When Does Antitrust Enforcement Discipline Common Ownership? A General Equilibrium Analysis*

Vishavdeep Sharma†

Abstract

This paper integrates antitrust enforcement into the general equilibrium oligopoly framework of Azar and Vives (2021a). In their model, common ownership by diversified institutional investors generates anti-competitive effects: shareholders who hold large stakes in rival firms prefer that managers internalize rivals' profits, which softens competition. Their framework abstracts from antitrust enforcement. I introduce a deterrence mechanism into their one-sector model: firms face expected fines for anti-competitive conduct, with damages redistributed to harmed workers under Clayton Act §4. I show that a unique threshold of portfolio diversification exists above which the enforcement channel dominates the anti-competitive channel, reversing the comparative static of their model. Calibrating the model at the same one-sector parameter values and projecting the empirical common-ownership trend through 2030 using a trend-stationary autoregressive specification, I find that the enforcement channel substantially moderates the anti-competitive prediction at empirical levels of common ownership, and that the projected trajectory approaches the reversal threshold within the forecast horizon. The findings inform the policy debate over whether existing antitrust mechanisms or new structural restrictions on institutional investor diversification are warranted.

Keywords: Common ownership; Antitrust enforcement; Clayton Act; General equilibrium oligopoly

JEL Classification: D43, G34, J42, K21, L41

*I thank Ian Sheldon for helpful comments. This research received no specific grant funding. I have no financial relationships or conflicts of interest related to the subject of this paper. All errors are my own.

†Department of Agricultural, Environmental, and Development Economics (AEDE), The Ohio State University, Columbus, OH 43210, United States. Email: sharma.1094@osu.edu

1 Introduction

Common ownership of product-market and labor-market competitors by diversified institutional investors rises substantially over the past four decades. The Big Three index fund families — BlackRock, Vanguard, and State Street — collectively hold equity stakes in nearly every S&P 500 firm, and the average degree to which large US firms internalize rivals' profits (the Edgeworth sympathy coefficient) nearly doubles since the mid-1980s (Antón et al., 2023; Backus et al., 2021). A growing literature predicts that this rise generates substantial anti-competitive effects (Azar et al., 2018; Azar and Vives, 2021a). The mechanism is straightforward: when shareholders hold large stakes in rival firms, they prefer that managers internalize rivals' profits, which softens competition.

A natural question follows: is this softening of competition legal under existing antitrust law? The specific policy debate over common ownership is active and unresolved (Harvard Law School Forum on Corporate Governance, 2025). Those who view it as illegal argue that horizontal shareholding violates §7 of the Clayton Act (Elhauge, 2016), that institutional investor diversification should face direct restrictions (Posner et al., 2017), and that empirical evidence from the US airline industry shows common ownership distorts ticket prices (Azar et al., 2018). The 2023 Merger Guidelines now treat common ownership and cross-holdings as a basis for closer antitrust review (U.S. Department of Justice and Federal Trade Commission, 2023). This unresolved debate motivates a closer look at the institutional features of antitrust enforcement, which the canonical common-ownership framework of Azar and Vives (2021a) abstracts from.[1]

Moreover, the anti-competitive effects of common ownership do not stop at the firm's product market. They pass through to the workers the firm employs. The evidence on this labor-market harm is substantial. For instance, workers in US manufacturing receive only sixty-five cents on each marginal dollar they generate, with plant-level markdowns averaging 1.53 (Yeh et al., 2022). This wedge is not innocuous: in general equilibrium, oligopsonistic wage-setting generates a 7.6 percent welfare loss and a 20.9 percent output gap (Berger et al., 2022). And these wedges are widespread: sixty percent of US local labor markets are highly concentrated by the DOJ–FTC standard (Azar et al., 2020), about 18 percent of US workers cannot freely join a competing employer because of noncompete clauses in their contracts (Starr et al., 2021), and 58 percent of major franchisors agree not to hire each other's workers (Krueger and Ashenfelter, 2022). In response, antitrust enforcement has caught up with this evidence. The 2023 Merger Guidelines treat employers as buyers of labor

[1] In their reply to discussants, Azar and Vives (2021b) describe their objective as "mostly methodological" and the one-sector model as "a device to illustrate ownership and market power mechanisms."

for the first time (Bredbenner and Hovenkamp, 2025). The Federal Trade Commission's (FTC) February 2025 Joint Labor Task Force has prosecuted similar restraints affecting tens of thousands of workers at Gateway Services, Rollins, and Adamas Amenity Services (Federal Trade Commission, 2025; Morrison and Foerster LLP, 2026). Whether antitrust tools should be used more aggressively against monopsony is contested (Albrecht et al., 2024; Dimick, 2023; Naidu et al., 2018; Posner, 2024; Prager, 2026). But the debate has paid little attention to common ownership itself: in US labor markets, common ownership more than doubles between 1999 and 2017, and it depresses earnings at locally competing employers (Azar et al., 2022b).

To answer this, I introduce antitrust enforcement into the general equilibrium oligopoly framework of Azar and Vives (2021a) in one sector. I add expected enforcement fines that firms pay for anti-competitive conduct, in the deterrence style of Becker (1968). The expected fine depends on the intensity of common ownership across rival firms and on the firm's labor-market markdown.[2] The modified model nests Azar and Vives (2021a) as the no-enforcement special case.

I make three contributions. First, I show that a unique reversal threshold exists under standard assumptions, above which equilibrium employment and real wages do not decrease as common ownership rises. The threshold depends on the enforcement penalty, the number of firms, and the labor-supply elasticity, and is strictly decreasing in the penalty. The reversal follows from a structural asymmetry. The same diversified shareholders who internalize rivals' profits also internalize rivals' fines; both effects operate through the Edgeworth sympathy coefficient that drives the Azar and Vives (2021a) anti-competitive prediction. Profit internalization softens competition; fine internalization disciplines it. The anti-competitive effect saturates at high diversification, but the enforcement effect persists, so the disciplining channel eventually dominates.[3]

Second, I introduce a redistribution mechanism in which collected fines flow to harmed workers as a uniform per-capita transfer, consistent with the structure of Clayton §4 damages and with the disposition of recoveries in In re High-Tech Employee Antitrust Litigation (2014).[4] This trans-

[2]Per-rival cross-holdings are observable through Securities and Exchange Commission Form 13F filings (Backus et al., 2021; U.S. Securities and Exchange Commission, 2024) and feed directly into the Modified Herfindahl–Hirschman Index used in the 2023 Merger Guidelines. The wage markdown is the conduct evidence used to compute damages in labor-market monopsony cases.

[3]Intuitively: at full common ownership, the managers weight a dollar at a rival firm the same as a dollar at their own firm and behave like a single cartel — no further common ownership can make them collude more, because caring more than equally about a rival makes no economic sense. The firms still exercise monopsony power, however, and continue to pay below the competitive wage. This is the conduct Clayton §4 punishes, and the fine scales with the size of the wage gap, which has no such ceiling. When a damages claim succeeds, the same investors pay the fine at every firm they own. The anti-competitive gain saturates; the enforcement cost does not.

[4]Approximately 64,000 software engineers received $415 million from Apple, Google, Adobe, Intel, and other defendants, illustrating this redistribution (In re High-Tech Employee Antitrust Litigation, 2014).

fer makes the framework genuinely general-equilibrium under enforcement: workers' labor supply responds to the augmented budget constraint, the real wage adjusts, and the markdown is endogenously redetermined. The modified equilibrium does not achieve the first-best allocation, but it lies strictly between the no-enforcement equilibrium of Azar and Vives (2021a) and the efficient benchmark, with the gap closing in enforcement penalty.

Third, I calibrate the model at Azar and Vives (2021a)'s one-sector parameter values, project the empirical common-ownership trend through 2030 using a trend-stationary autoregressive specification on the historical sympathy coefficient series, and quantify the welfare and efficiency consequences of enforcement at penalty intensities spanning statutory treble damages[5] alone through the upper bound implied by the empirical loss multiplier in the law-and-finance literature (Karpoff et al., 2008). At empirical levels of common ownership, the enforcement channel substantially moderates the anti-competitive prediction of the no-enforcement benchmark. At the upper-bound calibration, the equilibrium markdown rises by only two percent between 1985 and 2017, against forty-one percent in the no-enforcement baseline. The projected trajectory approaches the reversal threshold within the forecast horizon.

The remainder of the paper is organized as follows. Section 2 reviews related literature. Section 3 describes the institutional background on common ownership and antitrust enforcement. Section 4 develops the general equilibrium oligopoly model with antitrust enforcement. Section 5 characterizes the modified Cournot–Walras equilibrium, and establishes the reversal proposition. Section 6 provides the illustrative specifications under separable isoelastic preferences over consumption and leisure. Section 7 calibrates the model to US data over 1985–2017, projects the empirical ϕ through 2030, characterizes the reversal threshold, and quantifies welfare and efficiency at calibrated parameters. Section 8 discusses policy implications. Section 9 discusses extensions and directions for future research. Section 10 concludes.

2 Related Literature

This paper sits at the intersection of three literatures: the theory of common ownership, the law-and-economics literature on antitrust enforcement, and the empirical literature on market power and labor-market concentration.

The general equilibrium oligopoly framework with common ownership is developed by Azar and

[5] Treble damages award the harmed party three times the actual harm suffered, plus attorney's fees, under Clayton Act §4.

Vives (2021a), who construct a tractable Cournot–Walras equilibrium in which firms maximize a share-weighted average of owner indirect utilities. Their central result is that, under non-increasing returns, increased portfolio diversification raises the modified Herfindahl–Hirschman Index and depresses real wages, employment, and the labor share. Azar and Vives (2019) extend this analysis to a calibrated macroeconomic environment, attributing a substantial portion of post-1980 declines in the US labor share to rising effective concentration through common ownership. The Azar (2020) "trilemma" formalizes a related insight: portfolio diversification, shareholder representation, and competition cannot be simultaneously achieved within the existing institutional architecture. The contribution can be read as identifying a fourth lever — antitrust enforcement — that interacts with the trilemma constraint by disciplining the conduct that emerges when diversification and shareholder representation are jointly maximized. The theoretical apparatus rests on earlier partial-equilibrium work on managerial objectives under overlapping ownership: Rotemberg (1984) first proposes that managers maximize a weighted average of shareholder utilities, and O'Brien and Salop (2000) extend this formulation by separating cash-flow and control rights and introduce the modified Herfindahl–Hirschman Index that Azar and Vives (2021a) adopt. Recent empirical work by Azar and Vives (2024) finds, in airline industry data, that intra-industry common ownership raises prices while inter-industry common ownership reduces them. A related strand studies the contract-design margin: Antón et al. (2023) show theoretically and empirically that common owners induce flatter managerial incentive contracts. Their contract-design margin and the enforcement margin are complementary mechanisms — the former operates on the principal's choice of contract, the latter on the consequences of the conduct that emerges. I follow Azar and Vives (2021a) in treating the manager as a share-weighted aggregator of owner indirect utilities rather than as a separate agent with independent preferences, preserving comparability with the base paper.

The deterrence framework I integrate with common-ownership theory originates with Becker (1968) and is synthesized in Polinsky and Shavell (2007): expected sanctions equal the probability of detection times the magnitude of punishment. The specification adapts this framework to the common-ownership setting, with detection probability proportional to the product of two observable enforcement triggers identified in the 2023 US Department of Justice and Federal Trade Commission Merger Guidelines: the structural cross-holding intensity per rival firm, and the firm's observed labor-market wedge. The policy debate on common ownership has produced a sharp division. Posner et al. (2017) propose direct restrictions on institutional investor diversification in concentrated industries; Rock and Rubinfeld (2018) and Hemphill and Kahan (2020) push back

on both empirical and welfare grounds, arguing that such restrictions would impose substantial costs on retail investors and disrupt index investing. Intermediate positions are taken by Elhauge (2016) and Scott Morton and Hovenkamp (2018), who advocate aggressive enforcement of existing antitrust statutes against common-ownership conduct without new structural restrictions.[6] The framework provides a formal vehicle for evaluating these positions: I characterize the enforcement intensity at which existing legal mechanisms become sufficient to discipline common-ownership effects, and quantify how current US enforcement compares to this threshold. A complementary literature studies labor-market enforcement specifically: Naidu et al. (2018) and Marinescu and Hovenkamp (2019) argue for explicit antitrust action against employer monopsony, and Azar and Marinescu (2024) provide a recent synthesis of monopsony theory and policy. The contribution to this strand is to formalize the general-equilibrium feedback through which labor-market enforcement disciplines firms whose ownership structure would otherwise produce wage markdowns.

A substantial empirical literature documents rising market power in the US economy. De Loecker et al. (2020) document a large increase in markups since 1980, accompanied by a declining labor share; Nekarda and Ramey (2019) estimate a more modest increase using alternative methods. Autor et al. (2020) link these trends to the rise of "superstar" firms in concentrated industries, and Barkai (2020) documents the parallel decline in capital shares. On the labor-market side, Berger et al. (2022) estimate substantial labor-market markdowns at the firm level, and Azar et al. (2022a) document high concentration in US commuting-zone labor markets. The empirical evidence on common ownership specifically includes Azar et al. (2018) on airlines, Azar et al. (2022c) on banking, and Boller and Scott Morton (2020) on profit margins. Backus et al. (2021) provide foundational measurement infrastructure for empirical common-ownership research, and Antón et al. (2023) document that average pairwise profit weights for larger US firms reached approximately 0.60 by 2019. Bebchuk and Hirst (2019) document that the median combined Big Three stake in S&P 500 firms rose from 7.1 percent in 2000 to 21.9 percent by 2021. Dissenting evidence is summarized in Dennis et al. (2022), Rossi-Hansberg et al. (2021), and Karabarbounis and Neiman (2019), with Shapiro (2018) arguing that documented magnitudes are too modest to warrant substantial policy concern. Harvard Law School Forum on Corporate Governance (2025) document no robust relationship between firm-level common ownership and involvement in antitrust litigation, a finding that motivates the institutional focus on the deterrent effect of existing enforcement rather

[6] For a recent comprehensive review of common ownership and its implications for antitrust practice, see Ederer and Tecu (2025).

than on litigation volume.

3 Institutional Background

This section lays out the institutional context that motivates the framework. The plausibility of integrating antitrust enforcement into the Azar and Vives (2021a) framework rests on three preconditions: common ownership is empirically prevalent, antitrust enforcement operates through legally specified mechanisms tied to observable ownership structure, and recent enforcement practice treats common ownership and labor-market conduct as priorities. I document each in turn.

3.1 The Rise of Common Ownership

The world of dispersed corporate ownership described by Berle and Means (1932) no longer characterizes the US economy. Institutional investors dominate equity markets, and the asset-management industry has consolidated around a small number of large diversified investors. The Big Three — BlackRock, Vanguard, and State Street — collectively hold the largest combined position in approximately ninety percent of S&P 500 firms, with average aggregate stakes near twenty percent of each publicly traded US company (Fichtner et al., 2017). Index funds now control more equity than active funds for many of the largest US corporations (Bebchuk and Hirst, 2019).

A direct consequence is the prevalence of common ownership within concentrated industries. Azar et al. (2018) document that the four largest passive asset managers hold combined stakes exceeding ten percent in nearly every major US airline simultaneously; analogous patterns hold for banking, pharmaceuticals, and technology. Banal-Estañol et al. (2018) report that the average intra-industry sympathy coefficient for the largest 1,500 US firms by market capitalization rose from approximately 0.41 in 1985 to approximately 0.72 in 2017, nearly doubling over the period. In this paper, common-ownership parameter captures this trend.

The economic significance of common ownership rests on a key institutional feature: ownership stakes above five percent must be disclosed quarterly to the Securities and Exchange Commission via Form 13F filings (U.S. Securities and Exchange Commission, 2024). These filings are publicly available and enable third parties — including antitrust enforcers — to observe cross-holdings in real time. Per-rival cross-holdings are computable directly from public 13F data and feed into the modified Herfindahl–Hirschman indices that flag industries for scrutiny. The structural detection trigger in the model is grounded in this observability.

3.2 The Legal Architecture of Antitrust Enforcement

US antitrust law provides three statutory instruments that collectively constitute the deterrence framework underpinning the model: Sherman Act §§1 and 2, Clayton Act §4, and Clayton Act §7.

The Sherman Act of 1890 prohibits "contracts, combinations, or conspiracies in restraint of trade" (§1) and monopolization or attempts to monopolize (§2). Violations are prosecutable criminally, with maximum penalties of $100 million for corporations and $1 million plus ten years imprisonment for individuals. Although criminal antitrust prosecutions are rare in absolute terms, the personal exposure they create is severe and largely non-indemnifiable: standard directors-and-officers insurance policies exclude intentional antitrust violations (Baker, 2003).

The Clayton Act of 1914 broadened the enforcement architecture by creating a private right of action under §4. Any party injured by an antitrust violation may sue for treble damages plus reasonable attorney's fees. The treble multiplier serves three purposes recognized in the deterrence literature: it compensates for the low *ex ante* probability of detection, restoring expected sanctions to the level of actual harm; it incentivizes private enforcement, supplementing the limited budget of public agencies; and it compensates plaintiffs for litigation risk and legal costs. The §4 mechanism is central to the model in two ways. First, it provides the institutional foundation for the corporate fine: the magnitude of expected enforcement penalties is determined by the treble-damages structure together with the empirical multiplier of Karpoff et al. (2008), who show that total firm-level losses following enforcement actions are approximately seven to nine times the direct fine due to indirect costs including market-value declines and operational disruption. Second, it provides the redistribution channel: damages flow not to the government but directly to the harmed parties — consumers in product-market cases, workers in labor-market monopsony cases. This redistribution closes the resource balance in the general equilibrium framework.

Section 7 of the Clayton Act prohibits mergers and acquisitions whose effect "may be substantially to lessen competition." The Department of Justice and Federal Trade Commission apply §7 review to common-ownership acquisitions, including under the recent revision of the Horizontal Merger Guidelines (U.S. Department of Justice and Federal Trade Commission, 2023). Guideline §2.10 explicitly identifies common ownership as a structural feature warranting closer scrutiny in concentrated industries, with the Modified Herfindahl–Hirschman Index — which incorporates cross-holdings via the same sympathy coefficient that drives the Azar and Vives (2021a) anticompetitive prediction — serving as the primary structural indicator. The Guidelines' use of MHHI

validates the specification of the structural detection trigger as per-rival cross-holding intensity.

3.3 The Detection Function in Practice

The two arguments of the detection probability in the model correspond directly to the structural and conduct triggers used by enforcers in practice. The Modified HHI used in the 2023 Merger Guidelines is computed from 13F-disclosed cross-holdings and aggregated into an industry-level indicator of common ownership. Industries with Modified HHI above 2,500 are classified as highly concentrated and trigger investigation; industries above 1,800 face elevated scrutiny (Shapiro, 2018). As cross-holdings per rival rise, detection probability rises proportionally.[7]

Once an industry passes structural screening, enforcement focuses on observable conduct. Markups, price-cost margins, and wage markdowns serve as direct evidence of market power. In labor-market cases under Sherman §1, the wage markdown is the precise quantity that plaintiffs' economists estimate to compute damages (see, e.g., Edward Leamer's expert report in In re High-Tech Employee Antitrust Litigation, 2014). The larger the wedge, the more compelling the case for prosecution and the larger the recoverable treble damages.

The product form of detection probability and damages magnitude in the model captures the conjunction of these triggers, consistent with how DOJ and FTC investigations are actually structured. A firm with high cross-holdings but no observed wedge attracts little enforcement attention; a firm with a large wedge in a dispersed-ownership industry similarly does not. Only the conjunction generates substantial expected fines.[8]

3.4 Recent Enforcement: Labor Markets and the High-Tech Case

Antitrust enforcement against labor-market collusion has intensified substantially over the past decade. Between 2005 and 2009, senior executives of Apple, Google, Adobe, Intel, Intuit, Lucasfilm, and Pixar entered into secret agreements not to actively recruit one another's engineers, enforced

[7] No Clayton §4 case has yet been litigated to judgment on a pure common-ownership theory. The deterrence framework here relies on expected rather than realized enforcement: the 2023 Merger Guidelines establish MHHI-based structural review of common-ownership transactions, and the established Clayton §4 recovery in coordinated-employer cases such as *High-Tech Employee* ($415 million, 2015) establishes the damages template. Following Becker (1968), what disciplines firm conduct is the credible threat of prosecution, and absence of prosecutions is consistent with effective deterrence.

[8] The conjunctive logic is explicit in the 2023 Merger Guidelines' structural-plus-conduct screening architecture: structural concentration raises the prior on anticompetitive intent, and conduct evidence raises the likelihood of successful prosecution (U.S. Department of Justice and Federal Trade Commission, 2023). Kwoka (2014) documents empirically that merger challenges scale with both structural and conduct indicators, supporting the multiplicative form. Alternative additive specifications would imply that either trigger alone generates substantial enforcement risk, which is inconsistent with observed agency triage.

through direct chief-executive intervention when violations occurred. The Department of Justice investigated the conduct beginning in 2010 and filed civil charges under Sherman §1, resulting in consent decrees prohibiting the practice. A class action under Clayton §4 followed, brought on behalf of approximately 64,000 affected software engineers (In re High-Tech Employee Antitrust Litigation, 2014). Plaintiffs' damages estimates — prepared by Edward Leamer — placed total wage suppression at approximately $3 billion; treble damages would have exceeded $9 billion. The case settled in 2015 for $415 million, paid directly to the affected engineers.

