Soft Information, Hard Decisions: AI Advising*

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Abstract

While large language models (LLMs) perform well on well-defined tasks, effective prompts are challenging when tasks depend on users' soft traits and latent preferences. We formalize this friction by introducing preference uncertainty—capturing soft information—into a cheap talk framework (Crawford and Sobel, 1982), model soft information communication with AI as the investor's optimal stopping problem with Brownian information flow and propose a novel empirical methodology to test theoretical predictions via LLM simulations. Compared with human advisors, LLMs are not subject to misaligned incentives, but soft information communication is inefficient due to losses from digitization and LLMs' limited memory. An investor generally prefers LLMs trained to be more "opinionated" than her own prior, except when she is most confused and prefers an aligned and equally confused LLM. Our empirical analysis simulates investor profiles based on the Survey of Consumer Finances and conducts role-structured LLM advising experiments—benchmarked against standard portfolio questionnaires—to test these model predictions.

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1 Introduction

The proliferation of artificial intelligence (AI) in advisory roles—spanning financial planning, healthcare, and education—has created a compelling paradox in professional services. While AI advisors demonstrate objective advantages including elimination of conflicts of interest, scalable delivery, and performance comparable to human professionals, their market penetration remains surprisingly limited. In financial markets, the paradox is especially pronounced: although robo-advisors create portfolios with historical returns comparable to those of professional benchmarks (Fieberg et al., 2024) and assist seasoned investors in minimizing behavioral errors (Guo et al., 2022), they account for less than 2% of global assets under management as of 2025 and are expected to experience modest growth in the near future.[1] This limited adoption occurs even as traditional human advisors face well-documented incentive problems—commission-driven recommendations, product pushing, and strategic exploitation of client biases (Bolton et al., 2007; Carlin and Manso, 2011; Chalmers and Reuter, 2020). The persistence of potentially biased human advisors over unbiased AI alternatives suggests that current AI systems cannot replicate crucial aspects of human advisory value.

Effective AI outputs rely on well-designed prompts, which are especially challenging for advisory tasks which often involve users' *soft traits*—-the subjective, often unarticulated preferences, constraints, and goals that clients themselves may not be able to communicate clearly or even fully understand (Liberti and Petersen, 2019). Unlike prediction tasks involving external outcomes such as asset returns, advisory excellence requires understanding the individual client's internal landscape of uncertainty. A human advisor can help clients clarify their preferences through interpreting their tone, facial expressions, and body languages; ask targeted follow-up questions; reframing choices using familiar analogies. However, large language models (LLMs) face fundamental architectural constraints that create a novel form of information loss distinct from the strategic distortions in human advising. When clients cannot articulate their needs through effective prompts, and when LLMs' limited memory (see, e.g., Sam Altman, "people want memory") prevents synthesis across extended conversations, the result is generic recommendations that overlook the nuanced requirements underlying high-stakes decisions.

This paper develops a formal framework for AI-based advising via LLMs, and validates model predictions through LLM-driven simulations that generate multi-turn, role-structured conversations. We introduce *preference uncertainty*—capturing soft information—into a canonical cheap talk framework à la Crawford and Sobel (1982), and model the communication of soft information as the investor's optimal stopping problem with Brownian in-

[1] See analysis in https://resoinsights.com/insight/robo-advisors-in-wealth-management/

formation flow. We identify a novel source of information loss stemming from the frictions inherent in digitizing soft information. The LLM is free of strategic bias, but gives recommendations based on its own misspecified understanding of the investor's preferences, where the misspecification arises endogenously from soft information communication. Consequently, the investor must debias this misunderstanding, and the black-box nature of AI outputs leads to information loss that hinders the investor from making fully informed choices.

In the model, the investor has a quadratic loss utility function and faces two layers of uncertainty. The first—uncertainty about the fundamental state of the world—is standard in the cheap talk literature. The second, which is novel in this paper, is uncertainty about the investor's own objective, which we call *preference uncertainty*, and it captures soft information. For example, the investor does not know whether to target equity or fixed income investment (preference uncertainty); for either potential goals, she does not observe the true asset fundamental (fundamental uncertainty).

This preference can only be learned through communication with an advisor, prior to receiving the advisor's unverifiable recommendation. Beyond modeling convenience, this setting reflects the fact that the investor is a non-expert and "confused": although she understands her own circumstances—such as income and tax status, years until retirement, and general risk attitudes in daily life—she does not know how to formulate the investment problem or how these characteristics map into an investment objective. She must therefore also learn about her own preferences through communication.

For comparison, we briefly discuss the case of a human advison. While the advisor is misaligned, soft information communication is efficient. The problem therefore reduces to standard cheap talk (Crawford and Sobel, 1982), in which strategic distortion limits the amount of credible information the advisor can transmit, leading to information loss.

In contrast, the LLM advisor is unbiased, but communication of soft information is inefficient. First, by its nature, digitization inevitably incurs information loss—a challenge that is amplified when the non-expert investor herself does not know what information is relevant to provide. Second, the limited memory of LLMs prevents synthesis across extended conversations. The Transformer architecture scales quadratically with input length, which makes long-context processing computationally costly and often impractical. Consequently, for advisory tasks involving repeated portfolio adjustments, incorporating extensive chat histories can substantially degrade performance, while emerging agentic memory systems rely on consolidation with inevitable information loss and thus remain far from the nuanced and persistent human memory that retains clients' subtle preferences.

We model the soft information communication with the LLM as the investor's optimal stopping problem with Brownian information flow. Before seeking a recommendation (stop-

ping), the investor discusses her situation, generating public signals about her underlying preferences that she also comes to learn. The LLM, however, only partially learns from the chat history to its limited memory. In the baseline case, consistent with the stateless context windows and knowledge reset of the Transformer architecture, we assume that the LLM only learns from the most recent prompt, which, in the continuous-time limit, implies that its belief remains fixed at the pretrained prior. We also consider an extension in which the LLM may randomly miss each signal. The investor decides when to stop the conversation and seek recommendation, weighing the informational value of continued learning against information costs.

When the investor stops communication, she receives a recommendation from the LLM, which is based on the LLM's posterior belief about the investor's preferences. Hence, the investor debiases the recommendation; for example, she adjusts the portfolio to be more equity-oriented if she is more aggressive than perceived by the LLM. In addition, the AI output is generated through a black-box process that offers little transparency about the underlying asset fundamentals. As a result, the investor only partially learns about the underlying equity and fixed-income fundamentals from the recommendation and is therefore subject to residual fundamental uncertainty as well. Taken together, when consulting an LLM, information loss also arises endogenously from inefficiencies in the communication of soft information.

We solve the investor's optimal communication policy in closed form for the baseline model. In equilibrium, the investor follows a threshold strategy: she continues interacting with the LLM while her belief about her preference remains within an intermediate range and stops once she becomes sufficiently confident about her preference—either above an upper threshold or below a lower one. This policy reflects a trade-off between the option value of learning and information costs.

A central model implication is that the optimal LLM pretraining for an investor is generally not one that simply mirrors her prior beliefs, but rather one that is deliberately more "opinionated" in the same direction. If the investor were to directly seek recommendations immediately—for example, an investor with extreme priors in the stopping region—she always prefers an aligned LLM that shares her belief. For intermediate priors, however, the option value of learning tilts her preference toward a more "opinionated" LLM that leans in the direction of her prior. Although her belief follows a martingale, the informational gain in the more extreme direction dominates, in which case an opinionated LLM is more informative about asset fundamentals. One exception is when she is most "confused" with a prior of 0.5, where she optimally prefers the LLM to be aligned and equally "confused."

We derive a set of testable implications that connect the model's primitives to observable

advising behavior and outcomes. In the paper, we focus on four simulation-testable hypotheses. H1 posits that the primary value of interacting with a short-memory LLM advisor is that investors learn about their own initially uncertain preferences; the LLM's recommendation may remain generic, but dialogue helps investors reduce "preference uncertainty" and choose portfolios closer to their true objectives. H2 predicts that higher impatience induces earlier termination of the conversation, yielding less tailored portfolios. H3 predicts that giving the LLM persistent access to memory improves recommendation quality and narrows the gap with a human advisor. H4 tests the model's prediction about optimal AI training: investors benefit from opinionated advisors whose recommendations are more extreme than their own prior beliefs, rather than merely aligned advisors.

We complement the theory with prompt-based LLM simulations that generate multi-turn, role-structured conversations approximating real-world advisory exchanges. We draw a subsample from the 2022 public-use Consumer Finances Survey (SCF) to generate n = 500 hypothetical investor profiles for simulation (fixed seed). Investor "ground truth" comes from the 11-question Investor Questionnaire provided by Vanguard; we obtain each investor profile's rule-based "optimal" stock/bond allocation, and use these as benchmarks in evaluation. The advisor is implemented with OpenAI's GPT-5 under strict prompting. We compare a memoryless advisor, which is fed only the most recent Q&A and thus approximates fixed priors due to context-length limits, with a memory-augmented advisor that can use full chat history, and a full-information counterfactual that observes the complete profile. Conversations conclude according to an optimal stopping rule, modeled as a 0.10 exogenous probability of termination per round. The investor assesses the costs and benefits in each round and may choose to end the conversation early, with a maximum of eleven questions permitted.

Our findings, drawn from 2,500 conversations (500 profiles each undergoing 5 interactions), reveal several key patterns. First, supporting H1, it's clear that the act of interacting itself leads to most of the observed improvements: accuracy rises by 14.1 percentage points even before any specific recommendations, and by 15.7 points afterward. Each additional round of exchange adds about 1.03 points to accuracy, while every extra word contributes roughly 0.017 points. Second, in line with H2, we find that ending conversations prematurely—outside the advisor's control—reduces accuracy by around 2.62 points. Even when accounting for total rounds, this drop remains substantial at about 0.96 points, indicating a real penalty for impatience. Third, as H3 predicts, access to memory significantly boosts recommendation quality: the system works best when it has full access to information, performs next best with full memory, and does worst with no memory at all. Fourth, to match the binary preference in the model, we focus on a subsample of 50 profiles from the tails

of the allocation distribution with 30,250 observations and confirm H4: the empirical optimal advisor recommendation curve qualitatively matches the theoretical prediction, with investors benefiting from advisors whose recommendations are more extreme than their own prior beliefs.

Methodologically, this paper introduces an innovative approach to testing economic theory by complementing our analytical framework with prompt-based LLM simulations. This technique allows us to generate dynamic, multi-turn conversations that approximate real-world advisory exchanges, bridging the critical gap between theoretical rigor and empirical realism. Unlike traditional laboratory experiments or analytical models, these simulations enable the direct observation of complex mechanisms, such as information acquisition, belief updating, and optimal stopping, as they unfold within a controlled yet realistic interactive setting. Although recent literature has begun to explore the potential of using LLMs as economic agents to study behavioral patterns and simulate empirical regularities in human decision-making (Horton, 2023; Ouyang et al., 2024), our paper is among the first to leverage LLM-driven simulations explicitly as a tool for testing new economic theories. This distinguishes our approach from recent work that uses LLMs to replicate known behavioral regularities or to simulate agents in strategic games (Anand et al., 2025; Lopez-Lira, 2025). Our two-sided LLM framework, where both the advisor and investor are simulated, enables us to test novel theoretical predictions about soft information transmission and optimal stopping that would be difficult to examine in traditional laboratory settings. This approach opens new possibilities for economic research in domains where communication and context-dependent human behavior play a central role, offering a promising path for validating theoretical predictions.

Literature. This paper contributes to two rising strands of research: Artificial intelligence in financial advising, and the technology trend that transforms soft information into hard data. More broadly, theoretical research on AI technologies has primarily focused on their effects on labor (e.g., Ide and Talamas, 2024), while this paper examines their specific application in providing information within advisory roles.[2]

AI Advising. We build on classic cheap talk and financial advising models and introduce preference uncertainty to capture soft information. We highlight the key features of AI advisors by comparing them with human advisors, who are misaligned à la the classic cheap talk (Crawford and Sobel, 1982). This seminal framework is later applied to analyst settings where reputational and underwriting incentives drive distorted advice—see Bénabou and

[2]Theoretical research on AI in finance is still emerging. For example, Chen and Han (2024) show supervised AI intensifies agency conflicts.

Laroque, 1992; Ottaviani and Sørensen, 2006; Rüdiger and Vigier, 2019.[3] Our paper explores AI advisors as a new alternative and identifies their limitations: while unbiased, AI systems face challenges in interpreting soft, contextual information.

Our findings contribute to the growing literature on financial technology and AI-driven advisory services, extending the review by Mo and Ouyang (2025). Prior work shows that robo-advisors can improve investor outcomes: D'Acunto et al. (2019) find better diversification and reduced behavioral biases, while Rossi and Utkus (2024) report improved indexing, Sharpe ratios, and lower fees, especially for under-diversified investors. Yet challenges remain. Chak et al. (2022) show that even when robo-advice enhances debt choices, low algorithmic trust can limit uptake. Similarly, Andries et al. (2024) find that human advisors' effectiveness varies with available information. Recent LLM advances offer new potential: Lu et al. (2023) and Fieberg et al. (2024) show LLMs can generate effective, personalized investment advice. Field evidence from Guo et al. (2022) finds experienced, risk-averse investors benefit most from conversational AI advisors. Building on this, our paper develops a theoretical model of AI vs. human advising, validated through prompt-based simulations where an LLM engages in iterative dialogue with an investor who chooses when to stop.

This raises the question of whether human experts still offer distinct value in an era of automation. While robo-advisors excel at technical tasks, human advisors provide complementary "soft" services like emotional support and trust-building, which aid prudent decision-making (Linnainmaa et al., 2018; Gennaioli et al., 2015). Human discretion remains valuable: Costello et al. (2020) show it improves outcomes by incorporating private context, and Greig et al. (2024) find that hybrid platforms enhance investor retention and confidence. Similarly, Cao et al. (2024) find that while AI excels in forecasts, humans outperform in tasks needing institutional insight, with combined approaches performing best in uncertain, data-scarce settings. Our findings reinforce that human advisors play a complementary role, especially where nuance, context, and trust matter.

Hardening soft information. Our paper is related to the literature of soft versus hard information, as well as the rise of Big Data and machine learning technologies that transform soft, subjective information into hard, objective data. The literature on soft vs. hard information (e.g., Stein, 2002; Liberti and Petersen, 2019) emphasizes that hard information is verifiable and thus transferable within organizations, while soft information is often non-verifiable and modeled as cheap talk (e.g., Bertomeu and Marinovic, 2016; Corrao, 2023). Recent advances in Big Data and AI have made it possible to digitize even contextual, traditionally soft information. He et al. (2024) examines this technology trend in the context of credit market

[3]Cheap talk is also applied in corporate-governance settings where boards or proxy advisors offer non-binding recommendations (e.g., Levit and Malenko, 2011; Malenko and Malenko, 2019).

competition. In our paper, investor communication with the LLM is exactly the process that hardens a fraction of the soft information. However, we emphasize that LLMs face memory constraints that limit their ability to fully process and transform soft information.

The remainder of the paper proceeds as follows. Section 2 introduces the model. Section 3 characterizes equilibrium under human advising and the equilibrium under AI advising separately. Section 4 presents our empirical analysis. Section 5 concludes and discusses practical and policy implications as well as avenues for future research.

2 The Model

A decision maker seeks advice from a better informed advisor—either a human or an AI advisor (large language model, or LLM)—before taking action. To fix ideas, we use the context of financial advising throughout the paper and refer to the decision maker as the "investor." However, the model applies to a broad range of advisory applications.

2.1 Agents

We first introduce the agents' objectives and decision making given their information. The investor's dynamic communication with the LLM will be separately introduced in Section 2.2.

2.1.1 Investor

We build on the cheap talk framework (Crawford and Sobel, 1982) to characterize the advisory problem. Our key innovation is to model the *quality of prompts* for AI, which becomes critical when the task involves investors' soft traits and effective advising requires the communication of soft information.

Specifically, in addition to the standard uncertainty about the realization of states, we introduce a second layer of uncertainty about the investor's optimization problem to capture soft information, which we refer to as "preference uncertainty." To characterize this friction in the simplest possible way, we assume that the investor has a quadratic loss utility function and chooses an action a to match the realization of a fundamental state. However, it is uncertain which fundamental—either $\tilde{\theta}_1$ or $\tilde{\theta}_0$—is the relevant target. That is, the investor's objective function is either

$$-\mathbb{E}(a-\tilde{\theta}_1)^2,$$

or

$$-\mathbb{E}(a-\tilde{\theta}_0)^2.$$

We introduce $\omega \in \{1, 0\}$ to refer to the investor's underlying preference: $\omega = 1$ for $\tilde{\theta}_1$ and $\omega = 0$ for $\tilde{\theta}_0$. We interpret preference uncertainty as soft information, and refer to the two terms interchangeably in the paper.

We assume that ω is unobservable to all agents, including the investor, and can be learned only through communication conveying the investor's soft traits. In financial advising, although the investor understands her own circumstances—such as income and tax status, years until retirement, and general risk attitudes in daily life—she does not know how to formulate the investment problem or how these characteristics map into ω, for example into equity ($\tilde{\theta}_1$) versus fixed income ($\tilde{\theta}_0$) investment. To fix ideas, throughout the paper we use this example of "equity vs fixed income" throughout the paper. More broadly, this setting applies when inexperienced decision makers face complex, personalized choices, such as selecting medical insurance or self-diagnosing medical conditions.

As we discuss below, only human interactions fully elicit soft information ω. By contrast, when interacting with an LLM, soft information must be digitized into prompts, which inevitably entails information loss, and the LLM's limited memory further prevents synthesis across extended exchanges.

As standard in the literature, the realizations of $\tilde{\theta}_1$ and $\tilde{\theta}_0$, denoted by θ_1 and θ_0, are unobservable to the investor but observable to the advisors. Throughout the paper, we use $\tilde{\theta}_i$ to denote random variables and θ_i to denote their realizations. We refer to uncertainty about θ_1 and θ_0 as *fundamental uncertainty*, to distinguish it from *preference uncertainty*, which concerns ω. We assume that $\tilde{\theta}_1$ and $\tilde{\theta}_0$ are independent normal random variables,

$$\tilde{\theta}_1 \sim N(\mu_1, \sigma_\epsilon^2), \quad \tilde{\theta}_0 \sim N(\mu_0, \sigma_\epsilon^2),$$

where μ_i is the mean and σ_ϵ^2 is the variance of $\tilde{\theta}_i$ for $i \in \{1, 0\}$.

We introduce $p \in [0, 1]$ as the investor's belief about her preference:

$$p \equiv \mathbb{P}^i(\omega = 1). \tag{1}$$

Then, the investor's utility is

$$U(a, p, \theta_1, \theta_0) = -p\,(a - \theta_1)^2 - (1 - p)\,(a - \theta_0)^2\,. \tag{2}$$

We assume that, prior to making a decision, the investor seeks advice from either a human advisor or an LLM. We do not explicitly analyze the investor's optimal choice of her advisor. The paper focuses on AI advising, while the case of a human advisor is discussed briefly for comparison and to sheds light on contexts where each advisor may be better suited.

2.1.2 Human advisor

A human advisor can efficiently collect soft information ω through human interactions (Liberti and Petersen, 2019), but has misaligned incentives (Crawford and Sobel, 1982).

Formally, when the investor consults a human advisor, a public signal perfectly reveals the investor's preference ω. As explained, the investor possesses preference-related information (e.g., career status and risk appetite) but does not know how it maps into ω. The human advisor understands this mapping and elicits the relevant information through interaction—by interpreting tone, facial expressions, and body languages; ask targeted follow-up questions; reframing choices using familiar analogies; and iteratively help the client clarify her preferences when she herself is uncertain how to express them—thereby uncovering ω. As a result of this conversation, both parties know the target fundamental asset is $\tilde{\theta}_\omega$.

Because the paper focuses on AI advising, this formulation rules out strategic communication by the advisor about the investor's preference ω in order to induce a preferred action for simplicity. This assumption is also consistent with industry practice: an initial consultation elicits preferences and assigns the investor to a specialist—one skilled at $\tilde{\theta}_1$ or $\tilde{\theta}_0$. Because these specialists are distinct individuals, there is no scope for strategic communication about ω.

Once ω is revealed, the advising problem with a human advisor reduces to a classic cheap talk framework à la Crawford and Sobel (1982). The advisor is biased by $b > 0$, with utility function

$$U^h(a, \theta_\omega) = -(a - (\theta_\omega + b))^2. \tag{3}$$

This bias reflects commission-based incentives that tilt the advisor toward recommending greater investment. As a result, the advisor's preferred action is $\theta_\omega + b$, which exceeds the asset's fundamental value θ_ω.

The human advisor perfectly observes the realized θ_ω and sends an unverifiable recommendation m^h (with the superscript "h" denoting "human").[4] Upon receiving m^h, the investor updates her belief about θ_ω and chooses an action $a(m^h)$ to maximize her expected payoff as defined in Eq. (2),

$$\max_a \mathbb{E}[U(a, \omega, \theta_\omega)|m^h] = -\mathbb{E}[(a - \tilde{\theta}_\omega)^2|m^h]. \tag{4}$$

This action $a(m^h)$ in turn determines the advisor's payoff in (3).

[4]In practice, recommendations could be partially verifiable due to fiduciary duties or ex post scrutiny. As long as misreporting is not always detected and penalties do not increase with the magnitude of the lie (Kartik, 2009), the key logic of Crawford and Sobel (1982) applies.

