Carbon Pricing, Capital Composition, and Firm Investment

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April 2026

Abstract

Carbon pricing does not only tax emissions; it revalues the capital that produces them. This paper studies how this revaluation affects firm investment when the low-carbon transition requires changes in capital composition. I develop a dynamic investment model that distinguishes transition-oriented investment from total investment. The dirty-capital share links carbon exposure to investment through two channels: carbon pricing lowers the profitability of emissions-intensive capital, and it can also reduce collateralizability when lenders view dirty capital as exposed to transition risk and asset stranding. The model shows that carbon pricing creates countervailing forces: it raises the value of reducing dirty-capital exposure, but it can also tighten current financing conditions when dirty capital becomes less collateralizable. These countervailing forces and the recursive evolution of capital composition govern a self-consistent transition-investment rule. I estimate the model using an Euler-root simulated method of moments on a sample of EU ETS firms. The estimates support the main structure: transition investment is adjustment-costly, transition-investment effectiveness is positive but limited, and the composite collateral wedge improves the joint fit of the Euler and other selected moments. The results suggest that stronger carbon-price incentives need not mechanically translate into higher transition investment when dirty-capital firms face weaker collateral capacity. The paper provides a dynamic-investment perspective on asset stranding, transition inertia, and the complementarity between carbon pricing and transition finance.

Acknowledgements

I acknowledge that this research received no external financial support. I have no relevant or material financial relationships, outside activities, or other potential conflicts of interest to disclose. I conducted this research as part of my doctoral research at University College London.

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1 INTRODUCTION

The continued accumulation of GHG emissions is a key driver of anthropogenic climate change and has been treated as a global externality that requires policy intervention. Carbon pricing is a central market-based instrument for addressing this externality (Sterner et al., 2019). It internalizes environmental costs and decentralizes abatement decisions by allowing regulated firms to choose their own compliance paths (Montgomery, 1972; Rubin, 1996; Stavins, 2003; Gillingham and Stock, 2018). Together with subsidies, it forms an important part of the optimal climate-policy mix (Fischer and Newell, 2008; Acemoglu et al., 2012). Its effectiveness, however, depends on how firms respond. Since the low-carbon transition requires changes in technology, production processes, and capital composition, investment is a primary margin through which carbon pricing translates into real economic adjustment.

The investment effects of carbon pricing remain unclear. Firms may first respond through lower-adjustment-cost margins, such as carbon leakage through relocation, outsourcing, or reallocation of carbon-intensive activities, and cost pass-through to customers. These responses can partly insulate firms from carbon costs, but their effectiveness varies across sectors and market structures (Weyl and Fabinger, 2013; Ganapati et al., 2020; Colmer et al., 2024). When these margins are incomplete, firms must adjust production plans and investment decisions (Hintermann, 2016; Ganapati et al., 2020; Colmer et al., 2024). Empirically, several studies find limited adverse effects of carbon pricing on real activity, consistent with pass-through or up-front abatement investment that lowers long-run compliance costs (Marin et al., 2018; Dechezleprêtre et al., 2023; Colmer et al., 2024). Brown et al. (2025) even document a positive relationship between carbon costs and investment spending. In contrast, other studies argue that carbon pricing can reduce investment when pass-through is limited, internal finance tightens, or aggregate demand falls (Jacob and Zerwer, 2024; Matzner and Steininger, 2024). These mixed findings suggest that carbon pricing affects investment through heterogeneous firm-level margins rather than through a single average effect.

The investment margin itself is multidimensional. A firm can respond to carbon pricing by contracting carbon-intensive activity, by reallocating investment toward cleaner uses, or by doing both. These responses have different implications. A fall in total investment may reflect a contractionary real effect of climate policy. An increase in transition investment may indicate active adjustment toward cleaner capital. An increase in the transition share may reflect genuine decarbonization investment, but it may also occur because transition investment falls less than total investment. This distinction motivates the central question of this paper: how does carbon pricing affect total investment, transition-oriented investment, and the allocation between them?

A natural explanation for the mixed evidence is that carbon pricing affects investment through multiple channels. The first is an operating channel. Carbon pricing converts emissions into compliance costs, reduces cash flows, and changes the expected profitability of installed capital (Marin et al., 2018; Dechezleprêtre et al., 2023; Colmer et al., 2024; Brown et al., 2025; Jacob and Zerwer, 2024; Matzner and Steininger, 2024). The second is a financing channel. Carbon exposure can reduce financing flexibility and amplify the impact of carbon risk on investment

(Bartram et al., 2022; Jacob and Zerwer, 2024; Martinsson et al., 2024). These channels correspond to two sources of investment finance: internal funds generated by operations and external funds supported by collateral capacity. Existing work identifies both channels, but they are typically studied separately. A unified framework is needed to understand how they jointly determine total investment, transition investment, and the transition share.

The dynamic nature of carbon pricing further strengthens this need. Carbon prices are volatile and depend on future policy stringency, market conditions, and regulatory commitments. Firms therefore respond not only to contemporaneous compliance costs but also to expected future carbon-price paths. In dynamic neoclassical investment models, uncertainty shifts optimal investment by changing expected future profitability, marginal q, and the distribution of future returns (Hayashi, 1982; Abel, 1983; Bloom, 2009). In real-options models, uncertainty can delay irreversible, project-type investment by increasing the value of waiting (McDonald and Siegel, 1986; Dixit and Pindyck, 1994; Bernanke, 1983; Kaboski, 2005). This logic is particularly relevant for transition investment, which can be lumpy and project-like (Pindyck, 2000; Fried et al., 2022; Fuchs et al., 2024; Gavriilidis et al., 2026). Financing constraints add another dynamic force: expectations of tighter future borrowing conditions can induce precautionary or preemptive investment (Hennessy et al., 2007; Bolton et al., 2019). These literatures point to the same requirement: the investment response to carbon pricing should be studied in a dynamic framework that links operating exposure, collateral capacity, and investment allocation.

This paper proposes capital composition as the state variable that links these forces. Carbon pricing does not only tax current emissions. It revalues the capital stock that produces those emissions. Dirty capital becomes less valuable in operation because it generates current and future compliance costs. It may also become less valuable as collateral if lenders view carbon-intensive assets as more exposed to transition risk and asset stranding. Motivated by collateral-based amplification in Kiyotaki and Moore (1997), I model borrowing capacity as depending on the dirty-capital share. Capital composition therefore connects the operating channel and the financing channel in one dynamic mechanism. A firm with more dirty capital faces higher operating exposure and, when dirty capital is less collateralizable, tighter borrowing capacity. This creates the possibility of a transition trap: firms that need capital most to decarbonize may also face the tightest financing conditions.

I develop a dynamic investment model centered on this capital-composition mechanism. The firm owns one aggregate capital stock and chooses total investment. It also allocates investment between ordinary and transition-oriented uses. Ordinary investment maintains and expands productive capacity. Transition investment lowers the future dirty-capital share and therefore reduces future exposure to carbon costs and improves future collateral capacity. These two margins are nested. Transition investment is valuable because it changes future exposure, but it must be financed from the same dynamic resources that support ordinary investment. The model keeps the standard value-maximization logic of dynamic investment while allowing carbon pricing to affect both the level and the composition of investment.

The model delivers a countervailing-effect mechanism. Greater exposure to carbon pricing raises

the value of transition investment because reducing the dirty-capital share lowers future emissions costs and improves future collateral capacity. At the same time, greater exposure can tighten current financing conditions by reducing the collateral value of existing dirty capital. The net investment response depends on which force dominates. If future operating and financing gains dominate current financing tightening, carbon pricing can raise transition investment. If current financing tightening dominates, carbon pricing can reduce investment even when transition investment would be valuable in the long run. This provides a corporate-finance interpretation of transition inertia: dirty capital can make transition valuable while also weakening the financing capacity needed to undertake it.

The model has three main implications. First, total investment, transition investment, and the transition share need not move in the same direction. A higher transition share does not necessarily imply that transition investment rises in absolute terms; it may reflect reallocation toward transition uses within a smaller total investment budget. Second, transition investment has countervailing effects on total investment. When transition investment is costly, the firm may increase total investment to accommodate more transition investment without crowding out ordinary investment. When the continuation value of reducing the dirty-capital share is sufficiently large, the firm can support transition investment through reallocation rather than through a larger total investment budget. Third, transition investment is a dynamic allocation rule rather than a separate side decision. Current transition investment changes future capital composition; future capital composition changes operating and financing continuation values; and these continuation values feed back into the current transition choice. This recursive structure can generate either a unique self-consistent transition rule or multiple self-consistent rules. Multiplicity provides a theoretical interpretation of transition inertia and limited policy effectiveness: a firm may remain in a low-transition region even when a high-transition rule is also self-consistent.

I take the model to firm-level data using an Euler-root simulated method of moments. The empirical implementation treats capital expenditures as total investment and uses CDP *Investment Required* as a noisy observation-layer proxy for transition-investment demand. The estimates support the main structure: transition investment is substantially more adjustment-costly than ordinary investment, transition effectiveness is positive but limited, and a composite collateral wedge improves the joint fit of Euler and other selected moments. The selected specification delivers stable interior Euler roots throughout the estimation sample.

This paper makes three contributions. First, it contributes to the literature on the real effects of carbon pricing by distinguishing total investment, transition investment, and the transition share of investment. This distinction helps explain why carbon pricing may reduce total investment while increasing the share of investment allocated to transition uses, and why investment responses can differ across firms. Second, it provides a capital-composition perspective on asset stranding and transition risk. The model identifies the dirty-capital share as the state variable linking carbon exposure to firm investment through both operating values and collateral capacity in a dynamic setting. This structure delivers a recursive self-consistent transition rule. The discussion of the uniqueness–multiplicity boundary provides a model-based interpretation of transition inertia and a

possible investment trap. Third, it provides structural evidence for the countervailing mechanism. The Euler-root SMM estimates support high transition adjustment costs, positive but limited transition effectiveness, and an economically relevant composite collateral wedge. The results suggest that carbon pricing and transition finance may be complements: carbon pricing raises the value of reducing dirty-capital exposure, while transition finance can relax the collateral constraints that prevent firms from acting on that incentive.

2 MODEL

This section develops a unified model linking carbon pricing to firm investment through capital composition. The firm operates in an infinite-horizon neoclassical investment environment. It uses capital and labor to produce output, accumulates capital over time, and finances investment with internal funds and collateral-supported borrowing. The key state variable is the dirty-capital share. Dirty capital generates emissions and is exposed to carbon pricing; transition investment lowers the future dirty-capital share. The model therefore has a nested structure: the firm chooses total investment and then allocates investment between ordinary and transition-oriented uses. This structure allows carbon pricing to affect both the level and the composition of investment through operating values and collateral capacity.

2.1 Economic Environment

Investment and capital decomposition. The firm allocates total investment I_t between ordinary investment I_t^o and transition investment I_t^{tr}:

$$I_t = I_t^o + I_t^{tr}, \qquad I_t^{tr} \in [0, I_t], \tag{1}$$

$$s_t \equiv \frac{K_t^d}{K_t} \in [0, 1]. \tag{2}$$

Here, K_t is aggregate capital, K_t^d is dirty capital, and s_t is the dirty-capital share. Transition investment changes capital composition by reducing the future share of dirty capital. I summarize this mechanism with a Markov transition law:

$$s_{t+1} = \mathcal{S}\left(s_t, \frac{I_t^{tr}}{K_t}, \varepsilon_{t+1}^s\right), \tag{3}$$

$$\mathcal{S}_s \equiv \frac{\partial s_{t+1}}{\partial s_t} \geq 0, \qquad \mathcal{S}_{tr} \equiv \frac{\partial s_{t+1}}{\partial (I_t^{tr}/K_t)} < 0. \tag{4}$$

The shock ε_{t+1}^s captures transition shocks that are not known when the firm chooses investment at t but affect next-period capital composition. It may reflect uncertainty about abatement efficiency, technology implementation, or organizational adaptation. The condition $\mathcal{S}_s \geq 0$ captures persistence in dirty-capital composition. The condition $\mathcal{S}_{tr} < 0$ means that higher transition-investment intensity lowers the future dirty-capital share. The magnitude $|\mathcal{S}_{tr}|$ therefore measures transition-investment efficiency: how much the future dirty-capital share falls when transition investment

rises by one unit relative to capital.

All Scope 1 emissions are generated by dirty capital. Let $\mathbf{u}_t$ denote exogenous operating states that affect emissions. Since dirty capital is

$$K_t^d \equiv s_t K_t, \tag{5}$$

Scope 1 emissions are given by

$$CE_t^{(1)} = C\left(s_t K_t, \mathbf{u}_t\right), \tag{6}$$

$$\kappa_t^d \equiv \frac{\partial CE_t^{(1)}}{\partial K_t^d} > 0. \tag{7}$$

Here, κ_t^d is the marginal Scope 1 emissions generated by one additional unit of dirty capital. The marginal Scope 1 emissions intensity of total capital is obtained by holding s_t and $\mathbf{u}_t$ fixed when differentiating with respect to K_t:

$$\kappa_t \equiv \left.\frac{\partial CE_t^{(1)}}{\partial K_t}\right|_{s_t,\mathbf{u}_t} = \frac{\partial C(s_t K_t, \mathbf{u}_t)}{\partial (s_t K_t)} \frac{\partial (s_t K_t)}{\partial K_t} = s_t \kappa_t^d > 0, \tag{8}$$

$$\left.\frac{\partial CE_t^{(1)}}{\partial s_t}\right|_{K_t,\mathbf{u}_t} = \frac{\partial C(s_t K_t, \mathbf{u}_t)}{\partial (s_t K_t)} \frac{\partial (s_t K_t)}{\partial s_t} = K_t \kappa_t^d > 0. \tag{9}$$

Equations (8)–(9) make capital composition the object linking emissions exposure to investment. A higher dirty-capital share raises the emissions intensity of total capital and increases the marginal effect of capital composition on emissions.

Operational dynamics. Following Hayashi (1982), Gomes (2001), and Liu et al. (2009), I embed this capital-composition structure in a dynamic investment model based on Hayashi q theory. Capital accumulates according to the standard law of motion, and output is produced using capital K_t and labor L_t:

$$K_{t+1} = (1-\delta)K_t + I_t, \qquad \delta \in (0,1), \tag{10}$$

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha}, \qquad \alpha \in (0,1). \tag{11}$$

Consistent with Pedersen (2026), who treats carbon tax terms as additive components of firm cash flows, I assume that carbon compliance costs enter static operating profits as an additively separable wedge. The marginal profitability of capital is therefore

$$\pi_t \equiv \frac{\partial \Pi_t}{\partial K_t} = \alpha \frac{Y_t}{K_t} - P_t \kappa_t. \tag{12}$$

Equation (12) is the operating channel. A higher carbon price raises the marginal emissions cost $P_t \kappa_t$ and lowers the marginal profitability of capital. Since $\kappa_t = s_t \kappa_t^d$, the strength of this channel

depends on capital composition.

