Sustainable Investing with Real-Asset Trades: Incentives to Own and Transform Polluting Assets

Agostino Capponi*, Jay Sethuraman†, Felipe Verastegui‡

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Abstract

How do private decisions to buy and sell high-carbon real assets affect investment in socially valuable pollution-reduction opportunities? We analyze firms' optimal reform and secondary-market trading decisions in an environment where real-asset reallocation and transformation markets coexist. When only a reallocation market is available, value-aligned investors unintentionally weaken firms' incentives to undertake asset reforms, generating inefficiencies. Introducing a real-asset transformation market alters these incentives. Polluting firms post transformation contracts allowing other firms to contribute towards reform and claim transformation rights. This broadens investor participation, increasing real-asset reform and improving efficiency. We show that, unlike policy alternatives such as a carbon tax or an emissions-trading system, a real-asset transformation market increases surplus for the firms that own and transform pollutive assets. Our results underscore how marketable transformation rights and use-of-proceeds contracts can support efficient, incentive-compatible reform of high-carbon assets.

*Columbia University, Industrial Engineering and Operations Research and Columbia Business School, Email: ac3827@columbia.edu

†Columbia University, Industrial Engineering and Operations Research, Email: js1353@columbia.edu

‡Columbia University, Industrial Engineering and Operations Research, Email: fav2114@columbia.edu

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1 Introduction

The rise of sustainable investing has created a paradox in markets for high-carbon assets: as some environmentally-conscious investors divest from polluting firms, firms themselves have an incentive to offload pollutive real assets in secondary markets, which often flow to less transparent buyers with weaker environmental commitments, potentially undermining rather than advancing decarbonization efforts. Between 2014 and 2024, approximately USD 90 billion in petroleum assets moved from publicly listed to private companies[1]. Empirical analysis documents secondary sales of high-carbon real assets as a common response to environmental pressures. As Duchin et al. 2025 observe, such trades can "remove the option to engage with firms to reduce pollution". They further question whether such trades can have a net negative effect on the environment. This is the key motivating question for our work.

The tension between real asset trades and decarbonization efforts can be particularly relevant for industries in which real assets are long-lived. Here, asset owners commonly face discrete corporate choices, such as buying or selling real assets: natural gas processing plants, steel mills and coal power plants often change hands during their productive lifespans. In fact, secondary trades and the reallocation of real assets via over-the-counter deals play an important role in corporate restructuring processes for productive firms. Notable examples from the energy sector include RWE's 2015 decision to spin off its renewable business into a separate entity, and E.ON's 2016 choice to split and create Uniper to retain ownership over all fossil fuel generation assets while the restructured E.ON shifted its focus toward clean energy sources. Another example from that same year is ENGIE announcing a €15 billion portfolio rotation program, explicitly targeting the sale of coal and oil & gas assets over a three-year period as a key component of the company's strategic shift toward low-carbon energy. We refer to such processes as *real-asset reallocation*.

At the same time, asset owners make discrete investment decisions over their real assets that alter their characteristics, which we refer to as *real-asset transformations*. Some transformations are privately profitable. For example, installing high-efficiency boilers in steel plants improves fuel efficiency and provides fuel savings, and adding compressor units to natural gas facilities increases throughput and reduces downtime. Other transformations, however, entail net private

[1]These trade patterns have been discussed at length in industry reports, e.g. see Woodroffe, 2024.

costs despite generating large social benefits. Replacing coke-based reduction equipment with electric arc furnaces significantly reduces steel-making emissions, but is privately costly. Similarly, installing comprehensive methane leak detection equipment—fixed sensors, mobile monitoring, or satellite systems—can reveal higher-than-anticipated pollution levels, creating compliance costs for natural gas operators, yet it identifies opportunities for emission reductions that are socially valuable.

In some sectors of the economy, transforming and upgrading real assets potentially requires the involvement of many stakeholders. A prominent example are the retrofits of residential and commercial buildings, which generally include thermal insulation and renewing heating equipments, as well as other energy-efficiency related upgrades. A market-based innovation in this sector are residential and commercial property assessed clean energy loans (also called PACE and C-PACE financing programs). Homeowners, who have ownership of the underlying asset, value the upgrade itself and seek financial support from PACE lending-firms to finance the real-asset transformation. In addition to earning interest on the loans, the lending-firm gets credited towards social and environmental corporate targets, because they enable the energy-efficiency related upgrade. Thus, in exchange for their participation, the lending-firm can claim *rights to impact* — a corporate feature that is valued by investors who include socially responsible investment targets in their strategies. As more homeowners compete to secure such loans, and more lending-firms compete to originate them, a market for real-asset transformation emerges. Recent records of the U.S. Environmental Protection Agency (EPA) show that by 2025 more than 30 states had passed legislation to enable PACE and C-PACE programs with active involvement of the local governments.[2]

In this paper, we study how *coexisting markets for real-asset reallocation and transformation* can shape corporate incentives to own and reform high-carbon assets. We address three main research questions. First, from the perspective of investors, we examine how financial market structure influences the effectiveness of different sustainable investment strategies. Second, from the standpoint of corporate organization, we investigate the conditions under which real-asset transformation decisions are most likely to occur and how these decisions interact with firms' real-asset reallocation choices. Finally, from the viewpoint of financial-market regulation, we explore which types of contracts arise in equilibrium when we allow for co-existing markets, how they are

[2] A descriptive note on PACE and C-PACE programs can be found on this EPA website (2025).

priced, and how market composition affects firms incentives as well as the mechanisms through which transformation markets can enhance efficiency.

1.1 Main Results

We study an equilibrium model of risk-averse portfolio allocation and corporate decision-making, which incorporates both a secondary market for real assets and trading of contracts governing real-asset transformation. We consider an economy with three types of investors: unconstrained, value-aligned, and impact-aligned. Value-aligned investors will not buy shares of firms that operate unreformed pollutive assets, whereas impact-aligned investors will only invest in those firms that actively engage in environmental reform. Firms act to maximize their share price.[3] Firms that own pollutive assets face a corporate choice problem. They must choose among operating an unreformed pollutive real asset, engaging in secondary trading, or implementing a costly reform which reduces the environmental impact of the real asset.

First, we show that the equilibrium corporate choices for pollutive firms exhibit an *investor-separable sub-market structure*. In equilibrium, every firm that owns a pollutive asset caters to a specific investor type. In turn, each investor type only buys shares in the pollutive firms participating in their corresponding sub-market. Importantly, if the transaction costs of secondary trading are smaller than the cost of reform, value-aligned investors will only buy shares in firms that sell their pollutive assets. Firms operating unreformed assets are directly ineligible due to the investor's value-aligned strategy. However, value-aligned investors will also not buy shares in firms that engage in real-asset reform, because in equilibrium these trade at a higher price than the shares of firms that engage in secondary trading. Buying shares in firms that reform their real-assets then becomes a strictly dominated portfolio choice for the value-aligned investors.

Thus, our first main result shows that the paradox in markets for high-carbon assets is obtained as an equilibrium outcome. Value-aligned investors only exclude firms that operate unreformed assets, which provides firms with an incentive to sell rather than reform. This increases the flow of pollutive assets that go towards the outside private buyer. Hence, there is a *disincentive to own pollutive assets* driven by value-aligned investors. An increase in the number of such investors

[3]Consistently with corporate finance literature, firms maximize their equity value, given by the share price multiplied by the number of outstanding shares. We take the latter as constant.

reduces the number of firms that engage in real-asset reform, as well as reducing the impact-aligned investor's utility. Note that if the transaction costs of secondary trading are larger than the cost of reform, value-aligned and impact-aligned investors would co-invest in firms that engage in real-asset reform. A regulator could try to induce this alignment in equilibrium outcomes by imposing a tax on secondary-market trades for high-carbon assets — yet this has a negative effect on pollutive-firm surplus and on value-aligned investor utility across pollutive sub-markets.

Since constraining real-asset reallocation markets may be an impractical solution to this paradox, we turn our attention to alternative market-based approaches to curb the disincentive to own pollutive assets without directly affecting the secondary trading channel. In particular, we study the effects of introducing a separate market for *real-asset transformation contracts*. A transformation contract specifies a single real-asset, plus a reform action and a requested monetary contribution. It is issued by the firm that owns the unreformed pollutive asset individualized by the contract. When a counterparty buys into the contract, it contributes the specified amount and acquires an "*impact claim*" upon real-asset reform. The issuing firm commits to enabling reform upon the transfer of the requested monetary contribution, providing the remainder payment to finance the reform action if necessary. We emphasize that impact ownership is ambiguous in practice. Our model makes these rights explicit and tradable to prevent excessive impact claims stemming from individual reform actions. [4]

The introduction of a separate market for real-asset transformation contracts fundamentally changes the equilibrium structure. Value-aligned investors will not invest in firms that operate unreformed pollutive assets, but do not care about impact claims. Thus, polluting firms may have an incentive to post transformation contracts ceding impact claims to become value-eligible at a cost lower than the full cost of reform. At the same time, because impact-aligned investors only invest in firms that engage in environmental reforms, *firms that do not own pollutive assets* may have an incentive to buy into transformation contracts, claiming impact-alignment without increasing emissions ownership. This mechanism expands investor participation on both sides and enhances efficiency via improved risk-sharing. Thus, our second main result is that introducing an explicit market for transformation curbs the divestment-decarbonization tension. Our findings point to reduction-rights contracts and use-of-proceeds forward agreements as financing structures

[4]Relaxing these conditions would make the impact-investor strategy less meaningful.

that can support efficient, incentive-compatible reform of high-carbon assets.

Finally, we turn our attention to the welfare effects of co-existing markets for real-asset reallocation and transformation. Our last main result highlights that the introduction of a market for real-asset transformation not only increases real-asset reform, reducing the social costs of pollution, but it is also *surplus-enhancing for all clean and pollutive firms*. Recall that firms want to maximize their share prices, yet face investors who are constrained by their value-aligned and impact-aligned strategies. Satisfying investor constraints — by selling their pollutive assets or engaging in real-asset reform — can enhance investor competition and increase firm share price, but doing so is costly for the firms. A market for real-asset transformation provides a cost-efficient alternative for firms to better accommodate sustainable investing constraints. This makes attracting investors less costly for the firms, and has a positive effect on share prices *across all firms in the economy*. This is a significant point of departure from other policy alternatives — such as a carbon tax or an emissions-trading scheme — which also increase real-asset reform but, in doing so, reduce the surplus of firms that own and transform pollutive assets.

Our results contribute to ongoing debates over regulatory responses to secondary trading of high-carbon assets. Proposed interventions include enhanced transparency requirements for secondary trades, granting regulators approval rights over asset transfers, or even public ownership of high-carbon assets. However, screening trades without observable counterfactuals poses significant challenges for regulators, and empirical evidence from coal power plant reallocation decisions casts doubt on whether public ownership leads to greater environmental reform (Darmouni and Zhang, 2024). The mechanisms we study suggest an alternative: rather than restricting reallocation markets, introducing markets for transformation contracts can mitigate the divestment-decarbonization tension while preserving other beneficial aspects of the secondary trading channel.

1.2 Related Literature

This paper contributes to multiple strands of the literature on sustainable finance, decentralized financial markets, real-asset corporate choices, and the moral preferences of socially responsible investors. To the best of our knowledge, we are the first to propose a theory of how the presence of secondary trading channels for real assets subject to environmental reform — or the coexistence of markets for real-asset reallocation and transformation — can shape the outcomes of sustainable

investing practices.

Several studies have focused on the financial implications of socially responsible investing (e.g. Hong and Kacperczyk 2009, Pedersen et al. 2021, Ľuboš Pástor et al. 2021). We contribute to this strand of the literature by showing that the introduction of a market for real-asset transformation enhances investor diversification and market efficiency. In contrast with Ľuboš Pástor et al. 2021, where sustainable investors' preferences are well-aligned with positive social impact, in our setting, some investors do not differentiate between secondary trading of pollutive assets and real-asset reform, with only the latter producing measurable social impact. We show that the introduction of transformation contracts partially restores this alignment.

A growing number of papers focus on climate risk, which is generally decomposed into transition risks, such as the risk of regulatory shocks on pollution; and physical risks, such as increased frequency and intensity of natural disasters. These works study how climate-risk sharing can affect asset prices across financial markets (e.g. Giglio et al. 2021, Bolton and Kacperczyk 2021). In our paper, asset prices are determined by the interaction between socially responsible investor preferences, endogenous corporate choices, and financial market structure. We assume that some investors apply exclusionary screening based on direct emissions, in line with the empirical evidence presented in Bolton and Kacperczyk 2021 — while others screen for firms that engage in environmental reform actions. We then discuss reform-action contract pricing and study how these prices affect investor returns and firms' cost of capital. Consistent with the climate-risk pricing literature, we find that firms that directly reform assets trade at a premium compared to their unreformed counterparts. This premium *increases* when transformation and reallocation markets coexist.

A separate line of research has focused on developing models to capture the real outcomes of socially responsible investing (e.g. Oehmke and Opp 2022, Huang and Kopytov 2023, Sauzet 2024, Oehmke and Opp 2025, Gupta et al. 2025, Green and Roth 2025). A common finding across these papers, mirrored here, is that environmental reform of real assets requires the presence of *impact* investors (c.f. Oehmke and Opp 2025) — when reform is costly, the presence of value-aligned investors results only in corporate reallocation decisions, without any uptake of environmental reforms. We show that when reallocation and transformation markets coexist, impact investors provide stronger incentives for firms to adopt costly reform actions, and these incentives are less negatively affected by the presence of value-aligned investors who do not distinguish between sec-

ondary trades and real-asset reform. In contrast to papers that focus on entrepreneurial financing needs for new investment projects (Oehmke and Opp 2025, Green and Roth 2025), our model assumes an existing fixed-size economy to emphasize current – or legacy – real assets, which continue to play an important role in many productive industries. In that sense, our paper is closest to the work of Heinkel et al. 2001 on optimal corporate choices in the presence of socially driven investors. We enhance their model, however, to allow for heterogeneous strategies by socially driven investors, secondary trading, and a market for real-asset transformation in which firms trade contracts over reform actions.

Our paper also sheds light on implementability. We show that when there are sufficiently many impact-aligned investors, trading reform options suffices, and otherwise, trading reform obligations is optimal. Related works include Pedersen et al. 2021, who find that investor optimal portfolios are implemented using four-fund separation by including an ESG-tangency portfolio. Landier and Lovo 2024 show that when there are many value-aligned investors, a single-fund is optimal for a socially responsible intermediary, and otherwise, a dual-fund approach that targets clean sectors on one hand, and direct reductions on the other, maximizes assets under management. Oehmke and Opp 2025 find that a firm-optimal financing agreement can be implemented by selling a green bond to the socially responsible fund and a regular bond to financial investors.

Finally, the moral preferences of socially driven investors is a core discussion point in this literature. Empirical work by Bonnefon et al. 2025 suggests that warm-glow effects — the non-pecuniary value of holding a portfolio in which firms do not engage in negatively perceived activities — is a more prominent concern than the social consequences of investment decisions for responsible asset managers. Our paper contributes to the discussion by analyzing how the presence of investors driven by warm-glow negatively affects sustainable investing outcomes, limiting the impact that consequentialist investors can have over firms' environmental reform decisions. The conclusions from our model are in line with the empirical findings of Duchin et al. 2025, who identify secondary trading as an economically meaningful response from firms facing environmental pressure. In our model, investors with warm-glow preferences create a disincentive for firms to invest in pollution reduction when secondary trading is available. Instead, firms are incentivized to trade away the pollutive asset, removing the opportunity for impact-seeking investors to engage in environmental reform. A market for real-asset transformation alleviates the disincentives to own pollutive assets.

Our findings point towards the decentralized trading of reduction options as a market mechanism that can improve efficiency when secondary-trading frictions are present.

2 Institutional Evidence

In this section we discuss two empirical settings which provide partial evidence for the key economic mechanism that we study, that is, institutionalized markets for real-asset transformation contracts.

2.1 Associated Gas Flaring in the Oil and Gas Sector

The global oil industry produces approximately 100 million oil barrels per day. A standard barrel holds 40 U.S. gallons of oil, a commodity which is extracted from one of the many oil fields located across large productive regions such as the Permian or the Appalachian Basin in the United States, the Delta Basin in Niger, or the West Siberian Basin in Russia. The equipment necessary for oil extraction and production includes drilling rigs, processing facilities, and pipelines with pumps, compressors, valves, and monitoring systems. During the extraction and production of oil resources, associated gas is obtained as a secondary output. Generally, if there is no direct way for the firm to utilize this gas, it is burned to avoid the direct release of methane into the atmosphere, in a process known as flaring. This procedure releases carbon dioxide, and when combustion is incomplete, also releases methane into the atmosphere. A report by the World Bank (2022) estimates that flaring and related emissions represent around 12% of all greenhouse gases released by the global energy sector, and that the total volume of natural gas flared yearly is comparable to the energy needed to power the entirety of Sub-Saharan Africa.

Because of the large social costs of venting, as well as the large private opportunity cost of not utilizing the associated gas after a recovery process, organizations such as the United Nations, the International Energy Agency and the World Bank have created special interest groups around flaring and methane reduction investments. Estimates by McKinsey (2024) of the upfront cost required to finance flaring reduction projects which are cost-neutral or even financially beneficial at $200 Bn. USD, with 60% of this spending corresponding directly to infrastructure upgrades which connect flaring facilities to gas networks or demand centers. Given such estimates, it is perhaps surprising that many of these privately profitable projects do not get implemented by oil

and gas real-asset owners. Reasons cited by the World Bank (2022) include financial constraints, investment cycles, and the de-prioritization of gas flaring reduction investments in lieu of other core oil and gas infrastructure projects, which provide larger margins for the firms. Even though pressure from environmentally-minded investors could drive the asset owners to invest in pollution reduction actions, it can also trigger real-asset sales. An analysis by the Environmental Defense Fund (2022) on Upstream Oil and Gas M&A data from Refinitive between 2017 and 2021 showed that during this time window, 298 deals worth more than $140 Bn. USD transferred real assets from companies with flaring reduction commitments to companies without (119 deals were recorded with the opposite change direction). Similarly, 211 deals worth more than $110 Bn. USD transferred real assets from companies with methane reduction goals to companies without (88 deals were recorded with the opposite change direction).