2.1.3 LLM

Seeking advice from large language models (LLMs) is convenient and represents a central aspect of AI use. For example, nearly half of ChatGPT usage involves practical guidance or seeking information (Chatterji et al., 2025).

Unlike human advisors, LLMs are not biased: they are trained to minimize prediction errors, and any developer objectives beyond this—such as boosting subscriptions—are likely orthogonal to recommendation bias in specific applications. However, successful AI advising relies on *effective prompts*, which are challenging when investors' soft traits are involved. The communication of soft information ω with AI is subject to two frictions.

First, by its nature, digitizing soft information inevitably incurs information loss (Liberti and Petersen, 2019). This problem is exacerbated when the investor is a non-expert who struggles to formulate portfolio decisions: she may not know which information is relevant to provide, and the inclusion of irrelevant or misleading details can degrade the quality of interactions, resulting in a "garbage in, garbage out" outcome.

Second, LLMs operate within limited, largely stateless context windows, and this short memory prevents them from aggregating all relevant information across prompts to uncover soft information.[5] This limitation arises because the Transformer architecture underlying most LLMs scales quadratically with input length, making long-context processing computationally costly and often impractical. The issue is particularly acute for advising tasks involving soft traits: in repeated portfolio adjustments, including extensive chat histories to remind the AI of investor preferences can substantially degrade performance (Liu et al., 2023), while emerging Agentic memory systems rely on coarse summarization and remain far from the nuanced, persistent memory humans use to recollect clients' subtle preferences.

Suppose the LLM observes the fundamental realizations θ_1 and θ_0, and let $\hat{p} \equiv \mathbb{P}^L(\omega = 1) \in [0,1]$ denote the LLM's belief about ω. Both the investor's belief $p = \mathbb{P}^i(\omega = 1)$ and the LLM's belief $\hat{p}$ are endogenously determined by the investor's optimal stopping communication with the LLM, later introduced in Section 2.2. The investor can infer $\hat{p}$ from the LLM's pretrained prior $\hat{p}_0$ and its belief updates, but not vice versa: p is soft information and is not observed by the LLM advisor. The unbiased LLM's payoff U^L equals the investor's payoff as conjectured by the LLM under belief $\hat{p}$:

$$U^L(a, \hat{p}, \theta_1, \theta_0) = U(a, \hat{p}, \theta_1, \theta_0) = -\hat{p}\,(a - \theta_1)^2 - (1 - \hat{p})\,(a - \theta_0)^2\,. \tag{5}$$

[5]Each response is newly generated based solely on the current context, without incorporating any algorithmic memory of previous responses. Certain LLMs such as ChatGPT may include prior texts into the working memory, creating the impression of memory.

Maximizing this payoff, the LLM recommends

$$m^L = \hat{p}\theta_1 + (1-\hat{p})\theta_0. \tag{6}$$

The investor understands that m^L is based on the LLM's conjecture about the investor's preferences, and therefore treats m^L as a noisy signal of θ_1 and θ_0 as given in Eq. (6). She then chooses her optimal action $a(m^L, \hat{p})$ to maximize her expected utility, given her belief that $\omega = 1$ with probability p:

$$g(p, \hat{p}|m^L) \equiv \max_a \ \mathbb{E}[U(a, p, \theta_1, \theta_0)|\, m^L, \hat{p}] = -p\mathbb{E}[(a-\theta_1)^2|m^L, \hat{p}] - (1-p)\mathbb{E}[(a-\theta_0)^2|m^L, \hat{p}]. \tag{7}$$

For example, if $\hat{p} = 1$, the recommendation fully tilts toward θ_1, $m^L = \theta_1$. The investor therefore debiases the recommendation and shifts her action toward $\tilde{\theta}_0$; at the same time, she perfectly learns θ_1 but obtains no information about θ_0.

Let $a^*(m^L, \hat{p})$ denote the optimal action, and let $g(p, \hat{p}|m^L)$ denote the resulting investor payoff. Then, for any pair of the investor's belief p and the LLM's belief $\hat{p}$, the investor's expected utility from seeking a recommendation is

$$g(p, \hat{p}) \equiv \mathbb{E}[g(p, \hat{p}|m^L)] = \mathbb{E}[U(a^*(m^L, \hat{p}), p, \theta_1, \theta_0)]. \tag{8}$$

The investor seeks a recommendation when communication stops. Thus, Eq. (8) captures the investor's stopping value at any belief pair $(p, \hat{p})$ in the communication process introduced next.

Remark 1. One may argue that the LLM should directly report the values of θ_1 and θ_0, so that the investor can take the first-best action. However, in practice, it is infeasible to report θ_1 and θ_0 to the investor due to the "black-box" nature of AI outputs. Our simple two-state structure is an abstraction of the LLM's complex underlying algorithms, and "reporting θ_1 and θ_0" corresponds to providing intermediate reasoning steps, from which a typical user would be unable to infer θ_1 and θ_0 in a meaningful way.

2.2 Communication with the LLM

The investor can engage in multiple rounds of communication with the LLM to discuss about her needs ω. Communication is in continuous time, starting at $t = 0$ with an infinite horizon. Each round lasts an interval dt, during which information about the investor's preference ω is revealed through a Brownian diffusion process introduced below.

Communication may end exogenously with a Poisson shock of intensity λ, capturing

the possibility that the investor becomes distracted. At each time t, conditional on the Poisson shock not having occurred, the investor chooses whether to stop communication. If she continues during the interval $[t, t+dt)$, she incurs a cost of $cdt > 0$. Whenever the communication stops—either endogenously by the investor or exogenously by a Poisson event—the investor receives recommendation m^L as defined in (6) and the game stops.

If at time t the game has not yet ended exogenously and investor chooses to stop communication, her payoff is

$$-ct + g(p_t, \hat{p}_t),$$

where p_t and $\hat{p}_t$ are the investor's and the LLM's belief about ω at time t, respectively, and $g(p, \hat{p})$ is the stopping value defined in (8).

Remark 2. (Impatience) There are multiple ways to introduce the cost of communication, which do not affect the qualitative result of our analysis. We use a Poisson shock rather than a discount rate primarily to capture the practical motivation that AI consultation is less engaging and more likely to be disrupted. Moreover, using a discount rate would require adjusting utility levels, since the quadratic loss utility is always negative, which could lead the investor to never stop to avoid realizing this negative payoff. The additional flow cost c is needed because Poisson stopping per se does not penalize continued communication; without c, the value function becomes convex and the investor never stops endogenously.

Signals about soft information ω. During communication, information about the investor's true preference ω—where $\omega = 1$ corresponds to matching $\tilde{\theta}_1$ (equity) and $\omega = 0$ to $\tilde{\theta}_0$ (fixed income)—is revealed through a continuous-time signal process modeled as a Brownian diffusion with drift ω. Specifically, the signal process evolves according to

$$ds_t = \omega dt + \sigma dB_t, \tag{9}$$

where $B = \{B_t, \mathcal{F}_t, 0 \leq t \leq \infty\}$ is standard Brownian motion on the canonical probability space. The signal ds_t in (9) is more informative if σ is small. At each time t, the entire history of signals, $\{s_\tau, 0 \leq \tau \leq t\}$, is publicly observable. However, as will be discussed later, the LLM only partially learns from the signals due to its limited memory.

2.2.1 Belief updating

Investor's belief p. The investor holds a prior belief p_0 that $\omega = 1$. At time t, the investor's belief p_t is conditioned on the history of past communication, or the filtration $\mathcal{F}_t^s$ generated from $\{s_\tau, 0 \leq \tau \leq t\}$. Let f_t^ω denote the density of s_t conditional on ω, which is normally

distributed with mean ωt and variance $\sigma^2 t$, i.e., $s_t \sim N(\theta t, \sigma^2 t)$. The investor's posterior belief p_t satisfies the Bayes' rule

$$p_t = \frac{p_0 f_t^1(s_t)}{p_0 f_t^1(s_t) + (1-p_0) f_t^0(s_t)}. \tag{10}$$

This implies that the log-likelihood ratio $z_t \equiv \frac{p_t}{1-p_t}$ evolves according to a simple process, from which we derive the investor's belief process $\{p_t\}$. From Eq. (10):

$$z_t = z_0 + \ln \frac{f^1(s_t)}{f^0(s_t)} = z_0 + \frac{1}{\sigma^2}(s_t - \frac{t}{2}), \tag{11}$$

and thus

$$dz_t = \frac{1}{\sigma^2}\left(ds_t - \frac{1}{2}dt\right).$$

From the investor's perspective, $\omega = 1$ with probability p_t, so the signal is released according to $ds_t = p_t dt + \sigma dB_t$, where $p_t = \frac{e^{z_t}}{1+e^{z_t}}$. Therefore, the evolvement of investor's belief measured in log-likelihood ratio is

$$dz_t = \frac{1}{\sigma^2}\left(\frac{e^{z_t}}{1+e^{z_t}} - \frac{1}{2}\right)dt + \frac{1}{\sigma}dB_t. \tag{12}$$

Using Ito's Lemma and $z_t(p_t) = \ln \frac{p_t}{1-p_t}$, we show that p_t evolves according to the standard binary state Brownian signal formula (for details, see Appendix A.1),

$$dp_t = \frac{p_t(1-p_t)}{\sigma}dB_t. \tag{13}$$

There are a few thing worth noting. First, the belief process $\{p_t\}$ is a martingale without a drift term, reflecting prior consistency—the expectation of posterior beliefs equals the prior. Second, the term $\frac{1}{\sigma}$ is the signal-to-noise ratio: the numerator equals the gap in drifts when $\omega = 1$ versus when $\omega = 0$. Finally, the belief p_t is absorbing at $p_t = 0$ or 1: once the investor becomes certain about ω, there is nothing further to learn.

LLM's belief $\hat{p}$: baseline. As discussed in Section 2.1.3, LLMs have a short memory and their knowledge is reset after each prompt. Consistent with this feature, our baseline model assumes that the LLM advisor exhibits *one-shot memory*: it updates its belief solely based on signal ds_{t^-} from the most recent round of communication. In Appendix A.1, we use a standard Binomial approximation to show that, in the continuous-time setting, one-shot memory implies that the LLM's belief remains effectively fixed at its prior or pre-training

$\hat{p}_0$:

$$\hat{p}_t = \mathbb{E}[\omega = 1 | ds_{t^-}] = \hat{p}_0. \tag{14}$$

Although the LLM's belief remains constant, it still plays an important role in the model because the pre-training prior can vary.

The baseline captures the following scenario in practice: users who are uncertain about their needs and unable to articulate their soft traits provide prompts that convey little information to the LLM, which then generates recommendations based on a "typical" customer profile—or its pretrained prior $\hat{p}_0$. Meanwhile, the user herself gradually learns about her own preference through communication, as reflected in the evolution of p_t.

LLM's belief $\hat{p}$: extension. We also consider an extension with built-in LLM memory functions. In practice, the LLM can incorporate chat histories into the working memory (for example, ChatGPT within a single chat), or relying on long-term agentic memory. However, the former leads to excessive input length which significantly hurts performance,[6] while the latter relies on summarization of past interactions. Both therefore result in information loss.

In the model, we assume that the LLM only *partially* learning from the signals. In each interaction, the LLM absorbs the signal ds_t with probability $\kappa \in [0, 1)$ and updates its belief $\hat{p}_t$; otherwise, with probability $1 - \kappa$, it misses this signal and does not update. We assume that the events of missing the signal are independent over time.

Consider the LLM's belief as measured in the log-likelihood ratio, $\hat{z}_t \equiv \ln \frac{\hat{p}_t}{1-\hat{p}_t}$. Importantly, if the LLM misses signal ds_t, this is equivalent to receiving an alternative signal $d\tilde{s}_t$ with infinite noise. In this case, the belief update—which takes the same form of Eq. (12)—is

$$d\hat{z}_t = \lim_{\tilde{\sigma}\to\infty} \frac{1}{\tilde{\sigma}^2}\left(d\tilde{s}_t - \frac{1}{2}dt\right) = 0.$$

If instead the LLM absorbs ds_t, it shares the same belief update as the investor, $d\hat{z}_t = dz_t$.

Therefore, when measured in log-likelihood ratio, the LLM's belief update is exactly κ fraction of that of the investor,[7]

$$d\hat{z}_t = \kappa dz_t. \tag{15}$$

[6] The Transformer architecture underlying most LLMs correlates every input token (in the context window) with the whole universe of tokens, making long inputs computationally intensive and less efficient.

[7] One can consider the following microfoundation. Let $\{X_i\}$ be iid Bernoulli random variables with $\mathbb{P}(X_i = 1) = \kappa$. Then

$$\hat{z}_t - \hat{z}_0 = \int_0^t \mathbf{1}_{X_s} dz_t = \lim_{n\to\infty} \sum_{s=0}^{tn-1} \left[\lim_{K\to\infty} \sum_{k=1}^{K} \mathbf{1}_{X_{sk}} \frac{1}{K}\left(z_{\frac{s+1}{n}} - z_{\frac{s}{n}}\right)\right] = \lim_{n\to\infty} \sum_{s=0}^{tn-1} \kappa\left(z_{\frac{s+1}{n}} - z_{\frac{s}{n}}\right) = \kappa \int_0^t dz_t,$$

where the second last equation applies the Law of Large Numbers.

Equivalently,

$$\hat{z}_t = \hat{z}_0 + \int_0^t d\hat{z}_s = \hat{z}_0 + \kappa(z_t - z_0). \tag{16}$$

Eq. (16) implies that the LLM's belief $\hat{p}_t$ is a function of the investor's belief p_t.

Note that this extension nests the baseline model as the special case $\kappa = 0$: the LLM never learns from the conversation, and its belief remains fixed at prior q_0. At the other extreme, when $\kappa = 1$, the LLM shares the same belief as the principal. For the extension case, we assume that κ is sufficiently small to be close to the baseline model.

Remark 3. The investor's belief p_t is a sufficient statistics for her own information. However, she cannot summarize this belief in a prompt that can be conveyed to the LLM. This limitation reflects the nature of soft information: while the investor understands p_t herself, she is unable to articulate and digitize it in her prompt. Moreover, if she were able to partially convey p_t to the LLM, the setting would closely resemble our extension in which the LLM partially learns from the entire communication history.

Remark 4. (Asymmetric information) In our model, the investor is rational whereas the LLM is not. Given the LLM's learning in (16), the investor conjectures the LLM's posterior belief $\hat{p}_t(p_t)$ based on her own belief p_t. The LLM, however, is a "machine" rather than a strategic agent: it updates its belief mechanically but does not infer the investor's belief p_t from (16). If both parties were rational, asymmetric information would not arise in this extension.

2.3 Equilibrium Definition

Consultation with the human advisor. Suppose the investor consults with the human advisor and her preference is revealed as ω.

Definition 1. When the advisor is human, an equilibrium consists of the advisor's signaling rules, denoted by $\pi(m^h|\theta_\omega)$, and an action rule for the investor $a(m^h)$, such that:

(i) for any realization of θ_ω, $\int \pi(m^h|\theta_\omega)dm^h = 1$, and if m^{h*} is on the support of $\pi(\cdot|\theta_\omega)$, then m^* solves $\max_{m^h} U^h(a(m^h), \theta_\omega)$ where $U^h(a, \theta_\omega)$ is given in Eq. (3).

(ii) for each m^h, $a(m^h)$ solves $\max_a \mathbb{E}[\, U(a, \omega, \theta_\omega)]$ as in Eq. (4).

Consultation with the LLM. Given the evolvement of her belief $\{p_t\}$, the investor faces an optimal stopping problem

$$(SP) \quad \sup_{\tau \geq 0} \mathbb{E}^\omega \left\{ \int_0^\tau e^{-\lambda t}[-c + \lambda g(p_t, \hat{p}_t(p_t))]dt + e^{-\lambda\tau} g(p_\tau, \hat{p}_\tau(p_\tau)) \right\},$$

where her belief p_t evolves according to Eq. (13). With probability $e^{-\lambda\tau}$, the game has not stopped by time τ, allowing the investor to end communication voluntarily and receive $g(p_\tau, \hat{p}_\tau)$ in Eq. (8). At any earlier time $t < \tau$, the probability that the game has not yet ended is $e^{-\lambda t}$; during the next time interval dt, the investor incurs communication cost cdt, and with probability λdt, the game ends exogenously and she receives $g(p_t, \hat{p}_t)$.

Definition 2. When the investor consults the LLM, an equilibrium consists of the investor's stopping rule τ, the LLM's recommendation $m^L(\hat{p})$ and the investor's action $a(m^L)$ such that:

(i) Stopping time τ solves the investor's communication problem (SP);

(ii) The process of p_t is given by (12), and the process of $\hat{p}_t$ is implicitly given by (16), with the baseline case of $\kappa = 0$ and $\hat{p}_t = \hat{p}_0$;

(iii) Given any belief $\hat{p} \in [0, 1]$, the LLM's recommendation $m^L(\hat{p})$ is given by Eq. (6) which solves $\max_{m^L} U^L(m^L, \hat{p}, \theta_1, \theta_0)$;

(iv) Given any pair of beliefs $(p, \hat{p})$, the investor's action $a(m^L)$ solves $\max_a \mathbb{E}[U(a, p, \theta_1, \theta_0)|m^L(\hat{p})]$.

3 Equilibrium

We briefly characterize the equilibrium under the human advisor in Section 3.1 as a benchmark for comparison with AI advising. In Section 3.2, we construct the equilibrium with the LLM advisor and solve it in closed form.

3.1 Human advisor: cheap talk

When consulting a human advisor, communication about the soft information is efficient and ω becomes public. However, as in the standard cheap talk literature (Crawford and Sobel, 1982), misaligned objectives between the investor and advisor lead to information loss about the fundamental state θ_ω. Since the human advisor is biased with $b > 0$ and his message m^h is non-verifiable, he has an incentive to exaggerate the fundamental state θ_ω by sending a higher message m^h to induce a higher action a. Anticipating this, the investor rationally discounts the overly optimistic messages. In equilibrium, only partial information about $\tilde{\theta}$ can be credibly communicated. The following proposition, as a direct application of Theorem 1 of Crawford and Sobel (1982), characterizes the equilibrium.

Proposition 1. *Suppose b is sufficiently small. There exists a positive integer $N(b)$ such that for every N with $1 \leq N \leq N(b)$, there exists at least one equilibrium where the support*

of θ_ω is partitioned into intervals by $-\infty = \theta_0 < \theta_1 < \cdots < \theta_N = \infty$, and the human advisor sends a distinct message m_i^h for each interval $[\theta_{i-1}, \theta_i)$. At each threshold θ_i, the advisor is indifferent between adjacent messages:

$$\theta_i = \frac{a(m_i^h) + a(m_{i+1}^h)}{2} - b. \tag{17}$$

Given message m_i^h, the investor takes action

$$a_i = \mathbb{E}[\theta \mid \theta \in [\theta_{i-1}, \theta_i)] = \mu_\omega + \sigma_\epsilon \cdot \frac{\phi(\theta_{i-1}) - \phi(\theta_i)}{\Phi(\theta_i) - \Phi(\theta_{i-1})}, \tag{18}$$

where ϕ and Φ are respectively the PDF and CDF of fundamental state $\theta_\omega \sim N(\mu_\omega, \sigma_\epsilon^2)$.

Proposition 1 shows that the equilibrium takes the form of a partitional signal structure. For each distinct message m_i corresponding to an interval $[\theta_{i-1}, \theta_i)$, the investor's optimal action in Eq. (18) is the conditional expectation—the truncated normal mean over the interval, reflecting her quadratic loss utility in (2). In equilibrium, the advisor must be indifferent at each threshold θ_i, which requires $U^h(a(m_i^h), \theta_i) = U^h(a(m_{i+1}^h), \theta_i)$, or equivalently (17).

We focus on the equilibrium with the most informative signal structure (i.e., the highest number of partitions N). Since preference uncertainty is resolved, the investor's expected utility is depends solely on the residual fundamental uncertainty about θ_ω:

$$\mathbb{E}[U(a(m^h), \omega, \theta_\omega)] \equiv -\sum_{i=1}^{N} \mathbb{P}([\theta_{i-1}, \theta_i)) \cdot \mathrm{Var}(\theta_\omega \mid [\theta_{i-1}, \theta_i)),$$

where $\mathrm{Var}(\theta_\omega \mid [\theta_{i-1}, \theta_i))$ is the variance of a truncated normal distribution over $[\theta_{i-1}, \theta_i]$.[8] The investor enjoys a higher payoff if more information about θ_ω is transmitted, and the extent of information loss is determined by the magnitude of the advisor's bias b. As b increases, the number of partitions decreases, reducing the investor's payoff. In the limit as $b \to \infty$, communication becomes uninformative (babbling equilibrium), and the principal learns only her preference.

[8] The explicit expression is

$$\mathrm{Var}(\theta_\omega \mid [\theta_{i-1}, \theta_i)) = \sigma_\epsilon^2 \left[1 + \frac{\frac{\theta_{i-1}-\mu_\omega}{\sigma_\epsilon} \cdot \phi(\theta_{i-1}) - \frac{\theta_i-\mu_\omega}{\sigma_\epsilon} \cdot \phi(\theta_i)}{\Phi(\theta_i) - \Phi(\theta_{i-1})} - \left(\frac{\phi(\theta_{i-1}) - \phi(\theta_i)}{\Phi(\theta_i) - \Phi(\theta_{i-1})}\right)^2\right],$$

where ϕ and Φ denote the PDF and CDF of $\tilde{\theta}_\omega \sim N(\mu_\omega, \sigma_\epsilon^2)$, respectively.