This case is paradigmatic for the framework. Common-ownership-style coordination among horizontally competing employers reduced wage competition, generating a labor-market markdown. Detection occurred through the conjunction of observable structure (CEO-level coordination across firms) and conduct (documented wage suppression) — precisely the dual trigger in the model. Damages flowed from the firms to the harmed workers, corresponding to the corporate loss and worker recovery in the framework.

The High-Tech case has been followed by sustained enforcement attention to labor-market practices. The 2016 *Antitrust Guidance for Human Resource Professionals* (U.S. Department of Justice and Federal Trade Commission, 2016) announced criminal prosecution for naked no-poach agreements — a substantial escalation. Subsequent cases targeted no-poach agreements in fast-food franchising (multistate settlements including Jimmy John's and McDonald's), animation studios (*In re Animation Workers Antitrust Litigation*, $170 million settlement), and hospital nursing markets (Benmelech et al., 2022). The 2021 Executive Order on Promoting Competition in the American Economy directed federal agencies to increase scrutiny of labor-market concentration (The White House, 2021). The 2023 Merger Guidelines elevated labor-market effects to the same standing as product-market effects in merger review (U.S. Department of Justice and Federal Trade Commission, 2023). This trajectory is important because labor-market enforcement under Sherman §1 and Clayton §4 is precisely the channel through which common-ownership-induced wage suppression becomes costly. As enforcement priorities shift toward labor markets, the expected corporate cost of common-ownership conduct rises, strengthening the disciplining channel I identify.

4 The Model

I extend the general equilibrium oligopoly framework of Azar and Vives (2021a) (henceforth AV) to incorporate antitrust enforcement under Clayton Act §4. The economy retains the two-agent

structure, workers & owners, and I introduce no additional agent type. Antitrust enforcement enters the model in two ways: firms face expected fines that reduce their profits, and the recovered fines are redistributed to workers as compensatory damages under Clayton Act § 4. The manager remains a pure share-weighted aggregator of owner indirect utilities, exactly as in AV.[9] The modification is parsimonious: a single new parameter, the enforcement penalty multiplier $K \geq 0$, governs enforcement intensity. The model nests AV at $K = 0$.

4.1 Agents and Preferences

The economy consists of a finite number J of firms and a continuum of two types of agents, workers and owners, denoted by the sets I^W and I^O respectively. Each set has unit mass.

Each worker has a time endowment T and supplies labor $L_i \in [0, T]$ in exchange for a wage w. There are two goods: a consumption good with price p and leisure with price w. Worker preferences over consumption and leisure are represented by the twice continuously differentiable utility function $U(C_i, L_i)$ satisfying $U_C > 0, U_L < 0, U_{CC} < 0, U_{LL} < 0$, and $U_{CL} \leq 0$.[10] The last condition implies that the marginal utility of consumption is non-increasing in labor supply.

Owners hold all of the firms' shares. Following AV, I assume that owners are divided uniformly into J groups, with owners in group j holding a share $1 - \phi + \phi/J$ of firm j and a share ϕ/J of each rival firm $k \neq j$, where $\phi \in [0, 1]$. The parameter ϕ measures the level of portfolio diversification, or (quasi-)indexation, in the economy. Each owner derives utility solely from consumption: $U^O(C_i) = C_i$. The financial wealth of an owner i in group j is, before enforcement,

$$W_i = \frac{1 - \phi + \phi/J}{1/J} \pi_j + \sum_{k \neq j} \phi \, \pi_k,$$

where π_k denotes the profits of firm k. I modify this expression below to account for enforcement.

4.2 Production

Firms use only labor as an input. Each firm $j \in \{1, \ldots, J\}$ has access to a twice continuously differentiable production function $F(L_j)$ with $F' > 0$ and $F(0) \geq 0$. I assume non-increasing

[9] I follow AV in not introducing the manager as a separate agent with preferences distinct from those of owners. The "firm's manager" is a share-weighted aggregator of owner indirect utilities; it has no independent objective and does not appear in market clearing. Its sole role is to determine the firm's labor demand consistent with the aggregated preferences of its shareholders.

[10] Here U_C denotes the partial derivative of U with respect to C, and U_{CL} denotes the cross-partial derivative of U with respect to C and L.

returns throughout ($F'' \leq 0$). Firm j's gross profits are

$$\pi_j = p\, F(L_j) - w\, L_j.$$

4.3 Antitrust Enforcement under Clayton Section 4

I introduce antitrust enforcement following the canonical Becker (1968) deterrence framework: each firm faces an expected sanction equal to the probability of detection times the magnitude of penalty if detected. Both objects are grounded in the operative US antitrust framework, in particular Clayton Act § 4 treble damages.

Definition 1. *The detection probability of firm j is a twice continuously differentiable function on the interior of its domain, $\rho : \left[0, \frac{1}{J}\right] \times [0, \bar{\mu}] \longrightarrow [0, 1]$:*

$$\rho_j = \rho\left(\frac{\phi}{J},\, \mu_j\right), \tag{1}$$

where $\mu_j = \big(F'(L_j) - \omega\big)/\omega$ is firm j's labor-market markdown, $\omega = w/p$ is the real wage, and $\bar{\mu} < \infty$ is an upper bound on the admissible wedge. The function ρ satisfies:

(A1) *$\partial\rho/\partial(\phi/J) > 0$ and $\partial\rho/\partial\mu_j > 0$ on the interior of the domain.*

(A2) $\lim_{\phi \to 1} \partial\rho/\partial(\phi/J) > 0, \quad \forall\, \mu_j > 0.$

(A3) $\partial^2\rho/\partial(\phi/J)\,\partial\mu_j \geq 0.$

The two arguments of ρ_j correspond to the two empirical triggers identified by the US Department of Justice and the Federal Trade Commission's 2023 Merger Guidelines §2.10 (U.S. Department of Justice and Federal Trade Commission, 2023). The first argument, ϕ/J, is the average cross-holding by each shareholder in any given rival firm; this is the structural object observable through 13F filings and is the basis for modified-HHI–type concentration measures. The second argument, μ_j, is the firm's observed labor-market wedge, which constitutes the conduct evidence cited in Section 1 and Section 2 cases under the Sherman Act.

Assumption A1 reflects the empirical content of the 2023 Merger Guidelines' screening thresholds. Industries with a Modified Herfindahl–Hirschman Index above 2,500 trigger investigation, and those above 1,800 face elevated scrutiny; both thresholds are monotone increasing in cross-holding intensity. Analogously, the size of a firm's observed wedge is a direct input into plaintiffs' damages

estimates in labor-market cases (see Edward Leamer's expert report in In re High-Tech Employee Antitrust Litigation, 2014), so larger wedges generate both higher detection probabilities through agency triage and larger recoverable damages through private enforcement under Clayton §4. Both arguments of ρ are therefore strictly monotone in the interior of the domain.

Assumption A2 is a regularity condition beyond monotonicity: a function can be strictly increasing in its first argument and yet have vanishing derivative at the boundary — for instance, $\rho(\phi/J, \mu) = (\mu/\bar{\mu})\left[1 - (1 - J \cdot \phi/J)^2\right]$ is strictly increasing in ϕ/J but satisfies $\partial\rho/\partial(\phi/J) \to 0$ as $\phi/J \to 1/J$.[11] A2 rules out exactly this saturation case, capturing the requirement that enforcement responsiveness does not "max out" at high per-rival concentration. The canonical specification $\rho(\phi/J, \mu) = \min\{1, (\phi/J)\mu\}$ satisfies A2 automatically, since $\partial\rho/\partial(\phi/J) = \mu$ at any interior point with $(\phi/J)\mu < 1$, which evaluates to $\mu > 0$ at $\phi = 1$ for any positive markdown. At the calibrated parameters $\mu^* \in (0.7, 1.6)$ and $J = 4$ of Section 7, the interior condition $\mu^*/J < 1$ holds with substantial margin, so A2 is satisfied at the empirically relevant equilibrium. Economically, A2 captures the fact that enforcement responsiveness scales with the conduct variable μ: as long as firms practice positive markdowns, agency detection responds to marginal increases in per-rival cross-holding, regardless of how diversified ownership is. Since $\mu^* > 0$ at any interior oligopsony equilibrium, A2 holds structurally rather than as an exogenous imposition.

Two strands of evidence support A2. First, US antitrust law is conduct-based: Sherman §1, Clayton §4, and FTC §5 penalize observed anti-competitive behavior — wage coordination, price-fixing, monopsony — rather than ownership structure directly. Under the canonical specification, (A2) holds whenever firms commit the anti-competitive acts these statutes penalize, a condition well-documented in recent labor-market enforcement.[12] Second, enforcement intensity scales continuously with industry concentration across the observed range, without plateau: Kwoka (2014) documents strictly increasing merger challenge probabilities in post-merger HHI, and the 2023 DOJ/FTC Merger Guidelines §2.10 (U.S. Department of Justice and Federal Trade Commission, 2023) explicitly recognize horizontal shareholding as a consideration in merger review — the first formal policy acknowledgment that agencies respond to common ownership directly. Elhauge (2016) and Posner et al. (2017) argue that existing conduct-based authorities already reach common-

[11] The function depends on J as a structural parameter through $\phi = J \cdot (\phi/J)$; we suppress this in the notation since J is fixed throughout.

[12] The In re High-Tech Employee Antitrust Litigation (2014) prosecution of Apple, Google, Adobe, and Intel for no-poach agreements proceeded on conduct evidence despite substantial common institutional ownership across these firms; DOJ no-poach settlements in fast-food franchises (2018–2020) and animation (2017), and ongoing wage-coordination investigations in meatpacking (2022–), follow the same conduct-triggered logic.

ownership harms through this same channel.

Assumption A3 is consistent with the conjunctive logic of agency triage: a firm with both large cross-holdings and a large wedge attracts disproportionate enforcement attention relative to what either trigger would generate independently.[13]

4.3.1 Damages under Clayton Section 4

Damages under Clayton Section 4 are conditional on detection. I construct the damages magnitude from its statutory primitives. In a labor-market monopsony case, the harm each employed worker suffers is the gap between the competitive wage they should have been paid — the marginal product of labor $F'(L_j)$, and the wage they actually received, w. In real terms this per-worker underpayment is $F'(L_j) - \omega \;=\; \mu_j\,\omega$, which follows from the definition of the markdown $\mu_j = \big(F'(L_j) - \omega\big)/\omega$. Expressed in nominal units, per-worker harm equals $\mu_j\, w$. Aggregating across the L_j workers employed by firm j yields total compensatory harm equal to $\mu_j\, w\, L_j$. Clayton §4 mandates treble damages on compensatory harm, contributing a statutory multiplier $K = 3$.[14] The total damages magnitude is therefore

$$\mathcal{D}_j \;=\; K\,\mu_j\, w\, L_j, \tag{2}$$

in which $\mu_j\, w$ is per-worker harm, L_j is the size of the harmed class of workers, and K is the statutory multiplier. The parameter K is thus the penalty multiplier, with $K = 0$ corresponding to the no-enforcement benchmark of AV.

4.3.2 Expected Fine

Combining the general detection probability from Definition 1 with the damages magnitude in (2), the expected fine facing firm j is[15]

$$\tau_j \;=\; \rho\!\left(\frac{\phi}{J},\, \mu_j\right) \cdot K\,\mu_j\, w\, L_j. \tag{3}$$

The expected fine is increasing in the markdown μ_j through two distinct channels. The damages component $\mathcal{D}_j$ scales with μ_j mechanically under Clayton §4: larger per-worker underpayments generate larger statutory recoveries, which is a legal implication rather than a modeling choice.

[13] The conjunctive logic is explicit in the 2023 Merger Guidelines' structural-plus-conduct screening architecture, in which structural concentration raises the prior on anticompetitive intent and conduct evidence raises the likelihood of successful prosecution; the joint product determines enforcement priority.

[14] $K = 3$ is a lower bound on the penalty multiplier.

[15] In real (consumption-good) units, $\tau_j/p \;=\; \rho(\phi/J,\, \mu_j) \cdot K\,\mu_j\,\omega\, L_j$.

The detection component ρ scales with μ_j through assumption (A1): firms with larger wedges face higher agency scrutiny as a matter of triage practice. The expected fine therefore embeds both a statutory channel and an institutional-behavior channel, each operating through the observable wedge.

4.3.3 Net Profits

Firm j's profits net of expected enforcement are

$$\pi_j^{\text{net}} = \pi_j - \tau_j = p\,F(L_j) - w\,L_j \;-\; \rho\!\left(\frac{\phi}{J},\,\mu_j\right)\cdot K\,\mu_j\,w\,L_j. \tag{4}$$

The owner wealth expression accordingly becomes

$$W_i = \frac{1-\phi+\phi/J}{1/J}\big(\pi_j - \tau_j\big) + \sum_{k\neq j}\phi\big(\pi_k - \tau_k\big).$$

4.4 Clayton Section 4 Recovery

Antitrust fines do not represent deadweight loss in the framework. Under Clayton Act § 4, treble damages are paid directly to the parties harmed by the violation (Polinsky and Shavell, 2007). In labor-market monopsony cases, the harmed parties are workers underpaid by the markdown μ_j. Recent enforcement provides a concrete illustration: approximately 64,000 software engineers received $415 million in damages from Apple, Google, Adobe, Intel, and other defendants who had coordinated to suppress wage competition during 2005–2009 (settled 2015) (In re High-Tech Employee Antitrust Litigation, 2014).

I model this recovery by assuming that aggregate fines are redistributed in full to workers as a uniform per-capita transfer:

$$T^W = \sum_{j=1}^{J}\tau_j. \tag{5}$$

The worker's budget constraint is then

$$p\,C_i = w\,L_i + T^W. \tag{6}$$

Equation (5) reflects the simplifying assumption that the recovery rate $\theta = T^W/\sum_j \tau_j$ equals one. In practice, $\theta < 1$ because class-action procedures involve attorney fees, administrative costs,

and incomplete claims participation.[16] Relaxing $\theta = 1$ would require modeling the recipients of the residual $(1-\theta)\sum_j \tau_j$ — plaintiffs' counsel, claims administrators, and *cy pres* recipients — as additional agents with their own preferences and consumption. This extension moves the framework from the two-agent structure of AV to a multi-agent general equilibrium and is beyond the scope of the present paper. I adopt $\theta = 1$ for tractability and note that the qualitative results of Section 5 are invariant to the recovery rate because the firm-side cost of fines, which drives the reversal mechanism, does not depend on where the fines ultimately land.

This redistribution preserves the resource balance of the economy: total income equals total production. Specifically,

$$\int_{I^W \cup I^O} p\, C_i\, di \;=\; w\, L + \sum_{j=1}^{J} (\pi_j - \tau_j) + T^W \;=\; p \sum_{j=1}^{J} F(L_j),$$

where $L = \sum_j L_j$ is aggregate employment.

4.5 Manager's Objective

The manager of firm j aggregates owner indirect utilities with weights proportional to ownership shares, as in AV. The indirect utility of owner i in group j is $V^O(p, w; W_i) = W_i/p$, which depends linearly on the modified profits $\pi_k^{\text{net}} = \pi_k - \tau_k$. After applying AV's aggregation algebra to the modified wealth expression and regrouping terms, the manager's objective simplifies to

$$u_j^M = \frac{\pi_j - \tau_j}{p} + \lambda(\phi, J) \sum_{k \neq j} \frac{\pi_k - \tau_k}{p}, \tag{7}$$

where the Edgeworth sympathy coefficient $\lambda(\phi, J)$ is identical to that of AV:

$$\lambda(\phi, J) = \frac{(2-\phi)\phi}{(1-\phi)^2 J + (2-\phi)\phi}. \tag{8}$$

Two observations are essential. *(i) Nesting.* When $K = 0$, expected fines vanish ($\tau_j = 0$ for all j), and (7) reduces to the manager's objective in AV: $u_j^M = \frac{\pi_j}{p} + \lambda(\phi, J) \sum_{k \neq j} \frac{\pi_k}{p}$. The model thus nests AV exactly. *(ii) Mechanism.* The same sympathy coefficient λ that channels AV's anti-competitive effect now also channels a pro-competitive enforcement effect. Diversified shareholders

[16] Attorney fees in Clayton §4 class actions typically run 25–33% of settlement value under federal rules, and incomplete class participation further reduces recovery. In *In re High-Tech Employee Antitrust Litigation*, workers recovered approximately 75–80% of nominal damages after legal fees.

internalize not only rivals' profits π_k but also rivals' expected fines τ_k. The first internalization softens competition; the second disciplines it. As ϕ rises, both effects strengthen — but they push in opposite directions. The relative strength of these two channels determines whether equilibrium markdowns rise or fall with diversification. This is the central mechanism I develop in Section 5.[17]

The reversal that follows is not a mechanical consequence of opposing forces: λ saturates as $\phi \to 1$ while ρ does not (Assumption A2), and it is this asymmetry — not the existence of an opposing channel per se — that locates the threshold in the empirically relevant range.

4.6 Equilibrium Concept

I follow AV's notion of imperfectly competitive equilibrium with shareholder representation, modified to incorporate regulatory enforcement.

Definition 2. *Given production plans $(L_1, \ldots, L_J)$, a competitive equilibrium relative to these plans is a price system and allocation $\left[\{w, p\}; \{C_i, L_i\}_{i \in I^W}, \{C_i\}_{i \in I^O}\right]$ such that:*

1. *For each $i \in I^W$, (C_i, L_i) maximizes $U(C_i, L_i)$ subject to $p\, C_i \leq w\, L_i + T^W$; for each $i \in I^O$, $C_i = W_i/p$ where $W_i = \frac{1-\phi+\phi/J}{1/J}(\pi_j - \tau_j) + \sum_{k \neq j} \phi(\pi_k - \tau_k)$.*
2. *Labor market clears: $\int_{I^W} L_i \, di = \sum_{j=1}^{J} L_j$.*
3. *Goods market clears: $\int_{I^W \cup I^O} C_i \, di = \sum_{j=1}^{J} F(L_j)$.*

A price function assigns prices $\{w, p\}$ to each labor demand vector $L = (L_1, \ldots, L_J)$. The economy is in equilibrium when every firm's employment and production plans coincide with the expectations of all other firms, given the price function.

Definition 3. *A Cournot–Walras equilibrium with shareholder representation and enforcement is a price function $(W(\cdot), P(\cdot))$, an allocation $\left(\{C_i^*, L_i\}_{i \in I^W}, \{C_i^*\}_{i \in I^O}\right)$, and a vector of production plans L^* such that:*

1. *$\left[W(L^*), P(L^*); \{C_i^*, L_i\}_{i \in I^W}, \{C_i^*\}_{i \in I^O}\right]$ is a competitive equilibrium relative to L^*.*

[17] The manager's objective in (7) is a mathematical aggregation of shareholder utilities. It says that once fines enter firm profits, diversified shareholders automatically internalize rivals' fines through the same weighting that makes them internalize rivals' profits. Whether common owners in the real world actually translate this internalization into operational decisions — through voting, engagement, or pressure on management — is a separate empirical question that I do not address (see, e.g., Azar, 2020). The model follows AV in treating the aggregation as a behavioral premise: I assume that the manager acts as if this internalization occurs. Because fines enter the manager's objective symmetrically with profits, the behavioral premise is identical to AV's; the reversal result and AV's anti-competitive result rest on the same assumption.

2. *The vector L^* is a pure-strategy Nash equilibrium of the game in which players are the J firms, the strategy space of firm j is $[0, T]$, and the firm's payoff function is*

$$u_j^M = \frac{\pi_j - \tau_j}{p} + \lambda(\phi, J) \sum_{k \neq j} \frac{\pi_k - \tau_k}{p},$$

with $p = P(L)$, $w = W(L)$, $\pi_j = pF(L_j) - wL_j$, and $\tau_j = \rho(\phi/J, \mu_j) \cdot K \, \mu_j \, w \, L_j$.

As in AV, the manager's objective depends only on the real wage $\omega = w/p$, which is invariant to numéraire normalization. The enforcement extension preserves this property, since $\tau_j/p = \rho(\phi/J, \mu_j) \cdot K \, \mu_j \, \omega \, L_j$ depends only on the real wage ω and on real quantities.

4.7 Characterization of Equilibrium

Given firms' production plans, I derive the real wage under a competitive equilibrium by assuming that workers maximize their utility subject to the budget constraint that now incorporates the Clayton §4 transfer.