In this context, an innovative approach which resembles a regulatory-enabled market for real-asset transformation rights is the recent Nigerian Gas Flare Commercialization Programme, deployed by the Nigerian Upstream Petroleum Regulatory Commision in 2022. This market-based policy enabled third-party investors to bid on access to flare sites in Nigeria including the acquisition of a *connection agreement*, a contract between the winning bider and the real-asset owner allowing the bidder to "*[. . .] engineer, procure and construct the gas connection infrastructure at its own expense and under criteria acceptable to the oil producer*" as well as a *gas supply agreement*, by which the bidder acquires the right to commercialize the recovered associated gas at a price stipulated in the bid. Our theory results provide support to such market mechanisms as an effective way to increase real-asset reform, decrease secondary market trades, and enable broader investor-level diversification for firms that do not own pollutive assets but engage in such pollution reduction investments.

2.2 Property Assessed Clean Energy Loans in the Buildings Sector

Property Assessed Clean Energy (PACE) loans are a rapidly growing financial instrument primarily available in the United States with emerging programs in Canada, Australia and Europe that targets financing energy-efficiency and clean energy infrastructure upgrades for existing buildings. According to Pace Nation, between 2010 and 2024, more than 3,000 commercial buildings and more than 300,000 residential buildings have financed infrastructure upgrades via PACE loans.

These instruments have several important worth highlighting. First, PACE loans feature a unique regulatory-enabled structure, in which the government places a special assessment lien on the real asset tied to the loan repayments. Payments are then collected via the property tax. This prevents the need to accelerate PACE obligations even in the case of foreclosure or sale: under PACE financing, the lien stays with the asset, and the subsequent owner becomes responsible for future assessment payments. Second, PACE-eligibility is explicitly determined by the program administrator, which usually corresponds to state or local governments. In most cases, the key eligibility requirement is that the upgrade focuses on energy efficiency or renewable energy. This provides de facto certification of the loan's positive social impact by the governments, which PACE lenders can then harness to enjoy non-pecuniary benefits arising from their lending activity. For instance, in a recent brief by Climate Bonds Initiative (2019), it is shown that PACE lenders refinance their loans by issuing green or climate-themed bonds, which are sought by investors following sustainability-aligned strategies. In theory, this should enable more favorable financing conditions for PACE lenders than otherwise available in the market.

Although PACE instruments have been designed specifically to enable socially valuable upgrades in residential and commercial buildings, the implementation of PACE programs provides a useful benchmark for the development of transformation markets in productive sectors. A series of reports by Berkeley Lab with the U.S. Department of Energy (2018, 2019) discuss the roll-out of required regulation by drawing on different experiences of PACE adoption in states such as New York, California, Texas, Conneticut and Washingtonn D.C. On one hand, the regulatory-backed definition of PACE-eligibility can include specific energy savings and impact measurement requirements, as well as external audits or certifications (e.g. LEEDS, Energy Star, among others). While this can increase the cost of origination, it can also provide more credibility to the social impact of the loan. On the other hand, placing a special assessment lien on the financed asset tied to repayments enables debt seniority, and supports contract enforcement after the upgrade has been installed. Moreover, because the payment obligation stays with the asset, secondary trades are effectively decoupled from the incentives to engage in socially valuable real-asset reform activities.

3 Model

We now turn our attention to a theoretical model of the effects of co-existing markets for real-asset reallocation and transformation in the presence of sustainable investors. We consider a static equilibrium model of risk-averse portfolio allocation and corporate decision-making as follows:

Firms There are N firms selling a single divisible unit of shares each. Firms make corporate decisions to maximize their equity value — the unit share times a share price determined in equilibrium — minus any costs arising from their corporate choices. There are two types of firms:

(i) c-firms, endowed with a clean real asset that generates a normally distributed cash flow with mean μ_c and variance σ_c^2; and

(ii) d-firms, endowed with a pollutive (*dirty*) real asset that generates a normally distributed cash flow with mean μ_d and variance σ_d^2.

The number of c-firms is N_c with $0 \leq N_c \leq N$ ($N_d := N - N_c$). Covariance between the cash flows of c and d is σ_{cd}. Firm surplus is denoted S_f with:

$$S_f = P_f - C_f$$

where P_f is the equilibrium share price and C_f is the cost of the corporate choices made by f.

Investors There are I risk averse investors who are partitioned into three types:

(i) Value-aligned (v) investors, who exclude firms that operate unreformed d assets;

(ii) Impact-aligned (m) investors, who invest in firms that reduce environmental damage; and

(iii) Neutral (n) investors, who are unconstrained.

The number of investors of each type are, respectively, I_v, I_m, and I_n, with $I_v + I_m + I_n = I$. We denote the relative market share of a subset of investor types by $\lambda_\Theta = \frac{\sum_{\theta \in \Theta} I_\theta}{I}$ for any subset $\Theta \subseteq \{m, v, n\}$ with which $\lambda_n + \lambda_v + \lambda_m = \lambda_{nvm} = 1$. All investors have utility functions that exhibit constant absolute risk aversion (CARA), with (a common) risk tolerance parameter τ.[5]

[5]Note that g-investors do not distinguish between secondary trading and real-asset reforms.

Corporate Choices A firm endowed with a c-asset initially does not face a corporate choice problem and is always acceptable for value-aligned investors. We denote this state for the c-firm as choice A. On the other hand, a firm endowed with a d-asset has the opportunity to reduce the environmental damage caused by its operation by engaging in *real-asset reform* a cost of K (R-choice).[6] It can also choose to operate the *unreformed d-asset as is* at no added cost (U-choice). Finally, a d-firm can *sell its d-asset in a secondary market* (S-choice). When the firm decides to sell in the secondary market, we assume that the asset is acquired by an outside private buyer which provides a payment equal to the risky cash-flow of the d-asset minus a transaction cost T. When $K > T$, secondary trading introduces a gap: a firm choosing to offload the asset becomes eligible for a v-investor but not for an m-investor. We assume that this is the case. We denote the share price of a firm that makes corporate choice $y \in \{U, S, R, A\}$ as P_y.

Portfolio Allocation Investors buy shares in firms depending on their corporate choices. The number of shares that a type θ investor buys in firms making corporate choice y is denoted as $x_{\theta y}$. Investor surplus corresponds to the risk-adjusted returns of their allocation minus the cost of buying the shares, given by the position and the equilibrium share price.

Riskless Asset There exists a riskless asset in perfectly elastic supply with a rate of return normalized to zero. Borrowing is allowed but short selling of firm shares is prohibited.

Market Mechanism for Real-Asset Transformation With only a reallocation market, firms are allowed to keep or sell their d-asset, but cannot trade contracts over the reform actions. In other words, the opportunity to reduce environmental damage is non-transferable and thus inseparable from the d-asset itself. We relax this constraint by introducing a *real-asset transformation market*. Formally, we augment the reallocation market with an exchange in which d-firms can post – and c-firms can acquire – *transformation contracts*. A transformation contract specifies a non-negative amount π to be contributed by the c-firm towards a real-asset transformation action, in exchange

[6] We assume that the reform action is verifiable and contractible. This assumption is more plausible in industries where the physical outcomes of reform are directly measurable (e.g infrastructure or energy efficiency upgrades) as compared to industries where impact is less measurable or observed with delays, such as reforestation or biodiversity conservation.

for the *impact-rights* of the reform action.[7] We denote the d-firm choice of issuing a transformation contract actions as β, and the c-firm choice of buying into such a contract as α.

Mechanism 1 (Market Mechanism for Real-Asset Transformation)**.** *The market mechanism for real-asset transformation can be described as follows:*

1. ***Posting.*** *d-firm posts a transformation contract with requested contribution π.*
2. ***Accepting.*** *c-firm decides whether to enter a posted contract.*
3. ***Transfer.*** *Upon accepting, c commits to a transfer of π to d, and d commits to transfer impact claims upon the d-asset reform.*
4. ***Disclosure.*** *Contracts are publicly observable to all firms and investors.*

The transformation market allows c-firms to actively reduce environmental damage (becoming m-eligible), as well as provide an alternative for d-firms to not operate an unreformed d-asset. Figure 1 summarizes how investor-eligibility changes upon the introduction of a transformation market.[8]

Investor-Eligibility with a Transformation Market

	U	S	β	R	A	α
n	✓	✓	✓	✓	✓	✓
v	×	✓	✓	✓	✓	✓
m	×	×	×	✓	×	✓

Figure 1: Eligibility with Co-Existing Markets

Solution Concept. We focus on competitive equilibrium outcomes. A competitive equilibrium is a set of share prices $\mathbf{P} \in \mathbb{R}^{N_c+N_d}_{\geq 0}$, investor portfolio allocations $\mathbf{x} \in \mathbb{R}^{I}_{\geq 0}$, firm corporate choices

[7] In Section 1.1 we highlight the ambiguity of impact ownership in practice. A precondition for well-defined impact rights is that the environmental reform action is verifiable and contractible, which maps well to infrastructure upgrades, as is the case of associated gas recovery or energy efficiency upgrades in buildings.

[8] In Section 5 we show that the transformation market Pareto-improves d-firms surplus. This observation can be used to support enforcement of transformation contracts. Suppose that d-firms can strategically default on a transformation contract and this is known by the c-firms. Then, the transformation market unravels, which makes d-firms worse off. Hence, commitment and enforceability are ex-ante desirable for the d-firms. For example, in the case of PACE loans, payments are serviced by governments via property tax through the special assessment lien.

$\mathbf{y} \in \{U, S, R, A, \beta, \alpha\}^{N_c+N_d}$ and vector of real-asset transformation contract prices $\pi \in \mathbb{R}^{\mathbf{N_d}}$ which jointly satisfy investor and firm rationality and incentive-compatibility and market clearing:

(i) Market for firm shares and portfolio allocation clears.

(ii) Market for secondary-trades and corporate choices clears.

(iii) Market for real-asset transformation contracts clears.

The number of firms that make each corporate choice $y \in \{U, S, R, A, \beta, \alpha\}$ is denoted by $N_y \geq 0$ and is also determined in equilibrium. Our welfare measure is then the sum of the private surplus of firms and investors, which we contrast against the number of polluting assets that *do not* get reformed – either because they are sold or because they are operated as unreformed assets. Thus, even though we do not explicitly model a social cost of pollution, we interpret the number $N_R + N_S$ as an important pollution baseline across our results.

Note: Absent a separate market mechanism for real-asset transformation and removing the secondary trading channel makes the s and g-investors indistinguishable.[9] In this case, our model collapses to the that of Heinkel et al. 2001.

4 Corporate Choices and Share Prices in Equilibrium

We now characterize firms' share prices and optimal corporate choices in equilibrium. We provide solutions both in the case of a market for real-asset reallocation only and in the case where markets for reallocation and transformation coexist. We outline the main proof ideas and defer detailed computations to the Appendix.

4.1 Market for Real-Asset Reallocation Only

First, we study competitive equilibrium outcomes in the case where only a market for asset reallocation exists. In this case, the c-firms are automatically eligible for v and n-investors, but remain ineligible for the m-investors, as they own real assets that do not feature any pollution reduction opportunities. On the other hand, d-firms have three choices: (i) they can operate an unreformed pollutive asset, which makes them ineligible for v and m-investors; (ii) reform the pollutive asset,

[9] We can recover this as a limiting case by taking $T \to K$ and breaking ties in favor of environmental reform.

which makes them eligible for both v and m-investors; or (iii) trade the pollutive asset. In the last case, firms transfer their d-asset to an outside private buyer, and receive a payment that equals the risky cash-flow of the asset minus the transaction cost T. As we will see, a key feature of the equilibrium structure is *investor-separability* over the d-firm corporate choices: as a consequence of the surplus maximization objective under sustainable investing constraints, each investor-type will only buy shares in firms that belong to individual sub-market in which all d-firms make the same firm-optimal corporate choice.

First, because the number of c and d-firms is fixed, the following corporate choice market clearing conditions must hold in equilibrium:

$$N_A^* = N_c, \quad N_U^* + N_S^* + N_R^* = N_d. \tag{1}$$

In the optimal corporate choice problem faced by a d-firm, secondary trading and real-asset reform involve a positive cost. Thus, they will only be *firm* incentive-compatible when the resulting share price increase weakly offsets the action's cost. Because the share price decreases in the number of firms of a given type, it must be that, in equilibrium, the increase in share price equals the action's cost. This is analogous to a free entry condition in the corporate choice problem: if the share price premium from taking an action exceeded its cost, more firms would adopt it, driving the premium down until equality is restored. We refer to this as the *no-entry* and *no-exit* condition.

By this observation, the following relationships between share prices must hold in equilibrium:

$$P_S^* - P_U^* = T, \quad P_R^* - P_U^* = K. \tag{2}$$

Now, consider the optimal portfolio allocation problem faced by some investor i. Because $K \geq T$, it follows from (2) that, in equilibrium, $P_U^* < P_S^* < P_R^*$. This implies that n-investors (v-investors) will only buy shares from firms of type A and U (A and S) due to *investor* incentive-compatibility: the risk-return profiles of U, S and R firms are equivalent, but in equilibrium, the share price of an S-firms must be larger than that of a U-firm. Hence, it is never optimal for an n-investor to buy shares of an S or R-firm. By the same argument, it will never be optimal for a v-investor to buy shares in a R-firm. Then, at the portfolio allocation level, market clearing

imposes the following conditions involving the portfolio allocation decisions by investors:

$$I_n x^*_{nU} = N^*_U, \quad I_v x^*_{vS} = N^*_S, \quad I_m x^*_{mR} = N^*_R, \tag{3a}$$

$$I_n x^*_{nA} + I_v x^*_{vA} = N^*_A, \tag{3b}$$

The equilibrium structure is well summarized by the clearing conditions in (3). Observe that while there is a single c-firm sub-market – $(n \cup v, A)$ – there are three distinct d-firm sub-markets: (n, U), (v, A) and (m, R). Moreover, d-firm sub-markets are separable across investor types. It follows that, fixing the equilibrium capital allocation, the number of firms in each category scales proportionally to the respective investor population. A large number of value-aligned investors, then, would result in more firms participating in the secondary-trading market.

Our first Lemma characterizes the equilibrium corporate choices:

Lemma 4.1. *In a case with a market for real-asset reallocation only, there is a unique equilibrium, in which d-firms and investors participate in three separate sub-markets: (1) unconstrained investors only buy U-firm shares, (2) value-aligned investors only buy S-firm shares, and (3) impact-aligned investors only buy R-firm shares. The number of d-firms choosing each corporate action $\ell \in \{U, S, R\}$ in equilibrium can be written as a baseline quantity plus an allocative distortion:*

$$N^*_\ell = \max\{0, I_\theta x^*_{\theta,\ell}\} = \max\{0, \lambda_\theta(N_d + \delta^*_\ell)\},$$

for $(\theta, \ell) \in \{(n, U), (v, S), (m, R)\}$. Define $\phi := \sigma_c^2\sigma_d^2 - \sigma_{cd}^2$. The allocative distortions in equilibrium are given by

$$\begin{aligned}
\delta^*_U &= \frac{\tau}{\phi}\sigma_c^2 I_v T + \frac{\tau}{\sigma_d^2} I_m K - \frac{\sigma_{cd}}{\sigma_d^2}\,\xi^*_U, \\
\delta_{S^*} &= -\frac{\tau}{\phi}\sigma_c^2 I_{nm} T + \frac{\tau}{\sigma_d^2} I_m K - \frac{\sigma_{cd}}{\sigma_d^2}\,\xi^*_S, \\
\delta^*_R &= \frac{\tau}{\phi}\sigma_c^2 I_v T - \frac{\tau}{\sigma_d^2} I_{nv} K + \frac{\sigma_{cd}}{\sigma_d^2}\xi^*_R,
\end{aligned}$$

satisfying $\sum_{(\theta,\ell)} \lambda_\theta \delta^*_\ell = 0$. Covariance terms are non-negative, $\xi^*_\ell \geq 0$, as they are proportional to equilibrium portfolio positions in the c-firm sub-market, i.e., $\xi^*_U = \xi^*_S = I_m x^*_{nA}$ and $\xi^*_R = I_{nv} x^*_{nA}$. Full derivations are provided in Appendix A.1.

Proof. See Appendix A.1. □

Lemma 4.1 provides insights into the size of each sub-market in equilibrium. First, if allocative distortion terms δ_ℓ^* are small, then d-firms opt into each sub-market in proportion to investor composition weights, λ_θ for $\theta \in \{n, v, m\}$. Since $\lambda_n + \lambda_v + \lambda_m = 1$, an increase in the number of value-aligned investors incentivizes more d-firms to sell their pollutive assets in the secondary-market, reducing the size of both the sub-market of firms that engage in real-asset reform, and the sub-market of firms which operate unreformed d-assets. Second, the allocative distortions in equilibrium are driven by the cost of corporate actions plus correlation terms. Intuitively, a larger cost of reform K results in fewer firms choosing reform and more firms choosing secondary trades, and a larger cost of transaction T results in fewer firms choosing secondary-trades and more firms choosing reform. Finally, increases in both T or K result in more firms choosing to operate unreformed d-assets. Third, under large allocative distortions, an increase in the number of value-aligned investors has an additional negative effect on the number of firms that engage in real-asset reform. Adding to the initial effect of a smaller baseline $\frac{\partial \lambda_m N_d}{\partial I_v} < 0$, adding value-aligned investors also yields a larger allocative penalty $\frac{\partial \xi_R^*}{\partial I_v} < 0$. Because fewer firms engage in real-asset reform, an impact-aligned investor buys fewer shares upon the introduction of an additional value-aligned investor, reducing overall impact investor's utility.

Next, we characterize the equilibrium share prices for each firm category:

Lemma 4.2. *In a case with a market for real-asset reallocation only, the unique equilibrium share prices of c-firms and d-firm choosing each $\ell \in \{U, S, R\}$ can be decomposed into the mean return minus shared risk-adjusted penalty terms and a price wedge:*

$$P_f^* = \mu_c - R_c - \omega_f^*, \quad P_\ell^* = \mu_d - R_d - \omega_\ell^*.$$

where $R_d := \frac{N_d\sigma_d^2 + N_c\sigma_{cd}}{I\tau}$, $R_c := \frac{N_c\sigma_c^2 + N_d\sigma_{cd}}{I\tau}$. Price wedges for d-firms are given by

$$\omega_U^* = \lambda_m K + \lambda_v T$$
$$\omega_S^* = \lambda_m (K - T) - \lambda_n T$$
$$\omega_R^* = -\lambda_v (K - T) - \lambda_n K$$

and the price wedge for the c-firms is given by

$$\omega_A^* = \frac{\sigma_c^2}{\tau}\frac{\lambda_m}{\lambda_{vn}}\frac{N_c}{I} + \frac{\sigma_{cd}}{\phi}z_A^*$$

The equilibrium covariance price wedge term z_A^* is derived in the Appendix.