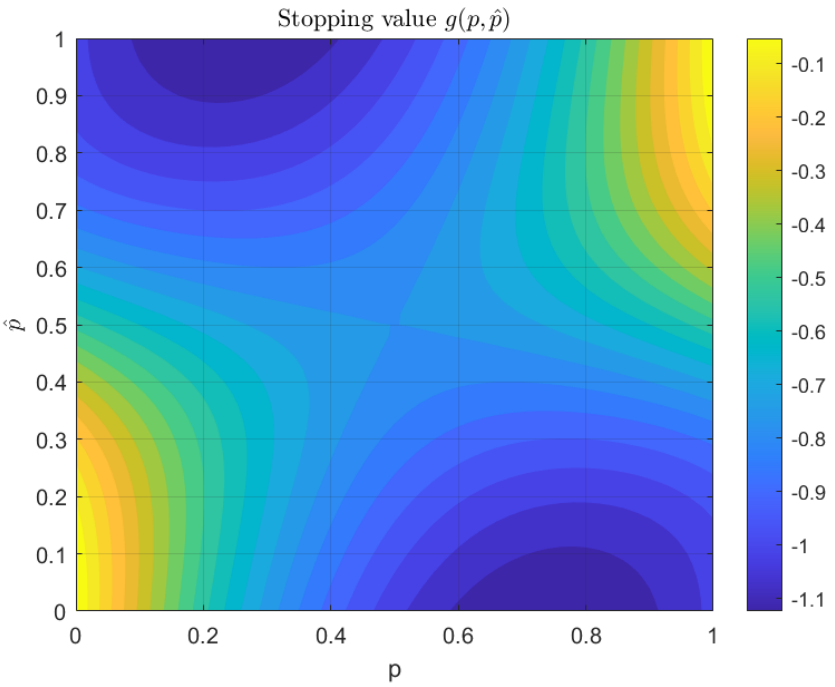


Figure 1: **Investor's stopping value** $g(p, \hat{p})$**.** Figure shows the investor's expected utility when seeking recommendation, $g(p, \hat{p})$, as a function of the investor's belief p in the horizontal axis and the LLM's belief $\hat{p}$ in the vertical axis. Parameters: $\mu_1 = 1,\ \mu_0 = 0,\ \sigma_\epsilon = 1$.

3.2 AI advising: equilibrium construction

We begin by analyzing the investor's expected payoff given any pair of beliefs $(p, \hat{p})$, highlighting the effect of inefficient communication of the soft information ω. We then construct the equilibrium of interest and provide a closed-form solution.

3.2.1 Stopping value.

Recall that the investor's expected payoff when seeking recommendation is given in (8) in Section 2.1.3. The following lemma characterizes its value.

Lemma 1. *(Stopping value) Given any pair of the investor's belief p and the LLM's belief $\hat{p}$ about $\omega = 1$, the investor's expected payoff when seeking recommendation is*

$$g(p, \hat{p}) = -\sigma_\epsilon^2 \frac{p(1-\hat{p})^2 + (1-p)\hat{p}^2}{\hat{p}^2 + (1-\hat{p})^2} - p(1-p)\left[(\mu_1 - \mu_0)^2 + \frac{(2\hat{p}-1)^2\sigma_\epsilon^2}{\hat{p}^2 + (1-\hat{p})^2}\right]. \tag{19}$$

Figure 1 provides an illustration of $g(p, \hat{p})$. The core friction of AI advising arises from the residual preference uncertainty—inefficient communication of the soft information ω. When both the investor and the LLM are certain and agree on ω—that is, $p = \hat{p} = 1$ or $p = \hat{p} = 0$—the investor enjoys the highest possible payoff, $g(p, \hat{p}) = 0$.

Otherwise, we rewrite (19) as follows to clarify inefficiency affects the investor's payoff:

$$g(p,\hat{p}) = -\underbrace{\left[p\,\mathrm{Var}(\tilde{\theta}_1 \mid m^L) + (1-p)\,\mathrm{Var}(\tilde{\theta}_2 \mid m^L)\right]}_{MSE} - p(1-p)\mathbb{E}[\Delta\mu^2_{\theta|m^L}|m^L], \tag{20}$$

where $\Delta\mu_{\theta|m^L} = \mathbb{E}[\tilde{\theta}_1|m^L] - \mathbb{E}[\tilde{\theta}_0|m^L]$ is the gap in the conditional mean. The last term captures the direct cost of preference uncertainty. This cost is small when the distributions of $\tilde{\theta}_1$ and $\tilde{\theta}_0$ are close—so that distinguishing between them is unnecessary (i.e., $\Delta\mu_{\theta|m^L}$ is small), or when the investor herself is certain about her preference (i.e., p is close to 1 or 0) and can adjust her action accordingly towards $\tilde{\theta}_\omega$.

In addition, even though the LLM is unbiased, the investor is also subject to residual fundamental uncertainty that arises from the preference uncertainty, as captured by the first two terms in (20) . Since the LLM's recommendation m^L in (6) is a weighted average of potential fundamentals, it is a noisy signal for either fundamental θ_1 or θ_0. (Remark 1 gives a short discussion why the LLM does not send (θ_1, θ_0) as message directly.)

As illustrated in Figure 1, the investor's payoff is higher when she is more certain about her preference, that is, as p approaches 0 or 1. This creates an incentive to continue communication about ω before ultimately seeking a recommendation. In the dynamic communication problem analyzed next, the investor trades off the benefit of learning more about ω against the ongoing cost of communication.

In addition, the principal's payoff is higher when the LLM's belief is more aligned with hers—along the 45-degree line in Figure 1. In the baseline case, where the LLM has only one-shot memory based on ds_{t^-} so its belief remains fixed at its pre-trained value $\hat{p}_t = \hat{p}_0$, the investor's payoff is high when she is the typical customer in the LLM's pre-training. In the extension case, where the LLM partially learns from the communication, the investor would enjoy a higher payoff as alignment improves over time.

Note that $g(p,\hat{p})$ is the stopping value in the dynamic communication. Since $\hat{p} = \hat{p}_0$ in the baseline and $\hat{p}$ is a function of p in the extension, we can express the stopping value as a function of the investor's belief: $g(p) \equiv g(p, \hat{p}(p))$.

3.2.2 Value function and optimal policy

The dynamic communication problem is stationary and the value function of the investor depends only on her belief p. Let $V(p)$ denote the investor's value in state p. We conjecture that there exits $0 \le \underline{p} < \overline{p} \le 1$ such that the investor continues the communication with the LLM if $p \in (\underline{p}, \overline{p})$.

Outside of the continuation region, $p \notin (\underline{p}, \overline{p})$, the investor immediately seeks recommen-

dation from the LLM and the game stops. The value function is simply the stopping value $g(p) \equiv g(p, \hat{p}(p))$ in Lemma 1.

For all $p \in (\underline{p}, \overline{p})$, the investor incurs cost c to initiate another round of interaction with the LLM. With probability λdt, the communication ends exogenously and she receives the recommendation immediately. Otherwise, the investor receives her continuation payoff. Hence,

$$V(p) = -cdt + \lambda dt g(p) + (1 - \lambda dt)\mathbb{E}_p[V(p + dp)]. \tag{21}$$

Applying the Ito's formula to the right-hand side of (21) gives

$$V(p) \approx -cdt + \lambda dt g(p) + (1 - \lambda dt)\mathbb{E}_p[V(p) + V'(p)dp + \frac{1}{2}V''(p)(dp)^2].$$

Using the law of motion is given in (13), and taking the limit of $dt \to 0$ gives a linear second-order differential equation for the investor's value function in the continuation region:

$$-c + \lambda \left(g\left(p\right) - V\left(p\right)\right) + \frac{p^2\left(1-p\right)^2}{2\sigma^2}V''\left(p\right) = 0. \tag{22}$$

All solutions of the differential equation take the following form:

$$V(p) = Q(p) + C_1 p^{\frac{1}{2}+\gamma}(1-p)^{\frac{1}{2}-\gamma} + C_2 p^{\frac{1}{2}-\gamma}(1-p)^{\frac{1}{2}+\gamma}, \tag{23}$$

where $\gamma \equiv \sqrt{2\sigma^2\lambda + \frac{1}{4}}$. The term $Q(p)$ is one particular solution to (22), and under the baseline case where $\hat{p} = \hat{p}_0$, we provide the closed-form $Q(p)$ in Appendix A.3. The two constants C_1 and C_2 are yet to be determined.

The constants are pinned down by the value-matching conditions,

$$V(\underline{p}) = g(\underline{p}), \tag{24}$$

$$V(\overline{p}) = g(\overline{p}). \tag{25}$$

Finally, the two equilibrium belief thresholds $\underline{p}$ and $\overline{p}$ satisfy two smooth pasting conditions, which are required such that the investor's strategy solves (SP). Specifically,

$$V'(\underline{p}) = g'(\underline{p}), \tag{26}$$

$$V'(\overline{p}) = g'(\overline{p}). \tag{27}$$

Baseline case: $\kappa = 0$. In the baseline case, the LLM has only one-shot memory and makes its recommendation based solely on ds_{t^-}, and its belief remains fixed under the dynamic setting: $\hat{p}_t = \mathbb{E}[\omega = 1 | ds_{t^-}] = \hat{p}_0$.

In this case, we can show that the investor's optimal stopping thresholds $\underline{p}$ and $\overline{p}$ are symmetric with respect to $\frac{1}{2}$. To see this, $v(p) = V(p) - g(p)$ captures the option value of waiting. The differential equation (22) could be rewritten as

$$\lambda v(p) = \frac{p^2(1-p)^2}{2\sigma^2} g''(p) - c + \frac{p^2(1-p)^2}{2\sigma^2} v''(p).$$

Importantly, $g(p)$ is a quadratic function of p so $g''(p)$ is a constant. It is clear to see that both the flow benefit of waiting, $\frac{p^2(1-p)^2}{2\sigma^2} g''(p) - c$, and the volatility $\frac{p^2(1-p)^2}{2\sigma^2}$ are symmetric for p around $\frac{1}{2}$. Intuitively, when the LLM's belief is fixed, the labels $\omega = 1$ or 0 for the investor's preference type are interchangeable. In contrast, in the extension case where $\kappa > 0$, this symmetry breaks down: the flow benefit of waiting depends on how the LLM's belief evolves, which in tern depends on its prior.

Remark 5. The quadratic form of $g(p)$ arises in many other applications as well. When an economic agent's payoff is linear in her information, as she also chooses an action based on that information, the resulting value function becomes quadratic in the information.

The following proposition summarizes the equilibrium in the baseline case.

Proposition 2. *When* $q_2 \equiv (\mu_1 - \mu_0)^2 + \frac{(2\hat{p}-1)^2\sigma_\epsilon^2}{\hat{p}^2+(1-\hat{p})^2} > 16c\sigma^2$, *there exists a unique pair of stopping thresholds* $\underline{p}$ *and* $\overline{p}$ *that are symmetric around* 0.5, *such that*

1. *When* $p \notin (\underline{p}, \overline{p})$, *the investor stops communication and receives* $g(p, \hat{p})$ *in* (19);

2. *When* $p \in (\underline{p}, \overline{p})$, *the investor communicates and her value is*

$$V(p) = g(p) - \frac{c}{\gamma} - \frac{q_2}{\gamma}\left[U_1(p)I_2(p) - U_2(p)I_1(p)\right] + C(\underline{p})\left[U_1(p) + U_2(p)\right], \tag{28}$$

where $U_1(p) \equiv p^{\frac{1}{2}+\gamma}(1-p)^{\frac{1}{2}-\gamma}$, $U_2(p) \equiv p^{\frac{1}{2}-\gamma}(1-p)^{\frac{1}{2}+\gamma}$, $I_1(p) \equiv \int_{\frac{1}{2}}^{p} U_1(s)ds$, $I_2(p) \equiv \int_{\frac{1}{2}}^{p} U_2(s)ds$, *and* $\gamma = \sqrt{2\sigma^2\lambda + \frac{1}{4}}$. *The constant is given by*

$$C(\underline{p}) = \frac{\frac{c}{\lambda} + \frac{q_2}{\gamma}\left[U_1\left(\underline{p}\right) I_2\left(\underline{p}\right) - U_2\left(\underline{p}\right) I_1\left(\underline{p}\right)\right]}{U_1\left(\underline{p}\right) + U_2\left(\underline{p}\right)}.$$

The optimal stopping thresholds are uniquely determined by $\overline{p} = 1 - \underline{p}$ *and*

$$\frac{d}{dp}\left\{\frac{\frac{c}{\lambda} + \frac{q_2}{\gamma}\left[U_1(p) I_2(p) - U_2(p) I_1(p)\right]}{U_1(p) + U_2(p)}\right\}\Bigg|_{p=\underline{p}} = 0, \quad \text{where } \underline{p} \in (0, 0.5), \tag{29}$$

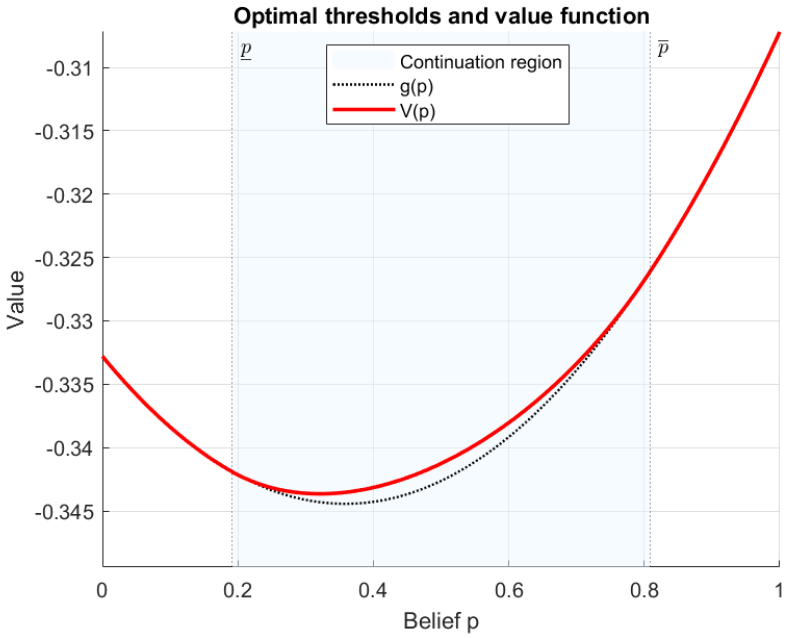


Figure 2: **Optimal policy and value function (baseline $\kappa = 0$).** The figure shows the continuation region $p \in (\underline{p}, \overline{p})$ (shaded area), the investor's value function $V(p)$ (red line) and stopping value $g(p) \equiv g(p, \hat{p}_0)$ (black dotted). Parameters: $\mu_1 = 0.3,\ \mu_0 = 0,\ \sigma_\epsilon = 0.8,\ \sigma = 0.3,\ \lambda = 0.3,\ c = 0.045,\ \hat{p}_0 = 0.51$.

Figure 2 illustrates the equilibrium. There are a few things worth noting. Since the LLM's belief is fixed at $\hat{p}_t = \hat{p}_0 > \frac{1}{2}$, the investor's stopping value $g(p)$ and value function $V(p)$ are tilted upwards for $p > \frac{1}{2}$, where her belief p is more aligned with the LLM's. However, the optimal stopping thresholds $\underline{p}, \overline{p}$ are symmetric around $p = \frac{1}{2}$. In the shaded continuation region, the investor's value function $V(p)$ lies above the immediate stopping payoff $g(p)$, so she chooses to continue communication. Notably, even if the preference uncertainty is fully resolved, that is $p = 1$ or $p = 0$, the investor's payoff $U < 0$ due to the residual fundamental uncertainty: since the LLM does not learn, its recommendation m^L is a noisy signal of the true fundamental θ_ω.

LLM's partial learning: the case of $\kappa > 0$. In this case, for each signal s_t generated in the communication, the LLM absorbs the signal with probability $\kappa > 0$. The baseline case where the LLM only has a one-shot memory could be nested as $\kappa = 0$. For this extension, we consider a sufficiently small κ, under which the structure of the continuation region in Proposition 2 remains robust.

As discussed in Section 2.2.1, the update in the LLM's log-likelihood ratio $\hat{z}_t \equiv \ln \frac{q_t}{1-q_t}$ is κ fraction of that of the investor. The following lemma summarizes the LLM's belief process.

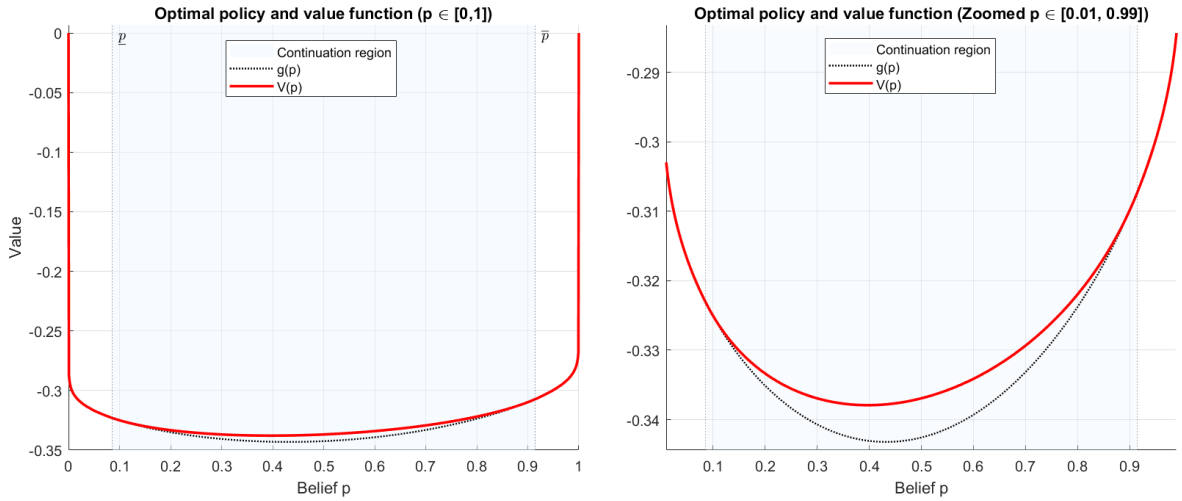


Figure 3: **Optimal policy and value function (extension $\kappa > 0$).** The left panel shows the full plot for $p \in [0,1]$ and the right panel shows the zoomed plot $p \in (0.01, 0.99)$. The figure shows the continuation region $p \in (\underline{p}, \overline{p})$ (shaded area), the investor's value function $V(p)$ (red line) and stopping value $g(p) \equiv g(p, \hat{p}_0)$ (black dotted). Parameters: $\mu_1 = 0.3,\ \mu_0 = 0,\ \sigma_\epsilon = 0.8,\ \sigma = 0.3,\ \lambda = 0.4,\ c = 0.06,\ \ln \frac{\hat{p}_0}{1-\hat{p}_0} = 0.03,\ \kappa = 0.019.$

Lemma 2. *The log-likelihood ratio of the LLM's belief* $\hat{z}_t \equiv \ln \frac{q_t}{1-q_t}$ *satisfies*

$$\hat{z}_t = \hat{z}_0 + \kappa(z_t - z_0).$$

Accordingly, the LLM's belief is a function of the investor's belief p_t*:*

$$\hat{p}_t(p_t) = \frac{\frac{\hat{p}_0}{1-\hat{p}_0}\left[\frac{(p_t/p_0)}{(1-p_t)/(1-p_0)}\right]^\kappa}{1 + \frac{\hat{p}_0}{1-\hat{p}_0}\left[\frac{(p_t/p_0)}{(1-p_t)/(1-p_0)}\right]^\kappa}.$$

Under a sufficiently small κ, the investor continues communication when $p \in (\underline{p}, \overline{p})$ and stops immediately otherwise. Since the LLM also partially learns from the communication, the optimal thresholds $\underline{p}$ and $\overline{p}$ are no longer symmetric around $\frac{1}{2}$ when $\hat{p}_0 \neq \frac{1}{2}$. Instead, learning is skewed towards the LLM's prior: for example, if $\hat{p}_0 > \frac{1}{2}$, we have $\overline{p} - \frac{1}{2} > \frac{1}{2} - \underline{p}$.

For the remainder of the equilibrium, the investor's value in the continuation region is given in (22). The two value matching conditions (24) and (25), and the two smooth pasting conditions (26) and (27) determine the constants C_1, C_2 and the optimal policies $\underline{p}$, $\overline{p}$.

