by increasing idea creation, while implementation success s^* affects balanced-growth-path volatility through the residual failure term $1 - s^*$. Therefore, the model separates the source of long-run growth from the force that smooths growth fluctuations.

Corollary 3 (Knife-edge condition for finite non-zero BGP growth) *Suppose the idea-arrival rate contains the knowledge-stock term $A_t^{\bar{\chi}}$. Holding the stationary allocation fixed, a finite non-zero balanced-growth-path expected growth rate is obtained only in the knife-edge case $\bar{\chi} = 0$. If $\bar{\chi} < 0$, expected growth vanishes asymptotically. If $\bar{\chi} > 0$, expected growth becomes explosive.*

If knowledge accumulation makes future idea creation progressively harder, growth eventually fades out. If accumulated knowledge makes future idea creation progressively easier, growth accelerates without bound. The knife-edge case $\bar{\chi} = 0$ therefore isolates the model's main mechanism: growth-cycle entanglement generated by imperfect implementation rather than by scale effects from the existing knowledge stock.

2.4 Three-sector Labor Allocation

The previous section treats idea creation and idea implementation as given. I now endogenize the allocation of labor across the three departments of the economy. The representative household supplies labor inelastically, and total labor is normalized to one:

$$\bar{L} = L_{Y,t} + L_{z,t} + L_{x,t} = 1$$

where $L_{Y,t}$ denotes final-good labor, $L_{z,t}$ denotes idea-creation labor, and $L_{x,t}$ denotes idea-implementation labor.

The final-good sector is perfectly competitive and produces according to

$$Y_t = A_t q_t L_{Y,t}$$

Therefore, the normalized marginal product of final-good labor is

$$\bar{w}_{Y,t} \equiv \frac{w_{Y,t}}{A_t} = q_t$$

This condition pins down the payoff to labor in the final-good sector. The remaining labor-market conditions are determined by the private incentives of the innovative firm that creates and implements ideas.

2.4.1 Labor allocation in a decentralized economy

I first consider a decentralized economy in which labor allocation is determined by private incentives. A representative innovative firm owns the stock of queued ideas and chooses how much labor to allocate to idea creation and idea implementation.

The total private monopoly rent generated by one successfully implemented idea is assumed to be proportional to the productivity frontier:

$$\pi_t = A_t \bar{\pi}, \qquad \bar{\pi} > 0. \tag{17}$$

Hence, conditional on the stock of queued ideas M_t, the expected contemporaneous private payoff from implementation is $\bar{\pi} s_t M_t$ in productivity-normalized units.

Let the firm's value function be $J(M_t, \omega_t, A_t)$, and define the normalized value function as

$$j(M_t, \omega_t) \equiv \frac{J(M_t, \omega_t, A_t)}{A_t}$$

Normalized wages in the idea-creation and idea-implementation sectors are defined as

$$\bar{w}_{z,t} \equiv \frac{w_{z,t}}{A_t}, \qquad \bar{w}_{x,t} \equiv \frac{w_{x,t}}{A_t}$$

The normalized dynamic problem of the innovative firm is

$$j(M_t, \omega_t) = \max_{L_{z,t}, L_{x,t} \geq 0} \left\{ \bar{\pi} s_t M_t - \bar{w}_{z,t} L_{z,t} - \bar{w}_{x,t} L_{x,t} + \tilde{\beta}_t \mathbb{E}_t \left[j(M_{t+1}, \omega_{t+1}) \right] \right\}, \tag{18}$$

where $\tilde{\beta}_t$ is the growth-adjusted discount factor. Under the binomial implementation process

$$\tilde{\beta}_t = \beta \left[1 + s_t (e^{\gamma} - 1) \right]^{M_t}$$

The derivation of this expression, together with the first-order conditions below, is provided in Appendix 6.1.

The private shadow value of an additional queued idea is denoted by

$$j_{M,t} \equiv \frac{\partial j(M_t, \omega_t)}{\partial M_t}$$

Under the stationary certainty-equivalent approximation, the stationary shadow value is

$$j_M^* = \frac{\bar{\pi} s^*}{1 - \tilde{\beta}^* (1 - s^*)} \tag{19}$$

For this object to be finite, I impose the following transversality condition.

Assumption 1 (Transversality) *The stationary allocation satisfies*

$$\tilde{\beta}^* (1 - s^*) < 1. \tag{20}$$

This condition ensures that the private value of a surviving queued idea does not explode over

time. Although $\tilde{\beta}^*$ may exceed one because future payoffs are growth-adjusted, a queued idea survives into the next period only with probability $1 - s^*$. Therefore, the product $\tilde{\beta}^*(1 - s^*)$ governs the effective persistence of private option value.

The stationary idea-creation labor condition is

$$\bar{w}_z^* = \tilde{\beta}^* j_M^* \phi (z^*)^\theta \nu (L_z^*)^{\nu - 1} \tag{21}$$

Equivalently,

$$L_z^* = \left(\frac{\tilde{\beta}^* \phi \nu \bar{\pi} s^* (z^*)^\theta}{\bar{w}_z^* \left[1 - \tilde{\beta}^* (1 - s^*) \right]} \right)^{\frac{1}{1-\nu}} \tag{22}$$

The stationary implementation-labor condition is

$$\bar{w}_x^* = M^* \eta q^* e^{-\eta q^* L_x^*} \left[\bar{\pi} - \tilde{\beta}^* j_M^* \right] \tag{23}$$

Equivalently,

$$L_x^* = \frac{1}{\eta q^*} \ln \left(\frac{M^* \eta q^* \left[\bar{\pi} - \tilde{\beta}^* j_M^* \right]}{\bar{w}_x^*} \right) \tag{24}$$

The idea-creation condition says that research labor is chosen by comparing its wage cost with the discounted shadow value of expanding the queue of future implementation opportunities. The implementation condition says that engineering labor is chosen by comparing its wage cost with the net private surplus from converting queued ideas into realized productivity. Therefore, both labor demands are governed by the same shadow value j_M^*, which measures the private value of an additional unimplemented idea.

Assumption 2 (Interior labor allocation) *The stationary implementation-labor demand is interior:*

$$0 < L_x^* < 1 \tag{25}$$

Economically, this assumption requires implementation labor to be privately valuable enough

to be positive, but not so valuable that the economy allocates all labor to implementation.

2.4.2 Dynamic interaction between idea creation and implementation

The previous Subsection characterizes the stationary labor-market conditions in the decentralized economy. I now examine how idea creation and idea implementation interact dynamically. The central question is whether a higher stationary level of implementation labor L_x^* raises or lowers the stationary demand for idea-creation labor L_z^*.

From the stationary idea-creation condition, idea-creation labor depends on implementation labor through the implementation success rate,

$$s^* = 1 - e^{-\eta q^* L_x^*} \tag{26}$$

Since

$$\frac{\partial s^*}{\partial L_x^*} = \eta q^* (1 - s^*) > 0$$

a rise in implementation labor increases the probability that queued ideas are successfully implemented. Therefore, the sign of dL_z^*/dL_x^* is determined by how L_z^* responds to the implementation success rate s^*.

Proposition 2 (Dynamic complementarity between idea creation and implementation) *In the decentralized equilibrium, stationary idea-creation labor and stationary implementation labor are dynamically complementary,*

$$\frac{dL_z^*}{dL_x^*} > 0$$

if and only if

$$1 - \tilde{\beta}(s^*) + s^* \frac{G'(s^*)}{G(s^*)} > 0 \tag{27}$$

They are dynamically substitutable,

$$\frac{dL_z^*}{dL_x^*} < 0$$

if and only if

$$1 - \tilde{\beta}(s^*) + s^* \frac{G'(s^*)}{G(s^*)} < 0 \tag{28}$$

The knife-edge case is given by equality.

The proposition shows that stronger implementation affects research incentives through two opposing forces. First, a higher s^* raises the probability that newly created ideas will eventually be implemented. This raises the private value of idea creation and encourages more research labor. Second, a higher s^* also means that queued ideas are depleted more quickly. This weakens the continuation value of keeping ideas in the queue. Dynamic complementarity holds when the first force dominates the second.

Under the binomial implementation structure,

$$G(s^*) = [1 + s^*(e^\gamma - 1)]^{M^*}$$

The general complementarity condition can then be written as the following explicit threshold:

$$1 + (M^* + 1)s^*(e^\gamma - 1) > \beta \left[1 + s^*(e^\gamma - 1)\right]^{M^*+1} \tag{29}$$

Conversely, idea creation and implementation are dynamically substitutable if

$$1 + (M^* + 1)s^*(e^\gamma - 1) < \beta \left[1 + s^*(e^\gamma - 1)\right]^{M^*+1} \tag{30}$$

The knife-edge case is obtained when the two sides are equal.

The left-hand side captures the marginal increase in the value of research opportunities when

implementation becomes more effective. The right-hand side captures the growth-adjusted discounted continuation value of queued ideas. Complementarity arises when better implementation raises the current and future value of idea creation strongly enough to offset the reduced option value of keeping ideas unimplemented.

For sufficiently small productivity jumps, $e^{\gamma} - 1 \approx \gamma$. In this case, the complementarity condition reduces approximately to

$$\frac{dL_z^*}{dL_x^*} > 0 \quad \Longleftrightarrow \quad \beta < 1 \tag{31}$$

Thus, when productivity jumps are small and the firm discounts the future, stronger implementation capacity tends to encourage idea creation. If the firm places sufficiently high value on the continuation value of queued ideas, the complementarity force may weaken or reverse.

2.4.3 Two opposing forces in balanced-growth-path volatility

The previous Subsection shows that idea creation and implementation may be dynamically complementary or dynamically substitutable. I now examine how this interaction affects balanced-growth-path volatility.

On the balanced growth path, conditional productivity-growth variance can be written as

$$Var^* = \gamma^2(1 - s^*)\lambda^* \tag{32}$$

Using the idea-arrival rate,

$$\lambda^* = \phi(z^*)^{\theta}(L_z^*)^{\nu}$$

balanced-growth-path volatility becomes

$$Var^* = \gamma^2(1-s^*)\phi(z^*)^\theta (L_z^*)^\nu \tag{33}$$

Equation (33) shows that implementation labor affects volatility through two channels. First, a rise in L_x^* increases the implementation success rate,

$$s^* = 1 - e^{-\eta q^* L_x^*}$$

which lowers the residual implementation-risk term $1 - s^*$. Second, because L_x^* may affect L_z^*, stronger implementation may also change the arrival rate of new ideas and therefore the scale of implementation events.