Proof. See Appendix A.1 □

Recall that firms maximize their share price minus the cost of their corporate choice. There are two key takeaways from Lemma 4.2. First, the shares of d-firms that directly engage in real-asset reform trade at a premium ($\omega_R^* < 0$) and the shares of a d-firm that operates an unreformed d-asset trade at a discount ($\omega_U^* > 0$) compared to the benchmark without sustainable investing constraints. The sign of the equilibrium price wedge for secondary-traders ω_S^* is ambiguous, and depends on the relative ratio of impact-aligned to unconstrained investors compared to $K/(K-T)$. When the population of impact investors is sufficiently small, $\omega_S^* < 0$. In this case, shares of secondary seller firms trade at a lower premium than R firms. Additionally, a higher share price implies a lower cost of capital in equilibrium.[10] Thus, the cost of capital of a secondary seller firm (direct reform firm) decreases as the transaction cost T (cost of reform K) increases. This is due to the fact that, in equilibrium, the increase in the share price for a S or R firm must offset the cost of trade T or the cost of reform K. Thus, a higher share of v or n investors results in a *higher* equilibrium share price (and lower cost of capital) for R firms.

The second takeaway is that the price wedge faced by the c-firms persists even in the absence of sustainable investing cost terms T or K. This is because the wedge comes from imperfect risk-sharing: as m-investors will only buy shares in firms that engage in environmental reform, under a market for real-asset reallocation only, these investors do not allocate any resources in the c-firm sub-market. The impact of such imperfect risk-sharing is larger when the c-asset cash flow is more volatile and when risk-tolerance τ is small, which gets directly reflected in the share prices.

[10] The cost of capital of a firm f can be formally defined as $\frac{\mu_f}{P_f} - 1$

4.2 Co-Existing Markets for Real-Asset Reallocation and Transformation

Now we study competitive equilibria upon the introduction of a separate market mechanism where firms trade contracts over real-asset transformation actions. Because there is no heterogeneity in the cost of reform actions, it must be that all posted transformation contracts request the same c-firm contribution in equilibrium. We can focus on the case in which, when a d-firm decides to post a transformation-contract, it does so at the clearing transformation-contract price $\pi \in \mathbb{R}$. It turns out that this market clearing price is critical to determine the structure of the equilibrium outcomes. We distinguish between a *surplus-extraction equilibrium*, where the transformation contract price is strictly larger than the cost of reform $\pi > K$, and a *cost-sharing equilibrium*, where $\pi \leq K$. In a cost-sharing equilibria, d-firms contribute a to finance the real-asset reform action, but make no impact claims. We next discuss these two equilibria:

(i) **Surplus-Extraction Equilibrium**: Suppose that the market clearing contract price is weakly larger than the cost of reform, meaning that the d-firm can extract surplus from the c-firm beyond the reform cost. Then, it must be the case that *all pollutive assets get reformed*. This is because such a contract price makes the corporate choice of operating an unreformed pollutive asset strictly dominated by posting a transformation contract. Moreover, *secondary trading unravels*. Secondary trading enables v-eligibility at a cost of T, whereas a transformation contract also makes the d-firm v-eligible, albeit with an additional revenue of $\pi - K \geq 0$.

(ii) **Cost-Sharing Equilibrium**: Suppose that the market clearing contract price is strictly smaller than the cost of reform. Then trading a transformation contract implements a cost-sharing mechanism because the issuing d-firm pays a reminder of $K - \pi \geq 0$ to finance the reform action and fulfill the contract. This implies a lower bound on the contract clearing price, $\pi \geq K - T$, for transformation contracts get effectively traded in equilibrium. To see why this is the case, note that if $\pi = K - T$, the d-firm contributes a payment of T towards the reform action and cedes the impact claim to the c-firm, becoming *indirectly* reformed. At this price, posting a transformation contract is *equivalent* to secondary trading – in such equilibria, both corporate choices co-exist. For any clearing price π strictly larger than $K - T$, secondary trading unravels: selling the d-asset in the secondary-market becomes

strictly dominated by posting a transformation contract. Finally, note that when $\pi < K$ there is no clear dominance between entering the transformation market and operating an unreformed pollutive asset. Thus, in a cost-sharing equilibrium *all pollutive assets do not necessarily get reformed.*

We analyze each case separately to account for the specific corporate-choice structures.

4.2.1 Surplus-Extraction Equilibrium

Suppose $\pi^* > K$. Since a transformation market is available, c-firms now also face a corporate choice problem. They can choose not participate in the transformation market (A-choice) or to enter a transformation contract allowing them to claim impact, in which case they contribute π towards the reform action (α-choice). In this equilibrium, the corporate choices which are sustained for d-firms are also two. They either directly engage in real-asset reform or participate in the transformation market by posting a transformation contract and becoming indirectly reformed (β-choice). Operating an unreformed pollutive asset, as well as engaging in secondary trading, are strictly dominated choices in equilibrium, so that $N_U^* = N_S^* = 0$. Because the number of c and d-firms is fixed, the corporate choice market clearing conditions in equilibrium become:

$$N_A^* + N_\alpha^* = N_c, \quad N_R^* + N_\beta^* = N_d. \tag{4}$$

Consider the optimal corporate choice faced by a c-firm. Again, because entering a transformation contract involves a positive cost for the firm, this action will only satisfy firm incentive-compatibility when the resulting share price weakly offsets the action's cost. For the d-firm, reducing environmental impact entails a direct cost as well as an *opportunity cost*: keeping the option to reduce implies that the d-firm foregoes a potential payoff of $\pi - K \geq 0$ in the transformation market. In equilibrium, the respective changes in share prices must equal these costs to guarantee a no entry & no exit condition across sub-markets. It follows that in equilibrium we must have:

$$P_\alpha^* - \pi^* = P_A^*, \quad P_R^* - K = P_\beta^* + \pi^* - K, \tag{5}$$

where P_β and P_α correspond to the share price of a firm making corporate choices β and α respectively. Compared to (2), the transformation market implies that the change in share price is not ex-ante fixed as it depends on the clearing price of the transformation market π, determined at equilibrium.

Consider, now, the optimal portfolio allocation faced by investors. From (5) we get that prices can be sorted in equilibrium: $P_\alpha > P_A$ and $P_R > P_\beta$. This implies that n and v-investors will only buy shares from firms of type A and β.[11] Conversely, m-investors will only buy shares from firms of type α and R due to the impact constraint. Market clearing then imposes the following conditions:

$$I_n x^*_{n\beta} + I_v x^*_{v\beta} = N^*_\beta, \quad I_m x^*_{mR} = N^*_R \tag{6a}$$

$$I_n x^*_{nA} + I_v x^*_{vA} = N^*_A, \quad I_m x^*_{m\alpha} = N^*_\alpha \tag{6b}$$

Compared to a case where there is only a market for real-asset reallocation (c.f. (3)), clearing conditions in (6) show a different sub-market structure. In the surplus-extraction equilibrium, there are two c-firm sub-markets, $(n \cup v, A)$ and (m, α) and two d-firm sub-markets, $(n \cup v, \beta)$ and (s, R). Our next Lemma characterizes equilibrium corporate choices under this structure as a function of the transformation-market clearing price π^*:

Lemma 4.3. *Suppose $\pi^* > K$. In equilibrium, investors participate in two c-firm and d-firm sub-markets: (1) unconstrained and value-aligned investors only buy A-firm and β-firm shares, and (2) impact-aligned investors only buy α-firm and R-firm shares. The number of c and d-firms choosing each corporate action $f \in \{A, \alpha\}$ and $\ell \in \{\beta, R\}$ can be written as*

$$N^*_f = \max\{0, \lambda_\theta(N_c + \delta^*_f)\}, \quad N^*_\ell = \max\{0, \lambda_\theta(N_d + \delta^*_\ell)\}$$

[11] In other words, the allocation problems of n and v investors become indistinguishable in the options-trading equilibrium.

for $(\theta, f) \in (n \cup v, A), (m, \alpha)$ *and* $(\theta, \ell) \in (n \cup v, \beta), (m, R)$*. Allocative distortions are given by:*

$$\delta_A^* = I_m \pi^* \frac{\tau}{\phi}(\sigma_d^2 - \sigma_{cd}), \qquad \delta_\beta^* = I_m \pi^* \frac{\tau}{\phi}(\sigma_c^2 - \sigma_{cd}),$$
$$\delta_\alpha^* = -I_{nv} \pi^* \frac{\tau}{\phi}(\sigma_d^2 - \sigma_{cd}), \qquad \delta_R^* = -I_{nv} \pi^* \frac{\tau}{\phi}(\sigma_c^2 - \sigma_{cd}),$$

and satisfy $\sum_{(\theta,f)} \lambda_\theta \delta_f^* = 0$ *and* $\sum_{(\theta,\ell)} \lambda_\theta \delta_\ell^* = 0$*.*

Proof. See Appendix A.2. □

The unconstrained and value-aligned investors hold equivalent positions in equilibrium in both c and d-firm sub-markets. This implies that, in the surplus-extraction equilibrium, the value-aligned investing constraint is slack. Moreover, unconstrained and value-aligned investors buy more shares than impact-aligned investors across both c and d-firm sub-markets, as implied by the sign of the allocative distortions in Lemma 4.3. Note that the magnitude of the allocative distortion terms depends directly on the transformation-contract clearing price π^*, with both quantities jointly determined in equilibrium. Holding everything else fixed, a larger transformation-contract price results in a higher number of *indirectly reformed firms* and a lower number of *directly reformed firms* — however, as long as $\pi^* > K$, all pollutive assets get reformed through either channel. A corollary and important point of departure from Lemma 4.1 is that the marginal effect of an additional value-aligned investor has no effect on the number of assets that become reformed in equilibrium. However, our next Lemma shows that adding an additional value-aligned investor decreases the transformation-contract clearing price, lowering the volume of trades in the real-asset transformation market.

Lemma 4.4. *In a surplus-extraction equilibrium, the transformation-contract clearing price is:*

$$\pi^* = \Big[\frac{N_c}{I_{nv}} - \frac{N_d}{I_m}\Big] \frac{\phi}{\psi \tau}$$

where $\psi := \sigma_c^2 + \sigma_d^2 - 2\sigma_{cd} \geq 0$*. Thus, a surplus-extraction equilibrium exists if and only if*

$$\frac{N_c}{I_{nv}} \geq \frac{N_d}{I_m} + K \frac{\psi \tau}{\phi}.$$

Proof. See Appendix A.2 □

Holding the cash flow distributions fixed, a surplus-extraction equilibrium exists when there is a sufficiently large (small) number of c-firms (d-firms) and m-investors ($n \cup v$-investors), the cost of reform K is sufficiently small, and investor are sufficiently risk averse. Combining Lemma 4.4 with Equation (5) we see that the transformation-contract price corresponds to the share price premia from m-eligibility, equivalent across c and d-firm sub-markets:

$$P_\alpha^* - P_A^* = P_R^* - P_\beta^* = \frac{\phi}{\psi\tau}\left[\frac{N_c}{I_{nv}} - \frac{N_d}{I_m}\right]$$

Intuitively, the market clearing transformation-contract price captures two effects:

(i) Enhancement in risk-sharing from improved diversification.

(ii) Supply-demand balance of reform actions in the transformation market.

For the risk-sharing component, note that the equilibrium transformation-contract price is the largest when risk tolerance τ is low and cash flow variance is high. Moreover, the price increases with the number of m-investors, and decreases with the number of v and n-investors. As the number of investors following an impact-aligned strategy becomes large, m-eligibility is more valuable for c-firms who are ex-ante restricted to investments only from value-aligned or unconstrained investors. The diversification enhancements yield higher equilibrium share prices, which make transformation contracts desirable for c-firms. The second key determinant for the transformation-contract price is the supply and demand balance of transformation opportunities in the transformation-market. In particular, the clearing price decreases with the number of d-firms as this implies a higher ex-ante availability of reform actions in the market. Conversely, the clearing price increases with the number of c-firms, as this creates more competition between potential buyers in the transformation-market.

Finally, we characterize the surplus-extraction equilibrium share prices:

Lemma 4.5. *In the surplus-extraction equilibrium, the share price of c-firm type f and a d-firm type ℓ can be written as a common risk-adjusted term minus a price wedge:*

$$P_f^* = \mu_c - R_c - \omega_f^*, \quad P_\ell^* = \mu_d - R_d - \omega_\ell^*$$

with equilibrium price wedges given by

$$\omega_A^* = \omega_\beta^* = \lambda_m \pi^* = \lambda_m \Big[\frac{N_c}{I_{nv}} - \frac{N_d}{I_m}\Big] \frac{\phi}{\psi\tau};$$
$$\omega_\alpha^* = \omega_R^* = -\lambda_{nv} \pi^* = -\lambda_{nv} \Big[\frac{N_c}{I_{nv}} - \frac{N_d}{I_m}\Big] \frac{\phi}{\psi\tau}$$

Proof. See Appendix A.2 □

From Lemma 4.5 we see that the shares of d-firms that engage in environmental reform trade at a premium which is larger than the premium under a market for reallocation only (c.f. Lemma 4.2). Note that this premium is the same as the one faced by the c-firms that participate in the transformation market ($\omega_\alpha^* = \omega_R^*$) — and this symmetry also holds for the case of the d-firms that sell reform-options in the transformation market and the passive c-firms, $\omega_A^* = \omega_\beta^*$, with the caveat that the shares from these firms trade at a discount ($\omega_\beta^* < 0$). Importantly, note that as opposed to Lemma 4.2, in the surplus-extraction equilibrium the share price wedges do not depend on the cost of reform K nor the cost of secondary trades T. As long as the conditions in Lemma 4.4 hold, these costs get fully absorbed by the c-firms, and the share price wedges are instead driven by the risk-sharing effects of the transformation market.

4.2.2 Cost-Sharing Equilibrium

Fix $\pi \leq K$. In this case, posting a transformation contract entails a positive cost for d-firms, which must contribute the non-negative remainder payment of $K - \pi \geq 0$ to finance the reform action. Hence, the transformation contract implements a cost-sharing mechanism between the d-firm and the c-firm, by which they jointly cover the full cost of reform. The c-firm benefits by claiming impact, becoming m-eligible. The d-firm, benefits because it ceases to operate an unreformed pollutive asset upon reform, which makes it v-eligible. It follows that to sustain a positive volume of contract trades in the transformation market, the c-firm contribution π can never fall below a threshold of $\underline{\pi} := K - T$. For $\pi < \underline{\pi}$, secondary trading strictly dominates posting a transformation contract for the d-firm, because it enables v-eligibility at a strictly lower cost. In what follows, we focus on the interior case $\pi \in (\underline{\pi}, K)$ as it provides a more intuitive description of the equilibrium structure. We defer the corner cases — namely where $\pi = \underline{\pi}$ and $\pi = K$ — to the Appendix.

In a cost-sharing equilibrium with $\pi \in (\underline{\pi}, K)$, the c-firm optimally decides between the A-choice and the α-choice. The d-firms decide between the R-choice and the β-choice, but as opposed to a surplus-extraction equilibrium, now operating the unreformed d-asset may also be optimal in equilibrium. Because $\pi > \underline{\pi}$, the secondary market unravels.[12] Thus, a key difference with the surplus-extraction equilibrium is that not all pollutive assets are necessarily getting reformed. The equilibrium corporate choice market clearing conditions become:

$$N_A^* + N_\alpha^* = N_c, \quad N_U^* + N_R^* + N_\beta^* = N_d. \tag{7}$$

The no-entry & no-exit condition across c and d-firm sub-markets imply that the following relationships across share prices must hold in equilibrium for any $\pi \in (\underline{\pi}, K)$:

$$P_\alpha^* - \pi = P_A^*, \qquad P_R^* - K = P_U^*, \qquad P_\beta^* - (K - \pi) = P_U^*. \tag{8}$$

Note that these conditions extend those of a surplus-extraction equilibrium to account for the firms that choose to operate unreformed d-assets. Equation (8) imposes a condition which mirrors that of a case where there is only a market for real-asset reallocation (c.f. (2)) with the transformation market substituting secondary trading. Thus, for the d-firms we have that $P_U^* < P_\beta^* < P_R^*$, and for the c-firms, we have that $P_\alpha^* > P_A^*$. This implies that n-investors only buy shares from U and A firms, v-investors only buy shares from A and β firms, and m-investors only buy shares from α and R firms. At the portfolio allocation level, market clearing imposes the following conditions in equilibrium:

$$I_n x_{nU}^* = N_U^*, \quad I_v x_{v\beta}^* = N_\beta^*, \quad I_m x_{mR}^* = N_R^* \tag{9a}$$

$$I_n x_{nA}^* + I_v x_{vA}^* = N_A^*, \quad I_m x_{m\alpha}^* = N_\alpha^* \tag{9b}$$

From (9) we observe that, in this case, the number of sub-markets is five. The sub-market structure for c-firms is that of a surplus-extraction equilibrium: $(n \cup v, A)$ and (m, α). For d-firm, the sub-market structure is that of a case where there is only a market for real-asset reallocation — (n, U),

[12] Note that when the transformation-contract price hits the lower bound, $\pi = K - T$, it might be the case that some d-firms also engage in secondary trading (S) – we discuss this case in the Appendix

(v, β) and (m, R) — but the transformation market effectively replaces secondary trading.

Our next Lemma characterizes equilibrium corporate choices for any transformation-contract clearing price $\pi^* \in (\underline{\pi}, K)$:

Lemma 4.6. *Suppose $\pi^* \in (\underline{\pi}, K)$. In equilibrium, investors participate in five sub-markets: (1) unconstrained and value-aligned investors buy shares in A-firms, (2) impact-aligned investors buy shares in α-firms, (3) unconstrained investors buy shares in U-firms, (4) value-aligned investors buy shares in β-firms, and (5) impact-aligned investors buy shares in R-firms. The number of c and d-firms making each corporate choice $f \in \{A, \alpha\}$ can be written as*

$$N_A^* = \max\{0, \lambda_{nv}(N_c + \delta_{nA}^* + \delta_{vA}^*)\}, \quad N_\alpha^* = \max\{0, \lambda_m(N_c + \delta_{m\alpha}^*)\}.$$

Allocative distortions are given by:

$$\delta_{nA}^* = \frac{\tau}{\phi}[\sigma_d^2 I_m \pi^* - \sigma_{cd}\xi_{nA}^*], \quad \delta_{vA}^* = \frac{\tau}{\phi}[\sigma_d^2 I_m \pi^* - \sigma_{cd}\xi_{vA}^*], \quad \delta_{m\alpha}^* = \frac{\tau}{\phi}[-\sigma_d^2 I_{nv}\pi^* + \sigma_{cd}\xi_{m\alpha}^*],$$

with $\sum_{(\theta,f)} I_\theta \delta_f^* = 0$. Equilibrium covariance distortion terms ξ_{if}^* are derived in the Appendix. Similarly, the number of d-firms making each corporate choice $\ell \in \{U, \beta, R\}$ can be written as:

$$N_\ell^* = \max\{0, \lambda_\theta(N_d + \delta_\ell^*)\}.$$

Allocative distortions are given by:

$$\begin{aligned}
\delta_U^* &= \frac{\tau}{\phi}[\sigma_c^2\big(I_v(K - \pi^*) + I_m K\big) - \sigma_{cd}\xi_U^*] \\
\delta_\beta^* &= \frac{\tau}{\phi}[\sigma_c^2\big(I_m\pi^* - I_n(K - \pi^*)\big) - \sigma_{cd}\xi_\beta^*] \\
\delta_R^* &= \frac{\tau}{\phi}[\sigma_c^2\big(-I_v\pi^* - I_n K\big) + \sigma_{cd}\xi_R^*]
\end{aligned}$$

and satisfy $\sum_{(\theta,\ell)} I_\theta \delta_\ell^* = 0$. Equilibrium covariance distortion terms ξ_ℓ^* are derived in the Appendix.