Figure 3 provides an illustration of the equilibrium. As shown in the left panel, the investor's attains the highest possible payoff, $U = 0$, when she is certain about her preference—that is, when $p = 0$ or 1. If the investor perfectly learns ω, the LLM's posterior belief is also $\hat{p} = 0$ or 1 (an infinite z implies an infinite $\hat{z}$). However, the investor stops learning before reaching $p = 0$ or 1 in equilibrium. As illustrated by Figure 3, the investor has a higher

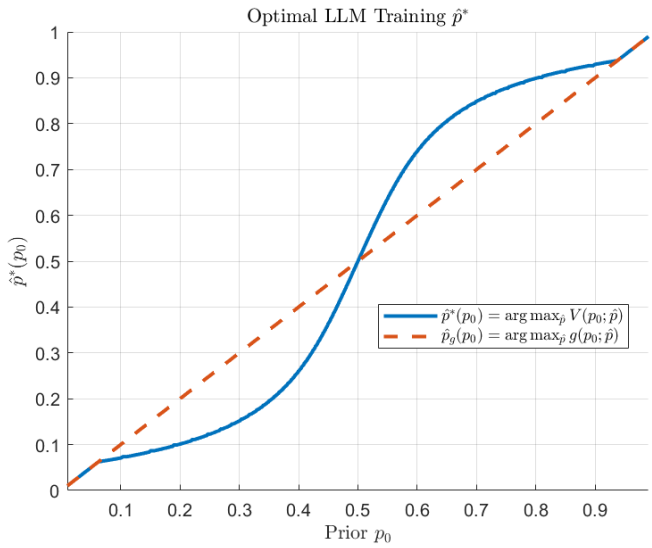


Figure 4: **Optimal LLM Training (baseline $\kappa = 0$).** The figure plots the optimal pretraining of LLM, $\hat{p}^*$, for uses with different prior beliefs p_0. The dashed red line plots a benchmark case of the optimal LLM training if the user does not communicate and immediately seeks recommendation. Parameters: $\mu_1 = 0.4,\ \mu_0 = 0,\ \sigma_\epsilon = 0.2,\ \sigma = 0.3,\ \lambda = 0.3,\ c = 0.045,\ \kappa = 0$.

incentive to communicate and stops at more extreme $\underline{p}, \overline{p}$ when the LLM partially learns from past communication.

3.3 Optimal AI Training

Developers can train LLMs using targeted data or alignment methods to affect LLM's understanding of the representative users—captured in the model as its prior belief $\hat{p}_0$. Given this, for an investor with a prior belief p_0, what choice of LLM training maximizes her value? Does she benefit more from an LLM whose prior closely aligns with her own belief, or from one that holds a more "opinionated" stance than she does? Intuitively, an investor perceives recommendations from a more "aligned" LLM as less noisy. However, consulting a more specialized or opinionated LLM results in a more engaged conversation, during which greater uncertainty about her needs, ω, can be resolved.

We examine the baseline case where the LLM has one-shot memory and does not update its belief, so that $\hat{p}$ is fixed at the pretraining level $\hat{p}_0$. The optimal LLM belief is plotted in Figure 4 and Proposition 3 presents the formal results.

Proposition 3. *There exist threshold prior beliefs $\underline{p}_0$ and $\overline{p}_0$, satisfying $\underline{p}_0 + \overline{p}_0 = 1$, such that for a user with prior belief $p_0 \in [0, 1]$, the optimal LLM training choice $\hat{p}^*(p_0) \equiv$*

$\arg\max_{\hat{p}} V(p_0;\hat{p})$ *satisfies*

$$\hat{p}^*(p_0)\begin{cases} > p_0, & \text{if } p_0 \in (0.5, \overline{p}_0), \\ < p_0, & \text{if } p_0 \in \left(\underline{p}_0, 0.5\right), \\ = p_0, & \text{if } p_0 = 0.5, \text{ or } p_0 \notin \left(\underline{p}_0, \overline{p}_0\right). \end{cases} \tag{30}$$

At these threshold priors, we have $\underline{p}(\hat{p} = \underline{p}_0) = \underline{p}_0$ *and* $\overline{p}(\hat{p} = \overline{p}_0) = \overline{p}_0$,

First, consider the benchmark case where the investor does not communicate with the LLM about her needs and seeks recommendation immediately. Her resulting value is $g(p_0;\hat{p})$ given in (19). Without any option value of learning, her most preferred LLM training is exactly $\hat{p} = p_0$ that aligns with her belief, as illustrated by the red dashed line in Figure 4.

The option value of learning tilts the investor's preference toward a more "opinionated" LLM—one that leans in the direction of her prior belief. For example, in Figure 4, when the investor's prior is $p_0 = 0.2$, which leans towards $\omega = 0$, her most preferred LLM belief is the aligned prior of 0.2 if she immediately exercises. However, if she engages in communication to clarify her needs, she prefers a more opinionated LLM with a prior of 0.1. When investors hold extreme priors with $p_0 \notin (\underline{p}_0, \overline{p}_0)$, she chooses not to communicate and prefers an aligned LLM with a prior of $\hat{p} = p_0$.

When the investor is most uncertain, with a prior of $p_0 = 0$, the option value of learning is highest. However, we can show analytically that this option value is dominated by the perceived noise of the recommendations from an opinionated or specialized LLM. In this case, the investor prefers the LLM to be equally "confused," so that $\hat{p}^*(p_0 = 0.5) = 0.5$.

3.4 Comparative Statics of AI Advising

We present a few comparatives statics based on the baseline equilibrium in Proposition 2.

Communication cost c**.** The investor trades off the gain from continuing communication to learn about ω against the communication cost. As illustrated in Figure 5, when the communication cost c is larger, the principal learns less as suggested by a higher $\underline{p}$ and a lower $\overline{p}$—she stops communication when she is less sure about ω.

Preference uncertainty $\Delta\mu$**.** We discuss how preference uncertainty affects AI advising. The inefficient communication of soft information ω is the core friction when the investor consults an LLM.

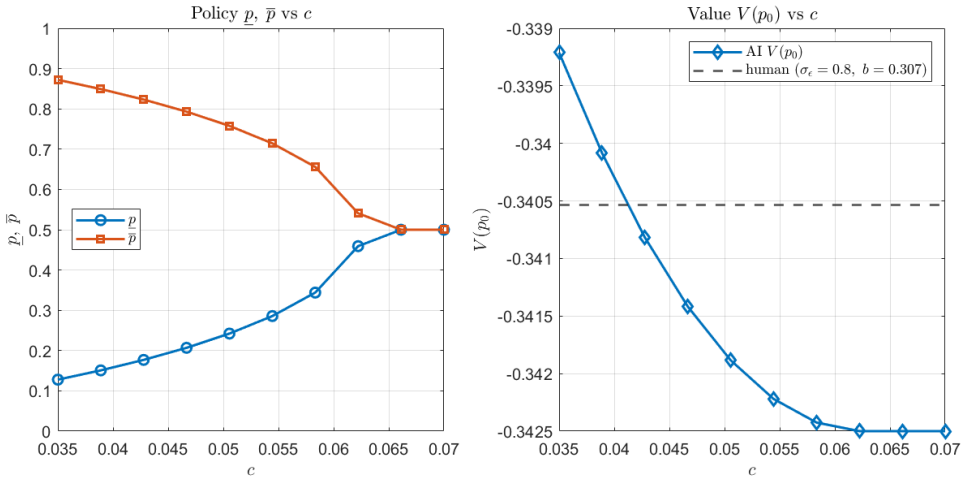


Figure 5: **The effects of communication cost** c**.** The left panel plots the optimal stopping thresholds $\underline{p}, \overline{p}$ as a function of c, and the right panel shows the investor's value at prior $V(p_0)$ as a function of c. Baseline parameters: $\mu_1 = 0.3,\ \mu_0 = 0,\ \sigma_\epsilon = 0.8,\ \sigma = 0.3,\ \lambda = 0.3,\ p_0 = \hat{p}_0 = 0.5,\ \kappa = 0$.

First, we discuss how the magnitude of preference uncertainty affects the equilibrium. When the distributions of $\tilde{\theta}_1$ and $\tilde{\theta}_0$ are close to each other, preference uncertainty is less consequential, and receiving an impersonalized recommendation from the LLM causes little harm. Recall that $\tilde{\theta}_1 \sim N(\mu_1, \sigma_\epsilon^2)$ and $\tilde{\theta}_0 \sim N(\mu_0, \sigma_\epsilon^2)$ are independent normal random variables. In our numerical exercise, we fix the value of μ_1 and vary μ_0 for $\mu_0 < \mu_1$. In Figure 6, the higher is μ_0—or the smaller is $\Delta\mu \equiv \mu_1 - \mu_0$, the investor learns less about ω (the optimal stopping thresholds become less extreme) and enjoys a higher value given the less consequential preference uncertainty. Intuitively, when the task is simple and personalization matters less, the investor has less incentive to figure out her exact preferences and she will still be better off.

Therefore, tasks without distinct contingencies are more suitable for AI advising. For example, tourism planning is easier, as the destination choice often reflects stereotypical preferences. In contrast, insurance or medical consultations involve much more soft information, where identifying the correct ω is critical.

We also examine the effects of the LLM's pretraining model or its belief $p_t = \hat{p}_0$. As illustrated in Figure 7, the principal learns more information if the LLM's belief is more extreme. Intuitively, if the LLM is trained towards a specific customer type, the investor benefits from learning more—either because she matches that type or needs to be more informed herself to adjust the action on her own.

Fundamental uncertainty σ_ϵ^2**.** Last, we analyze the effects fundamental uncertainty, which is captured by the variance σ_ϵ^2 of the fundamental state, $\tilde{\theta}_1$ or $\tilde{\theta}_0$. When the advisor

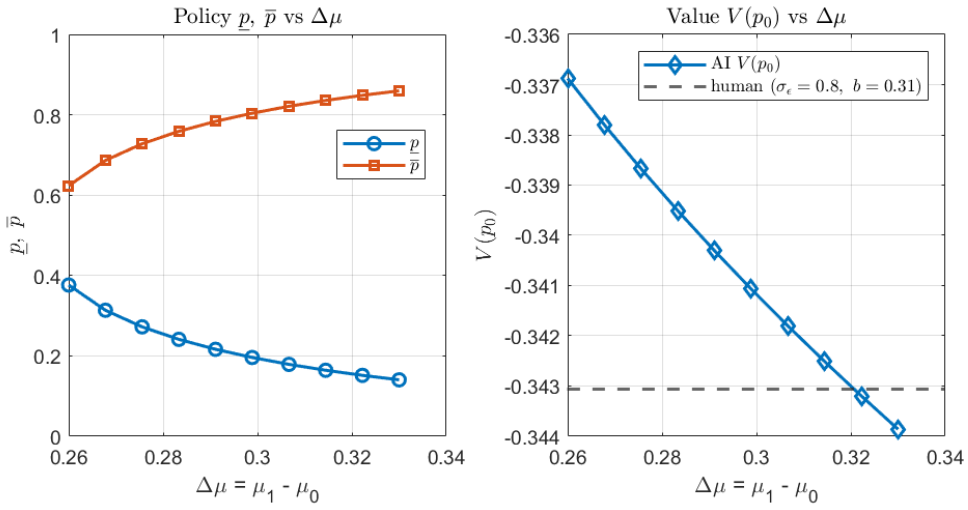


Figure 6: **The effects of preference uncertainty $\Delta\mu$.** The left panel plots the optimal stopping thresholds $\underline{p}, \overline{p}$ as a function of $\Delta\mu \equiv \mu_1 - \mu_0$, and the right panel shows the investor's value at prior $V(p_0)$ as a function of $\Delta\mu$. Baseline parameters: $c = 0.045,\ \mu_0 = 0,\ \sigma_\epsilon = 0.8,\ \sigma = 0.3,\ \lambda = 0.3,\ p_0 = \hat{p}_0 = 0.5,\ \kappa = 0$.

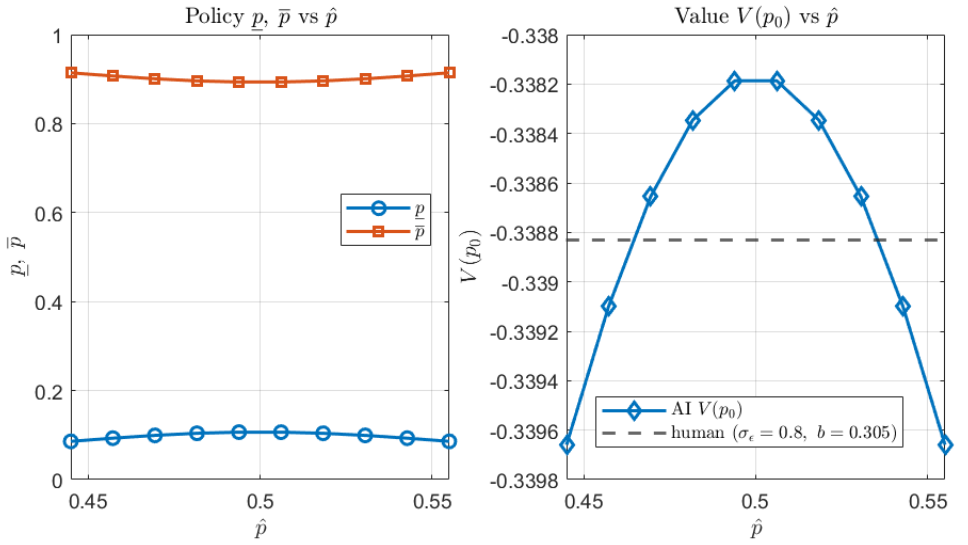


Figure 7: **The effects of the LLM's belief $\hat{p}$.** The left panel plots the optimal stopping thresholds $\underline{p}, \overline{p}$ as a function of $\hat{p}$, and the right panel shows the investor's value at prior $V(p_0)$ as a function of $\hat{p}$. Baseline parameters: $\mu_1 = 0.3,\ \mu_0 = 0,\ c = 0.045,\ \sigma_\epsilon = 0.8,\ \sigma = 0.3,\ \lambda = 0.3,\ p_0 = 0.5,\ \kappa = 0$.

is a human, the investor's utility is determined by the residual fundamental uncertainty—so she enjoys a higher payoff when σ_ϵ^2 is small.

When it comes to AI advising, the investor is still subject to residual fundamental uncertainty that arises from inefficient communication of preference ω, under which the LLM's recommendation is a noisy signal of the true fundamental. Hence, in AI advising, the investor also enjoys a higher payoff when σ_ϵ^2 is small, as shown in the lower panel of Figure 8.

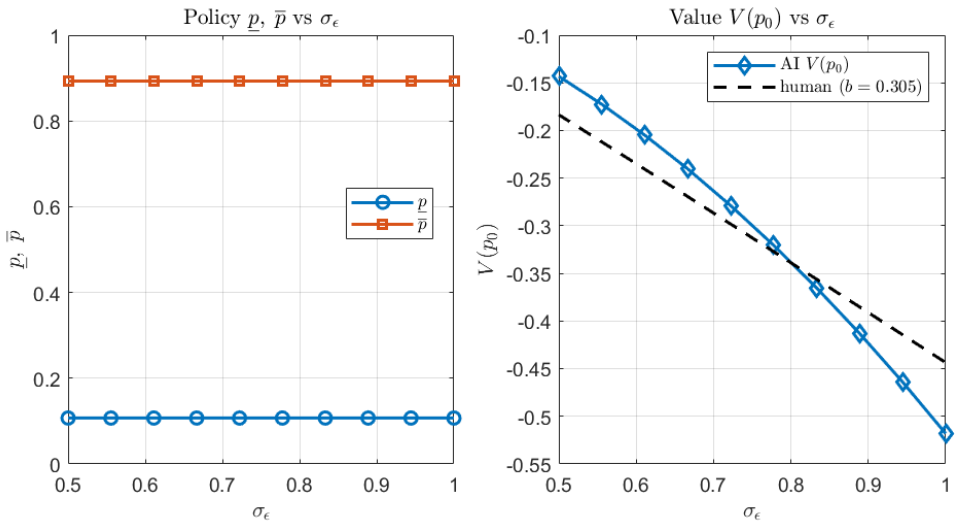


Figure 8: **The effects of fundamental uncertainty σ_ϵ.** The left panel plots the optimal stopping thresholds $\underline{p}, \overline{p}$ as a function of σ_ϵ, and the right panel shows the investor's value at prior $V(p_0)$ as a function of σ_ϵ. Baseline parameters: $\mu_1 = 0.3,\ \mu_0 = 0,\ c = 0.045,\ \sigma = 0.3,\ \lambda = 0.3,\ p_0 = \hat{p} = 0.5,\ \kappa = 0$.

Interestingly, as shown in the upper panel, the investor's optimal policies, $\underline{p}$ and $\overline{p}$, do not vary with fundamental uncertainty. This suggests that the inefficiency in AI advising comes from preference uncertainty, and fundamental uncertainty does not affect the endogenous communication about preference ω.

4 Empirical Analysis

In this section, we test model implications using prompt-based simulations in LLMs.

4.1 Testable Hypotheses

Building on the theoretical model and its comparative statics, we derive testable hypotheses that connect the model's primitives to observable investor behavior and advice outcomes. Each hypothesis articulates a directional prediction, identifies the underlying mechanism from the theory, and outlines an empirical strategy for validation.

In a controlled setting, we can simulate the advising process with an LLM using prompt response logs and resulting portfolio recommendations to directly test several model predictions. In particular, these experiments allow us to control the agents' *information sets*. There are additional testable hypotheses motivated by the theory that require observational data (such as records of advisor choice and performance from a financial platform); we list those in Appendix B.1.

H1 (Investor Learning and Decision Quality): The primary value of interacting with a memory-less LLM advisor stems from the investor clarifying their own initially uncertain preferences. A deeper interaction allows the investor to reduce their own "preference uncertainty," leading to a final investment decision that is better aligned with their true objectives, even if the LLM's output remains generic.

The model posits that the investor is initially "confused" and does not fully understand her own needs summarized by ω. The dialogue with the LLM provides signals $\{ds_t\}$ that allow the investor to update her own beliefs about her type, moving her closer to certainty—that is, her belief about ω, p approaches 1 or 0. Because we control the LLM's information set to be the last prompt (short memory), its recommendation does not adapt to a deeper conversation. However, the now better-informed investor can make a more appropriate final decision based on her sharpened posterior belief.

H2 (Investor Impatience and Early Termination): Investors who face a higher opportunity cost of time break off LLM conversations sooner, which corresponds to, and accept portfolios less tailored to their preferences.

In our model, an impatient investor may experience a Poisson "impatience shock" that prompts them to end the communication early. This premature truncation of the dialogue results in the investor gathering less information, which in turn leads to a portfolio decision that is less tailored to the investor's true preferences. To test this hypothesis, we examine the relationship between conversation length, which reflects the Poisson parameter λ, and the investor's final portfolio choice.

H3 (Memory Augmentation and Advisor Performance): Providing the LLM advisor with a form of persistent memory about past interactions (a larger κ) will improve its advice quality.

The theoretical motivation suggests that human advisors naturally retain and recall earlier parts of conversations and the client's background, unlike a standard LLM which may lose mid-conversation context due to lack of memory. Enhancing the LLM with tools such as automatically generated summaries of past chat history should help reduce this information loss, enabling the LLM to better understand and address the investor's needs.

H4 (Optimal AI Training): An investor generally benefits from an AI advisor whose recommendation style is more opinionated, meaning more extreme than her own prior belief, except when she is most uncertain about her type.

Consider a retail investor who suspects she may be more risk-tolerant than average, but is not entirely sure. Should she consult a general-purpose robo advisor, or one specialized for aggressive investors? Counterintuitively, our model (Proposition 3) suggests she is better off with the aggressive specialist. Here is why. She will gain if she truly belongs to the

aggressive type, as the specialist's recommendations will be well-suited; however, she will lose if she turns out to be conservative after all, as the mismatch with the advisor aggressive stance widens. The first effect dominates because an extreme advisor's recommendation is more informative about underlying asset fundamentals.

The same logic applies in reverse. An investor who leans conservative benefits from consulting an advisor trained on even more conservative clients. The key insight is that, for an investor subject to both preference and fundamental uncertainty, the option value of learning primarily reduces the fundamental uncertainty when interacting with an opinionated LLM, while preference uncertainty remains unchanged ex ante because belief updating follows a martingale. The exception is the investor who is genuinely uncertain and has no directional leaning. For her, an equally uncertain, neutral advisor is optimal, since there is no prior inclination to amplify.

4.2 Hypothesis Testing with LLM Simulations

To complement our analytical framework, we use prompt-based LLM simulations, a methodological innovation that enables the testing of complex economic theories in realistic, interactive settings. Unlike traditional laboratory experiments or analytical models, these simulations can generate dynamic, multi-turn conversations that closely approximate real-world advisory relationships. This approach bridges the critical gap between theoretical rigor and empirical realism, allowing us to observe how theoretical mechanisms like information acquisition and belief updating operate in practice. This section describes how we construct investor profiles, implement an LLM-based advisor, and map the discrete simulation environment back to the continuous-time model.

4.2.1 Data and Investor Profiles

The baseline for "optimal" advice in our simulations comes from the Vanguard Investor Questionnaire, a widely used 11-question survey that generates personalized asset-allocation recommendations based on an individual's investment horizon, financial stability and risk tolerance.[9] For example, the questionnaire recommends investors with short time horizons to hold a smaller fraction of equity in their portfolio, while those with longer horizons to take on more risks. Similarly, it notes that a stable income stream allows investors to tolerate greater market volatility and therefore merits a higher equity weight.

In our empirical construction, we draw a subsample from the 2022 public-use Consumer

[9]The service is available at the website: https://investor.vanguard.com/tools-calculators/investor-questionnaire/questions. The questionnaires are listed in Appendix B.2.

Finances Survey (SCF) to generate $n = 500$ hypothetical investor profiles for simulation. To maintain representativeness, selection is conducted at the household level using probabilities proportional to the final analysis weights of the survey, reflecting the dual-frame design of the SCF combining an area-probability sample and a list sample that oversamples high-income households. After selecting households, an imputed record per household is retained at random so that the final dataset contains exactly one observation per household while still reflecting imputation uncertainty. We fix the random seed prior to sampling to ensure that the draw is fully reproducible across runs. This design preserves fidelity to the underlying probability structure of the SCF while producing a compact and stable subsample suitable for repeated LLM experiments.