Theorem 4 (Two-force decomposition of balanced-growth-path volatility) *The response of balanced-growth-path volatility to stationary implementation labor is*

$$\frac{d\ln Var^*}{dL_x^*} = \underbrace{-\eta q^*}_{\textit{direct stabilizing effect}} + \underbrace{\nu \frac{d\ln L_z^*}{dL_x^*}}_{\textit{indirect idea-expansion effect}} \tag{34}$$

The first term is the direct stabilizing effect. More implementation labor raises the probability that queued ideas are successfully implemented and therefore lowers residual implementation risk. The second term is the indirect idea-expansion effect. If stronger implementation encourages more idea creation, then the arrival rate of new ideas rises, increasing the scale of implementation events and potentially raising volatility.

Proposition 3 (Volatility under dynamic complementarity) *Suppose idea creation and implementation are dynamically complementary, so that*

$$\frac{dL_z^*}{dL_x^*} > 0$$

Then the effect of implementation labor on balanced-growth-path volatility is ambiguous. In particular,

$$\frac{d\ln Var^*}{dL_x^*} > 0 \quad \Longleftrightarrow \quad \nu \frac{d\ln L_z^*}{dL_x^*} > \eta q^* \tag{35}$$

whereas

$$\frac{d\ln Var^*}{dL_x^*} < 0 \quad \Longleftrightarrow \quad \nu \frac{d\ln L_z^*}{dL_x^*} < \eta q^* \tag{36}$$

The knife-edge case is

$$\nu \frac{d\ln L_z^*}{dL_x^*} = \eta q^* \tag{37}$$

Under dynamic complementarity, implementation labor has a non-monotonic effect on volatility. On the one hand, it directly stabilizes the economy by increasing the probability of successful implementation. On the other hand, it may stimulate additional idea creation, raising the arrival rate of new ideas and increasing the scale of implementation events. Hence, volatility rises if the indirect idea-expansion effect dominates the direct stabilizing effect, and falls if the direct stabilizing effect dominates.

Under dynamic substitutability,

$$\frac{dL_z^*}{dL_x^*} < 0$$

In this case, both channels reduce volatility. More implementation labor directly lowers residual implementation risk and indirectly reduces idea arrivals by lowering idea-creation labor.

Finally, recall that balanced-growth-path expected productivity growth is

$$g_A^{BGP} = \gamma\phi(z^*)^\theta (L_z^*)^\nu \tag{38}$$

Therefore, when idea creation and implementation are dynamically complementary,

$$\frac{dL_z^*}{dL_x^*} > 0 \quad \Longrightarrow \quad \frac{dg_A^{BGP}}{dL_x^*} > 0$$

Stronger implementation labor can therefore raise balanced-growth-path growth by inducing more idea creation. Its effect on volatility, however, remains ambiguous because volatility depends both on the scale of idea creation and on residual implementation risk. The model can therefore generate a positive, negative, or non-monotonic relationship between growth and volatility, depending on the relative strength of the two forces.

3 Empirical Evidence

3.1 First-stage Construction of AR(2) Growth Innovations

To estimate the growth—volatility relationship, I first construct a measure of unexpected GDP per capita growth. Let annual GDP per capita growth for country i in year t be defined as

$$g_{i,t} = 100\left[\ln(GDPpc_{i,t}) - \ln(GDPpc_{i,t-1})\right]$$

The purpose of the first stage is to remove the predictable component of growth and obtain a residual measure of growth innovations. Therefore, I estimate a set of autoregressive forecasting equations with different lag lengths. Specifically, I consider candidate AR(p) models of the form

$$g_{i,t} = \alpha_i + \delta_t + \sum_{\ell=1}^{p} \rho_\ell g_{i,t-\ell} + \boldsymbol{Z}'_{i,t-1}\boldsymbol{\phi} + \varepsilon_{i,t}^{(p)}, \qquad p \in \mathcal{P} \tag{39}$$

where α_i denotes country fixed effects, δ_t denotes year fixed effects, and p is the autoregressive lag order. The lagged dependent variables capture predictable persistence in GDP per capita

growth. The vector of lagged macroeconomic controls is given by

$$\boldsymbol{Z}_{i,t-1} = \begin{bmatrix} \ln(y_{i,t-1}) \\ s^I_{i,t-1} \\ h_{i,t-1} \\ n_{i,t-1} \end{bmatrix}$$

Here, $\ln(y_{i,t-1})$ is lagged log GDP per capita, $s^I_{i,t-1}$ is lagged investment share, $h_{i,t-1}$ is lagged human capital, and $n_{i,t-1}$ is lagged population growth. The inclusion of country fixed effects removes time-invariant country-specific factors, while year fixed effects remove common shocks affecting all countries in a given year.

The lagged macroeconomic controls that vector $\boldsymbol{Z}_{i,t-1}$ contains are specified by Levine and Renelt (1992) for cross-country growth analysis, and $\varepsilon^{(p)}_{i,t}$ is the forecast error from the AR(p) model. The empirical analysis uses annual country-level macroeconomic data from two main sources. GDP per capita is obtained from Federal Reserve Economic Data (FRED), Federal Reserve Bank of St. Louis (Federal Reserve Bank of St. Louis, 2026). Investment share, human capital, and population are obtained from Penn World Table version 11.0 (Feenstra et al., 2015, 2025). Following Feenstra et al. (2015), Penn World Table provides internationally comparable data on income, output, inputs, and productivity. Population growth is computed from the PWT population series as

$$n_{i,t} = 100 \left[\ln(\text{pop}_{i,t}) - \ln(\text{pop}_{i,t-1}) \right].$$

In the empirical implementation, the candidate lag orders are

$$\mathcal{P} = \{0, 1, 2, 3, 4\}$$

I compare these models using the Akaike Information Criterion (AIC) and the Bayesian In-

formation Criterion (BIC). The lag-selection results are reported in Table 1.

Table 1: Lag Selection for First-stage Growth Forecasting Equation

Model	Lag order p	N	RSS	k	AIC	BIC
AR(0)	0.000	2062.000	116 415.300	114.000	8545.052	9187.035
AR(1)	1.000	2062.000	113 463.000	115.000	8494.085	9141.700
AR(2)	2.000	2062.000	112 370.000	116.000	**8476.124**	**9129.370**
AR(3)	3.000	2062.000	112 349.700	117.000	8477.751	9136.629
AR(4)	4.000	2062.000	112 300.000	118.000	8478.840	9143.349

Notes: Smaller AIC and BIC values indicate better model fit after penalizing model complexity. p denotes the autoregressive lag order, N denotes the number of observations, RSS denotes the residual sum of squares, and k denotes the number of estimated parameters. The preferred specification is AR(2), which minimises both AIC and BIC.

Both AIC and BIC select the AR(2) specification. Although the AR(4) model has a slightly lower residual sum of squares (RSS), it uses additional parameters and is therefore penalised more heavily by the information criteria. I therefore use the residuals from the AR(2) model to construct the innovation-volatility measure.

Given the lag-selection results, the first-stage forecasting equation used in the baseline construction is

$$g_{i,t} = \alpha_i + \delta_t + \rho_1 g_{i,t-1} + \rho_2 g_{i,t-2} + \boldsymbol{Z}'_{i,t-1}\boldsymbol{\phi} + \varepsilon_{i,t} \tag{40}$$

Let $\widehat{g}_{i,t}$ denote the fitted value from equation (40). The estimated residual is

$$\widehat{\varepsilon}_{i,t} = g_{i,t} - \widehat{g}_{i,t} \tag{41}$$

This residual is interpreted as the unexpected innovation to GDP per capita growth. It is the component of growth that cannot be predicted by lagged growth, observable lagged macroeconomic fundamentals, country fixed effects, or year fixed effects.

For each country i and five-year block b, I then construct innovation volatility as the standard deviation of the AR(2) residuals within that block:

$$\sigma_{i,b} = sd\left(\widehat{\varepsilon}_{i,t}\right)_{t\in b} \tag{42}$$

Equivalently, this can be written as

$$\sigma_{i,b} = \sqrt{\frac{1}{N_b - 1}\sum_{t\in b}\left(\widehat{\varepsilon}_{i,t} - \overline{\widehat{\varepsilon}}_{i,b}\right)^2} \tag{43}$$

where N_b is the number of annual observations in block b, and

$$\overline{\widehat{\varepsilon}}_{i,b} = \frac{1}{N_b}\sum_{t\in b}\widehat{\varepsilon}_{i,t} \tag{44}$$

This residual-based volatility measure is preferred to raw growth volatility because it isolates unpredictable growth fluctuations rather than mixing together predictable convergence dynamics, growth persistence, lagged macroeconomic fundamentals, and common global time effects. In the regression tables below, this measure is reported as $\sigma_{i,b}^{innov}$. The baseline second-stage Ramey-style regression without the income interaction is

$$\bar{g}_{i,b} = \alpha_i + \delta_b + \sum_{r=1}^{m}\beta_r\sigma_{i,b}^r + \boldsymbol{\theta}'\mathbf{X}_{i,b} + u_{i,b}, \qquad m \in \{1,2,3\} \tag{45}$$

In equation (45), i indexes countries and b indexes five-year blocks. The dependent variable $\bar{g}_{i,b}$ denotes the average growth rate of GDP per capita in country i during block b. The variable $\sigma_{i,b}$ denotes residual-based growth volatility, constructed from the forecast errors obtained from the first-stage autoregressive forecasting equation. The polynomial term $\sum_{r=1}^{m}\beta_r\sigma_{i,b}^r$ allows the growth-volatility relationship to take different functional forms. When $m = 1$, the model is linear in volatility; when $m = 2$, the model is quadratic; and when $m = 3$, the model is cubic. The vector $\mathbf{X}_{i,b}$ contains the contemporaneous macroeconomic controls as described earlier, and $\boldsymbol{\theta}$ is the corresponding coefficient vector. The terms α_i and δ_b denote

country fixed effects and block fixed effects, respectively. Country fixed effects absorb time-invariant country-specific factors, while block fixed effects absorb common shocks affecting all countries in the same five-year period. The second-stage error term is denoted by $u_{i,b}$.