Proof. See Appendix A.3. □

A key difference in Lemma 4.6 as compared to Lemma 4.1 is that in the cost-sharing equilibrium, value-aligned investors incentivize d-firms to engage in the real-asset transformation market via the

cost-sharing mechanism instead of using the secondary trading channel, which unravels as long as $\pi^* > \underline{\pi}$. The c-firm corporate choices, on the other hand, follow a similar structure to the surplus-extraction equilibrium (c.f. Lema 4.3) with N_α^* firms becoming m-eligible via indirect real-asset reforms. Again, the effect of adding one more value-aligned investor affects the transformation-contract price, which we discuss in our next Lemma.

Lemma 4.7. *In the cost-sharing equilibrium, the transformation-contract price is given by*

$$\pi^* = \frac{\psi}{\psi + \gamma_1} \frac{\phi}{\tau\psi} [\frac{N_c}{I_v} - \frac{N_d}{I_m}] + \frac{\gamma_2}{\psi + \gamma_1} K$$

where

$$\gamma_1 := \lambda_n (\frac{\sigma_c^2}{\lambda_m} + \frac{\sigma_d^2}{\lambda_v}); \quad \gamma_2 := \lambda_n (\frac{\sigma_c^2}{\lambda_m} + \frac{\sigma_{cd}}{\lambda_v})$$

An interior cost-sharing equilibrium exists if and only if $\pi^* \in (\underline{\pi}, K)$.

Proof. See Appendix A.3. □

Note that the transformation contract price differs from that in the surplus-extraction equilibrium in Lemma 4.4. However, the result in Lemma 4.7 collapses to that in Lemma 4.4 when the population of unconstrained investors goes to zero. This shows the differential role of n-investors in each case: in a surplus-extraction equilibrium, n-investors buy shares of d-firms that engage in the market for real-asset transformation due to their increased revenue from the transformation contract. Conversely, in a cost-sharing equilibrium, when a d-firm posts and accepts a transformation contract , this precludes participation from n-investors, which can make the cost-sharing mechanism unattractive. On the other hand, comparative statics with respect to value-aligned and impact-aligned investors, as well as c-firms and d-firms are equivalent: the clearing payment goes down if there are more m-investors and c-firms, and goes up if there are more v-investors or d-firms.

Lemma 4.8. *In the cost-sharing equilibrium, the share price of c-firms type f and d-firms type ℓ can be written as a common risk adjusted term minus a price wedge*

$$P_f^* = \mu_c - R_c - \omega_f^*, \quad P_\ell^* = \mu_d - R_d - \omega_\ell^*$$

with equilibrium price wedges for the two c-firm types given by

$$\omega_A^* = \lambda_m \pi, \quad \omega_\alpha^* = -\lambda_{nv} \pi,$$

and equilibrium price wedge for the three d-firm types given by

$$\omega_U^* = \lambda_m K + \lambda_v (K - \pi^*),$$
$$\omega_\beta^* = \lambda_m \pi^* - \lambda_n (K - \pi^*),$$
$$\omega_R^* = -\lambda_v \pi^* - \lambda_n K.$$

Proof. See Appendix A.3. □

The result in Lemma 4.8 again shows that the c-firm sub-market structure resembles the surplus-extraction equilibrium: the price wedges for c-firms follow the same relationship to the transformation-contract price as in Lemma 4.5. On the other hand, price wedges for d-firms follow the case of a market for real-asset reallocation, with the transaction cost T replaced by d-firm transformation contract payment $K - \pi^*$, which is weakly smaller than T.

5 Welfare Effects of Co-Existing Markets

Having characterized competitive equilibrium outcomes under each market specification and contracting structures, we turn our attention to welfare effects. We focus on three aspects: real-asset reform and secondary trading, firm surplus and investor surplus. We derive the welfare results for a case featuring symmetric firm types ($N_c = N_d = N$), and uncorrelated real-asset returns with equal variance. Our main insights hold when we account for covariance and firm type asymmetries. We discuss the effect of removing these assumptions in the Appendix.

First, consider the perspective of a social planner. When pollution externalities are sufficiently large, real-asset reform is socially efficient, but it is seldom the case that firms or markets internalize such effects. In our model, neither firms nor investors internalize any damages from pollution. Impact-aligned investors can enhance efficiency by providing incentives for share-price maximizing firms to engage in real-asset reform—but in the case of a market for real-asset reallocation only,

these incentives are curbed by the presence of value-aligned investors, who instead favor d-firms that engage in secondary market trades. Our first theorem discusses how the introduction of a separate market for real-asset transformation can strengthen the incentives to own and transform pollutive assets — and change the marginal effect of value-aligned investor participation in equilibrium.

Theorem 5.1 (Incentives to Own and Transform Pollutive Assets)**.** *In the case of symmetric firms and uncorrelated real-asset returns with equal variance:*

- *The introduction of a transformation market increases real-asset reform and decreases secondary trades.*
- *The introduction of a transformation market changes the direction of the marginal effect of adding one value-aligned investor for sufficiently many firms.*

Proof. See Appendix A.4 □

Theorem 5.1 demonstrates that real-asset reform incentives are most effective under co-existing markets for real-asset reallocation and transformation. When firms are allowed to contract over reform actions, c-firms indirectly contribute towards d-asset reform under the incentive to become eligible for m-investors – a channel which is absent in the case of a market for real-asset reallocation only. In the cost-sharing equilibrium, the cost of reform is divided between a c-firm, who claims impact upon reform, and a d-firm, who ceases to operate an unreformed pollutive asset and thus has an alternative to secondary trading. For sufficiently many firms N, in both the surplus-extraction and cost-sharing equilibrium, the marginal effect of an additional v-investors is positive on the total number of reformed assets, since the d-firms prefer to engage in the reform action exchange rather than using the secondary trading channel.

Second, we turn our attention to the effect of a real-asset transformation market on firm surplus. One of the main challenges of environmental reform is that common policy approaches, such as carbon taxes and cap-and-trade programs, may impose large costs on productive firms. These costs might affect firm competitiveness or be passed through to consumers, which may pose concerns for an environmental regulator. Our second theorem illustrates that, upon the introduction of a market for real-asset transformation, there is a weak Pareto improvement in firm surplus. Importantly, the transformation market stands to benefit both c and d-firms. We extend the model to consider a

tax on secondary trades, a carbon tax and an emissions-trading scheme, and show that all of these policy alternatives can also increase real-asset reform, but produce a *decrease* in d-firm surplus.

Theorem 5.2 (Policy Comparison and Pareto-Improvement on Firm Surplus)**.** *In the case of symmetric firms and uncorrelated real-asset returns with equal variance:*

- *Introducing a separate transformation market increases d-firm surplus and c-firm surplus, yet the increase in real-asset reform may be moderate.*
- *Policies such as a tax on secondary-trades, a carbon-tax, or an emissions-trading scheme, can increase real-asset reform beyond the transformation market, yet decrease d-firm surplus.*

Proof. See Appendix A.5. □

Theorem 5.2 shows that co-existing markets not only support incentive-compatible real-asset reform, but also unambiguously enhance firm surplus. The no-entry conditions, which are consequence of the endogenous corporate choice (c.f. Equations 2, 5 and 8) imply that c and d-firms share surplus symmetrically in each equilibrium. This metric can be pinned down as the share price of a d firm operating an unreformed real asset, or passive c firm that does not take any additional actions under reallocation only and cost-sharing equilibria. The shares of d firms that operates unreformed d assets trade at a *discount* in both cases – but the discount is *smaller* under cost-sharing than under allocation only. The same is true for a passive c-firm: the share price discount is smaller under the cost-sharing equilibrium. In the surplus-extraction equilibrium, d-firm surplus can be pinned down as the equilibrium share price of a reformed firm minus the direct cost of reform. This quantity is again greater than its counterpart under reallocation only. What supports higher equilibrium share prices when reallocation and transformation markets co-exist? In our model, the transformation market provides a channel for firms to broaden investor participation at lower costs than under reallocation only. This enables more investor competition, yielding higher firm surplus. In Appendix C we study policy alternatives such as (1) a tax on secondary trades, (2) a carbon tax, and (3) an emissions-trading system. We show that although these alternatives can increase real-asset reform beyond the effects of a real-asset transformation market, they all decrease d-firm surplus. This is a key point of departure from a dedicated market for real-asset transformation.

Finally, we discuss the effects of coexisting markets on investor surplus separating across the c and d sub-markets. The increase in participation (and investor competition) brought by co-existing markets induces a surplus redistribution effect across investor-types.

Theorem 5.3 (Investor Surplus Redistribution)**.** *The introduction of a separate transformation market for reform rights has the following effects on investor surplus:*

- *In c-firm sub-markets, v-surplus decreases and m-surplus increases.*
- *In d-firm sub-markets, v-surplus increases and m-surplus decreases.*

The unconstrained n-investor surplus decreases in both sub-markets unless there is a surplus-extraction equilibrium with a sufficiently high transformation contract price.

Proof. See Appendix A.6 □

The Sharpe ratio representation of the indirect investor utility in Theorem 5.3 is useful, as it shows, for instance, that with a market for asset reallocation only, n and v investors engage in larger rent-extraction over clean sub-markets due to m-investor constraints. Because the ownership of pollution is a necessary condition for positive environmental impact, m-investors avoid participation altogether, creating an advantage for n and v investors. This advantage disappears as soon as the transformation market is introduced. Conversely, the introduction of a transformation market can enhance v surplus in dirty sub-markets, providing a cost-efficient alternative for firms to become v-eligible instead of paying secondary transaction costs. Finally, in a market for asset reallocation only, n investors (who are unconstrained) take advantage of sustainable investing constraints to engage in higher d-firm sub-market rent-extraction, but this is can be curbed with the introduction of a transformation market – unless there is a surplus-extraction equilibrium with a sufficiently high transformation-contract price, where v investor constraints are not active anymore (n and v become indistinguishable) allowing both types to exploit the m-investor constraint.

6 Concluding Remarks

How do real-asset trading decisions affect environmental reform in the presence of socially motivated investors? Our paper addresses this question by laying out a theory of co-existing markets for real-asset reallocation and transformation in equilibrium with corporate choices and portfolio allocation

decisions. We show that introducing a transformation market leads to equilibrium structures where pollutive firms extract surplus from, or share costs with, other firms who benefit from participating in real-asset reform investments. Pollutive firms achieve this by issuing transformation contracts against their pollutive real assets – which feature pollution reduction opportunities – and pricing them as the anticipated change in a firms' share price associated with the impact claim steming from the real-asset reform action. When there is not enough demand to support a surplus-extraction equilibrium, transformation contracts are used by pollutive firms as a cost-sharing mechanism. In such an outcome, the transformation contract price is weakly smaller than the cost of reform – the pollutive firm contributes a remainder payment to finance the reform action. Both mechanisms expand investor participation and decrease the use of the secondary trading channel, enhancing efficiency and increasing the number of reformed real assets. This gains are not realized in a case where there is only a market for real-asset reallocation, as value-aligned investors incentivize firms to sell their pollutive assets to private outside buyers, removing the opportunity to reduce pollution from the market.

The main insight that arises from our theory is that a dedicated market for real-asset transformation decouples the disincentives to own pollutive real assets from the incentives to reform them. Transformation contracts provide an alternative to secondary trading for those firms which do not wish to operate unreformed pollutive assets, but are also not willing to pay the full cost of reform. This market mechanism *alleviates* the cost of satisfying sustainable investors constraints – leading to a better diversification as well as higher share prices, which deliver higher firm surplus, in equilibrium. Alternative regulatory interventions, such as a carbon tax, a tax on secondary trades, or an emissions-trading system, can also increase reform and reduce secondary-trading, but do so at the expense of firm surplus, and thus might be harder to implement for a constrained environmental regulator.

Our work suggests several direction for future research. First, our results highlight reform contracts across heterogeneous firms — such as clean and pollutive firms — as relevant structures for real-asset transformation under a heterogeneous investor base. There is some incipient empirical evidence on the use of forward reform contracts for methane abatement (Palacios et al. 2025). Cooperation agreements across heterogeneous firms are frequently mentioned as valuable contractual arrangements for real-asset transformation in the oil sector. For instance, non-operated joint

ventures (NOJVs), in which public firms with environmental targets help other companies, such as state-owned oil & gas companies, to reduce their emissions intensity resemble our transformation contracts (EDF 2022). An empirical analysis of such contracts would be important to better understand how corporate incentives interact with real-asset transformation decisions.

Second, our model implies that firms who enter contracts for real-asset reform actions should see increased investor participation via two different channels. A firm engaging in indirect reform actions over real assets becomes more attractive for an investor who directly values emissions reductions (impact-aligned investing). Furthermore, a firm whose real assets get indirectly reformed becomes more attractive for investors who value low balance-sheet emissions (value-aligned investing). As each of these channels is independent, our theory suggests that joint reform contracts could offer a window into whether sustainable investors and financial intermediaries follow impact or value-aligned strategies when it comes to real-asset reallocation. What are the adopted sustainable investing strategies is ultimately an empirical question, but has important implications for the theory of sustainable finance (Landier and Lovo 2024, Oehmke and Opp 2025).

Third, our model suggests that whenever secondary trading is costly, impact and value investor's strategies are de facto aligned. However, when secondary trading is available at a cost lower than the cost of real-asset reform, the presence of value-aligned investors imposes a double negative externality: corporate reallocation decisions makes achieving impact more costly for the impact investor, and make reform less appealing for firms that own pollutive assets. In such equilibria, secondary trading effectively "*removes the option to engage with firms to reduce pollution*" in the sense of Duchin et al. 2025. In such outcomes, adopting an impact-aligned investing strategy can have a high cost and little to no impact, suggesting that less investors would be willing to engage in reform-oriented strategies. It would be relevant to empirically distinguish these cases from other settings where secondary trading might not have negative effects on investor participation or real-asset reform.

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A Proofs

A.1 Market for Real-Asset Reallocation Only

For an investor type θ buying shares in c-firm type f and d-firm type ℓ, utility is given by

$$U_\theta(x_{\theta f}, x_{\theta \ell}) = x_{\theta f}\mu_c + x_{\theta \ell}\mu_d - \frac{x_{\theta f}^2\sigma_c^2 + x_{\theta \ell}^2\sigma_d^2 + 2x_{\theta f}x_{\theta \ell}\sigma_{cd}}{2\tau} - P_f x_{\theta f} - P_\ell x_{\theta \ell}. \tag{10}$$

Let $\pi_f := \mu_c - P_f$ and $\pi_\ell := \mu_d - P_\ell$ be the mean excess return for buying the shares of c-firm making corporate choice f or d-firm making corporate choice ℓ. Taking the first-order conditions over (13) gives investor-optimality conditions:

$$\tau\pi_f = x_{if}\sigma_c^2 + x_{i\ell}\sigma_{cd} \tag{11a}$$

$$\tau\pi_\ell = x_{i\ell}\sigma_d^2 + x_{if}\sigma_{cd} \tag{11b}$$

Solving for x_{if} and $x_{i\ell}$ using $\phi := \sigma_c^2\sigma_d^2 - \sigma_{cd}^2$ yields:

$$x_{if} = \frac{\tau}{\phi}[\sigma_d^2\pi_f - \sigma_{cd}\pi_\ell], \tag{12a}$$

$$x_{i\ell} = \frac{\tau}{\phi}[\sigma_c^2\pi_\ell - \sigma_{cd}\pi_f]. \tag{12b}$$

provided that $\phi > 0$. Holding mean excess returns fixed, a higher σ_{cd} reduces diversification benefits and changes the optimal portfolio through the cross-hedging terms. While the above results apply to the unconstrained and value-aligned investor, they do not apply directly to the impact-aligned investors as they do not buy shares in c-firms. In that case, utility is given directly by:

$$U_m(x_{mR}) = x_{mR}(\mu_d - P_R) - \frac{x_{mR}^2\sigma_d^2}{2\tau}. \tag{13}$$

First-order conditions are:

$$x_{mR} = \frac{\tau}{\sigma_d^2}\pi_R \tag{14}$$

Recall that the active sub-market pairs in equilibrium are $\{n, A, U\}$, $\{v, A, S\}$ and $\{m, \emptyset, R\}$. Plugging (12) into the market clearing condition (3), using that the number of c and d-firms is fixed by

(1) yields:

$$I_n[\sigma_d^2\pi_A - \sigma_{cd}\pi_U] + I_v[\sigma_d^2\pi_A - \sigma_{cd}\pi_S] = \frac{\phi}{\tau}N_c \tag{15a}$$

$$I_n[\sigma_c^2\pi_U - \sigma_{cd}\pi_A] + I_v[\sigma_c^2\pi_S - \sigma_{cd}\pi_A] + I_m\frac{\phi}{\sigma_d^2}[\,\pi_R] = \frac{\phi}{\tau}N_d \tag{15b}$$

Because of the no entry condition (2) the following relationship between the mean excess returns holds in equilibrium:

$$\pi_S = \pi_U - T, \quad \pi_R = \pi_U - K. \tag{16}$$

We can use (16) to rewrite (15) and recover the unconstrained investor positions:

$$I_n[\sigma_d^2\pi_A - \sigma_{cd}\pi_U] + I_v[\sigma_d^2\pi_A - \sigma_{cd}\pi_U + \sigma_{cd}T] = \frac{\phi}{\tau}N_c \tag{17a}$$

$$I_n[\sigma_c^2\pi_U - \sigma_{cd}\pi_A] + I_v[\sigma_c^2\pi_U - \sigma_c^2T - \sigma_{cd}\pi_A] + I_m\frac{\phi}{\sigma_d^2}[\pi_U - K] = \frac{\phi}{\tau}N_d \tag{17b}$$