We then take each simulated profile and use it to complete the Vanguard questionnaire on the website, recording the recommended stock/bond allocation for each hypothetical investor. Each profile in our benchmark is thus defined by: (i) a list of question-answer pairs that can be expressed in natural language, which correspond to the investor's underlying preference ω, and (ii) the recommended allocation between equity vs fixed income, returned by the Vanguard algorithm, which maps to the ideal recommendation $\omega\theta_1 + (1-\omega)\theta_0$ without any frictions. In this way, we obtain a consistent set of simulated investors, grounded in established industry practice.

Table 1 presents the correlation structure of the questionnaire responses across our SCF-based investor profiles. The matrix reveals several important patterns that validate both our sampling approach and the questionnaire's internal consistency. First, the strong positive correlations among risk tolerance questions (Q6-Q8, Q10-Q11) with coefficients ranging from 0.457 to 0.865 demonstrate that households provide coherent responses across different risk scenarios. The particularly high correlation between Q10 and Q11 (0.865), which both measure willingness to accept short-term losses, confirms that investors have stable risk preferences that manifest consistently across similar questions. Second, the high correlation between time horizon questions (Q1-Q2: 0.712) indicates natural alignment between investment timeline and planning horizon in our SCF households. Third, the negative correlations between independent thinking (Q9) and risk tolerance measures suggest that investors who rely more on their own judgment tend to be more conservative, possibly reflecting awareness of their own knowledge limitations. Finally, the moderate correlations between financial stability indicators (Q4-Q5) and other dimensions confirm that income stability and emergency preparedness represent distinct but related aspects of financial security. These correlation patterns, emerging from actual household data rather than random generation, provide confidence that our profiles capture realistic interdependencies among investor characteristics.

Table 1: Correlation Matrix of Investor Questionnaire Responses

This table presents the correlation matrix for responses to the 11-question Vanguard Investor Questionnaire based on 500 SCF household profiles. Each question captures different aspects of investor preferences and constraints: Q1-Q2 measure investment time horizon and planning period; Q3 captures past investment experience; Q4-Q5 assess income stability and emergency fund adequacy; Q6-Q8 evaluate risk tolerance through hypothetical portfolio performance scenarios; Q9 indicates independent thinking in financial decision making; Q10-Q11 gauge willingness to accept short-term losses for potential long-term gains. All correlations are calculated using Pearson correlation coefficients on the ordinal response scales.

	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11
Q1	1.000										
Q2	0.712	1.000									
Q3	0.425	0.505	1.000								
Q4	0.493	0.659	0.148	1.000							
Q5	0.153	0.145	0.310	-0.017	1.000						
Q6	0.443	0.441	0.405	0.267	0.042	1.000					
Q7	0.386	0.401	0.440	0.215	-0.001	0.700	1.000				
Q8	0.171	0.186	0.242	0.115	-0.068	0.457	0.645	1.000			
Q9	-0.062	0.001	0.125	-0.124	0.209	-0.229	-0.294	-0.184	1.000		
Q10	0.424	0.452	0.501	0.240	0.009	0.862	0.774	0.527	-0.248	1.000	
Q11	0.478	0.518	0.570	0.313	0.020	0.779	0.853	0.601	-0.277	0.865	1.000

4.2.2 Conversation with the LLM Advisor

For each investor profile we simulate a conversation between the investor and an LLM-based advisor. The LLM we use is the then-state-of-the-art OpenAI GPT-5, accessed via an API at temperature 0.75. Conversations are orchestrated through system and developer messages that clearly specify each agent's role and information set. The investor's system prompt states that she is preparing to set up an investment portfolio and will interact with a financial advisor. It also includes the full text of her questionnaire profile. As explained below, the investor chats with the LLM based on this questionnaire but makes portfolio decisions without it. The advisor's system prompt specifies that it is a financial advisor whose objective is to determine the client's optimal allocation between equities and bonds. It is instructed to remain in information-gathering mode and to ask diagnostic questions about financial situation, investment experience, time horizon and risk preferences until the conversation ends. Importantly, the system prompt explicitly forbids the advisor from providing any recommendations before being told to do so, ensuring that all intermediate messages consist only of questions.

The interaction unfolds in discrete rounds:

1. **Eliciting the prior.** Before any questions are asked, the investor reports her prior preferred equity allocation. She is prompted to return her intended equity percentage as

`JSON`, and this value is logged. This step parallels the initial belief p_0 in our theoretical model.

2. **Question–answer loop.** A developer message instructs the advisor to "ask your client one question that will help you identify their optimal split between equities and bonds." The advisor generates a question, which we append to the conversation. The investor's developer prompt tells her to answer concisely. She answers truthfully *according to her profile questionnaires*, and the answer is appended to both the investor and advisor message lists. In subsequent rounds the developer instructs the advisor to "ask your client another question," so that exactly one question is asked per round. The conversation thus alternates between one question from the advisor and one answer from the investor.

3. **Termination decision.** After each round, the simulation consults the pre-drawn termination schedule (described below) to determine whether the conversation should stop. If the investor terminates, the question–answer loop ends; otherwise the advisor is prompted to ask another question.

4. **Recommendations.** Upon termination the advisor is asked to provide equity-allocation recommendations in two ways. First, to simulate the memoryless case, the advisor receives only the most recent question and answer and is instructed to return the optimal equity allocation for the individual. Second, to simulate memory, the advisor receives the entire chat history (including all simulated questions and answers) and is instructed to produce an optimal equity allocation. In both cases, the advisor's output must be a `JSON` snippet specifying the recommended equity percentage; no narrative explanation is allowed. For the full-information advisor, the advisor is given the complete investor profile up front and asked once for the optimal equity allocation.

5. **Final decision.** Finally, the investor is told that "the advisor recommends an equity allocation of $x\,\%$" and is prompted to choose her final allocation. Like the prior, the final allocation is returned as `JSON`. This step captures the investor's own action in the theoretical model after observing the advisor's message.

Throughout the conversation, we log every prompt and response. The strict formatting (e.g., `JSON` outputs, one question per round, no unsolicited recommendations) reflects an attempt to control the LLM's behavior and reduce hallucinations. The advisor's inability to update its belief in the memoryless condition comes from receiving only a single question–answer pair as input; this effectively resets its context window at every termination event.

We implement three variants of the LLM advisor to isolate the effect of memory:

- **Advisor without memory.** Motivated by the architecture of current LLMs, we simulate an advisor that cannot recall previous signals. After each prompt, the LLM uses only the most recent question–answer pair as context to recommend an allocation. This design captures the short context windows of Transformer models, whose self-attention complexity scales quadratically in the input length and cannot reliably capture the entire query history. Wang and Sun (2025) show that even when longer contexts are available, retrieval accuracy can deteriorate rapidly due to interference from earlier inputs; the probability of recalling the most recent key–value pair declines log-linearly as similar distractors accumulate. Our "no-memory" LLM therefore approximates our baseline model that the advisor updates its prior based on only the last signal, $\hat{p}_t = \mathbb{E}[\omega = 1 | ds_{t^-}]$. (In the model, the advisor's belief remains fixed at a pretraining prior under the continuous time limit.)

- **Advisor with memory.** In this scenario, the LLM is fed the entire conversation history as context. It has access to previous answers and can refine its recommendation as it learns more about the investor. This scenario is our best approximation of an unbiased human advisor with a transcript of the conversation. This aligns with our model extension with $\kappa = 1$: both the investor and the advisor update their belief based on the full history of signals $\{s_t\}$ before termination; if they share the same prior, then $\hat{p}_t = p_t$.

- **Advisor with full information.** This counterfactual LLM receives the investor's entire questionnaire profile at once. It directly observes ω and effectively faces no preference uncertainty. This case can also mimic a human advisor who perfectly elicits soft information instantaneously, with no bias ($b = 0$). The full-information advisor's recommendation provides an upper bound on achievable accuracy. Note that the advisor's recommendation may still fall short of the frictionless Vanguard benchmark because of fundamental uncertainty. While the model assumes that advisors observe the fundamental realizations θ_1 and θ_0, in practice, the LLM only observes a noisy signal of these fundamentals.

4.2.3 Termination and Cost of Communication

We cap the interaction at a maximum of eleven rounds to mirror the finite length of the Vanguard questionnaire. However, an investor does not necessarily complete all eleven rounds, as the dialogue can end sooner based on one of two mechanisms: exogenous or endogenous termination. To model impatience or external factors that cut a conversation short, we introduce an exogenous termination probability of 0.10 per round. Before the simulation begins,

we conduct a Bernoulli draw for each round to determine if an "impatience shock" occurs. If a shock is scheduled for a given round, the conversation stops automatically after that round's question and answer are complete. If no exogenous shock occurs, the investor agent makes an active, endogenous decision to continue or stop. After answering the advisor's question, the investor is prompted with a decision frame that explicitly asks them to weigh the costs and benefits of more interaction: *"Your time is valuable, so each round of communication with the advisor carries a cost. You should choose to continue interacting with the advisor if and only if your expected informational gain from an additional round of communication exceeds your subjective cost of communication. Would you like to continue or terminate the conversation...?"* This prompt directly simulates the optimal stopping trade-off.

This dual-termination mechanism serves as a discrete analogue of the continuous-time optimal stopping problem in our theoretical model. Each question-answer round in the simulation corresponds to a small increment of time dt. The exogenous termination probability models the Poisson shock intensity λ, while the cost of continuing the conversation mirrors the flow cost c. The endogenous decision to terminate corresponds to the investor choosing the optimal stopping time τ once her belief p_t makes further interaction suboptimal. The theoretical problem is expressed as:

$$(SP) \quad \sup_{\tau \geq 0} \mathbb{E}^{\omega} \left\{ \int_0^{\tau} e^{-\lambda t} [-c + \lambda g(p_t, \hat{p}_t(p_t))] dt + e^{-\lambda \tau} g(p_\tau, \hat{p}_\tau(p_\tau)) \right\},$$

where p_t is the investor's belief about her type and $g(\cdot)$ is the payoff from acting.

4.3 Empirical Results and Hypothesis Testing

We now present the empirical results from our LLM simulations to test the three main hypotheses derived from the theoretical model. Our analysis uses data from 500 simulated investor profiles, each with 5 iterations, yielding 2,500 total observations. The results provide strong support for the model's predictions about the role of investor learning, the impact of communication costs, and the benefits of memory augmentation in AI advising.

4.3.1 Testing H1: Investor Learning and Decision Quality

The first hypothesis posits that LLM advising help investor clarify their own preferences. Table 2 presents the regression results testing this hypothesis. Across measures, the data show notable gains in investment accuracy that arise from the advisory process itself.

Investors improved significantly even before receiving any recommendations. As shown in Columns (1) and (2), accuracy increased by 14.4 percentage points before any advice,

Table 2: Investor Learning and Decision Quality

This table reports regression results examining factors influencing the accuracy of investors' investment choices, measured at interim and final stages, as well as accuracy improvements. Columns (1)-(2) display improvements in investment choice accuracy at pre- and post-recommendation stages. Column (3) specifically illustrates investors' interim investment choice accuracy, while Columns (4)-(8) present investors' final investment choice accuracy. Accuracy here is defined as 100 minus the deviation from the optimal portfolio allocation, expressed in percentage points. Independent variables include the number of interaction rounds (# Rounds, Total # Rounds), total words exchanged, and separately, the number of words contributed by the advisor and the investor. Standard errors, clustered by investor profile, are shown in parentheses. Profile fixed effects are incorporated in Columns (3)-(8) to control for investor-specific characteristics. ***, **, and * denote the 1%, 5%, and 10% confidence level, respectively.

	Accuracy Improvement		Investor's Accuracy					
	Pre-Rec	Post-Rec	Interim			Final		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
# Rounds			0.727***					
			(0.079)					
Total # Rounds				0.771***				
				(0.120)				
Total # Words					0.012***			
					(0.002)			
Total # Advisor's Words						0.020***		0.029***
						(0.003)		(0.006)
Total # Investor's Words							0.026***	-0.018*
							(0.006)	(0.009)
Constant	14.4***	17.6***						
	(0.717)	(0.887)						
Profile FE	N	N	Y	Y	Y	Y	Y	Y
Clustered by Profile	Y	Y	Y	Y	Y	Y	Y	Y
Observations	2,500	2,500	8,673	2,500	2,500	2,500	2,500	2,500
Adjusted R^2	-	-	0.693	0.923	0.922	0.922	0.921	0.922
Note:								$^*p < 0.1$; $^{**}p < 0.05$; $^{***}p < 0.01$

reaching 17.6 percentage points after the advisor's input. The 3.2 point boost from recommendations, while meaningful, suggests that much of the learning comes from the interactive process itself rather than solely from the advice received.

Interaction intensity is vital for accuracy improvement. Results from Columns (3) to (4) show that each additional round between advisor and investor increased interim accuracy by 0.727 points and final accuracy by 0.771 points. This implies iterative exchanges help investors gradually refine their understanding of risk and preferences through structured dialogue.

The actual words exchanged drive learning as well. Column (5) shows that every additional word spoken by either party increases accuracy by 0.012 points. Columns (6) and (7) reveal interesting dynamics when examining advisor and investor contributions separately. Advisor words improve accuracy by 0.020 points per word, while investor words show an even stronger effect at 0.026 points per word when considered in isolation. This suggests that articulating and reflecting on one's own beliefs is particularly valuable for preference clarification.

However, the multivariate model in Column (8) reveals important nuance. When both advisor and investor word counts are included simultaneously, advisor words show an even stronger positive effect, while investor words surprisingly become negative though only marginally significant. This reversal suggests potential multicollinearity or that once the advisor's structuring role is accounted for, excessive investor verbalization may indicate confusion rather than clarity. The advisor's words appear to be the critical factor in guiding productive self-reflection, helping investors articulate their preferences efficiently rather than meandering through unfocused self-expression.

The consistently strong effects of structured dialogue, combined with the substantial but not dominant role of final recommendations, suggest the primary benefit of LLM advising lies in facilitating investor self-discovery through guided interaction. For AI advisory systems, this means the greatest value comes from designing sophisticated questioning frameworks that help users systematically explore and clarify their preferences, rather than focusing solely on recommendation algorithms.

4.3.2 Testing H2: Investor Impatience and Early Termination

The second hypothesis examines whether investor impatience, manifested through early conversation termination, undermines the advisor's ability to provide accurate recommendations. Table 3 presents regression results analyzing the factors that influence the accuracy of the AI advisor's investment recommendations.

The results demonstrate a strong positive relationship between interaction intensity and

Table 3: Investor Impatience and Early Termination

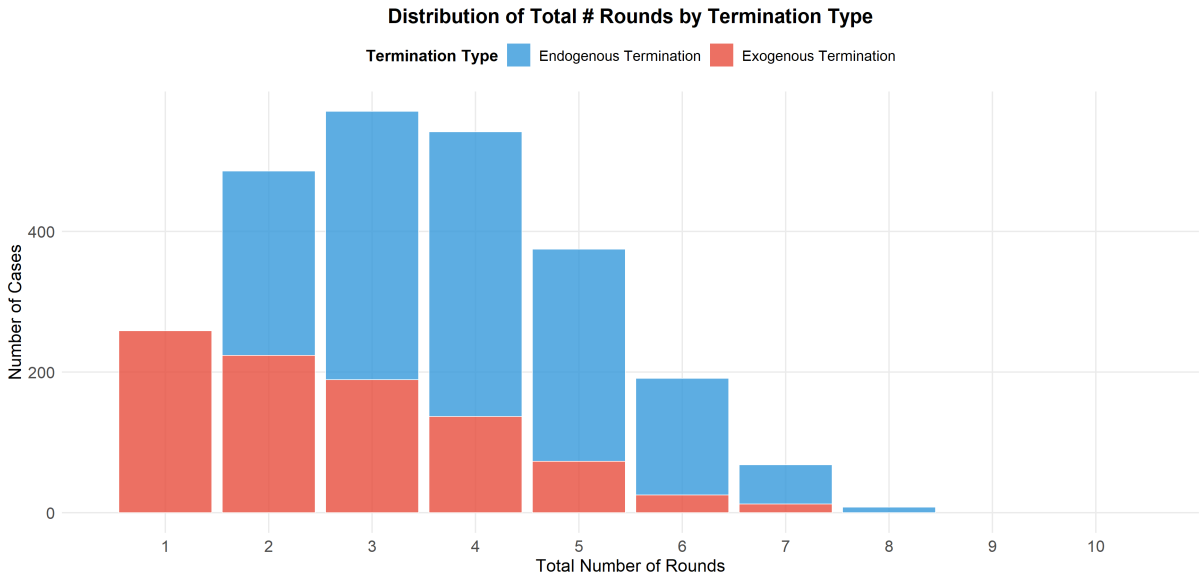
This table reports regression results examining factors influencing the accuracy of the AI advisor's investment recommendations. All columns present the accuracy of the advisor's recommendations, measured as 100 minus the deviation from the optimal portfolio allocation, expressed in percentage points. Columns (1)-(4) examine individual factors in isolation, while Column (5) presents a specification including both advisor and investor word counts. Column (6) examines the effect of exogenous termination. Independent variables include the total number of interaction rounds (Total # Rounds), total words exchanged, separately the number of words contributed by the advisor and the investor, and an indicator for whether the conversation was terminated exogenously rather than reaching a natural conclusion. Standard errors, clustered by investor profile, are shown in parentheses. Profile fixed effects are incorporated in all columns to control for investor-specific characteristics. ***, **, and * denote the 1%, 5%, and 10% confidence level, respectively.

	Accuracy of Advisor's Recommendation					
	(1)	(2)	(3)	(4)	(5)	(6)
Total # Rounds	0.931***					
	(0.131)					
Total # Words		0.016***				
		(0.002)				
Total # Advisor's Words			0.025***		0.021***	
			(0.004)		(0.006)	
Total # Investor's Words				0.040***	0.008	
				(0.006)	(0.010)	
$\mathbb{I}\{$Exogenous Termination$\}$						-0.988***
						(0.351)
Profile FE	Y	Y	Y	Y	Y	Y
Clustered by Profile	Y	Y	Y	Y	Y	Y
Observations	2,500	2,500	2,500	2,500	2,500	2,500
Adjusted R^2	0.498	0.497	0.497	0.494	0.497	0.480
Note:						$^*p < 0.1$; $^{**}p < 0.05$; $^{***}p < 0.01$

advisor recommendation accuracy. Column (1) shows that each additional round of interaction increases the advisor's recommendation accuracy by 0.931 percentage points, indicating that extended dialogue enables the advisor to better understand investor preferences and circumstances. This finding is reinforced by the word-count analyses in Columns (2)-(4).

The decomposition of word contributions reveals interesting dynamics in the advisory relationship. When examined separately, advisor words (Column 3) increase recommendation accuracy by 0.025 points per word, while investor words (Column 4) show an even larger effect of 0.040 points per word. However, when both are included simultaneously in Column (5), the advisor's words remain highly significant at 0.021 points per word, while the investor's words become statistically insignificant. This pattern, aligned with the findings in Table 2, suggests that while investor input is valuable for providing information, the advisor's ability to process, synthesize, and respond to that information is the critical factor in generating accurate recommendations.

Figure 9: Endogenous and Exogenous Terminations



This stacked bar chart shows the distribution of conversation rounds by termination type across the dataset. The x-axis represents the total number of question rounds, while the y-axis shows the absolute count of cases. Each bar is divided into two categories: Endogenous Termination (blue) and Exogenous Termination (red). Endogenous termination occurs when the conversation naturally concludes based on the conversation flow, while exogenous termination happens due to external shocks.

The distribution of the total number of rounds in Figure 9 distinguishes between two types of conversation endings, highlighting key dynamics of information acquisition. Endogenous terminations occur when investors stop early because their beliefs have sufficiently

converged, representing optimal stopping in response to the tradeoff between information gained and the costs incurred. In contrast, exogenous terminations happen either due to a random stopping probability or upon reaching the eleven-question cap, independent of belief convergence, though the maximum length of 11 rounds is never actually reached. The distribution reveals that exogenous termination is more frequent in the first few rounds, while endogenous termination becomes the majority after the third round. This indicates that the investor actively weighs the tradeoff between gaining more information from conversation and the cost of communication. Moreover, just a few rounds of conversation are already quite helpful for the investor to become satisfied.

The central finding regarding investor impatience emerges from Column (6), which shows that exogenous termination—conversations ended artificially due to external shocks rather than reaching natural completion—reduces advisor recommendation accuracy by 0.988 percentage points. This nearly full percentage point penalty demonstrates that premature conversation endings significantly impair the advisor's performance.