I allow for convex quadratic adjustment costs for both ordinary and transition investment:

$$\Phi^{tr}(I_t^{tr}, K_t) = \frac{a^{tr}}{2}\left(\frac{I_t^{tr}}{K_t}\right)^2 K_t, \qquad a^{tr} > 0, \tag{13}$$

$$\Phi^{o}(I_t^{o}, K_t) = \frac{a^{o}}{2}\left(\frac{I_t^{o}}{K_t}\right)^2 K_t, \qquad a^{o} > 0. \tag{14}$$

Total adjustment cost is

$$\Phi(I_t^o, I_t^{tr}, K_t) = \Phi^o(I_t^o, K_t) + \Phi^{tr}(I_t^{tr}, K_t). \tag{15}$$

The parameters a^o and a^{tr} govern the marginal adjustment costs of ordinary and transition investment. They measure capital-adjustment frictions and provide a model-based way to interpret transition inertia.

I adopt the tax convention in Liu et al. (2009): adjustment costs are tax deductible, capital expenditures are not, and firms receive a depreciation tax shield. Firms issue one-period debt. Let B_t denote the face value of debt outstanding at the beginning of period t and due in t, and let B_{t+1} denote new debt issued in period t and due in $t+1$. Let R_t be the gross borrowing rate between t and $t+1$. The firm's payout is

$$D_t = (1-\tau_t^c)\big[\Pi_t - \Phi(I_t^o, I_t^{tr}, K_t)\big] - I_t + \tau_t^c \delta K_t + B_{t+1} - R_t B_t + \tau_t^c (R_t - 1) B_t, \tag{16}$$

where τ_t^c is the statutory corporate income tax rate.

Financial constraints. The financing channel enters through collateral-based borrowing. Analogous to the credit-cycle mechanism in Kiyotaki and Moore (1997), the firm faces the one-period collateral constraint

$$B_{t+1} \le \lambda_t K_{t+1}. \tag{17}$$

The term λ_t is borrowing capacity per unit of installed capital, or the collateral ratio. To formalize the financing channel, I let λ_t depend on the carbon price P_t, the dirty-capital share s_t, and asset tangibility $Tang_t$. This specification captures limited collateralizability of dirty capital: a higher dirty-capital share can weaken borrowing capacity when lenders view carbon-intensive assets as more exposed to transition risk and asset stranding.

$$\lambda_t\big(P_t, s_t, Tang_t\big) \in [0, 1].$$

This specification captures the idea that dirty capital may become less collateralizable when lenders view carbon-intensive assets as exposed to transition risk and asset stranding. In the empirical implementation, direct measures of dirty-capital composition are not observed. I therefore proxy s_t with capital-scaled emissions intensity. This proxy is consistent with the model under a simple

measurement restriction. Since $CE_t^{(1)} = C(s_t K_t, \mathbf{u}_t)$, if dirty capital has locally constant emissions intensity, $CE_t^{(1)} = \bar{\kappa}_t^d s_t K_t$. Hence

$$\frac{CE_t^{(1)}}{K_t} = \bar{\kappa}_t^d s_t.$$

When cross-firm variation in $\bar{\kappa}_t^d$ is limited or controlled for by sector and technology states, capital-scaled emissions intensity is proportional to the dirty-capital share.

Consistent with the collateral-constraint literature (e.g., Rampini and Viswanathan, 2013), I impose limited liability and rule out frictionless equity issuance by requiring non-negative payouts:

$$D_T \geq 0, \qquad \forall T \geq t. \tag{18}$$

This restriction makes the shadow value of internal funds well defined and gives collateral capacity a direct role in shaping investment.

2.2 Firm Problem

The representative firm chooses total investment, transition investment, borrowing, capital, and the dirty-capital share to maximize cum-dividend equity value:

$$V_t = \max \ \mathbb{E}_t\left[\sum_{T=t}^{\infty} M_{t,T} D_T\right] \quad \text{s.t.} \quad (3),\ (10),\ (17),\ (18). \tag{19}$$

The primitive investment controls are total investment I_t and transition investment I_t^{tr}. Ordinary investment is the residual $I_t^o = I_t - I_t^{tr}$. Let q_t denote the multiplier on capital accumulation, ξ_t the multiplier on the dirty-capital-share transition law, μ_t the multiplier on the collateral constraint, and ρ_t the multiplier on the payout constraint. The Lagrangian is

$$\begin{aligned}\mathcal{L} = \mathbb{E}_t\Bigg[\sum_{T=t}^{\infty} M_{t,T}\Bigg\{ & D_T + q_T\Big((1-\delta)K_T + I_T - K_{T+1}\Big) + \xi_T\left(\mathcal{S}\Big(s_T, \frac{I_T^{tr}}{K_T}, \varepsilon_{T+1}^s\Big) - s_{T+1}\right) \\ & + \mu_T\big(\lambda_T K_{T+1} - B_{T+1}\big) + \rho_T D_T\Bigg\}\Bigg].\end{aligned} \tag{20}$$

A two-block decision-making. The firm's decision problem has two connected blocks. The first block is the total-investment and financing block: the firm chooses the investment budget, capital accumulation, and borrowing. The second block is the investment-allocation block: the firm allocates total investment between ordinary and transition uses. The two blocks are nested because transition investment changes the future dirty-capital share, and the dirty-capital share changes future operating values and collateral capacity.

The total-investment block is described by the first-order conditions with respect to current

total investment I_t, next-period capital K_{t+1}, and borrowing B_{t+1}:

$$q_t = (1+\rho_t)\left[1+(1-\tau_t^c)a^o\frac{I_t - I_t^{tr}}{K_t}\right], \tag{21a}$$

$$\begin{aligned} q_t = \phi_t + \mathbb{E}_t\Bigg[M_{t,t+1}\Bigg\{(1+\rho_{t+1})\Bigg[(1-\tau_{t+1}^c)\Bigg(\alpha\frac{Y_{t+1}}{K_{t+1}} - P_{t+1}\kappa_{t+1} \\ + \frac{a^o}{2}\left(\frac{I_{t+1}-I_{t+1}^{tr}}{K_{t+1}}\right)^2 + \frac{a^{tr}}{2}\left(\frac{I_{t+1}^{tr}}{K_{t+1}}\right)^2\Bigg) + \tau_{t+1}^c\delta\Bigg] + (1-\delta)q_{t+1} \\ - \xi_{t+1}\mathcal{S}_{tr}\left(s_{t+1}, \frac{I_{t+1}^{tr}}{K_{t+1}}, \varepsilon_{t+2}^s\right)\frac{I_{t+1}^{tr}}{K_{t+1}^2}\Bigg\}\Bigg], \end{aligned} \tag{21b}$$

$$\mu_t = (1+\rho_t) + \mathbb{E}_t\left[M_{t,t+1}(1+\rho_{t+1})\left(R_{t+1}(\tau_{t+1}^c - 1) - \tau_{t+1}^c\right)\right]. \tag{21c}$$

Equations (21a)–(21b) characterize the determinants of marginal q under optimal investment and financing. Ordinary investment affects current q_t through the marginal adjustment cost. Expected future ordinary and transition investment affect q_t through the continuation value of capital. Equation (21c) characterizes the shadow value of relaxing the borrowing constraint. When the collateral constraint binds, $\mu_t > 0$, the collateral wedge

$$\phi_t \equiv \lambda_t \mu_t$$

enters the capital FOC as an additive wedge. This is the financing channel: collateral capacity changes the marginal value of installed capital in addition to its productive value.

The investment-allocation block determines the transition pace and the evolution of capital composition. The first-order conditions with respect to current transition investment I_t^{tr} and next-period dirty-capital share s_{t+1} are

$$(1+\rho_t)(1-\tau_t^c)\left[a^{tr}\frac{I_t^{tr}}{K_t} - a^o\frac{I_t - I_t^{tr}}{K_t}\right] = \frac{\xi_t}{K_t}\mathcal{S}_{tr}\left(s_t, \frac{I_t^{tr}}{K_t}, \varepsilon_{t+1}^s\right), \tag{22a}$$

$$\begin{aligned} \xi_t = \mathbb{E}_t\Bigg[M_{t,t+1}\Bigg(-(1+\rho_{t+1})(1-\tau_{t+1}^c)P_{t+1}\frac{\partial CE_{t+1}^{(1)}}{\partial s_{t+1}} + \mu_{t+1}K_{t+2}\lambda_s(P_{t+1}, s_{t+1}, Tang_{t+1}) \\ + \xi_{t+1}\mathcal{S}_s\left(s_{t+1}, \frac{I_{t+1}^{tr}}{K_{t+1}}, \varepsilon_{t+2}^s\right)\Bigg)\Bigg]. \end{aligned} \tag{22b}$$

Equation (22a) is the optimal allocation rule between ordinary and transition investment. Holding total investment fixed, a marginal increase in transition investment reallocates resources away from ordinary investment. The left-hand side is the difference in marginal adjustment costs. The right-hand side is the marginal effect of transition investment on the dirty-capital-share state. Equation (22b) characterizes the shadow value ξ_t of reducing future dirty-capital exposure. This value has three components: an operating component from lower future emissions costs, a financing

component from improved future collateral capacity, and a persistence component because capital composition remains a state variable.

2.3 Euler equations

The first-order conditions can be rearranged into Euler equations that describe optimal investment dynamics. This step removes latent multipliers where possible and expresses the model in terms of continuation values, which are more economically interpretable. These Euler equations also provide the structural restrictions used in the empirical analysis.

Ordinary-investment Euler. Using the one-period-ahead analogues of Equations (21a) and (22a) to substitute for q_{t+1} and ξ_{t+1} in Equation (21b), define the one-period-ahead continuation value of installed capital as

$$\mathcal{C}^K_{t+1} \equiv (1+\rho_{t+1})\Bigg(1+(1-\tau^c_{t+1})\Bigg[\alpha\frac{Y_{t+1}}{K_{t+1}} - P_{t+1}\kappa_{t+1} + \frac{a^o}{2}\left(\frac{I_{t+1}}{K_{t+1}}\right)^2 - \frac{a^o+a^{tr}}{2}\left(\frac{I^{tr}_{t+1}}{K_{t+1}}\right)^2 + (1-\delta)a^o\frac{I_{t+1}-I^{tr}_{t+1}}{K_{t+1}} - \delta\Bigg]\Bigg). \tag{23}$$

Combining Equations (21a) and (21b) yields the ordinary-investment Euler equation:

$$(1+\rho_t)\left[1+(1-\tau^c_t)a^o\frac{I_t - I^{tr}_t}{K_t}\right] = \phi_t + \mathbb{E}_t\left[M_{t,t+1}\mathcal{C}^K_{t+1}\right]. \tag{24}$$

Equation (24) shows that ordinary investment and transition investment are nested. The current marginal cost of ordinary investment is equated to the collateral wedge plus the discounted continuation value of capital. That continuation value depends on future total investment and on the future allocation between ordinary and transition investment.

Investment-allocation Euler. The transition-allocation block delivers an Euler equation for the allocation between ordinary and transition investment. Rearranging Equation (22a) gives

$$\xi_t = \frac{K_t}{\mathcal{S}_{tr}\left(s_t, \frac{I^{tr}_t}{K_t}, \varepsilon^s_{t+1}\right)}(1+\rho_t)(1-\tau^c_t)\left[a^{tr}\frac{I^{tr}_t}{K_t} - a^o\frac{I_t - I^{tr}_t}{K_t}\right]. \tag{25}$$

Substituting Equation (25) and its one-period-ahead analogue into Equation (22b), define the one-period-ahead continuation value of capital composition as

$$\begin{aligned}\mathcal{C}^s_{t+1} \equiv (1+\rho_{t+1})(1-\tau^c_{t+1})\Bigg[&- P_{t+1}\frac{\partial CE^{(1)}_{t+1}}{\partial s_{t+1}} \\ &+ \frac{K_{t+1}\mathcal{S}_s\left(s_{t+1}, \frac{I^{tr}_{t+1}}{K_{t+1}}, \varepsilon^s_{t+2}\right)}{\mathcal{S}_{tr}\left(s_{t+1}, \frac{I^{tr}_{t+1}}{K_{t+1}}, \varepsilon^s_{t+2}\right)}\left(a^{tr}\frac{I^{tr}_{t+1}}{K_{t+1}} - a^o\frac{I_{t+1}-I^{tr}_{t+1}}{K_{t+1}}\right)\Bigg] \\ &+ \mu_{t+1}K_{t+2}\lambda_s(P_{t+1}, s_{t+1}, Tang_{t+1}).\end{aligned} \tag{26}$$

Combining Equations (22a)–(22b) yields the investment-allocation Euler equation:

$$(1+\rho_t)(1-\tau^c_t)\left[a^{tr}\frac{I^{tr}_t}{K_t} - a^o\frac{I_t - I^{tr}_t}{K_t}\right] = \mathbb{E}_t\left[M_{t,t+1}\frac{\mathcal{S}_{tr}\left(s_t, \frac{I^{tr}_t}{K_t}, \varepsilon^s_{t+1}\right)}{K_t}\mathcal{C}^s_{t+1}\right]. \tag{27}$$

Equation (27) shows that the current wedge between transition and ordinary investment is pinned down by the discounted continuation value of changing the future dirty-capital share. The compact object $\mathcal{C}^s_{t+1}$ summarizes the three sources of this continuation value: future emissions costs, future collateral capacity, and persistence in dirty-capital composition. $\mathcal{C}^s_{t+1} < 0$ is justifiable in the economically relevant region that a higher dirty-capital share lowers firm value. However, its sign is not pinned down mechanically by the model, because these components can have offsetting effects.

Transition-investment shares and adjustment costs. Suppose that the economically relevant region satisfies $\mathcal{S}_{tr} < 0$ and $\mathcal{C}^s_{t+1} < 0$. The first condition means that transition investment lowers the future dirty-capital share. The second means that a higher future dirty-capital share lowers continuation value, through higher future emissions costs, weaker future collateral capacity, or persistence in dirty-capital composition. Under these conditions, the right-hand side of Equation (27) is positive. The investment-allocation Euler then implies

$$a^{tr}\frac{I^{tr}_t}{K_t} > a^o\frac{I^o_t}{K_t}, \qquad \frac{I^o_t}{K_t} \equiv \frac{I_t}{K_t} - \frac{I^{tr}_t}{K_t}.$$

Equivalently,

$$\frac{I^{tr}_t}{I^o_t} > \frac{a^o}{a^{tr}}, \qquad \frac{I^{tr}_t}{I_t} > \frac{a^o}{a^o + a^{tr}}. \tag{28}$$

Equation (28) is therefore a conditional implication. It shows that, when transition investment improves future capital composition in value terms, the optimal transition-investment share is disciplined by relative adjustment costs. Since transition investment is often more project-like and may involve sunk expenditures, it is natural to allow $a^{tr} > a^o$, which implies

$$\frac{a^o}{a^o + a^{tr}} < \frac{1}{2}.$$

Therefore, transition investment need not exceed ordinary investment in levels. It must instead be sufficiently large relative to ordinary investment after weighting the two margins by their adjustment-cost slopes.