To examine whether the volatility–growth relationship differs across stages of development, I further interact volatility with average log GDP per capita during the 5-year block. This specification is motivated by the development-volatility literature. Koren and Tenreyro (2007) show that poorer countries tend to experience more volatile GDP growth because they are less diversified and are more exposed to severe aggregate shocks. Acemoglu and Zilibotti (1997) provide a related theoretical mechanism, arguing that economies at early stages of development face limited diversification opportunities and incomplete markets, causing poorer economies to suffer higher growth volatility and lower productivity. These studies suggest that the effect of macroeconomic volatility on growth should not be assumed to be homogeneous across countries at different income levels. Instead, development level may condition both the strength and shape of the volatility–growth relationship. Therefore, the corresponding second-stage specification with the income interaction is

$$\bar{g}_{i,b} = \alpha_i + \delta_b + \sum_{r=1}^{m} \beta_r \sigma_{i,b}^r + \psi \left[\sigma_{i,b} \times \overline{\ln(y)}_{i,b} \right] + \boldsymbol{\theta}' \mathbf{X}_{i,b} + u_{i,b}, \qquad m \in \{1, 2, 3\} \tag{46}$$

In equation (46), $\overline{\ln(y)}_{i,b}$ denotes the average log GDP per capita of country i during block b. The interaction term $\sigma_{i,b} \times \overline{\ln(y)}_{i,b}$ allows the association between volatility and growth to vary with the income level of the country. The coefficient ψ captures whether the marginal relationship between growth volatility and average growth is different for poorer and richer economies. All other notation is defined as in equation (45).

In the main second-stage Ramey-style regression model, the control variables exhibit consistent and economically meaningful effects across all specifications. $\ln(y_{i,b})$ enters with a

negative and highly significant coefficient, indicating strong evidence of conditional convergence. $s^I_{i,b}$ is positive and highly significant, consistent with standard growth theory. $h_{i,b}$ is positive and highly significant, but not in the two-way fixed effect model.

The coefficient on $\sigma_{i,b}$ is negative and statistically significant across the specified models. There is little evidence suggesting that the coefficient on quadratic term of $\sigma_{i,b}$ ($\sigma^2_{i,b}$) is statistically significant. The coefficient on the cubic term of $\sigma_{i,b}$ ($\sigma^3_{i,b}$) is marginally significant at the 10% significance level only in the two-way fixed effect model. In addition, it is close to zero. Based on the main regression model, it is suggested that the linear model in volatility explains the relationship between volatility and growth better than the other two.

In the second-stage Ramey-style regression model with the income interaction term as a robustness check, the control variables behave in the same way as in the main model.

The coefficients on $\sigma_{i,b}$ remain negative and statistically significant across models specified. However, they become more negative after the inclusion of the income interaction term. Although the coefficients on the quadratic term of $\sigma_{i,b}$ ($\sigma^2_{i,b}$) are statistically significant at the 5% significance level in the model without controls and marginally significant at the 10% significance level in the two-way fixed effect model, they are close to zero. Similarly, the coefficient on the cubic term of $\sigma_{i,b}$ ($\sigma^3_{i,b}$) are marginally significant at the 10% significance level only in the two-way fixed effect model, and it is again close to zero. The coefficients of the income interaction term ($\sigma_{i,b} \times \overline{\ln(y)}_{i,b}$) are positive and statistically significant in most models, which adds nuance to the volatility–growth implications: the negative association of volatility with growth becomes weaker as income rises. In short, the negative volatility–growth relationship remains robust even after cross-country developmental heterogeneity is considered.

The complementarity between idea-creation labor and idea-implementation labor provides a coherent theoretical framework for understanding the volatility–growth relationship. In the model, high volatility can be interpreted as reflecting low labor allocation to idea implementation. Because idea creation and idea implementation are complementary, this reduction also lowers employment in the idea-creation sector, ultimately depressing long-run growth.

Table 2: Linear Ramey-style Growth–Volatility Models

	(1) No controls	(2) Controls	(3) Country FE	(4) Time FE	(5) Two-way FE
$\sigma_{i,b}$	-0.194***	-0.185***	-0.208***	-0.191***	-0.218***
	(0.015)	(0.013)	(0.021)	(0.016)	(0.019)
	[-12.93]	[-14.23]	[-9.90]	[-11.94]	[-11.47]
$s^I_{i,b}$		18.072***	20.668***	13.165***	16.368***
		(2.892)	(3.930)	(2.606)	(3.621)
		[6.25]	[5.26]	[5.05]	[4.52]
$n_{i,b}$		-0.529	0.397	-0.366	0.583
		(0.425)	(0.768)	(0.413)	(0.714)
		[-1.24]	[0.52]	[-0.89]	[0.82]
h_b		2.425***	5.790***	2.420***	1.804
		(0.614)	(1.693)	(0.625)	(1.539)
		[3.95]	[3.42]	[3.87]	[1.17]
$\ln(y_{i,b})$		-2.201***	-4.387***	-1.604***	-5.357***
		(0.219)	(0.549)	(0.231)	(0.555)
		[-10.05]	[-7.99]	[-6.94]	[-9.65]
Observations	1028	1028	1028	1028	1028
Adjusted R^2	0.105	0.204	0.271	0.386	0.456
Controls	No	Yes	Yes	Yes	Yes
Country FE	No	No	Yes	No	Yes
Time FE	No	No	No	Yes	Yes

Notes: Each cell reports the coefficient, standard error in parentheses, and approximate t-statistic in square brackets. t-statistics are computed from the rounded coefficients and standard errors in the Stata output. $^{*}p < 0.10$, $^{**}p < 0.05$, $^{***}p < 0.01$.

Table 3: Quadratic Ramey-style Growth–Volatility Models

	(1) No controls	(2) Controls	(3) Country FE	(4) Time FE	(5) Two-way FE
$\sigma_{i,b}$	-0.135**	-0.140**	-0.146***	-0.156**	-0.197***
	(0.061)	(0.061)	(0.054)	(0.070)	(0.061)
	[-2.21]	[-2.30]	[-2.70]	[-2.23]	[-3.23]
$\sigma^2_{i,b}$	-0.000	-0.000	-0.000*	-0.000	-0.000
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
	[–]	[–]	[–]	[–]	[–]
$s^I_{i,b}$		18.550***	21.394***	13.452***	16.563***
		(2.747)	(3.869)	(2.478)	(3.590)
		[6.75]	[5.53]	[5.43]	[4.61]
$n_{i,b}$		-0.530	0.426	-0.362	0.593
		(0.429)	(0.777)	(0.417)	(0.717)
		[-1.24]	[0.55]	[-0.87]	[0.83]
$h_{i,b}$		2.416***	5.785***	2.404***	1.787
		(0.610)	(1.643)	(0.622)	(1.532)
		[3.96]	[3.52]	[3.86]	[1.17]
$\ln(y_{i,b})$		-2.166***	-4.354***	-1.561***	-5.320***
		(0.233)	(0.527)	(0.256)	(0.523)
		[-9.30]	[-8.26]	[-6.10]	[-10.17]
Observations	1028	1028	1028	1028	1028
Adjusted R^2	0.108	0.205	0.274	0.386	0.455
Controls	No	Yes	Yes	Yes	Yes
Country FE	No	No	Yes	No	Yes
Time FE	No	No	No	Yes	Yes

Notes: Each cell reports the coefficient, standard error in parentheses, and approximate t-statistic in square brackets. [−−] means the t-statistic cannot be recovered from the rounded output because both the coefficient and standard error are reported as 0.000. $^{*}p < 0.10$, $^{**}p < 0.05$, $^{***}p < 0.01$.

Table 4: Cubic Ramey-style Growth–Volatility Models

	(1) No controls	(2) Controls	(3) Country FE	(4) Time FE	(5) Two-way FE
$\sigma_{i,b}$	-0.189***	-0.211***	-0.199***	-0.272***	-0.325***
	(0.063)	(0.055)	(0.061)	(0.070)	(0.066)
	[-3.00]	[-3.84]	[-3.26]	[-3.89]	[-4.92]
$\sigma_{i,b}^2$	0.001	0.002	0.001	0.003	0.003*
	(0.002)	(0.002)	(0.002)	(0.002)	(0.002)
	[0.50]	[1.00]	[0.50]	[1.50]	[1.50]
$\sigma_{i,b}^3$	-0.000	-0.000	-0.000	-0.000	-0.000*
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
	[–]	[–]	[–]	[–]	[–]
$s_{i,b}^I$		18.538***	21.352***	13.465***	16.545***
		(2.724)	(3.849)	(2.442)	(3.546)
		[6.81]	[5.55]	[5.51]	[4.67]
n_b		-0.519	0.433	-0.350	0.603
		(0.430)	(0.776)	(0.418)	(0.712)
		[-1.21]	[0.56]	[-0.84]	[0.85]
$h_{i,b}$		2.404***	5.723***	2.403***	1.811
		(0.609)	(1.622)	(0.622)	(1.528)
		[3.95]	[3.53]	[3.86]	[1.19]
$\ln(y_{i,b})$		-2.170***	-4.333***	-1.586***	-5.298***
		(0.233)	(0.521)	(0.254)	(0.513)
		[-9.31]	[-8.32]	[-6.24]	[-10.33]
Observations	1028	1028	1028	1028	1028
Adjusted R^2	0.108	0.206	0.274	0.389	0.459
Controls	No	Yes	Yes	Yes	Yes
Country FE	No	No	Yes	No	Yes
Time FE	No	No	No	Yes	Yes

Notes: Each cell reports the coefficient, standard error in parentheses, and approximate t-statistic in square brackets. $[--]$ means the t-statistic cannot be recovered from the rounded output because both the coefficient and standard error are reported as 0.000. $^{*}p < 0.10$, $^{**}p < 0.05$, $^{***}p < 0.01$.