Worker's problem. Each worker $i \in I^W$ maximizes $U(C_i, L_i)$ subject to

$$p \, C_i \leq w \, L_i + T^W,$$

where $T^W = \sum_{j=1}^{J} \tau_j$ is the per-capita transfer from redistributed antitrust fines under Section 4.4. The budget constraint is always binding because $U_C > 0$. Substituting into the utility function, the worker's problem becomes

$$\max_{L_i \in [0,T]} \; U\left(\omega \, L_i + \frac{T^W}{p}, \; L_i\right),$$

where $\omega = w/p$ is the real wage. The first-order condition for an interior solution is

$$U_C\left(\omega \, L_i + \frac{T^W}{p}, \, L_i\right) \cdot \omega \; + \; U_L\left(\omega \, L_i + \frac{T^W}{p}, \, L_i\right) \; = \; 0. \tag{9}$$

Because individual workers are measure zero relative to the unit mass I^W, each worker takes the aggregate transfer T^W as parametric.[18] The assumptions on U (twice continuously differentiable,

[18] More generally, inverse labor supply may depend on the per-capita transfer T^W/p through an income effect. The analysis below abstracts from this channel by treating enforcement transfers as ex post lump-sum redistribution. Modeling redistribution explicitly reflects the statutory architecture of Clayton §4, under which treble damages flow

$U_{CC} < 0$, $U_{LL} < 0$, $U_{CL} \leq 0$) ensure that the second-order condition holds, so (9) implicitly defines an individual labor supply function $L_i = h(\omega; T^W/p)$.

Aggregate labor supply. Aggregating over the continuum of identical workers,

$$L \;=\; \int_{I^W} L_i \, di \;=\; h(\omega; T^W/p).$$

Let η denote the elasticity of aggregate labor supply with respect to the real wage. I maintain the following assumption throughout, consistent with AV and with the empirical evidence surveyed in Chetty et al. (2011), who find a long-run elasticity of aggregate hours with respect to the wage of approximately 0.59.

(A4) $\partial h/\partial \omega > 0, \quad \forall \omega \in [0, \infty)$.

Assumption A4, aggregate labor supply is strictly increasing in the real wage, and the range of h is $[0, T]$, which guarantees the existence of an increasing inverse labor supply function h^{-1} assigning a real wage to every admissible labor supply level.

Labor market clearing. In a competitive equilibrium relative to firms' production plans $L = (L_1, \ldots, L_J)$, aggregate labor supply equals aggregate labor demand:

$$\sum_{j=1}^{J} L_j \;=\; \int_{I^W} L_i \, di \;=\; L. \tag{10}$$

Any competitive equilibrium real wage must therefore satisfy $\omega = h^{-1}(L)$ when $L = \sum_{j=1}^{J} L_j < T$ (interior case), or $\omega \geq h^{-1}(T)$ when $L = T$ (full-employment corner). I focus on the interior case and adopt the price function

$$\omega(L) \;=\; h^{-1}\left(\sum_{j=1}^{J} L_j\right), \tag{11}$$

which assigns a real wage to every vector of firms' production plans.

to harmed parties; in the present labor-market setting these harmed parties are workers. It also preserves resource balance in the two-agent structure inherited from AV. Under quasi-linear preferences ($U_{CL} = 0$), the boundary case admitted by the model's assumptions on U, the abstraction is exact: labor supply is independent of T^W/p and the analytical results hold without approximation. Moreover, the reversal mechanism itself is driven by the firm-side asymmetry between λ saturating at $\phi = 1$ and ρ remaining responsive (Assumption A2), and does not rely on the particular ex post redistribution rule used in the baseline. I relax this assumption in Section 7 to examine numerically how the reversal pattern is affected through the transfer T^W/p.

The manager's first-order condition. With the real-wage function (11) in hand, I can now state firm j's problem. The manager chooses L_j to maximize the payoff function of Definition 3:

$$\max_{L_j \in [0,T]} u_j^M \;=\; \frac{\pi_j - \tau_j}{p} \;+\; \lambda(\phi, J) \sum_{k \neq j} \frac{\pi_k - \tau_k}{p}, \tag{12}$$

where $\pi_j = pF(L_j) - wL_j$, $\tau_j = \rho(\phi/J, \mu_j) \cdot K\, \mu_j\, w\, L_j$, and both π and τ are evaluated at $p = P(L)$, $w = W(L)$ consistent with (11).

Dividing the payoff by p and using the real-wage representation, the problem reduces to maximization over real quantities:

$$\max_{L_j \in [0,T]} \Big[F(L_j) - \omega\, L_j\Big] \;-\; \rho\Big(\tfrac{\phi}{J}, \mu_j\Big) \cdot K\, \mu_j\, \omega\, L_j \;+\; \lambda(\phi, J) \sum_{k \neq j} \Big[F(L_k) - \omega\, L_k - \rho\Big(\tfrac{\phi}{J}, \mu_k\Big) \cdot K\, \mu_k\, \omega\, L_k\Big], \tag{13}$$

subject to $\omega = \omega(L)$ given by (11) and $\mu_j = \big(F'(L_j) - \omega\big)/\omega$.

Firm j's choice of L_j affects the real wage ω through the aggregate labor-demand vector, which in turn affects both its own profits and the profits of rivals (through the term in λ). Firm j also affects its own expected fine τ_j directly through both L_j and the induced change in its own markdown μ_j. However, firm j's choice of L_j does not directly affect rivals' markdowns μ_k (for $k \neq j$) at fixed L_k, since μ_k depends on L_k and on ω, and the effect of L_j on μ_k operates only through the aggregate wage ω. Taking the first-order condition and simplifying:

$$\begin{aligned} F'(L_j) - \omega(L) - \omega'(L)\Big[L_j + \lambda(\phi, J) \sum_{k \neq j} L_k\Big] \\ - \frac{1}{p} \frac{\partial \tau_j}{\partial L_j} \;-\; \lambda(\phi, J) \sum_{k \neq j} \frac{1}{p} \frac{\partial \tau_k}{\partial L_j} \;=\; 0, \end{aligned} \tag{14}$$

where $\omega'(L) = dh^{-1}/dL$ is the slope of the inverse labor supply, $\partial \tau_j / \partial L_j$ captures the total effect of firm j's employment choice on its own expected fine (through L_j directly, through μ_j, and through ω), and $\partial \tau_k / \partial L_j$ for $k \neq j$ captures the indirect effect on rival k's expected fine through the equilibrium real wage.

The first-order condition (14) admits a clean economic interpretation. Recall that u_j^M aggregates the indirect utilities of firm j's owners — that is, their real wealth W_i/p weighted by ownership shares. The FOC identifies the value of L_j that maximizes this aggregated owner welfare. Its five terms capture the distinct channels through which firm j's labor demand affects owners' real

wealth.

The first two terms, $F'(L_j) - \omega$, are the wedge between the marginal product of labor and the real wage. The third term, involving $\omega'(L)$, reflects the aggregate wage channel: owners of firm j also hold shares in rival firms (through common ownership ϕ), and rivals' profits depend on ω, which in turn depends on firm j's labor demand. The sympathy coefficient λ quantifies how strongly owners' aggregated welfare weights rivals' profit effects.

The fourth and fifth terms are new to the framework. The own derivative $\partial\tau_j/\partial L_j$ has ambiguous sign in general: hiring more labor enlarges the harmed class but reduces the firm's own markdown through higher ω and lower $F'(L_j)$ under decreasing returns; the net sign depends on parameters. The cross derivative $\partial\tau_k/\partial L_j$ for $k \neq j$ is unambiguously negative: firm j's hiring bids up the aggregate real wage, which lowers rival k's markdown μ_k at given L_k, and therefore lowers rival k's expected fine τ_k. Diversified shareholders of firm j internalize this pecuniary benefit to rivals through λ, exactly as they internalize rivals' profit effects.

The decomposition makes transparent the dual role of λ emphasized in Section 4.5: the same sympathy coefficient channels both an anti-competitive effect (through $\omega'(L)$ on rivals' profits) and a pro-competitive enforcement effect (through $\partial\tau_k/\partial L_j$ on rivals' fines).

5 Existence, Uniqueness, and Characterization

I now state the central characterization result of the paper, which extends Proposition 1 of AV to the enforcement setting. Let $E_{\omega'} = -\omega''(L)\,L/\omega'(L)$ denote the elasticity of the inverse labor supply slope.

Lemma 1 (Cross-derivative bound)**.** *Let $E_{\omega'} < 1$. Define*

$$\bar{K}_{\text{cross}}(\phi, J, \eta, \alpha) \;=\; \frac{(1+\lambda)\,\omega'(L) - \dfrac{1+\lambda(J-1)}{J} L\,|\omega''(L)|}{\left|\partial^2\tilde{\tau}_j/\partial L_j\,\partial L_m\right| + \lambda\left[\left|\partial^2\tilde{\tau}_m/\partial L_j\,\partial L_m\right| + (J-2)\left|\partial^2\tilde{\tau}_k/\partial L_j\,\partial L_m\right|\right]}, \tag{15}$$

where

$$\tilde{\tau}_\ell \;=\; \frac{\tau_\ell}{pK} \;=\; \rho(\phi/J, \mu_\ell)\,\mu_\ell\,\omega(L)\,L_\ell$$

is firm ℓ's fine kernel in real units net of the statutory multiplier, and k is any index with $k \neq j, m$. Then for all $K \in [0, \bar{K}_{\text{cross}}]$, the cross-derivative $\partial^2 u_j^M/\partial L_j\,\partial L_m \leq 0$ for all $m \neq j$ at the symmetric allocation.

Proof. Firm j's aggregated owner objective is

$$u_j^M \;=\; \frac{\pi_j - \tau_j}{p} + \lambda \sum_{k \neq j} \frac{\pi_k - \tau_k}{p}.$$

Differentiating twice, once with respect to L_j and once with respect to L_m for some $m \neq j$, and separating profit and fine contributions:

$$\frac{\partial^2 u_j^M}{\partial L_j \, \partial L_m} \;=\; \frac{\partial^2}{\partial L_j \, \partial L_m}\left[\frac{\pi_j}{p} + \lambda \sum_{k \neq j} \frac{\pi_k}{p}\right] \;-\; \frac{\partial^2}{\partial L_j \, \partial L_m}\left[\frac{\tau_j}{p} + \lambda \sum_{k \neq j} \frac{\tau_k}{p}\right]. \tag{16}$$

Profit contribution. Profits in real terms are $\pi_k/p = F(L_k) - \omega(L) L_k$, where $\omega(L) = h^{-1}(L)$ depends on aggregate labor $L = \sum_i L_i$.

Differentiating π_j/p with respect to L_j gives $F'(L_j) - \omega(L) - \omega'(L) L_j$. Differentiating again with respect to L_m, since $F'(L_j)$ is independent of L_m while $\omega(L)$ and $\omega'(L)$ depend on L_m through L:

$$\frac{\partial^2}{\partial L_j \, \partial L_m} \frac{\pi_j}{p} \;=\; -\omega'(L) - \omega''(L)\, L_j. \tag{17}$$

For the sum $\lambda \sum_{k \neq j} \pi_k/p$, $F(L_k)$ does not depend on L_j, so $\partial(\pi_k/p)/\partial L_j = -\omega'(L) L_k$ and

$$\frac{\partial}{\partial L_j} \sum_{k \neq j} \frac{\pi_k}{p} \;=\; -\omega'(L) \sum_{k \neq j} L_k.$$

Differentiating again with respect to L_m, separating $k = m$ from $k \neq j, m$: for $k = m$, the term contributes both $-\omega''(L) L_m$ and $-\omega'(L)$ (from $\partial L_m / \partial L_m = 1$); for $k \neq j, m$, only $-\omega''(L) L_k$. Summing:

$$\frac{\partial^2}{\partial L_j \, \partial L_m} \sum_{k \neq j} \frac{\pi_k}{p} \;=\; -\omega''(L) \sum_{k \neq j} L_k \;-\; \omega'(L). \tag{18}$$

Combining (17) and (18):

$$\frac{\partial^2}{\partial L_j \, \partial L_m}\left[\frac{\pi_j}{p} + \lambda \sum_{k \neq j} \frac{\pi_k}{p}\right] \;=\; -(1+\lambda)\, \omega'(L) - \Big(L_j + \lambda \sum_{k \neq j} L_k\Big) \omega''(L). \tag{19}$$

Under A4, $\omega'(L) > 0$. The assumption $E_{\omega'} < 1$ ensures (19) is strictly negative at the symmetric allocation regardless of the sign of $\omega''(L)$: when $\omega''(L) \geq 0$, both terms are non-positive; when $\omega''(L) < 0$, $E_{\omega'} < 1$ implies $|\omega''(L)| L < \omega'(L)$, controlling the second term against the leading $-(1+\lambda)\omega'(L)$.

Fine contribution. Expected fines $\tau_k = \rho(\phi/J, \mu_k) \cdot K\mu_k w L_k = pK\,\tilde{\tau}_k$ are linear in K, so their cross-derivatives scale linearly with K. The remaining factors are bounded since $\rho \in [0,1]$, $\mu \leq \bar{\mu}$, and all other quantities are finite at a symmetric interior allocation. The index $k = m$ is structurally distinct from $k \neq j, m$: for $k = m$, τ_m depends on L_m both directly (through L_m and μ_m) and through $\omega(L)$, whereas for $k \neq j, m$, τ_k depends on L_m only through $\omega(L)$. By symmetry across the indices $k \neq j, m$,

$$\sum_{k \neq j} \frac{\partial^2 \tau_k}{\partial L_j\, \partial L_m} \;=\; \frac{\partial^2 \tau_m}{\partial L_j\, \partial L_m} + (J-2) \cdot \left.\frac{\partial^2 \tau_k}{\partial L_j\, \partial L_m}\right|_{k \neq j,m}.$$

Signing the cross-derivative. At the symmetric allocation, $L_j + \lambda \sum_{k \neq j} L_k = \frac{1+\lambda(J-1)}{J} L$. Substituting profit and fine contributions into (16), a sufficient condition for $\partial^2 u_j^M / \partial L_j\, \partial L_m \leq 0$ is

$$(1+\lambda)\,\omega'(L) - \frac{1+\lambda(J-1)}{J} L\,|\omega''(L)| \;\geq\; \frac{1}{p}\left|\partial^2 \tau_j / \partial L_j\, \partial L_m\right| + \frac{\lambda}{p}\Big[\left|\partial^2 \tau_m / \partial L_j\, \partial L_m\right| + (J-2)\left|\partial^2 \tau_k / \partial L_j\, \partial L_m\right|\Big]. \tag{20}$$

Since $\lambda \in [0,1]$, $\frac{1+\lambda(J-1)}{J} \leq 1+\lambda$, so $E_{\omega'} < 1$ implies the left-hand side is strictly positive.

Deriving the bound. Writing $\tau_\ell = pK\,\tilde{\tau}_\ell$, we have $\frac{1}{p}\partial^2 \tau_\ell / \partial L_j\, \partial L_m = K \cdot \partial^2 \tilde{\tau}_\ell / \partial L_j\, \partial L_m$, so the right-hand side of (20) is linear in K with coefficient depending only on structural parameters. Dividing through, the sufficient condition becomes $K \leq \bar{K}_{\text{cross}}$ where

$$\bar{K}_{\text{cross}}(\phi, J, \eta, \alpha) \;=\; \frac{(1+\lambda)\,\omega'(L) - \dfrac{1+\lambda(J-1)}{J} L\,|\omega''(L)|}{\left|\partial^2 \tilde{\tau}_j / \partial L_j\, \partial L_m\right| + \lambda\Big[\left|\partial^2 \tilde{\tau}_m / \partial L_j\, \partial L_m\right| + (J-2)\left|\partial^2 \tilde{\tau}_k / \partial L_j\, \partial L_m\right|\Big]},$$

matching (15). □

Lemma 2 (Own-second-derivative bound)**.** *Let $E_{\omega'} < 1$ and $F'' \leq 0$. Define*

$$\bar{K}_{\text{own}}(\phi, J, \eta, \alpha) \;=\; \frac{\left|F''(L_j)\right| + \big(2 - H(\phi, J)\, E_{\omega'}\big)\omega'(L)}{\left|\partial^2 \tilde{\tau}_j / \partial L_j^2\right| + \lambda(J-1)\left|\partial^2 \tilde{\tau}_k / \partial L_j^2\right|}, \tag{21}$$

where $H(\phi, J) = [1 + \lambda(\phi, J)(J-1)]/J$ is the modified Herfindahl–Hirschman Index and

$$\tilde{\tau}_\ell \;=\; \frac{\tau_\ell}{pK} \;=\; \rho(\phi/J, \mu_\ell)\, \mu_\ell\, \omega(L)\, L_\ell$$

is firm ℓ's fine kernel in real units. Then for all $K \in [0, \bar{K}_{\text{own}})$, the payoff u_j^M is strictly concave in L_j at the symmetric allocation.

Proof. Firm j's aggregated owner objective is

$$u_j^M \;=\; \frac{\pi_j - \tau_j}{p} + \lambda \sum_{k \neq j} \frac{\pi_k - \tau_k}{p}.$$

Differentiating twice with respect to L_j and separating profit and fine contributions:

$$\frac{\partial^2 u_j^M}{\partial L_j^2} \;=\; \frac{\partial^2}{\partial L_j^2}\left[\frac{\pi_j}{p} + \lambda \sum_{k \neq j} \frac{\pi_k}{p}\right] \;-\; \frac{\partial^2}{\partial L_j^2}\left[\frac{\tau_j}{p} + \lambda \sum_{k \neq j} \frac{\tau_k}{p}\right]. \tag{22}$$

Profit contribution. Profits in real terms are $\pi_k/p = F(L_k) - \omega(L)L_k$, where $\omega(L) = h^{-1}(L)$ depends on aggregate labor $L = \sum_i L_i$.

Differentiating π_j/p with respect to L_j gives $F'(L_j) - \omega(L) - \omega'(L)L_j$. Differentiating again, since $F'(L_j)$ depends on L_j alone while $\omega(L)$ and $\omega'(L)$ depend on L_j through L:

$$\frac{\partial^2}{\partial L_j^2}\frac{\pi_j}{p} \;=\; F''(L_j) - 2\omega'(L) - \omega''(L)\, L_j, \tag{23}$$

where the two $-\omega'(L)$ terms come from differentiating $-\omega(L)$ and the explicit L_j factor in $-\omega'(L)L_j$.

For the sum $\lambda \sum_{k \neq j} \pi_k/p$, $\partial(\pi_k/p)/\partial L_j = -\omega'(L)L_k$. Differentiating again, only $\omega'(L)$ depends on L_j (since L_k does not for $k \neq j$):

$$\frac{\partial^2}{\partial L_j^2} \sum_{k \neq j} \frac{\pi_k}{p} \;=\; -\omega''(L) \sum_{k \neq j} L_k. \tag{24}$$

Combining (23) and (24):

$$\frac{\partial^2}{\partial L_j^2}\left[\frac{\pi_j}{p} + \lambda \sum_{k \neq j} \frac{\pi_k}{p}\right] \;=\; F''(L_j) - 2\omega'(L) - \Big(L_j + \lambda \sum_{k \neq j} L_k\Big)\omega''(L). \tag{25}$$

At the symmetric allocation, $L_j + \lambda \sum_{k \neq j} L_k = L\, H(\phi, J)$. Since $E_{\omega'} = |\omega''(L)|L/\omega'(L)$, we have $-\omega''(L)L \leq E_{\omega'}\omega'(L)$, with equality when $\omega''(L) \leq 0$. Therefore

$$\frac{\partial^2}{\partial L_j^2}\left[\frac{\pi_j}{p} + \lambda \sum_{k \neq j} \frac{\pi_k}{p}\right]\Bigg|_{\text{sym}} \;\leq\; F''(L_j) - 2\omega'(L) + H(\phi, J)\, E_{\omega'}\, \omega'(L). \tag{26}$$

Under $F'' \leq 0$, $E_{\omega'} < 1$, and $H \in [1/J, 1]$, the right-hand side is bounded above by $-(2 - HE_{\omega'})\omega'(L) < -\omega'(L) < 0$, so the profit contribution is strictly negative with magnitude at least $|F''(L_j)| + (2 - HE_{\omega'})\omega'(L)$.

Fine contribution. Expected fines $\tau_k = \rho(\phi/J, \mu_k) \cdot K\mu_k w L_k = pK\,\tilde{\tau}_k$ are linear in K, so their own second derivatives scale linearly with K. The remaining factors are bounded since $\rho \in [0,1]$, $\mu \leq \bar{\mu}$, and all other quantities are finite at a symmetric interior allocation. By symmetry, $\partial^2\tau_k/\partial L_j^2$ takes a common value across all $k \neq j$ (each depends on L_j only through $\omega(L)$), so $\sum_{k\neq j} \partial^2\tau_k/\partial L_j^2 = (J-1)\cdot\partial^2\tau_k/\partial L_j^2$ at any representative $k \neq j$.

Signing the own second derivative. A sufficient condition for $\partial^2 u_j^M/\partial L_j^2 < 0$ is that the magnitude of the profit contribution strictly exceeds that of the fine contribution:

$$\big|F''(L_j)\big| + \big(2 - H(\phi,J)\,E_{\omega'}\big)\omega'(L) \;>\; \frac{1}{p}\big|\partial^2\tau_j/\partial L_j^2\big| + \lambda(J-1)\cdot\frac{1}{p}\big|\partial^2\tau_k/\partial L_j^2\big|.$$

The left-hand side is strictly positive under $F'' \leq 0$ and $E_{\omega'} < 1$.