Combined Euler. Combining the ordinary-investment Euler and the investment-allocation Euler eliminates the ordinary-investment margin and yields an optimality condition for transition-investment dynamics:

$$(1+\rho_t)\left[1+(1-\tau_t^c)a^{tr}\frac{I_t^{tr}}{K_t}\right]=\phi_t+\mathbb{E}_t\left[M_{t,t+1}\left(\mathcal{C}_{t+1}^K+\frac{\mathcal{S}_{tr}\left(s_t,\frac{I_t^{tr}}{K_t},\varepsilon_{t+1}^s\right)}{K_t}\mathcal{C}_{t+1}^s\right)\right]. \tag{29}$$

Equation (29) is the key Euler equation for transition investment. It compresses the information in the ordinary-investment Euler and the investment-allocation Euler into a single condition for transition-investment dynamics. Economically, it implies that transition investment is shaped by the current collateral premium, the continuation value of installed capital, and the continuation value of future capital composition. Because current transition investment affects the future dirty-capital share, and the future dirty-capital share affects both operating values and collateral capacity, the optimal transition-investment rule has a recursive fixed-point structure.

This structure is the basis for the theoretical implications in the next section. It is also empirically useful as an Euler restriction. Since it reduces the two-dimensional investment system—total investment and the allocation between ordinary and transition uses—to a one-dimensional optimality condition for the transition-investment margin, the structural estimation can focus on one Euler restriction while still preserving the interaction between total investment, transition investment, operating exposure, and collateral capacity.

3 THEORETICAL IMPLICATION

This section studies the model implications of the capital-composition mechanism. The combined Euler equation (29) gives a compact condition for the transition-investment rule. I use this condition to characterize optimal transition investment, its link with total investment, the boundary between uniqueness and multiplicity, and the local comparative statics of the main primitives.

3.1 Optimal Investment

Let the model-implied total-investment and transition-investment rates be

$$x_t\equiv\left(\frac{I_t}{K_t}\right),\qquad x_t^{tr}\equiv\left(\frac{I_t^{tr}}{K_t}\right). \tag{30}$$

The main object of interest is the optimal transition-investment rule, which is the policy target of carbon pricing. Because transition investment changes future capital composition, the rule is a forward-looking policy rule that trades off current adjustment costs against future operating and

financing gains.

Optimal transition investment. Equation (29) can be rewritten as the following recursive fixed-point condition:

$$x_t^{tr*} = F\big(x_t^{tr*}; \mathbf{\Sigma}_t\big) \equiv \frac{\phi_t - (1+\rho_t) + \mathbb{E}_t\left[M_{t,t+1}\widetilde{\mathcal{C}}_{t+1}(x_t^{tr*})\right]}{(1+\rho_t)(1-\tau_t^c)a^{tr}}\,, \tag{31}$$

where

$$\widetilde{\mathcal{C}}_{t+1}(x_t^{tr*}) \equiv \mathcal{C}_{t+1}^K + \frac{\mathcal{S}_{tr}\big(s_t, x_t^{tr*}, \varepsilon_{t+1}^s\big)}{K_t}\mathcal{C}_{t+1}^s\,. \tag{32}$$

Here, $\mathbf{\Sigma}_t$ denotes the state variables observed by the firm at the beginning of period t. Equation (31) shows that the optimal transition-investment intensity is pinned down by a recursive fixed-point problem. Economically, it suggests the marginal effectiveness of transition investment, $\mathcal{S}_{tr}\big(s_t, x_t^{tr}, \varepsilon_{t+1}^s\big)$, depends on the transition intensity chosen today. If the fixed-point problem admits an interior solution, that solution is also an Euler root of the combined Euler equation. Marginal optimality and recursive self-consistency are therefore two representations of the same transition-investment rule.

The additive structure of Equation (31) decomposes optimal transition investment intensity into three components:

$$x_t^{tr,cfin} = \frac{\phi_t}{(1+\rho_t)(1-\tau_t^c)a^{tr}}\,, \tag{33a}$$

$$x_t^{tr,s}(x_t^{tr*}) = \frac{\mathbb{E}_t\left[M_{t,t+1}\frac{\mathcal{S}_{tr}(x_t^{tr*})}{K_t}\mathcal{C}_{t+1}^s\right]}{(1+\rho_t)(1-\tau_t^c)a^{tr}}\,, \tag{33b}$$

$$x_t^{tr,K} = \frac{\mathbb{E}_t\left[M_{t,t+1}\mathcal{C}_{t+1}^K\right]}{(1+\rho_t)(1-\tau_t^c)a^{tr}}\,, \tag{33c}$$

$$\text{where,} \qquad x_t^{tr*} = x_t^{tr,cfin} + x_t^{tr,s}(x_t^{tr*}) + x_t^{tr,K} - \frac{1}{(1-\tau_t^c)a^{tr}}\,.$$

The decomposition shows where carbon pricing enters the transition decision. The contemporaneous financing term captures the collateral wedge attached to current capital stock. The capital continuation term captures the standard dynamic-investment value of capital. The composition continuation term captures the value of changing future dirty-capital share. Equation (33b) is the key feature in the model: it is where future emissions costs, future collateral capacity, and persistence in capital composition are bundled into the current transition choice.

A complementary representation comes from adding the total-investment FOC (21a) and the transition-investment FOC (22a):

$$(1+\rho_t)\Big[1 + (1-\tau_t^c)a^{tr}x_t^{tr*}\Big] = q_t + \frac{\xi_t}{K_t}\mathcal{S}_{tr}\Big(s_t, x_t^{tr*}, \varepsilon_{t+1}^s\Big). \tag{34}$$

Rearranging Equation (34) gives an equivalent compact representation of the optimal transition-investment intensity:

$$\begin{aligned} x_t^{tr*} &= f\big(x_t^{tr*}; \mathbf{\Sigma}_t\big) \\ &= \frac{1}{(1+\rho_t)(1-\tau_t^c)a^{tr}}\, q_t\big(x_t^{tr*}\big) \;+\; \frac{\mathcal{S}_{tr}\Big(s_t, x_t^{tr*}, \varepsilon_{t+1}^s\Big)}{K_t(1+\rho_t)(1-\tau_t^c)a^{tr}}\, \xi_t \;-\; \frac{1}{(1-\tau_t^c)a^{tr}}\,. \end{aligned} \tag{35}$$

Equation (35) gives a transparent interpretation in shadow-value terms. Transition investment is valuable because it contributes to capital through the two-channel mechanism, and also because it changes future capital composition. When a higher dirty-capital share lowers continuation value, $\xi_t < 0$. Since $\mathcal{S}_{tr} < 0$, a marginal increase in transition investment intensity raises value by lowering future dirty-capital exposure. This is the channel through which transition investment protects the firm from future operating and financing costs.

Optimal total investment. The optimal total-investment rate (intensity) is determined jointly with the transition-investment rate. Rearranging the ordinary-investment Euler (24) gives

$$x_t{}^* = x_t^{tr*} + \mathcal{V}_t^K\,, \tag{36}$$

where

$$\mathcal{V}_t^K \equiv \frac{\phi_t + \mathbb{E}_t\left[M_{t,t+1}\mathcal{C}_{t+1}^K\right] - (1+\rho_t)}{(1+\rho_t)(1-\tau_t^c)a^o}\,. \tag{37}$$

Equation (36) shows that total investment rate combines the transition component and a level component, $\mathcal{V}_t^K$, governed by the collateral wedge and the continuation value of installed capital. This representation makes clear that transition investment is nested inside the same capital-accumulation decision, rather than a separate margin outside the investment budget.

The same total-investment rate can also be characterized from the investment-allocation Euler (27):

$$x_t{}^* = \frac{a^{tr}+a^o}{a^o}\, x_t^{tr*} - \mathcal{V}_t^s\big(x_t^{tr*}\big)\,, \tag{38}$$

where

$$\mathcal{V}_t^s\big(x_t^{tr*}\big) \equiv \frac{\mathbb{E}_t\left[M_{t,t+1}\mathcal{S}_{tr}\Big(s_t, x_t^{tr*}, \varepsilon_{t+1}^s\Big)\mathcal{C}_{t+1}^s\right]}{(1+\rho_t)(1-\tau_t^c)a^o K_t}\,. \tag{39}$$

Model implication 1: transition investment's countervailing effects on total investment. Equation (38) explicitly shows that $x_t{}^*$ is disciplined by two forces brought by x_t^{tr*}. More transition investment can require a larger total investment budget if the firm wants to protect ordinary investment from being crowded out. This is the allocation-cost force. Meanwhile, transition investment changes future capital composition. If lowering the dirty-capital share generates large future operating or financing gains, the firm can support a larger transition component through reallocation rather than through expanding the total investment budget. This is the composition-

value force. The effect of transition investment on total investment therefore depends on which force dominates: the firm may expand total investment to accommodate transition, or it may protect transition investment within a fixed or shrinking investment budget by crowding out ordinary investment.

Equivalence of optimality and self-consistency. Define the Euler residual of the combined Euler equation:

$$\mathcal{E}_t(z;\mathbf{\Sigma}_t) \equiv \phi_t + \mathbb{E}_t\Big[M_{t,t+1}\widetilde{\mathcal{C}}_{t+1}(z;\mathbf{\Sigma}_t,\varepsilon^s_{t+1})\Big] - (1+\rho_t)\Big[1+(1-\tau^c_t)a^{tr}z\Big]\,, \tag{40}$$

where

$$\widetilde{\mathcal{C}}_{t+1}(z;\mathbf{\Sigma}_t,\varepsilon^s_{t+1}) = \mathcal{C}^K_{t+1} + \frac{\mathcal{S}_{tr}(s_t,z,\varepsilon^s_{t+1})}{K_t}\mathcal{C}^s_{t+1}\,. \tag{41}$$

Here, $z \in [0, x_t]$ is a candidate transition-investment intensity, $\mathbf{\Sigma}_t$ denotes a given state vector in period t. A transition-investment solution is a root of this residual. If $\mathcal{E}_t(z;\mathbf{\Sigma}_t)$ is continuous on the feasible interval and the endpoint values satisfy the intermediate-value endpoint conditions, the Intermediate Value Theorem implies that there exists

$$x^{tr*}_t \in [0, x_t] \quad \text{such that} \quad \mathcal{E}_t(x^{tr*}_t;\Sigma_t) = 0\,.$$

If the endpoint inequalities are strict, at least one such root lies in the interior, $x^{tr*}_t \in (0, x_t)$. Appendix A.1 gives the full statement of the regularity conditions and proof.

The same Euler root admits an equivalent recursive fixed-point condition:

$$\mathcal{E}_t(x^{tr*}_t;\Sigma_t) = 0 \quad \Longleftrightarrow \quad x^{tr*}_t = F_t(x^{tr*}_t;\Sigma_t)\,.$$

This equivalence implies an investment perspective of thinking the low-carbon-transition inertia: the firm does not a one-time decision. It chooses a transition intensity that must be self-consistent with the continuation values generated by the future dirty-capital share. Current transition investment changes future capital composition; future composition changes operating and financing continuation values; and those continuation values feed back into the current transition choice.

3.2 Boundary Condition of Optimality

The next step is to discuss when the transition-investment rule is unique. Uniqueness is meaningful as it gives a single policy rule for each state, while multiplicity means that the same firm state can support more than one self-consistent transition path. Hence, multiplicity provides a model-based characterization of transition inertia: a firm may remain in a low-transition region even when a high-transition rule is also self-consistent.

Let θ denote the primitives of the model, so that the Euler residual can be written as $\mathcal{E}_t(z;\mathbf{\Sigma}_t,\theta)$. Consistent with standard optimality analysis, the boundary between uniqueness and multiplicity is characterized by a tangency condition.

Proposition 1 (Boundary between uniqueness and multiplicity)**.** *Suppose that $\mathcal{E}$ is continuously differentiable in (z, θ), and suppose also that for each admissible θ, there exists at least one interior solution of the transition-investment Euler equation. Define $\bar{\theta}$ as a candidate primitive value on the boundary between the unique-interior-solution region and the multiple-interior-solution region. Then there exists an interior point $\bar{z} \in (0, x_t)$ such that*

$$\mathcal{E}_t(\bar{z}\,;\,\mathbf{\Sigma}_t, \bar{\theta}) = 0\,, \qquad \frac{\partial \mathcal{E}_t(\bar{z}\,;\,\mathbf{\Sigma}_t, \bar{\theta})}{\partial z} = 0\,. \tag{42}$$

The proof is in Appendix A.2. Proposition 1 is the primary boundary result. It is useful because it turns the uniqueness–multiplicity boundary into a slope-balance condition, which connects transition costs, operating continuation values, financing continuation values, and persistence in capital composition.

Slope decomposition of the Euler residual. To interpret the boundary, decompose the dirty-capital-composition continuation term as

$$\mathcal{C}^{s}_{t+1} = \mathcal{C}^{s,\text{op}}_{t+1} + \mathcal{C}^{s,\text{fin}}_{t+1} + \mathcal{C}^{s,\text{pers}}_{t+1}\,, \tag{43}$$

where

$$\mathcal{C}^{s,\text{op}}_{t+1} \equiv -(1+\rho_{t+1})(1-\tau^{c}_{t+1})P_{t+1}\frac{\partial CE^{(1)}_{t+1}}{\partial s_{t+1}}, \tag{44a}$$

$$\mathcal{C}^{s,\text{fin}}_{t+1} \equiv \mu_{t+1}K_{t+2}\lambda_s(P_{t+1}, s_{t+1}, Tang_{t+1}), \tag{44b}$$

$$\mathcal{C}^{s,\text{pers}}_{t+1} \equiv \frac{K_{t+1}\mathcal{S}_s\big(s_{t+1}, x^{tr}_{t+1}, \varepsilon^{s}_{t+2}\big)}{\mathcal{S}_{tr}\big(s_{t+1}, x^{tr}_{t+1}, \varepsilon^{s}_{t+2}\big)}(1+\rho_{t+1})(1-\tau^{c}_{t+1})\Big[a^{tr}x^{tr}_{t+1} - a^{o}(x_{t+1} - x^{tr}_{t+1})\Big]. \tag{44c}$$

The first term is the operating value of reducing future emissions exposure. The second term is the financing value of improving future collateral capacity. The third term captures the recursive persistence of capital composition. This decomposition shows the source of the model's countervailing forces: transition investment can improve future states, but the strength of that feedback depends on how current transition investment changes future capital composition.

Substituting Equation (41) into Equation (40) and differentiating with respect to z gives

$$\frac{\partial \mathcal{E}_t(z\,;\,\mathbf{\Sigma}_t, \theta)}{\partial z} = -(1+\rho_t)(1-\tau^{c}_{t})a^{tr} + \Gamma^{K}_{t}(z;\mathbf{\Sigma}_t) + \Gamma^{\text{op}}_{t}(z;\mathbf{\Sigma}_t) + \Gamma^{\text{fin}}_{t}(z;\mathbf{\Sigma}_t) + \Gamma^{\text{pers}}_{t}(z;\mathbf{\Sigma}_t). \tag{45}$$

$$\Gamma_t^K(z;\mathbf{\Sigma}_t) \equiv \frac{\partial}{\partial z}\mathbb{E}_t\Big[M_{t,t+1}\mathcal{C}_{t+1}^K\Big], \tag{46a}$$

$$\Gamma_t^{\text{op}}(z;\mathbf{\Sigma}_t) \equiv \frac{\partial}{\partial z}\mathbb{E}_t\Big[M_{t,t+1}\frac{\mathcal{S}_{tr}(s_t,z,\varepsilon_{t+1}^s)}{K_t}\mathcal{C}_{t+1}^{s,\text{op}}\Big], \tag{46b}$$

$$\Gamma_t^{\text{fin}}(z;\mathbf{\Sigma}_t) \equiv \frac{\partial}{\partial z}\mathbb{E}_t\Big[M_{t,t+1}\frac{\mathcal{S}_{tr}(s_t,z,\varepsilon_{t+1}^s)}{K_t}\mathcal{C}_{t+1}^{s,\text{fin}}\Big], \tag{46c}$$

$$\Gamma_t^{\text{pers}}(z;\mathbf{\Sigma}_t) \equiv \frac{\partial}{\partial z}\mathbb{E}_t\Big[M_{t,t+1}\frac{\mathcal{S}_{tr}(s_t,z,\varepsilon_{t+1}^s)}{K_t}\mathcal{C}_{t+1}^{s,\text{pers}}\Big]. \tag{46d}$$

Equation (45) decomposes the slope of the transition-investment Euler residual. The first term is the current convex-cost discipline. It is negative and governed by a^{tr}. The remaining terms are continuation-value feedbacks. They measure whether a marginal increase in current transition investment strengthens future operating values, future collateral capacity, and future transition incentives enough to offset the current cost slope.