Using (12) and market clearing we can now derive the c-firm sub-market positions:

$$x_{nA}^* = \frac{1}{I_{vn}}[N_c - \frac{\tau}{\phi}I_v\sigma_{cd}T] \tag{18a}$$

$$x_{vA}^* = \frac{1}{I_{vn}}[N_c + \frac{\tau}{\phi}I_n\sigma_{cd}T] \tag{18b}$$

It will be useful to write these positions as:

$$x_{nA}^* = \frac{1}{I}[N_c + \delta_{nA}^*], \qquad \delta_{nA}^* := \frac{\lambda_m}{\lambda_{vn}}N_c - (1 + \frac{\lambda_m}{\lambda_{vn}})\frac{\tau}{\phi}I_v\sigma_{cd}T \tag{18c}$$

$$x_{vA}^* = \frac{1}{I}[N_c + \delta_{vA}^*], \qquad \delta_{vA}^* := \frac{\lambda_m}{\lambda_{vn}}N_c + (1 + \frac{\lambda_m}{\lambda_{vn}})\frac{\tau}{\phi}I_n\sigma_{cd}T \tag{18d}$$

Note that x_{nA}^* (x_{vA}^*) is decreasing (increasing) in σ_{cd}. From (16) we can derive the d-firm position expression for the value-aligned investors as a function of the unconstrained investor position:

$$x_{vS}^* = x_{nU}^* - \frac{\tau}{\phi}\sigma_c^2T \tag{19a}$$

$$x_{mR}^* = x_{nU}^* + \frac{\sigma_{cd}}{\sigma_d^2}x_{nA}^* - \frac{\tau}{\sigma_d^2}K \tag{19b}$$

From these relations we can recover d-firm sub-market investor positions:

$$x^*_{nU} = \frac{1}{I}[N_d + \frac{\tau}{\sigma_d^2} I_m K + \frac{\tau}{\phi}\sigma_c^2 I_v T - \frac{\sigma_{cd}}{\sigma_d^2} I_m x^*_{nA}] \tag{20a}$$

$$x^*_{vS} = \frac{1}{I}[N_d + \frac{\tau}{\sigma_d^2} I_m K - \frac{\tau}{\phi}\sigma_c^2 I_{nm} T - \frac{\sigma_{cd}}{\sigma_d^2} I_m x^*_{nA}] \tag{20b}$$

$$x^*_{mR} = \frac{1}{I}[N_d - \frac{\tau}{\sigma_d^2} I_{nv} K + \frac{\tau}{\phi}\sigma_c^2 I_v T + \frac{\sigma_{cd}}{\sigma_d^2} I_{nv} x^*_{nA}] \tag{20c}$$

Note that x^*_{mR} is increasing and concave in σ_{cd} for small values of σ_{cd}. From (20) we can observe that correlation terms are driven by the mean excess return in the c-firm sub-market. Becase n and v-investors allocate across d and c-firm sub-markets, a positive correlation implies that higher values of π_A lead to a smaller allocation in their respective d-firm sub-markets U and S. This indirectly benefits m-investors, as more d-firms choose to engage in real-asset reform. We can rewrite the d-firm allocations as the average position plus the allocative distortions by defining $\delta^*_\ell := I x^*_{\theta\ell} - N_d$ for all $(\theta, \ell) \in (n, U), (v, S), (m, R)$:

$$\delta^*_U := \frac{\tau}{\phi}\sigma_c^2 I_v T + \frac{\tau}{\sigma_d^2} I_m K - \frac{\sigma_{cd}}{\sigma_d^2}\,\xi^*_U, \qquad \xi^*_U =: I_m x_{nA*} \tag{21a}$$

$$\delta_{S^*} := -\frac{\tau}{\phi}\sigma_c^2 I_{nm} T + \frac{\tau}{\sigma_d^2} I_m K - \frac{\sigma_{cd}}{\sigma_d^2}\,\xi^*_S, \qquad \xi^*_S := I_m x_{nA*} \tag{21b}$$

$$\delta^*_R := \frac{\tau}{\phi}\sigma_c^2 I_v T - \frac{\tau}{\sigma_d^2} I_{nv} K + \frac{\sigma_{cd}}{\sigma_d^2}\,\xi^*_R, \qquad \xi^*_R := I_{nv} x_{nA*} \tag{21c}$$

To compute share-prices, we can use (11) to derive:

$$P^*_U = \mu_d - \frac{1}{\tau}[\sigma_d^2 x^*_{nU} + \sigma_{cd} x^*_{nA}] \tag{22a}$$

$$P^*_A = \mu_c - \frac{1}{\tau}[\sigma_c^2 x^*_{nA} + \sigma_{cd} x^*_{nU}] \tag{22b}$$

Directly replacing yields d-firm share prices:

$$P^*_U = \mu_d - R_d - \lambda_m K - \lambda_v T \tag{23a}$$

$$P^*_S = P^*_U + T = \mu_d - R_d - \lambda_m K + \lambda_{nv} T \tag{23b}$$

$$P^*_R = P^*_U + K = \mu_d - R_d + \lambda_{nv} K - \lambda_v T \tag{23c}$$

and c-firm share-prices:

$$P_A^* = \mu_c - \frac{1}{I\tau}\left[\sigma_c^2(N_c + \delta_{nA}^*) + \sigma_{cd}(N_d + \delta_U^*)\right] \tag{23d}$$

$$= \mu_c - R_c - \frac{1}{I\tau}\left[\sigma_c^2\delta_{nA}^* + \sigma_{cd}\delta_U^*\right], \tag{23e}$$

which completes the equilibrium characterization.

A.2 Surplus-Extraction Equilibria with Co-Existing Markets

Recall that active sub-markets are $(n \cup v, A, \beta)$ and (m, α, R). Taking first order conditions over active capital allocation decisions then yields investor-optimality conditions:

$$x_{\theta f}^* = \frac{\tau}{\phi}[\pi_f\sigma_d^2 - \pi_\ell\sigma_{cd}] \quad \forall\,(\theta, f, \ell) \in (n \cup v, A, \beta), (m, \alpha, R), \tag{24a}$$

$$x_{\theta \ell}^* = \frac{\tau}{\phi}[\pi_\ell\sigma_c^2 - \pi_f\sigma_{cd}] \quad \forall\,(\theta, f, \ell) \in (n \cup v, A, \beta), (m, \alpha, R). \tag{24b}$$

Plugging (24) into market-clearing conditions (4) yields:

$$I_{vn}[\sigma_c^2\pi_\beta - \sigma_{cd}\pi_A] + I_m[\sigma_c^2\pi_R - \sigma_{cd}\pi_\alpha] = N_d\frac{\phi}{\tau} \tag{25a}$$

$$I_{vn}[\sigma_d^2\pi_A - \sigma_{cd}\pi_\beta] + I_m[\sigma_d^2\pi_\alpha - \sigma_{cd}\pi_R] = N_c\frac{\phi}{\tau} \tag{25b}$$

By the no-entry & no-exit condition (5) on share prices we know that:

$$\pi_R = \pi_\beta - \pi; \quad \pi_\alpha = \pi_A - \pi, \tag{26}$$

with π the transformation-market clearing price, satisfying $\pi > K$. Using (26) to rewrite (25) in terms of π_β and π_A:

$$\sigma_c^2\pi_\beta - \sigma_{cd}\pi_A = \frac{1}{I}[N_d\frac{\phi}{\tau} + I_m\sigma_c^2\pi - I_m\sigma_{cd}\pi] \tag{27a}$$

$$\sigma_d^2\pi_A - \sigma_{cd}\pi_\beta = \frac{1}{I}[N_c\frac{\phi}{\tau} + I_m\sigma_d^2\pi - I_m\sigma_{cd}\pi] \tag{27b}$$

Then, using (24), we write the positions of n and v investors:

$$x^*_{n\beta}(\pi) := \frac{1}{I}\big[N_d + \delta^*_\beta(\pi)\big], \qquad \delta^*_\beta(\pi) := \frac{\tau}{\phi}(I_m\sigma_c^2\pi - I_m\sigma_{cd}\pi) \tag{28a}$$

$$x^*_{nA}(\pi) := \frac{1}{I}\big[N_c + \delta^*_A(\pi)\big], \qquad \delta^*_A(\pi) := \frac{\tau}{\phi}(I_m\sigma_d^2\pi - I_m\sigma_{cd}\pi) \tag{28b}$$

But the corporate choice market-clearing condition (4) implies that:

$$x^*_{mR}(\pi) = \frac{N_d}{I_m} - \frac{I_{vn}}{I_m}x^*_{n\beta}(\pi), \tag{29a}$$

$$x^*_{m\alpha}(\pi) = \frac{N_c}{I_m} - \frac{I_{vn}}{I_m}x^*_{nA}(\pi). \tag{29b}$$

Replacing and rearranging yields the m investor positions:

$$x^*_{mR}(\pi) = \frac{1}{I}\big[N_d + \delta^*_R(\pi)\big], \qquad \delta^*_R(\pi) := -\frac{\tau}{\phi}(I_{nv}\sigma_c^2\pi - I_{nv}\sigma_{cd}\pi) \tag{30a}$$

$$x^*_{m\alpha}(\pi) = \frac{1}{I}\big[N_c + \delta^*_\alpha(\pi)\big], \qquad \delta^*_\alpha(\pi) := -\frac{\tau}{\phi}(I_{nv}\sigma_d^2\pi - I_{nv}\sigma_{cd}\pi). \tag{30b}$$

It is straight forward to derive the equilibrium sub-market sizes under optimal corporate choices since $N^*_\alpha(\pi) = I_m x^*_{m\alpha}$, $N^*_A(\pi) = I_{nv}x^*_{nA}$, $N^*_R(\pi) = I_m x^*_{mR}$ and $N^*_\beta(\pi) = I_{nv}x^*_{n\beta}$. For equilibrium share prices we can use (24) to explicitly compute excess rents in equilibrium. For example, for β and A-firms:

$$\pi^*_\beta(\pi) = \frac{\sigma_d^2}{\tau}x^*_{n\beta}(\pi) + \frac{\sigma_{cd}}{\tau}x^*_{nA}(\pi) \tag{31}$$

$$\pi^*_A(\pi) = \frac{\sigma_c^2}{\tau}x^*_{nA}(\pi) + \frac{\sigma_{cd}}{\tau}x^*_{n\beta}(\pi) \tag{32}$$

It follows that:

$$P^*_\beta(\pi) = \mu_d - \frac{1}{\tau}[\sigma_d^2 x^*_{n\beta}(\pi) + \sigma_{cd}x^*_{nA}(\pi)] \tag{33a}$$

$$P^*_A(\pi) = \mu_c - \frac{1}{\tau}[\sigma_c^2 x^*_{nA}(\pi) + \sigma_{cd}x^*_{n\beta}(\pi)] \tag{33b}$$

From (28) we compute the inner sums:

$$\sigma_d^2 x_{n\beta}^*(\pi) + \sigma_{cd} x_{nA}^*(\pi) = \frac{1}{I}\left[\sigma_d^2 N_d + \sigma_{cd} N_c + \frac{\tau}{\phi}(I_m \sigma_c^2 \sigma_d^2 \pi - I_m \sigma_{cd}^2 \pi)\right] \tag{34a}$$

$$= \tau(R_d + \lambda_m \pi) \tag{34b}$$

$$\sigma_c^2 x_{nA}^*(\pi) + \sigma_{cd} x_{n\beta}^*(\pi) = \frac{1}{I}\left[\sigma_c^2 N_c + \sigma_{cd} N_d + \frac{\tau}{\phi}(I_m \sigma_c^2 \sigma_d^2 \pi - I_m \sigma_{cd}^2 \pi)\right] \tag{34c}$$

$$= \tau(R_c + \lambda_m \pi) \tag{34d}$$

Equilibrium share prices as a function of the transformation contract price for A and β-firms are:

$$P_\beta^*(\pi) = \mu_d - R_d - \omega_\beta^*(\pi), \qquad \omega_\beta^*(\pi) = \lambda_m \pi, \tag{35a}$$

$$P_A^*(\pi) = \mu_c - R_c - \omega_A^*(\pi), \qquad \omega_A^*(\pi) = \lambda_m \pi \tag{35b}$$

and the no-entry condition (5) lets us recover the equilibrium share prices for α and R-firms:

$$P_R^*(\pi) = \mu_d - R_d - \omega_R^*(\pi), \qquad \omega_R^*(\pi) = -\lambda_{nv} \pi \tag{35c}$$

$$P_\alpha^*(\pi) = \mu_c - R_c - \omega_\alpha^*(\pi), \qquad \omega_\alpha^*(\pi) = -\lambda_{nv} \pi \tag{35d}$$

Finally, we can use the transformation market clearing condition to solve for the transformation-contract clearing price. Recall that the relative market share for any subset of investor types Θ is denoted as λ_Θ. Then:

$$N_\alpha^*(\pi^*) = N_\beta^*(\pi^*) \tag{36a}$$

$$\iff I_m x_{m\alpha}^*(\pi^*) = I_{nv} x_{n\beta}^*(\pi^*) \tag{36b}$$

$$\iff \lambda_m(N_c + \delta_\alpha^*(\pi^*)) = \lambda_{nv}(N_d + \delta_\beta^*(\pi^*)) \tag{36c}$$

$$\iff \lambda_m N_c - \lambda_{nv} N_d = \lambda_{nv}\delta_\beta^*(\pi^*) - \lambda_m \delta_\alpha^*(\pi^*) = \pi^* \frac{\tau}{\phi}[\lambda_m I_{nv}(\sigma_d^2 - \sigma_{cd}) + \lambda_{nv} I_m(\sigma_c^2 - \sigma_{cd})] \tag{36d}$$

$$\iff \pi^* = \frac{\lambda_m N_c - \lambda_{nv} N_d}{\lambda_m I_{nv}(\sigma_d^2 - \sigma_{cd}) + \lambda_{nv} I_m(\sigma_c^2 - \sigma_{cd})} \frac{\phi}{\tau}, \tag{36e}$$

Simplifying the sum in the denominator yields the expression in the main body of the paper :

$$\lambda_m I_{nv}[\sigma_d^2 - \sigma_{cd}] + \lambda_{nv} I_m[\sigma_c^2 - \sigma_{cd}] = I(\lambda_m \lambda_{nv}[\sigma_c^2 + \sigma_d^2 - 2\sigma_{cd}]) = I\lambda_m \lambda_{nv}\psi \tag{36f}$$

So that the transformation-contract clearing price can be written as:

$$\pi^* = \big[\frac{N_c}{I_{nv}} - \frac{N_d}{I_m}\big]\frac{\phi}{\psi\tau} \tag{36g}$$

A.3 Co-Existing Markets: Cost-Sharing Equilibria

Recall that active sub-markets are (n, A, U), (v, A, β) and (m, α, R). First order conditions over active capital allocation decisions then yields investor-optimality conditions:

$$x^*_{\theta f} = \frac{\tau}{\phi}[\pi_f \sigma_d^2 - \pi_\ell \sigma_{cd}] \quad \forall\,(\theta, f, \ell) \in (n, A, U), (v, A, \beta), (m, \alpha, R), \tag{37a}$$

$$x^*_{\theta \ell} = \frac{\tau}{\phi}[\pi_\ell \sigma_c^2 - \pi_f \sigma_{cd}] \quad \forall\,(\theta, f, \ell) \in (n, A, U), (v, A, \beta), (m, \alpha, R). \tag{37b}$$

Combining (37) with corporate-choice clearing (7) and portfolio allocation clearing (9) yields:

$$I_n[\sigma_d^2\pi_A - \sigma_{cd}\pi_U] + I_v[\sigma_d^2\pi_A - \sigma_{cd}\pi_\beta] + I_m[\sigma_d^2\pi_\alpha - \sigma_{cd}\pi_R] = N_c\frac{\phi}{\tau} \tag{38}$$

$$I_n[\sigma_c^2\pi_U - \sigma_{cd}\pi_A] + I_v[\sigma_c^2\pi_\beta - \sigma_{cd}\pi_A] + I_m[\sigma_c^2\pi_R - \sigma_{cd}\pi_\alpha] = N_d\frac{\phi}{\tau} \tag{39}$$

The no-entry conditions (8) imply that:

$$\pi_U - (K - \pi) = \pi_\beta, \quad \pi_U - K = \pi_R, \quad \pi_A - \pi = \pi_\alpha \tag{40}$$

Using (40) to rewrite (38) in terms of π_A and π_U:

$$I_n[\sigma_d^2\pi_A - \sigma_{cd}\pi_U] + I_v[\sigma_d^2\pi_A - \sigma_{cd}(\pi_U - (K - \pi))] + I_m[\sigma_d^2(\pi_A - \pi) - \sigma_{cd}(\pi_U - K)] = N_c\frac{\phi}{\tau} \tag{41a}$$

$$I_n[\sigma_c^2\pi_U - \sigma_{cd}\pi_A] + I_v[\sigma_c^2(\pi_U - (K - \pi)) - \sigma_{cd}\pi_A] + I_m[\sigma_c^2(\pi_U - K) - \sigma_{cd}(\pi_A - \pi)] = N_d\frac{\phi}{\tau} \tag{41b}$$

We can rearrange (41) to recover the unconstrained investor positions:

$$[\sigma_d^2\pi_A - \sigma_{cd}\pi_U] = \frac{1}{I}\big(N_c\frac{\phi}{\tau} + I_m\sigma_d^2\pi - I_m\sigma_{cd}K - I_v\sigma_{cd}(K-\pi)\big) \tag{42a}$$

$$[\sigma_c^2\pi_U - \sigma_{cd}\pi_A] = \frac{1}{I}\big(N_d\frac{\phi}{\tau} + I_v\sigma_c^2(K-\pi) + I_m\sigma_c^2K - I_m\sigma_{cd}\pi\big) \tag{42b}$$

We can write the equilibrium positions as a function of π:

$$x_{nA}^*(\pi) = \frac{1}{I}\big[N_c + \frac{\tau}{\phi}[I_m\sigma_d^2\pi - I_m\sigma_{cd}K - I_v\sigma_{cd}(K-\pi)]\big]$$
$$x_{nU}^*(\pi) = \frac{1}{I}\big[N_d + \frac{\tau}{\phi}[I_v\sigma_c^2(K-\pi) + I_m\sigma_c^2K - I_m\sigma_{cd}\pi]\big]$$