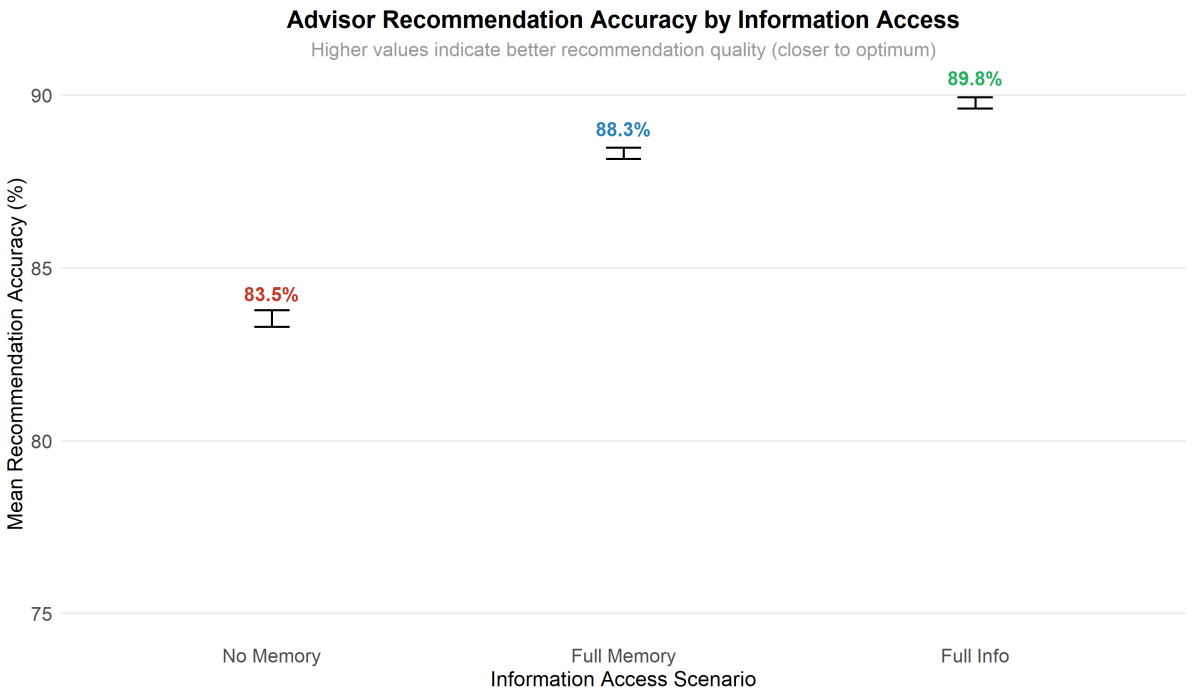
These results provide strong support for H2, indicating that investor impatience creates meaningful costs in advisory quality. The consistent negative impact of early termination, combined with the positive effects of extended interaction, suggests that the full advisory process requires adequate time and engagement to function effectively. For AI advisory systems, this highlights the importance of designing mechanisms that encourage sustained engagement and discourage premature exit from the advisory process.

4.3.3 Testing H3: Memory Augmentation and Advisor Performance

The third hypothesis examines whether providing LLM advisors with persistent memory improves their performance relative to memoryless systems. Figure 10 presents the results testing the benefits of memory augmentation.

The visualization demonstrates a clear hierarchy in recommendation quality based on information access: scenarios where the advisor has access to all available information including the investor's complete profile achieve the highest accuracy, followed by scenarios where the advisor has access to the complete conversation history, while scenarios where the advisor only sees the current question perform the worst. This strongly supports Hypothesis 3, illustrating that both conversation memory and comprehensive profile information significantly enhance AI advisor performance. These results underscore that memory augmentation and full access to user information are both valuable for generating accurate investment recommendations.

Figure 10: Memory Augmentation and Advisor Performance



This bar chart compares AI advisor recommendation accuracy across three information access scenarios: No Memory (advisor only sees the current question), Full Memory (advisor has access to the complete conversation history), and Full Info (advisor has access to all available information including the investor's complete profile). The y-axis shows mean recommendation accuracy as a percentage, with higher values indicating better recommendation quality and closer to the optimal equity allocation. Error bars represent standard errors.

4.3.4 Testing H4: Optimal AI Training and Advisor Priors

The fourth hypothesis examines whether investors benefit from AI advisors whose recommendations are more opinionated than their own beliefs, rather than merely aligned. To test this prediction, we implement a weighted pool analysis that directly maps to the theoretical structure of the model.

The key insight is to partition investor profiles into two pools based on their true preference type, then compute expected payoffs by weighting these pools according to the investor prior belief. We define two groups. The conservative pool consists of investors whose Vanguard optimal allocation is 0% equity, comprising 25 profiles and 15,125 observations. The aggressive pool consists of investors whose Vanguard optimal allocation is 80 to 100% equity, also comprising 25 profiles and 15,125 observations.

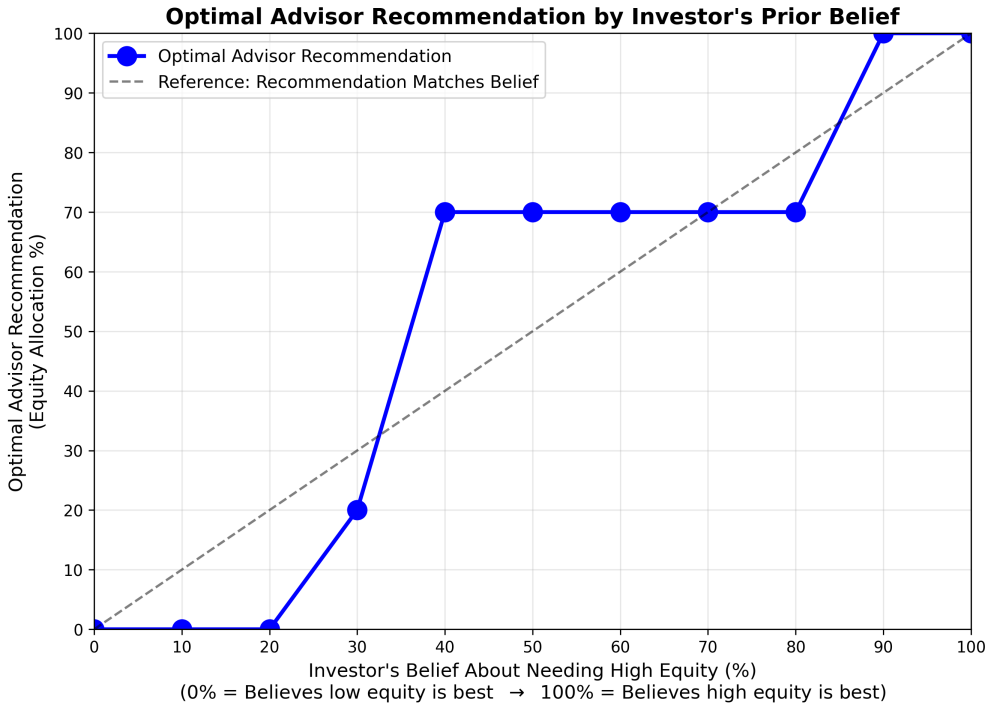
For each hypothetical investor with prior belief p_0 ranging from 0 to 100% in 10 percentage point increments, we compute the expected accuracy for each advisor recommendation level. The expected accuracy is a weighted average of accuracy in the aggressive pool, weighted by p_0, and accuracy in the conservative pool, weighted by $1 - p_0$. The optimal advisor recommendation for each investor prior is then the recommendation level that maximizes this expected accuracy. This methodology directly implements the theoretical expectation operator and allows us to trace out the empirical analog of Figure 4.

We use simulation data from 50 investor profiles drawn from the tails of the Vanguard allocation distribution. These include the 25 profiles with the lowest optimal allocations, all recommending 0% equity, and the 25 profiles with the highest optimal allocations (18 profiles at 80% and 7 profiles at 100%, yielding a mean of 85.6% equity). Each profile is simulated for 605 iterations, yielding 30,250 total observations. The advisor recommendation in each simulation serves as a proxy for the LLM prior, which we bin into 10 percentage point increments.

Figure 11 presents the main finding. The empirical optimal advisor recommendation exhibits a striking pattern that qualitatively matches the theoretical prediction in Figure 4.

For conservative investors whose prior belief is 20% or lower, the optimal recommendation is 0% equity, which is more conservative than their own belief. These investors benefit from an advisor who strongly reinforces their conservative inclination. For moderate to aggressive investors with prior beliefs between 40% and 80%, the optimal recommendation is approximately 70% equity, which exceeds their own belief for most of this range. The curve lies above the 45 degree line, indicating that these investors benefit from advisors who are more aggressive than themselves. For highly aggressive investors with prior beliefs of 90% or higher, the optimal recommendation is 100% equity, more extreme than their already high belief.

Figure 11: Optimal AI Advisor Recommendation by Investor Prior Belief



This figure plots the optimal advisor recommendation, measured as equity allocation percentage, against the investor prior belief about needing high equity. The x-axis represents the investor belief, where 0% indicates certainty of needing low equity and 100% indicates certainty of needing high equity. The blue line shows the empirically optimal advisor recommendation for each belief level. The dashed diagonal line represents the reference case where the advisor recommendation exactly matches the investor belief. When the blue line lies above the diagonal, the optimal advisor recommends a higher equity allocation than the investor own belief, indicating that opinionated advisors outperform aligned ones. Data consists of 50 profiles with 605 iterations each, totaling 30,250 observations from LLM simulated advisor investor conversations.

The step function pattern in Figure 11, rather than the smooth curve in the theoretical Figure 4, reflects two empirical features. First, advisor recommendations are binned into discrete 10 percentage point increments. Second, there is limited data support for recommendation levels that strongly contradict investor profiles, since advisors rarely recommend 0% equity to aggressive investors. Despite these limitations, the qualitative pattern strongly supports Hypothesis 4. Investors benefit from opinionated AI advisors whose recommendations are more extreme than their own beliefs, consistent with the option value of learning emphasized in the theoretical model.

4.3.5 Summary of Empirical Findings

The empirical results provide strong support for all four hypotheses derived from the theoretical model. H1 demonstrates that the primary value of LLM advising comes from investors clarifying their own preferences through self-reflection rather than receiving increasingly personalized recommendations, with each additional round improving accuracy by 1.03 percentage points and each word exchanged contributing 0.017 points to accuracy. H2 reveals that investor impatience, manifested through early conversation termination, significantly undermines advisor performance, with exogenous termination reducing recommendation accuracy by 2.62 percentage points. H3 shows that memory augmentation substantially improves AI advisor performance, with scenarios providing full information access achieving the highest accuracy, followed by full memory access, while no-memory scenarios perform worst. H4 confirms the model prediction about optimal AI training. Using a focused subsample of 50 profiles from the tails of the allocation distribution with 605 iterations each, we find that investors benefit from opinionated advisors whose recommendations are more extreme than their own prior beliefs, with the empirical optimal recommendation curve qualitatively matching the theoretical prediction in Proposition 3.

The ability to generate large-scale, realistic simulations (2,500 conversations across 500 profiles, plus 30,250 conversations across 50 tail profiles for H4) while maintaining the complexity of individual interactions provides unprecedented opportunities to test theories that depend on realistic communication patterns and adaptive behavior. By allowing researchers to observe mechanisms like optimal stopping in a practical setting, this methodology opens new possibilities for economic research in domains where human behavior is context-dependent and communication plays a central role. It offers a promising path forward for both theoretical development and the design of practical AI systems.

5 Conclusion

This study models the interaction between human and AI financial advisors as a dynamic optimal stopping game under two layers of uncertainty. We find that human advisors excel at interpreting soft, subjective information and clarifying ambiguous investor goals, while AI advisors provide unbiased, scalable recommendations but struggle to process unspoken preferences. This "soft information gap" limits the efficiency of AI-driven advice in complex decision-making contexts. Our results show that advisor value depends on the clarity of investor preferences. When goals are uncertain or evolving, human advisors' interpretive strengths can outweigh their incentive biases. Conversely, when preferences are well-defined, unbiased AI advisors can match or outperform humans. Moreover, in contexts where human advice is heavily biased, AI guidance offers a clear advantage.

This work contributes a theoretical framework that integrates strategic communication, advisor incentives, and the role of soft information, extending classic models and complementing recent empirical findings on robo-advisors and generative AI. By formalizing the inefficiencies created by digitizing soft information, we highlight key trade-offs in advisory relationships and set the stage for deeper integration of AI in finance.

A key methodological innovation of this study is the use of LLM simulations to operationalize and test our theoretical framework. By simulating realistic advisory interactions, we demonstrate how LLMs can serve as scalable, controlled environments for theory validation and refinement. This approach not only bridges theoretical and empirical analysis but also opens new avenues for using AI-driven simulations in behavioral finance and decision sciences.

Practically, our findings support a hybrid advisory approach. Novice or uncertain investors may benefit from initial human interaction, while experienced clients with clear goals can rely on AI platforms for efficient, unbiased advice. Enhancing AI systems with better memory and context-handling could further reduce information loss. Policymakers, meanwhile, should ensure that AI-driven advice remains transparent, unbiased, and aligned with fiduciary standards.

Future research could explore hybrid human–AI models, adaptive AI systems with extended memory, and applications in other fields such as medical or legal advising. Our framework provides a foundation for advancing the theory and practice of financial advice in the age of AI.

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A Technical Appendices

A.1 Additional Details for Section 2.2

In this part, we provide omitted details for the communication with the LLM.

Derivation of Eq. (13) . Since $p_t(z_t) = \frac{e^{z_t}}{1+e^{z_t}}$, Ito's Lemma implies

$$\begin{aligned} dp_t =& \left(\frac{e^{z_t}}{1+e^{z_t}}\right)'_{z_t} dz_t + \frac{1}{2}\left(\frac{e^{z_t}}{1+e^{z_t}}\right)''_{z_t} (dz_t)^2 \\ =& \frac{e^{z_t}}{(1+e^{z_t})^2} dz_t + \frac{1}{2}\frac{e^{z_t}}{(1+e^{z_t})^2}\left(1 - 2\frac{e^{z_t}}{1+e^{z_t}}\right)(dz_t)^2 \\ =& p_t(1-p_t)\left[\frac{1}{\sigma^2}(p_t - \frac{1}{2})dt + \frac{1}{\sigma} dB_t\right] + \frac{1}{2} p_t(1-p_t)(1-2p_t)\frac{dt}{\sigma^2} \\ =& \frac{p_t(1-p_t)}{\sigma^2} dB_t. \end{aligned}$$

The third equation uses the evolvement of z_t in (12), $(dz_t)^2 = \frac{(dB_t)^2}{\sigma}$ and $p_t(z_t) = \frac{e^{z_t}}{1+e^{z_t}}$.

Derivation of $\hat{p}_t = \mathbb{E}(\omega = 1 | ds_{t^-})$ (baseline). We use the following Binomial approximation of the process of Brownian motion. The investor observes the whole binomial tree whereas the LLM updates its belief only based on the last signal. Specifically, The probability that the particle goes up is

$$\pi = 0.5 + \frac{\omega}{2\sigma}\sqrt{\Delta t},$$

and the particle goes down is

$$1 - \pi = 0.5 - \frac{\omega}{2\sigma}\sqrt{\Delta t},$$

and the size of jump is

$$u = \sigma\sqrt{\Delta t}, \quad d = -\sigma\sqrt{\Delta t}.$$

For a particle that starts from zero, we have

$$\pi u + (1-\pi)\, d = (2\pi - 1)\, u = \frac{\omega}{\sigma}\sqrt{\Delta t} \times \sigma\sqrt{\Delta t} = \omega \Delta t,$$

which means it has a drift of $\omega \Delta t$.

Suppose the particle goes up in the last round. The posterior belief of the LLM is

$$\hat{p}_t = \frac{\pi(\omega = 1)\hat{p}_0}{\pi(\omega = 1)\hat{p}_0 + \pi(\omega = 0)(1 - \hat{p}_0)} = \frac{\left(0.5 + \frac{1}{2\sigma}\sqrt{\Delta t}\right)\hat{p}_0}{\left(0.5 + \frac{1}{2\sigma}\sqrt{\Delta t}\right)\hat{p}_0 + 0.5(1 - \hat{p}_0)}.$$

Under the continuous time setting where $\Delta t \to 0$, the above equation becomes $\hat{p}_t = \lim_{\Delta t \to 0} \frac{\left(0.5 + \frac{1}{2\sigma}\sqrt{\Delta t}\right)\hat{p}_0}{\left(0.5 + \frac{1}{2\sigma}\sqrt{\Delta t}\right)\hat{p}_0 + 0.5(1 - \hat{p}_0)} = \hat{p}_0$.

A.2 Proof of Lemma 1

Proof. The investor understands that the recommendation m^L is based on the LLM's belief $\hat{p}$ and satisfies

$$m^L(\hat{p}) = \hat{p}\theta_1 + (1 - \hat{p})\theta_0. \tag{31}$$

The investor chooses her optimal action $a(m^L)$ to solve

$$\max_a \mathbb{E}[U(a, p, \tilde{\theta}_1, \tilde{\theta}_0)] = -p\mathbb{E}[(a - \tilde{\theta}_1)^2 | m^L] - (1 - p)\mathbb{E}[(a - \tilde{\theta}_0)^2 | m^L].$$

Her optimal action is

$$a(m^L) = p\, \mathbb{E}[\tilde{\theta}_1 \mid m^L] + (1 - p)\, \mathbb{E}[\tilde{\theta}_0 \mid m^L]. \tag{32}$$

From the investor's perspective, $\tilde{\theta}_1$, $\tilde{\theta}_0$ and m^L in (31) are normal random variables. Hence, the conditional expectation of states given recommendation m^L are

$$\mu_{\theta_1|m} \equiv \mathbb{E}[\tilde{\theta}_1 \mid m^L] = \mu_1 + \frac{\hat{p}}{\hat{p}^2 + (1 - \hat{p})^2}\Big[m^L - \hat{p}\mu_1 - (1 - \hat{p})\mu_0\Big], \tag{33}$$

$$\mu_{\theta_0|m} \equiv \mathbb{E}[\tilde{\theta}_0 \mid m^L] = \mu_0 + \frac{1 - \hat{p}}{\hat{p}^2 + (1 - \hat{p})^2}\Big[m^L - \hat{p}\mu_1 - (1 - \hat{p})\mu_0\Big]. \tag{34}$$

The investor's expected utility given any recommendation m^L is then

$$\begin{aligned}\mathbb{E}[U(a(m^L),p,\tilde{\theta}_1,\tilde{\theta}_0)|m^L] &= -a(m^L)^2 + 2a(m^L)\left[p\mu_{\theta_1|m} + (1-p)\mu_{\theta_0|m}\right] - p\mathbb{E}[\tilde{\theta}_1^2|m^L] - (1-p)\mathbb{E}[\tilde{\theta}_0^2|m^L] \\ &= -\underbrace{\left[p\,\mathrm{Var}(\tilde{\theta}_1 \mid m^L) + (1-p)\,\mathrm{Var}(\tilde{\theta}_0 \mid m^L)\right]}_{MSE} - p(1-p)(\mu_{\theta_1|m} - \mu_{\theta_0|m})^2.\end{aligned}$$

The conditional expectations $\mu_{\theta_1|m}, \mu_{\theta_0|m}$ are given in (33) and (34). The conditional variance $\mathrm{Var}(\tilde{\theta}_1 \mid m^L) = \mathrm{Var}(\tilde{\theta}_1) - \frac{\mathrm{Cov}(\tilde{\theta}_1, m^L)^2}{\mathrm{Var}(m^L)} = \frac{(1-\hat{p})^2\sigma_\epsilon^2}{\hat{p}^2+(1-\hat{p})^2}$ and $\mathrm{Var}(\tilde{\theta}_0 \mid m^L) = \frac{\hat{p}^2\sigma_\epsilon^2}{\hat{p}^2+(1-\hat{p})^2}$. Now we take expectation over $m^L(\hat{p})$ in (31) to calculate the investor's expected utility when she seeks recommendation,

$$\begin{aligned}g(p,\hat{p}) =& \mathbb{E}[\mathbb{E}[U(a(m^L),p,\tilde{\theta}_1,\tilde{\theta}_0)|m^L]] \\ =& -\sigma_\epsilon^2\frac{p(1-\hat{p})^2 + (1-p)\hat{p}^2}{\hat{p}^2+(1-\hat{p})^2} - p(1-p)\mathbb{E}\left\{\mu_1 - \mu_0 + \frac{2\hat{p}-1}{\hat{p}^2+(1-\hat{p})^2}\left[m^L - \hat{p}\mu_1 - (1-\hat{p})\mu_0\right]\right\} \\ =& -\sigma_\epsilon^2\frac{p(1-\hat{p})^2 + (1-p)\hat{p}^2}{\hat{p}^2+(1-\hat{p})^2} - p(1-p)\left[(\mu_1-\mu_0)^2 + \frac{(2\hat{p}-1)^2\sigma_\epsilon^2}{\hat{p}^2+(1-\hat{p})^2}\right].\end{aligned}$$

Note that $g(p,\hat{p})$ is a quadratic function in p. □

A.3 Proof of Proposition 2

Proof. The investor continues if $p \in (\underline{p}, \overline{p})$ and stops to seek final recommendation if $p = \underline{p}$ or $\overline{p}$.