Model implication 2: uniqueness of self-consistent transition investment choice. A sufficient condition for the Euler residual to be strictly decreasing on the feasible interval is

$$\Gamma_t^K(z;\mathbf{\Sigma}_t)+\Gamma_t^{\text{op}}(z;\mathbf{\Sigma}_t)+\Gamma_t^{\text{fin}}(z;\mathbf{\Sigma}_t)+\Gamma_t^{\text{pers}}(z;\mathbf{\Sigma}_t) \;<\; (1+\rho_t)(1-\tau_t^c)a^{tr}\,, \qquad \forall z\in(0,x_t). \tag{47}$$

Under this condition, the current transition-cost slope dominates the continuation-value feedback. The Euler residual is globally single crossing, so the transition-investment Euler equation has at most one interior root.

Multiplicity can arise only when this dominance fails. At the boundary between uniqueness and multiplicity, Proposition 1 and Equation (45) imply

$$\Gamma_t^K(\bar{z};\mathbf{\Sigma}_t)+\Gamma_t^{\text{op}}(\bar{z};\mathbf{\Sigma}_t)+\Gamma_t^{\text{fin}}(\bar{z};\mathbf{\Sigma}_t)+\Gamma_t^{\text{pers}}(\bar{z};\mathbf{\Sigma}_t) \;=\; (1+\rho_t)(1-\tau_t^c)a^{tr}\,. \tag{48}$$

The uniqueness–multiplicity boundary suggests that a unique transition rule obtains when current convex transition costs dominate future composition benefits. When the inequality fails, continuation-value feedback can locally dominate the current marginal cost slope. In that region, a higher transition-investment intensity may raise the marginal continuation benefit faster than it raises the current marginal cost. The model can then support multiple self-consistent transition rules, which potentially triggers transition inertia: the same current state can sustain a low-transition rule even when a high-transition rule is also self-consistent.

The boundary condition also gives empirical content to the theory. The data do not only discipline the average investment response to carbon pricing. They also discipline the structural forces that shape the Euler residual: the current transition-cost slope governed by a^{tr} and the continuation-value feedback generated by operating exposure, collateral capacity, and composition

persistence. Structural estimation therefore provides a way to assess whether observed firm states are in a unique-solution region or close to a region where multiple transition rules may arise.

3.3 Local Comparative Statics

This section studies local comparative statics on the unique-interior-solution region. This restriction avoids the selection problem created by multiplicity and gives a transparent sign logic. Let $x_t^{tr*}(\theta)$ denote the unique interior transition-investment solution. The Implicit Function Theorem gives

$$\frac{\partial x_t^{tr*}(\theta)}{\partial \theta_j} = -\frac{\mathcal{E}_{\theta_j}\Big(x_t^{tr*}(\theta);\, \mathbf{\Sigma}_t, \theta\Big)}{\mathcal{E}_z\Big(x_t^{tr*}(\theta);\, \mathbf{\Sigma}_t, \theta\Big)}\,, \quad \mathcal{E}_z \neq 0\,, \tag{49}$$

The denominator is the local slope of the Euler residual. On the unique-solution region, $\mathcal{E}_{t,z} < 0$, so the sign of the response of transition investment is determined by the sign of the numerator $\mathcal{E}_{t,\theta_j}$. Because the optimal investment rule is described by the Euler residual, which is the difference between the marginal benefit (MB) and the marginal cost (MC) of transition investment:

$$\frac{\partial x_t^{tr*}}{\partial \theta_j} = -\frac{MB_{t,\theta_j}\big(x_t^{tr*}; \mathbf{\Sigma}_t, \theta_j\big) - MC_{t,\theta_j}\big(x_t^{tr*}; \mathbf{\Sigma}_t, \theta_j\big)}{MB_{t,z}\big(x_t^{tr*}; \mathbf{\Sigma}_t, \theta_j\big) - MC_{t,z}\big(x_t^{tr*}; \mathbf{\Sigma}_t, \theta_j\big)}\,. \tag{50}$$

The numerator measures how the primitive shifts the net marginal incentive to allocate investment toward transition uses. The denominator measures the local slope of that net marginal incentive with respect to transition-investment intensity itself. It is the same object that appears in the uniqueness–multiplicity boundary. When $\mathcal{E}_{t,z} < 0$, the investment rule is locally single crossing. In this case, the marginal response of x_t^{tr*} to a primitive depends on how much that primitive shifts the firm's net marginal benefit of transition investment.

Considering the paper's mechanism is built around capital composition and the two transmission channels, I focus on

$$a^{tr}, \qquad P_t, \qquad s_t, \qquad \eta_\lambda\,.$$

The parameter a^{tr} denotes the marginal cost of transition investment. The carbon price P_t revalues dirty capital. The dirty-capital share s_t measures the firm's exposure to this revaluation. Additionally, let η_λ capture how strongly dirty capital reduces collateral capacity:

$$\lambda_t = \lambda(P_t, s_t, Tang_t;\, \eta_\lambda)\,,$$

where a larger η_λ corresponds to a stronger collateral penalty for dirty capital.

Proposition 2 (Local comparative statics.)**.** *Suppose the regularity conditions in Appendix A.3 hold. On the unique-solution region, the unique interior transition-investment rule is continuously*

differentiable and satisfies Equation (49), then

$$\frac{\partial x_t^{tr*}}{\partial a^{tr}} < 0\,. \tag{51}$$

For P_t, s_t, and η_λ, the local responses are determined by countervailing effects:

$$\frac{\partial x_t^{tr*}}{\partial P_t} = -\frac{\mathcal{E}_{t,P_t}}{\mathcal{E}_{t,z}} = -\frac{\phi_{t,P_t} + \mathbb{E}_t\left[M_{t,t+1}\left(\mathcal{C}^K_{t+1,P_t} + \frac{\mathcal{S}_{tr,P_t}(s_t,z,\varepsilon^s_{t+1})}{K_t}\mathcal{C}^s_{t+1} + \frac{\mathcal{S}_{tr}(s_t,z,\varepsilon^s_{t+1})}{K_t}\mathcal{C}^s_{t+1,P_t}\right)\right]}{\mathcal{E}_{t,z}}\,, \tag{52a}$$

$$\frac{\partial x_t^{tr*}}{\partial s_t} = -\frac{\mathcal{E}_{t,s_t}}{\mathcal{E}_{t,z}} = -\frac{\phi_{t,s_t} + \mathbb{E}_t\left[M_{t,t+1}\left(\mathcal{C}^K_{t+1,s_t} + \frac{\mathcal{S}_{tr,s}(s_t,z,\varepsilon^s_{t+1})}{K_t}\mathcal{C}^s_{t+1} + \frac{\mathcal{S}_{tr}(s_t,z,\varepsilon^s_{t+1})}{K_t}\mathcal{C}^s_{t+1,s_t}\right)\right]}{\mathcal{E}_{t,z}}\,, \tag{52b}$$

$$\frac{\partial x_t^{tr*}}{\partial \eta_\lambda} = -\frac{\mathcal{E}_{t,\eta_\lambda}}{\mathcal{E}_{t,z}} = -\frac{\phi_{t,\eta_\lambda} + \mathbb{E}_t\left[M_{t,t+1}\left(\mathcal{C}^K_{t+1,\eta_\lambda} + \frac{\mathcal{S}_{tr}(s_t,z,\varepsilon^s_{t+1})}{K_t}\mathcal{C}^s_{t+1,\eta_\lambda}\right)\right]}{\mathcal{E}_{t,z}}, \tag{52c}$$

Proof. Equation (49) follows from the Implicit Function Theorem. For a^{tr} at any interior root $z = x_t^{tr*} > 0$:

$$\mathcal{E}_{t,a^{tr}}(z;\mathbf{\Sigma}_t) = -(1+\rho_t)(1-\tau_t^c)z < 0$$

Since $\mathcal{E}_{t,z} < 0$ on the unique-solution region, Equation (49) implies Equation (51). The expressions in Equations (52a)–(52c) follow directly from Equation (49). □

Model implication 3: countervailing forces of carbon pricing. The sign result for a^{tr} is clean. A higher transition-adjustment cost raises the current marginal cost of allocating investment toward transition uses and therefore lowers the optimal transition-investment intensity.

The results for the other primitives are generally ambiguous because they depend on which force dominates locally. A higher carbon price raises transition investment when the future value of reducing emissions exposure and improving collateral capacity dominates the contemporaneous financing tightening. In that case, $\mathcal{E}_{t,P_t} > 0$ and $\partial x_t^{tr*}/\partial P_t > 0$. If current financing tightening dominates the future composition-value gain, carbon pricing lowers transition investment. Thus, carbon pricing can make transition more valuable while also weakening the current financing capacity needed to implement it.

A higher dirty-capital share creates the same tension. It gives the firm more to gain from transition because more capital is exposed to carbon costs and potential collateral deterioration. But it also worsens the current balance sheet when dirty capital is less collateralizable. The sign of $\partial x_t^{tr*}/\partial s_t$ depends on whether the future transition-value effect dominates the current balance-sheet deterioration effect.

The parameter η_λ isolates the financing channel when the transition law itself does not depend

on η_λ. A larger η_λ makes dirty capital less collateralizable. This raises the future financing value of reducing the dirty-capital share, but it also tightens current borrowing capacity. The response of transition investment therefore depends on whether the future collateral-capacity gain dominates the current financing wedge.

These sign ambiguities are not a weakness of the model. They are the model's main implication. Carbon pricing can push firms toward transition through future composition benefits, but it can also pull them away from transition by weakening current financing capacity. Which force dominates depends on firm states.

The same primitives also affect total investment and the transition share. Using the level-allocation representation in Equation (36), define the transition-investment share as

$$\omega_t^* \equiv \frac{x_t^{tr*}}{x_t{}^*} \,. \tag{53}$$

Differentiating Equations (36) and (53) gives

$$\frac{\partial x_t{}^*}{\partial \theta_j} = \left[1 + \mathcal{V}_{t,z}^K\left(x_t^{tr*}; \mathbf{\Sigma}_t, \theta_j\right)\right] \frac{\partial x_t^{tr*}}{\partial \theta_j} + \mathcal{V}_{t,\theta_j}^K\left(x_t^{tr*}; \mathbf{\Sigma}_t, \theta_j\right), \tag{54a}$$

$$\frac{\partial \omega_t^*}{\partial \theta_j} = \frac{1}{x_t{}^*}\left[\frac{\partial x_t^{tr*}}{\partial \theta_j} - \omega_t^* \frac{\partial x_t{}^*}{\partial \theta_j}\right] . \tag{54b}$$

Equation (54a) separates the total-investment response into an allocation component and a direct level component. Equation (54b) then translates level responses into investment composition. This distinction is central for interpreting the real effects of carbon pricing. A higher carbon price may reduce total investment intensity while increasing the transition share. That pattern does not necessarily mean that transition investment rises in absolute terms; it may mean that transition investment is protected within a shrinking investment budget. Conversely, if the composition-value effect is strong enough, carbon pricing may raise transition investment in absolute terms even while depressing ordinary investment. The model therefore explains why carbon pricing can appear contractionary on one margin and transition-inducing on another.

4 EMPIRICAL ANALYSIS

This section takes the capital-composition model to firm-level data. The empirical exercise tests whether the Euler-root investment rules implied by the model can reproduce the joint behavior of total investment, reported transition-investment needs, emissions exposure, profitability, and financing states. I analyze the model structurally using the Simulated Method of Moments (SMM) based on the Euler-root restriction. The estimator recovers the parameters that govern adjustment costs, transition effectiveness, and the collateral wedge, and then asks whether the simulated economy supports the mechanism.

The main measurement constraint is that transition investment is not observed as a clean annual structural control. The CDP *Investment Required* variable records reported project-level or planned

investment needs. I therefore treat it as a noisy observation-layer proxy for transition-investment demand. Since the raw right tail of *Investment Required* is extreme, the SMM objective targets robust features of this object rather than raw cross-firm levels. This choice keeps the estimation focused on the transition-allocation margin rather than on a small number of very large disclosure values.

4.1 Data and Empirical Objects

Sample construction and data sources. The estimation sample combines firm-level accounting data, emissions disclosures, carbon prices, financing variables, and transition-investment disclosures. The empirical setting is the EU ETS, which is one of the most largest and active emissions trading system in the world. This setting is useful because carbon prices are observed at high frequency, while exposed firms can be linked to accounting and emissions data. The baseline estimation sample is an annual firm-year panel covering fiscal years 2015–2024, because emissions and transition-investment disclosures are reported at the firm-year level.

Firm-level accounting variables are drawn from Compustat Global annual fundamentals. Emissions and transition-investment variables are drawn from CDP climate-change disclosures. Carbon prices are measured using EUA futures prices from LSEG Workspace/Refinitiv's EEX EU Allowances continuation series.[1] The baseline carbon-price proxy is the within-year average of daily settlement prices from the first continuation series. Alternative continuation series and end-of-period prices are used as robustness variants.

The baseline sample consists of non-financial firms headquartered in EU ETS-region countries over fiscal years 2015–2024. Financial firms are excluded because their investment and balance-sheet dynamics are not comparable to the non-financial investment problem in the model. Utilities are retained because they are directly exposed to carbon pricing and are central to the transition mechanism. The estimation panel contains 2,794 firm-years and 622 firms. Missing CDP values are not filled with zero.

Empirical objects. The accounting data provide the observed total-investment decision. I use capital expenditures, $CAPEX_{i,t}$, as the empirical proxy for total investment, and measure total-investment intensity as capital expenditures scaled by lagged net property, plant, and equipment,

$$\frac{CAPEX_{i,t}}{K_{i,t-1}}.$$

This object maps to the model's total investment-to-capital ratio, I_t/K_t, not to ordinary investment. Ordinary investment is not directly observed; it is the residual model-implied decision, $I_t^o = I_t - I_t^{tr}$. This distinction is important for interpretation. The data directly discipline the total-investment

[1] Accounting statements are reported in firms' reporting currencies, while EUA prices are denominated in euros. Most empirical objects used in the estimation are ratios, so currency units cancel by construction. When monetary variables from different sources are combined, I express them in a common currency using the corresponding annual average exchange rate before constructing carbon-cost exposure and scaled transition-investment measures.

rule, while the transition-investment rule is disciplined through a separate observation layer.