Table 5: Linear Model with Income-Weighted Volatility

	(1) No controls	(2) Controls	(3) Country FE	(4) Time FE	(5) Two-way FE
$\sigma_{i,b}$	-0.149	-0.503***	-0.513***	-0.458***	-0.455***
	(0.105)	(0.088)	(0.119)	(0.097)	(0.125)
	[-1.42]	[-5.72]	[-4.31]	[-4.72]	[-3.64]
$\sigma_{i,b} \times \overline{\ln(y)}_{i,b}$	-0.008	0.057***	0.055**	0.048**	0.043*
	(0.020)	(0.019)	(0.024)	(0.021)	(0.025)
	[-0.40]	[3.00]	[2.29]	[2.29]	[1.72]
$s^I_{i,b}$		18.382***	20.868***	13.349***	16.253***
		(2.705)	(3.797)	(2.448)	(3.490)
		[6.80]	[5.50]	[5.45]	[4.66]
$n_{i,b}$		-0.548	0.462	-0.378	0.634
		(0.417)	(0.750)	(0.406)	(0.701)
		[-1.31]	[0.62]	[-0.93]	[0.90]
$h_{i,b}$		2.468***	5.867***	2.432***	1.739
		(0.604)	(1.616)	(0.609)	(1.483)
		[4.09]	[3.63]	[3.99]	[1.17]
$\ln(y_{i,b})$		-2.556***	-4.701***	-1.893***	-5.549***
		(0.222)	(0.535)	(0.239)	(0.502)
		[-11.51]	[-8.79]	[-7.92]	[-11.05]
Observations	1028	1028	1028	1028	1028
Adjusted R^2	0.104	0.217	0.282	0.395	0.462
Controls	No	Yes	Yes	Yes	Yes
Country FE	No	No	Yes	No	Yes
Time FE	No	No	No	Yes	Yes

Notes: Each cell reports the coefficient, standard error in parentheses, and approximate t-statistic in square brackets. Income-weighted volatility is defined as $vol \times avg_ln_gdppc$. $^{*}p < 0.10$, $^{**}p < 0.05$, $^{***}p < 0.01$.

Table 6: Quadratic Model with Income-Weighted Volatility

	(1) No controls	(2) Controls	(3) Country FE	(4) Time FE	(5) Two-way FE
	0.030	-0.775***	-0.685***	-0.705***	-0.777***
$\sigma_{i,b}$	(0.102)	(0.192)	(0.259)	(0.199)	(0.249)
	[0.29]	[-4.04]	[-2.64]	[-3.54]	[-3.12]
	-0.000**	0.000	0.000	0.000	0.001*
$\sigma_{i,b}^2$	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
	[–]	[–]	[–]	[–]	[–]
	-0.025	0.089***	0.076**	0.077***	0.082**
$\sigma_{i,b} \times \overline{\ln(y)}_{i,b}$	(0.016)	(0.027)	(0.038)	(0.028)	(0.037)
	[-1.56]	[3.30]	[2.00]	[2.75]	[2.22]
		17.612***	20.297***	12.785***	15.167***
$s_{i,b}^I$		(2.644)	(3.691)	(2.414)	(3.422)
		[6.66]	[5.50]	[5.30]	[4.43]
		-0.558	0.461	-0.395	0.629
$n_{i,b}$		(0.407)	(0.733)	(0.398)	(0.669)
		[-1.37]	[0.63]	[-0.99]	[0.94]
		2.511***	5.901***	2.477***	1.767
$h_{i,b}$		(0.614)	(1.635)	(0.616)	(1.481)
		[4.09]	[3.61]	[4.02]	[1.19]
		-2.830***	-4.851***	-2.171***	-5.912***
$\ln(y_{i,b})$		(0.281)	(0.571)	(0.289)	(0.539)
		[-10.07]	[-8.50]	[-7.51]	[-10.97]
Observations	1028	1028	1028	1028	1028
Adjusted R^2	0.110	0.220	0.283	0.397	0.466
Controls	No	Yes	Yes	Yes	Yes
Country FE	No	No	Yes	No	Yes
Time FE	No	No	No	Yes	Yes

Notes: Each cell reports the coefficient, standard error in parentheses, and approximate t-statistic in square brackets. Income-weighted volatility is defined as $vol \times avg_ln_gdppc$. $[--]$ means the t-statistic cannot be recovered from the rounded output because both the coefficient and standard error are reported as 0.000. $^{*}p < 0.10$, $^{**}p < 0.05$, $^{***}p < 0.01$.

Table 7: Cubic Model with Income-Weighted Volatility

	(1) No controls	(2) Controls	(3) Country FE	(4) Time FE	(5) Two-way FE
$\sigma_{i,b}$	-0.024	-0.889***	-0.770***	-0.874***	-0.954***
	(0.118)	(0.209)	(0.267)	(0.227)	(0.256)
	[-0.20]	[-4.25]	[-2.88]	[-3.85]	[-3.73]
$\sigma_{i,b}^2$	0.001	0.003	0.002	0.004	0.004**
	(0.002)	(0.002)	(0.002)	(0.003)	(0.002)
	[0.50]	[1.50]	[1.00]	[1.33]	[2.00]
$\sigma_{i,b}^3$	-0.000	-0.000	-0.000	-0.000	-0.000*
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
	[–]	[–]	[–]	[–]	[–]
$\sigma_{i,b} \times \overline{\ln(y)}_{i,b}$	-0.025	0.092***	0.078**	0.082***	0.087**
	(0.015)	(0.027)	(0.037)	(0.029)	(0.036)
	[-1.67]	[3.41]	[2.11]	[2.83]	[2.42]
$s_{i,b}^I$		17.564***	20.209***	12.762***	15.072***
		(2.605)	(3.674)	(2.361)	(3.383)
		[6.74]	[5.50]	[5.41]	[4.46]
$n_{i,b}$		-0.544	0.470	-0.383	0.642
		(0.408)	(0.729)	(0.398)	(0.660)
		[-1.33]	[0.64]	[-0.96]	[0.97]
$h_{i,b}$		2.499***	5.824***	2.480***	1.794
		(0.612)	(1.616)	(0.614)	(1.475)
		[4.08]	[3.60]	[4.04]	[1.22]
$\ln(y_{i,b})$		-2.858***	-4.839***	-2.235***	-5.918***
		(0.281)	(0.569)	(0.284)	(0.537)
		[-10.17]	[-8.50]	[-7.87]	[-11.02]
Observations	1028	1028	1028	1028	1028
Adjusted R^2	0.110	0.221	0.283	0.401	0.470
Controls	No	Yes	Yes	Yes	Yes
Country FE	No	No	Yes	No	Yes
Time FE	No	No	No	Yes	Yes

Notes: Each cell reports the coefficient, standard error in parentheses, and approximate t-statistic in square brackets. Income-weighted volatility is defined as $vol \times avg_ln_gdppc$. [−−] means the t-statistic cannot be recovered from the rounded output because both the coefficient and standard error are reported as 0.000. $^{*}p < 0.10$, $^{**}p < 0.05$, $^{***}p < 0.01$.

4 Calibration

4.1 Calibration Methods

In addition to the theoretical framework and the empirical evidence on the growth–volatility relationship, I also conduct a calibrated numerical experiment to examine the transitional dynamics implied by the model. The purpose of the calibration is not to match a set of empirical moments directly, but to provide an internally consistent benchmark economy in which the model's growth–volatility mechanism can be clearly illustrated. In particular, the calibration is designed to generate an interior balanced growth path, satisfy the transversality condition for the shadow value of queued ideas, and trace the transition after a one-time permanent jump in the idea type.

As assumed earlier, the economy is normalized with aggregate labor supply equal to 1. Ideas and machines are both constrained to lie in the bounded type space (0, 1]. I first solve the pre-shock balanced growth path using an initial idea type of 0.8. I then impose a one-time permanent technology improvement at period 20, raising the idea type to the upper bound (1) of the type space. The post-shock balanced growth path is then solved under the new idea type.

The benchmark parameter values are chosen as follows. The discount factor is $\beta = 0.9$, the productivity gain per successfully implemented idea is $\gamma = 0.012$, the baseline productivity of idea creation is $\phi = 0.4$, the elasticity of idea creation with respect to research labor is $\nu = 0.6$, and the elasticity with respect to the idea type is $\theta = 0.35$. The implementation-efficiency parameter is $\eta = 2$, the mismatch severity parameter is $\kappa = 2.5$, and the curvature parameter is $\psi = 2$. The machine adjustment speed is $\rho = 0.15$, implying that machines adapt gradually towards the new idea type. The knowledge-stock parameter is set to $\bar{\chi} = 0$, corresponding to the knife-edge case under which the model generates finite

non-zero balanced-growth-path growth. The normalized monopoly payoff per successfully implemented idea is set to $\bar{\pi} = 2.5$.

For each idea type, I first solve the corresponding balanced growth path. The pre-shock balanced growth path is solved under $z_0 = 0.8$, while the post-shock balanced growth path is solved under $z_1 = 1$.

The transition path is computed using a fixed-point algorithm over the full path of labor allocation. The simulation horizon is $T = 250$, with an additional tail of 120 periods to ensure convergence. Starting from a range of points between the old and new balanced growth paths, the model is simulated forward to obtain the implied paths of machine type, mismatch, compatibility, implementation success, idea arrivals, the queue of unimplemented ideas, productivity, and output. Given these forward paths, the shadow value of queued ideas is computed backward. Labor allocation is then updated using the model's labor–market conditions. To improve the numerical stability, the update is damped with weights 0.2, and the algorithm is repeated until the maximum change in the labor paths falls below 10^{-10}.

To interpret the transition, I report the log deviation of output from its pre-shock balanced-growth trend. In addition, I complement the deterministic expected transition with Monte Carlo simulations. Since idea arrivals and implementation success are stochastic, a single realized path may be noisy and unrepresentative. Therefore, I simulate 500 Monte Carlo replications and report the mean, median, and 10th–to–90th percentile bands of realized outcomes. I also compare the theoretical conditional standard deviation of growth with the cross-replication standard deviation from the Monte Carlo simulations.