From the first-order conditions now we can also derive the value-aligned investor position:

$$\begin{aligned}
x_{vA}^*(\pi) &= \frac{\tau}{\phi}[\sigma_d^2\pi_A - \sigma_{cd}\pi_\beta] = \frac{\tau}{\phi}[\sigma_d^2\pi_A - \sigma_{cd}\pi_U + \sigma_{cd}(K-\pi)] \\
&= x_{nA}^*(\pi) + \frac{\tau}{\phi}[\sigma_{cd}(K-\pi)] \\
&= \frac{1}{I}\big[N_c + \frac{\tau}{\phi}[I_m\sigma_d^2\pi - I_m\sigma_{cd}K + I_{nm}\sigma_{cd}(K-\pi)]\big] \\
x_{v\beta}^*(\pi) &= \frac{\tau}{\phi}[\sigma_c^2\pi_\beta - \sigma_{cd}\pi_A] = \frac{\tau}{\phi}[\sigma_c^2(\pi_U - (K-\pi)) - \sigma_{cd}\pi_A] \\
&= x_{nU}^*(\pi) - \frac{\tau}{\phi}[\sigma_c^2(K-\pi)] \\
&= \frac{1}{I}\big[N_d + \frac{\tau}{\phi}[-I_{nm}\sigma_c^2(K-\pi) + I_m\sigma_c^2K - I_m\sigma_{cd}\pi]\big]
\end{aligned}$$

Similarly for the impact-aligned investor:

$$\begin{aligned}
x_{m\alpha}^*(\pi) &= \frac{\tau}{\phi}[\sigma_d^2\pi_\alpha - \sigma_{cd}\pi_R] = \frac{\tau}{\phi}[\sigma_d^2(\pi_A - \pi) - \sigma_{cd}(\pi_U - K)] \\
&= x_{nA}^*(\pi) - \frac{\tau}{\phi}[\sigma_d^2\pi - \sigma_{cd}K] \\
&= \frac{1}{I}\big[N_c + \frac{\tau}{\phi}[-I_{nv}\sigma_d^2\pi + I_{nv}\sigma_{cd}K - I_v\sigma_{cd}(K-\pi)]\big] \\
x_{mR}^*(\pi) &= \frac{\tau}{\phi}[\sigma_c^2\pi_R - \sigma_{cd}\pi_\alpha] = \frac{\tau}{\phi}[\sigma_c^2(\pi_U - K) - \sigma_{cd}(\pi_A - \pi)] \\
&= x_{nU}^*(\pi) - \frac{\tau}{\phi}[\sigma_c^2K - \sigma_{cd}\pi] \\
&= \frac{1}{I}\big[N_d + \frac{\tau}{\phi}[I_v\sigma_c^2(K-\pi) - I_{nv}\sigma_c^2K + I_{nv}\sigma_{cd}\pi]\big]
\end{aligned}$$

It is straight forward to derive the sub-market equilibrium sizes under optimal corporate choices using that $N^*_\alpha(\pi) = I_m x^*_{m\alpha}$, $N^*_A(\pi) = I_n x^*_{nA} + I_v x^*_{vA}$, $N^*_R(\pi) = I_m x^*_{mR}$, $N^*_\beta(\pi) = I_v x^*_{v\beta}$ and $N^*_U(\pi) = I_n x^*_{nU}(\pi)$. This observation yields the expressions for allocative distortions and covariance terms, which we apply directly in the statement of Lemma 4.6:

$$\begin{aligned}
\delta^*_{nA} &= \frac{\tau}{\phi}[\sigma_d^2 I_m \pi - \sigma_{cd}\xi^*_{nA}], & \xi^*_{nA} &= I_m K + I_v(K-\pi)\\
\delta^*_{vA} &= \frac{\tau}{\phi}[\sigma_d^2 I_m \pi - \sigma_{cd}\xi^*_{vA}], & \xi^*_{vA} &= I_m K - I_{nm}(K-\pi)\\
\delta^*_{m\alpha} &= \frac{\tau}{\phi}[-\sigma_d^2 I_{nv} \pi + \sigma_{cd}\xi^*_{m\alpha}], & \xi^*_{m\alpha} &= I_{nv} K - I_v(K-\pi),\\
\delta^*_{U} &= \frac{\tau}{\phi}[\sigma_c^2\big(I_v(K-\pi) + I_m K\big) - \sigma_{cd}\xi^*_{U}], & \xi^*_{U} &= I_m \pi,\\
\delta^*_{\beta} &= \frac{\tau}{\phi}[\sigma_c^2\big(I_m \pi - I_n(K-\pi)\big) - \sigma_{cd}\xi^*_{\beta}], & \xi^*_{\beta} &= I_m \pi,\\
\delta^*_{R} &= \frac{\tau}{\phi}[\sigma_c^2\big(-I_v \pi - I_n K\big) + \sigma_{cd}\xi^*_{R}], & \xi^*_{R} &= I_{nv} \pi.
\end{aligned}$$

Note that holding π constant, x^*_{mR} is increasing in σ_{cd}. To compute equilibrium prices we can rearrange (42) and derive mean excess return in equilibrium. For example, for A and U-firm shares:

$$\pi^*_U(\pi) = \frac{\sigma_d^2}{\tau} x^*_{nU}(\pi) + \frac{\sigma_{cd}}{\tau} x^*_{nA}(\pi) \tag{46a}$$

$$\pi^*_A(\pi) = \frac{\sigma_c^2}{\tau} x^*_{nA}(\pi) + \frac{\sigma_{cd}}{\tau} x^*_{nU}(\pi) \tag{46b}$$

It follows that:

$$P^*_U(\pi) = \mu_d - \frac{1}{\tau}[\sigma_d^2 x^*_{nU}(\pi) + \sigma_{cd} x^*_{nA}(\pi)] \tag{47a}$$

$$P^*_A(\pi) = \mu_c - \frac{1}{\tau}[\sigma_c^2 x^*_{nA}(\pi) + \sigma_{cd} x^*_{nU}(\pi)] \tag{47b}$$

Computing the inner sums:

$$\sigma_d^2 x^*_{nU}(\pi) + \sigma_{cd} x^*_{nA}(\pi) = \tau R_d + \frac{\sigma_d^2\, \delta^*_U + \sigma_{cd}\, \delta^*_{nA}}{I} \tag{48a}$$

$$\sigma_c^2 x^*_{nA}(\pi) + \sigma_{cd} x^*_{nU}(\pi) = \tau R_c + \frac{\sigma_c^2\, \delta^*_{nA} + \sigma_{cd}\, \delta^*_U}{I} \tag{48b}$$

We can write the prices in closed form as follows:

$$P_U^*(\pi) = \mu_d - R_d - \lambda_m K - \lambda_v(K - \pi) \tag{49a}$$

$$P_\beta^*(\pi) = P_U^*(\pi) + K - \pi = \mu_d - R_d - \lambda_m K + \lambda_{nm}(K - \pi) \tag{49b}$$

$$P_R^*(\pi) = P_U^*(\pi) + K = \mu_d - R_d + \lambda_{nv} K - \lambda_m(K - \pi), \tag{49c}$$

$$P_A^*(\pi) = \mu_c - R_c - \lambda_m \pi \tag{49d}$$

$$P_\alpha(\pi) = \mu_c - R_c + \lambda_{nv}\pi \tag{49e}$$

Finally the market clearing condition for the transformation market imposes:

$$N_\beta^*(\pi) = N_\alpha^*(\pi) \tag{50a}$$

$$\iff I_v x_{v\beta}^*(\pi) = I_m x_{m\alpha}^*(\pi) \tag{50b}$$

Expanding the left side yields:

$$\lambda_v[N_d + \frac{\tau}{\phi}[\sigma_c^2\big(I_m\pi - I_n(K - \pi)\big) - \sigma_{cd} I_m \pi]] \tag{50c}$$

Expanding the right side yields:

$$\lambda_m[N_c + \frac{\tau}{\phi}[-\sigma_d^2 I_{nv}\pi + \sigma_{cd}\big(I_n K + I_v \pi\big)]] \tag{50d}$$

We can define:

$$C(\pi) = [I_n\sigma_c^2 K - \pi(I_n\sigma_c^2 + I_m\sigma_c^2 - I_m\sigma_{cd})] \tag{50e}$$

$$D(\pi) = [I_n\sigma_{cd} K - \pi(I_{nv}\sigma_d^2 - I_v\sigma_{cd})] \tag{50f}$$

The market clearing condition becomes:

$$\frac{\phi}{\tau}[\lambda_v N_d - \lambda_m N_c] = \lambda_m D(\pi) + \lambda_v C(\pi), \tag{50g}$$

which yields:

$$\frac{\phi}{\tau}[\lambda_v N_d - \lambda_m N_c] - I_n(\lambda_m \sigma_{cd} K + \lambda_v \sigma_c^2 K) = -\pi[\lambda_m(I_{nv}\sigma_d^2 - I_v \sigma_{cd}) + \lambda_v(I_n \sigma_c^2 + I_m \sigma_c^2 - I_m \sigma_{cd})]. \tag{50h}$$

From which we get that:

$$\pi = \frac{\frac{\phi}{\tau}[\lambda_m N_c - \lambda_v N_d] + I_n(\lambda_m \sigma_{cd} K + \lambda_v \sigma_c^2 K)}{[\lambda_m(I_{nv}\sigma_d^2 - I_v \sigma_{cd}) + \lambda_v(I_n \sigma_c^2 + I_m \sigma_c^2 - I_m \sigma_{cd})]} \tag{51}$$

Note that the denominator can be written as:

$$I[\lambda_m \lambda_{nv} \sigma_d^2 - \lambda_m \lambda_v \sigma_{cd} + \lambda_v \lambda_n \sigma_c^2 + \lambda_v \lambda_m \sigma_c^2 - \lambda_v \lambda_m \sigma_{cd}] \tag{52}$$

$$= I[\lambda_m \lambda_n \sigma_d^2 + \lambda_m \lambda_v \sigma_d^2 + \lambda_v \lambda_n \sigma_c^2 + \lambda_v \lambda_m \sigma_c^2 - 2\lambda_v \lambda_m \sigma_{cd}] \tag{53}$$

$$= I[\lambda_m \lambda_v(\sigma_d^2 + \sigma_c^2 - 2\sigma_{cd}) + \lambda_m \lambda_n \sigma_d^2 + \lambda_v \lambda_n \sigma_c^2] \tag{54}$$

$$= I[\lambda_m \lambda_v \psi + \lambda_m \lambda_n \sigma_d^2 + \lambda_v \lambda_n \sigma_c^2] \tag{55}$$

$$= I\lambda_m \lambda_v[\psi + \frac{\lambda_n}{\lambda_v}\sigma_d^2 + \frac{\lambda_n}{\lambda_m}\sigma_c^2] \tag{56}$$

$$= I\lambda_m \lambda_v[\psi + \lambda_n\big(\frac{\sigma_d^2}{\lambda_v} + \frac{\sigma_c^2}{\lambda_m}\big)] \tag{57}$$

$$= I\lambda_m \lambda_v[\psi + \gamma_1] \tag{58}$$

And the second numerator can be written as:

$$KI_n(\lambda_m \sigma_{cd} + \lambda_v \sigma_c^2) = K\lambda_m \lambda_v I\lambda_n(\frac{\sigma_c^2}{\lambda_m} + \frac{\sigma_{cd}}{\lambda_v}) \tag{59}$$

$$= K\lambda_m \lambda_v I\gamma_2 \tag{60}$$

This yields:

$$\pi^* = \frac{\psi}{\psi + \gamma_1}\frac{\phi}{\tau\psi}[\frac{N_c}{I_v} - \frac{N_d}{I_m}] + \frac{\gamma_2}{\psi + \gamma_1}K \tag{61}$$

Note that the first term in π is decreasing in σ_{cd}^2 and the second term is increasing in σ_{cd}.

A.4 Proof of Theorem 5.1

In the case of a market for real-asset reallocation only, denoted with (ro), the number of reformed real assets in equilibrium is given by:

$$A_R^{*(\text{ro})} = N_R^{*(ro)} = \lambda_m(N + \delta_R^{*(ro)}) = \lambda_m N - \lambda_m \frac{\tau}{\sigma^2} I_v(K - T) - \lambda_m \frac{\tau}{\sigma^2} I_n K$$

In the case where markets for real-asset reallocation and transformation coexist, we must consider all regimes. We denote the case of a surplus-extraction equilibrium with (ts). In this case, all pollutive assets get reformed:

$$A_R^{*(\text{ts})} = N > A_R^{*(ro)}$$

and the difference is given by:

$$A_R^{*(\text{ts})} - A_R^{*(\text{ro})} = \lambda_{nv} N + \frac{\tau}{\sigma^2} \lambda_m \big(I_v(K - T) - I_n K \big) > 0.$$

We denote the case of a cost-sharing equilibrium with (tc). In this case, the number of reformed real assets is the sum of the directly reformed and indirectly reformed assets in equilibrium:

$$\begin{aligned} A_R^{*(\text{tc})} &= N_\beta^{*(\text{tc})} + N_R^{*(\text{tc})} \\ &= \lambda_v(N + \frac{\tau}{\sigma^2}(I_m \pi^{*(\text{tc})} - I_n(K - \pi^{*(\text{tc})})) + \lambda_m(N - \frac{\tau}{\sigma^2}(I_v \pi^{*(\text{tc})} + I_n K)) \\ &= \lambda_{vm} N - \frac{\tau}{\sigma^2} I_n \big(\lambda_v(K - \pi^{*(\text{tc})}) + \lambda_m K \big) \\ &= \lambda_m N - \lambda_m \frac{\tau}{\sigma^2} I_n K + \lambda_v N - \frac{\tau}{\sigma^2} \lambda_v I_n (K - \pi^{*(\text{tc})}), \end{aligned}$$

and the difference is given by:

$$A_R^{*(\text{tc})} - A_R^{*(\text{ro})} = \lambda_v N - \frac{\tau}{\sigma^2} \lambda_v \, I_n K + \frac{\tau}{\sigma^2} \lambda_v \, I_n \pi^{*(\text{tc})} \geq 0$$

where the last inequality is directly implied by $N_\beta^{*(\text{tc})} \geq 0$. Secondary trades unravel in the surplus-extraction and cost-sharing equilibrium. Finally, consider the effect of adding a value-aligned investor on the number of reformed firms. First, we compare the number of unreformed firms in

(ro) and (cs):

$$N_U^{*(\text{ro})} = I_n x_{nU}^{*(\text{ro})} = \frac{I_n}{I}\big[N + \frac{\tau}{\sigma^2}\big(I_m K + I_v T\big)\big]$$
$$N_U^{*(\text{cs})} = I_n x_{nU}^{*(\text{cs})}(\pi^{*(\text{cs})}) = \frac{I_n}{I}\big[N + \frac{\tau}{\sigma^2}\big(I_v(K - \pi^{*(\text{cs})}) + I_m K\big)\big]$$

Introducing an additional value-aligned investor increases the denominator by one unit which creates downward pressure in the baseline share of unreformed firms. At the same time, there is an increase in the average number of shares bought by the unconstrained investor – but this increase is weakly smaller in the cost-sharing equilibria compared to the base case for any value of $\pi^{*(\text{cs})} \in [K - T, K]$. From this, we conclude that, if there is an increase in the number of unreformed firms, this increase is larger under (ro) than under (cs). Now, consider the number of firms that engage in secondary trades in the base case, versus the number of firms that engage in the transformation market:

$$N_S^{*(\text{ro})} = I_v x_{vS}^{*(\text{ro})} = \frac{I_v}{I}\big[N + \frac{\tau}{\sigma_d^2}\big(I_m(K - T) - I_n T\big)\big]$$
$$N_\beta^{*(\text{cs})} = I_v x_{v\beta}^{*(\text{cs})}(\pi^{*(\text{cs})}) = \frac{I_v}{I}\big[N + \frac{\tau}{\sigma^2}\big(I_m \pi^{*(\text{cs})} - I_n(K - \pi^{*(\text{cs})})\big)\big]$$

Introducing an additional value-aligned investor brings the market share $\frac{I_v}{I}$ closer to 1. Moreover, the inner expression is weakly larger with the transformation market for any value of $\pi \in [K-T, K]$. Additionaly, in (ro), the real assets corresponding to firms in $N_S^{*(ro)}$ are unreformed. By contrast, in (cs), the assets owned by firms in $N_\beta^{*(\text{cs})}$ get reformed.

Note that this result does not depend on the symmetry assumption, as it holds for any number of d-firms. If the c and d-asset cash flows are correlated ($\sigma_{cd} \neq 0$), there are two cases. A large positive correlation ($\sigma_{cd} > 0$) results in n and v-investors taking smaller positions in both (ro) and (cs), which can reduce the gap across equilibria. By contrast, a large negative correlation coefficient ($\sigma_{cd} < 0$) results in n and v-investors taking larger positions in both (ro) and (cs), which can increase this gap. In this case, a sufficient condition for the result to hold is that there are sufficiently many firms in the economy to maintain an interior solution.

A.5 Proof of Theorem 5.2

Proof. First, consider aggregate firm surplus in a case where there is a market for real-asset reallocation only. Our first claim is that, for each firm type $\gamma \in \{c, d\}$, aggregate surplus can be characterized as:

$$W_c^{*(\text{ro})} = N(\mu - R - \frac{\sigma^2}{\tau}\frac{\lambda_m}{\lambda_{nv}}\frac{N}{I}), \qquad W_d^{*(\text{ro})} = N(\mu - R - \lambda_m K - \lambda_v T)$$

To see why, first, consider c-firms. Under this market structure, c-firms do not face a corporate choice problem and take no costly actions. Thus, their surplus is directly given by the share price of an A-firm, derived in Lemma 4.2. Because all c-firm shares sell at the same price, $P_A^{*(\text{ro})}$, it suffices to add this price over the N firms to obtain $W_c^{*(\text{ro})}$. On the other hand, d-firms face a corporate choice between operating an unreformed pollutive asset (U-choice), engaging in real-asset reform (R-choice) or selling their d-asset in the secondary market (S-choice). However, since optimal corporate choices are endogenous and determined at equilibrium, we have by condition (2) that *all d-firms must obtain the same equilibrium surplus.* Moreover, this quantity must be equal to the share price of a U-firm, $P_U^{*(\text{ro})}$, which in equilibrium equals the share price of a R-firm minus the cost of reform, and the share price of an S-firm minus the transaction cost. By this argument, it suffices to add the share price of a firm that operates an unreformed d-asset in Lemma 4.2 over the N firms to obtain $W_d^{*(\text{ro})}$. This concludes the proof of the characterization.