The CDP transition-investment variable is not treated as realized transition investment. CDP *Investment Required* records firms' reported investment needs or planned investment requirements for emissions-reduction initiatives. It is informative about transition demand, but it is not a clean annual realization of I_t^{tr}. I therefore use it to target robust features of the transition-observation layer, such as incidence, median within-firm indexes, growth among continuing reporters, and high-minus-low carbon-intensity gaps. This avoids putting a noisy disclosure object directly inside the Euler residual.

Carbon exposure is measured using emissions intensity. The model's state variable is the dirty-capital share s_t. Because firm-level dirty capital is not directly observed, I use capital-scaled emissions intensity as its empirical proxy. The logic follows directly from the model. If Scope 1 emissions are generated by dirty capital and dirty capital has locally stable emissions intensity, then

$$CE_{i,t}^{(1)} = \bar{\kappa}_{i,t}^{d} s_{i,t} K_{i,t}, \qquad \frac{CE_{i,t}^{(1)}}{K_{i,t}} = \bar{\kappa}_{i,t}^{d} s_{i,t}.$$

After controlling for sector and technology states, emissions per unit of capital provides a disciplined way to rank firms by dirty-capital exposure. I also use total emissions intensity, based on Scope 1 plus Scope 2 emissions, to capture broader carbon exposure when Scope 2 electricity-related emissions are relevant for firms' transition risk.

Financing states are measured using leverage, liquidity, and tangibility. These variables discipline the collateral-channel moments and help identify the composite collateral wedge that enters the optimality conditions. The empirical implementation therefore follows the model's economic structure: carbon exposure affects investment through operating profitability and through the collateral value of capital.

Table 1: Estimation sample and empirical objects

Object	Empirical measure	Role in estimation
Total-investment intensity	$CAPEX/K_{t-1}$	Total-investment rule
Transition observation	CDP *Investment Required*	Noisy observation layer
Dirty-capital exposure	Capital-scaled emissions intensity	Proxy for $s_{i,t}$
Operating profitability	Profitability-to-capital	Operating channel
Collateral capacity	Tangibility, leverage, liquidity	Financing channel
Carbon price	EUA price	Policy state
Firm-years	2,794	SMM sample
Firms	622	SMM sample
Positive IR observations	1,706	Transition-observation layer

Notes: IR denotes CDP *Investment Required*. The table summarizes the key empirical objects used in the Euler-root SMM. Capital expenditures measure total investment, and $CAPEX/K_{t-1}$ measures total-investment intensity. CDP *Investment Required* is not treated as realized annual structural transition investment. Missing CDP values are not filled with zero. Positive IR observations are counted after imposing the full SMM estimation-sample restrictions and therefore differ from the broader descriptive count in Table 2.

Table 2 reports compact summary statistics for the main variables used in the empirical analysis. Two features are noticeable for the estimation. First, total-investment intensity displays substantial dispersion, which helps discipline the total-investment rule. Second, CDP *Investment Required* is much more right-skewed than the accounting variables. This supports treating it as a transition-observation object rather than as realized annual structural transition investment.

Table 2: Summary statistics for key estimation variables

Variable	N	Mean	SD	p25	Median	p75
$CAPEX/K_{t-1}$	3,054	0.213	0.194	0.104	0.163	0.245
$CAPEX/Assets$	3,822	0.039	0.033	0.015	0.030	0.051
CDP *Investment Required*/Assets	2,498	0.003	0.012	0.000	0.000	0.001
CDP *Investment Required*/CAPEX	2,485	0.163	0.752	0.001	0.007	0.034
Positive CDP *Investment Required*	2,498	0.869	0.338	1.000	1.000	1.000
Operating profitability/Capital	3,840	0.114	0.067	0.076	0.104	0.143
Carbon-cost exposure/Sales	3,711	0.006	0.017	0.000	0.000	0.003
Book leverage	3,845	0.281	0.158	0.166	0.267	0.374
Cash/Assets	3,845	0.114	0.089	0.053	0.092	0.147
Tangibility	3,845	0.251	0.190	0.099	0.210	0.362
Interest expense/Debt	3,813	0.042	0.038	0.023	0.033	0.050

Notes: The table reports selected summary statistics for the non-financial EU ETS sample. Ratio, exposure, profitability, and financing variables are winsorized at the 1st and 99th percentiles after construction within the restricted sample. CDP transition-investment variables are not zero-filled when undisclosed. Observation counts differ across variables because CDP disclosure, accounting coverage, and denominator restrictions vary across empirical objects. Summary statistics are reported for the broader restricted non-financial EU ETS sample before imposing all SMM moment-availability restrictions.

4.2 Descriptive Investment Patterns

Figures 1–3 organize the descriptive evidence around the model's core objects: carbon prices, total investment, transition-observation measures, and carbon-intensity groups. The figures are descriptive rather than causal. They illustrate that investment adjustment is not a single outcome. Carbon prices, total-investment intensity, capital accumulation, and reported transition-investment needs move differently over time and across exposure groups.

Figure 1 places EUA prices next to within-firm investment dynamics. EUA prices were relatively low before 2018 and rose sharply after 2020. Indexed CAPEX adjusts more gradually. By contrast, indexed CDP *Investment Required* is more volatile and rises sharply for firms that report positive transition-investment needs. Since the investment series are normalized within firm, the figure does not compare raw monetary levels across currencies. It shows instead that the high-carbon-price period coincides with large relative increases in reported transition-investment requirements among reporters.

Figure 2 sorts firm-years into low, medium, and high terciles of lagged capital-scaled emissions intensity within each fiscal year. The grouping is predetermined relative to year-t outcomes and reduces the influence of changing sample composition on exposure rankings. The capital-index

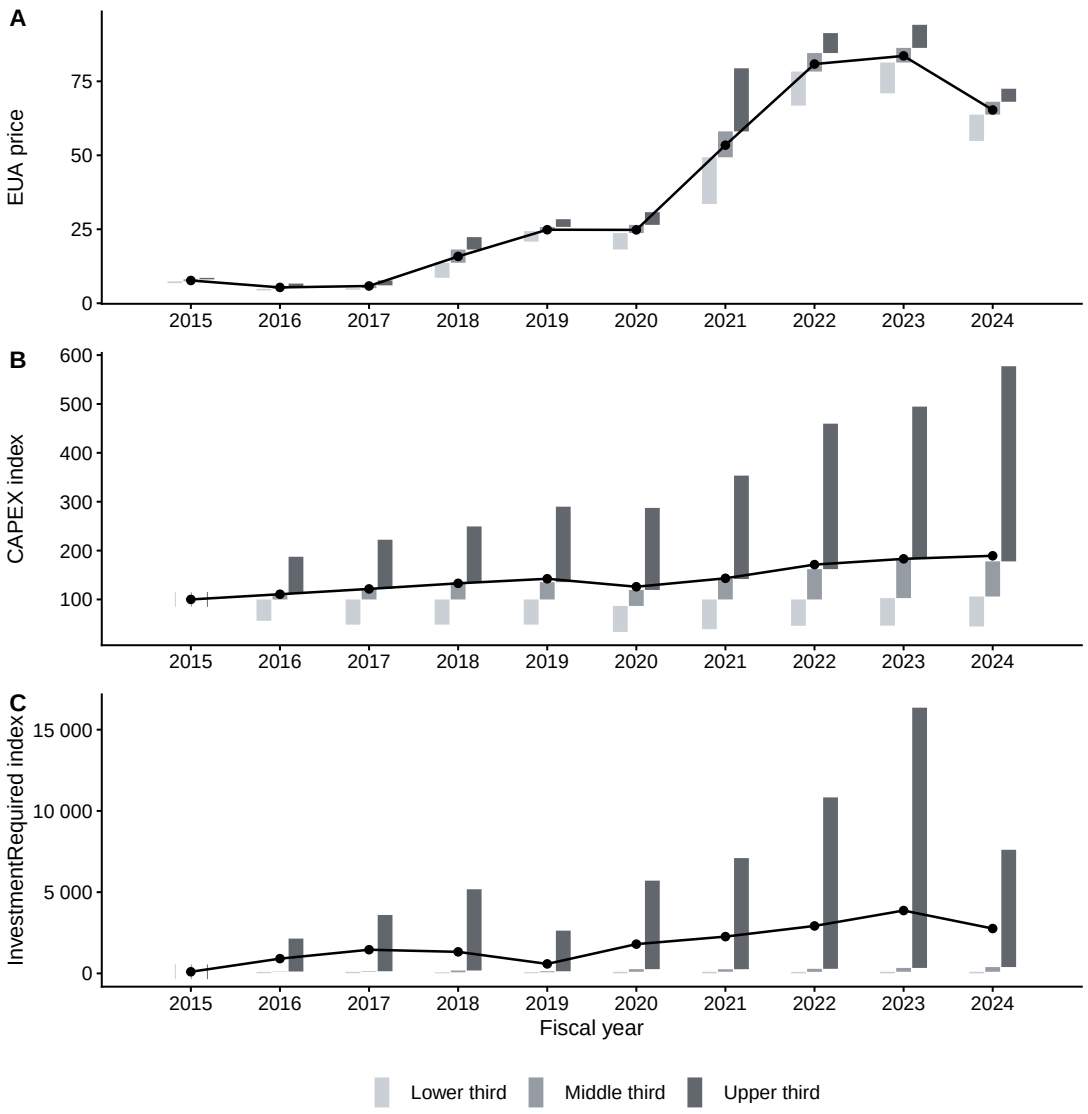


Figure 1: Carbon prices and relative investment dynamics

Notes: The sample is the non-financial location-based EU ETS sample over fiscal years 2015–2024. Panel A plots the annual mean EUA price from the first continuation series. Panel B plots indexed CAPEX. Panel C plots indexed CDP *Investment Required*. Indexed outcomes set each firm's first positive observation in 2015–2024 to 100 and are winsorized at the 1st and 99th percentiles for plotting. Line and point series show group-year means. Missing CDP values are not filled with zero.

path of high-carbon-intensity firms is lower in later years, while CAPEX-index and investment-rate differences vary over time. The observed heterogeneity indicates that investment dynamics differ systematically across exposure groups.

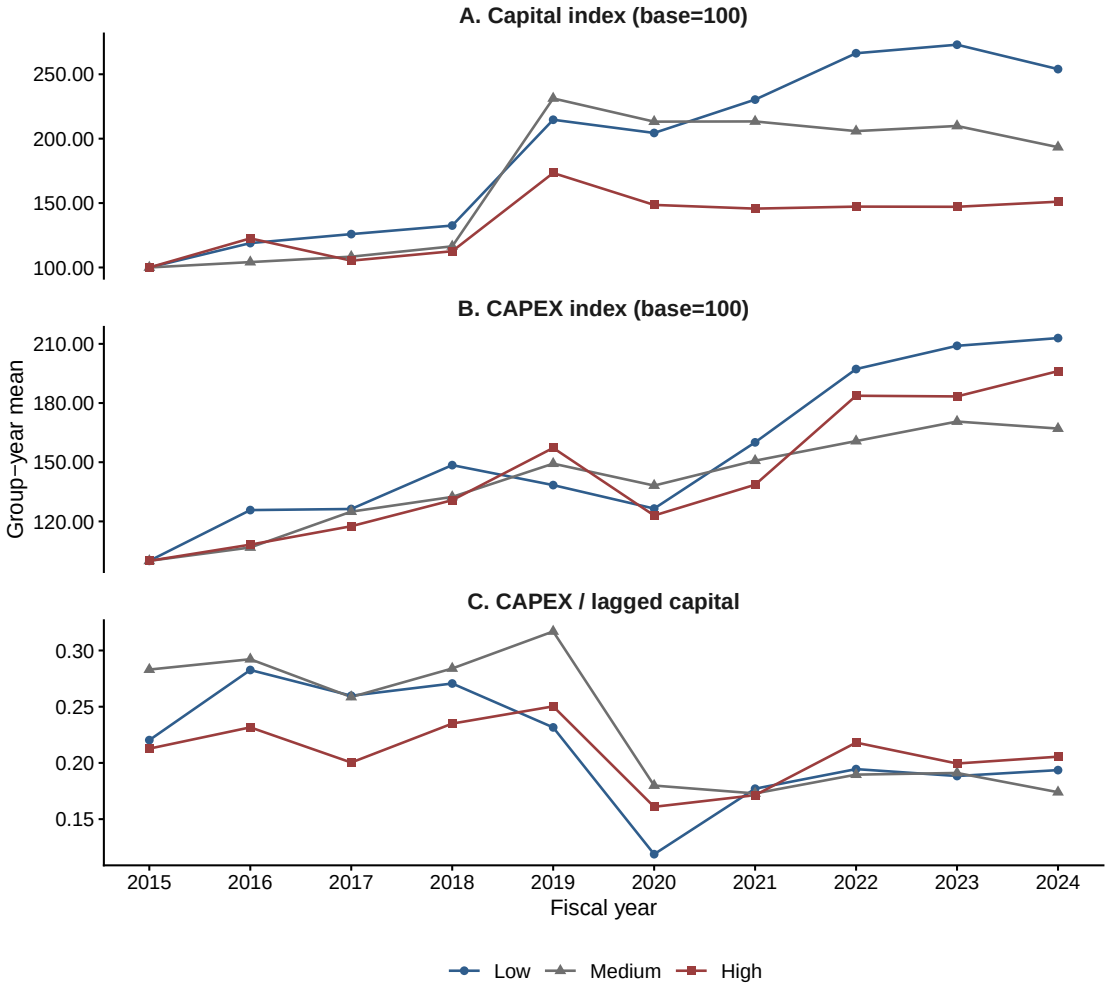


Figure 2: Investment and capital dynamics by carbon-intensity tercile

Notes: The sample is the non-financial location-based EU ETS sample over fiscal years 2015–2024. In each year, firm-years with available lagged carbon intensity are sorted into low, medium, and high terciles. Lagged carbon intensity is Scope 1 plus Scope 2 emissions divided by lagged net PPE. Panels A and B plot indexed capital and CAPEX. Panel C plots CAPEX divided by lagged net PPE. Indexed variables set each firm's first positive observation in 2015–2024 to 100 and are winsorized at the 1st and 99th percentiles for plotting. Line and point series show group-year means.

Figure 3 relates annual EUA prices to high-minus-low gaps between high- and low-carbon-intensity firms. Panel A shows a negative relation: in higher-price years, high-carbon-intensity firms have lower relative capital accumulation. Panel C shows the opposite pattern for the investment rate: high-carbon-intensity firms invest more relative to lagged capital in higher-price years. Panel B is less patterned, indicating that indexed CAPEX levels reflect scale, timing, and macroeconomic forces beyond carbon prices alone. The contrast across panels is informative. Higher carbon prices

are associated with weaker capital-stock growth among more exposed firms, but not necessarily with identically lower investment flows relative to the existing capital stock. This is why the empirical analysis distinguishes capital accumulation, total-investment intensity, and transition-observation moments.