Deriving the bound. Writing $\tau_\ell = pK\,\tilde{\tau}_\ell$, we have $\frac{1}{p}\partial^2\tau_\ell/\partial L_j^2 = K\cdot\partial^2\tilde{\tau}_\ell/\partial L_j^2$, so the right-hand side is linear in K with coefficient depending only on structural parameters. Dividing through, the sufficient condition becomes $K < \bar{K}_{\text{own}}$ where

$$\bar{K}_{\text{own}}(\phi,J,\eta,\alpha) \;=\; \frac{\big|F''(L_j)\big| + \big(2 - H(\phi,J)\,E_{\omega'}\big)\omega'(L)}{\big|\partial^2\tilde{\tau}_j/\partial L_j^2\big| + \lambda(J-1)\big|\partial^2\tilde{\tau}_k/\partial L_j^2\big|},$$

matching (21). □

Lemma 3 (Best-response-slope bound)**.** *Let $E_{\omega'} < 1$, $F'' \leq 0$, and $\lambda(\phi,J) \in [0,1]$. Define*

$$D_{\text{own}}(\phi,J,\eta,\alpha) \;=\; \big|\partial^2\tilde{\tau}_j/\partial L_j^2\big| + \lambda(J-1)\big|\partial^2\tilde{\tau}_k/\partial L_j^2\big|,$$

$$D_{\text{cross}}(\phi,J,\eta,\alpha) \;=\; \big|\partial^2\tilde{\tau}_j/\partial L_j\,\partial L_m\big| + \lambda\Big[\big|\partial^2\tilde{\tau}_m/\partial L_j\,\partial L_m\big| + (J-2)\big|\partial^2\tilde{\tau}_k/\partial L_j\,\partial L_m\big|\Big],$$

where $k \neq j, m$ and $\tilde{\tau}_\ell = \tau_\ell/(pK) = \rho(\phi/J,\mu_\ell)\,\mu_\ell\,\omega(L)\,L_\ell$, and let $D_{\text{slope}} = D_{\text{own}} + D_{\text{cross}}$. Define

$$\bar{K}_{\text{slope}}(\phi,J,\eta,\alpha) \;=\; \frac{\big|F''(L_j)\big| + \big(1 - \lambda(\phi,J)\big)\omega'(L)}{D_{\text{slope}}(\phi,J,\eta,\alpha)}. \tag{27}$$

Then for all $K < \bar{K}_{\text{slope}}$, the own second derivative strictly dominates the cross-derivative in magnitude at the symmetric allocation:

$$\left|\frac{\partial^2 u_j^M}{\partial L_j^2}\right| \;>\; \left|\frac{\partial^2 u_j^M}{\partial L_j\,\partial L_m}\right|,$$

so firm j's best-response slope with respect to rival employment lies in $(-1,0]$ at the symmetric

allocation.

Proof. At the symmetric allocation, the exact profit-side own and cross second derivatives from (25) and (19) satisfy

$$\left|\tfrac{\partial^2}{\partial L_j^2}[\pi_j/p+\lambda\sum_{k\neq j}\pi_k/p]\right| - \left|\tfrac{\partial^2}{\partial L_j\,\partial L_m}[\pi_j/p+\lambda\sum_{k\neq j}\pi_k/p]\right| \;=\; \left|F''(L_j)\right| + (1-\lambda)\omega'(L),$$

where the $\omega''(L)$ terms cancel because both derivatives carry the common factor $LH(\phi,J)\omega''(L)$. Writing $\tau_\ell = pK\,\tilde{\tau}_\ell$, the fine contributions to the own and cross second derivatives are bounded in magnitude by $K\,D_{\text{own}}$ and $K\,D_{\text{cross}}$ respectively. Worst-case dominance — own contribution reduced by $K\,D_{\text{own}}$, cross contribution increased by $K\,D_{\text{cross}}$ — yields the sufficient condition

$$\left|F''(L_j)\right| + (1-\lambda)\omega'(L) \;>\; K\big(D_{\text{own}} + D_{\text{cross}}\big) \;=\; K\,D_{\text{slope}},$$

which is equivalent to $K < \bar{K}_{\text{slope}}$. Under this condition, $|\partial^2 u_j^M/\partial L_j^2| > |\partial^2 u_j^M/\partial L_j\,\partial L_m|$, and the best-response slope $-(\partial^2 u_j^M/\partial L_j\,\partial L_m)/(\partial^2 u_j^M/\partial L_j^2)$ lies in $(-1,0]$. □

Proposition 1. *Let $E_{\omega'} < 1$, $F'' \le 0$, and $K < \bar{K}(\phi,J,\eta,\alpha)$, where*

$$\bar{K}(\phi,J,\eta,\alpha) \;=\; \min\big\{\bar{K}_{\text{cross}}(\phi,J,\eta,\alpha),\; \bar{K}_{\text{own}}(\phi,J,\eta,\alpha),\; \bar{K}_{\text{slope}}(\phi,J,\eta,\alpha)\big\}.$$

Then the game among firms is one of strategic substitutes with strictly concave own payoffs, and admits a unique symmetric pure-strategy Nash equilibrium $L_j^* = L^*/J$ for all j.

Proof. Under $K < \bar{K}$, Lemmas 1–3 imply, at the symmetric allocation,

$$\frac{\partial^2 u_j^M}{\partial L_j\,\partial L_m} \le 0, \qquad \frac{\partial^2 u_j^M}{\partial L_j^2} < 0, \qquad \left|\frac{\partial^2 u_j^M}{\partial L_j^2}\right| > \left|\frac{\partial^2 u_j^M}{\partial L_j\,\partial L_m}\right|.$$

Thus the game has strategic substitutes, strictly concave own payoffs, and best-response slope in $(-1,0]$. Existence follows from Vives (1999), Theorem 2.7, and uniqueness from Theorem 2.8. Symmetry of the problem — identical firms with symmetric ownership structure — implies the unique equilibrium satisfies $L_j^* = L^*/J$ for all j. □

Remark 1. *The bound $\bar{K}(\phi,J,\eta,\alpha)$ from Proposition 1 is a technical regularity condition rather than the central substantive restriction on enforcement intensity; it identifies the parameter region in which the strategic-substitutes, strict-concavity, and best-response-slope conditions are guaranteed.*

Section 7 verifies that $K < \bar{K}$ with substantial margin at the calibrated parameter values, so the reversal arises within the parameter region where existence and uniqueness are guaranteed.

Lemma 4. *Let the real expected fine be*

$$\frac{\tau_k}{p} \;=\; \rho\!\left(\frac{\phi}{J}, \mu_k\right) K \mu_k\, \omega(L)\, L_k, \qquad \mu_k = \frac{F'(L_k) - \omega(L)}{\omega(L)},$$

for $k \neq j$. Then

$$\frac{\partial(\tau_k/p)}{\partial L_j} \;=\; K L_k\, \omega'(L)\big[-\rho_\mu\, \mu_k(1+\mu_k) - \rho\big] \leq 0.$$

Proof. Since $k \neq j$, L_k is fixed. Differentiating:

$$\frac{\partial(\tau_k/p)}{\partial L_j} \;=\; K L_k\big[\rho_{L_j} \mu_k \omega + \rho\, \mu_{k,L_j} \omega + \rho \mu_k \omega'(L)\big].$$

From $\mu_k = F'(L_k)/\omega - 1$ and L_k fixed, $\mu_{k,L_j} = -(1+\mu_k)\omega'(L)/\omega$, so $\rho_{L_j} = \rho_\mu \mu_{k,L_j}$. Substituting:

$$\frac{\partial(\tau_k/p)}{\partial L_j} \;=\; K L_k\, \omega'(L)\big[-\rho_\mu \mu_k(1+\mu_k) - \rho\big].$$

Since $K > 0$, $L_k > 0$, $\omega'(L) > 0$, $\rho_\mu \geq 0$, $\rho \geq 0$, $\mu_k \geq 0$, and $1 + \mu_k > 0$ at any interior oligopsony allocation, the bracket is ≤ 0, with strict inequality if $\rho > 0$ or $\rho_\mu \mu_k > 0$. □

(A5) $L_j\, \omega(L) \left|\frac{\partial \mu_j}{\partial L_j}\right| \left(\frac{\partial^2 \rho}{\partial(\phi/J)\, \partial \mu_j}\, \mu_j + \frac{\partial \rho}{\partial(\phi/J)}\right) \;>\; \frac{\partial \rho}{\partial(\phi/J)}\, \mu_j\, \omega(L) \;+\; L_j\, \frac{\partial \rho}{\partial(\phi/J)}\, \mu_j\, \omega'(L).$

(A6) The output elasticity of labor $s(L) = F'(L/J) \cdot L/[J \cdot F(L/J)]$ is non-decreasing in L on the range of equilibrium employments $\{L^*(\phi) : \phi \in (0,1)\}$.

Assumption A5 states that the conduct margin dominates: enforcement responds primarily to the wedge between marginal product and wages rather than to scale alone. The key term is the complementarity $\partial^2 \rho / \partial(\phi/J)\, \partial \mu_j$, which captures how the detection effect of an increase in cross-holdings amplifies when the markdown is also large. This interpretation is consistent with modern antitrust practice, where enforcement typically conditions on both structural exposure and conduct-based indicators, rather than relying on firm size alone. Whether the complementarity is sufficiently strong to satisfy Assumption A5 is ultimately an empirical question, which I leave for future work.

Assumption A6 is a mild regularity condition on the curvature of the production function. Economically, it requires that the share of output accruing to the marginal unit of labor does

not sharply decline as aggregate employment expands — a property that holds under virtually all empirically used production specifications. Under Cobb–Douglas production $F(L) = AL^{\alpha}$, $s(L) = \alpha$ is constant and A6 holds trivially. Under constant returns to scale, $s(L) = 1$ identically. More generally, in standard single-input production specifications used in the labor and growth literatures, $s(L)$ is constant or weakly increasing over the relevant range. The assumption rules out only pathological cases in which labor's marginal contribution to output drops sharply with scale — a property that is inconsistent with the standard properties of production functions used in the labor and growth literatures. In the calibration of Section 7, which adopts Cobb–Douglas production, A6 is satisfied by construction.

Lemma 5. *Suppose Assumption A5 holds. Let* $\tau_j = \rho(\phi/J, \mu_j)\, K\, \mu_j\, \omega(L)\, L_j$, *where* $\mu_j = (F'(L_j) - \omega(L))/\omega(L)$. *Then*

$$\frac{\partial^2 \tau_j}{\partial \phi\, \partial L_j} \;=\; \frac{K}{J}\left[\frac{\partial \rho}{\partial(\phi/J)}\,\mu_j\,\omega(L) + L_j\,\omega(L)\,\frac{\partial \mu_j}{\partial L_j}\left(\frac{\partial^2 \rho}{\partial(\phi/J)\,\partial \mu_j}\,\mu_j + \frac{\partial \rho}{\partial(\phi/J)}\right) + L_j\,\frac{\partial \rho}{\partial(\phi/J)}\,\mu_j\,\omega'(L)\right] < 0. \tag{28}$$

Proof. Differentiating $\tau_j = \rho(\phi/J, \mu_j)\, K\, \mu_j\, \omega(L)\, L_j$ with respect to L_j via the product rule (treating ρ with μ_j depending on L_j directly and through $\omega(L)$):

$$\frac{\partial \tau_j}{\partial L_j} \;=\; K\left[\rho\,\mu_j\,\omega(L) \;+\; L_j\left(\frac{\partial \rho}{\partial \mu_j}\,\frac{\partial \mu_j}{\partial L_j}\,\mu_j\,\omega(L) + \rho\,\frac{\partial \mu_j}{\partial L_j}\,\omega(L) + \rho\,\mu_j\,\omega'(L)\right)\right].$$

Differentiating with respect to ϕ: since μ_j, $\partial \mu_j/\partial L_j$, $\omega(L)$, and $\omega'(L)$ do not depend directly on ϕ, only ρ and $\partial \rho/\partial \mu_j$ vary with ϕ. Using $\partial \rho/\partial \phi = (1/J)\,\partial \rho/\partial(\phi/J)$ and $\partial(\partial \rho/\partial \mu_j)/\partial \phi = (1/J)\,\partial^2 \rho/\partial(\phi/J)\,\partial \mu_j$:

$$\frac{\partial^2 \tau_j}{\partial \phi\, \partial L_j} \;=\; \frac{K}{J}\left[\frac{\partial \rho}{\partial(\phi/J)}\,\mu_j\,\omega(L) + L_j\left(\frac{\partial^2 \rho}{\partial(\phi/J)\,\partial \mu_j}\,\frac{\partial \mu_j}{\partial L_j}\,\mu_j\,\omega(L) + \frac{\partial \rho}{\partial(\phi/J)}\,\frac{\partial \mu_j}{\partial L_j}\,\omega(L) + \frac{\partial \rho}{\partial(\phi/J)}\,\mu_j\,\omega'(L)\right)\right],$$

which after factoring out $\partial \mu_j/\partial L_j$ in the middle two terms equals (28). The first and third terms in (28) are positive, while the middle term is negative since $\partial \mu_j/\partial L_j < 0$ at an interior oligopsony equilibrium. Assumption A5 ensures the magnitude of the middle term exceeds the sum of the positive terms, so $\partial^2 \tau_j/\partial \phi\, \partial L_j < 0$. □

Lemma 6. *Suppose Assumptions A1 and A3 hold. Let* $\tau_k = \rho(\phi/J, \mu_k)\, K\, \mu_k\, \omega(L)\, L_k$ *for* $k \neq j$,

where $\mu_k = (F'(L_k) - \omega(L))/\omega(L)$. *Then*

$$\frac{\partial^2 \tau_k}{\partial \phi \, \partial L_j} \;=\; \frac{K\, L_k\, \omega'(L)}{J} \left[-\frac{\partial^2 \rho}{\partial(\phi/J)\, \partial \mu_k}\, \mu_k\, (1+\mu_k) \;-\; \frac{\partial \rho}{\partial(\phi/J)} \right] \;<\; 0. \tag{29}$$

Proof. From the chain-rule expansion of $\partial \tau_k / \partial L_j$ (Lemma 4), using $\partial \mu_k / \partial L_j = -(1+\mu_k)\omega'(L)/\omega(L)$ for $k \neq j$:

$$\frac{\partial \tau_k}{\partial L_j} \;=\; K\, L_k\, \omega'(L) \left[-\frac{\partial \rho}{\partial \mu_k}\, \mu_k\, (1+\mu_k) \;-\; \rho \right].$$

Differentiating with respect to ϕ: the prefactor $K\, L_k\, \omega'(L)$ does not depend on ϕ; inside the bracket, μ_k does not depend on ϕ (since L_k and $\omega(L)$ are both independent of ϕ at fixed L_j), so only ρ and $\partial \rho / \partial \mu_k$ vary with ϕ. Using $\partial \rho / \partial \phi = (1/J)\, \partial \rho / \partial(\phi/J)$ and $\partial(\partial \rho / \partial \mu_k)/\partial \phi = (1/J)\, \partial^2 \rho / \partial(\phi/J)\, \partial \mu_k$:

$$\frac{\partial^2 \tau_k}{\partial \phi \, \partial L_j} \;=\; \frac{K\, L_k\, \omega'(L)}{J} \left[-\frac{\partial^2 \rho}{\partial(\phi/J)\, \partial \mu_k}\, \mu_k\, (1+\mu_k) \;-\; \frac{\partial \rho}{\partial(\phi/J)} \right].$$

Both terms in the bracket are non-positive: the first by Assumption A3 ($\partial^2 \rho / \partial(\phi/J)\, \partial \mu_k \geq 0$) together with $\mu_k \geq 0$ and $(1+\mu_k) > 0$; the second by Assumption A1 ($\partial \rho / \partial(\phi/J) > 0$). The strict positivity of $\partial \rho / \partial(\phi/J)$ ensures the bracket is strictly negative. Combined with $K > 0$, $L_k > 0$, $\omega'(L) > 0$, and $J > 0$, the cross-partial is strictly negative. □

Lemma 7. *Suppose Assumptions A1, A3, and A5 hold. Let*

$$\mathfrak{E}(\phi, J, K, \mu^*) \;=\; \frac{1}{\omega(L^*)} \left[\frac{1}{p} \frac{\partial \tau_j}{\partial L_j} + \lambda(\phi, J)(J-1)\, \frac{1}{p} \frac{\partial \tau_k}{\partial L_j} \right].$$

Then

$$\left. \frac{\partial \mathfrak{E}}{\partial \phi} \right|_{\mu^*} \;=\; \frac{1}{\omega(L^*)\, p} \left[\frac{\partial^2 \tau_j}{\partial \phi \, \partial L_j} + (J-1) \left(\frac{\partial \lambda}{\partial \phi} \cdot \frac{\partial \tau_k}{\partial L_j} + \lambda(\phi, J) \cdot \frac{\partial^2 \tau_k}{\partial \phi \, \partial L_j} \right) \right] < 0. \tag{30}$$

Proof. Since L^* is fixed under the hold-μ^*-fixed operation, $\omega(L^*)$ is constant with respect to ϕ. Hence

$$\left. \frac{\partial \mathfrak{E}}{\partial \phi} \right|_{\mu^*} \;=\; \frac{1}{\omega(L^*)\, p} \frac{\partial}{\partial \phi} \left[\frac{\partial \tau_j}{\partial L_j} + \lambda(\phi, J)(J-1) \frac{\partial \tau_k}{\partial L_j} \right].$$

Applying the product rule,

$$\left. \frac{\partial \mathfrak{E}}{\partial \phi} \right|_{\mu^*} \;=\; \frac{1}{\omega(L^*)\, p} \left[\frac{\partial^2 \tau_j}{\partial \phi \, \partial L_j} + (J-1) \left(\frac{\partial \lambda}{\partial \phi} \frac{\partial \tau_k}{\partial L_j} + \lambda(\phi, J) \frac{\partial^2 \tau_k}{\partial \phi \, \partial L_j} \right) \right].$$

The sympathy coefficient is monotone in common ownership: $\partial\lambda/\partial\phi > 0$ on $\phi \in (0,1)$ (AV). By Lemma 4, $\partial\tau_k/\partial L_j < 0$, so $(\partial\lambda/\partial\phi)(\partial\tau_k/\partial L_j) < 0$. By Lemmas 5 and 6, $\partial^2\tau_j/\partial\phi\,\partial L_j < 0$ and $\partial^2\tau_k/\partial\phi\,\partial L_j < 0$. Combined with $\omega(L^*) > 0$, $p > 0$, and $J - 1 > 0$, the bracket is strictly negative, so $\partial\mathfrak{E}/\partial\phi|_{\mu^*} < 0$. □

Lemma 8. *Suppose Assumptions A1, A3, and A5 hold. Let*

$$\mathfrak{E}(\phi, J, K, \mu^*) \;=\; \frac{1}{\omega(L^*)}\left[\frac{1}{p}\frac{\partial\tau_j}{\partial L_j} + \lambda(\phi, J)(J-1)\,\frac{1}{p}\frac{\partial\tau_k}{\partial L_j}\right].$$

Holding μ^* fixed, the partial derivative of $\mathfrak{E}$ with respect to J is

$$\left.\frac{\partial\mathfrak{E}}{\partial J}\right|_{\mu^*} \;=\; \frac{1}{\omega(L^*)\,p}\left[\frac{\partial}{\partial J}\left(\frac{\partial\tau_j}{\partial L_j}\right) \;+\; \left((J-1)\frac{\partial\lambda}{\partial J} + \lambda(\phi, J)\right)\frac{\partial\tau_k}{\partial L_j} \;+\; \lambda(\phi, J)(J-1)\,\frac{\partial}{\partial J}\left(\frac{\partial\tau_k}{\partial L_j}\right)\right]. \quad (31)$$

The three components have mixed signs: the first and third are positive (via the identities $\partial/\partial J(\partial\tau_j/\partial L_j) = -(\phi/J)\,\partial^2\tau_j/\partial\phi\,\partial L_j$ and $\partial/\partial J(\partial\tau_k/\partial L_j) = -(\phi/J)\,\partial^2\tau_k/\partial\phi\,\partial L_j$ applied to Lemmas 5 and 6), while the second is negative. The overall sign of $\partial\mathfrak{E}/\partial J|_{\mu^*}$ is therefore parameter-dependent.