Second, note that the same logic can be applied to a case where transformation markets and reallocation markets co-exist. In the surplus-extraction equilibrium, the aggregate c-firm utility is $W_c^{*(\text{ts})} = NP_A^{*(\text{ts})}$. Because all d-firms get reformed in this equilibrium, the aggregate d-firm utility corresponds to $W_d^{*(\text{ts})} = N(P_R^{*(\text{ts})} - K)$. From Lemma 4.5 we obtain:

$$W_c^{*(\text{ts})} = N(\mu - R - \lambda_m \pi^{*(\text{ts})}), \qquad W_d^{*(\text{ts})} = N(\mu - R - \lambda_m K + \lambda_{nv}(\pi^{*(\text{ts})} - K))$$

Similarly, in the cost-sharing equilibrium, c-firm and d-firm utility are given respectively by $W_c^{*(\text{tc})} = NP_A^{*(\text{tc})}$ and $W_d^{*(\text{tc})} = NP_U^{*(\text{tc})}$. From Lemma 4.8 we obtain:

$$W_c^{*(\text{tc})} = N(\mu - R - \lambda_m \pi^{*(\text{tc})}), \qquad W_d^{*(\text{tc})} = N(\mu - R - \lambda_m K - \lambda_v(K - \pi^{*(\text{tc})}))$$

Finally, we can compare surplus across equilibria. For d-firms, note that $W_d^{*(\text{ts})} > W_d^{*(\text{tc})}$. Moreover, for any $\pi^{*(\text{tc})} \in [K-T, K]$, we have $K - \pi^{*(\text{tc})} \leq T$ so that $W_d^{*(\text{tc})} \geq W_d^{*(\text{ro})}$. For c-firms, note that $W_c^{*(\text{tc})} > W_c^{*(\text{ts})}$. Replacing $\pi^{*(\text{ts})} = \frac{\sigma}{2\tau}[\frac{N}{I_{vn}} - \frac{N}{I_m}]$ into $W_c^{*(\text{ts})}$ gets that $W_c^{*(\text{ts})} \geq W_c^{*(\text{ro})}$. Note that this argument does not depend on the symmetry assumption, as it holds for any number of clean or dirty firms N. Additionally, the d-firm result does not depend on the uncorrelated cash-flow assumption, as this only enters share prices via the shared risk penalty term, R. Finally, for the c-firms, removing the uncorrelated cash-flow assumption changes the wedge of the price shares.

In addition to the Pareto-improvement property of transformation markets, we show in Appendix C that $W_d^{*(\text{ro})}$ can decrease upon the introduction of a tax on secondary trades, a carbon tax, and an emissions-trading scheme. Thus, unlike standard environmental policy responses, enabling transformation markets can benefit both environmental regulation and firm surplus.

□

A.6 Proof of Theorem 5.3

Proof. First, we show that it suffices to focus on the Sharpe ratios faced by each investor – the net excess return relative to volatility in equilibrium conditional on each sustainable investing strategy. Under uncorrelated cash-flows with equal variance, an investor of type θ buying shares in c-firm making corporate choice f and d-firm making corporate choice l has a utility function of:

$$U_\theta(x_\theta) = x_{\theta f}(\mu_c - P_f) + x_{\theta \ell}(\mu_d - P_\ell) - \frac{x_{\theta f}^2\sigma^2 + x_{\theta \ell}^2\sigma^2}{2\tau}$$

Then, first-order conditions impose $x_{\theta f}^* = \frac{\tau}{\sigma^2}(\mu - P_f^*)$ and $x_{\theta \ell}^* = \frac{\tau}{\sigma^2}(\mu - P_\ell^*)$. We can now define the equilibrium Sharpe ratio which is obtained from buying shares of a firm making corporate choice k as $S_k^* := \frac{\mu - P_k}{\sigma^2}$. Note that $x_{\theta k}^* = \tau S_k^*$. Plugging this relationship back into the utility function yields an expression for the investor's indirect utility:

$$\begin{aligned}\hat{U}_\theta &= \tau S_f^*(\mu - P_f^*) + \tau S_\ell^*(\mu - P_\ell^*) - \frac{(\tau S_f^*\sigma)^2 + (\tau S_\ell^*\sigma)^2}{2\tau} \\ &= \frac{\tau}{2}\sigma^2\big(S_f^{*2} + S_\ell^{*2}\big)\end{aligned}$$

Second, we analyze how market structure affects the equilibrium Sharpe ratios for each investor type $\theta \in \{n, v, m\}$. We begin by discussing c-firm sub-markets. Across all equilibria it holds that the n and v investor only buy shares in A-firms. Computing $S_A^{*(\mathrm{e})}$ for $(\mathrm{e}) \in \{(\mathrm{ro}), (\mathrm{ts}), (\mathrm{tc})\}$ using Lemmas 4.2, 4.5 and 4.8:

$$
\begin{aligned}
S_A^{*(\mathrm{ro})} &= \frac{1}{\sigma^2}(R + \lambda_m \frac{N}{I_{nv}} \frac{\sigma^2}{\tau}) \\
S_A^{*(\mathrm{ts})} &= \frac{1}{\sigma^2}(R + \lambda_m \pi^{*(\mathrm{ts})}) = \frac{1}{\sigma^2}(R + \lambda_m (\frac{N}{I_{nv}} - \frac{N}{I_m}) \frac{\sigma^2}{2\tau}) \\
S_A^{*(\mathrm{tc})} &= \frac{1}{\sigma^2}(R + \lambda_m \pi^{*(\mathrm{tc})})
\end{aligned}
$$

Observe that $S_A^{*(\mathrm{ro})} > S_A^{*(\mathrm{ts})} > S_A^{*(\mathrm{tc})}$. Upon the introduction of a transformation market, the n and v investor surplus from c-firm sub-markets decreases due to the increased competition coming from the m-investors participation. As more c-firms choose to engage in the transformation market (α-corporate choice) the sub-market available for the n and v investors becomes smaller. Finally, upon the introduction of a transformation market, m-investor surplus in c-firm sub-markets trivially increases: without it, m-participation is precluded, as their impact-aligned investment strategy prevents them from buying shares in c-firms which do not own pollution reduction opportunities.

Third, we analyze d-firm sub-markets. Since d-firm sub-markets are separable across investor types in some equilibria, we must do a case by case analysis. We begin with the v-investor. By Lemma 4.1, we have that in (ro), the v-investor only buys shares in S-firms . Conversely, by Lemmas 4.3 and 4.6, we have that in the surplus-extraction and cost-sharing equilibrium the v-investor only buys shares in in β-firms. Using Lemmas 4.2, 4.5 and 4.8 we obtain that the equilibrium Sharpe ratios in each case correspond to:

$$
\begin{aligned}
S_S^{*(\mathrm{ro})} &= \frac{1}{\sigma^2}(R + \lambda_m(K - T) - \lambda_n T), \\
S_\beta^{*(\mathrm{ts})} &= \frac{1}{\sigma^2}(R + \lambda_m \pi^{*(\mathrm{ts})}) \\
S_\beta^{*(\mathrm{tc})} &= \frac{1}{\sigma^2}(R + \lambda_m \pi^{*(\mathrm{tc})} - \lambda_n (K - \pi^{*(\mathrm{tc})}))
\end{aligned}
$$

Observe that $S_\beta^{*(\mathrm{ts})} > S_\beta^{*(\mathrm{tc})} > S_S^{*(\mathrm{ro})}$. Value-aligned investor surplus in d-firm sub-markets increases upon the introduction of a transformation market. Intuitively, in this case, the value-aligned

strategy becomes easier to comply with for firms. In addition to the secondary-trading channel, now the d-firm can request c-firm contributions to become indirectly reformed. By the same rationale, however, it follows that the m-investor will hold a less competitive position, as more d-firms will optimally participate in the transformation market, leaving less d-firms to directly engage in real-asset reform, reducing m-investor surplus across d-firm sub-markets. In fact, using the equilibrium pricing Lemmas:

$$\begin{aligned}
S_R^{*(\mathrm{ro})} &= \frac{1}{\sigma^2}(R - \lambda_v(K-T) - \lambda_n K),\\
S_R^{*(\mathrm{ts})} &= \frac{1}{\sigma^2}(R - \lambda_{nv}\pi^{*(\mathrm{ts})}),\\
S_R^{*(\mathrm{tc})} &= \frac{1}{\sigma^2}(R - \lambda_v\pi^{*(\mathrm{tc})} - \lambda_n K),
\end{aligned}$$

which shows that $S_R^{*(\mathrm{ro})} \geq S_R^{*(\mathrm{tc})} \geq S_R^{*(\mathrm{ts})}$. Finally, in the case of an n-investor, the equilibrium Sharpe ratio in d-firm sub-markets decreases under cost-sharing — but may increase in surplus-extraction for a sufficiently high transformation-contract price:

$$\begin{aligned}
S_U^{*(\mathrm{ro})} &= \frac{1}{\sigma^2}\big(R + \lambda_m K + \lambda_v T\big),\\
S_\beta^{*(\mathrm{ts})} &= \frac{1}{\sigma^2}(R + \lambda_m\pi^{*(\mathrm{ts})})\\
S_U^{*(\mathrm{tc})} &= \frac{1}{\sigma^2}(R + \lambda_m K + \lambda_n(K - \pi^{*(\mathrm{tc})}))
\end{aligned}$$

In general, the transformation-market provides a cost-efficient alternative for firms to alleviate the v-investor constraints in the d-firm sub-markets and the m-investor constraints in the c-firm sub-markets. For d-firm sub-markets, this prevents rent-extraction from the n-investors. The caveat is that in a surplus-extraction equilibria, the transformation-market may end up producing higher rents for the n-investors via the c-firm contribution. □

B Additional Characterizations

In this Section, we study competitive equilibria for the cases in which the transformation-contract clearing price takes the critical values of $K - T$ and K.

B.1 Cost-Sharing Equilibrium with $\pi^* = K - T$

Here, the active sub-market tuples are $\{n, A, U\}$, $\{v, A, G\}$ and $\{m, \alpha, R\}$, where $G = S \cup \beta$. Using first-order conditions and corporate-choice market clearing gives:

$$I_n[\sigma_c^2 \pi_U - \sigma_{cd}\pi_A] + I_v[\sigma_c^2 \pi_G - \sigma_{cd}\pi_A] + I_m[\sigma_c^2 \pi_R - \sigma_{cd}\pi_\alpha] = N_d \frac{\phi}{\tau} \tag{62}$$

$$I_n[\sigma_d^2 \pi_A - \sigma_{cd}\pi_U] + I_v[\sigma_d^2 \pi_A - \sigma_{cd}\pi_G] + I_m[\sigma_d^2 \pi_\alpha - \sigma_{cd}\pi_R] = N_c \frac{\phi}{\tau} \tag{63}$$

We can use the no-entry condition to derive relations between excess mean returns:

$$\pi_G = \pi_U - T; \quad \pi_R = \pi_U - K; \quad \pi_\alpha = \pi_A - (K - T) \tag{64}$$

Using these relations we can rearrange the first two equations to solve for the unconstrained investor's optimal position:

$$I_n[\sigma_c^2 \pi_U - \sigma_{cd}\pi_A] + I_v[\sigma_c^2 (\pi_U - T) - \sigma_{cd}\pi_A] + I_m[\sigma_c^2 (\pi_U - K) - \sigma_{cd}(\pi_A - (K - T))] = N_d \frac{\phi}{\tau} \tag{65}$$

$$I_n[\sigma_d^2 \pi_A - \sigma_{cd}\pi_U] + I_v[\sigma_d^2 \pi_A - \sigma_{cd}(\pi_U - T)] + I_m[\sigma_d^2 (\pi_A - (K - T)) - \sigma_{cd}(\pi_U - K)] = N_c \frac{\phi}{\tau} \tag{66}$$

With the above we can derive the following system for π_U and π_A:

$$\sigma_c^2 \pi_U - \sigma_{cd}\pi_A = \frac{1}{I}\big(N_d \frac{\phi}{\tau} + I_v \sigma_c^2 T + I_m \sigma_c^2 K - I_m \sigma_{cd}(K - T)\big) \tag{67}$$

$$\sigma_d^2 \pi_A - \sigma_{cd}\pi_U = \frac{1}{I}\big(N_c \frac{\phi}{\tau} + I_m \sigma_d^2 (K - T) - I_v \sigma_{cd} T - I_m \sigma_{cd} K\big) \tag{68}$$

From first-order conditions, we can directly compute the optimal positions:

$$x_{nU}^* = \frac{1}{I}\big[N_d + \frac{\tau}{\phi}\big(I_v \sigma_c^2 T + I_m \sigma_c^2 K - I_m \sigma_{cd}(K - T)\big)\big] \tag{69}$$

$$x_{nA}^* = \frac{1}{I}\big[N_c + \frac{\tau}{\phi}\big(I_m \sigma_d^2 (K - T) - I_v \sigma_{cd} T - I_m \sigma_{cd} K\big)\big] \tag{70}$$

The no-entry condition also provides a connection between the position of the value-aligned investor and the unconstrained investor:

$$x^*_{vA}(\pi) = \frac{\tau}{\phi}[\sigma_d^2\pi_A - \sigma_{cd}\pi_G] = \frac{\tau}{\phi}[\sigma_d^2\pi_A - \sigma_{cd}\pi_U + \sigma_{cd}T] = x^*_{nA}(\pi) + \frac{\tau}{\phi}\sigma_{cd}T$$
$$x^*_{vG}(\pi) = \frac{\tau}{\phi}[\sigma_c^2\pi_G - \sigma_{cd}\pi_A] = \frac{\tau}{\phi}[\sigma_c^2(\pi_U - T) + \sigma_{cd}\pi_A] = x^*_{nU}(\pi) - \frac{\tau}{\phi}\sigma_c^2 T$$

Similarly for the impact-aligned investor:

$$x^*_{m\alpha}(\pi) = \frac{\tau}{\phi}[\sigma_d^2\pi_\alpha - \sigma_{cd}\pi_R] = \frac{\tau}{\phi}[\sigma_d^2(\pi_A - (K-T)) - \sigma_{cd}(\pi_U - K)] = x^*_{nA}(\pi) - \frac{\tau}{\phi}[\sigma_d^2(K-T) - \sigma_{cd}K]$$
$$x^*_{mR}(\pi) = \frac{\tau}{\phi}[\sigma_c^2\pi_R - \sigma_{cd}\pi_\alpha] = \frac{\tau}{\phi}[\sigma_c^2(\pi_U - K) - \sigma_{cd}(\pi_A - (K-T))] = x^*_{nU}(\pi) - \frac{\tau}{\phi}[\sigma_c^2 K - \sigma_{cd}(K-T)]$$

It is straight forward to derive the sub-market equilibrium sizes under optimal corporate choices using $N^*_\alpha = I_m x^*_{m\alpha}$, $N^*_A = I_{vA}x^*_{vA} + I_{nA}x^*_{nA}$, $N^*_R = I_m x^*_{mR}$, $N^*_G = I_v x^*_{vG}$ and $N^*_U = I_n x^*_{nU}$. Moreover, we have that $N^*_\beta = N^*_\alpha$ so that $N^*_S = N^*_G - N^*_\alpha$. To compute the equilibrium prices we can analyze excess mean returns in equilibrium:

$$\pi^*_U = \frac{\sigma_d^2}{\tau}x^*_{nU} + \frac{\sigma_{cd}}{\tau}x^*_{nA} \tag{71}$$
$$\pi^*_A = \frac{\sigma_c^2}{\tau}x^*_{nA} + \frac{\sigma_{cd}}{\tau}x^*_{nU} \tag{72}$$

This yields direct expressions for the equilibrium share prices:

$$P^*_U = \mu_d - \frac{1}{\tau}[\sigma_d^2 x^*_{nU} + \sigma_{cd}x^*_{nA}] \tag{73}$$
$$P^*_A = \mu_c - \frac{1}{\tau}[\sigma_c^2 x^*_{nA} + \sigma_{cd}x^*_{nU}] \tag{74}$$

This results in the following distribution of corporate chocies in equilibrium:

$$N^*_R = \frac{I_m}{I}\big[N_d - KI_{nv}\frac{\tau}{\phi}(\sigma_c^2 - \sigma_{cd}) + T(I_v\sigma_c^2 - I_v\sigma_{cd} - I_n\sigma_{cd})\frac{\tau}{\phi}\big]$$
$$N^*_G = \frac{I_v}{I}\big[N_d + (K-T)I_m\frac{\tau}{\phi}(\sigma_c^2 - \sigma_{cd}) - TI_n\frac{\tau}{\phi}\sigma_c^2\big]$$
$$N^*_\alpha = \frac{I_m}{I}\big[N_c + (K-T)I_v\frac{\tau}{\phi}(\sigma_{cd} - \sigma_d^2) + KI_n\frac{\tau}{\phi}(\sigma_{cd} - \sigma_d^2) + TI_n\frac{\tau}{\phi}\sigma_d^2\big]$$

B.2 Cost-Sharing Equilibrium with $\pi^* = K$

In this case, because the c-firm contribution perfectly offsets the cost of reform, it must be the case that $P_U = P_\beta$ in equilibrium. In other words, becoming eligible for a v-investor is inconsequential for a d-firm. By the same argument, it must be the case that $N_S = 0$ because otherwise d-firms would be willing to pay at least T to become eligible for a v-investor – a contradiction with the previous statement. At the same time, for the c-firms, it must be the case that $P_\alpha = P_A + K$. With this in mind, we would have that n-investors only buy shares in firms type $D := \{U, \beta\}$ and A, v-investors only buy shares in firms type β and A, and m-investors only buy shares in firms type R and α. Taking first order conditions over investor capital allocation decisions for every investor-firm pair in the active sub-markets yields the optimality conditions:

$$
\begin{aligned}
x_{nA} &= \frac{\tau}{\phi}[(\mu_c - P_A)\sigma_d^2 - (\mu_d - P_D)\sigma_{cd}] \\
x_{nD} &= \frac{\tau}{\phi}[(\mu_d - P_D)\sigma_c^2 - (\mu_c - P_A)\sigma_{cd}] \\
x_{vA} &= \frac{\tau}{\phi}[(\mu_c - P_A)\sigma_d^2 - (\mu_d - P_D)\sigma_{cd}] \\
x_{v\beta} &= \frac{\tau}{\phi}[(\mu_d - P_D)\sigma_c^2 - (\mu_c - P_A)\sigma_{cd}] \\
x_{m\alpha} &= \frac{\tau}{\phi}[(\mu_c - P_\alpha)\sigma_d^2 - (\mu_d - P_R)\sigma_{cd}] \\
x_{mR} &= \frac{\tau}{\phi}[(\mu_d - P_R)\sigma_c^2 - (\mu_c - P_\alpha)\sigma_{cd}]
\end{aligned}
$$

with $\phi = \sigma_c^2\sigma_d^2 - \sigma_{cd}^2$ and $x_{nD} = (x_{nU} + x_{n\beta})$. The condition for this equilibrium to hold with positive contract trading volume is by $N_\alpha^* = N_\beta^* > 0$. Solving the resulting system yields the following equilibrium corporate choices are given by:

$$
\begin{aligned}
N_U^* &= \frac{I_{nv}}{I}(N_d - N_c\frac{I_m}{I_{nv}} + I_m K\tau\frac{\psi}{\phi}) \\
N_R^* &= \frac{I_m}{I}(N_d - I_{nv}K\tau\frac{\sigma_c^2 - \sigma_{cd}}{\phi}) \\
N_\alpha^* &= N_\beta^* = \frac{I_m}{I}(N_c - I_{nv}K\tau\frac{\sigma_d^2 - \sigma_{cd}}{\phi}) \\
N_A^* &= \frac{I_{nv}}{I}(N_c + I_m K\tau\frac{\sigma_d^2 - \sigma_{cd}}{\phi})
\end{aligned}
$$

The equilibrium share prices are given by:

$$
\begin{aligned}
P_A^* &= \mu_c - N_c\frac{\sigma_c^2}{I\tau} - N_d\frac{\sigma_{cd}}{I\tau} - K\frac{I_m}{I} \\
P_\alpha^* &= \mu_c - N_c\frac{\sigma_c^2}{I\tau} - N_d\frac{\sigma_{cd}}{I\tau} + K\frac{I_{nv}}{I} \\
P_R^* &= \mu_d - N_c\frac{\sigma_{cd}}{I\tau} - N_d\frac{\sigma_d^2}{I\tau} + K\frac{I_{nv}}{I} \\
P_U^* &= P_\beta^* = \mu_d - N_c\frac{\sigma_{cd}}{I\tau} - N_d\frac{\sigma_d^2}{I\tau} - K\frac{I_m}{I}
\end{aligned}
$$

C Policy Alternatives for Environmental Reform

In this section, we provide an analysis of policy alternatives including a tax on secondary trades of high-carbon assets, a carbon tax and an emissions trading scheme. While all of these alternatives increase real-asset reform, as opposed to a market for real-asset transformation, they decrease d-firm surplus. This is a key point of departure from the introduction of a real-asset transformation market.