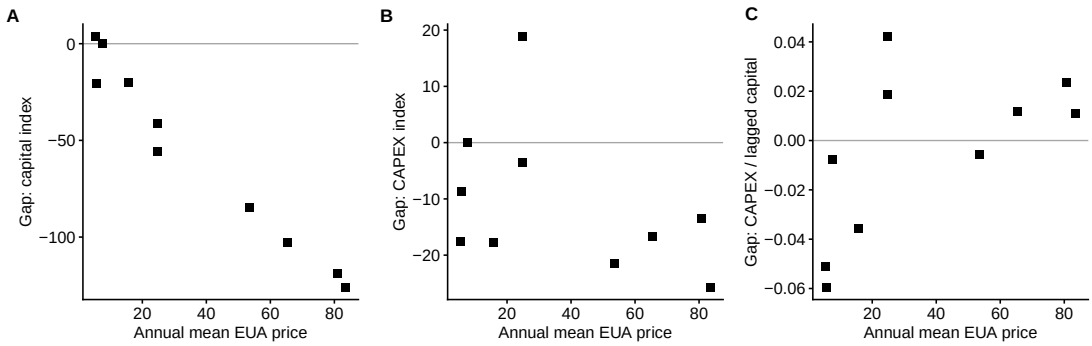


Figure 3: Carbon prices and high-minus-low carbon-intensity gaps

Notes: Each point in the figure is one year. The x-axis is the annual mean EUA price from the first continuation series. The y-axis is the high-minus-low gap in group-year means, where high and low groups are within-year terciles of lagged carbon intensity. Panel A uses the capital index from Figure 2, Panel B uses the CAPEX index from Figure 2, and Panel C uses CAPEX divided by lagged net PPE. The figure is descriptive and not causal.

Table 3: Descriptive comparison by baseline carbon-intensity tercile

	Low			Medium			High			
Variable	N	Mean	Median	N	Mean	Median	N	Mean	Median	H–L mean
Investment rate	847	0.2483	0.1733	1,079	0.2242	0.1751	1,119	0.1758	0.1452	-0.0725
Investment required / capital	616	0.0014	0.0001	880	0.0020	0.0001	998	0.0059	0.0004	0.0045
Carbon-cost exposure / sales	1,066	0.0004	0.0000	1,284	0.0008	0.0004	1,361	0.0157	0.0055	0.0152
Operating profitability / capital	1,099	0.1218	0.1122	1,326	0.1165	0.1075	1,385	0.1049	0.0982	-0.0169
Book leverage	1,100	0.2525	0.2250	1,327	0.2785	0.2763	1,388	0.3065	0.2938	0.0540
Cash / assets	1,100	0.1270	0.0891	1,327	0.1132	0.0955	1,388	0.1040	0.0906	-0.0230
Tangibility	1,100	0.1630	0.1195	1,327	0.2056	0.1594	1,388	0.3661	0.3557	0.2031

The sample is the non-financial location-based EU ETS sample over 2015–2024. Low, medium, and high groups are terciles of each firm's first available total emissions intensity, defined as Scope 1 plus Scope 2 emissions divided by capital within this restricted sample. The H–L mean column reports high minus low group mean differences.

Table 3 shows that high-carbon-intensity firms differ from low-carbon-intensity firms along both exposure and financing dimensions. Their average carbon-cost exposure relative to sales is much larger, and their reported required transition investment relative to capital is several times the low group's mean. Meanwhile, high-intensity firms have lower investment rates, lower operating profitability, higher leverage, lower cash holdings, and higher tangibility. These differences are consistent with the model's central mechanism. High-exposure firms have stronger incentives to reduce dirty-capital exposure, but they also have lower internal profitability, lower cash holdings, and higher leverage. Although these firms have higher tangibility, the empirical analysis asks whether their carbon-intensive physical assets remain fully collateralizable under carbon pricing.

4.3 Euler-root SMM

The analytical Euler equations allow for an SMM based on Euler roots. This keeps the structure of the model while avoiding the need to solve the full dynamic program. The simulated transition margin is therefore generated by the model's fixed-point restriction, not by an auxiliary reduced-form policy rule.

For firm i in year t, write the empirical analogue of the combined Euler residual as

$$\mathcal{E}_{i,t}(z;\mathbf{\Sigma}_{i,t},\theta)=\phi_{i,t}+\mathbb{E}_t\Big[M_{t,t+1}\widetilde{\mathcal{C}}_{i,t+1}(z;\mathbf{\Sigma}_{i,t},\theta)\Big]-(1+\rho_{i,t})\Big[1+(1-\tau^c_{i,t})a^{tr}z\Big], \tag{55}$$

where z is a candidate transition-investment intensity, $\mathbf{\Sigma}_{i,t}$ collects predetermined firm states, and θ denotes the structural parameter vector. The estimator solves

$$\mathcal{E}_{i,t}(z;\mathbf{\Sigma}_{i,t},\theta)=0 \qquad \Rightarrow \qquad x^{tr,*}_{i,t}(\theta)=z. \tag{56}$$

Given this transition root, optimal total investment rate is recovered from Equation (36):

$$x^*_{i,t}(\theta)=x^{tr,*}_{i,t}(\theta)+\mathcal{V}^K_{i,t}(\theta).$$

The SMM estimator chooses parameters to match moments of the observed panel and the simulated panel:

$$\widehat{\theta}=\arg\min_{\theta\in\Theta}\Big[m^{data}-m^{sim}(\theta)\Big]'W\Big[m^{data}-m^{sim}(\theta)\Big]. \tag{57}$$

The moment vector is organized around the model-implied investment rules generated by the Euler-root structure. Total-investment moments target $CAPEX/K_{t-1}$, its year and regime dynamics, and its relationship with profitability. Transition-observation moments target key features of CDP *Investment Required* disclosures, including the incidence of positive required investment, median within-firm required-investment indexes, growth in required investment among continuing reporters, and high-minus-low carbon-intensity gaps. I also include collateral-channel moments to discipline the model's core mechanism of limited collateralizability. These moments use leverage, liquidity, tangibility, and finance-conditioned carbon-price interactions to identify the composite collateral wedge $\phi_{i,t}$. The moment choices therefore stem from the same economic objects as the model: operating profitability, dirty-capital exposure, collateral capacity, and the nested allocation between ordinary and transition uses.

Although the Euler equations could in principle be estimated directly by GMM, I do not use direct Euler-GMM as the baseline for two reasons. First, the transition-investment control is not directly observed. Treating CDP *Investment Required* as realized annual $I^{tr}_{i,t}$ would put a noisy disclosure object directly inside the Euler residual. Second, *Investment Required* has an extreme right tail. In the estimation sample, the observed $IRIndex$ has a 95th percentile of 82.363, a mean of 243.645, skewness of 22.457, and a top-one-percent contribution of 0.935. These moments are not natural targets for a smooth annual Euler control. I therefore keep them as diagnostics and target robust features of the transition-observation layer.

The collateral wedge is estimated as a composite object. In the baseline specification, $\phi_{i,t}$ is parameterized directly using centered carbon-price, emissions-intensity, tangibility, and financing-pressure variables. I do not separately identify $\lambda_{i,t}$ and $\mu_{i,t}$ in the baseline because the collateral wedge is the object that enters the optimality conditions. This deliberate restriction reflects the fact that the data and moments are more credible for disciplining the composite collateral wedge than for separately recovering the collateral ratio and the borrowing-constraint multiplier. The empirical design therefore retains the logic of Euler-equation estimation but implements it through SMM.

4.4 Main Results

The selected specification has the lowest objective among admissible candidate specifications and satisfies the key economic and numerical diagnostics. This selection criterion is crucial because a low SMM objective alone is not sufficient for structural credibility. The selected specification also produces stable interior roots for all 2,794 firm-years, has no boundary parameters, retains the collateral block, and has a full-rank local Jacobian at the reported tolerances. These diagnostics support the Euler-root implementation as an empirical restriction rather than only as a theoretical construction.

For sampling uncertainty, I use a firm-block full re-estimation bootstrap with 300 replications.[2] All 300 replications converge. Across the bootstrap distribution, the mean interior-root share is 100%, the maximum boundary share is 0%, and the maximum near-boundary share is 0%. The original objective is 1.1174, and the bootstrap median objective is 1.4292. These results support local sampling-stability inference for the selected specification.

The estimates shown in Table 4 support the main economic structure of the model. The transition-adjustment-cost parameter is much larger than the ordinary-adjustment-cost parameter: $a^{tr} = 24.221$ compared with $a^{o} = 6.475$. The ratio a^{tr}/a^{o} is 3.741, and the bootstrap probability that $a^{tr} > a^{o}$ is 100%. This is a strong inference result for the paper's first empirical implication: transition investment is not a simple relabeling of ordinary investment. It is a more adjustment-costly margin. This supports the interpretation that transition investment is organizationally costly and difficult to scale smoothly. It also connects directly to Equation (28): transition investment does not need to exceed ordinary investment in levels to be economically important, because the two margins have different adjustment-cost slopes.

The estimated transition-effectiveness parameter is $\chi = 0.171$, with bootstrap probability $Pr(\chi > 0) = 100\%$. In the implemented transition law, this positive value means that transition investment reduces future dirty-capital intensity, but the effect is limited. This provides a potential explanation for transition inertia through low transition-investment efficiency. Even when firms undertake transition-oriented investment, capital composition changes gradually.

The mean local value of $\mathcal{S}_{tr}$ is negative, which is the sign required for transition investment to

[2]Each replication resamples firms with replacement, retains each sampled firm's full time series, recomputes the empirical moments, re-solves the Euler roots, reconstructs model-implied choices, and locally re-estimates the selected SMM specification.

Table 4: Selected Euler-root SMM Parameters with Bootstrap Inference

Parameter	Estimate	Boot. s.e.	95% boot. CI	5–95% range	Sign prob.	Boundary	Identifying block
Panel A. Adjustment costs and transition technology							
a^o	6.475	0.174	[6.149, 6.815]	[6.161, 6.805]	100.0%	0.0% / 0.0%	Euler cost scale
a^{tr}	24.221	0.938	[21.984, 25.435]	[22.214, 25.435]	100.0%	0.0% / 0.0%	Transition Euler
χ	0.171	0.011	[0.152, 0.189]	[0.155, 0.187]	100.0%	0.0% / 0.0%	Dirty-capital transition
$a^{tr} - a^o$	17.747	0.994	[15.370, 19.274]	[15.618, 18.960]	100.0%	0.0% / 0.0%	Derived restriction
a^{tr}/a^o	3.741	0.193	[3.272, 4.127]	[3.351, 3.941]	100.0%	0.0% / 0.0%	Derived restriction
Panel B. Carbon and transition-benefit parameters							
b_P	2.097	0.072	[1.938, 2.207]	[1.966, 2.177]		0.0% / 0.0%	Transition-benefit block
b_S	4.046	0.086	[3.886, 4.217]	[3.945, 4.183]		0.0% / 0.0%	Transition-benefit block
Panel C. Composite collateral wedge index							
ϕ_0	-3.280	0.039	[-3.330, -3.147]	[-3.292, -3.164]		0.0% / 0.0%	Collateral wedge
ϕ_T	-1.215	0.064	[-1.353, -1.109]	[-1.335, -1.140]		0.0% / 0.0%	Collateral wedge
ϕ_P	-1.215	0.065	[-1.408, -1.143]	[-1.361, -1.143]		0.0% / 0.0%	Collateral wedge
ϕ_S	-1.118	0.065	[-1.279, -1.014]	[-1.245, -1.046]		0.0% / 0.0%	Collateral wedge
ϕ_F	-0.915	0.071	[-1.046, -0.784]	[-1.024, -0.820]		0.0% / 0.0%	Collateral wedge
ϕ_{PF}	-1.115	0.044	[-1.271, -1.086]	[-1.262, -1.115]		0.0% / 0.0%	Collateral wedge
ϕ_{SF}	0.910	0.050	[0.849, 1.090]	[0.899, 1.066]		0.0% / 0.0%	Collateral wedge
Panel D. Regime shifter: 2020/post-2020 and payout/internal-funds wedges							
ψ_{2020}	-0.076	0.003	[-0.080, -0.071]	[-0.079, -0.071]	100.0%	0.0% / 0.0%	Regime shifter
ψ_{post}	-0.044	0.002	[-0.048, -0.041]	[-0.048, -0.041]	100.0%	0.0% / 0.0%	Regime shifter
ρ_0	-0.360	0.086	[-0.470, -0.153]	[-0.456, -0.183]		0.0% / 0.0%	Payout/internal funds

Notes: The table reports the selected Euler-root SMM specification. Bootstrap standard errors are firm-block full re-estimation bootstrap standard deviations across 300 replications. Percentile intervals use the 2.5th and 97.5th percentiles. Boundary reports boundary / near-boundary shares, where near-boundary means within one percent of the admissible parameter interval. Sign probabilities report the probability that economically meaningful one-sided restrictions hold across bootstrap replications.

lower the future dirty-capital share. The mean $\mathcal{C}^s$ is also negative, consistent with the economically relevant region in which a higher dirty-capital share lowers continuation value. These results support the maintained sign restrictions used in the theoretical analysis.

The local Euler-slope diagnostic is also consistent with stable roots. The mean and median local slope are both equal to -18.406. This is not a full decomposition of every slope component in Equation (45), but it is informative. The current transition-cost slope dominates locally on average, so the simulated roots are not generated by corner classifications. Echoing the theory, the estimated firm states are in a locally stable region rather than mechanically on the uniqueness–multiplicity boundary.

Figure 1 shows a sharp drop in firms' investment intensity in 2020. I therefore include regime shifters to absorb broad investment shocks in and after 2020, including COVID-19 disruptions, the energy crisis, and broader post-2020 macroeconomic shocks. The estimates of the 2020 and post-2020 total-investment shifters are negative, $\psi_{2020} = -0.076$ and $\psi_{post} = -0.044$. The bootstrap probability that each is negative is 100%. These parameters are the controls that prevent the estimator from mechanically attributing broad post-2020 investment shifts to carbon prices or collateral wedges.

Model fit. The selected model fits the total-investment moments reasonably well. The mean $CAPEX/K_{t-1}$ is 0.210 in the data and 0.184 in the simulation, with a standardized error of -0.820. The persistence moment is 0.697 in the data and 0.628 in the simulation. The model underpredicts the lower tail of investment: the 25th-percentile moment is 0.103 in the data and

0.152 in the simulation, with a standardized error of 1.973. This mismatch is visible, but it is not the dominant source of objective loss.

The profitability moments are economically central because they discipline the total-investment Euler. The correlation between investment and profitability is 0.360 in the data and 0.496 in the simulation. The rank-slope profitability moment is 0.066 in the data and 0.020 in the simulation. These results suggest that the model preserves the profitability-investment relation in broad terms, although it understates the rank-slope magnitude. Recovering this operating rule is necessary before interpreting the collateral wedge, because the financing channel is identified relative to the model-implied operating response.

Table 5: Selected moment fit

Moment	Data	Simulation	Std. error	Block
Mean $CAPEX/K_{t-1}$	0.210	0.184	−0.820	Investment
Persistence of $CAPEX/K_{t-1}$	0.697	0.628	−0.859	Investment
P25 of $CAPEX/K_{t-1}$	0.103	0.152	1.973	Investment
Investment-profitability correlation	0.360	0.496	1.701	Profitability
Rank-slope profitability	0.066	0.020	−1.818	Profitability
Median IRIndex 2024	1.400	0.990	−2.444	Transition
Median IRIndex 2023	1.335	1.000	−2.092	Transition
High-s, high-P, high-FP IR gap	−0.031	0.026	2.866	Financing
High-CI high-P IR gap	−0.032	0.012	2.232	Carbon exposure
Finance-pressure median IR gap	−0.233	−0.014	2.189	Financing

Notes: IRIndex denotes the within-firm indexed CDP *Investment Required* measure. FP denotes financing pressure. The table reports selected targeted moments rather than the full moment vector.