Proof. Throughout this partial derivative, L^*, L_j, L_k, and μ^* are held fixed, so the fine terms depend on J only through ϕ/J (via ρ and ρ_μ). Since L^* is fixed, $\omega(L^*)$ is also constant with respect to J. Hence

$$\left.\frac{\partial\mathfrak{E}}{\partial J}\right|_{\mu^*} \;=\; \frac{1}{\omega(L^*)\,p}\cdot\frac{\partial}{\partial J}\left[\frac{\partial\tau_j}{\partial L_j} \;+\; \lambda(\phi, J)(J-1)\,\frac{\partial\tau_k}{\partial L_j}\right].$$

Differentiating term by term, the first piece gives $\partial(\partial\tau_j/\partial L_j)/\partial J$. For the second piece, applying the product rule to the three factors $\lambda(\phi, J)$, $(J-1)$, and $\partial\tau_k/\partial L_j$:

$$\frac{\partial}{\partial J}\left[\lambda(\phi, J)(J-1)\frac{\partial\tau_k}{\partial L_j}\right] \;=\; \frac{\partial\lambda}{\partial J}(J-1)\frac{\partial\tau_k}{\partial L_j} \;+\; \lambda(\phi, J)\cdot 1\cdot\frac{\partial\tau_k}{\partial L_j} \;+\; \lambda(\phi, J)(J-1)\frac{\partial}{\partial J}\left(\frac{\partial\tau_k}{\partial L_j}\right).$$

Grouping the first two terms via the common factor $\partial\tau_k/\partial L_j$:

$$\left((J-1)\frac{\partial\lambda}{\partial J} + \lambda(\phi, J)\right)\frac{\partial\tau_k}{\partial L_j} \;+\; \lambda(\phi, J)(J-1)\frac{\partial}{\partial J}\left(\frac{\partial\tau_k}{\partial L_j}\right).$$

Combining with the first piece gives (31). The identities in the lemma statement follow because ρ and $\partial\rho/\partial\mu$ depend on J only through ϕ/J (under the held-fixed convention above), so chain rule gives $\partial/\partial J = -(\phi/J^2)\,\partial/\partial(\phi/J)$ while $\partial/\partial\phi = (1/J)\,\partial/\partial(\phi/J)$, yielding the ratio $-\phi/J$.

By Lemma 5, $\partial^2\tau_j/\partial\phi\,\partial L_j < 0$ under Assumption A5; by Lemma 6, $\partial^2\tau_k/\partial\phi\,\partial L_j < 0$ under Assumptions A1 and A3. Multiplying by $-\phi/J < 0$ (since $\phi \in (0,1)$, $J > 0$) gives the claimed positive signs for the first and third components. The second component is negative because $(J-1)\partial\lambda/\partial J + \lambda(\phi, J) = \lambda/D > 0$ under AV's sympathy function, where $D = (1-\phi)^2 J + (2-\phi)\phi$,[19] and $\partial\tau_k/\partial L_j < 0$ by Lemma 4. □

Remark 2. *The ϕ-dependence of μ^* differs structurally from the J-dependence. For ϕ, the saturation property $\partial\lambda/\partial\phi|_{\phi=1} = 0$ combined with the non-saturation of ρ (Assumption A2) produces a decisive sign asymmetry at $\phi = 1$, which by continuity delivers a sign change in $d\mu^*/d\phi$ on $(0,1)$. No analogous asymmetry arises in J: the derivative $\partial\mathfrak{E}/\partial J$ contains opposing components, as shown in Lemma 8, and their relative magnitudes are parameter-dependent. Whether μ^* is monotonic in J at empirically relevant parameters is therefore a quantitative question, addressed numerically in Section 7.*

Proposition 2. *Suppose Assumptions A1–A6 hold. Let $E_{\omega'} < 1$ and $F'' \leq 0$. There exists a non-empty interval*

$$\mathcal{K} = (0, \bar{K}_0), \qquad \bar{K}_0 \leq \inf_{\phi\in(0,1)} \bar{K}(\phi, J, \eta, \alpha),$$

such that for all $K \in \mathcal{K}$, the interior symmetric equilibrium with aggregate employment $L^* \in (0, T)$ satisfies the following:

(a) The equilibrium markdown $\mu^* = \big(F'(L^*/J) - \omega(L^*)\big)/\omega(L^*)$ satisfies

$$\mu^* \;=\; \frac{H(\phi, J)}{\eta(L^*)} \;+\; \frac{1}{\omega(L^*)}\left[\frac{1}{p}\frac{\partial\tau_j}{\partial L_j} + \lambda(\phi, J)(J-1)\,\frac{1}{p}\frac{\partial\tau_k}{\partial L_j}\right], \tag{32}$$

where $H(\phi, J) = [1 + \lambda(\phi, J)(J-1)]/J$ is the modified Herfindahl–Hirschman Index and the bracketed term is the enforcement contribution.

(b) Both L^* and $\omega(L^*)$ are non-monotonic in ϕ.

(c) The labor share of income $\omega(L^*)L^*/\big(J\,F(L^*/J)\big)$ is non-monotonic in ϕ.

Proof. Part (a). The first-order condition (14) evaluated at firm j is

$$F'(L_j) - \omega(L) - \omega'(L)\Big[L_j + \lambda(\phi, J)\sum_{k\neq j} L_k\Big] - \frac{1}{p}\frac{\partial\tau_j}{\partial L_j} - \lambda(\phi, J)\sum_{k\neq j}\frac{1}{p}\frac{\partial\tau_k}{\partial L_j} \;=\; 0.$$

[19] Direct computation: $(J-1)\partial\lambda/\partial J + \lambda = \lambda[D - (J-1)(1-\phi)^2]/D$, and $D - (J-1)(1-\phi)^2 = (1-\phi)^2 + (2-\phi)\phi = 1$ by expansion.

I evaluate each term at the symmetric equilibrium where $L_k = L^*/J$ for all k and $\omega = \omega(L^*)$.

Profit-side terms. The coefficient of $\omega'(L^*)$ inside the bracket becomes

$$L_j + \lambda(\phi, J)\sum_{k\neq j} L_k \;=\; \frac{L^*}{J} + \lambda(\phi, J)\frac{(J-1)L^*}{J} \;=\; \frac{L^*}{J}\big[1 + \lambda(\phi, J)(J-1)\big] \;=\; L^*\, H(\phi, J).$$

Fine-side terms. By symmetry, $\partial\tau_k/\partial L_j$ takes a common value across all $k \neq j$, so $\sum_{k\neq j} \partial\tau_k/\partial L_j = (J-1)\,\partial\tau_k/\partial L_j$ for any representative $k \neq j$.

Substituting both simplifications into the FOC:

$$F'(L^*/J) - \omega(L^*) - \omega'(L^*)\, L^*\, H(\phi, J) - \frac{1}{p}\frac{\partial\tau_j}{\partial L_j} - \lambda(\phi, J)(J-1)\,\frac{1}{p}\frac{\partial\tau_k}{\partial L_j} \;=\; 0. \tag{33}$$

Dividing by $\omega(L^*)$. Dividing (33) through by $\omega(L^*)$ and rearranging:

$$\frac{F'(L^*/J) - \omega(L^*)}{\omega(L^*)} \;=\; \frac{\omega'(L^*)\, L^*}{\omega(L^*)}\, H(\phi, J) + \frac{1}{\omega(L^*)}\left[\frac{1}{p}\frac{\partial\tau_j}{\partial L_j} + \lambda(\phi, J)(J-1)\,\frac{1}{p}\frac{\partial\tau_k}{\partial L_j}\right].$$

The left-hand side equals μ^* by definition. For the right-hand side, $\omega'(L^*)L^*/\omega(L^*)$ equals $1/\eta(L^*)$ by the definition of the labor-supply elasticity.[20] Substituting yields (32), completing the proof of part (a).

Part (b). Define the enforcement contribution

$$\mathfrak{E}(\phi, J, K, \mu^*) \;=\; \frac{1}{\omega(L^*)}\left[\frac{1}{p}\frac{\partial\tau_j}{\partial L_j} + \lambda(\phi, J)(J-1)\,\frac{1}{p}\frac{\partial\tau_k}{\partial L_j}\right],$$

so that (32) reads $\mu^* = H(\phi, J)/\eta(L^*) + \mathfrak{E}$. Implicit differentiation with respect to ϕ, treating $\eta(L^*)$ as locally smooth in ϕ (its variation through L^* contributes higher-order terms dominated by the principal channels at the boundaries, as in AV's Proposition 1), yields

$$\left[1 - \frac{\partial\mathfrak{E}}{\partial\mu^*}\right]\frac{d\mu^*}{d\phi} \;=\; \frac{1}{\eta(L^*)}\frac{\partial H}{\partial\phi} \;+\; \left.\frac{\partial\mathfrak{E}}{\partial\phi}\right|_{\mu^*} + (\text{higher-order terms in } \partial\eta/\partial\phi),$$

and under the regularity condition $1 - \partial\mathfrak{E}/\partial\mu^* > 0$, the sign of $d\mu^*/d\phi$ matches the sign of the principal right-hand side.

By direct computation, $\partial H/\partial\phi = [(J-1)/J]\,\partial\lambda/\partial\phi > 0$ for $\phi \in (0, 1)$, with $\partial\lambda/\partial\phi|_{\phi=1} = 0$ (saturation), so $\partial H/\partial\phi|_{\phi=1} = 0$. By Lemma 7, $\partial\mathfrak{E}/\partial\phi|_{\mu^*} < 0$ throughout $(0, 1)$, with persistent

[20] The identity $\omega'(L)L/\omega(L) = 1/\eta$ follows from the definition $\eta = (dL/d\omega)(\omega/L)$ applied to the inverse labor supply function $\omega = h^{-1}(L)$.

strictly negative magnitude at $\phi = 1$ by Assumption A2 (non-saturation of ρ). The two principal terms therefore push in opposite directions.

The reversal is established for K in a non-empty subinterval $\mathcal{K} \subset [0, \bar{K})$. Enforcement derivatives scale linearly in K, so for K small enough the AV sympathy term $[(J-1)/(J\eta(L^*))] \cdot \partial\lambda/\partial\phi|_{\phi=0} > 0$ exceeds $|\partial\mathfrak{E}/\partial\phi|_{\phi=0,\mu^*}|$ in magnitude, so $d\mu^*/d\phi > 0$ near $\phi = 0$. Conversely, the saturation $\partial H/\partial\phi|_{\phi=1} = 0$ combined with Assumption A2 ensures $d\mu^*/d\phi < 0$ near $\phi = 1$ for any $K > 0$. By continuity, for $K \in \mathcal{K}$ there exists $\bar{\phi}(K) \in (0,1)$ such that $d\mu^*/d\phi = 0$ at $\bar{\phi}(K)$ and changes sign there, establishing non-monotonicity of μ^* in ϕ.

Since $\mu^* = (F'(L^*/J) - \omega(L^*))/\omega(L^*)$ and $\omega(L^*) = h^{-1}(L^*)$ make μ^* a strictly decreasing transformation of L^* under $F'' \leq 0$, non-monotonicity of μ^* in ϕ implies non-monotonicity of L^* and $\omega(L^*)$ in ϕ.

Part (c). Labor share satisfies $LS = s(L^*)/(1+\mu^*)$, where $s(L) = F'(L/J) \cdot (L/J)/F(L/J)$ is the output elasticity of labor at the symmetric allocation. Under Assumption A6, $s(L)$ is non-decreasing in L. Since μ^* is a strictly decreasing transformation of L^* under $F'' \leq 0$, μ^* rising is equivalent to L^* falling, which implies $s(L^*)$ weakly falls. Combined with $1/(1+\mu^*)$ falling, this yields LS strictly falling. Conversely, when μ^* falls, L^* rises, $s(L^*)$ weakly rises, $1/(1+\mu^*)$ rises, and LS strictly rises. Hence LS is strictly decreasing in μ^*. Since μ^* is non-monotonic in ϕ by part (b), LS is non-monotonic in ϕ with the opposite sign pattern. □

Corollary 1. *In the absence of enforcement, the model reduces to the benchmark of AV:*

$$\mu^* = \frac{H(\phi, J)}{\eta(L^*)}.$$

Proof. At $K = 0$, $\tau_j = \tau_k = 0$ for all j, k, so both fine derivatives in (32) vanish. The remaining term is $H(\phi, J)/\eta(L^*)$, which matches Proposition 1 of AV. In other words, without enforcement, there are no expected fines, so the enforcement contribution in (32) is zero and only the AV term remains. □

(A7) The function $\mathfrak{G}(\phi) = (1/\eta(L^*))\,\partial H/\partial\phi + \partial\mathfrak{E}/\partial\phi|_{\mu^*}$ is continuous and quasi-concave on $(0,1)$, and is not identically zero on any subinterval of $(0,1)$.

Assumption A7 is a quasi-concavity regularity condition of the kind used throughout the monotone comparative statics literature (Milgrom and Shannon, 1994). It rules out oscillatory behavior

of $\mathfrak{G}(\phi)$ and guarantees that the interaction between the AV sympathy channel and the enforcement channel produces a single reversal point.[21] Intuitively, the AV sympathy term $(1/\eta)\,\partial H/\partial\phi$ is hump-shaped in ϕ (rising as common ownership strengthens the anti-competitive channel, then falling as λ saturates near the boundary), while the enforcement term $\partial\mathfrak{E}/\partial\phi$ is monotone negative; their sum inherits a single-peaked shape, which A7 formalizes.

Proposition 3. *Suppose Assumptions A1–A7 hold and $K \in \mathcal{K}$ as in Proposition 2. There exists a unique threshold $\bar{\phi}(J, K, \eta, \alpha) \in (0,1)$ such that $d\mu^*/d\phi > 0$ for $\phi \in (0, \bar{\phi})$ and $d\mu^*/d\phi < 0$ for $\phi \in (\bar{\phi}, 1)$. Equivalently, L^* and $\omega(L^*)$ are decreasing in ϕ below $\bar{\phi}$ and increasing above. The threshold solves $\mathfrak{G}(\bar{\phi}) = 0$.*

Proof. From the proof of Proposition 2(b), for $K \in \mathcal{K}$, $\mathfrak{G}(\phi) > 0$ near $\phi = 0$ (the AV sympathy term dominates) and $\mathfrak{G}(\phi) < 0$ near $\phi = 1$ (the AV term vanishes by saturation while the enforcement term persists by Assumption A2). By continuity and the boundary signs, the intermediate value theorem implies that there exists at least one $\bar{\phi} \in (0,1)$ such that $\mathfrak{G}(\bar{\phi}) = 0$. Quasi-concavity (Assumption A7) implies that the upper contour set $\{\phi \in (0,1) : \mathfrak{G}(\phi) \geq 0\}$ is an interval. Since $\mathfrak{G}(\phi) > 0$ near $\phi = 0$ and $\mathfrak{G}(\phi) < 0$ near $\phi = 1$, this interval has a unique right endpoint. The additional condition in Assumption A7 that $\mathfrak{G}$ is not identically zero on any subinterval rules out a flat interval of zeros. Hence there is a unique crossing point $\bar{\phi} \in (0,1)$ with $\mathfrak{G}(\bar{\phi}) = 0$, $\mathfrak{G}(\phi) > 0$ for $\phi < \bar{\phi}$, and $\mathfrak{G}(\phi) < 0$ for $\phi > \bar{\phi}$. The sign pattern of $d\mu^*/d\phi$ matches $\mathfrak{G}(\phi)$ as in Proposition 2(b). □

Corollary 2. *Suppose the conditions of Proposition 3 hold and the crossing is transversal, i.e. $\mathfrak{G}_\phi(\bar{\phi}; K) < 0$. Then the reversal threshold is strictly decreasing in the enforcement penalty: $\partial\bar{\phi}/\partial K < 0$.*

Proof. Implicit differentiation of $\mathfrak{G}(\bar{\phi}; K) = 0$ with respect to K yields $\partial\bar{\phi}/\partial K = -\mathfrak{G}_K/\mathfrak{G}_\phi$. By transversality, $\mathfrak{G}_\phi(\bar{\phi}) < 0$. Since $\mathfrak{E}$ is linear in K, $\mathfrak{G}_K = (1/K)\,\partial\mathfrak{E}/\partial\phi < 0$ by Lemma 7. Both numerator and denominator are negative, so $\partial\bar{\phi}/\partial K < 0$. □

Remark 3. *The non-monotonicity of L^*, $\omega(L^*)$, and the labor share in ϕ reflects the opposing AV sympathy channel (anti-competitive, scaling with $\partial H/\partial\phi > 0$) and the Clayton §4 enforce-*

[21] The condition is not restrictive at empirically relevant parameters: Section 7 verifies numerically that $\mathfrak{G}(\phi)$ is quasi-concave under the canonical detection function $\rho = \min\{1, (\phi/J)\mu\}$ at the calibrated values $(\alpha, \eta, J) = (0.5, 0.5, 4)$ for $K \in \{3, 9, 27\}$.

ment channel (pro-competitive, signed by Lemma 7). At $\phi = 1$, the sympathy channel saturates ($\partial\lambda/\partial\phi|_{\phi=1} = 0$) while the enforcement channel persists by Assumption A2, producing the reversal.

Remark 4. *In the corner case of constant returns to scale and full cartelization ($F'' = 0$ and $\phi = 1$, so $\lambda = 1$), the regularity bound $\bar{K}_{\text{slope}}$ vanishes and the equilibrium concept of Proposition 1 is delicate; the model inherits the equilibrium multiplicity of AV, with asymmetric equilibria sharing the same aggregate employment co-existing with the symmetric one. Since $\mathfrak{E}(0) = 0$ and Lemma 7 implies $\partial\mathfrak{E}/\partial\phi < 0$, the enforcement contribution is strictly negative at $\phi = 1$ at any such equilibrium, so $\mu^* < 1/\eta = \mu_{AV}^{cartel}$. Because firms operate along the upward-sloping labor supply $\omega = h^{-1}(L)$, this lower markdown translates into strictly higher aggregate employment and strictly higher real wages relative to AV's cartel–monopsony benchmark.*[22]

6 Illustrative Specification

The reversal mechanism is established generally in Section 5. I now work through a parametric special case that makes the equilibrium tractable and isolates two distinct channels through which enforcement operates: the firm-side response of labor demand and the worker-side response of labor supply through redistributed fines. I develop each channel in turn in Sections 6.1 and 6.2.

I assume separable isoelastic preferences over consumption and leisure,

$$U(C_i, L_i) \;=\; \frac{C_i^{1-\sigma}}{1-\sigma} \;-\; \chi\frac{L_i^{1+\xi}}{1+\xi}, \tag{34}$$

with $\sigma \in (0,1)$ and $\chi, \xi > 0$, and Cobb–Douglas production $F(L_j) = A\,L_j^{\alpha}$ with $A > 0$ and $\alpha \in (0,1)$. The labor supply elasticity is $\eta = (1-\sigma)/(\xi+\sigma) > 0$, and returns to scale are strictly decreasing.

Let $\Xi \equiv T^W/p$ denote the per-capita transfer in real units. At symmetric equilibrium, Ξ is determined endogenously by

$$\Xi \;=\; \rho(\phi/J, \mu^*)\, K\, \mu^*\, \omega(L^*)\, L^*. \tag{35}$$

The worker FOC for an interior solution,

$$\chi\, L_i^{\xi} \;=\; \omega\,(\omega L_i + \Xi)^{-\sigma}, \tag{36}$$

[22] I do not discuss two results from AV that are outside this paper's scope: (i) the markdown admits a product-market markup representation $\tilde{\mu} = \mu/(1+\mu)$; (ii) existence and uniqueness extend to moderately increasing returns ($F'' > 0$) under AV Remark 1 combined with $K \le \bar{K}$.

together with the firm FOC (14) at symmetry and the transfer identity (35) forms a system in $(L^*, \omega^*, \mu^*, \Xi)$. The system admits a clean closed-form solution under quasi-linear preferences, where the income-effect channel through Ξ vanishes; under the full separable isoelastic specification, the channel is active and the labor supply curve shifts in response to enforcement.

6.1 Equilibrium under Quasi-Linear Preferences

I first solve the equilibrium holding the labor supply curve fixed, abstracting from the income-effect feedback through Ξ. This isolates the firm-side enforcement channel and matches the boundary case of quasi-linear preferences ($\sigma = 0$, $U_{CL} = 0$), under which the abstraction is exact. The transfer Ξ drops out of (36), and the worker FOC reduces to $\chi L^{\xi} = \omega$. Inverting,

$$\omega(L) \;=\; \chi^{1/(1-\sigma)}\, L^{1/\eta}, \tag{37}$$

with $\omega'(L)L/\omega(L) = 1/\eta$ and $E_{\omega'} = 1 - 1/\eta < 1$.

For K in the range verified numerically in Section 7, the firm objective is strictly concave and Proposition 1 applies: a unique symmetric equilibrium exists. Substituting (37) and F into (32) and solving for aggregate employment,

$$L^* \;=\; \left(\chi^{-1/(1-\sigma)}\, J^{1-\alpha}\, \frac{A\,\alpha}{1 + H(\phi, J)/\eta + \mathfrak{E}^*}\right)^{1/(1-\alpha+1/\eta)}, \tag{38}$$

where $\mathfrak{E}^* = \mathfrak{E}(\phi, J, K, \mu^*)$.[23]

Figure 1 illustrates the firm-side mechanism. Under no enforcement ($K = 0$), the AV sympathy channel shifts labor demand inward as ϕ rises from 0 to 1, reducing equilibrium employment from $L^*_{\phi=0}$ to $L^*_{\phi=1}$ (the AV cartel benchmark). At $\phi = 1$, the sympathy channel saturates ($\partial\lambda/\partial\phi|_{\phi=1} = 0$) while the enforcement channel remains active. Any positive K therefore shifts labor demand outward relative to the AV cartel, raising equilibrium employment and real wages: $L^*_{\phi=1,K=0} < L^*_{\phi=1,K>0} < L^e$, where L^e denotes the first-best allocation. The labor supply curve L^S is held fixed across panels, abstracting from the income-effect feedback through Ξ.