Table 8: Benchmark calibration

Parameter	Value	Description
β	0.90	Discount factor
γ	0.012	Productivity gain from one successfully implemented idea
ϕ	0.40	Baseline productivity of idea creation
ν	0.60	Elasticity of idea creation with respect to research labor
θ	0.35	Elasticity of idea creation with respect to idea type
η	2.00	Efficiency of implementation labor
κ	2.50	Severity of mismatch losses
ψ	2.00	Curvature of mismatch losses
ρ	0.15	Speed of machine adaptation
$\bar{\chi}$	0.00	Knowledge-stock effect in idea creation
$\bar{\pi}$	2.50	Normalized monopoly payoff from one implemented idea
$\bar{L}$	1.00	Total labor supply
z_0	0.80	Pre-shock idea type
z_1	1.00	Post-shock idea type
$\Delta z/z_0$	25%	Size of the permanent idea-type improvement
t_{shock}	20	Period of the one-time permanent technology shock
$(\underline{z}, \bar{z})$	$(10^{-8}, 1]$	Numerically implemented bounded idea-type space
$(\underline{x}, \bar{x})$	$(10^{-8}, 1]$	Numerically implemented bounded machine-type space

4.2 Calibration Results

As shown in Figures 1 and 2, the one-time permanent jump in the idea type initially raises the mismatch level, ω_t. This temporarily reduces machine–idea compatibility (q_t) and lowers the success rate of idea implementation (s_t). The labor dynamics in Figure 3 reflect two forces. First, there is a temporary adjustment motive. Immediately after the shock, the idea type jumps while the machine type adjusts only gradually, so mismatch rises and compatibility falls. Since

$$s_t = 1 - e^{-\eta q_t L_{x,t}}, \qquad \frac{\partial s_t}{\partial L_{x,t}} = \eta q_t e^{-\eta q_t L_{x,t}}$$

the fall in q_t reduces the marginal effectiveness of implementation labor. This temporarily lowers the payoff from $L_{x,t}$. Because ideas are also harder to implement, the shadow value of creating additional ideas falls, so $L_{z,t}$ declines as well.

Second, there is a permanent level effect. In the new balanced growth path, compatibility recovers to $q^* = 1$, while the permanently higher idea type raises the productivity of idea creation. This can be seen from the stationary idea-creation condition

$$\bar{w}_z^* = \tilde{\beta}^* j_M^* \phi (z^*)^\theta \nu (L_z^*)^{\nu-1}$$

A higher z^* raises the marginal return to L_z^*, supporting a higher long-run level of idea-creation labor. At the same time, since

$$\lambda^* = \phi (z^*)^\theta (L_z^*)^\nu, \qquad M^* = \frac{\lambda^*}{s^*}$$

the larger long-run flow of ideas increases the stock of implementation opportunities. This raises the demand for implementation labor through

$$\bar{w}_x^* = M^* \eta q^* \exp(-\eta q^* L_x^*) \left[\bar{\pi} - \tilde{\beta}^* j_M^* \right]$$

Hence, once the temporary mismatch has faded, both $L_{z,t}$ and $L_{x,t}$ converge to higher post-shock levels. By the aggregate labor constraint, $L_{Y,t}$ remains lower in the new balanced growth path.

Figure 4 presents the economy's log deviation dynamics from the original balanced-growth-path trend. The log deviation is measured by

$$\ln \left(\frac{Y_{new}^{BGP}}{Y_{old}^{BGP}} \right)$$

Following the upward jump in the idea type, the economy initially experiences a severe slump due to the rise in mismatch and the sharp fall in machine–idea compatibility. After this temporary downturn, output gradually recovers, catches up with the shock-unaffected output level at approximately $t = 180$, and exceeds it thereafter. This recovery is driven

by the gradual convergence of the machine type toward the new idea type, which restores compatibility and improves implementation effectiveness. Moreover, since the slope of the log deviation measures the difference between the shock-affected output growth rate and the counterfactual shock-unaffected balanced-growth-path output growth rate, the positive slope indicates that output grows faster under the shock-affected path than under the original balanced growth path. This long-run acceleration is generated by the higher employment of idea-creation labor in the new balanced growth path. The implication is that technological advancement may be costly in the short run because the economy requires re-organization to adjust to the new technological environment. However, once machines and organizational structures gradually adapt, the long-run benefits begin to emerge. The economy eventually moves on to a higher growth path and performs better than it would have without the technological advancement.