Step 1. We solve for the value function in the continuation region in closed form. We rewrite the differential equation in log-likelihood ratio $z = \ln\frac{p}{1-p}$, so that $p(z) = \frac{e^z}{1+e^z}$, $1-p(z) = \frac{1}{1+e^z}$. Note that

$$p'(z) = \frac{1}{z'(p)} = \frac{1}{\frac{1}{p} + \frac{1}{1-p}} = p(1-p). \tag{35}$$

Define $W(z) \equiv V(p(z))$. Then

$$W_z = V'(p)p'(z) = V'(p)p(1-p), \tag{36}$$

or equivalently $V'(p) = \frac{W_z}{p(1-p)}$. Then second derivative

$$V''(p) = \frac{d}{dp}\left[\frac{W_z}{p(1-p)}\right] = z'(p)\frac{d}{dz}\left[\frac{W_z}{p(1-p)}\right] = \frac{1}{p^2(1-p)^2}\left[W_{zz} - (1-2p)W_z\right].$$

Hence, the differential equation (22) becomes

$$-c + \lambda(g(p) - W(z)) + \frac{1}{2\sigma^2}[W_{zz} - (1-2p)W_z] = 0. \tag{37}$$

Now we substitute $W(z) = u(z)\hat{W}(z)$ and choose $u(z)$ to remove the W_z term in (37). Then

$$W_z = u_z\hat{W} + u\hat{W}_z, \quad W_{zz} = u_{zz}\hat{W} + 2u_z\hat{W}_z + u\hat{W}_{zz},$$

Using them in (37), we have:

$$-c + \lambda[g(p(z)) - u\hat{W}] + \frac{1}{2\sigma^2}\left\{u\hat{W}_{zz} + [2u_z - (1-2p)\,u]\,\hat{W}_z + [u_{zz} - (1-2p(z))u_z]\,\hat{W}\right\} = 0.$$

To eliminate the $\hat{W}_z$ term, impose

$$2u_z - (1-2p)u = 0 \;\Leftrightarrow\; \frac{u_p}{u} = \frac{1-2p}{p'(z)} = \frac{1}{2}\left(\frac{1}{p} - \frac{1}{1-p}\right). \tag{38}$$

Integrate w.r.t. p, we need to set

$$u(z) = \sqrt{p(z)(1-p(z))},$$

and then the differential equation becomes

$$\hat{W}_{zz} + \left[\frac{u_{zz} - (1-2p)u_z}{u} - 2\lambda\sigma^2\right]\hat{W} + \frac{2\sigma^2}{u}\left[\lambda g(p) - c\right] = 0. \tag{39}$$

From the definition of u function in (38), we know that $\frac{u_z}{u} = \frac{1-2p}{2}$. Then

$$\frac{d}{dz}\left(\frac{u_z}{u}\right) = \frac{u_{zz}}{u} - \left(\frac{u_z}{u}\right)^2,$$

which implies the coefficient in front of $\hat{W}$ in (39) is a constant:

$$\begin{aligned}\frac{u_{zz}}{u} - \frac{(1-2p)u_z}{u} - 2\lambda\sigma^2 &= p'(z)\frac{d}{dp}\left(\frac{u_z}{u}\right) + \left(\frac{u_z}{u}\right)^2 - \frac{(1-2p)u_z}{u} - 2\lambda\sigma^2 \\ &= p(1-p)\cdot(-1) + \left(\frac{1-2p}{2}\right)^2 - \frac{(1-2p)^2}{2} - 2\lambda\sigma^2 = -\frac{1}{4} - 2\lambda\sigma^2.\end{aligned}$$

Therefore, with the substitution of $z \equiv \ln\frac{p}{1-p}$ and $\hat{W}(z) \equiv \frac{W(z)}{\sqrt{p(z)(1-p(z)}} = \frac{V(p(z))}{\sqrt{p(z)(1-p(z))}}$, the

original differential equation in (22) could be rewritten as

$$\hat{W}_{zz} - (\frac{1}{4} + 2\lambda\sigma^2)\hat{W} + 2\sigma^2[\lambda g(p) - c] = 0.$$

Let $\gamma = \sqrt{\frac{1}{4} + 2\sigma^2\lambda}$. The homogeneous solution of the above differential equation is

$$\hat{W}_{homo}(z) = Ae^{z\gamma} + Be^{-z\gamma}.$$

Hence, using $e^z = \frac{p}{1-p}$, the homogeneous solution of the original differential equation (22) is

$$V_{homo}(p) = W_{homo}(p(z)) = u(p)\hat{W}_{homo}(z) = C_1 p^{\gamma+\frac{1}{2}}(1-p)^{-\gamma+\frac{1}{2}} + C_2 p^{-\gamma+\frac{1}{2}}(1-p)^{\gamma+\frac{1}{2}}.$$

Let $U_1(p) \equiv p^{\gamma+\frac{1}{2}}(1-p)^{-\gamma+\frac{1}{2}}$ and $U_2(p) = p^{-\gamma+\frac{1}{2}}(1-p)^{\gamma+\frac{1}{2}}$ denote the roots, and let q_0, q_1, q_2 denote the coefficients for stopping value $g(p) = q_0 + q_1 p + q_2 p^2$ in (19). Conjecture the following particular solution

$$V_{part}(p) = q(p) = g(p) - \frac{c}{\lambda} + 2q_2\left[U_1(p)\int_{k_1}^{p}\frac{U_2(s)}{W(s)}ds - U_2(p)\int_{k_2}^{p}\frac{U_1(s)}{W(s)}ds\right], \tag{40}$$

where $W(p) \equiv U_1(p)U_2'(p) - U_1'(p)U_2(p)$ is the Wronskian term, and k_1, k_2 are constants with degree of freedom (they will be jointly determined with C_1 and C_2). Now we check that the particular solution satisfies the ODE in (22); that is,

$$\frac{2\lambda\sigma^2}{p^2(1-p)^2}\left[q(p) - g(p) - \frac{c}{\lambda}\right] = q''(p). \tag{41}$$

The left hand side is

$$\frac{2\lambda\sigma^2}{p^2(1-p)^2}\left[q(p) - g(p) - \frac{c}{\lambda}\right] = \frac{2\lambda\sigma^2}{p^2(1-p)^2}\cdot 2q_2\left[U_1(p)\int_{k_1}^{p}\frac{U_2(s)}{W(s)}ds - U_2(p)\int_{k_2}^{p}\frac{U_1(s)}{W(s)}ds\right].$$

The right hand side is

$$\begin{aligned}
q''(p) &= 2q_2 + 2q_2\left[U_1'(p)\int_{k_1}^{p}\frac{U_2(s)}{W(s)}ds + U_1(p)\frac{U_2(p)}{W(p)} - U_2'(p)\int_{k_2}^{p}\frac{U_1(s)}{W(s)}ds - U_2(p)\frac{U_1(p)}{W(p)}\right]' \\
&= 2q_2 + 2q_2\left[U_1''(p)\int_{k_1}^{p}\frac{U_2(s)}{W(s)}ds + U_1'(p)\frac{U_2(p)}{W(p)} - U_2''(p)\int_{k_2}^{p}\frac{U_1(s)}{W(s)}ds - U_2'(p)\frac{U_1(p)}{W(p)}\right] \\
&= 2q_2\left[U_1''(p)\int_{k_1}^{p}\frac{U_2(s)}{W(s)}ds - U_2''(p)\int_{k_2}^{p}\frac{U_1(s)}{W(s)}ds\right].
\end{aligned}$$

Hence,

$$\frac{2\lambda\sigma^2}{p^2(1-p)^2}\left[q\left(p\right)-g\left(p\right)\right]-q''\left(p\right)=2q_2\int_{k_1}^{p}\frac{U_2(s)}{W(s)}ds\left[\frac{2\lambda\sigma^2}{p^2(1-p)^2}U_1(p)-U_1''(p)\right]$$
$$-2q_2\int_{k_2}^{p}\frac{U_1(s)}{W(s)}ds\left[\frac{2\lambda\sigma^2}{p^2(1-p)^2}U_2(p)-U_2''(p)\right]=0.$$

The last inequality holds because $U_1(p)$ and $U_2(p)$ are the roots of the homogeneous part of the ODE in (22).

In addition, the Wronskian term is a constant,

$$W(p)=\left(\frac{1}{2}-\gamma\right)(1-p)-\left(\gamma+\frac{1}{2}\right)p-\left(\gamma+\frac{1}{2}\right)(1-p)+\left(-\gamma+\frac{1}{2}\right)p=-2\gamma.$$

Therefore, the value function is

$$V(p)=g\left(p\right)-\frac{c}{\lambda}-\frac{q_2}{\gamma}\left[U_1\left(p\right)\int_{k_1}^{p}U_2\left(s\right)ds-U_2\left(p\right)\int_{k_2}^{p}U_1\left(s\right)ds\right]+C_1\left(k_1,k_2\right)U_1\left(p\right)+C_2\left(k_1,k_2\right)U_2\left(p\right).$$

Step 2. We show that in the continuation region, $V(p)-g(p)$ is symmetric around $\frac{1}{2}$, and the optimal stopping thresholds satisfy $\underline{p}+\overline{p}=1$.

To see this, we consider the following adjusted value function

$$v\left(p\right)=V\left(p\right)-g\left(p\right), \tag{42}$$

which captures the option value of waiting. When waiting is strictly optimal, $v\left(p\right)>0$. We rewrite the HJB in the continuation region, (22), and boundary conditions in terms of v. Plugging $V\left(p\right)=v\left(p\right)+g\left(p\right)$ in (22) we have

$$\lambda v\left(p\right)=\frac{p^2\left(1-p\right)^2}{2\sigma^2}g''\left(p\right)-c+\frac{p^2\left(1-p\right)^2}{2\sigma^2}v''\left(p\right).$$

Importantly, $g(p)$ is a quadratic function of p so $g''\left(p\right)$ is a constant.

First, note that both the flow benefit of waiting $\frac{p^2(1-p)^2}{2\sigma^2}g''\left(p\right)-c$ and the volatility $\frac{p^2(1-p)^2}{2\sigma^2}$ are larger when p is close to $\frac{1}{2}$ and smaller when p is close to 0 or 1. Hence, we conjecture that the principal continues if $p\in(\underline{p},\overline{p})$ and stops to seek final recommendation if $p=\underline{p}$ or $\overline{p}$. The boundary conditions in terms of $v(\cdot)$ are

$$v(\underline{p})=0,v'(\underline{p})=0,v(\overline{p})=0,v'(\overline{p})=0.$$

Second, both the flow benefit of waiting $\frac{p^2(1-p)^2}{2\sigma^2}g''\left(p\right)-c$ and the volatility $\frac{p^2(1-p)^2}{2\sigma^2}$ are

symmetric for p around 0.5. Intuitively, the underlying $\theta = 0$ and 1 indicates borrower type (matching $\tilde{\theta}_i$) and are interchangeable. Therefore, $\underline{p}+\overline{p} = 1$. Also, the symmetry means that there exists a critical upper bound learning cost $c < \bar{c}$ under which $v(\frac{1}{2}) > 0$ and the above waiting solution holds.

Step 3. We specify $k_1 = k_2 = \frac{1}{2}$ and show that $C_1 = C_2$. Note that for any $p \in [0, 1]$,

$$U_1(1-p) = U_2(p), \qquad U_2(1-p) = U_1(p), \tag{43}$$

$$U_1'(1-p) = -U_2'(p), \qquad U_2'(1-p) = -U_1'(p). \tag{44}$$

Let $I_1(p) \equiv \int_{0.5}^{p} U_1(s)ds$ and $I_2(p) \equiv \int_{0.5}^{p} U_2(s)ds$. The two value matching conditions are

$$0 = V\left(\underline{p}\right) - g\left(\underline{p}\right) = -\frac{c}{\lambda} - \frac{q_2}{\gamma}\left[U_1\left(\underline{p}\right)\int_{0.5}^{\underline{p}} U_2(s)\,ds - U_2\left(\underline{p}\right)\int_{0.5}^{\underline{p}} U_1(s)\,ds\right] + C_1U_1\left(\underline{p}\right) + C_2U_2\left(\underline{p}\right), \tag{45}$$

$$\begin{aligned} 0 = V\left(1-\underline{p}\right) - g\left(1-\underline{p}\right) = &-\frac{c}{\lambda} - \frac{q_2}{\gamma}\left[U_1\left(1-\underline{p}\right)\int_{0.5}^{1-\underline{p}} U_2(s)\,ds - U_2\left(1-\underline{p}\right)\int_{0.5}^{1-\underline{p}} U_1(s)\,ds\right] \\ &+ C_1U_1\left(1-\underline{p}\right) + C_2U_2\left(1-\underline{p}\right). \end{aligned} \tag{46}$$

Using (43) and (44) in (46),

$$\begin{aligned} 0 =& -\frac{q_2}{\gamma}\left[U_2\left(\underline{p}\right)\int_{0.5}^{1-\underline{p}} U_1(1-s)\,ds - U_1\left(\underline{p}\right)\int_{0.5}^{1-\underline{p}} U_2(1-s)\,ds\right] + C_1U_2\left(\underline{p}\right) + C_2U_1\left(\underline{p}\right) \\ \underbrace{=}_{t=1-s} & -\frac{q_2}{\gamma}\left[-U_2\left(\underline{p}\right)\int_{0.5}^{\underline{p}} U_1(t)\,dt + U_1\left(\underline{p}\right)\int_{0.5}^{\underline{p}} U_2(t)\,dt\right] + C_1U_2\left(\underline{p}\right) + C_2U_1\left(\underline{p}\right) \\ =& -C_1U_1\left(\underline{p}\right) - C_2U_2\left(\underline{p}\right) + C_1U_2\left(\underline{p}\right) + C_2U_1\left(\underline{p}\right) \\ =& (C_1 - C_2)\left[U_2\left(\underline{p}\right) - U_1\left(\underline{p}\right)\right]. \end{aligned}$$

The second equality follows from changing integrand from s to $t = 1 - s$, and the third equality uses Eq. (45). Since $\underline{p} \neq 0.5$ whenever the investor learns, we have $U_1(\underline{p}) \neq U_2(\underline{p})$ and so

$$C_1 = C_2 = C.$$

Step 4. We solve for the remaining unknowns C and $\underline{p}$.

Let $I_1(p) \equiv \int_{0.5}^{p} U_1(s)ds$ and $I_2(p) \equiv \int_{0.5}^{p} U_2(s)ds$. Using $C_1 = C_2 = C$, the value matching conditions imply

$$C\left(\underline{p}\right) = \frac{\frac{c}{\lambda} + \frac{q_2}{\gamma}\left[U_1\left(\underline{p}\right)I_2\left(\underline{p}\right) - U_2\left(\underline{p}\right)I_1\left(\underline{p}\right)\right]}{U_1\left(\underline{p}\right) + U_2\left(\underline{p}\right)}. \tag{47}$$

The smooth pasting condition then pins down $\underline{p}$.

$$0 = V'\left(\underline{p}\right) - g'\left(\underline{p}\right) = -\frac{q_2}{\gamma}\left[U_1'\left(\underline{p}\right) I_2\left(\underline{p}\right) - U_2'\left(\underline{p}\right) I_1\left(\underline{p}\right)\right] + C\left(\underline{p}\right)\left[U_1'\left(\underline{p}\right) + U_2'\left(\underline{p}\right)\right]. \quad (48)$$

Using the expression of $C\left(\underline{p}\right)$ in Eq. (47) in (48), we have

$$\frac{d}{dp}\left\{\frac{\frac{c}{\lambda} + \frac{q_2}{\gamma}\left[U_1(p) I_2(p) - U_2(p) I_1(p)\right]}{U_1(p) + U_2(p)}\right\}\Bigg|_{p=\underline{p}} = 0, \quad \text{where } \underline{p} \in (0, 0.5). \quad (49)$$

Step 5. We argue that there exists a unique solution to the optimal stopping problem. To show this, we show that there exists a unique $\underline{p} \in (0, 0.5)$ that solves Eq. (49). Note that

$$\begin{aligned}
&\operatorname{sgn}\left\{\frac{d}{dp}\left\{\frac{\frac{c}{\lambda} + \frac{q_2}{\gamma}\left[U_1(p) I_2(p) - U_2(p) I_1(p)\right]}{U_1(p) + U_2(p)}\right\}\right\} \\
=&\operatorname{sgn}\left\{\underbrace{\frac{q_2}{\gamma}\left(U_1' I_2 - U_2' I_1\right)\left(U_1 + U_2\right) - \left[\frac{c}{\lambda} + \frac{q_2}{\gamma}\left(U_1 I_2 - U_2 I_1\right)\right]\left(U_1' + U_2'\right)}_{\equiv M(p)}\right\}, \quad (50)
\end{aligned}$$

and we discuss the sign of $M(p)$ over $p \in [0, 0.5]$. Recall that $U_1(p) = p^{\gamma+\frac{1}{2}}(1-p)^{-\gamma+\frac{1}{2}}$, $U_2(p) = p^{\frac{1}{2}-\gamma}(1-p)^{\gamma+\frac{1}{2}}$ and $I_1(p) = \int_{\frac{1}{2}}^{p} s^{\gamma+\frac{1}{2}}(1-s)^{-\gamma+\frac{1}{2}}\,ds$, $I_2(p) = \int_{\frac{1}{2}}^{p} s^{\frac{1}{2}-\gamma}(1-s)^{\gamma+\frac{1}{2}}\,ds$. Then we have

$$\begin{aligned}
U_1'(p) &= \left(\gamma + \frac{1}{2}\right)\left(\frac{p}{1-p}\right)^{\gamma-\frac{1}{2}} - \left(-\gamma + \frac{1}{2}\right)\left(\frac{p}{1-p}\right)^{\gamma+\frac{1}{2}}, \\
U_2'(p) &= \left(\frac{1}{2} - \gamma\right)\left(\frac{p}{1-p}\right)^{-\gamma-\frac{1}{2}} - \left(\gamma + \frac{1}{2}\right)\left(\frac{p}{1-p}\right)^{\frac{1}{2}-\gamma}.
\end{aligned}$$

We evaluate these terms at the two endpoints $p = 0$ and $\frac{1}{2}$. Given that $\gamma = \sqrt{\frac{1}{4} + 2\sigma^2\lambda} > \frac{1}{2}$, we have $U_1(0) = 0, U_2(0) = \infty$, $U_1'(0) = 0$ and $U_2'(0) = -\infty$. Also, $U_1(0.5) = U_2(0.5) = 0.5$, $U_1'(0.5) = 2\gamma$ and $U_2'(0.5) = -2\gamma$. Then we evaluate $M(p)$ at these endpoints,

$$\begin{aligned}
&M(0) = -\frac{q_2}{\gamma}U_2'(0)I_1(0)U_2(0) - \left(\frac{c}{\lambda} - \frac{q_2}{\gamma}U_2(0)I_1(0)\right)U_2'(0) = -\frac{c}{\lambda}U_2'(0) = \infty, \\
&M(\frac{1}{2}) = 0.
\end{aligned}$$

We make then examine the monotonicity of $M(p)$ over $p \in [0, 0.5]$. We calculate

$$\begin{aligned}
M'(p) =& \frac{q_2}{\gamma}\left(U_1'' I_2 - U_2'' I_1 + 2\gamma\right)\left(U_1 + U_2\right) + \frac{q_2}{\gamma}\left(U_1' I_2 - U_2' I_1\right)\left(U_1' + U_2'\right) \\
& - \frac{q_2}{\gamma}\left(U_1' I_2 - U_2' I_1\right)\left(U_1' + U_2'\right) - \left[\frac{c}{\lambda} + \frac{q_2}{\gamma}\left(U_1 I_2 - U_2 I_1\right)\right]\left(U_1'' + U_2''\right) \\
=& 2q_2\left(U_1 + U_2\right) + \frac{q_2}{\gamma}\left(I_1 + I_2\right)\left[U_1'' U_2 - U_2'' U_1\right] - \frac{c}{\lambda}\left(U_1'' + U_2''\right) \\
=& 2q_2\left(U_1 + U_2\right) + \frac{q_2}{\gamma}\left(I_1 + I_2\right)\frac{2\sigma^2\lambda}{p^2\left(1-p\right)^2}\left[U_1 U_2 - U_2 U_1\right] - \frac{c}{\lambda}\frac{2\sigma^2\lambda}{p^2\left(1-p\right)^2}\left(U_1 + U_2\right) \\
=& 2\left(U_1 + U_2\right)\left[q_2 - \frac{c\sigma^2}{p^2\left(1-p\right)^2}\right],
\end{aligned}$$

where the second equation follows because $U_1(p)$ and $U_2(p)$ are the roots of the homogeneous ODE and so $\frac{p^2(1-p)^2}{2\sigma^2}U_i''(p) = \lambda U_i(p)$ for $i \in \{1, 2\}$. Since $U_1(p) \geq 0$, $U_2(p) \geq 0$, the sign of $M'(p)$ depends on $q_2 - \frac{c\sigma^2}{p^2(1-p)^2}$ and this term is increasing over $p \in [0, 0.5]$.

There are two cases depending on the sign of $q_2 - \frac{c\sigma^2}{p^2(1-p)^2}$ at $p = 0.5$. First, if it's negative, that is $\frac{q_2}{c\sigma^2} \leq 16$, $M'(p) < 0$ for $p \in [0, 0.5]$ and so $M(p)$ is decreasing in this region. Combined with $M(0) = \infty$ and $M(0.5) = 0$ at the two endpoints, $M(p)$ is always positive over $[0, 0.5)$. Hence, there is no solution for $\underline{p}$, which satisfies $M(\underline{p}) = 0$ for $p \in (0, 0.5)$. Second, if $\frac{q_2}{c\sigma^2} > 16$, $M(p)$ first decreases and then increases over $[0, 0.5]$. Combined with $M(0) = \infty$ and $M(0.5) = 0$ at the two endpoints, $M(p)$ crosses zero only once over $[0, 0.5)$, which pins down the unique solution of $\underline{p}$.