[23] At $K = 0$, $\mathfrak{E}^* = 0$ and (38) reduces to $L^* = \left(\chi^{-1/(1-\sigma)} J^{1-\alpha}\, A\alpha/(1+H/\eta)\right)^{1/(1-\alpha+1/\eta)}$, the aggregate employment expression of AV in this specification.

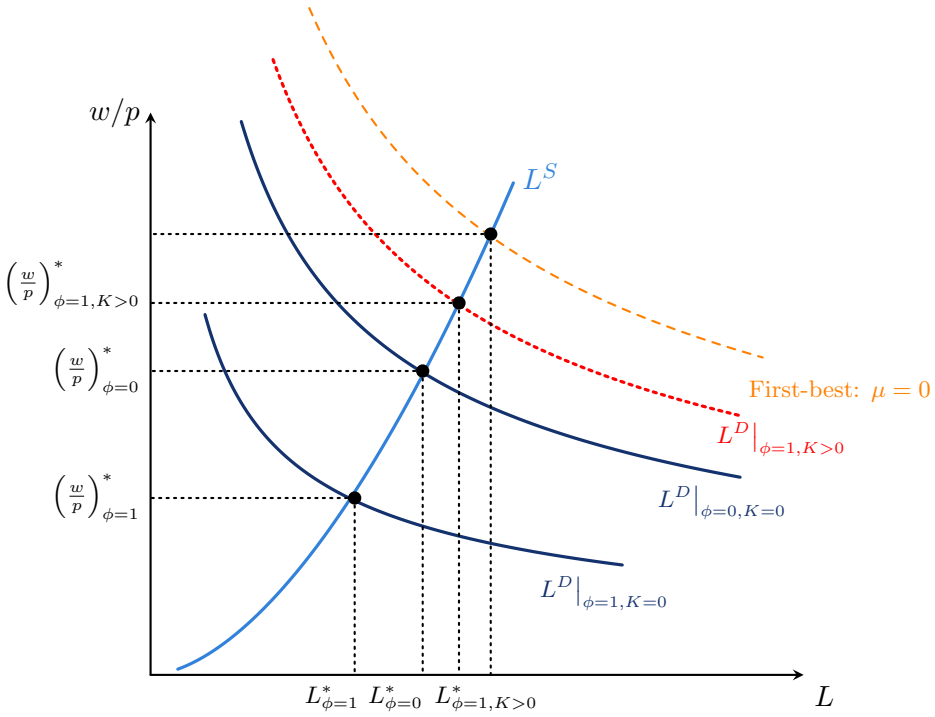


Figure 1. Labor market equilibrium illustrating the firm-side enforcement channel. The two solid dark blue curves show labor demand under no enforcement at $\phi = 0$ (outer) and $\phi = 1$ (inner, AV cartel). The red dotted curve shows the model at $\phi = 1$ with $K > 0$. The orange dashed curve is the first-best allocation ($\mu = 0$); it is not an equilibrium of the strategic oligopsony but bounds equilibrium employment from above, and its intersection with L^S defines $(L^e, (w/p)^e)$. The labor supply curve L^S is held fixed across panels, abstracting from the income-effect feedback through Ξ; the labor supply response is restored in Section 6.2. Parameters: $A = 6$, $J = 4$, $\alpha = 0.5$, $\xi = 0.5$, $\sigma = 0.5$, $\chi = 0.5$, $K = 27$; MHHI $H = 0.25$ at $\phi = 0$ and $H = 1$ at $\phi = 1$.

6.2 Equilibrium with the Redistribution Channel

I now restore the income-effect feedback through Ξ. With separable isoelastic preferences, the worker FOC (36) couples labor supply to the transfer, and the labor supply curve shifts with enforcement. Equilibrium is determined jointly by four conditions: the worker FOC, the firm FOC at symmetry, the transfer identity (35), and labor market clearing.

6.2.1 Augmented System

At symmetric equilibrium, the four equations characterizing $(L^*, \omega^*, \mu^*, \Xi^*)$ are

$$\chi\,(L^*)^{\xi} \;=\; \omega^*\big(\omega^* L^* + \Xi^*\big)^{-\sigma}, \tag{39}$$

$$\mu^* \;=\; \frac{H(\phi, J)}{\eta(L^*)} + \mathfrak{E}(\phi, J, K, \mu^*), \tag{40}$$

$$\Xi^* \;=\; \rho(\phi/J, \mu^*)\, K\, \mu^*\, \omega^*\, L^*, \tag{41}$$

$$F'(L^*/J) \;=\; \omega^*\,(1+\mu^*). \tag{42}$$

The system has no closed-form solution. I characterize the equilibrium analytically through a first-order expansion in K around the AV baseline ($K = 0$, $\Xi^* = 0$) at each ϕ. The resulting first-order coefficients depend on ϕ, allowing me to trace how the demand-side and supply-side channels evolve as common ownership rises toward saturation.

6.2.2 First-Order Expansion in K

Let (L_0, ω_0, μ_0) denote the AV baseline equilibrium at $K = 0$. Expand

$$L^*(K) \;=\; L_0 + K\,L_1 + O(K^2), \quad \omega^*(K) \;=\; \omega_0 + K\,\omega_1 + O(K^2), \quad \Xi^*(K) \;=\; K\,\Xi_1 + O(K^2). \tag{43}$$

Substituting (43) into (41) and matching the leading-order term yields

$$\Xi_1 \;=\; \rho_0\,\mu_0\,\omega_0\,L_0, \qquad \rho_0 \equiv \rho(\phi/J, \mu_0). \tag{44}$$

Linearizing the worker FOC (39) and the firm FOC (40) around the baseline, and solving the resulting 2×2 system, gives a decomposition

$$L_1 \;=\; L_1^{\text{demand}} + L_1^{\text{supply}}, \qquad \omega_1 \;=\; \omega_1^{\text{demand}} + \omega_1^{\text{supply}}, \tag{45}$$

where the demand-side components capture the firm-side enforcement effect (through the markdown response μ_1) holding labor supply fixed, and the supply-side components capture the income effect of Ξ_1 on labor supply holding demand fixed.

Proposition 4. *Under the conditions of Proposition 2 and at first order in K, with $\phi > 0$:*

(i) $L_1^{\text{demand}} > 0$ and $L_1^{\text{supply}} < 0$: the demand-side channel raises equilibrium employment, while

the supply-side channel lowers it.

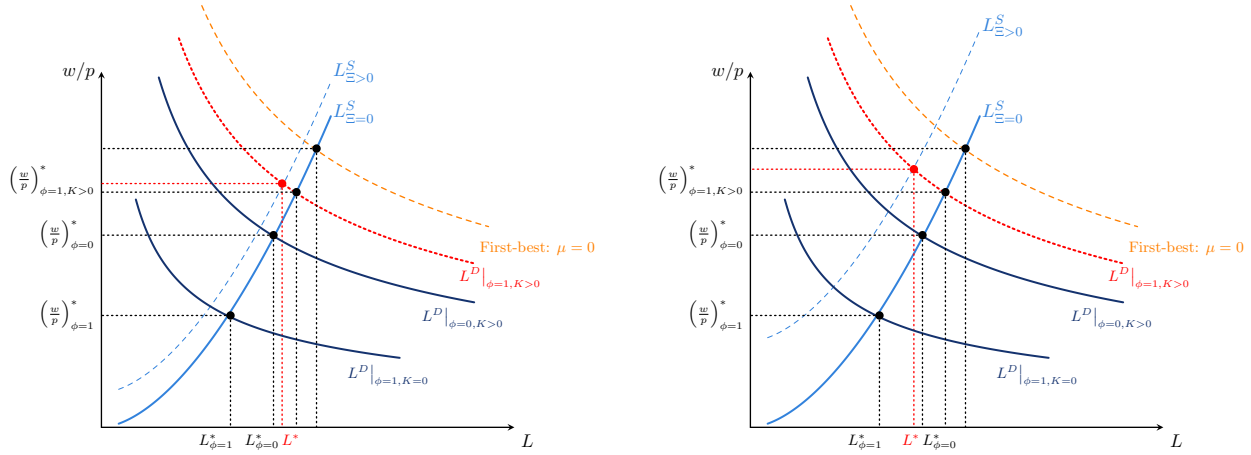
(ii) $\omega_1^{\text{demand}} > 0$ *and* $\omega_1^{\text{supply}} > 0$*: both channels raise the equilibrium real wage.*

Proof. Because $\mathfrak{E}$ is linear in K, write $\mathfrak{E} = K\mathfrak{E}_1 + O(K^2)$. Under the canonical specification $\rho(0,\mu) = 0$, $\mathfrak{E}_1(0) = 0$. Lemma 7 implies $\partial\mathfrak{E}_1/\partial\phi < 0$, so $\mathfrak{E}_1(\phi) < 0$ for $\phi > 0$. Under the constant-elasticity labor supply of (37), η is independent of L^*, so the first-order expansion of the firm FOC (40) yields $\mu_1 = \mathfrak{E}_1(\phi) < 0$. A lower markdown raises $F'(L^*/J)$ relative to ω^* at the existing supply curve, shifting the equilibrium upward and to the right along L^S, so $L_1^{\text{demand}} > 0$ and $\omega_1^{\text{demand}} > 0$. For the supply-side components, $\Xi_1 > 0$ enters the worker FOC (39) through the consumption argument of $U_C = C^{-\sigma}$. Higher Ξ raises consumption, lowers U_C, and reduces labor supply at any given ω (the income effect, signed by $\sigma > 0$). At unchanged labor demand, the inward shift in supply lowers L^* and raises ω^*, so $L_1^{\text{supply}} < 0$ and $\omega_1^{\text{supply}} > 0$. □

The proposition delivers the central observation of this section: *redistribution amplifies the wage response and dampens the employment response.* Both channels raise wages; they push employment in opposite directions. The net employment effect L_1 depends on which channel dominates — a quantitative question addressed in Section 7. The net wage effect $\omega_1 > 0$ is unambiguous.

Figure 2 depicts the equilibrium with the redistribution channel active. Each panel compares the low-common-ownership equilibrium E_0 ($\phi = 0$) with the high-common-ownership equilibrium E_1 ($\phi = 1$, $K > 0$, $\Xi > 0$), holding the enforcement multiplier K fixed.[24] For $K \in \mathcal{K}$ at the calibrated parameters of Section 7, the equilibrium markdown satisfies $\mu^*(\phi = 1, K) < \mu^*(\phi = 0)$, so net labor demand at $\phi = 1$ lies outward of demand at $\phi = 0$. Redistribution shifts labor supply inward from $L^S_{\Xi=0}$ to $L^S_{\Xi>0}$. The wage ω^* rises unambiguously; L^* rises when the demand-side channel dominates (panel (a)) and falls when the supply-side channel dominates (panel (b)), consistent with Proposition 4.

[24] The detection probability $\rho(0,\mu) = 0$ reflects the legal scope of Clayton §4, which targets coordinated conduct arising from common ownership rather than the standalone oligopsony markdowns present even under independent ownership. Hence $\Xi = 0$ at $\phi = 0$ regardless of K, and E_0 at $(\phi = 0, K > 0)$ coincides with the AV benchmark at $(\phi = 0, K = 0)$.



(a) Demand-side channel dominates: L^* rises.

(b) Supply-side channel dominates: L^* falls.

Figure 2. Equilibrium response to rising common ownership under fixed enforcement multiplier K. Each panel compares the low-common-ownership equilibrium E_0 ($\phi = 0$, $K > 0$, $\Xi = 0$) with the high-common-ownership equilibrium E_1 ($\phi = 1$, $K > 0$, $\Xi > 0$). At $\phi = 0$, the detection probability $\rho(0, \mu) = 0$ under the model's assumption that Clayton §4 detection requires positive common ownership; the per-capita transfer Ξ therefore vanishes and E_0 coincides with the AV benchmark at $\phi = 0$. At $\phi = 1$, enforcement is active and redistribution shifts labor supply inward from $L^S_{\Xi=0}$ to $L^S_{\Xi>0}$. For $K \in \mathcal{K}$ at the calibrated parameters of Section 7, $\mu^*(\phi = 1, K) < \mu^*(\phi = 0)$, so net labor demand at $\phi = 1$ lies outward of demand at $\phi = 0$. The equilibrium real wage ω^* rises in both regimes; aggregate employment L^* rises when the demand-side channel dominates (panel (a)) and falls when the supply-side channel dominates (panel (b)).

6.3 Welfare and the Efficiency Gap

The first-order expansion of Section 6.2 characterizes how enforcement and redistribution shift equilibrium $(L^*, \omega^*, \mu^*, \Xi^*)$. I now translate these shifts into welfare and efficiency consequences for workers, relative to the AV cartel benchmark at $(\phi = 1, K = 0)$.[25]

Worker indirect utility is

$$V(\omega, \Xi) \;=\; \max_{L} U(\omega L + \Xi,\, L) \;=\; \frac{(\omega L^* + \Xi)^{1-\sigma}}{1-\sigma} \;-\; \chi \frac{(L^*)^{1+\xi}}{1+\xi},$$

where L^* solves the worker FOC at (ω, Ξ). By the envelope theorem,

$$\frac{\partial V}{\partial \omega} = U_C\, L^* > 0, \qquad \frac{\partial V}{\partial \Xi} = U_C > 0.$$

Wage gains and transfer income both raise welfare; the labor adjustment cancels by optimality.

Proposition 5 (Welfare gain from enforcement)**.** *Under the conditions of Proposition 4, worker*

[25] The analysis is positive rather than normative: it tracks worker indirect utility $V(\omega^*, \Xi^*)$ across enforcement regimes at fixed J, and does not optimize over the number of firms or aggregate welfare weights as in AV's planner exercise (AV Section 5.1). Owner welfare falls under enforcement through the fine term $\tau^* > 0$; its quantification is straightforward but not the focus here. A brief remark on total surplus appears at the end of this subsection.

indirect utility under enforcement and redistribution exceeds that at the AV cartel benchmark, at first order in K:

$$V^* - V^{\mathrm{AV}} \;=\; K\,U_{C,0}\,\big(L_0\,\omega_1 + \Xi_1\big) + O(K^2) > 0,$$

where $U_{C,0} = (\omega_0 L_0)^{-\sigma}$ is marginal utility of consumption at the AV cartel.

Proof. Differentiate $V(\omega^*(K), \Xi^*(K))$ with respect to K at $K = 0$:

$$\left.\frac{dV}{dK}\right|_{K=0} = \left.\frac{\partial V}{\partial \omega}\right|_0 \cdot \omega_1 + \left.\frac{\partial V}{\partial \Xi}\right|_0 \cdot \Xi_1 = U_{C,0}\big(L_0\,\omega_1 + \Xi_1\big).$$

By Proposition 4, $\omega_1 > 0$ at $\phi = 1$. By (44), $\Xi_1 = \rho_0 \mu_0 \omega_0 L_0 > 0$ at $\phi = 1$, since $\rho(0, \mu) = 0$ and $\rho_{\phi/J} > 0$ imply $\rho(1/J, \mu_0) > 0$. With $U_{C,0} > 0$ and $L_0 > 0$, the first-order term is strictly positive, so $V^* > V^{\mathrm{AV}}$ for $K > 0$ small. □

Proposition 6 (Markdown efficiency gap closure)**.** *Define the markdown efficiency gap as $G(\mu) = \mu/(1+\mu) \in [0, 1)$, the fractional wedge between the marginal product of labor and the real wage. Under the conditions of Proposition 4, the gap closes at first order in K:*

$$G(\mu^*) - G(\mu^{\mathrm{AV}}) \;=\; \frac{\mu_1}{(1+\mu_0)^2}\,K + O(K^2) < 0,$$

where $\mu_1 = \mathfrak{E}_1(\phi) < 0$ for $\phi > 0$ by Proposition 4. The first-order closure rate is independent of the redistribution rule.

Proof. At first order, $\mu^* = \mu_0 + K\mu_1 + O(K^2)$. Differentiate $G(\mu) = \mu/(1+\mu)$ to obtain $G'(\mu) = 1/(1+\mu)^2 > 0$. By the chain rule,

$$G(\mu^*) - G(\mu_0) \;=\; G'(\mu_0)\,K\mu_1 + O(K^2) \;=\; \frac{K\mu_1}{(1+\mu_0)^2} + O(K^2).$$

Since $\mu_1 < 0$ at $\phi > 0$, the gap closes (becomes smaller).

Independence from the redistribution rule follows from the firm FOC: $\mu_1 = \mathfrak{E}_1(\phi)$ depends only on baseline (μ_0, ω_0, L_0) at $K = 0$, $\Xi = 0$, which are themselves determined by (ϕ, J, η) and not by the redistribution rule. The first-order markdown response is therefore identical across redistribution regimes; differences enter only at $O(K^2)$. □

Proposition 7 (Redistribution decomposition)**.** *Under the conditions of Proposition 4, the worker*

welfare gain at first order in K decomposes as

$$V^* - V^{\mathrm{AV}} \;=\; \underbrace{K\,U_{C,0}\,L_0\,\omega_1^{\mathrm{demand}}}_{\textit{firm-side gain}} \;+\; \underbrace{K\,U_{C,0}\,\left(L_0\,\omega_1^{\mathrm{supply}} + \Xi_1\right)}_{\textit{redistribution gain}} + O(K^2). \tag{46}$$

Both components are strictly positive at $\phi > 0$. Furthermore, the employment gap $1 - L^*/L^e$ behaves asymmetrically across channels: the firm-side channel narrows the gap ($L_1^{\mathrm{demand}} > 0$), while the redistribution channel widens it ($L_1^{\mathrm{supply}} < 0$).

Proof. Decompose the welfare expression of Proposition 5 using $\omega_1 = \omega_1^{\mathrm{demand}} + \omega_1^{\mathrm{supply}}$ from (45):

$$L_0\,\omega_1 + \Xi_1 \;=\; L_0\,\omega_1^{\mathrm{demand}} + L_0\,\omega_1^{\mathrm{supply}} + \Xi_1.$$

The firm-side component $L_0\,\omega_1^{\mathrm{demand}}$ is the welfare gain from the wage rise driven by markdown reduction; it is positive because $\omega_1^{\mathrm{demand}} > 0$ at $\phi > 0$ by Proposition 4. The redistribution component $L_0\,\omega_1^{\mathrm{supply}} + \Xi_1$ is the welfare gain from the wage rise driven by the inward labor supply shift plus the direct transfer income. Both $\omega_1^{\mathrm{supply}} > 0$ and $\Xi_1 > 0$ at $\phi > 0$, so this component is strictly positive.

For the employment gap, $L^* = L_0 + K(L_1^{\mathrm{demand}} + L_1^{\mathrm{supply}}) + O(K^2)$. By Proposition 4, $L_1^{\mathrm{demand}} > 0$ and $L_1^{\mathrm{supply}} < 0$ at $\phi > 0$. Since the firm-side channel narrows the gap (closer to first-best, since L^* rises toward L^e), while the redistribution channel widens it (further from first-best, since L^* falls). The net employment gap depends on the relative magnitudes. □

Remark 5 (Distributional vs. efficiency consequences)**.** *The welfare gain in Proposition 5 combines two distinct effects: a transfer of resources from owners (who pay fines) to workers (who receive them), and an efficiency gain from the markdown reduction (Proposition 6), which brings employment closer to the first-best level. The transfer component is a pure redistribution and does not by itself increase total surplus. The efficiency component reflects a genuine reduction in the deadweight loss of oligopsony. The relative magnitudes of these two effects are quantified in Section 7.*

Remark 6 (Cross-ϕ welfare)**.** *The cross-ϕ welfare comparison $V(\phi = 1, K > 0)$ versus $V(\phi = 0)$ is quantitative rather than analytical: it depends on whether the post-threshold welfare gains (for $\phi > \bar{\phi}(K)$) outweigh the pre-threshold losses (for $\phi < \bar{\phi}(K)$). Section 7 verifies numerically that the gain dominates at calibrated parameters with $K \in \{3, 9, 27\}$, so workers under full common ownership with enforcement are net better off than under no common ownership.*

Table 1. Summary of welfare and efficiency results: AV cartel vs. enforcement.

Dimension	Welfare & Efficiency
Workers' welfare (P5)	Workers' indirect utility rises with enforcement multiplier K at first order: $V^* > V^{\text{AV}}$. The gain decomposes into wage rise and direct transfer income.
Efficiency / markdown gap (P6)	Markdown falls toward the first-best level: $G(\mu^*) < G(\mu^{\text{AV}})$. The closure rate is independent of the redistribution rule.
Channel decomposition (P7)	Both channels raise workers' welfare through the wage rise; the redistribution channel adds a direct income gain through Ξ. The two channels move L^* in opposite directions: the firm-side narrows the employment gap, while the redistribution channel widens it.