C.1 Tax on Secondary Trades of High-Carbon Assets

Suppose that a regulator imposes a tax on secondary trading of high-carbon assets, denoted by $\rho_S > 0$. First, assume that the d-firm seller fully covers the tax, so that they effectively face a higher transaction cost, $T' := T + \rho_S > T$. Consider the number of reformed assets in Lemma 4.1:

$$
A_R^{*,(\mathrm{ro})} = N_R^{*,(\mathrm{ro})} = \lambda_m N - \lambda_s\frac{\tau}{\sigma^2}(I_n K + I_v(K - T')) \implies \frac{\partial A_R^{*,(\mathrm{ro})}}{\partial \rho_S} = \lambda_m\frac{\tau}{\sigma^2}I_v > 0
$$

which shows that having a larger tax increases real-asset reform. Now, consider d-firm surplus:

$$
W_d^{*,(\mathrm{ro})} = N(\mu - R - \lambda_m K - \lambda_v T') \implies \frac{\partial W_d^{*,(\mathrm{ro})}}{\partial \rho_S} = -N\lambda_v
$$

It follows that increasing ρ_S decreases d-firm surplus. Finally, for Sharpe ratios in each d-firm sub-market we have that:

$$S_U^{*,(\text{ro})} = \frac{1}{\sigma^2}(R + m_s K + m_g T') \implies \frac{\partial S_U^{*,(\text{ro})}}{\partial \rho_S} = \frac{\lambda_v}{\sigma^2}$$
$$S_S^{*,(\text{ro})} = \frac{1}{\sigma^2}(R + \lambda_m(K - T) - \lambda_n T) \implies \frac{\partial S_S^{*,(\text{ro})}}{\partial \rho_S} = -\frac{\lambda_m + \lambda_n}{\sigma^2}$$
$$S_R^{*,(\text{ro})} = \frac{1}{\sigma^2}(R - \lambda_v(K - T) - \lambda_n K) \implies \frac{\partial S_R^{*,(\text{ro})}}{\partial \rho_S} = \frac{\lambda_v}{\sigma^2}$$

Increasing the tax on secondary-trades of high-carbon assets increases n-investor and m-investor surplus, and decreases v-investor surplus in d-firm sub-markets.

C.2 Carbon Tax Without A Rebate

Suppose that a regulator imposes a carbon tax $\rho_d > 0$ on firms that operate unreformed pollutive assets. We assume that the outside private buyer passes the carbon-tax to the secondary seller, so that upon trading its d-asset, it receives a payment equal to the risky cash-flow minus the transaction cost T *and* the carbon tax ρ_d. [13] We can characterize the new equilibrium outcome. The market clearing condition in the base case is the same:

$$N_A = N_c, \quad N_U + N_R + N_S = N_d.$$

Optimal corporate choice for a d-firm yields a modified version of (2):

$$P_U - \rho_d = P_S - T - \rho_d = P_R - K,$$

or $P_U = P_S - T = P_R - (K - \rho_d)$. For a sufficiently large carbon tax $K - \rho_d < T$, real-asset reform dominates secondary trading. Suppose that $K - \rho_d \geq T$. Then, $P_U^* < P_S^* < P_R^*$, leading to the

[13] Removing this assumption could result in the carbon-tax increasing secondary-trade, thus becoming a less effective tool for environmental reform.

market clearing condition in (3):

$$I_n x_{nU} = N_U, \quad I_g x_{gS} = N_S, \quad I_s x_{sR} = N_R;$$
$$I_n x_{nA} + I_g x_{gA} = N_A.$$

Solving for the number of reformed real assets as a function of the carbon tax yields:

$$\begin{aligned} A_R^{*(\mathrm{ro})}(\rho_d) &= \lambda_m N - \lambda_m \frac{\tau}{\sigma^2}(I_n(K-\rho_d) + I_v(K-\rho_d-T)) \\ &= \lambda_m N - \lambda_m \frac{\tau}{\sigma^2}(I_n(K + I_v(K-\rho_d-T)) + \lambda_m \frac{\tau}{\sigma^2} I_{nv}\rho_d \\ &= A_R^{*(\mathrm{ro})}(0) + \lambda_m \frac{\tau}{\sigma^2} I_{nv}\rho_d > A_R^{*(\mathrm{ro})}(0) \end{aligned}$$

which shows that the carbon tax increases real-asset reform. On the other hand, all d-firms perceive a symmetric utility of $P_U^*(\rho_d) - \rho_d$ which implies that:

$$W_d^{*(\mathrm{ro})}(\rho_d) = N(\mu - R - \lambda_m(K-\rho_d) - \lambda_v T - \rho_d) \tag{76}$$
$$= N(\mu - R - \lambda_m K - \lambda_v T - \lambda_{nv}\rho_d) \tag{77}$$
$$= W_d^{*(\mathrm{ro})}(0) - N\lambda_{nv}\rho_d \tag{78}$$

We conclude that the carbon tax reduces d-firm surplus.

C.3 Emissions Trading System

Suppose now that a regulator introduces an emissions trading system.

Mechanism 2 (Emissions-Trading Scheme)**.** *The mechanism for pollution-permit allocation and trading is as follows:*

1. *Initially, the regulator allocates q emission permits to each one of the d-firms.*
2. *The d-firms can buy or sell pollution permits to accomodate their corporate choice.*
   - *An unreformed pollutive asset requires $q_U = q_D$ permits.*
   - *Secondary buyer does not reform so we assume it asks $q_S = q_D$ permits from the seller.*

- *A reformed pollutive asset requires $q_R < q_D$ permits.*

Note that for a sufficiently large initial allocation $q > q_D$ the equilibrium remains unchanged. Suppose instead that $q_D > q > q_R$. First, we will see that the emissions trading system implements a transfer between the $U \cup S$ and R-firms. Let $q_b = q_D - q$ be the number of permits that a U or S firm must buy given their corporate choice. Let $q_s = q - q_R$ be the number of permits that a R firm can sell given their corporate choice. Let η be the pollution-permit clearing price. In equilibrium, we must have that:

$$P_U^* - \eta q_b = P_S^* - T - \eta q_b = P_R^* - K + \eta q_s$$

which can be written as:

$$P_U^* = P_S^* - T = P_R^* - (K - \eta(q_s + q_b))$$

Again if η is large (that is, q is very small), real-asset reform strictly dominates secondary trading. Suppose that this is not the case. Then, in equilibrium, $P_U < P_S < P_R$. Market clearing for pollution permits imposes that:

$$N_U^* q_b + N_S^* q_b \leq N_R^* q_s \iff (q_D - q)(N_U^* + N_S^*) \leq (N_R^*)(q - q_R)$$

When the constraint does not bind, $q_s N_R^* > q_b N_U^* + q_b N_S^*$, the pollution permit price must be zero and the equilibrium is unchanged as compared to the base case. Suppose that q is in the intermediate regime where the constraint binds in equilibrium, but secondary trading is still attractive. Then:

$$\begin{aligned}(q_D - q)(N_U^* + N_S^*) = (N_R^*)(q - q_R) &\iff q_D(N_U^* + N_S^*) + q_R(N_R^*) = Nq \\ &\iff \frac{q_D}{q}\frac{N_U + N_S}{N} + \frac{q_R}{q}\frac{N_R}{N} = 1\end{aligned}$$

Because of corporate choice endogeneity, each d-firms perceives a symmetric utility of:

$$P_U^* - q_b\eta = P_S^* - T - q_b\eta = P_R^* - K + q_s\eta \iff P_U^* = P_S^* - T = P_R^* - K + (q_s\eta + q_b\eta)$$
$$\iff P_U^* = P_S^* - T = P_R^* - K + \eta(q_R + q_D)$$

For $P_U^* < P_S^* < P_R^*$ we get the same d-firm surplus effect as with a carbon tax:

$$\begin{aligned} W_d^{*(\text{ro})} &= N(\mu - R - \lambda_m(K - \eta(q_R + q_D)) - \lambda_v T - \eta q_D) \\ &= N(\mu - R - \lambda_m K + \lambda_m \eta(q_R + q_D) - \lambda_v T - \eta q_D) \\ &= N(\mu - R - \lambda_m K - \lambda_v T) + N\eta(\lambda_m q_R - (\lambda_v + \lambda_n)q_D) \end{aligned}$$

Now we show that the last term is indeed negative in the relevant regions, and that η increases as q decreases, to conclude that $W_d^{*(\text{ro})}$ decreases upon the introduction of an emissions-trading scheme. Suppose $\lambda_m q_R - (\lambda_v + \lambda_n)q_D > 0$, then:

$$\lambda_m q_R > (\lambda_v + \lambda_n)q_D$$

which implies that:

$$\lambda_m > \lambda_v + \lambda_n$$

Empirical evidence suggests that impact-aligned investment is smaller in magnitude than value-aligned investment across public markets (Bonnefon et al. 2025). Thus, the sum of the market shares of unconstrained and value-aligned investors being smaller than impact-aligned investors is an unlikely regime. If we assume instead that $\lambda_m \leq \lambda_v + \lambda_n$ we get that $\lambda_m q_r - (\lambda_v + \lambda_n)q_D < 0$. In this case, for any positive market-clearing price in the emissions-trading scheme $\eta^* > 0$, d-firm surplus *decreases* :

$$W_d^{*(\text{ro})}(\eta^*) = W_d^{*(\text{ro})}(0) - N\eta^*((\lambda_v + \lambda_n)q_D - \lambda_m q_R) < W_d^{*(\text{ro})}(0)$$

Finally, we show that η^* is increasing in the initial permit allocation, q. The market-clearing

condition for pollution permits is:

$$(N_U^*(\eta) + N_S^*(\eta))(q_b - q) = N_R^*(\eta)(q - q_s)$$

Let $\zeta := \eta(q_s + q_b)$. Then the allocative distortions in equilibrium corresponding to Lemma 4.1 are given by:

$$\begin{aligned}
\xi_U^* &= I_m(K - \zeta)\frac{\tau}{\sigma^2} + I_v T\frac{\tau}{\sigma^2} \\
\xi_S^* &= I_m(K - \zeta - T)\frac{\tau}{\sigma^2} - I_n T\frac{\tau}{\sigma^2} \\
\xi_R^* &= -I_n(K - \zeta)\frac{\tau}{\sigma^2} - I_v(K - \zeta - T)\frac{\tau}{\sigma^2}
\end{aligned}$$

Computing the sum of unreformed and secondary-seller firms, which correspond to the permit-buying firms, gives:

$$\begin{aligned}
N_U^*(\eta) + N_S^*(\eta) &= \lambda_n(N + I_m(K - \zeta)\frac{\tau}{\sigma^2} + I_v T\frac{\tau}{\sigma^2}) + \lambda_v(N + I_m(K - \zeta - T)\frac{\tau}{\sigma^2} - I_n T\frac{\tau}{\sigma^2}) \\
&= \lambda_{nv} N + \frac{\tau}{\sigma^2}\big(\lambda_n I_m(K - \zeta) + \lambda_n I_v T + \lambda_v I_m(K - \zeta - T) - \lambda_v I_n T\big) \\
&= \lambda_{nv} N + \frac{\tau}{\sigma^2} I_m\big(\lambda_n K + \lambda_v(K - T)\big) - \frac{\tau}{\sigma^2} I \lambda_m \lambda_{nv} \zeta
\end{aligned}$$

On the other side of the market, the number of reformed firms, which correspond to the permit-seller firms, is given by:

$$\begin{aligned}
N_R^*(\eta) &= \lambda_m N - \frac{\tau}{\sigma^2}\lambda_m\big(I_n(K - \zeta) + I_v(K - \zeta - T)\big) \\
&= \lambda_m N - \frac{\tau}{\sigma^2}\lambda_m\big(I_n K - I_v(K - T)\big) + \frac{\tau}{\sigma^2} I \lambda_m \lambda_{nv} \zeta
\end{aligned}$$

Defining the terms which do not depend on η as $G := N_U^*(0) + N_S^*(0)$ and $H := N_R^*(0)$, as well as $L := \frac{\tau}{\sigma^2} I \lambda_m \lambda_{nv} \zeta$, we can rewrite the market-clearing condition as:

$$\frac{N_U^*(\eta) + N_S^*(\eta)}{N_R^*(\eta)} = \frac{G - L\zeta}{H + L\zeta} = \frac{q - q_R}{q_D - q}$$

Solving the equation for ζ we get that:

$$\begin{aligned}(G - L\zeta)(q_D - q) &= (H + L\zeta)(q - q_R) \\ \implies \zeta^* &= \frac{G(q_D - q) - H(q - q_R)}{L[q_D - q_R]}\end{aligned}$$

From this expression we can conclude that the permit clearing price η increases linearly as as q decreases. Note also that $G + H = N$ so the slope is:

$$\frac{\partial \zeta^*}{\partial q} = -\frac{N}{I\lambda_m\lambda_{nv}(q_D - q_R)}\frac{\sigma^2}{\tau}$$

Using that $\eta^* = \frac{\zeta^*}{q_s + q_b} = \frac{\zeta^*}{q_D - q_R}$ yields:

$$\frac{\partial \eta^*}{\partial q} = -\frac{N}{Im_s m_{gn}(q_D - q_R)^2}\frac{\sigma^2}{\tau}$$

Plugging the expressions back to compute the rate of change of d-firm surplus with respect to q we get that:

$$\frac{\partial W_d^{*(\text{ro})}}{\partial q} = \frac{\partial W_d^{*(\text{ro})}}{\partial \eta^*}\frac{\partial \eta^*}{\partial q} = \frac{N^2(\lambda_{nv}q_D - \lambda_m q_R)}{I\lambda_m\lambda_{nv}(q_D - q_R)^2}\frac{\sigma^2}{\tau}$$

which shows that decreasing q can increase real-asset reform but it decreases d-firm surplus.

C.4 Carbon Tax with a Rebate To All Firms And Investors

Suppose that a regulator imposes a carbon tax $\rho_d > 0$ on firms that operate unreformed pollutive assets and redistributes the collected revenue as a lump-sum transfer to all firms and investors. We assume that the outside private buyer passes the carbon-tax to the secondary seller, so that upon trading its d-asset, it receives a payment equal to the risky cash-flow minus the transaction cost T *and* the carbon tax ρ_d. We can characterize the new equilibrium outcome. The market clearing condition in the base case is the same:

$$N_A = N_c, \quad N_U + N_R + N_S = N_d.$$

The optimal corporate choice for a d-firm yields a modified version of (2):

$$P_U - \rho_d + \nu\rho_d = P_S - T - \rho_d + \nu\rho_d = P_R - K + \nu\rho_d,$$

where $\nu = \frac{N_S^* + N_U^*}{2N+I}$ which implies $P_U = P_S - T = P_R - (K - \rho_d)$. For a sufficiently large carbon tax $K - \rho_d < T$, real-asset reform dominates secondary trading. Suppose that $K - \rho_d \geq T$. Then, $P_U^* < P_S^* < P_R^*$, leading to the market clearing condition in (3):

$$\begin{aligned} &I_n x_{nU} = N_U, \quad I_g x_{gS} = N_S, \quad I_s x_{sR} = N_R; \\ &I_n x_{nA} + I_g x_{gA} = N_A. \end{aligned}$$

Solving for the number of reformed real assets as a function of the carbon tax yields:

$$\begin{aligned} A_R^{*(\mathrm{ro})}(\rho_d) &= \lambda_m N - \lambda_m \frac{\tau}{\sigma^2}(I_n(K - \rho_d) + I_v(K - \rho_d - T)) \\ &= \lambda_m N - \lambda_m \frac{\tau}{\sigma^2}(I_n(K + I_v(K - \rho_d - T)) + \lambda_m \frac{\tau}{\sigma^2} I_{nv}\rho_d \\ &= A_R^{*(\mathrm{ro})}(0) + \lambda_m \frac{\tau}{\sigma^2} I_{nv}\rho_d > A_R^{*(\mathrm{ro})}(0) \end{aligned}$$

which shows that the carbon tax increases real-asset reform. On the other hand, all d-firms perceive a symmetric utility of $P_U^*(\rho_d) - \rho_d + \nu\rho_d$ which implies that:

$$W_d^{*(\mathrm{ro})}(\rho_d) = N(\mu - R - \lambda_m(K - \rho_d) - \lambda_v T - \rho_d(1 - \nu)) \tag{80}$$

$$= N(\mu - R - \lambda_m K - \lambda_v T - \lambda_{nv}\rho_d + \nu\rho_d) \tag{81}$$

$$= W_d^{*(\mathrm{ro})}(0) - N\lambda_{nv}\rho_d + N\nu\rho_d \tag{82}$$

We conclude that the carbon tax reduces d-firm surplus as long as $\nu \leq \lambda_{nv}$ which holds for sufficiently many firms or investors.