To summarize, in the continuation region, the value function is

$$V(p) = g(p) - \frac{c}{\gamma} - \frac{q_2}{\gamma}\left[U_1(p) I_2(p) - U_2(p) I_1(p)\right] + \frac{\frac{c}{\lambda} + \frac{q_2}{\gamma}\left[U_1\left(\underline{p}\right) I_2\left(\underline{p}\right) - U_2\left(\underline{p}\right) I_1\left(\underline{p}\right)\right]}{U_1\left(\underline{p}\right) + U_2\left(\underline{p}\right)} \cdot \left[U_1(p) + U_2(p)\right], \tag{51}$$

where $\underline{p} \in (0, 0.5)$ is uniquely determined by (49) if $\frac{q_2}{c\sigma^2} < 16$, $U_1(p) = p^{\gamma+\frac{1}{2}}\left(1-p\right)^{-\gamma+\frac{1}{2}}$ and $U_2(p) = p^{\frac{1}{2}-\gamma}\left(1-p\right)^{\gamma+\frac{1}{2}}$. The coefficients are $q_2 = (\mu_1 - \mu_0)^2 + \frac{(2\hat{p}-1)^2\sigma_\epsilon^2}{\hat{p}^2+(1-\hat{p})^2}$ and $\gamma = \sqrt{\frac{1}{4} + 2\sigma^2\lambda}$. □

A.4 Proof of Proposition 3

Proof. We find the optimal $\hat{p}$.

1. Stopping region. Introduce $x(\hat{p}) = \frac{\hat{p}}{1-\hat{p}}$. We can rewrite the investor's stopping value

in (19) as

$$\begin{aligned} g(p,\hat{p}) &= -\sigma_\epsilon^2 \frac{p(1-\hat{p})^2+(1-p)\hat{p}^2}{\hat{p}^2+(1-\hat{p})^2} - p(1-p)\left[(\mu_1^2-\mu_0)^2 + \frac{(2\hat{p}-1)^2\sigma_\epsilon^2}{\hat{p}^2+(1-\hat{p})^2}\right] \\ &= -\sigma_\epsilon^2 \frac{p+(1-p)\left(\frac{\hat{p}}{1-\hat{p}}\right)^2}{\left(\frac{\hat{p}}{1-\hat{p}}\right)^2+1} - p(1-p)\left[(\mu_1-\mu_0)^2 + \sigma_\epsilon^2 \frac{\left(\frac{\hat{p}}{1-\hat{p}}-1\right)^2}{\left(\frac{\hat{p}}{1-\hat{p}}\right)^2+1}\right] \\ &= -\sigma_\epsilon^2 \frac{2p-1}{x^2+1} - \sigma_\epsilon^2(1-p) - \sigma_\epsilon^2 p(1-p)\left[\left(\frac{\mu_1-\mu_0}{\sigma_\epsilon}\right)^2 + 1 - \frac{2x}{x^2+1}\right]. \end{aligned}$$

Take derivative with respect to x,

$$\operatorname{Sgn}\left\{\frac{\partial g(x,p)}{\partial x}\right\} = \operatorname{Sgn}\left\{(2p-1)\,x + p(1-p)\left(-x^2+1\right)\right\}.$$

The first-order condition for the optimal $x(p)$ then implies

$$-p\,(1-p)\,x^2 + (2p-1)\,x + p\,(1-p) = 0.$$

There are two roots,

$$x_1 = -\frac{1-p}{p}, \quad x_2 = \frac{p}{1-p},$$

and we take the positive root $x^*(p) = \frac{p}{1-p}$. Given the definition of $x = \frac{\hat{p}}{1-\hat{p}}$, we know that in the stopping region $p \notin [\underline{p}, \overline{p}]$,

$$\hat{p}^*(p) = p.$$

2. Continuation region. We rewrite (51) to highlight its dependence on $\hat{p}$, noting that $q_2(\hat{p})$ is a function of $\hat{p}$ and $\underline{p}$ is affected by $q_2(\hat{p})$:

$$\begin{aligned} V(p;\hat{p}) =& g(p;\hat{p}) - \frac{c}{\gamma} - \frac{q_2(\hat{p})}{\gamma}\left[U_1(p)I_2(p) - U_2(p)I_1(p)\right] \\ &+ \frac{\frac{c}{\lambda} + \frac{q_2(\hat{p})}{\gamma}\left[U_1\left(\underline{p}\right)I_2\left(\underline{p}(\hat{p})\right) - U_2\left(\underline{p}(\hat{p})\right)I_1\left(\underline{p}(\hat{p})\right)\right]}{U_1\left(\underline{p}(\hat{p})\right) + U_2\left(\underline{p}(\hat{p})\right)} \cdot \left[U_1(p) + U_2(p)\right] \end{aligned} \tag{52}$$

Introduce $H(p) \equiv U_1(p)I_2(p) - U_2(p)I_1(p)$ and $S(p) \equiv U_1(p) + U_2(p)$. Take derivative with

respect to $\hat{p}$,

$$\begin{aligned}\frac{dV(p;\hat{p})}{d\hat{p}} =& \frac{dg(p;\hat{p})}{d\hat{p}} - \frac{q_2'(\hat{p})}{\gamma}H(p) + \frac{q_2'(\hat{p})}{\gamma}\frac{H(\underline{p}(\hat{p}))}{S(\underline{p}(\hat{p}))}\cdot S(p) + \underbrace{\frac{d}{d\underline{p}}\left\{\frac{\frac{c}{\lambda}+\frac{q_2(\hat{p})}{\gamma}H(\underline{p}(\hat{p}))}{S(\underline{p}(\hat{p}))}\right\}}_{=0,\text{ (Envelope Theorem)}}\cdot S(p)\cdot\frac{d\underline{p}(\hat{p})}{d\hat{p}} \\ =& \frac{dg(p;\hat{p})}{d\hat{p}} - \frac{q_2'(\hat{p})}{\gamma}H(p) + \frac{q_2'(\hat{p})}{\gamma}\frac{H(\underline{p}(\hat{p}))}{S(\underline{p}(\hat{p}))}\cdot S(p), \end{aligned} \tag{53}$$

where we used the Envelope Theorem so that the indirect effect through $\underline{p}$ is zero (see (49).)

We can rewrite $g(p;\hat{p})$ in (19) as

$$\begin{aligned} g\left(p;\hat{p}\right) &= -\sigma_\epsilon^2\frac{p+(1-p)\left(\frac{\hat{p}}{1-\hat{p}}\right)^2}{\left(\frac{\hat{p}}{1-\hat{p}}\right)^2+1} - p(1-p)\left[(\mu_1-\mu_0)^2 + \frac{(\frac{\hat{p}}{1-\hat{p}}-1)^2\sigma_\epsilon^2}{\left(\frac{\hat{p}}{1-\hat{p}}\right)^2+1}\right] \\ &= -\sigma_\epsilon^2(1-p) - \sigma_\epsilon^2\frac{2p-1}{x(\hat{p})^2+1} - p(1-p)\underbrace{\left[(\mu_1-\mu_0)^2+\sigma_\epsilon^2 - \frac{2x(\hat{p})\sigma_\epsilon^2}{x(\hat{p})^2+1}\right]}_{q_2(\hat{p})}. \end{aligned} \tag{54}$$

Then

$$\begin{aligned}\frac{dg(p;\hat{p})}{d\hat{p}} &= \frac{2\sigma_\epsilon^2}{(1+\hat{p})^2(x(\hat{p})^2+1)^2}\cdot\left[(2p-1)x(\hat{p}) - p(1-p)(x(\hat{p})^2-1)\right], \\ q_2'(\hat{p}) &= \frac{2\sigma_\epsilon^2}{(1+\hat{p})^2(x(\hat{p})^2+1)^2}\cdot(x(\hat{p})^2-1).\end{aligned}$$

Using the above results in (53),

$$\frac{dV(p;\hat{p})}{d\hat{p}} = \frac{2\sigma_\epsilon^2}{(1+\hat{p})^2(x(\hat{p})^2+1)^2}\cdot\left\{(2p-1)x(\hat{p}) - p(1-p)(x(\hat{p})^2-1) - \frac{x(\hat{p})^2-1}{\gamma}\left[H(p) - \frac{H(\underline{p}(\hat{p}))}{S(\underline{p}(\hat{p}))}\cdot S(p)\right]\right. \tag{55}$$

Optimal $\hat{p}$ for $p_0 = 0.5$. In this case,

$$\frac{dV(p;\hat{p})}{d\hat{p}} = \frac{2\sigma_\epsilon^2}{(1+\hat{p})^2(x(\hat{p})^2+1)^2}\cdot\frac{x(\hat{p})^2-1}{\gamma}\cdot\left[-\frac{\gamma}{4} + \frac{H(\underline{p}(\hat{p}))}{S(\underline{p}(\hat{p}))}\right]. \tag{56}$$

We intend to show that $-\frac{\gamma}{4} + \frac{H(\underline{p}(\hat{p}))}{S(\underline{p}(\hat{p}))} < 0$ for any $\hat{p}$. Note that $\frac{H(p)}{S(p)}$ decreases in p when

$p \in (0, 0.5)$:

$$\left(\frac{H(p)}{S(p)}\right)' = \frac{(U_1' I_2 - U_2' I_1)(U_1 + U_2) - (U_1 I_2 - U_2 I_1)(U_1' + U_2')}{(U_1 + U_2)^2}$$
$$= \frac{(U_1' U_2 - U_1 U_2')(I_1 + I_2)}{(U_1 + U_2)^2} = \frac{2\gamma(I_1 + I_2)}{(U_1 + U_2)^2} < 0.$$

The inequality holds because $I_1 = \int_{0.5}^{p} U_1(s)ds < 0$ and $I_2 = \int_{0.5}^{p} U_2(s)ds < 0$. Hence, for any $\hat{p}$ and resulting $\underline{p} \in (0, 0.5)$, we have

$$\frac{H(\underline{p}(\hat{p}))}{S(\underline{p}(\hat{p}))} < \frac{H(0)}{S(0)} = \int_0^{\frac{1}{2}} s^{\gamma+\frac{1}{2}}(1-s)^{\frac{1}{2}-\gamma} ds.$$

In addition, we can show that $\frac{H(0;\gamma)}{G(0;\gamma)}$ decreases in γ for $\gamma = \sqrt{\frac{1}{4} + 2\sigma^2\lambda} > \frac{1}{2}$. To see this,

$$\frac{d}{d\gamma}\left[\int_0^{\frac{1}{2}} s^{\gamma+\frac{1}{2}}(1-s)^{\frac{1}{2}-\gamma} ds\right] = \int_0^{\frac{1}{2}} s^{\gamma+\frac{1}{2}}(1-s)^{\frac{1}{2}-\gamma} \ln\frac{s}{1-s} ds < 0.$$

The inequality holds because for $s \in (0, \frac{1}{2})$, we have $0 < \frac{s}{1-s} < 1$ and $\ln\frac{s}{1-s} < 0$. This means that

$$\int_0^{\frac{1}{2}} s^{\gamma+\frac{1}{2}}(1-s)^{\frac{1}{2}-\gamma} ds < \int_0^{\frac{1}{2}} s ds = \frac{1}{8}.$$

As a result, the key term in (56) is negative,

$$-\frac{\gamma}{4} + \frac{H(\underline{p}(\hat{p}))}{S(\underline{p}(\hat{p}))} < -\frac{\gamma}{4} + \int_0^{\frac{1}{2}} s^{\gamma+\frac{1}{2}}(1-s)^{\frac{1}{2}-\gamma} ds < -\frac{\gamma}{4} + \frac{1}{8} < 0.$$

For any $\hat{p} \in (0, \frac{1}{2})$, we have $x = \frac{\hat{p}}{1+\hat{p}} < 1$ and $x^2 - 1 < 0$, so the derivative in (56) is always positive—that is, value improves as $\hat{p}$ increases in $(0, \frac{1}{2})$. For any $\hat{p} \in (\frac{1}{2}, 1)$, we have $x = \frac{\hat{p}}{\hat{p}+1} > 1$ and $x^2 - 1 > 0$, so the derivative in (56) is always negative—value decreases as $\hat{p}$ increases in this region. Therefore, when $p_0 = 0.5$, the optimal LLM training is $\hat{p}^*(0.5) = 0.5$.

Option value of learning. Note that (55) can be decomposed as

$$\frac{dV(p;\hat{p})}{d\hat{p}} = \frac{dg(p;\hat{p})}{d\hat{p}} - \underbrace{\frac{2\sigma_\epsilon^2 S(p)}{\gamma(1+\hat{p})^2(x(\hat{p})^2+1)^2}}_{+} \cdot \underbrace{\left[x(\hat{p})^2 - 1\right]}_{+(-) \text{ if } \hat{p}>(<)0} \left[\frac{H(p)}{S(p)} - \frac{H(\underline{p}(\hat{p}))}{S(\underline{p}(\hat{p}))}\right]. \tag{57}$$

The first term maximizes $g(p;\hat{p}$—the optimal LLM training if the investor immediately seeks recommendation, and the second term corresponds to the option value of learning.

We have shown in the previous part that $\frac{H(p)}{S(p)}$ decreases in p when $p \in (0, 0.5)$. Similarly,

considering the sign flip of $I_1 + I_2 > 0$ when $p \in (0.5, 1)$, we know that $\frac{H(p)}{S(p)}$ increases in p when $p \in (0.5, 1)$. Hence, in the continuation region where $p \in (\underline{p}, \overline{p})$, we have

$$\frac{H(p)}{S(p)} - \frac{H(\underline{p}(\hat{p}))}{S(\underline{p}(\hat{p}))} < 0.$$

In addition, since $x(\hat{p}) = \frac{\hat{p}}{1-\hat{p}}$, we have $x(\hat{p})^2 - 1 > 0$ if $\hat{p} > 0.5$ and $x(\hat{p})^2 - 1 < 0$ if $\hat{p} < 0.5$. Taken together, we have $\hat{p}^*(p_0) > p_0$ if $p_0 \in (0.5, \overline{p}(\hat{p}))$ and $\hat{p}^*(p_0) < p_0$ if $p_0 \in (\underline{p}(\hat{p}), 0.5)$.

□

B Appendices for Empirical Exercise

B.1 Hypotheses Testable via Observational Data

The second set of hypotheses involves predictions that can be examined using real-world observational data, such as records from a financial platform offering both human and LLM-based advising. In our context, Yingmi Wealth's Qieman (meaning "Hold On"), which is an intelligent investment advisory platform in China, provides a useful example: some investors on the platform may choose to consult a human financial advisor, while others use an AI advisor. By leveraging data on these choices, alongside investor characteristics and outcomes, we can potentially test how the model's mechanisms play out in practice.

H4 (Preference Uncertainty and Advisor Choice): Investors who are more uncertain about their own financial goals or risk preferences are more likely to choose a human advisor over an LLM-based advisor.

The theoretical setup suggests that when investors are unsure about their preferences, such as unclear risk tolerance or retirement goals, human advisors may be better equipped to interpret nuanced soft information through interactive conversation compared to AI-based tools. This advantage is further supported by the model's ability to account for uncertainty in understanding investor preferences, which suggests that human advisors are more effective in dynamically clarifying the needs of investors with less self-awareness.

To test this hypothesis, we propose analyzing observational data on advisor selection based on investors' self-reported uncertainty. One approach is to use survey or onboarding data to construct a metric of ex ante preference uncertainty. This metric could be derived from variability in responses to risk tolerance questions or the absence of a clear investment goal. Using such data, a regression or discrete-choice model could be estimated to determine the probability of selecting a human advisor versus an AI advisor as a function of this uncertainty metric. The prediction is that investors with greater preference uncertainty will exhibit a significantly higher likelihood of opting for human advisors.

H5 (Extreme Investor Types and AI Adoption): Investors with extreme initial risk profiles are more likely to opt for the LLM advisor, especially those whose risk assessment falls in the very conservative or very aggressive ends of the distribution.

The intuition from the model suggests that for extreme investor types, those who are either highly risk-averse or highly risk-seeking, the potential downside from any misalignment in the AI's recommendations is relatively smaller. This arises because even if the AI's recommendations are slightly off, they will still be close to the investor's true preferences or optimal strategy. On the other hand, human advisors with biases—such as those stemming from sales commissions—introduce additional "cheap-talk" costs. This inherent impartiality

of the LLM makes its advice more appealing to investors situated at the spectrum's extremes.

To test this hypothesis, empirical analysis can be conducted by studying how investors' risk profiles correlate with their choice of advisor. Specifically, using data on initial risk scores, such as scores derived from risk assessment questionnaires, we can examine whether investors who fall into the lowest or highest quantiles of risk tolerance are more likely to opt for the LLM over human advisors. A probit or logit regression can be used where the decision to adopt the LLM is regressed on indicators for "very low risk tolerance" and "very high risk tolerance," controlling for other relevant factors. Positive coefficients on these indicators would support the hypothesis.

H6 (Commission Incentives and Advisor Choice): The adoption of the LLM advisor will be higher in settings where human financial advisors are paid on commission, as opposed to a fee or salary basis.

The model suggests that when human advisors have incentives to influence clients, such as earning commissions from selling specific products, their advice becomes more biased and less credible. In such situations, the informational value of human advisors' cheap-talk messages decreases, leading investors to favor the unbiased AI advisors. To test this theory, researchers could explore variations in compensation structures either across different regions or over time. For example, comparing adoption rates of LLM advisors in settings where some branches of a financial platform rely mainly on commission-based pay while others use fixed salaries.

A regression analysis can be employed with the proportion of investors opting for AI advisors as the dependent variable. The primary independent variable would measure the strength of commission-based incentives for human advisors. It is expected that stronger commission incentives or a shift toward a commission-heavy compensation model would correlate with increased AI advisor adoption. Establishing that investors choose LLM advisors significantly more often in high-commission contexts would offer evidence supporting the hypothesis.

H7 (Investor Experience and Advisor Performance): Among more experienced or financially sophisticated investors, those who effectively know their type with greater precision, the performance gap between the LLM and a human advisor is smaller and may even reverse in favor of the LLM.

The idea is that experienced investors can convey their objectives and constraints more clearly or already understand them well, so a human advisor's ability to uncover soft information becomes less critical. According to the model, when the investor's type is already known or obvious, the human advisor's traditional edge in interpreting the investor's needs vanishes, leaving only the downside of the human's potential bias versus the AI's objectiv-

ity. In such cases, the neutral LLM advisor could perform just as well or better in aligning recommendations with the investor's true preferences.

To test this hypothesis, observational data can be analyzed, focusing on variables related to investor experience, such as years of investment experience, trading volume, or financial literacy scores. This data would also need measures of advice outcomes, like realized portfolio returns, risk-adjusted performance, or consistency with stated goals. An empirical strategy could involve a regression analysis, where an interaction term between investor experience and advisor type measures the relative effectiveness of AI and human advisors. Specifically, the hypothesis predicts that the interaction term (Experience $\times$ LLM) will have a positive coefficient in the regression, indicating that the performance of AI advisors improves as investor experience increases.

B.2 Vanguard Questionnaire

1. Once you start withdrawing money from your investments, you plan to spend it over a period of...
   (a) 2 years or less
   (b) 3-5 years
   (c) 6-10 years
   (d) 11-15 years
   (e) More than 15 years
2. When making a long-term investment, you plan to keep the money invested for...
   (a) 1-2 years
   (b) 3-4 years
   (c) 5-6 years
   (d) 7-8 years
   (e) More than 8 years
3. When it comes to investing in stock or bond mutual funds or ETFs (or individual stocks or bonds) you would describe yourself as...
   (a) Very inexperienced
   (b) Somewhat inexperienced

(c) Somewhat experienced

(d) Experienced

(e) Very experienced

4. You plan to begin taking money from your investments in...

   (a) 1 year or less

   (b) 1-2 years

   (c) 3-5 years

   (d) 6-10 years

   (e) 11-15 years

   (f) More than 15 years

5. Your current and future income sources (for example, salary, social security, pensions) are...

   (a) Very unstable

   (b) Unstable

   (c) Somewhat stable

   (d) Stable

   (e) Very stable

6. From September 2008 through October 2008, bonds lost 4%. If you owned a bond investment that lost 4% in two months, you would...

   (a) Sell all the remaining investment

   (b) Sell a portion of the remaining investment

   (c) Hold onto the investment and sell nothing

   (d) Buy more of the remaining investment

7. The below table shows the greatest 1-year loss and the highest 1-year gain on 3 different hypothetical investments of $10,000.

   - Investment A (gain $593; loss -$164)
   - Investment B (gain $1,921; loss -$1,020)
   - Investment C (gain $4,229; loss -$3,639)

Given the potential gain or loss in any 1 year, you would invest your money in...

(a) minimal volatility

(b) moderate volatility

(c) most volatility

8. During market declines, you tend to sell portions of your riskier assets and invest the money in safer assets. **(R)**

(a) Strongly disagree

(b) Disagree

(c) Somewhat agree

(d) Agree

(e) Strongly agree

9. You would invest in a mutual fund or ETF (exchange-traded fund) based solely on a brief conversation with a friend, co-worker, or relative. **(R)**

(a) Strongly disagree

(b) Disagree

(c) Somewhat agree

(d) Agree

(e) Strongly agree

10. From September 2008 through November 2008, stocks lost over 31%. If you owned a stock investment that lost about 31% in three months, you would...

(a) Sell all the remaining investment

(b) Sell a portion of the remaining investment

(c) Hold onto the investment and sell nothing

(d) Buy more of the remaining investment

11. Generally, you prefer an investment with little or no ups and downs in value, and you are willing to accept the lower returns these investments may make. **(R)**

(a) Strongly disagree

(b) Disagree

(c) Somewhat agree

(d) Agree

(e) Strongly agree

Note: Questions marked with (R) are reverse-scored items where higher numerical responses indicate lower risk tolerance.