7 Calibration and Empirical Projection

I calibrate the model at the parameter values of AV, project the empirical ϕ forward from the AV calibration window using a trend-stationary AR(1) specification, and characterize the equilibrium markdown across the empirical range and the reversal threshold $\bar{\phi}(K)$. The enforcement intensity K is calibrated using the Clayton §4 statutory framework combined with the Karpoff et al. (2008) multiplier. Following the convention of AV Section 3.4, the calibration is illustrative rather than macroeconomic: it shows how the enforcement channel operates at empirically plausible parameter values, not how the one-sector model matches US aggregate markdowns.[26]

For the quantitative exercises I adopt the multiplicative specification

$$\rho\left(\frac{\phi}{J}, \mu_j\right) \;=\; \min\left\{1,\; \frac{\phi}{J}\,\mu_j\right\}, \tag{47}$$

which satisfies (A1)–(A3) and has the minimum parametric structure consistent with the conjunctive triage logic of U.S. Department of Justice and Federal Trade Commission (2023). The min operator ensures $\rho_j \in [0,1]$ for all parameter combinations, and the specification nests the no-enforcement benchmark of AV at $\phi = 0$ or $\mu_j = 0$.[27]

[26] The calibration is parametrically flexible: alternative choices for the Karpoff multiplier, the ρ specification, or the AV structural parameters shift the quantitative magnitudes without overturning the qualitative reversal mechanism. The exercise should therefore be read as identifying the order of magnitude at which existing enforcement begins to discipline common-ownership conduct, not as a structural test of the mechanism. A formal falsification test would require cross-industry variation in K, discussed in Section 9.

[27] Alternative specifications consistent with (A1)–(A3), including additive forms $\rho = a_1(\phi/J) + a_2\mu_j$ with appropriate bounds and constant-elasticity-of-substitution aggregators, yield qualitatively identical reversal results.

7.1 Parameter Choices

I adopt the parameter values from AV Section 3.4: $A = 6$, $J = 4$, $\alpha = 0.5$, $\xi = 0.5$, $\sigma = 0.5$, $\chi = 0.5$. The implied labor supply elasticity is $\eta = (1 - \sigma)/(\xi + \sigma) = 0.5$. AV locate these values within the existing empirical literature on labor supply elasticity, returns to scale, and product-market substitution; I take their calibration as given and focus on the enforcement channel rather than re-deriving the structural parameters.

The empirical range of ϕ is inferred from AV Figure 2, which reports average intra-sector Edgeworth sympathy coefficients λ for the 1,500 largest US firms by market capitalization over 1985–2017: $\lambda_{1985} \approx 0.41$ and $\lambda_{2017} \approx 0.72$. Inverting equation (8) for $J = 4$ yields $\phi_{1985} \approx 0.486$ and $\phi_{2017} \approx 0.702$, so the empirical range over the sample is approximately $[0.49, 0.70]$.[28] The structural parameter ϕ depends on J through equation (8), but the empirically observed λ is portable across calibrations with different J, and the equilibrium implications for μ^* are governed by λ rather than ϕ directly.

The enforcement parameter K aggregates two empirically documented multipliers. Clayton Act §4 provides for treble damages, contributing $K_{\text{treble}} = 3$. Karpoff et al. (2008) document that the total firm-level loss following enforcement actions — including direct fines, market-value declines, increased compliance costs, and operational disruption — is approximately seven to nine times the direct fine; I adopt the midpoint $K_{\text{Karpoff}} = 9$ as the baseline upper bound, with sensitivity analysis at $K_{\text{Karpoff}} = 7$ in Section 7.6. The total empirical multiplier is $K_{\text{empirical}} = K_{\text{treble}} \times K_{\text{Karpoff}} = 27$. I report results for $K \in \{0, 3, 9, 27\}$ to span the range from no enforcement (the AV baseline) through statutory treble damages alone to full empirically calibrated enforcement.[29]

7.2 Empirical Projection of Common Ownership

To project ϕ beyond the AV calibration window, I extrapolate the historical λ series forward and invert to ϕ. I use the intra-sector series from AV Figure 2 because the model is one-sector: the relevant rival-profit weight is the within-market weight. The inter-sector series captures cross-

[28] The intra-sector λ values in AV's Figure 2 are computed for publicly traded firms only, without the adjustment for privately held firms. For an economy-wide calibration, AV Section 4.4 adjust these values down to an aggregate intra-sector $\lambda \approx 0.124$ in 2017. The calibration here captures common-ownership effects within the publicly traded sector, appropriate for a one-sector theoretical model focused on the segment of the economy where common ownership is most concentrated.

[29] The calibrated $K = 27$ is consistent with the value implied by inverting the equilibrium markdown equation (32) at the empirical 2017 values of ϕ, J, and η. Matching the average US wage markdown of approximately 0.7 estimated by Berger et al. (2022) requires $K \in [22, 28]$, depending on which point estimate of μ^* is used. The agreement is reassuring but not a structural estimate: a full identification of K from cross-firm variation in enforcement intensity and ownership structure is left to future work.

industry common ownership, which has no role in the one-sector framework and belongs to a multi-sector extension.

AV derive the closed-form sympathy coefficient under proportional control,

$$\lambda(\phi, J) = \frac{(2-\phi)\phi}{(1-\phi)^2 J + (2-\phi)\phi},$$

which is strictly increasing in ϕ on $(0,1)$ for fixed J, with $\lambda(0,J) = 0$ and $\lambda(1,J) = 1$. The inverse $\phi(\lambda, J)$ is therefore well-defined; I solve $\lambda(\phi, J) = \hat{\lambda}_t$ for ϕ numerically.

I model λ as a trend-stationary AR(1) process,

$$\lambda_t = \alpha + \beta\, t + \rho\big(\lambda_{t-1} - \alpha - \beta(t-1)\big) + \varepsilon_t, \qquad \varepsilon_t \sim N(0, \sigma^2).$$

The augmented Dickey–Fuller test with deterministic trend rejects the unit-root null at the 1% level, and the KPSS test with trend fails to reject trend-stationarity, so trend-stationarity is appropriate. I select the AR(1) order by BIC over alternatives including AR(2) on levels, random walk with and without drift, and ARIMA$(p,1,q)$ for $p, q \in \{0,1,2\}$; trend-stationary AR(1) attains the lowest BIC.[30]

Estimation results and diagnostics are reported in Tables 2 and 3. The trend coefficient $\hat{\beta} = 0.0089$ per year is the dominant source of forecast variation and is highly significant ($t = 9.43$, $p < 0.001$). The AR(1) coefficient $\hat{\rho} = 0.303$ is significant at the 5% level ($t = 2.21$, $p = 0.027$); it captures residual persistence after detrending. Residual diagnostics — the Ljung–Box test at lags 4 and 8, and the Jarque–Bera test — all fail to reject the null of i.i.d. normal residuals.

Table 2. Trend-stationary AR(1) estimation: $\lambda_t = \alpha + \beta t + \rho(\lambda_{t-1} - \alpha - \beta(t-1)) + \varepsilon_t$.

Coefficient	Estimate	Std. error	t-statistic	p-value
Constant (α)	0.449	0.016	27.64	< 0.001
Trend (β, per year)	0.0089	0.0009	9.43	< 0.001
AR(1) coefficient (ρ)	0.303	0.137	2.21	0.027
Innovation variance (σ^2)	0.0010	0.000	3.69	< 0.001
Sample: 1985–2017, $n = 33$				
AIC $= -126.56$, BIC $= -120.58$, log-likelihood $= 67.28$				

[30] Random walk with drift was an alternative, but the drift is not statistically distinguishable from zero ($t = 1.41$) and the resulting forecast confidence intervals are uninformatively wide. AR(1) on levels without trend is rejected by the ADF test ($p = 0.38$) and produces a spurious mean-reverting forecast inconsistent with the visual upward trend.

Table 3. Stationarity and residual diagnostics.

(a) Stationarity tests on the level series

Test	Statistic	p-value
ADF (constant)	-1.79	0.385
ADF (constant + trend)	-4.07	0.007
KPSS (constant)	0.90	0.010
KPSS (constant + trend)	0.08	> 0.10

(b) Residual diagnostics

Test	Statistic	p-value
Ljung–Box (lag 4)	2.05	0.726
Ljung–Box (lag 8)	5.19	0.737
Jarque–Bera	1.40	0.498

The forecast for 2030 is $\hat{\lambda}_{2030} = 0.859$ with 95% confidence interval $[0.79, 0.92]$.[31] Inversion at $J = 4$ yields $\hat{\phi}_{2030} = 0.802$ with 95% confidence interval $[0.754, 0.858]$. Both the point estimate and the entire confidence interval lie above the reversal threshold $\bar{\phi}(K = 27) \approx 0.77$ established below in Section 7.4. The threshold is crossed by the point forecast in 2025–2026 and by the lower 95% bound by the end of the forecast horizon; the regime in which existing antitrust enforcement actively counteracts further common-ownership growth is reached within the next decade under the maintained trend.

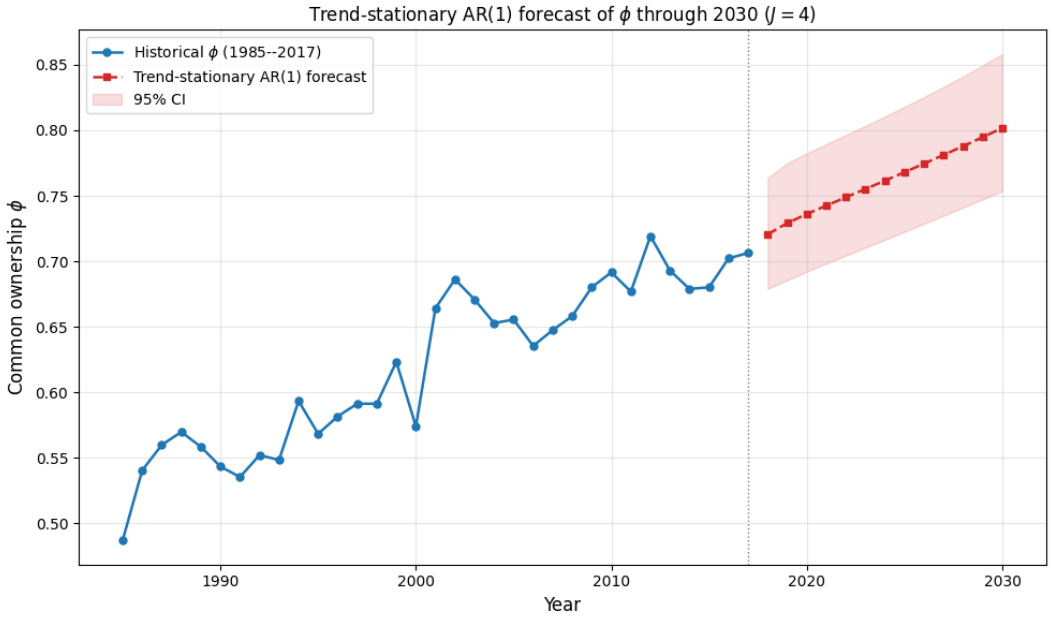


Figure 3. Trend-stationary AR(1) forecast of common ownership ϕ through 2030. The historical series (1985–2017) is the inverted intra-sector λ from AV Figure 2; the AR(1) is fit on the λ series and inverted to ϕ at $J = 4$. Shaded region is the 95% confidence interval.

[31] Non-parametric bootstrap confidence intervals (residual resampling, 5,000 replications) yield $\hat{\lambda}_{2030} = 0.86$ with 95% CI $[0.80, 0.94]$, within 4% of the parametric width reported in the text. The forecast and policy conclusions are unchanged.

7.3 Equilibrium Markdowns Across Diversification Levels

I solve the augmented equilibrium system of Section 6.2 for ϕ ranging from 0.05 to 0.99. Figure 4 displays the equilibrium markdown μ^* as a function of ϕ for each calibrated enforcement intensity.

Two patterns emerge. First, calibrated enforcement substantially reduces the equilibrium markdown across the empirical range. At the 2017 level $\phi = 0.702$, the markdown falls from 1.58 under no enforcement to 0.71 under statutory treble damages alone (a 55 percent reduction), to 0.36 at $K = 9$ (77 percent), and to 0.15 at $K = 27$ (91 percent). At the 1985 level $\phi = 0.486$, full Karpoff calibration reduces the markdown from 1.12 to 0.14, an 87 percent reduction. Second, the comparative static of μ^* with respect to ϕ is dramatically attenuated at calibrated enforcement intensities. Over the empirical 1985–2017 endpoints, μ^* rises by 41 percent at $K = 0$, by 16 percent at $K = 3$, by 8 percent at $K = 9$, and by only 2 percent at $K = 27$. At the upper-bound calibration, the anti-competitive comparative static is essentially neutralized over the empirical sample.

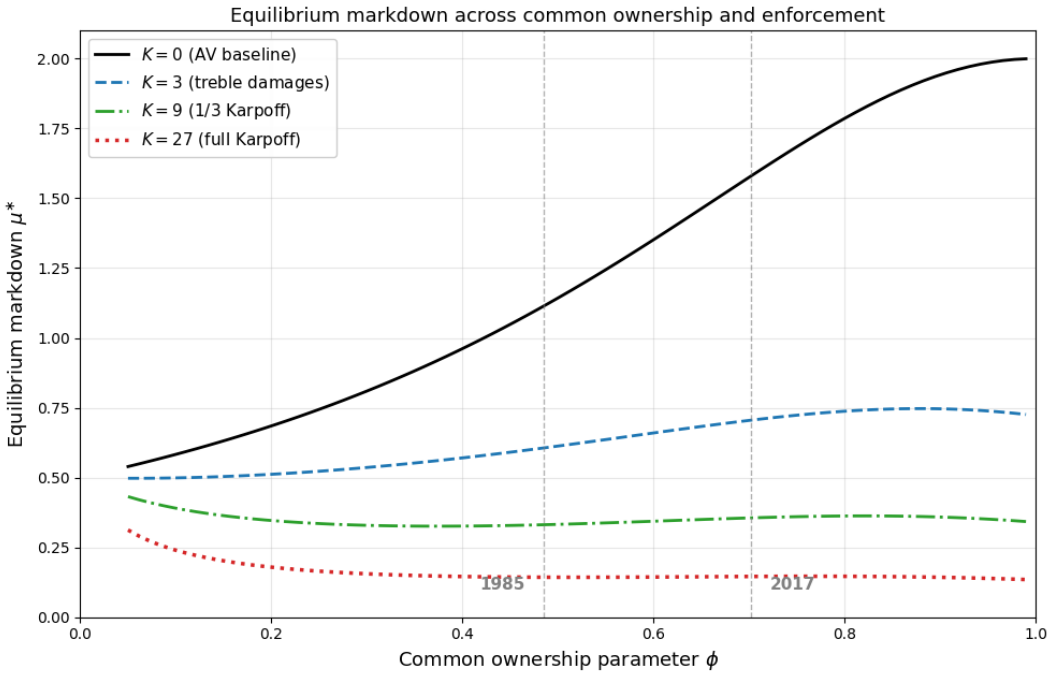


Figure 4. Equilibrium markdown μ^* as a function of common ownership ϕ for various enforcement intensities K. Solved from the augmented system of Section 6.2 with full redistribution channel; the markdown response to enforcement is independent of the redistribution rule at first order (Proposition 6). Vertical dashed lines mark the empirical ϕ values for 1985 ($\phi = 0.486$) and 2017 ($\phi = 0.702$). At $K = 0$, μ^* is monotonically increasing in ϕ. At calibrated enforcement intensities, μ^* is substantially lower in level and exhibits a much flatter dependence on ϕ across the empirical range. Parameters: $A = 6$, $J = 4$, $\alpha = 0.5$, $\eta = 0.5$.

7.4 The Reversal Threshold

Proposition 3 establishes that for each $K > 0$, there exists a threshold $\bar{\phi}(K)$ above which the equilibrium markdown is weakly decreasing in ϕ. At the calibrated parameter values, $\bar{\phi}(K = 27) \approx 0.77$, with smaller thresholds at higher enforcement intensities ($\bar{\phi}(K = 9) \approx 0.82$, $\bar{\phi}(K = 3) \approx 0.88$).

Whether the empirical ϕ has crossed this threshold depends on where the trajectory stands relative to it. Both the 1985 value $\phi = 0.486$ and the 2017 value $\phi = 0.702$ lie below $\bar{\phi}(K = 27)$, so the conventional anti-competitive channel remains operative over the historical sample. The AR(1) point forecast for 2030 ($\phi = 0.802$) lies above the threshold, and the forecast trajectory crosses $\bar{\phi}(K = 27)$ around 2025–2026. The 95% confidence interval $[0.754, 0.858]$ for ϕ_{2030} is also above the threshold, with the lower bound reaching it within the forecast horizon.

These quantitative conclusions are sensitive to the assumed ρ specification, the Karpoff multiplier, and the AV structural parameters. The qualitative finding — that calibrated enforcement generates a reversal threshold within the empirically relevant range of common ownership — is robust to plausible variations explored in Section 7.6, but the precise year of threshold crossing should be read as illustrative rather than predictive.

7.5 Welfare and Efficiency at Calibrated Values

I quantify the welfare and efficiency results of Section 6.3 at the calibrated parameters, solving the augmented system from Section 6.2 numerically. I report results at full common ownership $\phi = 1$, where the AV cartel benchmark and the enforcement model are most cleanly comparable; results at the empirical 2017 value $\phi = 0.702$ and the AR(1) 2030 forecast $\phi = 0.802$ are reported in Table 4 and tell the same qualitative story.

Table 4. Welfare and efficiency at calibrated parameters.

ϕ	K	V^*	$V^* - V^{\mathrm{AV}}$	$G(\mu^*)$	$G(\mu^{\mathrm{AV}}) - G(\mu^*)$
4*1.000	0	2.32	0.00	0.667	0.000
	3	4.22	+1.89	0.419	+0.247
	9	4.56	+2.23	0.254	+0.413
	27	4.59	+2.27	0.118	+0.549
4*0.702 (2017)	0	2.54	0.00	0.612	0.000
	3	3.95	+1.41	0.414	+0.199
	9	4.36	+1.82	0.262	+0.350
	27	4.49	+1.95	0.127	+0.485
4*0.802 (2030 fcst)	0	2.43	0.00	0.641	0.000
	3	4.05	+1.63	0.425	+0.217
	9	4.45	+2.02	0.266	+0.375
	27	4.54	+2.12	0.128	+0.514

V^* is worker indirect utility from the augmented system;
$G(\mu) = \mu/(1+\mu)$ is the markdown efficiency gap.
V^{AV} and $G(\mu^{\mathrm{AV}})$ correspond to the row with $K = 0$ at the same ϕ.

Three observations match the analytical predictions of Section 6.3.

First, worker welfare rises substantially with enforcement. At $\phi = 1$ and full Karpoff calibration ($K = 27$), worker indirect utility nearly doubles relative to the AV cartel benchmark ($V^* = 4.59$ versus $V^{\mathrm{AV}} = 2.32$, a gain of $+2.27$). The bulk of the gain is captured at moderate enforcement: $K = 3$ (statutory treble damages alone) delivers $+1.89$, or 83 percent of the full gain; $K = 9$ delivers $+2.23$, or 98 percent. Beyond $K = 9$, the marginal welfare gain from additional enforcement is small. At the empirical 2017 calibration $\phi = 0.702$ the welfare gain pattern is similar in shape but smaller in magnitude, since the AV cartel benchmark itself has lower markdown.

Second, the markdown efficiency gap closes substantially. Proposition 6 establishes that $G(\mu^*)$ falls toward zero with enforcement; the calibration quantifies the magnitude. At $\phi = 1$, full enforcement closes the gap from 0.667 (AV cartel) to 0.118, an 82 percent reduction. At the 2017 empirical level $\phi = 0.702$, the gap closes from 0.612 to 0.127, a 79 percent reduction.

Third, the redistribution channel contributes a substantial share of the welfare gain. Comparing the augmented system (with redistribution) to a no-redistribution counterfactual that holds the firm-side markdown reduction fixed but suppresses the transfer Ξ, at $\phi = 1$ and $K = 27$ the redistribution channel adds $+0.43$ to worker welfare ($V_{\mathrm{w}}^* = 4.59$ versus $V_{\mathrm{NR}}^* = 4.16$), about 19 percent of the total welfare gain. The remaining 81 percent comes from the firm-side wage rise. Consistent with Proposition 7, the redistribution channel raises welfare while reducing equilibrium

employment ($L^*_{\text{w}} = 3.24$ versus $L^*_{\text{NR}} = 3.39$), illustrating the welfare-employment tradeoff at the level of the calibrated economy.

The cross-ϕ welfare comparison from Remark 6 also holds numerically: at calibrated $K \in \{3, 9, 27\}$, worker welfare under full common ownership exceeds welfare under no common ownership. At $K = 27$, $V(\phi = 1) = 4.59$ compared with $V(\phi \to 0) = 3.52$, a gain of $+1.07$; the gain is positive at all calibrated $K > 0$ and reverses sign only at the no-enforcement benchmark $K = 0$ (where $V(\phi = 1) = 2.32$, below $V(\phi \to 0) = 3.52$ by -1.20). Enforcement is therefore quantitatively sufficient to make full common ownership welfare-improving relative to no common ownership at the calibrated parameter values.