The remaining moment errors are also informative. In the data, high-carbon firms in high-price states do not mechanically report more positive required investment, especially when financing pressure is high. This pattern is consistent with the countervailing mechanism in the theory. Carbon pricing raises the future value of reducing dirty-capital exposure, but the same firms may face current financing pressure that dampens transition investment.

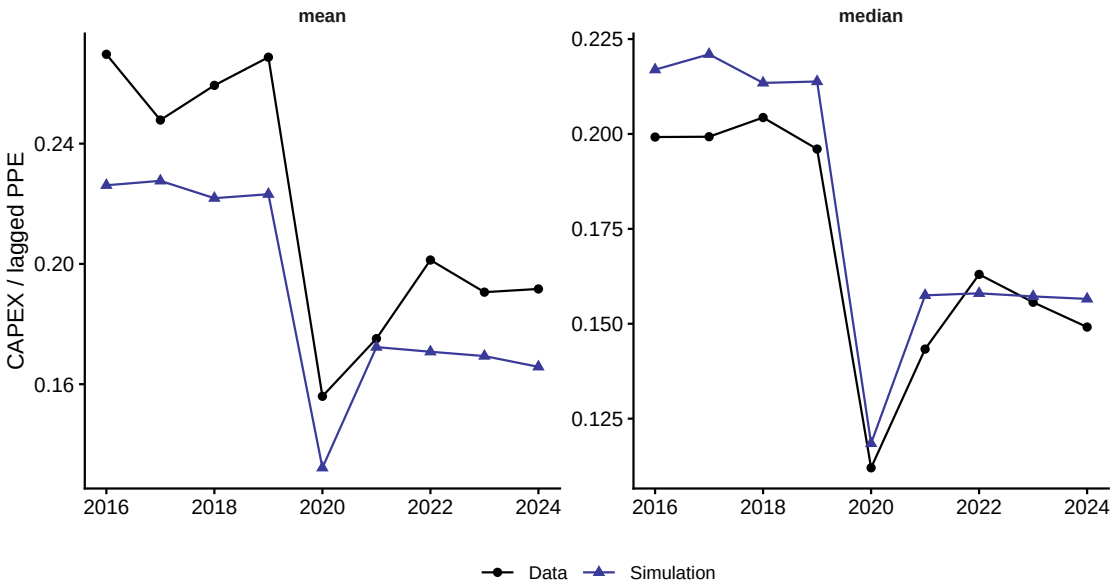


Figure 4: Model fit for total investment intensity

Notes: The figure compares annual mean and median $CAPEX/K_{t-1}$ in the data and in the simulated panel.

4.5 The Limited Collateralizability Channel

The limited-collateralizability channel is the main mechanism that distinguishes this model from a standard carbon-cost investment model. The evidence supports including a collateral wedge, but the interpretation should remain at the composite-wedge level. The selected direct-ϕ specification fits better than specifications that remove the collateral block or separately decompose λ and μ. [3]

Table 6: Collateral specification comparison

Specification	Objective	Root behavior	Collateral block
Selected direct ϕ	1.117	100% interior stable	Included
No collateral block	1.615	100% interior stable	Excluded
Reduced ϕ interactions	1.749	100% interior stable	Included
Finance-pressure ϕ	1.880	100% interior stable	Included
Decomposed $\lambda\mu$	3.004	100% interior stable	Included

Notes: The table reports selected collateral specifications from the SMM 2.2 ladder. The selected specification parameterizes the composite collateral wedge directly.

This comparison supports the financing channel at the level of the composite wedge, although it does not imply that the data separately identify collateralizability and the borrowing-constraint multiplier. A disciplined interpretation is that the current empirical design is informative about whether a financing wedge induced by limited collateralizability improves the joint fit of total in-

[3] In objective terms, the selected specification has objective 1.117. The no-collateral specification has objective 1.615, and the decomposed $\lambda\mu$ specification has objective 3.004.

vestment intensity, profitability, and transition-observation moments. A cleaner separation between λ and μ would require stronger measurement or additional moments.

The signs of the local diagnostics are nevertheless economically meaningful. The mean collateral wedge is positive, the mean composition continuation value is negative, and transition investment lowers the future dirty-capital share on average. These signs place the estimated model in the region emphasized by the theory: dirty capital creates future exposure, transition investment reduces that exposure, and collateral capacity affects the investment rule through the financing wedge.

4.6 Diagnostics and External Validation

To justify the empirical results and discipline the interpretation scope, the diagnostics are necessary. First, the root diagnostics are strong. The selected specification produces interior stable roots for every estimation firm-year, and the 300-replication bootstrap preserves this property. Second, bootstrap inference is credible, but it remains local. The bootstrap uses local re-estimation of the selected specification rather than a global model-selection bootstrap. Third, validation using future emissions intensity is directionally supportive. Firms in the high model-implied transition tercile experience a future Scope 1 emissions-intensity decline of 0.078, compared with 0.037 for firms in the low tercile. For Scope 1 plus Scope 2 intensity, the corresponding values are 0.095 and 0.061. These patterns suggest that the model-implied $x^{tr,*}$ contains information about subsequent reductions in emissions intensity. I therefore interpret $x^{tr,*}$ as a structurally disciplined proxy for transition-investment pressure, rather than as a direct measure of realized transition CAPEX.

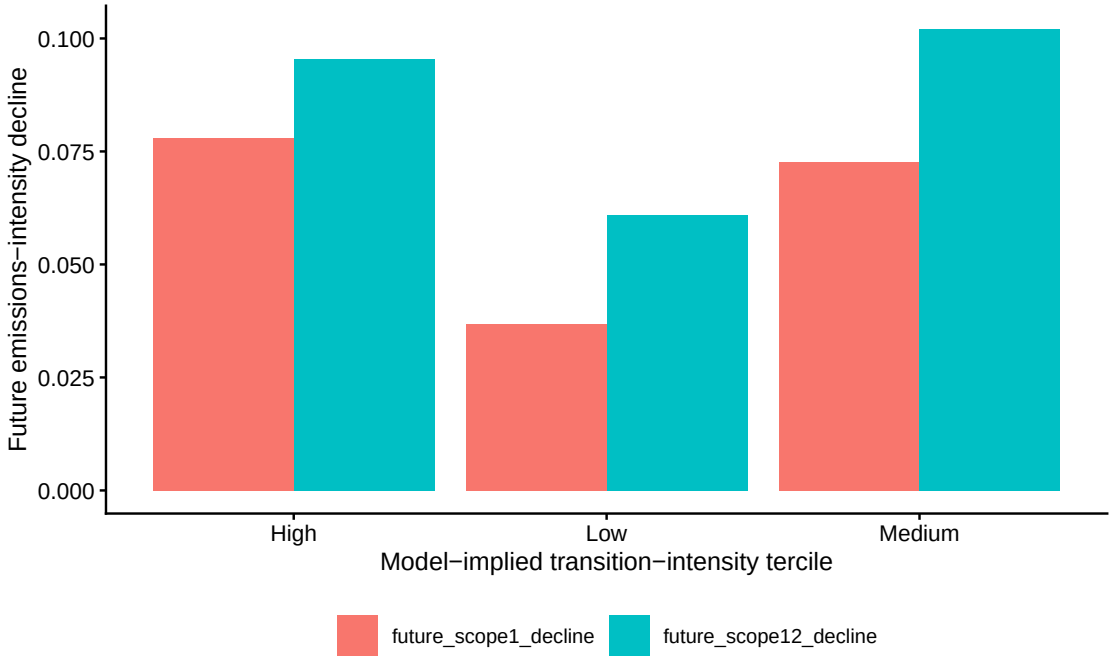


Figure 5: External validation using future emissions-intensity changes

Notes: The figure compares future emissions-intensity changes across terciles of model-implied transition investment.

The empirical conclusion can be stated in a disciplined way: the model that separates total investment from transition allocation, embeds capital composition in operating and collateral values, and treats CDP required investment as a noisy observation-layer object can reproduce many robust investment and transition moments in the data. The strongest evidence is for the Euler-root structure, the high adjustment cost of transition investment, gradual transition effectiveness, and the usefulness of a stable composite collateral wedge. The limited weaker evidence concerns the separate structural interpretation of λ and μ and the raw right tail of reported required investment. Overall, this identified pattern is consistent with the central mechanism of our paper: carbon pricing raises the future value of reducing dirty-capital exposure, but current financing conditions can dampen the investment needed to do so.

4.7 Next-step Analysis

The current estimates establish the baseline analysis. The next step is to use this Euler-root SMM baseline to study where the capital-composition mechanism is strongest and how investment changes under alternative policy and financing environments. This requires three extensions: additional SMM-specific robustness checks, heterogeneity analysis, and model-based counterfactuals.

Robustness checks. The robustness analysis should be framed around the Euler-root SMM estimator. The current implementation already provides evidence of local sampling stability for the selected specification: all bootstrap replications converge, all Euler roots remain interior and stable, and no boundary or near-boundary estimates arise. The next step is to test whether the main economic conclusions remain stable under alternative SMM implementations. In particular, I will examine alternative moment sets and alternative treatments of the transition-observation layer.

A second robustness step is to use nested SMM specifications as an exclusion-style diagnostic for the collateral-based financing channel. In the Euler-root SMM framework, the analogue of an exclusion test is not a single test statistic but a comparison of nested structural specifications and moment-block fit. The key question is whether the collateral block improves the model's ability to match financing-state and carbon-exposure moments without generating boundary estimates, unstable roots, or weaker external validation. I will therefore compare the baseline model with an operating-channel-only specification, a no-collateral benchmark, reduced and richer specifications of the composite collateral wedge, and a decomposed specification that attempts to separate collateralizability from the shadow value of the collateral constraint.

Heterogeneity analysis. Heterogeneous investment responses to carbon pricing are the motivating stylized fact of the paper. A natural next step is therefore to examine where the capital-composition mechanism is strongest. The baseline model predicts that carbon-intensive firms have a stronger incentive to reduce dirty capital, but that this incentive may be dampened when financing pressure is high. The key heterogeneity test is therefore not only whether high-carbon firms invest differently, but whether the model-implied transition response varies across the joint distribution of

carbon exposure and financing pressure. The first step is to use the baseline estimates to compute model-implied objects by group:

$$x_{i,t}^{tr,*}, \qquad x_{i,t}^{*}, \qquad x_{i,t}^{o,*} = x_{i,t}^{*} - x_{i,t}^{tr,*}, \qquad \omega_{i,t}^{tr,*}, \qquad \phi_{i,t}, \qquad \mathcal{C}_{i,t}^{K}, \qquad \mathcal{C}_{i,t}^{s}.$$

I will compare these objects across carbon-intensity terciles, financing-pressure terciles, tangibility groups, carbon-price regimes, sectors, and high-carbon/high-price/high-financing-pressure cells. This approach keeps the estimated structural model fixed and uses it to locate where the countervailing mechanism is quantitatively strongest. It is a feasible first step before repeated subsample Euler-root SMM estimation, which can become unstable when sample size and moment information are limited.

A further heterogeneity exercise will compare model-implied and observed moments within each group. The relevant empirical objects include observed total investment intensity $CAPEX_{i,t}/K_{i,t-1}$, the incidence of positive CDP *Investment Required*, median $IRIndex$, and future emissions-intensity declines. This group-level validation asks whether the baseline model fits high-exposure and financially constrained firms as well as it fits the average firm. If this step is informative, I will consider more demanding specifications that allow selected parameters to vary by firm state, such as group-specific a^{tr}, transition effectiveness χ, or selected ϕ-parameters. These extensions can reveal whether adjustment costs, transition effectiveness, or collateral-wedge mechanisms differ across firms. They can also help assess whether heterogeneous firms are located in different uniqueness–multiplicity regions, which may be useful to understand the heterogeneous transition inertia across various firm characteristics.

Counterfactual analysis. The advantage of the structural approach is that counterfactuals can be constructed by changing primitives and re-solving the Euler-root problem. The regime shifters should also be handled explicitly in the counterfactual design, so that investment changes are not mechanically attributed to carbon pricing when they reflect broader post-2020 investment shocks. For each counterfactual environment, I plan to solve

$$\mathcal{E}_{i,t}\big(z; \Sigma_{i,t}, \widehat{\theta}, \mathrm{cf}\big) = 0 \tag{58}$$

and recover the counterfactual transition-investment rule, total-investment rule, transition share, collateral wedge, and next-period dirty-capital state:

$$x_{i,t,\mathrm{cf}}^{tr,*}, \qquad x_{i,t,\mathrm{cf}}^{*}, \qquad \omega_{i,t,\mathrm{cf}}^{tr,*}, \qquad \phi_{i,t,\mathrm{cf}}, \qquad s_{i,t+1,\mathrm{cf}}.$$

The first set of counterfactuals will vary the carbon-price path, for example by imposing a one-standard-deviation increase in P_t, a 25 or 50 percent increase, or a smoother price path with the same average level. These exercises will report both total investment and transition investment rates. This distinction is essential because a higher carbon price can raise the value of transition investment while also tightening financing conditions for dirty-capital firms.

Given the central role of limited collateralizability, a core counterfactual is to shut down or weaken the collateral deterioration associated with dirty capital. This exercise compares the baseline economy with an economy in which the carbon- and dirty-capital sensitivities of the composite collateral wedge are removed or reduced. It directly quantifies the incremental role of the financing channel. If transition investment rises when the collateral penalty is relaxed, especially among high-exposure and high-financing-pressure firms, the result would support a potential transition-trap mechanism: firms may have valuable transition opportunities but insufficient collateral capacity to implement them. A related policy counterfactual is to improve collateralizability, either by reducing the financing penalty attached to dirty capital or by increasing collateral support for transition investment. This exercise maps naturally to transition-finance policies that relax balance-sheet constraints without directly changing the carbon price.

A final set of counterfactuals focuses on transition technology. Reducing a^{tr} captures lower implementation costs, cleaner technology standardization, subsidies, or learning-by-doing. Increasing χ captures higher transition effectiveness, meaning that a given amount of transition investment reduces the future dirty-capital share more strongly. These counterfactuals speak to policy design beyond the carbon price. They ask whether transition inertia is primarily due to weak price incentives, high transition adjustment costs, limited transition effectiveness, or financing constraints.

Overall, these next steps are intended to move from baseline estimation to mechanism quantification. Based on the current Euler-root framework, which is empirically viable and locally stable, the next empirical task is to determine which firms are closest to the transition-trap mechanism, whether the collateral channel materially changes investment allocation, and how alternative carbon-price paths and transition-finance policies affect total investment, transition investment, and the transition share.

5 CONCLUSION

This paper studies how carbon pricing affects firm investment when transition requires changes in capital composition. The central mechanism is that carbon pricing revalues dirty capital. Dirty capital lowers operating value because it generates current and future emissions costs. It can also reduce collateral capacity when lenders view carbon-intensive assets as exposed to transition risk and asset stranding. Carbon pricing therefore affects investment through two connected channels: an operating channel and a collateral-based financing channel.