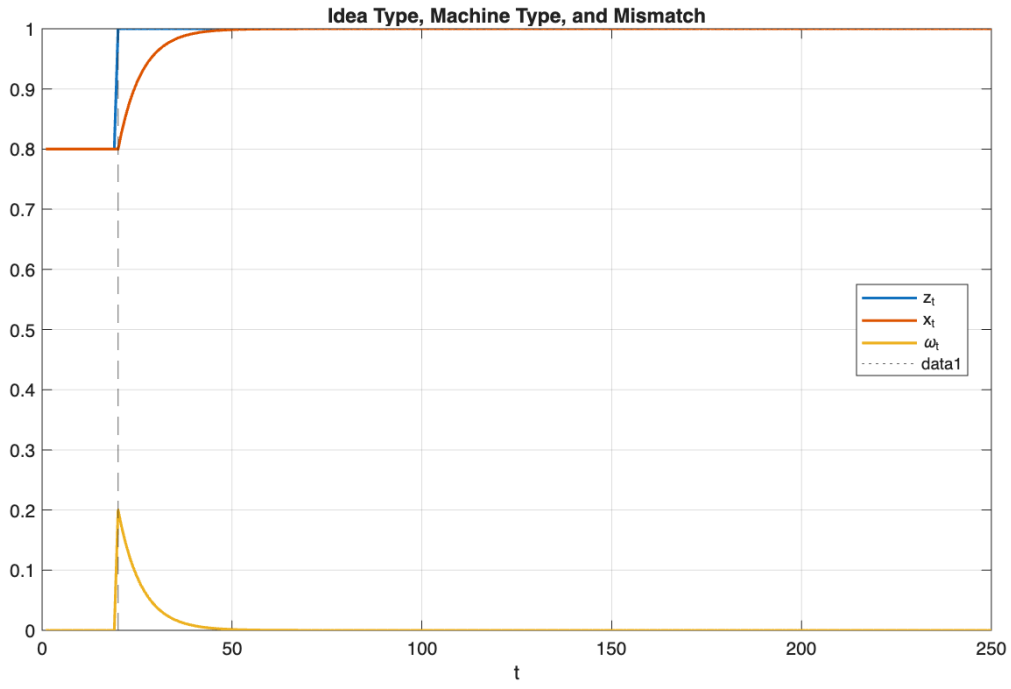


Figure 1: Idea type, machine type, and mismatch following the shock

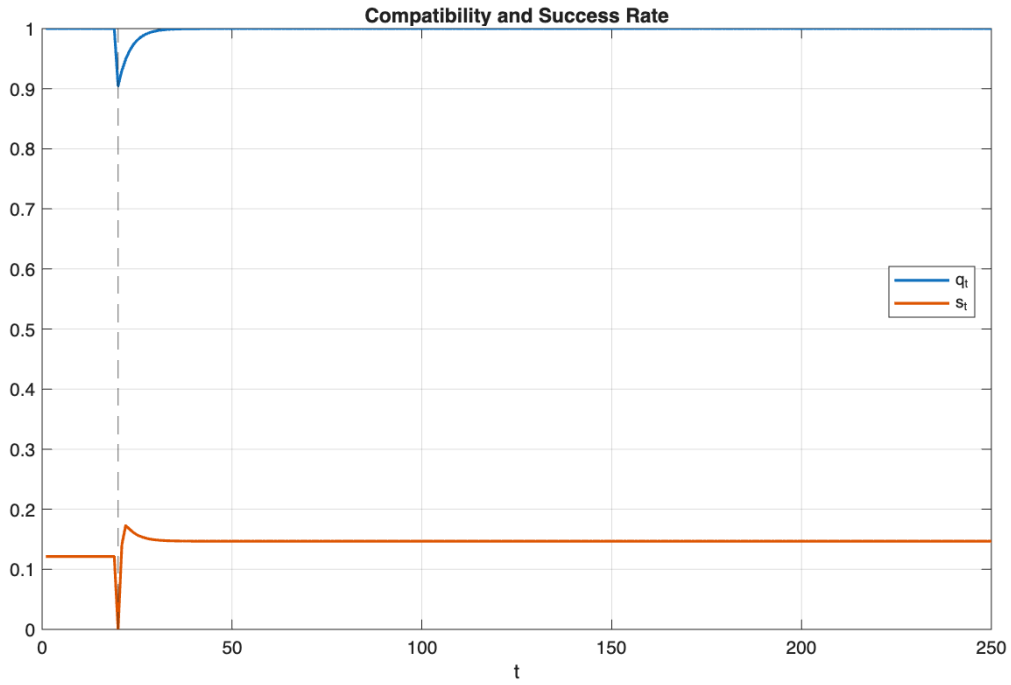


Figure 2: Compatibility and Success Rate

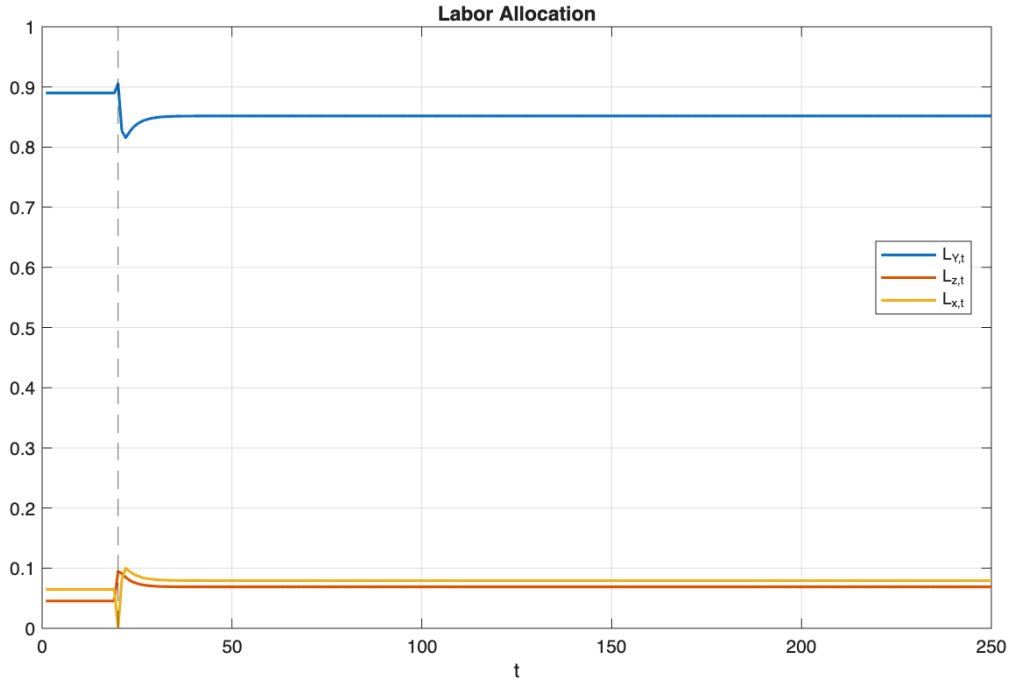


Figure 3: Dynamics of Labor Allocation

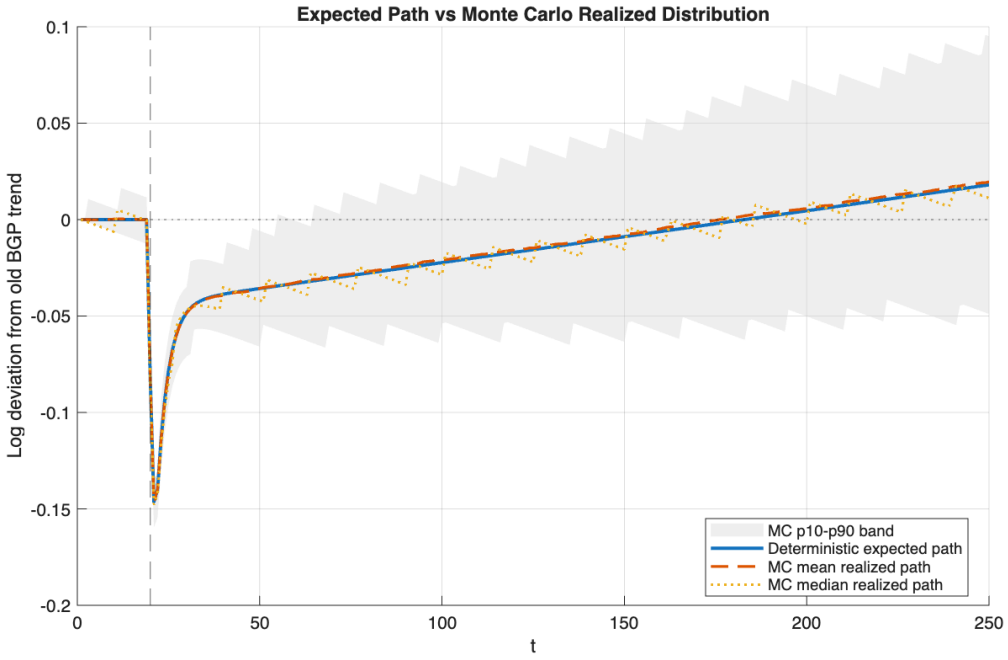


Figure 4: Log Deviation from the Old Balanced-Growth-Path Trend

5 Policy Implications

This section discusses the policy implications of the model. The model provides a theoretical framework for three main policy insights. First, innovation policy must be supported by appropriate institutional and organizational reorganization. Second, technological transitions may require temporary adjustment support to reduce short-run output loss. Third, stabilization policies can simultaneously reduce growth volatility and promote growth by supporting sustained innovation and re-organization.

5.1 Innovation Policies Should Support Re-organization

The model suggests that research and development (R&D) subsidies alone are incomplete. Cohen and Levinthal (1990) show that firms need absorptive capacity to recognize, assimilate, and exploit new knowledge; Bartel and Lichtenberg (1987) show that educated workers are especially important when new technologies create uncertainty and require adjustment;

and Bresnahan et al. (2002) show that technology becomes productive only when combined with complementary workplace reorganization and skilled labor. If technological progress is promoted without complementary institutional re-organization, the resulting increase in ideas or technologies may merely accumulate as implementation backlogs, and innovation policy may raise volatility by increasing unrealized technological opportunities, without generating a corresponding improvement in long-run growth. Therefore, policies should target the complementary conditions that allow technologies to be implemented effectively. This includes building technical human capital through education and training, supporting firm-level adoption through subsidies and advisory services, improving management capability and organizational upgrading, and strengthening technology diffusion through public extension services, university–industry transfer, and applied research institutes.

5.2 Technological Transitions Require Temporary Adjustment Support

Another implication stems from the earlier calibration results. In the model, a permanent improvement in the technological level can initially raise mismatch because the institutions are not immediately compatible with the new technological requirements. As a result, compatibility and implementation success may fall temporarily, generating a short-run slowdown before the long-run benefits of the new technology emerge. Policy should therefore treat early productivity slowdowns after major technological breakthroughs as adjustment problems rather than as evidence of technological failure. Temporary support for re-training, complementary investment, technical assistance, and infrastructure upgrading can shorten the mismatch period and accelerate the realization of productivity gains. This argument is consistent with empirical evidence that major technological transitions can generate short-run adjustment costs even when they raise productivity in the long run. Acemoglu and Restrepo (2020) show that robot adoption reduced employment and wages in exposed U.S. local labor markets, suggesting that workers and firms may face substantial transitional costs when

production technologies change. Similarly, Dauth et al. (2021) find that robot adoption in Germany displaced manufacturing employment, although aggregate employment losses were partly offset by job creation in the service sector. Graetz and Michaels (2018) also show that industrial robots contributed to productivity and value-added growth, which reinforces the view that the appropriate policy response is not to resist technological progress, but to reduce the frictions that delay successful adjustment. Therefore, temporary adjustment support should be interpreted as a transitional policy that helps workers, firms, and institutions adapt to the new technological environment, rather than as protection for obsolete technologies.

5.3 Stabilization Policies Can Be Growth Policies

The literature suggests that the effect of stabilization policy on long-run growth is theoretically ambiguous. Stabilization may promote growth when volatility disrupts learning-by-doing, human capital accumulation, investment, innovation, or technology adoption (Martin and Rogers, 1997; Aghion et al., 2010, 2014). However, it may also reduce growth when cyclical fluctuations generate productivity-enhancing learning, restructuring, or experimentation (Blackburn, 1999; Blackburn and Pelloni, 2005). This model proposes a theoretical framework which, through the conditional complementarity between the idea-creation labor and idea-implementation labor, suggests that stabilization policies can promote growth. The welfare effects of stabilization policy go beyond stabilizing unemployment. When stabilization policies prevent productivity-enhancing re-organization investment from being excessively reduced during downturns, they also preserve the economy's growth potential.

6 Conclusion

This paper develops a theoretical framework that extends the standard Schumpeterian-style growth model by incorporating the implementation of ideas, or technologies, alongside cre-

ative destruction. Innovation inevitably generates volatility through Schumpeterian waves of creative destruction. However, such volatility can be stabilized when the economy simultaneously allocates more labor to the implementation of new ideas and technologies. The conditional complementarity between idea-creation and idea-implementation labor provides a mechanism for explaining the negative growth–volatility relationship. In this framework, high volatility reflects insufficient employment in the implementation of new ideas and technologies. Because innovation and implementation are complementary, lower implementation effort leads to weaker innovation activities and lower long-run growth. Lastly, calibration results suggest that the economy may undergo a temporary recession before the stronger productivity effects of new technologies materialize. This provides a policy rationale for temporary adjustment support, through which the government can reduce transitional frictions and mitigate the short-run costs associated with technological adoption.

Tables

Table 9: List of Countries in the Sample

Country	Growth		Country	Growth		Country	Growth	
	Mean	SD		Mean	SD		Mean	SD
Algeria	4.940	13.430	Honduras	4.190	6.350	Panama	5.830	6.490
Argentina	4.620	23.290	Hungary	6.930	8.890	Paraguay	5.900	12.520
Australia	5.680	9.550	Iceland	6.370	13.970	Peru	5.460	17.830
Austria	6.500	9.810	India	5.390	8.340	Philippines	4.200	11.890
Bangladesh	5.450	14.580	Iran (Islamic Republic of)	5.160	18.280	Poland	7.720	10.050
Barbados	6.830	8.180	Iraq	5.420	92.130	Portugal	6.820	10.020
Belgium	5.960	9.940	Ireland	7.970	8.760	Senegal	2.750	10.910
Bolivia (Plurinational State of)	5.930	10.790	Israel	5.680	9.980	Sierra Leone	2.660	19.890
Botswana	7.760	11.760	Italy	6.110	9.900	Singapore	8.410	9.280
Brazil	6.010	15.640	Jamaica	4.570	12.010	Slovak Republic	7.010	9.630
Canada	5.040	6.990	Japan	6.660	11.230	Slovenia	3.610	11.710
Chile	5.550	17.590	Jordan	3.630	11.050	South Africa	3.850	12.380
China, Hong Kong SAR	7.580	7.790	Kenya	4.660	10.400	South Korea	8.590	15.130
Colombia	5.240	10.500	Latvia	8.250	11.970	Spain	7.000	11.150
Costa Rica	6.020	11.090	Lesotho	4.720	13.650	Sri Lanka	5.170	10.730
Cyprus	7.550	12.430	Liberia	2.650	18.170	Sudan	3.420	32.620
Czech	6.300	12.120	Lithuania	9.090	11.270	Sweden	5.160	10.280
Denmark	6.180	9.450	Luxembourg	6.470	10.310	Switzerland	6.260	9.590
Dominican Republic	6.260	15.710	Malawi	1.840	19.690	Syrian Arab Republic	2.820	20.190
Ecuador	4.290	13.100	Malaysia	6.120	10.930	Thailand	6.740	9.030
El Salvador	5.130	5.210	Malta	7.360	9.250	Togo	3.700	11.540
Estonia	8.070	10.780	Mauritius	6.120	11.760	Trinidad and Tobago	5.340	13.910
Eswatini	5.050	14.390	Mexico	5.820	14.990	Tunisia	4.810	8.130
Fiji	4.860	10.740	Mozambique	2.400	13.330	Turkiye	6.210	15.290
Finland	6.020	10.620	Myanmar	6.150	18.680	U.R. of Tanzania: Mainland	2.370	13.390
France	5.610	9.720	Nepal	5.260	9.950	UK	5.680	9.240
Germany	6.120	10.060	Netherlands	6.360	9.630	USA	5.210	2.810
Ghana	4.150	15.160	New Zealand	4.830	11.320	Uganda	4.600	17.810
Greece	6.060	9.830	Nicaragua	4.790	20.370	Uruguay	6.110	18.080
Guatemala	4.940	7.840	Niger	2.550	13.420	Venezuela (Bolivarian Republic of)	0.220	5.090
Guyana	7.480	19.080	Norway	6.510	10.570	Zambia	2.850	16.840
Haiti	5.050	13.580	Pakistan	4.460	11.100	Zimbabwe	3.290	19.210

Notes: Mean growth is the country-specific average annual growth rate of GDP per capita. Growth SD is the within-country standard deviation of annual growth. Annual growth is computed as $g_{ct} = 100[\ln(y_{ct}) - \ln(y_{c,t-1})]$, where y_{ct} is GDP per capita. Both variables are expressed in percentage points.