7.6 Comparison and Robustness

The natural benchmark is the one-sector model of AV, corresponding to $K = 0$. Under the no-enforcement benchmark, μ^* rises by 41 percent from 1985 to 2017; under the model with full Karpoff calibration ($K = 27$), the same rise in common ownership produces only a 2 percent change. AV's multi-sector calibration (their Section 4.4) predicts a smaller markup increase of approximately 8.5 percent over the same period, owing to inter-sector pecuniary externalities and the privately-held-firm adjustment; this is not directly comparable to the one-sector calibration here, and extending the enforcement mechanism to the multi-sector framework is left for future work. For empirical context, AV (Section 3.4) compare their implied markup trajectory to alternative empirical series; I take their comparison as given and do not reproduce it here.

The qualitative results are robust to plausible variations. With K_{Karpoff} at the lower bound of 7, the corresponding $K = 21$ yields $\bar{\phi}(21) \approx 0.78$, only slightly higher than at $K = 27$. Setting $\alpha = 1$ (constant returns to scale) instead of the baseline $\alpha = 0.5$ raises the threshold modestly to $\bar{\phi} \approx 0.82$, since the firm-side enforcement correction shrinks. Increasing the number of firms from $J = 4$ to $J = 6$ at fixed $K = 27$ raises the threshold more substantially, to $\bar{\phi} \approx 0.90$, reflecting the reduced per-rival cross-holding ϕ/J that triggers detection. The reversal mechanism and the welfare gain from enforcement are preserved across all variations; the precise threshold depends on the calibrated number of firms and on the assumed enforcement multiplier, and should be interpreted as illustrative.

8 Policy Implications

The calibration results in Section 7 speak directly to the policy debate over common ownership in US equity markets. Three positions have emerged. Posner et al. (2017) (henceforth PSW) propose direct structural restrictions on institutional investor diversification, limiting large diversified investors to holdings in at most one firm per oligopolistic industry. Rock and Rubinfeld (2018) caution that such restrictions would impose substantial costs on retail investors and disrupt index investing without clear evidence of net welfare gains. Elhauge (2016) and Scott Morton and Hovenkamp (2018) advocate aggressive enforcement of existing antitrust laws against common-ownership conduct, without new structural restrictions.

The framework here formalizes the third position. The PSW proposal targets the structural parameter ϕ directly, restricting diversification to lower the sympathy coefficient $\lambda(\phi, J)$. The calibration suggests an alternative path: the substantial markdown reduction documented in Section 7.3 at the empirical ϕ operates through the existing legal architecture, without modifying institutional investor portfolios. The welfare quantification of Section 7.5 shows that this reduction translates into a substantial welfare gain for workers (+2.27 units of indirect utility at $\phi = 1$, $K = 27$, relative to the AV cartel benchmark; +1.95 units at the 2017 calibration). To the extent that the calibrated reduction approximates the discipline imposed by existing enforcement, statutory mechanisms may deliver welfare gains comparable to those targeted by PSW's structural proposal while avoiding the costs of disrupting diversification.[32]

The forward-looking analysis in Section 7.4, combined with the empirical projection in Section 7.2, suggests that the case for new structural restrictions weakens, rather than strengthens, as ϕ rises. Existing Clayton §4 enforcement becomes more potent in absolute terms because the fine τ_j scales with μ_j^2, and it becomes increasingly potent as a share of the anti-competitive effect that needs to be disciplined. The AR(1) forecast places $\phi_{2030} \approx 0.80$, above the calibrated reversal threshold $\bar{\phi}(K = 27) \approx 0.77$, with the entire 95% confidence interval at or above the threshold. PSW argue for new structural restrictions *because* common ownership has risen; the framework here suggests that as common ownership continues to rise, existing enforcement becomes increasingly adequate, and the marginal benefit of new structural restrictions correspondingly declines.

[32] Following AV's distinction between first-best (social planner solving for optimal allocation) and second-best (planner controlling structural parameters ϕ, J), the analysis here is positioned as second-best in the enforcement dimension: I take K as institutionally given by existing antitrust statutes and characterize how it disciplines common-ownership conduct. The first-best characterization of K^* — the welfare-maximizing enforcement intensity that trades off markdown-reduction gains against enforcement costs — requires a Polinsky and Shavell (2007)-style analysis of detection costs and is left for future work.

The substitute framing does not preclude a complementary one. AV (their Proposition 6) establishes that reducing ϕ and increasing J are complementary policy tools. The enforcement parameter K adds a third lever; K and ϕ are complements in disciplining markdowns, since both reduce μ^*. A pragmatic policy program might combine moderate increases in enforcement intensity with selective structural review of the most extreme cases of common ownership in highly concentrated industries.

Three conditions qualify these implications. First, the analysis is positive rather than normative: I do not characterize optimal enforcement intensity, which requires a social planner analysis along the lines of AV Section 5. Second, the calibrated $K = 27$ is grounded in published estimates but is not structurally estimated within the model; a structural estimation of K from firm-level enforcement data would tighten the quantitative implications. Third, the reversal mechanism operates through diversified shareholders' internalization of rivals' fines, captured by the share-weighted aggregation of owner indirect utilities following AV. Whether real-world common owners translate this internalization into operational decisions through corporate governance is an empirical question I do not address. The policy implications are therefore conditional on this internalization assumption: the potential for enforcement-driven discipline grows with common ownership, but its actual realization depends on whether common owners' governance practices translate fine internalization into operational guidance for portfolio firms.

9 Discussion and Extensions

The framework establishes a clean enforcement mechanism within the one-sector model of AV, with quantified welfare and efficiency implications and an empirical projection of common ownership through 2030. Several extensions would enrich the quantitative and policy implications.

The natural first extension is to the multi-sector framework of AV Section 4, which distinguishes between intra-industry common ownership ϕ' and economy-wide common ownership ϕ, with corresponding sympathy parameters λ_{intra} and λ_{inter}. AV show that the equilibrium markdown can be non-monotone in ϕ even without enforcement, due to a pro-competitive pecuniary externality across sectors. In the multi-sector setting, both this externality and the enforcement channel push toward lower markdowns at high ϕ, suggesting that the reversal threshold is even lower. Because the enforcement channel operates primarily through intra-industry sympathy while the pecuniary externality operates through economy-wide diversification, the two are complementary rather than

competing. Characterizing their joint behavior requires a separate analysis.

A second extension allows for heterogeneous firms and ownership. Heterogeneous CRS technologies generate the superstar reallocation result of AV Proposition 2, in which increased common ownership shifts market share from low-markup to high-markup firms. The enforcement mechanism would interact naturally with this reallocation: high-markdown firms face disproportionately larger detection probabilities and expected fines, attenuating the reallocation. On the ownership side, real-world cross-holdings are asymmetric, with specific large investors holding distinct stakes; the O'Brien and Salop (2000) framework with a $J \times J$ matrix of profit weights captures this heterogeneity at the cost of analytical tractability. The enforcement mechanism extends qualitatively to such settings, with detection probability depending on observable concentration of cross-holdings rather than on the symmetric scalar ϕ/J.

The model is also static and abstracts from capital. Azar and Vives (2019) extend AV to include savers, capital, and the real interest rate, showing that common ownership can depress investment alongside its labor-market effects. Embedding the enforcement channel in this richer framework would let antitrust fines discipline firm behavior through reduced cash flows available for reinvestment. A dynamic version of the model would also allow study of how regulators update enforcement priorities over time and how firms anticipate future enforcement intensity — including the credibility of enforcement commitments.

The static framework is a deliberate scope choice. It preserves analytical tractability and matches AV's modeling tradition, allowing the enforcement mechanism to be attributed cleanly to a single channel rather than confounded with transition dynamics. The reversal threshold and welfare results are comparative-statics properties of steady-state equilibria; they characterize qualitative features of long-run outcomes rather than transition paths. The redistribution transfer Ξ is the expected per-period flow in steady state, integrating over the probability of detection; it is not a one-time wealth transfer, and the welfare gain $V^* - V^{\text{AV}}$ is a steady-state level difference between regimes. A dynamic extension would refine quantitative magnitudes — through anticipation, enforcement credibility, and capital accumulation — without overturning these qualitative features.

The empirical projection of Section 7.2 extrapolates the historical λ series under a single trend-stationary specification estimated on 33 annual observations. Updating this projection with post-2017 data, when an updated intra-sector λ series becomes available, would refine the quantitative timing of threshold crossing. The qualitative conclusion — that common ownership is approaching

the reversal regime under maintained trend growth — depends on the trend-stationarity of the underlying process and on the AV sympathy formula linking λ to ϕ.

Finally, the strongest test of the framework would exploit cross-industry variation in enforcement intensity. The model predicts smaller anti-competitive effects of common ownership in industries facing more aggressive antitrust scrutiny. Recent intensification of labor-market enforcement following the High-Tech case and subsequent no-poach settlements provides natural variation across industries and over time. A complementary direction endogenizes K itself: a welfare-maximizing regulator would trade off the benefits of disciplining anti-competitive behavior against the costs of enforcement, following Polinsky and Shavell (2007) in partial equilibrium and extending it to the general-equilibrium setting here.

These extensions would refine the quantitative implications without overturning the central mechanism: diversified shareholders internalize rivals' fines through the same coefficient λ that internalizes rivals' profits, and at empirically relevant parameters, this internalization substantially moderates the anti-competitive prediction of the no-enforcement benchmark.

10 Conclusion

This paper integrates antitrust enforcement into the general equilibrium oligopoly framework of Azar and Vives (2021a). The modification is parsimonious: a single enforcement intensity parameter, grounded in US Clayton §4 treble damages and the Karpoff et al. (2008) multiplier, captures the deterrent effect of existing antitrust mechanisms on common-ownership conduct. The model nests AV at zero enforcement and admits a clean characterization of the modified equilibrium markdown through the same sympathy coefficient that channels the anti-competitive effect.

The substantive insight is that the same diversified shareholders who internalize rivals' profits also internalize rivals' fines. As common ownership rises, both channels strengthen but push in opposite directions: the anti-competitive sympathy effect saturates at full diversification, while the enforcement channel persists. This asymmetry establishes a unique reversal threshold above which the equilibrium markdown is weakly decreasing in common ownership. At empirical levels, calibrated enforcement substantially moderates the anti-competitive prediction; the empirical projection in Section 7.2 places the empirical ϕ near the reversal threshold within the forecast horizon.

The framework speaks to the policy debate over institutional investor diversification. Posner

et al. (2017) propose direct structural restrictions; the analysis suggests an alternative reading: existing Clayton §4 enforcement already disciplines common-ownership conduct, and as common ownership rises, this discipline becomes more potent rather than less adequate. The same coefficient that channels common ownership into rivals' profits also channels it into rivals' fines. As common ownership rises, the existing legal architecture becomes more, not less, capable of disciplining the conduct it is designed to restrain.

References

Albrecht, B. C., Auer, D., and Manne, G. A. (2024). Labor Monopsony and Antitrust Enforcement: A Cautionary Tale. ICLE white paper, International Center for Law and Economics. SSRN 4818412.

Antón, M., Ederer, F., Giné, M., and Schmalz, M. (2023). Common Ownership, Competition, and Top Management Incentives. *Journal of Political Economy*, 131(5):1294–1355.

Autor, D., Dorn, D., Katz, L. F., Patterson, C., and Van Reenen, J. (2020). The Fall of the Labor Share and the Rise of Superstar Firms. *Quarterly Journal of Economics*, 135(2):645–709.

Azar, J. (2020). The Common Ownership Trilemma. *University of Chicago Law Review*, 87(2):263–296.

Azar, J. and Marinescu, I. (2024). Monopsony Power in the Labor Market: From Theory to Policy. *Annual Review of Economics*, 16:491–518.

Azar, J., Marinescu, I., and Steinbaum, M. (2022a). Labor Market Concentration. *Journal of Human Resources*, 57(S):S167–S199.

Azar, J., Marinescu, I., Steinbaum, M., and Taska, B. (2020). Concentration in US Labor Markets: Evidence from Online Vacancy Data. *Labour Economics*, 66:101886.

Azar, J., Qiu, Y., and Sojourner, A. (2022b). Common Ownership in Labor Markets. Upjohn Institute Working Paper 22-368, W.E. Upjohn Institute for Employment Research.

Azar, J., Raina, S., and Schmalz, M. (2022c). Ultimate Ownership and Bank Competition. *Financial Management*, 51(1):227–269.

Azar, J., Schmalz, M. C., and Tecu, I. (2018). Anticompetitive Effects of Common Ownership. *Journal of Finance*, 73(4):1513–1565.

Azar, J. and Vives, X. (2019). Common Ownership and the Secular Stagnation Hypothesis. *AEA Papers and Proceedings*, 109:322–326.

Azar, J. and Vives, X. (2021a). General Equilibrium Oligopoly and Ownership Structure. *Econometrica*, 89(3):999–1048.

Azar, J. and Vives, X. (2021b). Reply to: Comments on "General Equilibrium Oligopoly and Ownership Structure". *Econometrica*, 89(3):1061–1063.

Azar, J. and Vives, X. (2024). Revisiting the Anticompetitive Effects of Common Ownership. Working paper, IESE Business School.

Backus, M., Conlon, C., and Sinkinson, M. (2021). Common Ownership in America: 1980–2017. *American Economic Journal: Microeconomics*, 13(3):273–308.

Baker, D. I. (2003). The Use of Criminal Law Remedies to Deter and Punish Cartels and Bid-Rigging. *George Washington Law Review*, 69:693–714.

Banal-Estañol, A., Seldeslachts, J., and Vives, X. (2018). Diversification, Common Ownership, and Strategic Incentives. *AEA Papers and Proceedings*, 108:561–564.

Barkai, S. (2020). Declining Labor and Capital Shares. *Journal of Finance*, 75(5):2421–2463.

Bebchuk, L. A. and Hirst, S. (2019). Index Funds and the Future of Corporate Governance: Theory, Evidence, and Policy. *Columbia Law Review*, 119(8):2029–2145.

Becker, G. S. (1968). Crime and Punishment: An Economic Approach. *Journal of Political Economy*, 76(2):169–217.

Benmelech, E., Bergman, N. K., and Kim, H. (2022). Strong Employers and Weak Employees: How Does Employer Concentration Affect Wages? *Journal of Human Resources*, 57(S):S200–S250.

Berger, D., Herkenhoff, K., and Mongey, S. (2022). Labor Market Power. *American Economic Review*, 112(4):1147–1193.

Berle, A. A. and Means, G. C. (1932). *The Modern Corporation and Private Property*. Macmillan, New York.

Boller, L. and Scott Morton, F. (2020). Testing the Theory of Common Stock Ownership. *NBER Working Paper*, (27515).

Bredbenner, M. and Hovenkamp, H. (2025). Trump Antitrust Officials Should Stay Focused on Labor Harms. The Regulatory Review, Penn Program on Regulation. Accessed May 2026.

Chetty, R., Guren, A., Manoli, D., and Weber, A. (2011). Are Micro and Macro Labor Supply Elasticities Consistent? A Review of Evidence on the Intensive and Extensive Margins. *American Economic Review*, 101(3):471–475.

De Loecker, J., Eeckhout, J., and Unger, G. (2020). The Rise of Market Power and the Macroeconomic Implications. *Quarterly Journal of Economics*, 135(2):561–644.

Dennis, P. J., Gerardi, K., and Schenone, C. (2022). Common Ownership Does Not Have Anti-Competitive Effects in the Airline Industry. *Journal of Finance*, 77(5):2765–2798.

Dimick, M. (2023). Conflict of Laws? Tensions Between Antitrust and Labor Law. *University of Chicago Law Review*, 90(2):379–436.

Ederer, F. and Tecu, I. (2025). Common Ownership: A Guide for Antitrust Practitioners. In *Antitrust Economics for Lawyers*, chapter 7. LexisNexis. Forthcoming.

Elhauge, E. (2016). Horizontal Shareholding. *Harvard Law Review*, 129:1267–1317.

Federal Trade Commission (2025). FTC Continues Enforcement Action Streak Against Anticompetitive No-Hire Agreements. Press Release, Bureau of Competition (Adamas Amenity Services LLC). Order finalized February 12, 2026.

Fichtner, J., Heemskerk, E. M., and Garcia-Bernardo, J. (2017). Hidden Power of the Big Three? Passive Index Funds, Re-Concentration of Corporate Ownership, and New Financial Risk. *Business and Politics*, 19(2):298–326.

Harvard Law School Forum on Corporate Governance (2025). Does Common Ownership Raise Antitrust Concerns? https://corpgov.law.harvard.edu/2025/01/06/does-common-ownership-raise-antitrust-concerns/. Accessed April 2026.

Hemphill, C. S. and Kahan, M. (2020). The Strategies of Anticompetitive Common Ownership. *Yale Law Journal*, 129(5):1392–1459.

In re High-Tech Employee Antitrust Litigation (2014). Class Action Settlement, Case No. 11-CV-2509-LHK. N.D. Cal., settled 2015.

Karabarbounis, L. and Neiman, B. (2019). Accounting for Factorless Income. *NBER Macroeconomics Annual*, 33(1):167–228.

Karpoff, J. M., Lee, D. S., and Martin, G. S. (2008). The Cost to Firms of Cooking the Books. *Journal of Financial and Quantitative Analysis*, 43(3):581–611.

Krueger, A. B. and Ashenfelter, O. (2022). Theory and Evidence on Employer Collusion in the Franchise Sector. *Journal of Human Resources*, 57(S):S324–S348.

Kwoka, J. E. (2014). *Mergers, Merger Control, and Remedies: A Retrospective Analysis of U.S. Policy*. MIT Press.

Marinescu, I. and Hovenkamp, H. J. (2019). Anticompetitive Mergers in Labor Markets. *Indiana Law Journal*, 94(3):1031–1063.

Milgrom, P. and Shannon, C. (1994). Monotone Comparative Statics. *Econometrica*, 62(1):157–180.

Morrison and Foerster LLP (2026). FTC Steps Up Targeted Enforcement Against Employee Non-competes. Client Alert.

Naidu, S., Posner, E. A., and Weyl, E. G. (2018). Antitrust Remedies for Labor Market Power. *Harvard Law Review*, 132:536–601.

Nekarda, C. J. and Ramey, V. A. (2019). The Cyclical Behavior of the Price-Cost Markup. *NBER Working Paper*, (19099).

O'Brien, D. P. and Salop, S. C. (2000). Competitive Effects of Partial Ownership: Financial Interest and Corporate Control. *Antitrust Law Journal*, 67(3):559–614.

Polinsky, A. M. and Shavell, S. (2007). The Theory of Public Enforcement of Law. *Handbook of Law and Economics*, 1:403–454.

Posner, E. A. (2024). The New Labor Antitrust. *Antitrust Law Journal*, 86(2):503–600. SSRN 4575258.

Posner, E. A., Scott Morton, F. M., and Weyl, E. G. (2017). A Proposal to Limit the Anti-Competitive Power of Institutional Investors. *Antitrust Law Journal*, 81:669–728.

Prager, E. (2026). Antitrust Enforcement in Labor Markets. *Journal of Economic Perspectives*, 40(1):115–138.

Rock, E. B. and Rubinfeld, D. L. (2018). Antitrust for Institutional Investors. *Antitrust Law Journal*, 82(1):221–278.

Rossi-Hansberg, E., Sarte, P.-D., and Trachter, N. (2021). Diverging Trends in National and Local Concentration. *NBER Macroeconomics Annual*, 35:115–150.

Rotemberg, J. J. (1984). Financial Transaction Costs and Industrial Performance. *MIT Sloan Working Paper*, (1554-84).

Scott Morton, F. M. and Hovenkamp, H. J. (2018). Horizontal Shareholding and Antitrust Policy. *Yale Law Journal*, 127:2026–2047.

Shapiro, C. (2018). Antitrust in a Time of Populism. *International Journal of Industrial Organization*, 61:714–748.

Starr, E. P., Prescott, J. J., and Bishara, N. D. (2021). Noncompete Agreements in the US Labor Force. *Journal of Law and Economics*, 64(1):53–84.

The White House (2021). Executive Order 14036: Promoting Competition in the American Economy. Executive Order, July 9, 2021.

U.S. Department of Justice and Federal Trade Commission (2023). Merger Guidelines. U.S. Department of Justice and Federal Trade Commission. December 18, 2023.

U.S. Department of Justice and Federal Trade Commission (2016). Antitrust Guidance for Human Resource Professionals. U.S. Department of Justice and Federal Trade Commission.

U.S. Securities and Exchange Commission (2024). Form 13F — Reports Filed by Institutional Investment Managers. https://www.sec.gov/divisions/investment/13ffaq.htm.

Vives, X. (1999). *Oligopoly Pricing: Old Ideas and New Tools*. MIT Press, Cambridge, MA.

Yeh, C., Macaluso, C., and Hershbein, B. (2022). Monopsony in the US Labor Market. *American Economic Review*, 112(7):2099–2138.