I develop a dynamic model that distinguishes total investment, transition investment, and the transition share of investment. This distinction allows the model to capture the different responses of investment margins to carbon exposure. The model shows that carbon pricing has countervailing effects. It raises the future value of reducing dirty-capital exposure, but it can also tighten current financing capacity when existing dirty capital becomes less collateralizable. Transition investment is characterized by a recursive self-consistent allocation rule. Theoretically, multiple self-consistent rules can arise within the same state when the future benefits of improving capital composition are strong enough to dominate the current cost discipline. This multiplicity provides a model-based

interpretation of a potential transition trap, even though the estimated sample is locally stable on average and does not mechanically lie on the uniqueness–multiplicity boundary.

The Euler-root SMM estimates support the main structure. The selected specification produces stable interior roots for all estimation firm-years, and a 300-replication firm-block bootstrap supports local sampling stability. The transition-adjustment-cost parameter is substantially larger than the ordinary-adjustment-cost parameter, implying that transition investment is a more adjustment-costly margin. The transition-effectiveness parameter is positive but limited, implying gradual changes in capital composition. Specifications with the composite collateral wedge improve the joint fit of total investment, profitability, financing states, and transition-observation moments. This evidence supports the financing channel at the level of the composite wedge, which is the key mechanism of the paper: limited collateralizability can alter investment margins by changing the collateral value of dirty capital.

These findings imply that stronger carbon-price incentives do not mechanically translate into higher transition investment. When dirty capital weakens collateral capacity, firms with valuable transition opportunities may lack the financing capacity to implement them. This provides a dynamic-investment interpretation of transition inertia and suggests a role for transition-finance policies that improve collateral capacity or relax balance-sheet constraints. The next step is to study heterogeneity across carbon exposure, financing pressure, tangibility, and sectors, and to use counterfactuals to quantify how investment responds to alternative carbon-price paths and transition-finance policies. The framework therefore shows how carbon pricing, capital composition, asset stranding, and investment allocation can be studied within a single dynamic investment problem.

A Proofs and Derivations

A.1 Existence and interiority of the transition-investment rule

This appendix provides mathematical details behind the existence statement in the main text. The discussion is organized around the application of the Intermediate Value Theorem (IVT) to the Euler residual.

Assumption 1 (Feasibility and Continuity)**.** *Hold an admissible current state $\mathbf{\Sigma}_t$ and an admissible total-investment intensity $x_t \in (0, \infty)$ fixed. Assume:*

(i) The feasible transition-investment intensity set is the nonempty compact interval:

$$z \in [0, x_t]$$

(ii) The continuation-value term is finite and continuous in the candidate transition-investment intensity:

$$z \mapsto \mathbb{E}_t\left[M_{t,t+1}\widetilde{\mathcal{C}}_{t+1}\left(z; \mathbf{\Sigma}_t, \varepsilon^s_{t+1}\right)\right] \quad \text{is continuous on} \quad [0, x_t].$$

Assumption 2 (Well-definedness)**.** *The current marginal-cost coefficient of z is well defined and strictly positive:*

$$(1 + \rho_t)(1 - \tau^c_t)a^{tr} > 0\,; \tag{59}$$

Assumption 3 (Intermediate-value endpoint conditions)**.** *For the same current state $\mathbf{\Sigma}_t$ and feasible interval $[0, x_t]$, the Euler residual satisfies*

$$\mathcal{E}_t(0; \mathbf{\Sigma}_t) \geq 0, \qquad \mathcal{E}_t(x_t; \mathbf{\Sigma}_t) \leq 0.$$

For an interior root, these endpoint inequalities are strict:

$$\mathcal{E}_t(0; \mathbf{\Sigma}_t) > 0, \qquad \mathcal{E}_t(x_t; \mathbf{\Sigma}_t) < 0.$$

Proposition 3 (Existence of an Euler root and recursive fixed point)**.** *Suppose Assumptions 1 to 3 hold. Then there exists $x^{tr*}_t \in [0, x_t]$ such that*

$$\mathcal{E}_t(x^{tr*}_t; \mathbf{\Sigma}_t) = 0.$$

If the endpoint inequalities in Assumption 3 are strict, then there exists an interior root $x^{tr*}_t \in (0, x_t)$. The Euler root is equivalent to the fixed-point

$$\mathcal{E}_t(x^{tr*}_t; \mathbf{\Sigma}_t) = 0 \quad \Longleftrightarrow \quad x^{tr,*}_t = F_t(x^{tr*}_t; \mathbf{\Sigma}_t).$$

Proof. Fix $(\mathbf{\Sigma}_t, x_t)$. By Assumption 1-(ii), since the remaining terms in $\mathcal{E}_t(z; \mathbf{\Sigma}_t)$ are affine in z, the Euler residual is continuous on the feasible interval.

Assumption 3 implies that zero lies between the two endpoint values, if either endpoint value is zero, the corresponding endpoint is an Euler root. Otherwise,

$$\mathcal{E}_t(0;\mathbf{\Sigma}_t) > 0 > \mathcal{E}_t(x_t;\mathbf{\Sigma}_t),$$

and the IVT implies that there exists $x_t^{tr*} \in (0, x_t)$ such that

$$\mathcal{E}_t(x_t^{tr*};\mathbf{\Sigma}_t) = 0.$$

Under strict endpoint inequalities, the same argument directly gives an interior root. □

Remark 1 (Interpretation of the endpoint conditions)**.** *The endpoint conditions are restrictions on the net marginal benefit of transition allocation at the two ends of the feasible interval. Since the Euler residual can be written as*

$$\mathcal{E}_t(z;\mathbf{\Sigma}_t) = MB_t(z;\mathbf{\Sigma}_t) - MC_t(z;\mathbf{\Sigma}_t)\,, \tag{60}$$

the condition $\mathcal{E}_t(0;\mathbf{\Sigma}_t) \geq 0$ requires that the marginal benefit of initiating transition investment weakly exceeds its marginal cost. The condition $\mathcal{E}_t(x_t;\mathbf{\Sigma}_t) \leq 0$ requires that, at the upper allocation bound, the marginal cost of further transition allocation weakly exceeds its marginal benefit. These endpoint restrictions are economically reasonable to be imposed to apply the IVT.

A.2 Proof of the boundary tangency result

Proof of Proposition 1. Fix the current state $\mathbf{\Sigma}_t$. Let $\bar{\theta}$ be a primitive value on the boundary between the region with a unique interior Euler root and the region with multiple interior Euler roots. By assumption, at least one interior root exists at $\bar{\theta}$.

Suppose, to the contrary, that every interior root at $\bar{\theta}$ is simple. That is, for every $z \in (0, x_t)$ satisfying

$$\mathcal{E}_t(z;\mathbf{\Sigma}_t,\bar{\theta}) = 0,$$

we have

$$\mathcal{E}_{t,z}(z;\mathbf{\Sigma}_t,\bar{\theta}) \neq 0.$$

By the Implicit Function Theorem, each simple root can be locally continued as a continuously differentiable function of the primitive vector. More precisely, for each simple root z_i^*, there exist neighborhoods $\mathcal{N}_i^z$ of z_i^* and $\mathcal{N}_i^\theta$ of $\bar{\theta}$, and a continuously differentiable function $z_i(\theta)$, such that

$$z_i(\bar{\theta}) = z_i^*, \qquad \mathcal{E}_t(z_i(\theta);\mathbf{\Sigma}_t,\theta) = 0 \quad \text{for all} \quad \theta \in \mathcal{N}_i^\theta.$$

Moreover, this root is the unique solution in $\mathcal{N}_i^z$.

Simple roots are isolated, and hence the relevant interior roots are locally finite on compact subsets of the feasible interval. Since Proposition 1 concerns interior roots, changes in the number

of roots through the endpoints are excluded by the maintained interiority requirement. Therefore, in a sufficiently small neighborhood of $\bar{\theta}$, all relevant interior roots are covered by finitely many local neighborhoods of the form described above.

Taking the intersection of the corresponding parameter neighborhoods gives a neighborhood $\mathcal{N}^{\theta}$ of $\bar{\theta}$ on which every interior root continues uniquely and no new interior root appears locally. Hence the number of interior Euler roots is locally constant in $\mathcal{N}^{\theta}$.

This contradicts the fact that $\bar{\theta}$ lies on the boundary between a unique-interior-root region and a multiple-interior-root region. Therefore, at least one interior root at $\bar{\theta}$ must fail to be simple. Hence there exists $\bar{z} \in (0, x_t)$ such that

$$\mathcal{E}_t(\bar{z}; \mathbf{\Sigma}_t, \bar{\theta}) = 0, \qquad \mathcal{E}_{t,z}(\bar{z}; \mathbf{\Sigma}_t, \bar{\theta}) = 0.$$

This is the tangency condition in Equation (42). □

A.3 Regularity for local comparative statics

Assumption 4 (Local comparative-statics regularity)**.** *Fix a current state $\mathbf{\Sigma}_t$. Let θ denote a vector of primitives and state shifters. Assume:*

(i) The Euler residual $\mathcal{E}_t(z; \mathbf{\Sigma}_t, \theta)$ is continuously differentiable in (z, θ) on a neighborhood of the relevant interior root.

(ii) For each θ in the region of interest, there exists a unique interior solution $x_t^{tr*}(\theta) \in (0, x_t)$ satisfying

$$\mathcal{E}_t(x_t^{tr*}(\theta); \mathbf{\Sigma}_t, \theta) = 0.$$

(iii) At the interior solution,

$$\mathcal{E}_{t,z}(x_t^{tr*}(\theta); \mathbf{\Sigma}_t, \theta) \neq 0.$$

(iv) On the unique-solution region used in the main text,

$$\mathcal{E}_{t,z}(x_t^{tr*}(\theta); \mathbf{\Sigma}_t, \theta) < 0.$$

Proof of Proposition 2. The unique interior transition-investment rule satisfies

$$\mathcal{E}_t(x_t^{tr*}(\theta); \mathbf{\Sigma}_t, \theta) = 0.$$

By Assumption 4-(i), the Euler residual is continuously differentiable in (z, θ). By Assumption 4-(iii), its derivative with respect to the choice variable is nonzero at the root. The Implicit Function Theorem therefore implies that the root can be represented locally as a continuously differentiable function of θ. Differentiating the identity

$$\mathcal{E}_t(x_t^{tr*}(\theta); \mathbf{\Sigma}_t, \theta) = 0$$

with respect to the component θ_j gives

$$\mathcal{E}_{t,z}(x_t^{tr*}(\theta);\mathbf{\Sigma}_t,\theta)\frac{\partial x_t^{tr*}(\theta)}{\partial\theta_j}+\mathcal{E}_{t,\theta_j}(x_t^{tr*}(\theta);\mathbf{\Sigma}_t,\theta)=0.$$

Solving for the derivative gives

$$\frac{\partial x_t^{tr*}(\theta)}{\partial\theta_j}=-\frac{\mathcal{E}_{t,\theta_j}\left(x_t^{tr*}(\theta);\mathbf{\Sigma}_t,\theta\right)}{\mathcal{E}_{t,z}\left(x_t^{tr*}(\theta);\mathbf{\Sigma}_t,\theta\right)}.$$

This proves Equation (49).

For a^{tr}, suppose that the adjustment-cost parameter enters the Euler residual only through the current marginal-cost term

$$-(1+\rho_t)(1-\tau_t^c)a^{tr}z.$$

Then, at any interior root $z=x_t^{tr*}>0$,

$$\mathcal{E}_{t,a^{tr}}(z;\mathbf{\Sigma}_t)=-(1+\rho_t)(1-\tau_t^c)z<0.$$

By Assumption 4-(iv), $\mathcal{E}_{t,z}<0$ on the unique-solution region. Hence Equation (49) implies

$$\frac{\partial x_t^{tr*}}{\partial a^{tr}}<0.$$

This proves Equation (51).

It remains to verify the expressions for P_t, s_t, and η_λ. Write the Euler residual as

$$\mathcal{E}_t(z;\mathbf{\Sigma}_t,\theta)=\phi_t+\mathbb{E}_t\left[M_{t,t+1}\left(\mathcal{C}_{t+1}^K+\frac{\mathcal{S}_{tr}(s_t,z,\varepsilon_{t+1}^s)}{K_t}\mathcal{C}_{t+1}^s\right)\right]-(1+\rho_t)\left[1+(1-\tau_t^c)a^{tr}z\right].$$

For a primitive θ_j that does not directly change the current marginal-cost coefficient, differentiating the residual gives

$$\mathcal{E}_{t,\theta_j}=\phi_{t,\theta_j}+\mathbb{E}_t\left[M_{t,t+1}\left(\mathcal{C}_{t+1,\theta_j}^K+\frac{\mathcal{S}_{tr,\theta_j}(s_t,z,\varepsilon_{t+1}^s)}{K_t}\mathcal{C}_{t+1}^s+\frac{\mathcal{S}_{tr}(s_t,z,\varepsilon_{t+1}^s)}{K_t}\mathcal{C}_{t+1,\theta_j}^s\right)\right].$$

Setting $\theta_j=P_t$ gives Equation (52a). Setting $\theta_j=s_t$ gives Equation (52b), with $\mathcal{S}_{tr,s}$ denoting the derivative of $\mathcal{S}_{tr}$ with respect to the current dirty-capital share. If η_λ affects the collateral function but not the transition technology, then $\mathcal{S}_{tr,\eta_\lambda}=0$. Setting $\theta_j=\eta_\lambda$ therefore gives Equation (52c). □

A.4 Derivation of Equations (54)

From Equation (36), write the total-investment rule as

$$x_t{}^* = x_t^{tr*} + \mathcal{V}_t^K(x_t^{tr*}; \mathbf{\Sigma}_t, \theta_j).$$

Differentiating with respect to θ_j gives

$$\frac{\partial x_t{}^*}{\partial \theta_j} = \frac{\partial x_t^{tr*}}{\partial \theta_j} + \mathcal{V}_{t,z}^K(x_t^{tr*}; \mathbf{\Sigma}_t, \theta_j)\frac{\partial x_t^{tr*}}{\partial \theta_j} + \mathcal{V}_{t,\theta_j}^K(x_t^{tr*}; \mathbf{\Sigma}_t, \theta_j),$$

which is Equation (54a).

Next, by definition,

$$\omega_t^* = \frac{x_t^{tr*}}{x_t{}^*}.$$

Using the quotient rule,

$$\frac{\partial \omega_t^*}{\partial \theta_j} = \frac{1}{x_t{}^*}\frac{\partial x_t^{tr*}}{\partial \theta_j} - \frac{x_t^{tr*}}{(x_t{}^*)^2}\frac{\partial x_t{}^*}{\partial \theta_j}.$$

Since $\omega_t^* = x_t^{tr*}/x_t{}^*$, this becomes

$$\frac{\partial \omega_t^*}{\partial \theta_j} = \frac{1}{x_t{}^*}\left[\frac{\partial x_t^{tr*}}{\partial \theta_j} - \omega_t^*\frac{\partial x_t{}^*}{\partial \theta_j}\right],$$

which is Equation (54b).

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