Appendix

6.1 Derivations for the Decentralized Labor-Allocation Problem

This appendix provides the algebraic derivations behind the decentralized labor-allocation conditions in Subsection 2.4.1.

6.1.1 Growth-adjusted discount factor

Under the binomial implementation process,

$$N_t^{\text{impl}} \mid M_t \sim \text{Binomial}(M_t, s_t)$$

The growth-adjustment term is

$$G_t \equiv \mathbb{E}_t \left[e^{\gamma N_t^{\text{impl}}} \mid M_t \right]$$

Using the moment-generating function of a binomial random variable,

$$G_t = \left[(1 - s_t) + s_t e^{\gamma} \right]^{M_t} = \left[1 + s_t (e^{\gamma} - 1) \right]^{M_t}$$

Hence, the growth-adjusted discount factor is

$$\tilde{\beta}_t \equiv \beta G_t = \beta \left[1 + s_t (e^{\gamma} - 1) \right]^{M_t}$$

6.1.2 Shadow value of a queued idea

The normalized Bellman equation is

$$j(M_t, \omega_t) = \max_{L_{z,t}, L_{x,t} \geq 0} \left\{ \bar{\pi} s_t M_t - \bar{w}_{z,t} L_{z,t} - \bar{w}_{x,t} L_{x,t} + \tilde{\beta}_t \mathbb{E}_t \left[j(M_{t+1}, \omega_{t+1}) \right] \right\}$$

The expected law of motion for the idea queue is

$$\mathbb{E}_t[M_{t+1}] = (1 - s_t) M_t + \lambda_t$$

Under the stationary certainty-equivalent approximation, the shadow value of an additional queued idea satisfies

$$j_{M,t} = \bar{\pi} s_t + \tilde{\beta}_t (1 - s_t) \mathbb{E}_t[j_{M,t+1}]$$

At a stationary allocation,

$$j_{M,t} = j_M^*, \qquad s_t = s^*, \qquad \tilde{\beta}_t = \tilde{\beta}^*$$

Therefore,

$$j_M^* = \bar{\pi} s^* + \tilde{\beta}^*(1 - s^*) j_M^*$$

Rearranging gives

$$j_M^* = \frac{\bar{\pi} s^*}{1 - \tilde{\beta}^*(1 - s^*)}$$

The denominator is positive under the transversality condition

$$\tilde{\beta}^*(1 - s^*) < 1$$

6.1.3 Idea-creation labor condition

The idea-arrival rate is

$$\lambda_t = \phi z_t^{\theta} L_{z,t}^{\nu} A_t^{\bar{\chi}}$$

Therefore,

$$\frac{\partial \lambda_t}{\partial L_{z,t}} = \phi z_t^{\theta} \nu L_{z,t}^{\nu-1} A_t^{\bar{\chi}}$$

The first-order condition with respect to idea-creation labor is

$$\bar{w}_{z,t} = \tilde{\beta}_t \mathbb{E}_t[j_{M,t+1}] \frac{\partial \lambda_t}{\partial L_{z,t}}$$

Substituting the marginal effect of $L_{z,t}$ on λ_t gives

$$\bar{w}_{z,t} = \tilde{\beta}_t \mathbb{E}_t[j_{M,t+1}] \phi z_t^{\theta} \nu L_{z,t}^{\nu-1} A_t^{\bar{\chi}}$$

On a balanced growth path with $\bar{\chi} = 0$, this becomes

$$\bar{w}_z^* = \tilde{\beta}^* j_M^* \phi (z^*)^{\theta} \nu (L_z^*)^{\nu-1}$$

Using

$$j_M^* = \frac{\bar{\pi} s^*}{1 - \tilde{\beta}^*(1 - s^*)}$$

we obtain

$$\bar{w}_z^* = \tilde{\beta}^* \frac{\bar{\pi} s^*}{1 - \tilde{\beta}^*(1 - s^*)} \phi (z^*)^{\theta} \nu (L_z^*)^{\nu-1}$$

Solving for L_z^* gives

$$L_z^* = \left(\frac{\tilde{\beta}^* \phi \nu \bar{\pi} s^* (z^*)^\theta}{\bar{w}_z^* \left[1 - \tilde{\beta}^* (1 - s^*) \right]} \right)^{\frac{1}{1-\nu}}$$

6.1.4 Implementation-labor condition

The implementation success rate is

$$s_t = 1 - e^{-\eta q_t L_{x,t}}$$

Therefore,

$$\frac{\partial s_t}{\partial L_{x,t}} = \eta q_t e^{-\eta q_t L_{x,t}}$$

The first-order condition with respect to implementation labor is

$$\bar{w}_{x,t} = M_t \frac{\partial s_t}{\partial L_{x,t}} \left[\bar{\pi} - \tilde{\beta}_t \mathbb{E}_t [j_{M,t+1}] \right]$$

Substituting the derivative of s_t gives

$$\bar{w}_{x,t} = M_t \eta q_t e^{-\eta q_t L_{x,t}} \left[\bar{\pi} - \tilde{\beta}_t \mathbb{E}_t [j_{M,t+1}] \right]$$

At a stationary allocation,

$$\bar{w}_x^* = M^* \eta q^* e^{-\eta q^* L_x^*} \left[\bar{\pi} - \tilde{\beta}^* j_M^* \right]$$

Rearranging,

$$e^{-\eta q^* L_x^*} = \frac{\bar{w}_x^*}{M^* \eta q^* \left[\bar{\pi} - \tilde{\beta}^* j_M^* \right]}$$

Taking logs gives

$$L_x^* = \frac{1}{\eta q^*} \ln \left(\frac{M^* \eta q^* \left[\bar{\pi} - \tilde{\beta}^* j_M^* \right]}{\bar{w}_x^*} \right)$$

Using the stationary queue condition,

$$M^* = \frac{\lambda^*}{s^*} = \frac{\phi (z^*)^\theta (L_z^*)^\nu}{s^*}$$

and the stationary shadow value,

$$j_M^* = \frac{\bar{\pi} s^*}{1 - \tilde{\beta}^*(1 - s^*)}$$

the implementation-labor expression can also be written as

$$L_x^* = \frac{1}{\eta q^*} \ln \left(\frac{\phi (z^*)^\theta (L_z^*)^\nu \eta q^* \bar{\pi} (1 - \tilde{\beta}^*)}{s^* \bar{w}_x^* \left[1 - \tilde{\beta}^* (1 - s^*) \right]} \right)$$

6.2 Derivation of the Dynamic Complementarity Condition

This appendix derives the condition under which stationary idea-creation labor L_z^* is increasing in stationary implementation labor L_x^* in Subsection 2.4.2.

From the stationary idea-creation labor condition,

$$L_z^* = \left(\frac{\tilde{\beta}^* \phi \nu \bar{\pi} s^* (z^*)^\theta}{\bar{w}_z^* \left[1 - \tilde{\beta}^* (1 - s^*) \right]} \right)^{\frac{1}{1-\nu}}$$

Holding ϕ, ν, $\bar{\pi}$, z^*, and $\bar{w}_z^*$ fixed, the dependence of L_z^* on L_x^* operates through s^* and $\tilde{\beta}^*$. Write

$$\tilde{\beta}^* = \tilde{\beta}(s^*) = \beta G(s^*)$$

Define

$$D(s^*) \equiv 1 - \tilde{\beta}(s^*)(1 - s^*)$$

and

$$C \equiv \frac{\phi \nu \bar{\pi} (z^*)^\theta}{\bar{w}_z^*}$$

Then

$$L_z^* = \left(C \frac{\tilde{\beta}(s^*) s^*}{D(s^*)} \right)^{\frac{1}{1-\nu}}$$

Taking logs,

$$\ln L_z^* = \frac{1}{1 - \nu} \left[\ln C + \ln \tilde{\beta}(s^*) + \ln s^* - \ln D(s^*) \right]$$

Therefore,

$$\frac{d \ln L_z^*}{d s^*} = \frac{1}{1 - \nu} \left[\frac{\tilde{\beta}'(s^*)}{\tilde{\beta}(s^*)} + \frac{1}{s^*} - \frac{D'(s^*)}{D(s^*)} \right]$$

Since

$$D(s^*) = 1 - \tilde{\beta}(s^*)(1 - s^*)$$

we have

$$D'(s^*) = -\tilde{\beta}'(s^*)(1 - s^*) + \tilde{\beta}(s^*)$$

Hence,

$$\frac{d \ln L_z^*}{ds^*} = \frac{1}{1-\nu}\left[\frac{\tilde{\beta}'(s^*)}{\tilde{\beta}(s^*)} + \frac{1}{s^*} - \frac{-\tilde{\beta}'(s^*)(1-s^*) + \tilde{\beta}(s^*)}{D(s^*)}\right]$$

Using

$$\tilde{\beta}(s^*) = \beta G(s^*)$$

we obtain

$$\frac{\tilde{\beta}'(s^*)}{\tilde{\beta}(s^*)} = \frac{G'(s^*)}{G(s^*)}$$

After simplification, the sign of $d \ln L_z^*/ds^*$ is determined by

$$1 - \tilde{\beta}(s^*) + s^* \frac{G'(s^*)}{G(s^*)}$$

Therefore,

$$\frac{dL_z^*}{ds^*} > 0 \quad \Longleftrightarrow \quad 1 - \tilde{\beta}(s^*) + s^* \frac{G'(s^*)}{G(s^*)} > 0$$

Since

$$s^* = 1 - e^{-\eta q^* L_x^*}$$

we have

$$\frac{ds^*}{dL_x^*} = \eta q^* e^{-\eta q^* L_x^*} = \eta q^* (1 - s^*) > 0$$

Therefore,

$$\frac{dL_z^*}{dL_x^*} > 0 \quad \Longleftrightarrow \quad \frac{dL_z^*}{ds^*} > 0$$

This proves the general complementarity condition.

Under the binomial implementation structure,

$$G(s^*) = [1 + s^*(e^\gamma - 1)]^{M^*}$$

Taking logs,

$$\ln G(s^*) = M^* \ln [1 + s^*(e^\gamma - 1)]$$

Therefore,

$$\frac{G'(s^*)}{G(s^*)} = \frac{M^*(e^\gamma - 1)}{1 + s^*(e^\gamma - 1)}$$

Substituting this into the general complementarity condition gives

$$1 - \beta\left[1 + s^*(e^\gamma - 1)\right]^{M^*} + \frac{s^* M^*(e^\gamma - 1)}{1 + s^*(e^\gamma - 1)} > 0$$

Multiplying both sides by $1 + s^*(e^\gamma - 1) > 0$, we obtain

$$1 + s^*(e^\gamma - 1) - \beta\left[1 + s^*(e^\gamma - 1)\right]^{M^*+1} + s^* M^*(e^\gamma - 1) > 0$$

Combining terms gives

$$1 + (M^* + 1)s^*(e^\gamma - 1) > \beta\left[1 + s^*(e^\gamma - 1)\right]^{M^*+1}$$

This is the explicit complementarity threshold.

For small γ,

$$e^\gamma - 1 \approx \gamma$$

The knife-edge condition becomes

$$1 + (M^* + 1)\gamma s^* = \beta(1 + \gamma s^*)^{M^*+1}$$

Using the first-order approximation

$$(1 + \gamma s^*)^{M^*+1} \approx 1 + (M^* + 1)\gamma s^*$$

the knife-edge condition reduces to

$$\beta = 1$$

Therefore, to a first-order approximation,

$$\frac{dL_z^*}{dL_x^*} > 0 \quad \Longleftrightarrow \quad \beta < 1$$

6.3 Derivation of the Two-Force Volatility Decomposition

This appendix derives the two-force decomposition of balanced-growth-path volatility in Subsection 2.4.3.

From the conditional variance expression,

$$Var_t(\Delta \ln A_{t+1} \mid M_t) = \gamma^2 s_t(1-s_t)M_t$$

On the balanced growth path,

$$Var^* = \gamma^2 s^*(1-s^*)M^*$$

Using the stationary queue condition,

$$M^* = \frac{\lambda^*}{s^*}$$

we obtain

$$Var^* = \gamma^2 s^*(1-s^*)\frac{\lambda^*}{s^*} = \gamma^2(1-s^*)\lambda^*$$

Since

$$\lambda^* = \phi(z^*)^\theta (L_z^*)^\nu$$

balanced-growth-path volatility is

$$Var^* = \gamma^2(1-s^*)\phi(z^*)^\theta (L_z^*)^\nu$$

Taking logs gives

$$\ln Var^* = \ln(\gamma^2) + \ln(1-s^*) + \ln\phi + \theta \ln z^* + \nu \ln L_z^*$$

Holding γ, ϕ, z^*, θ, and ν fixed, differentiation with respect to L_x^* gives

$$\frac{d\ln Var^*}{dL_x^*} = \frac{d\ln(1-s^*)}{dL_x^*} + \nu\frac{d\ln L_z^*}{dL_x^*}$$

The implementation success rate is

$$s^* = 1 - e^{-\eta q^* L_x^*}$$

Therefore,

$$1 - s^* = e^{-\eta q^* L_x^*}$$

Taking logs,

$$\ln(1 - s^*) = -\eta q^* L_x^*$$

Hence,

$$\frac{d \ln(1 - s^*)}{dL_x^*} = -\eta q^*$$

Substituting this expression into the derivative of $\ln Var^*$ gives

$$\frac{d \ln Var^*}{dL_x^*} = -\eta q^* + \nu \frac{d \ln L_z^*}{dL_x^*}$$

This is the two-force decomposition.

The first term captures the direct stabilizing effect of implementation labor. The second term captures the indirect idea-expansion effect through the response of idea-creation labor.

6.4 Representative Household and Intertemporal Optimization

This appendix derives the household Euler equation and its stationary implication in Subsection **??**.

The representative household owns firms, consumes the final good, supplies labor inelastically, and trades a one-period risk-free bond. The household's preferences are

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \ln C_t$$

where C_t denotes final-good consumption and $\beta \in (0, 1)$.

Let A_{t+1}^B denote holdings of a one-period risk-free bond purchased at time t. Let Q_t be the time-t price of one unit of the bond that pays one unit of the final good at time $t + 1$. The unnormalized household budget constraint is

$$C_t + Q_t A_{t+1}^B \leq W_{Y,t} L_{Y,t} + W_{z,t} L_{z,t} + W_{x,t} L_{x,t} + \Pi_t^Y + \Pi_t^I + A_t^B$$

Since the final-good sector is perfectly competitive, equilibrium final-good profits satisfy

$$\Pi_t^Y = 0.$$

Define productivity-normalized variables:

$$c_t \equiv \frac{C_t}{A_t}, \qquad a_{t+1} \equiv \frac{A_{t+1}^B}{A_t},$$

$$\bar{w}_{Y,t} \equiv \frac{W_{Y,t}}{A_t}, \qquad \bar{w}_{z,t} \equiv \frac{W_{z,t}}{A_t}, \qquad \bar{w}_{x,t} \equiv \frac{W_{x,t}}{A_t}, \qquad \bar{\Pi}_t^I \equiv \frac{\Pi_t^I}{A_t}$$

Let

$$\Gamma_{t+1} \equiv \frac{A_{t+1}}{A_t}$$

In the present model,

$$\Gamma_{t+1} = e^{\gamma N_t^{\text{impl}}}$$

Dividing the unnormalized budget constraint by A_t, and noting that

$$\frac{A_t^B}{A_t} = \frac{A_t^B}{A_{t-1}} \frac{A_{t-1}}{A_t} = \frac{a_t}{\Gamma_t}$$

the normalized budget constraint becomes

$$c_t + Q_t a_{t+1} \leq \bar{w}_{Y,t} L_{Y,t} + \bar{w}_{z,t} L_{z,t} + \bar{w}_{x,t} L_{x,t} + \bar{\Pi}_t^I + \frac{a_t}{\Gamma_t}$$

The representative household solves

$$\max_{\{c_t, a_{t+1}\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \ln c_t$$

subject to

$$c_t + Q_t a_{t+1} \leq \bar{w}_{Y,t} L_{Y,t} + \bar{w}_{z,t} L_{z,t} + \bar{w}_{x,t} L_{x,t} + \bar{\Pi}_t^I + \frac{a_t}{\Gamma_t}$$

and a standard no-Ponzi or transversality condition.

Let λ_t denote the multiplier on the normalized budget constraint. The Lagrangian is

$$\mathcal{L} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\ln c_t + \lambda_t \left(\bar{w}_{Y,t} L_{Y,t} + \bar{w}_{z,t} L_{z,t} + \bar{w}_{x,t} L_{x,t} + \bar{\Pi}_t^I + \frac{a_t}{\Gamma_t} - c_t - Q_t a_{t+1} \right) \right]$$

The first-order condition with respect to consumption is

$$\frac{1}{c_t} = \lambda_t$$

The first-order condition with respect to bond holdings is

$$Q_t \lambda_t = \beta \mathbb{E}_t \left[\frac{\lambda_{t+1}}{\Gamma_{t+1}} \right]$$

Substituting $\lambda_t = 1/c_t$, we obtain

$$Q_t \frac{1}{c_t} = \beta \mathbb{E}_t \left[\frac{1}{c_{t+1} \Gamma_{t+1}} \right]$$

Therefore, the household Euler equation is

$$Q_t = \beta \mathbb{E}_t \left[\frac{c_t}{c_{t+1} \Gamma_{t+1}} \right]$$

Let the gross risk-free interest rate be

$$R_t^f \equiv \frac{1}{Q_t}$$

Then

$$R_t^f = \left(\beta \mathbb{E}_t \left[\frac{c_t}{c_{t+1} \Gamma_{t+1}} \right] \right)^{-1}$$

Using

$$\Gamma_{t+1} = e^{\gamma N_t^{\text{impl}}}$$

the risk-free rate can also be written as

$$R_t^f = \left(\beta \mathbb{E}_t \left[\frac{c_t}{c_{t+1}} e^{-\gamma N_t^{\text{impl}}} \right] \right)^{-1}$$

Under a stationary normalized allocation,

$$c_t = c_{t+1} = c^*$$

Therefore,

$$Q^* = \beta \mathbb{E} \left[\frac{1}{\Gamma_{t+1}} \right]$$

and

$$R^{f,*} = \left(\beta \mathbb{E} \left[\frac{1}{\Gamma_{t+1}} \right] \right)^{-1}$$

If frontier growth is deterministic on the balanced growth path, then

$$\Gamma_{t+1} = \Gamma^*$$

Hence,

$$Q^* = \frac{\beta}{\Gamma^*}$$

and

$$R^{f,*} = \frac{\Gamma^*}{\beta}$$

Taking logs,

$$\ln R^{f,*} = \ln \Gamma^* - \ln \beta$$

Define

$$g_A^* \equiv \ln \Gamma^*$$

Then

$$\ln R^{f,*} = g_A^* - \ln \beta$$

For small net interest rates, $\ln R^{f,*} \approx r^{f,*}$, so

$$r^{f,*} \approx g_A^* - \ln \beta$$

Since balanced-growth-path productivity growth in the model is

$$g_A^* = \gamma \lambda^*$$

we obtain

$$r^{f,*} \approx \gamma \lambda^* - \ln \beta$$

Thus, the stationary risk-free rate is pinned down by time preference and the balanced-growth-path growth rate of the productivity frontier. Faster frontier growth raises the equilibrium risk-free rate, holding β fixed.

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